Computer Algebra
Independent Integration Tests

Summer 2023 edition

0-Independent-test-suites/10-Timofeev-Problems

Nasser M. Abbasi

September 5, 2023 Compiled on September 5, 2023 at 10:16am

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [705]. This is test number [10].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (705)	0.00 (0)
Mathematica	100.00 (705)	0.00 (0)
Fricas	93.90 (662)	6.10 (43)
Maple	93.05 (656)	6.95 (49)
Giac	83.69 (590)	16.31 (115)
Maxima	80.14 (565)	19.86 (140)
Mupad	76.88 (542)	23.12 (163)
Sympy	65.25 (460)	34.75 (245)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

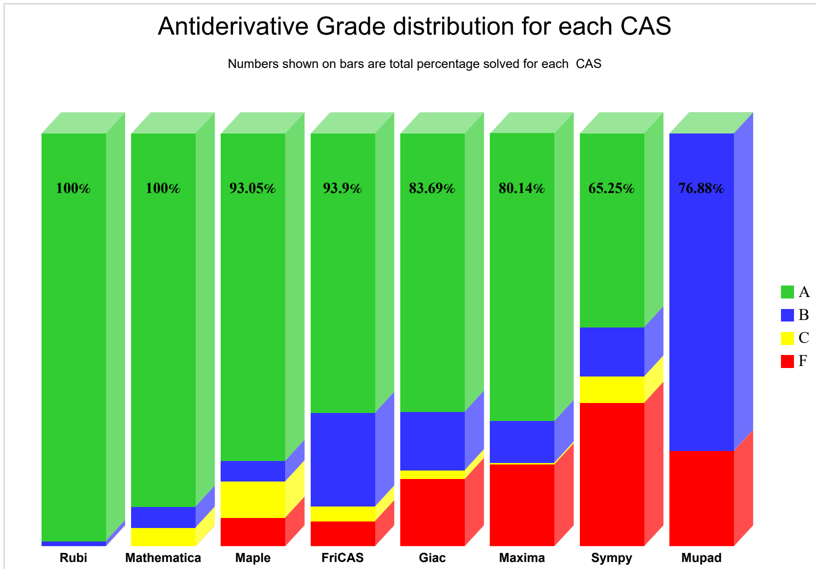
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

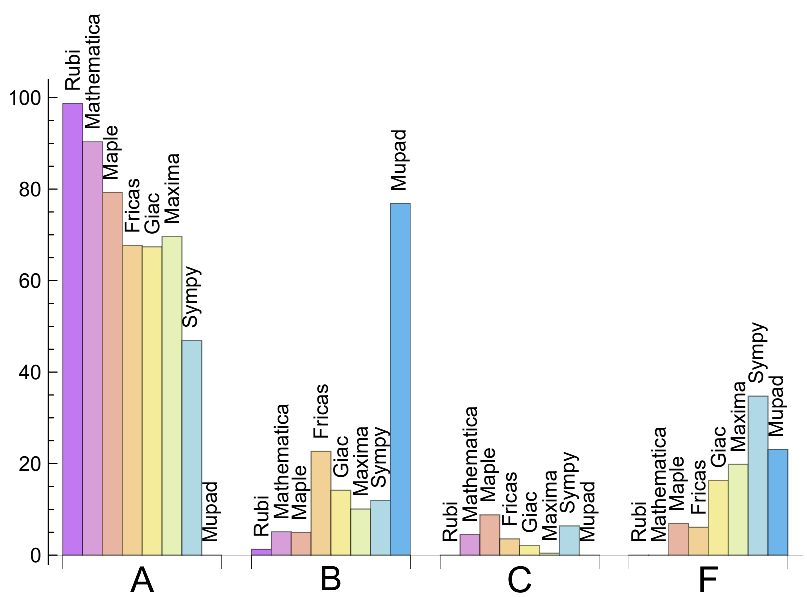
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.723	1.277	0.000	0.000
Mathematica	90.355	5.106	4.539	0.000
Maple	79.291	4.965	8.794	6.950
Maxima	69.645	10.071	0.426	19.858
Fricas	67.660	22.695	3.546	6.099
Giac	67.376	14.184	2.128	16.312
Sympy	46.950	11.915	6.383	34.752
Mupad	0.000	76.879	0.000	23.121

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	43	79.07	11.63	9.30
Maple	49	100.00	0.00	0.00
Giac	115	94.78	4.35	0.87
Maxima	140	87.86	2.86	9.29
Mupad	163	0.00	100.00	0.00
Sympy	245	75.51	22.86	1.63

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.07
Maxima	0.24
Giac	0.30
Mupad	0.33
Mathematica	0.35
Fricas	0.41
Maple	0.91
Sympy	1.76

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	42.33	1.06	28.00	0.85
Mathematica	51.15	1.12	37.00	1.00
Rubi	51.38	1.05	38.00	1.00
Maxima	58.40	1.48	33.00	0.91
Giac	61.68	1.44	35.00	0.93
Sympy	120.98	2.49	36.00	1.04
Fricas	234.57	2.44	40.50	1.12
Maple	267.75	2.02	30.00	0.87

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

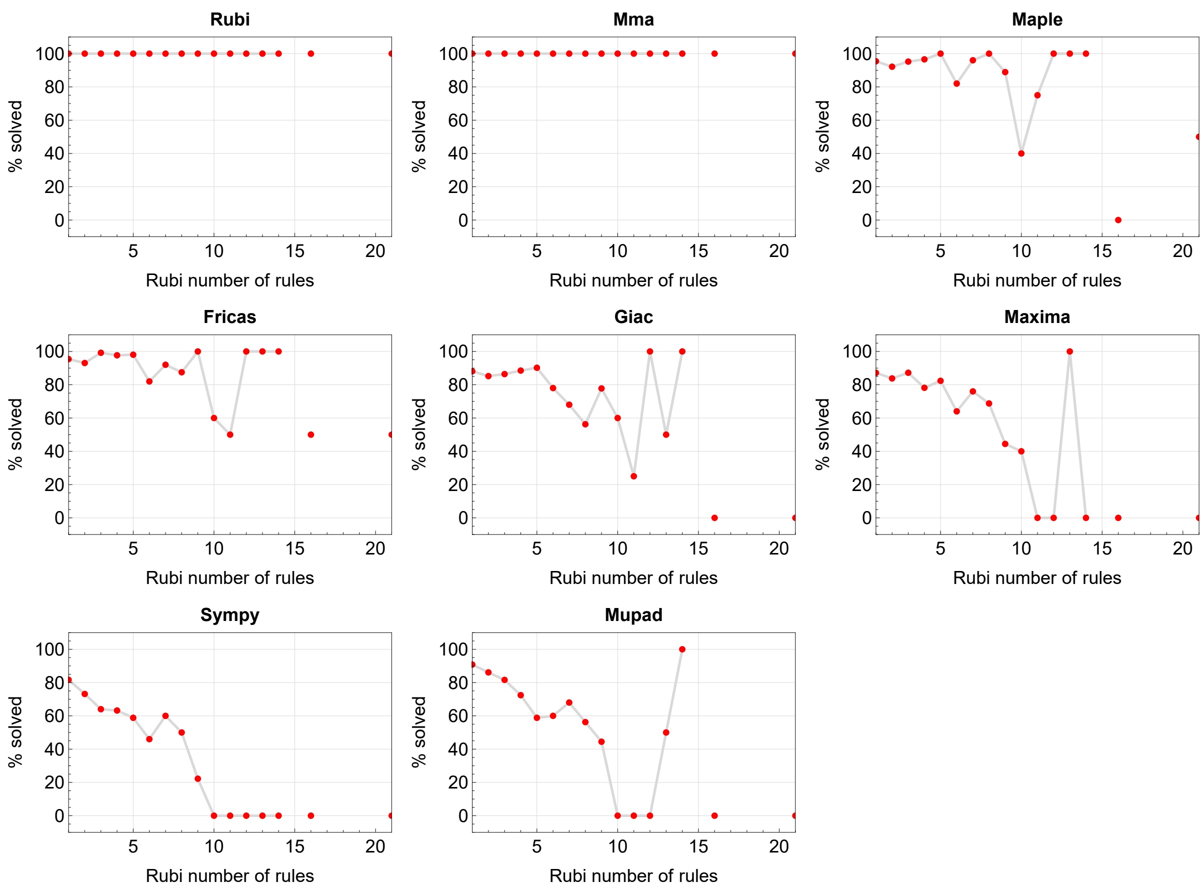


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

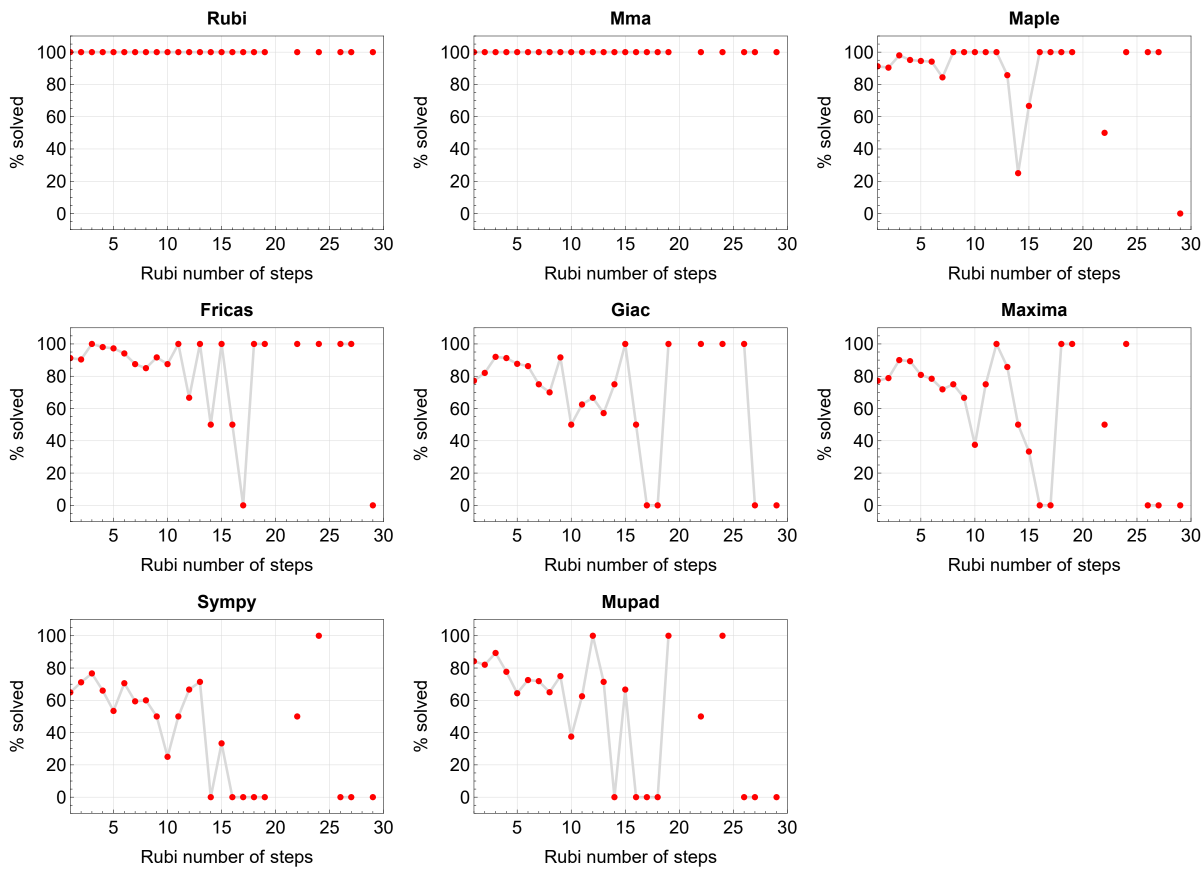


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

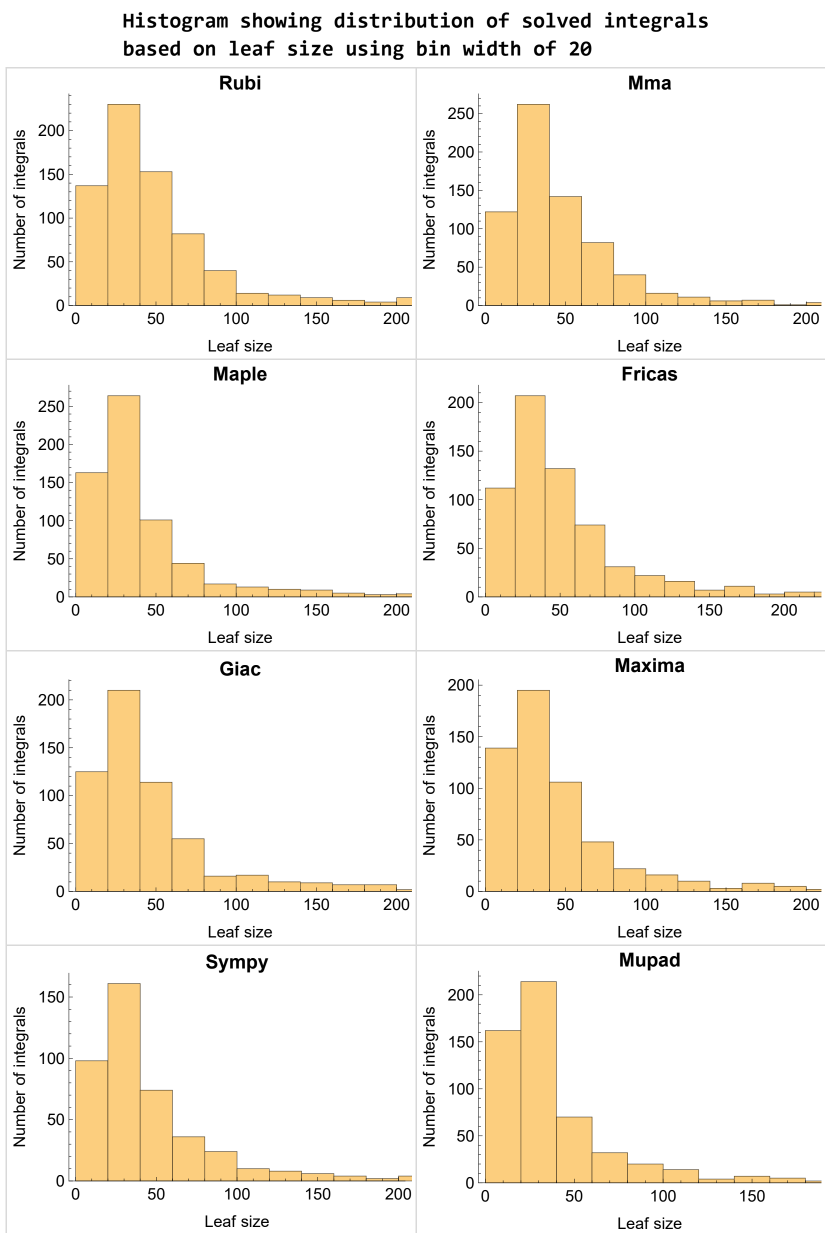


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

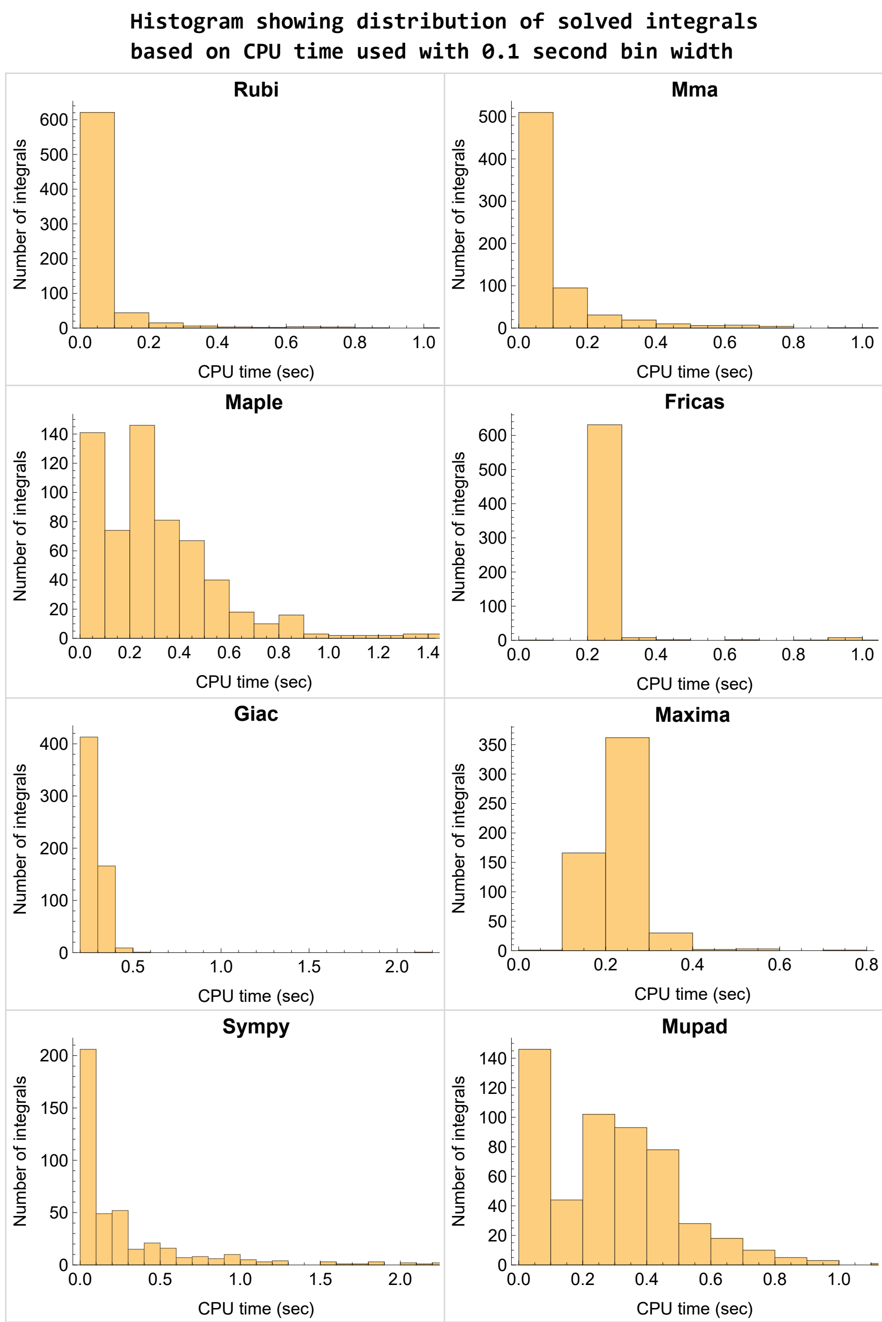


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

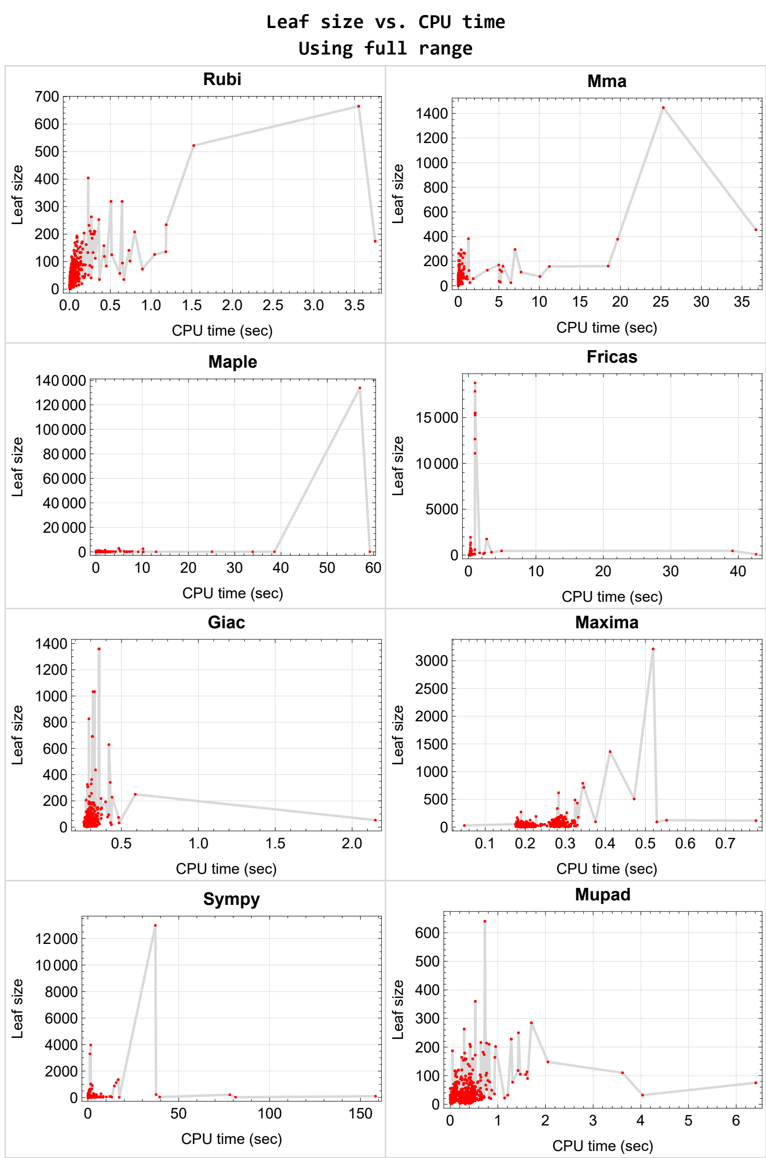


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{417, 426, 446\}$

Maple $\{217, 294, 298, 313, 319, 413, 416, 417, 643, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 704\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
x, aa = expr.operator(), expr.operands()
if x is None:
    return 1
else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

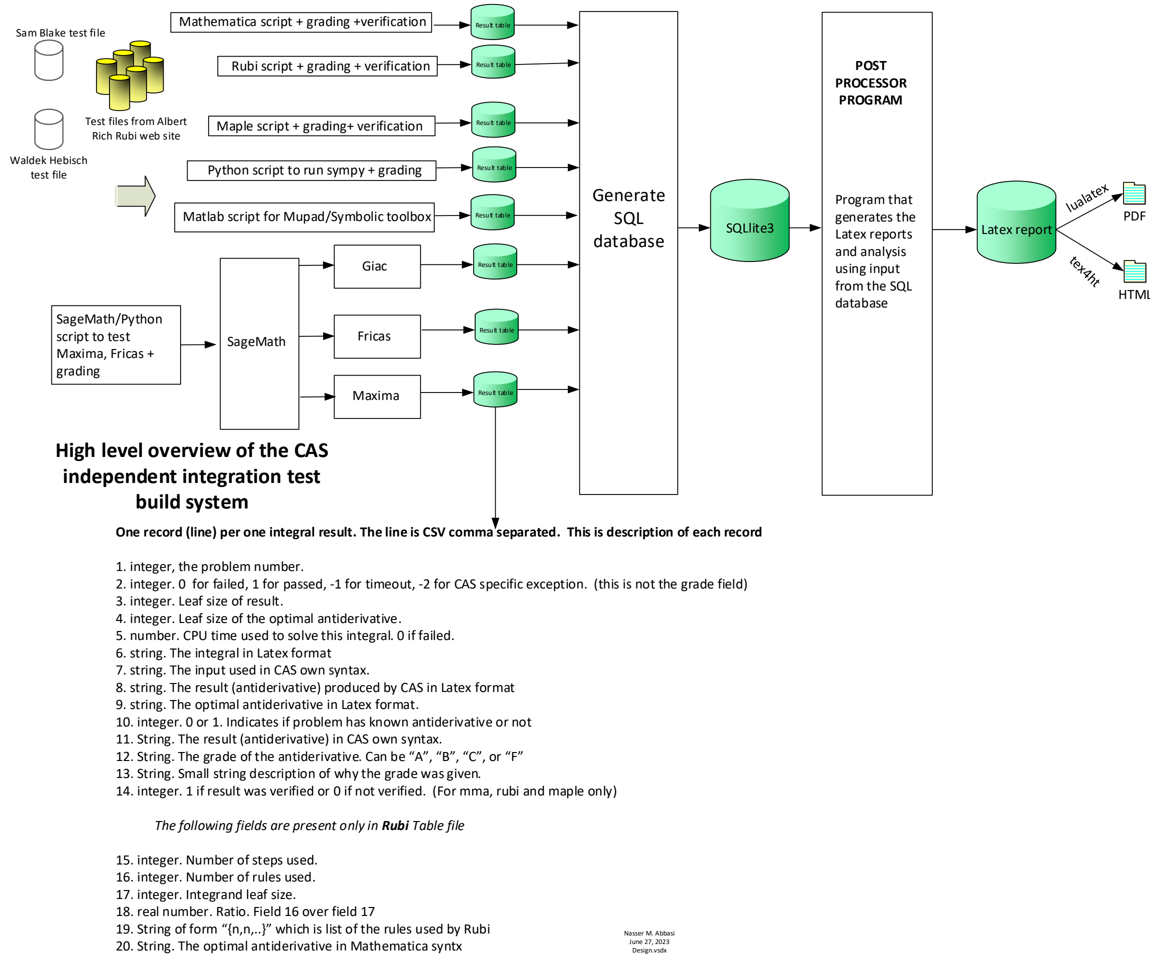
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705 }

B grade { 226, 228, 232, 335, 377, 413, 416, 447, 695 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 51, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 238, 239, 240, 241, 242, 243, 244, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 383, 385, 386, 387, 388, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 418, 419, 420, 421, 422, 423, 425, 427, 428, 429, 430, 431, 432, 433, 435, 436, 437, 438, 441, 442, 443, 447, 449, 450, 451, 453, 454, 455, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 556, 558, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 574, 576, 577, 578, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702 }

B grade { 4, 41, 52, 53, 56, 76, 99, 236, 237, 249, 254, 311, 322, 323, 338, 357, 361, 384, 389, 439, 444, 445, 456, 488, 553, 554, 555, 557, 559, 573, 575, 579, 592, 622, 623, 689 }

C grade { 37, 39, 50, 113, 193, 198, 222, 245, 246, 247, 248, 312, 328, 343, 365, 382, 399, 401, 416, 417, 424, 426, 434, 440, 446, 448, 452, 677, 678, 703, 704, 705 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 140, 141, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 218, 219, 220, 223, 224, 225, 227, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 250, 251, 252, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 296, 299, 300, 301, 307, 308, 309, 310, 311, 312, 314, 315, 317, 318, 320, 321, 322, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 412, 419, 420, 421, 422, 425, 426, 428, 430, 439, 440, 441, 448, 451, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 493, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 522, 524, 525, 526, 527, 528, 530, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 590, 591, 593, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 659, 660, 663, 666, 667, 668, 669, 670, 671, 673, 675, 676, 677, 678, 679, 680, 681, 682, 695, 696, 697, 698, 700, 701, 702 }

B grade { 13, 246, 247, 248, 253, 260, 367, 374, 383, 413, 416, 423, 424, 427, 429, 431, 432, 433, 434, 435, 436, 437, 438, 456, 457, 586, 587, 588, 592, 595, 661, 662, 665, 672, 674 }

C grade { 41, 119, 135, 136, 137, 138, 139, 142, 143, 144, 174, 177, 214, 217, 226, 228, 232, 249, 294, 295, 297, 298, 302, 303, 304, 305, 306, 313, 316, 319, 387, 392, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 417, 492, 523, 589, 643, 658, 664, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 704 }

F normal fail { 67, 126, 133, 145, 193, 198, 221, 222, 328, 329, 352, 414, 415, 418, 442, 443, 444, 445, 446, 447, 449, 450, 452, 453, 454, 455, 500, 506, 511, 516, 521, 529, 532, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 632, 699, 703, 705 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 58, 59, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 140, 141, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 194, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 225, 227, 230, 231, 232, 233, 234, 238, 241, 243, 250, 251, 252, 258, 260, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 282, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 312, 313, 316, 318, 320, 325, 326, 327, 330, 331, 332, 333, 334, 336, 339, 341, 344, 345, 346, 347, 348, 349, 350, 351, 356, 359, 362, 364, 366, 368, 370, 373, 374, 375, 376, 378, 380, 381, 385, 386, 387, 395, 396, 398, 409, 412, 414, 415, 418, 419, 420, 422, 425, 426, 428, 436, 437, 439, 440, 441, 448, 450, 454, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 492, 493, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 576, 583, 585, 590, 598, 599, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 626, 627, 628, 629, 630, 631, 633, 634, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 658, 659, 660, 663, 664, 666, 667, 668, 669, 670, 671, 673, 676, 677, 679, 680, 681, 682, 684, 685, 686, 687, 688, 690, 691, 692, 693, 694, 695, 696, 697, 699, 700, 705 }

B grade { 3, 4, 5, 19, 30, 31, 36, 37, 41, 53, 54, 55, 56, 57, 62, 63, 76, 99, 149, 159, 161, 165, 180, 187, 195, 196, 197, 202, 223, 224, 226, 228, 229, 235, 236, 237, 239, 240, 242, 244, 248, 249, 253, 254, 255, 256, 257, 259, 268, 269, 276, 281, 283, 284, 303, 311, 314, 319, 321, 322, 323, 324, 328, 335, 337, 338, 340, 342, 343, 353, 354, 355, 357, 358, 360, 361, 363, 365, 367, 369, 371, 372, 377, 379, 382, 383, 384, 388, 389, 390, 391, 392, 393, 394, 400, 402, 403, 404, 405, 406, 407, 408, 410, 411, 423, 424, 427, 429, 430, 431, 432, 433, 434, 435, 438, 443, 447, 451, 452, 453, 456, 457, 490, 491, 505, 510, 572, 573, 574, 575, 577, 578, 579, 580, 581, 582, 584, 586, 587, 588, 589, 591, 592, 593, 594, 595, 596, 597, 600, 601, 602, 603, 625, 635, 661, 662, 701, 702, 703, 704 }

C grade { 135, 136, 137, 138, 139, 142, 143, 144, 174, 177, 221, 222, 245, 246, 247, 298, 315, 317, 397, 399, 401, 413, 416, 421, 632 }

F normal fail { 126, 133, 145, 193, 198, 352, 500, 506, 511, 516, 521, 529, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 616, 657, 665, 672, 674, 675, 678, 683, 689, 698 }

F(-1) timedout fail { 442, 444, 445, 449, 455 }

F(-2) exception fail { 329, 417, 446, 543 }

Maxima

A grade { 2, 3, 6, 7, 9, 10, 11, 12, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 229, 235, 236, 237, 243, 250, 251, 252, 253, 254, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 282, 284, 285, 286, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 310, 312, 313, 316, 326, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 370, 371, 372, 375, 376, 377, 378, 379, 380, 381, 382, 392, 396, 397, 398, 400, 401, 412, 414, 415, 418, 419, 420, 422, 424, 428, 439, 444, 445, 448, 450, 453, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 475, 476, 477, 478, 479, 480, 481, 483, 484, 485, 486, 490, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 585, 590, 591, 598, 600, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 638, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 658, 659, 660, 661, 662, 663, 664, 666, 667, 669, 670, 671, 673, 676, 677, 678, 679, 680, 681, 682, 685, 686, 687, 690, 692, 694, 697, 698, 700, 702, 704, 705 }

B grade { 1, 4, 5, 8, 13, 19, 31, 41, 62, 76, 81, 82, 86, 128, 159, 240, 241, 257, 311, 318, 322, 323, 367, 369, 373, 374, 384, 385, 386, 387, 388, 389, 393, 394, 399, 423, 425, 429, 430, 431, 433, 437, 449, 451, 456, 474, 482, 488, 489, 491, 492, 493, 577, 578, 579, 580, 581, 584, 586, 588, 589, 593, 594, 596, 597, 599, 601, 643, 691, 695, 696 }

C grade { 421, 426, 457 }

F normal fail { 126, 133, 145, 193, 198, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 238, 239, 242, 244, 245, 246, 247, 248, 249, 255, 256, 258, 259, 260, 279, 281, 283, 287, 288, 289, 290, 291, 306, 307, 308, 309, 314, 315, 317, 319, 320, 321, 324, 325, 327, 328, 329, 352, 383, 390, 391, 395, 402, 403, 404, 405, 406, 407, 408, 409, 410, 413, 416, 417, 432, 434, 435, 438, 440, 441, 442, 443, 447, 452, 454, 455, 473, 500, 506, 511, 516, 521, 529, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 592, 595, 637, 650, 657, 665, 668, 672, 674, 675, 683, 684, 688, 689, 693, 699, 701, 703 }

F(-1) timedout fail { 411, 427, 436, 446 }

F(-2) exception fail { 69, 149, 194, 195, 196, 197, 487, 494, 495, 583, 587, 603, 621 }

Giac

A grade { 2, 6, 7, 8, 10, 11, 12, 14, 15, 16, 17, 18, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 227, 229, 230, 231, 241, 250, 251, 252, 253, 260, 261, 262, 263, 264, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278, 282, 284, 285, 286, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 310, 312, 313, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 344, 345, 346, 347, 348, 349, 350, 351, 354, 355, 356, 358, 359, 360, 361, 362, 363, 364, 366, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 385, 386, 387, 390, 391, 392, 393, 395, 396, 397, 398, 400, 401, 419, 420, 422, 423, 424, 425, 428, 429, 430, 431, 433, 440, 441, 442, 443, 444, 445, 450, 451, 452, 454, 455, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 492, 493, 496, 497, 498, 499, 501, 502, 507, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 534, 535, 536, 538, 539, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 574, 583, 584, 585, 587, 589, 592, 593, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 609, 610, 611, 612, 613, 614, 618, 619, 621, 622, 624, 627, 628, 629, 630, 631, 633, 634, 637, 638, 639, 640, 641, 642, 643, 644, 646, 647, 650, 651, 652, 653, 654, 655, 656, 659, 660, 661, 662, 663, 664, 667, 668, 669, 670, 671, 676, 677, 686, 687, 688, 691, 696, 697, 700, 701, 705 }

B grade { 1, 3, 4, 5, 9, 13, 19, 22, 52, 54, 55, 56, 58, 73, 99, 128, 197, 220, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 254, 255, 256, 257, 258, 259, 265, 266, 274, 279, 280, 281, 283, 287, 308, 311, 322, 323, 338, 342, 343, 353, 357, 365, 367, 374, 383, 384, 388, 389, 394, 438, 447, 449, 456, 474, 476, 487, 488, 489, 570, 571, 572, 573, 575, 576, 577, 578, 579, 580, 581, 586, 588, 590, 591, 595, 596, 597, 617, 620, 635, 645, 649, 658, 666, 685, 690, 702, 704 }

C grade { 79, 421, 437, 457, 494, 495, 503, 504, 505, 508, 509, 510, 537, 695, 703 }

F normal fail { 126, 133, 145, 154, 193, 198, 221, 223, 224, 225, 226, 228, 232, 233, 234, 291, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 327, 328, 329, 352, 399, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 426, 427, 432, 434, 435, 436, 439, 446, 448, 453, 473, 490, 491, 500, 506, 511, 516, 521, 529, 532, 533, 540, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 594, 608, 615, 616, 632, 636, 648, 657, 665, 672, 673, 674, 675, 678, 679, 680, 681, 682, 683, 684, 689, 692, 693, 694, 698, 699 }

F(-1) timedout fail { 222, 249, 623, 625, 626 }

F(-2) exception fail { 86 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 229, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 266, 268, 269, 271, 272, 273, 274, 282, 284, 285, 286, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 306, 307, 308, 309, 311, 312, 313, 318, 322, 323, 326, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 397, 398, 399, 400, 401, 408, 409, 410, 412, 414, 415, 419, 422, 425, 430, 431, 437, 439, 440, 441, 442, 443, 444, 451, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 642, 643, 644, 647, 648, 669, 670, 671, 673, 676, 677, 678, 679, 680, 681, 682, 696, 697, 700, 704 }

C grade { }

F normal fail { }

F(-1) timedout fail { 69, 86, 126, 133, 145, 193, 198, 221, 222, 226, 227, 228, 230, 231, 232, 243, 244, 245, 246, 247, 248, 249, 257, 264, 265, 267, 270, 275, 276, 277, 278, 279, 280, 281, 283, 287, 288, 290, 291, 304, 305, 310, 314, 315, 316, 317, 319, 320, 321, 324, 325, 327, 328, 329, 394, 395, 396, 402, 403, 404, 405, 406, 407, 411, 413, 416, 417, 418, 420, 421, 423, 424, 426, 427, 428, 429, 432, 433, 434, 435, 436, 438, 445, 446, 447, 448, 449, 450, 452, 453, 454, 455, 456, 457, 474, 490, 491, 492, 493, 500, 506, 511, 532, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 592, 595, 616, 641, 645, 646, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 672, 674, 675, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 698, 699, 701, 702, 703, 705 }

F(-2) exception fail { }

Sympy

A grade { 5, 6, 7, 9, 10, 11, 14, 15, 16, 17, 18, 20, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 49, 53, 54, 55, 56, 57, 60, 63, 65, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 120, 121, 124, 128, 129, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 212, 213, 235, 236, 237, 240, 241, 251, 261, 262, 263, 266, 267, 271, 272, 293, 300, 322, 323, 330, 331, 332, 333, 334, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 355, 356, 357, 358, 359, 361, 363, 364, 365, 366, 367, 368, 371, 372, 377, 385, 387, 441, 458, 459, 460, 463, 464, 465, 466, 471, 472, 476, 479, 480, 483, 484, 485, 486, 496, 497, 498, 499, 501, 502, 507, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 528, 534, 535, 537, 538, 539, 541, 542, 543, 546, 564, 565, 566, 567, 568, 569, 570, 571, 575, 577, 578, 582, 584, 596, 597, 598, 599, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 641, 642, 644, 646, 647, 648, 651, 652, 653, 654, 656, 659, 660, 661, 662, 663, 664, 666, 667, 668, 669, 671, 676, 677, 679, 680, 681, 699, 700 }

B grade { 1, 3, 4, 8, 12, 13, 19, 21, 30, 31, 36, 41, 48, 62, 89, 149, 159, 194, 195, 196, 197, 211, 214, 252, 253, 254, 273, 299, 312, 335, 340, 353, 354, 369, 370, 376, 378, 379, 382, 383, 388, 389, 396, 467, 468, 475, 487, 488, 489, 493, 494, 503, 504, 505, 508, 509, 510, 527, 530, 531, 547, 548, 572, 573, 574, 576, 580, 583, 585, 587, 588, 589, 590, 591, 602, 603, 604, 608, 609, 621, 655, 670, 697, 704 }

C grade { 2, 50, 51, 52, 79, 114, 118, 119, 122, 123, 125, 126, 127, 130, 132, 133, 134, 145, 175, 207, 215, 217, 250, 292, 294, 295, 296, 297, 298, 301, 302, 303, 304, 305, 313, 316, 461, 462, 470, 477, 536, 544, 545, 586, 616 }

F normal fail { 58, 59, 61, 66, 69, 193, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 238, 239, 242, 243, 244, 245, 246, 247, 248, 249, 255, 256, 257, 258, 259, 264, 265, 268, 269, 270, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 306, 307, 308, 309, 310, 311, 314, 315, 317, 319, 320, 321, 324, 325, 326, 327, 328, 329, 352, 360, 362, 373, 374, 380, 381, 384, 386, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 413, 427, 429, 430, 431, 434, 435, 436, 437, 439, 440, 442, 443, 444, 445, 448, 451, 452, 453, 456, 457, 469, 478, 481, 482, 490, 491, 492, 500, 506, 511, 516, 521, 529, 532, 533, 540, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 579, 581, 592, 594, 595, 600, 601, 605, 606, 607, 622, 623, 624, 625, 626, 627, 633, 643, 645, 649, 650, 658, 665, 672, 678, 682, 683, 690, 692, 694, 695, 696, 698, 701, 705 }

F(-1) timedout fail { 64, 198, 216, 222, 260, 318, 375, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 428, 432, 433, 438, 446, 447, 449, 450, 454, 455, 473, 474, 593, 640, 657, 684, 685, 686, 687, 688, 689, 691, 693, 702, 703 }

F(-2) exception fail { 495, 673, 674, 675 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	26	31	25	20	33	14
N.S.	1	1.00	1.00	1.86	2.21	1.79	1.43	2.36	1.00
time (sec)	N/A	0.006	0.005	0.189	0.199	0.254	0.058	0.280	0.055

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	26	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	1.86	1.00	1.00
time (sec)	N/A	0.003	0.004	0.220	0.283	0.243	0.054	0.276	0.041

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	18	17	26	27	40	11
N.S.	1	1.00	1.00	1.38	1.31	2.00	2.08	3.08	0.85
time (sec)	N/A	0.005	0.003	0.125	0.182	0.297	0.067	0.279	0.252

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	23	8	19	23	22	21	7
N.S.	1	1.00	2.09	0.73	1.73	2.09	2.00	1.91	0.64
time (sec)	N/A	0.003	0.033	0.135	0.192	0.259	0.059	0.271	0.079

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	21	27	29	22	29	24
N.S.	1	1.00	1.00	1.40	1.80	1.93	1.47	1.93	1.60
time (sec)	N/A	0.004	0.009	0.293	0.214	0.255	0.092	0.280	0.243

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	4	4	3	4	12
N.S.	1	1.00	1.00	1.50	2.00	2.00	1.50	2.00	6.00
time (sec)	N/A	0.005	0.002	0.069	0.194	0.254	0.020	0.313	0.331

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	6	6	5	6	6
N.S.	1	1.00	1.00	1.25	1.50	1.50	1.25	1.50	1.50
time (sec)	N/A	0.007	0.002	0.052	0.196	0.261	0.037	0.284	0.253

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	27	4	7	4	4
N.S.	1	1.00	1.00	0.83	4.50	0.67	1.17	0.67	0.67
time (sec)	N/A	0.018	0.026	0.102	0.231	0.248	0.319	0.293	0.205

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	9	9	3	30	4
N.S.	1	1.00	0.67	0.56	1.00	1.00	0.33	3.33	0.44
time (sec)	N/A	0.007	0.003	0.059	0.192	0.247	0.086	0.287	0.211

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	10	10	7	8	6
N.S.	1	1.00	0.67	0.75	0.83	0.83	0.58	0.67	0.50
time (sec)	N/A	0.009	0.002	0.069	0.204	0.240	0.163	0.285	0.002

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	12	15	14	13
N.S.	1	1.00	1.00	1.08	1.08	1.00	1.25	1.17	1.08
time (sec)	N/A	0.018	0.048	0.237	0.226	0.252	0.092	0.306	0.220

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	15	31	15	15
N.S.	1	1.00	1.00	1.07	1.00	1.00	2.07	1.00	1.00
time (sec)	N/A	0.026	0.013	0.692	0.288	0.282	0.222	0.288	0.049

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	33	33	26	44	35	15
N.S.	1	1.00	1.00	2.20	2.20	1.73	2.93	2.33	1.00
time (sec)	N/A	0.023	0.013	0.500	0.212	0.252	0.252	0.275	0.204

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	21	32	17	48
N.S.	1	1.00	1.00	1.06	1.00	1.24	1.88	1.00	2.82
time (sec)	N/A	0.030	0.012	6.745	0.224	0.265	0.872	0.298	0.597

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	20	23	34	21	48
N.S.	1	1.00	1.00	1.05	1.05	1.21	1.79	1.11	2.53
time (sec)	N/A	0.039	0.013	6.177	0.226	0.275	0.914	0.280	0.592

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	22	22	19	18	18	34	18	58
N.S.	1	1.22	1.22	1.06	1.00	1.00	1.89	1.00	3.22
time (sec)	N/A	0.039	0.018	10.257	0.214	0.265	0.872	0.279	0.462

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	22	22	19	19	19	32	20	58
N.S.	1	1.22	1.22	1.06	1.06	1.06	1.78	1.11	3.22
time (sec)	N/A	0.035	0.016	9.131	0.207	0.270	0.937	0.287	0.470

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	19	14	13	31	61	46	26
N.S.	1	1.00	0.46	0.34	0.32	0.76	1.49	1.12	0.63
time (sec)	N/A	0.017	0.109	0.151	0.306	0.271	0.223	0.299	0.257

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	15	15	15	16	15
N.S.	1	1.00	1.00	1.00	2.50	2.50	2.50	2.67	2.50
time (sec)	N/A	0.015	0.001	0.033	0.221	0.251	0.048	0.284	0.002

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	3	4	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	1.00	1.33	1.00
time (sec)	N/A	0.010	0.001	0.025	0.197	0.243	0.035	0.271	0.206

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	15	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.015	0.005	0.103	0.285	0.268	0.068	0.293	0.411

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	7	8	7	7	7	22	7
N.S.	1	1.00	0.78	0.89	0.78	0.78	0.78	2.44	0.78
time (sec)	N/A	0.015	0.034	0.040	0.210	0.235	0.039	0.325	0.261

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	9	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.78	1.00	1.00
time (sec)	N/A	0.017	0.035	0.033	0.201	0.241	0.041	0.285	0.278

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	22	24	22	23	24
N.S.	1	1.00	1.00	0.88	0.88	0.96	0.88	0.92	0.96
time (sec)	N/A	0.019	0.026	0.066	0.209	0.248	0.151	0.294	0.047

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	21	20	22	31	21	22
N.S.	1	1.00	1.00	0.70	0.67	0.73	1.03	0.70	0.73
time (sec)	N/A	0.085	0.034	0.027	0.196	0.237	0.933	0.274	0.036

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	10	10	8	11	8
N.S.	1	1.00	1.00	0.90	1.00	1.00	0.80	1.10	0.80
time (sec)	N/A	0.005	0.003	0.155	0.200	0.244	0.024	0.287	0.086

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	24	36	40	42	37	47
N.S.	1	1.00	1.00	0.51	0.77	0.85	0.89	0.79	1.00
time (sec)	N/A	0.021	0.034	0.194	0.284	0.248	0.045	0.274	0.138

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	23	23	27	21	21
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.90	0.70	0.70
time (sec)	N/A	0.007	0.016	0.190	0.289	0.255	0.044	0.278	0.042

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	24	22	17	13
N.S.	1	1.00	1.00	0.67	0.62	1.14	1.05	0.81	0.62
time (sec)	N/A	0.007	0.052	0.249	0.199	0.246	0.128	0.301	0.190

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	24	26	11	11
N.S.	1	1.00	1.00	0.80	0.73	1.60	1.73	0.73	0.73
time (sec)	N/A	0.008	0.002	0.237	0.210	0.254	0.127	0.282	0.059

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	186	89	138	18	118
N.S.	1	1.00	1.00	0.95	8.86	4.24	6.57	0.86	5.62
time (sec)	N/A	0.038	0.088	0.240	0.299	0.273	0.560	0.286	1.426

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.005	0.002	0.059	0.197	0.245	0.022	0.269	0.029

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.006	0.002	0.063	0.198	0.241	0.016	0.271	0.027

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	12
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	1.50
time (sec)	N/A	0.010	0.002	0.125	0.203	0.249	0.017	0.303	0.030

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	15	14	22	15	14	11
N.S.	1	1.00	1.00	1.36	1.27	2.00	1.36	1.27	1.00
time (sec)	N/A	0.012	0.010	0.308	0.216	0.238	0.033	0.290	0.099

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	6	11	9	18	12	9	6
N.S.	1	1.00	0.86	1.57	1.29	2.57	1.71	1.29	0.86
time (sec)	N/A	0.018	0.003	0.287	0.201	0.247	0.024	0.293	0.254

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	28	17	12	23	19	18	10
N.S.	1	1.00	2.00	1.21	0.86	1.64	1.36	1.29	0.71
time (sec)	N/A	0.007	0.017	0.052	0.273	0.240	0.025	0.435	0.232

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	26	19	12	20	17	22	18
N.S.	1	1.00	1.62	1.19	0.75	1.25	1.06	1.38	1.12
time (sec)	N/A	0.013	0.017	0.059	0.287	0.247	0.058	0.304	0.331

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	29	11	12	19	10	12	12
N.S.	1	1.00	2.90	1.10	1.20	1.90	1.00	1.20	1.20
time (sec)	N/A	0.027	0.014	0.183	0.274	0.240	0.143	0.291	0.285

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	25	10	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.79	0.71	1.00	1.00
time (sec)	N/A	0.062	0.009	0.438	0.279	0.248	0.325	0.296	0.297

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	15	28	24	29	12	12
N.S.	1	1.00	2.27	1.36	2.55	2.18	2.64	1.09	1.09
time (sec)	N/A	0.021	0.046	0.101	0.290	0.251	0.230	0.292	0.243

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	31	17	23	14	8	12	10
N.S.	1	1.00	1.94	1.06	1.44	0.88	0.50	0.75	0.62
time (sec)	N/A	0.025	0.094	0.093	0.272	0.268	0.211	0.292	0.180

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	33	19	18	22	19	18	18
N.S.	1	1.00	1.32	0.76	0.72	0.88	0.76	0.72	0.72
time (sec)	N/A	0.025	0.033	0.052	0.197	0.242	0.046	0.286	0.238

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	13	12	15	8
N.S.	1	1.00	1.00	0.67	0.62	0.62	0.57	0.71	0.38
time (sec)	N/A	0.004	0.004	0.224	0.208	0.249	0.042	0.292	0.242

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.017	0.008	0.078	0.271	0.228	0.043	0.277	0.068

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	47	29	35	45	42	44	34
N.S.	1	1.00	0.92	0.57	0.69	0.88	0.82	0.86	0.67
time (sec)	N/A	0.016	0.027	0.283	0.271	0.260	0.050	0.277	0.129

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	25	29	25	25
N.S.	1	1.00	1.00	0.79	0.76	0.76	0.88	0.76	0.76
time (sec)	N/A	0.009	0.022	0.136	0.200	0.232	0.056	0.290	0.246

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	18	68	73	18	19
N.S.	1	1.00	1.00	0.66	0.62	2.34	2.52	0.62	0.66
time (sec)	N/A	0.012	0.025	0.181	0.289	0.244	0.377	0.284	0.217

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	17	22	19	37	19	20
N.S.	1	1.00	0.74	0.63	0.81	0.70	1.37	0.70	0.74
time (sec)	N/A	0.011	0.019	0.178	0.284	0.236	0.135	0.300	0.035

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	28	19	18	25	29	10	18
N.S.	1	1.00	1.27	0.86	0.82	1.14	1.32	0.45	0.82
time (sec)	N/A	0.007	0.127	0.256	0.281	0.243	0.498	0.309	0.069

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	12	26	22	20	24
N.S.	1	1.00	1.00	0.95	0.55	1.18	1.00	0.91	1.09
time (sec)	N/A	0.010	0.026	0.438	0.286	0.261	0.498	0.287	0.311

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	53	37	34	25	22	43	21
N.S.	1	1.00	2.30	1.61	1.48	1.09	0.96	1.87	0.91
time (sec)	N/A	0.012	0.051	0.208	0.210	0.243	0.522	0.305	0.554

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	49	35	12	40	7	37	26
N.S.	1	1.00	2.33	1.67	0.57	1.90	0.33	1.76	1.24
time (sec)	N/A	0.010	0.041	0.221	0.201	0.245	0.508	0.310	0.107

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	21	7	8	30	8	26	6
N.S.	1	1.00	1.75	0.58	0.67	2.50	0.67	2.17	0.50
time (sec)	N/A	0.005	0.082	0.250	0.275	0.246	0.229	0.292	0.243

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	33	15	16	38	20	53	26
N.S.	1	1.00	1.74	0.79	0.84	2.00	1.05	2.79	1.37
time (sec)	N/A	0.009	0.086	0.796	0.286	0.264	0.260	0.298	0.382

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	40	7	6	16	5	25	6
N.S.	1	1.00	5.00	0.88	0.75	2.00	0.62	3.12	0.75
time (sec)	N/A	0.002	0.044	0.424	0.308	0.244	0.192	0.286	0.219

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	36	22	21	43	22	21	40
N.S.	1	1.00	1.33	0.81	0.78	1.59	0.81	0.78	1.48
time (sec)	N/A	0.009	0.128	0.254	0.323	0.278	0.358	0.289	0.419

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	25	33	39	0	71	28
N.S.	1	1.00	0.94	0.78	1.03	1.22	0.00	2.22	0.88
time (sec)	N/A	0.008	0.108	0.250	0.300	0.258	0.000	0.305	0.427

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	23	21	16	17	17	0	28	19
N.S.	1	1.10	1.00	0.76	0.81	0.81	0.00	1.33	0.90
time (sec)	N/A	0.004	0.139	0.276	0.298	0.245	0.000	0.318	0.301

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	54	21	33	39	22	31	22
N.S.	1	1.00	1.93	0.75	1.18	1.39	0.79	1.11	0.79
time (sec)	N/A	0.117	0.160	0.216	0.221	0.271	0.299	0.292	0.361

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	48	17	14	13	31	0	46	26
N.S.	1	1.30	0.46	0.38	0.35	0.84	0.00	1.24	0.70
time (sec)	N/A	0.023	0.541	0.155	0.313	0.258	0.000	0.285	0.294

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	21	11	47	32	27	11	10
N.S.	1	1.00	1.50	0.79	3.36	2.29	1.93	0.79	0.71
time (sec)	N/A	0.029	0.064	0.200	0.221	0.255	0.499	0.273	0.281

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	4	15	45	15	17	3
N.S.	1	1.00	0.82	0.36	1.36	4.09	1.36	1.55	0.27
time (sec)	N/A	0.030	0.007	0.063	0.218	0.252	0.061	0.280	0.392

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	19	19	15	14	14	0	14	14
N.S.	1	1.06	1.06	0.83	0.78	0.78	0.00	0.78	0.78
time (sec)	N/A	0.045	0.021	0.184	0.213	0.267	0.000	0.269	0.424

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	15	17	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.83	0.94	0.78	0.78
time (sec)	N/A	0.054	0.024	0.257	0.216	0.269	0.795	0.259	0.376

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	0	15	37
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.00	0.88	2.18
time (sec)	N/A	0.015	0.036	0.132	0.296	0.251	0.000	0.279	0.355

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	0	18	16	19	16	16
N.S.	1	1.00	1.10	0.00	0.90	0.80	0.95	0.80	0.80
time (sec)	N/A	0.021	0.039	0.000	0.215	0.256	0.281	0.273	0.411

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	15	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.25	1.00	1.00
time (sec)	N/A	0.024	0.008	0.546	0.221	0.256	0.833	0.268	0.296

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	0	38	0	15	0
N.S.	1	1.00	1.00	0.90	0.00	0.90	0.00	0.36	0.00
time (sec)	N/A	0.029	0.049	0.247	0.000	0.256	0.000	0.285	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.018	0.009	0.168	0.305	0.254	0.411	0.260	0.394

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	17	22	22	22	17
N.S.	1	1.00	1.00	0.82	0.61	0.79	0.79	0.79	0.61
time (sec)	N/A	0.008	0.000	0.021	0.211	0.239	0.040	0.287	0.034

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	11	13	11	15	13	9
N.S.	1	1.00	1.00	0.65	0.76	0.65	0.88	0.76	0.53
time (sec)	N/A	0.005	0.003	0.034	0.210	0.252	0.040	0.269	0.229

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	38	38	29	28	28	26	70	40
N.S.	1	1.06	1.06	0.81	0.78	0.78	0.72	1.94	1.11
time (sec)	N/A	0.021	0.009	0.108	0.212	0.257	0.048	0.285	0.413

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	15	18	17	15	21
N.S.	1	1.00	1.00	0.89	0.79	0.95	0.89	0.79	1.11
time (sec)	N/A	0.006	0.001	0.302	0.204	0.262	0.022	0.276	0.002

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	23	18	25	31	22	26
N.S.	1	1.00	0.88	0.68	0.53	0.74	0.91	0.65	0.76
time (sec)	N/A	0.026	0.001	0.318	0.207	0.255	0.017	0.270	0.024

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	71	26	42	69	46	38	33
N.S.	1	1.00	2.73	1.00	1.62	2.65	1.77	1.46	1.27
time (sec)	N/A	0.012	0.014	0.329	0.210	0.250	0.069	0.269	0.261

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	14	12	11	17	17	11	11
N.S.	1	1.00	0.61	0.52	0.48	0.74	0.74	0.48	0.48
time (sec)	N/A	0.009	0.015	0.125	0.207	0.248	0.163	0.277	0.022

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	19	21	26	19	19
N.S.	1	1.00	0.81	0.74	0.70	0.78	0.96	0.70	0.70
time (sec)	N/A	0.008	0.053	0.132	0.206	0.257	0.093	0.270	0.030

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	21	24	20	107	329	20
N.S.	1	1.00	0.65	0.68	0.77	0.65	3.45	10.61	0.65
time (sec)	N/A	0.010	0.022	0.161	0.215	0.243	0.269	0.304	0.030

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	11	10	13	15	13	13
N.S.	1	1.00	1.00	0.65	0.59	0.76	0.88	0.76	0.76
time (sec)	N/A	0.004	0.003	0.073	0.207	0.259	0.127	0.271	0.240

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	94	22	15	12	35
N.S.	1	1.00	1.00	1.08	7.83	1.83	1.25	1.00	2.92
time (sec)	N/A	0.018	0.019	0.565	0.286	0.246	17.259	0.282	0.506

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	107	21	19	23	13
N.S.	1	1.00	1.00	1.33	7.13	1.40	1.27	1.53	0.87
time (sec)	N/A	0.014	0.006	0.031	0.296	0.248	0.073	0.270	0.026

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	33	39	22	38	20
N.S.	1	1.00	1.00	0.95	1.50	1.77	1.00	1.73	0.91
time (sec)	N/A	0.014	0.001	0.013	0.290	0.268	0.980	0.267	0.023

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	23	22	23	22
N.S.	1	1.00	1.00	0.96	0.92	0.92	0.88	0.92	0.88
time (sec)	N/A	0.032	0.003	0.040	0.284	0.246	0.074	0.284	0.041

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	24	19	19	19	19
N.S.	1	1.00	1.00	0.87	1.04	0.83	0.83	0.83	0.83
time (sec)	N/A	0.049	0.019	0.234	0.292	0.249	0.108	0.282	0.294

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	57	49	45	78	30	58	0	0
N.S.	1	1.50	1.29	1.18	2.05	0.79	1.53	0.00	0.00
time (sec)	N/A	0.034	0.056	0.053	0.300	0.257	4.078	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	21	21	22	21	21
N.S.	1	1.00	1.00	0.84	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.005	0.003	0.161	0.209	0.235	0.018	0.286	0.046

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	29	29	29	34	29	29
N.S.	1	1.00	1.00	0.74	0.74	0.74	0.87	0.74	0.74
time (sec)	N/A	0.011	0.001	0.238	0.201	0.232	0.021	0.274	0.036

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	29	23	27	78	23	23
N.S.	1	1.00	1.00	1.26	1.00	1.17	3.39	1.00	1.00
time (sec)	N/A	0.008	0.037	0.865	0.212	0.248	12.029	0.279	0.817

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	27	21	22	22	20	23	20
N.S.	1	1.00	0.90	0.70	0.73	0.73	0.67	0.77	0.67
time (sec)	N/A	0.010	0.010	0.168	0.214	0.258	0.026	0.282	0.037

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	35	27	26	32	34	26	32
N.S.	1	1.00	0.85	0.66	0.63	0.78	0.83	0.63	0.78
time (sec)	N/A	0.010	0.013	0.182	0.298	0.238	0.036	0.278	0.044

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	15	15	14	17	8
N.S.	1	1.00	1.00	0.67	0.71	0.71	0.67	0.81	0.38
time (sec)	N/A	0.005	0.004	0.226	0.220	0.244	0.042	0.268	0.095

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	29	28	28	36	28	30
N.S.	1	1.00	0.97	0.88	0.85	0.85	1.09	0.85	0.91
time (sec)	N/A	0.013	0.008	0.710	0.291	0.255	0.047	0.273	0.046

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.014	0.005	0.301	0.284	0.239	0.045	0.283	0.048

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	39	38	38	46	38	40
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.98	0.81	0.85
time (sec)	N/A	0.051	0.019	0.411	0.293	0.263	0.046	0.275	0.284

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	17	17	17	20	17
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.68	0.80	0.68
time (sec)	N/A	0.029	0.007	0.039	0.202	0.239	0.054	0.279	0.261

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	26	25	25	26	28	25
N.S.	1	1.00	0.88	0.79	0.76	0.76	0.79	0.85	0.76
time (sec)	N/A	0.039	0.019	0.053	0.213	0.242	0.117	0.276	0.109

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	61	37	26	25	25	29	29	29
N.S.	1	1.65	1.00	0.70	0.68	0.68	0.78	0.78	0.78
time (sec)	N/A	0.032	0.008	0.047	0.203	0.256	0.077	0.290	0.064

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	69	26	43	51	60	59	25
N.S.	1	1.00	2.23	0.84	1.39	1.65	1.94	1.90	0.81
time (sec)	N/A	0.011	0.020	0.060	0.287	0.251	0.289	0.291	0.098

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	26	25	25	26	29	15
N.S.	1	1.00	1.00	0.63	0.61	0.61	0.63	0.71	0.37
time (sec)	N/A	0.019	0.008	0.259	0.206	0.242	0.075	0.273	0.272

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	16	17	22	29	17	18	22
N.S.	1	1.00	0.64	0.68	0.88	1.16	0.68	0.72	0.88
time (sec)	N/A	0.009	0.011	0.180	0.206	0.227	0.040	0.284	0.205

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	32	39	31	45	32
N.S.	1	1.00	1.00	0.92	0.89	1.08	0.86	1.25	0.89
time (sec)	N/A	0.012	0.016	0.171	0.204	0.252	0.030	0.278	0.038

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	37	25	30	45	31	27	30
N.S.	1	1.00	0.90	0.61	0.73	1.10	0.76	0.66	0.73
time (sec)	N/A	0.034	0.022	0.042	0.216	0.244	0.064	0.266	0.066

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	29	45	26	37	29
N.S.	1	1.00	1.00	1.04	1.07	1.67	0.96	1.37	1.07
time (sec)	N/A	0.028	0.018	0.193	0.206	0.271	0.052	0.294	0.269

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	47	31	29	45	32	35	31
N.S.	1	1.00	1.21	0.79	0.74	1.15	0.82	0.90	0.79
time (sec)	N/A	0.028	0.019	0.211	0.220	0.252	0.063	0.282	0.054

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	40	65	37	40	40
N.S.	1	1.00	0.87	0.76	0.87	1.41	0.80	0.87	0.87
time (sec)	N/A	0.017	0.018	0.049	0.206	0.239	0.068	0.290	0.245

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	23	23	22	24	29
N.S.	1	1.00	1.00	0.83	0.79	0.79	0.76	0.83	1.00
time (sec)	N/A	0.025	0.008	0.072	0.313	0.237	0.055	0.268	0.052

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	21	21	22	23	27
N.S.	1	1.00	1.00	0.81	0.78	0.78	0.81	0.85	1.00
time (sec)	N/A	0.020	0.009	0.233	0.326	0.242	0.080	0.292	0.051

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	32	26	25	25	29	27	17
N.S.	1	1.00	1.33	1.08	1.04	1.04	1.21	1.12	0.71
time (sec)	N/A	0.008	0.012	0.049	0.309	0.242	0.061	0.279	0.065

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	32	44	39	33	49
N.S.	1	1.00	1.00	0.80	0.78	1.07	0.95	0.80	1.20
time (sec)	N/A	0.068	0.025	0.053	0.312	0.238	0.063	0.291	0.336

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	36	57	37	62	38
N.S.	1	1.00	0.87	0.76	0.78	1.24	0.80	1.35	0.83
time (sec)	N/A	0.187	0.021	0.276	0.330	0.241	0.089	0.288	0.223

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	37	31	36	59	36	32	42
N.S.	1	1.00	0.79	0.66	0.77	1.26	0.77	0.68	0.89
time (sec)	N/A	0.030	0.028	0.253	0.295	0.245	0.076	0.279	0.050

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	73	54	53	53	70	53	47
N.S.	1	1.00	1.09	0.81	0.79	0.79	1.04	0.79	0.70
time (sec)	N/A	0.028	0.064	0.030	0.295	0.241	0.086	0.294	0.116

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	67	46	54	58	306	64	73
N.S.	1	1.00	1.40	0.96	1.12	1.21	6.38	1.33	1.52
time (sec)	N/A	0.039	0.020	0.244	0.298	0.239	0.286	0.297	0.181

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	46	38	43	66	46	39	44
N.S.	1	1.00	0.79	0.66	0.74	1.14	0.79	0.67	0.76
time (sec)	N/A	0.054	0.027	0.369	0.285	0.239	0.078	0.284	0.229

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	47	36	35	35	44	35	35
N.S.	1	1.00	0.92	0.71	0.69	0.69	0.86	0.69	0.69
time (sec)	N/A	0.232	0.029	0.316	0.291	0.243	0.205	0.271	0.301

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	34	33	33	32	33	33
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.78	0.80	0.80
time (sec)	N/A	0.264	0.010	0.243	0.198	0.245	0.076	0.277	0.062

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	52	51	49	45	73	50	64
N.S.	1	1.00	0.93	0.91	0.88	0.80	1.30	0.89	1.14
time (sec)	N/A	0.032	0.016	0.226	0.280	0.240	0.058	0.308	0.463

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	50	43	49	43	71	50	68
N.S.	1	1.00	0.89	0.77	0.88	0.77	1.27	0.89	1.21
time (sec)	N/A	0.025	0.010	0.207	0.287	0.238	0.050	0.280	0.123

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	11	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.92	0.83
time (sec)	N/A	0.002	0.003	0.193	0.193	0.236	0.038	0.287	0.032

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	23	18	19	22	18
N.S.	1	1.00	1.00	0.95	1.05	0.82	0.86	1.00	0.82
time (sec)	N/A	0.007	0.006	0.200	0.201	0.230	0.096	0.325	0.291

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	60	57	53	83	58	88
N.S.	1	1.00	0.95	0.95	0.90	0.84	1.32	0.92	1.40
time (sec)	N/A	0.031	0.019	0.230	0.276	0.238	0.071	0.296	0.319

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	68	60	57	62	80	58	86
N.S.	1	1.00	1.05	0.92	0.88	0.95	1.23	0.89	1.32
time (sec)	N/A	0.034	0.021	0.234	0.285	0.241	0.091	0.299	0.301

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	34	31	33	29	40	29
N.S.	1	1.00	1.00	1.03	0.94	1.00	0.88	1.21	0.88
time (sec)	N/A	0.023	0.007	0.207	0.198	0.235	0.125	0.271	0.086

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	74	66	66	68	90	67	99
N.S.	1	1.00	1.01	0.90	0.90	0.93	1.23	0.92	1.36
time (sec)	N/A	0.042	0.017	0.227	0.284	0.242	0.098	0.273	0.112

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	0	92	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.00	0.00	0.00
time (sec)	N/A	0.009	0.142	0.000	0.000	0.000	1.229	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	38	33	32	26	37	34	18
N.S.	1	1.00	1.41	1.22	1.19	0.96	1.37	1.26	0.67
time (sec)	N/A	0.006	0.012	0.249	0.288	0.244	0.065	0.277	0.093

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	27	29	26	24	30	13
N.S.	1	1.00	1.00	1.80	1.93	1.73	1.60	2.00	0.87
time (sec)	N/A	0.008	0.005	0.205	0.191	0.239	0.063	0.287	0.054

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	25	20	19	26	20
N.S.	1	1.00	1.00	0.96	1.04	0.83	0.79	1.08	0.83
time (sec)	N/A	0.011	0.008	0.203	0.196	0.231	0.120	0.284	0.338

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	46	41	40	36	44	42	31
N.S.	1	1.00	1.31	1.17	1.14	1.03	1.26	1.20	0.89
time (sec)	N/A	0.010	0.012	0.256	0.285	0.243	0.089	0.285	0.270

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	50	42	37	41	34	38	22
N.S.	1	1.00	1.92	1.62	1.42	1.58	1.31	1.46	0.85
time (sec)	N/A	0.013	0.009	0.216	0.210	0.236	0.098	0.275	0.260

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	48	41	40	45	48	42	31
N.S.	1	1.00	1.30	1.11	1.08	1.22	1.30	1.14	0.84
time (sec)	N/A	0.011	0.015	0.254	0.292	0.242	0.104	0.281	0.081

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	44	0	0	0	95	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.11	0.00	0.00
time (sec)	N/A	0.008	0.234	0.000	0.000	0.000	0.497	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	29	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.93	0.87	0.87
time (sec)	N/A	0.007	0.005	0.237	0.278	0.245	0.068	0.322	0.262

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	79	24	98	96	19	114	33
N.S.	1	1.00	0.72	0.22	0.90	0.88	0.17	1.05	0.30
time (sec)	N/A	0.056	0.034	0.232	0.295	0.243	0.059	0.301	0.112

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	204	55	180	11094	39	177	174
N.S.	1	1.00	1.01	0.27	0.90	55.19	0.19	0.88	0.87
time (sec)	N/A	0.301	0.183	0.230	0.293	0.962	0.058	0.290	0.714

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	204	60	160	18781	41	177	182
N.S.	1	1.00	1.01	0.30	0.80	93.44	0.20	0.88	0.91
time (sec)	N/A	0.264	0.067	0.211	0.285	0.985	0.054	0.304	0.690

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	204	58	160	12656	41	177	202
N.S.	1	1.00	1.01	0.29	0.80	62.97	0.20	0.88	1.00
time (sec)	N/A	0.289	0.072	0.210	0.298	0.964	0.062	0.316	0.954

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	204	73	180	17865	39	177	202
N.S.	1	1.00	1.01	0.36	0.90	88.88	0.19	0.88	1.00
time (sec)	N/A	0.290	0.041	0.213	0.285	0.972	0.062	0.288	0.427

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	11	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.92	0.83
time (sec)	N/A	0.003	0.003	0.204	0.196	0.227	0.042	0.298	0.257

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	23	18	19	22	18
N.S.	1	1.00	1.00	0.95	1.05	0.82	0.86	1.00	0.82
time (sec)	N/A	0.007	0.007	0.205	0.201	0.229	0.119	0.291	0.342

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	172	76	192	15275	48	185	210
N.S.	1	1.00	0.82	0.36	0.92	73.09	0.23	0.89	1.00
time (sec)	N/A	0.312	0.180	0.233	0.290	0.977	0.080	0.317	0.415

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	174	78	173	15499	51	185	210
N.S.	1	1.00	0.82	0.37	0.82	73.45	0.24	0.88	1.00
time (sec)	N/A	0.305	0.171	0.227	0.290	1.018	0.097	0.303	0.818

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	175	81	172	15501	51	185	214
N.S.	1	1.00	0.83	0.38	0.82	73.46	0.24	0.88	1.01
time (sec)	N/A	0.253	0.129	0.226	0.283	0.973	0.091	0.320	0.763

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	0	92	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.00	0.00	0.00
time (sec)	N/A	0.010	0.263	0.000	0.000	0.000	8.408	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	21	10	27	9	8	27	9
N.S.	1	1.00	0.60	0.29	0.77	0.26	0.23	0.77	0.26
time (sec)	N/A	0.367	0.008	0.240	0.282	0.239	0.051	0.269	0.034

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	44	54	71	63	44	45
N.S.	1	1.00	0.85	0.73	0.90	1.18	1.05	0.73	0.75
time (sec)	N/A	0.018	0.031	0.812	0.274	0.251	0.072	0.285	0.081

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	30	28	38	52	36	28	27
N.S.	1	1.00	0.70	0.65	0.88	1.21	0.84	0.65	0.63
time (sec)	N/A	0.012	0.014	0.234	0.274	0.253	0.059	0.287	0.227

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	88	103	0	448	323	92	159
N.S.	1	1.00	0.98	1.14	0.00	4.98	3.59	1.02	1.77
time (sec)	N/A	0.061	0.085	0.536	0.000	0.267	0.541	0.282	0.336

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	33	50	31	33	41
N.S.	1	1.00	1.00	0.92	0.87	1.32	0.82	0.87	1.08
time (sec)	N/A	0.025	0.015	0.322	0.278	0.241	0.050	0.289	0.218

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	41	42	58	53	43	60
N.S.	1	1.00	0.86	0.72	0.74	1.02	0.93	0.75	1.05
time (sec)	N/A	0.025	0.024	0.205	0.280	0.238	0.063	0.277	0.090

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	29	31	40	32	28	29
N.S.	1	1.00	0.92	0.81	0.86	1.11	0.89	0.78	0.81
time (sec)	N/A	0.024	0.017	0.234	0.268	0.226	0.063	0.278	0.217

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	78	39	40	40	46	40	52
N.S.	1	1.00	1.59	0.80	0.82	0.82	0.94	0.82	1.06
time (sec)	N/A	0.029	0.017	0.221	0.282	0.246	0.056	0.293	0.101

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	20	13	14	14	13	14	0	14
N.S.	1	1.54	1.00	1.08	1.08	1.00	1.08	0.00	1.08
time (sec)	N/A	0.035	0.028	0.290	0.204	0.246	0.784	0.000	0.370

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	38	35	34	34	42	34	34
N.S.	1	1.00	0.93	0.85	0.83	0.83	1.02	0.83	0.83
time (sec)	N/A	0.035	0.013	0.053	0.280	0.245	0.048	0.429	0.228

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	25	24	24	24	25	22
N.S.	1	1.00	1.00	0.78	0.75	0.75	0.75	0.78	0.69
time (sec)	N/A	0.020	0.007	0.051	0.189	0.249	0.049	0.311	0.087

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	17	17	15	19	16
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.60	0.76	0.64
time (sec)	N/A	0.015	0.006	0.048	0.191	0.242	0.040	0.303	0.614

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	24	22	32	46	29	23	22
N.S.	1	1.00	0.67	0.61	0.89	1.28	0.81	0.64	0.61
time (sec)	N/A	0.017	0.015	0.197	0.188	0.240	0.043	0.303	0.042

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	24	22	72	72	70	22	29
N.S.	1	1.00	0.53	0.49	1.60	1.60	1.56	0.49	0.64
time (sec)	N/A	0.012	0.008	0.198	0.192	0.232	0.061	0.284	0.100

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	41	34	48	67	42	55	43
N.S.	1	1.00	0.75	0.62	0.87	1.22	0.76	1.00	0.78
time (sec)	N/A	0.033	0.014	0.191	0.188	0.238	0.051	0.274	0.053

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	38	59	41	43	31
N.S.	1	1.00	1.06	0.97	1.06	1.64	1.14	1.19	0.86
time (sec)	N/A	0.014	0.024	0.207	0.197	0.240	0.056	0.287	0.043

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	31	28	31	48	29	40	34
N.S.	1	1.00	0.89	0.80	0.89	1.37	0.83	1.14	0.97
time (sec)	N/A	0.012	0.019	0.237	0.188	0.250	0.053	0.312	0.257

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	46	40	50	85	51	42	45
N.S.	1	1.00	0.75	0.66	0.82	1.39	0.84	0.69	0.74
time (sec)	N/A	0.013	0.025	0.243	0.205	0.246	0.069	0.284	0.137

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	36	36	44	62	42	34	39
N.S.	1	1.00	0.71	0.71	0.86	1.22	0.82	0.67	0.76
time (sec)	N/A	0.012	0.018	0.546	0.282	0.240	0.071	0.277	0.254

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	60	105	58	47	55
N.S.	1	1.00	1.00	0.83	1.11	1.94	1.07	0.87	1.02
time (sec)	N/A	0.019	0.018	0.196	0.191	0.235	0.069	0.279	0.060

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	42	53	32	40	53	49	48	33
N.S.	1	1.17	1.47	0.89	1.11	1.47	1.36	1.33	0.92
time (sec)	N/A	0.015	0.040	0.226	0.275	0.244	0.036	0.301	0.273

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	40	31	41	59	39	31	40
N.S.	1	1.00	0.69	0.53	0.71	1.02	0.67	0.53	0.69
time (sec)	N/A	0.011	0.021	0.237	0.291	0.241	0.051	0.281	0.077

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	36	45	42	36	36
N.S.	1	1.00	1.00	0.79	0.84	1.05	0.98	0.84	0.84
time (sec)	N/A	0.021	0.037	0.856	0.289	0.244	0.061	0.279	0.222

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	62	37	47	68	58	51	34
N.S.	1	1.00	1.44	0.86	1.09	1.58	1.35	1.19	0.79
time (sec)	N/A	0.020	0.041	0.313	0.291	0.232	0.061	0.289	0.262

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	25	27	30	38	24	40	32
N.S.	1	1.00	0.68	0.73	0.81	1.03	0.65	1.08	0.86
time (sec)	N/A	0.010	0.013	0.247	0.284	0.239	0.059	0.309	0.039

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	39	26	29	28	38	26	32	28
N.S.	1	1.22	0.81	0.91	0.88	1.19	0.81	1.00	0.88
time (sec)	N/A	0.020	0.017	0.183	0.205	0.230	0.047	0.485	0.223

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	19	20	11	11
N.S.	1	1.00	1.00	0.92	0.85	1.46	1.54	0.85	0.85
time (sec)	N/A	0.003	0.005	0.217	0.205	0.230	0.115	0.275	0.229

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	46	45	57	81	51	56	52
N.S.	1	1.00	0.85	0.83	1.06	1.50	0.94	1.04	0.96
time (sec)	N/A	0.026	0.027	0.221	0.195	0.245	0.219	0.310	0.115

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	134	75	147	227	66	150	76
N.S.	1	1.00	0.85	0.48	0.94	1.45	0.42	0.96	0.48
time (sec)	N/A	0.087	0.112	0.273	0.275	0.239	0.205	0.294	0.148

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	75	53	60	78	78	50	53
N.S.	1	1.00	1.17	0.83	0.94	1.22	1.22	0.78	0.83
time (sec)	N/A	0.027	0.051	0.279	0.288	0.236	0.226	0.294	0.263

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	29	34	45	27	30	29
N.S.	1	1.00	0.85	0.74	0.87	1.15	0.69	0.77	0.74
time (sec)	N/A	0.024	0.016	0.215	0.190	0.225	0.058	0.320	0.051

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	293	47	335	1751	37	250	285
N.S.	1	1.00	0.92	0.15	1.05	5.49	0.12	0.78	0.89
time (sec)	N/A	0.509	0.351	0.247	0.280	2.695	0.113	0.590	1.704

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	65	39	56	73	58	55	41
N.S.	1	1.00	1.10	0.66	0.95	1.24	0.98	0.93	0.69
time (sec)	N/A	0.016	0.074	0.222	0.301	0.232	0.070	0.292	0.275

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	29	31	40	32	28	29
N.S.	1	1.00	0.92	0.81	0.86	1.11	0.89	0.78	0.81
time (sec)	N/A	0.020	0.016	0.228	0.292	0.237	0.059	0.301	0.004

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	31	38	65	36	35	33
N.S.	1	1.00	0.84	0.82	1.00	1.71	0.95	0.92	0.87
time (sec)	N/A	0.050	0.025	0.050	0.221	0.226	0.057	0.288	0.070

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	65	50	50	73	60	60	63
N.S.	1	1.00	1.02	0.78	0.78	1.14	0.94	0.94	0.98
time (sec)	N/A	0.062	0.032	0.225	0.301	0.238	0.098	0.307	0.111

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	54	59	97	68	72	73
N.S.	1	1.00	1.00	0.86	0.94	1.54	1.08	1.14	1.16
time (sec)	N/A	0.106	0.039	0.471	0.279	0.248	0.083	0.303	0.284

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	32	36	55	34	33	25
N.S.	1	1.00	1.07	0.74	0.84	1.28	0.79	0.77	0.58
time (sec)	N/A	0.014	0.020	0.226	0.198	0.234	0.068	0.290	0.059

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	18	43	43	39	18	18
N.S.	1	1.00	0.74	0.67	1.59	1.59	1.44	0.67	0.67
time (sec)	N/A	0.016	0.011	0.210	0.202	0.229	0.067	0.302	0.119

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	34	39	55	36	40	34
N.S.	1	1.00	0.96	0.74	0.85	1.20	0.78	0.87	0.74
time (sec)	N/A	0.022	0.018	0.200	0.185	0.225	0.045	0.306	0.220

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	50	39	35	37	56	34	43	33
N.S.	1	1.14	0.89	0.80	0.84	1.27	0.77	0.98	0.75
time (sec)	N/A	0.012	0.024	0.200	0.189	0.230	0.061	0.288	0.050

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	60	105	58	47	55
N.S.	1	1.00	1.00	0.83	1.11	1.94	1.07	0.87	1.02
time (sec)	N/A	0.012	0.009	0.187	0.186	0.233	0.069	0.278	0.002

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	32	39	31	45	32
N.S.	1	1.00	1.00	0.92	0.89	1.08	0.86	1.25	0.89
time (sec)	N/A	0.011	0.015	0.182	0.182	0.237	0.025	0.299	0.002

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	38	38	38	39	39	37
N.S.	1	1.00	0.93	0.86	0.86	0.86	0.89	0.89	0.84
time (sec)	N/A	0.029	0.013	0.074	0.182	0.241	0.021	0.299	0.049

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	91	89	91	91	100	98	88
N.S.	1	1.00	0.95	0.93	0.95	0.95	1.04	1.02	0.92
time (sec)	N/A	0.091	0.030	0.362	0.194	0.234	0.022	0.277	0.218

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	167	165	171	171	189	188	164
N.S.	1	1.00	1.00	0.99	1.02	1.02	1.13	1.13	0.98
time (sec)	N/A	0.148	0.032	0.342	0.180	0.223	0.030	0.288	0.242

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	263	264	273	273	313	307	263
N.S.	1	1.00	1.00	1.00	1.04	1.04	1.19	1.17	1.00
time (sec)	N/A	0.268	0.061	0.350	0.189	0.246	0.036	0.282	0.294

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	159	159	267	0	0	0	0	0	0
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.076	0.673	0.000	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	66	63	0	203	246	60	155
N.S.	1	1.00	1.02	0.97	0.00	3.12	3.78	0.92	2.38
time (sec)	N/A	0.035	0.046	0.463	0.000	0.248	0.401	0.293	0.308

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	88	103	0	447	323	92	159
N.S.	1	1.00	0.99	1.16	0.00	5.02	3.63	1.03	1.79
time (sec)	N/A	0.051	0.073	0.464	0.000	0.259	0.522	0.292	0.304

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	127	155	0	1104	622	194	360
N.S.	1	1.00	0.98	1.19	0.00	8.49	4.78	1.49	2.77
time (sec)	N/A	0.061	0.127	0.457	0.000	0.269	0.996	0.289	0.528

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	168	206	0	1950	1027	363	640
N.S.	1	1.00	0.97	1.19	0.00	11.27	5.94	2.10	3.70
time (sec)	N/A	0.095	0.190	0.478	0.000	0.298	1.528	0.306	0.728

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	264	0	0	0	0	0	0
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.065	0.791	0.000	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	31	41	52	46	46	36
N.S.	1	1.00	0.90	0.63	0.84	1.06	0.94	0.94	0.73
time (sec)	N/A	0.014	0.026	0.263	0.262	0.232	0.041	0.294	0.176

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	70	56	67	97	76	61	53
N.S.	1	1.00	1.15	0.92	1.10	1.59	1.25	1.00	0.87
time (sec)	N/A	0.018	0.051	0.261	0.278	0.239	0.066	0.290	0.249

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	28	30	45	31	32	25
N.S.	1	1.00	0.92	0.78	0.83	1.25	0.86	0.89	0.69
time (sec)	N/A	0.009	0.017	0.248	0.182	0.230	0.050	0.282	0.052

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	60	90	165	88	62	65
N.S.	1	1.00	1.00	0.69	1.03	1.90	1.01	0.71	0.75
time (sec)	N/A	0.022	0.027	0.262	0.194	0.245	0.092	0.305	0.095

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	70	62	69	116	80	67	77
N.S.	1	1.00	0.86	0.77	0.85	1.43	0.99	0.83	0.95
time (sec)	N/A	0.051	0.035	0.849	0.263	0.238	0.102	0.311	0.115

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	87	60	90	165	90	62	90
N.S.	1	1.00	0.84	0.58	0.87	1.59	0.87	0.60	0.87
time (sec)	N/A	0.065	0.025	0.253	0.179	0.246	0.076	0.299	0.242

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	99	64	94	173	90	66	85
N.S.	1	1.00	0.97	0.63	0.92	1.70	0.88	0.65	0.83
time (sec)	N/A	0.039	0.052	0.284	0.178	0.236	0.091	0.292	0.246

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	39	39	37	47	37
N.S.	1	1.00	1.00	0.88	0.98	0.98	0.92	1.18	0.92
time (sec)	N/A	0.021	0.006	0.179	0.189	0.242	0.114	0.293	0.249

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	62	55	83	113	102	58	79
N.S.	1	1.00	0.75	0.66	1.00	1.36	1.23	0.70	0.95
time (sec)	N/A	0.043	0.025	0.292	0.261	0.233	0.323	0.288	0.149

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	43	32	31	37	42	31	31
N.S.	1	1.00	0.88	0.65	0.63	0.76	0.86	0.63	0.63
time (sec)	N/A	0.043	0.025	0.203	0.207	0.234	0.235	0.277	0.217

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	32	31	31	48	31	31
N.S.	1	1.00	1.00	0.58	0.56	0.56	0.87	0.56	0.56
time (sec)	N/A	0.192	0.193	0.216	0.195	0.241	1.214	0.312	0.066

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	19	18	18
N.S.	1	1.00	1.00	0.86	0.82	0.82	0.86	0.82	0.82
time (sec)	N/A	0.007	0.016	0.092	0.192	0.236	0.052	0.279	0.134

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	18	13	12	11	22	12	11
N.S.	1	1.00	1.20	0.87	0.80	0.73	1.47	0.80	0.73
time (sec)	N/A	0.005	0.017	0.040	0.217	0.233	0.486	0.275	0.031

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	21	20	19	20	21	19
N.S.	1	1.00	0.96	0.84	0.80	0.76	0.80	0.84	0.76
time (sec)	N/A	0.019	0.021	0.179	0.189	0.233	0.056	0.313	0.252

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	31	26	25	25	27	26	25
N.S.	1	1.00	0.94	0.79	0.76	0.76	0.82	0.79	0.76
time (sec)	N/A	0.022	0.356	0.065	0.190	0.239	0.525	0.309	0.291

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	17	19	19	134	19	16
N.S.	1	1.00	0.83	0.59	0.66	0.66	4.62	0.66	0.55
time (sec)	N/A	0.009	0.033	0.201	0.184	0.232	0.678	0.276	0.575

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	36	30	55	63	3966	49	43
N.S.	1	1.00	0.69	0.58	1.06	1.21	76.27	0.94	0.83
time (sec)	N/A	0.008	0.054	0.214	0.186	0.237	1.557	0.316	0.049

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	70	59	111	125	0	104	96
N.S.	1	1.00	0.59	0.50	0.94	1.06	0.00	0.88	0.81
time (sec)	N/A	0.025	0.094	0.310	0.190	0.240	0.000	0.299	0.233

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	104	104	90	85	105	86	12993	82	120
N.S.	1	1.00	0.87	0.82	1.01	0.83	124.93	0.79	1.15
time (sec)	N/A	0.034	0.242	0.642	0.291	0.254	37.158	0.314	0.244

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	64	39	43	32	0	29	43
N.S.	1	1.00	1.68	1.03	1.13	0.84	0.00	0.76	1.13
time (sec)	N/A	0.012	0.094	0.187	0.279	0.234	0.000	0.311	0.219

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	95	99	141	130	73	0	103	140
N.S.	1	1.03	1.08	1.53	1.41	0.79	0.00	1.12	1.52
time (sec)	N/A	0.055	0.380	0.161	0.294	0.238	0.000	0.311	0.377

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	47	64	0	65	0	74	95
N.S.	1	1.00	0.87	1.19	0.00	1.20	0.00	1.37	1.76
time (sec)	N/A	0.070	0.119	0.276	0.000	0.245	0.000	0.303	0.646

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	304	319	157	0	0	367	0	0	0
N.S.	1	1.05	0.52	0.00	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.645	11.239	0.000	0.000	0.263	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	292	522	455	0	0	785	0	0	0
N.S.	1	1.79	1.56	0.00	0.00	2.69	0.00	0.00	0.00
time (sec)	N/A	1.525	36.725	0.000	0.000	0.289	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	29	25	22	0	47	0	0	25
N.S.	1	1.16	1.00	0.88	0.00	1.88	0.00	0.00	1.00
time (sec)	N/A	0.012	0.022	0.229	0.000	0.234	0.000	0.000	0.259

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	30	25	22	0	69	0	0	25
N.S.	1	1.20	1.00	0.88	0.00	2.76	0.00	0.00	1.00
time (sec)	N/A	0.020	0.022	0.830	0.000	0.234	0.000	0.000	0.298

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	63	30	27	0	77	0	0	30
N.S.	1	1.19	0.57	0.51	0.00	1.45	0.00	0.00	0.57
time (sec)	N/A	0.021	0.051	1.448	0.000	0.226	0.000	0.000	0.257

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	188	123	370	0	128	0	0	0
N.S.	1	2.81	1.84	5.52	0.00	1.91	0.00	0.00	0.00
time (sec)	N/A	0.087	0.132	0.532	0.000	0.233	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	133	96	86	0	142	0	83	0
N.S.	1	1.09	0.79	0.70	0.00	1.16	0.00	0.68	0.00
time (sec)	N/A	0.225	0.120	0.762	0.000	0.250	0.000	0.344	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	404	229	1374	0	280	0	0	0
N.S.	1	2.69	1.53	9.16	0.00	1.87	0.00	0.00	0.00
time (sec)	N/A	0.229	0.550	5.166	0.000	0.244	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	39	26	51	64	0	23	27
N.S.	1	1.00	0.91	0.60	1.19	1.49	0.00	0.53	0.63
time (sec)	N/A	0.006	0.231	0.276	0.224	0.236	0.000	0.287	0.052

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	35	0	62	0	34	0
N.S.	1	1.00	0.88	0.83	0.00	1.48	0.00	0.81	0.00
time (sec)	N/A	0.045	0.025	0.154	0.000	0.242	0.000	0.275	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	58	71	0	138	0	75	0
N.S.	1	1.00	0.42	0.51	0.00	0.99	0.00	0.54	0.00
time (sec)	N/A	0.090	0.064	0.165	0.000	0.242	0.000	0.281	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	188	123	446	0	128	0	0	0
N.S.	1	2.51	1.64	5.95	0.00	1.71	0.00	0.00	0.00
time (sec)	N/A	0.094	0.131	0.566	0.000	0.238	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	23	20	0	22	0	0	24
N.S.	1	1.00	0.79	0.69	0.00	0.76	0.00	0.00	0.83
time (sec)	N/A	0.027	0.020	0.070	0.000	0.231	0.000	0.000	0.054

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	33	29	0	44	0	0	37
N.S.	1	1.00	0.36	0.32	0.00	0.48	0.00	0.00	0.40
time (sec)	N/A	0.071	0.051	0.083	0.000	0.255	0.000	0.000	0.092

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	32	15	16	33	20	36	16
N.S.	1	1.00	1.68	0.79	0.84	1.74	1.05	1.89	0.84
time (sec)	N/A	0.013	0.096	0.348	0.273	0.239	0.253	0.295	0.238

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	23	5	8	29	3	24	4
N.S.	1	1.00	2.88	0.62	1.00	3.62	0.38	3.00	0.50
time (sec)	N/A	0.005	0.082	0.293	0.279	0.268	0.245	0.275	0.237

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	33	12	11	40	12	31	11
N.S.	1	1.00	2.75	1.00	0.92	3.33	1.00	2.58	0.92
time (sec)	N/A	0.004	0.093	0.256	0.263	0.247	0.256	0.283	0.264

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	33	24	0	23	0	51	79
N.S.	1	1.00	1.06	0.77	0.00	0.74	0.00	1.65	2.55
time (sec)	N/A	0.006	0.144	0.582	0.000	0.244	0.000	0.286	0.583

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	40	22	0	54	0	57	61
N.S.	1	1.00	1.29	0.71	0.00	1.74	0.00	1.84	1.97
time (sec)	N/A	0.008	0.137	0.245	0.000	0.240	0.000	0.273	0.533

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	112	48	37	42	78
N.S.	1	1.00	1.00	0.88	4.67	2.00	1.54	1.75	3.25
time (sec)	N/A	0.015	0.060	0.339	0.274	0.240	2.274	0.266	0.831

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	101	32	24	20	83
N.S.	1	1.00	1.00	0.84	4.04	1.28	0.96	0.80	3.32
time (sec)	N/A	0.020	0.039	0.458	0.287	0.244	2.288	0.288	0.792

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	57	37	0	72	0	74	107
N.S.	1	1.00	1.33	0.86	0.00	1.67	0.00	1.72	2.49
time (sec)	N/A	0.021	0.164	0.598	0.000	0.249	0.000	0.312	0.115

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	49	54	74	0	109	0
N.S.	1	1.00	1.00	0.79	0.87	1.19	0.00	1.76	0.00
time (sec)	N/A	0.035	0.232	0.556	0.271	0.256	0.000	0.316	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	90	69	0	151	0	164	0
N.S.	1	1.00	1.10	0.84	0.00	1.84	0.00	2.00	0.00
time (sec)	N/A	0.103	0.373	0.783	0.000	0.252	0.000	0.323	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	102	53	0	237	0	165	0
N.S.	1	1.00	1.62	0.84	0.00	3.76	0.00	2.62	0.00
time (sec)	N/A	0.049	0.198	1.438	0.000	0.254	0.000	0.326	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	103	128	0	105	0	152	0
N.S.	1	1.00	1.84	2.29	0.00	1.88	0.00	2.71	0.00
time (sec)	N/A	0.037	0.184	1.034	0.000	0.253	0.000	0.304	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	123	158	0	229	0	257	0
N.S.	1	1.00	1.76	2.26	0.00	3.27	0.00	3.67	0.00
time (sec)	N/A	0.056	0.266	1.649	0.000	0.260	0.000	0.306	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	89	124	192	0	1339	0	629	0
N.S.	1	1.11	1.55	2.40	0.00	16.74	0.00	7.86	0.00
time (sec)	N/A	0.097	1.314	0.941	0.000	0.337	0.000	0.418	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	85	82	0	81	0	0	0
N.S.	1	1.00	2.24	2.16	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	0.019	0.574	0.537	0.000	0.234	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	57	40	48	42	153	36	35
N.S.	1	1.00	0.88	0.62	0.74	0.65	2.35	0.55	0.54
time (sec)	N/A	0.013	0.174	0.287	0.275	0.245	4.218	0.279	0.031

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	28	25	37	31	41	51	25
N.S.	1	1.00	0.57	0.51	0.76	0.63	0.84	1.04	0.51
time (sec)	N/A	0.009	0.050	0.234	0.280	0.263	1.065	0.289	0.035

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	28	25	37	44	139	26	187
N.S.	1	1.00	0.57	0.51	0.76	0.90	2.84	0.53	3.82
time (sec)	N/A	0.006	0.058	0.218	0.186	0.241	2.934	0.298	0.048

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	354	10	62	58	11	154
N.S.	1	1.00	1.00	29.50	0.83	5.17	4.83	0.92	12.83
time (sec)	N/A	0.041	0.020	0.075	0.188	0.243	0.738	0.272	0.275

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	26	11	10	37	29	22	24
N.S.	1	1.00	2.17	0.92	0.83	3.08	2.42	1.83	2.00
time (sec)	N/A	0.012	0.163	0.224	0.274	0.250	4.842	0.300	0.234

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	48	23	0	67	0	64	77
N.S.	1	1.00	1.78	0.85	0.00	2.48	0.00	2.37	2.85
time (sec)	N/A	0.011	0.125	0.266	0.000	0.250	0.000	0.296	0.180

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	55	38	0	83	0	101	117
N.S.	1	1.00	1.15	0.79	0.00	1.73	0.00	2.10	2.44
time (sec)	N/A	0.013	0.141	0.297	0.000	0.242	0.000	0.301	0.099

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	54	57	107	77	0	72	0
N.S.	1	1.00	1.32	1.39	2.61	1.88	0.00	1.76	0.00
time (sec)	N/A	0.019	0.191	0.454	0.271	0.234	0.000	0.324	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	50	0	50	0	123	115
N.S.	1	1.00	1.00	1.06	0.00	1.06	0.00	2.62	2.45
time (sec)	N/A	0.020	0.234	0.621	0.000	0.238	0.000	0.294	0.137

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	108	82	0	160	0	135	159
N.S.	1	1.00	1.23	0.93	0.00	1.82	0.00	1.53	1.81
time (sec)	N/A	0.194	0.333	1.450	0.000	0.248	0.000	0.319	0.474

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	106	654	0	170	0	176	216
N.S.	1	1.00	0.78	4.81	0.00	1.25	0.00	1.29	1.59
time (sec)	N/A	1.183	0.377	0.040	0.000	0.244	0.000	0.300	0.645

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	68	44	63	51	78	45	56
N.S.	1	1.00	1.06	0.69	0.98	0.80	1.22	0.70	0.88
time (sec)	N/A	0.017	0.133	0.470	0.275	0.240	0.252	0.280	0.106

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	76	57	81	60	92	54	75
N.S.	1	1.00	0.85	0.64	0.91	0.67	1.03	0.61	0.84
time (sec)	N/A	0.027	0.141	0.306	0.269	0.246	0.252	0.315	0.340

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	84	65	101	68	102	63	96
N.S.	1	1.00	0.74	0.58	0.89	0.60	0.90	0.56	0.85
time (sec)	N/A	0.032	0.153	0.323	0.276	0.238	0.275	0.311	0.139

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	78	35	54	62	0	71	0
N.S.	1	1.00	1.59	0.71	1.10	1.27	0.00	1.45	0.00
time (sec)	N/A	0.030	0.185	0.468	0.277	0.252	0.000	0.321	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	50	66	64	0	147	0
N.S.	1	1.00	0.99	0.63	0.84	0.81	0.00	1.86	0.00
time (sec)	N/A	0.032	0.325	0.563	0.268	0.238	0.000	0.301	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	20	10	11	18	14	34	12
N.S.	1	1.00	1.67	0.83	0.92	1.50	1.17	2.83	1.00
time (sec)	N/A	0.006	0.071	0.408	0.275	0.240	0.201	0.298	0.265

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	33	48	39	41	39	0
N.S.	1	1.00	0.89	0.62	0.91	0.74	0.77	0.74	0.00
time (sec)	N/A	0.021	0.107	0.418	0.278	0.232	0.237	0.329	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	22	34	0	15	13
N.S.	1	1.00	1.00	0.84	1.16	1.79	0.00	0.79	0.68
time (sec)	N/A	0.002	0.132	0.431	0.188	0.233	0.000	0.298	0.032

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	22	28	0	13	15
N.S.	1	1.00	1.00	0.82	1.29	1.65	0.00	0.76	0.88
time (sec)	N/A	0.003	0.129	0.428	0.189	0.234	0.000	0.308	0.026

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	47	33	47	64	0	38	0
N.S.	1	1.00	0.84	0.59	0.84	1.14	0.00	0.68	0.00
time (sec)	N/A	0.020	0.155	0.422	0.267	0.238	0.000	0.290	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	52	38	56	44	46	44	61
N.S.	1	1.00	0.80	0.58	0.86	0.68	0.71	0.68	0.94
time (sec)	N/A	0.019	0.096	0.419	0.271	0.234	0.232	0.310	0.133

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	52	38	56	44	85	44	43
N.S.	1	1.00	0.95	0.69	1.02	0.80	1.55	0.80	0.78
time (sec)	N/A	0.014	0.153	0.424	0.271	0.257	0.308	0.306	0.233

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	62	48	77	54	168	54	56
N.S.	1	1.00	0.84	0.65	1.04	0.73	2.27	0.73	0.76
time (sec)	N/A	0.015	0.255	0.420	0.274	0.246	0.451	0.288	0.074

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	31	37	52	0	67	31
N.S.	1	1.00	0.87	0.82	0.97	1.37	0.00	1.76	0.82
time (sec)	N/A	0.011	0.108	0.428	0.282	0.240	0.000	0.290	0.029

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	42	42	50	63	0	84	0
N.S.	1	1.00	0.74	0.74	0.88	1.11	0.00	1.47	0.00
time (sec)	N/A	0.021	0.120	0.428	0.283	0.256	0.000	0.290	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	45	41	58	94	0	80	0
N.S.	1	1.00	0.73	0.66	0.94	1.52	0.00	1.29	0.00
time (sec)	N/A	0.019	0.189	0.437	0.270	0.251	0.000	0.303	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	52	46	71	107	0	117	0
N.S.	1	1.00	0.66	0.58	0.90	1.35	0.00	1.48	0.00
time (sec)	N/A	0.036	0.201	0.517	0.279	0.239	0.000	0.287	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	18	22	25	30	0	32	0
N.S.	1	1.00	0.82	1.00	1.14	1.36	0.00	1.45	0.00
time (sec)	N/A	0.008	0.115	0.422	0.269	0.238	0.000	0.310	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	85	69	0	110	0	147	0
N.S.	1	1.00	0.99	0.80	0.00	1.28	0.00	1.71	0.00
time (sec)	N/A	0.249	0.281	0.453	0.000	0.262	0.000	0.339	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	71	56	61	92	0	149	0
N.S.	1	1.00	1.15	0.90	0.98	1.48	0.00	2.40	0.00
time (sec)	N/A	0.037	0.270	0.481	0.284	0.250	0.000	0.346	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	84	64	0	174	0	235	0
N.S.	1	1.00	1.11	0.84	0.00	2.29	0.00	3.09	0.00
time (sec)	N/A	0.053	0.611	0.836	0.000	0.243	0.000	0.293	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	26	21	28	22	0	30	19
N.S.	1	1.00	0.72	0.58	0.78	0.61	0.00	0.83	0.53
time (sec)	N/A	0.092	0.127	0.374	0.279	0.243	0.000	0.291	0.307

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	95	63	0	147	0	148	0
N.S.	1	1.00	1.09	0.72	0.00	1.69	0.00	1.70	0.00
time (sec)	N/A	0.163	0.539	1.822	0.000	0.251	0.000	0.300	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	39	38	76	98	0	33	69
N.S.	1	1.00	0.67	0.66	1.31	1.69	0.00	0.57	1.19
time (sec)	N/A	0.010	0.313	0.441	0.186	0.238	0.000	0.287	0.279

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	59	73	0	27	29
N.S.	1	1.00	0.70	0.64	1.26	1.55	0.00	0.57	0.62
time (sec)	N/A	0.006	0.217	0.325	0.177	0.246	0.000	0.293	0.057

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	59	51	0	39	29
N.S.	1	1.00	0.70	0.64	1.26	1.09	0.00	0.83	0.62
time (sec)	N/A	0.005	0.227	0.369	0.189	0.244	0.000	0.320	0.238

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	43	40	0	39	0	60	0
N.S.	1	1.00	1.48	1.38	0.00	1.34	0.00	2.07	0.00
time (sec)	N/A	0.028	0.291	0.072	0.000	0.255	0.000	0.292	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	59	54	52	0	63	0	66	0
N.S.	1	1.31	1.20	1.16	0.00	1.40	0.00	1.47	0.00
time (sec)	N/A	0.022	0.133	0.082	0.000	0.248	0.000	0.283	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	64	55	0	54	0	54	71
N.S.	1	1.00	0.81	0.70	0.00	0.68	0.00	0.68	0.90
time (sec)	N/A	0.104	0.206	0.049	0.000	0.244	0.000	0.279	0.074

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	72	80	0	99	0	105	0
N.S.	1	1.00	0.90	1.00	0.00	1.24	0.00	1.31	0.00
time (sec)	N/A	0.275	0.241	0.438	0.000	0.255	0.000	0.304	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	159	140	0	155	0	0	0
N.S.	1	1.00	1.01	0.89	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.422	5.512	0.023	0.000	0.253	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	31	30	38	28	131	29	29
N.S.	1	1.00	0.76	0.73	0.93	0.68	3.20	0.71	0.71
time (sec)	N/A	0.005	0.047	0.273	0.277	0.240	1.881	0.287	0.045

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	17	10	11	11
N.S.	1	1.00	1.00	0.92	0.85	1.31	0.77	0.85	0.85
time (sec)	N/A	0.004	0.033	0.023	0.192	0.246	0.231	0.298	0.480

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	71	71	85	67	76	80	541	76	90
N.S.	1	1.00	1.20	0.94	1.07	1.13	7.62	1.07	1.27
time (sec)	N/A	0.023	0.127	0.589	0.273	0.249	1.294	0.334	0.228

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	23	18	28	29	178	49	23
N.S.	1	1.00	0.58	0.45	0.70	0.72	4.45	1.22	0.58
time (sec)	N/A	0.007	0.027	0.315	0.189	0.236	0.877	0.282	0.217

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	53	52	52	51	53	36
N.S.	1	1.00	1.00	1.10	1.08	1.08	1.06	1.10	0.75
time (sec)	N/A	0.014	0.049	0.289	0.267	0.252	1.018	2.151	0.928

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	36	20	45	25	3303	63	45
N.S.	1	1.00	0.52	0.29	0.65	0.36	47.87	0.91	0.65
time (sec)	N/A	0.020	0.028	0.292	0.183	0.238	1.122	0.270	0.332

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	C	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	193	193	120	85	157	120	44	142	77
N.S.	1	1.00	0.62	0.44	0.81	0.62	0.23	0.74	0.40
time (sec)	N/A	0.091	0.358	0.329	0.280	0.263	2.753	0.311	1.310

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	26	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	2.00	0.69	0.69
time (sec)	N/A	0.002	0.014	0.342	0.188	0.248	0.115	0.298	0.306

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	12	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.92	0.69	0.69
time (sec)	N/A	0.003	0.018	0.302	0.177	0.236	0.181	0.294	0.345

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	82	67	64	66	42	65	76
N.S.	1	1.00	1.39	1.14	1.08	1.12	0.71	1.10	1.29
time (sec)	N/A	0.036	0.085	7.366	0.269	0.240	0.467	0.298	0.485

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	83	76	66	79	34	67	92
N.S.	1	1.00	1.19	1.09	0.94	1.13	0.49	0.96	1.31
time (sec)	N/A	0.031	0.126	6.638	0.270	0.240	0.681	0.273	0.392

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	20	83	146	41	83	18
N.S.	1	1.00	1.00	0.29	1.22	2.15	0.60	1.22	0.26
time (sec)	N/A	0.022	0.235	2.963	0.273	2.234	0.547	0.287	0.484

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	82	19	130	105	41	110	0
N.S.	1	1.00	0.88	0.20	1.40	1.13	0.44	1.18	0.00
time (sec)	N/A	0.024	0.389	2.229	0.269	0.257	0.916	0.313	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	82	20	129	105	39	109	0
N.S.	1	1.00	0.88	0.22	1.39	1.13	0.42	1.17	0.00
time (sec)	N/A	0.025	0.360	2.187	0.270	0.254	1.028	0.291	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	129	137	15	0	99	0	69	29
N.S.	1	1.39	1.47	0.16	0.00	1.06	0.00	0.74	0.31
time (sec)	N/A	0.063	0.704	2.615	0.000	0.349	0.000	0.297	0.402

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	114	69	51	0	87	0	66	27
N.S.	1	0.90	0.55	0.40	0.00	0.69	0.00	0.52	0.21
time (sec)	N/A	0.094	0.196	0.422	0.000	42.644	0.000	0.368	0.325

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	41	28	0	49	0	70	35
N.S.	1	1.00	1.21	0.82	0.00	1.44	0.00	2.06	1.03
time (sec)	N/A	0.024	0.254	0.856	0.000	0.261	0.000	0.301	0.553

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	41	0	47	0	49	43
N.S.	1	1.00	0.95	0.71	0.00	0.81	0.00	0.84	0.74
time (sec)	N/A	0.034	0.116	1.102	0.000	0.237	0.000	0.285	0.331

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	57	52	52	81	0	112	0
N.S.	1	1.00	0.80	0.73	0.73	1.14	0.00	1.58	0.00
time (sec)	N/A	0.045	0.137	0.217	0.270	0.245	0.000	0.285	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	52	23	52	59	0	69	49
N.S.	1	1.00	2.48	1.10	2.48	2.81	0.00	3.29	2.33
time (sec)	N/A	0.024	0.135	0.208	0.175	0.258	0.000	0.332	0.874

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	75	14	13	13	36	13	21
N.S.	1	1.00	4.41	0.82	0.76	0.76	2.12	0.76	1.24
time (sec)	N/A	0.008	10.073	0.326	0.180	0.243	0.183	0.310	0.387

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	46	46	40	116	34	35	221	34	34
N.S.	1	1.00	0.87	2.52	0.74	0.76	4.80	0.74	0.74
time (sec)	N/A	0.162	0.061	0.358	0.183	0.236	4.670	0.277	0.449

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	110	93	0	232	0	0	0
N.S.	1	1.00	1.41	1.19	0.00	2.97	0.00	0.00	0.00
time (sec)	N/A	0.015	0.357	3.120	0.000	1.651	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	76	100	0	225	0	0	0
N.S.	1	1.00	0.54	0.71	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.053	0.297	1.813	0.000	2.394	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	90	29	94	86	71	0	0
N.S.	1	1.00	1.43	0.46	1.49	1.37	1.13	0.00	0.00
time (sec)	N/A	0.009	0.264	1.628	0.272	0.247	0.966	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	87	0	313	0	0	0
N.S.	1	1.00	1.00	1.18	0.00	4.23	0.00	0.00	0.00
time (sec)	N/A	0.020	0.412	2.648	0.000	3.411	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	59	26	23	73	40	0	0	23
N.S.	1	1.23	0.54	0.48	1.52	0.83	0.00	0.00	0.48
time (sec)	N/A	0.009	1.437	0.371	0.185	0.242	0.000	0.000	0.282

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	90	90	180	2517	0	458	0	0	0
N.S.	1	1.00	2.00	27.97	0.00	5.09	0.00	0.00	0.00
time (sec)	N/A	0.028	0.490	10.174	0.000	4.911	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	0	18	0	0	0
N.S.	1	1.00	1.00	0.83	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.025	0.006	0.704	0.000	0.263	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	0	42	0	0	0
N.S.	1	1.00	1.00	0.96	0.00	1.83	0.00	0.00	0.00
time (sec)	N/A	0.026	0.004	0.887	0.000	0.275	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	23	41	18	57	49	14	51	17
N.S.	1	1.44	2.56	1.12	3.56	3.06	0.88	3.19	1.06
time (sec)	N/A	0.018	0.092	0.320	0.268	0.239	3.626	0.289	0.152

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	23	39	18	57	47	14	51	17
N.S.	1	1.44	2.44	1.12	3.56	2.94	0.88	3.19	1.06
time (sec)	N/A	0.018	0.091	0.317	0.266	0.243	2.135	0.277	0.252

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	31	0	45	0	0	0
N.S.	1	1.00	1.00	1.19	0.00	1.73	0.00	0.00	0.00
time (sec)	N/A	0.030	0.261	1.207	0.000	0.268	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	0	13	0	0	0
N.S.	1	1.00	1.00	0.93	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.026	0.200	0.861	0.000	0.268	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	22	14	0	0	14
N.S.	1	1.00	1.00	0.94	1.38	0.88	0.00	0.00	0.88
time (sec)	N/A	0.014	0.448	0.314	0.238	0.257	0.000	0.000	0.057

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	65	78	0	252	0	0	0
N.S.	1	1.00	0.88	1.05	0.00	3.41	0.00	0.00	0.00
time (sec)	N/A	0.142	0.769	2.507	0.000	0.383	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	96	0	0	62	0	0	0
N.S.	1	1.00	4.36	0.00	0.00	2.82	0.00	0.00	0.00
time (sec)	N/A	0.048	0.366	0.000	0.000	0.631	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.201	0.000	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.006	0.002	0.055	0.223	0.248	0.016	0.266	0.201

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	9	10	8	9	9
N.S.	1	1.00	1.00	1.00	0.82	0.91	0.73	0.82	0.82
time (sec)	N/A	0.006	0.001	0.411	0.206	0.251	0.019	0.294	0.035

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	17	16	19	24	16	16
N.S.	1	1.00	0.92	0.71	0.67	0.79	1.00	0.67	0.67
time (sec)	N/A	0.009	0.002	0.382	0.199	0.242	0.018	0.316	0.037

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	23	24	25	36	22	22
N.S.	1	1.00	0.88	0.68	0.71	0.74	1.06	0.65	0.65
time (sec)	N/A	0.018	0.003	0.388	0.200	0.244	0.016	0.301	0.039

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	38	29	30	31	48	28	28
N.S.	1	1.00	0.86	0.66	0.68	0.70	1.09	0.64	0.64
time (sec)	N/A	0.017	0.011	0.447	0.211	0.250	0.019	0.283	0.029

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	64	21	15	23	37	99	14	20
N.S.	1	3.20	1.05	0.75	1.15	1.85	4.95	0.70	1.00
time (sec)	N/A	0.019	0.052	0.442	0.202	0.257	0.128	0.275	0.323

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	22	23	22	39	23	22
N.S.	1	1.00	1.00	0.71	0.74	0.71	1.26	0.74	0.71
time (sec)	N/A	0.011	0.054	0.591	0.198	0.251	0.086	0.357	0.269

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	27	18	20	37	32	20	27
N.S.	1	1.00	1.29	0.86	0.95	1.76	1.52	0.95	1.29
time (sec)	N/A	0.008	0.012	0.418	0.213	0.246	0.018	0.285	0.224

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	95	32	54	93	60	112	44
N.S.	1	1.00	2.64	0.89	1.50	2.58	1.67	3.11	1.22
time (sec)	N/A	0.017	0.016	0.460	0.196	0.257	0.074	0.305	0.274

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	37	33	40	66	33	51
N.S.	1	1.00	1.00	0.90	0.80	0.98	1.61	0.80	1.24
time (sec)	N/A	0.014	0.011	0.688	0.190	0.242	0.019	0.275	0.230

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	40	51	70	388	53	35
N.S.	1	1.00	1.00	1.00	1.28	1.75	9.70	1.32	0.88
time (sec)	N/A	0.015	0.016	0.506	0.195	0.252	0.526	0.298	0.657

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	24	19	18	18	31	18	18
N.S.	1	1.00	1.09	0.86	0.82	0.82	1.41	0.82	0.82
time (sec)	N/A	0.012	0.001	0.029	0.266	0.256	0.025	0.291	0.035

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	23	26	22	40	19	37	26
N.S.	1	1.00	1.15	1.30	1.10	2.00	0.95	1.85	1.30
time (sec)	N/A	0.023	0.013	0.058	0.184	0.254	0.042	0.292	0.274

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	40	25	30	70	20	53	24
N.S.	1	1.00	1.25	0.78	0.94	2.19	0.62	1.66	0.75
time (sec)	N/A	0.016	0.022	0.075	0.264	0.251	0.067	0.302	0.090

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	46	35	24	37	56	34	38
N.S.	1	1.00	0.82	0.62	0.43	0.66	1.00	0.61	0.68
time (sec)	N/A	0.053	0.041	0.444	0.177	0.252	0.018	0.283	0.045

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	55	26	25	25	27	25	25
N.S.	1	1.00	1.67	0.79	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.022	0.102	0.252	0.181	0.252	0.022	0.275	0.229

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	40	38	44	42	36
N.S.	1	1.00	1.00	0.80	0.87	0.83	0.96	0.91	0.78
time (sec)	N/A	0.021	0.014	0.529	0.177	0.265	0.038	0.277	0.035

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	53	28	37	55	44	26	27
N.S.	1	1.00	1.29	0.68	0.90	1.34	1.07	0.63	0.66
time (sec)	N/A	0.030	0.052	0.705	0.197	0.239	0.023	0.292	0.125

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	14	11	10	19	14	10	18
N.S.	1	1.00	0.58	0.46	0.42	0.79	0.58	0.42	0.75
time (sec)	N/A	0.022	0.002	0.066	0.179	0.255	0.020	0.272	0.056

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	22	17	16	31	31	16	32
N.S.	1	1.00	0.48	0.37	0.35	0.67	0.67	0.35	0.70
time (sec)	N/A	0.052	0.002	0.418	0.179	0.250	0.021	0.311	0.040

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	30	23	24	43	46	22	44
N.S.	1	1.00	0.44	0.34	0.35	0.63	0.68	0.32	0.65
time (sec)	N/A	0.070	0.012	0.481	0.192	0.260	0.021	0.273	0.028

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	38	29	30	55	61	28	56
N.S.	1	1.00	0.42	0.32	0.33	0.61	0.68	0.31	0.62
time (sec)	N/A	0.091	0.040	0.543	0.182	0.279	0.021	0.277	0.031

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	58	0	0	0	0	0	52
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.76
time (sec)	N/A	0.027	0.065	0.000	0.000	0.000	0.000	0.000	0.766

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	47	25	41	71	54	132	24
N.S.	1	1.00	1.47	0.78	1.28	2.22	1.69	4.12	0.75
time (sec)	N/A	0.019	0.037	0.479	0.193	0.258	0.569	0.305	0.339

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	14	17	6	6
N.S.	1	1.00	1.00	0.88	0.75	1.75	2.12	0.75	0.75
time (sec)	N/A	0.015	0.003	0.170	0.183	0.242	0.018	0.338	0.029

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	14	22	15	14	11
N.S.	1	1.00	1.00	0.91	1.27	2.00	1.36	1.27	1.00
time (sec)	N/A	0.017	0.002	0.122	0.188	0.244	0.036	0.302	0.349

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.009	0.002	0.270	0.182	0.247	0.020	0.279	0.343

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	71	36	38	68	41	47	24
N.S.	1	1.00	2.73	1.38	1.46	2.62	1.58	1.81	0.92
time (sec)	N/A	0.030	0.034	0.134	0.194	0.247	0.068	0.264	0.337

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	14	30	15	14	13
N.S.	1	1.00	1.00	1.29	0.82	1.76	0.88	0.82	0.76
time (sec)	N/A	0.020	0.002	0.165	0.184	0.247	0.038	0.280	0.072

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	24	20	19	20	39	20	22
N.S.	1	1.00	0.77	0.65	0.61	0.65	1.26	0.65	0.71
time (sec)	N/A	0.022	0.077	0.332	0.193	0.248	13.120	0.293	1.142

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	22	14	13	27	0	13	32
N.S.	1	1.00	1.05	0.67	0.62	1.29	0.00	0.62	1.52
time (sec)	N/A	0.021	0.063	0.227	0.188	0.252	0.000	0.299	1.209

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	95	52	54	93	56	44	57
N.S.	1	1.00	2.50	1.37	1.42	2.45	1.47	1.16	1.50
time (sec)	N/A	0.049	0.037	0.230	0.185	0.254	0.066	0.296	0.316

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	74	76	74	82	0	95	75
N.S.	1	1.00	0.97	1.00	0.97	1.08	0.00	1.25	0.99
time (sec)	N/A	0.047	0.034	0.777	0.191	0.258	0.000	0.371	6.411

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	84	127	76	115	178	100	149	133
N.S.	1	0.95	1.44	0.86	1.31	2.02	1.14	1.69	1.51
time (sec)	N/A	0.451	3.584	59.217	0.277	0.287	158.344	0.293	0.410

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	74	46	54	54	148	54	94
N.S.	1	1.00	1.06	0.66	0.77	0.77	2.11	0.77	1.34
time (sec)	N/A	0.132	0.326	0.690	0.193	0.267	0.205	0.286	0.428

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	60	33	36	71	39	75	75
N.S.	1	1.00	1.82	1.00	1.09	2.15	1.18	2.27	2.27
time (sec)	N/A	0.055	0.048	0.125	0.281	0.262	0.219	0.296	0.576

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	20	21	14	26	24	14	26
N.S.	1	1.00	1.25	1.31	0.88	1.62	1.50	0.88	1.62
time (sec)	N/A	0.044	0.039	33.879	0.284	0.268	7.045	0.321	0.341

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	22	21	27	24	23	10
N.S.	1	1.00	1.00	1.83	1.75	2.25	2.00	1.92	0.83
time (sec)	N/A	0.028	0.015	0.622	0.183	0.274	0.429	0.276	0.291

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	18	20	13	13
N.S.	1	1.00	1.00	0.82	0.76	1.06	1.18	0.76	0.76
time (sec)	N/A	0.011	0.007	0.211	0.186	0.248	0.120	0.270	0.255

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	31	54	43	75	38	33
N.S.	1	1.00	1.00	1.19	2.08	1.65	2.88	1.46	1.27
time (sec)	N/A	0.029	0.056	38.577	0.186	0.280	6.979	0.271	0.258

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	29	30	31	139	28	36
N.S.	1	1.00	1.00	0.76	0.79	0.82	3.66	0.74	0.95
time (sec)	N/A	0.033	0.013	0.530	0.198	0.257	1.594	0.287	0.321

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	35	33	43	22	21	21
N.S.	1	1.00	1.00	1.75	1.65	2.15	1.10	1.05	1.05
time (sec)	N/A	0.049	0.015	25.099	0.193	0.282	8.365	0.277	0.070

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	19	19	25	14	13	35
N.S.	1	1.00	1.00	1.58	1.58	2.08	1.17	1.08	2.92
time (sec)	N/A	0.022	0.013	2.118	0.195	0.280	2.490	0.274	0.372

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	65	39	35	208	64	0	75	47
N.S.	1	1.02	0.61	0.55	3.25	1.00	0.00	1.17	0.73
time (sec)	N/A	0.078	0.065	0.441	0.291	0.286	0.000	0.289	0.368

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	17	17	27	92	17	0	24	25
N.S.	1	1.55	1.55	2.45	8.36	1.55	0.00	2.18	2.27
time (sec)	N/A	0.028	0.016	0.882	0.184	0.278	0.000	0.284	0.716

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	12	9	11	0	9	5
N.S.	1	1.00	1.00	1.71	1.29	1.57	0.00	1.29	0.71
time (sec)	N/A	0.034	0.125	12.969	0.273	0.277	0.000	0.273	0.290

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	83	33	43	27	38	107	78	23
N.S.	1	1.57	0.62	0.81	0.51	0.72	2.02	1.47	0.43
time (sec)	N/A	0.089	0.091	0.629	0.263	0.277	4.582	0.281	0.241

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	94	22	20	39	66	39	37	17
N.S.	1	2.85	0.67	0.61	1.18	2.00	1.18	1.12	0.52
time (sec)	N/A	0.060	0.062	0.188	0.265	0.277	0.264	0.310	0.087

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	9	8	7	21	219	20	16
N.S.	1	1.00	0.33	0.30	0.26	0.78	8.11	0.74	0.59
time (sec)	N/A	0.021	0.111	0.809	0.273	0.280	4.411	0.287	0.321

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	41	31	28	46	102	29	38
N.S.	1	1.00	1.46	1.11	1.00	1.64	3.64	1.04	1.36
time (sec)	N/A	0.038	0.046	0.191	0.264	0.265	0.231	0.277	0.347

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	58	39	38	61	0	78	120
N.S.	1	1.00	0.87	0.58	0.57	0.91	0.00	1.16	1.79
time (sec)	N/A	0.045	0.204	0.324	0.272	0.270	0.000	0.278	0.476

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	54	29	28	58	0	61	77
N.S.	1	1.00	0.98	0.53	0.51	1.05	0.00	1.11	1.40
time (sec)	N/A	0.037	1.136	0.207	0.269	0.273	0.000	0.311	0.413

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	58	45	44	77	252	39	48
N.S.	1	1.00	1.38	1.07	1.05	1.83	6.00	0.93	1.14
time (sec)	N/A	0.082	1.840	0.093	0.265	0.259	0.238	0.286	0.365

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	20	0	19	76	21	7
N.S.	1	1.00	1.00	2.22	0.00	2.11	8.44	2.33	0.78
time (sec)	N/A	0.019	0.011	0.897	0.000	0.259	1.799	0.277	0.395

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	35	13	129	33	0	49	12
N.S.	1	1.00	2.33	0.87	8.60	2.20	0.00	3.27	0.80
time (sec)	N/A	0.012	0.089	0.463	0.274	0.274	0.000	0.310	0.129

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	28	21	128	26	22	20	9
N.S.	1	1.00	1.65	1.24	7.53	1.53	1.29	1.18	0.53
time (sec)	N/A	0.041	0.032	0.974	0.267	0.270	0.465	0.277	0.275

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	81	19	0	24	15
N.S.	1	1.00	1.00	0.86	3.86	0.90	0.00	1.14	0.71
time (sec)	N/A	0.031	0.020	2.367	0.268	0.266	0.000	0.282	0.122

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	27	129	19	17	24	17
N.S.	1	1.00	1.00	1.29	6.14	0.90	0.81	1.14	0.81
time (sec)	N/A	0.019	0.013	0.428	0.267	0.261	0.448	0.307	0.316

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	28	171	50	294	48	27
N.S.	1	1.00	1.00	1.08	6.58	1.92	11.31	1.85	1.04
time (sec)	N/A	0.024	0.028	0.322	0.281	0.258	3.242	0.290	0.501

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	70	28	171	50	294	48	27
N.S.	1	1.00	2.69	1.08	6.58	1.92	11.31	1.85	1.04
time (sec)	N/A	0.029	0.073	0.596	0.283	0.269	7.256	0.280	0.355

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	25	22	0	34	0	17	21
N.S.	1	1.00	1.56	1.38	0.00	2.12	0.00	1.06	1.31
time (sec)	N/A	0.009	0.017	0.514	0.000	0.257	0.000	0.309	0.276

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	27	31	0	35	0	29	23
N.S.	1	1.00	1.59	1.82	0.00	2.06	0.00	1.71	1.35
time (sec)	N/A	0.014	0.017	0.391	0.000	0.255	0.000	0.316	0.276

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	16	9	41	55	0	41	13
N.S.	1	1.00	0.59	0.33	1.52	2.04	0.00	1.52	0.48
time (sec)	N/A	0.012	0.012	0.156	0.310	0.253	0.000	0.275	0.061

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	33	17	101	58	0	16	28
N.S.	1	1.00	1.10	0.57	3.37	1.93	0.00	0.53	0.93
time (sec)	N/A	0.015	0.039	0.356	0.324	0.249	0.000	0.344	0.269

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	61	52	433	107	0	100	0
N.S.	1	1.00	1.15	0.98	8.17	2.02	0.00	1.89	0.00
time (sec)	N/A	0.032	0.188	0.510	0.330	0.268	0.000	0.308	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	76	47	0	71	0	72	0
N.S.	1	1.00	1.04	0.64	0.00	0.97	0.00	0.99	0.00
time (sec)	N/A	0.035	1.068	0.487	0.000	0.262	0.000	0.302	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	42	43	40	230	43	0
N.S.	1	1.00	0.65	0.76	0.78	0.73	4.18	0.78	0.00
time (sec)	N/A	0.116	0.097	0.493	0.181	0.269	37.509	0.325	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	40	49	80	73	0	80	65
N.S.	1	1.00	0.41	0.50	0.82	0.74	0.00	0.82	0.66
time (sec)	N/A	0.060	0.074	0.108	0.277	0.259	0.000	0.272	0.129

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	69	69	53	52	54	0	52	67
N.S.	1	1.21	1.21	0.93	0.91	0.95	0.00	0.91	1.18
time (sec)	N/A	0.049	0.062	0.096	0.269	0.259	0.000	0.332	0.747

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	73	130	3213	151	0	0	63
N.S.	1	1.00	0.84	1.49	36.93	1.74	0.00	0.00	0.72
time (sec)	N/A	0.102	0.113	0.287	0.519	0.256	0.000	0.000	0.468

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	30	31	30	82	0	31	105
N.S.	1	1.00	0.75	0.78	0.75	2.05	0.00	0.78	2.62
time (sec)	N/A	0.104	5.196	0.447	0.181	0.260	0.000	0.289	1.470

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	133	62	94	117	603	0	111	228
N.S.	1	1.58	0.74	1.12	1.39	7.18	0.00	1.32	2.71
time (sec)	N/A	0.288	0.319	0.083	0.279	0.933	0.000	0.293	1.283

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	266	0	137	0	0	0
N.S.	1	1.00	1.00	8.58	0.00	4.42	0.00	0.00	0.00
time (sec)	N/A	0.011	0.059	0.375	0.000	0.279	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	29	98	0	137	0	0	0
N.S.	1	1.00	0.94	3.16	0.00	4.42	0.00	0.00	0.00
time (sec)	N/A	0.017	0.044	0.411	0.000	0.265	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	171	0	151	0	0	0
N.S.	1	1.00	0.91	3.80	0.00	3.36	0.00	0.00	0.00
time (sec)	N/A	0.031	0.050	2.493	0.000	0.273	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	43	396	0	76	0	0	0
N.S.	1	1.00	0.91	8.43	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.074	0.095	5.267	0.000	0.279	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	510	0	181	0	0	0
N.S.	1	1.00	0.82	8.36	0.00	2.97	0.00	0.00	0.00
time (sec)	N/A	0.068	0.133	6.089	0.000	0.278	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	1108	0	205	0	0	0
N.S.	1	1.00	0.92	18.16	0.00	3.36	0.00	0.00	0.00
time (sec)	N/A	0.068	0.107	0.729	0.000	0.268	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	508	0	39	0	0	18
N.S.	1	1.00	1.00	31.75	0.00	2.44	0.00	0.00	1.12
time (sec)	N/A	0.016	0.062	0.654	0.000	0.269	0.000	0.000	0.621

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	2946	0	32	0	0	20
N.S.	1	1.00	0.65	95.03	0.00	1.03	0.00	0.00	0.65
time (sec)	N/A	0.032	0.058	4.941	0.000	0.262	0.000	0.000	0.429

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	121	0	43	0	0	29
N.S.	1	1.00	0.83	4.17	0.00	1.48	0.00	0.00	1.00
time (sec)	N/A	0.036	0.050	0.375	0.000	0.251	0.000	0.000	0.511

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-1)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	95	111	761	0	136	0	0	0
N.S.	1	1.40	1.63	11.19	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	0.650	7.736	1.306	0.000	0.285	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	29	19	16	6	15	0	0	15
N.S.	1	1.53	1.00	0.84	0.32	0.79	0.00	0.00	0.79
time (sec)	N/A	0.086	0.010	0.876	0.300	0.257	0.000	0.000	0.594

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	92	204	66	299	0	628	0	0	0
N.S.	1	2.22	0.72	3.25	0.00	6.83	0.00	0.00	0.00
time (sec)	N/A	0.179	0.132	7.826	0.000	0.416	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	47	47	63	0	13	29	0	0	32
N.S.	1	1.00	1.34	0.00	0.28	0.62	0.00	0.00	0.68
time (sec)	N/A	0.054	0.175	0.000	0.280	0.247	0.000	0.000	4.036

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	35	0	77	101	0	0	110
N.S.	1	1.00	0.50	0.00	1.10	1.44	0.00	0.00	1.57
time (sec)	N/A	0.150	0.062	0.000	0.288	0.269	0.000	0.000	3.615

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	108	234	75	247	0	479	0	0	0
N.S.	1	2.17	0.69	2.29	0.00	4.44	0.00	0.00	0.00
time (sec)	N/A	1.187	0.502	3.679	0.000	0.459	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	364	665	378	133928	0	0	0	0	0
N.S.	1	1.83	1.04	367.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.551	19.658	57.052	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	141	58	0	60	56	0	0	0
N.S.	1	1.13	0.46	0.00	0.48	0.45	0.00	0.00	0.00
time (sec)	N/A	0.729	0.937	0.000	0.315	0.266	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	67	61	56	55	108	0	55	43
N.S.	1	0.92	0.84	0.77	0.75	1.48	0.00	0.75	0.59
time (sec)	N/A	0.034	0.168	1.084	0.290	0.287	0.000	0.304	0.137

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	48	54	53	88	0	41	0
N.S.	1	1.00	0.70	0.78	0.77	1.28	0.00	0.59	0.00
time (sec)	N/A	0.038	0.082	2.151	0.271	0.263	0.000	0.292	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	F(-1)	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	61	47	36	123	0	41	0
N.S.	1	1.00	1.05	0.81	0.62	2.12	0.00	0.71	0.00
time (sec)	N/A	0.042	0.081	0.261	0.285	0.264	0.000	0.285	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	37	44	43	51	0	30	28
N.S.	1	1.00	0.67	0.80	0.78	0.93	0.00	0.55	0.51
time (sec)	N/A	0.043	0.130	0.336	0.209	0.298	0.000	0.293	0.705

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	58	716	100	0	38	0
N.S.	1	1.00	1.00	1.49	18.36	2.56	0.00	0.97	0.00
time (sec)	N/A	0.050	0.110	0.742	0.346	0.288	0.000	0.287	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	63	105	69	131	0	40	0
N.S.	1	1.00	1.31	2.19	1.44	2.73	0.00	0.83	0.00
time (sec)	N/A	0.051	0.453	0.493	0.293	0.272	0.000	0.286	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	28	38	192	42	0	33	28
N.S.	1	1.00	0.57	0.78	3.92	0.86	0.00	0.67	0.57
time (sec)	N/A	0.095	0.096	0.356	0.227	0.271	0.000	0.274	0.627

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	111	119	295	131	115	127	0	0	0
N.S.	1	1.07	2.66	1.18	1.04	1.14	0.00	0.00	0.00
time (sec)	N/A	0.427	7.001	1.760	0.317	0.876	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	156	233	0	195	0	0	0
N.S.	1	1.00	1.39	2.08	0.00	1.74	0.00	0.00	0.00
time (sec)	N/A	0.315	0.719	1.150	0.000	0.298	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	38	25	26	0	35	0
N.S.	1	1.00	0.88	1.15	0.76	0.79	0.00	1.06	0.00
time (sec)	N/A	0.049	0.077	0.471	0.189	0.277	0.000	0.289	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	62	488	77	0	27	0
N.S.	1	1.00	0.97	1.88	14.79	2.33	0.00	0.82	0.00
time (sec)	N/A	0.017	0.040	0.472	0.324	0.267	0.000	0.289	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	55	790	103	0	48	29
N.S.	1	1.00	0.89	1.00	14.36	1.87	0.00	0.87	0.53
time (sec)	N/A	0.023	0.209	0.401	0.344	0.281	0.000	0.302	0.486

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	39	90	39	0	22	12
N.S.	1	1.00	1.00	2.44	5.62	2.44	0.00	1.38	0.75
time (sec)	N/A	0.011	0.058	0.582	0.306	0.255	0.000	0.293	0.437

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	100	0	118	0	0	0
N.S.	1	1.00	1.00	2.04	0.00	2.41	0.00	0.00	0.00
time (sec)	N/A	0.041	0.127	0.603	0.000	0.279	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	91	62	180	1359	163	0	55	0
N.S.	1	1.05	0.71	2.07	15.62	1.87	0.00	0.63	0.00
time (sec)	N/A	0.144	0.426	2.613	0.412	0.291	0.000	0.287	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	115	217	0	130	0	0	0
N.S.	1	1.00	1.69	3.19	0.00	1.91	0.00	0.00	0.00
time (sec)	N/A	0.042	0.266	7.204	0.000	0.269	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	79	124	0	115	0	0	0
N.S.	1	1.00	1.98	3.10	0.00	2.88	0.00	0.00	0.00
time (sec)	N/A	0.022	0.137	1.365	0.000	0.275	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	131	151	0	97	0	0	0
N.S.	1	1.00	1.39	1.61	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.138	0.600	4.208	0.000	0.332	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	36	67	63	33	0	135	20
N.S.	1	1.00	0.92	1.72	1.62	0.85	0.00	3.46	0.51
time (sec)	N/A	0.121	0.093	2.726	0.205	0.270	0.000	0.283	0.853

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	129	1498	0	257	0	121	0
N.S.	1	1.00	1.77	20.52	0.00	3.52	0.00	1.66	0.00
time (sec)	N/A	0.895	5.136	2.001	0.000	0.347	0.000	0.328	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	116	62	47	84	0	0	113
N.S.	1	1.00	2.04	1.09	0.82	1.47	0.00	0.00	1.98
time (sec)	N/A	0.617	5.350	3.319	0.282	0.275	0.000	0.000	1.612

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	49	61	0	50	0	52	90
N.S.	1	1.00	0.74	0.92	0.00	0.76	0.00	0.79	1.36
time (sec)	N/A	0.046	0.081	0.074	0.000	0.275	0.000	0.271	1.623

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	71	41	0	76	46	36	172
N.S.	1	1.00	1.31	0.76	0.00	1.41	0.85	0.67	3.19
time (sec)	N/A	0.045	0.661	0.056	0.000	0.283	2.317	0.268	0.525

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	105	0	0	0	0	186	250
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	1.40	1.88
time (sec)	N/A	0.125	0.157	0.000	0.000	0.000	0.000	0.308	1.433

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	69	0	0	117	0	79	101
N.S.	1	1.00	1.00	0.00	0.00	1.70	0.00	1.14	1.46
time (sec)	N/A	0.063	0.171	0.000	0.000	0.628	0.000	0.281	0.750

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F(-1)	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	52	52	256	0	71	0	0	73	46
N.S.	1	1.00	4.92	0.00	1.37	0.00	0.00	1.40	0.88
time (sec)	N/A	0.056	0.323	0.000	0.281	0.000	0.000	0.482	0.438

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	245	0	74	0	0	76	0
N.S.	1	1.00	4.54	0.00	1.37	0.00	0.00	1.41	0.00
time (sec)	N/A	0.069	0.302	0.000	0.287	0.000	0.000	0.410	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-1)	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	133	174	1447	0	0	0	0	0	0
N.S.	1	1.31	10.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.753	25.298	0.000	0.000	0.000	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	F	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	208	169	0	0	271	0	193	0
N.S.	1	2.08	1.69	0.00	0.00	2.71	0.00	1.93	0.00
time (sec)	N/A	0.799	4.996	0.000	0.000	0.303	0.000	0.397	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	47	127	136	185	0	0	0
N.S.	1	1.00	0.42	1.13	1.21	1.65	0.00	0.00	0.00
time (sec)	N/A	0.115	0.038	0.688	0.283	0.285	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	162	154	0	145	0	0	146	0
N.S.	1	1.71	1.62	0.00	1.53	0.00	0.00	1.54	0.00
time (sec)	N/A	0.204	0.103	0.000	0.301	0.000	0.000	0.372	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	49	30	0	37	46	0	37	0
N.S.	1	1.00	0.61	0.00	0.76	0.94	0.00	0.76	0.00
time (sec)	N/A	0.075	0.358	0.000	0.199	0.307	0.000	0.281	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	26	86	35	0	25	43
N.S.	1	1.00	1.00	1.30	4.30	1.75	0.00	1.25	2.15
time (sec)	N/A	0.150	0.271	0.184	0.259	0.266	0.000	0.329	0.635

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	27	35	97	0	0	93	0	40	0
N.S.	1	1.30	3.59	0.00	0.00	3.44	0.00	1.48	0.00
time (sec)	N/A	0.666	0.422	0.000	0.000	0.956	0.000	0.310	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	126	89	0	100	461	0	0	0
N.S.	1	1.25	0.88	0.00	0.99	4.56	0.00	0.00	0.00
time (sec)	N/A	1.045	0.452	0.000	0.296	39.137	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	20	0	0	26	0	25	0
N.S.	1	1.00	0.80	0.00	0.00	1.04	0.00	1.00	0.00
time (sec)	N/A	0.041	0.081	0.000	0.000	0.271	0.000	0.293	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	154	153	0	0	0	0	120	0
N.S.	1	1.51	1.50	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.131	0.106	0.000	0.000	0.000	0.000	0.275	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	18	45	78	259	50	0	85	0
N.S.	1	1.06	2.65	4.59	15.24	2.94	0.00	5.00	0.00
time (sec)	N/A	0.014	0.078	0.693	0.304	0.284	0.000	0.291	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	52	143	507	115	0	138	0
N.S.	1	1.00	1.62	4.47	15.84	3.59	0.00	4.31	0.00
time (sec)	N/A	0.033	0.088	6.694	0.472	0.267	0.000	0.365	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	27	30	24	32	24
N.S.	1	1.00	1.00	0.77	0.87	0.97	0.77	1.03	0.77
time (sec)	N/A	0.011	0.004	0.288	0.203	0.249	0.048	0.275	0.341

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	29	30	32	32	30	30
N.S.	1	1.00	1.00	0.74	0.77	0.82	0.82	0.77	0.77
time (sec)	N/A	0.010	0.010	0.282	0.281	0.266	0.055	0.263	0.036

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	40	34	46	73	41	42	44
N.S.	1	1.00	0.69	0.59	0.79	1.26	0.71	0.72	0.76
time (sec)	N/A	0.023	0.012	0.304	0.211	0.262	0.061	0.256	0.085

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	46	35	33	65	984	35	34
N.S.	1	1.00	0.88	0.67	0.63	1.25	18.92	0.67	0.65
time (sec)	N/A	0.016	0.045	0.420	0.282	0.253	1.810	0.263	0.550

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	49	41	42	47	165	46	46
N.S.	1	1.00	0.80	0.67	0.69	0.77	2.70	0.75	0.75
time (sec)	N/A	0.021	0.039	0.434	0.278	0.255	3.434	0.273	0.488

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	15	15	14	15	15	14	24
N.S.	1	1.00	0.94	0.94	0.88	0.94	0.94	0.88	1.50
time (sec)	N/A	0.002	0.002	0.014	0.209	0.259	0.016	0.258	0.498

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	36	30	35	49	32	36	35
N.S.	1	1.00	0.78	0.65	0.76	1.07	0.70	0.78	0.76
time (sec)	N/A	0.016	0.010	0.253	0.201	0.252	0.042	0.267	0.059

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	23	21	36	36	32	21	36
N.S.	1	1.00	0.58	0.52	0.90	0.90	0.80	0.52	0.90
time (sec)	N/A	0.024	0.006	0.292	0.216	0.253	0.050	0.267	0.070

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	17	19	21	41	19	20
N.S.	1	1.00	0.93	0.63	0.70	0.78	1.52	0.70	0.74
time (sec)	N/A	0.010	0.016	0.299	0.197	0.240	2.710	0.261	0.439

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	25	20	28	33	82	26	21
N.S.	1	1.00	0.66	0.53	0.74	0.87	2.16	0.68	0.55
time (sec)	N/A	0.012	0.019	0.295	0.193	0.245	0.794	0.261	0.461

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	24	20	25	34	61	19	99
N.S.	1	1.00	0.73	0.61	0.76	1.03	1.85	0.58	3.00
time (sec)	N/A	0.003	0.031	0.250	0.203	0.238	0.856	0.275	0.323

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	26	23	51	61	0	21	22
N.S.	1	1.00	0.60	0.53	1.19	1.42	0.00	0.49	0.51
time (sec)	N/A	0.004	0.153	0.351	0.192	0.250	0.000	0.283	0.314

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	28	23	35	40	153	62	24
N.S.	1	1.00	0.60	0.49	0.74	0.85	3.26	1.32	0.51
time (sec)	N/A	0.006	0.036	0.291	0.284	0.247	1.175	0.278	0.459

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	19	16	16	15	24	16	17
N.S.	1	1.00	0.68	0.57	0.57	0.54	0.86	0.57	0.61
time (sec)	N/A	0.004	0.014	0.254	0.195	0.248	0.808	0.267	0.039

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	49	47	53	74	49	58	51
N.S.	1	1.00	0.94	0.90	1.02	1.42	0.94	1.12	0.98
time (sec)	N/A	0.020	0.018	0.267	0.202	0.242	0.072	0.277	0.362

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	0	21	0	0	15
N.S.	1	1.00	1.00	0.88	0.00	0.84	0.00	0.00	0.60
time (sec)	N/A	0.038	6.487	0.260	0.000	0.390	0.000	0.000	0.501

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	35	28	79	69	0	98	0
N.S.	1	1.00	0.70	0.56	1.58	1.38	0.00	1.96	0.00
time (sec)	N/A	0.013	0.191	0.336	0.284	0.247	0.000	0.320	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	40	22	21	41	49	29	54
N.S.	1	1.00	1.67	0.92	0.88	1.71	2.04	1.21	2.25
time (sec)	N/A	0.004	0.054	0.323	0.286	0.241	0.181	0.273	0.325

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	46	32	45	45	32	77	33
N.S.	1	1.00	1.15	0.80	1.12	1.12	0.80	1.92	0.82
time (sec)	N/A	0.007	0.056	0.473	0.274	0.264	0.376	0.275	0.042

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	28	25	37	44	291	35	179
N.S.	1	1.00	0.57	0.51	0.76	0.90	5.94	0.71	3.65
time (sec)	N/A	0.005	0.169	0.298	0.180	0.275	2.995	0.284	0.305

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	31	28	59	47	0	35	29
N.S.	1	1.00	0.66	0.60	1.26	1.00	0.00	0.74	0.62
time (sec)	N/A	0.005	0.135	0.408	0.189	0.244	0.000	0.274	0.358

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	32	32	34	32	32
N.S.	1	1.00	1.00	0.92	0.89	0.89	0.94	0.89	0.89
time (sec)	N/A	0.009	0.002	0.381	0.184	0.261	0.021	0.305	0.034

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	30	29	29	34	29	29
N.S.	1	1.00	1.00	0.77	0.74	0.74	0.87	0.74	0.74
time (sec)	N/A	0.010	0.001	0.327	0.182	0.223	0.021	0.265	0.030

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	59	73	0	27	29
N.S.	1	1.00	0.70	0.64	1.26	1.55	0.00	0.57	0.62
time (sec)	N/A	0.006	0.195	0.377	0.182	0.253	0.000	0.279	0.414

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	33	30	76	73	0	41	29
N.S.	1	1.00	0.73	0.67	1.69	1.62	0.00	0.91	0.64
time (sec)	N/A	0.014	0.310	0.372	0.185	0.234	0.000	0.295	0.184

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	67	56	55	57	112	55	69
N.S.	1	1.00	0.81	0.67	0.66	0.69	1.35	0.66	0.83
time (sec)	N/A	0.071	0.039	0.530	0.195	0.242	0.368	0.278	0.423

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	51	50	49	52	92	49	59
N.S.	1	1.00	0.70	0.68	0.67	0.71	1.26	0.67	0.81
time (sec)	N/A	0.061	0.067	0.531	0.193	0.244	0.254	0.312	0.375

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	70	67	66	72	192	66	88
N.S.	1	1.00	0.67	0.64	0.63	0.69	1.83	0.63	0.84
time (sec)	N/A	0.079	0.059	0.842	0.197	0.257	0.547	0.265	0.418

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	39	32	35	36	53	35	40
N.S.	1	1.00	0.89	0.73	0.80	0.82	1.20	0.80	0.91
time (sec)	N/A	0.031	0.074	0.448	0.201	0.244	0.178	0.255	0.097

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	47	0	45	507	206	56
N.S.	1	1.00	1.00	1.42	0.00	1.36	15.36	6.24	1.70
time (sec)	N/A	0.054	0.025	0.333	0.000	0.249	0.683	0.270	0.635

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	104	43	619	47	551	341	35
N.S.	1	1.00	3.47	1.43	20.63	1.57	18.37	11.37	1.17
time (sec)	N/A	0.030	0.114	0.295	0.283	0.242	0.643	0.427	0.561

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	132	15	128	53	16
N.S.	1	1.00	1.00	0.81	8.25	0.94	8.00	3.31	1.00
time (sec)	N/A	0.014	0.024	0.381	0.182	0.243	0.410	0.273	0.384

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	57	66	113	0	0	0
N.S.	1	1.00	0.92	0.92	1.06	1.82	0.00	0.00	0.00
time (sec)	N/A	0.063	0.023	0.305	0.297	0.264	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	59	210	138	0	0	0
N.S.	1	1.00	0.92	1.00	3.56	2.34	0.00	0.00	0.00
time (sec)	N/A	0.052	0.011	0.149	0.289	0.258	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	14	44	78	13	0	10	0
N.S.	1	1.00	1.17	3.67	6.50	1.08	0.00	0.83	0.00
time (sec)	N/A	0.085	0.228	3.049	0.278	0.249	0.000	0.276	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	20	69	19	66	39	0
N.S.	1	1.00	0.95	1.00	3.45	0.95	3.30	1.95	0.00
time (sec)	N/A	0.031	0.288	0.505	0.199	0.246	0.716	0.286	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	22	42	325	22
N.S.	1	1.00	1.00	1.05	0.00	1.00	1.91	14.77	1.00
time (sec)	N/A	0.021	0.021	0.079	0.000	0.237	0.307	0.278	0.415

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	41	46	42	0	52	0	436	34
N.S.	1	1.21	1.35	1.24	0.00	1.53	0.00	12.82	1.00
time (sec)	N/A	0.176	0.041	0.087	0.000	0.249	0.000	0.331	0.446

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	11	7	7	4
N.S.	1	1.00	1.00	0.89	0.78	1.22	0.78	0.78	0.44
time (sec)	N/A	0.004	0.008	0.027	0.187	0.228	0.033	0.272	0.050

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	21	17	24	8
N.S.	1	1.00	1.00	0.77	0.73	0.95	0.77	1.09	0.36
time (sec)	N/A	0.016	0.010	0.035	0.200	0.245	0.042	0.262	0.055

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	24	23	24	24	25	23
N.S.	1	1.00	0.97	0.77	0.74	0.77	0.77	0.81	0.74
time (sec)	N/A	0.016	0.027	0.046	0.196	0.231	0.054	0.272	0.349

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	37	29	28	31	31	36	28
N.S.	1	1.00	1.03	0.81	0.78	0.86	0.86	1.00	0.78
time (sec)	N/A	0.018	0.041	0.049	0.199	0.248	0.067	0.285	0.335

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	52	45	0	0	0	0	0	0
N.S.	1	1.08	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.052	0.000	0.000	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	34	44	41	39	53	46	41
N.S.	1	1.00	0.79	1.02	0.95	0.91	1.23	1.07	0.95
time (sec)	N/A	0.020	0.061	0.181	0.203	0.249	0.114	0.281	0.499

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	27	27	26	29	27	26
N.S.	1	1.00	1.00	1.00	1.00	0.96	1.07	1.00	0.96
time (sec)	N/A	0.009	0.013	0.040	0.228	0.244	0.059	0.281	0.400

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	55	51	62	250	691	68
N.S.	1	1.00	1.00	1.04	0.96	1.17	4.72	13.04	1.28
time (sec)	N/A	0.065	0.043	0.070	0.192	0.255	0.461	0.313	0.410

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	65	84	77	130	665	1033	81
N.S.	1	1.00	0.82	1.06	0.97	1.65	8.42	13.08	1.03
time (sec)	N/A	0.081	0.072	0.117	0.193	0.244	2.049	0.315	0.439

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	80	109	99	205	1350	1359	106
N.S.	1	1.00	0.82	1.11	1.01	2.09	13.78	13.87	1.08
time (sec)	N/A	0.095	0.077	0.118	0.202	0.248	16.753	0.354	0.443

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	80	73	0	0	0	0	0	0
N.S.	1	1.11	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	0.030	0.000	0.000	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	28	28	28	29	28	27
N.S.	1	1.00	1.00	1.00	1.00	1.00	1.04	1.00	0.96
time (sec)	N/A	0.007	0.010	0.036	0.227	0.242	0.059	0.268	0.330

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	55	51	64	248	691	69
N.S.	1	1.00	1.00	1.04	0.96	1.21	4.68	13.04	1.30
time (sec)	N/A	0.063	0.064	0.051	0.208	0.239	0.444	0.310	0.384

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	66	84	77	131	663	1033	81
N.S.	1	1.00	0.84	1.06	0.97	1.66	8.39	13.08	1.03
time (sec)	N/A	0.066	0.103	0.076	0.206	0.267	2.051	0.325	0.441

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	80	109	99	207	1348	1359	106
N.S.	1	1.00	0.82	1.11	1.01	2.11	13.76	13.87	1.08
time (sec)	N/A	0.079	0.107	0.082	0.204	0.245	16.580	0.355	0.403

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	82	75	0	0	0	0	0	0
N.S.	1	1.11	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	19	15	15	15
N.S.	1	1.00	1.00	1.07	1.00	1.27	1.00	1.00	1.00
time (sec)	N/A	0.005	0.008	0.028	0.198	0.234	0.036	0.286	0.360

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	35	31	31	29	44	30	26
N.S.	1	1.00	1.06	0.94	0.94	0.88	1.33	0.91	0.79
time (sec)	N/A	0.013	0.017	0.029	0.194	0.240	0.049	0.287	0.352

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	45	41	46	39	70	40	34
N.S.	1	1.00	0.90	0.82	0.92	0.78	1.40	0.80	0.68
time (sec)	N/A	0.013	0.020	0.042	0.196	0.254	0.061	0.292	0.359

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	53	50	61	47	87	48	42
N.S.	1	1.00	0.82	0.77	0.94	0.72	1.34	0.74	0.65
time (sec)	N/A	0.017	0.022	0.042	0.193	0.253	0.067	0.329	0.366

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	0	0	0	55
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.38
time (sec)	N/A	0.016	0.040	0.000	0.000	0.000	0.000	0.000	0.361

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	21	19	16	16
N.S.	1	1.00	1.00	1.06	1.00	1.31	1.19	1.00	1.00
time (sec)	N/A	0.003	0.005	0.023	0.194	0.229	0.037	0.307	0.338

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	35	31	31	29	44	30	27
N.S.	1	1.00	1.06	0.94	0.94	0.88	1.33	0.91	0.82
time (sec)	N/A	0.011	0.018	0.030	0.221	0.237	0.050	0.300	0.345

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	45	41	46	39	70	40	35
N.S.	1	1.00	0.90	0.82	0.92	0.78	1.40	0.80	0.70
time (sec)	N/A	0.012	0.021	0.042	0.199	0.228	0.064	0.346	0.350

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	53	50	61	47	87	48	43
N.S.	1	1.00	0.82	0.77	0.94	0.72	1.34	0.74	0.66
time (sec)	N/A	0.014	0.023	0.046	0.200	0.241	0.071	0.299	0.352

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0	57
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.30
time (sec)	N/A	0.015	0.040	0.000	0.000	0.000	0.000	0.000	0.342

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	38	23	23	22	15	26	22
N.S.	1	1.00	1.58	0.96	0.96	0.92	0.62	1.08	0.92
time (sec)	N/A	0.012	0.019	0.030	0.202	0.242	0.048	0.269	0.359

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	123	97	26	100	311	22	116	104
N.S.	1	1.23	0.97	0.26	1.00	3.11	0.22	1.16	1.04
time (sec)	N/A	0.071	0.081	0.060	0.277	0.262	0.075	0.271	1.583

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	15	12	11	11	8	11	11
N.S.	1	1.00	1.25	1.00	0.92	0.92	0.67	0.92	0.92
time (sec)	N/A	0.015	0.014	0.032	0.193	0.252	0.030	0.279	0.049

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	38	37	39	22	37	39
N.S.	1	1.00	0.96	0.81	0.79	0.83	0.47	0.79	0.83
time (sec)	N/A	0.041	0.043	0.044	0.299	0.245	0.067	0.286	0.426

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	29	28	28	48	29	28
N.S.	1	1.00	0.95	0.74	0.72	0.72	1.23	0.74	0.72
time (sec)	N/A	0.050	0.049	0.053	0.286	0.252	0.094	0.278	0.094

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	32	37	94	32	29
N.S.	1	1.00	1.00	1.10	1.07	1.23	3.13	1.07	0.97
time (sec)	N/A	0.027	0.046	0.075	0.190	0.243	0.421	0.288	0.457

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	57	56	56	61	57	33
N.S.	1	1.00	1.00	1.06	1.04	1.04	1.13	1.06	0.61
time (sec)	N/A	0.015	0.030	0.066	0.268	0.236	0.535	0.293	0.408

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	75
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.27
time (sec)	N/A	0.024	0.036	0.000	0.000	0.000	0.000	0.000	0.481

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	22	15	7	18	37	18	14
N.S.	1	1.00	1.22	0.83	0.39	1.00	2.06	1.00	0.78
time (sec)	N/A	0.021	0.028	0.037	0.205	0.242	0.257	0.272	0.492

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	24	17	20	20	37	20	16
N.S.	1	1.00	1.20	0.85	1.00	1.00	1.85	1.00	0.80
time (sec)	N/A	0.021	0.029	0.037	0.205	0.237	0.274	0.277	0.520

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	0	39	46	0	0	0
N.S.	1	1.00	1.05	0.00	0.98	1.15	0.00	0.00	0.00
time (sec)	N/A	0.066	0.113	0.000	0.277	0.237	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	57	54	0	0	0	0	0	0
N.S.	1	1.54	1.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	0.032	0.000	0.000	0.000	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	44	37	49	30	49	65	30
N.S.	1	1.00	0.60	0.51	0.67	0.41	0.67	0.89	0.41
time (sec)	N/A	0.034	0.040	0.032	0.207	0.249	0.915	0.322	0.136

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	23	21	19	19	20	19	21
N.S.	1	1.00	0.52	0.48	0.43	0.43	0.45	0.43	0.48
time (sec)	N/A	0.025	0.015	0.029	0.197	0.247	0.031	0.276	0.027

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	26	27	7	23	32	27	22
N.S.	1	1.00	0.67	0.69	0.18	0.59	0.82	0.69	0.56
time (sec)	N/A	0.028	0.024	0.068	0.222	0.247	0.732	0.272	0.364

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	29	28	27	27	37	826	27
N.S.	1	1.00	0.66	0.64	0.61	0.61	0.84	18.77	0.61
time (sec)	N/A	0.015	0.015	0.033	0.200	0.229	0.048	0.288	0.058

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	18	9	8	18	9
N.S.	1	1.00	1.00	0.83	1.50	0.75	0.67	1.50	0.75
time (sec)	N/A	0.032	0.021	0.036	0.205	0.240	0.033	0.278	0.058

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	25	25	33	24	40	26
N.S.	1	1.00	0.97	0.78	0.78	1.03	0.75	1.25	0.81
time (sec)	N/A	0.055	0.074	0.043	0.282	0.237	0.046	0.285	0.422

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	21	12	0	0	12
N.S.	1	1.00	1.00	1.33	1.40	0.80	0.00	0.00	0.80
time (sec)	N/A	0.047	0.176	0.267	0.254	0.242	0.000	0.000	0.522

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	19	21	26	19	19
N.S.	1	1.00	0.81	0.74	0.70	0.78	0.96	0.70	0.70
time (sec)	N/A	0.007	0.030	0.105	0.204	0.246	0.172	0.291	0.040

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	18	39	21	29	39	19
N.S.	1	1.00	0.74	0.51	1.11	0.60	0.83	1.11	0.54
time (sec)	N/A	0.097	0.072	0.257	0.201	0.250	0.235	0.269	0.111

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	37	32	31	0	70	39	33
N.S.	1	1.00	0.65	0.56	0.54	0.00	1.23	0.68	0.58
time (sec)	N/A	0.021	0.079	0.247	0.290	0.000	0.462	0.264	0.044

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	39	37	45	37	265	43	37
N.S.	1	1.00	0.72	0.69	0.83	0.69	4.91	0.80	0.69
time (sec)	N/A	0.019	0.027	0.262	0.214	0.251	0.434	0.268	0.058

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	64	60	73	65	638	63	47
N.S.	1	1.00	0.78	0.73	0.89	0.79	7.78	0.77	0.57
time (sec)	N/A	0.032	0.122	0.336	0.218	0.249	1.216	0.283	0.063

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	36	28	27	42	76	33	39
N.S.	1	1.00	0.46	0.35	0.34	0.53	0.96	0.42	0.49
time (sec)	N/A	0.038	0.056	0.381	0.193	0.244	0.444	0.272	0.384

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	21	19	27	40	70	24	18
N.S.	1	1.00	0.58	0.53	0.75	1.11	1.94	0.67	0.50
time (sec)	N/A	0.033	0.033	0.191	0.208	0.243	0.522	0.277	0.432

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	21	19	27	50	99	24	18
N.S.	1	1.00	0.58	0.53	0.75	1.39	2.75	0.67	0.50
time (sec)	N/A	0.029	0.040	0.266	0.192	0.255	0.536	0.280	0.452

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	85	97	0	0	0	0	0	0
N.S.	1	1.47	1.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.178	0.000	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	90	0	0	0	0	0	0
N.S.	1	1.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.141	0.000	0.000	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	77	66	0	0	0	0	0	0
N.S.	1	1.51	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	0.036	0.000	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.009	0.000	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	84	0	0	0	0	0	0
N.S.	1	1.00	3.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.025	0.115	0.000	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	61	0	0	0	0	0	0
N.S.	1	1.00	2.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.025	0.614	0.000	0.000	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	61	0	0	0	0	0	0
N.S.	1	1.00	2.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.622	0.000	0.000	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	12	11	22	12	0	10	8
N.S.	1	1.00	0.80	0.73	1.47	0.80	0.00	0.67	0.53
time (sec)	N/A	0.024	0.106	0.243	0.267	0.251	0.000	0.289	0.470

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	45	100	0	0	0	0	0	0
N.S.	1	1.10	2.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.087	0.617	0.000	0.000	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	8	22	11	0	7	7
N.S.	1	1.00	1.00	0.67	1.83	0.92	0.00	0.58	0.58
time (sec)	N/A	0.023	0.098	0.181	0.272	0.250	0.000	0.286	0.464

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	44	87	0	0	0	0	0	0
N.S.	1	1.05	2.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.090	0.435	0.000	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	48	72	0	0	0	0	0	0
N.S.	1	1.04	1.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.104	0.659	0.000	0.000	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	13	19	22	24	0	20	24
N.S.	1	1.00	0.93	1.36	1.57	1.71	0.00	1.43	1.71
time (sec)	N/A	0.018	0.046	0.251	0.273	0.249	0.000	0.290	0.502

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	43	47	73	0	0	0	0	0	0
N.S.	1	1.09	1.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.097	0.172	0.000	0.000	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	18	22	24	0	21	20
N.S.	1	1.00	1.00	1.38	1.69	1.85	0.00	1.62	1.54
time (sec)	N/A	0.016	0.044	0.248	0.276	0.252	0.000	0.297	0.528

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	18	16	17	17	27	15	17
N.S.	1	1.00	0.60	0.53	0.57	0.57	0.90	0.50	0.57
time (sec)	N/A	0.028	0.013	0.085	0.202	0.245	0.149	0.296	0.068

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	25	22	26	26	48	25	21
N.S.	1	1.00	0.50	0.44	0.52	0.52	0.96	0.50	0.42
time (sec)	N/A	0.080	0.027	0.110	0.208	0.241	0.285	0.296	0.404

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	38	31	33	37	80	33	39
N.S.	1	1.00	0.51	0.41	0.44	0.49	1.07	0.44	0.52
time (sec)	N/A	0.111	0.028	0.130	0.214	0.255	0.373	0.282	0.456

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	253	72	59	77	72	202	73	83
N.S.	1	1.35	0.39	0.32	0.41	0.39	1.08	0.39	0.44
time (sec)	N/A	0.359	0.099	0.345	0.219	0.250	1.006	0.262	0.359

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	40	40	41	41	85	39	51
N.S.	1	1.00	0.46	0.46	0.47	0.47	0.98	0.45	0.59
time (sec)	N/A	0.124	0.056	0.128	0.197	0.241	0.295	0.270	0.447

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	76	78	77	72	202	73	83
N.S.	1	1.00	0.41	0.42	0.42	0.39	1.09	0.39	0.45
time (sec)	N/A	0.277	0.147	0.859	0.201	0.259	1.020	0.290	0.639

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	11	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	5.50	1.00
time (sec)	N/A	0.004	0.002	0.025	0.187	0.243	0.064	0.286	0.020

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	11	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	5.50	1.00
time (sec)	N/A	0.004	0.002	0.029	0.215	0.234	0.066	0.287	0.025

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	18	7	11	3
N.S.	1	1.00	1.00	1.33	1.00	6.00	2.33	3.67	1.00
time (sec)	N/A	0.003	0.000	0.045	0.221	0.243	0.046	0.285	0.021

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	7	4	3	18	12	12	3
N.S.	1	1.00	2.33	1.33	1.00	6.00	4.00	4.00	1.00
time (sec)	N/A	0.004	0.000	0.064	0.199	0.252	0.133	0.285	0.030

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	8	7	5	5
N.S.	1	1.00	1.00	1.33	1.00	2.67	2.33	1.67	1.67
time (sec)	N/A	0.003	0.002	0.195	0.189	0.236	0.178	0.328	0.023

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	17	6	5	17	5	14	5
N.S.	1	1.00	3.40	1.20	1.00	3.40	1.00	2.80	1.00
time (sec)	N/A	0.003	0.013	0.035	0.201	0.247	0.121	0.286	0.013

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	16	10	24	24	10
N.S.	1	1.00	1.00	0.79	1.14	0.71	1.71	1.71	0.71
time (sec)	N/A	0.006	0.006	0.079	0.195	0.236	0.064	0.279	0.028

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	23	18	35	42	29	37	15
N.S.	1	1.00	1.21	0.95	1.84	2.21	1.53	1.95	0.79
time (sec)	N/A	0.010	0.012	0.490	0.213	0.236	0.182	0.290	0.029

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	38	68	10	26	12
N.S.	1	1.00	1.14	0.93	2.71	4.86	0.71	1.86	0.86
time (sec)	N/A	0.010	0.005	0.037	0.219	0.242	0.078	0.284	0.072

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	47	11	45	211	0	45	16
N.S.	1	1.00	2.94	0.69	2.81	13.19	0.00	2.81	1.00
time (sec)	N/A	0.011	0.012	0.373	0.211	0.230	0.000	0.272	0.392

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	61	461	422	60	22
N.S.	1	1.00	1.00	0.81	2.35	17.73	16.23	2.31	0.85
time (sec)	N/A	0.016	0.004	0.965	0.284	0.245	1.168	0.277	0.074

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	35	257	0	43	35
N.S.	1	1.00	1.00	0.94	1.94	14.28	0.00	2.39	1.94
time (sec)	N/A	0.018	0.006	6.589	0.293	0.246	0.000	0.279	0.442

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	20	0	359	41	0	120
N.S.	1	1.00	0.87	0.65	0.00	11.58	1.32	0.00	3.87
time (sec)	N/A	0.025	0.054	0.200	0.000	0.237	39.538	0.000	0.177

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	42	41	36	0	175	126	32	43
N.S.	1	1.02	1.00	0.88	0.00	4.27	3.07	0.78	1.05
time (sec)	N/A	0.042	0.031	0.102	0.000	0.249	1.829	0.274	0.183

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	16	15	49	58	14	14	14
N.S.	1	1.00	0.64	0.60	1.96	2.32	0.56	0.56	0.56
time (sec)	N/A	0.016	0.009	0.057	0.207	0.229	0.163	0.280	0.395

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	49	42	41	42	146	43	35
N.S.	1	1.00	1.26	1.08	1.05	1.08	3.74	1.10	0.90
time (sec)	N/A	0.038	0.086	0.050	0.201	0.247	0.223	0.272	0.146

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	98	76	288	1129	79	109
N.S.	1	1.00	1.00	3.16	2.45	9.29	36.42	2.55	3.52
time (sec)	N/A	0.028	0.452	0.316	0.320	0.250	15.576	0.273	0.766

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	74	0	388	874	50	106
N.S.	1	1.00	1.00	2.11	0.00	11.09	24.97	1.43	3.03
time (sec)	N/A	0.029	0.077	0.219	0.000	0.270	14.555	0.273	0.399

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	46	69	113	908	48	63
N.S.	1	1.00	0.96	1.84	2.76	4.52	36.32	1.92	2.52
time (sec)	N/A	0.016	0.270	0.447	0.281	0.257	2.479	0.271	0.519

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	37	44	70	74	102	22	22
N.S.	1	1.00	1.12	1.33	2.12	2.24	3.09	0.67	0.67
time (sec)	N/A	0.110	5.033	0.312	0.286	0.248	0.518	0.291	0.467

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	42	44	116	48	40
N.S.	1	1.00	1.00	0.77	1.40	1.47	3.87	1.60	1.33
time (sec)	N/A	0.030	0.055	2.092	0.194	0.249	0.939	0.284	0.502

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	42	111	139	48	40
N.S.	1	1.00	1.00	0.77	1.40	3.70	4.63	1.60	1.33
time (sec)	N/A	0.027	0.038	2.012	0.275	0.242	0.938	0.287	0.559

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	102	160	167	0	376	0	90	0
N.S.	1	1.48	2.32	2.42	0.00	5.45	0.00	1.30	0.00
time (sec)	N/A	0.743	18.500	1.328	0.000	0.260	0.000	0.335	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	26	30	125	474	0	38	57
N.S.	1	1.00	0.70	0.81	3.38	12.81	0.00	1.03	1.54
time (sec)	N/A	0.032	0.070	0.180	0.212	0.260	0.000	0.322	0.143

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	21	30	177	161	0	0	47
N.S.	1	1.00	0.72	1.03	6.10	5.55	0.00	0.00	1.62
time (sec)	N/A	0.072	0.038	0.236	0.332	0.262	0.000	0.000	0.525

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	63	0	482	0	58	0
N.S.	1	1.00	1.00	4.20	0.00	32.13	0.00	3.87	0.00
time (sec)	N/A	0.015	0.009	0.490	0.000	0.258	0.000	0.299	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	49	93	17	51	21
N.S.	1	1.00	1.00	1.50	3.06	5.81	1.06	3.19	1.31
time (sec)	N/A	0.014	0.015	0.043	0.313	0.254	0.064	0.299	0.431

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	28	53	95	22	53	27
N.S.	1	1.00	1.00	1.75	3.31	5.94	1.38	3.31	1.69
time (sec)	N/A	0.015	0.015	0.067	0.238	0.250	0.263	0.280	0.399

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	23	14	13	20	26	11	16
N.S.	1	1.00	1.15	0.70	0.65	1.00	1.30	0.55	0.80
time (sec)	N/A	0.057	0.043	0.418	0.198	0.253	0.187	0.311	0.063

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	20	16	35	20	12	14	17
N.S.	1	1.00	1.33	1.07	2.33	1.33	0.80	0.93	1.13
time (sec)	N/A	0.115	0.061	0.186	0.193	0.240	0.149	0.293	0.342

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	22	75	0	24	24
N.S.	1	1.00	1.00	0.75	1.10	3.75	0.00	1.20	1.20
time (sec)	N/A	0.014	0.009	0.514	0.201	0.242	0.000	0.288	0.373

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	9	75	102	0	10	19
N.S.	1	1.00	1.00	0.69	5.77	7.85	0.00	0.77	1.46
time (sec)	N/A	0.012	0.008	0.619	0.202	0.250	0.000	0.286	0.348

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	16	12	6	6
N.S.	1	1.00	1.00	0.78	0.67	1.78	1.33	0.67	0.67
time (sec)	N/A	0.013	0.001	0.596	0.194	0.228	0.211	0.275	0.365

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	19	18	13	0	25	32	16	14
N.S.	1	1.46	1.38	1.00	0.00	1.92	2.46	1.23	1.08
time (sec)	N/A	0.026	0.020	0.555	0.000	0.242	0.225	0.287	0.112

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	10	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	10.00	1.00	1.00
time (sec)	N/A	0.011	0.000	0.509	0.199	0.221	0.174	0.277	0.320

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	36	17	16	26	0	17	16
N.S.	1	1.00	1.64	0.77	0.73	1.18	0.00	0.77	0.73
time (sec)	N/A	0.018	0.021	0.092	0.205	0.242	0.000	0.285	0.064

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	18	12	11	21	0	11	11
N.S.	1	1.00	1.38	0.92	0.85	1.62	0.00	0.85	0.85
time (sec)	N/A	0.022	0.018	0.187	0.197	0.232	0.000	0.279	0.351

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	20	12	11	22	0	11	11
N.S.	1	1.00	1.33	0.80	0.73	1.47	0.00	0.73	0.73
time (sec)	N/A	0.030	0.018	0.234	0.204	0.237	0.000	0.286	0.050

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	19	19	26	25	56	0	32
N.S.	1	1.00	0.73	0.73	1.00	0.96	2.15	0.00	1.23
time (sec)	N/A	0.008	0.007	0.030	0.200	0.241	0.275	0.000	0.450

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	30	41	42	45	155	84	43
N.S.	1	1.00	0.71	0.98	1.00	1.07	3.69	2.00	1.02
time (sec)	N/A	0.016	0.008	0.045	0.203	0.247	0.355	0.289	0.396

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	21	23	22	17	34	22	17
N.S.	1	1.00	0.62	0.68	0.65	0.50	1.00	0.65	0.50
time (sec)	N/A	0.015	0.004	0.085	0.198	0.247	0.724	0.275	0.037

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	25	25	22	24	21
N.S.	1	1.00	1.00	0.89	0.89	0.89	0.79	0.86	0.75
time (sec)	N/A	0.008	0.004	0.024	0.204	0.256	0.047	0.300	0.375

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	81	58	69	69	71	71	71
N.S.	1	1.00	1.21	0.87	1.03	1.03	1.06	1.06	1.06
time (sec)	N/A	0.028	0.012	0.302	0.204	0.233	0.062	0.285	0.439

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	36	23	26	23	20
N.S.	1	1.00	1.00	1.04	1.57	1.00	1.13	1.00	0.87
time (sec)	N/A	0.011	0.004	0.021	0.205	0.238	0.049	0.281	0.422

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	48	66	48	51	52	51
N.S.	1	1.00	1.00	0.80	1.10	0.80	0.85	0.87	0.85
time (sec)	N/A	0.071	0.008	0.030	0.211	0.236	0.069	0.299	0.458

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	31	8	34	32	0	29
N.S.	1	1.00	1.00	0.72	0.19	0.79	0.74	0.00	0.67
time (sec)	N/A	0.045	0.025	0.036	0.225	0.236	0.280	0.000	0.330

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	30	32	25	0	177	0	0
N.S.	1	1.00	1.03	1.10	0.86	0.00	6.10	0.00	0.00
time (sec)	N/A	0.018	0.004	0.356	0.197	0.000	3.779	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	30	38	34	24	138	35
N.S.	1	1.00	0.93	1.03	1.31	1.17	0.83	4.76	1.21
time (sec)	N/A	0.012	0.011	0.278	0.195	0.248	0.109	0.307	0.475

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	15	12	22
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.25	1.00	1.83
time (sec)	N/A	0.012	0.004	0.036	0.188	0.247	0.369	0.314	0.385

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	22	36	19	19
N.S.	1	1.00	1.00	1.05	1.00	1.16	1.89	1.00	1.00
time (sec)	N/A	0.019	0.007	0.063	0.189	0.243	0.465	0.287	0.501

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	30	11
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	2.73	1.00
time (sec)	N/A	0.016	0.020	0.029	0.197	0.230	0.048	0.269	0.345

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	0	27	71	22	22
N.S.	1	1.00	1.00	1.04	0.00	1.17	3.09	0.96	0.96
time (sec)	N/A	0.023	0.007	0.101	0.000	0.242	5.322	0.264	0.487

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	46	15	7	18	0	18	14
N.S.	1	1.00	2.88	0.94	0.44	1.12	0.00	1.12	0.88
time (sec)	N/A	0.027	0.014	0.026	0.202	0.230	0.000	0.282	0.473

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	50	17	20	20	0	0	16
N.S.	1	1.00	2.78	0.94	1.11	1.11	0.00	0.00	0.89
time (sec)	N/A	0.030	0.015	0.028	0.194	0.236	0.000	0.000	0.449

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	7	25	0	10	16
N.S.	1	1.00	1.00	0.94	0.39	1.39	0.00	0.56	0.89
time (sec)	N/A	0.035	0.015	0.027	0.286	0.248	0.000	0.301	0.656

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	37	13	44	0	0	27
N.S.	1	1.00	1.00	1.68	0.59	2.00	0.00	0.00	1.23
time (sec)	N/A	0.064	0.011	0.027	0.192	0.245	0.000	0.000	0.638

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	39	37	27	0	0	22
N.S.	1	1.00	1.00	1.62	1.54	1.12	0.00	0.00	0.92
time (sec)	N/A	0.069	0.013	0.026	0.185	0.256	0.000	0.000	0.689

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	43	13	27	0	21	25
N.S.	1	1.00	1.00	1.87	0.57	1.17	0.00	0.91	1.09
time (sec)	N/A	0.068	0.012	0.024	0.279	0.249	0.000	0.332	0.659

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	10	11	8
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.91	1.00	0.73
time (sec)	N/A	0.005	0.004	0.027	0.194	0.246	0.065	0.331	0.390

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	15	20	24	20	15
N.S.	1	1.00	1.00	1.05	0.75	1.00	1.20	1.00	0.75
time (sec)	N/A	0.012	0.005	0.046	0.191	0.243	0.094	0.290	0.500

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	30	22	29	36	29	29
N.S.	1	1.00	1.00	1.03	0.76	1.00	1.24	1.00	1.00
time (sec)	N/A	0.015	0.005	0.057	0.193	0.239	0.124	0.297	0.409

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	39	29	38	48	38	38
N.S.	1	1.00	1.00	1.03	0.76	1.00	1.26	1.00	1.00
time (sec)	N/A	0.017	0.007	0.060	0.194	0.256	0.151	0.309	0.420

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	29	15	24	0	24
N.S.	1	1.00	1.00	0.00	1.21	0.62	1.00	0.00	1.00
time (sec)	N/A	0.022	0.023	0.000	0.048	0.074	0.703	0.000	0.479

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	0	5	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.00	1.25	1.00
time (sec)	N/A	0.017	0.012	0.197	0.191	0.244	0.000	0.275	0.433

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	10	9	12	7	9	9
N.S.	1	1.00	1.00	1.43	1.29	1.71	1.00	1.29	1.29
time (sec)	N/A	0.049	0.003	0.663	0.207	0.253	0.681	0.297	0.430

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	46	10	38	8
N.S.	1	1.00	1.00	1.09	1.00	4.18	0.91	3.45	0.73
time (sec)	N/A	0.011	0.006	4.227	0.192	0.233	0.195	0.290	0.374

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	0	16
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.78	0.00	1.78
time (sec)	N/A	0.016	0.004	1.256	0.190	0.232	1.632	0.000	0.514

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	20	22	22
N.S.	1	1.00	1.00	0.88	0.00	0.85	0.77	0.85	0.85
time (sec)	N/A	0.006	0.014	0.017	0.000	0.251	2.612	0.307	0.083

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	27	26	25	26	26	27	25
N.S.	1	1.00	0.77	0.74	0.71	0.74	0.74	0.77	0.71
time (sec)	N/A	0.011	0.009	0.046	0.191	0.234	0.063	0.315	0.068

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	24	24	20	26	27	20	24
N.S.	1	1.00	0.75	0.75	0.62	0.81	0.84	0.62	0.75
time (sec)	N/A	0.055	0.019	0.083	0.208	0.237	81.280	0.290	0.438

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	42	43	42	42	0	43	31
N.S.	1	1.00	0.81	0.83	0.81	0.81	0.00	0.83	0.60
time (sec)	N/A	0.031	0.021	0.052	0.196	0.259	0.000	0.278	0.613

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	26	25	24	42	26	0
N.S.	1	1.00	1.00	0.87	0.83	0.80	1.40	0.87	0.00
time (sec)	N/A	0.030	0.006	0.384	0.196	0.239	0.662	0.302	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	29	27	25	39	46	26	148
N.S.	1	1.00	0.97	0.90	0.83	1.30	1.53	0.87	4.93
time (sec)	N/A	0.046	0.047	10.178	0.204	0.245	10.177	0.293	2.051

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	28	28	32	164	56	32	0	43	39
N.S.	1	1.00	1.14	5.86	2.00	1.14	0.00	1.54	1.39
time (sec)	N/A	0.028	0.078	0.668	0.204	0.245	0.000	0.307	0.644

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	56	55	86	53	88	36	164
N.S.	1	1.00	0.93	0.92	1.43	0.88	1.47	0.60	2.73
time (sec)	N/A	0.107	0.108	0.592	0.273	0.250	2.715	0.351	0.938

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	52	52	51	44	0	104	0
N.S.	1	1.00	0.80	0.80	0.78	0.68	0.00	1.60	0.00
time (sec)	N/A	0.072	0.022	0.122	0.273	0.256	0.000	0.304	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	42	37	47	36	54	57	0
N.S.	1	1.00	0.69	0.61	0.77	0.59	0.89	0.93	0.00
time (sec)	N/A	0.070	0.014	0.392	0.279	0.256	0.135	0.305	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	37	42	44	36	44	41	41
N.S.	1	1.00	0.70	0.79	0.83	0.68	0.83	0.77	0.77
time (sec)	N/A	0.090	0.010	0.096	0.285	0.242	0.138	0.286	0.413

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	48	64	53	53	0	44
N.S.	1	1.00	0.92	0.79	1.05	0.87	0.87	0.00	0.72
time (sec)	N/A	0.098	0.011	0.139	0.278	0.247	0.230	0.000	0.109

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	42	56	95	35	0	106	0
N.S.	1	1.00	0.67	0.89	1.51	0.56	0.00	1.68	0.00
time (sec)	N/A	0.052	0.026	0.108	0.375	0.264	0.000	0.306	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	92	174	0	77	0	137	0
N.S.	1	1.00	0.62	1.18	0.00	0.52	0.00	0.93	0.00
time (sec)	N/A	0.107	0.041	0.464	0.000	0.261	0.000	0.313	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	31	30	26	31	27	0
N.S.	1	1.00	0.88	0.91	0.88	0.76	0.91	0.79	0.00
time (sec)	N/A	0.024	0.006	0.322	0.271	0.256	0.535	0.307	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	33	30	26	31	27	0
N.S.	1	1.00	0.88	0.97	0.88	0.76	0.91	0.79	0.00
time (sec)	N/A	0.029	0.011	0.302	0.280	0.257	0.607	0.320	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	26	28	22	27	37	22	0
N.S.	1	1.00	0.87	0.93	0.73	0.90	1.23	0.73	0.00
time (sec)	N/A	0.027	0.016	0.385	0.275	0.241	0.162	0.325	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	42	54	50	39	53	50	0
N.S.	1	1.00	0.71	0.92	0.85	0.66	0.90	0.85	0.00
time (sec)	N/A	0.040	0.019	0.439	0.280	0.245	0.280	0.320	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	37	27	37	63	34	0
N.S.	1	1.00	0.95	1.00	0.73	1.00	1.70	0.92	0.00
time (sec)	N/A	0.025	0.011	0.403	0.293	0.256	0.378	0.294	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	77	49	47	88	60	0
N.S.	1	1.00	0.77	1.26	0.80	0.77	1.44	0.98	0.00
time (sec)	N/A	0.066	0.032	0.500	0.296	0.264	6.920	0.305	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	119	155	0	0	0	0	0
N.S.	1	1.00	1.25	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.110	0.204	0.591	0.000	0.000	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	36	201	35	44	0	135	0
N.S.	1	1.00	0.88	4.90	0.85	1.07	0.00	3.29	0.00
time (sec)	N/A	0.042	0.023	0.589	0.274	0.260	0.000	0.345	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	32	32	26	26	27	0
N.S.	1	1.00	0.82	0.94	0.94	0.76	0.76	0.79	0.00
time (sec)	N/A	0.043	0.007	0.296	0.276	0.242	0.105	0.327	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	43	54	52	39	53	50	0
N.S.	1	1.00	0.70	0.89	0.85	0.64	0.87	0.82	0.00
time (sec)	N/A	0.075	0.014	0.453	0.285	0.255	0.212	0.328	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	46	25	44	24	27	0
N.S.	1	1.00	1.00	2.42	1.32	2.32	1.26	1.42	0.00
time (sec)	N/A	0.029	0.009	0.306	0.279	0.254	3.930	0.300	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	32	47	25	44	24	27	0
N.S.	1	1.00	1.88	2.76	1.47	2.59	1.41	1.59	0.00
time (sec)	N/A	0.024	0.027	0.316	0.276	0.259	4.144	0.292	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	45	63	48	61	78	54	0
N.S.	1	1.00	0.73	1.02	0.77	0.98	1.26	0.87	0.00
time (sec)	N/A	0.028	0.046	0.319	0.295	0.266	12.590	0.316	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	40	102	45	57	37	40	0
N.S.	1	1.00	1.11	2.83	1.25	1.58	1.03	1.11	0.00
time (sec)	N/A	0.057	0.034	0.632	0.279	0.263	6.021	0.295	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	112	430	0	0	0	0	0
N.S.	1	1.00	1.81	6.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.101	0.156	0.691	0.000	0.000	0.000	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	38	43	42	36	49	95	0
N.S.	1	1.00	0.70	0.80	0.78	0.67	0.91	1.76	0.00
time (sec)	N/A	0.066	0.025	0.269	0.276	0.251	5.063	0.333	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	59	52	41	78	53	0
N.S.	1	1.00	0.76	0.89	0.79	0.62	1.18	0.80	0.00
time (sec)	N/A	0.062	0.031	0.470	0.278	0.245	0.309	0.304	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	60	69	0	49	66	60	0
N.S.	1	1.00	0.82	0.95	0.00	0.67	0.90	0.82	0.00
time (sec)	N/A	0.116	0.015	0.342	0.000	0.253	0.205	0.305	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	21	22	26	19	31	26	21
N.S.	1	1.00	0.66	0.69	0.81	0.59	0.97	0.81	0.66
time (sec)	N/A	0.027	0.012	0.326	0.276	0.258	0.205	0.298	0.090

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	36	37	39	38	88	34	26
N.S.	1	1.00	0.82	0.84	0.89	0.86	2.00	0.77	0.59
time (sec)	N/A	0.036	0.013	0.342	0.279	0.244	0.312	0.282	0.453

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	24	19	19	19	19
N.S.	1	1.00	1.00	0.87	1.04	0.83	0.83	0.83	0.83
time (sec)	N/A	0.038	0.003	0.281	0.278	0.251	0.110	0.318	0.368

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	57	126	0	0	0	0	0
N.S.	1	1.00	0.85	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.079	0.018	0.444	0.000	0.000	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	29	40	26	0	0	23
N.S.	1	1.00	0.82	0.85	1.18	0.76	0.00	0.00	0.68
time (sec)	N/A	0.035	0.017	0.509	0.293	0.240	0.000	0.000	0.072

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	64	137	0	0	0	0	0
N.S.	1	1.00	0.81	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.088	0.043	0.349	0.000	0.000	0.000	0.000	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	70	147	0	0	0	0	0
N.S.	1	1.00	0.79	1.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.174	0.137	0.539	0.000	0.000	0.000	0.000	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	21	26	19	25	22
N.S.	1	1.00	1.00	1.05	0.95	1.18	0.86	1.14	1.00
time (sec)	N/A	0.026	0.003	0.080	0.300	0.243	0.109	0.285	0.070

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	59	30	31	26	34	31	30
N.S.	1	1.00	1.90	0.97	1.00	0.84	1.10	1.00	0.97
time (sec)	N/A	0.018	0.006	0.076	0.294	0.245	0.216	0.276	0.378

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	81	79	71	0	0	0	53
N.S.	1	1.00	1.29	1.25	1.13	0.00	0.00	0.00	0.84
time (sec)	N/A	0.069	0.004	0.355	0.323	0.000	0.000	0.000	0.513

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	27	29	22	0	24
N.S.	1	1.00	1.00	0.89	0.96	1.04	0.79	0.00	0.86
time (sec)	N/A	0.046	0.006	0.342	0.281	0.258	0.154	0.000	0.089

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	38	34	36	38	32	0	31
N.S.	1	1.00	0.97	0.87	0.92	0.97	0.82	0.00	0.79
time (sec)	N/A	0.053	0.011	0.090	0.270	0.242	0.185	0.000	0.073

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	56	57	71	54	61	0	51
N.S.	1	1.00	0.93	0.95	1.18	0.90	1.02	0.00	0.85
time (sec)	N/A	0.059	0.011	0.123	0.285	0.255	0.236	0.000	0.117

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	82	47	78	94	51	0	0	56
N.S.	1	1.04	0.59	0.99	1.19	0.65	0.00	0.00	0.71
time (sec)	N/A	0.101	0.034	0.555	0.277	0.238	0.000	0.000	0.404

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	107	116	116	194	0	0	0	0	0
N.S.	1	1.08	1.08	1.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.136	0.163	0.658	0.000	0.000	0.000	0.000	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	106	133	86	274	0	51	0	0	0
N.S.	1	1.25	0.81	2.58	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.140	0.232	0.504	0.000	0.253	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	41	41	48	85	27	23	0	75	0
N.S.	1	1.00	1.17	2.07	0.66	0.56	0.00	1.83	0.00
time (sec)	N/A	0.039	0.029	0.485	0.281	0.258	0.000	0.338	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	65	67	67	127	48	75	0	58	0
N.S.	1	1.03	1.03	1.95	0.74	1.15	0.00	0.89	0.00
time (sec)	N/A	0.023	0.078	0.698	0.249	0.271	0.000	0.347	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	51	53	61	134	46	68	0	53	0
N.S.	1	1.04	1.20	2.63	0.90	1.33	0.00	1.04	0.00
time (sec)	N/A	0.046	0.070	0.493	0.243	0.253	0.000	0.333	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	82	84	75	101	0	69	0	64	0
N.S.	1	1.02	0.91	1.23	0.00	0.84	0.00	0.78	0.00
time (sec)	N/A	0.061	0.177	0.361	0.000	0.266	0.000	0.324	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	175	232	383	246	0	0	0	0	0
N.S.	1	1.33	2.19	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.239	1.248	0.832	0.000	0.000	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	23	23	35	56	17	16	0	50	0
N.S.	1	1.00	1.52	2.43	0.74	0.70	0.00	2.17	0.00
time (sec)	N/A	0.043	0.026	0.352	0.295	0.248	0.000	0.316	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	70	91	79	204	123	81	0	105	0
N.S.	1	1.30	1.13	2.91	1.76	1.16	0.00	1.50	0.00
time (sec)	N/A	0.084	0.086	0.721	0.553	0.255	0.000	0.348	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	74	101	76	102	58	37	0	0	0
N.S.	1	1.36	1.03	1.38	0.78	0.50	0.00	0.00	0.00
time (sec)	N/A	0.138	0.040	0.329	0.286	0.247	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	133	172	84	114	0	59	0	0	0
N.S.	1	1.29	0.63	0.86	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.156	0.176	0.529	0.000	0.251	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	110	146	92	147	93	57	0	0	0
N.S.	1	1.33	0.84	1.34	0.85	0.52	0.00	0.00	0.00
time (sec)	N/A	0.161	0.052	0.540	0.528	0.259	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	B	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	125	99	86	103	51	0	94	0
N.S.	1	2.27	1.80	1.56	1.87	0.93	0.00	1.71	0.00
time (sec)	N/A	0.518	0.107	0.078	0.294	0.267	0.000	0.417	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	71	66	89	58	0	49	36
N.S.	1	1.00	1.78	1.65	2.22	1.45	0.00	1.22	0.90
time (sec)	N/A	0.033	0.054	0.079	0.283	0.261	0.000	0.320	0.426

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	32	31	50	153	32	31
N.S.	1	1.00	0.90	0.82	0.79	1.28	3.92	0.82	0.79
time (sec)	N/A	0.023	0.021	0.356	0.289	0.248	0.235	0.296	0.408

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	105	102	118	0	0	0	0
N.S.	1	1.00	0.86	0.84	0.97	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	0.019	0.391	0.322	0.000	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	14	27	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.50	0.96	0.00	0.00
time (sec)	N/A	0.024	0.012	0.000	0.000	0.256	0.434	0.000	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	25	25	26	25	25
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.84	0.81	0.81
time (sec)	N/A	0.028	0.017	0.294	0.199	0.250	0.566	0.278	0.637

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	61	70	0	90	0	58	0
N.S.	1	1.00	1.07	1.23	0.00	1.58	0.00	1.02	0.00
time (sec)	N/A	0.022	0.069	0.286	0.000	0.249	0.000	0.301	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	140	72	76	116	125	0	228	0
N.S.	1	1.71	0.88	0.93	1.41	1.52	0.00	2.78	0.00
time (sec)	N/A	0.058	0.053	0.306	0.776	0.262	0.000	0.439	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	49	66	0	0	519	0	218	0
N.S.	1	1.00	1.35	0.00	0.00	10.59	0.00	4.45	0.00
time (sec)	N/A	0.101	0.153	0.000	0.000	0.282	0.000	0.367	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	36	36	144	850	54	423	214	70	103
N.S.	1	1.00	4.00	23.61	1.50	11.75	5.94	1.94	2.86
time (sec)	N/A	0.102	0.116	0.367	0.288	0.259	78.092	0.285	0.629

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	109	0	16	26	0	29	0
N.S.	1	1.00	3.89	0.00	0.57	0.93	0.00	1.04	0.00
time (sec)	N/A	0.056	0.400	0.000	0.296	0.255	0.000	0.316	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [397] had the largest ratio of [1.33299999999999996]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	14	0.071
2	A	1	1	1.00	13	0.077
3	A	1	1	1.00	5	0.200
4	A	2	2	1.00	10	0.200
5	A	1	1	1.00	12	0.083
6	A	2	2	1.00	5	0.400
7	A	2	2	1.00	5	0.400
8	A	2	1	1.00	7	0.143
9	A	1	1	1.00	6	0.167
10	A	1	1	1.00	8	0.125
11	A	2	2	1.00	12	0.167
12	A	2	2	1.00	17	0.118
13	A	2	2	1.00	18	0.111
14	A	3	2	1.00	19	0.105
15	A	3	2	1.00	20	0.100
16	A	3	2	1.22	19	0.105
17	A	3	2	1.22	20	0.100
18	A	2	2	1.00	10	0.200
19	A	2	2	1.00	13	0.154
20	A	2	2	1.00	8	0.250
21	A	2	1	1.00	12	0.083
22	A	2	2	1.00	12	0.167
23	A	2	2	1.00	14	0.143
24	A	3	2	1.00	16	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	4	2	1.00	22	0.091
26	A	2	1	1.00	13	0.077
27	A	3	2	1.00	15	0.133
28	A	3	3	1.00	15	0.200
29	A	1	1	1.00	9	0.111
30	A	1	1	1.00	9	0.111
31	A	4	3	1.00	10	0.300
32	A	2	2	1.00	4	0.500
33	A	2	2	1.00	4	0.500
34	A	2	2	1.00	7	0.286
35	A	2	1	1.00	7	0.143
36	A	3	2	1.00	9	0.222
37	A	2	2	1.00	8	0.250
38	A	2	2	1.00	8	0.250
39	A	4	2	1.00	9	0.222
40	A	4	4	1.00	9	0.444
41	A	2	2	1.00	9	0.222
42	A	2	2	1.00	11	0.182
43	A	3	2	1.00	19	0.105
44	A	3	2	1.00	10	0.200
45	A	3	3	1.00	16	0.188
46	A	3	2	1.00	14	0.143
47	A	4	3	1.00	11	0.273
48	A	2	2	1.00	13	0.154
49	A	3	2	1.00	13	0.154
50	A	3	3	1.00	15	0.200
51	A	3	3	1.00	17	0.176
52	A	3	3	1.00	17	0.176
53	A	3	3	1.00	15	0.200
54	A	2	2	1.00	12	0.167
55	A	2	2	1.00	14	0.143
56	A	2	2	1.00	11	0.182
57	A	3	3	1.00	18	0.167
58	A	2	2	1.00	16	0.125
59	A	1	1	1.10	18	0.056

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	7	5	1.00	17	0.294
61	A	2	2	1.30	10	0.200
62	A	3	1	1.00	11	0.091
63	A	2	1	1.00	17	0.059
64	A	3	2	1.06	19	0.105
65	A	3	2	1.00	19	0.105
66	A	3	3	1.00	11	0.273
67	A	3	3	1.00	17	0.176
68	A	1	1	1.00	12	0.083
69	A	1	1	1.00	24	0.042
70	A	1	1	1.00	16	0.062
71	A	2	2	1.00	6	0.333
72	A	1	1	1.00	6	0.167
73	A	5	4	1.06	12	0.333
74	A	2	1	1.00	4	0.250
75	A	4	3	1.00	9	0.333
76	A	3	2	1.00	4	0.500
77	A	1	1	1.00	8	0.125
78	A	1	1	1.00	10	0.100
79	A	1	1	1.00	6	0.167
80	A	1	1	1.00	3	0.333
81	A	3	4	1.00	8	0.500
82	A	3	3	1.00	6	0.500
83	A	4	4	1.00	6	0.667
84	A	3	3	1.00	4	0.750
85	A	4	4	1.00	13	0.308
86	A	6	6	1.50	12	0.500
87	A	2	1	1.00	11	0.091
88	A	2	1	1.00	16	0.062
89	A	1	1	1.00	15	0.067
90	A	2	1	1.00	11	0.091
91	A	3	2	1.00	13	0.154
92	A	3	2	1.00	12	0.167
93	A	4	4	1.00	16	0.250
94	A	5	5	1.00	14	0.357

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	7	6	1.00	29	0.207
96	A	3	2	1.00	19	0.105
97	A	2	1	1.00	39	0.026
98	A	12	5	1.65	21	0.238
99	A	3	2	1.00	20	0.100
100	A	2	1	1.00	21	0.048
101	A	2	1	1.00	11	0.091
102	A	2	1	1.00	9	0.111
103	A	2	1	1.00	25	0.040
104	A	2	1	1.00	24	0.042
105	A	3	2	1.00	19	0.105
106	A	2	1	1.00	19	0.053
107	A	5	4	1.00	19	0.210
108	A	5	4	1.00	16	0.250
109	A	3	3	1.00	14	0.214
110	A	6	5	1.00	33	0.152
111	A	5	4	1.00	21	0.190
112	A	5	4	1.00	21	0.190
113	A	9	5	1.00	10	0.500
114	A	10	9	1.00	17	0.529
115	A	6	5	1.00	26	0.192
116	A	6	2	1.00	29	0.069
117	A	3	2	1.00	30	0.067
118	A	6	6	1.00	9	0.667
119	A	6	6	1.00	11	0.546
120	A	1	1	1.00	13	0.077
121	A	4	4	1.00	13	0.308
122	A	7	7	1.00	13	0.538
123	A	7	7	1.00	13	0.538
124	A	3	2	1.00	13	0.154
125	A	8	7	1.00	13	0.538
126	A	1	1	1.00	15	0.067
127	A	3	3	1.00	11	0.273
128	A	2	2	1.00	13	0.154
129	A	4	4	1.00	15	0.267

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	4	4	1.00	15	0.267
131	A	3	3	1.00	15	0.200
132	A	4	4	1.00	15	0.267
133	A	1	1	1.00	17	0.059
134	A	2	2	1.00	11	0.182
135	A	9	6	1.00	13	0.462
136	A	6	6	1.00	9	0.667
137	A	6	6	1.00	11	0.546
138	A	6	6	1.00	13	0.462
139	A	6	6	1.00	13	0.462
140	A	1	1	1.00	13	0.077
141	A	4	4	1.00	13	0.308
142	A	7	7	1.00	13	0.538
143	A	7	7	1.00	13	0.538
144	A	7	7	1.00	13	0.538
145	A	1	1	1.00	15	0.067
146	A	22	8	1.00	13	0.615
147	A	4	3	1.00	10	0.300
148	A	4	4	1.00	16	0.250
149	A	3	3	1.00	19	0.158
150	A	5	5	1.00	26	0.192
151	A	7	7	1.00	7	1.000
152	A	4	4	1.00	18	0.222
153	A	7	7	1.00	9	0.778
154	A	5	4	1.54	18	0.222
155	A	5	5	1.00	18	0.278
156	A	5	4	1.00	16	0.250
157	A	4	3	1.00	16	0.188
158	A	2	1	1.00	16	0.062
159	A	2	1	1.00	9	0.111
160	A	3	2	1.00	15	0.133
161	A	3	2	1.00	11	0.182
162	A	2	1	1.00	11	0.091
163	A	5	3	1.00	10	0.300
164	A	4	3	1.00	10	0.300

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	2	1	1.00	11	0.091
166	A	4	3	1.17	11	0.273
167	A	5	3	1.00	11	0.273
168	A	3	3	1.00	18	0.167
169	A	3	3	1.00	18	0.167
170	A	4	4	1.00	11	0.364
171	A	3	3	1.22	21	0.143
172	A	1	1	1.00	13	0.077
173	A	3	2	1.00	13	0.154
174	A	12	8	1.00	13	0.615
175	A	5	4	1.00	13	0.308
176	A	3	2	1.00	13	0.154
177	A	8	8	1.00	13	0.615
178	A	5	4	1.00	16	0.250
179	A	4	4	1.00	18	0.222
180	A	2	1	1.00	44	0.023
181	A	7	6	1.00	16	0.375
182	A	7	6	1.00	22	0.273
183	A	2	1	1.00	15	0.067
184	A	3	2	1.00	13	0.154
185	A	3	2	1.00	13	0.154
186	A	2	1	1.14	11	0.091
187	A	2	1	1.00	11	0.091
188	A	2	1	1.00	9	0.111
189	A	2	1	1.00	17	0.059
190	A	2	1	1.00	19	0.053
191	A	2	1	1.00	19	0.053
192	A	2	1	1.00	19	0.053
193	A	2	2	1.00	19	0.105
194	A	4	4	1.00	19	0.210
195	A	3	3	1.00	19	0.158
196	A	4	4	1.00	19	0.210
197	A	5	4	1.00	19	0.210
198	A	2	2	1.00	21	0.095
199	A	3	2	1.00	14	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	4	4	1.00	18	0.222
201	A	4	3	1.00	14	0.214
202	A	7	3	1.00	10	0.300
203	A	7	6	1.00	16	0.375
204	A	8	5	1.00	14	0.357
205	A	7	5	1.00	20	0.250
206	A	3	2	1.00	14	0.143
207	A	6	4	1.00	13	0.308
208	A	9	4	1.00	24	0.167
209	A	5	3	1.00	33	0.091
210	A	4	3	1.00	11	0.273
211	A	2	1	1.00	13	0.077
212	A	4	3	1.00	21	0.143
213	A	5	4	1.00	19	0.210
214	A	3	2	1.00	17	0.118
215	A	5	4	1.00	11	0.364
216	A	9	4	1.00	13	0.308
217	A	8	5	1.00	11	0.454
218	A	3	3	1.00	15	0.200
219	A	4	4	1.03	19	0.210
220	A	6	6	1.00	27	0.222
221	A	33	16	1.05	52	0.308
222	A	46	21	1.79	56	0.375
223	A	2	2	1.16	15	0.133
224	A	2	2	1.20	15	0.133
225	A	3	3	1.19	15	0.200
226	B	3	3	2.81	13	0.231
227	A	9	9	1.09	19	0.474
228	B	6	6	2.69	17	0.353
229	A	2	2	1.00	12	0.167
230	A	4	4	1.00	17	0.235
231	A	7	6	1.00	17	0.353
232	B	3	3	2.51	17	0.176
233	A	3	3	1.00	17	0.176
234	A	5	4	1.00	17	0.235

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	2	2	1.00	14	0.143
236	A	2	2	1.00	14	0.143
237	A	2	2	1.00	14	0.143
238	A	2	2	1.00	19	0.105
239	A	2	2	1.00	19	0.105
240	A	3	3	1.00	22	0.136
241	A	3	3	1.00	22	0.136
242	A	5	4	1.00	17	0.235
243	A	5	5	1.00	21	0.238
244	A	9	7	1.00	20	0.350
245	A	5	5	1.00	24	0.208
246	A	5	4	1.00	21	0.190
247	A	5	4	1.00	30	0.133
248	A	5	4	1.11	32	0.125
249	A	2	2	1.00	30	0.067
250	A	4	3	1.00	15	0.200
251	A	3	2	1.00	13	0.154
252	A	3	2	1.00	11	0.182
253	A	3	2	1.00	20	0.100
254	A	2	2	1.00	18	0.111
255	A	4	4	1.00	17	0.235
256	A	3	3	1.00	17	0.176
257	A	5	5	1.00	20	0.250
258	A	4	4	1.00	24	0.167
259	A	15	9	1.00	33	0.273
260	A	32	14	1.00	44	0.318
261	A	4	4	1.00	16	0.250
262	A	5	5	1.00	18	0.278
263	A	6	5	1.00	18	0.278
264	A	5	5	1.00	19	0.263
265	A	4	4	1.00	24	0.167
266	A	2	2	1.00	10	0.200
267	A	4	4	1.00	14	0.286
268	A	1	1	1.00	10	0.100
269	A	1	1	1.00	12	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	4	4	1.00	14	0.286
271	A	5	5	1.00	14	0.357
272	A	4	3	1.00	10	0.300
273	A	5	3	1.00	10	0.300
274	A	3	3	1.00	14	0.214
275	A	4	4	1.00	14	0.286
276	A	4	4	1.00	14	0.286
277	A	5	5	1.00	14	0.357
278	A	2	2	1.00	16	0.125
279	A	10	7	1.00	22	0.318
280	A	6	6	1.00	18	0.333
281	A	6	6	1.00	28	0.214
282	A	4	4	1.00	34	0.118
283	A	14	10	1.00	24	0.417
284	A	3	2	1.00	12	0.167
285	A	2	2	1.00	14	0.143
286	A	2	2	1.00	14	0.143
287	A	5	4	1.00	16	0.250
288	A	3	2	1.31	14	0.143
289	A	7	6	1.00	23	0.261
290	A	26	8	1.00	29	0.276
291	A	36	8	1.00	31	0.258
292	A	4	3	1.00	11	0.273
293	A	1	1	1.00	15	0.067
294	A	6	6	1.00	13	0.462
295	A	2	1	1.00	13	0.077
296	A	6	6	1.00	17	0.353
297	A	3	2	1.00	15	0.133
298	A	13	9	1.00	17	0.529
299	A	1	1	1.00	13	0.077
300	A	1	1	1.00	13	0.077
301	A	5	5	1.00	15	0.333
302	A	6	6	1.00	13	0.462
303	A	5	5	1.00	15	0.333
304	A	6	5	1.00	15	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	6	6	1.00	15	0.400
306	A	6	6	1.39	13	0.462
307	A	8	6	0.90	13	0.462
308	A	3	3	1.00	22	0.136
309	A	5	5	1.00	16	0.312
310	A	5	5	1.00	18	0.278
311	A	3	3	1.00	23	0.130
312	A	1	1	1.00	23	0.043
313	A	9	4	1.00	39	0.103
314	A	1	1	1.00	17	0.059
315	A	10	7	1.00	17	0.412
316	A	2	2	1.00	15	0.133
317	A	5	5	1.00	17	0.294
318	A	3	2	1.23	17	0.118
319	A	3	3	1.00	32	0.094
320	A	2	2	1.00	24	0.083
321	A	2	2	1.00	24	0.083
322	A	6	6	1.44	18	0.333
323	A	6	6	1.44	18	0.333
324	A	2	2	1.00	27	0.074
325	A	2	2	1.00	27	0.074
326	A	1	1	1.00	21	0.048
327	A	1	1	1.00	44	0.023
328	A	2	2	1.00	27	0.074
329	A	2	2	1.00	31	0.065
330	A	2	2	1.00	4	0.500
331	A	2	1	1.00	4	0.250
332	A	3	2	1.00	4	0.500
333	A	4	2	1.00	4	0.500
334	A	5	2	1.00	4	0.500
335	B	3	2	3.20	14	0.143
336	A	2	1	1.00	14	0.071
337	A	2	1	1.00	4	0.250
338	A	4	2	1.00	4	0.500
339	A	2	1	1.00	4	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	2	2	1.00	12	0.167
341	A	4	2	1.00	4	0.500
342	A	3	2	1.00	4	0.500
343	A	3	2	1.00	14	0.143
344	A	6	3	1.00	9	0.333
345	A	3	2	1.00	9	0.222
346	A	4	3	1.00	7	0.429
347	A	3	2	1.00	9	0.222
348	A	3	3	1.00	9	0.333
349	A	5	3	1.00	9	0.333
350	A	7	3	1.00	9	0.333
351	A	9	3	1.00	9	0.333
352	A	1	1	1.00	13	0.077
353	A	3	2	1.00	23	0.087
354	A	2	2	1.00	9	0.222
355	A	2	1	1.00	7	0.143
356	A	2	2	1.00	7	0.286
357	A	3	3	1.00	9	0.333
358	A	3	2	1.00	9	0.222
359	A	3	2	1.00	11	0.182
360	A	3	2	1.00	11	0.182
361	A	4	3	1.00	9	0.333
362	A	3	3	1.00	29	0.103
363	A	8	5	0.95	22	0.227
364	A	15	7	1.00	17	0.412
365	A	4	3	1.00	17	0.176
366	A	4	2	1.00	9	0.222
367	A	4	3	1.00	7	0.429
368	A	1	1	1.00	7	0.143
369	A	4	4	1.00	9	0.444
370	A	6	2	1.00	9	0.222
371	A	4	3	1.00	9	0.333
372	A	3	1	1.00	9	0.111
373	A	7	6	1.02	11	0.546
374	A	5	5	1.55	9	0.556

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	5	4	1.00	16	0.250
376	A	3	3	1.57	14	0.214
377	B	3	3	2.85	12	0.250
378	A	2	1	1.00	16	0.062
379	A	6	4	1.00	10	0.400
380	A	4	3	1.00	9	0.333
381	A	3	2	1.00	9	0.222
382	A	4	4	1.00	21	0.190
383	A	2	1	1.00	9	0.111
384	A	2	2	1.00	7	0.286
385	A	4	3	1.00	9	0.333
386	A	4	3	1.00	9	0.333
387	A	5	5	1.00	7	0.714
388	A	4	2	1.00	7	0.286
389	A	4	2	1.00	9	0.222
390	A	1	1	1.00	10	0.100
391	A	1	1	1.00	12	0.083
392	A	2	2	1.00	10	0.200
393	A	2	2	1.00	12	0.167
394	A	3	3	1.00	12	0.250
395	A	3	2	1.00	14	0.143
396	A	4	2	1.00	32	0.062
397	A	11	8	1.00	6	1.333
398	A	9	9	1.21	8	1.125
399	A	6	5	1.00	12	0.417
400	A	4	2	1.00	32	0.062
401	A	19	13	1.58	13	1.000
402	A	1	1	1.00	11	0.091
403	A	1	1	1.00	11	0.091
404	A	2	2	1.00	11	0.182
405	A	6	5	1.00	16	0.312
406	A	4	3	1.00	13	0.231
407	A	4	3	1.00	13	0.231
408	A	1	1	1.00	13	0.077
409	A	2	2	1.00	13	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
410	A	3	3	1.00	11	0.273
411	A	6	4	1.40	35	0.114
412	A	5	4	1.53	11	0.364
413	B	13	9	2.22	11	0.818
414	A	5	3	1.00	13	0.231
415	A	4	2	1.00	13	0.154
416	B	27	11	2.17	27	0.407
417	A	66	21	1.83	41	0.512
418	A	13	4	1.13	28	0.143
419	A	5	3	0.92	15	0.200
420	A	5	3	1.00	18	0.167
421	A	5	4	1.00	20	0.200
422	A	4	3	1.00	20	0.150
423	A	3	3	1.00	19	0.158
424	A	4	3	1.00	22	0.136
425	A	4	3	1.00	23	0.130
426	A	18	13	1.07	33	0.394
427	A	27	6	1.00	39	0.154
428	A	5	3	1.00	17	0.176
429	A	3	3	1.00	11	0.273
430	A	5	4	1.00	11	0.364
431	A	1	1	1.00	11	0.091
432	A	6	6	1.00	13	0.462
433	A	11	9	1.05	28	0.321
434	A	7	6	1.00	12	0.500
435	A	4	4	1.00	12	0.333
436	A	10	10	1.00	22	0.454
437	A	5	4	1.00	23	0.174
438	A	16	12	1.00	31	0.387
439	A	10	7	1.00	48	0.146
440	A	7	5	1.00	15	0.333
441	A	6	5	1.00	15	0.333
442	A	6	6	1.00	19	0.316
443	A	7	7	1.00	15	0.467
444	A	6	6	1.00	19	0.316

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
445	A	6	6	1.00	20	0.300
446	A	29	16	1.31	61	0.262
447	B	22	11	2.08	29	0.379
448	A	7	7	1.00	20	0.350
449	A	14	10	1.71	15	0.667
450	A	4	3	1.00	19	0.158
451	A	2	2	1.00	33	0.061
452	A	15	9	1.30	31	0.290
453	A	14	10	1.25	52	0.192
454	A	4	3	1.00	15	0.200
455	A	14	10	1.51	15	0.667
456	A	3	3	1.06	11	0.273
457	A	6	5	1.00	11	0.454
458	A	3	2	1.00	11	0.182
459	A	4	2	1.00	11	0.182
460	A	3	2	1.00	11	0.182
461	A	5	4	1.00	13	0.308
462	A	6	4	1.00	13	0.308
463	A	1	1	1.00	7	0.143
464	A	3	2	1.00	11	0.182
465	A	4	3	1.00	19	0.158
466	A	3	2	1.00	13	0.154
467	A	3	2	1.00	13	0.154
468	A	2	2	1.00	11	0.182
469	A	2	2	1.00	12	0.167
470	A	3	2	1.00	13	0.154
471	A	2	1	1.00	13	0.077
472	A	3	2	1.00	11	0.182
473	A	3	3	1.00	23	0.130
474	A	2	2	1.00	19	0.105
475	A	2	2	1.00	15	0.133
476	A	3	2	1.00	15	0.133
477	A	3	2	1.00	11	0.182
478	A	2	2	1.00	14	0.143
479	A	2	1	1.00	12	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
480	A	2	1	1.00	16	0.062
481	A	2	2	1.00	20	0.100
482	A	2	2	1.00	25	0.080
483	A	9	4	1.00	8	0.500
484	A	8	4	1.00	8	0.500
485	A	13	4	1.00	8	0.500
486	A	4	4	1.00	10	0.400
487	A	6	5	1.00	10	0.500
488	A	5	4	1.00	8	0.500
489	A	3	3	1.00	8	0.375
490	A	8	8	1.00	8	1.000
491	A	7	7	1.00	6	1.167
492	A	2	2	1.00	18	0.111
493	A	3	3	1.00	15	0.200
494	A	2	2	1.00	11	0.182
495	A	9	4	1.21	22	0.182
496	A	3	1	1.00	11	0.091
497	A	4	3	1.00	13	0.231
498	A	3	2	1.00	13	0.154
499	A	4	3	1.00	13	0.231
500	A	4	4	1.08	13	0.308
501	A	4	3	1.00	15	0.200
502	A	3	1	1.00	11	0.091
503	A	6	2	1.00	13	0.154
504	A	7	2	1.00	13	0.154
505	A	8	2	1.00	13	0.154
506	A	2	2	1.11	13	0.154
507	A	3	1	1.00	13	0.077
508	A	6	2	1.00	15	0.133
509	A	7	2	1.00	15	0.133
510	A	8	2	1.00	15	0.133
511	A	2	2	1.11	15	0.133
512	A	2	1	1.00	7	0.143
513	A	3	2	1.00	9	0.222
514	A	3	2	1.00	9	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
515	A	3	2	1.00	9	0.222
516	A	2	2	1.00	9	0.222
517	A	2	1	1.00	9	0.111
518	A	3	2	1.00	11	0.182
519	A	3	2	1.00	11	0.182
520	A	3	2	1.00	11	0.182
521	A	2	2	1.00	11	0.182
522	A	4	4	1.00	11	0.364
523	A	7	7	1.23	15	0.467
524	A	3	2	1.00	13	0.154
525	A	5	5	1.00	24	0.208
526	A	6	5	1.00	29	0.172
527	A	2	2	1.00	21	0.095
528	A	6	6	1.00	15	0.400
529	A	2	2	1.00	15	0.133
530	A	3	3	1.00	17	0.176
531	A	3	3	1.00	19	0.158
532	A	3	3	1.00	39	0.077
533	A	4	4	1.54	17	0.235
534	A	3	2	1.00	21	0.095
535	A	4	2	1.00	11	0.182
536	A	3	2	1.00	11	0.182
537	A	3	2	1.00	9	0.222
538	A	5	3	1.00	12	0.250
539	A	6	6	1.00	13	0.462
540	A	1	1	1.00	25	0.040
541	A	1	1	1.00	10	0.100
542	A	6	4	1.00	21	0.190
543	A	2	2	1.00	16	0.125
544	A	2	2	1.00	10	0.200
545	A	2	2	1.00	10	0.200
546	A	3	3	1.00	16	0.188
547	A	4	3	1.00	14	0.214
548	A	4	3	1.00	22	0.136
549	A	5	3	1.47	10	0.300

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
550	A	1	1	1.00	10	0.100
551	A	2	2	1.51	10	0.200
552	A	2	2	1.00	10	0.200
553	A	2	2	1.00	12	0.167
554	A	2	2	1.00	10	0.200
555	A	2	2	1.00	12	0.167
556	A	1	1	1.00	18	0.056
557	A	7	6	1.10	16	0.375
558	A	1	1	1.00	14	0.071
559	A	7	6	1.05	16	0.375
560	A	7	6	1.04	18	0.333
561	A	1	1	1.00	16	0.062
562	A	7	6	1.09	14	0.429
563	A	1	1	1.00	16	0.062
564	A	4	3	1.00	7	0.429
565	A	11	5	1.00	9	0.556
566	A	11	5	1.00	11	0.454
567	A	31	8	1.35	15	0.533
568	A	11	5	1.00	13	0.385
569	A	24	6	1.00	17	0.353
570	A	1	1	1.00	2	0.500
571	A	1	1	1.00	2	0.500
572	A	1	1	1.00	2	0.500
573	A	1	1	1.00	2	0.500
574	A	1	1	1.00	2	0.500
575	A	1	1	1.00	2	0.500
576	A	2	2	1.00	4	0.500
577	A	2	1	1.00	4	0.250
578	A	3	2	1.00	4	0.500
579	A	2	2	1.00	4	0.500
580	A	3	2	1.00	4	0.500
581	A	4	3	1.00	7	0.429
582	A	3	2	1.00	11	0.182
583	A	2	2	1.02	8	0.250
584	A	2	2	1.00	6	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
585	A	2	2	1.00	8	0.250
586	A	2	2	1.00	14	0.143
587	A	2	2	1.00	15	0.133
588	A	3	3	1.00	10	0.300
589	A	5	3	1.00	23	0.130
590	A	5	2	1.00	11	0.182
591	A	5	2	1.00	15	0.133
592	A	8	4	1.48	31	0.129
593	A	3	3	1.00	15	0.200
594	A	5	3	1.00	21	0.143
595	A	2	2	1.00	11	0.182
596	A	3	3	1.00	6	0.500
597	A	3	3	1.00	6	0.500
598	A	13	7	1.00	16	0.438
599	A	8	4	1.00	13	0.308
600	A	3	3	1.00	10	0.300
601	A	3	3	1.00	10	0.300
602	A	2	2	1.00	13	0.154
603	A	3	3	1.46	13	0.231
604	A	2	2	1.00	11	0.182
605	A	4	3	1.00	12	0.250
606	A	3	2	1.00	14	0.143
607	A	3	2	1.00	18	0.111
608	A	1	1	1.00	6	0.167
609	A	2	2	1.00	8	0.250
610	A	2	2	1.00	10	0.200
611	A	2	1	1.00	8	0.125
612	A	4	4	1.00	10	0.400
613	A	6	2	1.00	14	0.143
614	A	13	8	1.00	16	0.500
615	A	5	3	1.00	8	0.375
616	A	2	2	1.00	10	0.200
617	A	2	2	1.00	10	0.200
618	A	2	2	1.00	8	0.250
619	A	2	2	1.00	12	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
620	A	2	2	1.00	12	0.167
621	A	2	2	1.00	14	0.143
622	A	3	2	1.00	16	0.125
623	A	3	2	1.00	18	0.111
624	A	3	2	1.00	18	0.111
625	A	4	3	1.00	20	0.150
626	A	4	3	1.00	22	0.136
627	A	4	3	1.00	22	0.136
628	A	1	1	1.00	7	0.143
629	A	3	2	1.00	9	0.222
630	A	4	2	1.00	9	0.222
631	A	5	2	1.00	9	0.222
632	A	3	2	1.00	9	0.222
633	A	3	3	1.00	8	0.375
634	A	3	1	1.00	8	0.125
635	A	2	2	1.00	6	0.333
636	A	2	3	1.00	6	0.500
637	A	2	2	1.00	14	0.143
638	A	3	2	1.00	8	0.250
639	A	8	6	1.00	20	0.300
640	A	6	6	1.00	14	0.429
641	A	4	4	1.00	8	0.500
642	A	4	3	1.00	8	0.375
643	A	4	5	1.00	14	0.357
644	A	6	7	1.00	12	0.583
645	A	5	5	1.00	8	0.625
646	A	5	5	1.00	8	0.625
647	A	10	7	1.00	8	0.875
648	A	13	8	1.00	8	1.000
649	A	5	5	1.00	8	0.625
650	A	10	5	1.00	8	0.625
651	A	3	3	1.00	14	0.214
652	A	3	3	1.00	14	0.214
653	A	2	1	1.00	15	0.067
654	A	6	5	1.00	14	0.357

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
655	A	3	2	1.00	15	0.133
656	A	4	5	1.00	17	0.294
657	A	10	7	1.00	17	0.412
658	A	4	3	1.00	17	0.176
659	A	3	3	1.00	17	0.176
660	A	5	3	1.00	17	0.176
661	A	2	2	1.00	15	0.133
662	A	2	2	1.00	15	0.133
663	A	4	4	1.00	14	0.286
664	A	3	5	1.00	17	0.294
665	A	8	6	1.00	17	0.353
666	A	4	4	1.00	17	0.235
667	A	5	4	1.00	17	0.235
668	A	6	4	1.00	19	0.210
669	A	3	3	1.00	11	0.273
670	A	4	3	1.00	11	0.273
671	A	4	4	1.00	13	0.308
672	A	8	8	1.00	13	0.615
673	A	2	2	1.00	13	0.154
674	A	8	8	1.00	13	0.615
675	A	17	11	1.00	13	0.846
676	A	8	8	1.00	11	0.727
677	A	3	2	1.00	11	0.182
678	A	12	6	1.00	13	0.462
679	A	7	7	1.00	13	0.538
680	A	8	7	1.00	8	0.875
681	A	11	8	1.00	13	0.615
682	A	4	4	1.04	15	0.267
683	A	9	7	1.08	15	0.467
684	A	11	8	1.25	15	0.533
685	A	4	4	1.00	15	0.267
686	A	4	6	1.03	12	0.500
687	A	4	5	1.04	15	0.333
688	A	5	6	1.02	15	0.400
689	A	16	11	1.33	15	0.733

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
690	A	2	3	1.00	15	0.200
691	A	5	8	1.30	15	0.533
692	A	6	4	1.36	17	0.235
693	A	11	9	1.29	17	0.529
694	A	8	5	1.33	17	0.294
695	B	8	7	2.27	16	0.438
696	A	4	4	1.00	16	0.250
697	A	5	4	1.00	8	0.500
698	A	5	5	1.00	13	0.385
699	A	2	2	1.00	24	0.083
700	A	2	3	1.00	21	0.143
701	A	5	5	1.00	12	0.417
702	A	7	6	1.71	10	0.600
703	A	5	6	1.00	8	0.750
704	A	6	6	1.00	10	0.600
705	A	5	6	1.00	7	0.857

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{1}{a^2-b^2x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{a^2-b^2x^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{ab}$$

```
[Out] arctanh(b*x/a)/a/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {214}

$$\int \frac{1}{a^2-b^2x^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{ab}$$

```
[In] Int[(a^2 - b^2*x^2)^(-1),x]
```

```
[Out] ArcTanh[(b*x)/a]/(a*b)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\text{integral}=\frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{ab}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2-b^2x^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{ab}$$

```
[In] Integrate[(a^2 - b^2*x^2)^(-1),x]
[Out] ArcTanh[(b*x)/a]/(a*b)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

method	result	size
parallelrisch	$-\frac{\ln(bx-a)-\ln(bx+a)}{2ab}$	26
default	$-\frac{\ln(-bx+a)}{2ba}+\frac{\ln(bx+a)}{2ab}$	31
norman	$-\frac{\ln(-bx+a)}{2ba}+\frac{\ln(bx+a)}{2ab}$	31
risch	$-\frac{\ln(-bx+a)}{2ba}+\frac{\ln(bx+a)}{2ab}$	31

```
[In] int(1/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
[Out] -1/2*(ln(b*x-a)-ln(b*x+a))/a/b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{a^2-b^2x^2}\,dx=\frac{\log(bx+a)-\log(bx-a)}{2\,ab}$$

```
[In] integrate(1/(-b^2*x^2+a^2),x, algorithm="fricas")
[Out] 1/2*(log(b*x + a) - log(b*x - a))/(a*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{a^2 - b^2x^2}\,dx = -\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{2} - \frac{\log\left(\frac{a}{b}+x\right)}{2}}{ab}$$

```
[In] integrate(1/(-b**2*x**2+a**2),x)
[Out] -(log(-a/b + x)/2 - log(a/b + x)/2)/(a*b)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{1}{a^2 - b^2x^2}\,dx = \frac{\log\left(bx+a\right)}{2\,ab} - \frac{\log\left(bx-a\right)}{2\,ab}$$

```
[In] integrate(1/(-b^2*x^2+a^2),x, algorithm="maxima")
[Out] 1/2*log(b*x + a)/(a*b) - 1/2*log(b*x - a)/(a*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{1}{a^2 - b^2x^2}\,dx = \frac{\log\left(|bx+a|\right)}{2\,ab} - \frac{\log\left(|bx-a|\right)}{2\,ab}$$

```
[In] integrate(1/(-b^2*x^2+a^2),x, algorithm="giac")
[Out] 1/2*log(abs(b*x + a))/(a*b) - 1/2*log(abs(b*x - a))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 - b^2x^2}\,dx = \frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{a\,b}$$

```
[In] int(1/(a^2 - b^2*x^2),x)
```

```
[Out] atanh((b*x)/a)/(a*b)
```

3.2 $\int \frac{1}{a^2+b^2x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{a^2+b^2x^2}\,dx = \frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

```
[Out] arctan(b*x/a)/a/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{211\}$

$$\int \frac{1}{a^2+b^2x^2}\,dx = \frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

```
[In] Int[(a^2 + b^2*x^2)^(-1),x]
```

```
[Out] ArcTan[(b*x)/a]/(a*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\text{integral} = \frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + b^2x^2} \, dx = \frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

```
[In] Integrate[(a^2 + b^2*x^2)^(-1),x]
[Out] ArcTan[(b*x)/a]/(a*b)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$	15
risch	$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$	15
parallelrisch	$-\frac{i\ln(-ia+bx)-i\ln(ia+bx)}{2ab}$	34

```
[In] int(1/(b^2*x^2+a^2),x,method=_RETURNVERBOSE)
[Out] arctan(b*x/a)/a/b
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + b^2x^2} \, dx = \frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

```
[In] integrate(1/(b^2*x^2+a^2),x, algorithm="fricas")
[Out] arctan(b*x/a)/(a*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{1}{a^2 + b^2x^2} \, dx = \frac{-\frac{i \log\left(-\frac{ia}{b} + x\right)}{2} + \frac{i \log\left(\frac{ia}{b} + x\right)}{2}}{ab}$$

```
[In] integrate(1/(b**2*x**2+a**2),x)
[Out] (-I*log(-I*a/b + x)/2 + I*log(I*a/b + x)/2)/(a*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + b^2x^2} \, dx = \frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

```
[In] integrate(1/(b^2*x^2+a^2),x, algorithm="maxima")
[Out] arctan(b*x/a)/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + b^2x^2} \, dx = \frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

```
[In] integrate(1/(b^2*x^2+a^2),x, algorithm="giac")
[Out] arctan(b*x/a)/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + b^2x^2}\,dx = \frac{\operatorname{atan}\left(\frac{b\,x}{a}\right)}{a\,b}$$

```
[In] int(1/(a^2 + b^2*x^2),x)
[Out] atan((b*x)/a)/(a*b)
```

3.3 $\int \sec(2ax)\,dx$

Optimal result

Integrand size = 5, antiderivative size = 13

$$\int \sec(2ax)\,dx = \frac{\operatorname{arctanh}(\sin(2ax))}{2a}$$

```
[Out] 1/2*arctanh(sin(2*a*x))/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3855}

$$\int \sec(2ax)\,dx = \frac{\operatorname{arctanh}(\sin(2ax))}{2a}$$

```
[In] Int[Sec[2*a*x],x]
```

```
[Out] ArcTanh[Sin[2*a*x]]/(2*a)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\operatorname{arctanh}(\sin(2ax))}{2a}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sec(2ax)\,dx = \frac{\operatorname{arctanh}(\sin(2ax))}{2a}$$

```
[In] Integrate[Sec[2*a*x],x]
[Out] ArcTanh[Sin[2*a*x]]/(2*a)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{\ln(\sec(2ax)+\tan(2ax))}{2a}$	18
default	$\frac{\ln(\sec(2ax)+\tan(2ax))}{2a}$	18
parallelrisch	$\frac{-\ln(\tan(ax)-1)+\ln(\tan(ax)+1)}{2a}$	23
norman	$-\frac{\ln(\tan(ax)-1)}{2a}+\frac{\ln(\tan(ax)+1)}{2a}$	26
risch	$\frac{\ln(\mathrm{e}^{2iax}+i)}{2a}-\frac{\ln(\mathrm{e}^{2iax}-i)}{2a}$	32

```
[In] int(sec(2*a*x),x,method=_RETURNVERBOSE)
[Out] 1/2/a*ln(sec(2*a*x)+tan(2*a*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(11) = 22.

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \sec(2ax)\,dx = \frac{\log\left(\sin\left(2\,ax\right)+1\right)-\log\left(-\sin\left(2\,ax\right)+1\right)}{4\,a}$$

```
[In] integrate(sec(2*a*x),x, algorithm="fricas")
[Out] 1/4*(log(sin(2*a*x) + 1) - log(-sin(2*a*x) + 1))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \sec(2ax)\,dx = \begin{cases} \frac{-\frac{\log(\sin(2ax)-1)}{2}+\frac{\log(\sin(2ax)+1)}{2}}{2a} & \text{for } a \neq 0 \\ x & \text{otherwise} \end{cases}$$

```
[In] integrate(sec(2*a*x),x)
[Out] Piecewise(((-log(sin(2*a*x) - 1)/2 + log(sin(2*a*x) + 1)/2)/(2*a), Ne(a, 0)
), (x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \sec(2ax)\,dx = \frac{\log(\sec(2\,ax)+\tan(2\,ax))}{2\,a}$$

```
[In] integrate(sec(2*a*x),x, algorithm="maxima")
[Out] 1/2*log(sec(2*a*x) + tan(2*a*x))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(11) = 22$.

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.08

$$\int \sec(2ax)\,dx = \frac{\log\left(\left|\frac{1}{\sin(2\,ax)}+\sin(2\,ax)+2\right|\right)-\log\left(\left|\frac{1}{\sin(2\,ax)}+\sin(2\,ax)-2\right|\right)}{8\,a}$$

```
[In] integrate(sec(2*a*x),x, algorithm="giac")
[Out] 1/8*(log(abs(1/sin(2*a*x) + sin(2*a*x) + 2)) - log(abs(1/sin(2*a*x) + sin(2
*a*x) - 2)))/a
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sec(2ax)\,dx = \frac{\operatorname{atanh}(\sin(2\,a\,x))}{2\,a}$$

[In] `int(1/cos(2*a*x),x)`

[Out] `atanh(sin(2*a*x))/(2*a)`

3.4 $\int \frac{1}{4} \csc\left(\frac{x}{3}\right) dx$

Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \frac{1}{4} \csc\left(\frac{x}{3}\right) dx = -\frac{3}{4} \operatorname{arctanh}\left(\cos\left(\frac{x}{3}\right)\right)$$

```
[Out] -3/4*arctanh(cos(1/3*x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {12, 3855}

$$\int \frac{1}{4} \csc\left(\frac{x}{3}\right) dx = -\frac{3}{4} \operatorname{arctanh}\left(\cos\left(\frac{x}{3}\right)\right)$$

```
[In] Int[Csc[x/3]/4,x]
```

```
[Out] (-3*ArcTanh[Cos[x/3]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\int \csc\left(\frac{x}{3}\right)\,dx\\ &= -\frac{3}{4}\text{arctanh}\left(\cos\left(\frac{x}{3}\right)\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = \frac{1}{4}\left(-3\log\left(\cos\left(\frac{x}{6}\right)\right)+3\log\left(\sin\left(\frac{x}{6}\right)\right)\right)$$

```
[In] Integrate[Csc[x/3]/4,x]
[Out] (-3*Log[Cos[x/6]] + 3*Log[Sin[x/6]])/4
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
norman	$\frac{3\ln\left(\tan\left(\frac{x}{6}\right)\right)}{4}$	8
parallelrisch	$\frac{3\ln\left(\tan\left(\frac{x}{6}\right)\right)}{4}$	8
derivativedivides	$\frac{3\ln\left(\csc\left(\frac{x}{3}\right)-\cot\left(\frac{x}{3}\right)\right)}{4}$	15
default	$\frac{3\ln\left(\csc\left(\frac{x}{3}\right)-\cot\left(\frac{x}{3}\right)\right)}{4}$	15
risch	$\frac{3\ln\left({e}^{\frac{ix}{3}}-1\right)}{4}-\frac{3\ln\left({e}^{\frac{ix}{3}}+1\right)}{4}$	22

```
[In] int(1/4/sin(1/3*x),x,method=_RETURNVERBOSE)
[Out] 3/4*ln(tan(1/6*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(7) = 14$.

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = -\frac{3}{8}\log\left(\frac{1}{2}\cos\left(\frac{1}{3}x\right)+\frac{1}{2}\right)+\frac{3}{8}\log\left(-\frac{1}{2}\cos\left(\frac{1}{3}x\right)+\frac{1}{2}\right)$$

```
[In] integrate(1/4/sin(1/3*x),x, algorithm="fricas")
[Out] -3/8*log(1/2*cos(1/3*x) + 1/2) + 3/8*log(-1/2*cos(1/3*x) + 1/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = \frac{3\log\left(\cos\left(\frac{x}{3}\right)-1\right)}{8}-\frac{3\log\left(\cos\left(\frac{x}{3}\right)+1\right)}{8}$$

```
[In] integrate(1/4/sin(1/3*x),x)
[Out] 3*log(cos(x/3) - 1)/8 - 3*log(cos(x/3) + 1)/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = -\frac{3}{8}\log\left(\cos\left(\frac{1}{3}x\right)+1\right)+\frac{3}{8}\log\left(\cos\left(\frac{1}{3}x\right)-1\right)$$

```
[In] integrate(1/4/sin(1/3*x),x, algorithm="maxima")
[Out] -3/8*log(cos(1/3*x) + 1) + 3/8*log(cos(1/3*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(7) = 14$.

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = -\frac{3}{8}\log\left(\cos\left(\frac{1}{3}\,x\right)+1\right)+\frac{3}{8}\log\left(-\cos\left(\frac{1}{3}\,x\right)+1\right)$$

```
[In] integrate(1/4/sin(1/3*x),x, algorithm="giac")
[Out] -3/8*log(cos(1/3*x) + 1) + 3/8*log(-cos(1/3*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = \frac{3\,\ln\left(\tan\left(\frac{x}{6}\right)\right)}{4}$$

```
[In] int(1/(4*sin(x/3)),x)
[Out] (3*log(tan(x/6)))/4
```

3.5 $\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{1}{2}\text{arctanh}\left(\sin\left(\frac{\pi}{4}+2x\right)\right)$$

```
[Out] -1/2*arctanh(sin(1/4*Pi+2*x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3855}

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{1}{2}\text{arctanh}\left(\sin\left(2x+\frac{\pi}{4}\right)\right)$$

```
[In] Int[-Sec[Pi/4 + 2*x],x]
```

```
[Out] -1/2*ArcTanh[Sin[Pi/4 + 2*x]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{1}{2}\text{arctanh}\left(\sin\left(\frac{\pi}{4}+2x\right)\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{1}{2}\operatorname{arctanh}\left(\sin\left(\frac{\pi}{4}+2x\right)\right)$$

[In] Integrate[-Sec[Pi/4 + 2*x],x]

[Out] -1/2*ArcTanh[Sin[Pi/4 + 2*x]]

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

method	result	size
derivativedivides	$-\frac{\ln\left(\sec\left(\frac{\pi}{4}+2x\right)+\tan\left(\frac{\pi}{4}+2x\right)\right)}{2}$	21
default	$-\frac{\ln\left(\sec\left(\frac{\pi}{4}+2x\right)+\tan\left(\frac{\pi}{4}+2x\right)\right)}{2}$	21
norman	$\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)+1\right)}{2}$	24
parallelrisch	$-\ln\left(\frac{1}{\sqrt{\tan\left(\frac{\pi}{8}+x\right)-1}}\right)-\ln\left(\sqrt{\tan\left(\frac{\pi}{8}+x\right)+1}\right)$	28
risch	$-\frac{\ln\left(e^{\frac{i(\pi+8x)}{4}}+i\right)}{2}+\frac{\ln\left(e^{\frac{i(\pi+8x)}{4}}-i\right)}{2}$	32

[In] int(-1/cos(1/4*Pi+2*x),x,method=_RETURNVERBOSE)

[Out] -1/2*ln(sec(1/4*Pi+2*x)+tan(1/4*Pi+2*x))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(\frac{1}{4}\pi+2x\right)+1\right)$$

[In] integrate(-1/cos(1/4*pi+2*x),x, algorithm="fricas")

[Out] -1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(-sin(1/4*pi + 2*x) + 1)

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx=\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)-1\right)}{2}-\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)+1\right)}{2}$$

```
[In] integrate(-1/cos(1/4*pi+2*x),x)
[Out] log(tan(x + pi/8) - 1)/2 - log(tan(x + pi/8) + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx=-\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)+1\right)+\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)-1\right)$$

```
[In] integrate(-1/cos(1/4*pi+2*x),x, algorithm="maxima")
[Out] -1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(sin(1/4*pi + 2*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(11) = 22.

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx=-\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(\frac{1}{4}\pi+2x\right)+1\right)$$

```
[In] integrate(-1/cos(1/4*pi+2*x),x, algorithm="giac")
[Out] -1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(-sin(1/4*pi + 2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx=-\frac{\ln\left(\frac{\sin\left(\frac{\Pi}{4}+2x\right)+1}{\cos\left(\frac{\Pi}{4}+2x\right)}\right)}{2}$$

```
[In] int(-1/cos(Pi/4 + 2*x),x)
[Out] -log((sin(Pi/4 + 2*x) + 1)/cos(Pi/4 + 2*x))/2
```

3.6 $\int \sec(x)\tan(x)\,dx$

Optimal result

Integrand size = 5, antiderivative size = 2

$$\int \sec(x)\tan(x)\,dx = \sec(x)$$

```
[Out] sec(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2686, 8}

$$\int \sec(x)\tan(x)\,dx = \sec(x)$$

```
[In] Int[Sec[x]*Tan[x],x]
```

```
[Out] Sec[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int 1\,dx, x, \sec(x)\right)$$
$$= \sec(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec(x)\tan(x)\,dx = \sec(x)$$

```
[In] Integrate[Sec[x]*Tan[x],x]
[Out] Sec[x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
derivativedivides	$\sec(x)$	3
default	$\sec(x)$	3
risch	$\frac{2\,e^{ix}}{e^{2ix}+1}$	17

```
[In] int(sec(x)*tan(x),x,method=_RETURNVERBOSE)
[Out] sec(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

```
[In] integrate(sec(x)*tan(x),x, algorithm="fricas")
[Out] 1/cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos{(x)}}$$

```
[In] integrate(sec(x)*tan(x),x)
[Out] 1/cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos{(x)}}$$

```
[In] integrate(sec(x)*tan(x),x, algorithm="maxima")
[Out] 1/cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos{(x)}}$$

```
[In] integrate(sec(x)*tan(x),x, algorithm="giac")
[Out] 1/cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 6.00

$$\int \sec(x)\tan(x)\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)^2 - 1}$$

```
[In] int(tan(x)/cos(x),x)
[Out] -2/(tan(x/2)^2 - 1)
```

3.7 $\int \cot(x)\csc(x)\,dx$

Optimal result

Integrand size = 5, antiderivative size = 4

$$\int \cot(x)\csc(x)\,dx = -\csc(x)$$

```
[Out] -csc(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2686, 8}

$$\int \cot(x)\csc(x)\,dx = -\csc(x)$$

```
[In] Int[Cot[x]*Csc[x],x]
[Out] -Csc[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int 1\,dx, x, \csc(x)\right) \\
&= -\csc(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cot(x)\csc(x)\,dx = -\csc(x)$$

```
[In] Integrate[Cot[x]*Csc[x],x]
[Out] -Csc[x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$-\csc\left(x\right)$	5
default	$-\csc\left(x\right)$	5
risch	$-\frac{2i e^{ix}}{e^{2ix}-1}$	18

```
[In] int(csc(x)*cot(x),x,method=_RETURNVERBOSE)
[Out] -csc(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin\left(x\right)}$$

```
[In] integrate(cot(x)*csc(x),x, algorithm="fricas")
[Out] -1/sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin(x)}$$

```
[In] integrate(cot(x)*csc(x),x)
[Out] -1/sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin(x)}$$

```
[In] integrate(cot(x)*csc(x),x, algorithm="maxima")
[Out] -1/sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin(x)}$$

```
[In] integrate(cot(x)*csc(x),x, algorithm="giac")
[Out] -1/sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin(x)}$$

```
[In] int(cot(x)/sin(x),x)
[Out] -1/sin(x)
```

3.8 $\int \csc(2x)\tan(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 6

$$\int \csc(2x)\tan(x)\,dx = \frac{\tan(x)}{2}$$

```
[Out] 1/2*tan(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {8}

$$\int \csc(2x)\tan(x)\,dx = \frac{\tan(x)}{2}$$

```
[In] Int[Csc[2*x]*Tan[x],x]
```

```
[Out] Tan[x]/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{2}\,dx, x, \tan(x)\right) \\ &= \frac{\tan(x)}{2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \csc(2x)\tan(x)\,dx = \frac{\tan(x)}{2}$$

[In] Integrate[Csc[2*x]*Tan[x],x]

[Out] Tan[x]/2

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\tan(x)}{2}$	5
default	$\frac{\tan(x)}{2}$	5
norman	$\frac{\tan(x)}{2}$	5
parallelrisch	$\frac{\tan(x)}{2}$	5
risch	$\frac{i}{e^{2ix}+1}$	13

[In] int(tan(x)/sin(2*x),x,method=_RETURNVERBOSE)

[Out] 1/2*tan(x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \csc(2x)\tan(x)\,dx = \frac{1}{2}\tan\left(x\right)$$

[In] integrate(tan(x)/sin(2*x),x, algorithm="fricas")

[Out] 1/2*tan(x)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.32 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \csc(2x)\tan(x)\,dx = \frac{\sin(x)}{2\cos(x)}$$

```
[In] integrate(tan(x)/sin(2*x),x)
[Out] sin(x)/(2*cos(x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(4) = 8.

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 4.50

$$\int \csc(2x)\tan(x)\,dx = \frac{\sin(2x)}{\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1}$$

```
[In] integrate(tan(x)/sin(2*x),x, algorithm="maxima")
[Out] sin(2*x)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \csc(2x)\tan(x)\,dx = \frac{1}{2}\tan(x)$$

```
[In] integrate(tan(x)/sin(2*x),x, algorithm="giac")
[Out] 1/2*tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \csc(2x)\tan(x)\,dx = \frac{\tan(x)}{2}$$

```
[In] int(tan(x)/sin(2*x),x)
[Out] tan(x)/2
```

3.9 $\int \frac{1}{1+\cos(x)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \frac{1}{1+\cos(x)}\,dx=\frac{\sin(x)}{1+\cos(x)}$$

```
[Out] sin(x)/(1+cos(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = $\{2727\}$

$$\int \frac{1}{1+\cos(x)}\,dx=\frac{\sin(x)}{\cos(x)+1}$$

```
[In] Int[(1 + Cos[x])^(-1),x]
```

```
[Out] Sin[x]/(1 + Cos[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral}=\frac{\sin(x)}{1+\cos(x)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

```
[In] Integrate[(1 + Cos[x])^(-1),x]
[Out] Tan[x/2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

method	result	size
default	$\tan\left(\frac{x}{2}\right)$	5
norman	$\tan\left(\frac{x}{2}\right)$	5
parallelrisch	$\tan\left(\frac{x}{2}\right)$	5
risch	$\frac{2i}{e^{ix}+1}$	13

```
[In] int(1/(cos(x)+1),x,method=_RETURNVERBOSE)
[Out] tan(1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{\cos(x)+1}$$

```
[In] integrate(1/(1+cos(x)),x, algorithm="fricas")
[Out] sin(x)/(cos(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.33

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

```
[In] integrate(1/(1+cos(x)),x)
[Out] tan(x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{\cos(x)+1}$$

```
[In] integrate(1/(1+cos(x)),x, algorithm="maxima")
[Out] sin(x)/(cos(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(9) = 18.

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 3.33

$$\int \frac{1}{1+\cos(x)}\,dx = -\frac{2\tan\left(\frac{1}{2}x\right)}{(x^2+1)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

```
[In] integrate(1/(1+cos(x)),x, algorithm="giac")
[Out] -2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

```
[In] int(1/(cos(x) + 1),x)
[Out] tan(x/2)
```

3.10 $\int \frac{1}{1-\cos(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -sin(x)/(1-cos(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2727}

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\sin(x)}{1-\cos(x)}$$

```
[In] Int[(1 - Cos[x])^(-1),x]
```

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx=-\cot\left(\frac{x}{2}\right)$$

```
[In] Integrate[(1 - Cos[x])^(-1),x]
[Out] -Cot[x/2]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
norman	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
parallelrisch	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
risch	$-\frac{2i}{e^{ix}-1}$	13

```
[In] int(1/(1-cos(x)),x,method=_RETURNVERBOSE)
[Out] -1/tan(1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)}\,dx=-\frac{\cos(x)+1}{\sin(x)}$$

```
[In] integrate(1/(1-cos(x)),x, algorithm="fricas")
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{x}{2}\right)}$$

```
[In] integrate(1/(1-cos(x)),x)
[Out] -1/tan(x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\cos\left(x\right)+1}{\sin\left(x\right)}$$

```
[In] integrate(1/(1-cos(x)),x, algorithm="maxima")
[Out] -(cos(x) + 1)/sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

```
[In] integrate(1/(1-cos(x)),x, algorithm="giac")
[Out] -1/tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{1-\cos(x)}\,dx = -\cot\left(\frac{x}{2}\right)$$

```
[In] int(-1/(cos(x) - 1),x)
[Out] -cot(x/2)
```

3.11 $\int \frac{\sin(x)}{a-b\cos(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sin(x)}{a-b\cos(x)}\,dx = \frac{\log(a-b\cos(x))}{b}$$

```
[Out] ln(a-b*cos(x))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2747, 31}

$$\int \frac{\sin(x)}{a-b\cos(x)}\,dx = \frac{\log(a-b\cos(x))}{b}$$

```
[In] Int[Sin[x]/(a - b*Cos[x]),x]
```

```
[Out] Log[a - b*Cos[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, -b\cos(x)\right)}{b}$$
$$= \frac{\log(a - b\cos(x))}{b}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a - b\cos(x)}\,dx = \frac{\log(a - b\cos(x))}{b}$$

```
[In] Integrate[Sin[x]/(a - b*Cos[x]),x]
[Out] Log[a - b*Cos[x]]/b
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\ln(a-b\cos(x))}{b}$	13
default	$\frac{\ln(a-b\cos(x))}{b}$	13
parallelrisch	$\frac{-\ln\left(\frac{1}{\cos(x)+1}\right)+\ln\left(\frac{a-b\cos(x)}{\cos(x)+1}\right)}{b}$	30
risch	$-\frac{ix}{b}+\frac{\ln\left(\mathrm{e}^{2ix}-\frac{2a\,\mathrm{e}^{ix}}{b}+1\right)}{b}$	32
norman	$\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+b\left(\tan^2\left(\frac{x}{2}\right)\right)+a-b\right)}{b}-\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{b}$	42

```
[In] int(sin(x)/(a-b*cos(x)),x,method=_RETURNVERBOSE)
[Out] ln(a-b*cos(x))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a-b\cos(x)}\,dx=\frac{\log\left(-b\cos\left(x\right)+a\right)}{b}$$

```
[In] integrate(sin(x)/(a-b*cos(x)),x, algorithm="fricas")
[Out] log(-b*cos(x) + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\sin(x)}{a-b\cos(x)}\,dx=\begin{cases}\frac{\log\left(-\frac{a}{b}+\cos\left(x\right)\right)}{b} & \text{for}\ b\neq 0\\ -\frac{\cos\left(x\right)}{a} & \text{otherwise}\end{cases}$$

```
[In] integrate(sin(x)/(a-b*cos(x)),x)
[Out] Piecewise((log(-a/b + cos(x))/b, Ne(b, 0)), (-cos(x)/a, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sin(x)}{a-b\cos(x)}\,dx=\frac{\log\left(b\cos\left(x\right)-a\right)}{b}$$

```
[In] integrate(sin(x)/(a-b*cos(x)),x, algorithm="maxima")
[Out] log(b*cos(x) - a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sin(x)}{a - b\cos(x)}\,dx = \frac{\log\left(\left|b\cos\left(x\right) - a\right|\right)}{b}$$

```
[In] integrate(sin(x)/(a-b*cos(x)),x, algorithm="giac")
[Out] log(abs(b*cos(x) - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sin(x)}{a - b\cos(x)}\,dx = \frac{\ln\left(b\,\cos\left(x\right) - a\right)}{b}$$

```
[In] int(sin(x)/(a - b*cos(x)),x)
[Out] log(b*cos(x) - a)/b
```

3.12 $\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx=\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

```
[Out] arctan(b*sin(x)/a)/a/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3269, 211}

$$\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx=\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

```
[In] Int[Cos[x]/(a^2 + b^2*Sin[x]^2),x]
[Out] ArcTan[(b*Sin[x])/a]/(a*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
```

```
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{a^2+b^2x^2}\,dx, x, \sin(x)\right) \\ &= \frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx = \frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

```
[In] Integrate[Cos[x]/(a^2 + b^2*Sin[x]^2),x]
[Out] ArcTan[(b*Sin[x])/a]/(a*b)
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$	16
default	$\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$	16
parallelrisch	$-\frac{i\left(-\ln\left(\frac{-ib\sin(x)+a}{\cos(x)+1}\right)+\ln\left(\frac{ib\sin(x)+a}{\cos(x)+1}\right)\right)}{2ab}$	45
risch	$-\frac{i\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}-1\right)}{2ab}+\frac{i\ln\left(e^{2ix}-\frac{2a\,e^{ix}}{b}-1\right)}{2ab}$	58

```
[In] int(cos(x)/(a^2+b^2*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] arctan(b*sin(x)/a)/a/b
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2 + b^2 \sin^2(x)} \, dx = \frac{\arctan\left(\frac{b \sin(x)}{a}\right)}{ab}$$

```
[In] integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="fricas")
[Out] arctan(b*sin(x)/a)/(a*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(10) = 20.

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{\cos(x)}{a^2 + b^2 \sin^2(x)} \, dx = \begin{cases} \frac{\tilde{\infty}}{\sin(x)} & \text{for } a = 0 \wedge b = 0 \\ -\frac{1}{b^2 \sin(x)} & \text{for } a = 0 \\ \frac{\sin(x)}{a^2} & \text{for } b = 0 \\ \frac{\operatorname{atan}\left(\frac{b \sin(x)}{a}\right)}{ab} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(x)/(a**2+b**2*sin(x)**2),x)
[Out] Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (-1/(b**2*sin(x)), Eq(a, 0)),
(sin(x)/a**2, Eq(b, 0)), (atan(b*sin(x)/a)/(a*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2 + b^2 \sin^2(x)} \, dx = \frac{\arctan\left(\frac{b \sin(x)}{a}\right)}{ab}$$

```
[In] integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="maxima")
[Out] arctan(b*sin(x)/a)/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2 + b^2 \sin^2(x)} \, dx = \frac{\arctan\left(\frac{b \sin(x)}{a}\right)}{ab}$$

```
[In] integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="giac")
[Out] arctan(b*sin(x)/a)/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2 + b^2 \sin^2(x)} \, dx = \frac{\mathrm{atan}\left(\frac{b \sin(x)}{a}\right)}{a\,b}$$

```
[In] int(cos(x)/(b^2*sin(x)^2 + a^2),x)
[Out] atan((b*sin(x))/a)/(a*b)
```

3.13 $\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

```
[Out] arctanh(b*sin(x)/a)/a/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3269, 214}

$$\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

```
[In] Int[Cos[x]/(a^2 - b^2*Sin[x]^2),x]
```

```
[Out] ArcTanh[(b*Sin[x])/a]/(a*b)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
```

```
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{a^2 - b^2x^2}\,dx, x, \sin(x)\right)\\ &= \frac{\text{arctanh}\left(\frac{b\sin(x)}{a}\right)}{ab}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2 - b^2\sin^2(x)}\,dx = \frac{\text{arctanh}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

```
[In] Integrate[Cos[x]/(a^2 - b^2*Sin[x]^2),x]
[Out] ArcTanh[(b*Sin[x])/a]/(a*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(15) = 30.

Time = 0.50 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

method	result	size
derivativedivides	$\frac{\ln(a+b\sin(x))}{2ba} - \frac{\ln(-b\sin(x)+a)}{2ba}$	33
default	$\frac{\ln(a+b\sin(x))}{2ba} - \frac{\ln(-b\sin(x)+a)}{2ba}$	33
parallelrisch	$\frac{-\ln\left(\frac{-b\sin(x)+a}{\cos(x)+1}\right)+\ln\left(\frac{a+b\sin(x)}{\cos(x)+1}\right)}{2ab}$	41
norman	$-\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)+a\right)}{2ba} + \frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)}{2ba}$	54
risch	$\frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{b}-1\right)}{2ba} - \frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{b}-1\right)}{2ba}$	58

```
[In] int(cos(x)/(a^2-b^2*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/2/b/a*ln(a+b*sin(x))-1/2/b/a*ln(-b*sin(x)+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{\cos(x)}{a^2 - b^2 \sin^2(x)} \, dx = \frac{\log\left(b \sin\left(x\right) + a\right) - \log\left(-b \sin\left(x\right) + a\right)}{2 \, a b}$$

```
[In] integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="fricas")
[Out] 1/2*(log(b*sin(x) + a) - log(-b*sin(x) + a))/(a*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(10) = 20.

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.93

$$\int \frac{\cos(x)}{a^2 - b^2 \sin^2(x)} \, dx = \begin{cases} \frac{\tilde{\infty}}{\sin\left(x\right)} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{1}{b^{2} \sin\left(x\right)} & \text{for}\: a = 0 \\ \frac{\sin\left(x\right)}{a^{2}} & \text{for}\: b = 0 \\ - \frac{\log\left(- \frac{a}{b} + \sin\left(x\right)\right)}{2ab} + \frac{\log\left(\frac{a}{b} + \sin\left(x\right)\right)}{2ab} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(x)/(a**2-b**2*sin(x)**2),x)
[Out] Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (1/(b**2*sin(x)), Eq(a, 0)), (
sin(x)/a**2, Eq(b, 0)), (-log(-a/b + sin(x))/(2*a*b) + log(a/b + sin(x))/(2
*a*b), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(15) = 30.

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \frac{\cos(x)}{a^2 - b^2 \sin^2(x)} \, dx = \frac{\log\left(b \sin\left(x\right) + a\right)}{2 \, a b} - \frac{\log\left(b \sin\left(x\right) - a\right)}{2 \, a b}$$

```
[In] integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="maxima")
[Out] 1/2*log(b*sin(x) + a)/(a*b) - 1/2*log(b*sin(x) - a)/(a*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(15) = 30.

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \frac{\cos(x)}{a^2 - b^2 \sin^2(x)}\, dx = \frac{\log\left(|b \sin\left(x\right) + a|\right)}{2\, ab} - \frac{\log\left(|b \sin\left(x\right) - a|\right)}{2\, ab}$$

```
[In] integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="giac")
[Out] 1/2*log(abs(b*sin(x) + a))/(a*b) - 1/2*log(abs(b*sin(x) - a))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2 - b^2 \sin^2(x)}\, dx = \frac{\mathrm{atanh}\left(\frac{b\, \sin(x)}{a}\right)}{a\, b}$$

```
[In] int(-cos(x)/(b^2*sin(x)^2 - a^2),x)
[Out] atanh((b*sin(x))/a)/(a*b)
```

3.14 $\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx = \frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

```
[Out] ln(a^2+b^2*sin(x)^2)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {12, 266}

$$\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx = \frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

```
[In] Int[Sin[2*x]/(a^2 + b^2*Sin[x]^2),x]
```

```
[Out] Log[a^2 + b^2*Sin[x]^2]/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{2x}{a^2+b^2x^2}\,dx, x, \sin(x)\right)$$
$$= 2\text{Subst}\left(\int \frac{x}{a^2+b^2x^2}\,dx, x, \sin(x)\right)$$
$$= \frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx = \frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

```
[In] Integrate[Sin[2*x]/(a^2 + b^2*Sin[x]^2),x]
[Out] Log[a^2 + b^2*Sin[x]^2]/b^2
```

Maple [A] (verified)

Time = 6.74 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\ln\left(a^2+b^2\left(\sin^2(x)\right)\right)}{b^2}$	18
default	$\frac{\ln\left(a^2+b^2\left(\sin^2(x)\right)\right)}{b^2}$	18
risch	$-\frac{2ix}{b^2}+\frac{\ln\left(e^{4ix}-\frac{2\left(2a^2+b^2\right)e^{2ix}}{b^2}+1\right)}{b^2}$	40

```
[In] int(sin(2*x)/(a^2+b^2*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] ln(a^2+b^2*sin(x)^2)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{\sin(2x)}{a^2 + b^2 \sin^2(x)}\, dx = \frac{\log\left(-b^2 \cos\left(x\right)^2 + a^2 + b^2\right)}{b^2}$$

```
[In] integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="fricas")
[Out] log(-b^2*cos(x)^2 + a^2 + b^2)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int \frac{\sin(2x)}{a^2 + b^2 \sin^2(x)}\, dx = 2\left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ \frac{\log\left(a^2 + b^2 \sin^2(x)\right)}{2b^2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(sin(2*x)/(a**2+b**2*sin(x)**2),x)
[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (log(a**2 + b**2*sin(x)**2)
/(2*b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2 + b^2 \sin^2(x)}\, dx = \frac{\log\left(b^2 \sin\left(x\right)^2 + a^2\right)}{b^2}$$

```
[In] integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="maxima")
[Out] log(b^2*sin(x)^2 + a^2)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2 + b^2 \sin^2(x)} \, dx = \frac{\log\left(b^2 \sin{(x)}^2 + a^2\right)}{b^2}$$

```
[In] integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="giac")
[Out] log(b^2*sin(x)^2 + a^2)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.82

$$\int \frac{\sin(2x)}{a^2 + b^2 \sin^2(x)} \, dx = \frac{\mathrm{atan}\left(\frac{b^2 \sin(x)^2}{a^2 \cos(x)^2 \, 2\mathrm{i} + a^2 \sin(x)^2 \, 2\mathrm{i} + b^2 \sin(x)^2 \, 1\mathrm{i}}\right) \, 2\mathrm{i}}{b^2}$$

```
[In] int(sin(2*x)/(b^2*sin(x)^2 + a^2),x)
[Out] (atan((b^2*sin(x)^2)/(a^2*cos(x)^2*2i + a^2*sin(x)^2*2i + b^2*sin(x)^2*1i))
*2i)/b^2
```

3.15 $\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 19

$$\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx=-\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

```
[Out] -ln(a^2-b^2*sin(x)^2)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {12, 266}

$$\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx=-\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

```
[In] Int[Sin[2*x]/(a^2 - b^2*Sin[x]^2),x]
```

```
[Out] -(Log[a^2 - b^2*Sin[x]^2]/b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{2x}{a^2-b^2x^2}\,dx,x,\sin(x)\right)\\ &= 2\text{Subst}\left(\int \frac{x}{a^2-b^2x^2}\,dx,x,\sin(x)\right)\\ &= -\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx = -\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

```
[In] Integrate[Sin[2*x]/(a^2 - b^2*Sin[x]^2),x]
[Out] -(Log[a^2 - b^2*Sin[x]^2]/b^2)
```

Maple [A] (verified)

Time = 6.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\frac{\ln\left(a^2-b^2\left(\sin^2(x)\right)\right)}{b^2}$	20
default	$-\frac{\ln\left(a^2-b^2\left(\sin^2(x)\right)\right)}{b^2}$	20
risch	$\frac{2ix}{b^2}-\frac{\ln\left(e^{4ix}+\frac{2\left(2a^2-b^2\right)e^{2ix}}{b^2}+1\right)}{b^2}$	43

```
[In] int(sin(2*x)/(a^2-b^2*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] -ln(a^2-b^2*sin(x)^2)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{\sin(2x)}{a^2 - b^2\sin^2(x)}\,dx = -\frac{\log\left(b^2\cos\left(x\right)^2 + a^2 - b^2\right)}{b^2}$$

```
[In] integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="fricas")
[Out] -log(b^2*cos(x)^2 + a^2 - b^2)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{\sin(2x)}{a^2 - b^2\sin^2(x)}\,dx = 2\left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ -\frac{\log(a^2 - b^2\sin^2(x))}{2b^2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(sin(2*x)/(a**2-b**2*sin(x)**2),x)
[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (-log(a**2 - b**2*sin(x)**2
)/(2*b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\sin(2x)}{a^2 - b^2\sin^2(x)}\,dx = -\frac{\log\left(b^2\sin\left(x\right)^2 - a^2\right)}{b^2}$$

```
[In] integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="maxima")
[Out] -log(b^2*sin(x)^2 - a^2)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\sin(2x)}{a^2 - b^2 \sin^2(x)}\, dx = -\frac{\log\left(\left|b^2 \sin{(x)}^2 - a^2\right|\right)}{b^2}$$

```
[In] integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="giac")
[Out] -log(abs(b^2*sin(x)^2 - a^2))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

$$\int \frac{\sin(2x)}{a^2 - b^2 \sin^2(x)}\, dx = \frac{\operatorname{atan}\left(\frac{b^2 \sin(x)^2}{a^2 \cos(x)^2\, 2\mathrm{i} + a^2 \sin(x)^2\, 2\mathrm{i} - b^2 \sin(x)^2\, 1\mathrm{i}}\right)\, 2\mathrm{i}}{b^2}$$

```
[In] int(-sin(2*x)/(b^2*sin(x)^2 - a^2),x)
[Out] (atan((b^2*sin(x)^2)/(a^2*cos(x)^2*2i + a^2*sin(x)^2*2i - b^2*sin(x)^2*1i))
*2i)/b^2
```

3.16 $\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx = -\frac{\log\left(a^2+b^2\cos^2(x)\right)}{b^2}$$

```
[Out] -ln(a^2+b^2*cos(x)^2)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {12, 266}

$$\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx = -\frac{\log\left(a^2-b^2\sin^2(x)+b^2\right)}{b^2}$$

```
[In] Int[Sin[2*x]/(a^2 + b^2*Cos[x]^2),x]
```

```
[Out] -(Log[a^2 + b^2 - b^2*Sin[x]^2]/b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{2x}{a^2+b^2-b^2x^2}\,dx, x, \sin(x)\right) \\ &= 2\text{Subst}\left(\int \frac{x}{a^2+b^2-b^2x^2}\,dx, x, \sin(x)\right) \\ &= -\frac{\log\left(a^2+b^2-b^2\sin^2(x)\right)}{b^2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx = -\frac{\log\left(a^2+b^2-b^2\sin^2(x)\right)}{b^2}$$

```
[In] Integrate[Sin[2*x]/(a^2 + b^2*Cos[x]^2),x]
[Out] -(Log[a^2 + b^2 - b^2*Sin[x]^2]/b^2)
```

Maple [A] (verified)

Time = 10.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{\ln\left(a^2+b^2\left(\cos^2(x)\right)\right)}{b^2}$	19
default	$-\frac{\ln\left(a^2+b^2\left(\cos^2(x)\right)\right)}{b^2}$	19
risch	$\frac{2ix}{b^2}-\frac{\ln\left(e^{4ix}+\frac{2\left(2a^2+b^2\right)e^{2ix}}{b^2}+1\right)}{b^2}$	41

```
[In] int(sin(2*x)/(a^2+b^2*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] -ln(a^2+b^2*cos(x)^2)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2 + b^2\cos^2(x)}\,dx = -\frac{\log\left(b^2\cos\left(x\right)^2 + a^2\right)}{b^2}$$

```
[In] integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="fricas")
[Out] -log(b^2*cos(x)^2 + a^2)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\sin(2x)}{a^2 + b^2\cos^2(x)}\,dx = 2\left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for}\ b^2 = 0 \\ -\frac{\log\left(a^2 + b^2\cos^2(x)\right)}{2b^2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(sin(2*x)/(a**2+b**2*cos(x)**2),x)
[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (-log(a**2 + b**2*cos(x)**2
)/(2*b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2 + b^2\cos^2(x)}\,dx = -\frac{\log\left(b^2\cos\left(x\right)^2 + a^2\right)}{b^2}$$

```
[In] integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="maxima")
[Out] -log(b^2*cos(x)^2 + a^2)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2 + b^2\cos^2(x)}\,dx = -\frac{\log\left(b^2\cos\left(x\right)^2 + a^2\right)}{b^2}$$

```
[In] integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="giac")
[Out] -log(b^2*cos(x)^2 + a^2)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.22

$$\int \frac{\sin(2x)}{a^2 + b^2\cos^2(x)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{b^2}{2\,a^2+b^2\cos(x)^2+b^2} - \frac{b^2\cos(x)^2}{2\,a^2+b^2\cos(x)^2+b^2}\right)}{b^2}$$

```
[In] int(sin(2*x)/(b^2*cos(x)^2 + a^2),x)
[Out] (2*atanh(b^2/(b^2*cos(x)^2 + 2*a^2 + b^2) - (b^2*cos(x)^2)/(b^2*cos(x)^2 +
2*a^2 + b^2)))/b^2
```

3.17 $\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 18

$$\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx = \frac{\log\left(a^2-b^2\cos^2(x)\right)}{b^2}$$

```
[Out] ln(a^2-b^2*cos(x)^2)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {12, 266}

$$\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx = \frac{\log\left(a^2+b^2\sin^2(x)-b^2\right)}{b^2}$$

```
[In] Int[Sin[2*x]/(a^2 - b^2*Cos[x]^2),x]
```

```
[Out] Log[a^2 - b^2 + b^2*Sin[x]^2]/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{2x}{a^2-b^2+b^2x^2}\,dx, x, \sin(x)\right)\\
&= 2\text{Subst}\left(\int \frac{x}{a^2-b^2+b^2x^2}\,dx, x, \sin(x)\right)\\
&= \frac{\log\left(a^2-b^2+b^2\sin^2(x)\right)}{b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx = \frac{\log\left(a^2-b^2+b^2\sin^2(x)\right)}{b^2}$$

```
[In] Integrate[Sin[2*x]/(a^2 - b^2*Cos[x]^2),x]
[Out] Log[a^2 - b^2 + b^2*Sin[x]^2]/b^2
```

Maple [A] (verified)

Time = 9.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\ln\left(a^2-b^2\left(\cos^2(x)\right)\right)}{b^2}$	19
default	$\frac{\ln\left(a^2-b^2\left(\cos^2(x)\right)\right)}{b^2}$	19
risch	$-\frac{2ix}{b^2}+\frac{\ln\left(\mathrm{e}^{4ix}-\frac{2\left(2a^2-b^2\right)\mathrm{e}^{2ix}}{b^2}+1\right)}{b^2}$	42

```
[In] int(sin(2*x)/(a^2-b^2*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] ln(a^2-b^2*cos(x)^2)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sin(2x)}{a^2 - b^2\cos^2(x)}\,dx = \frac{\log\left(b^2\cos(x)^2 - a^2\right)}{b^2}$$

```
[In] integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="fricas")
[Out] log(b^2*cos(x)^2 - a^2)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \frac{\sin(2x)}{a^2 - b^2\cos^2(x)}\,dx = 2\left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ \frac{\log(a^2 - b^2\cos^2(x))}{2b^2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(sin(2*x)/(a**2-b**2*cos(x)**2),x)
[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (log(a**2 - b**2*cos(x)**2)
/(2*b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sin(2x)}{a^2 - b^2\cos^2(x)}\,dx = \frac{\log\left(b^2\cos(x)^2 - a^2\right)}{b^2}$$

```
[In] integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="maxima")
[Out] log(b^2*cos(x)^2 - a^2)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin(2x)}{a^2 - b^2\cos^2(x)}\,dx = \frac{\log\left(\left|b^2\cos\left(x\right)^2 - a^2\right|\right)}{b^2}$$

```
[In] integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="giac")
[Out] log(abs(b^2*cos(x)^2 - a^2))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.22

$$\int \frac{\sin(2x)}{a^2 - b^2\cos^2(x)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{b^2}{-2\,a^2+b^2\,\cos(x)^2+b^2} - \frac{b^2\,\cos(x)^2}{-2\,a^2+b^2\,\cos(x)^2+b^2}\right)}{b^2}$$

```
[In] int(-sin(2*x)/(b^2*cos(x)^2 - a^2),x)
[Out] -(2*atanh(b^2/(b^2*cos(x)^2 - 2*a^2 + b^2) - (b^2*cos(x)^2)/(b^2*cos(x)^2 -
 2*a^2 + b^2)))/b^2
```

3.18 $\int \frac{1}{4-\cos^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \frac{1}{4-\cos^2(x)}\,dx=\frac{x}{2\sqrt{3}}+\frac{\arctan\left(\frac{\cos(x)\sin(x)}{3+2\sqrt{3}+\sin^2(x)}\right)}{2\sqrt{3}}$$

```
[Out] 1/6*x*3^(1/2)+1/6*arctan(cos(x)*sin(x)/(3+sin(x)^2+2*3^(1/2)))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3260, 209}

$$\int \frac{1}{4-\cos^2(x)}\,dx=\frac{\arctan\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+2\sqrt{3}+3}\right)}{2\sqrt{3}}+\frac{x}{2\sqrt{3}}$$

```
[In] Int[(4 - Cos[x]^2)^(-1),x]
```

```
[Out] x/(2*Sqrt[3]) + ArcTan[(Cos[x]*Sin[x])/(3 + 2*Sqrt[3] + Sin[x]^2)]/(2*Sqrt[
3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{4+3x^2}\,dx, x, \cot(x)\right)$$

$$= \frac{x}{2\sqrt{3}} + \frac{\arctan\left(\frac{\cos(x)\sin(x)}{3+2\sqrt{3}+\sin^2(x)}\right)}{2\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

$$\int \frac{1}{4-\cos^2(x)}\,dx = \frac{\arctan\left(\frac{2\tan(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[In] Integrate[(4 - Cos[x]^2)^(-1),x]
[Out] ArcTan[(2*Tan[x])/Sqrt[3]]/(2*Sqrt[3])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.34

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{2\tan(x)\sqrt{3}}{3}\right)}{6}$	14
risch	$\frac{i\sqrt{3}\,\ln\left(e^{2ix}-4\sqrt{3}-7\right)}{12}-\frac{i\sqrt{3}\,\ln\left(e^{2ix}+4\sqrt{3}-7\right)}{12}$	40

```
[In] int(1/(4-cos(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/6*3^(1/2)*arctan(2/3*tan(x)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{4-\cos^2(x)}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{7\,\sqrt{3}\cos\left(x\right)^2 - 4\,\sqrt{3}}{12\,\cos\left(x\right)\sin\left(x\right)}\right)$$

```
[In] integrate(1/(4-cos(x)^2),x, algorithm="fricas")
[Out] -1/12*sqrt(3)*arctan(1/12*(7*sqrt(3)*cos(x)^2 - 4*sqrt(3))/(cos(x)*sin(x)))
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

$$\int \frac{1}{4-\cos^2(x)}\,dx = \frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right) + \pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6} + \frac{\sqrt{3}\left(\operatorname{atan}\left(\sqrt{3}\tan\left(\frac{x}{2}\right)\right) + \pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6}$$

```
[In] integrate(1/(4-cos(x)**2),x)
[Out] sqrt(3)*(atan(sqrt(3)*tan(x/2)/3) + pi*floor((x/2 - pi/2)/pi))/6 + sqrt(3)*
(atan(sqrt(3)*tan(x/2)) + pi*floor((x/2 - pi/2)/pi))/6
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.32

$$\int \frac{1}{4-\cos^2(x)}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\tan\left(x\right)\right)$$

```
[In] integrate(1/(4-cos(x)^2),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*arctan(2/3*sqrt(3)*tan(x))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{1}{4-\cos^2(x)}\,dx = \frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)-2\sin(2x)}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)+2}\right)\right)$$

```
[In] integrate(1/(4-cos(x)^2),x, algorithm="giac")
[Out] 1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) - 2*sin(2*x))/(sqrt(3)*cos(2*x)
+ sqrt(3) - 2*cos(2*x) + 2)))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{4-\cos^2(x)}\,dx = \frac{\sqrt{3}\,(x-\mathrm{atan}(\tan(x)))}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\tan(x)}{3}\right)}{6}$$

```
[In] int(-1/(cos(x)^2 - 4),x)
[Out] (3^(1/2)*(x - atan(tan(x))))/6 + (3^(1/2)*atan((2*3^(1/2)*tan(x))/3))/6
```

3.19 $\int \frac{e^x}{-1+e^{2x}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 6

$$\int \frac{e^x}{-1+e^{2x}} \, dx = -\operatorname{arctanh}(e^x)$$

```
[Out] -arctanh(exp(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2281, 213}

$$\int \frac{e^x}{-1+e^{2x}} \, dx = -\operatorname{arctanh}(e^x)$$

```
[In] Int[E^x/(-1 + E^(2*x)),x]
```

```
[Out] -ArcTanh[E^x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
```

```
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^x\right) \\ &= -\text{arctanh}(e^x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{-1+e^{2x}}\,dx = -\text{arctanh}(e^x)$$

```
[In] Integrate[E^x/(-1 + E^(2*x)),x]
[Out] -ArcTanh[E^x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
default	$-\operatorname{arctanh}\left(\mathrm{e}^{x}\right)$	6
norman	$\frac{\ln\left(-1+\mathrm{e}^{x}\right)}{2}-\frac{\ln\left(1+\mathrm{e}^{x}\right)}{2}$	16
risch	$\frac{\ln\left(-1+\mathrm{e}^{x}\right)}{2}-\frac{\ln\left(1+\mathrm{e}^{x}\right)}{2}$	16

```
[In] int(exp(x)/(exp(2*x)-1),x,method=_RETURNVERBOSE)
[Out] -arctanh(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = -\frac{1}{2}\log\left(e^x+1\right)+\frac{1}{2}\log\left(e^x-1\right)$$

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="fricas")
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = \frac{\log\left(e^x-1\right)}{2}-\frac{\log\left(e^x+1\right)}{2}$$

```
[In] integrate(exp(x)/(-1+exp(2*x)),x)
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = -\frac{1}{2}\log\left(e^x+1\right)+\frac{1}{2}\log\left(e^x-1\right)$$

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="maxima")
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(5) = 10$.

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{e^x}{-1+e^{2x}}\,dx = -\frac{1}{2}\log\left(e^x+1\right) + \frac{1}{2}\log\left(|e^x-1|\right)$$

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="giac")
[Out] -1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = \frac{\ln\left(\mathrm{e}^x-1\right)}{2} - \frac{\ln\left(\mathrm{e}^x+1\right)}{2}$$

```
[In] int(exp(x)/(exp(2*x) - 1),x)
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

3.20 $\int \frac{1}{x \log(x)} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 3

$$\int \frac{1}{x \log(x)} \, dx = \log(\log(x))$$

```
[Out] ln(ln(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2339, 29}

$$\int \frac{1}{x \log(x)} \, dx = \log(\log(x))$$

```
[In] Int[1/(x*Log[x]),x]
[Out] Log[Log[x]]
```

<u>Rule 29</u>

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

<u>Rule 2339</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, \log(x)\right)\\ &= \log(\log(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log(\log(x))$$

```
[In] Integrate[1/(x*Log[x]),x]
[Out] Log[Log[x]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\ln(\ln(x))$	4
default	$\ln(\ln(x))$	4
norman	$\ln(\ln(x))$	4
risch	$\ln(\ln(x))$	4
parallelrisch	$\ln(\ln(x))$	4

```
[In] int(1/x/ln(x),x,method=_RETURNVERBOSE)
[Out] ln(ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/log(x),x, algorithm="fricas")
[Out] log(log(x))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/ln(x),x)
[Out] log(log(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/log(x),x, algorithm="maxima")
[Out] log(log(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\log(x)}\,dx = \log\left(\left|\log\left(x\right)\right|\right)$$

```
[In] integrate(1/x/log(x),x, algorithm="giac")
[Out] log(abs(log(x)))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \ln\left(\ln\left(x\right)\right)$$

```
[In] int(1/(x*log(x)),x)
[Out] log(log(x))
```

3.21 $\int \frac{1}{x(1+\log^2(x))} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 3

$$\int \frac{1}{x\left(1+\log^2(x)\right)} \, dx = \arctan(\log(x))$$

```
[Out] arctan(ln(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {209}

$$\int \frac{1}{x\left(1+\log^2(x)\right)} \, dx = \arctan(\log(x))$$

```
[In] Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \log(x)\right) \\ &= \arctan(\log(x)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx=\arctan(\log(x))$$

```
[In] Integrate[1/(x*(1 + Log[x]^2)),x]
[Out] ArcTan[Log[x]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arctan\left(\ln\left(x\right)\right)$	4
default	$\arctan\left(\ln\left(x\right)\right)$	4
risch	$\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$	20
parallelrisch	$\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$	20

```
[In] int(1/x/(1+ln(x)^2),x,method=_RETURNVERBOSE)
[Out] arctan(ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx=\arctan\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
[Out] arctan(log(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \operatorname{RootSum}\left(4z^2+1,\left(i\mapsto i\log\left(2i+\log\left(x\right)\right)\right)\right)$$

```
[In] integrate(1/x/(1+ln(x)**2),x)
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
[Out] arctan(log(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="giac")
[Out] arctan(log(x))
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \operatorname{atan}(\ln(x))$$

```
[In] int(1/(x*(log(x)^2 + 1)),x)
[Out] atan(log(x))
```

3.22 $\int \frac{1}{x(1-\log(x))} dx$

Optimal result

Integrand size = 12, antiderivative size = 9

$$\int \frac{1}{x(1-\log(x))} dx = -\log(1-\log(x))$$

```
[Out] -ln(1-ln(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2339, 29}

$$\int \frac{1}{x(1-\log(x))} dx = -\log(1-\log(x))$$

```
[In] Int[1/(x*(1 - Log[x])),x]
```

```
[Out] -Log[1 - Log[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int \frac{1}{x}\,dx, x, 1-\log(x)\right) \\ &= -\log(1-\log(x)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1-\log(x))}\,dx = -\log(-1+\log(x))$$

```
[In] Integrate[1/(x*(1 - Log[x])),x]
[Out] -Log[-1 + Log[x]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
norman	$-\ln(-1+\ln(x))$	8
risch	$-\ln(-1+\ln(x))$	8
parallelrisch	$-\ln(-1+\ln(x))$	8
derivativedivides	$-\ln(1-\ln(x))$	10
default	$-\ln(1-\ln(x))$	10

```
[In] int(1/x/(1-ln(x)),x,method=_RETURNVERBOSE)
[Out] -ln(-1+ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1-\log(x))}\,dx = -\log(\log(x)-1)$$

```
[In] integrate(1/x/(1-log(x)),x, algorithm="fricas")
[Out] -log(log(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1-\log(x))}\,dx = -\log\left(\log\left(x\right)-1\right)$$

```
[In] integrate(1/x/(1-ln(x)),x)
[Out] -log(log(x) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1-\log(x))}\,dx = -\log\left(\log\left(x\right)-1\right)$$

```
[In] integrate(1/x/(1-log(x)),x, algorithm="maxima")
[Out] -log(log(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.44

$$\int \frac{1}{x(1-\log(x))}\,dx = -\frac{1}{2}\log\left(\frac{1}{4}\pi^2(\mathrm{sgn}(x)-1)^2+\left(\log\left(|x|\right)-1\right)^2\right)$$

```
[In] integrate(1/x/(1-log(x)),x, algorithm="giac")
[Out] -1/2*log(1/4*pi^2*(sgn(x) - 1)^2 + (log(abs(x)) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1-\log(x))}\,dx = -\ln\left(\ln\left(x\right)-1\right)$$

```
[In] int(-1/(x*(log(x) - 1)),x)
[Out] -log(log(x) - 1)
```

3.23 $\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 9

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(1+\log\left(\frac{x}{a}\right)\right)$$

```
[Out] ln(1+ln(x/a))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2339, 29}

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(\log\left(\frac{x}{a}\right)+1\right)$$

```
[In] Int[1/(x*(1 + Log[x/a])),x]
```

```
[Out] Log[1 + Log[x/a]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{x}\,dx, x, 1+\log\left(\frac{x}{a}\right)\right)$$
$$= \log\left(1+\log\left(\frac{x}{a}\right)\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(1+\log\left(\frac{x}{a}\right)\right)$$

```
[In] Integrate[1/(x*(1 + Log[x/a])),x]
[Out] Log[1 + Log[x/a]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$	10
default	$\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$	10
norman	$\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$	10
risch	$\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$	10
parallelrisch	$\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$	10

```
[In] int(1/x/(1+ln(x/a)),x,method=_RETURNVERBOSE)
[Out] ln(1+ln(x/a))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(\log\left(\frac{x}{a}\right)+1\right)$$

```
[In] integrate(1/x/(1+log(x/a)),x, algorithm="fricas")
[Out] log(log(x/a) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(\log\left(\frac{x}{a}\right)+1\right)$$

```
[In] integrate(1/x/(1+ln(x/a)),x)
[Out] log(log(x/a) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(\log\left(\frac{x}{a}\right)+1\right)$$

```
[In] integrate(1/x/(1+log(x/a)),x, algorithm="maxima")
[Out] log(log(x/a) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(\log\left(\frac{x}{a}\right)+1\right)$$

```
[In] integrate(1/x/(1+log(x/a)),x, algorithm="giac")
[Out] log(log(x/a) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \ln\left(\ln\left(\frac{x}{a}\right)+1\right)$$

```
[In] int(1/(x*(log(x/a) + 1)),x)
[Out] log(log(x/a) + 1)
```

3.24 $\int \frac{(1-\sqrt{x}+x)^2}{x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{(1-\sqrt{x}+x)^2}{x^2} dx = -\frac{1}{x}+\frac{4}{\sqrt{x}}-4\sqrt{x}+x+3\log(x)$$

```
[Out] -1/x+x+3*ln(x)+4/x^(1/2)-4*x^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1371, 712}

$$\int \frac{(1-\sqrt{x}+x)^2}{x^2} dx = x-4\sqrt{x}+\frac{4}{\sqrt{x}}-\frac{1}{x}+3\log(x)$$

```
[In] Int[(1 - Sqrt[x] + x)^2/x^2,x]
```

```
[Out] -x^(-1) + 4/Sqrt[x] - 4*Sqrt[x] + x + 3*Log[x]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{(1-x+x^2)^2}{x^3}\,dx, x, \sqrt{x}\right)\\ &= 2\text{Subst}\left(\int \left(-2+\frac{1}{x^3}-\frac{2}{x^2}+\frac{3}{x}+x\right)\,dx, x, \sqrt{x}\right)\\ &= -\frac{1}{x}+\frac{4}{\sqrt{x}}-4\sqrt{x}+x+3\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(1-\sqrt{x}+x)^2}{x^2}\,dx = -\frac{1}{x}+\frac{4}{\sqrt{x}}-4\sqrt{x}+x+3\log(x)$$

```
[In] Integrate[(1 - Sqrt[x] + x)^2/x^2,x]
[Out] -x^(-1) + 4/Sqrt[x] - 4*Sqrt[x] + x + 3*Log[x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{1}{x}+x+3\ln(x)+\frac{4}{\sqrt{x}}-4\sqrt{x}$	22
default	$-\frac{1}{x}+x+3\ln(x)+\frac{4}{\sqrt{x}}-4\sqrt{x}$	22
trager	$\frac{(-1+x)(1+x)}{x}-\frac{4(-1+x)}{\sqrt{x}}-3\ln\left(\frac{1}{x}\right)$	26

```
[In] int((1+x-x^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
[Out] -1/x+x+3*ln(x)+4/x^(1/2)-4*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx=\frac{x^2+6\,x\log\left(\sqrt{x}\right)-4\,(x-1)\sqrt{x}-1}{x}$$

```
[In] integrate((1+x-x^(1/2))^2/x^2,x, algorithm="fricas")
[Out] (x^2 + 6*x*log(sqrt(x)) - 4*(x - 1)*sqrt(x) - 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx=-4\sqrt{x}+x+3\log\left(x\right)-\frac{1}{x}+\frac{4}{\sqrt{x}}$$

```
[In] integrate((1+x-x**(1/2))**2/x**2,x)
[Out] -4*sqrt(x) + x + 3*log(x) - 1/x + 4/sqrt(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx=x-4\,\sqrt{x}+\frac{4\,\sqrt{x}-1}{x}+3\,\log\left(x\right)$$

```
[In] integrate((1+x-x^(1/2))^2/x^2,x, algorithm="maxima")
[Out] x - 4*sqrt(x) + (4*sqrt(x) - 1)/x + 3*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx=x-4\,\sqrt{x}+\frac{4\,\sqrt{x}-1}{x}+3\,\log\left(|x|\right)$$

```
[In] integrate((1+x-x^(1/2))^2/x^2,x, algorithm="giac")
[Out] x - 4*sqrt(x) + (4*sqrt(x) - 1)/x + 3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx = x+6\,\ln\left(\sqrt{x}\right)+\frac{4\,\sqrt{x}-1}{x}-4\,\sqrt{x}$$

```
[In] int((x - x^(1/2) + 1)^2/x^2,x)
[Out] x + 6*log(x^(1/2)) + (4*x^(1/2) - 1)/x - 4*x^(1/2)
```

3.25 $\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 30

$$\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx=4\sqrt{x}-\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}+2\log(x)$$

```
[Out] -3/2*x^(2/3)-6/7*x^(7/6)+2*ln(x)+4*x^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1598, 1834}

$$\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx=-\frac{6x^{7/6}}{7}-\frac{3x^{2/3}}{2}+4\sqrt{x}+2\log(x)$$

```
[In] Int[((2 - x^(2/3))*(Sqrt[x] + x))/x^(3/2),x]
```

```
[Out] 4*Sqrt[x] - (3*x^(2/3))/2 - (6*x^(7/6))/7 + 2*Log[x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
```

```
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\left(1+\sqrt{x}\right)\left(2-x^{2/3}\right)}{x}\,dx \\
&= -\left(6\text{Subst}\left(\int \frac{\left(1+x^3\right)\left(-2+x^4\right)}{x}\,dx, x, \sqrt[6]{x}\right)\right) \\
&= -\left(6\text{Subst}\left(\int \left(-\frac{2}{x}-2x^2+x^3+x^6\right)\,dx, x, \sqrt[6]{x}\right)\right) \\
&= 4\sqrt{x}-\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}+2\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$
\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx = 4\sqrt{x}-\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}+2\log(x)
$$

```
[In] Integrate[((2 - x^(2/3))*(Sqrt[x] + x))/x^(3/2),x]
[Out] 4*Sqrt[x] - (3*x^(2/3))/2 - (6*x^(7/6))/7 + 2*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{3x^{\frac{2}{3}}}{2}-\frac{6x^{\frac{7}{6}}}{7}+2\ln\left(x\right)+4\sqrt{x}$	21
default	$-\frac{3x^{\frac{2}{3}}}{2}-\frac{6x^{\frac{7}{6}}}{7}+2\ln\left(x\right)+4\sqrt{x}$	21

```
[In] int((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x,method=_RETURNVERBOSE)
[Out] -3/2*x^(2/3)-6/7*x^(7/6)+2*ln(x)+4*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{(2-x^{2/3})(\sqrt{x}+x)}{x^{3/2}}\,dx = -\frac{6}{7}\,x^{\frac{7}{6}} - \frac{3}{2}\,x^{\frac{2}{3}} + 4\,\sqrt{x} + 12\,\log\left(x^{\frac{1}{6}}\right)$$

```
[In] integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x, algorithm="fricas")
[Out] -6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 12*log(x^(1/6))
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{(2-x^{2/3})(\sqrt{x}+x)}{x^{3/2}}\,dx = -\frac{6x^{\frac{7}{6}}}{7} - \frac{3x^{\frac{2}{3}}}{2} + 4\sqrt{x} + 6\log\left(\sqrt[3]{x}\right)$$

```
[In] integrate((2-x**(2/3))*(x+x**(1/2))/x**(3/2),x)
[Out] -6*x**(7/6)/7 - 3*x**(2/3)/2 + 4*sqrt(x) + 6*log(x**(1/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{(2-x^{2/3})(\sqrt{x}+x)}{x^{3/2}}\,dx = -\frac{6}{7}\,x^{\frac{7}{6}} - \frac{3}{2}\,x^{\frac{2}{3}} + 4\,\sqrt{x} + 2\,\log\left(x\right)$$

```
[In] integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x, algorithm="maxima")
[Out] -6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{(2-x^{2/3})(\sqrt{x}+x)}{x^{3/2}}\,dx = -\frac{6}{7}\,x^{\frac{7}{6}} - \frac{3}{2}\,x^{\frac{2}{3}} + 4\,\sqrt{x} + 2\,\log\left(|x|\right)$$

```
[In] integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x, algorithm="giac")
[Out] -6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx = 12\,\ln\left(x^{1/6}\right)+4\,\sqrt{x}-\frac{3\,x^{2/3}}{2}-\frac{6\,x^{7/6}}{7}$$

```
[In] int(-((x^(2/3) - 2)*(x + x^(1/2)))/x^(3/2),x)
[Out] 12*log(x^(1/6)) + 4*x^(1/2) - (3*x^(2/3))/2 - (6*x^(7/6))/7
```

3.26 $\int \frac{-1+2x}{3+2x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{-1+2x}{3+2x}\,dx = x-2\log(3+2x)$$

[Out] x-2*ln(3+2*x)

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\int \frac{-1+2x}{3+2x}\,dx = x-2\log(2x+3)$$

[In] Int[(-1 + 2*x)/(3 + 2*x),x]

[Out] x - 2*Log[3 + 2*x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(1-\frac{4}{3+2x}\right)\,dx\\ &= x-2\log(3+2x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x}{3+2x}\,dx = x-2\log(3+2x)$$

```
[In] Integrate[(-1 + 2*x)/(3 + 2*x),x]
[Out] x - 2*Log[3 + 2*x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$x-2\ln\left(\frac{3}{2}+x\right)$	9
default	$x-2\ln\left(3+2x\right)$	11
norman	$x-2\ln\left(3+2x\right)$	11
meijerg	$-2\ln\left(1+\frac{2x}{3}\right)+x$	11
risch	$x-2\ln\left(3+2x\right)$	11

```
[In] int((2*x-1)/(3+2*x),x,method=_RETURNVERBOSE)
[Out] x-2*ln(3/2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x}{3+2x}\,dx = x-2\,\log\left(2\,x+3\right)$$

```
[In] integrate((-1+2*x)/(3+2*x),x, algorithm="fricas")
[Out] x - 2*log(2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{-1+2x}{3+2x}\,dx = x - 2\log(2x+3)$$

```
[In] integrate((-1+2*x)/(3+2*x),x)
[Out] x - 2*log(2*x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x}{3+2x}\,dx = x - 2\log(2x+3)$$

```
[In] integrate((-1+2*x)/(3+2*x),x, algorithm="maxima")
[Out] x - 2*log(2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{-1+2x}{3+2x}\,dx = x - 2\log(|2x+3|)$$

```
[In] integrate((-1+2*x)/(3+2*x),x, algorithm="giac")
[Out] x - 2*log(abs(2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{-1+2x}{3+2x}\,dx = x - 2\ln\left(x+\frac{3}{2}\right)$$

```
[In] int((2*x - 1)/(2*x + 3),x)
[Out] x - 2*log(x + 3/2)
```

3.27 $\int \frac{-5+2x}{-2+3x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{-5+2x}{-2+3x^2} dx = \frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(\sqrt{6}+3x\right)$$

```
[Out] 1/12*ln(-3*x+6^(1/2))*(4-5*6^(1/2))+1/12*ln(3*x+6^(1/2))*(4+5*6^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {647, 31}

$$\int \frac{-5+2x}{-2+3x^2} dx = \frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(3x+\sqrt{6}\right)$$

```
[In] Int[(-5 + 2*x)/(-2 + 3*x^2),x]
```

```
[Out] ((4 - 5*Sqrt[6])*Log[Sqrt[6] - 3*x])/12 + ((4 + 5*Sqrt[6])*Log[Sqrt[6] + 3*
x])/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

```
(-a)*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\left(4-5\sqrt{6}\right)\int \frac{1}{-\sqrt{6}+3x}\,dx+\frac{1}{4}\left(4+5\sqrt{6}\right)\int \frac{1}{\sqrt{6}+3x}\,dx\\ &= \frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(\sqrt{6}+3x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{-5+2x}{-2+3x^2}\,dx=\frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(\sqrt{6}+3x\right)$$

```
[In] Integrate[(-5 + 2*x)/(-2 + 3*x^2),x]
[Out] ((4 - 5*Sqrt[6])*Log[Sqrt[6] - 3*x])/12 + ((4 + 5*Sqrt[6])*Log[Sqrt[6] + 3*
x])/12
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{\ln(3x^2-2)}{3}+\frac{5\sqrt{6}\,\operatorname{arctanh}\left(\frac{x\sqrt{6}}{2}\right)}{6}$	24
meijerg	$\frac{5\sqrt{6}\,\operatorname{arctanh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{6}+\frac{\ln\left(1-\frac{3x^2}{2}\right)}{3}$	27
risch	$\frac{\ln\left(3x+\sqrt{6}\right)}{3}+\frac{5\ln\left(3x+\sqrt{6}\right)\sqrt{6}}{12}+\frac{\ln\left(3x-\sqrt{6}\right)}{3}-\frac{5\ln\left(3x-\sqrt{6}\right)\sqrt{6}}{12}$	52

```
[In] int((-5+2*x)/(3*x^2-2),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(3*x^2-2)+5/6*6^(1/2)*arctanh(1/2*x*6^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{-5+2x}{-2+3x^2}\,dx = \frac{5}{12}\sqrt{6}\log\left(\frac{3\,x^2+2\,\sqrt{6}x+2}{3\,x^2-2}\right)+\frac{1}{3}\,\log\left(3\,x^2-2\right)$$

```
[In] integrate((-5+2*x)/(3*x^2-2),x, algorithm="fricas")
[Out] 5/12*sqrt(6)*log((3*x^2 + 2*sqrt(6)*x + 2)/(3*x^2 - 2)) + 1/3*log(3*x^2 - 2
)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{-5+2x}{-2+3x^2}\,dx = \left(\frac{1}{3}-\frac{5\sqrt{6}}{12}\right)\log\left(x-\frac{\sqrt{6}}{3}\right)+\left(\frac{1}{3}+\frac{5\sqrt{6}}{12}\right)\log\left(x+\frac{\sqrt{6}}{3}\right)$$

```
[In] integrate((-5+2*x)/(3*x**2-2),x)
[Out] (1/3 - 5*sqrt(6)/12)*log(x - sqrt(6)/3) + (1/3 + 5*sqrt(6)/12)*log(x + sqrt
(6)/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{-5+2x}{-2+3x^2}\,dx = -\frac{5}{12}\sqrt{6}\log\left(\frac{3\,x-\sqrt{6}}{3\,x+\sqrt{6}}\right)+\frac{1}{3}\,\log\left(3\,x^2-2\right)$$

```
[In] integrate((-5+2*x)/(3*x^2-2),x, algorithm="maxima")
[Out] -5/12*sqrt(6)*log((3*x - sqrt(6))/(3*x + sqrt(6))) + 1/3*log(3*x^2 - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{-5+2x}{-2+3x^2}\,dx = \frac{1}{12}\left(5\sqrt{6}+4\right)\log\left(\left|x+\frac{1}{3}\sqrt{6}\right|\right) - \frac{1}{12}\left(5\sqrt{6}-4\right)\log\left(\left|x-\frac{1}{3}\sqrt{6}\right|\right)$$

```
[In] integrate((-5+2*x)/(3*x^2-2),x, algorithm="giac")
[Out] 1/12*(5*sqrt(6) + 4)*log(abs(x + 1/3*sqrt(6))) - 1/12*(5*sqrt(6) - 4)*log(a
bs(x - 1/3*sqrt(6)))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{-5+2x}{-2+3x^2}\,dx = \frac{\ln\left(x-\frac{\sqrt{6}}{3}\right)}{3} + \frac{\ln\left(x+\frac{\sqrt{6}}{3}\right)}{3} - \frac{5\sqrt{6}\ln\left(x-\frac{\sqrt{6}}{3}\right)}{12} + \frac{5\sqrt{6}\ln\left(x+\frac{\sqrt{6}}{3}\right)}{12}$$

```
[In] int((2*x - 5)/(3*x^2 - 2),x)
[Out] log(x - 6^(1/2)/3)/3 + log(x + 6^(1/2)/3)/3 - (5*6^(1/2)*log(x - 6^(1/2)/3)
)/12 + (5*6^(1/2)*log(x + 6^(1/2)/3))/12
```

3.28 $\int \frac{-5+2x}{2+3x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{-5+2x}{2+3x^2} dx = -\frac{5\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}} + \frac{1}{3}\log\left(2+3x^2\right)$$

```
[Out] 1/3*ln(3*x^2+2)-5/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {649, 209, 266}

$$\int \frac{-5+2x}{2+3x^2} dx = \frac{1}{3}\log\left(3x^2+2\right) - \frac{5\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

```
[In] Int[(-5 + 2*x)/(2 + 3*x^2),x]
```

```
[Out] (-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{x}{2+3x^2}\,dx - 5\int \frac{1}{2+3x^2}\,dx\\ &= -\frac{5\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}} + \frac{1}{3}\log\left(2+3x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{-5+2x}{2+3x^2}\,dx = -\frac{5\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}} + \frac{1}{3}\log\left(2+3x^2\right)$$

```
[In] Integrate[(-5 + 2*x)/(2 + 3*x^2),x]
[Out] (-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\ln(3x^2+2)}{3} - \frac{5\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$	24
risch	$\frac{\ln(9x^2+6)}{3} - \frac{5\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$	24
meijerg	$-\frac{5\sqrt{6}\,\arctan\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{6} + \frac{\ln\left(1+\frac{3x^2}{2}\right)}{3}$	27

```
[In] int((-5+2*x)/(3*x^2+2),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(3*x^2+2)-5/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{-5+2x}{2+3x^2} \, dx = -\frac{5}{6}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{1}{3}\,\log\left(3\,x^2+2\right)$$

```
[In] integrate((-5+2*x)/(3*x^2+2),x, algorithm="fricas")
[Out] -5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{-5+2x}{2+3x^2} \, dx = \frac{\log\left(x^2+\frac{2}{3}\right)}{3} - \frac{5\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

```
[In] integrate((-5+2*x)/(3*x**2+2),x)
[Out] log(x**2 + 2/3)/3 - 5*sqrt(6)*atan(sqrt(6)*x/2)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{-5+2x}{2+3x^2} \, dx = -\frac{5}{6}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{1}{3}\,\log\left(3\,x^2+2\right)$$

```
[In] integrate((-5+2*x)/(3*x^2+2),x, algorithm="maxima")
[Out] -5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(3*x^2 + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{-5+2x}{2+3x^2} \, dx = -\frac{5}{6}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{1}{3}\,\log\left(x^2+\frac{2}{3}\right)$$

```
[In] integrate((-5+2*x)/(3*x^2+2),x, algorithm="giac")
[Out] -5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(x^2 + 2/3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{-5+2x}{2+3x^2}\,dx = \frac{\ln\left(x^2+\frac{2}{3}\right)}{3} - \frac{5\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

```
[In] int((2*x - 5)/(3*x^2 + 2),x)
[Out] log(x^2 + 2/3)/3 - (5*6^(1/2)*atan((6^(1/2)*x)/2))/6
```

3.29 $\int \sin\left(\frac{x}{4}\right) \sin(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 21

$$\int \sin\left(\frac{x}{4}\right) \sin(x) \, dx = \frac{2}{3} \sin\left(\frac{3x}{4}\right) - \frac{2}{5} \sin\left(\frac{5x}{4}\right)$$

```
[Out] 2/3*sin(3/4*x)-2/5*sin(5/4*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4367}

$$\int \sin\left(\frac{x}{4}\right) \sin(x) \, dx = \frac{2}{3} \sin\left(\frac{3x}{4}\right) - \frac{2}{5} \sin\left(\frac{5x}{4}\right)$$

```
[In] Int[Sin[x/4]*Sin[x],x]
[Out] (2*Sin[(3*x)/4])/3 - (2*Sin[(5*x)/4])/5
```

Rule 4367

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2}{3} \sin\left(\frac{3x}{4}\right) - \frac{2}{5} \sin\left(\frac{5x}{4}\right)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx = \frac{2}{3}\sin\left(\frac{3x}{4}\right) - \frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

```
[In] Integrate[Sin[x/4]*Sin[x],x]
[Out] (2*Sin[(3*x)/4])/3 - (2*Sin[(5*x)/4])/5
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{2\sin\left(\frac{3x}{4}\right)}{3}-\frac{2\sin\left(\frac{5x}{4}\right)}{5}$	14
parallelrisch	$\frac{2\sin\left(\frac{3x}{4}\right)}{3}-\frac{2\sin\left(\frac{5x}{4}\right)}{5}$	14
derivativedivides	$-\frac{32\left(\sin^5\left(\frac{x}{4}\right)\right)}{5}+\frac{16\left(\sin^3\left(\frac{x}{4}\right)\right)}{3}$	18
default	$-\frac{32\left(\sin^5\left(\frac{x}{4}\right)\right)}{5}+\frac{16\left(\sin^3\left(\frac{x}{4}\right)\right)}{3}$	18
norman	$\frac{-\frac{8\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{8}\right)\right)}{15}+\frac{32\left(\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{8}\right)}{15}+\frac{8\tan\left(\frac{x}{2}\right)}{15}-\frac{32\tan\left(\frac{x}{8}\right)}{15}}{\left(1+\tan^2\left(\frac{x}{8}\right)\right)\left(1+\tan^2\left(\frac{x}{2}\right)\right)}$	59

```
[In] int(sin(1/4*x)*sin(x),x,method=_RETURNVERBOSE)
[Out] 2/3*sin(3/4*x)-2/5*sin(5/4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx = -\frac{16}{15}\left(6\,\cos\left(\frac{1}{4}\,x\right)^4 - 7\,\cos\left(\frac{1}{4}\,x\right)^2 + 1\right)\sin\left(\frac{1}{4}\,x\right)$$

```
[In] integrate(sin(1/4*x)*sin(x),x, algorithm="fricas")
[Out] -16/15*(6*cos(1/4*x)^4 - 7*cos(1/4*x)^2 + 1)*sin(1/4*x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \sin \left(\frac{x}{4}\right) \sin(x) \, dx = -\frac{16 \sin \left(\frac{x}{4}\right) \cos \left(x\right)}{15} + \frac{4 \sin \left(x\right) \cos \left(\frac{x}{4}\right)}{15}$$

```
[In] integrate(sin(1/4*x)*sin(x),x)
[Out] -16*sin(x/4)*cos(x)/15 + 4*sin(x)*cos(x/4)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sin \left(\frac{x}{4}\right) \sin(x) \, dx = -\frac{2}{5} \sin \left(\frac{5}{4} x\right) + \frac{2}{3} \sin \left(\frac{3}{4} x\right)$$

```
[In] integrate(sin(1/4*x)*sin(x),x, algorithm="maxima")
[Out] -2/5*sin(5/4*x) + 2/3*sin(3/4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin \left(\frac{x}{4}\right) \sin(x) \, dx = -\frac{32}{5} \sin \left(\frac{1}{4} x\right)^{5} + \frac{16}{3} \sin \left(\frac{1}{4} x\right)^{3}$$

```
[In] integrate(sin(1/4*x)*sin(x),x, algorithm="giac")
[Out] -32/5*sin(1/4*x)^5 + 16/3*sin(1/4*x)^3
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sin \left(\frac{x}{4}\right) \sin(x) \, dx = \frac{2 \sin \left(\frac{3 x}{4}\right)}{3} - \frac{2 \sin \left(\frac{5 x}{4}\right)}{5}$$

```
[In] int(sin(x/4)*sin(x),x)
[Out] (2*sin((3*x)/4))/3 - (2*sin((5*x)/4))/5
```

3.30 $\int \cos(3x)\cos(4x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \cos(3x)\cos(4x)\,dx = \frac{\sin(x)}{2} + \frac{1}{14}\sin(7x)$$

```
[Out] 1/2*sin(x)+1/14*sin(7*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4368}

$$\int \cos(3x)\cos(4x)\,dx = \frac{\sin(x)}{2} + \frac{1}{14}\sin(7x)$$

```
[In] Int[Cos[3*x]*Cos[4*x],x]
```

```
[Out] Sin[x]/2 + Sin[7*x]/14
```

Rule 4368

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\sin(x)}{2} + \frac{1}{14}\sin(7x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(3x)\cos(4x)\,dx = \frac{\sin(x)}{2} + \frac{1}{14}\sin(7x)$$

```
[In] Integrate[Cos[3*x]*Cos[4*x],x]
[Out] Sin[x]/2 + Sin[7*x]/14
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sin(x)}{2}+\frac{\sin(7x)}{14}$	12
risch	$\frac{\sin(x)}{2}+\frac{\sin(7x)}{14}$	12
parallelrisch	$\frac{\sin(x)}{2}+\frac{\sin(7x)}{14}$	12
norman	$\frac{-\frac{8\tan(2x)\left(\tan^2\left(\frac{3x}{2}\right)\right)}{7}+\frac{6\left(\tan^2(2x)\right)\tan\left(\frac{3x}{2}\right)}{7}+\frac{8\tan(2x)}{7}-\frac{6\tan\left(\frac{3x}{2}\right)}{7}}{\left(1+\tan^2\left(\frac{3x}{2}\right)\right)\left(1+\tan^2(2x)\right)}$	59

```
[In] int(cos(3*x)*cos(4*x),x,method=_RETURNVERBOSE)
[Out] 1/2*sin(x)+1/14*sin(7*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \cos(3x)\cos(4x)\,dx = \frac{1}{7}\left(32\cos(x)^6 - 40\cos(x)^4 + 12\cos(x)^2 + 3\right)\sin(x)$$

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="fricas")
[Out] 1/7*(32*cos(x)^6 - 40*cos(x)^4 + 12*cos(x)^2 + 3)*sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \cos(3x)\cos(4x)\,dx = -\frac{3\sin{(3x)}\cos{(4x)}}{7} + \frac{4\sin{(4x)}\cos{(3x)}}{7}$$

```
[In] integrate(cos(3*x)*cos(4*x),x)
[Out] -3*sin(3*x)*cos(4*x)/7 + 4*sin(4*x)*cos(3*x)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x)\cos(4x)\,dx = \frac{1}{14}\sin{(7x)} + \frac{1}{2}\sin{(x)}$$

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="maxima")
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x)\cos(4x)\,dx = \frac{1}{14}\sin{(7x)} + \frac{1}{2}\sin{(x)}$$

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="giac")
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x)\cos(4x)\,dx = \frac{\sin{(7x)}}{14} + \frac{\sin{(x)}}{2}$$

```
[In] int(cos(3*x)*cos(4*x),x)
[Out] sin(7*x)/14 + sin(x)/2
```

3.31 $\int -\tan(a-x)\tan(x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 21

$$\int -\tan(a-x)\tan(x)\,dx = -x+\cot(a)\log(\cos(a-x))-\cot(a)\log(\cos(x))$$

```
[Out] -x-cot(a)*ln(cos(x))+cot(a)*ln(cos(a-x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4708, 4706, 3556}

$$\int -\tan(a-x)\tan(x)\,dx = \cot(a)\log(\cos(a-x))-\cot(a)\log(\cos(x))-x$$

```
[In] Int[-(Tan[a - x]*Tan[x]),x]
```

```
[Out] -x + Cot[a]*Log[Cos[a - x]] - Cot[a]*Log[Cos[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4706

```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[-Csc[
(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
 - a*d, 0]
```

Rule 4708

```
Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(-b)*
(x/d), x] + Dist[(b/d)*Cos[(b*c - a*d)/d], Int[Sec[a + b*x]*Sec[c + d*x], x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -x+\cos(a)\int \sec(a-x)\sec(x)\,dx\\ &= -x+\cot(a)\int \tan(a-x)\,dx+\cot(a)\int \tan(x)\,dx\\ &= -x+\cot(a)\log(\cos(a-x))-\cot(a)\log(\cos(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int -\tan(a-x)\tan(x)\,dx = -x+\cot(a)\log(\cos(a-x))-\cot(a)\log(\cos(x))$$

```
[In] Integrate[-(Tan[a - x]*Tan[x]),x]
[Out] -x + Cot[a]*Log[Cos[a - x]] - Cot[a]*Log[Cos[x]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\ln(1+\tan(a)\tan(x))}{\tan(a)}-\arctan\left(\tan\left(x\right)\right)$	20
default	$\frac{\ln(1+\tan(a)\tan(x))}{\tan(a)}-\arctan\left(\tan\left(x\right)\right)$	20
risch	$-x+\frac{i\ln\left(\mathrm{e}^{2ia}+\mathrm{e}^{2ix}\right)\mathrm{e}^{2ia}}{\mathrm{e}^{2ia}-1}+\frac{i\ln\left(\mathrm{e}^{2ia}+\mathrm{e}^{2ix}\right)}{\mathrm{e}^{2ia}-1}-\frac{i\ln\left(\mathrm{e}^{2ix}+1\right)\mathrm{e}^{2ia}}{\mathrm{e}^{2ia}-1}-\frac{i\ln\left(\mathrm{e}^{2ix}+1\right)}{\mathrm{e}^{2ia}-1}$	103

```
[In] int(-tan(x)*tan(a-x),x,method=_RETURNVERBOSE)
[Out] 1/tan(a)*ln(1+tan(a)*tan(x))-arctan(tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(21) = 42$.

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.24

$$\int -\tan(a-x)\tan(x)\,dx$$
$$= \frac{(\cos(2a)+1)\log\left(-\frac{(\cos(2a)-1)\tan(x)^2-2\sin(2a)\tan(x)-\cos(2a)-1}{(\cos(2a)+1)\tan(x)^2+\cos(2a)+1}\right)-(\cos(2a)+1)\log\left(\frac{1}{\tan(x)^2+1}\right)-2x\sin(2}{2\sin(2a)}$$

```
[In] integrate(-tan(x)*tan(a-x),x, algorithm="fricas")
```

```
[Out] 1/2*((cos(2*a) + 1)*log(-((cos(2*a) - 1)*tan(x)^2 - 2*sin(2*a)*tan(x) - cos
(2*a) - 1)/((cos(2*a) + 1)*tan(x)^2 + cos(2*a) + 1)) - (cos(2*a) + 1)*log(1
/(tan(x)^2 + 1)) - 2*x*sin(2*a))/sin(2*a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(19) = 38$.

Time = 0.56 (sec) , antiderivative size = 138, normalized size of antiderivative = 6.57

$$\int -\tan(a-x)\tan(x)\,dx$$
$$= -\left(\begin{cases}\frac{2x\tan(a)}{2\tan^2(a)+2}-\frac{2\log\left(\tan(x)+\frac{1}{\tan(a)}\right)}{2\tan^2(a)+2}+\frac{\log(\tan^2(x)+1)}{2\tan^2(a)+2} & \text{for } a\neq 0\\ \frac{\log(\tan^2(x)+1)}{2} & \text{otherwise}\end{cases}\right)\tan(a)$$
$$+\begin{cases}-\frac{2x\tan(a)}{2\tan^3(a)+2\tan(a)}+\frac{2\log\left(\tan(x)+\frac{1}{\tan(a)}\right)}{2\tan^3(a)+2\tan(a)}+\frac{\log(\tan^2(x)+1)\tan^2(a)}{2\tan^3(a)+2\tan(a)} & \text{for } a\neq 0\\ -x+\tan(x) & \text{otherwise}\end{cases}$$

```
[In] integrate(-tan(x)*tan(a-x),x)
```

```
[Out] -Piecewise((2*x*tan(a)/(2*tan(a)**2 + 2) - 2*log(tan(x) + 1/tan(a))/(2*tan(
a)**2 + 2) + log(tan(x)**2 + 1)/(2*tan(a)**2 + 2), Ne(a, 0)), (log(tan(x)**
2 + 1)/2, True))*tan(a) + Piecewise((-2*x*tan(a)/(2*tan(a)**3 + 2*tan(a)) +
 2*log(tan(x) + 1/tan(a))/(2*tan(a)**3 + 2*tan(a)) + log(tan(x)**2 + 1)*tan
(a)**2/(2*tan(a)**3 + 2*tan(a)), Ne(a, 0)), (-x + tan(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(21) = 42$.

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 8.86

$$\int -\tan(a-x)\tan(x)\,dx = -\frac{(\cos(2a)^2+\sin(2a)^2-2\cos(2a)+1)x+(\cos(2a)^2+\sin(2a)^2-1)\arctan(\sin(2a)+\sin(2x),\cos$$

[In] integrate(-tan(x)*tan(a-x),x, algorithm="maxima")

[Out] -((cos(2*a)^2 + sin(2*a)^2 - 2*cos(2*a) + 1)*x + (cos(2*a)^2 + sin(2*a)^2 - 1)*arctan2(sin(2*a) + sin(2*x), cos(2*a) + cos(2*x)) - (cos(2*a)^2 + sin(2*a)^2 - 1)*arctan2(sin(2*x), cos(2*x) + 1) - log(cos(2*a)^2 + 2*cos(2*a)*cos(2*x) + cos(2*x)^2 + sin(2*a)^2 + 2*sin(2*a)*sin(2*x) + sin(2*x)^2)*sin(2*a) + log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*sin(2*a))/(cos(2*a)^2 + sin(2*a)^2 - 2*cos(2*a) + 1)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int -\tan(a-x)\tan(x)\,dx = -x + \frac{\log(|\tan(a)\tan(x)+1|)}{\tan(a)}$$

[In] integrate(-tan(x)*tan(a-x),x, algorithm="giac")

[Out] -x + log(abs(tan(a)*tan(x) + 1))/tan(a)

Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 118, normalized size of antiderivative = 5.62

$$\int -\tan(a-x)\tan(x)\,dx = -x - \frac{\frac{\sin(2a)\,\ln\left(\sin(2a+x)^2\,2i+\sin(2a)^2\,2i-\sin(x)^2\,2i+\sin(4a)-\sin(2x)+\sin(4a+2x)\right)}{2} - \frac{\sin(2a)\,\ln\left(\sin(2a)\,\left(2\sin(a)^2-1\right)-\sin(2a)^2\,1i+\sin(2}{2}}{\sin(a)^2}$$

[In] int(-tan(a - x)*tan(x),x)

[Out] - x - ((sin(2*a)*log(sin(4*a) - sin(2*x) + sin(4*a + 2*x) - sin(x)^2*2i + sin(2*a + x)^2*2i + sin(2*a)^2*2i))/2 - (sin(2*a)*log(sin(2*a)*(2*sin(a)^2 - 1) - sin(2*a)^2*1i + sin(2*a)*(2*sin(x)^2 - 1) - sin(2*a)*sin(2*x)*1i))/2)/sin(a)^2

3.32 $\int \sin^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{2}\sin(x)\cos(x)$$

```
[In] Int[Sin[x]^2,x]
```

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\text{integral} = -\frac{1}{2}\cos(x)\sin(x) + \frac{\int 1\,dx}{2}$$
$$= \frac{x}{2} - \frac{1}{2}\cos(x)\sin(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{4}\sin(2x)$$

```
[In] Integrate[Sin[x]^2,x]
[Out] x/2 - Sin[2*x]/4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
meijerg	$\frac{\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}}-\frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$	22
norman	$\frac{\tan^3\left(\frac{x}{2}\right)+x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}-\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

```
[In] int(sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = -\frac{1}{2}\,\cos{(x)}\sin{(x)} + \frac{1}{2}\,x$$

```
[In] integrate(sin(x)^2,x, algorithm="fricas")
[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{\sin{(x)}\cos{(x)}}{2}$$

```
[In] integrate(sin(x)**2,x)
[Out] x/2 - sin(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{1}{2}\,x - \frac{1}{4}\,\sin{(2\,x)}$$

```
[In] integrate(sin(x)^2,x, algorithm="maxima")
[Out] 1/2*x - 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{1}{2}\,x - \frac{1}{4}\,\sin{(2\,x)}$$

```
[In] integrate(sin(x)^2,x, algorithm="giac")
[Out] 1/2*x - 1/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{\sin(2\,x)}{4}$$

```
[In] int(sin(x)^2,x)
[Out] x/2 - sin(2*x)/4
```

3.33 $\int \cos^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\sin(x)\cos(x)$$

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\cos(x)\sin(x) + \frac{\int 1\,dx}{2}$$
$$= \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{4}\sin(2x)$$

```
[In] Integrate[Cos[x]^2,x]
[Out] x/2 + Sin[2*x]/4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2} + \frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2} + \frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2} + \frac{\sin(2x)}{4}$	11
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

```
[In] int(cos(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

```
[In] integrate(cos(x)**2,x)
[Out] x/2 + sin(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}x + \frac{1}{4}\sin(2x)$$

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
[Out] 1/2*x + 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}x + \frac{1}{4}\sin(2x)$$

```
[In] integrate(cos(x)^2,x, algorithm="giac")
[Out] 1/2*x + 1/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(2\,x)}{4}$$

```
[In] int(cos(x)^2,x)
[Out] x/2 + sin(2*x)/4
```

3.34 $\int \cos^3(x) \sin(x) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \cos^3(x) \sin(x) \, dx = -\frac{1}{4} \cos^4(x)$$

```
[Out] -1/4*cos(x)^4
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2645, 30}

$$\int \cos^3(x) \sin(x) \, dx = -\frac{1}{4} \cos^4(x)$$

```
[In] Int[Cos[x]^3*Sin[x],x]
```

```
[Out] -1/4*Cos[x]^4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int x^3\,dx, x, \cos(x)\right) \\ &= -\frac{1}{4}\cos^4(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cos^3(x)\sin(x)\,dx = -\frac{1}{4}\cos^4(x)$$

```
[In] Integrate[Cos[x]^3*Sin[x],x]
[Out] -1/4*Cos[x]^4
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{(\cos^4(x))}{4}$	7
default	$-\frac{(\cos^4(x))}{4}$	7
risch	$-\frac{\cos(4x)}{32}-\frac{\cos(2x)}{8}$	14
parallelrisch	$-\frac{\cos(4x)}{32}+\frac{5}{32}-\frac{\cos(2x)}{8}$	15
norman	$\frac{2(\tan^2(\frac{x}{2}))+2(\tan^6(\frac{x}{2}))}{(1+\tan^2(\frac{x}{2}))^4}$	29
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{8}+\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(4x)}{\sqrt{\pi}}\right)}{32}$	38

```
[In] int(cos(x)^3*sin(x),x,method=_RETURNVERBOSE)
[Out] -1/4*cos(x)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos^3(x)\sin(x)\,dx = -\frac{1}{4}\cos(x)^4$$

```
[In] integrate(cos(x)^3*sin(x),x, algorithm="fricas")
[Out] -1/4*cos(x)^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \cos^3(x)\sin(x)\,dx = -\frac{\cos^4(x)}{4}$$

```
[In] integrate(cos(x)**3*sin(x),x)
[Out] -cos(x)**4/4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos^3(x)\sin(x)\,dx = -\frac{1}{4}\cos(x)^4$$

```
[In] integrate(cos(x)^3*sin(x),x, algorithm="maxima")
[Out] -1/4*cos(x)^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos^3(x)\sin(x)\,dx = -\frac{1}{4}\cos(x)^4$$

```
[In] integrate(cos(x)^3*sin(x),x, algorithm="giac")
[Out] -1/4*cos(x)^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \cos^3(x)\sin(x)\,dx = -\frac{\sin(x)^2\left(\sin(x)^2-2\right)}{4}$$

```
[In] int(cos(x)^3*sin(x),x)
[Out] -(sin(x)^2*(sin(x)^2 - 2))/4
```

3.35 $\int \cot^3(x)\csc(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \cot^3(x)\csc(x)\,dx = \csc(x) - \frac{\csc^3(x)}{3}$$

```
[Out] -1/3/sin(x)^3+1/sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{2686\}$

$$\int \cot^3(x)\csc(x)\,dx = \csc(x) - \frac{\csc^3(x)}{3}$$

```
[In] Int[Cot[x]^3*Csc[x],x]
```

```
[Out] Csc[x] - Csc[x]^3/3
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \csc(x)\right)\\ &= \csc(x) - \frac{\csc^3(x)}{3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc(x)\,dx = \csc(x) - \frac{\csc^3(x)}{3}$$

```
[In] Integrate[Cot[x]^3*Csc[x],x]
[Out] Csc[x] - Csc[x]^3/3
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

method	result	size
parallelrisch	$-\frac{(\csc^3(x))(-1+3\cos(2x))}{6}$	15
default	$-\frac{\cos^4(x)}{3\sin(x)^3}+\frac{\cos^4(x)}{3\sin(x)}+\frac{(2+\cos^2(x))\sin(x)}{3}$	32
norman	$\frac{-\frac{1}{24}+\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}-\frac{\left(\tan^6\left(\frac{x}{2}\right)\right)}{24}}{\tan\left(\frac{x}{2}\right)^3}$	34
risch	$\frac{2i(3\,e^{5ix}-2\,e^{3ix}+3\,e^{ix})}{3(e^{2ix}-1)^3}$	35

```
[In] int(cos(x)^3/sin(x)^4,x,method=_RETURNVERBOSE)
[Out] -1/6*csc(x)^3*(-1+3*cos(2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\,\cos(x)^2-2}{3\,\left(\cos(x)^2-1\right)\sin(x)}$$

```
[In] integrate(cos(x)^3/sin(x)^4,x, algorithm="fricas")
[Out] 1/3*(3*cos(x)^2 - 2)/((cos(x)^2 - 1)*sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \cot^3(x)\csc(x)\,dx = -\frac{1-3\sin^2{(x)}}{3\sin^3{(x)}}$$

```
[In] integrate(cos(x)**3/sin(x)**4,x)
[Out] -(1 - 3*sin(x)**2)/(3*sin(x)**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\,\sin{(x)}^2-1}{3\,\sin{(x)}^3}$$

```
[In] integrate(cos(x)^3/sin(x)^4,x, algorithm="maxima")
[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\,\sin{(x)}^2-1}{3\,\sin{(x)}^3}$$

```
[In] integrate(cos(x)^3/sin(x)^4,x, algorithm="giac")
[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc(x)\,dx = \frac{\sin{(x)}^2-\frac{1}{3}}{\sin{(x)}^3}$$

```
[In] int(cos(x)^3/sin(x)^4,x)
[Out] (sin(x)^2 - 1/3)/sin(x)^3
```

3.36 $\int \csc^2(x)\sec^2(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 7

$$\int \csc^2(x)\sec^2(x)\,dx = -\cot(x)+\tan(x)$$

```
[Out] -cot(x)+tan(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2700, 14}

$$\int \csc^2(x)\sec^2(x)\,dx = \tan(x)-\cot(x)$$

```
[In] Int[Csc[x]^2*Sec[x]^2,x]
```

```
[Out] -Cot[x] + Tan[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx,x,\tan(x)\right)\\ &= \text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx,x,\tan(x)\right)\\ &= -\cot(x)+\tan(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \csc^2(x)\sec^2(x)\,dx = -2\cot(2x)$$

```
[In] Integrate[Csc[x]^2*Sec[x]^2,x]
[Out] -2*Cot[2*x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

method	result	size
parallelrisch	$-2\cot(x)+\sec(x)\csc(x)$	11
default	$\frac{1}{\cos(x)\sin(x)}-2\cot(x)$	15
risch	$-\frac{4i}{(e^{2ix}+1)(e^{2ix}-1)}$	22
norman	$\frac{\frac{1}{2}-3\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)\tan\left(\frac{x}{2}\right)}$	36

```
[In] int(1/cos(x)^2/sin(x)^2,x,method=_RETURNVERBOSE)
[Out] -2*cot(x)+sec(x)*csc(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(7) = 14$.

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.57

$$\int \csc^2(x)\sec^2(x)\,dx = -\frac{2\cos(x)^2 - 1}{\cos(x)\sin(x)}$$

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="fricas")
[Out] -(2*cos(x)^2 - 1)/(cos(x)*sin(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(5) = 10$.

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int \csc^2(x)\sec^2(x)\,dx = -\frac{2\cos(2x)}{\sin(2x)}$$

```
[In] integrate(1/cos(x)**2/sin(x)**2,x)
[Out] -2*cos(2*x)/sin(2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc^2(x)\sec^2(x)\,dx = -\frac{1}{\tan(x)} + \tan(x)$$

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="maxima")
[Out] -1/tan(x) + tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc^2(x)\sec^2(x)\,dx = -\frac{1}{\tan(x)} + \tan(x)$$

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="giac")
[Out] -1/tan(x) + tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \csc^2(x)\sec^2(x)\,dx = -2\cot(2\,x)$$

```
[In] int(1/(cos(x)^2*sin(x)^2),x)
[Out] -2*cot(2*x)
```

3.37 $\int \cot^2\left(\frac{3x}{4}\right) dx$

Optimal result

Integrand size = 8, antiderivative size = 14

$$\int \cot^2\left(\frac{3x}{4}\right) dx = -x - \frac{4}{3}\cot\left(\frac{3x}{4}\right)$$

```
[Out] -x-4/3*cot(3/4*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3554, 8}

$$\int \cot^2\left(\frac{3x}{4}\right) dx = -x - \frac{4}{3}\cot\left(\frac{3x}{4}\right)$$

```
[In] Int[Cot[(3*x)/4]^2,x]
```

```
[Out] -x - (4*Cot[(3*x)/4])/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\text{integral} = -\frac{4}{3}\cot\left(\frac{3x}{4}\right) - \int 1\,dx$$
$$= -x - \frac{4}{3}\cot\left(\frac{3x}{4}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx = -\frac{4}{3}\cot\left(\frac{3x}{4}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2\left(\frac{3x}{4}\right)\right)$$

```
[In] Integrate[Cot[(3*x)/4]^2,x]
[Out] (-4*Cot[(3*x)/4]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[(3*x)/4]^2])/3
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

method	result	size
norman	$\frac{-\frac{4}{3}-x\tan\left(\frac{3x}{4}\right)}{\tan\left(\frac{3x}{4}\right)}$	17
risch	$-x-\frac{8i}{3\left(e^{\frac{3ix}{2}}-1\right)}$	17
derivativedivides	$-\frac{4\cot\left(\frac{3x}{4}\right)}{3}+\frac{2\pi}{3}-\frac{4\,\mathrm{arccot}\left(\cot\left(\frac{3x}{4}\right)\right)}{3}$	18
default	$-\frac{4\cot\left(\frac{3x}{4}\right)}{3}+\frac{2\pi}{3}-\frac{4\,\mathrm{arccot}\left(\cot\left(\frac{3x}{4}\right)\right)}{3}$	18
parallelrisch	$\frac{-3x\tan\left(\frac{3x}{4}\right)-4}{3\tan\left(\frac{3x}{4}\right)}$	18

```
[In] int(cot(3/4*x)^2,x,method=_RETURNVERBOSE)
[Out] (-4/3-x*tan(3/4*x))/tan(3/4*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx = -\frac{3\,x\sin\left(\frac{3}{2}\,x\right) + 4\,\cos\left(\frac{3}{2}\,x\right) + 4}{3\,\sin\left(\frac{3}{2}\,x\right)}$$

```
[In] integrate(cot(3/4*x)^2,x, algorithm="fricas")
[Out] -1/3*(3*x*sin(3/2*x) + 4*cos(3/2*x) + 4)/sin(3/2*x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx = -x - \frac{4\cos\left(\frac{3x}{4}\right)}{3\sin\left(\frac{3x}{4}\right)}$$

```
[In] integrate(cot(3/4*x)**2,x)
[Out] -x - 4*cos(3*x/4)/(3*sin(3*x/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx = -x - \frac{4}{3\,\tan\left(\frac{3}{4}\,x\right)}$$

```
[In] integrate(cot(3/4*x)^2,x, algorithm="maxima")
[Out] -x - 4/3/tan(3/4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx = -x - \frac{2}{3\,\tan\left(\frac{3}{8}\,x\right)} + \frac{2}{3}\,\tan\left(\frac{3}{8}\,x\right)$$

```
[In] integrate(cot(3/4*x)^2,x, algorithm="giac")
[Out] -x - 2/3/tan(3/8*x) + 2/3*tan(3/8*x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx = -x - \frac{4\cot\left(\frac{3\,x}{4}\right)}{3}$$

```
[In] int(cot((3*x)/4)^2,x)
[Out] - x - (4*cot((3*x)/4))/3
```

3.38 $\int (1+\tan(2x))^2\,dx$

Optimal result

Integrand size = 8, antiderivative size = 16

$$\int (1+\tan(2x))^2\,dx = -\log(\cos(2x)) + \frac{1}{2}\tan(2x)$$

```
[Out] -ln(cos(2*x))+1/2*tan(2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3558, 3556}

$$\int (1+\tan(2x))^2\,dx = \frac{1}{2}\tan(2x) - \log(\cos(2x))$$

```
[In] Int[(1 + Tan[2*x])^2,x]
```

```
[Out] -Log[Cos[2*x]] + Tan[2*x]/2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\tan(2x) + 2\int \tan(2x)\,dx$$
$$= -\log(\cos(2x)) + \frac{1}{2}\tan(2x)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int (1+\tan(2x))^2\,dx = x - \frac{1}{2}\arctan(\tan(2x)) - \log(\cos(2x)) + \frac{1}{2}\tan(2x)$$

```
[In] Integrate[(1 + Tan[2*x])^2,x]
[Out] x - ArcTan[Tan[2*x]]/2 - Log[Cos[2*x]] + Tan[2*x]/2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$\frac{\tan(2x)}{2} + \frac{\ln(1+\tan^2(2x))}{2}$	19
default	$\frac{\tan(2x)}{2} + \frac{\ln(1+\tan^2(2x))}{2}$	19
norman	$\frac{\tan(2x)}{2} + \frac{\ln(1+\tan^2(2x))}{2}$	19
parallelrisch	$\frac{\tan(2x)}{2} + \frac{\ln(1+\tan^2(2x))}{2}$	19
parts	$x + \frac{\tan(2x)}{2} - \frac{\arctan(\tan(2x))}{2} + \frac{\ln(1+\tan^2(2x))}{2}$	27
risch	$2ix + \frac{i}{e^{4ix}+1} - \ln\left(e^{4ix}+1\right)$	28

```
[In] int((1+tan(2*x))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*tan(2*x)+1/2*ln(1+tan(2*x)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (1+\tan(2x))^2\,dx = -\frac{1}{2}\,\log\left(\frac{1}{\tan\left(2\,x\right)^2+1}\right)+\frac{1}{2}\,\tan\left(2\,x\right)$$

```
[In] integrate((1+tan(2*x))^2,x, algorithm="fricas")
[Out] -1/2*log(1/(tan(2*x)^2 + 1)) + 1/2*tan(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (1+\tan(2x))^2\,dx = \frac{\log\left(\tan^2\left(2x\right)+1\right)}{2}+\frac{\tan\left(2x\right)}{2}$$

```
[In] integrate((1+tan(2*x))**2,x)
[Out] log(tan(2*x)**2 + 1)/2 + tan(2*x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (1+\tan(2x))^2\,dx = \log\left(\sec\left(2\,x\right)\right)+\frac{1}{2}\,\tan\left(2\,x\right)$$

```
[In] integrate((1+tan(2*x))^2,x, algorithm="maxima")
[Out] log(sec(2*x)) + 1/2*tan(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (1+\tan(2x))^2\,dx = -\frac{1}{2}\,\log\left(\frac{4}{\tan\left(2\,x\right)^2+1}\right)+\frac{1}{2}\,\tan\left(2\,x\right)$$

```
[In] integrate((1+tan(2*x))^2,x, algorithm="giac")
[Out] -1/2*log(4/(tan(2*x)^2 + 1)) + 1/2*tan(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (1 + \tan(2x))^2 \, dx = \frac{\tan(2\,x)}{2} + \frac{\ln\left(\tan(2\,x)^2 + 1\right)}{2}$$

```
[In] int((tan(2*x) + 1)^2,x)
[Out] tan(2*x)/2 + log(tan(2*x)^2 + 1)/2
```

3.39 $\int (-\cot(x)+\tan(x))^2\,dx$

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int (-\cot(x)+\tan(x))^2\,dx=-4x-\cot(x)+\tan(x)$$

```
[Out] -4*x-cot(x)+tan(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {472, 209}

$$\int (-\cot(x)+\tan(x))^2\,dx=-4x+\tan(x)-\cot(x)$$

```
[In] Int[(-Cot[x] + Tan[x])^2,x]
```

```
[Out] -4*x - Cot[x] + Tan[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

```
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1-x^2)^2}{x^2(1+x^2)}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{4}{1+x^2}\right)\,dx, x, \tan(x)\right)\\
&= -\cot(x)+\tan(x)-4\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\
&= -4x-\cot(x)+\tan(x)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int (-\cot(x)+\tan(x))^2\,dx = -2x-\arctan(\tan(x)) - \cot(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(x)\right)+\tan(x)$$

```
[In] Integrate[(-Cot[x] + Tan[x])^2,x]
[Out] -2*x - ArcTan[Tan[x]] - Cot[x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[x]^2] +
 Tan[x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$-4x-\cot(x)+\tan(x)$	11
parallelrisch	$-4x-\cot(x)+\tan(x)$	11
norman	$\frac{-1+\tan^2(x)-4x\tan(x)}{\tan(x)}$	17
parts	$-\cot(x)+\frac{\pi}{2}-\operatorname{arccot}(\cot(x))+\tan(x)-\arctan(\tan(x))-2x$	24
risch	$-4x-\frac{4i}{(e^{2ix}+1)(e^{2ix}-1)}$	26

```
[In] int((-cot(x)+tan(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4*x-cot(x)+tan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int (-\cot(x) + \tan(x))^2\, dx = -\frac{4\, x \tan\left(x\right) - \tan\left(x\right)^2 + 1}{\tan\left(x\right)}$$

```
[In] integrate((-cot(x)+tan(x))^2,x, algorithm="fricas")
[Out] -(4*x*tan(x) - tan(x)^2 + 1)/tan(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (-\cot(x) + \tan(x))^2\, dx = -4x + \tan\left(x\right) - \frac{1}{\tan\left(x\right)}$$

```
[In] integrate((-cot(x)+tan(x))**2,x)
[Out] -4*x + tan(x) - 1/tan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (-\cot(x) + \tan(x))^2\, dx = -4\, x - \frac{1}{\tan\left(x\right)} + \tan\left(x\right)$$

```
[In] integrate((-cot(x)+tan(x))^2,x, algorithm="maxima")
[Out] -4*x - 1/tan(x) + tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (-\cot(x) + \tan(x))^2 \, dx = -4\,x - \frac{1}{\tan{(x)}} + \tan{(x)}$$

```
[In] integrate((-cot(x)+tan(x))^2,x, algorithm="giac")
[Out] -4*x - 1/tan(x) + tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (-\cot(x) + \tan(x))^2 \, dx = \tan(x) - 4\,x - \frac{1}{\tan{(x)}}$$

```
[In] int((cot(x) - tan(x))^2,x)
[Out] tan(x) - 4*x - 1/tan(x)
```

3.40 $\int (-\sec(x)+\tan(x))^2 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int (-\sec(x)+\tan(x))^2 \, dx = -x - \frac{2\cos(x)}{1+\sin(x)}$$

```
[Out] -x-2*cos(x)/(1+sin(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4476, 2749, 2759, 8}

$$\int (-\sec(x)+\tan(x))^2 \, dx = -x - \frac{2\cos(x)}{\sin(x)+1}$$

```
[In] Int[(-Sec[x] + Tan[x])^2,x]
```

```
[Out] -x - (2*Cos[x])/(1 + Sin[x])
```

<u>Rule 8</u>

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

<u>Rule 2749</u>

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sec^2(x)(-1+\sin(x))^2\,dx\\ &= \int \frac{\cos^2(x)}{(-1-\sin(x))^2}\,dx\\ &= -\frac{2\cos(x)}{1+\sin(x)} - \int 1\,dx\\ &= -x - \frac{2\cos(x)}{1+\sin(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (-\sec(x)+\tan(x))^2\,dx = -\arctan(\tan(x)) - 2\sec(x) + 2\tan(x)$$

```
[In] Integrate[(-Sec[x] + Tan[x])^2,x]
[Out] -ArcTan[Tan[x]] - 2*Sec[x] + 2*Tan[x]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$-x+2\tan\left(x\right)-\frac{2}{\cos\left(x\right)}$	15
parts	$2\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)-2\sec\left(x\right)$	15
risch	$-x-\frac{4}{i+e^{ix}}$	17

[In] int((-sec(x)+tan(x))^2,x,method=_RETURNVERBOSE)

[Out] -x+2*tan(x)-2/cos(x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int (-\sec(x)+\tan(x))^2\,dx=-\frac{(x+2)\cos\left(x\right)+(x-2)\sin\left(x\right)+x+2}{\cos\left(x\right)+\sin\left(x\right)+1}$$

[In] integrate((-sec(x)+tan(x))^2,x, algorithm="fricas")

[Out] -((x + 2)*cos(x) + (x - 2)*sin(x) + x + 2)/(cos(x) + sin(x) + 1)

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (-\sec(x)+\tan(x))^2\,dx=-x+2\tan\left(x\right)-2\sec\left(x\right)$$

[In] integrate((-sec(x)+tan(x))**2,x)

[Out] -x + 2*tan(x) - 2*sec(x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (-\sec(x)+\tan(x))^2\,dx = -x - \frac{2}{\cos(x)} + 2\tan(x)$$

```
[In] integrate((-sec(x)+tan(x))^2,x, algorithm="maxima")
[Out] -x - 2/cos(x) + 2*tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (-\sec(x)+\tan(x))^2\,dx = -x - \frac{4}{\tan\left(\frac{1}{2}x\right)+1}$$

```
[In] integrate((-sec(x)+tan(x))^2,x, algorithm="giac")
[Out] -x - 4/(tan(1/2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (-\sec(x)+\tan(x))^2\,dx = -x - \frac{4}{\tan\left(\frac{x}{2}\right)+1}$$

```
[In] int((tan(x) - 1/cos(x))^2,x)
[Out] - x - 4/(tan(x/2) + 1)
```

3.41 $\int \frac{\sin(x)}{1+\sin(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = x+\frac{\cos(x)}{1+\sin(x)}$$

```
[Out] x+cos(x)/(1+sin(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2814, 2727}

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = x+\frac{\cos(x)}{\sin(x)+1}$$

```
[In] Int[Sin[x]/(1 + Sin[x]),x]
```

```
[Out] x + Cos[x]/(1 + Sin[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = x - \int \frac{1}{1+\sin(x)}\,dx$$
$$= x + \frac{\cos(x)}{1+\sin(x)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = x - \frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

```
[In] Integrate[Sin[x]/(1 + Sin[x]),x]
[Out] x - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

method	result	size
risch	$x+\frac{2}{i+e^{ix}}$	15
default	$2\arctan\left(\tan\left(\frac{x}{2}\right)\right)+\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	19
parallelrisch	$\frac{x\tan\left(\frac{x}{2}\right)+2+x}{1+\tan\left(\frac{x}{2}\right)}$	19
norman	$\frac{x+x\tan\left(\frac{x}{2}\right)+x\left(\tan^{2}\left(\frac{x}{2}\right)\right)+2\left(\tan^{2}\left(\frac{x}{2}\right)\right)+\left(\tan^{3}\left(\frac{x}{2}\right)\right)x+2}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}$	53

```
[In] int(sin(x)/(sin(x)+1),x,method=_RETURNVERBOSE)
[Out] x+2/(I+exp(I*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = \frac{(x+1)\cos(x)+(x-1)\sin(x)+x+1}{\cos(x)+\sin(x)+1}$$

```
[In] integrate(sin(x)/(1+sin(x)),x, algorithm="fricas")
[Out] ((x + 1)*cos(x) + (x - 1)*sin(x) + x + 1)/(cos(x) + sin(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(8) = 16$.

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.64

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = \frac{x\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)+1} + \frac{x}{\tan\left(\frac{x}{2}\right)+1} + \frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

```
[In] integrate(sin(x)/(1+sin(x)),x)
[Out] x*tan(x/2)/(tan(x/2) + 1) + x/(tan(x/2) + 1) + 2/(tan(x/2) + 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = \frac{2}{\frac{\sin(x)}{\cos(x)+1}+1} + 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

```
[In] integrate(sin(x)/(1+sin(x)),x, algorithm="maxima")
[Out] 2/(sin(x)/(cos(x) + 1) + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = x + \frac{2}{\tan\left(\frac{1}{2}\,x\right)+1}$$

```
[In] integrate(sin(x)/(1+sin(x)),x, algorithm="giac")
[Out] x + 2/(tan(1/2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = x + \frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

```
[In] int(sin(x)/(sin(x) + 1),x)
[Out] x + 2/(tan(x/2) + 1)
```

3.42 $\int \frac{\cos(x)}{1-\cos(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{\cos(x)}{1-\cos(x)}\,dx = -x-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -x-sin(x)/(1-cos(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2814, 2727}

$$\int \frac{\cos(x)}{1-\cos(x)}\,dx = -x-\frac{\sin(x)}{1-\cos(x)}$$

```
[In] Int[Cos[x]/(1 - Cos[x]),x]
[Out] -x - Sin[x]/(1 - Cos[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -x + \int \frac{1}{1-\cos(x)}\,dx \\
&= -x - \frac{\sin(x)}{1-\cos(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$
\int \frac{\cos(x)}{1-\cos(x)}\,dx = -\csc(x)\left(1+\cos(x)+2\arcsin\left(\sqrt{\sin^2\left(\frac{x}{2}\right)}\right)\sqrt{\sin^2(x)}\right)
$$

```
[In] Integrate[Cos[x]/(1 - Cos[x]),x]
[Out] -(Csc[x]*(1 + Cos[x] + 2*ArcSin[Sqrt[Sin[x/2]^2]]*Sqrt[Sin[x]^2]))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)-\frac{1}{\tan\left(\frac{x}{2}\right)}$	17
risch	$-x-\frac{2i}{{e}^{ix}-1}$	17
parallelrisch	$\frac{-x\tan\left(\frac{x}{2}\right)-1}{\tan\left(\frac{x}{2}\right)}$	17
norman	$\frac{-1-\left(\tan^{2}\left(\frac{x}{2}\right)\right)-x\tan\left(\frac{x}{2}\right)-\left(\tan^{3}\left(\frac{x}{2}\right)\right)x}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)}$	44

```
[In] int(cos(x)/(1-cos(x)),x,method=_RETURNVERBOSE)
[Out] -2*arctan(tan(1/2*x))-1/tan(1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\cos(x)}{1-\cos(x)}\, dx = -\frac{x \sin{(x)} + \cos{(x)} + 1}{\sin{(x)}}$$

```
[In] integrate(cos(x)/(1-cos(x)),x, algorithm="fricas")
[Out] -(x*sin(x) + cos(x) + 1)/sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{\cos(x)}{1-\cos(x)}\, dx = -x - \frac{1}{\tan{\left(\frac{x}{2}\right)}}$$

```
[In] integrate(cos(x)/(1-cos(x)),x)
[Out] -x - 1/tan(x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{\cos(x)}{1-\cos(x)}\, dx = -\frac{\cos{(x)} + 1}{\sin{(x)}} - 2 \arctan\left(\frac{\sin{(x)}}{\cos{(x)} + 1}\right)$$

```
[In] integrate(cos(x)/(1-cos(x)),x, algorithm="maxima")
[Out] -(cos(x) + 1)/sin(x) - 2*arctan(sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\cos(x)}{1-\cos(x)}\, dx = -x - \frac{1}{\tan{\left(\frac{1}{2}\, x\right)}}$$

```
[In] integrate(cos(x)/(1-cos(x)),x, algorithm="giac")
[Out] -x - 1/tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\cos(x)}{1-\cos(x)}\,dx = -x - \cot\left(\frac{x}{2}\right)$$

```
[In] int(-cos(x)/(cos(x) - 1),x)
[Out] - x - cot(x/2)
```

3.43 $\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx=2e^{-x/2}-6e^{x/2}+e^x+3x$$

```
[Out] 2/exp(1/2*x)-6*exp(1/2*x)+exp(x)+3*x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2280, 45}

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx=3x+2e^{-x/2}-6e^{x/2}+e^x$$

```
[In] Int[(-1 + E^(x/2))^3/E^(x/2),x]
```

```
[Out] 2/E^(x/2) - 6*E^(x/2) + E^x + 3*x
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2280

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[
```

```
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Den
ominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= 2\text{Subst}\left(\int \frac{(-1+x)^3}{x^2}\,dx, x, e^{x/2}\right)\\ &= 2\text{Subst}\left(\int \left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx, x, e^{x/2}\right)\\ &= 2e^{-x/2}-6e^{x/2}+e^x+3x \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx = e^{-x/2}\left(2-6e^x+e^{3x/2}\right)+6\log\left(e^{x/2}\right)$$

```
[In] Integrate[(-1 + E^(x/2))^3/E^(x/2),x]
[Out] (2 - 6*E^x + E^((3*x)/2))/E^(x/2) + 6*Log[E^(x/2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
risch	$\mathrm{e}^{x}+3x-6\,\mathrm{e}^{\frac{x}{2}}+2\,\mathrm{e}^{-\frac{x}{2}}$	19
parts	$\mathrm{e}^{x}+3x-6\,\mathrm{e}^{\frac{x}{2}}+2\,\mathrm{e}^{-\frac{x}{2}}$	25
derivativedivides	$\mathrm{e}^{x}-6\,\mathrm{e}^{\frac{x}{2}}+6\ln\left(\mathrm{e}^{\frac{x}{2}}\right)+2\,\mathrm{e}^{-\frac{x}{2}}$	29
default	$\mathrm{e}^{x}-6\,\mathrm{e}^{\frac{x}{2}}+6\ln\left(\mathrm{e}^{\frac{x}{2}}\right)+2\,\mathrm{e}^{-\frac{x}{2}}$	29
norman	$\left(2+\mathrm{e}^{\frac{3x}{2}}-6\,\mathrm{e}^{x}+3x\,\mathrm{e}^{\frac{x}{2}}\right)\mathrm{e}^{-\frac{x}{2}}$	31
parallelrisch	$-\left(-2-\mathrm{e}^{\frac{3x}{2}}-6\ln\left(\mathrm{e}^{\frac{x}{2}}\right)\mathrm{e}^{\frac{x}{2}}+6\,\mathrm{e}^{x}\right)\mathrm{e}^{-\frac{x}{2}}$	38

```
[In] int((-1+exp(1/2*x))^3/exp(1/2*x),x,method=_RETURNVERBOSE)
[Out] exp(x)+3*x-6*exp(1/2*x)+2*exp(-1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx=\left(3\,xe^{\left(\frac{1}{2}\,x\right)}+e^{\left(\frac{3}{2}\,x\right)}-6\,e^{x}+2\right)e^{\left(-\frac{1}{2}\,x\right)}$$

```
[In] integrate((-1+exp(1/2*x))^3/exp(1/2*x),x, algorithm="fricas")
[Out] (3*x*e^(1/2*x) + e^(3/2*x) - 6*e^x + 2)*e^(-1/2*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx=3x-6e^{\frac{x}{2}}+e^{x}+2e^{-\frac{x}{2}}$$

```
[In] integrate((-1+exp(1/2*x))**3/exp(1/2*x),x)
[Out] 3*x - 6*exp(x/2) + exp(x) + 2*exp(-x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx=3\,x-6\,e^{\left(\frac{1}{2}\,x\right)}+2\,e^{\left(-\frac{1}{2}\,x\right)}+e^{x}$$

```
[In] integrate((-1+exp(1/2*x))^3/exp(1/2*x),x, algorithm="maxima")
[Out] 3*x - 6*e^(1/2*x) + 2*e^(-1/2*x) + e^x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx=3\,x-6\,e^{\left(\frac{1}{2}\,x\right)}+2\,e^{\left(-\frac{1}{2}\,x\right)}+e^{x}$$

```
[In] integrate((-1+exp(1/2*x))^3/exp(1/2*x),x, algorithm="giac")
[Out] 3*x - 6*e^(1/2*x) + 2*e^(-1/2*x) + e^x
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx=3\,x+2\,\mathrm{e}^{-\frac{x}{2}}-6\,\mathrm{e}^{x/2}+\mathrm{e}^{x}$$

```
[In] int(exp(-x/2)*(exp(x/2) - 1)^3,x)
[Out] 3*x + 2*exp(-x/2) - 6*exp(x/2) + exp(x)
```

3.44 $\int \frac{1}{5-6x+x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \frac{1}{5-6x+x^2} dx = -\frac{1}{4}\log(1-x) + \frac{1}{4}\log(5-x)$$

```
[Out] -1/4*ln(1-x)+1/4*ln(5-x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {630, 31}

$$\int \frac{1}{5-6x+x^2} dx = \frac{1}{4}\log(5-x) - \frac{1}{4}\log(1-x)$$

```
[In] Int[(5 - 6*x + x^2)^(-1),x]
```

```
[Out] -1/4*Log[1 - x] + Log[5 - x]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\int \frac{1}{-5+x}\,dx - \frac{1}{4}\int \frac{1}{-1+x}\,dx\\ &= -\frac{1}{4}\log(1-x) + \frac{1}{4}\log(5-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{5-6x+x^2}\,dx = -\frac{1}{4}\log(1-x) + \frac{1}{4}\log(5-x)$$

```
[In] Integrate[(5 - 6*x + x^2)^(-1),x]
[Out] -1/4*Log[1 - x] + Log[5 - x]/4
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{\ln(-1+x)}{4}+\frac{\ln(x-5)}{4}$	14
norman	$-\frac{\ln(-1+x)}{4}+\frac{\ln(x-5)}{4}$	14
risch	$-\frac{\ln(-1+x)}{4}+\frac{\ln(x-5)}{4}$	14
parallelrisch	$-\frac{\ln(-1+x)}{4}+\frac{\ln(x-5)}{4}$	14

```
[In] int(1/(x^2-6*x+5),x,method=_RETURNVERBOSE)
[Out] -1/4*ln(-1+x)+1/4*ln(x-5)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{5-6x+x^2}\,dx = -\frac{1}{4}\,\log\left(x-1\right) + \frac{1}{4}\,\log\left(x-5\right)$$

```
[In] integrate(1/(x^2-6*x+5),x, algorithm="fricas")
[Out] -1/4*log(x - 1) + 1/4*log(x - 5)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{1}{5-6x+x^2}\,dx = \frac{\log(x-5)}{4} - \frac{\log(x-1)}{4}$$

```
[In] integrate(1/(x**2-6*x+5),x)
[Out] log(x - 5)/4 - log(x - 1)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{5-6x+x^2}\,dx = -\frac{1}{4}\log(x-1) + \frac{1}{4}\log(x-5)$$

```
[In] integrate(1/(x^2-6*x+5),x, algorithm="maxima")
[Out] -1/4*log(x - 1) + 1/4*log(x - 5)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{5-6x+x^2}\,dx = -\frac{1}{4}\log(|x-1|) + \frac{1}{4}\log(|x-5|)$$

```
[In] integrate(1/(x^2-6*x+5),x, algorithm="giac")
[Out] -1/4*log(abs(x - 1)) + 1/4*log(abs(x - 5))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{1}{5-6x+x^2}\,dx = -\frac{\operatorname{atanh}\left(\frac{x}{2}-\frac{3}{2}\right)}{2}$$

```
[In] int(1/(x^2 - 6*x + 5),x)
[Out] -atanh(x/2 - 3/2)/2
```

3.45 $\int \frac{x^2}{13-6x^3+x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \frac{x^2}{13-6x^3+x^6} \, dx = \frac{1}{6} \arctan \left(\frac{1}{2}(-3+x^3) \right)$$

```
[Out] 1/6*arctan(1/2*x^3-3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1366, 632, 210}

$$\int \frac{x^2}{13-6x^3+x^6} \, dx = \frac{1}{6} \arctan \left(\frac{1}{2}(x^3-3) \right)$$

```
[In] Int[x^2/(13 - 6*x^3 + x^6),x]
[Out] ArcTan[(-3 + x^3)/2]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{13-6x+x^2}\,dx, x, x^3\right)\\ &= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{-16-x^2}\,dx, x, 2(-3+x^3)\right)\right)\\ &= \frac{1}{6}\arctan\left(\frac{1}{2}(-3+x^3)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{13-6x^3+x^6}\,dx = \frac{1}{6}\arctan\left(\frac{1}{2}(-3+x^3)\right)$$

```
[In] Integrate[x^2/(13 - 6*x^3 + x^6),x]
[Out] ArcTan[(-3 + x^3)/2]/6
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\arctan\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$	11
risch	$\frac{\arctan\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$	11
parallelrisch	$\frac{i\ln(x^3+2i-3)}{12}-\frac{i\ln(x^3-2i-3)}{12}$	24

```
[In] int(x^2/(x^6-6*x^3+13),x,method=_RETURNVERBOSE)
[Out] 1/6*arctan(1/2*x^3-3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{13-6x^3+x^6}\,dx=\frac{1}{6}\,\arctan\left(\frac{1}{2}\,x^3-\frac{3}{2}\right)$$

```
[In] integrate(x^2/(x^6-6*x^3+13),x, algorithm="fricas")
[Out] 1/6*arctan(1/2*x^3 - 3/2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{13-6x^3+x^6}\,dx=\frac{\operatorname{atan}\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$$

```
[In] integrate(x**2/(x**6-6*x**3+13),x)
[Out] atan(x**3/2 - 3/2)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{13-6x^3+x^6}\,dx=\frac{1}{6}\,\arctan\left(\frac{1}{2}\,x^3-\frac{3}{2}\right)$$

```
[In] integrate(x^2/(x^6-6*x^3+13),x, algorithm="maxima")
[Out] 1/6*arctan(1/2*x^3 - 3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{13-6x^3+x^6}\,dx=\frac{1}{6}\,\arctan\left(\frac{1}{2}\,x^3-\frac{3}{2}\right)$$

```
[In] integrate(x^2/(x^6-6*x^3+13),x, algorithm="giac")
[Out] 1/6*arctan(1/2*x^3 - 3/2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{13-6x^3+x^6}\,dx = \frac{\operatorname{atan}\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$$

```
[In] int(x^2/(x^6 - 6*x^3 + 13),x)
[Out] atan(x^3/2 - 3/2)/6
```

3.46 $\int \frac{2+x}{-1-4x+x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{2+x}{-1-4x+x^2} \, dx = \frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(2-\sqrt{5}-x\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(2+\sqrt{5}-x\right)$$

```
[Out] 1/10*ln(2-x-5^(1/2))*(5-4*5^(1/2))+1/10*ln(2-x+5^(1/2))*(5+4*5^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {646, 31}

$$\int \frac{2+x}{-1-4x+x^2} \, dx = \frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(-x-\sqrt{5}+2\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(-x+\sqrt{5}+2\right)$$

```
[In] Int[(2 + x)/(-1 - 4*x + x^2),x]
```

```
[Out] ((5 - 4*Sqrt[5])*Log[2 - Sqrt[5] - x])/10 + ((5 + 4*Sqrt[5])*Log[2 + Sqrt[5
] - x])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
```

```
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{10}\left(-5+4\sqrt{5}\right)\int \frac{1}{-2+\sqrt{5}+x}\,dx\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\int \frac{1}{-2-\sqrt{5}+x}\,dx\\ &= \frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(2-\sqrt{5}-x\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(2+\sqrt{5}-x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{2+x}{-1-4x+x^2}\,dx=\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(2+\sqrt{5}-x\right)+\frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(-2+\sqrt{5}+x\right)$$

```
[In] Integrate[(2 + x)/(-1 - 4*x + x^2),x]
[Out] ((5 + 4*Sqrt[5])*Log[2 + Sqrt[5] - x])/10 + ((5 - 4*Sqrt[5])*Log[-2 + Sqrt[
5] + x])/10
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{\ln\left(x^2-4x-1\right)}{2}-\frac{4\sqrt{5}\,\operatorname{arctanh}\left(\frac{(-4+2x)\sqrt{5}}{10}\right)}{5}$	29
risch	$\frac{\ln\left(x-2-\sqrt{5}\right)}{2}+\frac{2\ln\left(x-2-\sqrt{5}\right)\sqrt{5}}{5}+\frac{\ln\left(x-2+\sqrt{5}\right)}{2}-\frac{2\ln\left(x-2+\sqrt{5}\right)\sqrt{5}}{5}$	48

```
[In] int((2+x)/(x^2-4*x-1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2-4*x-1)-4/5*5^(1/2)*arctanh(1/10*(-4+2*x)*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{2+x}{-1-4x+x^2}\,dx = \frac{2}{5}\sqrt{5}\log\left(\frac{x^2-2\sqrt{5}(x-2)-4x+9}{x^2-4x-1}\right)+\frac{1}{2}\log\left(x^2-4x-1\right)$$

```
[In] integrate((2+x)/(x^2-4*x-1),x, algorithm="fricas")
[Out] 2/5*sqrt(5)*log((x^2 - 2*sqrt(5)*(x - 2) - 4*x + 9)/(x^2 - 4*x - 1)) + 1/2*
log(x^2 - 4*x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{2+x}{-1-4x+x^2}\,dx = \left(\frac{1}{2}-\frac{2\sqrt{5}}{5}\right)\log\left(x-2+\sqrt{5}\right)+\left(\frac{1}{2}+\frac{2\sqrt{5}}{5}\right)\log\left(x-\sqrt{5}-2\right)$$

```
[In] integrate((2+x)/(x**2-4*x-1),x)
[Out] (1/2 - 2*sqrt(5)/5)*log(x - 2 + sqrt(5)) + (1/2 + 2*sqrt(5)/5)*log(x - sqrt
(5) - 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{2+x}{-1-4x+x^2}\,dx = \frac{2}{5}\sqrt{5}\log\left(\frac{x-\sqrt{5}-2}{x+\sqrt{5}-2}\right)+\frac{1}{2}\log\left(x^2-4x-1\right)$$

```
[In] integrate((2+x)/(x^2-4*x-1),x, algorithm="maxima")
[Out] 2/5*sqrt(5)*log((x - sqrt(5) - 2)/(x + sqrt(5) - 2)) + 1/2*log(x^2 - 4*x -
1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{2+x}{-1-4x+x^2}\,dx = \frac{2}{5}\sqrt{5}\log\left(\frac{|2x-2\sqrt{5}-4|}{|2x+2\sqrt{5}-4|}\right)+\frac{1}{2}\log\left(|x^2-4x-1|\right)$$

```
[In] integrate((2+x)/(x^2-4*x-1),x, algorithm="giac")
```

```
[Out] 2/5*sqrt(5)*log(abs(2*x - 2*sqrt(5) - 4)/abs(2*x + 2*sqrt(5) - 4)) + 1/2*lo
g(abs(x^2 - 4*x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{2+x}{-1-4x+x^2}\,dx = \ln\left(x-\sqrt{5}-2\right)\left(\frac{2\sqrt{5}}{5}+\frac{1}{2}\right)-\ln\left(x+\sqrt{5}-2\right)\left(\frac{2\sqrt{5}}{5}-\frac{1}{2}\right)$$

```
[In] int(-(x + 2)/(4*x - x^2 + 1),x)
```

```
[Out] log(x - 5^(1/2) - 2)*((2*5^(1/2))/5 + 1/2) - log(x + 5^(1/2) - 2)*((2*5^(1/
2))/5 - 1/2)
```

3.47 $\int \frac{1}{1+\sqrt[3]{1+x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = -3\sqrt[3]{1+x}+\frac{3}{2}(1+x)^{2/3}+3\log\left(1+\sqrt[3]{1+x}\right)$$

```
[Out] -3*(1+x)^(1/3)+3/2*(1+x)^(2/3)+3*ln(1+(1+x)^(1/3))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {253, 196, 45}

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = \frac{3}{2}(x+1)^{2/3}-3\sqrt[3]{x+1}+3\log\left(\sqrt[3]{x+1}+1\right)$$

```
[In] Int[(1 + (1 + x)^(1/3))^(-1),x]
```

```
[Out] -3*(1 + x)^(1/3) + (3*(1 + x)^(2/3))/2 + 3*Log[1 + (1 + x)^(1/3)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
```

```
IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{1+\sqrt[3]{x}}\,dx, x, 1+x\right)\\
&= 3\text{Subst}\left(\int \frac{x^2}{1+x}\,dx, x, \sqrt[3]{1+x}\right)\\
&= 3\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x}\right)\,dx, x, \sqrt[3]{1+x}\right)\\
&= -3\sqrt[3]{1+x}+\frac{3}{2}(1+x)^{2/3}+3\log\left(1+\sqrt[3]{1+x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = \frac{3}{2}\sqrt[3]{1+x}\left(-2+\sqrt[3]{1+x}\right)+3\log\left(1+\sqrt[3]{1+x}\right)$$

```
[In] Integrate[(1 + (1 + x)^(1/3))^(-1),x]
[Out] (3*(1 + x)^(1/3)*(-2 + (1 + x)^(1/3)))/2 + 3*Log[1 + (1 + x)^(1/3)]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-3(1+x)^{\frac{1}{3}}+\frac{3(1+x)^{\frac{2}{3}}}{2}+3\ln\left(1+(1+x)^{\frac{1}{3}}\right)$
trager	$-3(1+x)^{\frac{1}{3}}+\frac{3(1+x)^{\frac{2}{3}}}{2}+\ln\left(-3(1+x)^{\frac{2}{3}}-3(1+x)^{\frac{1}{3}}-x-2\right)$
default	$\ln(2+x)+\frac{3(1+x)^{\frac{2}{3}}}{2}+2\ln\left(1+(1+x)^{\frac{1}{3}}\right)-\ln\left((1+x)^{\frac{2}{3}}-(1+x)^{\frac{1}{3}}+1\right)-3(1+x)^{\frac{1}{3}}$

```
[In] int(1/(1+(1+x)^(1/3)),x,method=_RETURNVERBOSE)
```

[Out] -3*(1+x)^(1/3)+3/2*(1+x)^(2/3)+3*ln(1+(1+x)^(1/3))

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = \frac{3}{2}\,(x+1)^{\frac{2}{3}} - 3\,(x+1)^{\frac{1}{3}} + 3\,\log\left((x+1)^{\frac{1}{3}} + 1\right)$$

[In] integrate(1/(1+(1+x)^(1/3)),x, algorithm="fricas")

[Out] 3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = \frac{3(x+1)^{\frac{2}{3}}}{2} - 3\sqrt[3]{x+1} + 3\log\left(\sqrt[3]{x+1} + 1\right)$$

[In] integrate(1/(1+(1+x)**(1/3)),x)

[Out] 3*(x + 1)**(2/3)/2 - 3*(x + 1)**(1/3) + 3*log((x + 1)**(1/3) + 1)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = \frac{3}{2}\,(x+1)^{\frac{2}{3}} - 3\,(x+1)^{\frac{1}{3}} + 3\,\log\left((x+1)^{\frac{1}{3}} + 1\right)$$

[In] integrate(1/(1+(1+x)^(1/3)),x, algorithm="maxima")

[Out] 3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = \frac{3}{2}\,(x+1)^{\frac{2}{3}} - 3\,(x+1)^{\frac{1}{3}} + 3\,\log\left((x+1)^{\frac{1}{3}} + 1\right)$$

```
[In] integrate(1/(1+(1+x)^(1/3)),x, algorithm="giac")
[Out] 3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = 3\,\ln\left((x+1)^{1/3}+1\right) - 3\,(x+1)^{1/3} + \frac{3\,(x+1)^{2/3}}{2}$$

```
[In] int(1/((x + 1)^(1/3) + 1),x)
[Out] 3*log((x + 1)^(1/3) + 1) - 3*(x + 1)^(1/3) + (3*(x + 1)^(2/3))/2
```

3.48 $\int \frac{1}{\sqrt{x}(b+ax)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx=\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] 2*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {65, 211}

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx=\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[In] Int[1/(Sqrt[x]*(b + a*x)),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)$$
$$= \frac{2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[In] Integrate[1/(Sqrt[x]*(b + a*x)),x]
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19
default	$\frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19

```
[In] int(1/(a*x+b)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{ax-b-2\sqrt{-ab}\sqrt{x}}{ax+b}\right)}{ab}, -\frac{2\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{a\sqrt{x}}\right)}{ab}\right]$$

```
[In] integrate(1/(a*x+b)/x^(1/2),x, algorithm="fricas")
[Out] [-sqrt(-a*b)*log((a*x - b - 2*sqrt(-a*b)*sqrt(x))/(a*x + b))/(a*b), -2*sqrt
(a*b)*arctan(sqrt(a*b)/(a*sqrt(x)))/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(27) = 54$.

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.52

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a=0 \wedge b=0 \\ \frac{2\sqrt{x}}{b} & \text{for } a=0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b=0 \\ \frac{\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{a\sqrt{-\frac{b}{a}}} - \frac{\log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{a\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*x+b)/x**(1/2),x)
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b, Eq(a, 0)), (-2/
(a*sqrt(x)), Eq(b, 0)), (log(sqrt(x) - sqrt(-b/a))/(a*sqrt(-b/a)) - log(sqr
t(x) + sqrt(-b/a))/(a*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

```
[In] integrate(1/(a*x+b)/x^(1/2),x, algorithm="maxima")
[Out] 2*arctan(a*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

```
[In] integrate(1/(a*x+b)/x^(1/2),x, algorithm="giac")
[Out] 2*arctan(a*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[In] int(1/(x^(1/2)*(b + a*x)),x)
[Out] (2*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(a^(1/2)*b^(1/2))
```

3.49 $\int x^3\sqrt{1+x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int x^3\sqrt{1+x^2}\,dx=-\frac{1}{3}\left(1+x^2\right)^{3/2}+\frac{1}{5}\left(1+x^2\right)^{5/2}$$

```
[Out] -1/3*(x^2+1)^(3/2)+1/5*(x^2+1)^(5/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\int x^3\sqrt{1+x^2}\,dx=\frac{1}{5}\left(x^2+1\right)^{5/2}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[In] Int[x^3*Sqrt[1 + x^2],x]
```

```
[Out] -1/3*(1 + x^2)^(3/2) + (1 + x^2)^(5/2)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x\sqrt{1+x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\sqrt{1+x}+(1+x)^{3/2}\right)\,dx, x, x^2\right)\\ &= -\frac{1}{3}\left(1+x^2\right)^{3/2}+\frac{1}{5}\left(1+x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{1+x^2}\,dx = \frac{1}{15}\left(1+x^2\right)^{3/2}\left(-2+3x^2\right)$$

```
[In] Integrate[x^3*Sqrt[1 + x^2],x]
[Out] ((1 + x^2)^(3/2)*(-2 + 3*x^2))/15
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{\left(x^2+1\right)^{\frac{3}{2}}\left(3x^2-2\right)}{15}$	17
pseudoelliptic	$\frac{\left(x^2+1\right)^{\frac{3}{2}}\left(3x^2-2\right)}{15}$	17
risch	$\frac{\left(3x^4+x^2-2\right)\sqrt{x^2+1}}{15}$	20
trager	$\left(\frac{1}{5}x^4+\frac{1}{15}x^2-\frac{2}{15}\right)\sqrt{x^2+1}$	21
default	$\frac{x^2\left(x^2+1\right)^{\frac{3}{2}}}{5}-\frac{2\left(x^2+1\right)^{\frac{3}{2}}}{15}$	23
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(x^2+1\right)^{\frac{3}{2}}\left(-3x^2+2\right)}{15}}{4\sqrt{\pi}}$	31

```
[In] int(x^3*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/15*(x^2+1)^(3/2)*(3*x^2-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^3\sqrt{1+x^2}\,dx = \frac{1}{15}\left(3\,x^4+x^2-2\right)\sqrt{x^2+1}$$

```
[In] integrate(x^3*(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*x^4 + x^2 - 2)*sqrt(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int x^3\sqrt{1+x^2}\,dx = \frac{x^4\sqrt{x^2+1}}{5}+\frac{x^2\sqrt{x^2+1}}{15}-\frac{2\sqrt{x^2+1}}{15}$$

```
[In] integrate(x**3*(x**2+1)**(1/2),x)
[Out] x**4*sqrt(x**2 + 1)/5 + x**2*sqrt(x**2 + 1)/15 - 2*sqrt(x**2 + 1)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^3\sqrt{1+x^2}\,dx = \frac{1}{5}\left(x^2+1\right)^{\frac{3}{2}}x^2-\frac{2}{15}\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x^3*(x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/5*(x^2 + 1)^(3/2)*x^2 - 2/15*(x^2 + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^3\sqrt{1+x^2}\,dx = \frac{1}{5}\left(x^2+1\right)^{\frac{5}{2}}-\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x^3*(x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/5*(x^2 + 1)^(5/2) - 1/3*(x^2 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{1+x^2}\,dx = \sqrt{x^2+1}\left(\frac{x^4}{5}+\frac{x^2}{15}-\frac{2}{15}\right)$$

```
[In] int(x^3*(x^2 + 1)^(1/2),x)
[Out] (x^2 + 1)^(1/2)*(x^2/15 + x^4/5 - 2/15)
```

3.50 $\int \frac{x}{\sqrt{a^4-x^4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \frac{x}{\sqrt{a^4-x^4}} \, dx = \frac{1}{2}\arctan\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)$$

```
[Out] 1/2*arctan(x^2/(a^4-x^4)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {281, 223, 209}

$$\int \frac{x}{\sqrt{a^4-x^4}} \, dx = \frac{1}{2}\arctan\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)$$

```
[In] Int[x/Sqrt[a^4 - x^4],x]
```

```
[Out] ArcTan[x^2/Sqrt[a^4 - x^4]]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a^4-x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x^2}{\sqrt{a^4-x^4}}\right)\\ &= \frac{1}{2}\arctan\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{x}{\sqrt{a^4-x^4}}\,dx = -\frac{1}{2}i\log\left(ix^2+\sqrt{a^4-x^4}\right)$$

```
[In] Integrate[x/Sqrt[a^4 - x^4],x]
[Out] (-1/2*I)*Log[I*x^2 + Sqrt[a^4 - x^4]]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\arctan\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)}{2}$	19
elliptic	$\frac{\arctan\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)}{2}$	19
pseudoelliptic	$-\frac{i\ln\left(ix^2+\sqrt{a^4-x^4}\right)}{2}$	23

```
[In] int(x/(a^4-x^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x^2/(a^4-x^4)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{x}{\sqrt{a^4-x^4}}\,dx = -\arctan\left(-\frac{a^2-\sqrt{a^4-x^4}}{x^2}\right)$$

```
[In] integrate(x/(a^4-x^4)^(1/2),x, algorithm="fricas")
[Out] -arctan(-(a^2 - sqrt(a^4 - x^4))/x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{x}{\sqrt{a^4-x^4}}\,dx = \begin{cases} -\frac{i\operatorname{acosh}\left(\frac{x^2}{a^2}\right)}{2} & \text{for } \left|\frac{x^4}{a^4}\right| > 1 \\ \frac{\operatorname{asin}\left(\frac{x^2}{a^2}\right)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x/(a**4-x**4)**(1/2),x)
[Out] Piecewise((-I*acosh(x**2/a**2)/2, Abs(x**4/a**4) > 1), (asin(x**2/a**2)/2,
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{a^4-x^4}}\,dx = -\frac{1}{2}\arctan\left(\frac{\sqrt{a^4-x^4}}{x^2}\right)$$

```
[In] integrate(x/(a^4-x^4)^(1/2),x, algorithm="maxima")
[Out] -1/2*arctan(sqrt(a^4 - x^4)/x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.45

$$\int \frac{x}{\sqrt{a^4 - x^4}}\,dx = \frac{1}{2}\arcsin\left(\frac{x^2}{a^2}\right)$$

```
[In] integrate(x/(a^4-x^4)^(1/2),x, algorithm="giac")
[Out] 1/2*arcsin(x^2/a^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{a^4 - x^4}}\,dx = \frac{\operatorname{atan}\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)}{2}$$

```
[In] int(x/(a^4 - x^4)^(1/2),x)
[Out] atan(x^2/(a^4 - x^4)^(1/2))/2
```

3.51 $\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 22

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}$$

```
[Out] arctan((-a^2+x^2)^(1/2)/a)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {272, 65, 209}

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{x^2-a^2}}{a}\right)}{a}$$

```
[In] Int[1/(x*Sqrt[-a^2 + x^2]),x]
```

```
[Out] ArcTan[Sqrt[-a^2 + x^2]/a]/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{-a^2+x}}\,dx, x, x^2\right) \\ &= \text{Subst}\left(\int \frac{1}{a^2+x^2}\,dx, x, \sqrt{-a^2+x^2}\right) \\ &= \frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}$$

```
[In] Integrate[1/(x*Sqrt[-a^2 + x^2]),x]
[Out] ArcTan[Sqrt[-a^2 + x^2]/a]/a
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}$	21
default	$-\frac{\ln\left(\frac{-2a^2+2\sqrt{-a^2}\,\sqrt{-a^2+x^2}}{x}\right)}{\sqrt{-a^2}}$	41

```
[In] int(1/(-a^2+x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] arctan((-a^2+x^2)^(1/2)/a)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \frac{2\arctan\left(-\frac{x-\sqrt{-a^2+x^2}}{a}\right)}{a}$$

```
[In] integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="fricas")
[Out] 2*arctan(-(x - sqrt(-a^2 + x^2))/a)/a
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \begin{cases} \frac{i\operatorname{acosh}\left(\frac{a}{x}\right)}{a} & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ -\frac{\operatorname{asin}\left(\frac{a}{x}\right)}{a} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/(-a**2+x**2)**(1/2),x)
[Out] Piecewise((I*acosh(a/x)/a, Abs(a**2/x**2) > 1), (-asin(a/x)/a, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = -\frac{\arcsin\left(\frac{a}{|x|}\right)}{a}$$

```
[In] integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] -arcsin(a/abs(x))/a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}$$

```
[In] integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(-a^2 + x^2)/a)/a
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{x^2-a^2}}{\sqrt{a^2}}\right)}{\sqrt{a^2}}$$

```
[In] int(1/(x*(x^2 - a^2)^(1/2)),x)
[Out] atan((x^2 - a^2)^(1/2)/(a^2)^(1/2))/(a^2)^(1/2)
```

3.52 $\int \frac{1}{x\sqrt{a^2-x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

```
[Out] -arctanh((a^2-x^2)^(1/2)/a)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {272, 65, 212}

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

```
[In] Int[1/(x*Sqrt[a^2 - x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[a^2 - x^2]/a]/a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a^2-xx}}\,dx, x, x^2\right)\\ &= -\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, \sqrt{a^2-x^2}\right)\\ &= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 53 vs. 2(23) = 46.

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{\log\left(a+\sqrt{a^2-x^2}\right)}{2a} + \frac{\log\left(-a^2+a\sqrt{a^2-x^2}\right)}{2a}$$

```
[In] Integrate[1/(x*Sqrt[a^2 - x^2]),x]
[Out] -1/2*Log[a + Sqrt[a^2 - x^2]]/a + Log[-a^2 + a*Sqrt[a^2 - x^2]]/(2*a)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

method	result	size
default	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2-x^2}}{x}\right)}{\sqrt{a^2}}$	37
pseudoelliptic	$\frac{\ln\left(-a+\sqrt{a^2-x^2}\right)-\ln\left(a+\sqrt{a^2-x^2}\right)}{2a}$	39

```
[In] int(1/x/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] -1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2-x^2)^(1/2))/x)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx=\frac{\log\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)}{a}$$

[In] integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="fricas")

[Out] log(-(a - sqrt(a^2 - x^2))/x)/a

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx=\begin{cases}-\frac{\operatorname{acosh}\left(\frac{a}{x}\right)}{a} & \text{for } \left|\frac{a^2}{x^2}\right|>1\\ \frac{i\operatorname{asin}\left(\frac{a}{x}\right)}{a} & \text{otherwise}\end{cases}$$

[In] integrate(1/x/(a**2-x**2)**(1/2),x)

[Out] Piecewise((-acosh(a/x)/a, Abs(a**2/x**2) > 1), (I*asin(a/x)/a, True))

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx=-\frac{\log\left(\frac{2\,a^2}{|x|}+\frac{2\,\sqrt{a^2-x^2}a}{|x|}\right)}{a}$$

[In] integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="maxima")

[Out] -log(2*a^2/abs(x) + 2*sqrt(a^2 - x^2)*a/abs(x))/a

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \frac{1}{x\sqrt{a^2 - x^2}}\,dx = -\frac{\log\left(\left|a + \sqrt{a^2 - x^2}\right|\right)}{2\,a} + \frac{\log\left(\left|-a + \sqrt{a^2 - x^2}\right|\right)}{2\,a}$$

```
[In] integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="giac")
[Out] -1/2*log(abs(a + sqrt(a^2 - x^2)))/a + 1/2*log(abs(-a + sqrt(a^2 - x^2)))/a
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{a^2 - x^2}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{a^2 - x^2}}{a}\right)}{a}$$

```
[In] int(1/(x*(a^2 - x^2)^(1/2)),x)
[Out] -atanh((a^2 - x^2)^(1/2)/a)/a
```

3.53 $\int \frac{1}{x\sqrt{a^2+x^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{x\sqrt{a^2+x^2}} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}$$

```
[Out] -arctanh((a^2+x^2)^(1/2)/a)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {272, 65, 213}

$$\int \frac{1}{x\sqrt{a^2+x^2}} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}$$

```
[In] Int[1/(x*Sqrt[a^2 + x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[a^2 + x^2]/a]/a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a^2+x}}\,dx, x, x^2\right)\\ &= \text{Subst}\left(\int \frac{1}{-a^2+x^2}\,dx, x, \sqrt{a^2+x^2}\right)\\ &= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 49 vs. $2(21) = 42$.

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{\log\left(a+\sqrt{a^2+x^2}\right)}{2a} + \frac{\log\left(-a^2+a\sqrt{a^2+x^2}\right)}{2a}$$

```
[In] Integrate[1/(x*Sqrt[a^2 + x^2]),x]
[Out] -1/2*Log[a + Sqrt[a^2 + x^2]]/a + Log[-a^2 + a*Sqrt[a^2 + x^2]]/(2*a)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

method	result	size
default	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2+x^2}}{x}\right)}{\sqrt{a^2}}$	35
pseudoelliptic	$\frac{\ln\left(-a+\sqrt{a^2+x^2}\right)-\ln\left(a+\sqrt{a^2+x^2}\right)}{2a}$	35

```
[In] int(1/x/(a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2+x^2)^(1/2))/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(19) = 38$.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{\log\left(a-x+\sqrt{a^2+x^2}\right)-\log\left(-a-x+\sqrt{a^2+x^2}\right)}{a}$$

```
[In] integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="fricas")
[Out] -(log(a - x + sqrt(a^2 + x^2)) - log(-a - x + sqrt(a^2 + x^2)))/a
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.33

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{a}$$

```
[In] integrate(1/x/(a**2+x**2)**(1/2),x)
[Out] -asinh(a/x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{\operatorname{arsinh}\left(\frac{a}{|x|}\right)}{a}$$

```
[In] integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] -arcsinh(a/abs(x))/a
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{\log\left(a+\sqrt{a^2+x^2}\right)}{2\,a} + \frac{\log\left(-a+\sqrt{a^2+x^2}\right)}{2\,a}$$

```
[In] integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="giac")
[Out] -1/2*log(a + sqrt(a^2 + x^2))/a + 1/2*log(-a + sqrt(a^2 + x^2))/a
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{a^2+x^2}}{\sqrt{-a^2}}\right)}{\sqrt{-a^2}}$$

```
[In] int(1/(x*(a^2 + x^2)^(1/2)),x)
[Out] atan((a^2 + x^2)^(1/2)/(-a^2)^(1/2))/(-a^2)^(1/2)
```

3.54 $\int \frac{1}{\sqrt{2+x-x^2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = -\arcsin\left(\frac{1}{3}(1-2x)\right)$$

```
[Out] arcsin(-1/3+2/3*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {633, 222}

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = -\arcsin\left(\frac{1}{3}(1-2x)\right)$$

```
[In] Int[1/Sqrt[2 + x - x^2],x]
```

```
[Out] -ArcSin[(1 - 2*x)/3]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx, x, 1-2x\right)\right)$$
$$= -\arcsin\left(\frac{1}{3}(1-2x)\right)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{2+x-x^2}}{1+x}\right)$$

```
[In] Integrate[1/Sqrt[2 + x - x^2],x]
[Out] -2*ArcTan[Sqrt[2 + x - x^2]/(1 + x)]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

method	result	size
default	$\arcsin\left(-\frac{1}{3}+\frac{2x}{3}\right)$	7
trager	$\text{RootOf}\left(\_Z^2+1\right)\ln\left(-2\,\text{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-x^2+x+2}+\text{RootOf}\left(\_Z^2+1\right)\right)$	37

```
[In] int(1/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(-1/3+2/3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(6) = 12$.

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = -\arctan\left(\frac{\sqrt{-x^2+x+2}(2\,x-1)}{2\,(x^2-x-2)}\right)$$

```
[In] integrate(1/(-x^2+x+2)^(1/2),x, algorithm="fricas")
[Out] -arctan(1/2*sqrt(-x^2 + x + 2)*(2*x - 1)/(x^2 - x - 2))
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = \operatorname{asin}\left(\frac{2x}{3}-\frac{1}{3}\right)$$

```
[In] integrate(1/(-x**2+x+2)**(1/2),x)
[Out] asin(2*x/3 - 1/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = -\arcsin\left(-\frac{2}{3}\,x+\frac{1}{3}\right)$$

```
[In] integrate(1/(-x^2+x+2)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-2/3*x + 1/3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(6) = 12.

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = \frac{1}{4}\,\sqrt{-x^2+x+2}(2\,x-1)+\frac{9}{8}\,\arcsin\left(\frac{2}{3}\,x-\frac{1}{3}\right)$$

```
[In] integrate(1/(-x^2+x+2)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(-x^2 + x + 2)*(2*x - 1) + 9/8*arcsin(2/3*x - 1/3)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = \operatorname{asin}\left(\frac{2\,x}{3}-\frac{1}{3}\right)$$

```
[In] int(1/(x - x^2 + 2)^(1/2),x)
[Out] asin((2*x)/3 - 1/3)
```

3.55 $\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 19

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = -\frac{\operatorname{arcsinh}\left(\frac{2-3x}{\sqrt{11}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arcsinh(1/11*(2-3*x)*11^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {633, 221}

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = -\frac{\operatorname{arcsinh}\left(\frac{2-3x}{\sqrt{11}}\right)}{\sqrt{3}}$$

```
[In] Int[1/Sqrt[5 - 4*x + 3*x^2],x]
[Out] -(ArcSinh[(2 - 3*x)/Sqrt[11]]/Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{44}}}\,dx, x, -4+6x\right)}{2\sqrt{33}}$$
$$= -\frac{\text{arcsinh}\left(\frac{2-3x}{\sqrt{11}}\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = -\frac{\log\left(2-3x+\sqrt{3}\sqrt{5-4x+3x^2}\right)}{\sqrt{3}}$$

```
[In] Integrate[1/Sqrt[5 - 4*x + 3*x^2],x]
[Out] -(Log[2 - 3*x + Sqrt[3]*Sqrt[5 - 4*x + 3*x^2]]/Sqrt[3])
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\sqrt{3}\,\text{arcsinh}\left(\frac{3\sqrt{11}\left(x-\frac{2}{3}\right)}{11}\right)}{3}$	15
trager	$\frac{\text{RootOf}\left(\_Z^2-3\right)\ln\left(3\,\text{RootOf}\left(\_Z^2-3\right)x+3\sqrt{3x^2-4x+5}-2\,\text{RootOf}\left(\_Z^2-3\right)\right)}{3}$	42

```
[In] int(1/(3*x^2-4*x+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*3^(1/2)*arcsinh(3/11*11^(1/2)*(x-2/3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(16) = 32.

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(-2\,\sqrt{3}\sqrt{3\,x^2-4\,x+5}(3\,x-2)-18\,x^2+24\,x-19\right)$$

```
[In] integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log(-2*sqrt(3)*sqrt(3*x^2 - 4*x + 5)*(3*x - 2) - 18*x^2 + 24*x
- 19)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = \frac{\sqrt{3}\operatorname{asinh}\left(\frac{3\sqrt{11}\left(x-\frac{2}{3}\right)}{11}\right)}{3}$$

```
[In] integrate(1/(3*x**2-4*x+5)**(1/2),x)
[Out] sqrt(3)*asinh(3*sqrt(11)*(x - 2/3)/11)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = \frac{1}{3}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{11}\,\sqrt{11}(3\,x-2)\right)$$

```
[In] integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arcsinh(1/11*sqrt(11)*(3*x - 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(16) = 32$.

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.79

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = \frac{1}{6}\sqrt{3\,x^2-4\,x+5}(3\,x-2) - \frac{11}{18}\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-4\,x+5}\right)+2\right)$$

```
[In] integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="giac")
[Out] 1/6*sqrt(3*x^2 - 4*x + 5)*(3*x - 2) - 11/18*sqrt(3)*log(-sqrt(3)*(sqrt(3)*x
 - sqrt(3*x^2 - 4*x + 5)) + 2)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = \frac{\sqrt{3}\,\ln\left(\sqrt{3}\,\left(x-\frac{2}{3}\right)+\sqrt{3\,x^2-4\,x+5}\right)}{3}$$

```
[In] int(1/(3*x^2 - 4*x + 5)^(1/2),x)
[Out] (3^(1/2)*log(3^(1/2)*(x - 2/3) + (3*x^2 - 4*x + 5)^(1/2)))/3
```

3.56 $\int \frac{1}{\sqrt{x-x^2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{1}{\sqrt{x-x^2}} \, dx = -\arcsin(1-2x)$$

```
[Out] arcsin(-1+2*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {633, 222}

$$\int \frac{1}{\sqrt{x-x^2}} \, dx = -\arcsin(1-2x)$$

```
[In] Int[1/Sqrt[x - x^2],x]
[Out] -ArcSin[1 - 2*x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1-2x\right)$$
$$= -\arcsin(1-2x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(8) = 16.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 5.00

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = -\frac{2\sqrt{-1+x}\sqrt{x}\log\left(\sqrt{-1+x}-\sqrt{x}\right)}{\sqrt{-((-1+x)x)}}$$

```
[In] Integrate[1/Sqrt[x - x^2],x]
[Out] (-2*Sqrt[-1 + x]*Sqrt[x]*Log[Sqrt[-1 + x] - Sqrt[x]])/Sqrt[-((-1 + x)*x)]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\arcsin\left(2x-1\right)$	7
meijerg	$2\arcsin\left(\sqrt{x}\right)$	7
pseudoelliptic	$-2\arctan\left(\frac{\sqrt{-x(-1+x)}}{x}\right)$	16
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-2\operatorname{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-x^2+x}+\operatorname{RootOf}\left(\_Z^2+1\right)\right)$	36

```
[In] int(1/(-x^2+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(2*x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

```
[In] integrate(1/(-x^2+x)^(1/2),x, algorithm="fricas")
[Out] -2*arctan(sqrt(-x^2 + x)/x)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = \operatorname{asin}(2x-1)$$

```
[In] integrate(1/(-x**2+x)**(1/2),x)
[Out] asin(2*x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = \arcsin(2x-1)$$

```
[In] integrate(1/(-x^2+x)^(1/2),x, algorithm="maxima")
[Out] arcsin(2*x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(6) = 12$.

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 3.12

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = \frac{1}{4}\sqrt{-x^2+x}(2x-1) + \frac{1}{8}\arcsin(2x-1)$$

```
[In] integrate(1/(-x^2+x)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(-x^2 + x)*(2*x - 1) + 1/8*arcsin(2*x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = \operatorname{asin}(2\,x-1)$$

```
[In] int(1/(x - x^2)^(1/2),x)
[Out] asin(2*x - 1)
```

3.57 $\int \frac{1+2x}{\sqrt{2+x-x^2}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{1+2x}{\sqrt{2+x-x^2}} \, dx = -2\sqrt{2+x-x^2} - 2\arcsin\left(\frac{1}{3}(1-2x)\right)$$

```
[Out] 2*arcsin(-1/3+2/3*x)-2*(-x^2+x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {654, 633, 222}

$$\int \frac{1+2x}{\sqrt{2+x-x^2}} \, dx = -2\arcsin\left(\frac{1}{3}(1-2x)\right) - 2\sqrt{-x^2+x+2}$$

```
[In] Int[(1 + 2*x)/Sqrt[2 + x - x^2],x]
```

```
[Out] -2*Sqrt[2 + x - x^2] - 2*ArcSin[(1 - 2*x)/3]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -2\sqrt{2+x-x^2}+2\int\frac{1}{\sqrt{2+x-x^2}}\,dx\\ &= -2\sqrt{2+x-x^2}-\frac{2}{3}\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx,x,1-2x\right)\\ &= -2\sqrt{2+x-x^2}-2\arcsin\left(\frac{1}{3}(1-2x)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int\frac{1+2x}{\sqrt{2+x-x^2}}\,dx=-2\sqrt{2+x-x^2}-4\arctan\left(\frac{\sqrt{2+x-x^2}}{1+x}\right)$$

```
[In] Integrate[(1 + 2*x)/Sqrt[2 + x - x^2],x]
[Out] -2*Sqrt[2 + x - x^2] - 4*ArcTan[Sqrt[2 + x - x^2]/(1 + x)]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result
default	$2\arcsin\left(-\frac{1}{3}+\frac{2x}{3}\right)-2\sqrt{-x^2+x+2}$
risch	$\frac{2x^2-2x-4}{\sqrt{-x^2+x+2}}+2\arcsin\left(-\frac{1}{3}+\frac{2x}{3}\right)$
trager	$-2\sqrt{-x^2+x+2}+2\,\text{RootOf}\left(\_Z^2+1\right)\ln\left(-2\,\text{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-x^2+x+2}+\text{Root}\right.$

```
[In] int((1+2*x)/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arcsin(-1/3+2/3*x)-2*(-x^2+x+2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = -2\sqrt{-x^2+x+2} - 2\arctan\left(\frac{\sqrt{-x^2+x+2}(2x-1)}{2(x^2-x-2)}\right)$$

```
[In] integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-x^2 + x + 2) - 2*arctan(1/2*sqrt(-x^2 + x + 2)*(2*x - 1)/(x^2 - x
- 2))
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = -2\sqrt{-x^2+x+2} + 2\,\mathrm{asin}\left(\frac{2x}{3} - \frac{1}{3}\right)$$

```
[In] integrate((1+2*x)/(-x**2+x+2)**(1/2),x)
[Out] -2*sqrt(-x**2 + x + 2) + 2*asin(2*x/3 - 1/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = -2\sqrt{-x^2+x+2} - 2\arcsin\left(-\frac{2}{3}x + \frac{1}{3}\right)$$

```
[In] integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(-x^2 + x + 2) - 2*arcsin(-2/3*x + 1/3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = -2\sqrt{-x^2+x+2} + 2\arcsin\left(\frac{2}{3}x - \frac{1}{3}\right)$$

```
[In] integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-x^2 + x + 2) + 2*arcsin(2/3*x - 1/3)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = \operatorname{asin}\left(\frac{2\,x}{3} - \frac{1}{3}\right) - 2\sqrt{-x^2+x+2} - \ln\left(x\,1\mathrm{i} + \sqrt{-x^2+x+2} - \frac{1}{2}\mathrm{i}\right)\,1\mathrm{i}$$

```
[In] int((2*x + 1)/(x - x^2 + 2)^(1/2),x)
```

```
[Out] asin((2*x)/3 - 1/3) - log(x*1i + (x - x^2 + 2)^(1/2) - 1i/2)*1i - 2*(x - x^
2 + 2)^(1/2)
```

3.58 $\int \frac{1}{x\sqrt{2+x-x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{4+x}{2\sqrt{2}\sqrt{2+x-x^2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(1/4*(4+x)*2^(1/2)/(-x^2+x+2)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {738, 212}

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x+4}{2\sqrt{2}\sqrt{-x^2+x+2}}\right)}{\sqrt{2}}$$

```
[In] Int[1/(x*Sqrt[2 + x - x^2]),x]
```

```
[Out] -(ArcTanh[(4 + x)/(2*Sqrt[2]*Sqrt[2 + x - x^2])]/Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(2\text{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, \frac{4+x}{\sqrt{2+x-x^2}}\right)\right)\\ &= -\frac{\operatorname{arctanh}\left(\frac{4+x}{2\sqrt{2}\sqrt{2+x-x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{2+x-x^2}}{-2+x}\right)$$

```
[In] Integrate[1/(x*Sqrt[2 + x - x^2]),x]
[Out] Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[2 + x - x^2])/(-2 + x)]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{(4+x)\sqrt{2}}{4\sqrt{-x^2+x+2}}\right)\sqrt{2}}{2}$	25
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{-x^2+x+2}+4\operatorname{RootOf}\left(\_Z^2-2\right)}{x}\right)}{2}$	43

```
[In] int(1/x/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*arctanh(1/4*(4+x)*2^(1/2)/(-x^2+x+2)^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = \frac{1}{4}\,\sqrt{2}\log\left(-\frac{4\,\sqrt{2}\sqrt{-x^2+x+2}(x+4)+7\,x^2-16\,x-32}{x^2}\right)$$

```
[In] integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(4*sqrt(2)*sqrt(-x^2 + x + 2)*(x + 4) + 7*x^2 - 16*x - 32)
/x^2)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = \int \frac{1}{x\sqrt{-(x-2)(x+1)}}\,dx$$

```
[In] integrate(1/x/(-x**2+x+2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(x - 2)*(x + 1))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = -\frac{1}{2}\,\sqrt{2}\log\left(\frac{2\,\sqrt{2}\sqrt{-x^2+x+2}}{|x|}+\frac{4}{|x|}+1\right)$$

```
[In] integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(2*sqrt(2)*sqrt(-x^2 + x + 2)/abs(x) + 4/abs(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(24) = 48$.

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(2\sqrt{-x^2+x+2}-3\right)}{2x-1}-6\right|}{\left|4\sqrt{2}+\frac{2\left(2\sqrt{-x^2+x+2}-3\right)}{2x-1}-6\right|}\right)$$

```
[In] integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*(2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) -
6)/abs(4*sqrt(2) + 2*(2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) - 6))
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = -\frac{\sqrt{2}\,\ln\left(\frac{x+2\sqrt{2}\sqrt{-x^2+x+2}+4}{x}\right)}{2}$$

```
[In] int(1/(x*(x - x^2 + 2)^(1/2)),x)
[Out] -(2^(1/2)*log((x + 2*2^(1/2)*(x - x^2 + 2)^(1/2) + 4)/x))/2
```

3.59 $\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = \frac{2\sqrt{2+x-x^2}}{3(-2+x)}$$

```
[Out] 2/3*(-x^2+x+2)^(1/2)/(-2+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {664}

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = -\frac{2\sqrt{-x^2+x+2}}{3(2-x)}$$

```
[In] Int[1/((-2 + x)*Sqrt[2 + x - x^2]),x]
```

```
[Out] (-2*Sqrt[2 + x - x^2])/(3*(2 - x))
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{2+x-x^2}}{3(2-x)}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = \frac{2\sqrt{2+x-x^2}}{3(-2+x)}$$

```
[In] Integrate[1/((-2 + x)*Sqrt[2 + x - x^2]),x]
[Out] (2*Sqrt[2 + x - x^2])/(3*(-2 + x))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{2(1+x)}{3\sqrt{-x^2+x+2}}$	16
risch	$-\frac{2(1+x)}{3\sqrt{-x^2+x+2}}$	16
trager	$\frac{2\sqrt{-x^2+x+2}}{3(-2+x)}$	18
default	$\frac{2\sqrt{-(-2+x)^2+6-3x}}{3(-2+x)}$	22

```
[In] int(1/(-2+x)/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(1+x)/(-x^2+x+2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = \frac{2\,\sqrt{-x^2+x+2}}{3\,(x-2)}$$

```
[In] integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(-x^2 + x + 2)/(x - 2)
```

Sympy [F]

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = \int \frac{1}{\sqrt{-(x-2)(x+1)}\,(x-2)}\,dx$$

```
[In] integrate(1/(-2+x)/(-x**2+x+2)**(1/2),x)
[Out] Integral(1/(sqrt(-(x - 2)*(x + 1))*(x - 2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = \frac{2\sqrt{-x^2+x+2}}{3(x-2)}$$

```
[In] integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(-x^2 + x + 2)/(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = -\frac{4}{3\left(\frac{2\sqrt{-x^2+x+2}-3}{2x-1}+1\right)}$$

```
[In] integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="giac")
[Out] -4/3/((2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = \frac{2\sqrt{-x^2+x+2}}{3\,(x-2)}$$

```
[In] int(1/((x - 2)*(x - x^2 + 2)^(1/2)),x)
[Out] (2*(x - x^2 + 2)^(1/2))/(3*(x - 2))
```

3.60 $\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)} dx$

Optimal result

Integrand size = 17, antiderivative size = 28

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)} dx = -\text{arctanh}(\cos(x)) - \frac{1}{1-\cos(x)} - \frac{3\sin(x)}{1-\cos(x)}$$

```
[Out] -arctanh(cos(x))-1/(1-cos(x))-3*sin(x)/(1-cos(x))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {4486, 2727, 2746, 46, 213}

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)} dx = -\text{arctanh}(\cos(x)) - \frac{1}{1-\cos(x)} - \frac{3\sin(x)}{1-\cos(x)}$$

```
[In] Int[(Csc[x]*(2 + 3*Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] -ArcTanh[Cos[x]] - (1 - Cos[x])^(-1) - (3*Sin[x])/(1 - Cos[x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{3}{-1+\cos(x)} - \frac{2\csc(x)}{-1+\cos(x)}\right) dx \\
&= -\left(2\int \frac{\csc(x)}{-1+\cos(x)}\, dx\right) - 3\int \frac{1}{-1+\cos(x)}\, dx \\
&= -\frac{3\sin(x)}{1-\cos(x)} + 2\text{Subst}\left(\int \frac{1}{(-1-x)(-1+x)^2}\, dx, x, \cos(x)\right) \\
&= -\frac{3\sin(x)}{1-\cos(x)} + 2\text{Subst}\left(\int \left(-\frac{1}{2(-1+x)^2} + \frac{1}{2\left(-1+x^2\right)}\right) dx, x, \cos(x)\right) \\
&= -\frac{1}{1-\cos(x)} - \frac{3\sin(x)}{1-\cos(x)} + \text{Subst}\left(\int \frac{1}{-1+x^2}\, dx, x, \cos(x)\right) \\
&= -\text{arctanh}(\cos(x)) - \frac{1}{1-\cos(x)} - \frac{3\sin(x)}{1-\cos(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.93

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = \frac{1}{2}\csc^2\left(\frac{x}{2}\right)\left(-1-\log\left(\cos\left(\frac{x}{2}\right)\right)+\cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)-3\sin(x)\right)$$

```
[In] Integrate[(Csc[x]*(2 + 3*Sin[x]))/(1 - Cos[x]),x]
[Out] (Csc[x/2]^2*(-1 - Log[Cos[x/2]] + Cos[x]*(Log[Cos[x/2]] - Log[Sin[x/2]]) +
Log[Sin[x/2]] - 3*Sin[x]))/2
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
parallelrisch	$\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{\left(\cot^2\left(\frac{x}{2}\right)\right)}{2}-3\cot\left(\frac{x}{2}\right)$	21
default	$-\frac{3}{\tan\left(\frac{x}{2}\right)}+\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{1}{2\tan\left(\frac{x}{2}\right)^2}$	23
risch	$\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)\left(10\,\mathrm{e}^{ix}-9+3i\right)}{\left(\mathrm{e}^{ix}-1\right)^2}-\ln\left(\mathrm{e}^{ix}+1\right)+\ln\left(\mathrm{e}^{ix}-1\right)$	44
norman	$\frac{-\frac{1}{2}-\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-3\left(\tan^3\left(\frac{x}{2}\right)\right)-3\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)^2}+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	48

```
[In] int((2+3*sin(x))/(1-cos(x))/sin(x),x,method=_RETURNVERBOSE)
[Out] ln(tan(1/2*x))-1/2*cot(1/2*x)^2-3*cot(1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = -\frac{(\cos(x)-1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)-1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-6\sin(x)-2}{2\,(\cos(x)-1)}$$

```
[In] integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*((cos(x) - 1)*log(1/2*cos(x) + 1/2) - (cos(x) - 1)*log(-1/2*cos(x) + 1
/2) - 6*sin(x) - 2)/(cos(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = \log\left(\tan\left(\frac{x}{2}\right)\right) - \frac{3}{\tan\left(\frac{x}{2}\right)} - \frac{1}{2\tan^2\left(\frac{x}{2}\right)}$$

```
[In] integrate((2+3*sin(x))/(1-cos(x))/sin(x),x)
[Out] log(tan(x/2)) - 3/tan(x/2) - 1/(2*tan(x/2)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = -\frac{(\cos(x)+1)^2}{2\sin(x)^2} - \frac{3(\cos(x)+1)}{\sin(x)} + \log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

```
[In] integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="maxima")
[Out] -1/2*(cos(x) + 1)^2/sin(x)^2 - 3*(cos(x) + 1)/sin(x) + log(sin(x)/(cos(x) +
 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = -\frac{3\tan\left(\frac{1}{2}x\right)^2 + 6\tan\left(\frac{1}{2}x\right) + 1}{2\tan\left(\frac{1}{2}x\right)^2} + \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

```
[In] integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="giac")
[Out] -1/2*(3*tan(1/2*x)^2 + 6*tan(1/2*x) + 1)/tan(1/2*x)^2 + log(abs(tan(1/2*x))
)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right) - \frac{3\tan\left(\frac{x}{2}\right)+\frac{1}{2}}{\tan\left(\frac{x}{2}\right)^2}$$

```
[In] int(-(3*sin(x) + 2)/(sin(x)*(cos(x) - 1)),x)
[Out] log(tan(x/2)) - (3*tan(x/2) + 1/2)/tan(x/2)^2
```

3.61 $\int \frac{1}{2+3\cos^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 37

$$\int \frac{1}{2+3\cos^2(x)}\,dx=\frac{x}{\sqrt{10}}-\frac{\arctan\left(\frac{3\cos(x)\sin(x)}{2+\sqrt{10}+3\cos^2(x)}\right)}{\sqrt{10}}$$

```
[Out] 1/10*x*10^(1/2)-1/10*arctan(3*cos(x)*sin(x)/(2+3*cos(x)^2+10^(1/2)))*10^(1/
2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3260, 209}

$$\int \frac{1}{2+3\cos^2(x)}\,dx=\frac{x}{\sqrt{10}}-\frac{\arctan\left(\frac{\left(\sqrt{\frac{5}{2}}-1\right)\sin(x)\cos(x)}{\left(\sqrt{\frac{5}{2}}-1\right)\cos^2(x)+1}\right)}{\sqrt{10}}$$

```
[In] Int[(2 + 3*Cos[x]^2)^(-1),x]
```

```
[Out] x/Sqrt[10] - ArcTan[((-1 + Sqrt[5/2])*Cos[x]*Sin[x])/(1 + (-1 + Sqrt[5/2])*
Cos[x]^2)]/Sqrt[10]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{2+5x^2}\,dx, x, \cot(x)\right)$$

$$= \frac{x}{\sqrt{10}} - \frac{\arctan\left(\frac{\left(-1+\sqrt{\frac{5}{2}}\right)\cos(x)\sin(x)}{1+\left(-1+\sqrt{\frac{5}{2}}\right)\cos^2(x)}\right)}{\sqrt{10}}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.46

$$\int \frac{1}{2+3\cos^2(x)}\,dx = \frac{\arctan\left(\sqrt{\frac{2}{5}}\tan(x)\right)}{\sqrt{10}}$$

```
[In] Integrate[(2 + 3*Cos[x]^2)^(-1),x]
[Out] ArcTan[Sqrt[2/5]*Tan[x]]/Sqrt[10]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.38

method	result	size
default	$\frac{\sqrt{10}\,\arctan\left(\frac{\tan(x)\sqrt{10}}{5}\right)}{10}$	14
risch	$\frac{i\sqrt{10}\,\ln\left(e^{2ix}+\frac{2\sqrt{10}}{3}+\frac{7}{3}\right)}{20}-\frac{i\sqrt{10}\,\ln\left(e^{2ix}-\frac{2\sqrt{10}}{3}+\frac{7}{3}\right)}{20}$	40

```
[In] int(1/(3*cos(x)^2+2),x,method=_RETURNVERBOSE)
[Out] 1/10*10^(1/2)*arctan(1/5*tan(x)*10^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{2+3\cos^2(x)}\,dx = -\frac{1}{20}\,\sqrt{10}\arctan\left(\frac{7\,\sqrt{10}\cos\left(x\right)^2-2\,\sqrt{10}}{20\,\cos\left(x\right)\sin\left(x\right)}\right)$$

```
[In] integrate(1/(2+3*cos(x)^2),x, algorithm="fricas")
[Out] -1/20*sqrt(10)*arctan(1/20*(7*sqrt(10)*cos(x)^2 - 2*sqrt(10))/(cos(x)*sin(x
)))
```

Sympy [F]

$$\int \frac{1}{2+3\cos^2(x)}\,dx = \int \frac{1}{3\cos^2\left(x\right)+2}\,dx$$

```
[In] integrate(1/(2+3*cos(x)**2),x)
[Out] Integral(1/(3*cos(x)**2 + 2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.35

$$\int \frac{1}{2+3\cos^2(x)}\,dx = \frac{1}{10}\,\sqrt{10}\arctan\left(\frac{1}{5}\,\sqrt{10}\tan\left(x\right)\right)$$

```
[In] integrate(1/(2+3*cos(x)^2),x, algorithm="maxima")
[Out] 1/10*sqrt(10)*arctan(1/5*sqrt(10)*tan(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{1}{2+3\cos^2(x)}\,dx = \frac{1}{10}\,\sqrt{10}\left(x+\arctan\left(-\frac{\sqrt{10}\sin\left(2\,x\right)-2\,\sin\left(2\,x\right)}{\sqrt{10}\cos\left(2\,x\right)+\sqrt{10}-2\,\cos\left(2\,x\right)+2}\right)\right)$$

```
[In] integrate(1/(2+3*cos(x)^2),x, algorithm="giac")
[Out] 1/10*sqrt(10)*(x + arctan(-(sqrt(10)*sin(2*x) - 2*sin(2*x))/(sqrt(10)*cos(2
*x) + sqrt(10) - 2*cos(2*x) + 2)))
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{1}{2+3\cos^2(x)}\,dx = \frac{\sqrt{10}\,(x-\operatorname{atan}(\tan(x)))}{10} + \frac{\sqrt{10}\operatorname{atan}\left(\frac{\sqrt{10}\tan(x)}{5}\right)}{10}$$

```
[In] int(1/(3*cos(x)^2 + 2),x)
[Out] (10^(1/2)*(x - atan(tan(x))))/10 + (10^(1/2)*atan((10^(1/2)*tan(x))/5))/10
```

3.62 $\int \csc(2x)(1-\tan(x))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{1}{2}\log(\tan(x)) - \frac{\tan(x)}{2}$$

```
[Out] 1/2*ln(tan(x))-1/2*tan(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {12}

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{1}{2}\log(\tan(x)) - \frac{\tan(x)}{2}$$

```
[In] Int[Csc[2*x]*(1 - Tan[x]),x]
[Out] Log[Tan[x]]/2 - Tan[x]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{2}\left(-1+\frac{1}{x}\right)\,dx, x, \tan(x)\right) \\ &= \frac{1}{2}\text{Subst}\left(\int \left(-1+\frac{1}{x}\right)\,dx, x, \tan(x)\right) \\ &= \frac{1}{2}\log(\tan(x)) - \frac{\tan(x)}{2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \csc(2x)(1-\tan(x))\,dx = -\frac{1}{2}\log(\cos(x))+\frac{1}{2}\log(\sin(x))-\frac{\tan(x)}{2}$$

```
[In] Integrate[Csc[2*x]*(1 - Tan[x]),x]
[Out] -1/2*Log[Cos[x]] + Log[Sin[x]]/2 - Tan[x]/2
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\ln(\tan(x))}{2}-\frac{\tan(x)}{2}$	11
default	$\frac{\ln(\tan(x))}{2}-\frac{\tan(x)}{2}$	11
norman	$\frac{\ln(\tan(x))}{2}-\frac{\tan(x)}{2}$	11
parallelrisch	$\ln\left(\sqrt{\tan}(x)\right)-\frac{\tan(x)}{2}$	11
risch	$-\frac{i}{e^{2ix}+1}+\frac{\ln(e^{2ix}-1)}{2}-\frac{\ln(e^{2ix}+1)}{2}$	34

```
[In] int((1-tan(x))/sin(2*x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(tan(x))-1/2*tan(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(10) = 20.

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{1}{4}\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)-\frac{1}{4}\log\left(\frac{1}{\tan(x)^2+1}\right)-\frac{1}{2}\tan(x)$$

```
[In] integrate((1-tan(x))/sin(2*x),x, algorithm="fricas")
[Out] 1/4*log(tan(x)^2/(tan(x)^2 + 1)) - 1/4*log(1/(tan(x)^2 + 1)) - 1/2*tan(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{\log\left(\cos\left(2x\right)-1\right)}{4} - \frac{\log\left(\cos\left(2x\right)+1\right)}{4} - \frac{\sin\left(x\right)}{2\cos\left(x\right)}$$

```
[In] integrate((1-tan(x))/sin(2*x),x)
[Out] log(cos(2*x) - 1)/4 - log(cos(2*x) + 1)/4 - sin(x)/(2*cos(x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(10) = 20$.

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.36

$$\int \csc(2x)(1-\tan(x))\,dx = -\frac{\sin\left(2\,x\right)}{\cos\left(2\,x\right)^2+\sin\left(2\,x\right)^2+2\,\cos\left(2\,x\right)+1} - \frac{1}{4}\,\log\left(\cos\left(2\,x\right)+1\right) + \frac{1}{4}\,\log\left(\cos\left(2\,x\right)-1\right)$$

```
[In] integrate((1-tan(x))/sin(2*x),x, algorithm="maxima")
[Out] -sin(2*x)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - 1/4*log(cos(2*x) + 1
) + 1/4*log(cos(2*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{1}{2}\,\log\left(\left|\tan\left(x\right)\right|\right) - \frac{1}{2}\,\tan\left(x\right)$$

```
[In] integrate((1-tan(x))/sin(2*x),x, algorithm="giac")
[Out] 1/2*log(abs(tan(x))) - 1/2*tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{\ln(\tan(x))}{2} - \frac{\tan(x)}{2}$$

```
[In] int(-(tan(x) - 1)/sin(2*x),x)
[Out] log(tan(x))/2 - tan(x)/2
```

3.63 $\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 11

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{2}\text{arctanh}(2\cos(x)\sin(x))$$

```
[Out] 1/2*arctanh(2*cos(x)*sin(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {212}

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{2}\text{arctanh}(2\sin(x)\cos(x))$$

```
[In] Int[(1 + Tan[x]^2)/(1 - Tan[x]^2),x]
```

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tan(x)\right) \\ &= \frac{1}{2}\text{arctanh}(2\cos(x)\sin(x)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{2}\text{arctanh}(\sin(2x))$$

```
[In] Integrate[(1 + Tan[x]^2)/(1 - Tan[x]^2),x]
[Out] ArcTanh[Sin[2*x]]/2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.36

method	result	size
derivativedivides	$\operatorname{arctanh}(\tan(x))$	4
default	$\operatorname{arctanh}(\tan(x))$	4
norman	$-\frac{\ln(\tan(x)-1)}{2}+\frac{\ln(\tan(x)+1)}{2}$	16
parallelrisch	$-\frac{\ln(\tan(x)-1)}{2}+\frac{\ln(\tan(x)+1)}{2}$	16
risch	$-\frac{\ln(e^{2ix}-i)}{2}+\frac{\ln(e^{2ix}+i)}{2}$	24

```
[In] int((1+tan(x)^2)/(1-tan(x)^2),x,method=_RETURNVERBOSE)
[Out] arctanh(tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(9) = 18.

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 4.09

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{4}\log\left(\frac{\tan(x)^2+2\tan(x)+1}{\tan(x)^2+1}\right) - \frac{1}{4}\log\left(\frac{\tan(x)^2-2\tan(x)+1}{\tan(x)^2+1}\right)$$

```
[In] integrate((1+tan(x)^2)/(1-tan(x)^2),x, algorithm="fricas")
[Out] 1/4*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) - 1/4*log((tan(x)^2 - 2*t
an(x) + 1)/(tan(x)^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = -\frac{\log(\tan(x)-1)}{2} + \frac{\log(\tan(x)+1)}{2}$$

```
[In] integrate((1+tan(x)**2)/(1-tan(x)**2),x)
[Out] -log(tan(x) - 1)/2 + log(tan(x) + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{2}\log(\tan(x)+1) - \frac{1}{2}\log(\tan(x)-1)$$

```
[In] integrate((1+tan(x)^2)/(1-tan(x)^2),x, algorithm="maxima")
[Out] 1/2*log(tan(x) + 1) - 1/2*log(tan(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{2}\log(|\tan(x)+1|) - \frac{1}{2}\log(|\tan(x)-1|)$$

```
[In] integrate((1+tan(x)^2)/(1-tan(x)^2),x, algorithm="giac")
[Out] 1/2*log(abs(tan(x) + 1)) - 1/2*log(abs(tan(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.27

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = \operatorname{atanh}(\tan(x))$$

```
[In] int(-(tan(x)^2 + 1)/(tan(x)^2 - 1),x)
[Out] atanh(tan(x))
```

3.64 $\int (a^2-4\cos^2(x))^{3/4}\sin(2x)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\frac{1}{7}\left(a^2-4\cos^2(x)\right)^{7/4}$$

```
[Out] 1/7*(a^2-4*cos(x)^2)^(7/4)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {12, 267}

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\frac{1}{7}\left(a^2+4\sin^2(x)-4\right)^{7/4}$$

```
[In] Int[(a^2 - 4*Cos[x]^2)^(3/4)*Sin[2*x],x]
```

```
[Out] (-4 + a^2 + 4*Sin[x]^2)^(7/4)/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int 2x\left(-4+a^2+4x^2\right)^{3/4}\,dx, x, \sin(x)\right) \\ &= 2\text{Subst}\left(\int x\left(-4+a^2+4x^2\right)^{3/4}\,dx, x, \sin(x)\right) \\ &= \frac{1}{7}\left(-4+a^2+4\sin^2(x)\right)^{7/4} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx = \frac{1}{7}\left(-4+a^2+4\sin^2(x)\right)^{7/4}$$

```
[In] Integrate[(a^2 - 4*Cos[x]^2)^(3/4)*Sin[2*x],x]
[Out] (-4 + a^2 + 4*Sin[x]^2)^(7/4)/7
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\left(a^2-4\left(\cos^2(x)\right)\right)^{\frac{7}{4}}}{7}$	15
default	$\frac{\left(a^2-4\left(\cos^2(x)\right)\right)^{\frac{7}{4}}}{7}$	15

```
[In] int((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x,method=_RETURNVERBOSE)
[Out] 1/7*(a^2-4*cos(x)^2)^(7/4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\frac{1}{7}\left(a^2-4\cos\left(x\right)^2\right)^{\frac{7}{4}}$$

```
[In] integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="fricas")
[Out] 1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\text{Timed out}$$

```
[In] integrate((a**2-4*cos(x)**2)**(3/4)*sin(2*x),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\frac{1}{7}\left(a^2-4\cos\left(x\right)^2\right)^{\frac{7}{4}}$$

```
[In] integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="maxima")
[Out] 1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\frac{1}{7}\left(a^2-4\cos\left(x\right)^2\right)^{\frac{7}{4}}$$

```
[In] integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="giac")
[Out] 1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\frac{\left(a^2-4\cos\left(x\right)^2\right)^{7/4}}{7}$$

```
[In] int(sin(2*x)*(a^2 - 4*cos(x)^2)^(3/4),x)
[Out] (a^2 - 4*cos(x)^2)^(7/4)/7
```

3.65 $\int \frac{\sin(2x)}{\sqrt[3]{a^2 - 4\sin^2(x)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2 - 4\sin^2(x)}}\,dx = -\frac{3}{8}\left(a^2 - 4\sin^2(x)\right)^{2/3}$$

```
[Out] -3/8*(a^2-4*sin(x)^2)^(2/3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {12, 267}

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2 - 4\sin^2(x)}}\,dx = -\frac{3}{8}\left(a^2 - 4\sin^2(x)\right)^{2/3}$$

```
[In] Int[Sin[2*x]/(a^2 - 4*Sin[x]^2)^(1/3),x]
[Out] (-3*(a^2 - 4*Sin[x]^2)^(2/3))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{2x}{\sqrt[3]{a^2-4x^2}}\,dx, x, \sin(x)\right) \\ &= 2\text{Subst}\left(\int \frac{x}{\sqrt[3]{a^2-4x^2}}\,dx, x, \sin(x)\right) \\ &= -\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx = -\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}$$

```
[In] Integrate[Sin[2*x]/(a^2 - 4*Sin[x]^2)^(1/3),x]
[Out] (-3*(a^2 - 4*Sin[x]^2)^(2/3))/8
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{3\left(a^2-4\left(\sin^2(x)\right)\right)^{\frac{2}{3}}}{8}$	15
default	$-\frac{3\left(a^2-4\left(\sin^2(x)\right)\right)^{\frac{2}{3}}}{8}$	15

```
[In] int(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x,method=_RETURNVERBOSE)
[Out] -3/8*(a^2-4*sin(x)^2)^(2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx = -\frac{3}{8}\left(a^2+4\cos\left(x\right)^2-4\right)^{\frac{2}{3}}$$

```
[In] integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="fricas")
[Out] -3/8*(a^2 + 4*cos(x)^2 - 4)^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx = -\frac{3(a^2-4\sin^2\left(x\right))^{\frac{2}{3}}}{8}$$

```
[In] integrate(sin(2*x)/(a**2-4*sin(x)**2)**(1/3),x)
[Out] -3*(a**2 - 4*sin(x)**2)**(2/3)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx = -\frac{3}{8}\left(a^2-4\sin\left(x\right)^2\right)^{\frac{2}{3}}$$

```
[In] integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="maxima")
[Out] -3/8*(a^2 - 4*sin(x)^2)^(2/3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2 - 4\sin^2(x)}}\,dx = -\frac{3}{8}\left(a^2 - 4\,\sin\left(x\right)^2\right)^{\frac{2}{3}}$$

```
[In] integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="giac")
[Out] -3/8*(a^2 - 4*sin(x)^2)^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2 - 4\sin^2(x)}}\,dx = -\frac{3\left(a^2 - 4\sin\left(x\right)^2\right)^{2/3}}{8}$$

```
[In] int(sin(2*x)/(a^2 - 4*sin(x)^2)^(1/3),x)
[Out] -(3*(a^2 - 4*sin(x)^2)^(2/3))/8
```

3.66 $\int \frac{1}{\sqrt{-1+a^{2x}}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1}{\sqrt{-1+a^{2x}}} \, dx = \frac{\arctan\left(\sqrt{-1+a^{2x}}\right)}{\log(a)}$$

```
[Out] arctan((-1+a^(2*x))^(1/2))/ln(a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2320, 65, 209}

$$\int \frac{1}{\sqrt{-1+a^{2x}}} \, dx = \frac{\arctan\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

```
[In] Int[1/Sqrt[-1 + a^(2*x)],x]
```

```
[Out] ArcTan[Sqrt[-1 + a^(2*x)]]/Log[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x}\,dx, x, a^{2x}\right)}{2\log(a)}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+a^{2x}}\right)}{\log(a)}\\ &= \frac{\arctan\left(\sqrt{-1+a^{2x}}\right)}{\log(a)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \frac{\arctan\left(\sqrt{-1+a^{2x}}\right)}{\log(a)}$$

```
[In] Integrate[1/Sqrt[-1 + a^(2*x)],x]
[Out] ArcTan[Sqrt[-1 + a^(2*x)]]/Log[a]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\arctan\left(\sqrt{-1+a^{2x}}\right)}{\ln(a)}$	16
default	$\frac{\arctan\left(\sqrt{-1+a^{2x}}\right)}{\ln(a)}$	16

```
[In] int(1/(-1+a^(2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan((-1+a^(2*x))^(1/2))/ln(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \frac{\arctan\left(\sqrt{a^{2\,x}-1}\right)}{\log\left(a\right)}$$

```
[In] integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="fricas")
[Out] arctan(sqrt(a^(2*x) - 1))/log(a)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \int \frac{1}{\sqrt{a^{2x}-1}}\,dx$$

```
[In] integrate(1/(-1+a**(2*x))**(1/2),x)
[Out] Integral(1/sqrt(a**(2*x) - 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \frac{\arctan\left(\sqrt{a^{2\,x}-1}\right)}{\log\left(a\right)}$$

```
[In] integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="maxima")
[Out] arctan(sqrt(a^(2*x) - 1))/log(a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \frac{\arctan\left(\sqrt{a^{2\,x}-1}\right)}{\log\left(a\right)}$$

```
[In] integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(a^(2*x) - 1))/log(a)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = -\frac{a^x\,\mathrm{asin}\left(\frac{1}{a^x}\right)\,\sqrt{1-\frac{1}{a^{2\,x}}}}{\ln\left(a\right)\,\sqrt{a^{2\,x}-1}}$$

```
[In] int(1/(a^(2*x) - 1)^(1/2),x)
[Out] -(a^x*asin(1/a^x)*(1 - 1/a^(2*x))^(1/2))/(log(a)*(a^(2*x) - 1)^(1/2))
```

3.67 $\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = 2\text{arctanh}\left(\frac{e^{x/2}}{\sqrt{-1+e^x}}\right)$$

```
[Out] 2*arctanh(exp(1/2*x)/(-1+exp(x))^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2281, 223, 212}

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = 2\text{arctanh}\left(\frac{e^{x/2}}{\sqrt{e^x-1}}\right)$$

```
[In] Int[E^(x/2)/Sqrt[-1 + E^x],x]
```

```
[Out] 2*ArcTanh[E^(x/2)/Sqrt[-1 + E^x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= 2\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, e^{x/2}\right) \\ &= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{e^{x/2}}{\sqrt{-1+e^x}}\right) \\ &= 2\text{arctanh}\left(\frac{e^{x/2}}{\sqrt{-1+e^x}}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = -2\log\left(-e^{x/2}+\sqrt{-1+e^x}\right)$$

```
[In] Integrate[E^(x/2)/Sqrt[-1 + E^x],x]
[Out] -2*Log[-E^(x/2) + Sqrt[-1 + E^x]]
```

Maple [F]

$$\int \frac{e^{\frac{x}{2}}}{\sqrt{-1+e^x}}dx$$

```
[In] int(exp(1/2*x)/(-1+exp(x))^(1/2),x)
[Out] int(exp(1/2*x)/(-1+exp(x))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = -2\,\log\left(\sqrt{e^x-1}-e^{\left(\frac{1}{2}\,x\right)}\right)$$

```
[In] integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="fricas")
[Out] -2*log(sqrt(e^x - 1) - e^(1/2*x))
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = 2\log\left(2\sqrt{e^x-1}+2e^{\frac{x}{2}}\right)$$

```
[In] integrate(exp(1/2*x)/(-1+exp(x))**(1/2),x)
[Out] 2*log(2*sqrt(exp(x) - 1) + 2*exp(x/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = 2\,\log\left(2\,\sqrt{e^x-1}+2\,e^{\left(\frac{1}{2}\,x\right)}\right)$$

```
[In] integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="maxima")
[Out] 2*log(2*sqrt(e^x - 1) + 2*e^(1/2*x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = -2\,\log\left(-\sqrt{e^x-1}+e^{\left(\frac{1}{2}\,x\right)}\right)$$

```
[In] integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="giac")
[Out] -2*log(-sqrt(e^x - 1) + e^(1/2*x))
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = \ln\left(\mathrm{e}^x + \sqrt{\mathrm{e}^x}\,\sqrt{\mathrm{e}^x-1} - \frac{1}{2}\right)$$

```
[In] int(exp(x/2)/(exp(x) - 1)^(1/2),x)
[Out] log(exp(x) + exp(x)^(1/2)*(exp(x) - 1)^(1/2) - 1/2)
```

3.68 $\int \frac{\arctan(x)^n}{1+x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\arctan(x)^n}{1+x^2} \, dx = \frac{\arctan(x)^{1+n}}{1+n}$$

```
[Out] arctan(x)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5004}

$$\int \frac{\arctan(x)^n}{1+x^2} \, dx = \frac{\arctan(x)^{n+1}}{n+1}$$

```
[In] Int[ArcTan[x]^n/(1 + x^2),x]
```

```
[Out] ArcTan[x]^(1 + n)/(1 + n)
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\arctan(x)^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \frac{\arctan(x)^{1+n}}{1+n}$$

```
[In] Integrate[ArcTan[x]^n/(1 + x^2),x]
[Out] ArcTan[x]^(1 + n)/(1 + n)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\arctan(x)^{1+n}}{1+n}$	13
default	$\frac{\arctan(x)^{1+n}}{1+n}$	13
risch	$\frac{i(\ln(-ix+1)-\ln(ix+1))\left(\frac{i(\ln(-ix+1)-\ln(ix+1))}{2}\right)^n}{2+2n}$	48

```
[In] int(arctan(x)^n/(x^2+1),x,method=_RETURNVERBOSE)
[Out] arctan(x)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \frac{\arctan(x)^n \arctan(x)}{n+1}$$

```
[In] integrate(arctan(x)^n/(x^2+1),x, algorithm="fricas")
[Out] arctan(x)^n*arctan(x)/(n + 1)
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \begin{cases} \frac{\operatorname{atan}^{n+1}(x)}{n+1} & \text{for } n \neq -1 \\ \log(\operatorname{atan}(x)) & \text{otherwise} \end{cases}$$

```
[In] integrate(atan(x)**n/(x**2+1),x)
[Out] Piecewise((atan(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(atan(x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \frac{\arctan(x)^{n+1}}{n+1}$$

```
[In] integrate(arctan(x)^n/(x^2+1),x, algorithm="maxima")
[Out] arctan(x)^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \frac{\arctan(x)^{n+1}}{n+1}$$

```
[In] integrate(arctan(x)^n/(x^2+1),x, algorithm="giac")
[Out] arctan(x)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \frac{\operatorname{atan}(x)^{n+1}}{n+1}$$

```
[In] int(atan(x)^n/(x^2 + 1),x)
[Out] atan(x)^(n + 1)/(n + 1)
```

3.69 $\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 42

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \frac{2a\sqrt{1-\frac{x^2}{a^2}}\arcsin\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

```
[Out] 2/5*a*arcsin(x/a)^(5/2)*(1-x^2/a^2)^(1/2)/(a^2-x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {4737}

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \frac{2a\sqrt{1-\frac{x^2}{a^2}}\arcsin\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

```
[In] Int[ArcSin[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
[Out] (2*a*Sqrt[1 - x^2/a^2]*ArcSin[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

Rule 4737

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{2a\sqrt{1-\frac{x^2}{a^2}}\arcsin\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \frac{2a\sqrt{1-\frac{x^2}{a^2}}\arcsin\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

[In] Integrate[ArcSin[x/a]^(3/2)/Sqrt[a^2 - x^2],x]

[Out] (2*a*Sqrt[1 - x^2/a^2]*ArcSin[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{2\arcsin\left(\frac{x}{a}\right)^{\frac{5}{2}}\sqrt{\frac{a^2-x^2}{a^2}}\,a}{5\sqrt{a^2-x^2}}$	38

[In] int(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/5*arcsin(x/a)^(5/2)/(a^2-x^2)^(1/2)*((a^2-x^2)/a^2)^(1/2)*a

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \frac{2}{5}\sqrt{-\arctan\left(-\frac{x}{\sqrt{a^2-x^2}}\right)}\arctan\left(-\frac{x}{\sqrt{a^2-x^2}}\right)^2$$

[In] integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="fricas")

[Out] 2/5*sqrt(-arctan(-x/sqrt(a^2 - x^2)))*arctan(-x/sqrt(a^2 - x^2))^2

Sympy [F]

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \int \frac{\operatorname{asin}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\sqrt{-\left(-a+x\right)\left(a+x\right)}}\,dx$$

```
[In] integrate(asin(x/a)**(3/2)/(a**2-x**2)**(1/2),x)
[Out] Integral(asin(x/a)**(3/2)/sqrt(-(-a + x)*(a + x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.36

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \frac{2\,|a|\arcsin\left(\frac{x}{a}\right)^{\frac{5}{2}}}{5\,a}$$

```
[In] integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="giac")
[Out] 2/5*abs(a)*arcsin(x/a)^(5/2)/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \int \frac{\operatorname{asin}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$$

```
[In] int(asin(x/a)^(3/2)/(a^2 - x^2)^(1/2),x)
[Out] int(asin(x/a)^(3/2)/(a^2 - x^2)^(1/2), x)
```

3.70 $\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 8

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2\arccos(x)^2}$$

```
[Out] 1/2/arccos(x)^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4738}

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2\arccos(x)^2}$$

```
[In] Int[1/(Sqrt[1 - x^2]*ArcCos[x]^3),x]
```

```
[Out] 1/(2*ArcCos[x]^2)
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{2\arccos(x)^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx=\frac{1}{2\arccos(x)^2}$$

```
[In] Integrate[1/(Sqrt[1 - x^2]*ArcCos[x]^3),x]
[Out] 1/(2*ArcCos[x]^2)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{1}{2\arccos(x)^2}$	7
default	$\frac{1}{2\arccos(x)^2}$	7

```
[In] int(1/arccos(x)^3/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/arccos(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx=\frac{1}{2\,\arccos\left(x\right)^2}$$

```
[In] integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2/arccos(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2\operatorname{acos}^2(x)}$$

```
[In] integrate(1/acos(x)**3/(-x**2+1)**(1/2),x)
[Out] 1/(2*acos(x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2\arccos(x)^2}$$

```
[In] integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2/arccos(x)^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2\arccos(x)^2}$$

```
[In] integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2/arccos(x)^2
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2\operatorname{acos}(x)^2}$$

```
[In] int(1/(acos(x)^3*(1 - x^2)^(1/2)),x)
[Out] 1/(2*acos(x)^2)
```

3.71 $\int x \log^2(x)\, dx$

Optimal result

Integrand size = 6, antiderivative size = 28

$$\int x \log^2(x)\, dx = \frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)$$

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2342, 2341}

$$\int x \log^2(x)\, dx = \frac{x^2}{4} + \frac{1}{2}x^2\log^2(x) - \frac{1}{2}x^2\log(x)$$

```
[In] Int[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2\log^2(x) - \int x\log(x)\,dx$$
$$= \frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x\log^2(x)\,dx = \frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)$$

```
[In] Integrate[x*Log[x]^2,x]
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^2}{4} - \frac{x^2\ln(x)}{2} + \frac{x^2\ln(x)^2}{2}$	23
norman	$\frac{x^2}{4} - \frac{x^2\ln(x)}{2} + \frac{x^2\ln(x)^2}{2}$	23
risch	$\frac{x^2}{4} - \frac{x^2\ln(x)}{2} + \frac{x^2\ln(x)^2}{2}$	23
parallelrisch	$\frac{x^2}{4} - \frac{x^2\ln(x)}{2} + \frac{x^2\ln(x)^2}{2}$	23
parts	$\frac{x^2}{4} - \frac{x^2\ln(x)}{2} + \frac{x^2\ln(x)^2}{2}$	23

```
[In] int(x*ln(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right)^2 - \frac{1}{2}\,x^2\log\left(x\right) + \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(x)^2,x, algorithm="fricas")
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx = \frac{x^2\log\left(x\right)^2}{2} - \frac{x^2\log\left(x\right)}{2} + \frac{x^2}{4}$$

```
[In] integrate(x*ln(x)**2,x)
[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x\log^2(x)\,dx = \frac{1}{4}\,\left(2\,\log\left(x\right)^2 - 2\,\log\left(x\right) + 1\right)x^2$$

```
[In] integrate(x*log(x)^2,x, algorithm="maxima")
[Out] 1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right)^2 - \frac{1}{2}\,x^2\log\left(x\right) + \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(x)^2,x, algorithm="giac")
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x \log^2(x) \, dx = \frac{x^2 \left(2 \ln(x)^2 - 2 \ln(x) + 1\right)}{4}$$

```
[In] int(x*log(x)^2,x)
[Out] (x^2*(2*log(x)^2 - 2*log(x) + 1))/4
```

3.72 $\int \frac{\log(x)}{x^5} dx$

Optimal result

Integrand size = 6, antiderivative size = 17

$$\int \frac{\log(x)}{x^5} dx = -\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

```
[Out] -1/16/x^4-1/4*ln(x)/x^4
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{2341\}$

$$\int \frac{\log(x)}{x^5} dx = -\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

```
[In] Int[Log[x]/x^5,x]
```

```
[Out] -1/16*1/x^4 - Log[x]/(4*x^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{x^5} \, dx = -\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

[In] Integrate[Log[x]/x^5,x]

[Out] -1/16*1/x^4 - Log[x]/(4*x^4)

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

method	result	size
norman	$\frac{-\frac{1}{16}-\frac{\ln(x)}{4}}{x^4}$	11
parallelrisch	$-\frac{1+4\ln(x)}{16x^4}$	12
default	$-\frac{1}{16x^4}-\frac{\ln(x)}{4x^4}$	14
risch	$-\frac{1}{16x^4}-\frac{\ln(x)}{4x^4}$	14
parts	$-\frac{1}{16x^4}-\frac{\ln(x)}{4x^4}$	14

[In] int(ln(x)/x^5,x,method=_RETURNVERBOSE)

[Out] (-1/16-1/4*ln(x))/x^4

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\log(x)}{x^5} \, dx = -\frac{4 \log(x) + 1}{16 \, x^4}$$

[In] integrate(log(x)/x^5,x, algorithm="fricas")

[Out] -1/16*(4*log(x) + 1)/x^4

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\log(x)}{x^5} \, dx = -\frac{\log(x)}{4x^4} - \frac{1}{16x^4}$$

```
[In] integrate(ln(x)/x**5,x)
[Out] -log(x)/(4*x**4) - 1/(16*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\log(x)}{x^5} \, dx = -\frac{\log(x)}{4\,x^4} - \frac{1}{16\,x^4}$$

```
[In] integrate(log(x)/x^5,x, algorithm="maxima")
[Out] -1/4*log(x)/x^4 - 1/16/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\log(x)}{x^5} \, dx = -\frac{\log(x)}{4\,x^4} - \frac{1}{16\,x^4}$$

```
[In] integrate(log(x)/x^5,x, algorithm="giac")
[Out] -1/4*log(x)/x^4 - 1/16/x^4
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int \frac{\log(x)}{x^5} \, dx = -\frac{\ln(x) + \frac{1}{4}}{4\,x^4}$$

```
[In] int(log(x)/x^5,x)
[Out] -(log(x) + 1/4)/(4*x^4)
```

3.73 $\int x^2 \log \left(\frac{-1+x}{x}\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 36

$$\int x^2 \log \left(\frac{-1+x}{x}\right) dx = -\frac{x}{3} - \frac{x^2}{6} - \frac{1}{3}\log(-1+x) + \frac{1}{3}x^3 \log \left(\frac{-1+x}{x}\right)$$

```
[Out] -1/3*x-1/6*x^2-1/3*ln(-1+x)+1/3*x^3*ln((-1+x)/x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2511, 2505, 269, 45}

$$\int x^2 \log \left(\frac{-1+x}{x}\right) dx = \frac{1}{3}x^3 \log \left(1 - \frac{1}{x}\right) - \frac{x^2}{6} - \frac{x}{3} - \frac{1}{3}\log(1-x)$$

```
[In] Int[x^2*Log[(-1 + x)/x],x]
[Out] -1/3*x - x^2/6 + (x^3*Log[1 - x^(-1)])/3 - Log[1 - x]/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2511

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^2 \log\left(1-\frac{1}{x}\right)\,dx\\ &= \frac{1}{3}x^3\log\left(1-\frac{1}{x}\right)-\frac{1}{3}\int \frac{x}{1-\frac{1}{x}}\,dx\\ &= \frac{1}{3}x^3\log\left(1-\frac{1}{x}\right)-\frac{1}{3}\int \frac{x^2}{-1+x}\,dx\\ &= \frac{1}{3}x^3\log\left(1-\frac{1}{x}\right)-\frac{1}{3}\int \left(1+\frac{1}{-1+x}+x\right)\,dx\\ &= -\frac{x}{3}-\frac{x^2}{6}+\frac{1}{3}x^3\log\left(1-\frac{1}{x}\right)-\frac{1}{3}\log(1-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int x^2\log\left(\frac{-1+x}{x}\right)\,dx=-\frac{x}{3}-\frac{x^2}{6}-\frac{1}{3}\log(1-x)+\frac{1}{3}x^3\log\left(\frac{-1+x}{x}\right)$$

```
[In] Integrate[x^2*Log[(-1 + x)/x],x]
[Out] -1/3*x - x^2/6 - Log[1 - x]/3 + (x^3*Log[(-1 + x)/x])/3
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{x}{3}-\frac{x^{2}}{6}-\frac{\ln(-1+x)}{3}+\frac{x^{3}\ln\left(\frac{-1+x}{x}\right)}{3}$	29
parts	$-\frac{x}{3}-\frac{x^{2}}{6}-\frac{\ln(-1+x)}{3}+\frac{x^{3}\ln\left(\frac{-1+x}{x}\right)}{3}$	29
parallelrisch	$\frac{x^{3}\ln\left(\frac{-1+x}{x}\right)}{3}-\frac{1}{3}-\frac{x^{2}}{6}-\frac{\ln(x)}{3}-\frac{x}{3}-\frac{\ln\left(\frac{-1+x}{x}\right)}{3}$	38
derivativedivides	$-\frac{x^{2}}{6}-\frac{x}{3}+\frac{\ln\left(-\frac{1}{x}\right)}{3}+\frac{\ln\left(1-\frac{1}{x}\right)\left(1-\frac{1}{x}\right)\left(\left(1-\frac{1}{x}\right)^{2}+\frac{3}{x}\right)x^{3}}{3}$	53
default	$-\frac{x^{2}}{6}-\frac{x}{3}+\frac{\ln\left(-\frac{1}{x}\right)}{3}+\frac{\ln\left(1-\frac{1}{x}\right)\left(1-\frac{1}{x}\right)\left(\left(1-\frac{1}{x}\right)^{2}+\frac{3}{x}\right)x^{3}}{3}$	53

```
[In] int(x^2*ln((-1+x)/x),x,method=_RETURNVERBOSE)
[Out] -1/3*x-1/6*x^2-1/3*ln(-1+x)+1/3*x^3*ln((-1+x)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^2\log\left(\frac{-1+x}{x}\right)\,dx=\frac{1}{3}\,x^3\log\left(\frac{x-1}{x}\right)-\frac{1}{6}\,x^2-\frac{1}{3}\,x-\frac{1}{3}\,\log\left(x-1\right)$$

```
[In] integrate(x^2*log((-1+x)/x),x, algorithm="fricas")
[Out] 1/3*x^3*log((x - 1)/x) - 1/6*x^2 - 1/3*x - 1/3*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int x^2\log\left(\frac{-1+x}{x}\right)\,dx=\frac{x^3\log\left(\frac{x-1}{x}\right)}{3}-\frac{x^2}{6}-\frac{x}{3}-\frac{\log\left(x-1\right)}{3}$$

```
[In] integrate(x**2*ln((-1+x)/x),x)
[Out] x**3*log((x - 1)/x)/3 - x**2/6 - x/3 - log(x - 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^2 \log\left(\frac{-1+x}{x}\right)\,dx = \frac{1}{3}\,x^3 \log\left(\frac{x-1}{x}\right) - \frac{1}{6}\,x^2 - \frac{1}{3}\,x - \frac{1}{3}\,\log\left(x-1\right)$$

```
[In] integrate(x^2*log((-1+x)/x),x, algorithm="maxima")
[Out] 1/3*x^3*log((x - 1)/x) - 1/6*x^2 - 1/3*x - 1/3*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.94

$$\int x^2 \log\left(\frac{-1+x}{x}\right)\,dx = \frac{\frac{2\,(x-1)}{x} - 3}{6\left(\frac{x-1}{x} - 1\right)^2} - \frac{\log\left(\frac{x-1}{x}\right)}{3\left(\frac{x-1}{x} - 1\right)^3} - \frac{1}{3}\,\log\left(\frac{|x-1|}{|x|}\right) + \frac{1}{3}\,\log\left(\left|\frac{x-1}{x} - 1\right|\right)$$

```
[In] integrate(x^2*log((-1+x)/x),x, algorithm="giac")
[Out] 1/6*(2*(x - 1)/x - 3)/((x - 1)/x - 1)^2 - 1/3*log((x - 1)/x)/((x - 1)/x - 1
)^3 - 1/3*log(abs(x - 1)/abs(x)) + 1/3*log(abs((x - 1)/x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int x^2 \log\left(\frac{-1+x}{x}\right)\,dx = \frac{x^3\,\ln\left(\frac{x-1}{x}\right)}{3} - \frac{\ln\left(x\,(x-1)\right)}{6} - \frac{\ln\left(\frac{x-1}{x}\right)}{6} - \frac{x}{3} - \frac{x^2}{6}$$

```
[In] int(x^2*log((x - 1)/x),x)
[Out] (x^3*log((x - 1)/x))/3 - log(x*(x - 1))/6 - log((x - 1)/x)/6 - x/3 - x^2/6
```

3.74 $\int \cos^5(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 19

$$\int \cos^5(x)\,dx = \sin(x) - \frac{2\sin^3(x)}{3} + \frac{\sin^5(x)}{5}$$

```
[Out] sin(x)-2/3*sin(x)^3+1/5*sin(x)^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2713}

$$\int \cos^5(x)\,dx = \frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

```
[In] Int[Cos[x]^5,x]
```

```
[Out] Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, -\sin(x)\right)\\ &= \sin(x) - \frac{2\sin^3(x)}{3} + \frac{\sin^5(x)}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos^5(x)\,dx=\sin(x)-\frac{2\sin^3(x)}{3}+\frac{\sin^5(x)}{5}$$

[In] Integrate[Cos[x]^5,x]

[Out] Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\left(\frac{8}{3}+\cos^4(x)+\frac{4\left(\cos^2(x)\right)}{3}\right)\sin(x)}{5}$	17
risch	$\frac{5\sin(x)}{8}+\frac{\sin(5x)}{80}+\frac{5\sin(3x)}{48}$	18
parallelrisch	$\frac{5\sin(x)}{8}+\frac{\sin(5x)}{80}+\frac{5\sin(3x)}{48}$	18

[In] int(cos(x)^5,x,method=_RETURNVERBOSE)

[Out] 1/5*(8/3+cos(x)^4+4/3*cos(x)^2)*sin(x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \cos^5(x)\,dx=\frac{1}{15}\left(3\cos\left(x\right)^4+4\cos\left(x\right)^2+8\right)\sin\left(x\right)$$

[In] integrate(cos(x)^5,x, algorithm="fricas")

[Out] 1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \cos^5(x)\,dx = \frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

```
[In] integrate(cos(x)**5,x)
[Out] sin(x)**5/5 - 2*sin(x)**3/3 + sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cos^5(x)\,dx = \frac{1}{5}\sin(x)^5 - \frac{2}{3}\sin(x)^3 + \sin(x)$$

```
[In] integrate(cos(x)^5,x, algorithm="maxima")
[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cos^5(x)\,dx = \frac{1}{5}\sin(x)^5 - \frac{2}{3}\sin(x)^3 + \sin(x)$$

```
[In] integrate(cos(x)^5,x, algorithm="giac")
[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos^5(x)\,dx = \frac{\sin(x)\cos(x)^4}{5} + \frac{4\sin(x)\cos(x)^2}{15} + \frac{8\sin(x)}{15}$$

```
[In] int(cos(x)^5,x)
[Out] (8*sin(x))/15 + (4*cos(x)^2*sin(x))/15 + (cos(x)^4*sin(x))/5
```

3.75 $\int \cos^4(x)\sin^2(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 34

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{x}{16} + \frac{1}{16}\cos(x)\sin(x) + \frac{1}{24}\cos^3(x)\sin(x) - \frac{1}{6}\cos^5(x)\sin(x)$$

```
[Out] 1/16*x+1/16*cos(x)*sin(x)+1/24*cos(x)^3*sin(x)-1/6*cos(x)^5*sin(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2648, 2715, 8}

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{x}{16} - \frac{1}{6}\sin(x)\cos^5(x) + \frac{1}{24}\sin(x)\cos^3(x) + \frac{1}{16}\sin(x)\cos(x)$$

```
[In] Int[Cos[x]^4*Sin[x]^2,x]
```

```
[Out] x/16 + (Cos[x]*Sin[x])/16 + (Cos[x]^3*Sin[x])/24 - (Cos[x]^5*Sin[x])/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{6}\cos^5(x)\sin(x) + \frac{1}{6}\int \cos^4(x)\,dx \\
&= \frac{1}{24}\cos^3(x)\sin(x) - \frac{1}{6}\cos^5(x)\sin(x) + \frac{1}{8}\int \cos^2(x)\,dx \\
&= \frac{1}{16}\cos(x)\sin(x) + \frac{1}{24}\cos^3(x)\sin(x) - \frac{1}{6}\cos^5(x)\sin(x) + \frac{\int 1\,dx}{16} \\
&= \frac{x}{16} + \frac{1}{16}\cos(x)\sin(x) + \frac{1}{24}\cos^3(x)\sin(x) - \frac{1}{6}\cos^5(x)\sin(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{x}{16} + \frac{1}{64}\sin(2x) - \frac{1}{64}\sin(4x) - \frac{1}{192}\sin(6x)$$

```
[In] Integrate[Cos[x]^4*Sin[x]^2,x]
[Out] x/16 + Sin[2*x]/64 - Sin[4*x]/64 - Sin[6*x]/192
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result
risch	$\frac{x}{16}-\frac{\sin(6x)}{192}-\frac{\sin(4x)}{64}+\frac{\sin(2x)}{64}$
parallelrisch	$\frac{x}{16}-\frac{\sin(6x)}{192}-\frac{\sin(4x)}{64}+\frac{\sin(2x)}{64}$
default	$-\frac{(\cos^5(x))\sin(x)}{6}+\frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{24}+\frac{x}{16}$
norman	$\frac{\frac{x}{16}+\frac{47\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}-\frac{13\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{13\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}-\frac{47\left(\tan^9\left(\frac{x}{2}\right)\right)}{24}+\frac{\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{8}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{15x\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{5x\left(\tan^6\left(\frac{x}{2}\right)\right)}{4}+}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

```
[In] int(sin(x)^2*cos(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*x-1/192*sin(6*x)-1/64*sin(4*x)+1/64*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \cos^4(x)\sin^2(x)\,dx = -\frac{1}{48}\left(8\cos(x)^5 - 2\cos(x)^3 - 3\cos(x)\right)\sin(x) + \frac{1}{16}x$$

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="fricas")
[Out] -1/48*(8*cos(x)^5 - 2*cos(x)^3 - 3*cos(x))*sin(x) + 1/16*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{x}{16} - \frac{\sin(x)\cos^5(x)}{6} + \frac{\sin(x)\cos^3(x)}{24} + \frac{\sin(x)\cos(x)}{16}$$

```
[In] integrate(cos(x)**4*sin(x)**2,x)
[Out] x/16 - sin(x)*cos(x)**5/6 + sin(x)*cos(x)**3/24 + sin(x)*cos(x)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.53

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{1}{48}\sin(2x)^3 + \frac{1}{16}x - \frac{1}{64}\sin(4x)$$

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="maxima")
[Out] 1/48*sin(2*x)^3 + 1/16*x - 1/64*sin(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{1}{16}\,x - \frac{1}{192}\,\sin\left(6\,x\right) - \frac{1}{64}\,\sin\left(4\,x\right) + \frac{1}{64}\,\sin\left(2\,x\right)$$

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="giac")
[Out] 1/16*x - 1/192*sin(6*x) - 1/64*sin(4*x) + 1/64*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \cos^4(x)\sin^2(x)\,dx = \left(\frac{\cos\left(x\right)^3}{6} + \frac{\cos\left(x\right)}{8}\right)\,\sin\left(x\right)^3 - \frac{\cos\left(x\right)\,\sin\left(x\right)}{16} + \frac{x}{16}$$

```
[In] int(cos(x)^4*sin(x)^2,x)
[Out] x/16 - (cos(x)*sin(x))/16 + sin(x)^3*(cos(x)/8 + cos(x)^3/6)
```

3.76 $\int \csc^5(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 26

$$\int \csc^5(x)\,dx = -\frac{3}{8}\operatorname{arctanh}(\cos(x)) - \frac{3}{8}\cot(x)\csc(x) - \frac{1}{4}\cot(x)\csc^3(x)$$

```
[Out] -3/8*arctanh(cos(x))-3/8*cot(x)*csc(x)-1/4*cot(x)*csc(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3853, 3855}

$$\int \csc^5(x)\,dx = -\frac{3}{8}\operatorname{arctanh}(\cos(x)) - \frac{1}{4}\cot(x)\csc^3(x) - \frac{3}{8}\cot(x)\csc(x)$$

```
[In] Int[Csc[x]^5,x]
```

```
[Out] (-3*ArcTanh[Cos[x]])/8 - (3*Cot[x]*Csc[x])/8 - (Cot[x]*Csc[x]^3)/4
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{4}\cot(x)\csc^3(x) + \frac{3}{4}\int \csc^3(x)\,dx \\ &= -\frac{3}{8}\cot(x)\csc(x) - \frac{1}{4}\cot(x)\csc^3(x) + \frac{3}{8}\int \csc(x)\,dx \\ &= -\frac{3}{8}\text{arctanh}(\cos(x)) - \frac{3}{8}\cot(x)\csc(x) - \frac{1}{4}\cot(x)\csc^3(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 71 vs. 2(26) = 52.

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.73

$$\int \csc^5(x)\,dx = -\frac{3}{32}\csc^2\left(\frac{x}{2}\right) - \frac{1}{64}\csc^4\left(\frac{x}{2}\right) - \frac{3}{8}\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{3}{8}\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{3}{32}\sec^2\left(\frac{x}{2}\right) + \frac{1}{64}\sec^4\left(\frac{x}{2}\right)$$

```
[In] Integrate[Csc[x]^5,x]
[Out] (-3*Csc[x/2]^2)/32 - Csc[x/2]^4/64 - (3*Log[Cos[x/2]])/8 + (3*Log[Sin[x/2]]
)/8 + (3*Sec[x/2]^2)/32 + Sec[x/2]^4/64
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
default	$\left(-\frac{(\csc^3(x))}{4}-\frac{3\csc(x)}{8}\right)\cot(x)+\frac{3\ln(\csc(x)-\cot(x))}{8}$	26
parallelrisch	$\ln\left((\csc(x)-\cot(x))^{\frac{3}{8}}\right)+\frac{3(\cot^3(x))\csc(x)}{8}-\frac{5(\csc^3(x))\cot(x)}{8}$	28
norman	$\frac{-\frac{1}{64}-\frac{(\tan^2(\frac{x}{2}))}{8}+\frac{(\tan^6(\frac{x}{2}))}{8}+\frac{(\tan^8(\frac{x}{2}))}{64}}{\tan(\frac{x}{2})^4}+\frac{3\ln(\tan(\frac{x}{2}))}{8}$	42
risch	$\frac{3\,e^{7ix}-11\,e^{5ix}-11\,e^{3ix}+3\,e^{ix}}{4(e^{2ix}-1)^4}-\frac{3\ln(e^{ix}+1)}{8}+\frac{3\ln(e^{ix}-1)}{8}$	62

```
[In] int(1/sin(x)^5,x,method=_RETURNVERBOSE)
[Out] (-1/4*csc(x)^3-3/8*csc(x))*cot(x)+3/8*ln(csc(x)-cot(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(20) = 40$.

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.65

$$\int \csc^5(x)\,dx$$
$$= \frac{6\cos(x)^3 - 3\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + 3\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(-\frac{1}{2}\cos(x)\right.}{16\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)}$$

```
[In] integrate(1/sin(x)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(6*cos(x)^3 - 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*cos(x) + 1/2) + 3*
(cos(x)^4 - 2*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2) - 10*cos(x))/(cos(x)^4 -
 2*cos(x)^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \csc^5(x)\,dx = \frac{3\cos^3(x) - 5\cos(x)}{8\cos^4(x) - 16\cos^2(x) + 8} + \frac{3\log(\cos(x) - 1)}{16} - \frac{3\log(\cos(x) + 1)}{16}$$

```
[In] integrate(1/sin(x)**5,x)
```

```
[Out] (3*cos(x)**3 - 5*cos(x))/(8*cos(x)**4 - 16*cos(x)**2 + 8) + 3*log(cos(x) -
1)/16 - 3*log(cos(x) + 1)/16
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \csc^5(x)\,dx = \frac{3\cos(x)^3 - 5\cos(x)}{8\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)} - \frac{3}{16}\log(\cos(x) + 1) + \frac{3}{16}\log(\cos(x) - 1)$$

```
[In] integrate(1/sin(x)^5,x, algorithm="maxima")
```

```
[Out] 1/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) - 3/16*log(cos(x) +
 1) + 3/16*log(cos(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \csc^5(x)\,dx = \frac{3\,\cos(x)^3 - 5\,\cos(x)}{8\left(\cos(x)^2 - 1\right)^2} - \frac{3}{16}\,\log\left(\cos(x) + 1\right) + \frac{3}{16}\,\log\left(-\cos(x) + 1\right)$$

```
[In] integrate(1/sin(x)^5,x, algorithm="giac")
```

```
[Out] 1/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^2 - 1)^2 - 3/16*log(cos(x) + 1) + 3/16*
log(-cos(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \csc^5(x)\,dx = -\frac{3\,\mathrm{atanh}(\cos(x))}{8} - \frac{\frac{5\,\cos(x)}{8} - \frac{3\cos(x)^3}{8}}{\cos(x)^4 - 2\cos(x)^2 + 1}$$

```
[In] int(1/sin(x)^5,x)
```

```
[Out] - (3*atanh(cos(x)))/8 - ((5*cos(x))/8 - (3*cos(x)^3)/8)/(cos(x)^4 - 2*cos(x
)^2 + 1)
```

3.77 $\int e^{-x}\sin(x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 23

$$\int e^{-x}\sin(x)\,dx = -\frac{1}{2}e^{-x}\cos(x) - \frac{1}{2}e^{-x}\sin(x)$$

```
[Out] -1/2*cos(x)/exp(x)-1/2*sin(x)/exp(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4517}

$$\int e^{-x}\sin(x)\,dx = -\frac{1}{2}e^{-x}\sin(x) - \frac{1}{2}e^{-x}\cos(x)$$

```
[In] Int[Sin[x]/E^x,x]
```

```
[Out] -1/2*Cos[x]/E^x - Sin[x]/(2*E^x)
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{2}e^{-x}\cos(x) - \frac{1}{2}e^{-x}\sin(x)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int e^{-x}\sin(x)\,dx=-\frac{1}{2}e^{-x}(\cos(x)+\sin(x))$$

```
[In] Integrate[Sin[x]/E^x,x]
[Out] -1/2*(Cos[x] + Sin[x])/E^x
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

method	result	size
parallelrisch	$-\frac{(\cos(x)+\sin(x))\mathrm{e}^{-x}}{2}$	12
default	$-\frac{\mathrm{e}^{-x}\cos(x)}{2}-\frac{\mathrm{e}^{-x}\sin(x)}{2}$	18
norman	$\frac{\left(-\frac{1}{2}+\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\tan\left(\frac{x}{2}\right)\right)\mathrm{e}^{-x}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	32
risch	$-\frac{\mathrm{e}^{(-1+i)x}}{4}+\frac{i\mathrm{e}^{(-1+i)x}}{4}-\frac{\mathrm{e}^{(-1-i)x}}{4}-\frac{i\mathrm{e}^{(-1-i)x}}{4}$	36

```
[In] int(sin(x)/exp(x),x,method=_RETURNVERBOSE)
[Out] -1/2*(cos(x)+sin(x))*exp(-x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int e^{-x}\sin(x)\,dx=-\frac{1}{2}\cos\left(x\right)e^{\left(-x\right)}-\frac{1}{2}e^{\left(-x\right)}\sin\left(x\right)$$

```
[In] integrate(sin(x)/exp(x),x, algorithm="fricas")
[Out] -1/2*cos(x)*e^(-x) - 1/2*e^(-x)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int e^{-x}\sin(x)\,dx = -\frac{e^{-x}\sin(x)}{2} - \frac{e^{-x}\cos(x)}{2}$$

```
[In] integrate(sin(x)/exp(x),x)
[Out] -exp(-x)*sin(x)/2 - exp(-x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{-x}\sin(x)\,dx = -\frac{1}{2}\left(\cos(x) + \sin(x)\right)e^{(-x)}$$

```
[In] integrate(sin(x)/exp(x),x, algorithm="maxima")
[Out] -1/2*(cos(x) + sin(x))*e^(-x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{-x}\sin(x)\,dx = -\frac{1}{2}\left(\cos(x) + \sin(x)\right)e^{(-x)}$$

```
[In] integrate(sin(x)/exp(x),x, algorithm="giac")
[Out] -1/2*(cos(x) + sin(x))*e^(-x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{-x}\sin(x)\,dx = -\frac{e^{-x}\left(\cos(x) + \sin(x)\right)}{2}$$

```
[In] int(exp(-x)*sin(x),x)
[Out] -(exp(-x)*(cos(x) + sin(x)))/2
```

3.78 $\int e^{2x}\sin(3x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{2x}\sin(3x)\,dx = -\frac{3}{13}e^{2x}\cos(3x)+\frac{2}{13}e^{2x}\sin(3x)$$

```
[Out] -3/13*exp(2*x)*cos(3*x)+2/13*exp(2*x)*sin(3*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4517}

$$\int e^{2x}\sin(3x)\,dx = \frac{2}{13}e^{2x}\sin(3x)-\frac{3}{13}e^{2x}\cos(3x)$$

```
[In] Int[E^(2*x)*Sin[3*x],x]
```

```
[Out] (-3*E^(2*x)*Cos[3*x])/13 + (2*E^(2*x)*Sin[3*x])/13
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{3}{13}e^{2x}\cos(3x)+\frac{2}{13}e^{2x}\sin(3x)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{2x}\sin(3x)\,dx = \frac{1}{13}e^{2x}(-3\cos(3x)+2\sin(3x))$$

```
[In] Integrate[E^(2*x)*Sin[3*x],x]
[Out] (E^(2*x)*(-3*Cos[3*x] + 2*Sin[3*x]))/13
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{e^{2x}(-3\cos(3x)+2\sin(3x))}{13}$	20
default	$-\frac{3\,e^{2x}\cos(3x)}{13}+\frac{2\,e^{2x}\sin(3x)}{13}$	22
risch	$-\frac{3\,e^{(2+3i)x}}{26}-\frac{i e^{(2+3i)x}}{13}-\frac{3\,e^{(2-3i)x}}{26}+\frac{i e^{(2-3i)x}}{13}$	36
norman	$\frac{\frac{4\,e^{2x}\tan\left(\frac{3x}{2}\right)}{13}+\frac{3\,e^{2x}\left(\tan^{2}\left(\frac{3x}{2}\right)\right)}{13}-\frac{3\,e^{2x}}{13}}{1+\tan^{2}\left(\frac{3x}{2}\right)}$	41

```
[In] int(exp(2*x)*sin(3*x),x,method=_RETURNVERBOSE)
[Out] 1/13*exp(2*x)*(-3*cos(3*x)+2*sin(3*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{2x}\sin(3x)\,dx = -\frac{3}{13}\cos(3\,x)\,e^{(2\,x)}+\frac{2}{13}\,e^{(2\,x)}\sin(3\,x)$$

```
[In] integrate(exp(2*x)*sin(3*x),x, algorithm="fricas")
[Out] -3/13*cos(3*x)*e^(2*x) + 2/13*e^(2*x)*sin(3*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{2x}\sin(3x)\,dx = \frac{2e^{2x}\sin{(3x)}}{13} - \frac{3e^{2x}\cos{(3x)}}{13}$$

```
[In] integrate(exp(2*x)*sin(3*x),x)
[Out] 2*exp(2*x)*sin(3*x)/13 - 3*exp(2*x)*cos(3*x)/13
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{2x}\sin(3x)\,dx = -\frac{1}{13}\,(3\,\cos{(3\,x)} - 2\,\sin{(3\,x)})e^{(2\,x)}$$

```
[In] integrate(exp(2*x)*sin(3*x),x, algorithm="maxima")
[Out] -1/13*(3*cos(3*x) - 2*sin(3*x))*e^(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{2x}\sin(3x)\,dx = -\frac{1}{13}\,(3\,\cos{(3\,x)} - 2\,\sin{(3\,x)})e^{(2\,x)}$$

```
[In] integrate(exp(2*x)*sin(3*x),x, algorithm="giac")
[Out] -1/13*(3*cos(3*x) - 2*sin(3*x))*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{2x}\sin(3x)\,dx = -\frac{\mathrm{e}^{2\,x}\,(3\,\cos{(3\,x)} - 2\,\sin{(3\,x)})}{13}$$

```
[In] int(sin(3*x)*exp(2*x),x)
[Out] -(exp(2*x)*(3*cos(3*x) - 2*sin(3*x)))/13
```

3.79 $\int a^x \cos(x)\, dx$

Optimal result

Integrand size = 6, antiderivative size = 31

$$\int a^x \cos(x)\, dx = \frac{a^x \cos(x) \log(a)}{1+\log^2(a)} + \frac{a^x \sin(x)}{1+\log^2(a)}$$

```
[Out] a^x*cos(x)*ln(a)/(1+ln(a)^2)+a^x*sin(x)/(1+ln(a)^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4518}

$$\int a^x \cos(x)\, dx = \frac{a^x \sin(x)}{\log^2(a)+1} + \frac{a^x \log(a) \cos(x)}{\log^2(a)+1}$$

```
[In] Int[a^x*Cos[x],x]
```

```
[Out] (a^x*Cos[x]*Log[a])/(1 + Log[a]^2) + (a^x*Sin[x])/(1 + Log[a]^2)
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = \frac{a^x \cos(x) \log(a)}{1+\log^2(a)} + \frac{a^x \sin(x)}{1+\log^2(a)}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int a^x \cos(x)\,dx = \frac{a^x(\cos(x)\log(a) + \sin(x))}{1 + \log^2(a)}$$

```
[In] Integrate[a^x*Cos[x],x]
[Out] (a^x*(Cos[x]*Log[a] + Sin[x]))/(1 + Log[a]^2)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{a^x(\cos(x)\ln(a)+\sin(x))}{1+\ln(a)^2}$	21
risch	$\frac{a^x\cos(x)\ln(a)}{1+\ln(a)^2}+\frac{a^x\sin(x)}{1+\ln(a)^2}$	32
norman	$\frac{\frac{\ln(a)\mathrm{e}^{x\ln(a)}}{1+\ln(a)^2}+\frac{2\,\mathrm{e}^{x\ln(a)}\tan\left(\frac{x}{2}\right)}{1+\ln(a)^2}-\frac{\ln(a)\mathrm{e}^{x\ln(a)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(a)^2}}{1+\tan^2\left(\frac{x}{2}\right)}$	71

```
[In] int(a^x*cos(x),x,method=_RETURNVERBOSE)
[Out] a^x*(cos(x)*ln(a)+sin(x))/(1+ln(a)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int a^x \cos(x)\,dx = \frac{(\cos(x)\log(a) + \sin(x))a^x}{\log(a)^2 + 1}$$

```
[In] integrate(a^x*cos(x),x, algorithm="fricas")
[Out] (cos(x)*log(a) + sin(x))*a^x/(log(a)^2 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.45

$$\int a^x \cos(x)\,dx = \begin{cases} \frac{ixe^{-ix}\sin(x)}{2} + \frac{xe^{-ix}\cos(x)}{2} + \frac{ie^{-ix}\cos(x)}{2} & \text{for } a = e^{-i} \\ -\frac{ixe^{ix}\sin(x)}{2} + \frac{xe^{ix}\cos(x)}{2} - \frac{ie^{ix}\cos(x)}{2} & \text{for } a = e^{i} \\ \frac{a^x\log(a)\cos(x)}{\log(a)^2+1} + \frac{a^x\sin(x)}{\log(a)^2+1} & \text{otherwise} \end{cases}$$

```
[In] integrate(a**x*cos(x),x)
[Out] Piecewise((I*x*exp(-I*x)*sin(x)/2 + x*exp(-I*x)*cos(x)/2 + I*exp(-I*x)*cos(
x)/2, Eq(a, exp(-I))), (-I*x*exp(I*x)*sin(x)/2 + x*exp(I*x)*cos(x)/2 - I*ex
p(I*x)*cos(x)/2, Eq(a, exp(I))), (a**x*log(a)*cos(x)/(log(a)**2 + 1) + a**x
*sin(x)/(log(a)**2 + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int a^x \cos(x)\,dx = \frac{a^x\cos(x)\log(a) + a^x\sin(x)}{\log(a)^2 + 1}$$

```
[In] integrate(a^x*cos(x),x, algorithm="maxima")
[Out] (a^x*cos(x)*log(a) + a^x*sin(x))/(log(a)^2 + 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 329, normalized size of antiderivative = 10.61

$$\begin{aligned}\int a^x \cos(x)\,dx &= |a|^x\left(\frac{2\cos\left(\frac{1}{2}\pi x\operatorname{sgn}(a) - \frac{1}{2}\pi x + x\right)\log(|a|)}{(\pi - \pi\operatorname{sgn}(a) - 2)^2 + 4\log(|a|)^2} - \frac{(\pi - \pi\operatorname{sgn}(a) - 2)\sin\left(\frac{1}{2}\pi x\operatorname{sgn}(a) - \frac{1}{2}\pi x + x\right)}{(\pi - \pi\operatorname{sgn}(a) - 2)^2 + 4\log(|a|)^2}\right) \\ &+ |a|^x\left(\frac{2\cos\left(\frac{1}{2}\pi x\operatorname{sgn}(a) - \frac{1}{2}\pi x - x\right)\log(|a|)}{(\pi - \pi\operatorname{sgn}(a) + 2)^2 + 4\log(|a|)^2} - \frac{(\pi - \pi\operatorname{sgn}(a) + 2)\sin\left(\frac{1}{2}\pi x\operatorname{sgn}(a) - \frac{1}{2}\pi x - x\right)}{(\pi - \pi\operatorname{sgn}(a) + 2)^2 + 4\log(|a|)^2}\right) \\ &+ i|a|^x\left(\frac{ie^{\left(\frac{1}{2}i\pi x\operatorname{sgn}(a) - \frac{1}{2}i\pi x + ix\right)}}{-2i\pi + 2i\pi\operatorname{sgn}(a) + 4\log(|a|) + 4i} - \frac{ie^{\left(-\frac{1}{2}i\pi x\operatorname{sgn}(a) + \frac{1}{2}i\pi x - ix\right)}}{2i\pi - 2i\pi\operatorname{sgn}(a) + 4\log(|a|) - 4i}\right) \\ &+ i|a|^x\left(\frac{ie^{\left(\frac{1}{2}i\pi x\operatorname{sgn}(a) - \frac{1}{2}i\pi x - ix\right)}}{-2i\pi + 2i\pi\operatorname{sgn}(a) + 4\log(|a|) - 4i} - \frac{ie^{\left(-\frac{1}{2}i\pi x\operatorname{sgn}(a) + \frac{1}{2}i\pi x + ix\right)}}{2i\pi - 2i\pi\operatorname{sgn}(a) + 4\log(|a|) + 4i}\right)\end{aligned}$$

```
[In] integrate(a^x*cos(x),x, algorithm="giac")
```

```
[Out] abs(a)^x*(2*cos(1/2*pi*x*sgn(a) - 1/2*pi*x + x)*log(abs(a))/((pi - pi*sgn(a
) - 2)^2 + 4*log(abs(a))^2) - (pi - pi*sgn(a) - 2)*sin(1/2*pi*x*sgn(a) - 1/
2*pi*x + x)/((pi - pi*sgn(a) - 2)^2 + 4*log(abs(a))^2)) + abs(a)^x*(2*cos(1
/2*pi*x*sgn(a) - 1/2*pi*x - x)*log(abs(a))/((pi - pi*sgn(a) + 2)^2 + 4*log(
abs(a))^2) - (pi - pi*sgn(a) + 2)*sin(1/2*pi*x*sgn(a) - 1/2*pi*x - x)/((pi
- pi*sgn(a) + 2)^2 + 4*log(abs(a))^2)) + I*abs(a)^x*(I*e^(1/2*I*pi*x*sgn(a)
 - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sgn(a) + 4*log(abs(a)) + 4*I) - I*e^
(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*x - I*x)/(2*I*pi - 2*I*pi*sgn(a) + 4*log(abs
(a)) - 4*I)) + I*abs(a)^x*(I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x - I*x)/(-2*I
*pi + 2*I*pi*sgn(a) + 4*log(abs(a)) - 4*I) - I*e^(-1/2*I*pi*x*sgn(a) + 1/2*
I*pi*x + I*x)/(2*I*pi - 2*I*pi*sgn(a) + 4*log(abs(a)) + 4*I))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int a^x \cos(x)\,dx = \frac{a^x\left(\sin\left(x\right)+\ln\left(a\right)\,\cos\left(x\right)\right)}{{\ln\left(a\right)}^2+1}$$

```
[In] int(a^x*cos(x),x)
```

```
[Out] (a^x*(sin(x) + log(a)*cos(x)))/(log(a)^2 + 1)
```

3.80 $\int \cos(\log(x))\,dx$

Optimal result

Integrand size = 3, antiderivative size = 17

$$\int \cos(\log(x))\,dx = \frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

```
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{4564\}$

$$\int \cos(\log(x))\,dx = \frac{1}{2}x\sin(\log(x)) + \frac{1}{2}x\cos(\log(x))$$

```
[In] Int[Cos[Log[x]],x]
```

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rule 4564

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(\log(x))\,dx = \frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

```
[In] Integrate[Cos[Log[x]],x]
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{x(\cos(\ln(x))+\sin(\ln(x)))}{2}$	11
lookup	$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
default	$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
risch	$\left(\frac{1}{4}-\frac{i}{4}\right)x\,x^{i}+\left(\frac{1}{4}+\frac{i}{4}\right)x\,x^{-i}$	22

```
[In] int(cos(ln(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(cos(ln(x))+sin(ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x\cos\left(\log\left(x\right)\right) + \frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

```
[In] integrate(cos(log(x)),x, algorithm="fricas")
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cos(\log(x))\,dx = \frac{x\sin\left(\log\left(x\right)\right)}{2} + \frac{x\cos\left(\log\left(x\right)\right)}{2}$$

```
[In] integrate(cos(ln(x)),x)
[Out] x*sin(log(x))/2 + x*cos(log(x))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x(\cos\left(\log\left(x\right)\right) + \sin\left(\log\left(x\right)\right))$$

```
[In] integrate(cos(log(x)),x, algorithm="maxima")
[Out] 1/2*x*(cos(log(x)) + sin(log(x)))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x\cos\left(\log\left(x\right)\right) + \frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

```
[In] integrate(cos(log(x)),x, algorithm="giac")
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{\sqrt{2}\,x\,\sin\left(\frac{\pi}{4}+\ln\left(x\right)\right)}{2}$$

```
[In] int(cos(log(x)),x)
[Out] (2^(1/2)*x*sin(pi/4 + log(x)))/2
```

3.81 $\int \log(\cos(x)) \sec^2(x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \log(\cos(x)) \sec^2(x) \, dx = -x + \tan(x) + \log(\cos(x)) \tan(x)$$

```
[Out] -x+tan(x)+ln(cos(x))*tan(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3852, 8, 2634, 3554}

$$\int \log(\cos(x)) \sec^2(x) \, dx = -x + \tan(x) + \tan(x) \log(\cos(x))$$

```
[In] Int[Log[Cos[x]]*Sec[x]^2,x]
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \log(\cos(x))\tan(x) + \int \tan^2(x)\,dx \\ &= \tan(x) + \log(\cos(x))\tan(x) - \int 1\,dx \\ &= -x + \tan(x) + \log(\cos(x))\tan(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\sec^2(x)\,dx = -x + \tan(x) + \log(\cos(x))\tan(x)$$

```
[In] Integrate[Log[Cos[x]]*Sec[x]^2,x]
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result
parallelrisch	$-x+\tan\left(x\right)+\ln\left(\cos\left(x\right)\right)\tan\left(x\right)$
norman	$\frac{x-x\left(\tan^{2}\left(\frac{x}{2}\right)\right)-2\tan\left(\frac{x}{2}\right)\ln\left(\frac{1-\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}\right)-2\tan\left(\frac{x}{2}\right)}{\tan^{2}\left(\frac{x}{2}\right)-1}$
default	$-4i\left(\frac{\frac{e^{2ix}\ln\left(\left(e^{2ix}+1\right)e^{-ix}\right)}{2}-\frac{1}{2}}{e^{2ix}+1}-\frac{\ln\left(e^{2ix}+1\right)}{4}+\frac{\ln(2)}{2\,e^{2ix}+2}\right)$
risch	$-\frac{2i\ln\left(e^{ix}\right)}{e^{2ix}+1}+\frac{-\pi\,\operatorname{csgn}\left(i\left(e^{2ix}+1\right)\right)\operatorname{csgn}\left(i\cos(x)\right)^{2}+\pi\,\operatorname{csgn}\left(i\left(e^{2ix}+1\right)\right)\operatorname{csgn}\left(i\cos(x)\right)\operatorname{csgn}\left(ie^{-ix}\right)+\pi\operatorname{csgn}\left(i\cos(x)\right)^{3}-\pi\operatorname{csgn}(}{e^{2ix}+1}$

```
[In] int(ln(cos(x))*sec(x)^2,x,method=_RETURNVERBOSE)
[Out] -x+tan(x)+ln(cos(x))*tan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \log(\cos(x))\sec^2(x)\,dx = -\frac{x\cos(x) - \log(\cos(x))\sin(x) - \sin(x)}{\cos(x)}$$

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="fricas")
[Out] -(x*cos(x) - log(cos(x))*sin(x) - sin(x))/cos(x)
```

Sympy [A] (verification not implemented)

Time = 17.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \log(\cos(x))\sec^2(x)\,dx = -x + \log(\cos(x))\tan(x) + \frac{\sin(x)}{\cos(x)}$$

```
[In] integrate(ln(cos(x))*sec(x)**2,x)
[Out] -x + log(cos(x))*tan(x) + sin(x)/cos(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(12) = 24.

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 7.83

$$\int \log(\cos(x))\sec^2(x)\,dx = -\frac{2\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - \frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="maxima")
[Out] -2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x)
/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(x)
 + 1)^2 - 1)*(cos(x) + 1)) - 2*arctan(sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\sec^2(x)\,dx = \log\left(\cos\left(x\right)\right)\tan\left(x\right) - x + \tan\left(x\right)$$

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="giac")
[Out] log(cos(x))*tan(x) - x + tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.92

$$\int \log(\cos(x))\sec^2(x)\,dx = \tan(x) - 2\,x + \ln\left(\cos\left(x\right)\right)\,\tan(x) + \ln\left(\cos\left(x\right)\right)\,1\mathrm{i} - \ln\left(\cos\left(2\,x\right) + 1 + \sin\left(2\,x\right)\,1\mathrm{i}\right)\,1\mathrm{i}$$

```
[In] int(log(cos(x))/cos(x)^2,x)
[Out] log(cos(x))*1i - 2*x - log(cos(2*x) + sin(2*x)*1i + 1)*1i + tan(x) + log(co
s(x))*tan(x)
```

3.82 $\int x \tan^2(x) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 15

$$\int x \tan^2(x) \, dx = -\frac{x^2}{2} + \log(\cos(x)) + x \tan(x)$$

```
[Out] -1/2*x^2+ln(cos(x))+x*tan(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3801, 3556, 30}

$$\int x \tan^2(x) \, dx = -\frac{x^2}{2} + x \tan(x) + \log(\cos(x))$$

```
[In] Int[x*Tan[x]^2,x]
```

```
[Out] -1/2*x^2 + Log[Cos[x]] + x*Tan[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\tan(x) - \int x\,dx - \int \tan(x)\,dx\\ &= -\frac{x^2}{2} + \log(\cos(x)) + x\tan(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x\tan^2(x)\,dx = -\frac{x^2}{2} + \log(\cos(x)) + x\tan(x)$$

```
[In] Integrate[x*Tan[x]^2,x]
[Out] -1/2*x^2 + Log[Cos[x]] + x*Tan[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

method	result	size
norman	$x\tan\left(x\right)-\frac{x^{2}}{2}-\frac{\ln\left(1+\tan^{2}\left(x\right)\right)}{2}$	20
parallelrisch	$x\tan\left(x\right)-\frac{x^{2}}{2}-\frac{\ln\left(1+\tan^{2}\left(x\right)\right)}{2}$	20
risch	$-\frac{x^{2}}{2}-2ix+\frac{2ix}{{\mathrm e}^{2ix}+1}+\ln\left({\mathrm e}^{2ix}+1\right)$	32

```
[In] int(x*tan(x)^2,x,method=_RETURNVERBOSE)
[Out] x*tan(x)-1/2*x^2-1/2*ln(1+tan(x)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int x\tan^2(x)\,dx = -\frac{1}{2}\,x^2 + x\tan(x) + \frac{1}{2}\,\log\left(\frac{1}{\tan(x)^2+1}\right)$$

```
[In] integrate(x*tan(x)^2,x, algorithm="fricas")
[Out] -1/2*x^2 + x*tan(x) + 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int x\tan^2(x)\,dx = -\frac{x^2}{2} + x\tan(x) - \frac{\log(\tan^2(x)+1)}{2}$$

```
[In] integrate(x*tan(x)**2,x)
[Out] -x**2/2 + x*tan(x) - log(tan(x)**2 + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(13) = 26.

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 7.13

$$\int x\tan^2(x)\,dx =$$
$$-\frac{x^2\cos(2x)^2 + x^2\sin(2x)^2 + 2x^2\cos(2x) + x^2 - \left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right)\log\left(\cos(2x\right.}{2\left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right)}$$

```
[In] integrate(x*tan(x)^2,x, algorithm="maxima")
[Out] -1/2*(x^2*cos(2*x)^2 + x^2*sin(2*x)^2 + 2*x^2*cos(2*x) + x^2 - (cos(2*x)^2
+ sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1
) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int x \tan^2(x) \, dx = -\frac{1}{2} x^2 + x \tan{(x)} + \frac{1}{2} \log\left(\frac{4}{\tan{(x)}^2 + 1}\right)$$

```
[In] integrate(x*tan(x)^2,x, algorithm="giac")
[Out] -1/2*x^2 + x*tan(x) + 1/2*log(4/(tan(x)^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x \tan^2(x) \, dx = \ln{(\cos{(x)})} + x \tan(x) - \frac{x^2}{2}$$

```
[In] int(x*tan(x)^2,x)
[Out] log(cos(x)) + x*tan(x) - x^2/2
```

3.83 $\int \frac{\arcsin(x)}{x^2} dx$

Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \frac{\arcsin(x)}{x^2} dx = -\frac{\arcsin(x)}{x} - \operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

```
[Out] -arcsin(x)/x-arctanh((-x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4723, 272, 65, 212}

$$\int \frac{\arcsin(x)}{x^2} dx = -\frac{\arcsin(x)}{x} - \operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

```
[In] Int[ArcSin[x]/x^2,x]
```

```
[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arcsin(x)}{x} + \int \frac{1}{x\sqrt{1-x^2}}\,dx \\
&= -\frac{\arcsin(x)}{x} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, x^2\right) \\
&= -\frac{\arcsin(x)}{x} - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right) \\
&= -\frac{\arcsin(x)}{x} - \operatorname{arctanh}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{\arcsin(x)}{x} - \operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

```
[In] Integrate[ArcSin[x]/x^2,x]
[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{\arcsin(x)}{x}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)$	21
parts	$-\frac{\arcsin(x)}{x}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)$	21

```
[In] int(arcsin(x)/x^2,x,method=_RETURNVERBOSE)
[Out] -arcsin(x)/x-arctanh(1/(-x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{\arcsin(x)}{x^2}\,dx=-\frac{x\log\left(\sqrt{-x^2+1}+1\right)-x\log\left(\sqrt{-x^2+1}-1\right)+2\arcsin\left(x\right)}{2\,x}$$

```
[In] integrate(arcsin(x)/x^2,x, algorithm="fricas")
[Out] -1/2*(x*log(sqrt(-x^2 + 1) + 1) - x*log(sqrt(-x^2 + 1) - 1) + 2*arcsin(x))/
x
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(x)}{x^2}\,dx=\begin{cases}-\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for}\ \frac{1}{|x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise}\end{cases}-\frac{\operatorname{asin}\left(x\right)}{x}$$

```
[In] integrate(asin(x)/x**2,x)
[Out] Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) - asin(x)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{\arcsin(x)}{x} - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

```
[In] integrate(arcsin(x)/x^2,x, algorithm="maxima")
[Out] -arcsin(x)/x - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{\arcsin(x)}{x} - \frac{1}{2}\log\left(\sqrt{-x^2+1}+1\right) + \frac{1}{2}\log\left(-\sqrt{-x^2+1}+1\right)$$

```
[In] integrate(arcsin(x)/x^2,x, algorithm="giac")
[Out] -arcsin(x)/x - 1/2*log(sqrt(-x^2 + 1) + 1) + 1/2*log(-sqrt(-x^2 + 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\mathrm{atanh}\left(\frac{1}{\sqrt{1-x^2}}\right) - \frac{\mathrm{asin}(x)}{x}$$

```
[In] int(asin(x)/x^2,x)
[Out] - atanh(1/(1 - x^2)^(1/2)) - asin(x)/x
```

3.84 $\int \arcsin(x)^2 \, dx$

Optimal result

Integrand size = 4, antiderivative size = 25

$$\int \arcsin(x)^2 \, dx = -2x + 2\sqrt{1-x^2} \arcsin(x) + x \arcsin(x)^2$$

```
[Out] -2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4715, 4767, 8}

$$\int \arcsin(x)^2 \, dx = 2\sqrt{1-x^2} \arcsin(x) + x \arcsin(x)^2 - 2x$$

```
[In] Int[ArcSin[x]^2,x]
```

```
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4767

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\arcsin(x)^2 - 2\int \frac{x\arcsin(x)}{\sqrt{1-x^2}}\,dx\\ &= 2\sqrt{1-x^2}\arcsin(x) + x\arcsin(x)^2 - 2\int 1\,dx\\ &= -2x + 2\sqrt{1-x^2}\arcsin(x) + x\arcsin(x)^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \arcsin(x)^2\,dx = -2x + 2\sqrt{1-x^2}\arcsin(x) + x\arcsin(x)^2$$

```
[In] Integrate[ArcSin[x]^2,x]
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
default	$-2x + x\arcsin(x)^2 + 2\arcsin(x)\sqrt{-x^2+1}$	24

```
[In] int(arcsin(x)^2,x,method=_RETURNVERBOSE)
[Out] -2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2\,dx = x\arcsin{(x)}^2 + 2\sqrt{-x^2+1}\arcsin{(x)} - 2\,x$$

```
[In] integrate(arcsin(x)^2,x, algorithm="fricas")
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \arcsin(x)^2\,dx = x\,\mathrm{asin}^2{(x)} - 2x + 2\sqrt{1-x^2}\,\mathrm{asin}{(x)}$$

```
[In] integrate(asin(x)**2,x)
[Out] x*asin(x)**2 - 2*x + 2*sqrt(1 - x**2)*asin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2\,dx = x\arcsin{(x)}^2 + 2\sqrt{-x^2+1}\arcsin{(x)} - 2\,x$$

```
[In] integrate(arcsin(x)^2,x, algorithm="maxima")
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2\,dx = x\arcsin{(x)}^2 + 2\sqrt{-x^2+1}\arcsin{(x)} - 2\,x$$

```
[In] integrate(arcsin(x)^2,x, algorithm="giac")
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \arcsin(x)^2\,dx = 2\,\mathrm{asin}(x)\,\sqrt{1-x^2} + x\,\left(\mathrm{asin}(x)^2-2\right)$$

```
[In] int(asin(x)^2,x)
[Out] 2*asin(x)*(1 - x^2)^(1/2) + x*(asin(x)^2 - 2)
```

3.85 $\int \frac{x^2 \arctan(x)}{1+x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{1}{2} \log \left(1+x^2\right)$$

```
[Out] x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5036, 4930, 266, 5004}

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = -\frac{1}{2} \arctan(x)^2 + x \arctan(x) - \frac{1}{2} \log \left(x^2+1\right)$$

```
[In] Int[(x^2*ArcTan[x])/(1 + x^2),x]
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \arctan(x)\,dx - \int \frac{\arctan(x)}{1+x^2}\,dx\\ &= x\arctan(x) - \frac{\arctan(x)^2}{2} - \int \frac{x}{1+x^2}\,dx\\ &= x\arctan(x) - \frac{\arctan(x)^2}{2} - \frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2\arctan(x)}{1+x^2}\,dx = x\arctan(x) - \frac{\arctan(x)^2}{2} - \frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(x^2*ArcTan[x])/(1 + x^2),x]
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$x\arctan(x)-\frac{\arctan(x)^2}{2}-\frac{\ln(x^2+1)}{2}$	20
parallelrisch	$x\arctan(x)-\frac{\arctan(x)^2}{2}-\frac{\ln(x^2+1)}{2}$	20
parts	$x\arctan(x)-\frac{\arctan(x)^2}{2}-\frac{\ln(x^2+1)}{2}$	20
risch	$\frac{\ln(ix+1)^2}{8}+\frac{i\left(-x+\frac{i\ln(-ix+1)}{2}\right)\ln(ix+1)}{2}+\frac{\ln(-ix+1)^2}{8}+\frac{ix\ln(-ix+1)}{2}-\frac{\ln(x^2+1)}{2}$	67

```
[In] int(x^2*arctan(x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int\frac{x^2\arctan(x)}{1+x^2}\,dx=x\arctan(x)-\frac{1}{2}\arctan(x)^2-\frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="fricas")
[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int\frac{x^2\arctan(x)}{1+x^2}\,dx=x\,\mathrm{atan}(x)-\frac{\log(x^2+1)}{2}-\frac{\mathrm{atan}^2(x)}{2}$$

```
[In] integrate(x**2*atan(x)/(x**2+1),x)
[Out] x*atan(x) - log(x**2 + 1)/2 - atan(x)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = (x - \arctan(x)) \arctan(x) + \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2+1)$$

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="maxima")
[Out] (x - arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2+1)$$

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="giac")
[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = -\frac{\operatorname{atan}(x)^2}{2} + x \operatorname{atan}(x) - \frac{\ln(x^2+1)}{2}$$

```
[In] int((x^2*atan(x))/(x^2 + 1),x)
[Out] x*atan(x) - atan(x)^2/2 - log(x^2 + 1)/2
```

3.86 $\int \arccos\left(\sqrt{\frac{x}{1+x}}\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right) dx = (1+x)\left(\sqrt{\frac{1}{1+x}}\sqrt{\frac{x}{1+x}} + \arccos\left(\sqrt{\frac{x}{1+x}}\right)\right)$$

```
[Out] (1+x)*(arccos((x/(1+x))^(1/2))+(1/(1+x))^(1/2)*(x/(1+x))^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4925, 12, 6851, 52, 65, 209}

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right) dx = x \arccos\left(\sqrt{\frac{x}{x+1}}\right) - \frac{\sqrt{\frac{x}{(x+1)^2}}(x+1)\arctan\left(\sqrt{x}\right)}{\sqrt{x}} + \sqrt{\frac{x}{(x+1)^2}}(x+1)$$

```
[In] Int[ArcCos[Sqrt[x/(1 + x)]],x]
```

```
[Out] Sqrt[x/(1 + x)^2]*(1 + x) + x*ArcCos[Sqrt[x/(1 + x)]] - (Sqrt[x/(1 + x)^2]*
(1 + x)*ArcTan[Sqrt[x]])/Sqrt[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4925

```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\arccos\left(\sqrt{\frac{x}{1+x}}\right)+\int \frac{1}{2}\sqrt{\frac{x}{(1+x)^2}}\,dx\\
&= x\arccos\left(\sqrt{\frac{x}{1+x}}\right)+\frac{1}{2}\int \sqrt{\frac{x}{(1+x)^2}}\,dx\\
&= x\arccos\left(\sqrt{\frac{x}{1+x}}\right)+\frac{\left(\sqrt{\frac{x}{(1+x)^2}}(1+x)\right)\int \frac{\sqrt{x}}{1+x}\,dx}{2\sqrt{x}}
\end{aligned}$$

$$= \sqrt{\frac{x}{(1+x)^2}}(1+x) + x\arccos\left(\sqrt{\frac{x}{1+x}}\right) - \frac{\left(\sqrt{\frac{x}{(1+x)^2}}(1+x)\right)\int \frac{1}{\sqrt{x}(1+x)}\,dx}{2\sqrt{x}}$$

$$= \sqrt{\frac{x}{(1+x)^2}}(1+x)+x\arccos\left(\sqrt{\frac{x}{1+x}}\right)-\frac{\left(\sqrt{\frac{x}{(1+x)^2}}(1+x)\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{x}}$$

$$= \sqrt{\frac{x}{(1+x)^2}}(1+x) + x\arccos\left(\sqrt{\frac{x}{1+x}}\right) - \frac{\sqrt{\frac{x}{(1+x)^2}}(1+x)\arctan\left(\sqrt{x}\right)}{\sqrt{x}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right)\,dx = x\arccos\left(\sqrt{\frac{x}{1+x}}\right) + \frac{\sqrt{\frac{x}{(1+x)^2}}(1+x)\left(\sqrt{x}-\arctan\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

```
[In] Integrate[ArcCos[Sqrt[x/(1 + x)]],x]
```

```
[Out] x*ArcCos[Sqrt[x/(1 + x)]] + (Sqrt[x/(1 + x)^2]*(1 + x)*(Sqrt[x] - ArcTan[Sq
rt[x]]))/Sqrt[x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

method	result	size
default	$x\arccos\left(\sqrt{\frac{x}{1+x}}\right)-\frac{\sqrt{x}\,\sqrt{\frac{1}{1+x}}\,\left(\arctan\left(\sqrt{x}\right)-\sqrt{x}\right)}{\sqrt{\frac{x}{1+x}}}$	45
parts	$x\arccos\left(\sqrt{\frac{x}{1+x}}\right)-\frac{\sqrt{x}\,\sqrt{\frac{1}{1+x}}\,\left(\arctan\left(\sqrt{x}\right)-\sqrt{x}\right)}{\sqrt{\frac{x}{1+x}}}$	45

```
[In] int(arccos((x/(1+x))^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*arccos((x/(1+x))^(1/2))-1/(x/(1+x))^(1/2)*x^(1/2)*(1/(1+x))^(1/2)*(arctan
(x^(1/2))-x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right) dx = (x+1)\arccos\left(\sqrt{\frac{x}{x+1}}\right) + \sqrt{x+1}\sqrt{\frac{x}{x+1}}$$

```
[In] integrate(arccos((x/(1+x))^(1/2)),x, algorithm="fricas")
[Out] (x + 1)*arccos(sqrt(x/(x + 1))) + sqrt(x + 1)*sqrt(x/(x + 1))
```

Sympy [A] (verification not implemented)

Time = 4.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.53

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right) dx = x\,\mathrm{acos}\left(\sqrt{\frac{x}{x+1}}\right) - \begin{cases} -\frac{\sqrt{\frac{x}{x+1}}}{\sqrt{-\frac{x}{x+1}+1}} + \mathrm{asin}\left(\sqrt{\frac{x}{x+1}}\right) & \text{for } \sqrt{\frac{x}{x+1}} > -1 \wedge \sqrt{\frac{x}{x+1}} < 1 \end{cases}$$

```
[In] integrate(acos((x/(1+x))**(1/2)),x)
[Out] x*acos(sqrt(x/(x + 1))) - Piecewise((-sqrt(x/(x + 1))/sqrt(-x/(x + 1) + 1)
+ asin(sqrt(x/(x + 1))), (sqrt(x/(x + 1)) > -1) & (sqrt(x/(x + 1)) < 1)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(30) = 60$.

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right) dx = -\frac{\arccos\left(\sqrt{\frac{x}{x+1}}\right)}{\frac{x}{x+1}-1} - \frac{\sqrt{-\frac{x}{x+1}+1}}{2\left(\sqrt{\frac{x}{x+1}}+1\right)} - \frac{\sqrt{-\frac{x}{x+1}+1}}{2\left(\sqrt{\frac{x}{x+1}}-1\right)}$$

```
[In] integrate(arccos((x/(1+x))^(1/2)),x, algorithm="maxima")
[Out] -arccos(sqrt(x/(x + 1)))/(x/(x + 1) - 1) - 1/2*sqrt(-x/(x + 1) + 1)/(sqrt(x
/(x + 1)) + 1) - 1/2*sqrt(-x/(x + 1) + 1)/(sqrt(x/(x + 1)) - 1)
```

Giac [F(-2)]

Exception generated.

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arccos((x/(1+x))^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right)\,dx = \int \mathrm{acos}\left(\sqrt{\frac{x}{x+1}}\right)\,dx$$

```
[In] int(acos((x/(x + 1))^(1/2)),x)
```

```
[Out] int(acos((x/(x + 1))^(1/2)), x)
```

3.87 $\int (2x+3x^2)^3 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int (2x+3x^2)^3 \, dx = 2x^4 + \frac{36x^5}{5} + 9x^6 + \frac{27x^7}{7}$$

```
[Out] 2*x^4+36/5*x^5+9*x^6+27/7*x^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {625}

$$\int (2x+3x^2)^3 \, dx = \frac{27x^7}{7} + 9x^6 + \frac{36x^5}{5} + 2x^4$$

```
[In] Int[(2*x + 3*x^2)^3,x]
```

```
[Out] 2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

Rule 625

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int (8x^3+36x^4+54x^5+27x^6) \, dx \\ &= 2x^4 + \frac{36x^5}{5} + 9x^6 + \frac{27x^7}{7} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(2x+3x^2\right)^3\,dx=2x^4+\frac{36x^5}{5}+9x^6+\frac{27x^7}{7}$$

```
[In] Integrate[(2*x + 3*x^2)^3,x]
[Out] 2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{x^4(135x^3+315x^2+252x+70)}{35}$	21
default	$2x^4+\frac{36}{5}x^5+9x^6+\frac{27}{7}x^7$	22
norman	$2x^4+\frac{36}{5}x^5+9x^6+\frac{27}{7}x^7$	22
risch	$2x^4+\frac{36}{5}x^5+9x^6+\frac{27}{7}x^7$	22
parallelrisch	$2x^4+\frac{36}{5}x^5+9x^6+\frac{27}{7}x^7$	22

```
[In] int((3*x^2+2*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/35*x^4*(135*x^3+315*x^2+252*x+70)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(2x+3x^2\right)^3\,dx=\frac{27}{7}\,x^7+9\,x^6+\frac{36}{5}\,x^5+2\,x^4$$

```
[In] integrate((3*x^2+2*x)^3,x, algorithm="fricas")
[Out] 27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \left(2x+3x^2\right)^3\,dx=\frac{27x^7}{7}+9x^6+\frac{36x^5}{5}+2x^4$$

```
[In] integrate((3*x**2+2*x)**3,x)
[Out] 27*x**7/7 + 9*x**6 + 36*x**5/5 + 2*x**4
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(2x+3x^2\right)^3\,dx=\frac{27}{7}\,x^7+9\,x^6+\frac{36}{5}\,x^5+2\,x^4$$

```
[In] integrate((3*x^2+2*x)^3,x, algorithm="maxima")
[Out] 27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(2x+3x^2\right)^3\,dx=\frac{27}{7}\,x^7+9\,x^6+\frac{36}{5}\,x^5+2\,x^4$$

```
[In] integrate((3*x^2+2*x)^3,x, algorithm="giac")
[Out] 27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(2x+3x^2\right)^3\,dx=\frac{27\,x^7}{7}+9\,x^6+\frac{36\,x^5}{5}+2\,x^4$$

```
[In] int((2*x + 3*x^2)^3,x)
[Out] 2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

3.88 $\int (-1+x) \left(-1+2x+3x^2\right)^2 \, dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int (-1+x) \left(-1+2x+3x^2\right)^2 \, dx = -x+\frac{5x^2}{2}-\frac{2x^3}{3}-\frac{7x^4}{2}+\frac{3x^5}{5}+\frac{3x^6}{2}$$

```
[Out] -x+5/2*x^2-2/3*x^3-7/2*x^4+3/5*x^5+3/2*x^6
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {645}

$$\int (-1+x) \left(-1+2x+3x^2\right)^2 \, dx = \frac{3x^6}{2}+\frac{3x^5}{5}-\frac{7x^4}{2}-\frac{2x^3}{3}+\frac{5x^2}{2}-x$$

```
[In] Int[(-1 + x)*(-1 + 2*x + 3*x^2)^2,x]
[Out] -x + (5*x^2)/2 - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-1+5x-2x^2-14x^3+3x^4+9x^5\right) \, dx \\ &= -x+\frac{5x^2}{2}-\frac{2x^3}{3}-\frac{7x^4}{2}+\frac{3x^5}{5}+\frac{3x^6}{2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int (-1+x) \left(-1+2x+3x^2\right)^2 \, dx = -x+\frac{5x^2}{2}-\frac{2x^3}{3}-\frac{7x^4}{2}+\frac{3x^5}{5}+\frac{3x^6}{2}$$

```
[In] Integrate[(-1 + x)*(-1 + 2*x + 3*x^2)^2,x]
[Out] -x + (5*x^2)/2 - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{x(45x^5+18x^4-105x^3-20x^2+75x-30)}{30}$	29
default	$-x+\frac{5}{2}x^2-\frac{2}{3}x^3-\frac{7}{2}x^4+\frac{3}{5}x^5+\frac{3}{2}x^6$	30
norman	$-x+\frac{5}{2}x^2-\frac{2}{3}x^3-\frac{7}{2}x^4+\frac{3}{5}x^5+\frac{3}{2}x^6$	30
risch	$-x+\frac{5}{2}x^2-\frac{2}{3}x^3-\frac{7}{2}x^4+\frac{3}{5}x^5+\frac{3}{2}x^6$	30
parallelrisch	$-x+\frac{5}{2}x^2-\frac{2}{3}x^3-\frac{7}{2}x^4+\frac{3}{5}x^5+\frac{3}{2}x^6$	30

```
[In] int((-1+x)*(3*x^2+2*x-1)^2,x,method=_RETURNVERBOSE)
[Out] 1/30*x*(45*x^5+18*x^4-105*x^3-20*x^2+75*x-30)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+x) \left(-1+2x+3x^2\right)^2 \, dx = \frac{3}{2}\,x^6+\frac{3}{5}\,x^5-\frac{7}{2}\,x^4-\frac{2}{3}\,x^3+\frac{5}{2}\,x^2-x$$

```
[In] integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="fricas")
[Out] 3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=\frac{3x^6}{2}+\frac{3x^5}{5}-\frac{7x^4}{2}-\frac{2x^3}{3}+\frac{5x^2}{2}-x$$

```
[In] integrate((-1+x)*(3*x**2+2*x-1)**2,x)
[Out] 3*x**6/2 + 3*x**5/5 - 7*x**4/2 - 2*x**3/3 + 5*x**2/2 - x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=\frac{3}{2}\,x^6+\frac{3}{5}\,x^5-\frac{7}{2}\,x^4-\frac{2}{3}\,x^3+\frac{5}{2}\,x^2-x$$

```
[In] integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="maxima")
[Out] 3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=\frac{3}{2}\,x^6+\frac{3}{5}\,x^5-\frac{7}{2}\,x^4-\frac{2}{3}\,x^3+\frac{5}{2}\,x^2-x$$

```
[In] integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="giac")
[Out] 3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=\frac{3\,x^6}{2}+\frac{3\,x^5}{5}-\frac{7\,x^4}{2}-\frac{2\,x^3}{3}+\frac{5\,x^2}{2}-x$$

```
[In] int((x - 1)*(2*x + 3*x^2 - 1)^2,x)
[Out] (5*x^2)/2 - x - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

3.89 $\int x^{-1+k}\left(a+bx^k\right)^n\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int x^{-1+k}\left(a+bx^k\right)^n\,dx=\frac{\left(a+bx^k\right)^{1+n}}{bk(1+n)}$$

```
[Out] (a+b*x^k)^(1+n)/b/k/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {267}

$$\int x^{-1+k}\left(a+bx^k\right)^n\,dx=\frac{\left(a+bx^k\right)^{n+1}}{bk(n+1)}$$

```
[In] Int[x^(-1 + k)*(a + b*x^k)^n,x]
```

```
[Out] (a + b*x^k)^(1 + n)/(b*k*(1 + n))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{\left(a+bx^k\right)^{1+n}}{bk(1+n)}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+k}\left(a+bx^k\right)^n\,dx=\frac{\left(a+bx^k\right)^{1+n}}{bk(1+n)}$$

```
[In] Integrate[x^(-1 + k)*(a + b*x^k)^n,x]
[Out] (a + b*x^k)^(1 + n)/(b*k*(1 + n))
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

method	result	size
risch	$\frac{\left(a+b\,x^{k}\right)\left(a+b\,x^{k}\right)^{n}}{b(1+n)k}$	29

```
[In] int(x^(-1+k)*(a+b*x^k)^n,x,method=_RETURNVERBOSE)
[Out] (a+b*x^k)/b/(1+n)/k*(a+b*x^k)^n
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^{-1+k}\left(a+bx^k\right)^n\,dx=\frac{\left(bx^k+a\right)\left(bx^k+a\right)^n}{bkn+bk}$$

```
[In] integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="fricas")
[Out] (b*x^k + a)*(b*x^k + a)^n/(b*k*n + b*k)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(15) = 30$.

Time = 12.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.39

$$\int x^{-1+k}(a+bx^k)^n\,dx = \begin{cases} \frac{\log(x)}{a} & \text{for } b=0 \wedge k=0 \wedge n=-1 \\ \frac{a^n x x^{k-1}}{k} & \text{for } b=0 \\ (a+b)^n \log(x) & \text{for } k=0 \\ \frac{\log\left(\frac{a}{b}+x^k\right)}{bk} & \text{for } n=-1 \\ \frac{a(a+bx^k)^n}{bkn+bk} + \frac{bx^k(a+bx^k)^n}{bkn+bk} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(-1+k)*(a+b*x**k)**n,x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(k, 0) & Eq(n, -1)), (a**n*x*x**(k - 1)/k
, Eq(b, 0)), ((a + b)**n*log(x), Eq(k, 0)), (log(a/b + x**k)/(b*k), Eq(n, -
1)), (a*(a + b*x**k)**n/(b*k*n + b*k) + b*x**k*(a + b*x**k)**n/(b*k*n + b*k
), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+k}(a+bx^k)^n\,dx = \frac{(bx^k+a)^{n+1}}{bk(n+1)}$$

```
[In] integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="maxima")
```

```
[Out] (b*x^k + a)^(n + 1)/(b*k*(n + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+k}(a+bx^k)^n\,dx = \frac{(bx^k+a)^{n+1}}{bk(n+1)}$$

```
[In] integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="giac")
```

```
[Out] (b*x^k + a)^(n + 1)/(b*k*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+k}\left(a+bx^k\right)^n\,dx=\frac{\left(a+b\,x^k\right)^{n+1}}{b\,k\,\left(n+1\right)}$$

```
[In] int(x^(k - 1)*(a + b*x^k)^n,x)
[Out] (a + b*x^k)^(n + 1)/(b*k*(n + 1))
```

3.90 $\int \frac{x^3}{1+2x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \frac{x^3}{1+2x} \, dx = \frac{x}{8} - \frac{x^2}{8} + \frac{x^3}{6} - \frac{1}{16} \log(1+2x)$$

```
[Out] 1/8*x-1/8*x^2+1/6*x^3-1/16*ln(1+2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\int \frac{x^3}{1+2x} \, dx = \frac{x^3}{6} - \frac{x^2}{8} + \frac{x}{8} - \frac{1}{16} \log(2x+1)$$

```
[In] Int[x^3/(1 + 2*x),x]
```

```
[Out] x/8 - x^2/8 + x^3/6 - Log[1 + 2*x]/16
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{8} - \frac{x}{4} + \frac{x^2}{2} - \frac{1}{8(1+2x)}\right) \, dx \\ &= \frac{x}{8} - \frac{x^2}{8} + \frac{x^3}{6} - \frac{1}{16} \log(1+2x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{1+2x}\,dx = \frac{1}{96}\left(11+12x-12x^2+16x^3-6\log(1+2x)\right)$$

```
[In] Integrate[x^3/(1 + 2*x),x]
[Out] (11 + 12*x - 12*x^2 + 16*x^3 - 6*Log[1 + 2*x])/96
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

method	result	size
parallelrisch	$\frac{x^3}{6}-\frac{x^2}{8}+\frac{x}{8}-\frac{\ln\left(x+\frac{1}{2}\right)}{16}$	21
default	$\frac{x}{8}-\frac{x^2}{8}+\frac{x^3}{6}-\frac{\ln(1+2x)}{16}$	23
norman	$\frac{x}{8}-\frac{x^2}{8}+\frac{x^3}{6}-\frac{\ln(1+2x)}{16}$	23
meijerg	$\frac{x(16x^2-12x+12)}{96}-\frac{\ln(1+2x)}{16}$	23
risch	$\frac{x}{8}-\frac{x^2}{8}+\frac{x^3}{6}-\frac{\ln(1+2x)}{16}$	23

```
[In] int(x^3/(1+2*x),x,method=_RETURNVERBOSE)
[Out] 1/6*x^3-1/8*x^2+1/8*x-1/16*ln(x+1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{1+2x}\,dx = \frac{1}{6}\,x^3-\frac{1}{8}\,x^2+\frac{1}{8}\,x-\frac{1}{16}\,\log\left(2\,x+1\right)$$

```
[In] integrate(x^3/(1+2*x),x, algorithm="fricas")
[Out] 1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{1+2x}\,dx = \frac{x^3}{6} - \frac{x^2}{8} + \frac{x}{8} - \frac{\log(2x+1)}{16}$$

```
[In] integrate(x**3/(1+2*x),x)
[Out] x**3/6 - x**2/8 + x/8 - log(2*x + 1)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{1+2x}\,dx = \frac{1}{6}\,x^3 - \frac{1}{8}\,x^2 + \frac{1}{8}\,x - \frac{1}{16}\,\log(2\,x+1)$$

```
[In] integrate(x^3/(1+2*x),x, algorithm="maxima")
[Out] 1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(2*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{1+2x}\,dx = \frac{1}{6}\,x^3 - \frac{1}{8}\,x^2 + \frac{1}{8}\,x - \frac{1}{16}\,\log(|2\,x+1|)$$

```
[In] integrate(x^3/(1+2*x),x, algorithm="giac")
[Out] 1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(abs(2*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{1+2x}\,dx = \frac{x}{8} - \frac{\ln\left(x+\frac{1}{2}\right)}{16} - \frac{x^2}{8} + \frac{x^3}{6}$$

```
[In] int(x^3/(2*x + 1),x)
[Out] x/8 - log(x + 1/2)/16 - x^2/8 + x^3/6
```

3.91 $\int \frac{x^6}{2+3x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{x^6}{2+3x^2} \, dx = \frac{4x}{27} - \frac{2x^3}{27} + \frac{x^5}{15} - \frac{4}{27}\sqrt{\frac{2}{3}} \arctan\left(\sqrt{\frac{3}{2}}x\right)$$

```
[Out] 4/27*x-2/27*x^3+1/15*x^5-4/81*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {308, 209}

$$\int \frac{x^6}{2+3x^2} \, dx = -\frac{4}{27}\sqrt{\frac{2}{3}} \arctan\left(\sqrt{\frac{3}{2}}x\right) + \frac{x^5}{15} - \frac{2x^3}{27} + \frac{4x}{27}$$

```
[In] Int[x^6/(2 + 3*x^2),x]
```

```
[Out] (4*x)/27 - (2*x^3)/27 + x^5/15 - (4*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x])/27
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{4}{27} - \frac{2x^2}{9} + \frac{x^4}{3} - \frac{8}{27(2+3x^2)}\right) dx \\
&= \frac{4x}{27} - \frac{2x^3}{27} + \frac{x^5}{15} - \frac{8}{27}\int \frac{1}{2+3x^2}\, dx \\
&= \frac{4x}{27} - \frac{2x^3}{27} + \frac{x^5}{15} - \frac{4}{27}\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}x\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x^6}{2+3x^2}\, dx = \frac{1}{405}\left(60x - 30x^3 + 27x^5 - 20\sqrt{6}\arctan\left(\sqrt{\frac{3}{2}}x\right)\right)$$

```
[In] Integrate[x^6/(2 + 3*x^2),x]
[Out] (60*x - 30*x^3 + 27*x^5 - 20*Sqrt[6]*ArcTan[Sqrt[3/2]*x])/405
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{4x}{27}-\frac{2x^3}{27}+\frac{x^5}{15}-\frac{4\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{81}$	27
risch	$\frac{4x}{27}-\frac{2x^3}{27}+\frac{x^5}{15}-\frac{4\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{81}$	27
meijerg	$\frac{2\sqrt{2}\,\sqrt{3}\left(\frac{x\sqrt{2}\,\sqrt{3}\left(\frac{189}{4}x^4-\frac{105}{2}x^2+105\right)}{105}-2\arctan\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{81}$	43

```
[In] int(x^6/(3*x^2+2),x,method=_RETURNVERBOSE)
[Out] 4/27*x-2/27*x^3+1/15*x^5-4/81*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{x^6}{2+3x^2}\,dx = \frac{1}{15}\,x^5 - \frac{2}{27}\,x^3 - \frac{4}{81}\,\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{3}\sqrt{2}x\right) + \frac{4}{27}\,x$$

```
[In] integrate(x^6/(3*x^2+2),x, algorithm="fricas")
```

```
[Out] 1/15*x^5 - 2/27*x^3 - 4/81*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) +
4/27*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^6}{2+3x^2}\,dx = \frac{x^5}{15} - \frac{2x^3}{27} + \frac{4x}{27} - \frac{4\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}\right)}{81}$$

```
[In] integrate(x**6/(3*x**2+2),x)
```

```
[Out] x**5/15 - 2*x**3/27 + 4*x/27 - 4*sqrt(6)*atan(sqrt(6)*x/2)/81
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{2+3x^2}\,dx = \frac{1}{15}\,x^5 - \frac{2}{27}\,x^3 - \frac{4}{81}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{4}{27}\,x$$

```
[In] integrate(x^6/(3*x^2+2),x, algorithm="maxima")
```

```
[Out] 1/15*x^5 - 2/27*x^3 - 4/81*sqrt(6)*arctan(1/2*sqrt(6)*x) + 4/27*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{2+3x^2}\,dx = \frac{1}{15}\,x^5 - \frac{2}{27}\,x^3 - \frac{4}{81}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{4}{27}\,x$$

```
[In] integrate(x^6/(3*x^2+2),x, algorithm="giac")
[Out] 1/15*x^5 - 2/27*x^3 - 4/81*sqrt(6)*arctan(1/2*sqrt(6)*x) + 4/27*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{x^6}{2+3x^2}\,dx = \frac{4\,x}{27} - \frac{2\,x^3}{27} + \frac{x^5}{15} - \frac{4\,\sqrt{2}\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{81}$$

```
[In] int(x^6/(3*x^2 + 2),x)
[Out] (4*x)/27 - (2*x^3)/27 + x^5/15 - (4*2^(1/2)*3^(1/2)*atan((2^(1/2)*3^(1/2)*x
)/2))/81
```

3.92 $\int \frac{1}{2-7x+3x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{1}{2-7x+3x^2} dx = -\frac{1}{5}\log(1-3x) + \frac{1}{5}\log(2-x)$$

```
[Out] -1/5*ln(1-3*x)+1/5*ln(2-x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {630, 31}

$$\int \frac{1}{2-7x+3x^2} dx = \frac{1}{5}\log(2-x) - \frac{1}{5}\log(1-3x)$$

```
[In] Int[(2 - 7*x + 3*x^2)^(-1),x]
```

```
[Out] -1/5*Log[1 - 3*x] + Log[2 - x]/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3}{5}\int \frac{1}{-6+3x}\,dx - \frac{3}{5}\int \frac{1}{-1+3x}\,dx \\ &= -\frac{1}{5}\log(1-3x) + \frac{1}{5}\log(2-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{2-7x+3x^2}\,dx = -\frac{1}{5}\log(1-3x) + \frac{1}{5}\log(2-x)$$

```
[In] Integrate[(2 - 7*x + 3*x^2)^(-1),x]
[Out] -1/5*Log[1 - 3*x] + Log[2 - x]/5
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln(-2+x)}{5}-\frac{\ln\left(x-\frac{1}{3}\right)}{5}$	14
default	$-\frac{\ln(-1+3x)}{5}+\frac{\ln(-2+x)}{5}$	16
norman	$-\frac{\ln(-1+3x)}{5}+\frac{\ln(-2+x)}{5}$	16
risch	$-\frac{\ln(-1+3x)}{5}+\frac{\ln(-2+x)}{5}$	16

```
[In] int(1/(3*x^2-7*x+2),x,method=_RETURNVERBOSE)
[Out] 1/5*ln(-2+x)-1/5*ln(x-1/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{2-7x+3x^2}\,dx = -\frac{1}{5}\log\left(3\,x-1\right) + \frac{1}{5}\log\left(x-2\right)$$

```
[In] integrate(1/(3*x^2-7*x+2),x, algorithm="fricas")
[Out] -1/5*log(3*x - 1) + 1/5*log(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{1}{2-7x+3x^2}\,dx = \frac{\log(x-2)}{5} - \frac{\log\left(x-\frac{1}{3}\right)}{5}$$

```
[In] integrate(1/(3*x**2-7*x+2),x)
[Out] log(x - 2)/5 - log(x - 1/3)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{2-7x+3x^2}\,dx = -\frac{1}{5}\log(3x-1) + \frac{1}{5}\log(x-2)$$

```
[In] integrate(1/(3*x^2-7*x+2),x, algorithm="maxima")
[Out] -1/5*log(3*x - 1) + 1/5*log(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{2-7x+3x^2}\,dx = -\frac{1}{5}\log(|3x-1|) + \frac{1}{5}\log(|x-2|)$$

```
[In] integrate(1/(3*x^2-7*x+2),x, algorithm="giac")
[Out] -1/5*log(abs(3*x - 1)) + 1/5*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{1}{2-7x+3x^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{6x}{5}-\frac{7}{5}\right)}{5}$$

```
[In] int(1/(3*x^2 - 7*x + 2),x)
[Out] -(2*atanh((6*x)/5 - 7/5))/5
```

3.93 $\int \frac{-1+3x}{1-x+x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{-1+3x}{1-x+x^2} dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(1-x+x^2\right)$$

```
[Out] 3/2*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {648, 632, 210, 642}

$$\int \frac{-1+3x}{1-x+x^2} dx = \frac{3}{2}\log\left(x^2-x+1\right) - \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Int[(-1 + 3*x)/(1 - x + x^2),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + (3*Log[1 - x + x^2])/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int \frac{1}{1-x+x^2}\,dx+\frac{3}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\ &= \frac{3}{2}\log\left(1-x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{-1+3x}{1-x+x^2}\,dx=\frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(1-x+x^2\right)$$

```
[In] Integrate[(-1 + 3*x)/(1 - x + x^2),x]
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + (3*Log[1 - x + x^2])/2
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{3\ln\left(x^{2}-x+1\right)}{2}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	29
risch	$\frac{3\ln\left(4x^{2}-4x+4\right)}{2}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	31

```
[In] int((-1+3*x)/(x^2-x+1),x,method=_RETURNVERBOSE)
[Out] 3/2*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{3}{2}\,\log\left(x^2-x+1\right)$$

```
[In] integrate((-1+3*x)/(x^2-x+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{3\log\left(x^2-x+1\right)}{2}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((-1+3*x)/(x**2-x+1),x)
[Out] 3*log(x**2 - x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{3}{2}\,\log\left(x^2-x+1\right)$$

```
[In] integrate((-1+3*x)/(x^2-x+1),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{3}{2}\,\log\left(x^2-x+1\right)$$

```
[In] integrate((-1+3*x)/(x^2-x+1),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{3\,\ln\left(x^2-x+1\right)}{2} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] int((3*x - 1)/(x^2 - x + 1),x)
[Out] (3*log(x^2 - x + 1))/2 + (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

3.94 $\int \frac{x^2}{5+2x+x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{x^2}{5+2x+x^2} \, dx = x-\frac{3}{2}\arctan\left(\frac{1+x}{2}\right)-\log\left(5+2x+x^2\right)$$

```
[Out] x-3/2*arctan(1/2+1/2*x)-ln(x^2+2*x+5)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {717, 648, 632, 210, 642}

$$\int \frac{x^2}{5+2x+x^2} \, dx = -\frac{3}{2}\arctan\left(\frac{x+1}{2}\right)-\log\left(x^2+2x+5\right)+x$$

```
[In] Int[x^2/(5 + 2*x + x^2),x]
```

```
[Out] x - (3*ArcTan[(1 + x)/2])/2 - Log[5 + 2*x + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x+\int \frac{-5-2x}{5+2x+x^2}\,dx\\
&= x-3\int \frac{1}{5+2x+x^2}\,dx-\int \frac{2+2x}{5+2x+x^2}\,dx\\
&= x-\log\left(5+2x+x^2\right)+6\text{Subst}\left(\int \frac{1}{-16-x^2}\,dx,x,2+2x\right)\\
&= x-\frac{3}{2}\arctan\left(\frac{1+x}{2}\right)-\log\left(5+2x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{5+2x+x^2}\,dx=x-\frac{3}{2}\arctan\left(\frac{1+x}{2}\right)-\log\left(5+2x+x^2\right)$$

```
[In] Integrate[x^2/(5 + 2*x + x^2),x]
[Out] x - (3*ArcTan[(1 + x)/2])/2 - Log[5 + 2*x + x^2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$x-\frac{3\arctan\left(\frac{1}{2}+\frac{x}{2}\right)}{2}-\ln\left(x^{2}+2x+5\right)$	22
risch	$x-\frac{3\arctan\left(\frac{1}{2}+\frac{x}{2}\right)}{2}-\ln\left(x^{2}+2x+5\right)$	22
parallelrisch	$x-\ln\left(x+1-2i\right)+\frac{3i\ln\left(x+1-2i\right)}{4}-\ln\left(x+1+2i\right)-\frac{3i\ln\left(x+1+2i\right)}{4}$	37

```
[In] int(x^2/(x^2+2*x+5),x,method=_RETURNVERBOSE)
[Out] x-3/2*arctan(1/2+1/2*x)-ln(x^2+2*x+5)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{5+2x+x^2}\,dx = x-\frac{3}{2}\,\arctan\left(\frac{1}{2}\,x+\frac{1}{2}\right)-\log\left(x^{2}+2\,x+5\right)$$

```
[In] integrate(x^2/(x^2+2*x+5),x, algorithm="fricas")
[Out] x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{5+2x+x^2}\,dx = x-\log\left(x^{2}+2x+5\right)-\frac{3\,\mathrm{atan}\left(\frac{x}{2}+\frac{1}{2}\right)}{2}$$

```
[In] integrate(x**2/(x**2+2*x+5),x)
[Out] x - log(x**2 + 2*x + 5) - 3*atan(x/2 + 1/2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{5+2x+x^2}\,dx = x - \frac{3}{2}\,\arctan\left(\frac{1}{2}\,x + \frac{1}{2}\right) - \log\left(x^2 + 2\,x + 5\right)$$

```
[In] integrate(x^2/(x^2+2*x+5),x, algorithm="maxima")
[Out] x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{5+2x+x^2}\,dx = x - \frac{3}{2}\,\arctan\left(\frac{1}{2}\,x + \frac{1}{2}\right) - \log\left(x^2 + 2\,x + 5\right)$$

```
[In] integrate(x^2/(x^2+2*x+5),x, algorithm="giac")
[Out] x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{5+2x+x^2}\,dx = x - \ln\left(x^2 + 2\,x + 5\right) - \frac{3\,\mathrm{atan}\left(\frac{x}{2} + \frac{1}{2}\right)}{2}$$

```
[In] int(x^2/(2*x + x^2 + 5),x)
[Out] x - log(2*x + x^2 + 5) - (3*atan(x/2 + 1/2))/2
```

3.95 $\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 47

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2} \, dx = -\frac{x^2}{2}+x^3-\frac{\arctan\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}}+\frac{1}{4}\log\left(1-x+2x^2\right)$$

```
[Out] -1/2*x^2+x^3+1/4*ln(2*x^2-x+1)-1/14*arctan(1/7*(-4*x+1)*7^(1/2))*7^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1608, 1642, 648, 632, 210, 642}

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2} \, dx = -\frac{\arctan\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}}+x^3-\frac{x^2}{2}+\frac{1}{4}\log\left(2x^2-x+1\right)$$

```
[In] Int[(4*x^2 - 5*x^3 + 6*x^4)/(1 - x + 2*x^2),x]
```

```
[Out] -1/2*x^2 + x^3 - ArcTan[(1 - 4*x)/Sqrt[7]]/(2*Sqrt[7]) + Log[1 - x + 2*x^2]
/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2(4-5x+6x^2)}{1-x+2x^2}\,dx \\
&= \int \left(-x+3x^2+\frac{x}{1-x+2x^2}\right)\,dx \\
&= -\frac{x^2}{2}+x^3+\int \frac{x}{1-x+2x^2}\,dx \\
&= -\frac{x^2}{2}+x^3+\frac{1}{4}\int \frac{1}{1-x+2x^2}\,dx+\frac{1}{4}\int \frac{-1+4x}{1-x+2x^2}\,dx \\
&= -\frac{x^2}{2}+x^3+\frac{1}{4}\log\left(1-x+2x^2\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx,x,-1+4x\right) \\
&= -\frac{x^2}{2}+x^3-\frac{\arctan\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}}+\frac{1}{4}\log\left(1-x+2x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx=-\frac{x^2}{2}+x^3+\frac{\arctan\left(\frac{-1+4x}{\sqrt{7}}\right)}{2\sqrt{7}}+\frac{1}{4}\log\left(1-x+2x^2\right)$$

```
[In] Integrate[(4*x^2 - 5*x^3 + 6*x^4)/(1 - x + 2*x^2),x]
[Out] -1/2*x^2 + x^3 + ArcTan[(-1 + 4*x)/Sqrt[7]]/(2*Sqrt[7]) + Log[1 - x + 2*x^2
]/4
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

method	result	size
default	$x^3-\frac{x^2}{2}+\frac{\ln(2x^2-x+1)}{4}+\frac{\sqrt{7}\arctan\left(\frac{(-1+4x)\sqrt{7}}{7}\right)}{14}$	39
risch	$x^3-\frac{x^2}{2}+\frac{\ln(16x^2-8x+8)}{4}+\frac{\sqrt{7}\arctan\left(\frac{(-1+4x)\sqrt{7}}{7}\right)}{14}$	39

```
[In] int((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x,method=_RETURNVERBOSE)
[Out] x^3-1/2*x^2+1/4*ln(2*x^2-x+1)+1/14*7^(1/2)*arctan(1/7*(-1+4*x)*7^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx=x^3-\frac{1}{2}x^2+\frac{1}{14}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x-1)\right)+\frac{1}{4}\log\left(2x^2-x+1\right)$$

```
[In] integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="fricas")
[Out] x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*log(2*x^2
- x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx = x^3-\frac{x^2}{2}+\frac{\log\left(x^2-\frac{x}{2}+\frac{1}{2}\right)}{4}+\frac{\sqrt{7}\operatorname{atan}\left(\frac{4\sqrt{7}x}{7}-\frac{\sqrt{7}}{7}\right)}{14}$$

```
[In] integrate((6*x**4-5*x**3+4*x**2)/(2*x**2-x+1),x)
[Out] x**3 - x**2/2 + log(x**2 - x/2 + 1/2)/4 + sqrt(7)*atan(4*sqrt(7)*x/7 - sqrt
(7)/7)/14
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx = x^3-\frac{1}{2}x^2+\frac{1}{14}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x-1)\right)+\frac{1}{4}\log\left(2x^2-x+1\right)$$

```
[In] integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="maxima")
[Out] x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*log(2*x^2
- x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx = x^3-\frac{1}{2}x^2+\frac{1}{14}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x-1)\right)+\frac{1}{4}\log\left(2x^2-x+1\right)$$

```
[In] integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="giac")
[Out] x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*log(2*x^2
- x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx=\frac{\ln\left(2\,x^2-x+1\right)}{4}+\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{4\,\sqrt{7}\,x}{7}-\frac{\sqrt{7}}{7}\right)}{14}-\frac{x^2}{2}+x^3$$

```
[In] int((4*x^2 - 5*x^3 + 6*x^4)/(2*x^2 - x + 1),x)
```

```
[Out] log(2*x^2 - x + 1)/4 + (7^(1/2)*atan((4*7^(1/2)*x)/7 - 7^(1/2)/7))/14 - x^2
/2 + x^3
```

3.96 $\int \frac{-1+x+x^2}{-6x+x^2+x^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3} dx = \frac{1}{2}\log(2-x) + \frac{\log(x)}{6} + \frac{1}{3}\log(3+x)$$

```
[Out] 1/2*ln(2-x)+1/6*ln(x)+1/3*ln(3+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1608, 1642}

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3} dx = \frac{1}{2}\log(2-x) + \frac{\log(x)}{6} + \frac{1}{3}\log(x+3)$$

```
[In] Int[(-1 + x + x^2)/(-6*x + x^2 + x^3),x]
```

```
[Out] Log[2 - x]/2 + Log[x]/6 + Log[3 + x]/3
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{-1+x+x^2}{x\left(-6+x+x^2\right)}\,dx\\ &= \int \left(\frac{1}{2(-2+x)}+\frac{1}{6x}+\frac{1}{3(3+x)}\right)dx\\ &= \frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(3+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx = \frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(3+x)$$

```
[In] Integrate[(-1 + x + x^2)/(-6*x + x^2 + x^3),x]
[Out] Log[2 - x]/2 + Log[x]/6 + Log[3 + x]/3
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\ln(x)}{6}+\frac{\ln(3+x)}{3}+\frac{\ln(-2+x)}{2}$	18
norman	$\frac{\ln(x)}{6}+\frac{\ln(3+x)}{3}+\frac{\ln(-2+x)}{2}$	18
risch	$\frac{\ln(x)}{6}+\frac{\ln(3+x)}{3}+\frac{\ln(-2+x)}{2}$	18
parallelrisch	$\frac{\ln(x)}{6}+\frac{\ln(3+x)}{3}+\frac{\ln(-2+x)}{2}$	18

```
[In] int((x^2+x-1)/(x^3+x^2-6*x),x,method=_RETURNVERBOSE)
[Out] 1/6*ln(x)+1/3*ln(3+x)+1/2*ln(-2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx = \frac{1}{3}\log(x+3)+\frac{1}{2}\log(x-2)+\frac{1}{6}\log(x)$$

```
[In] integrate((x^2+x-1)/(x^3+x^2-6*x),x, algorithm="fricas")
[Out] 1/3*log(x + 3) + 1/2*log(x - 2) + 1/6*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx = \frac{\log(x)}{6}+\frac{\log(x-2)}{2}+\frac{\log(x+3)}{3}$$

```
[In] integrate((x**2+x-1)/(x**3+x**2-6*x),x)
[Out] log(x)/6 + log(x - 2)/2 + log(x + 3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx = \frac{1}{3}\log(x+3)+\frac{1}{2}\log(x-2)+\frac{1}{6}\log(x)$$

```
[In] integrate((x^2+x-1)/(x^3+x^2-6*x),x, algorithm="maxima")
[Out] 1/3*log(x + 3) + 1/2*log(x - 2) + 1/6*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx = \frac{1}{3}\log(|x+3|)+\frac{1}{2}\log(|x-2|)+\frac{1}{6}\log(|x|)$$

```
[In] integrate((x^2+x-1)/(x^3+x^2-6*x),x, algorithm="giac")
[Out] 1/3*log(abs(x + 3)) + 1/2*log(abs(x - 2)) + 1/6*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx=\frac{\ln(x-2)}{2}+\frac{\ln(x+3)}{3}+\frac{\ln(x)}{6}$$

```
[In] int((x + x^2 - 1)/(x^2 - 6*x + x^3),x)
[Out] log(x - 2)/2 + log(x + 3)/3 + log(x)/6
```

3.97 $\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 33

$$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx=\frac{9}{2}\log(a-x)-17\log(2a-x)+\frac{35}{2}\log(3a-x)$$

```
[Out] 9/2*ln(a-x)-17*ln(2*a-x)+35/2*ln(3*a-x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2099}

$$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx=\frac{9}{2}\log(a-x)-17\log(2a-x)+\frac{35}{2}\log(3a-x)$$

```
[In] Int[(11*a^2 - 7*a*x + 5*x^2)/(-6*a^3 + 11*a^2*x - 6*a*x^2 + x^3),x]
```

```
[Out] (9*Log[a - x])/2 - 17*Log[2*a - x] + (35*Log[3*a - x])/2
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(-\frac{9}{2(a-x)}+\frac{17}{2a-x}-\frac{35}{2(3a-x)}\right)\,dx\\&=\frac{9}{2}\log(a-x)-17\log(2a-x)+\frac{35}{2}\log(3a-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{11a^2 - 7ax + 5x^2}{-6a^3 + 11a^2x - 6ax^2 + x^3}\,dx = \frac{35}{2}\log(-3a + x) - 17\log(-2a + x) + \frac{9}{2}\log(-a + x)$$

```
[In] Integrate[(11*a^2 - 7*a*x + 5*x^2)/(-6*a^3 + 11*a^2*x - 6*a*x^2 + x^3),x]
[Out] (35*Log[-3*a + x])/2 - 17*Log[-2*a + x] + (9*Log[-a + x])/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{35\ln(-3a+x)}{2} - 17\ln(x-2a) + \frac{9\ln(-a+x)}{2}$	26
parallelrisch	$\frac{35\ln(-3a+x)}{2} - 17\ln(x-2a) + \frac{9\ln(-a+x)}{2}$	26
default	$\frac{9\ln(a-x)}{2} - 17\ln(-x+2a) + \frac{35\ln(3a-x)}{2}$	30
norman	$\frac{9\ln(a-x)}{2} - 17\ln(-x+2a) + \frac{35\ln(3a-x)}{2}$	30

```
[In] int((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x,method=_RETURNVERB
OSE)
[Out] 35/2*ln(-3*a+x)-17*ln(x-2*a)+9/2*ln(-a+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{11a^2 - 7ax + 5x^2}{-6a^3 + 11a^2x - 6ax^2 + x^3}\,dx = \frac{9}{2}\log(-a + x) - 17\log(-2a + x) + \frac{35}{2}\log(-3a + x)$$

```
[In] integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm="
fricas")
[Out] 9/2*log(-a + x) - 17*log(-2*a + x) + 35/2*log(-3*a + x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{11a^2 - 7ax + 5x^2}{-6a^3 + 11a^2x - 6ax^2 + x^3}\, dx = \frac{35\log\left(-3a + x\right)}{2} - 17\log\left(-2a + x\right) + \frac{9\log\left(-a + x\right)}{2}$$

```
[In] integrate((11*a**2-7*a*x+5*x**2)/(-6*a**3+11*a**2*x-6*a*x**2+x**3),x)
[Out] 35*log(-3*a + x)/2 - 17*log(-2*a + x) + 9*log(-a + x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{11a^2 - 7ax + 5x^2}{-6a^3 + 11a^2x - 6ax^2 + x^3}\, dx = \frac{9}{2}\log\left(-a + x\right) - 17\log\left(-2\,a + x\right) + \frac{35}{2}\log\left(-3\,a + x\right)$$

```
[In] integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm="
maxima")
[Out] 9/2*log(-a + x) - 17*log(-2*a + x) + 35/2*log(-3*a + x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{11a^2 - 7ax + 5x^2}{-6a^3 + 11a^2x - 6ax^2 + x^3}\, dx = \frac{9}{2}\log\left(|-a + x|\right) - 17\log\left(|-2\,a + x|\right) + \frac{35}{2}\log\left(|-3\,a + x|\right)$$

```
[In] integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm="
giac")
[Out] 9/2*log(abs(-a + x)) - 17*log(abs(-2*a + x)) + 35/2*log(abs(-3*a + x))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx=\frac{9\,\ln\left(x-a\right)}{2}-17\,\ln\left(x-2\,a\right)+\frac{35\,\ln\left(x-3\,a\right)}{2}$$

```
[In] int(-(11*a^2 - 7*a*x + 5*x^2)/(6*a*x^2 - 11*a^2*x + 6*a^3 - x^3),x)
[Out] (9*log(x - a))/2 - 17*log(x - 2*a) + (35*log(x - 3*a))/2
```

3.98 $\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx = -\frac{1}{3}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{2}{3}\log(1+x)-\frac{2}{3}\log(2+x)$$

```
[Out] -1/3*ln(1-x)+1/3*ln(2-x)+2/3*ln(1+x)-2/3*ln(2+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1687, 1175, 630, 31, 1121}

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx = \frac{1}{6}\log\left(1-x^2\right)-\frac{1}{6}\log\left(4-x^2\right)-\frac{1}{2}\log(1-x) +\frac{1}{2}\log(2-x)+\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x+2)$$

```
[In] Int[(2 - x + x^2)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -1/2*Log[1 - x] + Log[2 - x]/2 + Log[1 + x]/2 - Log[2 + x]/2 + Log[1 - x^2]
/6 - Log[4 - x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{x}{4-5x^2+x^4}\,dx+\int \frac{2+x^2}{4-5x^2+x^4}\,dx\\
&= \frac{1}{2}\int \frac{1}{2-3x+x^2}\,dx+\frac{1}{2}\int \frac{1}{2+3x+x^2}\,dx-\frac{1}{2}\text{Subst}\left(\int \frac{1}{4-5x+x^2}\,dx,x,x^2\right)\\
&= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{1}{-4+x}\,dx,x,x^2\right)\right)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,x^2\right)\\
&\quad +\frac{1}{2}\int \frac{1}{-2+x}\,dx-\frac{1}{2}\int \frac{1}{-1+x}\,dx+\frac{1}{2}\int \frac{1}{1+x}\,dx-\frac{1}{2}\int \frac{1}{2+x}\,dx\\
&= -\frac{1}{2}\log(1-x)+\frac{1}{2}\log(2-x)+\frac{1}{2}\log(1+x)-\frac{1}{2}\log(2+x)+\frac{1}{6}\log\left(1-x^2\right)-\frac{1}{6}\log\left(4-x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx=-\frac{1}{3}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{2}{3}\log(1+x)-\frac{2}{3}\log(2+x)$$

```
[In] Integrate[(2 - x + x^2)/(4 - 5*x^2 + x^4),x]
[Out] -1/3*Log[1 - x] + Log[2 - x]/3 + (2*Log[1 + x])/3 - (2*Log[2 + x])/3
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{\ln(-1+x)}{3}-\frac{2\ln(2+x)}{3}+\frac{2\ln(1+x)}{3}+\frac{\ln(-2+x)}{3}$	26
norman	$-\frac{\ln(-1+x)}{3}-\frac{2\ln(2+x)}{3}+\frac{2\ln(1+x)}{3}+\frac{\ln(-2+x)}{3}$	26
risch	$-\frac{\ln(-1+x)}{3}-\frac{2\ln(2+x)}{3}+\frac{2\ln(1+x)}{3}+\frac{\ln(-2+x)}{3}$	26
parallelrisch	$-\frac{\ln(-1+x)}{3}-\frac{2\ln(2+x)}{3}+\frac{2\ln(1+x)}{3}+\frac{\ln(-2+x)}{3}$	26

```
[In] int((x^2-x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
[Out] -1/3*ln(-1+x)-2/3*ln(2+x)+2/3*ln(1+x)+1/3*ln(-2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx=-\frac{2}{3}\log\left(x+2\right)+\frac{2}{3}\log\left(x+1\right)-\frac{1}{3}\log\left(x-1\right)+\frac{1}{3}\log\left(x-2\right)$$

```
[In] integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] -2/3*log(x + 2) + 2/3*log(x + 1) - 1/3*log(x - 1) + 1/3*log(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx = \frac{\log(x-2)}{3} - \frac{\log(x-1)}{3} + \frac{2\log(x+1)}{3} - \frac{2\log(x+2)}{3}$$

```
[In] integrate((x**2-x+2)/(x**4-5*x**2+4),x)
[Out] log(x - 2)/3 - log(x - 1)/3 + 2*log(x + 1)/3 - 2*log(x + 2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx = -\frac{2}{3}\log(x+2) + \frac{2}{3}\log(x+1) - \frac{1}{3}\log(x-1) + \frac{1}{3}\log(x-2)$$

```
[In] integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
[Out] -2/3*log(x + 2) + 2/3*log(x + 1) - 1/3*log(x - 1) + 1/3*log(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx = -\frac{2}{3}\log(|x+2|) + \frac{2}{3}\log(|x+1|) - \frac{1}{3}\log(|x-1|) + \frac{1}{3}\log(|x-2|)$$

```
[In] integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="giac")
[Out] -2/3*log(abs(x + 2)) + 2/3*log(abs(x + 1)) - 1/3*log(abs(x - 1)) + 1/3*log(
abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{64}{3\,(24\,x-16)}-\frac{5}{3}\right)}{3} + \frac{4\,\mathrm{atanh}\left(\frac{128}{3\,(48\,x+32)}+\frac{5}{3}\right)}{3}$$

```
[In] int((x^2 - x + 2)/(x^4 - 5*x^2 + 4),x)
[Out] (2*atanh(64/(3*(24*x - 16)) - 5/3))/3 + (4*atanh(128/(3*(48*x + 32)) + 5/3)
)/3
```

3.99 $\int \frac{-5+2x^2}{6-5x^2+x^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 31

$$\int \frac{-5+2x^2}{6-5x^2+x^4} \, dx = -\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -1/2*arctanh(1/2*x*2^(1/2))*2^(1/2)-1/3*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1180, 213}

$$\int \frac{-5+2x^2}{6-5x^2+x^4} \, dx = -\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Int[(-5 + 2*x^2)/(6 - 5*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[x/Sqrt[2]]/Sqrt[2]) - ArcTanh[x/Sqrt[3]]/Sqrt[3]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{-3+x^2}\,dx + \int \frac{1}{-2+x^2}\,dx\\ &= -\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 69 vs. $2(31) = 62$.

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.23

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx = \frac{1}{12}\left(3\sqrt{2}\log\left(\sqrt{2}-x\right) + 2\sqrt{3}\log\left(\sqrt{3}-x\right) - 3\sqrt{2}\log\left(\sqrt{2}+x\right) - 2\sqrt{3}\log\left(\sqrt{3}+x\right)\right)$$

```
[In] Integrate[(-5 + 2*x^2)/(6 - 5*x^2 + x^4),x]
[Out] (3*Sqrt[2]*Log[Sqrt[2] - x] + 2*Sqrt[3]*Log[Sqrt[3] - x] - 3*Sqrt[2]*Log[Sq
rt[2] + x] - 2*Sqrt[3]*Log[Sqrt[3] + x])/12
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}$	26
risch	$\frac{\sqrt{3}\,\ln\left(x-\sqrt{3}\right)}{6}-\frac{\sqrt{3}\,\ln\left(x+\sqrt{3}\right)}{6}+\frac{\sqrt{2}\,\ln\left(x-\sqrt{2}\right)}{4}-\frac{\sqrt{2}\,\ln\left(x+\sqrt{2}\right)}{4}$	50

```
[In] int((2*x^2-5)/(x^4-5*x^2+6),x,method=_RETURNVERBOSE)
[Out] -1/2*arctanh(1/2*x*2^(1/2))*2^(1/2)-1/3*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}x+2}{x^2-2}\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{x^2-2\sqrt{3}x+3}{x^2-3}\right)$$

```
[In] integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*x + 2)/(x^2 - 2)) + 1/6*sqrt(3)*log((x^2 -
 2*sqrt(3)*x + 3)/(x^2 - 3))
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx = \frac{\sqrt{2}\log\left(x-\sqrt{2}\right)}{4}-\frac{\sqrt{2}\log\left(x+\sqrt{2}\right)}{4}+\frac{\sqrt{3}\log\left(x-\sqrt{3}\right)}{6}-\frac{\sqrt{3}\log\left(x+\sqrt{3}\right)}{6}$$

```
[In] integrate((2*x**2-5)/(x**4-5*x**2+6),x)
```

```
[Out] sqrt(2)*log(x - sqrt(2))/4 - sqrt(2)*log(x + sqrt(2))/4 + sqrt(3)*log(x - s
qrt(3))/6 - sqrt(3)*log(x + sqrt(3))/6
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right)+\frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)$$

```
[In] integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*log((x - sqrt(3))/(x + sqrt(3))) + 1/4*sqrt(2)*log((x - sqrt(2)
)/(x + sqrt(2)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(25) = 50$.

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{|2x-2\sqrt{3}|}{|2x+2\sqrt{3}|}\right) + \frac{1}{4}\sqrt{2}\log\left(\frac{|2x-2\sqrt{2}|}{|2x+2\sqrt{2}|}\right)$$

```
[In] integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="giac")
[Out] 1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3))/abs(2*x + 2*sqrt(3))) + 1/4*sqrt(2)*lo
g(abs(2*x - 2*sqrt(2))/abs(2*x + 2*sqrt(2)))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{2}\right)}{2} - \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x}{3}\right)}{3}$$

```
[In] int((2*x^2 - 5)/(x^4 - 5*x^2 + 6),x)
[Out] - (2^(1/2)*atanh((2^(1/2)*x)/2))/2 - (3^(1/2)*atanh((3^(1/2)*x)/3))/3
```

3.100 $\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = -\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x) -\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

```
[Out] -1/6*ln(1-x)+1/2*ln(2-x)-1/2*ln(3-x)+1/6*ln(4-x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {186}

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = -\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x) -\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

```
[In] Int[1/((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] -1/6*Log[1 - x] + Log[2 - x]/2 - Log[3 - x]/2 + Log[4 - x]/6
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{6(-4+x)} - \frac{1}{2(-3+x)} + \frac{1}{2(-2+x)} - \frac{1}{6(-1+x)} \right) dx$$
$$= -\frac{1}{6}\log(1-x) + \frac{1}{2}\log(2-x) - \frac{1}{2}\log(3-x) + \frac{1}{6}\log(4-x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)} dx = -\frac{1}{6}\log(1-x) + \frac{1}{2}\log(2-x) - \frac{1}{2}\log(3-x) + \frac{1}{6}\log(4-x)$$

```
[In] Integrate[1/((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)),x]
[Out] -1/6*Log[1 - x] + Log[2 - x]/2 - Log[3 - x]/2 + Log[4 - x]/6
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

method	result	size
default	$-\frac{\ln(-1+x)}{6} - \frac{\ln(-3+x)}{2} + \frac{\ln(x-4)}{6} + \frac{\ln(-2+x)}{2}$	26
norman	$-\frac{\ln(-1+x)}{6} - \frac{\ln(-3+x)}{2} + \frac{\ln(x-4)}{6} + \frac{\ln(-2+x)}{2}$	26
risch	$-\frac{\ln(-1+x)}{6} - \frac{\ln(-3+x)}{2} + \frac{\ln(x-4)}{6} + \frac{\ln(-2+x)}{2}$	26
parallelrisch	$-\frac{\ln(-1+x)}{6} - \frac{\ln(-3+x)}{2} + \frac{\ln(x-4)}{6} + \frac{\ln(-2+x)}{2}$	26

```
[In] int(1/(x-4)/(-3+x)/(-2+x)/(-1+x),x,method=_RETURNVERBOSE)
[Out] -1/6*ln(-1+x)-1/2*ln(-3+x)+1/6*ln(x-4)+1/2*ln(-2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = -\frac{1}{6}\log(x-1) + \frac{1}{2}\log(x-2) - \frac{1}{2}\log(x-3) + \frac{1}{6}\log(x-4)$$

```
[In] integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
[Out] -1/6*log(x - 1) + 1/2*log(x - 2) - 1/2*log(x - 3) + 1/6*log(x - 4)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = \frac{\log(x-4)}{6} - \frac{\log(x-3)}{2} + \frac{\log(x-2)}{2} - \frac{\log(x-1)}{6}$$

```
[In] integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x)
[Out] log(x - 4)/6 - log(x - 3)/2 + log(x - 2)/2 - log(x - 1)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = -\frac{1}{6}\log(x-1) + \frac{1}{2}\log(x-2) - \frac{1}{2}\log(x-3) + \frac{1}{6}\log(x-4)$$

```
[In] integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
[Out] -1/6*log(x - 1) + 1/2*log(x - 2) - 1/2*log(x - 3) + 1/6*log(x - 4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = -\frac{1}{6}\log(|x-1|) + \frac{1}{2}\log(|x-2|) - \frac{1}{2}\log(|x-3|) + \frac{1}{6}\log(|x-4|)$$

```
[In] integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
[Out] -1/6*log(abs(x - 1)) + 1/2*log(abs(x - 2)) - 1/2*log(abs(x - 3)) + 1/6*log(
abs(x - 4))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.37

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = \operatorname{atanh}(2\,x-5) - \frac{\operatorname{atanh}\left(\frac{2\,x}{3}-\frac{5}{3}\right)}{3}$$

```
[In] int(1/((x - 1)*(x - 2)*(x - 3)*(x - 4)),x)
[Out] atanh(2*x - 5) - atanh((2*x)/3 - 5/3)/3
```

3.101 $\int \frac{1+x^2}{(-1+x)^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{1+x^2}{(-1+x)^3} \, dx = -\frac{1}{(1-x)^2} + \frac{2}{1-x} + \log(1-x)$$

```
[Out] -1/(1-x)^2+2/(1-x)+ln(1-x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {711}

$$\int \frac{1+x^2}{(-1+x)^3} \, dx = \frac{2}{1-x} - \frac{1}{(1-x)^2} + \log(1-x)$$

```
[In] Int[(1 + x^2)/(-1 + x)^3,x]
```

```
[Out] -(1 - x)^(-2) + 2/(1 - x) + Log[1 - x]
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{2}{(-1+x)^3} + \frac{2}{(-1+x)^2} + \frac{1}{-1+x}\right) dx \\ &= -\frac{1}{(1-x)^2} + \frac{2}{1-x} + \log(1-x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{1+x^2}{(-1+x)^3}\,dx = \frac{1-2x}{(-1+x)^2}+\log(-1+x)$$

```
[In] Integrate[(1 + x^2)/(-1 + x)^3,x]
[Out] (1 - 2*x)/(-1 + x)^2 + Log[-1 + x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

method	result	size
norman	$\frac{1-2x}{(-1+x)^2}+\ln(-1+x)$	17
risch	$\frac{1-2x}{(-1+x)^2}+\ln(-1+x)$	17
default	$\ln(-1+x)-\frac{2}{-1+x}-\frac{1}{(-1+x)^2}$	20
parallelrisch	$\frac{\ln(-1+x)x^2+1-2\ln(-1+x)x+\ln(-1+x)-2x}{(-1+x)^2}$	31
meijerg	$-\frac{x(2-x)}{2(1-x)^2}+\frac{x(-9x+6)}{6(1-x)^2}+\ln(1-x)$	38

```
[In] int((x^2+1)/(-1+x)^3,x,method=_RETURNVERBOSE)
[Out] (1-2*x)/(-1+x)^2+ln(-1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{1+x^2}{(-1+x)^3}\,dx = \frac{(x^2-2\,x+1)\log(x-1)-2\,x+1}{x^2-2\,x+1}$$

```
[In] integrate((x^2+1)/(-1+x)^3,x, algorithm="fricas")
[Out] ((x^2 - 2*x + 1)*log(x - 1) - 2*x + 1)/(x^2 - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1+x^2}{(-1+x)^3}\,dx = \frac{1-2x}{x^2-2x+1} + \log{(x-1)}$$

```
[In] integrate((x**2+1)/(-1+x)**3,x)
[Out] (1 - 2*x)/(x**2 - 2*x + 1) + log(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{(-1+x)^3}\,dx = -\frac{2\,x-1}{x^2-2\,x+1} + \log{(x-1)}$$

```
[In] integrate((x^2+1)/(-1+x)^3,x, algorithm="maxima")
[Out] -(2*x - 1)/(x^2 - 2*x + 1) + log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{1+x^2}{(-1+x)^3}\,dx = -\frac{2\,x-1}{(x-1)^2} + \log{(|x-1|)}$$

```
[In] integrate((x^2+1)/(-1+x)^3,x, algorithm="giac")
[Out] -(2*x - 1)/(x - 1)^2 + log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{(-1+x)^3}\,dx = \ln{(x-1)} - \frac{2\,x-1}{x^2-2\,x+1}$$

```
[In] int((x^2 + 1)/(x - 1)^3,x)
[Out] log(x - 1) - (2*x - 1)/(x^2 - 2*x + 1)
```

3.102 $\int \frac{x^5}{(3+x)^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 36

$$\int \frac{x^5}{(3+x)^2} dx = -108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405\log(3+x)$$

```
[Out] -108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$\int \frac{x^5}{(3+x)^2} dx = \frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + \frac{243}{x+3} + 405\log(x+3)$$

```
[In] Int[x^5/(3 + x)^2,x]
[Out] -108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3 + x) + 405*Log[3 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-108 + 27x - 6x^2 + x^3 - \frac{243}{(3+x)^2} + \frac{405}{3+x}\right) dx \\ &= -108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405\log(3+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(3+x)^2}\,dx = \frac{1}{4}\left(-2079 - 432x + 54x^2 - 8x^3 + x^4 + \frac{972}{3+x}\right) + 405\log(3+x)$$

```
[In] Integrate[x^5/(3 + x)^2,x]
```

```
[Out] (-2079 - 432*x + 54*x^2 - 8*x^3 + x^4 + 972/(3 + x))/4 + 405*Log[3 + x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
default	$-108x+\frac{27x^2}{2}-2x^3+\frac{x^4}{4}+\frac{243}{3+x}+405\ln(3+x)$	33
risch	$-108x+\frac{27x^2}{2}-2x^3+\frac{x^4}{4}+\frac{243}{3+x}+405\ln(3+x)$	33
norman	$\frac{-\frac{135}{2}x^2+\frac{15}{2}x^3-\frac{5}{4}x^4+\frac{1}{4}x^5+1215}{3+x}+405\ln(3+x)$	36
meijerg	$-\frac{9x\left(-\frac{1}{27}x^4+\frac{5}{27}x^3-\frac{10}{9}x^2+10x+60\right)}{4\left(1+\frac{x}{3}\right)}+405\ln\left(1+\frac{x}{3}\right)$	40
parallelrisch	$\frac{x^5-5x^4+30x^3+1620\ln(3+x)x-270x^2+4860+4860\ln(3+x)}{12+4x}$	41

```
[In] int(x^5/(3+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(3+x)^2}\,dx = \frac{x^5 - 5\,x^4 + 30\,x^3 - 270\,x^2 + 1620\,(x+3)\log(x+3) - 1296\,x + 972}{4\,(x+3)}$$

```
[In] integrate(x^5/(3+x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x^5 - 5*x^4 + 30*x^3 - 270*x^2 + 1620*(x + 3)*log(x + 3) - 1296*x + 97
2)/(x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(3+x)^2} \, dx = \frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + 405 \log(x+3) + \frac{243}{x+3}$$

```
[In] integrate(x**5/(3+x)**2,x)
[Out] x**4/4 - 2*x**3 + 27*x**2/2 - 108*x + 405*log(x + 3) + 243/(x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(3+x)^2} \, dx = \frac{1}{4} x^4 - 2 x^3 + \frac{27}{2} x^2 - 108 x + \frac{243}{x+3} + 405 \log(x+3)$$

```
[In] integrate(x^5/(3+x)^2,x, algorithm="maxima")
[Out] 1/4*x^4 - 2*x^3 + 27/2*x^2 - 108*x + 243/(x + 3) + 405*log(x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{x^5}{(3+x)^2} \, dx = -\frac{1}{4} (x+3)^4 \left(\frac{20}{x+3} - \frac{180}{(x+3)^2} + \frac{1080}{(x+3)^3} - 1 \right) + \frac{243}{x+3} + 405 \log(|x+3|)$$

```
[In] integrate(x^5/(3+x)^2,x, algorithm="giac")
[Out] -1/4*(x + 3)^4*(20/(x + 3) - 180/(x + 3)^2 + 1080/(x + 3)^3 - 1) + 243/(x +
 3) + 405*log(abs(x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(3+x)^2}\,dx = 405\,\ln(x+3) - 108\,x + \frac{243}{x+3} + \frac{27\,x^2}{2} - 2\,x^3 + \frac{x^4}{4}$$

```
[In] int(x^5/(x + 3)^2,x)
[Out] 405*log(x + 3) - 108*x + 243/(x + 3) + (27*x^2)/2 - 2*x^3 + x^4/4
```

3.103 $\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 41

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = -\frac{133}{8(3-x)^2}+\frac{407}{16(3-x)}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(1+x)$$

```
[Out] -133/8/(3-x)^2+407/16/(3-x)+313/64*ln(3-x)+7/64*ln(1+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2099}

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = \frac{407}{16(3-x)}-\frac{133}{8(3-x)^2}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(x+1)$$

```
[In] Int[(-2 + 5*x^3)/(-27 + 18*x^2 - 8*x^3 + x^4),x]
```

```
[Out] -133/(8*(3 - x)^2) + 407/(16*(3 - x)) + (313*Log[3 - x])/64 + (7*Log[1 + x]
)/64
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{133}{4(-3+x)^3}+\frac{407}{16(-3+x)^2}+\frac{313}{64(-3+x)}+\frac{7}{64(1+x)}\right)dx \\ &= -\frac{133}{8(3-x)^2}+\frac{407}{16(3-x)}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = -\frac{133}{8(-3+x)^2}-\frac{407}{16(-3+x)}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(1+x)$$

```
[In] Integrate[(-2 + 5*x^3)/(-27 + 18*x^2 - 8*x^3 + x^4),x]
[Out] -133/(8*(-3 + x)^2) - 407/(16*(-3 + x)) + (313*Log[3 - x])/64 + (7*Log[1 +
x])/64
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

method	result	size
norman	$\frac{-\frac{407x}{16}+\frac{955}{16}}{(-3+x)^2}+\frac{313\ln(-3+x)}{64}+\frac{7\ln(1+x)}{64}$	25
default	$\frac{7\ln(1+x)}{64}-\frac{133}{8(-3+x)^2}-\frac{407}{16(-3+x)}+\frac{313\ln(-3+x)}{64}$	28
risch	$\frac{-\frac{407x}{16}+\frac{955}{16}}{x^2-6x+9}+\frac{313\ln(-3+x)}{64}+\frac{7\ln(1+x)}{64}$	30
parallelrisch	$\frac{7\ln(1+x)x^2+313\ln(-3+x)x^2+3820-42\ln(1+x)x-1878\ln(-3+x)x+63\ln(1+x)+2817\ln(-3+x)-1628x}{64x^2-384x+576}$	62

```
[In] int((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x,method=_RETURNVERBOSE)
[Out] (-407/16*x+955/16)/(-3+x)^2+313/64*ln(-3+x)+7/64*ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = \frac{7\,(x^2-6\,x+9)\log(x+1)+313\,(x^2-6\,x+9)\log(x-3)-1628\,x+3820}{64\,(x^2-6\,x+9)}$$

```
[In] integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="fricas")
[Out] 1/64*(7*(x^2 - 6*x + 9)*log(x + 1) + 313*(x^2 - 6*x + 9)*log(x - 3) - 1628*
x + 3820)/(x^2 - 6*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = \frac{955-407x}{16x^2-96x+144}+\frac{313\log(x-3)}{64}+\frac{7\log(x+1)}{64}$$

```
[In] integrate((5*x**3-2)/(x**4-8*x**3+18*x**2-27),x)
[Out] (955 - 407*x)/(16*x**2 - 96*x + 144) + 313*log(x - 3)/64 + 7*log(x + 1)/64
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = -\frac{407\,x-955}{16\,(x^2-6\,x+9)}+\frac{7}{64}\,\log(x+1)+\frac{313}{64}\,\log(x-3)$$

```
[In] integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="maxima")
[Out] -1/16*(407*x - 955)/(x^2 - 6*x + 9) + 7/64*log(x + 1) + 313/64*log(x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = -\frac{407\,x-955}{16\,(x-3)^2}+\frac{7}{64}\,\log(|x+1|)+\frac{313}{64}\,\log(|x-3|)$$

```
[In] integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="giac")
[Out] -1/16*(407*x - 955)/(x - 3)^2 + 7/64*log(abs(x + 1)) + 313/64*log(abs(x - 3
))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = \frac{7\,\ln(x+1)}{64}+\frac{313\,\ln(x-3)}{64}-\frac{\frac{407\,x}{16}-\frac{955}{16}}{x^2-6\,x+9}$$

```
[In] int((5*x^3 - 2)/(18*x^2 - 8*x^3 + x^4 - 27),x)
[Out] (7*log(x + 1))/64 + (313*log(x - 3))/64 - ((407*x)/16 - 955/16)/(x^2 - 6*x
+ 9)
```

3.104 $\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx=\frac{99}{3+x}+\frac{181}{4+x}+264\log(3+x)-263\log(4+x)$$

```
[Out] 99/(3+x)+181/(4+x)+264*ln(3+x)-263*ln(4+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1634}

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx=\frac{99}{x+3}+\frac{181}{x+4}+264\log(x+3)-263\log(x+4)$$

```
[In] Int[(-9 + 3*x - 6*x^2 + x^3)/((3 + x)^2*(4 + x)^2),x]
[Out] 99/(3 + x) + 181/(4 + x) + 264*Log[3 + x] - 263*Log[4 + x]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\frac{99}{(3+x)^2}+\frac{264}{3+x}-\frac{181}{(4+x)^2}-\frac{263}{4+x}\right)dx \\ &= \frac{99}{3+x}+\frac{181}{4+x}+264\log(3+x)-263\log(4+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx=\frac{99}{3+x}+\frac{181}{4+x}+264\log(3+x)-263\log(4+x)$$

```
[In] Integrate[(-9 + 3*x - 6*x^2 + x^3)/((3 + x)^2*(4 + x)^2),x]
[Out] 99/(3 + x) + 181/(4 + x) + 264*Log[3 + x] - 263*Log[4 + x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{99}{3+x}+\frac{181}{4+x}+264\ln\left(3+x\right)-263\ln\left(4+x\right)$	28
norman	$\frac{280x+939}{(4+x)(3+x)}+264\ln\left(3+x\right)-263\ln\left(4+x\right)$	30
risch	$\frac{280x+939}{(4+x)(3+x)}+264\ln\left(3+x\right)-263\ln\left(4+x\right)$	30
parallelrisch	$\frac{264\ln(3+x)x^2-263\ln(4+x)x^2+939+1848\ln(3+x)x-1841\ln(4+x)x+3168\ln(3+x)-3156\ln(4+x)+280x}{(3+x)(4+x)}$	61

```
[In] int((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x,method=_RETURNVERBOSE)
[Out] 99/(3+x)+181/(4+x)+264*ln(3+x)-263*ln(4+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx=-\frac{263\left(x^2+7\,x+12\right)\log\left(x+4\right)-264\left(x^2+7\,x+12\right)\log\left(x+3\right)-280\,x-939}{x^2+7\,x+12}$$

```
[In] integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="fricas")
[Out] -(263*(x^2 + 7*x + 12)*log(x + 4) - 264*(x^2 + 7*x + 12)*log(x + 3) - 280*x
 - 939)/(x^2 + 7*x + 12)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx = \frac{280x+939}{x^2+7x+12}+264\log(x+3)-263\log(x+4)$$

```
[In] integrate((x**3-6*x**2+3*x-9)/(3+x)**2/(4+x)**2,x)
[Out] (280*x + 939)/(x**2 + 7*x + 12) + 264*log(x + 3) - 263*log(x + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx = \frac{280\,x+939}{x^2+7\,x+12}-263\,\log(x+4)+264\,\log(x+3)$$

```
[In] integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="maxima")
[Out] (280*x + 939)/(x^2 + 7*x + 12) - 263*log(x + 4) + 264*log(x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx = \frac{181}{x+4}-\frac{99}{\frac{1}{x+4}-1}+\log(|x+4|)+264\,\log\left(\left|-\frac{1}{x+4}+1\right|\right)$$

```
[In] integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="giac")
[Out] 181/(x + 4) - 99/(1/(x + 4) - 1) + log(abs(x + 4)) + 264*log(abs(-1/(x + 4)
 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx = 264\,\ln(x+3) - 263\,\ln(x+4) + \frac{280\,x+939}{x^2+7\,x+12}$$

```
[In] int((3*x - 6*x^2 + x^3 - 9)/((x + 3)^2*(x + 4)^2),x)
[Out] 264*log(x + 3) - 263*log(x + 4) + (280*x + 939)/(7*x + x^2 + 12)
```

3.105 $\int \frac{2+x^2+x^3}{x(-1+x^2)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2} dx = \frac{3+x}{2\left(1-x^2\right)} - \frac{3}{4}\log(1-x) + 2\log(x) - \frac{5}{4}\log(1+x)$$

```
[Out] 1/2*(3+x)/(-x^2+1)-3/4*ln(1-x)+2*ln(x)-5/4*ln(1+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1819, 815}

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2} dx = \frac{x+3}{2\left(1-x^2\right)} - \frac{3}{4}\log(1-x) + 2\log(x) - \frac{5}{4}\log(x+1)$$

```
[In] Int[(2 + x^2 + x^3)/(x*(-1 + x^2)^2),x]
```

```
[Out] (3 + x)/(2*(1 - x^2)) - (3*Log[1 - x])/4 + 2*Log[x] - (5*Log[1 + x])/4
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
```

```
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3+x}{2(1-x^2)}+\frac{1}{2}\int\frac{-4+x}{x(-1+x^2)}\,dx\\ &= \frac{3+x}{2(1-x^2)}+\frac{1}{2}\int\left(-\frac{3}{2(-1+x)}+\frac{4}{x}-\frac{5}{2(1+x)}\right)dx\\ &= \frac{3+x}{2(1-x^2)}-\frac{3}{4}\log(1-x)+2\log(x)-\frac{5}{4}\log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int\frac{2+x^2+x^3}{x(-1+x^2)^2}\,dx=\frac{1}{4}\left(-\frac{2}{-1+x}-\frac{4}{-1+x^2}+\log(1-x)+8\log(x)-\log(1+x)-4\log\left(1-x^2\right)\right)$$

```
[In] Integrate[(2 + x^2 + x^3)/(x*(-1 + x^2)^2),x]
[Out] (-2/(-1 + x) - 4/(-1 + x^2) + Log[1 - x] + 8*Log[x] - Log[1 + x] - 4*Log[1
- x^2])/4
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result	size
norman	$\frac{-\frac{x}{2}-\frac{3}{2}}{x^2-1}+2\ln(x)-\frac{3\ln(-1+x)}{4}-\frac{5\ln(1+x)}{4}$	31
risch	$\frac{-\frac{x}{2}-\frac{3}{2}}{x^2-1}+2\ln(x)-\frac{3\ln(-1+x)}{4}-\frac{5\ln(1+x)}{4}$	31
default	$-\frac{1}{-1+x}-\frac{3\ln(-1+x)}{4}+2\ln(x)+\frac{1}{2x+2}-\frac{5\ln(1+x)}{4}$	32
parallelrisch	$\frac{8x^2\ln(x)-3\ln(-1+x)x^2-5\ln(1+x)x^2-6-8\ln(x)+3\ln(-1+x)+5\ln(1+x)-2x}{4x^2-4}$	56
meijerg	$\frac{i\left(-\frac{ix}{-x^2+1}+i\operatorname{arctanh}(x)\right)}{2}+\frac{3x^2}{-2x^2+2}+1+2\ln(x)+i\pi-\ln(-x^2+1)$	71

```
[In] int((x^3+x^2+2)/x/(x^2-1)^2,x,method=_RETURNVERBOSE)
[Out] (-1/2*x-3/2)/(x^2-1)+2*ln(x)-3/4*ln(-1+x)-5/4*ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx = -\frac{5\left(x^2-1\right)\log\left(x+1\right)+3\left(x^2-1\right)\log\left(x-1\right)-8\left(x^2-1\right)\log\left(x\right)+2\,x+6}{4\left(x^2-1\right)}$$

```
[In] integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="fricas")
[Out] -1/4*(5*(x^2 - 1)*log(x + 1) + 3*(x^2 - 1)*log(x - 1) - 8*(x^2 - 1)*log(x)
+ 2*x + 6)/(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx = \frac{-x-3}{2x^2-2}+2\log\left(x\right)-\frac{3\log\left(x-1\right)}{4}-\frac{5\log\left(x+1\right)}{4}$$

```
[In] integrate((x**3+x**2+2)/x/(x**2-1)**2,x)
[Out] (-x - 3)/(2*x**2 - 2) + 2*log(x) - 3*log(x - 1)/4 - 5*log(x + 1)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx = -\frac{x+3}{2\left(x^2-1\right)}-\frac{5}{4}\,\log\left(x+1\right)-\frac{3}{4}\,\log\left(x-1\right)+2\,\log\left(x\right)$$

```
[In] integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="maxima")
[Out] -1/2*(x + 3)/(x^2 - 1) - 5/4*log(x + 1) - 3/4*log(x - 1) + 2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx=-\frac{x+3}{2\,(x+1)(x-1)}-\frac{5}{4}\,\log\left(|x+1|\right)-\frac{3}{4}\,\log\left(|x-1|\right)+2\,\log\left(|x|\right)$$

```
[In] integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="giac")
[Out] -1/2*(x + 3)/((x + 1)*(x - 1)) - 5/4*log(abs(x + 1)) - 3/4*log(abs(x - 1))
+ 2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx=2\,\ln\left(x\right)-\frac{5\,\ln\left(x+1\right)}{4}-\frac{3\,\ln\left(x-1\right)}{4}-\frac{\frac{x}{2}+\frac{3}{2}}{x^2-1}$$

```
[In] int((x^2 + x^3 + 2)/(x*(x^2 - 1)^2),x)
[Out] 2*log(x) - (5*log(x + 1))/4 - (3*log(x - 1))/4 - (x/2 + 3/2)/(x^2 - 1)
```

3.106 $\int \frac{1}{x^3-x^4-x^5+x^6} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{1}{x^3-x^4-x^5+x^6} \, dx=\frac{1}{2(1-x)}-\frac{1}{2x^2}-\frac{1}{x}-\frac{7}{4}\log(1-x)+2\log(x)-\frac{1}{4}\log(1+x)$$

```
[Out] 1/2/(1-x)-1/2/x^2-1/x-7/4*ln(1-x)+2*ln(x)-1/4*ln(1+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2083}

$$\int \frac{1}{x^3-x^4-x^5+x^6} \, dx=-\frac{1}{2x^2}+\frac{1}{2(1-x)}-\frac{1}{x}-\frac{7}{4}\log(1-x)+2\log(x)-\frac{1}{4}\log(x+1)$$

```
[In] Int[(x^3 - x^4 - x^5 + x^6)^(-1),x]
```

```
[Out] 1/(2*(1 - x)) - 1/(2*x^2) - x^(-1) - (7*Log[1 - x])/4 + 2*Log[x] - Log[1 + x]/4
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{2(-1+x)^2}-\frac{7}{4(-1+x)}+\frac{1}{x^3}+\frac{1}{x^2}+\frac{2}{x}-\frac{1}{4(1+x)}\right) \, dx \\ &= \frac{1}{2(1-x)}-\frac{1}{2x^2}-\frac{1}{x}-\frac{7}{4}\log(1-x)+2\log(x)-\frac{1}{4}\log(1+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3-x^4-x^5+x^6}\,dx=\frac{1}{4}\left(-\frac{2}{-1+x}-\frac{2}{x^2}-\frac{4}{x}-7\log(1-x)+8\log(x)-\log(1+x)\right)$$

```
[In] Integrate[(x^3 - x^4 - x^5 + x^6)^(-1),x]
[Out] (-2/(-1 + x) - 2/x^2 - 4/x - 7*Log[1 - x] + 8*Log[x] - Log[1 + x])/4
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{1}{2(-1+x)}-\frac{7\ln(-1+x)}{4}-\frac{1}{2x^2}-\frac{1}{x}+2\ln(x)-\frac{\ln(1+x)}{4}$	35
norman	$\frac{\frac{1}{2}-\frac{3}{2}x^2+\frac{1}{2}x}{x^2(-1+x)}+2\ln(x)-\frac{7\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}$	37
risch	$\frac{\frac{1}{2}-\frac{3}{2}x^2+\frac{1}{2}x}{x^2(-1+x)}+2\ln(x)-\frac{7\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}$	37
parallelrisch	$\frac{8x^3\ln(x)-7\ln(-1+x)x^3-\ln(1+x)x^3+2-8x^2\ln(x)+7\ln(-1+x)x^2+\ln(1+x)x^2-6x^2+2x}{4x^2(-1+x)}$	70

```
[In] int(1/(x^6-x^5-x^4+x^3),x,method=_RETURNVERBOSE)
[Out] -1/2/(-1+x)-7/4*ln(-1+x)-1/2/x^2-1/x+2*ln(x)-1/4*ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

$$\int \frac{1}{x^3-x^4-x^5+x^6}\,dx=-\frac{6\,x^2+(x^3-x^2)\log(x+1)+7\,(x^3-x^2)\log(x-1)-8\,(x^3-x^2)\log(x)-2\,x-2}{4\,(x^3-x^2)}$$

```
[In] integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="fricas")
[Out] -1/4*(6*x^2 + (x^3 - x^2)*log(x + 1) + 7*(x^3 - x^2)*log(x - 1) - 8*(x^3 -
x^2)*log(x) - 2*x - 2)/(x^3 - x^2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^3 - x^4 - x^5 + x^6}\, dx = 2\log(x) - \frac{7\log(x-1)}{4} - \frac{\log(x+1)}{4} + \frac{-3x^2 + x + 1}{2x^3 - 2x^2}$$

```
[In] integrate(1/(x**6-x**5-x**4+x**3),x)
[Out] 2*log(x) - 7*log(x - 1)/4 - log(x + 1)/4 + (-3*x**2 + x + 1)/(2*x**3 - 2*x*
*2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 - x^4 - x^5 + x^6}\, dx = -\frac{3\,x^2 - x - 1}{2\,(x^3 - x^2)} - \frac{1}{4}\,\log(x+1) - \frac{7}{4}\,\log(x-1) + 2\,\log(x)$$

```
[In] integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="maxima")
[Out] -1/2*(3*x^2 - x - 1)/(x^3 - x^2) - 1/4*log(x + 1) - 7/4*log(x - 1) + 2*log(
x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 - x^4 - x^5 + x^6}\, dx = -\frac{3\,x^2 - x - 1}{2\,(x-1)x^2} - \frac{1}{4}\,\log(|x+1|) - \frac{7}{4}\,\log(|x-1|) + 2\,\log(|x|)$$

```
[In] integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="giac")
[Out] -1/2*(3*x^2 - x - 1)/((x - 1)*x^2) - 1/4*log(abs(x + 1)) - 7/4*log(abs(x -
1)) + 2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 - x^4 - x^5 + x^6} \, dx = 2 \ln(x) - \frac{\ln(x+1)}{4} - \frac{7 \ln(x-1)}{4} - \frac{-\frac{3x^2}{2} + \frac{x}{2} + \frac{1}{2}}{x^2 - x^3}$$

```
[In] int(1/(x^3 - x^4 - x^5 + x^6),x)
[Out] 2*log(x) - log(x + 1)/4 - (7*log(x - 1))/4 - (x/2 - (3*x^2)/2 + 1/2)/(x^2 -
 x^3)
```

3.107 $\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 29

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx = x+\frac{x^2}{2}-\arctan(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] x+1/2*x^2-arctan(x)+ln(1-x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2099, 649, 209, 266}

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx = -\arctan(x)+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)+x+\log(1-x)$$

```
[In] Int[(1 + x^4)/(-1 + x - x^2 + x^3),x]
```

```
[Out] x + x^2/2 - ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(1+\frac{1}{-1+x}+x+\frac{-1-x}{1+x^2}\right) dx \\
&= x+\frac{x^2}{2}+\log(1-x)+\int \frac{-1-x}{1+x^2}\,dx \\
&= x+\frac{x^2}{2}+\log(1-x)-\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx \\
&= x+\frac{x^2}{2}-\arctan(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx = x+\frac{x^2}{2}-\arctan(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(1 + x^4)/(-1 + x - x^2 + x^3),x]
[Out] x + x^2/2 - ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
default	$x+\frac{x^2}{2}+\ln(-1+x)-\frac{\ln(x^2+1)}{2}-\arctan(x)$	24
risch	$x+\frac{x^2}{2}+\ln(-1+x)-\frac{\ln(x^2+1)}{2}-\arctan(x)$	24
parallelrisch	$\frac{x^2}{2}+x+\ln(-1+x)-\frac{\ln(x-i)}{2}+\frac{i\ln(x-i)}{2}-\frac{\ln(x+i)}{2}-\frac{i\ln(x+i)}{2}$	42

```
[In] int((x^4+1)/(x^3-x^2+x-1),x,method=_RETURNVERBOSE)
[Out] x+1/2*x^2+ln(-1+x)-1/2*ln(x^2+1)-arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx=\frac{1}{2}\,x^2+x-\arctan(x)-\frac{1}{2}\,\log\left(x^2+1\right)+\log(x-1)$$

```
[In] integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="fricas")
[Out] 1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx=\frac{x^2}{2}+x+\log(x-1)-\frac{\log(x^2+1)}{2}-\operatorname{atan}(x)$$

```
[In] integrate((x**4+1)/(x**3-x**2+x-1),x)
[Out] x**2/2 + x + log(x - 1) - log(x**2 + 1)/2 - atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx=\frac{1}{2}\,x^2+x-\arctan(x)-\frac{1}{2}\,\log\left(x^2+1\right)+\log(x-1)$$

```
[In] integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="maxima")
[Out] 1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx = \frac{1}{2}\,x^2 + x - \arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right) + \log\left(|x-1|\right)$$

```
[In] integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="giac")
[Out] 1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx = x+\ln\left(x-1\right)+\frac{x^2}{2}+\ln\left(x-\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

```
[In] int((x^4 + 1)/(x - x^2 + x^3 - 1),x)
[Out] x + log(x - 1) - log(x - 1i)*(1/2 - 1i/2) - log(x + 1i)*(1/2 + 1i/2) + x^2/
2
```

3.108 $\int \frac{1}{x(1+x)(1+x^2)} dx$

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{1}{x(1+x)(1+x^2)} dx = -\frac{\arctan(x)}{2} + \log(x) - \frac{1}{2}\log(1+x) - \frac{1}{4}\log(1+x^2)$$

```
[Out] -1/2*arctan(x)+ln(x)-1/2*ln(1+x)-1/4*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {908, 649, 209, 266}

$$\int \frac{1}{x(1+x)(1+x^2)} dx = -\frac{\arctan(x)}{2} - \frac{1}{4}\log(x^2+1) + \log(x) - \frac{1}{2}\log(x+1)$$

```
[In] Int[1/(x*(1 + x)*(1 + x^2)),x]
```

```
[Out] -1/2*ArcTan[x] + Log[x] - Log[1 + x]/2 - Log[1 + x^2]/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{x} - \frac{1}{2(1+x)} + \frac{-1-x}{2\left(1+x^2\right)}\right) dx \\
&= \log(x) - \frac{1}{2}\log(1+x) + \frac{1}{2}\int \frac{-1-x}{1+x^2}\,dx \\
&= \log(x) - \frac{1}{2}\log(1+x) - \frac{1}{2}\int \frac{1}{1+x^2}\,dx - \frac{1}{2}\int \frac{x}{1+x^2}\,dx \\
&= -\frac{\arctan(x)}{2} + \log(x) - \frac{1}{2}\log(1+x) - \frac{1}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x)\left(1+x^2\right)}\,dx = -\frac{\arctan(x)}{2} + \log(x) - \frac{1}{2}\log(1+x) - \frac{1}{4}\log\left(1+x^2\right)$$

```
[In] Integrate[1/(x*(1 + x)*(1 + x^2)),x]
[Out] -1/2*ArcTan[x] + Log[x] - Log[1 + x]/2 - Log[1 + x^2]/4
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\arctan(x)}{2}+\ln(x)-\frac{\ln(1+x)}{2}-\frac{\ln(x^2+1)}{4}$	22
risch	$-\frac{\arctan(x)}{2}+\ln(x)-\frac{\ln(1+x)}{2}-\frac{\ln(x^2+1)}{4}$	22
parallelrisch	$\ln(x)-\frac{\ln(1+x)}{2}-\frac{\ln(x-i)}{4}+\frac{i\ln(x-i)}{4}-\frac{\ln(x+i)}{4}-\frac{i\ln(x+i)}{4}$	40

```
[In] int(1/x/(1+x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2*arctan(x)+ln(x)-1/2*ln(1+x)-1/4*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1+x)(1+x^2)}\,dx = -\frac{1}{2}\arctan(x)-\frac{1}{4}\log(x^2+1)-\frac{1}{2}\log(x+1)+\log(x)$$

```
[In] integrate(1/x/(1+x)/(x^2+1),x, algorithm="fricas")
[Out] -1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(x + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{x(1+x)(1+x^2)}\,dx = \log(x)-\frac{\log(x+1)}{2}-\frac{\log(x^2+1)}{4}-\frac{\operatorname{atan}(x)}{2}$$

```
[In] integrate(1/x/(1+x)/(x**2+1),x)
[Out] log(x) - log(x + 1)/2 - log(x**2 + 1)/4 - atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1+x)\left(1+x^2\right)}\,dx = -\frac{1}{2}\arctan\left(x\right) - \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log\left(x+1\right) + \log\left(x\right)$$

```
[In] integrate(1/x/(1+x)/(x^2+1),x, algorithm="maxima")
[Out] -1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(x + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1+x)\left(1+x^2\right)}\,dx = -\frac{1}{2}\arctan\left(x\right) - \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log\left(|x+1|\right) + \log\left(|x|\right)$$

```
[In] integrate(1/x/(1+x)/(x^2+1),x, algorithm="giac")
[Out] -1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(abs(x + 1)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x)\left(1+x^2\right)}\,dx = \ln\left(x\right) - \frac{\ln\left(x+1\right)}{2} + \ln\left(x-\mathrm{i}\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

```
[In] int(1/(x*(x^2 + 1)*(x + 1)),x)
[Out] log(x) - log(x - 1i)*(1/4 - 1i/4) - log(x + 1i)*(1/4 + 1i/4) - log(x + 1)/2
```

3.109 $\int \frac{x^2}{-2+x^2+x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 24

$$\int \frac{x^2}{-2+x^2+x^4} \, dx = \frac{1}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) - \frac{\operatorname{arctanh}(x)}{3}$$

```
[Out] -1/3*arctanh(x)+1/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1144, 209, 213}

$$\int \frac{x^2}{-2+x^2+x^4} \, dx = \frac{1}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) - \frac{\operatorname{arctanh}(x)}{3}$$

```
[In] Int[x^2/(-2 + x^2 + x^4),x]
```

```
[Out] (Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - ArcTanh[x]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\int \frac{1}{-1+x^2}\,dx + \frac{2}{3}\int \frac{1}{2+x^2}\,dx\\ &= \frac{1}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) - \frac{\text{arctanh}(x)}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{-2+x^2+x^4}\,dx = \frac{1}{6}\left(2\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) + \log(1-x) - \log(1+x)\right)$$

```
[In] Integrate[x^2/(-2 + x^2 + x^4),x]
[Out] (2*Sqrt[2]*ArcTan[x/Sqrt[2]] + Log[1 - x] - Log[1 + x])/6
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\ln(-1+x)}{6}+\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}-\frac{\ln(1+x)}{6}$	26
risch	$\frac{\ln(-1+x)}{6}+\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}-\frac{\ln(1+x)}{6}$	26

```
[In] int(x^2/(x^4+x^2-2),x,method=_RETURNVERBOSE)
[Out] 1/6*ln(-1+x)+1/3*arctan(1/2*x*2^(1/2))*2^(1/2)-1/6*ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{-2+x^2+x^4}\,dx = \frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{6}\log(x+1) + \frac{1}{6}\log(x-1)$$

```
[In] integrate(x^2/(x^4+x^2-2),x, algorithm="fricas")
[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(x + 1) + 1/6*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{-2+x^2+x^4}\,dx = \frac{\log(x-1)}{6} - \frac{\log(x+1)}{6} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}$$

```
[In] integrate(x**2/(x**4+x**2-2),x)
[Out] log(x - 1)/6 - log(x + 1)/6 + sqrt(2)*atan(sqrt(2)*x/2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{-2+x^2+x^4}\,dx = \frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{6}\log(x+1) + \frac{1}{6}\log(x-1)$$

```
[In] integrate(x^2/(x^4+x^2-2),x, algorithm="maxima")
[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(x + 1) + 1/6*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{-2+x^2+x^4}\,dx = \frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{6}\log(|x+1|) + \frac{1}{6}\log(|x-1|)$$

```
[In] integrate(x^2/(x^4+x^2-2),x, algorithm="giac")
[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(abs(x + 1)) + 1/6*log(abs(x - 1
))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{-2+x^2+x^4} \, dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{3} - \frac{\mathrm{atanh}(x)}{3}$$

```
[In] int(x^2/(x^2 + x^4 - 2),x)
[Out] (2^(1/2)*atan((2^(1/2)*x)/2))/3 - atanh(x)/3
```

3.110 $\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4} \, dx$

Optimal result

Integrand size = 33, antiderivative size = 41

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4} \, dx = \frac{1}{1+x}+\frac{4}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{3}\log(1+x)+\frac{2}{3}\log\left(2+x^2\right)$$

```
[Out] 1/(1+x)-1/3*ln(1+x)+2/3*ln(x^2+2)+4/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1608, 2100, 649, 209, 266}

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4} \, dx = \frac{4}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{x+1}-\frac{1}{3}\log(x+1)$$

```
[In] Int[(6*x + 4*x^2 + x^3)/(2 + 4*x + 3*x^2 + 2*x^3 + x^4),x]
```

```
[Out] (1 + x)^(-1) + (4*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - Log[1 + x]/3 + (2*Log[2 +
x^2])/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 2100

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(6+4x+x^2)}{2+4x+3x^2+2x^3+x^4}\,dx \\
&= \int \left(-\frac{1}{(1+x)^2}-\frac{1}{3(1+x)}+\frac{4(2+x)}{3\,(2+x^2)}\right)dx \\
&= \frac{1}{1+x}-\frac{1}{3}\log(1+x)+\frac{4}{3}\int \frac{2+x}{2+x^2}\,dx \\
&= \frac{1}{1+x}-\frac{1}{3}\log(1+x)+\frac{4}{3}\int \frac{x}{2+x^2}\,dx+\frac{8}{3}\int \frac{1}{2+x^2}\,dx \\
&= \frac{1}{1+x}+\frac{4}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{3}\log(1+x)+\frac{2}{3}\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx=\frac{1}{1+x}+\frac{4}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{3}\log(1+x)+\frac{2}{3}\log\left(2+x^2\right)$$

```
[In] Integrate[(6*x + 4*x^2 + x^3)/(2 + 4*x + 3*x^2 + 2*x^3 + x^4),x]
[Out] (1 + x)^(-1) + (4*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - Log[1 + x]/3 + (2*Log[2 +
x^2])/3
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{1}{1+x}-\frac{\ln(1+x)}{3}+\frac{2\ln(x^2+2)}{3}+\frac{4\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}$	33
risch	$\frac{1}{1+x}-\frac{\ln(1+x)}{3}+\frac{2\ln(x^2+2)}{3}+\frac{4\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}$	33

```
[In] int((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x,method=_RETURNVERBOSE)
[Out] 1/(1+x)-1/3*ln(1+x)+2/3*ln(x^2+2)+4/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{4\sqrt{2}(x+1)\arctan\left(\frac{1}{2}\sqrt{2}x\right)+2(x+1)\log\left(x^2+2\right)-(x+1)\log\left(x+1\right)+3}{3\,(x+1)}$$

```
[In] integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="fricas")
[Out] 1/3*(4*sqrt(2)*(x + 1)*arctan(1/2*sqrt(2)*x) + 2*(x + 1)*log(x^2 + 2) - (x
+ 1)*log(x + 1) + 3)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = -\frac{\log(x+1)}{3}+\frac{2\log(x^2+2)}{3}+\frac{4\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}+\frac{1}{x+1}$$

```
[In] integrate((x**3+4*x**2+6*x)/(x**4+2*x**3+3*x**2+4*x+2),x)
[Out] -log(x + 1)/3 + 2*log(x**2 + 2)/3 + 4*sqrt(2)*atan(sqrt(2)*x/2)/3 + 1/(x +
1)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{4}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{x+1} + \frac{2}{3}\log\left(x^2+2\right)-\frac{1}{3}\log\left(x+1\right)$$

```
[In] integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="maxima")
[Out] 4/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/(x + 1) + 2/3*log(x^2 + 2) - 1/3*log(
x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{4}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{x+1} + \frac{2}{3}\log\left(x^2+2\right)-\frac{1}{3}\log\left(|x+1|\right)$$

```
[In] integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="giac")
[Out] 4/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/(x + 1) + 2/3*log(x^2 + 2) - 1/3*log(
abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{1}{x+1}-\frac{\ln\left(x+1\right)}{3}-\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(-\frac{2}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right) + \ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(\frac{2}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)$$

```
[In] int((6*x + 4*x^2 + x^3)/(4*x + 3*x^2 + 2*x^3 + x^4 + 2),x)
[Out] 1/(x + 1) - log(x + 1)/3 - log(x - 2^(1/2)*1i)*((2^(1/2)*2i)/3 - 2/3) + log
(x + 2^(1/2)*1i)*((2^(1/2)*2i)/3 + 2/3)
```

3.111 $\int \frac{x}{(1+x)(1+2x)^2(1+x^2)} dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)} dx = \frac{2}{5(1+2x)} + \frac{\arctan(x)}{50} - \frac{1}{2}\log(1+x) + \frac{16}{25}\log(1+2x) - \frac{7}{100}\log(1+x^2)$$

```
[Out] 2/5/(1+2*x)+1/50*arctan(x)-1/2*ln(1+x)+16/25*ln(1+2*x)-7/100*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6857, 649, 209, 266}

$$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)} dx = \frac{\arctan(x)}{50} - \frac{7}{100}\log(x^2+1) + \frac{2}{5(2x+1)} - \frac{1}{2}\log(x+1) + \frac{16}{25}\log(2x+1)$$

```
[In] Int[x/((1 + x)*(1 + 2*x)^2*(1 + x^2)),x]
```

```
[Out] 2/(5*(1 + 2*x)) + ArcTan[x]/50 - Log[1 + x]/2 + (16*Log[1 + 2*x])/25 - (7*L
og[1 + x^2])/100
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2(1+x)}-\frac{4}{5(1+2x)^2}+\frac{32}{25(1+2x)}+\frac{1-7x}{50\left(1+x^2\right)}\right) dx \\
&= \frac{2}{5(1+2x)}-\frac{1}{2}\log(1+x)+\frac{16}{25}\log(1+2x)+\frac{1}{50}\int \frac{1-7x}{1+x^2}\,dx \\
&= \frac{2}{5(1+2x)}-\frac{1}{2}\log(1+x)+\frac{16}{25}\log(1+2x)+\frac{1}{50}\int \frac{1}{1+x^2}\,dx-\frac{7}{50}\int \frac{x}{1+x^2}\,dx \\
&= \frac{2}{5(1+2x)}+\frac{\arctan(x)}{50}-\frac{1}{2}\log(1+x)+\frac{16}{25}\log(1+2x)-\frac{7}{100}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{x}{(1+x)(1+2x)^2\left(1+x^2\right)}\,dx = \frac{1}{100}\left(\frac{40}{1+2x}+2\arctan(x)-50\log(1+x)+64\log(1+2x)-7\log\left(1+x^2\right)\right)$$

```
[In] Integrate[x/((1 + x)*(1 + 2*x)^2*(1 + x^2)),x]
[Out] (40/(1 + 2*x) + 2*ArcTan[x] - 50*Log[1 + x] + 64*Log[1 + 2*x] - 7*Log[1 + x
^2])/100
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result
risch	$\frac{1}{5x+\frac{5}{2}}-\frac{\ln(1+x)}{2}-\frac{7\ln(x^2+1)}{100}+\frac{\arctan(x)}{50}+\frac{16\ln(1+2x)}{25}$
default	$\frac{2}{5(1+2x)}+\frac{\arctan(x)}{50}-\frac{\ln(1+x)}{2}+\frac{16\ln(1+2x)}{25}-\frac{7\ln(x^2+1)}{100}$
parallelrisch	$-\frac{2i\ln(x-i)x-2i\ln(x+i)x+i\ln(x-i)-i\ln(x+i)+100\ln(1+x)x-128\ln\left(x+\frac{1}{2}\right)x+14\ln(x-i)x+14\ln(x+i)x-40+50\ln(1+x)-}{100(1+2x)}$

```
[In] int(x/(1+x)/(1+2*x)^2/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/(x+1/2)-1/2*ln(1+x)-7/100*ln(x^2+1)+1/50*arctan(x)+16/25*ln(1+2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)}\,dx = \frac{2\,(2\,x+1)\arctan(x)-7\,(2\,x+1)\log(x^2+1)+64\,(2\,x+1)\log(2\,x+1)-50\,(2\,x+1)\log(x+1)+40}{100\,(2\,x+1)}$$

```
[In] integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/100*(2*(2*x + 1)*arctan(x) - 7*(2*x + 1)*log(x^2 + 1) + 64*(2*x + 1)*log(
2*x + 1) - 50*(2*x + 1)*log(x + 1) + 40)/(2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)}\,dx = \frac{16\log\left(x+\frac{1}{2}\right)}{25}-\frac{\log(x+1)}{2}-\frac{7\log(x^2+1)}{100}+\frac{\operatorname{atan}(x)}{50}+\frac{2}{10x+5}$$

```
[In] integrate(x/(1+x)/(1+2*x)**2/(x**2+1),x)
```

```
[Out] 16*log(x + 1/2)/25 - log(x + 1)/2 - 7*log(x**2 + 1)/100 + atan(x)/50 + 2/(1
0*x + 5)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{x}{(1+x)(1+2x)^2\,(1+x^2)}\,dx = \frac{2}{5\,(2\,x+1)} + \frac{1}{50}\,\arctan\left(x\right) - \frac{7}{100}\,\log\left(x^2+1\right) + \frac{16}{25}\,\log\left(2\,x+1\right) - \frac{1}{2}\,\log\left(x+1\right)$$

```
[In] integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] 2/5/(2*x + 1) + 1/50*arctan(x) - 7/100*log(x^2 + 1) + 16/25*log(2*x + 1) -
1/2*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{x}{(1+x)(1+2x)^2\,(1+x^2)}\,dx = \frac{2}{5\,(2\,x+1)} + \frac{1}{50}\,\arctan\left(-\frac{5}{2\,(2\,x+1)} + \frac{1}{2}\right) - \frac{7}{100}\,\log\left(-\frac{2}{2\,x+1} + \frac{5}{(2\,x+1)^2} + 1\right) - \frac{1}{2}\,\log\left(\left|-\frac{1}{2\,x+1} - 1\right|\right)$$

```
[In] integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/5/(2*x + 1) + 1/50*arctan(-5/2/(2*x + 1) + 1/2) - 7/100*log(-2/(2*x + 1)
+ 5/(2*x + 1)^2 + 1) - 1/2*log(abs(-1/(2*x + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1+x)(1+2x)^2\,(1+x^2)}\,dx = \frac{16\,\ln\left(x+\frac{1}{2}\right)}{25} - \frac{\ln\left(x+1\right)}{2} + \frac{1}{5\,\left(x+\frac{1}{2}\right)} + \ln\left(x-\mathrm{i}\right)\,\left(-\frac{7}{100}-\frac{1}{100}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\,\left(-\frac{7}{100}+\frac{1}{100}\mathrm{i}\right)$$

```
[In] int(x/((2*x + 1)^2*(x^2 + 1)*(x + 1)),x)
```

```
[Out] (16*log(x + 1/2))/25 - log(x + 1)/2 - log(x - 1i)*(7/100 + 1i/100) - log(x
+ 1i)*(7/100 - 1i/100) + 1/(5*(x + 1/2))
```

3.112 $\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)} dx$

Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)} dx = -\frac{1}{2(1-x)^2}+\frac{5}{2(1-x)}-\arctan(x)-\frac{3}{2}\log(1-x)+\frac{3}{4}\log\left(1+x^2\right)$$

```
[Out] -1/2/(1-x)^2+5/2/(1-x)-arctan(x)-3/2*ln(1-x)+3/4*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1643, 649, 209, 266}

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)} dx = -\arctan(x)+\frac{3}{4}\log\left(x^2+1\right)+\frac{5}{2(1-x)}-\frac{1}{2(1-x)^2}-\frac{3}{2}\log(1-x)$$

```
[In] Int[(-2 + x + 3*x^2)/((-1 + x)^3*(1 + x^2)),x]
```

```
[Out] -1/2*1/(1 - x)^2 + 5/(2*(1 - x)) - ArcTan[x] - (3*Log[1 - x])/2 + (3*Log[1
+ x^2])/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{(-1+x)^3}+\frac{5}{2(-1+x)^2}-\frac{3}{2(-1+x)}+\frac{-2+3x}{2\left(1+x^2\right)}\right)dx\\
&= -\frac{1}{2(1-x)^2}+\frac{5}{2(1-x)}-\frac{3}{2}\log(1-x)+\frac{1}{2}\int\frac{-2+3x}{1+x^2}\,dx\\
&= -\frac{1}{2(1-x)^2}+\frac{5}{2(1-x)}-\frac{3}{2}\log(1-x)+\frac{3}{2}\int\frac{x}{1+x^2}\,dx-\int\frac{1}{1+x^2}\,dx\\
&= -\frac{1}{2(1-x)^2}+\frac{5}{2(1-x)}-\arctan(x)-\frac{3}{2}\log(1-x)+\frac{3}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int\frac{-2+x+3x^2}{(-1+x)^3\left(1+x^2\right)}\,dx=\frac{1}{4}\left(-\frac{2}{(-1+x)^2}-\frac{10}{-1+x}-4\arctan(x)-6\log(-1+x)+3\log\left(1+x^2\right)\right)$$

```
[In] Integrate[(-2 + x + 3*x^2)/((-1 + x)^3*(1 + x^2)),x]
[Out] (-2/(-1 + x)^2 - 10/(-1 + x) - 4*ArcTan[x] - 6*Log[-1 + x] + 3*Log[1 + x^2]
)/4
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

method	result
risch	$\frac{-\frac{5x}{2}+2}{(-1+x)^2}+\frac{3\ln(x^2+1)}{4}-\arctan(x)-\frac{3\ln(-1+x)}{2}$
default	$-\frac{1}{2(-1+x)^2}-\frac{5}{2(-1+x)}-\frac{3\ln(-1+x)}{2}+\frac{3\ln(x^2+1)}{4}-\arctan(x)$
parallelrisch	$-\frac{-2i\ln(x-i)x^2+2i\ln(x+i)+6\ln(-1+x)x^2+2i\ln(x+i)x^2-3x^2\ln(x-i)+4i\ln(x-i)x-3\ln(x+i)x^2-3-12\ln(-1+x)x-2i\ln(}{4(-1+x)^2}$

```
[In] int((3*x^2+x-2)/(-1+x)^3/(x^2+1),x,method=_RETURNVERBOSE)
[Out] (-5/2*x+2)/(-1+x)^2+3/4*ln(x^2+1)-arctan(x)-3/2*ln(-1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = -\frac{4\,(x^2-2\,x+1)\arctan(x)-3\,(x^2-2\,x+1)\log(x^2+1)+6\,(x^2-2\,x+1)\log(x-1)+10\,x-8}{4\,(x^2-2\,x+1)}$$

```
[In] integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="fricas")
[Out] -1/4*(4*(x^2 - 2*x + 1)*arctan(x) - 3*(x^2 - 2*x + 1)*log(x^2 + 1) + 6*(x^2
 - 2*x + 1)*log(x - 1) + 10*x - 8)/(x^2 - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = \frac{4-5x}{2x^2-4x+2}-\frac{3\log(x-1)}{2}+\frac{3\log(x^2+1)}{4}-\operatorname{atan}(x)$$

```
[In] integrate((3*x**2+x-2)/(-1+x)**3/(x**2+1),x)
[Out] (4 - 5*x)/(2*x**2 - 4*x + 2) - 3*log(x - 1)/2 + 3*log(x**2 + 1)/4 - atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = -\frac{5\,x-4}{2\,(x^2-2\,x+1)} - \arctan(x) + \frac{3}{4}\,\log\left(x^2+1\right) - \frac{3}{2}\,\log(x-1)$$

```
[In] integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="maxima")
[Out] -1/2*(5*x - 4)/(x^2 - 2*x + 1) - arctan(x) + 3/4*log(x^2 + 1) - 3/2*log(x -
 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = -\frac{5\,x-4}{2\,(x-1)^2} - \arctan(x) + \frac{3}{4}\,\log\left(x^2+1\right) - \frac{3}{2}\,\log(|x-1|)$$

```
[In] integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="giac")
[Out] -1/2*(5*x - 4)/(x - 1)^2 - arctan(x) + 3/4*log(x^2 + 1) - 3/2*log(abs(x - 1
))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = -\frac{3\,\ln(x-1)}{2} - \frac{\frac{5\,x}{2}-2}{x^2-2\,x+1} + \ln(x-\mathrm{i})\,\left(\frac{3}{4}+\frac{1}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\,\left(\frac{3}{4}-\frac{1}{2}\mathrm{i}\right)$$

```
[In] int((x + 3*x^2 - 2)/((x^2 + 1)*(x - 1)^3),x)
[Out] log(x - 1i)*(3/4 + 1i/2) - (3*log(x - 1))/2 + log(x + 1i)*(3/4 - 1i/2) - ((
5*x)/2 - 2)/(x^2 - 2*x + 1)
```

3.113 $\int \frac{1}{1+x^2+x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 67

$$\int \frac{1}{1+x^2+x^4}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}\log\left(1-x+x^2\right) + \frac{1}{4}\log\left(1+x+x^2\right)$$

```
[Out] -1/4*ln(x^2-x+1)+1/4*ln(x^2+x+1)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/
6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1108, 648, 632, 210, 642}

$$\int \frac{1}{1+x^2+x^4}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}\log\left(x^2-x+1\right) + \frac{1}{4}\log\left(x^2+x+1\right)$$

```
[In] Int[(1 + x^2 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) - Log[1 - x + x^2]/4 + Log[1 + x + x^2]/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1-x}{1-x+x^2}\,dx + \frac{1}{2}\int \frac{1+x}{1+x+x^2}\,dx \\
&= \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{4}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+x+x^2}\,dx + \frac{1}{4}\int \frac{1+2x}{1+x+x^2}\,dx \\
&= -\frac{1}{4}\log\left(1-x+x^2\right) + \frac{1}{4}\log\left(1+x+x^2\right) \\
&\quad -\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) \\
&= -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}\log\left(1-x+x^2\right) + \frac{1}{4}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{i\left(\sqrt{1-i\sqrt{3}}\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)-\sqrt{1+i\sqrt{3}}\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)\right)}{\sqrt{6}}$$

```
[In] Integrate[(1 + x^2 + x^4)^(-1),x]
[Out] (I*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x)/2] - Sqrt[1 + I*Sqrt[3]]*
ArcTan[((I + Sqrt[3])*x)/2]))/Sqrt[6]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\ln\left(x^2-x+1\right)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln\left(x^2+x+1\right)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	54
risch	$\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln\left(4x^2+4x+4\right)}{4}-\frac{\ln\left(4x^2-4x+4\right)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	60

```
[In] int(1/(x^4+x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/4*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/4*ln(x^2+x+1)+1/
6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{1}{1+x^2+x^4}\,dx = -\frac{\log(x^2-x+1)}{4} + \frac{\log(x^2+x+1)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

```
[In] integrate(1/(x**4+x**2+1),x)
[Out] -log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 - s
qrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}\log\left(x^2+x+1\right) - \frac{1}{4}\log\left(x^2-x+1\right)$$

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}\log\left(x^2+x+1\right) - \frac{1}{4}\log\left(x^2-x+1\right)$$

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{1}{1+x^2+x^4}\,dx = \operatorname{atanh}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\operatorname{atanh}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

```
[In] int(1/(x^2 + x^4 + 1),x)
[Out] atanh((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) + atanh((2*x)/(3^(1/2)
*1i + 1))*((3^(1/2)*1i)/6 + 1/2)
```

3.114 $\int \frac{3+2x^3}{-9x+x^5} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{3+2x^3}{-9x+x^5} \, dx = \frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{3} + \frac{1}{12}\log\left(9-x^4\right)$$

```
[Out] -1/3*ln(x)+1/12*ln(-x^4+9)+1/3*arctan(1/3*x*3^(1/2))*3^(1/2)-1/3*arctanh(1/
3*x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1607, 1845, 272, 36, 31, 29, 304, 209, 212}

$$\int \frac{3+2x^3}{-9x+x^5} \, dx = \frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{12}\log\left(9-x^4\right) - \frac{\log(x)}{3}$$

```
[In] Int[(3 + 2*x^3)/(-9*x + x^5),x]
```

```
[Out] ArcTan[x/Sqrt[3]]/Sqrt[3] - ArcTanh[x/Sqrt[3]]/Sqrt[3] - Log[x]/3 + Log[9 -
 x^4]/12
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{3+2x^3}{x\,(-9+x^4)}\,dx \\
&= \int \left(\frac{3}{x\,(-9+x^4)}+\frac{2x^2}{-9+x^4}\right)dx \\
&= 2\int \frac{x^2}{-9+x^4}\,dx+3\int \frac{1}{x\,(-9+x^4)}\,dx \\
&= \frac{3}{4}\text{Subst}\left(\int \frac{1}{(-9+x)x}\,dx,x,x^4\right)-\int \frac{1}{3-x^2}\,dx+\int \frac{1}{3+x^2}\,dx \\
&= \frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{12}\text{Subst}\left(\int \frac{1}{-9+x}\,dx,x,x^4\right)-\frac{1}{12}\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right) \\
&= \frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{3}+\frac{1}{12}\log\left(9-x^4\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.40

$$
\int \frac{3+2x^3}{-9x+x^5}\,dx=\frac{1}{12}\left(4\sqrt{3}\arctan\left(\frac{x}{\sqrt{3}}\right)-4\log(x)+2\sqrt{3}\log\left(3-\sqrt{3}x\right)-2\sqrt{3}\log\left(3+\sqrt{3}x\right)+\log\left(9-x^4\right)\right)
$$

```
[In] Integrate[(3 + 2*x^3)/(-9*x + x^5),x]
[Out] (4*Sqrt[3]*ArcTan[x/Sqrt[3]] - 4*Log[x] + 2*Sqrt[3]*Log[3 - Sqrt[3]*x] - 2*
Sqrt[3]*Log[3 + Sqrt[3]*x] + Log[9 - x^4])/12
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\ln(x^2+3)}{12}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\ln(x)}{3}+\frac{\ln(x^2-3)}{12}-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}$	46
risch	$\frac{\ln(x^2+3)}{12}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln\left(x-\sqrt{3}\right)}{12}+\frac{\sqrt{3}\,\ln\left(x-\sqrt{3}\right)}{6}+\frac{\ln\left(x+\sqrt{3}\right)}{12}-\frac{\sqrt{3}\,\ln\left(x+\sqrt{3}\right)}{6}-\frac{\ln(x)}{3}$	68
meijerg	$-\frac{\ln(x)}{3}+\frac{\ln(3)}{6}-\frac{i\pi}{12}+\frac{\ln\left(1-\frac{x^4}{9}\right)}{12}+\frac{x^3\sqrt{3}\left(\ln\left(1-\frac{\sqrt{3}\,(x^4)^{\frac{1}{4}}}{3}\right)-\ln\left(1+\frac{\sqrt{3}\,(x^4)^{\frac{1}{4}}}{3}\right)+2\arctan\left(\frac{\sqrt{3}\,(x^4)^{\frac{1}{4}}}{3}\right)\right)}{6(x^4)^{\frac{3}{4}}}$	79

```
[In] int((2*x^3+3)/(x^5-9*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*ln(x^2+3)+1/3*arctan(1/3*x*3^(1/2))*3^(1/2)-1/3*ln(x)+1/12*ln(x^2-3)-1
/3*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{3+2x^3}{-9x+x^5}\,dx=\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{x^2-2\sqrt{3}x+3}{x^2-3}\right)+\frac{1}{12}\log\left(x^2+3\right)+\frac{1}{12}\log\left(x^2-3\right)-\frac{1}{3}\log\left(x\right)$$

```
[In] integrate((2*x^3+3)/(x^5-9*x),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*log((x^2 - 2*sqrt(3)*x + 3)
/(x^2 - 3)) + 1/12*log(x^2 + 3) + 1/12*log(x^2 - 3) - 1/3*log(x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 306, normalized size of antiderivative = 6.38

$$\int \frac{3+2x^3}{-9x+x^5}\,dx = -\frac{\log(x)}{3} + \left(\frac{1}{12} + \frac{\sqrt{3}i}{6}\right)\log\left(x + \frac{17413}{11544} - \frac{943\sqrt{3}i}{5772} + \frac{1368\left(\frac{1}{12}+\frac{\sqrt{3}i}{6}\right)^3}{481} + \frac{4158\left(\frac{1}{12}+\frac{\sqrt{3}i}{6}\right)^2}{481} - \frac{108000\left(\frac{1}{12}+\frac{\sqrt{3}i}{6}\right)^4}{481}\right) + \left(\frac{1}{12} - \frac{\sqrt{3}i}{6}\right)\log\left(x + \frac{17413}{11544} - \frac{108000\left(\frac{1}{12}-\frac{\sqrt{3}i}{6}\right)^4}{481} + \frac{4158\left(\frac{1}{12}-\frac{\sqrt{3}i}{6}\right)^2}{481} + \frac{1368\left(\frac{1}{12}-\frac{\sqrt{3}i}{6}\right)^3}{481} + \frac{943\sqrt{3}i}{5772}\right) + \left(\frac{1}{12} - \frac{\sqrt{3}}{6}\right)\log\left(x - \frac{108000\left(\frac{1}{12}-\frac{\sqrt{3}}{6}\right)^4}{481} + \frac{1368\left(\frac{1}{12}-\frac{\sqrt{3}}{6}\right)^3}{481} + \frac{943\sqrt{3}}{5772} + \frac{4158\left(\frac{1}{12}-\frac{\sqrt{3}}{6}\right)^2}{481} + \frac{17413}{11544}\right) + \left(\frac{1}{12} + \frac{\sqrt{3}}{6}\right)\log\left(x - \frac{108000\left(\frac{1}{12}+\frac{\sqrt{3}}{6}\right)^4}{481} - \frac{943\sqrt{3}}{5772} + \frac{1368\left(\frac{1}{12}+\frac{\sqrt{3}}{6}\right)^3}{481} + \frac{4158\left(\frac{1}{12}+\frac{\sqrt{3}}{6}\right)^2}{481} + \frac{17413}{11544}\right)$$

```
[In] integrate((2*x**3+3)/(x**5-9*x),x)
```

```
[Out] -log(x)/3 + (1/12 + sqrt(3)*I/6)*log(x + 17413/11544 - 943*sqrt(3)*I/5772 +
 1368*(1/12 + sqrt(3)*I/6)**3/481 + 4158*(1/12 + sqrt(3)*I/6)**2/481 - 1080
00*(1/12 + sqrt(3)*I/6)**4/481) + (1/12 - sqrt(3)*I/6)*log(x + 17413/11544
- 108000*(1/12 - sqrt(3)*I/6)**4/481 + 4158*(1/12 - sqrt(3)*I/6)**2/481 + 1
368*(1/12 - sqrt(3)*I/6)**3/481 + 943*sqrt(3)*I/5772) + (1/12 - sqrt(3)/6)*
log(x - 108000*(1/12 - sqrt(3)/6)**4/481 + 1368*(1/12 - sqrt(3)/6)**3/481 +
 943*sqrt(3)/5772 + 4158*(1/12 - sqrt(3)/6)**2/481 + 17413/11544) + (1/12 +
 sqrt(3)/6)*log(x - 108000*(1/12 + sqrt(3)/6)**4/481 - 943*sqrt(3)/5772 + 1
368*(1/12 + sqrt(3)/6)**3/481 + 4158*(1/12 + sqrt(3)/6)**2/481 + 17413/1154
4)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{3+2x^3}{-9x+x^5}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right) + \frac{1}{6}\,\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right) + \frac{1}{12}\,\log\left(x^2+3\right) + \frac{1}{12}\,\log\left(x^2-3\right) - \frac{1}{3}\,\log\left(x\right)$$

```
[In] integrate((2*x^3+3)/(x^5-9*x),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*log((x - sqrt(3))/(x + sqrt
(3))) + 1/12*log(x^2 + 3) + 1/12*log(x^2 - 3) - 1/3*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.33

$$\int \frac{3+2x^3}{-9x+x^5}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right) + \frac{1}{6}\,\sqrt{3}\log\left(\frac{|2\,x-2\,\sqrt{3}|}{|2\,x+2\,\sqrt{3}|}\right) + \frac{1}{12}\,\log\left(x^2+3\right) + \frac{1}{12}\,\log\left(|x^2-3|\right) - \frac{1}{3}\,\log\left(|x|\right)$$

```
[In] integrate((2*x^3+3)/(x^5-9*x),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3))/ab
s(2*x + 2*sqrt(3))) + 1/12*log(x^2 + 3) + 1/12*log(abs(x^2 - 3)) - 1/3*log(
abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{3+2x^3}{-9x+x^5}\,dx = \ln\left(x-\sqrt{3}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{12}\right) - \ln\left(x+\sqrt{3}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{12}\right) - \frac{\ln\left(x\right)}{3} - \ln\left(x-\sqrt{3}\,1\mathrm{i}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \ln\left(x+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

```
[In] int(-(2*x^3 + 3)/(9*x - x^5),x)
[Out] log(x - 3^(1/2))*(3^(1/2)/6 + 1/12) - log(x + 3^(1/2))*(3^(1/2)/6 - 1/12) -
 log(x)/3 - log(x - 3^(1/2)*1i)*((3^(1/2)*1i)/6 - 1/12) + log(x + 3^(1/2)*1
i)*((3^(1/2)*1i)/6 + 1/12)
```

3.115 $\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 58

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx = -\frac{83}{4(4-x)^2}+\frac{41}{4(4-x)}-\frac{3}{16}\arctan\left(1-\frac{x}{2}\right) -\frac{45}{16}\log(4-x)+\frac{45}{32}\log\left(8-4x+x^2\right)$$

```
[Out] -83/4/(4-x)^2+41/4/(4-x)+3/16*arctan(-1+1/2*x)-45/16*ln(4-x)+45/32*ln(x^2-4
*x+8)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1642, 648, 631, 210, 642}

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx = -\frac{3}{16}\arctan\left(1-\frac{x}{2}\right)+\frac{45}{32}\log\left(x^2-4x+8\right) +\frac{41}{4(4-x)}-\frac{83}{4(4-x)^2}-\frac{45}{16}\log(4-x)$$

```
[In] Int[(-20 + 8*x + 5*x^3)/((-4 + x)^3*(8 - 4*x + x^2)),x]
```

```
[Out] -83/(4*(4 - x)^2) + 41/(4*(4 - x)) - (3*ArcTan[1 - x/2])/16 - (45*Log[4 - x
])/16 + (45*Log[8 - 4*x + x^2])/32
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{83}{2(-4+x)^3}+\frac{41}{4(-4+x)^2}-\frac{45}{16(-4+x)}+\frac{3(-28+15x)}{16\left(8-4x+x^2\right)}\right) dx \\
&= -\frac{83}{4(4-x)^2}+\frac{41}{4(4-x)}-\frac{45}{16}\log(4-x)+\frac{3}{16}\int \frac{-28+15x}{8-4x+x^2}\,dx \\
&= -\frac{83}{4(4-x)^2}+\frac{41}{4(4-x)}-\frac{45}{16}\log(4-x)+\frac{3}{8}\int \frac{1}{8-4x+x^2}\,dx+\frac{45}{32}\int \frac{-4+2x}{8-4x+x^2}\,dx \\
&= -\frac{83}{4(4-x)^2}+\frac{41}{4(4-x)}-\frac{45}{16}\log(4-x)+\frac{45}{32}\log\left(8-4x+x^2\right) \\
&\quad +\frac{3}{16}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{x}{2}\right) \\
&= -\frac{83}{4(4-x)^2}+\frac{41}{4(4-x)}-\frac{3}{16}\arctan\left(1-\frac{x}{2}\right)-\frac{45}{16}\log(4-x)+\frac{45}{32}\log\left(8-4x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx = \frac{1}{32}\left(-\frac{664}{(-4+x)^2}-\frac{328}{-4+x}+6\arctan\left(\frac{1}{2}(-2+x)\right)\right.$$
$$\left.-90\log(-4+x)+45\log\left(8-4x+x^2\right)\right)$$

```
[In] Integrate[(-20 + 8*x + 5*x^3)/((-4 + x)^3*(8 - 4*x + x^2)),x]
```

```
[Out] (-664/(-4 + x)^2 - 328/(-4 + x) + 6*ArcTan[(-2 + x)/2] - 90*Log[-4 + x] + 4
5*Log[8 - 4*x + x^2])/32
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

method	result
risch	$\frac{-\frac{41x}{4}+\frac{81}{4}}{(x-4)^2}-\frac{45\ln(x-4)}{16}+\frac{45\ln(x^2-4x+8)}{32}+\frac{3\arctan\left(-1+\frac{x}{2}\right)}{16}$
default	$\frac{45\ln(x^2-4x+8)}{32}+\frac{3\arctan\left(-1+\frac{x}{2}\right)}{16}-\frac{83}{4(x-4)^2}-\frac{41}{4(x-4)}-\frac{45\ln(x-4)}{16}$
parallelrisch	$-\frac{-96i\ln(x-2+2i)-48i\ln(x-2-2i)x+96i\ln(x-2-2i)+48i\ln(x-2+2i)x+180\ln(x-4)x^2-90\ln(x-2-2i)x^2-90\ln(x-2+2i)x}{}$

```
[In] int((5*x^3+8*x-20)/(x-4)^3/(x^2-4*x+8),x,method=_RETURNVERBOSE)
```

```
[Out] (-41/4*x+81/4)/(x-4)^2-45/16*ln(x-4)+45/32*ln(x^2-4*x+8)+3/16*arctan(-1+1/2
*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx$$
$$=\frac{6\left(x^2-8x+16\right)\arctan\left(\frac{1}{2}x-1\right)+45\left(x^2-8x+16\right)\log\left(x^2-4x+8\right)-90\left(x^2-8x+16\right)\log\left(x-4\right)-}{32\left(x^2-8x+16\right)}$$

```
[In] integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="fricas")
```

```
[Out] 1/32*(6*(x^2 - 8*x + 16)*arctan(1/2*x - 1) + 45*(x^2 - 8*x + 16)*log(x^2 -
4*x + 8) - 90*(x^2 - 8*x + 16)*log(x - 4) - 328*x + 648)/(x^2 - 8*x + 16)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx = \frac{81-41x}{4x^2-32x+64} - \frac{45\log(x-4)}{16} + \frac{45\log(x^2-4x+8)}{32} + \frac{3\operatorname{atan}\left(\frac{x}{2}-1\right)}{16}$$

```
[In] integrate((5*x**3+8*x-20)/(-4+x)**3/(x**2-4*x+8),x)
[Out] (81 - 41*x)/(4*x**2 - 32*x + 64) - 45*log(x - 4)/16 + 45*log(x**2 - 4*x + 8
)/32 + 3*atan(x/2 - 1)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx = -\frac{41\,x-81}{4\,(x^2-8\,x+16)} + \frac{3}{16}\arctan\left(\frac{1}{2}\,x-1\right) + \frac{45}{32}\log\left(x^2-4\,x+8\right) - \frac{45}{16}\log\left(x-4\right)$$

```
[In] integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="maxima")
[Out] -1/4*(41*x - 81)/(x^2 - 8*x + 16) + 3/16*arctan(1/2*x - 1) + 45/32*log(x^2
- 4*x + 8) - 45/16*log(x - 4)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx = -\frac{41\,x-81}{4\,(x-4)^2} + \frac{3}{16}\arctan\left(\frac{1}{2}\,x-1\right) + \frac{45}{32}\log\left(x^2-4\,x+8\right) - \frac{45}{16}\log\left(|x-4|\right)$$

```
[In] integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="giac")
[Out] -1/4*(41*x - 81)/(x - 4)^2 + 3/16*arctan(1/2*x - 1) + 45/32*log(x^2 - 4*x +
 8) - 45/16*log(abs(x - 4))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{-20+8x+5x^3}{(-4+x)^3\,(8-4x+x^2)}\,dx = -\frac{45\,\ln(x-4)}{16} - \frac{\frac{41\,x}{4}-\frac{81}{4}}{x^2-8\,x+16} + \ln(x-2-2\mathrm{i})\,\left(\frac{45}{32}-\frac{3}{32}\mathrm{i}\right) + \ln(x-2+2\mathrm{i})\,\left(\frac{45}{32}+\frac{3}{32}\mathrm{i}\right)$$

```
[In] int((8*x + 5*x^3 - 20)/((x - 4)^3*(x^2 - 4*x + 8)),x)
```

```
[Out] log(x - (2 + 2i))*(45/32 - 3i/32) - (45*log(x - 4))/16 + log(x - (2 - 2i))*
(45/32 + 3i/32) - ((41*x)/4 - 81/4)/(x^2 - 8*x + 16)
```

3.116 $\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} dx$

Optimal result

Integrand size = 29, antiderivative size = 51

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} dx = -\frac{1}{12}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{6} - \frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/12*arctan(1/2*x)+1/6*arctan(x)-1/4*arctan(1/2*x*2^(1/2))*2^(1/2)+1/6*arc
tan(1/3*x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {6857, 209}

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} dx = -\frac{1}{12}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{6} - \frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[In] Int[1/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

```
[Out] -1/12*ArcTan[x/2] + ArcTan[x]/6 - ArcTan[x/Sqrt[2]]/(2*Sqrt[2]) + ArcTan[x/
Sqrt[3]]/(2*Sqrt[3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{6(1+x^2)} - \frac{1}{2(2+x^2)} + \frac{1}{2(3+x^2)} - \frac{1}{6(4+x^2)}\right) dx \\ &= \frac{1}{6}\int \frac{1}{1+x^2}\,dx - \frac{1}{6}\int \frac{1}{4+x^2}\,dx - \frac{1}{2}\int \frac{1}{2+x^2}\,dx + \frac{1}{2}\int \frac{1}{3+x^2}\,dx \\ &= -\frac{1}{12}\arctan\left(\frac{x}{2}\right) + \frac{\arctan(x)}{6} - \frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}} + \frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{12}\left(-\arctan\left(\frac{x}{2}\right) + 2\arctan(x) - 3\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) + 2\sqrt{3}\arctan\left(\frac{x}{\sqrt{3}}\right)\right)$$

```
[In] Integrate[1/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
[Out] (-ArcTan[x/2] + 2*ArcTan[x] - 3*Sqrt[2]*ArcTan[x/Sqrt[2]] + 2*Sqrt[3]*ArcTa
n[x/Sqrt[3]])/12
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\arctan\left(\frac{x}{2}\right)}{12}+\frac{\arctan(x)}{6}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{6}$	36
risch	$-\frac{\arctan\left(\frac{x}{2}\right)}{12}+\frac{\arctan(x)}{6}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{6}$	36

```
[In] int(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x,method=_RETURNVERBOSE)
[Out] -1/12*arctan(1/2*x)+1/6*arctan(x)-1/4*arctan(1/2*x*2^(1/2))*2^(1/2)+1/6*arc
tan(1/3*x*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{12}\arctan\left(\frac{1}{2}x\right) + \frac{1}{6}\arctan(x)$$

```
[In] integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/1
2*arctan(1/2*x) + 1/6*arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = -\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{12} + \frac{\operatorname{atan}(x)}{6} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

```
[In] integrate(1/(x**2+1)/(x**2+2)/(x**2+3)/(x**2+4),x)
[Out] -atan(x/2)/12 + atan(x)/6 - sqrt(2)*atan(sqrt(2)*x/2)/4 + sqrt(3)*atan(sqrt
(3)*x/3)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{12}\arctan\left(\frac{1}{2}x\right) + \frac{1}{6}\arctan(x)$$

```
[In] integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/1
2*arctan(1/2*x) + 1/6*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{12}\arctan\left(\frac{1}{2}x\right) + \frac{1}{6}\arctan(x)$$

```
[In] integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/1
2*arctan(1/2*x) + 1/6*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{\mathrm{atan}(x)}{6} - \frac{\mathrm{atan}\left(\frac{x}{2}\right)}{12} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}\right)}{6}$$

```
[In] int(1/((x^2 + 1)*(x^2 + 2)*(x^2 + 3)*(x^2 + 4)),x)
```

```
[Out] atan(x)/6 - atan(x/2)/12 - (2^(1/2)*atan((2^(1/2)*x)/2))/4 + (3^(1/2)*atan(
(3^(1/2)*x)/3))/6
```

3.117 $\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} dx$

Optimal result

Integrand size = 30, antiderivative size = 41

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} dx = \frac{1}{12}\log(1+x^2) - \frac{1}{4}\log(2+x^2) + \frac{1}{4}\log(3+x^2) - \frac{1}{12}\log(4+x^2)$$

```
[Out] 1/12*ln(x^2+1)-1/4*ln(x^2+2)+1/4*ln(x^2+3)-1/12*ln(x^2+4)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {6826, 186}

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} dx = \frac{1}{12}\log(x^2+1) - \frac{1}{4}\log(x^2+2) + \frac{1}{4}\log(x^2+3) - \frac{1}{12}\log(x^2+4)$$

```
[In] Int[x/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/12 - Log[2 + x^2]/4 + Log[3 + x^2]/4 - Log[4 + x^2]/12
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 6826

```
Int[(u_)*((c_.) + (d_.)*(v_))^(n_.)*((e_.) + (f_.)*(w_))^(p_.)*((a_.) + (b_
.)*(y_))^(m_.)*((g_.) + (h_.)*(z_))^(q_.), x_Symbol] :> With[{r = Derivativ
eDivides[y, u, x]}, Dist[r, Subst[Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(
g + h*x)^q, x], x, y], x] /;  !FalseQ[r]] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n, p, q}, x] && EqQ[v, y] && EqQ[w, y] && EqQ[z, y]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)(2+x)(3+x)(4+x)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{6(1+x)} - \frac{1}{2(2+x)} + \frac{1}{2(3+x)} - \frac{1}{6(4+x)}\right)dx, x, x^2\right)\\ &= \frac{1}{12}\log\left(1+x^2\right) - \frac{1}{4}\log\left(2+x^2\right) + \frac{1}{4}\log\left(3+x^2\right) - \frac{1}{12}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{12}\log\left(1+x^2\right) - \frac{1}{4}\log\left(2+x^2\right) + \frac{1}{4}\log\left(3+x^2\right) - \frac{1}{12}\log\left(4+x^2\right)$$

```
[In] Integrate[x/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
[Out] Log[1 + x^2]/12 - Log[2 + x^2]/4 + Log[3 + x^2]/4 - Log[4 + x^2]/12
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\ln(x^2+1)}{12} - \frac{\ln(x^2+2)}{4} + \frac{\ln(x^2+3)}{4} - \frac{\ln(x^2+4)}{12}$	34
norman	$\frac{\ln(x^2+1)}{12} - \frac{\ln(x^2+2)}{4} + \frac{\ln(x^2+3)}{4} - \frac{\ln(x^2+4)}{12}$	34
risch	$\frac{\ln(x^2+1)}{12} - \frac{\ln(x^2+2)}{4} + \frac{\ln(x^2+3)}{4} - \frac{\ln(x^2+4)}{12}$	34
parallelrisch	$\frac{\ln(x^2+1)}{12} - \frac{\ln(x^2+2)}{4} + \frac{\ln(x^2+3)}{4} - \frac{\ln(x^2+4)}{12}$	34

```
[In] int(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*ln(x^2+1)-1/4*ln(x^2+2)+1/4*ln(x^2+3)-1/12*ln(x^2+4)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = -\frac{1}{12}\log\left(x^2+4\right)+\frac{1}{4}\log\left(x^2+3\right) - \frac{1}{4}\log\left(x^2+2\right)+\frac{1}{12}\log\left(x^2+1\right)$$

```
[In] integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="fricas")
[Out] -1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*log(x^2 + 1
)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{\log\left(x^2+1\right)}{12}-\frac{\log\left(x^2+2\right)}{4} + \frac{\log\left(x^2+3\right)}{4}-\frac{\log\left(x^2+4\right)}{12}$$

```
[In] integrate(x/(x**2+1)/(x**2+2)/(x**2+3)/(x**2+4),x)
[Out] log(x**2 + 1)/12 - log(x**2 + 2)/4 + log(x**2 + 3)/4 - log(x**2 + 4)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = -\frac{1}{12}\log\left(x^2+4\right)+\frac{1}{4}\log\left(x^2+3\right) - \frac{1}{4}\log\left(x^2+2\right)+\frac{1}{12}\log\left(x^2+1\right)$$

```
[In] integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="maxima")
[Out] -1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*log(x^2 + 1
)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = -\frac{1}{12}\log\left(x^2+4\right)+\frac{1}{4}\log\left(x^2+3\right) -\frac{1}{4}\log\left(x^2+2\right)+\frac{1}{12}\log\left(x^2+1\right)$$

```
[In] integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="giac")
[Out] -1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*log(x^2 + 1
)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{\operatorname{atanh}\left(\frac{3072}{5\,(1280\,x^2+3072)}-\frac{1}{5}\right)}{2} - \frac{\operatorname{atanh}\left(\frac{1024}{405\left(\frac{640\,x^2}{243}+\frac{1024}{243}\right)}-\frac{3}{5}\right)}{6}$$

```
[In] int(x/((x^2 + 1)*(x^2 + 2)*(x^2 + 3)*(x^2 + 4)),x)
[Out] atanh(3072/(5*(1280*x^2 + 3072)) - 1/5)/2 - atanh(1024/(405*((640*x^2)/243
+ 1024/243)) - 3/5)/6
```

3.118 $\int \frac{1}{a^3+x^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 56

$$\int \frac{1}{a^3+x^3}\,dx=-\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^2}+\frac{\log(a+x)}{3a^2}-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}$$

```
[Out] 1/3*ln(a+x)/a^2-1/6*ln(a^2-a*x+x^2)/a^2-1/3*arctan(1/3*(a-2*x)/a*3^(1/2))/a
^2*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {206, 31, 648, 631, 210, 642}

$$\int \frac{1}{a^3+x^3}\,dx=-\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^2}-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}+\frac{\log(a+x)}{3a^2}$$

```
[In] Int[(a^3 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^2)) + Log[a + x]/(3*a^2) - Log[a
^2 - a*x + x^2]/(6*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{a+x}\,dx}{3a^2}+\frac{\int \frac{2a-x}{a^2-ax+x^2}\,dx}{3a^2}\\
&= \frac{\log(a+x)}{3a^2}-\frac{\int \frac{-a+2x}{a^2-ax+x^2}\,dx}{6a^2}+\frac{\int \frac{1}{a^2-ax+x^2}\,dx}{2a}\\
&= \frac{\log(a+x)}{3a^2}-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{a}\right)}{a^2}\\
&= -\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^2}+\frac{\log(a+x)}{3a^2}-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{1}{a^3+x^3}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right)+2\log(a+x)-\log\left(a^2-ax+x^2\right)}{6a^2}$$

```
[In] Integrate[(a^3 + x^3)^(-1),x]
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] + 2*Log[a + x] - Log[a^2 - a*x +
x^2])/(6*a^2)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{-\frac{\ln\left(a^2-ax+x^2\right)}{2}+\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3a^2}+\frac{\ln(a+x)}{3a^2}$	51
risch	$-\frac{\ln\left(4a^2-4ax+4x^2\right)}{6a^2}+\frac{\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3a^2}+\frac{\ln(a+x)}{3a^2}$	56

```
[In] int(1/(a^3+x^3),x,method=_RETURNVERBOSE)
[Out] 1/3/a^2*(-1/2*ln(a^2-a*x+x^2)+3^(1/2)*arctan(1/3*(-a+2*x)*3^(1/2)/a))+1/3*l
n(a+x)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{1}{a^3+x^3}\,dx=\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log\left(a^2-ax+x^2\right)+2\log\left(a+x\right)}{6a^2}$$

```
[In] integrate(1/(a^3+x^3),x, algorithm="fricas")
[Out] 1/6*(2*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a^2 - a*x + x^2) + 2*
log(a + x))/a^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int \frac{1}{a^3+x^3}\,dx = \frac{\frac{\log(a+x)}{3} + \left(-\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(3a\left(-\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)+x\right) + \left(-\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(3a\left(-\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)+x\right)}{a^2}$$

```
[In] integrate(1/(a**3+x**3),x)
```

```
[Out] (log(a + x)/3 + (-1/6 - sqrt(3)*I/6)*log(3*a*(-1/6 - sqrt(3)*I/6) + x) + (-
1/6 + sqrt(3)*I/6)*log(3*a*(-1/6 + sqrt(3)*I/6) + x))/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{1}{a^3+x^3}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^2} - \frac{\log\left(a^2-ax+x^2\right)}{6a^2} + \frac{\log(a+x)}{3a^2}$$

```
[In] integrate(1/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^2 - 1/6*log(a^2 - a*x + x^2)
/a^2 + 1/3*log(a + x)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{1}{a^3+x^3}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^2} - \frac{\log\left(a^2-ax+x^2\right)}{6a^2} + \frac{\log\left(|a+x|\right)}{3a^2}$$

```
[In] integrate(1/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^2 - 1/6*log(a^2 - a*x + x^2)
/a^2 + 1/3*log(abs(a + x))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14

$$\int \frac{1}{a^3+x^3}\,dx = \frac{\ln(a+x)}{3\,a^2} + \frac{\ln\left(x+\frac{a\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^2} - \frac{\ln\left(x-\frac{a\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^2}$$

```
[In] int(1/(a^3 + x^3),x)
[Out] log(a + x)/(3*a^2) + (log(x + (a*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*
a^2) - (log(x - (a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^2)
```

3.119 $\int \frac{x}{a^3+x^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 56

$$\int \frac{x}{a^3+x^3} dx = -\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a} - \frac{\log(a+x)}{3a} + \frac{\log(a^2-ax+x^2)}{6a}$$

```
[Out] -1/3*ln(a+x)/a+1/6*ln(a^2-a*x+x^2)/a-1/3*arctan(1/3*(a-2*x)/a*3^(1/2))/a*3^
(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {298, 31, 648, 631, 210, 642}

$$\int \frac{x}{a^3+x^3} dx = \frac{\log(a^2-ax+x^2)}{6a} - \frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a} - \frac{\log(a+x)}{3a}$$

```
[In] Int[x/(a^3 + x^3),x]
```

```
[Out] -(ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a)) - Log[a + x]/(3*a) + Log[a^2 -
 a*x + x^2]/(6*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{1}{a+x}\,dx}{3a}+\frac{\int \frac{a+x}{a^2-ax+x^2}\,dx}{3a}\\
&= -\frac{\log(a+x)}{3a}+\frac{1}{2}\int \frac{1}{a^2-ax+x^2}\,dx+\frac{\int \frac{-a+2x}{a^2-ax+x^2}\,dx}{6a}\\
&= -\frac{\log(a+x)}{3a}+\frac{\log\left(a^2-ax+x^2\right)}{6a}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{a}\right)}{a}\\
&= -\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a}-\frac{\log(a+x)}{3a}+\frac{\log\left(a^2-ax+x^2\right)}{6a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{x}{a^3+x^3}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right)-2\log(a+x)+\log\left(a^2-ax+x^2\right)}{6a}$$

```
[In] Integrate[x/(a^3 + x^3),x]
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x + 
x^2])/(6*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{\ln(a+x)}{3a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^2a^2-a\_Z+1\right)}\_R\ln\left(a^2\_R-a+x\right)\right)}{3}$	43
default	$\frac{\frac{\ln\left(a^2-ax+x^2\right)}{2}+\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3a}-\frac{\ln(a+x)}{3a}$	51

```
[In] int(x/(a^3+x^3),x,method=_RETURNVERBOSE)
[Out] -1/3*ln(a+x)/a+1/3*sum(_R*ln(_R*a^2-a+x),_R=RootOf(_Z^2*a^2-_Z*a+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{x}{a^3+x^3}\,dx=\frac{2\,\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)+\log\left(a^2-ax+x^2\right)-2\,\log\left(a+x\right)}{6\,a}$$

```
[In] integrate(x/(a^3+x^3),x, algorithm="fricas")
[Out] 1/6*(2*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + log(a^2 - a*x + x^2) - 2*
log(a + x))/a
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27

$$\int \frac{x}{a^3+x^3}\,dx = \frac{-\frac{\log(a+x)}{3}+\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)^2+x\right)+\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)^2+x\right)}{a}$$

```
[In] integrate(x/(a**3+x**3),x)
```

```
[Out] (-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(9*a*(1/6 - sqrt(3)*I/6)**2 + x) +
(1/6 + sqrt(3)*I/6)*log(9*a*(1/6 + sqrt(3)*I/6)**2 + x))/a
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{x}{a^3+x^3}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a}+\frac{\log(a^2-ax+x^2)}{6a}-\frac{\log(a+x)}{3a}$$

```
[In] integrate(x/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a + 1/6*log(a^2 - a*x + x^2)/a
 - 1/3*log(a + x)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{x}{a^3+x^3}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a}+\frac{\log(a^2-ax+x^2)}{6a}-\frac{\log(|a+x|)}{3a}$$

```
[In] integrate(x/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a + 1/6*log(a^2 - a*x + x^2)/a
 - 1/3*log(abs(a + x))/a
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{x}{a^3+x^3}\,dx = -\frac{\ln(a+x)}{3\,a} - \frac{\ln\left(x+\frac{a\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a} + \frac{\ln\left(x+\frac{a\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a}$$

```
[In] int(x/(a^3 + x^3),x)
```

```
[Out] (log(x + (a*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*a) - (log(x + (a*(3
^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(6*a) - log(a + x)/(3*a)
```

3.120 $\int \frac{x^2}{a^3+x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{x^2}{a^3+x^3} \, dx = \frac{1}{3} \log \left(a^3+x^3\right)$$

```
[Out] 1/3*ln(a^3+x^3)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {266}

$$\int \frac{x^2}{a^3+x^3} \, dx = \frac{1}{3} \log \left(a^3+x^3\right)$$

```
[In] Int[x^2/(a^3 + x^3),x]
```

```
[Out] Log[a^3 + x^3]/3
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\text{integral} = \frac{1}{3} \log \left(a^3+x^3\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a^3+x^3}\,dx=\frac{1}{3}\log\left(a^3+x^3\right)$$

```
[In] Integrate[x^2/(a^3 + x^3),x]
[Out] Log[a^3 + x^3]/3
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\ln\left(a^3+x^3\right)}{3}$	11
default	$\frac{\ln\left(a^3+x^3\right)}{3}$	11
risch	$\frac{\ln\left(a^3+x^3\right)}{3}$	11
norman	$\frac{\ln(a+x)}{3}+\frac{\ln\left(a^2-ax+x^2\right)}{3}$	22
parallelrisch	$\frac{\ln(a+x)}{3}+\frac{\ln\left(a^2-ax+x^2\right)}{3}$	22

```
[In] int(x^2/(a^3+x^3),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(a^3+x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a^3+x^3}\,dx=\frac{1}{3}\,\log\left(a^3+x^3\right)$$

```
[In] integrate(x^2/(a^3+x^3),x, algorithm="fricas")
[Out] 1/3*log(a^3 + x^3)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{a^3+x^3}\,dx = \frac{\log(a^3+x^3)}{3}$$

```
[In] integrate(x**2/(a**3+x**3),x)
[Out] log(a**3 + x**3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a^3+x^3}\,dx = \frac{1}{3}\log\left(a^3+x^3\right)$$

```
[In] integrate(x^2/(a^3+x^3),x, algorithm="maxima")
[Out] 1/3*log(a^3 + x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{a^3+x^3}\,dx = \frac{1}{3}\log\left(\left|a^3+x^3\right|\right)$$

```
[In] integrate(x^2/(a^3+x^3),x, algorithm="giac")
[Out] 1/3*log(abs(a^3 + x^3))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a^3+x^3}\,dx = \frac{\ln(a^3+x^3)}{3}$$

```
[In] int(x^2/(a^3 + x^3),x)
[Out] log(a^3 + x^3)/3
```

3.121 $\int \frac{1}{x(a^3+x^3)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{x(a^3+x^3)}\,dx = \frac{\log(x)}{a^3} - \frac{\log(a^3+x^3)}{3a^3}$$

```
[Out] ln(x)/a^3-1/3*ln(a^3+x^3)/a^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {272, 36, 29, 31}

$$\int \frac{1}{x(a^3+x^3)}\,dx = \frac{\log(x)}{a^3} - \frac{\log(a^3+x^3)}{3a^3}$$

```
[In] Int[1/(x*(a^3 + x^3)),x]
```

```
[Out] Log[x]/a^3 - Log[a^3 + x^3]/(3*a^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\,(a^3+x)}\,dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^3\right)}{3a^3} - \frac{\text{Subst}\left(\int \frac{1}{a^3+x}\,dx, x, x^3\right)}{3a^3}\\
&= \frac{\log(x)}{a^3} - \frac{\log\left(a^3+x^3\right)}{3a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(a^3+x^3)}\,dx = \frac{\log(x)}{a^3} - \frac{\log\left(a^3+x^3\right)}{3a^3}$$

```
[In] Integrate[1/(x*(a^3 + x^3)),x]
[Out] Log[x]/a^3 - Log[a^3 + x^3]/(3*a^3)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{\ln(x)}{a^3} - \frac{\ln(a^3+x^3)}{3a^3}$	21
parallelrisch	$\frac{3\ln(x)-\ln(a+x)-\ln(a^2-ax+x^2)}{3a^3}$	31
default	$\frac{\ln(x)}{a^3} - \frac{\ln(a^2-ax+x^2)}{3a^3} - \frac{\ln(a+x)}{3a^3}$	34
norman	$\frac{\ln(x)}{a^3} - \frac{\ln(a^2-ax+x^2)}{3a^3} - \frac{\ln(a+x)}{3a^3}$	34

```
[In] int(1/x/(a^3+x^3),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/a^3-1/3*ln(a^3+x^3)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x(a^3+x^3)}\,dx = -\frac{\log(a^3+x^3) - 3\log(x)}{3a^3}$$

```
[In] integrate(1/x/(a^3+x^3),x, algorithm="fricas")
[Out] -1/3*(log(a^3 + x^3) - 3*log(x))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{x(a^3+x^3)}\,dx = \frac{\log(x)}{a^3} - \frac{\log(a^3+x^3)}{3a^3}$$

```
[In] integrate(1/x/(a**3+x**3),x)
[Out] log(x)/a**3 - log(a**3 + x**3)/(3*a**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(a^3+x^3)}\,dx = -\frac{\log(a^3+x^3)}{3a^3} + \frac{\log(x^3)}{3a^3}$$

```
[In] integrate(1/x/(a^3+x^3),x, algorithm="maxima")
[Out] -1/3*log(a^3 + x^3)/a^3 + 1/3*log(x^3)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a^3+x^3)}\,dx = -\frac{\log(|a^3+x^3|)}{3a^3} + \frac{\log(|x|)}{a^3}$$

```
[In] integrate(1/x/(a^3+x^3),x, algorithm="giac")
[Out] -1/3*log(abs(a^3 + x^3))/a^3 + log(abs(x))/a^3
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x(a^3+x^3)}\,dx = -\frac{\ln(a^3+x^3) - 3\ln(x)}{3a^3}$$

```
[In] int(1/(x*(a^3 + x^3)),x)
[Out] -(log(a^3 + x^3) - 3*log(x))/(3*a^3)
```

3.122 $\int \frac{1}{x^2(a^3+x^3)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{1}{x^2(a^3+x^3)}\,dx = -\frac{1}{a^3x}+\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^4}+\frac{\log(a+x)}{3a^4}-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}$$

```
[Out] -1/a^3/x+1/3*ln(a+x)/a^4-1/6*ln(a^2-a*x+x^2)/a^4+1/3*arctan(1/3*(a-2*x)/a*3
^(1/2))/a^4*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {331, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^2(a^3+x^3)}\,dx = \frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^4}+\frac{\log(a+x)}{3a^4}-\frac{1}{a^3x}-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}$$

```
[In] Int[1/(x^2*(a^3 + x^3)),x]
```

```
[Out] -(1/(a^3*x)) + ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^4) + Log[a + x]/(3*
a^4) - Log[a^2 - a*x + x^2]/(6*a^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{a^3x} - \frac{\int \frac{x}{a^3+x^3}\,dx}{a^3}\\ &= -\frac{1}{a^3x} + \frac{\int \frac{1}{a+x}\,dx}{3a^4} - \frac{\int \frac{a+x}{a^2-ax+x^2}\,dx}{3a^4}\end{aligned}$$

$$= -\frac{1}{a^3x} + \frac{\log(a+x)}{3a^4} - \frac{\int \frac{-a+2x}{a^2-ax+x^2}\,dx}{6a^4} - \frac{\int \frac{1}{a^2-ax+x^2}\,dx}{2a^3}$$

$$= -\frac{1}{a^3x} + \frac{\log(a+x)}{3a^4} - \frac{\log(a^2-ax+x^2)}{6a^4} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{a}\right)}{a^4}$$

$$= -\frac{1}{a^3x} + \frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^4} + \frac{\log(a+x)}{3a^4} - \frac{\log(a^2-ax+x^2)}{6a^4}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2(a^3+x^3)}\,dx = -\frac{6a+2\sqrt{3}x\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right) - 2x\log(a+x) + x\log(a^2-ax+x^2)}{6a^4x}$$

```
[In] Integrate[1/(x^2*(a^3 + x^3)),x]
```

```
[Out] -1/6*(6*a + 2*Sqrt[3]*x*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*x*Log[a + x] + x
*Log[a^2 - a*x + x^2])/(a^4*x)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{1}{a^3x} + \frac{-\frac{\ln(a^2-ax+x^2)}{2} - \sqrt{3}\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3a^4} + \frac{\ln(a+x)}{3a^4}$	60
risch	$-\frac{1}{a^3x} - \frac{\ln(4a^2-4ax+4x^2)}{6a^4} + \frac{\arctan\left(\frac{(a-2x)\sqrt{3}}{3a}\right)\sqrt{3}}{3a^4} + \frac{\ln(-a-x)}{3a^4}$	66

```
[In] int(1/x^2/(a^3+x^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3/x+1/3/a^4*(-1/2*ln(a^2-a*x+x^2)-3^(1/2)*arctan(1/3*(-a+2*x)*3^(1/2)/
a))+1/3*ln(a+x)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2(a^3+x^3)}\,dx = -\frac{2\sqrt{3}x\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)+x\log\left(a^2-ax+x^2\right)-2x\log\left(a+x\right)+6a}{6a^4x}$$

```
[In] integrate(1/x^2/(a^3+x^3),x, algorithm="fricas")
[Out] -1/6*(2*sqrt(3)*x*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + x*log(a^2 - a*x + x^2)
 - 2*x*log(a + x) + 6*a)/(a^4*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^2(a^3+x^3)}\,dx = -\frac{1}{a^3x} + \frac{\frac{\log(a+x)}{3}+\left(-\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(-\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)^2+x\right)+\left(-\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(-\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)^2+x\right)}{a^4}$$

```
[In] integrate(1/x**2/(a**3+x**3),x)
[Out] -1/(a**3*x) + (log(a + x)/3 + (-1/6 - sqrt(3)*I/6)*log(9*a*(-1/6 - sqrt(3)*
I/6)**2 + x) + (-1/6 + sqrt(3)*I/6)*log(9*a*(-1/6 + sqrt(3)*I/6)**2 + x))/a
**4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2(a^3+x^3)}\,dx = -\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^4} - \frac{\log\left(a^2-ax+x^2\right)}{6a^4} + \frac{\log\left(a+x\right)}{3a^4} - \frac{1}{a^3x}$$

```
[In] integrate(1/x^2/(a^3+x^3),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^4 - 1/6*log(a^2 - a*x + x^2
)/a^4 + 1/3*log(a + x)/a^4 - 1/(a^3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\,(a^3+x^3)}\,dx = -\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)}{3\,a^4} - \frac{\log\left(a^2-ax+x^2\right)}{6\,a^4} + \frac{\log\left(|a+x|\right)}{3\,a^4} - \frac{1}{a^3x}$$

```
[In] integrate(1/x^2/(a^3+x^3),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^4 - 1/6*log(a^2 - a*x + x^2
)/a^4 + 1/3*log(abs(a + x))/a^4 - 1/(a^3*x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^2\,(a^3+x^3)}\,dx = \frac{\ln(a+x)}{3\,a^4} - \frac{1}{a^3\,x} + \frac{\ln\left(\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,a^4}{4}+x\,a^3\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^4} - \frac{\ln\left(\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,a^4}{4}+x\,a^3\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^4}$$

```
[In] int(1/(x^2*(a^3 + x^3)),x)
[Out] log(a + x)/(3*a^4) - 1/(a^3*x) + (log(a^3*x + (a^4*(3^(1/2)*1i - 1)^2)/4)*(
3^(1/2)*1i - 1))/(6*a^4) - (log(a^3*x + (a^4*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2
)*1i + 1))/(6*a^4)
```

3.123 $\int \frac{1}{x^3(a^3+x^3)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{1}{x^3(a^3+x^3)}\,dx=-\frac{1}{2a^3x^2}+\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5}-\frac{\log(a+x)}{3a^5}+\frac{\log(a^2-ax+x^2)}{6a^5}$$

```
[Out] -1/2/a^3/x^2-1/3*ln(a+x)/a^5+1/6*ln(a^2-a*x+x^2)/a^5+1/3*arctan(1/3*(a-2*x)
/a*3^(1/2))/a^5*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {331, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^3(a^3+x^3)}\,dx=\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5}-\frac{\log(a+x)}{3a^5}-\frac{1}{2a^3x^2}+\frac{\log(a^2-ax+x^2)}{6a^5}$$

```
[In] Int[1/(x^3*(a^3 + x^3)),x]
```

```
[Out] -1/2*1/(a^3*x^2) + ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^5) - Log[a + x]
/(3*a^5) + Log[a^2 - a*x + x^2]/(6*a^5)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2a^3x^2} - \frac{\int \frac{1}{a^3+x^3}\,dx}{a^3}\\ &= -\frac{1}{2a^3x^2} - \frac{\int \frac{1}{a+x}\,dx}{3a^5} - \frac{\int \frac{2a-x}{a^2-ax+x^2}\,dx}{3a^5}\end{aligned}$$

$$= -\frac{1}{2a^3x^2} - \frac{\log(a+x)}{3a^5} + \frac{\int \frac{-a+2x}{a^2-ax+x^2}\,dx}{6a^5} - \frac{\int \frac{1}{a^2-ax+x^2}\,dx}{2a^4}$$

$$= -\frac{1}{2a^3x^2} - \frac{\log(a+x)}{3a^5} + \frac{\log\left(a^2-ax+x^2\right)}{6a^5} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{a}\right)}{a^5}$$

$$= -\frac{1}{2a^3x^2} + \frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5} - \frac{\log(a+x)}{3a^5} + \frac{\log\left(a^2-ax+x^2\right)}{6a^5}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^3\left(a^3+x^3\right)}\,dx = -\frac{1}{2a^3x^2} - \frac{\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5} - \frac{\log(a+x)}{3a^5} + \frac{\log\left(a^2-ax+x^2\right)}{6a^5}$$

```
[In] Integrate[1/(x^3*(a^3 + x^3)),x]
```

```
[Out] -1/2*1/(a^3*x^2) - ArcTan[(-a + 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^5) - Log[a + x
]/(3*a^5) + Log[a^2 - a*x + x^2]/(6*a^5)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\frac{\ln\left(a^2-ax+x^2\right)}{2}-\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3a^5}-\frac{1}{2a^3x^2}-\frac{\ln(a+x)}{3a^5}$	60
risch	$-\frac{1}{2a^3x^2}-\frac{\ln(a+x)}{3a^5}+\frac{\left(\sum_{\_R=\text{RootOf}\left(a^{10}\_Z^2-a^5\_Z+1\right)}\_R\ln\left(\left(-4\_R^3a^{15}-3\right)x-a^6\_R\right)\right)}{3}$	62

```
[In] int(1/x^3/(a^3+x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^5*(1/2*ln(a^2-a*x+x^2)-3^(1/2)*arctan(1/3*(-a+2*x)*3^(1/2)/a))-1/2/a^
3/x^2-1/3*ln(a+x)/a^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3 (a^3 + x^3)} dx = -\frac{2\sqrt{3}x^2 \arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right) - x^2 \log\left(a^2 - ax + x^2\right) + 2x^2 \log\left(a + x\right) + 3a^2}{6a^5x^2}$$

```
[In] integrate(1/x^3/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x^2*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - x^2*log(a^2 - a*x +
x^2) + 2*x^2*log(a + x) + 3*a^2)/(a^5*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3 (a^3 + x^3)} dx = -\frac{1}{2a^3x^2} + \frac{-\frac{\log(a+x)}{3} + \left(\frac{1}{6} - \frac{\sqrt{3}i}{6}\right) \log\left(-3a\left(\frac{1}{6} - \frac{\sqrt{3}i}{6}\right) + x\right) + \left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) \log\left(-3a\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) + x\right)}{a^5}$$

```
[In] integrate(1/x**3/(a**3+x**3),x)
```

```
[Out] -1/(2*a**3*x**2) + (-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(-3*a*(1/6 - sqr
t(3)*I/6) + x) + (1/6 + sqrt(3)*I/6)*log(-3*a*(1/6 + sqrt(3)*I/6) + x))/a**
5
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3 (a^3 + x^3)} dx = -\frac{\sqrt{3} \arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^5} + \frac{\log\left(a^2 - ax + x^2\right)}{6a^5} - \frac{\log\left(a + x\right)}{3a^5} - \frac{1}{2a^3x^2}$$

```
[In] integrate(1/x^3/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^5 + 1/6*log(a^2 - a*x + x^2
)/a^5 - 1/3*log(a + x)/a^5 - 1/2/(a^3*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\left(a^3+x^3\right)}\,dx=-\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)}{3\,a^5}+\frac{\log\left(a^2-ax+x^2\right)}{6\,a^5}-\frac{\log\left(|a+x|\right)}{3\,a^5}-\frac{1}{2\,a^3x^2}$$

```
[In] integrate(1/x^3/(a^3+x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^5 + 1/6*log(a^2 - a*x + x^2
)/a^5 - 1/3*log(abs(a + x))/a^5 - 1/2/(a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^3\left(a^3+x^3\right)}\,dx=-\frac{\ln\left(a+x\right)}{3\,a^5}-\frac{1}{2\,a^3\,x^2}-\frac{\ln\left(\frac{3\,a^7\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\,a^6\,x\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^5}+\frac{\ln\left(\frac{3\,a^7\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}-3\,a^6\,x\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^5}$$

```
[In] int(1/(x^3*(a^3 + x^3)),x)
```

```
[Out] (log((3*a^7*(3^(1/2)*1i + 1))/2 - 3*a^6*x)*(3^(1/2)*1i + 1))/(6*a^5) - 1/(2
*a^3*x^2) - (log((3*a^7*(3^(1/2)*1i - 1))/2 + 3*a^6*x)*(3^(1/2)*1i - 1))/(6
*a^5) - log(a + x)/(3*a^5)
```

3.124 $\int \frac{1}{x^4(a^3+x^3)} dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{1}{x^4(a^3+x^3)} dx = -\frac{1}{3a^3x^3} - \frac{\log(x)}{a^6} + \frac{\log(a^3+x^3)}{3a^6}$$

```
[Out] -1/3/a^3/x^3-ln(x)/a^6+1/3*ln(a^3+x^3)/a^6
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 46}

$$\int \frac{1}{x^4(a^3+x^3)} dx = -\frac{\log(x)}{a^6} - \frac{1}{3a^3x^3} + \frac{\log(a^3+x^3)}{3a^6}$$

```
[In] Int[1/(x^4*(a^3 + x^3)),x]
```

```
[Out] -1/3*1/(a^3*x^3) - Log[x]/a^6 + Log[a^3 + x^3]/(3*a^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\,(a^3+x)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{a^3x^2}-\frac{1}{a^6x}+\frac{1}{a^6\,(a^3+x)}\right)\,dx, x, x^3\right)\\ &= -\frac{1}{3a^3x^3}-\frac{\log(x)}{a^6}+\frac{\log\left(a^3+x^3\right)}{3a^6}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\,(a^3+x^3)}\,dx = -\frac{1}{3a^3x^3}-\frac{\log(x)}{a^6}+\frac{\log\left(a^3+x^3\right)}{3a^6}$$

```
[In] Integrate[1/(x^4*(a^3 + x^3)),x]
[Out] -1/3*1/(a^3*x^3) - Log[x]/a^6 + Log[a^3 + x^3]/(3*a^6)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{1}{3a^3x^3}-\frac{\ln(x)}{a^6}+\frac{\ln\left(-a^3-x^3\right)}{3a^6}$	34
default	$-\frac{1}{3a^3x^3}-\frac{\ln(x)}{a^6}+\frac{\ln\left(a^2-ax+x^2\right)}{3a^6}+\frac{\ln(a+x)}{3a^6}$	43
norman	$-\frac{1}{3a^3x^3}-\frac{\ln(x)}{a^6}+\frac{\ln\left(a^2-ax+x^2\right)}{3a^6}+\frac{\ln(a+x)}{3a^6}$	43
parallelrisch	$-\frac{3x^3\ln(x)-\ln(a+x)x^3-\ln\left(a^2-ax+x^2\right)x^3+a^3}{3a^6x^3}$	46

```
[In] int(1/x^4/(a^3+x^3),x,method=_RETURNVERBOSE)
[Out] -1/3/a^3/x^3-ln(x)/a^6+1/3/a^6*ln(-a^3-x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (a^3 + x^3)} dx = \frac{x^3 \log (a^3 + x^3) - 3 x^3 \log (x) - a^3}{3 a^6 x^3}$$

```
[In] integrate(1/x^4/(a^3+x^3),x, algorithm="fricas")
[Out] 1/3*(x^3*log(a^3 + x^3) - 3*x^3*log(x) - a^3)/(a^6*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4 (a^3 + x^3)} dx = -\frac{1}{3a^3x^3} - \frac{\log (x)}{a^6} + \frac{\log (a^3 + x^3)}{3a^6}$$

```
[In] integrate(1/x**4/(a**3+x**3),x)
[Out] -1/(3*a**3*x**3) - log(x)/a**6 + log(a**3 + x**3)/(3*a**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4 (a^3 + x^3)} dx = \frac{\log (a^3 + x^3)}{3 a^6} - \frac{\log (x^3)}{3 a^6} - \frac{1}{3 a^3 x^3}$$

```
[In] integrate(1/x^4/(a^3+x^3),x, algorithm="maxima")
[Out] 1/3*log(a^3 + x^3)/a^6 - 1/3*log(x^3)/a^6 - 1/3/(a^3*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^4 (a^3 + x^3)} dx = \frac{\log (|a^3 + x^3|)}{3 a^6} - \frac{\log (|x|)}{a^6} - \frac{a^3 - x^3}{3 a^6 x^3}$$

```
[In] integrate(1/x^4/(a^3+x^3),x, algorithm="giac")
[Out] 1/3*log(abs(a^3 + x^3))/a^6 - log(abs(x))/a^6 - 1/3*(a^3 - x^3)/(a^6*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4 (a^3 + x^3)} \, dx = \frac{\ln (a^3 + x^3)}{3 a^6} - \frac{\ln (x)}{a^6} - \frac{1}{3 a^3 x^3}$$

```
[In] int(1/(x^4*(a^3 + x^3)),x)
[Out] log(a^3 + x^3)/(3*a^6) - log(x)/a^6 - 1/(3*a^3*x^3)
```

3.125 $\int \frac{1}{x^5(a^3+x^3)} dx$

Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \frac{1}{x^5(a^3+x^3)} dx = -\frac{1}{4a^3x^4} + \frac{1}{a^6x} - \frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7} - \frac{\log(a+x)}{3a^7} + \frac{\log(a^2-ax+x^2)}{6a^7}$$

```
[Out] -1/4/a^3/x^4+1/a^6/x-1/3*ln(a+x)/a^7+1/6*ln(a^2-a*x+x^2)/a^7-1/3*arctan(1/3
*(a-2*x)/a*3^(1/2))/a^7*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {331, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^5(a^3+x^3)} dx = -\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7} - \frac{\log(a+x)}{3a^7} + \frac{1}{a^6x} - \frac{1}{4a^3x^4} + \frac{\log(a^2-ax+x^2)}{6a^7}$$

```
[In] Int[1/(x^5*(a^3 + x^3)),x]
```

```
[Out] -1/4*1/(a^3*x^4) + 1/(a^6*x) - ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^7)
- Log[a + x]/(3*a^7) + Log[a^2 - a*x + x^2]/(6*a^7)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{4a^3x^4} - \frac{\int \frac{1}{x^2(a^3+x^3)}\,dx}{a^3} \\
&= -\frac{1}{4a^3x^4} + \frac{1}{a^6x} + \frac{\int \frac{x}{a^3+x^3}\,dx}{a^6}
\end{aligned}$$

$$= -\frac{1}{4a^3x^4} + \frac{1}{a^6x} - \frac{\int \frac{1}{a+x} dx}{3a^7} + \frac{\int \frac{a+x}{a^2-ax+x^2} dx}{3a^7}$$

$$= -\frac{1}{4a^3x^4} + \frac{1}{a^6x} - \frac{\log(a+x)}{3a^7} + \frac{\int \frac{-a+2x}{a^2-ax+x^2} dx}{6a^7} + \frac{\int \frac{1}{a^2-ax+x^2} dx}{2a^6}$$

$$= -\frac{1}{4a^3x^4} + \frac{1}{a^6x} - \frac{\log(a+x)}{3a^7} + \frac{\log(a^2-ax+x^2)}{6a^7} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1-\frac{2x}{a}\right)}{a^7}$$

$$= -\frac{1}{4a^3x^4} + \frac{1}{a^6x} - \frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7} - \frac{\log(a+x)}{3a^7} + \frac{\log(a^2-ax+x^2)}{6a^7}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^5(a^3+x^3)} dx = -\frac{1}{4a^3x^4} + \frac{1}{a^6x} + \frac{\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7} - \frac{\log(a+x)}{3a^7} + \frac{\log(a^2-ax+x^2)}{6a^7}$$

```
[In] Integrate[1/(x^5*(a^3 + x^3)),x]
[Out] -1/4*1/(a^3*x^4) + 1/(a^6*x) + ArcTan[(-a + 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^7)
 - Log[a + x]/(3*a^7) + Log[a^2 - a*x + x^2]/(6*a^7)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{1}{4a^3x^4} + \frac{1}{a^6x} + \frac{\frac{\ln(a^2-ax+x^2)}{2} + \sqrt{3}\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3a^7} - \frac{\ln(a+x)}{3a^7}$	66
risch	$\frac{\frac{x^3}{a^6} - \frac{1}{4a^3}}{x^4} - \frac{\ln(a+x)}{3a^7} + \frac{\left(\sum_{\_R=\text{RootOf}(a^{14}\_Z^2-a^7\_Z+1)} \_R\ln\left(\left(-4\_R^3a^{21}-3\right)x+a^{15}\_R^2\right)\right)}{3}$	72

```
[In] int(1/x^5/(a^3+x^3),x,method=_RETURNVERBOSE)
[Out] -1/4/a^3/x^4+1/a^6/x+1/3/a^7*(1/2*ln(a^2-a*x+x^2)+3^(1/2)*arctan(1/3*(-a+2*
x)*3^(1/2)/a))-1/3*ln(a+x)/a^7
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^5 (a^3+x^3)} \, dx = \frac{4 \sqrt{3} x^4 \arctan \left(-\frac{\sqrt{3}(a-2 x)}{3 a}\right) + 2 x^4 \log \left(a^2-a x+x^2\right) - 4 x^4 \log (a+x) - 3 a^4 + 12 a x^3}{12 a^7 x^4}$$

```
[In] integrate(1/x^5/(a^3+x^3),x, algorithm="fricas")
[Out] 1/12*(4*sqrt(3)*x^4*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + 2*x^4*log(a^2 - a*x
+ x^2) - 4*x^4*log(a + x) - 3*a^4 + 12*a*x^3)/(a^7*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^5 (a^3+x^3)} \, dx = \frac{-a^3+4x^3}{4a^6x^4} + \frac{-\frac{\log (a+x)}{3} + \left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right) \log \left(9a\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)^2 + x\right) + \left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right) \log \left(9a\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)^2 + x\right)}{a^7}$$

```
[In] integrate(1/x**5/(a**3+x**3),x)
[Out] (-a**3 + 4*x**3)/(4*a**6*x**4) + (-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(9
*a*(1/6 - sqrt(3)*I/6)**2 + x) + (1/6 + sqrt(3)*I/6)*log(9*a*(1/6 + sqrt(3)
*I/6)**2 + x))/a**7
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^5 (a^3+x^3)} \, dx = \frac{\sqrt{3} \arctan \left(-\frac{\sqrt{3}(a-2 x)}{3 a}\right)}{3 a^7} + \frac{\log \left(a^2-a x+x^2\right)}{6 a^7} - \frac{\log (a+x)}{3 a^7} - \frac{a^3-4 x^3}{4 a^6 x^4}$$

```
[In] integrate(1/x^5/(a^3+x^3),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^7 + 1/6*log(a^2 - a*x + x^2)
/a^7 - 1/3*log(a + x)/a^7 - 1/4*(a^3 - 4*x^3)/(a^6*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^5\left(a^3+x^3\right)}\,dx=\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)}{3\,a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6\,a^7}-\frac{\log\left(|a+x|\right)}{3\,a^7}-\frac{a^3-4\,x^3}{4\,a^6x^4}$$

```
[In] integrate(1/x^5/(a^3+x^3),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^7 + 1/6*log(a^2 - a*x + x^2)
/a^7 - 1/3*log(abs(a + x))/a^7 - 1/4*(a^3 - 4*x^3)/(a^6*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^5\left(a^3+x^3\right)}\,dx=-\frac{\frac{1}{4\,a^3}-\frac{x^3}{a^6}}{x^4}-\frac{\ln\left(a+x\right)}{3\,a^7}-\frac{\ln\left(\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2a^7}{4}+x\,a^6\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^7}+\frac{\ln\left(\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2a^7}{4}+x\,a^6\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^7}$$

```
[In] int(1/(x^5*(a^3 + x^3)),x)
[Out] (log(a^6*x + (a^7*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*a^7) - log(a
+ x)/(3*a^7) - (log(a^6*x + (a^7*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(
6*a^7) - (1/(4*a^3) - x^3/a^6)/x^4
```

3.126 $\int \frac{x^{-m}}{a^3+x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{x^{-m}}{a^3+x^3} dx = \frac{x^{1-m} \text{Hypergeometric2F1}\left(1, \frac{1-m}{3}, \frac{4-m}{3}, -\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

```
[Out] x^(1-m)*hypergeom([1, 1/3-1/3*m],[4/3-1/3*m],-x^3/a^3)/a^3/(1-m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {371}

$$\int \frac{x^{-m}}{a^3+x^3} dx = \frac{x^{1-m} \text{Hypergeometric2F1}\left(1, \frac{1-m}{3}, \frac{4-m}{3}, -\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

```
[In] Int[1/(x^m*(a^3 + x^3)),x]
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/3, (4 - m)/3, -(x^3/a^3)])/(a^3*(1
- m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{x^{1-m} \text{Hypergeometric2F1}\left(1, \frac{1-m}{3}, \frac{4-m}{3}, -\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{x^{-m}}{a^3+x^3}\,dx=-\frac{x^{1-m}\,\text{Hypergeometric2F1}\left(1,\frac{1}{3}-\frac{m}{3},\frac{4}{3}-\frac{m}{3},-\frac{x^3}{a^3}\right)}{a^3(-1+m)}$$

```
[In] Integrate[1/(x^m*(a^3 + x^3)),x]
[Out] -((x^(1 - m)*Hypergeometric2F1[1, 1/3 - m/3, 4/3 - m/3, -(x^3/a^3)])/(a^3*(
-1 + m)))
```

Maple [F]

$$\int \frac{x^{-m}}{a^3+x^3}dx$$

```
[In] int(1/(x^m)/(a^3+x^3),x)
[Out] int(1/(x^m)/(a^3+x^3),x)
```

Fricas [F]

$$\int \frac{x^{-m}}{a^3+x^3}\,dx=\int \frac{1}{(a^3+x^3)x^m}\,dx$$

```
[In] integrate(1/(x^m)/(a^3+x^3),x, algorithm="fricas")
[Out] integral(1/((a^3 + x^3)*x^m), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int \frac{x^{-m}}{a^3+x^3}\,dx=-\frac{mx^{1-m}\Phi\left(\frac{x^3e^{i\pi}}{a^3},1,\frac{1}{3}-\frac{m}{3}\right)\Gamma\left(\frac{1}{3}-\frac{m}{3}\right)}{9a^3\Gamma\left(\frac{4}{3}-\frac{m}{3}\right)}+\frac{x^{1-m}\Phi\left(\frac{x^3e^{i\pi}}{a^3},1,\frac{1}{3}-\frac{m}{3}\right)\Gamma\left(\frac{1}{3}-\frac{m}{3}\right)}{9a^3\Gamma\left(\frac{4}{3}-\frac{m}{3}\right)}$$

```
[In] integrate(1/(x**m)/(a**3+x**3),x)
[Out] -m*x**(1 - m)*lerchphi(x**3*exp_polar(I*pi)/a**3, 1, 1/3 - m/3)*gamma(1/3 -
 m/3)/(9*a**3*gamma(4/3 - m/3)) + x**(1 - m)*lerchphi(x**3*exp_polar(I*pi)/
a**3, 1, 1/3 - m/3)*gamma(1/3 - m/3)/(9*a**3*gamma(4/3 - m/3))
```

Maxima [F]

$$\int \frac{x^{-m}}{a^3 + x^3} \, dx = \int \frac{1}{(a^3 + x^3)x^m} \, dx$$

```
[In] integrate(1/(x^m)/(a^3+x^3),x, algorithm="maxima")
[Out] integrate(1/((a^3 + x^3)*x^m), x)
```

Giac [F]

$$\int \frac{x^{-m}}{a^3 + x^3} \, dx = \int \frac{1}{(a^3 + x^3)x^m} \, dx$$

```
[In] integrate(1/(x^m)/(a^3+x^3),x, algorithm="giac")
[Out] integrate(1/((a^3 + x^3)*x^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-m}}{a^3 + x^3} \, dx = \int \frac{1}{x^m \, (a^3 + x^3)} \, dx$$

```
[In] int(1/(x^m*(a^3 + x^3)),x)
[Out] int(1/(x^m*(a^3 + x^3)), x)
```

3.127 $\int \frac{1}{a^4-x^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \frac{1}{a^4-x^4}\,dx=\frac{\arctan\left(\frac{x}{a}\right)}{2a^3}+\frac{\operatorname{arctanh}\left(\frac{x}{a}\right)}{2a^3}$$

```
[Out] 1/2*arctan(x/a)/a^3+1/2*arctanh(x/a)/a^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {218, 212, 209}

$$\int \frac{1}{a^4-x^4}\,dx=\frac{\arctan\left(\frac{x}{a}\right)}{2a^3}+\frac{\operatorname{arctanh}\left(\frac{x}{a}\right)}{2a^3}$$

```
[In] Int[(a^4 - x^4)^(-1),x]
```

```
[Out] ArcTan[x/a]/(2*a^3) + ArcTanh[x/a]/(2*a^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{1}{a^2-x^2}\,dx}{2a^2} + \frac{\int \frac{1}{a^2+x^2}\,dx}{2a^2}\\ &= \frac{\arctan\left(\frac{x}{a}\right)}{2a^3} + \frac{\text{arctanh}\left(\frac{x}{a}\right)}{2a^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{1}{a^4-x^4}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{2a^3} - \frac{\log(a-x)}{4a^3} + \frac{\log(a+x)}{4a^3}$$

```
[In] Integrate[(a^4 - x^4)^(-1),x]
[Out] ArcTan[x/a]/(2*a^3) - Log[a - x]/(4*a^3) + Log[a + x]/(4*a^3)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

method	result	size
default	$-\frac{\ln(a-x)}{4a^3}+\frac{\arctan\left(\frac{x}{a}\right)}{2a^3}+\frac{\ln(a+x)}{4a^3}$	33
parallelrisch	$-\frac{i\ln(-ia+x)-i\ln(ia+x)+\ln(-a+x)-\ln(a+x)}{4a^3}$	39
risch	$\frac{\ln(a+x)}{4a^3}+\frac{\left(\sum_{\_R=\text{RootOf}\left(a^6\_Z^2+1\right)}\_R\ln\left(\_R a^4+x\right)\right)}{4}-\frac{\ln(-a+x)}{4a^3}$	47

```
[In] int(1/(a^4-x^4),x,method=_RETURNVERBOSE)
[Out] -1/4/a^3*ln(a-x)+1/2*arctan(x/a)/a^3+1/4*ln(a+x)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{a^4 - x^4}\,dx = \frac{2\arctan\left(\frac{x}{a}\right) + \log\left(a + x\right) - \log\left(-a + x\right)}{4\,a^3}$$

```
[In] integrate(1/(a^4-x^4),x, algorithm="fricas")
[Out] 1/4*(2*arctan(x/a) + log(a + x) - log(-a + x))/a^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{1}{a^4 - x^4}\,dx = -\frac{\frac{\log\left(-a+x\right)}{4} - \frac{\log\left(a+x\right)}{4} + \frac{i\log\left(-ia+x\right)}{4} - \frac{i\log\left(ia+x\right)}{4}}{a^3}$$

```
[In] integrate(1/(a**4-x**4),x)
[Out] -(log(-a + x)/4 - log(a + x)/4 + I*log(-I*a + x)/4 - I*log(I*a + x)/4)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{1}{a^4 - x^4}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{2\,a^3} + \frac{\log\left(a + x\right)}{4\,a^3} - \frac{\log\left(-a + x\right)}{4\,a^3}$$

```
[In] integrate(1/(a^4-x^4),x, algorithm="maxima")
[Out] 1/2*arctan(x/a)/a^3 + 1/4*log(a + x)/a^3 - 1/4*log(-a + x)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{1}{a^4 - x^4}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{2\,a^3} + \frac{\log\left(|a + x|\right)}{4\,a^3} - \frac{\log\left(|-a + x|\right)}{4\,a^3}$$

```
[In] integrate(1/(a^4-x^4),x, algorithm="giac")
[Out] 1/2*arctan(x/a)/a^3 + 1/4*log(abs(a + x))/a^3 - 1/4*log(abs(-a + x))/a^3
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{1}{a^4 - x^4}\,dx = \frac{\mathrm{atan}\left(\frac{x}{a}\right) + \mathrm{atanh}\left(\frac{x}{a}\right)}{2\,a^3}$$

```
[In] int(1/(a^4 - x^4),x)
[Out] (atan(x/a) + atanh(x/a))/(2*a^3)
```

3.128 $\int \frac{x}{a^4-x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x}{a^4-x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

```
[Out] 1/2*arctanh(x^2/a^2)/a^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {281, 212}

$$\int \frac{x}{a^4-x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

```
[In] Int[x/(a^4 - x^4),x]
```

```
[Out] ArcTanh[x^2/a^2]/(2*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{a^4 - x^2}\,dx, x, x^2\right)$$
$$= \frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{a^4 - x^4}\,dx = \frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

```
[In] Integrate[x/(a^4 - x^4),x]
[Out] ArcTanh[x^2/a^2]/(2*a^2)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

method	result	size
parallelrisch	$-\frac{\ln(-a+x)+\ln(a+x)-\ln(a^2+x^2)}{4a^2}$	27
default	$\frac{\ln(a^2+x^2)}{4a^2} - \frac{\ln(a^2-x^2)}{4a^2}$	30
risch	$-\frac{\ln(-a^2+x^2)}{4a^2} + \frac{\ln(a^2+x^2)}{4a^2}$	30
norman	$-\frac{\ln(a-x)}{4a^2} - \frac{\ln(a+x)}{4a^2} + \frac{\ln(a^2+x^2)}{4a^2}$	35

```
[In] int(x/(a^4-x^4),x,method=_RETURNVERBOSE)
[Out] -1/4*(ln(-a+x)+ln(a+x)-ln(a^2+x^2))/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{x}{a^4 - x^4} \, dx = \frac{\log(a^2 + x^2) - \log(-a^2 + x^2)}{4 a^2}$$

```
[In] integrate(x/(a^4-x^4),x, algorithm="fricas")
[Out] 1/4*(log(a^2 + x^2) - log(-a^2 + x^2))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{x}{a^4 - x^4} \, dx = -\frac{\frac{\log(-a^2+x^2)}{4} - \frac{\log(a^2+x^2)}{4}}{a^2}$$

```
[In] integrate(x/(a**4-x**4),x)
[Out] -(log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{x}{a^4 - x^4} \, dx = \frac{\log(a^2 + x^2)}{4 a^2} - \frac{\log(-a^2 + x^2)}{4 a^2}$$

```
[In] integrate(x/(a^4-x^4),x, algorithm="maxima")
[Out] 1/4*log(a^2 + x^2)/a^2 - 1/4*log(-a^2 + x^2)/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(13) = 26.

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

$$\int \frac{x}{a^4 - x^4} \, dx = \frac{\log(a^2 + x^2)}{4 a^2} - \frac{\log(|-a^2 + x^2|)}{4 a^2}$$

```
[In] integrate(x/(a^4-x^4),x, algorithm="giac")
[Out] 1/4*log(a^2 + x^2)/a^2 - 1/4*log(abs(-a^2 + x^2))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^4 - x^4} \, dx = \frac{\operatorname{atanh}\left(\frac{x^2}{a^2}\right)}{2\, a^2}$$

```
[In] int(x/(a^4 - x^4),x)
[Out] atanh(x^2/a^2)/(2*a^2)
```

3.129 $\int \frac{1}{x(a^4-x^4)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{1}{x(a^4-x^4)}\,dx=\frac{\log(x)}{a^4}-\frac{\log(a^4-x^4)}{4a^4}$$

```
[Out] ln(x)/a^4-1/4*ln(a^4-x^4)/a^4
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {272, 36, 31, 29}

$$\int \frac{1}{x(a^4-x^4)}\,dx=\frac{\log(x)}{a^4}-\frac{\log(a^4-x^4)}{4a^4}$$

```
[In] Int[1/(x*(a^4 - x^4)),x]
```

```
[Out] Log[x]/a^4 - Log[a^4 - x^4]/(4*a^4)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(a^4-x)\,x}\,dx, x, x^4\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{a^4-x}\,dx, x, x^4\right)}{4a^4} + \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)}{4a^4}\\ &= \frac{\log(x)}{a^4} - \frac{\log\left(a^4-x^4\right)}{4a^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(a^4-x^4)}\,dx = \frac{\log(x)}{a^4} - \frac{\log\left(-a^4+x^4\right)}{4a^4}$$

```
[In] Integrate[1/(x*(a^4 - x^4)),x]
[Out] Log[x]/a^4 - Log[-a^4 + x^4]/(4*a^4)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{\ln(x)}{a^4} - \frac{\ln(-a^4+x^4)}{4a^4}$	23
parallelrisch	$\frac{4\ln(x)-\ln(-a+x)-\ln(a+x)-\ln(a^2+x^2)}{4a^4}$	35
default	$-\frac{\ln(a-x)}{4a^4} - \frac{\ln(a^2+x^2)}{4a^4} + \frac{\ln(x)}{a^4} - \frac{\ln(a+x)}{4a^4}$	41
norman	$-\frac{\ln(a-x)}{4a^4} - \frac{\ln(a^2+x^2)}{4a^4} + \frac{\ln(x)}{a^4} - \frac{\ln(a+x)}{4a^4}$	41

```
[In] int(1/x/(a^4-x^4),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/a^4-1/4/a^4*ln(-a^4+x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\,(a^4-x^4)}\,dx = -\frac{\log\left(-a^4+x^4\right)-4\,\log\left(x\right)}{4\,a^4}$$

```
[In] integrate(1/x/(a^4-x^4),x, algorithm="fricas")
[Out] -1/4*(log(-a^4 + x^4) - 4*log(x))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\,(a^4-x^4)}\,dx = \frac{\log\left(x\right)}{a^4} - \frac{\log\left(-a^4+x^4\right)}{4a^4}$$

```
[In] integrate(1/x/(a**4-x**4),x)
[Out] log(x)/a**4 - log(-a**4 + x**4)/(4*a**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\,(a^4-x^4)}\,dx = -\frac{\log\left(-a^4+x^4\right)}{4\,a^4} + \frac{\log\left(x^4\right)}{4\,a^4}$$

```
[In] integrate(1/x/(a^4-x^4),x, algorithm="maxima")
[Out] -1/4*log(-a^4 + x^4)/a^4 + 1/4*log(x^4)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(a^4-x^4\right)}\,dx=\frac{\log\left(x^4\right)}{4\,a^4}-\frac{\log\left(\left|-a^4+x^4\right|\right)}{4\,a^4}$$

```
[In] integrate(1/x/(a^4-x^4),x, algorithm="giac")
[Out] 1/4*log(x^4)/a^4 - 1/4*log(abs(-a^4 + x^4))/a^4
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(a^4-x^4\right)}\,dx=-\frac{\ln\left(x^4-a^4\right)-4\,\ln\left(x\right)}{4\,a^4}$$

```
[In] int(1/(x*(a^4 - x^4)),x)
[Out] -(log(x^4 - a^4) - 4*log(x))/(4*a^4)
```

3.130 $\int \frac{1}{x^2(a^4-x^4)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{1}{x^2(a^4-x^4)}\,dx = -\frac{1}{a^4x}-\frac{\arctan\left(\frac{x}{a}\right)}{2a^5}+\frac{\operatorname{arctanh}\left(\frac{x}{a}\right)}{2a^5}$$

```
[Out] -1/a^4/x-1/2*arctan(x/a)/a^5+1/2*arctanh(x/a)/a^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {331, 304, 209, 212}

$$\int \frac{1}{x^2(a^4-x^4)}\,dx = -\frac{\arctan\left(\frac{x}{a}\right)}{2a^5}+\frac{\operatorname{arctanh}\left(\frac{x}{a}\right)}{2a^5}-\frac{1}{a^4x}$$

```
[In] Int[1/(x^2*(a^4 - x^4)),x]
```

```
[Out] -(1/(a^4*x)) - ArcTan[x/a]/(2*a^5) + ArcTanh[x/a]/(2*a^5)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{a^4x}+\frac{\int \frac{x^2}{a^4-x^4}\,dx}{a^4}\\
&= -\frac{1}{a^4x}+\frac{\int \frac{1}{a^2-x^2}\,dx}{2a^4}-\frac{\int \frac{1}{a^2+x^2}\,dx}{2a^4}\\
&= -\frac{1}{a^4x}-\frac{\arctan\left(\frac{x}{a}\right)}{2a^5}+\frac{\operatorname{arctanh}\left(\frac{x}{a}\right)}{2a^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^2\left(a^4-x^4\right)}\,dx=-\frac{1}{a^4x}-\frac{\arctan\left(\frac{x}{a}\right)}{2a^5}-\frac{\log(a-x)}{4a^5}+\frac{\log(a+x)}{4a^5}$$

```
[In] Integrate[1/(x^2*(a^4 - x^4)),x]
[Out] -(1/(a^4*x)) - ArcTan[x/a]/(2*a^5) - Log[a - x]/(4*a^5) + Log[a + x]/(4*a^5
)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

method	result	size
default	$-\frac{1}{a^{4} x}-\frac{\ln \left(a-x\right)}{4 a^{5}}-\frac{\arctan \left(\frac{x}{a}\right)}{2 a^{5}}+\frac{\ln \left(a+x\right)}{4 a^{5}}$	41
risch	$-\frac{1}{a^{4} x}-\frac{\arctan \left(\frac{x}{a}\right)}{2 a^{5}}-\frac{\ln \left(-a+x\right)}{4 a^{5}}+\frac{\ln \left(a+x\right)}{4 a^{5}}$	41
parallelrisch	$-\frac{-i \ln \left(-i a+x\right) x+i \ln \left(i a+x\right) x+\ln \left(-a+x\right) x-\ln \left(a+x\right) x+4 a}{4 a^{5} x}$	50

```
[In] int(1/x^2/(a^4-x^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^4/x-1/4/a^5*ln(a-x)-1/2*arctan(x/a)/a^5+1/4*ln(a+x)/a^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2\left(a^4-x^4\right)}\,dx=-\frac{2\,x\arctan\left(\frac{x}{a}\right)-x\log\left(a+x\right)+x\log\left(-a+x\right)+4\,a}{4\,a^5x}$$

```
[In] integrate(1/x^2/(a^4-x^4),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x*arctan(x/a) - x*log(a + x) + x*log(-a + x) + 4*a)/(a^5*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^2\left(a^4-x^4\right)}\,dx=-\frac{1}{a^4x}-\frac{\frac{\log\left(-a+x\right)}{4}-\frac{\log\left(a+x\right)}{4}-\frac{i\log\left(-ia+x\right)}{4}+\frac{i\log\left(ia+x\right)}{4}}{a^5}$$

```
[In] integrate(1/x**2/(a**4-x**4),x)
```

```
[Out] -1/(a**4*x) - (log(-a + x)/4 - log(a + x)/4 - I*log(-I*a + x)/4 + I*log(I*a
 + x)/4)/a**5
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2 (a^4 - x^4)} dx = -\frac{\arctan\left(\frac{x}{a}\right)}{2 a^5} + \frac{\log(a + x)}{4 a^5} - \frac{\log(-a + x)}{4 a^5} - \frac{1}{a^4 x}$$

```
[In] integrate(1/x^2/(a^4-x^4),x, algorithm="maxima")
[Out] -1/2*arctan(x/a)/a^5 + 1/4*log(a + x)/a^5 - 1/4*log(-a + x)/a^5 - 1/(a^4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 (a^4 - x^4)} dx = -\frac{\arctan\left(\frac{x}{a}\right)}{2 a^5} + \frac{\log(|a + x|)}{4 a^5} - \frac{\log(|-a + x|)}{4 a^5} - \frac{1}{a^4 x}$$

```
[In] integrate(1/x^2/(a^4-x^4),x, algorithm="giac")
[Out] -1/2*arctan(x/a)/a^5 + 1/4*log(abs(a + x))/a^5 - 1/4*log(abs(-a + x))/a^5 -
 1/(a^4*x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 (a^4 - x^4)} dx = \frac{\mathrm{atanh}\left(\frac{x}{a}\right)}{2 a^5} - \frac{\mathrm{atan}\left(\frac{x}{a}\right)}{2 a^5} - \frac{1}{a^4 x}$$

```
[In] int(1/(x^2*(a^4 - x^4)),x)
[Out] atanh(x/a)/(2*a^5) - atan(x/a)/(2*a^5) - 1/(a^4*x)
```

3.131 $\int \frac{1}{x^3(a^4-x^4)} dx$

Optimal result

Integrand size = 15, antiderivative size = 26

$$\int \frac{1}{x^3 (a^4 - x^4)} dx = -\frac{1}{2a^4x^2} + \frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^6}$$

```
[Out] -1/2/a^4/x^2+1/2*arctanh(x^2/a^2)/a^6
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {281, 331, 212}

$$\int \frac{1}{x^3 (a^4 - x^4)} dx = \frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^6} - \frac{1}{2a^4x^2}$$

```
[In] Int[1/(x^3*(a^4 - x^4)),x]
```

```
[Out] -1/2*1/(a^4*x^2) + ArcTanh[x^2/a^2]/(2*a^6)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a^4-x^2\right)}\,dx, x, x^2\right)\\ &= -\frac{1}{2a^4x^2}+\frac{\text{Subst}\left(\int \frac{1}{a^4-x^2}\,dx, x, x^2\right)}{2a^4}\\ &= -\frac{1}{2a^4x^2}+\frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^6}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{1}{x^3\left(a^4-x^4\right)}\,dx = -\frac{1}{2a^4x^2}-\frac{\log(a-x)}{4a^6}-\frac{\log(a+x)}{4a^6}+\frac{\log\left(a^2+x^2\right)}{4a^6}$$

```
[In] Integrate[1/(x^3*(a^4 - x^4)),x]
[Out] -1/2*1/(a^4*x^2) - Log[a - x]/(4*a^6) - Log[a + x]/(4*a^6) + Log[a^2 + x^2]
/(4*a^6)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

method	result	size
risch	$-\frac{1}{2a^4x^2}+\frac{\ln\left(-a^2-x^2\right)}{4a^6}-\frac{\ln\left(a^2-x^2\right)}{4a^6}$	42
default	$-\frac{\ln(a-x)}{4a^6}+\frac{\ln\left(a^2+x^2\right)}{4a^6}-\frac{1}{2a^4x^2}-\frac{\ln(a+x)}{4a^6}$	43
norman	$-\frac{\ln(a-x)}{4a^6}+\frac{\ln\left(a^2+x^2\right)}{4a^6}-\frac{1}{2a^4x^2}-\frac{\ln(a+x)}{4a^6}$	43
parallelrisch	$-\frac{\ln(-a+x)x^2+\ln(a+x)x^2-x^2\ln\left(a^2+x^2\right)+2a^2}{4a^6x^2}$	46

```
[In] int(1/x^3/(a^4-x^4),x,method=_RETURNVERBOSE)
[Out] -1/2/a^4/x^2+1/4/a^6*ln(-a^2-x^2)-1/4/a^6*ln(a^2-x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^3 (a^4 - x^4)} \, dx = \frac{x^2 \log(a^2 + x^2) - x^2 \log(-a^2 + x^2) - 2\, a^2}{4\, a^6 x^2}$$

```
[In] integrate(1/x^3/(a^4-x^4),x, algorithm="fricas")
[Out] 1/4*(x^2*log(a^2 + x^2) - x^2*log(-a^2 + x^2) - 2*a^2)/(a^6*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^3 (a^4 - x^4)} \, dx = -\frac{1}{2a^4x^2} - \frac{\frac{\log(-a^2+x^2)}{4} - \frac{\log(a^2+x^2)}{4}}{a^6}$$

```
[In] integrate(1/x**3/(a**4-x**4),x)
[Out] -1/(2*a**4*x**2) - (log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**6
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^3 (a^4 - x^4)} \, dx = \frac{\log(a^2 + x^2)}{4\, a^6} - \frac{\log(-a^2 + x^2)}{4\, a^6} - \frac{1}{2\, a^4 x^2}$$

```
[In] integrate(1/x^3/(a^4-x^4),x, algorithm="maxima")
[Out] 1/4*log(a^2 + x^2)/a^6 - 1/4*log(-a^2 + x^2)/a^6 - 1/2/(a^4*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^3 (a^4 - x^4)} \, dx = \frac{\log (a^2 + x^2)}{4 \, a^6} - \frac{\log (|-a^2 + x^2|)}{4 \, a^6} - \frac{1}{2 \, a^4 x^2}$$

```
[In] integrate(1/x^3/(a^4-x^4),x, algorithm="giac")
[Out] 1/4*log(a^2 + x^2)/a^6 - 1/4*log(abs(-a^2 + x^2))/a^6 - 1/2/(a^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 (a^4 - x^4)} \, dx = \frac{\operatorname{atanh}\left(\frac{x^2}{a^2}\right)}{2 \, a^6} - \frac{1}{2 \, a^4 \, x^2}$$

```
[In] int(1/(x^3*(a^4 - x^4)),x)
[Out] atanh(x^2/a^2)/(2*a^6) - 1/(2*a^4*x^2)
```

3.132 $\int \frac{1}{x^4(a^4-x^4)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{x^4(a^4-x^4)}\,dx=-\frac{1}{3a^4x^3}+\frac{\arctan\left(\frac{x}{a}\right)}{2a^7}+\frac{\operatorname{arctanh}\left(\frac{x}{a}\right)}{2a^7}$$

```
[Out] -1/3/a^4/x^3+1/2*arctan(x/a)/a^7+1/2*arctanh(x/a)/a^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {331, 218, 212, 209}

$$\int \frac{1}{x^4(a^4-x^4)}\,dx=\frac{\arctan\left(\frac{x}{a}\right)}{2a^7}+\frac{\operatorname{arctanh}\left(\frac{x}{a}\right)}{2a^7}-\frac{1}{3a^4x^3}$$

```
[In] Int[1/(x^4*(a^4 - x^4)),x]
```

```
[Out] -1/3*1/(a^4*x^3) + ArcTan[x/a]/(2*a^7) + ArcTanh[x/a]/(2*a^7)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{3a^4x^3} + \frac{\int \frac{1}{a^4-x^4}\,dx}{a^4} \\
&= -\frac{1}{3a^4x^3} + \frac{\int \frac{1}{a^2-x^2}\,dx}{2a^6} + \frac{\int \frac{1}{a^2+x^2}\,dx}{2a^6} \\
&= -\frac{1}{3a^4x^3} + \frac{\arctan\left(\frac{x}{a}\right)}{2a^7} + \frac{\operatorname{arctanh}\left(\frac{x}{a}\right)}{2a^7}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$
\int \frac{1}{x^4\left(a^4-x^4\right)}\,dx = -\frac{1}{3a^4x^3} + \frac{\arctan\left(\frac{x}{a}\right)}{2a^7} - \frac{\log(a-x)}{4a^7} + \frac{\log(a+x)}{4a^7}
$$

```
[In] Integrate[1/(x^4*(a^4 - x^4)),x]
[Out] -1/3*1/(a^4*x^3) + ArcTan[x/a]/(2*a^7) - Log[a - x]/(4*a^7) + Log[a + x]/(4
*a^7)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

method	result	size
default	$-\frac{\ln(a-x)}{4a^7}+\frac{\arctan\left(\frac{x}{a}\right)}{2a^7}-\frac{1}{3a^4x^3}+\frac{\ln(a+x)}{4a^7}$	41
parallelrisch	$-\frac{3i\ln(-ia+x)x^3-3i\ln(ia+x)x^3+3\ln(-a+x)x^3-3\ln(a+x)x^3+4a^3}{12a^7x^3}$	61
risch	$-\frac{1}{3a^4x^3}-\frac{\ln(a-x)}{4a^7}+\frac{\ln(-a-x)}{4a^7}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{14}\_Z^2+1\right)}\_R\ln\left(\left(-5\_R^4a^{28}+4\right)x-a^8\_R\right)\right)}{4}$	71

```
[In] int(1/x^4/(a^4-x^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^7*ln(a-x)+1/2*arctan(x/a)/a^7-1/3/a^4/x^3+1/4*ln(a+x)/a^7
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^4\left(a^4-x^4\right)}\,dx=\frac{6\,x^3\arctan\left(\frac{x}{a}\right)+3\,x^3\log\left(a+x\right)-3\,x^3\log\left(-a+x\right)-4\,a^3}{12\,a^7x^3}$$

```
[In] integrate(1/x^4/(a^4-x^4),x, algorithm="fricas")
```

```
[Out] 1/12*(6*x^3*arctan(x/a) + 3*x^3*log(a + x) - 3*x^3*log(-a + x) - 4*a^3)/(a^
7*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^4\left(a^4-x^4\right)}\,dx=-\frac{1}{3a^4x^3}-\frac{\frac{\log\left(-a+x\right)}{4}-\frac{\log\left(a+x\right)}{4}+\frac{i\log\left(-ia+x\right)}{4}-\frac{i\log\left(ia+x\right)}{4}}{a^7}$$

```
[In] integrate(1/x**4/(a**4-x**4),x)
```

```
[Out] -1/(3*a**4*x**3) - (log(-a + x)/4 - log(a + x)/4 + I*log(-I*a + x)/4 - I*lo
g(I*a + x)/4)/a**7
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (a^4 - x^4)}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{2\,a^7} + \frac{\log\left(a + x\right)}{4\,a^7} - \frac{\log\left(-a + x\right)}{4\,a^7} - \frac{1}{3\,a^4 x^3}$$

```
[In] integrate(1/x^4/(a^4-x^4),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x/a)/a^7 + 1/4*log(a + x)/a^7 - 1/4*log(-a + x)/a^7 - 1/3/(a^4*x
^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4 (a^4 - x^4)}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{2\,a^7} + \frac{\log\left(|a + x|\right)}{4\,a^7} - \frac{\log\left(|-a + x|\right)}{4\,a^7} - \frac{1}{3\,a^4 x^3}$$

```
[In] integrate(1/x^4/(a^4-x^4),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x/a)/a^7 + 1/4*log(abs(a + x))/a^7 - 1/4*log(abs(-a + x))/a^7 -
1/3/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4 (a^4 - x^4)}\,dx = \frac{\mathrm{atan}\left(\frac{x}{a}\right)}{2\,a^7} + \frac{\mathrm{atanh}\left(\frac{x}{a}\right)}{2\,a^7} - \frac{1}{3\,a^4\,x^3}$$

```
[In] int(1/(x^4*(a^4 - x^4)),x)
```

```
[Out] atan(x/a)/(2*a^7) + atanh(x/a)/(2*a^7) - 1/(3*a^4*x^3)
```

3.133 $\int \frac{x^{-m}}{a^4-x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{x^{-m}}{a^4-x^4}\,dx=\frac{x^{1-m}\,\text{Hypergeometric2F1}\left(1,\frac{1-m}{4},\frac{5-m}{4},\frac{x^4}{a^4}\right)}{a^4(1-m)}$$

```
[Out] x^(1-m)*hypergeom([1, 1/4-1/4*m],[5/4-1/4*m],x^4/a^4)/a^4/(1-m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {371}

$$\int \frac{x^{-m}}{a^4-x^4}\,dx=\frac{x^{1-m}\,\text{Hypergeometric2F1}\left(1,\frac{1-m}{4},\frac{5-m}{4},\frac{x^4}{a^4}\right)}{a^4(1-m)}$$

```
[In] Int[1/(x^m*(a^4 - x^4)),x]
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/4, (5 - m)/4, x^4/a^4])/(a^4*(1 - m
))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\text{integral}=\frac{x^{1-m}\,\text{Hypergeometric2F1}\left(1,\frac{1-m}{4},\frac{5-m}{4},\frac{x^4}{a^4}\right)}{a^4(1-m)}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{x^{-m}}{a^4 - x^4} \, dx = -\frac{x^{1-m} \text{Hypergeometric2F1}\left(1, \frac{1}{4} - \frac{m}{4}, \frac{5}{4} - \frac{m}{4}, \frac{x^4}{a^4}\right)}{a^4(-1+m)}$$

```
[In] Integrate[1/(x^m*(a^4 - x^4)),x]
[Out] -((x^(1 - m)*Hypergeometric2F1[1, 1/4 - m/4, 5/4 - m/4, x^4/a^4])/(a^4*(-1
+ m)))
```

Maple [F]

$$\int \frac{x^{-m}}{a^4 - x^4} dx$$

```
[In] int(1/(x^m)/(a^4-x^4),x)
[Out] int(1/(x^m)/(a^4-x^4),x)
```

Fricas [F]

$$\int \frac{x^{-m}}{a^4 - x^4} \, dx = \int \frac{1}{(a^4 - x^4)x^m} \, dx$$

```
[In] integrate(1/(x^m)/(a^4-x^4),x, algorithm="fricas")
[Out] integral(1/((a^4 - x^4)*x^m), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.11

$$\int \frac{x^{-m}}{a^4 - x^4} \, dx = -\frac{m x^{1-m} \Phi\left(\frac{x^4 e^{2i\pi}}{a^4}, 1, \frac{1}{4} - \frac{m}{4}\right) \Gamma\left(\frac{1}{4} - \frac{m}{4}\right)}{16 a^4 \Gamma\left(\frac{5}{4} - \frac{m}{4}\right)} + \frac{x^{1-m} \Phi\left(\frac{x^4 e^{2i\pi}}{a^4}, 1, \frac{1}{4} - \frac{m}{4}\right) \Gamma\left(\frac{1}{4} - \frac{m}{4}\right)}{16 a^4 \Gamma\left(\frac{5}{4} - \frac{m}{4}\right)}$$

```
[In] integrate(1/(x**m)/(a**4-x**4),x)
[Out] -m*x**(1 - m)*lerchphi(x**4*exp_polar(2*I*pi)/a**4, 1, 1/4 - m/4)*gamma(1/4
 - m/4)/(16*a**4*gamma(5/4 - m/4)) + x**(1 - m)*lerchphi(x**4*exp_polar(2*I
*pi)/a**4, 1, 1/4 - m/4)*gamma(1/4 - m/4)/(16*a**4*gamma(5/4 - m/4))
```

Maxima [F]

$$\int \frac{x^{-m}}{a^4 - x^4} \, dx = \int \frac{1}{(a^4 - x^4)x^m} \, dx$$

[In] integrate(1/(x^m)/(a^4-x^4),x, algorithm="maxima")

[Out] integrate(1/((a^4 - x^4)*x^m), x)

Giac [F]

$$\int \frac{x^{-m}}{a^4 - x^4} \, dx = \int \frac{1}{(a^4 - x^4)x^m} \, dx$$

[In] integrate(1/(x^m)/(a^4-x^4),x, algorithm="giac")

[Out] integrate(1/((a^4 - x^4)*x^m), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-m}}{a^4 - x^4} \, dx = \int \frac{1}{x^m \, (a^4 - x^4)} \, dx$$

[In] int(1/(x^m*(a^4 - x^4)),x)

[Out] int(1/(x^m*(a^4 - x^4)), x)

3.134 $\int \frac{x}{a^4+x^4} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{x}{a^4+x^4} \, dx = \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$$

```
[Out] 1/2*arctan(x^2/a^2)/a^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {281, 209}

$$\int \frac{x}{a^4+x^4} \, dx = \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$$

```
[In] Int[x/(a^4 + x^4),x]
```

```
[Out] ArcTan[x^2/a^2]/(2*a^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{a^4+x^2}\,dx, x, x^2\right)$$
$$= \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{a^4+x^4}\,dx = \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$$

```
[In] Integrate[x/(a^4 + x^4),x]
[Out] ArcTan[x^2/a^2]/(2*a^2)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$	14
risch	$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$	14
parallelrisch	$-\frac{i\ln\left(-ia^2+x^2\right)-i\ln\left(ia^2+x^2\right)}{4a^2}$	35

```
[In] int(x/(a^4+x^4),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x^2/a^2)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^4 + x^4} \, dx = \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

```
[In] integrate(x/(a^4+x^4),x, algorithm="fricas")
[Out] 1/2*arctan(x^2/a^2)/a^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{x}{a^4 + x^4} \, dx = \frac{-\frac{i\log\left(-ia^2+x^2\right)}{4} + \frac{i\log\left(ia^2+x^2\right)}{4}}{a^2}$$

```
[In] integrate(x/(a**4+x**4),x)
[Out] (-I*log(-I*a**2 + x**2)/4 + I*log(I*a**2 + x**2)/4)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^4 + x^4} \, dx = \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

```
[In] integrate(x/(a^4+x^4),x, algorithm="maxima")
[Out] 1/2*arctan(x^2/a^2)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^4 + x^4} \, dx = \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2 \, a^2}$$

```
[In] integrate(x/(a^4+x^4),x, algorithm="giac")
[Out] 1/2*arctan(x^2/a^2)/a^2
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^4 + x^4} \, dx = \frac{\operatorname{atan}\left(\frac{x^2}{a^2}\right)}{2 \, a^2}$$

```
[In] int(x/(a^4 + x^4),x)
[Out] atan(x^2/a^2)/(2*a^2)
```

3.135 $\int \frac{x^2}{a^4+x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 109

$$\int \frac{x^2}{a^4+x^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a}+\frac{\arctan\left(1+\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a} + \frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}-\frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}$$

```
[Out] -1/4*arctan(1-x*2^(1/2)/a)/a*2^(1/2)+1/4*arctan(1+x*2^(1/2)/a)/a*2^(1/2)+1/
8*ln(a^2+x^2-a*x*2^(1/2))/a*2^(1/2)-1/8*ln(a^2+x^2+a*x*2^(1/2))/a*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}-\frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{4\sqrt{2}a} -\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a}+\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{2\sqrt{2}a}$$

```
[In] Int[x^2/(a^4 + x^4),x]
```

```
[Out] -1/2*ArcTan[1 - (Sqrt[2]*x)/a]/(Sqrt[2]*a) + ArcTan[1 + (Sqrt[2]*x)/a]/(2*S
qrt[2]*a) + Log[a^2 - Sqrt[2]*a*x + x^2]/(4*Sqrt[2]*a) - Log[a^2 + Sqrt[2]*
a*x + x^2]/(4*Sqrt[2]*a)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}\,dx\right)+\frac{1}{2}\int \frac{a^2+x^2}{a^4+x^4}\,dx$$

$$=\frac{1}{4}\int \frac{1}{a^2-\sqrt{2}ax+x^2}\,dx+\frac{1}{4}\int \frac{1}{a^2+\sqrt{2}ax+x^2}\,dx+\frac{\int \frac{\sqrt{2}a+2x}{-a^2-\sqrt{2}ax-x^2}\,dx}{4\sqrt{2}a}+\frac{\int \frac{\sqrt{2}a-2x}{-a^2+\sqrt{2}ax-x^2}\,dx}{4\sqrt{2}a}$$

$$= \frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{4\sqrt{2}a} - \frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{4\sqrt{2}a} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a} + \frac{\arctan\left(1+\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a} + \frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{4\sqrt{2}a} - \frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{-2\arctan\left(1-\frac{\sqrt{2}x}{a}\right)+2\arctan\left(1+\frac{\sqrt{2}x}{a}\right)+\log\left(a^2-\sqrt{2}ax+x^2\right)-\log\left(a^2+\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}$$

```
[In] Integrate[x^2/(a^4 + x^4),x]
[Out] (-2*ArcTan[1 - (Sqrt[2]*x)/a] + 2*ArcTan[1 + (Sqrt[2]*x)/a] + Log[a^2 - Sqr
t[2]*a*x + x^2] - Log[a^2 + Sqrt[2]*a*x + x^2])/(4*Sqrt[2]*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.22

method	result	size
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+a^4\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{4}$	24
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(a^4\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}{x^2+\left(a^4\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(a^4\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(a^4\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(a^4\right)^{\frac{1}{4}}}$	85

```
[In] int(x^2/(a^4+x^4),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4+a^4))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{1}{4}\left(-\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(a^4\left(-\frac{1}{a^4}\right)^{\frac{3}{4}}+x\right) - \frac{1}{4}i\left(-\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(i\,a^4\left(-\frac{1}{a^4}\right)^{\frac{3}{4}}+x\right) + \frac{1}{4}i\left(-\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(-i\,a^4\left(-\frac{1}{a^4}\right)^{\frac{3}{4}}+x\right) - \frac{1}{4}\left(-\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(-a^4\left(-\frac{1}{a^4}\right)^{\frac{3}{4}}+x\right)$$

```
[In] integrate(x^2/(a^4+x^4),x, algorithm="fricas")
```

```
[Out] 1/4*(-1/a^4)^(1/4)*log(a^4*(-1/a^4)^(3/4) + x) - 1/4*I*(-1/a^4)^(1/4)*log(I
*a^4*(-1/a^4)^(3/4) + x) + 1/4*I*(-1/a^4)^(1/4)*log(-I*a^4*(-1/a^4)^(3/4) +
 x) - 1/4*(-1/a^4)^(1/4)*log(-a^4*(-1/a^4)^(3/4) + x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.17

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{\operatorname{RootSum}\left(256t^4+1,\left(t\mapsto t\log\left(64t^3a+x\right)\right)\right)}{a}$$

```
[In] integrate(x**2/(a**4+x**4),x)
```

```
[Out] RootSum(256*_t**4 + 1, Lambda(_t, _t*log(64*_t**3*a + x)))/a
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a+2\,x\right)}{2\,a}\right)}{4\,a} + \frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a-2\,x\right)}{2\,a}\right)}{4\,a} - \frac{\sqrt{2}\log\left(\sqrt{2}ax+a^2+x^2\right)}{8\,a} + \frac{\sqrt{2}\log\left(-\sqrt{2}ax+a^2+x^2\right)}{8\,a}$$

```
[In] integrate(x^2/(a^4+x^4),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a + 2*x)/a)/a + 1/4*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*a - 2*x)/a)/a - 1/8*sqrt(2)*log(sqrt(2)*a*x + a^2 + x
^2)/a + 1/8*sqrt(2)*log(-sqrt(2)*a*x + a^2 + x^2)/a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{\sqrt{2}|a|\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}|a|+2\,x\right)}{2\,|a|}\right)}{4\,a^2} + \frac{\sqrt{2}|a|\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}|a|-2\,x\right)}{2\,|a|}\right)}{4\,a^2} - \frac{\sqrt{2}|a|\log\left(\sqrt{2}x|a|+x^2+|a|^2\right)}{8\,a^2} + \frac{\sqrt{2}|a|\log\left(-\sqrt{2}x|a|+x^2+|a|^2\right)}{8\,a^2}$$

```
[In] integrate(x^2/(a^4+x^4),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*abs(a)*arctan(1/2*sqrt(2)*(sqrt(2)*abs(a) + 2*x)/abs(a))/a^2 +
1/4*sqrt(2)*abs(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*abs(a) - 2*x)/abs(a))/a^2 -
 1/8*sqrt(2)*abs(a)*log(sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^2 + 1/8*sqrt(2
)*abs(a)*log(-sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^2
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.30

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,x}{a}\right) - (-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,x}{a}\right)}{2\,a}$$

```
[In] int(x^2/(a^4 + x^4),x)
```

```
[Out] ((-1)^(1/4)*atan(((-1)^(1/4)*x)/a) - (-1)^(1/4)*atanh(((-1)^(1/4)*x)/a))/(2
*a)
```

3.136 $\int \frac{1}{a^5+x^5}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 201

$$\begin{aligned}\int \frac{1}{a^5+x^5}\,dx =& -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^4} \\ &-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^4} \\ &+\frac{\log(a+x)}{5a^4}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^4} \\ &-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^4}\end{aligned}$$

```
[Out] 1/5*ln(a+x)/a^4-1/20*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(-5^(1/2)+1)/a^4-1/20
*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(5^(1/2)+1)/a^4-1/10*arctan(1/20*(-4*x+a*(
5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10-2*5^(1/2))^(1/2)/a^4-1/10*arctan((
-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)/a^4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {207, 648, 632, 210, 642, 31}

$$\int \frac{1}{a^5+x^5}\,dx = -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^4} - \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^4} + \frac{\log(a+x)}{5a^4} - \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^4} - \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^4}$$

```
[In] Int[(a^5 + x^5)^(-1),x]
```

```
[Out] -1/5*(Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqr
t[5])]*a)])/a^4 - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1
 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^4) + Log[a + x]/(5*a^4) - ((1 - Sqrt[5])
*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^4) - ((1 + Sqrt[5])*Log[a^2
- ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k -
1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; (r/(a*n)
)*Int[1/(r + s*x), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 1)/2}], x], x]
] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && PosQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{2\int \frac{a-\frac{1}{4}\left(1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a^4} + \frac{2\int \frac{a-\frac{1}{4}\left(1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a^4} + \frac{\int \frac{1}{a+x}\,dx}{5a^4}$$

$$= \frac{\log(a+x)}{5a^4} - \frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^4} - \frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^4} + \frac{\left(5-\sqrt{5}\right)\int \frac{1}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^3} + \frac{\left(5+\sqrt{5}\right)\int \frac{1}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^3}$$

$$= \frac{\log(a+x)}{5a^4} - \frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a^4} - \frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}ax+2x^2\right)}{20a^4} - \frac{\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\left(5-\sqrt{5}\right)a^2-x^2}\,dx, x, -\frac{1}{2}\left(1+\sqrt{5}\right)a+2x\right)}{10a^3} - \frac{\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\left(5+\sqrt{5}\right)a^2-x^2}\,dx, x, -\frac{1}{2}\left(1-\sqrt{5}\right)a+2x\right)}{10a^3}$$

$$= -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^4} - \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^4} + \frac{\log(a+x)}{5a^4} - \frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a^4} - \frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}ax+2x^2\right)}{20a^4}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{1}{a^5+x^5}\,dx = -\frac{-2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right) - 2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right) - 4\log(a+x) + \log}{}$$

```
[In] Integrate[(a^5 + x^5)^(-1),x]
```

```
[Out] -1/20*(-2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5
+ Sqrt[5])]*a)] - 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/
(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2
 + x^2] - Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 +
 Sqrt[5])*a*x)/2 + x^2] + Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/a
^4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.27

method	result	size
risch	$\frac{\ln(a+x)}{5a^4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{16}\_Z^4+a^{12}\_Z^3+a^8\_Z^2+a^4\_Z+1\right)}\_R\ln\left(\_R a^5+x\right)\right)}{5}$	55
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3+2\_R^2a-3a^2\_R+4a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2a^2\_R-a^3}}{5a^4}+\frac{\ln(a+x)}{5a^4}$	101

```
[In] int(1/(a^5+x^5),x,method=_RETURNVERBOSE)
[Out] 1/5*ln(a+x)/a^4+1/5*sum(_R*ln(_R*a^5+x),_R=RootOf(_Z^4*a^16+_Z^3*a^12+_Z^2*
a^8+_Z*a^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 11094, normalized size of antiderivative = 55.19

$$\int \frac{1}{a^5+x^5}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a^5+x^5),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.19

$$\int \frac{1}{a^5+x^5}\,dx = \frac{\frac{\log{(a+x)}}{5}+\operatorname{RootSum}{\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log{(5ta+x)}\right)\right)}}{a^4}$$

```
[In] integrate(1/(a**5+x**5),x)
[Out] (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda
(_t, _t*log(5*_t*a + x))))/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.90

$$\int \frac{1}{a^5+x^5}\,dx = \frac{\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\,\sqrt{5}+10}}\right)}{5\,a^4\sqrt{2\,\sqrt{5}+10}} + \frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\,\sqrt{5}+10}}\right)}{5\,a^4\sqrt{-2\,\sqrt{5}+10}} - \frac{\left(\sqrt{5}+3\right)\log\left(-ax\left(\sqrt{5}+1\right)+2\,a^2+2\,x^2\right)}{10\,a^4\left(\sqrt{5}+1\right)} - \frac{\left(\sqrt{5}-3\right)\log\left(ax\left(\sqrt{5}-1\right)+2\,a^2+2\,x^2\right)}{10\,a^4\left(\sqrt{5}-1\right)} + \frac{\log\left(a+x\right)}{5\,a^4}$$

```
[In] integrate(1/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*(sqrt(5) + 1)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
+ 10)))/(a^4*sqrt(2*sqrt(5) + 10)) + 1/5*sqrt(5)*(sqrt(5) - 1)*arctan(-(a*(
sqrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/(a^4*sqrt(-2*sqrt(5) + 10))
- 1/10*(sqrt(5) + 3)*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^4*(sqrt(5)
+ 1)) - 1/10*(sqrt(5) - 3)*log(a*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a^4*(sqr
t(5) - 1)) + 1/5*log(a + x)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

$$\int \frac{1}{a^5+x^5}\,dx = \frac{\sqrt{2\,\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\,\sqrt{5}+10}}\right)}{10\,a^4} + \frac{\sqrt{-2\,\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\,\sqrt{5}+10}}\right)}{10\,a^4} - \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20\,a^4} + \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20\,a^4} - \frac{\log\left(|a^4-a^3x+a^2x^2-ax^3+x^4|\right)}{20\,a^4} + \frac{\log\left(|a+x|\right)}{5\,a^4}$$

```
[In] integrate(1/(a^5+x^5),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
+ 10)))/a^4 + 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a
*sqrt(-2*sqrt(5) + 10)))/a^4 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x
 + x^2)/a^4 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^4 - 1/2
0*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^4 + 1/5*log(abs(a + x))/a
^4
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.87

$$\int \frac{1}{a^5+x^5}\,dx = \frac{\ln(a+x)}{5\,a^4} - \frac{\ln\left(x-\frac{a\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)}{4}\right)\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a^4} - \frac{\ln\left(x-\frac{a\left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}+1\right)}{4}\right)\left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}+1\right)}{20\,a^4} + \frac{\ln\left(x+\frac{a\left(\sqrt{5}+\sqrt{-2\,\sqrt{5}-10}-1\right)}{4}\right)\left(\sqrt{5}+\sqrt{-2\,\sqrt{5}-10}-1\right)}{20\,a^4} - \frac{\ln\left(x-\frac{a\left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)}{4}\right)\left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a^4}$$

```
[In] int(1/(a^5 + x^5),x)
```

```
[Out] log(a + x)/(5*a^4) - (log(x - (a*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/4)
*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^4) - (log(x - (a*((- 2*5^(1/
2) - 10)^(1/2) - 5^(1/2) + 1))/4)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))
/(20*a^4) + (log(x + (a*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/4)*(5^(1/
2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*a^4) - (log(x - (a*(5^(1/2) + (2*5^
(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^4)
```

3.137 $\int \frac{x}{a^5+x^5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 201

$$\begin{aligned}\int \frac{x}{a^5+x^5}\,dx &= \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^3} \\ &\quad - \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^3} \\ &\quad - \frac{\log(a+x)}{5a^3} + \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^3} \\ &\quad + \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^3}\end{aligned}$$

```
[Out] -1/5*ln(a+x)/a^3+1/20*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(-5^(1/2)+1)/a^3+1/20
*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(5^(1/2)+1)/a^3+1/10*arctan((-4*x+a*(-5^(
1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)/a^3-1/10*arctan(1/20*
(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10+2*5^(1/2))^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {299, 648, 632, 210, 642, 31}

$$\int \frac{x}{a^5+x^5}\,dx = \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^3} - \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^3} - \frac{\log(a+x)}{5a^3} + \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^3} + \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^3}$$

```
[In] Int[x/(a^5 + x^5),x]
```

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqrt[5])
]*a)])/(5*a^3) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1
+ Sqrt[5])*a - 4*x))/(2*a)])/(5*a^3) - Log[a + x]/(5*a^3) + ((1 + Sqrt[5])*
Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^3) + ((1 - Sqrt[5])*Log[a^2 -
 ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; (-(-r)^(m + 1)/(a*n*s^m))*Int[1/(r + s*x), x]
 + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Free
Q[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a
```

```
/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\int \frac{\frac{1}{4}\left(1-\sqrt{5}\right)a-\frac{1}{4}\left(-1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a^3}+\frac{2\int \frac{\frac{1}{4}\left(1+\sqrt{5}\right)a-\frac{1}{4}\left(-1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a^3}-\frac{\int \frac{1}{a+x}\,dx}{5a^3}\\
&= -\frac{\log(a+x)}{5a^3}+\frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^3}+\frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^3}\\
&\quad-\frac{\int \frac{1}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{2\sqrt{5}a^2}+\frac{\int \frac{1}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{2\sqrt{5}a^2}\\
&= -\frac{\log(a+x)}{5a^3}+\frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a^3}\\
&\quad+\frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}ax+2x^2\right)}{20a^3}\\
&\quad-\frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\left(5-\sqrt{5}\right)a^2-x^2}\,dx,x,-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x\right)}{\sqrt{5}a^2}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\left(5+\sqrt{5}\right)a^2-x^2}\,dx,x,-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x\right)}{\sqrt{5}a^2}
\end{aligned}$$

$$= \frac{\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{a^3} - \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^3} - \frac{\log(a+x)}{5a^3} + \frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a^3} + \frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}ax+2x^2\right)}{20a^3}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{x}{a^5+x^5}\,dx = \frac{-2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)+2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right)-4\log(a+x)+\log\left(a^2\right.}{}$$

```
[In] Integrate[x/(a^5 + x^5),x]
```

```
[Out] (-2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 + Sqrt[
5])]*a)] + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[
10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2
] + Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 + Sqrt[
5])*a*x)/2 + x^2] - Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^3
)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.30

method	result	size
risch	$-\frac{\ln(a+x)}{5a^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{12}\_Z^4-a^9\_Z^3+a^6\_Z^2-a^3\_Z+1\right)}\_R\ln\left(-a^{10}\_R^3+x\right)\right)}{5}$	60
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(\_R^3-2\_R^2a+3a^2\_R+a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2a^2\_R-a^3}}{5a^3}-\frac{\ln(a+x)}{5a^3}$	97

```
[In] int(x/(a^5+x^5),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*ln(a+x)/a^3+1/5*sum(_R*ln(-_R^3*a^10+x),_R=RootOf(_Z^4*a^12-_Z^3*a^9+_
Z^2*a^6-_Z*a^3+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 18781, normalized size of antiderivative = 93.44

$$\int \frac{x}{a^5+x^5}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.20

$$\int \frac{x}{a^5+x^5}\,dx = \frac{-\frac{\log{(a+x)}}{5}+\operatorname{RootSum}{\left(625t^4-125t^3+25t^2-5t+1,\left(t\mapsto t\log{\left(-125t^3a+x\right)}\right)\right)}}{a^3}$$

```
[In] integrate(x/(a**5+x**5),x)
```

```
[Out] (-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, Lambd
a(_t, _t*log(-125*_t**3*a + x))))/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int \frac{x}{a^5+x^5}\,dx = -\frac{2\sqrt{5}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\,\sqrt{5}+10}}\right)}{5\,a^3\sqrt{2\,\sqrt{5}+10}} + \frac{2\sqrt{5}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\,\sqrt{5}+10}}\right)}{5\,a^3\sqrt{-2\,\sqrt{5}+10}} - \frac{\log\left(a+x\right)}{5\,a^3} - \frac{\log\left(-ax\left(\sqrt{5}+1\right)+2\,a^2+2\,x^2\right)}{5\,a^3\left(\sqrt{5}+1\right)} + \frac{\log\left(ax\left(\sqrt{5}-1\right)+2\,a^2+2\,x^2\right)}{5\,a^3\left(\sqrt{5}-1\right)}$$

```
[In] integrate(x/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a^3*
sqrt(2*sqrt(5) + 10)) + 2/5*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sqrt
(-2*sqrt(5) + 10)))/(a^3*sqrt(-2*sqrt(5) + 10)) - 1/5*log(a + x)/a^3 - 1/5*
log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^3*(sqrt(5) + 1)) + 1/5*log(a*x*(
sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a^3*(sqrt(5) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

$$\int \frac{x}{a^5+x^5}\,dx = -\frac{\sqrt{-2\,\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\,\sqrt{5}+10}}\right)}{10\,a^3} + \frac{\sqrt{2\,\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\,\sqrt{5}+10}}\right)}{10\,a^3} - \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20\,a^3} + \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20\,a^3} + \frac{\log\left(\left|a^4-a^3x+a^2x^2-ax^3+x^4\right|\right)}{20\,a^3} - \frac{\log\left(\left|a+x\right|\right)}{5\,a^3}$$

```
[In] integrate(x/(a^5+x^5),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5
) + 10)))/a^3 + 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(
a*sqrt(-2*sqrt(5) + 10)))/a^3 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*
x + x^2)/a^3 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^3 + 1/
20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^3 - 1/5*log(abs(a + x))/
a^3
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91

$$\int \frac{x}{a^5+x^5}\,dx=\frac{\ln\left(x-\frac{a\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^3}-\frac{\ln\left(a+x\right)}{5\,a^3}+\frac{\ln\left(x-\frac{a\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)^3}{64}\right)\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{20\,a^3}-\frac{\ln\left(x+\frac{a\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)^3}{64}\right)\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)}{20\,a^3}+\frac{\ln\left(x-\frac{a\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^3}$$

```
[In] int(x/(a^5 + x^5),x)
```

```
[Out] (log(x - (a*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^(
1/2) - 10)^(1/2) + 1))/(20*a^3) - log(a + x)/(5*a^3) + (log(x - (a*((- 2*5^
(1/2) - 10)^(1/2) - 5^(1/2) + 1)^3)/64)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2)
 + 1))/(20*a^3) - (log(x + (a*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1)^3)/6
4)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*a^3) + (log(x - (a*(5^(1/2
) + (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) +
1))/(20*a^3)
```

3.138 $\int \frac{x^2}{a^5+x^5}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 201

$$\int \frac{x^2}{a^5+x^5}\,dx = \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^2} - \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^2} + \frac{\log(a+x)}{5a^2} - \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^2} - \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^2}$$

```
[Out] 1/5*ln(a+x)/a^2-1/20*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(-5^(1/2)+1)/a^2-1/20*
ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(5^(1/2)+1)/a^2+1/10*arctan((-4*x+a*(-5^(1
/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)/a^2-1/10*arctan(1/20*(
-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10+2*5^(1/2))^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {299, 648, 632, 210, 642, 31}

$$\int \frac{x^2}{a^5+x^5}\,dx=\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^2}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^2}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^2}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^2}+\frac{\log(a+x)}{5a^2}$$

```
[In] Int[x^2/(a^5 + x^5),x]
```

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqrt[5])
]*a)])/(5*a^2) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1
+ Sqrt[5])*a - 4*x))/(2*a)])/(5*a^2) + Log[a + x]/(5*a^2) - ((1 + Sqrt[5])*
Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^2) - ((1 - Sqrt[5])*Log[a^2 -
 ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; (-(-r)^(m + 1)/(a*n*s^m))*Int[1/(r + s*x), x]
 + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Free
Q[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a
```

```
/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)a-\frac{1}{4}\left(1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a^2}+\frac{2\int \frac{\frac{1}{4}\left(-1+\sqrt{5}\right)a-\frac{1}{4}\left(1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a^2}+\frac{\int \frac{1}{a+x}\,dx}{5a^2}\\
&= \frac{\log(a+x)}{5a^2}-\frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^2}-\frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^2}\\
&\quad -\frac{\int \frac{1}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{2\sqrt{5}a}+\frac{\int \frac{1}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{2\sqrt{5}a}\\
&= \frac{\log(a+x)}{5a^2}-\frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a^2}\\
&\quad -\frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}ax+2x^2\right)}{20a^2}\\
&\quad -\frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\left(5-\sqrt{5}\right)a^2-x^2}\,dx,x,-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x\right)}{\sqrt{5}a}\\
&\quad +\frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\left(5+\sqrt{5}\right)a^2-x^2}\,dx,x,-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x\right)}{\sqrt{5}a}
\end{aligned}$$

$$= \frac{\sqrt{\frac{2}{5(5+\sqrt{5})}}\arctan\left(\frac{(1-\sqrt{5})a-4x}{\sqrt{2(5+\sqrt{5})}a}\right)}{a^2} - \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})}\arctan\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}((1+\sqrt{5})a-4x)}{2a}\right)}{5a^2} + \frac{\log(a+x)}{5a^2} - \frac{(1-\sqrt{5})\log(2a^2-ax-\sqrt{5}ax+2x^2)}{20a^2} - \frac{(1+\sqrt{5})\log(2a^2-ax+\sqrt{5}ax+2x^2)}{20a^2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{x^2}{a^5+x^5}\,dx = -\frac{2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{(-1+\sqrt{5})a+4x}{\sqrt{2(5+\sqrt{5})}a}\right) - 2\sqrt{2(5+\sqrt{5})}\arctan\left(\frac{-((1+\sqrt{5})a)+4x}{\sqrt{10-2\sqrt{5}}a}\right) - 4\log(a+x) + \log(}{}$$

```
[In] Integrate[x^2/(a^5 + x^5),x]
```

```
[Out] -1/20*(2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 +
Sqrt[5])]*a)] - 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(
Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2
+ x^2] + Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 +
Sqrt[5])*a*x)/2 + x^2] - Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/a^
2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{\ln(a+x)}{5a^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^8\_Z^4+a^6\_Z^3+a^4\_Z^2+a^2\_Z+1\right)}\_R\ln\left(x\_R^3a^5+1\right)\right)}{5}$	58
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3+2\_R^2a+2a^2\_R-a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2a^2\_R-a^3}}{5a^2}+\frac{\ln(a+x)}{5a^2}$	101

[In] int(x^2/(a^5+x^5),x,method=_RETURNVERBOSE)

[Out] 1/5*ln(a+x)/a^2+1/5*sum(_R*ln(_R^3*a^5*x+1),_R=RootOf(_Z^4*a^8+_Z^3*a^6+_Z^2*a^4+_Z*a^2+1))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 12656, normalized size of antiderivative = 62.97

$$\int\frac{x^2}{a^5+x^5}\,dx=\text{Too large to display}$$

[In] integrate(x^2/(a^5+x^5),x, algorithm="fricas")

[Out] Too large to include

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.20

$$\int\frac{x^2}{a^5+x^5}\,dx=\frac{\frac{\log(a+x)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(25t^2a+x\right)\right)\right)}{a^2}$$

[In] integrate(x**2/(a**5+x**5),x)

[Out] (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda(_t, _t*log(25*_t**2*a + x))))/a**2

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{a^5+x^5}\,dx = -\frac{2\sqrt{5}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{5a^2\sqrt{2\sqrt{5}+10}} + \frac{2\sqrt{5}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{5a^2\sqrt{-2\sqrt{5}+10}} + \frac{\log(a+x)}{5a^2} + \frac{\log\left(-ax\left(\sqrt{5}+1\right)+2a^2+2x^2\right)}{5a^2\left(\sqrt{5}+1\right)} - \frac{\log\left(ax\left(\sqrt{5}-1\right)+2a^2+2x^2\right)}{5a^2\left(\sqrt{5}-1\right)}$$

```
[In] integrate(x^2/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a^2*
sqrt(2*sqrt(5) + 10)) + 2/5*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sqrt
(-2*sqrt(5) + 10)))/(a^2*sqrt(-2*sqrt(5) + 10)) + 1/5*log(a + x)/a^2 + 1/5*
log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) + 1)) - 1/5*log(a*x*(
sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{a^5+x^5}\,dx = -\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^2} + \frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^2} + \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^2} - \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20a^2} - \frac{\log\left(|a^4-a^3x+a^2x^2-ax^3+x^4|\right)}{20a^2} + \frac{\log\left(|a+x|\right)}{5a^2}$$

```
[In] integrate(x^2/(a^5+x^5),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5
) + 10)))/a^2 + 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(
a*sqrt(-2*sqrt(5) + 10)))/a^2 + 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*
x + x^2)/a^2 - 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^2 - 1/
20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^2 + 1/5*log(abs(a + x))/
a^2
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a^5+x^5}\,dx = \frac{\ln(a+x)}{5\,a^2} + \frac{\ln\left(a^5+\frac{x\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)^3 a^4}{64}\right)\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)}{20\,a^2} - \frac{\ln\left(a^5-\frac{a^4\,x\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^2} - \frac{\ln\left(a^5-\frac{a^4\,x\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^2} - \frac{\ln\left(a^5-\frac{a^4\,x\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)^3}{64}\right)\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{20\,a^2}$$

```
[In] int(x^2/(a^5 + x^5),x)
```

```
[Out] log(a + x)/(5*a^2) + (log(a^5 + (a^4*x*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2)
- 1)^3)/64)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*a^2) - (log(a^5 -
 (a^4*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) + (2*5^(1/2)
 - 10)^(1/2) + 1))/(20*a^2) - (log(a^5 - (a^4*x*(5^(1/2) - (2*5^(1/2) - 10)
^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^2) - (log(
a^5 - (a^4*x*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1)^3)/64)*((- 2*5^(1/2)
- 10)^(1/2) - 5^(1/2) + 1))/(20*a^2)
```

3.139 $\int \frac{x^3}{a^5+x^5}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 201

$$\int \frac{x^3}{a^5+x^5}\,dx = -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a} - \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a} - \frac{\log(a+x)}{5a} + \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a} + \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a}$$

```
[Out] -1/5*ln(a+x)/a+1/20*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(-5^(1/2)+1)/a+1/20*ln
(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(5^(1/2)+1)/a-1/10*arctan(1/20*(-4*x+a*(5^(1/
2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10-2*5^(1/2))^(1/2)/a-1/10*arctan((-4*x+a*
(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)/a
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {299, 648, 632, 210, 642, 31}

$$\int \frac{x^3}{a^5+x^5}\,dx = \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a} + \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a} - \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a} - \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a} - \frac{\log(a+x)}{5a}$$

```
[In] Int[x^3/(a^5 + x^5),x]
```

```
[Out] -1/5*(Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqr
t[5])]*a)])/a - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1 +
 Sqrt[5])*a - 4*x))/(2*a)])/(5*a) - Log[a + x]/(5*a) + ((1 - Sqrt[5])*Log[a
^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a) + ((1 + Sqrt[5])*Log[a^2 - ((1 +
Sqrt[5])*a*x)/2 + x^2])/(20*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; (-(-r)^(m + 1)/(a*n*s^m))*Int[1/(r + s*x), x]
 + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Free
Q[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a
```

```
/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\int \frac{\frac{1}{4}\left(1+\sqrt{5}\right)a-\frac{1}{4}\left(-1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a}+\frac{2\int \frac{\frac{1}{4}\left(1-\sqrt{5}\right)a-\frac{1}{4}\left(-1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a}-\frac{\int \frac{1}{a+x}\,dx}{5a}\\
&= -\frac{\log(a+x)}{5a}+\frac{1}{20}\left(5-\sqrt{5}\right)\int \frac{1}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx\\
&\quad+\frac{1}{20}\left(5+\sqrt{5}\right)\int \frac{1}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx\\
&\quad+\frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a}+\frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a}\\
&= -\frac{\log(a+x)}{5a}+\frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a}\\
&\quad+\frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}ax+2x^2\right)}{20a}\\
&\quad+\frac{1}{10}\left(-5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\left(5-\sqrt{5}\right)a^2-x^2}\,dx,x,-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x\right)\\
&\quad-\frac{1}{10}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\left(5+\sqrt{5}\right)a^2-x^2}\,dx,x,-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x\right)
\end{aligned}$$

$$= -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a} - \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a} - \frac{\log(a+x)}{5a} + \frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a} + \frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}ax+2x^2\right)}{20a}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{a^5+x^5}\,dx = \frac{2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)+2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right)-4\log(a+x)+\log\left(a^2+\right.}{}$$

```
[In] Integrate[x^3/(a^5 + x^5),x]
```

```
[Out] (2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 + Sqrt[
5])]*a)] + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[1
0 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2]
 - Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 + Sqrt[5
])*a*x)/2 + x^2] + Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.36

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^4\_Z^4-a^3\_Z^3+\_Z^2a^2-a\_Z+1\right)}\_R\ln\left(\_R^3a^4-\_R^2a^3+a^2\_R-a+x\right)\right)}{5}-\frac{\ln(a+x)}{5a}$	73
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(\_R^3+3\_R^2a-2a^2\_R+a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2a^2\_R-a^3}}{5a}-\frac{\ln(a+x)}{5a}$	97

[In] int(x^3/(a^5+x^5),x,method=_RETURNVERBOSE)

[Out] 1/5*sum(_R*ln(_R^3*a^4-_R^2*a^3+_R*a^2-a+x),_R=RootOf(_Z^4*a^4-_Z^3*a^3+_Z^2*a^2-_Z*a+1))-1/5*ln(a+x)/a

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 17865, normalized size of antiderivative = 88.88

$$\int \frac{x^3}{a^5+x^5}\,dx = \text{Too large to display}$$

[In] integrate(x^3/(a^5+x^5),x, algorithm="fricas")

[Out] Too large to include

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.19

$$\int \frac{x^3}{a^5+x^5}\,dx = \frac{-\frac{\log(a+x)}{5}+\mathrm{RootSum}\left(625t^4-125t^3+25t^2-5t+1,\left(t\mapsto t\log\left(625t^4a+x\right)\right)\right)}{a}$$

[In] integrate(x**3/(a**5+x**5),x)

[Out] (-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, Lambda(_t, _t*log(625*_t**4*a + x))))/a

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{a^5+x^5}\,dx = \frac{\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\,\sqrt{5}+10}}\right)}{5\,a\sqrt{2\,\sqrt{5}+10}} + \frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\,\sqrt{5}+10}}\right)}{5\,a\sqrt{-2\,\sqrt{5}+10}} + \frac{\left(\sqrt{5}+3\right)\log\left(-ax\left(\sqrt{5}+1\right)+2\,a^2+2\,x^2\right)}{10\,a\left(\sqrt{5}+1\right)} + \frac{\left(\sqrt{5}-3\right)\log\left(ax\left(\sqrt{5}-1\right)+2\,a^2+2\,x^2\right)}{10\,a\left(\sqrt{5}-1\right)} - \frac{\log\left(a+x\right)}{5\,a}$$

```
[In] integrate(x^3/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*(sqrt(5) + 1)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
+ 10)))/(a*sqrt(2*sqrt(5) + 10)) + 1/5*sqrt(5)*(sqrt(5) - 1)*arctan(-(a*(sq
rt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/(a*sqrt(-2*sqrt(5) + 10)) + 1/
10*(sqrt(5) + 3)*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5) + 1))
+ 1/10*(sqrt(5) - 3)*log(a*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5) - 1
)) - 1/5*log(a + x)/a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{a^5+x^5}\,dx = \frac{\sqrt{2\,\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\,\sqrt{5}+10}}\right)}{10\,a} + \frac{\sqrt{-2\,\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\,\sqrt{5}+10}}\right)}{10\,a} + \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20\,a} - \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20\,a} + \frac{\log\left(|a^4-a^3x+a^2x^2-ax^3+x^4|\right)}{20\,a} - \frac{\log\left(|a+x|\right)}{5\,a}$$

```
[In] integrate(x^3/(a^5+x^5),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
+ 10)))/a + 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*s
qrt(-2*sqrt(5) + 10)))/a + 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x + x
^2)/a - 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a + 1/20*log(ab
s(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a - 1/5*log(abs(a + x))/a
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{a^5+x^5}\,dx = \frac{\ln\left(5\,a^{10}-\frac{5\,a^9\,x\,\left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)}{4}\right)\,\left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a} - \frac{\ln\left(5\,a^{10}+\frac{5\,x\,\left(\sqrt{5}+\sqrt{-2\,\sqrt{5}-10}-1\right)\,a^9}{4}\right)\,\left(\sqrt{5}+\sqrt{-2\,\sqrt{5}-10}-1\right)}{20\,a} - \frac{\ln\left(a+x\right)}{5\,a} + \frac{\ln\left(5\,a^{10}-\frac{5\,a^9\,x\,\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)}{4}\right)\,\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a} + \frac{\ln\left(5\,a^{10}-\frac{5\,a^9\,x\,\left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}+1\right)}{4}\right)\,\left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}+1\right)}{20\,a}$$

```
[In] int(x^3/(a^5 + x^5),x)
```

```
[Out] (log(5*a^10 - (5*a^9*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2)
+ (2*5^(1/2) - 10)^(1/2) + 1))/(20*a) - (log(5*a^10 + (5*a^9*x*(5^(1/2) + (
- 2*5^(1/2) - 10)^(1/2) - 1))/4)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/
(20*a) - log(a + x)/(5*a) + (log(5*a^10 - (5*a^9*x*(5^(1/2) - (2*5^(1/2) -
10)^(1/2) + 1))/4)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a) + (log(5*
a^10 - (5*a^9*x*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/4)*((- 2*5^(1/2)
- 10)^(1/2) - 5^(1/2) + 1))/(20*a)
```

3.140 $\int \frac{x^4}{a^5+x^5} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{x^4}{a^5+x^5} \, dx = \frac{1}{5} \log \left(a^5+x^5\right)$$

```
[Out] 1/5*ln(a^5+x^5)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {266}

$$\int \frac{x^4}{a^5+x^5} \, dx = \frac{1}{5} \log \left(a^5+x^5\right)$$

```
[In] Int[x^4/(a^5 + x^5),x]
```

```
[Out] Log[a^5 + x^5]/5
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\text{integral} = \frac{1}{5} \log \left(a^5+x^5\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{a^5+x^5}\,dx=\frac{1}{5}\log\left(a^5+x^5\right)$$

```
[In] Integrate[x^4/(a^5 + x^5),x]
[Out] Log[a^5 + x^5]/5
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\ln\left(a^{5}+x^{5}\right)}{5}$	11
default	$\frac{\ln\left(a^{5}+x^{5}\right)}{5}$	11
risch	$\frac{\ln\left(a^{5}+x^{5}\right)}{5}$	11
norman	$\frac{\ln\left(a+x\right)}{5}+\frac{\ln\left(a^{4}-a^{3}x+a^{2}x^{2}-a\,x^{3}+x^{4}\right)}{5}$	37
parallelrisch	$\frac{\ln\left(a+x\right)}{5}+\frac{\ln\left(a^{4}-a^{3}x+a^{2}x^{2}-a\,x^{3}+x^{4}\right)}{5}$	37

```
[In] int(x^4/(a^5+x^5),x,method=_RETURNVERBOSE)
[Out] 1/5*ln(a^5+x^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a^5+x^5}\,dx=\frac{1}{5}\,\log\left(a^5+x^5\right)$$

```
[In] integrate(x^4/(a^5+x^5),x, algorithm="fricas")
[Out] 1/5*log(a^5 + x^5)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{a^5+x^5}\,dx = \frac{\log\left(a^5+x^5\right)}{5}$$

```
[In] integrate(x**4/(a**5+x**5),x)
[Out] log(a**5 + x**5)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a^5+x^5}\,dx = \frac{1}{5}\log\left(a^5+x^5\right)$$

```
[In] integrate(x^4/(a^5+x^5),x, algorithm="maxima")
[Out] 1/5*log(a^5 + x^5)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{a^5+x^5}\,dx = \frac{1}{5}\log\left(\left|a^5+x^5\right|\right)$$

```
[In] integrate(x^4/(a^5+x^5),x, algorithm="giac")
[Out] 1/5*log(abs(a^5 + x^5))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a^5+x^5}\,dx = \frac{\ln\left(a^5+x^5\right)}{5}$$

```
[In] int(x^4/(a^5 + x^5),x)
[Out] log(a^5 + x^5)/5
```

3.141 $\int \frac{1}{x(a^5+x^5)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{x(a^5+x^5)}\,dx = \frac{\log(x)}{a^5} - \frac{\log(a^5+x^5)}{5a^5}$$

```
[Out] ln(x)/a^5-1/5*ln(a^5+x^5)/a^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {272, 36, 29, 31}

$$\int \frac{1}{x(a^5+x^5)}\,dx = \frac{\log(x)}{a^5} - \frac{\log(a^5+x^5)}{5a^5}$$

```
[In] Int[1/(x*(a^5 + x^5)),x]
```

```
[Out] Log[x]/a^5 - Log[a^5 + x^5]/(5*a^5)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{x\,(a^5+x)}\,dx, x, x^5\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^5\right)}{5a^5} - \frac{\text{Subst}\left(\int \frac{1}{a^5+x}\,dx, x, x^5\right)}{5a^5}\\ &= \frac{\log(x)}{a^5} - \frac{\log\left(a^5+x^5\right)}{5a^5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(a^5+x^5)}\,dx = \frac{\log(x)}{a^5} - \frac{\log\left(a^5+x^5\right)}{5a^5}$$

```
[In] Integrate[1/(x*(a^5 + x^5)),x]
[Out] Log[x]/a^5 - Log[a^5 + x^5]/(5*a^5)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{\ln(x)}{a^5} - \frac{\ln(a^5+x^5)}{5a^5}$	21
parallelrisch	$\frac{5\ln(x)-\ln(a+x)-\ln(a^4-a^3x+a^2x^2-a\,x^3+x^4)}{5a^5}$	46
default	$\frac{\ln(x)}{a^5} - \frac{\ln(a^4-a^3x+a^2x^2-a\,x^3+x^4)}{5a^5} - \frac{\ln(a+x)}{5a^5}$	49
norman	$\frac{\ln(x)}{a^5} - \frac{\ln(a^4-a^3x+a^2x^2-a\,x^3+x^4)}{5a^5} - \frac{\ln(a+x)}{5a^5}$	49

```
[In] int(1/x/(a^5+x^5),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/a^5-1/5*ln(a^5+x^5)/a^5
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\left(a^5+x^5\right)}\,dx=-\frac{\log\left(a^5+x^5\right)-5\,\log\left(x\right)}{5\,a^5}$$

```
[In] integrate(1/x/(a^5+x^5),x, algorithm="fricas")
[Out] -1/5*(log(a^5 + x^5) - 5*log(x))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a^5+x^5\right)}\,dx=\frac{\log\left(x\right)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}$$

```
[In] integrate(1/x/(a**5+x**5),x)
[Out] log(x)/a**5 - log(a**5 + x**5)/(5*a**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a^5+x^5\right)}\,dx=-\frac{\log\left(a^5+x^5\right)}{5\,a^5}+\frac{\log\left(x^5\right)}{5\,a^5}$$

```
[In] integrate(1/x/(a^5+x^5),x, algorithm="maxima")
[Out] -1/5*log(a^5 + x^5)/a^5 + 1/5*log(x^5)/a^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a^5+x^5)}\,dx = -\frac{\log(|a^5+x^5|)}{5\,a^5} + \frac{\log(|x|)}{a^5}$$

```
[In] integrate(1/x/(a^5+x^5),x, algorithm="giac")
[Out] -1/5*log(abs(a^5 + x^5))/a^5 + log(abs(x))/a^5
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x(a^5+x^5)}\,dx = -\frac{\ln(a^5+x^5) - 5\,\ln(x)}{5\,a^5}$$

```
[In] int(1/(x*(a^5 + x^5)),x)
[Out] -(log(a^5 + x^5) - 5*log(x))/(5*a^5)
```

3.142 $\int \frac{1}{x^2(a^5+x^5)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 209

$$\int \frac{1}{x^2(a^5+x^5)}\,dx = -\frac{1}{a^5x}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^6}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^6}+\frac{\log(a+x)}{5a^6}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^6}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^6}$$

```
[Out] -1/a^5/x+1/5*ln(a+x)/a^6-1/20*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(-5^(1/2)+1)
/a^6-1/20*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(5^(1/2)+1)/a^6+1/10*arctan(1/20*
(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10-2*5^(1/2))^(1/2)/a^6+1/10
*arctan((-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)/
a^6
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {331, 299, 648, 632, 210, 642, 31}

$$\int \frac{1}{x^2(a^5+x^5)}\,dx = \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})}\arctan\left(\frac{(1-\sqrt{5})a-4x}{\sqrt{2(5+\sqrt{5})}a}\right)}{5a^6} + \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})}\arctan\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}((1+\sqrt{5})a-4x)}{2a}\right)}{5a^6} + \frac{\log(a+x)}{5a^6} - \frac{1}{a^5x} - \frac{(1-\sqrt{5})\log\left(a^2-\frac{1}{2}(1-\sqrt{5})ax+x^2\right)}{20a^6} - \frac{(1+\sqrt{5})\log\left(a^2-\frac{1}{2}(1+\sqrt{5})ax+x^2\right)}{20a^6}$$

```
[In] Int[1/(x^2*(a^5 + x^5)),x]
```

```
[Out] -(1/(a^5*x)) + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[
2*(5 + Sqrt[5])]*a)])/(5*a^6) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sq
rt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^6) + Log[a + x]/(5*a^6) -
((1 - Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^6) - ((1 + Sqr
t[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^6)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; (-(-r)^(m + 1)/(a*n*s^m))*Int[1/(r + s*x), x]
 + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Free
Q[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a
```

```
/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{a^5x} - \frac{\int \frac{x^3}{a^5+x^5}\,dx}{a^5} \\
&= -\frac{1}{a^5x} + \frac{\int \frac{1}{a+x}\,dx}{5a^6} - \frac{2\int \frac{\frac{1}{4}\left(1+\sqrt{5}\right)a-\frac{1}{4}\left(-1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a^6} - \frac{2\int \frac{\frac{1}{4}\left(1-\sqrt{5}\right)a-\frac{1}{4}\left(-1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a^6} \\
&= -\frac{1}{a^5x} + \frac{\log(a+x)}{5a^6} - \frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^6} \\
&\quad - \frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^6} - \frac{\left(5-\sqrt{5}\right)\int \frac{1}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^5} \\
&\quad - \frac{\left(5+\sqrt{5}\right)\int \frac{1}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^5}
\end{aligned}$$

$$= -\frac{1}{a^5x} + \frac{\log(a+x)}{5a^6} - \frac{(1+\sqrt{5})\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a^6} - \frac{(1-\sqrt{5})\log\left(2a^2-ax+\sqrt{5}ax+2x^2\right)}{20a^6} + \frac{(5-\sqrt{5})\,\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\left(5-\sqrt{5}\right)a^2-x^2}\,dx, x, -\frac{1}{2}(1+\sqrt{5})\,a+2x\right)}{10a^5} + \frac{(5+\sqrt{5})\,\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\left(5+\sqrt{5}\right)a^2-x^2}\,dx, x, -\frac{1}{2}(1-\sqrt{5})\,a+2x\right)}{10a^5}$$

$$= -\frac{1}{a^5x} + \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^6} + \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^6} + \frac{\log(a+x)}{5a^6} - \frac{(1+\sqrt{5})\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a^6} - \frac{(1-\sqrt{5})\log\left(2a^2-ax+\sqrt{5}ax+2x^2\right)}{20a^6}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\left(a^5+x^5\right)}\,dx = -\frac{\frac{20a}{x} + 2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right) + 2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right) - 4\log(a+x) -}{20a^6}$$

```
[In] Integrate[1/(x^2*(a^5 + x^5)),x]
```

```
[Out] -1/20*((20*a)/x + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(
Sqrt[2*(5 + Sqrt[5])]*a)] + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5])*
a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] - (-1 + Sqrt[5])*Log[a^2
 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + (1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a
*x)/2 + x^2])/a^6
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

method	result	size
risch	$-\frac{1}{a^{5} x}+\frac{\left(\sum_{\_R =\operatorname{RootOf}\left(a^{24} \_Z^{4}+a^{18} \_Z^{3}+a^{12} \_Z^{2}+a^{6} \_Z +1\right)}\_R \ln \left(\left(6 \_R^{5} a^{30}-5\right) x +a^{25} \_R^{4}\right)\right)}{5}+\frac{\ln \left(a +x \right)}{5 a^{6}}$	76
default	$-\frac{1}{a^{5} x}+\frac{\sum_{\_R =\operatorname{RootOf}\left(\_Z^{4}-a \_Z^{3}+\_Z^{2} a^{2}-a^{3} \_Z +a^{4}\right)}\frac{\left(-\_R^{3}-3 \_R^{2} a +2 a^{2} \_R -a^{3}\right) \ln \left(x -\_R \right)}{4 \_R^{3}-3 \_R^{2} a +2 a^{2} \_R -a^{3}}}{5 a^{6}}+\frac{\ln \left(a +x \right)}{5 a^{6}}$	109

```
[In] int(1/x^2/(a^5+x^5),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^5/x+1/5*sum(_R*ln((6*_R^5*a^30-5)*x+a^25*_R^4),_R=RootOf(_Z^4*a^24+_Z^
3*a^18+_Z^2*a^12+_Z*a^6+1))+1/5*ln(a+x)/a^6
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 15275, normalized size of antiderivative = 73.09

$$\int \frac{1}{x^2 (a^5 + x^5)} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^2 (a^5 + x^5)} \, dx = -\frac{1}{a^5 x} + \frac{\frac{\log{(a+x)}}{5} + \operatorname{RootSum}{\left(625t^4 + 125t^3 + 25t^2 + 5t + 1, \left(t \mapsto t \log{\left(625t^4 a + x \right)}\right)\right)}}{a^6}$$

```
[In] integrate(1/x**2/(a**5+x**5),x)
```

```
[Out] -1/(a**5*x) + (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*
_t + 1, Lambda(_t, _t*log(625*_t**4*a + x))))/a**6
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\left(a^5+x^5\right)}\,dx = -\frac{\frac{2\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\sqrt{5}+10}}\right)}{a\sqrt{2\sqrt{5}+10}}+\frac{2\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\sqrt{5}+10}}\right)}{a\sqrt{-2\sqrt{5}+10}}+\frac{\left(\sqrt{5}+3\right)\log\left(-ax\left(\sqrt{5}+1\right)+2\,a^2+2\,x^2\right)}{a\left(\sqrt{5}+1\right)}+\frac{\left(\sqrt{5}-3\right)\mathrm{lo}}{}}{10\,a^5} - \frac{1}{a^5x}$$

[In] integrate(1/x^2/(a^5+x^5),x, algorithm="maxima")

```
[Out] -1/10*(2*sqrt(5)*(sqrt(5) + 1)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqr
t(5) + 10)))/(a*sqrt(2*sqrt(5) + 10)) + 2*sqrt(5)*(sqrt(5) - 1)*arctan(-(a*
(sqrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/(a*sqrt(-2*sqrt(5) + 10)) +
 (sqrt(5) + 3)*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5) + 1)) +
(sqrt(5) - 3)*log(a*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5) - 1)) - 2*
log(a + x)/a)/a^5 - 1/(a^5*x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\left(a^5+x^5\right)}\,dx = -\frac{\sqrt{2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\sqrt{5}+10}}\right)}{10\,a^6} - \frac{\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10\,a^6} - \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20\,a^6} + \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20\,a^6} - \frac{\log\left(|a^4-a^3x+a^2x^2-ax^3+x^4|\right)}{20\,a^6} + \frac{\log\left(|a+x|\right)}{5\,a^6} - \frac{1}{a^5x}$$

[In] integrate(1/x^2/(a^5+x^5),x, algorithm="giac")

```
[Out] -1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/a^6 - 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(
```

```
a*sqrt(-2*sqrt(5) + 10)))/a^6 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*
x + x^2)/a^6 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^6 - 1/
20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^6 + 1/5*log(abs(a + x))/
a^6 - 1/(a^5*x)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a^5+x^5)}\,dx = \frac{\ln(a+x)}{5a^6} - \frac{1}{a^5x} + \frac{\ln\left(5a^{30} + \frac{5x\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)a^{29}}{4}\right)\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)}{20a^6} - \frac{\ln\left(5a^{30} - \frac{5a^{29}x\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{4}\right)\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{20a^6} - \frac{\ln\left(5a^{30} - \frac{5a^{29}x\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{4}\right)\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{20a^6} - \frac{\ln\left(5a^{30} - \frac{5a^{29}x\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{4}\right)\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{20a^6}$$

```
[In] int(1/(x^2*(a^5 + x^5)),x)
```

```
[Out] log(a + x)/(5*a^6) - 1/(a^5*x) + (log(5*a^30 + (5*a^29*x*(5^(1/2) + (- 2*5^
(1/2) - 10)^(1/2) - 1))/4)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*a^
6) - (log(5*a^30 - (5*a^29*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^
(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^6) - (log(5*a^30 - (5*a^29*x*(5^
(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) +
 1))/(20*a^6) - (log(5*a^30 - (5*a^29*x*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2)
 + 1))/4)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a^6)
```

3.143 $\int \frac{1}{x^3(a^5+x^5)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 211

$$\int \frac{1}{x^3(a^5+x^5)}\,dx = -\frac{1}{2a^5x^2} - \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^7} + \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^7} - \frac{\log(a+x)}{5a^7} + \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^7} + \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^7}$$

```
[Out] -1/2/a^5/x^2-1/5*ln(a+x)/a^7+1/20*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(-5^(1/2)
+1)/a^7+1/20*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(5^(1/2)+1)/a^7-1/10*arctan((
-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)/a^7+1/10*
arctan(1/20*(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10+2*5^(1/2))^(1
/2)/a^7
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {331, 299, 648, 632, 210, 642, 31}

$$\int \frac{1}{x^3(a^5+x^5)}\,dx = -\frac{\sqrt{\frac{1}{2}(5-\sqrt{5})}\arctan\left(\frac{(1-\sqrt{5})a-4x}{\sqrt{2(5+\sqrt{5})}a}\right)}{5a^7} + \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})}\arctan\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}((1+\sqrt{5})a-4x)}{2a}\right)}{5a^7} - \frac{\log(a+x)}{5a^7} - \frac{1}{2a^5x^2} + \frac{(1+\sqrt{5})\log\left(a^2-\frac{1}{2}(1-\sqrt{5})ax+x^2\right)}{20a^7} + \frac{(1-\sqrt{5})\log\left(a^2-\frac{1}{2}(1+\sqrt{5})ax+x^2\right)}{20a^7}$$

```
[In] Int[1/(x^3*(a^5 + x^5)),x]
```

```
[Out] -1/2*1/(a^5*x^2) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)])/(5*a^7) + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^7) - Log[a + x]/(5*a^7) + ((1 + Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^7) + ((1 - Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^7)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k - 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; (-(-r)^(m + 1)/(a*n*s^m))*Int[1/(r + s*x), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a
```

```
/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{2a^5x^2} - \frac{\int \frac{x^2}{a^5+x^5}\,dx}{a^5} \\
&= -\frac{1}{2a^5x^2} - \frac{\int \frac{1}{a+x}\,dx}{5a^7} - \frac{2\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)a-\frac{1}{4}\left(1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a^7} - \frac{2\int \frac{\frac{1}{4}\left(-1+\sqrt{5}\right)a-\frac{1}{4}\left(1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a^7} \\
&= -\frac{1}{2a^5x^2} - \frac{\log(a+x)}{5a^7} + \frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^7} \\
&\quad + \frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^7} + \frac{\int \frac{1}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{2\sqrt{5}a^6} - \frac{\int \frac{1}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{2\sqrt{5}a^6}
\end{aligned}
$$

$$= -\frac{1}{2a^5x^2} - \frac{\log(a+x)}{5a^7} + \frac{(1-\sqrt{5})\log\left(2a^2 - ax - \sqrt{5}ax + 2x^2\right)}{20a^7} + \frac{(1+\sqrt{5})\log\left(2a^2 - ax + \sqrt{5}ax + 2x^2\right)}{20a^7} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{2}(5-\sqrt{5})a^2 - x^2}\,dx, x, -\frac{1}{2}(1+\sqrt{5})\,a + 2x\right)}{\sqrt{5}a^6} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{2}(5+\sqrt{5})a^2 - x^2}\,dx, x, -\frac{1}{2}(1-\sqrt{5})\,a + 2x\right)}{\sqrt{5}a^6}$$

$$= -\frac{1}{2a^5x^2} - \frac{\sqrt{\frac{2}{5(5+\sqrt{5})}}\arctan\left(\frac{(1-\sqrt{5})a - 4x}{\sqrt{2(5+\sqrt{5})}a}\right)}{a^7} + \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})}\arctan\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}\left((1+\sqrt{5})a - 4x\right)}{2a}\right)}{5a^7} - \frac{\log(a+x)}{5a^7} + \frac{(1-\sqrt{5})\log\left(2a^2 - ax - \sqrt{5}ax + 2x^2\right)}{20a^7} + \frac{(1+\sqrt{5})\log\left(2a^2 - ax + \sqrt{5}ax + 2x^2\right)}{20a^7}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3(a^5+x^5)}\,dx = -\frac{\frac{10a^2}{x^2} - 2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{(-1+\sqrt{5})a + 4x}{\sqrt{2(5+\sqrt{5})}a}\right) + 2\sqrt{2(5+\sqrt{5})}\arctan\left(\frac{-((1+\sqrt{5})a) + 4x}{\sqrt{10-2\sqrt{5}}a}\right) + 4\log(a+x)}{20a^7}$$

```
[In] Integrate[1/(x^3*(a^5 + x^5)),x]
```

```
[Out] -1/20*((10*a^2)/x^2 - 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x
)/(Sqrt[2*(5 + Sqrt[5])]*a)] + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[
5])*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] + 4*Log[a + x] - (1 + Sqrt[5])*Log[
a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + (-1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5
])*a*x)/2 + x^2])/a^7
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.37

method	result	size
risch	$-\frac{1}{2a^5x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{28}\_Z^4-a^{21}\_Z^3+a^{14}\_Z^2-a^7\_Z+1\right)}\_R\ln\left(\left(-6\_R^5a^{35}-5\right)x+a^{15}\_R^2\right)\right)}{5}-\frac{\ln(a+x)}{5a^7}$	78
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(\_R^3-2\_R^2a-2a^2\_R+a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2a^2\_R-a^3}}{5a^7}-\frac{1}{2a^5x^2}-\frac{\ln(a+x)}{5a^7}$	105

```
[In] int(1/x^3/(a^5+x^5),x,method=_RETURNVERBOSE)
[Out] -1/2/a^5/x^2+1/5*sum(_R*ln((-6*_R^5*a^35-5)*x+a^15*_R^2),_R=RootOf(_Z^4*a^2
8-_Z^3*a^21+_Z^2*a^14-_Z*a^7+1))-1/5*ln(a+x)/a^7
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 15499, normalized size of antiderivative = 73.45

$$\int \frac{1}{x^3\left(a^5+x^5\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^3/(a^5+x^5),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^3\left(a^5+x^5\right)}\,dx = -\frac{1}{2a^5x^2}+\frac{-\frac{\log(a+x)}{5}+\operatorname{RootSum}\left(625t^4-125t^3+25t^2-5t+1,\left(t\mapsto t\log\left(25t^2a+x\right)\right)\right)}{a^7}$$

```
[In] integrate(1/x**3/(a**5+x**5),x)
[Out] -1/(2*a**5*x**2) + (-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**
2 - 5*_t + 1, Lambda(_t, _t*log(25*_t**2*a + x))))/a**7
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3\left(a^5+x^5\right)}\,dx$$
$$=\frac{\frac{2\sqrt{5}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{a^2\sqrt{2\sqrt{5}+10}}-\frac{2\sqrt{5}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{a^2\sqrt{-2\sqrt{5}+10}}-\frac{\log(a+x)}{a^2}-\frac{\log\left(-ax\left(\sqrt{5}+1\right)+2a^2+2x^2\right)}{a^2\left(\sqrt{5}+1\right)}+\frac{\log\left(ax\left(\sqrt{5}-1\right)+2a^2+2x^2\right)}{a^2\left(\sqrt{5}-1\right)}}{5a^5}$$
$$-\frac{1}{2a^5x^2}$$

```
[In] integrate(1/x^3/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] 1/5*(2*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a^
2*sqrt(2*sqrt(5) + 10)) - 2*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sqrt
(-2*sqrt(5) + 10)))/(a^2*sqrt(-2*sqrt(5) + 10)) - log(a + x)/a^2 - log(-a*x
*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) + 1)) + log(a*x*(sqrt(5) - 1)
 + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) - 1)))/a^5 - 1/2/(a^5*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3\left(a^5+x^5\right)}\,dx=\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^7}$$
$$-\frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^7}$$
$$-\frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^7}$$
$$+\frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20a^7}$$
$$+\frac{\log\left(\left|a^4-a^3x+a^2x^2-ax^3+x^4\right|\right)}{20a^7}-\frac{\log\left(\left|a+x\right|\right)}{5a^7}-\frac{1}{2a^5x^2}$$

```
[In] integrate(1/x^3/(a^5+x^5),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/a^7 - 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a
```

```
*sqrt(-2*sqrt(5) + 10)))/a^7 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x
 + x^2)/a^7 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^7 + 1/2
0*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^7 - 1/5*log(abs(a + x))/a
^7 - 1/2/(a^5*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(a^5+x^5)}\,dx = \frac{\ln\left(a^{20}-\frac{a^{19}x\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^7} - \frac{1}{2\,a^5\,x^2} - \frac{\ln\left(a^{20}+\frac{x\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)^3a^{19}}{64}\right)\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)}{20\,a^7} - \frac{\ln(a+x)}{5\,a^7} + \frac{\ln\left(a^{20}-\frac{a^{19}x\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^7} + \frac{\ln\left(a^{20}-\frac{a^{19}x\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)^3}{64}\right)\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{20\,a^7}$$

```
[In] int(1/(x^3*(a^5 + x^5)),x)
```

```
[Out] (log(a^20 - (a^19*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2)
+ (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^7) - 1/(2*a^5*x^2) - (log(a^20 + (a^19
*x*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1)^3)/64)*(5^(1/2) + (- 2*5^(1/2)
- 10)^(1/2) - 1))/(20*a^7) - log(a + x)/(5*a^7) + (log(a^20 - (a^19*x*(5^(1
/2) - (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2)
+ 1))/(20*a^7) + (log(a^20 - (a^19*x*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) +
1)^3)/64)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a^7)
```

3.144 $\int \frac{1}{x^4(a^5+x^5)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 211

$$\int \frac{1}{x^4(a^5+x^5)}\,dx=-\frac{1}{3a^5x^3}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^8}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^8}+\frac{\log(a+x)}{5a^8}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^8}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^8}$$

```
[Out] -1/3/a^5/x^3+1/5*ln(a+x)/a^8-1/20*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(-5^(1/2)
+1)/a^8-1/20*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(5^(1/2)+1)/a^8-1/10*arctan((
-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)/a^8+1/10*
arctan(1/20*(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10+2*5^(1/2))^(1
/2)/a^8
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {331, 299, 648, 632, 210, 642, 31}

$$\int \frac{1}{x^4(a^5+x^5)}\,dx=-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^8}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^8}+\frac{\log(a+x)}{5a^8}-\frac{1}{3a^5x^3}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^8}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^8}$$

```
[In] Int[1/(x^4*(a^5 + x^5)),x]
```

```
[Out] -1/3*1/(a^5*x^3) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(S
qrt[2*(5 + Sqrt[5])]*a)])/(5*a^8) + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5
+ Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^8) + Log[a + x]/(5*a^8
) - ((1 + Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^8) - ((1 -
 Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^8)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; (-(-r)^(m + 1)/(a*n*s^m))*Int[1/(r + s*x), x]
 + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Free
Q[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a
```

```
/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{1}{3a^5x^3} - \frac{\int \frac{x}{a^5+x^5}\,dx}{a^5}$$

$$= -\frac{1}{3a^5x^3} + \frac{\int \frac{1}{a+x}\,dx}{5a^8} - \frac{2\int \frac{\frac{1}{4}\left(1-\sqrt{5}\right)a-\frac{1}{4}\left(-1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a^8} - \frac{2\int \frac{\frac{1}{4}\left(1+\sqrt{5}\right)a-\frac{1}{4}\left(-1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a^8}$$

$$= -\frac{1}{3a^5x^3} + \frac{\log(a+x)}{5a^8} - \frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^8} - \frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^8} + \frac{\int \frac{1}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{2\sqrt{5}a^7} - \frac{\int \frac{1}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{2\sqrt{5}a^7}$$

$$= -\frac{1}{3a^5x^3} + \frac{\log(a+x)}{5a^8} - \frac{(1-\sqrt{5})\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a^8} - \frac{(1+\sqrt{5})\log\left(2a^2-ax+\sqrt{5}ax+2x^2\right)}{20a^8} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\left(5-\sqrt{5}\right)a^2-x^2}\,dx, x, -\frac{1}{2}\left(1+\sqrt{5}\right)a+2x\right)}{\sqrt{5}a^7} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\left(5+\sqrt{5}\right)a^2-x^2}\,dx, x, -\frac{1}{2}\left(1-\sqrt{5}\right)a+2x\right)}{\sqrt{5}a^7}$$

$$= -\frac{1}{3a^5x^3} - \frac{\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{a^8} + \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^8} + \frac{\log(a+x)}{5a^8} - \frac{(1-\sqrt{5})\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a^8} - \frac{(1+\sqrt{5})\log\left(2a^2-ax+\sqrt{5}ax+2x^2\right)}{20a^8}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^4\left(a^5+x^5\right)}\,dx$$

$$= \frac{-\frac{20a^3}{x^3} + 6\sqrt{10-2\sqrt{5}}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right) - 6\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right) + 12\log(a+x) -}{60a^8}$$

```
[In] Integrate[1/(x^4*(a^5 + x^5)),x]
```

```
[Out] ((-20*a^3)/x^3 + 6*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sq
rt[2*(5 + Sqrt[5])]*a)] - 6*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])*a
) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] + 12*Log[a + x] - 3*(1 + Sqrt[5])*Log[a^
2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + 3*(-1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5
])*a*x)/2 + x^2])/(60*a^8)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.38

method	result	size
risch	$-\frac{1}{3a^5x^3}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{32}\_Z^4+a^{24}\_Z^3+a^{16}\_Z^2+a^8\_Z+1\right)}\_R\ln\left(\left(-6\_R^5a^{40}+5\right)x-a^{25}\_R^3\right)\right)}{5}+\frac{\ln(-a-x)}{5a^8}$	81
default	$-\frac{1}{3a^5x^3}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3+2\_R^2a-3a^2\_R-a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2a^2\_R-a^3}}{5a^8}+\frac{\ln(a+x)}{5a^8}$	109

```
[In] int(1/x^4/(a^5+x^5),x,method=_RETURNVERBOSE)
[Out] -1/3/a^5/x^3+1/5*sum(_R*ln((-6*_R^5*a^40+5)*x-a^25*_R^3),_R=RootOf(_Z^4*a^3
2+_Z^3*a^24+_Z^2*a^16+_Z*a^8+1))+1/5/a^8*ln(-a-x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 15501, normalized size of antiderivative = 73.46

$$\int \frac{1}{x^4(a^5+x^5)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^4/(a^5+x^5),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^4(a^5+x^5)}\,dx = -\frac{1}{3a^5x^3}+\frac{\frac{\log(a+x)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(125t^3a+x\right)\right)\right)}{a^8}$$

```
[In] integrate(1/x**4/(a**5+x**5),x)
[Out] -1/(3*a**5*x**3) + (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2
 + 5*_t + 1, Lambda(_t, _t*log(125*_t**3*a + x))))/a**8
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4(a^5+x^5)}\,dx = \frac{\frac{2\sqrt{5}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{a^3\sqrt{2\sqrt{5}+10}} - \frac{2\sqrt{5}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{a^3\sqrt{-2\sqrt{5}+10}} + \frac{\log(a+x)}{a^3} + \frac{\log\left(-ax(\sqrt{5}+1)+2a^2+2x^2\right)}{a^3(\sqrt{5}+1)} - \frac{\log\left(ax(\sqrt{5}-1)+2a^2+2x^2\right)}{a^3(\sqrt{5}-1)}}{5a^5} - \frac{1}{3a^5x^3}$$

```
[In] integrate(1/x^4/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] 1/5*(2*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a^
3*sqrt(2*sqrt(5) + 10)) - 2*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sqrt
(-2*sqrt(5) + 10)))/(a^3*sqrt(-2*sqrt(5) + 10)) + log(a + x)/a^3 + log(-a*x
*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^3*(sqrt(5) + 1)) - log(a*x*(sqrt(5) - 1)
 + 2*a^2 + 2*x^2)/(a^3*(sqrt(5) - 1)))/a^5 - 1/3/(a^5*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4(a^5+x^5)}\,dx = \frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^8} - \frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^8} + \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}(\sqrt{5}a+a)x+x^2\right)}{20a^8} - \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}(\sqrt{5}a-a)x+x^2\right)}{20a^8} - \frac{\log\left(|a^4-a^3x+a^2x^2-ax^3+x^4|\right)}{20a^8} + \frac{\log\left(|a+x|\right)}{5a^8} - \frac{1}{3a^5x^3}$$

```
[In] integrate(1/x^4/(a^5+x^5),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/a^8 - 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a
```

```
*sqrt(-2*sqrt(5) + 10)))/a^8 + 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x
 + x^2)/a^8 - 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^8 - 1/2
0*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^8 + 1/5*log(abs(a + x))/a
^8 - 1/3/(a^5*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^4 (a^5+x^5)}\,dx = \frac{\ln(a+x)}{5\,a^8} - \frac{1}{3\,a^5\,x^3} - \frac{\ln\left(a^{15}\,x - \frac{a^{16}\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a^8} - \frac{\ln\left(a^{15}\,x - \frac{a^{16}\left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}+1\right)^3}{64}\right)\left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}+1\right)}{20\,a^8} + \frac{\ln\left(\frac{\left(\sqrt{5}+\sqrt{-2\,\sqrt{5}-10}-1\right)^3 a^{16}}{64} + x\,a^{15}\right)\left(\sqrt{5}+\sqrt{-2\,\sqrt{5}-10}-1\right)}{20\,a^8} - \frac{\ln\left(a^{15}\,x - \frac{a^{16}\left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a^8}$$

```
[In] int(1/(x^4*(a^5 + x^5)),x)
```

```
[Out] log(a + x)/(5*a^8) - 1/(3*a^5*x^3) - (log(a^15*x - (a^16*(5^(1/2) - (2*5^(1
/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^8
) - (log(a^15*x - (a^16*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1)^3)/64)*((-
 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a^8) + (log(a^15*x + (a^16*(5^(1
/2) + (- 2*5^(1/2) - 10)^(1/2) - 1)^3)/64)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1
/2) - 1))/(20*a^8) - (log(a^15*x - (a^16*(5^(1/2) + (2*5^(1/2) - 10)^(1/2)
+ 1)^3)/64)*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^8)
```

3.145 $\int \frac{x^{-m}}{a^5+x^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{x^{-m}}{a^5+x^5} \, dx = \frac{x^{1-m} \text{Hypergeometric2F1}\left(1, \frac{1-m}{5}, \frac{6-m}{5}, -\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

```
[Out] x^(1-m)*hypergeom([1, 1/5-1/5*m],[6/5-1/5*m],-x^5/a^5)/a^5/(1-m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{371\}$

$$\int \frac{x^{-m}}{a^5+x^5} \, dx = \frac{x^{1-m} \text{Hypergeometric2F1}\left(1, \frac{1-m}{5}, \frac{6-m}{5}, -\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

```
[In] Int[1/(x^m*(a^5 + x^5)),x]
```

```
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/5, (6 - m)/5, -(x^5/a^5)])/(a^5*(1
- m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{x^{1-m} \text{Hypergeometric2F1}\left(1, \frac{1-m}{5}, \frac{6-m}{5}, -\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{x^{-m}}{a^5+x^5}\,dx=-\frac{x^{1-m}\,\text{Hypergeometric2F1}\left(1,\frac{1}{5}-\frac{m}{5},\frac{6}{5}-\frac{m}{5},-\frac{x^5}{a^5}\right)}{a^5(-1+m)}$$

```
[In] Integrate[1/(x^m*(a^5 + x^5)),x]
[Out] -((x^(1 - m)*Hypergeometric2F1[1, 1/5 - m/5, 6/5 - m/5, -(x^5/a^5)])/(a^5*(
-1 + m)))
```

Maple [F]

$$\int \frac{x^{-m}}{a^5+x^5}dx$$

```
[In] int(1/(x^m)/(a^5+x^5),x)
[Out] int(1/(x^m)/(a^5+x^5),x)
```

Fricas [F]

$$\int \frac{x^{-m}}{a^5+x^5}\,dx=\int \frac{1}{(a^5+x^5)x^m}\,dx$$

```
[In] integrate(1/(x^m)/(a^5+x^5),x, algorithm="fricas")
[Out] integral(1/((a^5 + x^5)*x^m), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int \frac{x^{-m}}{a^5+x^5}\,dx=-\frac{mx^{1-m}\Phi\left(\frac{x^5e^{i\pi}}{a^5},1,\frac{1}{5}-\frac{m}{5}\right)\Gamma\left(\frac{1}{5}-\frac{m}{5}\right)}{25a^5\Gamma\left(\frac{6}{5}-\frac{m}{5}\right)}+\frac{x^{1-m}\Phi\left(\frac{x^5e^{i\pi}}{a^5},1,\frac{1}{5}-\frac{m}{5}\right)\Gamma\left(\frac{1}{5}-\frac{m}{5}\right)}{25a^5\Gamma\left(\frac{6}{5}-\frac{m}{5}\right)}$$

```
[In] integrate(1/(x**m)/(a**5+x**5),x)
[Out] -m*x**(1 - m)*lerchphi(x**5*exp_polar(I*pi)/a**5, 1, 1/5 - m/5)*gamma(1/5 -
 m/5)/(25*a**5*gamma(6/5 - m/5)) + x**(1 - m)*lerchphi(x**5*exp_polar(I*pi)
/a**5, 1, 1/5 - m/5)*gamma(1/5 - m/5)/(25*a**5*gamma(6/5 - m/5))
```

Maxima [F]

$$\int \frac{x^{-m}}{a^5+x^5}\,dx=\int \frac{1}{(a^5+x^5)x^m}\,dx$$

```
[In] integrate(1/(x^m)/(a^5+x^5),x, algorithm="maxima")
[Out] integrate(1/((a^5 + x^5)*x^m), x)
```

Giac [F]

$$\int \frac{x^{-m}}{a^5+x^5}\,dx=\int \frac{1}{(a^5+x^5)x^m}\,dx$$

```
[In] integrate(1/(x^m)/(a^5+x^5),x, algorithm="giac")
[Out] integrate(1/((a^5 + x^5)*x^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-m}}{a^5+x^5}\,dx=\int \frac{1}{x^m\,(a^5+x^5)}\,dx$$

```
[In] int(1/(x^m*(a^5 + x^5)),x)
[Out] int(1/(x^m*(a^5 + x^5)), x)
```

3.146 $\int \frac{1+x^4}{1+x^6} dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{1+x^4}{1+x^6} dx = -\frac{1}{3}\arctan\left(\sqrt{3}-2x\right)+\frac{2\arctan(x)}{3}+\frac{1}{3}\arctan\left(\sqrt{3}+2x\right)$$

```
[Out] 2/3*arctan(x)+1/3*arctan(2*x-3^(1/2))+1/3*arctan(2*x+3^(1/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {1890, 215, 648, 632, 210, 642, 209, 301}

$$\int \frac{1+x^4}{1+x^6} dx = -\frac{1}{3}\arctan\left(\sqrt{3}-2x\right)+\frac{2\arctan(x)}{3}+\frac{1}{3}\arctan\left(2x+\sqrt{3}\right)$$

```
[In] Int[(1 + x^4)/(1 + x^6),x]
```

```
[Out] -1/3*ArcTan[Sqrt[3] - 2*x] + (2*ArcTan[x])/3 + ArcTan[Sqrt[3] + 2*x]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
```

```
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{1+x^6}+\frac{x^4}{1+x^6}\right) dx \\
&= \int \frac{1}{1+x^6}\,dx+\int \frac{x^4}{1+x^6}\,dx \\
&= \frac{1}{3}\int \frac{1-\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx+\frac{1}{3}\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx \\
&\quad +\frac{1}{3}\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx+\frac{1}{3}\int \frac{1+\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx+\frac{2}{3}\int \frac{1}{1+x^2}\,dx \\
&= \frac{2\arctan(x)}{3}+2\left(\frac{1}{12}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx\right)+2\left(\frac{1}{12}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx\right) \\
&= \frac{2\arctan(x)}{3}-2\left(\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x\right)\right) \\
&\quad -2\left(\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\sqrt{3}+2x\right)\right) \\
&= -\frac{1}{3}\arctan\left(\sqrt{3}-2x\right)+\frac{2\arctan(x)}{3}+\frac{1}{3}\arctan\left(\sqrt{3}+2x\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \frac{1+x^4}{1+x^6}\,dx=\frac{2\arctan(x)}{3}-\frac{1}{3}\arctan\left(\frac{x}{-1+x^2}\right)$$

```
[In] Integrate[(1 + x^4)/(1 + x^6),x]
[Out] (2*ArcTan[x])/3 - ArcTan[x/(-1 + x^2)]/3
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.29

method	result
risch	$\arctan(x)+\frac{\arctan(x^3)}{3}$
default	$\frac{2\arctan(x)}{3}+\frac{\arctan\left(2x-\sqrt{3}\right)}{3}+\frac{\arctan\left(2x+\sqrt{3}\right)}{3}$
parallelrisch	$\frac{i\ln(x+i)}{3}-\frac{i\ln(x-i)}{3}+\frac{i\ln(x^2+ix-1)}{6}-\frac{i\ln(x^2-ix-1)}{6}$
meijerg	$\frac{x^5\sqrt{3}\,\ln\left(1-\sqrt{3}\,(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{12(x^6)^{\frac{5}{6}}}+\frac{x^5\arctan\left(\frac{(x^6)^{\frac{1}{6}}}{2-\sqrt{3}\,(x^6)^{\frac{1}{6}}}\right)}{6(x^6)^{\frac{5}{6}}}+\frac{x^5\arctan\left((x^6)^{\frac{1}{6}}\right)}{3(x^6)^{\frac{5}{6}}}-\frac{x^5\sqrt{3}\,\ln\left(1+\sqrt{3}\,(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{12(x^6)^{\frac{5}{6}}}$

```
[In] int((x^4+1)/(x^6+1),x,method=_RETURNVERBOSE)
[Out] arctan(x)+1/3*arctan(x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.26

$$\int \frac{1+x^4}{1+x^6}\,dx=\frac{1}{3}\arctan\left(x^3\right)+\arctan(x)$$

```
[In] integrate((x^4+1)/(x^6+1),x, algorithm="fricas")
[Out] 1/3*arctan(x^3) + arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.23

$$\int \frac{1+x^4}{1+x^6}\,dx=\operatorname{atan}(x)+\frac{\operatorname{atan}(x^3)}{3}$$

```
[In] integrate((x**4+1)/(x**6+1),x)
[Out] atan(x) + atan(x**3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{1+x^4}{1+x^6}\,dx = \frac{1}{3}\arctan\left(2\,x+\sqrt{3}\right)+\frac{1}{3}\arctan\left(2\,x-\sqrt{3}\right)+\frac{2}{3}\arctan\left(x\right)$$

```
[In] integrate((x^4+1)/(x^6+1),x, algorithm="maxima")
[Out] 1/3*arctan(2*x + sqrt(3)) + 1/3*arctan(2*x - sqrt(3)) + 2/3*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{1+x^4}{1+x^6}\,dx = \frac{1}{3}\arctan\left(2\,x+\sqrt{3}\right)+\frac{1}{3}\arctan\left(2\,x-\sqrt{3}\right)+\frac{2}{3}\arctan\left(x\right)$$

```
[In] integrate((x^4+1)/(x^6+1),x, algorithm="giac")
[Out] 1/3*arctan(2*x + sqrt(3)) + 1/3*arctan(2*x - sqrt(3)) + 2/3*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.26

$$\int \frac{1+x^4}{1+x^6}\,dx = \frac{\mathrm{atan}(x^3)}{3}+\mathrm{atan}(x)$$

```
[In] int((x^4 + 1)/(x^6 + 1),x)
[Out] atan(x^3)/3 + atan(x)
```

3.147 $\int \frac{1}{(5+3x+x^2)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = \frac{3+2x}{22\,(5+3x+x^2)^2}+\frac{3(3+2x)}{121\,(5+3x+x^2)}+\frac{12\arctan\left(\frac{3+2x}{\sqrt{11}}\right)}{121\sqrt{11}}$$

```
[Out] 1/22*(3+2*x)/(x^2+3*x+5)^2+3/121*(3+2*x)/(x^2+3*x+5)+12/1331*arctan(1/11*(3
+2*x)*11^(1/2))*11^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {628, 632, 210}

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = \frac{12\arctan\left(\frac{2x+3}{\sqrt{11}}\right)}{121\sqrt{11}}+\frac{3(2x+3)}{121\,(x^2+3x+5)}+\frac{2x+3}{22\,(x^2+3x+5)^2}$$

```
[In] Int[(5 + 3*x + x^2)^(-3),x]
```

```
[Out] (3 + 2*x)/(22*(5 + 3*x + x^2)^2) + (3*(3 + 2*x))/(121*(5 + 3*x + x^2)) + (1
2*ArcTan[(3 + 2*x)/Sqrt[11]])/(121*Sqrt[11])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3+2x}{22\left(5+3x+x^2\right)^2}+\frac{3}{11}\int \frac{1}{\left(5+3x+x^2\right)^2}\,dx \\
&= \frac{3+2x}{22\left(5+3x+x^2\right)^2}+\frac{3(3+2x)}{121\left(5+3x+x^2\right)}+\frac{6}{121}\int \frac{1}{5+3x+x^2}\,dx \\
&= \frac{3+2x}{22\left(5+3x+x^2\right)^2}+\frac{3(3+2x)}{121\left(5+3x+x^2\right)}-\frac{12}{121}\text{Subst}\left(\int \frac{1}{-11-x^2}\,dx,x,3+2x\right) \\
&= \frac{3+2x}{22\left(5+3x+x^2\right)^2}+\frac{3(3+2x)}{121\left(5+3x+x^2\right)}+\frac{12\arctan\left(\frac{3+2x}{\sqrt{11}}\right)}{121\sqrt{11}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(5+3x+x^2\right)^3}\,dx=\frac{\frac{11(3+2x)\left(41+18x+6x^2\right)}{\left(5+3x+x^2\right)^2}+24\sqrt{11}\arctan\left(\frac{3+2x}{\sqrt{11}}\right)}{2662}$$

```
[In] Integrate[(5 + 3*x + x^2)^(-3),x]
```

```
[Out] ((11*(3 + 2*x)*(41 + 18*x + 6*x^2))/(5 + 3*x + x^2)^2 + 24*Sqrt[11]*ArcTan[
(3 + 2*x)/Sqrt[11]])/2662
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{\frac{6}{121}x^3+\frac{27}{121}x^2+\frac{68}{121}x+\frac{123}{242}}{\left(x^2+3x+5\right)^2}+\frac{12\arctan\left(\frac{(3+2x)\sqrt{11}}{11}\right)\sqrt{11}}{1331}$	44
default	$\frac{3+2x}{22\left(x^2+3x+5\right)^2}+\frac{\frac{9}{121}+\frac{6x}{121}}{x^2+3x+5}+\frac{12\arctan\left(\frac{(3+2x)\sqrt{11}}{11}\right)\sqrt{11}}{1331}$	52

```
[In] int(1/(x^2+3*x+5)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (6/121*x^3+27/121*x^2+68/121*x+123/242)/(x^2+3*x+5)^2+12/1331*arctan(1/11*(
3+2*x)*11^(1/2))*11^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(5+3x+x^2\right)^3}\,dx = \frac{132\,x^3+24\,\sqrt{11}\left(x^4+6\,x^3+19\,x^2+30\,x+25\right)\arctan\left(\frac{1}{11}\,\sqrt{11}(2\,x+3)\right)+594\,x^2+1496\,x+1353}{2662\,\left(x^4+6\,x^3+19\,x^2+30\,x+25\right)}$$

```
[In] integrate(1/(x^2+3*x+5)^3,x, algorithm="fricas")
```

```
[Out] 1/2662*(132*x^3 + 24*sqrt(11)*(x^4 + 6*x^3 + 19*x^2 + 30*x + 25)*arctan(1/1
1*sqrt(11)*(2*x + 3)) + 594*x^2 + 1496*x + 1353)/(x^4 + 6*x^3 + 19*x^2 + 30
*x + 25)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(5+3x+x^2\right)^3}\,dx = \frac{12x^3+54x^2+136x+123}{242x^4+1452x^3+4598x^2+7260x+6050} + \frac{12\sqrt{11}\,\mathrm{atan}\left(\frac{2\sqrt{11}x}{11}+\frac{3\sqrt{11}}{11}\right)}{1331}$$

```
[In] integrate(1/(x**2+3*x+5)**3,x)
```

```
[Out] (12*x**3 + 54*x**2 + 136*x + 123)/(242*x**4 + 1452*x**3 + 4598*x**2 + 7260*
x + 6050) + 12*sqrt(11)*atan(2*sqrt(11)*x/11 + 3*sqrt(11)/11)/1331
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = \frac{12}{1331}\,\sqrt{11}\arctan\left(\frac{1}{11}\,\sqrt{11}(2\,x+3)\right) + \frac{12\,x^3+54\,x^2+136\,x+123}{242\,(x^4+6\,x^3+19\,x^2+30\,x+25)}$$

```
[In] integrate(1/(x^2+3*x+5)^3,x, algorithm="maxima")
```

```
[Out] 12/1331*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 3)) + 1/242*(12*x^3 + 54*x^2 +
 136*x + 123)/(x^4 + 6*x^3 + 19*x^2 + 30*x + 25)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = \frac{12}{1331}\,\sqrt{11}\arctan\left(\frac{1}{11}\,\sqrt{11}(2\,x+3)\right) + \frac{12\,x^3+54\,x^2+136\,x+123}{242\,(x^2+3\,x+5)^2}$$

```
[In] integrate(1/(x^2+3*x+5)^3,x, algorithm="giac")
```

```
[Out] 12/1331*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 3)) + 1/242*(12*x^3 + 54*x^2 +
 136*x + 123)/(x^2 + 3*x + 5)^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = 6\left(x+\frac{3}{2}\right)\left(\frac{1}{121\,(x^2+3\,x+5)}+\frac{1}{66\,(x^2+3\,x+5)^2}\right) + \frac{12\,\sqrt{11}\,\mathrm{atan}\left(\frac{2\,\sqrt{11}\,\left(x+\frac{3}{2}\right)}{11}\right)}{1331}$$

```
[In] int(1/(3*x + x^2 + 5)^3,x)
```

```
[Out] 6*(x + 3/2)*(1/(121*(3*x + x^2 + 5)) + 1/(66*(3*x + x^2 + 5)^2)) + (12*11^(
1/2)*atan((2*11^(1/2)*(x + 3/2))/11))/1331
```

3.148 $\int \frac{1+x^2+x^4}{(1+x^2)^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 43

$$\int \frac{1+x^2+x^4}{(1+x^2)^4} \, dx = \frac{x}{6(1+x^2)^3} - \frac{x}{24(1+x^2)^2} + \frac{7x}{16(1+x^2)} + \frac{7\arctan(x)}{16}$$

```
[Out] 1/6*x/(x^2+1)^3-1/24*x/(x^2+1)^2+7/16*x/(x^2+1)+7/16*arctan(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1171, 393, 205, 209}

$$\int \frac{1+x^2+x^4}{(1+x^2)^4} \, dx = \frac{7\arctan(x)}{16} + \frac{7x}{16(x^2+1)} - \frac{x}{24(x^2+1)^2} + \frac{x}{6(x^2+1)^3}$$

```
[In] Int[(1 + x^2 + x^4)/(1 + x^2)^4,x]
```

```
[Out] x/(6*(1 + x^2)^3) - x/(24*(1 + x^2)^2) + (7*x)/(16*(1 + x^2)) + (7*ArcTan[x
])/16
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x}{6(1+x^2)^3} - \frac{1}{6}\int \frac{-5-6x^2}{(1+x^2)^3}\,dx \\
&= \frac{x}{6(1+x^2)^3} - \frac{x}{24(1+x^2)^2} + \frac{7}{8}\int \frac{1}{(1+x^2)^2}\,dx \\
&= \frac{x}{6(1+x^2)^3} - \frac{x}{24(1+x^2)^2} + \frac{7x}{16(1+x^2)} + \frac{7}{16}\int \frac{1}{1+x^2}\,dx \\
&= \frac{x}{6(1+x^2)^3} - \frac{x}{24(1+x^2)^2} + \frac{7x}{16(1+x^2)} + \frac{7\arctan(x)}{16}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{1+x^2+x^4}{(1+x^2)^4}\,dx=\frac{1}{48}\left(\frac{x(27+40x^2+21x^4)}{(1+x^2)^3}+21\arctan(x)\right)$$

```
[In] Integrate[(1 + x^2 + x^4)/(1 + x^2)^4,x]
[Out] ((x*(27 + 40*x^2 + 21*x^4))/(1 + x^2)^3 + 21*ArcTan[x])/48
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

method	result
default	$\frac{\frac{7}{16}x^5+\frac{5}{6}x^3+\frac{9}{16}x}{(x^2+1)^3}+\frac{7\arctan(x)}{16}$
risch	$\frac{\frac{7}{16}x^5+\frac{5}{6}x^3+\frac{9}{16}x}{(x^2+1)^3}+\frac{7\arctan(x)}{16}$
meijerg	$\frac{x(15x^4+40x^2+33)}{48(x^2+1)^3}+\frac{7\arctan(x)}{16}-\frac{x(-15x^4+40x^2+15)}{240(x^2+1)^3}-\frac{x(-3x^4-8x^2+3)}{48(x^2+1)^3}$
parallelrisch	$-\frac{21i\ln(x-i)x^6-21i\ln(x+i)x^6+63i\ln(x-i)x^4-63i\ln(x+i)x^4-42x^5+63i\ln(x-i)x^2-63i\ln(x+i)x^2-80x^3+21i\ln(x-i)-21i\ln}{96(x^2+1)^3}$

```
[In] int((x^4+x^2+1)/(x^2+1)^4,x,method=_RETURNVERBOSE)
[Out] (7/16*x^5+5/6*x^3+9/16*x)/(x^2+1)^3+7/16*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{1+x^2+x^4}{(1+x^2)^4}\,dx=\frac{21\,x^5+40\,x^3+21\,(x^6+3\,x^4+3\,x^2+1)\arctan(x)+27\,x}{48\,(x^6+3\,x^4+3\,x^2+1)}$$

```
[In] integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="fricas")
[Out] 1/48*(21*x^5 + 40*x^3 + 21*(x^6 + 3*x^4 + 3*x^2 + 1)*arctan(x) + 27*x)/(x^6
 + 3*x^4 + 3*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1+x^2+x^4}{(1+x^2)^4}\,dx = \frac{21x^5+40x^3+27x}{48x^6+144x^4+144x^2+48}+\frac{7\,\mathrm{atan}\,(x)}{16}$$

```
[In] integrate((x**4+x**2+1)/(x**2+1)**4,x)
[Out] (21*x**5 + 40*x**3 + 27*x)/(48*x**6 + 144*x**4 + 144*x**2 + 48) + 7*atan(x)
/16
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2+x^4}{(1+x^2)^4}\,dx = \frac{21\,x^5+40\,x^3+27\,x}{48\,(x^6+3\,x^4+3\,x^2+1)}+\frac{7}{16}\,\arctan\left(x\right)$$

```
[In] integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="maxima")
[Out] 1/48*(21*x^5 + 40*x^3 + 27*x)/(x^6 + 3*x^4 + 3*x^2 + 1) + 7/16*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \frac{1+x^2+x^4}{(1+x^2)^4}\,dx = \frac{21\,x^5+40\,x^3+27\,x}{48\,(x^2+1)^3}+\frac{7}{16}\,\arctan\left(x\right)$$

```
[In] integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="giac")
[Out] 1/48*(21*x^5 + 40*x^3 + 27*x)/(x^2 + 1)^3 + 7/16*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{1+x^2+x^4}{\left(1+x^2\right)^4}\,dx=\frac{7\,\mathrm{atan}(x)}{16}+\frac{\frac{7\,x^5}{16}+\frac{5\,x^3}{6}+\frac{9\,x}{16}}{\left(x^2+1\right)^3}$$

```
[In] int((x^2 + x^4 + 1)/(x^2 + 1)^4,x)
[Out] (7*atan(x))/16 + ((9*x)/16 + (5*x^3)/6 + (7*x^5)/16)/(x^2 + 1)^3
```

3.149 $\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx=-\frac{bB-Ac-(Ab-aB)x}{2(b^2-ac)(c+2bx+ax^2)}-\frac{(Ab-aB)\operatorname{arctanh}\left(\frac{b+ax}{\sqrt{b^2-ac}}\right)}{2(b^2-ac)^{3/2}}$$

```
[Out] 1/2*(-b*B+A*c+(A*b-B*a)*x)/(-a*c+b^2)/(a*x^2+2*b*x+c)-1/2*(A*b-B*a)*arctanh
((a*x+b)/(-a*c+b^2)^(1/2))/(-a*c+b^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {652, 632, 212}

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx=-\frac{(Ab-aB)\operatorname{arctanh}\left(\frac{ax+b}{\sqrt{b^2-ac}}\right)}{2(b^2-ac)^{3/2}}-\frac{-x(Ab-aB)-Ac+bB}{2(b^2-ac)(ax^2+2bx+c)}$$

```
[In] Int[(B + A*x)/(c + 2*b*x + a*x^2)^2,x]
```

```
[Out] -1/2*(b*B - A*c - (A*b - a*B)*x)/((b^2 - a*c)*(c + 2*b*x + a*x^2)) - ((A*b
- a*B)*ArcTanh[(b + a*x)/Sqrt[b^2 - a*c]])/(2*(b^2 - a*c)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{bB-Ac-(Ab-aB)x}{2\left(b^2-ac\right)\left(c+2bx+ax^2\right)}+\frac{(Ab-aB)\int \frac{1}{c+2bx+ax^2}\,dx}{2\left(b^2-ac\right)}\\
&= -\frac{bB-Ac-(Ab-aB)x}{2\left(b^2-ac\right)\left(c+2bx+ax^2\right)}-\frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{4(b^2-ac)-x^2}\,dx,x,2b+2ax\right)}{b^2-ac}\\
&= -\frac{bB-Ac-(Ab-aB)x}{2\left(b^2-ac\right)\left(c+2bx+ax^2\right)}-\frac{(Ab-aB)\text{arctanh}\left(\frac{b+ax}{\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{B+Ax}{\left(c+2bx+ax^2\right)^2}\,dx=\frac{\frac{-bB+Ac+Abx-aBx}{c+x(2b+ax)}+\frac{(Ab-aB)\arctan\left(\frac{b+ax}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2+ac}}}{2\left(b^2-ac\right)}$$

```
[In] Integrate[(B + A*x)/(c + 2*b*x + a*x^2)^2,x]
```

```
[Out] ((-(b*B) + A*c + A*b*x - a*B*x)/(c + x*(2*b + a*x)) + ((A*b - a*B)*ArcTan[(
b + a*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(2*(b^2 - a*c))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.14

method	result
default	$\frac{(-2Ab+2Ba)x+2bB-2Ac}{(4ac-4b^2)(a\,x^2+2bx+c)}+\frac{(-2Ab+2Ba)\arctan\left(\frac{2ax+2b}{2\sqrt{ac-b^2}}\right)}{(4ac-4b^2)\sqrt{ac-b^2}}$
risch	$\frac{-\frac{(Ab-Ba)x}{2(ac-b^2)}-\frac{Ac-bB}{2(ac-b^2)}}{a\,x^2+2bx+c}+\frac{\ln\left((-a^2c+a\,b^2)x-(-ac+b^2)^{\frac{3}{2}}-abc+b^3\right)Ab}{4(-ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left((-a^2c+a\,b^2)x-(-ac+b^2)^{\frac{3}{2}}-abc+b^3\right)Ba}{4(-ac+b^2)^{\frac{3}{2}}}-$

```
[In] int((A*x+B)/(a*x^2+2*b*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((-2*A*b+2*B*a)*x+2*b*B-2*A*c)/(4*a*c-4*b^2)/(a*x^2+2*b*x+c)+(-2*A*b+2*B*a)
/(4*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*a*x+2*b)/(a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(81) = 162.

Time = 0.27 (sec) , antiderivative size = 448, normalized size of antiderivative = 4.98

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx$$

$$=\left[-\frac{2\,Bb^3+2\,Aac^2-((Ba^2-Aab)x^2+(Ba-Ab)c+2\,(Bab-Ab^2)x)\sqrt{b^2-ac}\log\left(\frac{a^2x^2+2\,abx+2\,b^2-ac+2\,\sqrt{}}{ax^2+2\,bx+c}\right.}{4\,(b^4c-2\,ab^2c^2+a^2c^3+(ab^4-2\,a^2b^2c+a^3c^2)x^2+2\,(b}\right.$$

$$-\frac{Bb^3+Aac^2-((Ba^2-Aab)x^2+(Ba-Ab)c+2\,(Bab-Ab^2)x)\sqrt{-b^2+ac}\arctan\left(-\frac{\sqrt{-b^2+ac}(ax+b)}{b^2-ac}\right)-}{2\,(b^4c-2\,ab^2c^2+a^2c^3+(ab^4-2\,a^2b^2c+a^3c^2)x^2+2\,(b^5-2\,ab^3}$$

```
[In] integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*b^3 + 2*A*a*c^2 - ((B*a^2 - A*a*b)*x^2 + (B*a - A*b)*c + 2*(B*a*
b - A*b^2)*x)*sqrt(b^2 - a*c)*log((a^2*x^2 + 2*a*b*x + 2*b^2 - a*c + 2*sqrt
(b^2 - a*c)*(a*x + b))/(a*x^2 + 2*b*x + c)) - 2*(B*a*b + A*b^2)*c + 2*(B*a*
b^2 - A*b^3 - (B*a^2 - A*a*b)*c)*x)/(b^4*c - 2*a*b^2*c^2 + a^2*c^3 + (a*b^4
 - 2*a^2*b^2*c + a^3*c^2)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2*b*c^2)*x), -1/2*(B
*b^3 + A*a*c^2 - ((B*a^2 - A*a*b)*x^2 + (B*a - A*b)*c + 2*(B*a*b - A*b^2)*x
)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(a*x + b)/(b^2 - a*c)) - (B*a*b
 + A*b^2)*c + (B*a*b^2 - A*b^3 - (B*a^2 - A*a*b)*c)*x)/(b^4*c - 2*a*b^2*c^2
 + a^2*c^3 + (a*b^4 - 2*a^2*b^2*c + a^3*c^2)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2
*b*c^2)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(75) = 150$.

Time = 0.54 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.59

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx =$$

$$-\frac{\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)\log\left(x+\frac{-Ab^2+Bab-a^2c^2\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)+2ab^2c\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)-b^4\sqrt{-\frac{1}{(ac-b^2)^3}}(\cdots}{-Aab+Ba^2}\right.}{4}$$

$$+\frac{\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)\log\left(x+\frac{-Ab^2+Bab+a^2c^2\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)-2ab^2c\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)+b^4\sqrt{-\frac{1}{(ac-b^2)^3}}(\cdots}{-Aab+Ba^2}\right.}{4}$$

$$+\frac{-Ac+Bb+x(-Ab+Ba)}{2ac^2-2b^2c+x^2\cdot(2a^2c-2ab^2)+x\,(4abc-4b^3)}$$

```
[In] integrate((A*x+B)/(a*x**2+2*b*x+c)**2,x)
```

```
[Out] -sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a)*log(x + (-A*b**2 + B*a*b - a**2*c**2
*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a) + 2*a*b**2*c*sqrt(-1/(a*c - b**2)**3
)*(-A*b + B*a) - b**4*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a))/(-A*a*b + B*a*
*2))/4 + sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a)*log(x + (-A*b**2 + B*a*b + a
**2*c**2*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a) - 2*a*b**2*c*sqrt(-1/(a*c -
b**2)**3)*(-A*b + B*a) + b**4*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a))/(-A*a*
b + B*a**2))/4 + (-A*c + B*b + x*(-A*b + B*a))/(2*a*c**2 - 2*b**2*c + x**2*
(2*a**2*c - 2*a*b**2) + x*(4*a*b*c - 4*b**3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx = -\frac{(Ba-Ab)\arctan\left(\frac{ax+b}{\sqrt{-b^2+ac}}\right)}{2\,(b^2-ac)\sqrt{-b^2+ac}} - \frac{Bax-Abx+Bb-Ac}{2\,(ax^2+2\,bx+c)(b^2-ac)}$$

```
[In] integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a - A*b)*arctan((a*x + b)/sqrt(-b^2 + a*c))/((b^2 - a*c)*sqrt(-b^2
+ a*c)) - 1/2*(B*a*x - A*b*x + B*b - A*c)/((a*x^2 + 2*b*x + c)*(b^2 - a*c))
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.77

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx = \frac{\operatorname{atan}\left(\frac{2\,(a\,c-b^2)\left(\frac{\left(4\,b^3-4\,a\,b\,c\right)(A\,b-B\,a)}{8\,\left(a\,c-b^2\right)^{5/2}}-\frac{a\,x\,(A\,b-B\,a)}{2\,\left(a\,c-b^2\right)^{3/2}}\right)}{A\,b-B\,a}\right)(A\,b-B\,a)}{2\,(a\,c-b^2)^{3/2}} - \frac{\frac{A\,c-B\,b}{2\,(a\,c-b^2)}+\frac{x\,(A\,b-B\,a)}{2\,(a\,c-b^2)}}{a\,x^2+2\,b\,x+c}$$

```
[In] int((B + A*x)/(c + 2*b*x + a*x^2)^2,x)
```

```
[Out] (atan((2*(a*c - b^2)*(((4*b^3 - 4*a*b*c)*(A*b - B*a))/(8*(a*c - b^2)^(5/2))
 - (a*x*(A*b - B*a))/(2*(a*c - b^2)^(3/2))))/(A*b - B*a))*(A*b - B*a))/(2*(
a*c - b^2)^(3/2)) - ((A*c - B*b)/(2*(a*c - b^2)) + (x*(A*b - B*a))/(2*(a*c
- b^2)))/(c + 2*b*x + a*x^2)
```

3.150 $\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx = \frac{1-x}{5-4x+x^2} - 2\arctan(2-x) + \frac{5}{2}\log\left(5-4x+x^2\right)$$

```
[Out] (1-x)/(x^2-4*x+5)+2*arctan(-2+x)+5/2*ln(x^2-4*x+5)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1674, 648, 632, 210, 642}

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx = -2\arctan(2-x) + \frac{1-x}{x^2-4x+5} + \frac{5}{2}\log\left(x^2-4x+5\right)$$

```
[In] Int[(-41 + 55*x - 27*x^2 + 5*x^3)/(5 - 4*x + x^2)^2,x]
```

```
[Out] (1 - x)/(5 - 4*x + x^2) - 2*ArcTan[2 - x] + (5*Log[5 - 4*x + x^2])/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1-x}{5-4x+x^2}+\frac{1}{4}\int \frac{-32+20x}{5-4x+x^2}\,dx\\
&= \frac{1-x}{5-4x+x^2}+2\int \frac{1}{5-4x+x^2}\,dx+\frac{5}{2}\int \frac{-4+2x}{5-4x+x^2}\,dx\\
&= \frac{1-x}{5-4x+x^2}+\frac{5}{2}\log\left(5-4x+x^2\right)-4\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,-4+2x\right)\\
&= \frac{1-x}{5-4x+x^2}-2\arctan(2-x)+\frac{5}{2}\log\left(5-4x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx = \frac{1-x}{5-4x+x^2} - 2\arctan(2-x) + \frac{5}{2}\log\left(5-4x+x^2\right)$$

```
[In] Integrate[(-41 + 55*x - 27*x^2 + 5*x^3)/(5 - 4*x + x^2)^2,x]
[Out] (1 - x)/(5 - 4*x + x^2) - 2*ArcTan[2 - x] + (5*Log[5 - 4*x + x^2])/2
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result
default	$\frac{1-x}{x^2-4x+5} + 2\arctan\left(-2+x\right) + \frac{5\ln\left(x^2-4x+5\right)}{2}$
risch	$\frac{1-x}{x^2-4x+5} + 2\arctan\left(-2+x\right) + \frac{5\ln\left(x^2-4x+5\right)}{2}$
parallelrisch	$-\frac{-40i\ln(x-2-i)x-10i\ln(x-2+i)x^2-50i\ln(x-2+i)-25\ln(x-2-i)x^2+10i\ln(x-2-i)x^2-25\ln(x-2+i)x^2+40i\ln(x-2+i)x+}{10(x^2-4x+5)}$

```
[In] int((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x,method=_RETURNVERBOSE)
[Out] (1-x)/(x^2-4*x+5)+2*arctan(-2+x)+5/2*ln(x^2-4*x+5)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx = \frac{4\left(x^2-4\,x+5\right)\arctan\left(x-2\right)+5\left(x^2-4\,x+5\right)\log\left(x^2-4\,x+5\right)-2\,x+2}{2\left(x^2-4\,x+5\right)}$$

```
[In] integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="fricas")
[Out] 1/2*(4*(x^2 - 4*x + 5)*arctan(x - 2) + 5*(x^2 - 4*x + 5)*log(x^2 - 4*x + 5)
 - 2*x + 2)/(x^2 - 4*x + 5)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx = \frac{1-x}{x^2-4x+5} + \frac{5\log(x^2-4x+5)}{2} + 2\operatorname{atan}(x-2)$$

```
[In] integrate((5*x**3-27*x**2+55*x-41)/(x**2-4*x+5)**2,x)
[Out] (1 - x)/(x**2 - 4*x + 5) + 5*log(x**2 - 4*x + 5)/2 + 2*atan(x - 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx = -\frac{x-1}{x^2-4\,x+5} + 2\,\arctan(x-2) + \frac{5}{2}\,\log\left(x^2-4\,x+5\right)$$

```
[In] integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="maxima")
[Out] -(x - 1)/(x^2 - 4*x + 5) + 2*arctan(x - 2) + 5/2*log(x^2 - 4*x + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx = -\frac{x-1}{x^2-4\,x+5} + 2\,\arctan(x-2) + \frac{5}{2}\,\log\left(x^2-4\,x+5\right)$$

```
[In] integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="giac")
[Out] -(x - 1)/(x^2 - 4*x + 5) + 2*arctan(x - 2) + 5/2*log(x^2 - 4*x + 5)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx = 2\operatorname{atan}(x-2) + \frac{5\,\ln\left(x^2-4\,x+5\right)}{2} - \frac{x}{x^2-4\,x+5} + \frac{1}{x^2-4\,x+5}$$

```
[In] int((55*x - 27*x^2 + 5*x^3 - 41)/(x^2 - 4*x + 5)^2,x)
[Out] 2*atan(x - 2) + (5*log(x^2 - 4*x + 5))/2 - x/(x^2 - 4*x + 5) + 1/(x^2 - 4*x
 + 5)
```

3.151 $\int \frac{1}{(-1+x^3)^2} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 57

$$\int \frac{1}{(-1+x^3)^2} \, dx = \frac{x}{3(1-x^3)} + \frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2}{9}\log(1-x) + \frac{1}{9}\log\left(1+x+x^2\right)$$

```
[Out] 1/3*x/(-x^3+1)-2/9*ln(1-x)+1/9*ln(x^2+x+1)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*
3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {205, 206, 31, 648, 632, 210, 642}

$$\int \frac{1}{(-1+x^3)^2} \, dx = \frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{x}{3(1-x^3)} + \frac{1}{9}\log\left(x^2+x+1\right) - \frac{2}{9}\log(1-x)$$

```
[In] Int[(-1 + x^3)^(-2),x]
```

```
[Out] x/(3*(1 - x^3)) + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (2*Log[1 - x]
)/9 + Log[1 + x + x^2]/9
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x}{3(1-x^3)} - \frac{2}{3}\int \frac{1}{-1+x^3}\,dx \\ &= \frac{x}{3(1-x^3)} - \frac{2}{9}\int \frac{1}{-1+x}\,dx - \frac{2}{9}\int \frac{-2-x}{1+x+x^2}\,dx\end{aligned}$$

$$= \frac{x}{3(1-x^3)} - \frac{2}{9}\log(1-x) + \frac{1}{9}\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{3}\int \frac{1}{1+x+x^2}\,dx$$

$$= \frac{x}{3(1-x^3)} - \frac{2}{9}\log(1-x) + \frac{1}{9}\log\left(1+x+x^2\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right)$$

$$= \frac{x}{3(1-x^3)} + \frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2}{9}\log(1-x) + \frac{1}{9}\log\left(1+x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{1}{(-1+x^3)^2}\,dx = \frac{1}{9}\left(-\frac{3x}{-1+x^3} + 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 2\log(1-x) + \log\left(1+x+x^2\right)\right)$$

```
[In] Integrate[(-1 + x^3)^(-2),x]
[Out] ((-3*x)/(-1 + x^3) + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Log[1 - x] + L
og[1 + x + x^2])/9
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{x}{3(x^3-1)} - \frac{2\ln(-1+x)}{9} + \frac{\ln(x^2+x+1)}{9} + \frac{2\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{9}$	41
default	$-\frac{1}{9(-1+x)} - \frac{2\ln(-1+x)}{9} + \frac{-1+x}{9x^2+9x+9} + \frac{\ln(x^2+x+1)}{9} + \frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}$	53
meijerg	$-\frac{(-1)^{\frac{2}{3}}\left(\frac{3x(-1)^{\frac{1}{3}}}{-3x^3+3} - \frac{2x(-1)^{\frac{1}{3}}\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right) - \frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2} - \sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2+\left(x^3\right)^{\frac{1}{3}}}\right)\right)}{3\left(x^3\right)^{\frac{1}{3}}}\right)}{3}$	86

```
[In] int(1/(x^3-1)^2,x,method=_RETURNVERBOSE)
[Out] -1/3*x/(x^3-1)-2/9*ln(-1+x)+1/9*ln(x^2+x+1)+2/9*3^(1/2)*arctan(2/3*3^(1/2)*
(x+1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{1}{(-1+x^3)^2}\,dx = \frac{2\sqrt{3}(x^3-1)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+(x^3-1)\log\left(x^2+x+1\right)-2\left(x^3-1\right)\log\left(x-1\right)-3x}{9\left(x^3-1\right)}$$

```
[In] integrate(1/(x^3-1)^2,x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*(x^3 - 1)*arctan(1/3*sqrt(3)*(2*x + 1)) + (x^3 - 1)*log(x^2
+ x + 1) - 2*(x^3 - 1)*log(x - 1) - 3*x)/(x^3 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{1}{(-1+x^3)^2}\,dx = -\frac{x}{3x^3-3}-\frac{2\log\left(x-1\right)}{9}+\frac{\log\left(x^2+x+1\right)}{9}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate(1/(x**3-1)**2,x)
```

```
[Out] -x/(3*x**3 - 3) - 2*log(x - 1)/9 + log(x**2 + x + 1)/9 + 2*sqrt(3)*atan(2*s
qrt(3)*x/3 + sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{(-1+x^3)^2}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{x}{3\left(x^3-1\right)}+\frac{1}{9}\log\left(x^2+x+1\right)-\frac{2}{9}\log\left(x-1\right)$$

```
[In] integrate(1/(x^3-1)^2,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*x/(x^3 - 1) + 1/9*log(x^2 +
 x + 1) - 2/9*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-1+x^3)^2}\,dx = \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{x}{3\,(x^3-1)} + \frac{1}{9}\,\log\left(x^2+x+1\right) - \frac{2}{9}\,\log\left(|x-1|\right)$$

```
[In] integrate(1/(x^3-1)^2,x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*x/(x^3 - 1) + 1/9*log(x^2 +
 x + 1) - 2/9*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{1}{(-1+x^3)^2}\,dx = -\frac{2\,\ln\left(x-1\right)}{9} - \frac{x}{3\,\left(x^3-1\right)} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right) + \ln\left(2\,x+1+\sqrt{3}\,1\mathrm{i}\right)\,\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)$$

```
[In] int(1/(x^3 - 1)^2,x)
```

```
[Out] log(2*x + 3^(1/2)*1i + 1)*((3^(1/2)*1i)/9 + 1/9) - x/(3*(x^3 - 1)) - log(x
- (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1/9) - (2*log(x - 1))/9
```

3.152 $\int \frac{4+3x^4}{x^2(1+x^2)^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 36

$$\int \frac{4+3x^4}{x^2(1+x^2)^3} dx = -\frac{4}{x} - \frac{7x}{4(1+x^2)^2} - \frac{25x}{8(1+x^2)} - \frac{57\arctan(x)}{8}$$

```
[Out] -4/x-7/4*x/(x^2+1)^2-25/8*x/(x^2+1)-57/8*arctan(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1274, 467, 464, 209}

$$\int \frac{4+3x^4}{x^2(1+x^2)^3} dx = -\frac{57\arctan(x)}{8} - \frac{25x}{8(x^2+1)} - \frac{7x}{4(x^2+1)^2} - \frac{4}{x}$$

```
[In] Int[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

```
[Out] -4/x - (7*x)/(4*(1 + x^2)^2) - (25*x)/(8*(1 + x^2)) - (57*ArcTan[x])/8
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
 + 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1274

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Sym
bol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*x*((d + e*x^2)^(q + 1)/(2*e^(
2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m
*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*(-d)^(-m/2 + 1
)*e^(2*p)*(q + 1)*(a + c*x^4)^p - ((c*d^2 + a*e^2)^p/(e^(m/2)*x^m))*(d + e*
(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ[p, 0] && ILt
Q[q, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{7x}{4\left(1+x^2\right)^2}-\frac{1}{4}\int\frac{-16+9x^2}{x^2\left(1+x^2\right)^2}\,dx\\ &= -\frac{7x}{4\left(1+x^2\right)^2}-\frac{25x}{8\left(1+x^2\right)}+\frac{1}{8}\int\frac{32-25x^2}{x^2\left(1+x^2\right)}\,dx\\ &= -\frac{4}{x}-\frac{7x}{4\left(1+x^2\right)^2}-\frac{25x}{8\left(1+x^2\right)}-\frac{57}{8}\int\frac{1}{1+x^2}\,dx\\ &= -\frac{4}{x}-\frac{7x}{4\left(1+x^2\right)^2}-\frac{25x}{8\left(1+x^2\right)}-\frac{57\arctan(x)}{8}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx=-\frac{32+103x^2+57x^4}{8x\left(1+x^2\right)^2}-\frac{57\arctan(x)}{8}$$

```
[In] Integrate[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
[Out] -1/8*(32 + 103*x^2 + 57*x^4)/(x*(1 + x^2)^2) - (57*ArcTan[x])/8
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\frac{25}{8}x^3+\frac{39}{8}x}{(x^2+1)^2}-\frac{57\arctan(x)}{8}-\frac{4}{x}$	29
risch	$\frac{-\frac{57}{8}x^4-\frac{103}{8}x^2-4}{(x^2+1)^2 x}-\frac{57\arctan(x)}{8}$	29
meijerg	$-\frac{15x^4+25x^2+8}{2x(x^2+1)^2}-\frac{57\arctan(x)}{8}-\frac{x(-3x^2+3)}{8(x^2+1)^2}$	47
parallelrisch	$\frac{57i\ln(x-i)x^5-57i\ln(x+i)x^5-64+114i\ln(x-i)x^3-114i\ln(x+i)x^3-114x^4+57i\ln(x-i)x-57i\ln(x+i)x-206x^2}{16x(x^2+1)^2}$	87

```
[In] int((3*x^4+4)/x^2/(x^2+1)^3,x,method=_RETURNVERBOSE)
[Out] -(25/8*x^3+39/8*x)/(x^2+1)^2-57/8*arctan(x)-4/x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx=-\frac{57\,x^4+103\,x^2+57\,(x^5+2\,x^3+x)\arctan\left(x\right)+32}{8\,(x^5+2\,x^3+x)}$$

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="fricas")
[Out] -1/8*(57*x^4 + 103*x^2 + 57*(x^5 + 2*x^3 + x)*arctan(x) + 32)/(x^5 + 2*x^3
+ x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{4+3x^4}{x^2\,(1+x^2)^3}\,dx = \frac{-57x^4-103x^2-32}{8x^5+16x^3+8x} - \frac{57\,\mathrm{atan}\,(x)}{8}$$

```
[In] integrate((3*x**4+4)/x**2/(x**2+1)**3,x)
[Out] (-57*x**4 - 103*x**2 - 32)/(8*x**5 + 16*x**3 + 8*x) - 57*atan(x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{4+3x^4}{x^2\,(1+x^2)^3}\,dx = -\frac{57\,x^4+103\,x^2+32}{8\,(x^5+2\,x^3+x)} - \frac{57}{8}\,\arctan\,(x)$$

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="maxima")
[Out] -1/8*(57*x^4 + 103*x^2 + 32)/(x^5 + 2*x^3 + x) - 57/8*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{4+3x^4}{x^2\,(1+x^2)^3}\,dx = -\frac{25\,x^3+39\,x}{8\,(x^2+1)^2} - \frac{4}{x} - \frac{57}{8}\,\arctan\,(x)$$

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="giac")
[Out] -1/8*(25*x^3 + 39*x)/(x^2 + 1)^2 - 4/x - 57/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{4+3x^4}{x^2\,(1+x^2)^3}\,dx = -\frac{57\,\mathrm{atan}(x)}{8} - \frac{\frac{57\,x^4}{8}+\frac{103\,x^2}{8}+4}{x\,(x^2+1)^2}$$

```
[In] int((3*x^4 + 4)/(x^2*(x^2 + 1)^3),x)
[Out] - (57*atan(x))/8 - ((103*x^2)/8 + (57*x^4)/8 + 4)/(x*(x^2 + 1)^2)
```

3.153 $\int \frac{x}{1+x^6} dx$

Optimal result

Integrand size = 9, antiderivative size = 49

$$\int \frac{x}{1+x^6} dx = -\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{12}\log\left(1-x^2+x^4\right)$$

```
[Out] 1/6*ln(x^2+1)-1/12*ln(x^4-x^2+1)-1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {281, 206, 31, 648, 632, 210, 642}

$$\int \frac{x}{1+x^6} dx = -\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{6}\log\left(x^2+1\right) - \frac{1}{12}\log\left(x^4-x^2+1\right)$$

```
[In] Int[x/(1 + x^6),x]
```

```
[Out] -1/2*ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] + Log[1 + x^2]/6 - Log[1 - x^2 + x
^4]/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^3}\,dx,x,x^2\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)+\frac{1}{6}\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{6}\log\left(1+x^2\right)-\frac{1}{12}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^2\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{6}\log\left(1+x^2\right)-\frac{1}{12}\log\left(1-x^2+x^4\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right)
\end{aligned}$$

$$= -\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{12}\log\left(1-x^2+x^4\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \frac{x}{1+x^6}\,dx = \frac{1}{12}\Big(-2\sqrt{3}\arctan\left(\sqrt{3}-2x\right) - 2\sqrt{3}\arctan\left(\sqrt{3}+2x\right) + 2\log\left(1+x^2\right) - \log\left(1-\sqrt{3}x+x^2\right) - \log\left(1+\sqrt{3}x+x^2\right)\Big)$$

```
[In] Integrate[x/(1 + x^6),x]
[Out] (-2*Sqrt[3]*ArcTan[Sqrt[3] - 2*x] - 2*Sqrt[3]*ArcTan[Sqrt[3] + 2*x] + 2*Log
[1 + x^2] - Log[1 - Sqrt[3]*x + x^2] - Log[1 + Sqrt[3]*x + x^2])/12
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{\ln(x^2+1)}{6} - \frac{\ln(x^4-x^2+1)}{12} + \frac{\sqrt{3}\arctan\left(\frac{2\left(x^2-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$	39
default	$-\frac{\ln(x^4-x^2+1)}{12} + \frac{\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)\sqrt{3}}{6} + \frac{\ln(x^2+1)}{6}$	41
meijerg	$\frac{x^2\ln\left(1+(x^6)^{\frac{1}{3}}\right)}{6(x^6)^{\frac{1}{3}}} - \frac{x^2\ln\left(1-(x^6)^{\frac{1}{3}}+(x^6)^{\frac{2}{3}}\right)}{12(x^6)^{\frac{1}{3}}} + \frac{x^2\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{3}}}{2-(x^6)^{\frac{1}{3}}}\right)}{6(x^6)^{\frac{1}{3}}}$	80

```
[In] int(x/(x^6+1),x,method=_RETURNVERBOSE)
[Out] 1/6*ln(x^2+1)-1/12*ln(x^4-x^2+1)+1/6*3^(1/2)*arctan(2/3*(x^2-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x}{1+x^6}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right) - \frac{1}{12}\,\log\left(x^4-x^2+1\right) + \frac{1}{6}\,\log\left(x^2+1\right)$$

```
[In] integrate(x/(x^6+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/6
*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{x}{1+x^6}\,dx = \frac{\log\left(x^2+1\right)}{6} - \frac{\log\left(x^4-x^2+1\right)}{12} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3} - \frac{\sqrt{3}}{3}\right)}{6}$$

```
[In] integrate(x/(x**6+1),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**4 - x**2 + 1)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 -
 sqrt(3)/3)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x}{1+x^6}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right) - \frac{1}{12}\,\log\left(x^4-x^2+1\right) + \frac{1}{6}\,\log\left(x^2+1\right)$$

```
[In] integrate(x/(x^6+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/6
*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x}{1+x^6}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) - \frac{1}{12}\log\left(x^4-x^2+1\right) + \frac{1}{6}\log\left(x^2+1\right)$$

```
[In] integrate(x/(x^6+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/6
*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{x}{1+x^6}\,dx = \frac{\ln(x^2+1)}{6} - \ln\left(x^2 - \frac{\sqrt{3}\,1i}{2} - \frac{1}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}\,1i}{12}\right) + \ln\left(x^2 + \frac{\sqrt{3}\,1i}{2} - \frac{1}{2}\right)\left(-\frac{1}{12} + \frac{\sqrt{3}\,1i}{12}\right)$$

```
[In] int(x/(x^6 + 1),x)
```

```
[Out] log(x^2 + 1)/6 - log(x^2 - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12 + 1/12) +
 log((3^(1/2)*1i)/2 + x^2 - 1/2)*((3^(1/2)*1i)/12 - 1/12)
```

3.154 $\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 13

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx=\frac{\log(-nx+x^n)}{n}$$

```
[Out] ln(-n*x+x^n)/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1607, 528, 457, 78}

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx=\frac{\log(1-nx^{1-n})}{n}+\log(x)$$

```
[In] Int[(-1 + x^(-1 + n))/(-(n*x) + x^n),x]
```

```
[Out] Log[x] + Log[1 - n*x^(1 - n)]/n
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^{-n}(-1+x^{-1+n})}{1-nx^{1-n}}\,dx\\
&= \int \frac{1-x^{1-n}}{x\,(1-nx^{1-n})}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1-x}{x(1-nx)}\,dx, x, x^{1-n}\right)}{1-n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{1-n}{-1+nx}\right)\,dx, x, x^{1-n}\right)}{1-n}\\
&= \log(x)+\frac{\log\left(1-nx^{1-n}\right)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx = \frac{\log\left(-nx+x^n\right)}{n}$$

```
[In] Integrate[(-1 + x^(-1 + n))/(-(n*x) + x^n),x]
[Out] Log[-(n*x) + x^n]/n
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
risch	$\frac{\ln(-nx+x^n)}{n}$	14
norman	$\frac{\ln(nx-e^{n\ln(x)})}{n}$	17

```
[In] int((-1+x^(-1+n))/(-n*x+x^n),x,method=_RETURNVERBOSE)
[Out] ln(-n*x+x^n)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx=\frac{\log(-nx+x^n)}{n}$$

```
[In] integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="fricas")
[Out] log(-n*x + x^n)/n
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx=\begin{cases}\frac{\log\left(x-\frac{x^n}{n}\right)}{n} & \text{for } n\neq 0\\ -x+\log(x) & \text{otherwise}\end{cases}$$

```
[In] integrate((-1+x**(-1+n))/(-n*x+x**n),x)
[Out] Piecewise((log(x - x**n/n)/n, Ne(n, 0)), (-x + log(x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx = \frac{\log\left(nx-x^n\right)}{n}$$

```
[In] integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="maxima")
[Out] log(n*x - x^n)/n
```

Giac [F]

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx = \int -\frac{x^{n-1}-1}{nx-x^n}\,dx$$

```
[In] integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="giac")
[Out] integrate(-(x^(n - 1) - 1)/(n*x - x^n), x)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx = \frac{\ln\left(n\,x-x^n\right)}{n}$$

```
[In] int(-(x^(n - 1) - 1)/(n*x - x^n),x)
[Out] log(n*x - x^n)/n
```

3.155 $\int \frac{x^3}{1-2x^2+3x^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{x^3}{1-2x^2+3x^4} \, dx = -\frac{\arctan\left(\frac{1-3x^2}{\sqrt{2}}\right)}{6\sqrt{2}} + \frac{1}{12}\log\left(1-2x^2+3x^4\right)$$

```
[Out] 1/12*ln(3*x^4-2*x^2+1)-1/12*arctan(1/2*(-3*x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1128, 648, 632, 210, 642}

$$\int \frac{x^3}{1-2x^2+3x^4} \, dx = \frac{1}{12}\log\left(3x^4-2x^2+1\right) - \frac{\arctan\left(\frac{1-3x^2}{\sqrt{2}}\right)}{6\sqrt{2}}$$

```
[In] Int[x^3/(1 - 2*x^2 + 3*x^4),x]
```

```
[Out] -1/6*ArcTan[(1 - 3*x^2)/Sqrt[2]]/Sqrt[2] + Log[1 - 2*x^2 + 3*x^4]/12
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{1-2x+3x^2}\,dx, x, x^2\right)\\
&= \frac{1}{12}\text{Subst}\left(\int \frac{-2+6x}{1-2x+3x^2}\,dx, x, x^2\right)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx, x, x^2\right)\\
&= \frac{1}{12}\log\left(1-2x^2+3x^4\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2\left(-1+3x^2\right)\right)\\
&= -\frac{\arctan\left(\frac{1-3x^2}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{1}{12}\log\left(1-2x^2+3x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = \frac{1}{12}\left(\sqrt{2}\arctan\left(\frac{-1+3x^2}{\sqrt{2}}\right)+\log\left(1-2x^2+3x^4\right)\right)$$

```
[In] Integrate[x^3/(1 - 2*x^2 + 3*x^4),x]
[Out] (Sqrt[2]*ArcTan[(-1 + 3*x^2)/Sqrt[2]] + Log[1 - 2*x^2 + 3*x^4])/12
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\ln(3x^4-2x^2+1)}{12}+\frac{\sqrt{2}\,\arctan\left(\frac{(6x^2-2)\sqrt{2}}{4}\right)}{12}$	35
risch	$\frac{\ln(9x^4-6x^2+3)}{12}+\frac{\sqrt{2}\,\arctan\left(\frac{(3x^2-1)\sqrt{2}}{2}\right)}{12}$	35

```
[In] int(x^3/(3*x^4-2*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/12*ln(3*x^4-2*x^2+1)+1/12*2^(1/2)*arctan(1/4*(6*x^2-2)*2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = \frac{1}{12}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(3\,x^2-1)\right)+\frac{1}{12}\,\log\left(3\,x^4-2\,x^2+1\right)$$

```
[In] integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="fricas")
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = \frac{\log\left(x^4-\frac{2x^2}{3}+\frac{1}{3}\right)}{12}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{3\sqrt{2}x^2}{2}-\frac{\sqrt{2}}{2}\right)}{12}$$

```
[In] integrate(x**3/(3*x**4-2*x**2+1),x)
[Out] log(x**4 - 2*x**2/3 + 1/3)/12 + sqrt(2)*atan(3*sqrt(2)*x**2/2 - sqrt(2)/2)/
12
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = \frac{1}{12}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(3\,x^2-1\right)\right) + \frac{1}{12}\,\log\left(3\,x^4-2\,x^2+1\right)$$

```
[In] integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="maxima")
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2*x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = \frac{1}{12}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(3\,x^2-1\right)\right) + \frac{1}{12}\,\log\left(3\,x^4-2\,x^2+1\right)$$

```
[In] integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="giac")
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2*x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = \frac{\ln\left(x^4-\frac{2\,x^2}{3}+\frac{1}{3}\right)}{12} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}}{2}-\frac{3\,\sqrt{2}\,x^2}{2}\right)}{12}$$

```
[In] int(x^3/(3*x^4 - 2*x^2 + 1),x)
[Out] log(x^4 - (2*x^2)/3 + 1/3)/12 - (2^(1/2)*atan(2^(1/2)/2 - (3*2^(1/2)*x^2)/2
))/12
```

3.156 $\int \frac{x^5}{-4+x^2+3x^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{x^5}{-4+x^2+3x^4} dx = \frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(4+3x^2\right)$$

```
[Out] 1/6*x^2+1/14*ln(-x^2+1)-8/63*ln(3*x^2+4)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1128, 717, 646, 31}

$$\int \frac{x^5}{-4+x^2+3x^4} dx = \frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(3x^2+4\right)$$

```
[In] Int[x^5/(-4 + x^2 + 3*x^4),x]
```

```
[Out] x^2/6 + Log[1 - x^2]/14 - (8*Log[4 + 3*x^2])/63
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{-4+x+3x^2}\,dx, x, x^2\right)\\
&= \frac{x^2}{6}+\frac{1}{6}\text{Subst}\left(\int \frac{4-x}{-4+x+3x^2}\,dx, x, x^2\right)\\
&= \frac{x^2}{6}+\frac{3}{14}\text{Subst}\left(\int \frac{1}{-3+3x}\,dx, x, x^2\right)-\frac{8}{21}\text{Subst}\left(\int \frac{1}{4+3x}\,dx, x, x^2\right)\\
&= \frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(4+3x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{-4+x^2+3x^4}\,dx = \frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(4+3x^2\right)$$

```
[In] Integrate[x^5/(-4 + x^2 + 3*x^4),x]
[Out] x^2/6 + Log[1 - x^2]/14 - (8*Log[4 + 3*x^2])/63
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{x^2}{6}+\frac{\ln(x^2-1)}{14}-\frac{8\ln(3x^2+4)}{63}$	25
risch	$\frac{x^2}{6}+\frac{\ln(x^2-1)}{14}-\frac{8\ln(3x^2+4)}{63}$	25
parallelrisch	$\frac{x^2}{6}+\frac{\ln(-1+x)}{14}+\frac{\ln(1+x)}{14}-\frac{8\ln(x^2+\frac{4}{3})}{63}$	27
norman	$\frac{x^2}{6}+\frac{\ln(-1+x)}{14}+\frac{\ln(1+x)}{14}-\frac{8\ln(3x^2+4)}{63}$	29

```
[In] int(x^5/(3*x^4+x^2-4),x,method=_RETURNVERBOSE)
[Out] 1/6*x^2+1/14*ln(x^2-1)-8/63*ln(3*x^2+4)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{-4+x^2+3x^4}\,dx = \frac{1}{6}\,x^2 - \frac{8}{63}\,\log\left(3\,x^2+4\right) + \frac{1}{14}\,\log\left(x^2-1\right)$$

```
[In] integrate(x^5/(3*x^4+x^2-4),x, algorithm="fricas")
[Out] 1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{-4+x^2+3x^4}\,dx = \frac{x^2}{6} + \frac{\log\left(x^2-1\right)}{14} - \frac{8\log\left(x^2+\frac{4}{3}\right)}{63}$$

```
[In] integrate(x**5/(3*x**4+x**2-4),x)
[Out] x**2/6 + log(x**2 - 1)/14 - 8*log(x**2 + 4/3)/63
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{-4+x^2+3x^4}\,dx = \frac{1}{6}\,x^2 - \frac{8}{63}\,\log\left(3\,x^2+4\right) + \frac{1}{14}\,\log\left(x^2-1\right)$$

```
[In] integrate(x^5/(3*x^4+x^2-4),x, algorithm="maxima")
[Out] 1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{-4+x^2+3x^4}\,dx = \frac{1}{6}\,x^2 - \frac{8}{63}\,\log\left(3\,x^2+4\right) + \frac{1}{14}\,\log\left(\left|x^2-1\right|\right)$$

```
[In] integrate(x^5/(3*x^4+x^2-4),x, algorithm="giac")
[Out] 1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{x^5}{-4+x^2+3x^4}\,dx = \frac{\ln\left(x^2-1\right)}{14} - \frac{8\,\ln\left(x^2+\frac{4}{3}\right)}{63} + \frac{x^2}{6}$$

```
[In] int(x^5/(x^2 + 3*x^4 - 4),x)
[Out] log(x^2 - 1)/14 - (8*log(x^2 + 4/3))/63 + x^2/6
```

3.157 $\int \frac{x^2}{9-10x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{x^2}{9-10x^3+x^6}\,dx = -\frac{1}{24}\log\left(1-x^3\right)+\frac{1}{24}\log\left(9-x^3\right)$$

```
[Out] -1/24*ln(-x^3+1)+1/24*ln(-x^3+9)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1366, 630, 31}

$$\int \frac{x^2}{9-10x^3+x^6}\,dx = \frac{1}{24}\log\left(9-x^3\right)-\frac{1}{24}\log\left(1-x^3\right)$$

```
[In] Int[x^2/(9 - 10*x^3 + x^6),x]
```

```
[Out] -1/24*Log[1 - x^3] + Log[9 - x^3]/24
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{9-10x+x^2}\,dx, x, x^3\right)\\ &= \frac{1}{24}\text{Subst}\left(\int \frac{1}{-9+x}\,dx, x, x^3\right) - \frac{1}{24}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, x^3\right)\\ &= -\frac{1}{24}\log\left(1-x^3\right) + \frac{1}{24}\log\left(9-x^3\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{9-10x^3+x^6}\,dx = -\frac{1}{24}\log\left(1-x^3\right) + \frac{1}{24}\log\left(9-x^3\right)$$

```
[In] Integrate[x^2/(9 - 10*x^3 + x^6),x]
[Out] -1/24*Log[1 - x^3] + Log[9 - x^3]/24
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{\ln(x^3-1)}{24}+\frac{\ln(x^3-9)}{24}$	18
risch	$-\frac{\ln(x^3-1)}{24}+\frac{\ln(x^3-9)}{24}$	18
norman	$-\frac{\ln(-1+x)}{24}+\frac{\ln(x^3-9)}{24}-\frac{\ln(x^2+x+1)}{24}$	25
parallelrisch	$-\frac{\ln(-1+x)}{24}+\frac{\ln(x^3-9)}{24}-\frac{\ln(x^2+x+1)}{24}$	25

```
[In] int(x^2/(x^6-10*x^3+9),x,method=_RETURNVERBOSE)
[Out] -1/24*ln(x^3-1)+1/24*ln(x^3-9)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{9-10x^3+x^6}\,dx = -\frac{1}{24}\,\log\left(x^3-1\right)+\frac{1}{24}\,\log\left(x^3-9\right)$$

```
[In] integrate(x^2/(x^6-10*x^3+9),x, algorithm="fricas")
[Out] -1/24*log(x^3 - 1) + 1/24*log(x^3 - 9)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

$$\int \frac{x^2}{9-10x^3+x^6}\,dx = \frac{\log\left(x^3-9\right)}{24}-\frac{\log\left(x^3-1\right)}{24}$$

```
[In] integrate(x**2/(x**6-10*x**3+9),x)
[Out] log(x**3 - 9)/24 - log(x**3 - 1)/24
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{9-10x^3+x^6}\,dx = -\frac{1}{24}\,\log\left(x^3-1\right)+\frac{1}{24}\,\log\left(x^3-9\right)$$

```
[In] integrate(x^2/(x^6-10*x^3+9),x, algorithm="maxima")
[Out] -1/24*log(x^3 - 1) + 1/24*log(x^3 - 9)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{9-10x^3+x^6}\,dx = -\frac{1}{24}\,\log\left(\left|x^3-1\right|\right)+\frac{1}{24}\,\log\left(\left|x^3-9\right|\right)$$

```
[In] integrate(x^2/(x^6-10*x^3+9),x, algorithm="giac")
[Out] -1/24*log(abs(x^3 - 1)) + 1/24*log(abs(x^3 - 9))
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{x^2}{9-10x^3+x^6}\,dx = \frac{\operatorname{atanh}\left(\frac{81}{320\left(\frac{5x^3}{4}-\frac{9}{8}\right)}-\frac{41}{40}\right)}{12}$$

```
[In] int(x^2/(x^6 - 10*x^3 + 9),x)
[Out] atanh(81/(320*((5*x^3)/4 - 9/8)) - 41/40)/12
```

3.158 $\int \frac{1-4x^2+x^3}{(-2+x)^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int \frac{1-4x^2+x^3}{(-2+x)^4} \, dx = -\frac{7}{3(2-x)^3}+\frac{2}{(2-x)^2}+\frac{2}{2-x}+\log(2-x)$$

```
[Out] -7/3/(2-x)^3+2/(2-x)^2+2/(2-x)+ln(2-x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1864}

$$\int \frac{1-4x^2+x^3}{(-2+x)^4} \, dx = \frac{2}{2-x}+\frac{2}{(2-x)^2}-\frac{7}{3(2-x)^3}+\log(2-x)$$

```
[In] Int[(1 - 4*x^2 + x^3)/(-2 + x)^4,x]
```

```
[Out] -7/(3*(2 - x)^3) + 2/(2 - x)^2 + 2/(2 - x) + Log[2 - x]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\frac{7}{(-2+x)^4}-\frac{4}{(-2+x)^3}+\frac{2}{(-2+x)^2}+\frac{1}{-2+x}\right) \, dx \\ &= -\frac{7}{3(2-x)^3}+\frac{2}{(2-x)^2}+\frac{2}{2-x}+\log(2-x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx=\frac{-29+30x-6x^2}{3(-2+x)^3}+\log(-2+x)$$

```
[In] Integrate[(1 - 4*x^2 + x^3)/(-2 + x)^4,x]
[Out] (-29 + 30*x - 6*x^2)/(3*(-2 + x)^3) + Log[-2 + x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

method	result	size
norman	$\frac{-2x^2+10x-\frac{29}{3}}{(-2+x)^3}+\ln(-2+x)$	22
risch	$\frac{-2x^2+10x-\frac{29}{3}}{(-2+x)^3}+\ln(-2+x)$	22
default	$-\frac{2}{-2+x}+\frac{2}{(-2+x)^2}+\frac{7}{3(-2+x)^3}+\ln(-2+x)$	27
parallelrisch	$\frac{3\ln(-2+x)x^3-29-18\ln(-2+x)x^2+36\ln(-2+x)x-6x^2-24\ln(-2+x)+30x}{3(-2+x)^3}$	49
meijerg	$\frac{x(\frac{1}{4}x^2-\frac{3}{2}x+3)}{48(1-\frac{x}{2})^3}+\frac{x(\frac{11}{2}x^2-15x+12)}{24(1-\frac{x}{2})^3}+\ln\left(1-\frac{x}{2}\right)-\frac{x^3}{12(1-\frac{x}{2})^3}$	60

```
[In] int((x^3-4*x^2+1)/(-2+x)^4,x,method=_RETURNVERBOSE)
[Out] (-2*x^2+10*x-29/3)/(-2+x)^3+ln(-2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx=-\frac{6\,x^2-3\,(x^3-6\,x^2+12\,x-8)\log(x-2)-30\,x+29}{3\,(x^3-6\,x^2+12\,x-8)}$$

```
[In] integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="fricas")
[Out] -1/3*(6*x^2 - 3*(x^3 - 6*x^2 + 12*x - 8)*log(x - 2) - 30*x + 29)/(x^3 - 6*x
^2 + 12*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx = \frac{-6x^2+30x-29}{3x^3-18x^2+36x-24}+\log{(x-2)}$$

```
[In] integrate((x**3-4*x**2+1)/(-2+x)**4,x)
[Out] (-6*x**2 + 30*x - 29)/(3*x**3 - 18*x**2 + 36*x - 24) + log(x - 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx = -\frac{6\,x^2-30\,x+29}{3\,(x^3-6\,x^2+12\,x-8)}+\log{(x-2)}$$

```
[In] integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="maxima")
[Out] -1/3*(6*x^2 - 30*x + 29)/(x^3 - 6*x^2 + 12*x - 8) + log(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx = -\frac{6\,x^2-30\,x+29}{3\,(x-2)^3}+\log{(|x-2|)}$$

```
[In] integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="giac")
[Out] -1/3*(6*x^2 - 30*x + 29)/(x - 2)^3 + log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx = \ln{(x-2)}-\frac{2\,x^2-10\,x+\frac{29}{3}}{(x-2)^3}$$

```
[In] int((x^3 - 4*x^2 + 1)/(x - 2)^4,x)
[Out] log(x - 2) - (2*x^2 - 10*x + 29/3)/(x - 2)^3
```

3.159 $\int \frac{x^3}{(-1+x)^{12}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 45

$$\int \frac{x^3}{(-1+x)^{12}} \, dx = \frac{1}{11(1-x)^{11}} - \frac{3}{10(1-x)^{10}} + \frac{1}{3(1-x)^9} - \frac{1}{8(1-x)^8}$$

```
[Out] 1/11/(1-x)^11-3/10/(1-x)^10+1/3/(1-x)^9-1/8/(1-x)^8
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{45\}$

$$\int \frac{x^3}{(-1+x)^{12}} \, dx = -\frac{1}{8(1-x)^8} + \frac{1}{3(1-x)^9} - \frac{3}{10(1-x)^{10}} + \frac{1}{11(1-x)^{11}}$$

```
[In] Int[x^3/(-1 + x)^12,x]
[Out] 1/(11*(1 - x)^11) - 3/(10*(1 - x)^10) + 1/(3*(1 - x)^9) - 1/(8*(1 - x)^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{(-1+x)^{12}} + \frac{3}{(-1+x)^{11}} + \frac{3}{(-1+x)^{10}} + \frac{1}{(-1+x)^9}\right) dx \\ &= \frac{1}{11(1-x)^{11}} - \frac{3}{10(1-x)^{10}} + \frac{1}{3(1-x)^9} - \frac{1}{8(1-x)^8} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{(-1+x)^{12}} \, dx = \frac{1 - 11x + 55x^2 - 165x^3}{1320(-1+x)^{11}}$$

[In] Integrate[x^3/(-1 + x)^12,x]

[Out] (1 - 11*x + 55*x^2 - 165*x^3)/(1320*(-1 + x)^11)

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

method	result	size
norman	$\frac{-\frac{1}{8}x^3+\frac{1}{24}x^2-\frac{1}{120}x+\frac{1}{1320}}{(-1+x)^{11}}$	22
risch	$\frac{-\frac{1}{8}x^3+\frac{1}{24}x^2-\frac{1}{120}x+\frac{1}{1320}}{(-1+x)^{11}}$	22
gosper	$-\frac{165x^3-55x^2+11x-1}{1320(-1+x)^{11}}$	23
parallelrisch	$\frac{-165x^3+55x^2-11x+1}{1320(-1+x)^{11}}$	23
default	$-\frac{1}{3(-1+x)^9}-\frac{1}{11(-1+x)^{11}}-\frac{1}{8(-1+x)^8}-\frac{3}{10(-1+x)^{10}}$	30
meijerg	$\frac{x^4(-x^7+11x^6-55x^5+165x^4-330x^3+462x^2-462x+330)}{1320(1-x)^{11}}$	48

[In] int(x^3/(-1+x)^12,x,method=_RETURNVERBOSE)

[Out] 1/(-1+x)^11*(-1/8*x^3+1/24*x^2-1/120*x+1/1320)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(29) = 58.

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int \frac{x^3}{(-1+x)^{12}} \, dx = -\frac{165\,x^3 - 55\,x^2 + 11\,x - 1}{1320\,(x^{11} - 11\,x^{10} + 55\,x^9 - 165\,x^8 + 330\,x^7 - 462\,x^6 + 462\,x^5 - 330\,x^4 + 165\,x^3 - 55\,x^2 + 11\,x - 1)}$$

[In] integrate(x^3/(-1+x)^12,x, algorithm="fricas")

[Out] -1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 + 330*x^7 - 462*x^6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.56

$$\int \frac{x^3}{(-1+x)^{12}}\,dx = \frac{-165x^3+55x^2-11x+1}{1320x^{11}-14520x^{10}+72600x^9-217800x^8+435600x^7-609840x^6+609840x^5-435600x^4+217800x^3}$$

```
[In] integrate(x**3/(-1+x)**12,x)
```

```
[Out] (-165*x**3 + 55*x**2 - 11*x + 1)/(1320*x**11 - 14520*x**10 + 72600*x**9 - 2
17800*x**8 + 435600*x**7 - 609840*x**6 + 609840*x**5 - 435600*x**4 + 217800
*x**3 - 72600*x**2 + 14520*x - 1320)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(29) = 58$.

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int \frac{x^3}{(-1+x)^{12}}\,dx = -\frac{165\,x^3-55\,x^2+11\,x-1}{1320\,(x^{11}-11\,x^{10}+55\,x^9-165\,x^8+330\,x^7-462\,x^6+462\,x^5-330\,x^4+165\,x^3-55\,x^2+11\,x-1)}$$

```
[In] integrate(x^3/(-1+x)^12,x, algorithm="maxima")
```

```
[Out] -1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 +
330*x^7 - 462*x^6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

$$\int \frac{x^3}{(-1+x)^{12}}\,dx = -\frac{165\,x^3-55\,x^2+11\,x-1}{1320\,(x-1)^{11}}$$

```
[In] integrate(x^3/(-1+x)^12,x, algorithm="giac")
```

```
[Out] -1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x - 1)^11
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{(-1+x)^{12}}\,dx = -\frac{1}{8\,(x-1)^8} - \frac{1}{3\,(x-1)^9} - \frac{3}{10\,(x-1)^{10}} - \frac{1}{11\,(x-1)^{11}}$$

```
[In] int(x^3/(x - 1)^12,x)
[Out] - 1/(8*(x - 1)^8) - 1/(3*(x - 1)^9) - 3/(10*(x - 1)^10) - 1/(11*(x - 1)^11)
```

3.160 $\int \frac{-3x+x^4}{(1+2x)^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{-3x+x^4}{(1+2x)^5} dx = -\frac{25}{128(1+2x)^4} + \frac{7}{24(1+2x)^3} - \frac{3}{32(1+2x)^2} + \frac{1}{8(1+2x)} + \frac{1}{32}\log(1+2x)$$

```
[Out] -25/128/(1+2*x)^4+7/24/(1+2*x)^3-3/32/(1+2*x)^2+1/8/(1+2*x)+1/32*ln(1+2*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1607, 1634}

$$\int \frac{-3x+x^4}{(1+2x)^5} dx = \frac{1}{8(2x+1)} - \frac{3}{32(2x+1)^2} + \frac{7}{24(2x+1)^3} - \frac{25}{128(2x+1)^4} + \frac{1}{32}\log(2x+1)$$

```
[In] Int[(-3*x + x^4)/(1 + 2*x)^5,x]
```

```
[Out] -25/(128*(1 + 2*x)^4) + 7/(24*(1 + 2*x)^3) - 3/(32*(1 + 2*x)^2) + 1/(8*(1 +
 2*x)) + Log[1 + 2*x]/32
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
```

```
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{x(-3+x^3)}{(1+2x)^5}\,dx \\ &= \int \left(\frac{25}{16(1+2x)^5} - \frac{7}{4(1+2x)^4} + \frac{3}{8(1+2x)^3} - \frac{1}{4(1+2x)^2} + \frac{1}{16(1+2x)}\right)dx \\ &= -\frac{25}{128(1+2x)^4} + \frac{7}{24(1+2x)^3} - \frac{3}{32(1+2x)^2} + \frac{1}{8(1+2x)} + \frac{1}{32}\log(1+2x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx = \frac{49+368x+432x^2+384x^3+12(1+2x)^4\log(1+2x)}{384(1+2x)^4}$$

```
[In] Integrate[(-3*x + x^4)/(1 + 2*x)^5,x]
[Out] (49 + 368*x + 432*x^2 + 384*x^3 + 12*(1 + 2*x)^4*Log[1 + 2*x])/(384*(1 + 2*
x)^4)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{x^3+\frac{9}{8}x^2+\frac{23}{24}x+\frac{49}{384}}{(1+2x)^4}+\frac{\ln(1+2x)}{32}$	34
norman	$\frac{-\frac{37}{12}x^3-\frac{31}{16}x^2-\frac{1}{16}x-\frac{49}{24}x^4}{(1+2x)^4}+\frac{\ln(1+2x)}{32}$	37
default	$-\frac{25}{128(1+2x)^4}+\frac{7}{24(1+2x)^3}-\frac{3}{32(1+2x)^2}+\frac{1}{8+16x}+\frac{\ln(1+2x)}{32}$	46
meijerg	$-\frac{x(1000x^3+1040x^2+420x+60)}{960(1+2x)^4}+\frac{\ln(1+2x)}{32}-\frac{x^2(4x^2+8x+6)}{4(1+2x)^4}$	57
parallelrisch	$\frac{48\ln(x+\frac{1}{2})x^4+96\ln(x+\frac{1}{2})x^3-196x^4+72\ln(x+\frac{1}{2})x^2-296x^3+24\ln(x+\frac{1}{2})x-186x^2+3\ln(x+\frac{1}{2})-6x}{96(1+2x)^4}$	69

```
[In] int((x^4-3*x)/(1+2*x)^5,x,method=_RETURNVERBOSE)
[Out] 16*(1/16*x^3+9/128*x^2+23/384*x+49/6144)/(1+2*x)^4+1/32*ln(1+2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx = \frac{384\,x^3+432\,x^2+12\,(16\,x^4+32\,x^3+24\,x^2+8\,x+1)\log(2\,x+1)+368\,x+49}{384\,(16\,x^4+32\,x^3+24\,x^2+8\,x+1)}$$

```
[In] integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="fricas")
```

```
[Out] 1/384*(384*x^3 + 432*x^2 + 12*(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1)*log(2*x
+ 1) + 368*x + 49)/(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx = \frac{384x^3+432x^2+368x+49}{6144x^4+12288x^3+9216x^2+3072x+384} + \frac{\log(2x+1)}{32}$$

```
[In] integrate((x**4-3*x)/(1+2*x)**5,x)
```

```
[Out] (384*x**3 + 432*x**2 + 368*x + 49)/(6144*x**4 + 12288*x**3 + 9216*x**2 + 30
72*x + 384) + log(2*x + 1)/32
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx = \frac{384\,x^3+432\,x^2+368\,x+49}{384\,(16\,x^4+32\,x^3+24\,x^2+8\,x+1)} + \frac{1}{32}\,\log(2\,x+1)$$

```
[In] integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="maxima")
```

```
[Out] 1/384*(384*x^3 + 432*x^2 + 368*x + 49)/(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1)
 + 1/32*log(2*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx = \frac{1}{8\,(2\,x+1)} - \frac{3}{32\,(2\,x+1)^2} + \frac{7}{24\,(2\,x+1)^3} - \frac{25}{128\,(2\,x+1)^4} - \frac{1}{32}\,\log\left(\frac{|2\,x+1|}{2\,(2\,x+1)^2}\right)$$

```
[In] integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="giac")
```

```
[Out] 1/8/(2*x + 1) - 3/32/(2*x + 1)^2 + 7/24/(2*x + 1)^3 - 25/128/(2*x + 1)^4 -
1/32*log(1/2*abs(2*x + 1)/(2*x + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx = \frac{\ln\left(x+\frac{1}{2}\right)}{32} + \frac{\frac{x^3}{16}+\frac{9\,x^2}{128}+\frac{23\,x}{384}+\frac{49}{6144}}{x^4+2\,x^3+\frac{3\,x^2}{2}+\frac{x}{2}+\frac{1}{16}}$$

```
[In] int(-(3*x - x^4)/(2*x + 1)^5,x)
```

```
[Out] log(x + 1/2)/32 + ((23*x)/384 + (9*x^2)/128 + x^3/16 + 49/6144)/(x/2 + (3*x
^2)/2 + 2*x^3 + x^4 + 1/16)
```

3.161 $\int \frac{1}{(-1+x)^2(1+x)^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{1}{(-1+x)^2(1+x)^3} \, dx = \frac{1}{8(1-x)} - \frac{1}{8(1+x)^2} - \frac{1}{4(1+x)} + \frac{3\operatorname{arctanh}(x)}{8}$$

```
[Out] 1/8/(1-x)-1/8/(1+x)^2-1/4/(1+x)+3/8*arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {46, 213}

$$\int \frac{1}{(-1+x)^2(1+x)^3} \, dx = \frac{3\operatorname{arctanh}(x)}{8} + \frac{1}{8(1-x)} - \frac{1}{4(x+1)} - \frac{1}{8(x+1)^2}$$

```
[In] Int[1/((-1 + x)^2*(1 + x)^3),x]
[Out] 1/(8*(1 - x)) - 1/(8*(1 + x)^2) - 1/(4*(1 + x)) + (3*ArcTanh[x])/8
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{8(-1+x)^2}+\frac{1}{4(1+x)^3}+\frac{1}{4(1+x)^2}-\frac{3}{8\left(-1+x^2\right)}\right) dx \\
&= \frac{1}{8(1-x)}-\frac{1}{8(1+x)^2}-\frac{1}{4(1+x)}-\frac{3}{8}\int \frac{1}{-1+x^2}\, dx \\
&= \frac{1}{8(1-x)}-\frac{1}{8(1+x)^2}-\frac{1}{4(1+x)}+\frac{3\text{arctanh}(x)}{8}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$
\int \frac{1}{(-1+x)^2(1+x)^3}\, dx = \frac{1}{16}\left(\frac{4-6x-6x^2}{(-1+x)(1+x)^2}-3\log(-1+x)+3\log(1+x)\right)
$$

```
[In] Integrate[1/((-1 + x)^2*(1 + x)^3),x]
[Out] ((4 - 6*x - 6*x^2)/((-1 + x)*(1 + x)^2) - 3*Log[-1 + x] + 3*Log[1 + x])/16
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result
default	$-\frac{1}{8(-1+x)}-\frac{3\ln(-1+x)}{16}-\frac{1}{8(1+x)^2}-\frac{1}{4(1+x)}+\frac{3\ln(1+x)}{16}$
norman	$\frac{-\frac{3}{8}x-\frac{3}{8}x^2+\frac{1}{4}}{(-1+x)(1+x)^2}-\frac{3\ln(-1+x)}{16}+\frac{3\ln(1+x)}{16}$
risch	$\frac{-\frac{3}{8}x-\frac{3}{8}x^2+\frac{1}{4}}{(-1+x)(1+x)^2}-\frac{3\ln(-1+x)}{16}+\frac{3\ln(1+x)}{16}$
parallelrisch	$-\frac{3\ln(-1+x)x^3-3\ln(1+x)x^3-4+3\ln(-1+x)x^2-3\ln(1+x)x^2-3\ln(-1+x)x+3\ln(1+x)x+6x^2-3\ln(-1+x)+3\ln(1+x)+6x}{16(-1+x)(1+x)^2}$

```
[In] int(1/(-1+x)^2/(1+x)^3,x,method=_RETURNVERBOSE)
[Out] -1/8/(-1+x)-3/16*ln(-1+x)-1/8/(1+x)^2-1/4/(1+x)+3/16*ln(1+x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(26) = 52.

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx = -\frac{6\,x^2 - 3\,(x^3 + x^2 - x - 1)\log(x+1) + 3\,(x^3 + x^2 - x - 1)\log(x-1) + 6\,x - 4}{16\,(x^3 + x^2 - x - 1)}$$

```
[In] integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*x^2 - 3*(x^3 + x^2 - x - 1)*log(x + 1) + 3*(x^3 + x^2 - x - 1)*log
(x - 1) + 6*x - 4)/(x^3 + x^2 - x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx = \frac{-3x^2 - 3x + 2}{8x^3 + 8x^2 - 8x - 8} - \frac{3\log(x-1)}{16} + \frac{3\log(x+1)}{16}$$

```
[In] integrate(1/(-1+x)**2/(1+x)**3,x)
```

```
[Out] (-3*x**2 - 3*x + 2)/(8*x**3 + 8*x**2 - 8*x - 8) - 3*log(x - 1)/16 + 3*log(x
 + 1)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx = -\frac{3\,x^2 + 3\,x - 2}{8\,(x^3 + x^2 - x - 1)} + \frac{3}{16}\,\log(x+1) - \frac{3}{16}\,\log(x-1)$$

```
[In] integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*x^2 + 3*x - 2)/(x^3 + x^2 - x - 1) + 3/16*log(x + 1) - 3/16*log(x -
 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx = -\frac{1}{8\,(x-1)} + \frac{\frac{12}{x-1}+5}{32\,\left(\frac{2}{x-1}+1\right)^2} + \frac{3}{16}\,\log\left(\left|-\frac{2}{x-1}-1\right|\right)$$

```
[In] integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="giac")
```

```
[Out] -1/8/(x - 1) + 1/32*(12/(x - 1) + 5)/(2/(x - 1) + 1)^2 + 3/16*log(abs(-2/(x
 - 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx = \frac{3\,\mathrm{atanh}(x)}{8} + \frac{\frac{3\,x^2}{8}+\frac{3\,x}{8}-\frac{1}{4}}{-x^3-x^2+x+1}$$

```
[In] int(1/((x - 1)^2*(x + 1)^3),x)
```

```
[Out] (3*atanh(x))/8 + ((3*x)/8 + (3*x^2)/8 - 1/4)/(x - x^2 - x^3 + 1)
```

3.162 $\int \frac{1}{(5-6x)^2x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 35

$$\int \frac{1}{(5-6x)^2x^2}\,dx = \frac{6}{25(5-6x)}-\frac{1}{25x}-\frac{12}{125}\log(5-6x)+\frac{12\log(x)}{125}$$

```
[Out] 6/25/(5-6*x)-1/25/x-12/125*ln(5-6*x)+12/125*ln(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\int \frac{1}{(5-6x)^2x^2}\,dx = \frac{6}{25(5-6x)}-\frac{1}{25x}-\frac{12}{125}\log(5-6x)+\frac{12\log(x)}{125}$$

```
[In] Int[1/((5 - 6*x)^2*x^2),x]
[Out] 6/(25*(5 - 6*x)) - 1/(25*x) - (12*Log[5 - 6*x])/125 + (12*Log[x])/125
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{25x^2}+\frac{12}{125x}+\frac{36}{25(-5+6x)^2}-\frac{72}{125(-5+6x)}\right)dx \\ &= \frac{6}{25(5-6x)}-\frac{1}{25x}-\frac{12}{125}\log(5-6x)+\frac{12\log(x)}{125} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{(5-6x)^2x^2}\,dx=\frac{1}{125}\left(\frac{30}{5-6x}-\frac{5}{x}-12\log(5-6x)+12\log(x)\right)$$

```
[In] Integrate[1/((5 - 6*x)^2*x^2),x]
[Out] (30/(5 - 6*x) - 5/x - 12*Log[5 - 6*x] + 12*Log[x])/125
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{1}{25x}+\frac{12\ln(x)}{125}-\frac{6}{25(6x-5)}-\frac{12\ln(6x-5)}{125}$	28
risch	$\frac{-\frac{12x}{25}+\frac{1}{5}}{x(6x-5)}+\frac{12\ln(x)}{125}-\frac{12\ln(6x-5)}{125}$	31
norman	$\frac{\frac{1}{5}-\frac{72x^2}{125}}{x(6x-5)}+\frac{12\ln(x)}{125}-\frac{12\ln(6x-5)}{125}$	32
meijerg	$-\frac{1}{25x}+\frac{6}{125}+\frac{12\ln(x)}{125}+\frac{12\ln(2)}{125}+\frac{12\ln(3)}{125}-\frac{12\ln(5)}{125}+\frac{12i\pi}{125}+\frac{108x}{625\left(3-\frac{18x}{5}\right)}-\frac{12\ln\left(1-\frac{6x}{5}\right)}{125}$	46
parallelrisch	$\frac{72x^2\ln(x)-72\ln\left(x-\frac{5}{6}\right)x^2+25-60x\ln(x)+60\ln\left(x-\frac{5}{6}\right)x-72x^2}{125(6x-5)x}$	48

```
[In] int(1/(-6*x+5)^2/x^2,x,method=_RETURNVERBOSE)
[Out] -1/25/x+12/125*ln(x)-6/25/(6*x-5)-12/125*ln(6*x-5)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{1}{(5-6x)^2x^2}\,dx=-\frac{12\,(6\,x^2-5\,x)\log\left(6\,x-5\right)-12\,(6\,x^2-5\,x)\log\left(x\right)+60\,x-25}{125\,(6\,x^2-5\,x)}$$

```
[In] integrate(1/(5-6*x)^2/x^2,x, algorithm="fricas")
[Out] -1/125*(12*(6*x^2 - 5*x)*log(6*x - 5) - 12*(6*x^2 - 5*x)*log(x) + 60*x - 25
)/(6*x^2 - 5*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{1}{(5-6x)^2x^2}\,dx = \frac{5-12x}{150x^2-125x} + \frac{12\log(x)}{125} - \frac{12\log\left(x-\frac{5}{6}\right)}{125}$$

```
[In] integrate(1/(5-6*x)**2/x**2,x)
[Out] (5 - 12*x)/(150*x**2 - 125*x) + 12*log(x)/125 - 12*log(x - 5/6)/125
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{(5-6x)^2x^2}\,dx = -\frac{12\,x-5}{25\,(6\,x^2-5\,x)} - \frac{12}{125}\,\log(6\,x-5) + \frac{12}{125}\,\log(x)$$

```
[In] integrate(1/(5-6*x)^2/x^2,x, algorithm="maxima")
[Out] -1/25*(12*x - 5)/(6*x^2 - 5*x) - 12/125*log(6*x - 5) + 12/125*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{1}{(5-6x)^2x^2}\,dx = -\frac{6}{25\,(6\,x-5)} + \frac{6}{125\,\left(\frac{5}{6\,x-5}+1\right)} + \frac{12}{125}\,\log\left(\left|-\frac{5}{6\,x-5}-1\right|\right)$$

```
[In] integrate(1/(5-6*x)^2/x^2,x, algorithm="giac")
[Out] -6/25/(6*x - 5) + 6/125/(5/(6*x - 5) + 1) + 12/125*log(abs(-5/(6*x - 5) - 1
))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{1}{(5-6x)^2x^2}\,dx = \frac{1}{5\,x\,(6\,x-5)} - \frac{12}{25\,(6\,x-5)} - \frac{12\,\ln\left(\frac{6\,x-5}{x}\right)}{125}$$

```
[In] int(1/(x^2*(6*x - 5)^2),x)
[Out] 1/(5*x*(6*x - 5)) - 12/(25*(6*x - 5)) - (12*log((6*x - 5)/x))/125
```

3.163 $\int \frac{1}{(-3-2x+x^2)^3} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \frac{1}{(-3-2x+x^2)^3} \, dx = \frac{1-x}{16(3+2x-x^2)^2}+\frac{3(1-x)}{128(3+2x-x^2)}+\frac{3}{512}\log(3-x)-\frac{3}{512}\log(1+x)$$

```
[Out] 1/16*(1-x)/(-x^2+2*x+3)^2+3/128*(1-x)/(-x^2+2*x+3)+3/512*ln(3-x)-3/512*ln(1
+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {628, 630, 31}

$$\int \frac{1}{(-3-2x+x^2)^3} \, dx = \frac{3(1-x)}{128(-x^2+2x+3)}+\frac{1-x}{16(-x^2+2x+3)^2}+\frac{3}{512}\log(3-x)-\frac{3}{512}\log(x+1)$$

```
[In] Int[(-3 - 2*x + x^2)^(-3),x]
```

```
[Out] (1 - x)/(16*(3 + 2*x - x^2)^2) + (3*(1 - x))/(128*(3 + 2*x - x^2)) + (3*Log
[3 - x])/512 - (3*Log[1 + x])/512
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1-x}{16\left(3+2x-x^2\right)^2}-\frac{3}{16}\int \frac{1}{\left(-3-2x+x^2\right)^2}\,dx\\
&= \frac{1-x}{16\left(3+2x-x^2\right)^2}+\frac{3(1-x)}{128\left(3+2x-x^2\right)}+\frac{3}{128}\int \frac{1}{-3-2x+x^2}\,dx\\
&= \frac{1-x}{16\left(3+2x-x^2\right)^2}+\frac{3(1-x)}{128\left(3+2x-x^2\right)}+\frac{3}{512}\int \frac{1}{-3+x}\,dx-\frac{3}{512}\int \frac{1}{1+x}\,dx\\
&= \frac{1-x}{16\left(3+2x-x^2\right)^2}+\frac{3(1-x)}{128\left(3+2x-x^2\right)}+\frac{3}{512}\log(3-x)-\frac{3}{512}\log(1+x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(-3-2x+x^2\right)^3}\,dx=\frac{1}{512}\left(\frac{4\left(17-11x-9x^2+3x^3\right)}{\left(-3-2x+x^2\right)^2}+3\log(3-x)-3\log(1+x)\right)$$

```
[In] Integrate[(-3 - 2*x + x^2)^(-3),x]
[Out] ((4*(17 - 11*x - 9*x^2 + 3*x^3))/(-3 - 2*x + x^2)^2 + 3*Log[3 - x] - 3*Log[
1 + x])/512
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

method	result
norman	$\frac{-\frac{11}{128}x-\frac{9}{128}x^2+\frac{3}{128}x^3+\frac{17}{128}}{(x^2-2x-3)^2}+\frac{3\ln(-3+x)}{512}-\frac{3\ln(1+x)}{512}$
risch	$\frac{-\frac{11}{128}x-\frac{9}{128}x^2+\frac{3}{128}x^3+\frac{17}{128}}{(x^2-2x-3)^2}+\frac{3\ln(-3+x)}{512}-\frac{3\ln(1+x)}{512}$
default	$\frac{1}{128(1+x)^2}+\frac{3}{256(1+x)}-\frac{3\ln(1+x)}{512}-\frac{1}{128(-3+x)^2}+\frac{3}{256(-3+x)}+\frac{3\ln(-3+x)}{512}$
parallelrisch	$-\frac{3\ln(1+x)x^4-3\ln(-3+x)x^4-68-12\ln(1+x)x^3+12\ln(-3+x)x^3-6\ln(1+x)x^2+6\ln(-3+x)x^2-12x^3+36\ln(1+x)x-36\ln(\cdots}{512(x^2-2x-3)^2}$

```
[In] int(1/(x^2-2*x-3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-11/128*x-9/128*x^2+3/128*x^3+17/128)/(x^2-2*x-3)^2+3/512*ln(-3+x)-3/512*l
n(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = \frac{12\,x^3-36\,x^2-3\,(x^4-4\,x^3-2\,x^2+12\,x+9)\log(x+1)+3\,(x^4-4\,x^3-2\,x^2+12\,x+9)\log(x-3)-\cdots}{512\,(x^4-4\,x^3-2\,x^2+12\,x+9)}$$

```
[In] integrate(1/(x^2-2*x-3)^3,x, algorithm="fricas")
```

```
[Out] 1/512*(12*x^3 - 36*x^2 - 3*(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)*log(x + 1) + 3*
(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)*log(x - 3) - 44*x + 68)/(x^4 - 4*x^3 - 2*x
^2 + 12*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = \frac{3x^3-9x^2-11x+17}{128x^4-512x^3-256x^2+1536x+1152} + \frac{3\log(x-3)}{512} - \frac{3\log(x+1)}{512}$$

```
[In] integrate(1/(x**2-2*x-3)**3,x)
```

```
[Out] (3*x**3 - 9*x**2 - 11*x + 17)/(128*x**4 - 512*x**3 - 256*x**2 + 1536*x + 11
52) + 3*log(x - 3)/512 - 3*log(x + 1)/512
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = \frac{3\,x^3-9\,x^2-11\,x+17}{128\,(x^4-4\,x^3-2\,x^2+12\,x+9)} - \frac{3}{512}\,\log(x+1) + \frac{3}{512}\,\log(x-3)$$

```
[In] integrate(1/(x^2-2*x-3)^3,x, algorithm="maxima")
```

```
[Out] 1/128*(3*x^3 - 9*x^2 - 11*x + 17)/(x^4 - 4*x^3 - 2*x^2 + 12*x + 9) - 3/512*
log(x + 1) + 3/512*log(x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = \frac{3\,x^3-9\,x^2-11\,x+17}{128\,(x^2-2\,x-3)^2} - \frac{3}{512}\,\log(|x+1|) + \frac{3}{512}\,\log(|x-3|)$$

```
[In] integrate(1/(x^2-2*x-3)^3,x, algorithm="giac")
```

```
[Out] 1/128*(3*x^3 - 9*x^2 - 11*x + 17)/(x^2 - 2*x - 3)^2 - 3/512*log(abs(x + 1))
 + 3/512*log(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = -\frac{3\,\ln\left(\frac{x+1}{x-3}\right)}{512} - 6\left(\frac{1}{256\,(-x^2+2\,x+3)} + \frac{1}{96\,(-x^2+2\,x+3)^2}\right)(x-1)$$

```
[In] int(-1/(2*x - x^2 + 3)^3,x)
```

```
[Out] - (3*log((x + 1)/(x - 3)))/512 - 6*(1/(256*(2*x - x^2 + 3)) + 1/(96*(2*x -
x^2 + 3)^2))*(x - 1)
```

3.164 $\int \frac{1}{(13-4x+x^2)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 51

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = -\frac{2-x}{36(13-4x+x^2)^2}-\frac{2-x}{216(13-4x+x^2)}+\frac{1}{648}\arctan\left(\frac{1}{3}(-2+x)\right)$$

```
[Out] 1/36*(-2+x)/(x^2-4*x+13)^2+1/216*(-2+x)/(x^2-4*x+13)+1/648*arctan(-2/3+1/3*
x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {628, 632, 210}

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{1}{648}\arctan\left(\frac{x-2}{3}\right)-\frac{2-x}{216(x^2-4x+13)}-\frac{2-x}{36(x^2-4x+13)^2}$$

```
[In] Int[(13 - 4*x + x^2)^(-3),x]
```

```
[Out] -1/36*(2 - x)/(13 - 4*x + x^2)^2 - (2 - x)/(216*(13 - 4*x + x^2)) + ArcTan[
(-2 + x)/3]/648
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2-x}{36\left(13-4x+x^2\right)^2}+\frac{1}{12}\int\frac{1}{\left(13-4x+x^2\right)^2}\,dx\\
&= -\frac{2-x}{36\left(13-4x+x^2\right)^2}-\frac{2-x}{216\left(13-4x+x^2\right)}+\frac{1}{216}\int\frac{1}{13-4x+x^2}\,dx\\
&= -\frac{2-x}{36\left(13-4x+x^2\right)^2}-\frac{2-x}{216\left(13-4x+x^2\right)}-\frac{1}{108}\text{Subst}\left(\int\frac{1}{-36-x^2}\,dx,x,-4+2x\right)\\
&= -\frac{2-x}{36\left(13-4x+x^2\right)^2}-\frac{2-x}{216\left(13-4x+x^2\right)}+\frac{1}{648}\arctan\left(\frac{1}{3}(-2+x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

$$\int\frac{1}{\left(13-4x+x^2\right)^3}\,dx=\frac{1}{648}\left(\frac{3(-2+x)\left(19-4x+x^2\right)}{\left(13-4x+x^2\right)^2}+\arctan\left(\frac{1}{3}(-2+x)\right)\right)$$

```
[In] Integrate[(13 - 4*x + x^2)^(-3),x]
[Out] ((3*(-2 + x)*(19 - 4*x + x^2))/(13 - 4*x + x^2)^2 + ArcTan[(-2 + x)/3])/648
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\frac{1}{216}x^3-\frac{1}{36}x^2+\frac{1}{8}x-\frac{19}{108}}{(x^2-4x+13)^2}+\frac{\arctan\left(-\frac{2}{3}+\frac{x}{3}\right)}{648}$
default	$\frac{-4+2x}{72(x^2-4x+13)^2}+\frac{-4+2x}{432x^2-1728x+5616}+\frac{\arctan\left(-\frac{2}{3}+\frac{x}{3}\right)}{648}$
parallelrisch	$-\frac{7098i\ln(x-2-3i)x^2-28561i\ln(x-2+3i)+17576i\ln(x-2+3i)x+169i\ln(x-2-3i)x^4+1352i\ln(x-2+3i)x^3-1352i\ln(x-2-}{219024(x^2}$

```
[In] int(1/(x^2-4*x+13)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/216*x^3-1/36*x^2+1/8*x-19/108)/(x^2-4*x+13)^2+1/648*arctan(-2/3+1/3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{3\,x^3-18\,x^2+(x^4-8\,x^3+42\,x^2-104\,x+169)\arctan\left(\frac{1}{3}\,x-\frac{2}{3}\right)+81\,x-114}{648\,(x^4-8\,x^3+42\,x^2-104\,x+169)}$$

```
[In] integrate(1/(x^2-4*x+13)^3,x, algorithm="fricas")
```

```
[Out] 1/648*(3*x^3 - 18*x^2 + (x^4 - 8*x^3 + 42*x^2 - 104*x + 169)*arctan(1/3*x -
 2/3) + 81*x - 114)/(x^4 - 8*x^3 + 42*x^2 - 104*x + 169)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{x^3-6x^2+27x-38}{216x^4-1728x^3+9072x^2-22464x+36504}+\frac{\operatorname{atan}\left(\frac{x}{3}-\frac{2}{3}\right)}{648}$$

```
[In] integrate(1/(x**2-4*x+13)**3,x)
```

```
[Out] (x**3 - 6*x**2 + 27*x - 38)/(216*x**4 - 1728*x**3 + 9072*x**2 - 22464*x + 3
6504) + atan(x/3 - 2/3)/648
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{x^3-6\,x^2+27\,x-38}{216\,(x^4-8\,x^3+42\,x^2-104\,x+169)} + \frac{1}{648}\,\arctan\left(\frac{1}{3}\,x-\frac{2}{3}\right)$$

```
[In] integrate(1/(x^2-4*x+13)^3,x, algorithm="maxima")
```

```
[Out] 1/216*(x^3 - 6*x^2 + 27*x - 38)/(x^4 - 8*x^3 + 42*x^2 - 104*x + 169) + 1/64
8*arctan(1/3*x - 2/3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{x^3-6\,x^2+27\,x-38}{216\,(x^2-4\,x+13)^2} + \frac{1}{648}\,\arctan\left(\frac{1}{3}\,x-\frac{2}{3}\right)$$

```
[In] integrate(1/(x^2-4*x+13)^3,x, algorithm="giac")
```

```
[Out] 1/216*(x^3 - 6*x^2 + 27*x - 38)/(x^2 - 4*x + 13)^2 + 1/648*arctan(1/3*x - 2
/3)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{\mathrm{atan}\left(\frac{x}{3}-\frac{2}{3}\right)}{648} + 6\,(x-2)\,\left(\frac{1}{1296\,\left(x^2-4\,x+13\right)} + \frac{1}{216\,(x^2-4\,x+13)^2}\right)$$

```
[In] int(1/(x^2 - 4*x + 13)^3,x)
```

```
[Out] atan(x/3 - 2/3)/648 + 6*(x - 2)*(1/(1296*(x^2 - 4*x + 13)) + 1/(216*(x^2 -
4*x + 13)^2))
```

3.165 $\int \frac{1}{(2+x)^3(3+x)^4} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \frac{1}{(2+x)^3(3+x)^4} \, dx = -\frac{1}{2(2+x)^2}+\frac{4}{2+x}+\frac{1}{3(3+x)^3}+\frac{3}{2(3+x)^2}+\frac{6}{3+x}+10\log(2+x)-10\log(3+x)$$

```
[Out] -1/2/(2+x)^2+4/(2+x)+1/3/(3+x)^3+3/2/(3+x)^2+6/(3+x)+10*ln(2+x)-10*ln(3+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\int \frac{1}{(2+x)^3(3+x)^4} \, dx = \frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

```
[In] Int[1/((2 + x)^3*(3 + x)^4),x]
```

```
[Out] -1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x
) + 10*Log[2 + x] - 10*Log[3 + x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{(2+x)^3} - \frac{4}{(2+x)^2} + \frac{10}{2+x} - \frac{1}{(3+x)^4} - \frac{3}{(3+x)^3} - \frac{6}{(3+x)^2} - \frac{10}{3+x} \right) dx$$
$$= -\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10\log(2+x) - 10\log(3+x)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = -\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10\log(2+x) - 10\log(3+x)$$

```
[In] Integrate[1/((2 + x)^3*(3 + x)^4),x]
[Out] -1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x
) + 10*Log[2 + x] - 10*Log[3 + x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result
norman	$\frac{10x^4+105x^3+\frac{1225}{3}x^2+\frac{4175}{6}x+\frac{2627}{6}}{(2+x)^2(3+x)^3}+10\ln(2+x)-10\ln(3+x)$
risch	$\frac{10x^4+105x^3+\frac{1225}{3}x^2+\frac{4175}{6}x+\frac{2627}{6}}{(2+x)^2(3+x)^3}+10\ln(2+x)-10\ln(3+x)$
default	$-\frac{1}{2(2+x)^2}+\frac{4}{2+x}+\frac{1}{3(3+x)^3}+\frac{3}{2(3+x)^2}+\frac{6}{3+x}+10\ln(2+x)-10\ln(3+x)$
parallelrisch	$\frac{60\ln(2+x)x^5-60\ln(3+x)x^5+2627+780\ln(2+x)x^4-780\ln(3+x)x^4+4020\ln(2+x)x^3-4020\ln(3+x)x^3+60x^4+10260\ln(2+x)x^2}{6(2+x)^2(3+x)^3}$

```
[In] int(1/(2+x)^3/(3+x)^4,x,method=_RETURNVERBOSE)
[Out] (10*x^4+105*x^3+1225/3*x^2+4175/6*x+2627/6)/(2+x)^2/(3+x)^3+10*ln(2+x)-10*l
n(3+x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(48) = 96$.

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.94

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60\,x^4 + 630\,x^3 + 2450\,x^2 - 60\,(x^5 + 13\,x^4 + 67\,x^3 + 171\,x^2 + 216\,x + 108)\log(x+3) + 60\,(x^5 + 13\,x^4 + }{6\,(x^5 + 13\,x^4 + 67\,x^3 + 171\,x^2 + 216\,x + 108)}$$

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 - 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 21
6*x + 108)*log(x + 3) + 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108)*
log(x + 2) + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60x^4 + 630x^3 + 2450x^2 + 4175x + 2627}{6x^5 + 78x^4 + 402x^3 + 1026x^2 + 1296x + 648} + 10\log(x+2) - 10\log(x+3)$$

```
[In] integrate(1/(2+x)**3/(3+x)**4,x)
```

```
[Out] (60*x**4 + 630*x**3 + 2450*x**2 + 4175*x + 2627)/(6*x**5 + 78*x**4 + 402*x*
*3 + 1026*x**2 + 1296*x + 648) + 10*log(x + 2) - 10*log(x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60\,x^4 + 630\,x^3 + 2450\,x^2 + 4175\,x + 2627}{6\,(x^5 + 13\,x^4 + 67\,x^3 + 171\,x^2 + 216\,x + 108)} - 10\,\log(x+3) + 10\,\log(x+2)$$

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 +
171*x^2 + 216*x + 108) - 10*log(x + 3) + 10*log(x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60\,x^4+630\,x^3+2450\,x^2+4175\,x+2627}{6\,(x+3)^3(x+2)^2} - 10\,\log\left(|x+3|\right)+10\,\log\left(|x+2|\right)$$

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="giac")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/((x + 3)^3*(x + 2)^2) - 1
0*log(abs(x + 3)) + 10*log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{10\,x^4+105\,x^3+\frac{1225\,x^2}{3}+\frac{4175\,x}{6}+\frac{2627}{6}}{x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108} - 20\,\mathrm{atanh}(2\,x+5)$$

```
[In] int(1/((x + 2)^3*(x + 3)^4),x)
```

```
[Out] ((4175*x)/6 + (1225*x^2)/3 + 105*x^3 + 10*x^4 + 2627/6)/(216*x + 171*x^2 +
67*x^3 + 13*x^4 + x^5 + 108) - 20*atanh(2*x + 5)
```

3.166 $\int \frac{x^6}{(-2+x^2)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{x^6}{(-2+x^2)^2} dx = 4x + \frac{x^3}{3} - \frac{2x}{-2+x^2} - 5\sqrt{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 4*x+1/3*x^3-2*x/(x^2-2)-5*arctanh(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {294, 308, 213}

$$\int \frac{x^6}{(-2+x^2)^2} dx = -5\sqrt{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right) + \frac{5x^3}{6} + \frac{x^5}{2(2-x^2)} + 5x$$

```
[In] Int[x^6/(-2 + x^2)^2,x]
```

```
[Out] 5*x + (5*x^3)/6 + x^5/(2*(2 - x^2)) - 5*Sqrt[2]*ArcTanh[x/Sqrt[2]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^5}{2(2-x^2)}+\frac{5}{2}\int\frac{x^4}{-2+x^2}\,dx\\ &= \frac{x^5}{2(2-x^2)}+\frac{5}{2}\int\left(2+x^2+\frac{4}{-2+x^2}\right)dx\\ &= 5x+\frac{5x^3}{6}+\frac{x^5}{2(2-x^2)}+10\int\frac{1}{-2+x^2}\,dx\\ &= 5x+\frac{5x^3}{6}+\frac{x^5}{2(2-x^2)}-5\sqrt{2}\operatorname{arctanh}\left(\frac{x}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int\frac{x^6}{(-2+x^2)^2}\,dx=4x+\frac{x^3}{3}-\frac{2x}{-2+x^2}+\frac{5\log\left(\sqrt{2}-x\right)}{\sqrt{2}}-\frac{5\log\left(\sqrt{2}+x\right)}{\sqrt{2}}$$

```
[In] Integrate[x^6/(-2 + x^2)^2,x]
[Out] 4*x + x^3/3 - (2*x)/(-2 + x^2) + (5*Log[Sqrt[2] - x])/Sqrt[2] - (5*Log[Sqrt
[2] + x])/Sqrt[2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$4x+\frac{x^3}{3}-\frac{2x}{x^2-2}-5\ \mathrm{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	32
risch	$\frac{x^3}{3}+4x-\frac{2x}{x^2-2}+\frac{5\sqrt{2}\ \ln\left(x-\sqrt{2}\right)}{2}-\frac{5\sqrt{2}\ \ln\left(x+\sqrt{2}\right)}{2}$	44
meijerg	$i\sqrt{2}\left(-\frac{ix\sqrt{2}\left(-\frac{7}{2}x^4-35x^2+105\right)}{42\left(-\frac{x^2}{2}+1\right)}+5i\,\mathrm{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\right)$	46

```
[In] int(x^6/(x^2-2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4*x+1/3*x^3-2*x/(x^2-2)-5*arctanh(1/2*x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{x^6}{(-2+x^2)^2}\,dx=\frac{2\,x^5+20\,x^3+15\,\sqrt{2}(x^2-2)\log\left(\frac{x^2-2\,\sqrt{2}x+2}{x^2-2}\right)-60\,x}{6\,(x^2-2)}$$

```
[In] integrate(x^6/(x^2-2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^5 + 20*x^3 + 15*sqrt(2)*(x^2 - 2)*log((x^2 - 2*sqrt(2)*x + 2)/(x^2
 - 2)) - 60*x)/(x^2 - 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36

$$\int \frac{x^6}{(-2+x^2)^2}\,dx=\frac{x^3}{3}+4x-\frac{2x}{x^2-2}+\frac{5\sqrt{2}\log\left(x-\sqrt{2}\right)}{2}-\frac{5\sqrt{2}\log\left(x+\sqrt{2}\right)}{2}$$

```
[In] integrate(x**6/(x**2-2)**2,x)
```

```
[Out] x**3/3 + 4*x - 2*x/(x**2 - 2) + 5*sqrt(2)*log(x - sqrt(2))/2 - 5*sqrt(2)*lo
g(x + sqrt(2))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{x^6}{(-2+x^2)^2}\,dx = \frac{1}{3}\,x^3 + \frac{5}{2}\,\sqrt{2}\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right) + 4\,x - \frac{2\,x}{x^2-2}$$

```
[In] integrate(x^6/(x^2-2)^2,x, algorithm="maxima")
[Out] 1/3*x^3 + 5/2*sqrt(2)*log((x - sqrt(2))/(x + sqrt(2))) + 4*x - 2*x/(x^2 - 2
)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{x^6}{(-2+x^2)^2}\,dx = \frac{1}{3}\,x^3 + \frac{5}{2}\,\sqrt{2}\log\left(\frac{|2\,x-2\,\sqrt{2}|}{|2\,x+2\,\sqrt{2}|}\right) + 4\,x - \frac{2\,x}{x^2-2}$$

```
[In] integrate(x^6/(x^2-2)^2,x, algorithm="giac")
[Out] 1/3*x^3 + 5/2*sqrt(2)*log(abs(2*x - 2*sqrt(2))/abs(2*x + 2*sqrt(2))) + 4*x
- 2*x/(x^2 - 2)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{(-2+x^2)^2}\,dx = 4\,x - \frac{2\,x}{x^2-2} + \frac{x^3}{3} + \sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,1\mathrm{i}}{2}\right)\,5\mathrm{i}$$

```
[In] int(x^6/(x^2 - 2)^2,x)
[Out] 4*x + 2^(1/2)*atan((2^(1/2)*x*1i)/2)*5i - (2*x)/(x^2 - 2) + x^3/3
```

3.167 $\int \frac{x^8}{(4+x^2)^4} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 58

$$\int \frac{x^8}{(4+x^2)^4} \, dx = \frac{35x}{16} - \frac{x^7}{6(4+x^2)^3} - \frac{7x^5}{24(4+x^2)^2} - \frac{35x^3}{48(4+x^2)} - \frac{35}{8}\arctan\left(\frac{x}{2}\right)$$

```
[Out] 35/16*x-1/6*x^7/(x^2+4)^3-7/24*x^5/(x^2+4)^2-35/48*x^3/(x^2+4)-35/8*arctan(
1/2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {294, 327, 209}

$$\int \frac{x^8}{(4+x^2)^4} \, dx = -\frac{35}{8}\arctan\left(\frac{x}{2}\right) - \frac{x^7}{6(x^2+4)^3} - \frac{7x^5}{24(x^2+4)^2} - \frac{35x^3}{48(x^2+4)} + \frac{35x}{16}$$

```
[In] Int[x^8/(4 + x^2)^4,x]
```

```
[Out] (35*x)/16 - x^7/(6*(4 + x^2)^3) - (7*x^5)/(24*(4 + x^2)^2) - (35*x^3)/(48*(
4 + x^2)) - (35*ArcTan[x/2])/8
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^7}{6(4+x^2)^3}+\frac{7}{6}\int \frac{x^6}{(4+x^2)^3}\,dx\\
&= -\frac{x^7}{6(4+x^2)^3}-\frac{7x^5}{24(4+x^2)^2}+\frac{35}{24}\int \frac{x^4}{(4+x^2)^2}\,dx\\
&= -\frac{x^7}{6(4+x^2)^3}-\frac{7x^5}{24(4+x^2)^2}-\frac{35x^3}{48(4+x^2)}+\frac{35}{16}\int \frac{x^2}{4+x^2}\,dx\\
&= \frac{35x}{16}-\frac{x^7}{6(4+x^2)^3}-\frac{7x^5}{24(4+x^2)^2}-\frac{35x^3}{48(4+x^2)}-\frac{35}{4}\int \frac{1}{4+x^2}\,dx\\
&= \frac{35x}{16}-\frac{x^7}{6(4+x^2)^3}-\frac{7x^5}{24(4+x^2)^2}-\frac{35x^3}{48(4+x^2)}-\frac{35}{8}\arctan\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{x^8}{(4+x^2)^4}\,dx=\frac{x(1680+1120x^2+231x^4+12x^6)}{12(4+x^2)^3}-\frac{35}{8}\arctan\left(\frac{x}{2}\right)$$

```
[In] Integrate[x^8/(4 + x^2)^4,x]
[Out] (x*(1680 + 1120*x^2 + 231*x^4 + 12*x^6))/(12*(4 + x^2)^3) - (35*ArcTan[x/2]
)/8
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

method	result
risch	$x+\frac{\frac{29}{4}x^5+\frac{136}{3}x^3+76x}{(x^2+4)^3}-\frac{35\arctan\left(\frac{x}{2}\right)}{8}$
default	$x-\frac{16\left(-\frac{29}{64}x^5-\frac{17}{6}x^3-\frac{19}{4}x\right)}{(x^2+4)^3}-\frac{35\arctan\left(\frac{x}{2}\right)}{8}$
meijerg	$\frac{x\left(\frac{9}{4}x^6+\frac{693}{16}x^4+210x^2+315\right)}{144\left(1+\frac{x^2}{4}\right)^3}-\frac{35\arctan\left(\frac{x}{2}\right)}{8}$
parallelrisch	$\frac{-420i\ln(x+2i)x^6-26880i\ln(x+2i)+192x^7-5040i\ln(x+2i)x^4+20160i\ln(x-2i)x^2+3696x^5+5040i\ln(x-2i)x^4+420i\ln(x-2i}{192(x^2+4)^3}$

```
[In] int(x^8/(x^2+4)^4,x,method=_RETURNVERBOSE)
[Out] x+(29/4*x^5+136/3*x^3+76*x)/(x^2+4)^3-35/8*arctan(1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{x^8}{(4+x^2)^4}\,dx = \frac{24\,x^7+462\,x^5+2240\,x^3-105\,(x^6+12\,x^4+48\,x^2+64)\arctan\left(\frac{1}{2}\,x\right)+3360\,x}{24\,(x^6+12\,x^4+48\,x^2+64)}$$

```
[In] integrate(x^8/(x^2+4)^4,x, algorithm="fricas")
[Out] 1/24*(24*x^7 + 462*x^5 + 2240*x^3 - 105*(x^6 + 12*x^4 + 48*x^2 + 64)*arctan
(1/2*x) + 3360*x)/(x^6 + 12*x^4 + 48*x^2 + 64)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \frac{x^8}{(4+x^2)^4}\,dx = x+\frac{87x^5+544x^3+912x}{12x^6+144x^4+576x^2+768}-\frac{35\,\mathrm{atan}\left(\frac{x}{2}\right)}{8}$$

```
[In] integrate(x**8/(x**2+4)**4,x)
[Out] x + (87*x**5 + 544*x**3 + 912*x)/(12*x**6 + 144*x**4 + 576*x**2 + 768) - 35
*atan(x/2)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{x^8}{(4+x^2)^4}\,dx = x + \frac{87\,x^5 + 544\,x^3 + 912\,x}{12\,(x^6 + 12\,x^4 + 48\,x^2 + 64)} - \frac{35}{8}\,\arctan\left(\frac{1}{2}\,x\right)$$

```
[In] integrate(x^8/(x^2+4)^4,x, algorithm="maxima")
[Out] x + 1/12*(87*x^5 + 544*x^3 + 912*x)/(x^6 + 12*x^4 + 48*x^2 + 64) - 35/8*arc
tan(1/2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

$$\int \frac{x^8}{(4+x^2)^4}\,dx = x + \frac{87\,x^5 + 544\,x^3 + 912\,x}{12\,(x^2+4)^3} - \frac{35}{8}\,\arctan\left(\frac{1}{2}\,x\right)$$

```
[In] integrate(x^8/(x^2+4)^4,x, algorithm="giac")
[Out] x + 1/12*(87*x^5 + 544*x^3 + 912*x)/(x^2 + 4)^3 - 35/8*arctan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{x^8}{(4+x^2)^4}\,dx = x - \frac{35\,\mathrm{atan}\left(\frac{x}{2}\right)}{8} + \frac{\frac{29\,x^5}{4} + \frac{136\,x^3}{3} + 76\,x}{x^6 + 12\,x^4 + 48\,x^2 + 64}$$

```
[In] int(x^8/(x^2 + 4)^4,x)
[Out] x - (35*atan(x/2))/8 + (76*x + (136*x^3)/3 + (29*x^5)/4)/(48*x^2 + 12*x^4 +
 x^6 + 64)
```

3.168 $\int \frac{-4+7x}{(5+2x+3x^2)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 43

$$\int \frac{-4+7x}{(5+2x+3x^2)^2} dx = -\frac{39+19x}{28(5+2x+3x^2)} - \frac{19\arctan\left(\frac{1+3x}{\sqrt{14}}\right)}{28\sqrt{14}}$$

```
[Out] 1/28*(-39-19*x)/(3*x^2+2*x+5)-19/392*arctan(1/14*(1+3*x)*14^(1/2))*14^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {652, 632, 210}

$$\int \frac{-4+7x}{(5+2x+3x^2)^2} dx = -\frac{19\arctan\left(\frac{3x+1}{\sqrt{14}}\right)}{28\sqrt{14}} - \frac{19x+39}{28(3x^2+2x+5)}$$

```
[In] Int[(-4 + 7*x)/(5 + 2*x + 3*x^2)^2,x]
```

```
[Out] -1/28*(39 + 19*x)/(5 + 2*x + 3*x^2) - (19*ArcTan[(1 + 3*x)/Sqrt[14]])/(28*S
qrt[14])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{39+19x}{28\left(5+2x+3x^2\right)}-\frac{19}{28}\int\frac{1}{5+2x+3x^2}\,dx\\
&= -\frac{39+19x}{28\left(5+2x+3x^2\right)}+\frac{19}{14}\text{Subst}\left(\int\frac{1}{-56-x^2}\,dx,x,2+6x\right)\\
&= -\frac{39+19x}{28\left(5+2x+3x^2\right)}-\frac{19\arctan\left(\frac{1+3x}{\sqrt{14}}\right)}{28\sqrt{14}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int\frac{-4+7x}{\left(5+2x+3x^2\right)^2}\,dx=\frac{-39-19x}{28\left(5+2x+3x^2\right)}-\frac{19\arctan\left(\frac{1+3x}{\sqrt{14}}\right)}{28\sqrt{14}}$$

```
[In] Integrate[(-4 + 7*x)/(5 + 2*x + 3*x^2)^2,x]
[Out] (-39 - 19*x)/(28*(5 + 2*x + 3*x^2)) - (19*ArcTan[(1 + 3*x)/Sqrt[14]])/(28*S
qrt[14])
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{-\frac{19x}{84}-\frac{13}{28}}{x^2+\frac{2}{3}x+\frac{5}{3}}-\frac{19\arctan\left(\frac{(1+3x)\sqrt{14}}{14}\right)\sqrt{14}}{392}$	34
default	$\frac{-38x-78}{168x^2+112x+280}-\frac{19\sqrt{14}\,\arctan\left(\frac{(6x+2)\sqrt{14}}{28}\right)}{392}$	37

```
[In] int((-4+7*x)/(3*x^2+2*x+5)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-19/84*x-13/28)/(x^2+2/3*x+5/3)-19/392*arctan(1/14*(1+3*x)*14^(1/2))*14^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx=-\frac{19\sqrt{14}(3x^2+2x+5)\arctan\left(\frac{1}{14}\sqrt{14}(3x+1)\right)+266x+546}{392\,(3x^2+2x+5)}$$

```
[In] integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="fricas")
```

```
[Out] -1/392*(19*sqrt(14)*(3*x^2 + 2*x + 5)*arctan(1/14*sqrt(14)*(3*x + 1)) + 266
*x + 546)/(3*x^2 + 2*x + 5)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx=\frac{-19x-39}{84x^2+56x+140}-\frac{19\sqrt{14}\,\mathrm{atan}\left(\frac{3\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{392}$$

```
[In] integrate((-4+7*x)/(3*x**2+2*x+5)**2,x)
```

```
[Out] (-19*x - 39)/(84*x**2 + 56*x + 140) - 19*sqrt(14)*atan(3*sqrt(14)*x/14 + sq
rt(14)/14)/392
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx = -\frac{19}{392}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(3\,x+1)\right) - \frac{19\,x+39}{28\,(3\,x^2+2\,x+5)}$$

```
[In] integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="maxima")
[Out] -19/392*sqrt(14)*arctan(1/14*sqrt(14)*(3*x + 1)) - 1/28*(19*x + 39)/(3*x^2
+ 2*x + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx = -\frac{19}{392}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(3\,x+1)\right) - \frac{19\,x+39}{28\,(3\,x^2+2\,x+5)}$$

```
[In] integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="giac")
[Out] -19/392*sqrt(14)*arctan(1/14*sqrt(14)*(3*x + 1)) - 1/28*(19*x + 39)/(3*x^2
+ 2*x + 5)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx = -\frac{\frac{19\,x}{84}+\frac{13}{28}}{x^2+\frac{2\,x}{3}+\frac{5}{3}} - \frac{19\,\sqrt{14}\,\mathrm{atan}\left(\frac{3\,\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{392}$$

```
[In] int((7*x - 4)/(2*x + 3*x^2 + 5)^2,x)
[Out] - ((19*x)/84 + 13/28)/((2*x)/3 + x^2 + 5/3) - (19*14^(1/2)*atan((3*14^(1/2)
*x)/14 + 14^(1/2)/14))/392
```

3.169 $\int \frac{5-4x}{(-2-4x+3x^2)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 43

$$\int \frac{5-4x}{(-2-4x+3x^2)^2} dx = -\frac{18-7x}{20(2+4x-3x^2)} - \frac{7\text{arctanh}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

```
[Out] 1/20*(-18+7*x)/(-3*x^2+4*x+2)-7/200*arctanh(1/10*(2-3*x)*10^(1/2))*10^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {652, 632, 212}

$$\int \frac{5-4x}{(-2-4x+3x^2)^2} dx = -\frac{7\text{arctanh}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}} - \frac{18-7x}{20(-3x^2+4x+2)}$$

```
[In] Int[(5 - 4*x)/(-2 - 4*x + 3*x^2)^2,x]
```

```
[Out] -1/20*(18 - 7*x)/(2 + 4*x - 3*x^2) - (7*ArcTanh[(2 - 3*x)/Sqrt[10]])/(20*Sq
rt[10])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{18-7x}{20\left(2+4x-3x^2\right)}-\frac{7}{20}\int\frac{1}{-2-4x+3x^2}\,dx\\
&= -\frac{18-7x}{20\left(2+4x-3x^2\right)}+\frac{7}{10}\text{Subst}\left(\int\frac{1}{40-x^2}\,dx,x,-4+6x\right)\\
&= -\frac{18-7x}{20\left(2+4x-3x^2\right)}-\frac{7\text{arctanh}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$
\int\frac{5-4x}{\left(-2-4x+3x^2\right)^2}\,dx=\frac{18-7x}{20\left(-2-4x+3x^2\right)}-\frac{7\log\left(2+\sqrt{10}-3x\right)}{40\sqrt{10}}+\frac{7\log\left(-2+\sqrt{10}+3x\right)}{40\sqrt{10}}
$$

```
[In] Integrate[(5 - 4*x)/(-2 - 4*x + 3*x^2)^2,x]
[Out] (18 - 7*x)/(20*(-2 - 4*x + 3*x^2)) - (7*Log[2 + Sqrt[10] - 3*x])/(40*Sqrt[1
0]) + (7*Log[-2 + Sqrt[10] + 3*x])/(40*Sqrt[10])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{14x-36}{40(3x^2-4x-2)}+\frac{7\sqrt{10}\,\operatorname{arctanh}\left(\frac{(6x-4)\sqrt{10}}{20}\right)}{200}$	37
risch	$\frac{-\frac{7x}{60}+\frac{3}{10}}{x^2-\frac{4}{3}x-\frac{2}{3}}+\frac{7\sqrt{10}\,\ln\left(3x-2+\sqrt{10}\right)}{400}-\frac{7\sqrt{10}\,\ln\left(3x-2-\sqrt{10}\right)}{400}$	48

```
[In] int((5-4*x)/(3*x^2-4*x-2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*(14*x-36)/(3*x^2-4*x-2)+7/200*10^(1/2)*arctanh(1/20*(6*x-4)*10^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = \frac{7\sqrt{10}(3x^2-4x-2)\log\left(\frac{9x^2+2\sqrt{10}(3x-2)-12x+14}{3x^2-4x-2}\right)-140x+360}{400\,(3x^2-4x-2)}$$

```
[In] integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="fricas")
```

```
[Out] 1/400*(7*sqrt(10)*(3*x^2 - 4*x - 2)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12*
x + 14)/(3*x^2 - 4*x - 2)) - 140*x + 360)/(3*x^2 - 4*x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = -\frac{7x-18}{60x^2-80x-40}+\frac{7\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{400} - \frac{7\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{400}$$

```
[In] integrate((5-4*x)/(3*x**2-4*x-2)**2,x)
```

```
[Out] -(7*x - 18)/(60*x**2 - 80*x - 40) + 7*sqrt(10)*log(x - 2/3 + sqrt(10)/3)/40
0 - 7*sqrt(10)*log(x - sqrt(10)/3 - 2/3)/400
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = -\frac{7}{400}\sqrt{10}\log\left(\frac{3\,x-\sqrt{10}-2}{3\,x+\sqrt{10}-2}\right)-\frac{7\,x-18}{20\,(3\,x^2-4\,x-2)}$$

```
[In] integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="maxima")
```

```
[Out] -7/400*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2)) - 1/20*(7*x
- 18)/(3*x^2 - 4*x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = -\frac{7}{400}\sqrt{10}\log\left(\frac{|6\,x-2\,\sqrt{10}-4|}{|6\,x+2\,\sqrt{10}-4|}\right)-\frac{7\,x-18}{20\,(3\,x^2-4\,x-2)}$$

```
[In] integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="giac")
```

```
[Out] -7/400*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4)) -
1/20*(7*x - 18)/(3*x^2 - 4*x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = \frac{7\,\sqrt{10}\,\mathrm{atanh}\left(\sqrt{10}\,\left(\frac{3\,x}{10}-\frac{1}{5}\right)\right)}{200}+\frac{\frac{7\,x}{60}-\frac{3}{10}}{-x^2+\frac{4\,x}{3}+\frac{2}{3}}$$

```
[In] int(-(4*x - 5)/(4*x - 3*x^2 + 2)^2,x)
```

```
[Out] (7*10^(1/2)*atanh(10^(1/2)*((3*x)/10 - 1/5)))/200 + ((7*x)/60 - 3/10)/((4*x
)/3 - x^2 + 2/3)
```

3.170 $\int \frac{x^5}{(1+x^4)^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{x^5}{(1+x^4)^3} \, dx = -\frac{x^2}{8(1+x^4)^2} + \frac{x^2}{16(1+x^4)} + \frac{\arctan(x^2)}{16}$$

```
[Out] -1/8*x^2/(x^4+1)^2+1/16*x^2/(x^4+1)+1/16*arctan(x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {281, 294, 205, 209}

$$\int \frac{x^5}{(1+x^4)^3} \, dx = \frac{\arctan(x^2)}{16} + \frac{x^2}{16(x^4+1)} - \frac{x^2}{8(x^4+1)^2}$$

```
[In] Int[x^5/(1 + x^4)^3,x]
```

```
[Out] -1/8*x^2/(1 + x^4)^2 + x^2/(16*(1 + x^4)) + ArcTan[x^2]/16
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(1+x^2)^3}\,dx, x, x^2\right) \\
&= -\frac{x^2}{8(1+x^4)^2} + \frac{1}{8}\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, x^2\right) \\
&= -\frac{x^2}{8(1+x^4)^2} + \frac{x^2}{16(1+x^4)} + \frac{1}{16}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right) \\
&= -\frac{x^2}{8(1+x^4)^2} + \frac{x^2}{16(1+x^4)} + \frac{\arctan(x^2)}{16}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{x^5}{(1+x^4)^3}\,dx = \frac{1}{16}\left(\frac{x^2(-1+x^4)}{(1+x^4)^2} + \arctan(x^2)\right)$$

```
[In] Integrate[x^5/(1 + x^4)^3,x]
[Out] ((x^2*(-1 + x^4))/(1 + x^4)^2 + ArcTan[x^2])/16
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

method	result	size
meijerg	$-\frac{x^{2}\left(-3x^{4}+3\right)}{48\left(x^{4}+1\right)^{2}}+\frac{\arctan\left(x^{2}\right)}{16}$	27
risch	$\frac{\frac{1}{16}x^{6}-\frac{1}{16}x^{2}}{\left(x^{4}+1\right)^{2}}+\frac{\arctan\left(x^{2}\right)}{16}$	27
default	$\frac{\frac{1}{8}x^{6}-\frac{1}{8}x^{2}}{2\left(x^{4}+1\right)^{2}}+\frac{\arctan\left(x^{2}\right)}{16}$	28
parallelrisch	$-\frac{i\ln\left(x^{2}-i\right)x^{8}-i\ln\left(x^{2}+i\right)x^{8}+2i\ln\left(x^{2}-i\right)x^{4}-2i\ln\left(x^{2}+i\right)x^{4}-2x^{6}+i\ln\left(x^{2}-i\right)-i\ln\left(x^{2}+i\right)+2x^{2}}{32\left(x^{4}+1\right)^{2}}$	93

```
[In] int(x^5/(x^4+1)^3,x,method=_RETURNVERBOSE)
[Out] -1/48*x^2*(-3*x^4+3)/(x^4+1)^2+1/16*arctan(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{x^5}{\left(1+x^4\right)^3}\,dx=\frac{x^6-x^2+\left(x^8+2\,x^4+1\right)\arctan\left(x^2\right)}{16\,\left(x^8+2\,x^4+1\right)}$$

```
[In] integrate(x^5/(x^4+1)^3,x, algorithm="fricas")
[Out] 1/16*(x^6 - x^2 + (x^8 + 2*x^4 + 1)*arctan(x^2))/(x^8 + 2*x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \frac{x^5}{\left(1+x^4\right)^3}\,dx=\frac{x^6-x^2}{16x^8+32x^4+16}+\frac{\operatorname{atan}\left(x^2\right)}{16}$$

```
[In] integrate(x**5/(x**4+1)**3,x)
[Out] (x**6 - x**2)/(16*x**8 + 32*x**4 + 16) + atan(x**2)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{(1+x^4)^3}\,dx = \frac{x^6-x^2}{16\,(x^8+2\,x^4+1)} + \frac{1}{16}\,\arctan\left(x^2\right)$$

```
[In] integrate(x^5/(x^4+1)^3,x, algorithm="maxima")
[Out] 1/16*(x^6 - x^2)/(x^8 + 2*x^4 + 1) + 1/16*arctan(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(1+x^4)^3}\,dx = \frac{x^2-\frac{1}{x^2}}{16\left(\left(x^2-\frac{1}{x^2}\right)^2+4\right)} + \frac{1}{32}\,\arctan\left(\frac{x^4-1}{2\,x^2}\right)$$

```
[In] integrate(x^5/(x^4+1)^3,x, algorithm="giac")
[Out] 1/16*(x^2 - 1/x^2)/((x^2 - 1/x^2)^2 + 4) + 1/32*arctan(1/2*(x^4 - 1)/x^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(1+x^4)^3}\,dx = \frac{\mathrm{atan}(x^2)}{16} - \frac{\frac{x^2}{16}-\frac{x^6}{16}}{x^8+2\,x^4+1}$$

```
[In] int(x^5/(x^4 + 1)^3,x)
[Out] atan(x^2)/16 - (x^2/16 - x^6/16)/(2*x^4 + x^8 + 1)
```

3.171 $\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2} \, dx = \frac{1}{4(2+2x^2+x^4)} + \frac{1}{4}\log\left(2+2x^2+x^4\right)$$

```
[Out] 1/4/(x^4+2*x^2+2)+1/4*ln(x^4+2*x^2+2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1261, 700, 642}

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2} \, dx = \frac{1}{4}\log\left(x^4+2x^2+2\right) - \frac{(x^2+1)^2}{4(x^4+2x^2+2)}$$

```
[In] Int[(x*(1 + x^2)^3)/(2 + 2*x^2 + x^4)^2,x]
```

```
[Out] -1/4*(1 + x^2)^2/(2 + 2*x^2 + x^4) + Log[2 + 2*x^2 + x^4]/4
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 700

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))),
```

```
x] - Dist[d*e*((m - 1)/(b*(p + 1))), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)^3}{(2+2x+x^2)^2}\,dx, x, x^2\right)\\
&= -\frac{(1+x^2)^2}{4\,(2+2x^2+x^4)} + \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{2+2x+x^2}\,dx, x, x^2\right)\\
&= -\frac{(1+x^2)^2}{4\,(2+2x^2+x^4)} + \frac{1}{4}\log\left(2+2x^2+x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx = \frac{1}{4}\left(\frac{1}{1+(1+x^2)^2} + \log\left(1+\left(1+x^2\right)^2\right)\right)$$

```
[In] Integrate[(x*(1 + x^2)^3)/(2 + 2*x^2 + x^4)^2,x]
[Out] ((1 + (1 + x^2)^2)^(-1) + Log[1 + (1 + x^2)^2])/4
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{1}{4x^4+8x^2+8}+\frac{\ln(x^4+2x^2+2)}{4}$	29
norman	$\frac{1}{4x^4+8x^2+8}+\frac{\ln(x^4+2x^2+2)}{4}$	29
risch	$\frac{1}{4x^4+8x^2+8}+\frac{\ln(x^4+2x^2+2)}{4}$	29
parallelrisch	$\frac{\ln(x^4+2x^2+2)x^4+1+2\ln(x^4+2x^2+2)x^2+2\ln(x^4+2x^2+2)}{4x^4+8x^2+8}$	61

```
[In] int(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x,method=_RETURNVERBOSE)
[Out] 1/4/(x^4+2*x^2+2)+1/4*ln(x^4+2*x^2+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx=\frac{(x^4+2\,x^2+2)\log(x^4+2\,x^2+2)+1}{4\,(x^4+2\,x^2+2)}$$

```
[In] integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="fricas")
[Out] 1/4*((x^4 + 2*x^2 + 2)*log(x^4 + 2*x^2 + 2) + 1)/(x^4 + 2*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx=\frac{\log(x^4+2x^2+2)}{4}+\frac{1}{4x^4+8x^2+8}$$

```
[In] integrate(x*(x**2+1)**3/(x**4+2*x**2+2)**2,x)
[Out] log(x**4 + 2*x**2 + 2)/4 + 1/(4*x**4 + 8*x**2 + 8)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx = \frac{1}{4\,(x^4+2\,x^2+2)} + \frac{1}{4}\,\log\left(x^4+2\,x^2+2\right)$$

```
[In] integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="maxima")
[Out] 1/4/(x^4 + 2*x^2 + 2) + 1/4*log(x^4 + 2*x^2 + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx = \frac{1}{4\,(x^4+2\,x^2+2)} - \frac{1}{4}\,\log\left(\frac{1}{2\,(x^4+2\,x^2+2)}\right)$$

```
[In] integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="giac")
[Out] 1/4/(x^4 + 2*x^2 + 2) - 1/4*log(1/2/(x^4 + 2*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx = \frac{\ln\left(x^4+2\,x^2+2\right)}{4} + \frac{1}{4\,\left(x^4+2\,x^2+2\right)}$$

```
[In] int((x*(x^2 + 1)^3)/(2*x^2 + x^4 + 2)^2,x)
[Out] log(2*x^2 + x^4 + 2)/4 + 1/(4*(2*x^2 + x^4 + 2))
```

3.172 $\int \frac{x^3}{(a^4+x^4)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^3}{(a^4+x^4)^3} dx = -\frac{1}{8(a^4+x^4)^2}$$

```
[Out] -1/8/(a^4+x^4)^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$\int \frac{x^3}{(a^4+x^4)^3} dx = -\frac{1}{8(a^4+x^4)^2}$$

```
[In] Int[x^3/(a^4 + x^4)^3,x]
```

```
[Out] -1/8*1/(a^4 + x^4)^2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{8(a^4+x^4)^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a^4+x^4)^3}\,dx=-\frac{1}{8\,(a^4+x^4)^2}$$

```
[In] Integrate[x^3/(a^4 + x^4)^3,x]
[Out] -1/8*1/(a^4 + x^4)^2
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{1}{8(a^4+x^4)^2}$	12
derivativedivides	$-\frac{1}{8(a^4+x^4)^2}$	12
default	$-\frac{1}{8(a^4+x^4)^2}$	12
norman	$-\frac{1}{8(a^4+x^4)^2}$	12
risch	$-\frac{1}{8(a^4+x^4)^2}$	12
parallelrisch	$-\frac{1}{8(a^4+x^4)^2}$	12

```
[In] int(x^3/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
[Out] -1/8/(a^4+x^4)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{x^3}{(a^4+x^4)^3}\,dx=-\frac{1}{8\,(a^8+2\,a^4x^4+x^8)}$$

```
[In] integrate(x^3/(a^4+x^4)^3,x, algorithm="fricas")
[Out] -1/8/(a^8 + 2*a^4*x^4 + x^8)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{x^3}{(a^4+x^4)^3} \, dx = -\frac{1}{8a^8+16a^4x^4+8x^8}$$

```
[In] integrate(x**3/(a**4+x**4)**3,x)
[Out] -1/(8*a**8 + 16*a**4*x**4 + 8*x**8)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(a^4+x^4)^3} \, dx = -\frac{1}{8\,(a^4+x^4)^2}$$

```
[In] integrate(x^3/(a^4+x^4)^3,x, algorithm="maxima")
[Out] -1/8/(a^4 + x^4)^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(a^4+x^4)^3} \, dx = -\frac{1}{8\,(a^4+x^4)^2}$$

```
[In] integrate(x^3/(a^4+x^4)^3,x, algorithm="giac")
[Out] -1/8/(a^4 + x^4)^2
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(a^4+x^4)^3} \, dx = -\frac{1}{8\,(a^4+x^4)^2}$$

```
[In] int(x^3/(a^4 + x^4)^3,x)
[Out] -1/(8*(a^4 + x^4)^2)
```

3.173 $\int \frac{1}{x(a^4+x^4)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{1}{x(a^4+x^4)^3} \, dx = \frac{1}{8a^4(a^4+x^4)^2} + \frac{1}{4a^8(a^4+x^4)} + \frac{\log(x)}{a^{12}} - \frac{\log(a^4+x^4)}{4a^{12}}$$

```
[Out] 1/8/a^4/(a^4+x^4)^2+1/4/a^8/(a^4+x^4)+ln(x)/a^12-1/4*ln(a^4+x^4)/a^12
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 46}

$$\int \frac{1}{x(a^4+x^4)^3} \, dx = \frac{\log(x)}{a^{12}} + \frac{1}{8a^4(a^4+x^4)^2} - \frac{\log(a^4+x^4)}{4a^{12}} + \frac{1}{4a^8(a^4+x^4)}$$

```
[In] Int[1/(x*(a^4 + x^4)^3),x]
```

```
[Out] 1/(8*a^4*(a^4 + x^4)^2) + 1/(4*a^8*(a^4 + x^4)) + Log[x]/a^12 - Log[a^4 + x
^4]/(4*a^12)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\left(a^4+x\right)^3}\,dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{a^{12}x}-\frac{1}{a^4\left(a^4+x\right)^3}-\frac{1}{a^8\left(a^4+x\right)^2}-\frac{1}{a^{12}\left(a^4+x\right)}\right)dx, x, x^4\right)\\ &= \frac{1}{8a^4\left(a^4+x^4\right)^2}+\frac{1}{4a^8\left(a^4+x^4\right)}+\frac{\log(x)}{a^{12}}-\frac{\log\left(a^4+x^4\right)}{4a^{12}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx = \frac{\frac{3a^8+2a^4x^4}{\left(a^4+x^4\right)^2}+8\log(x)-2\log\left(a^4+x^4\right)}{8a^{12}}$$

```
[In] Integrate[1/(x*(a^4 + x^4)^3),x]
[Out] ((3*a^8 + 2*a^4*x^4)/(a^4 + x^4)^2 + 8*Log[x] - 2*Log[a^4 + x^4])/(8*a^12)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
norman	$\frac{\frac{3}{8a^4}+\frac{x^4}{4a^8}}{\left(a^4+x^4\right)^2}+\frac{\ln(x)}{a^{12}}-\frac{\ln\left(a^4+x^4\right)}{4a^{12}}$	45
risch	$\frac{\frac{3}{8a^4}+\frac{x^4}{4a^8}}{\left(a^4+x^4\right)^2}+\frac{\ln(x)}{a^{12}}-\frac{\ln\left(a^4+x^4\right)}{4a^{12}}$	45
default	$-\frac{\frac{\ln\left(a^4+x^4\right)}{2}-\frac{a^4}{2\left(a^4+x^4\right)}-\frac{a^8}{4\left(a^4+x^4\right)^2}}{2a^{12}}+\frac{\ln(x)}{a^{12}}$	52
parallelrisch	$\frac{8\ln(x)x^8+16\ln(x)x^4a^4+8\ln(x)a^8-2\ln\left(a^4+x^4\right)x^8-4\ln\left(a^4+x^4\right)x^4a^4-2\ln\left(a^4+x^4\right)a^8+2a^4x^4+3a^8}{8a^{12}\left(a^4+x^4\right)^2}$	95

```
[In] int(1/x/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
[Out] (3/8/a^4+1/4/a^8*x^4)/(a^4+x^4)^2+ln(x)/a^12-1/4*ln(a^4+x^4)/a^12
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx = \frac{3\,a^8+2\,a^4x^4-2\left(a^8+2\,a^4x^4+x^8\right)\log\left(a^4+x^4\right)+8\left(a^8+2\,a^4x^4+x^8\right)\log\left(x\right)}{8\left(a^{20}+2\,a^{16}x^4+a^{12}x^8\right)}$$

```
[In] integrate(1/x/(a^4+x^4)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*a^8 + 2*a^4*x^4 - 2*(a^8 + 2*a^4*x^4 + x^8)*log(a^4 + x^4) + 8*(a^8
+ 2*a^4*x^4 + x^8)*log(x))/(a^20 + 2*a^16*x^4 + a^12*x^8)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx = \frac{3a^4+2x^4}{8a^{16}+16a^{12}x^4+8a^8x^8}+\frac{\log\left(x\right)}{a^{12}}-\frac{\log\left(a^4+x^4\right)}{4a^{12}}$$

```
[In] integrate(1/x/(a**4+x**4)**3,x)
```

```
[Out] (3*a**4 + 2*x**4)/(8*a**16 + 16*a**12*x**4 + 8*a**8*x**8) + log(x)/a**12 -
log(a**4 + x**4)/(4*a**12)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx = \frac{3\,a^4+2\,x^4}{8\left(a^{16}+2\,a^{12}x^4+a^8x^8\right)}-\frac{\log\left(a^4+x^4\right)}{4\,a^{12}}+\frac{\log\left(x^4\right)}{4\,a^{12}}$$

```
[In] integrate(1/x/(a^4+x^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*a^4 + 2*x^4)/(a^16 + 2*a^12*x^4 + a^8*x^8) - 1/4*log(a^4 + x^4)/a^12
 + 1/4*log(x^4)/a^12
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx = -\frac{\log\left(a^4+x^4\right)}{4\,a^{12}} + \frac{\log\left(x^4\right)}{4\,a^{12}} + \frac{6\,a^8+8\,a^4x^4+3\,x^8}{8\left(a^4+x^4\right)^2a^{12}}$$

```
[In] integrate(1/x/(a^4+x^4)^3,x, algorithm="giac")
```

```
[Out] -1/4*log(a^4 + x^4)/a^12 + 1/4*log(x^4)/a^12 + 1/8*(6*a^8 + 8*a^4*x^4 + 3*x
^8)/((a^4 + x^4)^2*a^12)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx = \frac{\ln\left(x\right)}{a^{12}} + \frac{\frac{3}{8\,a^4}+\frac{x^4}{4\,a^8}}{a^8+2\,a^4\,x^4+x^8} - \frac{\ln\left(a^4+x^4\right)}{4\,a^{12}}$$

```
[In] int(1/(x*(a^4 + x^4)^3),x)
```

```
[Out] log(x)/a^12 + (3/(8*a^4) + x^4/(4*a^8))/(a^8 + x^8 + 2*a^4*x^4) - log(a^4 +
 x^4)/(4*a^12)
```

3.174 $\int \frac{1}{x^2(a^4+x^4)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 157

$$\int \frac{1}{x^2(a^4+x^4)^3}\,dx = -\frac{45}{32a^{12}x}+\frac{1}{8a^4x(a^4+x^4)^2}+\frac{9}{32a^8x(a^4+x^4)} + \frac{45\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}}-\frac{45\arctan\left(1+\frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}} - \frac{45\log\left(a^2-\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}+\frac{45\log\left(a^2+\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}$$

```
[Out] -45/32/a^12/x+1/8/a^4/x/(a^4+x^4)^2+9/32/a^8/x/(a^4+x^4)+45/128*arctan(1-x*
2^(1/2)/a)/a^13*2^(1/2)-45/128*arctan(1+x*2^(1/2)/a)/a^13*2^(1/2)-45/256*ln
(a^2+x^2-a*x*2^(1/2))/a^13*2^(1/2)+45/256*ln(a^2+x^2+a*x*2^(1/2))/a^13*2^(1
/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {296, 331, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{x^2(a^4+x^4)^3}\,dx = \frac{45\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}}-\frac{45\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{64\sqrt{2}a^{13}}-\frac{45}{32a^{12}x} + \frac{1}{8a^4x(a^4+x^4)^2}-\frac{45\log\left(a^2-\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}} + \frac{45\log\left(a^2+\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}+\frac{9}{32a^8x(a^4+x^4)}$$

```
[In] Int[1/(x^2*(a^4 + x^4)^3),x]
```

```
[Out] -45/(32*a^12*x) + 1/(8*a^4*x*(a^4 + x^4)^2) + 9/(32*a^8*x*(a^4 + x^4)) + (4
5*ArcTan[1 - (Sqrt[2]*x)/a])/(64*Sqrt[2]*a^13) - (45*ArcTan[1 + (Sqrt[2]*x)
/a])/(64*Sqrt[2]*a^13) - (45*Log[a^2 - Sqrt[2]*a*x + x^2])/(128*Sqrt[2]*a^1
3) + (45*Log[a^2 + Sqrt[2]*a*x + x^2])/(128*Sqrt[2]*a^13)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9\int \frac{1}{x^2\left(a^4+x^4\right)^2}\,dx}{8a^4}\\
&= \frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}+\frac{45\int \frac{1}{x^2\left(a^4+x^4\right)}\,dx}{32a^8}\\
&= -\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}-\frac{45\int \frac{x^2}{a^4+x^4}\,dx}{32a^{12}}\\
&= -\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}+\frac{45\int \frac{a^2-x^2}{a^4+x^4}\,dx}{64a^{12}}-\frac{45\int \frac{a^2+x^2}{a^4+x^4}\,dx}{64a^{12}}\\
&= -\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}-\frac{45\int \frac{\sqrt{2}a+2x}{-a^2-\sqrt{2}ax-x^2}\,dx}{128\sqrt{2}a^{13}}\\
&\quad -\frac{45\int \frac{\sqrt{2}a-2x}{-a^2+\sqrt{2}ax-x^2}\,dx}{128\sqrt{2}a^{13}}-\frac{45\int \frac{1}{a^2-\sqrt{2}ax+x^2}\,dx}{128a^{12}}-\frac{45\int \frac{1}{a^2+\sqrt{2}ax+x^2}\,dx}{128a^{12}}\\
&= -\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}\\
&\quad -\frac{45\log\left(a^2-\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}+\frac{45\log\left(a^2+\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}\\
&\quad -\frac{45\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}}+\frac{45\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}}
\end{aligned}$$

$$=-\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}+\frac{45\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}}-\frac{45\arctan\left(1+\frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}}-\frac{45\log\left(a^2-\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}+\frac{45\log\left(a^2+\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85

$$\int\frac{1}{x^2\left(a^4+x^4\right)^3}\,dx=$$
$$-\frac{\frac{256a}{x}+\frac{32a^5x^3}{(a^4+x^4)^2}+\frac{104ax^3}{a^4+x^4}-90\sqrt{2}\arctan\left(1-\frac{\sqrt{2}x}{a}\right)+90\sqrt{2}\arctan\left(1+\frac{\sqrt{2}x}{a}\right)+45\sqrt{2}\log\left(a^2-\sqrt{2}ax+\right.}{256a^{13}}$$

```
[In] Integrate[1/(x^2*(a^4 + x^4)^3),x]
```

```
[Out] -1/256*((256*a)/x + (32*a^5*x^3)/(a^4 + x^4)^2 + (104*a*x^3)/(a^4 + x^4) -
90*Sqrt[2]*ArcTan[1 - (Sqrt[2]*x)/a] + 90*Sqrt[2]*ArcTan[1 + (Sqrt[2]*x)/a]
 + 45*Sqrt[2]*Log[a^2 - Sqrt[2]*a*x + x^2] - 45*Sqrt[2]*Log[a^2 + Sqrt[2]*a
*x + x^2])/a^13
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.48

method	result	size
risch	$\frac{-\frac{45x^8}{32a^{12}}-\frac{81x^4}{32a^8}-\frac{1}{a^4}}{x(a^4+x^4)^2}+\frac{45\left(\sum_{\_R=\mathrm{RootOf}\left(a^{52}\_Z^4+1\right)}\_R\ln\left(\left(5\_R^4a^{52}+4\right)x+\_R^3a^{40}\right)\right)}{128}$	75
default	$-\frac{\frac{\frac{17}{32}a^4x^3+\frac{13}{32}x^7}{(a^4+x^4)^2}+\frac{45\sqrt{2}\left(\ln\left(\frac{x^2-(a^4)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}{x^2+(a^4)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(a^4)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(a^4)^{\frac{1}{4}}}-1\right)\right)}{256(a^4)^{\frac{1}{4}}}}{a^{12}}-\frac{1}{a^{12}x}$	124

```
[In] int(1/x^2/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-45/32/a^12*x^8-81/32/a^8*x^4-1/a^4)/x/(a^4+x^4)^2+45/128*sum(_R*ln((5*_R^
4*a^52+4)*x+_R^3*a^40),_R=RootOf(_Z^4*a^52+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^2 (a^4 + x^4)^3} \, dx = -\frac{128\, a^8 + 324\, a^4 x^4 + 180\, x^8 + 45\, (a^{20} x + 2\, a^{16} x^5 + a^{12} x^9) \left(-\frac{1}{a^{52}}\right)^{\frac{1}{4}} \log\left(a^{40} \left(-\frac{1}{a^{52}}\right)^{\frac{3}{4}} + x\right) + 45\, (-i\, a^{20} x -}{}$$

[In] integrate(1/x^2/(a^4+x^4)^3,x, algorithm="fricas")

```
[Out] -1/128*(128*a^8 + 324*a^4*x^4 + 180*x^8 + 45*(a^20*x + 2*a^16*x^5 + a^12*x^
9)*(-1/a^52)^(1/4)*log(a^40*(-1/a^52)^(3/4) + x) + 45*(-I*a^20*x - 2*I*a^16
*x^5 - I*a^12*x^9)*(-1/a^52)^(1/4)*log(I*a^40*(-1/a^52)^(3/4) + x) + 45*(I*
a^20*x + 2*I*a^16*x^5 + I*a^12*x^9)*(-1/a^52)^(1/4)*log(-I*a^40*(-1/a^52)^(
3/4) + x) - 45*(a^20*x + 2*a^16*x^5 + a^12*x^9)*(-1/a^52)^(1/4)*log(-a^40*(
-1/a^52)^(3/4) + x))/(a^20*x + 2*a^16*x^5 + a^12*x^9)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^2 (a^4 + x^4)^3} \, dx = \frac{-32a^8 - 81a^4x^4 - 45x^8}{32a^{20}x + 64a^{16}x^5 + 32a^{12}x^9} + \frac{\operatorname{RootSum}\left(268435456t^4 + 4100625, \left(t \mapsto t \log\left(-\frac{2097152t^3a}{91125} + x\right)\right)\right)}{a^{13}}$$

[In] integrate(1/x**2/(a**4+x**4)**3,x)

```
[Out] (-32*a**8 - 81*a**4*x**4 - 45*x**8)/(32*a**20*x + 64*a**16*x**5 + 32*a**12*
x**9) + RootSum(268435456*_t**4 + 4100625, Lambda(_t, _t*log(-2097152*_t**3
*a/91125 + x)))/a**13
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2 (a^4 + x^4)^3} \, dx = -\frac{32\, a^8 + 81\, a^4 x^4 + 45\, x^8}{32\, (a^{20} x + 2\, a^{16} x^5 + a^{12} x^9)} - \frac{45 \left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}(\sqrt{2}a + 2\,x)}{2\,a}\right)}{a} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}(\sqrt{2}a - 2\,x)}{2\,a}\right)}{a} - \frac{\sqrt{2}\log\left(\sqrt{2}ax + a^2 + x^2\right)}{a} + \frac{\sqrt{2}\log\left(-\sqrt{2}ax + a^2 + x^2\right)}{a}\right)}{256\, a^{12}}$$

```
[In] integrate(1/x^2/(a^4+x^4)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(32*a^8 + 81*a^4*x^4 + 45*x^8)/(a^20*x + 2*a^16*x^5 + a^12*x^9) - 45/
256*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a + 2*x)/a)/a + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*a - 2*x)/a)/a - sqrt(2)*log(sqrt(2)*a*x + a^2 + x^2)
/a + sqrt(2)*log(-sqrt(2)*a*x + a^2 + x^2)/a)/a^12
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2\left(a^4+x^4\right)^3}\,dx = -\frac{45\sqrt{2}|a|\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}|a|+2x\right)}{2|a|}\right)}{128\,a^{14}} - \frac{45\sqrt{2}|a|\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}|a|-2x\right)}{2|a|}\right)}{128\,a^{14}} + \frac{45\sqrt{2}|a|\log\left(\sqrt{2}x|a|+x^2+|a|^2\right)}{256\,a^{14}} - \frac{45\sqrt{2}|a|\log\left(-\sqrt{2}x|a|+x^2+|a|^2\right)}{256\,a^{14}} - \frac{17\,a^4x^3+13\,x^7}{32\left(a^4+x^4\right)^2a^{12}} - \frac{1}{a^{12}x}$$

```
[In] integrate(1/x^2/(a^4+x^4)^3,x, algorithm="giac")
```

```
[Out] -45/128*sqrt(2)*abs(a)*arctan(1/2*sqrt(2)*(sqrt(2)*abs(a) + 2*x)/abs(a))/a^
14 - 45/128*sqrt(2)*abs(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*abs(a) - 2*x)/abs(a
))/a^14 + 45/256*sqrt(2)*abs(a)*log(sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^14
 - 45/256*sqrt(2)*abs(a)*log(-sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^14 - 1/3
2*(17*a^4*x^3 + 13*x^7)/((a^4 + x^4)^2*a^12) - 1/(a^12*x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^2\left(a^4+x^4\right)^3}\,dx = \frac{45\left(-1\right)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}x}{a}\right)}{64\,a^{13}} - \frac{45\left(-1\right)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}x}{a}\right)}{64\,a^{13}} - \frac{\frac{1}{a^4}+\frac{81\,x^4}{32\,a^8}+\frac{45\,x^8}{32\,a^{12}}}{a^8\,x+2\,a^4\,x^5+x^9}$$

```
[In] int(1/(x^2*(a^4 + x^4)^3),x)
```

```
[Out] (45*(-1)^(1/4)*atanh(((-1)^(1/4)*x)/a))/(64*a^13) - (45*(-1)^(1/4)*atan(((-
1)^(1/4)*x)/a))/(64*a^13) - (1/a^4 + (81*x^4)/(32*a^8) + (45*x^8)/(32*a^12)
)/(a^8*x + x^9 + 2*a^4*x^5)
```

3.175 $\int \frac{1}{x^3\left(a^4+x^4\right)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{1}{x^3\left(a^4+x^4\right)^3}\,dx=-\frac{15}{16a^{12}x^2}+\frac{1}{8a^4x^2\left(a^4+x^4\right)^2}+\frac{5}{16a^8x^2\left(a^4+x^4\right)}-\frac{15\arctan\left(\frac{x^2}{a^2}\right)}{16a^{14}}$$

```
[Out] -15/16/a^12/x^2+1/8/a^4/x^2/(a^4+x^4)^2+5/16/a^8/x^2/(a^4+x^4)-15/16*arctan
(x^2/a^2)/a^14
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {281, 296, 331, 209}

$$\int \frac{1}{x^3\left(a^4+x^4\right)^3}\,dx=-\frac{15}{16a^{12}x^2}+\frac{1}{8a^4x^2\left(a^4+x^4\right)^2}-\frac{15\arctan\left(\frac{x^2}{a^2}\right)}{16a^{14}}+\frac{5}{16a^8x^2\left(a^4+x^4\right)}$$

```
[In] Int[1/(x^3*(a^4 + x^4)^3),x]
```

```
[Out] -15/(16*a^12*x^2) + 1/(8*a^4*x^2*(a^4 + x^4)^2) + 5/(16*a^8*x^2*(a^4 + x^4)
) - (15*ArcTan[x^2/a^2])/(16*a^14)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a^4+x^2\right)^3}\,dx, x, x^2\right)\\
&= \frac{1}{8a^4x^2\left(a^4+x^4\right)^2} + \frac{5\text{Subst}\left(\int \frac{1}{x^2\left(a^4+x^2\right)^2}\,dx, x, x^2\right)}{8a^4}\\
&= \frac{1}{8a^4x^2\left(a^4+x^4\right)^2} + \frac{5}{16a^8x^2\left(a^4+x^4\right)} + \frac{15\text{Subst}\left(\int \frac{1}{x^2\left(a^4+x^2\right)}\,dx, x, x^2\right)}{16a^8}\\
&= -\frac{15}{16a^{12}x^2} + \frac{1}{8a^4x^2\left(a^4+x^4\right)^2} + \frac{5}{16a^8x^2\left(a^4+x^4\right)} - \frac{15\text{Subst}\left(\int \frac{1}{a^4+x^2}\,dx, x, x^2\right)}{16a^{12}}\\
&= -\frac{15}{16a^{12}x^2} + \frac{1}{8a^4x^2\left(a^4+x^4\right)^2} + \frac{5}{16a^8x^2\left(a^4+x^4\right)} - \frac{15\arctan\left(\frac{x^2}{a^2}\right)}{16a^{14}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^3 (a^4+x^4)^3}\,dx = \frac{-\frac{a^2(8a^8+25a^4x^4+15x^8)}{x^2(a^4+x^4)^2}+15\arctan\left(1-\frac{\sqrt{2}x}{a}\right)+15\arctan\left(1+\frac{\sqrt{2}x}{a}\right)}{16a^{14}}$$

```
[In] Integrate[1/(x^3*(a^4 + x^4)^3),x]
```

```
[Out] (-((a^2*(8*a^8 + 25*a^4*x^4 + 15*x^8))/(x^2*(a^4 + x^4)^2)) + 15*ArcTan[1 -
 (Sqrt[2]*x)/a] + 15*ArcTan[1 + (Sqrt[2]*x)/a])/(16*a^14)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

method	result
default	$-\frac{\frac{\frac{9}{8}a^4x^2+\frac{7}{8}x^6}{(a^4+x^4)^2}+\frac{15\arctan\left(\frac{x^2}{a^2}\right)}{8a^2}}{2a^{12}}-\frac{1}{2a^{12}x^2}$
risch	$\frac{-\frac{15x^8}{16a^{12}}-\frac{25x^4}{16a^8}-\frac{1}{2a^4}}{x^2(a^4+x^4)^2}+\frac{15\left(\sum_{\_R=\operatorname{RootOf}(a^{28}\_Z^2+1)}\_R\ln\left(\left(-5\_R^2a^{28}-4\right)x^2-a^{16}\_R\right)\right)}{32}$
parallelrisch	$\frac{15i\ln(-ia^2+x^2)x^{10}+30i\ln(-ia^2+x^2)x^6a^4+15i\ln(-ia^2+x^2)x^2a^8-15i\ln(ia^2+x^2)x^{10}-30i\ln(ia^2+x^2)x^6a^4-15i\ln(ia^2+x^2)}{32a^{14}x^2(a^4+x^4)^2}$

```
[In] int(1/x^3/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^12*((9/8*a^4*x^2+7/8*x^6)/(a^4+x^4)^2+15/8*arctan(x^2/a^2)/a^2)-1/2/
a^12/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3 (a^4+x^4)^3}\,dx = -\frac{8\,a^{10}+25\,a^6x^4+15\,a^2x^8+15\,(a^8x^2+2\,a^4x^6+x^{10})\arctan\left(\frac{x^2}{a^2}\right)}{16\,(a^{22}x^2+2\,a^{18}x^6+a^{14}x^{10})}$$

```
[In] integrate(1/x^3/(a^4+x^4)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(8*a^10 + 25*a^6*x^4 + 15*a^2*x^8 + 15*(a^8*x^2 + 2*a^4*x^6 + x^10)*a
rctan(x^2/a^2))/(a^22*x^2 + 2*a^18*x^6 + a^14*x^10)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3 (a^4 + x^4)^3}\,dx = \frac{-8a^8 - 25a^4x^4 - 15x^8}{16a^{20}x^2 + 32a^{16}x^6 + 16a^{12}x^{10}} + \frac{\frac{15i\log\left(-ia^2+x^2\right)}{32} - \frac{15i\log\left(ia^2+x^2\right)}{32}}{a^{14}}$$

```
[In] integrate(1/x**3/(a**4+x**4)**3,x)
```

```
[Out] (-8*a**8 - 25*a**4*x**4 - 15*x**8)/(16*a**20*x**2 + 32*a**16*x**6 + 16*a**1
2*x**10) + (15*I*log(-I*a**2 + x**2)/32 - 15*I*log(I*a**2 + x**2)/32)/a**14
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3 (a^4 + x^4)^3}\,dx = -\frac{8\,a^8 + 25\,a^4x^4 + 15\,x^8}{16\,(a^{20}x^2 + 2\,a^{16}x^6 + a^{12}x^{10})} - \frac{15\,\arctan\left(\frac{x^2}{a^2}\right)}{16\,a^{14}}$$

```
[In] integrate(1/x^3/(a^4+x^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(8*a^8 + 25*a^4*x^4 + 15*x^8)/(a^20*x^2 + 2*a^16*x^6 + a^12*x^10) - 1
5/16*arctan(x^2/a^2)/a^14
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3 (a^4 + x^4)^3}\,dx = -\frac{9\,a^4x^2 + 7\,x^6}{16\,(a^4 + x^4)^2a^{12}} - \frac{15\,\arctan\left(\frac{x^2}{a^2}\right)}{16\,a^{14}} - \frac{1}{2\,a^{12}x^2}$$

```
[In] integrate(1/x^3/(a^4+x^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*(9*a^4*x^2 + 7*x^6)/((a^4 + x^4)^2*a^12) - 15/16*arctan(x^2/a^2)/a^14
 - 1/2/(a^12*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (a^4+x^4)^3}\,dx = -\frac{15\,\mathrm{atan}\left(\frac{x^2}{a^2}\right)}{16\,a^{14}} - \frac{\frac{a^{10}}{2}+\frac{25\,a^6\,x^4}{16}+\frac{15\,a^2\,x^8}{16}}{a^{14}\,x^2\,(a^4+x^4)^2}$$

```
[In] int(1/(x^3*(a^4 + x^4)^3),x)
```

```
[Out] - (15*atan(x^2/a^2))/(16*a^14) - (a^10/2 + (15*a^2*x^8)/16 + (25*a^6*x^4)/1
6)/(a^14*x^2*(a^4 + x^4)^2)
```

3.176 $\int \frac{x^{14}}{(3+2x^5)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{x^{14}}{(3+2x^5)^3} dx = -\frac{9}{80(3+2x^5)^2}+\frac{3}{20(3+2x^5)}+\frac{1}{40}\log\left(3+2x^5\right)$$

```
[Out] -9/80/(2*x^5+3)^2+3/20/(2*x^5+3)+1/40*ln(2*x^5+3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\int \frac{x^{14}}{(3+2x^5)^3} dx = \frac{3}{20(2x^5+3)}-\frac{9}{80(2x^5+3)^2}+\frac{1}{40}\log\left(2x^5+3\right)$$

```
[In] Int[x^14/(3 + 2*x^5)^3,x]
```

```
[Out] -9/(80*(3 + 2*x^5)^2) + 3/(20*(3 + 2*x^5)) + Log[3 + 2*x^5]/40
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}\text{Subst}\left(\int \frac{x^2}{(3+2x)^3}\,dx,x,x^5\right)\\
&= \frac{1}{5}\text{Subst}\left(\int \left(\frac{9}{4(3+2x)^3}-\frac{3}{2(3+2x)^2}+\frac{1}{4(3+2x)}\right)\,dx,x,x^5\right)\\
&= -\frac{9}{80\,(3+2x^5)^2}+\frac{3}{20\,(3+2x^5)}+\frac{1}{40}\log\left(3+2x^5\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = \frac{1}{80}\left(\frac{3(9+8x^5)}{(3+2x^5)^2}+2\log\left(3+2x^5\right)\right)$$

```
[In] Integrate[x^14/(3 + 2*x^5)^3,x]
[Out] ((3*(9 + 8*x^5))/(3 + 2*x^5)^2 + 2*Log[3 + 2*x^5])/80
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

method	result	size
norman	$\frac{\frac{3x^5}{10}+\frac{27}{80}}{(2x^5+3)^2}+\frac{\ln(2x^5+3)}{40}$	29
risch	$\frac{\frac{3x^5}{10}+\frac{27}{80}}{(2x^5+3)^2}+\frac{\ln(2x^5+3)}{40}$	30
meijerg	$-\frac{x^5(6x^5+6)}{360\left(1+\frac{2x^5}{3}\right)^2}+\frac{\ln\left(1+\frac{2x^5}{3}\right)}{40}$	33
default	$-\frac{9}{80(2x^5+3)^2}+\frac{3}{20(2x^5+3)}+\frac{\ln(2x^5+3)}{40}$	34
parallelrisch	$\frac{8\ln(x^5+\frac{3}{2})x^{10}+27+24\ln(x^5+\frac{3}{2})x^5+24x^5+18\ln(x^5+\frac{3}{2})}{80(2x^5+3)^2}$	49

```
[In] int(x^14/(2*x^5+3)^3,x,method=_RETURNVERBOSE)
[Out] (3/10*x^5+27/80)/(2*x^5+3)^2+1/40*ln(2*x^5+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = \frac{24\,x^5 + 2\,(4\,x^{10} + 12\,x^5 + 9)\log\left(2\,x^5 + 3\right) + 27}{80\,(4\,x^{10} + 12\,x^5 + 9)}$$

```
[In] integrate(x^14/(2*x^5+3)^3,x, algorithm="fricas")
[Out] 1/80*(24*x^5 + 2*(4*x^10 + 12*x^5 + 9)*log(2*x^5 + 3) + 27)/(4*x^10 + 12*x^
5 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = \frac{24x^5 + 27}{320x^{10} + 960x^5 + 720} + \frac{\log\left(2x^5 + 3\right)}{40}$$

```
[In] integrate(x**14/(2*x**5+3)**3,x)
[Out] (24*x**5 + 27)/(320*x**10 + 960*x**5 + 720) + log(2*x**5 + 3)/40
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = \frac{3\,(8\,x^5 + 9)}{80\,(4\,x^{10} + 12\,x^5 + 9)} + \frac{1}{40}\,\log\left(2\,x^5 + 3\right)$$

```
[In] integrate(x^14/(2*x^5+3)^3,x, algorithm="maxima")
[Out] 3/80*(8*x^5 + 9)/(4*x^10 + 12*x^5 + 9) + 1/40*log(2*x^5 + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = -\frac{3\,(x^{10}+x^5)}{20\,(2\,x^5+3)^2} + \frac{1}{40}\,\log\left(\left|2\,x^5+3\right|\right)$$

```
[In] integrate(x^14/(2*x^5+3)^3,x, algorithm="giac")
[Out] -3/20*(x^10 + x^5)/(2*x^5 + 3)^2 + 1/40*log(abs(2*x^5 + 3))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = \frac{\ln\left(x^5+\frac{3}{2}\right)}{40} + \frac{\frac{3\,x^5}{40}+\frac{27}{320}}{x^{10}+3\,x^5+\frac{9}{4}}$$

```
[In] int(x^14/(2*x^5 + 3)^3,x)
[Out] log(x^5 + 3/2)/40 + ((3*x^5)/40 + 27/320)/(3*x^5 + x^10 + 9/4)
```

3.177 $\int \frac{x^6}{(3+2x^5)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 319

$$\begin{aligned}\int \frac{x^6}{(3+2x^5)^3}\,dx = & -\frac{x^2}{20(3+2x^5)^2}+\frac{x^2}{150(3+2x^5)} \\ & -\frac{\sqrt{5+\sqrt{5}}\arctan\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}-\frac{2\,2^{7/10}x}{\sqrt[5]{3}\sqrt{5-\sqrt{5}}}\right)}{250\,2^{9/10}3^{3/5}} \\ & -\frac{\sqrt{5-\sqrt{5}}\arctan\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}+\frac{2\,2^{7/10}x}{\sqrt[5]{3}\sqrt{5+\sqrt{5}}}\right)}{250\,2^{9/10}3^{3/5}} \\ & -\frac{\log\left(\sqrt[5]{3}+\sqrt[5]{2}x\right)}{250\,2^{2/5}3^{3/5}}+\frac{\left(1+\sqrt{5}\right)\log\left(3^{2/5}-\frac{\sqrt[5]{3}\left(1-\sqrt{5}\right)x}{2^{4/5}}+2^{2/5}x^2\right)}{1000\,2^{2/5}3^{3/5}} \\ & +\frac{\left(1-\sqrt{5}\right)\log\left(3^{2/5}-\frac{\sqrt[5]{3}\left(1+\sqrt{5}\right)x}{2^{4/5}}+2^{2/5}x^2\right)}{1000\,2^{2/5}3^{3/5}}\end{aligned}$$

```
[Out] -1/20*x^2/(2*x^5+3)^2+1/150*x^2/(2*x^5+3)-1/1500*ln(3^(1/5)+2^(1/5)*x)*2^(3
/5)*3^(2/5)+1/6000*ln(2^(3/5)*3^(2/5)+2*x^2-1/2*3^(1/5)*2^(4/5)*x*(5^(1/2)+
1))*(-5^(1/2)+1)*2^(3/5)*3^(2/5)+1/6000*ln(2^(3/5)*3^(2/5)+2*x^2-1/2*3^(1/5
)*2^(4/5)*x*(-5^(1/2)+1))*(5^(1/2)+1)*2^(3/5)*3^(2/5)-1/1500*arctan(1/5*(25
-10*5^(1/2))^(1/2)+2/3*2^(7/10)*x*3^(4/5)/(5+5^(1/2))^(1/2))*(5-5^(1/2))^(1
/2)*2^(1/10)*3^(2/5)+1/1500*arctan(2/3*2^(7/10)*x*3^(4/5)/(5-5^(1/2))^(1/2)
-1/5*(25+10*5^(1/2))^(1/2))*(5+5^(1/2))^(1/2)*2^(1/10)*3^(2/5)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {294, 296, 299, 648, 632, 210, 642, 31}

$$\int \frac{x^6}{(3+2x^5)^3}\,dx = -\frac{\sqrt{5+\sqrt{5}}\arctan\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}-\frac{2\ 2^{7/10}x}{\sqrt[5]{3}\sqrt{5-\sqrt{5}}}\right)}{250\ 2^{9/10}3^{3/5}} - \frac{\sqrt{5-\sqrt{5}}\arctan\left(\frac{2\ 2^{7/10}x}{\sqrt[5]{3}\sqrt{5+\sqrt{5}}}+\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}\right)}{250\ 2^{9/10}3^{3/5}} + \frac{\left(1+\sqrt{5}\right)\log\left(2^{2/5}x^2-\frac{\sqrt[5]{3}\left(1-\sqrt{5}\right)x}{2^{4/5}}+3^{2/5}\right)}{1000\ 2^{2/5}3^{3/5}} + \frac{\left(1-\sqrt{5}\right)\log\left(2^{2/5}x^2-\frac{\sqrt[5]{3}\left(1+\sqrt{5}\right)x}{2^{4/5}}+3^{2/5}\right)}{1000\ 2^{2/5}3^{3/5}} + \frac{x^2}{150\left(2x^5+3\right)} - \frac{x^2}{20\left(2x^5+3\right)^2} - \frac{\log\left(\sqrt[5]{2}x+\sqrt[5]{3}\right)}{250\ 2^{2/5}3^{3/5}}$$

```
[In] Int[x^6/(3 + 2*x^5)^3,x]
```

```
[Out] -1/20*x^2/(3 + 2*x^5)^2 + x^2/(150*(3 + 2*x^5)) - (Sqrt[5 + Sqrt[5]]*ArcTan
[Sqrt[(5 + 2*Sqrt[5])/5] - (2*2^(7/10)*x)/(3^(1/5)*Sqrt[5 - Sqrt[5]])])/(25
0*2^(9/10)*3^(3/5)) - (Sqrt[5 - Sqrt[5]]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + (
2*2^(7/10)*x)/(3^(1/5)*Sqrt[5 + Sqrt[5]])])/(250*2^(9/10)*3^(3/5)) - Log[3^
(1/5) + 2^(1/5)*x]/(250*2^(2/5)*3^(3/5)) + ((1 + Sqrt[5])*Log[3^(2/5) - (3^
(1/5)*(1 - Sqrt[5])*x)/2^(4/5) + 2^(2/5)*x^2])/(1000*2^(2/5)*3^(3/5)) + ((1
 - Sqrt[5])*Log[3^(2/5) - (3^(1/5)*(1 + Sqrt[5])*x)/2^(4/5) + 2^(2/5)*x^2])
/(1000*2^(2/5)*3^(3/5))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; (-(-r)^(m + 1)/(a*n*s^m))*Int[1/(r + s*x), x]
 + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Free
Q[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a
/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{x^2}{20(3+2x^5)^2} + \frac{1}{10}\int \frac{x}{(3+2x^5)^2}\,dx$$

$$= -\frac{x^2}{20(3+2x^5)^2} + \frac{x^2}{150(3+2x^5)} + \frac{1}{50}\int \frac{x}{3+2x^5}\,dx$$

$$= -\frac{x^2}{20(3+2x^5)^2} + \frac{x^2}{150(3+2x^5)} - \frac{\int \frac{1}{\sqrt[5]{3}+\sqrt[5]{2}x}\,dx}{250\sqrt[5]{2}3^{3/5}} + \frac{\int \frac{\frac{1}{4}\sqrt[5]{3}(1-\sqrt{5})-\frac{(-1-\sqrt{5})x}{2\ 2^{4/5}}}{3^{2/5}-\frac{\sqrt[5]{3}(1-\sqrt{5})x}{2^{4/5}}+2^{2/5}x^2}\,dx}{125\sqrt[5]{2}3^{3/5}} + \frac{\int \frac{\frac{1}{4}\sqrt[5]{3}(1+\sqrt{5})-\frac{(-1+\sqrt{5})x}{2\ 2^{4/5}}}{3^{2/5}-\frac{\sqrt[5]{3}(1+\sqrt{5})x}{2^{4/5}}+2^{2/5}x^2}\,dx}{125\sqrt[5]{2}3^{3/5}}$$

$$= -\frac{x^2}{20(3+2x^5)^2} + \frac{x^2}{150(3+2x^5)} - \frac{\log\left(\sqrt[5]{3}+\sqrt[5]{2}x\right)}{250\ 2^{2/5}3^{3/5}} - \frac{\int \frac{1}{3^{2/5}-\frac{\sqrt[5]{3}(1-\sqrt{5})x}{2^{4/5}}+2^{2/5}x^2}\,dx}{100\sqrt[5]{2}3^{2/5}\sqrt{5}} + \frac{\int \frac{1}{3^{2/5}-\frac{\sqrt[5]{3}(1+\sqrt{5})x}{2^{4/5}}+2^{2/5}x^2}\,dx}{100\sqrt[5]{2}3^{2/5}\sqrt{5}} + \frac{(1-\sqrt{5})\int \frac{-\frac{\sqrt[5]{3}(1+\sqrt{5})}{2^{4/5}}+2\ 2^{2/5}x}{3^{2/5}-\frac{\sqrt[5]{3}(1+\sqrt{5})x}{2^{4/5}}+2^{2/5}x^2}\,dx}{1000\ 2^{2/5}3^{3/5}} + \frac{(1+\sqrt{5})\int \frac{-\frac{\sqrt[5]{3}(1-\sqrt{5})}{2^{4/5}}+2\ 2^{2/5}x}{3^{2/5}-\frac{\sqrt[5]{3}(1-\sqrt{5})x}{2^{4/5}}+2^{2/5}x^2}\,dx}{1000\ 2^{2/5}3^{3/5}}$$

$$= -\frac{x^2}{20(3+2x^5)^2} + \frac{x^2}{150(3+2x^5)} - \frac{\log\left(\sqrt[5]{3}+\sqrt[5]{2}x\right)}{250\ 2^{2/5}3^{3/5}} + \frac{(1-\sqrt{5})\log\left(2\ 3^{2/5}-\sqrt[5]{6}x-\sqrt{5}\sqrt[5]{6}x+2\ 2^{2/5}x^2\right)}{1000\ 2^{2/5}3^{3/5}} + \frac{(1+\sqrt{5})\log\left(2\ 3^{2/5}-\sqrt[5]{6}x+\sqrt{5}\sqrt[5]{6}x+2\ 2^{2/5}x^2\right)}{1000\ 2^{2/5}3^{3/5}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{3^{2/5}(5-\sqrt{5})}{2^{3/5}}-x^2}\,dx, x, -\frac{\sqrt[5]{3}(1+\sqrt{5})}{2^{4/5}}+2\ 2^{2/5}x\right)}{50\sqrt[5]{2}3^{2/5}\sqrt{5}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{3^{2/5}(5+\sqrt{5})}{2^{3/5}}-x^2}\,dx, x, -\frac{\sqrt[5]{3}(1-\sqrt{5})}{2^{4/5}}+2\ 2^{2/5}x\right)}{50\sqrt[5]{2}3^{2/5}\sqrt{5}}$$

$$= -\frac{x^2}{20(3+2x^5)^2} + \frac{x^2}{150(3+2x^5)} - \frac{\arctan\left(\frac{\sqrt[5]{3}(1+\sqrt{5})-4\sqrt[5]{2}x}{\sqrt[5]{3}\sqrt{2(5-\sqrt{5})}}\right)}{25\ 2^{9/10}3^{3/5}\sqrt{5(5-\sqrt{5})}} - \frac{\arctan\left(\frac{\sqrt[5]{3}\sqrt{3-\sqrt{5}}+2\ 2^{7/10}x}{\sqrt[5]{3}\sqrt{5+\sqrt{5}}}\right)}{25\ 2^{9/10}3^{3/5}\sqrt{5(5+\sqrt{5})}} - \frac{\log\left(\sqrt[5]{3}+\sqrt[5]{2}x\right)}{250\ 2^{2/5}3^{3/5}} + \frac{(1-\sqrt{5})\log\left(2\ 3^{2/5}-\sqrt[5]{6}x-\sqrt{5}\sqrt[5]{6}x+2\ 2^{2/5}x^2\right)}{1000\ 2^{2/5}3^{3/5}} + \frac{(1+\sqrt{5})\log\left(2\ 3^{2/5}-\sqrt[5]{6}x+\sqrt{5}\sqrt[5]{6}x+2\ 2^{2/5}x^2\right)}{1000\ 2^{2/5}3^{3/5}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{(3+2x^5)^3}\,dx = \frac{-\frac{300x^2}{(3+2x^5)^2} + \frac{40x^2}{3+2x^5} - 4\sqrt[10]{2}3^{2/5}\sqrt{5-\sqrt{5}}\arctan\left(\frac{-3+3\sqrt{5}+4\sqrt[5]{2}3^{4/5}x}{3\sqrt{2(5+\sqrt{5})}}\right) + 4\sqrt[10]{2}3^{2/5}\sqrt{5+\sqrt{5}}\arctan\left(\frac{-3(1+\sqrt{5})}{3}\right.}{}$$

```
[In] Integrate[x^6/(3 + 2*x^5)^3,x]
```

```
[Out] ((-300*x^2)/(3 + 2*x^5)^2 + (40*x^2)/(3 + 2*x^5) - 4*2^(1/10)*3^(2/5)*Sqrt[
5 - Sqrt[5]]*ArcTan[(-3 + 3*Sqrt[5] + 4*2^(1/5)*3^(4/5)*x)/(3*Sqrt[2*(5 + S
qrt[5])])] + 4*2^(1/10)*3^(2/5)*Sqrt[5 + Sqrt[5]]*ArcTan[(-3*(1 + Sqrt[5])
+ 4*2^(1/5)*3^(4/5)*x)/(3*Sqrt[10 - 2*Sqrt[5]])] - 4*2^(3/5)*3^(2/5)*Log[3
+ 2^(1/5)*3^(4/5)*x] + 2^(3/5)*3^(2/5)*(1 + Sqrt[5])*Log[3 + (3/2)^(4/5)*(-
1 + Sqrt[5])*x + 2^(2/5)*3^(3/5)*x^2] - 2^(3/5)*3^(2/5)*(-1 + Sqrt[5])*Log[
3 - (3/2)^(4/5)*(1 + Sqrt[5])*x + 2^(2/5)*3^(3/5)*x^2])/6000
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.15

method	result
risch	$\frac{\frac{1}{75}x^7-\frac{3}{100}x^2}{(2x^5+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(2\_Z^5+3)}\frac{\ln(x-\_R)}{\_R^3}\right)}{500}$
meijerg	$108^{\frac{4}{5}}\left(-\frac{x^2 3^{\frac{3}{5}}2^{\frac{2}{5}}\left(-\frac{28x^5}{3}+21\right)}{105\left(1+\frac{2x^5}{3}\right)^2}+\frac{2\,108^{\frac{1}{5}}x^2\left(-\frac{2^{\frac{3}{5}}3^{\frac{2}{5}}\ln\left(1+\frac{2^{\frac{1}{5}}3^{\frac{4}{5}}(x^5)^{\frac{1}{5}}}{3}\right)}{2(x^5)^{\frac{2}{5}}}-\frac{2^{\frac{3}{5}}3^{\frac{2}{5}}\cos\left(\frac{2\pi}{5}\right)\ln\left(1-\frac{2\cos\left(\frac{\pi}{5}\right)2^{\frac{1}{5}}3^{\frac{4}{5}}(x^5)^{\frac{1}{5}}}{3}+\frac{2^{\frac{2}{5}}3^{\frac{3}{5}}(x^5)^{\frac{2}{5}}}{3}\right)}{2(x^5)^{\frac{2}{5}}}+\cdots\right.}{\cdots}\right.$
default	$\frac{\frac{1}{75}x^7-\frac{3}{100}x^2}{(2x^5+3)^2}+\frac{48^{\frac{2}{5}}\ln\left(-x\sqrt{5}\,48^{\frac{1}{5}}-x48^{\frac{1}{5}}+48^{\frac{2}{5}}+4x^2\right)}{12000}-\frac{48^{\frac{2}{5}}\ln\left(-x\sqrt{5}\,48^{\frac{1}{5}}-x48^{\frac{1}{5}}+48^{\frac{2}{5}}+4x^2\right)\sqrt{5}}{12000}+\frac{48^{\frac{3}{5}}\arctan\left(-\frac{\sqrt{5}\,48\cdots}{\sqrt{10\,48^{\frac{2}{5}}-\cdots}}\right.}{\cdots}$

```
[In] int(x^6/(2*x^5+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4*(1/300*x^7-3/400*x^2)/(2*x^5+3)^2+1/500*sum(1/_R^3*ln(x-_R),_R=RootOf(2*_
Z^5+3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.69 (sec) , antiderivative size = 1751, normalized size of antiderivative = 5.49

$$\int \frac{x^6}{(3+2x^5)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6/(2*x^5+3)^3,x, algorithm="fricas")
```

```
[Out] 1/216000*(2880*x^7 - 2*108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*(sqrt(5)
+ I*sqrt(-2*sqrt(5) + 10) + 1)*log(-1/6912*108^(3/5)*(-1)^(2/5)*(108^(4/5)*
(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(1/5)
)*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 1/64*108^(2/5)*(-1)^(3/5)*(sq
rt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^3 - 1/6912*108^(4/5)*(-1)^(1/5)*(108^(
3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 4*108^(3/5)*(-1
)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) + 16*108^(3/5)*(-1)^(2/5))*
(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) + 1/16*108^(2/5)*(-1)^(3/5)*(sqrt(5
) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 1/4*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*
sqrt(-2*sqrt(5) + 10) + 1) + 108^(2/5)*(-1)^(3/5) + 6*x) - 2*108^(4/5)*(-1)
^(1/5)*(4*x^10 + 12*x^5 + 9)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)*log(1/
```

384*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^3 + x) + 8
108^(4/5)(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*log(-108^(2/5)*(-1)^(3/5) + 6*x
) - 6480*x^2 + (108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*(sqrt(5) + I*sqr
t(-2*sqrt(5) + 10) + 1) + 108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*(sqrt(
5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5
 + 9) - 24*sqrt(3)*(4*x^10 + 12*x^5 + 9)*sqrt(-1/864*108^(4/5)*(-1)^(1/5)*(
108^(4/5)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*
(-1)^(1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) - 3/16*108^(3/5)*(-1)^(
2/5)*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 3/16*108^(3/5)*(-1)^(2/5)*
(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 + 1/2*108^(3/5)*(-1)^(2/5)*(sqrt(
5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 3*108^(3/5)*(-1)^(2/5)))*log(1/768*108^
(3/5)*(-1)^(2/5)*(108^(4/5)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) +
 1) - 4*108^(4/5)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 + 1
/768*108^(4/5)*(-1)^(1/5)*(108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5
) + 10) + 1)^2 - 4*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10)
+ 1) + 16*108^(3/5)*(-1)^(2/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) - 9
/16*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 + 9/4*10
8^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) + 1/3456*(108^(4
/5)*(-1)^(1/5)*(108^(4/5)*sqrt(3)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) +
 10) + 1) - 4*108^(4/5)*sqrt(3)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) +
10) + 1) - 432*108^(3/5)*sqrt(3)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) +
10) + 1))*sqrt(-1/864*108^(4/5)*(-1)^(1/5)*(108^(4/5)*(-1)^(1/5)*(sqrt(5) -
 I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-
2*sqrt(5) + 10) + 1) - 3/16*108^(3/5)*(-1)^(2/5)*(sqrt(5) + I*sqrt(-2*sqrt(
5) + 10) + 1)^2 - 3/16*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) +
10) + 1)^2 + 1/2*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) +
1) - 3*108^(3/5)*(-1)^(2/5)) + 108*x) + (108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*
x^5 + 9)*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) + 108^(4/5)*(-1)^(1/5)*(4*
x^10 + 12*x^5 + 9)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-
1)^(1/5)*(4*x^10 + 12*x^5 + 9) + 24*sqrt(3)*(4*x^10 + 12*x^5 + 9)*sqrt(-1/8
64*108^(4/5)*(-1)^(1/5)*(108^(4/5)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5)
+ 10) + 1) - 4*108^(4/5)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1
) - 3/16*108^(3/5)*(-1)^(2/5)*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 3
/16*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 + 1/2*10
8^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 3*108^(3/5)*(-
1)^(2/5)))*log(1/768*108^(3/5)*(-1)^(2/5)*(108^(4/5)*(-1)^(1/5)*(sqrt(5) -
I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-2
*sqrt(5) + 10) + 1)^2 + 1/768*108^(4/5)*(-1)^(1/5)*(108^(3/5)*(-1)^(2/5)*(s
qrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 4*108^(3/5)*(-1)^(2/5)*(sqrt(5) -
 I*sqrt(-2*sqrt(5) + 10) + 1) + 16*108^(3/5)*(-1)^(2/5))*(sqrt(5) + I*sqrt(
-2*sqrt(5) + 10) + 1) - 9/16*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt
(5) + 10) + 1)^2 + 9/4*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) +
10) + 1) - 1/3456*(108^(4/5)*(-1)^(1/5)*(108^(4/5)*sqrt(3)*(-1)^(1/5)*(sqrt
(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*sqrt(3)*(-1)^(1/5))*(sqrt(
5) + I*sqrt(-2*sqrt(5) + 10) + 1) - 432*108^(3/5)*sqrt(3)*(-1)^(2/5)*(sqrt(

```
5) - I*sqrt(-2*sqrt(5) + 10) + 1))*sqrt(-1/864*108^(4/5)*(-1)^(1/5)*(108^(4
/5)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(
1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) - 3/16*108^(3/5)*(-1)^(2/5)*(
sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 3/16*108^(3/5)*(-1)^(2/5)*(sqrt(
5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 + 1/2*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I
*sqrt(-2*sqrt(5) + 10) + 1) - 3*108^(3/5)*(-1)^(2/5)) + 108*x))/(4*x^10 + 1
2*x^5 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.12

$$\int \frac{x^6}{(3+2x^5)^3}\,dx = \frac{4x^7-9x^2}{1200x^{10}+3600x^5+2700} + \operatorname{RootSum}\left(105468750000000t^5+1, \left(t\mapsto t\log\left(-281250000t^3+x\right)\right)\right)$$

```
[In] integrate(x**6/(2*x**5+3)**3,x)
[Out] (4*x**7 - 9*x**2)/(1200*x**10 + 3600*x**5 + 2700) + RootSum(105468750000000
*_t**5 + 1, Lambda(_t, _t*log(-281250000*_t**3 + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.05

$$\int \frac{x^6}{(3+2x^5)^3}\,dx = \frac{3^{\frac45}2^{\frac45}(\sqrt5-5)\arctan\left(\frac{3^{\frac45}2^{\frac45}\left(4\cdot2^{\frac25}x+\sqrt53^{\frac15}2^{\frac15}-3^{\frac15}2^{\frac15}\right)}{6\sqrt{2\sqrt5+10}}\right)}{750\left(\sqrt53^{\frac25}2^{\frac15}-3^{\frac25}2^{\frac15}\right)\sqrt{2\sqrt5+10}} + \frac{3^{\frac45}2^{\frac45}(\sqrt5+5)\arctan\left(\frac{3^{\frac45}2^{\frac45}\left(4\cdot2^{\frac25}x-\sqrt53^{\frac15}2^{\frac15}-3^{\frac15}2^{\frac15}\right)}{6\sqrt{-2\sqrt5+10}}\right)}{750\left(\sqrt53^{\frac25}2^{\frac15}+3^{\frac25}2^{\frac15}\right)\sqrt{-2\sqrt5+10}} - \frac{1}{1500}\cdot3^{\frac25}2^{\frac35}\log\left(2^{\frac15}x+3^{\frac15}\right) + \frac{4x^7-9x^2}{300(4x^{10}+12x^5+9)} - \frac{\log\left(2\cdot2^{\frac25}x^2-x\left(\sqrt53^{\frac15}2^{\frac15}+3^{\frac15}2^{\frac15}\right)+2\cdot3^{\frac25}\right)}{250\left(\sqrt53^{\frac35}2^{\frac25}+3^{\frac35}2^{\frac25}\right)} + \frac{\log\left(2\cdot2^{\frac25}x^2+x\left(\sqrt53^{\frac15}2^{\frac15}-3^{\frac15}2^{\frac15}\right)+2\cdot3^{\frac25}\right)}{250\left(\sqrt53^{\frac35}2^{\frac25}-3^{\frac35}2^{\frac25}\right)}$$

```
[In] integrate(x^6/(2*x^5+3)^3,x, algorithm="maxima")
```

```
[Out] 1/750*3^(4/5)*2^(4/5)*(sqrt(5) - 5)*arctan(1/6*3^(4/5)*2^(4/5)*(4*2^(2/5)*x
 + sqrt(5)*3^(1/5)*2^(1/5) - 3^(1/5)*2^(1/5))/sqrt(2*sqrt(5) + 10))/((sqrt(
5)*3^(2/5)*2^(1/5) - 3^(2/5)*2^(1/5))*sqrt(2*sqrt(5) + 10)) + 1/750*3^(4/5)
*2^(4/5)*(sqrt(5) + 5)*arctan(1/6*3^(4/5)*2^(4/5)*(4*2^(2/5)*x - sqrt(5)*3^
(1/5)*2^(1/5) - 3^(1/5)*2^(1/5))/sqrt(-2*sqrt(5) + 10))/((sqrt(5)*3^(2/5)*2
^(1/5) + 3^(2/5)*2^(1/5))*sqrt(-2*sqrt(5) + 10)) - 1/1500*3^(2/5)*2^(3/5)*l
og(2^(1/5)*x + 3^(1/5)) + 1/300*(4*x^7 - 9*x^2)/(4*x^10 + 12*x^5 + 9) - 1/2
50*log(2*2^(2/5)*x^2 - x*(sqrt(5)*3^(1/5)*2^(1/5) + 3^(1/5)*2^(1/5)) + 2*3^
(2/5))/(sqrt(5)*3^(3/5)*2^(2/5) + 3^(3/5)*2^(2/5)) + 1/250*log(2*2^(2/5)*x^
2 + x*(sqrt(5)*3^(1/5)*2^(1/5) - 3^(1/5)*2^(1/5)) + 2*3^(2/5))/(sqrt(5)*3^(
3/5)*2^(2/5) - 3^(3/5)*2^(2/5))
```

Giac [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.78

$$\int \frac{x^6}{(3+2x^5)^3}\,dx = -\frac{1}{3000}\left(\sqrt{5}\left(\frac{3}{2}\right)^{\frac{2}{5}}\sqrt{2\sqrt{5}+10}-\left(\frac{3}{2}\right)^{\frac{2}{5}}\sqrt{2\sqrt{5}+10}\right)\arctan\left(\frac{2\left(\frac{3}{2}\right)^{\frac{4}{5}}\left(\left(\frac{3}{2}\right)^{\frac{1}{5}}(\sqrt{5}-1)+4x\right)}{3\sqrt{2\sqrt{5}+10}}\right) + \frac{1}{3000}\left(\sqrt{5}\left(\frac{3}{2}\right)^{\frac{2}{5}}\sqrt{-2\sqrt{5}+10}+\left(\frac{3}{2}\right)^{\frac{2}{5}}\sqrt{-2\sqrt{5}+10}\right)\arctan\left(-\frac{2\left(\frac{3}{2}\right)^{\frac{4}{5}}\left(\left(\frac{3}{2}\right)^{\frac{1}{5}}(\sqrt{5}+1)-4x\right)}{3\sqrt{-2\sqrt{5}+10}}\right) - \frac{1}{6000}\left(\left(\frac{3}{2}\right)^{\frac{2}{5}}\left(\sqrt{5}-5\right)+\sqrt{5}\left(\frac{3}{2}\right)^{\frac{2}{5}}+3\left(\frac{3}{2}\right)^{\frac{2}{5}}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}\left(\frac{3}{2}\right)^{\frac{1}{5}}+\left(\frac{3}{2}\right)^{\frac{1}{5}}\right)+\left(\frac{3}{2}\right)^{\frac{2}{5}}\right) + \frac{1}{6000}\left(\left(\frac{3}{2}\right)^{\frac{2}{5}}\left(\sqrt{5}+5\right)+\sqrt{5}\left(\frac{3}{2}\right)^{\frac{2}{5}}-3\left(\frac{3}{2}\right)^{\frac{2}{5}}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}\left(\frac{3}{2}\right)^{\frac{1}{5}}-\left(\frac{3}{2}\right)^{\frac{1}{5}}\right)+\left(\frac{3}{2}\right)^{\frac{2}{5}}\right)-\frac{1}{750}\left(\frac{3}{2}\right)^{\frac{2}{5}}\log\left(\left|x+\left(\frac{3}{2}\right)^{\frac{1}{5}}\right|\right)+\frac{4x^7-9x^2}{300\left(2x^5+3\right)^2}$$

```
[In] integrate(x^6/(2*x^5+3)^3,x, algorithm="giac")
```

```
[Out] -1/3000*(sqrt(5)*(3/2)^(2/5)*sqrt(2*sqrt(5) + 10) - (3/2)^(2/5)*sqrt(2*sqrt
(5) + 10))*arctan(2/3*(3/2)^(4/5)*((3/2)^(1/5)*(sqrt(5) - 1) + 4*x)/sqrt(2*
sqrt(5) + 10)) + 1/3000*(sqrt(5)*(3/2)^(2/5)*sqrt(-2*sqrt(5) + 10) + (3/2)^
(2/5)*sqrt(-2*sqrt(5) + 10))*arctan(-2/3*(3/2)^(4/5)*((3/2)^(1/5)*(sqrt(5)
+ 1) - 4*x)/sqrt(-2*sqrt(5) + 10)) - 1/6000*((3/2)^(2/5)*(sqrt(5) - 5) + sq
rt(5)*(3/2)^(2/5) + 3*(3/2)^(2/5))*log(x^2 - 1/2*x*(sqrt(5)*(3/2)^(1/5) + (
```

```
3/2)^(1/5)) + (3/2)^(2/5)) + 1/6000*((3/2)^(2/5)*(sqrt(5) + 5) + sqrt(5)*(3
/2)^(2/5) - 3*(3/2)^(2/5))*log(x^2 + 1/2*x*(sqrt(5)*(3/2)^(1/5) - (3/2)^(1/
5)) + (3/2)^(2/5)) - 1/750*(3/2)^(2/5)*log(abs(x + (3/2)^(1/5))) + 1/300*(4
*x^7 - 9*x^2)/(2*x^5 + 3)^2
```

Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.89

$$\int \frac{x^6}{(3+2x^5)^3}\,dx$$
$$= \frac{3^{2/5}\ln\left(x-\frac{3^{1/5}\left(2\,2^{1/10}\sqrt{-\sqrt{5}-5}-2^{3/5}\left(\sqrt{5}-1\right)\right)^3}{256}\right)\left(2\,2^{1/10}\sqrt{-\sqrt{5}-5}-2^{3/5}\left(\sqrt{5}-1\right)\right)}{6000}$$
$$-\frac{\frac{3\,x^2}{400}-\frac{x^7}{300}}{x^{10}+3\,x^5+\frac{9}{4}}$$
$$-\frac{3^{2/5}\ln\left(x+\frac{3^{1/5}\left(2\,2^{1/10}\sqrt{-\sqrt{5}-5}+2^{3/5}\left(\sqrt{5}-1\right)\right)^3}{256}\right)\left(2\,2^{1/10}\sqrt{-\sqrt{5}-5}+2^{3/5}\left(\sqrt{5}-1\right)\right)}{6000}$$
$$-\frac{72^{1/5}\ln\left(x+\frac{72^{3/5}}{12}\right)}{1500}$$
$$+\frac{3^{2/5}\ln\left(x-\frac{3^{1/5}\left(2^{3/5}\left(\sqrt{5}+1\right)-2\,2^{1/10}\sqrt{\sqrt{5}-5}\right)^3}{256}\right)\left(2^{3/5}\left(\sqrt{5}+1\right)-2\,2^{1/10}\sqrt{\sqrt{5}-5}\right)}{6000}$$
$$+\frac{3^{2/5}\ln\left(x-\frac{3^{1/5}\left(2^{3/5}\left(\sqrt{5}+1\right)+2\,2^{1/10}\sqrt{\sqrt{5}-5}\right)^3}{256}\right)\left(2^{3/5}\left(\sqrt{5}+1\right)+2\,2^{1/10}\sqrt{\sqrt{5}-5}\right)}{6000}$$

```
[In] int(x^6/(2*x^5 + 3)^3,x)
```

```
[Out] (3^(2/5)*log(x - (3^(1/5)*(2*2^(1/10)*(- 5^(1/2) - 5)^(1/2) - 2^(3/5)*(5^(1
/2) - 1))^3)/256)*(2*2^(1/10)*(- 5^(1/2) - 5)^(1/2) - 2^(3/5)*(5^(1/2) - 1)
))/6000 - ((3*x^2)/400 - x^7/300)/(3*x^5 + x^10 + 9/4) - (3^(2/5)*log(x + (
3^(1/5)*(2*2^(1/10)*(- 5^(1/2) - 5)^(1/2) + 2^(3/5)*(5^(1/2) - 1))^3)/256)*
(2*2^(1/10)*(- 5^(1/2) - 5)^(1/2) + 2^(3/5)*(5^(1/2) - 1)))/6000 - (72^(1/5
)*log(x + 72^(3/5)/12))/1500 + (3^(2/5)*log(x - (3^(1/5)*(2^(3/5)*(5^(1/2)
+ 1) - 2*2^(1/10)*(5^(1/2) - 5)^(1/2))^3)/256)*(2^(3/5)*(5^(1/2) + 1) - 2*2
^(1/10)*(5^(1/2) - 5)^(1/2)))/6000 + (3^(2/5)*log(x - (3^(1/5)*(2^(3/5)*(5^
(1/2) + 1) + 2*2^(1/10)*(5^(1/2) - 5)^(1/2))^3)/256)*(2^(3/5)*(5^(1/2) + 1)
 + 2*2^(1/10)*(5^(1/2) - 5)^(1/2)))/6000
```

3.178 $\int \frac{9}{5x^2(3-2x^2)^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \frac{9}{5x^2(3-2x^2)^3} \, dx = -\frac{1}{8x} + \frac{3}{20x(3-2x^2)^2} + \frac{1}{8x(3-2x^2)} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}}$$

```
[Out] -1/8/x+3/20/x/(-2*x^2+3)^2+1/8/x/(-2*x^2+3)+1/24*arctanh(1/3*x*6^(1/2))*6^(
1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {12, 296, 331, 212}

$$\int \frac{9}{5x^2(3-2x^2)^3} \, dx = \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}} + \frac{1}{8x(3-2x^2)} + \frac{3}{20x(3-2x^2)^2} - \frac{1}{8x}$$

```
[In] Int[9/(5*x^2*(3 - 2*x^2)^3),x]
```

```
[Out] -1/8*1/x + 3/(20*x*(3 - 2*x^2)^2) + 1/(8*x*(3 - 2*x^2)) + ArcTanh[Sqrt[2/3]
*x]/(4*Sqrt[6])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{9}{5}\int \frac{1}{x^2\left(3-2x^2\right)^3}\,dx \\
&= \frac{3}{20x\left(3-2x^2\right)^2}+\frac{3}{4}\int \frac{1}{x^2\left(3-2x^2\right)^2}\,dx \\
&= \frac{3}{20x\left(3-2x^2\right)^2}+\frac{1}{8x\left(3-2x^2\right)}+\frac{3}{8}\int \frac{1}{x^2\left(3-2x^2\right)}\,dx \\
&= -\frac{1}{8x}+\frac{3}{20x\left(3-2x^2\right)^2}+\frac{1}{8x\left(3-2x^2\right)}+\frac{1}{4}\int \frac{1}{3-2x^2}\,dx \\
&= -\frac{1}{8x}+\frac{3}{20x\left(3-2x^2\right)^2}+\frac{1}{8x\left(3-2x^2\right)}+\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int \frac{9}{5x^2(3-2x^2)^3}\,dx = \frac{1}{240}\left(-\frac{12(12-25x^2+10x^4)}{x(3-2x^2)^2} - 5\sqrt{6}\log\left(\sqrt{6}-2x\right) + 5\sqrt{6}\log\left(\sqrt{6}+2x\right)\right)$$

```
[In] Integrate[9/(5*x^2*(3 - 2*x^2)^3),x]
[Out] ((-12*(12 - 25*x^2 + 10*x^4))/(x*(3 - 2*x^2)^2) - 5*Sqrt[6]*Log[Sqrt[6] - 2
*x] + 5*Sqrt[6]*Log[Sqrt[6] + 2*x])/240
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{1}{15x}-\frac{8\left(\frac{7}{16}x^3-\frac{27}{32}x\right)}{15(2x^2-3)^2}+\frac{\operatorname{arctanh}\left(\frac{x\sqrt{6}}{3}\right)\sqrt{6}}{24}$	39
meijerg	$\frac{i\sqrt{6}\left(\frac{i\sqrt{6}\left(\frac{20}{3}x^4-\frac{50}{3}x^2+8\right)}{4x\left(-\frac{2x^2}{3}+1\right)^2}-\frac{15i\operatorname{arctanh}\left(\frac{x\sqrt{2}\sqrt{3}}{3}\right)}{2}\right)}{180}$	51
risch	$\frac{-\frac{1}{2}x^4+\frac{5}{4}x^2-\frac{3}{5}}{(2x^2-3)^2x}+\frac{\sqrt{6}\ln\left(2x+\sqrt{6}\right)}{48}-\frac{\sqrt{6}\ln\left(2x-\sqrt{6}\right)}{48}$	56

```
[In] int(9/5/x^2/(-2*x^2+3)^3,x,method=_RETURNVERBOSE)
[Out] -1/15/x-8/15*(7/16*x^3-27/32*x)/(2*x^2-3)^2+1/24*arctanh(1/3*x*6^(1/2))*6^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \frac{9}{5x^2(3-2x^2)^3}\,dx = -\frac{120\,x^4 - 5\,\sqrt{6}(4\,x^5-12\,x^3+9\,x)\log\left(\frac{2\,x^2+2\,\sqrt{6}x+3}{2\,x^2-3}\right) - 300\,x^2 + 144}{240\,(4\,x^5-12\,x^3+9\,x)}$$

```
[In] integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="fricas")
[Out] -1/240*(120*x^4 - 5*sqrt(6)*(4*x^5 - 12*x^3 + 9*x)*log((2*x^2 + 2*sqrt(6)*x
 + 3)/(2*x^2 - 3)) - 300*x^2 + 144)/(4*x^5 - 12*x^3 + 9*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \frac{9}{5x^2\left(3-2x^2\right)^3}\,dx = -\frac{9\cdot(10x^4-25x^2+12)}{720x^5-2160x^3+1620x} - \frac{\sqrt{6}\log\left(x-\frac{\sqrt{6}}{2}\right)}{48} + \frac{\sqrt{6}\log\left(x+\frac{\sqrt{6}}{2}\right)}{48}$$

```
[In] integrate(9/5/x**2/(-2*x**2+3)**3,x)
```

```
[Out] -9*(10*x**4 - 25*x**2 + 12)/(720*x**5 - 2160*x**3 + 1620*x) - sqrt(6)*log(x
 - sqrt(6)/2)/48 + sqrt(6)*log(x + sqrt(6)/2)/48
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{9}{5x^2\left(3-2x^2\right)^3}\,dx = -\frac{1}{48}\sqrt{6}\log\left(\frac{2x-\sqrt{6}}{2x+\sqrt{6}}\right) - \frac{10x^4-25x^2+12}{20\left(4x^5-12x^3+9x\right)}$$

```
[In] integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -1/48*sqrt(6)*log((2*x - sqrt(6))/(2*x + sqrt(6))) - 1/20*(10*x^4 - 25*x^2
+ 12)/(4*x^5 - 12*x^3 + 9*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{9}{5x^2\left(3-2x^2\right)^3}\,dx = -\frac{1}{48}\sqrt{6}\log\left(\frac{\left|4x-2\sqrt{6}\right|}{\left|4x+2\sqrt{6}\right|}\right) - \frac{14x^3-27x}{60\left(2x^2-3\right)^2} - \frac{1}{15x}$$

```
[In] integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(6)*log(abs(4*x - 2*sqrt(6))/abs(4*x + 2*sqrt(6))) - 1/60*(14*x^3
 - 27*x)/(2*x^2 - 3)^2 - 1/15/x
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{9}{5x^2\left(3-2x^2\right)^3}\,dx = \frac{\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{6}\,x}{3}\right)}{24} - \frac{\frac{x^4}{8}-\frac{5\,x^2}{16}+\frac{3}{20}}{x^5-3\,x^3+\frac{9\,x}{4}}$$

```
[In] int(-9/(5*x^2*(2*x^2 - 3)^3),x)
[Out] (6^(1/2)*atanh((6^(1/2)*x)/3))/24 - (x^4/8 - (5*x^2)/16 + 3/20)/((9*x)/4 -
3*x^3 + x^5)
```

3.179 $\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 36

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{4}{x}-\frac{7x}{4(1+x^2)^2}-\frac{25x}{8(1+x^2)}-\frac{57\arctan(x)}{8}$$

```
[Out] -4/x-7/4*x/(x^2+1)^2-25/8*x/(x^2+1)-57/8*arctan(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1274, 467, 464, 209}

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{57\arctan(x)}{8}-\frac{25x}{8(x^2+1)}-\frac{7x}{4(x^2+1)^2}-\frac{4}{x}$$

```
[In] Int[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

```
[Out] -4/x - (7*x)/(4*(1 + x^2)^2) - (25*x)/(8*(1 + x^2)) - (57*ArcTan[x])/8
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1274

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Sym
bol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*x*((d + e*x^2)^(q + 1)/(2*e^(
2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m
*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*(-d)^(-m/2 + 1
)*e^(2*p)*(q + 1)*(a + c*x^4)^p - ((c*d^2 + a*e^2)^p/(e^(m/2)*x^m))*(d + e*
(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ[p, 0] && ILt
Q[q, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{7x}{4(1+x^2)^2} - \frac{1}{4}\int \frac{-16+9x^2}{x^2(1+x^2)^2}\,dx \\
&= -\frac{7x}{4(1+x^2)^2} - \frac{25x}{8(1+x^2)} + \frac{1}{8}\int \frac{32-25x^2}{x^2(1+x^2)}\,dx \\
&= -\frac{4}{x} - \frac{7x}{4(1+x^2)^2} - \frac{25x}{8(1+x^2)} - \frac{57}{8}\int \frac{1}{1+x^2}\,dx \\
&= -\frac{4}{x} - \frac{7x}{4(1+x^2)^2} - \frac{25x}{8(1+x^2)} - \frac{57\arctan(x)}{8}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx=-\frac{32+103x^2+57x^4}{8x\left(1+x^2\right)^2}-\frac{57\arctan(x)}{8}$$

```
[In] Integrate[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
[Out] -1/8*(32 + 103*x^2 + 57*x^4)/(x*(1 + x^2)^2) - (57*ArcTan[x])/8
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\frac{25}{8}x^3+\frac{39}{8}x}{\left(x^2+1\right)^2}-\frac{57\arctan(x)}{8}-\frac{4}{x}$	29
risch	$\frac{-\frac{57}{8}x^4-\frac{103}{8}x^2-4}{\left(x^2+1\right)^2 x}-\frac{57\arctan(x)}{8}$	29
meijerg	$-\frac{15x^4+25x^2+8}{2x\left(x^2+1\right)^2}-\frac{57\arctan(x)}{8}-\frac{x\left(-3x^2+3\right)}{8\left(x^2+1\right)^2}$	47
parallelrisch	$\frac{57i\ln(x-i)x^5-57i\ln(x+i)x^5-64+114i\ln(x-i)x^3-114i\ln(x+i)x^3-114x^4+57i\ln(x-i)x-57i\ln(x+i)x-206x^2}{16x\left(x^2+1\right)^2}$	87

```
[In] int((3*x^4+4)/x^2/(x^2+1)^3,x,method=_RETURNVERBOSE)
[Out] -(25/8*x^3+39/8*x)/(x^2+1)^2-57/8*arctan(x)-4/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx=-\frac{57\,x^4+103\,x^2+57\left(x^5+2\,x^3+x\right)\arctan\left(x\right)+32}{8\left(x^5+2\,x^3+x\right)}$$

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="fricas")
[Out] -1/8*(57*x^4 + 103*x^2 + 57*(x^5 + 2*x^3 + x)*arctan(x) + 32)/(x^5 + 2*x^3
+ x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = \frac{-57x^4-103x^2-32}{8x^5+16x^3+8x} - \frac{57\,\mathrm{atan}\,(x)}{8}$$

```
[In] integrate((3*x**4+4)/x**2/(x**2+1)**3,x)
[Out] (-57*x**4 - 103*x**2 - 32)/(8*x**5 + 16*x**3 + 8*x) - 57*atan(x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{57\,x^4+103\,x^2+32}{8\,(x^5+2\,x^3+x)} - \frac{57}{8}\,\arctan\,(x)$$

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="maxima")
[Out] -1/8*(57*x^4 + 103*x^2 + 32)/(x^5 + 2*x^3 + x) - 57/8*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{25\,x^3+39\,x}{8\,(x^2+1)^2} - \frac{4}{x} - \frac{57}{8}\,\arctan\,(x)$$

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="giac")
[Out] -1/8*(25*x^3 + 39*x)/(x^2 + 1)^2 - 4/x - 57/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{57\,\mathrm{atan}(x)}{8} - \frac{\frac{57\,x^4}{8}+\frac{103\,x^2}{8}+4}{x\,(x^2+1)^2}$$

```
[In] int((3*x^4 + 4)/(x^2*(x^2 + 1)^3),x)
[Out] - (57*atan(x))/8 - ((103*x^2)/8 + (57*x^4)/8 + 4)/(x*(x^2 + 1)^2)
```

3.180 $\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 38

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx=-\frac{3}{2(1-x)^2}+\frac{2}{1-x}+\frac{1}{1+x}+\log(1-x)-2\log(1+x)$$

```
[Out] -3/2/(1-x)^2+2/(1-x)+1/(1+x)+ln(1-x)-2*ln(1+x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2099}

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx=\frac{2}{1-x}+\frac{1}{x+1}-\frac{3}{2(1-x)^2}+\log(1-x)-2\log(x+1)$$

```
[In] Int[(5 - 3*x + 6*x^2 + 5*x^3 - x^4)/(-1 + x + 2*x^2 - 2*x^3 - x^4 + x^5),x]
[Out] -3/(2*(1 - x)^2) + 2/(1 - x) + (1 + x)^(-1) + Log[1 - x] - 2*Log[1 + x]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{3}{(-1+x)^3}+\frac{2}{(-1+x)^2}+\frac{1}{-1+x}-\frac{1}{(1+x)^2}-\frac{2}{1+x}\right)dx\\ &= -\frac{3}{2(1-x)^2}+\frac{2}{1-x}+\frac{1}{1+x}+\log(1-x)-2\log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx = -\frac{3}{2(-1+x)^2}-\frac{2}{-1+x}+\frac{1}{1+x} + \log(-1+x)-2\log(1+x)$$

```
[In] Integrate[(5 - 3*x + 6*x^2 + 5*x^3 - x^4)/(-1 + x + 2*x^2 - 2*x^3 - x^4 + x
^5),x]
```

```
[Out] -3/(2*(-1 + x)^2) - 2/(-1 + x) + (1 + x)^(-1) + Log[-1 + x] - 2*Log[1 + x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result
default	$\ln(-1+x)-\frac{3}{2(-1+x)^2}-\frac{2}{-1+x}+\frac{1}{1+x}-2\ln(1+x)$
norman	$\frac{-x^2-\frac{7}{2}x+\frac{3}{2}}{(-1+x)^2(1+x)}-2\ln(1+x)+\ln(-1+x)$
risch	$\frac{-x^2-\frac{7}{2}x+\frac{3}{2}}{x^3-x^2-x+1}-2\ln(1+x)+\ln(-1+x)$
parallelrisch	$\frac{2\ln(-1+x)x^3-4\ln(1+x)x^3+3-2\ln(-1+x)x^2+4\ln(1+x)x^2-2\ln(-1+x)x+4\ln(1+x)x-2x^2+2\ln(-1+x)-4\ln(1+x)-7x}{2x^3-2x^2-2x+2}$

```
[In] int((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x,method=_RETURNVERB
OSE)
```

```
[Out] ln(-1+x)-3/2/(-1+x)^2-2/(-1+x)+1/(1+x)-2*ln(1+x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(32) = 64.

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx = -\frac{2x^2+4(x^3-x^2-x+1)\log(x+1)-2(x^3-x^2-x+1)\log(x-1)+7x-3}{2(x^3-x^2-x+1)}$$

```
[In] integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm="
fricas")
```

```
[Out] -1/2*(2*x^2 + 4*(x^3 - x^2 - x + 1)*log(x + 1) - 2*(x^3 - x^2 - x + 1)*log(
x - 1) + 7*x - 3)/(x^3 - x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx = -\frac{2x^2+7x-3}{2x^3-2x^2-2x+2}+\log(x-1)-2\log(x+1)$$

```
[In] integrate((-x**4+5*x**3+6*x**2-3*x+5)/(x**5-x**4-2*x**3+2*x**2+x-1),x)
```

```
[Out] -(2*x**2 + 7*x - 3)/(2*x**3 - 2*x**2 - 2*x + 2) + log(x - 1) - 2*log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx = -\frac{2\,x^2+7\,x-3}{2\,(x^3-x^2-x+1)}-2\,\log(x+1)+\log(x-1)$$

```
[In] integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm="
maxima")
```

```
[Out] -1/2*(2*x^2 + 7*x - 3)/(x^3 - x^2 - x + 1) - 2*log(x + 1) + log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx = -\frac{2\,x^2+7\,x-3}{2\,(x+1)(x-1)^2}-2\,\log(|x+1|)+\log(|x-1|)$$

```
[In] integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm="
giac")
```

```
[Out] -1/2*(2*x^2 + 7*x - 3)/((x + 1)*(x - 1)^2) - 2*log(abs(x + 1)) + log(abs(x
- 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx = \ln (x-1)-2\,\ln (x+1)+\frac{x^2+\frac{7\,x}{2}-\frac{3}{2}}{-x^3+x^2+x-1}$$

```
[In] int((6*x^2 - 3*x + 5*x^3 - x^4 + 5)/(x + 2*x^2 - 2*x^3 - x^4 + x^5 - 1),x)
[Out] log(x - 1) - 2*log(x + 1) + ((7*x)/2 + x^2 - 3/2)/(x + x^2 - x^3 - 1)
```

3.181 $\int \frac{1+x^2}{x(1+x^3)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 64

$$\int \frac{1+x^2}{x(1+x^3)^2} dx = \frac{x(x-x^2)}{3(1+x^3)} - \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{4}{9}\log(1+x) - \frac{5}{18}\log\left(1-x+x^2\right)$$

```
[Out] 1/3*x*(-x^2+x)/(x^3+1)+ln(x)-4/9*ln(1+x)-5/18*ln(x^2-x+1)-1/9*arctan(1/3*(1
-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1843, 1848, 648, 632, 210, 642}

$$\int \frac{1+x^2}{x(1+x^3)^2} dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{5}{18}\log\left(x^2-x+1\right) + \frac{x(x-x^2)}{3(x^3+1)} + \log(x) - \frac{4}{9}\log(x+1)$$

```
[In] Int[(1 + x^2)/(x*(1 + x^3)^2),x]
```

```
[Out] (x*(x - x^2))/(3*(1 + x^3)) - ArcTan[(1 - 2*x)/Sqrt[3]]/(3*Sqrt[3]) + Log[x
] - (4*Log[1 + x])/9 - (5*Log[1 - x + x^2])/18
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(x-x^2)}{3\,(1+x^3)} - \frac{1}{3}\int \frac{-3-x^2}{x\,(1+x^3)}\,dx \\
&= \frac{x(x-x^2)}{3\,(1+x^3)} - \frac{1}{3}\int \left(-\frac{3}{x} + \frac{4}{3(1+x)} + \frac{-4+5x}{3\,(1-x+x^2)}\right)dx \\
&= \frac{x(x-x^2)}{3\,(1+x^3)} + \log(x) - \frac{4}{9}\log(1+x) - \frac{1}{9}\int \frac{-4+5x}{1-x+x^2}\,dx
\end{aligned}$$

$$= \frac{x(x-x^2)}{3\,(1+x^3)} + \log(x) - \frac{4}{9}\log(1+x) + \frac{1}{6}\int \frac{1}{1-x+x^2}\,dx - \frac{5}{18}\int \frac{-1+2x}{1-x+x^2}\,dx$$

$$= \frac{x(x-x^2)}{3\,(1+x^3)} + \log(x) - \frac{4}{9}\log(1+x) - \frac{5}{18}\log\left(1-x+x^2\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right)$$

$$= \frac{x(x-x^2)}{3\,(1+x^3)} - \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{4}{9}\log(1+x) - \frac{5}{18}\log\left(1-x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{1+x^2}{x\,(1+x^3)^2}\,dx = \frac{1}{18}\left(\frac{6(1+x^2)}{1+x^3} + 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 18\log(x) - 2\log(1+x) + \log\left(1-x+x^2\right) - 6\log\left(1+x^3\right)\right)$$

```
[In] Integrate[(1 + x^2)/(x*(1 + x^3)^2),x]
[Out] ((6*(1 + x^2))/(1 + x^3) + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 18*Log[x]
 - 2*Log[1 + x] + Log[1 - x + x^2] - 6*Log[1 + x^3])/18
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

method	result
risch	$\frac{\frac{x^2}{3}+\frac{1}{3}}{x^3+1} + \ln(x) - \frac{4\ln(1+x)}{9} - \frac{5\ln(x^2-x+1)}{18} + \frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{9}$
default	$-\frac{-1-x}{9(x^2-x+1)} - \frac{5\ln(x^2-x+1)}{18} + \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9} + \ln(x) + \frac{2}{9(1+x)} - \frac{4\ln(1+x)}{9}$
meijerg	$\frac{x^2}{3x^3+3} - \frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{9(x^3)^{\frac{2}{3}}} + \frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{18(x^3)^{\frac{2}{3}}} + \frac{x^2\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{9(x^3)^{\frac{2}{3}}} + \frac{1}{3} + \ln(x) - \frac{2x^3}{3(2x^3+2)} - \frac{\ln(x^3}{3}$

```
[In] int((x^2+1)/x/(x^3+1)^2,x,method=_RETURNVERBOSE)
[Out] (1/3*x^2+1/3)/(x^3+1)+ln(x)-4/9*ln(1+x)-5/18*ln(x^2-x+1)+1/9*3^(1/2)*arctan
(2/3*(x-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \frac{1+x^2}{x\left(1+x^3\right)^2}\,dx = \frac{2\sqrt{3}(x^3+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+6x^2-5(x^3+1)\log\left(x^2-x+1\right)-8(x^3+1)\log(x+1)+18(x^3}{18(x^3+1)}$$

```
[In] integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*(x^3 + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 6*x^2 - 5*(x^3 +
1)*log(x^2 - x + 1) - 8*(x^3 + 1)*log(x + 1) + 18*(x^3 + 1)*log(x) + 6)/(x^
3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{1+x^2}{x\left(1+x^3\right)^2}\,dx = \frac{x^2+1}{3x^3+3}+\log(x)-\frac{4\log(x+1)}{9}-\frac{5\log\left(x^2-x+1\right)}{18}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate((x**2+1)/x/(x**3+1)**2,x)
```

```
[Out] (x**2 + 1)/(3*x**3 + 3) + log(x) - 4*log(x + 1)/9 - 5*log(x**2 - x + 1)/18
+ sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int \frac{1+x^2}{x\left(1+x^3\right)^2}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{x^2+1}{3(x^3+1)}-\frac{5}{18}\log\left(x^2-x+1\right)-\frac{4}{9}\log(x+1)+\log(x)$$

```
[In] integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/3*(x^2 + 1)/(x^3 + 1) - 5/18*
log(x^2 - x + 1) - 4/9*log(x + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{1+x^2}{x\left(1+x^3\right)^2}\,dx = \frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{x^2+1}{3\,(x^2-x+1)(x+1)} - \frac{5}{18}\,\log\left(x^2-x+1\right) - \frac{4}{9}\,\log\left(|x+1|\right) + \log\left(|x|\right)$$

```
[In] integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/3*(x^2 + 1)/((x^2 - x + 1)*(x
 + 1)) - 5/18*log(x^2 - x + 1) - 4/9*log(abs(x + 1)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{1+x^2}{x\left(1+x^3\right)^2}\,dx = \ln(x) - \frac{4\,\ln(x+1)}{9} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{5}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{5}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right) + \frac{\frac{x^2}{3}+\frac{1}{3}}{x^3+1}$$

```
[In] int((x^2 + 1)/(x*(x^3 + 1)^2),x)
```

```
[Out] log(x) - (4*log(x + 1))/9 - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/18
+ 5/18) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/18 - 5/18) + (x^2/3 +
 1/3)/(x^3 + 1)
```

3.182 $\int \frac{-2-3x+x^2}{(1+x)^2(1+x+x^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{-2-3x+x^2}{(1+x)^2(1+x+x^2)^2} dx = -\frac{2}{1+x} - \frac{7+5x}{3(1+x+x^2)} - \frac{25\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \log(1+x) + \frac{1}{2}\log\left(1+x+x^2\right)$$

```
[Out] -2/(1+x)+1/3*(-7-5*x)/(x^2+x+1)-ln(1+x)+1/2*ln(x^2+x+1)-25/9*arctan(1/3*(1+
2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1660, 1642, 648, 632, 210, 642}

$$\int \frac{-2-3x+x^2}{(1+x)^2(1+x+x^2)^2} dx = -\frac{25\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{5x+7}{3(x^2+x+1)} + \frac{1}{2}\log\left(x^2+x+1\right) - \frac{2}{x+1} - \log(x+1)$$

```
[In] Int[(-2 - 3*x + x^2)/((1 + x)^2*(1 + x + x^2)^2),x]
```

```
[Out] -2/(1 + x) - (7 + 5*x)/(3*(1 + x + x^2)) - (25*ArcTan[(1 + 2*x)/Sqrt[3]])/(
3*Sqrt[3]) - Log[1 + x] + Log[1 + x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{7+5x}{3(1+x+x^2)} + \frac{1}{3}\int \frac{-8-19x-5x^2}{(1+x)^2(1+x+x^2)}\,dx$$

$$= -\frac{7+5x}{3(1+x+x^2)} + \frac{1}{3}\int \left(\frac{6}{(1+x)^2} - \frac{3}{1+x} + \frac{-11+3x}{1+x+x^2}\right) dx$$

$$= -\frac{2}{1+x} - \frac{7+5x}{3(1+x+x^2)} - \log(1+x) + \frac{1}{3}\int \frac{-11+3x}{1+x+x^2} dx$$

$$= -\frac{2}{1+x} - \frac{7+5x}{3(1+x+x^2)} - \log(1+x) + \frac{1}{2}\int \frac{1+2x}{1+x+x^2} dx - \frac{25}{6}\int \frac{1}{1+x+x^2} dx$$

$$= -\frac{2}{1+x} - \frac{7+5x}{3(1+x+x^2)} - \log(1+x) + \frac{1}{2}\log\left(1+x+x^2\right) + \frac{25}{3}\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2x\right)$$

$$= -\frac{2}{1+x} - \frac{7+5x}{3(1+x+x^2)} - \frac{25\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \log(1+x) + \frac{1}{2}\log\left(1+x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2} dx = -\frac{2}{1+x} - \frac{7+5x}{3(1+x+x^2)} - \frac{25\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \log(1+x) + \frac{1}{2}\log\left(1+x+x^2\right)$$

```
[In] Integrate[(-2 - 3*x + x^2)/((1 + x)^2*(1 + x + x^2)^2),x]
[Out] -2/(1 + x) - (7 + 5*x)/(3*(1 + x + x^2)) - (25*ArcTan[(1 + 2*x)/Sqrt[3]])/(
3*Sqrt[3]) - Log[1 + x] + Log[1 + x + x^2]/2
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{-\frac{5x}{3}-\frac{7}{3}}{x^2+x+1} + \frac{\ln(x^2+x+1)}{2} - \frac{25\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9} - \frac{2}{1+x} - \ln(1+x)$	54
risch	$\frac{-\frac{11}{3}x^2-6x-\frac{13}{3}}{(x^2+x+1)(1+x)} + \frac{\ln(4x^2+4x+4)}{2} - \frac{25\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9} - \ln(1+x)$	61

```
[In] int((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x,method=_RETURNVERBOSE)
[Out] (-5/3*x-7/3)/(x^2+x+1)+1/2*ln(x^2+x+1)-25/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(
1/2)-2/(1+x)-ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.54

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx = -\frac{50\sqrt{3}(x^3+2\,x^2+2\,x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+66\,x^2-9\,(x^3+2\,x^2+2\,x+1)\log\left(x^2+x+1\right)+1}{18\,(x^3+2\,x^2+2\,x+1)}$$

```
[In] integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(50*sqrt(3)*(x^3 + 2*x^2 + 2*x + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) + 6
6*x^2 - 9*(x^3 + 2*x^2 + 2*x + 1)*log(x^2 + x + 1) + 18*(x^3 + 2*x^2 + 2*x
+ 1)*log(x + 1) + 108*x + 78)/(x^3 + 2*x^2 + 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx = \frac{-11x^2-18x-13}{3x^3+6x^2+6x+3}-\log(x+1)+\frac{\log(x^2+x+1)}{2}-\frac{25\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate((x**2-3*x-2)/(1+x)**2/(x**2+x+1)**2,x)
```

```
[Out] (-11*x**2 - 18*x - 13)/(3*x**3 + 6*x**2 + 6*x + 3) - log(x + 1) + log(x**2
+ x + 1)/2 - 25*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx = -\frac{25}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)-\frac{11\,x^2+18\,x+13}{3\,(x^3+2\,x^2+2\,x+1)}+\frac{1}{2}\log\left(x^2+x+1\right)-\log(x+1)$$

```
[In] integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="maxima")
```

```
[Out] -25/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*(11*x^2 + 18*x + 13)/(x^3
 + 2*x^2 + 2*x + 1) + 1/2*log(x^2 + x + 1) - log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx = -\frac{25}{9}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(\frac{2}{x+1}-1\right)\right) + \frac{\frac{7}{x+1}-2}{3\left(\frac{1}{x+1}-\frac{1}{(x+1)^2}-1\right)} - \frac{2}{x+1} + \frac{1}{2}\log\left(-\frac{1}{x+1}+\frac{1}{(x+1)^2}+1\right)$$

```
[In] integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="giac")
```

```
[Out] -25/9*sqrt(3)*arctan(-1/3*sqrt(3)*(2/(x + 1) - 1)) + 1/3*(7/(x + 1) - 2)/(1
/(x + 1) - 1/(x + 1)^2 - 1) - 2/(x + 1) + 1/2*log(-1/(x + 1) + 1/(x + 1)^2
+ 1)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx = -\ln(x+1) - \frac{\frac{11x^2}{3}+6x+\frac{13}{3}}{x^3+2x^2+2x+1} + \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,25\mathrm{i}}{18}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,25\mathrm{i}}{18}\right)$$

```
[In] int(-(3*x - x^2 + 2)/((x + 1)^2*(x + x^2 + 1)^2),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*25i)/18 + 1/2) - (6*x + (11*x^2)/3
+ 13/3)/(2*x + 2*x^2 + x^3 + 1) - log(x + 1) - log(x + (3^(1/2)*1i)/2 + 1/2
)*((3^(1/2)*25i)/18 - 1/2)
```

3.183 $\int \frac{1}{(1-4x)^3(2-3x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{1}{10(1-4x)^2} - \frac{3}{25(1-4x)} - \frac{9}{125}\log(1-4x) + \frac{9}{125}\log(2-3x)$$

```
[Out] 1/10/(-4*x+1)^2-3/25/(-4*x+1)-9/125*ln(-4*x+1)+9/125*ln(2-3*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {46}

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = -\frac{3}{25(1-4x)} + \frac{1}{10(1-4x)^2} - \frac{9}{125}\log(1-4x) + \frac{9}{125}\log(2-3x)$$

```
[In] Int[1/((1 - 4*x)^3*(2 - 3*x)),x]
```

```
[Out] 1/(10*(1 - 4*x)^2) - 3/(25*(1 - 4*x)) - (9*Log[1 - 4*x])/125 + (9*Log[2 - 3
*x])/125
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{27}{125(-2+3x)} - \frac{4}{5(-1+4x)^3} - \frac{12}{25(-1+4x)^2} - \frac{36}{125(-1+4x)}\right) dx \\ &= \frac{1}{10(1-4x)^2} - \frac{3}{25(1-4x)} - \frac{9}{125}\log(1-4x) + \frac{9}{125}\log(2-3x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{-5+120x+18(1-4x)^2\log(8-12x)-18(1-4x)^2\log(-1+4x)}{250(1-4x)^2}$$

```
[In] Integrate[1/((1 - 4*x)^3*(2 - 3*x)),x]
[Out] (-5 + 120*x + 18*(1 - 4*x)^2*Log[8 - 12*x] - 18*(1 - 4*x)^2*Log[-1 + 4*x])/
(250*(1 - 4*x)^2)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{\frac{12x}{25}-\frac{1}{50}}{(-1+4x)^2}+\frac{9\ln(-2+3x)}{125}-\frac{9\ln(-1+4x)}{125}$	32
norman	$\frac{\frac{8}{25}x+\frac{8}{25}x^2}{(-1+4x)^2}+\frac{9\ln(-2+3x)}{125}-\frac{9\ln(-1+4x)}{125}$	35
default	$\frac{1}{10(-1+4x)^2}+\frac{3}{25(-1+4x)}-\frac{9\ln(-1+4x)}{125}+\frac{9\ln(-2+3x)}{125}$	36
parallelrisch	$\frac{144\ln(x-\frac{2}{3})x^2-144\ln(x-\frac{1}{4})x^2-72\ln(x-\frac{2}{3})x+72\ln(x-\frac{1}{4})x+40x^2+9\ln(x-\frac{2}{3})-9\ln(x-\frac{1}{4})+40x}{125(-1+4x)^2}$	63

```
[In] int(1/(-4*x+1)^3/(2-3*x),x,method=_RETURNVERBOSE)
[Out] 16*(3/100*x-1/800)/(-1+4*x)^2+9/125*ln(-2+3*x)-9/125*ln(-1+4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = -\frac{18\,(16\,x^2-8\,x+1)\log(4\,x-1)-18\,(16\,x^2-8\,x+1)\log(3\,x-2)-120\,x+5}{250\,(16\,x^2-8\,x+1)}$$

```
[In] integrate(1/(-4*x+1)^3/(2-3*x),x, algorithm="fricas")
[Out] -1/250*(18*(16*x^2 - 8*x + 1)*log(4*x - 1) - 18*(16*x^2 - 8*x + 1)*log(3*x
- 2) - 120*x + 5)/(16*x^2 - 8*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{24x-1}{800x^2-400x+50} + \frac{9\log\left(x-\frac{2}{3}\right)}{125} - \frac{9\log\left(x-\frac{1}{4}\right)}{125}$$

```
[In] integrate(1/(-4*x+1)**3/(2-3*x),x)
```

```
[Out] (24*x - 1)/(800*x**2 - 400*x + 50) + 9*log(x - 2/3)/125 - 9*log(x - 1/4)/12
5
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{24\,x-1}{50\,(16\,x^2-8\,x+1)} - \frac{9}{125}\,\log\left(4\,x-1\right) + \frac{9}{125}\,\log\left(3\,x-2\right)$$

```
[In] integrate(1/(-4*x+1)^3/(2-3*x),x, algorithm="maxima")
```

```
[Out] 1/50*(24*x - 1)/(16*x^2 - 8*x + 1) - 9/125*log(4*x - 1) + 9/125*log(3*x - 2
)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{24\,x-1}{50\,(4\,x-1)^2} - \frac{9}{125}\,\log\left(|4\,x-1|\right) + \frac{9}{125}\,\log\left(|3\,x-2|\right)$$

```
[In] integrate(1/(-4*x+1)^3/(2-3*x),x, algorithm="giac")
```

```
[Out] 1/50*(24*x - 1)/(4*x - 1)^2 - 9/125*log(abs(4*x - 1)) + 9/125*log(abs(3*x -
 2))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.58

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{\frac{3x}{100}-\frac{1}{800}}{x^2-\frac{x}{2}+\frac{1}{16}} - \frac{18\,\mathrm{atanh}\left(\frac{24x}{5}-\frac{11}{5}\right)}{125}$$

```
[In] int(1/((3*x - 2)*(4*x - 1)^3),x)
[Out] ((3*x)/100 - 1/800)/(x^2 - x/2 + 1/16) - (18*atanh((24*x)/5 - 11/5))/125
```

3.184 $\int \frac{x^3}{(2-5x^2)^7} dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{x^3}{(2-5x^2)^7} dx = \frac{1}{150(2-5x^2)^6} - \frac{1}{250(2-5x^2)^5}$$

```
[Out] 1/150/(-5*x^2+2)^6-1/250/(-5*x^2+2)^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\int \frac{x^3}{(2-5x^2)^7} dx = \frac{1}{150(2-5x^2)^6} - \frac{1}{250(2-5x^2)^5}$$

```
[In] Int[x^3/(2 - 5*x^2)^7,x]
```

```
[Out] 1/(150*(2 - 5*x^2)^6) - 1/(250*(2 - 5*x^2)^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(2-5x)^7}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{2}{5(-2+5x)^7}-\frac{1}{5(-2+5x)^6}\right)\,dx, x, x^2\right) \\
&= \frac{1}{150\,(2-5x^2)^6}-\frac{1}{250\,(2-5x^2)^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{(2-5x^2)^7}\,dx = \frac{-1+15x^2}{750\,(2-5x^2)^6}$$

```
[In] Integrate[x^3/(2 - 5*x^2)^7,x]
[Out] (-1 + 15*x^2)/(750*(2 - 5*x^2)^6)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
norman	$\frac{\frac{x^2}{50}-\frac{1}{750}}{(5x^2-2)^6}$	18
gosper	$\frac{15x^2-1}{750(5x^2-2)^6}$	19
risch	$\frac{\frac{x^2}{50}-\frac{1}{750}}{(5x^2-2)^6}$	19
default	$\frac{1}{150(5x^2-2)^6}+\frac{1}{250(5x^2-2)^5}$	24
meijerg	$\frac{x^4\left(\frac{625}{16}x^8-\frac{375}{4}x^6+\frac{375}{4}x^4-50x^2+15\right)}{7680\left(1-\frac{5x^2}{2}\right)^6}$	37
parallelrisch	$\frac{125x^{12}-300x^{10}+300x^8-160x^6+48x^4}{384(5x^2-2)^6}$	38

```
[In] int(x^3/(-5*x^2+2)^7,x,method=_RETURNVERBOSE)
[Out] (1/50*x^2-1/750)/(5*x^2-2)^6
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{x^3}{(2-5x^2)^7}\,dx = \frac{15\,x^2-1}{750\,(15625\,x^{12}-37500\,x^{10}+37500\,x^8-20000\,x^6+6000\,x^4-960\,x^2+64)}$$

```
[In] integrate(x^3/(-5*x^2+2)^7,x, algorithm="fricas")
[Out] 1/750*(15*x^2 - 1)/(15625*x^12 - 37500*x^10 + 37500*x^8 - 20000*x^6 + 6000*
x^4 - 960*x^2 + 64)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{x^3}{(2-5x^2)^7}\,dx = -\frac{1-15x^2}{11718750x^{12}-28125000x^{10}+28125000x^8-15000000x^6+4500000x^4-720000x^2+48000}$$

```
[In] integrate(x**3/(-5*x**2+2)**7,x)
[Out] -(1 - 15*x**2)/(11718750*x**12 - 28125000*x**10 + 28125000*x**8 - 15000000*
x**6 + 4500000*x**4 - 720000*x**2 + 48000)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{x^3}{(2-5x^2)^7}\,dx = \frac{15\,x^2-1}{750\,(15625\,x^{12}-37500\,x^{10}+37500\,x^8-20000\,x^6+6000\,x^4-960\,x^2+64)}$$

```
[In] integrate(x^3/(-5*x^2+2)^7,x, algorithm="maxima")
[Out] 1/750*(15*x^2 - 1)/(15625*x^12 - 37500*x^10 + 37500*x^8 - 20000*x^6 + 6000*
x^4 - 960*x^2 + 64)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{(2-5x^2)^7}\,dx = \frac{15\,x^2-1}{750\,(5\,x^2-2)^6}$$

```
[In] integrate(x^3/(-5*x^2+2)^7,x, algorithm="giac")
[Out] 1/750*(15*x^2 - 1)/(5*x^2 - 2)^6
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{(2-5x^2)^7}\,dx = \frac{15\,x^2-1}{750\,(5\,x^2-2)^6}$$

```
[In] int(-x^3/(5*x^2 - 2)^7,x)
[Out] (15*x^2 - 1)/(750*(5*x^2 - 2)^6)
```

3.185 $\int \frac{x^7}{(2-5x^2)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{x^7}{(2-5x^2)^3} \, dx = -\frac{x^2}{250} + \frac{2}{625(2-5x^2)^2} - \frac{6}{625(2-5x^2)} - \frac{3}{625} \log(2-5x^2)$$

```
[Out] -1/250*x^2+2/625/(-5*x^2+2)^2-6/625/(-5*x^2+2)-3/625*ln(-5*x^2+2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\int \frac{x^7}{(2-5x^2)^3} \, dx = -\frac{x^2}{250} - \frac{6}{625(2-5x^2)} + \frac{2}{625(2-5x^2)^2} - \frac{3}{625} \log(2-5x^2)$$

```
[In] Int[x^7/(2 - 5*x^2)^3,x]
```

```
[Out] -1/250*x^2 + 2/(625*(2 - 5*x^2)^2) - 6/(625*(2 - 5*x^2)) - (3*Log[2 - 5*x^2
])/625
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(2-5x)^3}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{125}-\frac{8}{125(-2+5x)^3}-\frac{12}{125(-2+5x)^2}-\frac{6}{125(-2+5x)}\right)dx, x, x^2\right)\\ &= -\frac{x^2}{250}+\frac{2}{625(2-5x^2)^2}-\frac{6}{625(2-5x^2)}-\frac{3}{625}\log\left(2-5x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(2-5x^2)^3}\,dx = -\frac{12-150x^4+125x^6+6(2-5x^2)^2\log(-2+5x^2)}{1250\,(2-5x^2)^2}$$

```
[In] Integrate[x^7/(2 - 5*x^2)^3,x]
[Out] -1/1250*(12 - 150*x^4 + 125*x^6 + 6*(2 - 5*x^2)^2*Log[-2 + 5*x^2])/(2 - 5*x
^2)^2
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

method	result	size
norman	$\frac{\frac{12}{125}x^2-\frac{1}{10}x^6-\frac{18}{625}}{(5x^2-2)^2}-\frac{3\ln(5x^2-2)}{625}$	34
risch	$-\frac{x^2}{250}+\frac{\frac{6x^2}{125}-\frac{2}{125}}{(5x^2-2)^2}-\frac{3\ln(5x^2-2)}{625}$	35
meijerg	$-\frac{x^2(25x^4-45x^2+12)}{1000\left(1-\frac{5x^2}{2}\right)^2}-\frac{3\ln\left(1-\frac{5x^2}{2}\right)}{625}$	38
default	$-\frac{x^2}{250}+\frac{6}{625(5x^2-2)}-\frac{3\ln(5x^2-2)}{625}+\frac{2}{625(5x^2-2)^2}$	39
parallelrisch	$-\frac{250x^6+300\ln(x^2-\frac{2}{5})x^4-450x^4-240\ln(x^2-\frac{2}{5})x^2+120x^2+48\ln(x^2-\frac{2}{5})}{2500(5x^2-2)^2}$	58

```
[In] int(x^7/(-5*x^2+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (12/125*x^2-1/10*x^6-18/625)/(5*x^2-2)^2-3/625*ln(5*x^2-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int \frac{x^7}{(2-5x^2)^3}\,dx = -\frac{125\,x^6-100\,x^4-40\,x^2+6\,(25\,x^4-20\,x^2+4)\log\left(5\,x^2-2\right)+20}{1250\,(25\,x^4-20\,x^2+4)}$$

```
[In] integrate(x^7/(-5*x^2+2)^3,x, algorithm="fricas")
[Out] -1/1250*(125*x^6 - 100*x^4 - 40*x^2 + 6*(25*x^4 - 20*x^2 + 4)*log(5*x^2 - 2
) + 20)/(25*x^4 - 20*x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(2-5x^2)^3}\,dx = -\frac{x^2}{250} - \frac{2-6x^2}{3125x^4-2500x^2+500} - \frac{3\log\left(5x^2-2\right)}{625}$$

```
[In] integrate(x**7/(-5*x**2+2)**3,x)
[Out] -x**2/250 - (2 - 6*x**2)/(3125*x**4 - 2500*x**2 + 500) - 3*log(5*x**2 - 2)/
625
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{x^7}{(2-5x^2)^3}\,dx = -\frac{1}{250}\,x^2 + \frac{2\,(3\,x^2-1)}{125\,(25\,x^4-20\,x^2+4)} - \frac{3}{625}\,\log\left(5\,x^2-2\right)$$

```
[In] integrate(x^7/(-5*x^2+2)^3,x, algorithm="maxima")
[Out] -1/250*x^2 + 2/125*(3*x^2 - 1)/(25*x^4 - 20*x^2 + 4) - 3/625*log(5*x^2 - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{x^7}{(2-5x^2)^3}\,dx = -\frac{1}{250}\,x^2 + \frac{225\,x^4 - 120\,x^2 + 16}{1250\,(5\,x^2-2)^2} - \frac{3}{625}\,\log\left(\left|5\,x^2-2\right|\right)$$

```
[In] integrate(x^7/(-5*x^2+2)^3,x, algorithm="giac")
[Out] -1/250*x^2 + 1/1250*(225*x^4 - 120*x^2 + 16)/(5*x^2 - 2)^2 - 3/625*log(abs(
5*x^2 - 2))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x^7}{(2-5x^2)^3}\,dx = \frac{\frac{6\,x^2}{3125}-\frac{2}{3125}}{x^4-\frac{4\,x^2}{5}+\frac{4}{25}} - \frac{3\,\ln\left(x^2-\frac{2}{5}\right)}{625} - \frac{x^2}{250}$$

```
[In] int(-x^7/(5*x^2 - 2)^3,x)
[Out] ((6*x^2)/3125 - 2/3125)/(x^4 - (4*x^2)/5 + 4/25) - (3*log(x^2 - 2/5))/625 -
 x^2/250
```

3.186 $\int \frac{1}{(-2+x)^3(1+x)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 44

$$\int \frac{1}{(-2+x)^3(1+x)^2} dx = -\frac{1}{18(-2+x)^2} + \frac{2}{27(-2+x)} + \frac{1}{27(1+x)} + \frac{1}{27}\log(-2+x) - \frac{1}{27}\log(1+x)$$

```
[Out] -1/18/(-2+x)^2+2/27/(-2+x)+1/27/(1+x)+1/27*ln(-2+x)-1/27*ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\int \frac{1}{(-2+x)^3(1+x)^2} dx = -\frac{2}{27(2-x)} + \frac{1}{27(x+1)} - \frac{1}{18(2-x)^2} + \frac{1}{27}\log(2-x) - \frac{1}{27}\log(x+1)$$

```
[In] Int[1/((-2 + x)^3*(1 + x)^2),x]
```

```
[Out] -1/18*1/(2 - x)^2 - 2/(27*(2 - x)) + 1/(27*(1 + x)) + Log[2 - x]/27 - Log[1
 + x]/27
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{9(-2+x)^3} - \frac{2}{27(-2+x)^2} + \frac{1}{27(-2+x)} - \frac{1}{27(1+x)^2} - \frac{1}{27(1+x)}\right) dx \\
&= -\frac{1}{18(2-x)^2} - \frac{2}{27(2-x)} + \frac{1}{27(1+x)} + \frac{1}{27}\log(2-x) - \frac{1}{27}\log(1+x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$
\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = \frac{1}{54}\left(\frac{3(-1-5x+2x^2)}{(-2+x)^2(1+x)} + 2\log(-2+x) - 2\log(1+x)\right)
$$

```
[In] Integrate[1/((-2 + x)^3*(1 + x)^2),x]
[Out] ((3*(-1 - 5*x + 2*x^2))/((-2 + x)^2*(1 + x)) + 2*Log[-2 + x] - 2*Log[1 + x]
)/54
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{1}{18(-2+x)^2}+\frac{2}{27(-2+x)}+\frac{1}{27+27x}+\frac{\ln(-2+x)}{27}-\frac{\ln(1+x)}{27}$	35
norman	$\frac{\frac{1}{9}x^2-\frac{5}{18}x-\frac{1}{18}}{(-2+x)^2(1+x)}+\frac{\ln(-2+x)}{27}-\frac{\ln(1+x)}{27}$	35
risch	$\frac{\frac{1}{9}x^2-\frac{5}{18}x-\frac{1}{18}}{(-2+x)^2(1+x)}+\frac{\ln(-2+x)}{27}-\frac{\ln(1+x)}{27}$	35
parallelrisch	$\frac{2\ln(-2+x)x^3-2\ln(1+x)x^3-3-6\ln(-2+x)x^2+6\ln(1+x)x^2+6x^2+8\ln(-2+x)-8\ln(1+x)-15x}{54(-2+x)^2(1+x)}$	71

```
[In] int(1/(-2+x)^3/(1+x)^2,x,method=_RETURNVERBOSE)
[Out] -1/18/(-2+x)^2+2/27/(-2+x)+1/27/(1+x)+1/27*ln(-2+x)-1/27*ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = \frac{6\,x^2-2\,(x^3-3\,x^2+4)\log(x+1)+2\,(x^3-3\,x^2+4)\log(x-2)-15\,x-3}{54\,(x^3-3\,x^2+4)}$$

[In] integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="fricas")

[Out] 1/54*(6*x^2 - 2*(x^3 - 3*x^2 + 4)*log(x + 1) + 2*(x^3 - 3*x^2 + 4)*log(x - 2) - 15*x - 3)/(x^3 - 3*x^2 + 4)

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = \frac{2x^2-5x-1}{18x^3-54x^2+72}+\frac{\log(x-2)}{27}-\frac{\log(x+1)}{27}$$

[In] integrate(1/(-2+x)**3/(1+x)**2,x)

[Out] (2*x**2 - 5*x - 1)/(18*x**3 - 54*x**2 + 72) + log(x - 2)/27 - log(x + 1)/27

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = \frac{2\,x^2-5\,x-1}{18\,(x^3-3\,x^2+4)}-\frac{1}{27}\,\log(x+1)+\frac{1}{27}\,\log(x-2)$$

[In] integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="maxima")

[Out] 1/18*(2*x^2 - 5*x - 1)/(x^3 - 3*x^2 + 4) - 1/27*log(x + 1) + 1/27*log(x - 2
)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = \frac{1}{27\,(x+1)} - \frac{\frac{18}{x+1}-5}{162\,\left(\frac{3}{x+1}-1\right)^2} + \frac{1}{27}\log\left(\left|-\frac{3}{x+1}+1\right|\right)$$

```
[In] integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="giac")
[Out] 1/27/(x + 1) - 1/162*(18/(x + 1) - 5)/(3/(x + 1) - 1)^2 + 1/27*log(abs(-3/(
x + 1) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{2\,x}{3}-\frac{1}{3}\right)}{27} - \frac{-\frac{x^2}{9}+\frac{5\,x}{18}+\frac{1}{18}}{x^3-3\,x^2+4}$$

```
[In] int(1/((x + 1)^2*(x - 2)^3),x)
[Out] - (2*atanh((2*x)/3 - 1/3))/27 - ((5*x)/18 - x^2/9 + 1/18)/(x^3 - 3*x^2 + 4)
```

3.187 $\int \frac{1}{(2+x)^3(3+x)^4} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \frac{1}{(2+x)^3(3+x)^4} \, dx = -\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10\log(2+x) - 10\log(3+x)$$

```
[Out] -1/2/(2+x)^2+4/(2+x)+1/3/(3+x)^3+3/2/(3+x)^2+6/(3+x)+10*ln(2+x)-10*ln(3+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$\int \frac{1}{(2+x)^3(3+x)^4} \, dx = \frac{4}{x+2} + \frac{6}{x+3} - \frac{1}{2(x+2)^2} + \frac{3}{2(x+3)^2} + \frac{1}{3(x+3)^3} + 10\log(x+2) - 10\log(x+3)$$

```
[In] Int[1/((2 + x)^3*(3 + x)^4),x]
```

```
[Out] -1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x
) + 10*Log[2 + x] - 10*Log[3 + x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{(2+x)^3} - \frac{4}{(2+x)^2} + \frac{10}{2+x} - \frac{1}{(3+x)^4} - \frac{3}{(3+x)^3} - \frac{6}{(3+x)^2} - \frac{10}{3+x} \right) dx$$
$$= -\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10\log(2+x) - 10\log(3+x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+x)^3(3+x)^4} \, dx = -\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10\log(2+x) - 10\log(3+x)$$

```
[In] Integrate[1/((2 + x)^3*(3 + x)^4),x]
[Out] -1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x
) + 10*Log[2 + x] - 10*Log[3 + x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result
norman	$\frac{10x^4+105x^3+\frac{1225}{3}x^2+\frac{4175}{6}x+\frac{2627}{6}}{(2+x)^2(3+x)^3} + 10\ln(2+x) - 10\ln(3+x)$
risch	$\frac{10x^4+105x^3+\frac{1225}{3}x^2+\frac{4175}{6}x+\frac{2627}{6}}{(2+x)^2(3+x)^3} + 10\ln(2+x) - 10\ln(3+x)$
default	$-\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10\ln(2+x) - 10\ln(3+x)$
parallelrisch	$\frac{60\ln(2+x)x^5-60\ln(3+x)x^5+2627+780\ln(2+x)x^4-780\ln(3+x)x^4+4020\ln(2+x)x^3-4020\ln(3+x)x^3+60x^4+10260\ln(2+x)}{6(2+x)^2(3+x)}$

```
[In] int(1/(2+x)^3/(3+x)^4,x,method=_RETURNVERBOSE)
[Out] (10*x^4+105*x^3+1225/3*x^2+4175/6*x+2627/6)/(2+x)^2/(3+x)^3+10*ln(2+x)-10*l
n(3+x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(48) = 96.

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.94

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60\,x^4+630\,x^3+2450\,x^2-60\,(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108)\log(x+3)+60\,(x^5+13\,x^4+6}{6\,(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108)}$$

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 - 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 21
6*x + 108)*log(x + 3) + 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108)*
log(x + 2) + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60x^4+630x^3+2450x^2+4175x+2627}{6x^5+78x^4+402x^3+1026x^2+1296x+648} + 10\log(x+2) - 10\log(x+3)$$

```
[In] integrate(1/(2+x)**3/(3+x)**4,x)
```

```
[Out] (60*x**4 + 630*x**3 + 2450*x**2 + 4175*x + 2627)/(6*x**5 + 78*x**4 + 402*x*
*3 + 1026*x**2 + 1296*x + 648) + 10*log(x + 2) - 10*log(x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60\,x^4+630\,x^3+2450\,x^2+4175\,x+2627}{6\,(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108)} - 10\,\log(x+3) + 10\,\log(x+2)$$

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 +
171*x^2 + 216*x + 108) - 10*log(x + 3) + 10*log(x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60\,x^4+630\,x^3+2450\,x^2+4175\,x+2627}{6\,(x+3)^3(x+2)^2} - 10\,\log\left(|x+3|\right)+10\,\log\left(|x+2|\right)$$

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="giac")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/((x + 3)^3*(x + 2)^2) - 1
0*log(abs(x + 3)) + 10*log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{10\,x^4+105\,x^3+\frac{1225\,x^2}{3}+\frac{4175\,x}{6}+\frac{2627}{6}}{x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108} - 20\,\mathrm{atanh}(2\,x+5)$$

```
[In] int(1/((x + 2)^3*(x + 3)^4),x)
```

```
[Out] ((4175*x)/6 + (1225*x^2)/3 + 105*x^3 + 10*x^4 + 2627/6)/(216*x + 171*x^2 +
67*x^3 + 13*x^4 + x^5 + 108) - 20*atanh(2*x + 5)
```

3.188 $\int \frac{x^5}{(3+x)^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 36

$$\int \frac{x^5}{(3+x)^2} dx = -108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405 \log(3+x)$$

```
[Out] -108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$\int \frac{x^5}{(3+x)^2} dx = \frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + \frac{243}{x+3} + 405 \log(x+3)$$

```
[In] Int[x^5/(3 + x)^2,x]
[Out] -108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3 + x) + 405*Log[3 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-108 + 27x - 6x^2 + x^3 - \frac{243}{(3+x)^2} + \frac{405}{3+x}\right) dx \\ &= -108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405 \log(3+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(3+x)^2}\,dx = \frac{1}{4}\left(-2079-432x+54x^2-8x^3+x^4+\frac{972}{3+x}\right)+405\log(3+x)$$

[In] Integrate[x^5/(3 + x)^2,x]

[Out] (-2079 - 432*x + 54*x^2 - 8*x^3 + x^4 + 972/(3 + x))/4 + 405*Log[3 + x]

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
default	$-108x+\frac{27x^2}{2}-2x^3+\frac{x^4}{4}+\frac{243}{3+x}+405\ln\left(3+x\right)$	33
risch	$-108x+\frac{27x^2}{2}-2x^3+\frac{x^4}{4}+\frac{243}{3+x}+405\ln\left(3+x\right)$	33
norman	$\frac{-\frac{135}{2}x^2+\frac{15}{2}x^3-\frac{5}{4}x^4+\frac{1}{4}x^5+1215}{3+x}+405\ln\left(3+x\right)$	36
meijerg	$-\frac{9x\left(-\frac{1}{27}x^4+\frac{5}{27}x^3-\frac{10}{9}x^2+10x+60\right)}{4\left(1+\frac{x}{3}\right)}+405\ln\left(1+\frac{x}{3}\right)$	40
parallelrisch	$\frac{x^5-5x^4+30x^3+1620\ln(3+x)x-270x^2+4860+4860\ln(3+x)}{12+4x}$	41

[In] int(x^5/(3+x)^2,x,method=_RETURNVERBOSE)

[Out] -108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(3+x)^2}\,dx = \frac{x^5-5\,x^4+30\,x^3-270\,x^2+1620\,(x+3)\log\left(x+3\right)-1296\,x+972}{4\,(x+3)}$$

[In] integrate(x^5/(3+x)^2,x, algorithm="fricas")

[Out] 1/4*(x^5 - 5*x^4 + 30*x^3 - 270*x^2 + 1620*(x + 3)*log(x + 3) - 1296*x + 97
2)/(x + 3)

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(3+x)^2}\,dx = \frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + 405\log{(x+3)} + \frac{243}{x+3}$$

```
[In] integrate(x**5/(3+x)**2,x)
[Out] x**4/4 - 2*x**3 + 27*x**2/2 - 108*x + 405*log(x + 3) + 243/(x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(3+x)^2}\,dx = \frac{1}{4}\,x^4 - 2\,x^3 + \frac{27}{2}\,x^2 - 108\,x + \frac{243}{x+3} + 405\,\log\left(x+3\right)$$

```
[In] integrate(x^5/(3+x)^2,x, algorithm="maxima")
[Out] 1/4*x^4 - 2*x^3 + 27/2*x^2 - 108*x + 243/(x + 3) + 405*log(x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{x^5}{(3+x)^2}\,dx = -\frac{1}{4}\,(x+3)^4\left(\frac{20}{x+3} - \frac{180}{(x+3)^2} + \frac{1080}{(x+3)^3} - 1\right) + \frac{243}{x+3} + 405\,\log\left(|x+3|\right)$$

```
[In] integrate(x^5/(3+x)^2,x, algorithm="giac")
[Out] -1/4*(x + 3)^4*(20/(x + 3) - 180/(x + 3)^2 + 1080/(x + 3)^3 - 1) + 243/(x +
 3) + 405*log(abs(x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(3+x)^2} \, dx = 405 \ln(x+3) - 108\,x + \frac{243}{x+3} + \frac{27\,x^2}{2} - 2\,x^3 + \frac{x^4}{4}$$

```
[In] int(x^5/(x + 3)^2,x)
[Out] 405*log(x + 3) - 108*x + 243/(x + 3) + (27*x^2)/2 - 2*x^3 + x^4/4
```

3.189 $\int (\mathbf{b1}+\mathbf{c1}x) (a+2bx+cx^2) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int (\mathrm{b1}+\mathrm{c1}x) (a+2bx+cx^2) \, dx = a\mathrm{b1}x+\frac{1}{2}(2b\mathrm{b1}+a\mathrm{c1})x^2+\frac{1}{3}(\mathrm{b1}c+2b\mathrm{c1})x^3+\frac{1}{4}c\mathrm{c1}x^4$$

```
[Out] a*b1*x+1/2*(a*c1+2*b*b1)*x^2+1/3*(2*b*c1+b1*c)*x^3+1/4*c*c1*x^4
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {645}

$$\int (\mathrm{b1}+\mathrm{c1}x) (a+2bx+cx^2) \, dx = \frac{1}{2}x^2(a\mathrm{c1}+2b\mathrm{b1})+a\mathrm{b1}x+\frac{1}{3}x^3(2b\mathrm{c1}+\mathrm{b1}c)+\frac{1}{4}c\mathrm{c1}x^4$$

```
[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2),x]
[Out] a*b1*x + ((2*b*b1 + a*c1)*x^2)/2 + ((b1*c + 2*b*c1)*x^3)/3 + (c*c1*x^4)/4
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(a\mathrm{b1}+(2b\mathrm{b1}+a\mathrm{c1})x+(\mathrm{b1}c+2b\mathrm{c1})x^2+c\mathrm{c1}x^3\right) \, dx \\ &= a\mathrm{b1}x+\frac{1}{2}(2b\mathrm{b1}+a\mathrm{c1})x^2+\frac{1}{3}(\mathrm{b1}c+2b\mathrm{c1})x^3+\frac{1}{4}c\mathrm{c1}x^4 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)\,dx=\frac{1}{12}x(6a(2\text{b1}+\text{c1}x)+x(4b(3\text{b1}+2\text{c1}x)+cx(4\text{b1}+3\text{c1}x)))$$

```
[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2),x]
[Out] (x*(6*a*(2*b1 + c1*x) + x*(4*b*(3*b1 + 2*c1*x) + c*x*(4*b1 + 3*c1*x))))/12
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result	size
norman	$\frac{c\,\text{c1}\,x^4}{4}+\left(\frac{2b\,\text{c1}}{3}+\frac{\text{b1}\,c}{3}\right)x^3+\left(\frac{a\,\text{c1}}{2}+b\,\text{b1}\right)x^2+a\,\text{b1}\,x$	38
default	$a\,\text{b1}\,x+\frac{(a\,\text{c1}+2b\,\text{b1})x^2}{2}+\frac{(2b\,\text{c1}+\text{b1}\,c)x^3}{3}+\frac{c\,\text{c1}\,x^4}{4}$	39
gosper	$\frac{1}{4}c\,\text{c1}\,x^4+\frac{2}{3}x^3b\,\text{c1}+\frac{1}{3}x^3\,\text{b1}\,c+\frac{1}{2}x^2a\,\text{c1}+x^2b\,\text{b1}+a\,\text{b1}\,x$	40
risch	$\frac{1}{4}c\,\text{c1}\,x^4+\frac{2}{3}x^3b\,\text{c1}+\frac{1}{3}x^3\,\text{b1}\,c+\frac{1}{2}x^2a\,\text{c1}+x^2b\,\text{b1}+a\,\text{b1}\,x$	40
parallelrisch	$\frac{1}{4}c\,\text{c1}\,x^4+\frac{2}{3}x^3b\,\text{c1}+\frac{1}{3}x^3\,\text{b1}\,c+\frac{1}{2}x^2a\,\text{c1}+x^2b\,\text{b1}+a\,\text{b1}\,x$	40

```
[In] int((c1*x+b1)*(c*x^2+2*b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/4*c*c1*x^4+(2/3*b*c1+1/3*b1*c)*x^3+(1/2*a*c1+b*b1)*x^2+a*b1*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)\,dx=\frac{1}{4}\,cc_1x^4+\frac{1}{3}\,(b_1c+2\,bc_1)x^3+ab_1x+\frac{1}{2}\,(2\,bb_1+ac_1)x^2$$

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="fricas")
[Out] 1/4*c*c1*x^4 + 1/3*(b1*c + 2*b*c1)*x^3 + a*b1*x + 1/2*(2*b*b1 + a*c1)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int (\text{b1} + \text{c1}x) \left(a + 2bx + cx^2\right) \, dx = ab_1x + \frac{cc_1x^4}{4} + x^3 \cdot \left(\frac{2bc_1}{3} + \frac{b_1c}{3}\right) + x^2 \left(\frac{ac_1}{2} + bb_1\right)$$

```
[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a),x)
[Out] a*b1*x + c*c1*x**4/4 + x**3*(2*b*c1/3 + b1*c/3) + x**2*(a*c1/2 + b*b1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int (\text{b1} + \text{c1}x) \left(a + 2bx + cx^2\right) \, dx = \frac{1}{4} cc_1x^4 + \frac{1}{3} (b_1c + 2 bc_1)x^3 + ab_1x + \frac{1}{2} (2 bb_1 + ac_1)x^2$$

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="maxima")
[Out] 1/4*c*c1*x^4 + 1/3*(b1*c + 2*b*c1)*x^3 + a*b1*x + 1/2*(2*b*b1 + a*c1)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int (\text{b1} + \text{c1}x) \left(a + 2bx + cx^2\right) \, dx = \frac{1}{4} cc_1x^4 + \frac{1}{3} b_1cx^3 + \frac{2}{3} bc_1x^3 + bb_1x^2 + \frac{1}{2} ac_1x^2 + ab_1x$$

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="giac")
[Out] 1/4*c*c1*x^4 + 1/3*b1*c*x^3 + 2/3*b*c1*x^3 + b*b1*x^2 + 1/2*a*c1*x^2 + a*b1
*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int (\text{b1} + \text{c1}x) \left(a + 2bx + cx^2\right) \, dx = \frac{c\,c_1\,x^4}{4} + \left(\frac{2\,b\,c_1}{3} + \frac{b_1\,c}{3}\right)\,x^3 + \left(\frac{a\,c_1}{2} + b\,b_1\right)\,x^2 + a\,b_1\,x$$

```
[In] int((b1 + c1*x)*(a + 2*b*x + c*x^2),x)
[Out] x^2*((a*c1)/2 + b*b1) + x^3*((2*b*c1)/3 + (b1*c)/3) + a*b1*x + (c*c1*x^4)/4
```

3.190 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 96

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^2\,dx = a^2\mathrm{b1}x+\frac{1}{2}a(4b\mathrm{b1}+a\mathrm{c1})x^2+\frac{2}{3}\left(2b^2\mathrm{b1}+a\mathrm{b1}c+2ab\mathrm{c1}\right)x^3 + \frac{1}{2}\left(2b\mathrm{b1}c+2b^2\mathrm{c1}+ac\mathrm{c1}\right)x^4 + \frac{1}{5}c(\mathrm{b1}c+4b\mathrm{c1})x^5+\frac{1}{6}c^2\mathrm{c1}x^6$$

```
[Out] a^2*b1*x+1/2*a*(a*c1+4*b*b1)*x^2+2/3*(2*a*b*c1+a*b1*c+2*b^2*b1)*x^3+1/2*(a*
c*c1+2*b^2*c1+2*b*b1*c)*x^4+1/5*c*(4*b*c1+b1*c)*x^5+1/6*c^2*c1*x^6
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {645}

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^2\,dx = a^2\mathrm{b1}x+\frac{1}{2}x^4\left(ac\mathrm{c1}+2b^2\mathrm{c1}+2b\mathrm{b1}c\right) + \frac{2}{3}x^3\left(2ab\mathrm{c1}+a\mathrm{b1}c+2b^2\mathrm{b1}\right)+\frac{1}{2}ax^2(a\mathrm{c1}+4b\mathrm{b1}) + \frac{1}{5}cx^5(4b\mathrm{c1}+\mathrm{b1}c)+\frac{1}{6}c^2\mathrm{c1}x^6$$

```
[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] a^2*b1*x + (a*(4*b*b1 + a*c1)*x^2)/2 + (2*(2*b^2*b1 + a*b1*c + 2*a*b*c1)*x^
3)/3 + ((2*b*b1*c + 2*b^2*c1 + a*c*c1)*x^4)/2 + (c*(b1*c + 4*b*c1)*x^5)/5 +
 (c^2*c1*x^6)/6
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2\text{b1} + a(4b\text{b1} + a\text{c1})x + 2\left(2b^2\text{b1} + a\text{b1}c + 2ab\text{c1}\right)x^2 \right.\\ &\qquad \left. + 2\left(2b\text{b1}c + 2b^2\text{c1} + ac\text{c1}\right)x^3 + c(\text{b1}c + 4b\text{c1})x^4 + c^2\text{c1}x^5\right) dx \\ &= a^2\text{b1}x + \frac{1}{2}a(4b\text{b1} + a\text{c1})x^2 + \frac{2}{3}\left(2b^2\text{b1} + a\text{b1}c + 2ab\text{c1}\right)x^3 \\ &\quad + \frac{1}{2}\left(2b\text{b1}c + 2b^2\text{c1} + ac\text{c1}\right)x^4 + \frac{1}{5}c(\text{b1}c + 4b\text{c1})x^5 + \frac{1}{6}c^2\text{c1}x^6\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\begin{aligned}\int (\text{b1} + \text{c1}x)\left(a + 2bx + cx^2\right)^2 dx &= \frac{1}{30}x\left(15a^2(2\text{b1} + \text{c1}x)\right. \\ &\quad + 5ax(4b(3\text{b1} + 2\text{c1}x) + cx(4\text{b1} + 3\text{c1}x)) \\ &\quad + x^2\left(10b^2(4\text{b1} + 3\text{c1}x) + 6bcx(5\text{b1} + 4\text{c1}x)\right. \\ &\quad \left.\left. + c^2x^2(6\text{b1} + 5\text{c1}x)\right)\right)\end{aligned}$$

```
[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] (x*(15*a^2*(2*b1 + c1*x) + 5*a*x*(4*b*(3*b1 + 2*c1*x) + c*x*(4*b1 + 3*c1*x)
) + x^2*(10*b^2*(4*b1 + 3*c1*x) + 6*b*c*x*(5*b1 + 4*c1*x) + c^2*x^2*(6*b1 +
 5*c1*x))))/30
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.93

method	result
norman	$\frac{c^2\,\text{c1}\,x^6}{6} + \left(\frac{4}{5}\,\text{c1}\,bc + \frac{1}{5}\,\text{b1}\,c^2\right)x^5 + \left(\frac{1}{2}ac\,\text{c1} + b^2\,\text{c1} + b\,\text{b1}\,c\right)x^4 + \left(\frac{4}{3}ab\,\text{c1} + \frac{2}{3}a\,\text{b1}\,c + \frac{4}{3}b^2\,\text{b1}\right)x^3 +$
default	$\frac{c^2\,\text{c1}\,x^6}{6} + \frac{(4\,\text{c1}\,bc + \text{b1}\,c^2)x^5}{5} + \frac{(4b\,\text{b1}\,c + \text{c1}\,(2ac + 4b^2))x^4}{4} + \frac{(\text{b1}\,(2ac + 4b^2) + 4ab\,\text{c1})x^3}{3} + \frac{(\text{c1}\,a^2 + 4\,\text{b1}\,ab)x^2}{2} + a^2\,\text{b1}$
gosper	$\frac{1}{6}c^2\,\text{c1}\,x^6 + \frac{4}{5}x^5\,\text{c1}\,bc + \frac{1}{5}x^5\,\text{b1}\,c^2 + \frac{1}{2}x^4ac\,\text{c1} + x^4b^2\,\text{c1} + x^4b\,\text{b1}\,c + \frac{4}{3}x^3ab\,\text{c1} + \frac{2}{3}x^3a\,\text{b1}\,c + \frac{4}{3}x^3$
risch	$\frac{1}{6}c^2\,\text{c1}\,x^6 + \frac{4}{5}x^5\,\text{c1}\,bc + \frac{1}{5}x^5\,\text{b1}\,c^2 + \frac{1}{2}x^4ac\,\text{c1} + x^4b^2\,\text{c1} + x^4b\,\text{b1}\,c + \frac{4}{3}x^3ab\,\text{c1} + \frac{2}{3}x^3a\,\text{b1}\,c + \frac{4}{3}x^3$
parallelrisch	$\frac{1}{6}c^2\,\text{c1}\,x^6 + \frac{4}{5}x^5\,\text{c1}\,bc + \frac{1}{5}x^5\,\text{b1}\,c^2 + \frac{1}{2}x^4ac\,\text{c1} + x^4b^2\,\text{c1} + x^4b\,\text{b1}\,c + \frac{4}{3}x^3ab\,\text{c1} + \frac{2}{3}x^3a\,\text{b1}\,c + \frac{4}{3}x^3$

```
[In] int((c1*x+b1)*(c*x^2+2*b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c^2*c1*x^6+(4/5*c1*b*c+1/5*b1*c^2)*x^5+(1/2*a*c*c1+b^2*c1+b*b1*c)*x^4+(
4/3*a*b*c1+2/3*a*b1*c+4/3*b^2*b1)*x^3+(1/2*c1*a^2+2*b1*a*b)*x^2+a^2*b1*x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^2\,dx=\frac{1}{6}\,c^2c_1x^6+\frac{1}{5}\left(b_1c^2+4\,bcc_1\right)x^5+\frac{1}{2}\left(2\,bb_1c+\left(2\,b^2+ac\right)c_1\right)x^4+a^2b_1x+\frac{2}{3}\left(2\,b^2b_1+ab_1c+2\,abc_1\right)x^3+\frac{1}{2}\left(4\,abb_1+a^2c_1\right)x^2$$

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*c^2*c1*x^6 + 1/5*(b1*c^2 + 4*b*c*c1)*x^5 + 1/2*(2*b*b1*c + (2*b^2 + a*c
)*c1)*x^4 + a^2*b1*x + 2/3*(2*b^2*b1 + a*b1*c + 2*a*b*c1)*x^3 + 1/2*(4*a*b*
b1 + a^2*c1)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^2\,dx=a^2b_1x+\frac{c^2c_1x^6}{6}+x^5\cdot\left(\frac{4bcc_1}{5}+\frac{b_1c^2}{5}\right)+x^4\left(\frac{acc_1}{2}+b^2c_1+bb_1c\right)+x^3\cdot\left(\frac{4abc_1}{3}+\frac{2ab_1c}{3}+\frac{4b^2b_1}{3}\right)+x^2\left(\frac{a^2c_1}{2}+2abb_1\right)$$

```
[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a)**2,x)
```

```
[Out] a**2*b1*x + c**2*c1*x**6/6 + x**5*(4*b*c*c1/5 + b1*c**2/5) + x**4*(a*c*c1/2
 + b**2*c1 + b*b1*c) + x**3*(4*a*b*c1/3 + 2*a*b1*c/3 + 4*b**2*b1/3) + x**2*
(a**2*c1/2 + 2*a*b*b1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\int (\text{b1} + \text{c1}x) \left(a + 2bx + cx^2\right)^2 \, dx = \frac{1}{6} c^2 c_1 x^6 + \frac{1}{5} \left(b_1 c^2 + 4 b c c_1\right) x^5 + \frac{1}{2} \left(2 b b_1 c + \left(2 b^2 + a c\right) c_1\right) x^4 + a^2 b_1 x + \frac{2}{3} \left(2 b^2 b_1 + a b_1 c + 2 a b c_1\right) x^3 + \frac{1}{2} \left(4 a b b_1 + a^2 c_1\right) x^2$$

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*c^2*c1*x^6 + 1/5*(b1*c^2 + 4*b*c*c1)*x^5 + 1/2*(2*b*b1*c + (2*b^2 + a*c
)*c1)*x^4 + a^2*b1*x + 2/3*(2*b^2*b1 + a*b1*c + 2*a*b*c1)*x^3 + 1/2*(4*a*b*
b1 + a^2*c1)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02

$$\int (\text{b1} + \text{c1}x) \left(a + 2bx + cx^2\right)^2 \, dx = \frac{1}{6} c^2 c_1 x^6 + \frac{1}{5} b_1 c^2 x^5 + \frac{4}{5} b c c_1 x^5 + b b_1 c x^4 + b^2 c_1 x^4 + \frac{1}{2} a c c_1 x^4 + \frac{4}{3} b^2 b_1 x^3 + \frac{2}{3} a b_1 c x^3 + \frac{4}{3} a b c_1 x^3 + 2 a b b_1 x^2 + \frac{1}{2} a^2 c_1 x^2 + a^2 b_1 x$$

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*c^2*c1*x^6 + 1/5*b1*c^2*x^5 + 4/5*b*c*c1*x^5 + b*b1*c*x^4 + b^2*c1*x^4
+ 1/2*a*c*c1*x^4 + 4/3*b^2*b1*x^3 + 2/3*a*b1*c*x^3 + 4/3*a*b*c1*x^3 + 2*a*b
*b1*x^2 + 1/2*a^2*c1*x^2 + a^2*b1*x
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

$$\int (\text{b1} + \text{c1}x) \left(a + 2bx + cx^2\right)^2 \, dx = x^3 \left(\frac{4\, b_1\, b^2}{3} + \frac{4\, a\, c_1\, b}{3} + \frac{2\, a\, b_1\, c}{3}\right) + x^4 \left(c_1\, b^2 + b_1\, c\, b + \frac{a\, c\, c_1}{2}\right) + x^2 \left(\frac{c_1\, a^2}{2} + 2\, b\, b_1\, a\right) + x^5 \left(\frac{b_1\, c^2}{5} + \frac{4\, b\, c_1\, c}{5}\right) + \frac{c^2\, c_1\, x^6}{6} + a^2\, b_1\, x$$

```
[In] int((b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x)
```

```
[Out] x^3*((4*b^2*b1)/3 + (4*a*b*c1)/3 + (2*a*b1*c)/3) + x^4*(b^2*c1 + (a*c*c1)/2
 + b*b1*c) + x^2*((a^2*c1)/2 + 2*a*b*b1) + x^5*((b1*c^2)/5 + (4*b*c*c1)/5)
+ (c^2*c1*x^6)/6 + a^2*b1*x
```

3.191 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 167

$$\begin{aligned}\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^3\,dx &= a^3\mathrm{b1}x+\frac{1}{2}a^2(6b\mathrm{b1}+a\mathrm{c1})x^2+a\left(4b^2\mathrm{b1}+a\mathrm{b1}c+2ab\mathrm{c1}\right)x^3\\ &\quad+\frac{1}{4}\left(8b^3\mathrm{b1}+12ab\mathrm{b1}c+12ab^2\mathrm{c1}+3a^2c\mathrm{c1}\right)x^4\\ &\quad+\frac{1}{5}\left(12b^2\mathrm{b1}c+3a\mathrm{b1}c^2+8b^3\mathrm{c1}+12abc\mathrm{c1}\right)x^5\\ &\quad+\frac{1}{2}c\left(2b\mathrm{b1}c+4b^2\mathrm{c1}+ac\mathrm{c1}\right)x^6\\ &\quad+\frac{1}{7}c^2(\mathrm{b1}c+6b\mathrm{c1})x^7+\frac{1}{8}c^3\mathrm{c1}x^8\end{aligned}$$

```
[Out] a^3*b1*x+1/2*a^2*(a*c1+6*b*b1)*x^2+a*(2*a*b*c1+a*b1*c+4*b^2*b1)*x^3+1/4*(3*
a^2*c*c1+12*a*b^2*c1+12*a*b*b1*c+8*b^3*b1)*x^4+1/5*(12*a*b*c*c1+3*a*b1*c^2+
8*b^3*c1+12*b^2*b1*c)*x^5+1/2*c*(a*c*c1+4*b^2*c1+2*b*b1*c)*x^6+1/7*c^2*(6*b
*c1+b1*c)*x^7+1/8*c^3*c1*x^8
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used

$= \{645\}$

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^3 \, dx = a^3\mathrm{b1}x + \frac{1}{4}x^4\left(3a^2c\mathrm{c1} + 12ab^2\mathrm{c1} + 12ab\mathrm{b1}c + 8b^3\mathrm{b1}\right) + \frac{1}{2}a^2x^2(a\mathrm{c1} + 6b\mathrm{b1}) + \frac{1}{2}cx^6\left(ac\mathrm{c1} + 4b^2\mathrm{c1} + 2b\mathrm{b1}c\right) + ax^3\left(2ab\mathrm{c1} + a\mathrm{b1}c + 4b^2\mathrm{b1}\right) + \frac{1}{5}x^5\left(12abc\mathrm{c1} + 3a\mathrm{b1}c^2 + 8b^3\mathrm{c1} + 12b^2\mathrm{b1}c\right) + \frac{1}{7}c^2x^7(6b\mathrm{c1} + \mathrm{b1}c) + \frac{1}{8}c^3\mathrm{c1}x^8$$

```
[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] a^3*b1*x + (a^2*(6*b*b1 + a*c1)*x^2)/2 + a*(4*b^2*b1 + a*b1*c + 2*a*b*c1)*x
^3 + ((8*b^3*b1 + 12*a*b*b1*c + 12*a*b^2*c1 + 3*a^2*c*c1)*x^4)/4 + ((12*b^2
*b1*c + 3*a*b1*c^2 + 8*b^3*c1 + 12*a*b*c*c1)*x^5)/5 + (c*(2*b*b1*c + 4*b^2*
c1 + a*c*c1)*x^6)/2 + (c^2*(b1*c + 6*b*c1)*x^7)/7 + (c^3*c1*x^8)/8
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\text{integral} = \int \left(a^3\mathrm{b1} + a^2(6b\mathrm{b1} + a\mathrm{c1})x + 3a\left(4b^2\mathrm{b1} + a\mathrm{b1}c + 2ab\mathrm{c1}\right)x^2 + \left(8b^3\mathrm{b1} + 12ab\mathrm{b1}c + 12ab^2\mathrm{c1} + 3a^2c\mathrm{c1}\right)x^3 + \left(12b^2\mathrm{b1}c + 3a\mathrm{b1}c^2 + 8b^3\mathrm{c1} + 12abc\mathrm{c1}\right)x^4 + 3c\left(2b\mathrm{b1}c + 4b^2\mathrm{c1} + ac\mathrm{c1}\right)x^5 + c^2(\mathrm{b1}c + 6b\mathrm{c1})x^6 + c^3\mathrm{c1}x^7\right) dx$$

$$= a^3\mathrm{b1}x + \frac{1}{2}a^2(6b\mathrm{b1} + a\mathrm{c1})x^2 + a\left(4b^2\mathrm{b1} + a\mathrm{b1}c + 2ab\mathrm{c1}\right)x^3 + \frac{1}{4}\left(8b^3\mathrm{b1} + 12ab\mathrm{b1}c + 12ab^2\mathrm{c1} + 3a^2c\mathrm{c1}\right)x^4 + \frac{1}{5}\left(12b^2\mathrm{b1}c + 3a\mathrm{b1}c^2 + 8b^3\mathrm{c1} + 12abc\mathrm{c1}\right)x^5 + \frac{1}{2}c\left(2b\mathrm{b1}c + 4b^2\mathrm{c1} + ac\mathrm{c1}\right)x^6 + \frac{1}{7}c^2(\mathrm{b1}c + 6b\mathrm{c1})x^7 + \frac{1}{8}c^3\mathrm{c1}x^8$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^3\,dx = a^3\mathrm{b1}x+\frac{1}{2}a^2(6b\mathrm{b1}+a\mathrm{c1})x^2+a\left(4b^2\mathrm{b1}+a\mathrm{b1}c+2ab\mathrm{c1}\right)x^3 + \frac{1}{4}\left(8b^3\mathrm{b1}+12ab\mathrm{b1}c+12ab^2\mathrm{c1}+3a^2c\mathrm{c1}\right)x^4 + \frac{1}{5}\left(12b^2\mathrm{b1}c+3a\mathrm{b1}c^2+8b^3\mathrm{c1}+12abc\mathrm{c1}\right)x^5 + \frac{1}{2}c\left(2b\mathrm{b1}c+4b^2\mathrm{c1}+ac\mathrm{c1}\right)x^6 + \frac{1}{7}c^2(\mathrm{b1}c+6b\mathrm{c1})x^7+\frac{1}{8}c^3\mathrm{c1}x^8$$

```
[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] a^3*b1*x + (a^2*(6*b*b1 + a*c1)*x^2)/2 + a*(4*b^2*b1 + a*b1*c + 2*a*b*c1)*x
^3 + ((8*b^3*b1 + 12*a*b*b1*c + 12*a*b^2*c1 + 3*a^2*c*c1)*x^4)/4 + ((12*b^2
*b1*c + 3*a*b1*c^2 + 8*b^3*c1 + 12*a*b*c*c1)*x^5)/5 + (c*(2*b*b1*c + 4*b^2*
c1 + a*c*c1)*x^6)/2 + (c^2*(b1*c + 6*b*c1)*x^7)/7 + (c^3*c1*x^8)/8
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

method	result
norman	$\frac{c^3\,\mathrm{c1}\,x^8}{8}+\left(\frac{6}{7}\,\mathrm{c1}\,b\,c^2+\frac{1}{7}\,\mathrm{b1}\,c^3\right)x^7+\left(\frac{1}{2}a\,c^2\,\mathrm{c1}+2b^2c\,\mathrm{c1}+\mathrm{b1}\,b\,c^2\right)x^6+\left(\frac{12}{5}abc\,\mathrm{c1}+\frac{3}{5}a\,\mathrm{b1}\,c^2+\frac{8}{5}b\right.$
gosper	$\frac{1}{8}c^3\,\mathrm{c1}\,x^8+\frac{6}{7}x^7\,\mathrm{c1}\,b\,c^2+\frac{1}{7}x^7\,\mathrm{b1}\,c^3+\frac{1}{2}x^6a\,c^2\,\mathrm{c1}+2x^6b^2c\,\mathrm{c1}+x^6\,\mathrm{b1}\,b\,c^2+\frac{12}{5}x^5abc\,\mathrm{c1}+\frac{3}{5}x^5a\,\mathrm{b1}$
risch	$\frac{1}{8}c^3\,\mathrm{c1}\,x^8+\frac{6}{7}x^7\,\mathrm{c1}\,b\,c^2+\frac{1}{7}x^7\,\mathrm{b1}\,c^3+\frac{1}{2}x^6a\,c^2\,\mathrm{c1}+2x^6b^2c\,\mathrm{c1}+x^6\,\mathrm{b1}\,b\,c^2+\frac{12}{5}x^5abc\,\mathrm{c1}+\frac{3}{5}x^5a\,\mathrm{b1}$
parallelrisch	$\frac{1}{8}c^3\,\mathrm{c1}\,x^8+\frac{6}{7}x^7\,\mathrm{c1}\,b\,c^2+\frac{1}{7}x^7\,\mathrm{b1}\,c^3+\frac{1}{2}x^6a\,c^2\,\mathrm{c1}+2x^6b^2c\,\mathrm{c1}+x^6\,\mathrm{b1}\,b\,c^2+\frac{12}{5}x^5abc\,\mathrm{c1}+\frac{3}{5}x^5a\,\mathrm{b1}$
default	$\frac{c^3\,\mathrm{c1}\,x^8}{8}+\frac{(6\,\mathrm{c1}\,b\,c^2+\mathrm{b1}\,c^3)x^7}{7}+\frac{(6\,\mathrm{b1}\,b\,c^2+\mathrm{c1}(c^2a+8b^2c+c(2ac+4b^2)))x^6}{6}+\frac{(\mathrm{b1}(c^2a+8b^2c+c(2ac+4b^2))+\mathrm{c1}(8abc+2b($

```
[In] int((c1*x+b1)*(c*x^2+2*b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c^3*c1*x^8+(6/7*c1*b*c^2+1/7*b1*c^3)*x^7+(1/2*a*c^2*c1+2*b^2*c*c1+b1*b*
c^2)*x^6+(12/5*a*b*c*c1+3/5*a*b1*c^2+8/5*b^3*c1+12/5*b^2*b1*c)*x^5+(3/4*a^2
*c*c1+3*a*b^2*c1+3*a*b*b1*c+2*b^3*b1)*x^4+(2*a^2*b*c1+a^2*b1*c+4*a*b^2*b1)*
x^3+(1/2*c1*a^3+3*b1*a^2*b)*x^2+a^3*b1*x
```

Fricas [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^3 \, dx = \frac{1}{8} c^3 c_1 x^8 + \frac{1}{7} \left(b_1 c^3 + 6 \, b c^2 c_1\right) x^7 + \frac{1}{2} \left(2 \, b b_1 c^2 + \left(4 \, b^2 c + a c^2\right) c_1\right) x^6 + \frac{1}{5} \left(12 \, b^2 b_1 c + 3 \, a b_1 c^2 + 4 \, \left(2 \, b^3 + 3 \, a b c\right) c_1\right) x^5 + a^3 b_1 x + \frac{1}{4} \left(8 \, b^3 b_1 + 12 \, a b b_1 c + 3 \, \left(4 \, a b^2 + a^2 c\right) c_1\right) x^4 + \left(4 \, a b^2 b_1 + a^2 b_1 c + 2 \, a^2 b c_1\right) x^3 + \frac{1}{2} \left(6 \, a^2 b b_1 + a^3 c_1\right) x^2$$

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*c^3*c1*x^8 + 1/7*(b1*c^3 + 6*b*c^2*c1)*x^7 + 1/2*(2*b*b1*c^2 + (4*b^2*c
 + a*c^2)*c1)*x^6 + 1/5*(12*b^2*b1*c + 3*a*b1*c^2 + 4*(2*b^3 + 3*a*b*c)*c1)
*x^5 + a^3*b1*x + 1/4*(8*b^3*b1 + 12*a*b*b1*c + 3*(4*a*b^2 + a^2*c)*c1)*x^4
 + (4*a*b^2*b1 + a^2*b1*c + 2*a^2*b*c1)*x^3 + 1/2*(6*a^2*b*b1 + a^3*c1)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.13

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^3 \, dx = a^3 b_1 x + \frac{c^3 c_1 x^8}{8} + x^7 \cdot \left(\frac{6bc^2 c_1}{7} + \frac{b_1 c^3}{7}\right) + x^6 \left(\frac{ac^2 c_1}{2} + 2b^2 c c_1 + b b_1 c^2\right) + x^5 \cdot \left(\frac{12abcc_1}{5} + \frac{3ab_1 c^2}{5} + \frac{8b^3 c_1}{5} + \frac{12b^2 b_1 c}{5}\right) + x^4 \cdot \left(\frac{3a^2 c c_1}{4} + 3ab^2 c_1 + 3abb_1 c + 2b^3 b_1\right) + x^3 \cdot \left(2a^2 b c_1 + a^2 b_1 c + 4ab^2 b_1\right) + x^2 \left(\frac{a^3 c_1}{2} + 3a^2 b b_1\right)$$

```
[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a)**3,x)
```

```
[Out] a**3*b1*x + c**3*c1*x**8/8 + x**7*(6*b*c**2*c1/7 + b1*c**3/7) + x**6*(a*c**
2*c1/2 + 2*b**2*c*c1 + b*b1*c**2) + x**5*(12*a*b*c*c1/5 + 3*a*b1*c**2/5 + 8
*b**3*c1/5 + 12*b**2*b1*c/5) + x**4*(3*a**2*c*c1/4 + 3*a*b**2*c1 + 3*a*b*b1
*c + 2*b**3*b1) + x**3*(2*a**2*b*c1 + a**2*b1*c + 4*a*b**2*b1) + x**2*(a**3
*c1/2 + 3*a**2*b*b1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^3 \, dx = \frac{1}{8} c^3 c_1 x^8 + \frac{1}{7} \left(b_1 c^3 + 6 \, b c^2 c_1\right) x^7 + \frac{1}{2} \left(2 \, b b_1 c^2 + \left(4 \, b^2 c + a c^2\right) c_1\right) x^6 + \frac{1}{5} \left(12 \, b^2 b_1 c + 3 \, a b_1 c^2 + 4 \left(2 \, b^3 + 3 \, a b c\right) c_1\right) x^5 + a^3 b_1 x + \frac{1}{4} \left(8 \, b^3 b_1 + 12 \, a b b_1 c + 3 \left(4 \, a b^2 + a^2 c\right) c_1\right) x^4 + \left(4 \, a b^2 b_1 + a^2 b_1 c + 2 \, a^2 b c_1\right) x^3 + \frac{1}{2} \left(6 \, a^2 b b_1 + a^3 c_1\right) x^2$$

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^3*c1*x^8 + 1/7*(b1*c^3 + 6*b*c^2*c1)*x^7 + 1/2*(2*b*b1*c^2 + (4*b^2*c
 + a*c^2)*c1)*x^6 + 1/5*(12*b^2*b1*c + 3*a*b1*c^2 + 4*(2*b^3 + 3*a*b*c)*c1)
*x^5 + a^3*b1*x + 1/4*(8*b^3*b1 + 12*a*b*b1*c + 3*(4*a*b^2 + a^2*c)*c1)*x^4
 + (4*a*b^2*b1 + a^2*b1*c + 2*a^2*b*c1)*x^3 + 1/2*(6*a^2*b*b1 + a^3*c1)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.13

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^3 \, dx = \frac{1}{8} c^3 c_1 x^8 + \frac{1}{7} b_1 c^3 x^7 + \frac{6}{7} b c^2 c_1 x^7 + b b_1 c^2 x^6 + 2 \, b^2 c c_1 x^6 + \frac{1}{2} a c^2 c_1 x^6 + \frac{12}{5} b^2 b_1 c x^5 + \frac{3}{5} a b_1 c^2 x^5 + \frac{8}{5} b^3 c_1 x^5 + \frac{12}{5} a b c c_1 x^5 + 2 \, b^3 b_1 x^4 + 3 \, a b b_1 c x^4 + 3 \, a b^2 c_1 x^4 + \frac{3}{4} a^2 c c_1 x^4 + 4 \, a b^2 b_1 x^3 + a^2 b_1 c x^3 + 2 \, a^2 b c_1 x^3 + 3 \, a^2 b b_1 x^2 + \frac{1}{2} a^3 c_1 x^2 + a^3 b_1 x$$

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*c^3*c1*x^8 + 1/7*b1*c^3*x^7 + 6/7*b*c^2*c1*x^7 + b*b1*c^2*x^6 + 2*b^2*c
*c1*x^6 + 1/2*a*c^2*c1*x^6 + 12/5*b^2*b1*c*x^5 + 3/5*a*b1*c^2*x^5 + 8/5*b^3
*c1*x^5 + 12/5*a*b*c*c1*x^5 + 2*b^3*b1*x^4 + 3*a*b*b1*c*x^4 + 3*a*b^2*c1*x^
4 + 3/4*a^2*c*c1*x^4 + 4*a*b^2*b1*x^3 + a^2*b1*c*x^3 + 2*a^2*b*c1*x^3 + 3*a
^2*b*b1*x^2 + 1/2*a^3*c1*x^2 + a^3*b1*x
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.98

$$\int (\text{b1} + \text{c1}x)\left(a + 2bx + cx^2\right)^3 \, dx = x^7\left(\frac{b_1\,c^3}{7} + \frac{6\,b\,c_1\,c^2}{7}\right) + x^3\left(2\,c_1\,a^2\,b + b_1\,c\,a^2 + 4\,b_1\,a\,b^2\right) + x^6\left(2\,c_1\,b^2\,c + b_1\,b\,c^2 + \frac{a\,c_1\,c^2}{2}\right) + x^4\left(\frac{3\,c\,c_1\,a^2}{4} + 3\,c_1\,a\,b^2 + 3\,b_1\,c\,a\,b + 2\,b_1\,b^3\right) + x^5\left(\frac{8\,c_1\,b^3}{5} + \frac{12\,b_1\,b^2\,c}{5} + \frac{12\,a\,c_1\,b\,c}{5} + \frac{3\,a\,b_1\,c^2}{5}\right) + x^2\left(\frac{c_1\,a^3}{2} + 3\,b\,b_1\,a^2\right) + \frac{c^3\,c_1\,x^8}{8} + a^3\,b_1\,x$$

```
[In] int((b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x)
```

```
[Out] x^7*((b1*c^3)/7 + (6*b*c^2*c1)/7) + x^3*(4*a*b^2*b1 + 2*a^2*b*c1 + a^2*b1*c
) + x^6*((a*c^2*c1)/2 + b*b1*c^2 + 2*b^2*c*c1) + x^4*(2*b^3*b1 + 3*a*b^2*c1
 + (3*a^2*c*c1)/4 + 3*a*b*b1*c) + x^5*((8*b^3*c1)/5 + (3*a*b1*c^2)/5 + (12*
b^2*b1*c)/5 + (12*a*b*c*c1)/5) + x^2*((a^3*c1)/2 + 3*a^2*b*b1) + (c^3*c1*x^
8)/8 + a^3*b1*x
```

3.192 $\int (\mathbf{b1}+\mathbf{c1}x)\left(a+2bx+cx^2\right)^4\,dx$

Optimal result

Integrand size = 19, antiderivative size = 263

$$\begin{aligned}\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx &= a^4\mathrm{b1}x+\frac{1}{2}a^3(8b\mathrm{b1}+a\mathrm{c1})x^2+\frac{4}{3}a^2\left(6b^2\mathrm{b1}+a\mathrm{b1}c+2ab\mathrm{c1}\right)x^3\\&\quad+a\left(8b^3\mathrm{b1}+6ab\mathrm{b1}c+6ab^2\mathrm{c1}+a^2c\mathrm{c1}\right)x^4+\frac{2}{5}\left(8b^4\mathrm{b1}\right.\\&\quad\left.+24ab^2\mathrm{b1}c+3a^2\mathrm{b1}c^2+16ab^3\mathrm{c1}+12a^2bc\mathrm{c1}\right)x^5\\&\quad+\frac{1}{3}\left(16b^3\mathrm{b1}c+12ab\mathrm{b1}c^2+8b^4\mathrm{c1}+24ab^2c\mathrm{c1}+3a^2c^2\mathrm{c1}\right)x^6\\&\quad+\frac{4}{7}c\left(6b^2\mathrm{b1}c+a\mathrm{b1}c^2+8b^3\mathrm{c1}+6abc\mathrm{c1}\right)x^7\\&\quad+\frac{1}{2}c^2\left(2b\mathrm{b1}c+6b^2\mathrm{c1}+ac\mathrm{c1}\right)x^8\\&\quad+\frac{1}{9}c^3(\mathrm{b1}c+8b\mathrm{c1})x^9+\frac{1}{10}c^4\mathrm{c1}x^{10}\end{aligned}$$

```
[Out] a^4*b1*x+1/2*a^3*(a*c1+8*b*b1)*x^2+4/3*a^2*(2*a*b*c1+a*b1*c+6*b^2*b1)*x^3+a
*(a^2*c*c1+6*a*b^2*c1+6*a*b*b1*c+8*b^3*b1)*x^4+2/5*(12*a^2*b*c*c1+3*a^2*b1*
c^2+16*a*b^3*c1+24*a*b^2*b1*c+8*b^4*b1)*x^5+1/3*(3*a^2*c^2*c1+24*a*b^2*c*c1
+12*a*b*b1*c^2+8*b^4*c1+16*b^3*b1*c)*x^6+4/7*c*(6*a*b*c*c1+a*b1*c^2+8*b^3*c
1+6*b^2*b1*c)*x^7+1/2*c^2*(a*c*c1+6*b^2*c1+2*b*b1*c)*x^8+1/9*c^3*(8*b*c1+b1
*c)*x^9+1/10*c^4*c1*x^10
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {645}

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^4\,dx = a^4\text{b1}x+\frac{1}{2}a^3x^2(a\text{c1}+8b\text{b1})+\frac{4}{3}a^2x^3\left(2ab\text{c1}+a\text{b1}c+6b^2\text{b1}\right) + ax^4\left(a^2c\text{c1}+6ab^2\text{c1}+6ab\text{b1}c+8b^3\text{b1}\right) + \frac{1}{3}x^6\left(3a^2c^2\text{c1}+24ab^2c\text{c1}+12ab\text{b1}c^2+8b^4\text{c1}+16b^3\text{b1}c\right) + \frac{2}{5}x^5\left(12a^2bc\text{c1}+3a^2\text{b1}c^2+16ab^3\text{c1}+24ab^2\text{b1}c + 8b^4\text{b1}\right)+\frac{1}{2}c^2x^8\left(ac\text{c1}+6b^2\text{c1}+2b\text{b1}c\right) + \frac{4}{7}cx^7\left(6abc\text{c1}+a\text{b1}c^2+8b^3\text{c1}+6b^2\text{b1}c\right) + \frac{1}{9}c^3x^9(8b\text{c1}+\text{b1}c)+\frac{1}{10}c^4\text{c1}x^{10}$$

```
[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x]
```

```
[Out] a^4*b1*x + (a^3*(8*b*b1 + a*c1)*x^2)/2 + (4*a^2*(6*b^2*b1 + a*b1*c + 2*a*b*
c1)*x^3)/3 + a*(8*b^3*b1 + 6*a*b*b1*c + 6*a*b^2*c1 + a^2*c*c1)*x^4 + (2*(8*
b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 + 16*a*b^3*c1 + 12*a^2*b*c*c1)*x^5)/5
 + ((16*b^3*b1*c + 12*a*b*b1*c^2 + 8*b^4*c1 + 24*a*b^2*c*c1 + 3*a^2*c^2*c1)
*x^6)/3 + (4*c*(6*b^2*b1*c + a*b1*c^2 + 8*b^3*c1 + 6*a*b*c*c1)*x^7)/7 + (c^
2*(2*b*b1*c + 6*b^2*c1 + a*c*c1)*x^8)/2 + (c^3*(b1*c + 8*b*c1)*x^9)/9 + (c^
4*c1*x^10)/10
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\text{integral} = \int \left(a^4\text{b1}+a^3(8b\text{b1}+a\text{c1})x+4a^2\left(6b^2\text{b1}+a\text{b1}c+2ab\text{c1}\right)x^2 + 4a\left(8b^3\text{b1}+6ab\text{b1}c+6ab^2\text{c1}+a^2c\text{c1}\right)x^3 + 2\left(8b^4\text{b1}+24ab^2\text{b1}c+3a^2\text{b1}c^2+16ab^3\text{c1}+12a^2bc\text{c1}\right)x^4 + 2\left(16b^3\text{b1}c+12ab\text{b1}c^2+8b^4\text{c1}+24ab^2c\text{c1}+3a^2c^2\text{c1}\right)x^5 + 4c\left(6b^2\text{b1}c+a\text{b1}c^2+8b^3\text{c1}+6abc\text{c1}\right)x^6+4c^2\left(2b\text{b1}c+6b^2\text{c1}+ac\text{c1}\right)x^7 + c^3(\text{b1}c+8b\text{c1})x^8+c^4\text{c1}x^9\right)dx$$

$$= a^4\text{b1}x + \frac{1}{2}a^3(8b\text{b1} + a\text{c1})x^2 + \frac{4}{3}a^2\left(6b^2\text{b1} + a\text{b1}c + 2ab\text{c1}\right)x^3 + a\left(8b^3\text{b1} + 6ab\text{b1}c + 6ab^2\text{c1} + a^2c\text{c1}\right)x^4 + \frac{2}{5}\left(8b^4\text{b1} + 24ab^2\text{b1}c + 3a^2\text{b1}c^2 + 16ab^3\text{c1} + 12a^2bc\text{c1}\right)x^5 + \frac{1}{3}\left(16b^3\text{b1}c + 12ab\text{b1}c^2 + 8b^4\text{c1} + 24ab^2c\text{c1} + 3a^2c^2\text{c1}\right)x^6 + \frac{4}{7}c\left(6b^2\text{b1}c + a\text{b1}c^2 + 8b^3\text{c1} + 6abc\text{c1}\right)x^7 + \frac{1}{2}c^2\left(2b\text{b1}c + 6b^2\text{c1} + ac\text{c1}\right)x^8 + \frac{1}{9}c^3(\text{b1}c + 8b\text{c1})x^9 + \frac{1}{10}c^4\text{c1}x^{10}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^4\,dx = a^4\text{b1}x + \frac{1}{2}a^3(8b\text{b1} + a\text{c1})x^2 + \frac{4}{3}a^2\left(6b^2\text{b1} + a\text{b1}c + 2ab\text{c1}\right)x^3 + a\left(8b^3\text{b1} + 6ab\text{b1}c + 6ab^2\text{c1} + a^2c\text{c1}\right)x^4 + \frac{2}{5}\left(8b^4\text{b1} + 24ab^2\text{b1}c + 3a^2\text{b1}c^2 + 16ab^3\text{c1} + 12a^2bc\text{c1}\right)x^5 + \frac{1}{3}\left(16b^3\text{b1}c + 12ab\text{b1}c^2 + 8b^4\text{c1} + 24ab^2c\text{c1} + 3a^2c^2\text{c1}\right)x^6 + \frac{4}{7}c\left(6b^2\text{b1}c + a\text{b1}c^2 + 8b^3\text{c1} + 6abc\text{c1}\right)x^7 + \frac{1}{2}c^2\left(2b\text{b1}c + 6b^2\text{c1} + ac\text{c1}\right)x^8 + \frac{1}{9}c^3(\text{b1}c + 8b\text{c1})x^9 + \frac{1}{10}c^4\text{c1}x^{10}$$

```
[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x]
```

```
[Out] a^4*b1*x + (a^3*(8*b*b1 + a*c1)*x^2)/2 + (4*a^2*(6*b^2*b1 + a*b1*c + 2*a*b*
c1)*x^3)/3 + a*(8*b^3*b1 + 6*a*b*b1*c + 6*a*b^2*c1 + a^2*c*c1)*x^4 + (2*(8*
b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 + 16*a*b^3*c1 + 12*a^2*b*c*c1)*x^5)/5
 + ((16*b^3*b1*c + 12*a*b*b1*c^2 + 8*b^4*c1 + 24*a*b^2*c*c1 + 3*a^2*c^2*c1)
*x^6)/3 + (4*c*(6*b^2*b1*c + a*b1*c^2 + 8*b^3*c1 + 6*a*b*c*c1)*x^7)/7 + (c^
2*(2*b*b1*c + 6*b^2*c1 + a*c*c1)*x^8)/2 + (c^3*(b1*c + 8*b*c1)*x^9)/9 + (c^
4*c1*x^10)/10
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00

method	result
norman	$\frac{c^4 \,\mathrm{c1}\, x^{10}}{10}+\left(\frac{8}{9}\,\mathrm{c1}\, b\, c^3+\frac{1}{9}\,\mathrm{b1}\ c^4\right) x^9+\left(\frac{1}{2} a\, c^3\,\mathrm{c1}+3 b^2 c^2\,\mathrm{c1}+\mathrm{b1}\, b\, c^3\right) x^8+\left(\frac{24}{7} a b\, c^2\,\mathrm{c1}+\frac{4}{7} a\,\mathrm{b1}\ c^3\right.$
gosper	$a^4\,\mathrm{b1}\, x+\frac{8}{9} x^9\,\mathrm{c1}\, b\, c^3+\frac{1}{2} x^2\,\mathrm{c1}\ a^4+\frac{1}{9} x^9\,\mathrm{b1}\ c^4+\frac{8}{3} x^6 b^4\,\mathrm{c1}+\frac{16}{5} x^5 b^4\,\mathrm{b1}+\frac{1}{2} x^8 a\, c^3\,\mathrm{c1}+3 x^8 b^2 c^2\,\mathrm{c1}$
risch	$a^4\,\mathrm{b1}\, x+\frac{8}{9} x^9\,\mathrm{c1}\, b\, c^3+\frac{1}{2} x^2\,\mathrm{c1}\ a^4+\frac{1}{9} x^9\,\mathrm{b1}\ c^4+\frac{8}{3} x^6 b^4\,\mathrm{c1}+\frac{16}{5} x^5 b^4\,\mathrm{b1}+\frac{1}{2} x^8 a\, c^3\,\mathrm{c1}+3 x^8 b^2 c^2\,\mathrm{c1}$
parallelrisch	$a^4\,\mathrm{b1}\, x+\frac{8}{9} x^9\,\mathrm{c1}\, b\, c^3+\frac{1}{2} x^2\,\mathrm{c1}\ a^4+\frac{1}{9} x^9\,\mathrm{b1}\ c^4+\frac{8}{3} x^6 b^4\,\mathrm{c1}+\frac{16}{5} x^5 b^4\,\mathrm{b1}+\frac{1}{2} x^8 a\, c^3\,\mathrm{c1}+3 x^8 b^2 c^2\,\mathrm{c1}$
default	$\frac{c^4\,\mathrm{c1}\ x^{10}}{10}+\frac{(8\,\mathrm{c1}\, b\, c^3+\mathrm{b1}\ c^4) x^9}{9}+\frac{(8\,\mathrm{b1}\, b\, c^3+\mathrm{c1}(2(2ac+4b^2)c^2+16b^2c^2)) x^8}{8}+\frac{(\mathrm{b1}(2(2ac+4b^2)c^2+16b^2c^2)+\mathrm{c1}(8ab\, c^2+}{7}$

```
[In] int((c1*x+b1)*(c*x^2+2*b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*c^4*c1*x^10+(8/9*c1*b*c^3+1/9*b1*c^4)*x^9+(1/2*a*c^3*c1+3*b^2*c^2*c1+b
1*b*c^3)*x^8+(24/7*a*b*c^2*c1+4/7*a*b1*c^3+32/7*b^3*c*c1+24/7*b^2*b1*c^2)*x
^7+(a^2*c^2*c1+8*a*b^2*c*c1+4*a*b*b1*c^2+8/3*b^4*c1+16/3*b^3*b1*c)*x^6+(24/
5*a^2*b*c*c1+6/5*a^2*b1*c^2+32/5*a*b^3*c1+48/5*a*b^2*b1*c+16/5*b^4*b1)*x^5+
(a^3*c*c1+6*a^2*b^2*c1+6*a^2*b*b1*c+8*a*b^3*b1)*x^4+(8/3*c1*a^3*b+4/3*a^3*b
1*c+8*a^2*b^2*b1)*x^3+(1/2*c1*a^4+4*b1*a^3*b)*x^2+a^4*b1*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.04

$$\begin{aligned}\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx &= \frac{1}{10}c^4c_1x^{10}+\frac{1}{9}\left(b_1c^4+8\,bc^3c_1\right)x^9+\frac{1}{2}\left(2\,bb_1c^3+\left(6\,b^2c^2+ac^3\right)c_1\right)x^8\\&\quad+\frac{4}{7}\left(6\,b^2b_1c^2+ab_1c^3+2\left(4\,b^3c+3\,abc^2\right)c_1\right)x^7\\&\quad+\frac{1}{3}\left(16\,b^3b_1c+12\,abb_1c^2+\left(8\,b^4+24\,ab^2c+3\,a^2c^2\right)c_1\right)x^6+a^4b_1x\\&\quad+\frac{2}{5}\left(8\,b^4b_1+24\,ab^2b_1c+3\,a^2b_1c^2+4\left(4\,ab^3+3\,a^2bc\right)c_1\right)x^5\\&\quad+\left(8\,ab^3b_1+6\,a^2bb_1c+\left(6\,a^2b^2+a^3c\right)c_1\right)x^4\\&\quad+\frac{4}{3}\left(6\,a^2b^2b_1+a^3b_1c+2\,a^3bc_1\right)x^3+\frac{1}{2}\left(8\,a^3bb_1+a^4c_1\right)x^2\end{aligned}$$

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/10*c^4*c1*x^10 + 1/9*(b1*c^4 + 8*b*c^3*c1)*x^9 + 1/2*(2*b*b1*c^3 + (6*b^2
*c^2 + a*c^3)*c1)*x^8 + 4/7*(6*b^2*b1*c^2 + a*b1*c^3 + 2*(4*b^3*c + 3*a*b*c
```

```
^2)*c1)*x^7 + 1/3*(16*b^3*b1*c + 12*a*b*b1*c^2 + (8*b^4 + 24*a*b^2*c + 3*a^
2*c^2)*c1)*x^6 + a^4*b1*x + 2/5*(8*b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 +
4*(4*a*b^3 + 3*a^2*b*c)*c1)*x^5 + (8*a*b^3*b1 + 6*a^2*b*b1*c + (6*a^2*b^2 +
 a^3*c)*c1)*x^4 + 4/3*(6*a^2*b^2*b1 + a^3*b1*c + 2*a^3*b*c1)*x^3 + 1/2*(8*a
^3*b*b1 + a^4*c1)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.19

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx = a^4b_1x+\frac{c^4c_1x^{10}}{10}+x^9\cdot\left(\frac{8bc^3c_1}{9}+\frac{b_1c^4}{9}\right) + x^8\left(\frac{ac^3c_1}{2}+3b^2c^2c_1+bb_1c^3\right)+x^7 \cdot\left(\frac{24abc^2c_1}{7}+\frac{4ab_1c^3}{7}+\frac{32b^3cc_1}{7}+\frac{24b^2b_1c^2}{7}\right) + x^6\left(a^2c^2c_1+8ab^2cc_1+4abb_1c^2+\frac{8b^4c_1}{3}+\frac{16b^3b_1c}{3}\right)+x^5 \cdot\left(\frac{24a^2bcc_1}{5}+\frac{6a^2b_1c^2}{5}+\frac{32ab^3c_1}{5}+\frac{48ab^2b_1c}{5}+\frac{16b^4b_1}{5}\right) + x^4\left(a^3cc_1+6a^2b^2c_1+6a^2bb_1c+8ab^3b_1\right)+x^3 \cdot\left(\frac{8a^3bc_1}{3}+\frac{4a^3b_1c}{3}+8a^2b^2b_1\right)+x^2\left(\frac{a^4c_1}{2}+4a^3bb_1\right)$$

```
[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a)**4,x)
```

```
[Out] a**4*b1*x + c**4*c1*x**10/10 + x**9*(8*b*c**3*c1/9 + b1*c**4/9) + x**8*(a*c
**3*c1/2 + 3*b**2*c**2*c1 + b*b1*c**3) + x**7*(24*a*b*c**2*c1/7 + 4*a*b1*c*
*3/7 + 32*b**3*c*c1/7 + 24*b**2*b1*c**2/7) + x**6*(a**2*c**2*c1 + 8*a*b**2*
c*c1 + 4*a*b*b1*c**2 + 8*b**4*c1/3 + 16*b**3*b1*c/3) + x**5*(24*a**2*b*c*c1
/5 + 6*a**2*b1*c**2/5 + 32*a*b**3*c1/5 + 48*a*b**2*b1*c/5 + 16*b**4*b1/5) +
 x**4*(a**3*c*c1 + 6*a**2*b**2*c1 + 6*a**2*b*b1*c + 8*a*b**3*b1) + x**3*(8*
a**3*b*c1/3 + 4*a**3*b1*c/3 + 8*a**2*b**2*b1) + x**2*(a**4*c1/2 + 4*a**3*b*
b1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.04

$$\begin{aligned}\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx &= \frac{1}{10}\,c^4c_1x^{10}+\frac{1}{9}\left(b_1c^4+8\,bc^3c_1\right)x^9+\frac{1}{2}\left(2\,bb_1c^3+\left(6\,b^2c^2+ac^3\right)c_1\right)x^8\\&+\frac{4}{7}\left(6\,b^2b_1c^2+ab_1c^3+2\left(4\,b^3c+3\,abc^2\right)c_1\right)x^7\\&+\frac{1}{3}\left(16\,b^3b_1c+12\,abb_1c^2+\left(8\,b^4+24\,ab^2c+3\,a^2c^2\right)c_1\right)x^6+a^4b_1x\\&+\frac{2}{5}\left(8\,b^4b_1+24\,ab^2b_1c+3\,a^2b_1c^2+4\left(4\,ab^3+3\,a^2bc\right)c_1\right)x^5\\&+\left(8\,ab^3b_1+6\,a^2bb_1c+\left(6\,a^2b^2+a^3c\right)c_1\right)x^4\\&+\frac{4}{3}\left(6\,a^2b^2b_1+a^3b_1c+2\,a^3bc_1\right)x^3+\frac{1}{2}\left(8\,a^3bb_1+a^4c_1\right)x^2\end{aligned}$$

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/10*c^4*c1*x^10 + 1/9*(b1*c^4 + 8*b*c^3*c1)*x^9 + 1/2*(2*b*b1*c^3 + (6*b^2
*c^2 + a*c^3)*c1)*x^8 + 4/7*(6*b^2*b1*c^2 + a*b1*c^3 + 2*(4*b^3*c + 3*a*b*c
^2)*c1)*x^7 + 1/3*(16*b^3*b1*c + 12*a*b*b1*c^2 + (8*b^4 + 24*a*b^2*c + 3*a^
2*c^2)*c1)*x^6 + a^4*b1*x + 2/5*(8*b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 +
4*(4*a*b^3 + 3*a^2*b*c)*c1)*x^5 + (8*a*b^3*b1 + 6*a^2*b*b1*c + (6*a^2*b^2 +
 a^3*c)*c1)*x^4 + 4/3*(6*a^2*b^2*b1 + a^3*b1*c + 2*a^3*b*c1)*x^3 + 1/2*(8*a
^3*b*b1 + a^4*c1)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.17

$$\begin{aligned}\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx &= \frac{1}{10}\,c^4c_1x^{10}+\frac{1}{9}\,b_1c^4x^9+\frac{8}{9}\,bc^3c_1x^9+bb_1c^3x^8+3\,b^2c^2c_1x^8\\&+\frac{1}{2}\,ac^3c_1x^8+\frac{24}{7}\,b^2b_1c^2x^7+\frac{4}{7}\,ab_1c^3x^7+\frac{32}{7}\,b^3cc_1x^7\\&+\frac{24}{7}\,abc^2c_1x^7+\frac{16}{3}\,b^3b_1cx^6+4\,abb_1c^2x^6+\frac{8}{3}\,b^4c_1x^6\\&+8\,ab^2cc_1x^6+a^2c^2c_1x^6+\frac{16}{5}\,b^4b_1x^5+\frac{48}{5}\,ab^2b_1cx^5\\&+\frac{6}{5}\,a^2b_1c^2x^5+\frac{32}{5}\,ab^3c_1x^5+\frac{24}{5}\,a^2bcc_1x^5+8\,ab^3b_1x^4\\&+6\,a^2bb_1cx^4+6\,a^2b^2c_1x^4+a^3cc_1x^4+8\,a^2b^2b_1x^3\\&+\frac{4}{3}\,a^3b_1cx^3+\frac{8}{3}\,a^3bc_1x^3+4\,a^3bb_1x^2+\frac{1}{2}\,a^4c_1x^2+a^4b_1x\end{aligned}$$

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/10*c^4*c1*x^10 + 1/9*b1*c^4*x^9 + 8/9*b*c^3*c1*x^9 + b*b1*c^3*x^8 + 3*b^2
*c^2*c1*x^8 + 1/2*a*c^3*c1*x^8 + 24/7*b^2*b1*c^2*x^7 + 4/7*a*b1*c^3*x^7 + 3
2/7*b^3*c*c1*x^7 + 24/7*a*b*c^2*c1*x^7 + 16/3*b^3*b1*c*x^6 + 4*a*b*b1*c^2*x
^6 + 8/3*b^4*c1*x^6 + 8*a*b^2*c*c1*x^6 + a^2*c^2*c1*x^6 + 16/5*b^4*b1*x^5 +
 48/5*a*b^2*b1*c*x^5 + 6/5*a^2*b1*c^2*x^5 + 32/5*a*b^3*c1*x^5 + 24/5*a^2*b*
c*c1*x^5 + 8*a*b^3*b1*x^4 + 6*a^2*b*b1*c*x^4 + 6*a^2*b^2*c1*x^4 + a^3*c*c1*
x^4 + 8*a^2*b^2*b1*x^3 + 4/3*a^3*b1*c*x^3 + 8/3*a^3*b*c1*x^3 + 4*a^3*b*b1*x
^2 + 1/2*a^4*c1*x^2 + a^4*b1*x
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx = x^9\left(\frac{b_1\,c^4}{9}+\frac{8\,b\,c_1\,c^3}{9}\right)+x^3\left(\frac{8\,c_1\,a^3\,b}{3}+\frac{4\,b_1\,c\,a^3}{3}+8\,b_1\,a^2\,b^2\right)+x^8\left(3\,c_1\,b^2\,c^2+b_1\,b\,c^3+\frac{a\,c_1\,c^3}{2}\right)+x^5\left(\frac{24\,c_1\,a^2\,b\,c}{5}+\frac{6\,b_1\,a^2\,c^2}{5}+\frac{32\,c_1\,a\,b^3}{5}+\frac{48\,b_1\,a\,b^2\,c}{5}+\frac{16\,b_1\,b^4}{5}\right)+x^6\left(c_1\,a^2\,c^2+8\,c_1\,a\,b^2\,c+4\,b_1\,a\,b\,c^2+\frac{8\,c_1\,b^4}{3}+\frac{16\,b_1\,b^3\,c}{3}\right)+x^4\left(c\,c_1\,a^3+6\,c_1\,a^2\,b^2+6\,b_1\,c\,a^2\,b+8\,b_1\,a\,b^3\right)+x^7\left(\frac{32\,c_1\,b^3\,c}{7}+\frac{24\,b_1\,b^2\,c^2}{7}+\frac{24\,a\,c_1\,b\,c^2}{7}+\frac{4\,a\,b_1\,c^3}{7}\right)+x^2\left(\frac{c_1\,a^4}{2}+4\,b\,b_1\,a^3\right)+\frac{c^4\,c_1\,x^{10}}{10}+a^4\,b_1\,x$$

```
[In] int((b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x)
```

```
[Out] x^9*((b1*c^4)/9 + (8*b*c^3*c1)/9) + x^3*(8*a^2*b^2*b1 + (8*a^3*b*c1)/3 + (4
*a^3*b1*c)/3) + x^8*(3*b^2*c^2*c1 + (a*c^3*c1)/2 + b*b1*c^3) + x^5*((16*b^4
*b1)/5 + (6*a^2*b1*c^2)/5 + (32*a*b^3*c1)/5 + (48*a*b^2*b1*c)/5 + (24*a^2*b
*c*c1)/5) + x^6*((8*b^4*c1)/3 + a^2*c^2*c1 + (16*b^3*b1*c)/3 + 4*a*b*b1*c^2
 + 8*a*b^2*c*c1) + x^4*(6*a^2*b^2*c1 + 8*a*b^3*b1 + a^3*c*c1 + 6*a^2*b*b1*c
) + x^7*((24*b^2*b1*c^2)/7 + (4*a*b1*c^3)/7 + (32*b^3*c*c1)/7 + (24*a*b*c^2
*c1)/7) + x^2*((a^4*c1)/2 + 4*a^3*b*b1) + (c^4*c1*x^10)/10 + a^4*b1*x
```

3.193 $\int (\mathbf{b1}+\mathbf{c1}x)\left(a+2bx+cx^2\right)^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 159

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^n\,dx=\frac{\mathrm{c1}(a+2bx+cx^2)^{1+n}}{2c(1+n)}-\frac{2^n(\mathrm{b1}c-b\mathrm{c1})\left(-\frac{b-\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-1-n}(a+2bx+cx^2)^{1+n}\,\mathrm{Hypergeometric2F1}\left(-n,1+n,2+n,\frac{b+\sqrt{b^2-ac}}{2\sqrt{b^2-ac}}\right.}{c\sqrt{b^2-ac}(1+n)}$$

```
[Out] 1/2*c1*(c*x^2+2*b*x+a)^(1+n)/c/(1+n)-2^n*(-b*c1+b1*c)*(c*x^2+2*b*x+a)^(1+n)
*hypergeom([-n, 1+n],[2+n],1/2*(b+c*x+(-a*c+b^2)^(1/2))/(-a*c+b^2)^(1/2))*(
(-b-c*x+(-a*c+b^2)^(1/2))/(-a*c+b^2)^(1/2))^(-1-n)/c/(1+n)/(-a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {654, 638}

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^n\,dx=\frac{\mathrm{c1}(a+2bx+cx^2)^{n+1}}{2c(n+1)}-\frac{2^n(\mathrm{b1}c-b\mathrm{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{-n-1}(a+2bx+cx^2)^{n+1}\,\mathrm{Hypergeometric2F1}\left(-n,n+1,n+2,\frac{b+cx+\sqrt{\ldots}}{2\sqrt{b^2-\ldots}}\right.}{c(n+1)\sqrt{b^2-ac}}$$

```
[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] (c1*(a + 2*b*x + c*x^2)^(1 + n))/(2*c*(1 + n)) - (2^n*(b1*c - b*c1)*(-((b -
 Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 - n)*(a + 2*b*x + c*x^2)^(1 +
 n)*Hypergeometric2F1[-n, 1 + n, 2 + n, (b + Sqrt[b^2 - a*c] + c*x)/(2*Sqrt
[b^2 - a*c])])/(c*Sqrt[b^2 - a*c]*(1 + n))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{c1}(a+2bx+cx^2)^{1+n}}{2c(1+n)} + \frac{(2\text{b1}c-2b\text{c1})\int (a+2bx+cx^2)^n\,dx}{2c}$$

$$= \frac{\text{c1}(a+2bx+cx^2)^{1+n}}{2c(1+n)} - \frac{2^n(\text{b1}c-b\text{c1})\left(-\frac{b-\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-1-n}(a+2bx+cx^2)^{1+n}\,\text{Hypergeometric2F1}\left(-n,1+n,2+n,\frac{b+}{}\right.}{c\sqrt{b^2-ac}(1+n)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.67 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.68

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^n\,dx$$

$$= \frac{1}{2}(a+x(2b+cx))^n\left(\text{c1}x^2\left(\frac{b-\sqrt{b^2-ac}+cx}{b-\sqrt{b^2-ac}}\right)^{-n}\left(\frac{b+\sqrt{b^2-ac}+cx}{b+\sqrt{b^2-ac}}\right)^{-n}\text{AppellF1}\left(2,-n,-n,3,-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{-b+\sqrt{b^2-ac}}\right) + \frac{2^{1+n}\text{b1}\left(b-\sqrt{b^2-ac}+cx\right)\left(\frac{b+\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-n}\text{Hypergeometric2F1}\left(-n,1+n,2+n,\frac{-b+\sqrt{b^2-ac}-cx}{2\sqrt{b^2-ac}}\right)}{c(1+n)}\right)$$

```
[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] ((a + x*(2*b + c*x))^n*((c1*x^2*AppellF1[2, -n, -n, 3, -((c*x)/(b + Sqrt[b^
2 - a*c])), (c*x)/(-b + Sqrt[b^2 - a*c])])/(((b - Sqrt[b^2 - a*c] + c*x)/(b
 - Sqrt[b^2 - a*c]))^n*((b + Sqrt[b^2 - a*c] + c*x)/(b + Sqrt[b^2 - a*c]))^
n) + (2^(1 + n)*b1*(b - Sqrt[b^2 - a*c] + c*x)*Hypergeometric2F1[-n, 1 + n,
 2 + n, (-b + Sqrt[b^2 - a*c] - c*x)/(2*Sqrt[b^2 - a*c])])/(c*(1 + n)*((b +
 Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c])^n)))/2
```

Maple [F]

$$\int (\mathrm{c1}\,x + \mathrm{b1})\left(c\,x^2 + 2bx + a\right)^n dx$$

```
[In] int((c1*x+b1)*(c*x^2+2*b*x+a)^n,x)
```

```
[Out] int((c1*x+b1)*(c*x^2+2*b*x+a)^n,x)
```

Fricas [F]

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^n dx = \int (c_1x + b_1)\left(cx^2 + 2\,bx + a\right)^n dx$$

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

Sympy [F]

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^n dx = \int (b_1 + c_1x)\left(a + 2bx + cx^2\right)^n dx$$

```
[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a)**n,x)
```

```
[Out] Integral((b1 + c1*x)*(a + 2*b*x + c*x**2)**n, x)
```

Maxima [F]

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^n dx = \int (c_1x + b_1)\left(cx^2 + 2\,bx + a\right)^n dx$$

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

Giac [F]

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^n \, dx = \int (c_1x + b_1)\left(cx^2 + 2\,bx + a\right)^n dx$$

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="giac")
[Out] integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^n \, dx = \int (b_1 + c_1\,x) \, \left(c\,x^2 + 2\,b\,x + a\right)^n dx$$

```
[In] int((b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x)
[Out] int((b1 + c1*x)*(a + 2*b*x + c*x^2)^n, x)
```

3.194 $\int \frac{\mathbf{b1}+\mathbf{c1}x}{a+2bx+cx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{a+2bx+cx^2}\,dx=-\frac{(\mathrm{b1}c-b\mathrm{c1})\mathrm{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}}+\frac{\mathrm{c1}\log\left(a+2bx+cx^2\right)}{2c}$$

```
[Out] 1/2*c1*ln(c*x^2+2*b*x+a)/c-(-b*c1+b1*c)*arctanh((c*x+b)/(-a*c+b^2)^(1/2))/c
/(-a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {648, 632, 212, 642}

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{a+2bx+cx^2}\,dx=\frac{\mathrm{c1}\log\left(a+2bx+cx^2\right)}{2c}-\frac{(\mathrm{b1}c-b\mathrm{c1})\mathrm{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}}$$

```
[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2),x]
```

```
[Out] -(((b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(c*Sqrt[b^2 - a*c])) +
 (c1*Log[a + 2*b*x + c*x^2])/(2*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{c1}\int \frac{2b+2cx}{a+2bx+cx^2}\,dx}{2c} + \frac{(2\text{b1}c-2b\text{c1})\int \frac{1}{a+2bx+cx^2}\,dx}{2c}\\
&= \frac{\text{c1}\log\left(a+2bx+cx^2\right)}{2c} - \frac{(2\text{b1}c-2b\text{c1})\text{Subst}\left(\int \frac{1}{4(b^2-ac)-x^2}\,dx, x, 2b+2cx\right)}{c}\\
&= -\frac{(\text{b1}c-b\text{c1})\text{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}} + \frac{\text{c1}\log\left(a+2bx+cx^2\right)}{2c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{\text{b1}+\text{c1}x}{a+2bx+cx^2}\,dx = \frac{(\text{b1}c-b\text{c1})\arctan\left(\frac{b+cx}{\sqrt{-b^2+ac}}\right)}{c\sqrt{-b^2+ac}} + \frac{\text{c1}\log\left(a+2bx+cx^2\right)}{2c}$$

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2),x]
[Out] ((b1*c - b*c1)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/(c*Sqrt[-b^2 + a*c]) + (
c1*Log[a + 2*b*x + c*x^2])/(2*c)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

method	result
default	$\frac{\mathrm{c1}\ln\left(c\,x^{2}+2bx+a\right)}{2c}+\frac{\left(\mathrm{b1}-\frac{\mathrm{c1}\,b}{c}\right)\arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^{2}}}\right)}{\sqrt{ac-b^{2}}}$
risch	$\frac{\ln\left(-abc\,\mathrm{c1}+a\,\mathrm{b1}\;c^{2}+b^{3}\,\mathrm{c1}-b^{2}\,\mathrm{b1}\,c-\sqrt{-\left(b\,\mathrm{c1}-\mathrm{b1}\,c\right)^{2}\left(ac-b^{2}\right)}\,cx-\sqrt{-\left(b\,\mathrm{c1}-\mathrm{b1}\,c\right)^{2}\left(ac-b^{2}\right)}\,b\right)a\,\mathrm{c1}}{2ac-2b^{2}}-\frac{\ln\left(-abc\,\mathrm{c1}+a\,\mathrm{b1}\;c^{2}+b^{3}\,\mathrm{c1}\right.}{}$

[In] int((c1*x+b1)/(c*x^2+2*b*x+a),x,method=_RETURNVERBOSE)

[Out] 1/2*c1*ln(c*x^2+2*b*x+a)/c+(b1-c1*b/c)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.12

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{a+2bx+cx^2}\,dx$$
$$=\left[\frac{(b^2-ac)c_1\log\left(cx^2+2\,bx+a\right)-\sqrt{b^2-ac}(b_1c-bc_1)\log\left(\frac{c^2x^2+2\,bcx+2\,b^2-ac+2\,\sqrt{b^2-ac}(cx+b)}{cx^2+2\,bx+a}\right)}{2\,(b^2c-ac^2)},\ \frac{(b^2-ac)c_1\,\mathrm{l}}{}\right.$$

[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="fricas")

[Out] [1/2*((b^2 - a*c)*c1*log(c*x^2 + 2*b*x + a) - sqrt(b^2 - a*c)*(b1*c - b*c1)*log((c^2*x^2 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*x + a)))/(b^2*c - a*c^2), 1/2*((b^2 - a*c)*c1*log(c*x^2 + 2*b*x + a) - 2*sqrt(-b^2 + a*c)*(b1*c - b*c1)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(b^2 - a*c)))/(b^2*c - a*c^2)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(53) = 106.

Time = 0.40 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.78

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{a+2bx+cx^2}\,dx = \left(\frac{c_1}{2c} - \frac{\sqrt{-ac+b^2}(bc_1-b_1c)}{2c(ac-b^2)}\right)\log\left(x+\frac{-2ac\left(\frac{c_1}{2c}-\frac{\sqrt{-ac+b^2}(bc_1-b_1c)}{2c(ac-b^2)}\right)+ac_1+2b^2\left(\frac{c_1}{2c}-\frac{\sqrt{-ac+b^2}(bc_1-b_1c)}{2c(ac-b^2)}\right)-bb_1}{bc_1-b_1c}\right) + \left(\frac{c_1}{2c} + \frac{\sqrt{-ac+b^2}(bc_1-b_1c)}{2c(ac-b^2)}\right)\log\left(x+\frac{-2ac\left(\frac{c_1}{2c}+\frac{\sqrt{-ac+b^2}(bc_1-b_1c)}{2c(ac-b^2)}\right)+ac_1+2b^2\left(\frac{c_1}{2c}+\frac{\sqrt{-ac+b^2}(bc_1-b_1c)}{2c(ac-b^2)}\right)-bb_1}{bc_1-b_1c}\right)$$

```
[In] integrate((c1*x+b1)/(c*x**2+2*b*x+a),x)
```

```
[Out] (c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2)))*log(x + (-2
*a*c*(c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2))) + a*c1
 + 2*b**2*(c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2))) -
 b*b1)/(b*c1 - b1*c)) + (c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a
*c - b**2)))*log(x + (-2*a*c*(c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2
*c*(a*c - b**2))) + a*c1 + 2*b**2*(c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*
c)/(2*c*(a*c - b**2))) - b*b1)/(b*c1 - b1*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{a+2bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{\text{b1} + \text{c1}x}{a + 2bx + cx^2}\,dx = \frac{c_1 \log\left(cx^2 + 2\,bx + a\right)}{2\,c} + \frac{(b_1c - bc_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2+ac}c}$$

[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="giac")

[Out] 1/2*c1*log(c*x^2 + 2*b*x + a)/c + (b1*c - b*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/(sqrt(-b^2 + a*c)*c)

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.38

$$\int \frac{\text{b1} + \text{c1}x}{a + 2bx + cx^2}\,dx = \frac{b_1\,\mathrm{atan}\left(\frac{b}{\sqrt{a\,c-b^2}} + \frac{c\,x}{\sqrt{a\,c-b^2}}\right)}{\sqrt{a\,c-b^2}} - \frac{2\,b^2\,c_1\,\ln\left(c\,x^2 + 2\,b\,x + a\right)}{4\,a\,c^2 - 4\,b^2\,c} + \frac{2\,a\,c\,c_1\,\ln\left(c\,x^2 + 2\,b\,x + a\right)}{4\,a\,c^2 - 4\,b^2\,c} - \frac{b\,c_1\,\mathrm{atan}\left(\frac{b}{\sqrt{a\,c-b^2}} + \frac{c\,x}{\sqrt{a\,c-b^2}}\right)}{c\,\sqrt{a\,c-b^2}}$$

[In] int((b1 + c1*x)/(a + 2*b*x + c*x^2),x)

[Out] (b1*atan(b/(a*c - b^2)^(1/2) + (c*x)/(a*c - b^2)^(1/2)))/(a*c - b^2)^(1/2) - (2*b^2*c1*log(a + 2*b*x + c*x^2))/(4*a*c^2 - 4*b^2*c) + (2*a*c*c1*log(a + 2*b*x + c*x^2))/(4*a*c^2 - 4*b^2*c) - (b*c1*atan(b/(a*c - b^2)^(1/2) + (c*x)/(a*c - b^2)^(1/2)))/(c*(a*c - b^2)^(1/2))

3.195 $\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^2}\,dx = -\frac{b\mathrm{b1}-a\mathrm{c1}+(\mathrm{b1}c-b\mathrm{c1})x}{2(b^2-ac)(a+2bx+cx^2)}+\frac{(\mathrm{b1}c-b\mathrm{c1})\operatorname{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{2(b^2-ac)^{3/2}}$$

```
[Out] 1/2*(-b*b1+a*c1-(-b*c1+b1*c)*x)/(-a*c+b^2)/(c*x^2+2*b*x+a)+1/2*(-b*c1+b1*c)
*arctanh((c*x+b)/(-a*c+b^2)^(1/2))/(-a*c+b^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {652, 632, 212}

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^2}\,dx = \frac{(\mathrm{b1}c-b\mathrm{c1})\operatorname{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{2(b^2-ac)^{3/2}}-\frac{-a\mathrm{c1}+x(\mathrm{b1}c-b\mathrm{c1})+b\mathrm{b1}}{2(b^2-ac)(a+2bx+cx^2)}$$

```
[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] -1/2*(b*b1 - a*c1 + (b1*c - b*c1)*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)) + ((
b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(2*(b^2 - a*c)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\text{integral} = -\frac{b\text{b1} - a\text{c1} + (\text{b1}c - b\text{c1})x}{2(b^2 - ac)(a + 2bx + cx^2)} - \frac{(\text{b1}c - b\text{c1}) \int \frac{1}{a+2bx+cx^2} \, dx}{2(b^2 - ac)}$$

$$= -\frac{b\text{b1} - a\text{c1} + (\text{b1}c - b\text{c1})x}{2(b^2 - ac)(a + 2bx + cx^2)} + \frac{(\text{b1}c - b\text{c1})\text{Subst}\left(\int \frac{1}{4(b^2-ac)-x^2} \, dx, x, 2b + 2cx\right)}{b^2 - ac}$$

$$= -\frac{b\text{b1} - a\text{c1} + (\text{b1}c - b\text{c1})x}{2(b^2 - ac)(a + 2bx + cx^2)} + \frac{(\text{b1}c - b\text{c1})\text{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{2(b^2 - ac)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^2} \, dx = \frac{\frac{-b\text{b1}+a\text{c1}-\text{b1}cx+b\text{c1}x}{a+x(2b+cx)} + \frac{(-\text{b1}c+b\text{c1}) \arctan\left(\frac{b+cx}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2+ac}}}{2(b^2 - ac)}$$

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] ((-(b*b1) + a*c1 - b1*c*x + b*c1*x)/(a + x*(2*b + c*x)) + ((-(b1*c) + b*c1)
*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(2*(b^2 - a*c))
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

method	result
default	$\frac{(-2b\,\mathrm{c1}+2\,\mathrm{b1}\,c)x+2b\,\mathrm{b1}-2a\,\mathrm{c1}}{(4ac-4b^2)(c\,x^2+2bx+a)}+\frac{(-2b\,\mathrm{c1}+2\,\mathrm{b1}\,c)\arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^2}}\right)}{(4ac-4b^2)\sqrt{ac-b^2}}$
risch	$\frac{-\frac{(b\,\mathrm{c1}-\mathrm{b1}\,c)x}{2(ac-b^2)}-\frac{a\,\mathrm{c1}-b\,\mathrm{b1}}{2(ac-b^2)}}{c\,x^2+2bx+a}+\frac{\ln\left((-c^2a+b^2c)x-(-ac+b^2)^{\frac{3}{2}}-abc+b^3\right)b\,\mathrm{c1}}{4(-ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left((-c^2a+b^2c)x-(-ac+b^2)^{\frac{3}{2}}-abc+b^3\right)\mathrm{b1}\,c}{4(-ac+b^2)^{\frac{3}{2}}}-\ldots$

```
[In] int((c1*x+b1)/(c*x^2+2*b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)+(-2*b*c1+2
*b1*c)/(4*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(81) = 162.

Time = 0.26 (sec) , antiderivative size = 447, normalized size of antiderivative = 5.02

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^2}\,dx$$
$$=\left[-\frac{2\,b^3b_1-2\,abb_1c-(ab_1c-abc_1+(b_1c^2-bcc_1)x^2+2\,(bb_1c-b^2c_1)x)\sqrt{b^2-ac}\log\left(\frac{c^2x^2+2\,bcx+2\,b^2-ac+2\,\sqrt{b^2\ldots}}{cx^2+2\,bx+a}\right.}{4\,(ab^4-2\,a^2b^2c+a^3c^2+(b^4c-2\,ab^2c^2+a^2c^3)x^2+2\,(b^5-\ldots}\right.$$
$$-\frac{b^3b_1-abb_1c-(ab_1c-abc_1+(b_1c^2-bcc_1)x^2+2\,(bb_1c-b^2c_1)x)\sqrt{-b^2+ac}\arctan\left(-\frac{\sqrt{-b^2+ac}(cx+b)}{b^2-ac}\right)-\ldots}{2\,(ab^4-2\,a^2b^2c+a^3c^2+(b^4c-2\,ab^2c^2+a^2c^3)x^2+2\,(b^5-2\,ab^3c-\ldots}$$

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*b^3*b1 - 2*a*b*b1*c - (a*b1*c - a*b*c1 + (b1*c^2 - b*c*c1)*x^2 + 2
*(b*b1*c - b^2*c1)*x)*sqrt(b^2 - a*c)*log((c^2*x^2 + 2*b*c*x + 2*b^2 - a*c
+ 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*x + a)) - 2*(a*b^2 - a^2*c)*c1
+ 2*(b^2*b1*c - a*b1*c^2 - (b^3 - a*b*c)*c1)*x)/(a*b^4 - 2*a^2*b^2*c + a^3*
c^2 + (b^4*c - 2*a*b^2*c^2 + a^2*c^3)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2*b*c^2)
*x), -1/2*(b^3*b1 - a*b*b1*c - (a*b1*c - a*b*c1 + (b1*c^2 - b*c*c1)*x^2 + 2
*(b*b1*c - b^2*c1)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(
b^2 - a*c)) - (a*b^2 - a^2*c)*c1 + (b^2*b1*c - a*b1*c^2 - (b^3 - a*b*c)*c1)
*x)/(a*b^4 - 2*a^2*b^2*c + a^3*c^2 + (b^4*c - 2*a*b^2*c^2 + a^2*c^3)*x^2 +
2*(b^5 - 2*a*b^3*c + a^2*b*c^2)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(75) = 150.

Time = 0.52 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.63

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^2}\,dx$$
$$= \frac{\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1 - b_1c)\log\left(x + \frac{-a^2c^2\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)+2ab^2c\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)-b^4\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)+b^2c_1-bb_1c}{bcc_1-b_1c^2}\right)}{4}$$
$$- \frac{\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1 - b_1c)\log\left(x + \frac{a^2c^2\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)-2ab^2c\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)+b^4\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)+b^2c_1-bb_1c}{bcc_1-b_1c^2}\right)}{4}$$
$$+ \frac{-ac_1 + bb_1 + x(-bc_1 + b_1c)}{2a^2c - 2ab^2 + x^2\cdot(2ac^2 - 2b^2c) + x\,(4abc - 4b^3)}$$

```
[In] integrate((c1*x+b1)/(c*x**2+2*b*x+a)**2,x)
```

```
[Out] sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c)*log(x + (-a**2*c**2*sqrt(-1/(a*c - b
**2)**3)*(b*c1 - b1*c) + 2*a*b**2*c*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c)
- b**4*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) + b**2*c1 - b*b1*c)/(b*c*c1 -
 b1*c**2))/4 - sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c)*log(x + (a**2*c**2*sq
rt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) - 2*a*b**2*c*sqrt(-1/(a*c - b**2)**3)*
(b*c1 - b1*c) + b**4*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) + b**2*c1 - b*b
1*c)/(b*c*c1 - b1*c**2))/4 + (-a*c1 + b*b1 + x*(-b*c1 + b1*c))/(2*a**2*c -
2*a*b**2 + x**2*(2*a*c**2 - 2*b**2*c) + x*(4*a*b*c - 4*b**3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^2}\,dx=-\frac{(b_1c-bc_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{2\,(b^2-ac)\sqrt{-b^2+ac}}-\frac{b_1cx-bc_1x+bb_1-ac_1}{2\,(cx^2+2\,bx+a)(b^2-ac)}$$

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b1*c - b*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^2 - a*c)*sqrt(-b^
2 + a*c)) - 1/2*(b1*c*x - b*c1*x + b*b1 - a*c1)/((c*x^2 + 2*b*x + a)*(b^2 -
 a*c))
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.79

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^2}\,dx=\frac{\mathrm{atan}\left(\frac{2\left(\frac{\left(4\,b^3-4\,a\,b\,c\right)\,\left(b\,c_1-b_1\,c\right)}{8\,{\left(a\,c-b^2\right)}^{5/2}}-\frac{c\,x\,\left(b\,c_1-b_1\,c\right)}{2\,{\left(a\,c-b^2\right)}^{3/2}}\right)\,\left(a\,c-b^2\right)}{b\,c_1-b_1\,c}\right)\,\left(b\,c_1-b_1\,c\right)}{2\,{\left(a\,c-b^2\right)}^{3/2}}-\frac{\frac{a\,c_1-b\,b_1}{2\,\left(a\,c-b^2\right)}+\frac{x\,\left(b\,c_1-b_1\,c\right)}{2\,\left(a\,c-b^2\right)}}{c\,x^2+2\,b\,x+a}$$

```
[In] int((b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x)
```

```
[Out] (atan((2*(((4*b^3 - 4*a*b*c)*(b*c1 - b1*c))/(8*(a*c - b^2)^(5/2)) - (c*x*(b
*c1 - b1*c))/(2*(a*c - b^2)^(3/2)))*(a*c - b^2))/(b*c1 - b1*c))*(b*c1 - b1*
c))/(2*(a*c - b^2)^(3/2)) - ((a*c1 - b*b1)/(2*(a*c - b^2)) + (x*(b*c1 - b1*
c))/(2*(a*c - b^2)))/(a + 2*b*x + c*x^2)
```

3.196 $\int \frac{\mathbf{b1}+\mathbf{c1}x}{(a+2bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^3}\,dx = -\frac{b\mathrm{b1}-a\mathrm{c1}+(\mathrm{b1}c-b\mathrm{c1})x}{4(b^2-ac)(a+2bx+cx^2)^2}+\frac{3(\mathrm{b1}c-b\mathrm{c1})(b+cx)}{8(b^2-ac)^2(a+2bx+cx^2)} - \frac{3c(\mathrm{b1}c-b\mathrm{c1})\operatorname{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{8(b^2-ac)^{5/2}}$$

```
[Out] 1/4*(-b*b1+a*c1-(-b*c1+b1*c)*x)/(-a*c+b^2)/(c*x^2+2*b*x+a)^2+3/8*(-b*c1+b1*
c)*(c*x+b)/(-a*c+b^2)^2/(c*x^2+2*b*x+a)-3/8*c*(-b*c1+b1*c)*arctanh((c*x+b)/
(-a*c+b^2)^(1/2))/(-a*c+b^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {652, 628, 632, 212}

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^3}\,dx = -\frac{3c(\mathrm{b1}c-b\mathrm{c1})\operatorname{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{8(b^2-ac)^{5/2}} + \frac{3(b+cx)(\mathrm{b1}c-b\mathrm{c1})}{8(b^2-ac)^2(a+2bx+cx^2)} - \frac{-a\mathrm{c1}+x(\mathrm{b1}c-b\mathrm{c1})+b\mathrm{b1}}{4(b^2-ac)(a+2bx+cx^2)^2}$$

```
[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] -1/4*(b*b1 - a*c1 + (b1*c - b*c1)*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)^2) +
(3*(b1*c - b*c1)*(b + c*x))/(8*(b^2 - a*c)^2*(a + 2*b*x + c*x^2)) - (3*c*(b
1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(8*(b^2 - a*c)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\text{b1} - a\text{c1} + (\text{b1}c - b\text{c1})x}{4\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)^2} - \frac{(3(\text{b1}c - b\text{c1}))\int \frac{1}{\left(a+2bx+cx^2\right)^2}\,dx}{4\left(b^2 - ac\right)} \\
&= -\frac{b\text{b1} - a\text{c1} + (\text{b1}c - b\text{c1})x}{4\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)^2} + \frac{3(\text{b1}c - b\text{c1})(b + cx)}{8\left(b^2 - ac\right)^2\left(a + 2bx + cx^2\right)} + \frac{(3c(\text{b1}c - b\text{c1}))\int \frac{1}{a+2bx+cx^2}\,dx}{8\left(b^2 - ac\right)^2} \\
&= -\frac{b\text{b1} - a\text{c1} + (\text{b1}c - b\text{c1})x}{4\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)^2} + \frac{3(\text{b1}c - b\text{c1})(b + cx)}{8\left(b^2 - ac\right)^2\left(a + 2bx + cx^2\right)} \\
&\quad - \frac{(3c(\text{b1}c - b\text{c1}))\text{Subst}\left(\int \frac{1}{4(b^2-ac)-x^2}\,dx, x, 2b + 2cx\right)}{4\left(b^2 - ac\right)^2} \\
&= -\frac{b\text{b1} - a\text{c1} + (\text{b1}c - b\text{c1})x}{4\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)^2} + \frac{3(\text{b1}c - b\text{c1})(b + cx)}{8\left(b^2 - ac\right)^2\left(a + 2bx + cx^2\right)} \\
&\quad - \frac{3c(\text{b1}c - b\text{c1})\text{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{8\left(b^2 - ac\right)^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^3} \, dx = \frac{\frac{2(b^2-ac)(-b\text{b1}+a\text{c1}-\text{b1}cx+b\text{c1}x)}{(a+x(2b+cx))^2} + \frac{3(\text{b1}c-b\text{c1})(b+cx)}{a+x(2b+cx)} + \frac{3c(\text{b1}c-b\text{c1})\arctan\left(\frac{b+cx}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2+ac}}}{8\left(b^2-ac\right)^2}$$

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] ((2*(b^2 - a*c)*(-(b*b1) + a*c1 - b1*c*x + b*c1*x))/(a + x*(2*b + c*x))^2 +
 (3*(b1*c - b*c1)*(b + c*x))/(a + x*(2*b + c*x)) + (3*c*(b1*c - b*c1)*ArcTa
n[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(8*(b^2 - a*c)^2)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.19

method	result
default	$\frac{(-2b\,\text{c1}+2\,\text{b1}\,c)x+2b\,\text{b1}-2a\,\text{c1}}{2(4ac-4b^2)(c\,x^2+2bx+a)^2}+\frac{3(-2b\,\text{c1}+2\,\text{b1}\,c)\left(\frac{2cx+2b}{\left(4ac-4b^2\right)\left(c\,x^2+2bx+a\right)}+\frac{2c\arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^2}}\right)}{\left(4ac-4b^2\right)\sqrt{ac-b^2}}\right)}{2(4ac-4b^2)}$
risch	$\frac{-\frac{3c^2(b\,\text{c1}-\text{b1}\,c)x^3}{8\left(a^2c^2-2a\,b^2c+b^4\right)}-\frac{9bc(b\,\text{c1}-\text{b1}\,c)x^2}{8\left(a^2c^2-2a\,b^2c+b^4\right)}-\frac{\left(5ac+4b^2\right)(b\,\text{c1}-\text{b1}\,c)x}{8\left(a^2c^2-2a\,b^2c+b^4\right)}-\frac{2a^2c\,\text{c1}+a\,b^2\,\text{c1}-5ab\,\text{b1}\,c+2b^3\,\text{b1}}{8\left(a^2c^2-2a\,b^2c+b^4\right)}}{(c\,x^2+2bx+a)^2}-\frac{3c\ln\Big((a^2c^3-2a\,b^2c^2+b^4c)x}{16}$

```
[In] int((c1*x+b1)/(c*x^2+2*b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)^2+3/2*
(-2*b*c1+2*b1*c)/(4*a*c-4*b^2)*((2*c*x+2*b)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)+2
*c/(4*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(120) = 240$.

Time = 0.27 (sec) , antiderivative size = 1104, normalized size of antiderivative = 8.49

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^3}\,dx$$

$$= \left[-\frac{4\,b^5b_1 - 14\,ab^3b_1c + 10\,a^2bb_1c^2 - 6\,(b^2b_1c^3 - ab_1c^4 - (b^3c^2 - abc^3)c_1)x^3 - 18\,(b^3b_1c^2 - abb_1c^3 - (b^4c - a}{16\,(a^2b^6 - 3\,a^3b}\right.$$

$$-\frac{2\,b^5b_1 - 7\,ab^3b_1c + 5\,a^2bb_1c^2 - 3\,(b^2b_1c^3 - ab_1c^4 - (b^3c^2 - abc^3)c_1)x^3 - 9\,(b^3b_1c^2 - abb_1c^3 - (b^4c - ab^2c^2}{8\,(a^2b^6 - 3\,a^3b^4c + 3\,a^4}$$

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*b^5*b1 - 14*a*b^3*b1*c + 10*a^2*b*b1*c^2 - 6*(b^2*b1*c^3 - a*b1*c
^4 - (b^3*c^2 - a*b*c^3)*c1)*x^3 - 18*(b^3*b1*c^2 - a*b*b1*c^3 - (b^4*c - a
*b^2*c^2)*c1)*x^2 + 3*(a^2*b1*c^2 - a^2*b*c*c1 + (b1*c^4 - b*c^3*c1)*x^4 +
4*(b*b1*c^3 - b^2*c^2*c1)*x^3 + 2*(2*b^2*b1*c^2 + a*b1*c^3 - (2*b^3*c + a*b
*c^2)*c1)*x^2 + 4*(a*b*b1*c^2 - a*b^2*c*c1)*x)*sqrt(b^2 - a*c)*log((c^2*x^2
 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*x + a)
) + 2*(a*b^4 + a^2*b^2*c - 2*a^3*c^2)*c1 - 2*(4*b^4*b1*c + a*b^2*b1*c^2 - 5
*a^2*b1*c^3 - (4*b^5 + a*b^3*c - 5*a^2*b*c^2)*c1)*x)/(a^2*b^6 - 3*a^3*b^4*c
 + 3*a^4*b^2*c^2 - a^5*c^3 + (b^6*c^2 - 3*a*b^4*c^3 + 3*a^2*b^2*c^4 - a^3*c
^5)*x^4 + 4*(b^7*c - 3*a*b^5*c^2 + 3*a^2*b^3*c^3 - a^3*b*c^4)*x^3 + 2*(2*b^
8 - 5*a*b^6*c + 3*a^2*b^4*c^2 + a^3*b^2*c^3 - a^4*c^4)*x^2 + 4*(a*b^7 - 3*a
^2*b^5*c + 3*a^3*b^3*c^2 - a^4*b*c^3)*x), -1/8*(2*b^5*b1 - 7*a*b^3*b1*c + 5
*a^2*b*b1*c^2 - 3*(b^2*b1*c^3 - a*b1*c^4 - (b^3*c^2 - a*b*c^3)*c1)*x^3 - 9*
(b^3*b1*c^2 - a*b*b1*c^3 - (b^4*c - a*b^2*c^2)*c1)*x^2 + 3*(a^2*b1*c^2 - a^
2*b*c*c1 + (b1*c^4 - b*c^3*c1)*x^4 + 4*(b*b1*c^3 - b^2*c^2*c1)*x^3 + 2*(2*b
^2*b1*c^2 + a*b1*c^3 - (2*b^3*c + a*b*c^2)*c1)*x^2 + 4*(a*b*b1*c^2 - a*b^2*
c*c1)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(b^2 - a*c)) +
 (a*b^4 + a^2*b^2*c - 2*a^3*c^2)*c1 - (4*b^4*b1*c + a*b^2*b1*c^2 - 5*a^2*b1
*c^3 - (4*b^5 + a*b^3*c - 5*a^2*b*c^2)*c1)*x)/(a^2*b^6 - 3*a^3*b^4*c + 3*a^
4*b^2*c^2 - a^5*c^3 + (b^6*c^2 - 3*a*b^4*c^3 + 3*a^2*b^2*c^4 - a^3*c^5)*x^4
 + 4*(b^7*c - 3*a*b^5*c^2 + 3*a^2*b^3*c^3 - a^3*b*c^4)*x^3 + 2*(2*b^8 - 5*a
*b^6*c + 3*a^2*b^4*c^2 + a^3*b^2*c^3 - a^4*c^4)*x^2 + 4*(a*b^7 - 3*a^2*b^5*
c + 3*a^3*b^3*c^2 - a^4*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. 2(117) = 234.

Time = 1.00 (sec) , antiderivative size = 622, normalized size of antiderivative = 4.78

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^3}\,dx$$

$$= \frac{3c\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1 - b_1c)\log\left(x + \frac{-3a^3c^4\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)+9a^2b^2c^3\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)-9ab^4c^2\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c}{3bc^2c_1-3b_1c^3}\right.}{16}$$

$$- \frac{3c\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1 - b_1c)\log\left(x + \frac{3a^3c^4\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)-9a^2b^2c^3\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)+9ab^4c^2\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b}{3bc^2c_1-3b_1c^3}\right.}{16}$$

$$+ \frac{-2a^2cc_1 - ab^2c_1 + 5abb_1c - 2b^3b_1 + x^3(-3bc^2c_1 + 3b_1c^3) + x^2(-9b^2cc_1 + 9bb_1c^2) + x}{8a^4c^2 - 16a^3b^2c + 8a^2b^4 + x^4\cdot(8a^2c^4 - 16ab^2c^3 + 8b^4c^2) + x^3\cdot(32a^2bc^3 - 64ab^3c^2 + 32b^5c) + x^2\cdot(16a}$$

```
[In] integrate((c1*x+b1)/(c*x**2+2*b*x+a)**3,x)
```

```
[Out] 3*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c)*log(x + (-3*a**3*c**4*sqrt(-1/(a
*c - b**2)**5)*(b*c1 - b1*c) + 9*a**2*b**2*c**3*sqrt(-1/(a*c - b**2)**5)*(b
*c1 - b1*c) - 9*a*b**4*c**2*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 3*b**6
*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 3*b**2*c*c1 - 3*b*b1*c**2)/(3*b
*c**2*c1 - 3*b1*c**3))/16 - 3*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c)*log(
x + (3*a**3*c**4*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) - 9*a**2*b**2*c**3*
sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 9*a*b**4*c**2*sqrt(-1/(a*c - b**2)
**5)*(b*c1 - b1*c) - 3*b**6*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 3*b*
*2*c*c1 - 3*b*b1*c**2)/(3*b*c**2*c1 - 3*b1*c**3))/16 + (-2*a**2*c*c1 - a*b*
*2*c1 + 5*a*b*b1*c - 2*b**3*b1 + x**3*(-3*b*c**2*c1 + 3*b1*c**3) + x**2*(-9
*b**2*c*c1 + 9*b*b1*c**2) + x*(-5*a*b*c*c1 + 5*a*b1*c**2 - 4*b**3*c1 + 4*b*
*2*b1*c))/(8*a**4*c**2 - 16*a**3*b**2*c + 8*a**2*b**4 + x**4*(8*a**2*c**4 -
 16*a*b**2*c**3 + 8*b**4*c**2) + x**3*(32*a**2*b*c**3 - 64*a*b**3*c**2 + 32
*b**5*c) + x**2*(16*a**3*c**3 - 48*a*b**4*c + 32*b**6) + x*(32*a**3*b*c**2
- 64*a**2*b**3*c + 32*a*b**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^3} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.49

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^3} \, dx = \frac{3\,(b_1c^2 - bcc_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{8\,(b^4 - 2\,ab^2c + a^2c^2)\sqrt{-b^2+ac}} + \frac{3\,b_1c^3x^3 - 3\,bc^2c_1x^3 + 9\,bb_1c^2x^2 - 9\,b^2cc_1x^2 + 4\,b^2b_1cx + 5\,ab_1c^2x - 4\,b^3c_1x - 5\,abcc_1x - 2\,b^3b_1 + 5\,abb_1c}{8\,(b^4 - 2\,ab^2c + a^2c^2)(cx^2 + 2\,bx + a)^2}$$

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*(b1*c^2 - b*c*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^4 - 2*a*b^2*c
+ a^2*c^2)*sqrt(-b^2 + a*c)) + 1/8*(3*b1*c^3*x^3 - 3*b*c^2*c1*x^3 + 9*b*b1*
c^2*x^2 - 9*b^2*c*c1*x^2 + 4*b^2*b1*c*x + 5*a*b1*c^2*x - 4*b^3*c1*x - 5*a*b
*c*c1*x - 2*b^3*b1 + 5*a*b*b1*c - a*b^2*c1 - 2*a^2*c*c1)/((b^4 - 2*a*b^2*c
+ a^2*c^2)*(c*x^2 + 2*b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.77

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^3} \, dx = \frac{3\,c\,\mathrm{atan}\left(\frac{8\left(\frac{3\,c^2\,x\,(b\,c_1-b_1\,c)}{8\,{(a\,c-b^2)}^{5/2}}+\frac{3\,c\,(b\,c_1-b_1\,c)\,(16\,a^2\,b\,c^2-32\,a\,b^3\,c+16\,b^5)}{128\,{(a\,c-b^2)}^{5/2}\,(a^2\,c^2-2\,a\,b^2\,c+b^4)}\right)(a^2\,c^2-2\,a\,b^2\,c+b^4)}{3\,b_1\,c^2-3\,b\,c\,c_1}\right)(b\,c_1-b_1\,c)}{8\,{(a\,c-b^2)}^{5/2}} - \frac{\frac{2\,c\,c_1\,a^2+c_1\,a\,b^2-5\,b_1\,c\,a\,b+2\,b_1\,b^3}{8\,(a^2\,c^2-2\,a\,b^2\,c+b^4)}+\frac{x\,(4\,b^2+5\,a\,c)\,(b\,c_1-b_1\,c)}{8\,(a^2\,c^2-2\,a\,b^2\,c+b^4)}+\frac{3\,c^2\,x^3\,(b\,c_1-b_1\,c)}{8\,(a^2\,c^2-2\,a\,b^2\,c+b^4)}+\frac{9\,b\,c\,x^2\,(b\,c_1-b_1\,c)}{8\,(a^2\,c^2-2\,a\,b^2\,c+b^4)}}{a^2+x^2\,(4\,b^2+2\,a\,c)+c^2\,x^4+4\,a\,b\,x+4\,b\,c\,x^3}$$

```
[In] int((b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x)
```

```
[Out] (3*c*atan((8*((3*c^2*x*(b*c1 - b1*c))/(8*(a*c - b^2)^(5/2)) + (3*c*(b*c1 -
b1*c)*(16*b^5 + 16*a^2*b*c^2 - 32*a*b^3*c))/(128*(a*c - b^2)^(5/2)*(b^4 + a
^2*c^2 - 2*a*b^2*c)))*(b^4 + a^2*c^2 - 2*a*b^2*c))/(3*b1*c^2 - 3*b*c*c1))*(
b*c1 - b1*c))/(8*(a*c - b^2)^(5/2)) - ((2*b^3*b1 + a*b^2*c1 + 2*a^2*c*c1 -
5*a*b*b1*c)/(8*(b^4 + a^2*c^2 - 2*a*b^2*c)) + (x*(5*a*c + 4*b^2)*(b*c1 - b1
*c))/(8*(b^4 + a^2*c^2 - 2*a*b^2*c)) + (3*c^2*x^3*(b*c1 - b1*c))/(8*(b^4 +
a^2*c^2 - 2*a*b^2*c)) + (9*b*c*x^2*(b*c1 - b1*c))/(8*(b^4 + a^2*c^2 - 2*a*b
^2*c)))/(a^2 + x^2*(2*a*c + 4*b^2) + c^2*x^4 + 4*a*b*x + 4*b*c*x^3)
```

3.197 $\int \frac{\mathbf{b1}+\mathbf{c1}x}{(a+2bx+cx^2)^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 173

$$\int \frac{\text{b1}+\text{c1}x}{(a+2bx+cx^2)^4}\,dx = -\frac{b\text{b1}-a\text{c1}+(\text{b1}c-b\text{c1})x}{6(b^2-ac)(a+2bx+cx^2)^3}+\frac{5(\text{b1}c-b\text{c1})(b+cx)}{24(b^2-ac)^2(a+2bx+cx^2)^2} - \frac{5c(\text{b1}c-b\text{c1})(b+cx)}{16(b^2-ac)^3(a+2bx+cx^2)}+\frac{5c^2(\text{b1}c-b\text{c1})\text{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{16(b^2-ac)^{7/2}}$$

```
[Out] 1/6*(-b*b1+a*c1-(-b*c1+b1*c)*x)/(-a*c+b^2)/(c*x^2+2*b*x+a)^3+5/24*(-b*c1+b1
*c)*(c*x+b)/(-a*c+b^2)^2/(c*x^2+2*b*x+a)^2-5/16*c*(-b*c1+b1*c)*(c*x+b)/(-a*
c+b^2)^3/(c*x^2+2*b*x+a)+5/16*c^2*(-b*c1+b1*c)*arctanh((c*x+b)/(-a*c+b^2)^(
1/2))/(-a*c+b^2)^(7/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {652, 628, 632, 212}

$$\int \frac{\text{b1}+\text{c1}x}{(a+2bx+cx^2)^4}\,dx = \frac{5c^2(\text{b1}c-b\text{c1})\text{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{16(b^2-ac)^{7/2}}-\frac{5c(b+cx)(\text{b1}c-b\text{c1})}{16(b^2-ac)^3(a+2bx+cx^2)} +\frac{5(b+cx)(\text{b1}c-b\text{c1})}{24(b^2-ac)^2(a+2bx+cx^2)^2}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{6(b^2-ac)(a+2bx+cx^2)^3}$$

```
[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x]
```

```
[Out] -1/6*(b*b1 - a*c1 + (b1*c - b*c1)*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)^3) +
(5*(b1*c - b*c1)*(b + c*x))/(24*(b^2 - a*c)^2*(a + 2*b*x + c*x^2)^2) - (5*c
```

```
*(b1*c - b*c1)*(b + c*x))/(16*(b^2 - a*c)^3*(a + 2*b*x + c*x^2)) + (5*c^2*(
b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(16*(b^2 - a*c)^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\text{b1}-a\text{c1}+(\text{b1}c-b\text{c1})x}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3}-\frac{(5(\text{b1}c-b\text{c1}))\int\frac{1}{(a+2bx+cx^2)^3}\,dx}{6\left(b^2-ac\right)}\\
&= -\frac{b\text{b1}-a\text{c1}+(\text{b1}c-b\text{c1})x}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3}+\frac{5(\text{b1}c-b\text{c1})(b+cx)}{24\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)^2}\\
&\quad+\frac{(5c(\text{b1}c-b\text{c1}))\int\frac{1}{(a+2bx+cx^2)^2}\,dx}{8\left(b^2-ac\right)^2}\\
&= -\frac{b\text{b1}-a\text{c1}+(\text{b1}c-b\text{c1})x}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3}+\frac{5(\text{b1}c-b\text{c1})(b+cx)}{24\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)^2}\\
&\quad-\frac{5c(\text{b1}c-b\text{c1})(b+cx)}{16\left(b^2-ac\right)^3\left(a+2bx+cx^2\right)}-\frac{\left(5c^2(\text{b1}c-b\text{c1})\right)\int\frac{1}{a+2bx+cx^2}\,dx}{16\left(b^2-ac\right)^3}
\end{aligned}
$$

$$= -\frac{b\mathrm{b1} - a\mathrm{c1} + (\mathrm{b1}c - b\mathrm{c1})x}{6\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)^3} + \frac{5(\mathrm{b1}c - b\mathrm{c1})(b + cx)}{24\left(b^2 - ac\right)^2\left(a + 2bx + cx^2\right)^2} - \frac{5c(\mathrm{b1}c - b\mathrm{c1})(b + cx)}{16\left(b^2 - ac\right)^3\left(a + 2bx + cx^2\right)} + \frac{\left(5c^2(\mathrm{b1}c - b\mathrm{c1})\right)\mathrm{Subst}\left(\int \frac{1}{4(b^2 - ac) - x^2}\,dx, x, 2b + 2cx\right)}{8\left(b^2 - ac\right)^3}$$

$$= -\frac{b\mathrm{b1} - a\mathrm{c1} + (\mathrm{b1}c - b\mathrm{c1})x}{6\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)^3} + \frac{5(\mathrm{b1}c - b\mathrm{c1})(b + cx)}{24\left(b^2 - ac\right)^2\left(a + 2bx + cx^2\right)^2} - \frac{5c(\mathrm{b1}c - b\mathrm{c1})(b + cx)}{16\left(b^2 - ac\right)^3\left(a + 2bx + cx^2\right)} + \frac{5c^2(\mathrm{b1}c - b\mathrm{c1})\mathrm{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{16\left(b^2 - ac\right)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.97

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{\left(a + 2bx + cx^2\right)^4}\,dx$$

$$= \frac{\frac{8(b^2-ac)^2(-b\mathrm{b1}+a\mathrm{c1}-\mathrm{b1}cx+b\mathrm{c1}x)}{(a+x(2b+cx))^3} - \frac{10(b^2-ac)(-\mathrm{b1}c+b\mathrm{c1})(b+cx)}{(a+x(2b+cx))^2} + \frac{15c(-\mathrm{b1}c+b\mathrm{c1})(b+cx)}{a+x(2b+cx)} + \frac{15c^2(-\mathrm{b1}c+b\mathrm{c1})\arctan\left(\frac{b+cx}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2+ac}}}{48\left(b^2 - ac\right)^3}$$

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x]
[Out] ((8*(b^2 - a*c)^2*(-(b*b1) + a*c1 - b1*c*x + b*c1*x))/(a + x*(2*b + c*x))^3
 - (10*(b^2 - a*c)*(-(b1*c) + b*c1)*(b + c*x))/(a + x*(2*b + c*x))^2 + (15*
c*(-(b1*c) + b*c1)*(b + c*x))/(a + x*(2*b + c*x)) + (15*c^2*(-(b1*c) + b*c1
)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(48*(b^2 - a*c)^3)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.19

method	result
default	$\frac{(-2b\,\mathrm{c1}+2\,\mathrm{b1}\,c)x+2b\,\mathrm{b1}-2a\,\mathrm{c1}}{3(4ac-4b^2)(c\,x^2+2bx+a)^3} + \frac{5(-2b\,\mathrm{c1}+2\,\mathrm{b1}\,c)\left(\frac{2cx+2b}{2\left(4ac-4b^2\right)\left(c\,x^2+2bx+a\right)^2} + \frac{3c\left(\frac{2cx+2b}{\left(4ac-4b^2\right)\left(c\,x^2+2bx+a\right)} + \frac{2c\arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^2}}\right)}{\left(4ac-4b^2\right)\sqrt{ac-b^2}}\right.}{4ac-4b^2}\right.}{3(4ac-4b^2)}$
risch	$\frac{-\frac{5c^4(b\,\mathrm{c1}-\mathrm{b1}\,c)x^5}{16\left(a^3c^3-3b^2c^2a^2+3a\,b^4c-b^6\right)} - \frac{25c^3(b\,\mathrm{c1}-\mathrm{b1}\,c)b\,x^4}{16\left(a^3c^3-3b^2c^2a^2+3a\,b^4c-b^6\right)} - \frac{5\left(4ac+11b^2\right)c^2(b\,\mathrm{c1}-\mathrm{b1}\,c)x^3}{24\left(a^3c^3-3b^2c^2a^2+3a\,b^4c-b^6\right)} - \frac{5b\left(4ac+b^2\right)c(b\,\mathrm{c1}-\mathrm{b1}\,c)x^2}{8\left(a^3c^3-3b^2c^2a^2+3a\,b^4c-b^6\right)} - \frac{(11a^2b\,c^2}{}}{(c\,x^2+2bx+a)^3}$

```
[In] int((c1*x+b1)/(c*x^2+2*b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)^3+5/3*
(-2*b*c1+2*b1*c)/(4*a*c-4*b^2)*(1/2*(2*c*x+2*b)/(4*a*c-4*b^2)/(c*x^2+2*b*x+
a)^2+3*c/(4*a*c-4*b^2)*((2*c*x+2*b)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)+2*c/(4*a*
c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 962 vs. $2(161) = 322$.

Time = 0.30 (sec) , antiderivative size = 1950, normalized size of antiderivative = 11.27

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*b^7*b1 - 68*a*b^5*b1*c + 118*a^2*b^3*b1*c^2 - 66*a^3*b*b1*c^3 +
30*(b^2*b1*c^5 - a*b1*c^6 - (b^3*c^4 - a*b*c^5)*c1)*x^5 + 150*(b^3*b1*c^4 -
 a*b*b1*c^5 - (b^4*c^3 - a*b^2*c^4)*c1)*x^4 + 20*(11*b^4*b1*c^3 - 7*a*b^2*b
1*c^4 - 4*a^2*b1*c^5 - (11*b^5*c^2 - 7*a*b^3*c^3 - 4*a^2*b*c^4)*c1)*x^3 + 6
0*(b^5*b1*c^2 + 3*a*b^3*b1*c^3 - 4*a^2*b*b1*c^4 - (b^6*c + 3*a*b^4*c^2 - 4*
a^2*b^2*c^3)*c1)*x^2 - 15*(a^3*b1*c^3 - a^3*b*c^2*c1 + (b1*c^6 - b*c^5*c1)*
x^6 + 6*(b*b1*c^5 - b^2*c^4*c1)*x^5 + 3*(4*b^2*b1*c^4 + a*b1*c^5 - (4*b^3*c
^3 + a*b*c^4)*c1)*x^4 + 4*(2*b^3*b1*c^3 + 3*a*b*b1*c^4 - (2*b^4*c^2 + 3*a*b
^2*c^3)*c1)*x^3 + 3*(4*a*b^2*b1*c^3 + a^2*b1*c^4 - (4*a*b^3*c^2 + a^2*b*c^3
)*c1)*x^2 + 6*(a^2*b*b1*c^3 - a^2*b^2*c^2*c1)*x)*sqrt(b^2 - a*c)*log((c^2*x
^2 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*x +
a)) + 2*(2*a*b^6 - 11*a^2*b^4*c + a^3*b^2*c^2 + 8*a^4*c^3)*c1 - 6*(4*b^6*b1
*c - 22*a*b^4*b1*c^2 + 7*a^2*b^2*b1*c^3 + 11*a^3*b1*c^4 - (4*b^7 - 22*a*b^5
*c + 7*a^2*b^3*c^2 + 11*a^3*b*c^3)*c1)*x)/(a^3*b^8 - 4*a^4*b^6*c + 6*a^5*b^
4*c^2 - 4*a^6*b^2*c^3 + a^7*c^4 + (b^8*c^3 - 4*a*b^6*c^4 + 6*a^2*b^4*c^5 -
4*a^3*b^2*c^6 + a^4*c^7)*x^6 + 6*(b^9*c^2 - 4*a*b^7*c^3 + 6*a^2*b^5*c^4 - 4
*a^3*b^3*c^5 + a^4*b*c^6)*x^5 + 3*(4*b^10*c - 15*a*b^8*c^2 + 20*a^2*b^6*c^3
 - 10*a^3*b^4*c^4 + a^5*c^6)*x^4 + 4*(2*b^11 - 5*a*b^9*c + 10*a^3*b^5*c^3 -
 10*a^4*b^3*c^4 + 3*a^5*b*c^5)*x^3 + 3*(4*a*b^10 - 15*a^2*b^8*c + 20*a^3*b^
6*c^2 - 10*a^4*b^4*c^3 + a^6*c^5)*x^2 + 6*(a^2*b^9 - 4*a^3*b^7*c + 6*a^4*b^
5*c^2 - 4*a^5*b^3*c^3 + a^6*b*c^4)*x), -1/48*(8*b^7*b1 - 34*a*b^5*b1*c + 59
*a^2*b^3*b1*c^2 - 33*a^3*b*b1*c^3 + 15*(b^2*b1*c^5 - a*b1*c^6 - (b^3*c^4 -
a*b*c^5)*c1)*x^5 + 75*(b^3*b1*c^4 - a*b*b1*c^5 - (b^4*c^3 - a*b^2*c^4)*c1)*
x^4 + 10*(11*b^4*b1*c^3 - 7*a*b^2*b1*c^4 - 4*a^2*b1*c^5 - (11*b^5*c^2 - 7*a
*b^3*c^3 - 4*a^2*b*c^4)*c1)*x^3 + 30*(b^5*b1*c^2 + 3*a*b^3*b1*c^3 - 4*a^2*b
*b1*c^4 - (b^6*c + 3*a*b^4*c^2 - 4*a^2*b^2*c^3)*c1)*x^2 - 15*(a^3*b1*c^3 -
a^3*b*c^2*c1 + (b1*c^6 - b*c^5*c1)*x^6 + 6*(b*b1*c^5 - b^2*c^4*c1)*x^5 + 3*
(4*b^2*b1*c^4 + a*b1*c^5 - (4*b^3*c^3 + a*b*c^4)*c1)*x^4 + 4*(2*b^3*b1*c^3
```

```
+ 3*a*b*b1*c^4 - (2*b^4*c^2 + 3*a*b^2*c^3)*c1)*x^3 + 3*(4*a*b^2*b1*c^3 + a^
2*b1*c^4 - (4*a*b^3*c^2 + a^2*b*c^3)*c1)*x^2 + 6*(a^2*b*b1*c^3 - a^2*b^2*c^
2*c1)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(b^2 - a*c)) +
 (2*a*b^6 - 11*a^2*b^4*c + a^3*b^2*c^2 + 8*a^4*c^3)*c1 - 3*(4*b^6*b1*c - 22
*a*b^4*b1*c^2 + 7*a^2*b^2*b1*c^3 + 11*a^3*b1*c^4 - (4*b^7 - 22*a*b^5*c + 7*
a^2*b^3*c^2 + 11*a^3*b*c^3)*c1)*x)/(a^3*b^8 - 4*a^4*b^6*c + 6*a^5*b^4*c^2 -
 4*a^6*b^2*c^3 + a^7*c^4 + (b^8*c^3 - 4*a*b^6*c^4 + 6*a^2*b^4*c^5 - 4*a^3*b
^2*c^6 + a^4*c^7)*x^6 + 6*(b^9*c^2 - 4*a*b^7*c^3 + 6*a^2*b^5*c^4 - 4*a^3*b^
3*c^5 + a^4*b*c^6)*x^5 + 3*(4*b^10*c - 15*a*b^8*c^2 + 20*a^2*b^6*c^3 - 10*a
^3*b^4*c^4 + a^5*c^6)*x^4 + 4*(2*b^11 - 5*a*b^9*c + 10*a^3*b^5*c^3 - 10*a^4
*b^3*c^4 + 3*a^5*b*c^5)*x^3 + 3*(4*a*b^10 - 15*a^2*b^8*c + 20*a^3*b^6*c^2 -
 10*a^4*b^4*c^3 + a^6*c^5)*x^2 + 6*(a^2*b^9 - 4*a^3*b^7*c + 6*a^4*b^5*c^2 -
 4*a^5*b^3*c^3 + a^6*b*c^4)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(158) = 316$.

Time = 1.53 (sec) , antiderivative size = 1027, normalized size of antiderivative = 5.94

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^4}\,dx$$
$$= \frac{5c^2\sqrt{-\frac{1}{(ac-b^2)^7}}(bc_1 - b_1c)\log\left(x + \frac{-5a^4c^6\sqrt{-\frac{1}{(ac-b^2)^7}}(bc_1-b_1c)+20a^3b^2c^5\sqrt{-\frac{1}{(ac-b^2)^7}}(bc_1-b_1c)-30a^2b^4c^4\sqrt{-\frac{1}{(ac-b^2)^7}}(bc_1-}{5bc^3c_1-5b_1}\right.}{32}$$
$$- \frac{5c^2\sqrt{-\frac{1}{(ac-b^2)^7}}(bc_1 - b_1c)\log\left(x + \frac{5a^4c^6\sqrt{-\frac{1}{(ac-b^2)^7}}(bc_1-b_1c)-20a^3b^2c^5\sqrt{-\frac{1}{(ac-b^2)^7}}(bc_1-b_1c)+30a^2b^4c^4\sqrt{-\frac{1}{(ac-b^2)^7}}(bc_1}{5bc^3c_1-5}\right.}{32}$$
$$+ \frac{-8a^3c^2c_1 - 9a^2b^2cc_1 + 33a^2bb_1c^2 + 2ab^4c_1 - 26ab^3b_1c + 8b}{48a^6c^3 - 144a^5b^2c^2 + 144a^4b^4c - 48a^3b^6 + x^6\cdot(48a^3c^6 - 144a^2b^2c^5 + 144ab^4c^4 - 48b^6c^3) + x^5\cdot(288a^3bc}$$

```
[In] integrate((c1*x+b1)/(c*x**2+2*b*x+a)**4,x)
```

```
[Out] 5*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c)*log(x + (-5*a**4*c**6*sqrt(-1
/(a*c - b**2)**7)*(b*c1 - b1*c) + 20*a**3*b**2*c**5*sqrt(-1/(a*c - b**2)**7
)*(b*c1 - b1*c) - 30*a**2*b**4*c**4*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c)
+ 20*a*b**6*c**3*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) - 5*b**8*c**2*sqrt(
-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 5*b**2*c**2*c1 - 5*b*b1*c**3)/(5*b*c**3
*c1 - 5*b1*c**4))/32 - 5*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c)*log(x
+ (5*a**4*c**6*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) - 20*a**3*b**2*c**5*s
qrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 30*a**2*b**4*c**4*sqrt(-1/(a*c - b*
*2)**7)*(b*c1 - b1*c) - 20*a*b**6*c**3*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*
c) + 5*b**8*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 5*b**2*c**2*c1 -
```

```
5*b*b1*c**3)/(5*b*c**3*c1 - 5*b1*c**4))/32 + (-8*a**3*c**2*c1 - 9*a**2*b**2
*c*c1 + 33*a**2*b*b1*c**2 + 2*a*b**4*c1 - 26*a*b**3*b1*c + 8*b**5*b1 + x**5
*(-15*b*c**4*c1 + 15*b1*c**5) + x**4*(-75*b**2*c**3*c1 + 75*b*b1*c**4) + x*
*3*(-40*a*b*c**3*c1 + 40*a*b1*c**4 - 110*b**3*c**2*c1 + 110*b**2*b1*c**3) +
 x**2*(-120*a*b**2*c**2*c1 + 120*a*b*b1*c**3 - 30*b**4*c*c1 + 30*b**3*b1*c*
*2) + x*(-33*a**2*b*c**2*c1 + 33*a**2*b1*c**3 - 54*a*b**3*c*c1 + 54*a*b**2*
b1*c**2 + 12*b**5*c1 - 12*b**4*b1*c))/(48*a**6*c**3 - 144*a**5*b**2*c**2 +
144*a**4*b**4*c - 48*a**3*b**6 + x**6*(48*a**3*c**6 - 144*a**2*b**2*c**5 +
144*a*b**4*c**4 - 48*b**6*c**3) + x**5*(288*a**3*b*c**5 - 864*a**2*b**3*c**
4 + 864*a*b**5*c**3 - 288*b**7*c**2) + x**4*(144*a**4*c**5 + 144*a**3*b**2*
c**4 - 1296*a**2*b**4*c**3 + 1584*a*b**6*c**2 - 576*b**8*c) + x**3*(576*a**
4*b*c**4 - 1344*a**3*b**3*c**3 + 576*a**2*b**5*c**2 + 576*a*b**7*c - 384*b*
*9) + x**2*(144*a**5*c**4 + 144*a**4*b**2*c**3 - 1296*a**3*b**4*c**2 + 1584
*a**2*b**6*c - 576*a*b**8) + x*(288*a**5*b*c**3 - 864*a**4*b**3*c**2 + 864*
a**3*b**5*c - 288*a**2*b**7))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^4}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(161) = 322$.

Time = 0.31 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.10

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^4}\, dx = -\frac{5\,(b_1c^3 - bc^2c_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{16\,(b^6 - 3\,ab^4c + 3\,a^2b^2c^2 - a^3c^3)\sqrt{-b^2+ac}} - \frac{15\,b_1c^5x^5 - 15\,bc^4c_1x^5 + 75\,bb_1c^4x^4 - 75\,b^2c^3c_1x^4 + 110\,b^2b_1c^3x^3 + 40\,ab_1c^4x^3 - 110\,b^3c^2c_1x^3 - 40\,abc^3c}{}$$

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="giac")
```

```
[Out] -5/16*(b1*c^3 - b*c^2*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^6 - 3*a*b^
4*c + 3*a^2*b^2*c^2 - a^3*c^3)*sqrt(-b^2 + a*c)) - 1/48*(15*b1*c^5*x^5 - 15
```

```
*b*c^4*c1*x^5 + 75*b*b1*c^4*x^4 - 75*b^2*c^3*c1*x^4 + 110*b^2*b1*c^3*x^3 +
40*a*b1*c^4*x^3 - 110*b^3*c^2*c1*x^3 - 40*a*b*c^3*c1*x^3 + 30*b^3*b1*c^2*x^
2 + 120*a*b*b1*c^3*x^2 - 30*b^4*c*c1*x^2 - 120*a*b^2*c^2*c1*x^2 - 12*b^4*b1
*c*x + 54*a*b^2*b1*c^2*x + 33*a^2*b1*c^3*x + 12*b^5*c1*x - 54*a*b^3*c*c1*x
- 33*a^2*b*c^2*c1*x + 8*b^5*b1 - 26*a*b^3*b1*c + 33*a^2*b*b1*c^2 + 2*a*b^4*
c1 - 9*a^2*b^2*c*c1 - 8*a^3*c^2*c1)/((b^6 - 3*a*b^4*c + 3*a^2*b^2*c^2 - a^3
*c^3)*(c*x^2 + 2*b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 640, normalized size of antiderivative = 3.70

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^4}\,dx$$

$$= \frac{\frac{5\,c^4\,x^5\,(b\,c_1-b_1\,c)}{16\,(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+b^6)}-\frac{-8\,c_1\,a^3\,c^2-9\,c_1\,a^2\,b^2\,c+33\,b_1\,a^2\,b\,c^2+2\,c_1\,a\,b^4-26\,b_1\,a\,b^3\,c+8\,b_1\,b^5}{48\,(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+b^6)}+\frac{x\,(b\,c_1-b_1\,c)\,(11\,a^2\,c^2+18\,a\,b^2\,c-4}{16\,(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+b^6}}{x^3\,(8\,b^3+12\,a\,c\,b)+x^2\,(3\,c\,a^2+12\,a\,b^2)+x^4\,(12\,b^2\,c+3}$$

$$-\frac{5\,c^2\,\mathrm{atan}\left(\frac{16\left(\frac{5\,c^3\,x\,(b\,c_1-b_1\,c)}{16\,(a\,c-b^2)^{7/2}}+\frac{5\,c^2\,(b\,c_1-b_1\,c)\,(-32\,a^3\,b\,c^3+96\,a^2\,b^3\,c^2-96\,a\,b^5\,c+32\,b^7)}{512\,(a\,c-b^2)^{7/2}\,(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+b^6)}\right)(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+b^6)}{5\,b_1\,c^3-5\,b\,c^2\,c_1}\right)(b\,c_1-b_1\,c}{16\,(a\,c-b^2)^{7/2}}$$

```
[In] int((b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x)
```

```
[Out] ((5*c^4*x^5*(b*c1 - b1*c))/(16*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c))
 - (8*b^5*b1 - 8*a^3*c^2*c1 + 2*a*b^4*c1 - 26*a*b^3*b1*c + 33*a^2*b*b1*c^2
- 9*a^2*b^2*c*c1)/(48*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) + (x*(b*
c1 - b1*c)*(11*a^2*c^2 - 4*b^4 + 18*a*b^2*c))/(16*(b^6 - a^3*c^3 + 3*a^2*b^
2*c^2 - 3*a*b^4*c)) + (5*c*x^3*(4*a*c^2 + 11*b^2*c)*(b*c1 - b1*c))/(24*(b^6
 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) + (5*c*x^2*(b^3 + 4*a*b*c)*(b*c1 -
 b1*c))/(8*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) + (25*b*c^3*x^4*(b*
c1 - b1*c))/(16*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)))/(x^3*(8*b^3 +
 12*a*b*c) + x^2*(12*a*b^2 + 3*a^2*c) + x^4*(3*a*c^2 + 12*b^2*c) + a^3 + c^
3*x^6 + 6*b*c^2*x^5 + 6*a^2*b*x) - (5*c^2*atan((16*((5*c^3*x*(b*c1 - b1*c))
/(16*(a*c - b^2)^(7/2)) + (5*c^2*(b*c1 - b1*c)*(32*b^7 - 32*a^3*b*c^3 + 96*
a^2*b^3*c^2 - 96*a*b^5*c))/(512*(a*c - b^2)^(7/2)*(b^6 - a^3*c^3 + 3*a^2*b^
2*c^2 - 3*a*b^4*c)))*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c))/(5*b1*c^3
 - 5*b*c^2*c1))*(b*c1 - b1*c))/(16*(a*c - b^2)^(7/2))
```

3.198 $\int (\mathbf{b1}+\mathbf{c1}x)\left(a+2bx+cx^2\right)^{-n}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 169

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^{-n}\,dx=\frac{\mathrm{c1}(a+2bx+cx^2)^{1-n}}{2c(1-n)} - \frac{2^{-n}(\mathrm{b1}c-b\mathrm{c1})\left(-\frac{b-\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-1+n}(a+2bx+cx^2)^{1-n}\,\mathrm{Hypergeometric2F1}\left(1-n,n,2-n,\frac{b+\sqrt{b^2-ac}}{2\sqrt{b^2-a}}\right.}{c\sqrt{b^2-ac}(1-n)}$$

```
[Out] 1/2*c1*(c*x^2+2*b*x+a)^(1-n)/c/(1-n)-(-b*c1+b1*c)*(c*x^2+2*b*x+a)^(1-n)*hyp
ergeom([n, 1-n],[2-n],1/2*(b+c*x+(-a*c+b^2)^(1/2))/(-a*c+b^2)^(1/2))*((-b-c
*x+(-a*c+b^2)^(1/2))/(-a*c+b^2)^(1/2))^(-1+n)/(2^n)/c/(1-n)/(-a*c+b^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {654, 638}

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^{-n}\,dx=\frac{\mathrm{c1}(a+2bx+cx^2)^{1-n}}{2c(1-n)} - \frac{2^{-n}(\mathrm{b1}c-b\mathrm{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{n-1}(a+2bx+cx^2)^{1-n}\,\mathrm{Hypergeometric2F1}\left(1-n,n,2-n,\frac{b+cx+\sqrt{b^2}}{2\sqrt{b^2-a}}\right.}{c(1-n)\sqrt{b^2-ac}}$$

```
[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] (c1*(a + 2*b*x + c*x^2)^(1 - n))/(2*c*(1 - n)) - ((b1*c - b*c1)*(-((b - Sqr
t[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 + n)*(a + 2*b*x + c*x^2)^(1 - n)*
```

```
Hypergeometric2F1[1 - n, n, 2 - n, (b + Sqrt[b^2 - a*c] + c*x)/(2*Sqrt[b^2
- a*c])])/(2^n*c*Sqrt[b^2 - a*c]*(1 - n))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{c1}(a+2bx+cx^2)^{1-n}}{2c(1-n)} + \frac{(2\text{b1}c - 2b\text{c1})\int (a+2bx+cx^2)^{-n}\,dx}{2c}$$

$$= \frac{\text{c1}(a+2bx+cx^2)^{1-n}}{2c(1-n)} - \frac{2^{-n}(\text{b1}c - b\text{c1})\left(-\frac{b-\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-1+n}(a+2bx+cx^2)^{1-n}\text{Hypergeometric2F1}\left(1-n,n,2-n,\frac{b+}{}\right.}{c\sqrt{b^2-ac}(1-n)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.79 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.56

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^{-n}\,dx = \frac{1}{2}(a+x(2b+cx))^{-n}\left(\text{c1}x^2\left(\frac{b-\sqrt{b^2-ac}+cx}{b-\sqrt{b^2-ac}}\right)^n\left(\frac{b+\sqrt{b^2-ac}+cx}{b+\sqrt{b^2-ac}}\right)^n\text{AppellF1}\left(2,n,n,3,-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{-b+\sqrt{b^2-ac}}\right) - \frac{2^{1-n}\text{b1}\left(b-\sqrt{b^2-ac}+cx\right)\left(\frac{b+\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^n\text{Hypergeometric2F1}\left(1-n,n,2-n,\frac{-b+\sqrt{b^2-ac}-cx}{2\sqrt{b^2-ac}}\right)}{c(-1+n)}\right)$$

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] (c1*x^2*((b - Sqrt[b^2 - a*c] + c*x)/(b - Sqrt[b^2 - a*c]))^n*((b + Sqrt[b^
2 - a*c] + c*x)/(b + Sqrt[b^2 - a*c]))^n*AppellF1[2, n, n, 3, -((c*x)/(b +
Sqrt[b^2 - a*c])), (c*x)/(-b + Sqrt[b^2 - a*c])] - (2^(1 - n)*b1*(b - Sqrt[
b^2 - a*c] + c*x)*((b + Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c])^n*Hypergeom
etric2F1[1 - n, n, 2 - n, (-b + Sqrt[b^2 - a*c] - c*x)/(2*Sqrt[b^2 - a*c])]
)/(c*(-1 + n)))/(2*(a + x*(2*b + c*x))^n)
```

Maple [F]

$$\int (\mathrm{c1}\, x + \mathrm{b1}) \left(c\, x^2 + 2bx + a\right)^{-n} dx$$

```
[In] int((c1*x+b1)/((c*x^2+2*b*x+a)^n),x)
[Out] int((c1*x+b1)/((c*x^2+2*b*x+a)^n),x)
```

Fricas [F]

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^{-n} \, dx = \int \frac{c_1 x + b_1}{(cx^2 + 2\,bx + a)^n} \, dx$$

```
[In] integrate((c1*x+b1)/((c*x^2+2*b*x+a)^n),x, algorithm="fricas")
[Out] integral((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^{-n} \, dx = \text{Timed out}$$

```
[In] integrate((c1*x+b1)/((c*x**2+2*b*x+a)**n),x)
[Out] Timed out
```

Maxima [F]

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^{-n} \, dx = \int \frac{c_1 x + b_1}{(cx^2 + 2\,bx + a)^n} \, dx$$

```
[In] integrate((c1*x+b1)/((c*x^2+2*b*x+a)^n),x, algorithm="maxima")
[Out] integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

Giac [F]

$$\int (\text{b1} + \text{c1}x)\left(a + 2bx + cx^2\right)^{-n} dx = \int \frac{c_1x + b_1}{(cx^2 + 2\,bx + a)^n}\, dx$$

```
[In] integrate((c1*x+b1)/((c*x^2+2*b*x+a)^n),x, algorithm="giac")
[Out] integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (\text{b1} + \text{c1}x)\left(a + 2bx + cx^2\right)^{-n} dx = \int \frac{b_1 + c_1\,x}{(c\,x^2 + 2\,b\,x + a)^n}\, dx$$

```
[In] int((b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x)
[Out] int((b1 + c1*x)/(a + 2*b*x + c*x^2)^n, x)
```

3.199 $\int \frac{x}{3+6x+2x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int \frac{x}{3+6x+2x^2} dx = \frac{1}{4}\left(1-\sqrt{3}\right)\log\left(3-\sqrt{3}+2x\right)+\frac{1}{4}\left(1+\sqrt{3}\right)\log\left(3+\sqrt{3}+2x\right)$$

```
[Out] 1/4*ln(3+2*x-3^(1/2))*(1-3^(1/2))+1/4*ln(3+2*x+3^(1/2))*(1+3^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {646, 31}

$$\int \frac{x}{3+6x+2x^2} dx = \frac{1}{4}\left(1-\sqrt{3}\right)\log\left(2x-\sqrt{3}+3\right)+\frac{1}{4}\left(1+\sqrt{3}\right)\log\left(2x+\sqrt{3}+3\right)$$

```
[In] Int[x/(3 + 6*x + 2*x^2),x]
```

```
[Out] ((1 - Sqrt[3])*Log[3 - Sqrt[3] + 2*x])/4 + ((1 + Sqrt[3])*Log[3 + Sqrt[3] +
 2*x])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
```

```
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{1}{3-\sqrt{3}+2x}\,dx+\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{1}{3+\sqrt{3}+2x}\,dx \\ &= \frac{1}{4}\left(1-\sqrt{3}\right)\log\left(3-\sqrt{3}+2x\right)+\frac{1}{4}\left(1+\sqrt{3}\right)\log\left(3+\sqrt{3}+2x\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{x}{3+6x+2x^2}\,dx=\frac{1}{4}\left(-\left(\left(-1+\sqrt{3}\right)\log\left(-3+\sqrt{3}-2x\right)\right)+\left(1+\sqrt{3}\right)\log\left(3+\sqrt{3}+2x\right)\right)$$

```
[In] Integrate[x/(3 + 6*x + 2*x^2),x]
[Out] (-((-1 + Sqrt[3])*Log[-3 + Sqrt[3] - 2*x]) + (1 + Sqrt[3])*Log[3 + Sqrt[3]
+ 2*x])/4
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{\ln(2x^2+6x+3)}{4}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(4x+6)\sqrt{3}}{6}\right)}{2}$	31
risch	$\frac{\ln\left(3+2x+\sqrt{3}\right)}{4}+\frac{\ln\left(3+2x+\sqrt{3}\right)\sqrt{3}}{4}+\frac{\ln\left(3+2x-\sqrt{3}\right)}{4}-\frac{\ln\left(3+2x-\sqrt{3}\right)\sqrt{3}}{4}$	56

```
[In] int(x/(2*x^2+6*x+3),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(2*x^2+6*x+3)+1/2*3^(1/2)*arctanh(1/6*(4*x+6)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{x}{3+6x+2x^2}\,dx = \frac{1}{4}\,\sqrt{3}\log\left(\frac{2\,x^2+\sqrt{3}(2\,x+3)+6\,x+6}{2\,x^2+6\,x+3}\right)+\frac{1}{4}\,\log\left(2\,x^2+6\,x+3\right)$$

```
[In] integrate(x/(2*x^2+6*x+3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*log((2*x^2 + sqrt(3)*(2*x + 3) + 6*x + 6)/(2*x^2 + 6*x + 3)) +
1/4*log(2*x^2 + 6*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{x}{3+6x+2x^2}\,dx = \left(\frac{1}{4}-\frac{\sqrt{3}}{4}\right)\log\left(x-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)+\left(\frac{1}{4}+\frac{\sqrt{3}}{4}\right)\log\left(x+\frac{\sqrt{3}}{2}+\frac{3}{2}\right)$$

```
[In] integrate(x/(2*x**2+6*x+3),x)
```

```
[Out] (1/4 - sqrt(3)/4)*log(x - sqrt(3)/2 + 3/2) + (1/4 + sqrt(3)/4)*log(x + sqrt
(3)/2 + 3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{x}{3+6x+2x^2}\,dx = -\frac{1}{4}\,\sqrt{3}\log\left(\frac{2\,x-\sqrt{3}+3}{2\,x+\sqrt{3}+3}\right)+\frac{1}{4}\,\log\left(2\,x^2+6\,x+3\right)$$

```
[In] integrate(x/(2*x^2+6*x+3),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(3)*log((2*x - sqrt(3) + 3)/(2*x + sqrt(3) + 3)) + 1/4*log(2*x^2 +
 6*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{x}{3+6x+2x^2}\,dx = -\frac{1}{4}\sqrt{3}\log\left(\frac{|4\,x-2\,\sqrt{3}+6|}{|4\,x+2\,\sqrt{3}+6|}\right)+\frac{1}{4}\,\log\left(|2\,x^2+6\,x+3|\right)$$

```
[In] integrate(x/(2*x^2+6*x+3),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(3)*log(abs(4*x - 2*sqrt(3) + 6)/abs(4*x + 2*sqrt(3) + 6)) + 1/4*l
og(abs(2*x^2 + 6*x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{x}{3+6x+2x^2}\,dx = \ln\left(x+\frac{\sqrt{3}}{2}+\frac{3}{2}\right)\left(\frac{\sqrt{3}}{4}+\frac{1}{4}\right)-\ln\left(x-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)\left(\frac{\sqrt{3}}{4}-\frac{1}{4}\right)$$

```
[In] int(x/(6*x + 2*x^2 + 3),x)
```

```
[Out] log(x + 3^(1/2)/2 + 3/2)*(3^(1/2)/4 + 1/4) - log(x - 3^(1/2)/2 + 3/2)*(3^(1
/2)/4 - 1/4)
```

3.200 $\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx = \frac{5+4x}{4(3+6x+2x^2)^2} - \frac{3+2x}{2(3+6x+2x^2)} + \frac{\operatorname{arctanh}\left(\frac{3+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/4*(5+4*x)/(2*x^2+6*x+3)^2+1/2*(-3-2*x)/(2*x^2+6*x+3)+1/3*arctanh(1/3*(3+2
*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {652, 628, 632, 212}

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx = \frac{\operatorname{arctanh}\left(\frac{2x+3}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2x+3}{2(2x^2+6x+3)} + \frac{4x+5}{4(2x^2+6x+3)^2}$$

```
[In] Int[(-3 + 2*x)/(3 + 6*x + 2*x^2)^3,x]
```

```
[Out] (5 + 4*x)/(4*(3 + 6*x + 2*x^2)^2) - (3 + 2*x)/(2*(3 + 6*x + 2*x^2)) + ArcTa
nh[(3 + 2*x)/Sqrt[3]]/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{5+4x}{4\left(3+6x+2x^2\right)^2}+3\int \frac{1}{\left(3+6x+2x^2\right)^2}\,dx \\
&= \frac{5+4x}{4\left(3+6x+2x^2\right)^2}-\frac{3+2x}{2\left(3+6x+2x^2\right)}-\int \frac{1}{3+6x+2x^2}\,dx \\
&= \frac{5+4x}{4\left(3+6x+2x^2\right)^2}-\frac{3+2x}{2\left(3+6x+2x^2\right)}+2\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,6+4x\right) \\
&= \frac{5+4x}{4\left(3+6x+2x^2\right)^2}-\frac{3+2x}{2\left(3+6x+2x^2\right)}+\frac{\operatorname{arctanh}\left(\frac{3+2x}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{-3+2x}{\left(3+6x+2x^2\right)^3}\,dx=\frac{1}{12}\left(-\frac{3\left(13+44x+36x^2+8x^3\right)}{\left(3+6x+2x^2\right)^2}-2\sqrt{3}\log\left(-3+\sqrt{3}-2x\right)+2\sqrt{3}\log\left(3+\sqrt{3}+2x\right)\right)$$

```
[In] Integrate[(-3 + 2*x)/(3 + 6*x + 2*x^2)^3,x]
[Out] ((-3*(13 + 44*x + 36*x^2 + 8*x^3))/(3 + 6*x + 2*x^2)^2 - 2*Sqrt[3]*Log[-3 +
 Sqrt[3] - 2*x] + 2*Sqrt[3]*Log[3 + Sqrt[3] + 2*x])/12
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{-24x-30}{24(2x^2+6x+3)^2}-\frac{4x+6}{4(2x^2+6x+3)}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(4x+6)\sqrt{3}}{6}\right)}{3}$	56
risch	$\frac{-2x^3-9x^2-11x-\frac{13}{4}}{(2x^2+6x+3)^2}+\frac{\ln\left(3+2x+\sqrt{3}\right)\sqrt{3}}{6}-\frac{\ln\left(3+2x-\sqrt{3}\right)\sqrt{3}}{6}$	61

```
[In] int((2*x-3)/(2*x^2+6*x+3)^3,x,method=_RETURNVERBOSE)
[Out] -1/24*(-24*x-30)/(2*x^2+6*x+3)^2-1/4*(4*x+6)/(2*x^2+6*x+3)+1/3*3^(1/2)*arct
anh(1/6*(4*x+6)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx = -\frac{24\,x^3-2\,\sqrt{3}(4\,x^4+24\,x^3+48\,x^2+36\,x+9)\log\left(\frac{2\,x^2+\sqrt{3}(2\,x+3)+6\,x+6}{2\,x^2+6\,x+3}\right)+108\,x^2+132\,x+39}{12\,(4\,x^4+24\,x^3+48\,x^2+36\,x+9)}$$

```
[In] integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="fricas")
[Out] -1/12*(24*x^3 - 2*sqrt(3)*(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)*log((2*x^2 +
 sqrt(3)*(2*x + 3) + 6*x + 6)/(2*x^2 + 6*x + 3)) + 108*x^2 + 132*x + 39)/(4
*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx = \frac{-8x^3-36x^2-44x-13}{16x^4+96x^3+192x^2+144x+36} - \frac{\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)}{6} + \frac{\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}+\frac{3}{2}\right)}{6}$$

```
[In] integrate((-3+2*x)/(2*x**2+6*x+3)**3,x)
[Out] (-8*x**3 - 36*x**2 - 44*x - 13)/(16*x**4 + 96*x**3 + 192*x**2 + 144*x + 36)
 - sqrt(3)*log(x - sqrt(3)/2 + 3/2)/6 + sqrt(3)*log(x + sqrt(3)/2 + 3/2)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx = -\frac{1}{6}\sqrt{3}\log\left(\frac{2\,x-\sqrt{3}+3}{2\,x+\sqrt{3}+3}\right) - \frac{8\,x^3+36\,x^2+44\,x+13}{4\,(4\,x^4+24\,x^3+48\,x^2+36\,x+9)}$$

```
[In] integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log((2*x - sqrt(3) + 3)/(2*x + sqrt(3) + 3)) - 1/4*(8*x^3 + 36
*x^2 + 44*x + 13)/(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx = -\frac{1}{6}\sqrt{3}\log\left(\frac{|4\,x-2\,\sqrt{3}+6|}{|4\,x+2\,\sqrt{3}+6|}\right) - \frac{8\,x^3+36\,x^2+44\,x+13}{4\,(2\,x^2+6\,x+3)^2}$$

```
[In] integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(abs(4*x - 2*sqrt(3) + 6)/abs(4*x + 2*sqrt(3) + 6)) - 1/4*(
8*x^3 + 36*x^2 + 44*x + 13)/(2*x^2 + 6*x + 3)^2
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\sqrt{3}\,\left(\frac{2\,x}{3}+1\right)\right)}{3} - \frac{\frac{x^3}{2}+\frac{9\,x^2}{4}+\frac{11\,x}{4}+\frac{13}{16}}{x^4+6\,x^3+12\,x^2+9\,x+\frac{9}{4}}$$

```
[In] int((2*x - 3)/(6*x + 2*x^2 + 3)^3,x)
```

```
[Out] (3^(1/2)*atanh(3^(1/2)*((2*x)/3 + 1)))/3 - ((11*x)/4 + (9*x^2)/4 + x^3/2 +
13/16)/(9*x + 12*x^2 + 6*x^3 + x^4 + 9/4)
```

3.201 $\int \frac{-1+x}{(4+5x+x^2)^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 36

$$\int \frac{-1+x}{(4+5x+x^2)^2} \, dx = \frac{13+7x}{9(4+5x+x^2)} + \frac{7}{27}\log(1+x) - \frac{7}{27}\log(4+x)$$

```
[Out] 1/9*(13+7*x)/(x^2+5*x+4)+7/27*ln(1+x)-7/27*ln(4+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {652, 630, 31}

$$\int \frac{-1+x}{(4+5x+x^2)^2} \, dx = \frac{7x+13}{9(x^2+5x+4)} + \frac{7}{27}\log(x+1) - \frac{7}{27}\log(x+4)$$

```
[In] Int[(-1 + x)/(4 + 5*x + x^2)^2,x]
[Out] (13 + 7*x)/(9*(4 + 5*x + x^2)) + (7*Log[1 + x])/27 - (7*Log[4 + x])/27
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{13+7x}{9\left(4+5x+x^2\right)}+\frac{7}{9}\int \frac{1}{4+5x+x^2}\,dx\\ &= \frac{13+7x}{9\left(4+5x+x^2\right)}+\frac{7}{27}\int \frac{1}{1+x}\,dx-\frac{7}{27}\int \frac{1}{4+x}\,dx\\ &= \frac{13+7x}{9\left(4+5x+x^2\right)}+\frac{7}{27}\log(1+x)-\frac{7}{27}\log(4+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{-1+x}{\left(4+5x+x^2\right)^2}\,dx=\frac{1}{27}\left(\frac{39+21x}{4+5x+x^2}+7\log(1+x)-7\log(4+x)\right)$$

```
[In] Integrate[(-1 + x)/(4 + 5*x + x^2)^2,x]
[Out] ((39 + 21*x)/(4 + 5*x + x^2) + 7*Log[1 + x] - 7*Log[4 + x])/27
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{5}{9(4+x)}-\frac{7\ln(4+x)}{27}+\frac{2}{9(1+x)}+\frac{7\ln(1+x)}{27}$	28
norman	$\frac{\frac{7x}{9}+\frac{13}{9}}{x^2+5x+4}+\frac{7\ln(1+x)}{27}-\frac{7\ln(4+x)}{27}$	30
risch	$\frac{\frac{7x}{9}+\frac{13}{9}}{x^2+5x+4}+\frac{7\ln(1+x)}{27}-\frac{7\ln(4+x)}{27}$	30
parallelrisch	$\frac{7\ln(1+x)x^2-7\ln(4+x)x^2+39+35\ln(1+x)x-35\ln(4+x)x+28\ln(1+x)-28\ln(4+x)+21x}{27x^2+135x+108}$	62

```
[In] int((-1+x)/(x^2+5*x+4)^2,x,method=_RETURNVERBOSE)
[Out] 5/9/(4+x)-7/27*ln(4+x)+2/9/(1+x)+7/27*ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{-1+x}{(4+5x+x^2)^2}\,dx = -\frac{7\,(x^2+5\,x+4)\log(x+4)-7\,(x^2+5\,x+4)\log(x+1)-21\,x-39}{27\,(x^2+5\,x+4)}$$

```
[In] integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="fricas")
[Out] -1/27*(7*(x^2 + 5*x + 4)*log(x + 4) - 7*(x^2 + 5*x + 4)*log(x + 1) - 21*x -
 39)/(x^2 + 5*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{-1+x}{(4+5x+x^2)^2}\,dx = \frac{7x+13}{9x^2+45x+36}+\frac{7\log(x+1)}{27}-\frac{7\log(x+4)}{27}$$

```
[In] integrate((-1+x)/(x**2+5*x+4)**2,x)
[Out] (7*x + 13)/(9*x**2 + 45*x + 36) + 7*log(x + 1)/27 - 7*log(x + 4)/27
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{-1+x}{(4+5x+x^2)^2}\,dx = \frac{7\,x+13}{9\,(x^2+5\,x+4)}-\frac{7}{27}\,\log(x+4)+\frac{7}{27}\,\log(x+1)$$

```
[In] integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="maxima")
[Out] 1/9*(7*x + 13)/(x^2 + 5*x + 4) - 7/27*log(x + 4) + 7/27*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{-1+x}{(4+5x+x^2)^2}\,dx = \frac{7\,x+13}{9\,(x^2+5\,x+4)} - \frac{7}{27}\,\log\left(|x+4|\right) + \frac{7}{27}\,\log\left(|x+1|\right)$$

```
[In] integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="giac")
[Out] 1/9*(7*x + 13)/(x^2 + 5*x + 4) - 7/27*log(abs(x + 4)) + 7/27*log(abs(x + 1)
)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{-1+x}{(4+5x+x^2)^2}\,dx = \frac{\frac{7\,x}{9}+\frac{13}{9}}{x^2+5\,x+4} - \frac{14\,\mathrm{atanh}\left(\frac{2\,x}{3}+\frac{5}{3}\right)}{27}$$

```
[In] int((x - 1)/(5*x + x^2 + 4)^2,x)
[Out] ((7*x)/9 + 13/9)/(5*x + x^2 + 4) - (14*atanh((2*x)/3 + 5/3))/27
```

3.202 $\int \frac{1}{(2+3x+x^2)^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 87

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = \frac{-3-2x}{4(2+3x+x^2)^4}+\frac{7(3+2x)}{6(2+3x+x^2)^3}-\frac{35(3+2x)}{6(2+3x+x^2)^2}+\frac{35(3+2x)}{2+3x+x^2}+70\log(1+x)-70\log(2+x)$$

```
[Out] 1/4*(-3-2*x)/(x^2+3*x+2)^4+7/6*(3+2*x)/(x^2+3*x+2)^3-35/6*(3+2*x)/(x^2+3*x+
2)^2+35*(3+2*x)/(x^2+3*x+2)+70*ln(1+x)-70*ln(2+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {628, 630, 31}

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = \frac{35(2x+3)}{x^2+3x+2}-\frac{35(2x+3)}{6(x^2+3x+2)^2}+\frac{7(2x+3)}{6(x^2+3x+2)^3}-\frac{2x+3}{4(x^2+3x+2)^4}+70\log(x+1)-70\log(x+2)$$

```
[In] Int[(2 + 3*x + x^2)^(-5),x]
```

```
[Out] -1/4*(3 + 2*x)/(2 + 3*x + x^2)^4 + (7*(3 + 2*x))/(6*(2 + 3*x + x^2)^3) - (3
5*(3 + 2*x))/(6*(2 + 3*x + x^2)^2) + (35*(3 + 2*x))/(2 + 3*x + x^2) + 70*Lo
g[1 + x] - 70*Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3+2x}{4\left(2+3x+x^2\right)^4}-\frac{7}{2}\int\frac{1}{\left(2+3x+x^2\right)^4}\,dx\\
&= -\frac{3+2x}{4\left(2+3x+x^2\right)^4}+\frac{7(3+2x)}{6\left(2+3x+x^2\right)^3}+\frac{35}{3}\int\frac{1}{\left(2+3x+x^2\right)^3}\,dx\\
&= -\frac{3+2x}{4\left(2+3x+x^2\right)^4}+\frac{7(3+2x)}{6\left(2+3x+x^2\right)^3}-\frac{35(3+2x)}{6\left(2+3x+x^2\right)^2}-35\int\frac{1}{\left(2+3x+x^2\right)^2}\,dx\\
&= -\frac{3+2x}{4\left(2+3x+x^2\right)^4}+\frac{7(3+2x)}{6\left(2+3x+x^2\right)^3}-\frac{35(3+2x)}{6\left(2+3x+x^2\right)^2}\\
&\quad+\frac{35(3+2x)}{2+3x+x^2}+70\int\frac{1}{2+3x+x^2}\,dx\\
&= -\frac{3+2x}{4\left(2+3x+x^2\right)^4}+\frac{7(3+2x)}{6\left(2+3x+x^2\right)^3}-\frac{35(3+2x)}{6\left(2+3x+x^2\right)^2}\\
&\quad+\frac{35(3+2x)}{2+3x+x^2}+70\int\frac{1}{1+x}\,dx-70\int\frac{1}{2+x}\,dx\\
&= -\frac{3+2x}{4\left(2+3x+x^2\right)^4}+\frac{7(3+2x)}{6\left(2+3x+x^2\right)^3}-\frac{35(3+2x)}{6\left(2+3x+x^2\right)^2}\\
&\quad+\frac{35(3+2x)}{2+3x+x^2}+70\log(1+x)-70\log(2+x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = \frac{-3-2x}{4(2+3x+x^2)^4} + \frac{7(3+2x)}{6(2+3x+x^2)^3} - \frac{35(3+2x)}{6(2+3x+x^2)^2} + \frac{35(3+2x)}{2+3x+x^2} + 70\log(1+x) - 70\log(2+x)$$

```
[In] Integrate[(2 + 3*x + x^2)^(-5),x]
```

```
[Out] (-3 - 2*x)/(4*(2 + 3*x + x^2)^4) + (7*(3 + 2*x))/(6*(2 + 3*x + x^2)^3) - (3
5*(3 + 2*x))/(6*(2 + 3*x + x^2)^2) + (35*(3 + 2*x))/(2 + 3*x + x^2) + 70*Lo
g[1 + x] - 70*Log[2 + x]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

method	result
norman	$\frac{70x^7+735x^6+4098x+9093x^2+\frac{9730}{3}x^5+\frac{15575}{2}x^4+\frac{32942}{3}x^3+\frac{3105}{4}}{(x^2+3x+2)^4}+70\ln(1+x)-70\ln(2+x)$
risch	$\frac{70x^7+735x^6+4098x+9093x^2+\frac{9730}{3}x^5+\frac{15575}{2}x^4+\frac{32942}{3}x^3+\frac{3105}{4}}{(x^2+3x+2)^4}+70\ln(1+x)-70\ln(2+x)$
default	$\frac{1}{4(2+x)^4}+\frac{5}{3(2+x)^3}+\frac{15}{2(2+x)^2}+\frac{35}{2+x}-70\ln(2+x)-\frac{1}{4(1+x)^4}+\frac{5}{3(1+x)^3}-\frac{15}{2(1+x)^2}+\frac{35}{1+x}+70\ln($
parallelrisch	$9315+8820x^6+49176x+38920x^5+13440\ln(1+x)-13440\ln(2+x)+93450x^4+131768x^3+109116x^2+840x^7-80640\ln(2+x)x+$

```
[In] int(1/(x^2+3*x+2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (70*x^7+735*x^6+4098*x+9093*x^2+9730/3*x^5+15575/2*x^4+32942/3*x^3+3105/4)/
(x^2+3*x+2)^4+70*ln(1+x)-70*ln(2+x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(81) = 162.

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.90

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = \frac{840\,x^7+8820\,x^6+38920\,x^5+93450\,x^4+131768\,x^3+109116\,x^2-840\,(x^8+12\,x^7+62\,x^6+180\,x^5+32}{12\,(x^8+12\,x}$$

```
[In] integrate(1/(x^2+3*x+2)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 109116*x^2
- 840*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x
 + 16)*log(x + 2) + 840*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^
3 + 248*x^2 + 96*x + 16)*log(x + 1) + 49176*x + 9315)/(x^8 + 12*x^7 + 62*x^
6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = \frac{840x^7+8820x^6+38920x^5+93450x^4+131768x^3+109116x^2+49176x+9315}{12x^8+144x^7+744x^6+2160x^5+3852x^4+4320x^3+2976x^2+1152x+192} + 70\log(x+1) - 70\log(x+2)$$

```
[In] integrate(1/(x**2+3*x+2)**5,x)
```

```
[Out] (840*x**7 + 8820*x**6 + 38920*x**5 + 93450*x**4 + 131768*x**3 + 109116*x**2
 + 49176*x + 9315)/(12*x**8 + 144*x**7 + 744*x**6 + 2160*x**5 + 3852*x**4 +
 4320*x**3 + 2976*x**2 + 1152*x + 192) + 70*log(x + 1) - 70*log(x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = \frac{840\,x^7+8820\,x^6+38920\,x^5+93450\,x^4+131768\,x^3+109116\,x^2+49176\,x+9315}{12\,(x^8+12\,x^7+62\,x^6+180\,x^5+321\,x^4+360\,x^3+248\,x^2+96\,x+16)} - 70\,\log(x+2) + 70\,\log(x+1)$$

```
[In] integrate(1/(x^2+3*x+2)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 109116*x^2
+ 49176*x + 9315)/(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 24
8*x^2 + 96*x + 16) - 70*log(x + 2) + 70*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \frac{1}{(2+3x+x^2)^5} \, dx = \frac{840 \, x^7 + 8820 \, x^6 + 38920 \, x^5 + 93450 \, x^4 + 131768 \, x^3 + 109116 \, x^2 + 49176 \, x + 9315}{12 \, (x^2 + 3 \, x + 2)^4} - 70 \, \log \left(|x+2|\right) + 70 \, \log \left(|x+1|\right)$$

```
[In] integrate(1/(x^2+3*x+2)^5,x, algorithm="giac")
```

```
[Out] 1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 109116*x^2
+ 49176*x + 9315)/(x^2 + 3*x + 2)^4 - 70*log(abs(x + 2)) + 70*log(abs(x + 1
))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.75

$$\int \frac{1}{(2+3x+x^2)^5} \, dx = 70 \, \ln \left(\frac{x+1}{x+2}\right) + 70 \left(x + \frac{3}{2}\right) \left(\frac{1}{x^2 + 3 \, x + 2} - \frac{1}{6 \, (x^2 + 3 \, x + 2)^2} + \frac{1}{30 \, (x^2 + 3 \, x + 2)^3} - \frac{1}{140 \, (x^2 + 3 \, x + 2)^4}\right)$$

```
[In] int(1/(3*x + x^2 + 2)^5,x)
```

```
[Out] 70*log((x + 1)/(x + 2)) + 70*(x + 3/2)*(1/(3*x + x^2 + 2) - 1/(6*(3*x + x^2
 + 2)^2) + 1/(30*(3*x + x^2 + 2)^3) - 1/(140*(3*x + x^2 + 2)^4))
```

3.203 $\int \frac{1}{x^3(7-6x+2x^2)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{1}{x^3(7-6x+2x^2)^2}\,dx = -\frac{1}{490x^2}-\frac{69}{1715x}-\frac{2-3x}{35x^2(7-6x+2x^2)}-\frac{234\arctan\left(\frac{3-2x}{\sqrt{5}}\right)}{12005\sqrt{5}}+\frac{80\log(x)}{2401}-\frac{40\log(7-6x+2x^2)}{2401}$$

```
[Out] -1/490/x^2-69/1715/x+1/35*(-2+3*x)/x^2/(2*x^2-6*x+7)+80/2401*ln(x)-40/2401*
ln(2*x^2-6*x+7)-234/60025*arctan(1/5*(3-2*x)*5^(1/2))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {754, 814, 648, 632, 210, 642}

$$\int \frac{1}{x^3(7-6x+2x^2)^2}\,dx = -\frac{234\arctan\left(\frac{3-2x}{\sqrt{5}}\right)}{12005\sqrt{5}}-\frac{2-3x}{35x^2(2x^2-6x+7)}-\frac{1}{490x^2}-\frac{40\log(2x^2-6x+7)}{2401}-\frac{69}{1715x}+\frac{80\log(x)}{2401}$$

```
[In] Int[1/(x^3*(7 - 6*x + 2*x^2)^2),x]
```

```
[Out] -1/490*1/x^2 - 69/(1715*x) - (2 - 3*x)/(35*x^2*(7 - 6*x + 2*x^2)) - (234*Ar
cTan[(3 - 2*x)/Sqrt[5]])/(12005*Sqrt[5]) + (80*Log[x])/2401 - (40*Log[7 - 6
*x + 2*x^2])/2401
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{2-3x}{35x^2\left(7-6x+2x^2\right)}+\frac{1}{140}\int\frac{4+36x}{x^3\left(7-6x+2x^2\right)}\,dx$$

$$= -\frac{2-3x}{35x^2(7-6x+2x^2)} + \frac{1}{140}\int \left(\frac{4}{7x^3} + \frac{276}{49x^2} + \frac{1600}{343x} - \frac{8(-717+400x)}{343(7-6x+2x^2)}\right) dx$$

$$= -\frac{1}{490x^2} - \frac{69}{1715x} - \frac{2-3x}{35x^2(7-6x+2x^2)} + \frac{80\log(x)}{2401} - \frac{2\int \frac{-717+400x}{7-6x+2x^2} dx}{12005}$$

$$= -\frac{1}{490x^2} - \frac{69}{1715x} - \frac{2-3x}{35x^2(7-6x+2x^2)} + \frac{80\log(x)}{2401} - \frac{40\int \frac{-6+4x}{7-6x+2x^2} dx}{2401} + \frac{234\int \frac{1}{7-6x+2x^2} dx}{12005}$$

$$= -\frac{1}{490x^2} - \frac{69}{1715x} - \frac{2-3x}{35x^2(7-6x+2x^2)} + \frac{80\log(x)}{2401} - \frac{40\log(7-6x+2x^2)}{2401} - \frac{468\text{Subst}\left(\int \frac{1}{-20-x^2} dx, x, -6+4x\right)}{12005}$$

$$= -\frac{1}{490x^2} - \frac{69}{1715x} - \frac{2-3x}{35x^2(7-6x+2x^2)} - \frac{234\arctan\left(\frac{3-2x}{\sqrt{5}}\right)}{12005\sqrt{5}} + \frac{80\log(x)}{2401} - \frac{40\log(7-6x+2x^2)}{2401}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3(7-6x+2x^2)^2} dx = \frac{-\frac{1225}{x^2} - \frac{4200}{x} - \frac{140(-41+9x)}{7-6x+2x^2} + 468\sqrt{5}\arctan\left(\frac{-3+2x}{\sqrt{5}}\right) + 4000\log(x) - 2000\log(7-6x+2x^2)}{120050}$$

```
[In] Integrate[1/(x^3*(7 - 6*x + 2*x^2)^2),x]
[Out] (-1225/x^2 - 4200/x - (140*(-41 + 9*x))/(7 - 6*x + 2*x^2) + 468*Sqrt[5]*Arc
Tan[(-3 + 2*x)/Sqrt[5]] + 4000*Log[x] - 2000*Log[7 - 6*x + 2*x^2])/120050
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{1}{98x^2} - \frac{12}{343x} + \frac{80\ln(x)}{2401} - \frac{4\left(\frac{63x}{20} - \frac{287}{20}\right)}{2401\left(x^2-3x+\frac{7}{2}\right)} - \frac{40\ln(2x^2-6x+7)}{2401} + \frac{234\sqrt{5}\,\arctan\left(\frac{(4x-6)\sqrt{5}}{10}\right)}{60025}$	62
risch	$\frac{-\frac{138}{1715}x^3 + \frac{407}{1715}x^2 - \frac{9}{49}x - \frac{1}{14}}{x^2(2x^2-6x+7)} - \frac{40\ln(4x^2-12x+14)}{2401} + \frac{234\sqrt{5}\,\arctan\left(\frac{(2x-3)\sqrt{5}}{5}\right)}{60025} + \frac{80\ln(x)}{2401}$	67

```
[In] int(1/x^3/(2*x^2-6*x+7)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/98/x^2-12/343/x+80/2401*ln(x)-4/2401*(63/20*x-287/20)/(x^2-3*x+7/2)-40/2
401*ln(2*x^2-6*x+7)+234/60025*5^(1/2)*arctan(1/10*(4*x-6)*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^3\left(7-6x+2x^2\right)^2}\,dx = -\frac{9660\,x^3-468\,\sqrt{5}(2\,x^4-6\,x^3+7\,x^2)\arctan\left(\frac{1}{5}\,\sqrt{5}(2\,x-3)\right)-28490\,x^2+2000\,(2\,x^4-6\,x^3+7\,x^2)\log}{120050\,(2\,x^4-6\,x^3+7\,x^2)}$$

```
[In] integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="fricas")
```

```
[Out] -1/120050*(9660*x^3 - 468*sqrt(5)*(2*x^4 - 6*x^3 + 7*x^2)*arctan(1/5*sqrt(5
)*(2*x - 3)) - 28490*x^2 + 2000*(2*x^4 - 6*x^3 + 7*x^2)*log(2*x^2 - 6*x + 7
) - 4000*(2*x^4 - 6*x^3 + 7*x^2)*log(x) + 22050*x + 8575)/(2*x^4 - 6*x^3 +
7*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^3\left(7-6x+2x^2\right)^2}\,dx = \frac{80\log\left(x\right)}{2401}-\frac{40\log\left(x^2-3x+\frac{7}{2}\right)}{2401}+\frac{234\sqrt{5}\operatorname{atan}\left(\frac{2\sqrt{5}x}{5}-\frac{3\sqrt{5}}{5}\right)}{60025}+\frac{-276x^3+814x^2-630x-245}{6860x^4-20580x^3+24010x^2}$$

```
[In] integrate(1/x**3/(2*x**2-6*x+7)**2,x)
```

```
[Out] 80*log(x)/2401 - 40*log(x**2 - 3*x + 7/2)/2401 + 234*sqrt(5)*atan(2*sqrt(5)
*x/5 - 3*sqrt(5)/5)/60025 + (-276*x**3 + 814*x**2 - 630*x - 245)/(6860*x**4
 - 20580*x**3 + 24010*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 (7-6x+2x^2)^2} dx = \frac{234}{60025} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5}(2x-3)\right) - \frac{276x^3 - 814x^2 + 630x + 245}{3430 (2x^4 - 6x^3 + 7x^2)} - \frac{40}{2401} \log(2x^2 - 6x + 7) + \frac{80}{2401} \log(x)$$

```
[In] integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="maxima")
```

```
[Out] 234/60025*sqrt(5)*arctan(1/5*sqrt(5)*(2*x - 3)) - 1/3430*(276*x^3 - 814*x^2
 + 630*x + 245)/(2*x^4 - 6*x^3 + 7*x^2) - 40/2401*log(2*x^2 - 6*x + 7) + 80
/2401*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (7-6x+2x^2)^2} dx = \frac{234}{60025} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5}(2x-3)\right) - \frac{276x^3 - 814x^2 + 630x + 245}{3430 (2x^2 - 6x + 7)x^2} - \frac{40}{2401} \log(2x^2 - 6x + 7) + \frac{80}{2401} \log(|x|)$$

```
[In] integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="giac")
```

```
[Out] 234/60025*sqrt(5)*arctan(1/5*sqrt(5)*(2*x - 3)) - 1/3430*(276*x^3 - 814*x^2
 + 630*x + 245)/((2*x^2 - 6*x + 7)*x^2) - 40/2401*log(2*x^2 - 6*x + 7) + 80
/2401*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3 (7-6x+2x^2)^2} \, dx = \frac{80 \ln(x)}{2401} - \frac{\frac{69 x^3}{1715} - \frac{407 x^2}{3430} + \frac{9 x}{98} + \frac{1}{28}}{x^4 - 3 x^3 + \frac{7 x^2}{2}} - \ln\left(x - \frac{3}{2} - \frac{\sqrt{5}\,1\mathrm{i}}{2}\right) \left(\frac{40}{2401} + \frac{\sqrt{5}\,117\mathrm{i}}{60025}\right) + \ln\left(x - \frac{3}{2} + \frac{\sqrt{5}\,1\mathrm{i}}{2}\right) \left(-\frac{40}{2401} + \frac{\sqrt{5}\,117\mathrm{i}}{60025}\right)$$

```
[In] int(1/(x^3*(2*x^2 - 6*x + 7)^2),x)
```

```
[Out] (80*log(x))/2401 - ((9*x)/98 - (407*x^2)/3430 + (69*x^3)/1715 + 1/28)/((7*x
^2)/2 - 3*x^3 + x^4) - log(x - (5^(1/2)*1i)/2 - 3/2)*((5^(1/2)*117i)/60025
+ 40/2401) + log(x + (5^(1/2)*1i)/2 - 3/2)*((5^(1/2)*117i)/60025 - 40/2401)
```

3.204 $\int \frac{x^9}{(2+3x+x^2)^5} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{x^9}{(2+3x+x^2)^5} \, dx = 735x + \frac{x^8(4+3x)}{4(2+3x+x^2)^4} - \frac{x^6(110+81x)}{12(2+3x+x^2)^3} + \frac{x^4(184+135x)}{2(2+3x+x^2)^2} - \frac{x^2(2206+1593x)}{2(2+3x+x^2)} - 1471\log(1+x) + 1472\log(2+x)$$

```
[Out] 735*x+1/4*x^8*(4+3*x)/(x^2+3*x+2)^4-1/12*x^6*(110+81*x)/(x^2+3*x+2)^3+1/2*x
^4*(184+135*x)/(x^2+3*x+2)^2-1/2*x^2*(2206+1593*x)/(x^2+3*x+2)-1471*ln(1+x)
+1472*ln(2+x)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {752, 832, 787, 646, 31}

$$\int \frac{x^9}{(2+3x+x^2)^5} \, dx = -\frac{(1593x+2206)x^2}{2(x^2+3x+2)} + \frac{(3x+4)x^8}{4(x^2+3x+2)^4} - \frac{(81x+110)x^6}{12(x^2+3x+2)^3} + \frac{(135x+184)x^4}{2(x^2+3x+2)^2} + 735x - 1471\log(x+1) + 1472\log(x+2)$$

```
[In] Int[x^9/(2 + 3*x + x^2)^5,x]
```

```
[Out] 735*x + (x^8*(4 + 3*x))/(4*(2 + 3*x + x^2)^4) - (x^6*(110 + 81*x))/(12*(2 +
 3*x + x^2)^3) + (x^4*(184 + 135*x))/(2*(2 + 3*x + x^2)^2) - (x^2*(2206 + 1
593*x))/(2*(2 + 3*x + x^2)) - 1471*Log[1 + x] + 1472*Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^8(4+3x)}{4(2+3x+x^2)^4} - \frac{1}{4}\int \frac{x^7(32+3x)}{(2+3x+x^2)^4}\,dx \\
&= \frac{x^8(4+3x)}{4(2+3x+x^2)^4} - \frac{x^6(110+81x)}{12(2+3x+x^2)^3} - \frac{1}{12}\int \frac{(-660-72x)x^5}{(2+3x+x^2)^3}\,dx \\
&= \frac{x^8(4+3x)}{4(2+3x+x^2)^4} - \frac{x^6(110+81x)}{12(2+3x+x^2)^3} + \frac{x^4(184+135x)}{2(2+3x+x^2)^2} - \frac{1}{24}\int \frac{x^3(8832+1476x)}{(2+3x+x^2)^2}\,dx \\
&= \frac{x^8(4+3x)}{4(2+3x+x^2)^4} - \frac{x^6(110+81x)}{12(2+3x+x^2)^3} + \frac{x^4(184+135x)}{2(2+3x+x^2)^2} \\
&\quad - \frac{x^2(2206+1593x)}{2(2+3x+x^2)} - \frac{1}{24}\int \frac{(-52944-17640x)x}{2+3x+x^2}\,dx \\
&= 735x + \frac{x^8(4+3x)}{4(2+3x+x^2)^4} - \frac{x^6(110+81x)}{12(2+3x+x^2)^3} + \frac{x^4(184+135x)}{2(2+3x+x^2)^2} \\
&\quad - \frac{x^2(2206+1593x)}{2(2+3x+x^2)} - \frac{1}{24}\int \frac{35280-24x}{2+3x+x^2}\,dx \\
&= 735x + \frac{x^8(4+3x)}{4(2+3x+x^2)^4} - \frac{x^6(110+81x)}{12(2+3x+x^2)^3} + \frac{x^4(184+135x)}{2(2+3x+x^2)^2} \\
&\quad - \frac{x^2(2206+1593x)}{2(2+3x+x^2)} - 1471\int \frac{1}{1+x}\,dx + 1472\int \frac{1}{2+x}\,dx \\
&= 735x + \frac{x^8(4+3x)}{4(2+3x+x^2)^4} - \frac{x^6(110+81x)}{12(2+3x+x^2)^3} + \frac{x^4(184+135x)}{2(2+3x+x^2)^2} \\
&\quad - \frac{x^2(2206+1593x)}{2(2+3x+x^2)} - 1471\log(1+x) + 1472\log(2+x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$
\begin{aligned}
\int \frac{x^9}{(2+3x+x^2)^5}\,dx &= \frac{514+513x}{4(2+3x+x^2)^4} + \frac{415+1998x}{12(2+3x+x^2)^3} + \frac{3(451+456x)}{4(2+3x+x^2)^2} \\
&\quad - \frac{2(1114+729x)}{2+3x+x^2} - 1471\log(1+x) + 1472\log(2+x)
\end{aligned}
$$

```
[In] Integrate[x^9/(2 + 3*x + x^2)^5,x]
```

```
[Out] (514 + 513*x)/(4*(2 + 3*x + x^2)^4) + (415 + 1998*x)/(12*(2 + 3*x + x^2)^3)
 + (3*(451 + 456*x))/(4*(2 + 3*x + x^2)^2) - (2*(1114 + 729*x))/(2 + 3*x +
x^2) - 1471*Log[1 + x] + 1472*Log[2 + x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

method	result
norman	$\frac{-229950x^3-85880x-67824x^5-15350x^6-1458x^7-\frac{651951}{4}x^4-\frac{571502}{3}x^2-\frac{48820}{3}}{(x^2+3x+2)^4}-1471\ln(1+x)+1472\ln(2+$
risch	$\frac{-229950x^3-85880x-67824x^5-15350x^6-1458x^7-\frac{651951}{4}x^4-\frac{571502}{3}x^2-\frac{48820}{3}}{(x^2+3x+2)^4}-1471\ln(1+x)+1472\ln(2+$
default	$-\frac{128}{(2+x)^4}-\frac{256}{3(2+x)^3}-\frac{384}{(2+x)^2}-\frac{1024}{2+x}+1472\ln(2+x)+\frac{1}{4(1+x)^4}-\frac{14}{3(1+x)^3}+\frac{48}{(1+x)^2}-\frac{434}{1+x}-147$
parallelrisch	$-\frac{195280+184200x^6+1030560x+813888x^5+282432\ln(1+x)-282624\ln(2+x)+1955853x^4+2759400x^3+2286008x^2+17496x}{}$

```
[In] int(x^9/(x^2+3*x+2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (-229950*x^3-85880*x-67824*x^5-15350*x^6-1458*x^7-651951/4*x^4-571502/3*x^2
-48820/3)/(x^2+3*x+2)^4-1471*ln(1+x)+1472*ln(2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.59

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = -\frac{17496\,x^7+184200\,x^6+813888\,x^5+1955853\,x^4+2759400\,x^3+2286008\,x^2-17664\,(x^8+12\,x^7+62\,x}{}$$

```
[In] integrate(x^9/(x^2+3*x+2)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 22
86008*x^2 - 17664*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 24
8*x^2 + 96*x + 16)*log(x + 2) + 17652*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 32
1*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)*log(x + 1) + 1030560*x + 195280)/(x^
8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = \frac{-17496x^7 - 184200x^6 - 813888x^5 - 1955853x^4 - 2759400x^3 - 2286008x^2 - 1030560x - 195280}{12x^8 + 144x^7 + 744x^6 + 2160x^5 + 3852x^4 + 4320x^3 + 2976x^2 + 1152x + 192} - 1471\log(x+1) + 1472\log(x+2)$$

```
[In] integrate(x**9/(x**2+3*x+2)**5,x)
```

```
[Out] (-17496*x**7 - 184200*x**6 - 813888*x**5 - 1955853*x**4 - 2759400*x**3 - 22
86008*x**2 - 1030560*x - 195280)/(12*x**8 + 144*x**7 + 744*x**6 + 2160*x**5
 + 3852*x**4 + 4320*x**3 + 2976*x**2 + 1152*x + 192) - 1471*log(x + 1) + 14
72*log(x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = -\frac{17496\,x^7 + 184200\,x^6 + 813888\,x^5 + 1955853\,x^4 + 2759400\,x^3 + 2286008\,x^2 + 1030560\,x + 195280}{12\,(x^8 + 12\,x^7 + 62\,x^6 + 180\,x^5 + 321\,x^4 + 360\,x^3 + 248\,x^2 + 96\,x + 16)} + 1472\,\log(x+2) - 1471\,\log(x+1)$$

```
[In] integrate(x^9/(x^2+3*x+2)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 22
86008*x^2 + 1030560*x + 195280)/(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4
+ 360*x^3 + 248*x^2 + 96*x + 16) + 1472*log(x + 2) - 1471*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.60

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = -\frac{17496\,x^7 + 184200\,x^6 + 813888\,x^5 + 1955853\,x^4 + 2759400\,x^3 + 2286008\,x^2 + 1030560\,x + 195280}{12\,(x+2)^4(x+1)^4} + 1472\,\log(|x+2|) - 1471\,\log(|x+1|)$$

```
[In] integrate(x^9/(x^2+3*x+2)^5,x, algorithm="giac")
```

```
[Out] -1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 22
86008*x^2 + 1030560*x + 195280)/((x + 2)^4*(x + 1)^4) + 1472*log(abs(x + 2)
) - 1471*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = 1472\,\ln(x+2)-1471\,\ln(x+1) - \frac{1458\,x^7+15350\,x^6+67824\,x^5+\frac{651951\,x^4}{4}+229950\,x^3+\frac{571502\,x^2}{3}+85880\,x+\frac{48820}{3}}{x^8+12\,x^7+62\,x^6+180\,x^5+321\,x^4+360\,x^3+248\,x^2+96\,x+16}$$

```
[In] int(x^9/(3*x + x^2 + 2)^5,x)
```

```
[Out] 1472*log(x + 2) - 1471*log(x + 1) - (85880*x + (571502*x^2)/3 + 229950*x^3
+ (651951*x^4)/4 + 67824*x^5 + 15350*x^6 + 1458*x^7 + 48820/3)/(96*x + 248*
x^2 + 360*x^3 + 321*x^4 + 180*x^5 + 62*x^6 + 12*x^7 + x^8 + 16)
```

3.205 $\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 102

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx = \frac{(1+2x)(7+6x)}{4(3+5x+2x^2)^4}+\frac{73+62x}{3(3+5x+2x^2)^3}-\frac{155(5+4x)}{3(3+5x+2x^2)^2}+\frac{620(5+4x)}{3+5x+2x^2}+2480\log(1+x)-2480\log(3+2x)$$

```
[Out] 1/4*(1+2*x)*(7+6*x)/(2*x^2+5*x+3)^4+1/3*(73+62*x)/(2*x^2+5*x+3)^3-155/3*(5+
4*x)/(2*x^2+5*x+3)^2+620*(5+4*x)/(2*x^2+5*x+3)+2480*ln(1+x)-2480*ln(3+2*x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {752, 652, 628, 630, 31}

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx = \frac{620(4x+5)}{2x^2+5x+3}-\frac{155(4x+5)}{3(2x^2+5x+3)^2}+\frac{62x+73}{3(2x^2+5x+3)^3}+\frac{(2x+1)(6x+7)}{4(2x^2+5x+3)^4}+2480\log(x+1)-2480\log(2x+3)$$

```
[In] Int[(1 + 2*x)^2/(3 + 5*x + 2*x^2)^5,x]
```

```
[Out] ((1 + 2*x)*(7 + 6*x))/(4*(3 + 5*x + 2*x^2)^4) + (73 + 62*x)/(3*(3 + 5*x + 2
*x^2)^3) - (155*(5 + 4*x))/(3*(3 + 5*x + 2*x^2)^2) + (620*(5 + 4*x))/(3 + 5
*x + 2*x^2) + 2480*Log[1 + x] - 2480*Log[3 + 2*x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(1+2x)(7+6x)}{4\left(3+5x+2x^2\right)^4}-\frac{1}{4}\int\frac{-28-72x}{\left(3+5x+2x^2\right)^4}\,dx\\ &= \frac{(1+2x)(7+6x)}{4\left(3+5x+2x^2\right)^4}+\frac{73+62x}{3\left(3+5x+2x^2\right)^3}+\frac{310}{3}\int\frac{1}{\left(3+5x+2x^2\right)^3}\,dx\\ &= \frac{(1+2x)(7+6x)}{4\left(3+5x+2x^2\right)^4}+\frac{73+62x}{3\left(3+5x+2x^2\right)^3}-\frac{155(5+4x)}{3\left(3+5x+2x^2\right)^2}-620\int\frac{1}{\left(3+5x+2x^2\right)^2}\,dx\end{aligned}$$

$$
\begin{aligned}
&= \frac{(1+2x)(7+6x)}{4(3+5x+2x^2)^4} + \frac{73+62x}{3(3+5x+2x^2)^3} - \frac{155(5+4x)}{3(3+5x+2x^2)^2} \\
&\quad + \frac{620(5+4x)}{3+5x+2x^2} + 2480\int \frac{1}{3+5x+2x^2}\,dx \\
&= \frac{(1+2x)(7+6x)}{4(3+5x+2x^2)^4} + \frac{73+62x}{3(3+5x+2x^2)^3} - \frac{155(5+4x)}{3(3+5x+2x^2)^2} \\
&\quad + \frac{620(5+4x)}{3+5x+2x^2} + 4960\int \frac{1}{2+2x}\,dx - 4960\int \frac{1}{3+2x}\,dx \\
&= \frac{(1+2x)(7+6x)}{4(3+5x+2x^2)^4} + \frac{73+62x}{3(3+5x+2x^2)^3} - \frac{155(5+4x)}{3(3+5x+2x^2)^2} \\
&\quad + \frac{620(5+4x)}{3+5x+2x^2} + 2480\log(1+x) - 2480\log(3+2x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97

$$
\begin{aligned}
\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx &= -\frac{11+10x}{4(3+5x+2x^2)^4} + \frac{31(5+4x)}{6(3+5x+2x^2)^3} - \frac{155(5+4x)}{3(3+5x+2x^2)^2} \\
&\quad + \frac{620(5+4x)}{3+5x+2x^2} + 2480\log(2(1+x)) - 2480\log(3+2x)
\end{aligned}
$$

```
[In] Integrate[(1 + 2*x)^2/(3 + 5*x + 2*x^2)^5,x]
```

```
[Out] -1/4*(11 + 10*x)/(3 + 5*x + 2*x^2)^4 + (31*(5 + 4*x))/(6*(3 + 5*x + 2*x^2)^
3) - (155*(5 + 4*x))/(3*(3 + 5*x + 2*x^2)^2) + (620*(5 + 4*x))/(3 + 5*x + 2
*x^2) + 2480*Log[2*(1 + x)] - 2480*Log[3 + 2*x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

method	result
norman	$\frac{19840x^7+173600x^6+1624648x^3+\frac{1428116}{3}x+\frac{1939360}{3}x^5+\frac{3552290}{3}x^2+\frac{3983500}{3}x^4+\frac{325799}{4}}{(2x^2+5x+3)^4}+2480\ln(1+x)-2480\ln($
risch	$\frac{19840x^7+173600x^6+1624648x^3+\frac{1428116}{3}x+\frac{1939360}{3}x^5+\frac{3552290}{3}x^2+\frac{3983500}{3}x^4+\frac{325799}{4}}{(2x^2+5x+3)^4}+2480\ln(1+x)-2480\ln($
default	$\frac{16}{(3+2x)^4}+\frac{256}{3(3+2x)^3}+\frac{328}{(3+2x)^2}+\frac{1360}{3+2x}-2480\ln(3+2x)-\frac{1}{4(1+x)^4}+\frac{14}{3(1+x)^3}-\frac{52}{(1+x)^2}+\frac{560}{1+x}+24$
parallelrisch	$3909588+8332800x^6+22849856x+31029760x^5+9642240\ln(1+x)+63736000x^4+77983104x^3+56836640x^2+952320x^7+828518$

```
[In] int((1+2*x)^2/(2*x^2+5*x+3)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (19840*x^7+173600*x^6+1624648*x^3+1428116/3*x+1939360/3*x^5+3552290/3*x^2+3
983500/3*x^4+325799/4)/(2*x^2+5*x+3)^4+2480*ln(1+x)-2480*ln(3+2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.70

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx$$
$$= \frac{238080\,x^7 + 2083200\,x^6 + 7757440\,x^5 + 15934000\,x^4 + 19495776\,x^3 + 14209160\,x^2 - 29760\,(16\,x^8 + 160\,}{}$$

```
[In] integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3
+ 14209160*x^2 - 29760*(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5 + 2641*x^4 +
2580*x^3 + 1566*x^2 + 540*x + 81)*log(2*x + 3) + 29760*(16*x^8 + 160*x^7 +
696*x^6 + 1720*x^5 + 2641*x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81)*log(x + 1
) + 5712464*x + 977397)/(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5 + 2641*x^4 +
 2580*x^3 + 1566*x^2 + 540*x + 81)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx$$
$$= \frac{238080x^7 + 2083200x^6 + 7757440x^5 + 15934000x^4 + 19495776x^3 + 14209160x^2 + 5712464x + 977397}{192x^8 + 1920x^7 + 8352x^6 + 20640x^5 + 31692x^4 + 30960x^3 + 18792x^2 + 6480x + 972}$$
$$+ 2480\log{(x+1)} - 2480\log{\left(x+\frac{3}{2}\right)}$$

```
[In] integrate((1+2*x)**2/(2*x**2+5*x+3)**5,x)
```

```
[Out] (238080*x**7 + 2083200*x**6 + 7757440*x**5 + 15934000*x**4 + 19495776*x**3
+ 14209160*x**2 + 5712464*x + 977397)/(192*x**8 + 1920*x**7 + 8352*x**6 + 2
0640*x**5 + 31692*x**4 + 30960*x**3 + 18792*x**2 + 6480*x + 972) + 2480*log
(x + 1) - 2480*log(x + 3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx = \frac{238080\,x^7+2083200\,x^6+7757440\,x^5+15934000\,x^4+19495776\,x^3+14209160\,x^2+5712464\,x+977397}{12\,(16\,x^8+160\,x^7+696\,x^6+1720\,x^5+2641\,x^4+2580\,x^3+1566\,x^2+540\,x+81)} - 2480\,\log\left(2\,x+3\right)+2480\,\log\left(x+1\right)$$

```
[In] integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3
+ 14209160*x^2 + 5712464*x + 977397)/(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5
 + 2641*x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81) - 2480*log(2*x + 3) + 2480*
log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx = \frac{238080\,x^7+2083200\,x^6+7757440\,x^5+15934000\,x^4+19495776\,x^3+14209160\,x^2+5712464\,x+977397}{12\,(2\,x^2+5\,x+3)^4} - 2480\,\log\left(|2\,x+3|\right)+2480\,\log\left(|x+1|\right)$$

```
[In] integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="giac")
```

```
[Out] 1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3
+ 14209160*x^2 + 5712464*x + 977397)/(2*x^2 + 5*x + 3)^4 - 2480*log(abs(2*x
 + 3)) + 2480*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx$$
$$= \frac{1240\,x^7+10850\,x^6+\frac{121210\,x^5}{3}+\frac{995875\,x^4}{12}+\frac{203081\,x^3}{2}+\frac{1776145\,x^2}{24}+\frac{357029\,x}{12}+\frac{325799}{64}}{x^8+10\,x^7+\frac{87\,x^6}{2}+\frac{215\,x^5}{2}+\frac{2641\,x^4}{16}+\frac{645\,x^3}{4}+\frac{783\,x^2}{8}+\frac{135\,x}{4}+\frac{81}{16}}$$
$$-4960\,\mathrm{atanh}(4\,x+5)$$

```
[In] int((2*x + 1)^2/(5*x + 2*x^2 + 3)^5,x)
```

```
[Out] ((357029*x)/12 + (1776145*x^2)/24 + (203081*x^3)/2 + (995875*x^4)/12 + (121
210*x^5)/3 + 10850*x^6 + 1240*x^7 + 325799/64)/((135*x)/4 + (783*x^2)/8 + (
645*x^3)/4 + (2641*x^4)/16 + (215*x^5)/2 + (87*x^6)/2 + 10*x^7 + x^8 + 81/1
6) - 4960*atanh(4*x + 5)
```

3.206 $\int \frac{(a-bx^2)^3}{x^7} dx$

Optimal result

Integrand size = 14, antiderivative size = 40

$$\int \frac{(a-bx^2)^3}{x^7} dx = -\frac{a^3}{6x^6} + \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} - b^3\log(x)$$

```
[Out] -1/6*a^3/x^6+3/4*a^2*b/x^4-3/2*a*b^2/x^2-b^3*ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {272, 45}

$$\int \frac{(a-bx^2)^3}{x^7} dx = -\frac{a^3}{6x^6} + \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} - b^3\log(x)$$

```
[In] Int[(a - b*x^2)^3/x^7,x]
```

```
[Out] -1/6*a^3/x^6 + (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) - b^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(a-bx)^3}{x^4}\,dx, x, x^2\right) \\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{3a^2b}{x^3}+\frac{3ab^2}{x^2}-\frac{b^3}{x}\right)\,dx, x, x^2\right) \\ &= -\frac{a^3}{6x^6}+\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}-b^3\log(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a-bx^2)^3}{x^7}\,dx = -\frac{a^3}{6x^6}+\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}-b^3\log(x)$$

```
[In] Integrate[(a - b*x^2)^3/x^7,x]
[Out] -1/6*a^3/x^6 + (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) - b^3*Log[x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^3}{6x^6}+\frac{3a^2b}{4x^4}-\frac{3a\,b^2}{2x^2}-b^3\ln\left(x\right)$	35
norman	$\frac{-\frac{1}{6}a^3+\frac{3}{4}a^2b\,x^2-\frac{3}{2}b^2a\,x^4}{x^6}-b^3\ln\left(x\right)$	37
risch	$\frac{-\frac{1}{6}a^3+\frac{3}{4}a^2b\,x^2-\frac{3}{2}b^2a\,x^4}{x^6}-b^3\ln\left(x\right)$	37
parallelrisch	$-\frac{12b^3\ln(x)x^6+18b^2a\,x^4-9a^2b\,x^2+2a^3}{12x^6}$	40

```
[In] int((-b*x^2+a)^3/x^7,x,method=_RETURNVERBOSE)
[Out] -1/6*a^3/x^6+3/4*a^2*b/x^4-3/2*a*b^2/x^2-b^3*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{(a-bx^2)^3}{x^7}\,dx = -\frac{12\,b^3x^6\log(x) + 18\,ab^2x^4 - 9\,a^2bx^2 + 2\,a^3}{12\,x^6}$$

```
[In] integrate((-b*x^2+a)^3/x^7,x, algorithm="fricas")
[Out] -1/12*(12*b^3*x^6*log(x) + 18*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a-bx^2)^3}{x^7}\,dx = -b^3\log(x) - \frac{2a^3 - 9a^2bx^2 + 18ab^2x^4}{12x^6}$$

```
[In] integrate((-b*x**2+a)**3/x**7,x)
[Out] -b**3*log(x) - (2*a**3 - 9*a**2*b*x**2 + 18*a*b**2*x**4)/(12*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{(a-bx^2)^3}{x^7}\,dx = -\frac{1}{2}\,b^3\log(x^2) - \frac{18\,ab^2x^4 - 9\,a^2bx^2 + 2\,a^3}{12\,x^6}$$

```
[In] integrate((-b*x^2+a)^3/x^7,x, algorithm="maxima")
[Out] -1/2*b^3*log(x^2) - 1/12*(18*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3)/x^6
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \frac{(a-bx^2)^3}{x^7}\,dx = -\frac{1}{2}\,b^3\log(x^2) + \frac{11\,b^3x^6 - 18\,ab^2x^4 + 9\,a^2bx^2 - 2\,a^3}{12\,x^6}$$

```
[In] integrate((-b*x^2+a)^3/x^7,x, algorithm="giac")
[Out] -1/2*b^3*log(x^2) + 1/12*(11*b^3*x^6 - 18*a*b^2*x^4 + 9*a^2*b*x^2 - 2*a^3)/
x^6
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a-bx^2)^3}{x^7}\,dx = -b^3\,\ln(x) - \frac{\frac{a^3}{6} - \frac{3\,a^2\,b\,x^2}{4} + \frac{3\,a\,b^2\,x^4}{2}}{x^6}$$

```
[In] int((a - b*x^2)^3/x^7,x)
[Out] - b^3*log(x) - (a^3/6 - (3*a^2*b*x^2)/4 + (3*a*b^2*x^4)/2)/x^6
```

3.207 $\int \frac{x^{13}}{(a^4+x^4)^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{x^{13}}{(a^4+x^4)^5} dx = -\frac{x^{10}}{16(a^4+x^4)^4} - \frac{5x^6}{96(a^4+x^4)^3} - \frac{5x^2}{128(a^4+x^4)^2} + \frac{5x^2}{256a^4(a^4+x^4)} + \frac{5\arctan\left(\frac{x^2}{a^2}\right)}{256a^6}$$

```
[Out] -1/16*x^10/(a^4+x^4)^4-5/96*x^6/(a^4+x^4)^3-5/128*x^2/(a^4+x^4)^2+5/256*x^2
/a^4/(a^4+x^4)+5/256*arctan(x^2/a^2)/a^6
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {281, 294, 205, 209}

$$\int \frac{x^{13}}{(a^4+x^4)^5} dx = -\frac{x^{10}}{16(a^4+x^4)^4} - \frac{5x^6}{96(a^4+x^4)^3} + \frac{5x^2}{256a^4(a^4+x^4)} - \frac{5x^2}{128(a^4+x^4)^2} + \frac{5\arctan\left(\frac{x^2}{a^2}\right)}{256a^6}$$

```
[In] Int[x^13/(a^4 + x^4)^5,x]
```

```
[Out] -1/16*x^10/(a^4 + x^4)^4 - (5*x^6)/(96*(a^4 + x^4)^3) - (5*x^2)/(128*(a^4 +
 x^4)^2) + (5*x^2)/(256*a^4*(a^4 + x^4)) + (5*ArcTan[x^2/a^2])/(256*a^6)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{(a^4+x^2)^5}\,dx, x, x^2\right)\\
&= -\frac{x^{10}}{16\,(a^4+x^4)^4}+\frac{5}{16}\text{Subst}\left(\int \frac{x^4}{(a^4+x^2)^4}\,dx, x, x^2\right)\\
&= -\frac{x^{10}}{16\,(a^4+x^4)^4}-\frac{5x^6}{96\,(a^4+x^4)^3}+\frac{5}{32}\text{Subst}\left(\int \frac{x^2}{(a^4+x^2)^3}\,dx, x, x^2\right)\\
&= -\frac{x^{10}}{16\,(a^4+x^4)^4}-\frac{5x^6}{96\,(a^4+x^4)^3}-\frac{5x^2}{128\,(a^4+x^4)^2}+\frac{5}{128}\text{Subst}\left(\int \frac{1}{(a^4+x^2)^2}\,dx, x, x^2\right)\\
&= -\frac{x^{10}}{16\,(a^4+x^4)^4}-\frac{5x^6}{96\,(a^4+x^4)^3}-\frac{5x^2}{128\,(a^4+x^4)^2}\\
&\quad+\frac{5x^2}{256a^4\,(a^4+x^4)}+\frac{5\text{Subst}\left(\int \frac{1}{a^4+x^2}\,dx, x, x^2\right)}{256a^4}\\
&= -\frac{x^{10}}{16\,(a^4+x^4)^4}-\frac{5x^6}{96\,(a^4+x^4)^3}-\frac{5x^2}{128\,(a^4+x^4)^2}+\frac{5x^2}{256a^4\,(a^4+x^4)}+\frac{5\arctan\left(\frac{x^2}{a^2}\right)}{256a^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = \frac{-\frac{a^2x^2(15a^{12}+55a^8x^4+73a^4x^8-15x^{12})}{(a^4+x^4)^4}+15\arctan\left(\frac{x^2}{a^2}\right)}{768a^6}$$

[In] Integrate[x^13/(a^4 + x^4)^5,x]

[Out] (-((a^2*x^2*(15*a^12 + 55*a^8*x^4 + 73*a^4*x^8 - 15*x^12))/(a^4 + x^4)^4) + 15*ArcTan[x^2/a^2])/(768*a^6)

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

method	result
risch	$\frac{-\frac{5a^8x^2}{256}-\frac{55a^4x^6}{768}-\frac{73x^{10}}{768}+\frac{5x^{14}}{256a^4}}{(a^4+x^4)^4}+\frac{5\arctan\left(\frac{x^2}{a^2}\right)}{256a^6}$
default	$\frac{\frac{5x^{14}}{128a^4}-\frac{73x^{10}}{384}-\frac{55a^4x^6}{384}-\frac{5a^8x^2}{128}}{2(a^4+x^4)^4}+\frac{5\arctan\left(\frac{x^2}{a^2}\right)}{256a^6}$
parallelrisch	$-\frac{-60i\ln(ia^2+x^2)x^4a^{12}+60i\ln(-ia^2+x^2)x^4a^{12}-60i\ln(ia^2+x^2)x^{12}a^4+15i\ln(-ia^2+x^2)x^{16}-90i\ln(ia^2+x^2)x^8a^8+90i\ln(-$

[In] int(x^13/(a^4+x^4)^5,x,method=_RETURNVERBOSE)

[Out] (-5/256*a^8*x^2-55/768*a^4*x^6-73/768*x^10+5/256/a^4*x^14)/(a^4+x^4)^4+5/25 6*arctan(x^2/a^2)/a^6

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = -\frac{15\,a^{14}x^2+55\,a^{10}x^6+73\,a^6x^{10}-15\,a^2x^{14}-15\,(a^{16}+4\,a^{12}x^4+6\,a^8x^8+4\,a^4x^{12}+x^{16})\arctan\left(\frac{x^2}{a^2}\right)}{768\,(a^{22}+4\,a^{18}x^4+6\,a^{14}x^8+4\,a^{10}x^{12}+a^6x^{16})}$$

[In] integrate(x^13/(a^4+x^4)^5,x, algorithm="fricas")

[Out] -1/768*(15*a^14*x^2 + 55*a^10*x^6 + 73*a^6*x^10 - 15*a^2*x^14 - 15*(a^16 + 4*a^12*x^4 + 6*a^8*x^8 + 4*a^4*x^12 + x^16)*arctan(x^2/a^2))/(a^22 + 4*a^18 *x^4 + 6*a^14*x^8 + 4*a^10*x^12 + a^6*x^16)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = \frac{-15a^{12}x^2-55a^8x^6-73a^4x^{10}+15x^{14}}{768a^{20}+3072a^{16}x^4+4608a^{12}x^8+3072a^8x^{12}+768a^4x^{16}} + \frac{-\frac{5i\log(-ia^2+x^2)}{512}+\frac{5i\log(ia^2+x^2)}{512}}{a^6}$$

```
[In] integrate(x**13/(a**4+x**4)**5,x)
```

```
[Out] (-15*a**12*x**2 - 55*a**8*x**6 - 73*a**4*x**10 + 15*x**14)/(768*a**20 + 307
2*a**16*x**4 + 4608*a**12*x**8 + 3072*a**8*x**12 + 768*a**4*x**16) + (-5*I*
log(-I*a**2 + x**2)/512 + 5*I*log(I*a**2 + x**2)/512)/a**6
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = -\frac{15\,a^{12}x^2+55\,a^8x^6+73\,a^4x^{10}-15\,x^{14}}{768\,(a^{20}+4\,a^{16}x^4+6\,a^{12}x^8+4\,a^8x^{12}+a^4x^{16})} + \frac{5\,\arctan\left(\frac{x^2}{a^2}\right)}{256\,a^6}$$

```
[In] integrate(x^13/(a^4+x^4)^5,x, algorithm="maxima")
```

```
[Out] -1/768*(15*a^12*x^2 + 55*a^8*x^6 + 73*a^4*x^10 - 15*x^14)/(a^20 + 4*a^16*x^
4 + 6*a^12*x^8 + 4*a^8*x^12 + a^4*x^16) + 5/256*arctan(x^2/a^2)/a^6
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = \frac{5\,\arctan\left(\frac{x^2}{a^2}\right)}{256\,a^6} - \frac{15\,a^{12}x^2+55\,a^8x^6+73\,a^4x^{10}-15\,x^{14}}{768\,(a^4+x^4)^4a^4}$$

```
[In] integrate(x^13/(a^4+x^4)^5,x, algorithm="giac")
```

```
[Out] 5/256*arctan(x^2/a^2)/a^6 - 1/768*(15*a^12*x^2 + 55*a^8*x^6 + 73*a^4*x^10 -
 15*x^14)/((a^4 + x^4)^4*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = \frac{5\,\mathrm{atan}\left(\frac{x^2}{a^2}\right)}{256\,a^6} - \frac{\frac{73\,x^{10}}{768}+\frac{55\,a^4\,x^6}{768}+\frac{5\,a^8\,x^2}{256}-\frac{5\,x^{14}}{256\,a^4}}{a^{16}+4\,a^{12}\,x^4+6\,a^8\,x^8+4\,a^4\,x^{12}+x^{16}}$$

```
[In] int(x^13/(a^4 + x^4)^5,x)
```

```
[Out] (5*atan(x^2/a^2))/(256*a^6) - ((73*x^10)/768 + (55*a^4*x^6)/768 + (5*a^8*x^
2)/256 - (5*x^14)/(256*a^4))/(a^16 + x^16 + 4*a^4*x^12 + 6*a^8*x^8 + 4*a^12
*x^4)
```

3.208 $\int (2\sqrt{x}-x)^2 x^{3/2}(1+x^2)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx=\frac{8x^{7/2}}{7}-x^4+\frac{2x^{9/2}}{9}+\frac{8x^{11/2}}{11}-\frac{2x^6}{3}+\frac{2x^{13/2}}{13}$$

```
[Out] 8/7*x^(7/2)-x^4+2/9*x^(9/2)+8/11*x^(11/2)-2/3*x^6+2/13*x^(13/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 1834, 272, 45}

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx=\frac{2x^{13/2}}{13}+\frac{8x^{11/2}}{11}+\frac{2x^{9/2}}{9}+\frac{8x^{7/2}}{7}-\frac{2x^6}{3}-x^4$$

```
[In] Int[(2*Sqrt[x] - x)^2*x^(3/2)*(1 + x^2),x]
```

```
[Out] (8*x^(7/2))/7 - x^4 + (2*x^(9/2))/9 + (8*x^(11/2))/11 - (2*x^6)/3 + (2*x^(1
3/2))/13
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(2-\sqrt{x}\right)^2 x^{5/2}\left(1+x^2\right)\,dx \\
&= \int \left(\left(-2+\sqrt{x}\right)^2 x^{5/2}+\left(-2+\sqrt{x}\right)^2 x^{9/2}\right)\,dx \\
&= \int \left(-2+\sqrt{x}\right)^2 x^{5/2}\,dx+\int \left(-2+\sqrt{x}\right)^2 x^{9/2}\,dx \\
&= 2\text{Subst}\left(\int (-2+x)^2x^6\,dx,x,\sqrt{x}\right)+2\text{Subst}\left(\int (-2+x)^2x^{10}\,dx,x,\sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \left(4x^6-4x^7+x^8\right)\,dx,x,\sqrt{x}\right)+2\text{Subst}\left(\int \left(4x^{10}-4x^{11}+x^{12}\right)\,dx,x,\sqrt{x}\right) \\
&= \frac{8x^{7/2}}{7}-x^4+\frac{2x^{9/2}}{9}+\frac{8x^{11/2}}{11}-\frac{2x^6}{3}+\frac{2x^{13/2}}{13}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx=\frac{10296x^{7/2}-9009x^4+2002x^{9/2}+6552x^{11/2}-6006x^6+1386x^{13/2}}{9009}$$

```
[In] Integrate[(2*Sqrt[x] - x)^2*x^(3/2)*(1 + x^2),x]
[Out] (10296*x^(7/2) - 9009*x^4 + 2002*x^(9/2) + 6552*x^(11/2) - 6006*x^6 + 1386*
x^(13/2))/9009
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{8x^{\frac{7}{2}}}{7}-x^{4}+\frac{2x^{\frac{9}{2}}}{9}+\frac{8x^{\frac{11}{2}}}{11}-\frac{2x^{6}}{3}+\frac{2x^{\frac{13}{2}}}{13}$	32
default	$\frac{8x^{\frac{7}{2}}}{7}-x^{4}+\frac{2x^{\frac{9}{2}}}{9}+\frac{8x^{\frac{11}{2}}}{11}-\frac{2x^{6}}{3}+\frac{2x^{\frac{13}{2}}}{13}$	32
trager	$-\frac{(2x^{5}+2x^{4}+5x^{3}+5x^{2}+5x+5)(-1+x)}{3}+\frac{2x^{\frac{7}{2}}(693x^{3}+3276x^{2}+1001x+5148)}{9009}$	52

```
[In] int(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x,method=_RETURNVERBOSE)
[Out] 8/7*x^(7/2)-x^4+2/9*x^(9/2)+8/11*x^(11/2)-2/3*x^6+2/13*x^(13/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx=-\frac{2}{3}\,x^6-x^4+\frac{2}{9009}\left(693\,x^6+3276\,x^5+1001\,x^4+5148\,x^3\right)\sqrt{x}$$

```
[In] integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="fricas")
[Out] -2/3*x^6 - x^4 + 2/9009*(693*x^6 + 3276*x^5 + 1001*x^4 + 5148*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx=\frac{2x^{\frac{13}{2}}}{13}+\frac{8x^{\frac{11}{2}}}{11}+\frac{2x^{\frac{9}{2}}}{9}+\frac{8x^{\frac{7}{2}}}{7}-\frac{2x^6}{3}-x^4$$

```
[In] integrate(x**(3/2)*(x**2+1)*(-x+2*x**(1/2))**2,x)
[Out] 2*x**(13/2)/13 + 8*x**(11/2)/11 + 2*x**(9/2)/9 + 8*x**(7/2)/7 - 2*x**6/3 -
x**4
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx = \frac{2}{13}\,x^{\frac{13}{2}} - \frac{2}{3}\,x^6 + \frac{8}{11}\,x^{\frac{11}{2}} + \frac{2}{9}\,x^{\frac{9}{2}} - x^4 + \frac{8}{7}\,x^{\frac{7}{2}}$$

```
[In] integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="maxima")
[Out] 2/13*x^(13/2) - 2/3*x^6 + 8/11*x^(11/2) + 2/9*x^(9/2) - x^4 + 8/7*x^(7/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx = \frac{2}{13}\,x^{\frac{13}{2}} - \frac{2}{3}\,x^6 + \frac{8}{11}\,x^{\frac{11}{2}} + \frac{2}{9}\,x^{\frac{9}{2}} - x^4 + \frac{8}{7}\,x^{\frac{7}{2}}$$

```
[In] integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="giac")
[Out] 2/13*x^(13/2) - 2/3*x^6 + 8/11*x^(11/2) + 2/9*x^(9/2) - x^4 + 8/7*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx = \frac{8\,x^{7/2}}{7} - \frac{2\,x^6}{3} - x^4 + \frac{2\,x^{9/2}}{9} + \frac{8\,x^{11/2}}{11} + \frac{2\,x^{13/2}}{13}$$

```
[In] int(x^(3/2)*(x - 2*x^(1/2))^2*(x^2 + 1),x)
[Out] (8*x^(7/2))/7 - (2*x^6)/3 - x^4 + (2*x^(9/2))/9 + (8*x^(11/2))/11 + (2*x^(1
3/2))/13
```

3.209 $\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 55

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right) dx = -\frac{45x^{43/15}}{43}+\frac{360x^{37/10}}{37}+\frac{60x^{113/30}}{113}-\frac{120x^{23/5}}{23}-\frac{x^{14/3}}{14}+\frac{8x^{11/2}}{11}$$

```
[Out] -45/43*x^(43/15)+360/37*x^(37/10)+60/113*x^(113/30)-120/23*x^(23/5)-1/14*x^
(14/3)+8/11*x^(11/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1607, 1598, 1834}

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right) dx = \frac{8x^{11/2}}{11}-\frac{x^{14/3}}{14}-\frac{120x^{23/5}}{23}+\frac{60x^{113/30}}{113}+\frac{360x^{37/10}}{37}-\frac{45x^{43/15}}{43}$$

```
[In] Int[(-3*x^(3/5) + x^(3/2))^2*(-1/3*x^(2/3) + 4*x^(3/2)),x]
```

```
[Out] (-45*x^(43/15))/43 + (360*x^(37/10))/37 + (60*x^(113/30))/113 - (120*x^(23/
5))/23 - x^(14/3)/14 + (8*x^(11/2))/11
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1834

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-3+x^{9/10}\right)^2 x^{6/5}\left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx\\
&= \int \left(-\frac{1}{3}+4x^{5/6}\right)\left(-3+x^{9/10}\right)^2 x^{28/15}\,dx\\
&= 30\text{Subst}\left(\int x^{85}\left(-\frac{1}{3}+4x^{25}\right)\left(-3+x^{27}\right)^2\,dx, x, \sqrt[30]{x}\right)\\
&= 30\text{Subst}\left(\int \left(-3x^{85}+36x^{110}+2x^{112}-24x^{137}-\frac{x^{139}}{3}+4x^{164}\right)dx, x, \sqrt[30]{x}\right)\\
&= -\frac{45x^{43/15}}{43}+\frac{360x^{37/10}}{37}+\frac{60x^{113/30}}{113}-\frac{120x^{23/5}}{23}-\frac{x^{14/3}}{14}+\frac{8x^{11/2}}{11}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2\left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)dx = -\frac{45x^{43/15}}{43}+\frac{360x^{37/10}}{37}+\frac{60x^{113/30}}{113}-\frac{120x^{23/5}}{23}-\frac{x^{14/3}}{14}+\frac{8x^{11/2}}{11}$$

```
[In] Integrate[(-3*x^(3/5) + x^(3/2))^2*(-1/3*x^(2/3) + 4*x^(3/2)),x]
[Out] (-45*x^(43/15))/43 + (360*x^(37/10))/37 + (60*x^(113/30))/113 - (120*x^(23/
5))/23 - x^(14/3)/14 + (8*x^(11/2))/11
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.58

method	result	size
derivativedivides	$-\frac{45x^{\frac{43}{15}}}{43}+\frac{360x^{\frac{37}{10}}}{37}+\frac{60x^{\frac{113}{30}}}{113}-\frac{120x^{\frac{23}{5}}}{23}-\frac{x^{\frac{14}{3}}}{14}+\frac{8x^{\frac{11}{2}}}{11}$	32
default	$-\frac{45x^{\frac{43}{15}}}{43}+\frac{360x^{\frac{37}{10}}}{37}+\frac{60x^{\frac{113}{30}}}{113}-\frac{120x^{\frac{23}{5}}}{23}-\frac{x^{\frac{14}{3}}}{14}+\frac{8x^{\frac{11}{2}}}{11}$	32

```
[In] int((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x,method=_RETURNVERBOSE
)
```

```
[Out] -45/43*x^(43/15)+360/37*x^(37/10)+60/113*x^(113/30)-120/23*x^(23/5)-1/14*x^
(14/3)+8/11*x^(11/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int (-3x^{3/5}+x^{3/2})^2\left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx=\frac{8}{11}\,x^{\frac{11}{2}}-\frac{1}{14}\,x^{\frac{14}{3}}-\frac{120}{23}\,x^{\frac{23}{5}}+\frac{60}{113}\,x^{\frac{113}{30}}+\frac{360}{37}\,x^{\frac{37}{10}}-\frac{45}{43}\,x^{\frac{43}{15}}$$

```
[In] integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="fri
cas")
```

```
[Out] 8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/30) + 360/3
7*x^(37/10) - 45/43*x^(43/15)
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2\left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx=\frac{60x^{\frac{113}{30}}}{113}-\frac{45x^{\frac{43}{15}}}{43}+\frac{360x^{\frac{37}{10}}}{37}-\frac{120x^{\frac{23}{5}}}{23}-\frac{x^{\frac{14}{3}}}{14}+\frac{8x^{\frac{11}{2}}}{11}$$

```
[In] integrate((-3*x**(3/5)+x**(3/2))**2*(-1/3*x**(2/3)+4*x**(3/2)),x)
```

```
[Out] 60*x**(113/30)/113 - 45*x**(43/15)/43 + 360*x**(37/10)/37 - 120*x**(23/5)/2
3 - x**(14/3)/14 + 8*x**(11/2)/11
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2\left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx=\frac{8}{11}\,x^{\frac{11}{2}}-\frac{1}{14}\,x^{\frac{14}{3}}-\frac{120}{23}\,x^{\frac{23}{5}}+\frac{60}{113}\,x^{\frac{113}{30}}+\frac{360}{37}\,x^{\frac{37}{10}}-\frac{45}{43}\,x^{\frac{43}{15}}$$

```
[In] integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="max
ima")
```

```
[Out] 8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/30) + 360/3
7*x^(37/10) - 45/43*x^(43/15)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2\left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx=\frac{8}{11}\,x^{\frac{11}{2}}-\frac{1}{14}\,x^{\frac{14}{3}}-\frac{120}{23}\,x^{\frac{23}{5}}+\frac{60}{113}\,x^{\frac{113}{30}}+\frac{360}{37}\,x^{\frac{37}{10}}-\frac{45}{43}\,x^{\frac{43}{15}}$$

```
[In] integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="gia
c")
```

```
[Out] 8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/30) + 360/3
7*x^(37/10) - 45/43*x^(43/15)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2\left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx=\frac{8\,x^{11/2}}{11}-\frac{x^{14/3}}{14}-\frac{120\,x^{23/5}}{23}+\frac{360\,x^{37/10}}{37}-\frac{45\,x^{43/15}}{43}+\frac{60\,x^{113/30}}{113}$$

```
[In] int(-(x^(3/2) - 3*x^(3/5))^2*(x^(2/3)/3 - 4*x^(3/2)),x)
```

```
[Out] (8*x^(11/2))/11 - x^(14/3)/14 - (120*x^(23/5))/23 + (360*x^(37/10))/37 - (4
5*x^(43/15))/43 + (60*x^(113/30))/113
```

3.210 $\int \frac{1}{1+\sqrt{1+x}} dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{1}{1+\sqrt{1+x}} dx = 2\sqrt{1+x} - 2\log\left(1+\sqrt{1+x}\right)$$

```
[Out] -2*ln(1+(1+x)^(1/2))+2*(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {253, 196, 45}

$$\int \frac{1}{1+\sqrt{1+x}} dx = 2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

```
[In] Int[(1 + Sqrt[1 + x])^(-1),x]
```

```
[Out] 2*Sqrt[1 + x] - 2*Log[1 + Sqrt[1 + x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
```

```
IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{1+\sqrt{x}}\,dx, x, 1+x\right)\\
&= 2\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, \sqrt{1+x}\right)\\
&= 2\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx, x, \sqrt{1+x}\right)\\
&= 2\sqrt{1+x}-2\log\left(1+\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\sqrt{1+x}-2\log\left(1+\sqrt{1+x}\right)$$

```
[In] Integrate[(1 + Sqrt[1 + x])^(-1),x]
[Out] 2*Sqrt[1 + x] - 2*Log[1 + Sqrt[1 + x]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-2\ln\left(1+\sqrt{1+x}\right)+2\sqrt{1+x}$	19
trager	$2\sqrt{1+x}-\ln\left(2\sqrt{1+x}+2+x\right)$	22
default	$2\sqrt{1+x}+\ln\left(-1+\sqrt{1+x}\right)-\ln\left(1+\sqrt{1+x}\right)-\ln\left(x\right)$	31
meijerg	$\frac{-4\sqrt{\pi}+4\sqrt{\pi}\,\sqrt{1+x}-4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+x}}{2}\right)}{2\sqrt{\pi}}$	37

```
[In] int(1/(1+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*ln(1+(1+x)^(1/2))+2*(1+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\,\sqrt{x+1} - 2\,\log\left(\sqrt{x+1}+1\right)$$

```
[In] integrate(1/(1+(1+x)^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

```
[In] integrate(1/(1+(1+x)**(1/2)),x)
[Out] 2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\,\sqrt{x+1} - 2\,\log\left(\sqrt{x+1}+1\right)$$

```
[In] integrate(1/(1+(1+x)^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

```
[In] integrate(1/(1+(1+x)^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\sqrt{x+1} - 2\ln\left(\sqrt{x+1}+1\right)$$

```
[In] int(1/((x + 1)^(1/2) + 1),x)
[Out] 2*(x + 1)^(1/2) - 2*log((x + 1)^(1/2) + 1)
```

3.211 $\int \frac{x}{1+\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x}{1+\sqrt{1+x}}\,dx = -x+\frac{2}{3}(1+x)^{3/2}$$

```
[Out] -x+2/3*(1+x)^(3/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {378}

$$\int \frac{x}{1+\sqrt{1+x}}\,dx = \frac{2}{3}(x+1)^{3/2}-x$$

```
[In] Int[x/(1 + Sqrt[1 + x]),x]
```

```
[Out] -x + (2*(1 + x)^(3/2))/3
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \left(-1+\sqrt{x}\right)\,dx,x,1+x\right)\\ &= -x+\frac{2}{3}(1+x)^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{x}{1+\sqrt{1+x}}\,dx = \frac{1}{3}(1+x)\left(-3+2\sqrt{1+x}\right)$$

```
[In] Integrate[x/(1 + Sqrt[1 + x]),x]
[Out] ((1 + x)*(-3 + 2*Sqrt[1 + x]))/3
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{2(1+x)^{\frac{3}{2}}}{3}-1-x$	13
default	$\frac{2(1+x)^{\frac{3}{2}}}{3}-1-x$	13
trager	$-x+\left(\frac{2}{3}+\frac{2x}{3}\right)\sqrt{1+x}$	16
meijerg	$\frac{-\frac{\sqrt{\pi}\,(12x+8)}{6}+\frac{\sqrt{\pi}\,(8+8x)\sqrt{1+x}}{6}}{2\sqrt{\pi}}$	32

```
[In] int(x/(1+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/3*(1+x)^(3/2)-1-x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{1+\sqrt{1+x}}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}}-x$$

```
[In] integrate(x/(1+(1+x)^(1/2)),x, algorithm="fricas")
[Out] 2/3*(x + 1)^(3/2) - x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{x}{1+\sqrt{1+x}}\,dx = \frac{2x\sqrt{x+1}}{3} - x + \frac{2\sqrt{x+1}}{3}$$

```
[In] integrate(x/(1+(1+x)**(1/2)),x)
[Out] 2*x*sqrt(x + 1)/3 - x + 2*sqrt(x + 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{x}{1+\sqrt{1+x}}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}} - x - 1$$

```
[In] integrate(x/(1+(1+x)^(1/2)),x, algorithm="maxima")
[Out] 2/3*(x + 1)^(3/2) - x - 1
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{x}{1+\sqrt{1+x}}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}} - x - 1$$

```
[In] integrate(x/(1+(1+x)^(1/2)),x, algorithm="giac")
[Out] 2/3*(x + 1)^(3/2) - x - 1
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{1+\sqrt{1+x}}\,dx = \frac{2\,(x+1)^{3/2}}{3} - x$$

```
[In] int(x/((x + 1)^(1/2) + 1),x)
[Out] (2*(x + 1)^(3/2))/3 - x
```

3.212 $\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x+4\sqrt{1+x}+4\log\left(1-\sqrt{1+x}\right)$$

```
[Out] x+4*ln(1-(1+x)^(1/2))+4*(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {442, 383, 78}

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x+4\sqrt{x+1}+4\log\left(1-\sqrt{x+1}\right)$$

```
[In] Int[(1 + Sqrt[1 + x])/(-1 + Sqrt[1 + x]),x]
```

```
[Out] x + 4*Sqrt[1 + x] + 4*Log[1 - Sqrt[1 + x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 442

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx, x, 1+x\right)\\
&= 2\text{Subst}\left(\int \frac{x(1+x)}{-1+x}\,dx, x, \sqrt{1+x}\right)\\
&= 2\text{Subst}\left(\int \left(2+\frac{2}{-1+x}+x\right)\,dx, x, \sqrt{1+x}\right)\\
&= x+4\sqrt{1+x}+4\log\left(1-\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = 1+x+4\sqrt{1+x}+4\log\left(-1+\sqrt{1+x}\right)$$

```
[In] Integrate[(1 + Sqrt[1 + x])/(-1 + Sqrt[1 + x]),x]
[Out] 1 + x + 4*Sqrt[1 + x] + 4*Log[-1 + Sqrt[1 + x]]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$1+x+4\sqrt{1+x}+4\ln\left(-1+\sqrt{1+x}\right)$	21
default	$1+x+4\sqrt{1+x}+4\ln\left(-1+\sqrt{1+x}\right)$	21
trager	$-1+x+4\sqrt{1+x}+2\ln\left(2\sqrt{1+x}-2-x\right)$	26

```
[In] int((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1+x+4*(1+x)^(1/2)+4*ln(-1+(1+x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x+4\,\sqrt{x+1}+4\,\log\left(\sqrt{x+1}-1\right)$$

```
[In] integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="fricas")
[Out] x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x+4\sqrt{x+1}+4\log\left(\sqrt{x+1}-1\right)$$

```
[In] integrate((1+(1+x)**(1/2))/(-1+(1+x)**(1/2)),x)
[Out] x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x+4\,\sqrt{x+1}+4\,\log\left(\sqrt{x+1}-1\right)+1$$

```
[In] integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="maxima")
[Out] x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1) + 1
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x + 4\sqrt{x+1} + 4\log\left(\left|\sqrt{x+1}-1\right|\right) + 1$$

```
[In] integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="giac")
[Out] x + 4*sqrt(x + 1) + 4*log(abs(sqrt(x + 1) - 1)) + 1
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x + 4\ln\left(\sqrt{x+1}-1\right) + 4\sqrt{x+1}$$

```
[In] int(((x + 1)^(1/2) + 1)/((x + 1)^(1/2) - 1),x)
[Out] x + 4*log((x + 1)^(1/2) - 1) + 4*(x + 1)^(1/2)
```

3.213 $\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 33

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 6\sqrt[6]{1+x}+3\sqrt[3]{1+x}+6\log\left(1-\sqrt[6]{1+x}\right)$$

```
[Out] 6*(1+x)^(1/6)+3*(1+x)^(1/3)+6*ln(1-(1+x)^(1/6))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2037, 1607, 272, 45}

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 3\sqrt[3]{x+1}+6\sqrt[6]{x+1}+6\log\left(1-\sqrt[6]{x+1}\right)$$

```
[In] Int[(-Sqrt[1 + x] + (1 + x)^(2/3))^(-1),x]
```

```
[Out] 6*(1 + x)^(1/6) + 3*(1 + x)^(1/3) + 6*Log[1 - (1 + x)^(1/6)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2037

```
Int[((a_.)*(u_)^(j_.) + (b_.)*(u_)^(n_.))^(p_), x_Symbol] :> Dist[1/Coeffic
ient[u, x, 1], Subst[Int[(a*x^j + b*x^n)^p, x], x, u], x] /; FreeQ[{a, b, j
, n, p}, x] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{-\sqrt{x}+x^{2/3}}\,dx, x, 1+x\right)\\
&= \text{Subst}\left(\int \frac{1}{\left(-1+\sqrt[6]{x}\right)\sqrt{x}}\,dx, x, 1+x\right)\\
&= 6\text{Subst}\left(\int \frac{x^2}{-1+x}\,dx, x, \sqrt[6]{1+x}\right)\\
&= 6\text{Subst}\left(\int \left(1+\frac{1}{-1+x}+x\right)\,dx, x, \sqrt[6]{1+x}\right)\\
&= 6\sqrt[6]{1+x}+3\sqrt[3]{1+x}+6\log\left(1-\sqrt[6]{1+x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 3\left(2\sqrt[6]{1+x}+\sqrt[3]{1+x}+2\log\left(-1+\sqrt[6]{1+x}\right)\right)$$

```
[In] Integrate[(-Sqrt[1 + x] + (1 + x)^(2/3))^(-1),x]
[Out] 3*(2*(1 + x)^(1/6) + (1 + x)^(1/3) + 2*Log[-1 + (1 + x)^(1/6)])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result
derivativedivides	$3(1+x)^{\frac{1}{3}}+6(1+x)^{\frac{1}{6}}+6\ln\left((1+x)^{\frac{1}{6}}-1\right)$
default	$6(1+x)^{\frac{1}{6}}+3(1+x)^{\frac{1}{3}}+\ln(x)-\ln\left((1+x)^{\frac{1}{3}}+(1+x)^{\frac{1}{6}}+1\right)+2\ln\left((1+x)^{\frac{1}{6}}-1\right)$

```
[In] int(1/((1+x)^(2/3)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 3*(1+x)^(1/3)+6*(1+x)^(1/6)+6*ln((1+x)^(1/6)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx=3\,(x+1)^{\frac{1}{3}}+6\,(x+1)^{\frac{1}{6}}+6\,\log\left((x+1)^{\frac{1}{6}}-1\right)$$

```
[In] integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="fricas")
[Out] 3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log((x + 1)^(1/6) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx=6\sqrt[6]{x+1}+3\sqrt[3]{x+1}+6\log\left(\sqrt[6]{x+1}-1\right)$$

```
[In] integrate(1/((1+x)**(2/3)-(1+x)**(1/2)),x)
[Out] 6*(x + 1)**(1/6) + 3*(x + 1)**(1/3) + 6*log((x + 1)**(1/6) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 3\,(x+1)^{\frac{1}{3}} + 6\,(x+1)^{\frac{1}{6}} + 6\,\log\left((x+1)^{\frac{1}{6}} - 1\right)$$

```
[In] integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="maxima")
[Out] 3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log((x + 1)^(1/6) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 3\,(x+1)^{\frac{1}{3}} + 6\,(x+1)^{\frac{1}{6}} + 6\,\log\left(\left|(x+1)^{\frac{1}{6}} - 1\right|\right)$$

```
[In] integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="giac")
[Out] 3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log(abs((x + 1)^(1/6) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 6\,\ln\left((x+1)^{1/6} - 1\right) + 3\,(x+1)^{1/3} + 6\,(x+1)^{1/6}$$

```
[In] int(-1/((x + 1)^(1/2) - (x + 1)^(2/3)),x)
[Out] 6*log((x + 1)^(1/6) - 1) + 3*(x + 1)^(1/3) + 6*(x + 1)^(1/6)
```

3.214 $\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 29

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}} \, dx = -3\left(1+\sqrt[4]{x}\right)^{4/3} + \frac{12}{7}\left(1+\sqrt[4]{x}\right)^{7/3}$$

```
[Out] -3*(1+x^(1/4))^(4/3)+12/7*(1+x^(1/4))^(7/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {272, 45}

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}} \, dx = \frac{12}{7}\left(\sqrt[4]{x}+1\right)^{7/3} - 3\left(\sqrt[4]{x}+1\right)^{4/3}$$

```
[In] Int[(1 + x^(1/4))^(1/3)/Sqrt[x],x]
```

```
[Out] -3*(1 + x^(1/4))^(4/3) + (12*(1 + x^(1/4))^(7/3))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = 4\text{Subst}\left(\int x\sqrt[3]{1+x}\,dx, x, \sqrt[4]{x}\right)$$
$$= 4\text{Subst}\left(\int \left(-\sqrt[3]{1+x} + (1+x)^{4/3}\right)\,dx, x, \sqrt[4]{x}\right)$$
$$= -3\left(1+\sqrt[4]{x}\right)^{4/3} + \frac{12}{7}\left(1+\sqrt[4]{x}\right)^{7/3}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{3}{7}\left(1+\sqrt[4]{x}\right)^{4/3}\left(-3+4\sqrt[4]{x}\right)$$

```
[In] Integrate[(1 + x^(1/4))^(1/3)/Sqrt[x],x]
[Out] (3*(1 + x^(1/4))^(4/3)*(-3 + 4*x^(1/4)))/7
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.59

method	result	size
meijerg	$2\sqrt{x}\,{}_2F_1\left(-\frac{1}{3},2;3;-x^{\frac{1}{4}}\right)$	17
derivativedivides	$-3\left(1+x^{\frac{1}{4}}\right)^{\frac{4}{3}}+\frac{12\left(1+x^{\frac{1}{4}}\right)^{\frac{7}{3}}}{7}$	20
default	$-3\left(1+x^{\frac{1}{4}}\right)^{\frac{4}{3}}+\frac{12\left(1+x^{\frac{1}{4}}\right)^{\frac{7}{3}}}{7}$	20

```
[In] int((1+x^(1/4))^(1/3)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*x^(1/2)*hypergeom([-1/3,2],[3],-x^(1/4))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{3}{7}\left(4\sqrt{x}+x^{\frac{1}{4}}-3\right)\left(x^{\frac{1}{4}}+1\right)^{\frac{1}{3}}$$

```
[In] integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="fricas")
[Out] 3/7*(4*sqrt(x) + x^(1/4) - 3)*(x^(1/4) + 1)^(1/3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(24) = 48.

Time = 0.68 (sec) , antiderivative size = 134, normalized size of antiderivative = 4.62

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{12x^{\frac{7}{4}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x} - \frac{6x^{\frac{5}{4}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x} + \frac{9x^{\frac{5}{4}}}{7x^{\frac{5}{4}}+7x} + \frac{15x^{\frac{3}{2}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x} - \frac{9x\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x} + \frac{9x}{7x^{\frac{5}{4}}+7x}$$

```
[In] integrate((1+x**(1/4))**(1/3)/x**(1/2),x)
[Out] 12*x**(7/4)*(x**(1/4) + 1)**(1/3)/(7*x**(5/4) + 7*x) - 6*x**(5/4)*(x**(1/4)
 + 1)**(1/3)/(7*x**(5/4) + 7*x) + 9*x**(5/4)/(7*x**(5/4) + 7*x) + 15*x**(3/
2)*(x**(1/4) + 1)**(1/3)/(7*x**(5/4) + 7*x) - 9*x*(x**(1/4) + 1)**(1/3)/(7*
x**(5/4) + 7*x) + 9*x/(7*x**(5/4) + 7*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{12}{7}\left(x^{\frac{1}{4}}+1\right)^{\frac{7}{3}} - 3\left(x^{\frac{1}{4}}+1\right)^{\frac{4}{3}}$$

```
[In] integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="maxima")
[Out] 12/7*(x^(1/4) + 1)^(7/3) - 3*(x^(1/4) + 1)^(4/3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{12}{7}\left(x^{\frac{1}{4}}+1\right)^{\frac{7}{3}} - 3\left(x^{\frac{1}{4}}+1\right)^{\frac{4}{3}}$$

```
[In] integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="giac")
[Out] 12/7*(x^(1/4) + 1)^(7/3) - 3*(x^(1/4) + 1)^(4/3)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{3\left(x^{1/4}+1\right)^{4/3}\left(4\,x^{1/4}-3\right)}{7}$$

```
[In] int((x^(1/4) + 1)^(1/3)/x^(1/2),x)
[Out] (3*(x^(1/4) + 1)^(4/3)*(4*x^(1/4) - 3))/7
```

3.215 $\int \frac{1}{x^3(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = \frac{15}{4\sqrt{1+x}} - \frac{1}{2x^2\sqrt{1+x}} + \frac{5}{4x\sqrt{1+x}} - \frac{15}{4}\operatorname{arctanh}\left(\sqrt{1+x}\right)$$

```
[Out] -15/4*arctanh((1+x)^(1/2))+15/4/(1+x)^(1/2)-1/2/x^2/(1+x)^(1/2)+5/4/x/(1+x)
^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {44, 53, 65, 213}

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = -\frac{15}{4}\operatorname{arctanh}\left(\sqrt{x+1}\right) - \frac{1}{2x^2\sqrt{x+1}} + \frac{5}{4x\sqrt{x+1}} + \frac{15}{4\sqrt{x+1}}$$

```
[In] Int[1/(x^3*(1 + x)^(3/2)),x]
```

```
[Out] 15/(4*Sqrt[1 + x]) - 1/(2*x^2*Sqrt[1 + x]) + 5/(4*x*Sqrt[1 + x]) - (15*ArcT
anh[Sqrt[1 + x]])/4
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{2x^2\sqrt{1+x}}-\frac{5}{4}\int\frac{1}{x^2(1+x)^{3/2}}\,dx\\
&= -\frac{1}{2x^2\sqrt{1+x}}+\frac{5}{4x\sqrt{1+x}}+\frac{15}{8}\int\frac{1}{x(1+x)^{3/2}}\,dx\\
&= \frac{15}{4\sqrt{1+x}}-\frac{1}{2x^2\sqrt{1+x}}+\frac{5}{4x\sqrt{1+x}}+\frac{15}{8}\int\frac{1}{x\sqrt{1+x}}\,dx\\
&= \frac{15}{4\sqrt{1+x}}-\frac{1}{2x^2\sqrt{1+x}}+\frac{5}{4x\sqrt{1+x}}+\frac{15}{4}\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\sqrt{1+x}\right)\\
&= \frac{15}{4\sqrt{1+x}}-\frac{1}{2x^2\sqrt{1+x}}+\frac{5}{4x\sqrt{1+x}}-\frac{15}{4}\text{arctanh}\left(\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = \frac{1}{4}\left(\frac{-2+5x+15x^2}{x^2\sqrt{1+x}} - 15\text{arctanh}\left(\sqrt{1+x}\right)\right)$$

```
[In] Integrate[1/(x^3*(1 + x)^(3/2)),x]
[Out] ((-2 + 5*x + 15*x^2)/(x^2*Sqrt[1 + x]) - 15*ArcTanh[Sqrt[1 + x]])/4
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

method	result
risch	$\frac{15x^2+5x-2}{4\sqrt{1+x}\,x^2} - \frac{15\,\text{arctanh}\left(\sqrt{1+x}\right)}{4}$
trager	$\frac{15x^2+5x-2}{4\sqrt{1+x}\,x^2} + \frac{15\ln\left(\frac{2\sqrt{1+x}-2-x}{x}\right)}{8}$
derivativedivides	$-\frac{1}{8\left(-1+\sqrt{1+x}\right)^2} + \frac{7}{8\left(-1+\sqrt{1+x}\right)} + \frac{15\ln\left(-1+\sqrt{1+x}\right)}{8} + \frac{1}{8\left(1+\sqrt{1+x}\right)^2} + \frac{7}{8\left(1+\sqrt{1+x}\right)} - \frac{15\ln\left(1+\sqrt{1+x}\right)}{8}$ -
default	$-\frac{1}{8\left(-1+\sqrt{1+x}\right)^2} + \frac{7}{8\left(-1+\sqrt{1+x}\right)} + \frac{15\ln\left(-1+\sqrt{1+x}\right)}{8} + \frac{1}{8\left(1+\sqrt{1+x}\right)^2} + \frac{7}{8\left(1+\sqrt{1+x}\right)} - \frac{15\ln\left(1+\sqrt{1+x}\right)}{8}$ -
meijerg	$\frac{-\frac{\sqrt{\pi}}{2x^2}+\frac{3\sqrt{\pi}}{2x}+\frac{15\left(\frac{47}{30}-2\ln(2)+\ln(x)\right)\sqrt{\pi}}{8}+\frac{\sqrt{\pi}\left(-47x^2-24x+8\right)}{16x^2}-\frac{\sqrt{\pi}\left(-60x^2-20x+8\right)}{16x^2\sqrt{1+x}}-\frac{15\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+x}}{2}\right)}{4}}{\sqrt{\pi}}$

```
[In] int(1/x^3/(1+x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/4*(15*x^2+5*x-2)/(1+x)^(1/2)/x^2-15/4*arctanh((1+x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = -\frac{15\,(x^3+x^2)\log\left(\sqrt{x+1}+1\right) - 15\,(x^3+x^2)\log\left(\sqrt{x+1}-1\right) - 2\,(15\,x^2+5\,x-2)\sqrt{x+1}}{8\,(x^3+x^2)}$$

```
[In] integrate(1/x^3/(1+x)^(3/2),x, algorithm="fricas")
[Out] -1/8*(15*(x^3 + x^2)*log(sqrt(x + 1) + 1) - 15*(x^3 + x^2)*log(sqrt(x + 1)
- 1) - 2*(15*x^2 + 5*x - 2)*sqrt(x + 1))/(x^3 + x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 3966, normalized size of antiderivative = 76.27

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x**3/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-30*(x + 1)**(17/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(
x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(
9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt
(x + 1)) - 15*I*pi*(x + 1)**(17/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2)
+ 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x +
 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 240*
(x + 1)**(15/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2)
+ 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x +
 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 120*
I*pi*(x + 1)**(15/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)*
*(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 2
24*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 840*(x + 1)**(13/2
)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)*
*(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 2
24*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 420*I*pi*(x + 1)**
(13/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*
(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5
/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 1680*(x + 1)**(11/2)*acoth(sqrt(
x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448
*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(
5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 840*I*pi*(x + 1)**(11/2)/(8*(x
+ 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/
2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x +
 1)**(3/2) + 8*sqrt(x + 1)) - 2100*(x + 1)**(9/2)*acoth(sqrt(x + 1))/(8*(x
+ 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/
2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x +
 1)**(3/2) + 8*sqrt(x + 1)) - 1050*I*pi*(x + 1)**(9/2)/(8*(x + 1)**(17/2) -
 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x +
1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8
*sqrt(x + 1)) + 1680*(x + 1)**(7/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) -
 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x +
1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8
*sqrt(x + 1)) + 840*I*pi*(x + 1)**(7/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(1
5/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448
*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) -
```

840*(x + 1)**(5/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15
/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*
(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) -
420*I*pi*(x + 1)**(5/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x +
1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2)
+ 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 240*(x + 1)**(3
/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1
)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) +
224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 120*I*pi*(x + 1)
(3/2)/(8*(x + 1)(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448
*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(
5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 30*sqrt(x + 1)*acoth(sqrt(x + 1
))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x +
1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2)
- 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 15*I*pi*sqrt(x + 1)/(8*(x + 1)**(17/
2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(
x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2)
+ 8*sqrt(x + 1)) + 30*(x + 1)**8/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) +
224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x +
1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 230*(
x + 1)**7/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 4
48*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)*
(5/2) - 64(x + 1)**(3/2) + 8*sqrt(x + 1)) + 766*(x + 1)**6/(8*(x + 1)**(1
7/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560
*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/
2) + 8*sqrt(x + 1)) - 1446*(x + 1)**5/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/
2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(
x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 1
690*(x + 1)**4/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2
) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x
+ 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 1250*(x + 1)**3/(8*(x +
1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2)
+ 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1
)**(3/2) + 8*sqrt(x + 1)) + 570*(x + 1)**2/(8*(x + 1)**(17/2) - 64*(x + 1)*
(15/2) + 224(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) -
448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)
) - 146*(x + 1)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/
2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x
+ 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 16/(8*(x + 1)**(17/2) -
64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x +
1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8
sqrt(x + 1)), Abs(x + 1) > 1), (-15(x + 1)**(17/2)*atanh(sqrt(x + 1))/(4*
(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(
11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(
x + 1)**(3/2) + 4*sqrt(x + 1)) + 120*(x + 1)**(15/2)*atanh(sqrt(x + 1))/(4*

```
(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(
11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(
x + 1)**(3/2) + 4*sqrt(x + 1)) - 420*(x + 1)**(13/2)*atanh(sqrt(x + 1))/(4*
(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(
11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(
x + 1)**(3/2) + 4*sqrt(x + 1)) + 840*(x + 1)**(11/2)*atanh(sqrt(x + 1))/(4*
(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(
11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(
x + 1)**(3/2) + 4*sqrt(x + 1)) - 1050*(x + 1)**(9/2)*atanh(sqrt(x + 1))/(4*
(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(
11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(
x + 1)**(3/2) + 4*sqrt(x + 1)) + 840*(x + 1)**(7/2)*atanh(sqrt(x + 1))/(4*(
x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(1
1/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x
 + 1)**(3/2) + 4*sqrt(x + 1)) - 420*(x + 1)**(5/2)*atanh(sqrt(x + 1))/(4*(x
 + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11
/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x
+ 1)**(3/2) + 4*sqrt(x + 1)) + 120*(x + 1)**(3/2)*atanh(sqrt(x + 1))/(4*(x
+ 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/
2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x +
 1)**(3/2) + 4*sqrt(x + 1)) - 15*sqrt(x + 1)*atanh(sqrt(x + 1))/(4*(x + 1)*
*(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) +
280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**
(3/2) + 4*sqrt(x + 1)) + 15*(x + 1)**8/(4*(x + 1)**(17/2) - 32*(x + 1)**(15
/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*
(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) -
115*(x + 1)**7/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2
) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x
+ 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) + 383*(x + 1)**6/(4*(x + 1
)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2)
+ 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)
**(3/2) + 4*sqrt(x + 1)) - 723*(x + 1)**5/(4*(x + 1)**(17/2) - 32*(x + 1)**
(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 2
24*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1))
 + 845*(x + 1)**4/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(1
3/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*
(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) - 625*(x + 1)**3/(4*(x
+ 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/
2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x +
 1)**(3/2) + 4*sqrt(x + 1)) + 285*(x + 1)**2/(4*(x + 1)**(17/2) - 32*(x + 1
)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2)
- 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x +
1)) - 73*(x + 1)/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13
/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(
x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) + 8/(4*(x + 1)**(17/2) -
```

```
32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x +
1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4
*sqrt(x + 1)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = \frac{15\,(x+1)^2-25\,x-17}{4\left((x+1)^{\frac{5}{2}}-2\,(x+1)^{\frac{3}{2}}+\sqrt{x+1}\right)} - \frac{15}{8}\,\log\left(\sqrt{x+1}+1\right)+\frac{15}{8}\,\log\left(\sqrt{x+1}-1\right)$$

```
[In] integrate(1/x^3/(1+x)^(3/2),x, algorithm="maxima")
[Out] 1/4*(15*(x + 1)^2 - 25*x - 17)/((x + 1)^(5/2) - 2*(x + 1)^(3/2) + sqrt(x +
1)) - 15/8*log(sqrt(x + 1) + 1) + 15/8*log(sqrt(x + 1) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = \frac{2}{\sqrt{x+1}}+\frac{7\,(x+1)^{\frac{3}{2}}-9\,\sqrt{x+1}}{4\,x^2} - \frac{15}{8}\,\log\left(\sqrt{x+1}+1\right)+\frac{15}{8}\,\log\left(\left|\sqrt{x+1}-1\right|\right)$$

```
[In] integrate(1/x^3/(1+x)^(3/2),x, algorithm="giac")
[Out] 2/sqrt(x + 1) + 1/4*(7*(x + 1)^(3/2) - 9*sqrt(x + 1))/x^2 - 15/8*log(sqrt(x
 + 1) + 1) + 15/8*log(abs(sqrt(x + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = -\frac{15\,\mathrm{atanh}\left(\sqrt{x+1}\right)}{4}-\frac{\frac{25\,x}{4}-\frac{15\,(x+1)^2}{4}+\frac{17}{4}}{\sqrt{x+1}-2\,(x+1)^{3/2}+(x+1)^{5/2}}$$

```
[In] int(1/(x^3*(x + 1)^(3/2)),x)
[Out] - (15*atanh((x + 1)^(1/2)))/4 - ((25*x)/4 - (15*(x + 1)^2)/4 + 17/4)/((x +
1)^(1/2) - 2*(x + 1)^(3/2) + (x + 1)^(5/2))
```

3.216 $\int \frac{1}{(1-x)^{7/2}x^5}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 118

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = \frac{3003}{320(1-x)^{5/2}}+\frac{1001}{64(1-x)^{3/2}}+\frac{3003}{64\sqrt{1-x}}-\frac{1}{4(1-x)^{5/2}x^4}-\frac{13}{24(1-x)^{5/2}x^3}-\frac{143}{96(1-x)^{5/2}x^2}-\frac{429}{64(1-x)^{5/2}x}-\frac{3003}{64}\operatorname{arctanh}\left(\sqrt{1-x}\right)$$

```
[Out] 3003/320/(1-x)^(5/2)+1001/64/(1-x)^(3/2)-1/4/(1-x)^(5/2)/x^4-13/24/(1-x)^(5
/2)/x^3-143/96/(1-x)^(5/2)/x^2-429/64/(1-x)^(5/2)/x-3003/64*arctanh((1-x)^(
1/2))+3003/64/(1-x)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {44, 53, 65, 212}

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = -\frac{3003}{64}\operatorname{arctanh}\left(\sqrt{1-x}\right)-\frac{1}{4(1-x)^{5/2}x^4}-\frac{13}{24(1-x)^{5/2}x^3}-\frac{143}{96(1-x)^{5/2}x^2}+\frac{3003}{64\sqrt{1-x}}-\frac{429}{64(1-x)^{5/2}x}+\frac{1001}{64(1-x)^{3/2}}+\frac{3003}{320(1-x)^{5/2}}$$

```
[In] Int[1/((1 - x)^(7/2)*x^5),x]
```

```
[Out] 3003/(320*(1 - x)^(5/2)) + 1001/(64*(1 - x)^(3/2)) + 3003/(64*Sqrt[1 - x])
- 1/(4*(1 - x)^(5/2)*x^4) - 13/(24*(1 - x)^(5/2)*x^3) - 143/(96*(1 - x)^(5/
2)*x^2) - 429/(64*(1 - x)^(5/2)*x) - (3003*ArcTanh[Sqrt[1 - x]])/64
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{4(1-x)^{5/2}x^4}+\frac{13}{8}\int\frac{1}{(1-x)^{7/2}x^4}\,dx\\
&= -\frac{1}{4(1-x)^{5/2}x^4}-\frac{13}{24(1-x)^{5/2}x^3}+\frac{143}{48}\int\frac{1}{(1-x)^{7/2}x^3}\,dx\\
&= -\frac{1}{4(1-x)^{5/2}x^4}-\frac{13}{24(1-x)^{5/2}x^3}-\frac{143}{96(1-x)^{5/2}x^2}+\frac{429}{64}\int\frac{1}{(1-x)^{7/2}x^2}\,dx\\
&= -\frac{1}{4(1-x)^{5/2}x^4}-\frac{13}{24(1-x)^{5/2}x^3}-\frac{143}{96(1-x)^{5/2}x^2}-\frac{429}{64(1-x)^{5/2}x}+\frac{3003}{128}\int\frac{1}{(1-x)^{7/2}x}\,dx\\
&= \frac{3003}{320(1-x)^{5/2}}-\frac{1}{4(1-x)^{5/2}x^4}-\frac{13}{24(1-x)^{5/2}x^3}\\
&\quad-\frac{143}{96(1-x)^{5/2}x^2}-\frac{429}{64(1-x)^{5/2}x}+\frac{3003}{128}\int\frac{1}{(1-x)^{5/2}x}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3003}{320(1-x)^{5/2}} + \frac{1001}{64(1-x)^{3/2}} - \frac{1}{4(1-x)^{5/2}x^4} - \frac{13}{24(1-x)^{5/2}x^3} \\
&\quad - \frac{143}{96(1-x)^{5/2}x^2} - \frac{429}{64(1-x)^{5/2}x} + \frac{3003}{128}\int \frac{1}{(1-x)^{3/2}x}\,dx \\
&= \frac{3003}{320(1-x)^{5/2}} + \frac{1001}{64(1-x)^{3/2}} + \frac{3003}{64\sqrt{1-x}} - \frac{1}{4(1-x)^{5/2}x^4} \\
&\quad - \frac{13}{24(1-x)^{5/2}x^3} - \frac{143}{96(1-x)^{5/2}x^2} - \frac{429}{64(1-x)^{5/2}x} + \frac{3003}{128}\int \frac{1}{\sqrt{1-x}x}\,dx \\
&= \frac{3003}{320(1-x)^{5/2}} + \frac{1001}{64(1-x)^{3/2}} + \frac{3003}{64\sqrt{1-x}} - \frac{1}{4(1-x)^{5/2}x^4} - \frac{13}{24(1-x)^{5/2}x^3} \\
&\quad - \frac{143}{96(1-x)^{5/2}x^2} - \frac{429}{64(1-x)^{5/2}x} - \frac{3003}{64}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x}\right) \\
&= \frac{3003}{320(1-x)^{5/2}} + \frac{1001}{64(1-x)^{3/2}} + \frac{3003}{64\sqrt{1-x}} - \frac{1}{4(1-x)^{5/2}x^4} - \frac{13}{24(1-x)^{5/2}x^3} \\
&\quad - \frac{143}{96(1-x)^{5/2}x^2} - \frac{429}{64(1-x)^{5/2}x} - \frac{3003}{64}\text{arctanh}\left(\sqrt{1-x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$
\int \frac{1}{(1-x)^{7/2}x^5}\,dx = -\frac{240 + 520x + 1430x^2 + 6435x^3 - 69069x^4 + 105105x^5 - 45045x^6 + 45045(1-x)^{5/2}x^4\text{arctanh}\left(\sqrt{1-x}\right)}{960(1-x)^{5/2}x^4}
$$

```
[In] Integrate[1/((1 - x)^(7/2)*x^5),x]
```

```
[Out] -1/960*(240 + 520*x + 1430*x^2 + 6435*x^3 - 69069*x^4 + 105105*x^5 - 45045*
x^6 + 45045*(1 - x)^(5/2)*x^4*ArcTanh[Sqrt[1 - x]])/((1 - x)^(5/2)*x^4)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

method	result
risch	$\frac{45045x^6-105105x^5+69069x^4-6435x^3-1430x^2-520x-240}{960x^4\sqrt{1-x}\,(-1+x)^2}-\frac{3003\,\mathrm{arctanh}\left(\sqrt{1-x}\right)}{64}$
trager	$-\frac{\left(45045x^6-105105x^5+69069x^4-6435x^3-1430x^2-520x-240\right)\sqrt{1-x}}{960(-1+x)^3x^4}-\frac{3003\ln\left(-\frac{2\sqrt{1-x}+2-x}{x}\right)}{128}$
pseudoelliptic	$\frac{\frac{3003x^4\sqrt{1-x}\,(-1+x)^2\ln\left(\sqrt{1-x}-1\right)}{128}-\frac{3003x^4\sqrt{1-x}\,(-1+x)^2\ln\left(\sqrt{1-x}+1\right)}{128}+\frac{3003x^6}{64}-\frac{7007x^5}{64}+\frac{23023x^4}{320}-\frac{429x^3}{64}-\frac{143x^2}{96}-\frac{13x}{24}-}{(1-x)^{\frac{5}{2}}\left(\sqrt{1-x}-1\right)^4\left(\sqrt{1-x}+1\right)^4}$
meijerg	$\frac{-\frac{\sqrt{\pi}}{4x^4}-\frac{7\sqrt{\pi}}{6x^3}-\frac{63\sqrt{\pi}}{16x^2}-\frac{231\sqrt{\pi}}{16x}+\frac{3003\left(\frac{329177}{180180}-2\ln(2)+\ln(x)+i\pi\right)\sqrt{\pi}}{128}+\frac{\sqrt{\pi}\left(-329177x^4+110880x^3+30240x^2+8960x+1920\right)}{7680x^4}-\frac{\sqrt{\pi}}{}}{\sqrt{\pi}}$
derivativedivides	$\frac{2}{5(1-x)^{\frac{5}{2}}}+\frac{10}{3(1-x)^{\frac{3}{2}}}+\frac{30}{\sqrt{1-x}}-\frac{1}{64\left(\sqrt{1-x}-1\right)^4}+\frac{17}{96\left(\sqrt{1-x}-1\right)^3}-\frac{159}{128\left(\sqrt{1-x}-1\right)^2}+\frac{1083}{128\left(\sqrt{1-x}-1\right)}+$
default	$\frac{2}{5(1-x)^{\frac{5}{2}}}+\frac{10}{3(1-x)^{\frac{3}{2}}}+\frac{30}{\sqrt{1-x}}-\frac{1}{64\left(\sqrt{1-x}-1\right)^4}+\frac{17}{96\left(\sqrt{1-x}-1\right)^3}-\frac{159}{128\left(\sqrt{1-x}-1\right)^2}+\frac{1083}{128\left(\sqrt{1-x}-1\right)}+$

```
[In] int(1/(1-x)^(7/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/960*(45045*x^6-105105*x^5+69069*x^4-6435*x^3-1430*x^2-520*x-240)/x^4/(1-x
)^(1/2)/(-1+x)^2-3003/64*arctanh((1-x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx =$$
$$-\frac{45045\left(x^7-3x^6+3x^5-x^4\right)\log\left(\sqrt{-x+1}+1\right)-45045\left(x^7-3x^6+3x^5-x^4\right)\log\left(\sqrt{-x+1}-1\right)+2\,(}{1920\left(x^7-3x^6+3x^5-x^4\right.}$$

```
[In] integrate(1/(1-x)^(7/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/1920*(45045*(x^7 - 3*x^6 + 3*x^5 - x^4)*log(sqrt(-x + 1) + 1) - 45045*(x
^7 - 3*x^6 + 3*x^5 - x^4)*log(sqrt(-x + 1) - 1) + 2*(45045*x^6 - 105105*x^5
 + 69069*x^4 - 6435*x^3 - 1430*x^2 - 520*x - 240)*sqrt(-x + 1))/(x^7 - 3*x^
6 + 3*x^5 - x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = \text{Timed out}$$

```
[In] integrate(1/(1-x)**(7/2)/x**5,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.94

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = \frac{45045\,(x-1)^6 + 165165\,(x-1)^5 + 219219\,(x-1)^4 + 119691\,(x-1)^3 + 18304\,(x-1)^2}{960\left((-x+1)^{\frac{13}{2}} - 4\,(-x+1)^{\frac{11}{2}} + 6\,(-x+1)^{\frac{9}{2}} - 4\,(-x+1)^{\frac{7}{2}} + (-x+1\right.}$$
$$-\frac{3003}{128}\log\left(\sqrt{-x+1}+1\right) + \frac{3003}{128}\log\left(\sqrt{-x+1}-1\right)$$

```
[In] integrate(1/(1-x)^(7/2)/x^5,x, algorithm="maxima")
[Out] 1/960*(45045*(x - 1)^6 + 165165*(x - 1)^5 + 219219*(x - 1)^4 + 119691*(x -
1)^3 + 18304*(x - 1)^2 - 1664*x + 2048)/((-x + 1)^(13/2) - 4*(-x + 1)^(11/2
) + 6*(-x + 1)^(9/2) - 4*(-x + 1)^(7/2) + (-x + 1)^(5/2)) - 3003/128*log(sq
rt(-x + 1) + 1) + 3003/128*log(sqrt(-x + 1) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.88

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = \frac{2\left(225\,(x-1)^2 - 25\,x + 28\right)}{15\,(x-1)^2\sqrt{-x+1}}$$
$$-\frac{3249\,(x-1)^3\sqrt{-x+1} + 10633\,(x-1)^2\sqrt{-x+1} - 11767\,(-x+1)^{\frac{3}{2}} + 4431\,\sqrt{-x+1}}{192\,x^4}$$
$$-\frac{3003}{128}\log\left(\sqrt{-x+1}+1\right) + \frac{3003}{128}\log\left(\left|\sqrt{-x+1}-1\right|\right)$$

```
[In] integrate(1/(1-x)^(7/2)/x^5,x, algorithm="giac")
[Out] 2/15*(225*(x - 1)^2 - 25*x + 28)/((x - 1)^2*sqrt(-x + 1)) - 1/192*(3249*(x
- 1)^3*sqrt(-x + 1) + 10633*(x - 1)^2*sqrt(-x + 1) - 11767*(-x + 1)^(3/2) +
 4431*sqrt(-x + 1))/x^4 - 3003/128*log(sqrt(-x + 1) + 1) + 3003/128*log(abs
(sqrt(-x + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = \frac{\frac{286\,(x-1)^2}{15}-\frac{26\,x}{15}+\frac{39897\,(x-1)^3}{320}+\frac{73073\,(x-1)^4}{320}+\frac{11011\,(x-1)^5}{64}+\frac{3003\,(x-1)^6}{64}+\frac{32}{15}}{(1-x)^{5/2}-4\,(1-x)^{7/2}+6\,(1-x)^{9/2}-4\,(1-x)^{11/2}+(1-x)^{13/2}} - \frac{3003\,\mathrm{atanh}\left(\sqrt{1-x}\right)}{64}$$

```
[In] int(1/(x^5*(1 - x)^(7/2)),x)
```

```
[Out] ((286*(x - 1)^2)/15 - (26*x)/15 + (39897*(x - 1)^3)/320 + (73073*(x - 1)^4)
/320 + (11011*(x - 1)^5)/64 + (3003*(x - 1)^6)/64 + 32/15)/((1 - x)^(5/2) -
 4*(1 - x)^(7/2) + 6*(1 - x)^(9/2) - 4*(1 - x)^(11/2) + (1 - x)^(13/2)) - (
3003*atanh((1 - x)^(1/2)))/64
```

3.217 $\int \frac{1}{(-1+x)^{2/3}x^5} dx$

Optimal result

Integrand size = 11, antiderivative size = 104

$$\int \frac{1}{(-1+x)^{2/3}x^5} dx = \frac{\sqrt[3]{-1+x}}{4x^4} + \frac{11\sqrt[3]{-1+x}}{36x^3} + \frac{11\sqrt[3]{-1+x}}{27x^2} + \frac{55\sqrt[3]{-1+x}}{81x} - \frac{110\arctan\left(\frac{1-2\sqrt[3]{-1+x}}{\sqrt{3}}\right)}{81\sqrt{3}} + \frac{55}{81}\log\left(1+\sqrt[3]{-1+x}\right) - \frac{55\log(x)}{243}$$

```
[Out] 1/4*(-1+x)^(1/3)/x^4+11/36*(-1+x)^(1/3)/x^3+11/27*(-1+x)^(1/3)/x^2+55/81*(-
1+x)^(1/3)/x+55/81*ln(1+(-1+x)^(1/3))-55/243*ln(x)-110/243*arctan(1/3*(1-2*
(-1+x)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {44, 60, 632, 210, 31}

$$\int \frac{1}{(-1+x)^{2/3}x^5} dx = -\frac{110\arctan\left(\frac{1-2\sqrt[3]{x-1}}{\sqrt{3}}\right)}{81\sqrt{3}} + \frac{\sqrt[3]{x-1}}{4x^4} + \frac{11\sqrt[3]{x-1}}{36x^3} + \frac{11\sqrt[3]{x-1}}{27x^2} + \frac{55\sqrt[3]{x-1}}{81x} + \frac{55}{81}\log\left(\sqrt[3]{x-1}+1\right) - \frac{55\log(x)}{243}$$

```
[In] Int[1/((-1 + x)^(2/3)*x^5),x]
```

```
[Out] (-1 + x)^(1/3)/(4*x^4) + (11*(-1 + x)^(1/3))/(36*x^3) + (11*(-1 + x)^(1/3))
/(27*x^2) + (55*(-1 + x)^(1/3))/(81*x) - (110*ArcTan[(1 - 2*(-1 + x)^(1/3))
/Sqrt[3]])/(81*Sqrt[3]) + (55*Log[1 + (-1 + x)^(1/3)])/81 - (55*Log[x])/243
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11}{12}\int \frac{1}{(-1+x)^{2/3}x^4}\,dx \\
&= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11\sqrt[3]{-1+x}}{36x^3}+\frac{22}{27}\int \frac{1}{(-1+x)^{2/3}x^3}\,dx \\
&= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11\sqrt[3]{-1+x}}{36x^3}+\frac{11\sqrt[3]{-1+x}}{27x^2}+\frac{55}{81}\int \frac{1}{(-1+x)^{2/3}x^2}\,dx \\
&= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11\sqrt[3]{-1+x}}{36x^3}+\frac{11\sqrt[3]{-1+x}}{27x^2}+\frac{55\sqrt[3]{-1+x}}{81x}+\frac{110}{243}\int \frac{1}{(-1+x)^{2/3}x}\,dx
\end{aligned}$$

$$= \frac{\sqrt[3]{-1+x}}{4x^4} + \frac{11\sqrt[3]{-1+x}}{36x^3} + \frac{11\sqrt[3]{-1+x}}{27x^2} + \frac{55\sqrt[3]{-1+x}}{81x} - \frac{55\log(x)}{243} + \frac{55}{81}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[3]{-1+x}\right) + \frac{55}{81}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[3]{-1+x}\right)$$

$$= \frac{\sqrt[3]{-1+x}}{4x^4} + \frac{11\sqrt[3]{-1+x}}{36x^3} + \frac{11\sqrt[3]{-1+x}}{27x^2} + \frac{55\sqrt[3]{-1+x}}{81x} + \frac{55}{81}\log\left(1+\sqrt[3]{-1+x}\right) - \frac{55\log(x)}{243} - \frac{110}{81}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[3]{-1+x}\right)$$

$$= \frac{\sqrt[3]{-1+x}}{4x^4} + \frac{11\sqrt[3]{-1+x}}{36x^3} + \frac{11\sqrt[3]{-1+x}}{27x^2} + \frac{55\sqrt[3]{-1+x}}{81x} - \frac{110\arctan\left(\frac{1-2\sqrt[3]{-1+x}}{\sqrt{3}}\right)}{81\sqrt{3}} + \frac{55}{81}\log\left(1+\sqrt[3]{-1+x}\right) - \frac{55\log(x)}{243}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \frac{1}{972}\left(\frac{3\sqrt[3]{-1+x}(81+99x+132x^2+220x^3)}{x^4} - 440\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{-1+x}}{\sqrt{3}}\right) + 440\log\left(1+\sqrt[3]{-1+x}\right) - 220\log\left(1-\sqrt[3]{-1+x}+(-1+x)^{2/3}\right)\right)$$

```
[In] Integrate[1/((-1 + x)^(2/3)*x^5),x]
```

```
[Out] ((3*(-1 + x)^(1/3)*(81 + 99*x + 132*x^2 + 220*x^3))/x^4 - 440*Sqrt[3]*ArcTa
n[(1 - 2*(-1 + x)^(1/3))/Sqrt[3]] + 440*Log[1 + (-1 + x)^(1/3)] - 220*Log[1
 - (-1 + x)^(1/3) + (-1 + x)^(2/3)])/972
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.82

method	result
meijerg	$\frac{(-\text{signum}(-1+x))^{\frac{2}{3}}\left(-\frac{\Gamma(\frac{2}{3})}{4x^4}-\frac{2\Gamma(\frac{2}{3})}{9x^3}-\frac{5\Gamma(\frac{2}{3})}{18x^2}-\frac{40\Gamma(\frac{2}{3})}{81x}+\frac{110\left(\frac{877}{1320}+\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+\ln(x)+i\pi\right)\Gamma(\frac{2}{3})}{243}+\frac{308\Gamma(\frac{2}{3})x_3F_2(1,1}{729}\right.}{\Gamma(\frac{2}{3})\,\text{signum}(-1+x)^{\frac{2}{3}}}$
risch	$\frac{220x^4-88x^3-33x^2-18x-81}{324x^4(-1+x)^{\frac{2}{3}}}+\frac{110(-\text{signum}(-1+x))^{\frac{2}{3}}\left(\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+\ln(x)+i\pi\right)\Gamma(\frac{2}{3})+\frac{2\Gamma(\frac{2}{3})x_3F_2(1,1,\frac{5}{3};2,2;x)}{3}\right.}{243\Gamma(\frac{2}{3})\,\text{signum}(-1+x)^{\frac{2}{3}}}$
derivativedivides	$-\frac{1}{324\left(1+(-1+x)^{\frac{1}{3}}\right)^4}-\frac{5}{243\left(1+(-1+x)^{\frac{1}{3}}\right)^3}-\frac{20}{243\left(1+(-1+x)^{\frac{1}{3}}\right)^2}-\frac{25}{81\left(1+(-1+x)^{\frac{1}{3}}\right)}+\frac{110\ln\left(1+(-1+x)\right.}{243}$
default	$-\frac{1}{324\left(1+(-1+x)^{\frac{1}{3}}\right)^4}-\frac{5}{243\left(1+(-1+x)^{\frac{1}{3}}\right)^3}-\frac{20}{243\left(1+(-1+x)^{\frac{1}{3}}\right)^2}-\frac{25}{81\left(1+(-1+x)^{\frac{1}{3}}\right)}+\frac{110\ln\left(1+(-1+x)\right.}{243}$
trager	$\frac{(220x^3+132x^2+99x+81)(-1+x)^{\frac{1}{3}}}{324x^4}-\frac{110\ln\left(-\frac{1152\,\text{RootOf}\left(2304\_Z^2+48\_Z+1\right)^2x-72\,\text{RootOf}\left(2304\_Z^2+48\_Z+1\right.}{}\right.}{}$

```
[In] int(1/(-1+x)^(2/3)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/GAMMA(2/3)/signum(-1+x)^(2/3)*(-signum(-1+x))^(2/3)*(-1/4*GAMMA(2/3)/x^4-
2/9*GAMMA(2/3)/x^3-5/18*GAMMA(2/3)/x^2-40/81*GAMMA(2/3)/x+110/243*(877/1320
+1/6*Pi*3^(1/2)-3/2*ln(3)+ln(x)+I*Pi)*GAMMA(2/3)+308/729*GAMMA(2/3)*x*hyper
geom([1,1,17/3],[2,6],x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \frac{440\sqrt{3}x^4\arctan\left(\frac{2}{3}\sqrt{3}(x-1)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-220\,x^4\log\left((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\right)+}{972\,x^4}$$

```
[In] integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="fricas")
```

```
[Out] 1/972*(440*sqrt(3)*x^4*arctan(2/3*sqrt(3)*(x - 1)^(1/3) - 1/3*sqrt(3)) - 22
0*x^4*log((x - 1)^(2/3) - (x - 1)^(1/3) + 1) + 440*x^4*log((x - 1)^(1/3) +
1) + 3*(220*x^3 + 132*x^2 + 99*x + 81)*(x - 1)^(1/3))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 37.16 (sec) , antiderivative size = 12993, normalized size of antiderivative = 124.93

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-1+x)**(2/3)/x**5,x)
```

```
[Out] -440*(x - 1)**(35/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(1/3)/
(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*
pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(
x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*
gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x -
1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamm
a(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8
/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) +
2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 440*(x - 1)**(35/3)*exp(I*pi/
3)*log(-(x - 1)**(1/3)*exp_polar(I*pi) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3
)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 1
60380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I
*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192
*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/
3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x -
 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamm
a(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*
exp(I*pi/3)*gamma(4/3)) - 440*(x - 1)**(35/3)*exp(2*I*pi/3)*log(-(x - 1)**(
1/3)*exp_polar(5*I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*
gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)*
*(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4
/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/
3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3)
+ 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*ex
p(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076
*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*ga
mma(4/3)) - 4840*(x - 1)**(32/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi/3) + 1)
*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(
32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3
) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*
exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1
347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(
I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380
*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*g
amma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 4840*(x - 1)**(32
```

```
/3)*exp(I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi) + 1)*gamma(1/3)/(2916*(
x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*g
amma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)*
*(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4
/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17
/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3)
 + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp
(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x
 - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) - 4840*(x - 1)**(32/3)*exp(2*I*pi/3)*l
og(-(x - 1)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3
)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 1
60380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I
*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192
*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/
3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x -
 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamm
a(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*
exp(I*pi/3)*gamma(4/3)) - 24200*(x - 1)**(29/3)*log(-(x - 1)**(1/3)*exp_pol
ar(I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 3
2076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*
pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(
x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)
*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x -
1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamm
a(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3
)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 24
200*(x - 1)**(29/3)*exp(I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi) + 1)*ga
mma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/
3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) +
 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp
(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347
192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*p
i/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x
 - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamm
a(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) - 24200*(x - 1)**(29/3
)*exp(2*I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(1/3)/(29
16*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/
3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x -
 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gam
ma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)*
*(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4
/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)
*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 291
6*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) - 72600*(x - 1)**(26/3)*log(-(x -
1)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/
```

```
3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x -
1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamm
a(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(
20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/
3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)
*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32
076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)
*gamma(4/3)) + 72600*(x - 1)**(26/3)*exp(I*pi/3)*log(-(x - 1)**(1/3)*exp_po
lar(I*pi) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32
076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*p
i/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x
 - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*
gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1
)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma
(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)
*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) - 726
00*(x - 1)**(26/3)*exp(2*I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(5*I*pi/3) +
1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)*
*(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4
/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3
)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) +
 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*ex
p(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 1603
80*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)
*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) - 145200*(x - 1)*
*(23/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)
**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4
/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3
)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) +
1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*ex
p(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 4811
40*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/
3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)*
*(2/3)*exp(I*pi/3)*gamma(4/3)) + 145200*(x - 1)**(23/3)*exp(I*pi/3)*log(-(x
 - 1)**(1/3)*exp_polar(I*pi) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi
/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x -
 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gam
ma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**
(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4
/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3
)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 3
2076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3
)*gamma(4/3)) - 145200*(x - 1)**(23/3)*exp(2*I*pi/3)*log(-(x - 1)**(1/3)*ex
p_polar(5*I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4
/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)
```

*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 9
62280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(
I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 96228
0*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/
3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1
)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3
)) - 203280*(x - 1)**(20/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi/3) + 1)*gamm
a(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)
*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 4
81140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I
*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 134719
2*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/
3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x -
 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(
4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 203280*(x - 1)**(20/3)
*exp(I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi) + 1)*gamma(1/3)/(2916*(x -
 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamm
a(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(2
6/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3)
 + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)
*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 4
81140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*
pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x -
1)**(2/3)*exp(I*pi/3)*gamma(4/3)) - 203280*(x - 1)**(20/3)*exp(2*I*pi/3)*lo
g(-(x - 1)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)
*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 16
0380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*
pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*
(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3
)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x -
1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma
(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*e
xp(I*pi/3)*gamma(4/3)) - 203280*(x - 1)**(17/3)*log(-(x - 1)**(1/3)*exp_pol
ar(I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 3
2076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*
pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(
x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)
gamma(4/3) + 1347192(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x -
1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamm
a(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3
)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 20
3280*(x - 1)**(17/3)*exp(I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi) + 1)*g
amma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32
/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3)
+ 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*ex

p(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 134 7192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I* pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gam ma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) - 203280*(x - 1)**(17 /3)*exp(2*I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*p i/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*g amma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma (4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/ 3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2 916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) - 145200*(x - 1)**(14/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I* pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*g amma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1) **(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma (4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11 /3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi /3)*gamma(4/3)) + 145200*(x - 1)**(14/3)*exp(I*pi/3)*log(-(x - 1)**(1/3)*ex p_polar(I*pi) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp (I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 96228 0*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi /3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*g amma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) - 145200*(x - 1)**(14/3)*exp(2*I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(5*I*pi/ 3) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*ga mma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)** (23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4 /3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/ 3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I* pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) - 72600*(x - 1)**(11/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gam ma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3

```
)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) +
481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I
*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x -
 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 72600*(x - 1)**(11/3)*exp(I*pi/3)*log(
-(x - 1)**(1/3)*exp_polar(I*pi) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I
*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(
x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*
gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1
)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamm
a(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(1
1/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3)
+ 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*p
i/3)*gamma(4/3)) - 72600*(x - 1)**(11/3)*exp(2*I*pi/3)*log(-(x - 1)**(1/3)*
exp_polar(5*I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma
(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/
3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) +
 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*ex
p(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962
280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*p
i/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x -
 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4
/3)) - 24200*(x - 1)**(8/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi/3) + 1)*gamm
a(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)
*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 4
81140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I
*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 134719
2*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/
3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x -
 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(
4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 24200*(x - 1)**(8/3)*e
xp(I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi) + 1)*gamma(1/3)/(2916*(x - 1
)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(
4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/
3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) +
 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*e
xp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481
140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi
/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)
**(2/3)*exp(I*pi/3)*gamma(4/3)) - 24200*(x - 1)**(8/3)*exp(2*I*pi/3)*log(-(
x - 1)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp
(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380
*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3
)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x -
 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*ga
mma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**
```

(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3
) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I
*pi/3)*gamma(4/3)) - 4840*(x - 1)**(5/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi
/3) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x
 - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*g
amma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)*
*(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(
4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14
/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3)
+ 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I
*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 4840*(x -
 1)**(5/3)*exp(I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi) + 1)*gamma(1/3)/
(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*
pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(
x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*
gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x -
1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamm
a(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8
/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) +
2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) - 4840*(x - 1)**(5/3)*exp(2*I*p
i/3)*log(-(x - 1)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)*
*(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/
3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)
*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1
347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp
(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 48114
0*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3
)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**
(2/3)*exp(I*pi/3)*gamma(4/3)) - 440*(x - 1)**(2/3)*log(-(x - 1)**(1/3)*exp_
polar(I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3)
+ 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp
(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 96228
0*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi
/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x
 - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*g
amma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(
5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) +
 440*(x - 1)**(2/3)*exp(I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi) + 1)*ga
mma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/
3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) +
 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp
(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347
192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*p
i/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x
 - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamm

a(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) - 440*(x - 1)**(2/3)*e
xp(2*I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(1/3)/(2916*
(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*
gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)
(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)(23/3)*exp(I*pi/3)*gamma(
4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(1
7/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3)
 + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*ex
p(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(
x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 660*(x - 1)**11*exp(I*pi/3)*gamma(1
/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*ex
p(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 4811
40*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi
/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(
x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*
gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)
(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)(5/3)*exp(I*pi/3)*gamma(4/3
) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 6996*(x - 1)**10*exp(I*pi
/3)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)
(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)(29/3)*exp(I*pi/3)*gamma(
4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/
3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3)
+ 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*e
xp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160
380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3
)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 33561*(x - 1)*
*9*exp(I*pi/3)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32
076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*p
i/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x
 - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*
gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1
)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma
(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)
*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 960
75*(x - 1)**8*exp(I*pi/3)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamm
a(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29
/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3)
+ 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*e
xp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 96
2280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*
pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x
- 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(
4/3)) + 181881*(x - 1)**7*exp(I*pi/3)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(
I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*
(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)

```
*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x -
1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gam
ma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(
11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3)
 + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*
pi/3)*gamma(4/3)) + 237951*(x - 1)**6*exp(I*pi/3)*gamma(1/3)/(2916*(x - 1)*
*(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/
3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)
*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1
347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp
(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 48114
0*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3
)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**
(2/3)*exp(I*pi/3)*gamma(4/3)) + 217791*(x - 1)**5*exp(I*pi/3)*gamma(1/3)/(2
916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi
/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x
- 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*ga
mma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)
**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(
4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3
)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 29
16*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 137601*(x - 1)**4*exp(I*pi/3)*g
amma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32
/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3)
+ 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*ex
p(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 134
7192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*
pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(
x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gam
ma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 57375*(x - 1)**3*ex
p(I*pi/3)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(
x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*
gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)
**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma
(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(1
4/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3)
 + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(
I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 14241*(x
 - 1)**2*exp(I*pi/3)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3
) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*e
xp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962
280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*
pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*
(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)
*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)*
```

```
*(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3))
 + 1596*(x - 1)*exp(I*pi/3)*gamma(1/3)/(2916*(x - 1)**(35/3)*exp(I*pi/3)*ga
mma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(
29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3
) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)
*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) +
962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(
I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(
x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamm
a(4/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \frac{110}{243}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\Big(2\,(x-1)^{\frac{1}{3}}-1\Big)\right) + \frac{220\,(x-1)^{\frac{10}{3}}+792\,(x-1)^{\frac{7}{3}}+1023\,(x-1)^{\frac{4}{3}}+532\,(x-1)^{\frac{1}{3}}}{324\,\big((x-1)^4+4\,(x-1)^3+6\,(x-1)^2+4\,x-3\big)} - \frac{55}{243}\,\log\Big((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\Big) + \frac{110}{243}\,\log\Big((x-1)^{\frac{1}{3}}+1\Big)$$

```
[In] integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="maxima")
```

```
[Out] 110/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x - 1)^(1/3) - 1)) + 1/324*(220*(x -
 1)^(10/3) + 792*(x - 1)^(7/3) + 1023*(x - 1)^(4/3) + 532*(x - 1)^(1/3))/((
x - 1)^4 + 4*(x - 1)^3 + 6*(x - 1)^2 + 4*x - 3) - 55/243*log((x - 1)^(2/3)
- (x - 1)^(1/3) + 1) + 110/243*log((x - 1)^(1/3) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \frac{110}{243}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\Big(2\,(x-1)^{\frac{1}{3}}-1\Big)\right) + \frac{220\,(x-1)^{\frac{10}{3}}+792\,(x-1)^{\frac{7}{3}}+1023\,(x-1)^{\frac{4}{3}}+532\,(x-1)^{\frac{1}{3}}}{324\,x^4} - \frac{55}{243}\,\log\Big((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\Big) + \frac{110}{243}\,\log\Big((x-1)^{\frac{1}{3}}+1\Big)$$

```
[In] integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="giac")
```

```
[Out] 110/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x - 1)^(1/3) - 1)) + 1/324*(220*(x -
 1)^(10/3) + 792*(x - 1)^(7/3) + 1023*(x - 1)^(4/3) + 532*(x - 1)^(1/3))/x^
4 - 55/243*log((x - 1)^(2/3) - (x - 1)^(1/3) + 1) + 110/243*log((x - 1)^(1/
3) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.15

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \frac{110\,\ln\left(\frac{12100\,(x-1)^{1/3}}{6561}+\frac{12100}{6561}\right)}{243} + \frac{\frac{133\,(x-1)^{1/3}}{81}+\frac{341\,(x-1)^{4/3}}{108}+\frac{22\,(x-1)^{7/3}}{9}+\frac{55\,(x-1)^{10/3}}{81}}{4\,x+6\,(x-1)^2+4\,(x-1)^3+(x-1)^4-3} - \ln\left(\frac{55}{27}-\frac{110\,(x-1)^{1/3}}{27}+\frac{\sqrt{3}\,55\mathrm{i}}{27}\right)\left(\frac{55}{243}+\frac{\sqrt{3}\,55\mathrm{i}}{243}\right)+\ln\left(\frac{110\,(x-1)^{1/3}}{27}-\frac{55}{27}+\frac{\sqrt{3}\,55\mathrm{i}}{27}\right)\left(-\frac{55}{243}+\frac{\sqrt{3}\,55\mathrm{i}}{243}\right)$$

```
[In] int(1/(x^5*(x - 1)^(2/3)),x)
```

```
[Out] (110*log((12100*(x - 1)^(1/3))/6561 + 12100/6561))/243 + ((133*(x - 1)^(1/3
))/81 + (341*(x - 1)^(4/3))/108 + (22*(x - 1)^(7/3))/9 + (55*(x - 1)^(10/3)
)/81)/(4*x + 6*(x - 1)^2 + 4*(x - 1)^3 + (x - 1)^4 - 3) - log((3^(1/2)*55i)
/27 - (110*(x - 1)^(1/3))/27 + 55/27)*((3^(1/2)*55i)/243 + 55/243) + log((1
10*(x - 1)^(1/3))/27 + (3^(1/2)*55i)/27 - 55/27)*((3^(1/2)*55i)/243 - 55/24
3)
```

3.218 $\int \sqrt{\frac{1-x}{1+x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sqrt{\frac{1-x}{1+x}}\,dx=\sqrt{\frac{1-x}{1+x}}(1+x)-2\arctan\left(\sqrt{\frac{1-x}{1+x}}\right)$$

```
[Out] -2*arctan(((1-x)/(1+x))^(1/2))+(1+x)*((1-x)/(1+x))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 294, 210}

$$\int \sqrt{\frac{1-x}{1+x}}\,dx=\sqrt{\frac{1-x}{x+1}}(x+1)-2\arctan\left(\sqrt{\frac{1-x}{x+1}}\right)$$

```
[In] Int[Sqrt[(1 - x)/(1 + x)],x]
[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(4\text{Subst}\left(\int \frac{x^2}{(-1-x^2)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)\\
&= \sqrt{\frac{1-x}{1+x}}(1+x) + 2\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\\
&= \sqrt{\frac{1-x}{1+x}}(1+x) - 2\arctan\left(\sqrt{\frac{1-x}{1+x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.68

$$
\int \sqrt{\frac{1-x}{1+x}}\,dx = \frac{\sqrt{\frac{1-x}{1+x}}\sqrt{1+x}\left(\sqrt{1-x^2} - 2\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)\right)}{\sqrt{1-x}}
$$

```
[In] Integrate[Sqrt[(1 - x)/(1 + x)],x]
[Out] (Sqrt[(1 - x)/(1 + x)]*Sqrt[1 + x]*(Sqrt[1 - x^2] - 2*ArcTan[Sqrt[1 - x^2]/
(-1 + x)]))/Sqrt[1 - x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

method	result
default	$\frac{\sqrt{-\frac{-1+x}{1+x}}\,(1+x)\left(\sqrt{-x^2+1}+\arcsin(x)\right)}{\sqrt{-(-1+x)(1+x)}}$
risch	$(1+x)\sqrt{-\frac{-1+x}{1+x}}-\frac{\arcsin(x)\sqrt{-\frac{-1+x}{1+x}}\,\sqrt{-(-1+x)(1+x)}}{-1+x}$
trager	$(1+x)\sqrt{-\frac{-1+x}{1+x}}+\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{-1+x}{1+x}}\,x+\mathrm{RootOf}\left(\_Z^2+1\right)\right.$

```
[In] int(((1-x)/(1+x))^(1/2),x,method=_RETURNVERBOSE)
[Out] (-(-1+x)/(1+x))^(1/2)*(1+x)/(-(-1+x)*(1+x))^(1/2)*((-x^2+1)^(1/2)+arcsin(x)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \sqrt{\frac{1-x}{1+x}}\,dx=(x+1)\sqrt{-\frac{x-1}{x+1}}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="fricas")
[Out] (x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Sympy [F]

$$\int \sqrt{\frac{1-x}{1+x}}\,dx=\int \sqrt{\frac{1-x}{x+1}}\,dx$$

```
[In] integrate(((1-x)/(1+x))**(1/2),x)
[Out] Integral(sqrt((1 - x)/(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1} - 2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-(x - 1)/(x
 + 1)))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \frac{1}{2}\,\pi\mathrm{sgn}(x+1) + \arcsin(x)\,\mathrm{sgn}(x+1) + \sqrt{-x^2+1}\mathrm{sgn}(x+1)$$

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="giac")
[Out] 1/2*pi*sgn(x + 1) + arcsin(x)*sgn(x + 1) + sqrt(-x^2 + 1)*sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right) - \frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

```
[In] int((-(x - 1)/(x + 1))^(1/2),x)
[Out] - 2*atan((-(x - 1)/(x + 1))^(1/2)) - (2*(-(x - 1)/(x + 1))^(1/2))/((x - 1)/
(x + 1) - 1)
```

3.219 $\int x\sqrt{\frac{-a+x}{b-x}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 92

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx = \frac{1}{4}(a-5b)(b-x)\sqrt{\frac{-a+x}{b-x}} + \frac{1}{2}(b-x)^2\sqrt{\frac{-a+x}{b-x}} - \frac{1}{4}(a-b)(a+3b)\arctan\left(\sqrt{\frac{-a+x}{b-x}}\right)$$

```
[Out] -1/4*(a-b)*(a+3*b)*arctan(((-a+x)/(b-x))^(1/2))+1/4*(a-5*b)*(b-x)*((-a+x)/(
b-x))^(1/2)+1/2*(b-x)^2*((-a+x)/(b-x))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1980, 466, 393, 209}

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx = -\frac{1}{4}(a-b)(a+3b)\arctan\left(\sqrt{-\frac{a-x}{b-x}}\right) + \frac{1}{2}(b-x)^2\sqrt{-\frac{a-x}{b-x}} + \frac{1}{4}(a-5b)(b-x)\sqrt{-\frac{a-x}{b-x}}$$

```
[In] Int[x*Sqrt[(-a + x)/(b - x)],x]
```

```
[Out] ((a - 5*b)*Sqrt[-((a - x)/(b - x))]*(b - x))/4 + (Sqrt[-((a - x)/(b - x))]*
(b - x)^2)/2 - ((a - b)*(a + 3*b)*ArcTan[Sqrt[-((a - x)/(b - x))]])/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left((2(a-b))\text{Subst}\left(\int \frac{x^2(a+bx^2)}{(1+x^2)^3}\,dx, x, \sqrt{\frac{-a+x}{b-x}}\right)\right)\\
&= \frac{1}{2}\sqrt{-\frac{a-x}{b-x}}(b-x)^2 - \frac{1}{2}(-a+b)\text{Subst}\left(\int \frac{-a+b-4bx^2}{(1+x^2)^2}\,dx, x, \sqrt{\frac{-a+x}{b-x}}\right)\\
&= \frac{1}{4}(a-5b)\sqrt{-\frac{a-x}{b-x}}(b-x) + \frac{1}{2}\sqrt{-\frac{a-x}{b-x}}(b-x)^2\\
&\quad - \frac{1}{4}((a-b)(a+3b))\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{-a+x}{b-x}}\right)\\
&= \frac{1}{4}(a-5b)\sqrt{-\frac{a-x}{b-x}}(b-x) + \frac{1}{2}\sqrt{-\frac{a-x}{b-x}}(b-x)^2 - \frac{1}{4}(a-b)(a+3b)\arctan\left(\sqrt{-\frac{a-x}{b-x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx = \frac{\sqrt{\frac{-a+x}{b-x}}\left((a-3b-2x)(b-x)\sqrt{-a+x}+(-a^2-2ab+3b^2)\sqrt{b-x}\arctan\left(\frac{\sqrt{-a+x}}{\sqrt{b-x}}\right)\right)}{4\sqrt{-a+x}}$$

```
[In] Integrate[x*Sqrt[(-a + x)/(b - x)],x]
```

```
[Out] (Sqrt[(-a + x)/(b - x)]*((a - 3*b - 2*x)*(b - x)*Sqrt[-a + x] + (-a^2 - 2*a
*b + 3*b^2)*Sqrt[b - x]*ArcTan[Sqrt[-a + x]/Sqrt[b - x]]))/(4*Sqrt[-a + x])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.53

method	result
risch	$\frac{(a-3b-2x)(b-x)\sqrt{-\frac{a-x}{b-x}}\,\sqrt{-(b-x)(a-x)}}{4\sqrt{-(-b+x)(-a+x)}}+\frac{\left(\frac{1}{4}ab-\frac{3}{8}b^2+\frac{1}{8}a^2\right)\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)\sqrt{-\frac{a-x}{b-x}}\,\sqrt{-(b-x)(a-x)}}{a-x}$
default	$\frac{\sqrt{-\frac{a-x}{b-x}}\,(b-x)\left(\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+ax+bx-x^2}}\right)a^2+2b\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+ax+bx-x^2}}\right)a-3\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+ax+bx-x^2}}\right)b^2+2\sqrt{-ab+ax+b}\right.}{8\sqrt{-(b-x)(a-x)}}$

```
[In] int(x*((-a+x)/(b-x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a-3*b-2*x)*(b-x)/(-(-b+x)*(-a+x))^(1/2)*(-(a-x)/(b-x))^(1/2)*(-(b-x)*(
a-x))^(1/2)+(1/4*a*b-3/8*b^2+1/8*a^2)*arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-
x^2)^(1/2))*(-(a-x)/(b-x))^(1/2)*(-(b-x)*(a-x))^(1/2)/(a-x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx = -\frac{1}{4}\left(a^2+2\,ab-3\,b^2\right)\arctan\left(\sqrt{-\frac{a-x}{b-x}}\right) + \frac{1}{4}\left(ab-3\,b^2-(a-b)x+2\,x^2\right)\sqrt{-\frac{a-x}{b-x}}$$

```
[In] integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(a^2 + 2*a*b - 3*b^2)*arctan(sqrt(-(a - x)/(b - x))) + 1/4*(a*b - 3*b^
2 - (a - b)*x + 2*x^2)*sqrt(-(a - x)/(b - x))
```

Sympy [F]

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx = \int x\sqrt{\frac{-a+x}{b-x}}\,dx$$

```
[In] integrate(x*((-a+x)/(b-x))**(1/2),x)
[Out] Integral(x*sqrt((-a + x)/(b - x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.41

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx = -\frac{1}{4}\left(a^2+2\,ab-3\,b^2\right)\arctan\left(\sqrt{-\frac{a-x}{b-x}}\right) - \frac{\left(a^2-6\,ab+5\,b^2\right)\left(-\frac{a-x}{b-x}\right)^{\frac{3}{2}}-\left(a^2+2\,ab-3\,b^2\right)\sqrt{-\frac{a-x}{b-x}}}{4\left(\frac{(a-x)^2}{(b-x)^2}-\frac{2\,(a-x)}{b-x}+1\right)}$$

```
[In] integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="maxima")
[Out] -1/4*(a^2 + 2*a*b - 3*b^2)*arctan(sqrt(-(a - x)/(b - x))) - 1/4*((a^2 - 6*a
*b + 5*b^2)*(-(a - x)/(b - x))^(3/2) - (a^2 + 2*a*b - 3*b^2)*sqrt(-(a - x)/
(b - x)))/((a - x)^2/(b - x)^2 - 2*(a - x)/(b - x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.12

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx = \frac{1}{8}\left(a^2\mathrm{sgn}(-b+x)+2\,ab\mathrm{sgn}(-b+x)-3\,b^2\mathrm{sgn}(-b+x)\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\mathrm{sgn}(-a+b) - \frac{1}{4}\sqrt{-ab+ax+bx-x^2}\left(a\mathrm{sgn}(-b+x)-3\,b\mathrm{sgn}(-b+x)-2\,x\mathrm{sgn}(-b+x)\right)$$

```
[In] integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="giac")
[Out] 1/8*(a^2*sgn(-b + x) + 2*a*b*sgn(-b + x) - 3*b^2*sgn(-b + x))*arcsin((a + b
 - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqrt(-a*b + a*x + b*x - x^2)*(a*sgn(-b +
 x) - 3*b*sgn(-b + x) - 2*x*sgn(-b + x))
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.52

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx=-\frac{\sqrt{-\frac{a-x}{b-x}}\,\left(\frac{a^2\,1\mathrm{i}}{4}+\frac{a\,b\,1\mathrm{i}}{2}-\frac{b^2\,3\mathrm{i}}{4}\right)\,1\mathrm{i}-{\left(-\frac{a-x}{b-x}\right)}^{3/2}\,\left(\frac{a^2\,1\mathrm{i}}{4}-\frac{a\,b\,3\mathrm{i}}{2}+\frac{b^2\,5\mathrm{i}}{4}\right)\,1\mathrm{i}}{\frac{{\left(a-x\right)}^2}{{\left(b-x\right)}^2}-\frac{2\,\left(a-x\right)}{b-x}+1}-\frac{\mathrm{atan}\left(\sqrt{-\frac{a-x}{b-x}}\right)\,\left(a-b\right)\,\left(a+3\,b\right)}{4}$$

```
[In] int(x*(-(a - x)/(b - x))^(1/2),x)
```

```
[Out] - ((-(a - x)/(b - x))^(1/2)*((a*b*1i)/2 + (a^2*1i)/4 - (b^2*3i)/4)*1i - (-(
a - x)/(b - x))^(3/2)*((a^2*1i)/4 - (a*b*3i)/2 + (b^2*5i)/4)*1i)/((a - x)^2
/(b - x)^2 - (2*(a - x))/(b - x) + 1) - (atan((-(a - x)/(b - x))^(1/2))*(a
- b)*(a + 3*b))/4
```

3.220 $\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)(-25+x^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 54

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\left(-25+x^2\right)}\,dx=\frac{1}{6}\arctan\left(\frac{1}{4}\sqrt{-5+x}\sqrt{3+x}\right)+\frac{\operatorname{arctanh}\left(\frac{\sqrt{5}\sqrt{3+x}}{\sqrt{-5+x}}\right)}{3\sqrt{5}}$$

```
[Out] 1/6*arctan(1/4*(-5+x)^(1/2)*(3+x)^(1/2))+1/15*arctanh(5^(1/2)*(3+x)^(1/2)/(
-5+x)^(1/2))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1600, 184, 94, 209, 95, 212}

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\left(-25+x^2\right)}\,dx=\frac{1}{6}\arctan\left(\frac{1}{4}\sqrt{x-5}\sqrt{x+3}\right)+\frac{\operatorname{arctanh}\left(\frac{\sqrt{5}\sqrt{x+3}}{\sqrt{x-5}}\right)}{3\sqrt{5}}$$

```
[In] Int[(Sqrt[-5 + x]*Sqrt[3 + x])/((-1 + x)*(-25 + x^2)),x]
```

```
[Out] ArcTan[(Sqrt[-5 + x]*Sqrt[3 + x])/4]/6 + ArcTanh[(Sqrt[5]*Sqrt[3 + x])/Sqrt
[-5 + x]]/(3*Sqrt[5])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 184

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_))/(((a_.) + (b_.)*(
x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[
(e + f*x)^(p - 1)*((g + h*x)^q/(a + b*x)), x], x] - Dist[(d*e - c*f)/(b*c -
 a*d), Int[(e + f*x)^(p - 1)*((g + h*x)^q/(c + d*x)), x], x] /; FreeQ[{a, b
, c, d, e, f, g, h, q}, x] && LtQ[0, p, 1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{3+x}}{\sqrt{-5+x}(-1+x)(5+x)}\,dx \\
&= \frac{1}{3}\int \frac{1}{\sqrt{-5+x}\sqrt{3+x}(5+x)}\,dx + \frac{2}{3}\int \frac{1}{\sqrt{-5+x}(-1+x)\sqrt{3+x}}\,dx \\
&= \frac{2}{3}\text{Subst}\left(\int \frac{1}{2-10x^2}\,dx, x, \frac{\sqrt{3+x}}{\sqrt{-5+x}}\right) + \frac{2}{3}\text{Subst}\left(\int \frac{1}{16+x^2}\,dx, x, \sqrt{-5+x}\sqrt{3+x}\right) \\
&= \frac{1}{6}\arctan\left(\frac{1}{4}\sqrt{-5+x}\sqrt{3+x}\right) + \frac{\operatorname{arctanh}\left(\frac{\sqrt{5}\sqrt{3+x}}{\sqrt{-5+x}}\right)}{3\sqrt{5}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\left(-25+x^2\right)}\,dx = \frac{1}{15}\left(-5\arctan\left(\frac{1}{\sqrt{\frac{-5+x}{3+x}}}\right)+\sqrt{5}\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{\frac{-5+x}{3+x}}}\right)\right)$$

[In] Integrate[(Sqrt[-5 + x]*Sqrt[3 + x])/((-1 + x)*(-25 + x^2)),x]

[Out] (-5*ArcTan[1/Sqrt[(-5 + x)/(3 + x)]] + Sqrt[5]*ArcTanh[Sqrt[5]/Sqrt[(-5 + x)/(3 + x)]])/15

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{\sqrt{x-5}\,\sqrt{3+x}\,\left(\sqrt{5}\,\operatorname{arctanh}\left(\frac{(5+3x)\sqrt{5}}{5\sqrt{x^2-2x-15}}\right)-5\arctan\left(\frac{4}{\sqrt{x^2-2x-15}}\right)\right)}{30\sqrt{x^2-2x-15}}$	64

[In] int((x-5)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x,method=_RETURNVERBOSE)

[Out] 1/30*(x-5)^(1/2)*(3+x)^(1/2)*(5^(1/2)*arctanh(1/5*(5+3*x)*5^(1/2)/(x^2-2*x-15)^(1/2))-5*arctan(4/(x^2-2*x-15)^(1/2)))/(x^2-2*x-15)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\left(-25+x^2\right)}\,dx = \frac{1}{30}\,\sqrt{5}\log\left(\frac{\sqrt{x+3}\sqrt{x-5}\left(3\,\sqrt{5}+5\right)+\sqrt{5}(3\,x+5)+9\,x+15}{x+5}\right) + \frac{1}{3}\,\arctan\left(\frac{1}{4}\,\sqrt{x+3}\sqrt{x-5}-\frac{1}{4}\,x+\frac{1}{4}\right)$$

[In] integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="fricas")

[Out] 1/30*sqrt(5)*log((sqrt(x + 3)*sqrt(x - 5)*(3*sqrt(5) + 5) + sqrt(5)*(3*x + 5) + 9*x + 15)/(x + 5)) + 1/3*arctan(1/4*sqrt(x + 3)*sqrt(x - 5) - 1/4*x + 1/4)

Sympy [F]

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\,(-25+x^2)}\,dx = \int \frac{\sqrt{x+3}}{\sqrt{x-5}\,(x-1)\,(x+5)}\,dx$$

```
[In] integrate((-5+x)**(1/2)*(3+x)**(1/2)/(-1+x)/(x**2-25),x)
[Out] Integral(sqrt(x + 3)/(sqrt(x - 5)*(x - 1)*(x + 5)), x)
```

Maxima [F]

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\,(-25+x^2)}\,dx = \int \frac{\sqrt{x+3}\sqrt{x-5}}{(x^2-25)(x-1)}\,dx$$

```
[In] integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="maxima")
[Out] integrate(sqrt(x + 3)*sqrt(x - 5)/((x^2 - 25)*(x - 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(36) = 72$.

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\,(-25+x^2)}\,dx = -\frac{1}{30}\sqrt{5}\log\left(\frac{\left(\sqrt{x+3}-\sqrt{x-5}\right)^2-4\sqrt{5}+12}{\left(\sqrt{x+3}-\sqrt{x-5}\right)^2+4\sqrt{5}+12}\right) - \frac{1}{3}\arctan\left(\frac{1}{8}\left(\sqrt{x+3}-\sqrt{x-5}\right)^2\right)$$

```
[In] integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="giac")
[Out] -1/30*sqrt(5)*log(((sqrt(x + 3) - sqrt(x - 5))^2 - 4*sqrt(5) + 12)/((sqrt(x
 + 3) - sqrt(x - 5))^2 + 4*sqrt(5) + 12)) - 1/3*arctan(1/8*(sqrt(x + 3) - s
qrt(x - 5))^2)
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\,(-25+x^2)}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{x+3}\,\sqrt{x-5}-2\,\sqrt{2}\,\sqrt{x-5}}{x-2\,\sqrt{2}\,\sqrt{x+3}+3}\right)}{3} - \frac{\sqrt{5}\,\operatorname{atanh}\left(-\frac{\sqrt{5}\,\sqrt{x+3}\,\sqrt{x-5}-2\,\sqrt{2}\,\sqrt{5}\,\sqrt{x-5}}{5\,x-10\,\sqrt{2}\,\sqrt{x+3}+15}\right)}{15}$$

```
[In] int(((x + 3)^(1/2)*(x - 5)^(1/2))/((x^2 - 25)*(x - 1)),x)
```

```
[Out] atan(((x + 3)^(1/2)*(x - 5)^(1/2) - 2*2^(1/2)*(x - 5)^(1/2))/(x - 2*2^(1/2)
*(x + 3)^(1/2) + 3))/3 - (5^(1/2)*atanh(-(5^(1/2)*(x + 3)^(1/2)*(x - 5)^(1/
2) - 2*2^(1/2)*5^(1/2)*(x - 5)^(1/2))/(5*x - 10*2^(1/2)*(x + 3)^(1/2) + 15)
))/15
```

3.221 $\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$

Optimal result

Integrand size = 52, antiderivative size = 304

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx = \frac{5}{16}(1-x)^{3/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4} + \frac{1}{24}(1-x)^{5/4}(1+x)^{3/4}+\frac{7(1-x^2)^{5/4}}{24\sqrt{1-x}}+\frac{x(1-x^2)^{5/4}}{6\sqrt{1-x}}+\frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4}-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{1+x}}\right)}{8\sqrt{2}}$$

```
[Out] 5/16*(1-x)^(3/4)*(1+x)^(1/4)-1/16*(1-x)^(1/4)*(1+x)^(3/4)+1/24*(1-x)^(5/4)*
(1+x)^(3/4)+3/16*arctan(-1+(1-x)^(1/4)*2^(1/2)/(1+x)^(1/4))*2^(1/2)+3/16*ar
ctan(1+(1-x)^(1/4)*2^(1/2)/(1+x)^(1/4))*2^(1/2)+1/16*ln(1-(1-x)^(1/4)*2^(1/
2)/(1+x)^(1/4)+(1-x)^(1/2)/(1+x)^(1/2))*2^(1/2)-1/16*ln(1+(1-x)^(1/4)*2^(1/
2)/(1+x)^(1/4)+(1-x)^(1/2)/(1+x)^(1/2))*2^(1/2)+7/24*(-x^2+1)^(5/4)/(1-x)^(
1/2)+1/6*x*(-x^2+1)^(5/4)/(1-x)^(1/2)+1/6*(-x^2+1)^(5/4)*(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.05, number of steps used = 33, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2128, 809, 689, 52, 65, 246, 217, 1179, 642, 1176, 631, 210, 1647, 807, 338, 303}

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right)}{8\sqrt{2}} + \frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1\right)}{8\sqrt{2}} + \frac{(1-x^2)^{9/4}}{3(1-x)^{3/2}} + \frac{1}{6}\sqrt{x+1}\left(1-x^2\right)^{5/4} + \frac{1}{6}(1-x)^{7/4}(x+1)^{5/4} + \frac{1}{24}(1-x)^{5/4}(x+1)^{3/4} - \frac{1}{16}\sqrt[4]{1-x}(x+1)^{3/4} + \frac{5}{24}(1-x)^{7/4}\sqrt[4]{x+1} - \frac{5}{48}(1-x)^{3/4}\sqrt[4]{x}$$

```
[In] Int[(x^2*Sqrt[1 + x]*(1 - x^2)^(1/4))/(Sqrt[1 - x]*(Sqrt[1 - x] - Sqrt[1 +
x])),x]
```

```
[Out] (-5*(1 - x)^(3/4)*(1 + x)^(1/4))/48 + (5*(1 - x)^(7/4)*(1 + x)^(1/4))/24 -
((1 - x)^(1/4)*(1 + x)^(3/4))/16 + ((1 - x)^(5/4)*(1 + x)^(3/4))/24 + ((1 -
 x)^(7/4)*(1 + x)^(5/4))/6 + (Sqrt[1 + x]*(1 - x^2)^(5/4))/6 + (1 - x^2)^(9
/4)/(3*(1 - x)^(3/2)) - (3*ArcTan[1 - (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)
])/(8*Sqrt[2]) + (3*ArcTan[1 + (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)])/(8*S
qrt[2]) + Log[1 + Sqrt[1 - x]/Sqrt[1 + x] - (Sqrt[2]*(1 - x)^(1/4))/(1 + x)
^(1/4)]/(8*Sqrt[2]) - Log[1 + Sqrt[1 - x]/Sqrt[1 + x] + (Sqrt[2]*(1 - x)^(1
/4))/(1 + x)^(1/4)]/(8*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 689

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && E
qQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m
, 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 2128

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\int x\sqrt{1+x}\sqrt[4]{1-x^2}\,dx\right) - \frac{1}{2}\int \frac{x(1+x)\sqrt[4]{1-x^2}}{\sqrt{1-x}}\,dx$$

$$= \frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4} - \frac{1}{12}\int \sqrt{1+x}\sqrt[4]{1-x^2}\,dx - \frac{1}{2}\int \frac{x\left(1-x^2\right)^{5/4}}{(1-x)^{3/2}}\,dx$$

$$= \frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4} + \frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}} - \frac{1}{12}\int \sqrt[4]{1-x}(1+x)^{3/4}\,dx - \frac{1}{2}\int \frac{\left(1-x^2\right)^{5/4}}{\sqrt{1-x}}\,dx$$

$$= \frac{1}{24}(1-x)^{5/4}(1+x)^{3/4} + \frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4} + \frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}} - \frac{1}{16}\int \frac{\sqrt[4]{1-x}}{\sqrt[4]{1+x}}\,dx - \frac{1}{2}\int (1-x)^{3/4}(1+x)^{5/4}\,dx$$

$$= -\frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4} + \frac{1}{24}(1-x)^{5/4}(1+x)^{3/4} + \frac{1}{6}(1-x)^{7/4}(1+x)^{5/4} + \frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4} + \frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}} - \frac{1}{32}\int \frac{1}{(1-x)^{3/4}\sqrt[4]{1+x}}\,dx - \frac{5}{12}\int (1$$

$$= \frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x} - \frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4} + \frac{1}{24}(1-x)^{5/4}(1+x)^{3/4} + \frac{1}{6}(1-x)^{7/4}(1+x)^{5/4} + \frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4} + \frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}} - \frac{5}{48}\int \frac{(1-x)^{3}}{(1+x)^{3}}$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x} + \frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x} - \frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4} + \frac{1}{24}(1-x)^{5/4}(1+x)^{3/4} + \frac{1}{6}(1-x)^{7/4}(1+x)^{5/4} + \frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4} + \frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}} + \frac{1}{8}\text{Subst}\left(\int \frac{}{1}\right.$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x} + \frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x} - \frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4} + \frac{1}{24}(1-x)^{5/4}(1+x)^{3/4} + \frac{1}{6}(1-x)^{7/4}(1+x)^{5/4} + \frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4} + \frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}} + \frac{1}{16}\text{Subst}\left(\int \frac{1}{1}\right.$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x} + \frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x} - \frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4} + \frac{1}{24}(1-x)^{5/4}(1+x)^{3/4} + \frac{1}{6}(1-x)^{7/4}(1+x)^{5/4} + \frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4} + \frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}} + \frac{1}{32}\text{Subst}\left(\int \frac{}{1}\right.$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x} + \frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x} - \frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4}$$

$$+\frac{1}{24}(1-x)^{5/4}(1+x)^{3/4}+\frac{1}{6}(1-x)^{7/4}(1+x)^{5/4}+\frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4}+\frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}}-\frac{\log\left(1+\frac{\sqrt{1-x}}{\sqrt{1+x}}\right.}{32\sqrt{}}$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x} + \frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x} - \frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4}$$

$$+\frac{1}{24}(1-x)^{5/4}(1+x)^{3/4}+\frac{1}{6}(1-x)^{7/4}(1+x)^{5/4}+\frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4}+\frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}}-\frac{\arctan\left(1-\frac{\sqrt{2}}{}\right.}{16\sqrt{2}}$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x} + \frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x} - \frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4}$$

$$+\frac{1}{24}(1-x)^{5/4}(1+x)^{3/4}+\frac{1}{6}(1-x)^{7/4}(1+x)^{5/4}+\frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4}+\frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}}-\frac{\arctan\left(1-\frac{\sqrt{2}}{}\right.}{16\sqrt{2}}$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x} + \frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x} - \frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4}$$

$$+\frac{1}{24}(1-x)^{5/4}(1+x)^{3/4}+\frac{1}{6}(1-x)^{7/4}(1+x)^{5/4}+\frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4}+\frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}}-\frac{3\arctan\left(1-\right.}{8\sqrt{2}}$$

Mathematica [A] (verified)

Time = 11.24 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.52

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$$

$$= -\frac{1}{48}\sqrt{1+x}\sqrt[4]{1-x^2}\left(-7+2x+8x^2-\frac{\sqrt{1-x^2}(29+22x+8x^2)}{1+x}\right) + \frac{3\arctan\left(\frac{\sqrt{2}\sqrt{1+x}\sqrt[4]{1-x^2}}{1+x-\sqrt{1-x^2}}\right)-2\operatorname{arctanh}\left(\frac{1+x+\sqrt{1-x^2}}{\sqrt{2}\sqrt{1+x}\sqrt[4]{1-x^2}}\right)}{8\sqrt{2}}$$

```
[In] Integrate[(x^2*Sqrt[1 + x]*(1 - x^2)^(1/4))/(Sqrt[1 - x]*(Sqrt[1 - x] - Sqr
t[1 + x])),x]
```

```
[Out] -1/48*(Sqrt[1 + x]*(1 - x^2)^(1/4)*(-7 + 2*x + 8*x^2 - (Sqrt[1 - x^2]*(29 +
 22*x + 8*x^2))/(1 + x))) + (3*ArcTan[(Sqrt[2]*Sqrt[1 + x]*(1 - x^2)^(1/4))
/(1 + x - Sqrt[1 - x^2])] - 2*ArcTanh[(1 + x + Sqrt[1 - x^2])/(Sqrt[2]*Sqrt
[1 + x]*(1 - x^2)^(1/4))])/(8*Sqrt[2])
```

Maple [F]

$$\int \frac{x^2(-x^2+1)^{\frac{1}{4}}\sqrt{1+x}}{\sqrt{1-x}\,\left(\sqrt{1-x}-\sqrt{1+x}\right)}dx$$

```
[In] int(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/2)),x)
[Out] int(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/2)),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.21

$$\begin{aligned}
&\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx \\
&= -\frac{1}{48}\left(8\,x^2+2\,x-7\right)\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}+\frac{1}{48}\left(8\,x^2+22\,x+29\right)\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{-x+1} \\
&\quad+\left(\frac{1}{64}i+\frac{1}{64}\right)\sqrt{2}\log\left(\frac{\sqrt{2}((i+1)\,x+i+1)+2\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}}{x+1}\right) \\
&\quad-\left(\frac{1}{64}i-\frac{1}{64}\right)\sqrt{2}\log\left(\frac{\sqrt{2}(-(i-1)\,x-i+1)+2\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}}{x+1}\right) \\
&\quad+\left(\frac{1}{64}i-\frac{1}{64}\right)\sqrt{2}\log\left(\frac{\sqrt{2}((i-1)\,x+i-1)+2\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}}{x+1}\right) \\
&\quad-\left(\frac{1}{64}i+\frac{1}{64}\right)\sqrt{2}\log\left(\frac{\sqrt{2}(-(i+1)\,x-i-1)+2\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}}{x+1}\right) \\
&\quad+\left(\frac{5}{64}i+\frac{5}{64}\right)\sqrt{2}\log\left(\frac{\sqrt{2}((i+1)\,x-i-1)+2\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{-x+1}}{x-1}\right) \\
&\quad-\left(\frac{5}{64}i-\frac{5}{64}\right)\sqrt{2}\log\left(\frac{\sqrt{2}(-(i-1)\,x+i-1)+2\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{-x+1}}{x-1}\right) \\
&\quad+\left(\frac{5}{64}i-\frac{5}{64}\right)\sqrt{2}\log\left(\frac{\sqrt{2}((i-1)\,x-i+1)+2\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{-x+1}}{x-1}\right) \\
&\quad-\left(\frac{5}{64}i+\frac{5}{64}\right)\sqrt{2}\log\left(\frac{\sqrt{2}(-(i+1)\,x+i+1)+2\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{-x+1}}{x-1}\right)
\end{aligned}$$

```
[In] integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/
2)),x, algorithm="fricas")
```

```
[Out] -1/48*(8*x^2 + 2*x - 7)*(-x^2 + 1)^(1/4)*sqrt(x + 1) + 1/48*(8*x^2 + 22*x +
 29)*(-x^2 + 1)^(1/4)*sqrt(-x + 1) + (1/64*I + 1/64)*sqrt(2)*log((sqrt(2)*(
(I + 1)*x + I + 1) + 2*(-x^2 + 1)^(1/4)*sqrt(x + 1))/(x + 1)) - (1/64*I - 1
/64)*sqrt(2)*log((sqrt(2)*(-(I - 1)*x - I + 1) + 2*(-x^2 + 1)^(1/4)*sqrt(x
+ 1))/(x + 1)) + (1/64*I - 1/64)*sqrt(2)*log((sqrt(2)*((I - 1)*x + I - 1) +
 2*(-x^2 + 1)^(1/4)*sqrt(x + 1))/(x + 1)) - (1/64*I + 1/64)*sqrt(2)*log((sq
rt(2)*(-(I + 1)*x - I - 1) + 2*(-x^2 + 1)^(1/4)*sqrt(x + 1))/(x + 1)) + (5/
64*I + 5/64)*sqrt(2)*log((sqrt(2)*((I + 1)*x - I - 1) + 2*(-x^2 + 1)^(1/4)*
sqrt(-x + 1))/(x - 1)) - (5/64*I - 5/64)*sqrt(2)*log((sqrt(2)*(-(I - 1)*x +
 I - 1) + 2*(-x^2 + 1)^(1/4)*sqrt(-x + 1))/(x - 1)) + (5/64*I - 5/64)*sqrt(
2)*log((sqrt(2)*((I - 1)*x - I + 1) + 2*(-x^2 + 1)^(1/4)*sqrt(-x + 1))/(x -
 1)) - (5/64*I + 5/64)*sqrt(2)*log((sqrt(2)*(-(I + 1)*x + I + 1) + 2*(-x^2
+ 1)^(1/4)*sqrt(-x + 1))/(x - 1))
```

Sympy [F]

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx=\int \frac{x^2\sqrt[4]{-(x-1)(x+1)}\sqrt{x+1}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{x+1}\right)}\,dx$$

```
[In] integrate(x**2*(-x**2+1)**(1/4)*(1+x)**(1/2)/(1-x)**(1/2)/((1-x)**(1/2)-(1+
x)**(1/2)),x)
```

```
[Out] Integral(x**2*(-(x - 1)*(x + 1))**(1/4)*sqrt(x + 1)/(sqrt(1 - x)*(sqrt(1 -
x) - sqrt(x + 1))), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx=\int -\frac{(-x^2+1)^{\frac{1}{4}}\sqrt{x+1}x^2}{\sqrt{-x+1}\left(\sqrt{x+1}-\sqrt{-x+1}\right)}\,dx$$

```
[In] integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/
2)),x, algorithm="maxima")
```

```
[Out] -integrate((-x^2 + 1)^(1/4)*sqrt(x + 1)*x^2/(sqrt(-x + 1)*(sqrt(x + 1) - sq
rt(-x + 1))), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\,(\sqrt{1-x}-\sqrt{1+x})}\,dx = \int -\frac{(-x^2+1)^{\frac{1}{4}}\sqrt{x+1}x^2}{\sqrt{-x+1}(\sqrt{x+1}-\sqrt{-x+1})}\,dx$$

```
[In] integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/
2)),x, algorithm="giac")
```

```
[Out] integrate(-(-x^2 + 1)^(1/4)*sqrt(x + 1)*x^2/(sqrt(-x + 1)*(sqrt(x + 1) - sq
rt(-x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\,(\sqrt{1-x}-\sqrt{1+x})}\,dx = -\int \frac{x^2\,(1-x^2)^{1/4}\,\sqrt{x+1}}{(\sqrt{x+1}-\sqrt{1-x})\,\sqrt{1-x}}\,dx$$

```
[In] int(-(x^2*(1 - x^2)^(1/4)*(x + 1)^(1/2))/(((x + 1)^(1/2) - (1 - x)^(1/2))*(
1 - x)^(1/2)),x)
```

```
[Out] -int((x^2*(1 - x^2)^(1/4)*(x + 1)^(1/2))/(((x + 1)^(1/2) - (1 - x)^(1/2))*(
1 - x)^(1/2)), x)
```

3.222 $\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 56, antiderivative size = 292

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx =$$

$$-\frac{1}{12}(1-3x)(1-x)^{2/3}\sqrt[3]{1+x}+\frac{1}{4}\sqrt{1-x}x\sqrt{1+x}-\frac{1}{4}(1-x)(3+x)+\frac{1}{12}\sqrt[3]{1-x}(1+x)^{2/3}(1+3x)+\frac{1}{12}\sqrt[6]{1-x}(1$$

```
[Out] -1/12*(1-3*x)*(1-x)^(2/3)*(1+x)^(1/3)-1/4*(1-x)*(3+x)+1/12*(1-x)^(1/3)*(1+x
)^(2/3)*(1+3*x)+1/12*(1-x)^(1/6)*(1+x)^(5/6)*(2+3*x)-1/12*(1-x)^(5/6)*(1+x)
^(1/6)*(10+3*x)+1/6*arctan((1+x)^(1/6)/(1-x)^(1/6))-5/6*arctan(((1-x)^(1/3)
-(1+x)^(1/3))/(1-x)^(1/6)/(1+x)^(1/6))-4/9*arctan(1/3*((1-x)^(1/3)-2*(1+x)^
(1/3))/(1-x)^(1/3)*3^(1/2))*3^(1/2)+1/18*arctanh((1-x)^(1/6)*(1+x)^(1/6)*3^
(1/2)/((1-x)^(1/3)+(1+x)^(1/3)))*3^(1/2)+1/4*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.79, number of steps used = 46, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6820, 6874, 52, 62, 531, 201, 222, 689, 904, 65, 246, 215, 648, 632, 210, 642, 209, 26, 21, 338, 301}

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx = \frac{\arcsin(x)}{4}$$

$$-\frac{2}{3}\arctan\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{x+1}}\right)+\frac{2\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{3\sqrt{3}}+\frac{1}{3}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-x}}{\sqrt[6]{x+1}}\right)-\frac{1}{3}\arctan\left(\frac{2\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+\sqrt{\right.$$

```
[In] Int[(Sqrt[1 - x]*x*(1 + x)^(2/3))/(-((1 - x)^(5/6)*(1 + x)^(1/3)) + (1 - x)
^(2/3)*Sqrt[1 + x]),x]
```

```
[Out] x/2 + x^2/4 - (7*(1 - x)^(5/6)*(1 + x)^(1/6))/12 + ((1 - x)^(2/3)*(1 + x)^(
1/3))/6 - ((1 - x)^(5/3)*(1 + x)^(1/3))/4 + ((1 - x)^(1/3)*(1 + x)^(2/3))/3
 - ((1 - x)^(4/3)*(1 + x)^(2/3))/4 + (5*(1 - x)^(1/6)*(1 + x)^(5/6))/12 - (
(1 - x)^(7/6)*(1 + x)^(5/6))/4 - ((1 - x)^(5/6)*(1 + x)^(7/6))/4 + (x*Sqrt[
1 - x^2])/4 + ArcSin[x]/4 - (2*ArcTan[(1 - x)^(1/6)/(1 + x)^(1/6)])/3 + (2*
ArcTan[1/Sqrt[3] - (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))])/(3*Sqrt[3])
+ ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))/(1 + x)^(1/6)]/3 - ArcTan[Sqrt[3] + (2
*(1 - x)^(1/6))/(1 + x)^(1/6)]/3 - (2*ArcTan[1/Sqrt[3] - (2*(1 + x)^(1/3))/
(Sqrt[3]*(1 - x)^(1/3))])/(3*Sqrt[3]) - Log[1 - x]/9 + Log[1 + x]/9 + Log[1
 + (1 - x)^(1/3)/(1 + x)^(1/3)]/3 - Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) - (
Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(12*Sqrt[3]) + Log[1 + (1 - x)^(1/3)/
(1 + x)^(1/3) + (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(12*Sqrt[3]) - Log[1
 + (1 + x)^(1/3)/(1 - x)^(1/3)]/3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
```

```
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 689

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && E
qQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m
, 0]
```

Rule 904

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p, x
] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m, 0] &&  !
IGtQ[n, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x\sqrt[3]{1+x}}{-\sqrt[3]{1-x}+\sqrt[6]{1-x^2}}\,dx \\
&= \int \left(\frac{1}{2}(1-x)^{2/3}\sqrt[3]{1+x}+\frac{1}{2}\sqrt[3]{1-x}\sqrt[3]{1+x}\sqrt[6]{1-x^2}+\frac{1}{2}\sqrt[3]{1+x}\sqrt[3]{1-x^2}\right. \\
&\qquad \left.+\frac{\sqrt[3]{1+x}\sqrt{1-x^2}}{2\sqrt[3]{1-x}}+\frac{\sqrt[3]{1+x}\left(1-x^2\right)^{2/3}}{2(1-x)^{2/3}}-\frac{\sqrt[3]{1+x}\left(1-x^2\right)^{5/6}}{2(-1+x)}\right)dx \\
&= \frac{1}{2}\int (1-x)^{2/3}\sqrt[3]{1+x}\,dx+\frac{1}{2}\int \sqrt[3]{1-x}\sqrt[3]{1+x}\sqrt[6]{1-x^2}\,dx+\frac{1}{2}\int \sqrt[3]{1+x}\sqrt[3]{1-x^2}\,dx \\
&\quad +\frac{1}{2}\int \frac{\sqrt[3]{1+x}\sqrt{1-x^2}}{\sqrt[3]{1-x}}\,dx+\frac{1}{2}\int \frac{\sqrt[3]{1+x}\left(1-x^2\right)^{2/3}}{(1-x)^{2/3}}\,dx-\frac{1}{2}\int \frac{\sqrt[3]{1+x}\left(1-x^2\right)^{5/6}}{-1+x}\,dx \\
&= -\frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}+\frac{1}{6}\int \frac{(1-x)^{2/3}}{(1+x)^{2/3}}\,dx \\
&\quad +\frac{1}{2}\int \sqrt[3]{1-x}(1+x)^{2/3}\,dx+\frac{1}{2}\int \sqrt[6]{1-x}(1+x)^{5/6}\,dx+\frac{1}{2}\int (1+x)\,dx-\frac{1}{2}\int \frac{(1-x)^{5/6}(1+x)^{7/6}}{-1+x}\,dx
\end{aligned}
$$

$$= \frac{x}{2} + \frac{x^2}{4} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}$$

$$-\frac{1}{4}(1-x)^{4/3}(1+x)^{2/3} - \frac{1}{4}(1-x)^{7/6}(1+x)^{5/6} + \frac{1}{4}x\sqrt{1-x^2} + \frac{2}{9}\int \frac{1}{\sqrt[3]{1-x}(1+x)^{2/3}}\,dx + \frac{1}{4}\int \frac{1}{\sqrt{1}}$$

$$= \frac{x}{2} + \frac{x^2}{4} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x} + \frac{1}{3}\sqrt[3]{1-x}(1+x)^{2/3}$$

$$-\frac{1}{4}(1-x)^{4/3}(1+x)^{2/3} + \frac{5}{12}\sqrt[6]{1-x}(1+x)^{5/6} - \frac{1}{4}(1-x)^{7/6}(1+x)^{5/6} - \frac{1}{4}(1-x)^{5/6}(1+x)^{7/6} + \frac{1}{4}x\sqrt{1}$$

$$= \frac{x}{2} + \frac{x^2}{4} - \frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}$$

$$+\frac{1}{3}\sqrt[3]{1-x}(1+x)^{2/3} - \frac{1}{4}(1-x)^{4/3}(1+x)^{2/3} + \frac{5}{12}\sqrt[6]{1-x}(1+x)^{5/6} - \frac{1}{4}(1-x)^{7/6}(1+x)^{5/6} - \frac{1}{4}(1-x)^{5}$$

$$= \frac{x}{2} + \frac{x^2}{4} - \frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}$$

$$+\frac{1}{3}\sqrt[3]{1-x}(1+x)^{2/3} - \frac{1}{4}(1-x)^{4/3}(1+x)^{2/3} + \frac{5}{12}\sqrt[6]{1-x}(1+x)^{5/6} - \frac{1}{4}(1-x)^{7/6}(1+x)^{5/6} - \frac{1}{4}(1-x)^{5}$$

$$= \frac{x}{2} + \frac{x^2}{4} - \frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}$$

$$+\frac{1}{3}\sqrt[3]{1-x}(1+x)^{2/3} - \frac{1}{4}(1-x)^{4/3}(1+x)^{2/3} + \frac{5}{12}\sqrt[6]{1-x}(1+x)^{5/6} - \frac{1}{4}(1-x)^{7/6}(1+x)^{5/6} - \frac{1}{4}(1-x)^{5}$$

$$= \frac{x}{2} + \frac{x^2}{4} - \frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}$$

$$+\frac{1}{3}\sqrt[3]{1-x}(1+x)^{2/3} - \frac{1}{4}(1-x)^{4/3}(1+x)^{2/3} + \frac{5}{12}\sqrt[6]{1-x}(1+x)^{5/6} - \frac{1}{4}(1-x)^{7/6}(1+x)^{5/6} - \frac{1}{4}(1-x)^{5}$$

$$= \frac{x}{2} + \frac{x^2}{4} - \frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}$$

$$+\frac{1}{3}\sqrt[3]{1-x}(1+x)^{2/3} - \frac{1}{4}(1-x)^{4/3}(1+x)^{2/3} + \frac{5}{12}\sqrt[6]{1-x}(1+x)^{5/6} - \frac{1}{4}(1-x)^{7/6}(1+x)^{5/6} - \frac{1}{4}(1-x)^{5}$$

$$= \frac{x}{2} + \frac{x^2}{4} - \frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}$$

$$+\frac{1}{3}\sqrt[3]{1-x}(1+x)^{2/3} - \frac{1}{4}(1-x)^{4/3}(1+x)^{2/3} + \frac{5}{12}\sqrt[6]{1-x}(1+x)^{5/6} - \frac{1}{4}(1-x)^{7/6}(1+x)^{5/6} - \frac{1}{4}(1-x)^{5}$$

$$= \frac{x}{2} + \frac{x^2}{4} - \frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x} + \frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x} - \frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}$$

$$+\frac{1}{3}\sqrt[3]{1-x}(1+x)^{2/3} - \frac{1}{4}(1-x)^{4/3}(1+x)^{2/3} + \frac{5}{12}\sqrt[6]{1-x}(1+x)^{5/6} - \frac{1}{4}(1-x)^{7/6}(1+x)^{5/6} - \frac{1}{4}(1-x)^{5/}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 36.73 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx =$$

$$-\frac{1}{12}\sqrt[3]{1+x}\left((1-3x)(1-x)^{2/3} - \frac{3\sqrt[3]{1-x}x(2+x)}{\sqrt[3]{1-x^2}} - 3\sqrt[3]{1-x}x\sqrt[6]{1-x^2} - (1+3x)\sqrt[3]{1-x^2} - \frac{(2+3x)\sqrt{1-x^2}}{\sqrt[3]{1-x}}\right.$$

```
[In] Integrate[(Sqrt[1 - x]*x*(1 + x)^(2/3))/(-((1 - x)^(5/6)*(1 + x)^(1/3)) + (
1 - x)^(2/3)*Sqrt[1 + x]),x]
```

```
[Out] -1/12*((1 + x)^(1/3)*((1 - 3*x)*(1 - x)^(2/3) - (3*(1 - x)^(1/3)*x*(2 + x))
/(1 - x^2)^(1/3) - 3*(1 - x)^(1/3)*x*(1 - x^2)^(1/6) - (1 + 3*x)*(1 - x^2)^
(1/3) - ((2 + 3*x)*Sqrt[1 - x^2])/(1 - x)^(1/3) + ((10 + 3*x)*(1 - x^2)^(5/
6))/(1 + x) - 4*2^(2/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (1 + x)/2])) + (9*
ArcSin[x] + 14*ArcTan[(1 + x)^(1/3)/(1 - x^2)^(1/6)] + 7*(1 + I*Sqrt[3])*Ar
cTan[((1 - I*Sqrt[3])*(1 + x)^(1/3))/(2*(1 - x^2)^(1/6))] + 7*(1 - I*Sqrt[3
])*ArcTan[((1 + I*Sqrt[3])*(1 + x)^(1/3))/(2*(1 - x^2)^(1/6))] + 8*Sqrt[3]*
ArcTan[(1 - (2*(1 - x^2)^(1/3))/(1 + x)^(2/3))/Sqrt[3]] - (15*2^(5/6)*Sqrt[
1 - x^2]*Hypergeometric2F1[1/6, 1/6, 7/6, (1 - x)/2])/((1 - x)^(1/3)*Sqrt[1
 + x]) - 8*Log[(1 + x)^(2/3) + (1 - x^2)^(1/3)] + 4*Log[(1 + x)^(1/3) + x*(
1 + x)^(1/3) - (1 + x)^(2/3)*(1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/36
```

Maple [F]

$$\int \frac{x(1+x)^{\frac{2}{3}}\sqrt{1-x}}{-(1-x)^{\frac{5}{6}}(1+x)^{\frac{1}{3}}+(1-x)^{\frac{2}{3}}\sqrt{1+x}}dx$$

```
[In] int(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^(
1/2)),x)
```

```
[Out] int(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^(
1/2)),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 785, normalized size of antiderivative = 2.69

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(
1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*x^2 + 1/12*(3*x + 2)*(x + 1)^(5/6)*(-x + 1)^(1/6) + 1/12*(3*x + 1)*(x +
 1)^(2/3)*(-x + 1)^(1/3) + 1/4*sqrt(x + 1)*x*sqrt(-x + 1) + 1/12*(3*x - 1)*
(x + 1)^(1/3)*(-x + 1)^(2/3) - 1/12*(3*x + 10)*(x + 1)^(1/6)*(-x + 1)^(5/6)
 - 1/72*sqrt(2)*sqrt(25*I*sqrt(3) + 25)*log((sqrt(2)*(x + 1)*sqrt(25*I*sqrt
(3) + 25) + 10*(x + 1)^(5/6)*(-x + 1)^(1/6))/(x + 1)) + 1/72*sqrt(2)*sqrt(2
5*I*sqrt(3) + 25)*log(-(sqrt(2)*(x + 1)*sqrt(25*I*sqrt(3) + 25) - 10*(x + 1
)^(5/6)*(-x + 1)^(1/6))/(x + 1)) - 1/72*sqrt(2)*sqrt(-25*I*sqrt(3) + 25)*lo
g((sqrt(2)*(x + 1)*sqrt(-25*I*sqrt(3) + 25) + 10*(x + 1)^(5/6)*(-x + 1)^(1/
6))/(x + 1)) + 1/72*sqrt(2)*sqrt(-25*I*sqrt(3) + 25)*log(-(sqrt(2)*(x + 1)*
sqrt(-25*I*sqrt(3) + 25) - 10*(x + 1)^(5/6)*(-x + 1)^(1/6))/(x + 1)) - 1/72
*sqrt(2)*sqrt(49*I*sqrt(3) + 49)*log((sqrt(2)*(x - 1)*sqrt(49*I*sqrt(3) + 4
9) + 14*(x + 1)^(1/6)*(-x + 1)^(5/6))/(x - 1)) + 1/72*sqrt(2)*sqrt(49*I*sqr
t(3) + 49)*log(-(sqrt(2)*(x - 1)*sqrt(49*I*sqrt(3) + 49) - 14*(x + 1)^(1/6)
*(-x + 1)^(5/6))/(x - 1)) - 1/72*sqrt(2)*sqrt(-49*I*sqrt(3) + 49)*log((sqrt
(2)*(x - 1)*sqrt(-49*I*sqrt(3) + 49) + 14*(x + 1)^(1/6)*(-x + 1)^(5/6))/(x
- 1)) + 1/72*sqrt(2)*sqrt(-49*I*sqrt(3) + 49)*log(-(sqrt(2)*(x - 1)*sqrt(-4
9*I*sqrt(3) + 49) - 14*(x + 1)^(1/6)*(-x + 1)^(5/6))/(x - 1)) - 2/9*sqrt(3)
*arctan(-1/3*(sqrt(3)*(x + 1) - 2*sqrt(3)*(x + 1)^(2/3)*(-x + 1)^(1/3))/(x
+ 1)) - 2/9*sqrt(3)*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(x + 1)^(1/3)*(
-x + 1)^(2/3))/(x - 1)) + 1/2*x - 5/18*arctan((-x + 1)^(1/6)/(x + 1)^(1/6))
 - 7/18*arctan((x + 1)^(1/6)*(-x + 1)^(5/6)/(x - 1)) - 1/2*arctan((sqrt(x +
 1)*sqrt(-x + 1) - 1)/x) - 2/9*log((x + (x + 1)^(2/3)*(-x + 1)^(1/3) + 1)/(
x + 1)) + 1/9*log((x - (x + 1)^(2/3)*(-x + 1)^(1/3) + (x + 1)^(1/3)*(-x + 1
)^(2/3) + 1)/(x + 1)) - 1/9*log((x - (x + 1)^(2/3)*(-x + 1)^(1/3) + (x + 1)
^(1/3)*(-x + 1)^(2/3) - 1)/(x - 1)) + 2/9*log(-(x - (x + 1)^(1/3)*(-x + 1)^
(2/3) - 1)/(x - 1))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx = \text{Timed out}$$

```
[In] integrate(x*(1+x)**(2/3)*(1-x)**(1/2)/(-(1-x)**(5/6)*(1+x)**(1/3)+(1-x)**(2
/3)*(1+x)**(1/2)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx = \int \frac{(x+1)^{\frac{2}{3}}x\sqrt{-x+1}}{\sqrt{x+1}(-x+1)^{\frac{2}{3}}-(x+1)^{\frac{1}{3}}(-x+1)^{\frac{5}{6}}}\,dx$$

```
[In] integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(
1+x)^(1/2)),x, algorithm="maxima")
[Out] integrate((x + 1)^(2/3)*x*sqrt(-x + 1)/(sqrt(x + 1)*(-x + 1)^(2/3) - (x + 1
)^(1/3)*(-x + 1)^(5/6)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx = \text{Timed out}$$

```
[In] integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(
1+x)^(1/2)),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx = \int \frac{x\sqrt{1-x}\,(x+1)^{2/3}}{(1-x)^{2/3}\sqrt{x+1}-(1-x)^{5/6}\,(x+1)^{1/3}}\,dx$$

```
[In] int((x*(1 - x)^(1/2)*(x + 1)^(2/3))/((1 - x)^(2/3)*(x + 1)^(1/2) - (1 - x)^
(5/6)*(x + 1)^(1/3)),x)
[Out] int((x*(1 - x)^(1/2)*(x + 1)^(2/3))/((1 - x)^(2/3)*(x + 1)^(1/2) - (1 - x)^
(5/6)*(x + 1)^(1/3)), x)
```

3.223 $\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = -\frac{3(-1+x)(1+x)}{2\sqrt[3]{(-1+x)^4(1+x)^2}}$$

```
[Out] -3/2*(-1+x)*(1+x)/((-1+x)^4*(1+x)^2)^(1/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6851, 37}

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = \frac{3(1-x)(x+1)}{2\sqrt[3]{(1-x)^4(x+1)^2}}$$

```
[In] Int[((-1 + x)^4*(1 + x)^2)^(-1/3),x]
[Out] (3*(1 - x)*(1 + x))/(2*((1 - x)^4*(1 + x)^2)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
```

```
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left((-1+x)^{4/3}(1+x)^{2/3}\right)\int \frac{1}{(-1+x)^{4/3}(1+x)^{2/3}}\,dx}{\sqrt[3]{(-1+x)^4(1+x)^2}}\\ &= \frac{3(1-x)(1+x)}{2\sqrt[3]{(1-x)^4(1+x)^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = -\frac{3(-1+x)(1+x)}{2\sqrt[3]{(-1+x)^4(1+x)^2}}$$

```
[In] Integrate[((-1 + x)^4*(1 + x)^2)^(-1/3),x]
[Out] (-3*(-1 + x)*(1 + x))/(2*((-1 + x)^4*(1 + x)^2)^(1/3))
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{3(-1+x)(1+x)}{2\left((-1+x)^4(1+x)^2\right)^{\frac{1}{3}}}$	22
risch	$-\frac{3(-1+x)(1+x)}{2\left((-1+x)^4(1+x)^2\right)^{\frac{1}{3}}}$	22
trager	$-\frac{3\left(x^6-2x^5-x^4+4x^3-x^2-2x+1\right)^{\frac{2}{3}}}{2(-1+x)^3(1+x)}$	43

```
[In] int(1/((-1+x)^4*(1+x)^2)^(1/3),x,method=_RETURNVERBOSE)
[Out] -3/2*(-1+x)*(1+x)/((-1+x)^4*(1+x)^2)^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(21) = 42$.

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = -\frac{3\,(x^6-2\,x^5-x^4+4\,x^3-x^2-2\,x+1)^{\frac{2}{3}}}{2\,(x^4-2\,x^3+2\,x-1)}$$

```
[In] integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="fricas")
[Out] -3/2*(x^6 - 2*x^5 - x^4 + 4*x^3 - x^2 - 2*x + 1)^(2/3)/(x^4 - 2*x^3 + 2*x -
 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = \int \frac{1}{\sqrt[3]{\left(x-1\right)^4\left(x+1\right)^2}}\,dx$$

```
[In] integrate(1/((-1+x)**4*(1+x)**2)**(1/3),x)
[Out] Integral(((x - 1)**4*(x + 1)**2)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = \int \frac{1}{\left((x+1)^2(x-1)^4\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="maxima")
[Out] integrate(((x + 1)^2*(x - 1)^4)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = \int \frac{1}{\left((x+1)^2(x-1)^4\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="giac")
[Out] integrate(((x + 1)^2*(x - 1)^4)^(-1/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = -\frac{3\left((x-1)^4\,(x+1)^2\right)^{2/3}}{2\,(x-1)^3\,\,(x+1)}$$

```
[In] int(1/((x - 1)^4*(x + 1)^2)^(1/3),x)
[Out] -(3*((x - 1)^4*(x + 1)^2)^(2/3))/(2*(x - 1)^3*(x + 1))
```

3.224 $\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \frac{4(-1+x)(2+x)}{3\sqrt[4]{(-1+x)^3(2+x)^5}}$$

```
[Out] 4/3*(-1+x)*(2+x)/((-1+x)^3*(2+x)^5)^(1/4)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6851, 37}

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = -\frac{4(1-x)(x+2)}{3\sqrt[4]{-(1-x)^3(x+2)^5}}$$

```
[In] Int[((-1 + x)^3*(2 + x)^5)^(-1/4),x]
[Out] (-4*(1 - x)*(2 + x))/(3*(-((1 - x)^3*(2 + x)^5))^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
```

```
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\text{integral} = \frac{\left((-1+x)^{3/4}(2+x)^{5/4}\right)\int \frac{1}{(-1+x)^{3/4}(2+x)^{5/4}}\,dx}{\sqrt[4]{(-1+x)^3(2+x)^5}}$$
$$= -\frac{4(1-x)(2+x)}{3\sqrt[4]{-(1-x)^3(2+x)^5}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \frac{4(-1+x)(2+x)}{3\sqrt[4]{(-1+x)^3(2+x)^5}}$$

```
[In] Integrate[((-1 + x)^3*(2 + x)^5)^(-1/4),x]
[Out] (4*(-1 + x)*(2 + x))/(3*((-1 + x)^3*(2 + x)^5)^(1/4))
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{4(-1+x)(2+x)}{3\left((-1+x)^3(2+x)^5\right)^{\frac{1}{4}}}$	22
risch	$\frac{4(-1+x)(2+x)}{3\left((-1+x)^3(2+x)^5\right)^{\frac{1}{4}}}$	22
trager	$\frac{4\left(x^8+7x^7+13x^6-11x^5-50x^4-8x^3+64x^2+16x-32\right)^{\frac{3}{4}}}{3(-1+x)^2(2+x)^4}$	53

```
[In] int(1/((-1+x)^3*(2+x)^5)^(1/4),x,method=_RETURNVERBOSE)
[Out] 4/3*(-1+x)*(2+x)/((-1+x)^3*(2+x)^5)^(1/4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(21) = 42$.

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.76

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \frac{4\left(x^8+7x^7+13x^6-11x^5-50x^4-8x^3+64x^2+16x-32\right)^{\frac{3}{4}}}{3\left(x^6+6x^5+9x^4-8x^3-24x^2+16\right)}$$

```
[In] integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x, algorithm="fricas")
[Out] 4/3*(x^8 + 7*x^7 + 13*x^6 - 11*x^5 - 50*x^4 - 8*x^3 + 64*x^2 + 16*x - 32)^(
3/4)/(x^6 + 6*x^5 + 9*x^4 - 8*x^3 - 24*x^2 + 16)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \int \frac{1}{\sqrt[4]{(x-1)^3\,(x+2)^5}}\,dx$$

```
[In] integrate(1/((-1+x)**3*(2+x)**5)**(1/4),x)
[Out] Integral(((x - 1)**3*(x + 2)**5)**(-1/4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \int \frac{1}{\left((x+2)^5(x-1)^3\right)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x, algorithm="maxima")
[Out] integrate(((x + 2)^5*(x - 1)^3)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \int \frac{1}{\left((x+2)^5(x-1)^3\right)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x, algorithm="giac")
[Out] integrate(((x + 2)^5*(x - 1)^3)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \frac{4\left((x-1)^3\,(x+2)^5\right)^{3/4}}{3\,(x-1)^2\,(x+2)^4}$$

```
[In] int(1/((x - 1)^3*(x + 2)^5)^(1/4),x)
[Out] (4*((x - 1)^3*(x + 2)^5)^(3/4))/(3*(x - 1)^2*(x + 2)^4)
```

3.225 $\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx=-\frac{3(-1+x)(1+x)}{8\sqrt[3]{(-1+x)^7(1+x)^2}}+\frac{9(-1+x)^2(1+x)}{16\sqrt[3]{(-1+x)^7(1+x)^2}}$$

```
[Out] -3/8*(-1+x)*(1+x)/((-1+x)^7*(1+x)^2)^(1/3)+9/16*(-1+x)^2*(1+x)/((-1+x)^7*(1
+x)^2)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6851, 47, 37}

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx=\frac{9(x+1)(1-x)^2}{16\sqrt[3]{-(1-x)^7(x+1)^2}}+\frac{3(x+1)(1-x)}{8\sqrt[3]{-(1-x)^7(x+1)^2}}$$

```
[In] Int[((-1 + x)^7*(1 + x)^2)^(-1/3),x]
```

```
[Out] (3*(1 - x)*(1 + x))/(8*(-((1 - x)^7*(1 + x)^2))^(1/3)) + (9*(1 - x)^2*(1 +
x))/(16*(-((1 - x)^7*(1 + x)^2))^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left((-1+x)^{7/3}(1+x)^{2/3}\right)\int \frac{1}{(-1+x)^{7/3}(1+x)^{2/3}}\,dx}{\sqrt[3]{(-1+x)^7(1+x)^2}}\\
&= \frac{3(1-x)(1+x)}{8\sqrt[3]{-(1-x)^7(1+x)^2}} - \frac{\left(3(-1+x)^{7/3}(1+x)^{2/3}\right)\int \frac{1}{(-1+x)^{4/3}(1+x)^{2/3}}\,dx}{8\sqrt[3]{(-1+x)^7(1+x)^2}}\\
&= \frac{3(1-x)(1+x)}{8\sqrt[3]{-(1-x)^7(1+x)^2}} + \frac{9(1-x)^2(1+x)}{16\sqrt[3]{-(1-x)^7(1+x)^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.57

$$
\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \frac{3(-1+x)(1+x)(-5+3x)}{16\sqrt[3]{(-1+x)^7(1+x)^2}}
$$

```
[In] Integrate[((-1 + x)^7*(1 + x)^2)^(-1/3),x]
[Out] (3*(-1 + x)*(1 + x)*(-5 + 3*x))/(16*((-1 + x)^7*(1 + x)^2)^(1/3))
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.51

method	result	size
gosper	$\frac{3(-1+x)(1+x)(3x-5)}{16\left((-1+x)^{7}(1+x)^{2}\right)^{\frac{1}{3}}}$	27
risch	$\frac{3(-1+x)(3x^{2}-2x-5)}{16\left((-1+x)^{7}(1+x)^{2}\right)^{\frac{1}{3}}}$	29
trager	$\frac{3(3x-5)\left(x^{9}-5x^{8}+8x^{7}-14x^{5}+14x^{4}-8x^{2}+5x-1\right)^{\frac{2}{3}}}{16(-1+x)^{6}(1+x)}$	53

```
[In] int(1/((-1+x)^7*(1+x)^2)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/16*(-1+x)*(1+x)*(3*x-5)/((-1+x)^7*(1+x)^2)^(1/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \frac{3\left(x^{9}-5x^{8}+8x^{7}-14x^{5}+14x^{4}-8x^{2}+5x-1\right)^{\frac{2}{3}}(3x-5)}{16\left(x^{7}-5x^{6}+9x^{5}-5x^{4}-5x^{3}+9x^{2}-5x+1\right)}$$

```
[In] integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="fricas")
[Out] 3/16*(x^9 - 5*x^8 + 8*x^7 - 14*x^5 + 14*x^4 - 8*x^2 + 5*x - 1)^(2/3)*(3*x -
 5)/(x^7 - 5*x^6 + 9*x^5 - 5*x^4 - 5*x^3 + 9*x^2 - 5*x + 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \int \frac{1}{\sqrt[3]{(x-1)^{7}(x+1)^{2}}}\,dx$$

```
[In] integrate(1/((-1+x)**7*(1+x)**2)**(1/3),x)
[Out] Integral(((x - 1)**7*(x + 1)**2)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \int \frac{1}{\left((x+1)^2(x-1)^7\right)^{\frac{1}{3}}}\,dx$$

[In] integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="maxima")

[Out] integrate(((x + 1)^2*(x - 1)^7)^(-1/3), x)

Giac [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \int \frac{1}{\left((x+1)^2(x-1)^7\right)^{\frac{1}{3}}}\,dx$$

[In] integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="giac")

[Out] integrate(((x + 1)^2*(x - 1)^7)^(-1/3), x)

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \frac{3\,(3\,x-5)\,\left((x-1)^7\,(x+1)^2\right)^{2/3}}{16\,(x-1)^6\,(x+1)}$$

[In] int(1/((x - 1)^7*(x + 1)^2)^(1/3),x)

[Out] (3*(3*x - 5)*((x - 1)^7*(x + 1)^2)^(2/3))/(16*(x - 1)^6*(x + 1))

3.226 $\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \sqrt{3}\arctan\left(\frac{1+\frac{2(-1+x)}{\sqrt[3]{(-1+x)^2(1+x)}}}{\sqrt{3}}\right) - \frac{1}{2}\log(1+x) - \frac{3}{2}\log\left(1-\frac{-1+x}{\sqrt[3]{(-1+x)^2(1+x)}}\right)$$

```
[Out] -1/2*ln(1+x)-3/2*ln(1+(1-x)/((-1+x)^2*(1+x))^(1/3))+arctan(1/3*(1+2*(-1+x)/
((-1+x)^2*(1+x))^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 188 vs. 2(67) = 134.

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.81, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2092, 2089, 62}

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = -\frac{(3-3x)^{2/3}\sqrt[3]{x+1}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x+1}}{\sqrt[6]{3}\sqrt[3]{3-3x}}\right)}{\sqrt[6]{3}\sqrt[3]{x^3-x^2-x+1}} - \frac{(3-3x)^{2/3}\sqrt[3]{x+1}\log\left(-\frac{8}{3}(x-1)\right)}{2\ 3^{2/3}\sqrt[3]{x^3-x^2-x+1}} - \frac{\sqrt[3]{3}(3-3x)^{2/3}\sqrt[3]{x+1}\log\left(\frac{\sqrt[3]{3}\sqrt[3]{x+1}}{\sqrt[3]{3-3x}}+1\right)}{2\sqrt[3]{x^3-x^2-x+1}}$$

```
[In] Int[((-1 + x)^2*(1 + x))^(-1/3),x]
```

```
[Out] -(((3 - 3*x)^(2/3)*(1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2*(1 + x)^(1/3))/(3^(1
/6)*(3 - 3*x)^(1/3))])/(3^(1/6)*(1 - x - x^2 + x^3)^(1/3))) - ((3 - 3*x)^(2
/3)*(1 + x)^(1/3)*Log[(-8*(-1 + x))/3])/(2*3^(2/3)*(1 - x - x^2 + x^3)^(1/3
)) - (3^(1/3)*(3 - 3*x)^(2/3)*(1 + x)^(1/3)*Log[1 + (3^(1/3)*(1 + x)^(1/3))
/(3 - 3*x)^(1/3)])/(2*(1 - x - x^2 + x^3)^(1/3))
```

Rule 62

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 2089

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\sqrt[3]{\frac{16}{27}-\frac{4x}{3}+x^3}}\,dx, x, -\frac{1}{3}+x\right)$$

$$= \frac{\left(4\ 2^{2/3}(1-x)^{2/3}\sqrt[3]{1+x}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{16}{9}-\frac{8x}{3}\right)^{2/3}\sqrt[3]{\frac{16}{9}+\frac{4x}{3}}}\,dx, x, -\frac{1}{3}+x\right)}{3\sqrt[3]{1-x-x^2+x^3}}$$

$$= -\frac{\sqrt{3}(1-x)^{2/3}\sqrt[3]{1+x}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1+x}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{\sqrt[3]{1-x-x^2+x^3}} - \frac{(1-x)^{2/3}\sqrt[3]{1+x}\log(1-x)}{2\sqrt[3]{1-x-x^2+x^3}} - \frac{3(1-x)^{2/3}\sqrt[3]{1+x}\log\left(\frac{\sqrt[3]{1-x}+\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\right)}{2\sqrt[3]{1-x-x^2+x^3}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \frac{(-1+x)^{2/3}\sqrt[3]{1+x}\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1+x}}{2\sqrt[3]{-1+x}+\sqrt[3]{1+x}}\right)-2\log\left(\sqrt[3]{-1+x}-\sqrt[3]{1+x}\right)+\log\left((-1+x\right.\right.}{2\sqrt[3]{(-1+x)^2(1+x)}}$$

```
[In] Integrate[((-1 + x)^2*(1 + x))^(-1/3),x]
```

```
[Out] ((-1 + x)^(2/3)*(1 + x)^(1/3)*(-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 + x)^(1/3))/(2
*(-1 + x)^(1/3) + (1 + x)^(1/3))] - 2*Log[(-1 + x)^(1/3) - (1 + x)^(1/3)] +
 Log[(-1 + x)^(2/3) + (-1 + x)^(1/3)*(1 + x)^(1/3) + (1 + x)^(2/3)]))/(2*((
-1 + x)^2*(1 + x))^(1/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.53 (sec) , antiderivative size = 370, normalized size of antiderivative = 5.52

method	result
trager	$-\ln\left(\frac{4\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2 x^2+3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3-x^2-x+1\right)^{\frac{2}{3}}-3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3-x^2-x+1\right)^{\frac{1}{3}}x\right.$

```
[In] int(1/((-1+x)^2*(1+x))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -ln((4*RootOf(_Z^2-_Z+1)^2*x^2+3*RootOf(_Z^2-_Z+1)*(x^3-x^2-x+1)^(2/3)-3*Ro
otOf(_Z^2-_Z+1)*(x^3-x^2-x+1)^(1/3)*x-4*RootOf(_Z^2-_Z+1)^2*x-4*RootOf(_Z^2
-_Z+1)*x^2+3*RootOf(_Z^2-_Z+1)*(x^3-x^2-x+1)^(1/3)+3*(x^3-x^2-x+1)^(1/3)*x+
2*RootOf(_Z^2-_Z+1)*x+x^2-3*(x^3-x^2-x+1)^(1/3)+2*RootOf(_Z^2-_Z+1)-1)/(-1+
x))+RootOf(_Z^2-_Z+1)*ln(-(2*RootOf(_Z^2-_Z+1)^2*x^2+3*RootOf(_Z^2-_Z+1)*(x
^3-x^2-x+1)^(2/3)-2*RootOf(_Z^2-_Z+1)^2*x-5*RootOf(_Z^2-_Z+1)*x^2-3*(x^3-x^
2-x+1)^(2/3)+3*(x^3-x^2-x+1)^(1/3)*x+6*RootOf(_Z^2-_Z+1)*x+2*x^2-3*(x^3-x^2
-x+1)^(1/3)-RootOf(_Z^2-_Z+1)-4*x+2)/(-1+x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(56) = 112$.

Time = 0.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = -\sqrt{3}\arctan\left(\frac{\sqrt{3}(x-1)+2\sqrt{3}(x^3-x^2-x+1)^{\frac{1}{3}}}{3\,(x-1)}\right) + \frac{1}{2}\log\left(\frac{x^2+(x^3-x^2-x+1)^{\frac{1}{3}}(x-1)-2\,x+(x^3-x^2-x+1)^{\frac{2}{3}}+1}{x^2-2\,x+1}\right) - \log\left(-\frac{x-(x^3-x^2-x+1)^{\frac{1}{3}}-1}{x-1}\right)$$

```
[In] integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(x^3 - x^2 - x + 1)^(1/3))
/(x - 1)) + 1/2*log((x^2 + (x^3 - x^2 - x + 1)^(1/3)*(x - 1) - 2*x + (x^3 -
 x^2 - x + 1)^(2/3) + 1)/(x^2 - 2*x + 1)) - log(-(x - (x^3 - x^2 - x + 1)^(
1/3) - 1)/(x - 1))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \int \frac{1}{\sqrt[3]{(x-1)^2\,(x+1)}}\,dx$$

```
[In] integrate(1/((-1+x)**2*(1+x))**(1/3),x)
```

```
[Out] Integral(((x - 1)**2*(x + 1))**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \int \frac{1}{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)*(x - 1)^2)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \int \frac{1}{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="giac")
[Out] integrate(((x + 1)*(x - 1)^2)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \int \frac{1}{\left((x-1)^2\,(x+1)\right)^{1/3}}\,dx$$

```
[In] int(1/((x - 1)^2*(x + 1))^(1/3),x)
[Out] int(1/((x - 1)^2*(x + 1))^(1/3), x)
```

3.227 $\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 122

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx=-\frac{4(-2+x)(1+x)}{3\sqrt{(-2+x)(1+x)^3}}+\frac{2\sqrt{-2+x}(1+x)^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{-2+x}}{\sqrt{3}}\right)}{\sqrt{(-2+x)(1+x)^3}}-\frac{\sqrt{2}\sqrt{-2+x}(1+x)^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{1+x}}{\sqrt{-2+x}}\right)}{\sqrt{(-2+x)(1+x)^3}}$$

```
[Out] -4/3*(-2+x)*(1+x)/((-2+x)*(1+x)^3)^(1/2)+2*(1+x)^(3/2)*arcsinh(1/3*(-2+x)^(
1/2)*3^(1/2))*(-2+x)^(1/2)/((-2+x)*(1+x)^3)^(1/2)-(1+x)^(3/2)*arctan(2^(1/2
)*(1+x)^(1/2)/(-2+x)^(1/2))*2^(1/2)*(-2+x)^(1/2)/((-2+x)*(1+x)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1607, 6851, 1628, 21, 132, 56, 221, 95, 210}

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx=\frac{2\sqrt{x-2}(x+1)^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{x-2}}{\sqrt{3}}\right)}{\sqrt{-\left((2-x)(x+1)^3\right)}}-\frac{\sqrt{2}\sqrt{x-2}(x+1)^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{x+1}}{\sqrt{x-2}}\right)}{\sqrt{-\left((2-x)(x+1)^3\right)}}+\frac{4(2-x)(x+1)}{3\sqrt{-\left((2-x)(x+1)^3\right)}}$$

```
[In] Int[(x^(-1) + x)/Sqrt[(-2 + x)*(1 + x)^3],x]
```

```
[Out] (4*(2 - x)*(1 + x))/(3*Sqrt[-((2 - x)*(1 + x)^3)]) + (2*Sqrt[-2 + x]*(1 + x
)^(3/2)*ArcSinh[Sqrt[-2 + x]/Sqrt[3]])/Sqrt[-((2 - x)*(1 + x)^3)] - (Sqrt[2
]*Sqrt[-2 + x]*(1 + x)^(3/2)*ArcTan[(Sqrt[2]*Sqrt[1 + x])/Sqrt[-2 + x]])/Sq
rt[-((2 - x)*(1 + x)^3)]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1+x^2}{x\sqrt{(-2+x)(1+x)^3}}\,dx \\
&= \frac{\left(\sqrt{-2+x}(1+x)^{3/2}\right)\int \frac{1+x^2}{\sqrt{-2+x}x(1+x)^{3/2}}\,dx}{\sqrt{(-2+x)(1+x)^3}} \\
&= \frac{4(2-x)(1+x)}{3\sqrt{-\left((2-x)(1+x)^3\right)}} - \frac{\left(2\sqrt{-2+x}(1+x)^{3/2}\right)\int \frac{-\frac{3}{2}-\frac{3x}{2}}{\sqrt{-2+x}x\sqrt{1+x}}\,dx}{3\sqrt{(-2+x)(1+x)^3}} \\
&= \frac{4(2-x)(1+x)}{3\sqrt{-\left((2-x)(1+x)^3\right)}} + \frac{\left(\sqrt{-2+x}(1+x)^{3/2}\right)\int \frac{\sqrt{1+x}}{\sqrt{-2+x}x}\,dx}{\sqrt{(-2+x)(1+x)^3}} \\
&= \frac{4(2-x)(1+x)}{3\sqrt{-\left((2-x)(1+x)^3\right)}} + \frac{\left(\sqrt{-2+x}(1+x)^{3/2}\right)\int \frac{1}{\sqrt{-2+x}\sqrt{1+x}}\,dx}{\sqrt{(-2+x)(1+x)^3}} \\
&\quad + \frac{\left(\sqrt{-2+x}(1+x)^{3/2}\right)\int \frac{1}{\sqrt{-2+x}x\sqrt{1+x}}\,dx}{\sqrt{(-2+x)(1+x)^3}}
\end{aligned}
$$

$$= \frac{4(2-x)(1+x)}{3\sqrt{-((2-x)(1+x)^3)}} + \frac{\left(2\sqrt{-2+x}(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{1}{-1-2x^2}\,dx, x, \frac{\sqrt{1+x}}{\sqrt{-2+x}}\right)}{\sqrt{(-2+x)(1+x)^3}} + \frac{\left(2\sqrt{-2+x}(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sqrt{-2+x}\right)}{\sqrt{(-2+x)(1+x)^3}}$$

$$= \frac{4(2-x)(1+x)}{3\sqrt{-((2-x)(1+x)^3)}} + \frac{2\sqrt{-2+x}(1+x)^{3/2}\text{arcsinh}\left(\frac{\sqrt{-2+x}}{\sqrt{3}}\right)}{\sqrt{-((2-x)(1+x)^3)}} - \frac{\sqrt{2}\sqrt{-2+x}(1+x)^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{1+x}}{\sqrt{-2+x}}\right)}{\sqrt{-((2-x)(1+x)^3)}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx = -\frac{(1+x)\left(-8+4x-3\sqrt{2}\sqrt{-2+x}\sqrt{1+x}\arctan\left(\frac{\sqrt{\frac{-2+x}{1+x}}}{\sqrt{2}}\right)-6\sqrt{-2+x}\sqrt{1+x}\text{arctanh}\left(\sqrt{\frac{-2+x}{1+x}}\right)\right)}{3\sqrt{(-2+x)(1+x)^3}}$$

```
[In] Integrate[(x^(-1) + x)/Sqrt[(-2 + x)*(1 + x)^3],x]
```

```
[Out] -1/3*((1 + x)*(-8 + 4*x - 3*Sqrt[2]*Sqrt[-2 + x]*Sqrt[1 + x]*ArcTan[Sqrt[(-
2 + x)/(1 + x)]/Sqrt[2]] - 6*Sqrt[-2 + x]*Sqrt[1 + x]*ArcTanh[Sqrt[(-2 + x)
/(1 + x)]]))/Sqrt[(-2 + x)*(1 + x)^3]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{4(-2+x)(1+x)}{3\sqrt{(-2+x)(1+x)^{3}}}+\frac{\left(\ln\left(x-\frac{1}{2}+\sqrt{x^{2}-x-2}\right)+\frac{\sqrt{2}\,\arctan\left(\frac{(-4-x)\sqrt{2}}{4\sqrt{x^{2}-x-2}}\right)}{2}\right)(1+x)\sqrt{(1+x)(-2+x)}}{\sqrt{(-2+x)(1+x)^{3}}}$
default	$-\frac{\left(3\sqrt{2}\,\arctan\left(\frac{(4+x)\sqrt{2}}{4\sqrt{x^{2}-x-2}}\right)x+3\sqrt{2}\,\arctan\left(\frac{(4+x)\sqrt{2}}{4\sqrt{x^{2}-x-2}}\right)-6\ln\left(x-\frac{1}{2}+\sqrt{x^{2}-x-2}\right)x-6\ln\left(x-\frac{1}{2}+\sqrt{x^{2}-x-2}\right)+8\sqrt{x^{2}-x-2}\right)\sqrt{(1+}}{6\sqrt{(-2+x)(1+x)^{3}}}$
trager	$-\frac{4\sqrt{x^{4}+x^{3}-3x^{2}-5x-2}}{3(1+x)^{2}}-\frac{\operatorname{RootOf}\left(\_Z^{2}+2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}+2\right)x^{2}+5\operatorname{RootOf}\left(\_Z^{2}+2\right)x+4\operatorname{RootOf}\left(\_Z^{2}+2\right)-4\sqrt{x^{4}+x^{3}-3x^{2}}}{x(1+x)}\right.}{2}$

[In] int((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x,method=_RETURNVERBOSE)

[Out] -4/3*(-2+x)*(1+x)/((-2+x)*(1+x)^3)^(1/2)+(ln(x-1/2+(x^2-x-2)^(1/2))+1/2*2^(1/2)*arctan(1/4*(-4-x)*2^(1/2)/(x^2-x-2)^(1/2)))/((-2+x)*(1+x)^3)^(1/2)*(1+x)*((1+x)*(-2+x))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.16

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx$$
$$=\frac{3\sqrt{2}(x^2+2x+1)\arctan\left(-\frac{\sqrt{2}(x^2+x)-\sqrt{2}\sqrt{x^4+x^3-3x^2-5x-2}}{2(x+1)}\right)-4x^2-3(x^2+2x+1)\log\left(-\frac{2x^2+x-2\sqrt{x^4+x^3-}}{x+1}\right.}{3(x^2+2x+1)}$$

[In] integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="fricas")

[Out] 1/3*(3*sqrt(2)*(x^2 + 2*x + 1)*arctan(-1/2*(sqrt(2)*(x^2 + x) - sqrt(2)*sqrt(x^4 + x^3 - 3*x^2 - 5*x - 2))/(x + 1)) - 4*x^2 - 3*(x^2 + 2*x + 1)*log(-(2*x^2 + x - 2*sqrt(x^4 + x^3 - 3*x^2 - 5*x - 2) - 1)/(x + 1)) - 8*x - 4*sqrt(x^4 + x^3 - 3*x^2 - 5*x - 2) - 4)/(x^2 + 2*x + 1)

Sympy [F]

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx=\int \frac{x^2+1}{x\sqrt{(x-2)\,(x+1)^3}}\,dx$$

[In] integrate((1/x+x)/((-2+x)*(1+x)**3)**(1/2),x)

[Out] Integral((x**2 + 1)/(x*sqrt((x - 2)*(x + 1)**3)), x)

Maxima [F]

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx=\int \frac{x+\frac{1}{x}}{\sqrt{(x+1)^3(x-2)}}\,dx$$

[In] integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="maxima")

[Out] integrate((x + 1/x)/sqrt((x + 1)^3*(x - 2)), x)

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.68

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx=\frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}(x-\sqrt{x^2-x-2})\right)}{\operatorname{sgn}(x+1)}-\frac{\log\left(\left|-2x+2\sqrt{x^2-x-2}+1\right|\right)}{\operatorname{sgn}(x+1)}-\frac{4}{(x-\sqrt{x^2-x-2}+1)\operatorname{sgn}(x+1)}$$

[In] integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="giac")

[Out] sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 - x - 2)))/sgn(x + 1) - log(abs(-2*x + 2*sqrt(x^2 - x - 2) + 1))/sgn(x + 1) - 4/((x - sqrt(x^2 - x - 2) + 1)*sgn(x + 1))

Mupad [F(-1)]

Timed out.

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx=\int \frac{x+\frac{1}{x}}{\sqrt{(x+1)^3\,(x-2)}}\,dx$$

```
[In] int((x + 1/x)/((x + 1)^3*(x - 2))^(1/2),x)
[Out] int((x + 1/x)/((x + 1)^3*(x - 2))^(1/2), x)
```

3.228 $\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 150

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = -\frac{\sqrt[3]{(-1+x)^2(1+x)}}{x} - \frac{\arctan\left(\frac{1-\frac{2(-1+x)}{\sqrt[3]{(-1+x)^2(1+x)}}}{\sqrt{3}}\right)}{\sqrt{3}} - \sqrt{3}\arctan\left(\frac{1+\frac{2(-1+x)}{\sqrt[3]{(-1+x)^2(1+x)}}}{\sqrt{3}}\right) + \frac{\log(x)}{6} - \frac{2}{3}\log(1+x) - \frac{3}{2}\log\left(1-\frac{-1+x}{\sqrt[3]{(-1+x)^2(1+x)}}\right) - \frac{1}{2}\log\left(1+\frac{-1+x}{\sqrt[3]{(-1+x)^2(1+x)}}\right)$$

```
[Out] -((-1+x)^2*(1+x))^(1/3)/x+1/6*ln(x)-2/3*ln(1+x)-3/2*ln(1+(1-x)/((-1+x)^2*(1
+x))^(1/3))-1/2*ln(1+(-1+x)/((-1+x)^2*(1+x))^(1/3))-1/3*arctan(1/3*(1-2*(-1
+x)/((-1+x)^2*(1+x))^(1/3))*3^(1/2))*3^(1/2)-arctan(1/3*(1+2*(-1+x)/((-1+x)
^2*(1+x))^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 404 vs. 2(150) = 300.

Time = 0.23 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.69, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2106, 2102, 99, 163, 62, 93}

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = -\frac{3\sqrt[6]{3}\sqrt[3]{x^3-x^2-x+1}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{3-3x}}{3^{5/6}\sqrt[3]{x+1}}\right)}{(3-3x)^{2/3}\sqrt[3]{x+1}} - \frac{\sqrt[6]{3}\sqrt[3]{x^3-x^2-x+1}\arctan\left(\frac{2\sqrt[3]{3-3x}}{3^{5/6}\sqrt[3]{x+1}}+\frac{1}{\sqrt{3}}\right)}{(3-3x)^{2/3}\sqrt[3]{x+1}} - \frac{\sqrt[3]{x^3-x^2-x+1}}{x} + \frac{\sqrt[3]{x^3-x^2-x+1}\log(x)}{2\sqrt[3]{3}(3-3x)^{2/3}\sqrt[3]{x+1}} - \frac{3^{2/3}\sqrt[3]{x^3-x^2-x+1}\log\left(\frac{4(x+1)}{3}\right)}{2(3-3x)^{2/3}\sqrt[3]{x+1}} - \frac{3\,3^{2/3}\sqrt[3]{x^3-x^2-x+1}\log\left(\frac{\sqrt[3]{3-3x}}{\sqrt[3]{3}\sqrt[3]{x+1}}+1\right)}{2(3-3x)^{2/3}\sqrt[3]{x+1}} - \frac{3^{2/3}\sqrt[3]{x^3-x^2-x+1}\log\left(\left(\frac{2}{3}\right)^{2/3}\sqrt[3]{3-3x}-\frac{2^{2/3}\sqrt[3]{x+1}}{\sqrt[3]{3}}\right)}{2(3-3x)^{2/3}\sqrt[3]{x+1}}$$

```
[In] Int[((-1 + x)^2*(1 + x))^(1/3)/x^2,x]
```

```
[Out] -((1 - x - x^2 + x^3)^(1/3)/x) - (3*3^(1/6)*(1 - x - x^2 + x^3)^(1/3)*ArcTa
n[1/Sqrt[3] - (2*(3 - 3*x)^(1/3))/(3^(5/6)*(1 + x)^(1/3))])/((3 - 3*x)^(2/3
)*(1 + x)^(1/3)) - (3^(1/6)*(1 - x - x^2 + x^3)^(1/3)*ArcTan[1/Sqrt[3] + (2
*(3 - 3*x)^(1/3))/(3^(5/6)*(1 + x)^(1/3))])/((3 - 3*x)^(2/3)*(1 + x)^(1/3))
 + ((1 - x - x^2 + x^3)^(1/3)*Log[x])/(2*3^(1/3)*(3 - 3*x)^(2/3)*(1 + x)^(1
/3)) - (3^(2/3)*(1 - x - x^2 + x^3)^(1/3)*Log[(4*(1 + x))/3])/(2*(3 - 3*x)^
(2/3)*(1 + x)^(1/3)) - (3*3^(2/3)*(1 - x - x^2 + x^3)^(1/3)*Log[1 + (3 - 3*
x)^(1/3)/(3^(1/3)*(1 + x)^(1/3))])/(2*(3 - 3*x)^(2/3)*(1 + x)^(1/3)) - (3^(
2/3)*(1 - x - x^2 + x^3)^(1/3)*Log[(2/3)^(2/3)*(3 - 3*x)^(1/3) - (2^(2/3)*(
1 + x)^(1/3))/3^(1/3)])/(2*(3 - 3*x)^(2/3)*(1 + x)^(1/3))
```

Rule 62

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
```

```
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 93

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqr
t[3])*q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))
]/(d*e - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*
(Log[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{
a, b, c, d, e, f}, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 2102

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Dist[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int
[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, e
, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2106

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\sqrt[3]{\frac{16}{27}-\frac{4x}{3}+x^3}}{\left(\frac{1}{3}+x\right)^2}\,dx, x, -\frac{1}{3}+x\right)$$

$$= \frac{\left(3\sqrt[3]{1-x-x^2+x^3}\right)\text{Subst}\left(\int \frac{\left(\frac{16}{9}-\frac{8x}{3}\right)^{2/3}\sqrt[3]{\frac{16}{9}+\frac{4x}{3}}}{\left(\frac{1}{3}+x\right)^2}\,dx, x, -\frac{1}{3}+x\right)}{4\ 2^{2/3}(1-x)^{2/3}\sqrt[3]{1+x}}$$

$$= -\frac{\sqrt[3]{1-x-x^2+x^3}}{x} + \frac{\left(3\sqrt[3]{1-x-x^2+x^3}\right)\text{Subst}\left(\int \frac{-\frac{64}{27}-\frac{32x}{9}}{\sqrt[3]{\frac{16}{9}-\frac{8x}{3}}\left(\frac{1}{3}+x\right)\left(\frac{16}{9}+\frac{4x}{3}\right)^{2/3}}\,dx, x, -\frac{1}{3}+x\right)}{4\ 2^{2/3}(1-x)^{2/3}\sqrt[3]{1+x}}$$

$$= -\frac{\sqrt[3]{1-x-x^2+x^3}}{x} - \frac{\left(4\sqrt[3]{2}\sqrt[3]{1-x-x^2+x^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{\frac{16}{9}-\frac{8x}{3}}\left(\frac{1}{3}+x\right)\left(\frac{16}{9}+\frac{4x}{3}\right)^{2/3}}\,dx, x, -\frac{1}{3}+x\right)}{9(1-x)^{2/3}\sqrt[3]{1+x}} - \frac{\left(4\sqrt[3]{2}\sqrt[3]{1-x-x^2+x^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{\frac{16}{9}-\frac{8x}{3}}\left(\frac{16}{9}+\frac{4x}{3}\right)^{2/3}}\,dx, x, -\frac{1}{3}+x\right)}{3(1-x)^{2/3}\sqrt[3]{1+x}}$$

$$= -\frac{\sqrt[3]{1-x-x^2+x^3}}{x} - \frac{\sqrt{3}\sqrt[3]{1-x-x^2+x^3}\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{(1-x)^{2/3}\sqrt[3]{1+x}}$$
$$- \frac{\sqrt[3]{1-x-x^2+x^3}\arctan\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{\sqrt{3}(1-x)^{2/3}\sqrt[3]{1+x}} + \frac{\sqrt[3]{1-x-x^2+x^3}\log(x)}{6(1-x)^{2/3}\sqrt[3]{1+x}}$$
$$- \frac{\sqrt[3]{1-x-x^2+x^3}\log(1+x)}{2(1-x)^{2/3}\sqrt[3]{1+x}} - \frac{\sqrt[3]{1-x-x^2+x^3}\log\left(\sqrt[3]{1-x}-\sqrt[3]{1+x}\right)}{2(1-x)^{2/3}\sqrt[3]{1+x}}$$
$$- \frac{3\sqrt[3]{1-x-x^2+x^3}\log\left(\frac{\sqrt[3]{1-x}+\sqrt[3]{1+x}}{\sqrt[3]{1+x}}\right)}{2(1-x)^{2/3}\sqrt[3]{1+x}}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = -\frac{(-1+x)^{4/3}(1+x)^{2/3}\left(18(-1+x)^{2/3}\sqrt[3]{1+x} - 6\sqrt{3}x\arctan\left(\frac{1-\frac{2}{\sqrt[3]{\frac{-1+x}{1+x}}}}{\sqrt{3}}\right) - 18\sqrt{3}x\arctan\left(\frac{1+\cdots}{\cdots}\right)\cdots\right)}{\cdots}$$

```
[In] Integrate[((-1 + x)^2*(1 + x))^(1/3)/x^2,x]
```

```
[Out] -1/18*((-1 + x)^(4/3)*(1 + x)^(2/3)*(18*(-1 + x)^(2/3)*(1 + x)^(1/3) - 6*Sq
rt[3]*x*ArcTan[(1 - 2/((-1 + x)/(1 + x))^(1/3))/Sqrt[3]] - 18*Sqrt[3]*x*Arc
Tan[(1 + 2/((-1 + x)/(1 + x))^(1/3))/Sqrt[3]] - 10*x*Log[2/(-1 + x)] - 3*x*
Log[1 + ((-1 + x)/(1 + x))^(-2/3) - ((-1 + x)/(1 + x))^(-1/3)] + 28*x*Log[-
1 + ((-1 + x)/(1 + x))^(-1/3)] + 6*x*Log[1 + ((-1 + x)/(1 + x))^(-1/3)] + x
*Log[1 + ((-1 + x)/(1 + x))^(-2/3) + ((-1 + x)/(1 + x))^(-1/3)]))/(x*((-1 +
 x)^2*(1 + x))^(2/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.17 (sec) , antiderivative size = 1374, normalized size of antiderivative = 9.16

method	result	size
trager	Expression too large to display	1374
risch	Expression too large to display	1846

```
[In] int(((-1+x)^2*(1+x))^(1/3)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(x^3-x^2-x+1)^(1/3)/x-1/3*ln(-(17542263127-67143384127*x+473641104429*x^5-
473641104429*x^4-315760736286*x^3+365361857286*x^2-148740584544*RootOf(4*_Z
^2-2*_Z+1)*x^3-20825761472*RootOf(4*_Z^2-2*_Z+1)*x^2+161302980208*RootOf(4*
_Z^2-2*_Z+1)*x+11734778880*RootOf(4*_Z^2-2*_Z+1)^2*x^5-11734778880*RootOf(4
*_Z^2-2*_Z+1)^2*x^4+223110876816*RootOf(4*_Z^2-2*_Z+1)*x^5-223110876816*Roo
tOf(4*_Z^2-2*_Z+1)*x^4+434621440*RootOf(4*_Z^2-2*_Z+1)^2+8263365808*RootOf(
4*_Z^2-2*_Z+1)+941414819418*(x^3-x^2-x+1)^(2/3)*RootOf(4*_Z^2-2*_Z+1)*x^3-1
170393085674*(x^3-x^2-x+1)^(1/3)*RootOf(4*_Z^2-2*_Z+1)*x^4-313804939806*(x^
3-x^2-x+1)^(2/3)*RootOf(4*_Z^2-2*_Z+1)*x^2+780262057116*(x^3-x^2-x+1)^(1/3)
*RootOf(4*_Z^2-2*_Z+1)*x^3-104601646602*RootOf(4*_Z^2-2*_Z+1)*(x^3-x^2-x+1)
^(2/3)*x+520174704744*(x^3-x^2-x+1)^(1/3)*RootOf(4*_Z^2-2*_Z+1)*x^2-8669578
4124*RootOf(4*_Z^2-2*_Z+1)*(x^3-x^2-x+1)^(1/3)*x-7823185920*RootOf(4*_Z^2-2
*_Z+1)^2*x^3-70191362560*RootOf(4*_Z^2-2*_Z+1)^2*x^2+77579927040*RootOf(4*_
Z^2-2*_Z+1)^2*x+4240338264*(x^3-x^2-x+1)^(1/3)-21673946031*(x^3-x^2-x+1)^(2
/3)+8480676528*(x^3-x^2-x+1)^(1/3)*x-585196542837*(x^3-x^2-x+1)^(2/3)*x^3+1
95065514279*(x^3-x^2-x+1)^(2/3)*x^2+65021838093*x*(x^3-x^2-x+1)^(2/3)+11448
9133128*(x^3-x^2-x+1)^(1/3)*x^4-76326088752*(x^3-x^2-x+1)^(1/3)*x^3+3486721
5534*RootOf(4*_Z^2-2*_Z+1)*(x^3-x^2-x+1)^(2/3)-50884059168*(x^3-x^2-x+1)^(1
/3)*x^2-43347892062*RootOf(4*_Z^2-2*_Z+1)*(x^3-x^2-x+1)^(1/3))/x/(-1+x))+2/
3*RootOf(4*_Z^2-2*_Z+1)*ln((18071341751+27297150257*x+487926227277*x^5-4879
26227277*x^4-325284151518*x^3+279915659510*x^2+803220480540*RootOf(4*_Z^2-2
*_Z+1)*x^3-503909876268*RootOf(4*_Z^2-2*_Z+1)*x^2-254687244242*RootOf(4*_Z^
2-2*_Z+1)*x+68875270272*RootOf(4*_Z^2-2*_Z+1)^2*x^5-68875270272*RootOf(4*_Z
^2-2*_Z+1)^2*x^4-1204830720810*RootOf(4*_Z^2-2*_Z+1)*x^5+1204830720810*Root
Of(4*_Z^2-2*_Z+1)*x^4+2550935936*RootOf(4*_Z^2-2*_Z+1)^2-44623360030*RootOf
(4*_Z^2-2*_Z+1)+941414819418*(x^3-x^2-x+1)^(2/3)*RootOf(4*_Z^2-2*_Z+1)*x^3+
228978266256*(x^3-x^2-x+1)^(1/3)*RootOf(4*_Z^2-2*_Z+1)*x^4-313804939806*(x^
3-x^2-x+1)^(2/3)*RootOf(4*_Z^2-2*_Z+1)*x^2-152652177504*(x^3-x^2-x+1)^(1/3)
*RootOf(4*_Z^2-2*_Z+1)*x^3-104601646602*RootOf(4*_Z^2-2*_Z+1)*(x^3-x^2-x+1)
^(2/3)*x-101768118336*(x^3-x^2-x+1)^(1/3)*RootOf(4*_Z^2-2*_Z+1)*x^2+1696135
3056*RootOf(4*_Z^2-2*_Z+1)*(x^3-x^2-x+1)^(1/3)*x-45916846848*RootOf(4*_Z^2-
2*_Z+1)^2*x^3-411976153664*RootOf(4*_Z^2-2*_Z+1)^2*x^2+455342064576*RootOf(
4*_Z^2-2*_Z+1)^2*x-21673946031*(x^3-x^2-x+1)^(1/3)+4240338264*(x^3-x^2-x+1)
^(2/3)-43347892062*(x^3-x^2-x+1)^(1/3)*x+114489133128*(x^3-x^2-x+1)^(2/3)*x
```

```
^3-38163044376*(x^3-x^2-x+1)^(2/3)*x^2-12721014792*x*(x^3-x^2-x+1)^(2/3)-58
5196542837*(x^3-x^2-x+1)^(1/3)*x^4+390131028558*(x^3-x^2-x+1)^(1/3)*x^3+348
67215534*RootOf(4*_Z^2-2*_Z+1)*(x^3-x^2-x+1)^(2/3)+260087352372*(x^3-x^2-x+
1)^(1/3)*x^2+8480676528*RootOf(4*_Z^2-2*_Z+1)*(x^3-x^2-x+1)^(1/3))/x/(-1+x)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(126) = 252$.

Time = 0.24 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = \frac{6\sqrt{3}x\arctan\left(\frac{\sqrt{3}(x-1)+2\sqrt{3}(x^3-x^2-x+1)^{\frac{1}{3}}}{3(x-1)}\right) - 2\sqrt{3}x\arctan\left(-\frac{\sqrt{3}(x-1)-2\sqrt{3}(x^3-x^2-x+1)^{\frac{1}{3}}}{3(x-1)}\right) + 3x\log\left(\frac{x^2+(x^3\cdots}{\cdots}\right)\cdots}{\cdots}$$

```
[In] integrate(((-1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*sqrt(3)*x*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(x^3 - x^2 - x + 1
)^(1/3))/(x - 1)) - 2*sqrt(3)*x*arctan(-1/3*(sqrt(3)*(x - 1) - 2*sqrt(3)*(x
^3 - x^2 - x + 1)^(1/3))/(x - 1)) + 3*x*log((x^2 + (x^3 - x^2 - x + 1)^(1/3
)*(x - 1) - 2*x + (x^3 - x^2 - x + 1)^(2/3) + 1)/(x^2 - 2*x + 1)) + x*log((
x^2 - (x^3 - x^2 - x + 1)^(1/3)*(x - 1) - 2*x + (x^3 - x^2 - x + 1)^(2/3) +
 1)/(x^2 - 2*x + 1)) - 2*x*log((x + (x^3 - x^2 - x + 1)^(1/3) - 1)/(x - 1))
 - 6*x*log(-(x - (x^3 - x^2 - x + 1)^(1/3) - 1)/(x - 1)) - 6*(x^3 - x^2 - x
 + 1)^(1/3))/x
```

Sympy [F]

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = \int \frac{\sqrt[3]{(x-1)^2\,(x+1)}}{x^2}\,dx$$

```
[In] integrate(((-1+x)**2*(1+x))**(1/3)/x**2,x)
```

```
[Out] Integral(((x - 1)**2*(x + 1))**(1/3)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = \int \frac{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}{x^2}\,dx$$

```
[In] integrate(((-1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="maxima")
[Out] integrate(((x + 1)*(x - 1)^2)^(1/3)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = \int \frac{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}{x^2}\,dx$$

```
[In] integrate(((-1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="giac")
[Out] integrate(((x + 1)*(x - 1)^2)^(1/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = \int \frac{\left((x-1)^2\,(x+1)\right)^{1/3}}{x^2}\,dx$$

```
[In] int(((x - 1)^2*(x + 1))^(1/3)/x^2,x)
[Out] int(((x - 1)^2*(x + 1))^(1/3)/x^2, x)
```

3.229 $\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \frac{1-x}{12(-3-2x+x^2)^{3/2}} - \frac{1-x}{24\sqrt{-3-2x+x^2}}$$

```
[Out] 1/12*(1-x)/(x^2-2*x-3)^(3/2)+1/24*(-1+x)/(x^2-2*x-3)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {628, 627}

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \frac{1-x}{12(x^2-2x-3)^{3/2}} - \frac{1-x}{24\sqrt{x^2-2x-3}}$$

```
[In] Int[(-3 - 2*x + x^2)^(-5/2),x]
```

```
[Out] (1 - x)/(12*(-3 - 2*x + x^2)^(3/2)) - (1 - x)/(24*Sqrt[-3 - 2*x + x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1-x}{12\left(-3-2x+x^2\right)^{3/2}}-\frac{1}{6}\int\frac{1}{\left(-3-2x+x^2\right)^{3/2}}\,dx\\ &= \frac{1-x}{12\left(-3-2x+x^2\right)^{3/2}}-\frac{1-x}{24\sqrt{-3-2x+x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int\frac{1}{\left(-3-2x+x^2\right)^{5/2}}\,dx=\frac{\sqrt{-3-2x+x^2}(5-3x-3x^2+x^3)}{24(-3+x)^2(1+x)^2}$$

```
[In] Integrate[(-3 - 2*x + x^2)^(-5/2),x]
[Out] (Sqrt[-3 - 2*x + x^2]*(5 - 3*x - 3*x^2 + x^3))/(24*(-3 + x)^2*(1 + x)^2)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.60

method	result	size
trager	$\frac{x^3-3x^2-3x+5}{24(x^2-2x-3)^{\frac{3}{2}}}$	26
risch	$\frac{x^3-3x^2-3x+5}{24(x^2-2x-3)^{\frac{3}{2}}}$	26
gosper	$\frac{(1+x)(-3+x)(x^3-3x^2-3x+5)}{24(x^2-2x-3)^{\frac{5}{2}}}$	32
default	$-\frac{-2+2x}{24(x^2-2x-3)^{\frac{3}{2}}}+\frac{-2+2x}{48\sqrt{x^2-2x-3}}$	36

```
[In] int(1/(x^2-2*x-3)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/24*(x^3-3*x^2-3*x+5)/(x^2-2*x-3)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(31) = 62$.

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \frac{x^4-4\,x^3-2\,x^2+(x^3-3\,x^2-3\,x+5)\sqrt{x^2-2\,x-3}+12\,x+9}{24\,(x^4-4\,x^3-2\,x^2+12\,x+9)}$$

```
[In] integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="fricas")
[Out] 1/24*(x^4 - 4*x^3 - 2*x^2 + (x^3 - 3*x^2 - 3*x + 5)*sqrt(x^2 - 2*x - 3) + 1
2*x + 9)/(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)
```

Sympy [F]

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \int \frac{1}{(x^2-2x-3)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(x**2-2*x-3)**(5/2),x)
[Out] Integral((x**2 - 2*x - 3)**(-5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \frac{x}{24\,\sqrt{x^2-2\,x-3}} - \frac{1}{24\,\sqrt{x^2-2\,x-3}} - \frac{x}{12\,(x^2-2\,x-3)^{\frac{3}{2}}} + \frac{1}{12\,(x^2-2\,x-3)^{\frac{3}{2}}}$$

```
[In] integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="maxima")
[Out] 1/24*x/sqrt(x^2 - 2*x - 3) - 1/24/sqrt(x^2 - 2*x - 3) - 1/12*x/(x^2 - 2*x -
 3)^(3/2) + 1/12/(x^2 - 2*x - 3)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.53

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \frac{((x-3)x-3)x+5}{24\,(x^2-2\,x-3)^{\frac{3}{2}}}$$

```
[In] integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="giac")
[Out] 1/24*(((x - 3)*x - 3)*x + 5)/(x^2 - 2*x - 3)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = -\frac{(4\,x-4)\,\left(-8\,x^2+16\,x+40\right)}{768\,(x^2-2\,x-3)^{3/2}}$$

```
[In] int(1/(x^2 - 2*x - 3)^(5/2),x)
[Out] -((4*x - 4)*(16*x - 8*x^2 + 40))/(768*(x^2 - 2*x - 3)^(3/2))
```

3.230 $\int \frac{1}{\sqrt{9+3x-5x^2+x^3}} dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx=\frac{(3-x)\sqrt{1+x}\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{2}\right)}{\sqrt{9+3x-5x^2+x^3}}$$

```
[Out] (3-x)*arctanh(1/2*(1+x)^(1/2))*(1+x)^(1/2)/(x^3-5*x^2+3*x+9)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2092, 2089, 65, 212}

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx=\frac{(3-x)\sqrt{x+1}\operatorname{arctanh}\left(\frac{\sqrt{x+1}}{2}\right)}{\sqrt{x^3-5x^2+3x+9}}$$

```
[In] Int[1/Sqrt[9 + 3*x - 5*x^2 + x^3],x]
```

```
[Out] ((3 - x)*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/Sqrt[9 + 3*x - 5*x^2 + x^3]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2089

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{\frac{128}{27}-\frac{16x}{3}+x^3}}\,dx, x, -\frac{5}{3}+x\right)\\
&= \frac{\left(128(3-x)\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)\sqrt{\frac{128}{9}+\frac{16x}{3}}}\,dx, x, -\frac{5}{3}+x\right)}{3\sqrt{3}\sqrt{9+3x-5x^2+x^3}}\\
&= \frac{\left(16(3-x)\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{\frac{128}{3}-2x^2}\,dx, x, \frac{4\sqrt{1+x}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{9+3x-5x^2+x^3}}\\
&= \frac{(3-x)\sqrt{1+x}\,\text{arctanh}\left(\frac{\sqrt{1+x}}{2}\right)}{\sqrt{9+3x-5x^2+x^3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$
\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx = -\frac{(-3+x)\sqrt{1+x}\,\text{arctanh}\left(\frac{\sqrt{1+x}}{2}\right)}{\sqrt{(-3+x)^2(1+x)}}
$$

```
[In] Integrate[1/Sqrt[9 + 3*x - 5*x^2 + x^3],x]
[Out] -(((-3 + x)*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/Sqrt[(-3 + x)^2*(1 + x)])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
trager	$-\frac{\ln\left(\frac{x^2+4\sqrt{x^3-5x^2+3x+9}+2x-15}{(-3+x)^2}\right)}{2}$	35
default	$\frac{(-3+x)\sqrt{1+x}\,(\ln(\sqrt{1+x}-2)-\ln(\sqrt{1+x}+2))}{2\sqrt{x^3-5x^2+3x+9}}$	45

```
[In] int(1/(x^3-5*x^2+3*x+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*ln((x^2+4*(x^3-5*x^2+3*x+9)^(1/2)+2*x-15)/(-3+x)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx = -\frac{1}{2}\log\left(\frac{2\,x+\sqrt{x^3-5\,x^2+3\,x+9}-6}{x-3}\right) + \frac{1}{2}\log\left(-\frac{2\,x-\sqrt{x^3-5\,x^2+3\,x+9}-6}{x-3}\right)$$

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="fricas")
[Out] -1/2*log((2*x + sqrt(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3)) + 1/2*log(-(2*x -
 sqrt(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3))
```

Sympy [F]

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-5x^2+3x+9}}\,dx$$

```
[In] integrate(1/(x**3-5*x**2+3*x+9)**(1/2),x)
[Out] Integral(1/sqrt(x**3 - 5*x**2 + 3*x + 9), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-5\,x^2+3\,x+9}}\,dx$$

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(x^3 - 5*x^2 + 3*x + 9), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx = -\frac{\log\left(\sqrt{x+1}+2\right)}{2\,\mathrm{sgn}\left(x-3\right)} + \frac{\log\left(\left|\sqrt{x+1}-2\right|\right)}{2\,\mathrm{sgn}\left(x-3\right)}$$

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="giac")
[Out] -1/2*log(sqrt(x + 1) + 2)/sgn(x - 3) + 1/2*log(abs(sqrt(x + 1) - 2))/sgn(x
- 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-5\,x^2+3\,x+9}}\,dx$$

```
[In] int(1/(3*x - 5*x^2 + x^3 + 9)^(1/2),x)
[Out] int(1/(3*x - 5*x^2 + x^3 + 9)^(1/2), x)
```

3.231 $\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 139

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx=\frac{(3-x)(1+x)}{8\left(9+3x-5x^2+x^3\right)^{3/2}}+\frac{5(3-x)^2(1+x)}{64\left(9+3x-5x^2+x^3\right)^{3/2}}-\frac{15(3-x)^3(1+x)}{256\left(9+3x-5x^2+x^3\right)^{3/2}}+\frac{15(3-x)^3(1+x)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{2}\right)}{512\left(9+3x-5x^2+x^3\right)^{3/2}}$$

```
[Out] 1/8*(3-x)*(1+x)/(x^3-5*x^2+3*x+9)^(3/2)+5/64*(3-x)^2*(1+x)/(x^3-5*x^2+3*x+9
)^(3/2)-15/256*(3-x)^3*(1+x)/(x^3-5*x^2+3*x+9)^(3/2)+15/512*(3-x)^3*(1+x)^(
3/2)*arctanh(1/2*(1+x)^(1/2))/(x^3-5*x^2+3*x+9)^(3/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2092, 2089, 44, 53, 65, 212}

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx=\frac{15(x+1)^{3/2}(3-x)^3\operatorname{arctanh}\left(\frac{\sqrt{x+1}}{2}\right)}{512\left(x^3-5x^2+3x+9\right)^{3/2}}-\frac{15(x+1)(3-x)^3}{256\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{5(x+1)(3-x)^2}{64\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{(x+1)(3-x)}{8\left(x^3-5x^2+3x+9\right)^{3/2}}$$

```
[In] Int[(9 + 3*x - 5*x^2 + x^3)^(-3/2),x]
```

```
[Out] ((3 - x)*(1 + x))/(8*(9 + 3*x - 5*x^2 + x^3)^(3/2)) + (5*(3 - x)^2*(1 + x))
/(64*(9 + 3*x - 5*x^2 + x^3)^(3/2)) - (15*(3 - x)^3*(1 + x))/(256*(9 + 3*x
```

```
- 5*x^2 + x^3)^(3/2)) + (15*(3 - x)^3*(1 + x)^(3/2)*ArcTanh[Sqrt[1 + x]/2])
/(512*(9 + 3*x - 5*x^2 + x^3)^(3/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2089

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\left(\frac{128}{27}-\frac{16x}{3}+x^3\right)^{3/2}}\,dx, x, -\frac{5}{3}+x\right)$$

$$= \frac{\left(2097152(3-x)^3(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^3\left(\frac{128}{9}+\frac{16x}{3}\right)^{3/2}}\,dx, x, -\frac{5}{3}+x\right)}{81\sqrt{3}\left(9+3x-5x^2+x^3\right)^{3/2}}$$

$$= \frac{(3-x)(1+x)}{8\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{\left(20480(3-x)^3(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^2\left(\frac{128}{9}+\frac{16x}{3}\right)^{3/2}}\,dx, x, -\frac{5}{3}+x\right)}{27\sqrt{3}\left(9+3x-5x^2+x^3\right)^{3/2}}$$

$$= \frac{(3-x)(1+x)}{8\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{5(3-x)^2(1+x)}{64\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{\left(80(3-x)^3(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)\left(\frac{128}{9}+\frac{16x}{3}\right)^{3/2}}\,dx, x, -\frac{5}{3}+x\right)}{3\sqrt{3}\left(9+3x-5x^2+x^3\right)^{3/2}}$$

$$= \frac{(3-x)(1+x)}{8\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{5(3-x)^2(1+x)}{64\left(9+3x-5x^2+x^3\right)^{3/2}} - \frac{15(3-x)^3(1+x)}{256\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{\left(5(3-x)^3(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)\sqrt{\frac{128}{9}+\frac{16x}{3}}}\,dx, x, -\frac{5}{3}+x\right)}{4\sqrt{3}\left(9+3x-5x^2+x^3\right)^{3/2}}$$

$$= \frac{(3-x)(1+x)}{8\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{5(3-x)^2(1+x)}{64\left(9+3x-5x^2+x^3\right)^{3/2}} - \frac{15(3-x)^3(1+x)}{256\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{\left(5\sqrt{3}(3-x)^3(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\frac{128}{3}-2x^2}\,dx, x, \frac{4\sqrt{1+x}}{\sqrt{3}}\right)}{32\left(9+3x-5x^2+x^3\right)^{3/2}}$$

$$= \frac{(3-x)(1+x)}{8\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{5(3-x)^2(1+x)}{64\left(9+3x-5x^2+x^3\right)^{3/2}} - \frac{15(3-x)^3(1+x)}{256\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{15(3-x)^3(1+x)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+x}}{2}\right)}{512\left(9+3x-5x^2+x^3\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.42

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = \frac{86-140x+30x^2-15(-3+x)^2\sqrt{1+x}\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{2}\right)}{512(-3+x)\sqrt{(-3+x)^2(1+x)}}$$

```
[In] Integrate[(9 + 3*x - 5*x^2 + x^3)^(-3/2),x]
[Out] (86 - 140*x + 30*x^2 - 15*(-3 + x)^2*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/(5
12*(-3 + x)*Sqrt[(-3 + x)^2*(1 + x)])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.51

method	result
risch	$\frac{15x^2-70x+43}{256(-3+x)\sqrt{(1+x)(-3+x)^2}}+\frac{\left(\frac{15\ln(\sqrt{1+x}-2)}{1024}-\frac{15\ln(\sqrt{1+x}+2)}{1024}\right)\sqrt{1+x}\,(-3+x)}{\sqrt{(1+x)(-3+x)^2}}$
trager	$\frac{(15x^2-70x+43)\sqrt{x^3-5x^2+3x+9}}{256(-3+x)^3(1+x)}+\frac{15\ln\left(\frac{-x^2+4\sqrt{x^3-5x^2+3x+9}-2x+15}{(-3+x)^2}\right)}{1024}$
default	$\frac{(-3+x)^3(1+x)\left(15(1+x)^{\frac{5}{2}}\ln(\sqrt{1+x}-2)-15(1+x)^{\frac{5}{2}}\ln(\sqrt{1+x}+2)-120(1+x)^{\frac{3}{2}}\ln(\sqrt{1+x}-2)+120(1+x)^{\frac{3}{2}}\ln(\sqrt{1+x}+2)+240\ln(\sqrt{1-\right.}{1024(\sqrt{1+x}+2)^2(\sqrt{1+x}-2)^2(x^3-5x^2+3x+9)^{\frac{3}{2}}}$

```
[In] int(1/(x^3-5*x^2+3*x+9)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/256*(15*x^2-70*x+43)/(-3+x)/((1+x)*(-3+x)^2)^(1/2)+(15/1024*ln((1+x)^(1/2
)-2)-15/1024*ln((1+x)^(1/2)+2))/((1+x)*(-3+x)^2)^(1/2)*(1+x)^(1/2)*(-3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx =$$

$$-\frac{15\,(x^4-8\,x^3+18\,x^2-27)\log\left(\frac{2\,x+\sqrt{x^3-5\,x^2+3\,x+9}-6}{x-3}\right)-15\,(x^4-8\,x^3+18\,x^2-27)\log\left(-\frac{2\,x-\sqrt{x^3-5\,x^2+3\,x+9}}{x-3}\right.}{1024\,(x^4-8\,x^3+18\,x^2-27)}$$

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1024*(15*(x^4 - 8*x^3 + 18*x^2 - 27)*log((2*x + sqrt(x^3 - 5*x^2 + 3*x +
 9) - 6)/(x - 3)) - 15*(x^4 - 8*x^3 + 18*x^2 - 27)*log(-(2*x - sqrt(x^3 - 5
*x^2 + 3*x + 9) - 6)/(x - 3)) - 4*sqrt(x^3 - 5*x^2 + 3*x + 9)*(15*x^2 - 70*
x + 43))/(x^4 - 8*x^3 + 18*x^2 - 27)
```

Sympy [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = \int \frac{1}{(x^3-5x^2+3x+9)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(x**3-5*x**2+3*x+9)**(3/2),x)
[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="maxima")
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = -\frac{15\log\left(\sqrt{x+1}+2\right)}{1024\,\mathrm{sgn}\,(x-3)} + \frac{15\log\left(\left|\sqrt{x+1}-2\right|\right)}{1024\,\mathrm{sgn}\,(x-3)} + \frac{1}{32\sqrt{x+1}\mathrm{sgn}\,(x-3)} + \frac{7(x+1)^{\frac{3}{2}}-36\sqrt{x+1}}{256\,(x-3)^2\mathrm{sgn}\,(x-3)}$$

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="giac")
[Out] -15/1024*log(sqrt(x + 1) + 2)/sgn(x - 3) + 15/1024*log(abs(sqrt(x + 1) - 2)
)/sgn(x - 3) + 1/32/(sqrt(x + 1)*sgn(x - 3)) + 1/256*(7*(x + 1)^(3/2) - 36*
sqrt(x + 1))/((x - 3)^2*sgn(x - 3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{3/2}}\,dx=\int \frac{1}{{\left(x^3-5\,x^2+3\,x+9\right)}^{3/2}} \,d x$$

```
[In] int(1/(3*x - 5*x^2 + x^3 + 9)^(3/2),x)
[Out] int(1/(3*x - 5*x^2 + x^3 + 9)^(3/2), x)
```

3.232 $\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 75

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx=\sqrt{3}\arctan\left(\frac{1+\frac{2(-3+x)}{\sqrt[3]{9+3x-5x^2+x^3}}}{\sqrt{3}}\right)-\frac{1}{2}\log(1+x)-\frac{3}{2}\log\left(1-\frac{-3+x}{\sqrt[3]{9+3x-5x^2+x^3}}\right)$$

```
[Out] -1/2*ln(1+x)-3/2*ln(1+(3-x)/(x^3-5*x^2+3*x+9)^(1/3))+arctan(1/3*(1+2*(-3+x)
/(x^3-5*x^2+3*x+9)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 188 vs. 2(75) = 150.

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.51, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2092, 2089, 62}

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx=-\frac{(9-3x)^{2/3}\sqrt[3]{x+1}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x+1}}{\sqrt[6]{3}\sqrt[3]{9-3x}}\right)}{\sqrt[6]{3}\sqrt[3]{x^3-5x^2+3x+9}}-\frac{(9-3x)^{2/3}\sqrt[3]{x+1}\log\left(-\frac{32}{3}(x-3)\right)}{2\,3^{2/3}\sqrt[3]{x^3-5x^2+3x+9}}-\frac{\sqrt[3]{3}(9-3x)^{2/3}\sqrt[3]{x+1}\log\left(\frac{\sqrt[3]{3}\sqrt[3]{x+1}}{\sqrt[3]{9-3x}}+1\right)}{2\sqrt[3]{x^3-5x^2+3x+9}}$$

```
[In] Int[(9 + 3*x - 5*x^2 + x^3)^(-1/3),x]
```

```
[Out] -(((9 - 3*x)^(2/3)*(1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2*(1 + x)^(1/3))/(3^(1
/6)*(9 - 3*x)^(1/3))])/(3^(1/6)*(9 + 3*x - 5*x^2 + x^3)^(1/3))) - ((9 - 3*x
)^(2/3)*(1 + x)^(1/3)*Log[(-32*(-3 + x))/3])/(2*3^(2/3)*(9 + 3*x - 5*x^2 +
x^3)^(1/3)) - (3^(1/3)*(9 - 3*x)^(2/3)*(1 + x)^(1/3)*Log[1 + (3^(1/3)*(1 +
x)^(1/3))/(9 - 3*x)^(1/3)])/(2*(9 + 3*x - 5*x^2 + x^3)^(1/3))
```

Rule 62

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 2089

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\sqrt[3]{\frac{128}{27}-\frac{16x}{3}+x^3}}\,dx, x, -\frac{5}{3}+x\right)$$

$$=\frac{\left(16\ 2^{2/3}(3-x)^{2/3}\sqrt[3]{1+x}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^{2/3}\sqrt[3]{\frac{128}{9}+\frac{16x}{3}}}\,dx, x, -\frac{5}{3}+x\right)}{3\sqrt[3]{9+3x-5x^2+x^3}}$$

$$= -\frac{\sqrt{3}(3-x)^{2/3}\sqrt[3]{1+x}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1+x}}{\sqrt{3}\sqrt[3]{3-x}}\right)}{\sqrt[3]{9+3x-5x^2+x^3}} - \frac{(3-x)^{2/3}\sqrt[3]{1+x}\log(3-x)}{2\sqrt[3]{9+3x-5x^2+x^3}} - \frac{3(3-x)^{2/3}\sqrt[3]{1+x}\log\left(\frac{\sqrt[3]{3-x}+\sqrt[3]{1+x}}{\sqrt[3]{3-x}}\right)}{2\sqrt[3]{9+3x-5x^2+x^3}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \frac{(-3+x)^{2/3}\sqrt[3]{1+x}\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1+x}}{2\sqrt[3]{-3+x}+\sqrt[3]{1+x}}\right)-2\log\left(\sqrt[3]{-3+x}-\sqrt[3]{1+x}\right)+\log\left((-3+x\right.\right.}{2\sqrt[3]{(-3+x)^2(1+x)}}$$

```
[In] Integrate[(9 + 3*x - 5*x^2 + x^3)^(-1/3),x]
```

```
[Out] ((-3 + x)^(2/3)*(1 + x)^(1/3)*(-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 + x)^(1/3))/(2
*(-3 + x)^(1/3) + (1 + x)^(1/3))] - 2*Log[(-3 + x)^(1/3) - (1 + x)^(1/3)] +
 Log[(-3 + x)^(2/3) + (-3 + x)^(1/3)*(1 + x)^(1/3) + (1 + x)^(2/3)]))/(2*((
-3 + x)^2*(1 + x))^(1/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 446, normalized size of antiderivative = 5.95

method	result
trager	$-\ln\left(-\frac{-16\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)^2 x^2+27\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)\left(x^3-5x^2+3x+9\right)^{\frac{2}{3}}+45\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)\left(x^3\right.}{}\right.$

```
[In] int(1/(x^3-5*x^2+3*x+9)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(-(-16*RootOf(_Z^2-3*_Z+9)^2*x^2+27*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9
)^(2/3)+45*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(1/3)*x+48*RootOf(_Z^2-3*_
Z+9)^2*x-24*RootOf(_Z^2-3*_Z+9)*x^2-216*(x^3-5*x^2+3*x+9)^(2/3)-135*RootOf(
_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(1/3)+81*(x^3-5*x^2+3*x+9)^(1/3)*x+156*RootO
f(_Z^2-3*_Z+9)*x-9*x^2-243*(x^3-5*x^2+3*x+9)^(1/3)-252*RootOf(_Z^2-3*_Z+9)+
90*x-189)/(-3+x))+1/3*RootOf(_Z^2-3*_Z+9)*ln((4*RootOf(_Z^2-3*_Z+9)^2*x^2+2
```

```
7*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(2/3)-72*RootOf(_Z^2-3*_Z+9)*(x^3-5
*x^2+3*x+9)^(1/3)*x-12*RootOf(_Z^2-3*_Z+9)^2*x+33*RootOf(_Z^2-3*_Z+9)*x^2+1
35*(x^3-5*x^2+3*x+9)^(2/3)+216*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(1/3)+
81*(x^3-5*x^2+3*x+9)^(1/3)*x-78*RootOf(_Z^2-3*_Z+9)*x-180*x^2-243*(x^3-5*x^
2+3*x+9)^(1/3)-63*RootOf(_Z^2-3*_Z+9)+792*x-756)/(-3+x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.71

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx$$
$$= -\sqrt{3}\arctan\left(\frac{\sqrt{3}(x-3)+2\sqrt{3}(x^3-5x^2+3x+9)^{\frac{1}{3}}}{3(x-3)}\right)$$
$$+\frac{1}{2}\log\left(\frac{x^2+(x^3-5x^2+3x+9)^{\frac{1}{3}}(x-3)-6x+(x^3-5x^2+3x+9)^{\frac{2}{3}}+9}{x^2-6x+9}\right)$$
$$-\log\left(-\frac{x-(x^3-5x^2+3x+9)^{\frac{1}{3}}-3}{x-3}\right)$$

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(1/3*(sqrt(3)*(x - 3) + 2*sqrt(3)*(x^3 - 5*x^2 + 3*x + 9)^(1
/3))/(x - 3)) + 1/2*log((x^2 + (x^3 - 5*x^2 + 3*x + 9)^(1/3)*(x - 3) - 6*x
+ (x^3 - 5*x^2 + 3*x + 9)^(2/3) + 9)/(x^2 - 6*x + 9)) - log(-(x - (x^3 - 5*
x^2 + 3*x + 9)^(1/3) - 3)/(x - 3))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{\sqrt[3]{x^3-5x^2+3x+9}}\,dx$$

```
[In] integrate(1/(x**3-5*x**2+3*x+9)**(1/3),x)
```

```
[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="maxima")
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="giac")
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{1/3}}\,dx$$

```
[In] int(1/(3*x - 5*x^2 + x^3 + 9)^(1/3),x)
[Out] int(1/(3*x - 5*x^2 + x^3 + 9)^(1/3), x)
```

3.233 $\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}} dx$

Optimal result

Integrand size = 17, antiderivative size = 29

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}} dx = \frac{3(3-x)(1+x)}{4(9+3x-5x^2+x^3)^{2/3}}$$

```
[Out] 3/4*(3-x)*(1+x)/(x^3-5*x^2+3*x+9)^(2/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2092, 2089, 37}

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}} dx = \frac{3(3-x)(x+1)}{4(x^3-5x^2+3x+9)^{2/3}}$$

```
[In] Int[(9 + 3*x - 5*x^2 + x^3)^(-2/3),x]
```

```
[Out] (3*(3 - x)*(1 + x))/(4*(9 + 3*x - 5*x^2 + x^3)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2089

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
```

```
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{\left(\frac{128}{27}-\frac{16x}{3}+x^3\right)^{2/3}}\,dx, x, -\frac{5}{3}+x\right)\\ &= \frac{\left(512\sqrt[3]{2}(3-x)^{4/3}(1+x)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^{4/3}\left(\frac{128}{9}+\frac{16x}{3}\right)^{2/3}}\,dx, x, -\frac{5}{3}+x\right)}{9\left(9+3x-5x^2+x^3\right)^{2/3}}\\ &= \frac{3(3-x)(1+x)}{4\left(9+3x-5x^2+x^3\right)^{2/3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{2/3}}\,dx = -\frac{3(-3+x)(1+x)}{4\left((-3+x)^2(1+x)\right)^{2/3}}$$

```
[In] Integrate[(9 + 3*x - 5*x^2 + x^3)^(-2/3),x]
[Out] (-3*(-3 + x)*(1 + x))/(4*((-3 + x)^2*(1 + x))^(2/3))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{3(-3+x)(1+x)}{4\left((1+x)(-3+x)^2\right)^{\frac{2}{3}}}$	20
trager	$-\frac{3(x^3-5x^2+3x+9)^{\frac{1}{3}}}{4(-3+x)}$	23
gosper	$-\frac{3(1+x)(-3+x)}{4(x^3-5x^2+3x+9)^{\frac{2}{3}}}$	24

```
[In] int(1/(x^3-5*x^2+3*x+9)^(2/3),x,method=_RETURNVERBOSE)
[Out] -3/4/((1+x)*(-3+x)^2)^(2/3)*(-3+x)*(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx = -\frac{3\,(x^3-5\,x^2+3\,x+9)^{\frac{1}{3}}}{4\,(x-3)}$$

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="fricas")
[Out] -3/4*(x^3 - 5*x^2 + 3*x + 9)^(1/3)/(x - 3)
```

Sympy [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx = \int \frac{1}{(x^3-5x^2+3x+9)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(x**3-5*x**2+3*x+9)**(2/3),x)
[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="maxima")
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="giac")
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-2/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx = -\frac{3\,(x^3-5\,x^2+3\,x+9)^{1/3}}{4\,(x-3)}$$

```
[In] int(1/(3*x - 5*x^2 + x^3 + 9)^(2/3),x)
[Out] -(3*(3*x - 5*x^2 + x^3 + 9)^(1/3))/(4*(x - 3))
```

3.234 $\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}} \, dx = \frac{3(3-x)(1+x)}{20(9+3x-5x^2+x^3)^{4/3}} + \frac{9(3-x)^2(1+x)}{80(9+3x-5x^2+x^3)^{4/3}} - \frac{27(3-x)^3(1+x)}{320(9+3x-5x^2+x^3)^{4/3}}$$

```
[Out] 3/20*(3-x)*(1+x)/(x^3-5*x^2+3*x+9)^(4/3)+9/80*(3-x)^2*(1+x)/(x^3-5*x^2+3*x+
9)^(4/3)-27/320*(3-x)^3*(1+x)/(x^3-5*x^2+3*x+9)^(4/3)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2092, 2089, 47, 37}

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}} \, dx = -\frac{27(x+1)(3-x)^3}{320(x^3-5x^2+3x+9)^{4/3}} + \frac{9(x+1)(3-x)^2}{80(x^3-5x^2+3x+9)^{4/3}} + \frac{3(x+1)(3-x)}{20(x^3-5x^2+3x+9)^{4/3}}$$

```
[In] Int[(9 + 3*x - 5*x^2 + x^3)^(-4/3),x]
```

```
[Out] (3*(3 - x)*(1 + x))/(20*(9 + 3*x - 5*x^2 + x^3)^(4/3)) + (9*(3 - x)^2*(1 +
x))/(80*(9 + 3*x - 5*x^2 + x^3)^(4/3)) - (27*(3 - x)^3*(1 + x))/(320*(9 + 3
*x - 5*x^2 + x^3)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 2089

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{\left(\frac{128}{27}-\frac{16x}{3}+x^3\right)^{4/3}}\,dx, x, -\frac{5}{3}+x\right)\\
&= \frac{\left(262144\ 2^{2/3}(3-x)^{8/3}(1+x)^{4/3}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^{8/3}\left(\frac{128}{9}+\frac{16x}{3}\right)^{4/3}}\,dx, x, -\frac{5}{3}+x\right)}{81\left(9+3x-5x^2+x^3\right)^{4/3}}\\
&= \frac{3(3-x)(1+x)}{20\left(9+3x-5x^2+x^3\right)^{4/3}}\\
&\quad+\frac{\left(4096\ 2^{2/3}(3-x)^{8/3}(1+x)^{4/3}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^{5/3}\left(\frac{128}{9}+\frac{16x}{3}\right)^{4/3}}\,dx, x, -\frac{5}{3}+x\right)}{45\left(9+3x-5x^2+x^3\right)^{4/3}}
\end{aligned}$$

$$= \frac{3(3-x)(1+x)}{20\left(9+3x-5x^2+x^3\right)^{4/3}}+\frac{9(3-x)^2(1+x)}{80\left(9+3x-5x^2+x^3\right)^{4/3}}$$
$$+\frac{\left(16\ 2^{2/3}(3-x)^{8/3}(1+x)^{4/3}\right)\text{Subst}\left(\int\frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^{2/3}\left(\frac{128}{9}+\frac{16x}{3}\right)^{4/3}}\,dx,x,-\frac{5}{3}+x\right)}{5\left(9+3x-5x^2+x^3\right)^{4/3}}$$
$$= \frac{3(3-x)(1+x)}{20\left(9+3x-5x^2+x^3\right)^{4/3}}+\frac{9(3-x)^2(1+x)}{80\left(9+3x-5x^2+x^3\right)^{4/3}}-\frac{27(3-x)^3(1+x)}{320\left(9+3x-5x^2+x^3\right)^{4/3}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.36

$$\int\frac{1}{\left(9+3x-5x^2+x^3\right)^{4/3}}\,dx=\frac{3(-3+x)(1+x)\left(29-42x+9x^2\right)}{320\left((-3+x)^2(1+x)\right)^{4/3}}$$

```
[In] Integrate[(9 + 3*x - 5*x^2 + x^3)^(-4/3),x]
[Out] (3*(-3 + x)*(1 + x)*(29 - 42*x + 9*x^2))/(320*((-3 + x)^2*(1 + x))^(4/3))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.32

method	result	size
risch	$\frac{\frac{27}{320}x^2-\frac{63}{160}x+\frac{87}{320}}{(-3+x)\left((1+x)(-3+x)^2\right)^{\frac{1}{3}}}$	29
gosper	$\frac{3(1+x)(-3+x)\left(9x^2-42x+29\right)}{320\left(x^3-5x^2+3x+9\right)^{\frac{4}{3}}}$	34
trager	$\frac{3\left(9x^2-42x+29\right)\left(x^3-5x^2+3x+9\right)^{\frac{2}{3}}}{320(-3+x)^3(1+x)}$	38

```
[In] int(1/(x^3-5*x^2+3*x+9)^(4/3),x,method=_RETURNVERBOSE)
[Out] 3/320*(9*x^2-42*x+29)/(-3+x)/((1+x)*(-3+x)^2)^(1/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.48

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \frac{3\,(x^3-5\,x^2+3\,x+9)^{\frac{2}{3}}(9\,x^2-42\,x+29)}{320\,(x^4-8\,x^3+18\,x^2-27)}$$

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="fricas")
[Out] 3/320*(x^3 - 5*x^2 + 3*x + 9)^(2/3)*(9*x^2 - 42*x + 29)/(x^4 - 8*x^3 + 18*x
^2 - 27)
```

Sympy [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \int \frac{1}{(x^3-5x^2+3x+9)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(x**3-5*x**2+3*x+9)**(4/3),x)
[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="maxima")
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="giac")
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-4/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.40

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \frac{3\,(9\,x^2-42\,x+29)\,\left(x^3-5\,x^2+3\,x+9\right)^{2/3}}{320\,\left(x+1\right)\,\left(x-3\right)^3}$$

```
[In] int(1/(3*x - 5*x^2 + x^3 + 9)^(4/3),x)
[Out] (3*(9*x^2 - 42*x + 29)*(3*x - 5*x^2 + x^3 + 9)^(2/3))/(320*(x + 1)*(x - 3)^
3)
```

3.235 $\int \frac{1}{\sqrt{4+3x-2x^2}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 19

$$\int \frac{1}{\sqrt{4+3x-2x^2}} \, dx = -\frac{\arcsin\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arcsin(1/41*(3-4*x)*41^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {633, 222}

$$\int \frac{1}{\sqrt{4+3x-2x^2}} \, dx = -\frac{\arcsin\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

```
[In] Int[1/Sqrt[4 + 3*x - 2*x^2],x]
[Out] -(ArcSin[(3 - 4*x)/Sqrt[41]]/Sqrt[2])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{41}}}\,dx, x, 3-4x\right)}{\sqrt{82}}$$
$$= -\frac{\arcsin\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = \sqrt{2}\arctan\left(\frac{\sqrt{2}x}{-2+\sqrt{4+3x-2x^2}}\right)$$

```
[In] Integrate[1/Sqrt[4 + 3*x - 2*x^2],x]
[Out] Sqrt[2]*ArcTan[(Sqrt[2]*x)/(-2 + Sqrt[4 + 3*x - 2*x^2])]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\sqrt{2}\,\arcsin\left(\frac{4\sqrt{41}\,\left(x-\frac{3}{4}\right)}{41}\right)}{2}$	15
trager	$-\frac{\text{RootOf}\left(\_Z^2+2\right)\ln\left(4\,\text{RootOf}\left(\_Z^2+2\right)x+4\sqrt{-2x^2+3x+4}-3\,\text{RootOf}\left(\_Z^2+2\right)\right)}{2}$	42

```
[In] int(1/(-2*x^2+3*x+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*2^(1/2)*arcsin(4/41*41^(1/2)*(x-3/4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(16) = 32$.

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = -\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2\,x^2+3\,x+4}-2\,\sqrt{2}}{2\,x}\right)$$

```
[In] integrate(1/(-2*x^2+3*x+4)^(1/2),x, algorithm="fricas")
[Out] -sqrt(2)*arctan(1/2*(sqrt(2)*sqrt(-2*x^2 + 3*x + 4) - 2*sqrt(2))/x)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = \frac{\sqrt{2}\,\mathrm{asin}\left(\frac{4\sqrt{41}\left(x-\frac{3}{4}\right)}{41}\right)}{2}$$

```
[In] integrate(1/(-2*x**2+3*x+4)**(1/2),x)
[Out] sqrt(2)*asin(4*sqrt(41)*(x - 3/4)/41)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = -\frac{1}{2}\,\sqrt{2}\arcsin\left(-\frac{1}{41}\,\sqrt{41}(4\,x-3)\right)$$

```
[In] integrate(1/(-2*x^2+3*x+4)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(2)*arcsin(-1/41*sqrt(41)*(4*x - 3))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = \frac{1}{8}\sqrt{-2\,x^2+3\,x+4}(4\,x-3)+\frac{41}{32}\sqrt{2}\arcsin\left(\frac{1}{41}\sqrt{41}(4\,x-3)\right)$$

```
[In] integrate(1/(-2*x^2+3*x+4)^(1/2),x, algorithm="giac")
[Out] 1/8*sqrt(-2*x^2 + 3*x + 4)*(4*x - 3) + 41/32*sqrt(2)*arcsin(1/41*sqrt(41)*(
4*x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = \frac{\sqrt{2}\,\mathrm{asin}\left(\frac{\sqrt{41}\,(4\,x-3)}{41}\right)}{2}$$

```
[In] int(1/(3*x - 2*x^2 + 4)^(1/2),x)
[Out] (2^(1/2)*asin((41^(1/2)*(4*x - 3))/41))/2
```

3.236 $\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 8

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = -\arcsin(2-x)$$

```
[Out] arcsin(-2+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 222}

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = -\arcsin(2-x)$$

```
[In] Int[1/Sqrt[-3 + 4*x - x^2],x]
```

```
[Out] -ArcSin[2 - x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, 4-2x\right)\right)$$
$$= -\arcsin(2-x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. $2(8) = 16$.

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.88

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{-3+4x-x^2}}{-1+x}\right)$$

```
[In] Integrate[1/Sqrt[-3 + 4*x - x^2],x]
[Out] -2*ArcTan[Sqrt[-3 + 4*x - x^2]/(-1 + x)]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

method	result	size
default	$\arcsin(-2+x)$	5
trager	$\text{RootOf}(\_Z^2+1)\ln\left(-\text{RootOf}(\_Z^2+1)\,x+\sqrt{-x^2+4x-3}+2\,\text{RootOf}(\_Z^2+1)\right)$	39

```
[In] int(1/(-x^2+4*x-3)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(-2+x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(4) = 8$.

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 3.62

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = -\arctan\left(\frac{\sqrt{-x^2+4\,x-3}(x-2)}{x^2-4\,x+3}\right)$$

```
[In] integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-x^2 + 4*x - 3)*(x - 2)/(x^2 - 4*x + 3))
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.38

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = \operatorname{asin}(x-2)$$

```
[In] integrate(1/(-x**2+4*x-3)**(1/2),x)
[Out] asin(x - 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = -\arcsin(-x+2)$$

```
[In] integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-x + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(4) = 8.

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = \frac{1}{2}\,\sqrt{-x^2+4\,x-3}(x-2)+\frac{1}{2}\,\arcsin(x-2)$$

```
[In] integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 4*x - 3)*(x - 2) + 1/2*arcsin(x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = \operatorname{asin}(x-2)$$

```
[In] int(1/(4*x - x^2 - 3)^(1/2),x)
[Out] asin(x - 2)
```

3.237 $\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 12

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = \frac{\arcsin(5+6x)}{\sqrt{3}}$$

```
[Out] 1/3*arcsin(5+6*x)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 222}

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = \frac{\arcsin(6x+5)}{\sqrt{3}}$$

```
[In] Int[1/Sqrt[-2 - 5*x - 3*x^2],x]
```

```
[Out] ArcSin[5 + 6*x]/Sqrt[3]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -5-6x\right)}{\sqrt{3}}$$
$$= \frac{\arcsin(5+6x)}{\sqrt{3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.75

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{-2-5x-3x^2}}{\sqrt{3}(1+x)}\right)}{\sqrt{3}}$$

```
[In] Integrate[1/Sqrt[-2 - 5*x - 3*x^2],x]
[Out] (-2*ArcTan[Sqrt[-2 - 5*x - 3*x^2]/(Sqrt[3]*(1 + x))])/Sqrt[3]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\arcsin(6x+5)\sqrt{3}}{3}$	12
trager	$\frac{\text{RootOf}\left(\_Z^2+3\right)\ln\left(-6\,\text{RootOf}\left(\_Z^2+3\right)x+6\sqrt{-3x^2-5x-2}-5\,\text{RootOf}\left(\_Z^2+3\right)\right)}{3}$	42

```
[In] int(1/(-3*x^2-5*x-2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*arcsin(6*x+5)*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.33

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2-5\,x-2}(6\,x+5)}{6\,(3\,x^2+5\,x+2)}\right)$$

```
[In] integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 - 5*x - 2)*(6*x + 5)/(3*x^2 + 5
*x + 2))
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = \frac{\sqrt{3}\operatorname{asin}(6x+5)}{3}$$

```
[In] integrate(1/(-3*x**2-5*x-2)**(1/2),x)
[Out] sqrt(3)*asin(6*x + 5)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = \frac{1}{3}\,\sqrt{3}\arcsin(6\,x+5)$$

```
[In] integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arcsin(6*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(11) = 22$.

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.58

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = \frac{1}{12}\sqrt{-3\,x^2-5\,x-2}(6\,x+5)+\frac{1}{72}\sqrt{3}\arcsin\left(6\,x+5\right)$$

```
[In] integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="giac")
[Out] 1/12*sqrt(-3*x^2 - 5*x - 2)*(6*x + 5) + 1/72*sqrt(3)*arcsin(6*x + 5)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}(6\,x+5)}{3}$$

```
[In] int(1/(- 5*x - 3*x^2 - 2)^(1/2),x)
[Out] (3^(1/2)*asin(6*x + 5))/3
```

3.238 $\int \frac{1}{\sqrt{1-x^2}(4+x^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

```
[Out] 1/10*arctan(1/2*x*5^(1/2)/(-x^2+1)^(1/2))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {385, 209}

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

```
[In] Int[1/(Sqrt[1 - x^2]*(4 + x^2)),x]
```

```
[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[1 - x^2])]/(2*Sqrt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{4+5x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)$$
$$= \frac{\arctan\left(\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = -\frac{\arctan\left(\frac{x\sqrt{5-5x^2}}{2(-1+x^2)}\right)}{2\sqrt{5}}$$

```
[In] Integrate[1/(Sqrt[1 - x^2]*(4 + x^2)),x]
[Out] -1/2*ArcTan[(x*Sqrt[5 - 5*x^2])/(2*(-1 + x^2))]/Sqrt[5]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
pseudoelliptic	$-\frac{\sqrt{5}\,\arctan\left(\frac{2\sqrt{5}\,\sqrt{-x^2+1}}{5x}\right)}{10}$	24
default	$-\frac{\sqrt{5}\,\arctan\left(\frac{\sqrt{-x^2+1}\,\sqrt{5}\,x}{2x^2-2}\right)}{10}$	29
trager	$\frac{\text{RootOf}\left(\_Z^2+5\right)\ln\left(\frac{-9\,\text{RootOf}\left(\_Z^2+5\right)x^2+20x\sqrt{-x^2+1}+4\,\text{RootOf}\left(\_Z^2+5\right)}{x^2+4}\right)}{20}$	50

```
[In] int(1/(x^2+4)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/10*5^(1/2)*arctan(2/5/x*5^(1/2)*(-x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = -\frac{1}{10}\,\sqrt{5}\arctan\left(\frac{2\,\sqrt{5}\sqrt{-x^2+1}}{5\,x}\right)$$

```
[In] integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/10*sqrt(5)*arctan(2/5*sqrt(5)*sqrt(-x^2 + 1)/x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = \int \frac{1}{\sqrt{-(x-1)\,(x+1)}\,(x^2+4)}\,dx$$

```
[In] integrate(1/(x**2+4)/(-x**2+1)**(1/2),x)
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*(x**2 + 4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = \int \frac{1}{(x^2+4)\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 4)*sqrt(-x^2 + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(21) = 42.

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = \frac{1}{20}\,\sqrt{5}\left(\pi\mathrm{sgn}(x) + 2\,\arctan\left(-\frac{\sqrt{5}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{5\,\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$

```
[In] integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/20*sqrt(5)*(pi*sgn(x) + 2*arctan(-1/5*sqrt(5)*x*((sqrt(-x^2 + 1) - 1)^2/x
^2 - 1)/(sqrt(-x^2 + 1) - 1)))
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.55

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = \frac{\sqrt{5}\,\ln\left(\frac{\frac{\sqrt{5}\,(-1+x\,2\mathrm{i})\,1\mathrm{i}}{5}-\sqrt{1-x^2}\,1\mathrm{i}}{x-2\mathrm{i}}\right)\,1\mathrm{i}}{20} - \frac{\sqrt{5}\,\ln\left(\frac{\frac{\sqrt{5}\,(1+x\,2\mathrm{i})\,1\mathrm{i}}{5}+\sqrt{1-x^2}\,1\mathrm{i}}{x+2\mathrm{i}}\right)\,1\mathrm{i}}{20}$$

```
[In] int(1/((1 - x^2)^(1/2)*(x^2 + 4)),x)
```

```
[Out] (5^(1/2)*log(((5^(1/2)*(x*2i - 1)*1i)/5 - (1 - x^2)^(1/2)*1i)/(x - 2i))*1i)
/20 - (5^(1/2)*log(((5^(1/2)*(x*2i + 1)*1i)/5 + (1 - x^2)^(1/2)*1i)/(x + 2i
))*1i)/20
```

3.239 $\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{15}x}{2\sqrt{1+4x^2}}\right)}{2\sqrt{15}}$$

```
[Out] 1/30*arctanh(1/2*x*15^(1/2)/(4*x^2+1)^(1/2))*15^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {385, 212}

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{15}x}{2\sqrt{4x^2+1}}\right)}{2\sqrt{15}}$$

```
[In] Int[1/((4 + x^2)*Sqrt[1 + 4*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[15]*x)/(2*Sqrt[1 + 4*x^2])]/(2*Sqrt[15])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{4-15x^2}\,dx, x, \frac{x}{\sqrt{1+4x^2}}\right)$$
$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{15}x}{2\sqrt{1+4x^2}}\right)}{2\sqrt{15}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{8+2x^2-x\sqrt{1+4x^2}}{2\sqrt{15}}\right)}{2\sqrt{15}}$$

```
[In] Integrate[1/((4 + x^2)*Sqrt[1 + 4*x^2]),x]
[Out] ArcTanh[(8 + 2*x^2 - x*Sqrt[1 + 4*x^2])/(2*Sqrt[15])]/(2*Sqrt[15])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{15}}{2\sqrt{4x^2+1}}\right)\sqrt{15}}{30}$	22
pseudoelliptic	$\frac{\sqrt{15}\,\operatorname{arctanh}\left(\frac{2\sqrt{4x^2+1}\,\sqrt{15}}{15x}\right)}{30}$	24
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-15\right)\ln\left(\frac{31\operatorname{RootOf}\left(\_Z^2-15\right)x^2+60\sqrt{4x^2+1}\,x+4\operatorname{RootOf}\left(\_Z^2-15\right)}{x^2+4}\right)}{60}$	50

```
[In] int(1/(x^2+4)/(4*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/30*arctanh(1/2*x*15^(1/2)/(4*x^2+1)^(1/2))*15^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(21) = 42$.

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = \frac{1}{60}\sqrt{15}\log\left(\frac{961\,x^2+8\sqrt{15}(31\,x^2+4)+4\sqrt{4\,x^2+1}(31\sqrt{15}x+120\,x)+124}{x^2+4}\right)$$

```
[In] integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/60*sqrt(15)*log((961*x^2 + 8*sqrt(15)*(31*x^2 + 4) + 4*sqrt(4*x^2 + 1)*(3
1*sqrt(15)*x + 120*x) + 124)/(x^2 + 4))
```

Sympy [F]

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = \int \frac{1}{(x^2+4)\sqrt{4x^2+1}}\,dx$$

```
[In] integrate(1/(x**2+4)/(4*x**2+1)**(1/2),x)
[Out] Integral(1/((x**2 + 4)*sqrt(4*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+1}(x^2+4)}\,dx$$

```
[In] integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(4*x^2 + 1)*(x^2 + 4)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(21) = 42$.

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.84

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = -\frac{1}{60}\sqrt{15}\log\left(\frac{\left(2\,x-\sqrt{4\,x^2+1}\right)^2-8\sqrt{15}+31}{\left(2\,x-\sqrt{4\,x^2+1}\right)^2+8\sqrt{15}+31}\right)$$

```
[In] integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/60*sqrt(15)*log(((2*x - sqrt(4*x^2 + 1))^2 - 8*sqrt(15) + 31)/((2*x - sq
rt(4*x^2 + 1))^2 + 8*sqrt(15) + 31))
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = -\frac{\sqrt{15}\left(\ln\left(x-2\mathrm{i}\right)-\ln\left(x+\frac{\sqrt{15}\sqrt{x^2+\frac{1}{4}}}{4}-\frac{1}{8}\mathrm{i}\right)\right)}{60} + \frac{\sqrt{15}\left(\ln\left(x+2\mathrm{i}\right)-\ln\left(x-\frac{\sqrt{15}\sqrt{x^2+\frac{1}{4}}}{4}+\frac{1}{8}\mathrm{i}\right)\right)}{60}$$

```
[In] int(1/((x^2 + 4)*(4*x^2 + 1)^(1/2)),x)
```

```
[Out] (15^(1/2)*(log(x + 2i) - log(x - (15^(1/2)*(x^2 + 1/4)^(1/2))/4 + 1i/8)))/6
0 - (15^(1/2)*(log(x - 2i) - log(x + (15^(1/2)*(x^2 + 1/4)^(1/2))/4 - 1i/8)
))/60
```

3.240 $\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 24

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(1/2*(-x^2+5)^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {455, 65, 213}

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[In] Int[x/((3 - x^2)*Sqrt[5 - x^2]),x]
```

```
[Out] ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(3-x)\sqrt{5-x}}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{5-x^2}\right)\\
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[In] Integrate[x/((3 - x^2)*Sqrt[5 - x^2]),x]
[Out] ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-x^{2}+5}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}$	21
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)x^{2}-7\operatorname{RootOf}\left(\_Z^{2}-2\right)-4\sqrt{-x^{2}+5}}{x^{2}-3}\right)}{4}$	48
default	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(4+2\sqrt{3}\left(x+\sqrt{3}\right)\right)\sqrt{2}}{4\sqrt{-\left(x+\sqrt{3}\right)^{2}+2\sqrt{3}\left(x+\sqrt{3}\right)+2}}\right)}{4}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(4-2\sqrt{3}\left(x-\sqrt{3}\right)\right)\sqrt{2}}{4\sqrt{-\left(x-\sqrt{3}\right)^{2}-2\sqrt{3}\left(x-\sqrt{3}\right)+2}}\right)}{4}$	100

```
[In] int(x/(-x^2+3)/(-x^2+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arctanh(1/2*(-x^2+5)^(1/2)*2^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{x^4-4\sqrt{2}(x^2-7)\sqrt{-x^2+5}-22\,x^2+89}{x^4-6\,x^2+9}\right)$$

```
[In] integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="fricas")
[Out] 1/8*sqrt(2)*log((x^4 - 4*sqrt(2)*(x^2 - 7)*sqrt(-x^2 + 5) - 22*x^2 + 89)/(x
^4 - 6*x^2 + 9))
```

Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = -\frac{\sqrt{2}\left(\log\left(\sqrt{5-x^2}-\sqrt{2}\right)-\log\left(\sqrt{5-x^2}+\sqrt{2}\right)\right)}{4}$$

```
[In] integrate(x/(-x**2+3)/(-x**2+5)**(1/2),x)
[Out] -sqrt(2)*(log(sqrt(5 - x**2) - sqrt(2)) - log(sqrt(5 - x**2) + sqrt(2)))/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.67

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{1}{12}\sqrt{3}\left(\sqrt{3}\sqrt{2}\log\left(\sqrt{3}+\frac{2\sqrt{2}\sqrt{-x^2+5}}{|2x+2\sqrt{3}|}+\frac{4}{|2x+2\sqrt{3}|}\right)+\sqrt{3}\sqrt{2}\log\left(-\sqrt{3}+\frac{2\sqrt{2}\sqrt{-x^2+5}}{|2x-2\sqrt{3}|}+\frac{}{|2x}\right.\right.$$

```
[In] integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*sqrt(2)*log(sqrt(3) + 2*sqrt(2)*sqrt(-x^2 + 5)/abs(2*
x + 2*sqrt(3)) + 4/abs(2*x + 2*sqrt(3))) + sqrt(3)*sqrt(2)*log(-sqrt(3) + 2
*sqrt(2)*sqrt(-x^2 + 5)/abs(2*x - 2*sqrt(3)) + 4/abs(2*x - 2*sqrt(3))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}+\sqrt{-x^2+5}\right)-\frac{1}{4}\sqrt{2}\log\left(\left|-\sqrt{2}+\sqrt{-x^2+5}\right|\right)$$

```
[In] integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(sqrt(2) + sqrt(-x^2 + 5)) - 1/4*sqrt(2)*log(abs(-sqrt(2) +
sqrt(-x^2 + 5)))
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.25

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{\sqrt{2}\left(\ln\left(\frac{\frac{\sqrt{2}\left(\sqrt{3}x+5\right)1i}{2}+\sqrt{5-x^2}\,1i}{x+\sqrt{3}}\right)+\ln\left(\frac{\frac{\sqrt{2}\left(\sqrt{3}x-5\right)1i}{2}-\sqrt{5-x^2}\,1i}{x-\sqrt{3}}\right)\right)}{4}$$

```
[In] int(-x/((x^2 - 3)*(5 - x^2)^(1/2)),x)
```

```
[Out] (2^(1/2)*(log(((2^(1/2)*(3^(1/2)*x + 5)*1i)/2 + (5 - x^2)^(1/2)*1i)/(x + 3^
(1/2))) + log(((2^(1/2)*(3^(1/2)*x - 5)*1i)/2 - (5 - x^2)^(1/2)*1i)/(x - 3^
(1/2)))))/4
```

3.241 $\int \frac{x}{\sqrt{3-x^2}(5-x^2)} dx$

Optimal result

Integrand size = 22, antiderivative size = 25

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{\arctan\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctan(1/2*(-x^2+3)^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {455, 65, 209}

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{\arctan\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[In] Int[x/(Sqrt[3 - x^2]*(5 - x^2)),x]
```

```
[Out] -(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3-x}(5-x)}\,dx, x, x^2\right)\\ &= -\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt{3-x^2}\right)\\ &= -\frac{\arctan\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{\arctan\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[In] Integrate[x/(Sqrt[3 - x^2]*(5 - x^2)),x]
[Out] -(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-x^2+3}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}$	21
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+2\right)x^2-\operatorname{RootOf}\left(\_Z^2+2\right)-4\sqrt{-x^2+3}}{x^2-5}\right)}{4}$	48
default	$-\frac{\sqrt{2}\,\arctan\left(\frac{\left(-4+2\sqrt{5}\,\left(x+\sqrt{5}\right)\right)\sqrt{2}}{4\sqrt{-\left(x+\sqrt{5}\right)^2+2\sqrt{5}\,\left(x+\sqrt{5}\right)-2}}\right)}{4}-\frac{\sqrt{2}\,\arctan\left(\frac{\left(-4-2\sqrt{5}\,\left(x-\sqrt{5}\right)\right)\sqrt{2}}{4\sqrt{-\left(x-\sqrt{5}\right)^2-2\sqrt{5}\,\left(x-\sqrt{5}\right)-2}}\right)}{4}$	100

```
[In] int(x/(-x^2+5)/(-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*arctan(1/2*(-x^2+3)^(1/2)*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^2-1)\sqrt{-x^2+3}}{4\,(x^2-3)}\right)$$

```
[In] integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="fricas")
[Out] -1/4*sqrt(2)*arctan(1/4*sqrt(2)*(x^2 - 1)*sqrt(-x^2 + 3)/(x^2 - 3))
```

Sympy [A] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{3-x^2}}{2}\right)}{2}$$

```
[In] integrate(x/(-x**2+5)/(-x**2+3)**(1/2),x)
[Out] -sqrt(2)*atan(sqrt(2)*sqrt(3 - x**2)/2)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(20) = 40.

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 4.04

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{1}{20}\sqrt{5}\left(\sqrt{5}\sqrt{2}\arcsin\left(\frac{2\sqrt{5}\sqrt{3}x}{3\,|2\,x+2\,\sqrt{5}|}+\frac{2\,\sqrt{3}}{|2\,x+2\,\sqrt{5}|}\right)-\sqrt{5}\sqrt{2}\arcsin\left(\frac{2\sqrt{5}\sqrt{3}x}{3\,|2\,x-2\,\sqrt{5}|}-\frac{2\,\sqrt{3}}{|2\,x-2\,\sqrt{5}|}\right.\right.$$

```
[In] integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="maxima")
[Out] -1/20*sqrt(5)*(sqrt(5)*sqrt(2)*arcsin(2/3*sqrt(5)*sqrt(3)*x/abs(2*x + 2*sqr
t(5)) + 2*sqrt(3)/abs(2*x + 2*sqrt(5))) - sqrt(5)*sqrt(2)*arcsin(2/3*sqrt(5
)*sqrt(3)*x/abs(2*x - 2*sqrt(5)) - 2*sqrt(3)/abs(2*x - 2*sqrt(5))))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x^2+3}\right)$$

```
[In] integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 3))
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.32

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,\left(\sqrt{5}\,x+3\right)}{2}+\sqrt{3-x^2}\,1\mathrm{i}}{x+\sqrt{5}}\right)\,1\mathrm{i}}{4}-\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,\left(\sqrt{5}\,x-3\right)}{2}-\sqrt{3-x^2}\,1\mathrm{i}}{x-\sqrt{5}}\right)\,1\mathrm{i}}{4}$$

```
[In] int(-x/((3 - x^2)^(1/2)*(x^2 - 5)),x)
[Out] - (2^(1/2)*log(((2^(1/2)*(5^(1/2)*x + 3))/2 + (3 - x^2)^(1/2)*1i)/(x + 5^(1
/2)))*1i)/4 - (2^(1/2)*log(((2^(1/2)*(5^(1/2)*x - 3))/2 - (3 - x^2)^(1/2)*1
i)/(x - 5^(1/2)))*1i)/4
```

3.242 $\int \frac{1}{\sqrt{2+x^2}(-1+x^4)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx=-\frac{1}{2}\arctan\left(\frac{x}{\sqrt{2+x^2}}\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{2+x^2}}\right)}{2\sqrt{3}}$$

```
[Out] -1/2*arctan(x/(x^2+2)^(1/2))-1/6*arctanh(x*3^(1/2)/(x^2+2)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1189, 385, 212, 209}

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx=-\frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^2+2}}\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{x^2+2}}\right)}{2\sqrt{3}}$$

```
[In] Int[1/(Sqrt[2 + x^2]*(-1 + x^4)),x]
```

```
[Out] -1/2*ArcTan[x/Sqrt[2 + x^2]] - ArcTanh[(Sqrt[3]*x)/Sqrt[2 + x^2]]/(2*Sqrt[3
])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[(-a)*c, 2]}, Dist[-c/(2*r), Int[(d + e*x^2)^q/(r - c*x^2), x], x] - Di
st[c/(2*r), Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e, q}
, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \frac{1}{(1-x^2)\sqrt{2+x^2}}\,dx\right) - \frac{1}{2}\int \frac{1}{(1+x^2)\sqrt{2+x^2}}\,dx \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{x}{\sqrt{2+x^2}}\right)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{2+x^2}}\right) \\
&= -\frac{1}{2}\arctan\left(\frac{x}{\sqrt{2+x^2}}\right) - \frac{\text{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{2+x^2}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = \frac{1}{6}\left(3\arctan\left(1+x^2-x\sqrt{2+x^2}\right) - \sqrt{3}\text{arctanh}\left(\frac{1-x^2+x\sqrt{2+x^2}}{\sqrt{3}}\right)\right)$$

```
[In] Integrate[1/(Sqrt[2 + x^2]*(-1 + x^4)),x]
[Out] (3*ArcTan[1 + x^2 - x*Sqrt[2 + x^2]] - Sqrt[3]*ArcTanh[(1 - x^2 + x*Sqrt[2
+ x^2])/Sqrt[3]])/6
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}\,\sqrt{x^{2}+2}}{3x}\right)}{6}+\frac{\arctan\left(\frac{\sqrt{x^{2}+2}}{x}\right)}{2}$
default	$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(2x+4)\sqrt{3}}{6\sqrt{(-1+x)^{2}+1+2x}}\right)}{12}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(-2x+4)\sqrt{3}}{6\sqrt{(1+x)^{2}+1-2x}}\right)}{12}-\frac{\arctan\left(\frac{x}{\sqrt{x^{2}+2}}\right)}{2}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\sqrt{x^{2}+2}\,x+\operatorname{RootOf}\left(\_Z^{2}+1\right)}{x^{2}+1}\right)}{4}+\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(-\frac{-2\operatorname{RootOf}\left(\_Z^{2}-3\right)x^{2}+3\sqrt{x^{2}+2}\,x-\mathrm{Ro}}{(-1+x)(1+x)}\right.}{12}$

[In] int(1/(x^4-1)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/6*3^(1/2)*arctanh(1/3*3^(1/2)*(x^2+2)^(1/2)/x)+1/2*arctan((x^2+2)^(1/2)/x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(31) = 62.

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{4\,x^{2}-\sqrt{3}(2\,x^{2}+1)-\sqrt{x^{2}+2}(2\,\sqrt{3}x-3\,x)+2}{x^{2}-1}\right) - \frac{1}{2}\arctan\left(-x^{2}+\sqrt{x^{2}+2}x-1\right)$$

[In] integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="fricas")

[Out] 1/12*sqrt(3)*log((4*x^2 - sqrt(3)*(2*x^2 + 1) - sqrt(x^2 + 2)*(2*sqrt(3)*x - 3*x) + 2)/(x^2 - 1)) - 1/2*arctan(-x^2 + sqrt(x^2 + 2)*x - 1)

Sympy [F]

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = \int \frac{1}{(x-1)\,(x+1)\,(x^2+1)\,\sqrt{x^2+2}}\,dx$$

[In] integrate(1/(x**4-1)/(x**2+2)**(1/2),x)

[Out] Integral(1/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(x**2 + 2)), x)

Maxima [F]

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = \int \frac{1}{(x^4-1)\sqrt{x^2+2}}\,dx$$

```
[In] integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^4 - 1)*sqrt(x^2 + 2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(31) = 62$.

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = -\frac{1}{12}\sqrt{3}\log\left(\frac{\left|2\left(x-\sqrt{x^2+2}\right)^2-4\sqrt{3}-8\right|}{\left|2\left(x-\sqrt{x^2+2}\right)^2+4\sqrt{3}-8\right|}\right) + \frac{1}{2}\arctan\left(\frac{1}{2}\left(x-\sqrt{x^2+2}\right)^2\right)$$

```
[In] integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="giac")
[Out] -1/12*sqrt(3)*log(abs(2*(x - sqrt(x^2 + 2))^2 - 4*sqrt(3) - 8)/abs(2*(x - s
qrt(x^2 + 2))^2 + 4*sqrt(3) - 8)) + 1/2*arctan(1/2*(x - sqrt(x^2 + 2))^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.49

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = \frac{\sqrt{3}\left(\ln(x-1)-\ln\left(x+\sqrt{3}\sqrt{x^2+2}+2\right)\right)}{12} - \frac{\sqrt{3}\left(\ln(x+1)-\ln\left(\sqrt{3}\sqrt{x^2+2}-x+2\right)\right)}{12} + \frac{\ln\left(\sqrt{x^2+2}+2-x\,1\mathrm{i}\right)\,1\mathrm{i}}{4} - \frac{\ln\left(\sqrt{x^2+2}+2+x\,1\mathrm{i}\right)\,1\mathrm{i}}{4} + \frac{\ln(x-\mathrm{i})\,1\mathrm{i}}{4} - \frac{\ln(x+1\mathrm{i})\,1\mathrm{i}}{4}$$

```
[In] int(1/((x^2 + 2)^(1/2)*(x^4 - 1)),x)
[Out] (log((x^2 + 2)^(1/2) - x*1i + 2)*1i)/4 - (log(x*1i + (x^2 + 2)^(1/2) + 2)*1
i)/4 + (log(x - 1i)*1i)/4 - (log(x + 1i)*1i)/4 + (3^(1/2)*(log(x - 1) - log
(x + 3^(1/2)*(x^2 + 2)^(1/2) + 2)))/12 - (3^(1/2)*(log(x + 1) - log(3^(1/2)
*(x^2 + 2)^(1/2) - x + 2)))/12
```

3.243 $\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{5+2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)}{2\sqrt{7}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(1/3*(x^2+2*x+4)^(1/2)*3^(1/2))*3^(1/2)-1/14*arctanh(1/7*(5+2*x
)*7^(1/2)/(x^2+2*x+4)^(1/2))*7^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1047, 738, 212, 702, 213}

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[In] Int[x/((-1 + x^2)*Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] -1/2*ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])]/Sqrt[7] - ArcTanh[Sqr
t[4 + 2*x + x^2]/Sqrt[3]]/(2*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 702

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{(-1+x)\sqrt{4+2x+x^2}}\,dx + \frac{1}{2}\int \frac{1}{(1+x)\sqrt{4+2x+x^2}}\,dx \\
&= 2\text{Subst}\left(\int \frac{1}{-12+4x^2}\,dx, x, \sqrt{4+2x+x^2}\right) - \text{Subst}\left(\int \frac{1}{28-x^2}\,dx, x, \frac{10+4x}{\sqrt{4+2x+x^2}}\right) \\
&= -\frac{\operatorname{arctanh}\left(\frac{10+4x}{2\sqrt{7}\sqrt{4+2x+x^2}}\right)}{2\sqrt{7}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{1+x-\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{1-x+\sqrt{4+2x+x^2}}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[In] Integrate[x/((-1 + x^2)*Sqrt[4 + 2*x + x^2]),x]
[Out] ArcTanh[(1 + x - Sqrt[4 + 2*x + x^2])/Sqrt[3]]/Sqrt[3] - ArcTanh[(1 - x + S
qrt[4 + 2*x + x^2])/Sqrt[7]]/Sqrt[7]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

method	result
default	$-\frac{\sqrt{7}\,\operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{14}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}}{\sqrt{(1+x)^2+3}}\right)}{6}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-7\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^2-7\right)x+7\sqrt{x^2+2x+4}+5\operatorname{RootOf}\left(\_Z^2-7\right)}{-1+x}\right)}{14}+\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{\sqrt{x^2+2x+4}-\operatorname{RootOf}\left(\right.}{1+x}\right.}{6}$

```
[In] int(x/(x^2-1)/(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/14*7^(1/2)*arctanh(1/14*(10+4*x)*7^(1/2)/((-1+x)^2+3+4*x)^(1/2))-1/6*3^(
1/2)*arctanh(3^(1/2)/((1+x)^2+3)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = \frac{1}{14}\sqrt{7}\log\left(\frac{\sqrt{7}(2x+5)+\sqrt{x^2+2x+4}(2\sqrt{7}-7)-4x-10}{x-1}\right) + \frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}-\sqrt{x^2+2x+4}}{x+1}\right)$$

```
[In] integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/14*sqrt(7)*log((sqrt(7)*(2*x + 5) + sqrt(x^2 + 2*x + 4)*(2*sqrt(7) - 7) -
 4*x - 10)/(x - 1)) + 1/6*sqrt(3)*log(-(sqrt(3) - sqrt(x^2 + 2*x + 4))/(x +
 1))
```

Sympy [F]

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = \int \frac{x}{(x-1)(x+1)\sqrt{x^2+2x+4}}\,dx$$

```
[In] integrate(x/(x**2-1)/(x**2+2*x+4)**(1/2),x)
[Out] Integral(x/((x - 1)*(x + 1)*sqrt(x**2 + 2*x + 4)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = -\frac{1}{14}\sqrt{7}\operatorname{arsinh}\left(\frac{4\sqrt{3}x}{3|2x-2|}+\frac{10\sqrt{3}}{3|2x-2|}\right) - \frac{1}{6}\sqrt{3}\operatorname{arsinh}\left(\frac{2\sqrt{3}}{|2x+2|}\right)$$

```
[In] integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="maxima")
[Out] -1/14*sqrt(7)*arcsinh(4/3*sqrt(3)*x/abs(2*x - 2) + 10/3*sqrt(3)/abs(2*x - 2
)) - 1/6*sqrt(3)*arcsinh(2*sqrt(3)/abs(2*x + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(48) = 96.

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.76

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = \frac{1}{14}\sqrt{7}\log\left(\frac{|-2x-2\sqrt{7}+2\sqrt{x^2+2x+4}+2|}{|-2x+2\sqrt{7}+2\sqrt{x^2+2x+4}+2|}\right) + \frac{1}{6}\sqrt{3}\log\left(-\frac{|-2x-2\sqrt{3}+2\sqrt{x^2+2x+4}-2|}{2\left(x-\sqrt{3}-\sqrt{x^2+2x+4}+1\right)}\right)$$

```
[In] integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="giac")
[Out] 1/14*sqrt(7)*log(abs(-2*x - 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)/abs(-2*x
 + 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)) + 1/6*sqrt(3)*log(-1/2*abs(-2*x
- 2*sqrt(3) + 2*sqrt(x^2 + 2*x + 4) - 2)/(x - sqrt(3) - sqrt(x^2 + 2*x + 4)
 + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = \int \frac{x}{(x^2-1)\,\sqrt{x^2+2\,x+4}}\,dx$$

```
[In] int(x/((x^2 - 1)*(2*x + x^2 + 4)^(1/2)),x)
[Out] int(x/((x^2 - 1)*(2*x + x^2 + 4)^(1/2)), x)
```

3.244 $\int \frac{1}{\sqrt{5+2x+x^2}(-8+x^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 82

$$\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx = -\frac{\arctan\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)}{4\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{7+3x}{\sqrt{13}\sqrt{5+2x+x^2}}\right)}{12\sqrt{13}} + \frac{1}{12}\operatorname{arctanh}\left(\sqrt{5+2x+x^2}\right)$$

```
[Out] 1/12*arctanh((x^2+2*x+5)^(1/2))-1/12*arctan(1/3*(1+x)*3^(1/2)/(x^2+2*x+5)^(
1/2))*3^(1/2)-1/156*arctanh(1/13*(7+3*x)*13^(1/2)/(x^2+2*x+5)^(1/2))*13^(1/
2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2099, 738, 212, 1039, 996, 210, 1038}

$$\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx = -\frac{\arctan\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{4\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}} + \frac{1}{12}\operatorname{arctanh}\left(\sqrt{x^2+2x+5}\right)$$

```
[In] Int[1/(Sqrt[5 + 2*x + x^2]*(-8 + x^3)),x]
```

```
[Out] -1/4*ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]/Sqrt[3] - ArcTanh[(7 + 3
*x)/(Sqrt[13]*Sqrt[5 + 2*x + x^2])]/(12*Sqrt[13]) + ArcTanh[Sqrt[5 + 2*x +
x^2]]/12
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1038

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1039

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
```

```
PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{12(-2+x)\sqrt{5+2x+x^2}}+\frac{-4-x}{12\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\right)dx\\
&= \frac{1}{12}\int \frac{1}{(-2+x)\sqrt{5+2x+x^2}}\,dx+\frac{1}{12}\int \frac{-4-x}{\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\,dx\\
&= -\left(\frac{1}{24}\int \frac{2+2x}{\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\,dx\right)\\
&\quad -\frac{1}{6}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{14+6x}{\sqrt{5+2x+x^2}}\right)\\
&\quad -\frac{1}{4}\int \frac{1}{\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\,dx\\
&= -\frac{\operatorname{arctanh}\left(\frac{7+3x}{\sqrt{13}\sqrt{5+2x+x^2}}\right)}{12\sqrt{13}}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{2-2x^2}\,dx,x,\sqrt{5+2x+x^2}\right)\\
&\quad +\text{Subst}\left(\int \frac{1}{-24-2x^2}\,dx,x,\frac{2+2x}{\sqrt{5+2x+x^2}}\right)\\
&= -\frac{\arctan\left(\frac{2+2x}{2\sqrt{3}\sqrt{5+2x+x^2}}\right)}{4\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{7+3x}{\sqrt{13}\sqrt{5+2x+x^2}}\right)}{12\sqrt{13}}+\frac{1}{12}\operatorname{arctanh}\left(\sqrt{5+2x+x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\begin{aligned}
\int \frac{1}{\sqrt{5+2x+x^2}\left(-8+x^3\right)}\,dx &= \frac{1}{156}\left(13\sqrt{3}\arctan\left(\frac{4+2x+x^2-(1+x)\sqrt{5+2x+x^2}}{\sqrt{3}}\right)\right.\\
&\quad +13\operatorname{arctanh}\left(\sqrt{5+2x+x^2}\right)\\
&\quad \left.-2\sqrt{13}\operatorname{arctanh}\left(\frac{2-x+\sqrt{5+2x+x^2}}{\sqrt{13}}\right)\right)
\end{aligned}$$

```
[In] Integrate[1/(Sqrt[5 + 2*x + x^2]*(-8 + x^3)),x]
```

```
[Out] (13*Sqrt[3]*ArcTan[(4 + 2*x + x^2 - (1 + x)*Sqrt[5 + 2*x + x^2])/Sqrt[3]] +
 13*ArcTanh[Sqrt[5 + 2*x + x^2]] - 2*Sqrt[13]*ArcTanh[(2 - x + Sqrt[5 + 2*x
 + x^2])/Sqrt[13]])/156
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\sqrt{13}\, \operatorname{arctanh}\left(\frac{(14+6x)\sqrt{13}}{26\sqrt{(-2+x)^{2}+1+6x}}\right)}{156}+\frac{\operatorname{arctanh}\left(\sqrt{x^{2}+2x+5}\right)}{12}-\frac{\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\,(2x+2)}{6\sqrt{x^{2}+2x+5}}\right)}{12}$
trager	$\operatorname{RootOf}\left(144\_Z^{2}-12\_Z+1\right) \ln\left(\frac{2880\operatorname{RootOf}\left(144\_Z^{2}-12\_Z+1\right)^{2}x-126\operatorname{RootOf}\left(144\_Z^{2}-12\_Z+1\right)\sqrt{x^{2}+2x}}{12\operatorname{RootOf}\left(1\right.}\right.$

[In] int(1/(x^3-8)/(x^2+2*x+5)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/156*13^(1/2)*arctanh(1/26*(14+6*x)*13^(1/2)/((-2+x)^2+1+6*x)^(1/2))+1/12*arctanh((x^2+2*x+5)^(1/2))-1/12*3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(1/2)*(2*x+2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(64) = 128$.

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sqrt{5+2x+x^{2}}\,(-8+x^{3})}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(x+2)+\frac{1}{3}\sqrt{3}\sqrt{x^{2}+2x+5}\right) - \frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{x^{2}+2x+5}\right) + \frac{1}{156}\sqrt{13}\log\left(\frac{\sqrt{13}(3x+7)+\sqrt{x^{2}+2x+5}(3\sqrt{13}-13)-9x-21}{x-2}\right) - \frac{1}{24}\log\left(x^{2}-\sqrt{x^{2}+2x+5}(x+2)+3x+6\right)+\frac{1}{24}\log\left(x^{2}-\sqrt{x^{2}+2x+5}x+x+4\right)$$

[In] integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")

[Out] 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x + 2) + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) - 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) + 1/156*sqrt(13)*log((sqrt(13)*(3*x + 7) + sqrt(x^2 + 2*x + 5)*(3*sqrt(13) - 13) - 9*x - 21)/(x - 2)) - 1/24*log(x^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) + 1/24*log(x^2 - sqrt(x^2 + 2*x + 5)*x + x + 4)

Sympy [F]

$$\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx = \int \frac{1}{(x-2)\,(x^2+2x+4)\,\sqrt{x^2+2x+5}}\,dx$$

```
[In] integrate(1/(x**3-8)/(x**2+2*x+5)**(1/2),x)
[Out] Integral(1/((x - 2)*(x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx = \int \frac{1}{(x^3-8)\sqrt{x^2+2\,x+5}}\,dx$$

```
[In] integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^3 - 8)*sqrt(x^2 + 2*x + 5)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(64) = 128.

Time = 0.32 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.00

$$\begin{aligned}\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx &= \frac{1}{12}\,\sqrt{3}\arctan\left(-\frac{1}{3}\,\sqrt{3}\left(x-\sqrt{x^2+2\,x+5}+2\right)\right)\\ &\quad-\frac{1}{12}\,\sqrt{3}\arctan\left(-\frac{1}{3}\,\sqrt{3}\left(x-\sqrt{x^2+2\,x+5}\right)\right)\\ &\quad+\frac{1}{156}\,\sqrt{13}\log\left(\frac{\left|-2\,x-2\,\sqrt{13}+2\,\sqrt{x^2+2\,x+5}+4\right|}{\left|-2\,x+2\,\sqrt{13}+2\,\sqrt{x^2+2\,x+5}+4\right|}\right)\\ &\quad-\frac{1}{24}\,\log\left(\left(x-\sqrt{x^2+2\,x+5}\right)^2+4\,x-4\,\sqrt{x^2+2\,x+5}\right.\\ &\qquad\left.+7\right)+\frac{1}{24}\,\log\left(\left(x-\sqrt{x^2+2\,x+5}\right)^2+3\right)\end{aligned}$$

```
[In] integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
[Out] 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) - 1/12*sqrt
(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5))) + 1/156*sqrt(13)*log(abs
(-2*x - 2*sqrt(13) + 2*sqrt(x^2 + 2*x + 5) + 4)/abs(-2*x + 2*sqrt(13) + 2*s
qrt(x^2 + 2*x + 5) + 4)) - 1/24*log((x - sqrt(x^2 + 2*x + 5))^2 + 4*x - 4*s
qrt(x^2 + 2*x + 5) + 7) + 1/24*log((x - sqrt(x^2 + 2*x + 5))^2 + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx = \int \frac{1}{(x^3-8)\,\sqrt{x^2+2\,x+5}}\,dx$$

```
[In] int(1/((x^3 - 8)*(2*x + x^2 + 5)^(1/2)),x)
[Out] int(1/((x^3 - 8)*(2*x + x^2 + 5)^(1/2)), x)
```

3.245 $\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{5+4x+4x^2}}{\sqrt{11}}\right)}{\sqrt{11}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{15}}(1+2x)}{\sqrt{5+4x+4x^2}}\right)}{\sqrt{165}}$$

```
[Out] 1/11*arctan(1/11*(4*x^2+4*x+5)^(1/2)*11^(1/2))*11^(1/2)-1/165*arctanh(1/15*
(1+2*x)*165^(1/2)/(4*x^2+4*x+5)^(1/2))*165^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1039, 996, 213, 1038, 210}

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{4x^2+4x+5}}{\sqrt{11}}\right)}{\sqrt{11}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{15}}(2x+1)}{\sqrt{4x^2+4x+5}}\right)}{\sqrt{165}}$$

```
[In] Int[x/((4 + x + x^2)*Sqrt[5 + 4*x + 4*x^2]),x]
```

```
[Out] ArcTan[Sqrt[5 + 4*x + 4*x^2]/Sqrt[11]]/Sqrt[11] - ArcTanh[(Sqrt[11/15]*(1 +
 2*x))/Sqrt[5 + 4*x + 4*x^2]]/Sqrt[165]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1038

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1039

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{8}\int \frac{4+8x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx - \frac{1}{2}\int \frac{1}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx \\
&= 4\text{Subst}\left(\int \frac{1}{-240+11x^2}\,dx, x, \frac{4+8x}{\sqrt{5+4x+4x^2}}\right) \\
&\quad - \text{Subst}\left(\int \frac{1}{-11-x^2}\,dx, x, \sqrt{5+4x+4x^2}\right) \\
&= \frac{\arctan\left(\frac{\sqrt{5+4x+4x^2}}{\sqrt{11}}\right)}{\sqrt{11}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{15}}(1+2x)}{\sqrt{5+4x+4x^2}}\right)}{\sqrt{165}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.62

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx = \frac{1}{2}\text{RootSum}\Big[69-108\#1+58\#1^2-4\#1^3 +\#1^4\&, \frac{-5\log\left(-2x+\sqrt{5+4x+4x^2}-\#1\right)+\log\left(-2x+\sqrt{5+4x+4x^2}-\#1\right)\#1^2}{-27+29\#1-3\#1^2+\#1^3}\&\Big]$$

```
[In] Integrate[x/((4 + x + x^2)*Sqrt[5 + 4*x + 4*x^2]),x]
```

```
[Out] RootSum[69 - 108*#1 + 58*#1^2 - 4*#1^3 + #1^4 & , (-5*Log[-2*x + Sqrt[5 + 4
*x + 4*x^2] - #1] + Log[-2*x + Sqrt[5 + 4*x + 4*x^2] - #1]*#1^2)/(-27 + 29*
#1 - 3*#1^2 + #1^3) & ]/2
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

method	result
default	$\frac{\arctan\left(\frac{\sqrt{4x^2+4x+5}\,\sqrt{11}}{11}\right)\sqrt{11}}{11}-\frac{\sqrt{165}\,\operatorname{arctanh}\left(\frac{\sqrt{165}\,(8x+4)}{60\sqrt{4x^2+4x+5}}\right)}{165}$
trager	$-\operatorname{RootOf}\left(27225\_Z^4+1155\_Z^2+16\right)\ln\left(\frac{3524400\operatorname{RootOf}\left(27225\_Z^4+1155\_Z^2+16\right)^5 x+111270\operatorname{RootOf}\left(27\right.}{}\right.$

```
[In] int(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*arctan(1/11*(4*x^2+4*x+5)^(1/2)*11^(1/2))*11^(1/2)-1/165*165^(1/2)*arc
tanh(1/60*165^(1/2)*(8*x+4)/(4*x^2+4*x+5)^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.76

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx$$
$$= \frac{1}{330}\sqrt{165}\sqrt{2i\sqrt{15}-14}\log\left(\sqrt{165}\sqrt{2i\sqrt{15}-14}\left(i\sqrt{15}+15\right)-480x-240i\sqrt{15}+240\sqrt{4x^2+4x+5}-240\right)$$
$$-\frac{1}{330}\sqrt{165}\sqrt{2i\sqrt{15}-14}\log\left(\sqrt{165}\sqrt{2i\sqrt{15}-14}\left(-i\sqrt{15}-15\right)-480x-240i\sqrt{15}+240\sqrt{4x^2+4x+5}-240\right)$$
$$-\frac{1}{330}\sqrt{165}\sqrt{-2i\sqrt{15}-14}\log\left(\sqrt{165}\left(i\sqrt{15}-15\right)\sqrt{-2i\sqrt{15}-14}-480x+240i\sqrt{15}+240\sqrt{4x^2+4x+5}-240\right)$$
$$+\frac{1}{330}\sqrt{165}\sqrt{-2i\sqrt{15}-14}\log\left(\sqrt{165}\left(-i\sqrt{15}+15\right)\sqrt{-2i\sqrt{15}-14}-480x+240i\sqrt{15}+240\sqrt{4x^2+4x+5}-240\right)$$

```
[In] integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/330*sqrt(165)*sqrt(2*I*sqrt(15) - 14)*log(sqrt(165)*sqrt(2*I*sqrt(15) - 1
4)*(I*sqrt(15) + 15) - 480*x - 240*I*sqrt(15) + 240*sqrt(4*x^2 + 4*x + 5) -
 240) - 1/330*sqrt(165)*sqrt(2*I*sqrt(15) - 14)*log(sqrt(165)*sqrt(2*I*sqrt
(15) - 14)*(-I*sqrt(15) - 15) - 480*x - 240*I*sqrt(15) + 240*sqrt(4*x^2 + 4
*x + 5) - 240) - 1/330*sqrt(165)*sqrt(-2*I*sqrt(15) - 14)*log(sqrt(165)*(I*
sqrt(15) - 15)*sqrt(-2*I*sqrt(15) - 14) - 480*x + 240*I*sqrt(15) + 240*sqrt
(4*x^2 + 4*x + 5) - 240) + 1/330*sqrt(165)*sqrt(-2*I*sqrt(15) - 14)*log(sqr
t(165)*(-I*sqrt(15) + 15)*sqrt(-2*I*sqrt(15) - 14) - 480*x + 240*I*sqrt(15)
 + 240*sqrt(4*x^2 + 4*x + 5) - 240)
```

Sympy [F]

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx = \int \frac{x}{(x^2+x+4)\sqrt{4x^2+4x+5}}\,dx$$

```
[In] integrate(x/(x**2+x+4)/(4*x**2+4*x+5)**(1/2),x)
[Out] Integral(x/((x**2 + x + 4)*sqrt(4*x**2 + 4*x + 5)), x)
```

Maxima [F]

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx = \int \frac{x}{\sqrt{4\,x^2+4\,x+5}(x^2+x+4)}\,dx$$

```
[In] integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(4*x^2 + 4*x + 5)*(x^2 + x + 4)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(52) = 104$.

Time = 0.33 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.62

$$\begin{aligned}\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx &= \frac{1}{165}\sqrt{165}\sqrt{15}\arctan\left(-\frac{2\,x-\sqrt{4\,x^2+4\,x+5}+1}{\sqrt{15}+\sqrt{11}}\right) \\ &\quad -\frac{1}{165}\sqrt{165}\sqrt{15}\arctan\left(-\frac{2\,x-\sqrt{4\,x^2+4\,x+5}+1}{\sqrt{15}-\sqrt{11}}\right) \\ &\quad -\frac{1}{330}\sqrt{165}\log\left(90000\left(2\,x-\sqrt{4\,x^2+4\,x+5}+1\right)^2 + 90000\left(\sqrt{15}+\sqrt{11}\right)^2\right) \\ &\quad +\frac{1}{330}\sqrt{165}\log\left(90000\left(2\,x-\sqrt{4\,x^2+4\,x+5}+1\right)^2 + 90000\left(\sqrt{15}-\sqrt{11}\right)^2\right)\end{aligned}$$

```
[In] integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="giac")
[Out] 1/165*sqrt(165)*sqrt(15)*arctan(-(2*x - sqrt(4*x^2 + 4*x + 5) + 1)/(sqrt(15
) + sqrt(11))) - 1/165*sqrt(165)*sqrt(15)*arctan(-(2*x - sqrt(4*x^2 + 4*x +
 5) + 1)/(sqrt(15) - sqrt(11))) - 1/330*sqrt(165)*log(90000*(2*x - sqrt(4*x
^2 + 4*x + 5) + 1)^2 + 90000*(sqrt(15) + sqrt(11))^2) + 1/330*sqrt(165)*log
(90000*(2*x - sqrt(4*x^2 + 4*x + 5) + 1)^2 + 90000*(sqrt(15) - sqrt(11))^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx = \int \frac{x}{\sqrt{4\,x^2+4\,x+5}\,\left(x^2+x+4\right)}\,dx$$

```
[In] int(x/((4*x + 4*x^2 + 5)^(1/2)*(x + x^2 + 4)),x)
[Out] int(x/((4*x + 4*x^2 + 5)^(1/2)*(x + x^2 + 4)), x)
```

3.246 $\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = -2\sqrt{2}\arctan\left(\frac{1-x}{\sqrt{2}\sqrt{1+x+x^2}}\right) + \sqrt{2}\text{arctanh}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x+x^2}}\right)$$

```
[Out] -2*arctan(1/2*(1-x)*2^(1/2)/(x^2+x+1)^(1/2))*2^(1/2)+arctanh(1/2*(1+x)*2^(1
/2)/(x^2+x+1)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1050, 1044, 213, 209}

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = \sqrt{2}\text{arctanh}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+x+1}}\right) - 2\sqrt{2}\arctan\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+x+1}}\right)$$

```
[In] Int[(3 + x)/((1 + x^2)*Sqrt[1 + x + x^2]),x]
```

```
[Out] -2*Sqrt[2]*ArcTan[(1 - x)/(Sqrt[2]*Sqrt[1 + x + x^2])] + Sqrt[2]*ArcTanh[(1
 + x)/(Sqrt[2]*Sqrt[1 + x + x^2])]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int \frac{-4-4x}{(1+x^2)\sqrt{1+x+x^2}}\,dx\right)+\frac{1}{2}\int \frac{2-2x}{(1+x^2)\sqrt{1+x+x^2}}\,dx\\ &= 4\text{Subst}\left(\int \frac{1}{-8+x^2}\,dx, x, \frac{-2-2x}{\sqrt{1+x+x^2}}\right)+16\text{Subst}\left(\int \frac{1}{32+x^2}\,dx, x, \frac{-4+4x}{\sqrt{1+x+x^2}}\right)\\ &= -2\sqrt{2}\arctan\left(\frac{1-x}{\sqrt{2}\sqrt{1+x+x^2}}\right)+\sqrt{2}\text{arctanh}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x+x^2}}\right)\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.84

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = \frac{1}{2}\text{RootSum}\Big[2-4\#1+2\#1^2 +\#1^4\&, \frac{2\log\left(-x+\sqrt{1+x+x^2}-\#1\right)-6\log\left(-x+\sqrt{1+x+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{1+x+x^2}\right.}{-1+\#1+\#1^3}$$

```
[In] Integrate[(3 + x)/((1 + x^2)*Sqrt[1 + x + x^2]),x]
```

```
[Out] RootSum[2 - 4*#1 + 2*#1^2 + #1^4 & , (2*Log[-x + Sqrt[1 + x + x^2] - #1] -
6*Log[-x + Sqrt[1 + x + x^2] - #1]*#1 + Log[-x + Sqrt[1 + x + x^2] - #1]*#1
^2)/(-1 + #1 + #1^3) & ]/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(46) = 92$.

Time = 1.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.29

method	result
default	$\frac{\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}\,\sqrt{2}\left(\operatorname{arctanh}\left(\frac{\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}\,\sqrt{2}}{2}\right)-2\arctan\left(\frac{\sqrt{2}\,(-1+x)}{\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}\,(-1-x)}\right)\right)}{\sqrt{\frac{\frac{(-1+x)^2}{(-1-x)^2}+3}{\left(\frac{-1+x}{-1-x}+1\right)^2}}\left(\frac{-1+x}{-1-x}+1\right)}$
trager	$\operatorname{RootOf}\left(4\_Z^4+12\_Z^2+25\right)\ln\left(-\frac{12\operatorname{RootOf}\left(4\_Z^4+12\_Z^2+25\right)^4 x+92x\operatorname{RootOf}\left(4\_Z^4+12\_Z^2+25\right)^2+64\,\mathrm{R}}{2x\operatorname{RootOf}\left(4\_Z\right.}\right.$

```
[In] int((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-1+x)^2/(-1-x)^2+3)^(1/2)*2^(1/2)*(arctanh(1/2*((-1+x)^2/(-1-x)^2+3)^(1/2
)*2^(1/2))-2*arctan(2^(1/2)/((-1+x)^2/(-1-x)^2+3)^(1/2)*(-1+x)/(-1-x)))/(((
-1+x)^2/(-1-x)^2+3)/((-1+x)/(-1-x)+1)^2)^(1/2)/((-1+x)/(-1-x)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.88

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = \frac{1}{2}\sqrt{8i-6}\log\left(-10\,x-(i-3)\,\sqrt{8i-6}+10\,\sqrt{x^2+x+1}+10i\right) - \frac{1}{2}\sqrt{8i-6}\log\left(-10\,x+(i-3)\,\sqrt{8i-6}+10\,\sqrt{x^2+x+1} + 10i\right) + \frac{1}{2}\sqrt{-8i-6}\log\left(-10\,x+(i+3)\,\sqrt{-8i-6} + 10\,\sqrt{x^2+x+1}-10i\right) - \frac{1}{2}\sqrt{-8i-6}\log\left(-10\,x - (i+3)\,\sqrt{-8i-6}+10\,\sqrt{x^2+x+1}-10i\right)$$

```
[In] integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(8*I - 6)*log(-10*x - (I - 3)*sqrt(8*I - 6) + 10*sqrt(x^2 + x + 1)
+ 10*I) - 1/2*sqrt(8*I - 6)*log(-10*x + (I - 3)*sqrt(8*I - 6) + 10*sqrt(x^2
 + x + 1) + 10*I) + 1/2*sqrt(-8*I - 6)*log(-10*x + (I + 3)*sqrt(-8*I - 6) +
 10*sqrt(x^2 + x + 1) - 10*I) - 1/2*sqrt(-8*I - 6)*log(-10*x - (I + 3)*sqrt
(-8*I - 6) + 10*sqrt(x^2 + x + 1) - 10*I)
```

Sympy [F]

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = \int \frac{x+3}{(x^2+1)\sqrt{x^2+x+1}}\,dx$$

```
[In] integrate((3+x)/(x**2+1)/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral((x + 3)/((x**2 + 1)*sqrt(x**2 + x + 1)), x)
```

Maxima [F]

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = \int \frac{x+3}{\sqrt{x^2+x+1}(x^2+1)}\,dx$$

```
[In] integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 3)/(sqrt(x^2 + x + 1)*(x^2 + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(44) = 88.

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.71

$$\begin{aligned}
&\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx \\
&= -\frac{1}{2}\sqrt{2}\left(\pi+4\arctan\left(-\left(x-\sqrt{x^2+x+1}\right)\left(\sqrt{2}+2\right)-\sqrt{2}-1\right)\right) \\
&\quad+\frac{1}{2}\sqrt{2}\left(\pi+4\arctan\left(\left(x-\sqrt{x^2+x+1}\right)\left(\sqrt{2}-2\right)+\sqrt{2}-1\right)\right) \\
&\quad-\frac{1}{2}\sqrt{2}\log\left(\left(x+\sqrt{2}-\sqrt{x^2+x+1}-1\right)^2+\left(x-\sqrt{x^2+x+1}+1\right)^2\right) \\
&\quad+\frac{1}{2}\sqrt{2}\log\left(\left(x-\sqrt{2}-\sqrt{x^2+x+1}-1\right)^2+\left(x-\sqrt{x^2+x+1}+1\right)^2\right)
\end{aligned}$$

```
[In] integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(pi + 4*arctan(-(x - sqrt(x^2 + x + 1))*(sqrt(2) + 2) - sqrt(2
) - 1)) + 1/2*sqrt(2)*(pi + 4*arctan((x - sqrt(x^2 + x + 1))*(sqrt(2) - 2)
+ sqrt(2) - 1)) - 1/2*sqrt(2)*log((x + sqrt(2) - sqrt(x^2 + x + 1) - 1)^2 +
 (x - sqrt(x^2 + x + 1) + 1)^2) + 1/2*sqrt(2)*log((x - sqrt(2) - sqrt(x^2 +
 x + 1) - 1)^2 + (x - sqrt(x^2 + x + 1) + 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = \int \frac{x+3}{(x^2+1)\sqrt{x^2+x+1}}\,dx$$

```
[In] int((x + 3)/((x^2 + 1)*(x + x^2 + 1)^(1/2)),x)
```

```
[Out] int((x + 3)/((x^2 + 1)*(x + x^2 + 1)^(1/2)), x)
```

3.247 $\int \frac{1+2x}{\sqrt{-1+6x+x^2}(4+4x+3x^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 70

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx = -\frac{5\arctan\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{-1+6x+x^2}}\right)}{6\sqrt{14}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{7}(1+x)}{\sqrt{-1+6x+x^2}}\right)}{3\sqrt{7}}$$

```
[Out] -1/21*arctanh((1+x)*7^(1/2)/(x^2+6*x-1)^(1/2))*7^(1/2)-5/84*arctan(1/4*(2-x
)*7^(1/2)*2^(1/2)/(x^2+6*x-1)^(1/2))*14^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1049, 1043, 213, 209}

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx = -\frac{5\arctan\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{x^2+6x-1}}\right)}{6\sqrt{14}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{7}(x+1)}{\sqrt{x^2+6x-1}}\right)}{3\sqrt{7}}$$

```
[In] Int[(1 + 2*x)/(Sqrt[-1 + 6*x + x^2]*(4 + 4*x + 3*x^2)),x]
```

```
[Out] (-5*ArcTan[(Sqrt[7/2]*(2 - x))/(2*Sqrt[-1 + 6*x + x^2])])/(6*Sqrt[14]) - Ar
cTanh[(Sqrt[7]*(1 + x))/Sqrt[-1 + 6*x + x^2]]/(3*Sqrt[7])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{42}\int \frac{-70-70x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx\right) \\
&\quad +\frac{1}{42}\int \frac{-28+14x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx \\
&= -\left(\frac{896}{3}\text{Subst}\left(\int \frac{1}{-200704+28x^2}\,dx, x, \frac{-224-224x}{\sqrt{-1+6x+x^2}}\right)\right) \\
&\quad -\frac{2800}{3}\text{Subst}\left(\int \frac{1}{627200+28x^2}\,dx, x, \frac{280-140x}{\sqrt{-1+6x+x^2}}\right) \\
&= -\frac{5\arctan\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{-1+6x+x^2}}\right)}{6\sqrt{14}}-\frac{\text{arctanh}\left(\frac{\sqrt{7}(1+x)}{\sqrt{-1+6x+x^2}}\right)}{3\sqrt{7}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.76

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx = \text{RootSum}\Big[171-104\#1+46\#1^2-8\#1^3 +3\#1^4\&, \frac{4\log\left(-x+\sqrt{-1+6x+x^2}-\#1\right)-\log\left(-x+\sqrt{-1+6x+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{-1+\right.}{-26+23\#1-6\#1^2+3\#1^3}$$

```
[In] Integrate[(1 + 2*x)/(Sqrt[-1 + 6*x + x^2]*(4 + 4*x + 3*x^2)),x]
```

```
[Out] RootSum[171 - 104*#1 + 46*#1^2 - 8*#1^3 + 3*#1^4 & , (4*Log[-x + Sqrt[-1 +
6*x + x^2] - #1] - Log[-x + Sqrt[-1 + 6*x + x^2] - #1]*#1 + Log[-x + Sqrt[-
1 + 6*x + x^2] - #1]*#1^2)/(-26 + 23*#1 - 6*#1^2 + 3*#1^3) & ]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(53) = 106.

Time = 1.65 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.26

method	result
default	$\frac{\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2}+15}\left(5\sqrt{14}\arctan\left(\frac{\sqrt{14}\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2}+15}\,(-2+x)}{4\left(\frac{2(-2+x)^2}{(-1-x)^2}-5\right)(-1-x)}\right)-4\sqrt{7}\,\text{arctanh}\left(\frac{\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2}+15}\,\sqrt{7}}{21}\right)\right)}{84\sqrt{-\frac{3\left(\frac{2(-2+x)^2}{(-1-x)^2}-5\right)}{\left(\frac{-2+x}{-1-x}+1\right)^2}}\left(\frac{-2+x}{-1-x}+1\right)}$
trager	$-\frac{672\ln\left(\frac{159860736\,\text{RootOf}\left(451584\_Z^4+7616\_Z^2+121\right)^5 x+2221632\,\text{RootOf}\left(451584\_Z^4+7616\_Z^2+121\right)^3 x-4044096\,\text{RootOf}\left(451584\_\right.}{}\right.}{}$

```
[In] int((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/84*(-6*(-2+x)^2/(-1-x)^2+15)^(1/2)*(5*14^(1/2)*arctan(1/4*14^(1/2)*(-6*(-
2+x)^2/(-1-x)^2+15)^(1/2)/(2*(-2+x)^2/(-1-x)^2-5)*(-2+x)/(-1-x))-4*7^(1/2)*
arctanh(1/21*(-6*(-2+x)^2/(-1-x)^2+15)^(1/2)*7^(1/2)))/(-3*(2*(-2+x)^2/(-1-
x)^2-5)/((-2+x)/(-1-x)+1)^2)^(1/2)/((-2+x)/(-1-x)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.27

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx$$
$$= \frac{1}{168}\sqrt{14}\sqrt{20i\sqrt{2}-17}\log\left(\sqrt{14}\sqrt{20i\sqrt{2}-17}\left(13i\sqrt{2}+16\right)-198x-132i\sqrt{2}+198\sqrt{x^2+6x-1}-132\right)$$
$$-\frac{1}{168}\sqrt{14}\sqrt{20i\sqrt{2}-17}\log\left(\sqrt{14}\sqrt{20i\sqrt{2}-17}\left(-13i\sqrt{2}-16\right)-198x-132i\sqrt{2}+198\sqrt{x^2+6x-1}-132\right)$$
$$-\frac{1}{168}\sqrt{14}\sqrt{-20i\sqrt{2}-17}\log\left(\sqrt{14}\left(13i\sqrt{2}-16\right)\sqrt{-20i\sqrt{2}-17}-198x+132i\sqrt{2}+198\sqrt{x^2+6x-1}-132\right)$$
$$+\frac{1}{168}\sqrt{14}\sqrt{-20i\sqrt{2}-17}\log\left(\sqrt{14}\left(-13i\sqrt{2}+16\right)\sqrt{-20i\sqrt{2}-17}-198x+132i\sqrt{2}+198\sqrt{x^2+6x-1}-132\right)$$

```
[In] integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/168*sqrt(14)*sqrt(20*I*sqrt(2) - 17)*log(sqrt(14)*sqrt(20*I*sqrt(2) - 17)
*(13*I*sqrt(2) + 16) - 198*x - 132*I*sqrt(2) + 198*sqrt(x^2 + 6*x - 1) - 13
2) - 1/168*sqrt(14)*sqrt(20*I*sqrt(2) - 17)*log(sqrt(14)*sqrt(20*I*sqrt(2)
- 17)*(-13*I*sqrt(2) - 16) - 198*x - 132*I*sqrt(2) + 198*sqrt(x^2 + 6*x - 1
) - 132) - 1/168*sqrt(14)*sqrt(-20*I*sqrt(2) - 17)*log(sqrt(14)*(13*I*sqrt(
2) - 16)*sqrt(-20*I*sqrt(2) - 17) - 198*x + 132*I*sqrt(2) + 198*sqrt(x^2 +
6*x - 1) - 132) + 1/168*sqrt(14)*sqrt(-20*I*sqrt(2) - 17)*log(sqrt(14)*(-13
*I*sqrt(2) + 16)*sqrt(-20*I*sqrt(2) - 17) - 198*x + 132*I*sqrt(2) + 198*sqr
t(x^2 + 6*x - 1) - 132)
```

Sympy [F]

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx=\int \frac{2x+1}{\sqrt{x^2+6x-1}\cdot(3x^2+4x+4)}\,dx$$

```
[In] integrate((1+2*x)/(3*x**2+4*x+4)/(x**2+6*x-1)**(1/2),x)
[Out] Integral((2*x + 1)/(sqrt(x**2 + 6*x - 1)*(3*x**2 + 4*x + 4)), x)
```

Maxima [F]

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx=\int \frac{2\,x+1}{(3\,x^2+4\,x+4)\sqrt{x^2+6\,x-1}}\,dx$$

```
[In] integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x + 1)/((3*x^2 + 4*x + 4)*sqrt(x^2 + 6*x - 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(51) = 102$.

Time = 0.31 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.67

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx=-\frac{5}{84}\sqrt{7}\sqrt{2}\left(\arctan(2)+\arctan\left(\frac{1}{8}\left(x-\sqrt{x^2+6\,x-1}\right)\left(\sqrt{14}+\sqrt{2}\right)+\frac{1}{8}\sqrt{14}+\frac{3}{8}\sqrt{2}\right)\right)+\frac{5}{84}\sqrt{7}\sqrt{2}\left(\arctan\left(\frac{1}{2}\right)+\arctan\left(-\frac{1}{8}\left(x-\sqrt{x^2+6\,x-1}\right)\left(\sqrt{14}-\sqrt{2}\right)-\frac{1}{8}\sqrt{14}+\frac{3}{8}\sqrt{2}\right)\right)+\frac{1}{42}\sqrt{7}\log\left(4\left(4\sqrt{7}\sqrt{2}+3\,x+\sqrt{7}-4\sqrt{2}-3\sqrt{x^2+6\,x-1}+2\right)^2+16\left(\sqrt{7}\sqrt{2}-3\,x-\sqrt{7}-\sqrt{2}+3\sqrt{x^2+6\,x-1}-2\right)^2\right)-\frac{1}{42}\sqrt{7}\log\left(4\left(4\sqrt{7}\sqrt{2}+3\,x-\sqrt{7}+4\sqrt{2}-3\sqrt{x^2+6\,x-1}+2\right)^2+16\left(\sqrt{7}\sqrt{2}-3\,x+\sqrt{7}+\sqrt{2}+3\sqrt{x^2+6\,x-1}-2\right)^2\right)$$

```
[In] integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="giac")
[Out] -5/84*sqrt(7)*sqrt(2)*(arctan(2) + arctan(1/8*(x - sqrt(x^2 + 6*x - 1))*(sq
rt(14) + sqrt(2)) + 1/8*sqrt(14) + 3/8*sqrt(2))) + 5/84*sqrt(7)*sqrt(2)*(ar
```

```
ctan(1/2) + arctan(-1/8*(x - sqrt(x^2 + 6*x - 1))*(sqrt(14) - sqrt(2)) - 1/
8*sqrt(14) + 3/8*sqrt(2))) + 1/42*sqrt(7)*log(4*(4*sqrt(7)*sqrt(2) + 3*x +
sqrt(7) - 4*sqrt(2) - 3*sqrt(x^2 + 6*x - 1) + 2)^2 + 16*(sqrt(7)*sqrt(2) -
3*x - sqrt(7) - sqrt(2) + 3*sqrt(x^2 + 6*x - 1) - 2)^2) - 1/42*sqrt(7)*log(
4*(4*sqrt(7)*sqrt(2) + 3*x - sqrt(7) + 4*sqrt(2) - 3*sqrt(x^2 + 6*x - 1) +
2)^2 + 16*(sqrt(7)*sqrt(2) - 3*x + sqrt(7) + sqrt(2) + 3*sqrt(x^2 + 6*x - 1
) - 2)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx=\int \frac{2\,x+1}{\sqrt{x^2+6\,x-1}\,\left(3\,x^2+4\,x+4\right)}\,dx$$

```
[In] int((2*x + 1)/((6*x + x^2 - 1)^(1/2)*(4*x + 3*x^2 + 4)),x)
[Out] int((2*x + 1)/((6*x + x^2 - 1)^(1/2)*(4*x + 3*x^2 + 4)), x)
```

3.248 $\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 80

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = -\frac{(2A+B)\arctan\left(\frac{\sqrt{35}(2-x)}{\sqrt{13-22x+10x^2}}\right)}{\sqrt{35}} - \frac{(A+B)\text{arctanh}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{13-22x+10x^2}}\right)}{2\sqrt{35}}$$

```
[Out] -1/35*(2*A+B)*arctan((2-x)*35^(1/2)/(10*x^2-22*x+13)^(1/2))*35^(1/2)-1/70*(
A+B)*arctanh(1/2*(1-x)*35^(1/2)/(10*x^2-22*x+13)^(1/2))*35^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1049, 1043, 212, 210}

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = -\frac{(2A+B)\arctan\left(\frac{\sqrt{35}(2-x)}{\sqrt{10x^2-22x+13}}\right)}{\sqrt{35}} - \frac{(A+B)\text{arctanh}\left(\frac{\sqrt{35}(-x(A+B)+A+B)}{2\sqrt{10x^2-22x+13}(A+B)}\right)}{2\sqrt{35}}$$

```
[In] Int[(B + A*x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

```
[Out] -(((2*A + B)*ArcTan[(Sqrt[35]*(2 - x))/Sqrt[13 - 22*x + 10*x^2]])/Sqrt[35])
 - ((A + B)*ArcTanh[(Sqrt[35]*(A + B - (A + B)*x))/(2*(A + B)*Sqrt[13 - 22*
x + 10*x^2])])/(2*Sqrt[35])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{70}\int \frac{140(A+B)-70(A+B)x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx\\ &\quad -\frac{1}{70}\int \frac{70(2A+B)-70(2A+B)x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx\end{aligned}$$

$$= (560(A + B)^2)\,\text{Subst}\left(\int \frac{1}{313600(A+B)^2 - 140x^2}\,dx, x, \frac{-140(A+B) + 140(A+B)x}{\sqrt{13-22x+10x^2}}\right) + (2240(2A + B)^2)\,\text{Subst}\left(\int \frac{1}{-1254400(2A+B)^2 - 140x^2}\,dx, x, \frac{1120(2A+B) - 560(2A+B)x}{\sqrt{13-22x+10x^2}}\right)$$

$$= -\frac{(2A+B)\arctan\left(\frac{\sqrt{35}(2-x)}{\sqrt{13-22x+10x^2}}\right)}{\sqrt{35}} - \frac{(A+B)\text{arctanh}\left(\frac{\sqrt{35}(A+B-(A+B)x)}{2(A+B)\sqrt{13-22x+10x^2}}\right)}{2\sqrt{35}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.55

$$\int \frac{B + Ax}{(17 - 18x + 5x^2)\sqrt{13 - 22x + 10x^2}}\,dx = \frac{((1+4i)A + (1+2i)B)\text{arctanh}\left(\frac{(4-i)\sqrt{10}-(2-i)\sqrt{10}x+(2-i)\sqrt{13-22x+10x^2}}{\sqrt{35}}\right) + ((1-4i)A + (1-2i)B)\text{arctanh}\left(\cdot\right.}{2\sqrt{35}}$$

```
[In] Integrate[(B + A*x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

```
[Out] (((1 + 4*I)*A + (1 + 2*I)*B)*ArcTanh[((4 - I)*Sqrt[10] - (2 - I)*Sqrt[10]*x
 + (2 - I)*Sqrt[13 - 22*x + 10*x^2])/Sqrt[35]] + ((1 - 4*I)*A + (1 - 2*I)*B
)*ArcTanh[((4 + I)*Sqrt[10] - (2 + I)*Sqrt[10]*x + (2 + I)*Sqrt[13 - 22*x +
 10*x^2])/Sqrt[35]])/(2*Sqrt[35])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(64) = 128.

Time = 0.94 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.40

method	result
default	$\frac{\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}\left(\text{arctanh}\left(\frac{2\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}}{35}\right)A - 4\arctan\left(\frac{\sqrt{35}\,(-2+x)}{\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,(1-x)}\right)A + \text{arctanh}\left(\frac{2\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}}{35}\right)B - 2\arctan\right.}{70\sqrt{\frac{\frac{(-2+x)^2}{(1-x)^2}+9}{\left(\frac{-2+x}{1-x}+1\right)^2}}\left(\frac{-2+x}{1-x}+1\right)}$

```
[In] int((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/70*((-2+x)^2/(1-x)^2+9)^(1/2)*35^(1/2)*(arctanh(2/35*((-2+x)^2/(1-x)^2+9)
^(1/2)*35^(1/2))*A-4*arctan(35^(1/2)/((-2+x)^2/(1-x)^2+9)^(1/2)*(-2+x)/(1-x
))*A+arctanh(2/35*((-2+x)^2/(1-x)^2+9)^(1/2)*35^(1/2))*B-2*arctan(35^(1/2)/
((-2+x)^2/(1-x)^2+9)^(1/2)*(-2+x)/(1-x))*B)/(((-2+x)^2/(1-x)^2+9)/((-2+x)/(
1-x)+1)^2)^(1/2)/((-2+x)/(1-x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1339 vs. $2(61) = 122$.

Time = 0.34 (sec) , antiderivative size = 1339, normalized size of antiderivative = 16.74

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/280*sqrt(35)*sqrt(-15*A^2 - 14*A*B - 3*B^2 + 4*sqrt(-4*A^4 - 12*A^3*B - 1
3*A^2*B^2 - 6*A*B^3 - B^4))*log(-(2380*A^4 + 6090*A^3*B + 5670*A^2*B^2 + 23
10*A*B^3 + 350*B^4 + (sqrt(35)*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 - 6*A*B^
3 - B^4)*(15*A + 7*B) - 2*sqrt(35)*(8*A^3 + 18*A^2*B + 13*A*B^2 + 3*B^3))*s
qrt(-15*A^2 - 14*A*B - 3*B^2 + 4*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 - 6*A*
B^3 - B^4))*sqrt(10*x^2 - 22*x + 13) - 71*(34*A^4 + 87*A^3*B + 81*A^2*B^2 +
 33*A*B^3 + 5*B^4)*x + 2*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 - 6*A*B^3 - B^
4)*(221*A^2 + 234*A*B + 65*B^2 - 11*(17*A^2 + 18*A*B + 5*B^2)*x))/x) - 1/28
0*sqrt(35)*sqrt(-15*A^2 - 14*A*B - 3*B^2 + 4*sqrt(-4*A^4 - 12*A^3*B - 13*A^
2*B^2 - 6*A*B^3 - B^4))*log(-(2380*A^4 + 6090*A^3*B + 5670*A^2*B^2 + 2310*A
*B^3 + 350*B^4 - (sqrt(35)*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 - 6*A*B^3 -
B^4)*(15*A + 7*B) - 2*sqrt(35)*(8*A^3 + 18*A^2*B + 13*A*B^2 + 3*B^3))*sqrt(
-15*A^2 - 14*A*B - 3*B^2 + 4*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 - 6*A*B^3
- B^4))*sqrt(10*x^2 - 22*x + 13) - 71*(34*A^4 + 87*A^3*B + 81*A^2*B^2 + 33*
A*B^3 + 5*B^4)*x + 2*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 - 6*A*B^3 - B^4)*(
221*A^2 + 234*A*B + 65*B^2 - 11*(17*A^2 + 18*A*B + 5*B^2)*x))/x) - 1/280*sq
rt(35)*sqrt(-15*A^2 - 14*A*B - 3*B^2 - 4*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^
2 - 6*A*B^3 - B^4))*log(-(2380*A^4 + 6090*A^3*B + 5670*A^2*B^2 + 2310*A*B^3
 + 350*B^4 + (sqrt(35)*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 - 6*A*B^3 - B^4)
*(15*A + 7*B) + 2*sqrt(35)*(8*A^3 + 18*A^2*B + 13*A*B^2 + 3*B^3))*sqrt(-15*
A^2 - 14*A*B - 3*B^2 - 4*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 - 6*A*B^3 - B^
4))*sqrt(10*x^2 - 22*x + 13) - 71*(34*A^4 + 87*A^3*B + 81*A^2*B^2 + 33*A*B^
3 + 5*B^4)*x - 2*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 - 6*A*B^3 - B^4)*(221*
A^2 + 234*A*B + 65*B^2 - 11*(17*A^2 + 18*A*B + 5*B^2)*x))/x) + 1/280*sqrt(3
5)*sqrt(-15*A^2 - 14*A*B - 3*B^2 - 4*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 -
6*A*B^3 - B^4))*log(-(2380*A^4 + 6090*A^3*B + 5670*A^2*B^2 + 2310*A*B^3 + 3
50*B^4 - (sqrt(35)*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 - 6*A*B^3 - B^4)*(15
```

```
*A + 7*B) + 2*sqrt(35)*(8*A^3 + 18*A^2*B + 13*A*B^2 + 3*B^3))*sqrt(-15*A^2
- 14*A*B - 3*B^2 - 4*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 - 6*A*B^3 - B^4))*
sqrt(10*x^2 - 22*x + 13) - 71*(34*A^4 + 87*A^3*B + 81*A^2*B^2 + 33*A*B^3 +
5*B^4)*x - 2*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 - 6*A*B^3 - B^4)*(221*A^2
+ 234*A*B + 65*B^2 - 11*(17*A^2 + 18*A*B + 5*B^2)*x))/x)
```

Sympy [F]

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx=\int \frac{Ax+B}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}\,dx$$

```
[In] integrate((A*x+B)/(5*x**2-18*x+17)/(10*x**2-22*x+13)**(1/2),x)
[Out] Integral((A*x + B)/((5*x**2 - 18*x + 17)*sqrt(10*x**2 - 22*x + 13)), x)
```

Maxima [F]

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx=\int \frac{Ax+B}{\sqrt{10\,x^2-22\,x+13}(5\,x^2-18\,x+17)}\,dx$$

```
[In] integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="maxi
ma")
[Out] integrate((A*x + B)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(61) = 122.

Time = 0.42 (sec) , antiderivative size = 629, normalized size of antiderivative = 7.86

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$$

$$= \frac{2\sqrt{35}(2A^2+3AB+B^2)\sqrt{A^2+2AB+B^2}\left(\arctan(3)+\arctan\left(-\frac{5\left(\sqrt{10}x-\sqrt{10x^2-22x+13}\right)\left(300\sqrt{14}-1129\right)-}{2329\sqrt{35}-4358\sqrt{10}}\right.\right.}{35\left(15A^2+14AB+3B^2-\sqrt{289A^4+612A^3B+494A^2B^2+180AB^3+25B^4}\right)}$$

$$-\frac{2\sqrt{35}(2A^2+3AB+B^2)\sqrt{A^2+2AB+B^2}\left(\arctan\left(\frac{1}{7}\right)+\arctan\left(-\frac{5\left(\sqrt{10}x-\sqrt{10x^2-22x+13}\right)\left(62556\sqrt{14}+2\right.}{496201\sqrt{35}+92}\right.\right.}{35\left(15A^2+14AB+3B^2-\sqrt{289A^4+612A^3B+494A^2B^2+180AB^3+2\right.}}$$

$$+\frac{1}{140}\sqrt{35}\sqrt{A^2+2AB+B^2}\log\left(25\left(546\sqrt{14}\left(\sqrt{10}x-\sqrt{10x^2-22x+13}\right)+2807\sqrt{10}x-234\sqrt{35}\right.\right.$$

$$+25\left(78\sqrt{14}\left(\sqrt{10}x-\sqrt{10x^2-22x+13}\right)+401\sqrt{10}x+48\sqrt{35}\sqrt{14}+208\sqrt{14}\sqrt{10}+141\sqrt{35}+611\right.$$

$$-\frac{1}{140}\sqrt{35}\sqrt{A^2+2AB+B^2}\log\left(625\left(18\sqrt{14}\left(\sqrt{10}x-\sqrt{10x^2-22x+13}\right)-75\sqrt{10}x+8\sqrt{35}\sqrt{14}-\right.\right.$$

$$+625\left(6\sqrt{14}\left(\sqrt{10}x-\sqrt{10x^2-22x+13}\right)-25\sqrt{10}x+6\sqrt{35}\sqrt{14}-18\sqrt{14}\sqrt{10}-25\sqrt{35}+75\sqrt{10}-\right.$$

```
[In] integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="giac
")
```

```
[Out] 2/35*sqrt(35)*(2*A^2 + 3*A*B + B^2)*sqrt(A^2 + 2*A*B + B^2)*(arctan(3) + ar
ctan(-(5*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13))*(300*sqrt(14) - 1129) - 76
58*sqrt(35) + 14361*sqrt(10))/(2329*sqrt(35) - 4358*sqrt(10))))/(15*A^2 + 1
4*A*B + 3*B^2 - sqrt(289*A^4 + 612*A^3*B + 494*A^2*B^2 + 180*A*B^3 + 25*B^4
)) - 2/35*sqrt(35)*(2*A^2 + 3*A*B + B^2)*sqrt(A^2 + 2*A*B + B^2)*(arctan(1/
7) + arctan(-(5*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13))*(62556*sqrt(14) + 2
45977) - 1617962*sqrt(35) - 3089577*sqrt(10))/(496201*sqrt(35) + 929846*sqr
t(10))))/(15*A^2 + 14*A*B + 3*B^2 - sqrt(289*A^4 + 612*A^3*B + 494*A^2*B^2
+ 180*A*B^3 + 25*B^4)) + 1/140*sqrt(35)*sqrt(A^2 + 2*A*B + B^2)*log(25*(546
*sqrt(14)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) + 2807*sqrt(10)*x - 234*s
qrt(35)*sqrt(14) - 1014*sqrt(14)*sqrt(10) - 1203*sqrt(35) - 5213*sqrt(10) -
 2807*sqrt(10*x^2 - 22*x + 13))^2 + 25*(78*sqrt(14)*(sqrt(10)*x - sqrt(10*x
^2 - 22*x + 13)) + 401*sqrt(10)*x + 48*sqrt(35)*sqrt(14) + 208*sqrt(14)*sqr
t(10) + 141*sqrt(35) + 611*sqrt(10) - 401*sqrt(10*x^2 - 22*x + 13))^2) - 1/
140*sqrt(35)*sqrt(A^2 + 2*A*B + B^2)*log(625*(18*sqrt(14)*(sqrt(10)*x - sqr
t(10*x^2 - 22*x + 13)) - 75*sqrt(10)*x + 8*sqrt(35)*sqrt(14) - 24*sqrt(14)*
sqrt(10) - 37*sqrt(35) + 111*sqrt(10) + 75*sqrt(10*x^2 - 22*x + 13))^2 + 62
5*(6*sqrt(14)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) - 25*sqrt(10)*x + 6*s
```

```
qrt(35)*sqrt(14) - 18*sqrt(14)*sqrt(10) - 25*sqrt(35) + 75*sqrt(10) + 25*sq
rt(10*x^2 - 22*x + 13))^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx=\int \frac{B+A\,x}{(5\,x^2-18\,x+17)\,\sqrt{10\,x^2-22\,x+13}}\,dx$$

```
[In] int((B + A*x)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)),x)
[Out] int((B + A*x)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)), x)
```

3.249 $\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{13-22x+10x^2}}\right)}{2\sqrt{35}}$$

```
[Out] 1/70*arctanh(1/2*(1-x)*35^(1/2)/(10*x^2-22*x+13)^(1/2))*35^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1043, 212}

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{10x^2-22x+13}}\right)}{2\sqrt{35}}$$

```
[In] Int[(-2 + x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
[Out] ArcTanh[(Sqrt[35]*(1 - x))/(2*Sqrt[13 - 22*x + 10*x^2])]/(2*Sqrt[35])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
```

```
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 8\text{Subst}\left(\int \frac{1}{64-140x^2}\,dx, x, \frac{2-2x}{\sqrt{13-22x+10x^2}}\right)\\ &= \frac{\operatorname{arctanh}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{13-22x+10x^2}}\right)}{2\sqrt{35}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 85 vs. $2(38) = 76$.

Time = 0.57 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.24

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{-135+145x-50x^2+\sqrt{10}(-9+5x)\sqrt{13-22x+10x^2}}{-20\sqrt{14}+10\sqrt{14}x-2\sqrt{35}\sqrt{13-22x+10x^2}}\right)}{2\sqrt{35}}$$

```
[In] Integrate[(-2 + x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
[Out] -1/2*ArcTanh[(-135 + 145*x - 50*x^2 + Sqrt[10]*(-9 + 5*x)*Sqrt[13 - 22*x +
10*x^2])/(-20*Sqrt[14] + 10*Sqrt[14]*x - 2*Sqrt[35]*Sqrt[13 - 22*x + 10*x^2
])]/Sqrt[35]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.54 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-35\right)\ln\left(-\frac{75\operatorname{RootOf}\left(\_Z^2-35\right)x^2-158\operatorname{RootOf}\left(\_Z^2-35\right)x+140\sqrt{10x^2-22x+13}\,x+87\operatorname{RootOf}\left(\_Z^2-35\right)-140\sqrt{10x^2}}{5x^2-18x+17}\right.}{140}$
default	$-\frac{\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}\operatorname{arctanh}\left(\frac{2\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}}{35}\right)}{70\sqrt{\frac{\frac{(-2+x)^2}{(1-x)^2}+9}{\left(\frac{-2+x}{1-x}+1\right)^2}}\left(\frac{-2+x}{1-x}+1\right)}$

[In] int((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/140*RootOf(_Z^2-35)*ln(-(75*RootOf(_Z^2-35)*x^2-158*RootOf(_Z^2-35)*x+140*(10*x^2-22*x+13)^(1/2)*x+87*RootOf(_Z^2-35)-140*(10*x^2-22*x+13)^(1/2))/(5*x^2-18*x+17))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(26) = 52$.

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.13

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$$
$$=\frac{1}{280}\sqrt{35}\log\left(\frac{11225\,x^4-47220\,x^3-8\sqrt{35}(75\,x^3-233\,x^2+245\,x-87)\sqrt{10\,x^2-22\,x+13}+75534\,x^2}{25\,x^4-180\,x^3+494\,x^2-612\,x+289}\right.$$

[In] integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="fricas")

[Out] 1/280*sqrt(35)*log((11225*x^4 - 47220*x^3 - 8*sqrt(35)*(75*x^3 - 233*x^2 + 245*x - 87)*sqrt(10*x^2 - 22*x + 13) + 75534*x^2 - 54372*x + 14849)/(25*x^4 - 180*x^3 + 494*x^2 - 612*x + 289))

Sympy [F]

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx=\int \frac{x-2}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}\,dx$$

[In] integrate((-2+x)/(5*x**2-18*x+17)/(10*x**2-22*x+13)**(1/2),x)

[Out] Integral((x - 2)/((5*x**2 - 18*x + 17)*sqrt(10*x**2 - 22*x + 13)), x)

Maxima [F]

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \int \frac{x-2}{\sqrt{10\,x^2-22\,x+13}(5\,x^2-18\,x+17)}\,dx$$

```
[In] integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="maxim
a")
[Out] integrate((x - 2)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \text{Timed out}$$

```
[In] integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="giac"
)
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \int \frac{x-2}{(5\,x^2-18\,x+17)\,\sqrt{10\,x^2-22\,x+13}}\,dx$$

```
[In] int((x - 2)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)),x)
[Out] int((x - 2)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)), x)
```

3.250 $\int x^4\sqrt{5-x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int x^4\sqrt{5-x^2}\,dx=-\frac{25}{16}x\sqrt{5-x^2}-\frac{5}{24}x^3\sqrt{5-x^2}+\frac{1}{6}x^5\sqrt{5-x^2}+\frac{125}{16}\arcsin\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] 125/16*arcsin(1/5*x*5^(1/2))-25/16*x*(-x^2+5)^(1/2)-5/24*x^3*(-x^2+5)^(1/2)
+1/6*x^5*(-x^2+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {285, 327, 222}

$$\int x^4\sqrt{5-x^2}\,dx=\frac{125}{16}\arcsin\left(\frac{x}{\sqrt{5}}\right)-\frac{25}{16}\sqrt{5-x^2}x+\frac{1}{6}\sqrt{5-x^2}x^5-\frac{5}{24}\sqrt{5-x^2}x^3$$

```
[In] Int[x^4*Sqrt[5 - x^2],x]
```

```
[Out] (-25*x*Sqrt[5 - x^2])/16 - (5*x^3*Sqrt[5 - x^2])/24 + (x^5*Sqrt[5 - x^2])/6
 + (125*ArcSin[x/Sqrt[5]])/16
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
```

```
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{6}x^5\sqrt{5-x^2}+\frac{5}{6}\int \frac{x^4}{\sqrt{5-x^2}}\,dx\\ &= -\frac{5}{24}x^3\sqrt{5-x^2}+\frac{1}{6}x^5\sqrt{5-x^2}+\frac{25}{8}\int \frac{x^2}{\sqrt{5-x^2}}\,dx\\ &= -\frac{25}{16}x\sqrt{5-x^2}-\frac{5}{24}x^3\sqrt{5-x^2}+\frac{1}{6}x^5\sqrt{5-x^2}+\frac{125}{16}\int \frac{1}{\sqrt{5-x^2}}\,dx\\ &= -\frac{25}{16}x\sqrt{5-x^2}-\frac{5}{24}x^3\sqrt{5-x^2}+\frac{1}{6}x^5\sqrt{5-x^2}+\frac{125}{16}\arcsin\left(\frac{x}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x^4\sqrt{5-x^2}\,dx=\frac{1}{48}x\sqrt{5-x^2}\left(-75-10x^2+8x^4\right)-\frac{125}{8}\arctan\left(\frac{x}{\sqrt{5}-\sqrt{5-x^2}}\right)$$

```
[In] Integrate[x^4*Sqrt[5 - x^2],x]
[Out] (x*Sqrt[5 - x^2]*(-75 - 10*x^2 + 8*x^4))/48 - (125*ArcTan[x/(Sqrt[5] - Sqrt
[5 - x^2])])/8
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{x(8x^4-10x^2-75)(x^2-5)}{48\sqrt{-x^2+5}}+\frac{125\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{16}$	40
pseudoelliptic	$-\frac{125\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)}{16}+\frac{(8x^5-10x^3-75x)\sqrt{-x^2+5}}{48}$	43
default	$-\frac{x^3(-x^2+5)^{\frac{3}{2}}}{6}-\frac{5x(-x^2+5)^{\frac{3}{2}}}{8}+\frac{25x\sqrt{-x^2+5}}{16}+\frac{125\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{16}$	49
meijerg	$\frac{125i\left(\frac{i\sqrt{\pi}\,x\sqrt{5}\,\left(-\frac{8}{5}x^4+2x^2+15\right)\sqrt{-\frac{x^2}{5}+1}}{300}-\frac{i\sqrt{\pi}\,\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{4}\right)}{4\sqrt{\pi}}$	52
trager	$\frac{x(8x^4-10x^2-75)\sqrt{-x^2+5}}{48}+\frac{125\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+5}+x\right)}{16}$	53

```
[In] int(x^4*(-x^2+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/48*x*(8*x^4-10*x^2-75)*(x^2-5)/(-x^2+5)^(1/2)+125/16*arcsin(1/5*x*5^(1/2
))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int x^4\sqrt{5-x^2}\,dx=\frac{1}{48}\left(8\,x^5-10\,x^3-75\,x\right)\sqrt{-x^2+5}-\frac{125}{16}\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)$$

```
[In] integrate(x^4*(-x^2+5)^(1/2),x, algorithm="fricas")
[Out] 1/48*(8*x^5 - 10*x^3 - 75*x)*sqrt(-x^2 + 5) - 125/16*arctan(sqrt(-x^2 + 5)/
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.22 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.35

$$\int x^4\sqrt{5-x^2}\,dx=\begin{cases}\frac{ix^7}{6\sqrt{x^2-5}}-\frac{25ix^5}{24\sqrt{x^2-5}}-\frac{25ix^3}{48\sqrt{x^2-5}}+\frac{125ix}{16\sqrt{x^2-5}}-\frac{125i\,\mathrm{acosh}\left(\frac{\sqrt{5}x}{5}\right)}{16} & \text{for } |x^2|>5\\ -\frac{x^7}{6\sqrt{5-x^2}}+\frac{25x^5}{24\sqrt{5-x^2}}+\frac{25x^3}{48\sqrt{5-x^2}}-\frac{125x}{16\sqrt{5-x^2}}+\frac{125\,\mathrm{asin}\left(\frac{\sqrt{5}x}{5}\right)}{16} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**4*(-x**2+5)**(1/2),x)
```

```
[Out] Piecewise((I*x**7/(6*sqrt(x**2 - 5)) - 25*I*x**5/(24*sqrt(x**2 - 5)) - 25*I
*x**3/(48*sqrt(x**2 - 5)) + 125*I*x/(16*sqrt(x**2 - 5)) - 125*I*acosh(sqrt(
5)*x/5)/16, Abs(x**2) > 5), (-x**7/(6*sqrt(5 - x**2)) + 25*x**5/(24*sqrt(5
- x**2)) + 25*x**3/(48*sqrt(5 - x**2)) - 125*x/(16*sqrt(5 - x**2)) + 125*as
in(sqrt(5)*x/5)/16, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int x^4\sqrt{5-x^2}\,dx = -\frac{1}{6}\left(-x^2+5\right)^{\frac{3}{2}}x^3 - \frac{5}{8}\left(-x^2+5\right)^{\frac{3}{2}}x + \frac{25}{16}\sqrt{-x^2+5}x + \frac{125}{16}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

```
[In] integrate(x^4*(-x^2+5)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 5)^(3/2)*x^3 - 5/8*(-x^2 + 5)^(3/2)*x + 25/16*sqrt(-x^2 + 5)*x
 + 125/16*arcsin(1/5*sqrt(5)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

$$\int x^4\sqrt{5-x^2}\,dx = \frac{1}{48}\left(2\left(4x^2-5\right)x^2-75\right)\sqrt{-x^2+5}x + \frac{125}{16}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

```
[In] integrate(x^4*(-x^2+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2 - 5)*x^2 - 75)*sqrt(-x^2 + 5)*x + 125/16*arcsin(1/5*sqrt(5)*
x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.54

$$\int x^4\sqrt{5-x^2}\,dx=\frac{125\,\mathrm{asin}\left(\frac{\sqrt{5}\,x}{5}\right)}{16}-\sqrt{5-x^2}\,\left(-\frac{x^5}{6}+\frac{5\,x^3}{24}+\frac{25\,x}{16}\right)$$

```
[In] int(x^4*(5 - x^2)^(1/2),x)
```

```
[Out] (125*asin((5^(1/2)*x)/5))/16 - (5 - x^2)^(1/2)*((25*x)/16 + (5*x^3)/24 - x^
5/6)
```

3.251 $\int \frac{1}{x^6\sqrt{2+x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx=-\frac{\sqrt{2+x^2}}{10x^5}+\frac{\sqrt{2+x^2}}{15x^3}-\frac{\sqrt{2+x^2}}{15x}$$

```
[Out] -1/10*(x^2+2)^(1/2)/x^5+1/15*(x^2+2)^(1/2)/x^3-1/15*(x^2+2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {277, 270}

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx=-\frac{\sqrt{x^2+2}}{15x}-\frac{\sqrt{x^2+2}}{10x^5}+\frac{\sqrt{x^2+2}}{15x^3}$$

```
[In] Int[1/(x^6*Sqrt[2 + x^2]),x]
```

```
[Out] -1/10*Sqrt[2 + x^2]/x^5 + Sqrt[2 + x^2]/(15*x^3) - Sqrt[2 + x^2]/(15*x)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{2+x^2}}{10x^5} - \frac{2}{5}\int \frac{1}{x^4\sqrt{2+x^2}}\,dx \\
&= -\frac{\sqrt{2+x^2}}{10x^5} + \frac{\sqrt{2+x^2}}{15x^3} + \frac{2}{15}\int \frac{1}{x^2\sqrt{2+x^2}}\,dx \\
&= -\frac{\sqrt{2+x^2}}{10x^5} + \frac{\sqrt{2+x^2}}{15x^3} - \frac{\sqrt{2+x^2}}{15x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = \frac{\sqrt{2+x^2}(-3+2x^2-2x^4)}{30x^5}$$

```
[In] Integrate[1/(x^6*Sqrt[2 + x^2]),x]
[Out] (Sqrt[2 + x^2]*(-3 + 2*x^2 - 2*x^4))/(30*x^5)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{\sqrt{x^2+2}\,(2x^4-2x^2+3)}{30x^5}$	25
trager	$-\frac{\sqrt{x^2+2}\,(2x^4-2x^2+3)}{30x^5}$	25
pseudoelliptic	$-\frac{\sqrt{x^2+2}\,(2x^4-2x^2+3)}{30x^5}$	25
meijerg	$-\frac{\sqrt{2}\,(\frac{2}{3}x^4-\frac{2}{3}x^2+1)\sqrt{1+\frac{x^2}{2}}}{10x^5}$	30
risch	$-\frac{2x^6+2x^4-x^2+6}{30x^5\sqrt{x^2+2}}$	30
default	$-\frac{\sqrt{x^2+2}}{10x^5}+\frac{\sqrt{x^2+2}}{15x^3}-\frac{\sqrt{x^2+2}}{15x}$	38

```
[In] int(1/x^6/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/30*(x^2+2)^(1/2)*(2*x^4-2*x^2+3)/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = -\frac{2\,x^5+(2\,x^4-2\,x^2+3)\sqrt{x^2+2}}{30\,x^5}$$

```
[In] integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/30*(2*x^5 + (2*x^4 - 2*x^2 + 3)*sqrt(x^2 + 2))/x^5
```

Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = -\frac{\sqrt{1+\frac{2}{x^2}}}{15}+\frac{\sqrt{1+\frac{2}{x^2}}}{15x^2}-\frac{\sqrt{1+\frac{2}{x^2}}}{10x^4}$$

```
[In] integrate(1/x**6/(x**2+2)**(1/2),x)
[Out] -sqrt(1 + 2/x**2)/15 + sqrt(1 + 2/x**2)/(15*x**2) - sqrt(1 + 2/x**2)/(10*x*
*4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = -\frac{\sqrt{x^2+2}}{15\,x}+\frac{\sqrt{x^2+2}}{15\,x^3}-\frac{\sqrt{x^2+2}}{10\,x^5}$$

```
[In] integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="maxima")
[Out] -1/15*sqrt(x^2 + 2)/x + 1/15*sqrt(x^2 + 2)/x^3 - 1/10*sqrt(x^2 + 2)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = \frac{32\left(5\left(x-\sqrt{x^2+2}\right)^4-5\left(x-\sqrt{x^2+2}\right)^2+2\right)}{15\left(\left(x-\sqrt{x^2+2}\right)^2-2\right)^5}$$

```
[In] integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="giac")
[Out] 32/15*(5*(x - sqrt(x^2 + 2))^4 - 5*(x - sqrt(x^2 + 2))^2 + 2)/((x - sqrt(x^
2 + 2))^2 - 2)^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = -\sqrt{x^2+2}\left(\frac{1}{15\,x}-\frac{1}{15\,x^3}+\frac{1}{10\,x^5}\right)$$

```
[In] int(1/(x^6*(x^2 + 2)^(1/2)),x)
[Out] -(x^2 + 2)^(1/2)*(1/(15*x) - 1/(15*x^3) + 1/(10*x^5))
```

3.252 $\int \frac{1}{(3+2x^2)^{7/2}} dx$

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{(3+2x^2)^{7/2}} dx = \frac{x}{15(3+2x^2)^{5/2}} + \frac{4x}{135(3+2x^2)^{3/2}} + \frac{8x}{405\sqrt{3+2x^2}}$$

```
[Out] 1/15*x/(2*x^2+3)^(5/2)+4/135*x/(2*x^2+3)^(3/2)+8/405*x/(2*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {198, 197}

$$\int \frac{1}{(3+2x^2)^{7/2}} dx = \frac{8x}{405\sqrt{2x^2+3}} + \frac{4x}{135(2x^2+3)^{3/2}} + \frac{x}{15(2x^2+3)^{5/2}}$$

```
[In] Int[(3 + 2*x^2)^(-7/2),x]
```

```
[Out] x/(15*(3 + 2*x^2)^(5/2)) + (4*x)/(135*(3 + 2*x^2)^(3/2)) + (8*x)/(405*Sqrt[
3 + 2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
```

```
0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x}{15\left(3+2x^2\right)^{5/2}}+\frac{4}{15}\int\frac{1}{\left(3+2x^2\right)^{5/2}}\,dx\\ &= \frac{x}{15\left(3+2x^2\right)^{5/2}}+\frac{4x}{135\left(3+2x^2\right)^{3/2}}+\frac{8}{135}\int\frac{1}{\left(3+2x^2\right)^{3/2}}\,dx\\ &= \frac{x}{15\left(3+2x^2\right)^{5/2}}+\frac{4x}{135\left(3+2x^2\right)^{3/2}}+\frac{8x}{405\sqrt{3+2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int\frac{1}{\left(3+2x^2\right)^{7/2}}\,dx=\frac{x(135+120x^2+32x^4)}{405\left(3+2x^2\right)^{5/2}}$$

```
[In] Integrate[(3 + 2*x^2)^(-7/2),x]
[Out] (x*(135 + 120*x^2 + 32*x^4))/(405*(3 + 2*x^2)^(5/2))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

method	result	size
gosper	$\frac{x\left(32x^4+120x^2+135\right)}{405\left(2x^2+3\right)^{\frac{5}{2}}}$	25
trager	$\frac{x\left(32x^4+120x^2+135\right)}{405\left(2x^2+3\right)^{\frac{5}{2}}}$	25
risch	$\frac{x\left(32x^4+120x^2+135\right)}{405\left(2x^2+3\right)^{\frac{5}{2}}}$	25
pseudoelliptic	$\frac{32x^5+120x^3+135x}{405\left(2x^2+3\right)^{\frac{5}{2}}}$	26
meijerg	$\frac{\sqrt{3}\,x\left(\frac{32}{9}x^4+\frac{40}{3}x^2+15\right)}{1215\left(1+\frac{2x^2}{3}\right)^{\frac{5}{2}}}$	28
default	$\frac{x}{15\left(2x^2+3\right)^{\frac{5}{2}}}+\frac{4x}{135\left(2x^2+3\right)^{\frac{3}{2}}}+\frac{8x}{405\sqrt{2x^2+3}}$	38

```
[In] int(1/(2*x^2+3)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/405*x*(32*x^4+120*x^2+135)/(2*x^2+3)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{1}{(3+2x^2)^{7/2}}\,dx = \frac{(32\,x^5+120\,x^3+135\,x)\sqrt{2\,x^2+3}}{405\,(8\,x^6+36\,x^4+54\,x^2+27)}$$

```
[In] integrate(1/(2*x^2+3)^(7/2),x, algorithm="fricas")
[Out] 1/405*(32*x^5 + 120*x^3 + 135*x)*sqrt(2*x^2 + 3)/(8*x^6 + 36*x^4 + 54*x^2 +
 27)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(42) = 84.

Time = 2.93 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

$$\int \frac{1}{(3+2x^2)^{7/2}}\,dx = \frac{32x^5}{1620x^4\sqrt{2x^2+3}+4860x^2\sqrt{2x^2+3}+3645\sqrt{2x^2+3}} + \frac{120x^3}{1620x^4\sqrt{2x^2+3}+4860x^2\sqrt{2x^2+3}+3645\sqrt{2x^2+3}} + \frac{135x}{1620x^4\sqrt{2x^2+3}+4860x^2\sqrt{2x^2+3}+3645\sqrt{2x^2+3}}$$

```
[In] integrate(1/(2*x**2+3)**(7/2),x)
[Out] 32*x**5/(1620*x**4*sqrt(2*x**2 + 3) + 4860*x**2*sqrt(2*x**2 + 3) + 3645*sqr
t(2*x**2 + 3)) + 120*x**3/(1620*x**4*sqrt(2*x**2 + 3) + 4860*x**2*sqrt(2*x*
*2 + 3) + 3645*sqrt(2*x**2 + 3)) + 135*x/(1620*x**4*sqrt(2*x**2 + 3) + 4860
*x**2*sqrt(2*x**2 + 3) + 3645*sqrt(2*x**2 + 3))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{(3+2x^2)^{7/2}}\,dx = \frac{8\,x}{405\,\sqrt{2\,x^2+3}} + \frac{4\,x}{135\,(2\,x^2+3)^{\frac{3}{2}}} + \frac{x}{15\,(2\,x^2+3)^{\frac{5}{2}}}$$

```
[In] integrate(1/(2*x^2+3)^(7/2),x, algorithm="maxima")
[Out] 8/405*x/sqrt(2*x^2 + 3) + 4/135*x/(2*x^2 + 3)^(3/2) + 1/15*x/(2*x^2 + 3)^(5
/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.53

$$\int \frac{1}{(3+2x^2)^{7/2}}\,dx = \frac{(8\,(4\,x^2+15)x^2+135)x}{405\,(2\,x^2+3)^{\frac{5}{2}}}$$

```
[In] integrate(1/(2*x^2+3)^(7/2),x, algorithm="giac")
[Out] 1/405*(8*(4*x^2 + 15)*x^2 + 135)*x/(2*x^2 + 3)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.82

$$\begin{aligned}\int \frac{1}{(3+2x^2)^{7/2}}\,dx &= \frac{2\,\sqrt{2}\,\sqrt{x^2+\frac{3}{2}}}{405\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{2}\right)} + \frac{2\,\sqrt{2}\,\sqrt{x^2+\frac{3}{2}}}{405\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{2}\right)} \\ &+ \frac{\sqrt{2}\,\sqrt{x^2+\frac{3}{2}}}{1440\,\left(-x^3+\frac{3\mathrm{i}\,\sqrt{6}\,x^2}{2}+\frac{9\,x}{2}-\frac{\sqrt{6}\,3\mathrm{i}}{4}\right)} + \frac{\sqrt{2}\,\sqrt{x^2+\frac{3}{2}}}{1440\,\left(-x^3-\frac{3\mathrm{i}\,\sqrt{6}\,x^2}{2}+\frac{9\,x}{2}+\frac{\sqrt{6}\,3\mathrm{i}}{4}\right)} \\ &+ \frac{\sqrt{2}\,\sqrt{6}\,\sqrt{x^2+\frac{3}{2}}\,19\mathrm{i}}{25920\,\left(x^2+1\mathrm{i}\,\sqrt{6}\,x-\frac{3}{2}\right)} + \frac{\sqrt{2}\,\sqrt{6}\,\sqrt{x^2+\frac{3}{2}}\,19\mathrm{i}}{25920\,\left(-x^2+1\mathrm{i}\,\sqrt{6}\,x+\frac{3}{2}\right)}\end{aligned}$$

```
[In] int(1/(2*x^2 + 3)^(7/2),x)
[Out] (2*2^(1/2)*(x^2 + 3/2)^(1/2))/(405*(x - (6^(1/2)*1i)/2)) + (2*2^(1/2)*(x^2
+ 3/2)^(1/2))/(405*(x + (6^(1/2)*1i)/2)) + (2^(1/2)*(x^2 + 3/2)^(1/2))/(144
0*((9*x)/2 - (6^(1/2)*3i)/4 + (6^(1/2)*x^2*3i)/2 - x^3)) + (2^(1/2)*(x^2 +
3/2)^(1/2))/(1440*((9*x)/2 + (6^(1/2)*3i)/4 - (6^(1/2)*x^2*3i)/2 - x^3)) +
(2^(1/2)*6^(1/2)*(x^2 + 3/2)^(1/2)*19i)/(25920*(6^(1/2)*x*1i + x^2 - 3/2))
+ (2^(1/2)*6^(1/2)*(x^2 + 3/2)^(1/2)*19i)/(25920*(6^(1/2)*x*1i - x^2 + 3/2)
)
```

3.253 $\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 12

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx = \log\left(a+\sqrt{1+x^2}\right)$$

```
[Out] ln(a+(x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2186, 31}

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx = \log\left(a+\sqrt{x^2+1}\right)$$

```
[In] Int[x/(1 + x^2 + a*Sqrt[1 + x^2]),x]
```

```
[Out] Log[a + Sqrt[1 + x^2]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+a\sqrt{1+x}}\,dx, x, x^2\right)\\ &= \text{Subst}\left(\int \frac{1}{a+x}\,dx, x, \sqrt{1+x^2}\right)\\ &= \log\left(a+\sqrt{1+x^2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx = \log\left(a+\sqrt{1+x^2}\right)$$

```
[In] Integrate[x/(1 + x^2 + a*Sqrt[1 + x^2]),x]
[Out] Log[a + Sqrt[1 + x^2]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(10) = 20.

Time = 0.08 (sec) , antiderivative size = 354, normalized size of antiderivative = 29.50

method	result
default	$\frac{\sqrt{x^2+1}}{a} - \frac{\sqrt{\left(x-\sqrt{(1+a)(a-1)}\right)^2+2\sqrt{(1+a)(a-1)}\left(x-\sqrt{(1+a)(a-1)}\right)+a^2}}{2a} + \frac{a\ln\left(\frac{2a^2+2\sqrt{(1+a)(a-1)}\left(x-\sqrt{(1+a)(a-1)}\right)+2\sqrt{a^2}}{}\right.}{}$

```
[In] int(x/(1+x^2+a*(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/a*(x^2+1)^(1/2)-1/2/a*((x-((1+a)*(a-1))^(1/2))^2+2*((1+a)*(a-1))^(1/2)*(x
-((1+a)*(a-1))^(1/2))+a^2)^(1/2)+1/2*a/(a^2)^(1/2)*ln((2*a^2+2*((1+a)*(a-1)
)^(1/2)*(x-((1+a)*(a-1))^(1/2))+2*(a^2)^(1/2)*((x-((1+a)*(a-1))^(1/2))^2+2*
((1+a)*(a-1))^(1/2)*(x-((1+a)*(a-1))^(1/2))+a^2)^(1/2))/(x-((1+a)*(a-1))^(1
/2)))-1/2/a*((x+((1+a)*(a-1))^(1/2))^2-2*((1+a)*(a-1))^(1/2)*(x+((1+a)*(a-1
))^(1/2))+a^2)^(1/2)+1/2*a/(a^2)^(1/2)*ln((2*a^2-2*((1+a)*(a-1))^(1/2)*(x+(
(1+a)*(a-1))^(1/2))+2*(a^2)^(1/2)*((x+((1+a)*(a-1))^(1/2))^2-2*((1+a)*(a-1)
)^(1/2)*(x+((1+a)*(a-1))^(1/2))+a^2)^(1/2))/(x+((1+a)*(a-1))^(1/2)))+1/2/a^
2*ln(-a^2+x^2+1)-1/2*(-a^2+1)/a^2*ln(-a^2+x^2+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(10) = 20$.

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 5.17

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx = \frac{1}{2}\log\left(-a^2+x^2+1\right) - \frac{1}{2}\log\left(ax+x^2-\sqrt{x^2+1}(a+x)+1\right) + \frac{1}{2}\log\left(-ax+x^2+\sqrt{x^2+1}(a-x)+1\right)$$

```
[In] integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-a^2 + x^2 + 1) - 1/2*log(a*x + x^2 - sqrt(x^2 + 1)*(a + x) + 1) +
1/2*log(-a*x + x^2 + sqrt(x^2 + 1)*(a - x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(10) = 20$.

Time = 0.74 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.83

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx = -a\left(-\frac{\log\left(2a+2\sqrt{x^2+1}\right)}{2a} + \frac{\log\left(-2\sqrt{x^2+1}\right)}{2a}\right) + \frac{\log\left(2a\sqrt{x^2+1}+2x^2+2\right)}{2}$$

```
[In] integrate(x/(1+x**2+a*(x**2+1)**(1/2)),x)
```

```
[Out] -a*(-log(2*a + 2*sqrt(x**2 + 1))/(2*a) + log(-2*sqrt(x**2 + 1))/(2*a)) + lo
g(2*a*sqrt(x**2 + 1) + 2*x**2 + 2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx = \log\left(a+\sqrt{x^2+1}\right)$$

```
[In] integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(a + sqrt(x^2 + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx = \log\left(\left|a+\sqrt{x^2+1}\right|\right)$$

```
[In] integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="giac")
[Out] log(abs(a + sqrt(x^2 + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 12.83

$$\begin{aligned}\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx &= \frac{\ln\left(x+\sqrt{a-1}\,\sqrt{a+1}\right)}{2}+\frac{\ln\left(x-\sqrt{a-1}\,\sqrt{a+1}\right)}{2}\\ &\quad-\frac{a\left(\ln\left(x+\sqrt{a-1}\,\sqrt{a+1}\right)-\ln\left(\sqrt{x^2+1}\,\sqrt{a^2}-x\,\sqrt{a-1}\,\sqrt{a+1}+1\right)\right)}{2\,\sqrt{(a-1)\,(a+1)+1}}\\ &\quad-\frac{a\left(\ln\left(x-\sqrt{a-1}\,\sqrt{a+1}\right)-\ln\left(\sqrt{x^2+1}\,\sqrt{a^2}+x\,\sqrt{a-1}\,\sqrt{a+1}+1\right)\right)}{2\,\sqrt{(a-1)\,(a+1)+1}}\end{aligned}$$

```
[In] int(x/(a*(x^2 + 1)^(1/2) + x^2 + 1),x)
[Out] log(x + (a - 1)^(1/2)*(a + 1)^(1/2))/2 + log(x - (a - 1)^(1/2)*(a + 1)^(1/2
))/2 - (a*(log(x + (a - 1)^(1/2)*(a + 1)^(1/2)) - log((x^2 + 1)^(1/2)*(a^2)
^(1/2) - x*(a - 1)^(1/2)*(a + 1)^(1/2) + 1)))/(2*((a - 1)*(a + 1) + 1)^(1/2
)) - (a*(log(x - (a - 1)^(1/2)*(a + 1)^(1/2)) - log((x^2 + 1)^(1/2)*(a^2)^(
1/2) + x*(a - 1)^(1/2)*(a + 1)^(1/2) + 1)))/(2*((a - 1)*(a + 1) + 1)^(1/2))
```

3.254 $\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = \frac{1}{\sqrt{1+x^2}} + \operatorname{arcsinh}(x)$$

```
[Out] arcsinh(x)+1/(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1828, 221}

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = \operatorname{arcsinh}(x) + \frac{1}{\sqrt{x^2+1}}$$

```
[In] Int[(1 - x + x^2)/(1 + x^2)^(3/2),x]
```

```
[Out] 1/Sqrt[1 + x^2] + ArcSinh[x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
```

```
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{\sqrt{1+x^2}} + \int \frac{1}{\sqrt{1+x^2}}\,dx \\ &= \frac{1}{\sqrt{1+x^2}} + \operatorname{arcsinh}(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = \frac{1}{\sqrt{1+x^2}} - \log\left(-x+\sqrt{1+x^2}\right)$$

```
[In] Integrate[(1 - x + x^2)/(1 + x^2)^(3/2),x]
[Out] 1/Sqrt[1 + x^2] - Log[-x + Sqrt[1 + x^2]]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$\operatorname{arcsinh}(x) + \frac{1}{\sqrt{x^2+1}}$	11
risch	$\operatorname{arcsinh}(x) + \frac{1}{\sqrt{x^2+1}}$	11
trager	$\frac{1}{\sqrt{x^2+1}} - \ln\left(-\sqrt{x^2+1}+x\right)$	23
meijerg	$\frac{x}{\sqrt{x^2+1}} + \frac{-\frac{\sqrt{\pi}\,x}{\sqrt{x^2+1}}+\sqrt{\pi}\,\operatorname{arcsinh}(x)}{\sqrt{\pi}} - \frac{\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{x^2+1}}}{\sqrt{\pi}}$	56

```
[In] int((x^2-x+1)/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(x)+1/(x^2+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.08

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = -\frac{(x^2+1)\log\left(-x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}}{x^2+1}$$

```
[In] integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="fricas")
[Out] -((x^2 + 1)*log(-x + sqrt(x^2 + 1)) - sqrt(x^2 + 1))/(x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 4.84 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = \frac{x^2\operatorname{asinh}(x)}{x^2+1}+\frac{\operatorname{asinh}(x)}{x^2+1}+\frac{1}{\sqrt{x^2+1}}$$

```
[In] integrate((x**2-x+1)/(x**2+1)**(3/2),x)
[Out] x**2*asinh(x)/(x**2 + 1) + asinh(x)/(x**2 + 1) + 1/sqrt(x**2 + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = \frac{1}{\sqrt{x^2+1}}+\operatorname{arsinh}(x)$$

```
[In] integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="maxima")
[Out] 1/sqrt(x^2 + 1) + arcsinh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = \frac{1}{\sqrt{x^2+1}} - \log\left(-x+\sqrt{x^2+1}\right)$$

```
[In] integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="giac")
[Out] 1/sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = \frac{\operatorname{asinh}(x)+x^2\operatorname{asinh}(x)+\sqrt{x^2+1}}{x^2+1}$$

```
[In] int((x^2 - x + 1)/(x^2 + 1)^(3/2),x)
[Out] (asinh(x) + x^2*asinh(x) + (x^2 + 1)^(1/2))/(x^2 + 1)
```

3.255 $\int \frac{\sqrt{1+x^2}}{2+x^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{\sqrt{1+x^2}}{2+x^2} \, dx = \operatorname{arcsinh}(x) - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}\sqrt{1+x^2}}\right)}{\sqrt{2}}$$

```
[Out] arcsinh(x)-1/2*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {399, 221, 385, 212}

$$\int \frac{\sqrt{1+x^2}}{2+x^2} \, dx = \operatorname{arcsinh}(x) - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{2}}$$

```
[In] Int[Sqrt[1 + x^2]/(2 + x^2),x]
[Out] ArcSinh[x] - ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{1+x^2}}\,dx - \int \frac{1}{\sqrt{1+x^2}\,(2+x^2)}\,dx\\ &= \operatorname{arcsinh}(x) - \operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \frac{x}{\sqrt{1+x^2}}\right)\\ &= \operatorname{arcsinh}(x) - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}\sqrt{1+x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2+x^2-x\sqrt{1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}} - \log\left(-x+\sqrt{1+x^2}\right)$$

```
[In] Integrate[Sqrt[1 + x^2]/(2 + x^2),x]
[Out] -(ArcTanh[(2 + x^2 - x*Sqrt[1 + x^2])/Sqrt[2]]/Sqrt[2]) - Log[-x + Sqrt[1 +
 x^2]]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
default	$\operatorname{arcsinh}\left(x\right)-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2\sqrt{x^{2}+1}}\right)\sqrt{2}}{2}$	23
pseudoelliptic	$\frac{\ln\left(\frac{x+\sqrt{x^{2}+1}}{x}\right)}{2}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x^{2}+1}}{x}\right)}{2}-\frac{\ln\left(\frac{\sqrt{x^{2}+1}-x}{x}\right)}{2}$	56
trager	$-\ln\left(-\sqrt{x^{2}+1}+x\right)-\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^{2}-2\right)x^{2}+4x\sqrt{x^{2}+1}+2\operatorname{RootOf}\left(\_Z^{2}-2\right)}{x^{2}+2}\right)}{4}$	63

```
[In] int((x^2+1)^(1/2)/(x^2+2),x,method=_RETURNVERBOSE)
[Out] arcsinh(x)-1/2*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(22) = 44$.

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.48

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{9\,x^{2}-2\,\sqrt{2}(3\,x^{2}+2)-2\,\sqrt{x^{2}+1}(3\,\sqrt{2}x-4\,x)+6}{x^{2}+2}\right) - \log\left(-x+\sqrt{x^{2}+1}\right)$$

```
[In] integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((9*x^2 - 2*sqrt(2)*(3*x^2 + 2) - 2*sqrt(x^2 + 1)*(3*sqrt(2)
*x - 4*x) + 6)/(x^2 + 2)) - log(-x + sqrt(x^2 + 1))
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = \int \frac{\sqrt{x^{2}+1}}{x^{2}+2}\,dx$$

```
[In] integrate((x**2+1)**(1/2)/(x**2+2),x)
[Out] Integral(sqrt(x**2 + 1)/(x**2 + 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{2+x^2} dx = \int \frac{\sqrt{x^2+1}}{x^2+2} dx$$

```
[In] integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 1)*x/(x^2 + 2) + integrate(sqrt(x^2 + 1)*x^4/(x^6 + 5*x^4 + 8*x^
2 + 4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(22) = 44$.

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.37

$$\int \frac{\sqrt{1+x^2}}{2+x^2} dx = \frac{1}{4}\sqrt{2}\log\left(\frac{\left(x-\sqrt{x^2+1}\right)^2-2\sqrt{2}+3}{\left(x-\sqrt{x^2+1}\right)^2+2\sqrt{2}+3}\right)-\log\left(-x+\sqrt{x^2+1}\right)$$

```
[In] integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(((x - sqrt(x^2 + 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 + 1)
)^2 + 2*sqrt(2) + 3)) - log(-x + sqrt(x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.85

$$\int \frac{\sqrt{1+x^2}}{2+x^2} dx = \operatorname{asinh}(x)+\frac{\sqrt{2}\left(\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)-\ln\left(1+\sqrt{2}\,x\,1\mathrm{i}+\sqrt{x^2+1}\,1\mathrm{i}\right)\right)}{4} - \frac{\sqrt{2}\left(\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)-\ln\left(1-\sqrt{2}\,x\,1\mathrm{i}+\sqrt{x^2+1}\,1\mathrm{i}\right)\right)}{4}$$

```
[In] int((x^2 + 1)^(1/2)/(x^2 + 2),x)
```

```
[Out] asinh(x) + (2^(1/2)*(log(x - 2^(1/2)*1i) - log(2^(1/2)*x*1i + (x^2 + 1)^(1/
2)*1i + 1)))/4 - (2^(1/2)*(log(x + 2^(1/2)*1i) - log((x^2 + 1)^(1/2)*1i - 2
^(1/2)*x*1i + 1)))/4
```

3.256 $\int \frac{1}{\sqrt{1+x^2}(2+x^2)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = -\frac{x\sqrt{1+x^2}}{4\,(2+x^2)}+\frac{3\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt{1+x^2}}\right)}{4\sqrt{2}}$$

```
[Out] 3/8*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)-1/4*x*(x^2+1)^(1/2)/(x^2+2
)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {390, 385, 212}

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = \frac{3\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2+1}}{4\,(x^2+2)}$$

```
[In] Int[1/(Sqrt[1 + x^2]*(2 + x^2)^2),x]
```

```
[Out] -1/4*(x*Sqrt[1 + x^2])/(2 + x^2) + (3*ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])])/(
4*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x\sqrt{1+x^2}}{4(2+x^2)} + \frac{3}{4}\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)}\,dx \\
&= -\frac{x\sqrt{1+x^2}}{4(2+x^2)} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \frac{x}{\sqrt{1+x^2}}\right) \\
&= -\frac{x\sqrt{1+x^2}}{4(2+x^2)} + \frac{3\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt{1+x^2}}\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = \frac{1}{8}\left(-\frac{2x\sqrt{1+x^2}}{2+x^2} + 3\sqrt{2}\text{arctanh}\left(\frac{2+x^2-x\sqrt{1+x^2}}{\sqrt{2}}\right)\right)$$

```
[In] Integrate[1/(Sqrt[1 + x^2]*(2 + x^2)^2),x]
```

```
[Out] ((-2*x*Sqrt[1 + x^2])/(2 + x^2) + 3*Sqrt[2]*ArcTanh[(2 + x^2 - x*Sqrt[1 + x
^2])/Sqrt[2]])/8
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{3\,\mathrm{arctanh}\left(\frac{x\sqrt{2}}{2\sqrt{x^2+1}}\right)\sqrt{2}}{8}-\frac{x\sqrt{x^2+1}}{4(x^2+2)}$	38
default	$\frac{x}{4\sqrt{x^2+1}\left(\frac{x^2}{x^2+1}-2\right)}+\frac{3\,\mathrm{arctanh}\left(\frac{x\sqrt{2}}{2\sqrt{x^2+1}}\right)\sqrt{2}}{8}$	46
pseudoelliptic	$\frac{(3x^2+6)\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x^2+1}}{x}\right)-2x\sqrt{x^2+1}}{8x^2+16}$	48
trager	$-\frac{x\sqrt{x^2+1}}{4(x^2+2)}+\frac{3\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2-2\right)x^2+4x\sqrt{x^2+1}+2\,\mathrm{RootOf}\left(\_Z^2-2\right)}{x^2+2}\right)}{16}$	66

```
[In] int(1/(x^2+2)^2/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 3/8*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)-1/4*x*(x^2+1)^(1/2)/(x^2+2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(37) = 74.

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = \frac{3\sqrt{2}(x^2+2)\log\left(\frac{9x^2+2\sqrt{2}(3x^2+2)+2\sqrt{x^2+1}\left(3\sqrt{2}x+4x\right)+6}{x^2+2}\right)-4x^2-4\sqrt{x^2+1}x-8}{16\,(x^2+2)}$$

```
[In] integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/16*(3*sqrt(2)*(x^2 + 2)*log((9*x^2 + 2*sqrt(2)*(3*x^2 + 2) + 2*sqrt(x^2 +
 1)*(3*sqrt(2)*x + 4*x) + 6)/(x^2 + 2)) - 4*x^2 - 4*sqrt(x^2 + 1)*x - 8)/(x
^2 + 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = \int \frac{1}{\sqrt{x^2+1}\,(x^2+2)^2}\,dx$$

```
[In] integrate(1/(x**2+2)**2/(x**2+1)**(1/2),x)
[Out] Integral(1/(sqrt(x**2 + 1)*(x**2 + 2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = \int \frac{1}{(x^2+2)^2\sqrt{x^2+1}}\,dx$$

```
[In] integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 2)^2*sqrt(x^2 + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(37) = 74$.

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.10

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = -\frac{3}{16}\sqrt{2}\log\left(\frac{\left(x-\sqrt{x^2+1}\right)^2-2\sqrt{2}+3}{\left(x-\sqrt{x^2+1}\right)^2+2\sqrt{2}+3}\right) - \frac{3\left(x-\sqrt{x^2+1}\right)^2+1}{2\left(\left(x-\sqrt{x^2+1}\right)^4+6\left(x-\sqrt{x^2+1}\right)^2+1\right)}$$

```
[In] integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="giac")
[Out] -3/16*sqrt(2)*log(((x - sqrt(x^2 + 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 +
1))^2 + 2*sqrt(2) + 3)) - 1/2*(3*(x - sqrt(x^2 + 1))^2 + 1)/((x - sqrt(x^2
+ 1))^4 + 6*(x - sqrt(x^2 + 1))^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.44

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = -\frac{3\sqrt{2}\left(\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)-\ln\left(1+\sqrt{2}\,x\,1\mathrm{i}+\sqrt{x^2+1}\,1\mathrm{i}\right)\right)}{16} + \frac{3\sqrt{2}\left(\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)-\ln\left(1-\sqrt{2}\,x\,1\mathrm{i}+\sqrt{x^2+1}\,1\mathrm{i}\right)\right)}{16} - \frac{\sqrt{x^2+1}}{8\,\left(x-\sqrt{2}\,1\mathrm{i}\right)} - \frac{\sqrt{x^2+1}}{8\,\left(x+\sqrt{2}\,1\mathrm{i}\right)}$$

```
[In] int(1/((x^2 + 1)^(1/2)*(x^2 + 2)^2),x)
```

```
[Out] (3*2^(1/2)*(log(x + 2^(1/2)*1i) - log((x^2 + 1)^(1/2)*1i - 2^(1/2)*x*1i + 1
)))/16 - (3*2^(1/2)*(log(x - 2^(1/2)*1i) - log(2^(1/2)*x*1i + (x^2 + 1)^(1/
2)*1i + 1)))/16 - (x^2 + 1)^(1/2)/(8*(x - 2^(1/2)*1i)) - (x^2 + 1)^(1/2)/(8
*(x + 2^(1/2)*1i))
```

3.257 $\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 41

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx=\operatorname{arctanh}\left(\frac{x}{\sqrt{-2+x^2}}\right)-\sqrt{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{-2+x^2}}\right)$$

```
[Out] arctanh(x/(x^2-2)^(1/2))-1/2*arctanh(1/3*x*6^(1/2)/(x^2-2)^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {494, 223, 212, 385, 213}

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx=\operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-2}}\right)-\sqrt{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{x^2-2}}\right)$$

```
[In] Int[x^2/((-6 + x^2)*Sqrt[-2 + x^2]),x]
```

```
[Out] ArcTanh[x/Sqrt[-2 + x^2]] - Sqrt[3/2]*ArcTanh[(Sqrt[2/3]*x)/Sqrt[-2 + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 6\int \frac{1}{(-6+x^2)\sqrt{-2+x^2}}\,dx + \int \frac{1}{\sqrt{-2+x^2}}\,dx\\ &= 6\text{Subst}\left(\int \frac{1}{-6+4x^2}\,dx, x, \frac{x}{\sqrt{-2+x^2}}\right) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-2+x^2}}\right)\\ &= \operatorname{arctanh}\left(\frac{x}{\sqrt{-2+x^2}}\right) - \sqrt{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{-2+x^2}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = -\sqrt{\frac{3}{2}}\operatorname{arctanh}\left(\frac{6-x^2+x\sqrt{-2+x^2}}{2\sqrt{6}}\right) - \log\left(-x+\sqrt{-2+x^2}\right)$$

```
[In] Integrate[x^2/((-6 + x^2)*Sqrt[-2 + x^2]),x]
[Out] -(Sqrt[3/2]*ArcTanh[(6 - x^2 + x*Sqrt[-2 + x^2])/(2*Sqrt[6])]) - Log[-x + S
qrt[-2 + x^2]]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$\frac{\ln\left(\frac{x+\sqrt{x^2-2}}{x}\right)}{2}-\frac{\sqrt{6}\,\operatorname{arctanh}\left(\frac{\sqrt{x^2-2}\,\sqrt{6}}{2x}\right)}{2}-\frac{\ln\left(\frac{\sqrt{x^2-2}-x}{x}\right)}{2}$
trager	$-\ln\left(x-\sqrt{x^2-2}\right)+\frac{\operatorname{RootOf}\left(\_Z^2-6\right)\ln\left(\frac{5\operatorname{RootOf}\left(\_Z^2-6\right)x^2-12\sqrt{x^2-2}\,x-6\operatorname{RootOf}\left(\_Z^2-6\right)}{x^2-6}\right)}{4}$
default	$\ln\left(x+\sqrt{x^2-2}\right)-\frac{\sqrt{6}\,\operatorname{arctanh}\left(\frac{8+2\left(x-\sqrt{6}\right)\sqrt{6}}{4\sqrt{\left(x-\sqrt{6}\right)^2+2\left(x-\sqrt{6}\right)\sqrt{6}+4}}\right)}{4}+\frac{\sqrt{6}\,\operatorname{arctanh}\left(\frac{8-2\left(x+\sqrt{6}\right)\sqrt{6}}{4\sqrt{\left(x+\sqrt{6}\right)^2-2\left(x+\sqrt{6}\right)\sqrt{6}+4}}\right)}{4}$

```
[In] int(x^2/(x^2-6)/(x^2-2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*ln((x+(x^2-2)^(1/2))/x)-1/2*6^(1/2)*arctanh(1/2*(x^2-2)^(1/2)/x*6^(1/2)
)-1/2*ln(((x^2-2)^(1/2)-x)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(30) = 60.

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.88

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx$$
$$=\frac{1}{4}\sqrt{3}\sqrt{2}\log\left(-\frac{2\sqrt{3}\sqrt{2}(5x^2-6)-25x^2+2\left(5\sqrt{3}\sqrt{2}x-12x\right)\sqrt{x^2-2}+30}{x^2-6}\right)$$
$$-\log\left(-x+\sqrt{x^2-2}\right)$$

```
[In] integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(3)*sqrt(2)*log(-(2*sqrt(3)*sqrt(2)*(5*x^2 - 6) - 25*x^2 + 2*(5*sqr
t(3)*sqrt(2)*x - 12*x)*sqrt(x^2 - 2) + 30)/(x^2 - 6)) - log(-x + sqrt(x^2 -
 2))
```

Sympy [F]

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = \int \frac{x^2}{(x^2-6)\sqrt{x^2-2}}\,dx$$

```
[In] integrate(x**2/(x**2-6)/(x**2-2)**(1/2),x)
[Out] Integral(x**2/((x**2 - 6)*sqrt(x**2 - 2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.61

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = \frac{1}{12}\sqrt{6}\left(2\sqrt{6}\log\left(x+\sqrt{x^2-2}\right) - 3\log\left(\sqrt{6}+\frac{4\sqrt{x^2-2}}{|2x-2\sqrt{6}|}+\frac{8}{|2x-2\sqrt{6}|}\right) + 3\log\left(-\sqrt{6}+\frac{4\sqrt{x^2-}}{|2x+2\sqrt{}}\right.\right.$$

```
[In] integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="maxima")
[Out] 1/12*sqrt(6)*(2*sqrt(6)*log(x + sqrt(x^2 - 2)) - 3*log(sqrt(6) + 4*sqrt(x^2
 - 2)/abs(2*x - 2*sqrt(6)) + 8/abs(2*x - 2*sqrt(6))) + 3*log(-sqrt(6) + 4*s
qrt(x^2 - 2)/abs(2*x + 2*sqrt(6)) + 8/abs(2*x + 2*sqrt(6))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(30) = 60$.

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = -\frac{1}{4}\sqrt{6}\log\left(\frac{\left|2\left(x-\sqrt{x^2-2}\right)^2-8\sqrt{6}-20\right|}{\left|2\left(x-\sqrt{x^2-2}\right)^2+8\sqrt{6}-20\right|}\right) - \frac{1}{2}\log\left(\left(x-\sqrt{x^2-2}\right)^2\right)$$

```
[In] integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="giac")
[Out] -1/4*sqrt(6)*log(abs(2*(x - sqrt(x^2 - 2))^2 - 8*sqrt(6) - 20)/abs(2*(x - s
qrt(x^2 - 2))^2 + 8*sqrt(6) - 20)) - 1/2*log((x - sqrt(x^2 - 2))^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = \int \frac{x^2}{\sqrt{x^2-2}\,(x^2-6)}\,dx$$

```
[In] int(x^2/((x^2 - 2)^(1/2)*(x^2 - 6)),x)
[Out] int(x^2/((x^2 - 2)^(1/2)*(x^2 - 6)), x)
```

3.258 $\int \frac{5+x^2}{\sqrt{1-x^2}(1+x^2)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \frac{x\sqrt{1-x^2}}{1+x^2} + 2\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$$

```
[Out] 2*arctan(x*2^(1/2)/(-x^2+1)^(1/2))*2^(1/2)+x*(-x^2+1)^(1/2)/(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {541, 12, 385, 209}

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = 2\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right) + \frac{\sqrt{1-x^2}x}{x^2+1}$$

```
[In] Int[(5 + x^2)/(Sqrt[1 - x^2]*(1 + x^2)^2),x]
```

```
[Out] (x*Sqrt[1 - x^2])/(1 + x^2) + 2*Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\sqrt{1-x^2}}{1+x^2} - \frac{1}{4}\int -\frac{16}{\sqrt{1-x^2}\,(1+x^2)}\,dx\\ &= \frac{x\sqrt{1-x^2}}{1+x^2} + 4\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx\\ &= \frac{x\sqrt{1-x^2}}{1+x^2} + 4\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)\\ &= \frac{x\sqrt{1-x^2}}{1+x^2} + 2\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \frac{x\sqrt{1-x^2}}{1+x^2} + 2\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$$

```
[In] Integrate[(5 + x^2)/(Sqrt[1 - x^2]*(1 + x^2)^2),x]
[Out] (x*Sqrt[1 - x^2])/(1 + x^2) + 2*Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

method	result	size
pseudoelliptic	$\frac{\left(-2 x^{2}-2\right) \sqrt{2}\, \arctan\left(\frac{\sqrt{2}\, \sqrt{-x^{2}+1}}{2 x}\right)+x \sqrt{-x^{2}+1}}{x^{2}+1}$	50
risch	$-\frac{x \left(x^{2}-1\right)}{\left(x^{2}+1\right) \sqrt{-x^{2}+1}}-2 \sqrt{2}\, \arctan\left(\frac{\sqrt{2}\, \sqrt{-x^{2}+1}\, x}{x^{2}-1}\right)$	53
trager	$\frac{x \sqrt{-x^{2}+1}}{x^{2}+1}-\operatorname{RootOf}\left(\_Z^{2}+2\right) \ln\left(\frac{3 \operatorname{RootOf}\left(\_Z^{2}+2\right) x^{2}+4 x \sqrt{-x^{2}+1}-\operatorname{RootOf}\left(\_Z^{2}+2\right)}{x^{2}+1}\right)$	69
default	$-2 \sqrt{2}\, \arctan\left(\frac{\sqrt{2}\, \sqrt{-x^{2}+1}\, x}{x^{2}-1}\right)-\frac{\sqrt{-x^{2}+1}\, x}{2 \left(x^{2}-1\right) \left(\frac{\left(-x^{2}+1\right) x^{2}}{\left(x^{2}-1\right)^{2}}+\frac{1}{2}\right)}$	70

```
[In] int((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((-2*x^2-2)*2^(1/2)*arctan(1/2/x*2^(1/2)*(-x^2+1)^(1/2))+x*(-x^2+1)^(1/2))/
(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = -\frac{2\,\sqrt{2}(x^2+1)\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}}{2\,x}\right)-\sqrt{-x^2+1}x}{x^2+1}$$

```
[In] integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(2*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 1)/x) - sqrt(-x^2 + 1)
*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \int \frac{x^2+5}{\sqrt{-(x-1)(x+1)}\,(x^2+1)^2}\,dx$$

```
[In] integrate((x**2+5)/(x**2+1)**2/(-x**2+1)**(1/2),x)
[Out] Integral((x**2 + 5)/(sqrt(-(x - 1)*(x + 1))*(x**2 + 1)**2), x)
```

Maxima [F]

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \int \frac{x^2+5}{(x^2+1)^2\sqrt{-x^2+1}}\,dx$$

```
[In] integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 + 5)/((x^2 + 1)^2*sqrt(-x^2 + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(39) = 78.

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.62

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \sqrt{2}\left(\pi\,\mathrm{sgn}(x)+2\arctan\left(-\frac{\sqrt{2}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right) - \frac{2\left(\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}\right)}{\left(\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}\right)^2+8}$$

```
[In] integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x^2 + 1) - 1)^2/x^2 -
1)/(sqrt(-x^2 + 1) - 1))) - 2*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1
)/x)/((x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)^2 + 8)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.45

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \sqrt{2}\ln\left(\frac{\frac{\sqrt{2}\,(-1+x\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\mathrm{i}}\right)1\mathrm{i} - \sqrt{2}\ln\left(\frac{\frac{\sqrt{2}\,(1+x\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-x^2}\,1\mathrm{i}}{x+1\mathrm{i}}\right)1\mathrm{i} + \frac{\sqrt{1-x^2}}{2\,(x-\mathrm{i})} + \frac{\sqrt{1-x^2}}{2\,(x+1\mathrm{i})}$$

```
[In] int((x^2 + 5)/((1 - x^2)^(1/2)*(x^2 + 1)^2),x)
[Out] 2^(1/2)*log(((2^(1/2)*(x*1i - 1)*1i)/2 - (1 - x^2)^(1/2)*1i)/(x - 1i))*1i -
 2^(1/2)*log(((2^(1/2)*(x*1i + 1)*1i)/2 + (1 - x^2)^(1/2)*1i)/(x + 1i))*1i
+ (1 - x^2)^(1/2)/(2*(x - 1i)) + (1 - x^2)^(1/2)/(2*(x + 1i))
```

3.259 $\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 88

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx = -x-4\sqrt{1-x^2}+5\arcsin(x)+\frac{25\arctan\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}} - \frac{25\arctan\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}}+20\log\left(5+\sqrt{1-x^2}\right)$$

```
[Out] -x+5*arcsin(x)+20*ln(5+(-x^2+1)^(1/2))+25/12*arctan(1/12*x*6^(1/2))*6^(1/2)
-25/12*arctan(5/12*x*6^(1/2)/(-x^2+1)^(1/2))*6^(1/2)-4*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6874, 1605, 196, 45, 6872, 209, 399, 222, 385}

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx = 5\arcsin(x)-\frac{25\arctan\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}}+\frac{25\arctan\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}} -4\sqrt{1-x^2}+20\log\left(\sqrt{1-x^2}+5\right)-x$$

```
[In] Int[(4*x - Sqrt[1 - x^2])/(5 + Sqrt[1 - x^2]),x]
```

```
[Out] -x - 4*Sqrt[1 - x^2] + 5*ArcSin[x] + (25*ArcTan[x/(2*Sqrt[6])])/(2*Sqrt[6])
 - (25*ArcTan[(5*x)/(2*Sqrt[6]*Sqrt[1 - x^2])])/(2*Sqrt[6]) + 20*Log[5 + Sq
rt[1 - x^2]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
```

```
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{4x}{5+\sqrt{1-x^2}}-\frac{\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\right) dx \\
&= 4\int \frac{x}{5+\sqrt{1-x^2}}\, dx-\int \frac{\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\, dx \\
&= -\left(2\text{Subst}\left(\int \frac{1}{5+\sqrt{x}}\, dx,x,1-x^2\right)\right)-\int \left(1-\frac{5}{5+\sqrt{1-x^2}}\right) dx \\
&= -x-4\text{Subst}\left(\int \frac{x}{5+x}\, dx,x,\sqrt{1-x^2}\right)+5\int \frac{1}{5+\sqrt{1-x^2}}\, dx \\
&= -x-4\text{Subst}\left(\int \left(1-\frac{5}{5+x}\right) dx,x,\sqrt{1-x^2}\right)+5\int \left(\frac{5}{24+x^2}-\frac{\sqrt{1-x^2}}{24+x^2}\right) dx \\
&= -x-4\sqrt{1-x^2}+20\log\left(5+\sqrt{1-x^2}\right)-5\int \frac{\sqrt{1-x^2}}{24+x^2}\, dx+25\int \frac{1}{24+x^2}\, dx \\
&= -x-4\sqrt{1-x^2}+\frac{25\arctan\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}+20\log\left(5+\sqrt{1-x^2}\right) \\
&\quad +5\int \frac{1}{\sqrt{1-x^2}}\, dx-125\int \frac{1}{\sqrt{1-x^2}\,(24+x^2)}\, dx \\
&= -x-4\sqrt{1-x^2}+5\arcsin(x)+\frac{25\arctan\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}} \\
&\quad +20\log\left(5+\sqrt{1-x^2}\right)-125\text{Subst}\left(\int \frac{1}{24+25x^2}\, dx,x,\frac{x}{\sqrt{1-x^2}}\right) \\
&= -x-4\sqrt{1-x^2}+5\arcsin(x)+\frac{25\arctan\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}} \\
&\quad -\frac{25\arctan\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}}+20\log\left(5+\sqrt{1-x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.23

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx = -x-4\sqrt{1-x^2}+10\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right) - \frac{25\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{-1+\sqrt{1-x^2}}\right)}{\sqrt{6}} - 20\log\left(-1+\sqrt{1-x^2}\right) + 20\log\left(-4-x^2+4\sqrt{1-x^2}\right)$$

```
[In] Integrate[(4*x - Sqrt[1 - x^2])/(5 + Sqrt[1 - x^2]),x]
```

```
[Out] -x - 4*Sqrt[1 - x^2] + 10*ArcTan[x/(-1 + Sqrt[1 - x^2])] - (25*ArcTan[(Sqrt
[3/2]*x)/(-1 + Sqrt[1 - x^2])])/Sqrt[6] - 20*Log[-1 + Sqrt[1 - x^2]] + 20*L
og[-4 - x^2 + 4*Sqrt[1 - x^2]]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

method	result
default	$\frac{25\arctan\left(\frac{x\sqrt{6}}{12}\right)\sqrt{6}}{12}+10\ln\left(x^2+24\right)-x+5\arcsin\left(x\right)+\frac{25\sqrt{6}\,\arctan\left(\frac{5\sqrt{6}\,\sqrt{-x^2+1}\,x}{12\left(x^2-1\right)}\right)}{12}-4\sqrt{-x^2+1}+20$
trager	Expression too large to display

```
[In] int((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 25/12*arctan(1/12*x*6^(1/2))*6^(1/2)+10*ln(x^2+24)-x+5*arcsin(x)+25/12*6^(1
/2)*arctan(5/12*6^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)-4*(-x^2+1)^(1/2)+20*arcta
nh(1/5*(-x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(66) = 132.

Time = 0.25 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.82

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx = \frac{25}{12}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}x\right)+\frac{25}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{-x^2+1}-\sqrt{6}}{2\,x}\right)+\frac{25}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{-x^2+1}-\sqrt{6}}{3\,x}\right)-x-4\sqrt{-x^2+1}-10\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+10\log\left(x^2+24\right)-10\log\left(-\frac{x^2+6\sqrt{-x^2+1}-6}{x^2}\right)+10\log\left(\frac{x^2-4\sqrt{-x^2+1}+4}{x^2}\right)$$

```
[In] integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 25/12*sqrt(6)*arctan(1/12*sqrt(6)*x) + 25/12*sqrt(6)*arctan(1/2*(sqrt(6)*sq
rt(-x^2 + 1) - sqrt(6))/x) + 25/12*sqrt(6)*arctan(1/3*(sqrt(6)*sqrt(-x^2 +
1) - sqrt(6))/x) - x - 4*sqrt(-x^2 + 1) - 10*arctan((sqrt(-x^2 + 1) - 1)/x)
 + 10*log(x^2 + 24) - 10*log(-(x^2 + 6*sqrt(-x^2 + 1) - 6)/x^2) + 10*log((x
^2 - 4*sqrt(-x^2 + 1) + 4)/x^2)
```

Sympy [F]

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx = \int \frac{4x-\sqrt{1-x^2}}{\sqrt{1-x^2}+5}\,dx$$

```
[In] integrate((4*x-(-x**2+1)**(1/2))/(5+(-x**2+1)**(1/2)),x)
```

```
[Out] Integral((4*x - sqrt(1 - x**2))/(sqrt(1 - x**2) + 5), x)
```

Maxima [F]

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx = \int \frac{4\,x-\sqrt{-x^2+1}}{\sqrt{-x^2+1}+5}\,dx$$

```
[In] integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] -x - 4*sqrt(-x^2 + 1) + 5*integrate(1/(sqrt(x + 1)*sqrt(-x + 1) + 5), x) +
20*log(sqrt(-x^2 + 1) + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(66) = 132$.

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.53

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx = \frac{25}{12}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}x\right) - \frac{25}{12}\sqrt{6}\arctan\left(-\frac{\sqrt{6}\left(\sqrt{-x^2+1}-1\right)}{3\,x}\right) - \frac{25}{12}\sqrt{6}\arctan\left(-\frac{\sqrt{6}\left(\sqrt{-x^2+1}-1\right)}{2\,x}\right) - x - 4\sqrt{-x^2+1} + 5\arcsin\left(x\right) + 10\log\left(x^2+24\right) - 10\log\left(\frac{3\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+2\right) + 10\log\left(\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+3\right)$$

```
[In] integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 25/12*sqrt(6)*arctan(1/12*sqrt(6)*x) - 25/12*sqrt(6)*arctan(-1/3*sqrt(6)*(s
qrt(-x^2 + 1) - 1)/x) - 25/12*sqrt(6)*arctan(-1/2*sqrt(6)*(sqrt(-x^2 + 1) -
 1)/x) - x - 4*sqrt(-x^2 + 1) + 5*arcsin(x) + 10*log(x^2 + 24) - 10*log(3*(
sqrt(-x^2 + 1) - 1)^2/x^2 + 2) + 10*log(2*(sqrt(-x^2 + 1) - 1)^2/x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.81

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx = 5\,\mathrm{asin}(x) - x - 4\sqrt{1-x^2} - \frac{\sqrt{24}\,\ln\left(\frac{\frac{2\sqrt{6}\,x}{5}+\sqrt{1-x^2}\,1\mathrm{i}+\frac{1}{5}\mathrm{i}}{x-\sqrt{6}\,2\mathrm{i}}\right)\,\left(125+\sqrt{24}\,100\mathrm{i}\right)\,1\mathrm{i}}{240} - \frac{\sqrt{24}\,\ln\left(\frac{-\frac{\sqrt{24}\,x}{5}+\sqrt{1-x^2}\,1\mathrm{i}+\frac{1}{5}\mathrm{i}}{x+\sqrt{24}\,1\mathrm{i}}\right)\,\left(-125+\sqrt{24}\,100\mathrm{i}\right)\,1\mathrm{i}}{240} - \frac{\sqrt{24}\,\ln\left(x-\sqrt{6}\,2\mathrm{i}\right)\,\left(25+\sqrt{24}\,20\mathrm{i}\right)\,1\mathrm{i}}{48} - \frac{\sqrt{24}\,\ln\left(x+\sqrt{24}\,1\mathrm{i}\right)\,\left(-25+\sqrt{24}\,20\mathrm{i}\right)\,1\mathrm{i}}{48}$$

```
[In] int((4*x - (1 - x^2)^(1/2))/((1 - x^2)^(1/2) + 5),x)
```

```
[Out] 5*asin(x) - x - 4*(1 - x^2)^(1/2) - (24^(1/2)*log(((2*6^(1/2)*x)/5 + (1 - x
^2)^(1/2)*1i + 1i/5)/(x - 6^(1/2)*2i))*(24^(1/2)*100i + 125)*1i)/240 - (24^
```

```
(1/2)*log(((1 - x^2)^(1/2)*1i - (24^(1/2)*x)/5 + 1i/5)/(x + 24^(1/2)*1i))*(
24^(1/2)*100i - 125)*1i)/240 - (24^(1/2)*log(x - 6^(1/2)*2i)*(24^(1/2)*20i
+ 25)*1i)/48 - (24^(1/2)*log(x + 24^(1/2)*1i)*(24^(1/2)*20i - 25)*1i)/48
```

3.260 $\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 136

$$\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx = \frac{8x}{9}-\frac{x^2}{6}+\frac{8\sqrt{1+x^2}}{9} -\frac{1}{6}x\sqrt{1+x^2}-\frac{41\operatorname{arcsinh}(x)}{54}+\frac{4}{27}\sqrt{2}\arctan\left(\frac{1+3x}{2\sqrt{2}}\right) +\frac{4}{27}\sqrt{2}\arctan\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right)+\frac{7}{27}\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{1+x^2}}\right)-\frac{7}{54}\log\left(3+2x+3x^2\right)$$

```
[Out] 8/9*x-1/6*x^2-41/54*arcsinh(x)+7/27*arctanh(1/2*(1-x)/(x^2+1)^(1/2))-7/54*l
n(3*x^2+2*x+3)+4/27*arctan(1/4*(1+3*x)*2^(1/2))*2^(1/2)+4/27*arctan(1/2*(1+
x)*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)+8/9*(x^2+1)^(1/2)-1/6*x*(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6874, 201, 221, 648, 632, 210, 642, 1034, 12, 1095, 1051, 1045, 212, 267}

$$\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx = -\frac{41\operatorname{arcsinh}(x)}{54} +\frac{4}{27}\sqrt{2}\arctan\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)+\frac{4}{27}\sqrt{2}\arctan\left(\frac{3x+1}{2\sqrt{2}}\right)+\frac{7}{27}\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{x^2+1}}\right) -\frac{x^2}{6}-\frac{1}{6}\sqrt{x^2+1}x+\frac{8\sqrt{x^2+1}}{9}-\frac{7}{54}\log\left(3x^2+2x+3\right)+\frac{8x}{9}$$

```
[In] Int[(x^2*(2 - Sqrt[1 + x^2]))/(Sqrt[1 + x^2]*(1 - x^3 + (1 + x^2)^(3/2))),x
]
```

```
[Out] (8*x)/9 - x^2/6 + (8*Sqrt[1 + x^2])/9 - (x*Sqrt[1 + x^2])/6 - (41*ArcSinh[x
])/54 + (4*Sqrt[2]*ArcTan[(1 + 3*x)/(2*Sqrt[2])])/27 + (4*Sqrt[2]*ArcTan[(1
 + x)/(Sqrt[2]*Sqrt[1 + x^2])])/27 + (7*ArcTanh[(1 - x)/(2*Sqrt[1 + x^2])])
/27 - (7*Log[3 + 2*x + 3*x^2])/54
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + c*x^2)^p*((d + e*x + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1045

```
Int[((g_) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b
 - 2*a*h)*(b^2 - 4*a*c) - b*d*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g
*c)*x, x]/Sqrt[d + f*x^2]], x] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^
2 - 4*a*c, 0] && EqQ[b*h^2*d - 2*g*h*(c*d - a*f) - g^2*b*f, 0]
```

Rule 1051

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + b^2*d*f, 2]}, Dist
[1/(2*q), Int[Simp[h*b*d - g*(c*d - a*f - q) + (h*(c*d - a*f + q) + g*b*f)*
x, x]/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[
h*b*d - g*(c*d - a*f + q) + (h*(c*d - a*f - q) + g*b*f)*x, x]/((a + b*x + c
*x^2)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b
^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1095

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dis
```

```
t[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x]
/; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{x^2}{1-x^3+\sqrt{1+x^2}+x^2\sqrt{1+x^2}}-\frac{2x^2}{\sqrt{1+x^2}\left(-1+x^3-(1+x^2)^{3/2}\right)}\right)dx\\
&=-\left(2\int \frac{x^2}{\sqrt{1+x^2}\left(-1+x^3-(1+x^2)^{3/2}\right)}\,dx\right)-\int \frac{x^2}{1-x^3+\sqrt{1+x^2}+x^2\sqrt{1+x^2}}\,dx\\
&=-\left(2\int \left(-\frac{1}{3}+\frac{2}{9\sqrt{1+x^2}}-\frac{x}{3\sqrt{1+x^2}}+\frac{2x}{3(3+2x+3x^2)}\right.\right.\\
&\qquad\left.\left.+\frac{3+5x}{9\sqrt{1+x^2}(3+2x+3x^2)}\right)dx\right)\\
&\quad-\int \left(-\frac{2}{9}+\frac{x}{3}+\frac{\sqrt{1+x^2}}{3}+\frac{-3-5x}{9(3+2x+3x^2)}-\frac{2x\sqrt{1+x^2}}{3(3+2x+3x^2)}\right)dx\\
&=\frac{8x}{9}-\frac{x^2}{6}-\frac{1}{9}\int \frac{-3-5x}{3+2x+3x^2}\,dx-\frac{2}{9}\int \frac{3+5x}{\sqrt{1+x^2}(3+2x+3x^2)}\,dx-\frac{1}{3}\int \sqrt{1+x^2}\,dx\\
&\quad-\frac{4}{9}\int \frac{1}{\sqrt{1+x^2}}\,dx+\frac{2}{3}\int \frac{x}{\sqrt{1+x^2}}\,dx+\frac{2}{3}\int \frac{x\sqrt{1+x^2}}{3+2x+3x^2}\,dx-\frac{4}{3}\int \frac{x}{3+2x+3x^2}\,dx\\
&=\frac{8x}{9}-\frac{x^2}{6}+\frac{8\sqrt{1+x^2}}{9}-\frac{1}{6}x\sqrt{1+x^2}-\frac{4\operatorname{arcsinh}(x)}{9}+\frac{1}{18}\int \frac{4-4x}{\sqrt{1+x^2}(3+2x+3x^2)}\,dx\\
&\quad-\frac{1}{18}\int \frac{16+16x}{\sqrt{1+x^2}(3+2x+3x^2)}\,dx+\frac{5}{54}\int \frac{2+6x}{3+2x+3x^2}\,dx\\
&\quad+\frac{4}{27}\int \frac{1}{3+2x+3x^2}\,dx-\frac{1}{6}\int \frac{1}{\sqrt{1+x^2}}\,dx-\frac{2}{9}\int \frac{2+6x}{3+2x+3x^2}\,dx\\
&\quad+\frac{2}{9}\int -\frac{2x^2}{\sqrt{1+x^2}(3+2x+3x^2)}\,dx+\frac{4}{9}\int \frac{1}{3+2x+3x^2}\,dx
\end{aligned}$$

$$= \frac{8x}{9} - \frac{x^2}{6} + \frac{8\sqrt{1+x^2}}{9} - \frac{1}{6}x\sqrt{1+x^2} - \frac{11\text{arcsinh}(x)}{18} - \frac{7}{54}\log(3+2x+3x^2) - \frac{8}{27}\text{Subst}\left(\int \frac{1}{-32-x^2}\,dx, x, 2+6x\right) - \frac{4}{9}\int \frac{x^2}{\sqrt{1+x^2}\,(3+2x+3x^2)}\,dx - \frac{8}{9}\text{Subst}\left(\int \frac{1}{-32-x^2}\,dx, x, 2+6x\right) - \frac{128}{9}\text{Subst}\left(\int \frac{1}{-4096-2x^2}\,dx, x, \frac{32+32x}{\sqrt{1+x^2}}\right) - \frac{1024}{9}\text{Subst}\left(\int \frac{1}{32768-2x^2}\,dx, x, \frac{-64+64x}{\sqrt{1+x^2}}\right)$$

$$= \frac{8x}{9} - \frac{x^2}{6} + \frac{8\sqrt{1+x^2}}{9} - \frac{1}{6}x\sqrt{1+x^2} - \frac{11\text{arcsinh}(x)}{18} + \frac{4}{27}\sqrt{2}\arctan\left(\frac{1+3x}{2\sqrt{2}}\right) + \frac{1}{9}\sqrt{2}\arctan\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right) + \frac{4}{9}\text{arctanh}\left(\frac{1-x}{2\sqrt{1+x^2}}\right) - \frac{7}{54}\log(3+2x+3x^2) - \frac{4}{27}\int \frac{1}{\sqrt{1+x^2}}\,dx - \frac{4}{27}\int \frac{-3-2x}{\sqrt{1+x^2}\,(3+2x+3x^2)}\,dx$$

$$= \frac{8x}{9} - \frac{x^2}{6} + \frac{8\sqrt{1+x^2}}{9} - \frac{1}{6}x\sqrt{1+x^2} - \frac{41\text{arcsinh}(x)}{54} + \frac{4}{27}\sqrt{2}\arctan\left(\frac{1+3x}{2\sqrt{2}}\right) + \frac{1}{9}\sqrt{2}\arctan\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right) + \frac{4}{9}\text{arctanh}\left(\frac{1-x}{2\sqrt{1+x^2}}\right) - \frac{7}{54}\log(3+2x+3x^2) - \frac{1}{27}\int \frac{-10-10x}{\sqrt{1+x^2}\,(3+2x+3x^2)}\,dx + \frac{1}{27}\int \frac{2-2x}{\sqrt{1+x^2}\,(3+2x+3x^2)}\,dx$$

$$= \frac{8x}{9} - \frac{x^2}{6} + \frac{8\sqrt{1+x^2}}{9} - \frac{1}{6}x\sqrt{1+x^2} - \frac{41\text{arcsinh}(x)}{54} + \frac{4}{27}\sqrt{2}\arctan\left(\frac{1+3x}{2\sqrt{2}}\right) + \frac{1}{9}\sqrt{2}\arctan\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right) + \frac{4}{9}\text{arctanh}\left(\frac{1-x}{2\sqrt{1+x^2}}\right) - \frac{7}{54}\log(3+2x+3x^2) - \frac{64}{27}\text{Subst}\left(\int \frac{1}{-1024-2x^2}\,dx, x, \frac{16+16x}{\sqrt{1+x^2}}\right) - \frac{800}{27}\text{Subst}\left(\int \frac{1}{12800-2x^2}\,dx, x, \frac{40-40x}{\sqrt{1+x^2}}\right)$$

$$= \frac{8x}{9} - \frac{x^2}{6} + \frac{8\sqrt{1+x^2}}{9} - \frac{1}{6}x\sqrt{1+x^2} - \frac{41\text{arcsinh}(x)}{54} + \frac{4}{27}\sqrt{2}\arctan\left(\frac{1+3x}{2\sqrt{2}}\right) + \frac{4}{27}\sqrt{2}\arctan\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right) + \frac{7}{27}\text{arctanh}\left(\frac{1-x}{2\sqrt{1+x^2}}\right) - \frac{7}{54}\log(3+2x+3x^2)$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

$$\int \frac{x^2(2-\sqrt{1+x^2})}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx = \frac{1}{54}\left(48x-9x^2 + 48\sqrt{1+x^2}-9x\sqrt{1+x^2}+16\sqrt{2}\arctan\left(\frac{1+x-\sqrt{1+x^2}}{\sqrt{2}}\right) + 55\log\left(-x+\sqrt{1+x^2}\right)-14\log\left(-2-x-x^2+(1+x)\sqrt{1+x^2}\right)\right)$$

```
[In] Integrate[(x^2*(2 - Sqrt[1 + x^2]))/(Sqrt[1 + x^2]*(1 - x^3 + (1 + x^2)^(3/
2))),x]
```

```
[Out] (48*x - 9*x^2 + 48*Sqrt[1 + x^2] - 9*x*Sqrt[1 + x^2] + 16*Sqrt[2]*ArcTan[(1
 + x - Sqrt[1 + x^2])/Sqrt[2]] + 55*Log[-x + Sqrt[1 + x^2]] - 14*Log[-2 - x
 - x^2 + (1 + x)*Sqrt[1 + x^2]])/54
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(99) = 198$.

Time = 0.04 (sec) , antiderivative size = 654, normalized size of antiderivative = 4.81

$$-\frac{x^2}{6}+\frac{8x}{9}-\frac{7\ln\left(3x^2+2x+3\right)}{54}+\frac{4\sqrt{2}\,\arctan\left(\frac{(6x+2)\sqrt{2}}{8}\right)}{27}-\frac{41\,\operatorname{arcsinh}(x)}{54}-\frac{\sqrt{2}\,\sqrt{\frac{2(1+x)^2}{(1-x)^2}+2}\left(-\sqrt{2}\,\arctan\left(\right.\right.}{12\sqrt{}}$$

```
[In] int(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x)
```

```
[Out] -1/6*x^2+8/9*x-7/54*ln(3*x^2+2*x+3)+4/27*2^(1/2)*arctan(1/8*(6*x+2)*2^(1/2)
)-41/54*arcsinh(x)-1/12*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-2^(1/2)*arcta
n(1/2*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/(1-x))+
5*arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/((1+x)/(1-x)+1
)^2)^(1/2)/((1+x)/(1-x)+1)+3/8*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-2^(1/2
)*arctan(1/2*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/
(1-x))+arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/((1+x)/(1
-x)+1)^2)^(1/2)/((1+x)/(1-x)+1)-1/6*x*(x^2+1)^(1/2)+8/9*(x^2+1)^(1/2)+1/216
*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(13*2^(1/2)*arctan(1/2*2^(1/2)*(2*(1+x
)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/(1-x))+43*arctanh((2*(1+x)^2
/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/((1+x)/(1-x)+1)^2)^(1/2)/((1+x)/(1
```

```
-x)+1)-1/36*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-11*2^(1/2)*arctan(1/2*2^(
1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/(1-x))+arctanh((
2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/((1+x)/(1-x)+1)^2)^(1/2)/
((1+x)/(1-x)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.25

$$\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+\left(1+x^2\right)^{3/2}\right)}\,dx = -\frac{1}{6}\,x^2-\frac{1}{18}\,\sqrt{x^2+1}(3\,x-16) + \frac{4}{27}\,\sqrt{2}\arctan\left(\frac{1}{4}\,\sqrt{2}(3\,x+1)\right)+\frac{4}{27}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}(3\,x-1)+\frac{3}{2}\,\sqrt{2}\sqrt{x^2+1}\right) - \frac{4}{27}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}(x+1)+\frac{1}{2}\,\sqrt{2}\sqrt{x^2+1}\right)+\frac{8}{9}\,x + \frac{7}{54}\,\log\left(3\,x^2-\sqrt{x^2+1}(3\,x-1)-x+2\right)-\frac{7}{54}\,\log\left(3\,x^2+2\,x+3\right) - \frac{7}{54}\,\log\left(x^2-\sqrt{x^2+1}(x+1)+x+2\right)+\frac{41}{54}\,\log\left(-x+\sqrt{x^2+1}\right)$$

```
[In] integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, algo
rithm="fricas")
```

```
[Out] -1/6*x^2 - 1/18*sqrt(x^2 + 1)*(3*x - 16) + 4/27*sqrt(2)*arctan(1/4*sqrt(2)*
(3*x + 1)) + 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(3*x - 1) + 3/2*sqrt(2)*sqrt(
x^2 + 1)) - 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(x + 1) + 1/2*sqrt(2)*sqrt(x^2
 + 1)) + 8/9*x + 7/54*log(3*x^2 - sqrt(x^2 + 1)*(3*x - 1) - x + 2) - 7/54*l
og(3*x^2 + 2*x + 3) - 7/54*log(x^2 - sqrt(x^2 + 1)*(x + 1) + x + 2) + 41/54
*log(-x + sqrt(x^2 + 1))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+\left(1+x^2\right)^{3/2}\right)}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(2-(x**2+1)**(1/2))/(1-x**3+(x**2+1)**(3/2))/(x**2+1)**(1/2)
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(2-\sqrt{1+x^2})}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx=\int \frac{x^2\left(\sqrt{x^2+1}-2\right)}{\left(x^3-(x^2+1)^{\frac{3}{2}}-1\right)\sqrt{x^2+1}}\,dx$$

```
[In] integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, algo
rithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x) + integrate(-1/2*(3*x^10 - 4*x^9 + 5*x^8 -
 2*x^7 + 15*x^6 + 6*x^5 + 9*x^4)/(2*x^13 + 7*x^11 - 4*x^10 + 11*x^9 - 11*x^
8 + 13*x^7 - 13*x^6 + 11*x^5 - 11*x^4 + 4*x^3 - 7*x^2 - 2*(x^12 + 3*x^10 -
2*x^9 + 3*x^8 - 6*x^7 + 2*x^6 - 6*x^5 + 3*x^4 - 2*x^3 + 3*x^2 + 1)*sqrt(x^2
 + 1) - 2), x) + 1/6*log(x^2 + x + 1) + 1/6*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.29

$$\begin{aligned}\int \frac{x^2(2-\sqrt{1+x^2})}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx &= -\frac{1}{6}x^2-\frac{1}{18}\sqrt{x^2+1}(3x-16)\\ &+\frac{4}{27}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(3x-3\sqrt{x^2+1}-1\right)\right)\\ &+\frac{4}{27}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}(3x+1)\right)-\frac{4}{27}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2+1}+1\right)\right)\\ &+\frac{8}{9}x+\frac{7}{54}\log\left(3\left(x-\sqrt{x^2+1}\right)^2-2x+2\sqrt{x^2+1}+1\right)\\ &-\frac{7}{54}\log\left(\left(x-\sqrt{x^2+1}\right)^2+2x-2\sqrt{x^2+1}+3\right)\\ &-\frac{7}{54}\log\left(3x^2+2x+3\right)+\frac{41}{54}\log\left(-x+\sqrt{x^2+1}\right)\end{aligned}$$

```
[In] integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, algo
rithm="giac")
```

```
[Out] -1/6*x^2 - 1/18*sqrt(x^2 + 1)*(3*x - 16) + 4/27*sqrt(2)*arctan(-1/2*sqrt(2)
*(3*x - 3*sqrt(x^2 + 1) - 1)) + 4/27*sqrt(2)*arctan(1/4*sqrt(2)*(3*x + 1))
- 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 1) + 1)) + 8/9*x + 7/54*
log(3*(x - sqrt(x^2 + 1))^2 - 2*x + 2*sqrt(x^2 + 1) + 1) - 7/54*log((x - sq
rt(x^2 + 1))^2 + 2*x - 2*sqrt(x^2 + 1) + 3) - 7/54*log(3*x^2 + 2*x + 3) + 4
1/54*log(-x + sqrt(x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.59

$$\int \frac{x^2(2-\sqrt{1+x^2})}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx = \frac{8\,x}{9}-\frac{41\,\mathrm{asinh}(x)}{54}-\left(\frac{x}{6}-\frac{8}{9}\right)\sqrt{x^2+1}-\frac{x^2}{6}$$
$$+\frac{\sqrt{2}\,\ln\left(x+\frac{1}{3}-\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)\left(-\frac{16}{27}+\frac{\sqrt{2}\,14\mathrm{i}}{27}\right)\,1\mathrm{i}}{8}+\frac{\sqrt{2}\,\ln\left(x+\frac{1}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)\left(\frac{16}{27}+\frac{\sqrt{2}\,14\mathrm{i}}{27}\right)\,1\mathrm{i}}{8}$$
$$+\frac{\sqrt{2}\left(\frac{4}{81}+\frac{\sqrt{2}\,44\mathrm{i}}{81}\right)\left(\ln\left(x+\frac{1}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)-\ln\left(1+\left(\frac{2}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}\right)\sqrt{x^2+1}-\frac{x}{3}-\frac{\sqrt{2}\,x\,2\mathrm{i}}{3}\right)\right)\,1\mathrm{i}}{8\sqrt{\left(\frac{1}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)^2+1}}$$
$$+\frac{\sqrt{2}\left(-\frac{4}{81}+\frac{\sqrt{2}\,44\mathrm{i}}{81}\right)\left(\ln\left(x+\frac{1}{3}-\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)-\ln\left(1-\left(-\frac{2}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}\right)\sqrt{x^2+1}-\frac{x}{3}+\frac{\sqrt{2}\,x\,2\mathrm{i}}{3}\right)\right)\,1\mathrm{i}}{8\sqrt{\left(-\frac{1}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)^2+1}}$$

```
[In] int(-(x^2*((x^2 + 1)^(1/2) - 2))/((x^2 + 1)^(1/2)*((x^2 + 1)^(3/2) - x^3 +
1)),x)
```

```
[Out] (8*x)/9 - (41*asinh(x))/54 - (x/6 - 8/9)*(x^2 + 1)^(1/2) - x^2/6 + (2^(1/2)
*log(x - (2^(1/2)*2i)/3 + 1/3)*((2^(1/2)*14i)/27 - 16/27)*1i)/8 + (2^(1/2)*
log(x + (2^(1/2)*2i)/3 + 1/3)*((2^(1/2)*14i)/27 + 16/27)*1i)/8 + (2^(1/2)*(
(2^(1/2)*44i)/81 + 4/81)*(log(x + (2^(1/2)*2i)/3 + 1/3) - log(((2^(1/2)*1i)
/3 + 2/3)*(x^2 + 1)^(1/2) - x/3 - (2^(1/2)*x*2i)/3 + 1))*1i)/(8*(((2^(1/2)*
2i)/3 + 1/3)^2 + 1)^(1/2)) + (2^(1/2)*((2^(1/2)*44i)/81 - 4/81)*(log(x - (2
^(1/2)*2i)/3 + 1/3) - log((2^(1/2)*x*2i)/3 - ((2^(1/2)*1i)/3 - 2/3)*(x^2 +
1)^(1/2) - x/3 + 1))*1i)/(8*(((2^(1/2)*2i)/3 - 1/3)^2 + 1)^(1/2))
```

3.261 $\int x\sqrt{2rx-x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 64

$$\int x\sqrt{2rx-x^2}\,dx = -\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}+r^3\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)$$

```
[Out] -1/3*(2*r*x-x^2)^(3/2)+r^3*arctan(x/(2*r*x-x^2)^(1/2))-1/2*r*(r-x)*(2*r*x-x
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {654, 626, 634, 209}

$$\int x\sqrt{2rx-x^2}\,dx = r^3\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}$$

```
[In] Int[x*Sqrt[2*r*x - x^2],x]
```

```
[Out] -1/2*(r*(r - x)*Sqrt[2*r*x - x^2]) - (2*r*x - x^2)^(3/2)/3 + r^3*ArcTan[x/S
qrt[2*r*x - x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{3}\left(2rx-x^2\right)^{3/2}+r\int\sqrt{2rx-x^2}\,dx\\
&= -\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}+\frac{1}{2}r^3\int\frac{1}{\sqrt{2rx-x^2}}\,dx\\
&= -\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}+r^3\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt{2rx-x^2}}\right)\\
&= -\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}+r^3\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int x\sqrt{2rx-x^2}\,dx=\frac{1}{6}\sqrt{-x(-2r+x)}\left(-3r^2-rx+2x^2+\frac{6r^3\log\left(-\sqrt{x}+\sqrt{-2r+x}\right)}{\sqrt{x}\sqrt{-2r+x}}\right)$$

```
[In] Integrate[x*Sqrt[2*r*x - x^2],x]
[Out] (Sqrt[-(x*(-2*r + x))]*(-3*r^2 - r*x + 2*x^2 + (6*r^3*Log[-Sqrt[x] + Sqrt[-
2*r + x]])/(Sqrt[x]*Sqrt[-2*r + x])))/6
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$-\arctan\left(\frac{\sqrt{x(2r-x)}}{x}\right)r^{3}-\frac{\sqrt{x(2r-x)}\,(r+x)\left(r-\frac{2x}{3}\right)}{2}$	44
risch	$-\frac{(3r^{2}+rx-2x^{2})x(2r-x)}{6\sqrt{-x(-2r+x)}}+\frac{r^{3}\arctan\left(\frac{x-r}{\sqrt{2rx-x^{2}}}\right)}{2}$	60
default	$-\frac{(2rx-x^{2})^{\frac{3}{2}}}{3}+r\left(-\frac{(2r-2x)\sqrt{2rx-x^{2}}}{4}+\frac{r^{2}\arctan\left(\frac{x-r}{\sqrt{2rx-x^{2}}}\right)}{2}\right)$	64

```
[In] int(x*(2*r*x-x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -arctan(1/x*(x*(2*r-x))^(1/2))*r^3-1/2*(x*(2*r-x))^(1/2)*(r+x)*(r-2/3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int x\sqrt{2rx-x^2}\,dx=-r^3\arctan\left(\frac{\sqrt{2\,rx-x^2}}{x}\right)-\frac{1}{6}\left(3\,r^2+rx-2\,x^2\right)\sqrt{2\,rx-x^2}$$

```
[In] integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="fricas")
[Out] -r^3*arctan(sqrt(2*r*x - x^2)/x) - 1/6*(3*r^2 + r*x - 2*x^2)*sqrt(2*r*x - x
^2)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int x\sqrt{2rx-x^2}\,dx=\frac{r^3\left(\begin{cases}-i\log\left(2r-2x+2i\sqrt{2rx-x^2}\right) & \text{for } r^2\neq 0\\ \frac{(-r+x)\log(-r+x)}{\sqrt{-(-r+x)^2}} & \text{otherwise}\end{cases}\right)}{2}+\sqrt{2rx-x^2}\left(-\frac{r^2}{2}-\frac{rx}{6}+\frac{x^2}{3}\right)$$

```
[In] integrate(x*(2*r*x-x**2)**(1/2),x)
[Out] r**3*Piecewise((-I*log(2*r - 2*x + 2*I*sqrt(2*r*x - x**2)), Ne(r**2, 0)), (
(-r + x)*log(-r + x)/sqrt(-(-r + x)**2), True))/2 + sqrt(2*r*x - x**2)*(-r*
*2/2 - r*x/6 + x**2/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int x\sqrt{2rx-x^2}\,dx = -\frac{1}{2}\,r^3\arcsin\left(\frac{r-x}{r}\right) - \frac{1}{2}\,\sqrt{2\,rx-x^2}r^2 + \frac{1}{2}\,\sqrt{2\,rx-x^2}rx - \frac{1}{3}\,\left(2\,rx-x^2\right)^{\frac{3}{2}}$$

```
[In] integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*r^3*arcsin((r - x)/r) - 1/2*sqrt(2*r*x - x^2)*r^2 + 1/2*sqrt(2*r*x - x
^2)*r*x - 1/3*(2*r*x - x^2)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.70

$$\int x\sqrt{2rx-x^2}\,dx = -\frac{1}{2}\,r^3\arcsin\left(\frac{r-x}{r}\right)\mathrm{sgn}(r) - \frac{1}{6}\,\left(3\,r^2+(r-2\,x)x\right)\sqrt{2\,rx-x^2}$$

```
[In] integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="giac")
[Out] -1/2*r^3*arcsin((r - x)/r)*sgn(r) - 1/6*(3*r^2 + (r - 2*x)*x)*sqrt(2*r*x -
x^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int x\sqrt{2rx-x^2}\,dx = -\frac{\sqrt{2\,r\,x-x^2}\,\left(12\,r^2+4\,r\,x-8\,x^2\right)}{24} - \frac{r^3\,\ln\left(x-r-\sqrt{x\,\left(2\,r-x\right)}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

```
[In] int(x*(2*r*x - x^2)^(1/2),x)
[Out] - ((2*r*x - x^2)^(1/2)*(4*r*x + 12*r^2 - 8*x^2))/24 - (r^3*log(x - r - (x*(
2*r - x))^(1/2)*1i)*1i)/2
```

3.262 $\int x^2\sqrt{2rx-x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int x^2\sqrt{2rx-x^2}\,dx=-\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2}-\frac{5}{12}r\left(2rx-x^2\right)^{3/2}-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}+\frac{5}{4}r^4\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)$$

```
[Out] -5/12*r*(2*r*x-x^2)^(3/2)-1/4*x*(2*r*x-x^2)^(3/2)+5/4*r^4*arctan(x/(2*r*x-x
^2)^(1/2))-5/8*r^2*(r-x)*(2*r*x-x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {684, 654, 626, 634, 209}

$$\int x^2\sqrt{2rx-x^2}\,dx=\frac{5}{4}r^4\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2}-\frac{5}{12}r\left(2rx-x^2\right)^{3/2}-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

```
[In] Int[x^2*Sqrt[2*r*x - x^2],x]
```

```
[Out] (-5*r^2*(r - x)*Sqrt[2*r*x - x^2])/8 - (5*r*(2*r*x - x^2)^(3/2))/12 - (x*(2
*r*x - x^2)^(3/2))/4 + (5*r^4*ArcTan[x/Sqrt[2*r*x - x^2]])/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{4}x\left(2rx-x^2\right)^{3/2}+\frac{1}{4}(5r)\int x\sqrt{2rx-x^2}\,dx \\
&= -\frac{5}{12}r\left(2rx-x^2\right)^{3/2}-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}+\frac{1}{4}\left(5r^2\right)\int \sqrt{2rx-x^2}\,dx \\
&= -\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2}-\frac{5}{12}r\left(2rx-x^2\right)^{3/2}-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}+\frac{1}{8}\left(5r^4\right)\int \frac{1}{\sqrt{2rx-x^2}}\,dx \\
&= -\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2}-\frac{5}{12}r\left(2rx-x^2\right)^{3/2} \\
&\quad -\frac{1}{4}x\left(2rx-x^2\right)^{3/2}+\frac{1}{4}\left(5r^4\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt{2rx-x^2}}\right) \\
&= -\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2}-\frac{5}{12}r\left(2rx-x^2\right)^{3/2}-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}+\frac{5}{4}r^4\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int x^2\sqrt{2rx-x^2}\,dx=\frac{1}{24}\sqrt{-x(-2r+x)}\left(-15r^3-5r^2x-2rx^2+6x^3+\frac{30r^4\log\left(-\sqrt{x}+\sqrt{-2r+x}\right)}{\sqrt{x}\sqrt{-2r+x}}\right)$$

```
[In] Integrate[x^2*Sqrt[2*r*x - x^2],x]
[Out] (Sqrt[-(x*(-2*r + x))]*(-15*r^3 - 5*r^2*x - 2*r*x^2 + 6*x^3 + (30*r^4*Log[-
Sqrt[x] + Sqrt[-2*r + x]])/(Sqrt[x]*Sqrt[-2*r + x])))/24
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$-\frac{5\arctan\left(\frac{\sqrt{x(2r-x)}}{x}\right)r^4}{4}-\frac{5\sqrt{x(2r-x)}\left(r^3+\frac{1}{3}r^2x+\frac{2}{15}r\,x^2-\frac{2}{5}x^3\right)}{8}$	57
risch	$-\frac{(15r^3+5r^2x+2r\,x^2-6x^3)x(2r-x)}{24\sqrt{-x(-2r+x)}}+\frac{5r^4\arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{8}$	69
default	$-\frac{x(2rx-x^2)^{\frac{3}{2}}}{4}+\frac{5r\left(-\frac{(2rx-x^2)^{\frac{3}{2}}}{3}+r\left(-\frac{(2r-2x)\sqrt{2rx-x^2}}{4}+\frac{r^2\arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{2}\right)\right)}{4}$	83

```
[In] int(x^2*(2*r*x-x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -5/4*arctan(1/x*(x*(2*r-x))^(1/2))*r^4-5/8*(x*(2*r-x))^(1/2)*(r^3+1/3*r^2*x
+2/15*r*x^2-2/5*x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int x^2\sqrt{2rx-x^2}\,dx=-\frac{5}{4}\,r^4\arctan\left(\frac{\sqrt{2\,rx-x^2}}{x}\right)-\frac{1}{24}\left(15\,r^3+5\,r^2x+2\,rx^2-6\,x^3\right)\sqrt{2\,rx-x^2}$$

```
[In] integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -5/4*r^4*arctan(sqrt(2*r*x - x^2)/x) - 1/24*(15*r^3 + 5*r^2*x + 2*r*x^2 - 6
*x^3)*sqrt(2*r*x - x^2)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int x^2\sqrt{2rx-x^2}\,dx = \frac{5r^4\left(\begin{cases} -i\log\left(2r-2x+2i\sqrt{2rx-x^2}\right) & \text{for } r^2\neq 0 \\ \frac{(-r+x)\log(-r+x)}{\sqrt{-(-r+x)^2}} & \text{otherwise} \end{cases}\right)}{8} + \sqrt{2rx-x^2}\left(-\frac{5r^3}{8}-\frac{5r^2x}{24}-\frac{rx^2}{12}+\frac{x^3}{4}\right)$$

```
[In] integrate(x**2*(2*r*x-x**2)**(1/2),x)
```

```
[Out] 5*r**4*Piecewise((-I*log(2*r - 2*x + 2*I*sqrt(2*r*x - x**2)), Ne(r**2, 0)),
 ((-r + x)*log(-r + x)/sqrt(-(-r + x)**2), True))/8 + sqrt(2*r*x - x**2)*(-
5*r**3/8 - 5*r**2*x/24 - r*x**2/12 + x**3/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int x^2\sqrt{2rx-x^2}\,dx = -\frac{5}{8}r^4\arcsin\left(\frac{r-x}{r}\right)-\frac{5}{8}\sqrt{2\,rx-x^2}r^3 + \frac{5}{8}\sqrt{2\,rx-x^2}r^2x-\frac{5}{12}\left(2\,rx-x^2\right)^{\frac{3}{2}}r-\frac{1}{4}\left(2\,rx-x^2\right)^{\frac{3}{2}}x$$

```
[In] integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/8*r^4*arcsin((r - x)/r) - 5/8*sqrt(2*r*x - x^2)*r^3 + 5/8*sqrt(2*r*x - x
^2)*r^2*x - 5/12*(2*r*x - x^2)^(3/2)*r - 1/4*(2*r*x - x^2)^(3/2)*x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int x^2\sqrt{2rx-x^2}\,dx = -\frac{5}{8}\,r^4\arcsin\left(\frac{r-x}{r}\right)\operatorname{sgn}(r) - \frac{1}{24}\left(15\,r^3+\left(5\,r^2+2\,(r-3\,x)x\right)x\right)\sqrt{2\,rx-x^2}$$

```
[In] integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -5/8*r^4*arcsin((r - x)/r)*sgn(r) - 1/24*(15*r^3 + (5*r^2 + 2*(r - 3*x)*x)*
x)*sqrt(2*r*x - x^2)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

$$\int x^2\sqrt{2rx-x^2}\,dx = -\frac{x\left(2\,r\,x-x^2\right)^{3/2}}{4} - \frac{5\,r\left(\frac{\sqrt{2\,r\,x-x^2}\,\left(12\,r^2+4\,r\,x-8\,x^2\right)}{24}+\frac{r^3\,\ln\left(x-r-\sqrt{x\,(2\,r-x)}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}\right)}{4}$$

```
[In] int(x^2*(2*r*x - x^2)^(1/2),x)
```

```
[Out] - (x*(2*r*x - x^2)^(3/2))/4 - (5*r*(((2*r*x - x^2)^(1/2)*(4*r*x + 12*r^2 -
8*x^2))/24 + (r^3*log(x - r - (x*(2*r - x))^(1/2)*1i)*1i)/2))/4
```

3.263 $\int x^3\sqrt{2rx-x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 113

$$\int x^3\sqrt{2rx-x^2}\,dx=-\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{7}{4}r^5\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)$$

```
[Out] -7/12*r^2*(2*r*x-x^2)^(3/2)-7/20*r*x*(2*r*x-x^2)^(3/2)-1/5*x^2*(2*r*x-x^2)^
(3/2)+7/4*r^5*arctan(x/(2*r*x-x^2)^(1/2))-7/8*r^3*(r-x)*(2*r*x-x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {684, 654, 626, 634, 209}

$$\int x^3\sqrt{2rx-x^2}\,dx=\frac{7}{4}r^5\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

```
[In] Int[x^3*Sqrt[2*r*x - x^2],x]
```

```
[Out] (-7*r^3*(r - x)*Sqrt[2*r*x - x^2])/8 - (7*r^2*(2*r*x - x^2)^(3/2))/12 - (7*
r*x*(2*r*x - x^2)^(3/2))/20 - (x^2*(2*r*x - x^2)^(3/2))/5 + (7*r^5*ArcTan[x
/Sqrt[2*r*x - x^2]])/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{1}{5}(7r)\int x^2\sqrt{2rx-x^2}\,dx\\
&= -\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{1}{4}\left(7r^2\right)\int x\sqrt{2rx-x^2}\,dx\\
&= -\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{1}{4}\left(7r^3\right)\int\sqrt{2rx-x^2}\,dx\\
&= -\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}\\
&\quad -\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{1}{8}\left(7r^5\right)\int\frac{1}{\sqrt{2rx-x^2}}\,dx
\end{aligned}
$$

$$= -\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2} - \frac{7}{12}r^2\left(2rx-x^2\right)^{3/2} - \frac{7}{20}rx\left(2rx-x^2\right)^{3/2} - \frac{1}{5}x^2\left(2rx-x^2\right)^{3/2} + \frac{1}{4}\left(7r^5\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{2rx-x^2}}\right)$$

$$= -\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2} - \frac{7}{12}r^2\left(2rx-x^2\right)^{3/2} - \frac{7}{20}rx\left(2rx-x^2\right)^{3/2} - \frac{1}{5}x^2\left(2rx-x^2\right)^{3/2} + \frac{7}{4}r^5\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{2rx-x^2}\,dx = \frac{1}{120}\sqrt{-x(-2r+x)}\left(-105r^4-35r^3x-14r^2x^2-6rx^3+24x^4 + \frac{210r^5\log\left(-\sqrt{x}+\sqrt{-2r+x}\right)}{\sqrt{x}\sqrt{-2r+x}}\right)$$

```
[In] Integrate[x^3*Sqrt[2*r*x - x^2],x]
```

```
[Out] (Sqrt[-(x*(-2*r + x))]*(-105*r^4 - 35*r^3*x - 14*r^2*x^2 - 6*r*x^3 + 24*x^4
 + (210*r^5*Log[-Sqrt[x] + Sqrt[-2*r + x]])/(Sqrt[x]*Sqrt[-2*r + x])))/120
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

method	result	size
pseudoelliptic	$-\frac{7\arctan\left(\frac{\sqrt{x(2r-x)}}{x}\right)r^5}{4} - \frac{7\sqrt{x(2r-x)}\left(r^4+\frac{1}{3}r^3x+\frac{2}{15}r^2x^2+\frac{2}{35}r\,x^3-\frac{8}{35}x^4\right)}{8}$	65
risch	$-\frac{\left(105r^4+35r^3x+14r^2x^2+6r\,x^3-24x^4\right)x(2r-x)}{120\sqrt{-x(-2r+x)}} + \frac{7r^5\arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{8}$	77
default	$-\frac{x^2\left(2rx-x^2\right)^{\frac{3}{2}}}{5} + \frac{7r\left(-\frac{x\left(2rx-x^2\right)^{\frac{3}{2}}}{4}+\frac{5r\left(-\frac{\left(2rx-x^2\right)^{\frac{3}{2}}}{3}+r\left(-\frac{(2r-2x)\sqrt{2rx-x^2}}{4}+\frac{r^2\arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{2}\right)\right)}{4}\right)}{5}$	104

```
[In] int(x^3*(2*r*x-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -7/4*arctan(1/x*(x*(2*r-x))^(1/2))*r^5-7/8*(x*(2*r-x))^(1/2)*(r^4+1/3*r^3*x
+2/15*r^2*x^2+2/35*r*x^3-8/35*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.60

$$\int x^3\sqrt{2rx-x^2}\,dx = -\frac{7}{4}\,r^5\arctan\left(\frac{\sqrt{2\,rx-x^2}}{x}\right) - \frac{1}{120}\left(105\,r^4+35\,r^3x+14\,r^2x^2+6\,rx^3-24\,x^4\right)\sqrt{2\,rx-x^2}$$

```
[In] integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -7/4*r^5*arctan(sqrt(2*r*x - x^2)/x) - 1/120*(105*r^4 + 35*r^3*x + 14*r^2*x
^2 + 6*r*x^3 - 24*x^4)*sqrt(2*r*x - x^2)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int x^3\sqrt{2rx-x^2}\,dx = \frac{7r^5\left(\begin{cases}-i\log\left(2r-2x+2i\sqrt{2rx-x^2}\right) & \text{for } r^2\neq 0\\ \frac{(-r+x)\log(-r+x)}{\sqrt{-(-r+x)^2}} & \text{otherwise}\end{cases}\right)}{8} + \sqrt{2rx-x^2}\left(-\frac{7r^4}{8}-\frac{7r^3x}{24}-\frac{7r^2x^2}{60}-\frac{rx^3}{20}+\frac{x^4}{5}\right)$$

```
[In] integrate(x**3*(2*r*x-x**2)**(1/2),x)
```

```
[Out] 7*r**5*Piecewise((-I*log(2*r - 2*x + 2*I*sqrt(2*r*x - x**2)), Ne(r**2, 0)),
 ((-r + x)*log(-r + x)/sqrt(-(-r + x)**2), True))/8 + sqrt(2*r*x - x**2)*(-
7*r**4/8 - 7*r**3*x/24 - 7*r**2*x**2/60 - r*x**3/20 + x**4/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int x^3\sqrt{2rx-x^2}\,dx = -\frac{7}{8}\,r^5\arcsin\left(\frac{r-x}{r}\right) - \frac{7}{8}\,\sqrt{2\,rx-x^2}r^4 + \frac{7}{8}\,\sqrt{2\,rx-x^2}r^3x - \frac{7}{12}\left(2\,rx-x^2\right)^{\frac{3}{2}}r^2 - \frac{7}{20}\left(2\,rx-x^2\right)^{\frac{3}{2}}rx - \frac{1}{5}\left(2\,rx-x^2\right)^{\frac{3}{2}}x^2$$

```
[In] integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -7/8*r^5*arcsin((r - x)/r) - 7/8*sqrt(2*r*x - x^2)*r^4 + 7/8*sqrt(2*r*x - x
^2)*r^3*x - 7/12*(2*r*x - x^2)^(3/2)*r^2 - 7/20*(2*r*x - x^2)^(3/2)*r*x - 1
/5*(2*r*x - x^2)^(3/2)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.56

$$\int x^3\sqrt{2rx-x^2}\,dx = -\frac{7}{8}\,r^5\arcsin\left(\frac{r-x}{r}\right)\operatorname{sgn}(r) - \frac{1}{120}\left(105\,r^4+\left(35\,r^3+2\left(7\,r^2+3\,(r-4\,x)x\right)x\right)x\right)\sqrt{2\,rx-x^2}$$

```
[In] integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -7/8*r^5*arcsin((r - x)/r)*sgn(r) - 1/120*(105*r^4 + (35*r^3 + 2*(7*r^2 + 3
*(r - 4*x)*x)*x)*x)*sqrt(2*r*x - x^2)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85

$$\int x^3\sqrt{2rx-x^2}\,dx = -\frac{7\,r\left(\frac{x\,\left(2\,r\,x-x^2\right)^{3/2}}{4}+\frac{5\,r\,\left(\frac{\sqrt{2\,r\,x-x^2}\,\left(12\,r^2+4\,r\,x-8\,x^2\right)}{24}+\frac{r^3\,\ln\left(x-r-\sqrt{x\,(2\,r-x)}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}\right)}{4}\right)}{5} - \frac{x^2\,\left(2\,r\,x-x^2\right)^{3/2}}{5}$$

```
[In] int(x^3*(2*r*x - x^2)^(1/2),x)
```

```
[Out] - (7*r*((x*(2*r*x - x^2)^(3/2))/4 + (5*r*(((2*r*x - x^2)^(1/2)*(4*r*x + 12*
r^2 - 8*x^2))/24 + (r^3*log(x - r - (x*(2*r - x))^(1/2)*1i)*1i)/2))/4))/5 -
 (x^2*(2*r*x - x^2)^(3/2))/5
```

3.264 $\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = -\frac{1}{2}\arctan\left(\sqrt{2x+x^2}\right) - \frac{\operatorname{arctanh}\left(\frac{1+2x}{\sqrt{3}\sqrt{2x+x^2}}\right)}{2\sqrt{3}}$$

```
[Out] -1/2*arctan((x^2+2*x)^(1/2))-1/6*arctanh(1/3*(1+2*x)*3^(1/2)/(x^2+2*x)^(1/2
))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {998, 702, 210, 738, 212}

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = -\frac{1}{2}\arctan\left(\sqrt{x^2+2x}\right) - \frac{\operatorname{arctanh}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+2x}}\right)}{2\sqrt{3}}$$

```
[In] Int[1/((-1 + x^2)*Sqrt[2*x + x^2]),x]
```

```
[Out] -1/2*ArcTan[Sqrt[2*x + x^2]] - ArcTanh[(1 + 2*x)/(Sqrt[3]*Sqrt[2*x + x^2])]
/(2*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 702

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 998

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int \frac{1}{(-1-x)\sqrt{2x+x^2}}\,dx+\frac{1}{2}\int \frac{1}{(-1+x)\sqrt{2x+x^2}}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{-4-4x^2}\,dx,x,\sqrt{2x+x^2}\right)-\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{2+4x}{\sqrt{2x+x^2}}\right)\\ &= -\frac{1}{2}\arctan\left(\sqrt{2x+x^2}\right)-\frac{\text{arctanh}\left(\frac{2+4x}{2\sqrt{3}\sqrt{2x+x^2}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\begin{aligned}&\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx\\ &= \frac{\sqrt{x}\sqrt{2+x}\left(3\arctan\left(1+x-\sqrt{x}\sqrt{2+x}\right)-\sqrt{3}\text{arctanh}\left(\frac{1-x+\sqrt{x}\sqrt{2+x}}{\sqrt{3}}\right)\right)}{3\sqrt{x(2+x)}}\end{aligned}$$

```
[In] Integrate[1/((-1 + x^2)*Sqrt[2*x + x^2]),x]
[Out] (Sqrt[x]*Sqrt[2 + x]*(3*ArcTan[1 + x - Sqrt[x]*Sqrt[2 + x]] - Sqrt[3]*ArcTa
nh[(1 - x + Sqrt[x]*Sqrt[2 + x])/Sqrt[3]]))/(3*Sqrt[x*(2 + x)])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}\,\sqrt{x(2+x)}}{3x}\right)}{3}+\arctan\left(\frac{\sqrt{x(2+x)}}{x}\right)$
default	$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(2+4x)\sqrt{3}}{6\sqrt{(-1+x)^{2}-1+4x}}\right)}{6}+\frac{\arctan\left(\frac{1}{\sqrt{(1+x)^{2}-1}}\right)}{2}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)+\sqrt{x^{2}+2x}}{1+x}\right)}{2}+\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(\frac{-2\operatorname{RootOf}\left(\_Z^{2}-3\right)x+3\sqrt{x^{2}+2x}-\operatorname{RootOf}}{-1+x}\right.}{6}$

```
[In] int(1/(x^2-1)/(x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*3^(1/2)*arctanh(1/3*3^(1/2)*(x*(2+x))^(1/2)/x)+arctan((x*(2+x))^(1/2)/
x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = \frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}(2x+1)+\sqrt{x^2+2x}(2\sqrt{3}-3)-4x-2}{x-1}\right) - \arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

```
[In] integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log(-(sqrt(3)*(2*x + 1) + sqrt(x^2 + 2*x)*(2*sqrt(3) - 3) - 4*x
 - 2)/(x - 1)) - arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Sympy [F]

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = \int \frac{1}{\sqrt{x(x+2)}\,(x-1)\,(x+1)}\,dx$$

```
[In] integrate(1/(x**2-1)/(x**2+2*x)**(1/2),x)
[Out] Integral(1/(sqrt(x*(x + 2))*(x - 1)*(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = -\frac{1}{6}\,\sqrt{3}\log\left(\frac{2\,\sqrt{3}\sqrt{x^2+2\,x}}{|2\,x-2|}+\frac{6}{|2\,x-2|}+2\right) + \frac{1}{2}\,\arcsin\left(\frac{2}{|2\,x+2|}\right)$$

```
[In] integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="maxima")
[Out] -1/6*sqrt(3)*log(2*sqrt(3)*sqrt(x^2 + 2*x)/abs(2*x - 2) + 6/abs(2*x - 2) +
2) + 1/2*arcsin(2/abs(2*x + 2))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(\frac{\left|-2\,x-2\,\sqrt{3}+2\,\sqrt{x^2+2\,x}+2\right|}{\left|-2\,x+2\,\sqrt{3}+2\,\sqrt{x^2+2\,x}+2\right|}\right) - \arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

```
[In] integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="giac")
[Out] 1/6*sqrt(3)*log(abs(-2*x - 2*sqrt(3) + 2*sqrt(x^2 + 2*x) + 2)/abs(-2*x + 2*
sqrt(3) + 2*sqrt(x^2 + 2*x) + 2)) - arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2\,x}\,\left(x^2-1\right)}\,dx$$

```
[In] int(1/((2*x + x^2)^(1/2)*(x^2 - 1)),x)
[Out] int(1/((2*x + x^2)^(1/2)*(x^2 - 1)), x)
```

3.265 $\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = -\frac{5\sqrt{2x-x^2}}{6(1+x)^2}-\frac{2\sqrt{2x-x^2}}{3(1+x)}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}\sqrt{2x-x^2}}\right)}{2\sqrt{3}}$$

```
[Out] 1/6*arctan(1/3*(1-2*x)*3^(1/2)/(-x^2+2*x)^(1/2))*3^(1/2)-5/6*(-x^2+2*x)^(1/
2)/(1+x)^2-2/3*(-x^2+2*x)^(1/2)/(1+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {848, 820, 738, 210}

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}\sqrt{2x-x^2}}\right)}{2\sqrt{3}}-\frac{2\sqrt{2x-x^2}}{3(x+1)}-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}$$

```
[In] Int[(-2 + 3*x)/((1 + x)^3*Sqrt[2*x - x^2]),x]
```

```
[Out] (-5*Sqrt[2*x - x^2])/(6*(1 + x)^2) - (2*Sqrt[2*x - x^2])/(3*(1 + x)) + ArcT
an[(1 - 2*x)/(Sqrt[3]*Sqrt[2*x - x^2])]/(2*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{5\sqrt{2x-x^2}}{6(1+x)^2}+\frac{1}{6}\int\frac{-7+5x}{(1+x)^2\sqrt{2x-x^2}}\,dx\\
&= -\frac{5\sqrt{2x-x^2}}{6(1+x)^2}-\frac{2\sqrt{2x-x^2}}{3(1+x)}-\frac{1}{2}\int\frac{1}{(1+x)\sqrt{2x-x^2}}\,dx\\
&= -\frac{5\sqrt{2x-x^2}}{6(1+x)^2}-\frac{2\sqrt{2x-x^2}}{3(1+x)}+\text{Subst}\left(\int\frac{1}{-12-x^2}\,dx,x,\frac{-2+4x}{\sqrt{2x-x^2}}\right)\\
&= -\frac{5\sqrt{2x-x^2}}{6(1+x)^2}-\frac{2\sqrt{2x-x^2}}{3(1+x)}-\frac{\arctan\left(\frac{-2+4x}{2\sqrt{3}\sqrt{2x-x^2}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = \frac{x(-18+x+4x^2)-2\sqrt{3}\sqrt{-2+x}\sqrt{x}(1+x)^2\text{arctanh}\left(\frac{1-\sqrt{-2+x}\sqrt{x}+x}{\sqrt{3}}\right)}{6\sqrt{-((-2+x)x)}(1+x)^2}$$

```
[In] Integrate[(-2 + 3*x)/((1 + x)^3*Sqrt[2*x - x^2]),x]
```

```
[Out] (x*(-18 + x + 4*x^2) - 2*Sqrt[3]*Sqrt[-2 + x]*Sqrt[x]*(1 + x)^2*ArcTanh[(1
- Sqrt[-2 + x]*Sqrt[x] + x)/Sqrt[3]])/(6*Sqrt[-((-2 + x)*x)]*(1 + x)^2)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$\frac{2\sqrt{3}\,(1+x)^2\arctan\left(\frac{\sqrt{3}\,\sqrt{-x(-2+x)}}{3x}\right)+(-4x-9)\sqrt{-x(-2+x)}}{6(1+x)^2}$	50
risch	$\frac{x(-2+x)(4x+9)}{6(1+x)^2\sqrt{-x(-2+x)}}-\frac{\sqrt{3}\,\arctan\left(\frac{(-2+4x)\sqrt{3}}{6\sqrt{-(1+x)^2+1+4x}}\right)}{6}$	56
trager	$-\frac{(4x+9)\sqrt{-x^2+2x}}{6(1+x)^2}-\frac{\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{-2\operatorname{RootOf}\left(\_Z^2+3\right)x+3\sqrt{-x^2+2x}+\operatorname{RootOf}\left(\_Z^2+3\right)}{1+x}\right)}{6}$	69
default	$-\frac{5\sqrt{-(1+x)^2+1+4x}}{6(1+x)^2}-\frac{2\sqrt{-(1+x)^2+1+4x}}{3(1+x)}-\frac{\sqrt{3}\,\arctan\left(\frac{(-2+4x)\sqrt{3}}{6\sqrt{-(1+x)^2+1+4x}}\right)}{6}$	74

```
[In] int((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*3^(1/2)*(1+x)^2*arctan(1/3*3^(1/2)*(-x*(-2+x))^(1/2)/x)+(-4*x-9)*(-x
*(-2+x))^(1/2))/(1+x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = \frac{2\sqrt{3}(x^2+2x+1)\arctan\left(\frac{\sqrt{3}\sqrt{-x^2+2x}}{3x}\right)-\sqrt{-x^2+2x}(4x+9)}{6(x^2+2x+1)}$$

```
[In] integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="fricas")
[Out] 1/6*(2*sqrt(3)*(x^2 + 2*x + 1)*arctan(1/3*sqrt(3)*sqrt(-x^2 + 2*x)/x) - sqr
t(-x^2 + 2*x)*(4*x + 9))/(x^2 + 2*x + 1)
```

Sympy [F]

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = \int \frac{3x-2}{\sqrt{-x(x-2)}\,(x+1)^3}\,dx$$

```
[In] integrate((-2+3*x)/(1+x)**3/(-x**2+2*x)**(1/2),x)
[Out] Integral((3*x - 2)/(sqrt(-x*(x - 2))*(x + 1)**3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = -\frac{1}{6}\sqrt{3}\arcsin\left(\frac{2x}{|x+1|}-\frac{1}{|x+1|}\right) - \frac{5\sqrt{-x^2+2x}}{6(x^2+2x+1)} - \frac{2\sqrt{-x^2+2x}}{3(x+1)}$$

```
[In] integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="maxima")
[Out] -1/6*sqrt(3)*arcsin(2*x/abs(x + 1) - 1/abs(x + 1)) - 5/6*sqrt(-x^2 + 2*x)/(
x^2 + 2*x + 1) - 2/3*sqrt(-x^2 + 2*x)/(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(64) = 128$.

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.86

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{-x^2+2x}-1\right)}{x-1}-1\right)\right) + \frac{\frac{34\left(\sqrt{-x^2+2x}-1\right)}{x-1}-\frac{39\left(\sqrt{-x^2+2x}-1\right)^2}{(x-1)^2}+\frac{18\left(\sqrt{-x^2+2x}-1\right)^3}{(x-1)^3}-26}{24\left(\frac{\sqrt{-x^2+2x}-1}{x-1}-\frac{\left(\sqrt{-x^2+2x}-1\right)^2}{(x-1)^2}-1\right)^2}$$

```
[In] integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 + 2*x) - 1)/(x - 1) - 1)) + 1/
24*(34*(sqrt(-x^2 + 2*x) - 1)/(x - 1) - 39*(sqrt(-x^2 + 2*x) - 1)^2/(x - 1)
^2 + 18*(sqrt(-x^2 + 2*x) - 1)^3/(x - 1)^3 - 26)/((sqrt(-x^2 + 2*x) - 1)/(x
 - 1) - (sqrt(-x^2 + 2*x) - 1)^2/(x - 1)^2 - 1)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = \int \frac{3\,x-2}{\sqrt{2\,x-x^2}\,{\left(x+1\right)}^3}\,dx$$

```
[In] int((3*x - 2)/((2*x - x^2)^(1/2)*(x + 1)^3),x)
[Out] int((3*x - 2)/((2*x - x^2)^(1/2)*(x + 1)^3), x)
```

3.266 $\int \frac{1}{\sqrt{1+x+x^2}} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 12

$$\int \frac{1}{\sqrt{1+x+x^2}} \, dx = \operatorname{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

```
[Out] arcsinh(1/3*(1+2*x)*3^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {633, 221}

$$\int \frac{1}{\sqrt{1+x+x^2}} \, dx = \operatorname{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[In] Int[1/Sqrt[1 + x + x^2],x]
```

```
[Out] ArcSinh[(1 + 2*x)/Sqrt[3]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, 1+2x\right)}{\sqrt{3}}$$
$$= \text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = -\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

```
[In] Integrate[1/Sqrt[1 + x + x^2],x]
[Out] -Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

method	result	size
default	$\text{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)$	10
trager	$-\ln\left(2\sqrt{x^2+x+1}-1-2x\right)$	19

```
[In] int(1/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(2/3*3^(1/2)*(x+1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = -\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

```
[In] integrate(1/(x^2+x+1)^(1/2),x, algorithm="fricas")
[Out] -log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = \operatorname{asinh}\left(\frac{2\sqrt{3}(x+\frac{1}{2})}{3}\right)$$

```
[In] integrate(1/(x**2+x+1)**(1/2),x)
[Out] asinh(2*sqrt(3)*(x + 1/2)/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = \operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

```
[In] integrate(1/(x^2+x+1)^(1/2),x, algorithm="maxima")
[Out] arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(11) = 22$.

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.83

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = \frac{1}{4}\sqrt{x^2+x+1}(2x+1) - \frac{3}{8}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

```
[In] integrate(1/(x^2+x+1)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(x^2 + x + 1)*(2*x + 1) - 3/8*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = \ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)$$

```
[In] int(1/(x + x^2 + 1)^(1/2),x)
[Out] log(x + (x + x^2 + 1)^(1/2) + 1/2)
```

3.267 $\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx=\frac{1}{3}x^2\sqrt{1+x+x^2}-\frac{1}{24}(1+10x)\sqrt{1+x+x^2}+\frac{7}{16}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

```
[Out] 7/16*arcsinh(1/3*(1+2*x)*3^(1/2))+1/3*x^2*(x^2+x+1)^(1/2)-1/24*(1+10*x)*(x^
2+x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {756, 793, 633, 221}

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx=\frac{7}{16}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{3}\sqrt{x^2+x+1}x^2-\frac{1}{24}(10x+1)\sqrt{x^2+x+1}$$

```
[In] Int[x^3/Sqrt[1 + x + x^2],x]
```

```
[Out] (x^2*Sqrt[1 + x + x^2])/3 - ((1 + 10*x)*Sqrt[1 + x + x^2])/24 + (7*ArcSinh[
(1 + 2*x)/Sqrt[3]])/16
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^2\sqrt{1+x+x^2}+\frac{1}{3}\int \frac{\left(-2-\frac{5x}{2}\right)x}{\sqrt{1+x+x^2}}\,dx \\
&= \frac{1}{3}x^2\sqrt{1+x+x^2}-\frac{1}{24}(1+10x)\sqrt{1+x+x^2}+\frac{7}{16}\int \frac{1}{\sqrt{1+x+x^2}}\,dx \\
&= \frac{1}{3}x^2\sqrt{1+x+x^2}-\frac{1}{24}(1+10x)\sqrt{1+x+x^2}+\frac{7\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+2x\right)}{16\sqrt{3}} \\
&= \frac{1}{3}x^2\sqrt{1+x+x^2}-\frac{1}{24}(1+10x)\sqrt{1+x+x^2}+\frac{7}{16}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx = \frac{1}{24}\sqrt{1+x+x^2}\left(-1-10x+8x^2\right)-\frac{7}{16}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

```
[In] Integrate[x^3/Sqrt[1 + x + x^2],x]
[Out] (Sqrt[1 + x + x^2]*(-1 - 10*x + 8*x^2))/24 - (7*Log[-1 - 2*x + 2*Sqrt[1 + x
 + x^2]])/16
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{\left(8x^2-10x-1\right)\sqrt{x^2+x+1}}{24}+\frac{7\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{16}$	33
trager	$\left(\frac{1}{3}x^2-\frac{5}{12}x-\frac{1}{24}\right)\sqrt{x^2+x+1}-\frac{7\ln\left(2\sqrt{x^2+x+1}-1-2x\right)}{16}$	39
default	$\frac{x^2\sqrt{x^2+x+1}}{3}-\frac{5x\sqrt{x^2+x+1}}{12}-\frac{\sqrt{x^2+x+1}}{24}+\frac{7\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{16}$	47

```
[In] int(x^3/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/24*(8*x^2-10*x-1)*(x^2+x+1)^(1/2)+7/16*arcsinh(2/3*3^(1/2)*(x+1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx = \frac{1}{24}\left(8\,x^2-10\,x-1\right)\sqrt{x^2+x+1}-\frac{7}{16}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

```
[In] integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="fricas")
[Out] 1/24*(8*x^2 - 10*x - 1)*sqrt(x^2 + x + 1) - 7/16*log(-2*x + 2*sqrt(x^2 + x
+ 1) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx = \left(\frac{x^2}{3}-\frac{5x}{12}-\frac{1}{24}\right)\sqrt{x^2+x+1}+\frac{7\operatorname{asinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{16}$$

```
[In] integrate(x**3/(x**2+x+1)**(1/2),x)
```

```
[Out] (x**2/3 - 5*x/12 - 1/24)*sqrt(x**2 + x + 1) + 7*asinh(2*sqrt(3)*(x + 1/2)/3
)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx = \frac{1}{3}\sqrt{x^2+x+1}x^2-\frac{5}{12}\sqrt{x^2+x+1}x-\frac{1}{24}\sqrt{x^2+x+1}+\frac{7}{16}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

```
[In] integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^2 + x + 1)*x^2 - 5/12*sqrt(x^2 + x + 1)*x - 1/24*sqrt(x^2 + x +
1) + 7/16*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx = \frac{1}{24}\left(2\left(4x-5\right)x-1\right)\sqrt{x^2+x+1}-\frac{7}{16}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

```
[In] integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(2*(4*x - 5)*x - 1)*sqrt(x^2 + x + 1) - 7/16*log(-2*x + 2*sqrt(x^2 + x
 + 1) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx = \int \frac{x^3}{\sqrt{x^2+x+1}}\,dx$$

```
[In] int(x^3/(x + x^2 + 1)^(1/2),x)
[Out] int(x^3/(x + x^2 + 1)^(1/2), x)
```

3.268 $\int \frac{1}{(1+x+x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{1}{(1+x+x^2)^{3/2}} \, dx = \frac{2(1+2x)}{3\sqrt{1+x+x^2}}$$

```
[Out] 2/3*(1+2*x)/(x^2+x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {627}

$$\int \frac{1}{(1+x+x^2)^{3/2}} \, dx = \frac{2(2x+1)}{3\sqrt{x^2+x+1}}$$

```
[In] Int[(1 + x + x^2)^(-3/2),x]
```

```
[Out] (2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{2(1+2x)}{3\sqrt{1+x+x^2}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(1+x+x^2\right)^{3/2}}\,dx=\frac{2(1+2x)}{3\sqrt{1+x+x^2}}$$

```
[In] Integrate[(1 + x + x^2)^(-3/2),x]
[Out] (2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{\frac{2}{3}+\frac{4x}{3}}{\sqrt{x^2+x+1}}$	16
default	$\frac{\frac{2}{3}+\frac{4x}{3}}{\sqrt{x^2+x+1}}$	16
trager	$\frac{\frac{2}{3}+\frac{4x}{3}}{\sqrt{x^2+x+1}}$	16
risch	$\frac{\frac{2}{3}+\frac{4x}{3}}{\sqrt{x^2+x+1}}$	16

```
[In] int(1/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(1+2*x)/(x^2+x+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(15) = 30.

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{1}{\left(1+x+x^2\right)^{3/2}}\,dx=\frac{2\left(2\,x^2+\sqrt{x^2+x+1}(2\,x+1)+2\,x+2\right)}{3\left(x^2+x+1\right)}$$

```
[In] integrate(1/(x^2+x+1)^(3/2),x, algorithm="fricas")
[Out] 2/3*(2*x^2 + sqrt(x^2 + x + 1)*(2*x + 1) + 2*x + 2)/(x^2 + x + 1)
```

Sympy [F]

$$\int \frac{1}{(1+x+x^2)^{3/2}}\,dx = \int \frac{1}{(x^2+x+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(x**2+x+1)**(3/2),x)
[Out] Integral((x**2 + x + 1)**(-3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{(1+x+x^2)^{3/2}}\,dx = \frac{4\,x}{3\,\sqrt{x^2+x+1}} + \frac{2}{3\,\sqrt{x^2+x+1}}$$

```
[In] integrate(1/(x^2+x+1)^(3/2),x, algorithm="maxima")
[Out] 4/3*x/sqrt(x^2 + x + 1) + 2/3/sqrt(x^2 + x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1+x+x^2)^{3/2}}\,dx = \frac{2\,(2\,x+1)}{3\,\sqrt{x^2+x+1}}$$

```
[In] integrate(1/(x^2+x+1)^(3/2),x, algorithm="giac")
[Out] 2/3*(2*x + 1)/sqrt(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{(1+x+x^2)^{3/2}}\,dx = \frac{4\,\left(x+\frac{1}{2}\right)}{3\,\sqrt{x^2+x+1}}$$

```
[In] int(1/(x + x^2 + 1)^(3/2),x)
[Out] (4*(x + 1/2))/(3*(x + x^2 + 1)^(1/2))
```

3.269 $\int \frac{x}{(1+x+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 17

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx = -\frac{2(2+x)}{3\sqrt{1+x+x^2}}$$

```
[Out] -2/3*(2+x)/(x^2+x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {650}

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx = -\frac{2(x+2)}{3\sqrt{x^2+x+1}}$$

```
[In] Int[x/(1 + x + x^2)^(3/2),x]
```

```
[Out] (-2*(2 + x))/(3*Sqrt[1 + x + x^2])
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = -\frac{2(2+x)}{3\sqrt{1+x+x^2}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx=-\frac{2(2+x)}{3\sqrt{1+x+x^2}}$$

```
[In] Integrate[x/(1 + x + x^2)^(3/2),x]
[Out] (-2*(2 + x))/(3*Sqrt[1 + x + x^2])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$-\frac{2(2+x)}{3\sqrt{x^2+x+1}}$	14
trager	$-\frac{2(2+x)}{3\sqrt{x^2+x+1}}$	14
risch	$-\frac{2(2+x)}{3\sqrt{x^2+x+1}}$	14
default	$-\frac{1}{\sqrt{x^2+x+1}}-\frac{1+2x}{3\sqrt{x^2+x+1}}$	27

```
[In] int(x/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(2+x)/(x^2+x+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.65

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx=-\frac{2\left(x^2+\sqrt{x^2+x+1}(x+2)+x+1\right)}{3\left(x^2+x+1\right)}$$

```
[In] integrate(x/(x^2+x+1)^(3/2),x, algorithm="fricas")
[Out] -2/3*(x^2 + sqrt(x^2 + x + 1)*(x + 2) + x + 1)/(x^2 + x + 1)
```

Sympy [F]

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx = \int \frac{x}{(x^2+x+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(x**2+x+1)**(3/2),x)
[Out] Integral(x/(x**2 + x + 1)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx = -\frac{2\,x}{3\,\sqrt{x^2+x+1}} - \frac{4}{3\,\sqrt{x^2+x+1}}$$

```
[In] integrate(x/(x^2+x+1)^(3/2),x, algorithm="maxima")
[Out] -2/3*x/sqrt(x^2 + x + 1) - 4/3/sqrt(x^2 + x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx = -\frac{2\,(x+2)}{3\,\sqrt{x^2+x+1}}$$

```
[In] integrate(x/(x^2+x+1)^(3/2),x, algorithm="giac")
[Out] -2/3*(x + 2)/sqrt(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx = -\frac{2\,x+4}{3\,\sqrt{x^2+x+1}}$$

```
[In] int(x/(x + x^2 + 1)^(3/2),x)
[Out] -(2*x + 4)/(3*(x + x^2 + 1)^(1/2))
```

3.270 $\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 56

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}} + \frac{1}{3}(5+2x)\sqrt{1+x+x^2} - \frac{3}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

```
[Out] -3/2*arcsinh(1/3*(1+2*x)*3^(1/2))-2/3*x^2*(2+x)/(x^2+x+1)^(1/2)+1/3*(5+2*x)
*(x^2+x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {752, 793, 633, 221}

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = -\frac{3}{2}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right) - \frac{2(x+2)x^2}{3\sqrt{x^2+x+1}} + \frac{1}{3}(2x+5)\sqrt{x^2+x+1}$$

```
[In] Int[x^3/(1 + x + x^2)^(3/2),x]
```

```
[Out] (-2*x^2*(2 + x))/(3*Sqrt[1 + x + x^2]) + ((5 + 2*x)*Sqrt[1 + x + x^2])/3 -
(3*ArcSinh[(1 + 2*x)/Sqrt[3]])/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}}+\frac{2}{3}\int \frac{x(4+2x)}{\sqrt{1+x+x^2}}\,dx\\
&= -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}}+\frac{1}{3}(5+2x)\sqrt{1+x+x^2}-\frac{3}{2}\int \frac{1}{\sqrt{1+x+x^2}}\,dx\\
&= -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}}+\frac{1}{3}(5+2x)\sqrt{1+x+x^2}-\frac{1}{2}\sqrt{3}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+2x\right)\\
&= -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}}+\frac{1}{3}(5+2x)\sqrt{1+x+x^2}-\frac{3}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = \frac{5+7x+3x^2}{3\sqrt{1+x+x^2}} + \frac{3}{2}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

```
[In] Integrate[x^3/(1 + x + x^2)^(3/2),x]
[Out] (5 + 7*x + 3*x^2)/(3*Sqrt[1 + x + x^2]) + (3*Log[-1 - 2*x + 2*Sqrt[1 + x + 
x^2]])/2
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{3x^2+7x+5}{3\sqrt{x^2+x+1}} - \frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{2}$	33
trager	$\frac{3x^2+7x+5}{3\sqrt{x^2+x+1}} + \frac{3\ln\left(2\sqrt{x^2+x+1}-1-2x\right)}{2}$	40
default	$\frac{x^2}{\sqrt{x^2+x+1}} + \frac{3x}{2\sqrt{x^2+x+1}} + \frac{5}{4\sqrt{x^2+x+1}} + \frac{\frac{5}{12}+\frac{5x}{6}}{\sqrt{x^2+x+1}} - \frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{2}$	61

```
[In] int(x^3/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(3*x^2+7*x+5)/(x^2+x+1)^(1/2)-3/2*arcsinh(2/3*3^(1/2)*(x+1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = \frac{19\,x^2 + 18\,(x^2+x+1)\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right) + 4\,(3\,x^2+7\,x+5)\sqrt{x^2+x}}{12\,(x^2+x+1)}$$

```
[In] integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="fricas")
[Out] 1/12*(19*x^2 + 18*(x^2 + x + 1)*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) + 4*(3*
x^2 + 7*x + 5)*sqrt(x^2 + x + 1) + 19*x + 19)/(x^2 + x + 1)
```

Sympy [F]

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = \int \frac{x^3}{(x^2+x+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/(x**2+x+1)**(3/2),x)
[Out] Integral(x**3/(x**2 + x + 1)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = \frac{x^2}{\sqrt{x^2+x+1}} + \frac{7\,x}{3\,\sqrt{x^2+x+1}} + \frac{5}{3\,\sqrt{x^2+x+1}} - \frac{3}{2}\,\operatorname{arsinh}\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)$$

```
[In] integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="maxima")
[Out] x^2/sqrt(x^2 + x + 1) + 7/3*x/sqrt(x^2 + x + 1) + 5/3/sqrt(x^2 + x + 1) - 3
/2*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = \frac{(3\,x+7)x+5}{3\,\sqrt{x^2+x+1}} + \frac{3}{2}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

```
[In] integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="giac")
[Out] 1/3*((3*x + 7)*x + 5)/sqrt(x^2 + x + 1) + 3/2*log(-2*x + 2*sqrt(x^2 + x + 1
) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = \int \frac{x^3}{(x^2+x+1)^{3/2}}\,dx$$

```
[In] int(x^3/(x + x^2 + 1)^(3/2),x)
[Out] int(x^3/(x + x^2 + 1)^(3/2), x)
```

3.271 $\int x^2\sqrt{1+x+x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int x^2\sqrt{1+x+x^2}\,dx = \frac{1}{64}(1+2x)\sqrt{1+x+x^2} - \frac{5}{24}\left(1+x+x^2\right)^{3/2} + \frac{1}{4}x\left(1+x+x^2\right)^{3/2} + \frac{3}{128}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

```
[Out] -5/24*(x^2+x+1)^(3/2)+1/4*x*(x^2+x+1)^(3/2)+3/128*arcsinh(1/3*(1+2*x)*3^(1/
2))+1/64*(1+2*x)*(x^2+x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {756, 654, 626, 633, 221}

$$\int x^2\sqrt{1+x+x^2}\,dx = \frac{3}{128}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{1}{4}x\left(x^2+x+1\right)^{3/2} - \frac{5}{24}\left(x^2+x+1\right)^{3/2} + \frac{1}{64}(2x+1)\sqrt{x^2+x+1}$$

```
[In] Int[x^2*Sqrt[1 + x + x^2],x]
```

```
[Out] ((1 + 2*x)*Sqrt[1 + x + x^2])/64 - (5*(1 + x + x^2)^(3/2))/24 + (x*(1 + x +
 x^2)^(3/2))/4 + (3*ArcSinh[(1 + 2*x)/Sqrt[3]])/128
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{1}{4}\int\left(-1-\frac{5x}{2}\right)\sqrt{1+x+x^2}\,dx\\
&= -\frac{5}{24}\left(1+x+x^2\right)^{3/2}+\frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{1}{16}\int\sqrt{1+x+x^2}\,dx\\
&= \frac{1}{64}(1+2x)\sqrt{1+x+x^2}-\frac{5}{24}\left(1+x+x^2\right)^{3/2}+\frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{3}{128}\int\frac{1}{\sqrt{1+x+x^2}}\,dx\\
&= \frac{1}{64}(1+2x)\sqrt{1+x+x^2}-\frac{5}{24}\left(1+x+x^2\right)^{3/2}\\
&\quad+\frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{1}{128}\sqrt{3}\,\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+2x\right)
\end{aligned}$$

$$=\frac{1}{64}(1+2x)\sqrt{1+x+x^2}-\frac{5}{24}\left(1+x+x^2\right)^{3/2}+\frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{3}{128}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int x^2\sqrt{1+x+x^2}\,dx=\frac{1}{192}\sqrt{1+x+x^2}\left(-37+14x+8x^2+48x^3\right)-\frac{3}{128}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

```
[In] Integrate[x^2*Sqrt[1 + x + x^2],x]
[Out] (Sqrt[1 + x + x^2]*(-37 + 14*x + 8*x^2 + 48*x^3))/192 - (3*Log[-1 - 2*x + 2
*Sqrt[1 + x + x^2]])/128
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

method	result	size
risch	$\frac{(48x^3+8x^2+14x-37)\sqrt{x^2+x+1}}{192}+\frac{3\ \text{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{128}$	38
trager	$\left(\frac{1}{4}x^3+\frac{1}{24}x^2+\frac{7}{96}x-\frac{37}{192}\right)\sqrt{x^2+x+1}-\frac{3\ln\left(2\sqrt{x^2+x+1}-1-2x\right)}{128}$	44
default	$\frac{x(x^2+x+1)^{\frac{3}{2}}}{4}-\frac{5(x^2+x+1)^{\frac{3}{2}}}{24}+\frac{(1+2x)\sqrt{x^2+x+1}}{64}+\frac{3\ \text{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{128}$	49

```
[In] int(x^2*(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/192*(48*x^3+8*x^2+14*x-37)*(x^2+x+1)^(1/2)+3/128*arcsinh(2/3*3^(1/2)*(x+1
/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int x^2\sqrt{1+x+x^2}\,dx = \frac{1}{192}\left(48\,x^3+8\,x^2+14\,x-37\right)\sqrt{x^2+x+1} - \frac{3}{128}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

```
[In] integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/192*(48*x^3 + 8*x^2 + 14*x - 37)*sqrt(x^2 + x + 1) - 3/128*log(-2*x + 2*s
qrt(x^2 + x + 1) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int x^2\sqrt{1+x+x^2}\,dx = \sqrt{x^2+x+1}\left(\frac{x^3}{4}+\frac{x^2}{24}+\frac{7x}{96}-\frac{37}{192}\right)+\frac{3\operatorname{asinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{128}$$

```
[In] integrate(x**2*(x**2+x+1)**(1/2),x)
```

```
[Out] sqrt(x**2 + x + 1)*(x**3/4 + x**2/24 + 7*x/96 - 37/192) + 3*asinh(2*sqrt(3)
*(x + 1/2)/3)/128
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int x^2\sqrt{1+x+x^2}\,dx = \frac{1}{4}\left(x^2+x+1\right)^{\frac{3}{2}}x - \frac{5}{24}\left(x^2+x+1\right)^{\frac{3}{2}} + \frac{1}{32}\sqrt{x^2+x+1}x + \frac{1}{64}\sqrt{x^2+x+1} + \frac{3}{128}\,\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)$$

```
[In] integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(x^2 + x + 1)^(3/2)*x - 5/24*(x^2 + x + 1)^(3/2) + 1/32*sqrt(x^2 + x +
1)*x + 1/64*sqrt(x^2 + x + 1) + 3/128*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int x^2\sqrt{1+x+x^2}\,dx = \frac{1}{192}\,(2\,(4\,(6\,x+1)x+7)x-37)\sqrt{x^2+x+1} - \frac{3}{128}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

```
[In] integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(2*(4*(6*x + 1)*x + 7)*x - 37)*sqrt(x^2 + x + 1) - 3/128*log(-2*x + 2
*sqrt(x^2 + x + 1) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int x^2\sqrt{1+x+x^2}\,dx = \frac{3\,\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{128} - \frac{\left(\frac{x}{2}+\frac{1}{4}\right)\,\sqrt{x^2+x+1}}{4} - \frac{5\,(8\,x^2+2\,x+5)\,\sqrt{x^2+x+1}}{192} + \frac{x\,(x^2+x+1)^{3/2}}{4}$$

```
[In] int(x^2*(x + x^2 + 1)^(1/2),x)
```

```
[Out] (3*log(x + (x + x^2 + 1)^(1/2) + 1/2))/128 - ((x/2 + 1/4)*(x + x^2 + 1)^(1/
2))/4 - (5*(2*x + 8*x^2 + 5)*(x + x^2 + 1)^(1/2))/192 + (x*(x + x^2 + 1)^(3
/2))/4
```

3.272 $\int (1+x+x^2)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 55

$$\int (1+x+x^2)^{3/2}\,dx = \frac{9}{64}(1+2x)\sqrt{1+x+x^2} + \frac{1}{8}(1+2x)\left(1+x+x^2\right)^{3/2} + \frac{27}{128}\operatorname{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

```
[Out] 1/8*(1+2*x)*(x^2+x+1)^(3/2)+27/128*arcsinh(1/3*(1+2*x)*3^(1/2))+9/64*(1+2*x
)*(x^2+x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {626, 633, 221}

$$\int (1+x+x^2)^{3/2}\,dx = \frac{27}{128}\operatorname{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{1}{8}(2x+1)\left(x^2+x+1\right)^{3/2} + \frac{9}{64}(2x+1)\sqrt{x^2+x+1}$$

```
[In] Int[(1 + x + x^2)^(3/2),x]
```

```
[Out] (9*(1 + 2*x)*Sqrt[1 + x + x^2])/64 + ((1 + 2*x)*(1 + x + x^2)^(3/2))/8 + (2
7*ArcSinh[(1 + 2*x)/Sqrt[3]])/128
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{8}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{9}{16}\int\sqrt{1+x+x^2}\,dx\\
&= \frac{9}{64}(1+2x)\sqrt{1+x+x^2}+\frac{1}{8}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{27}{128}\int\frac{1}{\sqrt{1+x+x^2}}\,dx\\
&= \frac{9}{64}(1+2x)\sqrt{1+x+x^2}+\frac{1}{8}(1+2x)\left(1+x+x^2\right)^{3/2}\\
&\quad+\frac{1}{128}\left(9\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+2x\right)\\
&= \frac{9}{64}(1+2x)\sqrt{1+x+x^2}+\frac{1}{8}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{27}{128}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int\left(1+x+x^2\right)^{3/2}\,dx=\frac{1}{64}\sqrt{1+x+x^2}\left(17+42x+24x^2+16x^3\right)-\frac{27}{128}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

```
[In] Integrate[(1 + x + x^2)^(3/2),x]
[Out] (Sqrt[1 + x + x^2]*(17 + 42*x + 24*x^2 + 16*x^3))/64 - (27*Log[-1 - 2*x + 2
*Sqrt[1 + x + x^2]])/128
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{(16x^3+24x^2+42x+17)\sqrt{x^2+x+1}}{64}+\frac{27\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{128}$	38
default	$\frac{(1+2x)\left(x^2+x+1\right)^{\frac{3}{2}}}{8}+\frac{9(1+2x)\sqrt{x^2+x+1}}{64}+\frac{27\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{128}$	43
trager	$\left(\frac{1}{4}x^3+\frac{3}{8}x^2+\frac{21}{32}x+\frac{17}{64}\right)\sqrt{x^2+x+1}+\frac{27\ln\left(1+2x+2\sqrt{x^2+x+1}\right)}{128}$	44

```
[In] int((x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/64*(16*x^3+24*x^2+42*x+17)*(x^2+x+1)^(1/2)+27/128*arcsinh(2/3*3^(1/2)*(x+
1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \left(1+x+x^2\right)^{3/2}\,dx = \frac{1}{64}\left(16\,x^3+24\,x^2+42\,x+17\right)\sqrt{x^2+x+1} - \frac{27}{128}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

```
[In] integrate((x^2+x+1)^(3/2),x, algorithm="fricas")
[Out] 1/64*(16*x^3 + 24*x^2 + 42*x + 17)*sqrt(x^2 + x + 1) - 27/128*log(-2*x + 2*
sqrt(x^2 + x + 1) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

$$\int \left(1+x+x^2\right)^{3/2}\,dx = \left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{x^2+x+1}+\left(\frac{x^2}{3}+\frac{x}{12}+\frac{5}{24}\right)\sqrt{x^2+x+1} + \sqrt{x^2+x+1}\left(\frac{x^3}{4}+\frac{x^2}{24}+\frac{7x}{96}-\frac{37}{192}\right)+\frac{27\,\mathrm{asinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{128}$$

```
[In] integrate((x**2+x+1)**(3/2),x)
[Out] (x/2 + 1/4)*sqrt(x**2 + x + 1) + (x**2/3 + x/12 + 5/24)*sqrt(x**2 + x + 1)
+ sqrt(x**2 + x + 1)*(x**3/4 + x**2/24 + 7*x/96 - 37/192) + 27*asinh(2*sqrt
(3)*(x + 1/2)/3)/128
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \left(1+x+x^2\right)^{3/2}\,dx = \frac{1}{4}\left(x^2+x+1\right)^{\frac{3}{2}}x+\frac{1}{8}\left(x^2+x+1\right)^{\frac{3}{2}} + \frac{9}{32}\sqrt{x^2+x+1}x+\frac{9}{64}\sqrt{x^2+x+1}+\frac{27}{128}\,\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)$$

```
[In] integrate((x^2+x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(x^2 + x + 1)^(3/2)*x + 1/8*(x^2 + x + 1)^(3/2) + 9/32*sqrt(x^2 + x + 1
)*x + 9/64*sqrt(x^2 + x + 1) + 27/128*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \left(1+x+x^2\right)^{3/2}\,dx = \frac{1}{64}\,(2\,(4\,(2\,x+3)x+21)x+17)\sqrt{x^2+x+1} - \frac{27}{128}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

```
[In] integrate((x^2+x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(2*(4*(2*x + 3)*x + 21)*x + 17)*sqrt(x^2 + x + 1) - 27/128*log(-2*x +
2*sqrt(x^2 + x + 1) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \left(1+x+x^2\right)^{3/2}\,dx = \frac{27\,\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{128} + \frac{\left(x+\frac{1}{2}\right)\,\left(x^2+x+1\right)^{3/2}}{4} + \frac{9\,\left(\frac{x}{2}+\frac{1}{4}\right)\,\sqrt{x^2+x+1}}{16}$$

```
[In] int((x + x^2 + 1)^(3/2),x)
```

```
[Out] (27*log(x + (x + x^2 + 1)^(1/2) + 1/2))/128 + ((x + 1/2)*(x + x^2 + 1)^(3/2
))/4 + (9*(x/2 + 1/4)*(x + x^2 + 1)^(1/2))/16
```

3.273 $\int (1+x+x^2)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 74

$$\int \left(1+x+x^2\right)^{5/2}\,dx=\frac{45}{512}(1+2x)\sqrt{1+x+x^2}+\frac{5}{64}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2}+\frac{135\operatorname{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)}{1024}$$

```
[Out] 5/64*(1+2*x)*(x^2+x+1)^(3/2)+1/12*(1+2*x)*(x^2+x+1)^(5/2)+135/1024*arcsinh(
1/3*(1+2*x)*3^(1/2))+45/512*(1+2*x)*(x^2+x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {626, 633, 221}

$$\int \left(1+x+x^2\right)^{5/2}\,dx=\frac{135\operatorname{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)}{1024}+\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}+\frac{5}{64}(2x+1)\left(x^2+x+1\right)^{3/2}+\frac{45}{512}(2x+1)\sqrt{x^2+x+1}$$

```
[In] Int[(1 + x + x^2)^(5/2),x]
```

```
[Out] (45*(1 + 2*x)*Sqrt[1 + x + x^2])/512 + (5*(1 + 2*x)*(1 + x + x^2)^(3/2))/64
 + ((1 + 2*x)*(1 + x + x^2)^(5/2))/12 + (135*ArcSinh[(1 + 2*x)/Sqrt[3]])/10
24
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2}+\frac{5}{8}\int\left(1+x+x^2\right)^{3/2}\,dx \\
&= \frac{5}{64}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2}+\frac{45}{128}\int\sqrt{1+x+x^2}\,dx \\
&= \frac{45}{512}(1+2x)\sqrt{1+x+x^2}+\frac{5}{64}(1+2x)\left(1+x+x^2\right)^{3/2} \\
&\quad+\frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2}+\frac{135\int\frac{1}{\sqrt{1+x+x^2}}\,dx}{1024} \\
&= \frac{45}{512}(1+2x)\sqrt{1+x+x^2}+\frac{5}{64}(1+2x)\left(1+x+x^2\right)^{3/2} \\
&\quad+\frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2}+\frac{\left(45\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+2x\right)}{1024} \\
&= \frac{45}{512}(1+2x)\sqrt{1+x+x^2}+\frac{5}{64}(1+2x)\left(1+x+x^2\right)^{3/2} \\
&\quad+\frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2}+\frac{135\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)}{1024}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \left(1+x+x^2\right)^{5/2} dx = \frac{\sqrt{1+x+x^2}(383+1142x+1256x^2+1264x^3+640x^4+256x^5)}{1536} - \frac{135\log\left(-1-2x+2\sqrt{1+x+x^2}\right)}{1024}$$

```
[In] Integrate[(1 + x + x^2)^(5/2),x]
[Out] (Sqrt[1 + x + x^2]*(383 + 1142*x + 1256*x^2 + 1264*x^3 + 640*x^4 + 256*x^5)
)/1536 - (135*Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]])/1024
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{(256x^5+640x^4+1264x^3+1256x^2+1142x+383)\sqrt{x^2+x+1}}{1536}+\frac{135\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{1024}$	48
trager	$\left(\frac{1}{6}x^5+\frac{5}{12}x^4+\frac{79}{96}x^3+\frac{157}{192}x^2+\frac{571}{768}x+\frac{383}{1536}\right)\sqrt{x^2+x+1}+\frac{135\ln\left(1+2x+2\sqrt{x^2+x+1}\right)}{1024}$	54
default	$\frac{(1+2x)\left(x^2+x+1\right)^{\frac{5}{2}}}{12}+\frac{5(1+2x)\left(x^2+x+1\right)^{\frac{3}{2}}}{64}+\frac{45(1+2x)\sqrt{x^2+x+1}}{512}+\frac{135\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{1024}$	58

```
[In] int((x^2+x+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/1536*(256*x^5+640*x^4+1264*x^3+1256*x^2+1142*x+383)*(x^2+x+1)^(1/2)+135/1
024*arcsinh(2/3*3^(1/2)*(x+1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.73

$$\int \left(1+x+x^2\right)^{5/2} dx = \frac{1}{1536}\left(256\,x^5+640\,x^4+1264\,x^3+1256\,x^2+1142\,x+383\right)\sqrt{x^2+x+1} - \frac{135}{1024}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

```
[In] integrate((x^2+x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(256*x^5 + 640*x^4 + 1264*x^3 + 1256*x^2 + 1142*x + 383)*sqrt(x^2 +
x + 1) - 135/1024*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(71) = 142$.

Time = 0.45 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.27

$$\int (1+x+x^2)^{5/2}\,dx = \left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{x^2+x+1}+2\left(\frac{x^2}{3}+\frac{x}{12}+\frac{5}{24}\right)\sqrt{x^2+x+1} +3\sqrt{x^2+x+1}\left(\frac{x^3}{4}+\frac{x^2}{24}+\frac{7x}{96}-\frac{37}{192}\right)+2\sqrt{x^2+x+1}\left(\frac{x^4}{5}+\frac{x^3}{40}+\frac{3x^2}{80}-\frac{27x}{320}+\frac{33}{640}\right) +\sqrt{x^2+x+1}\left(\frac{x^5}{6}+\frac{x^4}{60}+\frac{11x^3}{480}-\frac{47x^2}{960}+\frac{103x}{3840}+\frac{443}{7680}\right)+\frac{135\operatorname{asinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{1024}$$

```
[In] integrate((x**2+x+1)**(5/2),x)
```

```
[Out] (x/2 + 1/4)*sqrt(x**2 + x + 1) + 2*(x**2/3 + x/12 + 5/24)*sqrt(x**2 + x + 1
) + 3*sqrt(x**2 + x + 1)*(x**3/4 + x**2/24 + 7*x/96 - 37/192) + 2*sqrt(x**2
 + x + 1)*(x**4/5 + x**3/40 + 3*x**2/80 - 27*x/320 + 33/640) + sqrt(x**2 +
x + 1)*(x**5/6 + x**4/60 + 11*x**3/480 - 47*x**2/960 + 103*x/3840 + 443/768
0) + 135*asinh(2*sqrt(3)*(x + 1/2)/3)/1024
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int (1+x+x^2)^{5/2}\,dx = \frac{1}{6}\left(x^2+x+1\right)^{\frac{5}{2}}x+\frac{1}{12}\left(x^2+x+1\right)^{\frac{5}{2}} +\frac{5}{32}\left(x^2+x+1\right)^{\frac{3}{2}}x+\frac{5}{64}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{45}{256}\sqrt{x^2+x+1}x +\frac{45}{512}\sqrt{x^2+x+1}+\frac{135}{1024}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

```
[In] integrate((x^2+x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(x^2 + x + 1)^(5/2)*x + 1/12*(x^2 + x + 1)^(5/2) + 5/32*(x^2 + x + 1)^(
3/2)*x + 5/64*(x^2 + x + 1)^(3/2) + 45/256*sqrt(x^2 + x + 1)*x + 45/512*sqr
t(x^2 + x + 1) + 135/1024*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.73

$$\int (1+x+x^2)^{5/2}\, dx = \frac{1}{1536}\,(2\,(4\,(2\,(8\,(2\,x+5)x+79)x+157)x+571)x+383)\sqrt{x^2+x+1} - \frac{135}{1024}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

```
[In] integrate((x^2+x+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/1536*(2*(4*(2*(8*(2*x + 5)*x + 79)*x + 157)*x + 571)*x + 383)*sqrt(x^2 +
x + 1) - 135/1024*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int (1+x+x^2)^{5/2}\, dx = \frac{135\,\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{1024} + \frac{5\,\left(x+\frac{1}{2}\right)\,\left(x^2+x+1\right)^{3/2}}{32} + \frac{\left(x+\frac{1}{2}\right)\,\left(x^2+x+1\right)^{5/2}}{6} + \frac{45\,\left(\frac{x}{2}+\frac{1}{4}\right)\,\sqrt{x^2+x+1}}{128}$$

```
[In] int((x + x^2 + 1)^(5/2),x)
```

```
[Out] (135*log(x + (x + x^2 + 1)^(1/2) + 1/2))/1024 + (5*(x + 1/2)*(x + x^2 + 1)^
(3/2))/32 + ((x + 1/2)*(x + x^2 + 1)^(5/2))/6 + (45*(x/2 + 1/4)*(x + x^2 +
1)^(1/2))/128
```

3.274 $\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 38

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = -\frac{\sqrt{1+x+x^2}}{x}+\frac{1}{2}\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

```
[Out] 1/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-(x^2+x+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {744, 738, 212}

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = \frac{1}{2}\text{arctanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-\frac{\sqrt{x^2+x+1}}{x}$$

```
[In] Int[1/(x^2*Sqrt[1 + x + x^2]),x]
[Out] -(Sqrt[1 + x + x^2]/x) + ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{1+x+x^2}}{x} - \frac{1}{2}\int \frac{1}{x\sqrt{1+x+x^2}}\,dx \\
&= -\frac{\sqrt{1+x+x^2}}{x} + \text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{2+x}{\sqrt{1+x+x^2}}\right) \\
&= -\frac{\sqrt{1+x+x^2}}{x} + \frac{1}{2}\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = -\frac{\sqrt{1+x+x^2}}{x} - \text{arctanh}\left(x-\sqrt{1+x+x^2}\right)$$

```
[In] Integrate[1/(x^2*Sqrt[1 + x + x^2]),x]
[Out] -(Sqrt[1 + x + x^2]/x) - ArcTanh[x - Sqrt[1 + x + x^2]]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^{2}+x+1}}\right)}{2}-\frac{\sqrt{x^{2}+x+1}}{x}$	31
risch	$\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^{2}+x+1}}\right)}{2}-\frac{\sqrt{x^{2}+x+1}}{x}$	31
trager	$-\frac{\sqrt{x^{2}+x+1}}{x}+\frac{\ln\left(\frac{2\sqrt{x^{2}+x+1}+2+x}{x}\right)}{2}$	35

```
[In] int(1/x^2/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-(x^2+x+1)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = \frac{x\log\left(-x+\sqrt{x^2+x+1}+1\right)-x\log\left(-x+\sqrt{x^2+x+1}-1\right)-2\,x-2\,\sqrt{x^2+x+1}}{2\,x}$$

```
[In] integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*(x*log(-x + sqrt(x^2 + x + 1) + 1) - x*log(-x + sqrt(x^2 + x + 1) - 1)
- 2*x - 2*sqrt(x^2 + x + 1))/x
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x^2\sqrt{x^2+x+1}}\,dx$$

```
[In] integrate(1/x**2/(x**2+x+1)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(x**2 + x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = -\frac{\sqrt{x^2+x+1}}{x} + \frac{1}{2}\,\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|} + \frac{2\,\sqrt{3}}{3\,|x|}\right)$$

```
[In] integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(x^2 + x + 1)/x + 1/2*arcsinh(1/3*sqrt(3)*x/abs(x) + 2/3*sqrt(3)/abs(x
))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(30) = 60.

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = \frac{x-\sqrt{x^2+x+1}+2}{\left(x-\sqrt{x^2+x+1}\right)^2-1} + \frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) - \frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

```
[In] integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="giac")
[Out] (x - sqrt(x^2 + x + 1) + 2)/((x - sqrt(x^2 + x + 1))^2 - 1) + 1/2*log(abs(-
x + sqrt(x^2 + x + 1) + 1)) - 1/2*log(abs(-x + sqrt(x^2 + x + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\frac{x}{2}+1}{\sqrt{x^2+x+1}}\right)}{2} - \frac{\sqrt{x^2+x+1}}{x}$$

```
[In] int(1/(x^2*(x + x^2 + 1)^(1/2)),x)
[Out] atanh((x/2 + 1)/(x + x^2 + 1)^(1/2))/2 - (x + x^2 + 1)^(1/2)/x
```

3.275 $\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx=-\frac{\sqrt{1+x+x^2}}{2x^2}+\frac{3\sqrt{1+x+x^2}}{4x}+\frac{1}{8}\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

```
[Out] 1/8*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-1/2*(x^2+x+1)^(1/2)/x^2+3/4*(x^2+x+1
)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {758, 820, 738, 212}

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx=\frac{1}{8}\text{arctanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)+\frac{3\sqrt{x^2+x+1}}{4x}-\frac{\sqrt{x^2+x+1}}{2x^2}$$

```
[In] Int[1/(x^3*Sqrt[1 + x + x^2]),x]
```

```
[Out] -1/2*Sqrt[1 + x + x^2]/x^2 + (3*Sqrt[1 + x + x^2])/(4*x) + ArcTanh[(2 + x)/
(2*Sqrt[1 + x + x^2])]/8
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{1+x+x^2}}{2x^2} - \frac{1}{2}\int \frac{\frac{3}{2}+x}{x^2\sqrt{1+x+x^2}}\,dx \\
&= -\frac{\sqrt{1+x+x^2}}{2x^2} + \frac{3\sqrt{1+x+x^2}}{4x} - \frac{1}{8}\int \frac{1}{x\sqrt{1+x+x^2}}\,dx \\
&= -\frac{\sqrt{1+x+x^2}}{2x^2} + \frac{3\sqrt{1+x+x^2}}{4x} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{2+x}{\sqrt{1+x+x^2}}\right) \\
&= -\frac{\sqrt{1+x+x^2}}{2x^2} + \frac{3\sqrt{1+x+x^2}}{4x} + \frac{1}{8}\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \frac{(-2+3x)\sqrt{1+x+x^2}}{4x^2} - \frac{1}{4}\text{arctanh}\left(x-\sqrt{1+x+x^2}\right)$$

```
[In] Integrate[1/(x^3*Sqrt[1 + x + x^2]),x]
[Out] ((-2 + 3*x)*Sqrt[1 + x + x^2])/(4*x^2) - ArcTanh[x - Sqrt[1 + x + x^2]]/4
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

method	result	size
trager	$\frac{(-2+3x)\sqrt{x^2+x+1}}{4x^2}-\frac{\ln\left(\frac{-2-x+2\sqrt{x^2+x+1}}{x}\right)}{8}$	42
risch	$\frac{3x^3+x^2+x-2}{4x^2\sqrt{x^2+x+1}}+\frac{\text{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}$	42
default	$\frac{\text{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}-\frac{\sqrt{x^2+x+1}}{2x^2}+\frac{3\sqrt{x^2+x+1}}{4x}$	44

```
[In] int(1/x^3/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*(-2+3*x)/x^2*(x^2+x+1)^(1/2)-1/8*ln((-2-x+2*(x^2+x+1)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \frac{x^2\log\left(-x+\sqrt{x^2+x+1}+1\right)-x^2\log\left(-x+\sqrt{x^2+x+1}-1\right)+6\,x^2+2\,\sqrt{x^2+x+1}(3\,x-2)}{8\,x^2}$$

```
[In] integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="fricas")
[Out] 1/8*(x^2*log(-x + sqrt(x^2 + x + 1) + 1) - x^2*log(-x + sqrt(x^2 + x + 1) -
 1) + 6*x^2 + 2*sqrt(x^2 + x + 1)*(3*x - 2))/x^2
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x^3\sqrt{x^2+x+1}}\,dx$$

```
[In] integrate(1/x**3/(x**2+x+1)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(x**2 + x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \frac{3\sqrt{x^2+x+1}}{4\,x} - \frac{\sqrt{x^2+x+1}}{2\,x^2} + \frac{1}{8}\,\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|} + \frac{2\,\sqrt{3}}{3\,|x|}\right)$$

```
[In] integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="maxima")
[Out] 3/4*sqrt(x^2 + x + 1)/x - 1/2*sqrt(x^2 + x + 1)/x^2 + 1/8*arcsinh(1/3*sqrt(
3)*x/abs(x) + 2/3*sqrt(3)/abs(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \frac{\left(x-\sqrt{x^2+x+1}\right)^3 + 9\,x - 9\,\sqrt{x^2+x+1} + 8}{4\,\left(\left(x-\sqrt{x^2+x+1}\right)^2 - 1\right)^2} + \frac{1}{8}\,\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) - \frac{1}{8}\,\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

```
[In] integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="giac")
[Out] 1/4*((x - sqrt(x^2 + x + 1))^3 + 9*x - 9*sqrt(x^2 + x + 1) + 8)/((x - sqrt(
x^2 + x + 1))^2 - 1)^2 + 1/8*log(abs(-x + sqrt(x^2 + x + 1) + 1)) - 1/8*log
(abs(-x + sqrt(x^2 + x + 1) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x^3\,\sqrt{x^2+x+1}}\,dx$$

```
[In] int(1/(x^3*(x + x^2 + 1)^(1/2)),x)
[Out] int(1/(x^3*(x + x^2 + 1)^(1/2)), x)
```

3.276 $\int \frac{1}{x^2(1+x+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{1}{x^2(1+x+x^2)^{3/2}}\,dx = \frac{2(1-x)}{3x\sqrt{1+x+x^2}} - \frac{5\sqrt{1+x+x^2}}{3x} + \frac{3}{2}\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

```
[Out] 3/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))+2/3*(1-x)/x/(x^2+x+1)^(1/2)-5/3*(x^2
+x+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {754, 820, 738, 212}

$$\int \frac{1}{x^2(1+x+x^2)^{3/2}}\,dx = \frac{3}{2}\operatorname{arctanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right) + \frac{2(1-x)}{3x\sqrt{x^2+x+1}} - \frac{5\sqrt{x^2+x+1}}{3x}$$

```
[In] Int[1/(x^2*(1 + x + x^2)^(3/2)),x]
```

```
[Out] (2*(1 - x))/(3*x*Sqrt[1 + x + x^2]) - (5*Sqrt[1 + x + x^2])/(3*x) + (3*ArcT
anh[(2 + x)/(2*Sqrt[1 + x + x^2])])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(1-x)}{3x\sqrt{1+x+x^2}}+\frac{2}{3}\int \frac{\frac{5}{2}-x}{x^2\sqrt{1+x+x^2}}\,dx\\
&= \frac{2(1-x)}{3x\sqrt{1+x+x^2}}-\frac{5\sqrt{1+x+x^2}}{3x}-\frac{3}{2}\int \frac{1}{x\sqrt{1+x+x^2}}\,dx\\
&= \frac{2(1-x)}{3x\sqrt{1+x+x^2}}-\frac{5\sqrt{1+x+x^2}}{3x}+3\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{2+x}{\sqrt{1+x+x^2}}\right)\\
&= \frac{2(1-x)}{3x\sqrt{1+x+x^2}}-\frac{5\sqrt{1+x+x^2}}{3x}+\frac{3}{2}\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2 (1+x+x^2)^{3/2}}\,dx = \frac{-3-7x-5x^2}{3x\sqrt{1+x+x^2}} - 3\mathrm{arctanh}\left(x-\sqrt{1+x+x^2}\right)$$

```
[In] Integrate[1/(x^2*(1 + x + x^2)^(3/2)),x]
```

```
[Out] (-3 - 7*x - 5*x^2)/(3*x*Sqrt[1 + x + x^2]) - 3*ArcTanh[x - Sqrt[1 + x + x^2
]]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

method	result	size
risch	$-\frac{5x^2+7x+3}{3x\sqrt{x^2+x+1}}+\frac{3\,\mathrm{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{2}$	41
trager	$-\frac{5x^2+7x+3}{3x\sqrt{x^2+x+1}}-\frac{3\ln\left(\frac{-2-x+2\sqrt{x^2+x+1}}{x}\right)}{2}$	47
default	$-\frac{1}{x\sqrt{x^2+x+1}}-\frac{3}{2\sqrt{x^2+x+1}}-\frac{5(1+2x)}{6\sqrt{x^2+x+1}}+\frac{3\,\mathrm{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{2}$	56

```
[In] int(1/x^2/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(5*x^2+7*x+3)/x/(x^2+x+1)^(1/2)+3/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(46) = 92$.

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^2 (1+x+x^2)^{3/2}}\,dx = -\frac{10\,x^3+10\,x^2-9\,(x^3+x^2+x)\log\left(-x+\sqrt{x^2+x+1}+1\right)+9\,(x^3+x^2+x)\log\left(-x+\sqrt{x^2+x+1}-1\right)}{6\,(x^3+x^2+x)}$$

```
[In] integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(10*x^3 + 10*x^2 - 9*(x^3 + x^2 + x)*log(-x + sqrt(x^2 + x + 1) + 1) +
 9*(x^3 + x^2 + x)*log(-x + sqrt(x^2 + x + 1) - 1) + 2*(5*x^2 + 7*x + 3)*sq
rt(x^2 + x + 1) + 10*x)/(x^3 + x^2 + x)
```

Sympy [F]

$$\int \frac{1}{x^2 (1 + x + x^2)^{3/2}}\,dx = \int \frac{1}{x^2 (x^2 + x + 1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**2/(x**2+x+1)**(3/2),x)
[Out] Integral(1/(x**2*(x**2 + x + 1)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2 (1 + x + x^2)^{3/2}}\,dx = -\frac{5\,x}{3\,\sqrt{x^2 + x + 1}} - \frac{7}{3\,\sqrt{x^2 + x + 1}} - \frac{1}{\sqrt{x^2 + x + 1}x} + \frac{3}{2}\,\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|} + \frac{2\,\sqrt{3}}{3\,|x|}\right)$$

```
[In] integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="maxima")
[Out] -5/3*x/sqrt(x^2 + x + 1) - 7/3/sqrt(x^2 + x + 1) - 1/(sqrt(x^2 + x + 1)*x)
+ 3/2*arcsinh(1/3*sqrt(3)*x/abs(x) + 2/3*sqrt(3)/abs(x))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2 (1 + x + x^2)^{3/2}}\,dx = -\frac{2\,(x + 2)}{3\,\sqrt{x^2 + x + 1}} + \frac{x - \sqrt{x^2 + x + 1} + 2}{\left(x - \sqrt{x^2 + x + 1}\right)^2 - 1} + \frac{3}{2}\,\log\left(\left|-x + \sqrt{x^2 + x + 1} + 1\right|\right) - \frac{3}{2}\,\log\left(\left|-x + \sqrt{x^2 + x + 1} - 1\right|\right)$$

```
[In] integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="giac")
[Out] -2/3*(x + 2)/sqrt(x^2 + x + 1) + (x - sqrt(x^2 + x + 1) + 2)/((x - sqrt(x^2
 + x + 1))^2 - 1) + 3/2*log(abs(-x + sqrt(x^2 + x + 1) + 1)) - 3/2*log(abs(
-x + sqrt(x^2 + x + 1) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (1+x+x^2)^{3/2}} \, dx = \int \frac{1}{x^2 (x^2+x+1)^{3/2}} \, dx$$

```
[In] int(1/(x^2*(x + x^2 + 1)^(3/2)),x)
[Out] int(1/(x^2*(x + x^2 + 1)^(3/2)), x)
```

3.277 $\int \frac{1}{x^3(1+x+x^2)^{3/2}} dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int \frac{1}{x^3\left(1+x+x^2\right)^{3/2}} dx = \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}} - \frac{7\sqrt{1+x+x^2}}{6x^2} + \frac{37\sqrt{1+x+x^2}}{12x} - \frac{3}{8}\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

```
[Out] -3/8*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))+2/3*(1-x)/x^2/(x^2+x+1)^(1/2)-7/6*(
x^2+x+1)^(1/2)/x^2+37/12*(x^2+x+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {754, 848, 820, 738, 212}

$$\int \frac{1}{x^3\left(1+x+x^2\right)^{3/2}} dx = -\frac{3}{8}\operatorname{arctanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right) + \frac{2(1-x)}{3x^2\sqrt{x^2+x+1}} + \frac{37\sqrt{x^2+x+1}}{12x} - \frac{7\sqrt{x^2+x+1}}{6x^2}$$

```
[In] Int[1/(x^3*(1 + x + x^2)^(3/2)),x]
```

```
[Out] (2*(1 - x))/(3*x^2*Sqrt[1 + x + x^2]) - (7*Sqrt[1 + x + x^2])/(6*x^2) + (37
*Sqrt[1 + x + x^2])/(12*x) - (3*ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])])/8
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rubi steps

$$\text{integral} = \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}} + \frac{2}{3}\int \frac{\frac{7}{2}-2x}{x^3\sqrt{1+x+x^2}}\,dx$$

$$= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}} - \frac{7\sqrt{1+x+x^2}}{6x^2} - \frac{1}{3}\int \frac{\frac{37}{4}+\frac{7x}{2}}{x^2\sqrt{1+x+x^2}}\,dx$$

$$= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}} - \frac{7\sqrt{1+x+x^2}}{6x^2} + \frac{37\sqrt{1+x+x^2}}{12x} + \frac{3}{8}\int \frac{1}{x\sqrt{1+x+x^2}}\,dx$$

$$= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}} - \frac{7\sqrt{1+x+x^2}}{6x^2} + \frac{37\sqrt{1+x+x^2}}{12x} - \frac{3}{4}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{2+x}{\sqrt{1+x+x^2}}\right)$$

$$= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}} - \frac{7\sqrt{1+x+x^2}}{6x^2} + \frac{37\sqrt{1+x+x^2}}{12x} - \frac{3}{8}\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^3\left(1+x+x^2\right)^{3/2}}\,dx = \frac{-6+15x+23x^2+37x^3}{12x^2\sqrt{1+x+x^2}} + \frac{3}{4}\text{arctanh}\left(x-\sqrt{1+x+x^2}\right)$$

```
[In] Integrate[1/(x^3*(1 + x + x^2)^(3/2)),x]
```

```
[Out] (-6 + 15*x + 23*x^2 + 37*x^3)/(12*x^2*Sqrt[1 + x + x^2]) + (3*ArcTanh[x - S
qrt[1 + x + x^2]])/4
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

method	result	size
risch	$\frac{37x^3+23x^2+15x-6}{12\sqrt{x^2+x+1}\,x^2} - \frac{3\,\text{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}$	46
trager	$\frac{37x^3+23x^2+15x-6}{12\sqrt{x^2+x+1}\,x^2} - \frac{3\ln\left(\frac{2\sqrt{x^2+x+1}+2+x}{x}\right)}{8}$	50
default	$-\frac{1}{2x^2\sqrt{x^2+x+1}} + \frac{5}{4x\sqrt{x^2+x+1}} + \frac{3}{8\sqrt{x^2+x+1}} + \frac{\frac{37}{24}+\frac{37x}{12}}{\sqrt{x^2+x+1}} - \frac{3\,\text{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}$	69

```
[In] int(1/x^3/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(37*x^3+23*x^2+15*x-6)/(x^2+x+1)^(1/2)/x^2-3/8*arctanh(1/2*(2+x)/(x^2+
x+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^3 (1+x+x^2)^{3/2}}\,dx = \frac{74\,x^4+74\,x^3+74\,x^2-9\,(x^4+x^3+x^2)\log\left(-x+\sqrt{x^2+x+1}+1\right)+9\,(x^4+x^3}{24\,(x^4+x^3}$$

```
[In] integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(74*x^4 + 74*x^3 + 74*x^2 - 9*(x^4 + x^3 + x^2)*log(-x + sqrt(x^2 + x
+ 1) + 1) + 9*(x^4 + x^3 + x^2)*log(-x + sqrt(x^2 + x + 1) - 1) + 2*(37*x^3
 + 23*x^2 + 15*x - 6)*sqrt(x^2 + x + 1))/(x^4 + x^3 + x^2)
```

Sympy [F]

$$\int \frac{1}{x^3 (1+x+x^2)^{3/2}}\,dx = \int \frac{1}{x^3 (x^2+x+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**3/(x**2+x+1)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(x**2 + x + 1)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3 (1+x+x^2)^{3/2}}\,dx = \frac{37\,x}{12\,\sqrt{x^2+x+1}} + \frac{23}{12\,\sqrt{x^2+x+1}} + \frac{5}{4\,\sqrt{x^2+x+1}x} - \frac{1}{2\,\sqrt{x^2+x+1}x^2} - \frac{3}{8}\,\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|} + \frac{2\,\sqrt{3}}{3\,|x|}\right)$$

```
[In] integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 37/12*x/sqrt(x^2 + x + 1) + 23/12/sqrt(x^2 + x + 1) + 5/4/(sqrt(x^2 + x + 1
)*x) - 1/2/(sqrt(x^2 + x + 1)*x^2) - 3/8*arcsinh(1/3*sqrt(3)*x/abs(x) + 2/3
*sqrt(3)/abs(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^3 (1+x+x^2)^{3/2}}\,dx = \frac{2\,(2\,x+1)}{3\,\sqrt{x^2+x+1}} - \frac{3\,\left(x-\sqrt{x^2+x+1}\right)^3 + 8\,\left(x-\sqrt{x^2+x+1}\right)^2 - 13\,x + 13\,\sqrt{x^2+x+1} - 16}{4\,\left(\left(x-\sqrt{x^2+x+1}\right)^2-1\right)^2} - \frac{3}{8}\,\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) + \frac{3}{8}\,\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

```
[In] integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*x + 1)/sqrt(x^2 + x + 1) - 1/4*(3*(x - sqrt(x^2 + x + 1))^3 + 8*(x -
 sqrt(x^2 + x + 1))^2 - 13*x + 13*sqrt(x^2 + x + 1) - 16)/((x - sqrt(x^2 +
x + 1))^2 - 1)^2 - 3/8*log(abs(-x + sqrt(x^2 + x + 1) + 1)) + 3/8*log(abs(-
x + sqrt(x^2 + x + 1) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (1+x+x^2)^{3/2}}\,dx = \int \frac{1}{x^3\,{\left(x^2+x+1\right)}^{3/2}}\,dx$$

```
[In] int(1/(x^3*(x + x^2 + 1)^(3/2)),x)
```

```
[Out] int(1/(x^3*(x + x^2 + 1)^(3/2)), x)
```

3.278 $\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx=-\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)$$

```
[Out] -arctanh(1/2*(1-x)/(x^2+x+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {738, 212}

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx=-\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)$$

```
[In] Int[1/((1 + x)*Sqrt[1 + x + x^2]),x]
[Out] -ArcTanh[(1 - x)/(2*Sqrt[1 + x + x^2])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{1-x}{\sqrt{1+x+x^2}}\right)\right)$$
$$= -\text{arctanh}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = 2\text{arctanh}\left(1+x-\sqrt{1+x+x^2}\right)$$

```
[In] Integrate[1/((1 + x)*Sqrt[1 + x + x^2]),x]
[Out] 2*ArcTanh[1 + x - Sqrt[1 + x + x^2]]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

method	result	size
default	$-\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{(1+x)^2-x}}\right)$	22
trager	$-\ln\left(\frac{2\sqrt{x^2+x+1}+1-x}{1+x}\right)$	25

```
[In] int(1/(1+x)/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -arctanh(1/2*(1-x)/((1+x)^2-x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = -\log\left(-x+\sqrt{x^2+x+1}\right)+\log\left(-x+\sqrt{x^2+x+1}-2\right)$$

```
[In] integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 + x + 1)) + log(-x + sqrt(x^2 + x + 1) - 2)
```

Sympy [F]

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = \int \frac{1}{(x+1)\sqrt{x^2+x+1}}\,dx$$

```
[In] integrate(1/(1+x)/(x**2+x+1)**(1/2),x)
[Out] Integral(1/((x + 1)*sqrt(x**2 + x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = \operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x+1|}-\frac{\sqrt{3}}{3\,|x+1|}\right)$$

```
[In] integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="maxima")
[Out] arcsinh(1/3*sqrt(3)*x/abs(x + 1) - 1/3*sqrt(3)/abs(x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = -\log\left(\left|-x+\sqrt{x^2+x+1}\right|\right)+\log\left(\left|-x+\sqrt{x^2+x+1}-2\right|\right)$$

```
[In] integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="giac")
[Out] -log(abs(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x + 1) - 2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = \int \frac{1}{(x+1)\ \sqrt{x^2+x+1}}\,dx$$

```
[In] int(1/((x + 1)*(x + x^2 + 1)^(1/2)),x)
[Out] int(1/((x + 1)*(x + x^2 + 1)^(1/2)), x)
```

3.279 $\int \frac{1}{\sqrt{4+2x+x^2}(-x+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = \frac{1}{2}\operatorname{arctanh}\left(\frac{4+x}{2\sqrt{4+2x+x^2}}\right) - \frac{\operatorname{arctanh}\left(\frac{5+2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)}{2\sqrt{7}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] 1/2*arctanh(1/2*(4+x)/(x^2+2*x+4)^(1/2))-1/6*arctanh(1/3*(x^2+2*x+4)^(1/2)*
3^(1/2))*3^(1/2)-1/14*arctanh(1/7*(5+2*x)*7^(1/2)/(x^2+2*x+4)^(1/2))*7^(1/2
)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1607, 6857, 738, 212, 1047, 702, 213}

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = \frac{1}{2}\operatorname{arctanh}\left(\frac{x+4}{2\sqrt{x^2+2x+4}}\right) - \frac{\operatorname{arctanh}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[In] Int[1/(Sqrt[4 + 2*x + x^2]*(-x + x^3)),x]
```

```
[Out] ArcTanh[(4 + x)/(2*Sqrt[4 + 2*x + x^2])]/2 - ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqr
t[4 + 2*x + x^2])]/(2*Sqrt[7]) - ArcTanh[Sqrt[4 + 2*x + x^2]/Sqrt[3]]/(2*Sq
rt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 702

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x\left(-1+x^2\right)\sqrt{4+2x+x^2}}\,dx \\
&= \int \left(-\frac{1}{x\sqrt{4+2x+x^2}}+\frac{x}{\left(-1+x^2\right)\sqrt{4+2x+x^2}}\right)dx \\
&= -\int \frac{1}{x\sqrt{4+2x+x^2}}\,dx+\int \frac{x}{\left(-1+x^2\right)\sqrt{4+2x+x^2}}\,dx \\
&= \frac{1}{2}\int \frac{1}{(-1+x)\sqrt{4+2x+x^2}}\,dx+\frac{1}{2}\int \frac{1}{(1+x)\sqrt{4+2x+x^2}}\,dx \\
&\quad +2\text{Subst}\left(\int \frac{1}{16-x^2}\,dx,x,\frac{8+2x}{\sqrt{4+2x+x^2}}\right) \\
&= \frac{1}{2}\text{arctanh}\left(\frac{4+x}{2\sqrt{4+2x+x^2}}\right)+2\text{Subst}\left(\int \frac{1}{-12+4x^2}\,dx,x,\sqrt{4+2x+x^2}\right) \\
&\quad -\text{Subst}\left(\int \frac{1}{28-x^2}\,dx,x,\frac{10+4x}{\sqrt{4+2x+x^2}}\right) \\
&= \frac{1}{2}\text{arctanh}\left(\frac{4+x}{2\sqrt{4+2x+x^2}}\right)-\frac{\text{arctanh}\left(\frac{10+4x}{2\sqrt{7}\sqrt{4+2x+x^2}}\right)}{2\sqrt{7}}-\frac{\text{arctanh}\left(\frac{\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$
\int \frac{1}{\sqrt{4+2x+x^2}\left(-x+x^3\right)}\,dx = -\text{arctanh}\left(\frac{1}{2}\left(x-\sqrt{4+2x+x^2}\right)\right)+\frac{\text{arctanh}\left(\frac{1+x-\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\text{arctanh}\left(\frac{1-x+\sqrt{4+2x+x^2}}{\sqrt{7}}\right)}{\sqrt{7}}
$$

```
[In] Integrate[1/(Sqrt[4 + 2*x + x^2]*(-x + x^3)),x]
```

```
[Out] -ArcTanh[(x - Sqrt[4 + 2*x + x^2])/2] + ArcTanh[(1 + x - Sqrt[4 + 2*x + x^2
])/Sqrt[3]]/Sqrt[3] - ArcTanh[(1 - x + Sqrt[4 + 2*x + x^2])/Sqrt[7]]/Sqrt[7
]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

method	result
default	$\frac{\operatorname{arctanh}\left(\frac{8+2x}{4\sqrt{x^2+2x+4}}\right)}{2}-\frac{\sqrt{7}\,\operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{14}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}}{\sqrt{(1+x)^2+3}}\right)}{6}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{\sqrt{x^2+2x+4}-\operatorname{RootOf}\left(\_Z^2-3\right)}{1+x}\right)}{6}-\frac{\operatorname{RootOf}\left(\_Z^2-7\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^2-7\right)x+7\sqrt{x^2+2x+4}+5\operatorname{RootOf}\left(\_\right.}{-1+x}\right.}{14}$

[In] int(1/(x^3-x)/(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*arctanh(1/4*(8+2*x)/(x^2+2*x+4)^(1/2))-1/14*7^(1/2)*arctanh(1/14*(10+4*x)*7^(1/2)/((-1+x)^2+3+4*x)^(1/2))-1/6*3^(1/2)*arctanh(3^(1/2)/((1+x)^2+3)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx &= \frac{1}{14}\sqrt{7}\log\left(\frac{\sqrt{7}(2\,x+5)+\sqrt{x^2+2\,x+4}\left(2\,\sqrt{7}-7\right)-4\,x-10}{x-1}\right)\\ &\quad+\frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}-\sqrt{x^2+2\,x+4}}{x+1}\right)\\ &\quad+\frac{1}{2}\log\left(-x+\sqrt{x^2+2\,x+4}+2\right)-\frac{1}{2}\log\left(-x+\sqrt{x^2+2\,x+4}-2\right)\end{aligned}$$

[In] integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="fricas")

[Out] 1/14*sqrt(7)*log((sqrt(7)*(2*x + 5) + sqrt(x^2 + 2*x + 4)*(2*sqrt(7) - 7) - 4*x - 10)/(x - 1)) + 1/6*sqrt(3)*log(-(sqrt(3) - sqrt(x^2 + 2*x + 4))/(x + 1)) + 1/2*log(-x + sqrt(x^2 + 2*x + 4) + 2) - 1/2*log(-x + sqrt(x^2 + 2*x + 4) - 2)

Sympy [F]

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = \int \frac{1}{x\,(x-1)\,(x+1)\,\sqrt{x^2+2x+4}}\,dx$$

```
[In] integrate(1/(x**3-x)/(x**2+2*x+4)**(1/2),x)
[Out] Integral(1/(x*(x - 1)*(x + 1)*sqrt(x**2 + 2*x + 4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = \int \frac{1}{(x^3-x)\sqrt{x^2+2\,x+4}}\,dx$$

```
[In] integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^3 - x)*sqrt(x^2 + 2*x + 4)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(66) = 132$.

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.71

$$\begin{aligned}\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx &= \frac{1}{14}\sqrt{7}\log\left(\frac{\left|-2\,x-2\,\sqrt{7}+2\,\sqrt{x^2+2\,x+4}+2\right|}{\left|-2\,x+2\,\sqrt{7}+2\,\sqrt{x^2+2\,x+4}+2\right|}\right) \\ &\quad+\frac{1}{6}\sqrt{3}\log\left(-\frac{\left|-2\,x-2\,\sqrt{3}+2\,\sqrt{x^2+2\,x+4}-2\right|}{2\,(x-\sqrt{3}-\sqrt{x^2+2\,x+4}+1)}\right) \\ &\quad+\frac{1}{2}\log\left(\left|-x+\sqrt{x^2+2\,x+4}+2\right|\right) \\ &\quad-\frac{1}{2}\log\left(\left|-x+\sqrt{x^2+2\,x+4}-2\right|\right)\end{aligned}$$

```
[In] integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="giac")
[Out] 1/14*sqrt(7)*log(abs(-2*x - 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)/abs(-2*x
 + 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)) + 1/6*sqrt(3)*log(-1/2*abs(-2*x
- 2*sqrt(3) + 2*sqrt(x^2 + 2*x + 4) - 2)/(x - sqrt(3) - sqrt(x^2 + 2*x + 4)
 + 1)) + 1/2*log(abs(-x + sqrt(x^2 + 2*x + 4) + 2)) - 1/2*log(abs(-x + sqrt
(x^2 + 2*x + 4) - 2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = -\int \frac{1}{(x-x^3)\,\sqrt{x^2+2\,x+4}}\,dx$$

```
[In] int(-1/((x - x^3)*(2*x + x^2 + 4)^(1/2)),x)
[Out] -int(1/((x - x^3)*(2*x + x^2 + 4)^(1/2)), x)
```

3.280 $\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx = \frac{\sqrt{4+2x+x^2}}{1-x}+\operatorname{arcsinh}\left(\frac{1+x}{\sqrt{3}}\right)-\frac{2\operatorname{arctanh}\left(\frac{5+2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)}{\sqrt{7}}$$

```
[Out] arcsinh(1/3*(1+x)*3^(1/2))-2/7*arctanh(1/7*(5+2*x)*7^(1/2)/(x^2+2*x+4)^(1/2
))*7^(1/2)+(x^2+2*x+4)^(1/2)/(1-x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {746, 857, 633, 221, 738, 212}

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx = \operatorname{arcsinh}\left(\frac{x+1}{\sqrt{3}}\right)-\frac{2\operatorname{arctanh}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{\sqrt{7}}+\frac{\sqrt{x^2+2x+4}}{1-x}$$

```
[In] Int[Sqrt[4 + 2*x + x^2]/(-1 + x)^2,x]
```

```
[Out] Sqrt[4 + 2*x + x^2]/(1 - x) + ArcSinh[(1 + x)/Sqrt[3]] - (2*ArcTanh[(5 + 2*
x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])])/Sqrt[7]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{4+2x+x^2}}{1-x} + \frac{1}{2}\int \frac{2+2x}{(-1+x)\sqrt{4+2x+x^2}}\,dx$$

$$= \frac{\sqrt{4+2x+x^2}}{1-x} + 2\int \frac{1}{(-1+x)\sqrt{4+2x+x^2}}\,dx + \int \frac{1}{\sqrt{4+2x+x^2}}\,dx$$

$$= \frac{\sqrt{4+2x+x^2}}{1-x} - 4\text{Subst}\left(\int \frac{1}{28-x^2}\,dx, x, \frac{10+4x}{\sqrt{4+2x+x^2}}\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{12}}}\,dx, x, 2+2x\right)}{2\sqrt{3}}$$

$$= \frac{\sqrt{4+2x+x^2}}{1-x} + \operatorname{arcsinh}\left(\frac{1+x}{\sqrt{3}}\right) - \frac{2\operatorname{arctanh}\left(\frac{5+2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)}{\sqrt{7}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx = -\frac{\sqrt{4+2x+x^2}}{-1+x} - \frac{4\operatorname{arctanh}\left(\frac{1-x+\sqrt{4+2x+x^2}}{\sqrt{7}}\right)}{\sqrt{7}} - \log\left(-1-x+\sqrt{4+2x+x^2}\right)$$

```
[In] Integrate[Sqrt[4 + 2*x + x^2]/(-1 + x)^2,x]
[Out] -(Sqrt[4 + 2*x + x^2]/(-1 + x)) - (4*ArcTanh[(1 - x + Sqrt[4 + 2*x + x^2])/
Sqrt[7]])/Sqrt[7] - Log[-1 - x + Sqrt[4 + 2*x + x^2]]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{\sqrt{x^2+2x+4}}{-1+x}+\operatorname{arcsinh}\left(\frac{(1+x)\sqrt{3}}{3}\right)-\frac{2\sqrt{7}\,\operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{7}$
trager	$-\frac{\sqrt{x^2+2x+4}}{-1+x}+\ln\left(1+x+\sqrt{x^2+2x+4}\right)+\frac{2\operatorname{RootOf}\left(\_Z^2-7\right)\ln\left(-\frac{-2\operatorname{RootOf}\left(\_Z^2-7\right)x+7\sqrt{x^2+2x+4}-5\operatorname{RootOf}\left(\_\right.}{-1+x}\right.}{7}$
default	$-\frac{\left((-1+x)^2+3+4x\right)^{\frac{3}{2}}}{7(-1+x)}+\frac{2\sqrt{(-1+x)^2+3+4x}}{7}+\operatorname{arcsinh}\left(\frac{(1+x)\sqrt{3}}{3}\right)-\frac{2\sqrt{7}\,\operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{7}+\frac{(2x+2)\sqrt{(-}}{}$

```
[In] int((x^2+2*x+4)^(1/2)/(-1+x)^2,x,method=_RETURNVERBOSE)
[Out] -1/(-1+x)*(x^2+2*x+4)^(1/2)+arcsinh(1/3*(1+x)*3^(1/2))-2/7*7^(1/2)*arctanh(
1/14*(10+4*x)*7^(1/2)/((-1+x)^2+3+4*x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx = \frac{2\sqrt{7}(x-1)\log\left(\frac{\sqrt{7}(2x+5)+\sqrt{x^2+2x+4}\left(2\sqrt{7}-7\right)-4x-10}{x-1}\right)-7(x-1)\log\left(-x+\sqrt{x^2+2x+4}-1\right)-7x-7}{7(x-1)}$$

```
[In] integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/7*(2*sqrt(7)*(x - 1)*log((sqrt(7)*(2*x + 5) + sqrt(x^2 + 2*x + 4)*(2*sqrt
(7) - 7) - 4*x - 10)/(x - 1)) - 7*(x - 1)*log(-x + sqrt(x^2 + 2*x + 4) - 1)
 - 7*x - 7*sqrt(x^2 + 2*x + 4) + 7)/(x - 1)
```

Sympy [F]

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx = \int \frac{\sqrt{x^2+2x+4}}{(x-1)^2}\,dx$$

```
[In] integrate((x**2+2*x+4)**(1/2)/(-1+x)**2,x)
```

```
[Out] Integral(sqrt(x**2 + 2*x + 4)/(x - 1)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx = -\frac{2}{7}\sqrt{7}\,\mathrm{arsinh}\left(\frac{2\sqrt{3}x}{3|x-1|}+\frac{5\sqrt{3}}{3|x-1|}\right) - \frac{\sqrt{x^2+2x+4}}{x-1} + \mathrm{arsinh}\left(\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\right)$$

```
[In] integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="maxima")
```

```
[Out] -2/7*sqrt(7)*arcsinh(2/3*sqrt(3)*x/abs(x - 1) + 5/3*sqrt(3)/abs(x - 1)) - s
qrt(x^2 + 2*x + 4)/(x - 1) + arcsinh(1/3*sqrt(3)*x + 1/3*sqrt(3))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(53) = 106$.

Time = 0.35 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.40

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx = -\frac{2}{7}\sqrt{7}\log\left(\sqrt{7}\left(\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}+\frac{\sqrt{7}}{x-1}\right)+2\right)\operatorname{sgn}\left(\frac{1}{x-1}\right)+\log\left(\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}+\frac{\sqrt{7}}{x-1}+1\right)\operatorname{sgn}\left(\frac{1}{x-1}\right)-\log\left(\left|\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}+\frac{\sqrt{7}}{x-1}-1\right|\right)\operatorname{sgn}\left(\frac{1}{x-1}\right)-\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}\operatorname{sgn}\left(\frac{1}{x-1}\right)$$

```
[In] integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="giac")
```

```
[Out] -2/7*sqrt(7)*log(sqrt(7)*(sqrt(4/(x - 1) + 7/(x - 1)^2 + 1) + sqrt(7)/(x -
1)) + 2)*sgn(1/(x - 1)) + log(sqrt(4/(x - 1) + 7/(x - 1)^2 + 1) + sqrt(7)/(
x - 1) + 1)*sgn(1/(x - 1)) - log(abs(sqrt(4/(x - 1) + 7/(x - 1)^2 + 1) + sq
rt(7)/(x - 1) - 1))*sgn(1/(x - 1)) - sqrt(4/(x - 1) + 7/(x - 1)^2 + 1)*sgn(
1/(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx = \int \frac{\sqrt{x^2+2\,x+4}}{(x-1)^2}\,dx$$

```
[In] int((2*x + x^2 + 4)^(1/2)/(x - 1)^2,x)
```

```
[Out] int((2*x + x^2 + 4)^(1/2)/(x - 1)^2, x)
```

3.281 $\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 76

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = -\frac{(3-x)\sqrt{4+2x+x^2}}{4(3+2x+x^2)} - \frac{\arctan\left(\frac{1+x}{\sqrt{2}\sqrt{4+2x+x^2}}\right)}{4\sqrt{2}} + \operatorname{arctanh}\left(\sqrt{4+2x+x^2}\right)$$

```
[Out] arctanh((x^2+2*x+4)^(1/2))-1/8*arctan(1/2*(1+x)*2^(1/2)/(x^2+2*x+4)^(1/2))*
2^(1/2)-1/4*(3-x)*(x^2+2*x+4)^(1/2)/(x^2+2*x+3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1030, 1039, 996, 210, 1038, 212}

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = -\frac{\arctan\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+2x+4}}\right)}{4\sqrt{2}} + \operatorname{arctanh}\left(\sqrt{x^2+2x+4}\right) - \frac{\sqrt{x^2+2x+4}(3-x)}{4(x^2+2x+3)}$$

```
[In] Int[(3 + 2*x)/((3 + 2*x + x^2)^2*Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] -1/4*((3 - x)*Sqrt[4 + 2*x + x^2])/(3 + 2*x + x^2) - ArcTan[(1 + x)/(Sqrt[2
]*Sqrt[4 + 2*x + x^2])]/(4*Sqrt[2]) + ArcTanh[Sqrt[4 + 2*x + x^2]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1030

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c*
e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1038

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
```

```
EqQ[h*e - 2*g*f, 0]
```

Rule 1039

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(3-x)\sqrt{4+2x+x^2}}{4(3+2x+x^2)} + \frac{1}{8}\int \frac{-10-8x}{(3+2x+x^2)\sqrt{4+2x+x^2}}\,dx \\
&= -\frac{(3-x)\sqrt{4+2x+x^2}}{4(3+2x+x^2)} - \frac{1}{4}\int \frac{1}{(3+2x+x^2)\sqrt{4+2x+x^2}}\,dx \\
&\quad - \frac{1}{2}\int \frac{2+2x}{(3+2x+x^2)\sqrt{4+2x+x^2}}\,dx \\
&= -\frac{(3-x)\sqrt{4+2x+x^2}}{4(3+2x+x^2)} + 2\text{Subst}\left(\int \frac{1}{2-2x^2}\,dx, x, \sqrt{4+2x+x^2}\right) \\
&\quad + \text{Subst}\left(\int \frac{1}{-16-2x^2}\,dx, x, \frac{2+2x}{\sqrt{4+2x+x^2}}\right) \\
&= -\frac{(3-x)\sqrt{4+2x+x^2}}{4(3+2x+x^2)} - \frac{\arctan\left(\frac{2+2x}{2\sqrt{2}\sqrt{4+2x+x^2}}\right)}{4\sqrt{2}} + \text{arctanh}\left(\sqrt{4+2x+x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = \frac{1}{8}\left(\frac{2(-3+x)\sqrt{4+2x+x^2}}{3+2x+x^2} + \sqrt{2}\arctan\left(\frac{3+2x+x^2-(1+x)\sqrt{4+2x+x^2}}{\sqrt{2}}\right)\right) + \text{arctanh}\left(\sqrt{4+2x+x^2}\right)$$

```
[In] Integrate[(3 + 2*x)/((3 + 2*x + x^2)^2*Sqrt[4 + 2*x + x^2]),x]
[Out] ((2*(-3 + x)*Sqrt[4 + 2*x + x^2])/(3 + 2*x + x^2) + Sqrt[2]*ArcTan[(3 + 2*x
 + x^2 - (1 + x)*Sqrt[4 + 2*x + x^2])/Sqrt[2]])/8 + ArcTanh[Sqrt[4 + 2*x +
x^2]]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

method	result
risch	$\frac{(-3+x)\sqrt{x^2+2x+4}}{4x^2+8x+12}+\operatorname{arctanh}\left(\sqrt{x^2+2x+4}\right)-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(2x+2)}{4\sqrt{x^2+2x+4}}\right)}{8}$
default	$-\frac{1}{2\left(\sqrt{x^2+2x+4}+1\right)}+\frac{\ln\left(\sqrt{x^2+2x+4}+1\right)}{2}-\frac{1}{2\left(\sqrt{x^2+2x+4}-1\right)}-\frac{\ln\left(\sqrt{x^2+2x+4}-1\right)}{2}+\frac{\frac{3}{4}+\frac{3x}{4}}{\sqrt{x^2+2x+4}\left(\frac{(1+x)^2}{x^2+2x+4}+2\right)}-$ arct
trager	$\frac{(-3+x)\sqrt{x^2+2x+4}}{4x^2+8x+12}-3\ln\left(-\frac{48384\,\mathrm{RootOf}\left(384\_Z^2+128\_Z+11\right)^2 x+960\sqrt{x^2+2x+4}\,\mathrm{RootOf}\left(384\_Z^2+128\_Z+11\right)+153}{48\,\mathrm{RootOf}\left(384\_\right.}\right.$

[In] int((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4*(-3+x)/(x^2+2*x+3)*(x^2+2*x+4)^(1/2)+arctanh((x^2+2*x+4)^(1/2))-1/8*2^(1/2)*arctan(1/4*2^(1/2)/(x^2+2*x+4)^(1/2)*(2*x+2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(61) = 122.

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.29

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = \frac{\sqrt{2}(x^2+2\,x+3)\arctan\left(-\frac{1}{2}\sqrt{2}(x+2)+\frac{1}{2}\sqrt{2}\sqrt{x^2+2\,x+4}\right)-\sqrt{2}(x^2+2\,x+3)\arctan\left(-\frac{1}{2}\sqrt{2}x+\frac{1}{2}\sqrt{\right.}}{}$$

[In] integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="fricas")

[Out] 1/8*(sqrt(2)*(x^2 + 2*x + 3)*arctan(-1/2*sqrt(2)*(x + 2) + 1/2*sqrt(2)*sqrt(x^2 + 2*x + 4)) - sqrt(2)*(x^2 + 2*x + 3)*arctan(-1/2*sqrt(2)*x + 1/2*sqrt(2)*sqrt(x^2 + 2*x + 4)) + 2*x^2 - 4*(x^2 + 2*x + 3)*log(x^2 - sqrt(x^2 + 2*x + 4)*(x + 2) + 3*x + 5) + 4*(x^2 + 2*x + 3)*log(x^2 - sqrt(x^2 + 2*x + 4)*x + x + 3) + 2*sqrt(x^2 + 2*x + 4)*(x - 3) + 4*x + 6)/(x^2 + 2*x + 3)

Sympy [F]

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = \int \frac{2x+3}{(x^2+2x+3)^2\sqrt{x^2+2x+4}}\,dx$$

```
[In] integrate((3+2*x)/(x**2+2*x+3)**2/(x**2+2*x+4)**(1/2),x)
[Out] Integral((2*x + 3)/((x**2 + 2*x + 3)**2*sqrt(x**2 + 2*x + 4)), x)
```

Maxima [F]

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = \int \frac{2\,x+3}{\sqrt{x^2+2\,x+4}(x^2+2\,x+3)^2}\,dx$$

```
[In] integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x + 3)/(sqrt(x^2 + 2*x + 4)*(x^2 + 2*x + 3)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(61) = 122.

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.09

$$\begin{aligned}\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx &= \frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2+2\,x+4}+2\right)\right) \\ &- \frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2+2\,x+4}\right)\right) \\ &+ \frac{4\left(x-\sqrt{x^2+2\,x+4}\right)^3+13\left(x-\sqrt{x^2+2\,x+4}\right)^2+26\,x-26\sqrt{x^2+2\,x+4}+26}{2\left(\left(x-\sqrt{x^2+2\,x+4}\right)^4+4\left(x-\sqrt{x^2+2\,x+4}\right)^3+8\left(x-\sqrt{x^2+2\,x+4}\right)^2+8\,x-8\sqrt{x^2+2\,x+4}\right.} \\ &- \frac{1}{2}\log\left(\left(x-\sqrt{x^2+2\,x+4}\right)^2+4\,x-4\sqrt{x^2+2\,x+4}+6\right) \\ &+ \frac{1}{2}\log\left(\left(x-\sqrt{x^2+2\,x+4}\right)^2+2\right)\end{aligned}$$

```
[In] integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="giac")
[Out] 1/8*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 2*x + 4) + 2)) - 1/8*sqrt(2
)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 2*x + 4))) + 1/2*(4*(x - sqrt(x^2 + 2
*x + 4))^3 + 13*(x - sqrt(x^2 + 2*x + 4))^2 + 26*x - 26*sqrt(x^2 + 2*x + 4)
 + 26)/((x - sqrt(x^2 + 2*x + 4))^4 + 4*(x - sqrt(x^2 + 2*x + 4))^3 + 8*(x
- sqrt(x^2 + 2*x + 4))^2 + 8*x - 8*sqrt(x^2 + 2*x + 4) + 12) - 1/2*log((x -
 sqrt(x^2 + 2*x + 4))^2 + 4*x - 4*sqrt(x^2 + 2*x + 4) + 6) + 1/2*log((x - s
qrt(x^2 + 2*x + 4))^2 + 2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = \int \frac{2\,x+3}{\left(x^2+2\,x+3\right)^2\,\sqrt{x^2+2\,x+4}}\,dx$$

```
[In] int((2*x + 3)/((2*x + x^2 + 3)^2*(2*x + x^2 + 4)^(1/2)),x)
[Out] int((2*x + 3)/((2*x + x^2 + 3)^2*(2*x + x^2 + 4)^(1/2)), x)
```

3.282 $\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\left(-3+x+2x^2\right)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 36

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \sqrt{-3+2x+x^2}+\frac{\sqrt{-3+2x+x^2}}{2(1-x)}$$

```
[Out] (x^2+2*x-3)^(1/2)+1/2*(x^2+2*x-3)^(1/2)/(1-x)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1607, 1600, 1652, 664}

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \frac{\sqrt{x^2+2x-3}}{2(1-x)}+\sqrt{x^2+2x-3}$$

```
[In] Int[(3*x^2 + 2*x^3)/(Sqrt[-3 + 2*x + x^2]*(-3 + x + 2*x^2)),x]
```

```
[Out] Sqrt[-3 + 2*x + x^2] + Sqrt[-3 + 2*x + x^2]/(2*(1 - x))
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1652

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d - b
*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2(3+2x)}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx\\
&= \int \frac{x^2}{(-1+x)\sqrt{-3+2x+x^2}}\,dx\\
&= \sqrt{-3+2x+x^2}+\int \frac{1}{(-1+x)\sqrt{-3+2x+x^2}}\,dx\\
&= \sqrt{-3+2x+x^2}+\frac{\sqrt{-3+2x+x^2}}{2(1-x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx=\frac{(-3+2x)\sqrt{-3+2x+x^2}}{2(-1+x)}$$

```
[In] Integrate[(3*x^2 + 2*x^3)/(Sqrt[-3 + 2*x + x^2]*(-3 + x + 2*x^2)),x]
[Out] ((-3 + 2*x)*Sqrt[-3 + 2*x + x^2])/(2*(-1 + x))
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{(2x-3)(3+x)}{2\sqrt{x^2+2x-3}}$	21
trager	$\frac{(2x-3)\sqrt{x^2+2x-3}}{-2+2x}$	23
risch	$\frac{2x^2+3x-9}{2\sqrt{x^2+2x-3}}$	23
default	$\sqrt{x^2+2x-3}-\frac{\sqrt{(-1+x)^2-4+4x}}{2(-1+x)}$	31

```
[In] int((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(2*x-3)*(3+x)/(x^2+2*x-3)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \frac{\sqrt{x^2+2\,x-3}(2\,x-3)}{2\,(x-1)}$$

```
[In] integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="fricas"
)
[Out] 1/2*sqrt(x^2 + 2*x - 3)*(2*x - 3)/(x - 1)
```

Sympy [F]

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \int \frac{x^2}{\sqrt{(x-1)\,(x+3)}\,(x-1)}\,dx$$

```
[In] integrate((2*x**3+3*x**2)/(2*x**2+x-3)/(x**2+2*x-3)**(1/2),x)
[Out] Integral(x**2/(sqrt((x - 1)*(x + 3))*(x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \sqrt{x^2+2\,x-3}-\frac{\sqrt{x^2+2\,x-3}}{2\,(x-1)}$$

```
[In] integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="maxima"
)
```

```
[Out] sqrt(x^2 + 2*x - 3) - 1/2*sqrt(x^2 + 2*x - 3)/(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \sqrt{x^2+2\,x-3}+\frac{2}{x-\sqrt{x^2+2\,x-3}-1}$$

```
[In] integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 2*x - 3) + 2/(x - sqrt(x^2 + 2*x - 3) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \frac{\left(x-\frac{3}{2}\right)\,\sqrt{x^2+2\,x-3}}{x-1}$$

```
[In] int((3*x^2 + 2*x^3)/((x + 2*x^2 - 3)*(2*x + x^2 - 3)^(1/2)),x)
```

```
[Out] ((x - 3/2)*(2*x + x^2 - 3)^(1/2))/(x - 1)
```

3.283 $\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}} \, dx = -\frac{7}{4}\sqrt{2+x+x^2} + \frac{1}{2}x\sqrt{2+x+x^2} - \frac{1}{8}\text{arcsinh}\left(\frac{1+2x}{\sqrt{7}}\right) + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}\sqrt{2+x+x^2}}\right)}{\sqrt{3}} - \text{arctanh}\left(\sqrt{2+x+x^2}\right)$$

```
[Out] -1/8*arcsinh(1/7*(1+2*x)*7^(1/2))-arctanh((x^2+x+2)^(1/2))+1/3*arctan(1/3*(
1+2*x)*3^(1/2)/(x^2+x+2)^(1/2))*3^(1/2)-7/4*(x^2+x+2)^(1/2)+1/2*x*(x^2+x+2)
^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6860, 654, 633, 221, 756, 1039, 996, 210, 1038, 212}

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}} \, dx = -\frac{1}{8}\text{arcsinh}\left(\frac{2x+1}{\sqrt{7}}\right) + \frac{\arctan\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+x+2}}\right)}{\sqrt{3}} - \text{arctanh}\left(\sqrt{x^2+x+2}\right) + \frac{1}{2}\sqrt{x^2+x+2}x - \frac{7}{4}\sqrt{x^2+x+2}$$

```
[In] Int[(1 + x^4)/((1 + x + x^2)*Sqrt[2 + x + x^2]),x]
```

```
[Out] (-7*Sqrt[2 + x + x^2])/4 + (x*Sqrt[2 + x + x^2])/2 - ArcSinh[(1 + 2*x)/Sqrt
[7]]/8 + ArcTan[(1 + 2*x)/(Sqrt[3]*Sqrt[2 + x + x^2])]/Sqrt[3] - ArcTanh[Sq
rt[2 + x + x^2]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
```

```
]
```

Rule 1038

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1039

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{x}{\sqrt{2+x+x^2}}+\frac{x^2}{\sqrt{2+x+x^2}}+\frac{1+x}{(1+x+x^2)\sqrt{2+x+x^2}}\right)\,dx\\
&= -\int \frac{x}{\sqrt{2+x+x^2}}\,dx+\int \frac{x^2}{\sqrt{2+x+x^2}}\,dx+\int \frac{1+x}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx\\
&= -\sqrt{2+x+x^2}+\frac{1}{2}x\sqrt{2+x+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{2+x+x^2}}\,dx+\frac{1}{2}\int \frac{-2-\frac{3x}{2}}{\sqrt{2+x+x^2}}\,dx\\
&\quad +\frac{1}{2}\int \frac{1}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx+\frac{1}{2}\int \frac{1+2x}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx\\
&= -\frac{7}{4}\sqrt{2+x+x^2}+\frac{1}{2}x\sqrt{2+x+x^2}-\frac{5}{8}\int \frac{1}{\sqrt{2+x+x^2}}\,dx+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{7}}}\,dx,x,1+2x\right)}{2\sqrt{7}}\\
&\quad -\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,\frac{1+2x}{\sqrt{2+x+x^2}}\right)-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{2+x+x^2}\right)
\end{aligned}
$$

$$= -\frac{7}{4}\sqrt{2+x+x^2}+\frac{1}{2}x\sqrt{2+x+x^2}+\frac{1}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{7}}\right)+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}\sqrt{2+x+x^2}}\right)}{\sqrt{3}} - \text{arctanh}\left(\sqrt{2+x+x^2}\right)-\frac{5\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{7}}}dx,x,1+2x\right)}{8\sqrt{7}}$$

$$= -\frac{7}{4}\sqrt{2+x+x^2}+\frac{1}{2}x\sqrt{2+x+x^2}-\frac{1}{8}\text{arcsinh}\left(\frac{1+2x}{\sqrt{7}}\right) + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}\sqrt{2+x+x^2}}\right)}{\sqrt{3}}-\text{arctanh}\left(\sqrt{2+x+x^2}\right)$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int\frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = -\frac{\arctan\left(\frac{2+2x+2x^2-(1+2x)\sqrt{2+x+x^2}}{\sqrt{3}}\right)}{\sqrt{3}} - \text{arctanh}\left(\sqrt{2+x+x^2}\right)+\frac{1}{8}\left(2(-7+2x)\sqrt{2+x+x^2} + \log\left(-1-2x+2\sqrt{2+x+x^2}\right)\right)$$

```
[In] Integrate[(1 + x^4)/((1 + x + x^2)*Sqrt[2 + x + x^2]),x]
[Out] -(ArcTan[(2 + 2*x + 2*x^2 - (1 + 2*x)*Sqrt[2 + x + x^2])/Sqrt[3]]/Sqrt[3])
- ArcTanh[Sqrt[2 + x + x^2]] + (2*(-7 + 2*x)*Sqrt[2 + x + x^2] + Log[-1 - 2
*x + 2*Sqrt[2 + x + x^2]])/8
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{(2x-7)\sqrt{x^2+x+2}}{4}-\frac{\text{arcsinh}\left(\frac{2\sqrt{7}\left(x+\frac{1}{2}\right)}{7}\right)}{8}-\text{arctanh}\left(\sqrt{x^2+x+2}\right)+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3\sqrt{x^2+x+2}}\right)\sqrt{3}}{3}$	63
default	$\frac{x\sqrt{x^2+x+2}}{2}-\frac{7\sqrt{x^2+x+2}}{4}-\frac{\text{arcsinh}\left(\frac{2\sqrt{7}\left(x+\frac{1}{2}\right)}{7}\right)}{8}-\text{arctanh}\left(\sqrt{x^2+x+2}\right)+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3\sqrt{x^2+x+2}}\right)\sqrt{3}}{3}$	69
trager	Expression too large to display	1101

```
[In] int((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*x-7)*(x^2+x+2)^(1/2)-1/8*arcsinh(2/7*7^(1/2)*(x+1/2))-arctanh((x^2+x
+2)^(1/2))+1/3*arctan(1/3*(1+2*x)*3^(1/2)/(x^2+x+2)^(1/2))*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(70) = 140$.

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.69

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = \frac{1}{4}\sqrt{x^2+x+2}(2x-7) - \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(2x+3)+\frac{2}{3}\sqrt{3}\sqrt{x^2+x+2}\right) + \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(2x-1)+\frac{2}{3}\sqrt{3}\sqrt{x^2+x+2}\right) + \frac{1}{2}\log\left(2x^2-\sqrt{x^2+x+2}(2x+3)+4x+5\right) - \frac{1}{2}\log\left(2x^2-\sqrt{x^2+x+2}(2x-1)+3\right) + \frac{1}{8}\log\left(-2x+2\sqrt{x^2+x+2}-1\right)$$

```
[In] integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^2 + x + 2)*(2*x - 7) - 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x + 3)
 + 2/3*sqrt(3)*sqrt(x^2 + x + 2)) + 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x -
1) + 2/3*sqrt(3)*sqrt(x^2 + x + 2)) + 1/2*log(2*x^2 - sqrt(x^2 + x + 2)*(2*
x + 3) + 4*x + 5) - 1/2*log(2*x^2 - sqrt(x^2 + x + 2)*(2*x - 1) + 3) + 1/8*
log(-2*x + 2*sqrt(x^2 + x + 2) - 1)
```

Sympy [F]

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = \int \frac{x^4+1}{(x^2+x+1)\sqrt{x^2+x+2}}\,dx$$

```
[In] integrate((x**4+1)/(x**2+x+1)/(x**2+x+2)**(1/2),x)
```

```
[Out] Integral((x**4 + 1)/((x**2 + x + 1)*sqrt(x**2 + x + 2)), x)
```

Maxima [F]

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = \int \frac{x^4+1}{\sqrt{x^2+x+2}(x^2+x+1)}\,dx$$

```
[In] integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="maxima")
[Out] integrate((x^4 + 1)/(sqrt(x^2 + x + 2)*(x^2 + x + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(70) = 140$.

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.70

$$\begin{aligned}\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx &= \frac{1}{4}\sqrt{x^2+x+2}(2x-7) \\ &\quad - \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(2x-2\sqrt{x^2+x+2}+3\right)\right) \\ &\quad + \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(2x-2\sqrt{x^2+x+2}-1\right)\right) \\ &\quad + \frac{1}{2}\log\left(\left(x-\sqrt{x^2+x+2}\right)^2+3x-3\sqrt{x^2+x+2}+3\right) \\ &\quad - \frac{1}{2}\log\left(\left(x-\sqrt{x^2+x+2}\right)^2-x+\sqrt{x^2+x+2}+1\right) \\ &\quad + \frac{1}{8}\log\left(-2x+2\sqrt{x^2+x+2}-1\right)\end{aligned}$$

```
[In] integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(x^2 + x + 2)*(2*x - 7) - 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - 2*
sqrt(x^2 + x + 2) + 3)) + 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - 2*sqrt(x^2
 + x + 2) - 1)) + 1/2*log((x - sqrt(x^2 + x + 2))^2 + 3*x - 3*sqrt(x^2 + x
+ 2) + 3) - 1/2*log((x - sqrt(x^2 + x + 2))^2 - x + sqrt(x^2 + x + 2) + 1)
+ 1/8*log(-2*x + 2*sqrt(x^2 + x + 2) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = \int \frac{x^4+1}{(x^2+x+1)\,\sqrt{x^2+x+2}}\,dx$$

```
[In] int((x^4 + 1)/((x + x^2 + 1)*(x + x^2 + 2)^(1/2)),x)
[Out] int((x^4 + 1)/((x + x^2 + 1)*(x + x^2 + 2)^(1/2)), x)
```

3.284 $\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx=\frac{1+x}{15\,(4+2x+x^2)^{5/2}}+\frac{4(1+x)}{135\,(4+2x+x^2)^{3/2}}+\frac{8(1+x)}{405\sqrt{4+2x+x^2}}$$

```
[Out] 1/15*(1+x)/(x^2+2*x+4)^(5/2)+4/135*(1+x)/(x^2+2*x+4)^(3/2)+8/405*(1+x)/(x^2
+2*x+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {628, 627}

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx=\frac{8(x+1)}{405\sqrt{x^2+2x+4}}+\frac{4(x+1)}{135\,(x^2+2x+4)^{3/2}}+\frac{x+1}{15\,(x^2+2x+4)^{5/2}}$$

```
[In] Int[(4 + 2*x + x^2)^(-7/2),x]
```

```
[Out] (1 + x)/(15*(4 + 2*x + x^2)^(5/2)) + (4*(1 + x))/(135*(4 + 2*x + x^2)^(3/2)
) + (8*(1 + x))/(405*Sqrt[4 + 2*x + x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1+x}{15\left(4+2x+x^2\right)^{5/2}}+\frac{4}{15}\int\frac{1}{\left(4+2x+x^2\right)^{5/2}}\,dx\\&=\frac{1+x}{15\left(4+2x+x^2\right)^{5/2}}+\frac{4(1+x)}{135\left(4+2x+x^2\right)^{3/2}}+\frac{8}{135}\int\frac{1}{\left(4+2x+x^2\right)^{3/2}}\,dx\\&=\frac{1+x}{15\left(4+2x+x^2\right)^{5/2}}+\frac{4(1+x)}{135\left(4+2x+x^2\right)^{3/2}}+\frac{8(1+x)}{405\sqrt{4+2x+x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int\frac{1}{\left(4+2x+x^2\right)^{7/2}}\,dx=\frac{(1+x)\left(203+152x+108x^2+32x^3+8x^4\right)}{405\left(4+2x+x^2\right)^{5/2}}$$

```
[In] Integrate[(4 + 2*x + x^2)^(-7/2),x]
[Out] ((1 + x)*(203 + 152*x + 108*x^2 + 32*x^3 + 8*x^4))/(405*(4 + 2*x + x^2)^(5/
2))
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

method	result	size
gosper	$\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405(x^2+2x+4)^{\frac{5}{2}}}$	38
trager	$\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405(x^2+2x+4)^{\frac{5}{2}}}$	38
risch	$\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405(x^2+2x+4)^{\frac{5}{2}}}$	38
default	$\frac{2x+2}{30(x^2+2x+4)^{\frac{5}{2}}}+\frac{\frac{4}{135}+\frac{4x}{135}}{(x^2+2x+4)^{\frac{3}{2}}}+\frac{\frac{8}{405}+\frac{8x}{405}}{\sqrt{x^2+2x+4}}$	53

```
[In] int(1/(x^2+2*x+4)^(7/2),x,method=_RETURNVERBOSE)
```

[Out] 1/405*(8*x^5+40*x^4+140*x^3+260*x^2+355*x+203)/(x^2+2*x+4)^(5/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(46) = 92$.

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.69

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx = \frac{8\,x^6+48\,x^5+192\,x^4+448\,x^3+768\,x^2+(8\,x^5+40\,x^4+140\,x^3+260\,x^2+355\,x+}{405\,(x^6+6\,x^5+24\,x^4+56\,x^3+96\,x^2+96\,x+64}$$

[In] integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="fricas")

[Out] 1/405*(8*x^6 + 48*x^5 + 192*x^4 + 448*x^3 + 768*x^2 + (8*x^5 + 40*x^4 + 140
*x^3 + 260*x^2 + 355*x + 203)*sqrt(x^2 + 2*x + 4) + 768*x + 512)/(x^6 + 6*x
^5 + 24*x^4 + 56*x^3 + 96*x^2 + 96*x + 64)

Sympy [F]

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx = \int \frac{1}{(x^2+2x+4)^{\frac{7}{2}}}\,dx$$

[In] integrate(1/(x**2+2*x+4)**(7/2),x)

[Out] Integral((x**2 + 2*x + 4)**(-7/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx = \frac{8\,x}{405\,\sqrt{x^2+2\,x+4}} + \frac{8}{405\,\sqrt{x^2+2\,x+4}} + \frac{4\,x}{135\,(x^2+2\,x+4)^{\frac{3}{2}}} + \frac{4}{135\,(x^2+2\,x+4)^{\frac{3}{2}}} + \frac{x}{15\,(x^2+2\,x+4)^{\frac{5}{2}}} + \frac{1}{15\,(x^2+2\,x+4)^{\frac{5}{2}}}$$

[In] integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="maxima")

[Out] 8/405*x/sqrt(x^2 + 2*x + 4) + 8/405/sqrt(x^2 + 2*x + 4) + 4/135*x/(x^2 + 2*
x + 4)^(3/2) + 4/135/(x^2 + 2*x + 4)^(3/2) + 1/15*x/(x^2 + 2*x + 4)^(5/2) +
 1/15/(x^2 + 2*x + 4)^(5/2)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.57

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx = \frac{(4\,((2\,(x+5)x+35)x+65)x+355)x+203}{405\,(x^2+2\,x+4)^{\frac{5}{2}}}$$

```
[In] integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="giac")
[Out] 1/405*((4*((2*(x + 5)*x + 35)*x + 65)*x + 355)*x + 203)/(x^2 + 2*x + 4)^(5/
2)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx = \frac{51\,x+8\,x\,(x^2+2\,x+4)^2+8\,(x^2+2\,x+4)^2+12\,x^2+12\,x\,(x^2+2\,x+4)+75}{(x^2+2\,x+4)^{3/2}\,(405\,x^2+810\,x+1620)}$$

```
[In] int(1/(2*x + x^2 + 4)^(7/2),x)
[Out] (51*x + 8*x*(2*x + x^2 + 4)^2 + 8*(2*x + x^2 + 4)^2 + 12*x^2 + 12*x*(2*x +
x^2 + 4) + 75)/((2*x + x^2 + 4)^(3/2)*(810*x + 405*x^2 + 1620))
```

3.285 $\int \frac{1}{(1+8x+3x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{1}{(1+8x+3x^2)^{5/2}} \, dx = -\frac{4+3x}{39(1+8x+3x^2)^{3/2}} + \frac{2(4+3x)}{169\sqrt{1+8x+3x^2}}$$

```
[Out] 1/39*(-4-3*x)/(3*x^2+8*x+1)^(3/2)+2/169*(4+3*x)/(3*x^2+8*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {628, 627}

$$\int \frac{1}{(1+8x+3x^2)^{5/2}} \, dx = \frac{2(3x+4)}{169\sqrt{3x^2+8x+1}} - \frac{3x+4}{39(3x^2+8x+1)^{3/2}}$$

```
[In] Int[(1 + 8*x + 3*x^2)^(-5/2),x]
[Out] -1/39*(4 + 3*x)/(1 + 8*x + 3*x^2)^(3/2) + (2*(4 + 3*x))/(169*Sqrt[1 + 8*x +
 3*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
```

```
 3)/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{4+3x}{39\left(1+8x+3x^2\right)^{3/2}}-\frac{2}{13}\int\frac{1}{\left(1+8x+3x^2\right)^{3/2}}\,dx\\ &= -\frac{4+3x}{39\left(1+8x+3x^2\right)^{3/2}}+\frac{2(4+3x)}{169\sqrt{1+8x+3x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int\frac{1}{\left(1+8x+3x^2\right)^{5/2}}\,dx=\frac{(4+3x)\left(-7+48x+18x^2\right)}{507\left(1+8x+3x^2\right)^{3/2}}$$

```
[In] Integrate[(1 + 8*x + 3*x^2)^(-5/2),x]
[Out] ((4 + 3*x)*(-7 + 48*x + 18*x^2))/(507*(1 + 8*x + 3*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{54x^3+216x^2+171x-28}{507(3x^2+8x+1)^{\frac{3}{2}}}$	30
trager	$\frac{54x^3+216x^2+171x-28}{507(3x^2+8x+1)^{\frac{3}{2}}}$	30
risch	$\frac{54x^3+216x^2+171x-28}{507(3x^2+8x+1)^{\frac{3}{2}}}$	30
default	$-\frac{6x+8}{78(3x^2+8x+1)^{\frac{3}{2}}}+\frac{6x+8}{169\sqrt{3x^2+8x+1}}$	40

```
[In] int(1/(3*x^2+8*x+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/507*(54*x^3+216*x^2+171*x-28)/(3*x^2+8*x+1)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = -\frac{252\,x^4+1344\,x^3+1960\,x^2-(54\,x^3+216\,x^2+171\,x-28)\sqrt{3\,x^2+8\,x+1}+448\,x+28}{507\,(9\,x^4+48\,x^3+70\,x^2+16\,x+1)}$$

```
[In] integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="fricas")
[Out] -1/507*(252*x^4 + 1344*x^3 + 1960*x^2 - (54*x^3 + 216*x^2 + 171*x - 28)*sqr
t(3*x^2 + 8*x + 1) + 448*x + 28)/(9*x^4 + 48*x^3 + 70*x^2 + 16*x + 1)
```

Sympy [F]

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = \int \frac{1}{(3x^2+8x+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(3*x**2+8*x+1)**(5/2),x)
[Out] Integral((3*x**2 + 8*x + 1)**(-5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = \frac{6\,x}{169\,\sqrt{3\,x^2+8\,x+1}} + \frac{8}{169\,\sqrt{3\,x^2+8\,x+1}} - \frac{x}{13\,(3\,x^2+8\,x+1)^{\frac{3}{2}}} - \frac{4}{39\,(3\,x^2+8\,x+1)^{\frac{3}{2}}}$$

```
[In] integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="maxima")
[Out] 6/169*x/sqrt(3*x^2 + 8*x + 1) + 8/169/sqrt(3*x^2 + 8*x + 1) - 1/13*x/(3*x^2
 + 8*x + 1)^(3/2) - 4/39/(3*x^2 + 8*x + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.57

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = \frac{9\,(6\,(x+4)x+19)x-28}{507\,(3\,x^2+8\,x+1)^{\frac{3}{2}}}$$

```
[In] integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="giac")
[Out] 1/507*(9*(6*(x + 4)*x + 19)*x - 28)/(3*x^2 + 8*x + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = \frac{(12\,x+16)\,\left(72\,x^2+192\,x-28\right)}{8112\,(3\,x^2+8\,x+1)^{3/2}}$$

```
[In] int(1/(8*x + 3*x^2 + 1)^(5/2),x)
[Out] ((12*x + 16)*(192*x + 72*x^2 - 28))/(8112*(8*x + 3*x^2 + 1)^(3/2))
```

3.286 $\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = -\frac{2-3x}{57\,(5+4x-3x^2)^{3/2}} - \frac{2(2-3x)}{361\sqrt{5+4x-3x^2}}$$

```
[Out] 1/57*(-2+3*x)/(-3*x^2+4*x+5)^(3/2)-2/361*(2-3*x)/(-3*x^2+4*x+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {628, 627}

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = -\frac{2(2-3x)}{361\sqrt{-3x^2+4x+5}} - \frac{2-3x}{57\,(-3x^2+4x+5)^{3/2}}$$

```
[In] Int[(5 + 4*x - 3*x^2)^(-5/2),x]
```

```
[Out] -1/57*(2 - 3*x)/(5 + 4*x - 3*x^2)^(3/2) - (2*(2 - 3*x))/(361*Sqrt[5 + 4*x -
 3*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
```

```
3)/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2-3x}{57\left(5+4x-3x^2\right)^{3/2}}+\frac{2}{19}\int\frac{1}{\left(5+4x-3x^2\right)^{3/2}}\,dx\\ &= -\frac{2-3x}{57\left(5+4x-3x^2\right)^{3/2}}-\frac{2(2-3x)}{361\sqrt{5+4x-3x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int\frac{1}{\left(5+4x-3x^2\right)^{5/2}}\,dx=\frac{-98+99x+108x^2-54x^3}{1083\left(5+4x-3x^2\right)^{3/2}}$$

```
[In] Integrate[(5 + 4*x - 3*x^2)^(-5/2),x]
[Out] (-98 + 99*x + 108*x^2 - 54*x^3)/(1083*(5 + 4*x - 3*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{54x^3-108x^2-99x+98}{1083\left(-3x^2+4x+5\right)^{\frac{3}{2}}}$	30
default	$-\frac{-6x+4}{114\left(-3x^2+4x+5\right)^{\frac{3}{2}}}-\frac{-6x+4}{361\sqrt{-3x^2+4x+5}}$	40
trager	$-\frac{\left(54x^3-108x^2-99x+98\right)\sqrt{-3x^2+4x+5}}{1083\left(3x^2-4x-5\right)^2}$	42
risch	$\frac{54x^3-108x^2-99x+98}{1083\left(3x^2-4x-5\right)\sqrt{-3x^2+4x+5}}$	42

```
[In] int(1/(-3*x^2+4*x+5)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/1083/(-3*x^2+4*x+5)^(3/2)*(54*x^3-108*x^2-99*x+98)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = -\frac{(54\,x^3-108\,x^2-99\,x+98)\sqrt{-3\,x^2+4\,x+5}}{1083\,(9\,x^4-24\,x^3-14\,x^2+40\,x+25)}$$

```
[In] integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/1083*(54*x^3 - 108*x^2 - 99*x + 98)*sqrt(-3*x^2 + 4*x + 5)/(9*x^4 - 24*x
^3 - 14*x^2 + 40*x + 25)
```

Sympy [F]

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = \int \frac{1}{(-3x^2+4x+5)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(-3*x**2+4*x+5)**(5/2),x)
```

```
[Out] Integral((-3*x**2 + 4*x + 5)**(-5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = \frac{6\,x}{361\,\sqrt{-3\,x^2+4\,x+5}} - \frac{4}{361\,\sqrt{-3\,x^2+4\,x+5}} + \frac{x}{19\,(-3\,x^2+4\,x+5)^{\frac{3}{2}}} - \frac{2}{57\,(-3\,x^2+4\,x+5)^{\frac{3}{2}}}$$

```
[In] integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="maxima")
```

```
[Out] 6/361*x/sqrt(-3*x^2 + 4*x + 5) - 4/361/sqrt(-3*x^2 + 4*x + 5) + 1/19*x/(-3*
x^2 + 4*x + 5)^(3/2) - 2/57/(-3*x^2 + 4*x + 5)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = -\frac{(9\,(6\,(x-2)x-11)x+98)\sqrt{-3\,x^2+4\,x+5}}{1083\,(3\,x^2-4\,x-5)^2}$$

```
[In] integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="giac")
[Out] -1/1083*(9*(6*(x - 2)*x - 11)*x + 98)*sqrt(-3*x^2 + 4*x + 5)/(3*x^2 - 4*x -
 5)^2
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = \frac{(12\,x-8)\,(-72\,x^2+96\,x+196)}{17328\,(-3\,x^2+4\,x+5)^{3/2}}$$

```
[In] int(1/(4*x - 3*x^2 + 5)^(5/2),x)
[Out] ((12*x - 8)*(96*x - 72*x^2 + 196))/(17328*(4*x - 3*x^2 + 5)^(3/2))
```

3.287 $\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx=\frac{1}{1+x}-\frac{\sqrt{2+2x+x^2}}{1+x}+\operatorname{arcsinh}(1+x)$$

```
[Out] 1/(1+x)+arcsinh(1+x)-(x^2+2*x+2)^(1/2)/(1+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6874, 698, 633, 221}

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx=\operatorname{arcsinh}(x+1)-\frac{\sqrt{x^2+2x+2}}{x+1}+\frac{1}{x+1}$$

```
[In] Int[(1 + Sqrt[2 + 2*x + x^2])^(-1),x]
```

```
[Out] (1 + x)^(-1) - Sqrt[2 + 2*x + x^2]/(1 + x) + ArcSinh[1 + x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 698

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[b*(p/(d*e*(m + 1))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{1}{(1+x)^2}+\frac{\sqrt{2+2x+x^2}}{(1+x)^2}\right) dx\\ &= \frac{1}{1+x}+\int \frac{\sqrt{2+2x+x^2}}{(1+x)^2}\,dx\\ &= \frac{1}{1+x}-\frac{\sqrt{2+2x+x^2}}{1+x}+\int \frac{1}{\sqrt{2+2x+x^2}}\,dx\\ &= \frac{1}{1+x}-\frac{\sqrt{2+2x+x^2}}{1+x}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4}}}\,dx,x,2+2x\right)\\ &= \frac{1}{1+x}-\frac{\sqrt{2+2x+x^2}}{1+x}+\text{arcsinh}(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx=-\frac{-1+\sqrt{2+2x+x^2}+(1+x)\log\left(-1-x+\sqrt{2+2x+x^2}\right)}{1+x}$$

```
[In] Integrate[(1 + Sqrt[2 + 2*x + x^2])^(-1),x]
[Out] -((-1 + Sqrt[2 + 2*x + x^2] + (1 + x)*Log[-1 - x + Sqrt[2 + 2*x + x^2]])/(1
 + x))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

method	result	size
default	$-\frac{\left((1+x)^{2}+1\right)^{\frac{3}{2}}}{1+x}+(1+x)\sqrt{(1+x)^{2}+1}+\operatorname{arcsinh}(1+x)+\frac{1}{1+x}$	40
trager	$-\frac{x}{1+x}-\frac{\sqrt{x^{2}+2x+2}}{1+x}-\ln\left(\sqrt{x^{2}+2x+2}-1-x\right)$	45

```
[In] int(1/(1+(x^2+2*x+2)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/(1+x)*((1+x)^2+1)^(3/2)+(1+x)*((1+x)^2+1)^(1/2)+arcsinh(1+x)+1/(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx=-\frac{(x+1)\log\left(-x+\sqrt{x^2+2\,x+2}-1\right)+x+\sqrt{x^2+2\,x+2}}{x+1}$$

```
[In] integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="fricas")
[Out] -((x + 1)*log(-x + sqrt(x^2 + 2*x + 2) - 1) + x + sqrt(x^2 + 2*x + 2))/(x +
 1)
```

Sympy [F]

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx=\int \frac{1}{\sqrt{x^2+2x+2}+1}\,dx$$

```
[In] integrate(1/(1+(x**2+2*x+2)**(1/2)),x)
[Out] Integral(1/(sqrt(x**2 + 2*x + 2) + 1), x)
```

Maxima [F]

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2\,x+2}+1}\,dx$$

```
[In] integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 2*x + 2) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.07

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx = \frac{2}{\left(x-\sqrt{x^2+2\,x+2}\right)^2+2\,x-2\,\sqrt{x^2+2\,x+2}} + \frac{1}{x+1} - \log\left(-x+\sqrt{x^2+2\,x+2}-1\right)$$

```
[In] integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/((x - sqrt(x^2 + 2*x + 2))^2 + 2*x - 2*sqrt(x^2 + 2*x + 2)) + 1/(x + 1) -
 log(-x + sqrt(x^2 + 2*x + 2) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx = \frac{1}{x+1} + \int \frac{\sqrt{x^2+2\,x+2}}{\left(x+1\right)^2}\,dx$$

```
[In] int(1/((2*x + x^2 + 2)^(1/2) + 1),x)
```

```
[Out] 1/(x + 1) + int((2*x + x^2 + 2)^(1/2)/(x + 1)^2, x)
```

3.288 $\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx=-x+\sqrt{1+x+x^2}-\frac{3}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)+2\log\left(x+\sqrt{1+x+x^2}\right)$$

```
[Out] -x-3/2*arcsinh(1/3*(1+2*x)*3^(1/2))+2*ln(x+(x^2+x+1)^(1/2))+(x^2+x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2141, 907}

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx=\frac{3}{2\left(2\left(\sqrt{x^2+x+1}+x\right)+1\right)}+2\log\left(\sqrt{x^2+x+1}+x\right)-\frac{3}{2}\log\left(2\left(\sqrt{x^2+x+1}+x\right)+1\right)$$

```
[In] Int[(x + Sqrt[1 + x + x^2])^(-1),x]
```

```
[Out] 3/(2*(1 + 2*(x + Sqrt[1 + x + x^2]))) + 2*Log[x + Sqrt[1 + x + x^2]] - (3*L
og[1 + 2*(x + Sqrt[1 + x + x^2])])/2
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
```

)

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{1+x+x^2}{x(1+2x)^2}\,dx, x, x+\sqrt{1+x+x^2}\right)$$
$$= 2\text{Subst}\left(\int \left(\frac{1}{x} - \frac{3}{2(1+2x)^2} - \frac{3}{2(1+2x)}\right)\,dx, x, x+\sqrt{1+x+x^2}\right)$$
$$= \frac{3}{2\left(1+2\left(x+\sqrt{1+x+x^2}\right)\right)} + 2\log\left(x+\sqrt{1+x+x^2}\right) - \frac{3}{2}\log\left(1+2\left(x+\sqrt{1+x+x^2}\right)\right)$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = -x+\sqrt{1+x+x^2}+2\log\left(-2-x+\sqrt{1+x+x^2}\right) - \frac{1}{2}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

```
[In] Integrate[(x + Sqrt[1 + x + x^2])^(-1),x]
[Out] -x + Sqrt[1 + x + x^2] + 2*Log[-2 - x + Sqrt[1 + x + x^2]] - Log[-1 - 2*x +
 2*Sqrt[1 + x + x^2]]/2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

method	result	size
default	$\sqrt{(1+x)^2-x}-\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{2}-\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{(1+x)^2-x}}\right)-x+\ln(1+x)$	52
trager	$\sqrt{x^2+x+1}-x-\frac{\ln\left(\frac{2x^2\sqrt{x^2+x+1}+2x^3+8x\sqrt{x^2+x+1}+9x^2+14\sqrt{x^2+x+1}+12x+13}{(1+x)^4}\right)}{2}$	71

```
[In] int(1/(x+(x^2+x+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] ((1+x)^2-x)^(1/2)-1/2*arcsinh(2/3*3^(1/2)*(x+1/2))-arctanh(1/2*(1-x)/((1+x)
^2-x)^(1/2))-x+ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = -x+\sqrt{x^2+x+1}+\log(x+1)-\log\left(-x+\sqrt{x^2+x+1}\right) +\log\left(-x+\sqrt{x^2+x+1}-2\right)+\frac{1}{2}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

```
[In] integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="fricas")
[Out] -x + sqrt(x^2 + x + 1) + log(x + 1) - log(-x + sqrt(x^2 + x + 1)) + log(-x
+ sqrt(x^2 + x + 1) - 2) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Sympy [F]

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x+\sqrt{x^2+x+1}}\,dx$$

```
[In] integrate(1/(x+(x**2+x+1)**(1/2)),x)
[Out] Integral(1/(x + sqrt(x**2 + x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x+\sqrt{x^2+x+1}}\,dx$$

```
[In] integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(x + sqrt(x^2 + x + 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = -x+\sqrt{x^2+x+1}+\frac{1}{2}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)+\log\left(|x+1|\right)-\log\left(\left|-x+\sqrt{x^2+x+1}\right|\right)+\log\left(\left|-x+\sqrt{x^2+x+1}-2\right|\right)$$

```
[In] integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="giac")
[Out] -x + sqrt(x^2 + x + 1) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) + log(abs(
x + 1)) - log(abs(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x + 1)
 - 2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = \ln(x+1)-x+\int \frac{\sqrt{x^2+x+1}}{x+1}\,dx$$

```
[In] int(1/(x + (x + x^2 + 1)^(1/2)),x)
[Out] log(x + 1) - x + int((x + x^2 + 1)^(1/2)/(x + 1), x)
```

3.289 $\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = -\frac{x^3}{9}-\frac{x^4}{6}+\frac{1}{96}(1+2x)\sqrt{1+x+x^2}-\frac{5}{36}\left(1+x+x^2\right)^{3/2} +\frac{1}{6}x\left(1+x+x^2\right)^{3/2}+\frac{1}{64}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

```
[Out] -1/9*x^3-1/6*x^4-5/36*(x^2+x+1)^(3/2)+1/6*x*(x^2+x+1)^(3/2)+1/64*arcsinh(1/
3*(1+2*x)*3^(1/2))+1/96*(1+2*x)*(x^2+x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6874, 756, 654, 626, 633, 221}

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = \frac{1}{64}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{x^4}{6}-\frac{x^3}{9}+\frac{1}{6}\left(x^2+x+1\right)^{3/2}x -\frac{5}{36}\left(x^2+x+1\right)^{3/2}+\frac{1}{96}(2x+1)\sqrt{x^2+x+1}$$

```
[In] Int[x^2/(1 + 2*x + 2*Sqrt[1 + x + x^2]),x]
```

```
[Out] -1/9*x^3 - x^4/6 + ((1 + 2*x)*Sqrt[1 + x + x^2])/96 - (5*(1 + x + x^2)^(3/2
))/36 + (x*(1 + x + x^2)^(3/2))/6 + ArcSinh[(1 + 2*x)/Sqrt[3]]/64
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{x^2}{3}-\frac{2x^3}{3}+\frac{2}{3}x^2\sqrt{1+x+x^2}\right)\,dx\\
&= -\frac{x^3}{9}-\frac{x^4}{6}+\frac{2}{3}\int x^2\sqrt{1+x+x^2}\,dx\\
&= -\frac{x^3}{9}-\frac{x^4}{6}+\frac{1}{6}x\left(1+x+x^2\right)^{3/2}+\frac{1}{6}\int \left(-1-\frac{5x}{2}\right)\sqrt{1+x+x^2}\,dx
\end{aligned}$$

$$= -\frac{x^3}{9} - \frac{x^4}{6} - \frac{5}{36}\left(1+x+x^2\right)^{3/2} + \frac{1}{6}x\left(1+x+x^2\right)^{3/2} + \frac{1}{24}\int \sqrt{1+x+x^2}\,dx$$

$$= -\frac{x^3}{9} - \frac{x^4}{6} + \frac{1}{96}(1+2x)\sqrt{1+x+x^2} - \frac{5}{36}\left(1+x+x^2\right)^{3/2} + \frac{1}{6}x\left(1+x+x^2\right)^{3/2} + \frac{1}{64}\int \frac{1}{\sqrt{1+x+x^2}}\,dx$$

$$= -\frac{x^3}{9} - \frac{x^4}{6} + \frac{1}{96}(1+2x)\sqrt{1+x+x^2} - \frac{5}{36}\left(1+x+x^2\right)^{3/2} + \frac{1}{6}x\left(1+x+x^2\right)^{3/2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, 1+2x\right)}{64\sqrt{3}}$$

$$= -\frac{x^3}{9} - \frac{x^4}{6} + \frac{1}{96}(1+2x)\sqrt{1+x+x^2} - \frac{5}{36}\left(1+x+x^2\right)^{3/2} + \frac{1}{6}x\left(1+x+x^2\right)^{3/2} + \frac{1}{64}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = -\frac{1}{18}x^3(2+3x) + \frac{1}{288}\sqrt{1+x+x^2}\left(-37+14x+8x^2+48x^3\right) - \frac{1}{64}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

```
[In] Integrate[x^2/(1 + 2*x + 2*Sqrt[1 + x + x^2]),x]
```

```
[Out] -1/18*(x^3*(2 + 3*x)) + (Sqrt[1 + x + x^2]*(-37 + 14*x + 8*x^2 + 48*x^3))/2
88 - Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]/64
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

method	result	size
trager	$-\frac{(2+3x)x^3}{18} + \frac{\left(\frac{1}{2}x^3+\frac{1}{12}x^2+\frac{7}{48}x-\frac{37}{96}\right)\sqrt{x^2+x+1}}{3} + \frac{\ln\left(1+2x+2\sqrt{x^2+x+1}\right)}{64}$	55
default	$-\frac{x^3}{9} - \frac{x^4}{6} + \frac{x\left(x^2+x+1\right)^{\frac{3}{2}}}{6} - \frac{5\left(x^2+x+1\right)^{\frac{3}{2}}}{36} + \frac{(1+2x)\sqrt{x^2+x+1}}{96} + \frac{\text{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{64}$	59

```
[In] int(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*(2+3*x)*x^3+1/3*(1/2*x^3+1/12*x^2+7/48*x-37/96)*(x^2+x+1)^(1/2)+1/64*
ln(1+2*x+2*(x^2+x+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = -\frac{1}{6}\,x^4 - \frac{1}{9}\,x^3 + \frac{1}{288}\,\left(48\,x^3 + 8\,x^2 + 14\,x - 37\right)\sqrt{x^2+x+1} - \frac{1}{64}\,\log\left(-2\,x + 2\,\sqrt{x^2+x+1} - 1\right)$$

```
[In] integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/6*x^4 - 1/9*x^3 + 1/288*(48*x^3 + 8*x^2 + 14*x - 37)*sqrt(x^2 + x + 1) -
 1/64*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Sympy [F]

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = \int \frac{x^2}{2x+2\sqrt{x^2+x+1}+1}\,dx$$

```
[In] integrate(x**2/(1+2*x+2*(x**2+x+1)**(1/2)),x)
```

```
[Out] Integral(x**2/(2*x + 2*sqrt(x**2 + x + 1) + 1), x)
```

Maxima [F]

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = \int \frac{x^2}{2\,x+2\,\sqrt{x^2+x+1}+1}\,dx$$

```
[In] integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(2*x + 2*sqrt(x^2 + x + 1) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = -\frac{1}{6}\,x^4 - \frac{1}{9}\,x^3 + \frac{1}{288}\,(2\,(4\,(6\,x+1)x+7)x-37)\sqrt{x^2+x+1} - \frac{1}{64}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

```
[In] integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/6*x^4 - 1/9*x^3 + 1/288*(2*(4*(6*x + 1)*x + 7)*x - 37)*sqrt(x^2 + x + 1)
 - 1/64*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = \frac{\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{64} - \frac{\left(\frac{x}{2}+\frac{1}{4}\right)\,\sqrt{x^2+x+1}}{6} - \frac{x^3}{9} - \frac{x^4}{6} - \frac{5\,(8\,x^2+2\,x+5)\,\sqrt{x^2+x+1}}{288} + \frac{x\,(x^2+x+1)^{3/2}}{6}$$

```
[In] int(x^2/(2*x + 2*(x + x^2 + 1)^(1/2) + 1),x)
```

```
[Out] log(x + (x + x^2 + 1)^(1/2) + 1/2)/64 - ((x/2 + 1/4)*(x + x^2 + 1)^(1/2))/6
 - x^3/9 - x^4/6 - (5*(2*x + 8*x^2 + 5)*(x + x^2 + 1)^(1/2))/288 + (x*(x +
x^2 + 1)^(3/2))/6
```

3.290 $\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 80

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = x-3\sqrt{1+x+x^2}+\frac{5}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)+4\text{arctanh}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)-\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)+\log(x)-4\log(1+x)$$

```
[Out] x+5/2*arcsinh(1/3*(1+2*x)*3^(1/2))+4*arctanh(1/2*(1-x)/(x^2+x+1)^(1/2))-arc
tanh(1/2*(2+x)/(x^2+x+1)^(1/2))+ln(x)-4*ln(1+x)-3*(x^2+x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {6874, 748, 857, 633, 221, 738, 212, 6872}

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = \frac{5}{2}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)+4\text{arctanh}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)-\text{arctanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-3\sqrt{x^2+x+1}+x+\log(x)-4\log(x+1)$$

```
[In] Int[(-3*x + Sqrt[1 + x + x^2])/(-1 + Sqrt[1 + x + x^2]),x]
```

```
[Out] x - 3*Sqrt[1 + x + x^2] + (5*ArcSinh[(1 + 2*x)/Sqrt[3]])/2 + 4*ArcTanh[(1 -
 x)/(2*Sqrt[1 + x + x^2])] - ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])] + Log[x
] - 4*Log[1 + x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
```

```
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3x}{-1+\sqrt{1+x+x^2}}+\frac{\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\right)dx \\
&= -\left(3\int \frac{x}{-1+\sqrt{1+x+x^2}}\,dx\right)+\int \frac{\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx \\
&= -\left(3\int \left(\frac{1}{1+x}+\frac{\sqrt{1+x+x^2}}{1+x}\right)dx\right)+\int \left(1+\frac{1}{-1+\sqrt{1+x+x^2}}\right)dx \\
&= x-3\log(1+x)-3\int \frac{\sqrt{1+x+x^2}}{1+x}\,dx+\int \frac{1}{-1+\sqrt{1+x+x^2}}\,dx \\
&= x-3\sqrt{1+x+x^2}-3\log(1+x)+\frac{3}{2}\int \frac{-1+x}{(1+x)\sqrt{1+x+x^2}}\,dx \\
&\quad+\int \left(\frac{1}{-1-x}+\frac{1}{x}+\frac{\sqrt{1+x+x^2}}{x}-\frac{\sqrt{1+x+x^2}}{1+x}\right)dx \\
&= x-3\sqrt{1+x+x^2}+\log(x)-4\log(1+x)+\frac{3}{2}\int \frac{1}{\sqrt{1+x+x^2}}\,dx \\
&\quad-3\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx+\int \frac{\sqrt{1+x+x^2}}{x}\,dx-\int \frac{\sqrt{1+x+x^2}}{1+x}\,dx \\
&= x-3\sqrt{1+x+x^2}+\log(x)-4\log(1+x)-\frac{1}{2}\int \frac{-2-x}{x\sqrt{1+x+x^2}}\,dx \\
&\quad+\frac{1}{2}\int \frac{-1+x}{(1+x)\sqrt{1+x+x^2}}\,dx+6\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{1-x}{\sqrt{1+x+x^2}}\right) \\
&\quad+\frac{1}{2}\sqrt{3}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+2x\right) \\
&= x-3\sqrt{1+x+x^2}+\frac{3}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)+3\text{arctanh}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)+\log(x) \\
&\quad-4\log(1+x)+2\left(\frac{1}{2}\int \frac{1}{\sqrt{1+x+x^2}}\,dx\right)+\int \frac{1}{x\sqrt{1+x+x^2}}\,dx-\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= x-3\sqrt{1+x+x^2}+\frac{3}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)+3\text{arctanh}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right) \\
&\quad+\log(x)-4\log(1+x)+2\text{Subst}\left(\int\frac{1}{4-x^2}\,dx,x,\frac{1-x}{\sqrt{1+x+x^2}}\right) \\
&\quad-2\text{Subst}\left(\int\frac{1}{4-x^2}\,dx,x,\frac{2+x}{\sqrt{1+x+x^2}}\right)+2\frac{\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+2x\right)}{2\sqrt{3}} \\
&= x-3\sqrt{1+x+x^2}+\frac{5}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)+4\text{arctanh}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right) \\
&\quad-\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)+\log(x)-4\log(1+x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$
\begin{aligned}
\int\frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx &= x-3\sqrt{1+x+x^2}-8\log\left(-2-x+\sqrt{1+x+x^2}\right) \\
&\quad+2\log\left(-1-x+\sqrt{1+x+x^2}\right) \\
&\quad+\frac{1}{2}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)
\end{aligned}
$$

```
[In] Integrate[(-3*x + Sqrt[1 + x + x^2])/(-1 + Sqrt[1 + x + x^2]),x]
```

```
[Out] x - 3*Sqrt[1 + x + x^2] - 8*Log[-2 - x + Sqrt[1 + x + x^2]] + 2*Log[-1 - x
+ Sqrt[1 + x + x^2]] + Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]/2
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

method	result
default	$\ln(x)-4\ln(1+x)+x+\sqrt{x^2+x+1}+\frac{5\,\text{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{2}-\text{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)-4\sqrt{(1+x)$
trager	$-1+x-3\sqrt{x^2+x+1}+\frac{\ln\left(\frac{-8+3865870x^6-96x+1790544x^5+8\sqrt{x^2+x+1}+445596x^4-1526x^3-507x^2+2392341x^{10}+308624x^{12}+84}{}\right.}{}$

```
[In] int((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-4*ln(1+x)+x+(x^2+x+1)^(1/2)+5/2*arcsinh(2/3*3^(1/2)*(x+1/2))-arctanh(
1/2*(2+x)/(x^2+x+1)^(1/2))-4*((1+x)^2-x)^(1/2)+4*arctanh(1/2*(1-x)/((1+x)^2
-x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = x-3\sqrt{x^2+x+1}-4\log(x+1)+\log(x) - \log\left(-x+\sqrt{x^2+x+1}+1\right)+4\log\left(-x+\sqrt{x^2+x+1}\right) + \log\left(-x+\sqrt{x^2+x+1}-1\right)-4\log\left(-x+\sqrt{x^2+x+1}-2\right) - \frac{5}{2}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

```
[In] integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="fricas"
)
```

```
[Out] x - 3*sqrt(x^2 + x + 1) - 4*log(x + 1) + log(x) - log(-x + sqrt(x^2 + x + 1
) + 1) + 4*log(-x + sqrt(x^2 + x + 1)) + log(-x + sqrt(x^2 + x + 1) - 1) -
4*log(-x + sqrt(x^2 + x + 1) - 2) - 5/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Sympy [F]

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = -\int \frac{3x}{\sqrt{x^2+x+1}-1}\,dx - \int \left(-\frac{\sqrt{x^2+x+1}}{\sqrt{x^2+x+1}-1}\right)dx$$

```
[In] integrate((-3*x+(x**2+x+1)**(1/2))/(-1+(x**2+x+1)**(1/2)),x)
```

```
[Out] -Integral(3*x/(sqrt(x**2 + x + 1) - 1), x) - Integral(-sqrt(x**2 + x + 1)/(
sqrt(x**2 + x + 1) - 1), x)
```

Maxima [F]

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = \int -\frac{3\,x-\sqrt{x^2+x+1}}{\sqrt{x^2+x+1}-1}\,dx$$

```
[In] integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="maxima"
)
```

```
[Out] 3/4*x^2 + 1/2*x + integrate(-1/2*(3*x^3 + 2*x^2 - x)/(x^2 + x - 2*sqrt(x^2
+ x + 1) + 2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.31

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = x - 3\sqrt{x^2+x+1} - \frac{5}{2}\log\left(-2x+2\sqrt{x^2+x+1}-1\right) - 4\log\left(|x+1|\right) + \log\left(|x|\right) - \log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) + 4\log\left(\left|-x+\sqrt{x^2+x+1}\right|\right) + \log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right) - 4\log\left(\left|-x+\sqrt{x^2+x+1}-2\right|\right)$$

```
[In] integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="giac")
```

```
[Out] x - 3*sqrt(x^2 + x + 1) - 5/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) - 4*log(a
bs(x + 1)) + log(abs(x)) - log(abs(-x + sqrt(x^2 + x + 1) + 1)) + 4*log(abs
(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x + 1) - 1)) - 4*log(ab
s(-x + sqrt(x^2 + x + 1) - 2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = x - 4\ln\left(x+1\right) + \ln\left(x\right) - \int \frac{(3x-1)\sqrt{x^2+x+1}}{x\,(x+1)}\,dx$$

```
[In] int(-(3*x - (x + x^2 + 1)^(1/2))/((x + x^2 + 1)^(1/2) - 1),x)
```

```
[Out] x - 4*log(x + 1) + log(x) - int(((3*x - 1)*(x + x^2 + 1)^(1/2))/(x*(x + 1))
, x)
```

3.291 $\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 158

$$\begin{aligned}\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx &= -2\sqrt{1+x+x^2}+\frac{1}{4}(1+2x)\sqrt{1+x+x^2}\\ &\quad -2\sqrt{4+2x+x^2}+\frac{1}{2}(1+x)\sqrt{4+2x+x^2}\\ &\quad +\frac{11}{2}\operatorname{arcsinh}\left(\frac{1+x}{\sqrt{3}}\right)+\frac{43}{8}\operatorname{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)\\ &\quad -2\sqrt{7}\operatorname{arctanh}\left(\frac{1+5x}{2\sqrt{7}\sqrt{1+x+x^2}}\right)\\ &\quad +2\sqrt{7}\operatorname{arctanh}\left(\frac{1-2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)\end{aligned}$$

```
[Out] 11/2*arcsinh(1/3*(1+x)*3^(1/2))+43/8*arcsinh(1/3*(1+2*x)*3^(1/2))-2*arctanh
(1/14*(1+5*x)*7^(1/2)/(x^2+x+1)^(1/2))*7^(1/2)+2*arctanh(1/7*(1-2*x)*7^(1/2
)/(x^2+2*x+4)^(1/2))*7^(1/2)-2*(x^2+x+1)^(1/2)+1/4*(1+2*x)*(x^2+x+1)^(1/2)-
2*(x^2+2*x+4)^(1/2)+1/2*(1+x)*(x^2+2*x+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used

$= \{6874, 748, 857, 633, 221, 738, 212, 626\}$

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx = \frac{11}{2}\text{arcsinh}\left(\frac{x+1}{\sqrt{3}}\right)+\frac{43}{8}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right) - 2\sqrt{7}\text{arctanh}\left(\frac{5x+1}{2\sqrt{7}\sqrt{x^2+x+1}}\right) + 2\sqrt{7}\text{arctanh}\left(\frac{1-2x}{\sqrt{7}\sqrt{x^2+2x+4}}\right) + \frac{1}{2}\sqrt{x^2+2x+4}(x+1)+\frac{1}{4}(2x+1)\sqrt{x^2+x+1} - 2\sqrt{x^2+x+1}-2\sqrt{x^2+2x+4}$$

```
[In] Int[(1 + x)/(-Sqrt[1 + x + x^2] + Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] -2*Sqrt[1 + x + x^2] + ((1 + 2*x)*Sqrt[1 + x + x^2])/4 - 2*Sqrt[4 + 2*x + x
^2] + ((1 + x)*Sqrt[4 + 2*x + x^2])/2 + (11*ArcSinh[(1 + x)/Sqrt[3]])/2 + (
43*ArcSinh[(1 + 2*x)/Sqrt[3]])/8 - 2*Sqrt[7]*ArcTanh[(1 + 5*x)/(2*Sqrt[7]*S
qrt[1 + x + x^2])] + 2*Sqrt[7]*ArcTanh[(1 - 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^
2])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{\sqrt{1+x+x^2}-\sqrt{4+2x+x^2}}-\frac{x}{\sqrt{1+x+x^2}-\sqrt{4+2x+x^2}}\right) dx \\
&= -\int \frac{1}{\sqrt{1+x+x^2}-\sqrt{4+2x+x^2}}\, dx-\int \frac{x}{\sqrt{1+x+x^2}-\sqrt{4+2x+x^2}}\, dx \\
&= -\int \left(-\frac{\sqrt{1+x+x^2}}{3+x}-\frac{\sqrt{4+2x+x^2}}{3+x}\right) dx \\
&\quad -\int \left(-\sqrt{1+x+x^2}+\frac{3\sqrt{1+x+x^2}}{3+x}-\sqrt{4+2x+x^2}+\frac{3\sqrt{4+2x+x^2}}{3+x}\right) dx \\
&= -\left(3\int \frac{\sqrt{1+x+x^2}}{3+x}\, dx\right)-3\int \frac{\sqrt{4+2x+x^2}}{3+x}\, dx+\int \sqrt{1+x+x^2}\, dx \\
&\quad +\int \frac{\sqrt{1+x+x^2}}{3+x}\, dx+\int \sqrt{4+2x+x^2}\, dx+\int \frac{\sqrt{4+2x+x^2}}{3+x}\, dx
\end{aligned}$$

$$= -2\sqrt{1+x+x^2}+\frac{1}{4}(1+2x)\sqrt{1+x+x^2}-2\sqrt{4+2x+x^2}+\frac{1}{2}(1+x)\sqrt{4+2x+x^2}$$
$$+\frac{3}{8}\int\frac{1}{\sqrt{1+x+x^2}}\,dx-\frac{1}{2}\int\frac{1+5x}{(3+x)\sqrt{1+x+x^2}}\,dx-\frac{1}{2}\int\frac{-2+4x}{(3+x)\sqrt{4+2x+x^2}}\,dx$$
$$+\frac{3}{2}\int\frac{1+5x}{(3+x)\sqrt{1+x+x^2}}\,dx+\frac{3}{2}\int\frac{1}{\sqrt{4+2x+x^2}}\,dx+\frac{3}{2}\int\frac{-2+4x}{(3+x)\sqrt{4+2x+x^2}}\,dx$$

$$= -2\sqrt{1+x+x^2}+\frac{1}{4}(1+2x)\sqrt{1+x+x^2}-2\sqrt{4+2x+x^2}$$
$$+\frac{1}{2}(1+x)\sqrt{4+2x+x^2}-2\int\frac{1}{\sqrt{4+2x+x^2}}\,dx-\frac{5}{2}\int\frac{1}{\sqrt{1+x+x^2}}\,dx$$
$$+6\int\frac{1}{\sqrt{4+2x+x^2}}\,dx+7\int\frac{1}{(3+x)\sqrt{1+x+x^2}}\,dx$$
$$+7\int\frac{1}{(3+x)\sqrt{4+2x+x^2}}\,dx+\frac{15}{2}\int\frac{1}{\sqrt{1+x+x^2}}\,dx$$
$$-21\int\frac{1}{(3+x)\sqrt{1+x+x^2}}\,dx-21\int\frac{1}{(3+x)\sqrt{4+2x+x^2}}\,dx$$
$$+\frac{1}{8}\sqrt{3}\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+2x\right)+\frac{1}{4}\sqrt{3}\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{12}}}\,dx,x,2+2x\right)$$

$$= -2\sqrt{1+x+x^2}+\frac{1}{4}(1+2x)\sqrt{1+x+x^2}-2\sqrt{4+2x+x^2}$$
$$+\frac{1}{2}(1+x)\sqrt{4+2x+x^2}+\frac{3}{2}\text{arcsinh}\left(\frac{1+x}{\sqrt{3}}\right)$$
$$+\frac{3}{8}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)-14\text{Subst}\left(\int\frac{1}{28-x^2}\,dx,x,\frac{-1-5x}{\sqrt{1+x+x^2}}\right)$$
$$-14\text{Subst}\left(\int\frac{1}{28-x^2}\,dx,x,\frac{2-4x}{\sqrt{4+2x+x^2}}\right)$$
$$+42\text{Subst}\left(\int\frac{1}{28-x^2}\,dx,x,\frac{-1-5x}{\sqrt{1+x+x^2}}\right)$$
$$+42\text{Subst}\left(\int\frac{1}{28-x^2}\,dx,x,\frac{2-4x}{\sqrt{4+2x+x^2}}\right)-\frac{\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{12}}}\,dx,x,2+2x\right)}{\sqrt{3}}$$
$$-\frac{5\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+2x\right)}{2\sqrt{3}}+\sqrt{3}\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{12}}}\,dx,x,2+2x\right)$$
$$+\frac{1}{2}\left(5\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+2x\right)$$

$$
\begin{aligned}
&= -2\sqrt{1+x+x^2}+\frac{1}{4}(1+2x)\sqrt{1+x+x^2}-2\sqrt{4+2x+x^2} \\
&\quad+\frac{1}{2}(1+x)\sqrt{4+2x+x^2}+\frac{11}{2}\text{arcsinh}\left(\frac{1+x}{\sqrt{3}}\right)+\frac{43}{8}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right) \\
&\quad-2\sqrt{7}\text{arctanh}\left(\frac{1+5x}{2\sqrt{7}\sqrt{1+x+x^2}}\right)+2\sqrt{7}\text{arctanh}\left(\frac{1-2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 5.51 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01

$$
\begin{aligned}
\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx &= \frac{1}{8}\Bigg(-14\sqrt{1+x+x^2}+4x\sqrt{1+x+x^2} \\
&\quad-12\sqrt{4+2x+x^2}+4x\sqrt{4+2x+x^2} \\
&\quad-32\sqrt{7}\text{arctanh}\left(\frac{3+x-\sqrt{1+x+x^2}}{\sqrt{7}}\right) \\
&\quad-32\sqrt{7}\text{arctanh}\left(\frac{3+x-\sqrt{4+2x+x^2}}{\sqrt{7}}\right) \\
&\quad-43\log\left(-1-2x+2\sqrt{1+x+x^2}\right) \\
&\quad-44\log\left(-1-x+\sqrt{4+2x+x^2}\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[(1 + x)/(-Sqrt[1 + x + x^2] + Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] (-14*Sqrt[1 + x + x^2] + 4*x*Sqrt[1 + x + x^2] - 12*Sqrt[4 + 2*x + x^2] + 4
*x*Sqrt[4 + 2*x + x^2] - 32*Sqrt[7]*ArcTanh[(3 + x - Sqrt[1 + x + x^2])/Sqr
t[7]] - 32*Sqrt[7]*ArcTanh[(3 + x - Sqrt[4 + 2*x + x^2])/Sqrt[7]] - 43*Log[
-1 - 2*x + 2*Sqrt[1 + x + x^2]] - 44*Log[-1 - x + Sqrt[4 + 2*x + x^2]])/8
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.89

method	result
default	$-2\sqrt{(3+x)^2-5x-8}+\frac{43\,\text{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{8}+2\sqrt{7}\,\text{arctanh}\left(\frac{(-1-5x)\sqrt{7}}{14\sqrt{(3+x)^2-5x-8}}\right)-2\sqrt{(3+x)^2-}$

```
[In] int((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((3+x)^2-5*x-8)^(1/2)+43/8*arcsinh(2/3*3^(1/2)*(x+1/2))+2*7^(1/2)*arctan
h(1/14*(-1-5*x)*7^(1/2)/((3+x)^2-5*x-8)^(1/2))-2*((3+x)^2-4*x-5)^(1/2)+11/2
```

```
*arcsinh(1/3*(1+x)*3^(1/2))+2*7^(1/2)*arctanh(1/14*(2-4*x)*7^(1/2)/((3+x)^2
-4*x-5)^(1/2))+1/4*(1+2*x)*(x^2+x+1)^(1/2)+1/4*(2*x+2)*(x^2+2*x+4)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.98

$$\begin{aligned}
&\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx \\
&= \frac{1}{4}\sqrt{x^2+x+1}(2x-7)+\frac{1}{2}\sqrt{x^2+2x+4}(x-3) \\
&\quad + 2\sqrt{7}\log\left(\frac{2\sqrt{7}(5x+1)+2\sqrt{x^2+x+1}(5\sqrt{7}-14)-25x-5}{x+3}\right) \\
&\quad + 2\sqrt{7}\log\left(\frac{\sqrt{7}(2x-1)+\sqrt{x^2+2x+4}(2\sqrt{7}-7)-4x+2}{x+3}\right) \\
&\quad - \frac{11}{2}\log\left(-x+\sqrt{x^2+2x+4}-1\right)-\frac{43}{8}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)
\end{aligned}$$

```
[In] integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^2 + x + 1)*(2*x - 7) + 1/2*sqrt(x^2 + 2*x + 4)*(x - 3) + 2*sqrt(
7)*log((2*sqrt(7)*(5*x + 1) + 2*sqrt(x^2 + x + 1)*(5*sqrt(7) - 14) - 25*x -
 5)/(x + 3)) + 2*sqrt(7)*log((sqrt(7)*(2*x - 1) + sqrt(x^2 + 2*x + 4)*(2*sq
rt(7) - 7) - 4*x + 2)/(x + 3)) - 11/2*log(-x + sqrt(x^2 + 2*x + 4) - 1) - 4
3/8*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Sympy [F]

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx=\int \frac{x+1}{-\sqrt{x^2+x+1}+\sqrt{x^2+2x+4}}\,dx$$

```
[In] integrate((1+x)/(-(x**2+x+1)**(1/2)+(x**2+2*x+4)**(1/2)),x)
```

```
[Out] Integral((x + 1)/(-sqrt(x**2 + x + 1) + sqrt(x**2 + 2*x + 4)), x)
```

Maxima [F]

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx = \int \frac{x+1}{\sqrt{x^2+2\,x+4}-\sqrt{x^2+x+1}}\,dx$$

```
[In] integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="maxima")
[Out] integrate((x + 1)/(sqrt(x^2 + 2*x + 4) - sqrt(x^2 + x + 1)), x)
```

Giac [F]

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx = \int \frac{x+1}{\sqrt{x^2+2\,x+4}-\sqrt{x^2+x+1}}\,dx$$

```
[In] integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="giac")
[Out] integrate((x + 1)/(sqrt(x^2 + 2*x + 4) - sqrt(x^2 + x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx = \int -\frac{x+1}{\sqrt{x^2+x+1}-\sqrt{x^2+2\,x+4}}\,dx$$

```
[In] int(-(x + 1)/((x + x^2 + 1)^(1/2) - (2*x + x^2 + 4)^(1/2)),x)
[Out] int(-(x + 1)/((x + x^2 + 1)^(1/2) - (2*x + x^2 + 4)^(1/2)), x)
```

3.292 $\int \frac{1}{\sqrt{-1+x}x^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 41

$$\int \frac{1}{\sqrt{-1+x}x^3} dx = \frac{\sqrt{-1+x}}{2x^2} + \frac{3\sqrt{-1+x}}{4x} + \frac{3}{4}\arctan\left(\sqrt{-1+x}\right)$$

```
[Out] 3/4*arctan((-1+x)^(1/2))+1/2*(-1+x)^(1/2)/x^2+3/4*(-1+x)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {44, 65, 209}

$$\int \frac{1}{\sqrt{-1+x}x^3} dx = \frac{3}{4}\arctan\left(\sqrt{x-1}\right) + \frac{\sqrt{x-1}}{2x^2} + \frac{3\sqrt{x-1}}{4x}$$

```
[In] Int[1/(Sqrt[-1 + x]*x^3),x]
```

```
[Out] Sqrt[-1 + x]/(2*x^2) + (3*Sqrt[-1 + x])/(4*x) + (3*ArcTan[Sqrt[-1 + x]])/4
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{-1+x}}{2x^2}+\frac{3}{4}\int \frac{1}{\sqrt{-1+x}x^2}\,dx \\
&= \frac{\sqrt{-1+x}}{2x^2}+\frac{3\sqrt{-1+x}}{4x}+\frac{3}{8}\int \frac{1}{\sqrt{-1+x}x}\,dx \\
&= \frac{\sqrt{-1+x}}{2x^2}+\frac{3\sqrt{-1+x}}{4x}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x}\right) \\
&= \frac{\sqrt{-1+x}}{2x^2}+\frac{3\sqrt{-1+x}}{4x}+\frac{3}{4}\arctan\left(\sqrt{-1+x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx=\frac{1}{4}\left(\frac{\sqrt{-1+x}(2+3x)}{x^2}+3\arctan\left(\sqrt{-1+x}\right)\right)$$

```
[In] Integrate[1/(Sqrt[-1 + x]*x^3),x]
[Out] ((Sqrt[-1 + x]*(2 + 3*x))/x^2 + 3*ArcTan[Sqrt[-1 + x]])/4
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{3\arctan\left(\sqrt{-1+x}\right)}{4}+\frac{\sqrt{-1+x}}{2x^{2}}+\frac{3\sqrt{-1+x}}{4x}$
default	$\frac{3\arctan\left(\sqrt{-1+x}\right)}{4}+\frac{\sqrt{-1+x}}{2x^{2}}+\frac{3\sqrt{-1+x}}{4x}$
risch	$\frac{3x^{2}-x-2}{4x^{2}\sqrt{-1+x}}+\frac{3\arctan\left(\sqrt{-1+x}\right)}{4}$
trager	$\frac{(2+3x)\sqrt{-1+x}}{4x^{2}}-\frac{3\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{-1+x}-x+2}{x}\right)}{8}$
meijerg	$\frac{\sqrt{-\mathrm{signum}(-1+x)}\left(-\frac{\sqrt{\pi}}{2x^{2}}-\frac{\sqrt{\pi}}{2x}+\frac{3\left(\frac{7}{6}-2\ln(2)+\ln(x)+i\pi\right)\sqrt{\pi}}{8}+\frac{\sqrt{\pi}\left(-7x^{2}+8x+8\right)}{16x^{2}}-\frac{\sqrt{\pi}\,(12x+8)\sqrt{1-x}}{16x^{2}}-\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1-x}}{2}\right)}{4}\right)}{\sqrt{\pi}\,\sqrt{\mathrm{signum}(-1+x)}}$

```
[In] int(1/x^3/(-1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 3/4*arctan((-1+x)^(1/2))+1/2*(-1+x)^(1/2)/x^2+3/4*(-1+x)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx=\frac{3\,x^2\arctan\left(\sqrt{x-1}\right)+(3\,x+2)\sqrt{x-1}}{4\,x^2}$$

```
[In] integrate(1/x^3/(-1+x)^(1/2),x, algorithm="fricas")
[Out] 1/4*(3*x^2*arctan(sqrt(x - 1)) + (3*x + 2)*sqrt(x - 1))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.88 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.20

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx=\begin{cases}\frac{3i\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{4}-\frac{3i}{4\sqrt{x}\sqrt{-1+\frac{1}{x}}}+\frac{i}{4x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x}}}+\frac{i}{2x^{\frac{5}{2}}\sqrt{-1+\frac{1}{x}}} & \text{for}\ \frac{1}{|x|}>1\\ -\frac{3\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{4}+\frac{3}{4\sqrt{x}\sqrt{1-\frac{1}{x}}}-\frac{1}{4x^{\frac{3}{2}}\sqrt{1-\frac{1}{x}}}-\frac{1}{2x^{\frac{5}{2}}\sqrt{1-\frac{1}{x}}} & \text{otherwise}\end{cases}$$

```
[In] integrate(1/x**3/(-1+x)**(1/2),x)
[Out] Piecewise((3*I*acosh(1/sqrt(x))/4 - 3*I/(4*sqrt(x)*sqrt(-1 + 1/x)) + I/(4*x
**(3/2)*sqrt(-1 + 1/x)) + I/(2*x**(5/2)*sqrt(-1 + 1/x)), 1/Abs(x) > 1), (-3
*asin(1/sqrt(x))/4 + 3/(4*sqrt(x)*sqrt(1 - 1/x)) - 1/(4*x**(3/2)*sqrt(1 - 1
/x)) - 1/(2*x**(5/2)*sqrt(1 - 1/x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx = \frac{3\,(x-1)^{\frac{3}{2}}+5\,\sqrt{x-1}}{4\,\left((x-1)^2+2\,x-1\right)}+\frac{3}{4}\,\arctan\left(\sqrt{x-1}\right)$$

```
[In] integrate(1/x^3/(-1+x)^(1/2),x, algorithm="maxima")
[Out] 1/4*(3*(x - 1)^(3/2) + 5*sqrt(x - 1))/((x - 1)^2 + 2*x - 1) + 3/4*arctan(sq
rt(x - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx = \frac{3\,(x-1)^{\frac{3}{2}}+5\,\sqrt{x-1}}{4\,x^2}+\frac{3}{4}\,\arctan\left(\sqrt{x-1}\right)$$

```
[In] integrate(1/x^3/(-1+x)^(1/2),x, algorithm="giac")
[Out] 1/4*(3*(x - 1)^(3/2) + 5*sqrt(x - 1))/x^2 + 3/4*arctan(sqrt(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx = \frac{3\,\mathrm{atan}\left(\sqrt{x-1}\right)}{4}+\frac{3\,\sqrt{x-1}}{4\,x}+\frac{\sqrt{x-1}}{2\,x^2}$$

```
[In] int(1/(x^3*(x - 1)^(1/2)),x)
[Out] (3*atan((x - 1)^(1/2)))/4 + (3*(x - 1)^(1/2))/(4*x) + (x - 1)^(1/2)/(2*x^2)
```

3.293 $\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

```
[Out] -1/(1-3/x)^(1/3)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {267}

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

```
[In] Int[1/((1 - 3/x)^(4/3)*x^2),x]
[Out] -(1 - 3/x)^(-1/3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{1}{\sqrt[3]{\frac{-3+x}{x}}}$$

```
[In] Integrate[1/((1 - 3/x)^(4/3)*x^2),x]
[Out] -((-3 + x)/x)^(-1/3)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{1}{\left(1-\frac{3}{x}\right)^{\frac{1}{3}}}$	12
default	$-\frac{1}{\left(1-\frac{3}{x}\right)^{\frac{1}{3}}}$	12
risch	$-\frac{1}{\left(\frac{-3+x}{x}\right)^{\frac{1}{3}}}$	12
gosper	$-\frac{-3+x}{x\left(\frac{-3+x}{x}\right)^{\frac{4}{3}}}$	18
trager	$-\frac{x\left(-\frac{3-x}{x}\right)^{\frac{2}{3}}}{-3+x}$	21

```
[In] int(1/(1-3/x)^(4/3)/x^2,x,method=_RETURNVERBOSE)
[Out] -1/(1-3/x)^(1/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{x\left(\frac{x-3}{x}\right)^{\frac{2}{3}}}{x-3}$$

```
[In] integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="fricas")
[Out] -x*((x - 3)/x)^(2/3)/(x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

```
[In] integrate(1/(1-3/x)**(4/3)/x**2,x)
[Out] -1/(1 - 3/x)**(1/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{1}{\left(-\frac{3}{x}+1\right)^{\frac{1}{3}}}$$

```
[In] integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="maxima")
[Out] -1/(-3/x + 1)^(1/3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{1}{\left(\frac{x-3}{x}\right)^{\frac{1}{3}}}$$

```
[In] integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="giac")
[Out] -1/((x - 3)/x)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{1}{\left(1-\frac{3}{x}\right)^{1/3}}$$

```
[In] int(1/(x^2*(1 - 3/x)^(4/3)),x)
[Out] -1/(1 - 3/x)^(1/3)
```

3.294 $\int \frac{(-1+3x)^{4/3}}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \frac{(-1+3x)^{4/3}}{x^2} dx = 12\sqrt[3]{-1+3x} - \frac{(-1+3x)^{4/3}}{x} + 4\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{-1+3x}}{\sqrt{3}}\right) + 2\log(x) - 6\log\left(1+\sqrt[3]{-1+3x}\right)$$

```
[Out] 12*(-1+3*x)^(1/3)-(-1+3*x)^(4/3)/x+2*ln(x)-6*ln(1+(-1+3*x)^(1/3))+4*arctan(
1/3*(1-2*(-1+3*x)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {43, 52, 60, 632, 210, 31}

$$\int \frac{(-1+3x)^{4/3}}{x^2} dx = 4\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{3x-1}}{\sqrt{3}}\right) - \frac{(3x-1)^{4/3}}{x} + 12\sqrt[3]{3x-1} + 2\log(x) - 6\log\left(\sqrt[3]{3x-1}+1\right)$$

```
[In] Int[(-1 + 3*x)^(4/3)/x^2,x]
```

```
[Out] 12*(-1 + 3*x)^(1/3) - (-1 + 3*x)^(4/3)/x + 4*Sqrt[3]*ArcTan[(1 - 2*(-1 + 3*
x)^(1/3))/Sqrt[3]] + 2*Log[x] - 6*Log[1 + (-1 + 3*x)^(1/3)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(-1+3x)^{4/3}}{x}+4\int\frac{\sqrt[3]{-1+3x}}{x}\,dx\\
&= 12\sqrt[3]{-1+3x}-\frac{(-1+3x)^{4/3}}{x}-4\int\frac{1}{x(-1+3x)^{2/3}}\,dx
\end{aligned}
$$

$$= 12\sqrt[3]{-1+3x} - \frac{(-1+3x)^{4/3}}{x} + 2\log(x) - 6\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[3]{-1+3x}\right) - 6\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[3]{-1+3x}\right)$$

$$= 12\sqrt[3]{-1+3x} - \frac{(-1+3x)^{4/3}}{x} + 2\log(x) - 6\log\left(1+\sqrt[3]{-1+3x}\right) + 12\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[3]{-1+3x}\right)$$

$$= 12\sqrt[3]{-1+3x} - \frac{(-1+3x)^{4/3}}{x} + 4\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{-1+3x}}{\sqrt{3}}\right) + 2\log(x) - 6\log\left(1+\sqrt[3]{-1+3x}\right)$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \frac{(-1+3x)^{4/3}}{x^2}\,dx = \frac{\sqrt[3]{-1+3x}(1+9x)}{x} + 4\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{-1+3x}}{\sqrt{3}}\right) - 4\log\left(1+\sqrt[3]{-1+3x}\right) + 2\log\left(1-\sqrt[3]{-1+3x}+(-1+3x)^{2/3}\right)$$

```
[In] Integrate[(-1 + 3*x)^(4/3)/x^2,x]
[Out] ((-1 + 3*x)^(1/3)*(1 + 9*x))/x + 4*Sqrt[3]*ArcTan[(1 - 2*(-1 + 3*x)^(1/3))/
Sqrt[3]] - 4*Log[1 + (-1 + 3*x)^(1/3)] + 2*Log[1 - (-1 + 3*x)^(1/3) + (-1 +
 3*x)^(2/3)]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.59 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

method	result
meijerg	$-\frac{4\,\mathrm{signum}\left(x-\frac{1}{3}\right)^{\frac{4}{3}}\left(\frac{3\Gamma\left(\frac{2}{3}\right)}{4x}+3\left(2+\frac{\pi\sqrt{3}}{6}-\frac{\ln(3)}{2}+\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)-\frac{3\Gamma\left(\frac{2}{3}\right)x\,{}_3F_2\left(\frac{2}{3},1,1;2,3;3x\right)}{2}\right)}{3\Gamma\left(\frac{2}{3}\right)\left(-\,\mathrm{signum}\left(x-\frac{1}{3}\right)\right)^{\frac{4}{3}}}$
pseudoelliptic	$\frac{(27x+3)(-1+3x)^{\frac{1}{3}}-6x\left(2\sqrt{3}\,\arctan\left(\frac{\left(2(-1+3x)^{\frac{1}{3}}-1\right)\sqrt{3}}{3}\right)-\ln\left((-1+3x)^{\frac{2}{3}}-(-1+3x)^{\frac{1}{3}}+1\right)+2\ln\left(1+(-1+3x)^{\frac{1}{3}}\right)\right)}{\left((-1+3x)^{\frac{2}{3}}-(-1+3x)^{\frac{1}{3}}+1\right)\left(1+(-1+3x)^{\frac{1}{3}}\right)}$
derivativedivides	$9(-1+3x)^{\frac{1}{3}}+\frac{1+(-1+3x)^{\frac{1}{3}}}{(-1+3x)^{\frac{2}{3}}-(-1+3x)^{\frac{1}{3}}+1}+2\ln\left((-1+3x)^{\frac{2}{3}}-(-1+3x)^{\frac{1}{3}}+1\right)-4\sqrt{3}\,\mathrm{arc}$
default	$9(-1+3x)^{\frac{1}{3}}+\frac{1+(-1+3x)^{\frac{1}{3}}}{(-1+3x)^{\frac{2}{3}}-(-1+3x)^{\frac{1}{3}}+1}+2\ln\left((-1+3x)^{\frac{2}{3}}-(-1+3x)^{\frac{1}{3}}+1\right)-4\sqrt{3}\,\mathrm{arc}$
risch	$\frac{(-1+3x)^{\frac{1}{3}}}{x}+\frac{\left(-\frac{4(-1+3x)^{\frac{2}{3}}\left(-\,\mathrm{signum}\left(x-\frac{1}{3}\right)\right)^{\frac{2}{3}}\left(\left(\frac{\pi\sqrt{3}}{6}-\frac{\ln(3)}{2}+\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+2\Gamma\left(\frac{2}{3}\right)x\,{}_3F_2\left(1,1,\frac{5}{3};2,2;3x\right)\right)}{\left((-1+3x)^{2}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)\mathrm{signum}\left(x-\frac{1}{3}\right)^{\frac{2}{3}}}+\frac{9(-1+3x)}{}\right.}{(-1+3x)^{\frac{2}{3}}}$
trager	$\frac{(1+9x)(-1+3x)^{\frac{1}{3}}}{x}-4\ln\left(\frac{\mathrm{RootOf}\left(\_Z^{2}-\_Z+1\right)^{2}x+\mathrm{RootOf}\left(\_Z^{2}-\_Z+1\right)(-1+3x)^{\frac{2}{3}}-\mathrm{RootOf}\left(\_Z^{2}-\_Z\right.}{x}\right.$

[In] int((-1+3*x)^(4/3)/x^2,x,method=_RETURNVERBOSE)

[Out] -4/3/GAMMA(2/3)*signum(x-1/3)^(4/3)/(-signum(x-1/3))^(4/3)*(3/4*GAMMA(2/3)/
x+3*(2+1/6*Pi*3^(1/2)-1/2*ln(3)+ln(x)+I*Pi)*GAMMA(2/3)-3/2*GAMMA(2/3)*x*hyp
ergeom([2/3,1,1],[2,3],3*x))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

$$\int \frac{(-1+3x)^{4/3}}{x^2}\,dx =$$
$$-\frac{4\sqrt{3}x\arctan\left(\frac{2}{3}\sqrt{3}(3x-1)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-2x\log\left((3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1\right)+4x\log\left((3x-1)^{\frac{1}{3}}+1\right)}{x}$$

[In] integrate((-1+3*x)^(4/3)/x^2,x, algorithm="fricas")

[Out] -(4*sqrt(3)*x*arctan(2/3*sqrt(3)*(3*x - 1)^(1/3) - 1/3*sqrt(3)) - 2*x*log((
3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) + 4*x*log((3*x - 1)^(1/3) + 1) - (9*x
 + 1)*(3*x - 1)^(1/3))/x

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 541, normalized size of antiderivative = 7.62

$$\int \frac{(-1+3x)^{4/3}}{x^2}\,dx = \frac{189\cdot\sqrt[3]{3}\left(x-\frac{1}{3}\right)^{\frac{4}{3}}e^{\frac{i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)} + \frac{84\cdot\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{\frac{i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)} + \frac{84\left(x-\frac{1}{3}\right)\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)} - \frac{84\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{i\pi}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)} + \frac{84\left(x-\frac{1}{3}\right)e^{\frac{2i\pi}{3}}\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{\frac{5i\pi}{3}}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)} + \frac{28\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)} - \frac{28e^{\frac{i\pi}{3}}\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{i\pi}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)} + \frac{28e^{\frac{2i\pi}{3}}\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{\frac{5i\pi}{3}}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((-1+3*x)**(4/3)/x**2,x)
```

```
[Out] 189*3**(1/3)*(x - 1/3)**(4/3)*exp(I*pi/3)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/
3)*gamma(10/3) + 3*exp(I*pi/3)*gamma(10/3)) + 84*3**(1/3)*(x - 1/3)**(1/3)*
exp(I*pi/3)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)
*gamma(10/3)) + 84*(x - 1/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I*pi/
3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*gam
ma(10/3)) - 84*(x - 1/3)*exp(I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_pol
ar(I*pi) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/
3)*gamma(10/3)) + 84*(x - 1/3)*exp(2*I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)
*exp_polar(5*I*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) +
 3*exp(I*pi/3)*gamma(10/3)) + 28*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I
*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)
*gamma(10/3)) - 28*exp(I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I*p
i) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*gam
ma(10/3)) + 28*exp(2*I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(5*I*p
```

```
i/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*g
amma(10/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{(-1+3x)^{4/3}}{x^2}\,dx = -4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\Big(2\,(3\,x-1)^{\frac{1}{3}}-1\Big)\right)+9\,(3\,x-1)^{\frac{1}{3}} + \frac{(3\,x-1)^{\frac{1}{3}}}{x}+2\,\log\Big((3\,x-1)^{\frac{2}{3}}-(3\,x-1)^{\frac{1}{3}}+1\Big)-4\,\log\Big((3\,x-1)^{\frac{1}{3}}+1\Big)$$

```
[In] integrate((-1+3*x)^(4/3)/x^2,x, algorithm="maxima")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(3*x - 1)^(1/3) - 1)) + 9*(3*x - 1)^(1/3)
+ (3*x - 1)^(1/3)/x + 2*log((3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) - 4*log(
(3*x - 1)^(1/3) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{(-1+3x)^{4/3}}{x^2}\,dx = -4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\Big(2\,(3\,x-1)^{\frac{1}{3}}-1\Big)\right)+9\,(3\,x-1)^{\frac{1}{3}} + \frac{(3\,x-1)^{\frac{1}{3}}}{x}+2\,\log\Big((3\,x-1)^{\frac{2}{3}}-(3\,x-1)^{\frac{1}{3}}+1\Big)-4\,\log\Big((3\,x-1)^{\frac{1}{3}}+1\Big)$$

```
[In] integrate((-1+3*x)^(4/3)/x^2,x, algorithm="giac")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(3*x - 1)^(1/3) - 1)) + 9*(3*x - 1)^(1/3)
+ (3*x - 1)^(1/3)/x + 2*log((3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) - 4*log(
(3*x - 1)^(1/3) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

$$\int \frac{(-1+3x)^{4/3}}{x^2}\,dx = 9\,(3\,x-1)^{1/3}$$
$$-4\,\ln\left(144\,(3\,x-1)^{1/3}+144\right)+\frac{(3\,x-1)^{1/3}}{x}+\ln\left(18-36\,(3\,x-1)^{1/3}+\sqrt{3}\,18\mathrm{i}\right)\,\left(2+\sqrt{3}\,2\mathrm{i}\right)-\ln\left(36\,(3\,x-1)\right.$$

```
[In] int((3*x - 1)^(4/3)/x^2,x)
```

```
[Out] 9*(3*x - 1)^(1/3) - 4*log(144*(3*x - 1)^(1/3) + 144) + (3*x - 1)^(1/3)/x +
log(3^(1/2)*18i - 36*(3*x - 1)^(1/3) + 18)*(3^(1/2)*2i + 2) - log(3^(1/2)*1
8i + 36*(3*x - 1)^(1/3) - 18)*(3^(1/2)*2i - 2)
```

3.295 $\int (4-3x)^{4/3}x^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int (4-3x)^{4/3}x^2\,dx=-\frac{16}{63}(4-3x)^{7/3}+\frac{4}{45}(4-3x)^{10/3}-\frac{1}{117}(4-3x)^{13/3}$$

```
[Out] -16/63*(4-3*x)^(7/3)+4/45*(4-3*x)^(10/3)-1/117*(4-3*x)^(13/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\int (4-3x)^{4/3}x^2\,dx=-\frac{1}{117}(4-3x)^{13/3}+\frac{4}{45}(4-3x)^{10/3}-\frac{16}{63}(4-3x)^{7/3}$$

```
[In] Int[(4 - 3*x)^(4/3)*x^2,x]
[Out] (-16*(4 - 3*x)^(7/3))/63 + (4*(4 - 3*x)^(10/3))/45 - (4 - 3*x)^(13/3)/117
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{16}{9}(4-3x)^{4/3}-\frac{8}{9}(4-3x)^{7/3}+\frac{1}{9}(4-3x)^{10/3}\right)\,dx\\&=-\frac{16}{63}(4-3x)^{7/3}+\frac{4}{45}(4-3x)^{10/3}-\frac{1}{117}(4-3x)^{13/3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int (4-3x)^{4/3}x^2\,dx = -\frac{1}{455}(4-3x)^{7/3}\left(16+28x+35x^2\right)$$

```
[In] Integrate[(4 - 3*x)^(4/3)*x^2,x]
[Out] -1/455*((4 - 3*x)^(7/3)*(16 + 28*x + 35*x^2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.45

method	result	size
meijerg	$\frac{4\,2^{\frac{2}{3}}x^3\,{}_2F_1\left(-\frac{4}{3},3;4;\frac{3x}{4}\right)}{3}$	18
gosper	$-\frac{(4-3x)^{\frac{7}{3}}(35x^2+28x+16)}{455}$	20
pseudoelliptic	$-\frac{(35x^2+28x+16)(-4+3x)^2(4-3x)^{\frac{1}{3}}}{455}$	27
derivativedivides	$-\frac{16(4-3x)^{\frac{7}{3}}}{63}+\frac{4(4-3x)^{\frac{10}{3}}}{45}-\frac{(4-3x)^{\frac{13}{3}}}{117}$	29
default	$-\frac{16(4-3x)^{\frac{7}{3}}}{63}+\frac{4(4-3x)^{\frac{10}{3}}}{45}-\frac{(4-3x)^{\frac{13}{3}}}{117}$	29
trager	$\left(-\frac{9}{13}x^4+\frac{84}{65}x^3-\frac{32}{455}x^2-\frac{64}{455}x-\frac{256}{455}\right)(4-3x)^{\frac{1}{3}}$	29
risch	$\frac{(315x^4-588x^3+32x^2+64x+256)(-4+3x)}{455(4-3x)^{\frac{2}{3}}}$	35

```
[In] int((4-3*x)^(4/3)*x^2,x,method=_RETURNVERBOSE)
[Out] 4/3*2^(2/3)*x^3*hypergeom([-4/3,3],[4],3/4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int (4-3x)^{4/3}x^2\,dx = -\frac{1}{455}\left(315\,x^4 - 588\,x^3 + 32\,x^2 + 64\,x + 256\right)(-3\,x+4)^{\frac{1}{3}}$$

```
[In] integrate((4-3*x)^(4/3)*x^2,x, algorithm="fricas")
[Out] -1/455*(315*x^4 - 588*x^3 + 32*x^2 + 64*x + 256)*(-3*x + 4)^(1/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 178, normalized size of antiderivative = 4.45

$$\int (4-3x)^{4/3}x^2\,dx = \begin{cases} -\frac{9x^4\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{13} + \frac{84x^3\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{65} - \frac{32x^2\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455} - \frac{64x\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455} - \frac{256\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455} \\ -\frac{9x^4\sqrt[3]{4-3x}}{13} + \frac{84x^3\sqrt[3]{4-3x}}{65} - \frac{32x^2\sqrt[3]{4-3x}}{455} - \frac{64x\sqrt[3]{4-3x}}{455} - \frac{256\sqrt[3]{4-3x}}{455} \end{cases}$$

```
[In] integrate((4-3*x)**(4/3)*x**2,x)
[Out] Piecewise((-9*x**4*(3*x - 4)**(1/3)*exp(I*pi/3)/13 + 84*x**3*(3*x - 4)**(1/
3)*exp(I*pi/3)/65 - 32*x**2*(3*x - 4)**(1/3)*exp(I*pi/3)/455 - 64*x*(3*x -
4)**(1/3)*exp(I*pi/3)/455 - 256*(3*x - 4)**(1/3)*exp(I*pi/3)/455, Abs(x) >
4/3), (-9*x**4*(4 - 3*x)**(1/3)/13 + 84*x**3*(4 - 3*x)**(1/3)/65 - 32*x**2*
(4 - 3*x)**(1/3)/455 - 64*x*(4 - 3*x)**(1/3)/455 - 256*(4 - 3*x)**(1/3)/455
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int (4-3x)^{4/3}x^2\,dx = -\frac{1}{117}\,(-3\,x+4)^{\frac{13}{3}} + \frac{4}{45}\,(-3\,x+4)^{\frac{10}{3}} - \frac{16}{63}\,(-3\,x+4)^{\frac{7}{3}}$$

```
[In] integrate((4-3*x)^(4/3)*x^2,x, algorithm="maxima")
[Out] -1/117*(-3*x + 4)^(13/3) + 4/45*(-3*x + 4)^(10/3) - 16/63*(-3*x + 4)^(7/3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int (4-3x)^{4/3}x^2\,dx = -\frac{1}{117}\,(3\,x-4)^4(-3\,x+4)^{\frac{1}{3}} - \frac{4}{45}\,(3\,x-4)^3(-3\,x+4)^{\frac{1}{3}} - \frac{16}{63}\,(3\,x-4)^2(-3\,x+4)^{\frac{1}{3}}$$

```
[In] integrate((4-3*x)^(4/3)*x^2,x, algorithm="giac")
[Out] -1/117*(3*x - 4)^4*(-3*x + 4)^(1/3) - 4/45*(3*x - 4)^3*(-3*x + 4)^(1/3) - 1
6/63*(3*x - 4)^2*(-3*x + 4)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int (4-3x)^{4/3}x^2\,dx = -\frac{(4-3\,x)^{7/3}\,\left(1092\,x+35\,(3\,x-4)^2-416\right)}{4095}$$

```
[In] int(x^2*(4 - 3*x)^(4/3),x)
[Out] -((4 - 3*x)^(7/3)*(1092*x + 35*(3*x - 4)^2 - 416))/4095
```

3.296 $\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 4\left(1-2\sqrt[3]{x}\right)^{3/4} + 6\arctan\left(\sqrt[4]{1-2\sqrt[3]{x}}\right) - 6\text{arctanh}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)$$

```
[Out] 4*(1-2*x^(1/3))^(3/4)+6*arctan((1-2*x^(1/3))^(1/4))-6*arctanh((1-2*x^(1/3))
^(1/4))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {272, 52, 65, 304, 209, 212}

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 6\arctan\left(\sqrt[4]{1-2\sqrt[3]{x}}\right) - 6\text{arctanh}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right) + 4\left(1-2\sqrt[3]{x}\right)^{3/4}$$

```
[In] Int[(1 - 2*x^(1/3))^(3/4)/x,x]
```

```
[Out] 4*(1 - 2*x^(1/3))^(3/4) + 6*ArcTan[(1 - 2*x^(1/3))^(1/4)] - 6*ArcTanh[(1 -
2*x^(1/3))^(1/4)]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{(1-2x)^{3/4}}{x}\,dx, x, \sqrt[3]{x}\right)\\
&= 4\left(1-2\sqrt[3]{x}\right)^{3/4} + 3\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-2x}x}\,dx, x, \sqrt[3]{x}\right)\\
&= 4\left(1-2\sqrt[3]{x}\right)^{3/4} - 6\text{Subst}\left(\int \frac{x^2}{\frac{1}{2}-\frac{x^4}{2}}\,dx, x, \sqrt[4]{1-2\sqrt[3]{x}}\right)
\end{aligned}$$

$$
\begin{aligned}
&= 4\left(1-2\sqrt[3]{x}\right)^{3/4} \\
&\quad - 6\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1-2\sqrt[3]{x}}\right) + 6\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{1-2\sqrt[3]{x}}\right) \\
&= 4\left(1-2\sqrt[3]{x}\right)^{3/4} + 6\arctan\left(\sqrt[4]{1-2\sqrt[3]{x}}\right) - 6\text{arctanh}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$
\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 4\left(1-2\sqrt[3]{x}\right)^{3/4} + 6\arctan\left(\sqrt[4]{1-2\sqrt[3]{x}}\right) - 6\text{arctanh}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)
$$

```
[In] Integrate[(1 - 2*x^(1/3))^(3/4)/x,x]
[Out] 4*(1 - 2*x^(1/3))^(3/4) + 6*ArcTan[(1 - 2*x^(1/3))^(1/4)] - 6*ArcTanh[(1 - 
2*x^(1/3))^(1/4)]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

method	result
derivativedivides	$4\left(1-2x^{\frac{1}{3}}\right)^{\frac{3}{4}}+3\ln\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}-1\right)-3\ln\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}+1\right)+6\arctan\left(\left(1-2\right.\right.$
default	$4\left(1-2x^{\frac{1}{3}}\right)^{\frac{3}{4}}+3\ln\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}-1\right)-3\ln\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}+1\right)+6\arctan\left(\left(1-2\right.\right.$
meijerg	$-\frac{9\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)\left(-\frac{4\left(\frac{4}{3}-2\ln(2)-\frac{\pi}{2}+\frac{\ln(x)}{3}+i\pi\right)\pi\sqrt{2}}{3\Gamma\left(\frac{3}{4}\right)}+\frac{2\pi\sqrt{2}\,x^{\frac{1}{3}}\,{}_{3}F_{2}\left(\frac{1}{4},1,1;2,2;2x^{\frac{1}{3}}\right)}{\Gamma\left(\frac{3}{4}\right)}\right)}{8\pi}$

```
[In] int((1-2*x^(1/3))^(3/4)/x,x,method=_RETURNVERBOSE)
[Out] 4*(1-2*x^(1/3))^(3/4)+3*ln((1-2*x^(1/3))^(1/4)-1)-3*ln((1-2*x^(1/3))^(1/4)+
1)+6*arctan((1-2*x^(1/3))^(1/4))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 4\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{3}{4}} + 6\,\arctan\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right) + 3\,\log\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right)$$

```
[In] integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="fricas")
```

```
[Out] 4*(-2*x^(1/3) + 1)^(3/4) + 6*arctan((-2*x^(1/3) + 1)^(1/4)) - 3*log((-2*x^(
1/3) + 1)^(1/4) + 1) + 3*log((-2*x^(1/3) + 1)^(1/4) - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = -\frac{3\cdot 2^{\frac{3}{4}}\sqrt[4]{x}e^{\frac{3i\pi}{4}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{1}{2\sqrt[3]{x}}\right)}{\Gamma\left(\frac{1}{4}\right)}$$

```
[In] integrate((1-2*x**(1/3))**(3/4)/x,x)
```

```
[Out] -3*2**(3/4)*x**(1/4)*exp(3*I*pi/4)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,),
1/(2*x**(1/3)))/gamma(1/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 4\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{3}{4}} + 6\,\arctan\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right) + 3\,\log\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right)$$

```
[In] integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="maxima")
```

```
[Out] 4*(-2*x^(1/3) + 1)^(3/4) + 6*arctan((-2*x^(1/3) + 1)^(1/4)) - 3*log((-2*x^(
1/3) + 1)^(1/4) + 1) + 3*log((-2*x^(1/3) + 1)^(1/4) - 1)
```

Giac [A] (verification not implemented)

none

Time = 2.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 4\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{3}{4}} + 6\,\arctan\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right) + 3\,\log\left(\left|\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right|\right)$$

```
[In] integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="giac")
```

```
[Out] 4*(-2*x^(1/3) + 1)^(3/4) + 6*arctan((-2*x^(1/3) + 1)^(1/4)) - 3*log((-2*x^(
1/3) + 1)^(1/4) + 1) + 3*log(abs((-2*x^(1/3) + 1)^(1/4) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 6\,\mathrm{atan}\left(\left(1-2\,x^{1/3}\right)^{1/4}\right) - 6\,\mathrm{atanh}\left(\left(1-2\,x^{1/3}\right)^{1/4}\right) + 4\,\left(1-2\,x^{1/3}\right)^{3/4}$$

```
[In] int((1 - 2*x^(1/3))^(3/4)/x,x)
```

```
[Out] 6*atan((1 - 2*x^(1/3))^(1/4)) - 6*atanh((1 - 2*x^(1/3))^(1/4)) + 4*(1 - 2*x
^(1/3))^(3/4)
```

3.297 $\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx=-\frac{27}{2}\sqrt[4]{3-2\sqrt{x}}+\frac{27}{10}\left(3-2\sqrt{x}\right)^{5/4}-\frac{1}{2}\left(3-2\sqrt{x}\right)^{9/4}+\frac{1}{26}\left(3-2\sqrt{x}\right)^{13/4}$$

```
[Out] -27/2*(3-2*x^(1/2))^(1/4)+27/10*(3-2*x^(1/2))^(5/4)-1/2*(3-2*x^(1/2))^(9/4)
+1/26*(3-2*x^(1/2))^(13/4)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx=\frac{1}{26}\left(3-2\sqrt{x}\right)^{13/4}-\frac{1}{2}\left(3-2\sqrt{x}\right)^{9/4}+\frac{27}{10}\left(3-2\sqrt{x}\right)^{5/4}-\frac{27}{2}\sqrt[4]{3-2\sqrt{x}}$$

```
[In] Int[x/(3 - 2*Sqrt[x])^(3/4),x]
```

```
[Out] (-27*(3 - 2*Sqrt[x])^(1/4))/2 + (27*(3 - 2*Sqrt[x])^(5/4))/10 - (3 - 2*Sqrt
[x])^(9/4)/2 + (3 - 2*Sqrt[x])^(13/4)/26
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^3}{(3-2x)^{3/4}}\,dx, x, \sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int \left(\frac{27}{8(3-2x)^{3/4}} - \frac{27}{8}\sqrt[4]{3-2x} + \frac{9}{8}(3-2x)^{5/4} - \frac{1}{8}(3-2x)^{9/4}\right)dx, x, \sqrt{x}\right)$$
$$= -\frac{27}{2}\sqrt[4]{3-2\sqrt{x}} + \frac{27}{10}\left(3-2\sqrt{x}\right)^{5/4} - \frac{1}{2}\left(3-2\sqrt{x}\right)^{9/4} + \frac{1}{26}\left(3-2\sqrt{x}\right)^{13/4}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.52

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = -\frac{4}{65}\sqrt[4]{3-2\sqrt{x}}\left(144+24\sqrt{x}+10x+5x^{3/2}\right)$$

```
[In] Integrate[x/(3 - 2*Sqrt[x])^(3/4),x]
[Out] (-4*(3 - 2*Sqrt[x])^(1/4)*(144 + 24*Sqrt[x] + 10*x + 5*x^(3/2)))/65
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.29

method	result	size
meijerg	$\frac{3^{\frac{1}{4}} x^{2}\, {}_{2}F_{1}\left(\frac{3}{4},4;5;\frac{2\sqrt{x}}{3}\right)}{6}$	20
derivativedivides	$-\frac{27\left(3-2\sqrt{x}\right)^{\frac{1}{4}}}{2}+\frac{27\left(3-2\sqrt{x}\right)^{\frac{5}{4}}}{10}-\frac{\left(3-2\sqrt{x}\right)^{\frac{9}{4}}}{2}+\frac{\left(3-2\sqrt{x}\right)^{\frac{13}{4}}}{26}$	46
default	$-\frac{27\left(3-2\sqrt{x}\right)^{\frac{1}{4}}}{2}+\frac{27\left(3-2\sqrt{x}\right)^{\frac{5}{4}}}{10}-\frac{\left(3-2\sqrt{x}\right)^{\frac{9}{4}}}{2}+\frac{\left(3-2\sqrt{x}\right)^{\frac{13}{4}}}{26}$	46

```
[In] int(x/(3-2*x^(1/2))^(3/4),x,method=_RETURNVERBOSE)
[Out] 1/6*3^(1/4)*x^2*hypergeom([3/4,4],[5],2/3*x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.36

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = -\frac{4}{65}\left((5x+24)\sqrt{x}+10x+144\right)\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}$$

```
[In] integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="fricas")
[Out] -4/65*((5*x + 24)*sqrt(x) + 10*x + 144)*(-2*sqrt(x) + 3)^(1/4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 3303, normalized size of antiderivative = 47.87

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(3-2*x**(1/2))**(3/4),x)
[Out] Piecewise((1280*3**(1/4)*x**(25/2)*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-
37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(
17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9
+ 47385*3**(1/4)*x**8) + 26304*3**(1/4)*x**(23/2)*(2*sqrt(x) - 3)**(1/4)*ex
p(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 18954
0*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900
*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 200016*3**(1/4)*x**(21/2)*(2*sqrt(x
) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x
**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4
)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 331776*sqrt(3)*x**(
21/2)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1
/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/
4)*x**9 + 47385*3**(1/4)*x**8) - 2123820*3**(1/4)*x**(19/2)*(2*sqrt(x) - 3)
**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/
2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**1
0 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 2488320*sqrt(3)*x**(19/2)
/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x
**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x*
*9 + 47385*3**(1/4)*x**8) - 1609632*3**(1/4)*x**(17/2)*(2*sqrt(x) - 3)**(1/
4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) -
189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 3
15900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 1679616*sqrt(3)*x**(17/2)/(-37
```

440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17
/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 +
47385*3**(1/4)*x**8) - 8960*3**(1/4)*x**12*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*
pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1
/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/
4)*x**9 + 47385*3**(1/4)*x**8) - 18432*3**(1/4)*x**11*(2*sqrt(x) - 3)**(1/4
)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 1
89540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 31
5900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 36864*sqrt(3)*x**11/(-37440*3**
(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4
160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3
(1/4)*x8) + 965520*3**(1/4)*x**10*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)
/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x
(17/2) + 4160*3(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x*
*9 + 47385*3**(1/4)*x**8) + 1244160*sqrt(3)*x**10/(-37440*3**(1/4)*x**(21/2
) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x
11 + 140400*3(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8)
+ 2548584*3**(1/4)*x**9*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1
/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 416
0*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**
(1/4)*x**8) + 2799360*sqrt(3)*x**9/(-37440*3**(1/4)*x**(21/2) - 280800*3**(
1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3
(1/4)*x10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 419904*3**(1/
4)*x**8*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) -
280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11
 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 41
9904*sqrt(3)*x**8/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) -
189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 3
15900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8), Abs(sqrt(x)) > 3/2), (-1280*3**
(1/4)*x**(25/2)*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 280800*
3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 1404
00*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 26304*3**
(1/4)*x**(23/2)*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 280800*
3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 1404
00*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 200016*3*
*(1/4)*x**(21/2)*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 280800
*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140
400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 331776*s
qrt(3)*x**(21/2)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 1
89540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 31
5900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 2123820*3**(1/4)*x**(19/2)*(3 -
 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) -
 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 +
315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 2488320*sqrt(3)*x**(19/2)/(-3
7440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(1

```
7/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 +
 47385*3**(1/4)*x**8) + 1609632*3**(1/4)*x**(17/2)*(3 - 2*sqrt(x))**(1/4)/(
-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**
(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9
 + 47385*3**(1/4)*x**8) - 1679616*sqrt(3)*x**(17/2)/(-37440*3**(1/4)*x**(21
/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)
*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8
) + 8960*3**(1/4)*x**12*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) -
 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**1
1 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 1
8432*3**(1/4)*x**11*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 280
800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 +
140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 36864
*sqrt(3)*x**11/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189
540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 3159
00*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 965520*3**(1/4)*x**10*(3 - 2*sqrt
(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540
*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*
3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 1244160*sqrt(3)*x**10/(-37440*3**(1/
4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160
*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(
1/4)*x**8) - 2548584*3**(1/4)*x**9*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*
x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3*
*(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4
)*x**8) + 2799360*sqrt(3)*x**9/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)
*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1
/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 419904*3**(1/4)*x
**8*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**
(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*
x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 419904*sqrt(3)*x**8/(
-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**
(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9
 + 47385*3**(1/4)*x**8), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = \frac{1}{26}\left(-2\sqrt{x}+3\right)^{\frac{13}{4}} - \frac{1}{2}\left(-2\sqrt{x}+3\right)^{\frac{9}{4}} + \frac{27}{10}\left(-2\sqrt{x}+3\right)^{\frac{5}{4}} - \frac{27}{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}$$

```
[In] integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="maxima")
```

```
[Out] 1/26*(-2*sqrt(x) + 3)^(13/4) - 1/2*(-2*sqrt(x) + 3)^(9/4) + 27/10*(-2*sqrt(
x) + 3)^(5/4) - 27/2*(-2*sqrt(x) + 3)^(1/4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = -\frac{1}{26}\left(2\sqrt{x}-3\right)^{3}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}} - \frac{1}{2}\left(2\sqrt{x}-3\right)^{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}} + \frac{27}{10}\left(-2\sqrt{x}+3\right)^{\frac{5}{4}} - \frac{27}{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}$$

```
[In] integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="giac")
```

```
[Out] -1/26*(2*sqrt(x) - 3)^3*(-2*sqrt(x) + 3)^(1/4) - 1/2*(2*sqrt(x) - 3)^2*(-2*
sqrt(x) + 3)^(1/4) + 27/10*(-2*sqrt(x) + 3)^(5/4) - 27/2*(-2*sqrt(x) + 3)^(
1/4)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = \frac{27\left(3-2\sqrt{x}\right)^{5/4}}{10} - \frac{27\left(3-2\sqrt{x}\right)^{1/4}}{2} - \frac{\left(3-2\sqrt{x}\right)^{9/4}}{2} + \frac{\left(3-2\sqrt{x}\right)^{13/4}}{26}$$

```
[In] int(x/(3 - 2*x^(1/2))^(3/4),x)
```

```
[Out] (27*(3 - 2*x^(1/2))^(5/4))/10 - (27*(3 - 2*x^(1/2))^(1/4))/2 - (3 - 2*x^(1/
2))^(9/4)/2 + (3 - 2*x^(1/2))^(13/4)/26
```

3.298 $\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 193

$$\begin{aligned}\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx &= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x}-\frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}}\\ &\quad-\frac{5\arctan\left(1-\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}\right)}{2\sqrt{2}}+\frac{5\arctan\left(1+\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}\right)}{2\sqrt{2}}\\ &\quad-\frac{5\log\left(1-\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}+\sqrt{-1+2\sqrt{x}}\right)}{4\sqrt{2}}\\ &\quad+\frac{5\log\left(1+\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}+\sqrt{-1+2\sqrt{x}}\right)}{4\sqrt{2}}\end{aligned}$$

```
[Out] 5/4*arctan(-1+2^(1/2)*(-1+2*x^(1/2))^(1/4))*2^(1/2)+5/4*arctan(1+2^(1/2)*(-
1+2*x^(1/2))^(1/4))*2^(1/2)-5/8*ln(1-2^(1/2)*(-1+2*x^(1/2))^(1/4)+(-1+2*x^(
1/2))^(1/2))*2^(1/2)+5/8*ln(1+2^(1/2)*(-1+2*x^(1/2))^(1/4)+(-1+2*x^(1/2))^(
1/2))*2^(1/2)-5/2*(-1+2*x^(1/2))^(1/4)/x^(1/2)-(-1+2*x^(1/2))^(5/4)/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {272, 43, 65, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx = -\frac{5\arctan\left(1-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}\right)}{2\sqrt{2}} + \frac{5\arctan\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{2\sqrt{2}} - \frac{\left(2\sqrt{x}-1\right)^{5/4}}{x} - \frac{5\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{x}} - \frac{5\log\left(\sqrt{2\sqrt{x}-1}-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{4\sqrt{2}} + \frac{5\log\left(\sqrt{2\sqrt{x}-1}+\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{4\sqrt{2}}$$

```
[In] Int[(-1 + 2*Sqrt[x])^(5/4)/x^2,x]
```

```
[Out] -((-1 + 2*Sqrt[x])^(5/4)/x) - (5*(-1 + 2*Sqrt[x])^(1/4))/(2*Sqrt[x]) - (5*A
rcTan[1 - Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4)])/(2*Sqrt[2]) + (5*ArcTan[1 + Sqrt
[2]*(-1 + 2*Sqrt[x])^(1/4)])/(2*Sqrt[2]) - (5*Log[1 - Sqrt[2]*(-1 + 2*Sqrt[
x])^(1/4) + Sqrt[-1 + 2*Sqrt[x]]])/(4*Sqrt[2]) + (5*Log[1 + Sqrt[2]*(-1 + 2
*Sqrt[x])^(1/4) + Sqrt[-1 + 2*Sqrt[x]]])/(4*Sqrt[2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{(-1+2x)^{5/4}}{x^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x} + \frac{5}{2}\text{Subst}\left(\int \frac{\sqrt[4]{-1+2x}}{x^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x} - \frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}} + \frac{5}{4}\text{Subst}\left(\int \frac{1}{x(-1+2x)^{3/4}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x} - \frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}} + \frac{5}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{x^4}{2}}\,dx, x, \sqrt[4]{-1+2\sqrt{x}}\right)$$

$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x} - \frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}} + \frac{5}{4}\text{Subst}\left(\int \frac{1-x^2}{\frac{1}{2}+\frac{x^4}{2}}\,dx, x, \sqrt[4]{-1+2\sqrt{x}}\right) + \frac{5}{4}\text{Subst}\left(\int \frac{1+x^2}{\frac{1}{2}+\frac{x^4}{2}}\,dx, x, \sqrt[4]{-1+2\sqrt{x}}\right)$$

$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x} - \frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}} + \frac{5}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt[4]{-1+2\sqrt{x}}\right) + \frac{5}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt[4]{-1+2\sqrt{x}}\right) - \frac{5\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt[4]{-1+2\sqrt{x}}\right)}{4\sqrt{2}} - \frac{5\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt[4]{-1+2\sqrt{x}}\right)}{4\sqrt{2}}$$

$$= -\frac{(-1+2\sqrt{x})^{5/4}}{x} - \frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}} - \frac{5\log\left(1-\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}+\sqrt{-1+2\sqrt{x}}\right)}{4\sqrt{2}} + \frac{5\log\left(1+\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}+\sqrt{-1+2\sqrt{x}}\right)}{4\sqrt{2}} + \frac{5\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}\right)}{2\sqrt{2}} - \frac{5\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}\right)}{2\sqrt{2}}$$

$$= -\frac{(-1+2\sqrt{x})^{5/4}}{x} - \frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}} - \frac{5\arctan\left(1-\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}\right)}{2\sqrt{2}} + \frac{5\arctan\left(1+\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}\right)}{2\sqrt{2}} - \frac{5\log\left(1-\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}+\sqrt{-1+2\sqrt{x}}\right)}{4\sqrt{2}} + \frac{5\log\left(1+\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}+\sqrt{-1+2\sqrt{x}}\right)}{4\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

$$\int \frac{(-1+2\sqrt{x})^{5/4}}{x^2}\,dx = \frac{2(2-9\sqrt{x})\sqrt[4]{-1+2\sqrt{x}} + 5\sqrt{2}x\arctan\left(\frac{-1+\sqrt{-1+2\sqrt{x}}}{\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}}\right) + 5\sqrt{2}x\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\ldots}}{1+\ldots}\right.}{4x}$$

```
[In] Integrate[(-1 + 2*Sqrt[x])^(5/4)/x^2,x]
```

```
[Out] (2*(2 - 9*Sqrt[x])*(-1 + 2*Sqrt[x])^(1/4) + 5*Sqrt[2]*x*ArcTan[(-1 + Sqrt[-
1 + 2*Sqrt[x]])/(Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4))] + 5*Sqrt[2]*x*ArcTanh[(Sq
rt[2]*(-1 + 2*Sqrt[x])^(1/4))/(1 + Sqrt[-1 + 2*Sqrt[x]])])/(4*x)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.44

method	result
meijerg	$\frac{5\,\mathrm{signum}\left(-1+2\sqrt{x}\right)^{\frac{5}{4}}\left(-\frac{2\Gamma\left(\frac{3}{4}\right)}{5x}+\frac{2\Gamma\left(\frac{3}{4}\right)}{\sqrt{x}}+\frac{\left(-2\ln(2)+\frac{\pi}{2}-\frac{3}{2}+\frac{\ln(x)}{2}+i\pi\right)\Gamma\left(\frac{3}{4}\right)}{2}+\frac{\Gamma\left(\frac{3}{4}\right)\sqrt{x}\,{}_3F_2\left(1,1,\frac{7}{4};2,4;2\sqrt{x}\right)}{4}\right)}{2\Gamma\left(\frac{3}{4}\right)\left(-\mathrm{signum}\left(-1+2\sqrt{x}\right)\right)^{\frac{5}{4}}}$
derivativedivides	$\frac{-\frac{9\left(-1+2\sqrt{x}\right)^{\frac{5}{4}}}{4}-\frac{5\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}}{4}}{x}+\frac{5\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}+\sqrt{-1+2\sqrt{x}}}{1-\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}+\sqrt{-1+2\sqrt{x}}}\right)+2\arctan\left(1+\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}\right)+2\,\mathrm{arcta}\right.}{8}$
default	$\frac{-\frac{9\left(-1+2\sqrt{x}\right)^{\frac{5}{4}}}{4}-\frac{5\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}}{4}}{x}+\frac{5\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}+\sqrt{-1+2\sqrt{x}}}{1-\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}+\sqrt{-1+2\sqrt{x}}}\right)+2\arctan\left(1+\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}\right)+2\,\mathrm{arcta}\right.}{8}$

```
[In] int((-1+2*x^(1/2))^(5/4)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/2/GAMMA(3/4)*signum(-1+2*x^(1/2))^(5/4)/(-signum(-1+2*x^(1/2)))^(5/4)*(-2
/5*GAMMA(3/4)/x+2*GAMMA(3/4)/x^(1/2)+1/2*(-2*ln(2)+1/2*Pi-3/2+1/2*ln(x)+I*P
i)*GAMMA(3/4)+1/4*GAMMA(3/4)*x^(1/2)*hypergeom([1,1,7/4],[2,4],2*x^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

$$\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx = \frac{(5i+5)\ \sqrt{2}x\log\left((i+1)\ \sqrt{2}+2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)-(5i-5)\ \sqrt{2}x\log\left(-(i-1)\ \sqrt{2}\right.}{}$$

```
[In] integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/8*((5*I + 5)*sqrt(2)*x*log((I + 1)*sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4)) - (
5*I - 5)*sqrt(2)*x*log(-(I - 1)*sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4)) + (5*I -
 5)*sqrt(2)*x*log((I - 1)*sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4)) - (5*I + 5)*sq
rt(2)*x*log(-(I + 1)*sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4)) - 4*(9*sqrt(x) - 2)
*(2*sqrt(x) - 1)^(1/4))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.75 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.23

$$\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx = -\frac{4\cdot\sqrt[4]{2}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix} -\frac{5}{4},\frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{e^{2i\pi}}{2\sqrt{x}}\right)}{x^{\frac{3}{8}}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((-1+2*x**(1/2))**(5/4)/x**2,x)
```

```
[Out] -4*2**(1/4)*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), exp_polar(2*I*pi)/(2*sqrt
(x)))/(x**(3/8)*gamma(7/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.81

$$\begin{aligned}\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx &= \frac{5}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right) \\ &+ \frac{5}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right) \\ &+ \frac{5}{8}\sqrt{2}\log\left(\sqrt{2}(2\sqrt{x}-1)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1\right) \\ &- \frac{5}{8}\sqrt{2}\log\left(-\sqrt{2}(2\sqrt{x}-1)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1\right) - \frac{9\left(2\sqrt{x}-1\right)^{\frac{5}{4}}+5\left(2\sqrt{x}-1\right)^{\frac{1}{4}}}{\left(2\sqrt{x}-1\right)^2+4\sqrt{x}-1}\end{aligned}$$

```
[In] integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="maxima")
```

```
[Out] 5/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4))) + 5/4*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(2*sqrt(x) - 1)^(1/4))) + 5/8*sqrt(
2)*log(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 5/8*sqrt(
2)*log(-sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - (9*(2*sq
rt(x) - 1)^(5/4) + 5*(2*sqrt(x) - 1)^(1/4))/((2*sqrt(x) - 1)^2 + 4*sqrt(x)
- 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.74

$$\int \frac{(-1+2\sqrt{x})^{5/4}}{x^2}\,dx = \frac{5}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right) + \frac{5}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right) + \frac{5}{8}\sqrt{2}\log\left(\sqrt{2}(2\sqrt{x}-1)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1\right) - \frac{5}{8}\sqrt{2}\log\left(-\sqrt{2}(2\sqrt{x}-1)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1\right) - \frac{9\left(2\sqrt{x}-1\right)^{\frac{5}{4}}+5\left(2\sqrt{x}-1\right)^{\frac{1}{4}}}{4\,x}$$

```
[In] integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="giac")
```

```
[Out] 5/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4))) + 5/4*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(2*sqrt(x) - 1)^(1/4))) + 5/8*sqrt(
2)*log(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 5/8*sqrt(
2)*log(-sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 1/4*(9*(
2*sqrt(x) - 1)^(5/4) + 5*(2*sqrt(x) - 1)^(1/4))/x
```

Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.40

$$\int \frac{(-1+2\sqrt{x})^{5/4}}{x^2}\,dx = -\frac{5\left(2\sqrt{x}-1\right)^{1/4}}{4\,x} - \frac{9\left(2\sqrt{x}-1\right)^{5/4}}{4\,x} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(2\sqrt{x}-1\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{5}{4}+\frac{5}{4}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(2\sqrt{x}-1\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{5}{4}-\frac{5}{4}\mathrm{i}\right)$$

```
[In] int((2*x^(1/2) - 1)^(5/4)/x^2,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*(2*x^(1/2) - 1)^(1/4)*(1/2 - 1i/2))*(5/4 + 5i/4) - (9*
(2*x^(1/2) - 1)^(5/4))/(4*x) - (5*(2*x^(1/2) - 1)^(1/4))/(4*x) + 2^(1/2)*at
an(2^(1/2)*(2*x^(1/2) - 1)^(1/4)*(1/2 + 1i/2))*(5/4 - 5i/4)
```

3.299 $\int x^6 \sqrt[3]{1+x^7}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int x^6 \sqrt[3]{1+x^7}\,dx = \frac{3}{28}\left(1+x^7\right)^{4/3}$$

```
[Out] 3/28*(x^7+1)^(4/3)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {267}

$$\int x^6 \sqrt[3]{1+x^7}\,dx = \frac{3}{28}\left(x^7+1\right)^{4/3}$$

```
[In] Int[x^6*(1 + x^7)^(1/3),x]
```

```
[Out] (3*(1 + x^7)^(4/3))/28
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{3}{28}\left(1+x^7\right)^{4/3}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int x^6\sqrt[3]{1+x^7}\,dx=\frac{3}{28}\left(1+x^7\right)^{4/3}$$

```
[In] Integrate[x^6*(1 + x^7)^(1/3),x]
[Out] (3*(1 + x^7)^(4/3))/28
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{3(x^7+1)^{\frac{4}{3}}}{28}$	10
default	$\frac{3(x^7+1)^{\frac{4}{3}}}{28}$	10
risch	$\frac{3(x^7+1)^{\frac{4}{3}}}{28}$	10
pseudoelliptic	$\frac{3(x^7+1)^{\frac{4}{3}}}{28}$	10
trager	$\left(\frac{3}{28}+\frac{3x^7}{28}\right)\left(x^7+1\right)^{\frac{1}{3}}$	16
meijerg	$\frac{x^7{}_2F_1\left(-\frac{1}{3},1;2;-x^7\right)}{7}$	17
gosper	$\frac{3(1+x)\left(x^6-x^5+x^4-x^3+x^2-x+1\right)\left(x^7+1\right)^{\frac{1}{3}}}{28}$	37

```
[In] int(x^6*(x^7+1)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/28*(x^7+1)^(4/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^6\sqrt[3]{1+x^7}\,dx=\frac{3}{28}\left(x^7+1\right)^{\frac{4}{3}}$$

```
[In] integrate(x^6*(x^7+1)^(1/3),x, algorithm="fricas")
[Out] 3/28*(x^7 + 1)^(4/3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int x^6 \sqrt[3]{1+x^7}\,dx = \frac{3x^7\sqrt[3]{x^7+1}}{28} + \frac{3\sqrt[3]{x^7+1}}{28}$$

```
[In] integrate(x**6*(x**7+1)**(1/3),x)
[Out] 3*x**7*(x**7 + 1)**(1/3)/28 + 3*(x**7 + 1)**(1/3)/28
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^6 \sqrt[3]{1+x^7}\,dx = \frac{3}{28}\left(x^7+1\right)^{\frac{4}{3}}$$

```
[In] integrate(x^6*(x^7+1)^(1/3),x, algorithm="maxima")
[Out] 3/28*(x^7 + 1)^(4/3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^6 \sqrt[3]{1+x^7}\,dx = \frac{3}{28}\left(x^7+1\right)^{\frac{4}{3}}$$

```
[In] integrate(x^6*(x^7+1)^(1/3),x, algorithm="giac")
[Out] 3/28*(x^7 + 1)^(4/3)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^6 \sqrt[3]{1+x^7}\,dx = \frac{3\left(x^7+1\right)^{4/3}}{28}$$

```
[In] int(x^6*(x^7 + 1)^(1/3),x)
[Out] (3*(x^7 + 1)^(4/3))/28
```

3.300 $\int \frac{x^6}{(1+x^7)^{5/3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^6}{(1+x^7)^{5/3}} \, dx = -\frac{3}{14(1+x^7)^{2/3}}$$

```
[Out] -3/14/(x^7+1)^(2/3)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$\int \frac{x^6}{(1+x^7)^{5/3}} \, dx = -\frac{3}{14(x^7+1)^{2/3}}$$

```
[In] Int[x^6/(1 + x^7)^(5/3),x]
```

```
[Out] -3/(14*(1 + x^7)^(2/3))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{3}{14(1+x^7)^{2/3}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14\,(1+x^7)^{2/3}}$$

```
[In] Integrate[x^6/(1 + x^7)^(5/3),x]
[Out] -3/(14*(1 + x^7)^(2/3))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$	10
default	$-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$	10
trager	$-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$	10
risch	$-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$	10
pseudoelliptic	$-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$	10
meijerg	$\frac{x^7\,{}_2F_1\left(1,\frac{5}{3};2;-x^7\right)}{7}$	17
gosper	$-\frac{3(1+x)\left(x^6-x^5+x^4-x^3+x^2-x+1\right)}{14(x^7+1)^{\frac{5}{3}}}$	37

```
[In] int(x^6/(x^7+1)^(5/3),x,method=_RETURNVERBOSE)
[Out] -3/14/(x^7+1)^(2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14\,(x^7+1)^{\frac{2}{3}}}$$

```
[In] integrate(x^6/(x^7+1)^(5/3),x, algorithm="fricas")
[Out] -3/14/(x^7 + 1)^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14\,(x^7+1)^{\frac{2}{3}}}$$

```
[In] integrate(x**6/(x**7+1)**(5/3),x)
[Out] -3/(14*(x**7 + 1)**(2/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14\,(x^7+1)^{\frac{2}{3}}}$$

```
[In] integrate(x^6/(x^7+1)^(5/3),x, algorithm="maxima")
[Out] -3/14/(x^7 + 1)^(2/3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14\,(x^7+1)^{\frac{2}{3}}}$$

```
[In] integrate(x^6/(x^7+1)^(5/3),x, algorithm="giac")
[Out] -3/14/(x^7 + 1)^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14\,(x^7+1)^{2/3}}$$

```
[In] int(x^6/(x^7 + 1)^(5/3),x)
[Out] -3/(14*(x^7 + 1)^(2/3))
```

3.301 $\int \frac{1}{x(-27+2x^7)^{2/3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{1}{x(-27+2x^7)^{2/3}} \, dx = -\frac{\arctan\left(\frac{3-2\sqrt[3]{-27+2x^7}}{3\sqrt{3}}\right)}{21\sqrt{3}} - \frac{\log(x)}{18} + \frac{1}{42}\log\left(3+\sqrt[3]{-27+2x^7}\right)$$

```
[Out] -1/18*ln(x)+1/42*ln(3+(2*x^7-27)^(1/3))-1/63*arctan(1/9*(3-2*(2*x^7-27)^(1/
3))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 60, 632, 210, 31}

$$\int \frac{1}{x(-27+2x^7)^{2/3}} \, dx = -\frac{\arctan\left(\frac{3-2\sqrt[3]{2x^7-27}}{3\sqrt{3}}\right)}{21\sqrt{3}} + \frac{1}{42}\log\left(\sqrt[3]{2x^7-27}+3\right) - \frac{\log(x)}{18}$$

```
[In] Int[1/(x*(-27 + 2*x^7)^(2/3)),x]
```

```
[Out] -1/21*ArcTan[(3 - 2*(-27 + 2*x^7)^(1/3))/(3*Sqrt[3])]/Sqrt[3] - Log[x]/18 +
 Log[3 + (-27 + 2*x^7)^(1/3)]/42
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{7}\text{Subst}\left(\int \frac{1}{x(-27+2x)^{2/3}}\,dx, x, x^7\right)\\
&= -\frac{\log(x)}{18}+\frac{1}{42}\text{Subst}\left(\int \frac{1}{3+x}\,dx, x, \sqrt[3]{-27+2x^7}\right)\\
&\quad+\frac{1}{14}\text{Subst}\left(\int \frac{1}{9-3x+x^2}\,dx, x, \sqrt[3]{-27+2x^7}\right)\\
&= -\frac{\log(x)}{18}+\frac{1}{42}\log\left(3+\sqrt[3]{-27+2x^7}\right)-\frac{1}{7}\text{Subst}\left(\int \frac{1}{-27-x^2}\,dx, x, -3+2\sqrt[3]{-27+2x^7}\right)\\
&= -\frac{\arctan\left(\frac{3-2\sqrt[3]{-27+2x^7}}{3\sqrt{3}}\right)}{21\sqrt{3}}-\frac{\log(x)}{18}+\frac{1}{42}\log\left(3+\sqrt[3]{-27+2x^7}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\left(-27+2x^7\right)^{2/3}}\,dx = \frac{1}{126}\left(-2\sqrt{3}\arctan\left(\frac{3-2\sqrt[3]{-27+2x^7}}{3\sqrt{3}}\right)\right.$$
$$\left.+2\log\left(3+\sqrt[3]{-27+2x^7}\right)-\log\left(9-3\sqrt[3]{-27+2x^7}+\left(-27+2x^7\right)^{2/3}\right)\right)$$

```
[In] Integrate[1/(x*(-27 + 2*x^7)^(2/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(3 - 2*(-27 + 2*x^7)^(1/3))/(3*Sqrt[3])] + 2*Log[3 + (-2
7 + 2*x^7)^(1/3)] - Log[9 - 3*(-27 + 2*x^7)^(1/3) + (-27 + 2*x^7)^(2/3)])/1
26
```

Maple [A] (verified)

Time = 7.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$\frac{\ln\left(3+\left(2x^{7}-27\right)^{\frac{1}{3}}\right)}{63}-\frac{\ln\left(\left(2x^{7}-27\right)^{\frac{2}{3}}-3\left(2x^{7}-27\right)^{\frac{1}{3}}+9\right)}{126}+\frac{\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\,\left(2x^{7}-27\right)^{\frac{1}{3}}}{9}-\frac{\sqrt{3}}{3}\right)}{63}$
meijerg	$\frac{\left(-\operatorname{signum}\left(-1+\frac{2x^{7}}{27}\right)\right)^{\frac{2}{3}}\left(\left(\frac{\pi\sqrt{3}}{6}-\frac{9\ln(3)}{2}+7\ln(x)+\ln(2)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+\frac{4\Gamma\left(\frac{2}{3}\right)x^{7}{}_{3}F_{2}\left(1,1,\frac{5}{3};2,2;\frac{2x^{7}}{27}\right)}{81}\right)}{63\operatorname{signum}\left(-1+\frac{2x^{7}}{27}\right)^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}$
trager	$-\frac{\ln\left(-\frac{757355840490254039191854\,\mathrm{RootOf}\left(81\_Z^{2}+9\_Z+1\right)^{2}x^{7}+1193513320861007233414\,\mathrm{RootOf}\left(81\_Z^{2}+9\_Z+1\right)x^{7}-202}{}\right.}{}$

```
[In] int(1/x/(2*x^7-27)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/63*ln(3+(2*x^7-27)^(1/3))-1/126*ln((2*x^7-27)^(2/3)-3*(2*x^7-27)^(1/3)+9)
+1/63*3^(1/2)*arctan(2/9*3^(1/2)*(2*x^7-27)^(1/3)-1/3*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\,(-27+2x^7)^{2/3}}\,dx = \frac{1}{63}\,\sqrt{3}\arctan\left(\frac{2}{9}\,\sqrt{3}(2\,x^7-27)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right) - \frac{1}{126}\,\log\left((2\,x^7-27)^{\frac{2}{3}}-3\,(2\,x^7-27)^{\frac{1}{3}}+9\right)+\frac{1}{63}\,\log\left((2\,x^7-27)^{\frac{1}{3}}+3\right)$$

```
[In] integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="fricas")
[Out] 1/63*sqrt(3)*arctan(2/9*sqrt(3)*(2*x^7 - 27)^(1/3) - 1/3*sqrt(3)) - 1/126*l
og((2*x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*log((2*x^7 - 27)^(
1/3) + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\,(-27+2x^7)^{2/3}}\,dx = -\frac{\sqrt[3]{2}\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{27e^{2i\pi}}{2x^7}\right)}{14x^{\frac{14}{3}}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate(1/x/(2*x**7-27)**(2/3),x)
[Out] -2**(1/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), 27*exp_polar(2*I*pi)/(2*x**7
))/(14*x**(14/3)*gamma(5/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\,(-27+2x^7)^{2/3}}\,dx = \frac{1}{63}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}\left(2\,(2\,x^7-27)^{\frac{1}{3}}-3\right)\right) - \frac{1}{126}\,\log\left((2\,x^7-27)^{\frac{2}{3}}-3\,(2\,x^7-27)^{\frac{1}{3}}+9\right)+\frac{1}{63}\,\log\left((2\,x^7-27)^{\frac{1}{3}}+3\right)$$

```
[In] integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="maxima")
[Out] 1/63*sqrt(3)*arctan(1/9*sqrt(3)*(2*(2*x^7 - 27)^(1/3) - 3)) - 1/126*log((2*
x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*log((2*x^7 - 27)^(1/3) +
 3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(-27+2x^7\right)^{2/3}}\,dx = \frac{1}{63}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\left(2x^7-27\right)^{\frac{1}{3}}-3\right)\right) - \frac{1}{126}\log\left(\left(2x^7-27\right)^{\frac{2}{3}}-3\left(2x^7-27\right)^{\frac{1}{3}}+9\right) + \frac{1}{63}\log\left(\left|\left(2x^7-27\right)^{\frac{1}{3}}+3\right|\right)$$

```
[In] integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="giac")
```

```
[Out] 1/63*sqrt(3)*arctan(1/9*sqrt(3)*(2*(2*x^7 - 27)^(1/3) - 3)) - 1/126*log((2*
x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*log(abs((2*x^7 - 27)^(1/
3) + 3))
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(-27+2x^7\right)^{2/3}}\,dx = \frac{\ln\left(\frac{\left(2x^7-27\right)^{1/3}}{49}+\frac{3}{49}\right)}{63} - \ln\left(\frac{27}{14}-\frac{9\left(2x^7-27\right)^{1/3}}{7}+\frac{\sqrt{3}\,27\mathrm{i}}{14}\right)\left(\frac{1}{126}+\frac{\sqrt{3}\,1\mathrm{i}}{126}\right)+\ln\left(\frac{9\left(2x^7-27\right)^{1/3}}{7}-\frac{27}{14}+\frac{\sqrt{3}\,27\mathrm{i}}{14}\right)\left(-\frac{1}{126}+\frac{\sqrt{3}}{12\ldots}\right.$$

```
[In] int(1/(x*(2*x^7 - 27)^(2/3)),x)
```

```
[Out] log((2*x^7 - 27)^(1/3)/49 + 3/49)/63 - log((3^(1/2)*27i)/14 - (9*(2*x^7 - 2
7)^(1/3))/7 + 27/14)*((3^(1/2)*1i)/126 + 1/126) + log((3^(1/2)*27i)/14 + (9
*(2*x^7 - 27)^(1/3))/7 - 27/14)*((3^(1/2)*1i)/126 - 1/126)
```

3.302 $\int \frac{(1+x^7)^{2/3}}{x^8} dx$

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{(1+x^7)^{2/3}}{x^8} dx = -\frac{(1+x^7)^{2/3}}{7x^7} + \frac{2\arctan\left(\frac{1+2\sqrt[3]{1+x^7}}{\sqrt{3}}\right)}{7\sqrt{3}} - \frac{\log(x)}{3} + \frac{1}{7}\log\left(1-\sqrt[3]{1+x^7}\right)$$

```
[Out] -1/7*(x^7+1)^(2/3)/x^7-1/3*ln(x)+1/7*ln(1-(x^7+1)^(1/3))+2/21*arctan(1/3*(1
+2*(x^7+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {272, 43, 57, 632, 210, 31}

$$\int \frac{(1+x^7)^{2/3}}{x^8} dx = \frac{2\arctan\left(\frac{2\sqrt[3]{x^7+1}+1}{\sqrt{3}}\right)}{7\sqrt{3}} - \frac{(x^7+1)^{2/3}}{7x^7} + \frac{1}{7}\log\left(1-\sqrt[3]{x^7+1}\right) - \frac{\log(x)}{3}$$

```
[In] Int[(1 + x^7)^(2/3)/x^8,x]
```

```
[Out] -1/7*(1 + x^7)^(2/3)/x^7 + (2*ArcTan[(1 + 2*(1 + x^7)^(1/3))/Sqrt[3]])/(7*S
qrt[3]) - Log[x]/3 + Log[1 - (1 + x^7)^(1/3)]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{7}\text{Subst}\left(\int \frac{(1+x)^{2/3}}{x^2}\,dx, x, x^7\right)\\
&= -\frac{(1+x^7)^{2/3}}{7x^7} + \frac{2}{21}\text{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx, x, x^7\right)\\
&= -\frac{(1+x^7)^{2/3}}{7x^7} - \frac{\log(x)}{3}\\
&\quad - \frac{1}{7}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+x^7}\right) + \frac{1}{7}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1+x^7}\right)
\end{aligned}
$$

$$=-\frac{(1+x^7)^{2/3}}{7x^7}-\frac{\log(x)}{3}+\frac{1}{7}\log\left(1-\sqrt[3]{1+x^7}\right)-\frac{2}{7}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{1+x^7}\right)$$

$$=-\frac{(1+x^7)^{2/3}}{7x^7}+\frac{2\arctan\left(\frac{1+2\sqrt[3]{1+x^7}}{\sqrt{3}}\right)}{7\sqrt{3}}-\frac{\log(x)}{3}+\frac{1}{7}\log\left(1-\sqrt[3]{1+x^7}\right)$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19

$$\int\frac{(1+x^7)^{2/3}}{x^8}\,dx=\frac{1}{21}\left(-\frac{3(1+x^7)^{2/3}}{x^7}+2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1+x^7}}{\sqrt{3}}\right)+2\log\left(-1+\sqrt[3]{1+x^7}\right)-\log\left(1+\sqrt[3]{1+x^7}+(1+x^7)^{2/3}\right)\right)$$

```
[In] Integrate[(1 + x^7)^(2/3)/x^8,x]
[Out] ((-3*(1 + x^7)^(2/3))/x^7 + 2*Sqrt[3]*ArcTan[(1 + 2*(1 + x^7)^(1/3))/Sqrt[3
]] + 2*Log[-1 + (1 + x^7)^(1/3)] - Log[1 + (1 + x^7)^(1/3) + (1 + x^7)^(2/3
)])/21
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 6.64 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

method	result
meijerg	$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)x^{7}}-\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}-1+7\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}+\frac{\pi\sqrt{3}\,x^{7}\,{}_{3}F_{2}\left(1,1,\frac{4}{3};2,3;-x^{7}\right)}{9\Gamma\left(\frac{2}{3}\right)}\right)}{21\pi}$
risch	$-\frac{(x^{7}+1)^{\frac{2}{3}}}{7x^{7}}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+7\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}-\frac{2\pi\sqrt{3}\,x^{7}\,{}_{3}F_{2}\left(1,1,\frac{4}{3};2,2;-x^{7}\right)}{9\Gamma\left(\frac{2}{3}\right)}\right)}{21\pi}$
pseudoelliptic	$\frac{2\arctan\left(\frac{\left(1+2(x^{7}+1)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}\,x^{7}-\ln\left((x^{7}+1)^{\frac{2}{3}}+(x^{7}+1)^{\frac{1}{3}}+1\right)x^{7}+2\ln\left((x^{7}+1)^{\frac{1}{3}}-1\right)x^{7}-3(x^{7}+1)^{\frac{2}{3}}}{21\left((x^{7}+1)^{\frac{2}{3}}+(x^{7}+1)^{\frac{1}{3}}+1\right)\left((x^{7}+1)^{\frac{1}{3}}-1\right)}$
trager	$-\frac{(x^{7}+1)^{\frac{2}{3}}}{7x^{7}}-\frac{2\ln\left(\frac{3593313\,\text{RootOf}\left(9\_Z^{2}+3\_Z+1\right)^{2}x^{7}+3486414\,\text{RootOf}\left(9\_Z^{2}+3\_Z+1\right)x^{7}-106899x^{7}+6095754\left(x^{7}+1\right)^{\frac{2}{3}}\ldots}{}\right.}{}$

```
[In] int((x^7+1)^(2/3)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/Pi*3^(1/2)*GAMMA(2/3)*(Pi*3^(1/2)/GAMMA(2/3)/x^7-2/3*(-1/6*Pi*3^(1/2)
-3/2*ln(3)-1+7*ln(x))*Pi*3^(1/2)/GAMMA(2/3)+1/9*Pi*3^(1/2)/GAMMA(2/3)*x^7*h
ypergeom([1,1,4/3],[2,3],-x^7))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \frac{(1+x^7)^{2/3}}{x^8}\,dx = \frac{2\sqrt{3}x^7\arctan\left(\frac{2}{3}\sqrt{3}(x^7+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-x^7\log\left((x^7+1)^{\frac{2}{3}}+(x^7+1)^{\frac{1}{3}}+1\right)+2\,x^7\log}{21\,x^7}$$

```
[In] integrate((x^7+1)^(2/3)/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(2*sqrt(3)*x^7*arctan(2/3*sqrt(3)*(x^7 + 1)^(1/3) + 1/3*sqrt(3)) - x^7
*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) + 1) + 2*x^7*log((x^7 + 1)^(1/3) - 1
) - 3*(x^7 + 1)^(2/3))/x^7
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.49

$$\int \frac{(1+x^7)^{2/3}}{x^8}\,dx = -\frac{\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^7}\right)}{7x^{\frac{7}{3}}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((x**7+1)**(2/3)/x**8,x)
```

```
[Out] -gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(I*pi)/x**7)/(7*x**(7/3)*ga
mma(4/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{(1+x^7)^{2/3}}{x^8}\,dx &= \frac{2}{21}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^7+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^7+1\right)^{\frac{2}{3}}}{7\,x^7}\\ &-\frac{1}{21}\log\left(\left(x^7+1\right)^{\frac{2}{3}}+\left(x^7+1\right)^{\frac{1}{3}}+1\right)+\frac{2}{21}\log\left(\left(x^7+1\right)^{\frac{1}{3}}-1\right)\end{aligned}$$

```
[In] integrate((x^7+1)^(2/3)/x^8,x, algorithm="maxima")
```

```
[Out] 2/21*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^7 + 1)^(1/3) + 1)) - 1/7*(x^7 + 1)^(2
/3)/x^7 - 1/21*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) + 1) + 2/21*log((x^7 +
 1)^(1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \frac{(1+x^7)^{2/3}}{x^8}\,dx = \frac{2}{21}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\Big(2\left(x^7+1\right)^{\frac{1}{3}}+1\Big)\right) - \frac{\left(x^7+1\right)^{\frac{2}{3}}}{7\,x^7} - \frac{1}{21}\log\left(\left(x^7+1\right)^{\frac{2}{3}}+\left(x^7+1\right)^{\frac{1}{3}}+1\right) + \frac{2}{21}\log\left(\left|\left(x^7+1\right)^{\frac{1}{3}}-1\right|\right)$$

```
[In] integrate((x^7+1)^(2/3)/x^8,x, algorithm="giac")
```

```
[Out] 2/21*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^7 + 1)^(1/3) + 1)) - 1/7*(x^7 + 1)^(2
/3)/x^7 - 1/21*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) + 1) + 2/21*log(abs((x
^7 + 1)^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int \frac{(1+x^7)^{2/3}}{x^8}\,dx = \frac{2\,\ln\left(\frac{4\,(x^7+1)^{1/3}}{49}-\frac{4}{49}\right)}{21} + \ln\left(\frac{4\,(x^7+1)^{1/3}}{49} - 9\left(-\frac{1}{21}+\frac{\sqrt{3}\,1\mathrm{i}}{21}\right)^2\right)\left(-\frac{1}{21}+\frac{\sqrt{3}\,1\mathrm{i}}{21}\right) - \ln\left(\frac{4\,(x^7+1)^{1/3}}{49} - 9\left(\frac{1}{21}+\frac{\sqrt{3}\,1\mathrm{i}}{21}\right)^2\right)\left(\frac{1}{21}+\right.$$

```
[In] int((x^7 + 1)^(2/3)/x^8,x)
```

```
[Out] (2*log((4*(x^7 + 1)^(1/3))/49 - 4/49))/21 + log((4*(x^7 + 1)^(1/3))/49 - 9*
((3^(1/2)*1i)/21 - 1/21)^2)*((3^(1/2)*1i)/21 - 1/21) - log((4*(x^7 + 1)^(1/
3))/49 - 9*((3^(1/2)*1i)/21 + 1/21)^2)*((3^(1/2)*1i)/21 + 1/21) - (x^7 + 1)
^(2/3)/(7*x^7)
```

3.303 $\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = -\frac{\sqrt[4]{3+4x^4}}{x} - \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}}$$

```
[Out] -(4*x^4+3)^(1/4)/x-1/2*arctan(x*2^(1/2)/(4*x^4+3)^(1/4))*2^(1/2)+1/2*arctan
h(x*2^(1/2)/(4*x^4+3)^(1/4))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {283, 338, 304, 209, 212}

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = -\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{\sqrt{2}} - \frac{\sqrt[4]{4x^4+3}}{x}$$

```
[In] Int[(3 + 4*x^4)^(1/4)/x^2,x]
```

```
[Out] -((3 + 4*x^4)^(1/4)/x) - ArcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2] + Ar
cTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt[4]{3+4x^4}}{x} + 4\int \frac{x^2}{(3+4x^4)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{3+4x^4}}{x} + 4\text{Subst}\left(\int \frac{x^2}{1-4x^4}\,dx, x, \frac{x}{\sqrt[4]{3+4x^4}}\right) \\
&= -\frac{\sqrt[4]{3+4x^4}}{x} + \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt[4]{3+4x^4}}\right) - \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt[4]{3+4x^4}}\right) \\
&= -\frac{\sqrt[4]{3+4x^4}}{x} - \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx=-\frac{\sqrt[4]{3+4x^4}}{x}-\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}}$$

```
[In] Integrate[(3 + 4*x^4)^(1/4)/x^2,x]
[Out] -((3 + 4*x^4)^(1/4)/x) - ArcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2] + Ar
cTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 2.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.29

method	result
meijerg	$-\frac{3^{\frac{1}{4}}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};-\frac{4x^4}{3}\right)}{x}$
risch	$-\frac{(4x^4+3)^{\frac{1}{4}}}{x}+\frac{4\,3^{\frac{1}{4}}x^3\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^4}{3}\right)}{9}$
pseudoelliptic	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(4x^4+3)^{\frac{1}{4}}\sqrt{2}}{2x}\right)x+\sqrt{2}\,\arctan\left(\frac{(4x^4+3)^{\frac{1}{4}}\sqrt{2}}{2x}\right)x-2(4x^4+3)^{\frac{1}{4}}}{2x}$
trager	$-\frac{(4x^4+3)^{\frac{1}{4}}}{x}+\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(-4\operatorname{RootOf}\left(\_Z^2+2\right)\sqrt{4x^4+3}\,x^2+8\operatorname{RootOf}\left(\_Z^2+2\right)x^4-4(4x^4+3)^{\frac{3}{4}}x+8x^3(4\right.}{4}$

```
[In] int((4*x^4+3)^(1/4)/x^2,x,method=_RETURNVERBOSE)
[Out] -3^(1/4)/x*hypergeom([-1/4,-1/4],[3/4],-4/3*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(55) = 110.

Time = 2.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.15

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx=$$

$$-\frac{2\sqrt{2}x\arctan\left(\frac{4}{3}\sqrt{2}(4x^4+3)^{\frac{1}{4}}x^3+\frac{2}{3}\sqrt{2}(4x^4+3)^{\frac{3}{4}}x\right)-\sqrt{2}x\log\left(-256x^8-192x^4-4\sqrt{2}(16x^5+3\right.}{8x}$$

```
[In] integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(2)*x*arctan(4/3*sqrt(2)*(4*x^4 + 3)^(1/4)*x^3 + 2/3*sqrt(2)*(4
*x^4 + 3)^(3/4)*x) - sqrt(2)*x*log(-256*x^8 - 192*x^4 - 4*sqrt(2)*(16*x^5 +
 3*x)*(4*x^4 + 3)^(3/4) - 8*sqrt(2)*(16*x^7 + 9*x^3)*(4*x^4 + 3)^(1/4) - 16
*(8*x^6 + 3*x^2)*sqrt(4*x^4 + 3) - 9) + 8*(4*x^4 + 3)^(1/4))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = \frac{\sqrt[4]{3}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{4x^4e^{i\pi}}{3}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

```
[In] integrate((4*x**4+3)**(1/4)/x**2,x)
```

```
[Out] 3**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), 4*x**4*exp_polar(I*pi)/3)/
(4*x*gamma(3/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}(4x^4+3)^{\frac{1}{4}}}{2x}\right) - \frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{(4x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{(4x^4+3)^{\frac{1}{4}}}{x}}\right) - \frac{(4x^4+3)^{\frac{1}{4}}}{x}$$

```
[In] integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 1/4*sqrt(2)*log(-(sqr
t(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) - (4*x^4 + 3)^
(1/4)/x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}(4\,x^4+3)^{\frac{1}{4}}}{2\,x}\right) - \frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{(4\,x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{(4\,x^4+3)^{\frac{1}{4}}}{x}}\right) - \frac{(4\,x^4+3)^{\frac{1}{4}}}{x}$$

```
[In] integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="giac")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 1/4*sqrt(2)*log(-(sqr
t(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) - (4*x^4 + 3)^
(1/4)/x
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.26

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = -\frac{3^{1/4}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ -\frac{4\,x^4}{3}\right)}{x}$$

```
[In] int((4*x^4 + 3)^(1/4)/x^2,x)
[Out] -(3^(1/4)*hypergeom([-1/4, -1/4], 3/4, -(4*x^4)/3))/x
```

3.304 $\int x^2(3+4x^4)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 93

$$\int x^2\left(3+4x^4\right)^{5/4}\,dx=\frac{15}{32}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^3\left(3+4x^4\right)^{5/4}-\frac{45\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}+\frac{45\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}$$

```
[Out] 15/32*x^3*(4*x^4+3)^(1/4)+1/8*x^3*(4*x^4+3)^(5/4)-45/256*arctan(x*2^(1/2)/(
4*x^4+3)^(1/4))*2^(1/2)+45/256*arctanh(x*2^(1/2)/(4*x^4+3)^(1/4))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {285, 338, 304, 209, 212}

$$\int x^2\left(3+4x^4\right)^{5/4}\,dx=-\frac{45\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}}+\frac{45\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}}+\frac{1}{8}\left(4x^4+3\right)^{5/4}x^3+\frac{15}{32}\sqrt[4]{4x^4+3}x^3$$

```
[In] Int[x^2*(3 + 4*x^4)^(5/4),x]
```

```
[Out] (15*x^3*(3 + 4*x^4)^(1/4))/32 + (x^3*(3 + 4*x^4)^(5/4))/8 - (45*ArcTan[(Sqr
t[2]*x)/(3 + 4*x^4)^(1/4)])/(128*Sqrt[2]) + (45*ArcTanh[(Sqrt[2]*x)/(3 + 4*
x^4)^(1/4)])/(128*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{8}x^3\left(3+4x^4\right)^{5/4}+\frac{15}{8}\int x^2\sqrt[4]{3+4x^4}\,dx\\
&= \frac{15}{32}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^3\left(3+4x^4\right)^{5/4}+\frac{45}{32}\int \frac{x^2}{\left(3+4x^4\right)^{3/4}}\,dx\\
&= \frac{15}{32}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^3\left(3+4x^4\right)^{5/4}+\frac{45}{32}\text{Subst}\left(\int \frac{x^2}{1-4x^4}\,dx,x,\frac{x}{\sqrt[4]{3+4x^4}}\right)\\
&= \frac{15}{32}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^3\left(3+4x^4\right)^{5/4}+\frac{45}{128}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\frac{x}{\sqrt[4]{3+4x^4}}\right)\\
&\quad -\frac{45}{128}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\frac{x}{\sqrt[4]{3+4x^4}}\right)
\end{aligned}$$

$$=\frac{15}{32}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^3(3+4x^4)^{5/4}-\frac{45\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}+\frac{45\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int x^2(3+4x^4)^{5/4}\,dx=\frac{1}{32}x^3\sqrt[4]{3+4x^4}(27+16x^4)-\frac{45\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}+\frac{45\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}$$

```
[In] Integrate[x^2*(3 + 4*x^4)^(5/4),x]
[Out] (x^3*(3 + 4*x^4)^(1/4)*(27 + 16*x^4))/32 - (45*ArcTan[(Sqrt[2]*x)/(3 + 4*x^
4)^(1/4)])/(128*Sqrt[2]) + (45*ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(128
*Sqrt[2])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 2.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.20

method	result
meijerg	$3^{\frac{1}{4}}x^3{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^4}{3}\right)$
risch	$\frac{x^3(16x^4+27)(4x^4+3)^{\frac{1}{4}}}{32}+\frac{5\,3^{\frac{1}{4}}x^3{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^4}{3}\right)}{32}$
pseudoelliptic	$\frac{\frac{9x^7(4x^4+3)^{\frac{1}{4}}}{2}+\frac{243x^3(4x^4+3)^{\frac{1}{4}}}{32}+\frac{405\sqrt{2}\,\text{arctanh}\left(\frac{(4x^4+3)^{\frac{1}{4}}\sqrt{2}}{2x}\right)}{256}+\frac{405\sqrt{2}\,\arctan\left(\frac{(4x^4+3)^{\frac{1}{4}}\sqrt{2}}{2x}\right)}{256}}{\left(-2x^2+\sqrt{4x^4+3}\right)^2\left(2x^2+\sqrt{4x^4+3}\right)^2}$
trager	$\frac{x^3(16x^4+27)(4x^4+3)^{\frac{1}{4}}}{32}-\frac{45\,\text{RootOf}\left(\_Z^2-2\right)\ln\left(-4\sqrt{4x^4+3}\,\text{RootOf}\left(\_Z^2-2\right)x^2-8\,\text{RootOf}\left(\_Z^2-2\right)x^4+4(4x^4+3\right.}{512}$

```
[In] int(x^2*(4*x^4+3)^(5/4),x,method=_RETURNVERBOSE)
[Out] 3^(1/4)*x^3*hypergeom([-5/4,3/4],[7/4],-4/3*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$\int x^2 (3+4x^4)^{5/4}\,dx = \frac{45}{256}\sqrt{2}\arctan\left(\frac{\sqrt{2}(4\,x^4+3)^{\frac{1}{4}}}{2\,x}\right) + \frac{45}{512}\sqrt{2}\log\left(8\,x^4+4\,\sqrt{2}(4\,x^4+3)^{\frac{1}{4}}x^3+4\,\sqrt{4\,x^4+3}x^2+2\,\sqrt{2}(4\,x^4+3)^{\frac{3}{4}}x+3\right) + \frac{1}{32}\left(16\,x^7+27\,x^3\right)(4\,x^4+3)^{\frac{1}{4}}$$

```
[In] integrate(x^2*(4*x^4+3)^(5/4),x, algorithm="fricas")
[Out] 45/256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 45/512*sqrt(2)*log
(8*x^4 + 4*sqrt(2)*(4*x^4 + 3)^(1/4)*x^3 + 4*sqrt(4*x^4 + 3)*x^2 + 2*sqrt(2
)*(4*x^4 + 3)^(3/4)*x + 3) + 1/32*(16*x^7 + 27*x^3)*(4*x^4 + 3)^(1/4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.44

$$\int x^2 (3+4x^4)^{5/4}\,dx = \frac{3\cdot\sqrt[4]{3}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{4x^4e^{i\pi}}{3}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(x**2*(4*x**4+3)**(5/4),x)
[Out] 3*3**(1/4)*x**3*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), 4*x**4*exp_polar(I*pi
)/3)/(4*gamma(7/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.40

$$\int x^2 (3+4x^4)^{5/4}\,dx = \frac{45}{256}\sqrt{2}\arctan\left(\frac{\sqrt{2}(4\,x^4+3)^{\frac{1}{4}}}{2\,x}\right) - \frac{45}{512}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{(4\,x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{(4\,x^4+3)^{\frac{1}{4}}}{x}}\right) + \frac{9\left(\frac{20\,(4\,x^4+3)^{\frac{1}{4}}}{x}-\frac{9\,(4\,x^4+3)^{\frac{5}{4}}}{x^5}\right)}{32\left(\frac{8\,(4\,x^4+3)}{x^4}-\frac{(4\,x^4+3)^2}{x^8}-16\right)}$$

```
[In] integrate(x^2*(4*x^4+3)^(5/4),x, algorithm="maxima")
```

```
[Out] 45/256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 45/512*sqrt(2)*log
(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) + 9/32*(
20*(4*x^4 + 3)^(1/4)/x - 9*(4*x^4 + 3)^(5/4)/x^5)/(8*(4*x^4 + 3)/x^4 - (4*x
^4 + 3)^2/x^8 - 16)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.18

$$\int x^2 (3+4x^4)^{5/4} dx = \frac{1}{32} x^8 \left(\frac{9 (4 x^4+3)^{\frac{1}{4}} \left(\frac{3}{x^4}+4\right)}{x} - \frac{20 (4 x^4+3)^{\frac{1}{4}}}{x} \right) + \frac{45}{256} \sqrt{2} \arctan\left(\frac{\sqrt{2}(4 x^4+3)^{\frac{1}{4}}}{2 x}\right) - \frac{45}{512} \sqrt{2} \log\left(-\frac{\sqrt{2} - \frac{(4 x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2} + \frac{(4 x^4+3)^{\frac{1}{4}}}{x}}\right)$$

```
[In] integrate(x^2*(4*x^4+3)^(5/4),x, algorithm="giac")
```

```
[Out] 1/32*x^8*(9*(4*x^4 + 3)^(1/4)*(3/x^4 + 4)/x - 20*(4*x^4 + 3)^(1/4)/x) + 45/
256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 45/512*sqrt(2)*log(-(
sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x))
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (3+4x^4)^{5/4} dx = \int x^2 (4 x^4+3)^{5/4} dx$$

```
[In] int(x^2*(4*x^4 + 3)^(5/4),x)
```

```
[Out] int(x^2*(4*x^4 + 3)^(5/4), x)
```

3.305 $\int x^6 \sqrt[4]{3+4x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 93

$$\int x^6 \sqrt[4]{3+4x^4}\,dx = \frac{3}{128}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^7\sqrt[4]{3+4x^4}+\frac{27\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}}-\frac{27\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}}$$

```
[Out] 3/128*x^3*(4*x^4+3)^(1/4)+1/8*x^7*(4*x^4+3)^(1/4)+27/1024*arctan(x*2^(1/2)/
(4*x^4+3)^(1/4))*2^(1/2)-27/1024*arctanh(x*2^(1/2)/(4*x^4+3)^(1/4))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {285, 327, 338, 304, 209, 212}

$$\int x^6 \sqrt[4]{3+4x^4}\,dx = \frac{27\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}}-\frac{27\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}}+\frac{1}{8}\sqrt[4]{4x^4+3}x^7+\frac{3}{128}\sqrt[4]{4x^4+3}x^3$$

```
[In] Int[x^6*(3 + 4*x^4)^(1/4),x]
```

```
[Out] (3*x^3*(3 + 4*x^4)^(1/4))/128 + (x^7*(3 + 4*x^4)^(1/4))/8 + (27*ArcTan[(Sqr
t[2]*x)/(3 + 4*x^4)^(1/4)])/(512*Sqrt[2]) - (27*ArcTanh[(Sqrt[2]*x)/(3 + 4*
x^4)^(1/4)])/(512*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\text{integral} = \frac{1}{8}x^7\sqrt[4]{3+4x^4} + \frac{3}{8}\int \frac{x^6}{(3+4x^4)^{3/4}}\,dx$$

$$= \frac{3}{128}x^3\sqrt[4]{3+4x^4} + \frac{1}{8}x^7\sqrt[4]{3+4x^4} - \frac{27}{128}\int \frac{x^2}{(3+4x^4)^{3/4}}\,dx$$

$$= \frac{3}{128}x^3\sqrt[4]{3+4x^4} + \frac{1}{8}x^7\sqrt[4]{3+4x^4} - \frac{27}{128}\text{Subst}\left(\int \frac{x^2}{1-4x^4}\,dx, x, \frac{x}{\sqrt[4]{3+4x^4}}\right)$$

$$= \frac{3}{128}x^3\sqrt[4]{3+4x^4} + \frac{1}{8}x^7\sqrt[4]{3+4x^4} - \frac{27}{512}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt[4]{3+4x^4}}\right) + \frac{27}{512}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt[4]{3+4x^4}}\right)$$

$$= \frac{3}{128}x^3\sqrt[4]{3+4x^4} + \frac{1}{8}x^7\sqrt[4]{3+4x^4} + \frac{27\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}} - \frac{27\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int x^6\sqrt[4]{3+4x^4}\,dx = \frac{1}{128}x^3\sqrt[4]{3+4x^4}(3+16x^4) + \frac{27\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}} - \frac{27\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}}$$

```
[In] Integrate[x^6*(3 + 4*x^4)^(1/4),x]
```

```
[Out] (x^3*(3 + 4*x^4)^(1/4)*(3 + 16*x^4))/128 + (27*ArcTan[(Sqrt[2]*x)/(3 + 4*x^
4)^(1/4)])/(512*Sqrt[2]) - (27*ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(512
*Sqrt[2])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 2.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.22

method	result
meijerg	$\frac{3^{\frac{1}{4}} x^{7} {}_{2}F_{1}\left(-\frac{1}{4},\frac{7}{4};\frac{11}{4};-\frac{4x^{4}}{3}\right)}{7}$
risch	$\frac{x^{3}\left(16x^{4}+3\right)\left(4x^{4}+3\right)^{\frac{1}{4}}}{128}-\frac{3\,3^{\frac{1}{4}} x^{3} {}_{2}F_{1}\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^{4}}{3}\right)}{128}$
pseudoelliptic	$\frac{\frac{9x^{7}\left(4x^{4}+3\right)^{\frac{1}{4}}}{8}+\frac{27x^{3}\left(4x^{4}+3\right)^{\frac{1}{4}}}{128}-\frac{243\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(4x^{4}+3\right)^{\frac{1}{4}}\sqrt{2}}{2x}\right)}{1024}-\frac{243\sqrt{2}\,\arctan\left(\frac{\left(4x^{4}+3\right)^{\frac{1}{4}}\sqrt{2}}{2x}\right)}{1024}}{\left(-2x^{2}+\sqrt{4x^{4}+3}\right)^{2}\left(2x^{2}+\sqrt{4x^{4}+3}\right)^{2}}$
trager	$\frac{x^{3}\left(16x^{4}+3\right)\left(4x^{4}+3\right)^{\frac{1}{4}}}{128}+\frac{27\,\mathrm{RootOf}\left(\_Z^{2}-2\right)\ln\left(-4\sqrt{4x^{4}+3}\,\mathrm{RootOf}\left(\_Z^{2}-2\right)x^{2}-8\,\mathrm{RootOf}\left(\_Z^{2}-2\right)x^{4}+4\left(4x^{4}+3\right)\right.}{2048}$

[In] int(x^6*(4*x^4+3)^(1/4),x,method=_RETURNVERBOSE)

[Out] 1/7*3^(1/4)*x^7*hypergeom([-1/4,7/4],[11/4],-4/3*x^4)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$\int x^6\sqrt[4]{3+4x^4}\,dx = -\frac{27}{1024}\sqrt{2}\arctan\left(\frac{\sqrt{2}(4\,x^4+3)^{\frac{1}{4}}}{2\,x}\right) + \frac{27}{2048}\sqrt{2}\log\left(8\,x^4-4\,\sqrt{2}(4\,x^4+3)^{\frac{1}{4}}x^3+4\,\sqrt{4\,x^4+3}x^2 - 2\,\sqrt{2}(4\,x^4+3)^{\frac{3}{4}}x+3\right)+\frac{1}{128}\left(16\,x^7+3\,x^3\right)\left(4\,x^4+3\right)^{\frac{1}{4}}$$

```
[In] integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="fricas")
```

```
[Out] -27/1024*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 27/2048*sqrt(2)*
log(8*x^4 - 4*sqrt(2)*(4*x^4 + 3)^(1/4)*x^3 + 4*sqrt(4*x^4 + 3)*x^2 - 2*sqr
t(2)*(4*x^4 + 3)^(3/4)*x + 3) + 1/128*(16*x^7 + 3*x^3)*(4*x^4 + 3)^(1/4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

$$\int x^6\sqrt[4]{3+4x^4}\,dx = \frac{\sqrt[4]{3}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{4x^4e^{i\pi}}{3}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

```
[In] integrate(x**6*(4*x**4+3)**(1/4),x)
```

```
[Out] 3**(1/4)*x**7*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), 4*x**4*exp_polar(I*pi)
/3)/(4*gamma(11/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.39

$$\int x^6\sqrt[4]{3+4x^4}\,dx = -\frac{27}{1024}\sqrt{2}\arctan\left(\frac{\sqrt{2}(4x^4+3)^{\frac{1}{4}}}{2x}\right) + \frac{27}{2048}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{(4x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{(4x^4+3)^{\frac{1}{4}}}{x}}\right) - \frac{9\left(\frac{12\,(4x^4+3)^{\frac{1}{4}}}{x}+\frac{(4x^4+3)^{\frac{5}{4}}}{x^5}\right)}{128\left(\frac{8\,(4x^4+3)}{x^4}-\frac{(4x^4+3)^2}{x^8}-16\right)}$$

```
[In] integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="maxima")
```

```
[Out] -27/1024*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 27/2048*sqrt(2)*
log(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) - 9/1
28*(12*(4*x^4 + 3)^(1/4)/x + (4*x^4 + 3)^(5/4)/x^5)/(8*(4*x^4 + 3)/x^4 - (4
*x^4 + 3)^2/x^8 - 16)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.17

$$\int x^6\sqrt[4]{3+4x^4}\,dx = \frac{1}{128}x^8\left(\frac{(4x^4+3)^{\frac{1}{4}}\left(\frac{3}{x^4}+4\right)}{x}+\frac{12\,(4x^4+3)^{\frac{1}{4}}}{x}\right) - \frac{27}{1024}\sqrt{2}\arctan\left(\frac{\sqrt{2}(4x^4+3)^{\frac{1}{4}}}{2x}\right) + \frac{27}{2048}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{(4x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{(4x^4+3)^{\frac{1}{4}}}{x}}\right)$$

```
[In] integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="giac")
```

```
[Out] 1/128*x^8*((4*x^4 + 3)^(1/4)*(3/x^4 + 4)/x + 12*(4*x^4 + 3)^(1/4)/x) - 27/1
024*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 27/2048*sqrt(2)*log(-
(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x))
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \sqrt[4]{3+4x^4}\,dx = \int x^6\,\left(4\,x^4+3\right)^{1/4}\,dx$$

```
[In] int(x^6*(4*x^4 + 3)^(1/4),x)
[Out] int(x^6*(4*x^4 + 3)^(1/4), x)
```

3.306 $\int \sqrt[3]{x\left(1-x^2\right)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 93

$$\int \sqrt[3]{x\left(1-x^2\right)}\,dx=\frac{1}{2}x\sqrt[3]{x\left(1-x^2\right)}+\frac{\arctan\left(\frac{2x-\sqrt[3]{x\left(1-x^2\right)}}{\sqrt{3}\sqrt[3]{x\left(1-x^2\right)}}\right)}{2\sqrt{3}}+\frac{\log(x)}{12}-\frac{1}{4}\log\left(x+\sqrt[3]{x\left(1-x^2\right)}\right)$$

```
[Out] 1/2*x*(x*(-x^2+1))^(1/3)+1/12*ln(x)-1/4*ln(x+(x*(-x^2+1))^(1/3))+1/6*arctan
(1/3*(2*x-(x*(-x^2+1))^(1/3))/(x*(-x^2+1))^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.39, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2004, 2029, 2057, 335, 281, 337}

$$\int \sqrt[3]{x\left(1-x^2\right)}\,dx=-\frac{\left(1-x^2\right)^{2/3}x^{2/3}\arctan\left(\frac{1-\frac{2x^{2/3}}{\sqrt[3]{1-x^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\left(x-x^3\right)^{2/3}}+\frac{1}{2}\sqrt[3]{x-x^3}x-\frac{\left(1-x^2\right)^{2/3}x^{2/3}\log\left(x^{2/3}+\sqrt[3]{1-x^2}\right)}{4\left(x-x^3\right)^{2/3}}$$

```
[In] Int[(x*(1 - x^2))^(1/3),x]
```

```
[Out] (x*(x - x^3)^(1/3))/2 - (x^(2/3)*(1 - x^2)^(2/3)*ArcTan[(1 - (2*x^(2/3))/(1
 - x^2)^(1/3))/Sqrt[3]])/(2*Sqrt[3]*(x - x^3)^(2/3)) - (x^(2/3)*(1 - x^2)^(
2/3)*Log[x^(2/3) + (1 - x^2)^(1/3)])/(4*(x - x^3)^(2/3))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = \int \sqrt[3]{x-x^3}\,dx$$

$$= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{1}{3}\int \frac{x}{(x-x^3)^{2/3}}\,dx$$

$$= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{\left(x^{2/3}(1-x^2)^{2/3}\right)\int \frac{\sqrt[3]{x}}{(1-x^2)^{2/3}}\,dx}{3\left(x-x^3\right)^{2/3}}$$

$$= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{\left(x^{2/3}(1-x^2)^{2/3}\right)\text{Subst}\left(\int \frac{x^3}{(1-x^6)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\left(x-x^3\right)^{2/3}}$$

$$= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{\left(x^{2/3}(1-x^2)^{2/3}\right)\text{Subst}\left(\int \frac{x}{(1-x^3)^{2/3}}\,dx, x, x^{2/3}\right)}{2\left(x-x^3\right)^{2/3}}$$

$$= \frac{1}{2}x\sqrt[3]{x-x^3} - \frac{x^{2/3}(1-x^2)^{2/3}\arctan\left(\frac{1-\frac{2x^{2/3}}{\sqrt[3]{1-x^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\left(x-x^3\right)^{2/3}} - \frac{x^{2/3}(1-x^2)^{2/3}\log\left(x^{2/3}+\sqrt[3]{1-x^2}\right)}{4\left(x-x^3\right)^{2/3}}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.47

$$\int \sqrt[3]{x\left(1-x^2\right)}\,dx$$

$$= \frac{\sqrt[3]{x-x^3}\left(6x^{4/3}\sqrt[3]{-1+x^2}+2\sqrt{3}\arctan\left(\frac{\sqrt{3}x^{2/3}}{x^{2/3}+2\sqrt[3]{-1+x^2}}\right)+2\log\left(-x^{2/3}+\sqrt[3]{-1+x^2}\right)-\log\left(x^{4/3}+\right.\right.}{12\sqrt[3]{x}\sqrt[3]{-1+x^2}}$$

```
[In] Integrate[(x*(1 - x^2))^(1/3),x]
```

```
[Out] ((x - x^3)^(1/3)*(6*x^(4/3)*(-1 + x^2)^(1/3) + 2*Sqrt[3]*ArcTan[(Sqrt[3]*x^
(2/3))/(x^(2/3) + 2*(-1 + x^2)^(1/3))] + 2*Log[-x^(2/3) + (-1 + x^2)^(1/3)]
 - Log[x^(4/3) + x^(2/3)*(-1 + x^2)^(1/3) + (-1 + x^2)^(2/3)]))/(12*x^(1/3)
*(-1 + x^2)^(1/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 2.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.16

method	result
meijerg	$\frac{3x^{\frac{4}{3}}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^2\right)}{4}$
pseudoelliptic	$\frac{x\left(2\sqrt{3}\arctan\left(\frac{\left(-2\left(-x^3+x\right)^{\frac{1}{3}}+x\right)\sqrt{3}}{3x}\right)+6\left(-x^3+x\right)^{\frac{1}{3}}x-2\ln\left(\frac{\left(-x^3+x\right)^{\frac{1}{3}}+x}{x}\right)+\ln\left(\frac{\left(-x^3+x\right)^{\frac{2}{3}}-\left(-x^3+x\right)^{\frac{1}{3}}x+x^2}{x^2}\right)\right)}{12\left(\left(-x^3+x\right)^{\frac{1}{3}}+x\right)\left(\left(-x^3+x\right)^{\frac{2}{3}}-\left(-x^3+x\right)^{\frac{1}{3}}x+x^2\right)}$
trager	$\frac{\left(-x^3+x\right)^{\frac{1}{3}}x}{2}-\frac{\ln\left(4959\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)^2x^2+6768\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-x^3+x\right)^{\frac{2}{3}}+22833\,\mathrm{RootOf}\left(9\_\right.\right.}{}$
risch	Expression too large to display

[In] int((x*(-x^2+1))^(1/3),x,method=_RETURNVERBOSE)

[Out] 3/4*x^(4/3)*hypergeom([-1/3,2/3],[5/3],x^2)

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06

$$\int \sqrt[3]{x\left(1-x^2\right)}\,dx =$$

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{44032959556\sqrt{3}(-x^3+x)^{\frac{1}{3}}x-\sqrt{3}(16754327161\,x^2-2707204793)+10524305234\sqrt{3}(-}{81835897185\,x^2-1102302937}\right.$$

$$+\frac{1}{2}\left(-x^3+x\right)^{\frac{1}{3}}x-\frac{1}{12}\log\left(3\left(-x^3+x\right)^{\frac{1}{3}}x+3\left(-x^3+x\right)^{\frac{2}{3}}+1\right)$$

[In] integrate((x*(-x^2+1))^(1/3),x, algorithm="fricas")

[Out] -1/6*sqrt(3)*arctan((44032959556*sqrt(3)*(-x^3 + x)^(1/3)*x - sqrt(3)*(1675
4327161*x^2 - 2707204793) + 10524305234*sqrt(3)*(-x^3 + x)^(2/3))/(81835897
185*x^2 - 1102302937)) + 1/2*(-x^3 + x)^(1/3)*x - 1/12*log(3*(-x^3 + x)^(1/
3)*x + 3*(-x^3 + x)^(2/3) + 1)

Sympy [F]

$$\int \sqrt[3]{x\left(1-x^2\right)}\,dx = \int \sqrt[3]{x\left(1-x^2\right)}\,dx$$

[In] integrate((x*(-x**2+1))**(1/3),x)

[Out] Integral((x*(1 - x**2))**(1/3), x)

Maxima [F]

$$\int \sqrt[3]{x\left(1-x^2\right)}\,dx = \int \left(-\left(x^2-1\right)x\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((x*(-x^2+1))^(1/3),x, algorithm="maxima")
[Out] integrate((-(x^2 - 1)*x)^(1/3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.74

$$\int \sqrt[3]{x\left(1-x^2\right)}\,dx = \frac{1}{2}\,x^2\left(\frac{1}{x^2}-1\right)^{\frac{1}{3}} - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{1}{x^2}-1\right)^{\frac{1}{3}}-1\right)\right) + \frac{1}{12}\,\log\left(\left(\frac{1}{x^2}-1\right)^{\frac{2}{3}} - \left(\frac{1}{x^2}-1\right)^{\frac{1}{3}}+1\right) - \frac{1}{6}\,\log\left(\left|\left(\frac{1}{x^2}-1\right)^{\frac{1}{3}}+1\right|\right)$$

```
[In] integrate((x*(-x^2+1))^(1/3),x, algorithm="giac")
[Out] 1/2*x^2*(1/x^2 - 1)^(1/3) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(1/x^2 - 1)^(
1/3) - 1)) + 1/12*log((1/x^2 - 1)^(2/3) - (1/x^2 - 1)^(1/3) + 1) - 1/6*log(
abs((1/x^2 - 1)^(1/3) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.31

$$\int \sqrt[3]{x\left(1-x^2\right)}\,dx = \frac{3\,x\,\left(x-x^3\right)^{1/3}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\ \frac{5}{3};\ x^2\right)}{4\,\left(1-x^2\right)^{1/3}}$$

```
[In] int((-x*(x^2 - 1))^(1/3),x)
[Out] (3*x*(x - x^3)^(1/3)*hypergeom([-1/3, 2/3], 5/3, x^2))/(4*(1 - x^2)^(1/3))
```

3.307 $\int \sqrt{\left(1+\sqrt[3]{x}\right) x}\, dx$

Optimal result

Integrand size = 13, antiderivative size = 126

$$\int \sqrt{\left(1+\sqrt[3]{x}\right) x}\, dx = \frac{7}{64}\sqrt{\left(1+\sqrt[3]{x}\right) x} - \frac{21\sqrt{\left(1+\sqrt[3]{x}\right) x}}{128\sqrt[3]{x}} - \frac{7}{80}\sqrt[3]{x}\sqrt{\left(1+\sqrt[3]{x}\right) x} + \frac{3}{40}x^{2/3}\sqrt{\left(1+\sqrt[3]{x}\right) x} + \frac{3}{5}x\sqrt{\left(1+\sqrt[3]{x}\right) x} + \frac{21}{128}\text{arctanh}\left(\frac{x^{2/3}}{\sqrt{\left(1+\sqrt[3]{x}\right) x}}\right)$$

```
[Out] 21/128*arctanh(x^(2/3)/((1+x^(1/3))*x)^(1/2))+7/64*((1+x^(1/3))*x)^(1/2)-21
/128*((1+x^(1/3))*x)^(1/2)/x^(1/3)-7/80*x^(1/3)*((1+x^(1/3))*x)^(1/2)+3/40*
x^(2/3)*((1+x^(1/3))*x)^(1/2)+3/5*x*((1+x^(1/3))*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2004, 2029, 2049, 2035, 2054, 212}

$$\int \sqrt{\left(1+\sqrt[3]{x}\right) x}\, dx = \frac{21}{128}\text{arctanh}\left(\frac{x^{2/3}}{\sqrt{x^{4/3}+x}}\right) + \frac{3}{5}\sqrt{x^{4/3}+x}x + \frac{3}{40}\sqrt{x^{4/3}+x}x^{2/3} - \frac{7}{80}\sqrt{x^{4/3}+x}\sqrt[3]{x} + \frac{7}{64}\sqrt{x^{4/3}+x} - \frac{21\sqrt{x^{4/3}+x}}{128\sqrt[3]{x}}$$

```
[In] Int[Sqrt[(1 + x^(1/3))*x],x]
```

```
[Out] (7*Sqrt[x + x^(4/3)])/64 - (21*Sqrt[x + x^(4/3)])/(128*x^(1/3)) - (7*x^(1/3
)*Sqrt[x + x^(4/3)])/80 + (3*x^(2/3)*Sqrt[x + x^(4/3)])/40 + (3*x*Sqrt[x +
x^(4/3)])/5 + (21*ArcTanh[x^(2/3)/Sqrt[x + x^(4/3)]])/128
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2035

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[-2*(Sqrt
[a*x^j + b*x^n]/(b*(n - 2)*x^(n - 1))), x] - Dist[a*((2*n - j - 2)/(b*(n -
2))), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sqrt{x+x^{4/3}}\,dx \\
&= \frac{3}{5}x\sqrt{x+x^{4/3}} + \frac{1}{10}\int \frac{x}{\sqrt{x+x^{4/3}}}\,dx \\
&= \frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}} + \frac{3}{5}x\sqrt{x+x^{4/3}} - \frac{7}{80}\int \frac{x^{2/3}}{\sqrt{x+x^{4/3}}}\,dx \\
&= -\frac{7}{80}\sqrt[3]{x}\sqrt{x+x^{4/3}} + \frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}} + \frac{3}{5}x\sqrt{x+x^{4/3}} + \frac{7}{96}\int \frac{\sqrt[3]{x}}{\sqrt{x+x^{4/3}}}\,dx \\
&= \frac{7}{64}\sqrt{x+x^{4/3}} - \frac{7}{80}\sqrt[3]{x}\sqrt{x+x^{4/3}} + \frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}} + \frac{3}{5}x\sqrt{x+x^{4/3}} - \frac{7}{128}\int \frac{1}{\sqrt{x+x^{4/3}}}\,dx \\
&= \frac{7}{64}\sqrt{x+x^{4/3}} - \frac{21\sqrt{x+x^{4/3}}}{128\sqrt[3]{x}} - \frac{7}{80}\sqrt[3]{x}\sqrt{x+x^{4/3}} \\
&\quad + \frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}} + \frac{3}{5}x\sqrt{x+x^{4/3}} + \frac{7}{256}\int \frac{1}{\sqrt[3]{x}\sqrt{x+x^{4/3}}}\,dx \\
&= \frac{7}{64}\sqrt{x+x^{4/3}} - \frac{21\sqrt{x+x^{4/3}}}{128\sqrt[3]{x}} - \frac{7}{80}\sqrt[3]{x}\sqrt{x+x^{4/3}} + \frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}} \\
&\quad + \frac{3}{5}x\sqrt{x+x^{4/3}} + \frac{21}{128}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^{2/3}}{\sqrt{x+x^{4/3}}}\right) \\
&= \frac{7}{64}\sqrt{x+x^{4/3}} - \frac{21\sqrt{x+x^{4/3}}}{128\sqrt[3]{x}} - \frac{7}{80}\sqrt[3]{x}\sqrt{x+x^{4/3}} \\
&\quad + \frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}} + \frac{3}{5}x\sqrt{x+x^{4/3}} + \frac{21}{128}\text{arctanh}\left(\frac{x^{2/3}}{\sqrt{x+x^{4/3}}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.55

$$\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx = \frac{\sqrt{x+x^{4/3}}\left(-105+70\sqrt[3]{x}-56x^{2/3}+48x+384x^{4/3}\right)}{640\sqrt[3]{x}} + \frac{21}{128}\text{arctanh}\left(\frac{x^{2/3}}{\sqrt{x+x^{4/3}}}\right)$$

```
[In] Integrate[Sqrt[(1 + x^(1/3))*x],x]
```

```
[Out] (Sqrt[x + x^(4/3)]*(-105 + 70*x^(1/3) - 56*x^(2/3) + 48*x + 384*x^(4/3)))/(
640*x^(1/3)) + (21*ArcTanh[x^(2/3)/Sqrt[x + x^(4/3)]])/128
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

method	result
meijerg	$-\frac{3\left(\frac{\sqrt{\pi}\, x^{\frac{1}{6}}\left(-1152x^{\frac{4}{3}}-144x+168x^{\frac{2}{3}}-210x^{\frac{1}{3}}+315\right)\sqrt{x^{\frac{1}{3}}+1}}{2880}-\frac{7\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{1}{6}}\right)}{64}\right)}{2\sqrt{\pi}}$
derivativedivides	$\frac{\sqrt{\left(x^{\frac{1}{3}}+1\right)x}\left(768x^{\frac{2}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}-672x^{\frac{1}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}+560\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}-420\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}\,x^{\frac{1}{3}}-210\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}+105\ln\left(\right.}{1280x^{\frac{1}{3}}\sqrt{\left(x^{\frac{1}{3}}+1\right)x^{\frac{1}{3}}}}$
default	$\frac{\sqrt{\left(x^{\frac{1}{3}}+1\right)x}\left(768x^{\frac{2}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}-672x^{\frac{1}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}+560\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}-420\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}\,x^{\frac{1}{3}}-210\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}+105\ln\left(\right.}{1280x^{\frac{1}{3}}\sqrt{\left(x^{\frac{1}{3}}+1\right)x^{\frac{1}{3}}}}$

```
[In] int(((x^(1/3)+1)*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -3/2/Pi^(1/2)*(1/2880*Pi^(1/2)*x^(1/6)*(-1152*x^(4/3)-144*x+168*x^(2/3)-210
*x^(1/3)+315)*(x^(1/3)+1)^(1/2)-7/64*Pi^(1/2)*arcsinh(x^(1/6)))
```

Fricas [A] (verification not implemented)

none

Time = 42.64 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

$$\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx = \frac{35\,x\log\left(\frac{32\,x^2+48\,x^{\frac{5}{3}}+2\left(16\,x^{\frac{4}{3}}+16\,x+3\,x^{\frac{2}{3}}\right)\sqrt{x^{\frac{4}{3}}+x}+18\,x^{\frac{4}{3}}+x}{x}\right)+2\left(384\,x^2+3\,(16\,x-35)x^{\frac{2}{3}}-56\,x^{\frac{4}{3}}+70\,x\right)\sqrt{x}}{1280\,x}$$

```
[In] integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="fricas")
[Out] 1/1280*(35*x*x*log((32*x^2 + 48*x^(5/3) + 2*(16*x^(4/3) + 16*x + 3*x^(2/3))*s
qrt(x^(4/3) + x) + 18*x^(4/3) + x)/x) + 2*(384*x^2 + 3*(16*x - 35)*x^(2/3)
- 56*x^(4/3) + 70*x)*sqrt(x^(4/3) + x))/x
```

Sympy [F]

$$\int \sqrt{(1+\sqrt[3]{x})\,x}\,dx = \int \sqrt{x\,(\sqrt[3]{x}+1)}\,dx$$

```
[In] integrate(((1+x**(1/3))*x)**(1/2),x)
[Out] Integral(sqrt(x*(x**(1/3) + 1)), x)
```

Maxima [F]

$$\int \sqrt{(1+\sqrt[3]{x})\,x}\,dx = \int \sqrt{x\left(x^{\frac{1}{3}}+1\right)}\,dx$$

```
[In] integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x*(x^(1/3) + 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.52

$$\int \sqrt{(1+\sqrt[3]{x})\,x}\,dx$$
$$= \frac{1}{1280}\left(2\left(2\left(4\left(6\,x^{\frac{1}{3}}\left(8\,x^{\frac{1}{3}}+1\right)-7\right)x^{\frac{1}{3}}+35\right)x^{\frac{1}{3}}-105\right)\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}-105\,\log\left(\left|2\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}-2\,x^{\frac{1}{3}}-1\right|\right)\right.$$

```
[In] integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="giac")
[Out] 1/1280*(2*(2*(4*(6*x^(1/3)*(8*x^(1/3) + 1) - 7)*x^(1/3) + 35)*x^(1/3) - 105
)*sqrt(x^(2/3) + x^(1/3)) - 105*log(abs(2*sqrt(x^(2/3) + x^(1/3)) - 2*x^(1/
3) - 1)))*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.21

$$\int \sqrt{(1+\sqrt[3]{x})\,x}\,dx = \frac{2\,x\,\sqrt{x+x^{4/3}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{9}{2};\ \frac{11}{2};\ -x^{1/3}\right)}{3\,\sqrt{x^{1/3}+1}}$$

```
[In] int((x*(x^(1/3) + 1))^(1/2),x)
[Out] (2*x*(x + x^(4/3))^(1/2)*hypergeom([-1/2, 9/2], 11/2, -x^(1/3)))/(3*(x^(1/3
) + 1)^(1/2))
```

3.308 $\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 34

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{1+2x^4}{\sqrt{3}\sqrt{1+2x^8}}\right)}{4\sqrt{3}}$$

```
[Out] -1/12*arctanh(1/3*(2*x^4+1)*3^(1/2)/(2*x^8+1)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1483, 739, 212}

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2x^4+1}{\sqrt{3}\sqrt{2x^8+1}}\right)}{4\sqrt{3}}$$

```
[In] Int[x^3/((-1 + x^4)*Sqrt[1 + 2*x^8]),x]
```

```
[Out] -1/4*ArcTanh[(1 + 2*x^4)/(Sqrt[3]*Sqrt[1 + 2*x^8])]/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 1483

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{1+2x^2}}\,dx, x, x^4\right)\\ &= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{3-x^2}\,dx, x, \frac{1+2x^4}{\sqrt{1+2x^8}}\right)\right)\\ &= -\frac{\text{arctanh}\left(\frac{1+2x^4}{\sqrt{3}\sqrt{1+2x^8}}\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = -\frac{\text{arctanh}\left(\frac{1}{3}\left(\sqrt{6}-\sqrt{6}x^4+\sqrt{3+6x^8}\right)\right)}{2\sqrt{3}}$$

```
[In] Integrate[x^3/((-1 + x^4)*Sqrt[1 + 2*x^8]),x]
[Out] -1/2*ArcTanh[(Sqrt[6] - Sqrt[6]*x^4 + Sqrt[3 + 6*x^8])/3]/Sqrt[3]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$-\frac{\text{arctanh}\left(\frac{(2x^4+1)\sqrt{3}}{3\sqrt{2x^8+1}}\right)\sqrt{3}}{12}$	28
trager	$-\frac{\text{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{2\,\text{RootOf}\left(\_Z^2-3\right)x^4+3\sqrt{2x^8+1}+\text{RootOf}\left(\_Z^2-3\right)}{(-1+x)(1+x)(x^2+1)}\right)}{12}$	58

```
[In] int(x^3/(x^4-1)/(2*x^8+1)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] -1/12*arctanh(1/3*(2*x^4+1)*3^(1/2)/(2*x^8+1)^(1/2))*3^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{2\,x^4-\sqrt{3}(2\,x^4+1)-\sqrt{2\,x^8+1}(\sqrt{3}-3)+1}{x^4-1}\right)$$

[In] integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="fricas")

[Out] 1/12*sqrt(3)*log((2*x^4 - sqrt(3)*(2*x^4 + 1) - sqrt(2*x^8 + 1)*(sqrt(3) - 3) + 1)/(x^4 - 1))

Sympy [F]

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = \int \frac{x^3}{(x-1)\,(x+1)\,(x^2+1)\,\sqrt{2x^8+1}}\,dx$$

[In] integrate(x**3/(x**4-1)/(2*x**8+1)**(1/2),x)

[Out] Integral(x**3/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(2*x**8 + 1)), x)

Maxima [F]

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = \int \frac{x^3}{\sqrt{2\,x^8+1}(x^4-1)}\,dx$$

[In] integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="maxima")

[Out] integrate(x^3/(sqrt(2*x^8 + 1)*(x^4 - 1)), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(27) = 54$.

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = \frac{1}{12}\sqrt{3}\log\left(-\frac{\left|-2\sqrt{2}x^4-2\sqrt{3}+2\sqrt{2}+2\sqrt{2x^8+1}\right|}{2\left(\sqrt{2}x^4-\sqrt{3}-\sqrt{2}-\sqrt{2x^8+1}\right)}\right)$$

```
[In] integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="giac")
[Out] 1/12*sqrt(3)*log(-1/2*abs(-2*sqrt(2)*x^4 - 2*sqrt(3) + 2*sqrt(2) + 2*sqrt(2
*x^8 + 1))/(sqrt(2)*x^4 - sqrt(3) - sqrt(2) - sqrt(2*x^8 + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = -\frac{\sqrt{3}\left(\ln\left(x^4+\frac{\sqrt{2}\sqrt{3}\sqrt{x^8+\frac{1}{2}}}{2}+\frac{1}{2}\right)-\ln\left(x^4-1\right)\right)}{12}$$

```
[In] int(x^3/((x^4 - 1)*(2*x^8 + 1)^(1/2)),x)
[Out] -(3^(1/2)*(log(x^4 + (2^(1/2)*3^(1/2)*(x^8 + 1/2)^(1/2))/2 + 1/2) - log(x^4
 - 1)))/12
```

3.309 $\int x^9\sqrt{1+x^5+x^{10}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 58

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx=-\frac{1}{40}\left(1+2x^5\right)\sqrt{1+x^5+x^{10}}+\frac{1}{15}\left(1+x^5+x^{10}\right)^{3/2}-\frac{3}{80}\text{arcsinh}\left(\frac{1+2x^5}{\sqrt{3}}\right)$$

```
[Out] 1/15*(x^10+x^5+1)^(3/2)-3/80*arcsinh(1/3*(2*x^5+1)*3^(1/2))-1/40*(2*x^5+1)*
(x^10+x^5+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1371, 654, 626, 633, 221}

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx=-\frac{3}{80}\text{arcsinh}\left(\frac{2x^5+1}{\sqrt{3}}\right)+\frac{1}{15}\left(x^{10}+x^5+1\right)^{3/2}-\frac{1}{40}\left(2x^5+1\right)\sqrt{x^{10}+x^5+1}$$

```
[In] Int[x^9*Sqrt[1 + x^5 + x^10],x]
```

```
[Out] -1/40*((1 + 2*x^5)*Sqrt[1 + x^5 + x^10]) + (1 + x^5 + x^10)^(3/2)/15 - (3*A
rcSinh[(1 + 2*x^5)/Sqrt[3]])/80
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5}\text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx, x, x^5\right) \\
&= \frac{1}{15}\left(1+x^5+x^{10}\right)^{3/2} - \frac{1}{10}\text{Subst}\left(\int \sqrt{1+x+x^2}\,dx, x, x^5\right) \\
&= -\frac{1}{40}\left(1+2x^5\right)\sqrt{1+x^5+x^{10}} + \frac{1}{15}\left(1+x^5+x^{10}\right)^{3/2} - \frac{3}{80}\text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}}\,dx, x, x^5\right) \\
&= -\frac{1}{40}\left(1+2x^5\right)\sqrt{1+x^5+x^{10}} + \frac{1}{15}\left(1+x^5+x^{10}\right)^{3/2} \\
&\quad - \frac{1}{80}\sqrt{3}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, 1+2x^5\right) \\
&= -\frac{1}{40}\left(1+2x^5\right)\sqrt{1+x^5+x^{10}} + \frac{1}{15}\left(1+x^5+x^{10}\right)^{3/2} - \frac{3}{80}\text{arcsinh}\left(\frac{1+2x^5}{\sqrt{3}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx = \frac{1}{120}\sqrt{1+x^5+x^{10}}(5+2x^5+8x^{10}) + \frac{3}{80}\log\left(-1-2x^5+2\sqrt{1+x^5+x^{10}}\right)$$

```
[In] Integrate[x^9*Sqrt[1 + x^5 + x^10],x]
```

```
[Out] (Sqrt[1 + x^5 + x^10]*(5 + 2*x^5 + 8*x^10))/120 + (3*Log[-1 - 2*x^5 + 2*Sqr
t[1 + x^5 + x^10]])/80
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$-\frac{3\,\mathrm{arcsinh}\left(\frac{\left(2x^5+1\right)\sqrt{3}}{3}\right)}{80}+\frac{\left(8x^{10}+2x^5+5\right)\sqrt{x^{10}+x^5+1}}{120}$	41
trager	$\left(\frac{1}{15}x^{10}+\frac{1}{60}x^5+\frac{1}{24}\right)\sqrt{x^{10}+x^5+1}+\frac{3\ln\left(-2x^5+2\sqrt{x^{10}+x^5+1}-1\right)}{80}$	47
risch	$\frac{\left(8x^{10}+2x^5+5\right)\sqrt{x^{10}+x^5+1}}{120}-\frac{3\ln\left(2x^5+2\sqrt{x^{10}+x^5+1}+1\right)}{80}$	48

```
[In] int(x^9*(x^10+x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/80*arcsinh(1/3*(2*x^5+1)*3^(1/2))+1/120*(8*x^10+2*x^5+5)*(x^10+x^5+1)^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx = \frac{1}{120}\left(8\,x^{10}+2\,x^5+5\right)\sqrt{x^{10}+x^5+1} + \frac{3}{80}\log\left(-2\,x^5+2\,\sqrt{x^{10}+x^5+1}-1\right)$$

```
[In] integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/120*(8*x^10 + 2*x^5 + 5)*sqrt(x^10 + x^5 + 1) + 3/80*log(-2*x^5 + 2*sqrt(
x^10 + x^5 + 1) - 1)
```

Sympy [F]

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx=\int x^9\sqrt{(x^2+x+1)\,(x^8-x^7+x^5-x^4+x^3-x+1)}\,dx$$

```
[In] integrate(x**9*(x**10+x**5+1)**(1/2),x)
[Out] Integral(x**9*sqrt((x**2 + x + 1)*(x**8 - x**7 + x**5 - x**4 + x**3 - x + 1
)), x)
```

Maxima [F]

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx=\int \sqrt{x^{10}+x^5+1}x^9\,dx$$

```
[In] integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^10 + x^5 + 1)*x^9, x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx=\frac{1}{120}\sqrt{x^{10}+x^5+1}(2\,(4\,x^5+1)x^5+5)+\frac{3}{80}\log\left(-2\,x^5+2\,\sqrt{x^{10}+x^5+1}-1\right)$$

```
[In] integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="giac")
[Out] 1/120*sqrt(x^10 + x^5 + 1)*(2*(4*x^5 + 1)*x^5 + 5) + 3/80*log(-2*x^5 + 2*sq
rt(x^10 + x^5 + 1) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx=\frac{\sqrt{x^{10}+x^5+1}\,(8\,x^{10}+2\,x^5+5)}{120}-\frac{3\,\ln\left(\sqrt{x^{10}+x^5+1}+x^5+\frac{1}{2}\right)}{80}$$

```
[In] int(x^9*(x^5 + x^10 + 1)^(1/2),x)
[Out] ((x^5 + x^10 + 1)^(1/2)*(2*x^5 + 8*x^10 + 5))/120 - (3*log((x^5 + x^10 + 1)
^(1/2) + x^5 + 1/2))/80
```

3.310 $\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = -\frac{\sqrt{4+2x^2+x^4}}{16x^4}+\frac{3\sqrt{4+2x^2+x^4}}{64x^2}+\frac{1}{128}\text{arctanh}\left(\frac{4+x^2}{2\sqrt{4+2x^2+x^4}}\right)$$

```
[Out] 1/128*arctanh(1/2*(x^2+4)/(x^4+2*x^2+4)^(1/2))-1/16*(x^4+2*x^2+4)^(1/2)/x^4
+3/64*(x^4+2*x^2+4)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1128, 758, 820, 738, 212}

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \frac{1}{128}\text{arctanh}\left(\frac{x^2+4}{2\sqrt{x^4+2x^2+4}}\right)+\frac{3\sqrt{x^4+2x^2+4}}{64x^2}-\frac{\sqrt{x^4+2x^2+4}}{16x^4}$$

```
[In] Int[1/(x^5*Sqrt[4 + 2*x^2 + x^4]),x]
```

```
[Out] -1/16*Sqrt[4 + 2*x^2 + x^4]/x^4 + (3*Sqrt[4 + 2*x^2 + x^4])/(64*x^2) + ArcT
anh[(4 + x^2)/(2*Sqrt[4 + 2*x^2 + x^4])]/128
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt{4+2x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{4+2x^2+x^4}}{16x^4}-\frac{1}{16}\text{Subst}\left(\int \frac{3+x}{x^2\sqrt{4+2x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{4+2x^2+x^4}}{16x^4}+\frac{3\sqrt{4+2x^2+x^4}}{64x^2}-\frac{1}{64}\text{Subst}\left(\int \frac{1}{x\sqrt{4+2x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{4+2x^2+x^4}}{16x^4}+\frac{3\sqrt{4+2x^2+x^4}}{64x^2}+\frac{1}{32}\text{Subst}\left(\int \frac{1}{16-x^2}\,dx,x,\frac{2(4+x^2)}{\sqrt{4+2x^2+x^4}}\right)
\end{aligned}$$

$$= -\frac{\sqrt{4+2x^2+x^4}}{16x^4}+\frac{3\sqrt{4+2x^2+x^4}}{64x^2}+\frac{1}{128}\text{arctanh}\left(\frac{4+x^2}{2\sqrt{4+2x^2+x^4}}\right)$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \frac{1}{64}\left(\frac{(-4+3x^2)\sqrt{4+2x^2+x^4}}{x^4} - \text{arctanh}\left(\frac{1}{2}\left(x^2-\sqrt{4+2x^2+x^4}\right)\right)\right)$$

```
[In] Integrate[1/(x^5*Sqrt[4 + 2*x^2 + x^4]),x]
[Out] (((-4 + 3*x^2)*Sqrt[4 + 2*x^2 + x^4])/x^4 - ArcTanh[(x^2 - Sqrt[4 + 2*x^2 +
 x^4])/2])/64
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

method	result	size
trager	$\frac{(3x^2-4)\sqrt{x^4+2x^2+4}}{64x^4}+\frac{\ln\left(\frac{x^2+2\sqrt{x^4+2x^2+4}+4}{x^2}\right)}{128}$	52
default	$-\frac{\sqrt{x^4+2x^2+4}}{16x^4}+\frac{3\sqrt{x^4+2x^2+4}}{64x^2}+\frac{\text{arctanh}\left(\frac{2x^2+8}{4\sqrt{x^4+2x^2+4}}\right)}{128}$	60
risch	$\frac{3x^6+2x^4+4x^2-16}{64x^4\sqrt{x^4+2x^2+4}}+\frac{\text{arctanh}\left(\frac{2x^2+8}{4\sqrt{x^4+2x^2+4}}\right)}{128}$	60
elliptic	$-\frac{\sqrt{x^4+2x^2+4}}{16x^4}+\frac{3\sqrt{x^4+2x^2+4}}{64x^2}+\frac{\text{arctanh}\left(\frac{2x^2+8}{4\sqrt{x^4+2x^2+4}}\right)}{128}$	60
pseudoelliptic	$\frac{\text{arctanh}\left(\frac{x^2+4}{2\sqrt{x^4+2x^2+4}}\right)x^4+6x^2\sqrt{x^4+2x^2+4}-8\sqrt{x^4+2x^2+4}}{128x^4}$	62

```
[In] int(1/x^5/(x^4+2*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/64*(3*x^2-4)/x^4*(x^4+2*x^2+4)^(1/2)+1/128*ln((x^2+2*(x^4+2*x^2+4)^(1/2)+
4)/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \frac{x^4\log\left(-x^2+\sqrt{x^4+2\,x^2+4}+2\right)-x^4\log\left(-x^2+\sqrt{x^4+2\,x^2+4}-2\right)+6\,x^4+2\,\sqrt{x^4+2\,x^2+4}(3\,x^2-}{128\,x^4}$$

```
[In] integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/128*(x^4*log(-x^2 + sqrt(x^4 + 2*x^2 + 4) + 2) - x^4*log(-x^2 + sqrt(x^4
+ 2*x^2 + 4) - 2) + 6*x^4 + 2*sqrt(x^4 + 2*x^2 + 4)*(3*x^2 - 4))/x^4
```

Sympy [F]

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \int \frac{1}{x^5\sqrt{x^4+2x^2+4}}\,dx$$

```
[In] integrate(1/x**5/(x**4+2*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(x**4 + 2*x**2 + 4)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \frac{3\,\sqrt{x^4+2\,x^2+4}}{64\,x^2} - \frac{\sqrt{x^4+2\,x^2+4}}{16\,x^4} + \frac{1}{128}\,\operatorname{arsinh}\left(\frac{1}{3}\,\sqrt{3}+\frac{4\,\sqrt{3}}{3\,x^2}\right)$$

```
[In] integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/64*sqrt(x^4 + 2*x^2 + 4)/x^2 - 1/16*sqrt(x^4 + 2*x^2 + 4)/x^4 + 1/128*arc
sinh(1/3*sqrt(3) + 4/3*sqrt(3)/x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \frac{\left(x^2-\sqrt{x^4+2\,x^2+4}\right)^3+36\,x^2-36\,\sqrt{x^4+2\,x^2+4}+64}{32\,\left(\left(x^2-\sqrt{x^4+2\,x^2+4}\right)^2-4\right)^2} - \frac{1}{128}\,\log\left(x^2-\sqrt{x^4+2\,x^2+4}+2\right) + \frac{1}{128}\,\log\left(-x^2+\sqrt{x^4+2\,x^2+4}+2\right)$$

```
[In] integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*((x^2 - sqrt(x^4 + 2*x^2 + 4))^3 + 36*x^2 - 36*sqrt(x^4 + 2*x^2 + 4) +
 64)/((x^2 - sqrt(x^4 + 2*x^2 + 4))^2 - 4)^2 - 1/128*log(x^2 - sqrt(x^4 + 2
*x^2 + 4) + 2) + 1/128*log(-x^2 + sqrt(x^4 + 2*x^2 + 4) + 2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \int \frac{1}{x^5\,\sqrt{x^4+2\,x^2+4}}\,dx$$

```
[In] int(1/(x^5*(2*x^2 + x^4 + 4)^(1/2)),x)
```

```
[Out] int(1/(x^5*(2*x^2 + x^4 + 4)^(1/2)), x)
```

3.311 $\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 21

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = \operatorname{arctanh}\left(\frac{1+x^2}{\sqrt{1+3x^2+x^4}}\right)$$

```
[Out] arctanh((x^2+1)/(x^4+3*x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1265, 852, 212}

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = \operatorname{arctanh}\left(\frac{x^2+1}{\sqrt{x^4+3x^2+1}}\right)$$

```
[In] Int[(-1 + x^2)/(x*Sqrt[1 + 3*x^2 + x^4]),x]
```

```
[Out] ArcTanh[(1 + x^2)/Sqrt[1 + 3*x^2 + x^4]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 852

```
Int[((f_) + (g_.)*(x_))/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_
.)*(x_)^2]), x_Symbol] :> Dist[4*f*((a - d)/(b*d - a*e)), Subst[Int[1/(4*(a
 - d) - x^2), x], x, (2*(a - d) + (b - e)*x)/Sqrt[a + b*x + c*x^2]], x] /;
```

```
FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[4*c*(a - d) - (b - e)^2, 0] && EqQ[e
*f*(b - e) - 2*g*(b*d - a*e), 0] && NeQ[b*d - a*e, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{x\sqrt{1+3x+x^2}}\,dx, x, x^2\right)\\ &= 2\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{2(1+x^2)}{\sqrt{1+3x^2+x^4}}\right)\\ &= \text{arctanh}\left(\frac{1+x^2}{\sqrt{1+3x^2+x^4}}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 52 vs. 2(21) = 42.

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = -\text{arctanh}\left(x^2-\sqrt{1+3x^2+x^4}\right) - \frac{1}{2}\log\left(-3-2x^2+2\sqrt{1+3x^2+x^4}\right)$$

```
[In] Integrate[(-1 + x^2)/(x*Sqrt[1 + 3*x^2 + x^4]),x]
[Out] -ArcTanh[x^2 - Sqrt[1 + 3*x^2 + x^4]] - Log[-3 - 2*x^2 + 2*Sqrt[1 + 3*x^2 +
 x^4]]/2
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

method	result	size
trager	$\ln\left(\frac{x^2+\sqrt{x^4+3x^2+1}+1}{x}\right)$	23
default	$\frac{\ln\left(x^2+\frac{3}{2}+\sqrt{x^4+3x^2+1}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{3x^2+2}{2\sqrt{x^4+3x^2+1}}\right)}{2}$	46
elliptic	$\frac{\ln\left(x^2+\frac{3}{2}+\sqrt{x^4+3x^2+1}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{3x^2+2}{2\sqrt{x^4+3x^2+1}}\right)}{2}$	46
pseudoelliptic	$\frac{\ln\left(2x^2+3+2\sqrt{x^4+3x^2+1}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{3x^2+2}{2\sqrt{x^4+3x^2+1}}\right)}{2}$	50

```
[In] int((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln((x^2+(x^4+3*x^2+1)^(1/2)+1)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(19) = 38$.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.81

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = -\frac{1}{2}\log\left(4\,x^4+11\,x^2-\sqrt{x^4+3\,x^2+1}(4\,x^2+5)+5\right)+\frac{1}{2}\log\left(-x^2+\sqrt{x^4+3\,x^2+1}+1\right)$$

```
[In] integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*log(4*x^4 + 11*x^2 - sqrt(x^4 + 3*x^2 + 1)*(4*x^2 + 5) + 5) + 1/2*log(
-x^2 + sqrt(x^4 + 3*x^2 + 1) + 1)
```

Sympy [F]

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = \int \frac{(x-1)\,(x+1)}{x\sqrt{x^4+3x^2+1}}\,dx$$

```
[In] integrate((x**2-1)/x/(x**4+3*x**2+1)**(1/2),x)
[Out] Integral((x - 1)*(x + 1)/(x*sqrt(x**4 + 3*x**2 + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(19) = 38$.

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = \frac{1}{2}\log\left(2x^2+2\sqrt{x^4+3x^2+1}+3\right) + \frac{1}{2}\log\left(\frac{2\sqrt{x^4+3x^2+1}}{x^2}+\frac{2}{x^2}+3\right)$$

```
[In] integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*x^2 + 2*sqrt(x^4 + 3*x^2 + 1) + 3) + 1/2*log(2*sqrt(x^4 + 3*x^2 +
 1)/x^2 + 2/x^2 + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(19) = 38$.

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.29

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = -\frac{1}{2}\log\left(2x^2-2\sqrt{x^4+3x^2+1}+3\right) + \frac{1}{2}\log\left(-x^2+\sqrt{x^4+3x^2+1}+1\right) - \frac{1}{2}\log\left(-x^2+\sqrt{x^4+3x^2+1}-1\right)$$

```
[In] integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(2*x^2 - 2*sqrt(x^4 + 3*x^2 + 1) + 3) + 1/2*log(-x^2 + sqrt(x^4 + 3
*x^2 + 1) + 1) - 1/2*log(-x^2 + sqrt(x^4 + 3*x^2 + 1) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = \frac{\ln\left(\frac{1}{x^2}\right)}{2} + \frac{\ln\left(\sqrt{x^4+3x^2+1}+x^2+\frac{3}{2}\right)}{2} + \frac{\ln\left(\frac{2\sqrt{x^4+3x^2+1}}{3}+x^2+\frac{2}{3}\right)}{2}$$

```
[In] int((x^2 - 1)/(x*(3*x^2 + x^4 + 1)^(1/2)),x)
```

```
[Out] log(1/x^2)/2 + log((3*x^2 + x^4 + 1)^(1/2) + x^2 + 3/2)/2 + log((2*(3*x^2 +
 x^4 + 1)^(1/2))/3 + x^2 + 2/3)/2
```

3.312 $\int (-3x+2x^3)(-3x^2+x^4)^{3/5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 17

$$\int (-3x+2x^3)\left(-3x^2+x^4\right)^{3/5}\,dx=\frac{5}{16}\left(-3x^2+x^4\right)^{8/5}$$

```
[Out] 5/16*(x^4-3*x^2)^(8/5)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1602}

$$\int (-3x+2x^3)\left(-3x^2+x^4\right)^{3/5}\,dx=\frac{5}{16}\left(x^4-3x^2\right)^{8/5}$$

```
[In] Int[(-3*x + 2*x^3)*(-3*x^2 + x^4)^(3/5),x]
```

```
[Out] (5*(-3*x^2 + x^4)^(8/5))/16
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{5}{16}\left(-3x^2+x^4\right)^{8/5}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2 in optimal.

Time = 10.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 4.41

$$\int \left(-3x+2x^3\right)\left(-3x^2+x^4\right)^{3/5}\,dx=\frac{5\left(x^2(-3+x^2)\right)^{3/5}\left(-39x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{5},\frac{8}{5},\frac{13}{5},\frac{x^2}{3}\right)+16x^4\,\text{Hypergeometric2F1}\left(-\right.\right.}{208\left(1-\frac{x^2}{3}\right)^{3/5}}$$

```
[In] Integrate[(-3*x + 2*x^3)*(-3*x^2 + x^4)^(3/5),x]
```

```
[Out] (5*(x^2*(-3 + x^2))^(3/5)*(-39*x^2*Hypergeometric2F1[-3/5, 8/5, 13/5, x^2/3
] + 16*x^4*Hypergeometric2F1[-3/5, 13/5, 18/5, x^2/3]))/(208*(1 - x^2/3)^(3
/5))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{5\left(x^4-3x^2\right)^{\frac{8}{5}}}{16}$	14
gosper	$\frac{5\left(x^4-3x^2\right)^{\frac{3}{5}}x^2\left(x^2-3\right)}{16}$	22
trager	$\frac{5\left(x^4-3x^2\right)^{\frac{3}{5}}x^2\left(x^2-3\right)}{16}$	22
risch	$\frac{5x^2\left(x^2\left(x^2-3\right)\right)^{\frac{3}{5}}\left(x^2-3\right)}{16}$	22
pseudoelliptic	$\frac{5x^2\left(x^2\left(x^2-3\right)\right)^{\frac{3}{5}}\left(x^2-3\right)}{16}$	22
meijerg	$\frac{5\,3^{\frac{3}{5}}\,\mathrm{signum}\left(-1+\frac{x^2}{3}\right)^{\frac{3}{5}}x^{\frac{26}{5}}\,{}_2F_1\left(-\frac{3}{5},\frac{13}{5};\frac{18}{5};\frac{x^2}{3}\right)}{13\left(-\mathrm{signum}\left(-1+\frac{x^2}{3}\right)\right)^{\frac{3}{5}}}-\frac{15\,3^{\frac{3}{5}}\,\mathrm{signum}\left(-1+\frac{x^2}{3}\right)^{\frac{3}{5}}x^{\frac{16}{5}}\,{}_2F_1\left(-\frac{3}{5},\frac{8}{5};\frac{13}{5};\frac{x^2}{3}\right)}{16\left(-\mathrm{signum}\left(-1+\frac{x^2}{3}\right)\right)^{\frac{3}{5}}}$	84

```
[In] int((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x,method=_RETURNVERBOSE)
```

```
[Out] 5/16*(x^4-3*x^2)^(8/5)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(-3x+2x^3\right)\left(-3x^2+x^4\right)^{3/5}\,dx=\frac{5}{16}\,\left(x^4-3\,x^2\right)^{\frac{8}{5}}$$

```
[In] integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="fricas")
[Out] 5/16*(x^4 - 3*x^2)^(8/5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(14) = 28.

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int \left(-3x+2x^3\right)\left(-3x^2+x^4\right)^{3/5}\,dx=\frac{5x^4(x^4-3x^2)^{\frac{3}{5}}}{16}-\frac{15x^2(x^4-3x^2)^{\frac{3}{5}}}{16}$$

```
[In] integrate((2*x**3-3*x)*(x**4-3*x**2)**(3/5),x)
[Out] 5*x**4*(x**4 - 3*x**2)**(3/5)/16 - 15*x**2*(x**4 - 3*x**2)**(3/5)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(-3x+2x^3\right)\left(-3x^2+x^4\right)^{3/5}\,dx=\frac{5}{16}\,\left(x^4-3\,x^2\right)^{\frac{8}{5}}$$

```
[In] integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="maxima")
[Out] 5/16*(x^4 - 3*x^2)^(8/5)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (-3x+2x^3)\left(-3x^2+x^4\right)^{3/5}\,dx=\frac{5}{16}\left(x^4-3\,x^2\right)^{\frac{8}{5}}$$

```
[In] integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="giac")
[Out] 5/16*(x^4 - 3*x^2)^(8/5)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int (-3x+2x^3)\left(-3x^2+x^4\right)^{3/5}\,dx=\frac{5\,x^2\,\left(x^2-3\right)\,\left(x^4-3\,x^2\right)^{3/5}}{16}$$

```
[In] int(-(3*x - 2*x^3)*(x^4 - 3*x^2)^(3/5),x)
[Out] (5*x^2*(x^2 - 3)*(x^4 - 3*x^2)^(3/5))/16
```

3.313 $\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 46

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = -\frac{4}{27}\sqrt[4]{-1+3x^3} - \frac{4}{33}(-1+3x^3)^{11/12} + \frac{4}{243}(-1+3x^3)^{9/4}$$

```
[Out] -4/27*(3*x^3-1)^(1/4)-4/33*(3*x^3-1)^(11/12)+4/243*(3*x^3-1)^(9/4)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {6874, 272, 45, 267}

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = \frac{4}{243}(3x^3-1)^{9/4} - \frac{4}{33}(3x^3-1)^{11/12} - \frac{4}{27}\sqrt[4]{3x^3-1}$$

```
[In] Int[(-2*x^5 + 3*x^8 - x^2*(-1 + 3*x^3)^(2/3))/(-1 + 3*x^3)^(3/4),x]
```

```
[Out] (-4*(-1 + 3*x^3)^(1/4))/27 - (4*(-1 + 3*x^3)^(11/12))/33 + (4*(-1 + 3*x^3)^
(9/4))/243
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{2x^5}{\left(-1+3x^3\right)^{3/4}}+\frac{3x^8}{\left(-1+3x^3\right)^{3/4}}-\frac{x^2}{\sqrt[12]{-1+3x^3}}\right) dx \\
&= -\left(2\int \frac{x^5}{\left(-1+3x^3\right)^{3/4}}\, dx\right)+3\int \frac{x^8}{\left(-1+3x^3\right)^{3/4}}\, dx-\int \frac{x^2}{\sqrt[12]{-1+3x^3}}\, dx \\
&= -\frac{4}{33}\left(-1+3x^3\right)^{11/12} \\
&\quad -\frac{2}{3}\text{Subst}\left(\int \frac{x}{(-1+3x)^{3/4}}\, dx,x,x^3\right)+\text{Subst}\left(\int \frac{x^2}{(-1+3x)^{3/4}}\, dx,x,x^3\right) \\
&= -\frac{4}{33}\left(-1+3x^3\right)^{11/12}-\frac{2}{3}\text{Subst}\left(\int \left(\frac{1}{3(-1+3x)^{3/4}}+\frac{1}{3}\sqrt[4]{-1+3x}\right) dx,x,x^3\right) \\
&\quad +\text{Subst}\left(\int \left(\frac{1}{9(-1+3x)^{3/4}}+\frac{2}{9}\sqrt[4]{-1+3x}+\frac{1}{9}(-1+3x)^{5/4}\right) dx,x,x^3\right) \\
&= -\frac{4}{27}\sqrt[4]{-1+3x^3}-\frac{4}{33}\left(-1+3x^3\right)^{11/12}+\frac{4}{243}\left(-1+3x^3\right)^{9/4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx=-\frac{4\sqrt[4]{-1+3x^3}\left(88+66x^3-99x^6+81(-1+3x^3)^{2/3}\right)}{2673}$$

```
[In] Integrate[(-2*x^5 + 3*x^8 - x^2*(-1 + 3*x^3)^(2/3))/(-1 + 3*x^3)^(3/4),x]
[Out] (-4*(-1 + 3*x^3)^(1/4)*(88 + 66*x^3 - 99*x^6 + 81*(-1 + 3*x^3)^(2/3)))/2673
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.52

method	result
meijerg	$-\frac{\left(-\operatorname{signum}\left(3x^3-1\right)\right)^{\frac{3}{4}} x^6 {}_2F_1\left(\frac{3}{4},2;3;3x^3\right)}{3\operatorname{signum}\left(3x^3-1\right)^{\frac{3}{4}}}+\frac{\left(-\operatorname{signum}\left(3x^3-1\right)\right)^{\frac{3}{4}} x^9 {}_2F_1\left(\frac{3}{4},3;4;3x^3\right)}{3\operatorname{signum}\left(3x^3-1\right)^{\frac{3}{4}}}-\frac{\left(-\operatorname{signum}\left(3x^3-1\right)\right)^{\frac{1}{12}} x^3 {}_2F_1\left(\frac{1}{12},1;2;3x^3\right.}{3\operatorname{signum}\left(3x^3-1\right)^{\frac{1}{12}}}$

```
[In] int((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x,method=_RETURNVERB
OSE)
[Out] -1/3/signum(3*x^3-1)^(3/4)*(-signum(3*x^3-1))^(3/4)*x^6*hypergeom([3/4,2],[
3],3*x^3)+1/3/signum(3*x^3-1)^(3/4)*(-signum(3*x^3-1))^(3/4)*x^9*hypergeom(
[3/4,3],[4],3*x^3)-1/3/signum(3*x^3-1)^(1/12)*(-signum(3*x^3-1))^(1/12)*x^3
*hypergeom([1/12,1],[2],3*x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx=\frac{4}{243}\left(9\,x^6-6\,x^3-8\right)\left(3\,x^3-1\right)^{\frac{1}{4}}-\frac{4}{33}\left(3\,x^3-1\right)^{\frac{11}{12}}$$

```
[In] integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm="
fricas")
[Out] 4/243*(9*x^6 - 6*x^3 - 8)*(3*x^3 - 1)^(1/4) - 4/33*(3*x^3 - 1)^(11/12)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.67 (sec) , antiderivative size = 221, normalized size of antiderivative = 4.80

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = -\frac{4(3x^3-1)^{\frac{11}{12}}}{33} - 2\left(\begin{cases} \frac{4x^3\sqrt[4]{3x^3-1}}{45} + \frac{16\sqrt[4]{3x^3-1}}{135} & \text{for } |x^3| > \frac{1}{3} \\ -\frac{4x^3\sqrt[4]{1-3x^3}e^{-\frac{3i\pi}{4}}}{45} - \frac{16\sqrt[4]{1-3x^3}e^{-\frac{3i\pi}{4}}}{135} & \text{otherwise} \end{cases}\right) + 3\left(\begin{cases} \frac{4x^6\sqrt[4]{3x^3-1}}{81} + \frac{32x^3\sqrt[4]{3x^3-1}}{1215} + \frac{128\sqrt[4]{3x^3-1}}{3645} & \text{for } |x^3| > \frac{1}{3} \\ \frac{4x^6\sqrt[4]{1-3x^3}e^{\frac{i\pi}{4}}}{81} + \frac{32x^3\sqrt[4]{1-3x^3}e^{\frac{i\pi}{4}}}{1215} + \frac{128\sqrt[4]{1-3x^3}e^{\frac{i\pi}{4}}}{3645} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((-2*x**5+3*x**8-x**2*(3*x**3-1)**(2/3))/(3*x**3-1)**(3/4),x)
```

```
[Out] -4*(3*x**3 - 1)**(11/12)/33 - 2*Piecewise((4*x**3*(3*x**3 - 1)**(1/4)/45 +
16*(3*x**3 - 1)**(1/4)/135, Abs(x**3) > 1/3), (-4*x**3*(1 - 3*x**3)**(1/4)*
exp(-3*I*pi/4)/45 - 16*(1 - 3*x**3)**(1/4)*exp(-3*I*pi/4)/135, True)) + 3*P
iecewise((4*x**6*(3*x**3 - 1)**(1/4)/81 + 32*x**3*(3*x**3 - 1)**(1/4)/1215
+ 128*(3*x**3 - 1)**(1/4)/3645, Abs(x**3) > 1/3), (4*x**6*(1 - 3*x**3)**(1/
4)*exp(I*pi/4)/81 + 32*x**3*(1 - 3*x**3)**(1/4)*exp(I*pi/4)/1215 + 128*(1 -
 3*x**3)**(1/4)*exp(I*pi/4)/3645, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = \frac{4}{243}\left(3x^3-1\right)^{\frac{9}{4}} - \frac{4}{33}\left(3x^3-1\right)^{\frac{11}{12}} - \frac{4}{27}\left(3x^3-1\right)^{\frac{1}{4}}$$

```
[In] integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm="
maxima")
```

```
[Out] 4/243*(3*x^3 - 1)^(9/4) - 4/33*(3*x^3 - 1)^(11/12) - 4/27*(3*x^3 - 1)^(1/4)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = \frac{4}{243}\left(3\,x^3-1\right)^{\frac{9}{4}} - \frac{4}{33}\left(3\,x^3-1\right)^{\frac{11}{12}} - \frac{4}{27}\left(3\,x^3-1\right)^{\frac{1}{4}}$$

```
[In] integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm="
giac")
[Out] 4/243*(3*x^3 - 1)^(9/4) - 4/33*(3*x^3 - 1)^(11/12) - 4/27*(3*x^3 - 1)^(1/4)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = -\left(3\,x^3-1\right)^{1/4}\left(\frac{8\,x^3}{81}-\frac{4\,x^6}{27}+\frac{4\,\left(3\,x^3-1\right)^{2/3}}{33}+\frac{32}{243}\right)$$

```
[In] int(-(x^2*(3*x^3 - 1)^(2/3) + 2*x^5 - 3*x^8)/(3*x^3 - 1)^(3/4),x)
[Out] -(3*x^3 - 1)^(1/4)*((8*x^3)/81 - (4*x^6)/27 + (4*(3*x^3 - 1)^(2/3))/33 + 32
/243)
```

3.314 $\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3^{5/6}} - \frac{\log(-1+x^3)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{2+x^3}\right)}{2\sqrt[3]{3}}$$

```
[Out] -1/3*arctan(1/3*(1+2*3^(1/3)*x/(x^3+2)^(1/3))*3^(1/2))*3^(1/6)-1/18*ln(x^3-
1)*3^(2/3)+1/6*ln(3^(1/3)*x-(x^3+2)^(1/3))*3^(2/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {384}

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = -\frac{\arctan\left(\frac{\frac{2\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}} - \frac{\log(x^3-1)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{x^3+2}\right)}{2\sqrt[3]{3}}$$

```
[In] Int[1/((-1 + x^3)*(2 + x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[(1 + (2*3^(1/3)*x)/(2 + x^3)^(1/3))/Sqrt[3]]/3^(5/6)) - Log[-1 + x
^3]/(6*3^(1/3)) + Log[3^(1/3)*x - (2 + x^3)^(1/3)]/(2*3^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3^{5/6}} - \frac{\log(-1+x^3)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}x - \sqrt[3]{2+x^3}\right)}{2\sqrt[3]{3}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.41

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = \frac{-6\arctan\left(\frac{3^{5/6}x}{\sqrt[3]{3}x+2\sqrt[3]{2+x^3}}\right) + \sqrt{3}\left(2\log\left(-3x+3^{2/3}\sqrt[3]{2+x^3}\right) - \log\left(3x^2+3^{2/3}x\sqrt[3]{2+x^3}+\sqrt[3]{3}(2+x^3\right.\right.}{6\ 3^{5/6}}$$

```
[In] Integrate[1/((-1 + x^3)*(2 + x^3)^(1/3)),x]
[Out] (-6*ArcTan[(3^(5/6)*x)/(3^(1/3)*x + 2*(2 + x^3)^(1/3))] + Sqrt[3]*(2*Log[-3
*x + 3^(2/3)*(2 + x^3)^(1/3)] - Log[3*x^2 + 3^(2/3)*x*(2 + x^3)^(1/3) + 3^(
1/3)*(2 + x^3)^(2/3)]))/(6*3^(5/6))
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

method	result	size
pseudoelliptic	$\frac{3^{\frac{2}{3}}\ln\left(\frac{-3^{\frac{1}{3}}x+\left(x^{3}+2\right)^{\frac{1}{3}}}{x}\right)}{9}-\frac{3^{\frac{2}{3}}\ln\left(\frac{3^{\frac{2}{3}}x^{2}+3^{\frac{1}{3}}\left(x^{3}+2\right)^{\frac{1}{3}}x+\left(x^{3}+2\right)^{\frac{2}{3}}}{x^{2}}\right)}{18}+\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(2\,3^{\frac{2}{3}}\left(x^{3}+2\right)^{\frac{1}{3}}+3x\right)}{9x}\right)}{3}$	93
trager	Expression too large to display	818

```
[In] int(1/(x^3-1)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*3^(2/3)*ln((-3^(1/3)*x+(x^3+2)^(1/3))/x)-1/18*3^(2/3)*ln((3^(2/3)*x^2+3
^(1/3)*(x^3+2)^(1/3)*x+(x^3+2)^(2/3))/x^2)+1/3*3^(1/6)*arctan(1/9*3^(1/2)*(
2*3^(2/3)*(x^3+2)^(1/3)+3*x)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(59) = 118.

Time = 1.65 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.97

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = \frac{1}{27}\cdot 3^{\frac{2}{3}}\log\left(\frac{9\cdot 3^{\frac{1}{3}}(x^3+2)^{\frac{1}{3}}x^2-2\cdot 3^{\frac{2}{3}}(x^3-1)-9\,(x^3+2)^{\frac{2}{3}}x}{x^3-1}\right)-\frac{1}{54}\cdot 3^{\frac{2}{3}}\log\left(\frac{3\cdot 3^{\frac{2}{3}}(7\,x^4+2\,x)(x^3+2)^{\frac{2}{3}}+3^{\frac{1}{3}}(31\,x^6+46\,x^3+4)+9\,(5\,x^5+4\,x^2)(x^3+2)^{\frac{1}{3}}}{x^6-2\,x^3+1}\right)-\frac{1}{9}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{1}{6}}\left(12\cdot 3^{\frac{2}{3}}(7\,x^7-5\,x^4-2\,x)(x^3+2)^{\frac{2}{3}}-3^{\frac{1}{3}}(127\,x^9+402\,x^6+192\,x^3+8)-18\,(31\,x^8+46\right.}{3\,(251\,x^9+462\,x^6+24\,x^3-8)}\right.$$

```
[In] integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/27*3^(2/3)*log((9*3^(1/3)*(x^3 + 2)^(1/3)*x^2 - 2*3^(2/3)*(x^3 - 1) - 9*(
x^3 + 2)^(2/3)*x)/(x^3 - 1)) - 1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 2*x)*(x
^3 + 2)^(2/3) + 3^(1/3)*(31*x^6 + 46*x^3 + 4) + 9*(5*x^5 + 4*x^2)*(x^3 + 2)
^(1/3))/(x^6 - 2*x^3 + 1)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*
x^7 - 5*x^4 - 2*x)*(x^3 + 2)^(2/3) - 3^(1/3)*(127*x^9 + 402*x^6 + 192*x^3 +
 8) - 18*(31*x^8 + 46*x^5 + 4*x^2)*(x^3 + 2)^(1/3))/(251*x^9 + 462*x^6 + 24
*x^3 - 8))
```

Sympy [F]

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x-1)\sqrt[3]{x^3+2}\,(x^2+x+1)}\,dx$$

```
[In] integrate(1/(x**3-1)/(x**3+2)**(1/3),x)
```

```
[Out] Integral(1/((x - 1)*(x**3 + 2)**(1/3)*(x**2 + x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x^3+2)^{\frac{1}{3}}(x^3-1)}\,dx$$

```
[In] integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 2)^(1/3)*(x^3 - 1)), x)
```

Giac [F]

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x^3+2)^{\frac{1}{3}}(x^3-1)}\,dx$$

```
[In] integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="giac")
[Out] integrate(1/((x^3 + 2)^(1/3)*(x^3 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x^3-1)\,(x^3+2)^{1/3}}\,dx$$

```
[In] int(1/((x^3 - 1)*(x^3 + 2)^(1/3)),x)
[Out] int(1/((x^3 - 1)*(x^3 + 2)^(1/3)), x)
```

3.315 $\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 141

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}} - \frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}} + \frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}+\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}}$$

```
[Out] 1/4*arctan(-1+x*2^(1/2)/(x^4+2)^(1/4))*2^(1/2)+1/4*arctan(1+x*2^(1/2)/(x^4+
2)^(1/4))*2^(1/2)-1/8*ln(1-x*2^(1/2)/(x^4+2)^(1/4)+x^2/(x^4+2)^(1/2))*2^(1/
2)+1/8*ln(1+x*2^(1/2)/(x^4+2)^(1/4)+x^2/(x^4+2)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {385, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{2\sqrt{2}} + \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{2\sqrt{2}} - \frac{\log\left(-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+\frac{x^2}{\sqrt{x^4+2}}+1\right)}{4\sqrt{2}} + \frac{\log\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+\frac{x^2}{\sqrt{x^4+2}}+1\right)}{4\sqrt{2}}$$

```
[In] Int[1/((1 + x^4)*(2 + x^4)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[1 - (Sqrt[2]*x)/(2 + x^4)^(1/4)]/Sqrt[2] + ArcTan[1 + (Sqrt[2]*
x)/(2 + x^4)^(1/4)]/(2*Sqrt[2]) - Log[1 + x^2/Sqrt[2 + x^4] - (Sqrt[2]*x)/(
2 + x^4)^(1/4)]/(4*Sqrt[2]) + Log[1 + x^2/Sqrt[2 + x^4] + (Sqrt[2]*x)/(2 +
x^4)^(1/4)]/(4*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{x}{\sqrt[4]{2+x^4}}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{x}{\sqrt[4]{2+x^4}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{x}{\sqrt[4]{2+x^4}}\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{x}{\sqrt[4]{2+x^4}}\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{x}{\sqrt[4]{2+x^4}}\right)\\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}}\\
&= -\frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}} + \frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}+\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}}\\
&= -\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}}\\
&\quad - \frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}} + \frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}+\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.54

$$
\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}x\sqrt[4]{2+x^4}}{-x^2+\sqrt{2+x^4}}\right) + \operatorname{arctanh}\left(\frac{\sqrt{2}x\sqrt[4]{2+x^4}}{x^2+\sqrt{2+x^4}}\right)}{2\sqrt{2}}
$$

```
[In] Integrate[1/((1 + x^4)*(2 + x^4)^(1/4)),x]
[Out] (ArcTan[(Sqrt[2]*x*(2 + x^4)^(1/4))/(-x^2 + Sqrt[2 + x^4])] + ArcTanh[(Sqrt
[2]*x*(2 + x^4)^(1/4))/(x^2 + Sqrt[2 + x^4])])/(2*Sqrt[2])
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$-\frac{\sqrt{2}\left(\ln\left(\frac{-\left(x^{4}+2\right)^{\frac{1}{4}}\sqrt{2}\,x+x^{2}+\sqrt{x^{4}+2}}{\left(x^{4}+2\right)^{\frac{1}{4}}\sqrt{2}\,x+x^{2}+\sqrt{x^{4}+2}}\right)+2\arctan\left(\frac{\left(x^{4}+2\right)^{\frac{1}{4}}\sqrt{2}+x}{x}\right)+2\arctan\left(\frac{\left(x^{4}+2\right)^{\frac{1}{4}}\sqrt{2}-x}{x}\right)\right)}{8}$
trager	$\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}\ln\left(\frac{\left(x^{4}+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}x^{3}-\sqrt{x^{4}+2}\,\mathrm{RootOf}\left(\_Z^{4}+1\right)x^{2}+\left(x^{4}+2\right)^{\frac{3}{4}}x+\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}}{x^{4}+1}\right)}{4}+$

```
[In] int(1/(x^4+1)/(x^4+2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*2^(1/2)*(ln((-(x^4+2)^(1/4)*2^(1/2)*x+x^2+(x^4+2)^(1/2))/((x^4+2)^(1/4
)*2^(1/2)*x+x^2+(x^4+2)^(1/2)))+2*arctan(((x^4+2)^(1/4)*2^(1/2)+x)/x)+2*arc
tan(((x^4+2)^(1/4)*2^(1/2)-x)/x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.39 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.60

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = -\left(\frac{1}{16}i + \frac{1}{16}\right)\sqrt{2}\log\left(\frac{(i+1)\sqrt{2}(x^4+2)^{\frac{1}{4}}x^3 - 2i\sqrt{x^4+2}x^2 + (i-1)\sqrt{2}(x^4+2)^{\frac{3}{4}}x + 2}{x^4+1}\right) + \left(\frac{1}{16}i - \frac{1}{16}\right)\sqrt{2}\log\left(\frac{-(i-1)\sqrt{2}(x^4+2)^{\frac{1}{4}}x^3 + 2i\sqrt{x^4+2}x^2 - (i+1)\sqrt{2}(x^4+2)^{\frac{3}{4}}x + 2}{x^4+1}\right) - \left(\frac{1}{16}i - \frac{1}{16}\right)\sqrt{2}\log\left(\frac{(i-1)\sqrt{2}(x^4+2)^{\frac{1}{4}}x^3 + 2i\sqrt{x^4+2}x^2 + (i+1)\sqrt{2}(x^4+2)^{\frac{3}{4}}x + 2}{x^4+1}\right) + \left(\frac{1}{16}i + \frac{1}{16}\right)\sqrt{2}\log\left(\frac{-(i+1)\sqrt{2}(x^4+2)^{\frac{1}{4}}x^3 - 2i\sqrt{x^4+2}x^2 - (i-1)\sqrt{2}(x^4+2)^{\frac{3}{4}}x + 2}{x^4+1}\right)$$

```
[In] integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="fricas")
```

```
[Out] -(1/16*I + 1/16)*sqrt(2)*log(((I + 1)*sqrt(2)*(x^4 + 2)^(1/4)*x^3 - 2*I*sqr
t(x^4 + 2)*x^2 + (I - 1)*sqrt(2)*(x^4 + 2)^(3/4)*x + 2)/(x^4 + 1)) + (1/16*
I - 1/16)*sqrt(2)*log((-(I - 1)*sqrt(2)*(x^4 + 2)^(1/4)*x^3 + 2*I*sqrt(x^4
+ 2)*x^2 - (I + 1)*sqrt(2)*(x^4 + 2)^(3/4)*x + 2)/(x^4 + 1)) - (1/16*I - 1/
16)*sqrt(2)*log(((I - 1)*sqrt(2)*(x^4 + 2)^(1/4)*x^3 + 2*I*sqrt(x^4 + 2)*x^
2 + (I + 1)*sqrt(2)*(x^4 + 2)^(3/4)*x + 2)/(x^4 + 1)) + (1/16*I + 1/16)*sqr
t(2)*log((-(I + 1)*sqrt(2)*(x^4 + 2)^(1/4)*x^3 - 2*I*sqrt(x^4 + 2)*x^2 - (I
 - 1)*sqrt(2)*(x^4 + 2)^(3/4)*x + 2)/(x^4 + 1))
```

Sympy [F]

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \int \frac{1}{(x^4+1)\sqrt[4]{x^4+2}}\,dx$$

```
[In] integrate(1/(x**4+1)/(x**4+2)**(1/4),x)
```

```
[Out] Integral(1/((x**4 + 1)*(x**4 + 2)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \int \frac{1}{(x^4+2)^{\frac{1}{4}}(x^4+1)}\,dx$$

```
[In] integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 2)^(1/4)*(x^4 + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \int \frac{1}{(x^4+2)^{\frac{1}{4}}(x^4+1)}\,dx$$

```
[In] integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 2)^(1/4)*(x^4 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \int \frac{1}{(x^4+1)\,(x^4+2)^{1/4}}\,dx$$

```
[In] int(1/((x^4 + 1)*(x^4 + 2)^(1/4)),x)
[Out] int(1/((x^4 + 1)*(x^4 + 2)^(1/4)), x)
```

3.316 $\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx=\frac{1}{3}x\left(2+x^3\right)^{2/3}-\frac{5\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{5}{6}\log\left(-x+\sqrt[3]{2+x^3}\right)$$

```
[Out] 1/3*x*(x^3+2)^(2/3)+5/6*ln(-x+(x^3+2)^(1/3))-5/9*arctan(1/3*(1+2*x/(x^3+2)^
(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {396, 245}

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx=-\frac{5\arctan\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\left(x^3+2\right)^{2/3}x+\frac{5}{6}\log\left(\sqrt[3]{x^3+2}-x\right)$$

```
[In] Int[(-1 + x^3)/(2 + x^3)^(1/3),x]
```

```
[Out] (x*(2 + x^3)^(2/3))/3 - (5*ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]])/(3*
Sqrt[3]) + (5*Log[-x + (2 + x^3)^(1/3)])/6
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
```

```
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}x(2+x^3)^{2/3} - \frac{5}{3}\int \frac{1}{\sqrt[3]{2+x^3}}\,dx \\ &= \frac{1}{3}x(2+x^3)^{2/3} - \frac{5\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{5}{6}\log\left(-x+\sqrt[3]{2+x^3}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx = \frac{1}{18}\left(6x(2+x^3)^{2/3} - 10\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x+2\sqrt[3]{2+x^3}}\right) + 10\log\left(-x+\sqrt[3]{2+x^3}\right) - 5\log\left(x^2+x\sqrt[3]{2+x^3}+(2+x^3)^{2/3}\right)\right)$$

```
[In] Integrate[(-1 + x^3)/(2 + x^3)^(1/3),x]
```

```
[Out] (6*x*(2 + x^3)^(2/3) - 10*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(2 + x^3)^(1/3)
)] + 10*Log[-x + (2 + x^3)^(1/3)] - 5*Log[x^2 + x*(2 + x^3)^(1/3) + (2 + x^
3)^(2/3)])/18
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.46

method	result
risch	$\frac{x(x^3+2)^{\frac{2}{3}}}{3}-\frac{5\,2^{\frac{2}{3}}x\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{x^3}{2}\right)}{6}$
meijerg	$-\frac{2^{\frac{2}{3}}x\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{x^3}{2}\right)}{2}+\frac{2^{\frac{2}{3}}x^4\,{}_2F_1\left(\frac{1}{3},\frac{4}{3};\frac{7}{3};-\frac{x^3}{2}\right)}{8}$
pseudoelliptic	$\frac{6x(x^3+2)^{\frac{2}{3}}+10\sqrt{3}\,\arctan\left(\frac{\left(x+2\left(x^3+2\right)^{\frac{1}{3}}\right)\sqrt{3}}{3x}\right)-5\ln\left(\frac{\left(x^3+2\right)^{\frac{2}{3}}+x\left(x^3+2\right)^{\frac{1}{3}}+x^2}{x^2}\right)+10\ln\left(\frac{-x+\left(x^3+2\right)^{\frac{1}{3}}}{x}\right)}{9\left((x^3+2)^{\frac{2}{3}}+x(x^3+2)^{\frac{1}{3}}+x^2\right)\left(-x+(x^3+2)^{\frac{1}{3}}\right)}$
trager	$\frac{x(x^3+2)^{\frac{2}{3}}}{3}+\frac{5\ln\left(-8\,\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)^2x^3+6\,\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)(x^3+2)^{\frac{2}{3}}x-10\,\mathrm{RootOf}\left(4\_Z^2+2\_\right.\right.}{9}$

```
[In] int((x^3-1)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
[Out] 1/3*x*(x^3+2)^(2/3)-5/6*2^(2/3)*x*hypergeom([1/3,1/3],[4/3],-1/2*x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx=\frac{1}{3}\left(x^3+2\right)^{\frac{2}{3}}x+\frac{5}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}(x^3+2)^{\frac{1}{3}}}{3x}\right)+\frac{5}{9}\log\left(-\frac{x-(x^3+2)^{\frac{1}{3}}}{x}\right)-\frac{5}{18}\log\left(\frac{x^2+(x^3+2)^{\frac{1}{3}}x+(x^3+2)^{\frac{2}{3}}}{x^2}\right)$$

```
[In] integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="fricas")
[Out] 1/3*(x^3 + 2)^(2/3)*x + 5/9*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3
+ 2)^(1/3))/x) + 5/9*log(-(x - (x^3 + 2)^(1/3))/x) - 5/18*log((x^2 + (x^3 +
 2)^(1/3)*x + (x^3 + 2)^(2/3))/x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx=\frac{2^{\frac{2}{3}}x^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{x^3e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{7}{3}\right)}-\frac{2^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{x^3e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((x**3-1)/(x**3+2)**(1/3),x)
```

```
[Out] 2**(2/3)*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), x**3*exp_polar(I*pi)/2)/
(6*gamma(7/3)) - 2**(2/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_p
olar(I*pi)/2)/(6*gamma(4/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.49

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx = \frac{5}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\,(x^3+2)^{\frac{1}{3}}}{x}+1\right)\right)+\frac{2\,(x^3+2)^{\frac{2}{3}}}{3\,x^2\left(\frac{x^3+2}{x^3}-1\right)} - \frac{5}{18}\log\left(\frac{(x^3+2)^{\frac{1}{3}}}{x}+\frac{(x^3+2)^{\frac{2}{3}}}{x^2}+1\right)+\frac{5}{9}\log\left(\frac{(x^3+2)^{\frac{1}{3}}}{x}-1\right)$$

```
[In] integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="maxima")
```

```
[Out] 5/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 2)^(1/3)/x + 1)) + 2/3*(x^3 + 2)^(
2/3)/(x^2*((x^3 + 2)/x^3 - 1)) - 5/18*log((x^3 + 2)^(1/3)/x + (x^3 + 2)^(2/
3)/x^2 + 1) + 5/9*log((x^3 + 2)^(1/3)/x - 1)
```

Giac [F]

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx = \int \frac{x^3-1}{(x^3+2)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)/(x^3 + 2)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx = \int \frac{x^3-1}{(x^3+2)^{1/3}}\,dx$$

```
[In] int((x^3 - 1)/(x^3 + 2)^(1/3),x)
```

```
[Out] int((x^3 - 1)/(x^3 + 2)^(1/3), x)
```

3.317 $\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\, dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\, dx = \frac{x(1+x^4)^{3/4}}{8(2+x^4)} + \frac{3\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\ 2^{3/4}} + \frac{3\text{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\ 2^{3/4}}$$

```
[Out] 1/8*x*(x^4+1)^(3/4)/(x^4+2)+3/32*arctan(1/2*x*2^(3/4)/(x^4+1)^(1/4))*2^(1/4
)+3/32*arctanh(1/2*x*2^(3/4)/(x^4+1)^(1/4))*2^(1/4)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {386, 385, 218, 212, 209}

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\, dx = \frac{3\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{16\ 2^{3/4}} + \frac{3\text{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{16\ 2^{3/4}} + \frac{(x^4+1)^{3/4}x}{8(x^4+2)}$$

```
[In] Int[(1 + x^4)^(3/4)/(2 + x^4)^2,x]
```

```
[Out] (x*(1 + x^4)^(3/4))/(8*(2 + x^4)) + (3*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))])
/(16*2^(3/4)) + (3*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))])/(16*2^(3/4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(1+x^4)^{3/4}}{8\,(2+x^4)}+\frac{3}{8}\int\frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx\\
&=\frac{x(1+x^4)^{3/4}}{8\,(2+x^4)}+\frac{3}{8}\text{Subst}\left(\int\frac{1}{2-x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)\\
&=\frac{x(1+x^4)^{3/4}}{8\,(2+x^4)}+\frac{3\text{Subst}\left(\int\frac{1}{\sqrt{2}-x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)}{16\sqrt{2}}+\frac{3\text{Subst}\left(\int\frac{1}{\sqrt{2}+x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)}{16\sqrt{2}}\\
&=\frac{x(1+x^4)^{3/4}}{8\,(2+x^4)}+\frac{3\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\;2^{3/4}}+\frac{3\text{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\;2^{3/4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx = \frac{x(1+x^4)^{3/4}}{8\,(2+x^4)} + \frac{3\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\ 2^{3/4}} + \frac{3\text{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\ 2^{3/4}}$$

```
[In] Integrate[(1 + x^4)^(3/4)/(2 + x^4)^2,x]
[Out] (x*(1 + x^4)^(3/4))/(8*(2 + x^4)) + (3*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))])
/(16*2^(3/4)) + (3*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))])/(16*2^(3/4))
```

Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$\frac{-3(x^4+2)\left(2\arctan\left(\frac{2^{\frac{1}{4}}(x^4+1)^{\frac{1}{4}}}{x}\right)-\ln\left(\frac{-x2^{\frac{3}{4}}-2(x^4+1)^{\frac{1}{4}}}{x2^{\frac{3}{4}}-2(x^4+1)^{\frac{1}{4}}}\right)\right)2^{\frac{1}{4}}+8(x^4+1)^{\frac{3}{4}}x}{64x^4+128}$
trager	$\frac{(x^4+1)^{\frac{3}{4}}x}{8x^4+16}+\frac{3\,\text{RootOf}\left(\_Z^2+\text{RootOf}\left(\_Z^4-2\right)^2\right)\ln\left(-\frac{-2\sqrt{x^4+1}\ \text{RootOf}\left(\_Z^2+\text{RootOf}\left(\_Z^4-2\right)^2\right)\text{RootOf}\left(\_Z^4\right.}{}\right.}{}$
risch	$\frac{(x^4+1)^{\frac{3}{4}}x}{8x^4+16}-\frac{3\,\text{RootOf}\left(\_Z^2+\text{RootOf}\left(\_Z^4-2\right)^2\right)\ln\left(-\frac{2\sqrt{x^4+1}\ \text{RootOf}\left(\_Z^2+\text{RootOf}\left(\_Z^4-2\right)^2\right)\text{RootOf}\left(\_Z^4-\right.}{}\right.}{}$

```
[In] int((x^4+1)^(3/4)/(x^4+2)^2,x,method=_RETURNVERBOSE)
[Out] (-3*(x^4+2)*(2*arctan(1/x*2^(1/4)*(x^4+1)^(1/4))-ln((-x*2^(3/4)-2*(x^4+1)^(
1/4))/(x*2^(3/4)-2*(x^4+1)^(1/4))))*2^(1/4)+8*(x^4+1)^(3/4)*x)/(64*x^4+128)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.41 (sec) , antiderivative size = 313, normalized size of antiderivative = 4.23

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx = \frac{3\cdot 8^{\frac{3}{4}}(x^4+2)\log\left(\frac{8\sqrt{2}(x^4+1)^{\frac{1}{4}}x^3+8\cdot 8^{\frac{1}{4}}\sqrt{x^4+1}x^2+8^{\frac{3}{4}}(3x^4+2)+16(x^4+1)^{\frac{3}{4}}x}{x^4+2}\right)-3\cdot 8^{\frac{3}{4}}(-i\,x^4-2i)}{}$$

```
[In] integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="fricas")
```

```
[Out] 1/512*(3*8^(3/4)*(x^4 + 2)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 8*8^(1/4)*s
qrt(x^4 + 1)*x^2 + 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2)) -
 3*8^(3/4)*(-I*x^4 - 2*I)*log(-(8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 8*I*8^(1/4)
*sqrt(x^4 + 1)*x^2 - 8^(3/4)*(3*I*x^4 + 2*I) - 16*(x^4 + 1)^(3/4)*x)/(x^4 +
 2)) - 3*8^(3/4)*(I*x^4 + 2*I)*log(-(8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 - 8*I*8^
(1/4)*sqrt(x^4 + 1)*x^2 - 8^(3/4)*(-3*I*x^4 - 2*I) - 16*(x^4 + 1)^(3/4)*x)/
(x^4 + 2)) - 3*8^(3/4)*(x^4 + 2)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 - 8*8^(
1/4)*sqrt(x^4 + 1)*x^2 - 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 +
 2)) + 64*(x^4 + 1)^(3/4)*x)/(x^4 + 2)
```

Sympy [F]

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx = \int \frac{(x^4+1)^{\frac{3}{4}}}{(x^4+2)^2}\,dx$$

```
[In] integrate((x**4+1)**(3/4)/(x**4+2)**2,x)
[Out] Integral((x**4 + 1)**(3/4)/(x**4 + 2)**2, x)
```

Maxima [F]

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx = \int \frac{(x^4+1)^{\frac{3}{4}}}{(x^4+2)^2}\,dx$$

```
[In] integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="maxima")
[Out] integrate((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)
```

Giac [F]

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx = \int \frac{(x^4+1)^{\frac{3}{4}}}{(x^4+2)^2}\,dx$$

```
[In] integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="giac")
[Out] integrate((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx=\int \frac{(x^4+1)^{3/4}}{(x^4+2)^2}\,dx$$

```
[In] int((x^4 + 1)^(3/4)/(x^4 + 2)^2,x)
[Out] int((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)
```

3.318 $\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx=-\frac{5x(-2+x^5)}{33\,(3+x^5)^{11/5}}+\frac{5x}{297\,(3+x^5)^{6/5}}+\frac{97x}{891\sqrt[5]{3+x^5}}$$

```
[Out] -5/33*x*(x^5-2)/(x^5+3)^(11/5)+5/297*x/(x^5+3)^(6/5)+97/891*x/(x^5+3)^(1/5)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {386, 197}

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx=\frac{x(2-x^5)^2}{33\,(x^5+3)^{11/5}}+\frac{10x(2-x^5)}{297\,(x^5+3)^{6/5}}+\frac{100x}{891\sqrt[5]{x^5+3}}$$

```
[In] Int[(-2 + x^5)^2/(3 + x^5)^(16/5),x]
```

```
[Out] (x*(2 - x^5)^2)/(33*(3 + x^5)^(11/5)) + (10*x*(2 - x^5))/(297*(3 + x^5)^(6/
5)) + (100*x)/(891*(3 + x^5)^(1/5))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
```

```
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(2-x^5)^2}{33\,(3+x^5)^{11/5}} - \frac{20}{33}\int \frac{-2+x^5}{(3+x^5)^{11/5}}\,dx \\ &= \frac{x(2-x^5)^2}{33\,(3+x^5)^{11/5}} + \frac{10x(2-x^5)}{297\,(3+x^5)^{6/5}} + \frac{100}{297}\int \frac{1}{(3+x^5)^{6/5}}\,dx \\ &= \frac{x(2-x^5)^2}{33\,(3+x^5)^{11/5}} + \frac{10x(2-x^5)}{297\,(3+x^5)^{6/5}} + \frac{100x}{891\sqrt[5]{3+x^5}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.54

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = \frac{x(1188+462x^5+97x^{10})}{891\,(3+x^5)^{11/5}}$$

```
[In] Integrate[(-2 + x^5)^2/(3 + x^5)^(16/5),x]
[Out] (x*(1188 + 462*x^5 + 97*x^10))/(891*(3 + x^5)^(11/5))
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.48

method	result	size
gosper	$\frac{x(97x^{10}+462x^5+1188)}{891(x^5+3)^{\frac{11}{5}}}$	23
trager	$\frac{x(97x^{10}+462x^5+1188)}{891(x^5+3)^{\frac{11}{5}}}$	23
risch	$\frac{x(97x^{10}+462x^5+1188)}{891(x^5+3)^{\frac{11}{5}}}$	23
pseudoelliptic	$\frac{97x^{11}+462x^6+1188x}{891(x^5+3)^{\frac{11}{5}}}$	24
meijerg	$\frac{4\,3^{\frac{4}{5}}x(\frac{25}{9}x^{10}+\frac{55}{3}x^5+33)}{2673\left(1+\frac{x^5}{3}\right)^{\frac{11}{5}}}+\frac{3^{\frac{4}{5}}x^{11}}{891\left(1+\frac{x^5}{3}\right)^{\frac{11}{5}}}-\frac{2\,3^{\frac{4}{5}}x^6\left(11+\frac{5x^5}{3}\right)}{2673\left(1+\frac{x^5}{3}\right)^{\frac{11}{5}}}$	70

```
[In] int((x^5-2)^2/(x^5+3)^(16/5),x,method=_RETURNVERBOSE)
```

```
[Out] 1/891*x*(97*x^10+462*x^5+1188)/(x^5+3)^(11/5)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = \frac{(97\,x^{11}+462\,x^6+1188\,x)(x^5+3)^{\frac{4}{5}}}{891\,(x^{15}+9\,x^{10}+27\,x^5+27)}$$

```
[In] integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="fricas")
[Out] 1/891*(97*x^11 + 462*x^6 + 1188*x)*(x^5 + 3)^(4/5)/(x^15 + 9*x^10 + 27*x^5
+ 27)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = \text{Timed out}$$

```
[In] integrate((x**5-2)**2/(x**5+3)**(16/5),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(36) = 72.

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = -\frac{4\,x^{11}\left(\frac{11\,(x^5+3)}{x^5}-\frac{33\,(x^5+3)^2}{x^{10}}-3\right)}{891\,(x^5+3)^{\frac{11}{5}}}-\frac{2\,x^{11}\left(\frac{11\,(x^5+3)}{x^5}-6\right)}{297\,(x^5+3)^{\frac{11}{5}}}+\frac{x^{11}}{33\,(x^5+3)^{\frac{11}{5}}}$$

```
[In] integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="maxima")
[Out] -4/891*x^11*(11*(x^5 + 3)/x^5 - 33*(x^5 + 3)^2/x^10 - 3)/(x^5 + 3)^(11/5) -
 2/297*x^11*(11*(x^5 + 3)/x^5 - 6)/(x^5 + 3)^(11/5) + 1/33*x^11/(x^5 + 3)^(
11/5)
```

Giac [F]

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = \int \frac{(x^5-2)^2}{(x^5+3)^{\frac{16}{5}}}\,dx$$

[In] integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="giac")

[Out] integrate((x^5 - 2)^2/(x^5 + 3)^(16/5), x)

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.48

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = \frac{97\,x^{11}+462\,x^6+1188\,x}{891\,(x^5+3)^{11/5}}$$

[In] int((x^5 - 2)^2/(x^5 + 3)^(16/5),x)

[Out] (1188*x + 462*x^6 + 97*x^11)/(891*(x^5 + 3)^(11/5))

3.319 $\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 90

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}}{\sqrt{3}}\right)}{3^{5/6}} - \frac{\log\left(1-(1+x)^3\right)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}(1+x)-\sqrt[3]{2+(1+x)^3}\right)}{2\sqrt[3]{3}}$$

```
[Out] -1/3*arctan(1/3*(1+2*3^(1/3)*(1+x)/(2+(1+x)^3)^(1/3))*3^(1/2))*3^(1/6)-1/18
*ln(1-(1+x)^3)*3^(2/3)+1/6*ln(3^(1/3)*(1+x)-(2+(1+x)^3)^(1/3))*3^(2/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {443, 442, 384}

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = -\frac{\arctan\left(\frac{\frac{2\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}} - \frac{\log\left(1-(x+1)^3\right)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}(x+1)-\sqrt[3]{(x+1)^3+2}\right)}{2\sqrt[3]{3}}$$

```
[In] Int[1/((3*x + 3*x^2 + x^3)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[(1 + (2*3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3))/Sqrt[3]]/3^(5/6))
- Log[1 - (1 + x)^3]/(6*3^(1/3)) + Log[3^(1/3)*(1 + x) - (2 + (1 + x)^3)^(1
/3)]/(2*3^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 442

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rule 443

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[NormalizePseudoBinomial[u, x]^p
*NormalizePseudoBinomial[v, x]^q, x] /; FreeQ[{p, q}, x] && PseudoBinomialP
airQ[u, v, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(-1+(1+x)^3)\sqrt[3]{2+(1+x)^3}}\,dx$$

$$= \text{Subst}\left(\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx, x, 1+x\right)$$

$$= -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}}{\sqrt{3}}\right)}{3^{5/6}} - \frac{\log\left(1-(1+x)^3\right)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}(1+x)-\sqrt[3]{2+(1+x)^3}\right)}{2\sqrt[3]{3}}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.00

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \frac{\arctan\left(\frac{\sqrt{3}\sqrt[3]{3+3x+3x^2+x^3}}{2\sqrt[3]{3}+2\sqrt[3]{3}x+\sqrt[3]{3+3x+3x^2+x^3}}\right)}{3^{5/6}} + \frac{2\log\left(\sqrt[3]{3}+\sqrt[3]{3}x-\sqrt[3]{3+3x+3x^2+x^3}\right)-\log\left(3^{2/3}+2\,3^{2/3}x+3^{2/3}x^2+\sqrt[3]{3}(1+x)\sqrt[3]{3+3x+3x^2+\ldots}\right.}{6\sqrt[3]{3}}$$

```
[In] Integrate[1/((3*x + 3*x^2 + x^3)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*(3 + 3*x + 3*x^2 + x^3)^(1/3))/(2*3^(1/3) + 2*3^(1/3)*x + (
3 + 3*x + 3*x^2 + x^3)^(1/3))]/3^(5/6) + (2*Log[3^(1/3) + 3^(1/3)*x - (3 +
3*x + 3*x^2 + x^3)^(1/3)] - Log[3^(2/3) + 2*3^(2/3)*x + 3^(2/3)*x^2 + 3^(1/
3)*(1 + x)*(3 + 3*x + 3*x^2 + x^3)^(1/3) + (3 + 3*x + 3*x^2 + x^3)^(2/3)])/
(6*3^(1/3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 10.17 (sec) , antiderivative size = 2517, normalized size of antiderivative = 27.97

method	result	size
trager	Expression too large to display	2517

```
[In] int(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*ln(-(6809512275*(x^3+3*x^2+3*x+3)^(2/3)*x+2269837425*(x^3+3*x^2+3*x+3)
^(1/3)*RootOf(_Z^3-9)^2+19831665822*RootOf(_Z^3-9)*x+275418203481*RootOf(Ro
otOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2+275418203481*RootOf(RootOf(
_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x+15322002984*(x^3+3*x^2+3*x+3)^(2/3
)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2*x+1
5322002984*(x^3+3*x^2+3*x+3)^(2/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3
-9)+81*_Z^2)*RootOf(_Z^3-9)^2+2269837425*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^
3-9)^2*x^2+4539674850*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2*x+4596600895
2*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z
```

^2)*RootOf(_Z^3-9)+91806067827*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+
81*_Z^2)*x^3+6610555274*RootOf(_Z^3-9)*x^3-15589709631*RootOf(RootOf(_Z^3-9
)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2-1122547122*RootOf(R
ootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^2-155897096
31*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*
x-1122547122*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z
^3-9)^3*x+150700526433*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)
+10851288846*RootOf(_Z^3-9)+19831665822*RootOf(_Z^3-9)*x^2+6809512275*(x^3+
3*x^2+3*x+3)^(2/3)+2619276618*RootOf(_Z^3-9)^3*RootOf(RootOf(_Z^3-9)^2+9*_Z
*RootOf(_Z^3-9)+81*_Z^2)+36375989139*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z
^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2-374182374*RootOf(RootOf(_Z^3-9)^2+9*_Z*Ro
otOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3-5196569877*RootOf(RootOf(_Z^3-9)
^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+45966008952*(x^3+3*x
^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf
(_Z^3-9)*x^2+91932017904*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*
_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*x)/x/(x^2+3*x+3))*RootOf(_Z^3-9)-
ln(-(6809512275*(x^3+3*x^2+3*x+3)^(2/3)*x+2269837425*(x^3+3*x^2+3*x+3)^(1/3
)*RootOf(_Z^3-9)^2+19831665822*RootOf(_Z^3-9)*x+275418203481*RootOf(RootOf(
_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2+275418203481*RootOf(RootOf(_Z^3-
9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x+15322002984*(x^3+3*x^2+3*x+3)^(2/3)*Roo
tOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2*x+153220
02984*(x^3+3*x^2+3*x+3)^(2/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+8
1*_Z^2)*RootOf(_Z^3-9)^2+2269837425*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^
2*x^2+4539674850*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2*x+45966008952*(x^
3+3*x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*R
ootOf(_Z^3-9)+91806067827*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z
^2)*x^3+6610555274*RootOf(_Z^3-9)*x^3-15589709631*RootOf(RootOf(_Z^3-9)^2+9
*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2-1122547122*RootOf(RootOf
(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^2-15589709631*Ro
otOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x-112
2547122*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)
^3*x+150700526433*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+1085
1288846*RootOf(_Z^3-9)+19831665822*RootOf(_Z^3-9)*x^2+6809512275*(x^3+3*x^2
+3*x+3)^(2/3)+2619276618*RootOf(_Z^3-9)^3*RootOf(RootOf(_Z^3-9)^2+9*_Z*Root
Of(_Z^3-9)+81*_Z^2)+36375989139*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)
+81*_Z^2)^2*RootOf(_Z^3-9)^2-374182374*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(
_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3-5196569877*RootOf(RootOf(_Z^3-9)^2+9*
_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+45966008952*(x^3+3*x^2+3*
x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3
-9)*x^2+91932017904*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*Ro
otOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*x)/x/(x^2+3*x+3))*RootOf(RootOf(_Z^3-9
)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9
)+81*_Z^2)*ln((8512490709*(x^3+3*x^2+3*x+3)^(2/3)*x+2837496903*(x^3+3*x^2+3
*x+3)^(1/3)*RootOf(_Z^3-9)^2+11379999624*RootOf(_Z^3-9)*x+291007913112*Root
Of(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2+291007913112*RootOf(Ro

```
otOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x+15322002984*(x^3+3*x^2+3*x+3)
^(2/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^
2*x+15322002984*(x^3+3*x^2+3*x+3)^(2/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf
(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2+2837496903*(x^3+3*x^2+3*x+3)^(1/3)*RootO
f(_Z^3-9)^2*x^2+5674993806*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2*x+45966
008952*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+
81*_Z^2)*RootOf(_Z^3-9)+97002637704*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^
3-9)+81*_Z^2)*x^3+3793333208*RootOf(_Z^3-9)*x^3+15589709631*RootOf(RootOf(_
Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2+609642837*Root
Of(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^2+15589
709631*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9
)^2*x+609642837*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf
(_Z^3-9)^3*x+114324537294*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z
^2)+4470714138*RootOf(_Z^3-9)+11379999624*RootOf(_Z^3-9)*x^2+8512490709*(x^
3+3*x^2+3*x+3)^(2/3)-1422499953*RootOf(_Z^3-9)^3*RootOf(RootOf(_Z^3-9)^2+9*
_Z*RootOf(_Z^3-9)+81*_Z^2)-36375989139*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(
_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2+203214279*RootOf(RootOf(_Z^3-9)^2+9*_Z*
RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3+5196569877*RootOf(RootOf(_Z^3-
9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+45966008952*(x^3+3
*x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*Root
Of(_Z^3-9)*x^2+91932017904*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+
9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*x)/x/(x^2+3*x+3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(71) = 142$.

Time = 4.91 (sec) , antiderivative size = 458, normalized size of antiderivative = 5.09

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = -\frac{1}{54} \cdot 3^{\frac{2}{3}} \log\left(\frac{3\cdot 3^{\frac{2}{3}}(7x^4+28x^3+42x^2+30x+9)(x^3+3x^2+3x+3)^{\frac{2}{3}} + 3^{\frac{1}{3}}(31x^6+186x^5+465x^4+666}{x^6+6x^5+15x^4}\right.$$
$$+\frac{1}{27} \cdot 3^{\frac{2}{3}} \log\left(\frac{2\cdot 3^{\frac{2}{3}}(x^3+3x^2+3x) - 9\cdot 3^{\frac{1}{3}}(x^3+3x^2+3x+3)^{\frac{1}{3}}(x^2+2x+1) + 9(x^3+3x^2+3x+3)^{\frac{2}{3}}(x+}{x^3+3x^2+3x}\right.$$
$$-\frac{1}{9} \cdot 3^{\frac{1}{6}} \arctan\left(\frac{3^{\frac{1}{6}}\left(12\cdot 3^{\frac{2}{3}}(7x^7+49x^6+147x^5+240x^4+225x^3+117x^2+27x)(x^3+3x^2+3x+3)^{\frac{2}{3}} - \right.}{}\right.$$

```
[In] integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 28*x^3 + 42*x^2 + 30*x + 9)*(x^3 + 3*
x^2 + 3*x + 3)^(2/3) + 3^(1/3)*(31*x^6 + 186*x^5 + 465*x^4 + 666*x^3 + 603*
x^2 + 324*x + 81) + 9*(5*x^5 + 25*x^4 + 50*x^3 + 54*x^2 + 33*x + 9)*(x^3 +
3*x^2 + 3*x + 3)^(1/3))/(x^6 + 6*x^5 + 15*x^4 + 18*x^3 + 9*x^2)) + 1/27*3^(
2/3)*log((2*3^(2/3)*(x^3 + 3*x^2 + 3*x) - 9*3^(1/3)*(x^3 + 3*x^2 + 3*x + 3)
^(1/3)*(x^2 + 2*x + 1) + 9*(x^3 + 3*x^2 + 3*x + 3)^(2/3)*(x + 1))/(x^3 + 3*
x^2 + 3*x)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*x^7 + 49*x^6 +
147*x^5 + 240*x^4 + 225*x^3 + 117*x^2 + 27*x)*(x^3 + 3*x^2 + 3*x + 3)^(2/3)
 - 3^(1/3)*(127*x^9 + 1143*x^8 + 4572*x^7 + 11070*x^6 + 18414*x^5 + 22032*x
^4 + 18900*x^3 + 11178*x^2 + 4131*x + 729) - 18*(31*x^8 + 248*x^7 + 868*x^6
 + 1782*x^5 + 2400*x^4 + 2196*x^3 + 1332*x^2 + 486*x + 81)*(x^3 + 3*x^2 + 3
*x + 3)^(1/3))/(251*x^9 + 2259*x^8 + 9036*x^7 + 21546*x^6 + 34398*x^5 + 385
56*x^4 + 30348*x^3 + 16038*x^2 + 5103*x + 729))
```

Sympy [F]

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx=\int \frac{1}{x\left(x^2+3x+3\right)\sqrt[3]{x^3+3x^2+3x+3}}\,dx$$

```
[In] integrate(1/(x**3+3*x**2+3*x)/(x**3+3*x**2+3*x+3)**(1/3),x)
[Out] Integral(1/(x*(x**2 + 3*x + 3)*(x**3 + 3*x**2 + 3*x + 3)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx=\int \frac{1}{(x^3+3\,x^2+3\,x+3)^{\frac{1}{3}}(x^3+3\,x^2+3\,x)}\,dx$$

```
[In] integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^3 + 3*x^2 + 3*x)), x)
```

Giac [F]

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx=\int \frac{1}{(x^3+3\,x^2+3\,x+3)^{\frac{1}{3}}(x^3+3\,x^2+3\,x)}\,dx$$

```
[In] integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="giac")
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^3 + 3*x^2 + 3*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \int \frac{1}{\left(x^3+3\,x^2+3\,x\right)\,\left(x^3+3\,x^2+3\,x+3\right)^{1/3}}\,dx$$

```
[In] int(1/((3*x + 3*x^2 + x^3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)),x)
[Out] int(1/((3*x + 3*x^2 + x^3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)), x)
```

3.320 $\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 23

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1713, 209}

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

```
[In] Int[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1713

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
```

```
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)$$
$$= \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

```
[In] Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\arctan\left(\frac{x\sqrt{2}}{\sqrt{x^4+1}}\right)\sqrt{2}}{2}$	19
pseudoelliptic	$\frac{\arctan\left(\frac{x\sqrt{2}}{\sqrt{x^4+1}}\right)\sqrt{2}}{2}$	19
elliptic	$-\frac{\arctan\left(\frac{\sqrt{2}\,\sqrt{x^4+1}}{2x}\right)\sqrt{2}}{2}$	22
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)x-\sqrt{x^4+1}}{x^2+1}\right)}{2}$	37

```
[In] int((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)$$

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1))
```

Sympy [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{x^4+1}+\sqrt{x^4+1}}\,dx - \int \left(-\frac{1}{x^2\sqrt{x^4+1}+\sqrt{x^4+1}}\right)dx$$

```
[In] integrate((-x**2+1)/(x**2+1)/(x**4+1)**(1/2),x)
[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x) - Integral(-1/(x*
*2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x)
```

Maxima [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4+1}(x^2+1)}\,dx$$

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4+1}(x^2+1)}\,dx$$

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = -\int \frac{x^2-1}{(x^2+1)\sqrt{x^4+1}}\,dx$$

```
[In] int(-(x^2 - 1)/((x^2 + 1)*(x^4 + 1)^(1/2)),x)
[Out] -int((x^2 - 1)/((x^2 + 1)*(x^4 + 1)^(1/2)), x)
```

3.321 $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 23

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1713, 212}

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

```
[In] Int[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1713

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
```

```
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)$$
$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

```
[In] Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
elliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x^4+1}}{2x}\right)\sqrt{2}}{2}$	22
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x+\sqrt{x^4+1}}{(-1+x)(1+x)}\right)}{2}$	38
default	$\frac{\sqrt{2}\left(\operatorname{arctanh}\left(\frac{\left(x^2-x+1\right)\sqrt{2}}{\sqrt{x^4+1}}\right)-\operatorname{arctanh}\left(\frac{\left(x^2+x+1\right)\sqrt{2}}{\sqrt{x^4+1}}\right)\right)}{4}$	47
pseudoelliptic	$\frac{\sqrt{2}\left(\operatorname{arctanh}\left(\frac{\left(x^2-x+1\right)\sqrt{2}}{\sqrt{x^4+1}}\right)-\operatorname{arctanh}\left(\frac{\left(x^2+x+1\right)\sqrt{2}}{\sqrt{x^4+1}}\right)\right)}{4}$	47

```
[In] int((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arctanh(1/2/x*2^(1/2)*(x^4+1)^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}x+2x^2+1}{x^4-2x^2+1}\right)$$

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((x^4 + 2*sqrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2
+ 1))
```

Sympy [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx - \int \frac{1}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx$$

```
[In] integrate((x**2+1)/(-x**2+1)/(x**4+1)**(1/2),x)
[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x) - Integral(1/(x**
2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)
```

Maxima [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2+1}{\sqrt{x^4+1}(x^2-1)}\,dx$$

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2+1}{\sqrt{x^4+1}(x^2-1)}\,dx$$

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2+1}{(x^2-1)\sqrt{x^4+1}}\,dx$$

```
[In] int(-(x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)),x)
[Out] int(-(x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)), x)
```

3.322 $\int \frac{1+x^2}{x\sqrt{1+x^4}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 16

$$\int \frac{1+x^2}{x\sqrt{1+x^4}} \, dx = \operatorname{arctanh}\left(\frac{-1+x^2}{\sqrt{1+x^4}}\right)$$

```
[Out] arctanh((x^2-1)/(x^4+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1266, 858, 221, 272, 65, 213}

$$\int \frac{1+x^2}{x\sqrt{1+x^4}} \, dx = \frac{\operatorname{arcsinh}(x^2)}{2} - \frac{1}{2}\operatorname{arctanh}\left(\sqrt{x^4+1}\right)$$

```
[In] Int[(1 + x^2)/(x*Sqrt[1 + x^4]),x]
```

```
[Out] ArcSinh[x^2]/2 - ArcTanh[Sqrt[1 + x^4]]/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{x\sqrt{1+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x^2}}\,dx, x, x^2\right)\\
&= \frac{\text{arcsinh}(x^2)}{2} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^4\right)\\
&= \frac{\text{arcsinh}(x^2)}{2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^4}\right)\\
&= \frac{\text{arcsinh}(x^2)}{2} - \frac{1}{2}\text{arctanh}\left(\sqrt{1+x^4}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 41 vs. $2(16) = 32$.

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.56

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = \operatorname{arctanh}\left(1+2x^2-2\sqrt{1+x^4}\right)-\frac{1}{2}\log\left(1-x^2+\sqrt{1+x^4}\right)$$

```
[In] Integrate[(1 + x^2)/(x*Sqrt[1 + x^4]),x]
[Out] ArcTanh[1 + 2*x^2 - 2*Sqrt[1 + x^4]] - Log[1 - x^2 + Sqrt[1 + x^4]]/2
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\operatorname{arcsinh}(x^2)}{2}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	18
trager	$\ln\left(\frac{x^2+\sqrt{x^4+1}-1}{x}\right)$	18
elliptic	$\frac{\operatorname{arcsinh}(x^2)}{2}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	18
pseudoelliptic	$\frac{\operatorname{arcsinh}(x^2)}{2}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	18
meijerg	$\frac{\operatorname{arcsinh}(x^2)}{2}+\frac{(-2\ln(2)+4\ln(x))\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^4+1}}{2}\right)}{4\sqrt{\pi}}$	44

```
[In] int((x^2+1)/x/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsinh(x^2)-1/2*arctanh(1/(x^4+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = -\frac{1}{2}\log\left(2x^4-x^2-\sqrt{x^4+1}(2x^2-1)+1\right)+\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}-1\right)$$

```
[In] integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*log(2*x^4 - x^2 - sqrt(x^4 + 1)*(2*x^2 - 1) + 1) + 1/2*log(-x^2 + sqrt
(x^4 + 1) - 1)
```

Sympy [A] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = -\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2} + \frac{\operatorname{asinh}\left(x^2\right)}{2}$$

```
[In] integrate((x**2+1)/x/(x**4+1)**(1/2),x)
[Out] -asinh(x**(-2))/2 + asinh(x**2)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(14) = 28.

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = -\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right) + \frac{1}{4}\log\left(\sqrt{x^4+1}-1\right) + \frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right) - \frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

```
[In] integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1) + 1/4*log(sqrt(x^4
 + 1)/x^2 + 1) - 1/4*log(sqrt(x^4 + 1)/x^2 - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(14) = 28.

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = \frac{1}{2}\log\left(x^2-\sqrt{x^4+1}+1\right) - \frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}+1\right) - \frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}\right)$$

```
[In] integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="giac")
[Out] 1/2*log(x^2 - sqrt(x^4 + 1) + 1) - 1/2*log(-x^2 + sqrt(x^4 + 1) + 1) - 1/2*
log(-x^2 + sqrt(x^4 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = \frac{\operatorname{asinh}(x^2)}{2} - \frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

```
[In] int((x^2 + 1)/(x*(x^4 + 1)^(1/2)),x)
[Out] asinh(x^2)/2 - atanh((x^4 + 1)^(1/2))/2
```

3.323 $\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 16

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx = \operatorname{arctanh}\left(\frac{1+x^2}{\sqrt{1+x^4}}\right)$$

```
[Out] arctanh((x^2+1)/(x^4+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1266, 858, 221, 272, 65, 213}

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx = \frac{\operatorname{arcsinh}(x^2)}{2} + \frac{1}{2}\operatorname{arctanh}\left(\sqrt{x^4+1}\right)$$

```
[In] Int[(-1 + x^2)/(x*Sqrt[1 + x^4]),x]
```

```
[Out] ArcSinh[x^2]/2 + ArcTanh[Sqrt[1 + x^4]]/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{x\sqrt{1+x^2}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x^2}}\,dx, x, x^2\right) \\
&= \frac{\operatorname{arcsinh}(x^2)}{2} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^4\right) \\
&= \frac{\operatorname{arcsinh}(x^2)}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^4}\right) \\
&= \frac{\operatorname{arcsinh}(x^2)}{2} + \frac{1}{2}\operatorname{arctanh}\left(\sqrt{1+x^4}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. $2(16) = 32$.

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx = -\text{arctanh}\left(x^2-\sqrt{1+x^4}\right)-\frac{1}{2}\log\left(-x^2+\sqrt{1+x^4}\right)$$

```
[In] Integrate[(-1 + x^2)/(x*Sqrt[1 + x^4]),x]
[Out] -ArcTanh[x^2 - Sqrt[1 + x^4]] - Log[-x^2 + Sqrt[1 + x^4]]/2
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\operatorname{arcsinh}(x^2)}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	18
trager	$\ln\left(\frac{x^2+\sqrt{x^4+1}+1}{x}\right)$	18
elliptic	$\frac{\operatorname{arcsinh}(x^2)}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	18
pseudoelliptic	$\frac{\operatorname{arcsinh}(x^2)}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	18
meijerg	$\frac{\operatorname{arcsinh}(x^2)}{2}-\frac{(-2\ln(2)+4\ln(x))\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^4+1}}{2}\right)}{4\sqrt{\pi}}$	44

```
[In] int((x^2-1)/x/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsinh(x^2)+1/2*arctanh(1/(x^4+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.94

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx = -\frac{1}{2}\log\left(2\,x^4+x^2-\sqrt{x^4+1}(2\,x^2+1)+1\right)+\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}+1\right)$$

```
[In] integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*log(2*x^4 + x^2 - sqrt(x^4 + 1)*(2*x^2 + 1) + 1) + 1/2*log(-x^2 + sqrt
(x^4 + 1) + 1)
```

Sympy [A] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx=\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}+\frac{\operatorname{asinh}\left(x^2\right)}{2}$$

```
[In] integrate((x**2-1)/x/(x**4+1)**(1/2),x)
[Out] asinh(x**(-2))/2 + asinh(x**2)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(14) = 28.

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx=\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)-\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)+\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

```
[In] integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] 1/4*log(sqrt(x^4 + 1) + 1) - 1/4*log(sqrt(x^4 + 1) - 1) + 1/4*log(sqrt(x^4
+ 1)/x^2 + 1) - 1/4*log(sqrt(x^4 + 1)/x^2 - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(14) = 28.

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx=-\frac{1}{2}\log\left(x^2-\sqrt{x^4+1}+1\right)+\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}+1\right)-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}\right)$$

```
[In] integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="giac")
[Out] -1/2*log(x^2 - sqrt(x^4 + 1) + 1) + 1/2*log(-x^2 + sqrt(x^4 + 1) + 1) - 1/2
*log(-x^2 + sqrt(x^4 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx = \frac{\operatorname{asinh}(x^2)}{2} + \frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

```
[In] int((x^2 - 1)/(x*(x^4 + 1)^(1/2)),x)
[Out] asinh(x^2)/2 + atanh((x^4 + 1)^(1/2))/2
```

3.324 $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 26

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{1+x^2+x^4}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctanh(x*3^(1/2)/(x^4+x^2+1)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1712, 212}

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{x^4+x^2+1}}\right)}{\sqrt{3}}$$

```
[In] Int[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]),x]
[Out] ArcTanh[(Sqrt[3]*x)/Sqrt[1 + x^2 + x^4]]/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
```

```
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right)$$
$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{1+x^2+x^4}}\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{1+x^2+x^4}}\right)}{\sqrt{3}}$$

```
[In] Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]),x]
[Out] ArcTanh[(Sqrt[3]*x)/Sqrt[1 + x^2 + x^4]]/Sqrt[3]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

method	result	size
elliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^4+x^2+1}\,\sqrt{2}\,\sqrt{6}}{6x}\right)\sqrt{6}\,\sqrt{2}}{6}$	31
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-\operatorname{RootOf}\left(\_Z^2-3\right)x+\sqrt{x^4+x^2+1}}{(-1+x)(1+x)}\right)}{3}$	42
default	$\frac{\sqrt{3}\left(\operatorname{arctanh}\left(\frac{\left(2x^2-x+2\right)\sqrt{3}}{3\sqrt{x^4+x^2+1}}\right)-\operatorname{arctanh}\left(\frac{\left(2x^2+x+2\right)\sqrt{3}}{3\sqrt{x^4+x^2+1}}\right)\right)}{6}$	59
pseudoelliptic	$\frac{\sqrt{3}\left(\operatorname{arctanh}\left(\frac{\left(2x^2-x+2\right)\sqrt{3}}{3\sqrt{x^4+x^2+1}}\right)-\operatorname{arctanh}\left(\frac{\left(2x^2+x+2\right)\sqrt{3}}{3\sqrt{x^4+x^2+1}}\right)\right)}{6}$	59

```
[In] int((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*arctanh(1/6*(x^4+x^2+1)^(1/2)*2^(1/2)/x*6^(1/2))*6^(1/2)*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(21) = 42$.

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{x^4+2\sqrt{3}\sqrt{x^4+x^2+1}x+4x^2+1}{x^4-2x^2+1}\right)$$

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log((x^4 + 2*sqrt(3)*sqrt(x^4 + x^2 + 1)*x + 4*x^2 + 1)/(x^4 -
2*x^2 + 1))
```

Sympy [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{x^4+x^2+1}-\sqrt{x^4+x^2+1}}\,dx - \int \frac{1}{x^2\sqrt{x^4+x^2+1}-\sqrt{x^4+x^2+1}}\,dx$$

```
[In] integrate((x**2+1)/(-x**2+1)/(x**4+x**2+1)**(1/2),x)
[Out] -Integral(x**2/(x**2*sqrt(x**4 + x**2 + 1) - sqrt(x**4 + x**2 + 1)), x) - I
ntegral(1/(x**2*sqrt(x**4 + x**2 + 1) - sqrt(x**4 + x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = \int -\frac{x^2+1}{\sqrt{x^4+x^2+1}(x^2-1)}\,dx$$

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 + 1)/(sqrt(x^4 + x^2 + 1)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = \int -\frac{x^2+1}{\sqrt{x^4+x^2+1}(x^2-1)}\,dx$$

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 + 1)/(sqrt(x^4 + x^2 + 1)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = \int -\frac{x^2+1}{(x^2-1)\sqrt{x^4+x^2+1}}\,dx$$

```
[In] int(-(x^2 + 1)/((x^2 - 1)*(x^2 + x^4 + 1)^(1/2)),x)
[Out] int(-(x^2 + 1)/((x^2 - 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

3.325 $\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 15

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)$$

```
[Out] arctan(x/(x^4+x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1712, 209}

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

```
[In] Int[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
```

```
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right)\\ &= \arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)$$

```
[In] Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$	14
pseudoelliptic	$\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$	14
elliptic	$-\arctan\left(\frac{\sqrt{x^4+x^2+1}}{x}\right)$	18
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{x^4+x^2+1}}{x^2+1}\right)$	37

```
[In] int((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan(x/(x^4+x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
[Out] arctan(x/sqrt(x^4 + x^2 + 1))
```

Sympy [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{x^4+x^2+1}+\sqrt{x^4+x^2+1}}\,dx - \int \left(-\frac{1}{x^2\sqrt{x^4+x^2+1}+\sqrt{x^4+x^2+1}}\right)dx$$

```
[In] integrate((-x**2+1)/(x**2+1)/(x**4+x**2+1)**(1/2),x)
[Out] -Integral(x**2/(x**2*sqrt(x**4 + x**2 + 1) + sqrt(x**4 + x**2 + 1)), x) - I
ntegral(-1/(x**2*sqrt(x**4 + x**2 + 1) + sqrt(x**4 + x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\int \frac{x^2-1}{(x^2+1)\,\sqrt{x^4+x^2+1}}\,dx$$

```
[In] int(-(x^2 - 1)/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
[Out] -int((x^2 - 1)/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

3.326 $\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{1+x^2+x^4}}{x}$$

```
[Out] 1/x*(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1604}

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{x^4+x^2+1}}{x}$$

```
[In] Int[(-1 + x^4)/(x^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] Sqrt[1 + x^2 + x^4]/x
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+x^2+x^4}}{x}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{1+x^2+x^4}}{x}$$

```
[In] Integrate[(-1 + x^4)/(x^2*Sqrt[1 + x^2 + x^4]),x]
[Out] Sqrt[1 + x^2 + x^4]/x
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\sqrt{x^4+x^2+1}}{x}$	15
trager	$\frac{\sqrt{x^4+x^2+1}}{x}$	15
risch	$\frac{\sqrt{x^4+x^2+1}}{x}$	15
elliptic	$\frac{\sqrt{x^4+x^2+1}}{x}$	15
gosper	$\frac{\left(x^2+x+1\right)\left(x^2-x+1\right)}{\sqrt{x^4+x^2+1}\,x}$	29
pseudoelliptic	$\frac{\left(x^2+x+1\right)\left(x^2-x+1\right)}{\sqrt{x^4+x^2+1}\,x}$	29

```
[In] int((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/x*(x^4+x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{x^4+x^2+1}}{x}$$

```
[In] integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^4 + x^2 + 1)/x
```

Sympy [F]

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x-1)(x+1)(x^2+1)}{x^2\sqrt{(x^2-x+1)(x^2+x+1)}}\,dx$$

```
[In] integrate((x**4-1)/x**2/(x**4+x**2+1)**(1/2),x)
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/(x**2*sqrt((x**2 - x + 1)*(x**2 + x + 1
))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{x^2+x+1}\sqrt{x^2-x+1}}{x}$$

```
[In] integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 + x + 1)*sqrt(x^2 - x + 1)/x
```

Giac [F]

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^4-1}{\sqrt{x^4+x^2+1}x^2}\,dx$$

```
[In] integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate((x^4 - 1)/(sqrt(x^4 + x^2 + 1)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx=\frac{\sqrt{x^4+x^2+1}}{x}$$

```
[In] int((x^4 - 1)/(x^2*(x^2 + x^4 + 1)^(1/2)),x)
[Out] (x^2 + x^4 + 1)^(1/2)/x
```

3.327 $\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 74

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx=\frac{\arctan\left(\frac{a+2(1+a^2-b)x+ax^2}{\sqrt{2}\sqrt{1-b}\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\right)}{\sqrt{2}\sqrt{1-b}}$$

```
[Out] 1/2*arctan(1/2*(a+2*(a^2-b+1)*x+a*x^2)*2^(1/2)/(1-b)^(1/2)/(2*a*x^3+x^4+2*b
*x^2+2*a*x+1)^(1/2))*2^(1/2)/(1-b)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2109}

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx=\frac{\arctan\left(\frac{2x(a^2-b+1)+ax^2+a}{\sqrt{2}\sqrt{1-b}\sqrt{2ax^3+2ax+2bx^2+x^4+1}}\right)}{\sqrt{2}\sqrt{1-b}}$$

```
[In] Int[(1 - x^2)/((1 + 2*a*x + x^2)*Sqrt[1 + 2*a*x + 2*b*x^2 + 2*a*x^3 + x^4])
,x]
```

```
[Out] ArcTan[(a + 2*(1 + a^2 - b)*x + a*x^2)/(Sqrt[2]*Sqrt[1 - b]*Sqrt[1 + 2*a*x
+ 2*b*x^2 + 2*a*x^3 + x^4])]/(Sqrt[2]*Sqrt[1 - b])
```

Rule 2109

```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> Simp
[a*(f/(d*Rt[a^2*(2*a - c), 2]))*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b
*x^2)/(2*Rt[a^2*(2*a - c), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g, 0] &&
PosQ[a^2*(2*a - c)]
```

Rubi steps

$$\text{integral} = \frac{\arctan\left(\frac{a+2(1+a^2-b)x+ax^2}{\sqrt{2}\sqrt{1-b}\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\right)}{\sqrt{2}\sqrt{1-b}}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{-1+b}x}{1+2ax+x^2-\sqrt{1+2bx^2+x^4+2a(x+x^3)}}\right)}{\sqrt{-1+b}}$$

```
[In] Integrate[(1 - x^2)/((1 + 2*a*x + x^2)*Sqrt[1 + 2*a*x + 2*b*x^2 + 2*a*x^3 +
 x^4]),x]
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[-1 + b]*x)/(1 + 2*a*x + x^2 - Sqrt[1 + 2*b
*x^2 + x^4 + 2*a*(x + x^3)])]])/Sqrt[-1 + b])
```

Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{\ln(2)+\ln\left(\frac{\sqrt{2b-2}\,\sqrt{2a\,x^3+x^4+2x^2b+2ax+1}-a\,x^2+\left(-2a^2+2b-2\right)x-a}{2ax+x^2+1}\right)}{\sqrt{2b-2}}$	78
pseudoelliptic	$\frac{\ln(2)+\ln\left(\frac{\sqrt{2b-2}\,\sqrt{2a\,x^3+x^4+2x^2b+2ax+1}-a\,x^2+\left(-2a^2+2b-2\right)x-a}{2ax+x^2+1}\right)}{\sqrt{2b-2}}$	78
elliptic	Expression too large to display	258804

```
[In] int((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x,method=_RE
TURNVERBOSE)
[Out] 1/(2*b-2)^(1/2)*(ln(2)+ln(((2*b-2)^(1/2)*(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2
)-a*x^2+(-2*a^2+2*b-2)*x-a)/(2*a*x+x^2+1)))
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.41

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = \left[\frac{\sqrt{2}\log\left(\frac{4a^3x^3+(a^2+2b-2)x^4+4a^3x+2(2a^4+5a^2-2(2a^2+3)b+4b^2+2)x^2+a^2-\frac{2\sqrt{2}\sqrt{2ax^3+x^4+2bx^2+2ax+1}\left((ab-a)x^2+ab-2\left(a^2-\left(a^2\right.\right.\right.}{\sqrt{b-1}}}{4ax^3+x^4+2(2a^2+1)x^2+4ax+1}}{4\sqrt{b-1}}\right.$$

```
[In] integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log((4*a^3*x^3 + (a^2 + 2*b - 2)*x^4 + 4*a^3*x + 2*(2*a^4 + 5*
a^2 - 2*(2*a^2 + 3)*b + 4*b^2 + 2)*x^2 + a^2 - 2*sqrt(2)*sqrt(2*a*x^3 + x^4
 + 2*b*x^2 + 2*a*x + 1)*((a*b - a)*x^2 + a*b - 2*(a^2 - (a^2 + 2)*b + b^2 +
 1)*x - a)/sqrt(b - 1) + 2*b - 2)/(4*a*x^3 + x^4 + 2*(2*a^2 + 1)*x^2 + 4*a*
x + 1))/sqrt(b - 1), 1/2*sqrt(2)*sqrt(-1/(b - 1))*arctan(sqrt(2)*sqrt(2*a*x
^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(b - 1)*sqrt(-1/(b - 1))/(a*x^2 + 2*(a^2 -
b + 1)*x + a))]
```

Sympy [F]

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = -\int \frac{x^2}{2ax\sqrt{2ax^3+2ax+2bx^2+x^4+1}+x^2\sqrt{2ax^3+2ax+2bx^2+x^4+1}+\sqrt{2ax^3+2ax+2bx^2+x^4}} - \int \left(-\frac{1}{2ax\sqrt{2ax^3+2ax+2bx^2+x^4+1}+x^2\sqrt{2ax^3+2ax+2bx^2+x^4+1}+\sqrt{2ax^3+2ax+2bx^2+}}\right.$$

```
[In] integrate((-x**2+1)/(2*a*x+x**2+1)/(2*a*x**3+x**4+2*b*x**2+2*a*x+1)**(1/2),
x)
```

```
[Out] -Integral(x**2/(2*a*x*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + x**2*s
qrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + sqrt(2*a*x**3 + 2*a*x + 2*b*x
**2 + x**4 + 1)), x) - Integral(-1/(2*a*x*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2
+ x**4 + 1) + x**2*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + sqrt(2*a*
x**3 + 2*a*x + 2*b*x**2 + x**4 + 1)), x)
```

Maxima [F]

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{2\,ax^3+x^4+2\,bx^2+2\,ax+1}(2\,ax+x^2+1)}\,dx$$

```
[In] integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, alg
orithm="maxima")
[Out] -integrate((x^2 - 1)/(sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(2*a*x + x^
2 + 1)), x)
```

Giac [F]

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{2\,ax^3+x^4+2\,bx^2+2\,ax+1}(2\,ax+x^2+1)}\,dx$$

```
[In] integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, alg
orithm="giac")
[Out] integrate(-(x^2 - 1)/(sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(2*a*x + x^
2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = -\int \frac{x^2-1}{(x^2+2\,a\,x+1)\,\sqrt{x^4+2\,a\,x^3+2\,b\,x^2+2\,a\,x+1}}\,dx$$

```
[In] int(-(x^2 - 1)/((2*a*x + x^2 + 1)*(2*a*x + 2*a*x^3 + 2*b*x^2 + x^4 + 1)^(1/
2)),x)
[Out] -int((x^2 - 1)/((2*a*x + x^2 + 1)*(2*a*x + 2*a*x^3 + 2*b*x^2 + x^4 + 1)^(1/
2)), x)
```

3.328 $\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 22

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \arctan\left(\frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)$$

```
[Out] arctan(x/(-x^2+(x^4+1)^(1/2))^(1/2))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2153, 209}

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \arctan\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

```
[In] Int[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2153

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
```

```
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)\\ &= \arctan\left(\frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.36

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = i\text{arctanh}\left(\sqrt{2}+\sqrt{2}x^4-ix^3\sqrt{-x^2+\sqrt{1+x^4}} + \frac{\sqrt{1+x^4}\left(-2x^2+i\sqrt{2}x\sqrt{-x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}\right)$$

```
[In] Integrate[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
[Out] I*ArcTanh[Sqrt[2] + Sqrt[2]*x^4 - I*x^3*Sqrt[-x^2 + Sqrt[1 + x^4]] + (Sqrt[
1 + x^4]*(-2*x^2 + I*Sqrt[2]*x*Sqrt[-x^2 + Sqrt[1 + x^4]]))/Sqrt[2]]
```

Maple [F]

$$\int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}dx$$

```
[In] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
[Out] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(18) = 36$.

Time = 0.63 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.82

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{4}\arctan\left(\frac{4\left(10\,x^7-6\,x^3+(7\,x^5-x)\sqrt{x^4+1}\right)\sqrt{-x^2+\sqrt{x^4+1}}}{17\,x^8-46\,x^4+1}\right)$$

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
[Out] -1/4*arctan(4*(10*x^7 - 6*x^3 + (7*x^5 - x)*sqrt(x^4 + 1))*sqrt(-x^2 + sqrt
(x^4 + 1))/(17*x^8 - 46*x^4 + 1))
```

Sympy [F]

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{-x^2+\sqrt{x^4+1}}\,(x^4+1)}\,dx$$

```
[In] integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2),x)
[Out] Integral(1/(sqrt(-x**2 + sqrt(x**4 + 1))*(x**4 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

Giac [F]

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{\sqrt{x^4+1}-x^2}\,(x^4+1)}\,dx$$

```
[In] int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)),x)
[Out] int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)), x)
```

3.329 $\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 24

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \arctan\left(\frac{x}{\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\right)$$

```
[Out] arctan(x/(-x^2+(1+x^(2*n))^(1/n))^(1/2))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2153, 209}

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \arctan\left(\frac{x}{\sqrt{(x^{2n}+1)^{\frac{1}{n}}-x^2}}\right)$$

```
[In] Int[1/((1 + x^(2*n))*Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]),x]
```

```
[Out] ArcTan[x/Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2153

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{-x^2+\left(1+x^{2n}\right)^{\frac{1}{n}}}}\right) \\ &= \arctan\left(\frac{x}{\sqrt{-x^2+\left(1+x^{2n}\right)^{\frac{1}{n}}}}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(1+x^{2n}\right)\sqrt{-x^2+\left(1+x^{2n}\right)^{\frac{1}{n}}}}\,dx = \cot^{-1}\left(\frac{\sqrt{-x^2+\left(1+x^{2n}\right)^{\frac{1}{n}}}}{x}\right)$$

```
[In] Integrate[1/((1 + x^(2*n))*Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]),x]
[Out] ArcCot[Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]/x]
```

Maple [F]

$$\int \frac{1}{\left(1+x^{2n}\right)\sqrt{-x^2+\left(1+x^{2n}\right)^{\frac{1}{n}}}}dx$$

```
[In] int(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x)
[Out] int(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(1+x^{2n}\right)\sqrt{-x^2+\left(1+x^{2n}\right)^{\frac{1}{n}}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="fricas
")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \int \frac{1}{\sqrt{-x^2+(x^{2n}+1)^{\frac{1}{n}}}\,(x^{2n}+1)}\,dx$$

```
[In] integrate(1/(1+x**(2*n))/(-x**2+(1+x**(2*n))**(1/n))**(1/2),x)
[Out] Integral(1/(sqrt(-x**2 + (x**(2*n) + 1)**(1/n))*(x**(2*n) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \int \frac{1}{\sqrt{-x^2+(x^{2n}+1)^{\left(\frac{1}{n}\right)}}(x^{2n}+1)}\,dx$$

```
[In] integrate(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="maxima
")
[Out] integrate(1/(sqrt(-x^2 + (x^(2*n) + 1)^(1/n))*(x^(2*n) + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \int \frac{1}{\sqrt{-x^2+(x^{2n}+1)^{\left(\frac{1}{n}\right)}}(x^{2n}+1)}\,dx$$

```
[In] integrate(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^2 + (x^(2*n) + 1)^(1/n))*(x^(2*n) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \int \frac{1}{(x^{2n}+1)\sqrt{(x^{2n}+1)^{1/n}-x^2}}\,dx$$

```
[In] int(1/((x^(2*n) + 1)*((x^(2*n) + 1)^(1/n) - x^2)^(1/2)),x)
[Out] int(1/((x^(2*n) + 1)*((x^(2*n) + 1)^(1/n) - x^2)^(1/2)), x)
```

3.330 $\int \cos^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\sin(x)\cos(x)$$

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\cos(x)\sin(x) + \frac{\int 1\,dx}{2}\\ &= \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{4}\sin(2x)$$

```
[In] Integrate[Cos[x]^2,x]
[Out] x/2 + Sin[2*x]/4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

```
[In] int(cos(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}\,x$$

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

```
[In] integrate(cos(x)**2,x)
[Out] x/2 + sin(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\,x + \frac{1}{4}\sin(2\,x)$$

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
[Out] 1/2*x + 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\,x + \frac{1}{4}\sin(2\,x)$$

```
[In] integrate(cos(x)^2,x, algorithm="giac")
[Out] 1/2*x + 1/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(2\,x)}{4}$$

```
[In] int(cos(x)^2,x)
[Out] x/2 + sin(2*x)/4
```

3.331 $\int \cos^3(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 11

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin^3(x)}{3}$$

```
[Out] sin(x)-1/3*sin(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2713}

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin^3(x)}{3}$$

```
[In] Int[Cos[x]^3,x]
```

```
[Out] Sin[x] - Sin[x]^3/3
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(x)\right) \\ &= \sin(x) - \frac{\sin^3(x)}{3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin^3(x)}{3}$$

```
[In] Integrate[Cos[x]^3,x]
[Out] Sin[x] - Sin[x]^3/3
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{(2+\cos^2(x))\sin(x)}{3}$	11
risch	$\frac{3\sin(x)}{4} + \frac{\sin(3x)}{12}$	12
parallelrisch	$\frac{3\sin(x)}{4} + \frac{\sin(3x)}{12}$	12

```
[In] int(cos(x)^3,x,method=_RETURNVERBOSE)
[Out] 1/3*(2+cos(x)^2)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \cos^3(x)\,dx = \frac{1}{3}\left(\cos\left(x\right)^2 + 2\right)\sin\left(x\right)$$

```
[In] integrate(cos(x)^3,x, algorithm="fricas")
[Out] 1/3*(cos(x)^2 + 2)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \cos^3(x)\,dx = -\frac{\sin^3(x)}{3} + \sin(x)$$

```
[In] integrate(cos(x)**3,x)
[Out] -sin(x)**3/3 + sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = -\frac{1}{3}\sin(x)^3 + \sin(x)$$

```
[In] integrate(cos(x)^3,x, algorithm="maxima")
[Out] -1/3*sin(x)^3 + sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = -\frac{1}{3}\sin(x)^3 + \sin(x)$$

```
[In] integrate(cos(x)^3,x, algorithm="giac")
[Out] -1/3*sin(x)^3 + sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin(x)^3}{3}$$

```
[In] int(cos(x)^3,x)
[Out] sin(x) - sin(x)^3/3
```

3.332 $\int \sin^4(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 24

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{3}{8}\cos(x)\sin(x) - \frac{1}{4}\cos(x)\sin^3(x)$$

```
[Out] 3/8*x-3/8*cos(x)*sin(x)-1/4*cos(x)*sin(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{1}{4}\sin^3(x)\cos(x) - \frac{3}{8}\sin(x)\cos(x)$$

```
[In] Int[Sin[x]^4,x]
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{4}\cos(x)\sin^3(x) + \frac{3}{4}\int \sin^2(x)\,dx\\ &= -\frac{3}{8}\cos(x)\sin(x) - \frac{1}{4}\cos(x)\sin^3(x) + \frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8} - \frac{3}{8}\cos(x)\sin(x) - \frac{1}{4}\cos(x)\sin^3(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)$$

```
[In] Integrate[Sin[x]^4,x]
[Out] (3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{3x}{8}+\frac{\sin(4x)}{32}-\frac{\sin(2x)}{4}$	17
parallelrisch	$\frac{3x}{8}+\frac{\sin(4x)}{32}-\frac{\sin(2x)}{4}$	17
default	$-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}$	18
norman	$\frac{\frac{3x}{8}-\frac{11\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}+\frac{11\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{3\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{9x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}-\frac{3\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	82

```
[In] int(sin(x)^4,x,method=_RETURNVERBOSE)
[Out] 3/8*x+1/32*sin(4*x)-1/4*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \sin^4(x)\,dx = \frac{1}{8}\left(2\cos(x)^3 - 5\cos(x)\right)\sin(x) + \frac{3}{8}x$$

```
[In] integrate(sin(x)^4,x, algorithm="fricas")
[Out] 1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{\sin^3(x)\cos(x)}{4} - \frac{3\sin(x)\cos(x)}{8}$$

```
[In] integrate(sin(x)**4,x)
[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3}{8}x + \frac{1}{32}\sin(4x) - \frac{1}{4}\sin(2x)$$

```
[In] integrate(sin(x)^4,x, algorithm="maxima")
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3}{8}x + \frac{1}{32}\sin(4x) - \frac{1}{4}\sin(2x)$$

```
[In] integrate(sin(x)^4,x, algorithm="giac")
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3\,x}{8} - \frac{\sin\left(2\,x\right)}{4} + \frac{\sin\left(4\,x\right)}{32}$$

```
[In] int(sin(x)^4,x)
[Out] (3*x)/8 - sin(2*x)/4 + sin(4*x)/32
```

3.333 $\int \cos^6(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 34

$$\int \cos^6(x)\,dx=\frac{5x}{16}+\frac{5}{16}\cos(x)\sin(x)+\frac{5}{24}\cos^3(x)\sin(x)+\frac{1}{6}\cos^5(x)\sin(x)$$

```
[Out] 5/16*x+5/16*cos(x)*sin(x)+5/24*cos(x)^3*sin(x)+1/6*cos(x)^5*sin(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\int \cos^6(x)\,dx=\frac{5x}{16}+\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{24}\sin(x)\cos^3(x)+\frac{5}{16}\sin(x)\cos(x)$$

```
[In] Int[Cos[x]^6,x]
```

```
[Out] (5*x)/16 + (5*Cos[x]*Sin[x])/16 + (5*Cos[x]^3*Sin[x])/24 + (Cos[x]^5*Sin[x]
)/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{6}\cos^5(x)\sin(x) + \frac{5}{6}\int \cos^4(x)\,dx \\
&= \frac{5}{24}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x) + \frac{5}{8}\int \cos^2(x)\,dx \\
&= \frac{5}{16}\cos(x)\sin(x) + \frac{5}{24}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x) + \frac{5\int 1\,dx}{16} \\
&= \frac{5x}{16} + \frac{5}{16}\cos(x)\sin(x) + \frac{5}{24}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \cos^6(x)\,dx = \frac{5x}{16} + \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) + \frac{1}{192}\sin(6x)$$

```
[In] Integrate[Cos[x]^6,x]
[Out] (5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result
risch	$\frac{5x}{16}+\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}+\frac{15\sin(2x)}{64}$
parallelrisch	$\frac{5x}{16}+\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}+\frac{15\sin(2x)}{64}$
default	$\frac{\left(\cos^5(x)+\frac{5\left(\cos^3(x)\right)}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{6}+\frac{5x}{16}$
norman	$\frac{\frac{5x}{16}-\frac{5\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}+\frac{15\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}-\frac{15\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{5\left(\tan^9\left(\frac{x}{2}\right)\right)}{24}-\frac{11\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{8}+\frac{15x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{75x\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{25x\left(\tan^6\left(\frac{x}{2}\right)\right)}{4}+}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

```
[In] int(cos(x)^6,x,method=_RETURNVERBOSE)
[Out] 5/16*x+1/192*sin(6*x)+3/64*sin(4*x)+15/64*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \cos^6(x)\,dx = \frac{1}{48}\left(8\,\cos(x)^5 + 10\,\cos(x)^3 + 15\,\cos(x)\right)\sin(x) + \frac{5}{16}\,x$$

```
[In] integrate(cos(x)^6,x, algorithm="fricas")
[Out] 1/48*(8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x))*sin(x) + 5/16*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \cos^6(x)\,dx = \frac{5x}{16} + \frac{\sin(x)\cos^5(x)}{6} + \frac{5\sin(x)\cos^3(x)}{24} + \frac{5\sin(x)\cos(x)}{16}$$

```
[In] integrate(cos(x)**6,x)
[Out] 5*x/16 + sin(x)*cos(x)**5/6 + 5*sin(x)*cos(x)**3/24 + 5*sin(x)*cos(x)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \cos^6(x)\,dx = -\frac{1}{48}\,\sin(2\,x)^3 + \frac{5}{16}\,x + \frac{3}{64}\,\sin(4\,x) + \frac{1}{4}\,\sin(2\,x)$$

```
[In] integrate(cos(x)^6,x, algorithm="maxima")
[Out] -1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) + 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \cos^6(x)\,dx = \frac{5}{16}\,x + \frac{1}{192}\,\sin(6\,x) + \frac{3}{64}\,\sin(4\,x) + \frac{15}{64}\,\sin(2\,x)$$

```
[In] integrate(cos(x)^6,x, algorithm="giac")
[Out] 5/16*x + 1/192*sin(6*x) + 3/64*sin(4*x) + 15/64*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \cos^6(x)\,dx = \frac{5\,x}{16} + \frac{15\,\sin\left(2\,x\right)}{64} + \frac{3\,\sin\left(4\,x\right)}{64} + \frac{\sin\left(6\,x\right)}{192}$$

```
[In] int(cos(x)^6,x)
[Out] (5*x)/16 + (15*sin(2*x))/64 + (3*sin(4*x))/64 + sin(6*x)/192
```

3.334 $\int \sin^8(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 44

$$\int \sin^8(x)\,dx = \frac{35x}{128} - \frac{35}{128}\cos(x)\sin(x) - \frac{35}{192}\cos(x)\sin^3(x) - \frac{7}{48}\cos(x)\sin^5(x) - \frac{1}{8}\cos(x)\sin^7(x)$$

```
[Out] 35/128*x-35/128*cos(x)*sin(x)-35/192*cos(x)*sin(x)^3-7/48*cos(x)*sin(x)^5-1
/8*cos(x)*sin(x)^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\int \sin^8(x)\,dx = \frac{35x}{128} - \frac{1}{8}\sin^7(x)\cos(x) - \frac{7}{48}\sin^5(x)\cos(x) - \frac{35}{192}\sin^3(x)\cos(x) - \frac{35}{128}\sin(x)\cos(x)$$

```
[In] Int[Sin[x]^8,x]
```

```
[Out] (35*x)/128 - (35*Cos[x]*Sin[x])/128 - (35*Cos[x]*Sin[x]^3)/192 - (7*Cos[x]*
Sin[x]^5)/48 - (Cos[x]*Sin[x]^7)/8
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{8}\cos(x)\sin^7(x)+\frac{7}{8}\int\sin^6(x)\,dx\\
&= -\frac{7}{48}\cos(x)\sin^5(x)-\frac{1}{8}\cos(x)\sin^7(x)+\frac{35}{48}\int\sin^4(x)\,dx\\
&= -\frac{35}{192}\cos(x)\sin^3(x)-\frac{7}{48}\cos(x)\sin^5(x)-\frac{1}{8}\cos(x)\sin^7(x)+\frac{35}{64}\int\sin^2(x)\,dx\\
&= -\frac{35}{128}\cos(x)\sin(x)-\frac{35}{192}\cos(x)\sin^3(x)-\frac{7}{48}\cos(x)\sin^5(x)-\frac{1}{8}\cos(x)\sin^7(x)+\frac{35\int 1\,dx}{128}\\
&= \frac{35x}{128}-\frac{35}{128}\cos(x)\sin(x)-\frac{35}{192}\cos(x)\sin^3(x)-\frac{7}{48}\cos(x)\sin^5(x)-\frac{1}{8}\cos(x)\sin^7(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int\sin^8(x)\,dx=\frac{35x}{128}-\frac{7}{32}\sin(2x)+\frac{7}{128}\sin(4x)-\frac{1}{96}\sin(6x)+\frac{\sin(8x)}{1024}$$

```
[In] Integrate[Sin[x]^8,x]
[Out] (35*x)/128 - (7*Sin[2*x])/32 + (7*Sin[4*x])/128 - Sin[6*x]/96 + Sin[8*x]/10
24
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

method	result
risch	$\frac{35x}{128}+\frac{\sin(8x)}{1024}-\frac{\sin(6x)}{96}+\frac{7\sin(4x)}{128}-\frac{7\sin(2x)}{32}$
parallelrisch	$\frac{35x}{128}+\frac{\sin(8x)}{1024}-\frac{\sin(6x)}{96}+\frac{7\sin(4x)}{128}-\frac{7\sin(2x)}{32}$
default	$-\frac{\left(\sin^{7}(x)+\frac{7\left(\sin^{5}(x)\right)}{6}+\frac{35\left(\sin^{3}(x)\right)}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{35x}{128}$
norman	$\frac{35x}{128}-\frac{805\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{192}-\frac{2681\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{192}-\frac{5053\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{192}+\frac{5053\left(\tan^{9}\left(\frac{x}{2}\right)\right)}{192}+\frac{2681\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{192}+\frac{805\left(\tan^{13}\left(\frac{x}{2}\right)\right)}{192}+\frac{35\left(\tan^{15}\left(\frac{x}{2}\right)\right)}{64}+3$

```
[In] int(sin(x)^8,x,method=_RETURNVERBOSE)
[Out] 35/128*x+1/1024*sin(8*x)-1/96*sin(6*x)+7/128*sin(4*x)-7/32*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \sin^8(x)\,dx = \frac{1}{384}\left(48\,\cos\left(x\right)^{7}-200\,\cos\left(x\right)^{5}+326\,\cos\left(x\right)^{3}-279\,\cos\left(x\right)\right)\sin\left(x\right)+\frac{35}{128}\,x$$

```
[In] integrate(sin(x)^8,x, algorithm="fricas")
[Out] 1/384*(48*cos(x)^7 - 200*cos(x)^5 + 326*cos(x)^3 - 279*cos(x))*sin(x) + 35/
128*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \sin^8(x)\,dx = \frac{35x}{128}-\frac{\sin^{7}\left(x\right)\cos\left(x\right)}{8}-\frac{7\sin^{5}\left(x\right)\cos\left(x\right)}{48}-\frac{35\sin^{3}\left(x\right)\cos\left(x\right)}{192}-\frac{35\sin\left(x\right)\cos\left(x\right)}{128}$$

```
[In] integrate(sin(x)**8,x)
[Out] 35*x/128 - sin(x)**7*cos(x)/8 - 7*sin(x)**5*cos(x)/48 - 35*sin(x)**3*cos(x)
/192 - 35*sin(x)*cos(x)/128
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int \sin^8(x)\,dx = \frac{1}{24}\sin(2x)^3 + \frac{35}{128}x + \frac{1}{1024}\sin(8x) + \frac{7}{128}\sin(4x) - \frac{1}{4}\sin(2x)$$

```
[In] integrate(sin(x)^8,x, algorithm="maxima")
[Out] 1/24*sin(2*x)^3 + 35/128*x + 1/1024*sin(8*x) + 7/128*sin(4*x) - 1/4*sin(2*x
)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \sin^8(x)\,dx = \frac{35}{128}x + \frac{1}{1024}\sin(8x) - \frac{1}{96}\sin(6x) + \frac{7}{128}\sin(4x) - \frac{7}{32}\sin(2x)$$

```
[In] integrate(sin(x)^8,x, algorithm="giac")
[Out] 35/128*x + 1/1024*sin(8*x) - 1/96*sin(6*x) + 7/128*sin(4*x) - 7/32*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \sin^8(x)\,dx = \frac{35\,x}{128} - \frac{7\sin(2x)}{32} + \frac{7\sin(4x)}{128} - \frac{\sin(6x)}{96} + \frac{\sin(8x)}{1024}$$

```
[In] int(sin(x)^8,x)
[Out] (35*x)/128 - (7*sin(2*x))/32 + (7*sin(4*x))/128 - sin(6*x)/96 + sin(8*x)/10
24
```

3.335 $\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 20

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right) dx = \frac{3x}{8}+\frac{\cos(x)}{2}-\frac{1}{8}\cos(x)\sin(x)$$

```
[Out] 3/8*x+1/2*cos(x)-1/8*cos(x)*sin(x)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(20) = 40.

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.20, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2715, 8}

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right) dx = \frac{3x}{8}+\frac{1}{2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^3\left(\frac{x}{2}+\frac{\pi}{4}\right)+\frac{3}{4}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

```
[In] Int[Cos[Pi/4 + x/2]^4,x]
```

```
[Out] (3*x)/8 + (3*Cos[Pi/4 + x/2]*Sin[Pi/4 + x/2])/4 + (Cos[Pi/4 + x/2]^3*Sin[Pi
/4 + x/2])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

`*n]`

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\cos^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{3}{4}\int\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)\,dx\\
&= \frac{3}{4}\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{1}{2}\cos^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{3\int 1\,dx}{8}\\
&= \frac{3x}{8}+\frac{3}{4}\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{1}{2}\cos^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int\cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{1}{16}(3\pi+6x+8\cos(x)-2\cos(x)\sin(x))$$

```
[In] Integrate[Cos[Pi/4 + x/2]^4,x]
[Out] (3*Pi + 6*x + 8*Cos[x] - 2*Cos[x]*Sin[x])/16
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result
risch	$\frac{3x}{8}+\frac{\cos(x)}{2}-\frac{\sin(2x)}{16}$
parallelrisch	$\frac{3x}{8}+\frac{\cos(x)}{2}-\frac{\sin(2x)}{16}$
derivativedivides	$\frac{\left(\cos^3\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{3\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}+\frac{3\pi}{16}+\frac{3x}{8}$
default	$\frac{\left(\cos^3\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{3\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}+\frac{3\pi}{16}+\frac{3x}{8}$
norman	$\frac{\frac{3x}{8}-\frac{3\left(\tan^3\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{2}+\frac{3\left(\tan^5\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{2}-\frac{5\left(\tan^7\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{2}+\frac{3x\left(\tan^2\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{2}+\frac{9x\left(\tan^4\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{4}+\frac{3x\left(\tan^6\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{2}+\frac{3x\left(\tan^8\right.}{}}{\left(1+\tan^2\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)^4}$

```
[In] int(cos(1/4*Pi+1/2*x)^4,x,method=_RETURNVERBOSE)
[Out] 3/8*x+1/2*cos(x)-1/16*sin(2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = \frac{1}{4}\left(2\,\cos\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)^3+3\,\cos\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)+\frac{3}{8}\,x$$

```
[In] integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="fricas")
[Out] 1/4*(2*cos(1/4*pi + 1/2*x)^3 + 3*cos(1/4*pi + 1/2*x))*sin(1/4*pi + 1/2*x) +
 3/8*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(17) = 34$.

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.95

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = \frac{3x\sin^4\left(\frac{x}{2}+\frac{\pi}{4}\right)}{8}+\frac{3x\sin^2\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^2\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}+\frac{3x\cos^4\left(\frac{x}{2}+\frac{\pi}{4}\right)}{8}+\frac{3\sin^3\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}+\frac{5\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^3\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}$$

```
[In] integrate(cos(1/4*pi+1/2*x)**4,x)
[Out] 3*x*sin(x/2 + pi/4)**4/8 + 3*x*sin(x/2 + pi/4)**2*cos(x/2 + pi/4)**2/4 + 3*
x*cos(x/2 + pi/4)**4/8 + 3*sin(x/2 + pi/4)**3*cos(x/2 + pi/4)/4 + 5*sin(x/2
 + pi/4)*cos(x/2 + pi/4)**3/4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = \frac{3}{16}\,\pi+\frac{3}{8}\,x+\frac{1}{16}\,\sin\left(\pi+2\,x\right)+\frac{1}{2}\,\sin\left(\frac{1}{2}\,\pi+x\right)$$

```
[In] integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="maxima")
[Out] 3/16*pi + 3/8*x + 1/16*sin(pi + 2*x) + 1/2*sin(1/2*pi + x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{3}{8}\,x+\frac{1}{2}\,\cos\left(x\right)-\frac{1}{16}\,\sin\left(2\,x\right)$$

```
[In] integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="giac")
[Out] 3/8*x + 1/2*cos(x) - 1/16*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{3\,x}{8}+\frac{\sin\left(\Pi+2\,x\right)}{16}+\frac{\sin\left(\frac{\Pi}{2}+x\right)}{2}$$

```
[In] int(cos(Pi/4 + x/2)^4,x)
[Out] (3*x)/8 + sin(Pi + 2*x)/16 + sin(Pi/2 + x)/2
```

3.336 $\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx=-\frac{1}{3}\cos\left(\frac{\pi}{12}-3x\right)+\frac{1}{9}\cos^3\left(\frac{\pi}{12}-3x\right)$$

```
[Out] -1/3*sin(5/12*Pi+3*x)+1/9*sin(5/12*Pi+3*x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2713}

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx=\frac{1}{9}\cos^3\left(\frac{\pi}{12}-3x\right)-\frac{1}{3}\cos\left(\frac{\pi}{12}-3x\right)$$

```
[In] Int[-Sin[Pi/12 - 3*x]^3,x]
```

```
[Out] -1/3*Cos[Pi/12 - 3*x] + Cos[Pi/12 - 3*x]^3/9
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=-\left(\frac{1}{3}\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cos\left(\frac{\pi}{12}-3x\right)\right)\right)\\&=-\frac{1}{3}\cos\left(\frac{\pi}{12}-3x\right)+\frac{1}{9}\cos^3\left(\frac{\pi}{12}-3x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx=-\frac{1}{4}\cos\left(\frac{\pi}{12}-3x\right)+\frac{1}{36}\cos\left(3\left(\frac{\pi}{12}-3x\right)\right)$$

```
[In] Integrate[-Sin[Pi/12 - 3*x]^3,x]
[Out] -1/4*Cos[Pi/12 - 3*x] + Cos[3*(Pi/12 - 3*x)]/36
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{\sin\left(\frac{\pi}{4}+9x\right)}{36}-\frac{\sin\left(\frac{5\pi}{12}+3x\right)}{4}$	22
parallelrisch	$\frac{\sin\left(\frac{\pi}{4}+9x\right)}{36}-\frac{\sin\left(\frac{5\pi}{12}+3x\right)}{4}$	22
derivativedivides	$-\frac{\left(2+\cos^2\left(\frac{5\pi}{12}+3x\right)\right)\sin\left(\frac{5\pi}{12}+3x\right)}{9}$	23
default	$-\frac{\left(2+\cos^2\left(\frac{5\pi}{12}+3x\right)\right)\sin\left(\frac{5\pi}{12}+3x\right)}{9}$	23
norman	$\frac{-\frac{4\left(\tan^3\left(\frac{5\pi}{24}+\frac{3x}{2}\right)\right)}{9}-\frac{2\left(\tan^5\left(\frac{5\pi}{24}+\frac{3x}{2}\right)\right)}{3}-\frac{2\tan\left(\frac{5\pi}{24}+\frac{3x}{2}\right)}{3}}{\left(1+\tan^2\left(\frac{5\pi}{24}+\frac{3x}{2}\right)\right)^3}$	51

```
[In] int(-cos(5/12*Pi+3*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/36*sin(1/4*Pi+9*x)-1/4*sin(5/12*Pi+3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx=-\frac{1}{9}\left(\cos\left(\frac{5}{12}\pi+3x\right)^2+2\right)\sin\left(\frac{5}{12}\pi+3x\right)$$

```
[In] integrate(-cos(5/12*pi+3*x)^3,x, algorithm="fricas")
[Out] -1/9*(cos(5/12*pi + 3*x)^2 + 2)*sin(5/12*pi + 3*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx = -\frac{2\sin^3\left(3x+\frac{5\pi}{12}\right)}{9} - \frac{\sin\left(3x+\frac{5\pi}{12}\right)\cos^2\left(3x+\frac{5\pi}{12}\right)}{3}$$

```
[In] integrate(-cos(5/12*pi+3*x)**3,x)
[Out] -2*sin(3*x + 5*pi/12)**3/9 - sin(3*x + 5*pi/12)*cos(3*x + 5*pi/12)**2/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx = \frac{1}{9}\sin\left(\frac{5}{12}\pi+3x\right)^3 - \frac{1}{3}\sin\left(\frac{5}{12}\pi+3x\right)$$

```
[In] integrate(-cos(5/12*pi+3*x)^3,x, algorithm="maxima")
[Out] 1/9*sin(5/12*pi + 3*x)^3 - 1/3*sin(5/12*pi + 3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx = \frac{1}{9}\sin\left(\frac{5}{12}\pi+3x\right)^3 - \frac{1}{3}\sin\left(\frac{5}{12}\pi+3x\right)$$

```
[In] integrate(-cos(5/12*pi+3*x)^3,x, algorithm="giac")
[Out] 1/9*sin(5/12*pi + 3*x)^3 - 1/3*sin(5/12*pi + 3*x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx = \frac{\sin\left(\frac{5\Pi}{12}+3x\right)\left(\sin\left(\frac{5\Pi}{12}+3x\right)^2-3\right)}{9}$$

```
[In] int(-cos((5*Pi)/12 + 3*x)^3,x)
[Out] (sin((5*Pi)/12 + 3*x)*(sin((5*Pi)/12 + 3*x)^2 - 3))/9
```

3.337 $\int \csc^6(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 21

$$\int \csc^6(x)\,dx = -\cot(x) - \frac{2\cot^3(x)}{3} - \frac{\cot^5(x)}{5}$$

```
[Out] -cot(x)-2/3*cot(x)^3-1/5*cot(x)^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3852}

$$\int \csc^6(x)\,dx = -\frac{1}{5}\cot^5(x) - \frac{2\cot^3(x)}{3} - \cot(x)$$

```
[In] Int[Csc[x]^6,x]
```

```
[Out] -Cot[x] - (2*Cot[x]^3)/3 - Cot[x]^5/5
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, \cot(x)\right)\\ &= -\cot(x) - \frac{2\cot^3(x)}{3} - \frac{\cot^5(x)}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \csc^6(x)\,dx = -\frac{8\cot(x)}{15} - \frac{4}{15}\cot(x)\csc^2(x) - \frac{1}{5}\cot(x)\csc^4(x)$$

```
[In] Integrate[Csc[x]^6,x]
[Out] (-8*Cot[x])/15 - (4*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\left(-\frac{8}{15}-\frac{\left(\csc^4(x)\right)}{5}-\frac{4\left(\csc^2(x)\right)}{15}\right)\cot\left(x\right)$	18
parallelrisch	$-\frac{\cot(x)\left(\csc^4(x)\right)\left(8+\cos(4x)-6\cos(2x)\right)}{15}$	21
risch	$-\frac{16i\left(10\,e^{4ix}-5\,e^{2ix}+1\right)}{15\left(e^{2ix}-1\right)^5}$	29
norman	$\frac{-\frac{1}{160}-\frac{5\left(\tan^2\left(\frac{x}{2}\right)\right)}{96}-\frac{5\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{5\left(\tan^6\left(\frac{x}{2}\right)\right)}{16}+\frac{5\left(\tan^8\left(\frac{x}{2}\right)\right)}{96}+\frac{\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{160}}{\tan\left(\frac{x}{2}\right)^5}$	50

```
[In] int(1/sin(x)^6,x,method=_RETURNVERBOSE)
[Out] (-8/15-1/5*csc(x)^4-4/15*csc(x)^2)*cot(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \csc^6(x)\,dx = -\frac{8\,\cos\left(x\right)^5 - 20\,\cos\left(x\right)^3 + 15\,\cos\left(x\right)}{15\,\left(\cos\left(x\right)^4 - 2\,\cos\left(x\right)^2 + 1\right)\sin\left(x\right)}$$

```
[In] integrate(1/sin(x)^6,x, algorithm="fricas")
[Out] -1/15*(8*cos(x)^5 - 20*cos(x)^3 + 15*cos(x))/((cos(x)^4 - 2*cos(x)^2 + 1)*s
in(x))
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.52

$$\int \csc^6(x)\,dx = -\frac{8\cos(x)}{15\sin(x)} - \frac{4\cos(x)}{15\sin^3(x)} - \frac{\cos(x)}{5\sin^5(x)}$$

```
[In] integrate(1/sin(x)**6,x)
[Out] -8*cos(x)/(15*sin(x)) - 4*cos(x)/(15*sin(x)**3) - cos(x)/(5*sin(x)**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \csc^6(x)\,dx = -\frac{15\,\tan(x)^4 + 10\,\tan(x)^2 + 3}{15\,\tan(x)^5}$$

```
[In] integrate(1/sin(x)^6,x, algorithm="maxima")
[Out] -1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \csc^6(x)\,dx = -\frac{15\,\tan(x)^4 + 10\,\tan(x)^2 + 3}{15\,\tan(x)^5}$$

```
[In] integrate(1/sin(x)^6,x, algorithm="giac")
[Out] -1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \csc^6(x)\,dx = -\frac{8\,\cos(x)\,\sin(x)^4 + 4\,\cos(x)\,\sin(x)^2 + 3\,\cos(x)}{15\sin(x)^5}$$

```
[In] int(1/sin(x)^6,x)
[Out] -(3*cos(x) + 4*cos(x)*sin(x)^2 + 8*cos(x)*sin(x)^4)/(15*sin(x)^5)
```

3.338 $\int \csc^7(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 36

$$\int \csc^7(x)\,dx = -\frac{5}{16}\text{arctanh}(\cos(x)) - \frac{5}{16}\cot(x)\csc(x) - \frac{5}{24}\cot(x)\csc^3(x) - \frac{1}{6}\cot(x)\csc^5(x)$$

```
[Out] -5/16*arctanh(cos(x))-5/16*cot(x)*csc(x)-5/24*cot(x)*csc(x)^3-1/6*cot(x)*cs
c(x)^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3853, 3855}

$$\int \csc^7(x)\,dx = -\frac{5}{16}\text{arctanh}(\cos(x)) - \frac{1}{6}\cot(x)\csc^5(x) - \frac{5}{24}\cot(x)\csc^3(x) - \frac{5}{16}\cot(x)\csc(x)$$

```
[In] Int[Csc[x]^7,x]
```

```
[Out] (-5*ArcTanh[Cos[x]])/16 - (5*Cot[x]*Csc[x])/16 - (5*Cot[x]*Csc[x]^3)/24 - (
Cot[x]*Csc[x]^5)/6
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{6}\cot(x)\csc^5(x) + \frac{5}{6}\int \csc^5(x)\,dx \\
&= -\frac{5}{24}\cot(x)\csc^3(x) - \frac{1}{6}\cot(x)\csc^5(x) + \frac{5}{8}\int \csc^3(x)\,dx \\
&= -\frac{5}{16}\cot(x)\csc(x) - \frac{5}{24}\cot(x)\csc^3(x) - \frac{1}{6}\cot(x)\csc^5(x) + \frac{5}{16}\int \csc(x)\,dx \\
&= -\frac{5}{16}\text{arctanh}(\cos(x)) - \frac{5}{16}\cot(x)\csc(x) - \frac{5}{24}\cot(x)\csc^3(x) - \frac{1}{6}\cot(x)\csc^5(x)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 95 vs. $2(36) = 72$.

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.64

$$\begin{aligned}
\int \csc^7(x)\,dx = &-\frac{5}{64}\csc^2\left(\frac{x}{2}\right) - \frac{1}{64}\csc^4\left(\frac{x}{2}\right) - \frac{1}{384}\csc^6\left(\frac{x}{2}\right) - \frac{5}{16}\log\left(\cos\left(\frac{x}{2}\right)\right) \\
&+ \frac{5}{16}\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{5}{64}\sec^2\left(\frac{x}{2}\right) + \frac{1}{64}\sec^4\left(\frac{x}{2}\right) + \frac{1}{384}\sec^6\left(\frac{x}{2}\right)
\end{aligned}$$

```
[In] Integrate[Csc[x]^7,x]
[Out] (-5*Csc[x/2]^2)/64 - Csc[x/2]^4/64 - Csc[x/2]^6/384 - (5*Log[Cos[x/2]])/16
+ (5*Log[Sin[x/2]])/16 + (5*Sec[x/2]^2)/64 + Sec[x/2]^4/64 + Sec[x/2]^6/384
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$\left(-\frac{\left(\csc^5(x)\right)}{6}-\frac{5\left(\csc^3(x)\right)}{24}-\frac{5\csc(x)}{16}\right)\cot(x)+\frac{5\ln(\csc(x)-\cot(x))}{16}$	32
parallelrisch	$-\frac{75\left(\csc^6(x)\right)\left(\left(\cos(2x)-\frac{2\cos(4x)}{5}+\frac{\cos(6x)}{15}-\frac{2}{3}\right)\ln(\csc(x)-\cot(x))+\frac{44\cos(x)}{25}-\frac{34\cos(3x)}{45}+\frac{2\cos(5x)}{15}\right)}{512}$	51
norman	$\frac{-\frac{1}{384}-\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{128}-\frac{15\left(\tan^4\left(\frac{x}{2}\right)\right)}{128}+\frac{15\left(\tan^8\left(\frac{x}{2}\right)\right)}{128}+\frac{3\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{128}+\frac{\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{384}}{\tan\left(\frac{x}{2}\right)^6}+\frac{5\ln\left(\tan\left(\frac{x}{2}\right)\right)}{16}$	58
risch	$\frac{15\,e^{11ix}-85\,e^{9ix}+198\,e^{7ix}+198\,e^{5ix}-85\,e^{3ix}+15\,e^{ix}}{24\left(e^{2ix}-1\right)^6}-\frac{5\ln\left(e^{ix}+1\right)}{16}+\frac{5\ln\left(e^{ix}-1\right)}{16}$	76

```
[In] int(csc(x)^7,x,method=_RETURNVERBOSE)
[Out] (-1/6*csc(x)^5-5/24*csc(x)^3-5/16*csc(x))*cot(x)+5/16*ln(csc(x)-cot(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.58

$$\int \csc^7(x)\,dx = \frac{30\cos(x)^5 - 80\cos(x)^3 - 15\left(\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + 15\left(\cos(x)^6 - \right.}{96\left(\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1\right)}$$

```
[In] integrate(csc(x)^7,x, algorithm="fricas")
[Out] 1/96*(30*cos(x)^5 - 80*cos(x)^3 - 15*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 -
1)*log(1/2*cos(x) + 1/2) + 15*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*log(
-1/2*cos(x) + 1/2) + 66*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.67

$$\int \csc^7(x)\,dx = -\frac{-15\cos^5(x) + 40\cos^3(x) - 33\cos(x)}{48\cos^6(x) - 144\cos^4(x) + 144\cos^2(x) - 48} + \frac{5\log(\cos(x) - 1)}{32} - \frac{5\log(\cos(x) + 1)}{32}$$

```
[In] integrate(csc(x)**7,x)
[Out] -(-15*cos(x)**5 + 40*cos(x)**3 - 33*cos(x))/(48*cos(x)**6 - 144*cos(x)**4 +
 144*cos(x)**2 - 48) + 5*log(cos(x) - 1)/32 - 5*log(cos(x) + 1)/32
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \csc^7(x)\,dx = \frac{15\cos(x)^5 - 40\cos(x)^3 + 33\cos(x)}{48\left(\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1\right)} - \frac{5}{32}\log(\cos(x) + 1) + \frac{5}{32}\log(\cos(x) - 1)$$

```
[In] integrate(csc(x)^7,x, algorithm="maxima")
[Out] 1/48*(15*cos(x)^5 - 40*cos(x)^3 + 33*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos
(x)^2 - 1) - 5/32*log(cos(x) + 1) + 5/32*log(cos(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(28) = 56$.

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.11

$$\int \csc^7(x)\,dx = -\frac{\left(\frac{9(\cos(x)-1)}{\cos(x)+1} - \frac{45(\cos(x)-1)^2}{(\cos(x)+1)^2} + \frac{110(\cos(x)-1)^3}{(\cos(x)+1)^3} - 1\right)(\cos(x)+1)^3}{384(\cos(x)-1)^3} - \frac{15(\cos(x)-1)}{128(\cos(x)+1)} + \frac{3(\cos(x)-1)^2}{128(\cos(x)+1)^2} - \frac{(\cos(x)-1)^3}{384(\cos(x)+1)^3} + \frac{5}{32}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

```
[In] integrate(csc(x)^7,x, algorithm="giac")
```

```
[Out] -1/384*(9*(cos(x) - 1)/(cos(x) + 1) - 45*(cos(x) - 1)^2/(cos(x) + 1)^2 + 11
0*(cos(x) - 1)^3/(cos(x) + 1)^3 - 1)*(cos(x) + 1)^3/(cos(x) - 1)^3 - 15/128
*(cos(x) - 1)/(cos(x) + 1) + 3/128*(cos(x) - 1)^2/(cos(x) + 1)^2 - 1/384*(c
os(x) - 1)^3/(cos(x) + 1)^3 + 5/32*log(-(cos(x) - 1)/(cos(x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \csc^7(x)\,dx = \frac{\frac{5\cos(x)^5}{16} - \frac{5\cos(x)^3}{6} + \frac{11\cos(x)}{16}}{\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1} - \frac{5\,\mathrm{atanh}(\cos(x))}{16}$$

```
[In] int(1/sin(x)^7,x)
```

```
[Out] ((11*cos(x))/16 - (5*cos(x)^3)/6 + (5*cos(x)^5)/16)/(3*cos(x)^2 - 3*cos(x)^
4 + cos(x)^6 - 1) - (5*atanh(cos(x)))/16
```

3.339 $\int \sec^{12}(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 41

$$\int \sec^{12}(x)\,dx = \tan(x) + \frac{5\tan^3(x)}{3} + 2\tan^5(x) + \frac{10\tan^7(x)}{7} + \frac{5\tan^9(x)}{9} + \frac{\tan^{11}(x)}{11}$$

```
[Out] tan(x)+5/3*tan(x)^3+2*tan(x)^5+10/7*tan(x)^7+5/9*tan(x)^9+1/11*tan(x)^11
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{3852\}$

$$\int \sec^{12}(x)\,dx = \frac{\tan^{11}(x)}{11} + \frac{5\tan^9(x)}{9} + \frac{10\tan^7(x)}{7} + 2\tan^5(x) + \frac{5\tan^3(x)}{3} + \tan(x)$$

```
[In] Int[Sec[x]^12,x]
```

```
[Out] Tan[x] + (5*Tan[x]^3)/3 + 2*Tan[x]^5 + (10*Tan[x]^7)/7 + (5*Tan[x]^9)/9 + T
an[x]^11/11
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \left(1+5x^2+10x^4+10x^6+5x^8+x^{10}\right)\,dx, x, -\tan(x)\right)\\ &= \tan(x) + \frac{5\tan^3(x)}{3} + 2\tan^5(x) + \frac{10\tan^7(x)}{7} + \frac{5\tan^9(x)}{9} + \frac{\tan^{11}(x)}{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \sec^{12}(x)\,dx=\tan(x)+\frac{5\tan^3(x)}{3}+2\tan^5(x)+\frac{10\tan^7(x)}{7}+\frac{5\tan^9(x)}{9}+\frac{\tan^{11}(x)}{11}$$

```
[In] Integrate[Sec[x]^12,x]
```

```
[Out] Tan[x] + (5*Tan[x]^3)/3 + 2*Tan[x]^5 + (10*Tan[x]^7)/7 + (5*Tan[x]^9)/9 + T
an[x]^11/11
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

method	result	size
default	$-\left(-\frac{256}{693}-\frac{(\sec^{10}(x))}{11}-\frac{10(\sec^{8}(x))}{99}-\frac{80(\sec^{6}(x))}{693}-\frac{32(\sec^{4}(x))}{231}-\frac{128(\sec^{2}(x))}{693}\right)\tan(x)$	37
parallelrisch	$\frac{\tan(x)(\sec^{10}(x))(512+\cos(10x)+12\cos(8x)+67\cos(6x)+232\cos(4x)+562\cos(2x))}{1386}$	39
risch	$\frac{512i(462\,e^{10ix}+330\,e^{8ix}+165\,e^{6ix}+55\,e^{4ix}+11\,e^{2ix}+1)}{693(e^{2ix}+1)^{11}}$	50

```
[In] int(1/cos(x)^12,x,method=_RETURNVERBOSE)
```

```
[Out] -(-256/693-1/11*sec(x)^10-10/99*sec(x)^8-80/693*sec(x)^6-32/231*sec(x)^4-12
8/693*sec(x)^2)*tan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \sec^{12}(x)\,dx = \frac{(256\cos(x)^{10}+128\cos(x)^8+96\cos(x)^6+80\cos(x)^4+70\cos(x)^2+63)\sin(x)}{693\cos(x)^{11}}$$

```
[In] integrate(1/cos(x)^12,x, algorithm="fricas")
```

```
[Out] 1/693*(256*cos(x)^10 + 128*cos(x)^8 + 96*cos(x)^6 + 80*cos(x)^4 + 70*cos(x)
^2 + 63)*sin(x)/cos(x)^11
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \sec^{12}(x)\,dx = \frac{256\sin(x)}{693\cos(x)} + \frac{128\sin(x)}{693\cos^3(x)} + \frac{32\sin(x)}{231\cos^5(x)} + \frac{80\sin(x)}{693\cos^7(x)} + \frac{10\sin(x)}{99\cos^9(x)} + \frac{\sin(x)}{11\cos^{11}(x)}$$

```
[In] integrate(1/cos(x)**12,x)
```

```
[Out] 256*sin(x)/(693*cos(x)) + 128*sin(x)/(693*cos(x)**3) + 32*sin(x)/(231*cos(x
)**5) + 80*sin(x)/(693*cos(x)**7) + 10*sin(x)/(99*cos(x)**9) + sin(x)/(11*c
os(x)**11)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \sec^{12}(x)\,dx = \frac{1}{11}\tan(x)^{11} + \frac{5}{9}\tan(x)^9 + \frac{10}{7}\tan(x)^7 + 2\tan(x)^5 + \frac{5}{3}\tan(x)^3 + \tan(x)$$

```
[In] integrate(1/cos(x)^12,x, algorithm="maxima")
```

```
[Out] 1/11*tan(x)^11 + 5/9*tan(x)^9 + 10/7*tan(x)^7 + 2*tan(x)^5 + 5/3*tan(x)^3 +
 tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \sec^{12}(x)\,dx = \frac{1}{11}\tan(x)^{11} + \frac{5}{9}\tan(x)^9 + \frac{10}{7}\tan(x)^7 + 2\tan(x)^5 + \frac{5}{3}\tan(x)^3 + \tan(x)$$

```
[In] integrate(1/cos(x)^12,x, algorithm="giac")
```

```
[Out] 1/11*tan(x)^11 + 5/9*tan(x)^9 + 10/7*tan(x)^7 + 2*tan(x)^5 + 5/3*tan(x)^3 +
 tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \sec^{12}(x)\,dx$$
$$= \frac{256 \sin(x) \cos(x)^{10} + 128 \sin(x) \cos(x)^{8} + 96 \sin(x) \cos(x)^{6} + 80 \sin(x) \cos(x)^{4} + 70 \sin(x) \cos(x)^{2} +}{693 \cos(x)^{11}}$$

```
[In] int(1/cos(x)^12,x)
```

```
[Out] (63*sin(x) + 70*cos(x)^2*sin(x) + 80*cos(x)^4*sin(x) + 96*cos(x)^6*sin(x) +
 128*cos(x)^8*sin(x) + 256*cos(x)^10*sin(x))/(693*cos(x)^11)
```

3.340 $\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx=\frac{1}{6}\text{arctanh}\left(\sin\left(\frac{\pi}{4}+3x\right)\right)+\frac{1}{6}\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)$$

```
[Out] 1/6*arctanh(sin(1/4*Pi+3*x))+1/6*sec(1/4*Pi+3*x)*tan(1/4*Pi+3*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3853, 3855}

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx=\frac{1}{6}\text{arctanh}\left(\sin\left(3x+\frac{\pi}{4}\right)\right)+\frac{1}{6}\tan\left(3x+\frac{\pi}{4}\right)\sec\left(3x+\frac{\pi}{4}\right)$$

```
[In] Int[Sec[Pi/4 + 3*x]^3,x]
```

```
[Out] ArcTanh[Sin[Pi/4 + 3*x]]/6 + (Sec[Pi/4 + 3*x]*Tan[Pi/4 + 3*x])/6
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{6}\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)+\frac{1}{2}\int\csc\left(\frac{\pi}{4}-3x\right)\,dx\\ &= \frac{1}{6}\text{arctanh}\left(\sin\left(\frac{\pi}{4}+3x\right)\right)+\frac{1}{6}\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int\sec^3\left(\frac{\pi}{4}+3x\right)\,dx=\frac{1}{6}\text{arctanh}\left(\sin\left(\frac{\pi}{4}+3x\right)\right)+\frac{1}{6}\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)$$

```
[In] Integrate[Sec[Pi/4 + 3*x]^3,x]
[Out] ArcTanh[Sin[Pi/4 + 3*x]]/6 + (Sec[Pi/4 + 3*x]*Tan[Pi/4 + 3*x])/6
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)}{6}+\frac{\ln\left(\sec\left(\frac{\pi}{4}+3x\right)+\tan\left(\frac{\pi}{4}+3x\right)\right)}{6}$	40
default	$\frac{\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)}{6}+\frac{\ln\left(\sec\left(\frac{\pi}{4}+3x\right)+\tan\left(\frac{\pi}{4}+3x\right)\right)}{6}$	40
parallelrisch	$\frac{(1-\sin(6x))\ln\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)-1\right)+(\sin(6x)-1)\ln\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)+1\right)-2\sin\left(\frac{\pi}{4}+3x\right)}{6\sin(6x)-6}$	61
norman	$\frac{\left(\frac{\tan^3\left(\frac{\pi}{8}+\frac{3x}{2}\right)}{3}\right)+\frac{\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)}{3}}{\left(\tan^2\left(\frac{\pi}{8}+\frac{3x}{2}\right)-1\right)^2}-\frac{\ln\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)-1\right)}{6}+\frac{\ln\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)+1\right)}{6}$	66
risch	$-\frac{i\left((-1)^{\frac{3}{4}}e^{9ix}-(-1)^{\frac{1}{4}}e^{3ix}\right)}{3(ie^{6ix}+1)^2}-\frac{\ln\left((-1)^{\frac{1}{4}}e^{3ix}-i\right)}{6}+\frac{\ln\left((-1)^{\frac{1}{4}}e^{3ix}+i\right)}{6}$	67

```
[In] int(1/cos(1/4*Pi+3*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/6*sec(1/4*Pi+3*x)*tan(1/4*Pi+3*x)+1/6*ln(sec(1/4*Pi+3*x)+tan(1/4*Pi+3*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(30) = 60$.

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.75

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx = \frac{\cos\left(\frac{1}{4}\pi+3x\right)^2\log\left(\sin\left(\frac{1}{4}\pi+3x\right)+1\right)-\cos\left(\frac{1}{4}\pi+3x\right)^2\log\left(-\sin\left(\frac{1}{4}\pi+3x\right)+1\right)+2\sin\left(\frac{1}{4}\pi+3x\right)}{12\cos\left(\frac{1}{4}\pi+3x\right)^2}$$

```
[In] integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="fricas")
[Out] 1/12*(cos(1/4*pi + 3*x)^2*log(sin(1/4*pi + 3*x) + 1) - cos(1/4*pi + 3*x)^2*
log(-sin(1/4*pi + 3*x) + 1) + 2*sin(1/4*pi + 3*x))/cos(1/4*pi + 3*x)^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(29) = 58$.

Time = 0.53 (sec) , antiderivative size = 388, normalized size of antiderivative = 9.70

$$\begin{aligned}\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx = &-\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right)\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6} \\ &+\frac{2\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right)\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6} \\ &-\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6} \\ &+\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)+1\right)\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6} \\ &-\frac{2\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)+1\right)\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6} \\ &+\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)+1\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6} \\ &+\frac{2\tan^3\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6} \\ &+\frac{2\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}\end{aligned}$$

```
[In] integrate(1/cos(1/4*pi+3*x)**3,x)
[Out] -log(tan(3*x/2 + pi/8) - 1)*tan(3*x/2 + pi/8)**4/(6*tan(3*x/2 + pi/8)**4 -
12*tan(3*x/2 + pi/8)**2 + 6) + 2*log(tan(3*x/2 + pi/8) - 1)*tan(3*x/2 + pi/
```

```
8)**2/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) - log(tan(3*x/
2 + pi/8) - 1)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + log
(tan(3*x/2 + pi/8) + 1)*tan(3*x/2 + pi/8)**4/(6*tan(3*x/2 + pi/8)**4 - 12*t
an(3*x/2 + pi/8)**2 + 6) - 2*log(tan(3*x/2 + pi/8) + 1)*tan(3*x/2 + pi/8)**
2/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + log(tan(3*x/2 +
pi/8) + 1)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + 2*tan(3
*x/2 + pi/8)**3/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + 2*
tan(3*x/2 + pi/8)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx = -\frac{\sin\left(\frac{1}{4}\,\pi+3\,x\right)}{6\left(\sin\left(\frac{1}{4}\,\pi+3\,x\right)^2-1\right)}+\frac{1}{12}\,\log\left(\sin\left(\frac{1}{4}\,\pi+3\,x\right)+1\right) - \frac{1}{12}\,\log\left(\sin\left(\frac{1}{4}\,\pi+3\,x\right)-1\right)$$

```
[In] integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="maxima")
```

```
[Out] -1/6*sin(1/4*pi + 3*x)/(sin(1/4*pi + 3*x)^2 - 1) + 1/12*log(sin(1/4*pi + 3*
x) + 1) - 1/12*log(sin(1/4*pi + 3*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx = -\frac{\sin\left(\frac{1}{4}\,\pi+3\,x\right)}{6\left(\sin\left(\frac{1}{4}\,\pi+3\,x\right)^2-1\right)}+\frac{1}{12}\,\log\left(\sin\left(\frac{1}{4}\,\pi+3\,x\right)+1\right) - \frac{1}{12}\,\log\left(-\sin\left(\frac{1}{4}\,\pi+3\,x\right)+1\right)$$

```
[In] integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="giac")
```

```
[Out] -1/6*sin(1/4*pi + 3*x)/(sin(1/4*pi + 3*x)^2 - 1) + 1/12*log(sin(1/4*pi + 3*
x) + 1) - 1/12*log(-sin(1/4*pi + 3*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx=\frac{\ln\left(\tan\left(\frac{\Pi}{8}+\frac{3\,x}{2}+\frac{\pi}{4}\right)\right)}{6}+\frac{\tan\left(\frac{\Pi}{4}+3\,x\right)}{6\,\cos\left(\frac{\Pi}{4}+3\,x\right)}$$

```
[In] int(1/cos(Pi/4 + 3*x)^3,x)
[Out] log(tan(Pi/8 + (3*x)/2 + pi/4))/6 + tan(Pi/4 + 3*x)/(6*cos(Pi/4 + 3*x))
```

3.341 $\int \tan^6(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 22

$$\int \tan^6(x)\,dx = -x + \tan(x) - \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5}$$

```
[Out] -x+tan(x)-1/3*tan(x)^3+1/5*tan(x)^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$\int \tan^6(x)\,dx = -x + \frac{\tan^5(x)}{5} - \frac{\tan^3(x)}{3} + \tan(x)$$

```
[In] Int[Tan[x]^6,x]
```

```
[Out] -x + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan^5(x)}{5} - \int \tan^4(x)\,dx \\
&= -\frac{1}{3}\tan^3(x) + \frac{\tan^5(x)}{5} + \int \tan^2(x)\,dx \\
&= \tan(x) - \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5} - \int 1\,dx \\
&= -x + \tan(x) - \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \tan^6(x)\,dx = -\arctan(\tan(x)) + \tan(x) - \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5}$$

```
[In] Integrate[Tan[x]^6,x]
[Out] -ArcTan[Tan[x]] + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
norman	$-x+\tan\left(x\right)-\frac{\left(\tan^{3}\left(x\right)\right)}{3}+\frac{\left(\tan^{5}\left(x\right)\right)}{5}$	19
parallelrisch	$-x+\tan\left(x\right)-\frac{\left(\tan^{3}\left(x\right)\right)}{3}+\frac{\left(\tan^{5}\left(x\right)\right)}{5}$	19
derivativedivides	$\frac{\left(\tan^{5}\left(x\right)\right)}{5}-\frac{\left(\tan^{3}\left(x\right)\right)}{3}+\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)$	21
default	$\frac{\left(\tan^{5}\left(x\right)\right)}{5}-\frac{\left(\tan^{3}\left(x\right)\right)}{3}+\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)$	21
risch	$-x+\frac{2i\left(45\,\mathrm{e}^{8ix}+90\,\mathrm{e}^{6ix}+140\,\mathrm{e}^{4ix}+70\,\mathrm{e}^{2ix}+23\right)}{15\left(\mathrm{e}^{2ix}+1\right)^{5}}$	47

```
[In] int(tan(x)^6,x,method=_RETURNVERBOSE)
[Out] -x+tan(x)-1/3*tan(x)^3+1/5*tan(x)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\,dx = \frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

```
[In] integrate(tan(x)^6,x, algorithm="fricas")
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \tan^6(x)\,dx = -x + \frac{\sin^5(x)}{5\cos^5(x)} - \frac{\sin^3(x)}{3\cos^3(x)} + \frac{\sin(x)}{\cos(x)}$$

```
[In] integrate(tan(x)**6,x)
[Out] -x + sin(x)**5/(5*cos(x)**5) - sin(x)**3/(3*cos(x)**3) + sin(x)/cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\,dx = \frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

```
[In] integrate(tan(x)^6,x, algorithm="maxima")
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\,dx = \frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

```
[In] integrate(tan(x)^6,x, algorithm="giac")
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\,dx = \frac{\tan(x)^5}{5} - \frac{\tan(x)^3}{3} + \tan(x) - x$$

```
[In] int(tan(x)^6,x)
[Out] tan(x) - x - tan(x)^3/3 + tan(x)^5/5
```

3.342 $\int \cot^5(x) \, dx$

Optimal result

Integrand size = 4, antiderivative size = 20

$$\int \cot^5(x) \, dx = \frac{\cot^2(x)}{2} - \frac{\cot^4(x)}{4} + \log(\sin(x))$$

```
[Out] 1/2*cot(x)^2-1/4*cot(x)^4+ln(sin(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 3556}

$$\int \cot^5(x) \, dx = -\frac{1}{4} \cot^4(x) + \frac{\cot^2(x)}{2} + \log(\sin(x))$$

```
[In] Int[Cot[x]^5,x]
```

```
[Out] Cot[x]^2/2 - Cot[x]^4/4 + Log[Sin[x]]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{4}\cot^4(x) - \int \cot^3(x)\,dx \\
&= \frac{\cot^2(x)}{2} - \frac{\cot^4(x)}{4} + \int \cot(x)\,dx \\
&= \frac{\cot^2(x)}{2} - \frac{\cot^4(x)}{4} + \log(\sin(x))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \cot^5(x)\,dx = \frac{\cot^2(x)}{2} - \frac{\cot^4(x)}{4} + \log(\cos(x)) + \log(\tan(x))$$

```
[In] Integrate[Cot[x]^5,x]
[Out] Cot[x]^2/2 - Cot[x]^4/4 + Log[Cos[x]] + Log[Tan[x]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$-\frac{1}{4\tan(x)^4}+\ln(\tan(x))+\frac{1}{2\tan(x)^2}-\frac{\ln(1+\tan^2(x))}{2}$	26
default	$-\frac{1}{4\tan(x)^4}+\ln(\tan(x))+\frac{1}{2\tan(x)^2}-\frac{\ln(1+\tan^2(x))}{2}$	26
norman	$\frac{-\frac{1}{4}+\frac{(\tan^2(x))}{2}}{\tan(x)^4}-\frac{\ln(1+\tan^2(x))}{2}+\ln(\tan(x))$	27
parallelrisch	$\frac{4\ln(\tan(x))(\tan^4(x))-2\ln(1+\tan^2(x))(\tan^4(x))-1+2(\tan^2(x))}{4\tan(x)^4}$	37
risch	$-ix-\frac{4(e^{6ix}-e^{4ix}+e^{2ix})}{(e^{2ix}-1)^4}+\ln(e^{2ix}-1)$	43

```
[In] int(1/tan(x)^5,x,method=_RETURNVERBOSE)
[Out] -1/4/tan(x)^4+ln(tan(x))+1/2/tan(x)^2-1/2*ln(1+tan(x)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(16) = 32$.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \cot^5(x)\,dx = \frac{2\,\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)\tan(x)^4 + 3\,\tan(x)^4 + 2\,\tan(x)^2 - 1}{4\,\tan(x)^4}$$

```
[In] integrate(1/tan(x)^5,x, algorithm="fricas")
[Out] 1/4*(2*log(tan(x)^2/(tan(x)^2 + 1))*tan(x)^4 + 3*tan(x)^4 + 2*tan(x)^2 - 1)
/tan(x)^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \cot^5(x)\,dx = \frac{4\sin^2(x) - 1}{4\sin^4(x)} + \log(\sin(x))$$

```
[In] integrate(1/tan(x)**5,x)
[Out] (4*sin(x)**2 - 1)/(4*sin(x)**4) + log(sin(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \cot^5(x)\,dx = \frac{4\,\sin(x)^2 - 1}{4\,\sin(x)^4} + \frac{1}{2}\,\log\left(\sin(x)^2\right)$$

```
[In] integrate(1/tan(x)^5,x, algorithm="maxima")
[Out] 1/4*(4*sin(x)^2 - 1)/sin(x)^4 + 1/2*log(sin(x)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \cot^5(x)\,dx = -\frac{3\,\tan\left(x\right)^4 - 2\,\tan\left(x\right)^2 + 1}{4\,\tan\left(x\right)^4} - \frac{1}{2}\,\log\left(\tan\left(x\right)^2 + 1\right) + \frac{1}{2}\,\log\left(\tan\left(x\right)^2\right)$$

```
[In] integrate(1/tan(x)^5,x, algorithm="giac")
[Out] -1/4*(3*tan(x)^4 - 2*tan(x)^2 + 1)/tan(x)^4 - 1/2*log(tan(x)^2 + 1) + 1/2*l
og(tan(x)^2)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \cot^5(x)\,dx = \ln\left(\tan(x)\right) - \frac{\ln\left(\tan(x)^2 + 1\right)}{2} + \frac{\frac{\tan(x)^2}{2} - \frac{1}{4}}{\tan\left(x\right)^4}$$

```
[In] int(1/tan(x)^5,x)
[Out] log(tan(x)) - log(tan(x)^2 + 1)/2 + (tan(x)^2/2 - 1/4)/tan(x)^4
```

3.343 $\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = x+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)$$

```
[Out] x+3*cot(1/4*Pi+1/3*x)-cot(1/4*Pi+1/3*x)^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3554, 8}

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = x-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

```
[In] Int[Cot[Pi/4 + x/3]^4,x]
```

```
[Out] x + 3*Cot[Pi/4 + x/3] - Cot[Pi/4 + x/3]^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)-\int \tan^2\left(\frac{\pi}{4}-\frac{x}{3}\right)\,dx\\
&= 3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)+\int 1\,dx\\
&= x+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$
\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = -\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)
$$

```
[In] Integrate[Cot[Pi/4 + x/3]^4,x]
[Out] -(Cot[Pi/4 + x/3]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[Pi/4 + x/3]^2])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
parallelrisch	$x+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\left(\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)$	25
derivativedivides	$-\left(\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\frac{3\pi}{2}+3\operatorname{arccot}\left(\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)$	38
default	$-\left(\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\frac{3\pi}{2}+3\operatorname{arccot}\left(\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)$	38
norman	$\frac{-1+x\left(\tan^3\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)+3\left(\tan^2\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)}{\tan\left(\frac{\pi}{4}+\frac{x}{3}\right)^3}$	38
risch	$x+\frac{4i\left(-3\,\mathrm{e}^{\frac{4ix}{3}}-3i\mathrm{e}^{\frac{2ix}{3}}+2\right)}{\left(\mathrm{e}^{\frac{i(3\pi+4x)}{6}}-1\right)^3}$	38

```
[In] int(cot(1/4*Pi+1/3*x)^4,x,method=_RETURNVERBOSE)
[Out] x+3*cot(1/4*Pi+1/3*x)-cot(1/4*Pi+1/3*x)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(24) = 48$.

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.19

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = \frac{4\,\cos\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)^2+\left(x\cos\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)-x\right)\sin\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)+2\,\cos\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)-2}{\left(\cos\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)-1\right)\sin\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)}$$

```
[In] integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="fricas")
[Out] (4*cos(1/2*pi + 2/3*x)^2 + (x*cos(1/2*pi + 2/3*x) - x)*sin(1/2*pi + 2/3*x)
+ 2*cos(1/2*pi + 2/3*x) - 2)/((cos(1/2*pi + 2/3*x) - 1)*sin(1/2*pi + 2/3*x)
)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = x-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

```
[In] integrate(cot(1/4*pi+1/3*x)**4,x)
[Out] x - cot(x/3 + pi/4)**3 + 3*cot(x/3 + pi/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = \frac{3}{4}\,\pi+x+\frac{3\,\tan\left(\frac{1}{4}\,\pi+\frac{1}{3}\,x\right)^2-1}{\tan\left(\frac{1}{4}\,\pi+\frac{1}{3}\,x\right)^3}$$

```
[In] integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="maxima")
[Out] 3/4*pi + x + (3*tan(1/4*pi + 1/3*x)^2 - 1)/tan(1/4*pi + 1/3*x)^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(24) = 48$.

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = \frac{3}{4}\,\pi+\frac{1}{8}\,\tan\left(\frac{1}{8}\,\pi+\frac{1}{6}\,x\right)^3+x+\frac{15\,\tan\left(\frac{1}{8}\,\pi+\frac{1}{6}\,x\right)^2-1}{8\,\tan\left(\frac{1}{8}\,\pi+\frac{1}{6}\,x\right)^3}-\frac{15}{8}\,\tan\left(\frac{1}{8}\,\pi+\frac{1}{6}\,x\right)$$

```
[In] integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="giac")
[Out] 3/4*pi + 1/8*tan(1/8*pi + 1/6*x)^3 + x + 1/8*(15*tan(1/8*pi + 1/6*x)^2 - 1)
/tan(1/8*pi + 1/6*x)^3 - 15/8*tan(1/8*pi + 1/6*x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = -\cot\left(\frac{\Pi}{4}+\frac{x}{3}\right)^3+3\cot\left(\frac{\Pi}{4}+\frac{x}{3}\right)+x$$

```
[In] int(cot(Pi/4 + x/3)^4,x)
[Out] x + 3*cot(Pi/4 + x/3) - cot(Pi/4 + x/3)^3
```

3.344 $\int \cos^6(x)\sin^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 56

$$\int \cos^6(x)\sin^4(x)\,dx = \frac{3x}{256} + \frac{3}{256}\cos(x)\sin(x) + \frac{1}{128}\cos^3(x)\sin(x) + \frac{1}{160}\cos^5(x)\sin(x) - \frac{3}{80}\cos^7(x)\sin(x) - \frac{1}{10}\cos^7(x)\sin^3(x)$$

```
[Out] 3/256*x+3/256*cos(x)*sin(x)+1/128*cos(x)^3*sin(x)+1/160*cos(x)^5*sin(x)-3/8
0*cos(x)^7*sin(x)-1/10*cos(x)^7*sin(x)^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2648, 2715, 8}

$$\int \cos^6(x)\sin^4(x)\,dx = \frac{3x}{256} - \frac{1}{10}\sin^3(x)\cos^7(x) - \frac{3}{80}\sin(x)\cos^7(x) + \frac{1}{160}\sin(x)\cos^5(x) + \frac{1}{128}\sin(x)\cos^3(x) + \frac{3}{256}\sin(x)\cos(x)$$

```
[In] Int[Cos[x]^6*Sin[x]^4,x]
```

```
[Out] (3*x)/256 + (3*Cos[x]*Sin[x])/256 + (Cos[x]^3*Sin[x])/128 + (Cos[x]^5*Sin[x
])/160 - (3*Cos[x]^7*Sin[x])/80 - (Cos[x]^7*Sin[x]^3)/10
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{10}\cos^7(x)\sin^3(x) + \frac{3}{10}\int \cos^6(x)\sin^2(x)\,dx \\
&= -\frac{3}{80}\cos^7(x)\sin(x) - \frac{1}{10}\cos^7(x)\sin^3(x) + \frac{3}{80}\int \cos^6(x)\,dx \\
&= \frac{1}{160}\cos^5(x)\sin(x) - \frac{3}{80}\cos^7(x)\sin(x) - \frac{1}{10}\cos^7(x)\sin^3(x) + \frac{1}{32}\int \cos^4(x)\,dx \\
&= \frac{1}{128}\cos^3(x)\sin(x) + \frac{1}{160}\cos^5(x)\sin(x) - \frac{3}{80}\cos^7(x)\sin(x) \\
&\quad - \frac{1}{10}\cos^7(x)\sin^3(x) + \frac{3}{128}\int \cos^2(x)\,dx \\
&= \frac{3}{256}\cos(x)\sin(x) + \frac{1}{128}\cos^3(x)\sin(x) + \frac{1}{160}\cos^5(x)\sin(x) \\
&\quad - \frac{3}{80}\cos^7(x)\sin(x) - \frac{1}{10}\cos^7(x)\sin^3(x) + \frac{3\int 1\,dx}{256} \\
&= \frac{3x}{256} + \frac{3}{256}\cos(x)\sin(x) + \frac{1}{128}\cos^3(x)\sin(x) \\
&\quad + \frac{1}{160}\cos^5(x)\sin(x) - \frac{3}{80}\cos^7(x)\sin(x) - \frac{1}{10}\cos^7(x)\sin^3(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \cos^6(x)\sin^4(x)\,dx = \frac{3x}{256}+\frac{1}{512}\sin(2x)-\frac{1}{256}\sin(4x)-\frac{\sin(6x)}{1024}+\frac{\sin(8x)}{2048}+\frac{\sin(10x)}{5120}$$

```
[In] Integrate[Cos[x]^6*Sin[x]^4,x]
```

```
[Out] (3*x)/256 + Sin[2*x]/512 - Sin[4*x]/256 - Sin[6*x]/1024 + Sin[8*x]/2048 + S
in[10*x]/5120
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{3x}{256}+\frac{\sin(10x)}{5120}+\frac{\sin(8x)}{2048}-\frac{\sin(6x)}{1024}-\frac{\sin(4x)}{256}+\frac{\sin(2x)}{512}$	35
parallelrisch	$\frac{3x}{256}+\frac{\sin(10x)}{5120}+\frac{\sin(8x)}{2048}-\frac{\sin(6x)}{1024}-\frac{\sin(4x)}{256}+\frac{\sin(2x)}{512}$	35
default	$-\frac{(\cos^7(x))(\sin^3(x))}{10}-\frac{3(\cos^7(x))\sin(x)}{80}+\frac{\left(\cos^5(x)+\frac{5(\cos^3(x))}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{160}+\frac{3x}{256}$	42

```
[In] int(cos(x)^6*sin(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 3/256*x+1/5120*sin(10*x)+1/2048*sin(8*x)-1/1024*sin(6*x)-1/256*sin(4*x)+1/5
12*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int \cos^6(x)\sin^4(x)\,dx = \frac{1}{1280}\left(128\cos(x)^9-176\cos(x)^7+8\cos(x)^5+10\cos(x)^3+15\cos(x)\right)\sin(x) + \frac{3}{256}x$$

```
[In] integrate(cos(x)^6*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/1280*(128*cos(x)^9 - 176*cos(x)^7 + 8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x))
*sin(x) + 3/256*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \cos^6(x)\sin^4(x)\,dx = \frac{3x}{256} + \frac{\sin(x)\cos^9(x)}{10} - \frac{11\sin(x)\cos^7(x)}{80} + \frac{\sin(x)\cos^5(x)}{160} + \frac{\sin(x)\cos^3(x)}{128} + \frac{3\sin(x)\cos(x)}{256}$$

```
[In] integrate(cos(x)**6*sin(x)**4,x)
```

```
[Out] 3*x/256 + sin(x)*cos(x)**9/10 - 11*sin(x)*cos(x)**7/80 + sin(x)*cos(x)**5/1
60 + sin(x)*cos(x)**3/128 + 3*sin(x)*cos(x)/256
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.43

$$\int \cos^6(x)\sin^4(x)\,dx = \frac{1}{320}\sin(2x)^5 + \frac{3}{256}x + \frac{1}{2048}\sin(8x) - \frac{1}{256}\sin(4x)$$

```
[In] integrate(cos(x)^6*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 1/320*sin(2*x)^5 + 3/256*x + 1/2048*sin(8*x) - 1/256*sin(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int \cos^6(x)\sin^4(x)\,dx = \frac{3}{256}x + \frac{1}{5120}\sin(10x) + \frac{1}{2048}\sin(8x) - \frac{1}{1024}\sin(6x) - \frac{1}{256}\sin(4x) + \frac{1}{512}\sin(2x)$$

```
[In] integrate(cos(x)^6*sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/256*x + 1/5120*sin(10*x) + 1/2048*sin(8*x) - 1/1024*sin(6*x) - 1/256*sin(
4*x) + 1/512*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

$$\int \cos^6(x)\sin^4(x)\,dx = \left(\frac{\cos(x)^5}{10}+\frac{\cos(x)^3}{16}+\frac{\cos(x)}{32}\right)\sin(x)^5+\frac{3x}{256}-\frac{\sin(2x)}{128}+\frac{\sin(4x)}{1024}$$

```
[In] int(cos(x)^6*sin(x)^4,x)
[Out] (3*x)/256 - sin(2*x)/128 + sin(4*x)/1024 + sin(x)^5*(cos(x)/32 + cos(x)^3/1
6 + cos(x)^5/10)
```

3.345 $\int \cos^6(x)\sin^7(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \cos^6(x)\sin^7(x)\,dx = -\frac{1}{7}\cos^7(x) + \frac{\cos^9(x)}{3} - \frac{3\cos^{11}(x)}{11} + \frac{\cos^{13}(x)}{13}$$

```
[Out] -1/7*cos(x)^7+1/3*cos(x)^9-3/11*cos(x)^11+1/13*cos(x)^13
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2645, 276}

$$\int \cos^6(x)\sin^7(x)\,dx = \frac{\cos^{13}(x)}{13} - \frac{3\cos^{11}(x)}{11} + \frac{\cos^9(x)}{3} - \frac{\cos^7(x)}{7}$$

```
[In] Int[Cos[x]^6*Sin[x]^7,x]
```

```
[Out] -1/7*Cos[x]^7 + Cos[x]^9/3 - (3*Cos[x]^11)/11 + Cos[x]^13/13
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int x^6\left(1-x^2\right)^3\,dx,x,\cos(x)\right)\\&= -\text{Subst}\left(\int \left(x^6-3x^8+3x^{10}-x^{12}\right)\,dx,x,\cos(x)\right)\\&= -\frac{1}{7}\cos^7(x)+\frac{\cos^9(x)}{3}-\frac{3\cos^{11}(x)}{11}+\frac{\cos^{13}(x)}{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.67

$$\int \cos^6(x)\sin^7(x)\,dx = -\frac{5\cos(x)}{1024}-\frac{5\cos(3x)}{4096}+\frac{3\cos(5x)}{4096}+\frac{3\cos(7x)}{14336}-\frac{\cos(9x)}{6144}-\frac{\cos(11x)}{45056}+\frac{\cos(13x)}{53248}$$

```
[In] Integrate[Cos[x]^6*Sin[x]^7,x]
[Out] (-5*Cos[x])/1024 - (5*Cos[3*x])/4096 + (3*Cos[5*x])/4096 + (3*Cos[7*x])/143
36 - Cos[9*x]/6144 - Cos[11*x]/45056 + Cos[13*x]/53248
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{(\cos^7(x))}{7}+\frac{(\cos^9(x))}{3}-\frac{3(\cos^{11}(x))}{11}+\frac{(\cos^{13}(x))}{13}$	26
default	$-\frac{(\cos^7(x))}{7}+\frac{(\cos^9(x))}{3}-\frac{3(\cos^{11}(x))}{11}+\frac{(\cos^{13}(x))}{13}$	26
risch	$-\frac{5\cos(x)}{1024}+\frac{\cos(13x)}{53248}-\frac{\cos(11x)}{45056}-\frac{\cos(9x)}{6144}+\frac{3\cos(7x)}{14336}+\frac{3\cos(5x)}{4096}-\frac{5\cos(3x)}{4096}$	42
parallelrisch	$\frac{320}{3003}-\frac{5\cos(x)}{1024}+\frac{\cos(13x)}{53248}-\frac{\cos(11x)}{45056}-\frac{\cos(9x)}{6144}+\frac{3\cos(7x)}{14336}+\frac{3\cos(5x)}{4096}-\frac{5\cos(3x)}{4096}$	43

```
[In] int(cos(x)^6*sin(x)^7,x,method=_RETURNVERBOSE)
[Out] -1/7*cos(x)^7+1/3*cos(x)^9-3/11*cos(x)^11+1/13*cos(x)^13
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^6(x)\sin^7(x)\,dx = \frac{1}{13}\cos(x)^{13} - \frac{3}{11}\cos(x)^{11} + \frac{1}{3}\cos(x)^9 - \frac{1}{7}\cos(x)^7$$

```
[In] integrate(cos(x)^6*sin(x)^7,x, algorithm="fricas")
[Out] 1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \cos^6(x)\sin^7(x)\,dx = \frac{\cos^{13}(x)}{13} - \frac{3\cos^{11}(x)}{11} + \frac{\cos^9(x)}{3} - \frac{\cos^7(x)}{7}$$

```
[In] integrate(cos(x)**6*sin(x)**7,x)
[Out] cos(x)**13/13 - 3*cos(x)**11/11 + cos(x)**9/3 - cos(x)**7/7
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^6(x)\sin^7(x)\,dx = \frac{1}{13}\cos(x)^{13} - \frac{3}{11}\cos(x)^{11} + \frac{1}{3}\cos(x)^9 - \frac{1}{7}\cos(x)^7$$

```
[In] integrate(cos(x)^6*sin(x)^7,x, algorithm="maxima")
[Out] 1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^6(x)\sin^7(x)\,dx = \frac{1}{13}\cos(x)^{13} - \frac{3}{11}\cos(x)^{11} + \frac{1}{3}\cos(x)^9 - \frac{1}{7}\cos(x)^7$$

```
[In] integrate(cos(x)^6*sin(x)^7,x, algorithm="giac")
[Out] 1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^6(x)\sin^7(x)\,dx = \frac{\cos(x)^{13}}{13} - \frac{3\cos(x)^{11}}{11} + \frac{\cos(x)^9}{3} - \frac{\cos(x)^7}{7}$$

```
[In] int(cos(x)^6*sin(x)^7,x)
[Out] cos(x)^9/3 - cos(x)^7/7 - (3*cos(x)^11)/11 + cos(x)^13/13
```

3.346 $\int \sin^{10}(x)\tan(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 46

$$\int \sin^{10}(x)\tan(x)\,dx = \frac{5\cos^2(x)}{2} - \frac{5\cos^4(x)}{2} + \frac{5\cos^6(x)}{3} - \frac{5\cos^8(x)}{8} + \frac{\cos^{10}(x)}{10} - \log(\cos(x))$$

```
[Out] 5/2*cos(x)^2-5/2*cos(x)^4+5/3*cos(x)^6-5/8*cos(x)^8+1/10*cos(x)^10-ln(cos(x
))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2670, 272, 45}

$$\int \sin^{10}(x)\tan(x)\,dx = \frac{\cos^{10}(x)}{10} - \frac{5\cos^8(x)}{8} + \frac{5\cos^6(x)}{3} - \frac{5\cos^4(x)}{2} + \frac{5\cos^2(x)}{2} - \log(\cos(x))$$

```
[In] Int[Sin[x]^10*Tan[x],x]
```

```
[Out] (5*Cos[x]^2)/2 - (5*Cos[x]^4)/2 + (5*Cos[x]^6)/3 - (5*Cos[x]^8)/8 + Cos[x]^
10/10 - Log[Cos[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(1-x^2)^5}{x}\,dx, x, \cos(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^5}{x}\,dx, x, \cos^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-5+\frac{1}{x}+10x-10x^2+5x^3-x^4\right)\,dx, x, \cos^2(x)\right)\right)\\
&= \frac{5\cos^2(x)}{2}-\frac{5\cos^4(x)}{2}+\frac{5\cos^6(x)}{3}-\frac{5\cos^8(x)}{8}+\frac{\cos^{10}(x)}{10}-\log(\cos(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sin^{10}(x)\tan(x)\,dx = \frac{5\cos^2(x)}{2}-\frac{5\cos^4(x)}{2}+\frac{5\cos^6(x)}{3}-\frac{5\cos^8(x)}{8}+\frac{\cos^{10}(x)}{10}-\log(\cos(x))$$

```
[In] Integrate[Sin[x]^10*Tan[x],x]
[Out] (5*Cos[x]^2)/2 - (5*Cos[x]^4)/2 + (5*Cos[x]^6)/3 - (5*Cos[x]^8)/8 + Cos[x]^
10/10 - Log[Cos[x]]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result
default	$-\frac{\left(\sin^{10}(x)\right)}{10}-\frac{\left(\sin^{8}(x)\right)}{8}-\frac{\left(\sin^{6}(x)\right)}{6}-\frac{\left(\sin^{4}(x)\right)}{4}-\frac{\left(\sin^{2}(x)\right)}{2}-\ln\left(\cos\left(x\right)\right)$
risch	$ix+\frac{281\,\mathrm{e}^{2ix}}{1024}+\frac{281\,\mathrm{e}^{-2ix}}{1024}-\ln\left(\mathrm{e}^{2ix}+1\right)+\frac{\cos(10x)}{5120}-\frac{3\cos(8x)}{1024}+\frac{67\cos(6x)}{3072}-\frac{29\cos(4x)}{256}$
parallelrisch	$-\frac{469}{46080}+\ln\left(\frac{2}{\cos(x)+1}\right)-\ln\left(-\cot\left(x\right)+1+\csc\left(x\right)\right)-\ln\left(-\cot\left(x\right)+\csc\left(x\right)-1\right)+\frac{\cos(10x)}{5120}$

[In] int(sin(x)^11/cos(x),x,method=_RETURNVERBOSE)

[Out] -1/10*sin(x)^10-1/8*sin(x)^8-1/6*sin(x)^6-1/4*sin(x)^4-1/2*sin(x)^2-ln(cos(
x))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \sin^{10}(x)\tan(x)\,dx=\frac{1}{10}\cos\left(x\right)^{10}-\frac{5}{8}\cos\left(x\right)^{8}+\frac{5}{3}\cos\left(x\right)^{6}-\frac{5}{2}\cos\left(x\right)^{4}+\frac{5}{2}\cos\left(x\right)^{2}-\log\left(-\cos\left(x\right)\right)$$

[In] integrate(sin(x)^11/cos(x),x, algorithm="fricas")

[Out] 1/10*cos(x)^10 - 5/8*cos(x)^8 + 5/3*cos(x)^6 - 5/2*cos(x)^4 + 5/2*cos(x)^2
- log(-cos(x))

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \sin^{10}(x)\tan(x)\,dx=-\log\left(\cos\left(x\right)\right)+\frac{\cos^{10}\left(x\right)}{10}-\frac{5\cos^{8}\left(x\right)}{8}+\frac{5\cos^{6}\left(x\right)}{3}-\frac{5\cos^{4}\left(x\right)}{2}+\frac{5\cos^{2}\left(x\right)}{2}$$

[In] integrate(sin(x)**11/cos(x),x)

[Out] -log(cos(x)) + cos(x)**10/10 - 5*cos(x)**8/8 + 5*cos(x)**6/3 - 5*cos(x)**4/
2 + 5*cos(x)**2/2

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \sin^{10}(x)\tan(x)\,dx = -\frac{1}{10}\sin(x)^{10} - \frac{1}{8}\sin(x)^{8} - \frac{1}{6}\sin(x)^{6} - \frac{1}{4}\sin(x)^{4} - \frac{1}{2}\sin(x)^{2} - \frac{1}{2}\log\left(\sin(x)^{2} - 1\right)$$

```
[In] integrate(sin(x)^11/cos(x),x, algorithm="maxima")
[Out] -1/10*sin(x)^10 - 1/8*sin(x)^8 - 1/6*sin(x)^6 - 1/4*sin(x)^4 - 1/2*sin(x)^2
 - 1/2*log(sin(x)^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \sin^{10}(x)\tan(x)\,dx = -\frac{1}{10}\sin(x)^{10} - \frac{1}{8}\sin(x)^{8} - \frac{1}{6}\sin(x)^{6} - \frac{1}{4}\sin(x)^{4} - \frac{1}{2}\sin(x)^{2} - \frac{1}{2}\log\left(-\sin(x)^{2} + 1\right)$$

```
[In] integrate(sin(x)^11/cos(x),x, algorithm="giac")
[Out] -1/10*sin(x)^10 - 1/8*sin(x)^8 - 1/6*sin(x)^6 - 1/4*sin(x)^4 - 1/2*sin(x)^2
 - 1/2*log(-sin(x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin^{10}(x)\tan(x)\,dx = -\frac{\sin(x)^{10}}{10} - \frac{\sin(x)^{8}}{8} - \frac{\sin(x)^{6}}{6} - \frac{\sin(x)^{4}}{4} - \frac{\sin(x)^{2}}{2} - \ln(\cos(x))$$

```
[In] int(sin(x)^11/cos(x),x)
[Out] - log(cos(x)) - sin(x)^2/2 - sin(x)^4/4 - sin(x)^6/6 - sin(x)^8/8 - sin(x)^
10/10
```

3.347 $\int \csc^6(x)\sec^6(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 41

$$\int \csc^6(x)\sec^6(x)\,dx = -10\cot(x)-\frac{5\cot^3(x)}{3}-\frac{\cot^5(x)}{5}+10\tan(x)+\frac{5\tan^3(x)}{3}+\frac{\tan^5(x)}{5}$$

```
[Out] -10*cot(x)-5/3*cot(x)^3-1/5*cot(x)^5+10*tan(x)+5/3*tan(x)^3+1/5*tan(x)^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2700, 276}

$$\int \csc^6(x)\sec^6(x)\,dx = \frac{\tan^5(x)}{5}+\frac{5\tan^3(x)}{3}+10\tan(x)-\frac{1}{5}\cot^5(x)-\frac{5\cot^3(x)}{3}-10\cot(x)$$

```
[In] Int[Csc[x]^6*Sec[x]^6,x]
```

```
[Out] -10*Cot[x] - (5*Cot[x]^3)/3 - Cot[x]^5/5 + 10*Tan[x] + (5*Tan[x]^3)/3 + Tan
[x]^5/5
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{(1+x^2)^5}{x^6}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(10+\frac{1}{x^6}+\frac{5}{x^4}+\frac{10}{x^2}+5x^2+x^4\right)\,dx, x, \tan(x)\right)\\ &= -10\cot(x)-\frac{5\cot^3(x)}{3}-\frac{\cot^5(x)}{5}+10\tan(x)+\frac{5\tan^3(x)}{3}+\frac{\tan^5(x)}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.29

$$\int \csc^6(x)\sec^6(x)\,dx = -\frac{128\cot(x)}{15}-\frac{19}{15}\cot(x)\csc^2(x)-\frac{1}{5}\cot(x)\csc^4(x)+\frac{128\tan(x)}{15}+\frac{19}{15}\sec^2(x)\tan(x)+\frac{1}{5}\sec^4(x)\tan(x)$$

```
[In] Integrate[Csc[x]^6*Sec[x]^6,x]
[Out] (-128*Cot[x])/15 - (19*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5 + (128*Tan
[x])/15 + (19*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$-\frac{(\sec^5(x))(\csc^5(x))(\cos(10x)-5\cos(6x)+10\cos(2x))}{30}$	28
risch	$-\frac{512i(10\,\mathrm{e}^{8ix}-5\,\mathrm{e}^{4ix}+1)}{15(\mathrm{e}^{2ix}-1)^5(\mathrm{e}^{2ix}+1)^5}$	38
default	$\frac{1}{5\sin(x)^5\cos(x)^5}-\frac{2}{5\sin(x)^5\cos(x)^3}+\frac{16}{15\sin(x)^3\cos(x)^3}-\frac{32}{15\sin(x)^3\cos(x)}+\frac{128}{15\cos(x)\sin(x)}-\frac{256\cot(x)}{15}$	56

```
[In] int(1/cos(x)^6/sin(x)^6,x,method=_RETURNVERBOSE)
[Out] -1/30*sec(x)^5*csc(x)^5*(cos(10*x)-5*cos(6*x)+10*cos(2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \csc^6(x)\sec^6(x)\,dx = -\frac{256\,\cos\left(x\right)^{10}-640\,\cos\left(x\right)^{8}+480\,\cos\left(x\right)^{6}-80\,\cos\left(x\right)^{4}-10\,\cos\left(x\right)^{2}-3}{15\,\left(\cos\left(x\right)^{9}-2\,\cos\left(x\right)^{7}+\cos\left(x\right)^{5}\right)\sin\left(x\right)}$$

```
[In] integrate(1/cos(x)^6/sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/15*(256*cos(x)^10 - 640*cos(x)^8 + 480*cos(x)^6 - 80*cos(x)^4 - 10*cos(x
)^2 - 3)/((cos(x)^9 - 2*cos(x)^7 + cos(x)^5)*sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \csc^6(x)\sec^6(x)\,dx = -\frac{256\cos\left(2x\right)}{15\sin\left(2x\right)}-\frac{128\cos\left(2x\right)}{15\sin^3\left(2x\right)}-\frac{32\cos\left(2x\right)}{5\sin^5\left(2x\right)}$$

```
[In] integrate(1/cos(x)**6/sin(x)**6,x)
```

```
[Out] -256*cos(2*x)/(15*sin(2*x)) - 128*cos(2*x)/(15*sin(2*x)**3) - 32*cos(2*x)/(
5*sin(2*x)**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \csc^6(x)\sec^6(x)\,dx = \frac{1}{5}\,\tan\left(x\right)^{5}+\frac{5}{3}\,\tan\left(x\right)^{3}-\frac{150\,\tan\left(x\right)^{4}+25\,\tan\left(x\right)^{2}+3}{15\,\tan\left(x\right)^{5}}+10\,\tan\left(x\right)$$

```
[In] integrate(1/cos(x)^6/sin(x)^6,x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5 + 5/3*tan(x)^3 - 1/15*(150*tan(x)^4 + 25*tan(x)^2 + 3)/tan(x)^
5 + 10*tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \csc^6(x)\sec^6(x)\,dx = -\frac{32\left(15\,\tan\left(2\,x\right)^4+10\,\tan\left(2\,x\right)^2+3\right)}{15\,\tan\left(2\,x\right)^5}$$

```
[In] integrate(1/cos(x)^6/sin(x)^6,x, algorithm="giac")
[Out] -32/15*(15*tan(2*x)^4 + 10*tan(2*x)^2 + 3)/tan(2*x)^5
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \csc^6(x)\sec^6(x)\,dx = -\frac{32\left(\frac{\cos(2\,x)}{3}-\frac{\cos(6\,x)}{6}+\frac{\cos(10\,x)}{30}\right)}{\sin\left(2\,x\right)^5}$$

```
[In] int(1/(cos(x)^6*sin(x)^6),x)
[Out] -(32*(cos(2*x)/3 - cos(6*x)/6 + cos(10*x)/30))/sin(2*x)^5
```

3.348 $\int \cos^2(x) \sin^2(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \cos^2(x) \sin^2(x) \, dx = \frac{x}{8} + \frac{1}{8} \cos(x) \sin(x) - \frac{1}{4} \cos^3(x) \sin(x)$$

```
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2648, 2715, 8}

$$\int \cos^2(x) \sin^2(x) \, dx = \frac{x}{8} - \frac{1}{4} \sin(x) \cos^3(x) + \frac{1}{8} \sin(x) \cos(x)$$

```
[In] Int[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{4}\cos^3(x)\sin(x) + \frac{1}{4}\int \cos^2(x)\,dx \\ &= \frac{1}{8}\cos(x)\sin(x) - \frac{1}{4}\cos^3(x)\sin(x) + \frac{\int 1\,dx}{8} \\ &= \frac{x}{8} + \frac{1}{8}\cos(x)\sin(x) - \frac{1}{4}\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{x}{8} - \frac{1}{32}\sin(4x)$$

```
[In] Integrate[Cos[x]^2*Sin[x]^2,x]
[Out] x/8 - Sin[4*x]/32
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

method	result	size
risch	$\frac{x}{8}-\frac{\sin(4x)}{32}$	11
parallelrisch	$\frac{x}{8}-\frac{\sin(4x)}{32}$	11
default	$\frac{x}{8}+\frac{\cos(x)\sin(x)}{8}-\frac{(\cos^3(x))\sin(x)}{4}$	19
norman	$\frac{\frac{x}{8}+\frac{7\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}-\frac{7\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}-\frac{\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	82

```
[In] int(cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/8*x-1/32*sin(4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \cos^2(x)\sin^2(x)\,dx = -\frac{1}{8}\left(2\,\cos\left(x\right)^3 - \cos\left(x\right)\right)\sin\left(x\right) + \frac{1}{8}\,x$$

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{x}{8} - \frac{\sin\left(2x\right)\cos\left(2x\right)}{16}$$

```
[In] integrate(cos(x)**2*sin(x)**2,x)
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.42

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{1}{8}\,x - \frac{1}{32}\,\sin\left(4\,x\right)$$

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
[Out] 1/8*x - 1/32*sin(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.42

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{1}{8}\,x - \frac{1}{32}\,\sin\left(4\,x\right)$$

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
[Out] 1/8*x - 1/32*sin(4*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{\cos(x)\,\sin(x)^3}{4} - \frac{\cos(x)\,\sin(x)}{8} + \frac{x}{8}$$

```
[In] int(cos(x)^2*sin(x)^2,x)
[Out] x/8 - (cos(x)*sin(x))/8 + (cos(x)*sin(x)^3)/4
```

3.349 $\int \cos^4(x) \sin^4(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 46

$$\int \cos^4(x) \sin^4(x) \, dx = \frac{3x}{128} + \frac{3}{128} \cos(x) \sin(x) + \frac{1}{64} \cos^3(x) \sin(x) - \frac{1}{16} \cos^5(x) \sin(x) - \frac{1}{8} \cos^5(x) \sin^3(x)$$

```
[Out] 3/128*x+3/128*cos(x)*sin(x)+1/64*cos(x)^3*sin(x)-1/16*cos(x)^5*sin(x)-1/8*c
os(x)^5*sin(x)^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2648, 2715, 8}

$$\int \cos^4(x) \sin^4(x) \, dx = \frac{3x}{128} - \frac{1}{8} \sin^3(x) \cos^5(x) - \frac{1}{16} \sin(x) \cos^5(x) + \frac{1}{64} \sin(x) \cos^3(x) + \frac{3}{128} \sin(x) \cos(x)$$

```
[In] Int[Cos[x]^4*Sin[x]^4,x]
```

```
[Out] (3*x)/128 + (3*Cos[x]*Sin[x])/128 + (Cos[x]^3*Sin[x])/64 - (Cos[x]^5*Sin[x]
)/16 - (Cos[x]^5*Sin[x]^3)/8
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{3}{8}\int\cos^4(x)\sin^2(x)\,dx\\
&= -\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{1}{16}\int\cos^4(x)\,dx\\
&= \frac{1}{64}\cos^3(x)\sin(x)-\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{3}{64}\int\cos^2(x)\,dx\\
&= \frac{3}{128}\cos(x)\sin(x)+\frac{1}{64}\cos^3(x)\sin(x)-\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{3\int 1\,dx}{128}\\
&= \frac{3x}{128}+\frac{3}{128}\cos(x)\sin(x)+\frac{1}{64}\cos^3(x)\sin(x)-\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int\cos^4(x)\sin^4(x)\,dx=\frac{3x}{128}-\frac{1}{128}\sin(4x)+\frac{\sin(8x)}{1024}$$

```
[In] Integrate[Cos[x]^4*Sin[x]^4,x]
[Out] (3*x)/128 - Sin[4*x]/128 + Sin[8*x]/1024
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.37

method	result
risch	$\frac{3x}{128}+\frac{\sin(8x)}{1024}-\frac{\sin(4x)}{128}$
parallelrisch	$\frac{3x}{128}+\frac{\sin(8x)}{1024}-\frac{\sin(4x)}{128}$
default	$-\frac{(\cos^5(x))(\sin^3(x))}{8}-\frac{(\cos^5(x))\sin(x)}{16}+\frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{64}+\frac{3x}{128}$
norman	$\frac{\frac{3x}{128}-\frac{23\left(\tan^3\left(\frac{x}{2}\right)\right)}{64}+\frac{333\left(\tan^5\left(\frac{x}{2}\right)\right)}{64}-\frac{671\left(\tan^7\left(\frac{x}{2}\right)\right)}{64}+\frac{671\left(\tan^9\left(\frac{x}{2}\right)\right)}{64}-\frac{333\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{64}+\frac{23\left(\tan^{13}\left(\frac{x}{2}\right)\right)}{64}+\frac{3\left(\tan^{15}\left(\frac{x}{2}\right)\right)}{64}+\frac{3x(\tan^2(}{16}}{(1+}$

[In] int(cos(x)^4*sin(x)^4,x,method=_RETURNVERBOSE)

[Out] 3/128*x+1/1024*sin(8*x)-1/128*sin(4*x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{1}{128}\left(16\cos(x)^7-24\cos(x)^5+2\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{3}{128}x$$

[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="fricas")

[Out] 1/128*(16*cos(x)^7 - 24*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 3/128*x

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{3x}{128}-\frac{\sin^3(2x)\cos(2x)}{128}-\frac{3\sin(2x)\cos(2x)}{256}$$

[In] integrate(cos(x)**4*sin(x)**4,x)

[Out] 3*x/128 - sin(2*x)**3*cos(2*x)/128 - 3*sin(2*x)*cos(2*x)/256

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.35

$$\int \cos^4(x) \sin^4(x)\, dx = \frac{3}{128}\, x + \frac{1}{1024}\, \sin\left(8\, x\right) - \frac{1}{128}\, \sin\left(4\, x\right)$$

```
[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="maxima")
[Out] 3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.35

$$\int \cos^4(x) \sin^4(x)\, dx = \frac{3}{128}\, x + \frac{1}{1024}\, \sin\left(8\, x\right) - \frac{1}{128}\, \sin\left(4\, x\right)$$

```
[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="giac")
[Out] 3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \cos^4(x) \sin^4(x)\, dx = \left(\frac{\cos\left(x\right)^3}{8} + \frac{\cos\left(x\right)}{16}\right)\, \sin\left(x\right)^5 + \frac{3\, x}{128} - \frac{\sin\left(2\, x\right)}{64} + \frac{\sin\left(4\, x\right)}{512}$$

```
[In] int(cos(x)^4*sin(x)^4,x)
[Out] (3*x)/128 - sin(2*x)/64 + sin(4*x)/512 + sin(x)^5*(cos(x)/16 + cos(x)^3/8)
```

3.350 $\int \cos^6(x)\sin^6(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 68

$$\int \cos^6(x)\sin^6(x)\,dx = \frac{5x}{1024}+\frac{5\cos(x)\sin(x)}{1024}+\frac{5\cos^3(x)\sin(x)}{1536}+\frac{1}{384}\cos^5(x)\sin(x)-\frac{1}{64}\cos^7(x)\sin(x)-\frac{1}{24}\cos^7(x)\sin^3(x)-\frac{1}{12}\cos^7(x)\sin^5(x)$$

```
[Out] 5/1024*x+5/1024*cos(x)*sin(x)+5/1536*cos(x)^3*sin(x)+1/384*cos(x)^5*sin(x)-
1/64*cos(x)^7*sin(x)-1/24*cos(x)^7*sin(x)^3-1/12*cos(x)^7*sin(x)^5
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2648, 2715, 8}

$$\int \cos^6(x)\sin^6(x)\,dx = \frac{5x}{1024}-\frac{1}{12}\sin^5(x)\cos^7(x)-\frac{1}{24}\sin^3(x)\cos^7(x)-\frac{1}{64}\sin(x)\cos^7(x)+\frac{1}{384}\sin(x)\cos^5(x)+\frac{5\sin(x)\cos^3(x)}{1536}+\frac{5\sin(x)\cos(x)}{1024}$$

```
[In] Int[Cos[x]^6*Sin[x]^6,x]
```

```
[Out] (5*x)/1024 + (5*Cos[x]*Sin[x])/1024 + (5*Cos[x]^3*Sin[x])/1536 + (Cos[x]^5*
Sin[x])/384 - (Cos[x]^7*Sin[x])/64 - (Cos[x]^7*Sin[x]^3)/24 - (Cos[x]^7*Sin
[x]^5)/12
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{12}\cos^7(x)\sin^5(x)+\frac{5}{12}\int\cos^6(x)\sin^4(x)\,dx\\
&= -\frac{1}{24}\cos^7(x)\sin^3(x)-\frac{1}{12}\cos^7(x)\sin^5(x)+\frac{1}{8}\int\cos^6(x)\sin^2(x)\,dx\\
&= -\frac{1}{64}\cos^7(x)\sin(x)-\frac{1}{24}\cos^7(x)\sin^3(x)-\frac{1}{12}\cos^7(x)\sin^5(x)+\frac{1}{64}\int\cos^6(x)\,dx\\
&= \frac{1}{384}\cos^5(x)\sin(x)-\frac{1}{64}\cos^7(x)\sin(x)-\frac{1}{24}\cos^7(x)\sin^3(x)\\
&\quad-\frac{1}{12}\cos^7(x)\sin^5(x)+\frac{5}{384}\int\cos^4(x)\,dx\\
&= \frac{5\cos^3(x)\sin(x)}{1536}+\frac{1}{384}\cos^5(x)\sin(x)-\frac{1}{64}\cos^7(x)\sin(x)\\
&\quad-\frac{1}{24}\cos^7(x)\sin^3(x)-\frac{1}{12}\cos^7(x)\sin^5(x)+\frac{5}{512}\int\cos^2(x)\,dx\\
&= \frac{5\cos(x)\sin(x)}{1024}+\frac{5\cos^3(x)\sin(x)}{1536}+\frac{1}{384}\cos^5(x)\sin(x)\\
&\quad-\frac{1}{64}\cos^7(x)\sin(x)-\frac{1}{24}\cos^7(x)\sin^3(x)-\frac{1}{12}\cos^7(x)\sin^5(x)+\frac{5\int 1\,dx}{1024}\\
&= \frac{5x}{1024}+\frac{5\cos(x)\sin(x)}{1024}+\frac{5\cos^3(x)\sin(x)}{1536}+\frac{1}{384}\cos^5(x)\sin(x)\\
&\quad-\frac{1}{64}\cos^7(x)\sin(x)-\frac{1}{24}\cos^7(x)\sin^3(x)-\frac{1}{12}\cos^7(x)\sin^5(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.44

$$\int \cos^6(x)\sin^6(x)\,dx = \frac{5x}{1024} - \frac{15\sin(4x)}{8192} + \frac{3\sin(8x)}{8192} - \frac{\sin(12x)}{24576}$$

```
[In] Integrate[Cos[x]^6*Sin[x]^6,x]
[Out] (5*x)/1024 - (15*Sin[4*x])/8192 + (3*Sin[8*x])/8192 - Sin[12*x]/24576
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.34

method	result	s
risch	$\frac{5x}{1024}-\frac{\sin(12x)}{24576}+\frac{3\sin(8x)}{8192}-\frac{15\sin(4x)}{8192}$	2
parallelrisch	$\frac{5x}{1024}-\frac{\sin(12x)}{24576}+\frac{3\sin(8x)}{8192}-\frac{15\sin(4x)}{8192}$	2
default	$-\frac{(\sin^5(x))(\cos^7(x))}{12}-\frac{(\cos^7(x))(\sin^3(x))}{24}-\frac{(\cos^7(x))\sin(x)}{64}+\frac{\left(\cos^5(x)+\frac{5(\cos^3(x))}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{384}+\frac{5x}{1024}$	5

```
[In] int(sin(x)^6*cos(x)^6,x,method=_RETURNVERBOSE)
[Out] 5/1024*x-1/24576*sin(12*x)+3/8192*sin(8*x)-15/8192*sin(4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.63

$$\int \cos^6(x)\sin^6(x)\,dx = -\frac{1}{3072}\left(256\cos(x)^{11} - 640\cos(x)^9 + 432\cos(x)^7 - 8\cos(x)^5 - 10\cos(x)^3 - 15\cos(x)\right)\sin(x) + \frac{5}{1024}x$$

```
[In] integrate(cos(x)^6*sin(x)^6,x, algorithm="fricas")
[Out] -1/3072*(256*cos(x)^11 - 640*cos(x)^9 + 432*cos(x)^7 - 8*cos(x)^5 - 10*cos(
x)^3 - 15*cos(x))*sin(x) + 5/1024*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int \cos^6(x)\sin^6(x)\,dx = \frac{5x}{1024} - \frac{\sin^5(2x)\cos(2x)}{768} - \frac{5\sin^3(2x)\cos(2x)}{3072} - \frac{5\sin(2x)\cos(2x)}{2048}$$

```
[In] integrate(cos(x)**6*sin(x)**6,x)
[Out] 5*x/1024 - sin(2*x)**5*cos(2*x)/768 - 5*sin(2*x)**3*cos(2*x)/3072 - 5*sin(2
*x)*cos(2*x)/2048
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.35

$$\int \cos^6(x)\sin^6(x)\,dx = \frac{1}{6144}\sin(4x)^3 + \frac{5}{1024}x + \frac{3}{8192}\sin(8x) - \frac{1}{512}\sin(4x)$$

```
[In] integrate(cos(x)^6*sin(x)^6,x, algorithm="maxima")
[Out] 1/6144*sin(4*x)^3 + 5/1024*x + 3/8192*sin(8*x) - 1/512*sin(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.32

$$\int \cos^6(x)\sin^6(x)\,dx = \frac{5}{1024}x - \frac{1}{24576}\sin(12x) + \frac{3}{8192}\sin(8x) - \frac{15}{8192}\sin(4x)$$

```
[In] integrate(cos(x)^6*sin(x)^6,x, algorithm="giac")
[Out] 5/1024*x - 1/24576*sin(12*x) + 3/8192*sin(8*x) - 15/8192*sin(4*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

$$\int \cos^6(x)\sin^6(x)\,dx = \left(\frac{\cos(x)^5}{12} + \frac{\cos(x)^3}{24} + \frac{\cos(x)}{64}\right)\sin(x)^7 + \frac{5x}{1024} - \frac{15\sin(2x)}{4096} + \frac{3\sin(4x)}{4096} - \frac{\sin(6x)}{12288}$$

```
[In] int(cos(x)^6*sin(x)^6,x)
[Out] (5*x)/1024 - (15*sin(2*x))/4096 + (3*sin(4*x))/4096 - sin(6*x)/12288 + sin(
x)^7*(cos(x)/64 + cos(x)^3/24 + cos(x)^5/12)
```

3.351 $\int \cos^8(x)\sin^8(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 90

$$\int \cos^8(x)\sin^8(x)\,dx = \frac{35x}{32768}+\frac{35\cos(x)\sin(x)}{32768}+\frac{35\cos^3(x)\sin(x)}{49152}+\frac{7\cos^5(x)\sin(x)}{12288}+\frac{\cos^7(x)\sin(x)}{2048}-\frac{1}{256}\cos^9(x)\sin(x)-\frac{5}{384}\cos^9(x)\sin^3(x)-\frac{1}{32}\cos^9(x)\sin^5(x)-\frac{1}{16}\cos^9(x)\sin^7(x)$$

```
[Out] 35/32768*x+35/32768*cos(x)*sin(x)+35/49152*cos(x)^3*sin(x)+7/12288*cos(x)^5
*sin(x)+1/2048*cos(x)^7*sin(x)-1/256*cos(x)^9*sin(x)-5/384*cos(x)^9*sin(x)^
3-1/32*cos(x)^9*sin(x)^5-1/16*cos(x)^9*sin(x)^7
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2648, 2715, 8}

$$\int \cos^8(x)\sin^8(x)\,dx = \frac{35x}{32768}-\frac{1}{16}\sin^7(x)\cos^9(x)-\frac{1}{32}\sin^5(x)\cos^9(x)-\frac{5}{384}\sin^3(x)\cos^9(x)-\frac{1}{256}\sin(x)\cos^9(x)+\frac{\sin(x)\cos^7(x)}{2048}+\frac{7\sin(x)\cos^5(x)}{12288}+\frac{35\sin(x)\cos^3(x)}{49152}+\frac{35\sin(x)\cos(x)}{32768}$$

```
[In] Int[Cos[x]^8*Sin[x]^8,x]
```

```
[Out] (35*x)/32768 + (35*Cos[x]*Sin[x])/32768 + (35*Cos[x]^3*Sin[x])/49152 + (7*C
os[x]^5*Sin[x])/12288 + (Cos[x]^7*Sin[x])/2048 - (Cos[x]^9*Sin[x])/256 - (5
*Cos[x]^9*Sin[x]^3)/384 - (Cos[x]^9*Sin[x]^5)/32 - (Cos[x]^9*Sin[x]^7)/16
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{16}\cos^9(x)\sin^7(x)+\frac{7}{16}\int\cos^8(x)\sin^6(x)\,dx\\
&= -\frac{1}{32}\cos^9(x)\sin^5(x)-\frac{1}{16}\cos^9(x)\sin^7(x)+\frac{5}{32}\int\cos^8(x)\sin^4(x)\,dx\\
&= -\frac{5}{384}\cos^9(x)\sin^3(x)-\frac{1}{32}\cos^9(x)\sin^5(x)-\frac{1}{16}\cos^9(x)\sin^7(x)+\frac{5}{128}\int\cos^8(x)\sin^2(x)\,dx\\
&= -\frac{1}{256}\cos^9(x)\sin(x)-\frac{5}{384}\cos^9(x)\sin^3(x)\\
&\quad-\frac{1}{32}\cos^9(x)\sin^5(x)-\frac{1}{16}\cos^9(x)\sin^7(x)+\frac{1}{256}\int\cos^8(x)\,dx\\
&= \frac{\cos^7(x)\sin(x)}{2048}-\frac{1}{256}\cos^9(x)\sin(x)-\frac{5}{384}\cos^9(x)\sin^3(x)\\
&\quad-\frac{1}{32}\cos^9(x)\sin^5(x)-\frac{1}{16}\cos^9(x)\sin^7(x)+\frac{7\int\cos^6(x)\,dx}{2048}\\
&= \frac{7\cos^5(x)\sin(x)}{12288}+\frac{\cos^7(x)\sin(x)}{2048}-\frac{1}{256}\cos^9(x)\sin(x)-\frac{5}{384}\cos^9(x)\sin^3(x)\\
&\quad-\frac{1}{32}\cos^9(x)\sin^5(x)-\frac{1}{16}\cos^9(x)\sin^7(x)+\frac{35\int\cos^4(x)\,dx}{12288}\\
&= \frac{35\cos^3(x)\sin(x)}{49152}+\frac{7\cos^5(x)\sin(x)}{12288}+\frac{\cos^7(x)\sin(x)}{2048}-\frac{1}{256}\cos^9(x)\sin(x)\\
&\quad-\frac{5}{384}\cos^9(x)\sin^3(x)-\frac{1}{32}\cos^9(x)\sin^5(x)-\frac{1}{16}\cos^9(x)\sin^7(x)+\frac{35\int\cos^2(x)\,dx}{16384}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{35\cos(x)\sin(x)}{32768} + \frac{35\cos^3(x)\sin(x)}{49152} + \frac{7\cos^5(x)\sin(x)}{12288} + \frac{\cos^7(x)\sin(x)}{2048} \\
&\quad - \frac{1}{256}\cos^9(x)\sin(x) - \frac{5}{384}\cos^9(x)\sin^3(x) - \frac{1}{32}\cos^9(x)\sin^5(x) - \frac{1}{16}\cos^9(x)\sin^7(x) \\
&\quad + \frac{35\int 1\,dx}{32768} \\
&= \frac{35x}{32768} + \frac{35\cos(x)\sin(x)}{32768} + \frac{35\cos^3(x)\sin(x)}{49152} + \frac{7\cos^5(x)\sin(x)}{12288} + \frac{\cos^7(x)\sin(x)}{2048} \\
&\quad - \frac{1}{256}\cos^9(x)\sin(x) - \frac{5}{384}\cos^9(x)\sin^3(x) - \frac{1}{32}\cos^9(x)\sin^5(x) - \frac{1}{16}\cos^9(x)\sin^7(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.42

$$
\int \cos^8(x)\sin^8(x)\,dx = \frac{35x}{32768} - \frac{7\sin(4x)}{16384} + \frac{7\sin(8x)}{65536} - \frac{\sin(12x)}{49152} + \frac{\sin(16x)}{524288}
$$

```
[In] Integrate[Cos[x]^8*Sin[x]^8,x]
```

```
[Out] (35*x)/32768 - (7*Sin[4*x])/16384 + (7*Sin[8*x])/65536 - Sin[12*x]/49152 +
Sin[16*x]/524288
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.32

method	result
risch	$\frac{35x}{32768} + \frac{\sin(16x)}{524288} - \frac{\sin(12x)}{49152} + \frac{7\sin(8x)}{65536} - \frac{7\sin(4x)}{16384}$
parallelrisch	$\frac{35x}{32768} + \frac{\sin(16x)}{524288} - \frac{\sin(12x)}{49152} + \frac{7\sin(8x)}{65536} - \frac{7\sin(4x)}{16384}$
default	$-\frac{(\cos^9(x))(\sin^7(x))}{16} - \frac{(\cos^9(x))(\sin^5(x))}{32} - \frac{5(\sin^3(x))(\cos^9(x))}{384} - \frac{(\cos^9(x))\sin(x)}{256} + \frac{\left(\cos^7(x)+\frac{7\left(\cos^5(x)\right)}{6}+\frac{35\left(\cos\right.}{24}\right.}{2048}$

```
[In] int(cos(x)^8*sin(x)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 35/32768*x+1/524288*sin(16*x)-1/49152*sin(12*x)+7/65536*sin(8*x)-7/16384*si
n(4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.61

$$\int \cos^8(x)\sin^8(x)\,dx$$
$$= \frac{1}{98304}\left(6144\,\cos(x)^{15} - 21504\,\cos(x)^{13} + 25856\,\cos(x)^{11} - 10880\,\cos(x)^{9} + 48\,\cos(x)^{7} + 56\,\cos(x)^{5} + 7\right.$$
$$+ \frac{35}{32768}\,x$$

```
[In] integrate(cos(x)^8*sin(x)^8,x, algorithm="fricas")
```

```
[Out] 1/98304*(6144*cos(x)^15 - 21504*cos(x)^13 + 25856*cos(x)^11 - 10880*cos(x)^
9 + 48*cos(x)^7 + 56*cos(x)^5 + 70*cos(x)^3 + 105*cos(x))*sin(x) + 35/32768
*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.68

$$\int \cos^8(x)\sin^8(x)\,dx = \frac{35x}{32768} - \frac{\sin^7(2x)\cos(2x)}{4096} - \frac{7\sin^5(2x)\cos(2x)}{24576} - \frac{35\sin^3(2x)\cos(2x)}{98304} - \frac{35\sin(2x)\cos(2x)}{65536}$$

```
[In] integrate(cos(x)**8*sin(x)**8,x)
```

```
[Out] 35*x/32768 - sin(2*x)**7*cos(2*x)/4096 - 7*sin(2*x)**5*cos(2*x)/24576 - 35*
sin(2*x)**3*cos(2*x)/98304 - 35*sin(2*x)*cos(2*x)/65536
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.33

$$\int \cos^8(x)\sin^8(x)\,dx = \frac{1}{12288}\,\sin(4\,x)^3 + \frac{35}{32768}\,x + \frac{1}{524288}\,\sin(16\,x) + \frac{7}{65536}\,\sin(8\,x) - \frac{1}{2048}\,\sin(4\,x)$$

```
[In] integrate(cos(x)^8*sin(x)^8,x, algorithm="maxima")
```

```
[Out] 1/12288*sin(4*x)^3 + 35/32768*x + 1/524288*sin(16*x) + 7/65536*sin(8*x) - 1
/2048*sin(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.31

$$\int \cos^8(x)\sin^8(x)\,dx = \frac{35}{32768}\,x + \frac{1}{524288}\,\sin(16\,x) - \frac{1}{49152}\,\sin(12\,x) + \frac{7}{65536}\,\sin(8\,x) - \frac{7}{16384}\,\sin(4\,x)$$

```
[In] integrate(cos(x)^8*sin(x)^8,x, algorithm="giac")
```

```
[Out] 35/32768*x + 1/524288*sin(16*x) - 1/49152*sin(12*x) + 7/65536*sin(8*x) - 7/
16384*sin(4*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \cos^8(x)\sin^8(x)\,dx = \left(\frac{\cos(x)^7}{16} + \frac{\cos(x)^5}{32} + \frac{5\cos(x)^3}{384} + \frac{\cos(x)}{256}\right)\sin(x)^9 + \frac{35\,x}{32768} - \frac{7\,\sin(2\,x)}{8192} + \frac{7\,\sin(4\,x)}{32768} - \frac{\sin(6\,x)}{24576} + \frac{\sin(8\,x)}{262144}$$

```
[In] int(cos(x)^8*sin(x)^8,x)
```

```
[Out] (35*x)/32768 - (7*sin(2*x))/8192 + (7*sin(4*x))/32768 - sin(6*x)/24576 + si
n(8*x)/262144 + sin(x)^9*(cos(x)/256 + (5*cos(x)^3)/384 + cos(x)^5/32 + cos
(x)^7/16)
```

3.352 $\int \cos^{2m}(x)\sin^{2m}(x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \frac{\cos^{-1+2m}(x)\cos^2(x)^{\frac{1}{2}-m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1-2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\sin^2(x)\right)\sin^{1+2m}(x)}{1+2m}$$

```
[Out] cos(x)^(-1+2*m)*(cos(x)^2)^(1/2-m)*hypergeom([1/2+m, 1/2-m],[3/2+m],sin(x)^
2)*sin(x)^(1+2*m)/(1+2*m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2657}

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \frac{\sin^{2m+1}(x)\cos^{2m-1}(x)\cos^2(x)^{\frac{1}{2}-m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1-2m),\frac{1}{2}(2m+1),\frac{1}{2}(2m+3),\sin^2(x)\right)}{2m+1}$$

```
[In] Int[Cos[x]^(2*m)*Sin[x]^(2*m),x]
```

```
[Out] (Cos[x]^(-1 + 2*m)*(Cos[x]^2)^(1/2 - m)*Hypergeometric2F1[(1 - 2*m)/2, (1 +
 2*m)/2, (3 + 2*m)/2, Sin[x]^2]*Sin[x]^(1 + 2*m))/(1 + 2*m)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

integral
$$= \frac{\cos^{-1+2m}(x)\cos^2(x)^{\frac{1}{2}-m}\text{Hypergeometric2F1}\left(\frac{1}{2}(1-2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\sin^2(x)\right)\sin^{1+2m}(x)}{1+2m}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \frac{\cos^{-1+2m}(x)\cos^2(x)^{\frac{1}{2}-m}\text{Hypergeometric2F1}\left(\frac{1}{2}-m,\frac{1}{2}+m,\frac{3}{2}+m,\sin^2(x)\right)\sin^{1+2m}(x)}{1+2m}$$

```
[In] Integrate[Cos[x]^(2*m)*Sin[x]^(2*m),x]
[Out] (Cos[x]^(-1 + 2*m)*(Cos[x]^2)^(1/2 - m)*Hypergeometric2F1[1/2 - m, 1/2 + m,
 3/2 + m, Sin[x]^2]*Sin[x]^(1 + 2*m))/(1 + 2*m)
```

Maple [F]

$$\int \left(\cos^{2m}(x)\right)\left(\sin^{2m}(x)\right)dx$$

```
[In] int(cos(x)^(2*m)*sin(x)^(2*m),x)
[Out] int(cos(x)^(2*m)*sin(x)^(2*m),x)
```

Fricas [F]

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \int \cos(x)^{2m}\sin(x)^{2m}\,dx$$

```
[In] integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="fricas")
[Out] integral(cos(x)^(2*m)*sin(x)^(2*m), x)
```

Sympy [F]

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \int \sin^{2m}(x)\cos^{2m}(x)\,dx$$

```
[In] integrate(cos(x)**(2*m)*sin(x)**(2*m),x)
```

```
[Out] Integral(sin(x)**(2*m)*cos(x)**(2*m), x)
```

Maxima [F]

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \int \cos(x)^{2m}\sin(x)^{2m}\,dx$$

```
[In] integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)^(2*m)*sin(x)^(2*m), x)
```

Giac [F]

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \int \cos(x)^{2m}\sin(x)^{2m}\,dx$$

```
[In] integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="giac")
```

```
[Out] integrate(cos(x)^(2*m)*sin(x)^(2*m), x)
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = -\frac{\cos(x)^{2m+1}\sin(x)^{2m+1}\,{}_2F_1\left(\frac{1}{2}-m,m+\frac{1}{2};\ m+\frac{3}{2};\ \cos(x)^2\right)}{(2m+1)\,\left(\sin(x)^2\right)^{m+\frac{1}{2}}}$$

```
[In] int(cos(x)^(2*m)*sin(x)^(2*m),x)
```

```
[Out] -(cos(x)^(2*m + 1)*sin(x)^(2*m + 1)*hypergeom([1/2 - m, m + 1/2], m + 3/2,
cos(x)^2))/((2*m + 1)*(sin(x)^2)^(m + 1/2))
```

3.353 $\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx=-\frac{1}{4}\cot^2\left(\frac{\pi}{4}+2x\right)+\frac{1}{2}\log\left(\tan\left(\frac{\pi}{4}+2x\right)\right)$$

```
[Out] -1/4*cot(1/4*Pi+2*x)^2+1/2*ln(tan(1/4*Pi+2*x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2700, 14}

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx=\frac{1}{2}\log\left(\tan\left(2x+\frac{\pi}{4}\right)\right)-\frac{1}{4}\cot^2\left(2x+\frac{\pi}{4}\right)$$

```
[In] Int[Csc[Pi/4 + 2*x]^3*Sec[Pi/4 + 2*x],x]
```

```
[Out] -1/4*Cot[Pi/4 + 2*x]^2 + Log[Tan[Pi/4 + 2*x]]/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{x^3}\,dx, x, \tan\left(\frac{\pi}{4}+2x\right)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{1}{x}\right)\,dx, x, \tan\left(\frac{\pi}{4}+2x\right)\right)\\ &= -\frac{1}{4}\cot^2\left(\frac{\pi}{4}+2x\right)+\frac{1}{2}\log\left(\tan\left(\frac{\pi}{4}+2x\right)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{1}{4}\csc^2\left(\frac{\pi}{4}+2x\right)-\frac{1}{2}\log\left(\cos\left(\frac{1}{4}(\pi+8x)\right)\right) + \frac{1}{2}\log\left(\sin\left(\frac{\pi}{4}+2x\right)\right)$$

```
[In] Integrate[Csc[Pi/4 + 2*x]^3*Sec[Pi/4 + 2*x],x]
[Out] -1/4*Csc[Pi/4 + 2*x]^2 - Log[Cos[(Pi + 8*x)/4]]/2 + Log[Sin[Pi/4 + 2*x]]/2
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result
derivativedivides	$-\frac{1}{4\sin\left(\frac{\pi}{4}+2x\right)^2}+\frac{\ln\left(\tan\left(\frac{\pi}{4}+2x\right)\right)}{2}$
default	$-\frac{1}{4\sin\left(\frac{\pi}{4}+2x\right)^2}+\frac{\ln\left(\tan\left(\frac{\pi}{4}+2x\right)\right)}{2}$
risch	$\frac{i e^{4ix}}{\left(i e^{4ix}-1\right)^2}+\frac{\ln\left(i e^{4ix}-1\right)}{2}-\frac{\ln\left(i e^{4ix}+1\right)}{2}$
parallelrisch	$\ln\left(\sqrt{\tan}\left(\frac{\pi}{8}+x\right)\right)+\ln\left(\frac{1}{\sqrt{\tan\left(\frac{\pi}{8}+x\right)-1}}\right)+\ln\left(\frac{1}{\sqrt{\tan\left(\frac{\pi}{8}+x\right)+1}}\right)-\frac{\left(\tan^2\left(\frac{\pi}{8}+x\right)\right)}{16}-\frac{\left(\cot^2\left(\frac{\pi}{8}+x\right)\right)}{16}$
norman	$\frac{-\frac{1}{16}-\frac{\left(\tan^4\left(\frac{\pi}{8}+x\right)\right)}{16}}{\tan\left(\frac{\pi}{8}+x\right)^2}+\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)+1\right)}{2}$

```
[In] int(1/cos(1/4*Pi+2*x)/sin(1/4*Pi+2*x)^3,x,method=_RETURNVERBOSE)
[Out] -1/4/sin(1/4*Pi+2*x)^2+1/2*ln(tan(1/4*Pi+2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{\left(\cos\left(\frac{1}{4}\pi+2x\right)^2-1\right)\log\left(\cos\left(\frac{1}{4}\pi+2x\right)^2\right)-\left(\cos\left(\frac{1}{4}\pi+2x\right)^2-1\right)\log\left(-\frac{1}{4}\cos\left(\frac{1}{4}\pi+2x\right)^2+\frac{1}{4}\right)}{4\left(\cos\left(\frac{1}{4}\pi+2x\right)^2-1\right)}$$

```
[In] integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="fricas")
[Out] -1/4*((cos(1/4*pi + 2*x)^2 - 1)*log(cos(1/4*pi + 2*x)^2) - (cos(1/4*pi + 2*
x)^2 - 1)*log(-1/4*cos(1/4*pi + 2*x)^2 + 1/4) - 1)/(cos(1/4*pi + 2*x)^2 - 1
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(22) = 44.

Time = 0.57 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)-1\right)}{2}-\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)+1\right)}{2}+\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)\right)}{2}-\frac{\tan^2\left(x+\frac{\pi}{8}\right)}{16}-\frac{1}{16\tan^2\left(x+\frac{\pi}{8}\right)}$$

```
[In] integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)**3,x)
[Out] -log(tan(x + pi/8) - 1)/2 - log(tan(x + pi/8) + 1)/2 + log(tan(x + pi/8))/2
 - tan(x + pi/8)**2/16 - 1/(16*tan(x + pi/8)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{1}{4\sin\left(\frac{1}{4}\pi+2x\right)^2}-\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)^2-1\right)+\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)^2\right)$$

```
[In] integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="maxima")
[Out] -1/4/sin(1/4*pi + 2*x)^2 - 1/4*log(sin(1/4*pi + 2*x)^2 - 1) + 1/4*log(sin(1
/4*pi + 2*x)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(24) = 48$.

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 4.12

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{\left(\frac{4\left(\cos\left(\frac{1}{4}\pi+2x\right)-1\right)}{\cos\left(\frac{1}{4}\pi+2x\right)+1}-1\right)\left(\cos\left(\frac{1}{4}\pi+2x\right)+1\right)}{16\left(\cos\left(\frac{1}{4}\pi+2x\right)-1\right)} + \frac{\cos\left(\frac{1}{4}\pi+2x\right)-1}{16\left(\cos\left(\frac{1}{4}\pi+2x\right)+1\right)} + \frac{1}{4}\log\left(-\frac{\cos\left(\frac{1}{4}\pi+2x\right)-1}{\cos\left(\frac{1}{4}\pi+2x\right)+1}\right) - \frac{1}{2}\log\left(\left|-\frac{\cos\left(\frac{1}{4}\pi+2x\right)-1}{\cos\left(\frac{1}{4}\pi+2x\right)+1}-1\right|\right)$$

```
[In] integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="giac")
[Out] -1/16*(4*(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1) - 1)*(cos(1/4*pi +
 2*x) + 1)/(cos(1/4*pi + 2*x) - 1) + 1/16*(cos(1/4*pi + 2*x) - 1)/(cos(1/4*
pi + 2*x) + 1) + 1/4*log(-(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1))
- 1/2*log(abs(-(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx = \frac{\ln\left(\tan\left(\frac{\Pi}{4}+2x\right)\right)}{2} - \frac{1}{4\sin\left(\frac{\Pi}{4}+2x\right)^2}$$

```
[In] int(1/(cos(Pi/4 + 2*x)*sin(Pi/4 + 2*x)^3),x)
[Out] log(tan(Pi/4 + 2*x))/2 - 1/(4*sin(Pi/4 + 2*x)^2)
```

3.354 $\int \sec^2(x)\tan^2(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \sec^2(x)\tan^2(x)\,dx = \frac{\tan^3(x)}{3}$$

```
[Out] 1/3*tan(x)^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2687, 30}

$$\int \sec^2(x)\tan^2(x)\,dx = \frac{\tan^3(x)}{3}$$

```
[In] Int[Sec[x]^2*Tan[x]^2,x]
```

```
[Out] Tan[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^2\,dx, x, \tan(x)\right)\\ &= \frac{\tan^3(x)}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\tan^2(x)\,dx = \frac{\tan^3(x)}{3}$$

```
[In] Integrate[Sec[x]^2*Tan[x]^2,x]
[Out] Tan[x]^3/3
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\tan^3(x))}{3}$	7
default	$\frac{(\tan^3(x))}{3}$	7
risch	$-\frac{2i(3\,\mathrm{e}^{4ix}+1)}{3(\mathrm{e}^{2ix}+1)^3}$	22

```
[In] int(tan(x)^2*sec(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*tan(x)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \sec^2(x)\tan^2(x)\,dx = -\frac{(\cos(x)^2 - 1)\sin(x)}{3\cos(x)^3}$$

```
[In] integrate(sec(x)^2*tan(x)^2,x, algorithm="fricas")
[Out] -1/3*(cos(x)^2 - 1)*sin(x)/cos(x)^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$.

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \sec^2(x)\tan^2(x)\,dx = -\frac{\sin(x)}{3\cos(x)} + \frac{\sin(x)}{3\cos^3(x)}$$

```
[In] integrate(sec(x)**2*tan(x)**2,x)
[Out] -sin(x)/(3*cos(x)) + sin(x)/(3*cos(x)**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan^2(x)\,dx = \frac{1}{3}\tan(x)^3$$

```
[In] integrate(sec(x)^2*tan(x)^2,x, algorithm="maxima")
[Out] 1/3*tan(x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan^2(x)\,dx = \frac{1}{3}\tan(x)^3$$

```
[In] integrate(sec(x)^2*tan(x)^2,x, algorithm="giac")
[Out] 1/3*tan(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan^2(x)\,dx = \frac{\tan(x)^3}{3}$$

```
[In] int(tan(x)^2/cos(x)^2,x)
[Out] tan(x)^3/3
```

3.355 $\int \cot^3(x) \csc(x) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \cot^3(x) \csc(x) \, dx = \csc(x) - \frac{\csc^3(x)}{3}$$

```
[Out] csc(x)-1/3*csc(x)^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2686}

$$\int \cot^3(x) \csc(x) \, dx = \csc(x) - \frac{\csc^3(x)}{3}$$

```
[In] Int[Cot[x]^3*Csc[x],x]
[Out] Csc[x] - Csc[x]^3/3
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int \left(-1+x^2\right) \, dx, x, \csc(x)\right) \\ &= \csc(x) - \frac{\csc^3(x)}{3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc(x)\,dx = \csc(x) - \frac{\csc^3(x)}{3}$$

```
[In] Integrate[Cot[x]^3*Csc[x],x]
[Out] Csc[x] - Csc[x]^3/3
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\csc(x) - \frac{(\csc^3(x))}{3}$	10
default	$\csc(x) - \frac{(\csc^3(x))}{3}$	10
risch	$\frac{2i(3\,e^{5ix} - 2\,e^{3ix} + 3\,e^{ix})}{3(e^{2ix} - 1)^3}$	35

```
[In] int(cot(x)^3*csc(x),x,method=_RETURNVERBOSE)
[Out] csc(x)-1/3*csc(x)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(9) = 18$.

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\,\cos(x)^2 - 2}{3\,(\cos(x)^2 - 1)\sin(x)}$$

```
[In] integrate(cot(x)^3*csc(x),x, algorithm="fricas")
[Out] 1/3*(3*cos(x)^2 - 2)/((cos(x)^2 - 1)*sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \cot^3(x)\csc(x)\,dx = -\frac{1-3\sin^2(x)}{3\sin^3(x)}$$

```
[In] integrate(cot(x)**3*csc(x),x)
[Out] -(1 - 3*sin(x)**2)/(3*sin(x)**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\,\sin(x)^2-1}{3\,\sin(x)^3}$$

```
[In] integrate(cot(x)^3*csc(x),x, algorithm="maxima")
[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\,\sin(x)^2-1}{3\,\sin(x)^3}$$

```
[In] integrate(cot(x)^3*csc(x),x, algorithm="giac")
[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc(x)\,dx = \frac{\sin(x)^2-\frac{1}{3}}{\sin(x)^3}$$

```
[In] int(cot(x)^3/sin(x),x)
[Out] (sin(x)^2 - 1/3)/sin(x)^3
```

3.356 $\int \sec^3(x)\tan(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \sec^3(x)\tan(x)\,dx = \frac{\sec^3(x)}{3}$$

```
[Out] 1/3*sec(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2686, 30}

$$\int \sec^3(x)\tan(x)\,dx = \frac{\sec^3(x)}{3}$$

```
[In] Int[Sec[x]^3*Tan[x],x]
```

```
[Out] Sec[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^2\,dx, x, \sec(x)\right)\\ &= \frac{\sec^3(x)}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sec^3(x)\tan(x)\,dx = \frac{\sec^3(x)}{3}$$

```
[In] Integrate[Sec[x]^3*Tan[x],x]
[Out] Sec[x]^3/3
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sec^3(x))}{3}$	7
default	$\frac{(\sec^3(x))}{3}$	7
risch	$\frac{8\,e^{3ix}}{3(e^{2ix}+1)^3}$	17

```
[In] int(sec(x)^3*tan(x),x,method=_RETURNVERBOSE)
[Out] 1/3*sec(x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\,\cos(x)^3}$$

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="fricas")
[Out] 1/3/cos(x)^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\cos^3{(x)}}$$

```
[In] integrate(sec(x)**3*tan(x),x)
[Out] 1/(3*cos(x)**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\,\cos\left(x\right)^3}$$

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="maxima")
[Out] 1/3/cos(x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\,\cos\left(x\right)^3}$$

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="giac")
[Out] 1/3/cos(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\cos\left(x\right)^3}$$

```
[In] int(tan(x)/cos(x)^3,x)
[Out] 1/(3*cos(x)^3)
```

3.357 $\int \cot^2(x)\csc^3(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 26

$$\int \cot^2(x)\csc^3(x)\,dx = \frac{1}{8}\operatorname{arctanh}(\cos(x)) + \frac{1}{8}\cot(x)\csc(x) - \frac{1}{4}\cot(x)\csc^3(x)$$

```
[Out] 1/8*arctanh(cos(x))+1/8*cot(x)*csc(x)-1/4*cot(x)*csc(x)^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2691, 3853, 3855}

$$\int \cot^2(x)\csc^3(x)\,dx = \frac{1}{8}\operatorname{arctanh}(\cos(x)) - \frac{1}{4}\cot(x)\csc^3(x) + \frac{1}{8}\cot(x)\csc(x)$$

```
[In] Int[Cot[x]^2*Csc[x]^3,x]
```

```
[Out] ArcTanh[Cos[x]]/8 + (Cot[x]*Csc[x])/8 - (Cot[x]*Csc[x]^3)/4
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{4}\cot(x)\csc^3(x) - \frac{1}{4}\int \csc^3(x)\,dx\\ &= \frac{1}{8}\cot(x)\csc(x) - \frac{1}{4}\cot(x)\csc^3(x) - \frac{1}{8}\int \csc(x)\,dx\\ &= \frac{1}{8}\text{arctanh}(\cos(x)) + \frac{1}{8}\cot(x)\csc(x) - \frac{1}{4}\cot(x)\csc^3(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 71 vs. 2(26) = 52.

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.73

$$\begin{aligned}\int \cot^2(x)\csc^3(x)\,dx &= \frac{1}{32}\csc^2\left(\frac{x}{2}\right) - \frac{1}{64}\csc^4\left(\frac{x}{2}\right) + \frac{1}{8}\log\left(\cos\left(\frac{x}{2}\right)\right)\\ &\quad - \frac{1}{8}\log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{32}\sec^2\left(\frac{x}{2}\right) + \frac{1}{64}\sec^4\left(\frac{x}{2}\right)\end{aligned}$$

```
[In] Integrate[Cot[x]^2*Csc[x]^3,x]
```

```
[Out] Csc[x/2]^2/32 - Csc[x/2]^4/64 + Log[Cos[x/2]]/8 - Log[Sin[x/2]]/8 - Sec[x/2
]^2/32 + Sec[x/2]^4/64
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

method	result	size
default	$-\frac{\cos^3(x)}{4\sin(x)^4} - \frac{\cos^3(x)}{8\sin(x)^2} - \frac{\cos(x)}{8} - \frac{\ln(\csc(x)-\cot(x))}{8}$	36
risch	$-\frac{e^{7ix}+7e^{5ix}+7e^{3ix}+e^{ix}}{4(e^{2ix}-1)^4} - \frac{\ln(e^{ix}-1)}{8} + \frac{\ln(e^{ix}+1)}{8}$	58

```
[In] int(cot(x)^2*csc(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*cos(x)^3/sin(x)^4-1/8*cos(x)^3/sin(x)^2-1/8*cos(x)-1/8*ln(csc(x)-cot(x
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(20) = 40$.

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.62

$$\int \cot^2(x)\csc^3(x)\,dx = -\frac{2\cos(x)^3 - \left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(-\frac{1}{2}\cos(x)\right.}{16\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)}$$

```
[In] integrate(cot(x)^2*csc(x)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*cos(x)^3 - (cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*cos(x) + 1/2) + (co
s(x)^4 - 2*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2) + 2*cos(x))/(cos(x)^4 - 2*c
os(x)^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \cot^2(x)\csc^3(x)\,dx = \frac{-\cos^3(x) - \cos(x)}{8\cos^4(x) - 16\cos^2(x) + 8} - \frac{\log(\cos(x) - 1)}{16} + \frac{\log(\cos(x) + 1)}{16}$$

```
[In] integrate(cot(x)**2*csc(x)**3,x)
```

```
[Out] (-cos(x)**3 - cos(x))/(8*cos(x)**4 - 16*cos(x)**2 + 8) - log(cos(x) - 1)/16
 + log(cos(x) + 1)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \cot^2(x)\csc^3(x)\,dx = -\frac{\cos(x)^3 + \cos(x)}{8\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)} + \frac{1}{16}\log(\cos(x) + 1) - \frac{1}{16}\log(\cos(x) - 1)$$

```
[In] integrate(cot(x)^2*csc(x)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(cos(x)^3 + cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) + 1/16*log(cos(x) + 1)
 - 1/16*log(cos(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(20) = 40.

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int \cot^2(x)\csc^3(x)\,dx = -\frac{\frac{1}{\cos(x)} + \cos(x)}{8\left(\left(\frac{1}{\cos(x)} + \cos(x)\right)^2 - 4\right)} + \frac{1}{32}\log\left(\left|\frac{1}{\cos(x)} + \cos(x) + 2\right|\right) - \frac{1}{32}\log\left(\left|\frac{1}{\cos(x)} + \cos(x) - 2\right|\right)$$

```
[In] integrate(cot(x)^2*csc(x)^3,x, algorithm="giac")
```

```
[Out] -1/8*(1/cos(x) + cos(x))/((1/cos(x) + cos(x))^2 - 4) + 1/32*log(abs(1/cos(x
) + cos(x) + 2)) - 1/32*log(abs(1/cos(x) + cos(x) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \cot^2(x)\csc^3(x)\,dx = \frac{\tan\left(\frac{x}{2}\right)^4}{64} - \frac{1}{64\tan\left(\frac{x}{2}\right)^4} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8}$$

```
[In] int(cot(x)^2/sin(x)^3,x)
```

```
[Out] tan(x/2)^4/64 - 1/(64*tan(x/2)^4) - log(tan(x/2))/8
```

3.358 $\int \cot^3(x)\csc^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

```
[Out] 1/4*csc(x)^4-1/6*csc(x)^6
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2686, 14}

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

```
[In] Int[Cot[x]^3*Csc[x]^4,x]
```

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int x^3\left(-1+x^2\right)\,dx,x,\csc(x)\right)\\ &= -\text{Subst}\left(\int \left(-x^3+x^5\right)\,dx,x,\csc(x)\right)\\ &= \frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc^4(x)\,dx=\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

```
[In] Integrate[Cot[x]^3*Csc[x]^4,x]
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

method	result	size
default	$-\frac{\cos^4(x)}{6\sin(x)^6}-\frac{\cos^4(x)}{12\sin(x)^4}$	22
norman	$\frac{-\frac{1}{384}+\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{128}+\frac{3\left(\tan^8\left(\frac{x}{2}\right)\right)}{128}-\frac{\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{384}}{\tan\left(\frac{x}{2}\right)^6}$	34
risch	$\frac{4\,e^{8ix}+\frac{8\,e^{6ix}}{3}+4\,e^{4ix}}{\left(e^{2ix}-1\right)^6}$	34
parallelrisch	$\frac{-\left(\tan^{12}\left(\frac{x}{2}\right)\right)+9\left(\tan^8\left(\frac{x}{2}\right)\right)+9\left(\tan^4\left(\frac{x}{2}\right)\right)-1}{384\tan\left(\frac{x}{2}\right)^6}$	35

```
[In] int(cos(x)^3/sin(x)^7,x,method=_RETURNVERBOSE)
[Out] -1/6/sin(x)^6*cos(x)^4-1/12/sin(x)^4*cos(x)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(13) = 26$.

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\cos(x)^2 - 1}{12\left(\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1\right)}$$

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="fricas")
[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cot^3(x)\csc^4(x)\,dx = -\frac{2 - 3\sin^2(x)}{12\sin^6(x)}$$

```
[In] integrate(cos(x)**3/sin(x)**7,x)
[Out] -(2 - 3*sin(x)**2)/(12*sin(x)**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\sin(x)^2 - 2}{12\sin(x)^6}$$

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="maxima")
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\sin(x)^2 - 2}{12\sin(x)^6}$$

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="giac")
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cot^3(x) \csc^4(x) \, dx = \frac{\frac{\sin(x)^2}{4} - \frac{1}{6}}{\sin(x)^6}$$

```
[In] int(cos(x)^3/sin(x)^7,x)
[Out] (sin(x)^2/4 - 1/6)/sin(x)^6
```

3.359 $\int \sec^{\frac{13}{2}}(x) \sin^5(x) \, dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \sec^{\frac{13}{2}}(x) \sin^5(x) \, dx = \frac{2}{3} \sec^{\frac{3}{2}}(x) - \frac{4}{7} \sec^{\frac{7}{2}}(x) + \frac{2}{11} \sec^{\frac{11}{2}}(x)$$

```
[Out] 2/3*sec(x)^(3/2)-4/7*sec(x)^(7/2)+2/11*sec(x)^(11/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2702, 276}

$$\int \sec^{\frac{13}{2}}(x) \sin^5(x) \, dx = \frac{2}{11} \sec^{\frac{11}{2}}(x) - \frac{4}{7} \sec^{\frac{7}{2}}(x) + \frac{2}{3} \sec^{\frac{3}{2}}(x)$$

```
[In] Int[Sec[x]^(13/2)*Sin[x]^5,x]
[Out] (2*Sec[x]^(3/2))/3 - (4*Sec[x]^(7/2))/7 + (2*Sec[x]^(11/2))/11
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \sqrt{x}\left(-1+x^2\right)^2\,dx, x, \sec(x)\right)\\
&= \text{Subst}\left(\int \left(\sqrt{x}-2x^{5/2}+x^{9/2}\right)\,dx, x, \sec(x)\right)\\
&= \frac{2}{3}\sec^{\frac{3}{2}}(x)-\frac{4}{7}\sec^{\frac{7}{2}}(x)+\frac{2}{11}\sec^{\frac{11}{2}}(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{1}{924}(135+44\cos(2x)+77\cos(4x))\sec^{\frac{11}{2}}(x)$$

```
[In] Integrate[Sec[x]^(13/2)*Sin[x]^5,x]
[Out] ((135 + 44*Cos[2*x] + 77*Cos[4*x])*Sec[x]^(11/2))/924
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{2\left(\sec^{\frac{3}{2}}(x)\right)}{3}-\frac{4\left(\sec^{\frac{7}{2}}(x)\right)}{7}+\frac{2\left(\sec^{\frac{11}{2}}(x)\right)}{11}$	20
default	$\frac{2\left(\sec^{\frac{3}{2}}(x)\right)}{3}-\frac{4\left(\sec^{\frac{7}{2}}(x)\right)}{7}+\frac{2\left(\sec^{\frac{11}{2}}(x)\right)}{11}$	20

```
[In] int(sec(x)^(3/2)*tan(x)^5,x,method=_RETURNVERBOSE)
[Out] 2/3*sec(x)^(3/2)-4/7*sec(x)^(7/2)+2/11*sec(x)^(11/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2\left(77\cos(x)^4 - 66\cos(x)^2 + 21\right)}{231\cos(x)^{\frac{11}{2}}}$$

```
[In] integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="fricas")
[Out] 2/231*(77*cos(x)^4 - 66*cos(x)^2 + 21)/cos(x)^(11/2)
```

Sympy [A] (verification not implemented)

Time = 13.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2\tan^4(x)\sec^{\frac{3}{2}}(x)}{11} - \frac{16\tan^2(x)\sec^{\frac{3}{2}}(x)}{77} + \frac{64\sec^{\frac{3}{2}}(x)}{231}$$

```
[In] integrate(sec(x)**(3/2)*tan(x)**5,x)
[Out] 2*tan(x)**4*sec(x)**(3/2)/11 - 16*tan(x)**2*sec(x)**(3/2)/77 + 64*sec(x)**(
3/2)/231
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2}{3\cos(x)^{\frac{3}{2}}} - \frac{4}{7\cos(x)^{\frac{7}{2}}} + \frac{2}{11\cos(x)^{\frac{11}{2}}}$$

```
[In] integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="maxima")
[Out] 2/3/cos(x)^(3/2) - 4/7/cos(x)^(7/2) + 2/11/cos(x)^(11/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2\left(77\,\cos\left(x\right)^4 - 66\,\cos\left(x\right)^2 + 21\right)}{231\,\cos\left(x\right)^{\frac{11}{2}}}$$

```
[In] integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="giac")
[Out] 2/231*(77*cos(x)^4 - 66*cos(x)^2 + 21)/cos(x)^(11/2)
```

Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2\left(\frac{1}{\cos(x)}\right)^{11/2}\left(77\cos\left(x\right)^4 - 66\cos\left(x\right)^2 + 21\right)}{231}$$

```
[In] int(tan(x)^5*(1/cos(x))^(3/2),x)
[Out] (2*(1/cos(x))^(11/2)*(77*cos(x)^4 - 66*cos(x)^2 + 21))/231
```

3.360 $\int \sec^4(x) \tan^{\frac{3}{2}}(x) \, dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \sec^4(x) \tan^{\frac{3}{2}}(x) \, dx = \frac{2}{5} \tan^{\frac{5}{2}}(x) + \frac{2}{9} \tan^{\frac{9}{2}}(x)$$

```
[Out] 2/5*tan(x)^(5/2)+2/9*tan(x)^(9/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2687, 14}

$$\int \sec^4(x) \tan^{\frac{3}{2}}(x) \, dx = \frac{2}{9} \tan^{\frac{9}{2}}(x) + \frac{2}{5} \tan^{\frac{5}{2}}(x)$$

```
[In] Int[Sec[x]^4*Tan[x]^(3/2),x]
```

```
[Out] (2*Tan[x]^(5/2))/5 + (2*Tan[x]^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int x^{3/2}\left(1+x^2\right)\, dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(x^{3/2}+x^{7/2}\right)\, dx, x, \tan(x)\right)$$
$$= \frac{2}{5}\tan^{\frac{5}{2}}(x)+\frac{2}{9}\tan^{\frac{9}{2}}(x)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx = \frac{2}{45}(7+2\cos(2x))\sec^2(x)\tan^{\frac{5}{2}}(x)$$

```
[In] Integrate[Sec[x]^4*Tan[x]^(3/2),x]
[Out] (2*(7 + 2*Cos[2*x])*Sec[x]^2*Tan[x]^(5/2))/45
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2\left(\tan^{\frac{5}{2}}(x)\right)}{5}+\frac{2\left(\tan^{\frac{9}{2}}(x)\right)}{9}$	14
default	$\frac{2\left(\tan^{\frac{5}{2}}(x)\right)}{5}+\frac{2\left(\tan^{\frac{9}{2}}(x)\right)}{9}$	14

```
[In] int(sec(x)^4*tan(x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/5*tan(x)^(5/2)+2/9*tan(x)^(9/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx = -\frac{2\,\left(4\,\cos\left(x\right)^4+\cos\left(x\right)^2-5\right)\sqrt{\frac{\sin(x)}{\cos(x)}}}{45\,\cos\left(x\right)^4}$$

```
[In] integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="fricas")
[Out] -2/45*(4*cos(x)^4 + cos(x)^2 - 5)*sqrt(sin(x)/cos(x))/cos(x)^4
```

Sympy [F]

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx = \int \tan^{\frac{3}{2}}\left(x\right)\sec^4\left(x\right)\,dx$$

```
[In] integrate(sec(x)**4*tan(x)**(3/2),x)
[Out] Integral(tan(x)**(3/2)*sec(x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx = \frac{2}{9}\,\tan\left(x\right)^{\frac{9}{2}}+\frac{2}{5}\,\tan\left(x\right)^{\frac{5}{2}}$$

```
[In] integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="maxima")
[Out] 2/9*tan(x)^(9/2) + 2/5*tan(x)^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx = \frac{2}{9}\,\tan\left(x\right)^{\frac{9}{2}}+\frac{2}{5}\,\tan\left(x\right)^{\frac{5}{2}}$$

```
[In] integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="giac")
[Out] 2/9*tan(x)^(9/2) + 2/5*tan(x)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.52

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx = -\frac{4\sqrt{\sin(2x)}\,\left(2\cos(2x)^2+5\,\cos(2x)-7\right)}{45\,\left(\cos(2x)+1\right)^{5/2}}$$

```
[In] int(tan(x)^(3/2)/cos(x)^4,x)
[Out] -(4*sin(2*x)^(1/2)*(5*cos(2*x) + 2*cos(2*x)^2 - 7))/(45*(cos(2*x) + 1)^(5/2
))
```

3.361 $\int \cot^4(x) \csc^3(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \cot^4(x) \csc^3(x) \, dx = -\frac{1}{16} \text{arctanh}(\cos(x)) - \frac{1}{16} \cot(x) \csc(x) + \frac{1}{8} \cot(x) \csc^3(x) - \frac{1}{6} \cot^3(x) \csc^3(x)$$

```
[Out] -1/16*arctanh(cos(x))-1/16*cot(x)*csc(x)+1/8*cot(x)*csc(x)^3-1/6*cot(x)^3*c
sc(x)^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2691, 3853, 3855}

$$\int \cot^4(x) \csc^3(x) \, dx = -\frac{1}{16} \text{arctanh}(\cos(x)) - \frac{1}{6} \cot^3(x) \csc^3(x) + \frac{1}{8} \cot(x) \csc^3(x) - \frac{1}{16} \cot(x) \csc(x)$$

```
[In] Int[Cot[x]^4*Csc[x]^3,x]
```

```
[Out] -1/16*ArcTanh[Cos[x]] - (Cot[x]*Csc[x])/16 + (Cot[x]*Csc[x]^3)/8 - (Cot[x]^
3*Csc[x]^3)/6
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{1}{6}\cot^3(x)\csc^3(x) - \frac{1}{2}\int \cot^2(x)\csc^3(x)\,dx \\ &= \frac{1}{8}\cot(x)\csc^3(x) - \frac{1}{6}\cot^3(x)\csc^3(x) + \frac{1}{8}\int \csc^3(x)\,dx \\ &= -\frac{1}{16}\cot(x)\csc(x) + \frac{1}{8}\cot(x)\csc^3(x) - \frac{1}{6}\cot^3(x)\csc^3(x) + \frac{1}{16}\int \csc(x)\,dx \\ &= -\frac{1}{16}\text{arctanh}(\cos(x)) - \frac{1}{16}\cot(x)\csc(x) + \frac{1}{8}\cot(x)\csc^3(x) - \frac{1}{6}\cot^3(x)\csc^3(x) \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 95 vs. $2(38) = 76$.

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.50

$$\begin{aligned} \int \cot^4(x)\csc^3(x)\,dx = &-\frac{1}{64}\csc^2\left(\frac{x}{2}\right) + \frac{1}{64}\csc^4\left(\frac{x}{2}\right) - \frac{1}{384}\csc^6\left(\frac{x}{2}\right) - \frac{1}{16}\log\left(\cos\left(\frac{x}{2}\right)\right) \\ &+ \frac{1}{16}\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{1}{64}\sec^2\left(\frac{x}{2}\right) - \frac{1}{64}\sec^4\left(\frac{x}{2}\right) + \frac{1}{384}\sec^6\left(\frac{x}{2}\right) \end{aligned}$$

```
[In] Integrate[Cot[x]^4*Csc[x]^3,x]
[Out] -1/64*Csc[x/2]^2 + Csc[x/2]^4/64 - Csc[x/2]^6/384 - Log[Cos[x/2]]/16 + Log[
Sin[x/2]]/16 + Sec[x/2]^2/64 - Sec[x/2]^4/64 + Sec[x/2]^6/384
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

method	result	size
default	$-\frac{\cos^5(x)}{6\sin(x)^6}-\frac{\cos^5(x)}{24\sin(x)^4}+\frac{\cos^5(x)}{48\sin(x)^2}+\frac{(\cos^3(x))}{48}+\frac{\cos(x)}{16}+\frac{\ln(\csc(x)-\cot(x))}{16}$	52
risch	$\frac{3\,\mathrm{e}^{11ix}+47\,\mathrm{e}^{9ix}+78\,\mathrm{e}^{7ix}+78\,\mathrm{e}^{5ix}+47\,\mathrm{e}^{3ix}+3\,\mathrm{e}^{ix}}{24(\mathrm{e}^{2ix}-1)^6}+\frac{\ln(\mathrm{e}^{ix}-1)}{16}-\frac{\ln(\mathrm{e}^{ix}+1)}{16}$	76

```
[In] int(cot(x)^4*csc(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/6/sin(x)^6*cos(x)^5-1/24/sin(x)^4*cos(x)^5+1/48/sin(x)^2*cos(x)^5+1/48*c
os(x)^3+1/16*cos(x)+1/16*ln(csc(x)-cot(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(30) = 60.

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.45

$$\int \cot^4(x)\csc^3(x)\,dx = \frac{6\cos(x)^5+16\cos(x)^3-3\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+3\left(\cos(x)^6-3\cos(x)^4\right.}{96\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)}$$

```
[In] integrate(cot(x)^4*csc(x)^3,x, algorithm="fricas")
[Out] 1/96*(6*cos(x)^5 + 16*cos(x)^3 - 3*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
*log(1/2*cos(x) + 1/2) + 3*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*log(-1/
2*cos(x) + 1/2) - 6*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \cot^4(x)\csc^3(x)\,dx = -\frac{-3\cos^5(x)-8\cos^3(x)+3\cos(x)}{48\cos^6(x)-144\cos^4(x)+144\cos^2(x)-48} + \frac{\log(\cos(x)-1)}{32}-\frac{\log(\cos(x)+1)}{32}$$

```
[In] integrate(cot(x)**4*csc(x)**3,x)
[Out] -(-3*cos(x)**5 - 8*cos(x)**3 + 3*cos(x))/(48*cos(x)**6 - 144*cos(x)**4 + 14
4*cos(x)**2 - 48) + log(cos(x) - 1)/32 - log(cos(x) + 1)/32
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \cot^4(x)\csc^3(x)\,dx = \frac{3\cos(x)^5 + 8\cos(x)^3 - 3\cos(x)}{48\left(\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1\right)} - \frac{1}{32}\log(\cos(x)+1) + \frac{1}{32}\log(\cos(x)-1)$$

```
[In] integrate(cot(x)^4*csc(x)^3,x, algorithm="maxima")
```

```
[Out] 1/48*(3*cos(x)^5 + 8*cos(x)^3 - 3*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)
^2 - 1) - 1/32*log(cos(x) + 1) + 1/32*log(cos(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \cot^4(x)\csc^3(x)\,dx = \frac{3\cos(x)^5 + 8\cos(x)^3 - 3\cos(x)}{48\left(\cos(x)^2 - 1\right)^3} - \frac{1}{32}\log(\cos(x)+1) + \frac{1}{32}\log(-\cos(x)+1)$$

```
[In] integrate(cot(x)^4*csc(x)^3,x, algorithm="giac")
```

```
[Out] 1/48*(3*cos(x)^5 + 8*cos(x)^3 - 3*cos(x))/(cos(x)^2 - 1)^3 - 1/32*log(cos(x
) + 1) + 1/32*log(-cos(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int \cot^4(x)\csc^3(x)\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{16} + \frac{\frac{\tan\left(\frac{x}{2}\right)^4}{128} + \frac{\tan\left(\frac{x}{2}\right)^2}{128} - \frac{1}{384}}{\tan\left(\frac{x}{2}\right)^6} - \frac{\tan\left(\frac{x}{2}\right)^2}{128} - \frac{\tan\left(\frac{x}{2}\right)^4}{128} + \frac{\tan\left(\frac{x}{2}\right)^6}{384}$$

```
[In] int(cot(x)^4/sin(x)^3,x)
```

```
[Out] log(tan(x/2))/16 + (tan(x/2)^2/128 + tan(x/2)^4/128 - 1/384)/tan(x/2)^6 - t
an(x/2)^2/128 - tan(x/2)^4/128 + tan(x/2)^6/384
```

3.362 $\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 76

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=-\frac{1}{4}\operatorname{arctanh}\left(\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)-\frac{1}{4}\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{1}{2}\sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

```
[Out] -1/4*arctanh(sin(1/4*Pi+1/2*x))-1/4*sec(1/4*Pi+1/2*x)*tan(1/4*Pi+1/2*x)+1/2
*sec(1/4*Pi+1/2*x)^3*tan(1/4*Pi+1/2*x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2691, 3853, 3855}

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=-\frac{1}{4}\operatorname{arctanh}\left(\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)+\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)-\frac{1}{4}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

```
[In] Int[Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2]^2,x]
```

```
[Out] -1/4*ArcTanh[Sin[Pi/4 + x/2]] - (Sec[Pi/4 + x/2]*Tan[Pi/4 + x/2])/4 + (Sec[
Pi/4 + x/2]^3*Tan[Pi/4 + x/2])/2
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)-\frac{1}{4}\int\sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx\\ &= -\frac{1}{4}\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{1}{2}\sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)-\frac{1}{8}\int\csc\left(\frac{\pi}{4}-\frac{x}{2}\right)\,dx\\ &= -\frac{1}{4}\operatorname{arctanh}\left(\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)-\frac{1}{4}\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{1}{2}\sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97

$$\begin{aligned}\int\sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx &= -\frac{1}{4}\operatorname{arctanh}\left(\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)\\ &\quad-\frac{1}{4}\sec^2\left(\frac{1}{4}(\pi+2x)\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\\ &\quad+\frac{1}{2}\sec^4\left(\frac{1}{4}(\pi+2x)\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\end{aligned}$$

```
[In] Integrate[Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2]^2,x]
```

```
[Out] -1/4*ArcTanh[Sin[Pi/4 + x/2]] - (Sec[(Pi + 2*x)/4]^2*Sin[Pi/4 + x/2])/4 + (
Sec[(Pi + 2*x)/4]^4*Sin[Pi/4 + x/2])/2
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\sin^3(\frac{\pi}{4}+\frac{x}{2})}{2\cos(\frac{\pi}{4}+\frac{x}{2})^4}+\frac{\sin^3(\frac{\pi}{4}+\frac{x}{2})}{4\cos(\frac{\pi}{4}+\frac{x}{2})^2}+\frac{\sin(\frac{\pi}{4}+\frac{x}{2})}{4}-\frac{\ln(\sec(\frac{\pi}{4}+\frac{x}{2})+\tan(\frac{\pi}{4}+\frac{x}{2}))}{4}$	76
default	$\frac{\sin^3(\frac{\pi}{4}+\frac{x}{2})}{2\cos(\frac{\pi}{4}+\frac{x}{2})^4}+\frac{\sin^3(\frac{\pi}{4}+\frac{x}{2})}{4\cos(\frac{\pi}{4}+\frac{x}{2})^2}+\frac{\sin(\frac{\pi}{4}+\frac{x}{2})}{4}-\frac{\ln(\sec(\frac{\pi}{4}+\frac{x}{2})+\tan(\frac{\pi}{4}+\frac{x}{2}))}{4}$	76
risch	$\frac{i\left(-(-1)^{\frac{3}{4}}\mathrm{e}^{\frac{7ix}{2}}+7(-1)^{\frac{1}{4}}\mathrm{e}^{\frac{5ix}{2}}+7(-1)^{\frac{3}{4}}\mathrm{e}^{\frac{3ix}{2}}-(-1)^{\frac{1}{4}}\mathrm{e}^{\frac{ix}{2}}\right)}{2(i\mathrm{e}^{ix}+1)^4}-\frac{\ln\left(\mathrm{e}^{\frac{i(\pi+2x)}{4}}+i\right)}{4}+\frac{\ln\left(\mathrm{e}^{\frac{i(\pi+2x)}{4}}-i\right)}{4}$	88

[In] int(sec(1/4*Pi+1/2*x)^3*tan(1/4*Pi+1/2*x)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*sin(1/4*Pi+1/2*x)^3/cos(1/4*Pi+1/2*x)^4+1/4*sin(1/4*Pi+1/2*x)^3/cos(1/4*Pi+1/2*x)^2+1/4*sin(1/4*Pi+1/2*x)-1/4*ln(sec(1/4*Pi+1/2*x)+tan(1/4*Pi+1/2*x))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = -\frac{\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^4\log\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right)-\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^4\log\left(-\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right)+2\left(\cos\left(\frac{1}{4}\pi+\frac{1}{2}\right.\right.}{8\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^4}$$

[In] integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="fricas")

[Out] -1/8*(cos(1/4*pi + 1/2*x)^4*log(sin(1/4*pi + 1/2*x) + 1) - cos(1/4*pi + 1/2*x)^4*log(-sin(1/4*pi + 1/2*x) + 1) + 2*(cos(1/4*pi + 1/2*x)^2 - 2)*sin(1/4*pi + 1/2*x))/cos(1/4*pi + 1/2*x)^4

Sympy [F]

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = \int \tan^2\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)\,dx$$

[In] integrate(sec(1/4*pi+1/2*x)**3*tan(1/4*pi+1/2*x)**2,x)

[Out] Integral(tan(x/2 + pi/4)**2*sec(x/2 + pi/4)**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = \frac{\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)^3+\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)}{4\left(\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)^4-2\,\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)^2+1\right)} - \frac{1}{8}\,\log\left(\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)+1\right) + \frac{1}{8}\,\log\left(\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)-1\right)$$

```
[In] integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(sin(1/4*pi + 1/2*x)^3 + sin(1/4*pi + 1/2*x))/(sin(1/4*pi + 1/2*x)^4 -
2*sin(1/4*pi + 1/2*x)^2 + 1) - 1/8*log(sin(1/4*pi + 1/2*x) + 1) + 1/8*log(s
in(1/4*pi + 1/2*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.25

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = \frac{\frac{1}{\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)}+\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)}{4\left(\left(\frac{1}{\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)}+\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)\right)^2-4\right)} - \frac{1}{16}\,\log\left(\left|\frac{1}{\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)}+\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)+2\right|\right) + \frac{1}{16}\,\log\left(\left|\frac{1}{\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)}+\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)-2\right|\right)$$

```
[In] integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="giac")
```

```
[Out] 1/4*(1/sin(1/4*pi + 1/2*x) + sin(1/4*pi + 1/2*x))/((1/sin(1/4*pi + 1/2*x) +
 sin(1/4*pi + 1/2*x))^2 - 4) - 1/16*log(abs(1/sin(1/4*pi + 1/2*x) + sin(1/4
*pi + 1/2*x) + 2)) + 1/16*log(abs(1/sin(1/4*pi + 1/2*x) + sin(1/4*pi + 1/2*
x) - 2))
```

Mupad [B] (verification not implemented)

Time = 6.41 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{2\left(\frac{\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)^7}{4}+\frac{7\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)^5}{4}+\frac{7\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)^3}{4}+\frac{\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)}{4}\right)}{\left(\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)^2-1\right)^4}-\frac{\operatorname{atanh}\left(\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)\right)}{2}$$

```
[In] int(tan(Pi/4 + x/2)^2/cos(Pi/4 + x/2)^3,x)
```

```
[Out] (2*(tan(Pi/8 + x/4)/4 + (7*tan(Pi/8 + x/4)^3)/4 + (7*tan(Pi/8 + x/4)^5)/4 +
 tan(Pi/8 + x/4)^7/4))/(tan(Pi/8 + x/4)^2 - 1)^4 - atanh(tan(Pi/8 + x/4))/2
```

3.363 $\int \left(1+\cot^3(x)\right) \left(a \sec^2(x)-\sin(2 x)\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \left(1+\cot^3(x)\right) \left(a \sec^2(x)-\sin(2 x)\right)^2 \, dx=\frac{x}{2}+4 a x+2 \cos^2(x)+\cos^4(x)+4 a \cot(x)-\frac{1}{2} a^2 \cot^2(x)+(4-a) a \log(\cos(x))+\left(4+a^2\right) \log(\sin(x))+\frac{1}{2} \cos(x) \sin(x)-\cos^3(x) \sin(x)+a^2 \tan(x)+\frac{1}{3} a^2 \tan^3(x)$$

```
[Out] 1/2*x+4*a*x+2*cos(x)^2+cos(x)^4+4*a*cot(x)-1/2*a^2*cot(x)^2+(4-a)*a*ln(cos(
x))+(a^2+4)*ln(sin(x))+1/2*cos(x)*sin(x)-cos(x)^3*sin(x)+a^2*tan(x)+1/3*a^2
*tan(x)^3
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1819, 1816, 649, 209, 266}

$$\int \left(1+\cot^3(x)\right) \left(a \sec^2(x)-\sin(2 x)\right)^2 \, dx=\frac{1}{3} a^2 \tan^3(x)+a^2 \tan(x)-\frac{1}{2} a^2 \cot^2(x)+\left(a^2+4\right) \log(\tan(x))+\frac{1}{2}(8 a+1) x+4 a \cot(x)+4(a+1) \log(\cos(x))+\cos^4(x)(1-\tan(x))+\frac{1}{2} \cos^2(x)(\tan(x)+4)$$

```
[In] Int[(1 + Cot[x]^3)*(a*Sec[x]^2 - Sin[2*x])^2,x]
```

```
[Out] ((1 + 8*a)*x)/2 + 4*a*Cot[x] - (a^2*Cot[x]^2)/2 + 4*(1 + a)*Log[Cos[x]] + (
4 + a^2)*Log[Tan[x]] + Cos[x]^4*(1 - Tan[x]) + a^2*Tan[x] + (a^2*Tan[x]^3)/
3 + (Cos[x]^2*(4 + Tan[x]))/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(1+x^3)\left(a-2x+2ax^2+ax^4\right)^2}{x^3\left(1+x^2\right)^3}\,dx, x, \tan(x)\right)$$

$$= \cos^4(x)(1-\tan(x)) - \frac{1}{4}\text{Subst}\left(\int \frac{-4a^2+16ax-4\left(4+3a^2\right)x^2-4\left(1-4a+a^2\right)x^3+4(4-3a)ax^4-12a^2x^5+4(4-}{x^3\left(1+x^2\right)^2}\right.$$

$$= \cos^4(x)(1-\tan(x)) + \frac{1}{2}\cos^2(x)(4+\tan(x)) + \frac{1}{8}\text{Subst}\left(\int \frac{8a^2 - 32ax + 16(2+a^2)x^2 + 4(1+2a^2)x^3 - 8(4-a)ax^4 + 16a^2x^5 + 8a^2x^7}{x^3(1+x^2)}\,dx, x,$$

$$= \cos^4(x)(1-\tan(x)) + \frac{1}{2}\cos^2(x)(4+\tan(x)) + \frac{1}{8}\text{Subst}\left(\int \left(8a^2 + \frac{8a^2}{x^3} - \frac{32a}{x^2} + \frac{8(4+a^2)}{x} + 8a^2x^2 + \frac{4(1+8a-8(1+a)x)}{1+x^2}\right)dx, x, \tan(x)\right)$$

$$= 4a\cot(x) - \frac{1}{2}a^2\cot^2(x) + (4+a^2)\log(\tan(x)) + \cos^4(x)(1-\tan(x)) + a^2\tan(x) + \frac{1}{3}a^2\tan^3(x) + \frac{1}{2}\cos^2(x)(4+\tan(x)) + \frac{1}{2}\text{Subst}\left(\int \frac{1+8a-8(1+a)x}{1+x^2}\,dx, x, \tan(x)\right)$$

$$= 4a\cot(x) - \frac{1}{2}a^2\cot^2(x) + (4+a^2)\log(\tan(x)) + \cos^4(x)(1-\tan(x)) + a^2\tan(x) + \frac{1}{3}a^2\tan^3(x) + \frac{1}{2}\cos^2(x)(4+\tan(x)) - (4(1+a))\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \tan(x)\right) + \frac{1}{2}(1+8a)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)$$

$$= \frac{1}{2}(1+8a)x + 4a\cot(x) - \frac{1}{2}a^2\cot^2(x) + 4(1+a)\log(\cos(x)) + (4+a^2)\log(\tan(x)) + \cos^4(x)(1-\tan(x)) + a^2\tan(x) + \frac{1}{3}a^2\tan^3(x) + \frac{1}{2}\cos^2(x)(4+\tan(x))$$

Mathematica [A] (verified)

Time = 3.58 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.44

$$\int (1+\cot^3(x))\left(a\sec^2(x)-\sin(2x)\right)^2 dx = -\frac{2\cos^3(x)\sin(x)\left(-a\sec^2(x)+\sin(2x)\right)^2\left(-96a\cot^2(x) - 8a^2(2+\cos(2x))\sec^2(x) - 3\cot(x)(4x+32a\right.}{3(-4a}$$

```
[In] Integrate[(1 + Cot[x]^3)*(a*Sec[x]^2 - Sin[2*x])^2,x]
```

```
[Out] (-2*Cos[x]^3*Sin[x]*(-(a*Sec[x]^2) + Sin[2*x])^2*(-96*a*Cot[x]^2 - 8*a^2*(2
 + Cos[2*x])*Sec[x]^2 - 3*Cot[x]*(4*x + 32*a*x + 12*Cos[2*x] + Cos[4*x] - 4
*a^2*Csc[x]^2 + 32*a*Log[Cos[x]] - 8*a^2*Log[Cos[x]] + 32*Log[Sin[x]] + 8*a
^2*Log[Sin[x]] - Sin[4*x])))/(3*(-4*a + 2*Sin[2*x] + Sin[4*x])^2)
```

Maple [A] (verified)

Time = 59.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

method	result
parts	$-\frac{\sin(2x)\cos(2x)}{4}+\frac{x}{2}+\cos^4(x)+2(\cos^2(x))+4\ln(\sin(x))-4a(-x-\cot(x))+a^2\left(-\frac{1}{2\sin(x)^2}+\ln\right.$
default	$\cos^4(x)+2(\cos^2(x))+4\ln(\sin(x))-4a(-x-\cot(x))+a^2\left(-\frac{1}{2\sin(x)^2}+\ln(\tan(x))\right)+\frac{x}{2}+\cos$
risch	$\frac{x}{2}-\frac{ie^{-4ix}}{16}+4ax-4ix+\frac{e^{4ix}}{16}-4iax+\frac{3e^{2ix}}{4}+\frac{3e^{-2ix}}{4}+\frac{e^{-4ix}}{16}+\frac{ie^{4ix}}{16}+\frac{2a(12ie^{8ix}+3a e^{8ix}+6ia e^{6ix}+24ie^{6i}}{}$

```
[In] int((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*sin(2*x)*cos(2*x)+1/2*x+cos(x)^4+2*cos(x)^2+4*ln(sin(x))-4*a*(-x-cot(x
))+a^2*(-1/2/sin(x)^2+ln(tan(x)))-a^2*(-2/3-1/3*sec(x)^2)*tan(x)-4*a*ln(sec
(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(79) = 158.

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.02

$$\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx$$
$$=\frac{24\cos(x)^9+24\cos(x)^7+3(4(8a+1)x-27)\cos(x)^5+3(4a^2-4(8a+1)x+11)\cos(x)^3-12\left((a^2-\right.}{}$$

```
[In] integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(24*cos(x)^9 + 24*cos(x)^7 + 3*(4*(8*a + 1)*x - 27)*cos(x)^5 + 3*(4*a^
2 - 4*(8*a + 1)*x + 11)*cos(x)^3 - 12*((a^2 - 4*a)*cos(x)^5 - (a^2 - 4*a)*c
os(x)^3)*log(cos(x)^2) + 12*((a^2 + 4)*cos(x)^5 - (a^2 + 4)*cos(x)^3)*log(-
1/4*cos(x)^2 + 1/4) - 4*(6*cos(x)^8 - 9*cos(x)^6 - (4*a^2 - 24*a - 3)*cos(x
)^4 + 2*a^2*cos(x)^2 + 2*a^2)*sin(x))/(cos(x)^5 - cos(x)^3)
```

Sympy [A] (verification not implemented)

Time = 158.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.14

$$\int (1+\cot^3(x)) \left(a\sec^2(x)-\sin(2x)\right)^2 \, dx = -\frac{a^2\log\left(\sin^2(x)-1\right)}{2} + a^2\log(\sin(x)) + \frac{a^2\tan^3(x)}{3} + a^2\tan(x) - \frac{a^2}{2\sin^2(x)} + 4ax + 4a\log(\cos(x)) + \frac{4a\cos(x)}{\sin(x)} + \frac{x}{2} + 4\log(\sin(x)) + \sin^4(x) - 4\sin^2(x) - \frac{\sin(4x)}{8}$$

```
[In] integrate((1+cot(x)**3)*(a*sec(x)**2-sin(2*x))**2,x)
```

```
[Out] -a**2*log(sin(x)**2 - 1)/2 + a**2*log(sin(x)) + a**2*tan(x)**3/3 + a**2*tan
(x) - a**2/(2*sin(x)**2) + 4*a*x + 4*a*log(cos(x)) + 4*a*cos(x)/sin(x) + x/
2 + 4*log(sin(x)) + sin(x)**4 - 4*sin(x)**2 - sin(4*x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

$$\int (1+\cot^3(x)) \left(a\sec^2(x)-\sin(2x)\right)^2 \, dx = \frac{1}{3}\left(\tan(x)^3 + 3\tan(x)\right)a^2 - \frac{1}{2}a^2\left(\frac{1}{\sin(x)^2} + \log\left(\sin(x)^2-1\right) - \log\left(\sin(x)^2\right)\right) + 4a\left(x + \frac{1}{\tan(x)}\right) + 2a\log\left(-\sin(x)^2+1\right) + \frac{1}{2}x + \frac{1}{8}\cos(4x) + \frac{3}{2}\cos(2x) + 2\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right) + 2\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right) - \frac{1}{8}\sin(4x)$$

```
[In] integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(x)^3 + 3*tan(x))*a^2 - 1/2*a^2*(1/sin(x)^2 + log(sin(x)^2 - 1) - l
og(sin(x)^2)) + 4*a*(x + 1/tan(x)) + 2*a*log(-sin(x)^2 + 1) + 1/2*x + 1/8*c
os(4*x) + 3/2*cos(2*x) + 2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 2*log(
cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 1/8*sin(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.69

$$\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx=\frac{1}{3}\,a^2\tan\left(x\right)^3+a^2\tan\left(x\right)+\frac{1}{2}\,(8\,a+1)x-2\,(a+1)\log\left(\tan\left(x\right)^2+1\right)+\left(a^2+4\right)\log\left(\left|\tan\left(x\right)\right|\right)-\frac{a^2\tan\left(x\right)^6-4\,a\tan\left(x\right)^6+3\,a^2\tan\left(x\right)^4-8\,a\tan\left(x\right)^5-8\,a\tan\left(x\right)^4-\tan\left(x\right)^5+3\,a^2\tan\left(x\right)^2-16\,a\tan}{2\left(\tan\left(x\right)^3+\tan\left(x\right)\right)^2}$$

```
[In] integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*tan(x)^3 + a^2*tan(x) + 1/2*(8*a + 1)*x - 2*(a + 1)*log(tan(x)^2 +
1) + (a^2 + 4)*log(abs(tan(x))) - 1/2*(a^2*tan(x)^6 - 4*a*tan(x)^6 + 3*a^2*
tan(x)^4 - 8*a*tan(x)^5 - 8*a*tan(x)^4 - tan(x)^5 + 3*a^2*tan(x)^2 - 16*a*t
an(x)^3 - 4*tan(x)^4 - 4*a*tan(x)^2 + tan(x)^3 + a^2 - 8*a*tan(x) - 6*tan(x
)^2)/(tan(x)^3 + tan(x))^2
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.51

$$\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx=a^2\tan(x)-\frac{\tan(x)^4\left(\frac{a^2}{2}-2\right)-4\,a\tan(x)+\frac{a^2}{2}-\tan(x)^5\left(4\,a+\frac{1}{2}\right)-\tan(x)^3\left(8\,a-\frac{1}{2}\right)+\tan(x)^2\left(a^2-3\right)}{\tan\left(x\right)^6+2\tan\left(x\right)^4+\tan\left(x\right)^2}-\ln\left(\tan(x)-\mathrm{i}\right)\left(a\left(2+2\mathrm{i}\right)+2+\frac{1}{4}\mathrm{i}\right)-\ln\left(\tan(x)+1\mathrm{i}\right)\left(a\left(2-2\mathrm{i}\right)+2-\frac{1}{4}\mathrm{i}\right)+\frac{a^2\tan(x)^3}{3}+\ln\left(\tan(x)\right)\left(a^2+4\right)$$

```
[In] int((cot(x)^3 + 1)*(sin(2*x) - a/cos(x)^2)^2,x)
```

```
[Out] a^2*tan(x) - (tan(x)^4*(a^2/2 - 2) - 4*a*tan(x) + a^2/2 - tan(x)^5*(4*a + 1
/2) - tan(x)^3*(8*a - 1/2) + tan(x)^2*(a^2 - 3))/(tan(x)^2 + 2*tan(x)^4 + t
an(x)^6) - log(tan(x) - 1i)*(a*(2 + 2i) + (2 + 1i/4)) - log(tan(x) + 1i)*(a
*(2 - 2i) + (2 - 1i/4)) + (a^2*tan(x)^3)/3 + log(tan(x))*(a^2 + 4)
```

3.364 $\int (4-3\cos(x)) \left(1-\frac{\sin(x)}{2}\right)^4 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (4-3\cos(x)) \left(1-\frac{\sin(x)}{2}\right)^4 \, dx = \frac{227x}{32} + 10\cos(x) - 3\cos^2(x) - \frac{2\cos^3(x)}{3} - 3\sin(x) - \frac{99}{32}\cos(x)\sin(x) - \frac{3\sin^3(x)}{2} - \frac{1}{16}\cos(x)\sin^3(x) + \frac{3\sin^4(x)}{8} - \frac{3\sin^5(x)}{80}$$

```
[Out] 227/32*x+10*cos(x)-3*cos(x)^2-2/3*cos(x)^3-3*sin(x)-99/32*cos(x)*sin(x)-3/2
*sin(x)^3-1/16*cos(x)*sin(x)^3+3/8*sin(x)^4-3/80*sin(x)^5
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4486, 2717, 2747, 2748, 2715, 8, 655}

$$\int (4-3\cos(x)) \left(1-\frac{\sin(x)}{2}\right)^4 \, dx = \frac{227x}{32} - \frac{3}{80}\sin^5(x) + \frac{3\sin^4(x)}{8} - \frac{3\sin^3(x)}{2} - 3\sin(x) - \frac{2\cos^3(x)}{3} - 3\cos^2(x) + 10\cos(x) - \frac{1}{16}\sin^3(x)\cos(x) - \frac{99}{32}\sin(x)\cos(x)$$

```
[In] Int[(4 - 3*Cos[x])*(1 - Sin[x]/2)^4,x]
```

```
[Out] (227*x)/32 + 10*Cos[x] - 3*Cos[x]^2 - (2*Cos[x]^3)/3 - 3*Sin[x] - (99*Cos[x
]*Sin[x])/32 - (3*Sin[x]^3)/2 - (Cos[x]*Sin[x]^3)/16 + (3*Sin[x]^4)/8 - (3*
Sin[x]^5)/80
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(4-3\cos(x)+2(-4+3\cos(x))\sin(x)-\frac{3}{2}(-4+3\cos(x))\sin^2(x)\right. \\
&\qquad \left. +\frac{1}{2}(-4+3\cos(x))\sin^3(x)-\frac{1}{16}(-4+3\cos(x))\sin^4(x)\right)\,dx \\
&= 4x-\frac{1}{16}\int(-4+3\cos(x))\sin^4(x)\,dx+\frac{1}{2}\int(-4+3\cos(x))\sin^3(x)\,dx \\
&\quad -\frac{3}{2}\int(-4+3\cos(x))\sin^2(x)\,dx+2\int(-4+3\cos(x))\sin(x)\,dx-3\int\cos(x)\,dx \\
&= 4x-3\sin(x)-\frac{3\sin^3(x)}{2}-\frac{3\sin^5(x)}{80}-\frac{1}{54}\text{Subst}\left(\int(-4+x)\left(9-x^2\right)\,dx,x,3\cos(x)\right) \\
&\quad +\frac{1}{4}\int\sin^4(x)\,dx-\frac{2}{3}\text{Subst}\left(\int(-4+x)\,dx,x,3\cos(x)\right)+6\int\sin^2(x)\,dx \\
&= 4x+8\cos(x)-3\cos^2(x)-3\sin(x)-3\cos(x)\sin(x)-\frac{3\sin^3(x)}{2}-\frac{1}{16}\cos(x)\sin^3(x) \\
&\quad +\frac{3\sin^4(x)}{8}-\frac{3\sin^5(x)}{80}+\frac{2}{27}\text{Subst}\left(\int\left(9-x^2\right)\,dx,x,3\cos(x)\right)+\frac{3}{16}\int\sin^2(x)\,dx \\
&\quad +3\int 1\,dx \\
&= 7x+10\cos(x)-3\cos^2(x)-\frac{2\cos^3(x)}{3}-3\sin(x)-\frac{99}{32}\cos(x)\sin(x) \\
&\quad -\frac{3\sin^3(x)}{2}-\frac{1}{16}\cos(x)\sin^3(x)+\frac{3\sin^4(x)}{8}-\frac{3\sin^5(x)}{80}+\frac{3\int 1\,dx}{32} \\
&= \frac{227x}{32}+10\cos(x)-3\cos^2(x)-\frac{2\cos^3(x)}{3}-3\sin(x)-\frac{99}{32}\cos(x)\sin(x) \\
&\quad -\frac{3\sin^3(x)}{2}-\frac{1}{16}\cos(x)\sin^3(x)+\frac{3\sin^4(x)}{8}-\frac{3\sin^5(x)}{80}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\begin{aligned}
\int(4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4\,dx &= \frac{227x}{32}+\frac{19\cos(x)}{2}-\frac{27}{16}\cos(2x)-\frac{1}{6}\cos(3x) \\
&\quad +\frac{3}{64}\cos(4x)-\frac{531\sin(x)}{128}-\frac{25}{16}\sin(2x) \\
&\quad +\frac{99}{256}\sin(3x)+\frac{1}{128}\sin(4x)-\frac{3\sin(5x)}{1280}
\end{aligned}$$

```
[In] Integrate[(4 - 3*Cos[x])*(1 - Sin[x]/2)^4,x]
```

```
[Out] (227*x)/32 + (19*Cos[x])/2 - (27*Cos[2*x])/16 - Cos[3*x]/6 + (3*Cos[4*x])/6
4 - (531*Sin[x])/128 - (25*Sin[2*x])/16 + (99*Sin[3*x])/256 + Sin[4*x]/128
- (3*Sin[5*x])/1280
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.66

method	result
parts	$\frac{227x}{32}-\frac{3(\sin(x)-2)^5}{80}-3\cos(x)\sin(x)+\frac{2(2+\sin^2(x))\cos(x)}{3}-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{16}+8\cos(x)$
risch	$\frac{227x}{32}+\frac{19\cos(x)}{2}-\frac{531\sin(x)}{128}-\frac{3\sin(5x)}{1280}+\frac{3\cos(4x)}{64}+\frac{\sin(4x)}{128}-\frac{\cos(3x)}{6}+\frac{99\sin(3x)}{256}-\frac{27\cos(2x)}{16}-\frac{25\sin(2x)}{16}$ …
parallelrisch	$-\frac{409}{960}+\frac{227x}{32}-\frac{25\sin(2x)}{16}-\frac{3\sin(5x)}{1280}+\frac{99\sin(3x)}{256}+\frac{\sin(4x)}{128}-\frac{531\sin(x)}{128}+\frac{3\cos(4x)}{64}-\frac{27\cos(2x)}{16}-\frac{\cos(3x)}{6}$ …
default	$-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{16}+\frac{227x}{32}+\frac{2(2+\sin^2(x))\cos(x)}{3}-3\cos(x)\sin(x)+8\cos(x)-\frac{3(\sin^5(x))}{80}+$ …
norman	$\frac{28\left(\tan^8\left(\frac{x}{2}\right)\right)+114\left(\tan^6\left(\frac{x}{2}\right)\right)+\frac{268\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}+\frac{470\left(\tan^4\left(\frac{x}{2}\right)\right)}{3}+\frac{227x}{32}-\frac{391\left(\tan^3\left(\frac{x}{2}\right)\right)}{8}-\frac{306\left(\tan^5\left(\frac{x}{2}\right)\right)}{5}-\frac{185\left(\tan^7\left(\frac{x}{2}\right)\right)}{8}+\frac{3\left(\tan^9\left(\frac{x}{2}\right)\right)}{16}\ldots}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^5}$

```
[In] int((4-3*cos(x))*(1-1/2*sin(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 227/32*x-3/80*(sin(x)-2)^5-3*cos(x)*sin(x)+2/3*(2+sin(x)^2)*cos(x)-1/16*(si
n(x)^3+3/2*sin(x))*cos(x)+8*cos(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\begin{aligned}\int(4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4\,dx &= \frac{3}{8}\cos(x)^4-\frac{2}{3}\cos(x)^3-\frac{15}{4}\cos(x)^2\\ &\quad-\frac{1}{160}\left(6\cos(x)^4-10\cos(x)^3-252\cos(x)^2+505\cos(x)+726\right)\sin(x)\\ &\quad+\frac{227}{32}x+10\cos(x)\end{aligned}$$

```
[In] integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="fricas")
```

```
[Out] 3/8*cos(x)^4 - 2/3*cos(x)^3 - 15/4*cos(x)^2 - 1/160*(6*cos(x)^4 - 10*cos(x)
^3 - 252*cos(x)^2 + 505*cos(x) + 726)*sin(x) + 227/32*x + 10*cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.11

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4\,dx = \frac{3x\sin^4(x)}{32}+\frac{3x\sin^2(x)\cos^2(x)}{16}+3x\sin^2(x)+\frac{3x\cos^4(x)}{32}+3x\cos^2(x)+4x-\frac{3\sin^5(x)}{80}+\frac{3\sin^4(x)}{8}-\frac{5\sin^3(x)\cos(x)}{32}-\frac{3\sin^3(x)}{2}+2\sin^2(x)\cos(x)-\frac{3\sin(x)\cos^3(x)}{32}-3\sin(x)\cos(x)-3\sin(x)+\frac{4\cos^3(x)}{3}-3\cos^2(x)+8\cos(x)$$

```
[In] integrate((4-3*cos(x))*(1-1/2*sin(x))**4,x)
```

```
[Out] 3*x*sin(x)**4/32 + 3*x*sin(x)**2*cos(x)**2/16 + 3*x*sin(x)**2 + 3*x*cos(x)*
*4/32 + 3*x*cos(x)**2 + 4*x - 3*sin(x)**5/80 + 3*sin(x)**4/8 - 5*sin(x)**3*
cos(x)/32 - 3*sin(x)**3/2 + 2*sin(x)**2*cos(x) - 3*sin(x)*cos(x)**3/32 - 3*
sin(x)*cos(x) - 3*sin(x) + 4*cos(x)**3/3 - 3*cos(x)**2 + 8*cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4\,dx = -\frac{3}{80}\sin(x)^5+\frac{3}{8}\sin(x)^4-\frac{2}{3}\cos(x)^3-\frac{3}{2}\sin(x)^3-3\cos(x)^2+\frac{227}{32}x+10\cos(x)+\frac{1}{128}\sin(4x)-\frac{25}{16}\sin(2x)-3\sin(x)$$

```
[In] integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="maxima")
```

```
[Out] -3/80*sin(x)^5 + 3/8*sin(x)^4 - 2/3*cos(x)^3 - 3/2*sin(x)^3 - 3*cos(x)^2 +
227/32*x + 10*cos(x) + 1/128*sin(4*x) - 25/16*sin(2*x) - 3*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx = \frac{227}{32}\,x + \frac{3}{64}\,\cos\left(4\,x\right) - \frac{1}{6}\,\cos\left(3\,x\right) - \frac{27}{16}\,\cos\left(2\,x\right) + \frac{19}{2}\,\cos\left(x\right) - \frac{3}{1280}\,\sin\left(5\,x\right) + \frac{1}{128}\,\sin\left(4\,x\right) + \frac{99}{256}\,\sin\left(3\,x\right) - \frac{25}{16}\,\sin\left(2\,x\right) - \frac{531}{128}\,\sin\left(x\right)$$

```
[In] integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="giac")
```

```
[Out] 227/32*x + 3/64*cos(4*x) - 1/6*cos(3*x) - 27/16*cos(2*x) + 19/2*cos(x) - 3/
1280*sin(5*x) + 1/128*sin(4*x) + 99/256*sin(3*x) - 25/16*sin(2*x) - 531/128
*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx = -\frac{6\,\sin\left(\frac{x}{2}\right)\,\cos\left(\frac{x}{2}\right)^9}{5} + 6\cos\left(\frac{x}{2}\right)^8 + \frac{17\,\sin\left(\frac{x}{2}\right)\,\cos\left(\frac{x}{2}\right)^7}{5} - \frac{52\cos\left(\frac{x}{2}\right)^6}{3} + \frac{93\,\sin\left(\frac{x}{2}\right)\,\cos\left(\frac{x}{2}\right)^5}{10} + 2\cos\left(\frac{x}{2}\right)^4 - \frac{191\,\sin\left(\frac{x}{2}\right)\,\cos\left(\frac{x}{2}\right)^3}{8} + 28\cos\left(\frac{x}{2}\right)^2 + \frac{3\,\sin\left(\frac{x}{2}\right)\,\cos\left(\frac{x}{2}\right)}{16} + \frac{227\,x}{32}$$

```
[In] int(-(3*cos(x) - 4)*(sin(x)/2 - 1)^4,x)
```

```
[Out] (227*x)/32 - (191*cos(x/2)^3*sin(x/2))/8 + (93*cos(x/2)^5*sin(x/2))/10 + (1
7*cos(x/2)^7*sin(x/2))/5 - (6*cos(x/2)^9*sin(x/2))/5 + 28*cos(x/2)^2 + 2*co
s(x/2)^4 - (52*cos(x/2)^6)/3 + 6*cos(x/2)^8 + (3*cos(x/2)*sin(x/2))/16
```

3.365 $\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx = -\frac{285x}{2}+5(3-2\cot(x))^2+(3-2\cot(x))^3-42\cot(x)+4\log(\sin(x))$$

```
[Out] -285/2*x+5*(3-2*cot(x))^2+(3-2*cot(x))^3-42*cot(x)+4*ln(sin(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3609, 3606, 3556}

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx = -\frac{285x}{2}+(3-2\cot(x))^3+5(3-2\cot(x))^2-42\cot(x)+4\log(\sin(x))$$

```
[In] Int[(1/2 - 3*Cot[x])*(3 - 2*Cot[x])^3,x]
```

```
[Out] (-285*x)/2 + 5*(3 - 2*Cot[x])^2 + (3 - 2*Cot[x])^3 - 42*Cot[x] + 4*Log[Sin[
x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (3-2\cot(x))^3+\int\left(-\frac{9}{2}-10\cot(x)\right)(3-2\cot(x))^2\,dx\\
&= 5(3-2\cot(x))^2+(3-2\cot(x))^3+\int\left(-\frac{67}{2}-21\cot(x)\right)(3-2\cot(x))\,dx\\
&= -\frac{285x}{2}+5(3-2\cot(x))^2+(3-2\cot(x))^3-42\cot(x)+4\int\cot(x)\,dx\\
&= -\frac{285x}{2}+5(3-2\cot(x))^2+(3-2\cot(x))^3-42\cot(x)+4\log(\sin(x))
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.82

$$\int\left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx=\frac{27x}{2}+56\cot^2(x)-8\cot^3(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(x)\right)-180\cot(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(x)\right)+4\log(\cos(x))+4\log(\tan(x))$$

```
[In] Integrate[(1/2 - 3*Cot[x])*(3 - 2*Cot[x])^3,x]
[Out] (27*x)/2 + 56*Cot[x]^2 - 8*Cot[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[x
]^2] - 180*Cot[x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[x]^2] + 4*Log[Cos[x]
] + 4*Log[Tan[x]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

method	result
parallelrisch	$4\ln\left(\tan\left(x\right)\right)-2\ln\left(\sec^{2}\left(x\right)\right)-\frac{285x}{2}-8(\cot^{3}\left(x\right))-156\cot\left(x\right)+56(\cot^{2}\left(x\right))$
derivativedivides	$-8(\cot^{3}\left(x\right))+56(\cot^{2}\left(x\right))-156\cot\left(x\right)-2\ln\left(\cot^{2}\left(x\right)+1\right)+\frac{285\pi}{4}-\frac{285\,\mathrm{arccot}(\cot(x))}{2}$
default	$-8(\cot^{3}\left(x\right))+56(\cot^{2}\left(x\right))-156\cot\left(x\right)-2\ln\left(\cot^{2}\left(x\right)+1\right)+\frac{285\pi}{4}-\frac{285\,\mathrm{arccot}(\cot(x))}{2}$
norman	$\frac{-8-156(\tan^{2}(x))-\frac{285x\left(\tan^{3}(x)\right)}{2}+56\tan(x)}{\tan(x)^{3}}+4\ln\left(\tan\left(x\right)\right)-2\ln\left(1+\tan^{2}\left(x\right)\right)$
parts	$\frac{27x}{2}-156\cot\left(x\right)+78\pi-156\,\mathrm{arccot}\left(\cot\left(x\right)\right)+56(\cot^{2}\left(x\right))-56\ln\left(\cot^{2}\left(x\right)+1\right)-8$
risch	$-\frac{285x}{2}-4ix+\frac{\left(-\frac{224}{1873}-\frac{264i}{1873}\right)\left(1873\,\mathrm{e}^{4ix}-1260i\mathrm{e}^{2ix}-3358\,\mathrm{e}^{2ix}+1221+1036i\right)}{\left(\mathrm{e}^{2ix}-1\right)^{3}}+4\ln\left(\mathrm{e}^{2ix}-1\right)$

```
[In] int((1/2-3*cot(x))*(3-2*cot(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4*ln(tan(x))-2*ln(sec(x)^2)-285/2*x-8*cot(x)^3-156*cot(x)+56*cot(x)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(33) = 66$.

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.15

$$\int\left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx$$
$$=\frac{4\left(\cos\left(2\,x\right)-1\right)\log\left(-\frac{1}{2}\,\cos\left(2\,x\right)+\frac{1}{2}\right)\sin\left(2\,x\right)-296\,\cos\left(2\,x\right)^{2}-\left(285\,x\cos\left(2\,x\right)-285\,x+224\right)\sin\left(2\,x\right)}{2\left(\cos\left(2\,x\right)-1\right)\sin\left(2\,x\right)}$$

```
[In] integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*(cos(2*x) - 1)*log(-1/2*cos(2*x) + 1/2)*sin(2*x) - 296*cos(2*x)^2 -
(285*x*cos(2*x) - 285*x + 224)*sin(2*x) + 32*cos(2*x) + 328)/((cos(2*x) - 1
)*sin(2*x))
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int\left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx=-\frac{285x}{2}-2\log\left(\tan^2(x)+1\right)+4\log(\tan(x))-\frac{156}{\tan(x)}+\frac{56}{\tan^2(x)}-\frac{8}{\tan^3(x)}$$

```
[In] integrate((1/2-3*cot(x))*(3-2*cot(x))**3,x)
[Out] -285*x/2 - 2*log(tan(x)**2 + 1) + 4*log(tan(x)) - 156/tan(x) + 56/tan(x)**2
 - 8/tan(x)**3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int\left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx=-\frac{285}{2}\,x-\frac{4\left(39\,\tan(x)^2-14\,\tan(x)+2\right)}{\tan(x)^3}-2\,\log\left(\tan(x)^2+1\right)+4\,\log(\tan(x))$$

```
[In] integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="maxima")
[Out] -285/2*x - 4*(39*tan(x)^2 - 14*tan(x) + 2)/tan(x)^3 - 2*log(tan(x)^2 + 1) +
 4*log(tan(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(33) = 66.

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.27

$$\int\left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx=\tan\left(\frac{1}{2}\,x\right)^3+14\,\tan\left(\frac{1}{2}\,x\right)^2-\frac{285}{2}\,x-\frac{22\,\tan\left(\frac{1}{2}\,x\right)^3+225\,\tan\left(\frac{1}{2}\,x\right)^2-42\,\tan\left(\frac{1}{2}\,x\right)+3}{3\,\tan\left(\frac{1}{2}\,x\right)^3}-4\,\log\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)+4\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)+75\,\tan\left(\frac{1}{2}\,x\right)$$

```
[In] integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="giac")
```

```
[Out] tan(1/2*x)^3 + 14*tan(1/2*x)^2 - 285/2*x - 1/3*(22*tan(1/2*x)^3 + 225*tan(1
/2*x)^2 - 42*tan(1/2*x) + 3)/tan(1/2*x)^3 - 4*log(tan(1/2*x)^2 + 1) + 4*log
(abs(tan(1/2*x))) + 75*tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.27

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx = x\left(-\frac{285}{2}-4\mathrm{i}\right)+4\,\ln\left(\mathrm{e}^{x\,2\mathrm{i}}-1\right)+\frac{64\mathrm{i}}{3\,\mathrm{e}^{x\,2\mathrm{i}}-3\,\mathrm{e}^{x\,4\mathrm{i}}+\mathrm{e}^{x\,6\mathrm{i}}-1}+\frac{-224+96\mathrm{i}}{1+\mathrm{e}^{x\,4\mathrm{i}}-2\,\mathrm{e}^{x\,2\mathrm{i}}}+\frac{-224-264\mathrm{i}}{\mathrm{e}^{x\,2\mathrm{i}}-1}$$

```
[In] int((2*cot(x) - 3)^3*(3*cot(x) - 1/2),x)
```

```
[Out] 4*log(exp(x*2i) - 1) - x*(285/2 + 4i) + 64i/(3*exp(x*2i) - 3*exp(x*4i) + ex
p(x*6i) - 1) - (224 - 96i)/(exp(x*4i) - 2*exp(x*2i) + 1) - (224 + 264i)/(ex
p(x*2i) - 1)
```

3.366 $\int \cos(5x)\sec^5(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \cos(5x)\sec^5(x)\,dx = 16x - 15\tan(x) + \frac{5\tan^3(x)}{3}$$

```
[Out] 16*x-15*tan(x)+5/3*tan(x)^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1167, 209}

$$\int \cos(5x)\sec^5(x)\,dx = 16x + \frac{5\tan^3(x)}{3} - 15\tan(x)$$

```
[In] Int[Cos[5*x]*Sec[x]^5,x]
```

```
[Out] 16*x - 15*Tan[x] + (5*Tan[x]^3)/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
```

```
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1-10x^2+5x^4}{1+x^2}\,dx,x,\tan(x)\right)\\
&= \text{Subst}\left(\int \left(-15+5x^2+\frac{16}{1+x^2}\right)\,dx,x,\tan(x)\right)\\
&= -15\tan(x)+\frac{5\tan^3(x)}{3}+16\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\
&= 16x-15\tan(x)+\frac{5\tan^3(x)}{3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \cos(5x)\sec^5(x)\,dx = 16x-\frac{50\tan(x)}{3}+\frac{5}{3}\sec^2(x)\tan(x)$$

```
[In] Integrate[Cos[5*x]*Sec[x]^5,x]
[Out] 16*x - (50*Tan[x])/3 + (5*Sec[x]^2*Tan[x])/3
```

Maple [A] (verified)

Time = 33.88 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

method	result	size
default	$16x-5\left(-\frac{2}{3}-\frac{\left(\sec^2(x)\right)}{3}\right)\tan\left(x\right)-20\tan\left(x\right)$	21
risch	$16x-\frac{20i\left(6\,\mathrm{e}^{4ix}+9\,\mathrm{e}^{2ix}+5\right)}{3\left(\mathrm{e}^{2ix}+1\right)^3}$	33

```
[In] int(cos(5*x)/cos(x)^5,x,method=_RETURNVERBOSE)
[Out] 16*x-5*(-2/3-1/3*sec(x)^2)*tan(x)-20*tan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \cos(5x)\sec^5(x)\,dx = \frac{48\,x\cos{(x)}^3 - 5\left(10\,\cos{(x)}^2 - 1\right)\sin{(x)}}{3\,\cos{(x)}^3}$$

```
[In] integrate(cos(5*x)/cos(x)^5,x, algorithm="fricas")
[Out] 1/3*(48*x*cos(x)^3 - 5*(10*cos(x)^2 - 1)*sin(x))/cos(x)^3
```

Sympy [A] (verification not implemented)

Time = 7.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \cos(5x)\sec^5(x)\,dx = 16x - \frac{20\sin{(x)}}{\cos{(x)}} + \frac{5\tan^3{(x)}}{3} + 5\tan{(x)}$$

```
[In] integrate(cos(5*x)/cos(x)**5,x)
[Out] 16*x - 20*sin(x)/cos(x) + 5*tan(x)**3/3 + 5*tan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \cos(5x)\sec^5(x)\,dx = \frac{5}{3}\,\tan{(x)}^3 + 16\,x - 15\,\tan{(x)}$$

```
[In] integrate(cos(5*x)/cos(x)^5,x, algorithm="maxima")
[Out] 5/3*tan(x)^3 + 16*x - 15*tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \cos(5x)\sec^5(x)\,dx = \frac{5}{3}\,\tan{(x)}^3 + 16\,x - 15\,\tan{(x)}$$

```
[In] integrate(cos(5*x)/cos(x)^5,x, algorithm="giac")
[Out] 5/3*tan(x)^3 + 16*x - 15*tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \cos(5x)\sec^5(x)\,dx = \frac{48\,x\cos\left(x\right)^3 - 50\,\sin\left(x\right)\,\cos\left(x\right)^2 + 5\,\sin\left(x\right)}{3\cos\left(x\right)^3}$$

```
[In] int(cos(5*x)/cos(x)^5,x)
[Out] (5*sin(x) + 48*x*cos(x)^3 - 50*cos(x)^2*sin(x))/(3*cos(x)^3)
```

3.367 $\int \cos(4x)\sec(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \cos(4x)\sec(x)\,dx = \operatorname{arctanh}(\sin(x)) - \frac{8\sin^3(x)}{3}$$

```
[Out] arctanh(sin(x))-8/3*sin(x)^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4449, 1167, 212}

$$\int \cos(4x)\sec(x)\,dx = \operatorname{arctanh}(\sin(x)) - \frac{8\sin^3(x)}{3}$$

```
[In] Int[Cos[4*x]*Sec[x],x]
```

```
[Out] ArcTanh[Sin[x]] - (8*Sin[x]^3)/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
```

```
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4449

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1-8x^2+8x^4}{1-x^2}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(-8x^2+\frac{1}{1-x^2}\right)\,dx, x, \sin(x)\right)\\ &= -\frac{8}{3}\sin^3(x)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)\\ &= \operatorname{arctanh}(\sin(x))-\frac{8\sin^3(x)}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \cos(4x)\sec(x)\,dx = \operatorname{arctanh}(\sin(x))-\frac{8\sin^3(x)}{3}$$

```
[In] Integrate[Cos[4*x]*Sec[x],x]
[Out] ArcTanh[Sin[x]] - (8*Sin[x]^3)/3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

method	result	size
default	$\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)+\frac{8\left(2+\cos^2\left(x\right)\right)\sin\left(x\right)}{3}-8\sin\left(x\right)$	22
risch	$i\,\mathrm{e}^{ix}-i\,\mathrm{e}^{-ix}+\ln\left(i+\mathrm{e}^{ix}\right)-\ln\left(\mathrm{e}^{ix}-i\right)+\frac{2\sin\left(3x\right)}{3}$	44

```
[In] int(cos(4*x)/cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(sec(x)+tan(x))+8/3*(2+cos(x)^2)*sin(x)-8*sin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \cos(4x)\sec(x)\,dx = \frac{8}{3}\left(\cos\left(x\right)^2 - 1\right)\sin\left(x\right) + \frac{1}{2}\log\left(\sin\left(x\right) + 1\right) - \frac{1}{2}\log\left(-\sin\left(x\right) + 1\right)$$

```
[In] integrate(cos(4*x)/cos(x),x, algorithm="fricas")
[Out] 8/3*(cos(x)^2 - 1)*sin(x) + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \cos(4x)\sec(x)\,dx = -\frac{\log\left(\sin\left(x\right) - 1\right)}{2} + \frac{\log\left(\sin\left(x\right) + 1\right)}{2} - \frac{8\sin^3\left(x\right)}{3}$$

```
[In] integrate(cos(4*x)/cos(x),x)
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - 8*sin(x)**3/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \cos(4x)\sec(x)\,dx = -\frac{8}{3}\sin\left(x\right)^3 + \frac{1}{2}\log\left(\sin\left(x\right) + 1\right) - \frac{1}{2}\log\left(\sin\left(x\right) - 1\right)$$

```
[In] integrate(cos(4*x)/cos(x),x, algorithm="maxima")
[Out] -8/3*sin(x)^3 + 1/2*log(sin(x) + 1) - 1/2*log(sin(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \cos(4x)\sec(x)\,dx = -\frac{8}{3}\sin(x)^3 + \frac{1}{2}\log(\sin(x)+1) - \frac{1}{2}\log(-\sin(x)+1)$$

```
[In] integrate(cos(4*x)/cos(x),x, algorithm="giac")
[Out] -8/3*sin(x)^3 + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \cos(4x)\sec(x)\,dx = \operatorname{atanh}(\sin(x)) - \frac{8\sin(x)^3}{3}$$

```
[In] int(cos(4*x)/cos(x),x)
[Out] atanh(sin(x)) - (8*sin(x)^3)/3
```

3.368 $\int \cos(x)\cos(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

```
[Out] 1/6*sin(3*x)+1/10*sin(5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4368}

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

```
[In] Int[Cos[x]*Cos[4*x],x]
```

```
[Out] Sin[3*x]/6 + Sin[5*x]/10
```

Rule 4368

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

```
[In] Integrate[Cos[x]*Cos[4*x],x]
[Out] Sin[3*x]/6 + Sin[5*x]/10
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$	14
risch	$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$	14
parallelrisch	$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$	14
norman	$\frac{-\frac{8\tan(2x)\left(\tan^2\left(\frac{x}{2}\right)\right)}{15}+\frac{2\left(\tan^2(2x)\right)\tan\left(\frac{x}{2}\right)}{15}+\frac{8\tan(2x)}{15}-\frac{2\tan\left(\frac{x}{2}\right)}{15}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2(2x)\right)}$	59

```
[In] int(cos(x)*cos(4*x),x,method=_RETURNVERBOSE)
[Out] 1/6*sin(3*x)+1/10*sin(5*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{15}\left(24\cos(x)^4 - 8\cos(x)^2 - 1\right)\sin(x)$$

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="fricas")
[Out] 1/15*(24*cos(x)^4 - 8*cos(x)^2 - 1)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cos(x)\cos(4x)\,dx = -\frac{\sin(x)\cos(4x)}{15} + \frac{4\sin(4x)\cos(x)}{15}$$

```
[In] integrate(cos(x)*cos(4*x),x)
[Out] -sin(x)*cos(4*x)/15 + 4*sin(4*x)*cos(x)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{10}\sin(5x) + \frac{1}{6}\sin(3x)$$

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="maxima")
[Out] 1/10*sin(5*x) + 1/6*sin(3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{10}\sin(5x) + \frac{1}{6}\sin(3x)$$

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="giac")
[Out] 1/10*sin(5*x) + 1/6*sin(3*x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(4x)\,dx = \frac{\sin(3x)}{6} + \frac{\sin(5x)}{10}$$

```
[In] int(cos(4*x)*cos(x),x)
[Out] sin(3*x)/6 + sin(5*x)/10
```

3.369 $\int \cos(4x) \sec^5(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 26

$$\int \cos(4x) \sec^5(x) \, dx = \frac{35}{8} \text{arctanh}(\sin(x)) - \frac{29}{8} \sec(x) \tan(x) + \frac{1}{4} \sec^3(x) \tan(x)$$

```
[Out] 35/8*arctanh(sin(x))-29/8*sec(x)*tan(x)+1/4*sec(x)^3*tan(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4449, 1171, 393, 212}

$$\int \cos(4x) \sec^5(x) \, dx = \frac{35}{8} \text{arctanh}(\sin(x)) + \frac{1}{4} \tan(x) \sec^3(x) - \frac{29}{8} \tan(x) \sec(x)$$

```
[In] Int[Cos[4*x]*Sec[x]^5,x]
```

```
[Out] (35*ArcTanh[Sin[x]])/8 - (29*Sec[x]*Tan[x])/8 + (Sec[x]^3*Tan[x])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4449

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1-8x^2+8x^4}{(1-x^2)^3}\,dx, x, \sin(x)\right)\\
&= \frac{1}{4}\sec^3(x)\tan(x) - \frac{1}{4}\text{Subst}\left(\int \frac{-3+32x^2}{(1-x^2)^2}\,dx, x, \sin(x)\right)\\
&= -\frac{29}{8}\sec(x)\tan(x) + \frac{1}{4}\sec^3(x)\tan(x) + \frac{35}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)\\
&= \frac{35}{8}\text{arctanh}(\sin(x)) - \frac{29}{8}\sec(x)\tan(x) + \frac{1}{4}\sec^3(x)\tan(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cos(4x)\sec^5(x)\,dx = \frac{1}{8}\left(35\text{arctanh}(\sin(x)) - 27\sec^3(x)\tan(x) + 29\sec(x)\tan^3(x)\right)$$

```
[In] Integrate[Cos[4*x]*Sec[x]^5,x]
[Out] (35*ArcTanh[Sin[x]] - 27*Sec[x]^3*Tan[x] + 29*Sec[x]*Tan[x]^3)/8
```

Maple [A] (verified)

Time = 38.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

method	result	size
default	$-\left(-\frac{\left(\sec^{3}\left(x\right)\right)}{4}-\frac{3\sec\left(x\right)}{8}\right)\tan\left(x\right)+\frac{35\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)}{8}-4\sec\left(x\right)\tan\left(x\right)$	31
risch	$\frac{i\left(29\,{\mathrm e}^{7ix}+21\,{\mathrm e}^{5ix}-21\,{\mathrm e}^{3ix}-29\,{\mathrm e}^{ix}\right)}{4\left({\mathrm e}^{2ix}+1\right)^{4}}+\frac{35\ln\left(i+{\mathrm e}^{ix}\right)}{8}-\frac{35\ln\left({\mathrm e}^{ix}-i\right)}{8}$	65

```
[In] int(cos(4*x)/cos(x)^5,x,method=_RETURNVERBOSE)
[Out] -(-1/4*sec(x)^3-3/8*sec(x))*tan(x)+35/8*ln(sec(x)+tan(x))-4*sec(x)*tan(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(20) = 40.

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \cos(4x)\sec^5(x)\,dx = \frac{35\cos\left(x\right)^{4}\log\left(\sin\left(x\right)+1\right)-35\cos\left(x\right)^{4}\log\left(-\sin\left(x\right)+1\right)-2\left(29\cos\left(x\right)^{2}-2\right)\sin\left(x\right)}{16\cos\left(x\right)^{4}}$$

```
[In] integrate(cos(4*x)/cos(x)^5,x, algorithm="fricas")
[Out] 1/16*(35*cos(x)^4*log(sin(x) + 1) - 35*cos(x)^4*log(-sin(x) + 1) - 2*(29*co
s(x)^2 - 2)*sin(x))/cos(x)^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(27) = 54.

Time = 6.98 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.88

$$\int \cos(4x)\sec^5(x)\,dx = -\frac{35\log\left(\sin\left(x\right)-1\right)}{16}+\frac{35\log\left(\sin\left(x\right)+1\right)}{16}-\frac{3\sin^{3}\left(x\right)}{8\sin^{4}\left(x\right)-16\sin^{2}\left(x\right)+8}+\frac{5\sin\left(x\right)}{8\sin^{4}\left(x\right)-16\sin^{2}\left(x\right)+8}+\frac{8\sin\left(x\right)}{2\sin^{2}\left(x\right)-2}$$

```
[In] integrate(cos(4*x)/cos(x)**5,x)
[Out] -35*log(sin(x) - 1)/16 + 35*log(sin(x) + 1)/16 - 3*sin(x)**3/(8*sin(x)**4 -
 16*sin(x)**2 + 8) + 5*sin(x)/(8*sin(x)**4 - 16*sin(x)**2 + 8) + 8*sin(x)/(
2*sin(x)**2 - 2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(20) = 40$.

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08

$$\int \cos(4x)\sec^5(x)\,dx = \frac{5\sin(x)^3 - 3\sin(x)}{8\left(\sin(x)^4 - 2\sin(x)^2 + 1\right)} + \frac{3\sin(x)}{\sin(x)^2 - 1} + \frac{35}{16}\log(\sin(x) + 1) - \frac{35}{16}\log(\sin(x) - 1)$$

```
[In] integrate(cos(4*x)/cos(x)^5,x, algorithm="maxima")
```

```
[Out] 1/8*(5*sin(x)^3 - 3*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 3*sin(x)/(sin(x)^
2 - 1) + 35/16*log(sin(x) + 1) - 35/16*log(sin(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \cos(4x)\sec^5(x)\,dx = \frac{29\sin(x)^3 - 27\sin(x)}{8\left(\sin(x)^2 - 1\right)^2} + \frac{35}{16}\log(\sin(x)+1) - \frac{35}{16}\log(-\sin(x)+1)$$

```
[In] integrate(cos(4*x)/cos(x)^5,x, algorithm="giac")
```

```
[Out] 1/8*(29*sin(x)^3 - 27*sin(x))/(sin(x)^2 - 1)^2 + 35/16*log(sin(x) + 1) - 35
/16*log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \cos(4x)\sec^5(x)\,dx = \frac{35\,\mathrm{atanh}(\sin(x))}{8} - \frac{\frac{27\sin(x)}{8} - \frac{29\sin(x)^3}{8}}{\sin(x)^4 - 2\sin(x)^2 + 1}$$

```
[In] int(cos(4*x)/cos(x)^5,x)
```

```
[Out] (35*atanh(sin(x)))/8 - ((27*sin(x))/8 - (29*sin(x)^3)/8)/(sin(x)^4 - 2*sin(
x)^2 + 1)
```

3.370 $\int \cos^4(x) \cos(4x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \cos^4(x) \cos(4x) \, dx = \frac{x}{16} + \frac{1}{8} \sin(2x) + \frac{3}{32} \sin(4x) + \frac{1}{24} \sin(6x) + \frac{1}{128} \sin(8x)$$

```
[Out] 1/16*x+1/8*sin(2*x)+3/32*sin(4*x)+1/24*sin(6*x)+1/128*sin(8*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4439, 2717}

$$\int \cos^4(x) \cos(4x) \, dx = \frac{x}{16} + \frac{1}{8} \sin(2x) + \frac{3}{32} \sin(4x) + \frac{1}{24} \sin(6x) + \frac{1}{128} \sin(8x)$$

```
[In] Int[Cos[x]^4*Cos[4*x],x]
[Out] x/16 + Sin[2*x]/8 + (3*Sin[4*x])/32 + Sin[6*x]/24 + Sin[8*x]/128
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4439

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{16}+\frac{1}{4}\cos(2x)+\frac{3}{8}\cos(4x)+\frac{1}{4}\cos(6x)+\frac{1}{16}\cos(8x)\right)\,dx\\ &= \frac{x}{16}+\frac{1}{16}\int \cos(8x)\,dx+\frac{1}{4}\int \cos(2x)\,dx+\frac{1}{4}\int \cos(6x)\,dx+\frac{3}{8}\int \cos(4x)\,dx\\ &= \frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \cos^4(x)\cos(4x)\,dx=\frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)$$

```
[In] Integrate[Cos[x]^4*Cos[4*x],x]
[Out] x/16 + Sin[2*x]/8 + (3*Sin[4*x])/32 + Sin[6*x]/24 + Sin[8*x]/128
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{x}{16}+\frac{\sin(2x)}{8}+\frac{3\sin(4x)}{32}+\frac{\sin(6x)}{24}+\frac{\sin(8x)}{128}$	29
risch	$\frac{x}{16}+\frac{\sin(2x)}{8}+\frac{3\sin(4x)}{32}+\frac{\sin(6x)}{24}+\frac{\sin(8x)}{128}$	29
parallelrisch	$\frac{x}{16}+\frac{\sin(2x)}{8}+\frac{3\sin(4x)}{32}+\frac{\sin(6x)}{24}+\frac{\sin(8x)}{128}$	29

```
[In] int(cos(x)^4*cos(4*x),x,method=_RETURNVERBOSE)
[Out] 1/16*x+1/8*sin(2*x)+3/32*sin(4*x)+1/24*sin(6*x)+1/128*sin(8*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \cos^4(x)\cos(4x)\,dx = \frac{1}{48}\left(48\cos(x)^7 - 8\cos(x)^5 + 2\cos(x)^3 + 3\cos(x)\right)\sin(x) + \frac{1}{16}x$$

```
[In] integrate(cos(x)^4*cos(4*x),x, algorithm="fricas")
[Out] 1/48*(48*cos(x)^7 - 8*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 1/16*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(31) = 62.

Time = 1.59 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.66

$$\int \cos^4(x)\cos(4x)\,dx = \frac{x\sin^4(x)\cos(4x)}{16} - \frac{x\sin^3(x)\sin(4x)\cos(x)}{4} - \frac{3x\sin^2(x)\cos^2(x)\cos(4x)}{8} + \frac{x\sin(x)\sin(4x)\cos^3(x)}{4} + \frac{x\cos^4(x)\cos(4x)}{16} - \frac{\sin^4(x)\sin(4x)}{24} - \frac{5\sin^3(x)\cos(x)\cos(4x)}{48} - \frac{11\sin(x)\cos^3(x)\cos(4x)}{48} + \frac{7\sin(4x)\cos^4(x)}{24}$$

```
[In] integrate(cos(x)**4*cos(4*x),x)
[Out] x*sin(x)**4*cos(4*x)/16 - x*sin(x)**3*sin(4*x)*cos(x)/4 - 3*x*sin(x)**2*cos
(x)**2*cos(4*x)/8 + x*sin(x)*sin(4*x)*cos(x)**3/4 + x*cos(x)**4*cos(4*x)/16
 - sin(x)**4*sin(4*x)/24 - 5*sin(x)**3*cos(x)*cos(4*x)/48 - 11*sin(x)*cos(x
)**3*cos(4*x)/48 + 7*sin(4*x)*cos(x)**4/24
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \cos^4(x)\cos(4x)\,dx = -\frac{1}{6}\sin(2x)^3 + \frac{1}{16}x + \frac{1}{128}\sin(8x) + \frac{3}{32}\sin(4x) + \frac{1}{4}\sin(2x)$$

```
[In] integrate(cos(x)^4*cos(4*x),x, algorithm="maxima")
[Out] -1/6*sin(2*x)^3 + 1/16*x + 1/128*sin(8*x) + 3/32*sin(4*x) + 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \cos^4(x)\cos(4x)\,dx = \frac{1}{16}\,x + \frac{1}{128}\,\sin\left(8\,x\right) + \frac{1}{24}\,\sin\left(6\,x\right) + \frac{3}{32}\,\sin\left(4\,x\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

```
[In] integrate(cos(x)^4*cos(4*x),x, algorithm="giac")
```

```
[Out] 1/16*x + 1/128*sin(8*x) + 1/24*sin(6*x) + 3/32*sin(4*x) + 1/8*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \cos^4(x)\cos(4x)\,dx = \frac{x}{16} + \frac{\frac{\tan(x)^7}{16} + \frac{11\tan(x)^5}{48} + \frac{5\tan(x)^3}{48} + \frac{15\tan(x)}{16}}{\left(\tan\left(x\right)^2 + 1\right)^4}$$

```
[In] int(cos(4*x)*cos(x)^4,x)
```

```
[Out] x/16 + ((15*tan(x))/16 + (5*tan(x)^3)/48 + (11*tan(x)^5)/48 + tan(x)^7/16)/
(tan(x)^2 + 1)^4
```

3.371 $\int \cos(5x)\csc^5(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \cos(5x)\csc^5(x)\,dx = 6\csc^2(x) - \frac{\csc^4(x)}{4} + 16\log(\sin(x))$$

```
[Out] 6*csc(x)^2-1/4*csc(x)^4+16*ln(sin(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4451, 1261, 712}

$$\int \cos(5x)\csc^5(x)\,dx = -\frac{1}{4}\csc^4(x) + 6\csc^2(x) + 16\log(\sin(x))$$

```
[In] Int[Cos[5*x]*Csc[x]^5,x]
[Out] 6*Csc[x]^2 - Csc[x]^4/4 + 16*Log[Sin[x]]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 4451

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x(5-20x^2+16x^4)}{(1-x^2)^3}\,dx, x, \cos(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{5-20x+16x^2}{(1-x)^3}\,dx, x, \cos^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^3}-\frac{12}{(-1+x)^2}-\frac{16}{-1+x}\right)dx, x, \cos^2(x)\right)\right)\\
&= 6\csc^2(x)-\frac{\csc^4(x)}{4}+16\log(\sin(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \cos(5x)\csc^5(x)\,dx = 6\csc^2(x)-\frac{\csc^4(x)}{4}+16\log(\sin(x))$$

```
[In] Integrate[Cos[5*x]*Csc[x]^5,x]
[Out] 6*Csc[x]^2 - Csc[x]^4/4 + 16*Log[Sin[x]]
```

Maple [A] (verified)

Time = 25.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

method	result	size
default	$-\frac{5}{4\sin(x)^4}+\frac{5(\cos^4(x))}{\sin(x)^4}-4(\cot^4(x))+8(\cot^2(x))+16\ln(\sin(x))$	35
risch	$-16ix-\frac{4(6\,e^{6ix}-11\,e^{4ix}+6\,e^{2ix})}{(e^{2ix}-1)^4}+16\ln(e^{2ix}-1)$	49

```
[In] int(cos(5*x)/sin(x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -5/4/sin(x)^4+5/sin(x)^4*cos(x)^4-4*cot(x)^4+8*cot(x)^2+16*ln(sin(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \cos(5x)\csc^5(x)\,dx = -\frac{24\,\cos(x)^2 - 64\left(\cos(x)^4 - 2\,\cos(x)^2 + 1\right)\log\left(\frac{1}{2}\,\sin(x)\right) - 23}{4\left(\cos(x)^4 - 2\,\cos(x)^2 + 1\right)}$$

```
[In] integrate(cos(5*x)/sin(x)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(24*cos(x)^2 - 64*(cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*sin(x)) - 23)/(c
os(x)^4 - 2*cos(x)^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \cos(5x)\csc^5(x)\,dx = 8\log\left(\sin^2(x)\right) + \frac{6}{\sin^2(x)} - \frac{1}{4\sin^4(x)}$$

```
[In] integrate(cos(5*x)/sin(x)**5,x)
```

```
[Out] 8*log(sin(x)**2) + 6/sin(x)**2 - 1/(4*sin(x)**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \cos(5x)\csc^5(x)\,dx = \frac{5}{\sin(x)^2} + \frac{4\,\sin(x)^2 - 1}{4\,\sin(x)^4} + \frac{11}{2}\,\log\left(\sin(x)^2\right) + 5\,\log\left(\sin(x)\right)$$

```
[In] integrate(cos(5*x)/sin(x)^5,x, algorithm="maxima")
```

```
[Out] 5/sin(x)^2 + 1/4*(4*sin(x)^2 - 1)/sin(x)^4 + 11/2*log(sin(x)^2) + 5*log(sin
(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \cos(5x)\csc^5(x)\,dx = \frac{24\,\sin(x)^2 - 1}{4\,\sin(x)^4} + 16\,\log(|\sin(x)|)$$

```
[In] integrate(cos(5*x)/sin(x)^5,x, algorithm="giac")
[Out] 1/4*(24*sin(x)^2 - 1)/sin(x)^4 + 16*log(abs(sin(x)))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \cos(5x)\csc^5(x)\,dx = 8\,\ln\left(\sin(x)^2\right) + \frac{6\sin(x)^2 - \frac{1}{4}}{\sin(x)^4}$$

```
[In] int(cos(5*x)/sin(x)^5,x)
[Out] 8*log(sin(x)^2) + (6*sin(x)^2 - 1/4)/sin(x)^4
```

3.372 $\int \csc^4(x)\sin(4x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \csc^4(x)\sin(4x)\,dx = -2\csc^2(x) - 8\log(\sin(x))$$

```
[Out] -2*csc(x)^2-8*ln(sin(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{14\}$

$$\int \csc^4(x)\sin(4x)\,dx = -2\csc^2(x) - 8\log(\sin(x))$$

```
[In] Int[Csc[x]^4*Sin[4*x],x]
```

```
[Out] -2*Csc[x]^2 - 8*Log[Sin[x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{4-8x^2}{x^3}\,dx, x, \sin(x)\right) \\
&= \text{Subst}\left(\int \left(\frac{4}{x^3} - \frac{8}{x}\right)\,dx, x, \sin(x)\right) \\
&= -2\csc^2(x) - 8\log(\sin(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \csc^4(x)\sin(4x)\,dx = -2\csc^2(x) - 8\log(\sin(x))$$

```
[In] Integrate[Csc[x]^4*Sin[4*x],x]
[Out] -2*Csc[x]^2 - 8*Log[Sin[x]]
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

method	result	size
default	$\frac{2}{\sin(x)^2} - 4(\cot^2(x)) - 8\ln(\sin(x))$	19
risch	$8ix + \frac{8\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}-1)^2} - 8\ln(\mathrm{e}^{2ix} - 1)$	32

```
[In] int(sin(4*x)/sin(x)^4,x,method=_RETURNVERBOSE)
[Out] 2/sin(x)^2-4*cot(x)^2-8*ln(sin(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int \csc^4(x)\sin(4x)\,dx = -\frac{2\left(4\left(\cos(x)^2 - 1\right)\log\left(\frac{1}{2}\sin(x)\right) - 1\right)}{\cos(x)^2 - 1}$$

```
[In] integrate(sin(4*x)/sin(x)^4,x, algorithm="fricas")
[Out] -2*(4*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos(x)^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \csc^4(x)\sin(4x)\,dx = -8\log\left(\sin\left(x\right)\right) - \frac{2}{\sin^2\left(x\right)}$$

```
[In] integrate(sin(4*x)/sin(x)**4,x)
[Out] -8*log(sin(x)) - 2/sin(x)**2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \csc^4(x)\sin(4x)\,dx = -\frac{2}{\sin\left(x\right)^2} - 2\,\log\left(\sin\left(x\right)^2\right) - 4\,\log\left(\sin\left(x\right)\right)$$

```
[In] integrate(sin(4*x)/sin(x)^4,x, algorithm="maxima")
[Out] -2/sin(x)^2 - 2*log(sin(x)^2) - 4*log(sin(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \csc^4(x)\sin(4x)\,dx = -\frac{2}{\sin\left(x\right)^2} - 8\,\log\left(\left|\sin\left(x\right)\right|\right)$$

```
[In] integrate(sin(4*x)/sin(x)^4,x, algorithm="giac")
[Out] -2/sin(x)^2 - 8*log(abs(sin(x)))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.92

$$\int \csc^4(x)\sin(4x)\,dx = 8\,\ln\left({\tan\left(\frac{x}{2}\right)}^2+1\right) - 8\,\ln\left(\tan\left(\frac{x}{2}\right)\right) - \frac{1}{2\tan\left(\frac{x}{2}\right)^2} - \frac{{\tan\left(\frac{x}{2}\right)}^2}{2}$$

```
[In] int(sin(4*x)/sin(x)^4,x)
[Out] 8*log(tan(x/2)^2 + 1) - 8*log(tan(x/2)) - 1/(2*tan(x/2)^2) - tan(x/2)^2/2
```

3.373 $\int \frac{\cot(x)}{2+\sin(2x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 64

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = -\frac{x}{2\sqrt{3}}+\frac{\arctan\left(\frac{1-2\cos^2(x)}{2+\sqrt{3}+2\cos(x)\sin(x)}\right)}{2\sqrt{3}}+\frac{1}{2}\log(\sin(x))-\frac{1}{4}\log(1+\cos(x)\sin(x))$$

```
[Out] 1/2*ln(sin(x))-1/4*ln(1+cos(x)*sin(x))-1/6*x*3^(1/2)+1/6*arctan((1-2*cos(x)
^2)/(2+2*cos(x)*sin(x)+3^(1/2)))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {719, 29, 648, 632, 210, 642}

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = \frac{\arctan\left(\frac{1-2\cos^2(x)}{2\sin(x)\cos(x)+\sqrt{3}+2}\right)}{2\sqrt{3}}-\frac{x}{2\sqrt{3}}-\frac{1}{4}\log\left(\tan^2(x)+\tan(x)+1\right)+\frac{1}{2}\log(\tan(x))$$

```
[In] Int[Cot[x]/(2 + Sin[2*x]),x]
```

```
[Out] -1/2*x/Sqrt[3] + ArcTan[(1 - 2*Cos[x]^2)/(2 + Sqrt[3] + 2*Cos[x]*Sin[x])]/(
2*Sqrt[3]) + Log[Tan[x]]/2 - Log[1 + Tan[x] + Tan[x]^2]/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x\,(2+2x+2x^2)}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tan(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{-2-2x}{2+2x+2x^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\log(\tan(x))-\frac{1}{4}\text{Subst}\left(\int \frac{2+4x}{2+2x+2x^2}\,dx, x, \tan(x)\right)\\
&\quad -\frac{1}{2}\text{Subst}\left(\int \frac{1}{2+2x+2x^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\log(\tan(x))-\frac{1}{4}\log\left(1+\tan(x)+\tan^2(x)\right)+\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, 2+4\tan(x)\right)
\end{aligned}$$

$$= -\frac{x}{2\sqrt{3}} + \frac{\arctan\left(\frac{1-2\cos^2(x)}{2+\sqrt{3}+2\cos(x)\sin(x)}\right)}{2\sqrt{3}} + \frac{1}{2}\log(\tan(x)) - \frac{1}{4}\log\left(1+\tan(x)+\tan^2(x)\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = \frac{1}{12}\left(-2\sqrt{3}\arctan\left(\frac{1+2\tan(x)}{\sqrt{3}}\right) + 6\log(\sin(x)) - 3\log(2+\sin(2x))\right)$$

```
[In] Integrate[Cot[x]/(2 + Sin[2*x]),x]
[Out] (-2*Sqrt[3]*ArcTan[(1 + 2*Tan[x])/Sqrt[3]] + 6*Log[Sin[x]] - 3*Log[2 + Sin[
2*x]])/12
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\ln(\tan(x))}{2} - \frac{\ln\left(\tan^2(x)+\tan(x)+1\right)}{4} - \frac{\sqrt{3}\,\arctan\left(\frac{(2\tan(x)+1)\sqrt{3}}{3}\right)}{6}$	35
risch	$-\frac{\ln\left(e^{2ix}-i\sqrt{3}+2i\right)}{4} + \frac{i\ln\left(e^{2ix}-i\sqrt{3}+2i\right)\sqrt{3}}{12} - \frac{\ln\left(e^{2ix}+i\sqrt{3}+2i\right)}{4} - \frac{i\ln\left(e^{2ix}+i\sqrt{3}+2i\right)\sqrt{3}}{12} + \frac{\ln\left(e^{2ix}-1\right)}{2}$	88

```
[In] int(cos(x)/sin(x)/(2+sin(2*x)),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(tan(x))-1/4*ln(tan(x)^2+tan(x)+1)-1/6*3^(1/2)*arctan(1/3*(2*tan(x)+1
)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{\cot(x)}{2+\sin(2x)}\,dx = & -\frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)+\sqrt{3}}{3\left(2\cos(x)^2-1\right)}\right) \\ & -\frac{1}{8}\log\left(-\cos(x)^4+\cos(x)^2+2\cos(x)\sin(x)+1\right) \\ & +\frac{1}{4}\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right)\end{aligned}$$

```
[In] integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) + sqrt(3))/(2*cos(x)^2 -
1)) - 1/8*log(-cos(x)^4 + cos(x)^2 + 2*cos(x)*sin(x) + 1) + 1/4*log(-1/4*co
s(x)^2 + 1/4)
```

Sympy [F]

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx=\int \frac{\cos{(x)}}{(\sin{(2x)}+2)\sin{(x)}}\,dx$$

```
[In] integrate(cos(x)/sin(x)/(2+sin(2*x)),x)
[Out] Integral(cos(x)/((sin(2*x) + 2)*sin(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(51) = 102.

Time = 0.29 (sec) , antiderivative size = 208, normalized size of antiderivative = 3.25

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = -\frac{1}{24}\sqrt{3}\left(\sqrt{3}\log\left(-2\,(4\,\sin{(2\,x)}+1)\cos{(4\,x)}+\cos{(4\,x)}^2+16\,\cos{(2\,x)}^2+8\,\cos{(2\,x)}\sin{(4\,x)}+\sin{(4\right.\right.$$

```
[In] integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="maxima")
[Out] -1/24*sqrt(3)*(sqrt(3)*log(-2*(4*sin(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 16*c
os(2*x)^2 + 8*cos(2*x)*sin(4*x) + sin(4*x)^2 + 16*sin(2*x)^2 + 8*sin(2*x) +
 1) - 2*sqrt(3)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 2*sqrt(3)*log(cos
(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*arctan2(2*sqrt(3)*cos(2*x)/(cos(2*x)^2
 - 2*(sqrt(3) - 2)*sin(2*x) + sin(2*x)^2 - 4*sqrt(3) + 7), (cos(2*x)^2 + si
n(2*x)^2 + 4*sin(2*x) + 1)/(cos(2*x)^2 - 2*(sqrt(3) - 2)*sin(2*x) + sin(2*x
)^2 - 4*sqrt(3) + 7)))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\begin{aligned}\int \frac{\cot(x)}{2+\sin(2x)}\,dx &= -\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin{(2\,x)}-\cos{(2\,x)}-2\,\sin{(2\,x)}-1}{\sqrt{3}\cos{(2\,x)}+\sqrt{3}-2\,\cos{(2\,x)}+\sin{(2\,x)}+2}\right)\right)\\&\quad-\frac{1}{4}\log\left(\tan{(x)}^2+\tan{(x)}+1\right)+\frac{1}{2}\log\left(|\tan{(x)}|\right)\end{aligned}$$

```
[In] integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) - cos(2*x) - 2*sin(2*x) - 1)/(s
qrt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) + sin(2*x) + 2))) - 1/4*log(tan(x)^2
 + tan(x) + 1) + 1/2*log(abs(tan(x)))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = \frac{\ln(\tan(x))}{2} + \ln\left(\tan(x) + \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{4} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(\tan(x) + \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{4} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

```
[In] int(cos(x)/(sin(x)*(sin(2*x) + 2)),x)
```

```
[Out] log(tan(x))/2 + log(tan(x) - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/12 - 1/4)
- log(tan(x) + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/12 + 1/4)
```

3.374 $\int \cos(x)\cot(x)\sec(3x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \cos(x)\cot(x)\sec(3x)\,dx = -\frac{1}{2}\log\left(-4+\csc^2(x)\right)$$

```
[Out] -1/2*ln(-4+csc(x)^2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4441, 272, 36, 31, 29}

$$\int \cos(x)\cot(x)\sec(3x)\,dx = \log(\sin(x)) - \frac{1}{2}\log\left(1-4\sin^2(x)\right)$$

```
[In] Int[Cos[x]*Cot[x]*Sec[3*x],x]
[Out] Log[Sin[x]] - Log[1 - 4*Sin[x]^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x\,(1-4x^2)}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-4x)x}\,dx, x, \sin^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(x)\right) + 2\text{Subst}\left(\int \frac{1}{1-4x}\,dx, x, \sin^2(x)\right)\\
&= \log(\sin(x)) - \frac{1}{2}\log\left(1-4\sin^2(x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \cos(x)\cot(x)\sec(3x)\,dx = \log(\sin(x)) - \frac{1}{2}\log\left(1-4\sin^2(x)\right)$$

```
[In] Integrate[Cos[x]*Cot[x]*Sec[3*x],x]
[Out] Log[Sin[x]] - Log[1 - 4*Sin[x]^2]/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.

Time = 0.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

method	result	size
default	$-\frac{\ln(4(\cos^2(x))-3)}{2}+\frac{\ln(-1+\cos(x))}{2}+\frac{\ln(\cos(x)+1)}{2}$	27
risch	$\ln\left(e^{2ix}-1\right)-\frac{\ln(e^{4ix}-e^{2ix}+1)}{2}$	27

```
[In] int(cos(x)^2/cos(3*x)/sin(x),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(4*cos(x)^2-3)+1/2*ln(-1+cos(x))+1/2*ln(cos(x)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \cos(x)\cot(x)\sec(3x)\,dx = -\frac{1}{2}\log\left(4\cos(x)^2-3\right)+\log\left(\frac{1}{2}\sin(x)\right)$$

```
[In] integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="fricas")
[Out] -1/2*log(4*cos(x)^2 - 3) + log(1/2*sin(x))
```

Sympy [F]

$$\int \cos(x)\cot(x)\sec(3x)\,dx = \int \frac{\cos^2(x)}{\sin(x)\cos(3x)}\,dx$$

```
[In] integrate(cos(x)**2/cos(3*x)/sin(x),x)
[Out] Integral(cos(x)**2/(sin(x)*cos(3*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(9) = 18$.

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 8.36

$$\int \cos(x)\cot(x)\sec(3x)\,dx = -\frac{1}{4}\log\left(-2\left(\cos\left(2\,x\right)-1\right)\cos\left(4\,x\right)+\cos\left(4\,x\right)^{2}+\cos\left(2\,x\right)^{2}+\sin\left(4\,x\right)^{2}-2\,\sin\left(4\,x\right)\sin\left(2\,x\right)+\sin\left(2\,x\right)^{2}-2\,\cos\left(2\,x\right)+1\right)+\frac{1}{2}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\,\cos\left(x\right)+1\right)+\frac{1}{2}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\,\cos\left(x\right)+1\right)$$

```
[In] integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="maxima")
```

```
[Out] -1/4*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + sin(4*x)^2
- 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) + 1/2*log(cos(x)^2 + s
in(x)^2 + 2*cos(x) + 1) + 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(9) = 18$.

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \cos(x)\cot(x)\sec(3x)\,dx = \frac{1}{2}\log\left(-\cos\left(x\right)^{2}+1\right)-\frac{1}{2}\log\left(\left|4\,\cos\left(x\right)^{2}-3\right|\right)$$

```
[In] integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="giac")
```

```
[Out] 1/2*log(-cos(x)^2 + 1) - 1/2*log(abs(4*cos(x)^2 - 3))
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \cos(x)\cot(x)\sec(3x)\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{\ln\left({\tan\left(\frac{x}{2}\right)}^4-14\,{\tan\left(\frac{x}{2}\right)}^2+1\right)}{2}$$

```
[In] int(cos(x)^2/(cos(3*x)*sin(x)),x)
```

```
[Out] log(tan(x/2)) - log(tan(x/2)^4 - 14*tan(x/2)^2 + 1)/2
```

3.375 $\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 7

$$\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx = -\arctan(\cos(2x))$$

```
[Out] -arctan(cos(2*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {12, 1121, 631, 210}

$$\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx = -\arctan(\cos(2x))$$

```
[In] Int[Sin[2*x]/(Cos[x]^4 + Sin[x]^4),x]
```

```
[Out] -ArcTan[Cos[2*x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{2x}{1-2x^2+2x^4}\,dx, x, \sin(x)\right)\\ &= 2\text{Subst}\left(\int \frac{x}{1-2x^2+2x^4}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1-2x+2x^2}\,dx, x, \sin^2(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-2\sin^2(x)\right)\\ &= -\arctan\left(1-2\sin^2(x)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx = -\arctan(\cos(2x))$$

```
[In] Integrate[Sin[2*x]/(Cos[x]^4 + Sin[x]^4),x]
[Out] -ArcTan[Cos[2*x]]
```

Maple [A] (verified)

Time = 12.97 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

method	result	size
derivativedivides	$-\arctan\left(2(\cos^2(x))-1\right)$	12
default	$-\arctan\left(2(\cos^2(x))-1\right)$	12
risch	$-\frac{i\ln\left(e^{4ix}+2ie^{2ix}+1\right)}{2}+\frac{i\ln\left(e^{4ix}-2ie^{2ix}+1\right)}{2}$	40

```
[In] int(sin(2*x)/(cos(x)^4+sin(x)^4),x,method=_RETURNVERBOSE)
[Out] -arctan(2*cos(x)^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx = -\arctan\left(2\,\cos\left(x\right)^2-1\right)$$

```
[In] integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="fricas")
[Out] -arctan(2*cos(x)^2 - 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(2*x)/(cos(x)**4+sin(x)**4),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{\sin(2x)}{\cos^4(x) + \sin^4(x)}\,dx = \arctan\left(2\,\sin\left(x\right)^2 - 1\right)$$

```
[In] integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="maxima")
[Out] arctan(2*sin(x)^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{\sin(2x)}{\cos^4(x) + \sin^4(x)}\,dx = \arctan\left(2\,\sin\left(x\right)^2 - 1\right)$$

```
[In] integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="giac")
[Out] arctan(2*sin(x)^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{\sin(2x)}{\cos^4(x) + \sin^4(x)}\,dx = \mathrm{atan}\left(\tan(x)^2\right)$$

```
[In] int(sin(2*x)/(cos(x)^4 + sin(x)^4),x)
[Out] atan(tan(x)^2)
```

3.376 $\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx=\frac{x}{2\sqrt{3}}+\frac{\arctan\left(\frac{\cos(x)-\sqrt{3}\sin(x)}{2\left(2+\sqrt{3}\right)+\sqrt{3}\cos(x)+\sin(x)}\right)}{\sqrt{3}}$$

```
[Out] 1/6*x*3^(1/2)+1/3*arctan((cos(x)-sin(x)*3^(1/2))/(sin(x)+cos(x)*3^(1/2)+4+2
*3^(1/2)))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.57, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3203, 631, 210}

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx=\frac{\arctan\left(\frac{\left(3-4\sqrt{3}\right)\sin(x)+\left(4-\sqrt{3}\right)\cos(x)}{\left(4-\sqrt{3}\right)\sin(x)-\left(\left(3-4\sqrt{3}\right)\cos(x)\right)+2\left(5+2\sqrt{3}\right)}\right)}{\sqrt{3}}+\frac{x}{2\sqrt{3}}$$

```
[In] Int[(4 + Sqrt[3]*Cos[x] + Sin[x])^(-1),x]
```

```
[Out] x/(2*Sqrt[3]) + ArcTan[((4 - Sqrt[3])*Cos[x] + (3 - 4*Sqrt[3])*Sin[x])/(2*(
5 + 2*Sqrt[3]) - (3 - 4*Sqrt[3])*Cos[x] + (4 - Sqrt[3])*Sin[x])]/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{1}{4+\sqrt{3}+2x+(4-\sqrt{3})\,x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= -\left(2\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, 1+\left(4-\sqrt{3}\right)\tan\left(\frac{x}{2}\right)\right)\right)\\
&= \frac{x}{2\sqrt{3}}+\frac{\arctan\left(\frac{\left(4-\sqrt{3}\right)\cos(x)+\left(3-4\sqrt{3}\right)\sin(x)}{2\left(5+2\sqrt{3}\right)-\left(3-4\sqrt{3}\right)\cos(x)+\left(4-\sqrt{3}\right)\sin(x)}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx = -\frac{\arctan\left(\frac{-1+\left(-4+\sqrt{3}\right)\tan\left(\frac{x}{2}\right)}{2\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Integrate[(4 + Sqrt[3]*Cos[x] + Sin[x])^(-1),x]
[Out] -(ArcTan[(-1 + (-4 + Sqrt[3])*Tan[x/2])/(2*Sqrt[3])]/Sqrt[3])
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{52 \arctan\left(\frac{26 \tan\left(\frac{x}{2}\right)+2\sqrt{3}+8}{16\sqrt{3}+12}\right)}{\left(\sqrt{3}-4\right)\left(16\sqrt{3}+12\right)}$	43
risch	$\frac{i\sqrt{3}\,\ln\left(\frac{i\sqrt{3}}{2}+\sqrt{3}+\frac{3}{2}+i+{\mathrm e}^{i x}\right)}{6}-\frac{i\sqrt{3}\,\ln\left({\mathrm e}^{i x}+\sqrt{3}-\frac{3}{2}+i-\frac{i\sqrt{3}}{2}\right)}{6}$	52
parallelrisch	$-\frac{i\left(\ln\left(13\tan\left(\frac{x}{2}\right)+4-6 i+\left(1-8 i\right)\sqrt{3}\right)-\ln\left(13\tan\left(\frac{x}{2}\right)+4+6 i+\left(1+8 i\right)\sqrt{3}\right)\right)\left(4+\sqrt{3}\right)}{8\sqrt{3}+6}$	57

```
[In] int(1/(4+sin(x)+cos(x)*3^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -52/(3^(1/2)-4)/(16*3^(1/2)+12)*arctan((26*tan(1/2*x)+2*3^(1/2)+8)/(16*3^(1
/2)+12))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.72

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx$$
$$=\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{2\,\left(\left(4\,\sqrt{3}\cos\left(x\right)+3\right)\sin\left(x\right)+\sqrt{3}\cos\left(x\right)+3\right)}{3\,\left(4\,\cos\left(x\right)^{2}-3\right)}\right)$$

```
[In] integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(2/3*((4*sqrt(3)*cos(x) + 3)*sin(x) + sqrt(3)*cos(x) + 3)
/(4*cos(x)^2 - 3))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(48) = 96.

Time = 4.58 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx$$
$$=-\frac{13906891405206808\sqrt{3}\left(\operatorname{atan}\left(-\frac{\tan\left(\frac{x}{2}\right)}{2}+\frac{2\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}+\frac{\sqrt{3}}{6}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{-41720674215620424+24087442555831531\sqrt{3}}$$
$$+\frac{24087442555831531\left(\operatorname{atan}\left(-\frac{\tan\left(\frac{x}{2}\right)}{2}+\frac{2\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}+\frac{\sqrt{3}}{6}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{-41720674215620424+24087442555831531\sqrt{3}}$$

```
[In] integrate(1/(4+sin(x)+cos(x)*3**(1/2)),x)
```

```
[Out] -13906891405206808*sqrt(3)*(atan(-tan(x/2)/2 + 2*sqrt(3)*tan(x/2)/3 + sqrt(
3)/6) + pi*floor((x/2 - pi/2)/pi))/(-41720674215620424 + 24087442555831531*
sqrt(3)) + 24087442555831531*(atan(-tan(x/2)/2 + 2*sqrt(3)*tan(x/2)/3 + sqr
t(3)/6) + pi*floor((x/2 - pi/2)/pi))/(-41720674215620424 + 2408744255583153
1*sqrt(3))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.51

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{6}\sqrt{3}\left(\frac{(\sqrt{3}-4)\sin(x)}{\cos(x)+1}-1\right)\right)$$

```
[In] integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*((sqrt(3) - 4)*sin(x)/(cos(x) + 1) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx = \frac{\left(x+2\arctan\left(\frac{\sqrt{3}\cos(x)-8\sqrt{3}\sin(x)+\sqrt{3}+4\cos(x)+7\sin(x)+4}{8\sqrt{3}\cos(x)+\sqrt{3}\sin(x)+8\sqrt{3}-7\cos(x)+4\sin(x)+19}\right)\right)(\sqrt{3}+4)}{2(4\sqrt{3}+3)}$$

```
[In] integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*(x + 2*arctan((sqrt(3)*cos(x) - 8*sqrt(3)*sin(x) + sqrt(3) + 4*cos(x) +
 7*sin(x) + 4)/(8*sqrt(3)*cos(x) + sqrt(3)*sin(x) + 8*sqrt(3) - 7*cos(x) +
4*sin(x) + 19)))*(sqrt(3) + 4)/(4*sqrt(3) + 3)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.43

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx=-\frac{\sqrt{12}\,\mathrm{atan}\left(\frac{\sqrt{12}\,\left(\tan\left(\frac{x}{2}\right)\,\left(\sqrt{3}-4\right)-1\right)}{12}\right)}{6}$$

```
[In] int(1/(sin(x) + 3^(1/2)*cos(x) + 4),x)
[Out] -(12^(1/2)*atan((12^(1/2)*(tan(x/2)*(3^(1/2) - 4) - 1))/12))/6
```

3.377 $\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{23}(\cos(x)-\sin(x))}{8+3\cos(x)+3\sin(x)}\right)}{\sqrt{23}}$$

```
[Out] -1/23*arctanh((cos(x)-sin(x))*23^(1/2)/(8+3*cos(x)+3*sin(x)))*23^(1/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 94 vs. $2(33) = 66$.

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.85, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3203, 632, 212}

$$\begin{aligned}&\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx\\&=\frac{\log\left(\sqrt{23}\sin(x)-4\sin(x)-4\sqrt{23}\cos(x)+19\cos(x)+4\left(5-\sqrt{23}\right)\right)}{2\sqrt{23}}\\&\quad-\frac{\log\left(-\sqrt{23}\sin(x)-4\sin(x)+4\sqrt{23}\cos(x)+19\cos(x)+4\left(5+\sqrt{23}\right)\right)}{2\sqrt{23}}\end{aligned}$$

```
[In] Int[(3 + 4*Cos[x] + 4*Sin[x])^(-1),x]
```

```
[Out] -1/2*Log[4*(5 + Sqrt[23]) + 19*Cos[x] + 4*Sqrt[23]*Cos[x] - 4*Sin[x] - Sqrt
[23]*Sin[x]]/Sqrt[23] + Log[4*(5 - Sqrt[23]) + 19*Cos[x] - 4*Sqrt[23]*Cos[x
] - 4*Sin[x] + Sqrt[23]*Sin[x]]/(2*Sqrt[23])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{1}{7+8x-x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right) \\
&= -\left(4\text{Subst}\left(\int \frac{1}{92-x^2}\,dx, x, 8-2\tan\left(\frac{x}{2}\right)\right)\right) \\
&= -\frac{\log\left(4(5+\sqrt{23})+19\cos(x)+4\sqrt{23}\cos(x)-4\sin(x)-\sqrt{23}\sin(x)\right)}{2\sqrt{23}} \\
&\quad+\frac{\log\left(4(5-\sqrt{23})+19\cos(x)-4\sqrt{23}\cos(x)-4\sin(x)+\sqrt{23}\sin(x)\right)}{2\sqrt{23}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$
\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = \frac{2\text{arctanh}\left(\frac{-4+\tan\left(\frac{x}{2}\right)}{\sqrt{23}}\right)}{\sqrt{23}}
$$

```
[In] Integrate[(3 + 4*Cos[x] + 4*Sin[x])^(-1),x]
[Out] (2*ArcTanh[(-4 + Tan[x/2])/Sqrt[23]])/Sqrt[23]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{2\sqrt{23}\, \operatorname{arctanh}\left(\frac{\left(2\tan\left(\frac{x}{2}\right)-8\right)\sqrt{23}}{46}\right)}{23}$	20
risch	$\frac{\sqrt{23}\, \ln\left(\mathrm{e}^{ix}+\frac{3}{8}+\frac{3i}{8}-\frac{\sqrt{23}}{8}+\frac{i\sqrt{23}}{8}\right)}{23}-\frac{\sqrt{23}\, \ln\left(\mathrm{e}^{ix}+\frac{3}{8}+\frac{3i}{8}+\frac{\sqrt{23}}{8}-\frac{i\sqrt{23}}{8}\right)}{23}$	54

```
[In] int(1/(3+4*cos(x)+4*sin(x)),x,method=_RETURNVERBOSE)
[Out] 2/23*23^(1/2)*arctanh(1/46*(2*tan(1/2*x)-8)*23^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(29) = 58.

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = \frac{1}{46}\sqrt{23}\log\left(-\frac{6\sqrt{23}\cos(x)^2+8\left(\sqrt{23}-3\right)\cos(x)-2\left(4\sqrt{23}-7\cos(x)+12\right)\sin(x)-3\sqrt{23}-48}{8\left(4\cos(x)+3\right)\sin(x)+24\cos(x)+25}\right)$$

```
[In] integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="fricas")
[Out] 1/46*sqrt(23)*log(-(6*sqrt(23)*cos(x)^2 + 8*(sqrt(23) - 3)*cos(x) - 2*(4*sq
rt(23) - 7*cos(x) + 12)*sin(x) - 3*sqrt(23) - 48)/(8*(4*cos(x) + 3)*sin(x)
+ 24*cos(x) + 25))
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = \frac{\sqrt{23}\log\left(\tan\left(\frac{x}{2}\right)-4+\sqrt{23}\right)}{23}-\frac{\sqrt{23}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{23}-4\right)}{23}$$

```
[In] integrate(1/(3+4*cos(x)+4*sin(x)),x)
[Out] sqrt(23)*log(tan(x/2) - 4 + sqrt(23))/23 - sqrt(23)*log(tan(x/2) - sqrt(23)
 - 4)/23
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = -\frac{1}{23}\,\sqrt{23}\log\left(-\frac{\sqrt{23}-\frac{\sin(x)}{\cos(x)+1}+4}{\sqrt{23}+\frac{\sin(x)}{\cos(x)+1}-4}\right)$$

```
[In] integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/23*sqrt(23)*log(-(sqrt(23) - sin(x)/(cos(x) + 1) + 4)/(sqrt(23) + sin(x)
/(cos(x) + 1) - 4))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = -\frac{1}{23}\,\sqrt{23}\log\left(\frac{\left|-2\,\sqrt{23}+2\,\tan\left(\frac{1}{2}\,x\right)-8\right|}{\left|2\,\sqrt{23}+2\,\tan\left(\frac{1}{2}\,x\right)-8\right|}\right)$$

```
[In] integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="giac")
```

```
[Out] -1/23*sqrt(23)*log(abs(-2*sqrt(23) + 2*tan(1/2*x) - 8)/abs(2*sqrt(23) + 2*t
an(1/2*x) - 8))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.52

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = \frac{2\,\sqrt{23}\,\mathrm{atanh}\left(\frac{\sqrt{23}\,\left(\tan\left(\frac{x}{2}\right)-4\right)}{23}\right)}{23}$$

```
[In] int(1/(4*cos(x) + 4*sin(x) + 3),x)
```

```
[Out] (2*23^(1/2)*atanh((23^(1/2)*(tan(x/2) - 4))/23))/23
```

3.378 $\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx=\frac{x}{3}+\frac{1}{3}\arctan\left(\frac{2\cos(x)\sin(x)}{1+2\sin^2(x)}\right)$$

```
[Out] 1/3*x+1/3*arctan(2*cos(x)*sin(x)/(1+2*sin(x)^2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {209}

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx=\frac{1}{3}\arctan\left(\frac{2\sin(x)\cos(x)}{2\sin^2(x)+1}\right)+\frac{x}{3}$$

```
[In] Int[(4 - 3*Cos[x]^2 + 5*Sin[x]^2)^(-1),x]
```

```
[Out] x/3 + ArcTan[(2*Cos[x]*Sin[x])/(1 + 2*Sin[x]^2)]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{1+9x^2}\,dx,x,\tan(x)\right) \\ &= \frac{x}{3}+\frac{1}{3}\arctan\left(\frac{2\cos(x)\sin(x)}{1+2\sin^2(x)}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.33

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx=\frac{1}{3}\arctan(3\tan(x))$$

```
[In] Integrate[(4 - 3*Cos[x]^2 + 5*Sin[x]^2)^(-1),x]
[Out] ArcTan[3*Tan[x]]/3
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.30

method	result	size
default	$\frac{\arctan(3\tan(x))}{3}$	8
risch	$\frac{i\ln\left(e^{2ix}-2\right)}{6}-\frac{i\ln\left(e^{2ix}-\frac{1}{2}\right)}{6}$	24
parallelrisch	$-\frac{i\left(\ln\left(\frac{-3i\sin(x)-\cos(x)}{\cos(x)+1}\right)-\ln\left(\frac{3i\sin(x)-\cos(x)}{\cos(x)+1}\right)\right)}{6}$	43

```
[In] int(1/(4-3*cos(x)^2+5*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/3*arctan(3*tan(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx=-\frac{1}{6}\,\arctan\left(\frac{10\,\cos\left(x\right)^2-9}{6\,\cos\left(x\right)\sin\left(x\right)}\right)$$

```
[In] integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="fricas")
[Out] -1/6*arctan(1/6*(10*cos(x)^2 - 9)/(cos(x)*sin(x)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(22) = 44$.

Time = 4.41 (sec) , antiderivative size = 219, normalized size of antiderivative = 8.11

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = \frac{4478554083\sqrt{17-12\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{17-12\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}} + \frac{3166815962\sqrt{2}\sqrt{17-12\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{17-12\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}} + \frac{131836323\sqrt{12\sqrt{2}+17}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{12\sqrt{2}+17}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}} + \frac{93222358\sqrt{2}\sqrt{12\sqrt{2}+17}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{12\sqrt{2}+17}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}}$$

```
[In] integrate(1/(4-3*cos(x)**2+5*sin(x)**2),x)
```

```
[Out] 4478554083*sqrt(17 - 12*sqrt(2))*(atan(tan(x/2)/sqrt(17 - 12*sqrt(2))) + pi
*floor((x/2 - pi/2)/pi))/(2305195203 + 1630019160*sqrt(2)) + 3166815962*sqr
t(2)*sqrt(17 - 12*sqrt(2))*(atan(tan(x/2)/sqrt(17 - 12*sqrt(2))) + pi*floor
((x/2 - pi/2)/pi))/(2305195203 + 1630019160*sqrt(2)) + 131836323*sqrt(12*sq
rt(2) + 17)*(atan(tan(x/2)/sqrt(12*sqrt(2) + 17)) + pi*floor((x/2 - pi/2)/p
i))/(2305195203 + 1630019160*sqrt(2)) + 93222358*sqrt(2)*sqrt(12*sqrt(2) +
17)*(atan(tan(x/2)/sqrt(12*sqrt(2) + 17)) + pi*floor((x/2 - pi/2)/pi))/(230
5195203 + 1630019160*sqrt(2))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.26

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = \frac{1}{3}\arctan\left(3\tan\left(x\right)\right)$$

```
[In] integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(3*tan(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = \frac{1}{3}\,x - \frac{1}{3}\,\arctan\left(\frac{\sin(2\,x)}{\cos(2\,x)-2}\right)$$

```
[In] integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="giac")
[Out] 1/3*x - 1/3*arctan(sin(2*x)/(cos(2*x) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = \frac{x}{3} - \frac{\mathrm{atan}(\tan(x))}{3} + \frac{\mathrm{atan}(3\tan(x))}{3}$$

```
[In] int(1/(5*sin(x)^2 - 3*cos(x)^2 + 4),x)
[Out] x/3 - atan(tan(x))/3 + atan(3*tan(x))/3
```

3.379 $\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 28

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx=\frac{4x}{25}-\frac{3}{25}\log(2\cos(x)+\sin(x))+\frac{2}{5(2+\tan(x))}$$

```
[Out] 4/25*x-3/25*ln(2*cos(x)+sin(x))+2/5/(2+tan(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {815, 649, 209, 266}

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx=\frac{4x}{25}+\frac{2}{5(\tan(x)+2)}-\frac{3}{25}\log(\sin(x)+2\cos(x))$$

```
[In] Int[(4 + 4*Cot[x] + Tan[x])^(-1),x]
```

```
[Out] (4*x)/25 - (3*Log[2*Cos[x] + Sin[x]])/25 + 2/(5*(2 + Tan[x]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x}{(2+x)^2(1+x^2)}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{2}{5(2+x)^2}-\frac{3}{25(2+x)}+\frac{4+3x}{25(1+x^2)}\right)dx, x, \tan(x)\right)\\
&= -\frac{3}{25}\log(2+\tan(x))+\frac{2}{5(2+\tan(x))}+\frac{1}{25}\text{Subst}\left(\int \frac{4+3x}{1+x^2}\,dx, x, \tan(x)\right)\\
&= -\frac{3}{25}\log(2+\tan(x))+\frac{2}{5(2+\tan(x))}+\frac{3}{25}\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \tan(x)\right)\\
&\quad+\frac{4}{25}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\
&= \frac{4x}{25}-\frac{3}{25}\log(\cos(x))-\frac{3}{25}\log(2+\tan(x))+\frac{2}{5(2+\tan(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = \frac{-5+4x+\cot(x)(8x-6\log(2\cos(x)+\sin(x)))-3\log(2\cos(x)+\sin(x))}{25+50\cot(x)}$$

```
[In] Integrate[(4 + 4*Cot[x] + Tan[x])^(-1),x]
[Out] (-5 + 4*x + Cot[x]*(8*x - 6*Log[2*Cos[x] + Sin[x]]) - 3*Log[2*Cos[x] + Sin[
x]])/(25 + 50*Cot[x])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{3 \ln\left(1+\tan^{2}\left(x \right)\right)}{50}+\frac{4 \arctan \left(\tan \left(x \right)\right)}{25}+\frac{2}{5 \left(2+\tan \left(x \right)\right)}-\frac{3 \ln \left(2+\tan \left(x \right)\right)}{25}$	31
default	$\frac{3 \ln\left(1+\tan^{2}\left(x \right)\right)}{50}+\frac{4 \arctan \left(\tan \left(x \right)\right)}{25}+\frac{2}{5 \left(2+\tan \left(x \right)\right)}-\frac{3 \ln \left(2+\tan \left(x \right)\right)}{25}$	31
norman	$\frac{\frac{8 x}{25}+\frac{4 x \tan \left(x \right)}{25}+\frac{2}{5}}{2+\tan \left(x \right)}-\frac{3 \ln \left(2+\tan \left(x \right)\right)}{25}+\frac{3 \ln \left(1+\tan^{2}\left(x \right)\right)}{50}$	35
parallelrisch	$\frac{\left(3 \tan \left(x \right)+6\right) \ln \left(\sec^{2}\left(x \right)\right)+\left(-6 \tan \left(x \right)-12\right) \ln \left(2+\tan \left(x \right)\right)+8 x \tan \left(x \right)+16 x +20}{100+50 \tan \left(x \right)}$	44
risch	$\frac{4 x}{25}+\frac{3 i x}{25}+\frac{16}{25 \left(5 \,{\mathrm e}^{2 i x}+3+4 i\right)}-\frac{12 i}{25 \left(5 \,{\mathrm e}^{2 i x}+3+4 i\right)}-\frac{3 \ln \left({\mathrm e}^{2 i x}+\frac{3}{5}+\frac{4 i}{5}\right)}{25}$	52

```
[In] int(1/(4+4*cot(x)+tan(x)),x,method=_RETURNVERBOSE)
[Out] 3/50*ln(1+tan(x)^2)+4/25*arctan(tan(x))+2/5/(2+tan(x))-3/25*ln(2+tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = -\frac{3\left(\tan\left(x\right)+2\right)\log\left(\frac{\tan\left(x\right)^{2}+4\,\tan\left(x\right)+4}{\tan\left(x\right)^{2}+1}\right)-8\left(x-1\right)\tan\left(x\right)-16\,x-4}{50\left(\tan\left(x\right)+2\right)}$$

```
[In] integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="fricas")
[Out] -1/50*(3*(tan(x) + 2)*log((tan(x)^2 + 4*tan(x) + 4)/(tan(x)^2 + 1)) - 8*(x
- 1)*tan(x) - 16*x - 4)/(tan(x) + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(26) = 52$.

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.64

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = \frac{8x\tan(x)}{50\tan(x)+100}+\frac{16x}{50\tan(x)+100}-\frac{6\log(\tan(x)+2)\tan(x)}{50\tan(x)+100}-\frac{12\log(\tan(x)+2)}{50\tan(x)+100}+\frac{3\log(\tan^2(x)+1)\tan(x)}{50\tan(x)+100}+\frac{6\log(\tan^2(x)+1)}{50\tan(x)+100}+\frac{20}{50\tan(x)+100}$$

```
[In] integrate(1/(4+4*cot(x)+tan(x)),x)
```

```
[Out] 8*x*tan(x)/(50*tan(x) + 100) + 16*x/(50*tan(x) + 100) - 6*log(tan(x) + 2)*t
an(x)/(50*tan(x) + 100) - 12*log(tan(x) + 2)/(50*tan(x) + 100) + 3*log(tan(
x)**2 + 1)*tan(x)/(50*tan(x) + 100) + 6*log(tan(x)**2 + 1)/(50*tan(x) + 100
) + 20/(50*tan(x) + 100)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = \frac{4}{25}\,x+\frac{2}{5\,(\tan(x)+2)}+\frac{3}{50}\,\log\left(\tan(x)^2+1\right)-\frac{3}{25}\,\log(\tan(x)+2)$$

```
[In] integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] 4/25*x + 2/5/(tan(x) + 2) + 3/50*log(tan(x)^2 + 1) - 3/25*log(tan(x) + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = \frac{4}{25}\,x+\frac{2}{5\,(\tan(x)+2)}+\frac{3}{50}\,\log\left(\tan(x)^2+1\right)-\frac{3}{25}\,\log(|\tan(x)+2|)$$

```
[In] integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="giac")
```

```
[Out] 4/25*x + 2/5/(tan(x) + 2) + 3/50*log(tan(x)^2 + 1) - 3/25*log(abs(tan(x) +
2))
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = \frac{2}{5\,\left(\tan\left(x\right)+2\right)}-\frac{3\,\ln\left(\tan(x)+2\right)}{25}+\ln\left(\tan(x)-\mathrm{i}\right)\,\left(\frac{3}{50}-\frac{2}{25}\mathrm{i}\right)+\ln\left(\tan(x)+1\mathrm{i}\right)\,\left(\frac{3}{50}+\frac{2}{25}\mathrm{i}\right)$$

```
[In] int(1/(4*cot(x) + tan(x) + 4),x)
```

```
[Out] log(tan(x) - 1i)*(3/50 - 2i/25) - (3*log(tan(x) + 2))/25 + log(tan(x) + 1i)
*(3/50 + 2i/25) + 2/(5*(tan(x) + 2))
```

3.380 $\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 67

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \frac{8x}{15\sqrt{15}} - \frac{8\arctan\left(\frac{1-2\cos^2(x)}{4+\sqrt{15}+2\cos(x)\sin(x)}\right)}{15\sqrt{15}} + \frac{1+4\tan(x)}{15\left(2+\tan(x)+2\tan^2(x)\right)}$$

```
[Out] 8/225*x*15^(1/2)-8/225*arctan((1-2*cos(x)^2)/(4+2*cos(x)*sin(x)+15^(1/2)))*
15^(1/2)+1/15*(1+4*tan(x))/(2+tan(x)+2*tan(x)^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {628, 632, 210}

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = -\frac{8\arctan\left(\frac{1-2\cos^2(x)}{2\sin(x)\cos(x)+\sqrt{15}+4}\right)}{15\sqrt{15}} + \frac{8x}{15\sqrt{15}} + \frac{4\tan(x)+1}{15\left(2\tan^2(x)+\tan(x)+2\right)}$$

```
[In] Int[(2*Sec[x] + Sin[x])^(-2),x]
```

```
[Out] (8*x)/(15*Sqrt[15]) - (8*ArcTan[(1 - 2*Cos[x]^2)/(4 + Sqrt[15] + 2*Cos[x]*S
in[x])])/(15*Sqrt[15]) + (1 + 4*Tan[x])/(15*(2 + Tan[x] + 2*Tan[x]^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(2+x+2x^2)^2}\,dx, x, \tan(x)\right)\\
&= \frac{1+4\tan(x)}{15\left(2+\tan(x)+2\tan^2(x)\right)} + \frac{4}{15}\text{Subst}\left(\int \frac{1}{2+x+2x^2}\,dx, x, \tan(x)\right)\\
&= \frac{1+4\tan(x)}{15\left(2+\tan(x)+2\tan^2(x)\right)} - \frac{8}{15}\text{Subst}\left(\int \frac{1}{-15-x^2}\,dx, x, 1+4\tan(x)\right)\\
&= \frac{8x}{15\sqrt{15}} - \frac{8\arctan\left(\frac{1-2\cos^2(x)}{4+\sqrt{15}+2\cos(x)\sin(x)}\right)}{15\sqrt{15}} + \frac{1+4\tan(x)}{15\left(2+\tan(x)+2\tan^2(x)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \frac{\sec^2(x)(4+\sin(2x))\left(15(-15+\cos(2x))+8\sqrt{15}\arctan\left(\frac{1+4\tan(x)}{\sqrt{15}}\right)(4+\sin(2x))\right)}{900(2\sec(x)+\sin(x))^2}$$

```
[In] Integrate[(2*Sec[x] + Sin[x])^(-2),x]
```

```
[Out] (Sec[x]^2*(4 + Sin[2*x])*(15*(-15 + Cos[2*x]) + 8*Sqrt[15]*ArcTan[(1 + 4*Ta
n[x])/Sqrt[15]]*(4 + Sin[2*x])))/(900*(2*Sec[x] + Sin[x])^2)
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{1+4\tan(x)}{30+15\tan(x)+30(\tan^2(x))}+\frac{8\sqrt{15}\,\arctan\left(\frac{(1+4\tan(x))\sqrt{15}}{15}\right)}{225}$	39
risch	$\frac{\left(\frac{8}{3615}-\frac{2i}{241}\right)(241\,\mathrm{e}^{2ix}-15+4i)}{\mathrm{e}^{4ix}+8i\mathrm{e}^{2ix}-1}+\frac{4i\sqrt{15}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{15}+4i\right)}{225}-\frac{4i\sqrt{15}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{15}+4i\right)}{225}$	76

```
[In] int(1/(2*sec(x)+sin(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/15*(1+4*tan(x))/(2+tan(x)+2*tan(x)^2)+8/225*15^(1/2)*arctan(1/15*(1+4*tan
(x))*15^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \frac{4\left(\sqrt{15}\cos(x)\sin(x)+2\sqrt{15}\right)\arctan\left(\frac{8\sqrt{15}\cos(x)\sin(x)+\sqrt{15}}{15\left(2\cos(x)^2-1\right)}\right)+15\cos(x)^2-120}{225\left(\cos(x)\sin(x)+2\right)}$$

```
[In] integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="fricas")
[Out] 1/225*(4*(sqrt(15)*cos(x)*sin(x) + 2*sqrt(15))*arctan(1/15*(8*sqrt(15)*cos(
x)*sin(x) + sqrt(15))/(2*cos(x)^2 - 1)) + 15*cos(x)^2 - 120)/(cos(x)*sin(x)
 + 2)
```

Sympy [F]

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \int \frac{1}{(\sin(x)+2\sec(x))^2}\,dx$$

```
[In] integrate(1/(2*sec(x)+sin(x))**2,x)
[Out] Integral((sin(x) + 2*sec(x))**(-2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \frac{8}{225}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4\tan(x)+1)\right) + \frac{4\tan(x)+1}{15\left(2\tan(x)^2+\tan(x)+2\right)}$$

```
[In] integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="maxima")
[Out] 8/225*sqrt(15)*arctan(1/15*sqrt(15)*(4*tan(x) + 1)) + 1/15*(4*tan(x) + 1)/(
2*tan(x)^2 + tan(x) + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \frac{8}{225}\sqrt{15}\left(x+\arctan\left(-\frac{\sqrt{15}\sin(2x)-\cos(2x)-4\sin(2x)-1}{\sqrt{15}\cos(2x)+\sqrt{15}-4\cos(2x)+\sin(2x)+4}\right)\right) + \frac{4\tan(x)+1}{15\left(2\tan(x)^2+\tan(x)+2\right)}$$

```
[In] integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="giac")
[Out] 8/225*sqrt(15)*(x + arctan(-(sqrt(15)*sin(2*x) - cos(2*x) - 4*sin(2*x) - 1)
/(sqrt(15)*cos(2*x) + sqrt(15) - 4*cos(2*x) + sin(2*x) + 4))) + 1/15*(4*tan
(x) + 1)/(2*tan(x)^2 + tan(x) + 2)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.79

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \frac{4\sqrt{15}\left(2\,\mathrm{atan}\left(\frac{2\sqrt{15}\tan\left(\frac{x}{2}\right)^3}{15}-\frac{2\sqrt{15}\tan\left(\frac{x}{2}\right)^2}{15}+\frac{2\sqrt{15}\tan\left(\frac{x}{2}\right)}{5}+\frac{\sqrt{15}}{15}\right)-2\,\mathrm{atan}\left(\frac{\sqrt{15}}{15}-\frac{2\sqrt{15}\tan\left(\frac{x}{2}\right)}{15}\right)\right)}{225} - \frac{\frac{7\tan\left(\frac{x}{2}\right)^3}{30}+\frac{2\tan\left(\frac{x}{2}\right)^2}{15}-\frac{7\tan\left(\frac{x}{2}\right)}{30}}{\tan\left(\frac{x}{2}\right)^4-\tan\left(\frac{x}{2}\right)^3+2\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)+1}$$

```
[In] int(1/(sin(x) + 2/cos(x))^2,x)
```

```
[Out] (4*15^(1/2)*(2*atan((2*15^(1/2)*tan(x/2))/5 + 15^(1/2)/15 - (2*15^(1/2)*tan
(x/2)^2)/15 + (2*15^(1/2)*tan(x/2)^3)/15) - 2*atan(15^(1/2)/15 - (2*15^(1/2
)*tan(x/2))/15)))/225 - ((2*tan(x/2)^2)/15 - (7*tan(x/2))/30 + (7*tan(x/2)^
3)/30)/(tan(x/2) + 2*tan(x/2)^2 - tan(x/2)^3 + tan(x/2)^4 + 1)
```

3.381 $\int \frac{1}{(\cos(x)+2\sec(x))^2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 55

$$\int \frac{1}{(\cos(x)+2\sec(x))^2} \, dx = \frac{x}{6\sqrt{6}} - \frac{\arctan\left(\frac{\cos(x)\sin(x)}{2+\sqrt{6}+\cos^2(x)}\right)}{6\sqrt{6}} + \frac{\tan(x)}{6\left(3+2\tan^2(x)\right)}$$

```
[Out] 1/36*x*6^(1/2)-1/36*arctan(cos(x)*sin(x)/(2+cos(x)^2+6^(1/2)))*6^(1/2)+1/6*
tan(x)/(3+2*tan(x)^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {205, 209}

$$\int \frac{1}{(\cos(x)+2\sec(x))^2} \, dx = -\frac{\arctan\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{6}+2}\right)}{6\sqrt{6}} + \frac{x}{6\sqrt{6}} + \frac{\tan(x)}{6\left(2\tan^2(x)+3\right)}$$

```
[In] Int[(Cos[x] + 2*Sec[x])^(-2),x]
```

```
[Out] x/(6*Sqrt[6]) - ArcTan[(Cos[x]*Sin[x])/(2 + Sqrt[6] + Cos[x]^2)]/(6*Sqrt[6]
) + Tan[x]/(6*(3 + 2*Tan[x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{(3+2x^2)^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan(x)}{6\,(3+2\tan^2(x))} + \frac{1}{6}\text{Subst}\left(\int \frac{1}{3+2x^2}\,dx, x, \tan(x)\right)\\ &= \frac{x}{6\sqrt{6}} - \frac{\arctan\left(\frac{\cos(x)\sin(x)}{2+\sqrt{6}+\cos^2(x)}\right)}{6\sqrt{6}} + \frac{\tan(x)}{6\,(3+2\tan^2(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx = \frac{(5+\cos(2x))\sec^4(x)\left(\sqrt{6}\arctan\left(\sqrt{\frac{2}{3}}\tan(x)\right)(5+\cos(2x))+6\sin(2x)\right)}{144\,(1+2\sec^2(x))^2}$$

```
[In] Integrate[(Cos[x] + 2*Sec[x])^(-2),x]
[Out] ((5 + Cos[2*x])*Sec[x]^4*(Sqrt[6]*ArcTan[Sqrt[2/3]*Tan[x]]*(5 + Cos[2*x]) +
 6*Sin[2*x]))/(144*(1 + 2*Sec[x]^2)^2)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{\tan(x)}{18+12(\tan^2(x))}+\frac{\sqrt{6}\,\arctan\left(\frac{\tan(x)\sqrt{6}}{3}\right)}{36}$	29
risch	$\frac{i(5\,\mathrm{e}^{2ix}+1)}{3\,\mathrm{e}^{4ix}+30\,\mathrm{e}^{2ix}+3}+\frac{i\sqrt{6}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{6}+5\right)}{72}-\frac{i\sqrt{6}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{6}+5\right)}{72}$	68

```
[In] int(1/(cos(x)+2*sec(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/6*tan(x)/(3+2*tan(x)^2)+1/36*6^(1/2)*arctan(1/3*tan(x)*6^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx = -\frac{\left(\sqrt{6}\cos\left(x\right)^2+2\sqrt{6}\right)\arctan\left(\frac{5\sqrt{6}\cos(x)^2-2\sqrt{6}}{12\cos(x)\sin(x)}\right)-12\cos\left(x\right)\sin\left(x\right)}{72\left(\cos\left(x\right)^2+2\right)}$$

```
[In] integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="fricas")
[Out] -1/72*((sqrt(6)*cos(x)^2 + 2*sqrt(6))*arctan(1/12*(5*sqrt(6)*cos(x)^2 - 2*s
qrt(6))/(cos(x)*sin(x))) - 12*cos(x)*sin(x))/(cos(x)^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx = \int \frac{1}{\left(\cos\left(x\right)+2\sec\left(x\right)\right)^2}\,dx$$

```
[In] integrate(1/(cos(x)+2*sec(x))**2,x)
[Out] Integral((cos(x) + 2*sec(x))**(-2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.51

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx = \frac{1}{36}\sqrt{6}\arctan\left(\frac{1}{3}\sqrt{6}\tan\left(x\right)\right)+\frac{\tan\left(x\right)}{6\left(2\tan\left(x\right)^2+3\right)}$$

```
[In] integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="maxima")
[Out] 1/36*sqrt(6)*arctan(1/3*sqrt(6)*tan(x)) + 1/6*tan(x)/(2*tan(x)^2 + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx=\frac{1}{36}\sqrt{6}\left(x+\arctan\left(-\frac{\sqrt{6}\sin(2x)-2\sin(2x)}{\sqrt{6}\cos(2x)+\sqrt{6}-2\cos(2x)+2}\right)\right)+\frac{\tan(x)}{6\left(2\tan(x)^2+3\right)}$$

```
[In] integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="giac")
[Out] 1/36*sqrt(6)*(x + arctan(-(sqrt(6)*sin(2*x) - 2*sin(2*x))/(sqrt(6)*cos(2*x)
 + sqrt(6) - 2*cos(2*x) + 2))) + 1/6*tan(x)/(2*tan(x)^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx=\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\tan\left(\frac{x}{2}\right)^3}{4}+\frac{5\sqrt{6}\tan\left(\frac{x}{2}\right)}{12}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{6}\tan\left(\frac{x}{2}\right)}{4}\right)\right)}{72}+\frac{\frac{\tan\left(\frac{x}{2}\right)}{9}-\frac{\tan\left(\frac{x}{2}\right)^3}{9}}{\tan\left(\frac{x}{2}\right)^4+\frac{2\tan\left(\frac{x}{2}\right)^2}{3}+1}$$

```
[In] int(1/(cos(x) + 2/cos(x))^2,x)
[Out] (6^(1/2)*(2*atan((5*6^(1/2)*tan(x/2))/12 + (6^(1/2)*tan(x/2)^3)/4) + 2*atan
((6^(1/2)*tan(x/2))/4)))/72 + (tan(x/2)/9 - tan(x/2)^3/9)/((2*tan(x/2)^2)/3
 + tan(x/2)^4 + 1)
```

3.382 $\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 42

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{67x}{250}-\frac{28}{125}\log(\cos(x)+3\sin(x)) - \frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}$$

```
[Out] -67/250*x-28/125*ln(cos(x)+3*sin(x))-7/10/(1+3*tan(x))^2-29/50/(1+3*tan(x))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3709, 3610, 3612, 3611}

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{67x}{250}-\frac{29}{50(3\tan(x)+1)}-\frac{7}{10(3\tan(x)+1)^2} - \frac{28}{125}\log(3\sin(x)+\cos(x))$$

```
[In] Int[(5 - Tan[x] - 6*Tan[x]^2)/(1 + 3*Tan[x])^3,x]
```

```
[Out] (-67*x)/250 - (28*Log[Cos[x] + 3*Sin[x]])/125 - 7/(10*(1 + 3*Tan[x])^2) - 2
9/(50*(1 + 3*Tan[x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
```

```
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{7}{10(1+3\tan(x))^2}+\frac{1}{10}\int\frac{8-34\tan(x)}{(1+3\tan(x))^2}\,dx\\
&= -\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}+\frac{1}{100}\int\frac{-94-58\tan(x)}{1+3\tan(x)}\,dx\\
&= -\frac{67x}{250}-\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}-\frac{28}{125}\int\frac{3-\tan(x)}{1+3\tan(x)}\,dx\\
&= -\frac{67x}{250}-\frac{28}{125}\log(\cos(x)+3\sin(x))-\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = \frac{1}{500}\left((56+67i)\log(i-\tan(x))+(56-67i)\log(i+\tan(x)) - 112\log(1+3\tan(x)) - \frac{350}{(1+3\tan(x))^2} - \frac{290}{1+3\tan(x)}\right)$$

```
[In] Integrate[(5 - Tan[x] - 6*Tan[x]^2)/(1 + 3*Tan[x])^3,x]
```

```
[Out] ((56 + 67*I)*Log[I - Tan[x]] + (56 - 67*I)*Log[I + Tan[x]] - 112*Log[1 + 3*
Tan[x]] - 350/(1 + 3*Tan[x])^2 - 290/(1 + 3*Tan[x]))/500
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{14\ln\left(1+\tan^2(x)\right)}{125}-\frac{67\arctan(\tan(x))}{250}-\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}-\frac{28\ln(1+3\tan(x))}{125}$
default	$\frac{14\ln\left(1+\tan^2(x)\right)}{125}-\frac{67\arctan(\tan(x))}{250}-\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}-\frac{28\ln(1+3\tan(x))}{125}$
risch	$-\frac{67x}{250}+\frac{28ix}{125}+\frac{\left(-\frac{36}{24125}-\frac{621i}{48250}\right)\left(965\,\mathrm{e}^{2ix}-324+768i\right)}{\left(5\,\mathrm{e}^{2ix}-4+3i\right)^2}-\frac{28\ln\left(\mathrm{e}^{2ix}-\frac{4}{5}+\frac{3i}{5}\right)}{125}$
norman	$\frac{-\frac{87\tan(x)}{50}-\frac{67x}{250}-\frac{201x\tan(x)}{125}-\frac{603x\left(\tan^2(x)\right)}{250}-\frac{32}{25}}{(1+3\tan(x))^2}-\frac{28\ln(1+3\tan(x))}{125}+\frac{14\ln\left(1+\tan^2(x)\right)}{125}$
parallelrisch	$-\frac{4536\ln\left(\frac{1}{3}+\tan(x)\right)\left(\tan^2(x)\right)-2268\ln\left(1+\tan^2(x)\right)\left(\tan^2(x)\right)+5427x\left(\tan^2(x)\right)+2880+3024\ln\left(\frac{1}{3}+\tan(x)\right)\tan(x)-151}{2250(1+3\tan(x))^2}$

```
[In] int((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 14/125*ln(1+tan(x)^2)-67/250*arctan(tan(x))-7/10/(1+3*tan(x))^2-29/50/(1+3*
tan(x))-28/125*ln(1+3*tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(34) = 68$.

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.83

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{9\,(134\,x-1)\tan\left(x\right)^2+56\,\left(9\,\tan\left(x\right)^2+6\,\tan\left(x\right)+1\right)\log\left(\frac{9\,\tan(x)^2+6\,\tan(x)+1}{\tan(x)^2+1}\right)+12\,(67\,x+72)\tan\left(x\right)+134\,x+639}{500\,\left(9\,\tan\left(x\right)^2+6\,\tan\left(x\right)+1\right)}$$

```
[In] integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="fricas")
[Out] -1/500*(9*(134*x - 1)*tan(x)^2 + 56*(9*tan(x)^2 + 6*tan(x) + 1)*log((9*tan(
x)^2 + 6*tan(x) + 1)/(tan(x)^2 + 1)) + 12*(67*x + 72)*tan(x) + 134*x + 639)
/(9*tan(x)^2 + 6*tan(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(39) = 78$.

Time = 0.24 (sec) , antiderivative size = 252, normalized size of antiderivative = 6.00

$$\begin{aligned}\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = &-\frac{603x\tan^2(x)}{2250\tan^2(x)+1500\tan(x)+250} \\ &-\frac{402x\tan(x)}{2250\tan^2(x)+1500\tan(x)+250} \\ &-\frac{67x}{2250\tan^2(x)+1500\tan(x)+250} \\ &-\frac{504\log(3\tan(x)+1)\tan^2(x)}{2250\tan^2(x)+1500\tan(x)+250} \\ &-\frac{336\log(3\tan(x)+1)\tan(x)}{2250\tan^2(x)+1500\tan(x)+250} \\ &-\frac{56\log(3\tan(x)+1)}{2250\tan^2(x)+1500\tan(x)+250} \\ &+\frac{252\log(\tan^2(x)+1)\tan^2(x)}{2250\tan^2(x)+1500\tan(x)+250} \\ &+\frac{168\log(\tan^2(x)+1)\tan(x)}{2250\tan^2(x)+1500\tan(x)+250} \\ &+\frac{28\log(\tan^2(x)+1)}{2250\tan^2(x)+1500\tan(x)+250} \\ &-\frac{435\tan(x)}{2250\tan^2(x)+1500\tan(x)+250} \\ &-\frac{320}{2250\tan^2(x)+1500\tan(x)+250}\end{aligned}$$

```
[In] integrate((5-tan(x)-6*tan(x)**2)/(1+3*tan(x))**3,x)
```

```
[Out] -603*x*tan(x)**2/(2250*tan(x)**2 + 1500*tan(x) + 250) - 402*x*tan(x)/(2250*
tan(x)**2 + 1500*tan(x) + 250) - 67*x/(2250*tan(x)**2 + 1500*tan(x) + 250)
- 504*log(3*tan(x) + 1)*tan(x)**2/(2250*tan(x)**2 + 1500*tan(x) + 250) - 33
6*log(3*tan(x) + 1)*tan(x)/(2250*tan(x)**2 + 1500*tan(x) + 250) - 56*log(3*
tan(x) + 1)/(2250*tan(x)**2 + 1500*tan(x) + 250) + 252*log(tan(x)**2 + 1)*t
an(x)**2/(2250*tan(x)**2 + 1500*tan(x) + 250) + 168*log(tan(x)**2 + 1)*tan(
x)/(2250*tan(x)**2 + 1500*tan(x) + 250) + 28*log(tan(x)**2 + 1)/(2250*tan(x
)**2 + 1500*tan(x) + 250) - 435*tan(x)/(2250*tan(x)**2 + 1500*tan(x) + 250)
 - 320/(2250*tan(x)**2 + 1500*tan(x) + 250)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{67}{250}\,x - \frac{87\,\tan\left(x\right)+64}{50\,\left(9\,\tan\left(x\right)^2+6\,\tan\left(x\right)+1\right)} + \frac{14}{125}\,\log\left(\tan\left(x\right)^2+1\right) - \frac{28}{125}\,\log\left(3\,\tan\left(x\right)+1\right)$$

```
[In] integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="maxima")
```

```
[Out] -67/250*x - 1/50*(87*tan(x) + 64)/(9*tan(x)^2 + 6*tan(x) + 1) + 14/125*log(
tan(x)^2 + 1) - 28/125*log(3*tan(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{67}{250}\,x - \frac{87\,\tan\left(x\right)+64}{50\,\left(3\,\tan\left(x\right)+1\right)^2} + \frac{14}{125}\,\log\left(\tan\left(x\right)^2+1\right) - \frac{28}{125}\,\log\left(\left|3\,\tan\left(x\right)+1\right|\right)$$

```
[In] integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="giac")
```

```
[Out] -67/250*x - 1/50*(87*tan(x) + 64)/(3*tan(x) + 1)^2 + 14/125*log(tan(x)^2 +
1) - 28/125*log(abs(3*tan(x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{28\,\ln\left(\tan(x)+\frac{1}{3}\right)}{125} - \frac{\frac{29\tan(x)}{150}+\frac{32}{225}}{\tan\left(x\right)^2+\frac{2\tan(x)}{3}+\frac{1}{9}} + \ln\left(\tan(x)-\mathrm{i}\right)\left(\frac{14}{125}+\frac{67}{500}\mathrm{i}\right) + \ln\left(\tan(x)+1\mathrm{i}\right)\left(\frac{14}{125}-\frac{67}{500}\mathrm{i}\right)$$

```
[In] int(-(tan(x) + 6*tan(x)^2 - 5)/(3*tan(x) + 1)^3,x)
```

```
[Out] log(tan(x) - 1i)*(14/125 + 67i/500) - (28*log(tan(x) + 1/3))/125 + log(tan(
x) + 1i)*(14/125 - 67i/500) - ((29*tan(x))/150 + 32/225)/((2*tan(x))/3 + ta
n(x)^2 + 1/9)
```

3.383 $\int \cos^2(x) \sec(3x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 9

$$\int \cos^2(x) \sec(3x) \, dx = \frac{1}{2} \operatorname{arctanh}(2 \sin(x))$$

```
[Out] 1/2*arctanh(2*sin(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {212}

$$\int \cos^2(x) \sec(3x) \, dx = \frac{1}{2} \operatorname{arctanh}(2 \sin(x))$$

```
[In] Int[Cos[x]^2*Sec[3*x],x]
```

```
[Out] ArcTanh[2*Sin[x]]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{1-4x^2} \, dx, x, \sin(x)\right) \\ &= \frac{1}{2} \operatorname{arctanh}(2 \sin(x)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \cos^2(x) \sec(3x) \, dx = \frac{1}{2}\text{arctanh}(2\sin(x))$$

```
[In] Integrate[Cos[x]^2*Sec[3*x],x]
[Out] ArcTanh[2*Sin[x]]/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

method	result	size
default	$\frac{\ln(1+2\sin(x))}{4} - \frac{\ln(2\sin(x)-1)}{4}$	20
risch	$-\frac{\ln(-i e^{ix}+e^{2ix}-1)}{4} + \frac{\ln(i e^{ix}+e^{2ix}-1)}{4}$	38

```
[In] int(cos(x)^2/cos(3*x),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(1+2*sin(x))-1/4*ln(2*sin(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int \cos^2(x) \sec(3x) \, dx = \frac{1}{4} \log\left(2 \sin\left(x\right) + 1\right) - \frac{1}{4} \log\left(-2 \sin\left(x\right) + 1\right)$$

```
[In] integrate(cos(x)^2/cos(3*x),x, algorithm="fricas")
[Out] 1/4*log(2*sin(x) + 1) - 1/4*log(-2*sin(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(7) = 14$.

Time = 1.80 (sec) , antiderivative size = 76, normalized size of antiderivative = 8.44

$$\int \cos^2(x)\sec(3x)\,dx = -\frac{\log(\sin(3x)-1)}{12} + \frac{\log(\sin(3x)+1)}{12} - \frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{6} + \frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{6} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)}{12} + \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+1\right)}{12}$$

```
[In] integrate(cos(x)**2/cos(3*x),x)
[Out] -log(sin(3*x) - 1)/12 + log(sin(3*x) + 1)/12 - log(tan(x/2) - 1)/6 + log(ta
n(x/2) + 1)/6 - log(tan(x/2)**2 - 4*tan(x/2) + 1)/12 + log(tan(x/2)**2 + 4*
tan(x/2) + 1)/12
```

Maxima [F]

$$\int \cos^2(x)\sec(3x)\,dx = \int \frac{\cos(x)^2}{\cos(3x)}\,dx$$

```
[In] integrate(cos(x)^2/cos(3*x),x, algorithm="maxima")
[Out] integrate(cos(x)^2/cos(3*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(7) = 14$.

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33

$$\int \cos^2(x)\sec(3x)\,dx = \frac{1}{4}\log(|2\sin(x)+1|) - \frac{1}{4}\log(|2\sin(x)-1|)$$

```
[In] integrate(cos(x)^2/cos(3*x),x, algorithm="giac")
[Out] 1/4*log(abs(2*sin(x) + 1)) - 1/4*log(abs(2*sin(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cos^2(x) \sec(3x) \, dx = \frac{\operatorname{atanh}(2 \sin(x))}{2}$$

```
[In] int(cos(x)^2/cos(3*x),x)
[Out] atanh(2*sin(x))/2
```

3.384 $\int \sec(2x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \sec(2x)\sin(x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4442, 213}

$$\int \sec(2x)\sin(x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

```
[In] Int[Sec[2*x]*Sin[x],x]
[Out] ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 4442

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
```

```
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cos(x)\right)\\ &= \frac{\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \sec(2x)\sin(x)\,dx = \frac{\text{arctanh}\left(\sqrt{2}-\tan\left(\frac{x}{2}\right)\right)+\text{arctanh}\left(\sqrt{2}+\tan\left(\frac{x}{2}\right)\right)}{\sqrt{2}}$$

```
[In] Integrate[Sec[2*x]*Sin[x],x]
[Out] (ArcTanh[Sqrt[2] - Tan[x/2]] + ArcTanh[Sqrt[2] + Tan[x/2]])/Sqrt[2]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\text{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{2}$	13
risch	$\frac{\sqrt{2}\,\ln\left(e^{2ix}+\sqrt{2}\,e^{ix}+1\right)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-\sqrt{2}\,e^{ix}+1\right)}{4}$	47

```
[In] int(sin(x)/cos(2*x),x,method=_RETURNVERBOSE)
[Out] 1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \sec(2x)\sin(x)\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2 + 2\sqrt{2}\cos(x) + 1}{2\cos(x)^2 - 1}\right)$$

```
[In] integrate(sin(x)/cos(2*x),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))
```

Sympy [F]

$$\int \sec(2x)\sin(x)\,dx = \int \frac{\sin(x)}{\cos(2x)}\,dx$$

```
[In] integrate(sin(x)/cos(2*x),x)
[Out] Integral(sin(x)/cos(2*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 8.60

$$\begin{aligned}\int \sec(2x)\sin(x)\,dx = {} & \frac{1}{8}\sqrt{2}\log\Big(2\sqrt{2}\sin(2x)\sin(x) + 2\left(\sqrt{2}\cos(x) + 1\right)\cos(2x) \\ & + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 1\Big) \\ & - \frac{1}{8}\sqrt{2}\log\Big(-2\sqrt{2}\sin(2x)\sin(x) - 2\left(\sqrt{2}\cos(x) - 1\right)\cos(2x) \\ & + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 1\Big)\end{aligned}$$

```
[In] integrate(sin(x)/cos(2*x),x, algorithm="maxima")
[Out] 1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x)
 + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 1
) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*cos
(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x
) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.27

$$\int \sec(2x)\sin(x)\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2} - \frac{2(\cos(x)-1)}{\cos(x)+1} - 6\right|}{\left|4\sqrt{2} - \frac{2(\cos(x)-1)}{\cos(x)+1} - 6\right|}\right)$$

```
[In] integrate(sin(x)/cos(2*x),x, algorithm="giac")
[Out] 1/4*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*sqr
t(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \sec(2x)\sin(x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\cos(x)\right)}{2}$$

```
[In] int(sin(x)/cos(2*x),x)
[Out] (2^(1/2)*atanh(2^(1/2)*cos(x)))/2
```

3.385 $\int \sec(2x)\sin^2(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \sec(2x)\sin^2(x)\,dx = -\frac{x}{2} + \frac{1}{4}\text{arctanh}(2\cos(x)\sin(x))$$

```
[Out] -1/2*x+1/4*arctanh(2*cos(x)*sin(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {304, 209, 212}

$$\int \sec(2x)\sin^2(x)\,dx = \frac{1}{4}\text{arctanh}(2\sin(x)\cos(x)) - \frac{x}{2}$$

```
[In] Int[Sec[2*x]*Sin[x]^2,x]
```

```
[Out] -1/2*x + ArcTanh[2*Cos[x]*Sin[x]]/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tan(x)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\
&= -\frac{x}{2} + \frac{1}{4}\text{arctanh}(2\cos(x)\sin(x))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.65

$$\int \sec(2x)\sin^2(x)\,dx = -\frac{x}{2} - \frac{1}{4}\log(\cos(x)-\sin(x)) + \frac{1}{4}\log(\cos(x)+\sin(x))$$

```
[In] Integrate[Sec[2*x]*Sin[x]^2,x]
[Out] -1/2*x - Log[Cos[x] - Sin[x]]/4 + Log[Cos[x] + Sin[x]]/4
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

method	result	size
default	$-\frac{\ln(\tan(x)-1)}{4}-\frac{\arctan(\tan(x))}{2}+\frac{\ln(\tan(x)+1)}{4}$	21
risch	$-\frac{x}{2}-\frac{\ln\left(e^{2ix}-i\right)}{4}+\frac{\ln\left(e^{2ix}+i\right)}{4}$	27

```
[In] int(sin(x)^2/cos(2*x),x,method=_RETURNVERBOSE)
[Out] -1/4*ln(tan(x)-1)-1/2*arctan(tan(x))+1/4*ln(tan(x)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \sec(2x)\sin^2(x)\,dx = -\frac{1}{2}\,x + \frac{1}{8}\,\log\left(2\,\cos\left(x\right)\sin\left(x\right) + 1\right) - \frac{1}{8}\,\log\left(-2\,\cos\left(x\right)\sin\left(x\right) + 1\right)$$

```
[In] integrate(sin(x)^2/cos(2*x),x, algorithm="fricas")
[Out] -1/2*x + 1/8*log(2*cos(x)*sin(x) + 1) - 1/8*log(-2*cos(x)*sin(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \sec(2x)\sin^2(x)\,dx = -\frac{x}{2} - \frac{\log\left(\sin\left(2x\right) - 1\right)}{8} + \frac{\log\left(\sin\left(2x\right) + 1\right)}{8}$$

```
[In] integrate(sin(x)**2/cos(2*x),x)
[Out] -x/2 - log(sin(2*x) - 1)/8 + log(sin(2*x) + 1)/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 7.53

$$\begin{aligned}\int \sec(2x)\sin^2(x)\,dx =& -\frac{1}{2}\,x - \frac{1}{8}\,\log\left(2\,\cos\left(x\right)^2 + 2\,\sin\left(x\right)^2 + 2\,\sqrt{2}\cos\left(x\right) + 2\,\sqrt{2}\sin\left(x\right) + 2\right)\\ &+ \frac{1}{8}\,\log\left(2\,\cos\left(x\right)^2 + 2\,\sin\left(x\right)^2 + 2\,\sqrt{2}\cos\left(x\right) - 2\,\sqrt{2}\sin\left(x\right) + 2\right)\\ &+ \frac{1}{8}\,\log\left(2\,\cos\left(x\right)^2 + 2\,\sin\left(x\right)^2 - 2\,\sqrt{2}\cos\left(x\right) + 2\,\sqrt{2}\sin\left(x\right) + 2\right)\\ &- \frac{1}{8}\,\log\left(2\,\cos\left(x\right)^2 + 2\,\sin\left(x\right)^2 - 2\,\sqrt{2}\cos\left(x\right) - 2\,\sqrt{2}\sin\left(x\right) + 2\right)\end{aligned}$$

```
[In] integrate(sin(x)^2/cos(2*x),x, algorithm="maxima")
[Out] -1/2*x - 1/8*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) + 1/8*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sqrt(2)*s
in(x) + 2) + 1/8*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt(2)
*sin(x) + 2) - 1/8*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(
2)*sin(x) + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sec(2x)\sin^2(x)\,dx = -\frac{1}{2}\,x + \frac{1}{4}\,\log\left(\left|\tan\left(x\right) + 1\right|\right) - \frac{1}{4}\,\log\left(\left|\tan\left(x\right) - 1\right|\right)$$

```
[In] integrate(sin(x)^2/cos(2*x),x, algorithm="giac")
[Out] -1/2*x + 1/4*log(abs(tan(x) + 1)) - 1/4*log(abs(tan(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int \sec(2x)\sin^2(x)\,dx = \frac{\operatorname{atanh}(\tan(x))}{2} - \frac{x}{2}$$

```
[In] int(sin(x)^2/cos(2*x),x)
[Out] atanh(tan(x))/2 - x/2
```

3.386 $\int \sec(3x)\sin^3(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 21

$$\int \sec(3x)\sin^3(x)\,dx = \frac{1}{3}\log(\cos(x)) - \frac{1}{24}\log\left(3-4\cos^2(x)\right)$$

```
[Out] 1/3*ln(cos(x))-1/24*ln(3-4*cos(x)^2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4451, 457, 78}

$$\int \sec(3x)\sin^3(x)\,dx = \frac{1}{3}\log(\cos(x)) - \frac{1}{24}\log\left(3-4\cos^2(x)\right)$$

```
[In] Int[Sec[3*x]*Sin[x]^3,x]
[Out] Log[Cos[x]]/3 - Log[3 - 4*Cos[x]^2]/24
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4451

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{-1+x^2}{x\,(3-4x^2)}\,dx, x, \cos(x)\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{(3-4x)x}\,dx, x, \cos^2(x)\right)\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{3x}+\frac{1}{3(-3+4x)}\right)\,dx, x, \cos^2(x)\right)\right) \\
&= \frac{1}{3}\log(\cos(x)) - \frac{1}{24}\log\left(3-4\cos^2(x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sec(3x)\sin^3(x)\,dx = \frac{1}{3}\log(\cos(x)) - \frac{1}{24}\log\left(1-4\sin^2(x)\right)$$

```
[In] Integrate[Sec[3*x]*Sin[x]^3,x]
[Out] Log[Cos[x]]/3 - Log[1 - 4*Sin[x]^2]/24
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\ln(\cos(x))}{3}-\frac{\ln(4(\cos^2(x))-3)}{24}$	18
risch	$-\frac{ix}{4}+\frac{\ln(e^{2ix}+1)}{3}-\frac{\ln(e^{4ix}-e^{2ix}+1)}{24}$	33

```
[In] int(sin(x)^3/cos(3*x),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(cos(x))-1/24*ln(4*cos(x)^2-3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sec(3x)\sin^3(x)\,dx = -\frac{1}{24}\log\left(4\cos\left(x\right)^2-3\right)+\frac{1}{3}\log\left(-\cos\left(x\right)\right)$$

```
[In] integrate(sin(x)^3/cos(3*x),x, algorithm="fricas")
[Out] -1/24*log(4*cos(x)^2 - 3) + 1/3*log(-cos(x))
```

Sympy [F]

$$\int \sec(3x)\sin^3(x)\,dx = \int \frac{\sin^3\left(x\right)}{\cos\left(3x\right)}\,dx$$

```
[In] integrate(sin(x)**3/cos(3*x),x)
[Out] Integral(sin(x)**3/cos(3*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.86

$$\begin{aligned}\int \sec(3x)\sin^3(x)\,dx = &-\frac{1}{48}\log\left(-2\left(\cos\left(2x\right)-1\right)\cos\left(4x\right)+\cos\left(4x\right)^2+\cos\left(2x\right)^2\right.\\ &\left.+\sin\left(4x\right)^2-2\sin\left(4x\right)\sin\left(2x\right)+\sin\left(2x\right)^2-2\cos\left(2x\right)+1\right)\\ &+\frac{1}{6}\log\left(\cos\left(2x\right)^2+\sin\left(2x\right)^2+2\cos\left(2x\right)+1\right)\end{aligned}$$

```
[In] integrate(sin(x)^3/cos(3*x),x, algorithm="maxima")
[Out] -1/48*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + sin(4*x)^2
 - 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) + 1/6*log(cos(2*x)^2
+ sin(2*x)^2 + 2*cos(2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \sec(3x)\sin^3(x)\,dx = \frac{1}{6}\log\left(-\sin\left(x\right)^2+1\right) - \frac{1}{24}\log\left(\left|4\sin\left(x\right)^2-1\right|\right)$$

```
[In] integrate(sin(x)^3/cos(3*x),x, algorithm="giac")
[Out] 1/6*log(-sin(x)^2 + 1) - 1/24*log(abs(4*sin(x)^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sec(3x)\sin^3(x)\,dx = \frac{\ln\left(\cos\left(x\right)\right)}{3} - \frac{\ln\left(\cos\left(x\right)^2-\frac{3}{4}\right)}{24}$$

```
[In] int(sin(x)^3/cos(3*x),x)
[Out] log(cos(x))/3 - log(cos(x)^2 - 3/4)/24
```

3.387 $\int \cos(x)\csc(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \cos(x)\csc(3x)\,dx = \frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

```
[Out] 1/3*ln(sin(x))-1/6*ln(3-4*sin(x)^2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4441, 272, 36, 31, 29}

$$\int \cos(x)\csc(3x)\,dx = \frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

```
[In] Int[Cos[x]*Csc[3*x],x]
```

```
[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{x(3-4x^2)}\,dx, x, \sin(x)\right) \\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(3-4x)x}\,dx, x, \sin^2(x)\right) \\ &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(x)\right) + \frac{2}{3}\text{Subst}\left(\int \frac{1}{3-4x}\,dx, x, \sin^2(x)\right) \\ &= \frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3-4\sin^2(x)\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \cos(x)\csc(3x)\,dx = \frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

```
[In] Integrate[Cos[x]*Csc[3*x],x]
[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

method	result	size
risch	$\frac{\ln(e^{2ix}-1)}{3}-\frac{\ln(e^{4ix}+e^{2ix}+1)}{6}$	27
default	$\frac{\ln(\cos(x)+1)}{6}-\frac{\ln(2\cos(x)-1)}{6}+\frac{\ln(-1+\cos(x))}{6}-\frac{\ln(1+2\cos(x))}{6}$	34

```
[In] int(cos(x)/sin(3*x),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(exp(2*I*x)-1)-1/6*ln(exp(4*I*x)+exp(2*I*x)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \cos(x)\csc(3x)\,dx = -\frac{1}{6}\log\left(4\cos(x)^2-1\right)+\frac{1}{3}\log\left(\frac{1}{2}\sin(x)\right)$$

```
[In] integrate(cos(x)/sin(3*x),x, algorithm="fricas")
[Out] -1/6*log(4*cos(x)^2 - 1) + 1/3*log(1/2*sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos(x)\csc(3x)\,dx = -\frac{\log\left(4\sin^2(x)-3\right)}{6}+\frac{\log(\sin(x))}{3}$$

```
[In] integrate(cos(x)/sin(3*x),x)
[Out] -log(4*sin(x)**2 - 3)/6 + log(sin(x))/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 6.14

$$\begin{aligned}\int \cos(x)\csc(3x)\,dx = &-\frac{1}{12}\log\left(2\left(\cos\left(x\right)+1\right)\cos\left(2\,x\right)+\cos\left(2\,x\right)^{2}+\cos\left(x\right)^{2}+\sin\left(2\,x\right)^{2}\right.\\&\left.+2\,\sin\left(2\,x\right)\sin\left(x\right)+\sin\left(x\right)^{2}+2\,\cos\left(x\right)+1\right)\\&-\frac{1}{12}\log\left(-2\left(\cos\left(x\right)-1\right)\cos\left(2\,x\right)+\cos\left(2\,x\right)^{2}+\cos\left(x\right)^{2}+\sin\left(2\,x\right)^{2}\right.\\&\left.-2\,\sin\left(2\,x\right)\sin\left(x\right)+\sin\left(x\right)^{2}-2\,\cos\left(x\right)+1\right)\\&+\frac{1}{6}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\,\cos\left(x\right)+1\right)\\&+\frac{1}{6}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\,\cos\left(x\right)+1\right)\end{aligned}$$

```
[In] integrate(cos(x)/sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/12*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2*
sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) - 1/12*log(-2*(cos(x) - 1)*cos(2
*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2 - 2
*cos(x) + 1) + 1/6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/6*log(cos(x)
^2 + sin(x)^2 - 2*cos(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \cos(x)\csc(3x)\,dx = \frac{1}{6}\log\left(-\cos\left(x\right)^{2}+1\right)-\frac{1}{6}\log\left(\left|4\,\cos\left(x\right)^{2}-1\right|\right)$$

```
[In] integrate(cos(x)/sin(3*x),x, algorithm="giac")
```

```
[Out] 1/6*log(-cos(x)^2 + 1) - 1/6*log(abs(4*cos(x)^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos(x)\csc(3x)\,dx = \frac{\ln\left(\sin\left(x\right)\right)}{3} - \frac{\ln\left(\frac{1}{4} - \cos\left(x\right)^2\right)}{6}$$

```
[In] int(cos(x)/sin(3*x),x)
[Out] log(sin(x))/3 - log(1/4 - cos(x)^2)/6
```

3.388 $\int \csc(4x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 26

$$\int \csc(4x)\sin(x)\,dx = -\frac{1}{4}\operatorname{arctanh}(\sin(x)) + \frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

```
[Out] -1/4*arctanh(sin(x))+1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1107, 213}

$$\int \csc(4x)\sin(x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}} - \frac{1}{4}\operatorname{arctanh}(\sin(x))$$

```
[In] Int[Csc[4*x]*Sin[x],x]
```

```
[Out] -1/4*ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
```

```
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{4-12x^2+8x^4}\,dx, x, \sin(x)\right)\\
&= 2\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx, x, \sin(x)\right) - 2\text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx, x, \sin(x)\right)\\
&= -\frac{1}{4}\text{arctanh}(\sin(x)) + \frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \csc(4x)\sin(x)\,dx = -\frac{1}{4}\text{arctanh}(\sin(x)) + \frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

```
[In] Integrate[Csc[4*x]*Sin[x],x]
[Out] -1/4*ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\ln(\sin(x)-1)}{8}-\frac{\ln(\sin(x)+1)}{8}+\frac{\text{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{4}$	28
risch	$\frac{\ln(e^{ix}-i)}{4}-\frac{\ln(i+e^{ix})}{4}+\frac{\sqrt{2}\,\ln\left(e^{2ix}+i\sqrt{2}\,e^{ix}-1\right)}{8}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-i\sqrt{2}\,e^{ix}-1\right)}{8}$	72

```
[In] int(sin(x)/sin(4*x),x,method=_RETURNVERBOSE)
[Out] 1/8*ln(sin(x)-1)-1/8*ln(sin(x)+1)+1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \csc(4x)\sin(x)\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$$

```
[In] integrate(sin(x)/sin(4*x),x, algorithm="fricas")
[Out] 1/8*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(22) = 44$.

Time = 3.24 (sec) , antiderivative size = 294, normalized size of antiderivative = 11.31

$$\begin{aligned}\int \csc(4x)\sin(x)\,dx = {} & \frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right) - 1\right)}{110880\sqrt{2} + 156808} + \frac{39202\log\left(\tan\left(\frac{x}{2}\right) - 1\right)}{110880\sqrt{2} + 156808} \\ & - \frac{39202\log\left(\tan\left(\frac{x}{2}\right) + 1\right)}{110880\sqrt{2} + 156808} - \frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right) + 1\right)}{110880\sqrt{2} + 156808} \\ & + \frac{27720\log\left(\tan\left(\frac{x}{2}\right) - 1 + \sqrt{2}\right)}{110880\sqrt{2} + 156808} + \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right) - 1 + \sqrt{2}\right)}{110880\sqrt{2} + 156808} \\ & + \frac{27720\log\left(\tan\left(\frac{x}{2}\right) + 1 + \sqrt{2}\right)}{110880\sqrt{2} + 156808} + \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right) + 1 + \sqrt{2}\right)}{110880\sqrt{2} + 156808} \\ & - \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right) - \sqrt{2} - 1\right)}{110880\sqrt{2} + 156808} - \frac{27720\log\left(\tan\left(\frac{x}{2}\right) - \sqrt{2} - 1\right)}{110880\sqrt{2} + 156808} \\ & - \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right) - \sqrt{2} + 1\right)}{110880\sqrt{2} + 156808} - \frac{27720\log\left(\tan\left(\frac{x}{2}\right) - \sqrt{2} + 1\right)}{110880\sqrt{2} + 156808}\end{aligned}$$

```
[In] integrate(sin(x)/sin(4*x),x)
[Out] 27720*sqrt(2)*log(tan(x/2) - 1)/(110880*sqrt(2) + 156808) + 39202*log(tan(x
/2) - 1)/(110880*sqrt(2) + 156808) - 39202*log(tan(x/2) + 1)/(110880*sqrt(2
) + 156808) - 27720*sqrt(2)*log(tan(x/2) + 1)/(110880*sqrt(2) + 156808) + 2
7720*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*
log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(x/2)
+ 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2) + 1 +
 sqrt(2))/(110880*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2)
- 1)/(110880*sqrt(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) - 1)/(110880*
sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(
2) + 156808) - 27720*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(2) + 156808)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.58

$$\begin{aligned}\int \csc(4x)\sin(x)\,dx &= \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)\\&-\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)\\&+\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)\\&-\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)\\&-\frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)\\&+\frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)\end{aligned}$$

```
[In] integrate(sin(x)/sin(4*x),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^2 + 2
*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\begin{aligned}\int \csc(4x)\sin(x)\,dx &= -\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(x)\right|}{\left|2\sqrt{2}+4\sin(x)\right|}\right)\\&-\frac{1}{8}\log\left(\sin(x)+1\right)+\frac{1}{8}\log\left(-\sin(x)+1\right)\end{aligned}$$

```
[In] integrate(sin(x)/sin(4*x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \csc(4x)\sin(x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin\left(x\right)\right)}{4} - \frac{\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}$$

```
[In] int(sin(x)/sin(4*x),x)
[Out] (2^(1/2)*atanh(2^(1/2)*sin(x)))/4 - atanh(sin(x/2)/cos(x/2))/2
```

3.389 $\int \csc(4x)\sin^3(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 26

$$\int \csc(4x)\sin^3(x)\,dx = -\frac{1}{4}\text{arctanh}(\sin(x)) + \frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{4\sqrt{2}}$$

```
[Out] -1/4*arctanh(sin(x))+1/8*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1144, 213}

$$\int \csc(4x)\sin^3(x)\,dx = \frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{4\sqrt{2}} - \frac{1}{4}\text{arctanh}(\sin(x))$$

```
[In] Int[Csc[4*x]*Sin[x]^3,x]
```

```
[Out] -1/4*ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/(4*Sqrt[2])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
```

```
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{x^2}{4-12x^2+8x^4}\,dx, x, \sin(x)\right)\\ &= 2\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx, x, \sin(x)\right) - \text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx, x, \sin(x)\right)\\ &= -\frac{1}{4}\text{arctanh}(\sin(x)) + \frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 70 vs. $2(26) = 52$.

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.69

$$\int \csc(4x)\sin^3(x)\,dx = \frac{1}{16}\left(4\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) - 4\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right) + \sqrt{2}\left(-\log\left(\sqrt{2}-2\sin(x)\right)+\log\left(\sqrt{2}+2\sin(x)\right)\right)\right)$$

```
[In] Integrate[Csc[4*x]*Sin[x]^3,x]
[Out] (4*Log[Cos[x/2] - Sin[x/2]] - 4*Log[Cos[x/2] + Sin[x/2]] + Sqrt[2]*(-Log[Sq
rt[2] - 2*Sin[x]] + Log[Sqrt[2] + 2*Sin[x]]))/16
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\text{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{8}+\frac{\ln(\sin(x)-1)}{8}-\frac{\ln(\sin(x)+1)}{8}$	28
risch	$-\frac{\ln(i+e^{ix})}{4}+\frac{\ln(e^{ix}-i)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-i\sqrt{2}\,e^{ix}-1\right)}{16}+\frac{\sqrt{2}\,\ln\left(e^{2ix}+i\sqrt{2}\,e^{ix}-1\right)}{16}$	72

```
[In] int(sin(x)^3/sin(4*x),x,method=_RETURNVERBOSE)
[Out] 1/8*arctanh(sin(x)*2^(1/2))*2^(1/2)+1/8*ln(sin(x)-1)-1/8*ln(sin(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(18) = 36.

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \csc(4x)\sin^3(x)\,dx = \frac{1}{16}\sqrt{2}\log\left(-\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$$

```
[In] integrate(sin(x)^3/sin(4*x),x, algorithm="fricas")
[Out] 1/16*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1
/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(22) = 44.

Time = 7.26 (sec) , antiderivative size = 294, normalized size of antiderivative = 11.31

$$\int \csc(4x)\sin^3(x)\,dx = \frac{4093147632754948\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{16372590531019792+11577169790074752\sqrt{2}} + \frac{2894292447518688\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{4093147632754948\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{2894292447518688\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{16372590531019792+11577169790074752\sqrt{2}} + \frac{1447146223759344\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{16372590531019792+11577169790074752\sqrt{2}} + \frac{1023286908188737\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{16372590531019792+11577169790074752\sqrt{2}} + \frac{1447146223759344\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{16372590531019792+11577169790074752\sqrt{2}} + \frac{1023286908188737\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{1447146223759344\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{1023286908188737\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{1447146223759344\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{1023286908188737\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{16372590531019792+11577169790074752\sqrt{2}}$$

```
[In] integrate(sin(x)**3/sin(4*x),x)
```

```
[Out] 4093147632754948*log(tan(x/2) - 1)/(16372590531019792 + 11577169790074752*s
qrt(2)) + 2894292447518688*sqrt(2)*log(tan(x/2) - 1)/(16372590531019792 + 1
1577169790074752*sqrt(2)) - 4093147632754948*log(tan(x/2) + 1)/(16372590531
019792 + 11577169790074752*sqrt(2)) - 2894292447518688*sqrt(2)*log(tan(x/2)
 + 1)/(16372590531019792 + 11577169790074752*sqrt(2)) + 1447146223759344*lo
g(tan(x/2) - 1 + sqrt(2))/(16372590531019792 + 11577169790074752*sqrt(2)) +
 1023286908188737*sqrt(2)*log(tan(x/2) - 1 + sqrt(2))/(16372590531019792 +
11577169790074752*sqrt(2)) + 1447146223759344*log(tan(x/2) + 1 + sqrt(2))/(
16372590531019792 + 11577169790074752*sqrt(2)) + 1023286908188737*sqrt(2)*l
og(tan(x/2) + 1 + sqrt(2))/(16372590531019792 + 11577169790074752*sqrt(2))
- 1447146223759344*log(tan(x/2) - sqrt(2) - 1)/(16372590531019792 + 1157716
9790074752*sqrt(2)) - 1023286908188737*sqrt(2)*log(tan(x/2) - sqrt(2) - 1)/
```

```
(16372590531019792 + 11577169790074752*sqrt(2)) - 1447146223759344*log(tan(
x/2) - sqrt(2) + 1)/(16372590531019792 + 11577169790074752*sqrt(2)) - 10232
86908188737*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(16372590531019792 + 115771
69790074752*sqrt(2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.58

$$\int \csc(4x)\sin^3(x)\,dx = \frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) + \frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1\right) + \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1\right)$$

```
[In] integrate(sin(x)^3/sin(4*x),x, algorithm="maxima")
```

```
[Out] 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^2 + 2
*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \csc(4x)\sin^3(x)\,dx = -\frac{1}{16}\sqrt{2}\log\left(\frac{|-2\sqrt{2} + 4\sin(x)|}{|2\sqrt{2} + 4\sin(x)|}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$$

```
[In] integrate(sin(x)^3/sin(4*x),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1
/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \csc(4x)\sin^3(x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin\left(x\right)\right)}{8} - \frac{\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}$$

```
[In] int(sin(x)^3/sin(4*x),x)
[Out] (2^(1/2)*atanh(2^(1/2)*sin(x)))/8 - atanh(sin(x/2)/cos(x/2))/2
```

3.390 $\int \sqrt{1+\sin(2x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \sqrt{1+\sin(2x)}\,dx = -\frac{\cos(2x)}{\sqrt{1+\sin(2x)}}$$

```
[Out] -cos(2*x)/(1+sin(2*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2725}

$$\int \sqrt{1+\sin(2x)}\,dx = -\frac{\cos(2x)}{\sqrt{\sin(2x)+1}}$$

```
[In] Int[Sqrt[1 + Sin[2*x]],x]
[Out] -(Cos[2*x]/Sqrt[1 + Sin[2*x]])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\cos(2x)}{\sqrt{1+\sin(2x)}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \sqrt{1+\sin(2x)}\,dx = \frac{(-\cos(x)+\sin(x))\sqrt{1+\sin(2x)}}{\cos(x)+\sin(x)}$$

```
[In] Integrate[Sqrt[1 + Sin[2*x]],x]
[Out] ((-Cos[x] + Sin[x])*Sqrt[1 + Sin[2*x]])/(Cos[x] + Sin[x])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{(-1+\sin(2x))\sqrt{1+\sin(2x)}}{\cos(2x)}$	22

```
[In] int((1+sin(2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] (-1+sin(2*x))*(1+sin(2*x))^(1/2)/cos(2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \sqrt{1+\sin(2x)}\,dx = -\frac{(\cos(2x)-\sin(2x)+1)\sqrt{\sin(2x)+1}}{\cos(2x)+\sin(2x)+1}$$

```
[In] integrate((1+sin(2*x))^(1/2),x, algorithm="fricas")
[Out] -(cos(2*x) - sin(2*x) + 1)*sqrt(sin(2*x) + 1)/(cos(2*x) + sin(2*x) + 1)
```

Sympy [F]

$$\int \sqrt{1+\sin(2x)}\,dx = \int \sqrt{\sin{(2x)}+1}\,dx$$

```
[In] integrate((1+sin(2*x))**(1/2),x)
[Out] Integral(sqrt(sin(2*x) + 1), x)
```

Maxima [F]

$$\int \sqrt{1+\sin(2x)}\,dx = \int \sqrt{\sin{(2\,x)}+1}\,dx$$

```
[In] integrate((1+sin(2*x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sin(2*x) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \sqrt{1+\sin(2x)}\,dx = \sqrt{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+x\right)\right)\sin\left(-\frac{1}{4}\,\pi+x\right)$$

```
[In] integrate((1+sin(2*x))^(1/2),x, algorithm="giac")
[Out] sqrt(2)*sgn(cos(-1/4*pi + x))*sin(-1/4*pi + x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \sqrt{1+\sin(2x)}\,dx = \frac{\left(\sin{(2\,x)}-1\right)\,\sqrt{\sin{(2\,x)}+1}}{\cos{(2\,x)}}$$

```
[In] int((sin(2*x) + 1)^(1/2),x)
[Out] ((sin(2*x) - 1)*(sin(2*x) + 1)^(1/2))/cos(2*x)
```

3.391 $\int \sqrt{1-\sin(2x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 17

$$\int \sqrt{1-\sin(2x)}\,dx = \frac{\cos(2x)}{\sqrt{1-\sin(2x)}}$$

```
[Out] cos(2*x)/(1-sin(2*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2725}

$$\int \sqrt{1-\sin(2x)}\,dx = \frac{\cos(2x)}{\sqrt{1-\sin(2x)}}$$

```
[In] Int[Sqrt[1 - Sin[2*x]],x]
```

```
[Out] Cos[2*x]/Sqrt[1 - Sin[2*x]]
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\cos(2x)}{\sqrt{1-\sin(2x)}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \sqrt{1-\sin(2x)}\,dx=\frac{(\cos(x)+\sin(x))\sqrt{1-\sin(2x)}}{\cos(x)-\sin(x)}$$

```
[In] Integrate[Sqrt[1 - Sin[2*x]],x]
[Out] ((Cos[x] + Sin[x])*Sqrt[1 - Sin[2*x]])/(Cos[x] - Sin[x])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

method	result	size
default	$-\frac{(-1+\sin(2x))(1+\sin(2x))}{\cos(2x)\sqrt{1-\sin(2x)}}$	31

```
[In] int((1-sin(2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-1+sin(2*x))*(1+sin(2*x))/cos(2*x)/(1-sin(2*x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \sqrt{1-\sin(2x)}\,dx=\frac{(\cos(2x)+\sin(2x)+1)\sqrt{-\sin(2x)+1}}{\cos(2x)-\sin(2x)+1}$$

```
[In] integrate((1-sin(2*x))^(1/2),x, algorithm="fricas")
[Out] (cos(2*x) + sin(2*x) + 1)*sqrt(-sin(2*x) + 1)/(cos(2*x) - sin(2*x) + 1)
```

Sympy [F]

$$\int \sqrt{1-\sin(2x)}\,dx = \int \sqrt{1-\sin{(2x)}}\,dx$$

```
[In] integrate((1-sin(2*x))**(1/2),x)
[Out] Integral(sqrt(1 - sin(2*x)), x)
```

Maxima [F]

$$\int \sqrt{1-\sin(2x)}\,dx = \int \sqrt{-\sin{(2x)}+1}\,dx$$

```
[In] integrate((1-sin(2*x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-sin(2*x) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\begin{aligned}&\int \sqrt{1-\sin(2x)}\,dx\\&= -\sqrt{2}\left(\cos\left(-\frac{1}{4}\pi+x\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+x\right)\right)-\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+x\right)\right)\right)\end{aligned}$$

```
[In] integrate((1-sin(2*x))^(1/2),x, algorithm="giac")
[Out] -sqrt(2)*(cos(-1/4*pi + x)*sgn(sin(-1/4*pi + x)) - sgn(sin(-1/4*pi + x)))
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \sqrt{1-\sin(2x)}\,dx = \frac{\sqrt{1-\sin{(2x)}}\,(\sin{(2x)}+1)}{\cos{(2x)}}$$

```
[In] int((1 - sin(2*x))^(1/2),x)
[Out] ((1 - sin(2*x))^(1/2)*(sin(2*x) + 1))/cos(2*x)
```

3.392 $\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1+\cos(2x)}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(1/2*sin(2*x)*2^(1/2)/(1+cos(2*x))^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2728, 212}

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{\cos(2x)+1}}\right)}{\sqrt{2}}$$

```
[In] Int[1/Sqrt[1 + Cos[2*x]],x]
```

```
[Out] ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 + Cos[2*x]])]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, -\frac{\sin(2x)}{\sqrt{1+\cos(2x)}}\right)$$
$$= \frac{\operatorname{arctanh}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1+\cos(2x)}}\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{\operatorname{arctanh}(\sin(x))\cos(x)}{\sqrt{1+\cos(2x)}}$$

```
[In] Integrate[1/Sqrt[1 + Cos[2*x]],x]
[Out] (ArcTanh[Sin[x]]*Cos[x])/Sqrt[1 + Cos[2*x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.33

method	result	size
default	$\frac{\sqrt{2}\,\operatorname{am}^{-1}(x\mid 1)}{2}$	9
risch	$-\frac{\sqrt{2}\,\ln(e^{ix}-i)\cos(x)}{\sqrt{(e^{2ix}+1)^2 e^{-2ix}}}+\frac{\sqrt{2}\,\ln(i+e^{ix})\cos(x)}{\sqrt{(e^{2ix}+1)^2 e^{-2ix}}}$	67

```
[In] int(1/(1+cos(2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*2^(1/2)*InverseJacobiAM(x,1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{\cos(2x)^2 - 2\sqrt{2}\sqrt{\cos(2x)+1}\sin(2x) - 2\cos(2x) - 3}{\cos(2x)^2 + 2\cos(2x) + 1}\right)$$

```
[In] integrate(1/(1+cos(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(cos(2*x)^2 - 2*sqrt(2)*sqrt(cos(2*x) + 1)*sin(2*x) - 2*co
s(2*x) - 3)/(cos(2*x)^2 + 2*cos(2*x) + 1))
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \int \frac{1}{\sqrt{\cos(2x)+1}}\,dx$$

```
[In] integrate(1/(1+cos(2*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(cos(2*x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{1}{4}\sqrt{2}\log\left(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1\right) - \frac{1}{4}\sqrt{2}\log\left(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1\right)$$

```
[In] integrate(1/(1+cos(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*sqrt(2)*log(cos(x
)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{\sqrt{2}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)+2\right|\right)}{8\,\mathrm{sgn}(\cos(x))} - \frac{\sqrt{2}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)-2\right|\right)}{8\,\mathrm{sgn}(\cos(x))}$$

```
[In] integrate(1/(1+cos(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(1/sin(x) + sin(x) + 2))/sgn(cos(x)) - 1/8*sqrt(2)*log(a
bs(1/sin(x) + sin(x) - 2))/sgn(cos(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{\sqrt{2}\,\mathrm{asinh}\left(\frac{\sin(x)}{\cos(x)}\right)}{2}$$

```
[In] int(1/(cos(2*x) + 1)^(1/2),x)
```

```
[Out] (2^(1/2)*asinh(sin(x)/cos(x)))/2
```

3.393 $\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(1/2*sin(2*x)*2^(1/2)/(1-cos(2*x))^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2728, 212}

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}$$

```
[In] Int[1/Sqrt[1 - Cos[2*x]],x]
[Out] -(ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 - Cos[2*x]])]/Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \frac{\sin(2x)}{\sqrt{1-\cos(2x)}}\right)$$
$$= -\frac{\operatorname{arctanh}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = -\frac{\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sin(x)}{\sqrt{1-\cos(2x)}}$$

```
[In] Integrate[1/Sqrt[1 - Cos[2*x]],x]
[Out] -(((Log[Cos[x/2]] - Log[Sin[x/2]])*Sin[x])/Sqrt[1 - Cos[2*x]])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

method	result	size
default	$-\frac{\sin(x)\operatorname{arctanh}(\cos(x))\sqrt{2}}{\sqrt{2-2\cos(2x)}}$	17
risch	$-\frac{\sqrt{2}\,\ln(e^{ix}+1)\sin(x)}{\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}}+\frac{\sqrt{2}\,\ln(e^{ix}-1)\sin(x)}{\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}}$	67

```
[In] int(1/(1-cos(2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*sin(x)*arctanh(cos(x))*2^(1/2)/(sin(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.93

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{(\cos(2x)+3)\sin(2x)-2\left(\sqrt{2}\cos(2x)+\sqrt{2}\right)\sqrt{-\cos(2x)+1}}{(\cos(2x)-1)\sin(2x)}\right)$$

```
[In] integrate(1/(1-cos(2*x))^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log(-((cos(2*x) + 3)*sin(2*x) - 2*(sqrt(2)*cos(2*x) + sqrt(2))*
sqrt(-cos(2*x) + 1))/((cos(2*x) - 1)*sin(2*x)))
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = \int \frac{1}{\sqrt{1-\cos(2x)}}\,dx$$

```
[In] integrate(1/(1-cos(2*x))**(1/2),x)
[Out] Integral(1/sqrt(1 - cos(2*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(25) = 50$.

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.37

$$\begin{aligned}\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = & -\frac{1}{4}\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan(\sin(2x),\cos(2x))\right)^2 + \sin\left(\frac{1}{2}\arctan(\sin(2x),\cos(2x))\right)^2 + 2\cos\left(\frac{1}{2}\arctan(\sin(2x),\cos(2x))\right) + 1\right) \\ & + \frac{1}{4}\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan(\sin(2x),\cos(2x))\right)^2 + \sin\left(\frac{1}{2}\arctan(\sin(2x),\cos(2x))\right)^2 - 2\cos\left(\frac{1}{2}\arctan(\sin(2x),\cos(2x))\right) + 1\right)\end{aligned}$$

```
[In] integrate(1/(1-cos(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*log(cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(1/2*arctan2(s
in(2*x), cos(2*x)))^2 + 2*cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1) + 1/4*s
qrt(2)*log(cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(1/2*arctan2(sin(2*x
), cos(2*x)))^2 - 2*cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = \frac{\sqrt{2}\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{2\,\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

```
[In] integrate(1/(1-cos(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(abs(tan(1/2*x)))/sgn(sin(x))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = -\frac{\sqrt{2}\,\sin\left(2\,x\right)\,\mathrm{atanh}\left(\sqrt{\cos\left(x\right)^2}\right)}{2\,\sqrt{1-\cos\left(2\,x\right)^2}}$$

```
[In] int(1/(1 - cos(2*x))^(1/2),x)
```

```
[Out] -(2^(1/2)*sin(2*x)*atanh((cos(x)^2)^(1/2)))/(2*(1 - cos(2*x)^2)^(1/2))
```

3.394 $\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 53

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}}-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}$$

```
[Out] -1/6*sin(3*x)/(1-cos(3*x))^(3/2)-1/12*arctanh(1/2*sin(3*x)*2^(1/2)/(1-cos(3
*x))^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2729, 2728, 212}

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}}-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}$$

```
[In] Int[(1 - Cos[3*x])^(-3/2),x]
```

```
[Out] -1/6*ArcTanh[Sin[3*x]/(Sqrt[2]*Sqrt[1 - Cos[3*x]])]/Sqrt[2] - Sin[3*x]/(6*(
1 - Cos[3*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}+\frac{1}{4}\int\frac{1}{\sqrt{1-\cos(3x)}}\,dx\\ &= -\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}-\frac{1}{6}\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\frac{\sin(3x)}{\sqrt{1-\cos(3x)}}\right)\\ &= -\frac{\operatorname{arctanh}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}}-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int\frac{1}{(1-\cos(3x))^{3/2}}\,dx=-\frac{\left(\csc^2\left(\frac{3x}{4}\right)+4\log\left(\cos\left(\frac{3x}{4}\right)\right)-4\log\left(\sin\left(\frac{3x}{4}\right)\right)-\sec^2\left(\frac{3x}{4}\right)\right)\sin^3\left(\frac{3x}{2}\right)}{12(1-\cos(3x))^{3/2}}$$

```
[In] Integrate[(1 - Cos[3*x])^(-3/2),x]
[Out] -1/12*((Csc[(3*x)/4]^2 + 4*Log[Cos[(3*x)/4]] - 4*Log[Sin[(3*x)/4]] - Sec[(3
*x)/4]^2)*Sin[(3*x)/2]^3)/(1 - Cos[3*x])^(3/2)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{\left(\frac{\cos\left(\frac{3x}{2}\right)}{2}+\frac{\left(\ln\left(\cos\left(\frac{3x}{2}\right)+1\right)-\ln\left(\cos\left(\frac{3x}{2}\right)-1\right)\right)\left(\sin^2\left(\frac{3x}{2}\right)\right)}{4}\right)\sqrt{2}}{3\sin\left(\frac{3x}{2}\right)\sqrt{2-2\cos(3x)}}$	52

[In] int(1/(1-cos(3*x))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/6*(1/2*cos(3/2*x)+1/4*(ln(cos(3/2*x)+1)-ln(cos(3/2*x)-1))*sin(3/2*x)^2)/sin(3/2*x)*2^(1/2)/(sin(3/2*x)^2)^(1/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(42) = 84.

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = \frac{\left(\sqrt{2}\cos\left(3\,x\right)-\sqrt{2}\right)\log\left(-\frac{(\cos(3\,x)+3)\sin(3\,x)-2\left(\sqrt{2}\cos(3\,x)+\sqrt{2}\right)\sqrt{-\cos(3\,x)+1}}{(\cos(3\,x)-1)\sin(3\,x)}\right)\sin\left(3\,x\right)+}{24\left(\cos\left(3\,x\right)-1\right)\sin\left(3\,x\right)}$$

[In] integrate(1/(1-cos(3*x))^(3/2),x, algorithm="fricas")

[Out] 1/24*((sqrt(2)*cos(3*x) - sqrt(2))*log(-((cos(3*x) + 3)*sin(3*x) - 2*(sqrt(2)*cos(3*x) + sqrt(2))*sqrt(-cos(3*x) + 1))/((cos(3*x) - 1)*sin(3*x)))*sin(3*x) + 4*(cos(3*x) + 1)*sqrt(-cos(3*x) + 1))/((cos(3*x) - 1)*sin(3*x))

Sympy [F]

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = \int \frac{1}{\left(1-\cos\left(3x\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(1-cos(3*x))**(3/2),x)

[Out] Integral((1 - cos(3*x))**(-3/2), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(42) = 84.

Time = 0.33 (sec) , antiderivative size = 433, normalized size of antiderivative = 8.17

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = \frac{4\left(\sin\left(6\,x\right)-2\,\sin\left(3\,x\right)\right)\cos\left(\frac{3}{2}\,\pi+\frac{3}{2}\,\arctan\left(\sin\left(3\,x\right),\cos\left(3\,x\right)\right)\right)-4\left(\sin\left(6\,x\right)-2\right.}{}$$

[In] integrate(1/(1-cos(3*x))^(3/2),x, algorithm="maxima")

[Out] 1/12*(4*(sin(6*x) - 2*sin(3*x))*cos(3/2*pi + 3/2*arctan2(sin(3*x), cos(3*x))) - 4*(sin(6*x) - 2*sin(3*x))*cos(1/2*pi + 1/2*arctan2(sin(3*x), cos(3*x))) + (2*(2*cos(3*x) - 1)*cos(6*x) - cos(6*x)^2 - 4*cos(3*x)^2 - sin(6*x)^2 +

```
4*sin(6*x)*sin(3*x) - 4*sin(3*x)^2 + 4*cos(3*x) - 1)*log(cos(1/2*arctan2(s
in(3*x), cos(3*x)))^2 + sin(1/2*arctan2(sin(3*x), cos(3*x)))^2 + 2*cos(1/2*
arctan2(sin(3*x), cos(3*x))) + 1) - (2*(2*cos(3*x) - 1)*cos(6*x) - cos(6*x)
^2 - 4*cos(3*x)^2 - sin(6*x)^2 + 4*sin(6*x)*sin(3*x) - 4*sin(3*x)^2 + 4*cos
(3*x) - 1)*log(cos(1/2*arctan2(sin(3*x), cos(3*x)))^2 + sin(1/2*arctan2(sin
(3*x), cos(3*x)))^2 - 2*cos(1/2*arctan2(sin(3*x), cos(3*x))) + 1) - 4*(cos(
6*x) - 2*cos(3*x) + 1)*sin(3/2*pi + 3/2*arctan2(sin(3*x), cos(3*x))) + 4*(c
os(6*x) - 2*cos(3*x) + 1)*sin(1/2*pi + 1/2*arctan2(sin(3*x), cos(3*x))))/(s
qrt(2)*cos(6*x)^2 + 4*sqrt(2)*cos(3*x)^2 + sqrt(2)*sin(6*x)^2 - 4*sqrt(2)*s
in(6*x)*sin(3*x) + 4*sqrt(2)*sin(3*x)^2 - 2*(2*sqrt(2)*cos(3*x) - sqrt(2))*
cos(6*x) - 4*sqrt(2)*cos(3*x) + sqrt(2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(42) = 84$.

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.89

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = -\frac{\sqrt{2}\left(\frac{2\left(\cos\left(\frac{3}{2}x\right)-1\right)}{\cos\left(\frac{3}{2}x\right)+1}-1\right)\left(\cos\left(\frac{3}{2}x\right)+1\right)}{48\left(\cos\left(\frac{3}{2}x\right)-1\right)\operatorname{sgn}\left(\sin\left(\frac{3}{2}x\right)\right)} + \frac{\sqrt{2}\log\left(-\frac{\cos\left(\frac{3}{2}x\right)-1}{\cos\left(\frac{3}{2}x\right)+1}\right)}{24\operatorname{sgn}\left(\sin\left(\frac{3}{2}x\right)\right)} - \frac{\sqrt{2}\left(\cos\left(\frac{3}{2}x\right)-1\right)}{48\left(\cos\left(\frac{3}{2}x\right)+1\right)\operatorname{sgn}\left(\sin\left(\frac{3}{2}x\right)\right)}$$

```
[In] integrate(1/(1-cos(3*x))^(3/2),x, algorithm="giac")
[Out] -1/48*sqrt(2)*(2*(cos(3/2*x) - 1)/(cos(3/2*x) + 1) - 1)*(cos(3/2*x) + 1)/((
cos(3/2*x) - 1)*sgn(sin(3/2*x))) + 1/24*sqrt(2)*log(-(cos(3/2*x) - 1)/(cos(
3/2*x) + 1))/sgn(sin(3/2*x)) - 1/48*sqrt(2)*(cos(3/2*x) - 1)/((cos(3/2*x) +
 1)*sgn(sin(3/2*x)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = \int \frac{1}{{\left(1-\cos\left(3\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int(1/(1 - cos(3*x))^(3/2),x)
[Out] int(1/(1 - cos(3*x))^(3/2), x)
```

3.395 $\int \left(1-\sin \left(\frac{2x}{3}\right)\right)^{5/2} dx$

Optimal result

Integrand size = 14, antiderivative size = 73

$$\int \left(1-\sin \left(\frac{2x}{3}\right)\right)^{5/2} dx = \frac{32\cos \left(\frac{2x}{3}\right)}{5\sqrt{1-\sin \left(\frac{2x}{3}\right)}} + \frac{8}{5}\cos \left(\frac{2x}{3}\right)\sqrt{1-\sin \left(\frac{2x}{3}\right)} + \frac{3}{5}\cos \left(\frac{2x}{3}\right)\left(1-\sin \left(\frac{2x}{3}\right)\right)^{3/2}$$

```
[Out] 3/5*cos(2/3*x)*(1-sin(2/3*x))^(3/2)+32/5*cos(2/3*x)/(1-sin(2/3*x))^(1/2)+8/
5*cos(2/3*x)*(1-sin(2/3*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2726, 2725}

$$\int \left(1-\sin \left(\frac{2x}{3}\right)\right)^{5/2} dx = \frac{3}{5}\left(1-\sin \left(\frac{2x}{3}\right)\right)^{3/2}\cos \left(\frac{2x}{3}\right) + \frac{8}{5}\sqrt{1-\sin \left(\frac{2x}{3}\right)}\cos \left(\frac{2x}{3}\right) + \frac{32\cos \left(\frac{2x}{3}\right)}{5\sqrt{1-\sin \left(\frac{2x}{3}\right)}}$$

```
[In] Int[(1 - Sin[(2*x)/3])^(5/2),x]
```

```
[Out] (32*Cos[(2*x)/3])/(5*Sqrt[1 - Sin[(2*x)/3]]) + (8*Cos[(2*x)/3]*Sqrt[1 - Sin
[(2*x)/3]])/5 + (3*Cos[(2*x)/3]*(1 - Sin[(2*x)/3])^(3/2))/5
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3}{5}\cos\left(\frac{2x}{3}\right)\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}+\frac{8}{5}\int\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}dx \\
&= \frac{8}{5}\cos\left(\frac{2x}{3}\right)\sqrt{1-\sin\left(\frac{2x}{3}\right)}+\frac{3}{5}\cos\left(\frac{2x}{3}\right)\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}+\frac{32}{15}\int\sqrt{1-\sin\left(\frac{2x}{3}\right)}\,dx \\
&= \frac{32\cos\left(\frac{2x}{3}\right)}{5\sqrt{1-\sin\left(\frac{2x}{3}\right)}}+\frac{8}{5}\cos\left(\frac{2x}{3}\right)\sqrt{1-\sin\left(\frac{2x}{3}\right)}+\frac{3}{5}\cos\left(\frac{2x}{3}\right)\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int\left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}dx=\frac{\left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\left(150\cos\left(\frac{x}{3}\right)+25\cos(x)-3\cos\left(\frac{5x}{3}\right)+150\sin\left(\frac{x}{3}\right)-25\sin(x)-3\,\mathrm{s}\right.}{20\left(\cos\left(\frac{x}{3}\right)-\sin\left(\frac{x}{3}\right)\right)^{5}}$$

```
[In] Integrate[(1 - Sin[(2*x)/3])^(5/2),x]
[Out] ((1 - Sin[(2*x)/3])^(5/2)*(150*Cos[x/3] + 25*Cos[x] - 3*Cos[(5*x)/3] + 150*
Sin[x/3] - 25*Sin[x] - 3*Sin[(5*x)/3]))/(20*(Cos[x/3] - Sin[x/3])^5)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result	size
default	$-\frac{\left(-1+\sin\left(\frac{2x}{3}\right)\right)\left(\sin\left(\frac{2x}{3}\right)+1\right)\left(3\left(\sin^{2}\left(\frac{2x}{3}\right)\right)-14\sin\left(\frac{2x}{3}\right)+43\right)}{5\cos\left(\frac{2x}{3}\right)\sqrt{1-\sin\left(\frac{2x}{3}\right)}}$	47

[In] int((1-sin(2/3*x))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/5*(-1+sin(2/3*x))*(sin(2/3*x)+1)*(3*sin(2/3*x)^2-14*sin(2/3*x)+43)/cos(2/3*x)/(1-sin(2/3*x))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\,dx =$$
$$-\frac{\left(3\,\cos\left(\frac{2}{3}\,x\right)^{3}-11\,\cos\left(\frac{2}{3}\,x\right)^{2}+\left(3\,\cos\left(\frac{2}{3}\,x\right)^{2}+14\,\cos\left(\frac{2}{3}\,x\right)-32\right)\sin\left(\frac{2}{3}\,x\right)-46\,\cos\left(\frac{2}{3}\,x\right)-32\right)\sqrt{-\sin(}}{5\left(\cos\left(\frac{2}{3}\,x\right)-\sin\left(\frac{2}{3}\,x\right)+1\right)}$$

[In] integrate((1-sin(2/3*x))^(5/2),x, algorithm="fricas")

[Out] -1/5*(3*cos(2/3*x)^3 - 11*cos(2/3*x)^2 + (3*cos(2/3*x)^2 + 14*cos(2/3*x) - 32)*sin(2/3*x) - 46*cos(2/3*x) - 32)*sqrt(-sin(2/3*x) + 1)/(cos(2/3*x) - sin(2/3*x) + 1)

Sympy [F]

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\,dx = \int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{\frac{5}{2}}\,dx$$

[In] integrate((1-sin(2/3*x))**(5/2),x)

[Out] Integral((1 - sin(2*x/3))**(5/2), x)

Maxima [F]

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\,dx=\int \left(-\sin\left(\frac{2}{3}\,x\right)+1\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((1-sin(2/3*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-sin(2/3*x) + 1)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\,dx=$$
$$-\frac{1}{20}\,\sqrt{2}\left(150\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{3}\,x\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{3}\,x\right)\right)-25\,\cos\left(-\frac{3}{4}\,\pi+x\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{3}\,x\right)\right)\right.$$

```
[In] integrate((1-sin(2/3*x))^(5/2),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(2)*(150*cos(-1/4*pi + 1/3*x)*sgn(sin(-1/4*pi + 1/3*x)) - 25*cos(
-3/4*pi + x)*sgn(sin(-1/4*pi + 1/3*x)) + 3*cos(-5/4*pi + 5/3*x)*sgn(sin(-1/
4*pi + 1/3*x)) - 128*sgn(sin(-1/4*pi + 1/3*x)))
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\,dx=\int {\left(1-\sin\left(\frac{2\,x}{3}\right)\right)}^{5/2}\,dx$$

```
[In] int((1 - sin((2*x)/3))^(5/2),x)
```

```
[Out] int((1 - sin((2*x)/3))^(5/2), x)
```

3.396 $\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 55

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = \frac{3}{4\sqrt{1+2\sin(x)}} - \frac{4}{\sqrt[4]{1+2\sin(x)}} - \frac{1}{2}\sqrt{1+2\sin(x)} + \frac{1}{12}(1+2\sin(x))^{3/2}$$

```
[Out] -4/(1+2*sin(x))^(1/4)+1/12*(1+2*sin(x))^(3/2)+3/4/(1+2*sin(x))^(1/2)-1/2*(1
+2*sin(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4441, 14}

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = \frac{1}{12}(2\sin(x)+1)^{3/2} - \frac{1}{2}\sqrt{2\sin(x)+1} - \frac{4}{\sqrt[4]{2\sin(x)+1}} + \frac{3}{4\sqrt{2\sin(x)+1}}$$

```
[In] Int[(Cos[x]*(-Cos[x]^2 + 2*(1 + 2*Sin[x])^(1/4)))/(1 + 2*Sin[x])^(3/2),x]
```

```
[Out] 3/(4*Sqrt[1 + 2*Sin[x]]) - 4/(1 + 2*Sin[x])^(1/4) - Sqrt[1 + 2*Sin[x]]/2 +
(1 + 2*Sin[x])^(3/2)/12
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{-1+x^2+2\sqrt[4]{1+2x}}{(1+2x)^{3/2}}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-3+8x-2x^4+x^8}{x^3}\,dx, x, \sqrt[4]{1+2\sin(x)}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{3}{x^3}+\frac{8}{x^2}-2x+x^5\right)dx, x, \sqrt[4]{1+2\sin(x)}\right)\\
&= \frac{3}{4\sqrt{1+2\sin(x)}}-\frac{4}{\sqrt[4]{1+2\sin(x)}}-\frac{1}{2}\sqrt{1+2\sin(x)}+\frac{1}{12}(1+2\sin(x))^{3/2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$
\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = -\frac{-3+\cos(2x)+4\sin(x)+24\sqrt[4]{1+2\sin(x)}}{6\sqrt{1+2\sin(x)}}
$$

```
[In] Integrate[(Cos[x]*(-Cos[x]^2 + 2*(1 + 2*Sin[x])^(1/4)))/(1 + 2*Sin[x])^(3/2
),x]

[Out] -1/6*(-3 + Cos[2*x] + 4*Sin[x] + 24*(1 + 2*Sin[x])^(1/4))/Sqrt[1 + 2*Sin[x]
]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$-\frac{4}{(1+2\sin(x))^{\frac{1}{4}}}+\frac{(1+2\sin(x))^{\frac{3}{2}}}{12}+\frac{3}{4\sqrt{1+2\sin(x)}}-\frac{\sqrt{1+2\sin(x)}}{2}$	42
default	$-\frac{4}{(1+2\sin(x))^{\frac{1}{4}}}+\frac{(1+2\sin(x))^{\frac{3}{2}}}{12}+\frac{3}{4\sqrt{1+2\sin(x)}}-\frac{\sqrt{1+2\sin(x)}}{2}$	42
parts	$-\frac{4}{(1+2\sin(x))^{\frac{1}{4}}}+\frac{(1+2\sin(x))^{\frac{3}{2}}}{12}+\frac{3}{4\sqrt{1+2\sin(x)}}-\frac{\sqrt{1+2\sin(x)}}{2}$	42

```
[In] int(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -4/(1+2*sin(x))^(1/4)+1/12*(1+2*sin(x))^(3/2)+3/4/(1+2*sin(x))^(1/2)-1/2*(1
+2*sin(x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = -\frac{\left(\cos\left(x\right)^{2}+2\,\sin\left(x\right)-2\right)\sqrt{2\,\sin\left(x\right)+1}+12\,\left(2\,\sin\left(x\right)+1\right)^{\frac{3}{4}}}{3\,\left(2\,\sin\left(x\right)+1\right)}$$

```
[In] integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/3*((cos(x)^2 + 2*sin(x) - 2)*sqrt(2*sin(x) + 1) + 12*(2*sin(x) + 1)^(3/4
))/(2*sin(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(48) = 96.

Time = 37.51 (sec) , antiderivative size = 230, normalized size of antiderivative = 4.18

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = \frac{4(2\sin(x)+1)^{\frac{3}{4}}\sin^2(x)}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}} - \frac{2(2\sin(x)+1)^{\frac{3}{4}}\sin(x)}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}} + \frac{3(2\sin(x)+1)^{\frac{3}{4}}\cos^2(x)}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}} - \frac{2(2\sin(x)+1)^{\frac{3}{4}}}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}} - \frac{24\sin(x)}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}} - \frac{12}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}}$$

```
[In] integrate(cos(x)*(-cos(x)**2+2*(1+2*sin(x))**(1/4))/(1+2*sin(x))**(3/2),x)
```

```
[Out] 4*(2*sin(x) + 1)**(3/4)*sin(x)**2/(6*(2*sin(x) + 1)**(1/4)*sin(x) + 3*(2*si
n(x) + 1)**(1/4)) - 2*(2*sin(x) + 1)**(3/4)*sin(x)/(6*(2*sin(x) + 1)**(1/4)
*sin(x) + 3*(2*sin(x) + 1)**(1/4)) + 3*(2*sin(x) + 1)**(3/4)*cos(x)**2/(6*(
2*sin(x) + 1)**(1/4)*sin(x) + 3*(2*sin(x) + 1)**(1/4)) - 2*(2*sin(x) + 1)**
(3/4)/(6*(2*sin(x) + 1)**(1/4)*sin(x) + 3*(2*sin(x) + 1)**(1/4)) - 24*sin(x
)/(6*(2*sin(x) + 1)**(1/4)*sin(x) + 3*(2*sin(x) + 1)**(1/4)) - 12/(6*(2*sin
(x) + 1)**(1/4)*sin(x) + 3*(2*sin(x) + 1)**(1/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = \frac{1}{12}\,(2\,\sin(x)+1)^{\frac{3}{2}} - \frac{16\,(2\,\sin(x)+1)^{\frac{1}{4}}-3}{4\,\sqrt{2\,\sin(x)+1}} - \frac{1}{2}\,\sqrt{2\,\sin(x)+1}$$

```
[In] integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, alg
orithm="maxima")
```

```
[Out] 1/12*(2*sin(x) + 1)^(3/2) - 1/4*(16*(2*sin(x) + 1)^(1/4) - 3)/sqrt(2*sin(x)
 + 1) - 1/2*sqrt(2*sin(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = \frac{1}{12}\left(2\,\sin\left(x\right)+1\right)^{\frac{3}{2}} - \frac{16\left(2\,\sin\left(x\right)+1\right)^{\frac{1}{4}}-3}{4\,\sqrt{2\,\sin\left(x\right)+1}} - \frac{1}{2}\,\sqrt{2\,\sin\left(x\right)+1}$$

```
[In] integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, alg
orithm="giac")
```

```
[Out] 1/12*(2*sin(x) + 1)^(3/2) - 1/4*(16*(2*sin(x) + 1)^(1/4) - 3)/sqrt(2*sin(x)
 + 1) - 1/2*sqrt(2*sin(x) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = -\int -\frac{\cos\left(x\right)\,\left(2\,{\left(2\,\sin\left(x\right)+1\right)}^{1/4}-{\cos\left(x\right)}^2\right)}{{\left(2\,\sin\left(x\right)+1\right)}^{3/2}}\,dx$$

```
[In] int((cos(x)*(2*(2*sin(x) + 1)^(1/4) - cos(x)^2))/(2*sin(x) + 1)^(3/2),x)
```

```
[Out] -int(-(cos(x)*(2*(2*sin(x) + 1)^(1/4) - cos(x)^2))/(2*sin(x) + 1)^(3/2), x)
```

3.397 $\int \sqrt{\tan(x)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 98

$$\int \sqrt{\tan(x)}\,dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}} +\frac{\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}}$$

```
[Out] 1/2*arctan(-1+2^(1/2)*tan(x)^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*tan(x)^(1/
2))*2^(1/2)+1/4*ln(1-2^(1/2)*tan(x)^(1/2)+tan(x))*2^(1/2)-1/4*ln(1+2^(1/2)*
tan(x)^(1/2)+tan(x))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \sqrt{\tan(x)}\,dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}+\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}} +\frac{\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}$$

```
[In] Int[Sqrt[Tan[x]],x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[x
]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 +
Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= 2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)\\
&= -\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right) + \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(x)}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(x)}\right)\\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(x)}\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(x)}\right)}{2\sqrt{2}}\\
&= \frac{\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}\\
&= -\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}\\
&\quad + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.41

$$\int \sqrt{\tan(x)}\,dx = \frac{\left(\arctan\left(\sqrt[4]{-\tan^2(x)}\right) - \operatorname{arctanh}\left(\sqrt[4]{-\tan^2(x)}\right)\right)\sqrt[4]{-\tan(x)}}{\sqrt[4]{\tan(x)}}$$

```
[In] Integrate[Sqrt[Tan[x]],x]
```

```
[Out] ((ArcTan[(-Tan[x]^2)^(1/4)] - ArcTanh[(-Tan[x]^2)^(1/4)])*(-Tan[x])^(1/4))/
Tan[x]^(1/4)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.50

method	result	size
lookup	$\frac{\left(\sqrt{\tan}(x)\right)\cos(x)\sqrt{2}\arccos(\cos(x)-\sin(x))}{2\sqrt{\cos(x)\sin(x)}}-\frac{\sqrt{2}\ln\left(\cos(x)+\sqrt{2}\left(\sqrt{\tan}(x)\right)\cos(x)+\sin(x)\right)}{2}$	49
default	$\frac{\left(\sqrt{\tan}(x)\right)\cos(x)\sqrt{2}\arccos(\cos(x)-\sin(x))}{2\sqrt{\cos(x)\sin(x)}}-\frac{\sqrt{2}\ln\left(\cos(x)+\sqrt{2}\left(\sqrt{\tan}(x)\right)\cos(x)+\sin(x)\right)}{2}$	49
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\left(\sqrt{\tan}(x)\right)+\tan(x)}{1+\sqrt{2}\left(\sqrt{\tan}(x)\right)+\tan(x)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan}(x)\right)\right)+2\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan}(x)\right)\right)\right)}{4}$	62

```
[In] int(tan(x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*tan(x)^(1/2)/(cos(x)*sin(x))^(1/2)*cos(x)*2^(1/2)*arccos(cos(x)-sin(x))
-1/2*2^(1/2)*ln(cos(x)+2^(1/2)*tan(x)^(1/2)*cos(x)+sin(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\begin{aligned}\int \sqrt{\tan(x)}\,dx = &\left(\frac{1}{4}i - \frac{1}{4}\right)\sqrt{2}\log\left((i+1)\sqrt{2} + 2\sqrt{\tan(x)}\right) \\ &- \left(\frac{1}{4}i + \frac{1}{4}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2} + 2\sqrt{\tan(x)}\right) \\ &+ \left(\frac{1}{4}i + \frac{1}{4}\right)\sqrt{2}\log\left((i-1)\sqrt{2} + 2\sqrt{\tan(x)}\right) \\ &- \left(\frac{1}{4}i - \frac{1}{4}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2} + 2\sqrt{\tan(x)}\right)\end{aligned}$$

```
[In] integrate(tan(x)^(1/2),x, algorithm="fricas")
```

```
[Out] (1/4*I - 1/4)*sqrt(2)*log((I + 1)*sqrt(2) + 2*sqrt(tan(x))) - (1/4*I + 1/4)
*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*sqrt(tan(x))) + (1/4*I + 1/4)*sqrt(2)*log
((I - 1)*sqrt(2) + 2*sqrt(tan(x))) - (1/4*I - 1/4)*sqrt(2)*log(-(I + 1)*sqr
t(2) + 2*sqrt(tan(x)))
```

Sympy [F]

$$\int \sqrt{\tan(x)}\,dx = \int \sqrt{\tan{(x)}}\,dx$$

```
[In] integrate(tan(x)**(1/2),x)
```

```
[Out] Integral(sqrt(tan(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\begin{aligned}\int \sqrt{\tan(x)}\,dx = {} & \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan{(x)}}\right)\right) \\ & +\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan{(x)}}\right)\right) \\ & -\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan{(x)}}+\tan{(x)}+1\right) \\ & +\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan{(x)}}+\tan{(x)}+1\right)\end{aligned}$$

```
[In] integrate(tan(x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 1/2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 1/4*sqrt(2)*log(sqrt(2)*sqr
t(tan(x)) + tan(x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) +
1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \sqrt{\tan(x)}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)$$

```
[In] integrate(tan(x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 1/2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 1/4*sqrt(2)*log(sqrt(2)*sqr
t(tan(x)) + tan(x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) +
1)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.66

$$\int \sqrt{\tan(x)}\,dx = \frac{\sqrt{2}\left(\ln\left(\sqrt{2}\sqrt{\tan(x)}-\tan(x)-1\right)-\ln\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)\right)}{4} + \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\sqrt{\tan(x)}-1\right)+\operatorname{atan}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)\right)}{2}$$

```
[In] int(tan(x)^(1/2),x)
```

```
[Out] (2^(1/2)*(log(2^(1/2)*tan(x)^(1/2) - tan(x) - 1) - log(tan(x) + 2^(1/2)*tan
(x)^(1/2) + 1)))/4 + (2^(1/2)*(atan(2^(1/2)*tan(x)^(1/2) - 1) + atan(2^(1/2
)*tan(x)^(1/2) + 1)))/2
```

3.398 $\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 57

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = -\frac{1}{10}\sqrt{3}\arctan\left(\frac{1-2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right)+\frac{3}{20}\log\left(1+\tan^{\frac{2}{3}}(5x)\right)-\frac{1}{20}\log\left(1+\tan^2(5x)\right)$$

```
[Out] 3/20*ln(1+tan(5*x)^(2/3))-1/20*ln(1+tan(5*x)^2)-1/10*arctan(1/3*(1-2*tan(5*
x)^(2/3))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {3557, 335, 281, 206, 31, 648, 632, 210, 642}

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = -\frac{1}{10}\sqrt{3}\arctan\left(\frac{1-2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right)+\frac{1}{10}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)-\frac{1}{20}\log\left(\tan^{\frac{4}{3}}(5x)-\tan^{\frac{2}{3}}(5x)+1\right)$$

```
[In] Int[Tan[5*x]^(-1/3),x]
```

```
[Out] -1/10*(Sqrt[3]*ArcTan[(1 - 2*Tan[5*x]^(2/3))/Sqrt[3]]) + Log[1 + Tan[5*x]^(
2/3)]/10 - Log[1 - Tan[5*x]^(2/3) + Tan[5*x]^(4/3)]/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,(1+x^2)}\,dx, x, \tan(5x)\right)\\
&= \frac{3}{5}\text{Subst}\left(\int \frac{x}{1+x^6}\,dx, x, \sqrt[3]{\tan(5x)}\right)\\
&= \frac{3}{10}\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, \tan^{\frac{2}{3}}(5x)\right)\\
&= \frac{1}{10}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \tan^{\frac{2}{3}}(5x)\right) + \frac{1}{10}\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, \tan^{\frac{2}{3}}(5x)\right)\\
&= \frac{1}{10}\log\left(1+\tan^{\frac{2}{3}}(5x)\right) - \frac{1}{20}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \tan^{\frac{2}{3}}(5x)\right)\\
&\quad + \frac{3}{20}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tan^{\frac{2}{3}}(5x)\right)\\
&= \frac{1}{10}\log\left(1+\tan^{\frac{2}{3}}(5x)\right) - \frac{1}{20}\log\left(1-\tan^{\frac{2}{3}}(5x)+\tan^{\frac{4}{3}}(5x)\right)\\
&\quad - \frac{3}{10}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tan^{\frac{2}{3}}(5x)\right)\\
&= -\frac{1}{10}\sqrt{3}\arctan\left(\frac{1-2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right) + \frac{1}{10}\log\left(1+\tan^{\frac{2}{3}}(5x)\right)\\
&\quad - \frac{1}{20}\log\left(1-\tan^{\frac{2}{3}}(5x)+\tan^{\frac{4}{3}}(5x)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \frac{1}{10}\sqrt{3}\arctan\left(\frac{-1+2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right)+\frac{1}{10}\log\left(1+\tan^{\frac{2}{3}}(5x)\right) - \frac{1}{20}\log\left(1-\tan^{\frac{2}{3}}(5x)+\tan^{\frac{4}{3}}(5x)\right)$$

```
[In] Integrate[Tan[5*x]^(-1/3),x]
[Out] (Sqrt[3]*ArcTan[(-1 + 2*Tan[5*x]^(2/3))/Sqrt[3]])/10 + Log[1 + Tan[5*x]^(2/
3)]/10 - Log[1 - Tan[5*x]^(2/3) + Tan[5*x]^(4/3)]/20
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\ln\left(1+\tan^{\frac{2}{3}}(5x)\right)}{10}-\frac{\ln\left(\tan^{\frac{4}{3}}(5x)-\left(\tan^{\frac{2}{3}}(5x)\right)+1\right)}{20}+\frac{\sqrt{3}\arctan\left(\frac{\left(2\left(\tan^{\frac{2}{3}}(5x)\right)-1\right)\sqrt{3}}{3}\right)}{10}$	53
default	$\frac{\ln\left(1+\tan^{\frac{2}{3}}(5x)\right)}{10}-\frac{\ln\left(\tan^{\frac{4}{3}}(5x)-\left(\tan^{\frac{2}{3}}(5x)\right)+1\right)}{20}+\frac{\sqrt{3}\arctan\left(\frac{\left(2\left(\tan^{\frac{2}{3}}(5x)\right)-1\right)\sqrt{3}}{3}\right)}{10}$	53

```
[In] int(1/tan(5*x)^(1/3),x,method=_RETURNVERBOSE)
[Out] 1/10*ln(1+tan(5*x)^(2/3))-1/20*ln(tan(5*x)^(4/3)-tan(5*x)^(2/3)+1)+1/10*3^(
1/2)*arctan(1/3*(2*tan(5*x)^(2/3)-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \frac{1}{10}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\tan\left(5x\right)^{\frac{2}{3}}-\frac{1}{3}\sqrt{3}\right) - \frac{1}{20}\log\left(\tan\left(5x\right)^{\frac{4}{3}}-\tan\left(5x\right)^{\frac{2}{3}}+1\right)+\frac{1}{10}\log\left(\tan\left(5x\right)^{\frac{2}{3}}+1\right)$$

```
[In] integrate(1/tan(5*x)^(1/3),x, algorithm="fricas")
[Out] 1/10*sqrt(3)*arctan(2/3*sqrt(3)*tan(5*x)^(2/3) - 1/3*sqrt(3)) - 1/20*log(ta
n(5*x)^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3) + 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \int \frac{1}{\sqrt[3]{\tan{(5x)}}}\,dx$$

```
[In] integrate(1/tan(5*x)**(1/3),x)
[Out] Integral(tan(5*x)**(-1/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \frac{1}{10}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan{(5x)}^{\frac{2}{3}}-1\right)\right) - \frac{1}{20}\log\left(\tan{(5x)}^{\frac{4}{3}}-\tan{(5x)}^{\frac{2}{3}}+1\right) + \frac{1}{10}\log\left(\tan{(5x)}^{\frac{2}{3}}+1\right)$$

```
[In] integrate(1/tan(5*x)^(1/3),x, algorithm="maxima")
[Out] 1/10*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(5*x)^(2/3) - 1)) - 1/20*log(tan(5*x)
^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \frac{1}{10}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan{(5x)}^{\frac{2}{3}}-1\right)\right) - \frac{1}{20}\log\left(\tan{(5x)}^{\frac{4}{3}}-\tan{(5x)}^{\frac{2}{3}}+1\right) + \frac{1}{10}\log\left(\tan{(5x)}^{\frac{2}{3}}+1\right)$$

```
[In] integrate(1/tan(5*x)^(1/3),x, algorithm="giac")
[Out] 1/10*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(5*x)^(2/3) - 1)) - 1/20*log(tan(5*x)
^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \frac{\ln\left(81\tan(5\,x)^{2/3}+81\right)}{10} - \ln\left(81-162\tan(5\,x)^{2/3}+\sqrt{3}\,81\mathrm{i}\right)\left(\frac{1}{20}+\frac{\sqrt{3}\,1\mathrm{i}}{20}\right) + \ln\left(162\tan(5\,x)^{2/3}-81+\sqrt{3}\,81\mathrm{i}\right)\left(-\frac{1}{20}+\frac{\sqrt{3}\,1\mathrm{i}}{20}\right)$$

```
[In] int(1/tan(5*x)^(1/3),x)
```

```
[Out] log(81*tan(5*x)^(2/3) + 81)/10 - log(3^(1/2)*81i - 162*tan(5*x)^(2/3) + 81)
*((3^(1/2)*1i)/20 + 1/20) + log(3^(1/2)*81i + 162*tan(5*x)^(2/3) - 81)*((3^
(1/2)*1i)/20 - 1/20)
```

3.399 $\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 87

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = -\frac{9\arctan\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{4+3\tan(2x)}}\right)}{250\sqrt{2}} + \frac{13\operatorname{arctanh}\left(\frac{3+\tan(2x)}{\sqrt{2}\sqrt{4+3\tan(2x)}}\right)}{250\sqrt{2}} - \frac{3}{25\sqrt{4+3\tan(2x)}}$$

```
[Out] -9/500*arctan(1/2*(1-3*tan(2*x))*2^(1/2)/(4+3*tan(2*x))^(1/2))*2^(1/2)+13/5
00*arctanh(1/2*(3+tan(2*x))*2^(1/2)/(4+3*tan(2*x))^(1/2))*2^(1/2)-3/25/(4+3
*tan(2*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3564, 3617, 3616, 209, 213}

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = -\frac{9\arctan\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{250\sqrt{2}} + \frac{13\operatorname{arctanh}\left(\frac{\tan(2x)+3}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{250\sqrt{2}} - \frac{3}{25\sqrt{3\tan(2x)+4}}$$

```
[In] Int[(4 + 3*Tan[2*x])^(-3/2),x]
```

```
[Out] (-9*ArcTan[(1 - 3*Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*x]])])/(250*Sqrt[2])
+ (13*ArcTanh[(3 + Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*x]])])/(250*Sqrt[2])
 - 3/(25*Sqrt[4 + 3*Tan[2*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3}{25\sqrt{4+3\tan(2x)}}+\frac{1}{25}\int\frac{4-3\tan(2x)}{\sqrt{4+3\tan(2x)}}\,dx \\
&= -\frac{3}{25\sqrt{4+3\tan(2x)}}+\frac{1}{250}\int\frac{27+9\tan(2x)}{\sqrt{4+3\tan(2x)}}\,dx-\frac{1}{250}\int\frac{-13+39\tan(2x)}{\sqrt{4+3\tan(2x)}}\,dx
\end{aligned}
$$

$$= -\frac{3}{25\sqrt{4+3\tan(2x)}} - \frac{81}{250}\text{Subst}\left(\int \frac{1}{162+x^2}\,dx, x, \frac{9-27\tan(2x)}{\sqrt{4+3\tan(2x)}}\right) + \frac{1521}{250}\text{Subst}\left(\int \frac{1}{-27378+x^2}\,dx, x, \frac{-351-117\tan(2x)}{\sqrt{4+3\tan(2x)}}\right)$$

$$= -\frac{9\arctan\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{4+3\tan(2x)}}\right)}{250\sqrt{2}} + \frac{13\text{arctanh}\left(\frac{3+\tan(2x)}{\sqrt{2}\sqrt{4+3\tan(2x)}}\right)}{250\sqrt{2}} - \frac{3}{25\sqrt{4+3\tan(2x)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = -\frac{(3+4i)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\left(\frac{4}{25}-\frac{3i}{25}\right)(4+3\tan(2x))\right)+(3-4i)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2}\right.}{50\sqrt{4+3\tan(2x)}}$$

```
[In] Integrate[(4 + 3*Tan[2*x])^(-3/2),x]
```

```
[Out] -1/50*((3 + 4*I)*Hypergeometric2F1[-1/2, 1, 1/2, (4/25 - (3*I)/25)*(4 + 3*T
an[2*x])] + (3 - 4*I)*Hypergeometric2F1[-1/2, 1, 1/2, (4/25 + (3*I)/25)*(4
+ 3*Tan[2*x])])/Sqrt[4 + 3*Tan[2*x]]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.49

method	result
derivativedivides	$-\frac{3}{25\sqrt{4+3\tan(2x)}}+\frac{13\sqrt{2}\,\ln\left(9+3\tan(2x)+3\sqrt{4+3\tan(2x)}\,\sqrt{2}\right)}{1000}+\frac{9\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(2x)}+3\sqrt{2}\right)\sqrt{2}}{2}\right)}{500}-$
default	$-\frac{3}{25\sqrt{4+3\tan(2x)}}+\frac{13\sqrt{2}\,\ln\left(9+3\tan(2x)+3\sqrt{4+3\tan(2x)}\,\sqrt{2}\right)}{1000}+\frac{9\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(2x)}+3\sqrt{2}\right)\sqrt{2}}{2}\right)}{500}-$

```
[In] int(1/(4+3*tan(2*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/25/(4+3*tan(2*x))^(1/2)+13/1000*2^(1/2)*ln(9+3*tan(2*x)+3*(4+3*tan(2*x))
^(1/2)*2^(1/2))+9/500*2^(1/2)*arctan(1/2*(2*(4+3*tan(2*x))^(1/2)+3*2^(1/2))
*2^(1/2))-13/1000*2^(1/2)*ln(9+3*tan(2*x)-3*(4+3*tan(2*x))^(1/2)*2^(1/2))+9
/500*2^(1/2)*arctan(1/2*(2*(4+3*tan(2*x))^(1/2)-3*2^(1/2))*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.74

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = \frac{\sqrt{117i+44}(3\,\tan\left(2\,x\right)+4)\log\left(-(7i-24)\,\sqrt{117i+44}+125\,\sqrt{3\,\tan\left(2\,x\right)+4}\right)}{}$$

```
[In] integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/500*(sqrt(117*I + 44)*(3*tan(2*x) + 4)*log(-(7*I - 24)*sqrt(117*I + 44) +
 125*sqrt(3*tan(2*x) + 4)) - sqrt(117*I + 44)*(3*tan(2*x) + 4)*log((7*I - 2
4)*sqrt(117*I + 44) + 125*sqrt(3*tan(2*x) + 4)) + sqrt(-117*I + 44)*(3*tan(
2*x) + 4)*log((7*I + 24)*sqrt(-117*I + 44) + 125*sqrt(3*tan(2*x) + 4)) - sq
rt(-117*I + 44)*(3*tan(2*x) + 4)*log(-(7*I + 24)*sqrt(-117*I + 44) + 125*sq
rt(3*tan(2*x) + 4)) - 60*sqrt(3*tan(2*x) + 4))/(3*tan(2*x) + 4)
```

Sympy [F]

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = \int \frac{1}{\left(3\tan\left(2x\right)+4\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(4+3*tan(2*x))**(3/2),x)
```

```
[Out] Integral((3*tan(2*x) + 4)**(-3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3213 vs. 2(69) = 138.

Time = 0.52 (sec) , antiderivative size = 3213, normalized size of antiderivative = 36.93

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/18000*(2000*(3*cos(4*x) + sin(4*x))*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(
8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^3 + 2000*
(3*cos(4*x) + sin(4*x))*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*
x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))*sin(1/2*arctan2(-3*cos(8
*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4
))^2 - 2000*(cos(4*x) - 3*sin(4*x) - 3)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin
(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^3 - 80*(
```

```
48*cos(4*x) + 25*sin(4*x) - 27)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) +
8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4)) - 80*(25*(cos(4*
x) - 3*sin(4*x) - 3)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x)
+ 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 - 25*cos(4*x) + 48*sin(4*
x) + 75)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8
*x) + 8*cos(4*x) + 3*sin(8*x) + 4)) + 9*(18*(sqrt(2)*cos(1/2*arctan2(-3*cos
(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) +
 4))^2 + sqrt(2)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3,
 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2)*arctan2(1/3*25^(1/4)*(25*cos
(4*x)^4 + 25*sin(4*x)^4 + 64*cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) +
25)*sin(4*x)^2 + 48*sin(4*x)^3 + 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x
) + 1)*sin(4*x) + 64*cos(4*x) + 25)^(1/4)*sin(1/2*arctan2(-8/3*cos(4*x)^2 +
 2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos(4*
x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*cos(4*x) + 25/
9)) + cos(4*x) - 4/3*sin(4*x), 1/3*25^(1/4)*(25*cos(4*x)^4 + 25*sin(4*x)^4
+ 64*cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(
4*x)^3 + 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos
(4*x) + 25)^(1/4)*cos(1/2*arctan2(-8/3*cos(4*x)^2 + 2/9*(7*cos(4*x) + 16)*s
in(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos(4*x)^2 + 8/3*(2*cos(4*x) +
 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*cos(4*x) + 25/9)) - 4/3*cos(4*x) - sin
(4*x) - 4/3) + 18*(sqrt(2)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin
(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 + sqrt(2)*sin(1/2*a
rctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) +
 3*sin(8*x) + 4))^2)*arctan2(2/3*4^(1/4)*(4*cos(4*x)^4 + 4*sin(4*x)^4 + 16*
cos(4*x)^3 + (8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3 +
 33*cos(4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x) +
13)^(1/4)*sin(1/2*arctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3
, 16/9*cos(4*x)^2 - 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9))
 + 4/3*sin(4*x) + 1, 2/3*4^(1/4)*(4*cos(4*x)^4 + 4*sin(4*x)^4 + 16*cos(4*x)
^3 + (8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3 + 33*cos(
4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x) + 13)^(1/4
)*cos(1/2*arctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3, 16/9*c
os(4*x)^2 - 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9)) + 4/3*c
os(4*x) + 4/3) + 13*(sqrt(2)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*s
in(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 + sqrt(2)*sin(1/2
*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x)
 + 3*sin(8*x) + 4))^2)*log(-2/9*25^(1/4)*(25*cos(4*x)^4 + 25*sin(4*x)^4 + 6
4*cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x
)^3 + 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*
x) + 25)^(1/4)*(4*cos(4*x) + 3*sin(4*x) + 4)*cos(1/2*arctan2(-8/3*cos(4*x)^
2 + 2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos
(4*x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*cos(4*x) +
25/9)) + 5/9*sqrt(25*cos(4*x)^4 + 25*sin(4*x)^4 + 64*cos(4*x)^3 + 2*(25*cos
(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x)^3 + 78*cos(4*x)^2 + 48
*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*x) + 25)*cos(1/2*arctan2
```

```
(-8/3*cos(4*x)^2 + 2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*co
s(4*x), 7/9*cos(4*x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 3
2/9*cos(4*x) + 25/9))^2 + 2/9*25^(1/4)*(25*cos(4*x)^4 + 25*sin(4*x)^4 + 64*
cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x)^
3 + 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*x)
 + 25)^(1/4)*(3*cos(4*x) - 4*sin(4*x))*sin(1/2*arctan2(-8/3*cos(4*x)^2 + 2/
9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos(4*x)^
2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*cos(4*x) + 25/9))
 + 5/9*sqrt(25*cos(4*x)^4 + 25*sin(4*x)^4 + 64*cos(4*x)^3 + 2*(25*cos(4*x)^
2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x)^3 + 78*cos(4*x)^2 + 48*(cos(
4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*x) + 25)*sin(1/2*arctan2(-8/3*
cos(4*x)^2 + 2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x)
, 7/9*cos(4*x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*co
s(4*x) + 25/9))^2 + 25/9*cos(4*x)^2 + 25/9*sin(4*x)^2 + 32/9*cos(4*x) + 8/3
*sin(4*x) + 16/9) - 13*(sqrt(2)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) +
8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 + sqrt(2)*sin(
1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4
*x) + 3*sin(8*x) + 4))^2)*log(16/9*4^(1/4)*(4*cos(4*x)^4 + 4*sin(4*x)^4 + 1
6*cos(4*x)^3 + (8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3
 + 33*cos(4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x)
+ 13)^(1/4)*(cos(4*x) + 1)*cos(1/2*arctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8
/3*cos(4*x) + 8/3, 16/9*cos(4*x)^2 - 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*
sin(4*x) + 16/9)) + 8/9*sqrt(4*cos(4*x)^4 + 4*sin(4*x)^4 + 16*cos(4*x)^3 +
(8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3 + 33*cos(4*x)^
2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x) + 13)*cos(1/2*a
rctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3, 16/9*cos(4*x)^2 -
 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9))^2 + 4/9*4^(1/4)*(4
*cos(4*x)^4 + 4*sin(4*x)^4 + 16*cos(4*x)^3 + (8*cos(4*x)^2 + 16*cos(4*x) +
17)*sin(4*x)^2 + 12*sin(4*x)^3 + 33*cos(4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x
) + 1)*sin(4*x) + 34*cos(4*x) + 13)^(1/4)*(4*sin(4*x) + 3)*sin(1/2*arctan2(
32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3, 16/9*cos(4*x)^2 - 16/9*s
in(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9)) + 8/9*sqrt(4*cos(4*x)^4 +
 4*sin(4*x)^4 + 16*cos(4*x)^3 + (8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^
2 + 12*sin(4*x)^3 + 33*cos(4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*
x) + 34*cos(4*x) + 13)*sin(1/2*arctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*c
os(4*x) + 8/3, 16/9*cos(4*x)^2 - 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(
4*x) + 16/9))^2 + 16/9*cos(4*x)^2 + 16/9*sin(4*x)^2 + 32/9*cos(4*x) + 8/3*s
in(4*x) + 25/9))*(2*(32*cos(4*x) - 24*sin(4*x) + 7)*cos(8*x) + 25*cos(8*x)^
2 + 64*cos(4*x)^2 + 16*(3*cos(4*x) + 4*sin(4*x) + 3)*sin(8*x) + 25*sin(8*x)
^2 + 64*sin(4*x)^2 + 64*cos(4*x) + 48*sin(4*x) + 25)^(1/4))/((2*(32*cos(4*x
) - 24*sin(4*x) + 7)*cos(8*x) + 25*cos(8*x)^2 + 64*cos(4*x)^2 + 16*(3*cos(4
*x) + 4*sin(4*x) + 3)*sin(8*x) + 25*sin(8*x)^2 + 64*sin(4*x)^2 + 64*cos(4*x
) + 48*sin(4*x) + 25)^(1/4)*(cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*s
in(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 + sin(1/2*arctan2
(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin
```

```
(8*x) + 4))^2))
```

Giac [F]

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx=\int \frac{1}{(3\,\tan(2\,x)+4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="giac")
[Out] integrate((3*tan(2*x) + 4)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\begin{aligned}\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx &= -\frac{3}{25\,\sqrt{3\tan(2\,x)+4}}\\ &\quad+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{3\tan(2\,x)+4}\,\left(\frac{1}{10}-\frac{3}{10}\mathrm{i}\right)\right)\,\left(\frac{9}{500}+\frac{13}{500}\mathrm{i}\right)\\ &\quad+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{3\tan(2\,x)+4}\,\left(\frac{1}{10}+\frac{3}{10}\mathrm{i}\right)\right)\,\left(\frac{9}{500}-\frac{13}{500}\mathrm{i}\right)\end{aligned}$$

```
[In] int(1/(3*tan(2*x) + 4)^(3/2),x)
[Out] 2^(1/2)*atan(2^(1/2)*(3*tan(2*x) + 4)^(1/2)*(1/10 - 3i/10))*(9/500 + 13i/50
0) + 2^(1/2)*atan(2^(1/2)*(3*tan(2*x) + 4)^(1/2)*(1/10 + 3i/10))*(9/500 - 1
3i/500) - 3/(25*(3*tan(2*x) + 4)^(1/2))
```

3.400 $\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 40

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{1}{3}\log(4-3\tan(x)) + \frac{8}{3\sqrt{4-3\tan(x)}} + \frac{2}{3}\sqrt{4-3\tan(x)}$$

```
[Out] 1/3*ln(4-3*tan(x))+8/3/(4-3*tan(x))^(1/2)+2/3*(4-3*tan(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4427, 45}

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{2}{3}\sqrt{4-3\tan(x)} + \frac{8}{3\sqrt{4-3\tan(x)}} + \frac{1}{3}\log(4-3\tan(x))$$

```
[In] Int[(Sec[x]^2*(-Sqrt[4 - 3*Tan[x]] + 3*Tan[x]))/(4 - 3*Tan[x])^(3/2),x]
[Out] Log[4 - 3*Tan[x]]/3 + 8/(3*Sqrt[4 - 3*Tan[x]]) + (2*Sqrt[4 - 3*Tan[x]])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \left(\frac{3x}{(4-3x)^{3/2}}+\frac{1}{-4+3x}\right)\,dx,x,\tan(x)\right)\\
&= \frac{1}{3}\log(4-3\tan(x))+3\text{Subst}\left(\int \frac{x}{(4-3x)^{3/2}}\,dx,x,\tan(x)\right)\\
&= \frac{1}{3}\log(4-3\tan(x))+3\text{Subst}\left(\int \left(\frac{4}{3(4-3x)^{3/2}}-\frac{1}{3\sqrt{4-3x}}\right)\,dx,x,\tan(x)\right)\\
&= \frac{1}{3}\log(4-3\tan(x))+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{2}{3}\sqrt{4-3\tan(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx=\frac{1}{3}\left(\log(4-3\tan(x))+\frac{2(8-3\tan(x))}{\sqrt{4-3\tan(x)}}\right)$$

```
[In] Integrate[(Sec[x]^2*(-Sqrt[4 - 3*Tan[x]] + 3*Tan[x]))/(4 - 3*Tan[x])^(3/2),
x]
[Out] (Log[4 - 3*Tan[x]] + (2*(8 - 3*Tan[x]))/Sqrt[4 - 3*Tan[x]])/3
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\ln(4-3\tan(x))}{3}+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{2\sqrt{4-3\tan(x)}}{3}$	31
default	$\frac{\ln(4-3\tan(x))}{3}+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{2\sqrt{4-3\tan(x)}}{3}$	31

```
[In] int((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x,method=_RE
TURNVERBOSE)
[Out] 1/3*ln(4-3*tan(x))+8/3/(4-3*tan(x))^(1/2)+2/3*(4-3*tan(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(30) = 60.

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.05

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx=\frac{(4\cos(x)-3\sin(x))\log\left(\frac{7}{4}\cos(x)^2-6\cos(x)\sin(x)+\frac{9}{4}\right)}{6(}$$

```
[In] integrate((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x, alg
orithm="fricas")
[Out] 1/6*((4*cos(x) - 3*sin(x))*log(7/4*cos(x)^2 - 6*cos(x)*sin(x) + 9/4) - (4*c
os(x) - 3*sin(x))*log(cos(x)^2) + 4*sqrt((4*cos(x) - 3*sin(x))/cos(x))*(8*c
os(x) - 3*sin(x)))/(4*cos(x) - 3*sin(x))
```

Sympy [F]

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = -\int \frac{\sqrt{4-3\tan(x)}}{-3\sqrt{4-3\tan(x)}\cos^2(x)\tan(x)+4\sqrt{4-3\tan(x)}\cos^2(x)}\,dx - \int\left(-\frac{3\tan(x)}{-3\sqrt{4-3\tan(x)}\cos^2(x)\tan(x)+4\sqrt{4-3\tan(x)}\cos^2(x)}\right)dx$$

```
[In] integrate((-(4-3*tan(x))**(1/2)+3*tan(x))/cos(x)**2/(4-3*tan(x))**(3/2),x)
[Out] -Integral(sqrt(4 - 3*tan(x))/(-3*sqrt(4 - 3*tan(x))*cos(x)**2*tan(x) + 4*sq
rt(4 - 3*tan(x))*cos(x)**2), x) - Integral(-3*tan(x)/(-3*sqrt(4 - 3*tan(x))
*cos(x)**2*tan(x) + 4*sqrt(4 - 3*tan(x))*cos(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{2}{3}\sqrt{-3\,\tan(x)+4} + \frac{8}{3\sqrt{-3\,\tan(x)+4}} + \frac{1}{3}\,\log\left(-3\,\tan(x)+4\right)$$

```
[In] integrate((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x, alg
orithm="maxima")
```

```
[Out] 2/3*sqrt(-3*tan(x) + 4) + 8/3/sqrt(-3*tan(x) + 4) + 1/3*log(-3*tan(x) + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{2}{3}\sqrt{-3\,\tan(x)+4} + \frac{8}{3\sqrt{-3\,\tan(x)+4}} + \frac{1}{3}\,\log\left(|-3\,\tan(x)+4|\right)$$

```
[In] integrate((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x, alg
orithm="giac")
```

```
[Out] 2/3*sqrt(-3*tan(x) + 4) + 8/3/sqrt(-3*tan(x) + 4) + 1/3*log(abs(-3*tan(x) +
 4))
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.62

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{\ln\left(e^{x\,2i}\left(-\frac{16}{3}-4i\right)-\frac{16}{3}+4i\right)}{3} - \frac{\ln\left(e^{x\,2i}\left(\frac{16}{3}-4i\right)+\frac{16}{3}-4i\right)}{3} + \frac{2\,e^{x\,1i}\,\cos(x)\,\left(\frac{32\,e^{x\,1i}\,\cos(x)}{3}-4\,e^{x\,1i}\,\sin(x)\right)\,\sqrt{4-\frac{3\,\sin(x)}{\cos(x)}}}{8\,e^{x\,2i}+8\,\cos(2\,x)\,e^{x\,2i}-6\,\sin(2\,x)\,e^{x\,2i}}$$

```
[In] int((3*tan(x) - (4 - 3*tan(x))^(1/2))/(cos(x)^2*(4 - 3*tan(x))^(3/2)),x)
```

```
[Out] log(- exp(x*2i)*(16/3 + 4i) - (16/3 - 4i))/3 - log(exp(x*2i)*(16/3 - 4i) +
(16/3 - 4i))/3 + (2*exp(x*1i)*cos(x)*((32*exp(x*1i)*cos(x))/3 - 4*exp(x*1i)
*sin(x))*(4 - (3*sin(x))/cos(x))^(1/2))/(8*exp(x*2i) + 8*cos(2*x)*exp(x*2i)
 - 6*sin(2*x)*exp(x*2i))
```

3.401 $\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = -\frac{x}{2}+\frac{\arctan\left(\frac{1-\tan(x)}{\sqrt{2}\sqrt{\tan(x)}}\right)}{\sqrt{2}}+\frac{\operatorname{arctanh}\left(\frac{1+\tan(x)}{\sqrt{2}\sqrt{\tan(x)}}\right)}{\sqrt{2}} +\frac{1}{2}\log(\cos(x))+\log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}$$

```
[Out] -1/2*x+1/2*ln(cos(x))+ln(1-tan(x)^(1/2))+1/2*arctan(1/2*(1-tan(x))*2^(1/2)/
tan(x)^(1/2))*2^(1/2)+1/2*arctanh(1/2*(1+tan(x))*2^(1/2)/tan(x)^(1/2))*2^(1
/2)+1/(1-tan(x)^(1/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.58, number of steps used = 19, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3751, 6857, 1845, 303, 1176, 631, 210, 1179, 642, 1262, 649, 209, 266}

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}} - \frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}} - \frac{x}{2} + \frac{1}{1-\sqrt{\tan(x)}} + \log\left(1-\sqrt{\tan(x)}\right) - \frac{\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}} + \frac{\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}} + \frac{1}{2}\log(\cos(x))$$

```
[In] Int[Tan[x]/(-1 + Sqrt[Tan[x]])^2,x]
```

```
[Out] -1/2*x + ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt
[Tan[x]]]/Sqrt[2] + Log[Cos[x]]/2 + Log[1 - Sqrt[Tan[x]]] - Log[1 - Sqrt[2]
*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]
]/(2*Sqrt[2]) + (1 - Sqrt[Tan[x]])^(-1)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
```

```
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x}{\left(-1+\sqrt{x}\right)^2(1+x^2)}\,dx, x, \tan(x)\right)\\
&= 2\text{Subst}\left(\int \frac{x^3}{(-1+x)^2(1+x^4)}\,dx, x, \sqrt{\tan(x)}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{1}{2(-1+x)^2}+\frac{1}{2(-1+x)}-\frac{x(1+x)^2}{2(1+x^4)}\right)dx, x, \sqrt{\tan(x)}\right)\\
&= \log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}-\text{Subst}\left(\int \frac{x(1+x)^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)\\
&= \log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}-\text{Subst}\left(\int \left(\frac{2x^2}{1+x^4}+\frac{x(1+x^2)}{1+x^4}\right)dx, x, \sqrt{\tan(x)}\right)\\
&= \log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}-2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)\\
&\quad -\text{Subst}\left(\int \frac{x(1+x^2)}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)\\
&= \log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}-\frac{1}{2}\text{Subst}\left(\int \frac{1+x}{1+x^2}\,dx, x, \tan(x)\right)\\
&\quad +\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)-\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)
\end{aligned}$$

$$= \log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}-\frac{1}{2}\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\tan(x)\right)$$
$$-\frac{1}{2}\text{Subst}\left(\int\frac{x}{1+x^2}\,dx,x,\tan(x)\right)-\frac{1}{2}\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(x)}\right)$$
$$-\frac{1}{2}\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(x)}\right)$$
$$-\frac{\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(x)}\right)}{2\sqrt{2}}-\frac{\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(x)}\right)}{2\sqrt{2}}$$
$$= -\frac{x}{2}+\frac{1}{2}\log(\cos(x))+\log\left(1-\sqrt{\tan(x)}\right)$$
$$-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}}$$
$$+\frac{1}{1-\sqrt{\tan(x)}}-\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}$$
$$+\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}$$
$$= -\frac{x}{2}+\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}$$
$$+\frac{1}{2}\log(\cos(x))+\log\left(1-\sqrt{\tan(x)}\right)-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}}$$
$$+\frac{\log\left(1+\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}}+\frac{1}{1-\sqrt{\tan(x)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int\frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = -\frac{1}{2}\arctan(\tan(x))+\frac{1}{2}\log(\cos(x))$$
$$+\log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}$$
$$-\frac{2}{3}\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\tan^2(x)\right)\tan^{\frac{3}{2}}(x)$$

```
[In] Integrate[Tan[x]/(-1 + Sqrt[Tan[x]])^2,x]
```

[Out] -1/2*ArcTan[Tan[x]] + Log[Cos[x]]/2 + Log[1 - Sqrt[Tan[x]]] + (1 - Sqrt[Tan[x]])^(-1) - (2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[x]^2]*Tan[x]^(3/2))/3

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.12

method	result
derivativedivides	$-\frac{\arctan(\tan(x))}{2}-\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\,(\sqrt{\tan}(x))+\tan(x)}{1+\sqrt{2}\,(\sqrt{\tan}(x))+\tan(x)}\right)+2\arctan\left(1+\sqrt{2}\,(\sqrt{\tan}(x))\right)+2\arctan\left(-1+\sqrt{2}\,(\sqrt{\tan}(x))\right)\right)}{4}$
default	$-\frac{\arctan(\tan(x))}{2}-\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\,(\sqrt{\tan}(x))+\tan(x)}{1+\sqrt{2}\,(\sqrt{\tan}(x))+\tan(x)}\right)+2\arctan\left(1+\sqrt{2}\,(\sqrt{\tan}(x))\right)+2\arctan\left(-1+\sqrt{2}\,(\sqrt{\tan}(x))\right)\right)}{4}$

[In] int(tan(x)/(-1+tan(x)^(1/2))^2,x,method=_RETURNVERBOSE)

[Out] -1/2*arctan(tan(x))-1/4*2^(1/2)*(ln((1-2^(1/2)*tan(x)^(1/2)+tan(x))/(1+2^(1/2)*tan(x)^(1/2)+tan(x)))+2*arctan(1+2^(1/2)*tan(x)^(1/2))+2*arctan(-1+2^(1/2)*tan(x)^(1/2)))-1/4*ln(1+tan(x)^2)-1/(-1+tan(x)^(1/2))+ln(-1+tan(x)^(1/2))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 603, normalized size of antiderivative = 7.18

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = \text{Too large to display}$$

[In] integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="fricas")

[Out] -1/8*(2*(2*sqrt(-I) - I + 1)*(tan(x) - 1)*log(-1/2*(2*sqrt(-I) - I + 1)^2*(4*(-1)^(1/4) + 2*I + 1) - (2*(-1)^(1/4) + I + 1)^3 - ((2*(-1)^(1/4) + I + 1)^2 - 8*(-1)^(1/4) - 4*I - 3)*(2*sqrt(-I) - I + 1) + 4*(2*(-1)^(1/4) + I + 1)^2 + 6*sqrt(tan(x)) - 16*(-1)^(1/4) - 8*I - 9) + 2*(2*(-1)^(1/4) + I + 1)*(tan(x) - 1)*log((2*(-1)^(1/4) + I + 1)^3 - 7/2*(2*(-1)^(1/4) + I + 1)^2 + 6*sqrt(tan(x)) + 14*(-1)^(1/4) + 7*I + 14) - ((2*sqrt(-I) - I + 1)*(tan(x) - 1) + (2*(-1)^(1/4) + I + 1)*(tan(x) - 1) - 4*sqrt(-3/16*(2*sqrt(-I) - I + 1)^2 - 3/16*(2*(-1)^(1/4) + I + 1)^2 - 1/8*(2*sqrt(-I) - I + 1)*(2*(-1)^(1/4) + I - 3) + (-1)^(1/4) + 1/2*I - 1/2)*(tan(x) - 1) - 4*tan(x) + 4)*log(1/4*(2*sqrt(-I) - I + 1)^2*(4*(-1)^(1/4) + 2*I + 1) + 1/2*((2*(-1)^(1/4) + I + 1)^2 - 8*(-1)^(1/4) - 4*I - 3)*(2*sqrt(-I) - I + 1) - 1/4*(2*(-1)^(1/4) + I + 1)^2 + sqrt(-3/16*(2*sqrt(-I) - I + 1)^2 - 3/16*(2*(-1)^(1/4) + I + 1)^2 - 1/8*(2*sqrt(-I) - I + 1)*(2*(-1)^(1/4) + I - 3) + (-1)^(1/4) + 1/2*I

```
 - 1/2)*((2*sqrt(-I) - I + 1)*(4*(-1)^(1/4) + 2*I + 1) - 2*(-1)^(1/4) - I +
 1) + 6*sqrt(tan(x)) + (-1)^(1/4) + 1/2*I - 5/2) - ((2*sqrt(-I) - I + 1)*(t
an(x) - 1) + (2*(-1)^(1/4) + I + 1)*(tan(x) - 1) + 4*sqrt(-3/16*(2*sqrt(-I)
 - I + 1)^2 - 3/16*(2*(-1)^(1/4) + I + 1)^2 - 1/8*(2*sqrt(-I) - I + 1)*(2*(
-1)^(1/4) + I - 3) + (-1)^(1/4) + 1/2*I - 1/2)*(tan(x) - 1) - 4*tan(x) + 4)
*log(1/4*(2*sqrt(-I) - I + 1)^2*(4*(-1)^(1/4) + 2*I + 1) + 1/2*((2*(-1)^(1/
4) + I + 1)^2 - 8*(-1)^(1/4) - 4*I - 3)*(2*sqrt(-I) - I + 1) - 1/4*(2*(-1)^
(1/4) + I + 1)^2 - sqrt(-3/16*(2*sqrt(-I) - I + 1)^2 - 3/16*(2*(-1)^(1/4) +
 I + 1)^2 - 1/8*(2*sqrt(-I) - I + 1)*(2*(-1)^(1/4) + I - 3) + (-1)^(1/4) +
1/2*I - 1/2)*((2*sqrt(-I) - I + 1)*(4*(-1)^(1/4) + 2*I + 1) - 2*(-1)^(1/4)
- I + 1) + 6*sqrt(tan(x)) + (-1)^(1/4) + 1/2*I - 5/2) - 8*(tan(x) - 1)*log(
sqrt(tan(x)) - 1) + 8*sqrt(tan(x)) + 8)/(tan(x) - 1)
```

Sympy [F]

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = \int \frac{\tan(x)}{\left(\sqrt{\tan(x)}-1\right)^2}\,dx$$

```
[In] integrate(tan(x)/(-1+tan(x)**(1/2))**2,x)
[Out] Integral(tan(x)/(sqrt(tan(x)) - 1)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.39

$$\begin{aligned}\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx =& \frac{1}{4}\sqrt{2}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right) \\ &- \frac{1}{4}\sqrt{2}\left(\sqrt{2}+2\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right) \\ &- \frac{1}{8}\sqrt{2}\left(\sqrt{2}-2\right)\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right) \\ &- \frac{1}{8}\sqrt{2}\left(\sqrt{2}+2\right)\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right) \\ &- \frac{1}{\sqrt{\tan(x)}-1} + \log\left(\sqrt{\tan(x)}-1\right)\end{aligned}$$

```
[In] integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="maxima")
[Out] 1/4*sqrt(2)*(sqrt(2) - 2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) -
1/4*sqrt(2)*(sqrt(2) + 2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) -
```

```
 1/8*sqrt(2)*(sqrt(2) - 2)*log(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/8*sqr
t(2)*(sqrt(2) + 2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/(sqrt(tan(x)
) - 1) + log(sqrt(tan(x)) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.32

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = -\frac{1}{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right) - \frac{1}{2}\left(\sqrt{2}+1\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right) + \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right) - \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right) - \frac{1}{\sqrt{\tan(x)}-1} - \frac{1}{4}\log\left(\tan(x)^2+1\right) + \log\left(\left|\sqrt{\tan(x)}-1\right|\right)$$

```
[In] integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) - 1/2*(sq
rt(2) + 1)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) + 1/4*sqrt(2)*lo
g(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)
) + tan(x) + 1) - 1/(sqrt(tan(x)) - 1) - 1/4*log(tan(x)^2 + 1) + log(abs(sq
rt(tan(x)) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.71

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = \ln\left(612\sqrt{\tan(x)}-612\right) - \frac{1}{\sqrt{\tan(x)}-1} + \left(\sum_{k=1}^{4}\ln\left(4\sqrt{\tan(x)} + \mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^2\sqrt{\tan(x)}\,80 + \mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^3\sqrt{\tan(x)}\,448 + \mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^4\sqrt{\tan(x)}\,128 + 32\,\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^2 - 384\,\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^3 - 256\,\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^4 - \mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)\sqrt{\tan(x)}\,48 - 4\right)\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)\right)$$

```
[In] int(tan(x)/(tan(x)^(1/2) - 1)^2,x)
```

```
[Out] log(612*tan(x)^(1/2) - 612) - 1/(tan(x)^(1/2) - 1) + symsum(log(4*tan(x)^(1
/2) + 80*root(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^2*tan(x)^(1/2) + 448*ro
ot(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^3*tan(x)^(1/2) + 128*root(z^4 + z^
3 + z^2/2 - z/8 + 1/64, z, k)^4*tan(x)^(1/2) + 32*root(z^4 + z^3 + z^2/2 -
z/8 + 1/64, z, k)^2 - 384*root(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^3 - 25
6*root(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^4 - 48*root(z^4 + z^3 + z^2/2
- z/8 + 1/64, z, k)*tan(x)^(1/2) - 4)*root(z^4 + z^3 + z^2/2 - z/8 + 1/64,
z, k), k, 1, 4)
```

3.402 $\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = -\frac{1}{2}\arcsin(\cos(x)-\sin(x)) - \frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

```
[Out] -1/2*arcsin(cos(x)-sin(x))-1/2*ln(cos(x)+sin(x)+sin(2*x)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4391}

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = -\frac{1}{2}\arcsin(\cos(x)-\sin(x)) - \frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

```
[In] Int[Sin[x]/Sqrt[Sin[2*x]],x]
[Out] -1/2*ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

Rule 4391

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\text{integral} = -\frac{1}{2}\arcsin(\cos(x)-\sin(x)) - \frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \frac{1}{2}\left(-\arcsin(\cos(x)-\sin(x)) - \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\right)$$

```
[In] Integrate[Sin[x]/Sqrt[Sin[2*x]],x]
[Out] (-ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 266, normalized size of antiderivative = 8.58

method	result
default	$-\frac{\sqrt{-\frac{\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}}\,(\tan^2(\frac{x}{2})-1)\left(2\sqrt{1+\tan(\frac{x}{2})}\,\sqrt{-2\tan(\frac{x}{2})+2}\,\sqrt{-\tan(\frac{x}{2})}\,E\left(\sqrt{1+\tan(\frac{x}{2})},\frac{\sqrt{2}}{2}\right)(\tan^2(\frac{x}{2}))-\sqrt{1+\tan(\frac{x}{2})}\,\sqrt{-2\,\mathrm{ta}}\right.}{}$

```
[In] int(sin(x)/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(2*(1+tan(1/2*x)
)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x)
)^(1/2),1/2*2^(1/2))*tan(1/2*x)^2-(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1
/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2
*x)^2+2*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*El
lipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))-(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*
x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))
+2*tan(1/2*x)^4-2*tan(1/2*x)^2)/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(1+tan(
1/2*x)^2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 4.42

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \frac{1}{4}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right) - \frac{1}{4}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right) + \frac{1}{8}\log\left(-32\cos(x)^4 + 4\sqrt{2}\left(4\cos(x)^3-\left(4\cos(x)^2+1\right)\sin(x)-5\cos(x)\right)\sqrt{\cos(x)\sin(x)} + 32\cos(x)^2+16\cos(x)\sin(x)+1\right)$$

```
[In] integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))
/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/4*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin
(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) + 1/8*log(-32*cos(x)^4 + 4*sqrt(
2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) +
32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(x)/sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$$

```
[In] integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)/sqrt(sin(2*x)), x)
```

Giac [F]

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\sin\left(x\right)}{\sqrt{\sin\left(2\,x\right)}}\,dx$$

[In] integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="giac")

[Out] integrate(sin(x)/sqrt(sin(2*x)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\sin\left(x\right)}{\sqrt{\sin\left(2\,x\right)}}\,dx$$

[In] int(sin(x)/sin(2*x)^(1/2),x)

[Out] int(sin(x)/sin(2*x)^(1/2), x)

3.403 $\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = -\frac{1}{2}\arcsin(\cos(x)-\sin(x)) + \frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

```
[Out] -1/2*arcsin(cos(x)-sin(x))+1/2*ln(cos(x)+sin(x)+sin(2*x)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4390}

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right) - \frac{1}{2}\arcsin(\cos(x)-\sin(x))$$

```
[In] Int[Cos[x]/Sqrt[Sin[2*x]],x]
```

```
[Out] -1/2*ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\text{integral} = -\frac{1}{2}\arcsin(\cos(x)-\sin(x)) + \frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \frac{1}{2}\left(-\arcsin(\cos(x)-\sin(x))+\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\right)$$

```
[In] Integrate[Cos[x]/Sqrt[Sin[2*x]],x]
[Out] (-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.41 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.16

method	result	size
default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,\left(\tan^2\left(\frac{x}{2}\right)-1\right)\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,F\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)-1\right)}\,\sqrt{\tan^3\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)}}$	98

```
[In] int(cos(x)/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/(tan(1/2*x)*(tan(1/2*
x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(
1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(
1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 4.42

$$\begin{aligned}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx &= \frac{1}{4}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)\\ &\quad-\frac{1}{4}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right)\\ &\quad-\frac{1}{8}\log\Big(-32\cos(x)^4\\ &\quad+4\sqrt{2}\left(4\cos(x)^3-\left(4\cos(x)^2+1\right)\sin(x)-5\cos(x)\right)\sqrt{\cos(x)\sin(x)}\\ &\quad+32\cos(x)^2+16\cos(x)\sin(x)+1\Big)\end{aligned}$$

```
[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))
/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/4*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin
(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) - 1/8*log(-32*cos(x)^4 + 4*sqrt(
2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) +
32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(x)/sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

```
[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)/sqrt(sin(2*x)), x)
```

Giac [F]

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

```
[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(x)/sqrt(sin(2*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cos\left(x\right)}{\sqrt{\sin\left(2\,x\right)}}\,dx$$

```
[In] int(cos(x)/sin(2*x)^(1/2),x)
[Out] int(cos(x)/sin(2*x)^(1/2), x)
```

3.404 $\int \sin(x)\sqrt{\sin(2x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 45

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = -\frac{1}{4}\arcsin(\cos(x)-\sin(x)) + \frac{1}{4}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) - \frac{1}{2}\cos(x)\sqrt{\sin(2x)}$$

```
[Out] -1/4*arcsin(cos(x)-sin(x))+1/4*ln(cos(x)+sin(x)+sin(2*x)^(1/2))-1/2*cos(x)*
sin(2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4387, 4390}

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = -\frac{1}{4}\arcsin(\cos(x)-\sin(x)) - \frac{1}{2}\sqrt{\sin(2x)}\cos(x) + \frac{1}{4}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

```
[In] Int[Sin[x]*Sqrt[Sin[2*x]],x]
```

```
[Out] -1/4*ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/4 - (C
os[x]*Sqrt[Sin[2*x]])/2
```

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*
(g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
```

```
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\text{integral} = -\frac{1}{2}\cos(x)\sqrt{\sin(2x)} + \frac{1}{2}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$
$$= -\frac{1}{4}\arcsin(\cos(x)-\sin(x)) + \frac{1}{4}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) - \frac{1}{2}\cos(x)\sqrt{\sin(2x)}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = \frac{1}{4}\left(-\arcsin(\cos(x)-\sin(x)) + \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) - 2\cos(x)\sqrt{\sin(2x)}\right)$$

```
[In] Integrate[Sin[x]*Sqrt[Sin[2*x]],x]
[Out] (-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]] - 2*Cos[x
]*Sqrt[Sin[2*x]])/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 2.49 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.80

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,(\tan^2(\frac{x}{2})-1)\left(\sqrt{1+\tan(\frac{x}{2})}\,\sqrt{-2\tan(\frac{x}{2})+2}\,\sqrt{-\tan(\frac{x}{2})}\,F\left(\sqrt{1+\tan(\frac{x}{2})},\frac{\sqrt{2}}{2}\right)(\tan^2(\frac{x}{2}))+\sqrt{1+\tan(\frac{x}{2})}\,\sqrt{-2\tan(\frac{x}{2}}}{\sqrt{\tan(\frac{x}{2})(\tan^2(\frac{x}{2})-1)}\,\sqrt{\tan^3(\frac{x}{2})-\tan(\frac{x}{2})}\,(1+\tan^2(\frac{x}{2}))}$

```
[In] int(sin(x)*sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*((1+tan(1/2*x))^(1/2)
*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2)
,1/2*2^(1/2))*tan(1/2*x)^2+(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-t
an(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))+2*tan(1/2*x)^3
-2*tan(1/2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x)
)^(1/2)/(1+tan(1/2*x)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(35) = 70$.

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.36

$$\begin{aligned}\int \sin(x)\sqrt{\sin(2x)}\,dx &= -\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x) \\ &+ \frac{1}{8}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right) \\ &- \frac{1}{8}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right) - \frac{1}{16}\log\Big(-32\cos(x)^4 \\ &+ 4\sqrt{2}\left(4\cos(x)^3-\left(4\cos(x)^2+1\right)\sin(x)-5\cos(x)\right)\sqrt{\cos(x)\sin(x)}+32\cos(x)^2 \\ &+ 16\cos(x)\sin(x)+1\Big)\end{aligned}$$

```
[In] integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(cos(x)*sin(x))*cos(x) + 1/8*arctan(-(sqrt(2)*sqrt(cos(x)*
sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))/(cos(x)^2 + 2*cos(x)*sin(x) - 1)
) - 1/8*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x) -
 sin(x))) - 1/16*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1
)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 + 16*cos(x)*sin(x) +
 1)
```

Sympy [F(-1)]

Timed out.

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(x)*sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = \int \sqrt{\sin(2x)}\sin(x)\,dx$$

```
[In] integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sin(2*x))*sin(x), x)
```

Giac [F]

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = \int \sqrt{\sin(2x)}\sin(x)\,dx$$

```
[In] integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sin(2*x))*sin(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = \int \sqrt{\sin(2x)}\,\sin(x)\,dx$$

```
[In] int(sin(2*x)^(1/2)*sin(x),x)
[Out] int(sin(2*x)^(1/2)*sin(x), x)
```

3.405 $\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 47

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx=-\frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)+\frac{1}{2}\cos(x)\sqrt{\sin(2x)}+\frac{1}{2}\sin(x)\sqrt{\sin(2x)}$$

```
[Out] -1/2*ln(cos(x)+sin(x)+sin(2*x)^(1/2))+1/2*cos(x)*sin(2*x)^(1/2)+1/2*sin(x)*
sin(2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4486, 4386, 4391, 4387, 4390}

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx=\frac{1}{2}\sin(x)\sqrt{\sin(2x)}+\frac{1}{2}\sqrt{\sin(2x)}\cos(x)-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

```
[In] Int[(Cos[x] - Sin[x])*Sqrt[Sin[2*x]],x]
```

```
[Out] -1/2*Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]] + (Cos[x]*Sqrt[Sin[2*x]])/2 + (S
in[x]*Sqrt[Sin[2*x]])/2
```

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
```

```
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*
(g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4391

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\cos(x)\sqrt{\sin(2x)} - \sin(x)\sqrt{\sin(2x)}\right) dx \\
&= \int \cos(x)\sqrt{\sin(2x)}\,dx - \int \sin(x)\sqrt{\sin(2x)}\,dx \\
&= \frac{1}{2}\cos(x)\sqrt{\sin(2x)} + \frac{1}{2}\sin(x)\sqrt{\sin(2x)} - \frac{1}{2}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx + \frac{1}{2}\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx \\
&= -\frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) + \frac{1}{2}\cos(x)\sqrt{\sin(2x)} + \frac{1}{2}\sin(x)\sqrt{\sin(2x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx = \frac{1}{2}\Big(-\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) + \cos(x)\sqrt{\sin(2x)}+\sin(x)\sqrt{\sin(2x)}\Big)$$

```
[In] Integrate[(Cos[x] - Sin[x])*Sqrt[Sin[2*x]],x]
```

```
[Out] (-Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]] + Cos[x]*Sqrt[Sin[2*x]] + Sin[x]*Sq
rt[Sin[2*x]])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 5.27 (sec) , antiderivative size = 396, normalized size of antiderivative = 8.43

method	result
parts	$\frac{2\sqrt{-\frac{\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}}\,(\tan^2(\frac{x}{2})-1)\left(2\sqrt{(1+\tan(\frac{x}{2}))(\tan(\frac{x}{2})-1)\tan(\frac{x}{2})}\,\sqrt{1+\tan(\frac{x}{2})}\,\sqrt{-2\tan(\frac{x}{2})+2}\,\sqrt{-\tan(\frac{x}{2})}\,E\left(\sqrt{1+\tan(\frac{x}{2})},\frac{\sqrt{2}}{2}\right)-\right.}{\sqrt{\tan(\frac{x}{2})(\tan^2(\frac{x}{2})-1)}\,\sqrt{\tan^3(\frac{x}{2})-\tan}}$
default	$\sqrt{-\frac{\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}}\,(\tan^2(\frac{x}{2})-1)\left(4\sqrt{(1+\tan(\frac{x}{2}))(\tan(\frac{x}{2})-1)\tan(\frac{x}{2})}\,\sqrt{1+\tan(\frac{x}{2})}\,\sqrt{-2\tan(\frac{x}{2})+2}\,\sqrt{-\tan(\frac{x}{2})}\,E\left(\sqrt{1+\tan(\frac{x}{2})},\frac{\sqrt{2}}{2}\right)(\tan\right.$

```
[In] int((cos(x)-sin(x))*sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(2*((1+tan(1/2*x))*
(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/
2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))-(1+tan(1
/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1
/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)+
2*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^2)/(tan(1/2*x)*(tan(1/2*x)^2-1
))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/((1+tan(1/2*x))*(tan(1/2*x)-1)*tan
(1/2*x))^(1/2)-(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*((1+ta
n(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+ta
n(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2+(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*
x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))
+2*tan(1/2*x)^3-2*tan(1/2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(tan(1/2*
x)^3-tan(1/2*x))^(1/2)/(1+tan(1/2*x)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(35) = 70$.

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx = \frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x)) + \frac{1}{8}\log\Big(-32\cos(x)^4 + 4\sqrt{2}\big(4\cos(x)^3 - \left(4\cos(x)^2+1\right)\sin(x) - 5\cos(x)\big)\sqrt{\cos(x)\sin(x)} + 32\cos(x)^2 + 16\cos(x)\sin(x) + 1\Big)$$

```
[In] integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) + 1/8*log(-32*cos(x)^4 +
4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin
(x)) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx = \text{Timed out}$$

```
[In] integrate((cos(x)-sin(x))*sin(2*x)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx = \int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx$$

```
[In] integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="maxima")
[Out] integrate((cos(x) - sin(x))*sqrt(sin(2*x)), x)
```

Giac [F]

$$\int (\cos(x) - \sin(x))\sqrt{\sin(2x)}\,dx = \int (\cos(x) - \sin(x))\sqrt{\sin(2x)}\,dx$$

```
[In] integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="giac")
[Out] integrate((cos(x) - sin(x))*sqrt(sin(2*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (\cos(x) - \sin(x))\sqrt{\sin(2x)}\,dx = \int \sqrt{\sin(2x)}\,(\cos(x) - \sin(x))\,dx$$

```
[In] int(sin(2*x)^(1/2)*(cos(x) - sin(x)),x)
[Out] int(sin(2*x)^(1/2)*(cos(x) - sin(x)), x)
```

3.406 $\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)} dx = -\frac{1}{16}\arcsin(\cos(x)-\sin(x)) + \frac{1}{16}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) + \frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2x)}}$$

```
[Out] -1/16*arcsin(cos(x)-sin(x))+1/16*ln(cos(x)+sin(x)+sin(2*x)^(1/2))+1/5*sin(x
)^5/sin(2*x)^(5/2)-1/4*sin(x)/sin(2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4379, 4393, 4390}

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)} dx = -\frac{1}{16}\arcsin(\cos(x)-\sin(x)) + \frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2x)}} + \frac{1}{16}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

```
[In] Int[Sin[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] -1/16*ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/16 +
Sin[x]^5/(5*Sin[2*x]^(5/2)) - Sin[x]/(4*Sqrt[Sin[2*x]])
```

Rule 4379

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(p + 1))), x] + Dist[e^4*((m + p - 1)/(4*g^2*(p + 1))), Int[(e*
Sin[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d
, e, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m,
2] && LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{1}{4}\int \frac{\sin^3(x)}{\sin^{\frac{3}{2}}(2x)}\,dx \\
&= \frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2x)}} + \frac{1}{16}\int \csc(x)\sqrt{\sin(2x)}\,dx \\
&= \frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2x)}} + \frac{1}{8}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx \\
&= -\frac{1}{16}\arcsin(\cos(x)-\sin(x)) \\
&\quad + \frac{1}{16}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) + \frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \frac{1}{80}\Big(5\Big(-\arcsin(\cos(x)-\sin(x))+\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\Big) + 2\sec(x)\left(-6+\sec^2(x)\right)\sqrt{\sin(2x)}\Big)$$

```
[In] Integrate[Sin[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] (5*(-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]) + 2*S
ec[x]*(-6 + Sec[x]^2)*Sqrt[Sin[2*x]])/80
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 6.09 (sec) , antiderivative size = 510, normalized size of antiderivative = 8.36

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,(\tan^2(\frac{x}{2})-1)\left(5\sqrt{1+\tan(\frac{x}{2})}\,\sqrt{-2\tan(\frac{x}{2})+2}\,\sqrt{-\tan(\frac{x}{2})}\,F\left(\sqrt{1+\tan(\frac{x}{2})},\frac{\sqrt{2}}{2}\right)(\tan^{14}(\frac{x}{2}))+35\sqrt{1+\tan(\frac{x}{2})}\,\sqrt{-2\mathrm{ta}\right.}{}$

```
[In] int(sin(x)^7/sin(2*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2688*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(5*(1+tan(1/2*
x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*
x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^14+35*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)
+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*t
an(1/2*x)^12+10*tan(1/2*x)^15+105*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1
/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2
*x)^10+66*tan(1/2*x)^13+175*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-
tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^8-
1014*tan(1/2*x)^11+175*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1
/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^6+2002*
tan(1/2*x)^9+105*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))
^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^4-2002*tan(1/
2*x)^7+35*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*
EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2+1014*tan(1/2*x)^5+
5*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Elliptic
F((1+tan(1/2*x))^(1/2),1/2*2^(1/2))-66*tan(1/2*x)^3-10*tan(1/2*x))/(tan(1/2
*x)*(tan(1/2*x)^2-1))^(1/2)/(1+tan(1/2*x)^2)^7/(tan(1/2*x)^3-tan(1/2*x))^(1
/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(47) = 94$.

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.97

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \frac{10 \arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)\cos(x)^3 - 10 \arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right)\cos}{}$$

```
[In] integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/320*(10*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*s
in(x))/(cos(x)^2 + 2*cos(x)*sin(x) - 1))*cos(x)^3 - 10*arctan(-(2*sqrt(2)*s
qrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x) - sin(x)))*cos(x)^3 - 5*cos(x
)^3*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*
cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1) - 48*cos(
x)^3 - 8*sqrt(2)*(6*cos(x)^2 - 1)*sqrt(cos(x)*sin(x)))/cos(x)^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(x)**7/sin(2*x)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\sin(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)^7/sin(2*x)^(7/2), x)
```

Giac [F]

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\sin(x)^7}{\sin(2\,x)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="giac")
[Out] integrate(sin(x)^7/sin(2*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\sin(x)^7}{\sin(2\,x)^{7/2}}\,dx$$

```
[In] int(sin(x)^7/sin(2*x)^(7/2),x)
[Out] int(sin(x)^7/sin(2*x)^(7/2), x)
```

3.407 $\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = -\frac{1}{16}\arcsin(\cos(x)-\sin(x)) - \frac{1}{16}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) - \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} + \frac{\cos(x)}{4\sqrt{\sin(2x)}}$$

```
[Out] -1/16*arcsin(cos(x)-sin(x))-1/16*ln(cos(x)+sin(x)+sin(2*x)^(1/2))-1/5*cos(x
)^5/sin(2*x)^(5/2)+1/4*cos(x)/sin(2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4378, 4392, 4391}

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = -\frac{1}{16}\arcsin(\cos(x)-\sin(x)) - \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} + \frac{\cos(x)}{4\sqrt{\sin(2x)}} - \frac{1}{16}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

```
[In] Int[Cos[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] -1/16*ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/16 -
Cos[x]^5/(5*Sin[2*x]^(5/2)) + Cos[x]/(4*Sqrt[Sin[2*x]])
```

Rule 4378

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(p + 1))), x] + Dist[e^4*((m + p - 1)/(4*g^2*(p + 1))), Int[(e*Cos
[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e
, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 2]
&& LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

Rule 4391

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4392

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/cos[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{1}{4}\int \frac{\cos^3(x)}{\sin^{\frac{3}{2}}(2x)}\,dx \\
&= -\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} + \frac{\cos(x)}{4\sqrt{\sin(2x)}} + \frac{1}{16}\int \sec(x)\sqrt{\sin(2x)}\,dx \\
&= -\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} + \frac{\cos(x)}{4\sqrt{\sin(2x)}} + \frac{1}{8}\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx \\
&= -\frac{1}{16}\arcsin(\cos(x)-\sin(x)) \\
&\quad -\frac{1}{16}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) - \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} + \frac{\cos(x)}{4\sqrt{\sin(2x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \frac{1}{16}\Big(-\arcsin(\cos(x)-\sin(x)) - \log\Big(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\Big)\Big) + \left(\frac{3\csc(x)}{20} - \frac{\csc^3(x)}{40}\right)\sqrt{\sin(2x)}$$

```
[In] Integrate[Cos[x]^7/Sin[2*x]^(7/2),x]
[Out] (-ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/16 + ((3
*Csc[x])/20 - Csc[x]^3/40)*Sqrt[Sin[2*x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.73 (sec) , antiderivative size = 1108, normalized size of antiderivative = 18.16

method	result	size
default	Expression too large to display	1108

```
[In] int(cos(x)^7/sin(2*x)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/160*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(192*(tan(1/2*x)*(t
an(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/
2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*(ta
n(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^6-96*(tan(1/2*x)*(tan(1/2*x)^2-1))
^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Ell
ipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan
(1/2*x))^(1/2)*tan(1/2*x)^6-(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2
*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^10+96*(tan(1/2*x)*(tan(1/2
*x)^2-1))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^8-384*(tan(1/2*x
)*(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-ta
n(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))
*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^4+192*(tan(1/2*x)*(tan(1/2*x)^
2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2
)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*(tan(1/2*x)-1
)*tan(1/2*x))^(1/2)*tan(1/2*x)^4+3*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+
tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^8+48*(tan(1/2*x)^3-
tan(1/2*x))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*
x)^8-192*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2
)*tan(1/2*x)^6+192*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)
*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2)
```

```
,1/2*2^(1/2))*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2
-96*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)
+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(
(1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2+14*(tan(1/2*x)
*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*t
an(1/2*x)^6-144*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)
-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^6+96*tan(1/2*x)^4*(tan(1/2*x)^3-tan(1/2*x)
)^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)+14*tan(1/2*x)^4*(tan(1/2*x)*(ta
n(1/2*x)^2-1))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)+144*(
tan(1/2*x)^3-tan(1/2*x))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(
1/2)*tan(1/2*x)^4+3*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2*x))*(ta
n(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2-48*(tan(1/2*x)^3-tan(1/2*x))^(1/
2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2-(tan(1/2*x
)*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2))
/(tan(1/2*x)^2-1)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/(tan(1/2*x)-1)/((1+tan(1/
2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)/(1+tan(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(47) = 94$.

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.36

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \frac{10\left(\cos(x)^2-1\right)\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)\sin(x)-10\left(\cos(x)^2-1\right)\arctan\left(-\frac{2\sqrt{}}{}\right.}{}$$

```
[In] integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/320*(10*(cos(x)^2 - 1)*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin
(x)) + cos(x)*sin(x))/(cos(x)^2 + 2*cos(x)*sin(x) - 1))*sin(x) - 10*(cos(x)
^2 - 1)*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x) -
 sin(x)))*sin(x) + 5*(cos(x)^2 - 1)*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos(x)^
3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 +
 16*cos(x)*sin(x) + 1)*sin(x) + 8*sqrt(2)*(6*cos(x)^2 - 5)*sqrt(cos(x)*sin(
x)) + 48*(cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \text{Timed out}$$

```
[In] integrate(cos(x)**7/sin(2*x)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\cos(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="maxima")
[Out] integrate(cos(x)^7/sin(2*x)^(7/2), x)
```

Giac [F]

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\cos(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="giac")
[Out] integrate(cos(x)^7/sin(2*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\cos(x)^7}{\sin(2x)^{7/2}}\,dx$$

```
[In] int(cos(x)^7/sin(2*x)^(7/2),x)
[Out] int(cos(x)^7/sin(2*x)^(7/2), x)
```

3.408 $\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = -\frac{1}{5}\csc^5(x)\sin^{\frac{5}{2}}(2x)$$

```
[Out] -1/5*csc(x)^5*sin(2*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{4377\}$

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = -\frac{1}{5}\sin^{\frac{5}{2}}(2x)\csc^5(x)$$

```
[In] Int[Csc[x]^5*Sin[2*x]^(3/2),x]
```

```
[Out] -1/5*(Csc[x]^5*Sin[2*x]^(5/2))
```

Rule 4377

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{5}\csc^5(x)\sin^{\frac{5}{2}}(2x)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = -\frac{1}{5}\csc^5(x)\sin^{\frac{5}{2}}(2x)$$

[In] Integrate[Csc[x]^5*Sin[2*x]^(3/2),x]

[Out] -1/5*(Csc[x]^5*Sin[2*x]^(5/2))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.65 (sec) , antiderivative size = 508, normalized size of antiderivative = 31.75

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\left(96\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)-1\right)}\sqrt{1+\tan\left(\frac{x}{2}\right)}\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\sqrt{-\tan\left(\frac{x}{2}\right)}\,E\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)\sqrt{\left(1+\tan\left(\frac{x}{2}\right)\right)\left(\tan\left(\frac{x}{2}\right)-1\right)}\right.}{}$

[In] int(sin(2*x)^(3/2)/sin(x)^5,x,method=_RETURNVERBOSE)

[Out] 1/5*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(96*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2-48*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2-(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^6+40*tan(1/2*x)^4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)+tan(1/2*x)^4*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)+28*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^4+(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2-28*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2-(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = \frac{4\left(\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x)^2 + \left(\cos(x)^2 - 1\right)\sin(x)\right)}{5\left(\cos(x)^2 - 1\right)\sin(x)}$$

```
[In] integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="fricas")
[Out] 4/5*(sqrt(2)*sqrt(cos(x)*sin(x))*cos(x)^2 + (cos(x)^2 - 1)*sin(x))/((cos(x)
^2 - 1)*sin(x))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = \text{Timed out}$$

```
[In] integrate(sin(2*x)**(3/2)/sin(x)**5,x)
[Out] Timed out
```

Maxima [F]

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = \int \frac{\sin(2x)^{\frac{3}{2}}}{\sin(x)^5}\,dx$$

```
[In] integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="maxima")
[Out] integrate(sin(2*x)^(3/2)/sin(x)^5, x)
```

Giac [F]

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = \int \frac{\sin(2x)^{\frac{3}{2}}}{\sin(x)^5}\,dx$$

```
[In] integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="giac")
[Out] integrate(sin(2*x)^(3/2)/sin(x)^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = \frac{4\sqrt{\sin(2x)}\left(\sin(x)^2-1\right)}{5\sin(x)^3}$$

```
[In] int(sin(2*x)^(3/2)/sin(x)^5,x)
[Out] (4*sin(2*x)^(1/2)*(sin(x)^2 - 1))/(5*sin(x)^3)
```

3.409 $\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \frac{4}{5}\sec(x)\sqrt{\sin(2x)}+\frac{1}{5}\sec^3(x)\sqrt{\sin(2x)}$$

```
[Out] 4/5*sec(x)*sin(2*x)^(1/2)+1/5*sec(x)^3*sin(2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4384, 4376}

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \frac{1}{5}\sqrt{\sin(2x)}\sec^3(x)+\frac{4}{5}\sqrt{\sin(2x)}\sec(x)$$

```
[In] Int[Sec[x]^3/Sqrt[Sin[2*x]],x]
```

```
[Out] (4*Sec[x]*Sqrt[Sin[2*x]])/5 + (Sec[x]^3*Sqrt[Sin[2*x]])/5
```

Rule 4376

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4384

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
```

```
g*(m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Cos[a +
b*x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[
m + 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{5}\sec^3(x)\sqrt{\sin(2x)}+\frac{4}{5}\int\frac{\sec(x)}{\sqrt{\sin(2x)}}\,dx\\ &= \frac{4}{5}\sec(x)\sqrt{\sin(2x)}+\frac{1}{5}\sec^3(x)\sqrt{\sin(2x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int\frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx=\frac{1}{5}\sec(x)\left(4+\sec^2(x)\right)\sqrt{\sin(2x)}$$

```
[In] Integrate[Sec[x]^3/Sqrt[Sin[2*x]],x]
```

```
[Out] (Sec[x]*(4 + Sec[x]^2)*Sqrt[Sin[2*x]])/5
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 4.94 (sec) , antiderivative size = 2946, normalized size of antiderivative = 95.03

method	result	size
default	Expression too large to display	2946

```
[In] int(1/cos(x)^3/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(-56*tan(1/2*x)-96*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(
1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2+8*ta
n(1/2*x)^3-8*tan(1/2*x)^5+56*tan(1/2*x)^7-32*(1+tan(1/2*x))^(1/2)*(-2*tan(1
/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/
2))+6*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Elli
pticPi((1+tan(1/2*x))^(1/2),1/2-1/2*I,1/2*2^(1/2))+6*(1+tan(1/2*x))^(1/2)*(
-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),
1/2+1/2*I,1/2*2^(1/2))+12*ln(1/tan(1/2*x)*(tan(1/2*x)^2+2*(tan(1/2*x)^3-tan
(1/2*x))^(1/2)+2*tan(1/2*x)-1))*(tan(1/2*x)^3-tan(1/2*x))^(3/2)-24*arctan((
(tan(1/2*x)^3-tan(1/2*x))^(1/2)+tan(1/2*x))/tan(1/2*x))*(tan(1/2*x)^3-tan(1
/2*x))^(3/2)-12*ln(-1/tan(1/2*x)*(-tan(1/2*x)^2+2*(tan(1/2*x)^3-tan(1/2*x))
```

^(1/2)-2*tan(1/2*x)+1))*(tan(1/2*x)^3-tan(1/2*x))^(3/2)-24*arctan(((tan(1/2*x)^3-tan(1/2*x))^(1/2)-tan(1/2*x))/tan(1/2*x))*(tan(1/2*x)^3-tan(1/2*x))^(3/2)-3*ln(1/tan(1/2*x)*(tan(1/2*x)^2+2*(tan(1/2*x)^3-tan(1/2*x))^(1/2)+2*tan(1/2*x)-1))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)+6*arctan(((tan(1/2*x)^3-tan(1/2*x))^(1/2)+tan(1/2*x))/tan(1/2*x))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)+3*ln(-1/tan(1/2*x)*(-tan(1/2*x)^2+2*(tan(1/2*x)^3-tan(1/2*x))^(1/2)-2*tan(1/2*x)+1))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)+6*arctan(((tan(1/2*x)^3-tan(1/2*x))^(1/2)-tan(1/2*x))/tan(1/2*x))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)+6*I*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-6*I*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+6*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2-1/2*I,1/2*2^(1/2))*tan(1/2*x)^6+6*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2+1/2*I,1/2*2^(1/2))*tan(1/2*x)^6+18*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2-1/2*I,1/2*2^(1/2))*tan(1/2*x)^4+18*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2+1/2*I,1/2*2^(1/2))*tan(1/2*x)^4+18*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2-1/2*I,1/2*2^(1/2))*tan(1/2*x)^2+18*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2+1/2*I,1/2*2^(1/2))*tan(1/2*x)^2-18*I*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2+1/2*I,1/2*2^(1/2))*tan(1/2*x)^2+6*I*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2-1/2*I,1/2*2^(1/2))*tan(1/2*x)^6-6*I*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2+1/2*I,1/2*2^(1/2))*tan(1/2*x)^6+18*I*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2-1/2*I,1/2*2^(1/2))*tan(1/2*x)^4-18*I*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2+1/2*I,1/2*2^(1/2))*tan(1/2*x)^4+18*I*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2-1/2*I,1/2*2^(1/2))*tan(1/2*x)^2-32*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^6-96*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^4-12*ln(1/tan(1/2*x)*(tan(1/2*x)^2+2*(tan(1/2*x)^3-tan(1/2*x))^(1/2)+2*tan(1/2*x)-1))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^5+24*arctan(((tan(1/2*x)^3-tan(1/2*x))^(1/2)+tan(1/2*x))/tan(1/2*x))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^5+12*ln(-1/tan(1/2*x)*(-tan(1/2*x)^2+2*(tan(1/2*x)^3-tan(1/2*x))^(1/2)-2*tan(1/2*x)+1))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^5+24*arctan(((tan(1/2*x)^3-tan(1/2*x))^(1/2)-tan(1/2*x))/tan(1/2*x))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^5-3*ln(1/tan(1/2*x)*(tan(1/2*x)^2+2*(tan(1/2*x)^3-tan(1/2*x))^(1/2)+2*tan(1/2*x)-1))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^6+6*arctan(((tan(1/2*x)^3-tan(1/2*x))^(1/2)+

```
tan(1/2*x))/tan(1/2*x))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^6+3*ln(-
1/tan(1/2*x)*(-tan(1/2*x)^2+2*(tan(1/2*x)^3-tan(1/2*x))^(1/2)-2*tan(1/2*x)+
1))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^6+6*arctan(((tan(1/2*x)^3-ta
n(1/2*x))^(1/2)-tan(1/2*x))/tan(1/2*x))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan
(1/2*x)^6+12*ln(1/tan(1/2*x)*(tan(1/2*x)^2+2*(tan(1/2*x)^3-tan(1/2*x))^(1/2
)+2*tan(1/2*x)-1))*(tan(1/2*x)^3-tan(1/2*x))^(3/2)*tan(1/2*x)^2-9*ln(1/tan(
1/2*x)*(tan(1/2*x)^2+2*(tan(1/2*x)^3-tan(1/2*x))^(1/2)+2*tan(1/2*x)-1))*(ta
n(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^4-24*arctan(((tan(1/2*x)^3-tan(1/2*
x))^(1/2)+tan(1/2*x))/tan(1/2*x))*(tan(1/2*x)^3-tan(1/2*x))^(3/2)*tan(1/2*x
)^2+18*arctan(((tan(1/2*x)^3-tan(1/2*x))^(1/2)+tan(1/2*x))/tan(1/2*x))*(tan
(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^4-12*ln(-1/tan(1/2*x)*(-tan(1/2*x)^2
+2*(tan(1/2*x)^3-tan(1/2*x))^(1/2)-2*tan(1/2*x)+1))*(tan(1/2*x)^3-tan(1/2*x
))^(3/2)*tan(1/2*x)^2+9*ln(-1/tan(1/2*x)*(-tan(1/2*x)^2+2*(tan(1/2*x)^3-tan
(1/2*x))^(1/2)-2*tan(1/2*x)+1))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^
4-24*arctan(((tan(1/2*x)^3-tan(1/2*x))^(1/2)-tan(1/2*x))/tan(1/2*x))*(tan(1
/2*x)^3-tan(1/2*x))^(3/2)*tan(1/2*x)^2+18*arctan(((tan(1/2*x)^3-tan(1/2*x))
^(1/2)-tan(1/2*x))/tan(1/2*x))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^4
-9*ln(1/tan(1/2*x)*(tan(1/2*x)^2+2*(tan(1/2*x)^3-tan(1/2*x))^(1/2)+2*tan(1/
2*x)-1))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^2+18*arctan(((tan(1/2*x
)^3-tan(1/2*x))^(1/2)+tan(1/2*x))/tan(1/2*x))*(tan(1/2*x)^3-tan(1/2*x))^(1/
2)*tan(1/2*x)^2+9*ln(-1/tan(1/2*x)*(-tan(1/2*x)^2+2*(tan(1/2*x)^3-tan(1/2*x
))^(1/2)-2*tan(1/2*x)+1))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^2+18*a
rctan(((tan(1/2*x)^3-tan(1/2*x))^(1/2)-tan(1/2*x))/tan(1/2*x))*(tan(1/2*x)^
3-tan(1/2*x))^(1/2)*tan(1/2*x)^2+12*ln(1/tan(1/2*x)*(tan(1/2*x)^2+2*(tan(1/
2*x)^3-tan(1/2*x))^(1/2)+2*tan(1/2*x)-1))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*t
an(1/2*x)-24*arctan(((tan(1/2*x)^3-tan(1/2*x))^(1/2)+tan(1/2*x))/tan(1/2*x)
)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)-12*ln(-1/tan(1/2*x)*(-tan(1/2*
x)^2+2*(tan(1/2*x)^3-tan(1/2*x))^(1/2)-2*tan(1/2*x)+1))*(tan(1/2*x)^3-tan(1
/2*x))^(1/2)*tan(1/2*x)-24*arctan(((tan(1/2*x)^3-tan(1/2*x))^(1/2)-tan(1/2*
x))/tan(1/2*x))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x))*(tan(1/2*x)^2-1
)*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/(-tan(1/2*x)^2+2*(tan(1/2*x)^3-tan(1
/2*x))^(1/2)-2*tan(1/2*x)+1)/(tan(1/2*x)^2+2*(tan(1/2*x)^3-tan(1/2*x))^(1/2
)+2*tan(1/2*x)-1)/(1+tan(1/2*x)^2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/(tan(1/2
*x)*(tan(1/2*x)^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \frac{4\,\cos\left(x\right)^{3} + \sqrt{2}\left(4\,\cos\left(x\right)^{2} + 1\right)\sqrt{\cos\left(x\right)\sin\left(x\right)}}{5\,\cos\left(x\right)^{3}}$$

```
[In] integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(4*cos(x)^3 + sqrt(2)*(4*cos(x)^2 + 1)*sqrt(cos(x)*sin(x)))/cos(x)^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/cos(x)**3/sin(2*x)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{1}{\cos(x)^3\sqrt{\sin(2x)}}\,dx$$

```
[In] integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(cos(x)^3*sqrt(sin(2*x))), x)
```

Giac [F]

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{1}{\cos(x)^3\sqrt{\sin(2x)}}\,dx$$

```
[In] integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(cos(x)^3*sqrt(sin(2*x))), x)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \frac{\sqrt{\sin(2x)}\,(2\cos(2x)+3)}{5\cos(x)^3}$$

```
[In] int(1/(sin(2*x)^(1/2)*cos(x)^3),x)
[Out] (sin(2*x)^(1/2)*(2*cos(2*x) + 3))/(5*cos(x)^3)
```

3.410 $\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = -\frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)} + \frac{4\sin(x)}{3\sqrt{\sin(2x)}}$$

```
[Out] -2/3*cos(x)/sin(2*x)^(3/2)+4/3*sin(x)/sin(2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4393, 4388, 4377}

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = \frac{4\sin(x)}{3\sqrt{\sin(2x)}} - \frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)}$$

```
[In] Int[Csc[x]/Sin[2*x]^(3/2),x]
```

```
[Out] (-2*Cos[x])/(3*Sin[2*x]^(3/2)) + (4*Sin[x])/(3*Sqrt[Sin[2*x]])
```

Rule 4377

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4388

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Dist
```

```
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{\cos(x)}{\sin^{\frac{5}{2}}(2x)}\,dx \\ &= -\frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)} + \frac{4}{3}\int \frac{\sin(x)}{\sin^{\frac{3}{2}}(2x)}\,dx \\ &= -\frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)} + \frac{4\sin(x)}{3\sqrt{\sin(2x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = \left(-\frac{1}{6}\cot(x)\csc(x) + \frac{\sec(x)}{2}\right)\sqrt{\sin(2x)}$$

```
[In] Integrate[Csc[x]/Sin[2*x]^(3/2),x]
[Out] (-1/6*(Cot[x]*Csc[x]) + Sec[x]/2)*Sqrt[Sin[2*x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 4.17

method	result	size
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^{2}\left(\frac{x}{2}\right)-1}}\,\left(\tan^{2}\left(\frac{x}{2}\right)-1\right)\left(2\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,F\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)\tan\left(\frac{x}{2}\right)-\left(\tan^{4}\left(\frac{x}{2}\right)\right)+1\right)}{12\tan\left(\frac{x}{2}\right)\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan^{2}\left(\frac{x}{2}\right)-1\right)}\,\sqrt{\tan^{3}\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)}}$	121

```
[In] int(1/sin(x)/sin(2*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/tan(1/2*x)*(2*(
1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((
1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)-tan(1/2*x)^4+1)/(tan(1/2*x)*(ta
n(1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = \frac{4\,\cos\left(x\right)^{3} + \sqrt{2}\left(4\,\cos\left(x\right)^{2} - 3\right)\sqrt{\cos\left(x\right)\sin\left(x\right)} - 4\,\cos\left(x\right)}{6\,\left(\cos\left(x\right)^{3} - \cos\left(x\right)\right)}$$

```
[In] integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*cos(x)^3 + sqrt(2)*(4*cos(x)^2 - 3)*sqrt(cos(x)*sin(x)) - 4*cos(x))/
(cos(x)^3 - cos(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = \text{Timed out}$$

```
[In] integrate(1/sin(x)/sin(2*x)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = \int \frac{1}{\sin\left(2\,x\right)^{\frac{3}{2}}\sin\left(x\right)}\,dx$$

```
[In] integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sin(2*x)^(3/2)*sin(x)), x)
```

Giac [F]

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = \int \frac{1}{\sin\left(2\,x\right)^{\frac{3}{2}}\sin\left(x\right)}\,dx$$

```
[In] integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/(sin(2*x)^(3/2)*sin(x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = -\frac{\sqrt{\sin\left(2\,x\right)}\,\left(2\,\cos\left(2\,x\right)-1\right)}{6\,\left(\cos\left(x\right)-{\cos\left(x\right)}^3\right)}$$

```
[In] int(1/(sin(2*x)^(3/2)*sin(x)),x)
[Out] -(sin(2*x)^(1/2)*(2*cos(2*x) - 1))/(6*(cos(x) - cos(x)^3))
```

3.411 $\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 68

$$\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \frac{33}{32}\operatorname{arctanh}\left(\frac{1}{2}\sec(x)\sqrt{\sin(2x)}\right)-\frac{9\cos(x)}{16\sqrt{\sin(2x)}} -\frac{5\cos(x)\cot(x)}{24\sqrt{\sin(2x)}}+\frac{\cos(x)\cot^2(x)}{20\sqrt{\sin(2x)}}$$

```
[Out] 33/32*arctanh(1/2*sin(2*x)^(1/2)/cos(x))-9/16*cos(x)/sin(2*x)^(1/2)-5/24*co
s(x)*cot(x)/sin(2*x)^(1/2)+1/20*cos(x)*cot(x)^2/sin(2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4475, 1633, 65, 213}

$$\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \frac{33\sin^5(x)\operatorname{arctanh}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}+\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} -\frac{5\sin(x)\cos^4(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{9\sin^2(x)\cos^3(x)}{4\sin^{\frac{5}{2}}(2x)}$$

```
[In] Int[(Cos[x]^3*(Cos[2*x] - 3*Tan[x]))/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2))
,x]
```

```
[Out] Cos[x]^5/(5*Sin[2*x]^(5/2)) - (5*Cos[x]^4*Sin[x])/(6*Sin[2*x]^(5/2)) - (9*C
os[x]^3*Sin[x]^2)/(4*Sin[2*x]^(5/2)) + (33*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]]*Si
n[x]^5)/(4*Sqrt[2]*Sin[2*x]^(5/2)*Tan[x]^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1633

```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_)), x_Symbol] :> I
nt[ExpandIntegrand[1/Sqrt[c + d*x], Px*((c + d*x)^(n + 1/2)/(a + b*x)), x],
 x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0] && GtQ
[Expon[Px, x], 2]
```

Rule 4475

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[u*(Sin
[v/2]^(2*m)/(c*Tan[v/2])^m), x]}, Dist[(c*Sin[v])^m*((c*Tan[v/2])^m/Sin[v/2
]^(2*m)), Int[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sin^5(x)\int \frac{\csc^2(x)(\cos(2x)-3\tan(x))}{(\sin^2(x)-\sin(2x))\sqrt{\tan(x)}}\,dx}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\sin^5(x)\text{Subst}\left(\int \frac{-1+3x+x^2+3x^3}{(2-x)x^{7/2}}\,dx,x,\tan(x)\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\sin^5(x)\text{Subst}\left(\int \left(-\frac{1}{2x^{7/2}}+\frac{5}{4x^{5/2}}+\frac{9}{8x^{3/2}}-\frac{33}{8(-2+x)\sqrt{x}}\right)\,dx,x,\tan(x)\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{5\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{9\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)}\\
&\quad-\frac{(33\sin^5(x))\,\text{Subst}\left(\int \frac{1}{(-2+x)\sqrt{x}}\,dx,x,\tan(x)\right)}{8\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}
\end{aligned}
$$

$$= \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{5\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)} - \frac{9\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)} - \frac{(33\sin^5(x))\,\mathrm{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{\tan(x)}\right)}{4\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$= \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{5\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)} - \frac{9\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)} + \frac{33\mathrm{arctanh}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin^5(x)}{4\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

Mathematica [A] (verified)

Time = 7.74 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.63

$$\int \frac{\cos^3(x)(\cos(2x) - 3\tan(x))}{(\sin^2(x) - \sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \frac{\cos(x)\sqrt{\sin(2x)}\left(\frac{1}{15}\csc(x)\left(-147 - 50\cot(x) + 12\csc^2(x)\right) + \frac{33\arctan\left(\frac{\sqrt{\tan\left(\frac{x}{2}\right)}}{\sqrt{-1+\tan^2\left(\frac{x}{2}\right)}}\right)\sqrt{-\frac{\cos(x)}{2+2\cos(x)}}\sec(x)}{\sqrt{\tan\left(\frac{x}{2}\right)}}\right)(\cos(2}{16(\cos(x) + \cos(3x) - 6\sin(x))}$$

```
[In] Integrate[(Cos[x]^3*(Cos[2*x] - 3*Tan[x]))/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^
(5/2)),x]
```

```
[Out] (Cos[x]*Sqrt[Sin[2*x]]*((Csc[x]*(-147 - 50*Cot[x] + 12*Csc[x]^2))/15 + (33*
ArcTan[Sqrt[Tan[x/2]]/Sqrt[-1 + Tan[x/2]^2]]*Sqrt[-(Cos[x]/(2 + 2*Cos[x]))]
*Sec[x])/Sqrt[Tan[x/2]])*(Cos[2*x] - 3*Tan[x]))/(16*(Cos[x] + Cos[3*x] - 6*
Sin[x]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.31 (sec) , antiderivative size = 761, normalized size of antiderivative = 11.19

method	result	size
default	Expression too large to display	761

```
[In] int(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/3840*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(-3024*(tan(1/2*x)
*(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan
(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*
```

```
(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2+932*(tan(1/2*x)*(tan(1/2*x)^2
-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)
*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*(tan(1/2*x)-1)
*tan(1/2*x))^(1/2)*tan(1/2*x)^2+24*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+
tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^6+3*2^(1/2)*(tan(1/
2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((1+tan(1/2*x)
)*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*sum((34*_alpha^3+13*_alpha^2+34*_alpha-2
1)*(_alpha^3+2*_alpha-3)*(1+tan(1/2*x))^(1/2)*(-tan(1/2*x)+1)^(1/2)*(-tan(1
/2*x))^(1/2)/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*EllipticPi((1+tan(1/2*x))^
(1/2),-1/4*_alpha^3-1/2*_alpha+3/4,1/2*2^(1/2)),_alpha=RootOf(_Z^4+_Z^3+2*_
Z^2-_Z+1))*tan(1/2*x)^2+200*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2
*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^5-1920*tan(1/2*x)^4*(tan(1
/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)-24*tan(1/2*x)
^4*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1
/2*x))^(1/2)-552*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x
)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^4-24*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*
((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2+552*(tan(1/2*
x)^3-tan(1/2*x))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan
(1/2*x)^2-200*tan(1/2*x)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2*x)
)*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)+24*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(
(1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^
(1/2)/((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(52) = 104$.

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.00

$$\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = -\frac{495\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}(4\cos(x)+3\sin(x))+\frac{1}{2}\cos(x)^2+\frac{7}{2}\cos(x)\sin(x)+\right.}{}$$

```
[In] integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x
, algorithm="fricas")
```

```
[Out] -1/1920*(495*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x)
+ 3*sin(x)) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2)*sin(x) - 495*(cos(x)^
2 - 1)*log(1/2*cos(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(
x)*sin(x) + 1/2)*sin(x) + 4*sqrt(2)*(147*cos(x)^2 - 50*cos(x)*sin(x) - 135)
*sqrt(cos(x)*sin(x)) + 388*(cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)(\cos(2x) - 3\tan(x))}{(\sin^2(x) - \sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \text{Timed out}$$

```
[In] integrate(cos(x)**3*(cos(2*x)-3*tan(x))/(sin(x)**2-sin(2*x))/sin(2*x)**(5/2
),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)(\cos(2x) - 3\tan(x))}{(\sin^2(x) - \sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \text{Timed out}$$

```
[In] integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x
, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\cos^3(x)(\cos(2x) - 3\tan(x))}{(\sin^2(x) - \sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \int \frac{(\cos(2x) - 3\tan(x))\cos(x)^3}{(\sin(x)^2 - \sin(2x))\sin(2x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x
, algorithm="giac")
[Out] integrate((cos(2*x) - 3*tan(x))*cos(x)^3/((sin(x)^2 - sin(2*x))*sin(2*x)^(5
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)(\cos(2x) - 3\tan(x))}{(\sin^2(x) - \sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \int -\frac{\cos(x)^3(\cos(2x) - 3\tan(x))}{\sin(2x)^{5/2}(\sin(2x) - \sin(x)^2)}\,dx$$

```
[In] int(-(cos(x)^3*(cos(2*x) - 3*tan(x)))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)
),x)
[Out] int(-(cos(x)^3*(cos(2*x) - 3*tan(x)))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)
), x)
```

3.412 $\int \sqrt{\sec^4(x)\tan(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \frac{2}{3}\cos(x)\sin(x)\sqrt{\sec^4(x)\tan(x)}$$

```
[Out] 2/3*cos(x)*sin(x)*(sec(x)^4*tan(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2024, 1968, 1264, 30}

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \frac{2\tan^2(x)\sec^2(x)}{3\sqrt{\tan^5(x)+2\tan^3(x)+\tan(x)}}$$

```
[In] Int[Sqrt[Sec[x]^4*Tan[x]],x]
[Out] (2*Sec[x]^2*Tan[x]^2)/(3*Sqrt[Tan[x] + 2*Tan[x]^3 + Tan[x]^5])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
```

```
a*c, 0] &&  !IntegerQ[p]
```

Rule 1968

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.))^(p_
)*((A_) + (B_.)*(x_)^(q_)), x_Symbol] :> Dist[(a*x^j + b*x^k + c*x^n)^p/(x^
(j*p)*(a + b*x^(k - j) + c*x^(2*(k - j)))^p), Int[x^(m + j*p)*(A + B*x^(k -
 j))*(a + b*x^(k - j) + c*x^(2*(k - j)))^p, x], x] /; FreeQ[{a, b, c, A, B,
 j, k, m, p}, x] && EqQ[q, k - j] && EqQ[n, 2*k - j] &&  !IntegerQ[p] && Po
sQ[k - j]
```

Rule 2024

```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_), x_Symbol] :> Int[(f*x)^m*ExpandToSu
m[z, x]*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p}, x] && BinomialQ[z, x] &
& GeneralizedTrinomialQ[u, x] && EqQ[BinomialDegree[z, x] - GeneralizedTrin
omialDegree[u, x], 0] &&  !(BinomialMatchQ[z, x] && GeneralizedTrinomialMat
chQ[u, x])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x(1+x^2)}{\sqrt{x\left(1+x^2\right)^2}}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \frac{x(1+x^2)}{\sqrt{x+2x^3+x^5}}\,dx, x, \tan(x)\right)\\
&= \frac{\left(\sqrt{\tan(x)}\sqrt{1+2\tan^2(x)+\tan^4(x)}\right)\text{Subst}\left(\int \frac{\sqrt{x}(1+x^2)}{\sqrt{1+2x^2+x^4}}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)+2\tan^3(x)+\tan^5(x)}}\\
&= \frac{\left(\sec^2(x)\sqrt{\tan(x)}\right)\text{Subst}\left(\int \sqrt{x}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)+2\tan^3(x)+\tan^5(x)}}\\
&= \frac{2\sec^2(x)\tan^2(x)}{3\sqrt{\tan(x)+2\tan^3(x)+\tan^5(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \frac{2}{3}\cos(x)\sin(x)\sqrt{\sec^4(x)\tan(x)}$$

```
[In] Integrate[Sqrt[Sec[x]^4*Tan[x]],x]
[Out] (2*Cos[x]*Sin[x]*Sqrt[Sec[x]^4*Tan[x]])/3
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{2\cos(x)\sin(x)\sqrt{(\sec^4(x))\tan(x)}}{3}$	16

```
[In] int((sin(x)/cos(x)^5)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*cos(x)*sin(x)*(sec(x)^4*tan(x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \frac{2}{3}\sqrt{\frac{\sin(x)}{\cos(x)^5}}\cos(x)\sin(x)$$

```
[In] integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(sin(x)/cos(x)^5)*cos(x)*sin(x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \text{Timed out}$$

```
[In] integrate((sin(x)/cos(x)**5)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.32

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \frac{2}{3}\tan(x)^{\frac{3}{2}}$$

```
[In] integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="maxima")
[Out] 2/3*tan(x)^(3/2)
```

Giac [F]

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \int \sqrt{\frac{\sin(x)}{\cos(x)^5}}\,dx$$

```
[In] integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sin(x)/cos(x)^5), x)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \frac{\sin(2x)\sqrt{\frac{\sin(x)}{\cos(x)^5}}}{3}$$

```
[In] int((sin(x)/cos(x)^5)^(1/2),x)
[Out] (sin(2*x)*(sin(x)/cos(x)^5)^(1/2))/3
```

3.413 $\int \sqrt{\sin^4(x)\tan(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 92

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \frac{3\arctan\left(\frac{(1-\cot(x))\csc^2(x)\sqrt{\sin^4(x)\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}} + \frac{3\log\left(\cos(x)+\sin(x)-\sqrt{2}\cot(x)\csc(x)\sqrt{\sin^4(x)\tan(x)}\right)}{4\sqrt{2}} - \frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)}$$

```
[Out] 3/8*arctan(1/2*(1-cot(x))*csc(x)^2*(sin(x)^4*tan(x))^(1/2)*2^(1/2))*2^(1/2)
+3/8*ln(cos(x)+sin(x)-cot(x)*csc(x)*2^(1/2)*(sin(x)^4*tan(x))^(1/2))*2^(1/2
)-1/2*cot(x)*(sin(x)^4*tan(x))^(1/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 204 vs. 2(92) = 184.

Time = 0.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.22, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {6851, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = -\frac{3\sec^2(x)\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)\sqrt{\sin^4(x)\tan(x)}}{4\sqrt{2}\tan^{\frac{5}{2}}(x)} + \frac{3\sec^2(x)\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)\sqrt{\sin^4(x)\tan(x)}}{4\sqrt{2}\tan^{\frac{5}{2}}(x)} - \frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} + \frac{3\sec^2(x)\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)\sqrt{\sin^4(x)\tan(x)}}{8\sqrt{2}\tan^{\frac{5}{2}}(x)} - \frac{3\sec^2(x)\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)\sqrt{\sin^4(x)\tan(x)}}{8\sqrt{2}\tan^{\frac{5}{2}}(x)}$$

```
[In] Int[Sqrt[Sin[x]^4*Tan[x]],x]
```

```
[Out] -1/2*(Cot[x]*Sqrt[Sin[x]^4*Tan[x]]) - (3*ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]*S
ec[x]^2*Sqrt[Sin[x]^4*Tan[x]])/(4*Sqrt[2]*Tan[x]^(5/2)) + (3*ArcTan[1 + Sqr
t[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Sin[x]^4*Tan[x]])/(4*Sqrt[2]*Tan[x]^(5/2))
 + (3*Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Sin[x]^4*Tan[x]]
)/(8*Sqrt[2]*Tan[x]^(5/2)) - (3*Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[
x]^2*Sqrt[Sin[x]^4*Tan[x]])/(8*Sqrt[2]*Tan[x]^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\sqrt{\frac{x^5}{(1+x^2)^2}}}{1+x^2}\,dx, x, \tan(x)\right)$$

$$= \frac{\left(\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{x^{5/2}}{(1+x^2)^2}\,dx, x, \tan(x)\right)}{\tan^{\frac{5}{2}}(x)}$$

$$= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} + \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(x)\right)}{4\tan^{\frac{5}{2}}(x)}$$

$$= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} + \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)}{2\tan^{\frac{5}{2}}(x)}$$

$$= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} - \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)}{4\tan^{\frac{5}{2}}(x)} + \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)}{4\tan^{\frac{5}{2}}(x)}$$

$$= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} + \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(x)}\right)}{8\tan^{\frac{5}{2}}(x)} + \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(x)}\right)}{8\tan^{\frac{5}{2}}(x)} + \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(x)}\right)}{8\sqrt{2}\tan^{\frac{5}{2}}(x)} + \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(x)}\right)}{8\sqrt{2}\tan^{\frac{5}{2}}(x)}$$

$$
\begin{aligned}
&= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} \\
&\quad + \frac{3\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)\sec^2(x)\sqrt{\sin^4(x)\tan(x)}}{8\sqrt{2}\tan^{\frac{5}{2}}(x)} \\
&\quad - \frac{3\log\left(1+\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)\sec^2(x)\sqrt{\sin^4(x)\tan(x)}}{8\sqrt{2}\tan^{\frac{5}{2}}(x)} \\
&\quad + \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(x)}\right)}{4\sqrt{2}\tan^{\frac{5}{2}}(x)} \\
&\quad - \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(x)}\right)}{4\sqrt{2}\tan^{\frac{5}{2}}(x)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} - \frac{3\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)\sec^2(x)\sqrt{\sin^4(x)\tan(x)}}{4\sqrt{2}\tan^{\frac{5}{2}}(x)} \\
&\quad + \frac{3\arctan\left(1+\sqrt{2}\sqrt{\tan(x)}\right)\sec^2(x)\sqrt{\sin^4(x)\tan(x)}}{4\sqrt{2}\tan^{\frac{5}{2}}(x)} \\
&\quad + \frac{3\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)\sec^2(x)\sqrt{\sin^4(x)\tan(x)}}{8\sqrt{2}\tan^{\frac{5}{2}}(x)} \\
&\quad - \frac{3\log\left(1+\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)\sec^2(x)\sqrt{\sin^4(x)\tan(x)}}{8\sqrt{2}\tan^{\frac{5}{2}}(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72

$$
\begin{aligned}
\int\sqrt{\sin^4(x)\tan(x)}\,dx = -\frac{1}{8}\csc^3(x)\Big(&3\arcsin(\cos(x)-\sin(x)) \\
&+ 3\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) \\
&+ 2\sin(x)\sqrt{\sin(2x)}\Big)\sqrt{\sin(2x)}\sqrt{\sin^4(x)\tan(x)}
\end{aligned}
$$

```
[In] Integrate[Sqrt[Sin[x]^4*Tan[x]],x]
```

```
[Out] -1/8*(Csc[x]^3*(3*ArcSin[Cos[x] - Sin[x]] + 3*Log[Cos[x] + Sin[x] + Sqrt[Si
n[2*x]]] + 2*Sin[x]*Sqrt[Sin[2*x]])*Sqrt[Sin[2*x]]*Sqrt[Sin[x]^4*Tan[x]])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(73) = 146$.

Time = 7.83 (sec) , antiderivative size = 299, normalized size of antiderivative = 3.25

method	result
default	$\frac{\left(4\cos(x)\sin(x)\sqrt{2}\,\sqrt{-\frac{\cos(x)\sin(x)}{(\cos(x)+1)^2}}+4\sqrt{2}\,\sqrt{-\frac{\cos(x)\sin(x)}{(\cos(x)+1)^2}}\,\sin(x)-3\ln\left(-\frac{\cos(x)\cot(x)-2\cot(x)-2\sin(x)\sqrt{-\left(\cot^3(x)\right)+3\csc(x)\left(\cot^2(x)\right)-\cdots}}{-1\cdots}\right.\right.}{}$

```
[In] int((sin(x)^5/cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(4*cos(x)*sin(x)*2^(1/2)*(-cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)+4*2^(1/2)
*(-cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)*sin(x)-3*ln(-(cos(x)*cot(x)-2*cot(x)-2
*sin(x)*(-cot(x)^3+3*csc(x)*cot(x)^2-3*cot(x)*csc(x)^2+csc(x)^3-csc(x)+cot(
x))^(1/2)-2*cos(x)-sin(x)+csc(x)+2)/(-1+cos(x)))+3*ln(-(cos(x)*cot(x)-2*cot
(x)+2*sin(x)*(-cot(x)^3+3*csc(x)*cot(x)^2-3*cot(x)*csc(x)^2+csc(x)^3-csc(x)
+cot(x))^(1/2)-2*cos(x)-sin(x)+csc(x)+2)/(-1+cos(x)))+6*arctan((-2^(1/2)*(-
cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)*sin(x)+cos(x)-1)/(-1+cos(x)))-6*arctan((2
^(1/2)*(-cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)*sin(x)+cos(x)-1)/(-1+cos(x))))*(
sin(x)^4*tan(x))^(1/2)*cos(x)/(-1+cos(x))/(cos(x)+1)^2/(-cos(x)*sin(x)/(cos
(x)+1)^2)^(1/2)*32^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 628, normalized size of antiderivative = 6.83

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(-(3*I - 3)*sqrt(2)*log((2*I*cos(x)^4 - 3*I*cos(x)^2 + 2*(cos(x)^3 - c
os(x))*sin(x) + ((I + 1)*sqrt(2)*cos(x)^2 - (I - 1)*sqrt(2)*cos(x)*sin(x))*
sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) + I)/(cos(x)^2 - 1))*sin(x)
 + (3*I - 3)*sqrt(2)*log((2*I*cos(x)^4 - 3*I*cos(x)^2 + 2*(cos(x)^3 - cos(x
))*sin(x) + (-(I + 1)*sqrt(2)*cos(x)^2 + (I - 1)*sqrt(2)*cos(x)*sin(x))*sqr
t((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) + I)/(cos(x)^2 - 1))*sin(x) +
(3*I + 3)*sqrt(2)*log((-2*I*cos(x)^4 + 3*I*cos(x)^2 + 2*(cos(x)^3 - cos(x))
*sin(x) + (-(I - 1)*sqrt(2)*cos(x)^2 + (I + 1)*sqrt(2)*cos(x)*sin(x))*sqrt(
(cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - I)/(cos(x)^2 - 1))*sin(x) - (3
*I + 3)*sqrt(2)*log((-2*I*cos(x)^4 + 3*I*cos(x)^2 + 2*(cos(x)^3 - cos(x))*s
```

```
in(x) + ((I - 1)*sqrt(2)*cos(x)^2 - (I + 1)*sqrt(2)*cos(x)*sin(x))*sqrt((co
s(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - I)/(cos(x)^2 - 1))*sin(x) + (3*I
+ 3)*sqrt(2)*log((cos(x)^2 + ((I + 1)*sqrt(2)*cos(x)^2 - (I - 1)*sqrt(2)*co
s(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - 1)/(cos(x)^2
 - 1))*sin(x) - (3*I - 3)*sqrt(2)*log((cos(x)^2 + (-(I - 1)*sqrt(2)*cos(x)^
2 + (I + 1)*sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/
cos(x)) - 1)/(cos(x)^2 - 1))*sin(x) + (3*I - 3)*sqrt(2)*log((cos(x)^2 + ((I
 - 1)*sqrt(2)*cos(x)^2 - (I + 1)*sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*
cos(x)^2 + 1)*sin(x)/cos(x)) - 1)/(cos(x)^2 - 1))*sin(x) - (3*I + 3)*sqrt(2
)*log((cos(x)^2 + (-(I + 1)*sqrt(2)*cos(x)^2 + (I - 1)*sqrt(2)*cos(x)*sin(x
))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - 1)/(cos(x)^2 - 1))*sin
(x) - 32*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x))*cos(x))/sin(x)
```

Sympy [F]

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \int \sqrt{\frac{\sin^5(x)}{\cos(x)}}\,dx$$

```
[In] integrate((sin(x)**5/cos(x))**(1/2),x)
[Out] Integral(sqrt(sin(x)**5/cos(x)), x)
```

Maxima [F]

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \int \sqrt{\frac{\sin(x)^5}{\cos(x)}}\,dx$$

```
[In] integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sin(x)^5/cos(x)), x)
```

Giac [F]

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \int \sqrt{\frac{\sin(x)^5}{\cos(x)}}\,dx$$

```
[In] integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sin(x)^5/cos(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \int \sqrt{\frac{\sin(x)^5}{\cos(x)}}\,dx$$

```
[In] int((sin(x)^5/cos(x))^(1/2),x)
[Out] int((sin(x)^5/cos(x))^(1/2), x)
```

3.414 $\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \frac{3}{5}\cos^3(x)\sin(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)} + \frac{3}{11}\cos(x)\sin^3(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}$$

```
[Out] 3/5*cos(x)^3*sin(x)*(sec(x)^12*tan(x)^2)^(1/3)+3/11*cos(x)*sin(x)^3*(sec(x)
^12*tan(x)^2)^(1/3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1986, 15, 14}

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \frac{3}{5}\sin(x)\cos^3(x)\sqrt[3]{\tan^2(x)\sec^{12}(x)} + \frac{3}{11}\sin^3(x)\cos(x)\sqrt[3]{\tan^2(x)\sec^{12}(x)}$$

```
[In] Int[(Sec[x]^12*Tan[x]^2)^(1/3),x]
```

```
[Out] (3*Cos[x]^3*Sin[x]*(Sec[x]^12*Tan[x]^2)^(1/3))/5 + (3*Cos[x]*Sin[x]^3*(Sec[
x]^12*Tan[x]^2)^(1/3))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\sqrt[3]{x^2\left(1+x^2\right)^6}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= \frac{\left(\cos^4(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}\right)\text{Subst}\left(\int \sqrt[3]{x^2}\left(1+x^2\right)\,dx, x, \tan(x)\right)}{\sqrt[3]{\tan^2(x)}}\\
&= \frac{\left(\cos^4(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}\right)\text{Subst}\left(\int x^{2/3}\left(1+x^2\right)\,dx, x, \tan(x)\right)}{\tan^{\frac{2}{3}}(x)}\\
&= \frac{\left(\cos^4(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}\right)\text{Subst}\left(\int \left(x^{2/3}+x^{8/3}\right)\,dx, x, \tan(x)\right)}{\tan^{\frac{2}{3}}(x)}\\
&= \frac{3}{5}\cos^3(x)\sin(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}+\frac{3}{11}\cos(x)\sin^3(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \frac{3\cos(x)\sin(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}\left(-3+8\left(-\tan^2(x)\right)^{5/6}+3\cos(2x)\left(-1+\left(-\tan^2(x)\right)^{5/6}\right)\right)}{55\left(-\tan^2(x)\right)^{5/6}}$$

```
[In] Integrate[(Sec[x]^12*Tan[x]^2)^(1/3),x]
[Out] (3*Cos[x]*Sin[x]*(Sec[x]^12*Tan[x]^2)^(1/3)*(-3 + 8*(-Tan[x]^2)^(5/6) + 3*C
os[2*x]*(-1 + (-Tan[x]^2)^(5/6))))/(55*(-Tan[x]^2)^(5/6))
```

Maple [F]

$$\int \left(\frac{\sin^2(x)}{\cos(x)^{14}}\right)^{\frac{1}{3}} dx$$

```
[In] int((sin(x)^2/cos(x)^14)^(1/3),x)
[Out] int((sin(x)^2/cos(x)^14)^(1/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \frac{3}{55}\left(6\cos(x)^3 + 5\cos(x)\right)\left(-\frac{\cos(x)^2 - 1}{\cos(x)^{14}}\right)^{\frac{1}{3}}\sin(x)$$

```
[In] integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="fricas")
[Out] 3/55*(6*cos(x)^3 + 5*cos(x))*(-(cos(x)^2 - 1)/cos(x)^14)^(1/3)*sin(x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \text{Timed out}$$

```
[In] integrate((sin(x)**2/cos(x)**14)**(1/3),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.28

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \frac{3}{11}\tan(x)^{\frac{11}{3}} + \frac{3}{5}\tan(x)^{\frac{5}{3}}$$

```
[In] integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="maxima")
[Out] 3/11*tan(x)^(11/3) + 3/5*tan(x)^(5/3)
```

Giac [F]

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \int \left(\frac{\sin(x)^2}{\cos(x)^{14}}\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="giac")
[Out] integrate((sin(x)^2/cos(x)^14)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \frac{6\sin(2x)\,(1-\cos(2x))^{1/3}\,(3\cos(2x)+8)}{55\,(\cos(2x)+1)^{7/3}}$$

```
[In] int((sin(x)^2/cos(x)^14)^(1/3),x)
[Out] (6*sin(2*x)*(1 - cos(2*x))^(1/3)*(3*cos(2*x) + 8))/(55*(cos(2*x) + 1)^(7/3)
)
```

3.415 $\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = -\frac{4\cos^5(x)\sin(x)}{9\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}-\frac{8\cos^3(x)\sin^3(x)}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}+\frac{4\cos(x)\sin^5(x)}{7\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}$$

```
[Out] -4/9*cos(x)^5*sin(x)/(cos(x)^11*sin(x)^13)^(1/4)-8*cos(x)^3*sin(x)^3/(cos(x
)^11*sin(x)^13)^(1/4)+4/7*cos(x)*sin(x)^5/(cos(x)^11*sin(x)^13)^(1/4)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6851, 276}

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = \frac{4\sin^5(x)\cos(x)}{7\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}-\frac{4\sin(x)\cos^5(x)}{9\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}-\frac{8\sin^3(x)\cos^3(x)}{\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}$$

```
[In] Int[(Cos[x]^11*Sin[x]^13)^(-1/4),x]
```

```
[Out] (-4*Cos[x]^5*Sin[x])/(9*(Cos[x]^11*Sin[x]^13)^(1/4)) - (8*Cos[x]^3*Sin[x]^3
)/(Cos[x]^11*Sin[x]^13)^(1/4) + (4*Cos[x]*Sin[x]^5)/(7*(Cos[x]^11*Sin[x]^13
)^(1/4))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\sqrt[4]{\frac{x^{13}}{(1+x^2)^{12}}}\,(1+x^2)}\,dx, x, \tan(x)\right)$$

$$= \frac{\left(\cos^6(x)\tan^{\frac{13}{4}}(x)\right)\text{Subst}\left(\int \frac{(1+x^2)^2}{x^{13/4}}\,dx, x, \tan(x)\right)}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}$$

$$= \frac{\left(\cos^6(x)\tan^{\frac{13}{4}}(x)\right)\text{Subst}\left(\int \left(\frac{1}{x^{13/4}}+\frac{2}{x^{5/4}}+x^{3/4}\right)\,dx, x, \tan(x)\right)}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}$$

$$= -\frac{4\cos^5(x)\sin(x)}{9\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}} - \frac{8\cos^3(x)\sin^3(x)}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}} + \frac{4\cos(x)\sin^5(x)}{7\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = -\frac{4\cos(x)(15+8\cos(2x)-16\cos(4x))\sin(x)}{63\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}$$

```
[In] Integrate[(Cos[x]^11*Sin[x]^13)^(-1/4),x]
[Out] (-4*Cos[x]*(15 + 8*Cos[2*x] - 16*Cos[4*x])*Sin[x])/(63*(Cos[x]^11*Sin[x]^13
)^(1/4))
```

Maple [F]

$$\int \frac{1}{\left(\left(\cos^{11}\left(x\right)\right)\left(\sin^{13}\left(x\right)\right)\right)^{\frac{1}{4}}}dx$$

```
[In] int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
```

```
[Out] int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx$$
$$= \frac{4\left(128\cos\left(x\right)^{4} - 144\cos\left(x\right)^{2} + 9\right)\left(\left(\cos\left(x\right)^{23} - 6\cos\left(x\right)^{21} + 15\cos\left(x\right)^{19} - 20\cos\left(x\right)^{17} + 15\cos\left(x\right)^{15} -\right.\right.}{63\left(\cos\left(x\right)^{22} - 6\cos\left(x\right)^{20} + 15\cos\left(x\right)^{18} - 20\cos\left(x\right)^{16} + 15\cos\left(x\right)^{14} - 6\cos\left(x\right)^{12}\right.}$$

```
[In] integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/63*(128*cos(x)^4 - 144*cos(x)^2 + 9)*((cos(x)^23 - 6*cos(x)^21 + 15*cos(x
)^19 - 20*cos(x)^17 + 15*cos(x)^15 - 6*cos(x)^13 + cos(x)^11)*sin(x))^(3/4)
/(cos(x)^22 - 6*cos(x)^20 + 15*cos(x)^18 - 20*cos(x)^16 + 15*cos(x)^14 - 6*
cos(x)^12 + cos(x)^10)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(cos(x)**11*sin(x)**13)**(1/4),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = \frac{4}{23}\tan(x)^{\frac{23}{4}} + \frac{8}{15}\tan(x)^{\frac{15}{4}} + \frac{4}{7}\tan(x)^{\frac{7}{4}} - \frac{4\left(35\tan(x)^7 + 161\tan(x)^5 + 345\tan(x)^3 - 805\tan(x)\right)}{805\tan(x)^{\frac{5}{4}}} + \frac{4\left(21\tan(x)^7 + 135\tan(x)^5 - 945\tan(x)^3 - 35\tan(x)\right)}{315\tan(x)^{\frac{13}{4}}}$$

```
[In] integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="maxima")
[Out] 4/23*tan(x)^(23/4) + 8/15*tan(x)^(15/4) + 4/7*tan(x)^(7/4) - 4/805*(35*tan(
x)^7 + 161*tan(x)^5 + 345*tan(x)^3 - 805*tan(x))/tan(x)^(5/4) + 4/315*(21*t
an(x)^7 + 135*tan(x)^5 - 945*tan(x)^3 - 35*tan(x))/tan(x)^(13/4)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = \int \frac{1}{\left(\cos(x)^{11}\sin(x)^{13}\right)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="giac")
[Out] integrate((cos(x)^11*sin(x)^13)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = -\frac{2^{3/4}\left(-32\cos(2x)^2 + 8\cos(2x) + 31\right)\left(924\sin(2x) - 132\sin(4x) - 660\sin(6x) + 165\sin(8x) + 33\right.}{2016\,(\cos}$$

```
[In] int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
[Out] -(2^(3/4)*(8*cos(2*x) - 32*cos(2*x)^2 + 31)*(924*sin(2*x) - 132*sin(4*x) -
660*sin(6*x) + 165*sin(8*x) + 330*sin(10*x) - 110*sin(12*x) - 110*sin(14*x)
 + 44*sin(16*x) + 22*sin(18*x) - 10*sin(20*x) - 2*sin(22*x) + sin(24*x))^(3
/4))/(2016*(cos(2*x) - 1)^6*(cos(2*x) + 1)^5)
```

3.416 $\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 108

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx = -\sqrt{2}\log\left(\cos(x)+\sin(x)-\sqrt{2}\sec(x)\sqrt{\cos^3(x)\sin(x)}\right) - \frac{\arcsin(\cos(x)-\sin(x))\cos(x)\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}} - \frac{\operatorname{arctanh}(\sin(x))\cos(x)\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}} - \frac{\sin(2x)}{\sqrt{\cos^3(x)\sin(x)}}$$

```
[Out] -ln(cos(x)+sin(x)-sec(x)*2^(1/2)*(cos(x)^3*sin(x))^(1/2))*2^(1/2)-sin(2*x)/
(cos(x)^3*sin(x))^(1/2)-arcsin(cos(x)-sin(x))*cos(x)*sin(2*x)^(1/2)/(cos(x)
^3*sin(x))^(1/2)-arctanh(sin(x))*cos(x)*sin(2*x)^(1/2)/(cos(x)^3*sin(x))^(1
/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 234 vs. 2(108) = 216.

Time = 1.19 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.17, number of steps used = 27, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules

used = {6851, 6857, 221, 335, 217, 1179, 642, 1176, 631, 210, 327}

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx$$
$$= -\sqrt{2}\cot(x)\sec^2(x)^{3/2}\text{arcsinh}(\tan(x))\sqrt{\sin(x)\cos(x)}\sqrt{\sin(x)\cos^3(x)} - \frac{\sqrt{2}\sec^2(x)\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)\sqrt{\sin(x)\cos^3(x)}}{\sqrt{\tan(x)}} + \frac{\sqrt{2}\sec^2(x)\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)\sqrt{\sin(x)\cos^3(x)}}{\sqrt{\tan(x)}} - 2\sec^2(x)\sqrt{\sin(x)\cos^3(x)} - \frac{\sec^2(x)\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)\sqrt{\sin(x)\cos^3(x)}}{\sqrt{2}\sqrt{\tan(x)}} + \frac{\sec^2(x)\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)\sqrt{\sin(x)\cos^3(x)}}{\sqrt{2}\sqrt{\tan(x)}}$$

```
[In] Int[(Cos[2*x] - Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]],x]
```

```
[Out] -2*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]] - Sqrt[2]*ArcSinh[Tan[x]]*Cot[x]*(Sec[x]^
2)^(3/2)*Sqrt[Cos[x]*Sin[x]]*Sqrt[Cos[x]^3*Sin[x]] - (Sqrt[2]*ArcTan[1 - Sq
rt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] + (Sqrt[2]
*ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[
x]] - (Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]
])/(Sqrt[2]*Sqrt[Tan[x]]) + (Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^
2*Sqrt[Cos[x]^3*Sin[x]])/(Sqrt[2]*Sqrt[Tan[x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
```

```
[v, x] &&  !FreeQ[w, x]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\sqrt{\frac{x}{(1+x^2)^2}}\left(1-x^2-\frac{x}{\sqrt{\frac{x}{2+2x^2}}}\right)}{x}\,dx, x, \tan(x)\right)$$

$$= \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \frac{1-x^2-\frac{x}{\sqrt{\frac{x}{2+2x^2}}}}{\sqrt{x}(1+x^2)}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)}}$$

$$= \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \left(-\frac{\sqrt{2}\sqrt{\frac{x}{1+x^2}}}{\sqrt{x}}+\frac{1}{\sqrt{x}(1+x^2)}-\frac{x^{3/2}}{1+x^2}\right)\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)}}$$

$$= \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)}} - \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \frac{x^{3/2}}{1+x^2}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)}} - \frac{\left(\sqrt{2}\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \frac{\sqrt{\frac{x}{1+x^2}}}{\sqrt{x}}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)}}$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)} - \left(\sqrt{2}\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tan(x)\right) + \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)}} + \frac{\left(2\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)}{\sqrt{\tan(x)}}$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)}$$
$$-\sqrt{2}\text{arcsinh}(\tan(x))\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}\sqrt{\cos^3(x)\sin(x)}$$
$$+\frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(x)}\right)}{\sqrt{\tan(x)}}$$
$$+\frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\sqrt{\tan(x)}\right)}{\sqrt{\tan(x)}}$$
$$+\frac{\left(2\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{1}{1+x^4}\,dx,x,\sqrt{\tan(x)}\right)}{\sqrt{\tan(x)}}$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)}$$
$$-\sqrt{2}\text{arcsinh}(\tan(x))\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}\sqrt{\cos^3(x)\sin(x)}$$
$$+\frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(x)}\right)}{2\sqrt{\tan(x)}}$$
$$+\frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(x)}\right)}{2\sqrt{\tan(x)}}$$
$$+\frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(x)}\right)}{\sqrt{\tan(x)}}$$
$$+\frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\sqrt{\tan(x)}\right)}{\sqrt{\tan(x)}}$$
$$-\frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(x)}\right)}{2\sqrt{2}\sqrt{\tan(x)}}$$
$$-\frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(x)}\right)}{2\sqrt{2}\sqrt{\tan(x)}}$$

$$
\begin{aligned}
&= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)} \\
&\quad - \sqrt{2}\operatorname{arcsinh}(\tan(x))\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}\sqrt{\cos^3(x)\sin(x)} \\
&\quad - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)\sec^2(x)\sqrt{\cos^3(x)\sin(x)}}{2\sqrt{2}\sqrt{\tan(x)}} \\
&\quad + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)\sec^2(x)\sqrt{\cos^3(x)\sin(x)}}{2\sqrt{2}\sqrt{\tan(x)}} \\
&\quad + \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(x)}\right)}{2\sqrt{\tan(x)}} \\
&\quad + \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(x)}\right)}{2\sqrt{\tan(x)}} \\
&\quad - \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(x)}\right)}{2\sqrt{2}\sqrt{\tan(x)}} \\
&\quad - \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(x)}\right)}{2\sqrt{2}\sqrt{\tan(x)}} \\
&\quad + \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}\sqrt{\tan(x)}} \\
&\quad - \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}\sqrt{\tan(x)}}
\end{aligned}
$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)} - \sqrt{2}\text{arcsinh}(\tan(x))\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}\sqrt{\cos^3(x)\sin(x)} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)\sec^2(x)\sqrt{\cos^3(x)\sin(x)}}{\sqrt{2}\sqrt{\tan(x)}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(x)}\right)\sec^2(x)\sqrt{\cos^3(x)\sin(x)}}{\sqrt{2}\sqrt{\tan(x)}} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)\sec^2(x)\sqrt{\cos^3(x)\sin(x)}}{\sqrt{2}\sqrt{\tan(x)}} + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)\sec^2(x)\sqrt{\cos^3(x)\sin(x)}}{\sqrt{2}\sqrt{\tan(x)}} + \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}\sqrt{\tan(x)}} - \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}\sqrt{\tan(x)}}$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)} - \sqrt{2}\text{arcsinh}(\tan(x))\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}\sqrt{\cos^3(x)\sin(x)} - \frac{\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)\sec^2(x)\sqrt{\cos^3(x)\sin(x)}}{\sqrt{\tan(x)}} + \frac{\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\tan(x)}\right)\sec^2(x)\sqrt{\cos^3(x)\sin(x)}}{\sqrt{\tan(x)}} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)\sec^2(x)\sqrt{\cos^3(x)\sin(x)}}{\sqrt{2}\sqrt{\tan(x)}} + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)\sec^2(x)\sqrt{\cos^3(x)\sin(x)}}{\sqrt{2}\sqrt{\tan(x)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx$$
$$=\frac{-4\cos^3(x)\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\cos^2(x)\right)\sin(x)-3\cos(x)\sqrt[4]{\sin^2(x)}\left(2\sin(x)+\operatorname{arctanh}(\sin(x))\sqrt{s\right.}}{3\sqrt{\cos^3(x)\sin(x)}\sqrt[4]{\sin^2(x)}}$$

```
[In] Integrate[(Cos[2*x] - Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]],x]
```

```
[Out] (-4*Cos[x]^3*Hypergeometric2F1[3/4, 3/4, 7/4, Cos[x]^2]*Sin[x] - 3*Cos[x]*(
Sin[x]^2)^(1/4)*(2*Sin[x] + ArcTanh[Sin[x]]*Sqrt[Sin[2*x]]))/(3*Sqrt[Cos[x]
^3*Sin[x]]*(Sin[x]^2)^(1/4))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(92) = 184.

Time = 3.68 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.29

method	result
default	$-\frac{2\sin(x)\cos(x)}{\sqrt{(\cos^3(x))\sin(x)}}+\frac{2\sqrt{2}\cos(x)\sqrt{\cos(x)\sin(x)}\,\operatorname{arctanh}(-\csc(x)+\cot(x))}{\sqrt{(\cos^3(x))\sin(x)}}+\frac{\sqrt{\frac{\cos(x)\sin(x)}{(\cos(x)+1)^2}}\left(\ln\left(2\cot(x)\sqrt{2}\sqrt{\frac{\cos(x)\sin(x)}{(\cos(x)+1)^2}}+2c\right.\right.}{}$
parts	$-\frac{2\sin(x)\cos(x)}{\sqrt{(\cos^3(x))\sin(x)}}+\frac{2\sqrt{2}\cos(x)\sqrt{\cos(x)\sin(x)}\,\operatorname{arctanh}(-\csc(x)+\cot(x))}{\sqrt{(\cos^3(x))\sin(x)}}+\frac{\sqrt{\frac{\cos(x)\sin(x)}{(\cos(x)+1)^2}}\left(\ln\left(2\cot(x)\sqrt{2}\sqrt{\frac{\cos(x)\sin(x)}{(\cos(x)+1)^2}}+2c\right.\right.}{}$

```
[In] int((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -2*sin(x)*cos(x)/(cos(x)^3*sin(x))^(1/2)+2*2^(1/2)*cos(x)*(cos(x)*sin(x))^(
1/2)*arctanh(-csc(x)+cot(x))/(cos(x)^3*sin(x))^(1/2)+1/2*(cos(x)*sin(x)/(co
s(x)+1)^2)^(1/2)*(ln(2*cot(x)*2^(1/2)*(cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)+2*
csc(x)*2^(1/2)*(cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)+2+2*cot(x))+2*arctan((2^(
1/2)*(cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)*sin(x)-cos(x)+1)/(-1+cos(x)))-ln(-2
*cot(x)*2^(1/2)*(cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)-2*csc(x)*2^(1/2)*(cos(x)
*sin(x)/(cos(x)+1)^2)^(1/2)+2+2*cot(x))+2*arctan((2^(1/2)*(cos(x)*sin(x)/(c
os(x)+1)^2)^(1/2)*sin(x)+cos(x)-1)/(-1+cos(x))))/(cos(x)^3*sin(x))^(1/2)*(c
os(x)^2+cos(x))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 479, normalized size of antiderivative = 4.44

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx$$
$$= \frac{-(i-1)\sqrt{2}\cos(x)^2\log\left(\frac{2\cos(x)^3+2i\cos(x)^2\sin(x)+\sqrt{\cos(x)^3\sin(x)}\left((i+1)\sqrt{2}\cos(x)+(i-1)\sqrt{2}\sin(x)\right)-\cos(x)}{\cos(x)}\right)+(i-1)}{}$$

```
[In] integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/8*(-(I - 1)*sqrt(2)*cos(x)^2*log((2*cos(x)^3 + 2*I*cos(x)^2*sin(x) + sqrt
(cos(x)^3*sin(x))*((I + 1)*sqrt(2)*cos(x) + (I - 1)*sqrt(2)*sin(x)) - cos(x
))/cos(x)) + (I - 1)*sqrt(2)*cos(x)^2*log((2*cos(x)^3 + 2*I*cos(x)^2*sin(x)
 + sqrt(cos(x)^3*sin(x))*(-(I + 1)*sqrt(2)*cos(x) - (I - 1)*sqrt(2)*sin(x))
 - cos(x))/cos(x)) + (I + 1)*sqrt(2)*cos(x)^2*log((2*cos(x)^3 - 2*I*cos(x)^
2*sin(x) + sqrt(cos(x)^3*sin(x))*(-(I - 1)*sqrt(2)*cos(x) - (I + 1)*sqrt(2)
*sin(x)) - cos(x))/cos(x)) - (I + 1)*sqrt(2)*cos(x)^2*log((2*cos(x)^3 - 2*I
*cos(x)^2*sin(x) + sqrt(cos(x)^3*sin(x))*((I - 1)*sqrt(2)*cos(x) + (I + 1)*
sqrt(2)*sin(x)) - cos(x))/cos(x)) + (I - 1)*sqrt(2)*cos(x)^2*log((sqrt(cos(
x)^3*sin(x))*((I + 1)*sqrt(2)*cos(x) - (I - 1)*sqrt(2)*sin(x)) - cos(x))/co
s(x)) - (I + 1)*sqrt(2)*cos(x)^2*log((sqrt(cos(x)^3*sin(x))*(-(I - 1)*sqrt(
2)*cos(x) + (I + 1)*sqrt(2)*sin(x)) - cos(x))/cos(x)) + (I + 1)*sqrt(2)*cos
(x)^2*log((sqrt(cos(x)^3*sin(x))*((I - 1)*sqrt(2)*cos(x) - (I + 1)*sqrt(2)*
sin(x)) - cos(x))/cos(x)) - (I - 1)*sqrt(2)*cos(x)^2*log((sqrt(cos(x)^3*sin
(x))*(-(I + 1)*sqrt(2)*cos(x) + (I - 1)*sqrt(2)*sin(x)) - cos(x))/cos(x)) +
 4*sqrt(2)*cos(x)^2*log(-(cos(x)^4 - 2*cos(x)^2 + 2*sqrt(cos(x)^3*sin(x))*s
qrt(cos(x)*sin(x)))/cos(x)^4) - 16*sqrt(cos(x)^3*sin(x)))/cos(x)^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx = \text{Timed out}$$

```
[In] integrate((cos(2*x)-sin(2*x)**(1/2))/(cos(x)**3*sin(x))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos(2x) - \sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx = \int -\frac{\sqrt{\sin(2x)} - \cos(2x)}{\sqrt{\cos(x)^3\sin(x)}}\,dx$$

```
[In] integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="m
axima")
```

```
[Out] 1/2*sqrt(2)*integrate(2*((((cos(4*x) + 1)*cos(1/2*arctan2(sin(x), -cos(x) +
 1)) - sin(4*x)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin(
x), cos(x) + 1)) + (cos(1/2*arctan2(sin(x), -cos(x) + 1))*sin(4*x) + (cos(4
*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(1/2*arctan2(sin(x), cos
(x) + 1)))*cos(3/2*arctan2(sin(2*x), cos(2*x) + 1)) + ((cos(1/2*arctan2(sin
(x), -cos(x) + 1))*sin(4*x) + (cos(4*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x
) + 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - ((cos(4*x) + 1)*cos(1/2*arc
tan2(sin(x), -cos(x) + 1)) - sin(4*x)*sin(1/2*arctan2(sin(x), -cos(x) + 1))
)*sin(1/2*arctan2(sin(x), cos(x) + 1)))*sin(3/2*arctan2(sin(2*x), cos(2*x)
+ 1)))/((cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(3/4)*(cos(x)^2 + sin(x)
^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4)), x) -
1/2*sqrt(2)*integrate(-2*(((cos(1/2*arctan2(sin(x), -cos(x) + 1))*sin(4*x)
+ (cos(4*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin
(x), cos(x) + 1)) - ((cos(4*x) + 1)*cos(1/2*arctan2(sin(x), -cos(x) + 1)) -
 sin(4*x)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(1/2*arctan2(sin(x), co
s(x) + 1)))*cos(3/2*arctan2(sin(2*x), cos(2*x) + 1)) - (((cos(4*x) + 1)*cos
(1/2*arctan2(sin(x), -cos(x) + 1)) - sin(4*x)*sin(1/2*arctan2(sin(x), -cos(
x) + 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) + (cos(1/2*arctan2(sin(x), -
cos(x) + 1))*sin(4*x) + (cos(4*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)
))*sin(1/2*arctan2(sin(x), cos(x) + 1)))*sin(3/2*arctan2(sin(2*x), cos(2*x)
 + 1)))/((cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(3/4)*(cos(x)^2 + sin(x
)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4)), x) -
 1/2*sqrt(2)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + 1/2*sqrt(2)*log(cos(
x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Giac [F]

$$\int \frac{\cos(2x) - \sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx = \int -\frac{\sqrt{\sin(2x)} - \cos(2x)}{\sqrt{\cos(x)^3\sin(x)}}\,dx$$

```
[In] integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(-(sqrt(sin(2*x)) - cos(2*x))/sqrt(cos(x)^3*sin(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(2x) - \sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx = \int \frac{\cos(2\,x) - \sqrt{\sin(2\,x)}}{\sqrt{\cos(x)^3\,\sin(x)}}\,dx$$

```
[In] int((cos(2*x) - sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x)
[Out] int((cos(2*x) - sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2), x)
```

3.417 $\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 364

$$\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx$$
$$= -2\sqrt{2}\coth^{-1}\left(\frac{\cos(x)(\cos(x)+\sin(x))}{\sqrt{2}\sqrt{\cos^3(x)\sin(x)}}\right)+\sqrt[4]{2}\coth^{-1}\left(\frac{\cos(x)\left(\sqrt{2}\cos(x)+\sin(x)\right)}{2^{3/4}\sqrt{\cos^3(x)\sin(x)}}\right)$$
$$-\sqrt[4]{2}\coth^{-1}\left(\frac{\sqrt{2}+\tan(x)}{2^{3/4}\sqrt{\tan(x)}}\right)-2\sqrt{2}\arctan\left(\frac{\cos(x)(\cos(x)-\sin(x))}{\sqrt{2}\sqrt{\cos^3(x)\sin(x)}}\right)$$
$$+\sqrt[4]{2}\arctan\left(\frac{\cos(x)\left(\sqrt{2}\cos(x)-\sin(x)\right)}{2^{3/4}\sqrt{\cos^3(x)\sin(x)}}\right)-\sqrt[4]{2}\arctan\left(\frac{\sqrt{2}-\tan(x)}{2^{3/4}\sqrt{\tan(x)}}\right)+4\csc(x)\sec(x)\sqrt{\cos^3(x)\sin(x}$$

```
[Out] 2^(1/4)*arccoth(1/2*cos(x)*(sin(x)+cos(x)*2^(1/2))*2^(1/4)/(cos(x)^3*sin(x)
)^(1/2))-2^(1/4)*arccoth(1/2*(2^(1/2)+tan(x))*2^(1/4)/tan(x)^(1/2))+2^(1/4)
*arctan(1/2*cos(x)*(-sin(x)+cos(x)*2^(1/2))*2^(1/4)/(cos(x)^3*sin(x))^(1/2)
)-2^(1/4)*arctan(1/2*(2^(1/2)-tan(x))*2^(1/4)/tan(x)^(1/2))-2*arccoth(1/2*c
os(x)*(cos(x)+sin(x))*2^(1/2)/(cos(x)^3*sin(x))^(1/2))*2^(1/2)-2*arctan(1/2
*cos(x)*(cos(x)-sin(x))*2^(1/2)/(cos(x)^3*sin(x))^(1/2))*2^(1/2)+4*csc(x)*s
ec(x)*(cos(x)^3*sin(x))^(1/2)+1/4*csc(x)^2*ln(1+cos(x)^2)*sec(x)^2*(cos(x)^
3*sin(x))^(1/2)*(cos(x)*sin(x)^3)^(1/2)+1/2*csc(x)^2*ln(sin(x))*sec(x)^2*(c
os(x)^3*sin(x))^(1/2)*(cos(x)*sin(x)^3)^(1/2)+4/tan(x)^(1/2)-1/4*csc(x)^2*l
n(1+cos(x)^2)*(cos(x)*sin(x)^3)^(1/2)*tan(x)^(1/2)+1/2*csc(x)^2*ln(sin(x))*
(cos(x)*sin(x)^3)^(1/2)*tan(x)^(1/2)
```

Rubi [A] (verified)

Time = 3.55 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.83, number of steps used = 66, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.512$, Rules used = {6857, 6874, 6851, 331, 335, 303, 1176, 631, 210, 1179, 642, 477, 493, 6865, 15, 29, 272, 36, 31, 266, 455}

$$\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx = -\sqrt[4]{2}\arctan\left(1-\sqrt[4]{2}\sqrt{\tan(x)}\right) + \sqrt[4]{2}\arctan\left(\sqrt[4]{2}\sqrt{\tan(x)}+1\right) + \frac{\sqrt[4]{2}\sec^2(x)\arctan\left(1-\sqrt[4]{2}\sqrt{\tan(x)}\right)\sqrt{\sin(x)\cos^3(x)}}{\sqrt{\tan(x)}} - \frac{\sqrt[4]{2}\sec^2(x)\arctan\left(\sqrt[4]{2}\sqrt{\tan(x)}+1\right)\sqrt{\sin(x)\cos^3(x)}}{\sqrt{\tan(x)}} - \frac{2\sqrt{2}\sec^2(x)\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)\sqrt{\sin(x)\cos^3(x)}}{\sqrt{\tan(x)}} + \frac{2\sqrt{2}\sec^2(x)\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)\sqrt{\sin(x)\cos^3(x)}}{\sqrt{\tan(x)}} + \frac{4}{\sqrt{\tan(x)}} + \frac{\log\left(\tan(x)-2^{3/4}\sqrt{\tan(x)}+\sqrt{2}\right)}{2^{3/4}} - \frac{\log\left(\tan(x)+2^{3/4}\sqrt{\tan(x)}+\sqrt{2}\right)}{2^{3/4}} + 4\csc(x)\sec(x)\sqrt{\sin(x)\cos^3(x)} + \frac{\sqrt{2}\sec^2(x)\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)\sqrt{\sin(x)\cos^3(x)}}{\sqrt{\tan(x)}} - \frac{\sqrt{2}\sec^2}{}$$

```
[In] Int[(Sqrt[Cos[x]*Sin[x]^3] - 2*Sin[2*x])/(-Sqrt[Cos[x]^3*Sin[x]] + Sqrt[Tan
[x]]),x]
```

```
[Out] -(2^(1/4)*ArcTan[1 - 2^(1/4)*Sqrt[Tan[x]]]) + 2^(1/4)*ArcTan[1 + 2^(1/4)*Sq
rt[Tan[x]]] + Log[Sqrt[2] - 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]/2^(3/4) - Log[Sq
rt[2] + 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]/2^(3/4) + 4*Csc[x]*Sec[x]*Sqrt[Cos[x
]^3*Sin[x]] - (Csc[x]^2*Log[Sec[x]^2]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]]*Sqrt[C
os[x]*Sin[x]^3])/2 + Csc[x]^2*Log[Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[
x]]*Sqrt[Cos[x]*Sin[x]^3] + (Csc[x]^2*Log[2 + Tan[x]^2]*Sec[x]^2*Sqrt[Cos[x
]^3*Sin[x]]*Sqrt[Cos[x]*Sin[x]^3])/4 + (Log[Tan[x]]*Sec[x]^2*Sqrt[Cos[x]*Si
n[x]^3])/(2*Tan[x]^(3/2)) - (Log[2 + Tan[x]^2]*Sec[x]^2*Sqrt[Cos[x]*Sin[x]^
3])/(4*Tan[x]^(3/2)) + 4/Sqrt[Tan[x]] + (2^(1/4)*ArcTan[1 - 2^(1/4)*Sqrt[Ta
n[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] - (2^(1/4)*ArcTan[1 + 2
^(1/4)*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] - (2*Sqrt
[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[T
an[x]] + (2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3
*Sin[x]])/Sqrt[Tan[x]] + (Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Se
c[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] - (Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[
```

```
Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] - (Log[Sqrt[
2] - 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/(2^(3/4
)*Sqrt[Tan[x]]) + (Log[Sqrt[2] + 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sq
rt[Cos[x]^3*Sin[x]])/(2^(3/4)*Sqrt[Tan[x]])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\sqrt{\frac{x^3}{(1+x^2)^2}} - \frac{4x}{1+x^2}}{(1+x^2)\left(\sqrt{x} - \sqrt{\frac{x}{(1+x^2)^2}}\right)}\,dx, x, \tan(x)\right)$$

$$= \text{Subst}\left(\int \left(\frac{4x}{(1+x^2)^2\left(-\sqrt{x} + \sqrt{\frac{x}{(1+x^2)^2}}\right)} - \frac{\sqrt{\frac{x^3}{(1+x^2)^2}}}{(1+x^2)\left(-\sqrt{x} + \sqrt{\frac{x}{(1+x^2)^2}}\right)}\right)dx, x, \tan(x)\right)$$

$$= 4\text{Subst}\left(\int \frac{x}{(1+x^2)^2\left(-\sqrt{x} + \sqrt{\frac{x}{(1+x^2)^2}}\right)}\,dx, x, \tan(x)\right) - \text{Subst}\left(\int \frac{\sqrt{\frac{x^3}{(1+x^2)^2}}}{(1+x^2)\left(-\sqrt{x} + \sqrt{\frac{x}{(1+x^2)^2}}\right)}\,dx, x, \tan(x)\right)$$

$$= 4\text{Subst}\left(\int \left(-\frac{1}{2x^{3/2}} - \frac{\sqrt{\frac{x}{(1+x^2)^2}}}{2x^2} + \frac{\sqrt{x}}{2(2+x^2)} + \frac{\sqrt{\frac{x}{(1+x^2)^2}}}{2(2+x^2)}\right)dx, x, \tan(x)\right) - \frac{\left(\sec^2(x)\sqrt{\cos(x)\sin^3(x)}\right)\text{Subst}\left(\int \frac{x^{3/2}}{(1+x^2)^2\left(-\sqrt{x} + \sqrt{\frac{x}{(1+x^2)^2}}\right)}\,dx, x, \tan(x)\right)}{\tan^{\frac{3}{2}}(x)}$$

$$= \frac{4}{\sqrt{\tan(x)}} - 2\text{Subst}\left(\int \frac{\sqrt{\frac{x}{(1+x^2)^2}}}{x^2}\,dx, x, \tan(x)\right) + 2\text{Subst}\left(\int \frac{\sqrt{x}}{2+x^2}\,dx, x, \tan(x)\right) + 2\text{Subst}\left(\int \frac{\sqrt{\frac{x}{(1+x^2)^2}}}{2+x^2}\,dx, x, \tan(x)\right) - \frac{\left(2\sec^2(x)\sqrt{\cos(x)\sin^3(x)}\right)\text{Subst}\left(\int \frac{x^4}{(1+x^4)^2\left(-\sqrt{x^2} + \sqrt{\frac{x^2}{(1+x^4)^2}}\right)}\,dx, x, \sqrt{\tan(x)}\right)}{\tan^{\frac{3}{2}}(x)}$$

$$= \frac{4}{\sqrt{\tan(x)}} + 4\text{Subst}\left(\int \frac{x^2}{2+x^4}\,dx, x, \sqrt{\tan(x)}\right)$$

$$- \frac{\left(2\sec^2(x)\sqrt{\cos(x)\sin^3(x)}\right)\text{Subst}\left(\int \left(-\frac{1}{2\sqrt{x^2}} - \frac{\sqrt{\frac{x^2}{(1+x^4)^2}}}{2x^2} + \frac{(x^2)^{3/2}}{2(2+x^4)} + \frac{x^2\sqrt{\frac{x^2}{(1+x^4)^2}}}{2(2+x^4)}\right)dx, x, \sqrt{\tan}\right.}{\tan^{\frac{3}{2}}(x)}$$

$$- \frac{\left(2\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \frac{1}{x^{3/2}(1+x^2)}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)}}$$

$$+ \frac{\left(2\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{(1+x^2)(2+x^2)}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)}}$$

$$= 4\csc(x)\sec(x)\sqrt{\cos^3(x)\sin(x)} + \frac{4}{\sqrt{\tan(x)}}$$

$$- 2\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{2+x^4}\,dx, x, \sqrt{\tan(x)}\right) + 2\text{Subst}\left(\int \frac{\sqrt{2}+x^2}{2+x^4}\,dx, x, \sqrt{\tan(x)}\right)$$

$$+ \frac{\left(\sec^2(x)\sqrt{\cos(x)\sin^3(x)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x^2}}\,dx, x, \sqrt{\tan(x)}\right)}{\tan^{\frac{3}{2}}(x)}$$

$$+ \frac{\left(\sec^2(x)\sqrt{\cos(x)\sin^3(x)}\right)\text{Subst}\left(\int \frac{\sqrt{\frac{x^2}{(1+x^4)^2}}}{x^2}\,dx, x, \sqrt{\tan(x)}\right)}{\tan^{\frac{3}{2}}(x)}$$

$$- \frac{\left(\sec^2(x)\sqrt{\cos(x)\sin^3(x)}\right)\text{Subst}\left(\int \frac{(x^2)^{3/2}}{2+x^4}\,dx, x, \sqrt{\tan(x)}\right)}{\tan^{\frac{3}{2}}(x)}$$

$$- \frac{\left(\sec^2(x)\sqrt{\cos(x)\sin^3(x)}\right)\text{Subst}\left(\int \frac{x^2\sqrt{\frac{x^2}{(1+x^4)^2}}}{2+x^4}\,dx, x, \sqrt{\tan(x)}\right)}{\tan^{\frac{3}{2}}(x)}$$

$$+ \frac{\left(2\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)}}$$

$$+ \frac{\left(4\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \frac{x^2}{(1+x^4)(2+x^4)}\,dx, x, \sqrt{\tan(x)}\right)}{\sqrt{\tan(x)}}$$

$$= 4\csc(x)\sec(x)\sqrt{\cos^3(x)\sin(x)} + \frac{4}{\sqrt{\tan(x)}}$$
$$+ \frac{\text{Subst}\left(\int \frac{2^{3/4}+2x}{-\sqrt{2}-2^{3/4}x-x^2}\,dx, x, \sqrt{\tan(x)}\right)}{2^{3/4}} + \frac{\text{Subst}\left(\int \frac{2^{3/4}-2x}{-\sqrt{2}+2^{3/4}x-x^2}\,dx, x, \sqrt{\tan(x)}\right)}{2^{3/4}}$$
$$+ \left(\csc^2(x)\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\sqrt{\cos(x)\sin^3(x)}\right)\text{Subst}\left(\int \frac{1}{x(1+x^4)}\,dx, x, \sqrt{\tan(x)}\right) - \left(\text{cs}\right.$$

$$= \frac{\log\left(\sqrt{2}-2^{3/4}\sqrt{\tan(x)}+\tan(x)\right)}{2^{3/4}} - \frac{\log\left(\sqrt{2}+2^{3/4}\sqrt{\tan(x)}+\tan(x)\right)}{2^{3/4}}$$
$$+ 4\csc(x)\sec(x)\sqrt{\cos^3(x)\sin(x)} + \frac{\log(\tan(x))\sec^2(x)\sqrt{\cos(x)\sin^3(x)}}{2\tan^{\frac{3}{2}}(x)} - \frac{\log\left(2+\tan^2(x)\right)\sec^2(x)}{4\tan^{\frac{3}{2}}(x}$$

$$= -\sqrt[4]{2}\arctan\left(1-\sqrt[4]{2}\sqrt{\tan(x)}\right) + \sqrt[4]{2}\arctan\left(1+\sqrt[4]{2}\sqrt{\tan(x)}\right)$$
$$+ \frac{\log\left(\sqrt{2}-2^{3/4}\sqrt{\tan(x)}+\tan(x)\right)}{2^{3/4}} - \frac{\log\left(\sqrt{2}+2^{3/4}\sqrt{\tan(x)}+\tan(x)\right)}{2^{3/4}}$$
$$+ 4\csc(x)\sec(x)\sqrt{\cos^3(x)\sin(x)} + \frac{\log(\tan(x))\sec^2(x)\sqrt{\cos(x)\sin^3(x)}}{2\tan^{\frac{3}{2}}(x)} - \frac{\log\left(2+\tan^2(x)\right)\sec^2(x)}{4\tan^{\frac{3}{2}}(x}$$

$$= -\sqrt[4]{2}\arctan\left(1-\sqrt[4]{2}\sqrt{\tan(x)}\right) + \sqrt[4]{2}\arctan\left(1+\sqrt[4]{2}\sqrt{\tan(x)}\right)$$
$$+ \frac{\log\left(\sqrt{2}-2^{3/4}\sqrt{\tan(x)}+\tan(x)\right)}{2^{3/4}} - \frac{\log\left(\sqrt{2}+2^{3/4}\sqrt{\tan(x)}+\tan(x)\right)}{2^{3/4}}$$
$$+ 4\csc(x)\sec(x)\sqrt{\cos^3(x)\sin(x)} + \csc^2(x)\log(\cos(x))\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\sqrt{\cos(x)\sin^3(x)} + \frac{1}{2}$$

$$= -\sqrt[4]{2}\arctan\left(1-\sqrt[4]{2}\sqrt{\tan(x)}\right) + \sqrt[4]{2}\arctan\left(1+\sqrt[4]{2}\sqrt{\tan(x)}\right)$$
$$+ \frac{\log\left(\sqrt{2}-2^{3/4}\sqrt{\tan(x)}+\tan(x)\right)}{2^{3/4}} - \frac{\log\left(\sqrt{2}+2^{3/4}\sqrt{\tan(x)}+\tan(x)\right)}{2^{3/4}}$$
$$+ 4\csc(x)\sec(x)\sqrt{\cos^3(x)\sin(x)} + \csc^2(x)\log(\cos(x))\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\sqrt{\cos(x)\sin^3(x)} + \frac{1}{2}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 19.66 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx = 4\csc(x)\sec(x)\sqrt{\cos^3(x)\sin(x)} + \frac{\cot(x)\left(-2\log\left(\sec^2(x)\right)+2\log(\tan(x))+\log\left(2+\tan^2(x)\right)\right)\sqrt{\cos(x)\sin^3(x)}}{4\sqrt{\cos^3(x)\sin(x)}} + \frac{4}{\sqrt{\tan(x)}} + \frac{\left(2\arctan\left(1-\sqrt[4]{2}\sqrt{\tan(x)}\right)-2\arctan\left(1+\sqrt[4]{2}\sqrt{\tan(x)}\right)-4\sqrt[4]{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)+4\sqrt[4]{2}\,\mathrm{arc}\cdots\right.}{\cdots} + \frac{1}{4}\csc^2(x)\left(2\log(\tan(x))-\log\left(2+\tan^2(x)\right)\right)\sqrt{\cos(x)\sin^3(x)}\sqrt{\tan(x)} + \frac{4\sqrt{2}\cos^2(x)^{3/4}\,\mathrm{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{2\sin^2(x)}{3+\cos(2x)}\right)\tan^{\frac{3}{2}}(x)}{3(3+\cos(2x))^{3/4}}$$

```
[In] Integrate[(Sqrt[Cos[x]*Sin[x]^3] - 2*Sin[2*x])/(-Sqrt[Cos[x]^3*Sin[x]] + Sq
rt[Tan[x]]),x]
```

```
[Out] 4*Csc[x]*Sec[x]*Sqrt[Cos[x]^3*Sin[x]] + (Cot[x]*(-2*Log[Sec[x]^2] + 2*Log[T
an[x]] + Log[2 + Tan[x]^2])*Sqrt[Cos[x]*Sin[x]^3])/(4*Sqrt[Cos[x]^3*Sin[x]]
) + 4/Sqrt[Tan[x]] + ((2*ArcTan[1 - 2^(1/4)*Sqrt[Tan[x]]] - 2*ArcTan[1 + 2^
(1/4)*Sqrt[Tan[x]]] - 4*2^(1/4)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]] + 4*2^(1/4
)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]] + 2*2^(1/4)*Log[1 - Sqrt[2]*Sqrt[Tan[x]]
 + Tan[x]] - 2*2^(1/4)*Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]] - Log[2 - 2*2
^(1/4)*Sqrt[Tan[x]] + Sqrt[2]*Tan[x]] + Log[2 + 2*2^(1/4)*Sqrt[Tan[x]] + Sq
rt[2]*Tan[x]])*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/(2^(3/4)*Sqrt[Tan[x]]) + (Cs
c[x]^2*(2*Log[Tan[x]] - Log[2 + Tan[x]^2])*Sqrt[Cos[x]*Sin[x]^3]*Sqrt[Tan[x
]])/4 + (4*Sqrt[2]*(Cos[x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (2*Sin
[x]^2)/(3 + Cos[2*x])]*Tan[x]^(3/2))/(3*(3 + Cos[2*x])^(3/4))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 57.05 (sec) , antiderivative size = 133928, normalized size of antiderivative = 367.93

method	result	size
default	Expression too large to display	133928
parts	Expression too large to display	149826

```
[In] int((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+tan(x)^
(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cos(x)\sin^3(x)} - 2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)} + \sqrt{\tan(x)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+t
an(x)^(1/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   not i
nvertible
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(x)\sin^3(x)} - 2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)} + \sqrt{\tan(x)}}\,dx = \text{Timed out}$$

```
[In] integrate((-2*sin(2*x)+(cos(x)*sin(x)**3)**(1/2))/(-(cos(x)**3*sin(x))**(1/
2)+tan(x)**(1/2)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\cos(x)\sin^3(x)} - 2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)} + \sqrt{\tan(x)}}\,dx = \int -\frac{\sqrt{\cos(x)\sin(x)^3} - 2\sin(2x)}{\sqrt{\cos(x)^3\sin(x)} - \sqrt{\tan(x)}}\,dx$$

```
[In] integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+t
an(x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*integrate(-1/4*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(1/4)*(((((sqr
t(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*
sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(4*x) - (sqrt(2)*cos(3*x) + 2*sqrt(2)
*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2
)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x)
```

```
- sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(1/2*arctan2(
sin(x), -cos(x) + 1)) - ((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*c
os(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) +
(sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x)
+ 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2*sqrt
(2)*cos(2*x) - sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqr
t(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin(x),
 cos(x) + 1)) + (((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) -
 sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2
)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqr
t(2)*sin(2*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2*sqrt(2)*cos
(2*x) - sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*si
n(x))*cos(1/2*arctan2(sin(x), -cos(x) + 1)) + ((sqrt(2)*cos(3*x) + 2*sqrt(2
)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(
2)*sin(x))*cos(4*x) - (sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(
x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqr
t(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*
sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*s
in(1/2*arctan2(sin(x), cos(x) + 1)))*cos(1/2*arctan2(sin(2*x), cos(2*x) + 1
)) - ((((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*s
in(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2)*cos(3*x)
 + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2
*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) - sq
rt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(
1/2*arctan2(sin(x), -cos(x) + 1)) + ((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x)
 + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))
*cos(4*x) - (sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(
2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3
*x) - 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*si
n(2*x) - sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arc
tan2(sin(x), cos(x) + 1)) - (((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt
(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(4*
x) - (sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(
3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2
*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x)
- sqrt(2)*sin(x))*cos(1/2*arctan2(sin(x), -cos(x) + 1)) - ((sqrt(2)*cos(3*x
) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(
2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) +
sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*si
n(4*x) - sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) + sqrt(2)*s
in(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos
(x) + 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))*sin(1/2*arctan2(sin(2*x),
cos(2*x) + 1)))/((((2*(2*cos(2*x) + cos(x))*cos(3*x) + cos(3*x)^2 + 4*cos(2
*x)^2 + 4*cos(2*x)*cos(x) + cos(x)^2 + 2*(2*sin(2*x) + sin(x))*sin(3*x) + s
in(3*x)^2 + 4*sin(2*x)^2 + 4*sin(2*x)*sin(x) + sin(x)^2)*cos(1/2*arctan2(si
```

n(x), -cos(x) + 1))^2 + (2*(2*cos(2*x) + cos(x))*cos(3*x) + cos(3*x)^2 + 4*
cos(2*x)^2 + 4*cos(2*x)*cos(x) + cos(x)^2 + 2*(2*sin(2*x) + sin(x))*sin(3*x
) + sin(3*x)^2 + 4*sin(2*x)^2 + 4*sin(2*x)*sin(x) + sin(x)^2)*sin(1/2*arcta
n2(sin(x), -cos(x) + 1))^2)*cos(1/2*arctan2(sin(x), cos(x) + 1))^2 + ((2*(2
*cos(2*x) + cos(x))*cos(3*x) + cos(3*x)^2 + 4*cos(2*x)^2 + 4*cos(2*x)*cos(x
) + cos(x)^2 + 2*(2*sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 4*sin(2*x)^2
 + 4*sin(2*x)*sin(x) + sin(x)^2)*cos(1/2*arctan2(sin(x), -cos(x) + 1))^2 +
(2*(2*cos(2*x) + cos(x))*cos(3*x) + cos(3*x)^2 + 4*cos(2*x)^2 + 4*cos(2*x)*
cos(x) + cos(x)^2 + 2*(2*sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 4*sin(2
*x)^2 + 4*sin(2*x)*sin(x) + sin(x)^2)*sin(1/2*arctan2(sin(x), -cos(x) + 1))
^2)*sin(1/2*arctan2(sin(x), cos(x) + 1))^2)*(cos(x)^2 + sin(x)^2 + 2*cos(x)
 + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4)), x) + 2*integrate(1
/4*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(1/4)*(((((sqrt(2)*cos(3*x) -
 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x
) + sqrt(2)*sin(x))*cos(4*x) - (sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqr
t(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4
*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(
3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(1/2*arctan2(sin(x), -cos(x)
 + 1)) - ((sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)
*sin(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2)*cos(3*
x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin
(2*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) -
sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*si
n(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) +
 (((sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*
x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2)*cos(3*x) - 2*
sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) +
 sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) - sqrt(2)
*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(1/2*a
rctan2(sin(x), -cos(x) + 1)) + ((sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sq
rt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(
4*x) - (sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*si
n(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) +
 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x
) - sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(1/2*arctan2(
sin(x), cos(x) + 1)))*cos(1/2*arctan2(sin(2*x), cos(2*x) + 1)) - ((((sqrt(2
)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqr
t(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2)*cos(3*x) - 2*sqrt(2)*co
s(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*s
in(x))*sin(4*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) +
sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(1/2*arctan2(sin
(x), -cos(x) + 1)) + ((sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(
x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(4*x) - (sq
rt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2
*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)

```
*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2
)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin(x), co
s(x) + 1)) - (((sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sq
rt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(4*x) - (sqrt(2)*c
os(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2
)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*
x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x
))*cos(1/2*arctan2(sin(x), -cos(x) + 1)) - ((sqrt(2)*cos(3*x) - 2*sqrt(2)*c
os(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*
sin(x))*cos(4*x) + (sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x)
+ sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2
)*cos(3*x) + 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqr
t(2)*sin(2*x) + sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(
1/2*arctan2(sin(x), cos(x) + 1)))*sin(1/2*arctan2(sin(2*x), cos(2*x) + 1)))
/((((2*(2*cos(2*x) - cos(x))*cos(3*x) - cos(3*x)^2 - 4*cos(2*x)^2 + 4*cos(2
*x)*cos(x) - cos(x)^2 + 2*(2*sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - 4*s
in(2*x)^2 + 4*sin(2*x)*sin(x) - sin(x)^2)*cos(1/2*arctan2(sin(x), -cos(x) +
 1))^2 + (2*(2*cos(2*x) - cos(x))*cos(3*x) - cos(3*x)^2 - 4*cos(2*x)^2 + 4*
cos(2*x)*cos(x) - cos(x)^2 + 2*(2*sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2
- 4*sin(2*x)^2 + 4*sin(2*x)*sin(x) - sin(x)^2)*sin(1/2*arctan2(sin(x), -cos
(x) + 1))^2)*cos(1/2*arctan2(sin(x), cos(x) + 1))^2 + ((2*(2*cos(2*x) - cos
(x))*cos(3*x) - cos(3*x)^2 - 4*cos(2*x)^2 + 4*cos(2*x)*cos(x) - cos(x)^2 +
2*(2*sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - 4*sin(2*x)^2 + 4*sin(2*x)*s
in(x) - sin(x)^2)*cos(1/2*arctan2(sin(x), -cos(x) + 1))^2 + (2*(2*cos(2*x)
- cos(x))*cos(3*x) - cos(3*x)^2 - 4*cos(2*x)^2 + 4*cos(2*x)*cos(x) - cos(x)
^2 + 2*(2*sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - 4*sin(2*x)^2 + 4*sin(2
*x)*sin(x) - sin(x)^2)*sin(1/2*arctan2(sin(x), -cos(x) + 1))^2)*sin(1/2*arc
tan2(sin(x), cos(x) + 1))^2)*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(co
s(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4)), x) + 1/2*log(cos(x)^2 + sin(x)^2
+ 2*cos(x) + 1) + 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Giac [F]

$$\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx=\int -\frac{\sqrt{\cos\left(x\right)\sin\left(x\right)^{3}}-2\,\sin\left(2\,x\right)}{\sqrt{\cos\left(x\right)^{3}\sin\left(x\right)}-\sqrt{\tan\left(x\right)}}\,dx$$

```
[In] integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+t
an(x)^(1/2)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(x)\sin^3(x)} - 2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)} + \sqrt{\tan(x)}}\,dx = -\int \frac{2\,\sin\left(2\,x\right)-\sqrt{\cos\left(x\right)\,{\sin\left(x\right)}^3}}{\sqrt{\tan\left(x\right)}-\sqrt{{\cos\left(x\right)}^3\,\sin\left(x\right)}}\,dx$$

```
[In] int(-(2*sin(2*x) - (cos(x)*sin(x)^3)^(1/2))/(tan(x)^(1/2) - (cos(x)^3*sin(x
))^(1/2)),x)
[Out] -int((2*sin(2*x) - (cos(x)*sin(x)^3)^(1/2))/(tan(x)^(1/2) - (cos(x)^3*sin(x
))^(1/2)), x)
```

3.418 $\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{\left(\cos^5(x)\sin(x)\right)^{2/3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{\left(\cos^5(x)\sin(x)\right)^{2/3}}\,dx=-\frac{9\sin^4(x)}{10\left(\cos^5(x)\sin(x)\right)^{2/3}}-\frac{9}{4}\sec^8(x)\left(\cos^5(x)\sin(x)\right)^{4/3}+\frac{3}{2}\sqrt[3]{\cos^5(x)\sin(x)}\sqrt[3]{\sec^6(x)\tan(x)}+\frac{3}{4}\sqrt[3]{\cos^5(x)\sin(x)}\tan^2(x)\sqrt[3]{\sec^6(x)\tan(x)}+\frac{3}{14}\sqrt[3]{\cos^5(x)\sin(x)}\tan^4(x)\sqrt[3]{\sec^6(x)\tan(x)}$$

```
[Out] -9/10*sin(x)^4/(cos(x)^5*sin(x))^(2/3)-9/4*sec(x)^8*(cos(x)^5*sin(x))^(4/3)
+3/2*(cos(x)^5*sin(x))^(1/3)*(sec(x)^6*tan(x))^(1/3)+3/4*(cos(x)^5*sin(x))^
(1/3)*tan(x)^2*(sec(x)^6*tan(x))^(1/3)+3/14*(cos(x)^5*sin(x))^(1/3)*tan(x)^
4*(sec(x)^6*tan(x))^(1/3)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6851, 6865, 6874, 14}

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{\left(\cos^5(x)\sin(x)\right)^{2/3}}\,dx=-\frac{9\sin^4(x)}{10\left(\sin(x)\cos^5(x)\right)^{2/3}}-\frac{9\sin^2(x)\cos^2(x)}{4\left(\sin(x)\cos^5(x)\right)^{2/3}}+\frac{3\sin^5(x)\cos(x)\sqrt[3]{\tan(x)\sec^6(x)}}{14\left(\sin(x)\cos^5(x)\right)^{2/3}}+\frac{3\sin(x)\cos^5(x)\sqrt[3]{\tan(x)\sec^6(x)}}{2\left(\sin(x)\cos^5(x)\right)^{2/3}}+\frac{3\sin^3(x)\cos^3(x)\sqrt[3]{\tan(x)\sec^6(x)}}{4\left(\sin(x)\cos^5(x)\right)^{2/3}}$$

```
[In] Int[(-3*Tan[x] + (Sec[x]^6*Tan[x])^(1/3))/(Cos[x]^5*Sin[x])^(2/3),x]
```

```
[Out] (-9*Cos[x]^2*Sin[x]^2)/(4*(Cos[x]^5*Sin[x])^(2/3)) - (9*Sin[x]^4)/(10*(Cos[
x]^5*Sin[x])^(2/3)) + (3*Cos[x]^5*Sin[x]*(Sec[x]^6*Tan[x])^(1/3))/(2*(Cos[x
]^5*Sin[x])^(2/3)) + (3*Cos[x]^3*Sin[x]^3*(Sec[x]^6*Tan[x])^(1/3))/(4*(Cos[
x]^5*Sin[x])^(2/3)) + (3*Cos[x]*Sin[x]^5*(Sec[x]^6*Tan[x])^(1/3))/(14*(Cos[
x]^5*Sin[x])^(2/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{-3x+\sqrt[3]{x\,(1+x^2)^3}}{\left(\frac{x}{(1+x^2)^3}\right)^{2/3}(1+x^2)}\,dx, x, \tan(x)\right)$$

$$= \frac{\left(\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int \frac{(1+x^2)\left(-3x+\sqrt[3]{x\,(1+x^2)^3}\right)}{x^{2/3}}\,dx, x, \tan(x)\right)}{(\cos^5(x)\sin(x))^{2/3}}$$

$$= \frac{\left(3\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int (1+x^6)\left(-3x^3+\sqrt[3]{x^3\,(1+x^6)^3}\right)\,dx, x, \sqrt[3]{\tan(x)}\right)}{(\cos^5(x)\sin(x))^{2/3}}$$

$$=\frac{\left(3\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int\left(-3x^3+\sqrt[3]{x^3\left(1+x^6\right)^3}-x^6\left(3x^3-\sqrt[3]{\left(x+x^7\right)^3}\right)\right)\,dx,x,\sqrt[3]{\tan(x}\right.}{\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$=-\frac{9\cos^2(x)\sin^2(x)}{4\left(\cos^5(x)\sin(x)\right)^{2/3}}+\frac{\left(3\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int\sqrt[3]{x^3\left(1+x^6\right)^3}\,dx,x,\sqrt[3]{\tan(x)}\right)}{\left(\cos^5(x)\sin(x)\right)^{2/3}}-\frac{\left(3\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int x^6\left(3x^3-\sqrt[3]{\left(x+x^7\right)^3}\right)\,dx,x,\sqrt[3]{\tan(x)}\right)}{\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$=-\frac{9\cos^2(x)\sin^2(x)}{4\left(\cos^5(x)\sin(x)\right)^{2/3}}-\frac{\left(3\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int\left(3x^9-x^6\sqrt[3]{x^3\left(1+x^6\right)^3}\right)\,dx,x,\sqrt[3]{\tan(x)}\right)}{\left(\cos^5(x)\sin(x)\right)^{2/3}}+\frac{\left(3\cos^6(x)\sqrt[3]{\tan(x)}\sqrt[3]{\sec^6(x)\tan(x)}\right)\text{Subst}\left(\int x\left(1+x^6\right)\,dx,x,\sqrt[3]{\tan(x)}\right)}{\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$=-\frac{9\cos^2(x)\sin^2(x)}{4\left(\cos^5(x)\sin(x)\right)^{2/3}}-\frac{9\sin^4(x)}{10\left(\cos^5(x)\sin(x)\right)^{2/3}}+\frac{\left(3\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int x^6\sqrt[3]{x^3\left(1+x^6\right)^3}\,dx,x,\sqrt[3]{\tan(x)}\right)}{\left(\cos^5(x)\sin(x)\right)^{2/3}}+\frac{\left(3\cos^6(x)\sqrt[3]{\tan(x)}\sqrt[3]{\sec^6(x)\tan(x)}\right)\text{Subst}\left(\int\left(x+x^7\right)\,dx,x,\sqrt[3]{\tan(x)}\right)}{\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$=-\frac{9\cos^2(x)\sin^2(x)}{4\left(\cos^5(x)\sin(x)\right)^{2/3}}-\frac{9\sin^4(x)}{10\left(\cos^5(x)\sin(x)\right)^{2/3}}+\frac{3\cos^5(x)\sin(x)\sqrt[3]{\sec^6(x)\tan(x)}}{2\left(\cos^5(x)\sin(x)\right)^{2/3}}+\frac{3\cos^3(x)\sin^3(x)\sqrt[3]{\sec^6(x)\tan(x)}}{8\left(\cos^5(x)\sin(x)\right)^{2/3}}+\frac{\left(3\cos^6(x)\sqrt[3]{\tan(x)}\sqrt[3]{\sec^6(x)\tan(x)}\right)\text{Subst}\left(\int x^7\left(1+x^6\right)\,dx,x,\sqrt[3]{\tan(x)}\right)}{\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$= -\frac{9\cos^2(x)\sin^2(x)}{4(\cos^5(x)\sin(x))^{2/3}} - \frac{9\sin^4(x)}{10(\cos^5(x)\sin(x))^{2/3}} + \frac{3\cos^5(x)\sin(x)\sqrt[3]{\sec^6(x)\tan(x)}}{2(\cos^5(x)\sin(x))^{2/3}} + \frac{3\cos^3(x)\sin^3(x)\sqrt[3]{\sec^6(x)\tan(x)}}{8(\cos^5(x)\sin(x))^{2/3}} + \frac{\left(3\cos^6(x)\sqrt[3]{\tan(x)}\sqrt[3]{\sec^6(x)\tan(x)}\right)\text{Subst}\left(\int(x^7+x^{13})\,dx, x, \sqrt[3]{\tan(x)}\right)}{(\cos^5(x)\sin(x))^{2/3}}$$

$$= -\frac{9\cos^2(x)\sin^2(x)}{4(\cos^5(x)\sin(x))^{2/3}} - \frac{9\sin^4(x)}{10(\cos^5(x)\sin(x))^{2/3}} + \frac{3\cos^5(x)\sin(x)\sqrt[3]{\sec^6(x)\tan(x)}}{2(\cos^5(x)\sin(x))^{2/3}} + \frac{3\cos^3(x)\sin^3(x)\sqrt[3]{\sec^6(x)\tan(x)}}{4(\cos^5(x)\sin(x))^{2/3}} + \frac{3\cos(x)\sin^5(x)\sqrt[3]{\sec^6(x)\tan(x)}}{14(\cos^5(x)\sin(x))^{2/3}}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.46

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx = -\frac{3\sin(x)\left(924\sin(x)+252\sin(3x)-5(158\cos(x)+57\cos(3x)+9\cos(5x))\sqrt[3]{\sec^6(x)\tan(x)}\right)}{2240(\cos^5(x)\sin(x))^{2/3}}$$

```
[In] Integrate[(-3*Tan[x] + (Sec[x]^6*Tan[x])^(1/3))/(Cos[x]^5*Sin[x])^(2/3),x]
[Out] (-3*Sin[x]*(924*Sin[x] + 252*Sin[3*x] - 5*(158*Cos[x] + 57*Cos[3*x] + 9*Cos
[5*x])*(Sec[x]^6*Tan[x])^(1/3)))/(2240*(Cos[x]^5*Sin[x])^(2/3))
```

Maple [F]

$$\int \frac{\left(\frac{\sin(x)}{\cos(x)^7}\right)^{\frac{1}{3}} - 3\tan(x)}{((\cos^5(x))\sin(x))^{\frac{2}{3}}}dx$$

```
[In] int(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x)
[Out] int(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.45

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx = -\frac{3\left(\cos(x)^5\sin(x)\right)^{\frac{1}{3}}\left(21\left(3\cos(x)^2+2\right)\sin(x)-5\left(9\cos(x)^5+3\cos(x)^3+2\cos(x)\right)\left(\frac{\sin(x)}{\cos(x)^7}\right)^{\frac{1}{3}}\right)}{140\cos(x)^5}$$

```
[In] integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, alg
orithm="fricas")
```

```
[Out] -3/140*(cos(x)^5*sin(x))^(1/3)*(21*(3*cos(x)^2 + 2)*sin(x) - 5*(9*cos(x)^5
+ 3*cos(x)^3 + 2*cos(x))*(sin(x)/cos(x)^7)^(1/3))/cos(x)^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx = \text{Timed out}$$

```
[In] integrate(((sin(x)/cos(x)**7)**(1/3)-3*tan(x))/(cos(x)**5*sin(x))**(2/3),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx = -\frac{3}{20}\tan(x)^{\frac{20}{3}}-\frac{3}{7}\tan(x)^{\frac{14}{3}}-\frac{9}{10}\tan(x)^{\frac{10}{3}} - \frac{3}{8}\tan(x)^{\frac{8}{3}}-\frac{9}{4}\tan(x)^{\frac{4}{3}}+\frac{3\left(14\tan(x)^7+60\tan(x)^5+105\tan(x)^3+140\tan(x)\right)}{280\tan(x)^{\frac{1}{3}}}$$

```
[In] integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, alg
orithm="maxima")
```

```
[Out] -3/20*tan(x)^(20/3) - 3/7*tan(x)^(14/3) - 9/10*tan(x)^(10/3) - 3/8*tan(x)^(
8/3) - 9/4*tan(x)^(4/3) + 3/280*(14*tan(x)^7 + 60*tan(x)^5 + 105*tan(x)^3 +
 140*tan(x))/tan(x)^(1/3)
```

Giac [F]

$$\int \frac{-3\tan(x) + \sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx = \int \frac{\left(\frac{\sin(x)}{\cos(x)^7}\right)^{\frac{1}{3}} - 3\tan(x)}{(\cos(x)^5\sin(x))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, alg
orithm="giac")
[Out] integrate(((sin(x)/cos(x)^7)^(1/3) - 3*tan(x))/(cos(x)^5*sin(x))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-3\tan(x) + \sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx = \int -\frac{3\tan(x) - \left(\frac{\sin(x)}{\cos(x)^7}\right)^{1/3}}{(\cos(x)^5\sin(x))^{2/3}}\,dx$$

```
[In] int(-(3*tan(x) - (sin(x)/cos(x)^7)^(1/3))/(cos(x)^5*sin(x))^(2/3),x)
[Out] int(-(3*tan(x) - (sin(x)/cos(x)^7)^(1/3))/(cos(x)^5*sin(x))^(2/3), x)
```

3.419 $\int (1+2\cos^2(x))^{5/2}\sin(x)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int (1+2\cos^2(x))^{5/2}\sin(x)\,dx = -\frac{5\text{arcsinh}\left(\sqrt{2}\cos(x)\right)}{16\sqrt{2}} - \frac{5}{16}\cos(x)\sqrt{1+2\cos^2(x)} - \frac{5}{24}\cos(x)\left(1+2\cos^2(x)\right)^{3/2} - \frac{1}{6}\cos(x)\left(1+2\cos^2(x)\right)^{5/2}$$

```
[Out] -5/24*cos(x)*(1+2*cos(x)^2)^(3/2)-1/6*cos(x)*(1+2*cos(x)^2)^(5/2)-5/32*arcs
inh(cos(x)*2^(1/2))*2^(1/2)-5/16*cos(x)*(1+2*cos(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3269, 201, 221}

$$\int (1+2\cos^2(x))^{5/2}\sin(x)\,dx = -\frac{5\text{arcsinh}\left(\sqrt{2}\cos(x)\right)}{16\sqrt{2}} - \frac{1}{6}\cos(x)(\cos(2x)+2)^{5/2} - \frac{5}{24}\cos(x)(\cos(2x)+2)^{3/2} - \frac{5}{16}\cos(x)\sqrt{\cos(2x)+2}$$

```
[In] Int[(1 + 2*Cos[x]^2)^(5/2)*Sin[x],x]
```

```
[Out] (-5*ArcSinh[Sqrt[2]*Cos[x]])/(16*Sqrt[2]) - (5*Cos[x]*Sqrt[2 + Cos[2*x]])/1
6 - (5*Cos[x]*(2 + Cos[2*x])^(3/2))/24 - (Cos[x]*(2 + Cos[2*x])^(5/2))/6
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \left(1+2x^2\right)^{5/2}\,dx,x,\cos(x)\right)\\
&= -\frac{1}{6}\cos(x)(2+\cos(2x))^{5/2}-\frac{5}{6}\text{Subst}\left(\int \left(1+2x^2\right)^{3/2}\,dx,x,\cos(x)\right)\\
&= -\frac{5}{24}\cos(x)(2+\cos(2x))^{3/2}-\frac{1}{6}\cos(x)(2+\cos(2x))^{5/2}-\frac{5}{8}\text{Subst}\left(\int \sqrt{1+2x^2}\,dx,x,\cos(x)\right)\\
&= -\frac{5}{16}\cos(x)\sqrt{2+\cos(2x)}-\frac{5}{24}\cos(x)(2+\cos(2x))^{3/2}\\
&\quad -\frac{1}{6}\cos(x)(2+\cos(2x))^{5/2}-\frac{5}{16}\text{Subst}\left(\int \frac{1}{\sqrt{1+2x^2}}\,dx,x,\cos(x)\right)\\
&= -\frac{5\text{arcsinh}\left(\sqrt{2}\cos(x)\right)}{16\sqrt{2}}-\frac{5}{16}\cos(x)\sqrt{2+\cos(2x)}\\
&\quad -\frac{5}{24}\cos(x)(2+\cos(2x))^{3/2}-\frac{1}{6}\cos(x)(2+\cos(2x))^{5/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = \frac{1}{96}\left(-2\sqrt{2+\cos(2x)}(92\cos(x)+23\cos(3x)+2\cos(5x)) - 15\sqrt{2}\log\left(\sqrt{2}\cos(x)+\sqrt{2+\cos(2x)}\right)\right)$$

```
[In] Integrate[(1 + 2*Cos[x]^2)^(5/2)*Sin[x],x]
[Out] (-2*Sqrt[2 + Cos[2*x]]*(92*Cos[x] + 23*Cos[3*x] + 2*Cos[5*x]) - 15*Sqrt[2]*
Log[Sqrt[2]*Cos[x] + Sqrt[2 + Cos[2*x]]])/96
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

method	result	si
derivativedivides	$-\frac{5\cos(x)\left(1+2\left(\cos^2(x)\right)\right)^{\frac{3}{2}}}{24}-\frac{\cos(x)\left(1+2\left(\cos^2(x)\right)\right)^{\frac{5}{2}}}{6}-\frac{5\,\mathrm{arcsinh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{32}-\frac{5\cos(x)\sqrt{1+2\left(\cos^2(x)\right)}}{16}$	56
default	$-\frac{5\cos(x)\left(1+2\left(\cos^2(x)\right)\right)^{\frac{3}{2}}}{24}-\frac{\cos(x)\left(1+2\left(\cos^2(x)\right)\right)^{\frac{5}{2}}}{6}-\frac{5\,\mathrm{arcsinh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{32}-\frac{5\cos(x)\sqrt{1+2\left(\cos^2(x)\right)}}{16}$	56

```
[In] int((1+2*cos(x)^2)^(5/2)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -5/24*cos(x)*(1+2*cos(x)^2)^(3/2)-1/6*cos(x)*(1+2*cos(x)^2)^(5/2)-5/32*arcs
inh(cos(x)*2^(1/2))*2^(1/2)-5/16*cos(x)*(1+2*cos(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.48

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = -\frac{1}{48}\left(32\cos(x)^5+52\cos(x)^3+33\cos(x)\right)\sqrt{2\cos(x)^2+1} + \frac{5}{256}\sqrt{2}\log\left(2048\cos(x)^8+2048\cos(x)^6+640\cos(x)^4+64\cos(x)^2 - 8\left(128\sqrt{2}\cos(x)^7+96\sqrt{2}\cos(x)^5+20\sqrt{2}\cos(x)^3+\sqrt{2}\cos(x)\right)\sqrt{2\cos(x)^2+1} + 1\right)$$

```
[In] integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/48*(32*cos(x)^5 + 52*cos(x)^3 + 33*cos(x))*sqrt(2*cos(x)^2 + 1) + 5/256*
sqrt(2)*log(2048*cos(x)^8 + 2048*cos(x)^6 + 640*cos(x)^4 + 64*cos(x)^2 - 8*
(128*sqrt(2)*cos(x)^7 + 96*sqrt(2)*cos(x)^5 + 20*sqrt(2)*cos(x)^3 + sqrt(2)
*cos(x))*sqrt(2*cos(x)^2 + 1) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = \text{Timed out}$$

```
[In] integrate((1+2*cos(x)**2)**(5/2)*sin(x),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = -\frac{1}{6}\left(2\cos(x)^2+1\right)^{\frac{5}{2}}\cos(x) - \frac{5}{24}\left(2\cos(x)^2+1\right)^{\frac{3}{2}}\cos(x) - \frac{5}{32}\sqrt{2}\operatorname{arsinh}\left(\sqrt{2}\cos(x)\right) - \frac{5}{16}\sqrt{2\cos(x)^2+1}\cos(x)$$

```
[In] integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="maxima")
[Out] -1/6*(2*cos(x)^2 + 1)^(5/2)*cos(x) - 5/24*(2*cos(x)^2 + 1)^(3/2)*cos(x) - 5
/32*sqrt(2)*arcsinh(sqrt(2)*cos(x)) - 5/16*sqrt(2*cos(x)^2 + 1)*cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = -\frac{1}{48}\left(4\left(8\cos(x)^2+13\right)\cos(x)^2+33\right)\sqrt{2\cos(x)^2+1}\cos(x) + \frac{5}{32}\sqrt{2}\log\left(-\sqrt{2}\cos(x)+\sqrt{2\cos(x)^2+1}\right)$$

```
[In] integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="giac")
[Out] -1/48*(4*(8*cos(x)^2 + 13)*cos(x)^2 + 33)*sqrt(2*cos(x)^2 + 1)*cos(x) + 5/3
2*sqrt(2)*log(-sqrt(2)*cos(x) + sqrt(2*cos(x)^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.59

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = -\frac{5\sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\,\cos\left(x\right)\right)}{32} - \frac{\sqrt{2}\,\sqrt{\cos\left(x\right)^2+\frac{1}{2}}\,\left(\frac{4\cos(x)^5}{3}+\frac{13\cos(x)^3}{6}+\frac{11\,\cos(x)}{8}\right)}{2}$$

```
[In] int(sin(x)*(2*cos(x)^2 + 1)^(5/2),x)
```

```
[Out] - (5*2^(1/2)*asinh(2^(1/2)*cos(x)))/32 - (2^(1/2)*(cos(x)^2 + 1/2)^(1/2)*((
11*cos(x))/8 + (13*cos(x)^3)/6 + (4*cos(x)^5)/3))/2
```

3.420 $\int \cos(x)\left(5\cos^2(x)+\sin^2(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \cos(x)\left(5\cos^2(x)+\sin^2(x)\right)^{5/2}\,dx=\frac{625}{32}\arcsin\left(\frac{2\sin(x)}{\sqrt{5}}\right)+\frac{125}{16}\sin(x)\sqrt{5-4\sin^2(x)}+\frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2}+\frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}$$

```
[Out] 625/32*arcsin(2/5*sin(x)*5^(1/2))+25/24*sin(x)*(5-4*sin(x)^2)^(3/2)+1/6*sin
(x)*(5-4*sin(x)^2)^(5/2)+125/16*sin(x)*(5-4*sin(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4441, 201, 222}

$$\int \cos(x)\left(5\cos^2(x)+\sin^2(x)\right)^{5/2}\,dx=\frac{625}{32}\arcsin\left(\frac{2\sin(x)}{\sqrt{5}}\right)+\frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}+\frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2}+\frac{125}{16}\sin(x)\sqrt{5-4\sin^2(x)}$$

```
[In] Int[Cos[x]*(5*Cos[x]^2 + Sin[x]^2)^(5/2),x]
```

```
[Out] (625*ArcSin[(2*Sin[x])/Sqrt[5]])/32 + (125*Sin[x]*Sqrt[5 - 4*Sin[x]^2])/16
+ (25*Sin[x]*(5 - 4*Sin[x]^2)^(3/2))/24 + (Sin[x]*(5 - 4*Sin[x]^2)^(5/2))/6
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \left(5-4x^2\right)^{5/2}\,dx, x, \sin(x)\right)\\
&= \frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}+\frac{25}{6}\text{Subst}\left(\int \left(5-4x^2\right)^{3/2}\,dx, x, \sin(x)\right)\\
&= \frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2}\\
&\quad+\frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}+\frac{125}{8}\text{Subst}\left(\int \sqrt{5-4x^2}\,dx, x, \sin(x)\right)\\
&= \frac{125}{16}\sin(x)\sqrt{5-4\sin^2(x)}+\frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2}\\
&\quad+\frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}+\frac{625}{16}\text{Subst}\left(\int \frac{1}{\sqrt{5-4x^2}}\,dx, x, \sin(x)\right)\\
&= \frac{625}{32}\arcsin\left(\frac{2\sin(x)}{\sqrt{5}}\right)+\frac{125}{16}\sin(x)\sqrt{5-4\sin^2(x)}\\
&\quad+\frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2}+\frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

$$\int \cos(x)\left(5\cos^2(x)+\sin^2(x)\right)^{5/2}\,dx=\frac{1}{96}\left(1875\arcsin\left(\frac{2\sin(x)}{\sqrt{5}}\right)+2\sqrt{3+2\cos(2x)}(515\sin(x)+90\sin(3x)+8\sin(5x))\right)$$

```
[In] Integrate[Cos[x]*(5*Cos[x]^2 + Sin[x]^2)^(5/2),x]
[Out] (1875*ArcSin[(2*Sin[x])/Sqrt[5]] + 2*Sqrt[3 + 2*Cos[2*x]]*(515*Sin[x] + 90*
Sin[3*x] + 8*Sin[5*x]))/96
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

method	result	si
derivativedivides	$\frac{625\arcsin\left(\frac{2\sin(x)\sqrt{5}}{5}\right)}{32}+\frac{25\sin(x)\left(5-4\left(\sin^2(x)\right)\right)^{\frac{3}{2}}}{24}+\frac{\sin(x)\left(5-4\left(\sin^2(x)\right)\right)^{\frac{5}{2}}}{6}+\frac{125\sin(x)\sqrt{5-4\left(\sin^2(x)\right)}}{16}$	54
default	$\frac{625\arcsin\left(\frac{2\sin(x)\sqrt{5}}{5}\right)}{32}+\frac{25\sin(x)\left(5-4\left(\sin^2(x)\right)\right)^{\frac{3}{2}}}{24}+\frac{\sin(x)\left(5-4\left(\sin^2(x)\right)\right)^{\frac{5}{2}}}{6}+\frac{125\sin(x)\sqrt{5-4\left(\sin^2(x)\right)}}{16}$	54

```
[In] int(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 625/32*arcsin(2/5*sin(x)*5^(1/2))+25/24*sin(x)*(5-4*sin(x)^2)^(3/2)+1/6*sin
(x)*(5-4*sin(x)^2)^(5/2)+125/16*sin(x)*(5-4*sin(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \cos(x)\left(5\cos^2(x)+\sin^2(x)\right)^{5/2}\,dx=\frac{1}{48}\left(128\cos(x)^4+264\cos(x)^2+433\right)\sqrt{4\cos(x)^2+1}\sin(x)+\frac{625}{64}\arctan\left(\frac{4\left(8\cos(x)^2-3\right)\sqrt{4\cos(x)^2+1}\sin(x)-25\cos(x)\sin(x)}{64\cos(x)^4-23\cos(x)^2-16}\right)+\frac{625}{64}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

```
[In] integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/48*(128*cos(x)^4 + 264*cos(x)^2 + 433)*sqrt(4*cos(x)^2 + 1)*sin(x) + 625/
64*arctan((4*(8*cos(x)^2 - 3)*sqrt(4*cos(x)^2 + 1)*sin(x) - 25*cos(x)*sin(x
))/(64*cos(x)^4 - 23*cos(x)^2 - 16)) + 625/64*arctan(sin(x)/cos(x))
```

Sympy [F(-1)]

Timed out.

$$\int \cos(x) \left(5\cos^2(x) + \sin^2(x)\right)^{5/2} \, dx = \text{Timed out}$$

```
[In] integrate(cos(x)*(5*cos(x)**2+sin(x)**2)**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int \cos(x) \left(5\cos^2(x) + \sin^2(x)\right)^{5/2} \, dx = \frac{1}{6} \left(-4 \sin(x)^2 + 5\right)^{\frac{5}{2}} \sin(x) + \frac{25}{24} \left(-4 \sin(x)^2 + 5\right)^{\frac{3}{2}} \sin(x) + \frac{125}{16} \sqrt{-4 \sin(x)^2 + 5} \sin(x) + \frac{625}{32} \arcsin\left(\frac{2}{5} \sqrt{5} \sin(x)\right)$$

```
[In] integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="maxima")
[Out] 1/6*(-4*sin(x)^2 + 5)^(5/2)*sin(x) + 25/24*(-4*sin(x)^2 + 5)^(3/2)*sin(x) +
 125/16*sqrt(-4*sin(x)^2 + 5)*sin(x) + 625/32*arcsin(2/5*sqrt(5)*sin(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int \cos(x) \left(5\cos^2(x) + \sin^2(x)\right)^{5/2} \, dx = \frac{1}{48} \left(8 \left(16 \sin(x)^2 - 65\right) \sin(x)^2 + 825\right) \sqrt{-4 \sin(x)^2 + 5} \sin(x) + \frac{625}{32} \arcsin\left(\frac{2}{5} \sqrt{5} \sin(x)\right)$$

```
[In] integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="giac")
[Out] 1/48*(8*(16*sin(x)^2 - 65)*sin(x)^2 + 825)*sqrt(-4*sin(x)^2 + 5)*sin(x) + 6
25/32*arcsin(2/5*sqrt(5)*sin(x))
```

Mupad [F(-1)]

Timed out.

$$\int \cos(x) \left(5\cos^2(x)+\sin^2(x)\right)^{5/2} \, dx = \int \cos\left(x\right) \, \left(5\cos\left(x\right)^2+\sin\left(x\right)^2\right)^{5/2} \, dx$$

```
[In] int(cos(x)*(5*cos(x)^2 + sin(x)^2)^(5/2),x)
[Out] int(cos(x)*(5*cos(x)^2 + sin(x)^2)^(5/2), x)
```

3.421 $\int \cos(x) \left(-\cos^2(x) - 5\sin^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \cos(x) \left(-\cos^2(x) - 5\sin^2(x)\right)^{3/2} dx = \frac{3}{16}\arctan\left(\frac{2\sin(x)}{\sqrt{-1-4\sin^2(x)}}\right) - \frac{3}{8}\sin(x)\sqrt{-1-4\sin^2(x)} + \frac{1}{4}\sin(x)\left(-1-4\sin^2(x)\right)^{3/2}$$

```
[Out] 3/16*arctan(2*sin(x)/(-1-4*sin(x)^2)^(1/2))+1/4*sin(x)*(-1-4*sin(x)^2)^(3/2
)-3/8*sin(x)*(-1-4*sin(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4441, 201, 223, 209}

$$\int \cos(x) \left(-\cos^2(x) - 5\sin^2(x)\right)^{3/2} dx = \frac{3}{16}\arctan\left(\frac{2\sin(x)}{\sqrt{-4\sin^2(x)-1}}\right) + \frac{1}{4}\sin(x)\left(-4\sin^2(x)-1\right)^{3/2} - \frac{3}{8}\sin(x)\sqrt{-4\sin^2(x)-1}$$

```
[In] Int[Cos[x]*(-Cos[x]^2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] (3*ArcTan[(2*Sin[x])/Sqrt[-1 - 4*Sin[x]^2]])/16 - (3*Sin[x]*Sqrt[-1 - 4*Sin
[x]^2])/8 + (Sin[x]*(-1 - 4*Sin[x]^2)^(3/2))/4
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \left(-1-4x^2\right)^{3/2}\,dx, x, \sin(x)\right)\\
&= \frac{1}{4}\sin(x)\left(-1-4\sin^2(x)\right)^{3/2} - \frac{3}{4}\text{Subst}\left(\int \sqrt{-1-4x^2}\,dx, x, \sin(x)\right)\\
&= -\frac{3}{8}\sin(x)\sqrt{-1-4\sin^2(x)} + \frac{1}{4}\sin(x)\left(-1-4\sin^2(x)\right)^{3/2}\\
&\quad + \frac{3}{8}\text{Subst}\left(\int \frac{1}{\sqrt{-1-4x^2}}\,dx, x, \sin(x)\right)\\
&= -\frac{3}{8}\sin(x)\sqrt{-1-4\sin^2(x)} + \frac{1}{4}\sin(x)\left(-1-4\sin^2(x)\right)^{3/2}\\
&\quad + \frac{3}{8}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{\sin(x)}{\sqrt{-1-4\sin^2(x)}}\right)\\
&= \frac{3}{16}\arctan\left(\frac{2\sin(x)}{\sqrt{-1-4\sin^2(x)}}\right) - \frac{3}{8}\sin(x)\sqrt{-1-4\sin^2(x)} + \frac{1}{4}\sin(x)\left(-1-4\sin^2(x)\right)^{3/2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int \cos(x) \left(-\cos^2(x) -5\sin^2(x)\right)^{3/2} dx = \frac{\sqrt{-3+2\cos(2x)}\left(-3\text{arcsinh}(2\sin(x))+2\sqrt{3-2\cos(2x)}(-11\sin(x)+2\sin(3x))\right)}{16\sqrt{1+4\sin^2(x)}}$$

```
[In] Integrate[Cos[x]*(-Cos[x]^2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] (Sqrt[-3 + 2*Cos[2*x]]*(-3*ArcSinh[2*Sin[x]] + 2*Sqrt[3 - 2*Cos[2*x]]*(-11*
Sin[x] + 2*Sin[3*x])))/(16*Sqrt[1 + 4*Sin[x]^2])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{3\arctan\left(\frac{2\sin(x)}{\sqrt{-1-4\left(\sin^2(x)\right)}}\right)}{16}+\frac{\sin(x)\left(-1-4\left(\sin^2(x)\right)\right)^{\frac{3}{2}}}{4}-\frac{3\sin(x)\sqrt{-1-4\left(\sin^2(x)\right)}}{8}$	47
default	$\frac{3\arctan\left(\frac{2\sin(x)}{\sqrt{-1-4\left(\sin^2(x)\right)}}\right)}{16}+\frac{\sin(x)\left(-1-4\left(\sin^2(x)\right)\right)^{\frac{3}{2}}}{4}-\frac{3\sin(x)\sqrt{-1-4\left(\sin^2(x)\right)}}{8}$	47

```
[In] int(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/16*arctan(2*sin(x)/(-1-4*sin(x)^2)^(1/2))+1/4*sin(x)*(-1-4*sin(x)^2)^(3/2
)-3/8*sin(x)*(-1-4*sin(x)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.12

$$\int \cos(x) \left(-\cos^2(x) -5\sin^2(x)\right)^{3/2} dx = \frac{1}{128}\left(12i\,e^{(4i\,x)}\log\left(-\frac{1}{2}\sqrt{e^{(4i\,x)}-3\,e^{(2i\,x)}+1}\left(4\,e^{(2i\,x)}-5\right)+2\,e^{(4i\,x)}-\frac{11}{2}\,e^{(2i\,x)}+\frac{5}{2}\right)-\right.$$

```
[In] integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(12*I*e^(4*I*x)*log(-1/2*sqrt(e^(4*I*x) - 3*e^(2*I*x) + 1)*(4*e^(2*I*
x) - 5) + 2*e^(4*I*x) - 11/2*e^(2*I*x) + 5/2) - 12*I*e^(4*I*x)*log(sqrt(e^(
```

```
4*I*x) - 3*e^(2*I*x) + 1) - e^(2*I*x) - 1) - 8*(2*I*e^(6*I*x) - 11*I*e^(4*I
*x) + 11*I*e^(2*I*x) - 2*I)*sqrt(e^(4*I*x) - 3*e^(2*I*x) + 1) - 145*I*e^(4*
I*x))*e^(-4*I*x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(x)\left(-\cos^2(x)-5\sin^2(x)\right)^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(x)*(-cos(x)**2-5*sin(x)**2)**(3/2),x)
[Out] Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \cos(x)\left(-\cos^2(x)-5\sin^2(x)\right)^{3/2}\,dx = \frac{1}{4}\left(-4\sin(x)^2-1\right)^{\frac{3}{2}}\sin(x) - \frac{3}{8}\sqrt{-4\sin(x)^2-1}\,\sin(x) - \frac{3}{16}i\,\operatorname{arsinh}(2\sin(x))$$

```
[In] integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="maxima")
[Out] 1/4*(-4*sin(x)^2 - 1)^(3/2)*sin(x) - 3/8*sqrt(-4*sin(x)^2 - 1)*sin(x) - 3/1
6*I*arcsinh(2*sin(x))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \cos(x)\left(-\cos^2(x)-5\sin^2(x)\right)^{3/2}\,dx = -\frac{1}{8}i\left(8\sin(x)^2+5\right)\sqrt{4\sin(x)^2+1}\,\sin(x) + \frac{3}{16}i\log\left(\sqrt{4\sin(x)^2+1}-2\sin(x)\right)$$

```
[In] integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="giac")
[Out] -1/8*I*(8*sin(x)^2 + 5)*sqrt(4*sin(x)^2 + 1)*sin(x) + 3/16*I*log(sqrt(4*sin
(x)^2 + 1) - 2*sin(x))
```

Mupad [F(-1)]

Timed out.

$$\int \cos(x)\left(-\cos^2(x)-5\sin^2(x)\right)^{3/2}\,dx=\int \cos\left(x\right)\,\left(-\cos\left(x\right)^2-5\sin\left(x\right)^2\right)^{3/2}\,dx$$

```
[In] int(cos(x)*(- cos(x)^2 - 5*sin(x)^2)^(3/2),x)
[Out] int(cos(x)*(- cos(x)^2 - 5*sin(x)^2)^(3/2), x)
```

3.422 $\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx = \frac{\cos(x)}{10\,(-2+7\cos^2(x))^{5/2}} - \frac{\cos(x)}{15\,(-2+7\cos^2(x))^{3/2}} + \frac{\cos(x)}{15\sqrt{-2+7\cos^2(x)}}$$

```
[Out] 1/10*cos(x)/(-2+7*cos(x)^2)^(5/2)-1/15*cos(x)/(-2+7*cos(x)^2)^(3/2)+1/15*co
s(x)/(-2+7*cos(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4442, 198, 197}

$$\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx = \frac{\cos(x)}{15\sqrt{7\cos^2(x)-2}} - \frac{\cos(x)}{15\,(7\cos^2(x)-2)^{3/2}} + \frac{\cos(x)}{10\,(7\cos^2(x)-2)^{5/2}}$$

```
[In] Int[Sin[x]/(5*Cos[x]^2 - 2*Sin[x]^2)^(7/2),x]
```

```
[Out] Cos[x]/(10*(-2 + 7*Cos[x]^2)^(5/2)) - Cos[x]/(15*(-2 + 7*Cos[x]^2)^(3/2)) +
 Cos[x]/(15*Sqrt[-2 + 7*Cos[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4442

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{(-2+7x^2)^{7/2}}\,dx, x, \cos(x)\right)\\
&= \frac{\cos(x)}{10\,(-2+7\cos^2(x))^{5/2}} + \frac{2}{5}\text{Subst}\left(\int \frac{1}{(-2+7x^2)^{5/2}}\,dx, x, \cos(x)\right)\\
&= \frac{\cos(x)}{10\,(-2+7\cos^2(x))^{5/2}} - \frac{\cos(x)}{15\,(-2+7\cos^2(x))^{3/2}} - \frac{2}{15}\text{Subst}\left(\int \frac{1}{(-2+7x^2)^{3/2}}\,dx, x, \cos(x)\right)\\
&= \frac{\cos(x)}{10\,(-2+7\cos^2(x))^{5/2}} - \frac{\cos(x)}{15\,(-2+7\cos^2(x))^{3/2}} + \frac{\cos(x)}{15\sqrt{-2+7\cos^2(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

$$\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx = \frac{\cos(x)(67+56\cos(2x)+49\cos(4x))}{15\sqrt{2}(3+7\cos(2x))^{5/2}}$$

```
[In] Integrate[Sin[x]/(5*Cos[x]^2 - 2*Sin[x]^2)^(7/2),x]
[Out] (Cos[x]*(67 + 56*Cos[2*x] + 49*Cos[4*x]))/(15*Sqrt[2]*(3 + 7*Cos[2*x])^(5/2
))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\cos(x)}{10(-2+7(\cos^2(x)))^{\frac{5}{2}}}-\frac{\cos(x)}{15(-2+7(\cos^2(x)))^{\frac{3}{2}}}+\frac{\cos(x)}{15\sqrt{-2+7(\cos^2(x))}}$	44
default	$\frac{\cos(x)}{10(-2+7(\cos^2(x)))^{\frac{5}{2}}}-\frac{\cos(x)}{15(-2+7(\cos^2(x)))^{\frac{3}{2}}}+\frac{\cos(x)}{15\sqrt{-2+7(\cos^2(x))}}$	44

```
[In] int(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/10*cos(x)/(-2+7*cos(x)^2)^(5/2)-1/15*cos(x)/(-2+7*cos(x)^2)^(3/2)+1/15*co
s(x)/(-2+7*cos(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx = \frac{\left(98\,\cos\left(x\right)^{5}-70\,\cos\left(x\right)^{3}+15\,\cos\left(x\right)\right)\sqrt{7\,\cos\left(x\right)^{2}-2}}{30\,\left(343\,\cos\left(x\right)^{6}-294\,\cos\left(x\right)^{4}+84\,\cos\left(x\right)^{2}-8\right)}$$

```
[In] integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="fricas")
[Out] 1/30*(98*cos(x)^5 - 70*cos(x)^3 + 15*cos(x))*sqrt(7*cos(x)^2 - 2)/(343*cos(
x)^6 - 294*cos(x)^4 + 84*cos(x)^2 - 8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(x)/(5*cos(x)**2-2*sin(x)**2)**(7/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx = \frac{\cos{(x)}}{15\sqrt{7\cos{(x)}^2-2}} - \frac{\cos{(x)}}{15\left(7\cos{(x)}^2-2\right)^{\frac{3}{2}}} + \frac{\cos{(x)}}{10\left(7\cos{(x)}^2-2\right)^{\frac{5}{2}}}$$

```
[In] integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="maxima")
[Out] 1/15*cos(x)/sqrt(7*cos(x)^2 - 2) - 1/15*cos(x)/(7*cos(x)^2 - 2)^(3/2) + 1/1
0*cos(x)/(7*cos(x)^2 - 2)^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

$$\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx = \frac{\left(14\left(7\cos{(x)}^2-5\right)\cos{(x)}^2+15\right)\cos{(x)}}{30\left(7\cos{(x)}^2-2\right)^{\frac{5}{2}}}$$

```
[In] integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="giac")
[Out] 1/30*(14*(7*cos(x)^2 - 5)*cos(x)^2 + 15)*cos(x)/(7*cos(x)^2 - 2)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.51

$$\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx = \frac{\cos{(x)}\,\left(98\cos{(x)}^4-70\cos{(x)}^2+15\right)}{30\left(7\cos{(x)}^2-2\right)^{5/2}}$$

```
[In] int(sin(x)/(5*cos(x)^2 - 2*sin(x)^2)^(7/2),x)
[Out] (cos(x)*(98*cos(x)^4 - 70*cos(x)^2 + 15))/(30*(7*cos(x)^2 - 2)^(5/2))
```

3.423 $\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \frac{2\arcsin\left(\sqrt{\frac{5}{2}}\sin(x)\right)}{5\sqrt{5}} + \frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}$$

```
[Out] 2/25*arcsin(1/2*sin(x)*10^(1/2))*5^(1/2)+1/10*sin(x)/(2-5*sin(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4441, 393, 222}

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \frac{2\arcsin\left(\sqrt{\frac{5}{2}}\sin(x)\right)}{5\sqrt{5}} + \frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}$$

```
[In] Int[(Cos[x]*Cos[2*x])/(2 - 5*Sin[x]^2)^(3/2),x]
[Out] (2*ArcSin[Sqrt[5/2]*Sin[x]])/(5*Sqrt[5]) + Sin[x]/(10*Sqrt[2 - 5*Sin[x]^2])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1-2x^2}{(2-5x^2)^{3/2}}\,dx, x, \sin(x)\right)\\
&= \frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}} + \frac{2}{5}\text{Subst}\left(\int \frac{1}{\sqrt{2-5x^2}}\,dx, x, \sin(x)\right)\\
&= \frac{2\arcsin\left(\sqrt{\frac{5}{2}}\sin(x)\right)}{5\sqrt{5}} + \frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \frac{1}{50}\left(4\sqrt{5}\arcsin\left(\sqrt{\frac{5}{2}}\sin(x)\right) + \frac{5\sin(x)}{\sqrt{2-5\sin^2(x)}}\right)$$

```
[In] Integrate[(Cos[x]*Cos[2*x])/(2 - 5*Sin[x]^2)^(3/2),x]
[Out] (4*Sqrt[5]*ArcSin[Sqrt[5/2]*Sin[x]] + (5*Sin[x])/Sqrt[2 - 5*Sin[x]^2])/50
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(28) = 56.

Time = 0.74 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{20\sqrt{5}\,\arcsin\left(\frac{\sin(x)\sqrt{10}}{2}\right)(\cos^2(x))+5\sin(x)\sqrt{5(\cos^2(x))-3}-12\arcsin\left(\frac{\sin(x)\sqrt{10}}{2}\right)\sqrt{5}}{250(\cos^2(x))-150}$	58

```
[In] int(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/50/(5*cos(x)^2-3)*(20*5^(1/2)*arcsin(1/2*sin(x)*10^(1/2))*cos(x)^2+5*sin(
x)*(5*cos(x)^2-3)^(1/2)-12*arcsin(1/2*sin(x)*10^(1/2))*5^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(28) = 56$.

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.56

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = -\frac{\left(5\sqrt{5}\cos(x)^2 - 3\sqrt{5}\right)\arctan\left(\frac{\left(50\sqrt{5}\cos(x)^4 - 80\sqrt{5}\cos(x)^2 + 31\sqrt{5}\right)\sqrt{5\cos(x)^2 - 3}}{10\left(25\cos(x)^4 - 35\cos(x)^2 + 12\right)\sin(x)}\right) - 5\sqrt{5\cos(x)^2 - 3}\sin(x)}{50\left(5\cos(x)^2 - 3\right)}$$

```
[In] integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/50*((5*sqrt(5)*cos(x)^2 - 3*sqrt(5))*arctan(1/10*(50*sqrt(5)*cos(x)^4 -
80*sqrt(5)*cos(x)^2 + 31*sqrt(5))*sqrt(5*cos(x)^2 - 3)/((25*cos(x)^4 - 35*c
os(x)^2 + 12)*sin(x))) - 5*sqrt(5*cos(x)^2 - 3)*sin(x))/(5*cos(x)^2 - 3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(x)*cos(2*x)/(2-5*sin(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(28) = 56$.

Time = 0.35 (sec) , antiderivative size = 716, normalized size of antiderivative = 18.36

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/50*(5*cos(1/2*arctan2(5*sin(4*x) - 2*sin(2*x), 5*cos(4*x) - 2*cos(2*x) +
5))*sin(2*x) - 5*(cos(2*x) - 1)*sin(1/2*arctan2(5*sin(4*x) - 2*sin(2*x), 5*
```

```
cos(4*x) - 2*cos(2*x) + 5)) + 2*(-10*(2*cos(2*x) - 5)*cos(4*x) + 25*cos(4*x
)^2 + 4*cos(2*x)^2 + 25*sin(4*x)^2 - 20*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 -
20*cos(2*x) + 25)^(1/4)*(sqrt(5)*arctan2(1/12*sqrt(6)*(sqrt(6)*(25/36)^(1/4
)*(25*cos(2*x)^4 + 25*sin(2*x)^4 - 20*cos(2*x)^3 + 2*(25*cos(2*x)^2 - 10*co
s(2*x) - 23)*sin(2*x)^2 + 54*cos(2*x)^2 - 20*cos(2*x) + 25)^(1/4)*sin(1/2*a
rctan2(5/12*(5*cos(2*x) - 1)*sin(2*x), 25/24*cos(2*x)^2 - 25/24*sin(2*x)^2
- 5/12*cos(2*x) + 25/24)) + 5*sin(2*x)), 5/12*sqrt(6)*cos(2*x) + 1/2*(25/36
)^(1/4)*(25*cos(2*x)^4 + 25*sin(2*x)^4 - 20*cos(2*x)^3 + 2*(25*cos(2*x)^2 -
 10*cos(2*x) - 23)*sin(2*x)^2 + 54*cos(2*x)^2 - 20*cos(2*x) + 25)^(1/4)*cos
(1/2*arctan2(5/12*(5*cos(2*x) - 1)*sin(2*x), 25/24*cos(2*x)^2 - 25/24*sin(2
*x)^2 - 5/12*cos(2*x) + 25/24)) - 1/12*sqrt(6)) + sqrt(5)*arctan2(1/12*sqrt
(6)*(sqrt(6)*(1/36)^(1/4)*(cos(2*x)^4 + sin(2*x)^4 - 20*cos(2*x)^3 + 2*(cos
(2*x)^2 - 10*cos(2*x) + 1)*sin(2*x)^2 + 198*cos(2*x)^2 - 980*cos(2*x) + 240
1)^(1/4)*sin(1/2*arctan2(1/12*(cos(2*x) - 5)*sin(2*x), 1/24*cos(2*x)^2 - 1/
24*sin(2*x)^2 - 5/12*cos(2*x) + 49/24)) + sin(2*x)), 1/12*sqrt(6)*cos(2*x)
+ 1/2*(1/36)^(1/4)*(cos(2*x)^4 + sin(2*x)^4 - 20*cos(2*x)^3 + 2*(cos(2*x)^2
 - 10*cos(2*x) + 1)*sin(2*x)^2 + 198*cos(2*x)^2 - 980*cos(2*x) + 2401)^(1/4
)*cos(1/2*arctan2(1/12*(cos(2*x) - 5)*sin(2*x), 1/24*cos(2*x)^2 - 1/24*sin(
2*x)^2 - 5/12*cos(2*x) + 49/24)) - 5/12*sqrt(6))))/(-10*(2*cos(2*x) - 5)*co
s(4*x) + 25*cos(4*x)^2 + 4*cos(2*x)^2 + 25*sin(4*x)^2 - 20*sin(4*x)*sin(2*x
) + 4*sin(2*x)^2 - 20*cos(2*x) + 25)^(1/4)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \frac{2}{25}\sqrt{5}\arcsin\left(\frac{1}{2}\sqrt{10}\sin\left(x\right)\right) - \frac{\sqrt{-5\sin\left(x\right)^2+2}\sin\left(x\right)}{10\left(5\sin\left(x\right)^2-2\right)}$$

```
[In] integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/25*sqrt(5)*arcsin(1/2*sqrt(10)*sin(x)) - 1/10*sqrt(-5*sin(x)^2 + 2)*sin(x
)/(5*sin(x)^2 - 2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)\cos(2x)}{\left(2-5\sin^2(x)\right)^{3/2}}\,dx=\int \frac{\cos\left(2\,x\right)\,\cos\left(x\right)}{{\left(2-5\,{\sin\left(x\right)}^2\right)}^{3/2}}\,dx$$

```
[In] int((cos(2*x)*cos(x))/(2 - 5*sin(x)^2)^(3/2),x)
[Out] int((cos(2*x)*cos(x))/(2 - 5*sin(x)^2)^(3/2), x)
```

3.424 $\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx = -\frac{1}{2}\arcsin\left(\frac{2\cos(x)}{3}\right)-\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}+\frac{295\cos(x)}{243\sqrt{9-4\cos^2(x)}}$$

```
[Out] -1/2*arcsin(2/3*cos(x))-55/27*cos(x)/(9-4*cos(x)^2)^(3/2)+295/243*cos(x)/(9
-4*cos(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1171, 393, 222}

$$\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx = -\frac{1}{2}\arcsin\left(\frac{2\cos(x)}{3}\right)+\frac{295\cos(x)}{243\sqrt{9-4\cos^2(x)}}-\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}$$

```
[In] Int[Sin[5*x]/(5*Cos[x]^2 + 9*Sin[x]^2)^(5/2),x]
```

```
[Out] -1/2*ArcSin[(2*Cos[x])/3] - (55*Cos[x])/(27*(9 - 4*Cos[x]^2)^(3/2)) + (295*
Cos[x])/(243*Sqrt[9 - 4*Cos[x]^2])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1-12x^2+16x^4}{(9-4x^2)^{5/2}}\,dx, x, \cos(x)\right)\\
&= -\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}} + \frac{1}{27}\text{Subst}\left(\int \frac{52+108x^2}{(9-4x^2)^{3/2}}\,dx, x, \cos(x)\right)\\
&= -\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}} + \frac{295\cos(x)}{243\sqrt{9-4\cos^2(x)}} - \text{Subst}\left(\int \frac{1}{\sqrt{9-4x^2}}\,dx, x, \cos(x)\right)\\
&= -\frac{1}{2}\arcsin\left(\frac{2\cos(x)}{3}\right) - \frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}} + \frac{295\cos(x)}{243\sqrt{9-4\cos^2(x)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.31

$$\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx = \frac{2550\cos(x)-590\cos(3x)+243i(7-2\cos(2x))^{3/2}\log\left(2i\cos(x)+\sqrt{7-\right.}{486(7-2\cos(2x))^{3/2}}$$

```
[In] Integrate[Sin[5*x]/(5*Cos[x]^2 + 9*Sin[x]^2)^(5/2),x]
[Out] (2550*Cos[x] - 590*Cos[3*x] + (243*I)*(7 - 2*Cos[2*x])^(3/2)*Log[(2*I)*Cos[
x] + Sqrt[7 - 2*Cos[2*x]]])/(486*(7 - 2*Cos[2*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(36) = 72$.

Time = 0.49 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.19

method	result
derivativedivides	$-\frac{\arcsin\left(\frac{2\cos(x)}{3}\right)}{2}-\frac{55\sqrt{-4\left(\cos(x)-\frac{3}{2}\right)^{2}-12\cos(x)+18}}{2592\left(\cos(x)-\frac{3}{2}\right)^{2}}-\frac{295\sqrt{-\left(\cos(x)-\frac{3}{2}\right)^{2}-3\cos(x)+\frac{9}{2}}}{972\left(\cos(x)-\frac{3}{2}\right)}+\frac{55\sqrt{-4\left(\cos(x)+\frac{3}{2}\right.}}{2592(\cos}$
default	$-\frac{\arcsin\left(\frac{2\cos(x)}{3}\right)}{2}-\frac{55\sqrt{-4\left(\cos(x)-\frac{3}{2}\right)^{2}-12\cos(x)+18}}{2592\left(\cos(x)-\frac{3}{2}\right)^{2}}-\frac{295\sqrt{-\left(\cos(x)-\frac{3}{2}\right)^{2}-3\cos(x)+\frac{9}{2}}}{972\left(\cos(x)-\frac{3}{2}\right)}+\frac{55\sqrt{-4\left(\cos(x)+\frac{3}{2}\right.}}{2592(\cos}$

```
[In] int(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arcsin(2/3*cos(x))-55/2592/(cos(x)-3/2)^2*(-4*(cos(x)-3/2)^2-12*cos(x)
+18)^(1/2)-295/972/(cos(x)-3/2)*(-(cos(x)-3/2)^2-3*cos(x)+9/2)^(1/2)+55/259
2/(cos(x)+3/2)^2*(-4*(cos(x)+3/2)^2+12*cos(x)+18)^(1/2)-295/972/(cos(x)+3/2
)*(-(cos(x)+3/2)^2+3*cos(x)+9/2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(36) = 72$.

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.73

$$\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx = \frac{243\left(16\cos(x)^4-72\cos(x)^2+81\right)\arctan\left(-\frac{81\cos(x)\sin(x)-4\left(8\cos(x)^3-9\cos(\right.}{64\cos(x)^4-225\cos(x)}\right.}{}$$

```
[In] integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/972*(243*(16*cos(x)^4 - 72*cos(x)^2 + 81)*arctan(-(81*cos(x)*sin(x) - 4*(
8*cos(x)^3 - 9*cos(x))*sqrt(-4*cos(x)^2 + 9))/(64*cos(x)^4 - 225*cos(x)^2 +
 81)) - 243*(16*cos(x)^4 - 72*cos(x)^2 + 81)*arctan(sin(x)/cos(x)) - 80*(59
*cos(x)^3 - 108*cos(x))*sqrt(-4*cos(x)^2 + 9))/(16*cos(x)^4 - 72*cos(x)^2 +
 81)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(5x)}{(5\cos^2(x)+9\sin^2(x))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(5*x)/(5*cos(x)**2+9*sin(x)**2)**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int \frac{\sin(5x)}{(5\cos^2(x)+9\sin^2(x))^{5/2}}\,dx = -2\left(\frac{2\cos(x)^2}{(-4\cos(x)^2+9)^{\frac{3}{2}}} - \frac{3}{(-4\cos(x)^2+9)^{\frac{3}{2}}}\right)\cos(x) + \frac{52\cos(x)}{243\sqrt{-4\cos(x)^2+9}} + \frac{26\cos(x)}{27(-4\cos(x)^2+9)^{\frac{3}{2}}} - \frac{1}{2}\arcsin\left(\frac{2}{3}\cos(x)\right)$$

```
[In] integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="maxima")
[Out] -2*(2*cos(x)^2/(-4*cos(x)^2 + 9)^(3/2) - 3/(-4*cos(x)^2 + 9)^(3/2))*cos(x)
+ 52/243*cos(x)/sqrt(-4*cos(x)^2 + 9) + 26/27*cos(x)/(-4*cos(x)^2 + 9)^(3/2
) - 1/2*arcsin(2/3*cos(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{\sin(5x)}{(5\cos^2(x)+9\sin^2(x))^{5/2}}\,dx = -\frac{20(59\cos(x)^2-108)\sqrt{-4\cos(x)^2+9}\cos(x)}{243(4\cos(x)^2-9)^2} - \frac{1}{2}\arcsin\left(\frac{2}{3}\cos(x)\right)$$

```
[In] integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="giac")
[Out] -20/243*(59*cos(x)^2 - 108)*sqrt(-4*cos(x)^2 + 9)*cos(x)/(4*cos(x)^2 - 9)^2
 - 1/2*arcsin(2/3*cos(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx=\int \frac{\sin\left(5\,x\right)}{{\left(5\cos\left(x\right)^2+9\sin\left(x\right)^2\right)}^{5/2}}\,dx$$

```
[In] int(sin(5*x)/(5*cos(x)^2 + 9*sin(x)^2)^(5/2),x)
[Out] int(sin(5*x)/(5*cos(x)^2 + 9*sin(x)^2)^(5/2), x)
```

3.425 $\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx=-\frac{1}{4\left(-5+4\sin^2(x)\right)^{3/2}}-\frac{5}{8\sqrt{-5+4\sin^2(x)}}+\frac{1}{8}\sqrt{-5+4\sin^2(x)}$$

```
[Out] -1/4/(-5+4*sin(x)^2)^(3/2)-5/8/(-5+4*sin(x)^2)^(1/2)+1/8*(-5+4*sin(x)^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4441, 1261, 712}

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx=\frac{1}{8}\sqrt{4\sin^2(x)-5}-\frac{5}{8\sqrt{4\sin^2(x)-5}}-\frac{1}{4\left(4\sin^2(x)-5\right)^{3/2}}$$

```
[In] Int[(Cos[x]*Cos[2*x]*Sin[3*x])/(-5 + 4*Sin[x]^2)^(5/2),x]
```

```
[Out] -1/4*1/(-5 + 4*Sin[x]^2)^(3/2) - 5/(8*Sqrt[-5 + 4*Sin[x]^2]) + Sqrt[-5 + 4*
Sin[x]^2]/8
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x(3-10x^2+8x^4)}{(-5+4x^2)^{5/2}}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{3-10x+8x^2}{(-5+4x)^{5/2}}\,dx, x, \sin^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{3}{(-5+4x)^{5/2}}+\frac{5}{2(-5+4x)^{3/2}}+\frac{1}{2\sqrt{-5+4x}}\right)dx, x, \sin^2(x)\right)\\
&= -\frac{1}{4\left(-5+4\sin^2(x)\right)^{3/2}}-\frac{5}{8\sqrt{-5+4\sin^2(x)}}+\frac{1}{8}\sqrt{-5+4\sin^2(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$
\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx = \frac{12+11\cos(2x)+\cos(4x)}{4\left(-5+4\sin^2(x)\right)^{3/2}}
$$

```
[In] Integrate[(Cos[x]*Cos[2*x]*Sin[3*x])/(-5 + 4*Sin[x]^2)^(5/2),x]
[Out] (12 + 11*Cos[2*x] + Cos[4*x])/(4*(-5 + 4*Sin[x]^2)^(3/2))
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\sqrt{-4(\cos^2(x))-1}}{8}-\frac{1}{4(-4(\cos^2(x))-1)^{\frac{3}{2}}}-\frac{5}{8\sqrt{-4(\cos^2(x))-1}}$	38
default	$\frac{\sqrt{-4(\cos^2(x))-1}}{8}-\frac{1}{4(-4(\cos^2(x))-1)^{\frac{3}{2}}}-\frac{5}{8\sqrt{-4(\cos^2(x))-1}}$	38

```
[In] int(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/8*(-4*cos(x)^2-1)^(1/2)-1/4/(-4*cos(x)^2-1)^(3/2)-5/8/(-4*cos(x)^2-1)^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx=\frac{\left(4\,\cos\left(x\right)^{4}+7\,\cos\left(x\right)^{2}+1\right)\sqrt{-4\,\cos\left(x\right)^{2}-1}}{2\,\left(16\,\cos\left(x\right)^{4}+8\,\cos\left(x\right)^{2}+1\right)}$$

```
[In] integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="fric
as")
[Out] 1/2*(4*cos(x)^4 + 7*cos(x)^2 + 1)*sqrt(-4*cos(x)^2 - 1)/(16*cos(x)^4 + 8*co
s(x)^2 + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)**2)**(5/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(37) = 74$.

Time = 0.23 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.92

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{(-5+4\sin^2(x))^{5/2}}\,dx = -\frac{(\cos(11x)+14\cos(9x)+58\cos(7x)+94\cos(5x)+58\cos(3x)+15\cos(x))\cos\left(\frac{5}{2}\arctan(\sin(4x)+\right.}{8\left(2(3\cos(2x)+\right.}$$

```
[In] integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] -1/8*((cos(11*x) + 14*cos(9*x) + 58*cos(7*x) + 94*cos(5*x) + 58*cos(3*x) +
15*cos(x))*cos(5/2*arctan2(sin(4*x) + 3*sin(2*x), -cos(4*x) - 3*cos(2*x) -
1)) - (sin(11*x) + 14*sin(9*x) + 58*sin(7*x) + 94*sin(5*x) + 58*sin(3*x) +
13*sin(x))*sin(5/2*arctan2(sin(4*x) + 3*sin(2*x), -cos(4*x) - 3*cos(2*x) -
1)))/(2*(3*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 9*cos(2*x)^2 + sin(4*x)^2
+ 6*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) + 1)^(5/4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{(-5+4\sin^2(x))^{5/2}}\,dx = \frac{1}{8}\sqrt{4\sin(x)^2-5} - \frac{20\sin(x)^2-23}{8\left(4\sin(x)^2-5\right)^{\frac{3}{2}}}$$

```
[In] integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="giac
")
```

```
[Out] 1/8*sqrt(4*sin(x)^2 - 5) - 1/8*(20*sin(x)^2 - 23)/(4*sin(x)^2 - 5)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{(-5+4\sin^2(x))^{5/2}}\,dx = \frac{2\cos(2x)^2+11\cos(2x)+11}{4\left(-2\cos(2x)-3\right)^{3/2}}$$

```
[In] int((cos(2*x)*sin(3*x)*cos(x))/(4*sin(x)^2 - 5)^(5/2),x)
```

```
[Out] (11*cos(2*x) + 2*cos(2*x)^2 + 11)/(4*(- 2*cos(2*x) - 3)^(3/2))
```

3.426 $\int \frac{\csc^2(x)(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x))}{\sqrt{-5+\sin^2(x)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 111

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx = 2\arctan\left(\frac{\cos(x)}{\sqrt{-5+\sin^2(x)}}\right)-\frac{\arctan\left(\frac{\sqrt{5}\cos(x)}{\sqrt{-5+\sin^2(x)}}\right)}{\sqrt{5}}-\frac{2\arctan\left(\frac{\sqrt{-5+\sin^2(x)}}{\sqrt{5}}\right)}{\sqrt{5}} - 2\text{arctanh}\left(\frac{\sin(x)}{\sqrt{-5+\sin^2(x)}}\right)+2\sqrt{-5+\sin^2(x)}+\frac{2}{5}\csc(x)\sqrt{-5+\sin^2(x)}$$

```
[Out] 2*arctan(cos(x)/(-5+sin(x)^2)^(1/2))-2*arctanh(sin(x)/(-5+sin(x)^2)^(1/2))-
1/5*arctan(cos(x)*5^(1/2)/(-5+sin(x)^2)^(1/2))*5^(1/2)-2/5*arctan(1/5*(-5+s
in(x)^2)^(1/2)*5^(1/2))*5^(1/2)+2*(-5+sin(x)^2)^(1/2)+2/5*(-5+sin(x)^2)^(1/
2)/sin(x)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.394, Rules used = {4486, 4441, 462, 223, 212, 4451, 6857, 209, 267, 1024, 385, 455, 65}

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx = 2\arctan\left(\frac{\cos(x)}{\sqrt{-\cos^2(x)-4}}\right)-\frac{\arctan\left(\frac{\sqrt{5}\cos(x)}{\sqrt{-\cos^2(x)-4}}\right)}{\sqrt{5}}-\frac{2\arctan\left(\frac{\sqrt{-\cos^2(x)-4}}{\sqrt{5}}\right)}{\sqrt{5}} - 2\text{arctanh}\left(\frac{\sin(x)}{\sqrt{\sin^2(x)-5}}\right)+2\sqrt{-\cos^2(x)-4}+\frac{2}{5}\sqrt{\sin^2(x)-5}\csc(x)$$

```
[In] Int[(Csc[x]^2*(-2*Cos[x]^3*(-1 + Sin[x]) + Cos[2*x]*Sin[x]))/Sqrt[-5 + Sin[
x]^2],x]
```

```
[Out] 2*ArcTan[Cos[x]/Sqrt[-4 - Cos[x]^2]] - ArcTan[(Sqrt[5]*Cos[x])/Sqrt[-4 - Co
s[x]^2]]/Sqrt[5] - (2*ArcTan[Sqrt[-4 - Cos[x]^2]/Sqrt[5]])/Sqrt[5] - 2*ArcT
anh[Sin[x]/Sqrt[-5 + Sin[x]^2]] + 2*Sqrt[-4 - Cos[x]^2] + (2*Csc[x]*Sqrt[-5
 + Sin[x]^2])/5
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 4451

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2\cos(x)\cot^2(x)}{\sqrt{-5+\sin^2(x)}}+\frac{(-2\cos^3(x)+\cos(2x))\csc(x)}{\sqrt{-5+\sin^2(x)}}\right)dx\\
&= 2\int \frac{\cos(x)\cot^2(x)}{\sqrt{-5+\sin^2(x)}}\,dx+\int \frac{(-2\cos^3(x)+\cos(2x))\csc(x)}{\sqrt{-5+\sin^2(x)}}\,dx\\
&= 2\text{Subst}\left(\int \frac{1-x^2}{x^2\sqrt{-5+x^2}}\,dx,x,\sin(x)\right)-\text{Subst}\left(\int \frac{-1+2x^2-2x^3}{\sqrt{-4-x^2}\,(1-x^2)}\,dx,x,\cos(x)\right)\\
&= \frac{2}{5}\csc(x)\sqrt{-5+\sin^2(x)}-2\text{Subst}\left(\int \frac{1}{\sqrt{-5+x^2}}\,dx,x,\sin(x)\right)\\
&\quad-\text{Subst}\left(\int \left(-\frac{2}{\sqrt{-4-x^2}}+\frac{2x}{\sqrt{-4-x^2}}+\frac{1-2x}{\sqrt{-4-x^2}\,(1-x^2)}\right)dx,x,\cos(x)\right)\\
&= \frac{2}{5}\csc(x)\sqrt{-5+\sin^2(x)}+2\text{Subst}\left(\int \frac{1}{\sqrt{-4-x^2}}\,dx,x,\cos(x)\right)\\
&\quad-2\text{Subst}\left(\int \frac{x}{\sqrt{-4-x^2}}\,dx,x,\cos(x)\right)\\
&\quad-2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sin(x)}{\sqrt{-5+\sin^2(x)}}\right)\\
&\quad-\text{Subst}\left(\int \frac{1-2x}{\sqrt{-4-x^2}\,(1-x^2)}\,dx,x,\cos(x)\right)\\
&= -2\text{arctanh}\left(\frac{\sin(x)}{\sqrt{-5+\sin^2(x)}}\right)+2\sqrt{-4-\cos^2(x)}\\
&\quad+\frac{2}{5}\csc(x)\sqrt{-5+\sin^2(x)}+2\text{Subst}\left(\int \frac{x}{\sqrt{-4-x^2}\,(1-x^2)}\,dx,x,\cos(x)\right)\\
&\quad+2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\cos(x)}{\sqrt{-4-\cos^2(x)}}\right)\\
&\quad-\text{Subst}\left(\int \frac{1}{\sqrt{-4-x^2}\,(1-x^2)}\,dx,x,\cos(x)\right)\\
&= 2\arctan\left(\frac{\cos(x)}{\sqrt{-4-\cos^2(x)}}\right)-2\text{arctanh}\left(\frac{\sin(x)}{\sqrt{-5+\sin^2(x)}}\right)+2\sqrt{-4-\cos^2(x)}\\
&\quad+\frac{2}{5}\csc(x)\sqrt{-5+\sin^2(x)}+\text{Subst}\left(\int \frac{1}{\sqrt{-4-x}(1-x)}\,dx,x,\cos^2(x)\right)\\
&\quad-\text{Subst}\left(\int \frac{1}{1+5x^2}\,dx,x,\frac{\cos(x)}{\sqrt{-4-\cos^2(x)}}\right)
\end{aligned}$$

$$= 2\arctan\left(\frac{\cos(x)}{\sqrt{-4-\cos^2(x)}}\right) - \frac{\arctan\left(\frac{\sqrt{5}\cos(x)}{\sqrt{-4-\cos^2(x)}}\right)}{\sqrt{5}} - 2\text{arctanh}\left(\frac{\sin(x)}{\sqrt{-5+\sin^2(x)}}\right) + 2\sqrt{-4-\cos^2(x)} + \frac{2}{5}\csc(x)\sqrt{-5+\sin^2(x)} - 2\text{Subst}\left(\int\frac{1}{5+x^2}\,dx, x, \sqrt{-4-\cos^2(x)}\right)$$

$$= 2\arctan\left(\frac{\cos(x)}{\sqrt{-4-\cos^2(x)}}\right) - \frac{\arctan\left(\frac{\sqrt{5}\cos(x)}{\sqrt{-4-\cos^2(x)}}\right)}{\sqrt{5}} - \frac{2\arctan\left(\frac{\sqrt{-4-\cos^2(x)}}{\sqrt{5}}\right)}{\sqrt{5}} - 2\text{arctanh}\left(\frac{\sin(x)}{\sqrt{-5+\sin^2(x)}}\right) + 2\sqrt{-4-\cos^2(x)} + \frac{2}{5}\csc(x)\sqrt{-5+\sin^2(x)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.00 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.66

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx$$

$$= \frac{2\sqrt{2}(-2\cos^3(x)+\cos(2x)+2\cos^2(x)\cot(x))\left(18+2\cos(2x)+20\sqrt{2}\text{arctanh}\left(\frac{2\sqrt{2}\tan\left(\frac{x}{2}\right)}{\sqrt{-\left((9+\cos(2x))\sec^4\left(\frac{x}{2}\right)\right)}}\right)\text{co}\right.}{}$$

```
[In] Integrate[(Csc[x]^2*(-2*Cos[x]^3*(-1 + Sin[x]) + Cos[2*x]*Sin[x]))/Sqrt[-5
+ Sin[x]^2],x]
```

```
[Out] (2*Sqrt[2]*(-2*Cos[x]^3 + Cos[2*x] + 2*Cos[x]^2*Cot[x])*(18 + 2*Cos[2*x] +
20*Sqrt[2]*ArcTanh[(2*Sqrt[2]*Tan[x/2])/Sqrt[-((9 + Cos[2*x])*Sec[x/2]^4)]]
*Cos[x/2]^3*Sqrt[-((9 + Cos[2*x])*Sec[x/2]^4)]*Sin[x/2] + 85*Sin[x] + Sqrt[
10]*ArcTan[(Sqrt[10]*Cos[x])/Sqrt[-9 - Cos[2*x]]]*Sqrt[-9 - Cos[2*x]]*Sin[x
] + 2*Sqrt[10]*ArcTan[Sqrt[-9 - Cos[2*x]]/Sqrt[10]]*Sqrt[-9 - Cos[2*x]]*Sin
[x] + (10*I)*Sqrt[2]*Sqrt[-9 - Cos[2*x]]*Log[I*Sqrt[2]*Cos[x] + Sqrt[-9 - C
os[2*x]]]*Sin[x] + 5*Sin[3*x]))/(5*Sqrt[-9 - Cos[2*x]]*(-6*Cos[x] - 2*Cos[3
*x] + 2*Sin[x] + 2*Sin[2*x] - 2*Sin[3*x] + Sin[4*x]))
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.18

method	result
default	$-2\ln\left(\sin\left(x\right)+\sqrt{-5+\sin^{2}\left(x\right)}\right)+2\sqrt{-5+\sin^{2}\left(x\right)}+\frac{2\sqrt{-5+\sin^{2}(x)}}{5\sin(x)}+\frac{2\sqrt{5}\,\arctan\left(\frac{\sqrt{5}}{\sqrt{-5+\sin^{2}(x)}}\right)}{5}-$
parts	$-2\ln\left(\sin\left(x\right)+\sqrt{-5+\sin^{2}\left(x\right)}\right)+2\sqrt{-5+\sin^{2}\left(x\right)}+\frac{2\sqrt{-5+\sin^{2}(x)}}{5\sin(x)}+\frac{2\sqrt{5}\,\arctan\left(\frac{\sqrt{5}}{\sqrt{-5+\sin^{2}(x)}}\right)}{5}-$

```
[In] int((-2*cos(x)^3*(sin(x)-1)+sin(x)*cos(2*x))/sin(x)^2/(-5+sin(x)^2)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] -2*ln(sin(x)+(-5+sin(x)^2)^(1/2))+2*(-5+sin(x)^2)^(1/2)+2/5*(-5+sin(x)^2)^(
1/2)/sin(x)+2/5*5^(1/2)*arctan(5^(1/2)/(-5+sin(x)^2)^(1/2))-1/10*((-5+sin(x
)^2)*cos(x)^2)^(1/2)*(-5^(1/2)*arctan(1/5*(3*sin(x)^2-5)*5^(1/2)/(-cos(x)^4
-4*cos(x)^2)^(1/2))-10*arcsin(1+1/2*cos(x)^2))/cos(x)/(-5+sin(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.88 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.14

$$\int\frac{\csc^{2}(x)\left(-2\cos^{3}(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^{2}(x)}}\,dx=$$
$$-\frac{\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{-\cos(x)^{2}-4}}{\cos(x)+4}\right)\sin\left(x\right)-3\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{-\cos(x)^{2}-4}}{\cos(x)-4}\right)\sin\left(x\right)+20\arctan\left(\frac{\sqrt{-\cos(x)^{2}-4}}{\cos(x)}\right)\text{si}}{10\,\text{sin}}$$

```
[In] integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)^
(1/2),x, algorithm="fricas")
```

```
[Out] -1/10*(sqrt(5)*arctan(sqrt(5)*sqrt(-cos(x)^2 - 4)/(cos(x) + 4))*sin(x) - 3*
sqrt(5)*arctan(sqrt(5)*sqrt(-cos(x)^2 - 4)/(cos(x) - 4))*sin(x) + 20*arctan
(sqrt(-cos(x)^2 - 4)/cos(x))*sin(x) - 10*log(2*cos(x)^2 + 2*sqrt(-cos(x)^2
- 4)*sin(x) + 3)*sin(x) - 4*sqrt(-cos(x)^2 - 4)*(5*sin(x) + 1))/sin(x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx=\text{Timed out}$$

```
[In] integrate((-2*cos(x)**3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)**2/(-5+sin(x)**
2)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.04

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx$$
$$=\frac{2}{5}\sqrt{5}\arcsin\left(\frac{\sqrt{5}}{|\sin(x)|}\right)-\frac{1}{10}i\sqrt{5}\operatorname{arsinh}\left(\frac{\cos(x)}{2(\cos(x)+1)}-\frac{2}{\cos(x)+1}\right)$$
$$-\frac{1}{10}i\sqrt{5}\operatorname{arsinh}\left(-\frac{\cos(x)}{2(\cos(x)-1)}-\frac{2}{\cos(x)-1}\right)+2\sqrt{\sin(x)^2-5}$$
$$+\frac{2\sqrt{\sin(x)^2-5}}{5\sin(x)}-2i\operatorname{arsinh}\left(\frac{1}{2}\cos(x)\right)-2\log\left(2\sqrt{\sin(x)^2-5}+2\sin(x)\right)$$

```
[In] integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)^
(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(5)*arcsin(sqrt(5)/abs(sin(x))) - 1/10*I*sqrt(5)*arcsinh(1/2*cos(x)
/(cos(x) + 1) - 2/(cos(x) + 1)) - 1/10*I*sqrt(5)*arcsinh(-1/2*cos(x)/(cos(x
) - 1) - 2/(cos(x) - 1)) + 2*sqrt(sin(x)^2 - 5) + 2/5*sqrt(sin(x)^2 - 5)/si
n(x) - 2*I*arcsinh(1/2*cos(x)) - 2*log(2*sqrt(sin(x)^2 - 5) + 2*sin(x))
```

Giac [F]

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx$$
$$=\int -\frac{2(\sin(x)-1)\cos(x)^3-\cos(2x)\sin(x)}{\sqrt{\sin(x)^2-5}\sin(x)^2}\,dx$$

```
[In] integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)^
(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*(sin(x) - 1)*cos(x)^3 - cos(2*x)*sin(x))/(sqrt(sin(x)^2 - 5)*
sin(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx = \int \frac{\cos\left(2\,x\right)\,\sin\left(x\right)-2\cos\left(x\right)^3\,\left(\sin\left(x\right)-1\right)}{\sin\left(x\right)^2\,\sqrt{\sin\left(x\right)^2-5}}\,dx$$

```
[In] int((cos(2*x)*sin(x) - 2*cos(x)^3*(sin(x) - 1))/(sin(x)^2*(sin(x)^2 - 5)^(1
/2)),x)
```

```
[Out] int((cos(2*x)*sin(x) - 2*cos(x)^3*(sin(x) - 1))/(sin(x)^2*(sin(x)^2 - 5)^(1
/2)), x)
```

3.427 $\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 112

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = \frac{5\arcsin\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}}+\frac{3}{4}\arcsin\left(\frac{2\sin(x)}{\sqrt{3}}\right)-\frac{3}{4}\arctan\left(\frac{\sin(x)}{\sqrt{-1+4\cos^2(x)}}\right)-\frac{3}{4}\arctan\left(\frac{\sin(x)}{\sqrt{-1+8\cos^2(x)}}\right)-\frac{1}{2}\sqrt{-1+4\cos^2(x)}\sin(x)-\frac{1}{2}\sqrt{-1+8\cos^2(x)}\sin(x)$$

```
[Out] 3/4*arcsin(2/3*sin(x)*3^(1/2))-3/4*arctan(sin(x)/(-1+4*cos(x)^2)^(1/2))-3/4
*arctan(sin(x)/(-1+8*cos(x)^2)^(1/2))+5/8*arcsin(2/7*sin(x)*14^(1/2))*2^(1/
2)-1/2*sin(x)*(-1+4*cos(x)^2)^(1/2)-1/2*sin(x)*(-1+8*cos(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6874, 399, 222, 385, 210, 201}

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = \frac{5\arcsin\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}}+\frac{3}{4}\arcsin\left(\frac{2\sin(x)}{\sqrt{3}}\right)-\frac{3}{4}\arctan\left(\frac{\sin(x)}{\sqrt{7-8\sin^2(x)}}\right)-\frac{3}{4}\arctan\left(\frac{\sin(x)}{\sqrt{3-4\sin^2(x)}}\right)-\frac{1}{2}\sin(x)\sqrt{7-8\sin^2(x)}-\frac{1}{2}\sin(x)\sqrt{3-4\sin^2(x)}$$

```
[In] Int[Cos[3*x]/(-Sqrt[-1 + 8*Cos[x]^2] + Sqrt[3*Cos[x]^2 - Sin[x]^2]),x]
```

```
[Out] (5*ArcSin[2*Sqrt[2/7]*Sin[x]])/(4*Sqrt[2]) + (3*ArcSin[(2*Sin[x])/Sqrt[3]])
/4 - (3*ArcTan[Sin[x]/Sqrt[7 - 8*Sin[x]^2]])/4 - (3*ArcTan[Sin[x]/Sqrt[3 -
4*Sin[x]^2]])/4 - (Sin[x]*Sqrt[7 - 8*Sin[x]^2])/2 - (Sin[x]*Sqrt[3 - 4*Sin[
x]^2])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{-1+4x^2}{\sqrt{7-8x^2}-\sqrt{3-4x^2}}\,dx,x,\sin(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{7-8x^2}-\sqrt{3-4x^2}}+\frac{4x^2}{\sqrt{7-8x^2}-\sqrt{3-4x^2}}\right)\,dx,x,\sin(x)\right)\\
&= 4\text{Subst}\left(\int \frac{x^2}{\sqrt{7-8x^2}-\sqrt{3-4x^2}}\,dx,x,\sin(x)\right)\\
&\quad - \text{Subst}\left(\int \frac{1}{\sqrt{7-8x^2}-\sqrt{3-4x^2}}\,dx,x,\sin(x)\right)\\
&= 4\text{Subst}\left(\int \left(-\frac{1}{4}\sqrt{7-8x^2}-\frac{1}{4}\sqrt{3-4x^2}-\frac{\sqrt{7-8x^2}}{4(-1+x^2)}\right.\right.\\
&\qquad\left.\left.-\frac{\sqrt{3-4x^2}}{4(-1+x^2)}\right)\,dx,x,\sin(x)\right)\\
&\quad - \text{Subst}\left(\int \left(-\frac{\sqrt{7-8x^2}}{4(-1+x^2)}-\frac{\sqrt{3-4x^2}}{4(-1+x^2)}\right)\,dx,x,\sin(x)\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{7-8x^2}}{-1+x^2}\,dx,x,\sin(x)\right)+\frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{3-4x^2}}{-1+x^2}\,dx,x,\sin(x)\right)\\
&\quad - \text{Subst}\left(\int \sqrt{7-8x^2}\,dx,x,\sin(x)\right)-\text{Subst}\left(\int \sqrt{3-4x^2}\,dx,x,\sin(x)\right)\\
&\quad - \text{Subst}\left(\int \frac{\sqrt{7-8x^2}}{-1+x^2}\,dx,x,\sin(x)\right)-\text{Subst}\left(\int \frac{\sqrt{3-4x^2}}{-1+x^2}\,dx,x,\sin(x)\right)\\
&= -\frac{1}{2}\sin(x)\sqrt{7-8\sin^2(x)}-\frac{1}{2}\sin(x)\sqrt{3-4\sin^2(x)}\\
&\quad - \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{7-8x^2}\,(-1+x^2)}\,dx,x,\sin(x)\right)\\
&\quad - \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{3-4x^2}\,(-1+x^2)}\,dx,x,\sin(x)\right)\\
&\quad - \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3-4x^2}}\,dx,x,\sin(x)\right)-2\text{Subst}\left(\int \frac{1}{\sqrt{7-8x^2}}\,dx,x,\sin(x)\right)\\
&\quad - \frac{7}{2}\text{Subst}\left(\int \frac{1}{\sqrt{7-8x^2}}\,dx,x,\sin(x)\right)+4\text{Subst}\left(\int \frac{1}{\sqrt{3-4x^2}}\,dx,x,\sin(x)\right)\\
&\quad + 8\text{Subst}\left(\int \frac{1}{\sqrt{7-8x^2}}\,dx,x,\sin(x)\right)-\text{Subst}\left(\int \frac{1}{\sqrt{3-4x^2}}\,dx,x,\sin(x)\right)\\
&\quad + \text{Subst}\left(\int \frac{1}{\sqrt{7-8x^2}\,(-1+x^2)}\,dx,x,\sin(x)\right)\\
&\quad + \text{Subst}\left(\int \frac{1}{\sqrt{3-4x^2}\,(-1+x^2)}\,dx,x,\sin(x)\right)
\end{aligned}
$$

$$= -\frac{11\arcsin\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}} + 2\sqrt{2}\arcsin\left(2\sqrt{\frac{2}{7}}\sin(x)\right) + \frac{3}{4}\arcsin\left(\frac{2\sin(x)}{\sqrt{3}}\right) - \frac{1}{2}\sin(x)\sqrt{7-8\sin^2(x)} - \frac{1}{2}\sin(x)\sqrt{3-4\sin^2(x)} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \frac{\sin(x)}{\sqrt{7-8\sin^2(x)}}\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \frac{\sin(x)}{\sqrt{3-4\sin^2(x)}}\right) + \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \frac{\sin(x)}{\sqrt{7-8\sin^2(x)}}\right) + \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \frac{\sin(x)}{\sqrt{3-4\sin^2(x)}}\right)$$

$$= -\frac{11\arcsin\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}} + 2\sqrt{2}\arcsin\left(2\sqrt{\frac{2}{7}}\sin(x)\right) + \frac{3}{4}\arcsin\left(\frac{2\sin(x)}{\sqrt{3}}\right) - \frac{3}{4}\arctan\left(\frac{\sin(x)}{\sqrt{7-8\sin^2(x)}}\right) - \frac{3}{4}\arctan\left(\frac{\sin(x)}{\sqrt{3-4\sin^2(x)}}\right) - \frac{1}{2}\sin(x)\sqrt{7-8\sin^2(x)} - \frac{1}{2}\sin(x)\sqrt{3-4\sin^2(x)}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.39

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = \frac{1}{8}\left(5\sqrt{2}\arcsin\left(2\sqrt{\frac{2}{7}}\sin(x)\right) + 6\arcsin\left(\frac{2\sin(x)}{\sqrt{3}}\right) + 3\arctan\left(\frac{7-8\sin(x)}{\sqrt{3+4\cos(2x)}}\right) + 3\arctan\left(\frac{3-4\sin(x)}{\sqrt{1+2\cos(2x)}}\right) - 3\arctan\left(\frac{3+4\sin(x)}{\sqrt{1+2\cos(2x)}}\right) - 3\arctan\left(\frac{7+8\sin(x)}{\sqrt{3+4\cos(2x)}}\right) - 4\sqrt{1+2\cos(2x)}\sin(x) - 4\sqrt{3+4\cos(2x)}\sin(x)\right)$$

```
[In] Integrate[Cos[3*x]/(-Sqrt[-1 + 8*Cos[x]^2] + Sqrt[3*Cos[x]^2 - Sin[x]^2]),x
]
```

```
[Out] (5*Sqrt[2]*ArcSin[2*Sqrt[2/7]*Sin[x]] + 6*ArcSin[(2*Sin[x])/Sqrt[3]] + 3*Ar
cTan[(7 - 8*Sin[x])/Sqrt[3 + 4*Cos[2*x]]] + 3*ArcTan[(3 - 4*Sin[x])/Sqrt[1
```

```
+ 2*Cos[2*x]]] - 3*ArcTan[(3 + 4*Sin[x])/Sqrt[1 + 2*Cos[2*x]]] - 3*ArcTan[(
7 + 8*Sin[x])/Sqrt[3 + 4*Cos[2*x]]] - 4*Sqrt[1 + 2*Cos[2*x]]*Sin[x] - 4*Sqr
t[3 + 4*Cos[2*x]]*Sin[x])/8
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(84) = 168$.

Time = 1.15 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.08

method	result
default	$\frac{3\sqrt{-8(\sin(x)+1)^2+16\sin(x)+15}}{8}+\frac{5\arcsin\left(\frac{2\sin(x)\sqrt{14}}{7}\right)\sqrt{2}}{8}-\frac{3\arctan\left(\frac{14+16\sin(x)}{2\sqrt{-8(\sin(x)+1)^2+16\sin(x)+15}}\right)}{8}-\frac{3\sqrt{-8(\sin(x)-1)^2}}{8}$

```
[In] int(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,method=
_RETURNVERBOSE)
```

```
[Out] 3/8*(-8*(sin(x)+1)^2+16*sin(x)+15)^(1/2)+5/8*arcsin(2/7*sin(x)*14^(1/2))*2^
(1/2)-3/8*arctan(1/2*(14+16*sin(x))/(-8*(sin(x)+1)^2+16*sin(x)+15)^(1/2))-3
/8*(-8*(sin(x)-1)^2-16*sin(x)+15)^(1/2)+3/8*arctan(1/2*(14-16*sin(x))/(-8*(
sin(x)-1)^2-16*sin(x)+15)^(1/2))+3/8*(-4*(sin(x)+1)^2+8*sin(x)+7)^(1/2)+3/4
*arcsin(2/3*sin(x)*3^(1/2))-3/8*arctan(1/2*(6+8*sin(x))/(-4*(sin(x)+1)^2+8*
sin(x)+7)^(1/2))-3/8*(-4*(sin(x)-1)^2-8*sin(x)+7)^(1/2)+3/8*arctan(1/2*(6-8
*sin(x))/(-4*(sin(x)-1)^2-8*sin(x)+7)^(1/2))-1/2*sin(x)*(-8*sin(x)^2+7)^(1/
2)-1/2*sin(x)*(3-4*sin(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(84) = 168$.

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.74

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$$
$$= -\frac{5}{32}\sqrt{2}\arctan\left(\frac{\left(512\sqrt{2}\cos(x)^4-576\sqrt{2}\cos(x)^2+113\sqrt{2}\right)\sqrt{8\cos(x)^2-1}}{16\left(128\cos(x)^4-88\cos(x)^2+9\right)\sin(x)}\right)$$
$$-\frac{1}{2}\sqrt{8\cos(x)^2-1}\sin(x)-\frac{1}{2}\sqrt{4\cos(x)^2-1}\sin(x)$$
$$+\frac{3}{8}\arctan\left(\frac{4\left(8\cos(x)^2-5\right)\sqrt{4\cos(x)^2-1}\sin(x)-9\cos(x)\sin(x)}{64\cos(x)^4-71\cos(x)^2+16}\right)$$
$$+\frac{3}{8}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)+\frac{3}{8}\arctan\left(\frac{9\cos(x)^2-2}{2\sqrt{8\cos(x)^2-1}\sin(x)}\right)$$
$$+\frac{3}{4}\arctan\left(\frac{\sqrt{4\cos(x)^2-1}}{\sin(x)}\right)$$

```
[In] integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,
algorithm="fricas")
```

```
[Out] -5/32*sqrt(2)*arctan(1/16*(512*sqrt(2)*cos(x)^4 - 576*sqrt(2)*cos(x)^2 + 11
3*sqrt(2))*sqrt(8*cos(x)^2 - 1)/((128*cos(x)^4 - 88*cos(x)^2 + 9)*sin(x)))
- 1/2*sqrt(8*cos(x)^2 - 1)*sin(x) - 1/2*sqrt(4*cos(x)^2 - 1)*sin(x) + 3/8*a
rctan((4*(8*cos(x)^2 - 5)*sqrt(4*cos(x)^2 - 1)*sin(x) - 9*cos(x)*sin(x))/(6
4*cos(x)^4 - 71*cos(x)^2 + 16)) + 3/8*arctan(sin(x)/cos(x)) + 3/8*arctan(1/
2*(9*cos(x)^2 - 2)/(sqrt(8*cos(x)^2 - 1)*sin(x))) + 3/4*arctan(sqrt(4*cos(x
)^2 - 1)/sin(x))
```

Sympy [F]

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$$
$$= \int \frac{\cos(3x)}{\sqrt{-\sin^2(x)+3\cos^2(x)}-\sqrt{8\cos^2(x)-1}}\,dx$$

```
[In] integrate(cos(3*x)/(-(-1+8*cos(x)**2)**(1/2)+(3*cos(x)**2-sin(x)**2)**(1/2)
),x)
```

```
[Out] Integral(cos(3*x)/(sqrt(-sin(x)**2 + 3*cos(x)**2) - sqrt(8*cos(x)**2 - 1)),
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,
algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = \int -\frac{\cos(3x)}{\sqrt{8\cos(x)^2-1}-\sqrt{3\cos(x)^2-\sin(x)^2}}\,dx$$

```
[In] integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,
algorithm="giac")
```

```
[Out] integrate(-cos(3*x)/(sqrt(8*cos(x)^2 - 1) - sqrt(3*cos(x)^2 - sin(x)^2)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = -\int -\frac{\cos(3x)}{\sqrt{3\cos(x)^2-\sin(x)^2}-\sqrt{8\cos(x)^2-1}}\,dx$$

```
[In] int(cos(3*x)/((3*cos(x)^2 - sin(x)^2)^(1/2) - (8*cos(x)^2 - 1)^(1/2)),x)
```

```
[Out] -int(-cos(3*x)/((3*cos(x)^2 - sin(x)^2)^(1/2) - (8*cos(x)^2 - 1)^(1/2)), x)
```

3.428 $\int (2-3\sin^2(x))^{3/5}\sin(4x)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int (2-3\sin^2(x))^{3/5}\sin(4x)\,dx=\frac{5}{36}(2-3\sin^2(x))^{8/5}-\frac{20}{117}(2-3\sin^2(x))^{13/5}$$

```
[Out] 5/36*(2-3*sin(x)^2)^(8/5)-20/117*(2-3*sin(x)^2)^(13/5)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {12, 455, 45}

$$\int (2-3\sin^2(x))^{3/5}\sin(4x)\,dx=\frac{5}{36}(2-3\sin^2(x))^{8/5}-\frac{20}{117}(2-3\sin^2(x))^{13/5}$$

```
[In] Int[(2 - 3*Sin[x]^2)^(3/5)*Sin[4*x],x]
[Out] (5*(2 - 3*Sin[x]^2)^(8/5))/36 - (20*(2 - 3*Sin[x]^2)^(13/5))/117
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int 4x\left(2-3x^2\right)^{3/5}\left(1-2x^2\right)\,dx, x, \sin(x)\right)\\
&= 4\text{Subst}\left(\int x\left(2-3x^2\right)^{3/5}\left(1-2x^2\right)\,dx, x, \sin(x)\right)\\
&= 2\text{Subst}\left(\int (2-3x)^{3/5}(1-2x)\,dx, x, \sin^2(x)\right)\\
&= 2\text{Subst}\left(\int \left(-\frac{1}{3}(2-3x)^{3/5}+\frac{2}{3}(2-3x)^{8/5}\right)\,dx, x, \sin^2(x)\right)\\
&= \frac{5}{36}\left(2-3\sin^2(x)\right)^{8/5}-\frac{20}{117}\left(2-3\sin^2(x)\right)^{13/5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = -\frac{5(1+3\cos(2x))^{8/5}(-5+24\cos(2x))}{936\ 2^{3/5}}$$

```
[In] Integrate[(2 - 3*Sin[x]^2)^(3/5)*Sin[4*x],x]
[Out] (-5*(1 + 3*Cos[2*x])^(8/5)*(-5 + 24*Cos[2*x]))/(936*2^(3/5))
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{5\left(2-3\left(\sin^2(x)\right)\right)^{\frac{8}{5}}}{12}-\frac{5(16+48\cos(2x))^{\frac{13}{5}}}{239616}-\frac{5(16+48\cos(2x))^{\frac{8}{5}}}{4608}$	38

```
[In] int((2-3*sin(x)^2)^(3/5)*sin(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] 5/12*(2-3*sin(x)^2)^(8/5)-5/239616*(16+48*cos(2*x))^(13/5)-5/4608*(16+48*co
s(2*x))^(8/5)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = -\frac{5}{468}\left(144\,\cos\left(x\right)^4-135\,\cos\left(x\right)^2+29\right)\left(3\,\cos\left(x\right)^2-1\right)^{\frac{3}{5}}$$

```
[In] integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="fricas")
```

```
[Out] -5/468*(144*cos(x)^4 - 135*cos(x)^2 + 29)*(3*cos(x)^2 - 1)^(3/5)
```

Sympy [F(-1)]

Timed out.

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = \text{Timed out}$$

```
[In] integrate((2-3*sin(x)**2)**(3/5)*sin(4*x),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = -\frac{20}{117}\left(-3\,\sin\left(x\right)^2+2\right)^{\frac{13}{5}}+\frac{5}{36}\left(-3\,\sin\left(x\right)^2+2\right)^{\frac{8}{5}}$$

```
[In] integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="maxima")
```

```
[Out] -20/117*(-3*sin(x)^2 + 2)^(13/5) + 5/36*(-3*sin(x)^2 + 2)^(8/5)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = -\frac{20}{117}\left(3\,\sin\left(x\right)^2-2\right)^2\left(-3\,\sin\left(x\right)^2+2\right)^{\frac{3}{5}}+\frac{5}{36}\left(-3\,\sin\left(x\right)^2+2\right)^{\frac{8}{5}}$$

```
[In] integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="giac")
[Out] -20/117*(3*sin(x)^2 - 2)^2*(-3*sin(x)^2 + 2)^(3/5) + 5/36*(-3*sin(x)^2 + 2)
^(8/5)
```

Mupad [F(-1)]

Timed out.

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = \int \sin\left(4\,x\right)\,\left(2-3\sin\left(x\right)^2\right)^{3/5}\,dx$$

```
[In] int(sin(4*x)*(2 - 3*sin(x)^2)^(3/5),x)
[Out] int(sin(4*x)*(2 - 3*sin(x)^2)^(3/5), x)
```

3.429 $\int \cos(x)\sqrt{\cos(2x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \frac{\arcsin\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}} + \frac{1}{2}\sqrt{\cos(2x)}\sin(x)$$

```
[Out] 1/4*arcsin(sin(x)*2^(1/2))*2^(1/2)+1/2*sin(x)*cos(2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4441, 201, 222}

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \frac{\arcsin\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}} + \frac{1}{2}\sin(x)\sqrt{\cos(2x)}$$

```
[In] Int[Cos[x]*Sqrt[Cos[2*x]],x]
```

```
[Out] ArcSin[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + (Sqrt[Cos[2*x]]*Sin[x])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \sqrt{1-2x^2}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}\sqrt{\cos(2x)}\sin(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-2x^2}}\,dx, x, \sin(x)\right)\\ &= \frac{\arcsin\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}} + \frac{1}{2}\sqrt{\cos(2x)}\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \frac{1}{4}\left(\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right) + 2\sqrt{\cos(2x)}\sin(x)\right)$$

```
[In] Integrate[Cos[x]*Sqrt[Cos[2*x]],x]
[Out] (Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] + 2*Sqrt[Cos[2*x]]*Sin[x])/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(23) = 46.

Time = 0.47 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

method	result	size
default	$-\frac{\sqrt{(2(\cos^2(x))-1)(\sin^2(x))}\left(-\sqrt{2}\arcsin(4(\sin^2(x))-1)-4\sqrt{-2(\sin^4(x))+\sin^2(x)}\right)}{8\sin(x)\sqrt{2(\cos^2(x))-1}}$	62

```
[In] int(cos(x)*cos(2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/8*((2*cos(x)^2-1)*sin(x)^2)^(1/2)*(-2^(1/2)*arcsin(4*sin(x)^2-1)-4*(-2*s
in(x)^4+sin(x)^2)^(1/2))/sin(x)/(2*cos(x)^2-1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(23) = 46$.

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = -\frac{1}{16}\sqrt{2}\arctan\left(\frac{\left(32\sqrt{2}\cos\left(x\right)^{4}-48\sqrt{2}\cos\left(x\right)^{2}+17\sqrt{2}\right)\sqrt{2\cos\left(x\right)^{2}-1}}{8\left(8\cos\left(x\right)^{4}-10\cos\left(x\right)^{2}+3\right)\sin\left(x\right)}\right) + \frac{1}{2}\sqrt{2\cos\left(x\right)^{2}-1}\sin\left(x\right)$$

```
[In] integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="fricas")
[Out] -1/16*sqrt(2)*arctan(1/8*(32*sqrt(2)*cos(x)^4 - 48*sqrt(2)*cos(x)^2 + 17*sq
rt(2))*sqrt(2*cos(x)^2 - 1)/((8*cos(x)^4 - 10*cos(x)^2 + 3)*sin(x))) + 1/2*
sqrt(2*cos(x)^2 - 1)*sin(x)
```

Sympy [F]

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \int \cos\left(x\right)\sqrt{\cos\left(2x\right)}\,dx$$

```
[In] integrate(cos(x)*cos(2*x)**(1/2),x)
[Out] Integral(cos(x)*sqrt(cos(2*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(23) = 46$.

Time = 0.32 (sec) , antiderivative size = 488, normalized size of antiderivative = 14.79

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="maxima")
[Out] 1/16*sqrt(2)*(2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*a
rctan2(sin(4*x), cos(4*x) + 1))*sin(2*x) - (cos(2*x) - 1)*sin(1/2*arctan2(s
in(4*x), cos(4*x) + 1))) + arctan2(-(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) +
 1)^(1/4)*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(2*x) - cos(2*x)*sin
(1/2*arctan2(sin(4*x), cos(4*x) + 1))), (cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*
```

```
x) + 1)^(1/4)*(cos(2*x)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x)
*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))) + 1) - arctan2(-(cos(4*x)^2 + si
n(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))*
sin(2*x) - cos(2*x)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))), (cos(4*x)^2
+ sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(2*x)*cos(1/2*arctan2(sin(4*x), co
s(4*x) + 1)) + sin(2*x)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))) - 1) - ar
ctan2((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*sin(1/2*arctan2(sin(
4*x), cos(4*x) + 1)), (cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(
1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) + arctan2((cos(4*x)^2 + sin(4*x)^
2 + 2*cos(4*x) + 1)^(1/4)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1)), (cos(4*
x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x
) + 1)) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \frac{1}{4}\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(x\right)\right) + \frac{1}{2}\sqrt{-2\sin\left(x\right)^{2} + 1}\sin\left(x\right)$$

```
[In] integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(2)*arcsin(sqrt(2)*sin(x)) + 1/2*sqrt(-2*sin(x)^2 + 1)*sin(x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \int \sqrt{\cos\left(2\,x\right)}\,\cos\left(x\right)\,dx$$

```
[In] int(cos(2*x)^(1/2)*cos(x),x)
[Out] int(cos(2*x)^(1/2)*cos(x), x)
```

3.430 $\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 55

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = -\frac{3\operatorname{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{8\sqrt{2}} + \frac{3}{8}\cos(x)\sqrt{\cos(2x)} - \frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)$$

```
[Out] -1/4*cos(x)*cos(2*x)^(3/2)-3/16*arctanh(cos(x)*2^(1/2)/cos(2*x)^(1/2))*2^(1
/2)+3/8*cos(x)*cos(2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4442, 201, 223, 212}

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = -\frac{3\operatorname{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{8\sqrt{2}} - \frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x) + \frac{3}{8}\cos(x)\sqrt{\cos(2x)}$$

```
[In] Int[Cos[2*x]^(3/2)*Sin[x],x]
```

```
[Out] (-3*ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]])/(8*Sqrt[2]) + (3*Cos[x]*Sqrt[
Cos[2*x]])/8 - (Cos[x]*Cos[2*x]^(3/2))/4
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4442

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \left(-1+2x^2\right)^{3/2}\,dx, x, \cos(x)\right)\\
&= -\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)+\frac{3}{4}\text{Subst}\left(\int \sqrt{-1+2x^2}\,dx, x, \cos(x)\right)\\
&= \frac{3}{8}\cos(x)\sqrt{\cos(2x)}-\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)-\frac{3}{8}\text{Subst}\left(\int \frac{1}{\sqrt{-1+2x^2}}\,dx, x, \cos(x)\right)\\
&= \frac{3}{8}\cos(x)\sqrt{\cos(2x)}-\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)-\frac{3}{8}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\cos(x)}{\sqrt{\cos(2x)}}\right)\\
&= -\frac{3\text{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{8\sqrt{2}}+\frac{3}{8}\cos(x)\sqrt{\cos(2x)}-\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = -\frac{1}{8}\sqrt{\cos(2x)}(-2\cos(x)+\cos(3x))-\frac{3\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)}{8\sqrt{2}}$$

```
[In] Integrate[Cos[2*x]^(3/2)*Sin[x],x]
[Out] -1/8*(Sqrt[Cos[2*x]]*(-2*Cos[x] + Cos[3*x])) - (3*Log[Sqrt[2]*Cos[x] + Sqrt
[Cos[2*x]]])/(8*Sqrt[2])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{3 \ln\left(\cos(x)\sqrt{2}+\sqrt{2(\cos^2(x))-1}\right)\sqrt{2}}{16}-\frac{(\cos^3(x))\sqrt{2(\cos^2(x))-1}}{2}+\frac{5\cos(x)\sqrt{2(\cos^2(x))-1}}{8}$	55

```
[In] int(cos(2*x)^(3/2)*sin(x),x,method=_RETURNVERBOSE)
[Out] -3/16*ln(cos(x)*2^(1/2)+(2*cos(x)^2-1)^(1/2))*2^(1/2)-1/2*cos(x)^3*(2*cos(x
)^2-1)^(1/2)+5/8*cos(x)*(2*cos(x)^2-1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(39) = 78.

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.87

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = -\frac{1}{8}\left(4\cos(x)^3-5\cos(x)\right)\sqrt{2\cos(x)^2-1} + \frac{3}{128}\sqrt{2}\log\left(2048\cos(x)^8-2048\cos(x)^6+640\cos(x)^4-64\cos(x)^2 - 8\left(128\sqrt{2}\cos(x)^7-96\sqrt{2}\cos(x)^5+20\sqrt{2}\cos(x)^3-\sqrt{2}\cos(x)\right)\sqrt{2\cos(x)^2-1} + 1\right)$$

```
[In] integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="fricas")
[Out] -1/8*(4*cos(x)^3 - 5*cos(x))*sqrt(2*cos(x)^2 - 1) + 3/128*sqrt(2)*log(2048*
cos(x)^8 - 2048*cos(x)^6 + 640*cos(x)^4 - 64*cos(x)^2 - 8*(128*sqrt(2)*cos(
x)^7 - 96*sqrt(2)*cos(x)^5 + 20*sqrt(2)*cos(x)^3 - sqrt(2)*cos(x))*sqrt(2*c
os(x)^2 - 1) + 1)
```

Sympy [F]

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = \int \sin(x)\cos^{\frac{3}{2}}(2x)\,dx$$

```
[In] integrate(cos(2*x)**(3/2)*sin(x),x)
[Out] Integral(sin(x)*cos(2*x)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. 2(39) = 78.

Time = 0.34 (sec) , antiderivative size = 790, normalized size of antiderivative = 14.36

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = \text{Too large to display}$$

```
[In] integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="maxima")
[Out] -1/128*sqrt(2)*(4*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(((cos(4
*x) - 2)*cos(1/2*arctan2(sin(4*x), cos(4*x))) + sin(4*x)*sin(1/2*arctan2(si
n(4*x), cos(4*x))) + cos(4*x) - 2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))
 - (cos(1/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x) - (cos(4*x) - 2)*sin(1/2*
arctan2(sin(4*x), cos(4*x))) - sin(4*x))*sin(1/2*arctan2(sin(4*x), cos(4*x)
 + 1))) + 3*log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arct
an2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x)
+ 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + 2*(cos(4*x)^2 + sin(4*x)^
2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) - 3
*log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x
), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*sin(1/
2*arctan2(sin(4*x), cos(4*x) + 1))^2 - 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4
*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) + 3*log(((cos(
1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2
)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(1/2*arctan2(sin(4*x), c
os(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2)*sin(1/2*arctan2(sin(4
*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1) + 2*(
cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x),
cos(4*x) + 1))*cos(1/2*arctan2(sin(4*x), cos(4*x))) + sin(1/2*arctan2(sin(4
*x), cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*x)))) + 1) - 3*log(((co
s(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))
^2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(1/2*arctan2(sin(4*x),
 cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2)*sin(1/2*arctan2(sin
(4*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1) - 2
*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x)
, cos(4*x) + 1))*cos(1/2*arctan2(sin(4*x), cos(4*x))) + sin(1/2*arctan2(sin
(4*x), cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*x)))) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = -\frac{1}{8}\left(4\cos(x)^2-5\right)\sqrt{2\cos(x)^2-1}\cos(x) + \frac{3}{16}\sqrt{2}\log\left(\left|-\sqrt{2}\cos(x)+\sqrt{2\cos(x)^2-1}\right|\right)$$

```
[In] integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="giac")
[Out] -1/8*(4*cos(x)^2 - 5)*sqrt(2*cos(x)^2 - 1)*cos(x) + 3/16*sqrt(2)*log(abs(-s
qrt(2)*cos(x) + sqrt(2*cos(x)^2 - 1)))
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.53

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = -\frac{\cos(2x)^{3/2}\cos(x)\;{}_2F_1\left(-\frac{3}{2},\frac{1}{2};\;\frac{3}{2};\;\cos(2x)+1\right)}{\left(-\cos(2x)\right)^{3/2}}$$

```
[In] int(cos(2*x)^(3/2)*sin(x),x)
[Out] -(cos(2*x)^(3/2)*cos(x)*hypergeom([-3/2, 1/2], 3/2, cos(2*x) + 1))/(-cos(2*
x))^(3/2)
```

3.431 $\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

```
[Out] -1/3*cos(3*x)/cos(2*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4416}

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

```
[In] Int[Sin[x]/Cos[2*x]^(5/2),x]
[Out] -1/3*Cos[3*x]/Cos[2*x]^(3/2)
```

Rule 4416

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)], x_Symbol
] :> Simp[(-(m + 2))*(e*Cos[a + b*x])^(m + 1)*(Cos[(m + 1)*(a + b*x)]/(d*e*
(m + 1))), x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d
/b, Abs[m + 2]]
```

Rubi steps

$$\text{integral} = -\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

```
[In] Integrate[Sin[x]/Cos[2*x]^(5/2),x]
[Out] -1/3*Cos[3*x]/Cos[2*x]^(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(12) = 24.

Time = 0.58 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

method	result	size
default	$\frac{\sqrt{1-2(\sin^2(x))}\,\cos(x)(4(\sin^2(x))-1)}{12(\sin^4(x))-12(\sin^2(x))+3}$	39

```
[In] int(sin(x)/cos(2*x)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3/(4*sin(x)^4-4*sin(x)^2+1)*(1-2*sin(x)^2)^(1/2)*cos(x)*(4*sin(x)^2-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\left(4\cos(x)^3 - 3\cos(x)\right)\sqrt{2\cos(x)^2 - 1}}{3\left(4\cos(x)^4 - 4\cos(x)^2 + 1\right)}$$

```
[In] integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="fricas")
[Out] -1/3*(4*cos(x)^3 - 3*cos(x))*sqrt(2*cos(x)^2 - 1)/(4*cos(x)^4 - 4*cos(x)^2
+ 1)
```

Sympy [F]

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = \int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx$$

```
[In] integrate(sin(x)/cos(2*x)**(5/2),x)
[Out] Integral(sin(x)/cos(2*x)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(12) = 24.

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 5.62

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\sqrt{2}\sin\left(\frac{3}{2}\arctan\left(\sin\left(4x\right),\cos\left(4x\right)+1\right)\right)\sin\left(\frac{3}{2}\arctan\left(\sin\left(4x\right),\cos\left(4x\right)\right)\right)+\left(\sqrt{2}\cos\left(\frac{3}{2}\arctan\left(\sin\right.\right.\right.}{3\left(\cos\left(4x\right)^{2}+\sin\left(4x\right)^{2}+2\cos\left(4x\right)+1\right)}$$

```
[In] integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="maxima")
[Out] -1/3*(sqrt(2)*sin(3/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(3/2*arctan2(sin(
4*x), cos(4*x))) + (sqrt(2)*cos(3/2*arctan2(sin(4*x), cos(4*x))) + sqrt(2))
*cos(3/2*arctan2(sin(4*x), cos(4*x) + 1)))/(cos(4*x)^2 + sin(4*x)^2 + 2*cos
(4*x) + 1)^(3/4)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\left(4\cos\left(x\right)^{2}-3\right)\cos\left(x\right)}{3\left(2\cos\left(x\right)^{2}-1\right)^{\frac{3}{2}}}$$

```
[In] integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="giac")
[Out] -1/3*(4*cos(x)^2 - 3)*cos(x)/(2*cos(x)^2 - 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\cos(3\,x)}{3\cos(2\,x)^{3/2}}$$

```
[In] int(sin(x)/cos(2*x)^(5/2),x)
[Out] -cos(3*x)/(3*cos(2*x)^(3/2))
```

3.432 $\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = 2\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right) - \frac{5}{2}\arctan\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right) - \frac{1}{2}\sqrt{\cos(2x)}\sec(x)\tan(x)$$

```
[Out] -5/2*arctan(sin(x)/cos(2*x)^(1/2))+2*arcsin(sin(x)*2^(1/2))*2^(1/2)-1/2*sec
(x)*cos(2*x)^(1/2)*tan(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {4449, 424, 537, 222, 385, 209}

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = 2\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right) - \frac{5}{2}\arctan\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right) - \frac{1}{2}\sqrt{\cos(2x)}\tan(x)\sec(x)$$

```
[In] Int[Cos[2*x]^(3/2)*Sec[x]^3,x]
```

```
[Out] 2*Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - (5*ArcTan[Sin[x]/Sqrt[Cos[2*x]]])/2 - (S
qrt[Cos[2*x]]*Sec[x]*Tan[x])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4449

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{(1-2x^2)^{3/2}}{(1-x^2)^2}\,dx, x, \sin(x)\right) \\ &= -\frac{1}{2}\sqrt{\cos(2x)}\sec(x)\tan(x) - \frac{1}{2}\text{Subst}\left(\int \frac{-3+8x^2}{\sqrt{1-2x^2}\,(1-x^2)}\,dx, x, \sin(x)\right) \end{aligned}$$

$$= -\frac{1}{2}\sqrt{\cos(2x)}\sec(x)\tan(x) - \frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-2x^2}\,(1-x^2)}\,dx, x, \sin(x)\right) + 4\text{Subst}\left(\int \frac{1}{\sqrt{1-2x^2}}\,dx, x, \sin(x)\right)$$

$$= 2\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right) - \frac{1}{2}\sqrt{\cos(2x)}\sec(x)\tan(x) - \frac{5}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sin(x)}{\sqrt{\cos(2x)}}\right)$$

$$= 2\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right) - \frac{5}{2}\arctan\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right) - \frac{1}{2}\sqrt{\cos(2x)}\sec(x)\tan(x)$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = \frac{1}{2}\left(4\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right) - 5\arctan\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right) - \sqrt{\cos(2x)}\sec(x)\tan(x)\right)$$

```
[In] Integrate[Cos[2*x]^(3/2)*Sec[x]^3,x]
```

```
[Out] (4*Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - 5*ArcTan[Sin[x]/Sqrt[Cos[2*x]]] - Sqrt[
Cos[2*x]]*Sec[x]*Tan[x])/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(37) = 74.

Time = 0.60 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.04

method	result
default	$-\frac{\sqrt{(2(\cos^2(x))-1)(\sin^2(x))}\left(4\sqrt{2}\,\arcsin(4(\cos^2(x))-3)(\cos^2(x))-5\arctan\left(\frac{3\left(\cos^2(x)\right)-2}{2\sqrt{-2\left(\sin^4(x)\right)+\sin^2(x)}}\right)(\cos^2(x))+2\sqrt{-2(\sin^4(x)}\right.}{4\cos(x)^2\sin(x)\sqrt{2(\cos^2(x))-1}}$

```
[In] int(cos(2*x)^(3/2)/cos(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*((2*cos(x)^2-1)*sin(x)^2)^(1/2)*(4*2^(1/2)*arcsin(4*cos(x)^2-3)*cos(x)
^2-5*arctan(1/2*(3*cos(x)^2-2)/(-2*sin(x)^4+sin(x)^2)^(1/2))*cos(x)^2+2*(-2
*sin(x)^4+sin(x)^2)^(1/2))/cos(x)^2/sin(x)/(2*cos(x)^2-1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.41

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx =$$

$$-\frac{2\sqrt{2}\arctan\left(\frac{\left(32\sqrt{2}\cos(x)^4-48\sqrt{2}\cos(x)^2+17\sqrt{2}\right)\sqrt{2\cos(x)^2-1}}{8\left(8\cos(x)^4-10\cos(x)^2+3\right)\sin(x)}\right)\cos(x)^2 - 5\arctan\left(\frac{3\cos(x)^2-2}{2\sqrt{2\cos(x)^2-1}\sin(x)}\right)\cos(x)^2}{4\cos(x)^2}$$

```
[In] integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/8*(32*sqrt(2)*cos(x)^4 - 48*sqrt(2)*cos(x)^2 + 17*
sqrt(2))*sqrt(2*cos(x)^2 - 1)/((8*cos(x)^4 - 10*cos(x)^2 + 3)*sin(x)))*cos(
x)^2 - 5*arctan(1/2*(3*cos(x)^2 - 2)/(sqrt(2*cos(x)^2 - 1)*sin(x)))*cos(x)^
2 + 2*sqrt(2*cos(x)^2 - 1)*sin(x))/cos(x)^2
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = \text{Timed out}$$

```
[In] integrate(cos(2*x)**(3/2)/cos(x)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = \int \frac{\cos(2x)^{\frac{3}{2}}}{\cos(x)^3}\,dx$$

```
[In] integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(2*x)^(3/2)/cos(x)^3, x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = \int \frac{\cos(2x)^{\frac{3}{2}}}{\cos(x)^3}\,dx$$

```
[In] integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="giac")
[Out] integrate(cos(2*x)^(3/2)/cos(x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = \int \frac{\cos(2x)^{3/2}}{\cos(x)^3}\,dx$$

```
[In] int(cos(2*x)^(3/2)/cos(x)^3,x)
[Out] int(cos(2*x)^(3/2)/cos(x)^3, x)
```

3.433 $\int \frac{\sin^2(x)(3\sin^3(x)-\cos(x)\sin(4x))}{\cos^{\frac{7}{2}}(2x)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 87

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}}-\frac{11\cos(x)}{20\cos^{\frac{3}{2}}(2x)}-\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)}+\frac{63\cos(x)}{20\sqrt{\cos(2x)}}+\frac{3\cos(x)\sin^2(x)}{10\cos^{\frac{5}{2}}(2x)}$$

```
[Out] -11/20*cos(x)/cos(2*x)^(3/2)-2/3*cos(x)^3/cos(2*x)^(3/2)+3/10*cos(x)*sin(x)
^2/cos(2*x)^(5/2)-1/2*arctanh(cos(x)*2^(1/2)/cos(2*x)^(1/2))*2^(1/2)+63/20*
cos(x)/cos(2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {4462, 12, 463, 294, 223, 212, 4451, 386, 197}

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}}-\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)}+\frac{13\cos(x)}{5\sqrt{\cos(2x)}}+\frac{3\sin^4(x)\cos(x)}{5\cos^{\frac{5}{2}}(2x)}-\frac{4\sin^2(x)\cos(x)}{5\cos^{\frac{3}{2}}(2x)}$$

```
[In] Int[(Sin[x]^2*(3*Sin[x]^3 - Cos[x]*Sin[4*x]))/Cos[2*x]^(7/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]]/Sqrt[2]) - (2*Cos[x]^3)/(3*Cos[2
*x]^(3/2)) + (13*Cos[x])/(5*Sqrt[Cos[2*x]]) - (4*Cos[x]*Sin[x]^2)/(5*Cos[2*
x]^(3/2)) + (3*Cos[x]*Sin[x]^4)/(5*Cos[2*x]^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rule 4451

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 4462

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\int \frac{\sin^5(x)}{\cos^{\frac{7}{2}}(2x)}\,dx-\int \frac{\cos(x)\sin^2(x)\sin(4x)}{\cos^{\frac{7}{2}}(2x)}\,dx\\
&=-\left(3\text{Subst}\left(\int \frac{(1-x^2)^2}{(-1+2x^2)^{7/2}}\,dx,x,\cos(x)\right)\right)+\text{Subst}\left(\int \frac{4x^2(1-x^2)}{(-1+2x^2)^{5/2}}\,dx,x,\cos(x)\right)\\
&=\frac{3\cos(x)\sin^4(x)}{5\cos^{\frac{5}{2}}(2x)}+\frac{12}{5}\text{Subst}\left(\int \frac{1-x^2}{(-1+2x^2)^{5/2}}\,dx,x,\cos(x)\right)\\
&\quad+4\text{Subst}\left(\int \frac{x^2(1-x^2)}{(-1+2x^2)^{5/2}}\,dx,x,\cos(x)\right)\\
&=-\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)}-\frac{4\cos(x)\sin^2(x)}{5\cos^{\frac{3}{2}}(2x)}+\frac{3\cos(x)\sin^4(x)}{5\cos^{\frac{5}{2}}(2x)}\\
&\quad-\frac{8}{5}\text{Subst}\left(\int \frac{1}{(-1+2x^2)^{3/2}}\,dx,x,\cos(x)\right)\\
&\quad-2\text{Subst}\left(\int \frac{x^2}{(-1+2x^2)^{3/2}}\,dx,x,\cos(x)\right)\\
&=-\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)}+\frac{13\cos(x)}{5\sqrt{\cos(2x)}}-\frac{4\cos(x)\sin^2(x)}{5\cos^{\frac{3}{2}}(2x)}\\
&\quad+\frac{3\cos(x)\sin^4(x)}{5\cos^{\frac{5}{2}}(2x)}-\text{Subst}\left(\int \frac{1}{\sqrt{-1+2x^2}}\,dx,x,\cos(x)\right)\\
&=-\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)}+\frac{13\cos(x)}{5\sqrt{\cos(2x)}}-\frac{4\cos(x)\sin^2(x)}{5\cos^{\frac{3}{2}}(2x)}\\
&\quad+\frac{3\cos(x)\sin^4(x)}{5\cos^{\frac{5}{2}}(2x)}-\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\frac{\cos(x)}{\sqrt{\cos(2x)}}\right)
\end{aligned}
$$

$$= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}} - \frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)} + \frac{13\cos(x)}{5\sqrt{\cos(2x)}} - \frac{4\cos(x)\sin^2(x)}{5\cos^{\frac{3}{2}}(2x)} + \frac{3\cos(x)\sin^4(x)}{5\cos^{\frac{5}{2}}(2x)}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2(x)\left(3\sin^3(x) - \cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx$$
$$= \frac{250\cos(x) + 45\cos(3x) + 169\cos(5x) - 120\sqrt{2}\cos^{\frac{5}{2}}(2x)\log\left(\sqrt{2}\cos(x) + \sqrt{\cos(2x)}\right)}{240\cos^{\frac{5}{2}}(2x)}$$

```
[In] Integrate[(Sin[x]^2*(3*Sin[x]^3 - Cos[x]*Sin[4*x]))/Cos[2*x]^(7/2),x]
```

```
[Out] (250*Cos[x] + 45*Cos[3*x] + 169*Cos[5*x] - 120*Sqrt[2]*Cos[2*x]^(5/2)*Log[S
qrt[2]*Cos[x] + Sqrt[Cos[2*x]]])/(240*Cos[2*x]^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(65) = 130.

Time = 2.61 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.07

method	result
default	$-\frac{120\sqrt{2}\,\ln\left(\cos(x)\sqrt{2}+\sqrt{1-2(\sin^2(x))}\right)(\sin^6(x))+338\sqrt{1-2(\sin^2(x))}\,\cos(x)(\sin^4(x))-180\ln\left(\cos(x)\sqrt{2}+\sqrt{1-2(\sin^2(x))}\right)\sqrt{2}\,(\mathrm{s}}{30(8(}$
parts	$-\frac{\sqrt{1-2(\sin^2(x))}\,\cos(x)(43(\sin^4(x))-36(\sin^2(x))+8)}{5(8(\sin^6(x))-12(\sin^4(x))+6(\sin^2(x))-1)} - \frac{12\ln\left(\cos(x)\sqrt{2}+\sqrt{1-2(\sin^2(x))}\right)\sqrt{2}\,(\sin^4(x))+8\sqrt{1-2(\sin^2(x))}\,(\sin}{}$

```
[In] int((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/30/(8*sin(x)^6-12*sin(x)^4+6*sin(x)^2-1)*(120*2^(1/2)*ln(cos(x)*2^(1/2)+
(1-2*sin(x)^2)^(1/2))*sin(x)^6+338*(1-2*sin(x)^2)^(1/2)*cos(x)*sin(x)^4-180
*ln(cos(x)*2^(1/2)+(1-2*sin(x)^2)^(1/2))*2^(1/2)*sin(x)^4-276*(1-2*sin(x)^2
)^(1/2)*sin(x)^2*cos(x)+90*ln(cos(x)*2^(1/2)+(1-2*sin(x)^2)^(1/2))*2^(1/2)*
sin(x)^2+58*(1-2*sin(x)^2)^(1/2)*cos(x)-15*ln(cos(x)*2^(1/2)+(1-2*sin(x)^2)
^(1/2))*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(65) = 130$.

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.87

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx$$
$$= \frac{15\left(8\sqrt{2}\cos(x)^6-12\sqrt{2}\cos(x)^4+6\sqrt{2}\cos(x)^2-\sqrt{2}\right)\log\left(2048\cos(x)^8-2048\cos(x)^6+640\cos(x)^4\right.}{}$$

```
[In] integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorithm
="fricas")
```

```
[Out] 1/240*(15*(8*sqrt(2)*cos(x)^6 - 12*sqrt(2)*cos(x)^4 + 6*sqrt(2)*cos(x)^2 -
sqrt(2))*log(2048*cos(x)^8 - 2048*cos(x)^6 + 640*cos(x)^4 - 64*cos(x)^2 - 8
*(128*sqrt(2)*cos(x)^7 - 96*sqrt(2)*cos(x)^5 + 20*sqrt(2)*cos(x)^3 - sqrt(2
)*cos(x))*sqrt(2*cos(x)^2 - 1) + 1) + 16*(169*cos(x)^5 - 200*cos(x)^3 + 60*
cos(x))*sqrt(2*cos(x)^2 - 1))/(8*cos(x)^6 - 12*cos(x)^4 + 6*cos(x)^2 - 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx = \text{Timed out}$$

```
[In] integrate((3*sin(x)**3-cos(x)*sin(4*x))/cos(2*x)**(7/2)/csc(x)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1359 vs. $2(65) = 130$.

Time = 0.41 (sec) , antiderivative size = 1359, normalized size of antiderivative = 15.62

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx = \text{Too large to display}$$

```
[In] integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorithm
="maxima")
```

```
[Out] 1/48*(4*(4*sqrt(2)*sin(4*x)*sin(5/2*arctan2(sin(4*x), cos(4*x))) + 4*(sqrt(
2)*cos(4*x) + sqrt(2))*cos(5/2*arctan2(sin(4*x), cos(4*x))) + 3*sqrt(2)*cos
```

(8*x) + 7*sqrt(2)*cos(4*x) + 4*sqrt(2))*cos(5/2*arctan2(sin(4*x), cos(4*x)
+ 1)) + 12*sqrt(2)*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(3/2*a
rctan2(sin(4*x), cos(4*x) + 1)) - 12*(sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)
^2 + 2*sqrt(2)*cos(4*x) + sqrt(2))*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))
 - 4*(4*sqrt(2)*cos(5/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x) - 4*(sqrt(2)*
cos(4*x) + sqrt(2))*sin(5/2*arctan2(sin(4*x), cos(4*x))) - 3*sqrt(2)*sin(8*
x) - 7*sqrt(2)*sin(4*x))*sin(5/2*arctan2(sin(4*x), cos(4*x) + 1)) - 3*(cos(
4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*((sqrt(2)*cos(4*x)^2 + sqrt(2)*
sin(4*x)^2 + 2*sqrt(2)*cos(4*x) + sqrt(2))*log(sqrt(cos(4*x)^2 + sin(4*x)^2
 + 2*cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(4*
x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1)
)^2 + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(si
n(4*x), cos(4*x) + 1)) + 1) - (sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)^2 + 2*
sqrt(2)*cos(4*x) + sqrt(2))*log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) +
 1)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)
^2 + 2*cos(4*x) + 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 - 2*(cos(4*
x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x
) + 1)) + 1) + (sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)^2 + 2*sqrt(2)*cos(4*x
) + sqrt(2))*log(((cos(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2
(sin(4*x), cos(4*x)))^2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(
1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2
)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2
+ 2*cos(4*x) + 1) + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos
(1/2*arctan2(sin(4*x), cos(4*x) + 1))*cos(1/2*arctan2(sin(4*x), cos(4*x)))
+ sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*
x)))) + 1) - (sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)^2 + 2*sqrt(2)*cos(4*x)
+ sqrt(2))*log(((cos(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(s
in(4*x), cos(4*x)))^2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(1/
2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2)*
sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2 +
2*cos(4*x) + 1) - 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1
/2*arctan2(sin(4*x), cos(4*x) + 1))*cos(1/2*arctan2(sin(4*x), cos(4*x))) +
sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*x)
))) + 1)))/(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(5/4) + 1/20*(((15*co
s(8*x) + 70*cos(4*x) + 43)*cos(5/2*arctan2(sin(4*x), cos(4*x))) + 5*(3*sin(
8*x) + 14*sin(4*x))*sin(5/2*arctan2(sin(4*x), cos(4*x))) - 12)*cos(5/2*arct
an2(sin(4*x), cos(4*x) + 1)) + 15*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1
)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) - (5*(3*sin(8*x) + 14*sin(4*x))*
cos(5/2*arctan2(sin(4*x), cos(4*x))) - (15*cos(8*x) + 70*cos(4*x) + 43)*sin
(5/2*arctan2(sin(4*x), cos(4*x))))*sin(5/2*arctan2(sin(4*x), cos(4*x) + 1))
 + 40*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(3/2*arctan2(sin(4*
x), cos(4*x) + 1)))/((sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)^2 + 2*sqrt(2)*c
os(4*x) + sqrt(2))*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4))

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx=\frac{1}{2}\sqrt{2}\log\left(\left|-\sqrt{2}\cos\left(x\right)+\sqrt{2\cos\left(x\right)^2-1}\right|\right)+\frac{\left(\left(169\cos\left(x\right)^2-200\right)\cos\left(x\right)^2+60\right)\cos\left(x\right)}{15\left(2\cos\left(x\right)^2-1\right)^{\frac{5}{2}}}$$

```
[In] integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorithm
="giac")
```

```
[Out] 1/2*sqrt(2)*log(abs(-sqrt(2)*cos(x) + sqrt(2*cos(x)^2 - 1))) + 1/15*((169*c
os(x)^2 - 200)*cos(x)^2 + 60)*cos(x)/(2*cos(x)^2 - 1)^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx=\int \frac{\sin\left(x\right)^2\left(3\sin\left(x\right)^3-\sin\left(4x\right)\cos\left(x\right)\right)}{\cos\left(2x\right)^{7/2}}\,dx$$

```
[In] int((sin(x)^2*(3*sin(x)^3 - sin(4*x)*cos(x)))/cos(2*x)^(7/2),x)
```

```
[Out] int((sin(x)^2*(3*sin(x)^3 - sin(4*x)*cos(x)))/cos(2*x)^(7/2), x)
```

3.434 $\int (4-5\sec^2(x))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 68

$$\int (4-5\sec^2(x))^{3/2}\,dx = 8\arctan\left(\frac{2\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{7}{2}\sqrt{5}\arctan\left(\frac{\sqrt{5}\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{5}{2}\tan(x)\sqrt{-1-5\tan^2(x)}$$

```
[Out] 8*arctan(2*tan(x)/(-1-5*tan(x)^2)^(1/2))-7/2*arctan(5^(1/2)*tan(x)/(-1-5*ta
n(x)^2)^(1/2))*5^(1/2)-5/2*(-1-5*tan(x)^2)^(1/2)*tan(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4213, 427, 537, 223, 209, 385}

$$\int (4-5\sec^2(x))^{3/2}\,dx = 8\arctan\left(\frac{2\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right) - \frac{7}{2}\sqrt{5}\arctan\left(\frac{\sqrt{5}\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right) - \frac{5}{2}\tan(x)\sqrt{-5\tan^2(x)-1}$$

```
[In] Int[(4 - 5*Sec[x]^2)^(3/2),x]
```

```
[Out] 8*ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]] - (7*Sqrt[5]*ArcTan[(Sqrt[5]*Tan
[x])/Sqrt[-1 - 5*Tan[x]^2]])/2 - (5*Tan[x]*Sqrt[-1 - 5*Tan[x]^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\left(-1-5x^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= -\frac{5}{2}\tan(x)\sqrt{-1-5\tan^2(x)}+\frac{1}{2}\text{Subst}\left(\int \frac{-3-35x^2}{\sqrt{-1-5x^2}\,(1+x^2)}\,dx, x, \tan(x)\right)
\end{aligned}$$

$$= -\frac{5}{2}\tan(x)\sqrt{-1-5\tan^2(x)} + 16\text{Subst}\left(\int \frac{1}{\sqrt{-1-5x^2}\,(1+x^2)}\,dx, x, \tan(x)\right) - \frac{35}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1-5x^2}}\,dx, x, \tan(x)\right)$$

$$= -\frac{5}{2}\tan(x)\sqrt{-1-5\tan^2(x)} + 16\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{35}{2}\text{Subst}\left(\int \frac{1}{1+5x^2}\,dx, x, \frac{\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right)$$

$$= 8\arctan\left(\frac{2\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{7}{2}\sqrt{5}\arctan\left(\frac{\sqrt{5}\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{5}{2}\tan(x)\sqrt{-1-5\tan^2(x)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.69

$$\int \left(4-5\sec^2(x)\right)^{3/2}\,dx = -\frac{(-5+4\cos^2(x))\sec(x)\sqrt{4-5\sec^2(x)}\left(7\sqrt{5}\arctan\left(\frac{\sqrt{5}\sin(x)}{\sqrt{-3+2\cos(2x)}}\right)\cos^2(x)+16i\cos^2(x)\log\left(\sqrt{-3+2\cos(2x)}\right.\right.}{2(-3+2\cos(2x))^{3/2}}$$

```
[In] Integrate[(4 - 5*Sec[x]^2)^(3/2),x]
```

```
[Out] -1/2*((-5 + 4*Cos[x]^2)*Sec[x]*Sqrt[4 - 5*Sec[x]^2]*(7*Sqrt[5]*ArcTan[(Sqrt
[5]*Sin[x])/Sqrt[-3 + 2*Cos[2*x]]]*Cos[x]^2 + (16*I)*Cos[x]^2*Log[Sqrt[-3 +
 2*Cos[2*x]] + (2*I)*Sin[x]] + 5*Sqrt[-3 + 2*Cos[2*x]]*Sin[x]))/(-3 + 2*Cos
[2*x])^(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(54) = 108.

Time = 7.20 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.19

method	result
default	$-\frac{(4-5(\sec^2(x)))^{\frac{3}{2}}\left(7(\cos^3(x))\sqrt{5}\,\arctan\left(\frac{(4\sin(x)-1)\sqrt{5}}{5(\cos(x)+1)\sqrt{\frac{4\left(\cos^2(x)\right)-5}{(\cos(x)+1)^2}}}\right)+7(\cos^3(x))\sqrt{5}\,\arctan\left(\frac{(4\sin(x)+1)\sqrt{5}}{5(\cos(x)+1)\sqrt{\frac{4\left(\cos^2(x)\right)-5}{(\cos(x)+1)^2}}}\right)-32(c\cdots\right.}{4(4(\cos^2(x))-5)\sqrt{\frac{4\left(\cos^2(x)\right)\cdots}{(\cos(x)+1)\cdots}}\cdots}$

[In] int((4-5*sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/4*(4-5*sec(x)^2)^(3/2)/(4*cos(x)^2-5)/((4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2)/(cos(x)+1)*(7*cos(x)^3*5^(1/2)*arctan(1/5*(4*sin(x)-1)/(cos(x)+1)/((4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2)*5^(1/2))+7*cos(x)^3*5^(1/2)*arctan(1/5*(4*sin(x)+1)/(cos(x)+1)/((4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2)*5^(1/2))-32*cos(x)^3*arctan(2*sin(x)/(cos(x)+1)/((4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2))+10*cos(x)^2*sin(x)*((4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2)+10*cos(x)*sin(x)*((4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(54) = 108$.

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.91

$$\int \left(4-5\sec^2(x)\right)^{3/2}\,dx = \frac{7\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)}{5\sin(x)}\right)\cos(x)+8\arctan\left(\frac{4\left(8\cos(x)^3-9\cos(x)\right)\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\sin(x)\cdots}{64\cos(x)^4-143\cos(x)^2+80}\cdots\right.}{2\cos(x)}$$

[In] integrate((4-5*sec(x)^2)^(3/2),x, algorithm="fricas")

[Out] 1/2*(7*sqrt(5)*arctan(1/5*sqrt(5)*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*cos(x)/sin(x))*cos(x) + 8*arctan((4*(8*cos(x)^3 - 9*cos(x))*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*sin(x) + cos(x)*sin(x))/(64*cos(x)^4 - 143*cos(x)^2 + 80))*cos(x) - 8*arctan(sin(x)/cos(x))*cos(x) - 5*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*sin(x))/cos(x)

Sympy [F]

$$\int \left(4-5\sec^2(x)\right)^{3/2}\,dx=\int \left(4-5\sec^2{(x)}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((4-5*sec(x)**2)**(3/2),x)
[Out] Integral((4 - 5*sec(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(4-5\sec^2(x)\right)^{3/2}\,dx=\int \left(-5\,\sec\left(x\right)^2+4\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((4-5*sec(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((-5*sec(x)^2 + 4)^(3/2), x)
```

Giac [F]

$$\int \left(4-5\sec^2(x)\right)^{3/2}\,dx=\int \left(-5\,\sec\left(x\right)^2+4\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((4-5*sec(x)^2)^(3/2),x, algorithm="giac")
[Out] integrate((-5*sec(x)^2 + 4)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(4-5\sec^2(x)\right)^{3/2}\,dx=\int {\left(4-\frac{5}{{\cos\left(x\right)}^2}\right)}^{3/2}\,dx$$

```
[In] int((4 - 5/cos(x)^2)^(3/2),x)
[Out] int((4 - 5/cos(x)^2)^(3/2), x)
```

3.435 $\int \frac{1}{\left(4-5\sec^2(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{1}{\left(4-5\sec^2(x)\right)^{3/2}}\,dx=\frac{1}{8}\arctan\left(\frac{2\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right)-\frac{5\tan(x)}{4\sqrt{-1-5\tan^2(x)}}$$

```
[Out] 1/8*arctan(2*tan(x)/(-1-5*tan(x)^2)^(1/2))-5/4*tan(x)/(-1-5*tan(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4213, 390, 385, 209}

$$\int \frac{1}{\left(4-5\sec^2(x)\right)^{3/2}}\,dx=\frac{1}{8}\arctan\left(\frac{2\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)-\frac{5\tan(x)}{4\sqrt{-5\tan^2(x)-1}}$$

```
[In] Int[(4 - 5*Sec[x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]]/8 - (5*Tan[x])/(4*Sqrt[-1 - 5*Tan[
x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(-1-5x^2)^{3/2}(1+x^2)}\,dx, x, \tan(x)\right)\\
&= -\frac{5\tan(x)}{4\sqrt{-1-5\tan^2(x)}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{-1-5x^2}\,(1+x^2)}\,dx, x, \tan(x)\right)\\
&= -\frac{5\tan(x)}{4\sqrt{-1-5\tan^2(x)}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right)\\
&= \frac{1}{8}\arctan\left(\frac{2\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{5\tan(x)}{4\sqrt{-1-5\tan^2(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.98

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx = -\frac{(-3+2\cos(2x))^{3/2}\sec^3(x)\left(\text{arcsinh}(2\sin(x))(-3+2\cos(2x))+10\sqrt{3-2\cos(2x)}\sin(x)\right)}{8(4-5\sec^2(x))^{3/2}\sqrt{-(1+4\sin^2(x))^2}}$$

```
[In] Integrate[(4 - 5*Sec[x]^2)^(-3/2),x]
```

```
[Out] -1/8*((-3 + 2*Cos[2*x])^(3/2)*Sec[x]^3*(ArcSinh[2*Sin[x]]*(-3 + 2*Cos[2*x])
 + 10*Sqrt[3 - 2*Cos[2*x]]*Sin[x]))/((4 - 5*Sec[x]^2)^(3/2)*Sqrt[-(1 + 4*Si
n[x]^2)^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(32) = 64$.

Time = 1.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.10

method	result
default	$\frac{(\sec^3(x))(4(\cos^2(x))-5)\left(\arctan\left(\frac{2\sin(x)}{(\cos(x)+1)\sqrt{\frac{4(\cos^2(x))-5}{(\cos(x)+1)^2}}}\right)\sqrt{\frac{4(\cos^2(x))-5}{(\cos(x)+1)^2}}\cos(x)+\arctan\left(\frac{2\sin(x)}{(\cos(x)+1)\sqrt{\frac{4(\cos^2(x))-5}{(\cos(x)+1)^2}}}\right)\sqrt{\frac{4(\cos^2(}{(\cos(x)}}\right.}{8(4-5(\sec^2(x)))^{\frac{3}{2}}}$

```
[In] int(1/(4-5*sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sec(x)^3*(4*cos(x)^2-5)*(arctan(2*sin(x)/(cos(x)+1)/((4*cos(x)^2-5)/(co
s(x)+1)^2)^(1/2))*((4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2)*cos(x)+arctan(2*sin(x
)/(cos(x)+1)/((4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2))*((4*cos(x)^2-5)/(cos(x)+1
)^2)^(1/2)-10*sin(x))/(4-5*sec(x)^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(32) = 64$.

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.88

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx = -\frac{20\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)\sin(x) - (4\cos(x)^2-5)\arctan\left(\frac{4\left(8\cos(x)^3-9\cos(x)\right)\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\sin(x)+\cos(x)\sin(x)}{64\cos(x)^4-143\cos(x)^2+80}\right) + (4}{16\left(4\cos(x)^2-5\right)}$$

```
[In] integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/16*(20*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*cos(x)*sin(x) - (4*cos(x)^2 - 5)*
arctan((4*(8*cos(x)^3 - 9*cos(x))*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*sin(x) +
cos(x)*sin(x))/(64*cos(x)^4 - 143*cos(x)^2 + 80)) + (4*cos(x)^2 - 5)*arctan
(sin(x)/cos(x)))/(4*cos(x)^2 - 5)
```

Sympy [F]

$$\int \frac{1}{\left(4-5\sec^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(4-5\sec^2{(x)}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(4-5*sec(x)**2)**(3/2),x)
[Out] Integral((4 - 5*sec(x)**2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(4-5\sec^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(-5\,\sec\left(x\right)^2+4\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((-5*sec(x)^2 + 4)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(4-5\sec^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(-5\,\sec\left(x\right)^2+4\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="giac")
[Out] integrate((-5*sec(x)^2 + 4)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(4-5\sec^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(4-\frac{5}{\cos\left(x\right)^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(4 - 5/cos(x)^2)^(3/2),x)
[Out] int(1/(4 - 5/cos(x)^2)^(3/2), x)
```

3.436 $\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx = -\frac{1}{4}\operatorname{arctanh}\left(\frac{2\tan(x)}{\sqrt{1+5\tan^2(x)}}\right) - \frac{\cos(x)}{4\sqrt{1+5\tan^2(x)}} - \frac{5\cot(x)}{2\sqrt{1+5\tan^2(x)}} - \frac{1}{8}\cos(x)\sqrt{1+5\tan^2(x)} + \frac{9}{2}\cot(x)\sqrt{1+5\tan^2(x)}$$

```
[Out] -1/4*arctanh(2*tan(x)/(1+5*tan(x)^2)^(1/2))-1/4*cos(x)/(1+5*tan(x)^2)^(1/2)
-5/2*cot(x)/(1+5*tan(x)^2)^(1/2)-1/8*cos(x)*(1+5*tan(x)^2)^(1/2)+9/2*cot(x)
*(1+5*tan(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {4462, 12, 3751, 483, 597, 385, 212, 3745, 277, 197}

$$\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx = -\frac{1}{4}\operatorname{arctanh}\left(\frac{2\tan(x)}{\sqrt{5\tan^2(x)+1}}\right) - \frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}} + \frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}} + \frac{9}{2}\sqrt{5\tan^2(x)+1}\cot(x) - \frac{5\cot(x)}{2\sqrt{5\tan^2(x)+1}}$$

```
[In] Int[(-2*Cot[x]^2 + Sin[x])/(1 + 5*Tan[x]^2)^(3/2),x]
```

```
[Out] -1/4*ArcTanh[(2*Tan[x])/Sqrt[1 + 5*Tan[x]^2]] + Cos[x]/(4*Sqrt[-4 + 5*Sec[x
]^2]) - (5*Sec[x])/(8*Sqrt[-4 + 5*Sec[x]^2]) - (5*Cot[x])/(2*Sqrt[1 + 5*Tan
[x]^2]) + (9*Cot[x]*Sqrt[1 + 5*Tan[x]^2])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 4462

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int -\frac{2\cot^2(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx + \int \frac{\sin(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx \\
&= -\left(2\int \frac{\cot^2(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx\right) + \text{Subst}\left(\int \frac{1}{x^2\left(-4+5x^2\right)^{3/2}}\,dx, x, \sec(x)\right) \\
&= \frac{\cos(x)}{4\sqrt{-4+5\sec^2(x)}} - 2\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)\left(1+5x^2\right)^{3/2}}\,dx, x, \tan(x)\right) \\
&\quad + \frac{5}{2}\text{Subst}\left(\int \frac{1}{\left(-4+5x^2\right)^{3/2}}\,dx, x, \sec(x)\right) \\
&= \frac{\cos(x)}{4\sqrt{-4+5\sec^2(x)}} - \frac{5\sec(x)}{8\sqrt{-4+5\sec^2(x)}} - \frac{5\cot(x)}{2\sqrt{1+5\tan^2(x)}} \\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{-9-10x^2}{x^2\left(1+x^2\right)\sqrt{1+5x^2}}\,dx, x, \tan(x)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\cos(x)}{4\sqrt{-4+5\sec^2(x)}} - \frac{5\sec(x)}{8\sqrt{-4+5\sec^2(x)}} - \frac{5\cot(x)}{2\sqrt{1+5\tan^2(x)}} \\
&\quad + \frac{9}{2}\cot(x)\sqrt{1+5\tan^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{1+5x^2}}\,dx, x, \tan(x)\right) \\
&= \frac{\cos(x)}{4\sqrt{-4+5\sec^2(x)}} - \frac{5\sec(x)}{8\sqrt{-4+5\sec^2(x)}} - \frac{5\cot(x)}{2\sqrt{1+5\tan^2(x)}} \\
&\quad + \frac{9}{2}\cot(x)\sqrt{1+5\tan^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{\tan(x)}{\sqrt{1+5\tan^2(x)}}\right) \\
&= -\frac{1}{4}\text{arctanh}\left(\frac{2\tan(x)}{\sqrt{1+5\tan^2(x)}}\right) + \frac{\cos(x)}{4\sqrt{-4+5\sec^2(x)}} \\
&\quad - \frac{5\sec(x)}{8\sqrt{-4+5\sec^2(x)}} - \frac{5\cot(x)}{2\sqrt{1+5\tan^2(x)}} + \frac{9}{2}\cot(x)\sqrt{1+5\tan^2(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.39

$$
\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx =
$$

$$
-\frac{(-3+2\cos(2x))^{3/2}(-1+2\cot^2(x)\csc(x))\sin^2(x)\left(-2\text{arcsinh}(2\sin(x))(4+\csc^2(x))+(-2+164\csc(x\right.}{2\sqrt{-(3-2\cos(2x))^2}(5+\cot^2(x))(4+4\cos(2x)-3\sin(x)+\sin(3x))}
$$

```
[In] Integrate[(-2*Cot[x]^2 + Sin[x])/(1 + 5*Tan[x]^2)^(3/2),x]
```

```
[Out] -1/2*((-3 + 2*Cos[2*x])^(3/2)*(-1 + 2*Cot[x]^2*Csc[x])*Sin[x]^2*(-2*ArcSinh
[2*Sin[x]]*(4 + Csc[x]^2) + (-2 + 164*Csc[x] - 3*Csc[x]^2 + 16*Csc[x]^3)*Sq
rt[1 + 4*Sin[x]^2])*Tan[x])/(Sqrt[-(3 - 2*Cos[2*x])^2]*(5 + Cot[x]^2)*(4 +
4*Cos[2*x] - 3*Sin[x] + Sin[3*x])*Sqrt[1 + 5*Tan[x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(74) = 148.

Time = 4.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.61

method	result
default	$-\frac{(\sec^3(x))\csc(x)(4(\cos^2(x))-5)\left(-2\cos(x)\sin(x)\operatorname{arctanh}\left(\frac{2\sin(x)}{(\cos(x)+1)\sqrt{-\frac{4(\cos^2(x))-5}{(\cos(x)+1)^2}}}\right)\sqrt{-\frac{4(\cos^2(x))-5}{(\cos(x)+1)^2}}+2(\cos^2(x))\sin(x)-2\sin}{8(5(\sec^2(x))-4)^{\frac{3}{2}}}$
parts	$-\frac{8\cos(x)-30\sec(x)+25(\sec^3(x))}{8(5(\sec^2(x))-4)^{\frac{3}{2}}\left(-2+\sqrt{5}\right)^2\left(2+\sqrt{5}\right)^2}+\frac{(\sec^3(x))\csc(x)(4(\cos^2(x))-5)\left(\cos(x)\sin(x)\operatorname{arctanh}\left(\frac{2\sin(x)}{(\cos(x)+1)\sqrt{-\frac{4(\cos^2(x))-5}{(\cos(x)+1)^2}}}\right)\right.}{4}$

```
[In] int((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*sec(x)^3*csc(x)*(4*cos(x)^2-5)*(-2*cos(x)*sin(x)*arctanh(2*sin(x)/(cos
(x)+1)/(-(4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2))*(-(4*cos(x)^2-5)/(cos(x)+1)^2)
^(1/2)+2*cos(x)^2*sin(x)-2*sin(x)*arctanh(2*sin(x)/(cos(x)+1)/(-(4*cos(x)^2
-5)/(cos(x)+1)^2)^(1/2))*(-(4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2)-164*cos(x)^2-
5*sin(x)+180)/(5*sec(x)^2-4)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx=\frac{2\left(4\cos(x)^2-5\right)\log\left(\sqrt{-\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)-2\sin(x)\right)\sin(x)+\left(164\cos(x)^3\right.}{8\left(4\cos(x)^2-5\right)\sin}$$

```
[In] integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(4*cos(x)^2 - 5)*log(sqrt(-(4*cos(x)^2 - 5)/cos(x)^2)*cos(x) - 2*sin
(x))*sin(x) + (164*cos(x)^3 - (2*cos(x)^3 - 5*cos(x))*sin(x) - 180*cos(x))*
sqrt(-(4*cos(x)^2 - 5)/cos(x)^2))/((4*cos(x)^2 - 5)*sin(x))
```

Sympy [F]

$$\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx=-\int\left(-\frac{\sin(x)}{5\sqrt{5\tan^2(x)+1}\tan^2(x)+\sqrt{5\tan^2(x)+1}}\right)dx-\int\frac{2\cot^2(x)}{5\sqrt{5\tan^2(x)+1}\tan^2(x)+\sqrt{5\tan^2(x)+1}}\,dx$$

```
[In] integrate((-2*cot(x)**2+sin(x))/(1+5*tan(x)**2)**(3/2),x)
```

```
[Out] -Integral(-sin(x)/(5*sqrt(5*tan(x)**2 + 1)*tan(x)**2 + sqrt(5*tan(x)**2 + 1
)), x) - Integral(2*cot(x)**2/(5*sqrt(5*tan(x)**2 + 1)*tan(x)**2 + sqrt(5*t
an(x)**2 + 1)), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{-2\cot^2(x) + \sin(x)}{(1 + 5\tan^2(x))^{3/2}} \, dx = \text{Timed out}$$

```
[In] integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{-2\cot^2(x) + \sin(x)}{(1 + 5\tan^2(x))^{3/2}} \, dx = \int -\frac{2\cot(x)^2 - \sin(x)}{(5\tan(x)^2 + 1)^{\frac{3}{2}}} \, dx$$

```
[In] integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="giac")
[Out] integrate(-(2*cot(x)^2 - sin(x))/(5*tan(x)^2 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-2\cot^2(x) + \sin(x)}{(1 + 5\tan^2(x))^{3/2}} \, dx = \int \frac{\sin(x) - 2\cot(x)^2}{(5\tan(x)^2 + 1)^{3/2}} \, dx$$

```
[In] int((sin(x) - 2*cot(x)^2)/(5*tan(x)^2 + 1)^(3/2),x)
[Out] int((sin(x) - 2*cot(x)^2)/(5*tan(x)^2 + 1)^(3/2), x)
```

3.437 $\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = -\frac{2}{3}\sqrt{4-\cot^2(x)}\tan(x)-\frac{1}{3}\sqrt{4-\cot^2(x)}\tan^3(x)$$

```
[Out] -2/3*(4-cot(x)^2)^(1/2)*tan(x)-1/3*(4-cot(x)^2)^(1/2)*tan(x)^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {12, 445, 464, 197}

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = -\frac{1}{3}\tan^3(x)\sqrt{4-\cot^2(x)}-\frac{2}{3}\tan(x)\sqrt{4-\cot^2(x)}$$

```
[In] Int[((-3 + Cos[2*x])*Sec[x]^4)/Sqrt[4 - Cot[x]^2],x]
[Out] (-2*Sqrt[4 - Cot[x]^2]*Tan[x])/3 - (Sqrt[4 - Cot[x]^2]*Tan[x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{2(-1-2x^2)}{\sqrt{4-\frac{1}{x^2}}}\,dx, x, \tan(x)\right) \\
&= 2\text{Subst}\left(\int \frac{-1-2x^2}{\sqrt{4-\frac{1}{x^2}}}\,dx, x, \tan(x)\right) \\
&= 2\text{Subst}\left(\int \frac{\left(-2-\frac{1}{x^2}\right)x^2}{\sqrt{4-\frac{1}{x^2}}}\,dx, x, \tan(x)\right) \\
&= -\frac{1}{3}\sqrt{4-\cot^2(x)}\tan^3(x) - \frac{8}{3}\text{Subst}\left(\int \frac{1}{\sqrt{4-\frac{1}{x^2}}}\,dx, x, \tan(x)\right) \\
&= -\frac{2}{3}\sqrt{4-\cot^2(x)}\tan(x) - \frac{1}{3}\sqrt{4-\cot^2(x)}\tan^3(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = \frac{(3+\cos(2x))(-3+5\cos(2x))\csc(x)\sec^3(x)}{12\sqrt{4-\cot^2(x)}}$$

```
[In] Integrate[((-3 + Cos[2*x])*Sec[x]^4)/Sqrt[4 - Cot[x]^2],x]
[Out] ((3 + Cos[2*x])*(-3 + 5*Cos[2*x])*Csc[x]*Sec[x]^3)/(12*Sqrt[4 - Cot[x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(31) = 62.

Time = 2.73 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.72

method	result	size
default	$\frac{(\sec^3(x))\csc(x)(25(\cos^4(x))-10(\cos^2(x))-8)}{6\sqrt{-5(\cot^2(x))+4(\csc^2(x))}}-\frac{5\cot(x)-4\sec(x)\csc(x)}{2\sqrt{-5(\cot^2(x))+4(\csc^2(x))}}$	67

```
[In] int((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*sec(x)^3*csc(x)*(25*cos(x)^4-10*cos(x)^2-8)/(-5*cot(x)^2+4*csc(x)^2)^(1
/2)-1/2/(-5*cot(x)^2+4*csc(x)^2)^(1/2)*(5*cot(x)-4*sec(x)*csc(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = -\frac{(\cos(x)^2+1)\sqrt{\frac{5\cos(x)^2-4}{\cos(x)^2-1}}\sin(x)}{3\cos(x)^3}$$

```
[In] integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/3*(cos(x)^2 + 1)*sqrt((5*cos(x)^2 - 4)/(cos(x)^2 - 1))*sin(x)/cos(x)^3
```

Sympy [F]

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = \int \frac{\cos(2x)-3}{\sqrt{-(\cot(x)-2)(\cot(x)+2)}\cos^4(x)}\,dx$$

```
[In] integrate((-3+cos(2*x))/cos(x)**4/(4-cot(x)**2)**(1/2),x)
[Out] Integral((cos(2*x) - 3)/(sqrt(-(cot(x) - 2)*(cot(x) + 2))*cos(x)**4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(31) = 62.

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = -\frac{1}{48}\left(-\frac{1}{\tan(x)^2}+4\right)^{\frac{3}{2}}\tan(x)^3 + \frac{3}{16}\sqrt{-\frac{1}{\tan(x)^2}+4}\tan(x) - \frac{8\tan(x)^4+26\tan(x)^2-7}{8\sqrt{2\tan(x)+1}\sqrt{2\tan(x)-1}}$$

```
[In] integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/48*(-1/tan(x)^2 + 4)^(3/2)*tan(x)^3 + 3/16*sqrt(-1/tan(x)^2 + 4)*tan(x)
- 1/8*(8*tan(x)^4 + 26*tan(x)^2 - 7)/(sqrt(2*tan(x) + 1)*sqrt(2*tan(x) - 1)
)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.46

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = \frac{\frac{125\sqrt{5}\left(\frac{21\left(\sqrt{5}\sqrt{-5\cos(x)^2+4}-2\sqrt{5}\right)^2}{\cos(x)^2}+125\right)\cos(x)^3}{\left(\sqrt{5}\sqrt{-5\cos(x)^2+4}-2\sqrt{5}\right)^3} - \frac{\sqrt{5}\left(\sqrt{5}\sqrt{-5\cos(x)^2+4}-2\sqrt{5}\right)^3}{\cos(x)^3} - \frac{105\sqrt{5}\left(\sqrt{5}\sqrt{-5\cos(x)^2+4}-2\sqrt{5}\right)}{\cos(x)}}{2400\,\mathrm{sgn}(\sin(x))} + \frac{2}{3}i\,\mathrm{sgn}(\sin(x))$$

```
[In] integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/2400*(125*sqrt(5)*(21*(sqrt(5)*sqrt(-5*cos(x)^2 + 4) - 2*sqrt(5))^2/cos(x
)^2 + 125)*cos(x)^3/(sqrt(5)*sqrt(-5*cos(x)^2 + 4) - 2*sqrt(5))^3 - sqrt(5)
*(sqrt(5)*sqrt(-5*cos(x)^2 + 4) - 2*sqrt(5))^3/cos(x)^3 - 105*sqrt(5)*(sqrt
(5)*sqrt(-5*cos(x)^2 + 4) - 2*sqrt(5))/cos(x))/sgn(sin(x)) + 2/3*I*sgn(sin(
x))
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.51

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = -\frac{\tan(x)\,\left(\tan(x)^2+2\right)\,\sqrt{4-\frac{1}{\tan(x)^2}}}{3}$$

```
[In] int((cos(2*x) - 3)/(cos(x)^4*(4 - cot(x)^2)^(1/2)),x)
[Out] -(tan(x)*(tan(x)^2 + 2)*(4 - 1/tan(x)^2)^(1/2))/3
```

3.438 $\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))(5-4\sec^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 73

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))(5-4\sec^2(x))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}} - \frac{2}{15\sqrt{5-4\sec^2(x)}}$$

```
[Out] -1/18*arctanh(1/3*(5-4*sec(x)^2)^(1/2)*3^(1/2))*3^(1/2)-1/25*arctanh(1/5*(5
-4*sec(x)^2)^(1/2)*5^(1/2))*5^(1/2)-2/15/(5-4*sec(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {4458, 6857, 267, 272, 53, 65, 212, 528, 457, 87, 162, 213}

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))(5-4\sec^2(x))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}} - \frac{2}{15\sqrt{5-4\sec^2(x)}}$$

```
[In] Int[((3 + Sin[x]^2)*Tan[x]^3)/((-2 + Cos[x]^2)*(5 - 4*Sec[x]^2)^(3/2)),x]
```

```
[Out] -1/6*ArcTanh[Sqrt[5 - 4*Sec[x]^2]/Sqrt[3]]/Sqrt[3] - ArcTanh[Sqrt[5 - 4*Sec
[x]^2]/Sqrt[5]]/(5*Sqrt[5]) - 2/(15*Sqrt[5 - 4*Sec[x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 4458

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Dist[-(b*c*d^(n - 1))^(-1), Subst[Int[SubstF
or[(1 - d^2*x^2)^((n - 1)/2)/x^n, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(
a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b,
c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(1-x^2)\,(4-x^2)}{\left(5-\frac{4}{x^2}\right)^{3/2} x^3\,(-2+x^2)}\,dx, x, \cos(x)\right)\\
&= -\text{Subst}\left(\int \left(-\frac{2}{\left(5-\frac{4}{x^2}\right)^{3/2} x^3}+\frac{3}{2\left(5-\frac{4}{x^2}\right)^{3/2} x}-\frac{x}{2\left(5-\frac{4}{x^2}\right)^{3/2}(-2+x^2)}\right)\,dx, x, \cos(x)\right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\left(5-\frac{4}{x^2}\right)^{3/2}(-2+x^2)}\,dx, x, \cos(x)\right) \\
&\quad - \frac{3}{2}\text{Subst}\left(\int \frac{1}{\left(5-\frac{4}{x^2}\right)^{3/2}x}\,dx, x, \cos(x)\right) \\
&\quad + 2\text{Subst}\left(\int \frac{1}{\left(5-\frac{4}{x^2}\right)^{3/2}x^3}\,dx, x, \cos(x)\right) \\
&= -\frac{1}{2\sqrt{5-4\sec^2(x)}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(5-\frac{4}{x^2}\right)^{3/2}\left(1-\frac{2}{x^2}\right)x}\,dx, x, \cos(x)\right) \\
&\quad + \frac{3}{4}\text{Subst}\left(\int \frac{1}{(5-4x)^{3/2}x}\,dx, x, \sec^2(x)\right) \\
&= -\frac{1}{5\sqrt{5-4\sec^2(x)}} + \frac{3}{20}\text{Subst}\left(\int \frac{1}{\sqrt{5-4x}x}\,dx, x, \sec^2(x)\right) \\
&\quad - \frac{1}{4}\text{Subst}\left(\int \frac{1}{(5-4x)^{3/2}(1-2x)x}\,dx, x, \sec^2(x)\right) \\
&= -\frac{2}{15\sqrt{5-4\sec^2(x)}} + \frac{1}{120}\text{Subst}\left(\int \frac{-6-8x}{\sqrt{5-4x}(1-2x)x}\,dx, x, \sec^2(x)\right) \\
&\quad - \frac{3}{40}\text{Subst}\left(\int \frac{1}{\frac{5}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{5-4\sec^2(x)}\right) \\
&= -\frac{3\text{arctanh}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{10\sqrt{5}} - \frac{2}{15\sqrt{5-4\sec^2(x)}} \\
&\quad - \frac{1}{20}\text{Subst}\left(\int \frac{1}{\sqrt{5-4x}x}\,dx, x, \sec^2(x)\right) \\
&\quad - \frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{5-4x}(1-2x)}\,dx, x, \sec^2(x)\right) \\
&= -\frac{3\text{arctanh}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{10\sqrt{5}} - \frac{2}{15\sqrt{5-4\sec^2(x)}} \\
&\quad + \frac{1}{40}\text{Subst}\left(\int \frac{1}{\frac{5}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{5-4\sec^2(x)}\right) \\
&\quad + \frac{1}{12}\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{x^2}{2}}\,dx, x, \sqrt{5-4\sec^2(x)}\right) \\
&= -\frac{\text{arctanh}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{\text{arctanh}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}} - \frac{2}{15\sqrt{5-4\sec^2(x)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 5.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\left(5-4\sec^2(x)\right)^{3/2}}\,dx = -\frac{\sqrt{-\cos^2(x)}\left(60\sqrt{-\cos^2(x)}+9\text{arcsinh}\left(\frac{1}{2}\sqrt{5}\sqrt{-\cos^2(x)}\right)\sqrt{30-50\cos(2x)}+25\text{arctanh}\left(\frac{\sqrt{3}\sqrt{-\cos^2(x)}}{\sqrt{-1+5\sin^2(x)}}\right)\right.}{450\left(-1+5\sin^2(x)\right)}$$

```
[In] Integrate[((3 + Sin[x]^2)*Tan[x]^3)/((-2 + Cos[x]^2)*(5 - 4*Sec[x]^2)^(3/2)
),x]
```

```
[Out] -1/450*(Sqrt[-Cos[x]^2]*(60*Sqrt[-Cos[x]^2] + 9*ArcSinh[(Sqrt[5]*Sqrt[-Cos[
x]^2])/2]*Sqrt[30 - 50*Cos[2*x]] + 25*ArcTanh[(Sqrt[3]*Sqrt[-Cos[x]^2])/Sqr
t[-1 + 5*Sin[x]^2]]*Sqrt[-3 + 15*Sin[x]^2])*Sqrt[Sec[x]^2 - 5*Tan[x]^2])/(-
1 + 5*Sin[x]^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1497 vs. 2(55) = 110.

Time = 2.00 (sec) , antiderivative size = 1498, normalized size of antiderivative = 20.52

method	result	size
default	Expression too large to display	1498

```
[In] int((3+sin(x)^2)*tan(x)^3/(cos(x)^2-2)/(5-4*sec(x)^2)^(3/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] 3/5*sec(x)^3*(5*cos(x)^2-4)*(50*3^(1/2)*2^(1/2)*((5*cos(x)^2-4)/(cos(x)+1)^
2)^(1/2)*arctanh((5*cos(x)*2^(1/2)+4*2^(1/2)+10*cos(x)+4)/(cos(x)+1)/((5*co
s(x)^2-4)/(cos(x)+1)^2)^(1/2)/(2*3^(1/2)+6^(1/2)))*cos(x)+50*3^(1/2)*2^(1/2
)*((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)*arctanh((5*cos(x)*2^(1/2)+4*2^(1/2)-1
0*cos(x)-4)/(cos(x)+1)/((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)/(2*3^(1/2)-6^(1/
2)))*cos(x)-25*6^(1/2)*2^(1/2)*((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)*arctanh(
(5*cos(x)*2^(1/2)+4*2^(1/2)+10*cos(x)+4)/(cos(x)+1)/((5*cos(x)^2-4)/(cos(x)
+1)^2)^(1/2)/(2*3^(1/2)+6^(1/2)))*cos(x)+25*6^(1/2)*2^(1/2)*((5*cos(x)^2-4)
/(cos(x)+1)^2)^(1/2)*arctanh((5*cos(x)*2^(1/2)+4*2^(1/2)-10*cos(x)-4)/(cos(
x)+1)/((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)/(2*3^(1/2)-6^(1/2)))*cos(x)+100*3
^(1/2)*((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)*arctanh((5*cos(x)*2^(1/2)+4*2^(1
/2)+10*cos(x)+4)/(cos(x)+1)/((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)/(2*3^(1/2)+
6^(1/2)))*cos(x)-100*3^(1/2)*((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)*arctanh((5
*cos(x)*2^(1/2)+4*2^(1/2)-10*cos(x)-4)/(cos(x)+1)/((5*cos(x)^2-4)/(cos(x)+1
)^2)^(1/2)/(2*3^(1/2)-6^(1/2)))*cos(x)-50*6^(1/2)*((5*cos(x)^2-4)/(cos(x)+1
)^2)^(1/2)*arctanh((5*cos(x)*2^(1/2)+4*2^(1/2)+10*cos(x)+4)/(cos(x)+1)/((5*
```

```
cos(x)^2-4)/(cos(x)+1)^2)^(1/2)/(2*3^(1/2)+6^(1/2)))*cos(x)-50*6^(1/2)*((5*
cos(x)^2-4)/(cos(x)+1)^2)^(1/2)*arctanh((5*cos(x)*2^(1/2)+4*2^(1/2)-10*cos(
x)-4)/(cos(x)+1)/((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)/(2*3^(1/2)-6^(1/2)))*c
os(x)+72*5^(1/2)*((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)*arctanh(cos(x)/(cos(x)
+1)/((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)*5^(1/2))*cos(x)+50*arctanh((5*cos(x
)*2^(1/2)+4*2^(1/2)+10*cos(x)+4)/(cos(x)+1)/((5*cos(x)^2-4)/(cos(x)+1)^2)^(
1/2)/(2*3^(1/2)+6^(1/2)))*((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)*2^(1/2)*3^(1/
2)+50*arctanh((5*cos(x)*2^(1/2)+4*2^(1/2)-10*cos(x)-4)/(cos(x)+1)/((5*cos(x
)^2-4)/(cos(x)+1)^2)^(1/2)/(2*3^(1/2)-6^(1/2)))*((5*cos(x)^2-4)/(cos(x)+1)^
2)^(1/2)*2^(1/2)*3^(1/2)-25*arctanh((5*cos(x)*2^(1/2)+4*2^(1/2)+10*cos(x)+4
)/(cos(x)+1)/((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)/(2*3^(1/2)+6^(1/2)))*((5*c
os(x)^2-4)/(cos(x)+1)^2)^(1/2)*2^(1/2)*6^(1/2)+25*arctanh((5*cos(x)*2^(1/2)
+4*2^(1/2)-10*cos(x)-4)/(cos(x)+1)/((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)/(2*3
^(1/2)-6^(1/2)))*((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)*2^(1/2)*6^(1/2)+100*ar
ctanh((5*cos(x)*2^(1/2)+4*2^(1/2)+10*cos(x)+4)/(cos(x)+1)/((5*cos(x)^2-4)/(
cos(x)+1)^2)^(1/2)/(2*3^(1/2)+6^(1/2)))*((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)
*3^(1/2)-100*arctanh((5*cos(x)*2^(1/2)+4*2^(1/2)-10*cos(x)-4)/(cos(x)+1)/((
5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)/(2*3^(1/2)-6^(1/2)))*((5*cos(x)^2-4)/(cos
(x)+1)^2)^(1/2)*3^(1/2)-50*arctanh((5*cos(x)*2^(1/2)+4*2^(1/2)+10*cos(x)+4)
/(cos(x)+1)/((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)/(2*3^(1/2)+6^(1/2)))*((5*co
s(x)^2-4)/(cos(x)+1)^2)^(1/2)*6^(1/2)-50*arctanh((5*cos(x)*2^(1/2)+4*2^(1/2
)-10*cos(x)-4)/(cos(x)+1)/((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)/(2*3^(1/2)-6^
(1/2)))*((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)*6^(1/2)+72*arctanh(cos(x)/(cos(
x)+1)/((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)*5^(1/2))*((5*cos(x)^2-4)/(cos(x)+
1)^2)^(1/2)*5^(1/2)+240*cos(x))/(5-4*sec(x)^2)^(3/2)/(-5+2*5^(1/2))/(5+2*5^
(1/2))/(6+2*5^(1/2)+2^(1/2))/(6-2*5^(1/2)+2^(1/2))/(2*3^(1/2)+6^(1/2))/(-6-
2*5^(1/2)+2^(1/2))/(-6+2*5^(1/2)+2^(1/2))/(2*3^(1/2)-6^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(55) = 110$.

Time = 0.35 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.52

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\left(5-4\sec^2(x)\right)^{3/2}}\,dx =$$

$$-\frac{480\sqrt{\frac{5\cos(x)^2-4}{\cos(x)^2}}\cos(x)^2-18\left(5\sqrt{5}\cos(x)^2-4\sqrt{5}\right)\log\left(625\cos(x)^8-1000\cos(x)^6+500\cos(x)^4-80\right.}{}$$

```
[In] integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/3600*(480*sqrt((5*cos(x)^2 - 4)/cos(x)^2)*cos(x)^2 - 18*(5*sqrt(5)*cos(x
)^2 - 4*sqrt(5))*log(625*cos(x)^8 - 1000*cos(x)^6 + 500*cos(x)^4 - 80*cos(x
```

```
)^2 - (125*sqrt(5)*cos(x)^8 - 150*sqrt(5)*cos(x)^6 + 50*sqrt(5)*cos(x)^4 -
4*sqrt(5)*cos(x)^2)*sqrt((5*cos(x)^2 - 4)/cos(x)^2) + 2) - 25*(5*sqrt(3)*co
s(x)^2 - 4*sqrt(3))*log((1921*cos(x)^8 - 3464*cos(x)^6 + 2040*cos(x)^4 - 41
6*cos(x)^2 - 8*(62*sqrt(3)*cos(x)^8 - 87*sqrt(3)*cos(x)^6 + 36*sqrt(3)*cos(
x)^4 - 4*sqrt(3)*cos(x)^2)*sqrt((5*cos(x)^2 - 4)/cos(x)^2) + 16)/(cos(x)^8
- 8*cos(x)^6 + 24*cos(x)^4 - 32*cos(x)^2 + 16)))/(5*cos(x)^2 - 4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\left(5-4\sec^2(x)\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((3+sin(x)**2)*tan(x)**3/(-2+cos(x)**2)/(5-4*sec(x)**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\left(5-4\sec^2(x)\right)^{3/2}}\,dx = \int \frac{\left(\sin\left(x\right)^2+3\right)\tan\left(x\right)^3}{\left(\cos\left(x\right)^2-2\right)\left(-4\sec\left(x\right)^2+5\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algor
ithm="maxima")
[Out] integrate((sin(x)^2 + 3)*tan(x)^3/((cos(x)^2 - 2)*(-4*sec(x)^2 + 5)^(3/2)),
x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(55) = 110.

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.66

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\left(5-4\sec^2(x)\right)^{3/2}}\,dx = -\frac{5\sqrt{15}\sqrt{5}\log\left(-\frac{2\left(\left(\sqrt{5}\cos(x)-\sqrt{5\cos(x)^2-4}\right)^2-4\sqrt{15}-16\right)}{\left|2\left(\sqrt{5}\cos(x)-\sqrt{5\cos(x)^2-4}\right)^2+8\sqrt{15}-32\right|}\right)-18\sqrt{5}\log\left(\left(\sqrt{5}\cos\left(x\right)-\sqrt{5\cos\left(x\right)^2-4}\right)^2\right)}{900\,\mathrm{sgn}\left(\cos\left(x\right)\right)}$$

```
[In] integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algor
ithm="giac")
```

```
[Out] -1/900*(5*sqrt(15)*sqrt(5)*log(-2*((sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))^
2 - 4*sqrt(15) - 16)/abs(2*(sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))^2 + 8*sq
rt(15) - 32)) - 18*sqrt(5)*log((sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))^2) +
 120*cos(x)/sqrt(5*cos(x)^2 - 4))/sgn(cos(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\left(5-4\sec^2(x)\right)^{3/2}}\,dx=\int \frac{\tan(x)^3\left(\sin(x)^2+3\right)}{\left(\cos(x)^2-2\right)\left(5-\frac{4}{\cos(x)^2}\right)^{3/2}}\,dx$$

```
[In] int((tan(x)^3*(sin(x)^2 + 3))/((cos(x)^2 - 2)*(5 - 4/cos(x)^2)^(3/2)),x)
[Out] int((tan(x)^3*(sin(x)^2 + 3))/((cos(x)^2 - 2)*(5 - 4/cos(x)^2)^(3/2)), x)
```

3.439 $\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 57

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = -\frac{3}{4}\log(\tan(x))+\frac{3}{8}\log\left(4+9\tan^2(x)\right)-\frac{\cot(x)}{4\sqrt{4+9\tan^2(x)}}-\frac{7\tan(x)}{8\sqrt{4+9\tan^2(x)}}$$

```
[Out] -3/4*ln(tan(x))+3/8*ln(4+9*tan(x)^2)-1/4*cot(x)/(4+9*tan(x)^2)^(1/2)-7/8*ta
n(x)/(4+9*tan(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {6874, 197, 277, 272, 36, 29, 31}

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = -\frac{7\tan(x)}{8\sqrt{9\tan^2(x)+4}}+\frac{3}{8}\log\left(9\tan^2(x)+4\right)-\frac{3}{4}\log(\tan(x))-\frac{\cot(x)}{4\sqrt{9\tan^2(x)+4}}$$

```
[In] Int[(Csc[x]^2*(Sec[x]^2 - 3*Tan[x]*Sqrt[4*Sec[x]^2 + 5*Tan[x]^2]))/(4*Sec[x
]^2 + 5*Tan[x]^2)^(3/2),x]
```

```
[Out] (-3*Log[Tan[x]])/4 + (3*Log[4 + 9*Tan[x]^2])/8 - Cot[x]/(4*Sqrt[4 + 9*Tan[x
]^2]) - (7*Tan[x])/(8*Sqrt[4 + 9*Tan[x]^2])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1+x^2-3x\sqrt{4+9x^2}}{x^2\,(4+9x^2)^{3/2}}\,dx, x, \tan(x)\right)$$

$$= \text{Subst}\left(\int \left(\frac{1}{(4+9x^2)^{3/2}}+\frac{1}{x^2\,(4+9x^2)^{3/2}}-\frac{3}{x\,(4+9x^2)}\right)\,dx, x, \tan(x)\right)$$

$$= -\left(3\text{Subst}\left(\int \frac{1}{x(4+9x^2)}\,dx, x, \tan(x)\right)\right) + \text{Subst}\left(\int \frac{1}{(4+9x^2)^{3/2}}\,dx, x, \tan(x)\right) + \text{Subst}\left(\int \frac{1}{x^2(4+9x^2)^{3/2}}\,dx, x, \tan(x)\right)$$

$$= -\frac{\cot(x)}{4\sqrt{4+9\tan^2(x)}} + \frac{\tan(x)}{4\sqrt{4+9\tan^2(x)}} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{x(4+9x)}\,dx, x, \tan^2(x)\right) - \frac{9}{2}\text{Subst}\left(\int \frac{1}{(4+9x^2)^{3/2}}\,dx, x, \tan(x)\right)$$

$$= -\frac{\cot(x)}{4\sqrt{4+9\tan^2(x)}} - \frac{7\tan(x)}{8\sqrt{4+9\tan^2(x)}} - \frac{3}{8}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tan^2(x)\right) + \frac{27}{8}\text{Subst}\left(\int \frac{1}{4+9x}\,dx, x, \tan^2(x)\right)$$

$$= -\frac{3}{4}\log(\tan(x)) + \frac{3}{8}\log\left(4+9\tan^2(x)\right) - \frac{\cot(x)}{4\sqrt{4+9\tan^2(x)}} - \frac{7\tan(x)}{8\sqrt{4+9\tan^2(x)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 116 vs. 2(57) = 114.

Time = 5.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.04

$$\int \frac{\csc^2(x)\left(\sec^2(x) - 3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = \frac{5\cot(x) + 6\sqrt{\frac{13-5\cos(2x)}{1+\cos(2x)}}\log\left(1+7\tan^2\left(\frac{x}{2}\right)+\right.}{}$$

```
[In] Integrate[(Csc[x]^2*(Sec[x]^2 - 3*Tan[x]*Sqrt[4*Sec[x]^2 + 5*Tan[x]^2]))/(4
*Sec[x]^2 + 5*Tan[x]^2)^(3/2),x]
```

```
[Out] (5*Cot[x] + 6*Sqrt[(13 - 5*Cos[2*x])/(1 + Cos[2*x])]*Log[1 + 7*Tan[x/2]^2 +
 Tan[x/2]^4] - 9*Csc[x]*Sec[x] - 5*Tan[x] - 6*Sqrt[2]*Log[Tan[x/2]]*Sqrt[-5
 + 13*Sec[x]^2 + 5*Tan[x]^2])/(16*Sqrt[(13 - 5*Cos[2*x])/(1 + Cos[2*x])])
```

Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

method	result
parts	$-\frac{(\sec^3(x))\csc(x)(25(\cos^4(x))-80(\cos^2(x))+63)\sqrt{4}}{16(-5+9(\sec^2(x)))^{\frac{3}{2}}}-\frac{3\ln(\cos(x)+1)}{8}+\frac{3\ln(5(\cos^2(x))-9)}{8}-\frac{3\ln(-1+\cos(x))}{8}$
default	$-\frac{6(-5+9(\sec^2(x)))^{\frac{3}{2}}\ln(\csc(x)-\cot(x))-3(-5+9(\sec^2(x)))^{\frac{3}{2}}\ln\left(-\frac{5(\cos^2(x))-9}{(\cos(x)+1)^2}\right)+25\cot(x)-80\sec(x)\csc(x)+63(\sec^3(x))\csc(x)}{8(-5+9(\sec^2(x)))^{\frac{3}{2}}}$

[In] int((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec(x)^2+5*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/16*sec(x)^3*csc(x)*(25*cos(x)^4-80*cos(x)^2+63)/(-5+9*sec(x)^2)^(3/2)*4^(1/2)-3/8*ln(cos(x)+1)+3/8*ln(5*cos(x)^2-9)-3/8*ln(-1+cos(x))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.47

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = \frac{3\left(5\cos(x)^2-9\right)\log\left(-\frac{5}{4}\cos(x)^2+\frac{9}{4}\right)\sin(x)-}{}$$

[In] integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec(x)^2+5*tan(x)^2)^(3/2),x, algorithm="fricas")

[Out] 1/8*(3*(5*cos(x)^2 - 9)*log(-5/4*cos(x)^2 + 9/4)*sin(x) - 6*(5*cos(x)^2 - 9)*log(1/2*sin(x))*sin(x) - (5*cos(x)^3 - 7*cos(x))*sqrt(-(5*cos(x)^2 - 9)/cos(x)^2))/((5*cos(x)^2 - 9)*sin(x))

Sympy [F]

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = \int \frac{-3\sqrt{5\tan^2(x)+4\sec^2(x)}\tan(x)+\sec^2(x)}{\left(5\tan^2(x)+4\sec^2(x)\right)^{\frac{3}{2}}\sin^2(x)}$$

[In] integrate((sec(x)**2-3*(4*sec(x)**2+5*tan(x)**2)**(1/2)*tan(x))/sin(x)**2/(4*sec(x)**2+5*tan(x)**2)**(3/2),x)

[Out] Integral((-3*sqrt(5*tan(x)**2 + 4*sec(x)**2)*tan(x) + sec(x)**2)/((5*tan(x)**2 + 4*sec(x)**2)**(3/2)*sin(x)**2), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = -\frac{7\tan(x)}{8\sqrt{9\tan(x)^2+4}} - \frac{1}{4\sqrt{9\tan(x)^2+4}\tan(x)} + \frac{3}{8}\log\left(9\tan(x)^2+4\right) - \frac{3}{4}\log\left(\tan(x)\right)$$

```
[In] integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec
(x)^2+5*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -7/8*tan(x)/sqrt(9*tan(x)^2 + 4) - 1/4/(sqrt(9*tan(x)^2 + 4)*tan(x)) + 3/8*
log(9*tan(x)^2 + 4) - 3/4*log(tan(x))
```

Giac [F]

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = \int \frac{\sec(x)^2-3\sqrt{4\sec(x)^2+5\tan(x)^2}\tan(x}{\left(4\sec(x)^2+5\tan(x)^2\right)^{\frac{3}{2}}\sin(x)^2}$$

```
[In] integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec
(x)^2+5*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sec(x)^2 - 3*sqrt(4*sec(x)^2 + 5*tan(x)^2)*tan(x))/((4*sec(x)^2
+ 5*tan(x)^2)^(3/2)*sin(x)^2), x)
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.98

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = \frac{3\ln\left((\cos(2x)+\sin(2x)\ 1\mathrm{i})\ (5\cos(2x)-13\right.}{8} - \frac{3\ln\left(\cos(2x)\ 852930\mathrm{i}-852930\sin(2x)-852930\mathrm{i}\right)}{4} - \frac{\frac{18\sin(2x)\sqrt{13-5\cos(2x)}}{\sqrt{\cos(2x)+1}}-\frac{5\sin(4x)\sqrt{13-5\cos(2x)}}{\sqrt{\cos(2x)+1}}}{80\cos(2x)^2-288\cos(2x)+208}$$

```
[In] int((1/cos(x)^2 - 3*tan(x)*(4/cos(x)^2 + 5*tan(x)^2)^(1/2))/(sin(x)^2*(4/co
s(x)^2 + 5*tan(x)^2)^(3/2)),x)
```

```
[Out] (3*log((cos(2*x) + sin(2*x)*1i)*(5*cos(2*x) - 13)))/8 - (3*log(cos(2*x)*852
930i - 852930*sin(2*x) - 852930i))/4 - ((18*sin(2*x)*(13 - 5*cos(2*x))^(1/2
))/(cos(2*x) + 1)^(1/2) - (5*sin(4*x)*(13 - 5*cos(2*x))^(1/2))/(cos(2*x) +
1)^(1/2))/(80*cos(2*x)^2 - 288*cos(2*x) + 208)
```

3.440 $\int \tan(x) \left(1+5\tan^2(x)\right)^{5/2} dx$

Optimal result

Integrand size = 15, antiderivative size = 66

$$\int \tan(x) \left(1+5\tan^2(x)\right)^{5/2} dx = -32\arctan\left(\frac{1}{2}\sqrt{1+5\tan^2(x)}\right) + 16\sqrt{1+5\tan^2(x)} - \frac{4}{3}\left(1+5\tan^2(x)\right)^{3/2} + \frac{1}{5}\left(1+5\tan^2(x)\right)^{5/2}$$

```
[Out] -32*arctan(1/2*(1+5*tan(x)^2)^(1/2))+16*(1+5*tan(x)^2)^(1/2)-4/3*(1+5*tan(x
)^2)^(3/2)+1/5*(1+5*tan(x)^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3751, 455, 52, 65, 209}

$$\int \tan(x) \left(1+5\tan^2(x)\right)^{5/2} dx = -32\arctan\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right) + \frac{1}{5}\left(5\tan^2(x)+1\right)^{5/2} - \frac{4}{3}\left(5\tan^2(x)+1\right)^{3/2} + 16\sqrt{5\tan^2(x)+1}$$

```
[In] Int[Tan[x]*(1 + 5*Tan[x]^2)^(5/2),x]
```

```
[Out] -32*ArcTan[Sqrt[1 + 5*Tan[x]^2]/2] + 16*Sqrt[1 + 5*Tan[x]^2] - (4*(1 + 5*Ta
n[x]^2)^(3/2))/3 + (1 + 5*Tan[x]^2)^(5/2)/5
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x(1+5x^2)^{5/2}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1+5x)^{5/2}}{1+x}\,dx, x, \tan^2(x)\right)\\
&= \frac{1}{5}\left(1+5\tan^2(x)\right)^{5/2} - 2\text{Subst}\left(\int \frac{(1+5x)^{3/2}}{1+x}\,dx, x, \tan^2(x)\right)\\
&= -\frac{4}{3}\left(1+5\tan^2(x)\right)^{3/2} + \frac{1}{5}\left(1+5\tan^2(x)\right)^{5/2} + 8\text{Subst}\left(\int \frac{\sqrt{1+5x}}{1+x}\,dx, x, \tan^2(x)\right)
\end{aligned}$$

$$= 16\sqrt{1+5\tan^2(x)} - \frac{4}{3}\big(1+5\tan^2(x)\big)^{3/2} + \frac{1}{5}\big(1+5\tan^2(x)\big)^{5/2} - 32\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{1+5x}}\,dx, x, \tan^2(x)\right)$$

$$= 16\sqrt{1+5\tan^2(x)} - \frac{4}{3}\big(1+5\tan^2(x)\big)^{3/2} + \frac{1}{5}\big(1+5\tan^2(x)\big)^{5/2} - \frac{64}{5}\text{Subst}\left(\int \frac{1}{\frac{4}{5}+\frac{x^2}{5}}\,dx, x, \sqrt{1+5\tan^2(x)}\right)$$

$$= -32\arctan\left(\frac{1}{2}\sqrt{1+5\tan^2(x)}\right) + 16\sqrt{1+5\tan^2(x)} - \frac{4}{3}\big(1+5\tan^2(x)\big)^{3/2} + \frac{1}{5}\big(1+5\tan^2(x)\big)^{5/2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \tan(x)\big(1+5\tan^2(x)\big)^{5/2}\,dx = \frac{5\sqrt{5}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{2},-\frac{3}{2},\frac{4\cos^2(x)}{5}\right)\big(1+5\tan^2(x)\big)^{5/2}}{(3-2\cos(2x))^{5/2}}$$

```
[In] Integrate[Tan[x]*(1 + 5*Tan[x]^2)^(5/2),x]
[Out] (5*Sqrt[5]*Hypergeometric2F1[-5/2, -5/2, -3/2, (4*Cos[x]^2)/5]*(1 + 5*Tan[x
]^2)^(5/2))/(3 - 2*Cos[2*x])^(5/2)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{223\sqrt{1+5(\tan^2(x))}}{15} + 5(\tan^4(x))\sqrt{1+5(\tan^2(x))} - \frac{14(\tan^2(x))\sqrt{1+5(\tan^2(x))}}{3} - 32\arctan\Big(\surd$
default	$\frac{223\sqrt{1+5(\tan^2(x))}}{15} + 5(\tan^4(x))\sqrt{1+5(\tan^2(x))} - \frac{14(\tan^2(x))\sqrt{1+5(\tan^2(x))}}{3} - 32\arctan\Big(\surd$

```
[In] int(tan(x)*(1+5*tan(x)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 223/15*(1+5*tan(x)^2)^(1/2)+5*tan(x)^4*(1+5*tan(x)^2)^(1/2)-14/3*tan(x)^2*(
1+5*tan(x)^2)^(1/2)-32*arctan(1/2*(1+5*tan(x)^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \tan(x) \left(1+5\tan^2(x)\right)^{5/2} dx = \frac{1}{15} \left(75 \tan(x)^4 - 70 \tan(x)^2 + 223\right) \sqrt{5 \tan(x)^2 + 1} - 16 \arctan\left(\frac{5 \tan(x)^2 - 3}{4 \sqrt{5 \tan(x)^2 + 1}}\right)$$

```
[In] integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="fricas")
[Out] 1/15*(75*tan(x)^4 - 70*tan(x)^2 + 223)*sqrt(5*tan(x)^2 + 1) - 16*arctan(1/4
*(5*tan(x)^2 - 3)/sqrt(5*tan(x)^2 + 1))
```

Sympy [F]

$$\int \tan(x) \left(1+5\tan^2(x)\right)^{5/2} dx = \int \left(5\tan^2(x) + 1\right)^{\frac{5}{2}} \tan(x)\, dx$$

```
[In] integrate(tan(x)*(1+5*tan(x)**2)**(5/2),x)
[Out] Integral((5*tan(x)**2 + 1)**(5/2)*tan(x), x)
```

Maxima [F]

$$\int \tan(x) \left(1+5\tan^2(x)\right)^{5/2} dx = \int \left(5 \tan(x)^2 + 1\right)^{\frac{5}{2}} \tan(x)\, dx$$

```
[In] integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="maxima")
[Out] integrate((5*tan(x)^2 + 1)^(5/2)*tan(x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int \tan(x) \left(1+5\tan^2(x)\right)^{5/2} dx = \frac{1}{5} \left(5 \tan(x)^2 + 1\right)^{\frac{5}{2}} - \frac{4}{3} \left(5 \tan(x)^2 + 1\right)^{\frac{3}{2}} + 16 \sqrt{5 \tan(x)^2 + 1} - 32 \arctan\left(\frac{1}{2} \sqrt{5 \tan(x)^2 + 1}\right)$$

```
[In] integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/5*(5*tan(x)^2 + 1)^(5/2) - 4/3*(5*tan(x)^2 + 1)^(3/2) + 16*sqrt(5*tan(x)^
2 + 1) - 32*arctan(1/2*sqrt(5*tan(x)^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36

$$\int \tan(x)\left(1+5\tan^2(x)\right)^{5/2}\,dx=\frac{\sqrt{5}\,\sqrt{\tan(x)^2+\frac{1}{5}}\,\left(25\tan(x)^4-\frac{70\tan(x)^2}{3}+\frac{223}{3}\right)}{5}-\ln\left(\tan(x)-\frac{2\sqrt{5}\,\sqrt{\tan(x)^2+\frac{1}{5}}}{5}+\frac{1}{5}\mathrm{i}\right)16\mathrm{i}-\ln\left(\tan(x)+\frac{2\sqrt{5}\,\sqrt{\tan(x)^2+\frac{1}{5}}}{5}-\frac{1}{5}\mathrm{i}\right)16\mathrm{i}+\ln\left(\tan(x)-\mathrm{i}\right)16\mathrm{i}+\ln\left(\tan(x)+1\mathrm{i}\right)16\mathrm{i}$$

```
[In] int(tan(x)*(5*tan(x)^2 + 1)^(5/2),x)
```

```
[Out] log(tan(x) - 1i)*16i - log(tan(x) + (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 -
1i/5)*16i - log(tan(x) - (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 + 1i/5)*16i +
 log(tan(x) + 1i)*16i + (5^(1/2)*(tan(x)^2 + 1/5)^(1/2)*(25*tan(x)^4 - (70*
tan(x)^2)/3 + 223/3))/5
```

3.441 $\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx = \frac{1}{32}\arctan\left(\frac{1}{2}\sqrt{1+5\tan^2(x)}\right) - \frac{1}{12(1+5\tan^2(x))^{3/2}} + \frac{1}{16\sqrt{1+5\tan^2(x)}}$$

```
[Out] 1/32*arctan(1/2*(1+5*tan(x)^2)^(1/2))+1/16/(1+5*tan(x)^2)^(1/2)-1/12/(1+5*t
an(x)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3751, 455, 53, 65, 209}

$$\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx = \frac{1}{32}\arctan\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right) + \frac{1}{16\sqrt{5\tan^2(x)+1}} - \frac{1}{12(5\tan^2(x)+1)^{3/2}}$$

```
[In] Int[Tan[x]/(1 + 5*Tan[x]^2)^(5/2),x]
```

```
[Out] ArcTan[Sqrt[1 + 5*Tan[x]^2]/2]/32 - 1/(12*(1 + 5*Tan[x]^2)^(3/2)) + 1/(16*S
qrt[1 + 5*Tan[x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x}{(1+x^2)\,(1+5x^2)^{5/2}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)(1+5x)^{5/2}}\,dx, x, \tan^2(x)\right)
\end{aligned}$$

$$= -\frac{1}{12\left(1+5\tan^2(x)\right)^{3/2}} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{(1+x)(1+5x)^{3/2}}\,dx, x, \tan^2(x)\right)$$

$$= -\frac{1}{12\left(1+5\tan^2(x)\right)^{3/2}} + \frac{1}{16\sqrt{1+5\tan^2(x)}} + \frac{1}{32}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{1+5x}}\,dx, x, \tan^2(x)\right)$$

$$= -\frac{1}{12\left(1+5\tan^2(x)\right)^{3/2}} + \frac{1}{16\sqrt{1+5\tan^2(x)}} + \frac{1}{80}\text{Subst}\left(\int \frac{1}{\frac{4}{5}+\frac{x^2}{5}}\,dx, x, \sqrt{1+5\tan^2(x)}\right)$$

$$= \frac{1}{32}\arctan\left(\frac{1}{2}\sqrt{1+5\tan^2(x)}\right) - \frac{1}{12\left(1+5\tan^2(x)\right)^{3/2}} + \frac{1}{16\sqrt{1+5\tan^2(x)}}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

$$\int \frac{\tan(x)}{\left(1+5\tan^2(x)\right)^{5/2}}\,dx = \frac{(-3+2\cos(2x))\left(-6\cos(x)+8\cos(3x)-3(-3+2\cos(2x))^{3/2}\log\left(2\cos(x)+\sqrt{}\right.\right.}{96\left(1+5\tan^2(x)\right)^{5/2}}$$

```
[In] Integrate[Tan[x]/(1 + 5*Tan[x]^2)^(5/2),x]
```

```
[Out] ((-3 + 2*Cos[2*x])*(-6*Cos[x] + 8*Cos[3*x] - 3*(-3 + 2*Cos[2*x])^(3/2)*Log[
2*Cos[x] + Sqrt[-3 + 2*Cos[2*x]]])*Sec[x]^5)/(96*(1 + 5*Tan[x]^2)^(5/2))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{1+5\left(\tan^2(x)\right)}}{2}\right)}{32}+\frac{1}{16\sqrt{1+5\left(\tan^2(x)\right)}}-\frac{1}{12\left(1+5\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}$	41
default	$\frac{\arctan\left(\frac{\sqrt{1+5\left(\tan^2(x)\right)}}{2}\right)}{32}+\frac{1}{16\sqrt{1+5\left(\tan^2(x)\right)}}-\frac{1}{12\left(1+5\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}$	41

```
[In] int(tan(x)/(1+5*tan(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*arctan(1/2*(1+5*tan(x)^2)^(1/2))+1/16/(1+5*tan(x)^2)^(1/2)-1/12/(1+5*t
an(x)^2)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx = \frac{3\left(25\tan(x)^4+10\tan(x)^2+1\right)\arctan\left(\frac{5\tan(x)^2-3}{4\sqrt{5\tan(x)^2+1}}\right)+4\left(15\tan(x)^2-1\right)\sqrt{}}{192\left(25\tan(x)^4+10\tan(x)^2+1\right)}$$

```
[In] integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="fricas")
[Out] 1/192*(3*(25*tan(x)^4 + 10*tan(x)^2 + 1)*arctan(1/4*(5*tan(x)^2 - 3)/sqrt(5
*tan(x)^2 + 1)) + 4*(15*tan(x)^2 - 1)*sqrt(5*tan(x)^2 + 1))/(25*tan(x)^4 +
10*tan(x)^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{5\tan^2(x)+1}}{2}\right)}{32}+\frac{1}{16\sqrt{5\tan^2(x)+1}}-\frac{1}{12\left(5\tan^2(x)+1\right)^{\frac{3}{2}}}$$

```
[In] integrate(tan(x)/(1+5*tan(x)**2)**(5/2),x)
[Out] atan(sqrt(5*tan(x)**2 + 1)/2)/32 + 1/(16*sqrt(5*tan(x)**2 + 1)) - 1/(12*(5*
tan(x)**2 + 1)**(3/2))
```

Maxima [F]

$$\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx = \int \frac{\tan(x)}{\left(5\tan(x)^2+1\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(tan(x)/(5*tan(x)^2 + 1)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.67

$$\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx = \frac{15\,\tan\left(x\right)^2-1}{48\,\left(5\,\tan\left(x\right)^2+1\right)^{\frac{3}{2}}}+\frac{1}{32}\,\arctan\left(\frac{1}{2}\,\sqrt{5\,\tan\left(x\right)^2+1}\right)$$

```
[In] integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/48*(15*tan(x)^2 - 1)/(5*tan(x)^2 + 1)^(3/2) + 1/32*arctan(1/2*sqrt(5*tan(
x)^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.19

$$\begin{aligned}\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx &= \frac{\ln\left(\tan(x)-\frac{2\,\sqrt{5}\,\sqrt{\tan(x)^2+\frac{1}{5}}}{5}+\frac{1}{5}\mathrm{i}\right)\,1\mathrm{i}}{64}\\&+\frac{\ln\left(\tan(x)+\frac{2\,\sqrt{5}\,\sqrt{\tan(x)^2+\frac{1}{5}}}{5}-\frac{1}{5}\mathrm{i}\right)\,1\mathrm{i}}{64}-\frac{\ln\left(\tan(x)-\mathrm{i}\right)\,1\mathrm{i}}{64}\\&-\frac{\ln\left(\tan(x)+1\mathrm{i}\right)\,1\mathrm{i}}{64}-\frac{\sqrt{\tan\left(x\right)^2+\frac{1}{5}}\,1\mathrm{i}}{96\,\left(\tan\left(x\right)-\frac{\sqrt{5}\,1\mathrm{i}}{5}\right)}+\frac{\sqrt{\tan\left(x\right)^2+\frac{1}{5}}\,1\mathrm{i}}{96\,\left(\tan\left(x\right)+\frac{\sqrt{5}\,1\mathrm{i}}{5}\right)}\\&+\frac{\sqrt{5}\,\sqrt{\tan\left(x\right)^2+\frac{1}{5}}}{240\,\left(\tan\left(x\right)^2+\frac{2\mathrm{i}\,\sqrt{5}\,\tan(x)}{5}-\frac{1}{5}\right)}-\frac{\sqrt{5}\,\sqrt{\tan\left(x\right)^2+\frac{1}{5}}}{240\,\left(-\tan\left(x\right)^2+\frac{2\mathrm{i}\,\sqrt{5}\,\tan(x)}{5}+\frac{1}{5}\right)}\end{aligned}$$

```
[In] int(tan(x)/(5*tan(x)^2 + 1)^(5/2),x)
```

```
[Out] (log(tan(x) - (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 + 1i/5)*1i)/64 + (log(ta
n(x) + (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 - 1i/5)*1i)/64 - (log(tan(x) -
1i)*1i)/64 - (log(tan(x) + 1i)*1i)/64 - ((tan(x)^2 + 1/5)^(1/2)*1i)/(96*(ta
n(x) - (5^(1/2)*1i)/5)) + ((tan(x)^2 + 1/5)^(1/2)*1i)/(96*(tan(x) + (5^(1/2
)*1i)/5)) + (5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/(240*(tan(x)^2 + (5^(1/2)*tan(
x)*2i)/5 - 1/5)) - (5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/(240*((5^(1/2)*tan(x)*2
i)/5 - tan(x)^2 + 1/5))
```

3.442 $\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{a^3-b^3}} + \frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}} + \frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}$$

```
[Out] 1/2*ln(cos(x))/(a^3-b^3)^(1/3)+3/4*ln((a^3-b^3)^(1/3)-(a^3+b^3*tan(x)^2)^(1
/3))/(a^3-b^3)^(1/3)+1/2*arctan(1/3*(1+2*(a^3+b^3*tan(x)^2)^(1/3)/(a^3-b^3)
^(1/3))*3^(1/2))*3^(1/2)/(a^3-b^3)^(1/3)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used

$= \{3751, 455, 57, 631, 210, 31\}$

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{a^3-b^3}} + \frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}} + \frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}}$$

```
[In] Int[Tan[x]/(a^3 + b^3*Tan[x]^2)^(1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*Tan[x]^2)^(1/3))/(a^3 - b^3)^(1/3))/Sqrt
[3]])/(2*(a^3 - b^3)^(1/3)) + Log[Cos[x]]/(2*(a^3 - b^3)^(1/3)) + (3*Log[(a
^3 - b^3)^(1/3) - (a^3 + b^3*Tan[x]^2)^(1/3)])/(4*(a^3 - b^3)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt[3]{a^3+b^3x^2}}\,dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt[3]{a^3+b^3x}}\,dx, x, \tan^2(x)\right)$$

$$= \frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{(a^3-b^3)^{2/3}+\sqrt[3]{a^3-b^3}x+x^2}\,dx, x, \sqrt[3]{a^3+b^3\tan^2(x)}\right) - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt[3]{a^3-b^3}-x}\,dx, x, \sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}$$

$$= \frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}} + \frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}} - \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}\right)}{2\sqrt[3]{a^3-b^3}}$$

$$= \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{a^3-b^3}} + \frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}} + \frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.79

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}}{\sqrt{3}}\right)+2\log(\cos(x))+3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}$$

```
[In] Integrate[Tan[x]/(a^3 + b^3*Tan[x]^2)^(1/3),x]
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*Tan[x]^2)^(1/3))/(a^3 - b^3)^(1/3))/Sq
rt[3]] + 2*Log[Cos[x]] + 3*Log[(a^3 - b^3)^(1/3) - (a^3 + b^3*Tan[x]^2)^(1/
3)])/(4*(a^3 - b^3)^(1/3))
```

Maple [F]

$$\int \frac{\tan(x)}{(a^3+b^3(\tan^2(x)))^{\frac{1}{3}}}dx$$

```
[In] int(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x)
[Out] int(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx = \int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx$$

```
[In] integrate(tan(x)/(a**3+b**3*tan(x)**2)**(1/3),x)
[Out] Integral(tan(x)/(a**3 + b**3*tan(x)**2)**(1/3), x)
```

Maxima [F]

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx = \int \frac{\tan(x)}{\left(b^3\tan(x)^2+a^3\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="maxima")
[Out] integrate(tan(x)/(b^3*tan(x)^2 + a^3)^(1/3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.40

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx = \frac{3\left(a^3-b^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(b^3\tan(x)^2+a^3\right)^{\frac{1}{3}}+\left(a^3-b^3\right)^{\frac{1}{3}}\right)}{3\left(a^3-b^3\right)^{\frac{1}{3}}}\right)}{2\left(\sqrt{3}a^3-\sqrt{3}b^3\right)} - \frac{\log\left(\left(b^3\tan(x)^2+a^3\right)^{\frac{2}{3}}+\left(b^3\tan(x)^2+a^3\right)^{\frac{1}{3}}\left(a^3-b^3\right)^{\frac{1}{3}}+\left(a^3-b^3\right)^{\frac{2}{3}}\right)}{4\left(a^3-b^3\right)^{\frac{1}{3}}} + \frac{\log\left(\left|\left(b^3\tan(x)^2+a^3\right)^{\frac{1}{3}}-\left(a^3-b^3\right)^{\frac{1}{3}}\right|\right)}{2\left(a^3-b^3\right)^{\frac{1}{3}}}$$

```
[In] integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="giac")
[Out] 3/2*(a^3 - b^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b^3*tan(x)^2 + a^3)^(1/3) + (a
^3 - b^3)^(1/3))/(a^3 - b^3)^(1/3))/(sqrt(3)*a^3 - sqrt(3)*b^3) - 1/4*log((
b^3*tan(x)^2 + a^3)^(2/3) + (b^3*tan(x)^2 + a^3)^(1/3)*(a^3 - b^3)^(1/3) +
(a^3 - b^3)^(2/3))/(a^3 - b^3)^(1/3) + 1/2*log(abs((b^3*tan(x)^2 + a^3)^(1/
3) - (a^3 - b^3)^(1/3)))/(a^3 - b^3)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.88

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx = \frac{\ln\left(\frac{9\left(a^3+b^3\tan(x)^2\right)^{1/3}}{4}-\frac{9a^3-9b^3}{4(a-b)^{2/3}(a^2+ab+b^2)^{2/3}}\right)}{2(a-b)^{1/3}(a^2+ab+b^2)^{1/3}} + \frac{\ln\left(\frac{9\left(a^3+b^3\tan(x)^2\right)^{1/3}}{4}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2(9a^3-9b^3)}{16(a-b)^{2/3}(a^2+ab+b^2)^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4(a-b)^{1/3}(a^2+ab+b^2)^{1/3}} - \frac{\ln\left(\frac{9\left(a^3+b^3\tan(x)^2\right)^{1/3}}{4}-\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2(9a^3-9b^3)}{16(a-b)^{2/3}(a^2+ab+b^2)^{2/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4(a-b)^{1/3}(a^2+ab+b^2)^{1/3}}$$

```
[In] int(tan(x)/(b^3*tan(x)^2 + a^3)^(1/3),x)
```

```
[Out] log((9*(b^3*tan(x)^2 + a^3)^(1/3))/4 - (9*a^3 - 9*b^3)/(4*(a - b)^(2/3)*(a*
b + a^2 + b^2)^(2/3)))/(2*(a - b)^(1/3)*(a*b + a^2 + b^2)^(1/3)) + (log((9*
(b^3*tan(x)^2 + a^3)^(1/3))/4 - ((3^(1/2)*1i - 1)^2*(9*a^3 - 9*b^3))/(16*(a
 - b)^(2/3)*(a*b + a^2 + b^2)^(2/3)))*(3^(1/2)*1i - 1))/(4*(a - b)^(1/3)*(a
*b + a^2 + b^2)^(1/3)) - (log((9*(b^3*tan(x)^2 + a^3)^(1/3))/4 - ((3^(1/2)*
1i + 1)^2*(9*a^3 - 9*b^3))/(16*(a - b)^(2/3)*(a*b + a^2 + b^2)^(2/3)))*(3^(
1/2)*1i + 1))/(4*(a - b)^(1/3)*(a*b + a^2 + b^2)^(1/3))
```

3.443 $\int \tan(x) \left(1-7 \tan^2(x)\right)^{2/3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \tan(x) \left(1-7 \tan^2(x)\right)^{2/3} \, dx = 2\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{1-7\tan^2(x)}}{\sqrt{3}}\right) + 2\log(\cos(x)) + 3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right) + \frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}$$

```
[Out] 2*ln(cos(x))+3*ln(2-(1-7*tan(x)^2)^(1/3))+2*arctan(1/3*(1+(1-7*tan(x)^2)^(1
/3))*3^(1/2))*3^(1/2)+3/4*(1-7*tan(x)^2)^(2/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3751, 455, 52, 57, 632, 210, 31}

$$\int \tan(x) \left(1-7 \tan^2(x)\right)^{2/3} \, dx = 2\sqrt{3}\arctan\left(\frac{\sqrt[3]{1-7\tan^2(x)}+1}{\sqrt{3}}\right) + \frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3} + 3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right) + 2\log(\cos(x))$$

```
[In] Int[Tan[x]*(1 - 7*Tan[x]^2)^(2/3),x]
```

```
[Out] 2*Sqrt[3]*ArcTan[(1 + (1 - 7*Tan[x]^2)^(1/3))/Sqrt[3]] + 2*Log[Cos[x]] + 3*
Log[2 - (1 - 7*Tan[x]^2)^(1/3)] + (3*(1 - 7*Tan[x]^2)^(2/3))/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x(1-7x^2)^{2/3}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1-7x)^{2/3}}{1+x}\,dx, x, \tan^2(x)\right)\\
&= \frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3} + 4\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-7x}(1+x)}\,dx, x, \tan^2(x)\right)\\
&= 2\log(\cos(x)) + \frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3} - 3\text{Subst}\left(\int \frac{1}{2-x}\,dx, x, \sqrt[3]{1-7\tan^2(x)}\right)\\
&\quad + 6\text{Subst}\left(\int \frac{1}{4+2x+x^2}\,dx, x, \sqrt[3]{1-7\tan^2(x)}\right)\\
&= 2\log(\cos(x)) + 3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right) + \frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}\\
&\quad - 12\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, 2+2\sqrt[3]{1-7\tan^2(x)}\right)\\
&= 2\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{1-7\tan^2(x)}}{\sqrt{3}}\right) + 2\log(\cos(x))\\
&\quad + 3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right) + \frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int \tan(x)\left(1-7\tan^2(x)\right)^{2/3}\,dx &= 2\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{1-7\tan^2(x)}}{\sqrt{3}}\right)\\
&\quad + 2\log(\cos(x)) + 3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right) + \frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}
\end{aligned}$$

```
[In] Integrate[Tan[x]*(1 - 7*Tan[x]^2)^(2/3),x]
```

```
[Out] 2*Sqrt[3]*ArcTan[(1 + (1 - 7*Tan[x]^2)^(1/3))/Sqrt[3]] + 2*Log[Cos[x]] + 3*
Log[2 - (1 - 7*Tan[x]^2)^(1/3)] + (3*(1 - 7*Tan[x]^2)^(2/3))/4
```

Maple [F]

$$\int \tan(x)\left(1-7\left(\tan^2(x)\right)\right)^{\frac{2}{3}} dx$$

```
[In] int(tan(x)*(1-7*tan(x)^2)^(2/3),x)
[Out] int(tan(x)*(1-7*tan(x)^2)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(58) = 116.

Time = 0.63 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.70

$$\int \tan(x)\left(1-7\tan^2(x)\right)^{2/3} dx = 2\sqrt{3}\arctan\left(\frac{7\sqrt{3}\tan(x)^2+4\sqrt{3}(-7\tan(x)^2+1)^{\frac{2}{3}}-16\sqrt{3}(-7\tan(x)^2+1)^{\frac{1}{3}}-\sqrt{3}}{7\tan(x)^2-65}\right) + \frac{3}{4}\left(-7\tan(x)^2+1\right)^{\frac{2}{3}} + \log\left(\frac{7\tan(x)^2+6\left(-7\tan(x)^2+1\right)^{\frac{2}{3}}-12\left(-7\tan(x)^2+1\right)^{\frac{1}{3}}+7}{\tan(x)^2+1}\right)$$

```
[In] integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="fricas")
[Out] 2*sqrt(3)*arctan((7*sqrt(3)*tan(x)^2 + 4*sqrt(3)*(-7*tan(x)^2 + 1)^(2/3) -
16*sqrt(3)*(-7*tan(x)^2 + 1)^(1/3) - sqrt(3))/(7*tan(x)^2 - 65)) + 3/4*(-7*
tan(x)^2 + 1)^(2/3) + log((7*tan(x)^2 + 6*(-7*tan(x)^2 + 1)^(2/3) - 12*(-7*
tan(x)^2 + 1)^(1/3) + 7)/(tan(x)^2 + 1))
```

Sympy [F]

$$\int \tan(x)\left(1-7\tan^2(x)\right)^{2/3} dx = \int \left(1-7\tan^2(x)\right)^{\frac{2}{3}}\tan(x)\, dx$$

```
[In] integrate(tan(x)*(1-7*tan(x)**2)**(2/3),x)
[Out] Integral((1 - 7*tan(x)**2)**(2/3)*tan(x), x)
```

Maxima [F]

$$\int \tan(x)\left(1-7\tan^2(x)\right)^{2/3}\,dx=\int \left(-7\,\tan\left(x\right)^2+1\right)^{\frac{2}{3}}\tan\left(x\right)\,dx$$

```
[In] integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="maxima")
[Out] integrate((-7*tan(x)^2 + 1)^(2/3)*tan(x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \tan(x)\left(1-7\tan^2(x)\right)^{2/3}\,dx &= 2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\left(-7\,\tan\left(x\right)^2+1\right)^{\frac{1}{3}}+1\right)\right)\\&\quad+\frac{3}{4}\left(-7\,\tan\left(x\right)^2+1\right)^{\frac{2}{3}}-\log\left(\left(-7\,\tan\left(x\right)^2+1\right)^{\frac{2}{3}}+2\left(-7\,\tan\left(x\right)^2+1\right)^{\frac{1}{3}}+4\right)\\&\quad+2\,\log\left(\left|\left(-7\,\tan\left(x\right)^2+1\right)^{\frac{1}{3}}-2\right|\right)\end{aligned}$$

```
[In] integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="giac")
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*((-7*tan(x)^2 + 1)^(1/3) + 1)) + 3/4*(-7*tan(x
)^2 + 1)^(2/3) - log((-7*tan(x)^2 + 1)^(2/3) + 2*(-7*tan(x)^2 + 1)^(1/3) +
4) + 2*log(abs((-7*tan(x)^2 + 1)^(1/3) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.46

$$\int \tan(x)\left(1-7\tan^2(x)\right)^{2/3}\,dx=2\,\ln\left(144\left(1-7\tan(x)^2\right)^{1/3}-288\right)+\frac{3\left(1-7\tan(x)^2\right)^{2/3}}{4}$$
$$+\ln\left(144\left(1-7\tan(x)^2\right)^{1/3}-72\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)-\ln\left(144\left(1-7\tan(x)^2\right)^{1/3}-72\left(1+\sqrt{3}\,1\mathrm{i}\right.\right.$$

```
[In] int(tan(x)*(1 - 7*tan(x)^2)^(2/3),x)
[Out] 2*log(144*(1 - 7*tan(x)^2)^(1/3) - 288) + (3*(1 - 7*tan(x)^2)^(2/3))/4 + lo
g(144*(1 - 7*tan(x)^2)^(1/3) - 72*(3^(1/2)*1i - 1)^2)*(3^(1/2)*1i - 1) - lo
g(144*(1 - 7*tan(x)^2)^(1/3) - 72*(3^(1/2)*1i + 1)^2)*(3^(1/2)*1i + 1)
```

3.444 $\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx = -\frac{\arctan\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}$$

```
[Out] -arctan((a^4+b^4*csc(x)^2)^(1/4)/a)/a+arctanh((a^4+b^4*csc(x)^2)^(1/4)/a)/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {4224, 272, 65, 304, 209, 212}

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}-\frac{\arctan\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}$$

```
[In] Int[Cot[x]/(a^4 + b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] -(ArcTan[(a^4 + b^4*Csc[x]^2)^(1/4)/a]/a) + ArcTanh[(a^4 + b^4*Csc[x]^2)^(1
/4)/a]/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a^4+b^4x^2}}\,dx, x, \csc(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a^4+b^4x}}\,dx, x, \csc^2(x)\right)\right)\\
&= -\frac{2\text{Subst}\left(\int \frac{x^2}{-\frac{a^4}{b^4}+\frac{x^4}{b^4}}\,dx, x, \sqrt[4]{a^4+b^4\csc^2(x)}\right)}{b^4}
\end{aligned}
$$

$$= \text{Subst}\left(\int \frac{1}{a^2 - x^2}\,dx, x, \sqrt[4]{a^4 + b^4\csc^2(x)}\right) - \text{Subst}\left(\int \frac{1}{a^2 + x^2}\,dx, x, \sqrt[4]{a^4 + b^4\csc^2(x)}\right)$$

$$= -\frac{\arctan\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 256 vs. $2(52) = 104$.

Time = 0.32 (sec) , antiderivative size = 256, normalized size of antiderivative = 4.92

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 + b^4\csc^2(x)}}\,dx$$
$$= \frac{\sqrt[4]{-a^4 - 2b^4 + a^4\cos(2x)}\left(-2\arctan\left(1 - \frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{-b^4 - a^4\sin^2(x)}}\right) + 2\arctan\left(1 + \frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{-b^4 - a^4\sin^2(x)}}\right) -}{2\,2^{3/4}a\sqrt[4]{a^4 + b^4 \text{c}}}$$

```
[In] Integrate[Cot[x]/(a^4 + b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] ((-a^4 - 2*b^4 + a^4*Cos[2*x])^(1/4)*(-2*ArcTan[1 - (Sqrt[2]*a*Sqrt[Sin[x]]
)/(-b^4 - a^4*Sin[x]^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*a*Sqrt[Sin[x]])/(-b^
4 - a^4*Sin[x]^2)^(1/4)] - Log[1 + (a^2*Sin[x])/Sqrt[-b^4 - a^4*Sin[x]^2] -
 (Sqrt[2]*a*Sqrt[Sin[x]])/(-b^4 - a^4*Sin[x]^2)^(1/4)] + Log[1 + (a^2*Sin[x
])/Sqrt[-b^4 - a^4*Sin[x]^2] + (Sqrt[2]*a*Sqrt[Sin[x]])/(-b^4 - a^4*Sin[x]^
2)^(1/4)]))/(2*2^(3/4)*a*(a^4 + b^4*Csc[x]^2)^(1/4)*Sqrt[Sin[x]])
```

Maple [F]

$$\int \frac{\cot(x)}{\left(a^4 + b^4\left(\csc^2(x)\right)\right)^{\frac{1}{4}}}\,dx$$

```
[In] int(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x)
```

```
[Out] int(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx=\text{Timed out}$$

```
[In] integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx=\int \frac{\cot\left(x\right)}{\sqrt[4]{a^4+b^4\csc^2\left(x\right)}}\,dx$$

```
[In] integrate(cot(x)/(a**4+b**4*csc(x)**2)**(1/4),x)
[Out] Integral(cot(x)/(a**4 + b**4*csc(x)**2)**(1/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.37

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx=-\frac{\arctan\left(\frac{\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a}+\frac{\log\left(a+\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}-\frac{\log\left(-a+\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}$$

```
[In] integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="maxima")
[Out] -arctan((a^4 + b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(a + (a^4 + b^4/sin(x)^2)^
(1/4))/a - 1/2*log(-a + (a^4 + b^4/sin(x)^2)^(1/4))/a
```

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.40

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx = -\frac{\arctan\left(\frac{\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a} + \frac{\log\left(\left|a+\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right|\right)}{2\,a} - \frac{\log\left(\left|-a+\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right|\right)}{2\,a}$$

```
[In] integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="giac")
[Out] -arctan((a^4 + b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(abs(a + (a^4 + b^4/sin(x)
^2)^(1/4)))/a - 1/2*log(abs(-a + (a^4 + b^4/sin(x)^2)^(1/4)))/a
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx = -\frac{\mathrm{atan}\left(\frac{\left(\frac{b^4}{\sin(x)^2}+a^4\right)^{1/4}}{a}\right) - \mathrm{atanh}\left(\frac{\left(\frac{b^4}{\sin(x)^2}+a^4\right)^{1/4}}{a}\right)}{a}$$

```
[In] int(cot(x)/(b^4/sin(x)^2 + a^4)^(1/4),x)
[Out] -(atan((b^4/sin(x)^2 + a^4)^(1/4)/a) - atanh((b^4/sin(x)^2 + a^4)^(1/4)/a))
/a
```

3.445 $\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx=-\frac{\arctan\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}$$

```
[Out] -arctan((a^4-b^4*csc(x)^2)^(1/4)/a)/a+arctanh((a^4-b^4*csc(x)^2)^(1/4)/a)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4224, 272, 65, 304, 209, 212}

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}-\frac{\arctan\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}$$

```
[In] Int[Cot[x]/(a^4 - b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] -(ArcTan[(a^4 - b^4*Csc[x]^2)^(1/4)/a]/a) + ArcTanh[(a^4 - b^4*Csc[x]^2)^(1
/4)/a]/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a^4-b^4x^2}}\,dx, x, \csc(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a^4-b^4x}}\,dx, x, \csc^2(x)\right)\right)\\
&= \frac{2\text{Subst}\left(\int \frac{x^2}{\frac{a^4}{b^4}-\frac{x^4}{b^4}}\,dx, x, \sqrt[4]{a^4-b^4\csc^2(x)}\right)}{b^4}
\end{aligned}$$

$$= \text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, \sqrt[4]{a^4-b^4\csc^2(x)}\right) - \text{Subst}\left(\int \frac{1}{a^2+x^2}\,dx, x, \sqrt[4]{a^4-b^4\csc^2(x)}\right)$$

$$= -\frac{\arctan\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 245 vs. $2(54) = 108$.

Time = 0.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.54

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx$$

$$= \frac{\sqrt[4]{-a^4+2b^4+a^4\cos(2x)}\left(-2\arctan\left(1-\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{b^4-a^4\sin^2(x)}}\right)+2\arctan\left(1+\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{b^4-a^4\sin^2(x)}}\right)-\right.}{2\,2^{3/4}a\sqrt[4]{a^4-b^4\csc^2(}$$

```
[In] Integrate[Cot[x]/(a^4 - b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] ((-a^4 + 2*b^4 + a^4*Cos[2*x])^(1/4)*(-2*ArcTan[1 - (Sqrt[2]*a*Sqrt[Sin[x]]
)/(b^4 - a^4*Sin[x]^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*a*Sqrt[Sin[x]])/(b^4
- a^4*Sin[x]^2)^(1/4)] - Log[1 + (a^2*Sin[x])/Sqrt[b^4 - a^4*Sin[x]^2] - (S
qrt[2]*a*Sqrt[Sin[x]])/(b^4 - a^4*Sin[x]^2)^(1/4)] + Log[1 + (a^2*Sin[x])/S
qrt[b^4 - a^4*Sin[x]^2] + (Sqrt[2]*a*Sqrt[Sin[x]])/(b^4 - a^4*Sin[x]^2)^(1/
4)]))/(2*2^(3/4)*a*(a^4 - b^4*Csc[x]^2)^(1/4)*Sqrt[Sin[x]])
```

Maple [F]

$$\int \frac{\cot(x)}{\left(a^4-b^4\left(\csc^2(x)\right)\right)^{\frac{1}{4}}}dx$$

```
[In] int(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x)
```

```
[Out] int(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx=\text{Timed out}$$

```
[In] integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx=\int \frac{\cot(x)}{\sqrt[4]{(a^2-b^2\csc(x))\,(a^2+b^2\csc(x))}}\,dx$$

```
[In] integrate(cot(x)/(a**4-b**4*csc(x)**2)**(1/4),x)
[Out] Integral(cot(x)/((a**2 - b**2*csc(x))*(a**2 + b**2*csc(x)))**(1/4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx=-\frac{\arctan\left(\frac{\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a}+\frac{\log\left(a+\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}-\frac{\log\left(-a+\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}$$

```
[In] integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="maxima")
[Out] -arctan((a^4 - b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(a + (a^4 - b^4/sin(x)^2)^
(1/4))/a - 1/2*log(-a + (a^4 - b^4/sin(x)^2)^(1/4))/a
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx = -\frac{\arctan\left(\frac{\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a}+\frac{\log\left(\left|a+\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right|\right)}{2\,a}-\frac{\log\left(\left|-a+\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right|\right)}{2\,a}$$

```
[In] integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="giac")
[Out] -arctan((a^4 - b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(abs(a + (a^4 - b^4/sin(x)
^2)^(1/4)))/a - 1/2*log(abs(-a + (a^4 - b^4/sin(x)^2)^(1/4)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx=\int \frac{\cot(x)}{\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{1/4}}\,dx$$

```
[In] int(cot(x)/(a^4 - b^4/sin(x)^2)^(1/4),x)
[Out] int(cot(x)/(a^4 - b^4/sin(x)^2)^(1/4), x)
```

3.446 $\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx$

Optimal result

Integrand size = 61, antiderivative size = 133

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx=\sqrt{3}\arctan\left(\frac{1+2\sqrt[6]{1-3\sec^2(x)}}{\sqrt{3}}\right)+\frac{1}{4}\log\left(\sec^2(x)\right)-\frac{3}{2}\log\left(1-\sqrt[6]{1-3\sec^2(x)}\right)+\frac{1}{3}\log\left(1-\sqrt{1-3\sec^2(x)}\right)-\sqrt[6]{1-3\sec^2(x)}-\frac{1}{4}\left(1-3\sec^2(x)\right)^{2/3}+\frac{1}{2\left(1-\sqrt{1-3\sec^2(x)}\right)}$$

```
[Out] 1/4*ln(sec(x)^2)-3/2*ln(1-(1-3*sec(x)^2)^(1/6))+1/3*ln(1-(1-3*sec(x)^2)^(1/
2))-(1-3*sec(x)^2)^(1/6)-1/4*(1-3*sec(x)^2)^(2/3)+arctan(1/3*(1+2*(1-3*sec(
x)^2)^(1/6))*3^(1/2))*3^(1/2)+1/2/(1-(1-3*sec(x)^2)^(1/2))
```

Rubi [A] (verified)

Time = 3.75 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.31, number of steps used = 29, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules

used = {4446, 6874, 6816, 267, 6829, 348, 59, 632, 210, 31, 6820, 272, 43, 65, 212, 25}

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx=\sqrt{3}\arctan\left(\frac{2\sqrt[6]{1-3\sec^2(x)}+1}{\sqrt{3}}\right)$$
$$+\frac{1}{2}\text{arctanh}\left(\sqrt{1-3\sec^2(x)}\right)+\frac{\cos^2(x)}{6}-\frac{1}{4}\left(1-3\sec^2(x)\right)^{2/3}-\sqrt[6]{1-3\sec^2(x)}$$
$$-\frac{3}{2}\log\left(1-\sqrt[6]{1-3\sec^2(x)}\right)+\frac{1}{2}\log\left(1-\sqrt{1-3\sec^2(x)}\right)$$
$$+\frac{1}{6}\cos^2(x)\sqrt{1-3\sec^2(x)}+\frac{1}{3}\log\left(1-\sqrt{-\left((3-\cos^2(x))\sec^2(x)\right)}\right)$$

```
[In] Int[(Sec[x]^2*Tan[x]*((1 - 3*Sec[x]^2)^(1/3)*Sin[x]^2 + 3*Tan[x]^2))/((1 -
3*Sec[x]^2)^(5/6)*(1 - Sqrt[1 - 3*Sec[x]^2])),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + 2*(1 - 3*Sec[x]^2)^(1/6))/Sqrt[3]] + ArcTanh[Sqrt[1 - 3
*Sec[x]^2]]/2 + Cos[x]^2/6 + Log[1 - Sqrt[-((3 - Cos[x]^2)*Sec[x]^2)]]/3 -
(3*Log[1 - (1 - 3*Sec[x]^2)^(1/6)])/2 + Log[1 - Sqrt[1 - 3*Sec[x]^2]]/2 - (
1 - 3*Sec[x]^2)^(1/6) + (Cos[x]^2*Sqrt[1 - 3*Sec[x]^2])/6 - (1 - 3*Sec[x]^2
)^(2/3)/4
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4446

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
```

```
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rule 6816

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6829

```
Int[(u_.)*(v_)^(m_.)*((a_.) + (b_.)*(y_)^(n_))^(p_.), x_Symbol] :> Module[{
q, r}, Dist[q*r, Subst[Int[x^m*(a + b*x^n)^p, x], x, y], x] /;  !FalseQ[r =
 Divides[y^m, v^m, x]] &&  !FalseQ[q = DerivativeDivides[y, u, x]]] /; Free
Q[{a, b, m, n, p}, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(1-x^2)\left(3+\sqrt[3]{1-\frac{3}{x^2}}x^2\right)}{\left(1-\sqrt{1-\frac{3}{x^2}}\right)\left(1-\frac{3}{x^2}\right)^{5/6}x^5}\,dx, x, \cos(x)\right)$$

$$= -\text{Subst}\left(\int \left(\frac{-3-x^2\sqrt[3]{\frac{-3+x^2}{x^2}}}{\left(1-\frac{3}{x^2}\right)^{5/6}x^5\left(-1+\sqrt{\frac{-3+x^2}{x^2}}\right)} + \frac{3+x^2\sqrt[3]{\frac{-3+x^2}{x^2}}}{\left(1-\frac{3}{x^2}\right)^{5/6}x^3\left(-1+\sqrt{\frac{-3+x^2}{x^2}}\right)}\right) dx, x, \cos(x)\right)$$

$$= -\text{Subst}\left(\int \frac{-3-x^2\sqrt[3]{\frac{-3+x^2}{x^2}}}{\left(1-\frac{3}{x^2}\right)^{5/6}x^5\left(-1+\sqrt{\frac{-3+x^2}{x^2}}\right)}dx, x, \cos(x)\right)$$

$$- \text{Subst}\left(\int \frac{3+x^2\sqrt[3]{\frac{-3+x^2}{x^2}}}{\left(1-\frac{3}{x^2}\right)^{5/6}x^3\left(-1+\sqrt{\frac{-3+x^2}{x^2}}\right)}dx, x, \cos(x)\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{\sqrt{1-\frac{3}{x^2}}x^3\left(1-\sqrt{\frac{-3+x^2}{x^2}}\right)} - \frac{3}{\left(1-\frac{3}{x^2}\right)^{5/6}x^5\left(-1+\sqrt{\frac{-3+x^2}{x^2}}\right)}\right)dx, x, \cos(x)\right)$$

$$-\text{Subst}\left(\int \left(\frac{3}{\left(1-\frac{3}{x^2}\right)^{5/6}x^3\left(-1+\sqrt{\frac{-3+x^2}{x^2}}\right)} + \frac{1}{\sqrt{1-\frac{3}{x^2}}x\left(-1+\sqrt{\frac{-3+x^2}{x^2}}\right)}\right)dx, x, \cos(x)\right)$$

$$= 3\text{Subst}\left(\int \frac{1}{\left(1-\frac{3}{x^2}\right)^{5/6}x^5\left(-1+\sqrt{\frac{-3+x^2}{x^2}}\right)}dx, x, \cos(x)\right)$$

$$- 3\text{Subst}\left(\int \frac{1}{\left(1-\frac{3}{x^2}\right)^{5/6}x^3\left(-1+\sqrt{\frac{-3+x^2}{x^2}}\right)}dx, x, \cos(x)\right)$$

$$- \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{3}{x^2}}x^3\left(1-\sqrt{\frac{-3+x^2}{x^2}}\right)}dx, x, \cos(x)\right)$$

$$- \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{3}{x^2}}x\left(-1+\sqrt{\frac{-3+x^2}{x^2}}\right)}dx, x, \cos(x)\right)$$

$$= \frac{1}{3}\log\left(1-\sqrt{-\left(\left(3-\cos^2(x)\right)\sec^2(x)\right)}\right)$$
$$-\frac{1}{2}\text{Subst}\left(\int\frac{1}{\left(-1+\sqrt{x}\right)x^{5/6}}\,dx,x,\left(-3+\cos^2(x)\right)\sec^2(x)\right)$$
$$+3\text{Subst}\left(\int\left(-\frac{1}{3\left(1-\frac{3}{x^2}\right)^{5/6}x^3}-\frac{1}{3\sqrt[3]{1-\frac{3}{x^2}}x^3}\right)dx,x,\cos(x)\right)$$
$$-\text{Subst}\left(\int\frac{1}{-\frac{3}{x}+x-\sqrt{1-\frac{3}{x^2}}x}\,dx,x,\cos(x)\right)$$
$$= \frac{1}{3}\log\left(1-\sqrt{-\left(\left(3-\cos^2(x)\right)\sec^2(x)\right)}\right)$$
$$-\text{Subst}\left(\int\frac{1}{\left(1-\frac{3}{x^2}\right)^{5/6}x^3}\,dx,x,\cos(x)\right)-\text{Subst}\left(\int\frac{1}{\sqrt[3]{1-\frac{3}{x^2}}x^3}\,dx,x,\cos(x)\right)$$
$$-\text{Subst}\left(\int\frac{1}{(-1+x)x^{2/3}}\,dx,x,\sqrt{\left(-3+\cos^2(x)\right)\sec^2(x)}\right)$$
$$-\text{Subst}\left(\int\left(-\frac{x}{3}-\frac{1}{3}\sqrt{1-\frac{3}{x^2}}x+\frac{\sqrt{1-\frac{3}{x^2}}x}{3-x^2}\right)dx,x,\cos(x)\right)$$
$$= \frac{\cos^2(x)}{6}+\frac{1}{3}\log\left(1-\sqrt{-\left(\left(3-\cos^2(x)\right)\sec^2(x)\right)}\right)$$
$$+\frac{1}{2}\log\left(1-\sqrt{\left(-3+\cos^2(x)\right)\sec^2(x)}\right)-\sqrt[6]{1-3\sec^2(x)}$$
$$-\frac{1}{4}\left(1-3\sec^2(x)\right)^{2/3}+\frac{1}{3}\text{Subst}\left(\int\sqrt{1-\frac{3}{x^2}}x\,dx,x,\cos(x)\right)$$
$$+\frac{3}{2}\text{Subst}\left(\int\frac{1}{1-x}\,dx,x,\sqrt[6]{\left(-3+\cos^2(x)\right)\sec^2(x)}\right)$$
$$+\frac{3}{2}\text{Subst}\left(\int\frac{1}{1+x+x^2}\,dx,x,\sqrt[6]{\left(-3+\cos^2(x)\right)\sec^2(x)}\right)$$
$$-\text{Subst}\left(\int\frac{\sqrt{1-\frac{3}{x^2}}x}{3-x^2}\,dx,x,\cos(x)\right)$$

$$
\begin{aligned}
&= \frac{\cos^2(x)}{6} - \frac{3}{2}\log\left(1 - \sqrt[6]{-\left((3-\cos^2(x))\sec^2(x)\right)}\right) \\
&\quad + \frac{1}{3}\log\left(1 - \sqrt{-\left((3-\cos^2(x))\sec^2(x)\right)}\right) + \frac{1}{2}\log\left(1 - \sqrt{(-3+\cos^2(x))\sec^2(x)}\right) \\
&\quad - \sqrt[6]{1-3\sec^2(x)} - \frac{1}{4}\left(1-3\sec^2(x)\right)^{2/3} - \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt{1-3x}}{x^2}\,dx, x, \sec^2(x)\right) \\
&\quad - 3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[6]{(-3+\cos^2(x))\sec^2(x)}\right) \\
&\quad + \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{3}{x^2}}x}\,dx, x, \cos(x)\right) \\
&= \sqrt{3}\arctan\left(\frac{1+2\sqrt[6]{-\left((3-\cos^2(x))\sec^2(x)\right)}}{\sqrt{3}}\right) \\
&\quad + \frac{\cos^2(x)}{6} - \frac{3}{2}\log\left(1 - \sqrt[6]{-\left((3-\cos^2(x))\sec^2(x)\right)}\right) \\
&\quad + \frac{1}{3}\log\left(1 - \sqrt{-\left((3-\cos^2(x))\sec^2(x)\right)}\right) + \frac{1}{2}\log\left(1 - \sqrt{(-3+\cos^2(x))\sec^2(x)}\right) \\
&\quad - \sqrt[6]{1-3\sec^2(x)} + \frac{1}{6}\cos^2(x)\sqrt{1-3\sec^2(x)} - \frac{1}{4}\left(1-3\sec^2(x)\right)^{2/3} \\
&\quad + \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-3x}x}\,dx, x, \sec^2(x)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-3x}x}\,dx, x, \sec^2(x)\right) \\
&= \sqrt{3}\arctan\left(\frac{1+2\sqrt[6]{-\left((3-\cos^2(x))\sec^2(x)\right)}}{\sqrt{3}}\right) \\
&\quad + \frac{\cos^2(x)}{6} - \frac{3}{2}\log\left(1 - \sqrt[6]{-\left((3-\cos^2(x))\sec^2(x)\right)}\right) \\
&\quad + \frac{1}{3}\log\left(1 - \sqrt{-\left((3-\cos^2(x))\sec^2(x)\right)}\right) + \frac{1}{2}\log\left(1 - \sqrt{(-3+\cos^2(x))\sec^2(x)}\right) \\
&\quad - \sqrt[6]{1-3\sec^2(x)} + \frac{1}{6}\cos^2(x)\sqrt{1-3\sec^2(x)} - \frac{1}{4}\left(1-3\sec^2(x)\right)^{2/3} \\
&\quad - \frac{1}{6}\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{x^2}{3}}\,dx, x, \sqrt{1-3\sec^2(x)}\right) \\
&\quad + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{x^2}{3}}\,dx, x, \sqrt{1-3\sec^2(x)}\right) \\
&= \sqrt{3}\arctan\left(\frac{1+2\sqrt[6]{-\left((3-\cos^2(x))\sec^2(x)\right)}}{\sqrt{3}}\right) + \frac{1}{2}\text{arctanh}\left(\sqrt{1-3\sec^2(x)}\right) \\
&\quad + \frac{\cos^2(x)}{6} - \frac{3}{2}\log\left(1 - \sqrt[6]{-\left((3-\cos^2(x))\sec^2(x)\right)}\right) \\
&\quad + \frac{1}{3}\log\left(1 - \sqrt{-\left((3-\cos^2(x))\sec^2(x)\right)}\right) + \frac{1}{2}\log\left(1 - \sqrt{(-3+\cos^2(x))\sec^2(x)}\right) \\
&\quad - \sqrt[6]{1-3\sec^2(x)} + \frac{1}{6}\cos^2(x)\sqrt{1-3\sec^2(x)} - \frac{1}{4}\left(1-3\sec^2(x)\right)^{2/3}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 25.30 (sec) , antiderivative size = 1447, normalized size of antiderivative = 10.88

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx =$$

$$-\frac{}{3\left(-1+\sqrt{\frac{-5+\cos(2x)}{1+\cos(2x)}}\right)(1-3\sec^2(x))^{5/6}\left(6+\sqrt[3]{1-3\sec^2(x)}+\cos(2x)\sqrt[3]{1-3\sec^2(x)}\right)\left(12\csc(x)\sec(x\right.}$$

```
[In] Integrate[(Sec[x]^2*Tan[x]*((1 - 3*Sec[x]^2)^(1/3)*Sin[x]^2 + 3*Tan[x]^2))/
((1 - 3*Sec[x]^2)^(5/6)*(1 - Sqrt[1 - 3*Sec[x]^2])),x]
```

```
[Out] -1/3*((6 + ((-5 + Cos[2*x])/(1 + Cos[2*x]))^(1/3) + Cos[2*x]*((-5 + Cos[2*x
])/(1 + Cos[2*x]))^(1/3))*(3*Sec[x]^2 + (1 - 3*Sec[x]^2)^(1/3))*Tan[x]*(-2
- 3*Tan[x]^2)^(5/6)*(2 + 3*Tan[x]^2)*Sqrt[-(2 + 3*Tan[x]^2)^2]*(-1 + (-2 -
3*Tan[x]^2)^(1/3))*(1 + (-2 - 3*Tan[x]^2)^(1/3) + (-2 - 3*Tan[x]^2)^(2/3))*
(6*ArcTan[Sqrt[2 + 3*Tan[x]^2]]*Sqrt[-2 - 3*Tan[x]^2] - 5*Sqrt[2 + 3*Tan[x]
^2] - 4*ArcTanh[Sqrt[-2 - 3*Tan[x]^2]]*Sqrt[2 + 3*Tan[x]^2] + Cos[2*x]*Sqrt
[2 + 3*Tan[x]^2] + 5*Log[Sec[x]^2]*Sqrt[2 + 3*Tan[x]^2] - 9*Log[1 - (-2 - 3
*Tan[x]^2)^(1/3)]*Sqrt[2 + 3*Tan[x]^2] - 12*(-2 - 3*Tan[x]^2)^(1/6)*Sqrt[2
+ 3*Tan[x]^2] + 36*Hypergeometric2F1[1/6, 1, 7/6, -2 - 3*Tan[x]^2]*(-2 - 3*
Tan[x]^2)^(1/6)*Sqrt[2 + 3*Tan[x]^2] - 3*(-2 - 3*Tan[x]^2)^(2/3)*Sqrt[2 + 3
*Tan[x]^2] + Sqrt[-(2 + 3*Tan[x]^2)^2] + Cos[2*x]*Sqrt[-(2 + 3*Tan[x]^2)^2]
 - 6*ArcTan[(1 + 2*(-2 - 3*Tan[x]^2)^(1/3))/Sqrt[3]]*Sqrt[6 + 9*Tan[x]^2]))
/((-1 + Sqrt[(-5 + Cos[2*x])/(1 + Cos[2*x])])*(1 - 3*Sec[x]^2)^(5/6)*(6 + (
1 - 3*Sec[x]^2)^(1/3) + Cos[2*x]*(1 - 3*Sec[x]^2)^(1/3))*(12*Csc[x]*Sec[x]*
(-2 - 3*Tan[x]^2)^(5/6) + 12*Cos[2*x]*Csc[x]*Sec[x]*(-2 - 3*Tan[x]^2)^(5/6)
 - 88*Sin[2*x]*(-2 - 3*Tan[x]^2)^(5/6) - 16*Cot[x]^2*Sin[2*x]*(-2 - 3*Tan[x
]^2)^(5/6) + 48*Sec[x]^2*Tan[x]*(-2 - 3*Tan[x]^2)^(5/6) + 48*Cos[2*x]*Sec[x
]^2*Tan[x]*(-2 - 3*Tan[x]^2)^(5/6) - 180*Sin[2*x]*Tan[x]^2*(-2 - 3*Tan[x]^2
)^(5/6) + 63*Sec[x]^2*Tan[x]^3*(-2 - 3*Tan[x]^2)^(5/6) + 63*Cos[2*x]*Sec[x]
^2*Tan[x]^3*(-2 - 3*Tan[x]^2)^(5/6) - 162*Sin[2*x]*Tan[x]^4*(-2 - 3*Tan[x]^
2)^(5/6) + 27*Sec[x]^2*Tan[x]^5*(-2 - 3*Tan[x]^2)^(5/6) + 27*Cos[2*x]*Sec[x
]^2*Tan[x]^5*(-2 - 3*Tan[x]^2)^(5/6) - 54*Sin[2*x]*Tan[x]^6*(-2 - 3*Tan[x]^
2)^(5/6) - 24*Sec[x]^2*Tan[x]*Sqrt[2 + 3*Tan[x]^2]*Sqrt[-(2 + 3*Tan[x]^2)^2
] - 36*Sec[x]^2*Tan[x]^3*Sqrt[2 + 3*Tan[x]^2]*Sqrt[-(2 + 3*Tan[x]^2)^2] + 4
*Csc[x]*Sec[x]*(-2 - 3*Tan[x]^2)^(1/3)*Sqrt[2 + 3*Tan[x]^2]*Sqrt[-(2 + 3*Ta
n[x]^2)^2] + 6*Sec[x]^2*Tan[x]*(-2 - 3*Tan[x]^2)^(1/3)*Sqrt[2 + 3*Tan[x]^2]
*Sqrt[-(2 + 3*Tan[x]^2)^2] - 24*Sec[x]^2*Tan[x]*Sqrt[-2 - 3*Tan[x]^2]*Sqrt[
```

```
2 + 3*Tan[x]^2]*Sqrt[-(2 + 3*Tan[x]^2)^2] - 36*Sec[x]^2*Tan[x]^3*Sqrt[-2 -
3*Tan[x]^2]*Sqrt[2 + 3*Tan[x]^2]*Sqrt[-(2 + 3*Tan[x]^2)^2] - 20*Cot[x]*(-2
- 3*Tan[x]^2)^(5/6)*Sqrt[2 + 3*Tan[x]^2]*Sqrt[-(2 + 3*Tan[x]^2)^2] + 12*Csc
[x]*Sec[x]*(-2 - 3*Tan[x]^2)^(5/6)*Sqrt[2 + 3*Tan[x]^2]*Sqrt[-(2 + 3*Tan[x]
^2)^2] + 10*Sin[2*x]*(-2 - 3*Tan[x]^2)^(5/6)*Sqrt[2 + 3*Tan[x]^2]*Sqrt[-(2
+ 3*Tan[x]^2)^2] + 4*Cot[x]^2*Sin[2*x]*(-2 - 3*Tan[x]^2)^(5/6)*Sqrt[2 + 3*T
an[x]^2]*Sqrt[-(2 + 3*Tan[x]^2)^2] - 50*Tan[x]*(-2 - 3*Tan[x]^2)^(5/6)*Sqrt
[2 + 3*Tan[x]^2]*Sqrt[-(2 + 3*Tan[x]^2)^2] + 18*Sec[x]^2*Tan[x]*(-2 - 3*Tan
[x]^2)^(5/6)*Sqrt[2 + 3*Tan[x]^2]*Sqrt[-(2 + 3*Tan[x]^2)^2] + 6*Sin[2*x]*Ta
n[x]^2*(-2 - 3*Tan[x]^2)^(5/6)*Sqrt[2 + 3*Tan[x]^2]*Sqrt[-(2 + 3*Tan[x]^2)^
2] - 30*Tan[x]^3*(-2 - 3*Tan[x]^2)^(5/6)*Sqrt[2 + 3*Tan[x]^2]*Sqrt[-(2 + 3*
Tan[x]^2)^2]))
```

Maple [F]

$$\int \frac{\tan(x)\left((1-3(\sec^2(x)))^{\frac{1}{3}}(\sin^2(x))+3(\tan^2(x))\right)}{\cos(x)^2\left(1-3\left(\sec^2(x)\right)\right)^{\frac{5}{6}}\left(1-\sqrt{1-3\left(\sec^2(x)\right)}\right)}dx$$

```
[In] int(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*sec(x)^
2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x)
```

```
[Out] int(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*sec(x)^
2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*s
ec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Curve
 not irreducible after change of variable 0 -> infinity
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx = \text{Timed out}$$

```
[In] integrate(tan(x)*((1-3*sec(x)**2)**(1/3)*sin(x)**2+3*tan(x)**2)/cos(x)**2/(
1-3*sec(x)**2)**(5/6)/(1-(1-3*sec(x)**2)**(1/2)),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx = \text{Timed out}$$

```
[In] integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*s
ec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx = \int -\frac{\left(\left(-3\sec(x)^2+1\right)^{\frac{1}{3}}\sin(x)^2+3\tan(x)\right.}{\left(-3\sec(x)^2+1\right)^{\frac{5}{6}}\left(\sqrt{-3\sec(x)^2+1}-\right.}$$

```
[In] integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*s
ec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx = -\int \frac{\tan(x)\left(\sin(x)^2\left(1-\frac{3}{\cos(x)^2}\right)^{1/3}+3\tan(x)^2\right)}{\cos(x)^2\left(\sqrt{1-\frac{3}{\cos(x)^2}}-1\right)\left(1-\frac{3}{\cos(x)^2}\right)^{5/6}}\,dx$$

```
[In] int(-(tan(x)*(sin(x)^2*(1 - 3/cos(x)^2)^(1/3) + 3*tan(x)^2))/(cos(x)^2*((1
- 3/cos(x)^2)^(1/2) - 1)*(1 - 3/cos(x)^2)^(5/6)),x)
```

```
[Out] -int((tan(x)*(sin(x)^2*(1 - 3/cos(x)^2)^(1/3) + 3*tan(x)^2))/(cos(x)^2*((1
- 3/cos(x)^2)^(1/2) - 1)*(1 - 3/cos(x)^2)^(5/6)), x)
```

3.447 $\int \frac{\sec^2(x)(-\cos(2x)+2\tan^2(x))}{(\tan(x)\tan(2x))^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 100

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = 2\text{arctanh}\left(\frac{\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right) - \frac{11\text{arctanh}\left(\frac{\sqrt{2}\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right)}{4\sqrt{2}} + \frac{\tan(x)}{2(\tan(x)\tan(2x))^{3/2}} + \frac{2\tan^3(x)}{3(\tan(x)\tan(2x))^{3/2}} + \frac{3\tan(x)}{4\sqrt{\tan(x)\tan(2x)}}$$

```
[Out] 2*arctanh(tan(x)/(tan(x)*tan(2*x))^(1/2))-11/8*arctanh(2^(1/2)*tan(x)/(tan(
x)*tan(2*x))^(1/2))*2^(1/2)+3/4*tan(x)/(tan(x)*tan(2*x))^(1/2)+1/2*tan(x)/(
tan(x)*tan(2*x))^(3/2)+2/3*tan(x)^3/(tan(x)*tan(2*x))^(3/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 208 vs. 2(100) = 200.

Time = 0.80 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.08, number of steps used = 22, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules

used = {4482, 12, 1986, 15, 6857, 272, 43, 52, 65, 209, 455}

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = -\frac{11\tan(x)\arctan\left(\sqrt{\tan^2(x)-1}\right)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{\tan^2(x)-1}} + \frac{2\tan(x)\arctan\left(\frac{\sqrt{\tan^2(x)-1}}{\sqrt{2}}\right)}{\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{\tan^2(x)-1}} + \frac{(1-\tan^2(x))\tan(x)}{3\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{3\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{(1-\tan^2(x))\cot(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}}$$

```
[In] Int[(Sec[x]^2*(-Cos[2*x] + 2*Tan[x]^2))/(Tan[x]*Tan[2*x])^(3/2),x]
```

```
[Out] (3*Tan[x])/(4*Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]) + (Cot[x]*(1 - Tan[x]^
2))/(4*Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]) + (Tan[x]*(1 - Tan[x]^2))/(3*
Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]) - (11*ArcTan[Sqrt[-1 + Tan[x]^2]]*Ta
n[x])/(4*Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]*Sqrt[-1 + Tan[x]^2]) + (2*Ar
cTan[Sqrt[-1 + Tan[x]^2]/Sqrt[2]]*Tan[x])/(Sqrt[Tan[x]^2/(1 - Tan[x]^2)]*Sq
rt[-1 + Tan[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(-1+\sec(2x))^{3/2}}\,dx$$

$$= \text{Subst}\left(\int \frac{(1-x^2)(-1+3x^2+2x^4)}{2\sqrt{2}x^2\sqrt{\frac{x^2}{1-x^2}}\,(1+x^2)}\,dx, x, \tan(x)\right)$$

$$= \frac{\text{Subst}\left(\int \frac{(1-x^2)(-1+3x^2+2x^4)}{x^2\sqrt{\frac{x^2}{1-x^2}}(1+x^2)}\,dx, x, \tan(x)\right)}{2\sqrt{2}}$$

$$= \frac{\sqrt{\tan^2(x)}\text{Subst}\left(\int \frac{(1-x^2)^{3/2}(-1+3x^2+2x^4)}{(x^2)^{3/2}(1+x^2)}\,dx, x, \tan(x)\right)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$= \frac{\tan(x)\text{Subst}\left(\int \frac{(1-x^2)^{3/2}(-1+3x^2+2x^4)}{x^3(1+x^2)}\,dx, x, \tan(x)\right)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$= \frac{\tan(x)\text{Subst}\left(\int \left(-\frac{(1-x^2)^{3/2}}{x^3}+\frac{4(1-x^2)^{3/2}}{x}-\frac{2x(1-x^2)^{3/2}}{1+x^2}\right)dx, x, \tan(x)\right)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$= -\frac{\tan(x)\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{x^3}\,dx, x, \tan(x)\right)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} - \frac{\tan(x)\text{Subst}\left(\int \frac{x(1-x^2)^{3/2}}{1+x^2}\,dx, x, \tan(x)\right)}{\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} + \frac{\left(\sqrt{2}\tan(x)\right)\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{x}\,dx, x, \tan(x)\right)}{\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$= -\frac{\tan(x)\text{Subst}\left(\int \frac{(1-x)^{3/2}}{x^2}\,dx, x, \tan^2(x)\right)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} - \frac{\tan(x)\text{Subst}\left(\int \frac{(1-x)^{3/2}}{1+x}\,dx, x, \tan^2(x)\right)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} + \frac{\tan(x)\text{Subst}\left(\int \frac{(1-x)^{3/2}}{x}\,dx, x, \tan^2(x)\right)}{\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$= \frac{\cot(x)\left(1-\tan^2(x)\right)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{\tan(x)\left(1-\tan^2(x)\right)}{3\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\sqrt{2}\tan(x)\left(1-\tan^2(x)\right)}{3\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{(3\tan(x))\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx, x, \tan^2(x)\right)}{8\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} + \frac{\tan(x)\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx, x, \tan^2(x)\right)}{\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} - \frac{\tan(x)\text{Subst}\left(\int \frac{\sqrt{1-x}}{1+x}\,dx, x, \tan^2(x)\right)}{\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$= \frac{3\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\cot(x)\left(1-\tan^2(x)\right)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{\tan(x)\left(1-\tan^2(x)\right)}{3\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\sqrt{2}\tan(x)\left(1-\tan^2(x)\right)}{3\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{(3\tan(x))\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \tan^2(x)\right)}{8\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} + \frac{\tan(x)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \tan^2(x)\right)}{\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} - \frac{\left(\sqrt{2}\tan(x)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}(1+x)}\,dx, x, \tan^2(x)\right)}{\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$= \frac{3\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\cot(x)\left(1-\tan^2(x)\right)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{\tan(x)\left(1-\tan^2(x)\right)}{3\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\sqrt{2}\tan(x)\left(1-\tan^2(x)\right)}{3\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{(3\tan(x))\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\tan^2(x)}\right)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} - \frac{\left(\sqrt{2}\tan(x)\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\tan^2(x)}\right)}{\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} + \frac{\left(2\sqrt{2}\tan(x)\right)\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1-\tan^2(x)}\right)}{\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$= \frac{3\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{3\text{arctanh}\left(\sqrt{1-\tan^2(x)}\right)\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} - \frac{\sqrt{2}\text{arctanh}\left(\sqrt{1-\tan^2(x)}\right)\tan(x)}{\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} + \frac{2\text{arctanh}\left(\frac{\sqrt{1-\tan^2(x)}}{\sqrt{2}}\right)\tan(x)}{\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} + \frac{\cot(x)\left(1-\tan^2(x)\right)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{\tan(x)\left(1-\tan^2(x)\right)}{3\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\sqrt{2}\tan(x)\left(1-\tan^2(x)\right)}{3\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}}$$

Mathematica [A] (verified)

Time = 5.00 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.69

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = \frac{\left(-\cos(2x)+2\tan^2(x)\right)\left(\frac{4\sqrt{2}\left(-2\text{arctanh}\left(\sqrt{\frac{\cos(2x)}{1+\cos(2x)}}\right)+\sqrt{2}\text{arctanh}\left(\sqrt{1-\text{ta}\right.\right.}{\sqrt{1-\tan^2(x)}}$$

```
[In] Integrate[(Sec[x]^2*(-Cos[2*x] + 2*Tan[x]^2))/(Tan[x]*Tan[2*x])^(3/2),x]
[Out] ((-Cos[2*x] + 2*Tan[x]^2)*((4*Sqrt[2]*(-2*ArcTanh[Sqrt[Cos[2*x]/(1 + Cos[2*
x])]] + Sqrt[2]*ArcTanh[Sqrt[1 - Tan[x]^2]])*Cos[2*x]*Tan[x])/Sqrt[1 - Tan[
x]^2] - 3*ArcTan[Sqrt[-1 + Tan[x]^2]]*Cos[x]*Sin[x]*Sqrt[-1 + Tan[x]^2] + (
-3*Cot[x] - 4*Cos[x]*Sin[x] + (5 + 9*Cos[2*x])*Tan[x]^3)/3)*Tan[2*x]^2)/(2*
(-3 + 6*Cos[2*x] + Cos[4*x])*(Tan[x]*Tan[2*x])^(3/2))
```

Maple [F]

$$\int \frac{-\cos(2x)+2\left(\tan^2(x)\right)}{\cos(x)^2\left(\tan(x)\tan(2x)\right)^{\frac{3}{2}}}dx$$

```
[In] int((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x)
[Out] int((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(78) = 156$.

Time = 0.30 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.71

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx =$$

$$-\frac{24\left(\cos(x)^5-\cos(x)^3\right)\log\left(-\frac{4\sqrt{2}\left(8\cos(x)^5-6\cos(x)^3+\cos(x)\right)\sqrt{-\frac{\cos(x)^2-1}{2\cos(x)^2-1}}-\left(32\cos(x)^4-16\cos(x)^2+1\right)\sin(x)}{\sin(x)}\right)\sin(x) -}{}$$

```
[In] integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algori
thm="fricas")
```

```
[Out] -1/48*(24*(cos(x)^5 - cos(x)^3)*log(-(4*sqrt(2)*(8*cos(x)^5 - 6*cos(x)^3 +
cos(x))*sqrt(-(cos(x)^2 - 1)/(2*cos(x)^2 - 1)) - (32*cos(x)^4 - 16*cos(x)^2
 + 1)*sin(x))/sin(x))*sin(x) - 33*(sqrt(2)*cos(x)^5 - sqrt(2)*cos(x)^3)*log
(4*(sqrt(2)*(2*(3*sqrt(2) - 4)*cos(x)^3 - (3*sqrt(2) - 4)*cos(x))*sqrt(-(co
s(x)^2 - 1)/(2*cos(x)^2 - 1)) + (3*(2*sqrt(2) - 3)*cos(x)^2 - 2*sqrt(2) + 3
)*sin(x))/((cos(x)^2 - 1)*sin(x)))*sin(x) - 2*sqrt(2)*(22*cos(x)^6 - 47*cos
(x)^4 + 26*cos(x)^2 - 4)*sqrt(-(cos(x)^2 - 1)/(2*cos(x)^2 - 1)) - 44*(cos(x
)^5 - cos(x)^3)*sin(x))/((cos(x)^5 - cos(x)^3)*sin(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((-cos(2*x)+2*tan(x)**2)/cos(x)**2/(tan(x)*tan(2*x))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = \int \frac{2\,\tan\left(x\right)^2-\cos\left(2\,x\right)}{\left(\tan\left(2\,x\right)\tan\left(x\right)\right)^{\frac{3}{2}}\cos\left(x\right)^2}\,dx$$

```
[In] integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algori
thm="maxima")
```

```
[Out] integrate((2*tan(x)^2 - cos(2*x))/((tan(2*x)*tan(x))^(3/2)*cos(x)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(78) = 156$.

Time = 0.40 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.93

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = -\frac{\sqrt{2}\left(2\left(-\tan(x)^2+1\right)^{\frac{3}{2}}+3\sqrt{-\tan(x)^2+1}\right)}{12\,\mathrm{sgn}\left(\tan(x)^2-1\right)\mathrm{sgn}(\tan(x))} + \frac{11\sqrt{2}\log\left(\sqrt{-\tan(x)^2+1}+1\right)}{16\,\mathrm{sgn}\left(\tan(x)^2-1\right)\mathrm{sgn}(\tan(x))} - \frac{11\sqrt{2}\log\left(-\sqrt{-\tan(x)^2+1}+1\right)}{16\,\mathrm{sgn}\left(\tan(x)^2-1\right)\mathrm{sgn}(\tan(x))} + \frac{\log\left(\frac{\sqrt{2}-\sqrt{-\tan(x)^2+1}}{\sqrt{2}+\sqrt{-\tan(x)^2+1}}\right)}{\mathrm{sgn}\left(\tan(x)^2-1\right)\mathrm{sgn}(\tan(x))} - \frac{\sqrt{2}\sqrt{-\tan(x)^2+1}}{8\,\mathrm{sgn}\left(\tan(x)^2-1\right)\mathrm{sgn}(\tan(x))\tan(x)^2}$$

```
[In] integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algori
thm="giac")
```

```
[Out] -1/12*sqrt(2)*(2*(-tan(x)^2 + 1)^(3/2) + 3*sqrt(-tan(x)^2 + 1))/(sgn(tan(x)
^2 - 1)*sgn(tan(x))) + 11/16*sqrt(2)*log(sqrt(-tan(x)^2 + 1) + 1)/(sgn(tan(
x)^2 - 1)*sgn(tan(x))) - 11/16*sqrt(2)*log(-sqrt(-tan(x)^2 + 1) + 1)/(sgn(t
an(x)^2 - 1)*sgn(tan(x))) + log((sqrt(2) - sqrt(-tan(x)^2 + 1))/(sqrt(2) +
sqrt(-tan(x)^2 + 1)))/(sgn(tan(x)^2 - 1)*sgn(tan(x))) - 1/8*sqrt(2)*sqrt(-t
an(x)^2 + 1)/(sgn(tan(x)^2 - 1)*sgn(tan(x))*tan(x)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = -\int \frac{\cos(2x)-2\tan(x)^2}{\cos(x)^2\left(\tan(2x)\,\tan(x)\right)^{3/2}}\,dx$$

```
[In] int(-(cos(2*x) - 2*tan(x)^2)/(cos(x)^2*(tan(2*x)*tan(x))^(3/2)),x)
```

```
[Out] -int((cos(2*x) - 2*tan(x)^2)/(cos(x)^2*(tan(2*x)*tan(x))^(3/2)), x)
```

3.448 $\int \frac{\tan(x)}{(a^3-b^3\cos^n(x))^{4/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{\tan(x)}{(a^3-b^3\cos^n(x))^{4/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{a+2\sqrt[3]{a^3-b^3\cos^n(x)}}{\sqrt{3}a}\right)}{a^4n} - \frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}} + \frac{\log(\cos(x))}{2a^4} - \frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}$$

```
[Out] -3/a^3/n/(a^3-b^3*cos(x)^n)^(1/3)+1/2*ln(cos(x))/a^4-3/2*ln(a-(a^3-b^3*cos(
x)^n)^(1/3))/a^4/n-arctan(1/3*(a+2*(a^3-b^3*cos(x)^n)^(1/3))/a*3^(1/2))*3^(
1/2)/a^4/n
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3309, 272, 53, 57, 631, 210, 31}

$$\int \frac{\tan(x)}{(a^3-b^3\cos^n(x))^{4/3}}\,dx = \frac{\log(\cos(x))}{2a^4} - \frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}} - \frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{a^3-b^3\cos^n(x)}+a}{\sqrt{3}a}\right)}{a^4n} - \frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}$$

```
[In] Int[Tan[x]/(a^3 - b^3*Cos[x]^n)^(4/3),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*Cos[x]^n)^(1/3))/(Sqrt[3]*a)])/(a^4*n))
 - 3/(a^3*n*(a^3 - b^3*Cos[x]^n)^(1/3)) + Log[Cos[x]]/(2*a^4) - (3*Log[a -
(a^3 - b^3*Cos[x]^n)^(1/3)])/(2*a^4*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{x\left(a^3-b^3x^n\right)^{4/3}}\,dx, x, \cos(x)\right)\\
&= -\frac{\text{Subst}\left(\int \frac{1}{x\left(a^3-b^3x\right)^{4/3}}\,dx, x, \cos^n(x)\right)}{n}\\
&= -\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx, x, \cos^n(x)\right)}{a^3n}\\
&= -\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}} + \frac{\log(\cos(x))}{2a^4} + \frac{3\text{Subst}\left(\int \frac{1}{a-x}\,dx, x, \sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}\\
&\quad - \frac{3\text{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx, x, \sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^3n}\\
&= -\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}} + \frac{\log(\cos(x))}{2a^4} - \frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}\\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a^3-b^3\cos^n(x)}}{a}\right)}{a^4n}\\
&= -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a^3-b^3\cos^n(x)}}{a}}{\sqrt{3}}\right)}{a^4n} - \frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}\\
&\quad + \frac{\log(\cos(x))}{2a^4} - \frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.42

$$\int \frac{\tan(x)}{(a^3 - b^3 \cos^n(x))^{4/3}} \, dx = -\frac{3 \, \text{Hypergeometric2F1}\left(-\frac{1}{3}, 1, \frac{2}{3}, 1 - \frac{b^3 \cos^n(x)}{a^3}\right)}{a^3 n \sqrt[3]{a^3 - b^3 \cos^n(x)}}$$

[In] Integrate[Tan[x]/(a^3 - b^3*Cos[x]^n)^(4/3),x]

[Out] (-3*Hypergeometric2F1[-1/3, 1, 2/3, 1 - (b^3*Cos[x]^n)/a^3])/(a^3*n*(a^3 - b^3*Cos[x]^n)^(1/3))

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.13

method	result
derivativedivides	$-\frac{\frac{\ln\left(a-\left(a^3-b^3(\cos^n(x))\right)^{\frac{1}{3}}\right)}{a^4}+\frac{-\frac{\ln\left(a^2+a\left(a^3-b^3(\cos^n(x))\right)^{\frac{1}{3}}+\left(a^3-b^3(\cos^n(x))\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\left(a+2\left(a^3-b^3(\cos^n(x))\right)^{\frac{1}{3}}\right)}{3a}\right.}{a^4}}{n}$
default	$-\frac{\frac{\ln\left(a-\left(a^3-b^3(\cos^n(x))\right)^{\frac{1}{3}}\right)}{a^4}+\frac{-\frac{\ln\left(a^2+a\left(a^3-b^3(\cos^n(x))\right)^{\frac{1}{3}}+\left(a^3-b^3(\cos^n(x))\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\left(a+2\left(a^3-b^3(\cos^n(x))\right)^{\frac{1}{3}}\right)}{3a}\right.}{a^4}}{n}$

[In] int(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x,method=_RETURNVERBOSE)

[Out] -1/n*(1/a^4*ln(a-(a^3-b^3*cos(x)^n)^(1/3))+1/a^4*(-1/2*ln(a^2+a*(a^3-b^3*cos(x)^n)^(1/3)+(a^3-b^3*cos(x)^n)^(2/3))+3^(1/2)*arctan(1/3*(a+2*(a^3-b^3*cos(x)^n)^(1/3))/a*3^(1/2)))+3/a^3/(a^3-b^3*cos(x)^n)^(1/3))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.65

$$\int \frac{\tan(x)}{(a^3 - b^3\cos^n(x))^{4/3}}\,dx =$$
$$-\frac{2\left(\sqrt{3}b^3\cos(x)^n - \sqrt{3}a^3\right)\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}}{3a}\right) - \left(b^3\cos(x)^n - a^3\right)\log\left(a^2 + \left(-b^3\cos(x)^n + \right.\right.}{2\left(a^4b^3n\right.}$$

```
[In] integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="fricas")
[Out] -1/2*(2*(sqrt(3)*b^3*cos(x)^n - sqrt(3)*a^3)*arctan(1/3*(sqrt(3)*a + 2*sqrt
(3)*(-b^3*cos(x)^n + a^3)^(1/3))/a) - (b^3*cos(x)^n - a^3)*log(a^2 + (-b^3*
cos(x)^n + a^3)^(1/3)*a + (-b^3*cos(x)^n + a^3)^(2/3)) + 2*(b^3*cos(x)^n -
a^3)*log(-a + (-b^3*cos(x)^n + a^3)^(1/3)) - 6*(-b^3*cos(x)^n + a^3)^(2/3)*
a)/(a^4*b^3*n*cos(x)^n - a^7*n)
```

Sympy [F]

$$\int \frac{\tan(x)}{(a^3 - b^3\cos^n(x))^{4/3}}\,dx = \int \frac{\tan(x)}{(a^3 - b^3\cos^n(x))^{\frac{4}{3}}}\,dx$$

```
[In] integrate(tan(x)/(a**3-b**3*cos(x)**n)**(4/3),x)
[Out] Integral(tan(x)/(a**3 - b**3*cos(x)**n)**(4/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.21

$$\int \frac{\tan(x)}{(a^3 - b^3\cos^n(x))^{4/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^4n} + \frac{\log\left(a^2 + \left(-b^3\cos(x)^n + a^3\right)^{\frac{1}{3}}a + \left(-b^3\cos(x)^n + a^3\right)^{\frac{2}{3}}\right)}{2a^4n} - \frac{\log\left(-a + \left(-b^3\cos(x)^n + a^3\right)^{\frac{1}{3}}\right)}{a^4n} - \frac{3}{\left(-b^3\cos(x)^n + a^3\right)^{\frac{1}{3}}a^3n}$$

```
[In] integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*cos(x)^n + a^3)^(1/3))/a)/(a^4*n)
+ 1/2*log(a^2 + (-b^3*cos(x)^n + a^3)^(1/3)*a + (-b^3*cos(x)^n + a^3)^(2/3)
)/(a^4*n) - log(-a + (-b^3*cos(x)^n + a^3)^(1/3))/(a^4*n) - 3/((-b^3*cos(x)
^n + a^3)^(1/3)*a^3*n)
```

Giac [F]

$$\int \frac{\tan(x)}{(a^3 - b^3\cos^n(x))^{4/3}}\,dx = \int \frac{\tan\left(x\right)}{\left(-b^3\cos\left(x\right)^n + a^3\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(tan(x)/(-b^3*cos(x)^n + a^3)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(x)}{(a^3 - b^3\cos^n(x))^{4/3}}\,dx = \int \frac{\tan(x)}{\left(a^3 - b^3\cos\left(x\right)^n\right)^{4/3}}\,dx$$

```
[In] int(tan(x)/(a^3 - b^3*cos(x)^n)^(4/3),x)
```

```
[Out] int(tan(x)/(a^3 - b^3*cos(x)^n)^(4/3), x)
```

3.449 $\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx=\frac{\arctan\left(\frac{1-\sqrt[3]{1+2\cos^9(x)}}{\sqrt{3}\sqrt[6]{1+2\cos^9(x)}}\right)}{3\sqrt{3}}+\frac{1}{3}\text{arctanh}\left(\sqrt[6]{1+2\cos^9(x)}\right)-\frac{1}{9}\text{arctanh}\left(\sqrt{1+2\cos^9(x)}\right)-\frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6}$$

```
[Out] 1/3*arctanh((1+2*cos(x)^9)^(1/6))-1/9*arctanh((1+2*cos(x)^9)^(1/2))-2/15*(1
+2*cos(x)^9)^(5/6)+1/9*arctan(1/3*(1-(1+2*cos(x)^9)^(1/3))/(1+2*cos(x)^9)^(
1/6)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.71, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3309, 272, 52, 65, 302, 648, 632, 210, 642, 212}

$$\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx=\frac{\arctan\left(\frac{1-2\sqrt[6]{2\cos^9(x)+1}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\arctan\left(\frac{2\sqrt[6]{2\cos^9(x)+1}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{9}\text{arctanh}\left(\sqrt[6]{2\cos^9(x)+1}\right)-\frac{2}{15}\left(2\cos^9(x)+1\right)^{5/6}-\frac{1}{18}\log\left(\sqrt[3]{2\cos^9(x)+1}-\sqrt[6]{2\cos^9(x)+1}+1\right)+\frac{1}{18}\log\left(\sqrt[3]{2\cos^9(x)+1}+\sqrt[6]{2\cos^9(x)+1}+1\right)$$

```
[In] Int[(1 + 2*Cos[x]^9)^(5/6)*Tan[x],x]
```

```
[Out] ArcTan[(1 - 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]]/(3*Sqrt[3]) - ArcTan[(1 + 2*
(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]]/(3*Sqrt[3]) + (2*ArcTanh[(1 + 2*Cos[x]^9)^
(1/6)])/9 - (2*(1 + 2*Cos[x]^9)^(5/6))/15 - Log[1 - (1 + 2*Cos[x]^9)^(1/6)
+ (1 + 2*Cos[x]^9)^(1/3)]/18 + Log[1 + (1 + 2*Cos[x]^9)^(1/6) + (1 + 2*Cos[
x]^9)^(1/3)]/18
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
```

```
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(1+2x^9)^{5/6}}{x}\,dx, x, \cos(x)\right)\\
&= -\left(\frac{1}{9}\text{Subst}\left(\int \frac{(1+2x)^{5/6}}{x}\,dx, x, \cos^9(x)\right)\right)\\
&= -\frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6}-\frac{1}{9}\text{Subst}\left(\int \frac{1}{x\sqrt[6]{1+2x}}\,dx, x, \cos^9(x)\right)\\
&= -\frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6}-\frac{1}{3}\text{Subst}\left(\int \frac{x^4}{-\frac{1}{2}+\frac{x^6}{2}}\,dx, x, \sqrt[6]{1+2\cos^9(x)}\right)
\end{aligned}
$$

$$= -\frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6} + \frac{2}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[6]{1+2\cos^9(x)}\right)$$
$$+ \frac{2}{9}\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx, x, \sqrt[6]{1+2\cos^9(x)}\right)$$
$$+ \frac{2}{9}\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\,dx, x, \sqrt[6]{1+2\cos^9(x)}\right)$$
$$= \frac{2}{9}\text{arctanh}\left(\sqrt[6]{1+2\cos^9(x)}\right) - \frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6}$$
$$- \frac{1}{18}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[6]{1+2\cos^9(x)}\right)$$
$$+ \frac{1}{18}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \sqrt[6]{1+2\cos^9(x)}\right)$$
$$- \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[6]{1+2\cos^9(x)}\right)$$
$$- \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[6]{1+2\cos^9(x)}\right)$$
$$= \frac{2}{9}\text{arctanh}\left(\sqrt[6]{1+2\cos^9(x)}\right) - \frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6}$$
$$- \frac{1}{18}\log\left(1-\sqrt[6]{1+2\cos^9(x)}+\sqrt[3]{1+2\cos^9(x)}\right)$$
$$+ \frac{1}{18}\log\left(1+\sqrt[6]{1+2\cos^9(x)}+\sqrt[3]{1+2\cos^9(x)}\right)$$
$$+ \frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[6]{1+2\cos^9(x)}\right)$$
$$+ \frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[6]{1+2\cos^9(x)}\right)$$
$$= \frac{\arctan\left(\frac{1-2\sqrt[6]{1+2\cos^9(x)}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{1+2\sqrt[6]{1+2\cos^9(x)}}{\sqrt{3}}\right)}{3\sqrt{3}}$$
$$+ \frac{2}{9}\text{arctanh}\left(\sqrt[6]{1+2\cos^9(x)}\right) - \frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6}$$
$$- \frac{1}{18}\log\left(1-\sqrt[6]{1+2\cos^9(x)}+\sqrt[3]{1+2\cos^9(x)}\right)$$
$$+ \frac{1}{18}\log\left(1+\sqrt[6]{1+2\cos^9(x)}+\sqrt[3]{1+2\cos^9(x)}\right)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.62

$$\int (1+2\cos^9(x))^{5/6}\tan(x)\,dx=\frac{1}{90}\left(10\sqrt{3}\arctan\left(\frac{1-2\sqrt[6]{1+2\cos^9(x)}}{\sqrt{3}}\right)-10\sqrt{3}\arctan\left(\frac{1+2\sqrt[6]{1+2\cos^9(x)}}{\sqrt{3}}\right)+20\text{arctanh}\left(\sqrt[6]{1+2\cos^9(x)}\right)-12(1+2\cos^9(x))^{5/6}-5\log\left(1-\sqrt[6]{1+2\cos^9(x)}+\sqrt[3]{1+2\cos^9(x)}\right)+5\log\left(1+\sqrt[6]{1+2\cos^9(x)}+\sqrt[3]{1+2\cos^9(x)}\right)\right)$$

```
[In] Integrate[(1 + 2*Cos[x]^9)^(5/6)*Tan[x],x]
```

```
[Out] (10*Sqrt[3]*ArcTan[(1 - 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]] - 10*Sqrt[3]*Arc
Tan[(1 + 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]] + 20*ArcTanh[(1 + 2*Cos[x]^9)^(
1/6)] - 12*(1 + 2*Cos[x]^9)^(5/6) - 5*Log[1 - (1 + 2*Cos[x]^9)^(1/6) + (1 +
 2*Cos[x]^9)^(1/3)] + 5*Log[1 + (1 + 2*Cos[x]^9)^(1/6) + (1 + 2*Cos[x]^9)^(
1/3)])/90
```

Maple [F]

$$\int \left(1+2\left(\cos^9(x)\right)\right)^{\frac{5}{6}}\tan(x)\,dx$$

```
[In] int((1+2*cos(x)^9)^(5/6)*tan(x),x)
```

```
[Out] int((1+2*cos(x)^9)^(5/6)*tan(x),x)
```

Fricas [F(-1)]

Timed out.

$$\int (1+2\cos^9(x))^{5/6}\tan(x)\,dx=\text{Timed out}$$

```
[In] integrate((1+2*cos(x)^9)^(5/6)*tan(x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx=\text{Timed out}$$

```
[In] integrate((1+2*cos(x)**9)**(5/6)*tan(x),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(72) = 144$.

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.53

$$\begin{aligned}\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx &= -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos\left(x\right)^{9}+1\right)^{\frac{1}{6}}+1\right)\right)\\
&\quad-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos\left(x\right)^{9}+1\right)^{\frac{1}{6}}-1\right)\right)-\frac{2}{15}\left(2\cos\left(x\right)^{9}+1\right)^{\frac{5}{6}}\\
&\quad+\frac{1}{18}\log\left(\left(2\cos\left(x\right)^{9}+1\right)^{\frac{1}{3}}+\left(2\cos\left(x\right)^{9}+1\right)^{\frac{1}{6}}+1\right)\\
&\quad-\frac{1}{18}\log\left(\left(2\cos\left(x\right)^{9}+1\right)^{\frac{1}{3}}-\left(2\cos\left(x\right)^{9}+1\right)^{\frac{1}{6}}+1\right)\\
&\quad+\frac{1}{9}\log\left(\left(2\cos\left(x\right)^{9}+1\right)^{\frac{1}{6}}+1\right)-\frac{1}{9}\log\left(\left(2\cos\left(x\right)^{9}+1\right)^{\frac{1}{6}}-1\right)\end{aligned}$$

```
[In] integrate((1+2*cos(x)^9)^(5/6)*tan(x),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) + 1)) - 1/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) - 1)) - 2/15*(2*cos(x)^9 +
1)^(5/6) + 1/18*log((2*cos(x)^9 + 1)^(1/3) + (2*cos(x)^9 + 1)^(1/6) + 1) -
1/18*log((2*cos(x)^9 + 1)^(1/3) - (2*cos(x)^9 + 1)^(1/6) + 1) + 1/9*log((2*
cos(x)^9 + 1)^(1/6) + 1) - 1/9*log((2*cos(x)^9 + 1)^(1/6) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(72) = 144$.

Time = 0.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.54

$$\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right)\right) - \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}-1\right)\right) - \frac{2}{15}\left(2\cos(x)^9+1\right)^{\frac{5}{6}} + \frac{1}{18}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{3}}+\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right) - \frac{1}{18}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{3}}-\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right) + \frac{1}{9}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right) - \frac{1}{9}\log\left(\left|\left(2\cos(x)^9+1\right)^{\frac{1}{6}}-1\right|\right)$$

```
[In] integrate((1+2*cos(x)^9)^(5/6)*tan(x),x, algorithm="giac")
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) + 1)) - 1/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) - 1)) - 2/15*(2*cos(x)^9 +
1)^(5/6) + 1/18*log((2*cos(x)^9 + 1)^(1/3) + (2*cos(x)^9 + 1)^(1/6) + 1) -
1/18*log((2*cos(x)^9 + 1)^(1/3) - (2*cos(x)^9 + 1)^(1/6) + 1) + 1/9*log((2*
cos(x)^9 + 1)^(1/6) + 1) - 1/9*log(abs((2*cos(x)^9 + 1)^(1/6) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx = \int \tan(x)\,\left(2\cos(x)^9+1\right)^{5/6}\,dx$$

```
[In] int(tan(x)*(2*cos(x)^9 + 1)^(5/6),x)
[Out] int(tan(x)*(2*cos(x)^9 + 1)^(5/6), x)
```

3.450 $\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = \frac{4}{125\sqrt[3]{2-5\sin^3(x)}}+\frac{2}{125}\left(2-5\sin^3(x)\right)^{2/3}-\frac{1}{625}\left(2-5\sin^3(x)\right)^{5/3}$$

```
[Out] 4/125/(2-5*sin(x)^3)^(1/3)+2/125*(2-5*sin(x)^3)^(2/3)-1/625*(2-5*sin(x)^3)^
(5/3)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4419, 272, 45}

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = -\frac{1}{625}\left(2-5\sin^3(x)\right)^{5/3}+\frac{2}{125}\left(2-5\sin^3(x)\right)^{2/3}+\frac{4}{125\sqrt[3]{2-5\sin^3(x)}}$$

```
[In] Int[(Cos[x]*Sin[x]^8)/(2 - 5*Sin[x]^3)^(4/3),x]
```

```
[Out] 4/(125*(2 - 5*Sin[x]^3)^(1/3)) + (2*(2 - 5*Sin[x]^3)^(2/3))/125 - (2 - 5*Si
n[x]^3)^(5/3)/625
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^8}{(2-5x^3)^{4/3}}\,dx, x, \sin(x)\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(2-5x)^{4/3}}\,dx, x, \sin^3(x)\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{4}{25(2-5x)^{4/3}} - \frac{4}{25\sqrt[3]{2-5x}} + \frac{1}{25}(2-5x)^{2/3}\right)\,dx, x, \sin^3(x)\right)\\
&= \frac{4}{125\sqrt[3]{2-5\sin^3(x)}} + \frac{2}{125}\left(2-5\sin^3(x)\right)^{2/3} - \frac{1}{625}\left(2-5\sin^3(x)\right)^{5/3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

$$\int \frac{\cos(x)\sin^8(x)}{\left(2-5\sin^3(x)\right)^{4/3}}\,dx = \frac{36-30\sin^3(x)-25\sin^6(x)}{625\sqrt[3]{2-5\sin^3(x)}}$$

```
[In] Integrate[(Cos[x]*Sin[x]^8)/(2 - 5*Sin[x]^3)^(4/3),x]
[Out] (36 - 30*Sin[x]^3 - 25*Sin[x]^6)/(625*(2 - 5*Sin[x]^3)^(1/3))
```

Maple [F]

$$\int \frac{\cot(x)(\sin^9(x))}{(2-5(\sin^3(x)))^{\frac{4}{3}}}dx$$

[In] int(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x)

[Out] int(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = \frac{25\cos(x)^6 - 75\cos(x)^4 + 75\cos(x)^2 + 30\left(\cos(x)^2 - 1\right)\sin(x) + 11}{625\left(5\left(\cos(x)^2 - 1\right)\sin(x) + 2\right)^{\frac{1}{3}}}$$

[In] integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="fricas")

[Out] 1/625*(25*cos(x)^6 - 75*cos(x)^4 + 75*cos(x)^2 + 30*(cos(x)^2 - 1)*sin(x) + 11)/(5*(cos(x)^2 - 1)*sin(x) + 2)^(1/3)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = \text{Timed out}$$

[In] integrate(cot(x)*sin(x)**9/(2-5*sin(x)**3)**(4/3),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = -\frac{1}{625}\left(-5\sin(x)^3 + 2\right)^{\frac{5}{3}} + \frac{2}{125}\left(-5\sin(x)^3 + 2\right)^{\frac{2}{3}} + \frac{4}{125\left(-5\sin(x)^3 + 2\right)^{\frac{1}{3}}}$$

[In] integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="maxima")

[Out] -1/625*(-5*sin(x)^3 + 2)^(5/3) + 2/125*(-5*sin(x)^3 + 2)^(2/3) + 4/125/(-5*sin(x)^3 + 2)^(1/3)

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = -\frac{1}{625}\left(-5\,\sin\left(x\right)^3+2\right)^{\frac{5}{3}} + \frac{2}{125}\left(-5\,\sin\left(x\right)^3+2\right)^{\frac{2}{3}} + \frac{4}{125\left(-5\,\sin\left(x\right)^3+2\right)^{\frac{1}{3}}}$$

```
[In] integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="giac")
[Out] -1/625*(-5*sin(x)^3 + 2)^(5/3) + 2/125*(-5*sin(x)^3 + 2)^(2/3) + 4/125/(-5*
sin(x)^3 + 2)^(1/3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = \int \frac{\cot(x)\,\sin\left(x\right)^9}{\left(2-5\sin\left(x\right)^3\right)^{4/3}}\,dx$$

```
[In] int((cot(x)*sin(x)^9)/(2 - 5*sin(x)^3)^(4/3),x)
[Out] int((cot(x)*sin(x)^9)/(2 - 5*sin(x)^3)^(4/3), x)
```

3.451 $\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 20

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx=-\frac{3}{32}\left(1+\sqrt[3]{1-8\tan^2(x)}\right)^2$$

```
[Out] -3/32*(1+(1-8*tan(x)^2)^(1/3))^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {4427, 6818}

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx=-\frac{3}{32}\left(\sqrt[3]{1-8\tan^2(x)}+1\right)^2$$

```
[In] Int[(Sec[x]^2*Tan[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3),x
]
```

```
[Out] (-3*(1 + (1 - 8*Tan[x]^2)^(1/3))^2)/32
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
```

```
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{x\left(1+\sqrt[3]{1-8x^2}\right)}{(1-8x^2)^{2/3}}\,dx, x, \tan(x)\right) \\ &= -\frac{3}{32}\left(1+\sqrt[3]{1-8\tan^2(x)}\right)^2 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx = -\frac{3}{32}\left(1+\sqrt[3]{1-8\tan^2(x)}\right)^2$$

```
[In] Integrate[(Sec[x]^2*Tan[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(
2/3),x]
[Out] (-3*(1 + (1 - 8*Tan[x]^2)^(1/3))^2)/32
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$-\frac{3(1-8(\tan^2(x)))^{\frac{1}{3}}}{16}-\frac{3(1-8(\tan^2(x)))^{\frac{2}{3}}}{32}$	26
default	$-\frac{3(1-8(\tan^2(x)))^{\frac{1}{3}}}{16}-\frac{3(1-8(\tan^2(x)))^{\frac{2}{3}}}{32}$	26

```
[In] int(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,method=
_RETURNVERBOSE)
[Out] -3/16*(1-8*tan(x)^2)^(1/3)-3/32*(1-8*tan(x)^2)^(2/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx = -\frac{3}{32}\left(\frac{9\cos(x)^2-8}{\cos(x)^2}\right)^{\frac{2}{3}} - \frac{3}{16}\left(\frac{9\cos(x)^2-8}{\cos(x)^2}\right)^{\frac{1}{3}}$$

```
[In] integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="fricas")
[Out] -3/32*((9*cos(x)^2 - 8)/cos(x)^2)^(2/3) - 3/16*((9*cos(x)^2 - 8)/cos(x)^2)^
(1/3)
```

Sympy [F]

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx = \int \frac{\left(\sqrt[3]{1-8\tan^2(x)}+1\right)\tan(x)}{(1-8\tan^2(x))^{\frac{2}{3}}\cos^2(x)}\,dx$$

```
[In] integrate(tan(x)*(1+(1-8*tan(x)**2)**(1/3))/cos(x)**2/(1-8*tan(x)**2)**(2/3
),x)
[Out] Integral(((1 - 8*tan(x)**2)**(1/3) + 1)*tan(x)/((1 - 8*tan(x)**2)**(2/3)*co
s(x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 4.30

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx = -\frac{3\left(\frac{(9\sin(x)^2-1)(3\sin(x)-1)^{\frac{1}{3}}(\sin(x)+1)^{\frac{1}{3}}(\sin(x)-1)^{\frac{1}{3}}}{(3\sin(x)+1)^{\frac{1}{3}}} + \frac{2(9\sin(x)^2-1)(\sin(x)+1)^{\frac{2}{3}}(\sin(x)-1)^{\frac{2}{3}}}{(3\sin(x)+1)^{\frac{2}{3}}}\right)}{32(\sin(x)^2-1)(3\sin(x)-1)^{\frac{2}{3}}}$$

```
[In] integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="maxima")
[Out] -3/32*((9*sin(x)^2 - 1)*(3*sin(x) - 1)^(1/3)*(sin(x) + 1)^(1/3)*(sin(x) - 1
)^(1/3)/(3*sin(x) + 1)^(1/3) + 2*(9*sin(x)^2 - 1)*(sin(x) + 1)^(2/3)*(sin(x
) - 1)^(2/3)/(3*sin(x) + 1)^(2/3))/((sin(x)^2 - 1)*(3*sin(x) - 1)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx=-\frac{3}{32}\left(-8\,\tan\left(x\right)^2+1\right)^{\frac{2}{3}}-\frac{3}{16}\left(-8\,\tan\left(x\right)^2+1\right)^{\frac{1}{3}}$$

```
[In] integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="giac")
```

```
[Out] -3/32*(-8*tan(x)^2 + 1)^(2/3) - 3/16*(-8*tan(x)^2 + 1)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx= -\frac{3\left(\left(18\cos\left(x\right)^2-16\right)^{1/3}+2\left(2\cos\left(x\right)^2\right)^{1/3}\right)\,\left(18\cos\left(x\right)^2-16\right)^{1/3}}{32\left(2\cos\left(x\right)^2\right)^{2/3}}$$

```
[In] int((tan(x)*((1 - 8*tan(x)^2)^(1/3) + 1))/(cos(x)^2*(1 - 8*tan(x)^2)^(2/3))
,x)
```

```
[Out] -(3*((18*cos(x)^2 - 16)^(1/3) + 2*(2*cos(x)^2)^(1/3))*(18*cos(x)^2 - 16)^(1
/3))/(32*(2*cos(x)^2)^(2/3))
```

3.452 $\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 27

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = -\log(\tan(x)) + \frac{3}{2}\log\left(1-\sqrt[3]{1-8\tan^2(x)}\right)$$

```
[Out] -ln(tan(x))+3/2*ln(1-(1-8*tan(x)^2)^(1/3))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {4451, 6857, 528, 455, 59, 632, 210, 31, 57}

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = \frac{3}{2}\log\left(1-\sqrt[3]{9-8\sec^2(x)}\right)-\frac{1}{2}\log\left(1-\sec^2(x)\right)$$

```
[In] Int[(Csc[x]*Sec[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3),x]
[Out] -1/2*Log[1 - Sec[x]^2] + (3*Log[1 - (9 - 8*Sec[x]^2)^(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4451

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
```

```
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1+\sqrt[3]{9-\frac{8}{x^2}}}{\left(9-\frac{8}{x^2}\right)^{2/3} x\left(1-x^2\right)}\,dx, x, \cos(x)\right)\\
&= -\text{Subst}\left(\int \left(-\frac{1}{\left(9-\frac{8}{x^2}\right)^{2/3} x\left(-1+x^2\right)}-\frac{1}{\sqrt[3]{9-\frac{8}{x^2}}x\left(-1+x^2\right)}\right)dx, x, \cos(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{\left(9-\frac{8}{x^2}\right)^{2/3} x\left(-1+x^2\right)}\,dx, x, \cos(x)\right)\\
&\quad+\text{Subst}\left(\int \frac{1}{\sqrt[3]{9-\frac{8}{x^2}}x\left(-1+x^2\right)}\,dx, x, \cos(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{\left(9-\frac{8}{x^2}\right)^{2/3}\left(1-\frac{1}{x^2}\right)x^3}\,dx, x, \cos(x)\right)\\
&\quad+\text{Subst}\left(\int \frac{1}{\sqrt[3]{9-\frac{8}{x^2}}\left(1-\frac{1}{x^2}\right)x^3}\,dx, x, \cos(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(9-8x)^{2/3}(1-x)}\,dx, x, \sec^2(x)\right)\right)\\
&\quad-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{9-8x}(1-x)}\,dx, x, \sec^2(x)\right)\\
&= -\log(\tan(x))-2\left(\frac{3}{4}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{9-8\sec^2(x)}\right)\right)\\
&= \frac{3}{2}\log\left(1-\sqrt[3]{9-8\sec^2(x)}\right)-\log(\tan(x))
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.42 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = \frac{3\cos^2(x)\sqrt[3]{\sec^2(x)-9\tan^2(x)}\left(\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},\right.\right.}{}$$

[In] Integrate[(Csc[x]*Sec[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3),x]

[Out] (3*Cos[x]^2*(Sec[x]^2 - 9*Tan[x]^2)^(1/3)*(Hypergeometric2F1[2/3, 1, 5/3, (2*Cos[x]^2)/(-7 + 9*Cos[2*x])] + 2*Hypergeometric2F1[1/3, 1, 4/3, (2*Cos[x]^2)/(-7 + 9*Cos[2*x])]*(Sec[x]^2 - 9*Tan[x]^2)^(1/3)))/(-4 + 36*Sin[x]^2)

Maple [F]

$$\int \frac{\cot(x)\left(1+\left(1-8\left(\tan^2(x)\right)\right)^{\frac{1}{3}}\right)}{\cos(x)^2\left(1-8\left(\tan^2(x)\right)\right)^{\frac{2}{3}}}dx$$

[In] int(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x)

[Out] int(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(23) = 46.

Time = 0.96 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = -\frac{1}{2}\log\left(\frac{16\left(145\cos(x)^4-200\cos(x)^2+3\left(11\cos(x)^4-8\cos(x)^2\right)\left(\frac{9\cos(x)^2-8}{\cos(x)^2}\right)^{\frac{2}{3}}+3\left(19\cos(x)^4-16\right.\right.}{\cos(x)^4-2\cos(x)^2+1}\right.$$

[In] integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x, algorithm="fricas")

[Out] -1/2*log(16*(145*cos(x)^4 - 200*cos(x)^2 + 3*(11*cos(x)^4 - 8*cos(x)^2)*((9*cos(x)^2 - 8)/cos(x)^2)^(2/3) + 3*(19*cos(x)^4 - 16*cos(x)^2)*((9*cos(x)^2 - 8)/cos(x)^2)^(1/3) + 64)/(cos(x)^4 - 2*cos(x)^2 + 1))

Sympy [F]

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = \int \frac{\left(\sqrt[3]{1-8\tan^2{(x)}}+1\right)\cot{(x)}}{\left(1-8\tan^2{(x)}\right)^{\frac{2}{3}}\cos^2{(x)}}\,dx$$

```
[In] integrate(cot(x)*(1+(1-8*tan(x)**2)**(1/3))/cos(x)**2/(1-8*tan(x)**2)**(2/3
),x)
```

```
[Out] Integral(((1 - 8*tan(x)**2)**(1/3) + 1)*cot(x)/((1 - 8*tan(x)**2)**(2/3)*co
s(x)**2), x)
```

Maxima [F]

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = \int \frac{\left(\left(-8\tan{(x)}^2+1\right)^{\frac{1}{3}}+1\right)\cot{(x)}}{\left(-8\tan{(x)}^2+1\right)^{\frac{2}{3}}\cos{(x)}^2}\,dx$$

```
[In] integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="maxima")
```

```
[Out] integrate(((-8*tan(x)^2 + 1)^(1/3) + 1)*cot(x)/((-8*tan(x)^2 + 1)^(2/3)*cos
(x)^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = -\frac{1}{2}\log\left(\left(-8\tan{(x)}^2+1\right)^{\frac{2}{3}}+\left(-8\tan{(x)}^2+1\right)^{\frac{1}{3}}+1\right) + \log\left(\left|\left(-8\tan{(x)}^2+1\right)^{\frac{1}{3}}-1\right|\right)$$

```
[In] integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="giac")
```

```
[Out] -1/2*log((-8*tan(x)^2 + 1)^(2/3) + (-8*tan(x)^2 + 1)^(1/3) + 1) + log(abs((
-8*tan(x)^2 + 1)^(1/3) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx = \int \frac{\cot(x)\,\left(\left(1-8\tan(x)^2\right)^{1/3}+1\right)}{\cos(x)^2\,\left(1-8\tan(x)^2\right)^{2/3}}\,dx$$

```
[In] int((cot(x)*((1 - 8*tan(x)^2)^(1/3) + 1))/(cos(x)^2*(1 - 8*tan(x)^2)^(2/3))
,x)
[Out] int((cot(x)*((1 - 8*tan(x)^2)^(1/3) + 1))/(cos(x)^2*(1 - 8*tan(x)^2)^(2/3))
, x)
```

3.453 $\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$

Optimal result

Integrand size = 52, antiderivative size = 101

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx = -\frac{3\arctan\left(\frac{\sqrt[4]{-1+5\sin^2(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{-1+5\sin^2(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} + 2\sqrt[4]{-1+5\sin^2(x)}-\frac{\sqrt[4]{-1+5\sin^2(x)}}{2\left(2+\sqrt{-1+5\sin^2(x)}\right)}$$

```
[Out] 2*(-1+5*sin(x)^2)^(1/4)-3/2*arctan(1/2*(-1+5*sin(x)^2)^(1/4)*2^(1/2))*2^(1/
2)-1/4*arctanh(1/2*(-1+5*sin(x)^2)^(1/4)*2^(1/2))*2^(1/2)-1/2*(-1+5*sin(x)^
2)^(1/4)/(2+(-1+5*sin(x)^2)^(1/2))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.25, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4446, 6874, 6829, 348, 52, 65, 209, 481, 536, 213}

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$$
$$= -2\sqrt{2}\arctan\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)+\frac{\arctan\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{\sqrt{2}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}+2\sqrt[4]{4-5\cos^2(x)}-\frac{\sqrt[4]{4-5\cos^2(x)}}{2\left(\sqrt{4-5\cos^2(x)}+2\right)}$$

```
[In] Int[((5*Cos[x]^2 - Sqrt[-1 + 5*Sin[x]^2])*Tan[x])/((-1 + 5*Sin[x]^2)^(1/4)*
(2 + Sqrt[-1 + 5*Sin[x]^2])),x]
```

```
[Out] ArcTan[(4 - 5*Cos[x]^2)^(1/4)/Sqrt[2]]/Sqrt[2] - 2*Sqrt[2]*ArcTan[(4 - 5*Co
s[x]^2)^(1/4)/Sqrt[2]] - ArcTanh[(4 - 5*Cos[x]^2)^(1/4)/Sqrt[2]]/(2*Sqrt[2]
) + 2*(4 - 5*Cos[x]^2)^(1/4) - (4 - 5*Cos[x]^2)^(1/4)/(2*(2 + Sqrt[4 - 5*Co
s[x]^2]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4446

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rule 6829

```
Int[(u_.)*(v_)^(m_.)*((a_.) + (b_.)*(y_)^(n_))^(p_.), x_Symbol] :> Module[{
q, r}, Dist[q*r, Subst[Int[x^m*(a + b*x^n)^p, x], x, y], x] /;  !FalseQ[r =
 Divides[y^m, v^m, x]] &&  !FalseQ[q = DerivativeDivides[y, u, x]]] /; Free
Q[{a, b, m, n, p}, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{5x^2-\sqrt{4-5x^2}}{\sqrt[4]{4-5x^2}\left(2x+x\sqrt{4-5x^2}\right)}\,dx, x, \cos(x)\right)$$

$$= -\text{Subst}\left(\int \left(\frac{5x}{\sqrt[4]{4-5x^2}\left(2+\sqrt{4-5x^2}\right)} - \frac{\sqrt[4]{4-5x^2}}{x\left(2+\sqrt{4-5x^2}\right)}\right) dx, x, \cos(x)\right)$$

$$= -\left(5\text{Subst}\left(\int \frac{x}{\sqrt[4]{4-5x^2}\left(2+\sqrt{4-5x^2}\right)}\,dx, x, \cos(x)\right)\right) + \text{Subst}\left(\int \frac{\sqrt[4]{4-5x^2}}{x\left(2+\sqrt{4-5x^2}\right)}\,dx, x, \cos(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt[4]{4-5x}}{\left(2+\sqrt{4-5x}\right)x}\,dx, x, \cos^2(x)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(2+\sqrt{x}\right)\sqrt[4]{x}}\,dx, x, 4-5\cos^2(x)\right)$$

$$= 2\text{Subst}\left(\int \frac{x^4}{\left(-2+x^2\right)\left(2+x^2\right)^2}\,dx, x, \sqrt[4]{4-5\cos^2(x)}\right) + \text{Subst}\left(\int \frac{\sqrt{x}}{2+x}\,dx, x, \sqrt{4-5\cos^2(x)}\right)$$

$$= 2\sqrt[4]{4-5\cos^2(x)} - \frac{\sqrt[4]{4-5\cos^2(x)}}{2\left(2+\sqrt{4-5\cos^2(x)}\right)} + \frac{1}{4}\text{Subst}\left(\int \frac{-4+6x^2}{\left(-2+x^2\right)\left(2+x^2\right)}\,dx, x, \sqrt[4]{4-5\cos^2(x)}\right) - 2\text{Subst}\left(\int \frac{1}{\sqrt{x}(2+x)}\,dx, x, \sqrt{4-5\cos^2(x)}\right)$$

$$= 2\sqrt[4]{4-5\cos^2(x)} - \frac{\sqrt[4]{4-5\cos^2(x)}}{2\left(2+\sqrt{4-5\cos^2(x)}\right)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt[4]{4-5\cos^2(x)}\right) - 4\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt[4]{4-5\cos^2(x)}\right) + \text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt[4]{4-5\cos^2(x)}\right)$$

$$= \frac{\arctan\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{\sqrt{2}} - 2\sqrt{2}\arctan\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} + 2\sqrt[4]{4-5\cos^2(x)} - \frac{\sqrt[4]{4-5\cos^2(x)}}{2\left(2+\sqrt{4-5\cos^2(x)}\right)}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx = \frac{1}{4}\left(-6\sqrt{2}\arctan\left(\frac{\sqrt[4]{3-5\cos(2x)}}{2^{3/4}}\right) - \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt[4]{3-5\cos(2x)}}{2^{3/4}}\right) - 2\sqrt[4]{4-5\cos^2(x)}\left(-4+\frac{1}{2+\sqrt{4-5\cos^2(x)}}\right)\right)$$

```
[In] Integrate[((5*Cos[x]^2 - Sqrt[-1 + 5*Sin[x]^2])*Tan[x])/((-1 + 5*Sin[x]^2)^
(1/4)*(2 + Sqrt[-1 + 5*Sin[x]^2])),x]
```

```
[Out] (-6*Sqrt[2]*ArcTan[(3 - 5*Cos[2*x])^(1/4)/2^(3/4)] - Sqrt[2]*ArcTanh[(3 - 5
*Cos[2*x])^(1/4)/2^(3/4)] - 2*(4 - 5*Cos[x]^2)^(1/4)*(-4 + (2 + Sqrt[4 - 5*
Cos[x]^2])^(-1)))/4
```

Maple [F]

$$\int \frac{\left(5(\cos^2(x))-\sqrt{-1+5(\sin^2(x))}\right)\tan(x)}{\left(-1+5(\sin^2(x))\right)^{\frac{1}{4}}\left(2+\sqrt{-1+5(\sin^2(x))}\right)}dx$$

```
[In] int((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(2+(-1+
5*sin(x)^2)^(1/2)),x)
```

```
[Out] int((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(2+(-1+
5*sin(x)^2)^(1/2)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(81) = 162.

Time = 39.14 (sec) , antiderivative size = 461, normalized size of antiderivative = 4.56

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx = \frac{70\left(5\sqrt{2}\cos(x)^4-4\sqrt{2}\cos(x)^2\right)\arctan\left(-\frac{2\left(\left(5\sqrt{2}\cos(x)^2-4\sqrt{2}\right)\left(-5\cos(x)^2+4\right)^{\frac{3}{4}}-2\sqrt{2}\left(-5\cos(x)^2+4\right)^{\frac{5}{4}}\right)}{5\left(5\cos(x)^4-4\cos(x)^2\right)}\right)-50}{}$$

```
[In] integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(
2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/160*(70*(5*sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos(x)^2)*arctan(-2/5*((5*sqrt(2)
*cos(x)^2 - 4*sqrt(2))*(-5*cos(x)^2 + 4)^(3/4) - 2*sqrt(2)*(-5*cos(x)^2 + 4
)^(5/4))/(5*cos(x)^4 - 4*cos(x)^2)) - 50*(5*sqrt(2)*cos(x)^4 - 4*sqrt(2)*co
s(x)^2)*arctan(2/5*(sqrt(2)*(-5*cos(x)^2 + 4)^(3/4) + 2*sqrt(2)*(-5*cos(x)^
2 + 4)^(1/4))/cos(x)^2) + 35*(5*sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos(x)^2)*log(
-(125*cos(x)^6 - 1700*cos(x)^4 - 8*(15*sqrt(2)*cos(x)^2 - 16*sqrt(2))*(-5*c
os(x)^2 + 4)^(5/4) + 2560*cos(x)^2 + 4*(25*sqrt(2)*cos(x)^4 - 100*sqrt(2)*c
os(x)^2 + 64*sqrt(2))*(-5*cos(x)^2 + 4)^(3/4) - 16*(25*cos(x)^4 - 60*cos(x)
^2 + 32)*sqrt(-5*cos(x)^2 + 4) - 1024)/(5*cos(x)^6 - 4*cos(x)^4)) + 25*(5*s
qrt(2)*cos(x)^4 - 4*sqrt(2)*cos(x)^2)*log(-(25*cos(x)^4 - 320*cos(x)^2 - 4*
(5*sqrt(2)*cos(x)^2 - 16*sqrt(2))*(-5*cos(x)^2 + 4)^(3/4) - 16*(5*cos(x)^2
- 8)*sqrt(-5*cos(x)^2 + 4) - 8*(15*sqrt(2)*cos(x)^2 - 16*sqrt(2))*(-5*cos(x
)^2 + 4)^(1/4) + 256)/cos(x)^4) + 16*(5*cos(x)^2 - 2*(10*cos(x)^2 - 1)*sqrt
(-5*cos(x)^2 + 4) - 4)*(-5*cos(x)^2 + 4)^(3/4))/(5*cos(x)^4 - 4*cos(x)^2)
```

Sympy [F]

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx = \int \frac{\left(-\sqrt{5\sin^2(x)-1}+5\cos^2(x)\right)\tan(x)}{\left(\sqrt{5\sin^2(x)-1}+2\right)\sqrt[4]{5\sin^2(x)-1}}\,dx$$

```
[In] integrate((5*cos(x)**2-(-1+5*sin(x)**2)**(1/2))*tan(x)/(-1+5*sin(x)**2)**(1
/4)/(2+(-1+5*sin(x)**2)**(1/2)),x)
```

```
[Out] Integral((-sqrt(5*sin(x)**2 - 1) + 5*cos(x)**2)*tan(x)/((sqrt(5*sin(x)**2 -
 1) + 2)*(5*sin(x)**2 - 1)**(1/4)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$$
$$= -\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(5\sin(x)^2-1\right)^{\frac{1}{4}}\right)+\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-\left(5\sin(x)^2-1\right)^{\frac{1}{4}}}{\sqrt{2}+\left(5\sin(x)^2-1\right)^{\frac{1}{4}}}\right)$$
$$+2\left(5\sin(x)^2-1\right)^{\frac{1}{4}}-\frac{\left(5\sin(x)^2-1\right)^{\frac{1}{4}}}{2\left(\sqrt{5\sin(x)^2-1}+2\right)}$$

```
[In] integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(
2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(5*sin(x)^2 - 1)^(1/4)) + 1/8*sqrt(2)*log(-
(sqrt(2) - (5*sin(x)^2 - 1)^(1/4))/(sqrt(2) + (5*sin(x)^2 - 1)^(1/4))) + 2*
(5*sin(x)^2 - 1)^(1/4) - 1/2*(5*sin(x)^2 - 1)^(1/4)/(sqrt(5*sin(x)^2 - 1) +
 2)
```

Giac [F]

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx = \int \frac{\left(5\cos(x)^2-\sqrt{5\sin(x)^2-1}\right)\tan(x)}{\left(5\sin(x)^2-1\right)^{\frac{1}{4}}\left(\sqrt{5\sin(x)^2-1}+2\right)}\,dx$$

```
[In] integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(
2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate((5*cos(x)^2 - sqrt(5*sin(x)^2 - 1))*tan(x)/((5*sin(x)^2 - 1)^(1/4
)*(sqrt(5*sin(x)^2 - 1) + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$$
$$=\int \frac{\tan(x)\left(5\cos\left(x\right)^2-\sqrt{5\sin\left(x\right)^2-1}\right)}{\left(5\sin\left(x\right)^2-1\right)^{1/4}\left(\sqrt{5\sin\left(x\right)^2-1}+2\right)}\,dx$$

```
[In] int((tan(x)*(5*cos(x)^2 - (5*sin(x)^2 - 1)^(1/2)))/((5*sin(x)^2 - 1)^(1/4)*
((5*sin(x)^2 - 1)^(1/2) + 2)),x)
[Out] int((tan(x)*(5*cos(x)^2 - (5*sin(x)^2 - 1)^(1/2)))/((5*sin(x)^2 - 1)^(1/4)*
((5*sin(x)^2 - 1)^(1/2) + 2)), x)
```

3.454 $\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx = -\frac{3}{40} \cos^{\frac{5}{3}}(2x) - \frac{3}{64} \cos^{\frac{8}{3}}(2x)$$

```
[Out] -3/40*cos(2*x)^(5/3)-3/64*cos(2*x)^(8/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4442, 272, 45}

$$\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx = -\frac{3}{64} \cos^{\frac{8}{3}}(2x) - \frac{3}{40} \cos^{\frac{5}{3}}(2x)$$

```
[In] Int[Cos[x]^3*Cos[2*x]^(2/3)*Sin[x],x]
```

```
[Out] (-3*Cos[2*x]^(5/3))/40 - (3*Cos[2*x]^(8/3))/64
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4442

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int x^3\left(-1+2x^2\right)^{2/3}\,dx,x,\cos(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int x(-1+2x)^{2/3}\,dx,x,\cos^2(x)\right)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int\left(\frac{1}{2}(-1+2x)^{2/3}+\frac{1}{2}(-1+2x)^{5/3}\right)\,dx,x,\cos^2(x)\right)\right)\\ &= -\frac{3}{40}\left(-1+2\cos^2(x)\right)^{5/3}-\frac{3}{64}\left(-1+2\cos^2(x)\right)^{8/3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int\cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x)\,dx=-\frac{3}{320}\cos^{\frac{5}{3}}(2x)(8+5\cos(2x))$$

```
[In] Integrate[Cos[x]^3*Cos[2*x]^(2/3)*Sin[x],x]
[Out] (-3*Cos[2*x]^(5/3)*(8 + 5*Cos[2*x]))/320
```

Maple [F]

$$\int\left(\cos^4(x)\right)\left(\cos^{\frac{2}{3}}(2x)\right)\tan(x)\,dx$$

```
[In] int(cos(x)^4*cos(2*x)^(2/3)*tan(x),x)
[Out] int(cos(x)^4*cos(2*x)^(2/3)*tan(x),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx = -\frac{3}{320} \left(20 \cos (x)^4 - 4 \cos (x)^2 - 3\right) \left(2 \cos (x)^2 - 1\right)^{\frac{2}{3}}$$

```
[In] integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="fricas")
[Out] -3/320*(20*cos(x)^4 - 4*cos(x)^2 - 3)*(2*cos(x)^2 - 1)^(2/3)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx = \text{Timed out}$$

```
[In] integrate(cos(x)**4*cos(2*x)**(2/3)*tan(x),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx = \int \cos (2 x)^{\frac{2}{3}} \cos (x)^4 \tan (x) \, dx$$

```
[In] integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="maxima")
[Out] integrate(cos(2*x)^(2/3)*cos(x)^4*tan(x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx = -\frac{3}{64} \left(2 \cos (x)^2 - 1\right)^{\frac{8}{3}} - \frac{3}{40} \left(2 \cos (x)^2 - 1\right)^{\frac{5}{3}}$$

```
[In] integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="giac")
[Out] -3/64*(2*cos(x)^2 - 1)^(8/3) - 3/40*(2*cos(x)^2 - 1)^(5/3)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x)\,dx = \int \cos(2x)^{2/3}\cos(x)^4\tan(x)\,dx$$

```
[In] int(cos(2*x)^(2/3)*cos(x)^4*tan(x),x)
[Out] int(cos(2*x)^(2/3)*cos(x)^4*tan(x), x)
```

3.455 $\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 102

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \frac{\arctan\left(\frac{1-\sqrt{\cos(2x)}}{\sqrt{2}\sqrt[4]{\cos(2x)}}\right)}{\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{1+\sqrt{\cos(2x)}}{\sqrt{2}\sqrt[4]{\cos(2x)}}\right)}{\sqrt{2}} + \frac{7}{4}\sqrt[4]{\cos(2x)} - \frac{1}{5}\cos^{\frac{5}{4}}(2x) + \frac{1}{36}\cos^{\frac{9}{4}}(2x)$$

```
[Out] 7/4*cos(2*x)^(1/4)-1/5*cos(2*x)^(5/4)+1/36*cos(2*x)^(9/4)+1/2*arctan(1/2*(1
-cos(2*x)^(1/2))/cos(2*x)^(1/4)*2^(1/2))*2^(1/2)-1/2*arctanh(1/2*(1+cos(2*x
)^(1/2))/cos(2*x)^(1/4)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.51, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {4446, 457, 90, 65, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \frac{\arctan\left(1-\sqrt{2}\sqrt[4]{\cos(2x)}\right)}{\sqrt{2}} - \frac{\arctan\left(\sqrt{2}\sqrt[4]{\cos(2x)}+1\right)}{\sqrt{2}} + \frac{1}{36}\cos^{\frac{9}{4}}(2x) - \frac{1}{5}\cos^{\frac{5}{4}}(2x) + \frac{7}{4}\sqrt[4]{\cos(2x)} + \frac{\log\left(\sqrt{\cos(2x)}-\sqrt{2}\sqrt[4]{\cos(2x)}+1\right)}{2\sqrt{2}} - \frac{\log\left(\sqrt{\cos(2x)}+\sqrt{2}\sqrt[4]{\cos(2x)}+1\right)}{2\sqrt{2}}$$

```
[In] Int[(Sin[x]^6*Tan[x])/Cos[2*x]^(3/4),x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Cos[2*x]^(1/4)]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Cos[2*x]^(1
/4)]/Sqrt[2] + (7*Cos[2*x]^(1/4))/4 - Cos[2*x]^(5/4)/5 + Cos[2*x]^(9/4)/36
+ Log[1 - Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x]]]/(2*Sqrt[2]) - Log[1 + Sq
rt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x]]]/(2*Sqrt[2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4446

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(1-x^2)^3}{x\,(-1+2x^2)^{3/4}}\,dx, x, \cos(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^3}{x(-1+2x)^{3/4}}\,dx, x, \cos^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{7}{4(-1+2x)^{3/4}}+\frac{1}{x(-1+2x)^{3/4}}+\sqrt[4]{-1+2x}-\frac{1}{4}(-1+2x)^{5/4}\right)\,dx, x, \cos^2(x)\right)\right)\\
&= \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)}-\frac{1}{5}\left(-1+2\cos^2(x)\right)^{5/4}\\
&\quad+\frac{1}{36}\left(-1+2\cos^2(x)\right)^{9/4}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(-1+2x)^{3/4}}\,dx, x, \cos^2(x)\right)
\end{aligned}$$

$$= \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5}\left(-1+2\cos^2(x)\right)^{5/4} + \frac{1}{36}\left(-1+2\cos^2(x)\right)^{9/4} - \text{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{x^4}{2}}\,dx, x, \sqrt[4]{-1+2\cos^2(x)}\right)$$

$$= \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5}\left(-1+2\cos^2(x)\right)^{5/4} + \frac{1}{36}\left(-1+2\cos^2(x)\right)^{9/4} - \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{\frac{1}{2}+\frac{x^4}{2}}\,dx, x, \sqrt[4]{-1+2\cos^2(x)}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{\frac{1}{2}+\frac{x^4}{2}}\,dx, x, \sqrt[4]{-1+2\cos^2(x)}\right)$$

$$= \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5}\left(-1+2\cos^2(x)\right)^{5/4} + \frac{1}{36}\left(-1+2\cos^2(x)\right)^{9/4} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt[4]{-1+2\cos^2(x)}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt[4]{-1+2\cos^2(x)}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt[4]{-1+2\cos^2(x)}\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt[4]{-1+2\cos^2(x)}\right)}{2\sqrt{2}}$$

$$= \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5}\left(-1+2\cos^2(x)\right)^{5/4} + \frac{1}{36}\left(-1+2\cos^2(x)\right)^{9/4} + \frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+2\cos^2(x)}+\sqrt{-1+2\cos^2(x)}\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt[4]{-1+2\cos^2(x)}+\sqrt{-1+2\cos^2(x)}\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt[4]{-1+2\cos^2(x)}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt[4]{-1+2\cos^2(x)}\right)}{\sqrt{2}}$$

$$= \frac{\arctan\left(1-\sqrt{2}\sqrt[4]{-1+2\cos^2(x)}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt[4]{-1+2\cos^2(x)}\right)}{\sqrt{2}} + \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5}\left(-1+2\cos^2(x)\right)^{5/4} + \frac{1}{36}\left(-1+2\cos^2(x)\right)^{9/4} + \frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+2\cos^2(x)}+\sqrt{-1+2\cos^2(x)}\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt[4]{-1+2\cos^2(x)}+\sqrt{-1+2\cos^2(x)}\right)}{2\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.50

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \frac{1}{360}\Big(180\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt[4]{\cos(2x)}\right) - 180\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt[4]{\cos(2x)}\right) + 635\sqrt[4]{\cos(2x)} - 72\cos^{\frac{5}{4}}(2x) + 5\sqrt[4]{\cos(2x)}\cos(4x) + 90\sqrt{2}\log\left(1-\sqrt{2}\sqrt[4]{\cos(2x)}+\sqrt{\cos(2x)}\right) - 90\sqrt{2}\log\left(1+\sqrt{2}\sqrt[4]{\cos(2x)}+\sqrt{\cos(2x)}\right)\Big)$$

```
[In] Integrate[(Sin[x]^6*Tan[x])/Cos[2*x]^(3/4),x]
```

```
[Out] (180*Sqrt[2]*ArcTan[1 - Sqrt[2]*Cos[2*x]^(1/4)] - 180*Sqrt[2]*ArcTan[1 + Sq
rt[2]*Cos[2*x]^(1/4)] + 635*Cos[2*x]^(1/4) - 72*Cos[2*x]^(5/4) + 5*Cos[2*x]
^(1/4)*Cos[4*x] + 90*Sqrt[2]*Log[1 - Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x]
]] - 90*Sqrt[2]*Log[1 + Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x]]])/360
```

Maple [F]

$$\int \frac{\left(\sin^6(x)\right)\tan(x)}{\cos(2x)^{\frac{3}{4}}}dx$$

```
[In] int(sin(x)^6*tan(x)/cos(2*x)^(3/4),x)
```

```
[Out] int(sin(x)^6*tan(x)/cos(2*x)^(3/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(x)**6*tan(x)/cos(2*x)**(3/4),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \int \frac{\sin(x)^6\tan(x)}{\cos(2x)^{\frac{3}{4}}}\,dx$$

```
[In] integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="maxima")
[Out] integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx &= \frac{1}{36}\cos(2x)^{\frac{9}{4}} - \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\cos(2x)^{\frac{1}{4}}\right)\right)\\ &\quad - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\cos(2x)^{\frac{1}{4}}\right)\right)\\ &\quad - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\cos(2x)^{\frac{1}{4}}+\sqrt{\cos(2x)}+1\right)\\ &\quad + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\cos(2x)^{\frac{1}{4}}+\sqrt{\cos(2x)}+1\right)\\ &\quad - \frac{1}{5}\cos(2x)^{\frac{5}{4}} + \frac{7}{4}\cos(2x)^{\frac{1}{4}}\end{aligned}$$

```
[In] integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="giac")
```

```
[Out] 1/36*cos(2*x)^(9/4) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*cos(2*x)^
(1/4))) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*cos(2*x)^(1/4))) - 1
/4*sqrt(2)*log(sqrt(2)*cos(2*x)^(1/4) + sqrt(cos(2*x)) + 1) + 1/4*sqrt(2)*l
og(-sqrt(2)*cos(2*x)^(1/4) + sqrt(cos(2*x)) + 1) - 1/5*cos(2*x)^(5/4) + 7/4
*cos(2*x)^(1/4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \int \frac{\sin\left(x\right)^6\tan(x)}{\cos\left(2\,x\right)^{3/4}}\,dx$$

```
[In] int((sin(x)^6*tan(x))/cos(2*x)^(3/4),x)
```

```
[Out] int((sin(x)^6*tan(x))/cos(2*x)^(3/4), x)
```

3.456 $\int \sqrt{\tan(x)\tan(2x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \sqrt{\tan(x)\tan(2x)}\,dx = -\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right)$$

```
[Out] -arctanh(tan(x)/(tan(x)*tan(2*x))^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4482, 3859, 213}

$$\int \sqrt{\tan(x)\tan(2x)}\,dx = -\operatorname{arctanh}\left(\frac{\tan(2x)}{\sqrt{\sec(2x)-1}}\right)$$

```
[In] Int[Sqrt[Tan[x]*Tan[2*x]],x]
[Out] -ArcTanh[Tan[2*x]/Sqrt[-1 + Sec[2*x]]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d), Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sqrt{-1+\sec(2x)}\,dx \\ &= -\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -\frac{\tan(2x)}{\sqrt{-1+\sec(2x)}}\right) \\ &= -\text{arctanh}\left(\frac{\tan(2x)}{\sqrt{-1+\sec(2x)}}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 45 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.65

$$\int \sqrt{\tan(x)\tan(2x)}\,dx = -\frac{\text{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)\sqrt{\cos(2x)}\csc(x)\sqrt{\tan(x)\tan(2x)}}{\sqrt{2}}$$

```
[In] Integrate[Sqrt[Tan[x]*Tan[2*x]],x]
[Out] -((ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]]*Sqrt[Cos[2*x]]*Csc[x]*Sqrt[Tan[
x]*Tan[2*x]])/Sqrt[2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(15) = 30$.

Time = 0.69 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.59

method	result	size
default	$\frac{\sqrt{\frac{\sin^2(x)}{2\left(\cos^2(x)\right)-1}}\,\sin(x)\sqrt{\frac{2\left(\cos^2(x)\right)-1}{(\cos(x)+1)^2}}\,\text{arctanh}\left(\frac{\cos(x)\sqrt{2}}{(\cos(x)+1)\sqrt{\frac{2\left(\cos^2(x)\right)-1}{(\cos(x)+1)^2}}}\right)\sqrt{4}}{-2+2\cos(x)}$	78

```
[In] int((tan(x)*tan(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(sin(x)^2/(2*cos(x)^2-1))^(1/2)*sin(x)*((2*cos(x)^2-1)/(cos(x)+1)^2)^(1
/2)*arctanh(cos(x)/(cos(x)+1)/((2*cos(x)^2-1)/(cos(x)+1)^2)^(1/2)*2^(1/2))/
(-1+cos(x))*4^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.94

$$\int \sqrt{\tan(x)\tan(2x)}\,dx = \frac{1}{2}\log\left(-\frac{\tan{(x)}^3 - 2\sqrt{2}\left(\tan{(x)}^2 - 1\right)\sqrt{-\frac{\tan(x)^2}{\tan(x)^2-1}} - 3\tan{(x)}}{\tan{(x)}^3 + \tan{(x)}}\right)$$

```
[In] integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(tan(x)^3 - 2*sqrt(2)*(tan(x)^2 - 1)*sqrt(-tan(x)^2/(tan(x)^2 - 1)
) - 3*tan(x))/(tan(x)^3 + tan(x)))
```

Sympy [F]

$$\int \sqrt{\tan(x)\tan(2x)}\,dx = \int \sqrt{\tan{(x)}\tan{(2x)}}\,dx$$

```
[In] integrate((tan(x)*tan(2*x))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(x)*tan(2*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(15) = 30$.

Time = 0.30 (sec) , antiderivative size = 259, normalized size of antiderivative = 15.24

$$\int \sqrt{\tan(x)\tan(2x)}\,dx$$
$$= \frac{1}{4}\log\left(4\sqrt{\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1}\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)^2 + 4\sqrt{\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1}\sin\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)^2 + 8\left(\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right) + 4\right) - \frac{1}{4}\log\left(\cos(2x)^2+\sin(2x)^2 + \sqrt{\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1}\left(\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)^2+\sin\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)^2\right) + 2\left(\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1\right)^{\frac{1}{4}}\left(\cos(2x)\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)+\sin(2x)\sin\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)\right)\right)$$

```
[In] integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*log(4*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arctan2(si
n(4*x), cos(4*x) + 1))^2 + 4*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)
*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + 8*(cos(4*x)^2 + sin(4*x)^2 +
2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 4) - 1/4*l
og(cos(2*x)^2 + sin(2*x)^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)
*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sin(1/2*arctan2(sin(4*x), co
s(4*x) + 1))^2) + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(2
*x)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x)*sin(1/2*arctan2(sin
(4*x), cos(4*x) + 1))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(15) = 30.

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 5.00

$$\int \sqrt{\tan(x)\tan(2x)}\,dx$$
$$= \frac{1}{4}\sqrt{2}\left(\left(\sqrt{2}\log\left(\sqrt{2}+\sqrt{-\tan(x)^2+1}\right)-\sqrt{2}\log\left(\sqrt{2}-\sqrt{-\tan(x)^2+1}\right)\right)\operatorname{sgn}\left(\tan(x)^2-1\right)\operatorname{sgn}(\tan(x)) + \left(\sqrt{2}\log\left(\sqrt{2}+1\right)-\sqrt{2}\log\left(\sqrt{2}-1\right)\right)\operatorname{sgn}(\tan(x))\right)$$

```
[In] integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((sqrt(2)*log(sqrt(2) + sqrt(-tan(x)^2 + 1)) - sqrt(2)*log(sqrt
(2) - sqrt(-tan(x)^2 + 1)))*sgn(tan(x)^2 - 1)*sgn(tan(x)) + (sqrt(2)*log(sq
rt(2) + 1) - sqrt(2)*log(sqrt(2) - 1))*sgn(tan(x)))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\tan(x)\tan(2x)}\,dx = \int \sqrt{\tan\left(2\,x\right)\,\tan\left(x\right)}\,dx$$

```
[In] int((tan(2*x)*tan(x))^(1/2),x)
[Out] int((tan(2*x)*tan(x))^(1/2), x)
```

3.457 $\int \sqrt{\cot(2x)\tan(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = -\frac{\arcsin(\tan(x))}{\sqrt{2}} + \arctan\left(\frac{\sqrt{2}\tan(x)}{\sqrt{1-\tan^2(x)}}\right)$$

```
[Out] arctan(2^(1/2)*tan(x)/(1-tan(x)^2)^(1/2))-1/2*arcsin(tan(x))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {12, 399, 222, 385, 209}

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = \arctan\left(\frac{\sqrt{2}\tan(x)}{\sqrt{1-\tan^2(x)}}\right) - \frac{\arcsin(\tan(x))}{\sqrt{2}}$$

```
[In] Int[Sqrt[Cot[2*x]*Tan[x]],x]
[Out] -(ArcSin[Tan[x]]/Sqrt[2]) + ArcTan[(Sqrt[2]*Tan[x])/Sqrt[1 - Tan[x]^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{2}\,(1+x^2)}\,dx, x, \tan(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx, x, \tan(x)\right)}{\sqrt{2}}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tan(x)\right)}{\sqrt{2}} + \sqrt{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx, x, \tan(x)\right)\\
&= -\frac{\arcsin(\tan(x))}{\sqrt{2}} + \sqrt{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\tan(x)}{\sqrt{1-\tan^2(x)}}\right)\\
&= -\frac{\arcsin(\tan(x))}{\sqrt{2}} + \arctan\left(\frac{\sqrt{2}\tan(x)}{\sqrt{1-\tan^2(x)}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = \frac{\left(\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right) - \arctan\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right)\right)\cos(x)\sqrt{\cot(2x)\tan(x)}}{\sqrt{\cos(2x)}}$$

```
[In] Integrate[Sqrt[Cot[2*x]*Tan[x]],x]
[Out] ((Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - ArcTan[Sin[x]/Sqrt[Cos[2*x]]])*Cos[x]*Sq
rt[Cot[2*x]*Tan[x]])/Sqrt[Cos[2*x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(26) = 52.

Time = 6.69 (sec) , antiderivative size = 143, normalized size of antiderivative = 4.47

method	result
default	$\frac{\sqrt{2}\,\sqrt{2-(\sec^2(x))}\left(2\sqrt{2}\,\arctan\left(\frac{\sin(x)\sqrt{2}}{(\cos(x)+1)\sqrt{\frac{2\left(\cos^2(x)\right)-1}{(\cos(x)+1)^2}}}\right)-\arctan\left(\frac{1+2\sin(x)}{(\cos(x)+1)\sqrt{\frac{2\left(\cos^2(x)\right)-1}{(\cos(x)+1)^2}}}\right)-\arctan\left(\frac{2\sin(x)-1}{(\cos(x)+1)\sqrt{\frac{2\left(\cos^2(x)\right)-}{(\cos(x)+1)^2}}}\right.\right.}{4(\cos(x)+1)\sqrt{\frac{2\left(\cos^2(x)\right)-1}{(\cos(x)+1)^2}}}$

```
[In] int((cot(2*x)/cot(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*2^(1/2)*(2-sec(x)^2)^(1/2)*(2*2^(1/2)*arctan(sin(x)/(cos(x)+1)/((2*cos(
x)^2-1)/(cos(x)+1)^2)^(1/2)*2^(1/2))-arctan((1+2*sin(x))/(cos(x)+1)/((2*cos
(x)^2-1)/(cos(x)+1)^2)^(1/2))-arctan((2*sin(x)-1)/(cos(x)+1)/((2*cos(x)^2-1
)/(cos(x)+1)^2)^(1/2)))*cos(x)/(cos(x)+1)/((2*cos(x)^2-1)/(cos(x)+1)^2)^(1/
2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.59

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(3\,\cos(2\,x)^2+2\,\cos(2\,x)-1\right)\sqrt{\frac{\cos(2\,x)}{\cos(2\,x)+1}}}{4\,\cos(2\,x)\sin(2\,x)}\right) - \frac{1}{2}\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{2}\cos(2\,x)^2+\sqrt{2}\cos(2\,x)-\sqrt{2}\right)\sqrt{\frac{\cos(2\,x)}{\cos(2\,x)+1}}}{4\,\cos(2\,x)\sin(2\,x)}\right)$$

```
[In] integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*(3*cos(2*x)^2 + 2*cos(2*x) - 1)*sqrt(cos(2*x
)/(cos(2*x) + 1))/(cos(2*x)*sin(2*x))) - 1/2*arctan(1/4*sqrt(2)*(2*sqrt(2)*
cos(2*x)^2 + sqrt(2)*cos(2*x) - sqrt(2))*sqrt(cos(2*x)/(cos(2*x) + 1))/(cos
(2*x)*sin(2*x)))
```

Sympy [F]

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = \int \sqrt{\frac{\cot(2x)}{\cot(x)}}\,dx$$

```
[In] integrate((cot(2*x)/cot(x))**(1/2),x)
[Out] Integral(sqrt(cot(2*x)/cot(x)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 507, normalized size of antiderivative = 15.84

$$\int \sqrt{\cot(2x)\tan(x)}\,dx$$
$$= \frac{1}{4}\sqrt{2}\left(\sqrt{2}\arctan\left(\left(\cos(4\,x)^2+\sin(4\,x)^2+2\,\cos(4\,x)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(\sin(4\,x),\cos(4\,x)+1\right)\right)+\right.\right.$$

```
[In] integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*arctan2((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/
4)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x), (cos(4*x)^2 + sin(4
*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + co
s(2*x)) - 2*arctan2(((abs(2*e^(2*I*x) + 2)^4 + 16*cos(2*x)^4 + 16*sin(2*x)^
4 + 8*(cos(2*x)^2 - sin(2*x)^2 - 2*cos(2*x) + 1)*abs(2*e^(2*I*x) + 2)^2 - 6
4*cos(2*x)^3 + 32*(cos(2*x)^2 - 2*cos(2*x) + 1)*sin(2*x)^2 + 96*cos(2*x)^2
- 64*cos(2*x) + 16)^(1/4)*sin(1/2*arctan2(8*(cos(2*x) - 1)*sin(2*x)/abs(2*e
^(2*I*x) + 2)^2, (abs(2*e^(2*I*x) + 2)^2 + 4*cos(2*x)^2 - 4*sin(2*x)^2 - 8*
cos(2*x) + 4)/abs(2*e^(2*I*x) + 2)^2)) + 2*sin(2*x))/abs(2*e^(2*I*x) + 2),
((abs(2*e^(2*I*x) + 2)^4 + 16*cos(2*x)^4 + 16*sin(2*x)^4 + 8*(cos(2*x)^2 -
sin(2*x)^2 - 2*cos(2*x) + 1)*abs(2*e^(2*I*x) + 2)^2 - 64*cos(2*x)^3 + 32*(c
os(2*x)^2 - 2*cos(2*x) + 1)*sin(2*x)^2 + 96*cos(2*x)^2 - 64*cos(2*x) + 16)^
(1/4)*cos(1/2*arctan2(8*(cos(2*x) - 1)*sin(2*x)/abs(2*e^(2*I*x) + 2)^2, (ab
s(2*e^(2*I*x) + 2)^2 + 4*cos(2*x)^2 - 4*sin(2*x)^2 - 8*cos(2*x) + 4)/abs(2*
e^(2*I*x) + 2)^2)) + 2*cos(2*x) - 2)/abs(2*e^(2*I*x) + 2)))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 138, normalized size of antiderivative = 4.31

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = \frac{1}{2}\left(\pi - \sqrt{2}\arctan(-i) - \sqrt{2}\arctan\left(\sqrt{2}\right) - i\log\left(2\sqrt{2}+3\right)\right)\mathrm{sgn}(\sin(2x)) - \frac{\sqrt{2}\left(-i\sqrt{2}\log\left(2i\sqrt{2}+3i\right) - 2\arctan(-i)\right)\mathrm{sgn}(\cos(x)) + 2\left(\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(\frac{3\left(2\sqrt{2}\sqrt{-2\cos(x)^4+3\cos(\cdots}{4\cos(x)^2-3}\cdots\right.\right.\right.}{4\,\mathrm{sgn}(\cos(x))\,\mathrm{sgn}(\sin(2x))}$$

```
[In] integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(pi - sqrt(2)*arctan(-I) - sqrt(2)*arctan(sqrt(2)) - I*log(2*sqrt(2) +
3))*sgn(sin(2*x)) - 1/4*(sqrt(2)*(-I*sqrt(2)*log(2*I*sqrt(2) + 3*I) - 2*arc
tan(-I))*sgn(cos(x)) + 2*(sqrt(2)*arctan(1/4*sqrt(2)*(3*(2*sqrt(2)*sqrt(-2*
cos(x)^4 + 3*cos(x)^2 - 1) - 1)/(4*cos(x)^2 - 3) - 1)) + arcsin(4*cos(x)^2
- 3))*sgn(cos(x)))/(sgn(cos(x))*sgn(sin(2*x)))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = \int \sqrt{\frac{\cot(2\,x)}{\cot(x)}}\,dx$$

```
[In] int((cot(2*x)/cot(x))^(1/2),x)
[Out] int((cot(2*x)/cot(x))^(1/2), x)
```

3.458 $\int \frac{1}{x^5(5+x^2)} dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{1}{x^5(5+x^2)} dx = -\frac{1}{20x^4} + \frac{1}{50x^2} + \frac{\log(x)}{125} - \frac{1}{250}\log\left(5+x^2\right)$$

```
[Out] -1/20/x^4+1/50/x^2+1/125*ln(x)-1/250*ln(x^2+5)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {272, 46}

$$\int \frac{1}{x^5(5+x^2)} dx = -\frac{1}{20x^4} + \frac{1}{50x^2} - \frac{1}{250}\log\left(x^2+5\right) + \frac{\log(x)}{125}$$

```
[In] Int[1/(x^5*(5 + x^2)),x]
```

```
[Out] -1/20*1/x^4 + 1/(50*x^2) + Log[x]/125 - Log[5 + x^2]/250
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(5+x)}\,dx, x, x^2\right) \\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{5x^3} - \frac{1}{25x^2} + \frac{1}{125x} - \frac{1}{125(5+x)}\right)\,dx, x, x^2\right) \\ &= -\frac{1}{20x^4} + \frac{1}{50x^2} + \frac{\log(x)}{125} - \frac{1}{250}\log\left(5+x^2\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5\left(5+x^2\right)}\,dx = -\frac{1}{20x^4} + \frac{1}{50x^2} + \frac{\log(x)}{125} - \frac{1}{250}\log\left(5+x^2\right)$$

```
[In] Integrate[1/(x^5*(5 + x^2)),x]
[Out] -1/20*1/x^4 + 1/(50*x^2) + Log[x]/125 - Log[5 + x^2]/250
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{1}{20x^4}+\frac{1}{50x^2}+\frac{\ln(x)}{125}-\frac{\ln(x^2+5)}{250}$	24
norman	$\frac{-\frac{1}{20}+\frac{x^2}{50}}{x^4}+\frac{\ln(x)}{125}-\frac{\ln(x^2+5)}{250}$	25
risch	$\frac{-\frac{1}{20}+\frac{x^2}{50}}{x^4}+\frac{\ln(x)}{125}-\frac{\ln(x^2+5)}{250}$	25
meijerg	$-\frac{1}{20x^4}+\frac{1}{50x^2}+\frac{\ln(x)}{125}-\frac{\ln(5)}{250}-\frac{\ln\left(1+\frac{x^2}{5}\right)}{250}$	30
parallelrisch	$\frac{4x^4\ln(x)-2\ln(x^2+5)x^4-25+10x^2}{500x^4}$	31

```
[In] int(1/x^5/(x^2+5),x,method=_RETURNVERBOSE)
[Out] -1/20/x^4+1/50/x^2+1/125*ln(x)-1/250*ln(x^2+5)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^5(5+x^2)}\,dx = -\frac{2\,x^4\log\left(x^2+5\right)-4\,x^4\log\left(x\right)-10\,x^2+25}{500\,x^4}$$

```
[In] integrate(1/x^5/(x^2+5),x, algorithm="fricas")
[Out] -1/500*(2*x^4*log(x^2 + 5) - 4*x^4*log(x) - 10*x^2 + 25)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^5(5+x^2)}\,dx = \frac{\log\left(x\right)}{125}-\frac{\log\left(x^2+5\right)}{250}+\frac{2x^2-5}{100x^4}$$

```
[In] integrate(1/x**5/(x**2+5),x)
[Out] log(x)/125 - log(x**2 + 5)/250 + (2*x**2 - 5)/(100*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^5(5+x^2)}\,dx = \frac{2\,x^2-5}{100\,x^4}-\frac{1}{250}\,\log\left(x^2+5\right)+\frac{1}{250}\,\log\left(x^2\right)$$

```
[In] integrate(1/x^5/(x^2+5),x, algorithm="maxima")
[Out] 1/100*(2*x^2 - 5)/x^4 - 1/250*log(x^2 + 5) + 1/250*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^5(5+x^2)}\,dx = -\frac{3\,x^4-10\,x^2+25}{500\,x^4}-\frac{1}{250}\,\log\left(x^2+5\right)+\frac{1}{250}\,\log\left(x^2\right)$$

```
[In] integrate(1/x^5/(x^2+5),x, algorithm="giac")
[Out] -1/500*(3*x^4 - 10*x^2 + 25)/x^4 - 1/250*log(x^2 + 5) + 1/250*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^5 (5+x^2)} \, dx = \frac{\ln(x)}{125} - \frac{\ln(x^2+5)}{250} + \frac{\frac{x^2}{50} - \frac{1}{20}}{x^4}$$

```
[In] int(1/(x^5*(x^2 + 5)),x)
[Out] log(x)/125 - log(x^2 + 5)/250 + (x^2/50 - 1/20)/x^4
```

3.459 $\int \frac{1}{x^6(5+x^2)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{1}{x^6(5+x^2)}\,dx = -\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{\arctan\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

```
[Out] -1/25/x^5+1/75/x^3-1/125/x-1/625*arctan(1/5*x*5^(1/2))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {331, 209}

$$\int \frac{1}{x^6(5+x^2)}\,dx = -\frac{\arctan\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}-\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}$$

```
[In] Int[1/(x^6*(5 + x^2)),x]
```

```
[Out] -1/25*1/x^5 + 1/(75*x^3) - 1/(125*x) - ArcTan[x/Sqrt[5]]/(125*Sqrt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{25x^5} - \frac{1}{5}\int \frac{1}{x^4(5+x^2)}\,dx \\ &= -\frac{1}{25x^5} + \frac{1}{75x^3} + \frac{1}{25}\int \frac{1}{x^2(5+x^2)}\,dx \\ &= -\frac{1}{25x^5} + \frac{1}{75x^3} - \frac{1}{125x} - \frac{1}{125}\int \frac{1}{5+x^2}\,dx \\ &= -\frac{1}{25x^5} + \frac{1}{75x^3} - \frac{1}{125x} - \frac{\arctan\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6(5+x^2)}\,dx = -\frac{1}{25x^5} + \frac{1}{75x^3} - \frac{1}{125x} - \frac{\arctan\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

```
[In] Integrate[1/(x^6*(5 + x^2)),x]
[Out] -1/25*1/x^5 + 1/(75*x^3) - 1/(125*x) - ArcTan[x/Sqrt[5]]/(125*Sqrt[5])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{625}$	29
risch	$\frac{-\frac{1}{125}x^4+\frac{1}{75}x^2-\frac{1}{25}}{x^5}-\frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{625}$	30
meijerg	$\frac{\sqrt{5}\left(-\frac{2\sqrt{5}}{x}+\frac{10\sqrt{5}}{3x^3}-\frac{10\sqrt{5}}{x^5}-2\arctan\left(\frac{x\sqrt{5}}{5}\right)\right)}{1250}$	40

```
[In] int(1/x^6/(x^2+5),x,method=_RETURNVERBOSE)
[Out] -1/25/x^5+1/75/x^3-1/125/x-1/625*arctan(1/5*x*5^(1/2))*5^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^6(5+x^2)}\,dx = -\frac{3\sqrt{5}x^5\arctan\left(\frac{1}{5}\sqrt{5}x\right)+15\,x^4-25\,x^2+75}{1875\,x^5}$$

```
[In] integrate(1/x^6/(x^2+5),x, algorithm="fricas")
[Out] -1/1875*(3*sqrt(5)*x^5*arctan(1/5*sqrt(5)*x) + 15*x^4 - 25*x^2 + 75)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^6(5+x^2)}\,dx = -\frac{\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)}{625}+\frac{-3x^4+5x^2-15}{375x^5}$$

```
[In] integrate(1/x**6/(x**2+5),x)
[Out] -sqrt(5)*atan(sqrt(5)*x/5)/625 + (-3*x**4 + 5*x**2 - 15)/(375*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^6(5+x^2)}\,dx = -\frac{1}{625}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\frac{3\,x^4-5\,x^2+15}{375\,x^5}$$

```
[In] integrate(1/x^6/(x^2+5),x, algorithm="maxima")
[Out] -1/625*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/375*(3*x^4 - 5*x^2 + 15)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^6(5+x^2)}\,dx = -\frac{1}{625}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\frac{3\,x^4-5\,x^2+15}{375\,x^5}$$

```
[In] integrate(1/x^6/(x^2+5),x, algorithm="giac")
[Out] -1/625*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/375*(3*x^4 - 5*x^2 + 15)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^6 (5+x^2)} \, dx = -\frac{\frac{x^4}{125} - \frac{x^2}{75} + \frac{1}{25}}{x^5} - \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,x}{5}\right)}{625}$$

```
[In] int(1/(x^6*(x^2 + 5)),x)
[Out] - (x^4/125 - x^2/75 + 1/25)/x^5 - (5^(1/2)*atan((5^(1/2)*x)/5))/625
```

3.460 $\int \frac{1}{x\left(-4+x^2\right)^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 58

$$\int \frac{1}{x\left(-4+x^2\right)^4} dx=\frac{1}{24\left(4-x^2\right)^3}+\frac{1}{64\left(4-x^2\right)^2}+\frac{1}{128\left(4-x^2\right)}+\frac{\log(x)}{256}-\frac{1}{512}\log\left(4-x^2\right)$$

```
[Out] 1/24/(-x^2+4)^3+1/64/(-x^2+4)^2+1/128/(-x^2+4)+1/256*ln(x)-1/512*ln(-x^2+4)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {272, 46}

$$\int \frac{1}{x\left(-4+x^2\right)^4} dx=\frac{1}{128\left(4-x^2\right)}+\frac{1}{64\left(4-x^2\right)^2}+\frac{1}{24\left(4-x^2\right)^3}-\frac{1}{512}\log\left(4-x^2\right)+\frac{\log(x)}{256}$$

```
[In] Int[1/(x*(-4 + x^2)^4),x]
```

```
[Out] 1/(24*(4 - x^2)^3) + 1/(64*(4 - x^2)^2) + 1/(128*(4 - x^2)) + Log[x]/256 - 
Log[4 - x^2]/512
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-4+x)^4 x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{4(-4+x)^4} - \frac{1}{16(-4+x)^3} + \frac{1}{64(-4+x)^2} - \frac{1}{256(-4+x)} + \frac{1}{256x}\right) dx, x, x^2\right)\\ &= \frac{1}{24\,(4-x^2)^3} + \frac{1}{64\,(4-x^2)^2} + \frac{1}{128\,(4-x^2)} + \frac{\log(x)}{256} - \frac{1}{512}\log\left(4-x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{1}{x\,(-4+x^2)^4}\,dx = \frac{-\frac{4(88-30x^2+3x^4)}{(-4+x^2)^3} + 6\log(x) - 3\log\left(4-x^2\right)}{1536}$$

```
[In] Integrate[1/(x*(-4 + x^2)^4),x]
[Out] ((-4*(88 - 30*x^2 + 3*x^4))/(-4 + x^2)^3 + 6*Log[x] - 3*Log[4 - x^2])/1536
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.59

method	result
risch	$\frac{-\frac{1}{128}x^4+\frac{5}{64}x^2-\frac{11}{48}}{(x^2-4)^3} + \frac{\ln(x)}{256} - \frac{\ln(x^2-4)}{512}$
norman	$\frac{-\frac{1}{128}x^4+\frac{5}{64}x^2-\frac{11}{48}}{(x^2-4)^3} + \frac{\ln(x)}{256} - \frac{\ln(-2+x)}{512} - \frac{\ln(2+x)}{512}$
meijerg	$\frac{11}{3072} + \frac{\ln(x)}{256} - \frac{\ln(2)}{256} + \frac{i\pi}{512} + \frac{x^2\left(\frac{11}{16}x^4-\frac{27}{4}x^2+18\right)}{12288\left(1-\frac{x^2}{4}\right)^3} - \frac{\ln\left(1-\frac{x^2}{4}\right)}{512}$
default	$\frac{1}{1536(2+x)^3} + \frac{3}{2048(2+x)^2} + \frac{11}{4096(2+x)} - \frac{\ln(2+x)}{512} + \frac{\ln(x)}{256} - \frac{1}{1536(-2+x)^3} + \frac{3}{2048(-2+x)^2} - \frac{11}{4096(-2+x)} -$
parallelrisch	$\frac{6\ln(x)x^6-3\ln(-2+x)x^6-3\ln(2+x)x^6-352-72x^4\ln(x)+36\ln(-2+x)x^4+36\ln(2+x)x^4-12x^4+288x^2\ln(x)-144\ln(-2+x)x}{1536(x^2-4)^3}$

```
[In] int(1/x/(x^2-4)^4,x,method=_RETURNVERBOSE)
[Out] (-1/128*x^4+5/64*x^2-11/48)/(x^2-4)^3+1/256*ln(x)-1/512*ln(x^2-4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \frac{1}{x(-4+x^2)^4}\,dx = -\frac{12\,x^4-120\,x^2+3\,(x^6-12\,x^4+48\,x^2-64)\log(x^2-4)-6\,(x^6-12\,x^4+48\,x^2-64)\log(x)+352}{1536\,(x^6-12\,x^4+48\,x^2-64)}$$

```
[In] integrate(1/x/(x^2-4)^4,x, algorithm="fricas")
```

```
[Out] -1/1536*(12*x^4 - 120*x^2 + 3*(x^6 - 12*x^4 + 48*x^2 - 64)*log(x^2 - 4) - 6
*(x^6 - 12*x^4 + 48*x^2 - 64)*log(x) + 352)/(x^6 - 12*x^4 + 48*x^2 - 64)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{1}{x(-4+x^2)^4}\,dx = \frac{-3x^4+30x^2-88}{384x^6-4608x^4+18432x^2-24576}+\frac{\log(x)}{256}-\frac{\log(x^2-4)}{512}$$

```
[In] integrate(1/x/(x**2-4)**4,x)
```

```
[Out] (-3*x**4 + 30*x**2 - 88)/(384*x**6 - 4608*x**4 + 18432*x**2 - 24576) + log(
x)/256 - log(x**2 - 4)/512
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{1}{x(-4+x^2)^4}\,dx = -\frac{3\,x^4-30\,x^2+88}{384\,(x^6-12\,x^4+48\,x^2-64)}-\frac{1}{512}\,\log\left(x^2-4\right)+\frac{1}{512}\,\log\left(x^2\right)$$

```
[In] integrate(1/x/(x^2-4)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(3*x^4 - 30*x^2 + 88)/(x^6 - 12*x^4 + 48*x^2 - 64) - 1/512*log(x^2 -
 4) + 1/512*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

$$\int \frac{1}{x\left(-4+x^2\right)^4}\,dx = \frac{11\,x^6-156\,x^4+768\,x^2-1408}{3072\,\left(x^2-4\right)^3}+\frac{1}{512}\,\log\left(x^2\right)-\frac{1}{512}\,\log\left(\left|x^2-4\right|\right)$$

```
[In] integrate(1/x/(x^2-4)^4,x, algorithm="giac")
```

```
[Out] 1/3072*(11*x^6 - 156*x^4 + 768*x^2 - 1408)/(x^2 - 4)^3 + 1/512*log(x^2) - 1
/512*log(abs(x^2 - 4))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\left(-4+x^2\right)^4}\,dx = \frac{\ln\left(x\right)}{256}-\frac{\ln\left(x^2-4\right)}{512}-\frac{\frac{x^4}{128}-\frac{5\,x^2}{64}+\frac{11}{48}}{x^6-12\,x^4+48\,x^2-64}$$

```
[In] int(1/(x*(x^2 - 4)^4),x)
```

```
[Out] log(x)/256 - log(x^2 - 4)/512 - (x^4/128 - (5*x^2)/64 + 11/48)/(48*x^2 - 12
*x^4 + x^6 - 64)
```

3.461 $\int \frac{1}{x(-2+x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx=-\frac{1}{6\left(-2+x^2\right)^{3/2}}+\frac{1}{4\sqrt{-2+x^2}}+\frac{\arctan\left(\frac{\sqrt{-2+x^2}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -1/6/(x^2-2)^(3/2)+1/8*arctan(1/2*(x^2-2)^(1/2)*2^(1/2))*2^(1/2)+1/4/(x^2-2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {272, 53, 65, 209}

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{x^2-2}}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{1}{4\sqrt{x^2-2}}-\frac{1}{6\left(x^2-2\right)^{3/2}}$$

```
[In] Int[1/(x*(-2 + x^2)^(5/2)),x]
```

```
[Out] -1/6*1/(-2 + x^2)^(3/2) + 1/(4*Sqrt[-2 + x^2]) + ArcTan[Sqrt[-2 + x^2]/Sqrt
[2]]/(4*Sqrt[2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-2+x)^{5/2}x}\,dx, x, x^2\right) \\
&= -\frac{1}{6\left(-2+x^2\right)^{3/2}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{(-2+x)^{3/2}x}\,dx, x, x^2\right) \\
&= -\frac{1}{6\left(-2+x^2\right)^{3/2}} + \frac{1}{4\sqrt{-2+x^2}} + \frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}x}\,dx, x, x^2\right) \\
&= -\frac{1}{6\left(-2+x^2\right)^{3/2}} + \frac{1}{4\sqrt{-2+x^2}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt{-2+x^2}\right) \\
&= -\frac{1}{6\left(-2+x^2\right)^{3/2}} + \frac{1}{4\sqrt{-2+x^2}} + \frac{\arctan\left(\frac{\sqrt{-2+x^2}}{\sqrt{2}}\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx=\frac{-8+3x^2}{12\left(-2+x^2\right)^{3/2}}+\frac{\arctan\left(\frac{\sqrt{-2+x^2}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[In] Integrate[1/(x*(-2 + x^2)^(5/2)),x]
[Out] (-8 + 3*x^2)/(12*(-2 + x^2)^(3/2)) + ArcTan[Sqrt[-2 + x^2]/Sqrt[2]]/(4*Sqrt
[2])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{3x^2-8}{12(x^2-2)^{\frac{3}{2}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)}{8}$	35
default	$-\frac{1}{6(x^2-2)^{\frac{3}{2}}}+\frac{1}{4\sqrt{x^2-2}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)}{8}$	37
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{x^2-2}\,\sqrt{2}}{2}\right)\sqrt{2}\,(x^2-2)^{\frac{3}{2}}+2x^2-\frac{16}{3}}{8(x^2-2)^{\frac{3}{2}}}$	41
trager	$\frac{3x^2-8}{12(x^2-2)^{\frac{3}{2}}}-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{\sqrt{x^2-2}-\mathrm{RootOf}\left(\_Z^2+2\right)}{x}\right)}{8}$	47
meijerg	$\frac{\sqrt{2}\left(-\mathrm{signum}\left(-1+\frac{x^2}{2}\right)\right)^{\frac{5}{2}}\left(\frac{3\left(\frac{8}{3}-3\ln(2)+2\ln(x)+i\pi\right)\sqrt{\pi}}{4}-2\sqrt{\pi}+\frac{\sqrt{\pi}\,(-6x^2+16)}{8\left(-\frac{x^2}{2}+1\right)^{\frac{3}{2}}}-\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{x^2}{2}+1}}{2}\right)}{2}\right)}{12\sqrt{\pi}\,\mathrm{signum}\left(-1+\frac{x^2}{2}\right)^{\frac{5}{2}}}$	96

```
[In] int(1/x/(x^2-2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/12*(3*x^2-8)/(x^2-2)^(3/2)-1/8*2^(1/2)*arctan(1/(x^2-2)^(1/2)*2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx = \frac{3\sqrt{2}(x^4-4x^2+4)\arctan\left(-\frac{1}{2}\sqrt{2}x+\frac{1}{2}\sqrt{2}\sqrt{x^2-2}\right)+(3x^2-8)\sqrt{x^2-2}}{12\left(x^4-4x^2+4\right)}$$

```
[In] integrate(1/x/(x^2-2)^(5/2),x, algorithm="fricas")
[Out] 1/12*(3*sqrt(2)*(x^4 - 4*x^2 + 4)*arctan(-1/2*sqrt(2)*x + 1/2*sqrt(2)*sqrt(
x^2 - 2)) + (3*x^2 - 8)*sqrt(x^2 - 2))/(x^4 - 4*x^2 + 4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.81 (sec) , antiderivative size = 984, normalized size of antiderivative = 18.92

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx = \begin{cases} \frac{6ix^4\log(x)}{24\sqrt{2}x^4-96\sqrt{2}x^2+96\sqrt{2}} - \frac{3ix^4\log(x^2)}{24\sqrt{2}x^4-96\sqrt{2}x^2+96\sqrt{2}} - \frac{6x^4\operatorname{asin}\left(\frac{\sqrt{2}}{x}\right)}{24\sqrt{2}x^4-96\sqrt{2}x^2+96\sqrt{2}} + \frac{6\sqrt{2}x^2\sqrt{x^2-2}}{24\sqrt{2}x^4-96\sqrt{2}x^2+96\sqrt{2}} \\ -\frac{3ix^4\log(x^2)}{24\sqrt{2}x^4-96\sqrt{2}x^2+96\sqrt{2}} + \frac{6ix^4\log\left(\sqrt{1-\frac{x^2}{2}}+1\right)}{24\sqrt{2}x^4-96\sqrt{2}x^2+96\sqrt{2}} - \frac{3\pi x^4}{24\sqrt{2}x^4-96\sqrt{2}x^2+96\sqrt{2}} + \frac{3ix^4\log(2)}{24\sqrt{2}x^4-96\sqrt{2}x^2+9} \end{cases}$$

```
[In] integrate(1/x/(x**2-2)**(5/2),x)
[Out] Piecewise((6*I*x**4*log(x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2))
 - 3*I*x**4*log(x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 6*
x**4*asin(sqrt(2)/x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 6*s
qrt(2)*x**2*sqrt(x**2 - 2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2))
 - 24*I*x**2*log(x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 12*I
*x**2*log(x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 24*x**2*
asin(sqrt(2)/x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 16*sqrt(
2)*sqrt(x**2 - 2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 24*I*l
og(x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*I*log(x**2)/(24
*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 24*asin(sqrt(2)/x)/(24*sqrt
(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)), Abs(x**2) > 2), (-3*I*x**4*log(x*
*2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 6*I*x**4*log(sqrt(1
 - x**2/2) + 1)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 3*pi*x**4
/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 3*I*x**4*log(2)/(24*sqr
t(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 6*sqrt(2)*I*x**2*sqrt(2 - x**2)
/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 12*I*x**2*log(x**2)/(24
*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 24*I*x**2*log(sqrt(1 - x**2
/2) + 1)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 12*pi*x**2/(24*
sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*I*x**2*log(2)/(24*sqrt(2)
```

```
*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 16*sqrt(2)*I*sqrt(2 - x**2)/(24*sqr
t(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*I*log(x**2)/(24*sqrt(2)*x**4
 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 24*I*log(sqrt(1 - x**2/2) + 1)/(24*sqrt(
2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*pi/(24*sqrt(2)*x**4 - 96*sqrt(
2)*x**2 + 96*sqrt(2)) + 12*I*log(2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96
*sqrt(2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.63

$$\int \frac{1}{x\,(-2+x^2)^{5/2}}\,dx = -\frac{1}{8}\,\sqrt{2}\arcsin\left(\frac{\sqrt{2}}{|x|}\right) + \frac{1}{4\,\sqrt{x^2-2}} - \frac{1}{6\,(x^2-2)^{\frac{3}{2}}}$$

```
[In] integrate(1/x/(x^2-2)^(5/2),x, algorithm="maxima")
[Out] -1/8*sqrt(2)*arcsin(sqrt(2)/abs(x)) + 1/4/sqrt(x^2 - 2) - 1/6/(x^2 - 2)^(3/
2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int \frac{1}{x\,(-2+x^2)^{5/2}}\,dx = \frac{1}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\sqrt{x^2-2}\right) + \frac{3\,x^2-8}{12\,(x^2-2)^{\frac{3}{2}}}$$

```
[In] integrate(1/x/(x^2-2)^(5/2),x, algorithm="giac")
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 2)) + 1/12*(3*x^2 - 8)/(x^2 - 2)^
(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.65

$$\int \frac{1}{x\,(-2+x^2)^{5/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{x^2-2}}{2}\right)}{8} + \frac{\frac{x^2}{4}-\frac{2}{3}}{(x^2-2)^{3/2}}$$

```
[In] int(1/(x*(x^2 - 2)^(5/2)),x)
[Out] (2^(1/2)*atan((2^(1/2)*(x^2 - 2)^(1/2))/2))/8 + (x^2/4 - 2/3)/(x^2 - 2)^(3/
2)
```

3.462 $\int \frac{(-10+x^2)^{5/2}}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{(-10+x^2)^{5/2}}{x} dx = 100\sqrt{-10+x^2} - \frac{10}{3}(-10+x^2)^{3/2} + \frac{1}{5}(-10+x^2)^{5/2} - 100\sqrt{10}\arctan\left(\frac{\sqrt{-10+x^2}}{\sqrt{10}}\right)$$

```
[Out] -10/3*(x^2-10)^(3/2)+1/5*(x^2-10)^(5/2)-100*arctan(1/10*(x^2-10)^(1/2)*10^(
1/2))*10^(1/2)+100*(x^2-10)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {272, 52, 65, 209}

$$\int \frac{(-10+x^2)^{5/2}}{x} dx = -100\sqrt{10}\arctan\left(\frac{\sqrt{x^2-10}}{\sqrt{10}}\right) + \frac{1}{5}(x^2-10)^{5/2} - \frac{10}{3}(x^2-10)^{3/2} + 100\sqrt{x^2-10}$$

```
[In] Int[(-10 + x^2)^(5/2)/x,x]
```

```
[Out] 100*Sqrt[-10 + x^2] - (10*(-10 + x^2)^(3/2))/3 + (-10 + x^2)^(5/2)/5 - 100*
Sqrt[10]*ArcTan[Sqrt[-10 + x^2]/Sqrt[10]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(-10+x)^{5/2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{5}\left(-10+x^2\right)^{5/2} - 5\text{Subst}\left(\int \frac{(-10+x)^{3/2}}{x}\,dx, x, x^2\right)\\
&= -\frac{10}{3}\left(-10+x^2\right)^{3/2} + \frac{1}{5}\left(-10+x^2\right)^{5/2} + 50\text{Subst}\left(\int \frac{\sqrt{-10+x}}{x}\,dx, x, x^2\right)\\
&= 100\sqrt{-10+x^2} - \frac{10}{3}\left(-10+x^2\right)^{3/2} + \frac{1}{5}\left(-10+x^2\right)^{5/2} - 500\text{Subst}\left(\int \frac{1}{\sqrt{-10+x}x}\,dx, x, x^2\right)\\
&= 100\sqrt{-10+x^2} - \frac{10}{3}\left(-10+x^2\right)^{3/2}\\
&\quad + \frac{1}{5}\left(-10+x^2\right)^{5/2} - 1000\text{Subst}\left(\int \frac{1}{10+x^2}\,dx, x, \sqrt{-10+x^2}\right)\\
&= 100\sqrt{-10+x^2} - \frac{10}{3}\left(-10+x^2\right)^{3/2} + \frac{1}{5}\left(-10+x^2\right)^{5/2} - 100\sqrt{10}\arctan\left(\frac{\sqrt{-10+x^2}}{\sqrt{10}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx=\frac{1}{15}\sqrt{-10+x^2}\left(2300-110x^2+3x^4\right)-100\sqrt{10}\arctan\left(\frac{\sqrt{-10+x^2}}{\sqrt{10}}\right)$$

```
[In] Integrate[(-10 + x^2)^(5/2)/x,x]
[Out] (Sqrt[-10 + x^2]*(2300 - 110*x^2 + 3*x^4))/15 - 100*Sqrt[10]*ArcTan[Sqrt[-1
0 + x^2]/Sqrt[10]]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-100\arctan\left(\frac{\sqrt{x^2-10}\,\sqrt{10}}{10}\right)\sqrt{10}+\frac{\sqrt{x^2-10}\,(3x^4-110x^2+2300)}{15}$
default	$\frac{(x^2-10)^{\frac{5}{2}}}{5}-\frac{10(x^2-10)^{\frac{3}{2}}}{3}+100\sqrt{x^2-10}+100\sqrt{10}\,\arctan\left(\frac{\sqrt{10}}{\sqrt{x^2-10}}\right)$
trager	$\left(\frac{1}{5}x^4-\frac{22}{3}x^2+\frac{460}{3}\right)\sqrt{x^2-10}-100\,\mathrm{RootOf}\left(\_Z^2+10\right)\ln\left(\frac{\sqrt{x^2-10}+\mathrm{RootOf}\left(\_Z^2+10\right)}{x}\right)$
meijerg	$-\frac{375\sqrt{2}\sqrt{5}\,\mathrm{signum}\left(-1+\frac{x^2}{10}\right)^{\frac{5}{2}}\left(-\frac{8\left(\frac{46}{15}-3\ln(2)+2\ln(x)-\ln(5)+i\pi\right)\sqrt{\pi}}{15}+\frac{368\sqrt{\pi}}{225}-\frac{4\sqrt{\pi}\left(\frac{3}{25}x^4-\frac{22}{5}x^2+92\right)\sqrt{1-\frac{x^2}{10}}}{225}+\frac{16\sqrt{\pi}\,\ln\left(\cdots\right.}{\cdots}\right.}{4\sqrt{\pi}\left(-\mathrm{signum}\left(-1+\frac{x^2}{10}\right)\right)^{\frac{5}{2}}}$

```
[In] int((x^2-10)^(5/2)/x,x,method=_RETURNVERBOSE)
[Out] -100*arctan(1/10*(x^2-10)^(1/2)*10^(1/2))*10^(1/2)+1/15*(x^2-10)^(1/2)*(3*x
^4-110*x^2+2300)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = \frac{1}{15}\left(3\,x^4-110\,x^2+2300\right)\sqrt{x^2-10} - 200\,\sqrt{10}\arctan\left(-\frac{1}{10}\,\sqrt{10}x+\frac{1}{10}\,\sqrt{10}\sqrt{x^2-10}\right)$$

```
[In] integrate((x^2-10)^(5/2)/x,x, algorithm="fricas")
[Out] 1/15*(3*x^4 - 110*x^2 + 2300)*sqrt(x^2 - 10) - 200*sqrt(10)*arctan(-1/10*sq
rt(10)*x + 1/10*sqrt(10)*sqrt(x^2 - 10))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.43 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.70

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = \begin{cases} \frac{x^4\sqrt{x^2-10}}{5} - \frac{22x^2\sqrt{x^2-10}}{3} + \frac{460\sqrt{x^2-10}}{3} - 100\sqrt{10}i\log(x) + 50\sqrt{10}i\log(x^2) + 100\sqrt{10}\,\mathrm{as} \\ \frac{ix^4\sqrt{10-x^2}}{5} - \frac{22ix^2\sqrt{10-x^2}}{3} + \frac{460i\sqrt{10-x^2}}{3} + 50\sqrt{10}i\log(x^2) - 100\sqrt{10}i\log\left(\sqrt{1-\frac{x^2}{10}}+1\right. \end{cases}$$

```
[In] integrate((x**2-10)**(5/2)/x,x)
[Out] Piecewise((x**4*sqrt(x**2 - 10)/5 - 22*x**2*sqrt(x**2 - 10)/3 + 460*sqrt(x*
*2 - 10)/3 - 100*sqrt(10)*I*log(x) + 50*sqrt(10)*I*log(x**2) + 100*sqrt(10)
*asin(sqrt(10)/x), Abs(x**2) > 10), (I*x**4*sqrt(10 - x**2)/5 - 22*I*x**2*s
qrt(10 - x**2)/3 + 460*I*sqrt(10 - x**2)/3 + 50*sqrt(10)*I*log(x**2) - 100*
sqrt(10)*I*log(sqrt(1 - x**2/10) + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = \frac{1}{5}\left(x^2-10\right)^{\frac{5}{2}} - \frac{10}{3}\left(x^2-10\right)^{\frac{3}{2}} + 100\,\sqrt{10}\arcsin\left(\frac{\sqrt{10}}{|x|}\right) + 100\,\sqrt{x^2-10}$$

```
[In] integrate((x^2-10)^(5/2)/x,x, algorithm="maxima")
[Out] 1/5*(x^2 - 10)^(5/2) - 10/3*(x^2 - 10)^(3/2) + 100*sqrt(10)*arcsin(sqrt(10)
/abs(x)) + 100*sqrt(x^2 - 10)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = \frac{1}{5}\left(x^2-10\right)^{\frac{5}{2}} - \frac{10}{3}\left(x^2-10\right)^{\frac{3}{2}} - 100\sqrt{10}\arctan\left(\frac{1}{10}\sqrt{10}\sqrt{x^2-10}\right) + 100\sqrt{x^2-10}$$

```
[In] integrate((x^2-10)^(5/2)/x,x, algorithm="giac")
[Out] 1/5*(x^2 - 10)^(5/2) - 10/3*(x^2 - 10)^(3/2) - 100*sqrt(10)*arctan(1/10*sqr
t(10)*sqrt(x^2 - 10)) + 100*sqrt(x^2 - 10)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = 100\sqrt{x^2-10} - 100\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\sqrt{x^2-10}}{10}\right) - \frac{10\,(x^2-10)^{3/2}}{3} + \frac{(x^2-10)^{5/2}}{5}$$

```
[In] int((x^2 - 10)^(5/2)/x,x)
[Out] 100*(x^2 - 10)^(1/2) - 100*10^(1/2)*atan((10^(1/2)*(x^2 - 10)^(1/2))/10) -
(10*(x^2 - 10)^(3/2))/3 + (x^2 - 10)^(5/2)/5
```

3.463 $\int x^{1+2n} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int x^{1+2n} \, dx = \frac{x^{2(1+n)}}{2(1+n)}$$

```
[Out] 1/2*x^(2+2*n)/(1+n)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{30\}$

$$\int x^{1+2n} \, dx = \frac{x^{2(n+1)}}{2(n+1)}$$

```
[In] Int[x^(1 + 2*n),x]
```

```
[Out] x^(2*(1 + n))/(2*(1 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{x^{2(1+n)}}{2(1+n)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^{1+2n}\,dx=\frac{x^{2+2n}}{2+2n}$$

```
[In] Integrate[x^(1 + 2*n),x]
[Out] x^(2 + 2*n)/(2 + 2*n)
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x^{2+2n}}{2+2n}$	15
default	$\frac{x^{2+2n}}{2+2n}$	16
risch	$\frac{x\,x^{1+2n}}{2+2n}$	16
parallelrisch	$\frac{x\,x^{1+2n}}{2+2n}$	16
norman	$\frac{x\,\mathrm{e}^{(1+2n)\ln(x)}}{2+2n}$	18

```
[In] int(x^(1+2*n),x,method=_RETURNVERBOSE)
[Out] 1/2*x^(2+2*n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^{1+2n}\,dx=\frac{xx^{2\,n+1}}{2\,(n+1)}$$

```
[In] integrate(x^(1+2*n),x, algorithm="fricas")
[Out] 1/2*x*x^(2*n + 1)/(n + 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^{1+2n}\,dx = \begin{cases} \frac{x^{2n+2}}{2n+2} & \text{for } n \neq -1 \\ \log(x) & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(1+2*n),x)
[Out] Piecewise((x**(2*n + 2)/(2*n + 2), Ne(n, -1)), (log(x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{1+2n}\,dx = \frac{x^{2n+2}}{2(n+1)}$$

```
[In] integrate(x^(1+2*n),x, algorithm="maxima")
[Out] 1/2*x^(2*n + 2)/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{1+2n}\,dx = \frac{x^{2n+2}}{2(n+1)}$$

```
[In] integrate(x^(1+2*n),x, algorithm="giac")
[Out] 1/2*x^(2*n + 2)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x^{1+2n}\,dx = \begin{cases} \ln(x) & \text{if } n = -1 \\ \frac{x^{2n+2}}{2(n+1)} & \text{if } n \neq -1 \end{cases}$$

```
[In] int(x^(2*n + 1),x)
[Out] piecewise(n == -1, log(x), n ~= -1, x^(2*n + 2)/(2*(n + 1)))
```

3.464 $\int \frac{x^7}{(-5+x^2)^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{x^2}{2} - \frac{125}{4(5-x^2)^2} + \frac{75}{2(5-x^2)} + \frac{15}{2}\log(5-x^2)$$

```
[Out] 1/2*x^2-125/4/(-x^2+5)^2+75/2/(-x^2+5)+15/2*ln(-x^2+5)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {272, 45}

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{x^2}{2} + \frac{75}{2(5-x^2)} - \frac{125}{4(5-x^2)^2} + \frac{15}{2}\log(5-x^2)$$

```
[In] Int[x^7/(-5 + x^2)^3,x]
```

```
[Out] x^2/2 - 125/(4*(5 - x^2)^2) + 75/(2*(5 - x^2)) + (15*Log[5 - x^2])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(-5+x)^3}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(1+\frac{125}{(-5+x)^3}+\frac{75}{(-5+x)^2}+\frac{15}{-5+x}\right)\,dx, x, x^2\right)\\ &= \frac{x^2}{2}-\frac{125}{4\,(5-x^2)^2}+\frac{75}{2\,(5-x^2)}+\frac{15}{2}\log\left(5-x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{1}{4}\left(2x^2-\frac{125}{(-5+x^2)^2}-\frac{150}{-5+x^2}+30\log\left(-5+x^2\right)\right)$$

```
[In] Integrate[x^7/(-5 + x^2)^3,x]
[Out] (2*x^2 - 125/(-5 + x^2)^2 - 150/(-5 + x^2) + 30*Log[-5 + x^2])/4
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

method	result	size
norman	$\frac{-75x^2+\frac{1}{2}x^6+\frac{1125}{4}}{(x^2-5)^2}+\frac{15\ln(x^2-5)}{2}$	30
risch	$\frac{x^2}{2}+\frac{-\frac{75x^2}{2}+\frac{625}{4}}{(x^2-5)^2}+\frac{15\ln(x^2-5)}{2}$	30
default	$\frac{x^2}{2}+\frac{15\ln(x^2-5)}{2}-\frac{125}{4(x^2-5)^2}-\frac{75}{2(x^2-5)}$	33
meijerg	$\frac{x^2\left(\frac{4}{25}x^4-\frac{18}{5}x^2+12\right)}{8\left(-\frac{x^2}{5}+1\right)^2}+\frac{15\ln\left(-\frac{x^2}{5}+1\right)}{2}$	38
parallelrisch	$\frac{2x^6+30\ln(x^2-5)x^4+1125-300\ln(x^2-5)x^2-300x^2+750\ln(x^2-5)}{4(x^2-5)^2}$	52

```
[In] int(x^7/(x^2-5)^3,x,method=_RETURNVERBOSE)
[Out] (-75*x^2+1/2*x^6+1125/4)/(x^2-5)^2+15/2*ln(x^2-5)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{2\,x^6 - 20\,x^4 - 100\,x^2 + 30\,(x^4 - 10\,x^2 + 25)\log\left(x^2 - 5\right) + 625}{4\,(x^4 - 10\,x^2 + 25)}$$

```
[In] integrate(x^7/(x^2-5)^3,x, algorithm="fricas")
[Out] 1/4*(2*x^6 - 20*x^4 - 100*x^2 + 30*(x^4 - 10*x^2 + 25)*log(x^2 - 5) + 625)/
(x^4 - 10*x^2 + 25)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{x^2}{2} + \frac{625 - 150x^2}{4x^4 - 40x^2 + 100} + \frac{15\log\left(x^2 - 5\right)}{2}$$

```
[In] integrate(x**7/(x**2-5)**3,x)
[Out] x**2/2 + (625 - 150*x**2)/(4*x**4 - 40*x**2 + 100) + 15*log(x**2 - 5)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{1}{2}\,x^2 - \frac{25\,(6\,x^2 - 25)}{4\,(x^4 - 10\,x^2 + 25)} + \frac{15}{2}\,\log\left(x^2 - 5\right)$$

```
[In] integrate(x^7/(x^2-5)^3,x, algorithm="maxima")
[Out] 1/2*x^2 - 25/4*(6*x^2 - 25)/(x^4 - 10*x^2 + 25) + 15/2*log(x^2 - 5)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{1}{2}\,x^2 - \frac{5\,(9\,x^4-60\,x^2+100)}{4\,(x^2-5)^2} + \frac{15}{2}\,\log\left(\left|x^2-5\right|\right)$$

```
[In] integrate(x^7/(x^2-5)^3,x, algorithm="giac")
[Out] 1/2*x^2 - 5/4*(9*x^4 - 60*x^2 + 100)/(x^2 - 5)^2 + 15/2*log(abs(x^2 - 5))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{15\,\ln\left(x^2-5\right)}{2} - \frac{\frac{75\,x^2}{2}-\frac{625}{4}}{x^4-10\,x^2+25} + \frac{x^2}{2}$$

```
[In] int(x^7/(x^2 - 5)^3,x)
[Out] (15*log(x^2 - 5))/2 - ((75*x^2)/2 - 625/4)/(x^4 - 10*x^2 + 25) + x^2/2
```

3.465 $\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = \frac{1}{8(1-x^2)^4}+\frac{1}{3(1-x^2)^3}-\frac{3}{4(1-x^2)^2}$$

```
[Out] 1/8/(-x^2+1)^4+1/3/(-x^2+1)^3-3/4/(-x^2+1)^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1607, 457, 78}

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = -\frac{3}{4(1-x^2)^2}+\frac{1}{3(1-x^2)^3}+\frac{1}{8(1-x^2)^4}$$

```
[In] Int[(-4*x^3 + 3*x^5)/(-1 + x^2)^5,x]
```

```
[Out] 1/(8*(1 - x^2)^4) + 1/(3*(1 - x^2)^3) - 3/(4*(1 - x^2)^2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3(-4+3x^2)}{(-1+x^2)^5}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x(-4+3x)}{(-1+x)^5}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^5}+\frac{2}{(-1+x)^4}+\frac{3}{(-1+x)^3}\right)dx, x, x^2\right) \\
&= \frac{1}{8(1-x^2)^4}+\frac{1}{3(1-x^2)^3}-\frac{3}{4(1-x^2)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = \frac{-7+28x^2-18x^4}{24(-1+x^2)^4}$$

```
[In] Integrate[(-4*x^3 + 3*x^5)/(-1 + x^2)^5,x]
[Out] (-7 + 28*x^2 - 18*x^4)/(24*(-1 + x^2)^4)
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result
norman	$\frac{-\frac{3}{4}x^4+\frac{7}{6}x^2-\frac{7}{24}}{(x^2-1)^4}$
risch	$\frac{-\frac{3}{4}x^4+\frac{7}{6}x^2-\frac{7}{24}}{(x^2-1)^4}$
gosper	$-\frac{18x^4-28x^2+7}{24(x^2-1)^4}$
parallelrisch	$\frac{-18x^4+28x^2-7}{24(x^2-1)^4}$
meijerg	$-\frac{x^6(-x^2+4)}{8(-x^2+1)^4}+\frac{x^4(x^4-4x^2+6)}{6(-x^2+1)^4}$
default	$\frac{1}{128(-1+x)^4}-\frac{11}{192(-1+x)^3}-\frac{27}{256(-1+x)^2}+\frac{27}{256(-1+x)}+\frac{1}{128(1+x)^4}+\frac{11}{192(1+x)^3}-\frac{27}{256(1+x)^2}-\frac{27}{256(1+x)}$

```
[In] int((3*x^5-4*x^3)/(x^2-1)^5,x,method=_RETURNVERBOSE)
[Out] (-3/4*x^4+7/6*x^2-7/24)/(x^2-1)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = -\frac{18\,x^4-28\,x^2+7}{24\,(x^8-4\,x^6+6\,x^4-4\,x^2+1)}$$

```
[In] integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="fricas")
[Out] -1/24*(18*x^4 - 28*x^2 + 7)/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = \frac{-18x^4+28x^2-7}{24x^8-96x^6+144x^4-96x^2+24}$$

```
[In] integrate((3*x**5-4*x**3)/(x**2-1)**5,x)
[Out] (-18*x**4 + 28*x**2 - 7)/(24*x**8 - 96*x**6 + 144*x**4 - 96*x**2 + 24)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = -\frac{18\,x^4-28\,x^2+7}{24\,(x^8-4\,x^6+6\,x^4-4\,x^2+1)}$$

```
[In] integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="maxima")
[Out] -1/24*(18*x^4 - 28*x^2 + 7)/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = -\frac{18\,x^4-28\,x^2+7}{24\,(x^2-1)^4}$$

```
[In] integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="giac")
[Out] -1/24*(18*x^4 - 28*x^2 + 7)/(x^2 - 1)^4
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = -\frac{\frac{3\,x^4}{4}-\frac{7\,x^2}{6}+\frac{7}{24}}{x^8-4\,x^6+6\,x^4-4\,x^2+1}$$

```
[In] int(-(4*x^3 - 3*x^5)/(x^2 - 1)^5,x)
[Out] -((3*x^4)/4 - (7*x^2)/6 + 7/24)/(6*x^4 - 4*x^2 - 4*x^6 + x^8 + 1)
```

3.466 $\int x^3(1+x^2)^{9/14}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int x^3(1+x^2)^{9/14}\,dx=-\frac{7}{23}(1+x^2)^{23/14}+\frac{7}{37}(1+x^2)^{37/14}$$

```
[Out] -7/23*(x^2+1)^(23/14)+7/37*(x^2+1)^(37/14)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\int x^3(1+x^2)^{9/14}\,dx=\frac{7}{37}(x^2+1)^{37/14}-\frac{7}{23}(x^2+1)^{23/14}$$

```
[In] Int[x^3*(1 + x^2)^(9/14),x]
```

```
[Out] (-7*(1 + x^2)^(23/14))/23 + (7*(1 + x^2)^(37/14))/37
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x(1+x)^{9/14}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-(1+x)^{9/14}+(1+x)^{23/14}\right)\,dx, x, x^2\right)\\
&= -\frac{7}{23}\left(1+x^2\right)^{23/14}+\frac{7}{37}\left(1+x^2\right)^{37/14}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int x^3\left(1+x^2\right)^{9/14}\,dx=\frac{7}{851}\left(1+x^2\right)^{9/14}\left(-14+9x^2+23x^4\right)$$

```
[In] Integrate[x^3*(1 + x^2)^(9/14),x]
[Out] (7*(1 + x^2)^(9/14)*(-14 + 9*x^2 + 23*x^4))/851
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{7\left(x^2+1\right)^{\frac{23}{14}}\left(23x^2-14\right)}{851}$	17
meijerg	$\frac{x^4\,{}_2F_1\left(-\frac{9}{14},2;3;-x^2\right)}{4}$	17
pseudoelliptic	$\frac{7\left(x^2+1\right)^{\frac{23}{14}}\left(23x^2-14\right)}{851}$	17
trager	$\left(\frac{7}{37}x^4+\frac{63}{851}x^2-\frac{98}{851}\right)\left(x^2+1\right)^{\frac{9}{14}}$	21
risch	$\frac{7\left(x^2+1\right)^{\frac{9}{14}}\left(23x^4+9x^2-14\right)}{851}$	22

```
[In] int(x^3*(x^2+1)^(9/14),x,method=_RETURNVERBOSE)
[Out] 7/851*(x^2+1)^(23/14)*(23*x^2-14)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int x^3 \left(1+x^2\right)^{9/14} \, dx = \frac{7}{851} \left(23 \, x^{4} + 9 \, x^{2} - 14\right) \left(x^{2} + 1\right)^{\frac{9}{14}}$$

```
[In] integrate(x^3*(x^2+1)^(9/14),x, algorithm="fricas")
[Out] 7/851*(23*x^4 + 9*x^2 - 14)*(x^2 + 1)^(9/14)
```

Sympy [A] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int x^3 \left(1+x^2\right)^{9/14} \, dx = \frac{7x^4(x^2+1)^{\frac{9}{14}}}{37} + \frac{63x^2(x^2+1)^{\frac{9}{14}}}{851} - \frac{98(x^2+1)^{\frac{9}{14}}}{851}$$

```
[In] integrate(x**3*(x**2+1)**(9/14),x)
[Out] 7*x**4*(x**2 + 1)**(9/14)/37 + 63*x**2*(x**2 + 1)**(9/14)/851 - 98*(x**2 +
1)**(9/14)/851
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^3 \left(1+x^2\right)^{9/14} \, dx = \frac{7}{37} \left(x^{2} + 1\right)^{\frac{37}{14}} - \frac{7}{23} \left(x^{2} + 1\right)^{\frac{23}{14}}$$

```
[In] integrate(x^3*(x^2+1)^(9/14),x, algorithm="maxima")
[Out] 7/37*(x^2 + 1)^(37/14) - 7/23*(x^2 + 1)^(23/14)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^3 \left(1+x^2\right)^{9/14} \, dx = \frac{7}{37} \left(x^{2} + 1\right)^{\frac{37}{14}} - \frac{7}{23} \left(x^{2} + 1\right)^{\frac{23}{14}}$$

```
[In] integrate(x^3*(x^2+1)^(9/14),x, algorithm="giac")
[Out] 7/37*(x^2 + 1)^(37/14) - 7/23*(x^2 + 1)^(23/14)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^3 (1+x^2)^{9/14} \, dx = (x^2+1)^{9/14} \left(\frac{7 x^4}{37} + \frac{63 x^2}{851} - \frac{98}{851}\right)$$

```
[In] int(x^3*(x^2 + 1)^(9/14),x)
[Out] (x^2 + 1)^(9/14)*((63*x^2)/851 + (7*x^4)/37 - 98/851)
```

3.467 $\int \frac{x^5}{\left(-4+x^2\right)^{13/6}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{x^5}{\left(-4+x^2\right)^{13/6}}\,dx=-\frac{48}{7\left(-4+x^2\right)^{7/6}}-\frac{24}{\sqrt[6]{-4+x^2}}+\frac{3}{5}\left(-4+x^2\right)^{5/6}$$

```
[Out] -48/7/(x^2-4)^(7/6)-24/(x^2-4)^(1/6)+3/5*(x^2-4)^(5/6)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\int \frac{x^5}{\left(-4+x^2\right)^{13/6}}\,dx=\frac{3}{5}\left(x^2-4\right)^{5/6}-\frac{24}{\sqrt[6]{x^2-4}}-\frac{48}{7\left(x^2-4\right)^{7/6}}$$

```
[In] Int[x^5/(-4 + x^2)^(13/6),x]
```

```
[Out] -48/(7*(-4 + x^2)^(7/6)) - 24/(-4 + x^2)^(1/6) + (3*(-4 + x^2)^(5/6))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(-4+x)^{13/6}}\,dx, x, x^2\right) \\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{16}{(-4+x)^{13/6}} + \frac{8}{(-4+x)^{7/6}} + \frac{1}{\sqrt[6]{-4+x}}\right)\,dx, x, x^2\right) \\ &= -\frac{48}{7\left(-4+x^2\right)^{7/6}} - \frac{24}{\sqrt[6]{-4+x^2}} + \frac{3}{5}\left(-4+x^2\right)^{5/6} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{x^5}{\left(-4+x^2\right)^{13/6}}\,dx = \frac{3(1152-336x^2+7x^4)}{35\left(-4+x^2\right)^{7/6}}$$

```
[In] Integrate[x^5/(-4 + x^2)^(13/6),x]
[Out] (3*(1152 - 336*x^2 + 7*x^4))/(35*(-4 + x^2)^(7/6))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

method	result	size
pseudoelliptic	$\frac{\frac{3}{5}x^4-\frac{144}{5}x^2+\frac{3456}{35}}{(x^2-4)^{\frac{7}{6}}}$	20
trager	$\frac{\frac{3}{5}x^4-\frac{144}{5}x^2+\frac{3456}{35}}{(x^2-4)^{\frac{7}{6}}}$	22
risch	$\frac{\frac{3}{5}x^4-\frac{144}{5}x^2+\frac{3456}{35}}{(x^2-4)^{\frac{7}{6}}}$	22
gosper	$\frac{3(-2+x)(2+x)(7x^4-336x^2+1152)}{35(x^2-4)^{\frac{13}{6}}}$	28
meijerg	$\frac{2^{\frac{2}{3}}\left(-\operatorname{signum}\left(-1+\frac{x^2}{4}\right)\right)^{\frac{13}{6}} x^6\,{}_2F_1\left(\frac{13}{6},3;4;\frac{x^2}{4}\right)}{192\operatorname{signum}\left(-1+\frac{x^2}{4}\right)^{\frac{13}{6}}}$	42

```
[In] int(x^5/(x^2-4)^(13/6),x,method=_RETURNVERBOSE)
[Out] 3/5*(x^4-48*x^2+1152/7)/(x^2-4)^(7/6)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx = \frac{3\,(7\,x^4-336\,x^2+1152)(x^2-4)^{\frac{5}{6}}}{35\,(x^4-8\,x^2+16)}$$

```
[In] integrate(x^5/(x^2-4)^(13/6),x, algorithm="fricas")
[Out] 3/35*(7*x^4 - 336*x^2 + 1152)*(x^2 - 4)^(5/6)/(x^4 - 8*x^2 + 16)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(32) = 64.

Time = 0.79 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx = \frac{21x^4}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}} - \frac{1008x^2}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}} + \frac{3456}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}}$$

```
[In] integrate(x**5/(x**2-4)**(13/6),x)
[Out] 21*x**4/(35*x**2*(x**2 - 4)**(1/6) - 140*(x**2 - 4)**(1/6)) - 1008*x**2/(35
*x**2*(x**2 - 4)**(1/6) - 140*(x**2 - 4)**(1/6)) + 3456/(35*x**2*(x**2 - 4)
**(1/6) - 140*(x**2 - 4)**(1/6))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx = \frac{3}{5}\,(x^2-4)^{\frac{5}{6}} - \frac{24}{(x^2-4)^{\frac{1}{6}}} - \frac{48}{7\,(x^2-4)^{\frac{7}{6}}}$$

```
[In] integrate(x^5/(x^2-4)^(13/6),x, algorithm="maxima")
[Out] 3/5*(x^2 - 4)^(5/6) - 24/(x^2 - 4)^(1/6) - 48/7/(x^2 - 4)^(7/6)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx = \frac{3}{5}\left(x^2-4\right)^{\frac{5}{6}} - \frac{24\left(7x^2-26\right)}{7\left(x^2-4\right)^{\frac{7}{6}}}$$

```
[In] integrate(x^5/(x^2-4)^(13/6),x, algorithm="giac")
[Out] 3/5*(x^2 - 4)^(5/6) - 24/7*(7*x^2 - 26)/(x^2 - 4)^(7/6)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

$$\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx = \frac{3\left(7x^4-336x^2+1152\right)}{35\left(x^2-4\right)^{7/6}}$$

```
[In] int(x^5/(x^2 - 4)^(13/6),x)
[Out] (3*(7*x^4 - 336*x^2 + 1152))/(35*(x^2 - 4)^(7/6))
```

3.468 $\int \frac{1}{(1+2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{x}{3(1+2x^2)^{3/2}} + \frac{2x}{3\sqrt{1+2x^2}}$$

```
[Out] 1/3*x/(2*x^2+1)^(3/2)+2/3*x/(2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {198, 197}

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{2x}{3\sqrt{2x^2+1}} + \frac{x}{3(2x^2+1)^{3/2}}$$

```
[In] Int[(1 + 2*x^2)^(-5/2),x]
```

```
[Out] x/(3*(1 + 2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 + 2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
```

```
0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{x}{3\left(1+2x^2\right)^{3/2}}+\frac{2}{3}\int\frac{1}{\left(1+2x^2\right)^{3/2}}\,dx$$
$$= \frac{x}{3\left(1+2x^2\right)^{3/2}}+\frac{2x}{3\sqrt{1+2x^2}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int\frac{1}{\left(1+2x^2\right)^{5/2}}\,dx=\frac{3x+4x^3}{3\left(1+2x^2\right)^{3/2}}$$

```
[In] Integrate[(1 + 2*x^2)^(-5/2),x]
[Out] (3*x + 4*x^3)/(3*(1 + 2*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{x\left(4x^{2}+3\right)}{3\left(2x^{2}+1\right)^{\frac{3}{2}}}$	20
trager	$\frac{x\left(4x^{2}+3\right)}{3\left(2x^{2}+1\right)^{\frac{3}{2}}}$	20
meijerg	$\frac{x\left(4x^{2}+3\right)}{3\left(2x^{2}+1\right)^{\frac{3}{2}}}$	20
risch	$\frac{x\left(4x^{2}+3\right)}{3\left(2x^{2}+1\right)^{\frac{3}{2}}}$	20
pseudoelliptic	$\frac{4x^{3}+3x}{3\left(2x^{2}+1\right)^{\frac{3}{2}}}$	21
default	$\frac{x}{3\left(2x^{2}+1\right)^{\frac{3}{2}}}+\frac{2x}{3\sqrt{2x^{2}+1}}$	26

```
[In] int(1/(2*x^2+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x*(4*x^2+3)/(2*x^2+1)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{(4\,x^3+3\,x)\sqrt{2\,x^2+1}}{3\,(4\,x^4+4\,x^2+1)}$$

```
[In] integrate(1/(2*x^2+1)^(5/2),x, algorithm="fricas")
[Out] 1/3*(4*x^3 + 3*x)*sqrt(2*x^2 + 1)/(4*x^4 + 4*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(27) = 54.

Time = 0.86 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{4x^3}{6x^2\sqrt{2x^2+1}+3\sqrt{2x^2+1}} + \frac{3x}{6x^2\sqrt{2x^2+1}+3\sqrt{2x^2+1}}$$

```
[In] integrate(1/(2*x**2+1)**(5/2),x)
[Out] 4*x**3/(6*x**2*sqrt(2*x**2 + 1) + 3*sqrt(2*x**2 + 1)) + 3*x/(6*x**2*sqrt(2*
x**2 + 1) + 3*sqrt(2*x**2 + 1))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{2\,x}{3\,\sqrt{2\,x^2+1}} + \frac{x}{3\,(2\,x^2+1)^{\frac{3}{2}}}$$

```
[In] integrate(1/(2*x^2+1)^(5/2),x, algorithm="maxima")
[Out] 2/3*x/sqrt(2*x^2 + 1) + 1/3*x/(2*x^2 + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.58

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{(4\,x^2+3)x}{3\,(2\,x^2+1)^{\frac{3}{2}}}$$

```
[In] integrate(1/(2*x^2+1)^(5/2),x, algorithm="giac")
[Out] 1/3*(4*x^2 + 3)*x/(2*x^2 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.00

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{\sqrt{x^2+\frac{1}{2}}\,1\mathrm{i}}{24\,\left(-x^2+1\mathrm{i}\,\sqrt{2}\,x+\frac{1}{2}\right)} + \frac{\sqrt{x^2+\frac{1}{2}}\,1\mathrm{i}}{24\,\left(x^2+1\mathrm{i}\,\sqrt{2}\,x-\frac{1}{2}\right)} + \frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{6\,\left(x-\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)} + \frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{6\,\left(x+\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)}$$

```
[In] int(1/(2*x^2 + 1)^(5/2),x)
[Out] ((x^2 + 1/2)^(1/2)*1i)/(24*(2^(1/2)*x*1i - x^2 + 1/2)) + ((x^2 + 1/2)^(1/2)
*1i)/(24*(2^(1/2)*x*1i + x^2 - 1/2)) + (2^(1/2)*(x^2 + 1/2)^(1/2))/(6*(x -
(2^(1/2)*1i)/2)) + (2^(1/2)*(x^2 + 1/2)^(1/2))/(6*(x + (2^(1/2)*1i)/2))
```

3.469 $\int \frac{1}{(-1-2x+x^2)^{5/2}} dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{(-1-2x+x^2)^{5/2}} dx = \frac{1-x}{6(-1-2x+x^2)^{3/2}} - \frac{1-x}{6\sqrt{-1-2x+x^2}}$$

```
[Out] 1/6*(1-x)/(x^2-2*x-1)^(3/2)+1/6*(-1+x)/(x^2-2*x-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {628, 627}

$$\int \frac{1}{(-1-2x+x^2)^{5/2}} dx = \frac{1-x}{6(x^2-2x-1)^{3/2}} - \frac{1-x}{6\sqrt{x^2-2x-1}}$$

```
[In] Int[(-1 - 2*x + x^2)^(-5/2),x]
```

```
[Out] (1 - x)/(6*(-1 - 2*x + x^2)^(3/2)) - (1 - x)/(6*Sqrt[-1 - 2*x + x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1-x}{6\left(-1-2x+x^2\right)^{3/2}}-\frac{1}{3}\int\frac{1}{\left(-1-2x+x^2\right)^{3/2}}\,dx\\ &= \frac{1-x}{6\left(-1-2x+x^2\right)^{3/2}}-\frac{1-x}{6\sqrt{-1-2x+x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.60

$$\int\frac{1}{\left(-1-2x+x^2\right)^{5/2}}\,dx=\frac{2-3x^2+x^3}{6\left(-1-2x+x^2\right)^{3/2}}$$

```
[In] Integrate[(-1 - 2*x + x^2)^(-5/2),x]
[Out] (2 - 3*x^2 + x^3)/(6*(-1 - 2*x + x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.53

method	result	size
gosper	$\frac{x^3-3x^2+2}{6(x^2-2x-1)^{\frac{3}{2}}}$	23
trager	$\frac{x^3-3x^2+2}{6(x^2-2x-1)^{\frac{3}{2}}}$	23
risch	$\frac{x^3-3x^2+2}{6(x^2-2x-1)^{\frac{3}{2}}}$	23
default	$-\frac{-2+2x}{12(x^2-2x-1)^{\frac{3}{2}}}+\frac{-2+2x}{12\sqrt{x^2-2x-1}}$	36

```
[In] int(1/(x^2-2*x-1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/6*(x^3-3*x^2+2)/(x^2-2*x-1)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx = \frac{x^4-4\,x^3+2\,x^2+(x^3-3\,x^2+2)\sqrt{x^2-2\,x-1}+4\,x+1}{6\,(x^4-4\,x^3+2\,x^2+4\,x+1)}$$

```
[In] integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="fricas")
[Out] 1/6*(x^4 - 4*x^3 + 2*x^2 + (x^3 - 3*x^2 + 2)*sqrt(x^2 - 2*x - 1) + 4*x + 1)
/(x^4 - 4*x^3 + 2*x^2 + 4*x + 1)
```

Sympy [F]

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx = \int \frac{1}{(x^2-2x-1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(x**2-2*x-1)**(5/2),x)
[Out] Integral((x**2 - 2*x - 1)**(-5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx = \frac{x}{6\,\sqrt{x^2-2\,x-1}} - \frac{1}{6\,\sqrt{x^2-2\,x-1}} - \frac{x}{6\,(x^2-2\,x-1)^{\frac{3}{2}}} + \frac{1}{6\,(x^2-2\,x-1)^{\frac{3}{2}}}$$

```
[In] integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="maxima")
[Out] 1/6*x/sqrt(x^2 - 2*x - 1) - 1/6/sqrt(x^2 - 2*x - 1) - 1/6*x/(x^2 - 2*x - 1)
^(3/2) + 1/6/(x^2 - 2*x - 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.49

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx = \frac{(x-3)x^2+2}{6\,(x^2-2\,x-1)^{\frac{3}{2}}}$$

```
[In] integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="giac")
[Out] 1/6*((x - 3)*x^2 + 2)/(x^2 - 2*x - 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.51

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx = \frac{x^3-3\,x^2+2}{6\,(x^2-2\,x-1)^{3/2}}$$

```
[In] int(1/(x^2 - 2*x - 1)^(5/2),x)
[Out] (x^3 - 3*x^2 + 2)/(6*(x^2 - 2*x - 1)^(3/2))
```

3.470 $\int \frac{1}{x^4(-8+x^2)^{3/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}} dx = \frac{1}{24x^3\sqrt{-8+x^2}} + \frac{1}{48x\sqrt{-8+x^2}} - \frac{x}{192\sqrt{-8+x^2}}$$

```
[Out] 1/24/x^3/(x^2-8)^(1/2)+1/48/x/(x^2-8)^(1/2)-1/192*x/(x^2-8)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {277, 197}

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}} dx = -\frac{x}{192\sqrt{x^2-8}} + \frac{1}{48\sqrt{x^2-8}x} + \frac{1}{24\sqrt{x^2-8}x^3}$$

```
[In] Int[1/(x^4*(-8 + x^2)^(3/2)),x]
```

```
[Out] 1/(24*x^3*Sqrt[-8 + x^2]) + 1/(48*x*Sqrt[-8 + x^2]) - x/(192*Sqrt[-8 + x^2]
)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{24x^3\sqrt{-8+x^2}}+\frac{1}{6}\int\frac{1}{x^2\left(-8+x^2\right)^{3/2}}\,dx\\ &= \frac{1}{24x^3\sqrt{-8+x^2}}+\frac{1}{48x\sqrt{-8+x^2}}+\frac{1}{24}\int\frac{1}{\left(-8+x^2\right)^{3/2}}\,dx\\ &= \frac{1}{24x^3\sqrt{-8+x^2}}+\frac{1}{48x\sqrt{-8+x^2}}-\frac{x}{192\sqrt{-8+x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int\frac{1}{x^4\left(-8+x^2\right)^{3/2}}\,dx=\frac{8+4x^2-x^4}{192x^3\sqrt{-8+x^2}}$$

```
[In] Integrate[1/(x^4*(-8 + x^2)^(3/2)),x]
[Out] (8 + 4*x^2 - x^4)/(192*x^3*Sqrt[-8 + x^2])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.49

method	result	size
gosper	$-\frac{x^4-4x^2-8}{192x^3\sqrt{x^2-8}}$	23
trager	$-\frac{x^4-4x^2-8}{192x^3\sqrt{x^2-8}}$	23
risch	$-\frac{x^4-4x^2-8}{192x^3\sqrt{x^2-8}}$	23
pseudoelliptic	$-\frac{x^4-4x^2-8}{192x^3\sqrt{x^2-8}}$	23
default	$\frac{1}{24x^3\sqrt{x^2-8}}+\frac{1}{48x\sqrt{x^2-8}}-\frac{x}{192\sqrt{x^2-8}}$	36
meijerg	$-\frac{\sqrt{2}\left(-\operatorname{signum}\left(-1+\frac{x^2}{8}\right)\right)^{\frac{3}{2}}\left(-\frac{1}{8}x^4+\frac{1}{2}x^2+1\right)}{96\operatorname{signum}\left(-1+\frac{x^2}{8}\right)^{\frac{3}{2}}x^3\sqrt{1-\frac{x^2}{8}}}$	52

```
[In] int(1/x^4/(x^2-8)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/192*(x^4-4*x^2-8)/x^3/(x^2-8)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}}\,dx = -\frac{x^5-8\,x^3+(x^4-4\,x^2-8)\sqrt{x^2-8}}{192\,(x^5-8\,x^3)}$$

```
[In] integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="fricas")
[Out] -1/192*(x^5 - 8*x^3 + (x^4 - 4*x^2 - 8)*sqrt(x^2 - 8))/(x^5 - 8*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.26

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}}\,dx = \begin{cases} -\frac{ix^4\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{4ix^2\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{8i\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} & \text{for}\ \frac{1}{|x^2|} > \frac{1}{8} \\ -\frac{x^4\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{4x^2\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{8\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x**4/(x**2-8)**(3/2),x)
[Out] Piecewise((-I*x**4*sqrt(-1 + 8/x**2)/(192*x**4 - 1536*x**2) + 4*I*x**2*sqrt
(-1 + 8/x**2)/(192*x**4 - 1536*x**2) + 8*I*sqrt(-1 + 8/x**2)/(192*x**4 - 15
36*x**2), 1/Abs(x**2) > 1/8), (-x**4*sqrt(1 - 8/x**2)/(192*x**4 - 1536*x**2
) + 4*x**2*sqrt(1 - 8/x**2)/(192*x**4 - 1536*x**2) + 8*sqrt(1 - 8/x**2)/(19
2*x**4 - 1536*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}}\,dx = -\frac{x}{192\sqrt{x^2-8}} + \frac{1}{48\sqrt{x^2-8}x} + \frac{1}{24\sqrt{x^2-8}x^3}$$

```
[In] integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="maxima")
[Out] -1/192*x/sqrt(x^2 - 8) + 1/48/(sqrt(x^2 - 8)*x) + 1/24/(sqrt(x^2 - 8)*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}} \, dx = -\frac{x}{512 \sqrt{x^2-8}} - \frac{3 \left(x-\sqrt{x^2-8}\right)^4 + 96 \left(x-\sqrt{x^2-8}\right)^2 + 320}{96 \left(\left(x-\sqrt{x^2-8}\right)^2 + 8\right)^3}$$

```
[In] integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="giac")
[Out] -1/512*x/sqrt(x^2 - 8) - 1/96*(3*(x - sqrt(x^2 - 8))^4 + 96*(x - sqrt(x^2 -
 8))^2 + 320)/((x - sqrt(x^2 - 8))^2 + 8)^3
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}} \, dx = \frac{-x^4+4\,x^2+8}{192\,x^3\,\sqrt{x^2-8}}$$

```
[In] int(1/(x^4*(x^2 - 8)^(3/2)),x)
[Out] (4*x^2 - x^4 + 8)/(192*x^3*(x^2 - 8)^(1/2))
```

3.471 $\int \frac{(5+x^2)^2}{x^{13/3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{(5+x^2)^2}{x^{13/3}} \, dx = -\frac{15}{2x^{10/3}} - \frac{15}{2x^{4/3}} + \frac{3x^{2/3}}{2}$$

```
[Out] -15/2/x^(10/3)-15/2/x^(4/3)+3/2*x^(2/3)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$\int \frac{(5+x^2)^2}{x^{13/3}} \, dx = \frac{3x^{2/3}}{2} - \frac{15}{2x^{4/3}} - \frac{15}{2x^{10/3}}$$

```
[In] Int[(5 + x^2)^2/x^(13/3),x]
```

```
[Out] -15/(2*x^(10/3)) - 15/(2*x^(4/3)) + (3*x^(2/3))/2
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{25}{x^{13/3}} + \frac{10}{x^{7/3}} + \frac{1}{\sqrt[3]{x}}\right) dx \\ &= -\frac{15}{2x^{10/3}} - \frac{15}{2x^{4/3}} + \frac{3x^{2/3}}{2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx = \frac{3(-5-5x^2+x^4)}{2x^{10/3}}$$

```
[In] Integrate[(5 + x^2)^2/x^(13/3),x]
[Out] (3*(-5 - 5*x^2 + x^4))/(2*x^(10/3))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{\frac{3}{2}x^4-\frac{15}{2}-\frac{15}{2}x^2}{x^{\frac{10}{3}}}$	16
trager	$\frac{\frac{3}{2}x^4-\frac{15}{2}-\frac{15}{2}x^2}{x^{\frac{10}{3}}}$	16
risch	$\frac{\frac{3}{2}x^4-\frac{15}{2}-\frac{15}{2}x^2}{x^{\frac{10}{3}}}$	16
derivativedivides	$-\frac{15}{2x^{\frac{10}{3}}}-\frac{15}{2x^{\frac{4}{3}}}+\frac{3x^{\frac{2}{3}}}{2}$	17
default	$-\frac{15}{2x^{\frac{10}{3}}}-\frac{15}{2x^{\frac{4}{3}}}+\frac{3x^{\frac{2}{3}}}{2}$	17

```
[In] int((x^2+5)^2/x^(13/3),x,method=_RETURNVERBOSE)
[Out] 3/2*(x^4-5*x^2-5)/x^(10/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.54

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx = \frac{3\,(x^4-5\,x^2-5)}{2\,x^{\frac{10}{3}}}$$

```
[In] integrate((x^2+5)^2/x^(13/3),x, algorithm="fricas")
[Out] 3/2*(x^4 - 5*x^2 - 5)/x^(10/3)
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx = \frac{3x^{\frac{2}{3}}}{2} - \frac{15}{2x^{\frac{4}{3}}} - \frac{15}{2x^{\frac{10}{3}}}$$

```
[In] integrate((x**2+5)**2/x**(13/3),x)
[Out] 3*x**(2/3)/2 - 15/(2*x**(4/3)) - 15/(2*x**(10/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx = \frac{3}{2}\,x^{\frac{2}{3}} - \frac{15\,(x^2+1)}{2\,x^{\frac{10}{3}}}$$

```
[In] integrate((x^2+5)^2/x^(13/3),x, algorithm="maxima")
[Out] 3/2*x^(2/3) - 15/2*(x^2 + 1)/x^(10/3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx = \frac{3}{2}\,x^{\frac{2}{3}} - \frac{15\,(x^2+1)}{2\,x^{\frac{10}{3}}}$$

```
[In] integrate((x^2+5)^2/x^(13/3),x, algorithm="giac")
[Out] 3/2*x^(2/3) - 15/2*(x^2 + 1)/x^(10/3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx = -\frac{-3\,x^4+15\,x^2+15}{2\,x^{10/3}}$$

```
[In] int((x^2 + 5)^2/x^(13/3),x)
[Out] -(15*x^2 - 3*x^4 + 15)/(2*x^(10/3))
```

3.472 $\int \frac{1}{x^7(1+x^2)^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \frac{1}{x^7(1+x^2)^3}\,dx = -\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{3}{x^2}-\frac{1}{4(1+x^2)^2}-\frac{2}{1+x^2}-10\log(x)+5\log\left(1+x^2\right)$$

```
[Out] -1/6/x^6+3/4/x^4-3/x^2-1/4/(x^2+1)^2-2/(x^2+1)-10*ln(x)+5*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {272, 46}

$$\int \frac{1}{x^7(1+x^2)^3}\,dx = -\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{2}{x^2+1}-\frac{3}{x^2}-\frac{1}{4(x^2+1)^2}+5\log\left(x^2+1\right)-10\log(x)$$

```
[In] Int[1/(x^7*(1 + x^2)^3),x]
```

```
[Out] -1/6*1/x^6 + 3/(4*x^4) - 3/x^2 - 1/(4*(1 + x^2)^2) - 2/(1 + x^2) - 10*Log[x
] + 5*Log[1 + x^2]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(1+x)^3}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^4}-\frac{3}{x^3}+\frac{6}{x^2}-\frac{10}{x}+\frac{1}{(1+x)^3}+\frac{4}{(1+x)^2}+\frac{10}{1+x}\right)dx, x, x^2\right)\\ &= -\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{3}{x^2}-\frac{1}{4\left(1+x^2\right)^2}-\frac{2}{1+x^2}-10\log(x)+5\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^7\left(1+x^2\right)^3}\,dx = -\frac{2-5x^2+20x^4+90x^6+60x^8}{12x^6\left(1+x^2\right)^2}-10\log(x)+5\log\left(1+x^2\right)$$

```
[In] Integrate[1/(x^7*(1 + x^2)^3),x]
[Out] -1/12*(2 - 5*x^2 + 20*x^4 + 90*x^6 + 60*x^8)/(x^6*(1 + x^2)^2) - 10*Log[x]
+ 5*Log[1 + x^2]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

method	result
default	$-\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{3}{x^2}-\frac{1}{4(x^2+1)^2}-\frac{2}{x^2+1}-10\ln(x)+5\ln(x^2+1)$
norman	$\frac{-\frac{1}{6}-5x^8-\frac{15}{2}x^6+\frac{5}{12}x^2-\frac{5}{3}x^4}{x^6(x^2+1)^2}-10\ln(x)+5\ln(x^2+1)$
risch	$\frac{-\frac{1}{6}-5x^8-\frac{15}{2}x^6+\frac{5}{12}x^2-\frac{5}{3}x^4}{x^6(x^2+1)^2}-10\ln(x)+5\ln(x^2+1)$
meijerg	$-\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{3}{x^2}-\frac{9}{4}-10\ln(x)+\frac{x^2(9x^2+10)}{4(x^2+1)^2}+5\ln(x^2+1)$
parallelrisch	$-\frac{120x^{10}\ln(x)-60\ln(x^2+1)x^{10}+2+240\ln(x)x^8-120\ln(x^2+1)x^8+60x^8+120\ln(x)x^6-60x^6\ln(x^2+1)+90x^6+20x^4-5x^2}{12x^6(x^2+1)^2}$

```
[In] int(1/x^7/(x^2+1)^3,x,method=_RETURNVERBOSE)
[Out] -1/6/x^6+3/4/x^4-3/x^2-1/4/(x^2+1)^2-2/(x^2+1)-10*ln(x)+5*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^7 (1+x^2)^3} \, dx = -\frac{60 \, x^8 + 90 \, x^6 + 20 \, x^4 - 5 \, x^2 - 60 \, (x^{10} + 2 \, x^8 + x^6) \log \left(x^2 + 1\right) + 120 \, (x^{10} + 2 \, x^8 + x^6) \log \left(x\right) + 2}{12 \, (x^{10} + 2 \, x^8 + x^6)}$$

```
[In] integrate(1/x^7/(x^2+1)^3,x, algorithm="fricas")
[Out] -1/12*(60*x^8 + 90*x^6 + 20*x^4 - 5*x^2 - 60*(x^10 + 2*x^8 + x^6)*log(x^2 +
 1) + 120*(x^10 + 2*x^8 + x^6)*log(x) + 2)/(x^10 + 2*x^8 + x^6)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^7 (1+x^2)^3} \, dx = -10 \log \left(x\right) + 5 \log \left(x^2 + 1\right) + \frac{-60x^8 - 90x^6 - 20x^4 + 5x^2 - 2}{12x^{10} + 24x^8 + 12x^6}$$

```
[In] integrate(1/x**7/(x**2+1)**3,x)
[Out] -10*log(x) + 5*log(x**2 + 1) + (-60*x**8 - 90*x**6 - 20*x**4 + 5*x**2 - 2)/
(12*x**10 + 24*x**8 + 12*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^7 (1+x^2)^3} \, dx = -\frac{60 \, x^8 + 90 \, x^6 + 20 \, x^4 - 5 \, x^2 + 2}{12 \, (x^{10} + 2 \, x^8 + x^6)} + 5 \, \log \left(x^2 + 1\right) - 5 \, \log \left(x^2\right)$$

```
[In] integrate(1/x^7/(x^2+1)^3,x, algorithm="maxima")
[Out] -1/12*(60*x^8 + 90*x^6 + 20*x^4 - 5*x^2 + 2)/(x^10 + 2*x^8 + x^6) + 5*log(x
^2 + 1) - 5*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^7 (1+x^2)^3} \, dx = -\frac{30 \, x^4 + 68 \, x^2 + 39}{4 \, (x^2+1)^2} + \frac{110 \, x^6 - 36 \, x^4 + 9 \, x^2 - 2}{12 \, x^6} + 5 \, \log \left(x^2+1\right) - 5 \, \log \left(x^2\right)$$

```
[In] integrate(1/x^7/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -1/4*(30*x^4 + 68*x^2 + 39)/(x^2 + 1)^2 + 1/12*(110*x^6 - 36*x^4 + 9*x^2 -
2)/x^6 + 5*log(x^2 + 1) - 5*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^7 (1+x^2)^3} \, dx = 5 \, \ln \left(x^2+1\right) - 10 \, \ln \left(x\right) - \frac{5 \, x^8 + \frac{15 \, x^6}{2} + \frac{5 \, x^4}{3} - \frac{5 \, x^2}{12} + \frac{1}{6}}{x^{10} + 2 \, x^8 + x^6}$$

```
[In] int(1/(x^7*(x^2 + 1)^3),x)
```

```
[Out] 5*log(x^2 + 1) - 10*log(x) - ((5*x^4)/3 - (5*x^2)/12 + (15*x^6)/2 + 5*x^8 +
 1/6)/(x^6 + 2*x^8 + x^10)
```

3.473 $\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 25

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = -\frac{9\left(1+\frac{2}{x^2}\right)^{7/9}x}{10\sqrt{2+x^2}}$$

```
[Out] -9/10*(1+2/x^2)^(7/9)*x/(x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2016, 446, 270}

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = -\frac{9\left(\frac{2}{x^2}+1\right)^{7/9}x}{10\sqrt{x^2+2}}$$

```
[In] Int[((2 + x^2)/x^2)^(7/9)/(2 + x^2)^(3/2),x]
```

```
[Out] (-9*(1 + 2/x^2)^(7/9)*x)/(10*Sqrt[2 + x^2])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 446

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart[
q]), Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Rule 2016

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && BinomialQ[v, x] &&
!(BinomialMatchQ[u, x] && BinomialMatchQ[v, x])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\left(1+\frac{2}{x^2}\right)^{7/9}}{\left(2+x^2\right)^{3/2}}\,dx \\
&= \frac{\left(\left(1+\frac{2}{x^2}\right)^{7/9} x^{14/9}\right)\int \frac{1}{x^{14/9}\left(2+x^2\right)^{13/18}}\,dx}{\left(2+x^2\right)^{7/9}} \\
&= -\frac{9\left(1+\frac{2}{x^2}\right)^{7/9} x}{10\sqrt{2+x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 6.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{\left(2+x^2\right)^{3/2}}\,dx = -\frac{9\left(1+\frac{2}{x^2}\right)^{7/9} x}{10\sqrt{2+x^2}}$$

```
[In] Integrate[((2 + x^2)/x^2)^(7/9)/(2 + x^2)^(3/2),x]
[Out] (-9*(1 + 2/x^2)^(7/9)*x)/(10*Sqrt[2 + x^2])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{9x\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{10\sqrt{x^2+2}}$	22
risch	$-\frac{9x\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{10\sqrt{x^2+2}}$	22

```
[In] int((((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -9/10*x/(x^2+2)^(1/2)*((x^2+2)/x^2)^(7/9)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = -\frac{9\,x\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{10\,\sqrt{x^2+2}}$$

```
[In] integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="fricas")
[Out] -9/10*x*((x^2 + 2)/x^2)^(7/9)/sqrt(x^2 + 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(((x**2+2)/x**2)**(7/9)/(x**2+2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = \int \frac{\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{(x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = \int \frac{\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{(x^2+2)^{\frac{3}{2}}}\,dx$$

[In] integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="giac")

[Out] integrate(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = -\frac{9\,x\,(x^2+2)^{5/18}\,\left(\frac{1}{x^2}\right)^{7/9}}{10}$$

[In] int(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2),x)

[Out] -(9*x*(x^2 + 2)^(5/18)*(1/x^2)^(7/9))/10

3.474 $\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx=\frac{x^5}{7\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}+\frac{x^5}{175\left(\sqrt{10}-x^2\right)^{5/2}}$$

```
[Out] 1/70*x^5*10^(1/2)/(-x^2+10^(1/2))^(7/2)+1/175*x^5/(-x^2+10^(1/2))^(5/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {277, 270}

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx=\frac{x^5}{5\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}-\frac{x^7}{175\left(\sqrt{10}-x^2\right)^{7/2}}$$

```
[In] Int[x^4/(Sqrt[10] - x^2)^(9/2),x]
```

```
[Out] x^5/(5*Sqrt[10]*(Sqrt[10] - x^2)^(7/2)) - x^7/(175*(Sqrt[10] - x^2)^(7/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
```

```
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^5}{5\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}-\frac{1}{5}\sqrt{\frac{2}{5}}\int\frac{x^6}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx\\ &= \frac{x^5}{5\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}-\frac{x^7}{175\left(\sqrt{10}-x^2\right)^{7/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int\frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx=-\frac{x^5\left(-7\sqrt{10}+2x^2\right)}{350\left(\sqrt{10}-x^2\right)^{7/2}}$$

```
[In] Integrate[x^4/(Sqrt[10] - x^2)^(9/2),x]
[Out] -1/350*(x^5*(-7*Sqrt[10] + 2*x^2))/(Sqrt[10] - x^2)^(7/2)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.56

method	result
gosper	$\frac{x^{5}\left(-2x^{2}+7\sqrt{10}\right)}{350\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}$
pseudoelliptic	$\frac{x^{5}\left(-2x^{2}+7\sqrt{10}\right)}{350\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}$
meijerg	$\frac{10^{\frac{3}{4}}x^{5}\left(-\frac{\sqrt{2}\sqrt{5}\,x^{2}}{5}+7\right)}{35000\left(1-\frac{\sqrt{10}\,x^{2}}{10}\right)^{\frac{7}{2}}}$
risch	$\frac{2x^{7}-7\sqrt{10}\,x^{5}}{350\left(x^{2}-\sqrt{10}\right)^{3}\sqrt{-x^{2}+\sqrt{10}}}$
trager	$-\frac{2\sqrt{10}\left(\sqrt{10}\,x^{2}-35\right)x^{5}\sqrt{-x^{2}+\sqrt{10}}}{35\left(\sqrt{10}\,x^{2}-10\right)^{4}}$
default	$\frac{x^{3}}{4\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}-\frac{3\sqrt{10}\left(\frac{x}{6\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}-\frac{\sqrt{10}\left(\frac{x\sqrt{10}}{70\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}+\frac{3\sqrt{10}\left(\frac{x\sqrt{10}}{50\left(-x^{2}+\sqrt{10}\right)^{\frac{5}{2}}}+\frac{2\sqrt{10}\left(\frac{x\sqrt{10}}{30\left(-x^{2}+\sqrt{10}\right)^{\frac{3}{2}}}+\frac{\cdots}{15\sqrt{-x\cdots}}\right.}{25}\right.}{35}\right.}{6}\right.}{4}$

[In] int(x^4/(-x^2+10^(1/2))^(9/2),x,method=_RETURNVERBOSE)

[Out] 1/350*x^5*(-2*x^2+7*10^(1/2))/(-x^2+10^(1/2))^(7/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx = -\frac{\left(2\,x^{15}-160\,x^{11}-2600\,x^{7}+\sqrt{10}(x^{13}-340\,x^{9}-700\,x^{5})\right)\sqrt{-x^{2}+\sqrt{10}}}{350\,\left(x^{16}-40\,x^{12}+600\,x^{8}-4000\,x^{4}+10000\right)}$$

```
[In] integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="fricas")
[Out] -1/350*(2*x^15 - 160*x^11 - 2600*x^7 + sqrt(10)*(x^13 - 340*x^9 - 700*x^5))
*sqrt(-x^2 + sqrt(10))/(x^16 - 40*x^12 + 600*x^8 - 4000*x^4 + 10000)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**4/(-x**2+10**(1/2))**(9/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(36) = 72.

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx = \frac{x}{175\,\sqrt{-x^{2}+\sqrt{10}}}+\frac{\sqrt{10}x}{350\,\left(-x^{2}+\sqrt{10}\right)^{\frac{3}{2}}}+\frac{x^{3}}{4\,\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}+\frac{3\,x}{140\,\left(-x^{2}+\sqrt{10}\right)^{\frac{5}{2}}}-\frac{3\,\sqrt{10}x}{28\,\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}$$

```
[In] integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="maxima")
[Out] 1/175*x/sqrt(-x^2 + sqrt(10)) + 1/350*sqrt(10)*x/(-x^2 + sqrt(10))^(3/2) +
1/4*x^3/(-x^2 + sqrt(10))^(7/2) + 3/140*x/(-x^2 + sqrt(10))^(5/2) - 3/28*sq
rt(10)*x/(-x^2 + sqrt(10))^(7/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(36) = 72$.

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.96

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx = -\frac{16\left(7\left(\frac{x}{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}-\frac{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}{x}\right)^2+20\right)}{175\left(\frac{x}{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}-\frac{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}{x}\right)^7}$$

```
[In] integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="giac")
```

```
[Out] -16/175*(7*(x/(sqrt(-x^2 + sqrt(10)) - 10^(1/4)) - (sqrt(-x^2 + sqrt(10)) -
 10^(1/4))/x)^2 + 20)/(x/(sqrt(-x^2 + sqrt(10)) - 10^(1/4)) - (sqrt(-x^2 +
sqrt(10)) - 10^(1/4))/x)^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx = \int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx$$

```
[In] int(x^4/(10^(1/2) - x^2)^(9/2),x)
```

```
[Out] int(x^4/(10^(1/2) - x^2)^(9/2), x)
```

3.475 $\int \frac{x^2}{(3-x^2)^{3/2}}\, dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{x^2}{(3-x^2)^{3/2}}\, dx = \frac{x}{\sqrt{3-x^2}} - \arcsin\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] -arcsin(1/3*x*3^(1/2))+x/(-x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {294, 222}

$$\int \frac{x^2}{(3-x^2)^{3/2}}\, dx = \frac{x}{\sqrt{3-x^2}} - \arcsin\left(\frac{x}{\sqrt{3}}\right)$$

```
[In] Int[x^2/(3 - x^2)^(3/2),x]
```

```
[Out] x/Sqrt[3 - x^2] - ArcSin[x/Sqrt[3]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x}{\sqrt{3-x^2}}-\int\frac{1}{\sqrt{3-x^2}}\,dx\\ &= \frac{x}{\sqrt{3-x^2}}-\arcsin\left(\frac{x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int\frac{x^2}{(3-x^2)^{3/2}}\,dx=\frac{x}{\sqrt{3-x^2}}+2\arctan\left(\frac{x}{\sqrt{3}-\sqrt{3-x^2}}\right)$$

```
[In] Integrate[x^2/(3 - x^2)^(3/2),x]
[Out] x/Sqrt[3 - x^2] + 2*ArcTan[x/(Sqrt[3] - Sqrt[3 - x^2])]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

method	result	size
default	$-\arcsin\left(\frac{x\sqrt{3}}{3}\right)+\frac{x}{\sqrt{-x^2+3}}$	22
risch	$-\arcsin\left(\frac{x\sqrt{3}}{3}\right)+\frac{x}{\sqrt{-x^2+3}}$	22
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{-x^2+3}}{x}\right)\sqrt{-x^2+3}+x}{\sqrt{-x^2+3}}$	37
meijerg	$\frac{i\left(-\frac{i\sqrt{\pi}\,x\sqrt{3}}{3\sqrt{-\frac{x^2}{3}+1}}+i\sqrt{\pi}\,\arcsin\left(\frac{x\sqrt{3}}{3}\right)\right)}{\sqrt{\pi}}$	40
trager	$-\frac{x\sqrt{-x^2+3}}{x^2-3}+\text{RootOf}\left(\_Z^2+1\right)\ln\left(-\text{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+3}+x\right)$	48

```
[In] int(x^2/(-x^2+3)^(3/2),x,method=_RETURNVERBOSE)
[Out] -arcsin(1/3*x*3^(1/2))+x/(-x^2+3)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = \frac{(x^2-3)\arctan\left(\frac{\sqrt{-x^2+3}}{x}\right) - \sqrt{-x^2+3}\,x}{x^2-3}$$

```
[In] integrate(x^2/(-x^2+3)^(3/2),x, algorithm="fricas")
[Out] ((x^2 - 3)*arctan(sqrt(-x^2 + 3)/x) - sqrt(-x^2 + 3)*x)/(x^2 - 3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(19) = 38.

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = -\frac{x^2\operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)}{x^2-3} - \frac{x\sqrt{3-x^2}}{x^2-3} + \frac{3\operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)}{x^2-3}$$

```
[In] integrate(x**2/(-x**2+3)**(3/2),x)
[Out] -x**2*asin(sqrt(3)*x/3)/(x**2 - 3) - x*sqrt(3 - x**2)/(x**2 - 3) + 3*asin(s
qrt(3)*x/3)/(x**2 - 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = \frac{x}{\sqrt{-x^2+3}} - \arcsin\left(\frac{1}{3}\sqrt{3}x\right)$$

```
[In] integrate(x^2/(-x^2+3)^(3/2),x, algorithm="maxima")
[Out] x/sqrt(-x^2 + 3) - arcsin(1/3*sqrt(3)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = -\frac{\sqrt{-x^2+3}x}{x^2-3} - \arcsin\left(\frac{1}{3}\sqrt{3}x\right)$$

```
[In] integrate(x^2/(-x^2+3)^(3/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 3)*x/(x^2 - 3) - arcsin(1/3*sqrt(3)*x)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = -\mathrm{asin}\left(\frac{\sqrt{3}\,x}{3}\right) - \frac{\sqrt{3-x^2}}{2\,\left(x-\sqrt{3}\right)} - \frac{\sqrt{3-x^2}}{2\,\left(x+\sqrt{3}\right)}$$

```
[In] int(x^2/(3 - x^2)^(3/2),x)
[Out] - asin((3^(1/2)*x)/3) - (3 - x^2)^(1/2)/(2*(x - 3^(1/2))) - (3 - x^2)^(1/2)
/(2*(x + 3^(1/2)))
```

3.476 $\int \frac{(25-x^2)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{(25-x^2)^{3/2}}{x^4} dx = \frac{\sqrt{25-x^2}}{x} - \frac{(25-x^2)^{3/2}}{3x^3} + \arcsin\left(\frac{x}{5}\right)$$

```
[Out] -1/3*(-x^2+25)^(3/2)/x^3+arcsin(1/5*x)+(-x^2+25)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {283, 222}

$$\int \frac{(25-x^2)^{3/2}}{x^4} dx = \arcsin\left(\frac{x}{5}\right) + \frac{\sqrt{25-x^2}}{x} - \frac{(25-x^2)^{3/2}}{3x^3}$$

```
[In] Int[(25 - x^2)^(3/2)/x^4,x]
```

```
[Out] Sqrt[25 - x^2]/x - (25 - x^2)^(3/2)/(3*x^3) + ArcSin[x/5]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

`nomialQ[a, b, c, n, m, p, x]`

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(25-x^2)^{3/2}}{3x^3} - \int \frac{\sqrt{25-x^2}}{x^2}\,dx\\ &= \frac{\sqrt{25-x^2}}{x} - \frac{(25-x^2)^{3/2}}{3x^3} + \int \frac{1}{\sqrt{25-x^2}}\,dx\\ &= \frac{\sqrt{25-x^2}}{x} - \frac{(25-x^2)^{3/2}}{3x^3} + \arcsin\left(\frac{x}{5}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = \frac{\sqrt{25-x^2}(-25+4x^2)}{3x^3} - 2\arctan\left(\frac{\sqrt{25-x^2}}{5+x}\right)$$

```
[In] Integrate[(25 - x^2)^(3/2)/x^4,x]
[Out] (Sqrt[25 - x^2]*(-25 + 4*x^2))/(3*x^3) - 2*ArcTan[Sqrt[25 - x^2]/(5 + x)]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{4x^4-125x^2+625}{3x^3\sqrt{-x^2+25}}+\arcsin\left(\frac{x}{5}\right)$	32
meijerg	$-\frac{3i\left(-\frac{1000i\sqrt{\pi}\left(1-\frac{4x^2}{25}\right)\sqrt{-\frac{x^2}{25}+1}}{9x^3}+\frac{8i\sqrt{\pi}\,\arcsin\left(\frac{x}{5}\right)}{3}\right)}{8\sqrt{\pi}}$	43
trager	$\frac{(4x^2-25)\sqrt{-x^2+25}}{3x^3}+\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2+25}\right)$	50
pseudoelliptic	$\frac{-3\arctan\left(\frac{\sqrt{-x^2+25}}{x}\right)x^3+4\sqrt{-x^2+25}\,x^2-25\sqrt{-x^2+25}}{3x^3}$	51
default	$-\frac{(-x^2+25)^{\frac{5}{2}}}{75x^3}+\frac{2(-x^2+25)^{\frac{5}{2}}}{1875x}+\frac{2x(-x^2+25)^{\frac{3}{2}}}{1875}+\frac{\sqrt{-x^2+25}\,x}{25}+\arcsin\left(\frac{x}{5}\right)$	58

```
[In] int((-x^2+25)^(3/2)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*(4*x^4-125*x^2+625)/x^3/(-x^2+25)^(1/2)+arcsin(1/5*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = -\frac{6\,x^3\arctan\left(\frac{\sqrt{-x^2+25}-5}{x}\right)-(4\,x^2-25)\sqrt{-x^2+25}}{3\,x^3}$$

```
[In] integrate((-x^2+25)^(3/2)/x^4,x, algorithm="fricas")
[Out] -1/3*(6*x^3*arctan((sqrt(-x^2 + 25) - 5)/x) - (4*x^2 - 25)*sqrt(-x^2 + 25))
/x^3
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = \operatorname{asin}\left(\frac{x}{5}\right)+\frac{4\sqrt{25-x^2}}{3x}-\frac{25\sqrt{25-x^2}}{3x^3}$$

```
[In] integrate((-x**2+25)**(3/2)/x**4,x)
[Out] asin(x/5) + 4*sqrt(25 - x**2)/(3*x) - 25*sqrt(25 - x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = \frac{1}{25}\,\sqrt{-x^2+25}x+\frac{2\,(-x^2+25)^{\frac{3}{2}}}{75\,x}-\frac{(-x^2+25)^{\frac{5}{2}}}{75\,x^3}+\arcsin\left(\frac{1}{5}\,x\right)$$

```
[In] integrate((-x^2+25)^(3/2)/x^4,x, algorithm="maxima")
[Out] 1/25*sqrt(-x^2 + 25)*x + 2/75*(-x^2 + 25)^(3/2)/x - 1/75*(-x^2 + 25)^(5/2)/
x^3 + arcsin(1/5*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(32) = 64$.

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.92

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = -\frac{x^3\left(\frac{15\left(\sqrt{-x^2+25}-5\right)^2}{x^2}-1\right)}{24\left(\sqrt{-x^2+25}-5\right)^3} + \frac{5\left(\sqrt{-x^2+25}-5\right)}{8\,x} - \frac{\left(\sqrt{-x^2+25}-5\right)^3}{24\,x^3} + \arcsin\left(\frac{1}{5}\,x\right)$$

```
[In] integrate((-x^2+25)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/24*x^3*(15*(sqrt(-x^2 + 25) - 5)^2/x^2 - 1)/(sqrt(-x^2 + 25) - 5)^3 + 5/
8*(sqrt(-x^2 + 25) - 5)/x - 1/24*(sqrt(-x^2 + 25) - 5)^3/x^3 + arcsin(1/5*x
)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = \mathrm{asin}\left(\frac{x}{5}\right) + \frac{4\sqrt{25-x^2}}{3\,x} - \frac{25\sqrt{25-x^2}}{3\,x^3}$$

```
[In] int((25 - x^2)^(3/2)/x^4,x)
```

```
[Out] asin(x/5) + (4*(25 - x^2)^(1/2))/(3*x) - (25*(25 - x^2)^(1/2))/(3*x^3)
```

3.477 $\int \frac{1}{(1-2x^2)^{7/2}} dx$

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{(1-2x^2)^{7/2}} dx = \frac{x}{5(1-2x^2)^{5/2}} + \frac{4x}{15(1-2x^2)^{3/2}} + \frac{8x}{15\sqrt{1-2x^2}}$$

```
[Out] 1/5*x/(-2*x^2+1)^(5/2)+4/15*x/(-2*x^2+1)^(3/2)+8/15*x/(-2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {198, 197}

$$\int \frac{1}{(1-2x^2)^{7/2}} dx = \frac{8x}{15\sqrt{1-2x^2}} + \frac{4x}{15(1-2x^2)^{3/2}} + \frac{x}{5(1-2x^2)^{5/2}}$$

```
[In] Int[(1 - 2*x^2)^(-7/2),x]
```

```
[Out] x/(5*(1 - 2*x^2)^(5/2)) + (4*x)/(15*(1 - 2*x^2)^(3/2)) + (8*x)/(15*Sqrt[1 -
 2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
```

```
0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{x}{5\,(1-2x^2)^{5/2}}+\frac{4}{5}\int \frac{1}{(1-2x^2)^{5/2}}\,dx \\ &= \frac{x}{5\,(1-2x^2)^{5/2}}+\frac{4x}{15\,(1-2x^2)^{3/2}}+\frac{8}{15}\int \frac{1}{(1-2x^2)^{3/2}}\,dx \\ &= \frac{x}{5\,(1-2x^2)^{5/2}}+\frac{4x}{15\,(1-2x^2)^{3/2}}+\frac{8x}{15\sqrt{1-2x^2}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx=\frac{x(15-40x^2+32x^4)}{15\,(1-2x^2)^{5/2}}$$

```
[In] Integrate[(1 - 2*x^2)^(-7/2),x]
[Out] (x*(15 - 40*x^2 + 32*x^4))/(15*(1 - 2*x^2)^(5/2))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

method	result	size
gosper	$\frac{x\left(32x^4-40x^2+15\right)}{15\left(-2x^2+1\right)^{\frac{5}{2}}}$	25
meijerg	$\frac{x\left(32x^4-40x^2+15\right)}{15\left(-2x^2+1\right)^{\frac{5}{2}}}$	25
pseudoelliptic	$\frac{32x^5-40x^3+15x}{15\left(-2x^2+1\right)^{\frac{5}{2}}}$	26
trager	$-\frac{\left(32x^4-40x^2+15\right)x\sqrt{-2x^2+1}}{15\left(2x^2-1\right)^3}$	34
risch	$\frac{x\left(32x^4-40x^2+15\right)}{15\left(2x^2-1\right)^2\sqrt{-2x^2+1}}$	34
default	$\frac{x}{5\left(-2x^2+1\right)^{\frac{5}{2}}}+\frac{4x}{15\left(-2x^2+1\right)^{\frac{3}{2}}}+\frac{8x}{15\sqrt{-2x^2+1}}$	38

```
[In] int(1/(-2*x^2+1)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/15*x/(-2*x^2+1)^(5/2)*(32*x^4-40*x^2+15)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx = -\frac{(32\,x^5-40\,x^3+15\,x)\sqrt{-2\,x^2+1}}{15\,(8\,x^6-12\,x^4+6\,x^2-1)}$$

```
[In] integrate(1/(-2*x^2+1)^(7/2),x, algorithm="fricas")
[Out] -1/15*(32*x^5 - 40*x^3 + 15*x)*sqrt(-2*x^2 + 1)/(8*x^6 - 12*x^4 + 6*x^2 - 1
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.00 (sec) , antiderivative size = 291, normalized size of antiderivative = 5.94

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx = \begin{cases} -\frac{32ix^5}{60x^4\sqrt{2x^2-1}-60x^2\sqrt{2x^2-1}+15\sqrt{2x^2-1}} + \frac{40ix^3}{60x^4\sqrt{2x^2-1}-60x^2\sqrt{2x^2-1}+15\sqrt{2x^2-1}} - \frac{15}{60x^4\sqrt{2x^2-1}-60x^2\sqrt{}} \\ \frac{32x^5}{60x^4\sqrt{1-2x^2}-60x^2\sqrt{1-2x^2}+15\sqrt{1-2x^2}} - \frac{40x^3}{60x^4\sqrt{1-2x^2}-60x^2\sqrt{1-2x^2}+15\sqrt{1-2x^2}} + \frac{15x}{60x^4\sqrt{1-2x^2}-60x^2\sqrt{1}} \end{cases}$$

```
[In] integrate(1/(-2*x**2+1)**(7/2),x)
[Out] Piecewise((-32*I*x**5/(60*x**4*sqrt(2*x**2 - 1) - 60*x**2*sqrt(2*x**2 - 1)
+ 15*sqrt(2*x**2 - 1)) + 40*I*x**3/(60*x**4*sqrt(2*x**2 - 1) - 60*x**2*sqrt
(2*x**2 - 1) + 15*sqrt(2*x**2 - 1)) - 15*I*x/(60*x**4*sqrt(2*x**2 - 1) - 60
*x**2*sqrt(2*x**2 - 1) + 15*sqrt(2*x**2 - 1)), Abs(x**2) > 1/2), (32*x**5/(
60*x**4*sqrt(1 - 2*x**2) - 60*x**2*sqrt(1 - 2*x**2) + 15*sqrt(1 - 2*x**2))
- 40*x**3/(60*x**4*sqrt(1 - 2*x**2) - 60*x**2*sqrt(1 - 2*x**2) + 15*sqrt(1
- 2*x**2)) + 15*x/(60*x**4*sqrt(1 - 2*x**2) - 60*x**2*sqrt(1 - 2*x**2) + 15
*sqrt(1 - 2*x**2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx = \frac{8\,x}{15\,\sqrt{-2\,x^2+1}} + \frac{4\,x}{15\,(-2\,x^2+1)^{\frac{3}{2}}} + \frac{x}{5\,(-2\,x^2+1)^{\frac{5}{2}}}$$

```
[In] integrate(1/(-2*x^2+1)^(7/2),x, algorithm="maxima")
[Out] 8/15*x/sqrt(-2*x^2 + 1) + 4/15*x/(-2*x^2 + 1)^(3/2) + 1/5*x/(-2*x^2 + 1)^(5
/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(1-2x^2\right)^{7/2}}\,dx = -\frac{\left(8\,\left(4\,x^2-5\right)x^2+15\right)\sqrt{-2\,x^2+1}x}{15\,\left(2\,x^2-1\right)^3}$$

```
[In] integrate(1/(-2*x^2+1)^(7/2),x, algorithm="giac")
[Out] -1/15*(8*(4*x^2 - 5)*x^2 + 15)*sqrt(-2*x^2 + 1)*x/(2*x^2 - 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.65

$$\begin{aligned}\int \frac{1}{\left(1-2x^2\right)^{7/2}}\,dx &= \frac{19\,\sqrt{\frac{1}{2}-x^2}}{480\,\left(x^2-\sqrt{2}\,x+\frac{1}{2}\right)} \\ &- \frac{19\,\sqrt{\frac{1}{2}-x^2}}{480\,\left(x^2+\sqrt{2}\,x+\frac{1}{2}\right)} - \frac{\sqrt{2}\,\sqrt{\frac{1}{2}-x^2}}{160\,\left(x^3-\frac{3\,\sqrt{2}\,x^2}{2}+\frac{3\,x}{2}-\frac{\sqrt{2}}{4}\right)} \\ &- \frac{\sqrt{2}\,\sqrt{\frac{1}{2}-x^2}}{160\,\left(x^3+\frac{3\,\sqrt{2}\,x^2}{2}+\frac{3\,x}{2}+\frac{\sqrt{2}}{4}\right)} - \frac{2\,\sqrt{2}\,\sqrt{\frac{1}{2}-x^2}}{15\,\left(x-\frac{\sqrt{2}}{2}\right)} - \frac{2\,\sqrt{2}\,\sqrt{\frac{1}{2}-x^2}}{15\,\left(x+\frac{\sqrt{2}}{2}\right)}\end{aligned}$$

```
[In] int(1/(1 - 2*x^2)^(7/2),x)
[Out] (19*(1/2 - x^2)^(1/2))/(480*(x^2 - 2^(1/2)*x + 1/2)) - (19*(1/2 - x^2)^(1/2
))/(480*(2^(1/2)*x + x^2 + 1/2)) - (2^(1/2)*(1/2 - x^2)^(1/2))/(160*((3*x)/
2 - 2^(1/2)/4 - (3*2^(1/2)*x^2)/2 + x^3)) - (2^(1/2)*(1/2 - x^2)^(1/2))/(16
0*((3*x)/2 + 2^(1/2)/4 + (3*2^(1/2)*x^2)/2 + x^3)) - (2*2^(1/2)*(1/2 - x^2)
^(1/2))/(15*(x - 2^(1/2)/2)) - (2*2^(1/2)*(1/2 - x^2)^(1/2))/(15*(x + 2^(1/
2)/2))
```

3.478 $\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx=-\frac{3-x}{6(-7+6x-x^2)^{3/2}}-\frac{3-x}{6\sqrt{-7+6x-x^2}}$$

```
[Out] 1/6*(-3+x)/(-x^2+6*x-7)^(3/2)+1/6*(-3+x)/(-x^2+6*x-7)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {628, 627}

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx=-\frac{3-x}{6\sqrt{-x^2+6x-7}}-\frac{3-x}{6(-x^2+6x-7)^{3/2}}$$

```
[In] Int[(-7 + 6*x - x^2)^(-5/2),x]
```

```
[Out] -1/6*(3 - x)/(-7 + 6*x - x^2)^(3/2) - (3 - x)/(6*Sqrt[-7 + 6*x - x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{3-x}{6\left(-7+6x-x^2\right)^{3/2}}+\frac{1}{3}\int\frac{1}{\left(-7+6x-x^2\right)^{3/2}}\,dx\\ &= -\frac{3-x}{6\left(-7+6x-x^2\right)^{3/2}}-\frac{3-x}{6\sqrt{-7+6x-x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int\frac{1}{\left(-7+6x-x^2\right)^{5/2}}\,dx=-\frac{-18+24x-9x^2+x^3}{6\left(-7+6x-x^2\right)^{3/2}}$$

```
[In] Integrate[(-7 + 6*x - x^2)^(-5/2),x]
[Out] -1/6*(-18 + 24*x - 9*x^2 + x^3)/(-7 + 6*x - x^2)^(3/2)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{x^{3}-9x^{2}+24x-18}{6\left(-x^{2}+6x-7\right)^{\frac{3}{2}}}$	28
trager	$-\frac{\left(x^{3}-9x^{2}+24x-18\right)\sqrt{-x^{2}+6x-7}}{6\left(x^{2}-6x+7\right)^{2}}$	38
risch	$\frac{x^{3}-9x^{2}+24x-18}{6\left(x^{2}-6x+7\right)\sqrt{-x^{2}+6x-7}}$	38
default	$-\frac{-2x+6}{12\left(-x^{2}+6x-7\right)^{\frac{3}{2}}}-\frac{-2x+6}{12\sqrt{-x^{2}+6x-7}}$	40

```
[In] int(1/(-x^2+6*x-7)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/6/(-x^2+6*x-7)^(3/2)*(x^3-9*x^2+24*x-18)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = -\frac{(x^3-9\,x^2+24\,x-18)\sqrt{-x^2+6\,x-7}}{6\,(x^4-12\,x^3+50\,x^2-84\,x+49)}$$

```
[In] integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="fricas")
[Out] -1/6*(x^3 - 9*x^2 + 24*x - 18)*sqrt(-x^2 + 6*x - 7)/(x^4 - 12*x^3 + 50*x^2
- 84*x + 49)
```

Sympy [F]

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = \int \frac{1}{(-x^2+6x-7)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(-x**2+6*x-7)**(5/2),x)
[Out] Integral((-x**2 + 6*x - 7)**(-5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = \frac{x}{6\sqrt{-x^2+6\,x-7}} - \frac{1}{2\sqrt{-x^2+6\,x-7}} + \frac{x}{6\,(-x^2+6\,x-7)^{\frac{3}{2}}} - \frac{1}{2\,(-x^2+6\,x-7)^{\frac{3}{2}}}$$

```
[In] integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="maxima")
[Out] 1/6*x/sqrt(-x^2 + 6*x - 7) - 1/2/sqrt(-x^2 + 6*x - 7) + 1/6*x/(-x^2 + 6*x -
 7)^(3/2) - 1/2/(-x^2 + 6*x - 7)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = -\frac{(((x-9)x+24)x-18)\sqrt{-x^2+6\,x-7}}{6\,(x^2-6\,x+7)^2}$$

```
[In] integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="giac")
[Out] -1/6*(((x - 9)*x + 24)*x - 18)*sqrt(-x^2 + 6*x - 7)/(x^2 - 6*x + 7)^2
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = -\frac{(4\,x-12)\,(8\,x^2-48\,x+48)}{192\,(-x^2+6\,x-7)^{3/2}}$$

```
[In] int(1/(6*x - x^2 - 7)^(5/2),x)
[Out] -((4*x - 12)*(8*x^2 - 48*x + 48))/(192*(6*x - x^2 - 7)^(3/2))
```

3.479 $\int (1-2x-2x^2)^3 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 36

$$\int (1-2x-2x^2)^3 \, dx = x-3x^2+2x^3+4x^4-\frac{12x^5}{5}-4x^6-\frac{8x^7}{7}$$

```
[Out] x-3*x^2+2*x^3+4*x^4-12/5*x^5-4*x^6-8/7*x^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {625}

$$\int (1-2x-2x^2)^3 \, dx = -\frac{8x^7}{7}-4x^6-\frac{12x^5}{5}+4x^4+2x^3-3x^2+x$$

```
[In] Int[(1 - 2*x - 2*x^2)^3,x]
```

```
[Out] x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

Rule 625

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(1-6x+6x^2+16x^3-12x^4-24x^5-8x^6\right) \, dx \\ &= x-3x^2+2x^3+4x^4-\frac{12x^5}{5}-4x^6-\frac{8x^7}{7} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \left(1-2x-2x^2\right)^3 \, dx = x-3x^2+2x^3+4x^4-\frac{12x^5}{5}-4x^6-\frac{8x^7}{7}$$

```
[In] Integrate[(1 - 2*x - 2*x^2)^3,x]
[Out] x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
default	$x-3x^2+2x^3+4x^4-\frac{12}{5}x^5-4x^6-\frac{8}{7}x^7$	33
norman	$x-3x^2+2x^3+4x^4-\frac{12}{5}x^5-4x^6-\frac{8}{7}x^7$	33
risch	$x-3x^2+2x^3+4x^4-\frac{12}{5}x^5-4x^6-\frac{8}{7}x^7$	33
parallelrisch	$x-3x^2+2x^3+4x^4-\frac{12}{5}x^5-4x^6-\frac{8}{7}x^7$	33
gosper	$-\frac{x\left(40x^6+140x^5+84x^4-140x^3-70x^2+105x-35\right)}{35}$	34

```
[In] int((-2*x^2-2*x+1)^3,x,method=_RETURNVERBOSE)
[Out] x-3*x^2+2*x^3+4*x^4-12/5*x^5-4*x^6-8/7*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \left(1-2x-2x^2\right)^3 \, dx = -\frac{8}{7}\,x^7-4\,x^6-\frac{12}{5}\,x^5+4\,x^4+2\,x^3-3\,x^2+x$$

```
[In] integrate((-2*x^2-2*x+1)^3,x, algorithm="fricas")
[Out] -8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \left(1-2x-2x^2\right)^3\,dx=-\frac{8x^7}{7}-4x^6-\frac{12x^5}{5}+4x^4+2x^3-3x^2+x$$

```
[In] integrate((-2*x**2-2*x+1)**3,x)
[Out] -8*x**7/7 - 4*x**6 - 12*x**5/5 + 4*x**4 + 2*x**3 - 3*x**2 + x
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \left(1-2x-2x^2\right)^3\,dx=-\frac{8}{7}\,x^7-4\,x^6-\frac{12}{5}\,x^5+4\,x^4+2\,x^3-3\,x^2+x$$

```
[In] integrate((-2*x^2-2*x+1)^3,x, algorithm="maxima")
[Out] -8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \left(1-2x-2x^2\right)^3\,dx=-\frac{8}{7}\,x^7-4\,x^6-\frac{12}{5}\,x^5+4\,x^4+2\,x^3-3\,x^2+x$$

```
[In] integrate((-2*x^2-2*x+1)^3,x, algorithm="giac")
[Out] -8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \left(1-2x-2x^2\right)^3\,dx=-\frac{8\,x^7}{7}-4\,x^6-\frac{12\,x^5}{5}+4\,x^4+2\,x^3-3\,x^2+x$$

```
[In] int(-(2*x + 2*x^2 - 1)^3,x)
[Out] x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

3.480 $\int (-1+5x) \left(-1-x+x^2\right)^2 \, dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int (-1+5x) \left(-1-x+x^2\right)^2 \, dx = -x+\frac{3x^2}{2}+\frac{11x^3}{3}-\frac{3x^4}{4}-\frac{11x^5}{5}+\frac{5x^6}{6}$$

```
[Out] -x+3/2*x^2+11/3*x^3-3/4*x^4-11/5*x^5+5/6*x^6
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {645}

$$\int (-1+5x) \left(-1-x+x^2\right)^2 \, dx = \frac{5x^6}{6}-\frac{11x^5}{5}-\frac{3x^4}{4}+\frac{11x^3}{3}+\frac{3x^2}{2}-x$$

```
[In] Int[(-1 + 5*x)*(-1 - x + x^2)^2,x]
```

```
[Out] -x + (3*x^2)/2 + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-1+3x+11x^2-3x^3-11x^4+5x^5\right) \, dx \\ &= -x+\frac{3x^2}{2}+\frac{11x^3}{3}-\frac{3x^4}{4}-\frac{11x^5}{5}+\frac{5x^6}{6} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int (-1+5x)\left(-1-x+x^2\right)^2\,dx=-x+\frac{3x^2}{2}+\frac{11x^3}{3}-\frac{3x^4}{4}-\frac{11x^5}{5}+\frac{5x^6}{6}$$

```
[In] Integrate[(-1 + 5*x)*(-1 - x + x^2)^2,x]
[Out] -x + (3*x^2)/2 + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result	size
gosper	$-x+\frac{3}{2}x^2+\frac{11}{3}x^3-\frac{3}{4}x^4-\frac{11}{5}x^5+\frac{5}{6}x^6$	30
default	$-x+\frac{3}{2}x^2+\frac{11}{3}x^3-\frac{3}{4}x^4-\frac{11}{5}x^5+\frac{5}{6}x^6$	30
norman	$-x+\frac{3}{2}x^2+\frac{11}{3}x^3-\frac{3}{4}x^4-\frac{11}{5}x^5+\frac{5}{6}x^6$	30
risch	$-x+\frac{3}{2}x^2+\frac{11}{3}x^3-\frac{3}{4}x^4-\frac{11}{5}x^5+\frac{5}{6}x^6$	30
parallelrisch	$-x+\frac{3}{2}x^2+\frac{11}{3}x^3-\frac{3}{4}x^4-\frac{11}{5}x^5+\frac{5}{6}x^6$	30

```
[In] int((-1+5*x)*(x^2-x-1)^2,x,method=_RETURNVERBOSE)
[Out] -x+3/2*x^2+11/3*x^3-3/4*x^4-11/5*x^5+5/6*x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+5x)\left(-1-x+x^2\right)^2\,dx=\frac{5}{6}\,x^6-\frac{11}{5}\,x^5-\frac{3}{4}\,x^4+\frac{11}{3}\,x^3+\frac{3}{2}\,x^2-x$$

```
[In] integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="fricas")
[Out] 5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int (-1+5x)\left(-1-x+x^2\right)^2\,dx = \frac{5x^6}{6} - \frac{11x^5}{5} - \frac{3x^4}{4} + \frac{11x^3}{3} + \frac{3x^2}{2} - x$$

```
[In] integrate((-1+5*x)*(x**2-x-1)**2,x)
[Out] 5*x**6/6 - 11*x**5/5 - 3*x**4/4 + 11*x**3/3 + 3*x**2/2 - x
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+5x)\left(-1-x+x^2\right)^2\,dx = \frac{5}{6}\,x^6 - \frac{11}{5}\,x^5 - \frac{3}{4}\,x^4 + \frac{11}{3}\,x^3 + \frac{3}{2}\,x^2 - x$$

```
[In] integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="maxima")
[Out] 5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+5x)\left(-1-x+x^2\right)^2\,dx = \frac{5}{6}\,x^6 - \frac{11}{5}\,x^5 - \frac{3}{4}\,x^4 + \frac{11}{3}\,x^3 + \frac{3}{2}\,x^2 - x$$

```
[In] integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="giac")
[Out] 5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+5x)\left(-1-x+x^2\right)^2\,dx = \frac{5\,x^6}{6} - \frac{11\,x^5}{5} - \frac{3\,x^4}{4} + \frac{11\,x^3}{3} + \frac{3\,x^2}{2} - x$$

```
[In] int((5*x - 1)*(x - x^2 + 1)^2,x)
[Out] (3*x^2)/2 - x + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

3.481 $\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \frac{1-2x}{6(1-8x+2x^2)^{3/2}} - \frac{2(2-x)}{21\sqrt{1-8x+2x^2}}$$

```
[Out] 1/6*(1-2*x)/(2*x^2-8*x+1)^(3/2)-2/21*(2-x)/(2*x^2-8*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {652, 627}

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \frac{1-2x}{6(2x^2-8x+1)^{3/2}} - \frac{2(2-x)}{21\sqrt{2x^2-8x+1}}$$

```
[In] Int[(1 + 3*x)/(1 - 8*x + 2*x^2)^(5/2),x]
```

```
[Out] (1 - 2*x)/(6*(1 - 8*x + 2*x^2)^(3/2)) - (2*(2 - x))/(21*Sqrt[1 - 8*x + 2*x^
2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
```

```
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1-2x}{6\left(1-8x+2x^2\right)^{3/2}}-\frac{2}{3}\int\frac{1}{\left(1-8x+2x^2\right)^{3/2}}\,dx\\ &= \frac{1-2x}{6\left(1-8x+2x^2\right)^{3/2}}-\frac{2(2-x)}{21\sqrt{1-8x+2x^2}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int\frac{1+3x}{\left(1-8x+2x^2\right)^{5/2}}\,dx=\frac{-1+54x-48x^2+8x^3}{42\left(1-8x+2x^2\right)^{3/2}}$$

```
[In] Integrate[(1 + 3*x)/(1 - 8*x + 2*x^2)^(5/2),x]
[Out] (-1 + 54*x - 48*x^2 + 8*x^3)/(42*(1 - 8*x + 2*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{8x^3-48x^2+54x-1}{42(2x^2-8x+1)^{\frac{3}{2}}}$	30
trager	$\frac{8x^3-48x^2+54x-1}{42(2x^2-8x+1)^{\frac{3}{2}}}$	30
risch	$\frac{8x^3-48x^2+54x-1}{42(2x^2-8x+1)^{\frac{3}{2}}}$	30
default	$-\frac{4x-8}{12(2x^2-8x+1)^{\frac{3}{2}}}+\frac{4x-8}{42\sqrt{2x^2-8x+1}}-\frac{1}{2(2x^2-8x+1)^{\frac{3}{2}}}$	54

```
[In] int((1+3*x)/(2*x^2-8*x+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/42*(8*x^3-48*x^2+54*x-1)/(2*x^2-8*x+1)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = -\frac{4\,x^4-32\,x^3+68\,x^2-(8\,x^3-48\,x^2+54\,x-1)\sqrt{2\,x^2-8\,x+1}-16\,x+1}{42\,(4\,x^4-32\,x^3+68\,x^2-16\,x+1)}$$

```
[In] integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="fricas")
[Out] -1/42*(4*x^4 - 32*x^3 + 68*x^2 - (8*x^3 - 48*x^2 + 54*x - 1)*sqrt(2*x^2 - 8
*x + 1) - 16*x + 1)/(4*x^4 - 32*x^3 + 68*x^2 - 16*x + 1)
```

Sympy [F]

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \int \frac{3x+1}{(2x^2-8x+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((1+3*x)/(2*x**2-8*x+1)**(5/2),x)
[Out] Integral((3*x + 1)/(2*x**2 - 8*x + 1)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \frac{2\,x}{21\,\sqrt{2\,x^2-8\,x+1}} - \frac{4}{21\,\sqrt{2\,x^2-8\,x+1}} - \frac{x}{3\,(2\,x^2-8\,x+1)^{\frac{3}{2}}} + \frac{1}{6\,(2\,x^2-8\,x+1)^{\frac{3}{2}}}$$

```
[In] integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="maxima")
[Out] 2/21*x/sqrt(2*x^2 - 8*x + 1) - 4/21/sqrt(2*x^2 - 8*x + 1) - 1/3*x/(2*x^2 -
8*x + 1)^(3/2) + 1/6/(2*x^2 - 8*x + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.57

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \frac{2\,(4\,(x-6)x+27)x-1}{42\,(2\,x^2-8\,x+1)^{\frac{3}{2}}}$$

```
[In] integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="giac")
[Out] 1/42*(2*(4*(x - 6)*x + 27)*x - 1)/(2*x^2 - 8*x + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \frac{8\,x^3-48\,x^2+54\,x-1}{42\,(2\,x^2-8\,x+1)^{3/2}}$$

```
[In] int((3*x + 1)/(2*x^2 - 8*x + 1)^(5/2),x)
[Out] (54*x - 48*x^2 + 8*x^3 - 1)/(42*(2*x^2 - 8*x + 1)^(3/2))
```

3.482 $\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 45

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx = -\frac{4(1+x)}{15\,(1+2x-4x^2)^{3/2}} - \frac{7+122x}{75\sqrt{1+2x-4x^2}}$$

```
[Out] -4/15*(1+x)/(-4*x^2+2*x+1)^(3/2)+1/75*(-7-122*x)/(-4*x^2+2*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1674, 650}

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx = -\frac{4(x+1)}{15\,(-4x^2+2x+1)^{3/2}} - \frac{122x+7}{75\sqrt{-4x^2+2x+1}}$$

```
[In] Int[(-1 - 8*x + 8*x^3)/(1 + 2*x - 4*x^2)^(5/2),x]
```

```
[Out] (-4*(1 + x))/(15*(1 + 2*x - 4*x^2)^(3/2)) - (7 + 122*x)/(75*Sqrt[1 + 2*x -
4*x^2])
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral}&=-\frac{4(1+x)}{15\left(1+2x-4x^2\right)^{3/2}}-\frac{1}{30}\int\frac{46+60x}{\left(1+2x-4x^2\right)^{3/2}}\,dx\\&=-\frac{4(1+x)}{15\left(1+2x-4x^2\right)^{3/2}}-\frac{7+122x}{75\sqrt{1+2x-4x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int\frac{-1-8x+8x^3}{\left(1+2x-4x^2\right)^{5/2}}\,dx=-\frac{27+156x+216x^2-488x^3}{75\left(1+2x-4x^2\right)^{3/2}}$$

```
[In] Integrate[(-1 - 8*x + 8*x^3)/(1 + 2*x - 4*x^2)^(5/2),x]
[Out] -1/75*(27 + 156*x + 216*x^2 - 488*x^3)/(1 + 2*x - 4*x^2)^(3/2)
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{488x^3-216x^2-156x-27}{75(-4x^2+2x+1)^{\frac{3}{2}}}$	30
trager	$\frac{(488x^3-216x^2-156x-27)\sqrt{-4x^2+2x+1}}{75(4x^2-2x-1)^2}$	42
risch	$-\frac{488x^3-216x^2-156x-27}{75(4x^2-2x-1)\sqrt{-4x^2+2x+1}}$	42
default	$\frac{\frac{61}{240}-\frac{61x}{60}}{(-4x^2+2x+1)^{\frac{3}{2}}}+\frac{\frac{61}{150}-\frac{122x}{75}}{\sqrt{-4x^2+2x+1}}-\frac{49}{48(-4x^2+2x+1)^{\frac{3}{2}}}+\frac{2x^2}{(-4x^2+2x+1)^{\frac{3}{2}}}-\frac{x}{4(-4x^2+2x+1)^{\frac{3}{2}}}$	86

```
[In] int((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/75/(-4*x^2+2*x+1)^(3/2)*(488*x^3-216*x^2-156*x-27)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx = -\frac{432\,x^4-432\,x^3-108\,x^2-(488\,x^3-216\,x^2-156\,x-27)\sqrt{-4\,x^2+2\,x+1}+108\,x+27}{75\,(16\,x^4-16\,x^3-4\,x^2+4\,x+1)}$$

```
[In] integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="fricas")
[Out] -1/75*(432*x^4 - 432*x^3 - 108*x^2 - (488*x^3 - 216*x^2 - 156*x - 27)*sqrt(
-4*x^2 + 2*x + 1) + 108*x + 27)/(16*x^4 - 16*x^3 - 4*x^2 + 4*x + 1)
```

Sympy [F]

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx = \int \frac{8x^3-8x-1}{(-4x^2+2x+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((8*x**3-8*x-1)/(-4*x**2+2*x+1)**(5/2),x)
[Out] Integral((8*x**3 - 8*x - 1)/(-4*x**2 + 2*x + 1)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(37) = 74.

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx = -\frac{122\,x}{75\,\sqrt{-4\,x^2+2\,x+1}} + \frac{2\,x^2}{(-4\,x^2+2\,x+1)^{\frac{3}{2}}} + \frac{61}{150\,\sqrt{-4\,x^2+2\,x+1}} - \frac{19\,x}{15\,(-4\,x^2+2\,x+1)^{\frac{3}{2}}} - \frac{23}{30\,(-4\,x^2+2\,x+1)^{\frac{3}{2}}}$$

```
[In] integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="maxima")
[Out] -122/75*x/sqrt(-4*x^2 + 2*x + 1) + 2*x^2/(-4*x^2 + 2*x + 1)^(3/2) + 61/150/
sqrt(-4*x^2 + 2*x + 1) - 19/15*x/(-4*x^2 + 2*x + 1)^(3/2) - 23/30/(-4*x^2 +
 2*x + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx = \frac{(4\,(2\,(61\,x-27)x-39)x-27)\sqrt{-4\,x^2+2\,x+1}}{75\,(4\,x^2-2\,x-1)^2}$$

```
[In] integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="giac")
[Out] 1/75*(4*(2*(61*x - 27)*x - 39)*x - 27)*sqrt(-4*x^2 + 2*x + 1)/(4*x^2 - 2*x
- 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx = -\frac{-488\,x^3+216\,x^2+156\,x+27}{75\,(-4\,x^2+2\,x+1)^{3/2}}$$

```
[In] int(-(8*x - 8*x^3 + 1)/(2*x - 4*x^2 + 1)^(5/2),x)
[Out] -(156*x + 216*x^2 - 488*x^3 + 27)/(75*(2*x - 4*x^2 + 1)^(3/2))
```

3.483 $\int x^2 \cos^5(x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 83

$$\int x^2 \cos^5(x)\,dx = \frac{16}{15}x\cos(x)+\frac{8}{45}x\cos^3(x)+\frac{2}{25}x\cos^5(x)-\frac{298\sin(x)}{225}+\frac{8}{15}x^2\sin(x)+\frac{4}{15}x^2\cos^2(x)\sin(x)+\frac{1}{5}x^2\cos^4(x)\sin(x)+\frac{76\sin^3(x)}{675}-\frac{2\sin^5(x)}{125}$$

```
[Out] 16/15*x*cos(x)+8/45*x*cos(x)^3+2/25*x*cos(x)^5-298/225*sin(x)+8/15*x^2*sin(
x)+4/15*x^2*cos(x)^2*sin(x)+1/5*x^2*cos(x)^4*sin(x)+76/675*sin(x)^3-2/125*s
in(x)^5
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3392, 3377, 2717, 2713}

$$\int x^2 \cos^5(x)\,dx = \frac{8}{15}x^2\sin(x)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{4}{15}x^2\sin(x)\cos^2(x)-\frac{2\sin^5(x)}{125}+\frac{76\sin^3(x)}{675}-\frac{298\sin(x)}{225}+\frac{2}{25}x\cos^5(x)+\frac{8}{45}x\cos^3(x)+\frac{16}{15}x\cos(x)$$

```
[In] Int[x^2*Cos[x]^5,x]
```

```
[Out] (16*x*Cos[x])/15 + (8*x*Cos[x]^3)/45 + (2*x*Cos[x]^5)/25 - (298*Sin[x])/225
 + (8*x^2*Sin[x])/15 + (4*x^2*Cos[x]^2*Sin[x])/15 + (x^2*Cos[x]^4*Sin[x])/5
 + (76*Sin[x]^3)/675 - (2*Sin[x]^5)/125
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{25}x\cos^5(x)+\frac{1}{5}x^2\cos^4(x)\sin(x)-\frac{2}{25}\int\cos^5(x)\,dx+\frac{4}{5}\int x^2\cos^3(x)\,dx \\
&= \frac{8}{45}x\cos^3(x)+\frac{2}{25}x\cos^5(x)+\frac{4}{15}x^2\cos^2(x)\sin(x) \\
&\quad+\frac{1}{5}x^2\cos^4(x)\sin(x)+\frac{2}{25}\text{Subst}\left(\int\left(1-2x^2+x^4\right)\,dx,x,-\sin(x)\right) \\
&\quad-\frac{8}{45}\int\cos^3(x)\,dx+\frac{8}{15}\int x^2\cos(x)\,dx \\
&= \frac{8}{45}x\cos^3(x)+\frac{2}{25}x\cos^5(x)-\frac{2\sin(x)}{25}+\frac{8}{15}x^2\sin(x)+\frac{4}{15}x^2\cos^2(x)\sin(x)+\frac{1}{5}x^2\cos^4(x)\sin(x) \\
&\quad+\frac{4\sin^3(x)}{75}-\frac{2\sin^5(x)}{125}+\frac{8}{45}\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(x)\right)-\frac{16}{15}\int x\sin(x)\,dx \\
&= \frac{16}{15}x\cos(x)+\frac{8}{45}x\cos^3(x)+\frac{2}{25}x\cos^5(x)-\frac{58\sin(x)}{225}+\frac{8}{15}x^2\sin(x) \\
&\quad+\frac{4}{15}x^2\cos^2(x)\sin(x)+\frac{1}{5}x^2\cos^4(x)\sin(x)+\frac{76\sin^3(x)}{675}-\frac{2\sin^5(x)}{125}-\frac{16}{15}\int\cos(x)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{16}{15}x\cos(x) + \frac{8}{45}x\cos^3(x) + \frac{2}{25}x\cos^5(x) - \frac{298\sin(x)}{225} + \frac{8}{15}x^2\sin(x) \\
&\quad + \frac{4}{15}x^2\cos^2(x)\sin(x) + \frac{1}{5}x^2\cos^4(x)\sin(x) + \frac{76\sin^3(x)}{675} - \frac{2\sin^5(x)}{125}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$
\begin{aligned}
\int x^2\cos^5(x)\,dx &= \frac{5}{4}x\cos(x) + \frac{5}{72}x\cos(3x) + \frac{1}{200}x\cos(5x) + \frac{5}{8}\left(-2+x^2\right)\sin(x) \\
&\quad + \frac{5}{432}\left(-2+9x^2\right)\sin(3x) + \frac{\left(-2+25x^2\right)\sin(5x)}{2000}
\end{aligned}
$$

```
[In] Integrate[x^2*Cos[x]^5,x]
```

```
[Out] (5*x*Cos[x])/4 + (5*x*Cos[3*x])/72 + (x*Cos[5*x])/200 + (5*(-2 + x^2)*Sin[x
])/8 + (5*(-2 + 9*x^2)*Sin[3*x])/432 + ((-2 + 25*x^2)*Sin[5*x])/2000
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

method	result
risch	$\frac{5x\cos(x)}{4}+\frac{5(x^2-2)\sin(x)}{8}+\frac{x\cos(5x)}{200}+\frac{(25x^2-2)\sin(5x)}{2000}+\frac{5x\cos(3x)}{72}+\frac{5(9x^2-2)\sin(3x)}{432}$
parallelrisch	$\frac{(5625x^2-1250)\sin(3x)}{54000}+\frac{(675x^2-54)\sin(5x)}{54000}+\frac{5x^2\sin(x)}{8}+\frac{5x\cos(x)}{4}+\frac{5x\cos(3x)}{72}+\frac{x\cos(5x)}{200}-\frac{5\sin(x)}{4}$
default	$\frac{x^2\left(\frac{8}{3}+\cos^4(x)+\frac{4\left(\cos^2(x)\right)}{3}\right)\sin(x)}{5}+\frac{2\left(\cos^5(x)\right)x}{25}-\frac{2\left(\frac{8}{3}+\cos^4(x)+\frac{4\left(\cos^2(x)\right)}{3}\right)\sin(x)}{125}+\frac{8x\left(\cos^3(x)\right)}{45}-\frac{8(2+\cos^2(x}{135}$

```
[In] int(x^2*cos(x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 5/4*x*cos(x)+5/8*(x^2-2)*sin(x)+1/200*x*cos(5*x)+1/2000*(25*x^2-2)*sin(5*x)
+5/72*x*cos(3*x)+5/432*(9*x^2-2)*sin(3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

$$\int x^2\cos^5(x)\,dx = \frac{2}{25}\,x\cos\left(x\right)^5+\frac{8}{45}\,x\cos\left(x\right)^3+\frac{16}{15}\,x\cos\left(x\right) + \frac{1}{3375}\,\left(27\,\left(25\,x^2-2\right)\cos\left(x\right)^4+4\,\left(225\,x^2-68\right)\cos\left(x\right)^2+1800\,x^2-4144\right)\sin\left(x\right)$$

```
[In] integrate(x^2*cos(x)^5,x, algorithm="fricas")
```

```
[Out] 2/25*x*cos(x)^5 + 8/45*x*cos(x)^3 + 16/15*x*cos(x) + 1/3375*(27*(25*x^2 - 2
)*cos(x)^4 + 4*(225*x^2 - 68)*cos(x)^2 + 1800*x^2 - 4144)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int x^2\cos^5(x)\,dx = \frac{8x^2\sin^5\left(x\right)}{15}+\frac{4x^2\sin^3\left(x\right)\cos^2\left(x\right)}{3}+x^2\sin\left(x\right)\cos^4\left(x\right) + \frac{16x\sin^4\left(x\right)\cos\left(x\right)}{15}+\frac{104x\sin^2\left(x\right)\cos^3\left(x\right)}{45}+\frac{298x\cos^5\left(x\right)}{225} - \frac{4144\sin^5\left(x\right)}{3375}-\frac{1712\sin^3\left(x\right)\cos^2\left(x\right)}{675}-\frac{298\sin\left(x\right)\cos^4\left(x\right)}{225}$$

```
[In] integrate(x**2*cos(x)**5,x)
```

```
[Out] 8*x**2*sin(x)**5/15 + 4*x**2*sin(x)**3*cos(x)**2/3 + x**2*sin(x)*cos(x)**4
+ 16*x*sin(x)**4*cos(x)/15 + 104*x*sin(x)**2*cos(x)**3/45 + 298*x*cos(x)**5
/225 - 4144*sin(x)**5/3375 - 1712*sin(x)**3*cos(x)**2/675 - 298*sin(x)*cos(
x)**4/225
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

$$\int x^2\cos^5(x)\,dx = \frac{1}{200}\,x\cos\left(5\,x\right)+\frac{5}{72}\,x\cos\left(3\,x\right)+\frac{5}{4}\,x\cos\left(x\right)+\frac{1}{2000}\,\left(25\,x^2-2\right)\sin\left(5\,x\right) + \frac{5}{432}\,\left(9\,x^2-2\right)\sin\left(3\,x\right)+\frac{5}{8}\,\left(x^2-2\right)\sin\left(x\right)$$

```
[In] integrate(x^2*cos(x)^5,x, algorithm="maxima")
```

```
[Out] 1/200*x*cos(5*x) + 5/72*x*cos(3*x) + 5/4*x*cos(x) + 1/2000*(25*x^2 - 2)*sin
(5*x) + 5/432*(9*x^2 - 2)*sin(3*x) + 5/8*(x^2 - 2)*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

$$\int x^2 \cos^5(x)\,dx = \frac{1}{200}\,x\cos(5\,x) + \frac{5}{72}\,x\cos(3\,x) + \frac{5}{4}\,x\cos(x) + \frac{1}{2000}\,\left(25\,x^2 - 2\right)\sin(5\,x) + \frac{5}{432}\,\left(9\,x^2 - 2\right)\sin(3\,x) + \frac{5}{8}\,\left(x^2 - 2\right)\sin(x)$$

```
[In] integrate(x^2*cos(x)^5,x, algorithm="giac")
[Out] 1/200*x*cos(5*x) + 5/72*x*cos(3*x) + 5/4*x*cos(x) + 1/2000*(25*x^2 - 2)*sin
(5*x) + 5/432*(9*x^2 - 2)*sin(3*x) + 5/8*(x^2 - 2)*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int x^2 \cos^5(x)\,dx = \frac{8\,x\cos(x)^3}{45} - \frac{4144\,\sin(x)}{3375} + \frac{2\,x\cos(x)^5}{25} + \frac{8\,x^2\,\sin(x)}{15} - \frac{272\cos(x)^2\,\sin(x)}{3375} - \frac{2\cos(x)^4\,\sin(x)}{125} + \frac{16\,x\,\cos(x)}{15} + \frac{4\,x^2\cos(x)^2\,\sin(x)}{15} + \frac{x^2\cos(x)^4\,\sin(x)}{5}$$

```
[In] int(x^2*cos(x)^5,x)
[Out] (8*x*cos(x)^3)/45 - (4144*sin(x))/3375 + (2*x*cos(x)^5)/25 + (8*x^2*sin(x))
/15 - (272*cos(x)^2*sin(x))/3375 - (2*cos(x)^4*sin(x))/125 + (16*x*cos(x))/
15 + (4*x^2*cos(x)^2*sin(x))/15 + (x^2*cos(x)^4*sin(x))/5
```

3.484 $\int x^3 \sin^3(x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 73

$$\int x^3 \sin^3(x) \, dx = \frac{40}{9} x \cos(x) - \frac{2}{3} x^3 \cos(x) - \frac{40 \sin(x)}{9} + 2x^2 \sin(x) + \frac{2}{9} x \cos(x) \sin^2(x) - \frac{1}{3} x^3 \cos(x) \sin^2(x) - \frac{2 \sin^3(x)}{27} + \frac{1}{3} x^2 \sin^3(x)$$

```
[Out] 40/9*x*cos(x)-2/3*x^3*cos(x)-40/9*sin(x)+2*x^2*sin(x)+2/9*x*cos(x)*sin(x)^2
-1/3*x^3*cos(x)*sin(x)^2-2/27*sin(x)^3+1/3*x^2*sin(x)^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3392, 3377, 2717, 3391}

$$\int x^3 \sin^3(x) \, dx = -\frac{2}{3} x^3 \cos(x) - \frac{1}{3} x^3 \sin^2(x) \cos(x) + \frac{1}{3} x^2 \sin^3(x) + 2x^2 \sin(x) - \frac{2 \sin^3(x)}{27} - \frac{40 \sin(x)}{9} + \frac{40}{9} x \cos(x) + \frac{2}{9} x \sin^2(x) \cos(x)$$

```
[In] Int[x^3*Sin[x]^3,x]
```

```
[Out] (40*x*Cos[x])/9 - (2*x^3*Cos[x])/3 - (40*Sin[x])/9 + 2*x^2*Sin[x] + (2*x*Co
s[x]*Sin[x]^2)/9 - (x^3*Cos[x]*Sin[x]^2)/3 - (2*Sin[x]^3)/27 + (x^2*Sin[x]^
3)/3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{3}x^3\cos(x)\sin^2(x)+\frac{1}{3}x^2\sin^3(x)+\frac{2}{3}\int x^3\sin(x)\,dx-\frac{2}{3}\int x\sin^3(x)\,dx\\
&= -\frac{2}{3}x^3\cos(x)+\frac{2}{9}x\cos(x)\sin^2(x)-\frac{1}{3}x^3\cos(x)\sin^2(x)-\frac{2\sin^3(x)}{27}\\
&\quad+\frac{1}{3}x^2\sin^3(x)-\frac{4}{9}\int x\sin(x)\,dx+2\int x^2\cos(x)\,dx\\
&= \frac{4}{9}x\cos(x)-\frac{2}{3}x^3\cos(x)+2x^2\sin(x)+\frac{2}{9}x\cos(x)\sin^2(x)-\frac{1}{3}x^3\cos(x)\sin^2(x)\\
&\quad-\frac{2\sin^3(x)}{27}+\frac{1}{3}x^2\sin^3(x)-\frac{4}{9}\int\cos(x)\,dx-4\int x\sin(x)\,dx\\
&= \frac{40}{9}x\cos(x)-\frac{2}{3}x^3\cos(x)-\frac{4\sin(x)}{9}+2x^2\sin(x)+\frac{2}{9}x\cos(x)\sin^2(x)\\
&\quad-\frac{1}{3}x^3\cos(x)\sin^2(x)-\frac{2\sin^3(x)}{27}+\frac{1}{3}x^2\sin^3(x)-4\int\cos(x)\,dx\\
&= \frac{40}{9}x\cos(x)-\frac{2}{3}x^3\cos(x)-\frac{40\sin(x)}{9}+2x^2\sin(x)\\
&\quad+\frac{2}{9}x\cos(x)\sin^2(x)-\frac{1}{3}x^3\cos(x)\sin^2(x)-\frac{2\sin^3(x)}{27}+\frac{1}{3}x^2\sin^3(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int x^3 \sin^3(x)\,dx = \frac{1}{108}\left(-81x\left(-6+x^2\right)\cos(x)+3x\left(-2+3x^2\right)\cos(3x)\right.$$
$$\left.+243\left(-2+x^2\right)\sin(x)-\left(-2+9x^2\right)\sin(3x)\right)$$

```
[In] Integrate[x^3*Sin[x]^3,x]
```

```
[Out] (-81*x*(-6 + x^2)*Cos[x] + 3*x*(-2 + 3*x^2)*Cos[3*x] + 243*(-2 + x^2)*Sin[x
] - (-2 + 9*x^2)*Sin[3*x])/108
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

method	result
risch	$\left(-\frac{3}{4}x^3+\frac{9}{2}x\right)\cos\left(x\right)+\frac{9(x^2-2)\sin(x)}{4}+\left(\frac{1}{12}x^3-\frac{1}{18}x\right)\cos\left(3x\right)-\frac{(9x^2-2)\sin(3x)}{108}$
default	$-\frac{x^3(2+\sin^2(x))\cos(x)}{3}+2x^2\sin\left(x\right)-\frac{40\sin(x)}{9}+4x\cos\left(x\right)+\frac{x^2(\sin^3(x))}{3}+\frac{2x(2+\sin^2(x))\cos(x)}{9}-\frac{2(\sin^3(x))}{27}$
norman	$\frac{\frac{40x}{9}-\frac{2x^3}{3}-\frac{496\left(\tan^3\left(\frac{x}{2}\right)\right)}{27}-\frac{80\left(\tan^5\left(\frac{x}{2}\right)\right)}{9}+\frac{16x\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}-\frac{16x\left(\tan^4\left(\frac{x}{2}\right)\right)}{3}-\frac{40x\left(\tan^6\left(\frac{x}{2}\right)\right)}{9}+4x^2\tan\left(\frac{x}{2}\right)-2x^3\left(\tan^2\left(\frac{x}{2}\right)\right)+2x^3\left(\tan^4\left(\frac{x}{2}\right.\right.}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$

```
[In] int(x^3*sin(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/4*x^3+9/2*x)*cos(x)+9/4*(x^2-2)*sin(x)+(1/12*x^3-1/18*x)*cos(3*x)-1/108
*(9*x^2-2)*sin(3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int x^3 \sin^3(x)\,dx = \frac{1}{9}\left(3\,x^3-2\,x\right)\cos\left(x\right)^3-\frac{1}{3}\left(3\,x^3-14\,x\right)\cos\left(x\right)$$
$$-\frac{1}{27}\left(\left(9\,x^2-2\right)\cos\left(x\right)^2-63\,x^2+122\right)\sin\left(x\right)$$

```
[In] integrate(x^3*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/9*(3*x^3 - 2*x)*cos(x)^3 - 1/3*(3*x^3 - 14*x)*cos(x) - 1/27*((9*x^2 - 2)*
cos(x)^2 - 63*x^2 + 122)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int x^3 \sin^3(x)\,dx = -x^3 \sin^2(x)\cos(x) - \frac{2x^3\cos^3(x)}{3} + \frac{7x^2\sin^3(x)}{3} + 2x^2\sin(x)\cos^2(x) + \frac{14x\sin^2(x)\cos(x)}{3} + \frac{40x\cos^3(x)}{9} - \frac{122\sin^3(x)}{27} - \frac{40\sin(x)\cos^2(x)}{9}$$

```
[In] integrate(x**3*sin(x)**3,x)
```

```
[Out] -x**3*sin(x)**2*cos(x) - 2*x**3*cos(x)**3/3 + 7*x**2*sin(x)**3/3 + 2*x**2*s
in(x)*cos(x)**2 + 14*x*sin(x)**2*cos(x)/3 + 40*x*cos(x)**3/9 - 122*sin(x)**
3/27 - 40*sin(x)*cos(x)**2/9
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int x^3 \sin^3(x)\,dx = \frac{1}{36}\left(3x^3 - 2x\right)\cos(3x) - \frac{3}{4}\left(x^3 - 6x\right)\cos(x) - \frac{1}{108}\left(9x^2 - 2\right)\sin(3x) + \frac{9}{4}\left(x^2 - 2\right)\sin(x)$$

```
[In] integrate(x^3*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(3*x^3 - 2*x)*cos(3*x) - 3/4*(x^3 - 6*x)*cos(x) - 1/108*(9*x^2 - 2)*si
n(3*x) + 9/4*(x^2 - 2)*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int x^3 \sin^3(x)\,dx = \frac{1}{36}\left(3x^3 - 2x\right)\cos(3x) - \frac{3}{4}\left(x^3 - 6x\right)\cos(x) - \frac{1}{108}\left(9x^2 - 2\right)\sin(3x) + \frac{9}{4}\left(x^2 - 2\right)\sin(x)$$

```
[In] integrate(x^3*sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/36*(3*x^3 - 2*x)*cos(3*x) - 3/4*(x^3 - 6*x)*cos(x) - 1/108*(9*x^2 - 2)*si
n(3*x) + 9/4*(x^2 - 2)*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int x^3 \sin^3(x)\,dx = \frac{7\,x^2\,\sin(x)}{3} - \frac{2\,x\cos(x)^3}{9} - x^3\,\cos(x) - \frac{122\,\sin(x)}{27} + \frac{x^3\cos(x)^3}{3} + \frac{2\cos(x)^2\,\sin(x)}{27} + \frac{14\,x\,\cos(x)}{3} - \frac{x^2\cos(x)^2\,\sin(x)}{3}$$

```
[In] int(x^3*sin(x)^3,x)
```

```
[Out] (7*x^2*sin(x))/3 - (2*x*cos(x)^3)/9 - x^3*cos(x) - (122*sin(x))/27 + (x^3*c
os(x)^3)/3 + (2*cos(x)^2*sin(x))/27 + (14*x*cos(x))/3 - (x^2*cos(x)^2*sin(x
))/3
```

3.485 $\int x^2 \sin^6(x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 105

$$\int x^2 \sin^6(x)\,dx = -\frac{245x}{1152}+\frac{5x^3}{48}+\frac{245\cos(x)\sin(x)}{1152}-\frac{5}{16}x^2\cos(x)\sin(x)+\frac{5}{16}x\sin^2(x)+\frac{65\cos(x)\sin^3(x)}{1728}-\frac{5}{24}x^2\cos(x)\sin^3(x)+\frac{5}{48}x\sin^4(x)+\frac{1}{108}\cos(x)\sin^5(x)-\frac{1}{6}x^2\cos(x)\sin^5(x)+\frac{1}{18}x\sin^6(x)$$

```
[Out] -245/1152*x+5/48*x^3+245/1152*cos(x)*sin(x)-5/16*x^2*cos(x)*sin(x)+5/16*x*s
in(x)^2+65/1728*cos(x)*sin(x)^3-5/24*x^2*cos(x)*sin(x)^3+5/48*x*sin(x)^4+1/
108*cos(x)*sin(x)^5-1/6*x^2*cos(x)*sin(x)^5+1/18*x*sin(x)^6
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3392, 30, 2715, 8}

$$\int x^2 \sin^6(x)\,dx = \frac{5x^3}{48}-\frac{1}{6}x^2\sin^5(x)\cos(x)-\frac{5}{24}x^2\sin^3(x)\cos(x)-\frac{5}{16}x^2\sin(x)\cos(x)-\frac{245x}{1152}+\frac{1}{18}x\sin^6(x)+\frac{5}{48}x\sin^4(x)+\frac{5}{16}x\sin^2(x)+\frac{1}{108}\sin^5(x)\cos(x)+\frac{65\sin^3(x)\cos(x)}{1728}+\frac{245\sin(x)\cos(x)}{1152}$$

```
[In] Int[x^2*Sin[x]^6,x]
```

```
[Out] (-245*x)/1152 + (5*x^3)/48 + (245*Cos[x]*Sin[x])/1152 - (5*x^2*Cos[x]*Sin[x
])/16 + (5*x*Sin[x]^2)/16 + (65*Cos[x]*Sin[x]^3)/1728 - (5*x^2*Cos[x]*Sin[x
```

```
]^3)/24 + (5*x*Sin[x]^4)/48 + (Cos[x]*Sin[x]^5)/108 - (x^2*Cos[x]*Sin[x]^5)
/6 + (x*Sin[x]^6)/18
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{6}x^2\cos(x)\sin^5(x)+\frac{1}{18}x\sin^6(x)-\frac{1}{18}\int\sin^6(x)\,dx+\frac{5}{6}\int x^2\sin^4(x)\,dx\\
&= -\frac{5}{24}x^2\cos(x)\sin^3(x)+\frac{5}{48}x\sin^4(x)+\frac{1}{108}\cos(x)\sin^5(x)-\frac{1}{6}x^2\cos(x)\sin^5(x)\\
&\quad+\frac{1}{18}x\sin^6(x)-\frac{5}{108}\int\sin^4(x)\,dx-\frac{5}{48}\int\sin^4(x)\,dx+\frac{5}{8}\int x^2\sin^2(x)\,dx\\
&= -\frac{5}{16}x^2\cos(x)\sin(x)+\frac{5}{16}x\sin^2(x)+\frac{65\cos(x)\sin^3(x)}{1728}-\frac{5}{24}x^2\cos(x)\sin^3(x)\\
&\quad+\frac{5}{48}x\sin^4(x)+\frac{1}{108}\cos(x)\sin^5(x)-\frac{1}{6}x^2\cos(x)\sin^5(x)+\frac{1}{18}x\sin^6(x)\\
&\quad-\frac{5}{144}\int\sin^2(x)\,dx-\frac{5}{64}\int\sin^2(x)\,dx+\frac{5\int x^2\,dx}{16}-\frac{5}{16}\int\sin^2(x)\,dx
\end{aligned}
$$

$$= \frac{5x^3}{48} + \frac{245\cos(x)\sin(x)}{1152} - \frac{5}{16}x^2\cos(x)\sin(x) + \frac{5}{16}x\sin^2(x) + \frac{65\cos(x)\sin^3(x)}{1728} - \frac{5}{24}x^2\cos(x)\sin^3(x) + \frac{5}{48}x\sin^4(x) + \frac{1}{108}\cos(x)\sin^5(x) - \frac{1}{6}x^2\cos(x)\sin^5(x) + \frac{1}{18}x\sin^6(x) - \frac{5\int 1\,dx}{288} - \frac{5\int 1\,dx}{128} - \frac{5\int 1\,dx}{32}$$

$$= -\frac{245x}{1152} + \frac{5x^3}{48} + \frac{245\cos(x)\sin(x)}{1152} - \frac{5}{16}x^2\cos(x)\sin(x) + \frac{5}{16}x\sin^2(x) + \frac{65\cos(x)\sin^3(x)}{1728} - \frac{5}{24}x^2\cos(x)\sin^3(x) + \frac{5}{48}x\sin^4(x) + \frac{1}{108}\cos(x)\sin^5(x) - \frac{1}{6}x^2\cos(x)\sin^5(x) + \frac{1}{18}x\sin^6(x)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int x^2\sin^6(x)\,dx = \frac{1440x^3 - 3240x\cos(2x) + 324x\cos(4x) - 24x\cos(6x) - 1620(-1+2x^2)\sin(2x) + 81(-1+8x^2)\sin(4x)}{13824}$$

```
[In] Integrate[x^2*Sin[x]^6,x]
[Out] (1440*x^3 - 3240*x*Cos[2*x] + 324*x*Cos[4*x] - 24*x*Cos[6*x] - 1620*(-1 + 2
*x^2)*Sin[2*x] + 81*(-1 + 8*x^2)*Sin[4*x] - 4*(-1 + 18*x^2)*Sin[6*x])/13824
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

method	result
risch	$\frac{5x^3}{48} - \frac{x\cos(6x)}{576} - \frac{(18x^2-1)\sin(6x)}{3456} + \frac{3x\cos(4x)}{128} + \frac{3(8x^2-1)\sin(4x)}{512} - \frac{15x\cos(2x)}{64} - \frac{15(2x^2-1)\sin(2x)}{128}$
default	$x^2\left(-\frac{\left(\sin^5(x)+\frac{5(\sin^3(x))}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6} + \frac{5x}{16}\right) + \frac{x(\sin^6(x))}{18} + \frac{\left(\sin^5(x)+\frac{5(\sin^3(x))}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{108} + \frac{115x}{1152} +$

```
[In] int(x^2*sin(x)^6,x,method=_RETURNVERBOSE)
[Out] 5/48*x^3-1/576*x*cos(6*x)-1/3456*(18*x^2-1)*sin(6*x)+3/128*x*cos(4*x)+3/512
*(8*x^2-1)*sin(4*x)-15/64*x*cos(2*x)-15/128*(2*x^2-1)*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.69

$$\int x^2 \sin^6(x)\,dx = -\frac{1}{18}\,x\cos\left(x\right)^6 + \frac{13}{48}\,x\cos\left(x\right)^4 + \frac{5}{48}\,x^3 - \frac{11}{16}\,x\cos\left(x\right)^2 - \frac{1}{3456}\left(32\left(18\,x^2-1\right)\cos\left(x\right)^5 - 2\left(936\,x^2-97\right)\cos\left(x\right)^3 + 3\left(792\,x^2-299\right)\cos\left(x\right)\right)\sin\left(x\right) + \frac{299}{1152}\,x$$

```
[In] integrate(x^2*sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/18*x*cos(x)^6 + 13/48*x*cos(x)^4 + 5/48*x^3 - 11/16*x*cos(x)^2 - 1/3456*
(32*(18*x^2 - 1)*cos(x)^5 - 2*(936*x^2 - 97)*cos(x)^3 + 3*(792*x^2 - 299)*c
os(x))*sin(x) + 299/1152*x
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.83

$$\int x^2 \sin^6(x)\,dx = \frac{5x^3\sin^6\left(x\right)}{48} + \frac{5x^3\sin^4\left(x\right)\cos^2\left(x\right)}{16} + \frac{5x^3\sin^2\left(x\right)\cos^4\left(x\right)}{16} + \frac{5x^3\cos^6\left(x\right)}{48} - \frac{11x^2\sin^5\left(x\right)\cos\left(x\right)}{16} - \frac{5x^2\sin^3\left(x\right)\cos^3\left(x\right)}{6} - \frac{5x^2\sin\left(x\right)\cos^5\left(x\right)}{16} + \frac{299x\sin^6\left(x\right)}{1152} + \frac{35x\sin^4\left(x\right)\cos^2\left(x\right)}{384} - \frac{125x\sin^2\left(x\right)\cos^4\left(x\right)}{384} - \frac{245x\cos^6\left(x\right)}{1152} + \frac{299\sin^5\left(x\right)\cos\left(x\right)}{1152} + \frac{25\sin^3\left(x\right)\cos^3\left(x\right)}{54} + \frac{245\sin\left(x\right)\cos^5\left(x\right)}{1152}$$

```
[In] integrate(x**2*sin(x)**6,x)
```

```
[Out] 5*x**3*sin(x)**6/48 + 5*x**3*sin(x)**4*cos(x)**2/16 + 5*x**3*sin(x)**2*cos(
x)**4/16 + 5*x**3*cos(x)**6/48 - 11*x**2*sin(x)**5*cos(x)/16 - 5*x**2*sin(x
)**3*cos(x)**3/6 - 5*x**2*sin(x)*cos(x)**5/16 + 299*x*sin(x)**6/1152 + 35*x
*sin(x)**4*cos(x)**2/384 - 125*x*sin(x)**2*cos(x)**4/384 - 245*x*cos(x)**6/
1152 + 299*sin(x)**5*cos(x)/1152 + 25*sin(x)**3*cos(x)**3/54 + 245*sin(x)*c
os(x)**5/1152
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int x^2 \sin^6(x)\,dx = \frac{5}{48}\,x^3 - \frac{1}{576}\,x\cos(6\,x) + \frac{3}{128}\,x\cos(4\,x) - \frac{15}{64}\,x\cos(2\,x) - \frac{1}{3456}\,\left(18\,x^2 - 1\right)\sin(6\,x) + \frac{3}{512}\,\left(8\,x^2 - 1\right)\sin(4\,x) - \frac{15}{128}\,\left(2\,x^2 - 1\right)\sin(2\,x)$$

```
[In] integrate(x^2*sin(x)^6,x, algorithm="maxima")
```

```
[Out] 5/48*x^3 - 1/576*x*cos(6*x) + 3/128*x*cos(4*x) - 15/64*x*cos(2*x) - 1/3456*
(18*x^2 - 1)*sin(6*x) + 3/512*(8*x^2 - 1)*sin(4*x) - 15/128*(2*x^2 - 1)*sin
(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int x^2 \sin^6(x)\,dx = \frac{5}{48}\,x^3 - \frac{1}{576}\,x\cos(6\,x) + \frac{3}{128}\,x\cos(4\,x) - \frac{15}{64}\,x\cos(2\,x) - \frac{1}{3456}\,\left(18\,x^2 - 1\right)\sin(6\,x) + \frac{3}{512}\,\left(8\,x^2 - 1\right)\sin(4\,x) - \frac{15}{128}\,\left(2\,x^2 - 1\right)\sin(2\,x)$$

```
[In] integrate(x^2*sin(x)^6,x, algorithm="giac")
```

```
[Out] 5/48*x^3 - 1/576*x*cos(6*x) + 3/128*x*cos(4*x) - 15/64*x*cos(2*x) - 1/3456*
(18*x^2 - 1)*sin(6*x) + 3/512*(8*x^2 - 1)*sin(4*x) - 15/128*(2*x^2 - 1)*sin
(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

$$\int x^2 \sin^6(x)\,dx = \frac{15\,\sin(2\,x)}{128} - \frac{3\,\sin(4\,x)}{512} + \frac{\sin(6\,x)}{3456} - \frac{3\,x\,\left(2\sin(2\,x)^2 - 1\right)}{128} + \frac{x\,\left(2\sin(3\,x)^2 - 1\right)}{576} - \frac{15\,x^2\,\sin(2\,x)}{64} + \frac{3\,x^2\,\sin(4\,x)}{64} - \frac{x^2\,\sin(6\,x)}{192} + \frac{5\,x^3}{48} + \frac{15\,x\,\left(2\sin(x)^2 - 1\right)}{64}$$

[In] int(x^2*sin(x)^6,x)

[Out] (15*sin(2*x))/128 - (3*sin(4*x))/512 + sin(6*x)/3456 - (3*x*(2*sin(2*x)^2 - 1))/128 + (x*(2*sin(3*x)^2 - 1))/576 - (15*x^2*sin(2*x))/64 + (3*x^2*sin(4*x))/64 - (x^2*sin(6*x))/192 + (5*x^3)/48 + (15*x*(2*sin(x)^2 - 1))/64

3.486 $\int x^2 \cos(x) \sin^2(x) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 44

$$\int x^2 \cos(x) \sin^2(x) \, dx = \frac{4}{9} x \cos(x) - \frac{4 \sin(x)}{9} + \frac{2}{9} x \cos(x) \sin^2(x) - \frac{2 \sin^3(x)}{27} + \frac{1}{3} x^2 \sin^3(x)$$

```
[Out] 4/9*x*cos(x)-4/9*sin(x)+2/9*x*cos(x)*sin(x)^2-2/27*sin(x)^3+1/3*x^2*sin(x)^
3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3524, 3391, 3377, 2717}

$$\int x^2 \cos(x) \sin^2(x) \, dx = \frac{1}{3} x^2 \sin^3(x) - \frac{2 \sin^3(x)}{27} - \frac{4 \sin(x)}{9} + \frac{4}{9} x \cos(x) + \frac{2}{9} x \sin^2(x) \cos(x)$$

```
[In] Int[x^2*Cos[x]*Sin[x]^2,x]
```

```
[Out] (4*x*Cos[x])/9 - (4*Sin[x])/9 + (2*x*Cos[x]*Sin[x]^2)/9 - (2*Sin[x]^3)/27 +
 (x^2*Sin[x]^3)/3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^2\sin^3(x) - \frac{2}{3}\int x\sin^3(x)\,dx \\
&= \frac{2}{9}x\cos(x)\sin^2(x) - \frac{2\sin^3(x)}{27} + \frac{1}{3}x^2\sin^3(x) - \frac{4}{9}\int x\sin(x)\,dx \\
&= \frac{4}{9}x\cos(x) + \frac{2}{9}x\cos(x)\sin^2(x) - \frac{2\sin^3(x)}{27} + \frac{1}{3}x^2\sin^3(x) - \frac{4}{9}\int\cos(x)\,dx \\
&= \frac{4}{9}x\cos(x) - \frac{4\sin(x)}{9} + \frac{2}{9}x\cos(x)\sin^2(x) - \frac{2\sin^3(x)}{27} + \frac{1}{3}x^2\sin^3(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int x^2\cos(x)\sin^2(x)\,dx = \frac{1}{54}\left(27x\cos(x) - 3x\cos(3x) + \left(-26 + 9x^2 + \left(2 - 9x^2\right)\cos(2x)\right)\sin(x)\right)$$

```
[In] Integrate[x^2*Cos[x]*Sin[x]^2,x]
[Out] (27*x*Cos[x] - 3*x*Cos[3*x] + (-26 + 9*x^2 + (2 - 9*x^2)*Cos[2*x])*Sin[x])/
54
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{x^2(\sin^3(x))}{3}+\frac{2x(2+\sin^2(x))\cos(x)}{9}-\frac{2(\sin^3(x))}{27}-\frac{4\sin(x)}{9}$	32
risch	$\frac{x\cos(x)}{2}+\frac{(x^2-2)\sin(x)}{4}-\frac{x\cos(3x)}{18}-\frac{(9x^2-2)\sin(3x)}{108}$	36
parallelrisch	$\frac{x\cos(x)}{2}+\frac{x^2\sin(x)}{4}-\frac{\sin(x)}{2}-\frac{x\cos(3x)}{18}-\frac{x^2\sin(3x)}{12}+\frac{\sin(3x)}{54}$	40
norman	$\frac{\frac{4x}{9}-\frac{64\left(\tan^3\left(\frac{x}{2}\right)\right)}{27}-\frac{8\left(\tan^5\left(\frac{x}{2}\right)\right)}{9}+\frac{4x\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}-\frac{4x\left(\tan^4\left(\frac{x}{2}\right)\right)}{3}-\frac{4x\left(\tan^6\left(\frac{x}{2}\right)\right)}{9}+\frac{8\left(\tan^3\left(\frac{x}{2}\right)\right)x^2}{3}-\frac{8\tan\left(\frac{x}{2}\right)}{9}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$	76

```
[In] int(x^2*cos(x)*sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*x^2*sin(x)^3+2/9*x*(2+sin(x)^2)*cos(x)-2/27*sin(x)^3-4/9*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int x^2\cos(x)\sin^2(x)\,dx = -\frac{2}{9}\,x\cos\left(x\right)^3+\frac{2}{3}\,x\cos\left(x\right) - \frac{1}{27}\left(\left(9\,x^2-2\right)\cos\left(x\right)^2-9\,x^2+14\right)\sin\left(x\right)$$

```
[In] integrate(x^2*cos(x)*sin(x)^2,x, algorithm="fricas")
[Out] -2/9*x*cos(x)^3 + 2/3*x*cos(x) - 1/27*((9*x^2 - 2)*cos(x)^2 - 9*x^2 + 14)*s
in(x)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int x^2\cos(x)\sin^2(x)\,dx = \frac{x^2\sin^3\left(x\right)}{3}+\frac{2x\sin^2\left(x\right)\cos\left(x\right)}{3}+\frac{4x\cos^3\left(x\right)}{9} - \frac{14\sin^3\left(x\right)}{27}-\frac{4\sin\left(x\right)\cos^2\left(x\right)}{9}$$

```
[In] integrate(x**2*cos(x)*sin(x)**2,x)
[Out] x**2*sin(x)**3/3 + 2*x*sin(x)**2*cos(x)/3 + 4*x*cos(x)**3/9 - 14*sin(x)**3/
27 - 4*sin(x)*cos(x)**2/9
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int x^2 \cos(x)\sin^2(x)\,dx = -\frac{1}{18}\,x\cos\left(3\,x\right) + \frac{1}{2}\,x\cos\left(x\right) - \frac{1}{108}\left(9\,x^2 - 2\right)\sin\left(3\,x\right) + \frac{1}{4}\left(x^2 - 2\right)\sin\left(x\right)$$

```
[In] integrate(x^2*cos(x)*sin(x)^2,x, algorithm="maxima")
[Out] -1/18*x*cos(3*x) + 1/2*x*cos(x) - 1/108*(9*x^2 - 2)*sin(3*x) + 1/4*(x^2 - 2
)*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int x^2 \cos(x)\sin^2(x)\,dx = -\frac{1}{18}\,x\cos\left(3\,x\right) + \frac{1}{2}\,x\cos\left(x\right) - \frac{1}{108}\left(9\,x^2 - 2\right)\sin\left(3\,x\right) + \frac{1}{4}\left(x^2 - 2\right)\sin\left(x\right)$$

```
[In] integrate(x^2*cos(x)*sin(x)^2,x, algorithm="giac")
[Out] -1/18*x*cos(3*x) + 1/2*x*cos(x) - 1/108*(9*x^2 - 2)*sin(3*x) + 1/4*(x^2 - 2
)*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int x^2 \cos(x)\sin^2(x)\,dx = \frac{x^2\sin\left(x\right)^3}{3} + \frac{4\,x\cos\left(x\right)^3}{9} + \frac{2\,x\,\cos\left(x\right)\,\sin\left(x\right)^2}{3} - \frac{4\cos\left(x\right)^2\,\sin\left(x\right)}{9} - \frac{14\sin\left(x\right)^3}{27}$$

```
[In] int(x^2*cos(x)*sin(x)^2,x)
[Out] (4*x*cos(x)^3)/9 - (14*sin(x)^3)/27 + (x^2*sin(x)^3)/3 - (4*cos(x)^2*sin(x)
)/9 + (2*x*cos(x)*sin(x)^2)/3
```

3.487 $\int x\cos^2(x)\cot^2(x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 33

$$\int x\cos^2(x)\cot^2(x)\,dx=-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\cos(x)\sin(x)$$

```
[Out] -3/4*x^2-1/4*cos(x)^2-x*cot(x)+ln(sin(x))-1/2*x*cos(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4493, 3391, 30, 3801, 3556}

$$\int x\cos^2(x)\cot^2(x)\,dx=-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\sin(x)\cos(x)$$

```
[In] Int[x*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] (-3*x^2)/4 - Cos[x]^2/4 - x*Cot[x] + Log[Sin[x]] - (x*Cos[x]*Sin[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

```
]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4493

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int x\cos^2(x)\,dx + \int x\cot^2(x)\,dx \\
&= -\frac{1}{4}\cos^2(x) - x\cot(x) - \frac{1}{2}x\cos(x)\sin(x) - \frac{\int x\,dx}{2} - \int x\,dx + \int \cot(x)\,dx \\
&= -\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) + \log(\sin(x)) - \frac{1}{2}x\cos(x)\sin(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x\cos^2(x)\cot^2(x)\,dx = -\frac{3x^2}{4} - \frac{1}{8}\cos(2x) - x\cot(x) + \log(\sin(x)) - \frac{1}{4}x\sin(2x)$$

```
[In] Integrate[x*Cos[x]^2*Cot[x]^2,x]
[Out] (-3*x^2)/4 - Cos[2*x]/8 - x*Cot[x] + Log[Sin[x]] - (x*Sin[2*x])/4
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

method	result
parallelrisch	$-\frac{3x^2}{4}-\frac{3}{8}+\ln\left(\frac{\csc(x)}{2}-\frac{\cot(x)}{2}\right)-\ln\left(\frac{1}{\cos(x)+1}\right)+\frac{x\cot(x)\cos(2x)}{4}-\frac{5x\cot(x)}{4}-\frac{\cos(2x)}{8}$
risch	$-\frac{3x^2}{4}+\frac{i(i+2x)e^{2ix}}{16}-\frac{i(-i+2x)e^{-2ix}}{16}-2ix-\frac{2ix}{e^{2ix}-1}+\ln\left(e^{2ix}-1\right)$
norman	$\frac{\tan^3\left(\frac{x}{2}\right)-\frac{x}{2}-\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\frac{x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}-\frac{3x^2\tan\left(\frac{x}{2}\right)}{4}-\frac{3\left(\tan^3\left(\frac{x}{2}\right)\right)x^2}{2}-\frac{3\left(\tan^5\left(\frac{x}{2}\right)\right)x^2}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\tan\left(\frac{x}{2}\right)}-\ln\left(1+\tan^2\left(\right.\right.$

```
[In] int(x*cos(x)^4/sin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*x^2-3/8+ln(1/2*csc(x)-1/2*cot(x))-ln(1/(cos(x)+1))+1/4*x*cot(x)*cos(2*
x)-5/4*x*cot(x)-1/8*cos(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int x\cos^2(x)\cot^2(x)\,dx = \frac{4\,x\cos(x)^3-12\,x\cos(x)-\left(6\,x^2+2\,\cos(x)^2-1\right)\sin(x)+8\,\log\left(\frac{1}{2}\,\sin(x)\right)\sin(x)}{8\,\sin(x)}$$

```
[In] integrate(x*cos(x)^4/sin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*x*cos(x)^3 - 12*x*cos(x) - (6*x^2 + 2*cos(x)^2 - 1)*sin(x) + 8*log(1
/2*sin(x))*sin(x))/sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(32) = 64$.

Time = 0.68 (sec) , antiderivative size = 507, normalized size of antiderivative = 15.36

$$\int x\cos^2(x)\cot^2(x)\,dx = -\frac{3x^2\tan^5\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} - \frac{6x^2\tan^3\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} - \frac{3x^2\tan\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} + \frac{2x\tan^6\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} + \frac{6x\tan^4\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} - \frac{6x\tan^2\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} - \frac{2x}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} - \frac{4\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^5\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} - \frac{8\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^3\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} - \frac{4\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} + \frac{4\log\left(\tan\left(\frac{x}{2}\right)\right)\tan^5\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} + \frac{8\log\left(\tan\left(\frac{x}{2}\right)\right)\tan^3\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} + \frac{4\log\left(\tan\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} + \frac{4\tan^3\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)}$$

```
[In] integrate(x*cos(x)**4/sin(x)**2,x)
```

```
[Out] -3*x**2*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 6*x**2*t
an(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 3*x**2*tan(x/2)/(
4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 2*x*tan(x/2)**6/(4*tan(x/2)**
5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 6*x*tan(x/2)**4/(4*tan(x/2)**5 + 8*tan(x/
2)**3 + 4*tan(x/2)) - 6*x*tan(x/2)**2/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*ta
n(x/2)) - 2*x/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 4*log(tan(x/2)
```

```
**2 + 1)*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 8*log(t
an(x/2)**2 + 1)*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) -
4*log(tan(x/2)**2 + 1)*tan(x/2)/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)
) + 4*log(tan(x/2))*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)
) + 8*log(tan(x/2))*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)
) + 4*log(tan(x/2))*tan(x/2)/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) +
 4*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2))
```

Maxima [F(-2)]

Exception generated.

$$\int x\cos^2(x)\cot^2(x)\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*cos(x)^4/sin(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(27) = 54$.

Time = 0.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 6.24

$$\int x\cos^2(x)\cot^2(x)\,dx = -\frac{6\,x^2\tan\left(\frac{1}{2}\,x\right)^5 - 4\,x\tan\left(\frac{1}{2}\,x\right)^6 - 4\,\log\left(\frac{16\,\tan\left(\frac{1}{2}\,x\right)^2}{\tan\left(\frac{1}{2}\,x\right)^4 + 2\,\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)\tan\left(\frac{1}{2}\,x\right)^5 + 12\,x^2\tan\left(\frac{1}{2}\,x\right)^3 - 12\,x\tan}{}$$

```
[In] integrate(x*cos(x)^4/sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/8*(6*x^2*tan(1/2*x)^5 - 4*x*tan(1/2*x)^6 - 4*log(16*tan(1/2*x)^2/(tan(1/
2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^5 + 12*x^2*tan(1/2*x)^3 - 12*x*tan
(1/2*x)^4 + tan(1/2*x)^5 - 8*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*
x)^2 + 1))*tan(1/2*x)^3 + 6*x^2*tan(1/2*x) + 12*x*tan(1/2*x)^2 - 6*tan(1/2*
x)^3 - 4*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x
) + 4*x + tan(1/2*x))/(tan(1/2*x)^5 + 2*tan(1/2*x)^3 + tan(1/2*x))
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int x\cos^2(x)\cot^2(x)\,dx = \ln\left(\mathrm{e}^{x\,2\mathrm{i}}-1\right)-\mathrm{e}^{-x\,2\mathrm{i}}\left(\frac{1}{16}+\frac{x\,1\mathrm{i}}{8}\right)+\mathrm{e}^{x\,2\mathrm{i}}\left(-\frac{1}{16}+\frac{x\,1\mathrm{i}}{8}\right)-\frac{3\,x^2}{4}-x\,2\mathrm{i}-\frac{x\,2\mathrm{i}}{\mathrm{e}^{x\,2\mathrm{i}}-1}$$

```
[In] int((x*cos(x)^4)/sin(x)^2,x)
```

```
[Out] log(exp(x*2i) - 1) - x*2i - exp(-x*2i)*((x*1i)/8 + 1/16) + exp(x*2i)*((x*1i
)/8 - 1/16) - (x*2i)/(exp(x*2i) - 1) - (3*x^2)/4
```

3.488 $\int x\sec(x)\tan^3(x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 30

$$\int x\sec(x)\tan^3(x)\,dx = \frac{5}{6}\text{arctanh}(\sin(x)) - x\sec(x) + \frac{1}{3}x\sec^3(x) - \frac{1}{6}\sec(x)\tan(x)$$

```
[Out] 5/6*arctanh(sin(x))-x*sec(x)+1/3*x*sec(x)^3-1/6*sec(x)*tan(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2686, 4502, 3855, 3853}

$$\int x\sec(x)\tan^3(x)\,dx = \frac{5}{6}\text{arctanh}(\sin(x)) + \frac{1}{3}x\sec^3(x) - x\sec(x) - \frac{1}{6}\tan(x)\sec(x)$$

```
[In] Int[x*Sec[x]*Tan[x]^3,x]
```

```
[Out] (5*ArcTanh[Sin[x]])/6 - x*Sec[x] + (x*Sec[x]^3)/3 - (Sec[x]*Tan[x])/6
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4502

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Sec[a + b*x]^n*Tan[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m, 0] && (IntegerQ[n/2] || Int
egerQ[(p - 1)/2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -x\sec(x)+\frac{1}{3}x\sec^3(x)-\int\left(-\sec(x)+\frac{\sec^3(x)}{3}\right)dx\\
&= -x\sec(x)+\frac{1}{3}x\sec^3(x)-\frac{1}{3}\int\sec^3(x)\,dx+\int\sec(x)\,dx\\
&= \operatorname{arctanh}(\sin(x))-x\sec(x)+\frac{1}{3}x\sec^3(x)-\frac{1}{6}\sec(x)\tan(x)-\frac{1}{6}\int\sec(x)\,dx\\
&= \frac{5}{6}\operatorname{arctanh}(\sin(x))-x\sec(x)+\frac{1}{3}x\sec^3(x)-\frac{1}{6}\sec(x)\tan(x)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 104 vs. $2(30)=60$.

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.47

$$\begin{aligned}
\int x\sec(x)\tan^3(x)\,dx = -\frac{1}{24}\sec^3(x)\Bigg(&4x+12x\cos(2x)+5\cos(3x)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\\
&+15\cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)\\
&-5\cos(3x)\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+2\sin(2x)\Bigg)
\end{aligned}$$

```
[In] Integrate[x*Sec[x]*Tan[x]^3,x]
```

```
[Out] -1/24*(Sec[x]^3*(4*x + 12*x*Cos[2*x] + 5*Cos[3*x]*Log[Cos[x/2] - Sin[x/2]]
+ 15*Cos[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) - 5*Cos[3
*x]*Log[Cos[x/2] + Sin[x/2]] + 2*Sin[2*x]))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

method	result	si
parallelrisch	$\ln\left(\left(-\cot(x)+1+\csc(x)\right)^{\frac{5}{6}}\right)+\ln\left(\frac{1}{\left(-\cot(x)+\csc(x)-1\right)^{\frac{5}{6}}}\right)+\frac{x\left(\sec^3(x)\right)}{3}+\frac{(-6x-\tan(x))\sec(x)}{6}$	43
norman	$\frac{\frac{2x}{3}-\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)}{3}-2x\left(\tan^2\left(\frac{x}{2}\right)\right)-2x\left(\tan^4\left(\frac{x}{2}\right)\right)+\frac{2x\left(\tan^6\left(\frac{x}{2}\right)\right)}{3}+\frac{\tan\left(\frac{x}{2}\right)}{3}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)^3}-\frac{5\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{6}+\frac{5\ln\left(1+\tan\left(\frac{x}{2}\right)\right)}{6}$	76
risch	$-\frac{e^{ix}\left(6x\,e^{4ix}+4x\,e^{2ix}-ie^{4ix}+6x+i\right)}{3\left(e^{2ix}+1\right)^3}+\frac{5\ln\left(i+e^{ix}\right)}{6}-\frac{5\ln\left(e^{ix}-i\right)}{6}$	78

```
[In] int(x*sin(x)^3/cos(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] ln((-cot(x)+1+csc(x))^(5/6))+ln(1/(-cot(x)+csc(x)-1)^(5/6))+1/3*x*sec(x)^3+
1/6*(-6*x-tan(x))*sec(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.57

$$\int x\sec(x)\tan^3(x)\,dx$$
$$=\frac{5\cos(x)^3\log(\sin(x)+1)-5\cos(x)^3\log(-\sin(x)+1)-12\,x\cos(x)^2-2\cos(x)\sin(x)+4\,x}{12\cos(x)^3}$$

```
[In] integrate(x*sin(x)^3/cos(x)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(5*cos(x)^3*log(sin(x) + 1) - 5*cos(x)^3*log(-sin(x) + 1) - 12*x*cos(x
)^2 - 2*cos(x)*sin(x) + 4*x)/cos(x)^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(29) = 58.

Time = 0.64 (sec) , antiderivative size = 551, normalized size of antiderivative = 18.37

$$\int x\sec(x)\tan^3(x)\,dx = \frac{4x\tan^6\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} - \frac{12x\tan^4\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} - \frac{12x\tan^2\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} + \frac{4x}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} - \frac{5\log\left(\tan\left(\frac{x}{2}\right)-1\right)\tan^6\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} + \frac{15\log\left(\tan\left(\frac{x}{2}\right)-1\right)\tan^4\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} - \frac{15\log\left(\tan\left(\frac{x}{2}\right)-1\right)\tan^2\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} + \frac{5\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} + \frac{5\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan^6\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} - \frac{15\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan^4\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} + \frac{15\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} - \frac{5\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} - \frac{2\tan^5\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} + \frac{2\tan\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6}$$

```
[In] integrate(x*sin(x)**3/cos(x)**4,x)
```

```
[Out] 4*x*tan(x/2)**6/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 12*
x*tan(x/2)**4/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 12*x*
tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 4*x/(6*
tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 5*log(tan(x/2) - 1)*ta
n(x/2)**6/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 15*log(ta
n(x/2) - 1)*tan(x/2)**4/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 -
```

```
6) - 15*log(tan(x/2) - 1)*tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*
tan(x/2)**2 - 6) + 5*log(tan(x/2) - 1)/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18
*tan(x/2)**2 - 6) + 5*log(tan(x/2) + 1)*tan(x/2)**6/(6*tan(x/2)**6 - 18*tan
(x/2)**4 + 18*tan(x/2)**2 - 6) - 15*log(tan(x/2) + 1)*tan(x/2)**4/(6*tan(x/
2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 15*log(tan(x/2) + 1)*tan(x/2
)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 5*log(tan(x/2)
 + 1)/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 2*tan(x/2)**5
/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 2*tan(x/2)/(6*tan(
x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(24) = 48$.

Time = 0.28 (sec) , antiderivative size = 619, normalized size of antiderivative = 20.63

$$\int x\sec(x)\tan^3(x)\,dx = \text{Too large to display}$$

```
[In] integrate(x*sin(x)^3/cos(x)^4,x, algorithm="maxima")
```

```
[Out] -1/12*(48*x*sin(3*x)*sin(2*x) + 4*(6*x*cos(5*x) + 4*x*cos(3*x) + 6*x*cos(x)
 + sin(5*x) - sin(x))*cos(6*x) + 12*(6*x*cos(4*x) + 6*x*cos(2*x) + 2*x - si
n(4*x) - sin(2*x))*cos(5*x) + 12*(4*x*cos(3*x) + 6*x*cos(x) - sin(x))*cos(4
*x) + 16*(3*x*cos(2*x) + x)*cos(3*x) + 12*(6*x*cos(x) - sin(x))*cos(2*x) +
24*x*cos(x) - 5*(2*(3*cos(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*
(3*cos(2*x) + 1)*cos(4*x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin
(2*x))*sin(6*x) + sin(6*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(
2*x)^2 + 6*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + 5*(2*(3*
cos(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*(3*cos(2*x) + 1)*cos(4
*x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin(2*x))*sin(6*x) + sin(
6*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) +
1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + 4*(6*x*sin(5*x) + 4*x*sin(3*x)
 + 6*x*sin(x) - cos(5*x) + cos(x))*sin(6*x) + 4*(18*x*sin(4*x) + 18*x*sin(2
*x) + 3*cos(4*x) + 3*cos(2*x) + 1)*sin(5*x) + 12*(4*x*sin(3*x) + 6*x*sin(x)
 + cos(x))*sin(4*x) + 12*(6*x*sin(x) + cos(x))*sin(2*x) - 4*sin(x))/(2*(3*c
os(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*(3*cos(2*x) + 1)*cos(4*
x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin(2*x))*sin(6*x) + sin(6
*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) + 1
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(24) = 48.

Time = 0.43 (sec) , antiderivative size = 341, normalized size of antiderivative = 11.37

$$\int x\sec(x)\tan^3(x)\,dx$$
$$= \frac{8\,x\tan\left(\frac{1}{2}\,x\right)^6+5\,\log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2+2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^6-5\,\log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2-2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^6-2}{}$$

```
[In] integrate(x*sin(x)^3/cos(x)^4,x, algorithm="giac")
```

```
[Out] 1/12*(8*x*tan(1/2*x)^6 + 5*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2
*x)^2 + 1))*tan(1/2*x)^6 - 5*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1
/2*x)^2 + 1))*tan(1/2*x)^6 - 24*x*tan(1/2*x)^4 - 15*log(2*(tan(1/2*x)^2 + 2
*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4 + 15*log(2*(tan(1/2*x)^2
- 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4 - 4*tan(1/2*x)^5 - 24*
x*tan(1/2*x)^2 + 15*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 +
 1))*tan(1/2*x)^2 - 15*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^
2 + 1))*tan(1/2*x)^2 + 8*x - 5*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan
(1/2*x)^2 + 1)) + 5*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 +
 1)) + 4*tan(1/2*x))/(tan(1/2*x)^6 - 3*tan(1/2*x)^4 + 3*tan(1/2*x)^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int x\sec(x)\tan^3(x)\,dx = -\frac{x\cos\left(x\right)^2-\frac{x}{3}+\frac{\sin(2\,x)}{12}}{\cos\left(x\right)^3}-\frac{\mathrm{atan}(\cos\left(x\right)+\sin\left(x\right)\ 1\mathrm{i})\ 5\mathrm{i}}{3}$$

```
[In] int((x*sin(x)^3)/cos(x)^4,x)
```

```
[Out] - (atan(cos(x) + sin(x)*1i)*5i)/3 - (sin(2*x)/12 - x/3 + x*cos(x)^2)/cos(x)
^3
```

3.489 $\int x\sec^2(x)\tan(x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 16

$$\int x\sec^2(x)\tan(x)\,dx=\frac{1}{2}x\sec^2(x)-\frac{\tan(x)}{2}$$

```
[Out] 1/2*x*sec(x)^2-1/2*tan(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3842, 3852, 8}

$$\int x\sec^2(x)\tan(x)\,dx=\frac{1}{2}x\sec^2(x)-\frac{\tan(x)}{2}$$

```
[In] Int[x*Sec[x]^2*Tan[x],x]
```

```
[Out] (x*Sec[x]^2)/2 - Tan[x]/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3842

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x\sec^2(x) - \frac{1}{2}\int \sec^2(x)\,dx\\ &= \frac{1}{2}x\sec^2(x) + \frac{1}{2}\text{Subst}(\int 1\,dx, x, -\tan(x))\\ &= \frac{1}{2}x\sec^2(x) - \frac{\tan(x)}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x\sec^2(x)\tan(x)\,dx = \frac{1}{2}x\sec^2(x) - \frac{\tan(x)}{2}$$

```
[In] Integrate[x*Sec[x]^2*Tan[x],x]
[Out] (x*Sec[x]^2)/2 - Tan[x]/2
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x}{2\cos(x)^2} - \frac{\tan(x)}{2}$	13
parallelrisch	$\frac{x(\sec^2(x))}{2} - \frac{\tan(x)}{2}$	13
risch	$\frac{2x\,e^{2ix} - ie^{2ix} - i}{(e^{2ix}+1)^2}$	30
norman	$\frac{\tan^3(\frac{x}{2}) + x(\tan^2(\frac{x}{2})) + \frac{x}{2} + \frac{x(\tan^4(\frac{x}{2}))}{2} - \tan(\frac{x}{2})}{(\tan^2(\frac{x}{2})-1)^2}$	45

```
[In] int(x*sin(x)/cos(x)^3,x,method=_RETURNVERBOSE)
[Out] 1/2*x/cos(x)^2-1/2*tan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x\sec^2(x)\tan(x)\,dx = -\frac{\cos(x)\sin(x) - x}{2\cos(x)^2}$$

```
[In] integrate(x*sin(x)/cos(x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(cos(x)*sin(x) - x)/cos(x)^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(12) = 24.

Time = 0.41 (sec) , antiderivative size = 128, normalized size of antiderivative = 8.00

$$\int x\sec^2(x)\tan(x)\,dx = \frac{x\tan^4\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right) - 4\tan^2\left(\frac{x}{2}\right) + 2} + \frac{2x\tan^2\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right) - 4\tan^2\left(\frac{x}{2}\right) + 2} + \frac{x}{2\tan^4\left(\frac{x}{2}\right) - 4\tan^2\left(\frac{x}{2}\right) + 2} + \frac{2\tan^3\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right) - 4\tan^2\left(\frac{x}{2}\right) + 2} - \frac{2\tan\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right) - 4\tan^2\left(\frac{x}{2}\right) + 2}$$

```
[In] integrate(x*sin(x)/cos(x)**3,x)
```

```
[Out] x*tan(x/2)**4/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) + 2*x*tan(x/2)**2/(2*tan(
x/2)**4 - 4*tan(x/2)**2 + 2) + x/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) + 2*ta
n(x/2)**3/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) - 2*tan(x/2)/(2*tan(x/2)**4 -
 4*tan(x/2)**2 + 2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(12) = 24.

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 8.25

$$\int x\sec^2(x)\tan(x)\,dx = \frac{4\,x\cos(2\,x)^2 + 4\,x\sin(2\,x)^2 + (2\,x\cos(2\,x) + \sin(2\,x))\cos(4\,x) + 2\,x\cos(2\,x) + (2\,x\sin(2\,x) - \cos(2\,x)}{2\,(2\,\cos(2\,x) + 1)\cos(4\,x) + \cos(4\,x)^2 + 4\,\cos(2\,x)^2 + \sin(4\,x)^2 + 4\,\sin(4\,x)\sin(2\,x) + 4\,\sin(2\,x)}$$

```
[In] integrate(x*sin(x)/cos(x)^3,x, algorithm="maxima")
```

```
[Out] (4*x*cos(2*x)^2 + 4*x*sin(2*x)^2 + (2*x*cos(2*x) + sin(2*x))*cos(4*x) + 2*x
*cos(2*x) + (2*x*sin(2*x) - cos(2*x) - 1)*sin(4*x) - sin(2*x))/(2*(2*cos(2*
x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(
2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

$$\int x\sec^2(x)\tan(x)\,dx = \frac{x\tan\left(\frac{1}{2}x\right)^4 + 2\,x\tan\left(\frac{1}{2}x\right)^2 + 2\,\tan\left(\frac{1}{2}x\right)^3 + x - 2\,\tan\left(\frac{1}{2}x\right)}{2\left(\tan\left(\frac{1}{2}x\right)^4 - 2\,\tan\left(\frac{1}{2}x\right)^2 + 1\right)}$$

```
[In] integrate(x*sin(x)/cos(x)^3,x, algorithm="giac")
[Out] 1/2*(x*tan(1/2*x)^4 + 2*x*tan(1/2*x)^2 + 2*tan(1/2*x)^3 + x - 2*tan(1/2*x))
/(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x\sec^2(x)\tan(x)\,dx = \frac{2\,x - \sin\left(2\,x\right)}{4\cos\left(x\right)^2}$$

```
[In] int((x*sin(x))/cos(x)^3,x)
[Out] (2*x - sin(2*x))/(4*cos(x)^2)
```

3.490 $\int x \sin^2(x) \tan(x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 62

$$\int x \sin^2(x) \tan(x) \, dx = \frac{x}{4} + \frac{ix^2}{2} - x \log \left(1 + e^{2ix}\right) + \frac{1}{2} i \operatorname{PolyLog}\left(2, -e^{2ix}\right) - \frac{1}{4} \cos(x) \sin(x) - \frac{1}{2} x \sin^2(x)$$

```
[Out] 1/4*x+1/2*I*x^2-x*ln(1+exp(2*I*x))+1/2*I*polylog(2,-exp(2*I*x))-1/4*cos(x)*
sin(x)-1/2*x*sin(x)^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4492, 3524, 2715, 8, 3800, 2221, 2317, 2438}

$$\int x \sin^2(x) \tan(x) \, dx = \frac{1}{2} i \operatorname{PolyLog}\left(2, -e^{2ix}\right) + \frac{ix^2}{2} + \frac{x}{4} - x \log \left(1 + e^{2ix}\right) - \frac{1}{2} x \sin^2(x) - \frac{1}{4} \sin(x) \cos(x)$$

```
[In] Int[x*Sin[x]^2*Tan[x],x]
```

```
[Out] x/4 + (I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + (I/2)*PolyLog[2, -E^((2*I)*x)] -
 (Cos[x]*Sin[x])/4 - (x*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int x\cos(x)\sin(x)\,dx + \int x\tan(x)\,dx\\
&= \frac{ix^2}{2} - \frac{1}{2}x\sin^2(x) - 2i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx + \frac{1}{2}\int \sin^2(x)\,dx\\
&= \frac{ix^2}{2} - x\log\left(1+e^{2ix}\right) - \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x\sin^2(x) + \frac{\int 1\,dx}{4} + \int \log\left(1+e^{2ix}\right)\,dx\\
&= \frac{x}{4} + \frac{ix^2}{2} - x\log\left(1+e^{2ix}\right) - \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x\sin^2(x) - \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right)\\
&= \frac{x}{4} + \frac{ix^2}{2} - x\log\left(1+e^{2ix}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right) - \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x\sin^2(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int x\sin^2(x)\tan(x)\,dx = \frac{ix^2}{2} + \frac{1}{4}x\cos(2x) - x\log\left(1+e^{2ix}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right) - \frac{1}{8}\sin(2x)$$

```
[In] Integrate[x*Sin[x]^2*Tan[x],x]
[Out] (I/2)*x^2 + (x*Cos[2*x])/4 - x*Log[1 + E^((2*I)*x)] + (I/2)*PolyLog[2, -E^(
(2*I)*x)] - Sin[2*x]/8
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{ix^2}{2} + \frac{(i+2x)e^{2ix}}{16} + \frac{(-i+2x)e^{-2ix}}{16} - x\ln\left(e^{2ix}+1\right) + \frac{i\,\text{Li}_2\left(-e^{2ix}\right)}{2}$	57

```
[In] int(x*sin(x)^3/cos(x),x,method=_RETURNVERBOSE)
[Out] 1/2*I*x^2+1/16*(I+2*x)*exp(2*I*x)+1/16*(-I+2*x)*exp(-2*I*x)-x*ln(exp(2*I*x)
+1)+1/2*I*polylog(2,-exp(2*I*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(41) = 82.

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.82

$$\int x\sin^2(x)\tan(x)\,dx = \frac{1}{2}\,x\cos\left(x\right)^2 - \frac{1}{2}\,x\log\left(i\,\cos\left(x\right) + \sin\left(x\right) + 1\right) - \frac{1}{2}\,x\log\left(i\,\cos\left(x\right) - \sin\left(x\right) + 1\right) - \frac{1}{2}\,x\log\left(-i\,\cos\left(x\right) + \sin\left(x\right) + 1\right) - \frac{1}{2}\,x\log\left(-i\,\cos\left(x\right) - \sin\left(x\right) + 1\right) - \frac{1}{4}\,\cos\left(x\right)\sin\left(x\right) - \frac{1}{4}\,x - \frac{1}{2}i\,\mathrm{Li}_2(i\,\cos\left(x\right) + \sin\left(x\right)) + \frac{1}{2}i\,\mathrm{Li}_2(i\,\cos\left(x\right) - \sin\left(x\right)) + \frac{1}{2}i\,\mathrm{Li}_2(-i\,\cos\left(x\right) + \sin\left(x\right)) - \frac{1}{2}i\,\mathrm{Li}_2(-i\,\cos\left(x\right) - \sin\left(x\right))$$

```
[In] integrate(x*sin(x)^3/cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(x)^2 - 1/2*x*log(I*cos(x) + sin(x) + 1) - 1/2*x*log(I*cos(x) - si
n(x) + 1) - 1/2*x*log(-I*cos(x) + sin(x) + 1) - 1/2*x*log(-I*cos(x) - sin(x
) + 1) - 1/4*cos(x)*sin(x) - 1/4*x - 1/2*I*dilog(I*cos(x) + sin(x)) + 1/2*I
*dilog(I*cos(x) - sin(x)) + 1/2*I*dilog(-I*cos(x) + sin(x)) - 1/2*I*dilog(-
I*cos(x) - sin(x))
```

Sympy [F]

$$\int x\sin^2(x)\tan(x)\,dx = \int \frac{x\sin^3\left(x\right)}{\cos\left(x\right)}\,dx$$

```
[In] integrate(x*sin(x)**3/cos(x),x)
```

```
[Out] Integral(x*sin(x)**3/cos(x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int x\sin^2(x)\tan(x)\,dx = \frac{1}{2}i\,x^2 - i\,x\arctan\left(\sin\left(2\,x\right), \cos\left(2\,x\right) + 1\right) + \frac{1}{4}\,x\cos\left(2\,x\right) - \frac{1}{2}\,x\log\left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right) + 1\right) + \frac{1}{2}i\,\mathrm{Li}_2\left(-e^{(2i\,x)}\right) - \frac{1}{8}\,\sin\left(2\,x\right)$$

```
[In] integrate(x*sin(x)^3/cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*I*x^2 - I*x*arctan2(sin(2*x), cos(2*x) + 1) + 1/4*x*cos(2*x) - 1/2*x*lo
g(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + 1/2*I*dilog(-e^(2*I*x)) - 1/8
*sin(2*x)
```

Giac [F]

$$\int x\sin^2(x)\tan(x)\,dx = \int \frac{x\sin(x)^3}{\cos(x)}\,dx$$

```
[In] integrate(x*sin(x)^3/cos(x),x, algorithm="giac")
```

```
[Out] integrate(x*sin(x)^3/cos(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sin^2(x)\tan(x)\,dx = \int \frac{x\sin(x)^3}{\cos(x)}\,dx$$

```
[In] int((x*sin(x)^3)/cos(x),x)
```

```
[Out] int((x*sin(x)^3)/cos(x), x)
```

3.491 $\int x \tan^3(x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 59

$$\int x\tan^3(x)\,dx=\frac{x}{2}-\frac{ix^2}{2}+x\log\left(1+e^{2ix}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{\tan(x)}{2}+\frac{1}{2}x\tan^2(x)$$

```
[Out] 1/2*x-1/2*I*x^2+x*ln(1+exp(2*I*x))-1/2*I*polylog(2,-exp(2*I*x))-1/2*tan(x)+
1/2*x*tan(x)^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {3801, 3554, 8, 3800, 2221, 2317, 2438}

$$\int x\tan^3(x)\,dx=-\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{ix^2}{2}+\frac{x}{2}+x\log\left(1+e^{2ix}\right)+\frac{1}{2}x\tan^2(x)-\frac{\tan(x)}{2}$$

```
[In] Int[x*Tan[x]^3,x]
```

```
[Out] x/2 - (I/2)*x^2 + x*Log[1 + E^((2*I)*x)] - (I/2)*PolyLog[2, -E^((2*I)*x)] -
 Tan[x]/2 + (x*Tan[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x\tan^2(x) - \frac{1}{2}\int \tan^2(x)\,dx - \int x\tan(x)\,dx \\
&= -\frac{ix^2}{2} - \frac{\tan(x)}{2} + \frac{1}{2}x\tan^2(x) + 2i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx + \frac{\int 1\,dx}{2} \\
&= \frac{x}{2} - \frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) - \frac{\tan(x)}{2} + \frac{1}{2}x\tan^2(x) - \int \log\left(1+e^{2ix}\right)\,dx \\
&= \frac{x}{2} - \frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) - \frac{\tan(x)}{2} + \frac{1}{2}x\tan^2(x) + \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right)
\end{aligned}
$$

$$= \frac{x}{2} - \frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right) - \frac{\tan(x)}{2} + \frac{1}{2}x\tan^2(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x\tan^3(x)\,dx = -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right) + \frac{1}{2}x\sec^2(x) - \frac{\tan(x)}{2}$$

```
[In] Integrate[x*Tan[x]^3,x]
```

```
[Out] (-1/2*I)*x^2 + x*Log[1 + E^((2*I)*x)] - (I/2)*PolyLog[2, -E^((2*I)*x)] + (x
*Sec[x]^2)/2 - Tan[x]/2
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{ix^2}{2}+\frac{2x\,\mathrm{e}^{2ix}-i\mathrm{e}^{2ix}-i}{\left(\mathrm{e}^{2ix}+1\right)^2}+x\ln\left(\mathrm{e}^{2ix}+1\right)-\frac{i\,\mathrm{Li}_2\left(-\mathrm{e}^{2ix}\right)}{2}$	59

```
[In] int(x*sin(x)^3/cos(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*x^2+(2*x*exp(2*I*x)-I*exp(2*I*x)-I)/(exp(2*I*x)+1)^2+x*ln(exp(2*I*x)
+1)-1/2*I*polylog(2,-exp(2*I*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(38) = 76.

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.34

$$\int x\tan^3(x)\,dx$$

$$= \frac{x\cos(x)^2\log(i\,\cos(x)+\sin(x)+1) + x\cos(x)^2\log(i\,\cos(x)-\sin(x)+1) + x\cos(x)^2\log(-i\,\cos(x)+}{}$$

```
[In] integrate(x*sin(x)^3/cos(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos(x)^2*log(I*cos(x) + sin(x) + 1) + x*cos(x)^2*log(I*cos(x) - sin(
x) + 1) + x*cos(x)^2*log(-I*cos(x) + sin(x) + 1) + x*cos(x)^2*log(-I*cos(x)
 - sin(x) + 1) + I*cos(x)^2*dilog(I*cos(x) + sin(x)) - I*cos(x)^2*dilog(I*c
os(x) - sin(x)) - I*cos(x)^2*dilog(-I*cos(x) + sin(x)) + I*cos(x)^2*dilog(-
I*cos(x) - sin(x)) - cos(x)*sin(x) + x)/cos(x)^2
```

Sympy [F]

$$\int x\tan^3(x)\,dx = \int \frac{x\sin^3(x)}{\cos^3(x)}\,dx$$

```
[In] integrate(x*sin(x)**3/cos(x)**3,x)
```

```
[Out] Integral(x*sin(x)**3/cos(x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(38) = 76.

Time = 0.29 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.56

$$\int x\tan^3(x)\,dx = -\frac{x^2\cos(4x) + i\,x^2\sin(4x) + x^2 - 2\,(x\cos(4x) + 2\,x\cos(2x) + i\,x\sin(4x) + 2i\,x\sin(2x) + x)\arctan}{}$$

```
[In] integrate(x*sin(x)^3/cos(x)^3,x, algorithm="maxima")
```

```
[Out] -(x^2*cos(4*x) + I*x^2*sin(4*x) + x^2 - 2*(x*cos(4*x) + 2*x*cos(2*x) + I*x*
sin(4*x) + 2*I*x*sin(2*x) + x)*arctan2(sin(2*x), cos(2*x) + 1) + 2*(x^2 + 2
*I*x + 1)*cos(2*x) + (cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*sin(2*x) + 1
)*dilog(-e^(2*I*x)) - (-I*x*cos(4*x) - 2*I*x*cos(2*x) + x*sin(4*x) + 2*x*si
n(2*x) - I*x)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + 2*(I*x^2 - 2*
x + I)*sin(2*x) + 2)/(-2*I*cos(4*x) - 4*I*cos(2*x) + 2*sin(4*x) + 4*sin(2*x
) - 2*I)
```

Giac [F]

$$\int x\tan^3(x)\,dx = \int \frac{x\sin(x)^3}{\cos(x)^3}\,dx$$

```
[In] integrate(x*sin(x)^3/cos(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*sin(x)^3/cos(x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\tan^3(x)\,dx = \int \frac{x\sin(x)^3}{\cos(x)^3}\,dx$$

```
[In] int((x*sin(x)^3)/cos(x)^3,x)
[Out] int((x*sin(x)^3)/cos(x)^3, x)
```

3.492 $\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx=\frac{2}{1+\frac{\cot(x)}{x}}$$

```
[Out] 2/(1+cot(x)/x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6843, 32}

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx=\frac{2}{\frac{\cot(x)}{x}+1}$$

```
[In] Int[(2*x + Sin[2*x])/(Cos[x] + x*Sin[x])^2,x]
```

```
[Out] 2/(1 + Cot[x]/x)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6843

```
Int[(u_)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.), x_Symbol] :> With[{c 
= Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, Dist[c*p, Subst[Int[(b + a*x^p)
^m, x], x, v*w^(m*q + 1)], x] /; FreeQ[c, x]] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& EqQ[p + q*(m*p + 1), 0] && IntegerQ[p] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int \frac{1}{(1+x)^2}\,dx, x, \frac{\cot(x)}{x}\right)\right)$$
$$= \frac{2}{1+\frac{\cot(x)}{x}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx = \frac{2x\sin(x)}{\cos(x)+x\sin(x)}$$

```
[In] Integrate[(2*x + Sin[2*x])/(Cos[x] + x*Sin[x])^2,x]
[Out] (2*x*Sin[x])/(Cos[x] + x*Sin[x])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 3.67

method	result	size
risch	$-\frac{2i}{x+i}-\frac{4ix}{(x+i)(x\,\mathrm{e}^{2ix}-x+i\mathrm{e}^{2ix}+i)}$	44

```
[In] int((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x,method=_RETURNVERBOSE)
[Out] -2*I/(x+I)-4*I*x/(x+I)/(x*exp(2*I*x)-x+I*exp(2*I*x)+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx = -\frac{2\,\cos(x)}{x\sin(x)+\cos(x)}$$

```
[In] integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="fricas")
[Out] -2*cos(x)/(x*sin(x) + cos(x))
```

Sympy [F]

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx = \int \frac{2x+\sin{(2x)}}{(x\sin{(x)}+\cos{(x)})^2}\,dx$$

```
[In] integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))**2,x)
```

```
[Out] Integral((2*x + sin(2*x))/(x*sin(x) + cos(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 6.50

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx = -\frac{2\left(\cos{(2x)}^2+2x\sin{(2x)}+\sin{(2x)}^2+2\cos{(2x)}+1\right)}{(x^2+1)\cos{(2x)}^2+(x^2+1)\sin{(2x)}^2+x^2-2(x^2-1)\cos{(2x)}+4x\sin{(2x)}+1}$$

```
[In] integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(cos(2*x)^2 + 2*x*sin(2*x) + sin(2*x)^2 + 2*cos(2*x) + 1)/((x^2 + 1)*cos
(2*x)^2 + (x^2 + 1)*sin(2*x)^2 + x^2 - 2*(x^2 - 1)*cos(2*x) + 4*x*sin(2*x)
+ 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx = -\frac{2}{x\tan{(x)}+1}$$

```
[In] integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2/(x*tan(x) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx = \int \frac{2\,x+\sin\left(2\,x\right)}{{\left(\cos\left(x\right)+x\,\sin\left(x\right)\right)}^2} \,d x$$

```
[In] int((2*x + sin(2*x))/(cos(x) + x*sin(x))^2,x)
[Out] int((2*x + sin(2*x))/(cos(x) + x*sin(x))^2, x)
```

3.493 $\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = -\cot(x)+\frac{x\csc(x)}{x\cos(x)-\sin(x)}$$

```
[Out] -cot(x)+x*csc(x)/(x*cos(x)-sin(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4690, 3852, 8}

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = \frac{x\csc(x)}{x\cos(x)-\sin(x)}-\cot(x)$$

```
[In] Int[x^2/(x*Cos[x] - Sin[x])^2,x]
[Out] -Cot[x] + (x*Csc[x])/(x*Cos[x] - Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4690

```
Int[(x_)^2/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol
] :> Simp[x/(a*d*Sin[a*x]*(c*Sin[a*x] + d*x*Cos[a*x])), x] + Dist[1/d^2, In
t[1/Sin[a*x]^2, x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\csc(x)}{x\cos(x)-\sin(x)}+\int \csc^2(x)\,dx\\
&= \frac{x\csc(x)}{x\cos(x)-\sin(x)}-\text{Subst}\left(\int 1\,dx,x,\cot(x)\right)\\
&= -\cot(x)+\frac{x\csc(x)}{x\cos(x)-\sin(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx=\frac{\cos(x)+x\sin(x)}{x\cos(x)-\sin(x)}$$

```
[In] Integrate[x^2/(x*Cos[x] - Sin[x])^2,x]
[Out] (Cos[x] + x*Sin[x])/(x*Cos[x] - Sin[x])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{\cos(x)+x\sin(x)}{x\cos(x)-\sin(x)}$	20
risch	$\frac{2i(x-i)}{i e^{2ix}+x\,e^{2ix}-i+x}$	29
norman	$\frac{\tan^2\left(\frac{x}{2}\right)-2x\tan\left(\frac{x}{2}\right)-1}{x\left(\tan^2\left(\frac{x}{2}\right)\right)-x+2\tan\left(\frac{x}{2}\right)}$	37

```
[In] int(x^2/(x*cos(x)-sin(x))^2,x,method=_RETURNVERBOSE)
[Out] (cos(x)+x*sin(x))/(x*cos(x)-sin(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = \frac{x\sin(x)+\cos(x)}{x\cos(x)-\sin(x)}$$

```
[In] integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="fricas")
[Out] (x*sin(x) + cos(x))/(x*cos(x) - sin(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(15) = 30.

Time = 0.72 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.30

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = -\frac{2x\tan\left(\frac{x}{2}\right)}{x\tan^2\left(\frac{x}{2}\right)-x+2\tan\left(\frac{x}{2}\right)} + \frac{\tan^2\left(\frac{x}{2}\right)}{x\tan^2\left(\frac{x}{2}\right)-x+2\tan\left(\frac{x}{2}\right)} - \frac{1}{x\tan^2\left(\frac{x}{2}\right)-x+2\tan\left(\frac{x}{2}\right)}$$

```
[In] integrate(x**2/(x*cos(x)-sin(x))**2,x)
[Out] -2*x*tan(x/2)/(x*tan(x/2)**2 - x + 2*tan(x/2)) + tan(x/2)**2/(x*tan(x/2)**2
 - x + 2*tan(x/2)) - 1/(x*tan(x/2)**2 - x + 2*tan(x/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(20) = 40.

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.45

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = \frac{2\,(2\,x\cos(2\,x)+(x^2-1)\sin(2\,x))}{(x^2+1)\cos(2\,x)^2+(x^2+1)\sin(2\,x)^2+x^2+2\,(x^2-1)\cos(2\,x)-4\,x\sin(2\,x)+1}$$

```
[In] integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="maxima")
[Out] 2*(2*x*cos(2*x) + (x^2 - 1)*sin(2*x))/((x^2 + 1)*cos(2*x)^2 + (x^2 + 1)*sin
(2*x)^2 + x^2 + 2*(x^2 - 1)*cos(2*x) - 4*x*sin(2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = -\frac{2\,x\tan\left(\frac{1}{2}\,x\right)-\tan\left(\frac{1}{2}\,x\right)^2+1}{x\tan\left(\frac{1}{2}\,x\right)^2-x+2\,\tan\left(\frac{1}{2}\,x\right)}$$

```
[In] integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="giac")
[Out] -(2*x*tan(1/2*x) - tan(1/2*x)^2 + 1)/(x*tan(1/2*x)^2 - x + 2*tan(1/2*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = \int \frac{x^2}{\left(\sin\left(x\right)-x\,\cos\left(x\right)\right)^2}\,dx$$

```
[In] int(x^2/(sin(x) - x*cos(x))^2,x)
[Out] int(x^2/(sin(x) - x*cos(x))^2, x)
```

3.494 $\int a^{mx} b^{nx} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int a^{mx} b^{nx} \, dx = \frac{a^{mx} b^{nx}}{m \log(a) + n \log(b)}$$

```
[Out] a^(m*x)*b^(n*x)/(m*ln(a)+n*ln(b))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2325, 2225}

$$\int a^{mx} b^{nx} \, dx = \frac{a^{mx} b^{nx}}{m \log(a) + n \log(b)}$$

```
[In] Int[a^(m*x)*b^(n*x),x]
```

```
[Out] (a^(m*x)*b^(n*x))/(m*Log[a] + n*Log[b])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int e^{x(m\log(a)+n\log(b))}\,dx\\ &= \frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int a^{mx}b^{nx}\,dx = \frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}$$

```
[In] Integrate[a^(m*x)*b^(n*x),x]
[Out] (a^(m*x)*b^(n*x))/(m*Log[a] + n*Log[b])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{a^{mx}b^{nx}}{m\ln(a)+n\ln(b)}$	23
risch	$\frac{a^{mx}b^{nx}}{m\ln(a)+n\ln(b)}$	23
parallelrisch	$\frac{a^{mx}b^{nx}}{m\ln(a)+n\ln(b)}$	23
norman	$\frac{e^{mx\ln(a)}e^{nx\ln(b)}}{m\ln(a)+n\ln(b)}$	25
meijerg	$-\frac{1-e^{xn\ln(b)\left(1+\frac{m\ln(a)}{n\ln(b)}\right)}}{n\ln(b)\left(1+\frac{m\ln(a)}{n\ln(b)}\right)}$	48

```
[In] int(a^(m*x)*b^(n*x),x,method=_RETURNVERBOSE)
[Out] a^(m*x)*b^(n*x)/(m*ln(a)+n*ln(b))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int a^{mx}b^{nx}\,dx=\frac{a^{mx}b^{nx}}{m\log\left(a\right)+n\log\left(b\right)}$$

```
[In] integrate(a^(m*x)*b^(n*x),x, algorithm="fricas")
[Out] a^(m*x)*b^(n*x)/(m*log(a) + n*log(b))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(19) = 38.

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int a^{mx}b^{nx}\,dx=\begin{cases}\frac{a^{mx}b^{nx}}{m\log\left(a\right)+n\log\left(b\right)} & \text{for } m\neq-\frac{n\log\left(b\right)}{\log\left(a\right)}\\ b^{nx}xe^{-nx\log\left(b\right)} & \text{otherwise}\end{cases}$$

```
[In] integrate(a**(m*x)*b**(n*x),x)
[Out] Piecewise((a**(m*x)*b**(n*x)/(m*log(a) + n*log(b)), Ne(m, -n*log(b)/log(a))
), (b**(n*x)*x*exp(-n*x*log(b)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int a^{mx}b^{nx}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(a^(m*x)*b^(n*x),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((log(b)*n)/(log(a)*m)>0)', see `ass
ume?` f
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 325, normalized size of antiderivative = 14.77

$$\int a^{mx}b^{nx}\,dx$$
$$= 2\left(\frac{2\,(m\log(|a|) + n\log(|b|))\cos\left(-\frac{1}{2}\,\pi mx\mathrm{sgn}(a) - \frac{1}{2}\,\pi nx\mathrm{sgn}(b) + \frac{1}{2}\,\pi mx + \frac{1}{2}\,\pi nx\right)}{(\pi m\mathrm{sgn}\,(a) + \pi n\mathrm{sgn}\,(b) - \pi m - \pi n)^2 + 4\,(m\log(|a|) + n\log(|b|))^2} - \frac{(\pi m\mathrm{sgn}(a) + \pi n\mathrm{sg}}{(\pi m\mathrm{sgn}}\right.$$
$$+i\left(\frac{i\,e^{\left(\frac{1}{2}i\,\pi mx\mathrm{sgn}(a) + \frac{1}{2}i\,\pi nx\mathrm{sgn}(b) - \frac{1}{2}i\,\pi mx - \frac{1}{2}i\,\pi nx\right)}}{i\,\pi m\mathrm{sgn}\,(a) + i\,\pi n\mathrm{sgn}\,(b) - i\,\pi m - i\,\pi n + 2\,m\log(|a|) + 2\,n\log(|b|)} - \frac{i\,e^{\left(-\frac{1}{2}i\,\pi mx\mathrm{sgn}(a\right.}}{-i\,\pi m\mathrm{sgn}\,(a) - i\,\pi n\mathrm{sgn}\,(b) +}\right.$$

```
[In] integrate(a^(m*x)*b^(n*x),x, algorithm="giac")
```

```
[Out] 2*(2*(m*log(abs(a)) + n*log(abs(b)))*cos(-1/2*pi*m*x*sgn(a) - 1/2*pi*n*x*sg
n(b) + 1/2*pi*m*x + 1/2*pi*n*x)/((pi*m*sgn(a) + pi*n*sgn(b) - pi*m - pi*n)^
2 + 4*(m*log(abs(a)) + n*log(abs(b)))^2) - (pi*m*sgn(a) + pi*n*sgn(b) - pi*
m - pi*n)*sin(-1/2*pi*m*x*sgn(a) - 1/2*pi*n*x*sgn(b) + 1/2*pi*m*x + 1/2*pi*
n*x)/((pi*m*sgn(a) + pi*n*sgn(b) - pi*m - pi*n)^2 + 4*(m*log(abs(a)) + n*lo
g(abs(b)))^2))*e^((m*log(abs(a)) + n*log(abs(b)))*x) + I*(I*e^(1/2*I*pi*m*x
*sgn(a) + 1/2*I*pi*n*x*sgn(b) - 1/2*I*pi*m*x - 1/2*I*pi*n*x)/(I*pi*m*sgn(a)
 + I*pi*n*sgn(b) - I*pi*m - I*pi*n + 2*m*log(abs(a)) + 2*n*log(abs(b))) - I
*e^(-1/2*I*pi*m*x*sgn(a) - 1/2*I*pi*n*x*sgn(b) + 1/2*I*pi*m*x + 1/2*I*pi*n*
x)/(-I*pi*m*sgn(a) - I*pi*n*sgn(b) + I*pi*m + I*pi*n + 2*m*log(abs(a)) + 2*
n*log(abs(b))))*e^((m*log(abs(a)) + n*log(abs(b)))*x)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int a^{mx}b^{nx}\,dx = \frac{a^{m\,x}\,b^{n\,x}}{m\,\ln(a) + n\,\ln(b)}$$

```
[In] int(a^(m*x)*b^(n*x),x)
```

```
[Out] (a^(m*x)*b^(n*x))/(m*log(a) + n*log(b))
```

3.495 $\int a^{-x}b^{-x}(a^x-b^x)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 34

$$\int a^{-x}b^{-x}(a^x-b^x)^2\,dx=-2x+\frac{a^xb^{-x}-a^{-x}b^x}{\log(a)-\log(b)}$$

```
[Out] -2*x+(a^x/(b^x)-b^x/(a^x))/(ln(a)-ln(b))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2325, 6874, 2225, 8}

$$\int a^{-x}b^{-x}(a^x-b^x)^2\,dx=-\frac{a^{-x}b^x}{\log(a)-\log(b)}+\frac{a^xb^{-x}}{\log(a)-\log(b)}-2x$$

```
[In] Int[(a^x - b^x)^2/(a^x*b^x),x]
```

```
[Out] -2*x + a^x/(b^x*(Log[a] - Log[b])) - b^x/(a^x*(Log[a] - Log[b]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (a^x - b^x)^2 e^{-x(\log(a)+\log(b))} \, dx \\
&= \int \left(a^{2x} e^{-x(\log(a)+\log(b))} - 2a^x b^x e^{-x(\log(a)+\log(b))} + b^{2x} e^{-x(\log(a)+\log(b))}\right) \, dx \\
&= -\left(2 \int a^x b^x e^{-x(\log(a)+\log(b))} \, dx\right) + \int a^{2x} e^{-x(\log(a)+\log(b))} \, dx + \int b^{2x} e^{-x(\log(a)+\log(b))} \, dx \\
&= -(2 \int 1 \, dx) + \int e^{-x(\log(a)-\log(b))} \, dx + \int e^{x(\log(a)-\log(b))} \, dx \\
&= -2x + \frac{a^x b^{-x}}{\log(a) - \log(b)} - \frac{a^{-x} b^x}{\log(a) - \log(b)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int a^{-x} b^{-x} (a^x - b^x)^2 \, dx = -2x + \frac{e^{x(\log(a)-\log(b))}}{\log(a) - \log(b)} + \frac{e^{x(-\log(a)+\log(b))}}{-\log(a) + \log(b)}$$

```
[In] Integrate[(a^x - b^x)^2/(a^x*b^x),x]
[Out] -2*x + E^(x*(Log[a] - Log[b]))/(Log[a] - Log[b]) + E^(x*(-Log[a] + Log[b]))
/(-Log[a] + Log[b])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

method	result	size
risch	$-2x-\frac{a^{-x} b^{x}}{\ln\left(a\right)-\ln\left(b\right)}+\frac{a^{x} b^{-x}}{\ln\left(a\right)-\ln\left(b\right)}$	42
parallelrisch	$-\frac{\left(2 x \,a^{x} b^{x} \ln\left(a\right)-2 x \,a^{x} b^{x} \ln\left(b\right)-a^{2 x}+b^{2 x}\right) a^{-x} b^{-x}}{\ln\left(a\right)-\ln\left(b\right)}$	57
norman	$\left(\frac{{\mathrm e}^{2 x \ln\left(a\right)}}{\ln\left(a\right)-\ln\left(b\right)}-\frac{{\mathrm e}^{2 x \ln\left(b\right)}}{\ln\left(a\right)-\ln\left(b\right)}-2 x \,{\mathrm e}^{x \ln\left(a\right)} {\mathrm e}^{x \ln\left(b\right)}\right) {\mathrm e}^{-x \ln\left(a\right)} {\mathrm e}^{-x \ln\left(b\right)}$	65

```
[In] int((a^x-b^x)^2/(a^x)/(b^x),x,method=_RETURNVERBOSE)
[Out] -2*x-1/(ln(a)-ln(b))/(a^x)*b^x+a^x/(b^x)/(ln(a)-ln(b))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.53

$$\int a^{-x}b^{-x}(a^x-b^x)^2\,dx=-\frac{2\,(x\log\left(a\right)-x\log\left(b\right))a^xb^x-a^{2\,x}+b^{2\,x}}{a^xb^x(\log\left(a\right)-\log\left(b\right))}$$

```
[In] integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="fricas")
[Out] -(2*(x*log(a) - x*log(b))*a^x*b^x - a^(2*x) + b^(2*x))/(a^x*b^x*(log(a) - l
og(b)))
```

Sympy [F(-2)]

Exception generated.

$$\int a^{-x}b^{-x}(a^x-b^x)^2\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a**x-b**x)**2/(a**x)/(b**x),x)
[Out] Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [F(-2)]

Exception generated.

$$\int a^{-x}b^{-x}(a^x-b^x)^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-log(b)/log(a)>0)', see `assume?` f
or more
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 436, normalized size of antiderivative = 12.82

$$\int a^{-x}b^{-x}(a^x-b^x)^2\,dx = \text{Too large to display}$$

```
[In] integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="giac")
```

```
[Out] 2*(2*(log(abs(a)) - log(abs(b)))*cos(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((
pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2) - (pi*sgn(a) -
pi*sgn(b))*sin(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(b))
^2 + 4*(log(abs(a)) - log(abs(b)))^2))*e^(x*(log(abs(a)) - log(abs(b)))) +
I*(I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b))/(I*pi*sgn(a) - I*pi*sgn(b) +
 2*log(abs(a)) - 2*log(abs(b))) - I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*x*sgn(
b))/(-I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a)) - 2*log(abs(b))))*e^(x*(log
(abs(a)) - log(abs(b)))) - 2*(2*(log(abs(a)) - log(abs(b)))*cos(1/2*pi*x*sg
n(a) - 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(a
bs(b)))^2) - (pi*sgn(a) - pi*sgn(b))*sin(1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b))
/((pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2))*e^(-x*(log(
abs(a)) - log(abs(b)))) + I*(-I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b))/(
I*pi*sgn(a) - I*pi*sgn(b) - 2*log(abs(a)) + 2*log(abs(b))) + I*e^(-1/2*I*pi
*x*sgn(a) + 1/2*I*pi*x*sgn(b))/(-I*pi*sgn(a) + I*pi*sgn(b) - 2*log(abs(a))
+ 2*log(abs(b))))*e^(-x*(log(abs(a)) - log(abs(b)))) - 2*x
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int a^{-x}b^{-x}(a^x-b^x)^2\,dx=\frac{\frac{a^x}{b^x}-\frac{b^x}{a^x}}{\ln(a)-\ln(b)}-2\,x$$

```
[In] int((a^x - b^x)^2/(a^x*b^x),x)
[Out] (a^x/b^x - b^x/a^x)/(log(a) - log(b)) - 2*x
```

3.496 $\int (-e^{-x}+e^x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 9

$$\int (-e^{-x}+e^x)\,dx=e^{-x}+e^x$$

```
[Out] exp(-x)+exp(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2225}

$$\int (-e^{-x}+e^x)\,dx=e^{-x}+e^x$$

```
[In] Int[-E^(-x) + E^x,x]
```

```
[Out] E^(-x) + E^x
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\int e^{-x}\,dx+\int e^x\,dx \\ &= e^{-x}+e^x \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \left(-e^{-x}+e^x\right) \, dx = e^{-x}+e^x$$

```
[In] Integrate[-E^(-x) + E^x,x]
[Out] E^(-x) + E^x
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\mathrm{e}^{-x}+\mathrm{e}^{x}$	8
default	$\mathrm{e}^{-x}+\mathrm{e}^{x}$	8
risch	$\mathrm{e}^{-x}+\mathrm{e}^{x}$	8
parts	$\mathrm{e}^{-x}+\mathrm{e}^{x}$	8
meijerg	$-2+\mathrm{e}^{-x}+\mathrm{e}^{x}$	9
norman	$\left(1+\mathrm{e}^{2x}\right)\mathrm{e}^{-x}$	12
parallelrisch	$\left(1+\mathrm{e}^{2x}\right)\mathrm{e}^{-x}$	12

```
[In] int(-1/exp(x)+exp(x),x,method=_RETURNVERBOSE)
[Out] 1/exp(x)+exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \left(-e^{-x}+e^x\right) \, dx = \left(e^{(2\,x)}+1\right)e^{(-x)}$$

```
[In] integrate(-1/exp(x)+exp(x),x, algorithm="fricas")
[Out] (e^(2*x) + 1)*e^(-x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \left(-e^{-x}+e^{x}\right)\,dx = e^{x}+e^{-x}$$

```
[In] integrate(-1/exp(x)+exp(x),x)
[Out] exp(x) + exp(-x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \left(-e^{-x}+e^{x}\right)\,dx = e^{(-x)}+e^{x}$$

```
[In] integrate(-1/exp(x)+exp(x),x, algorithm="maxima")
[Out] e^(-x) + e^x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \left(-e^{-x}+e^{x}\right)\,dx = e^{(-x)}+e^{x}$$

```
[In] integrate(-1/exp(x)+exp(x),x, algorithm="giac")
[Out] e^(-x) + e^x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \left(-e^{-x}+e^{x}\right)\,dx = 2\cosh(x)$$

```
[In] int(exp(x) - exp(-x),x)
[Out] 2*cosh(x)
```

3.497 $\int \left(-e^{-x}+e^x\right)^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \left(-e^{-x}+e^x\right)^2 \, dx = -\frac{1}{2}e^{-2x}+\frac{e^{2x}}{2}-2x$$

```
[Out] -1/2/exp(2*x)+1/2*exp(2*x)-2*x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2320, 272, 45}

$$\int \left(-e^{-x}+e^x\right)^2 \, dx = -2x-\frac{e^{-2x}}{2}+\frac{e^{2x}}{2}$$

```
[In] Int[(-E^(-x) + E^x)^2,x]
```

```
[Out] -1/2*1/E^(2*x) + E^(2*x)/2 - 2*x
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{(1-x^2)^2}{x^3}\,dx, x, e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, e^{2x}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)\,dx, x, e^{2x}\right)\\ &= -\frac{1}{2}e^{-2x}+\frac{e^{2x}}{2}-2x\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(-e^{-x}+e^x\right)^2\,dx = -\frac{1}{2}e^{-2x}+\frac{e^{2x}}{2}-2x$$

```
[In] Integrate[(-E^(-x) + E^x)^2,x]
[Out] -1/2*1/E^(2*x) + E^(2*x)/2 - 2*x
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
risch	$-2x+\frac{e^{2x}}{2}-\frac{e^{-2x}}{2}$	17
parts	$-2x+\frac{e^{2x}}{2}-\frac{e^{-2x}}{2}$	17
derivativedivides	$\frac{e^{2x}}{2}-\frac{e^{-2x}}{2}-2\ln\left(e^{x}\right)$	19
default	$\frac{e^{2x}}{2}-\frac{e^{-2x}}{2}-2\ln\left(e^{x}\right)$	19
parallelrisch	$\frac{\left(-1+e^{4x}-4\,e^{2x}x\right)e^{-2x}}{2}$	20
norman	$\left(-\frac{1}{2}+\frac{e^{4x}}{2}-2\,e^{2x}x\right)e^{-2x}$	21

```
[In] int((-1/exp(x)+exp(x))^2,x,method=_RETURNVERBOSE)
[Out] -2*x+1/2*exp(2*x)-1/2*exp(-2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \left(-e^{-x}+e^{x}\right)^{2}\,dx=-\frac{1}{2}\left(4\,xe^{(2\,x)}-e^{(4\,x)}+1\right)e^{(-2\,x)}$$

```
[In] integrate((-1/exp(x)+exp(x))^2,x, algorithm="fricas")
[Out] -1/2*(4*x*e^(2*x) - e^(4*x) + 1)*e^(-2*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \left(-e^{-x}+e^{x}\right)^{2}\,dx=-2x+\frac{e^{2x}}{2}-\frac{e^{-2x}}{2}$$

```
[In] integrate((-1/exp(x)+exp(x))**2,x)
[Out] -2*x + exp(2*x)/2 - exp(-2*x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \left(-e^{-x}+e^{x}\right)^{2} dx = -2x + \frac{1}{2}e^{(2x)} - \frac{1}{2}e^{(-2x)}$$

```
[In] integrate((-1/exp(x)+exp(x))^2,x, algorithm="maxima")
[Out] -2*x + 1/2*e^(2*x) - 1/2*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(-e^{-x}+e^{x}\right)^{2} dx = \frac{1}{2}\left(2e^{(2x)}-1\right)e^{(-2x)} - 2x + \frac{1}{2}e^{(2x)}$$

```
[In] integrate((-1/exp(x)+exp(x))^2,x, algorithm="giac")
[Out] 1/2*(2*e^(2*x) - 1)*e^(-2*x) - 2*x + 1/2*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.36

$$\int \left(-e^{-x}+e^{x}\right)^{2} dx = \sinh(2x) - 2x$$

```
[In] int((exp(-x) - exp(x))^2,x)
[Out] sinh(2*x) - 2*x
```

3.498 $\int (-e^{-x}+e^x)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int (-e^{-x}+e^x)^3\,dx=\frac{e^{-3x}}{3}-3e^{-x}-3e^x+\frac{e^{3x}}{3}$$

```
[Out] 1/3/exp(3*x)-3/exp(x)-3*exp(x)+1/3*exp(3*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2320, 276}

$$\int (-e^{-x}+e^x)^3\,dx=\frac{e^{-3x}}{3}-3e^{-x}-3e^x+\frac{e^{3x}}{3}$$

```
[In] Int[(-E^(-x) + E^x)^3,x]
[Out] 1/(3*E^(3*x)) - 3/E^x - 3*E^x + E^(3*x)/3
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{(-1+x^2)^3}{x^4}\,dx, x, e^x\right)\\ &= \text{Subst}\left(\int \left(-3-\frac{1}{x^4}+\frac{3}{x^2}+x^2\right)dx, x, e^x\right)\\ &= \frac{e^{-3x}}{3}-3e^{-x}-3e^x+\frac{e^{3x}}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \left(-e^{-x}+e^x\right)^3\,dx = \frac{1}{3}e^{-3x}\left(1-9e^{2x}-9e^{4x}+e^{6x}\right)$$

```
[In] Integrate[(-E^(-x) + E^x)^3,x]
[Out] (1 - 9*E^(2*x) - 9*E^(4*x) + E^(6*x))/(3*E^(3*x))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{e^{3x}}{3}-3\,e^{x}-3\,e^{-x}+\frac{e^{-3x}}{3}$	24
default	$\frac{e^{3x}}{3}-3\,e^{x}-3\,e^{-x}+\frac{e^{-3x}}{3}$	24
risch	$\frac{e^{3x}}{3}-3\,e^{x}-3\,e^{-x}+\frac{e^{-3x}}{3}$	24
parts	$\frac{e^{3x}}{3}-3\,e^{x}-3\,e^{-x}+\frac{e^{-3x}}{3}$	24
meijerg	$\frac{16}{3}+\frac{e^{-3x}}{3}-3\,e^{-x}-3\,e^{x}+\frac{e^{3x}}{3}$	25
norman	$\left(\frac{1}{3}-3\,e^{2x}-3\,e^{4x}+\frac{e^{6x}}{3}\right)e^{-3x}$	26
parallelrisch	$-\frac{\left(-e^{6x}-1+9\,e^{4x}+9\,e^{2x}\right)e^{-3x}}{3}$	27

```
[In] int((-1/exp(x)+exp(x))^3,x,method=_RETURNVERBOSE)
[Out] 1/3*exp(x)^3-3*exp(x)-3/exp(x)+1/3/exp(x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \left(-e^{-x}+e^x\right)^3\,dx=\frac{1}{3}\left(e^{(6\,x)}-9\,e^{(4\,x)}-9\,e^{(2\,x)}+1\right)e^{(-3\,x)}$$

```
[In] integrate((-1/exp(x)+exp(x))^3,x, algorithm="fricas")
[Out] 1/3*(e^(6*x) - 9*e^(4*x) - 9*e^(2*x) + 1)*e^(-3*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \left(-e^{-x}+e^x\right)^3\,dx=\frac{e^{3x}}{3}-3e^x-3e^{-x}+\frac{e^{-3x}}{3}$$

```
[In] integrate((-1/exp(x)+exp(x))**3,x)
[Out] exp(3*x)/3 - 3*exp(x) - 3*exp(-x) + exp(-3*x)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \left(-e^{-x}+e^x\right)^3\,dx=\frac{1}{3}\,e^{(3\,x)}-3\,e^{(-x)}+\frac{1}{3}\,e^{(-3\,x)}-3\,e^x$$

```
[In] integrate((-1/exp(x)+exp(x))^3,x, algorithm="maxima")
[Out] 1/3*e^(3*x) - 3*e^(-x) + 1/3*e^(-3*x) - 3*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \left(-e^{-x}+e^x\right)^3\,dx=-\frac{1}{3}\left(9\,e^{(2\,x)}-1\right)e^{(-3\,x)}+\frac{1}{3}\,e^{(3\,x)}-3\,e^x$$

```
[In] integrate((-1/exp(x)+exp(x))^3,x, algorithm="giac")
[Out] -1/3*(9*e^(2*x) - 1)*e^(-3*x) + 1/3*e^(3*x) - 3*e^x
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \left(-e^{-x}+e^x\right)^3\,dx=\frac{\mathrm{e}^{-3\,x}}{3}-3\,\mathrm{e}^{-x}+\frac{\mathrm{e}^{3\,x}}{3}-3\,\mathrm{e}^x$$

```
[In] int(-(exp(-x) - exp(x))^3,x)
[Out] exp(-3*x)/3 - 3*exp(-x) + exp(3*x)/3 - 3*exp(x)
```

3.499 $\int (-e^{-x}+e^x)^4 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int (-e^{-x}+e^x)^4 \, dx = -\frac{1}{4}e^{-4x}+2e^{-2x}-2e^{2x}+\frac{e^{4x}}{4}+6x$$

```
[Out] -1/4/exp(4*x)+2/exp(2*x)-2*exp(2*x)+1/4*exp(4*x)+6*x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2320, 272, 45}

$$\int (-e^{-x}+e^x)^4 \, dx = 6x-\frac{e^{-4x}}{4}+2e^{-2x}-2e^{2x}+\frac{e^{4x}}{4}$$

```
[In] Int[(-E^(-x) + E^x)^4,x]
```

```
[Out] -1/4*1/E^(4*x) + 2/E^(2*x) - 2*E^(2*x) + E^(4*x)/4 + 6*x
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1-x^2)^4}{x^5}\,dx, x, e^x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^4}{x^3}\,dx, x, e^{2x}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-4+\frac{1}{x^3}-\frac{4}{x^2}+\frac{6}{x}+x\right)\,dx, x, e^{2x}\right)\\
&= -\frac{1}{4}e^{-4x}+2e^{-2x}-2e^{2x}+\frac{e^{4x}}{4}+6x
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \left(-e^{-x}+e^x\right)^4\,dx = \frac{1}{4}e^{-4x}\left(-1+8e^{2x}-8e^{6x}+e^{8x}\right)+6\log\left(e^x\right)$$

```
[In] Integrate[(-E^(-x) + E^x)^4,x]
[Out] (-1 + 8*E^(2*x) - 8*E^(6*x) + E^(8*x))/(4*E^(4*x)) + 6*Log[E^x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

method	result	size
risch	$6x+\frac{e^{4x}}{4}-2\,e^{2x}+2\,e^{-2x}-\frac{e^{-4x}}{4}$	29
parts	$6x+\frac{e^{4x}}{4}-2\,e^{2x}+2\,e^{-2x}-\frac{e^{-4x}}{4}$	29
derivativedivides	$\frac{e^{4x}}{4}-2\,e^{2x}+2\,e^{-2x}+6\ln\left(e^{x}\right)-\frac{e^{-4x}}{4}$	31
default	$\frac{e^{4x}}{4}-2\,e^{2x}+2\,e^{-2x}+6\ln\left(e^{x}\right)-\frac{e^{-4x}}{4}$	31
norman	$\left(-\frac{1}{4}+2\,e^{2x}-2\,e^{6x}+\frac{e^{8x}}{4}+6x\,e^{4x}\right)e^{-4x}$	33
parallelrisch	$-\frac{\left(-e^{8x}+1+8\,e^{6x}-24\ln\left(e^{x}\right)e^{4x}-8\,e^{2x}\right)e^{-4x}}{4}$	36

```
[In] int((-1/exp(x)+exp(x))^4,x,method=_RETURNVERBOSE)
[Out] 6*x+1/4*exp(4*x)-2*exp(2*x)+2*exp(-2*x)-1/4*exp(-4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \left(-e^{-x}+e^{x}\right)^{4}\,dx=\frac{1}{4}\left(24\,xe^{(4\,x)}+e^{(8\,x)}-8\,e^{(6\,x)}+8\,e^{(2\,x)}-1\right)e^{(-4\,x)}$$

```
[In] integrate((-1/exp(x)+exp(x))^4,x, algorithm="fricas")
[Out] 1/4*(24*x*e^(4*x) + e^(8*x) - 8*e^(6*x) + 8*e^(2*x) - 1)*e^(-4*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \left(-e^{-x}+e^{x}\right)^{4}\,dx=6x+\frac{e^{4x}}{4}-2e^{2x}+2e^{-2x}-\frac{e^{-4x}}{4}$$

```
[In] integrate((-1/exp(x)+exp(x))**4,x)
[Out] 6*x + exp(4*x)/4 - 2*exp(2*x) + 2*exp(-2*x) - exp(-4*x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \left(-e^{-x}+e^{x}\right)^{4}\,dx = 6\,x + \frac{1}{4}\,e^{(4\,x)} - 2\,e^{(2\,x)} + 2\,e^{(-2\,x)} - \frac{1}{4}\,e^{(-4\,x)}$$

```
[In] integrate((-1/exp(x)+exp(x))^4,x, algorithm="maxima")
[Out] 6*x + 1/4*e^(4*x) - 2*e^(2*x) + 2*e^(-2*x) - 1/4*e^(-4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \left(-e^{-x}+e^{x}\right)^{4}\,dx = -\frac{1}{4}\,\left(18\,e^{(4\,x)} - 8\,e^{(2\,x)} + 1\right)e^{(-4\,x)} + 6\,x + \frac{1}{4}\,e^{(4\,x)} - 2\,e^{(2\,x)}$$

```
[In] integrate((-1/exp(x)+exp(x))^4,x, algorithm="giac")
[Out] -1/4*(18*e^(4*x) - 8*e^(2*x) + 1)*e^(-4*x) + 6*x + 1/4*e^(4*x) - 2*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \left(-e^{-x}+e^{x}\right)^{4}\,dx = 6\,x + 2\,\mathrm{e}^{-2\,x} - 2\,\mathrm{e}^{2\,x} - \frac{\mathrm{e}^{-4\,x}}{4} + \frac{\mathrm{e}^{4\,x}}{4}$$

```
[In] int((exp(-x) - exp(x))^4,x)
[Out] 6*x + 2*exp(-2*x) - 2*exp(2*x) - exp(-4*x)/4 + exp(4*x)/4
```

3.500 $\int (-e^{-x}+e^x)^n \, dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int (-e^{-x}+e^x)^n \, dx = -\frac{(-e^{-x}+e^x)^n (1-e^{2x}) \text{Hypergeometric2F1}\left(1,\frac{2+n}{2},1-\frac{n}{2},e^{2x}\right)}{n}$$

```
[Out] -(-1/exp(x)+exp(x))^n*(1-exp(2*x))*hypergeom([1, 1+1/2*n],[1-1/2*n],exp(2*x
))/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2320, 2057, 372, 371}

$$\int (-e^{-x}+e^x)^n \, dx = -\frac{(e^x-e^{-x})^n (1-e^{2x})^{-n} \text{Hypergeometric2F1}\left(-n,-\frac{n}{2},1-\frac{n}{2},e^{2x}\right)}{n}$$

```
[In] Int[(-E^(-x) + E^x)^n,x]
```

```
[Out] -(((-E^(-x) + E^x)^n*Hypergeometric2F1[-n, -1/2*n, 1 - n/2, E^(2*x)])/((1 -
 E^(2*x))^n*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\left(-\frac{1}{x}+x\right)^n}{x}\,dx, x, e^x\right)\\
&= \left((e^x)^n\left(-e^{-x}+e^x\right)^n\left(-1+e^{2x}\right)^{-n}\right)\text{Subst}\left(\int x^{-1-n}\left(-1+x^2\right)^n\,dx, x, e^x\right)\\
&= \left((e^x)^n\left(-e^{-x}+e^x\right)^n\left(1-e^{2x}\right)^{-n}\right)\text{Subst}\left(\int x^{-1-n}\left(1-x^2\right)^n\,dx, x, e^x\right)\\
&= -\frac{\left(-e^{-x}+e^x\right)^n\left(1-e^{2x}\right)^{-n}\text{Hypergeometric2F1}\left(-n,-\frac{n}{2},1-\frac{n}{2},e^{2x}\right)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \left(-e^{-x}+e^x\right)^n\,dx = \frac{\left(-e^{-x}+e^x\right)^n\left(-1+e^{2x}\right)\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},1-\frac{n}{2},e^{2x}\right)}{n}$$

```
[In] Integrate[(-E^(-x) + E^x)^n,x]
[Out] ((-E^(-x) + E^x)^n*(-1 + E^(2*x))*Hypergeometric2F1[1, 1 + n/2, 1 - n/2, E^
(2*x)])/n
```

Maple [F]

$$\int \left(-\mathrm{e}^{-x}+\mathrm{e}^{x}\right)^{n}dx$$

```
[In] int((-1/exp(x)+exp(x))^n,x)
[Out] int((-1/exp(x)+exp(x))^n,x)
```

Fricas [F]

$$\int \left(-e^{-x}+e^{x}\right)^{n}\,dx=\int \left(-e^{(-x)}+e^{x}\right)^{n}dx$$

```
[In] integrate((-1/exp(x)+exp(x))^n,x, algorithm="fricas")
[Out] integral((-e^(-x) + e^x)^n, x)
```

Sympy [F]

$$\int \left(-e^{-x}+e^{x}\right)^{n}\,dx=\int \left(e^{x}-e^{-x}\right)^{n}\,dx$$

```
[In] integrate((-1/exp(x)+exp(x))**n,x)
[Out] Integral((exp(x) - exp(-x))**n, x)
```

Maxima [F]

$$\int \left(-e^{-x}+e^{x}\right)^{n}\,dx=\int \left(-e^{(-x)}+e^{x}\right)^{n}dx$$

```
[In] integrate((-1/exp(x)+exp(x))^n,x, algorithm="maxima")
[Out] integrate((-e^(-x) + e^x)^n, x)
```

Giac [F]

$$\int \left(-e^{-x}+e^{x}\right)^{n}\,dx=\int \left(-e^{(-x)}+e^{x}\right)^{n}\,dx$$

```
[In] integrate((-1/exp(x)+exp(x))^n,x, algorithm="giac")
[Out] integrate((-e^(-x) + e^x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-e^{-x}+e^{x}\right)^{n}\,dx=\int \left(\mathrm{e}^{x}-\mathrm{e}^{-x}\right)^{n}\,dx$$

```
[In] int((exp(x) - exp(-x))^n,x)
[Out] int((exp(x) - exp(-x))^n, x)
```

3.501 $\int \left(a^{-4x}-a^{2x}\right)^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \left(a^{-4x}-a^{2x}\right)^3 \, dx=3x-\frac{a^{-12x}}{12\log(a)}+\frac{a^{-6x}}{2\log(a)}-\frac{a^{6x}}{6\log(a)}$$

```
[Out] 3*x-1/12/(a^(12*x))/ln(a)+1/2/(a^(6*x))/ln(a)-1/6*a^(6*x)/ln(a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2320, 272, 45}

$$\int \left(a^{-4x}-a^{2x}\right)^3 \, dx=-\frac{a^{-12x}}{12\log(a)}+\frac{a^{-6x}}{2\log(a)}-\frac{a^{6x}}{6\log(a)}+3x$$

```
[In] Int[(a^(-4*x) - a^(2*x))^3,x]
```

```
[Out] 3*x - 1/(12*a^(12*x)*Log[a]) + 1/(2*a^(6*x)*Log[a]) - a^(6*x)/(6*Log[a])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^3)^3}{x^7}\,dx, x, a^{2x}\right)}{2\log(a)}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^3}{x^3}\,dx, x, a^{6x}\right)}{6\log(a)}\\
&= \frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^3}-\frac{3}{x^2}+\frac{3}{x}\right)\,dx, x, a^{6x}\right)}{6\log(a)}\\
&= 3x-\frac{a^{-12x}}{12\log(a)}+\frac{a^{-6x}}{2\log(a)}-\frac{a^{6x}}{6\log(a)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \left(a^{-4x}-a^{2x}\right)^3\,dx = -\frac{a^{-12x}-6a^{-6x}+2a^{6x}-36\log\left(a^x\right)}{12\log(a)}$$

```
[In] Integrate[(a^(-4*x) - a^(2*x))^3,x]
[Out] -1/12*(a^(-12*x) - 6/a^(6*x) + 2*a^(6*x) - 36*Log[a^x])/Log[a]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

method	result	size
risch	$3x-\frac{a^{6x}}{6\ln(a)}+\frac{a^{-6x}}{2\ln(a)}-\frac{a^{-12x}}{12\ln(a)}$	44
norman	$\left(-\frac{1}{12\ln(a)}+3x\,\mathrm{e}^{12x\ln(a)}+\frac{\mathrm{e}^{6x\ln(a)}}{2\ln(a)}-\frac{\mathrm{e}^{18x\ln(a)}}{6\ln(a)}\right)\mathrm{e}^{-12x\ln(a)}$	56
parallelrisch	$-\frac{(1-36a^{8x}a^{4x}x\ln(a)+2a^{12x}a^{6x}-6a^{2x}a^{4x})a^{-12x}}{12\ln(a)}$	63

```
[In] int((1/(a^(4*x))-a^(2*x))^3,x,method=_RETURNVERBOSE)
[Out] 3*x-1/6/ln(a)*(a^(2*x))^3+1/2/ln(a)/(a^(2*x))^3-1/12/ln(a)/(a^(2*x))^6
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int\left(a^{-4x}-a^{2x}\right)^3\,dx=\frac{36\,a^{12\,x}x\log\left(a\right)-2\,a^{18\,x}+6\,a^{6\,x}-1}{12\,a^{12\,x}\log\left(a\right)}$$

```
[In] integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="fricas")
[Out] 1/12*(36*a^(12*x)*x*log(a) - 2*a^(18*x) + 6*a^(6*x) - 1)/(a^(12*x)*log(a))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int\left(a^{-4x}-a^{2x}\right)^3\,dx=3x+\begin{cases}\frac{-24a^{6x}\log\left(a\right)^2+72a^{-6x}\log\left(a\right)^2-12a^{-12x}\log\left(a\right)^2}{144\log\left(a\right)^3} & \text{for }\log\left(a\right)^3\neq 0\\ -3x & \text{otherwise}\end{cases}$$

```
[In] integrate((1/(a**(4*x))-a**(2*x))**3,x)
[Out] 3*x + Piecewise(((-24*a**(6*x)*log(a)**2 + 72*log(a)**2/a**(6*x) - 12*log(a
)**2/a**(12*x))/(144*log(a)**3), Ne(log(a)**3, 0)), (-3*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \left(a^{-4x} - a^{2x}\right)^3 \, dx = 3\,x - \frac{a^{6\,x}}{6\,\log\left(a\right)} - \frac{1}{12\,a^{12\,x}\log\left(a\right)} + \frac{1}{2\,a^{6\,x}\log\left(a\right)}$$

```
[In] integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="maxima")
[Out] 3*x - 1/6*a^(6*x)/log(a) - 1/12/(a^(12*x)*log(a)) + 1/2/(a^(6*x)*log(a))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \left(a^{-4x} - a^{2x}\right)^3 \, dx = -\frac{2\,a^{6\,x} + \frac{9\,a^{12\,x} - 6\,a^{6\,x} + 1}{a^{12\,x}} - 6\,\log\left(a^{6\,x}\right)}{12\,\log\left(a\right)}$$

```
[In] integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="giac")
[Out] -1/12*(2*a^(6*x) + (9*a^(12*x) - 6*a^(6*x) + 1)/a^(12*x) - 6*log(a^(6*x)))/
log(a)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \left(a^{-4x} - a^{2x}\right)^3 \, dx = 3\,x + \frac{1}{2\,a^{6\,x}\,\ln\left(a\right)} - \frac{a^{6\,x}}{6\,\ln\left(a\right)} - \frac{1}{12\,a^{12\,x}\,\ln\left(a\right)}$$

```
[In] int(-(a^(2*x) - 1/a^(4*x))^3,x)
[Out] 3*x + 1/(2*a^(6*x)*log(a)) - a^(6*x)/(6*log(a)) - 1/(12*a^(12*x)*log(a))
```

3.502 $\int (a^{kx} + a^{lx}) \, dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int (a^{kx} + a^{lx}) \, dx = \frac{a^{kx}}{k \log(a)} + \frac{a^{lx}}{l \log(a)}$$

```
[Out] a^(k*x)/k/ln(a)+a^(l*x)/l/ln(a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2225}

$$\int (a^{kx} + a^{lx}) \, dx = \frac{a^{kx}}{k \log(a)} + \frac{a^{lx}}{l \log(a)}$$

```
[In] Int[a^(k*x) + a^(l*x),x]
```

```
[Out] a^(k*x)/(k*Log[a]) + a^(l*x)/(l*Log[a])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int a^{kx} \, dx + \int a^{lx} \, dx \\ &= \frac{a^{kx}}{k \log(a)} + \frac{a^{lx}}{l \log(a)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}+a^{lx}\right)\,dx=\frac{a^{kx}}{k\log(a)}+\frac{a^{lx}}{l\log(a)}$$

```
[In] Integrate[a^(k*x) + a^(l*x),x]
[Out] a^(k*x)/(k*Log[a]) + a^(l*x)/(l*Log[a])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{a^{kx}l+a^{lx}k}{k\ln(a)l}$	27
default	$\frac{a^{kx}}{k\ln(a)}+\frac{a^{lx}}{l\ln(a)}$	28
risch	$\frac{a^{kx}}{k\ln(a)}+\frac{a^{lx}}{l\ln(a)}$	28
parts	$\frac{a^{kx}}{k\ln(a)}+\frac{a^{lx}}{l\ln(a)}$	28
norman	$\frac{e^{kx\ln(a)}}{k\ln(a)}+\frac{e^{lx\ln(a)}}{l\ln(a)}$	30
meijerg	$-\frac{1-e^{kx\ln(a)}}{k\ln(a)}-\frac{1-e^{lx\ln(a)}}{l\ln(a)}$	40

```
[In] int(a^(k*x)+a^(l*x),x,method=_RETURNVERBOSE)
[Out] (a^(k*x)*l+a^(l*x)*k)/k/ln(a)/l
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \left(a^{kx}+a^{lx}\right)\,dx=\frac{a^{lx}k+a^{kx}l}{kl\log\left(a\right)}$$

```
[In] integrate(a^(k*x)+a^(l*x),x, algorithm="fricas")
[Out] (a^(l*x)*k + a^(k*x)*l)/(k*l*log(a))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \left(a^{kx}+a^{lx}\right)\,dx=\begin{cases}\frac{a^{kx}}{k\log\left(a\right)} & \text{for } k\log\left(a\right)\neq 0\\ x & \text{otherwise}\end{cases}+\begin{cases}\frac{a^{lx}}{l\log\left(a\right)} & \text{for } l\log\left(a\right)\neq 0\\ x & \text{otherwise}\end{cases}$$

```
[In] integrate(a**(k*x)+a**(l*x),x)
```

```
[Out] Piecewise((a**(k*x)/(k*log(a)), Ne(k*log(a), 0)), (x, True)) + Piecewise((a
**(l*x)/(l*log(a)), Ne(l*log(a), 0)), (x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}+a^{lx}\right)\,dx=\frac{a^{kx}}{k\log\left(a\right)}+\frac{a^{lx}}{l\log\left(a\right)}$$

```
[In] integrate(a^(k*x)+a^(l*x),x, algorithm="maxima")
```

```
[Out] a^(k*x)/(k*log(a)) + a^(l*x)/(l*log(a))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}+a^{lx}\right)\,dx=\frac{a^{kx}}{k\log\left(a\right)}+\frac{a^{lx}}{l\log\left(a\right)}$$

```
[In] integrate(a^(k*x)+a^(l*x),x, algorithm="giac")
```

```
[Out] a^(k*x)/(k*log(a)) + a^(l*x)/(l*log(a))
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \left(a^{kx}+a^{lx}\right)\,dx=\frac{a^{k\,x}\,l+a^{l\,x}\,k}{k\,l\,\ln\left(a\right)}$$

```
[In] int(a^(k*x) + a^(l*x),x)
[Out] (a^(k*x)*l + a^(l*x)*k)/(k*l*log(a))
```

3.503 $\int \left(a^{kx}+a^{lx}\right)^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \left(a^{kx}+a^{lx}\right)^2 \, dx=\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}+\frac{2a^{(k+l)x}}{(k+l)\log(a)}$$

```
[Out] 1/2*a^(2*k*x)/k/ln(a)+1/2*a^(2*l*x)/l/ln(a)+2*a^((k+l)*x)/(k+l)/ln(a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6874, 2225}

$$\int \left(a^{kx}+a^{lx}\right)^2 \, dx=\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

```
[In] Int[(a^(k*x) + a^(l*x))^2,x]
```

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) + (2*a^((k + l)*x))/((k + l
)*Log[a])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(e^{kx}+e^{lx}\right)^2 \, dx, x, x\log(a)\right)}{\log(a)} \\
&= \frac{\text{Subst}\left(\int \left(e^{2kx}+e^{2lx}+2e^{(k+l)x}\right) \, dx, x, x\log(a)\right)}{\log(a)} \\
&= \frac{\text{Subst}\left(\int e^{2kx} \, dx, x, x\log(a)\right)}{\log(a)}+\frac{\text{Subst}\left(\int e^{2lx} \, dx, x, x\log(a)\right)}{\log(a)}+\frac{2\text{Subst}\left(\int e^{(k+l)x} \, dx, x, x\log(a)\right)}{\log(a)} \\
&= \frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}+\frac{2a^{(k+l)x}}{(k+l)\log(a)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$
\int \left(a^{kx}+a^{lx}\right)^2 \, dx = \frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}+\frac{2a^{(k+l)x}}{(k+l)\log(a)}
$$

```
[In] Integrate[(a^(k*x) + a^(l*x))^2,x]
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) + (2*a^((k + l)*x))/((k + l
)*Log[a])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

method	result	size
risch	$\frac{a^{2kx}}{2k\ln(a)}+\frac{a^{2lx}}{2l\ln(a)}+\frac{2a^{kx}a^{lx}}{\ln(a)(k+l)}$	55
norman	$\frac{\mathrm{e}^{2kx\ln(a)}}{2k\ln(a)}+\frac{\mathrm{e}^{2lx\ln(a)}}{2l\ln(a)}+\frac{2\,\mathrm{e}^{kx\ln(a)}\mathrm{e}^{lx\ln(a)}}{\ln(a)(k+l)}$	59
parallelrisch	$\frac{a^{2kx}lk+a^{2kx}l^2+4a^{kx}a^{lx}kl+a^{2lx}k^2+a^{2lx}kl}{2\ln(a)kl(k+l)}$	75
meijerg	$-\frac{1-\mathrm{e}^{2kx\ln(a)}}{2k\ln(a)}-\frac{2\left(1-\mathrm{e}^{xl\ln(a)\left(1+\frac{k}{l}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)}-\frac{1-\mathrm{e}^{2lx\ln(a)}}{2l\ln(a)}$	77

```
[In] int((a^(k*x)+a^(l*x))^2,x,method=_RETURNVERBOSE)
[Out] 1/2/k/ln(a)*(a^(k*x))^2+1/2/l/ln(a)*(a^(l*x))^2+2/ln(a)/(k+l)*a^(k*x)*a^(l*
x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int \left(a^{kx}+a^{lx}\right)^2\,dx=\frac{4\,a^{kx}a^{lx}kl+(kl+l^2)a^{2\,kx}+(k^2+kl)a^{2\,lx}}{2\,(k^2l+kl^2)\log{(a)}}$$

```
[In] integrate((a^(k*x)+a^(l*x))^2,x, algorithm="fricas")
[Out] 1/2*(4*a^(k*x)*a^(l*x)*k*l + (k*l + l^2)*a^(2*k*x) + (k^2 + k*l)*a^(2*l*x))
/((k^2*l + k*l^2)*log(a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(41) = 82.

Time = 0.46 (sec) , antiderivative size = 250, normalized size of antiderivative = 4.72

$$\int \left(a^{kx}+a^{lx}\right)^2\,dx = \begin{cases} 4x \\ \frac{a^{2lx}}{2l\log{(a)}}+\frac{2a^{lx}}{l\log{(a)}}+x \\ \frac{a^{2lx}}{2l\log{(a)}}+2x-\frac{a^{-2lx}}{2l\log{(a)}} \\ \frac{a^{2kx}}{2k\log{(a)}}+\frac{2a^{kx}}{k\log{(a)}}+x \\ \frac{a^{2kx}kl}{2k^2l\log{(a)}+2kl^2\log{(a)}}+\frac{a^{2kx}l^2}{2k^2l\log{(a)}+2kl^2\log{(a)}}+\frac{4a^{kx}a^{lx}kl}{2k^2l\log{(a)}+2kl^2\log{(a)}}+\frac{a^{2lx}k^2}{2k^2l\log{(a)}+2kl^2\log{(a)}}+\frac{a^{2lx}kl}{2k^2l\log{(a)}+2kl^2\log{(a)}} \end{cases}$$

```
[In] integrate((a**(k*x)+a**(l*x))**2,x)
[Out] Piecewise((4*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))),
(a**(2*l*x)/(2*l*log(a)) + 2*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(2*l*x
)/(2*l*log(a)) + 2*x - 1/(2*a**(2*l*x)*l*log(a)), Eq(k, -l)), (a**(2*k*x)/(
2*k*log(a)) + 2*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (a**(2*k*x)*k*l/(2*k**2
*l*log(a) + 2*k*l**2*log(a)) + a**(2*k*x)*l**2/(2*k**2*l*log(a) + 2*k*l**2*
log(a)) + 4*a**(k*x)*a**(l*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**
(2*l*x)*k**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*k*l/(2*k**2*l
*log(a) + 2*k*l**2*log(a)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \left(a^{kx}+a^{lx}\right)^2 \, dx = \frac{2\, a^{kx+lx}}{(k+l)\log{(a)}} + \frac{a^{2\,kx}}{2\,k\log{(a)}} + \frac{a^{2\,lx}}{2\,l\log{(a)}}$$

```
[In] integrate((a^(k*x)+a^(l*x))^2,x, algorithm="maxima")
[Out] 2*a^(k*x + l*x)/((k + l)*log(a)) + 1/2*a^(2*k*x)/(k*log(a)) + 1/2*a^(2*l*x)
/(l*log(a))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 691, normalized size of antiderivative = 13.04

$$\int \left(a^{kx}+a^{lx}\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((a^(k*x)+a^(l*x))^2,x, algorithm="giac")
[Out] (2*k*cos(-pi*k*x*sgn(a) + pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2 + (pi*k*
sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-pi*k*x*sgn(a) + pi*k*x)/(4*k^
2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(2*k*x) + (2*l*cos(-pi*l*
x*sgn(a) + pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^
2) - (pi*l*sgn(a) - pi*l)*sin(-pi*l*x*sgn(a) + pi*l*x)/(4*l^2*log(abs(a))^2
 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(2*l*x) - 1/2*I*abs(a)^(2*k*x)*(-I*e^(I*
pi*k*x*sgn(a) - I*pi*k*x)/(I*pi*k*sgn(a) - I*pi*k + 2*k*log(abs(a))) + I*e^
(-I*pi*k*x*sgn(a) + I*pi*k*x)/(-I*pi*k*sgn(a) + I*pi*k + 2*k*log(abs(a))))
- 1/2*I*abs(a)^(2*l*x)*(-I*e^(I*pi*l*x*sgn(a) - I*pi*l*x)/(I*pi*l*sgn(a) -
I*pi*l + 2*l*log(abs(a))) + I*e^(-I*pi*l*x*sgn(a) + I*pi*l*x)/(-I*pi*l*sgn(
a) + I*pi*l + 2*l*log(abs(a)))) + 4*(2*(k*log(abs(a)) + l*log(abs(a)))*cos(
-1/2*pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*x)/((pi*k*sg
n(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2)
- (pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)*sin(-1/2*pi*k*x*sgn(a) - 1/2*pi
*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*x)/((pi*k*sgn(a) + pi*l*sgn(a) - pi*k -
 pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2))*e^((k*log(abs(a)) + l*log(
abs(a)))*x) + 2*I*(I*e^(1/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l*x*sgn(a) - 1/2*I*p
i*k*x - 1/2*I*pi*l*x)/(I*pi*k*sgn(a) + I*pi*l*sgn(a) - I*pi*k - I*pi*l + 2*
k*log(abs(a)) + 2*l*log(abs(a))) - I*e^(-1/2*I*pi*k*x*sgn(a) - 1/2*I*pi*l*x
*sgn(a) + 1/2*I*pi*k*x + 1/2*I*pi*l*x)/(-I*pi*k*sgn(a) - I*pi*l*sgn(a) + I*
pi*k + I*pi*l + 2*k*log(abs(a)) + 2*l*log(abs(a))))*e^((k*log(abs(a)) + l*l
og(abs(a)))*x)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

$$\int (a^{kx}+a^{lx})^2\,dx=\frac{a^{2\,k\,x}}{2\,k\,\ln(a)}+\frac{\frac{a^{2\,l\,x}\,k^2}{2}+l\left(2\,a^{k\,x+l\,x}\,k+\frac{a^{2\,l\,x}\,k}{2}\right)}{k\,l\,\ln(a)\,(k+l)}$$

[In] int((a^(k*x) + a^(l*x))^2,x)

[Out] a^(2*k*x)/(2*k*log(a)) + ((a^(2*l*x)*k^2)/2 + l*(2*a^(k*x + l*x)*k + (a^(2*l*x)*k)/2))/(k*l*log(a)*(k + l))

3.504 $\int \left(a^{kx}+a^{lx}\right)^3 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 79

$$\int \left(a^{kx}+a^{lx}\right)^3 \, dx=\frac{a^{3kx}}{3k\log(a)}+\frac{a^{3lx}}{3l\log(a)}+\frac{3a^{(2k+l)x}}{(2k+l)\log(a)}+\frac{3a^{(k+2l)x}}{(k+2l)\log(a)}$$

```
[Out] 1/3*a^(3*k*x)/k/ln(a)+1/3*a^(3*l*x)/l/ln(a)+3*a^((2*k+l)*x)/(2*k+l)/ln(a)+3
*a^((k+2*l)*x)/(k+2*l)/ln(a)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6874, 2225}

$$\int \left(a^{kx}+a^{lx}\right)^3 \, dx=\frac{3a^{x(2k+l)}}{\log(a)(2k+l)}+\frac{3a^{x(k+2l)}}{\log(a)(k+2l)}+\frac{a^{3kx}}{3k\log(a)}+\frac{a^{3lx}}{3l\log(a)}$$

```
[In] Int[(a^(k*x) + a^(l*x))^3,x]
```

```
[Out] a^(3*k*x)/(3*k*Log[a]) + a^(3*l*x)/(3*l*Log[a]) + (3*a^((2*k + l)*x))/((2*k
 + l)*Log[a]) + (3*a^((k + 2*l)*x))/((k + 2*l)*Log[a])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(e^{kx}+e^{lx}\right)^3\,dx,x,x\log(a)\right)}{\log(a)}\\
&= \frac{\text{Subst}\left(\int \left(e^{3kx}+e^{3lx}+3e^{(2k+l)x}+3e^{(k+2l)x}\right)\,dx,x,x\log(a)\right)}{\log(a)}\\
&= \frac{\text{Subst}\left(\int e^{3kx}\,dx,x,x\log(a)\right)}{\log(a)}+\frac{\text{Subst}\left(\int e^{3lx}\,dx,x,x\log(a)\right)}{\log(a)}\\
&\quad+\frac{3\text{Subst}\left(\int e^{(2k+l)x}\,dx,x,x\log(a)\right)}{\log(a)}+\frac{3\text{Subst}\left(\int e^{(k+2l)x}\,dx,x,x\log(a)\right)}{\log(a)}\\
&= \frac{a^{3kx}}{3k\log(a)}+\frac{a^{3lx}}{3l\log(a)}+\frac{3a^{(2k+l)x}}{(2k+l)\log(a)}+\frac{3a^{(k+2l)x}}{(k+2l)\log(a)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int \left(a^{kx}+a^{lx}\right)^3\,dx=\frac{\frac{a^{3kx}}{k}+\frac{a^{3lx}}{l}+\frac{9a^{(2k+l)x}}{2k+l}+\frac{9a^{(k+2l)x}}{k+2l}}{3\log(a)}$$

```
[In] Integrate[(a^(k*x) + a^(l*x))^3,x]
[Out] (a^(3*k*x)/k + a^(3*l*x)/l + (9*a^((2*k + l)*x))/(2*k + l) + (9*a^((k + 2*l
)*x))/(k + 2*l))/(3*Log[a])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06

method	result	s
risch	$\frac{a^{3kx}}{3k\ln(a)}+\frac{a^{3lx}}{3l\ln(a)}+\frac{3a^{kx}a^{2lx}}{\ln(a)(k+2l)}+\frac{3a^{2kx}a^{lx}}{\ln(a)(2k+l)}$	8
norman	$\frac{e^{3kx\ln(a)}}{3k\ln(a)}+\frac{e^{3lx\ln(a)}}{3l\ln(a)}+\frac{3e^{kx\ln(a)}e^{2lx\ln(a)}}{\ln(a)(k+2l)}+\frac{3e^{2kx\ln(a)}e^{lx\ln(a)}}{\ln(a)(2k+l)}$	9
meijerg	$-\frac{1-e^{3kx\ln(a)}}{3k\ln(a)}-\frac{3\left(1-e^{xk\ln(a)\left(2+\frac{l}{k}\right)}\right)}{k\ln(a)\left(2+\frac{l}{k}\right)}-\frac{3\left(1-e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{1}{1+\frac{k}{l}}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{1}{1+\frac{k}{l}}\right)}-\frac{1-e^{3lx\ln(a)}}{3l\ln(a)}$	1
parallelrisch	$\frac{2a^{3kx}k^2l+5a^{3kx}kl^2+2a^{3kx}l^3+9a^{2kx}a^{lx}k^2l+18a^{2kx}a^{lx}kl^2+18a^{kx}a^{2lx}k^2l+9a^{kx}a^{2lx}kl^2+2a^{3lx}k^3+5a^{3lx}k^2l+2a^{3lx}kl^2}{3\ln(a)kl(k+2l)(2k+l)}$	1

```
[In] int((a^(k*x)+a^(l*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/k/ln(a)*(a^(k*x))^3+1/3/l/ln(a)*(a^(l*x))^3+3/ln(a)/(k+2*l)*a^(k*x)*(a^
(l*x))^2+3/ln(a)/(2*k+l)*(a^(k*x))^2*a^(l*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.65

$$\int \left(a^{kx}+a^{lx}\right)^3 dx = \frac{9\,(2\,k^2l+kl^2)a^{kx}a^{2\,lx}+9\,(k^2l+2\,kl^2)a^{2\,kx}a^{lx}+(2\,k^2l+5\,kl^2+2\,l^3)a^{3\,kx}+(2\,k^3+5\,k^2l+2\,kl^2)a^{3\,lx}}{3\,(2\,k^3l+5\,k^2l^2+2\,kl^3)\log(a)}$$

```
[In] integrate((a^(k*x)+a^(l*x))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(9*(2*k^2*l + k*l^2)*a^(k*x)*a^(2*l*x) + 9*(k^2*l + 2*k*l^2)*a^(2*k*x)*
a^(l*x) + (2*k^2*l + 5*k*l^2 + 2*l^3)*a^(3*k*x) + (2*k^3 + 5*k^2*l + 2*k*l^
2)*a^(3*l*x))/((2*k^3*l + 5*k^2*l^2 + 2*k*l^3)*log(a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. 2(63) = 126.

Time = 2.05 (sec) , antiderivative size = 665, normalized size of antiderivative = 8.42

$$\int \left(a^{kx}+a^{lx}\right)^3 dx = \begin{cases} 8x \\ \frac{a^{3lx}}{3l\log(a)}+\frac{3a^{2lx}}{2l\log(a)}+\frac{3a^{lx}}{l\log(a)}+x \\ \frac{a^{3lx}}{3l\log(a)}+3x-\frac{a^{-3lx}}{l\log(a)}-\frac{a^{-6lx}}{6l\log(a)} \\ \frac{2a^{\frac{3lx}{2}}}{l\log(a)}+\frac{a^{3lx}}{3l\log(a)}+3x-\frac{2a^{-\frac{3lx}{2}}}{3l\log(a)} \\ \frac{a^{3kx}}{3k\log(a)}+\frac{3a^{2kx}}{2k\log(a)}+\frac{3a^{kx}}{k\log(a)}+x \\ \frac{2a^{3kx}k^2l}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)}+\frac{5a^{3kx}kl^2}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)}+\frac{2a^{3kx}l^3}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)}+\frac{}{6k^3l\log(a}\end{cases}$$

```
[In] integrate((a**(k*x)+a**(l*x))**3,x)
```

```
[Out] Piecewise((8*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))),
(a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(2*l*log(a)) + 3*a**(l*x)/(l*log(a)
) + x, Eq(k, 0)), (a**(3*l*x)/(3*l*log(a)) + 3*x - 1/(a**(3*l*x)*l*log(a))
- 1/(6*a**(6*l*x)*l*log(a)), Eq(k, -2*l)), (2*a**(3*l*x/2)/(l*log(a)) + a**
```

```
(3*l*x)/(3*l*log(a)) + 3*x - 2/(3*a**(3*l*x/2)*l*log(a)), Eq(k, -l/2)), (a*
*(3*k*x)/(3*k*log(a)) + 3*a**(2*k*x)/(2*k*log(a)) + 3*a**(k*x)/(k*log(a)) +
 x, Eq(l, 0)), (2*a**(3*k*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a)
+ 6*k*l**3*log(a)) + 5*a**(3*k*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*lo
g(a) + 6*k*l**3*log(a)) + 2*a**(3*k*x)*l**3/(6*k**3*l*log(a) + 15*k**2*l**2
*log(a) + 6*k*l**3*log(a)) + 9*a**(2*k*x)*a**(l*x)*k**2*l/(6*k**3*l*log(a)
+ 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a**(2*k*x)*a**(l*x)*k*l**2/(6
*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a**(k*x)*a**(2
*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 9*
a**(k*x)*a**(2*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**
3*log(a)) + 2*a**(3*l*x)*k**3/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*
l**3*log(a)) + 5*a**(3*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) +
 6*k*l**3*log(a)) + 2*a**(3*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log
(a) + 6*k*l**3*log(a)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int \left(a^{kx}+a^{lx}\right)^3\,dx=\frac{3\,a^{2\,kx+lx}}{(2\,k+l)\log{(a)}}+\frac{3\,a^{kx+2\,lx}}{(k+2\,l)\log{(a)}}+\frac{a^{3\,kx}}{3\,k\log{(a)}}+\frac{a^{3\,lx}}{3\,l\log{(a)}}$$

```
[In] integrate((a^(k*x)+a^(l*x))^3,x, algorithm="maxima")
```

```
[Out] 3*a^(2*k*x + l*x)/((2*k + l)*log(a)) + 3*a^(k*x + 2*l*x)/((k + 2*l)*log(a))
 + 1/3*a^(3*k*x)/(k*log(a)) + 1/3*a^(3*l*x)/(l*log(a))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1033, normalized size of antiderivative = 13.08

$$\int \left(a^{kx}+a^{lx}\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate((a^(k*x)+a^(l*x))^3,x, algorithm="giac")
```

```
[Out] 2/3*(2*k*cos(-3/2*pi*k*x*sgn(a) + 3/2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a)
)^2 + (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-3/2*pi*k*x*sgn(a)
 + 3/2*pi*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(3*k*
x) + 2/3*(2*l*cos(-3/2*pi*l*x*sgn(a) + 3/2*pi*l*x)*log(abs(a))/(4*l^2*log(a
bs(a))^2 + (pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-3/2*pi*l*x*s
gn(a) + 3/2*pi*l*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^
```

```
(3*l*x) + I*abs(a)^(3*k*x)*(I*e^(3/2*I*pi*k*x*sgn(a) - 3/2*I*pi*k*x)/(3*I*p
i*k*sgn(a) - 3*I*pi*k + 6*k*log(abs(a))) - I*e^(-3/2*I*pi*k*x*sgn(a) + 3/2*
I*pi*k*x)/(-3*I*pi*k*sgn(a) + 3*I*pi*k + 6*k*log(abs(a)))) + I*abs(a)^(3*l*
x)*(I*e^(3/2*I*pi*l*x*sgn(a) - 3/2*I*pi*l*x)/(3*I*pi*l*sgn(a) - 3*I*pi*l +
6*l*log(abs(a))) - I*e^(-3/2*I*pi*l*x*sgn(a) + 3/2*I*pi*l*x)/(-3*I*pi*l*sgn
(a) + 3*I*pi*l + 6*l*log(abs(a)))) + 6*(2*(2*k*log(abs(a)) + l*log(abs(a)))
*cos(-pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*l*x)/((2*pi*k*sgn
(a) + pi*l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a)) + l*log(abs(a)))^
2) - (2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)*sin(-pi*k*x*sgn(a) - 1/2
*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*l*x)/((2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*
k - pi*l)^2 + 4*(2*k*log(abs(a)) + l*log(abs(a)))^2))*e^((2*k*log(abs(a)) +
 l*log(abs(a)))*x) + 3*I*(I*e^(I*pi*k*x*sgn(a) + 1/2*I*pi*l*x*sgn(a) - I*pi
*k*x - 1/2*I*pi*l*x)/(2*I*pi*k*sgn(a) + I*pi*l*sgn(a) - 2*I*pi*k - I*pi*l +
 4*k*log(abs(a)) + 2*l*log(abs(a))) - I*e^(-I*pi*k*x*sgn(a) - 1/2*I*pi*l*x*
sgn(a) + I*pi*k*x + 1/2*I*pi*l*x)/(-2*I*pi*k*sgn(a) - I*pi*l*sgn(a) + 2*I*p
i*k + I*pi*l + 4*k*log(abs(a)) + 2*l*log(abs(a))))*e^((2*k*log(abs(a)) + l*
log(abs(a)))*x) + 6*(2*(k*log(abs(a)) + 2*l*log(abs(a)))*cos(-1/2*pi*k*x*sg
n(a) - pi*l*x*sgn(a) + 1/2*pi*k*x + pi*l*x)/((pi*k*sgn(a) + 2*pi*l*sgn(a) -
 pi*k - 2*pi*l)^2 + 4*(k*log(abs(a)) + 2*l*log(abs(a)))^2) - (pi*k*sgn(a) +
 2*pi*l*sgn(a) - pi*k - 2*pi*l)*sin(-1/2*pi*k*x*sgn(a) - pi*l*x*sgn(a) + 1/
2*pi*k*x + pi*l*x)/((pi*k*sgn(a) + 2*pi*l*sgn(a) - pi*k - 2*pi*l)^2 + 4*(k*
log(abs(a)) + 2*l*log(abs(a)))^2))*e^((k*log(abs(a)) + 2*l*log(abs(a)))*x)
+ 3*I*(I*e^(1/2*I*pi*k*x*sgn(a) + I*pi*l*x*sgn(a) - 1/2*I*pi*k*x - I*pi*l*x
)/(I*pi*k*sgn(a) + 2*I*pi*l*sgn(a) - I*pi*k - 2*I*pi*l + 2*k*log(abs(a)) +
4*l*log(abs(a))) - I*e^(-1/2*I*pi*k*x*sgn(a) - I*pi*l*x*sgn(a) + 1/2*I*pi*k
*x + I*pi*l*x)/(-I*pi*k*sgn(a) - 2*I*pi*l*sgn(a) + I*pi*k + 2*I*pi*l + 2*k*
log(abs(a)) + 4*l*log(abs(a))))*e^((k*log(abs(a)) + 2*l*log(abs(a)))*x)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \left(a^{kx}+a^{lx}\right)^3\,dx=\frac{3\,a^{k\,x}\,a^{2\,l\,x}}{k\,\ln\left(a\right)+2\,l\,\ln\left(a\right)}+\frac{3\,a^{2\,k\,x}\,a^{l\,x}}{2\,k\,\ln\left(a\right)+l\,\ln\left(a\right)}+\frac{a^{3\,k\,x}}{3\,k\,\ln\left(a\right)}+\frac{a^{3\,l\,x}}{3\,l\,\ln\left(a\right)}$$

```
[In] int((a^(k*x) + a^(l*x))^3,x)
```

```
[Out] (3*a^(k*x)*a^(2*l*x))/(k*log(a) + 2*l*log(a)) + (3*a^(2*k*x)*a^(l*x))/(2*k*
log(a) + l*log(a)) + a^(3*k*x)/(3*k*log(a)) + a^(3*l*x)/(3*l*log(a))
```

3.505 $\int \left(a^{kx}+a^{lx}\right)^4 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \left(a^{kx}+a^{lx}\right)^4 \, dx = \frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}+\frac{3a^{2(k+l)x}}{(k+l)\log(a)}+\frac{4a^{(3k+l)x}}{(3k+l)\log(a)}+\frac{4a^{(k+3l)x}}{(k+3l)\log(a)}$$

```
[Out] 1/4*a^(4*k*x)/k/ln(a)+1/4*a^(4*l*x)/l/ln(a)+3*a^(2*(k+l)*x)/(k+l)/ln(a)+4*a
^((3*k+l)*x)/(3*k+l)/ln(a)+4*a^((k+3*l)*x)/(k+3*l)/ln(a)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6874, 2225}

$$\int \left(a^{kx}+a^{lx}\right)^4 \, dx = \frac{3a^{2x(k+l)}}{\log(a)(k+l)}+\frac{4a^{x(3k+l)}}{\log(a)(3k+l)}+\frac{4a^{x(k+3l)}}{\log(a)(k+3l)}+\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}$$

```
[In] Int[(a^(k*x) + a^(l*x))^4,x]
```

```
[Out] a^(4*k*x)/(4*k*Log[a]) + a^(4*l*x)/(4*l*Log[a]) + (3*a^(2*(k + l)*x))/((k +
 l)*Log[a]) + (4*a^((3*k + l)*x))/((3*k + l)*Log[a]) + (4*a^((k + 3*l)*x))/
((k + 3*l)*Log[a])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(e^{kx}+e^{lx}\right)^4\,dx, x, x\log(a)\right)}{\log(a)}$$
$$= \frac{\text{Subst}\left(\int \left(e^{4kx}+e^{4lx}+6e^{2(k+l)x}+4e^{(3k+l)x}+4e^{(k+3l)x}\right)\,dx, x, x\log(a)\right)}{\log(a)}$$
$$= \frac{\text{Subst}\left(\int e^{4kx}\,dx, x, x\log(a)\right)}{\log(a)} + \frac{\text{Subst}\left(\int e^{4lx}\,dx, x, x\log(a)\right)}{\log(a)} + \frac{4\text{Subst}\left(\int e^{(3k+l)x}\,dx, x, x\log(a)\right)}{\log(a)} + \frac{4\text{Subst}\left(\int e^{(k+3l)x}\,dx, x, x\log(a)\right)}{\log(a)} + \frac{6\text{Subst}\left(\int e^{2(k+l)x}\,dx, x, x\log(a)\right)}{\log(a)}$$
$$= \frac{a^{4kx}}{4k\log(a)} + \frac{a^{4lx}}{4l\log(a)} + \frac{3a^{2(k+l)x}}{(k+l)\log(a)} + \frac{4a^{(3k+l)x}}{(3k+l)\log(a)} + \frac{4a^{(k+3l)x}}{(k+3l)\log(a)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \left(a^{kx}+a^{lx}\right)^4\,dx = \frac{\frac{a^{4kx}}{k}+\frac{a^{4lx}}{l}+\frac{12a^{2(k+l)x}}{k+l}+\frac{16a^{(3k+l)x}}{3k+l}+\frac{16a^{(k+3l)x}}{k+3l}}{4\log(a)}$$

```
[In] Integrate[(a^(k*x) + a^(l*x))^4,x]
[Out] (a^(4*k*x)/k + a^(4*l*x)/l + (12*a^(2*(k + l)*x))/(k + l) + (16*a^((3*k + l
)*x))/(3*k + l) + (16*a^((k + 3*l)*x))/(k + 3*l))/(4*Log[a])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

method	result
risch	$\frac{a^{4kx}}{4k\ln(a)}+\frac{4a^{3kx}a^{lx}}{\ln(a)(3k+l)}+\frac{3a^{2kx}a^{2lx}}{\ln(a)(k+l)}+\frac{4a^{kx}a^{3lx}}{\ln(a)(k+3l)}+\frac{a^{4lx}}{4l\ln(a)}$
meijerg	$-\frac{1-e^{4kx\ln(a)}}{4k\ln(a)}-\frac{4\left(1-e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2k}{l\left(1+\frac{k}{l}\right)}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2k}{l\left(1+\frac{k}{l}\right)}\right)}-\frac{3\left(1-e^{2xl\ln(a)\left(1+\frac{k}{l}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)}-\frac{4\left(1-e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2}{1+\frac{k}{l}}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2}{1+\frac{k}{l}}\right)}-$
parallelrisch	$\frac{3a^{4kx}k^3l+13a^{4kx}k^2l^2+13a^{4kx}kl^3+3a^{4kx}l^4+16a^{3kx}a^{lx}k^3l+64a^{3kx}a^{lx}k^2l^2+48a^{3kx}a^{lx}kl^3+36a^{2kx}a^{2lx}k^3l+120a^{2kx}a^{2lx}k^2l}{4\ln(a)k(3k+l)(k+l)(}$

```
[In] int((a^(k*x)+a^(l*x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/ln(a)/k*(a^(k*x))^4+4*(a^(k*x))^3/ln(a)/(3*k+l)*a^(l*x)+3*(a^(k*x))^2/l
n(a)/(k+l)*(a^(l*x))^2+4*a^(k*x)/ln(a)/(k+3*l)*(a^(l*x))^3+1/4/ln(a)/l*(a^(
l*x))^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(94) = 188.

Time = 0.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.09

$$\int \left(a^{kx}+a^{lx}\right)^4\,dx = \frac{16\,(3\,k^3l+4\,k^2l^2+kl^3)a^{kx}a^{3\,lx}+12\,(3\,k^3l+10\,k^2l^2+3\,kl^3)a^{2\,kx}a^{2\,lx}+16\,(k^3l+4\,k^2l^2+3\,kl^3)a^{3\,kx}a^{lx}+}{4\,(3\,k^4l+13\,k^3l^2+13\,k^2l^3+3\,kl^4)\,\mathrm{lo}}$$

```
[In] integrate((a^(k*x)+a^(l*x))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(16*(3*k^3*l + 4*k^2*l^2 + k*l^3)*a^(k*x)*a^(3*l*x) + 12*(3*k^3*l + 10*
k^2*l^2 + 3*k*l^3)*a^(2*k*x)*a^(2*l*x) + 16*(k^3*l + 4*k^2*l^2 + 3*k*l^3)*a
^(3*k*x)*a^(l*x) + (3*k^3*l + 13*k^2*l^2 + 13*k*l^3 + 3*l^4)*a^(4*k*x) + (3
*k^4 + 13*k^3*l + 13*k^2*l^2 + 3*k*l^3)*a^(4*l*x))/((3*k^4*l + 13*k^3*l^2 +
 13*k^2*l^3 + 3*k*l^4)*log(a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1350 vs. 2(82) = 164.

Time = 16.75 (sec) , antiderivative size = 1350, normalized size of antiderivative = 13.78

$$\int \left(a^{kx}+a^{lx}\right)^4\,dx = \text{Too large to display}$$

```
[In] integrate((a**(k*x)+a**(l*x))**4,x)
```

```
[Out] Piecewise((16*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))),
 (a**(4*l*x)/(4*l*log(a)) + 4*a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(l*log
(a)) + 4*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(4*l*x)/(4*l*log(a)) + 4*x
 - 3/(2*a**(4*l*x)*l*log(a)) - 1/(2*a**(8*l*x)*l*log(a)) - 1/(12*a**(12*l*x
)*l*log(a)), Eq(k, -3*l)), (a**(4*l*x)/(4*l*log(a)) + 2*a**(2*l*x)/(l*log(a
)) + 6*x - 2/(a**(2*l*x)*l*log(a)) - 1/(4*a**(4*l*x)*l*log(a)), Eq(k, -l)),
 (3*a**(8*l*x/3)/(2*l*log(a)) + 9*a**(4*l*x/3)/(2*l*log(a)) + a**(4*l*x)/(4
*l*log(a)) + 4*x - 3/(4*a**(4*l*x/3)*l*log(a)), Eq(k, -l/3)), (a**(4*k*x)/(
4*k*log(a)) + 4*a**(3*k*x)/(3*k*log(a)) + 3*a**(2*k*x)/(k*log(a)) + 4*a**(k
*x)/(k*log(a)) + x, Eq(l, 0)), (3*a**(4*k*x)*k**3*l/(12*k**4*l*log(a) + 52*
k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 13*a**(4*k*x)*
k**2*l**2/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 1
2*k*l**4*log(a)) + 13*a**(4*k*x)*k*l**3/(12*k**4*l*log(a) + 52*k**3*l**2*lo
g(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 3*a**(4*k*x)*l**4/(12*k**4
*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) +
 16*a**(3*k*x)*a**(l*x)*k**3*l/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52
*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 64*a**(3*k*x)*a**(l*x)*k**2*l**2/(1
2*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log
(a)) + 48*a**(3*k*x)*a**(l*x)*k*l**3/(12*k**4*l*log(a) + 52*k**3*l**2*log(a
) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 36*a**(2*k*x)*a**(2*l*x)*k**3
*l/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**
4*log(a)) + 120*a**(2*k*x)*a**(2*l*x)*k**2*l**2/(12*k**4*l*log(a) + 52*k**3
*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 36*a**(2*k*x)*a**(
2*l*x)*k*l**3/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a)
 + 12*k*l**4*log(a)) + 48*a**(k*x)*a**(3*l*x)*k**3*l/(12*k**4*l*log(a) + 52
*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 64*a**(k*x)*a
**(3*l*x)*k**2*l**2/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*
log(a) + 12*k*l**4*log(a)) + 16*a**(k*x)*a**(3*l*x)*k*l**3/(12*k**4*l*log(a
) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 3*a**(4
*l*x)*k**4/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) +
12*k*l**4*log(a)) + 13*a**(4*l*x)*k**3*l/(12*k**4*l*log(a) + 52*k**3*l**2*l
og(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 13*a**(4*l*x)*k**2*l**2/(
12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*lo
g(a)) + 3*a**(4*l*x)*k*l**3/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k*
*2*l**3*log(a) + 12*k*l**4*log(a)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \left(a^{kx}+a^{lx}\right)^4 \, dx = \frac{4\,a^{3\,kx+lx}}{(3\,k+l)\log(a)}+\frac{4\,a^{kx+3\,lx}}{(k+3\,l)\log(a)} + \frac{3\,a^{2\,kx+2\,lx}}{(k+l)\log(a)}+\frac{a^{4\,kx}}{4\,k\log(a)}+\frac{a^{4\,lx}}{4\,l\log(a)}$$

```
[In] integrate((a^(k*x)+a^(l*x))^4,x, algorithm="maxima")
```

```
[Out] 4*a^(3*k*x + l*x)/((3*k + l)*log(a)) + 4*a^(k*x + 3*l*x)/((k + 3*l)*log(a))
 + 3*a^(2*k*x + 2*l*x)/((k + l)*log(a)) + 1/4*a^(4*k*x)/(k*log(a)) + 1/4*a^
(4*l*x)/(l*log(a))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 1359, normalized size of antiderivative = 13.87

$$\int \left(a^{kx}+a^{lx}\right)^4\,dx=\text{Too large to display}$$

```
[In] integrate((a^(k*x)+a^(l*x))^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*k*cos(-2*pi*k*x*sgn(a) + 2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2
+ (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-2*pi*k*x*sgn(a) + 2*p
i*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(4*k*x) + 1/2
*(2*l*cos(-2*pi*l*x*sgn(a) + 2*pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2 + (
pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-2*pi*l*x*sgn(a) + 2*pi*l
*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(4*l*x) - 1/2*I*
abs(a)^(4*k*x)*(-I*e^(2*I*pi*k*x*sgn(a) - 2*I*pi*k*x)/(2*I*pi*k*sgn(a) - 2*
I*pi*k + 4*k*log(abs(a))) + I*e^(-2*I*pi*k*x*sgn(a) + 2*I*pi*k*x)/(-2*I*pi*
k*sgn(a) + 2*I*pi*k + 4*k*log(abs(a)))) - 1/2*I*abs(a)^(4*l*x)*(-I*e^(2*I*p
i*l*x*sgn(a) - 2*I*pi*l*x)/(2*I*pi*l*sgn(a) - 2*I*pi*l + 4*l*log(abs(a))) +
 I*e^(-2*I*pi*l*x*sgn(a) + 2*I*pi*l*x)/(-2*I*pi*l*sgn(a) + 2*I*pi*l + 4*l*l
og(abs(a)))) + 8*(2*(3*k*log(abs(a)) + l*log(abs(a)))*cos(-3/2*pi*k*x*sgn(a
) - 1/2*pi*l*x*sgn(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(a) + pi*l*sgn
(a) - 3*pi*k - pi*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^2) - (3*pi*k*s
gn(a) + pi*l*sgn(a) - 3*pi*k - pi*l)*sin(-3/2*pi*k*x*sgn(a) - 1/2*pi*l*x*sg
n(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(a) + pi*l*sgn(a) - 3*pi*k - pi
*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^2))*e^((3*k*log(abs(a)) + l*log
(abs(a)))*x) + 4*I*(I*e^(3/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l*x*sgn(a) - 3/2*I*
pi*k*x - 1/2*I*pi*l*x)/(3*I*pi*k*sgn(a) + I*pi*l*sgn(a) - 3*I*pi*k - I*pi*l
 + 6*k*log(abs(a)) + 2*l*log(abs(a))) - I*e^(-3/2*I*pi*k*x*sgn(a) - 1/2*I*p
i*l*x*sgn(a) + 3/2*I*pi*k*x + 1/2*I*pi*l*x)/(-3*I*pi*k*sgn(a) - I*pi*l*sgn(
a) + 3*I*pi*k + I*pi*l + 6*k*log(abs(a)) + 2*l*log(abs(a))))*e^((3*k*log(ab
s(a)) + l*log(abs(a)))*x) + 8*(2*(k*log(abs(a)) + 3*l*log(abs(a)))*cos(-1/2
*pi*k*x*sgn(a) - 3/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 3/2*pi*l*x)/((pi*k*sgn(a)
 + 3*pi*l*sgn(a) - pi*k - 3*pi*l)^2 + 4*(k*log(abs(a)) + 3*l*log(abs(a)))^2
) - (pi*k*sgn(a) + 3*pi*l*sgn(a) - pi*k - 3*pi*l)*sin(-1/2*pi*k*x*sgn(a) -
3/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 3/2*pi*l*x)/((pi*k*sgn(a) + 3*pi*l*sgn(a)
- pi*k - 3*pi*l)^2 + 4*(k*log(abs(a)) + 3*l*log(abs(a)))^2))*e^((k*log(abs(
a)) + 3*l*log(abs(a)))*x) + 4*I*(I*e^(1/2*I*pi*k*x*sgn(a) + 3/2*I*pi*l*x*sg
```

```
n(a) - 1/2*I*pi*k*x - 3/2*I*pi*l*x)/(I*pi*k*sgn(a) + 3*I*pi*l*sgn(a) - I*pi
*k - 3*I*pi*l + 2*k*log(abs(a)) + 6*l*log(abs(a))) - I*e^(-1/2*I*pi*k*x*sgn
(a) - 3/2*I*pi*l*x*sgn(a) + 1/2*I*pi*k*x + 3/2*I*pi*l*x)/(-I*pi*k*sgn(a) -
3*I*pi*l*sgn(a) + I*pi*k + 3*I*pi*l + 2*k*log(abs(a)) + 6*l*log(abs(a))))*e
^((k*log(abs(a)) + 3*l*log(abs(a)))*x) + 6*(2*(k*log(abs(a)) + l*log(abs(a)
))*cos(-pi*k*x*sgn(a) - pi*l*x*sgn(a) + pi*k*x + pi*l*x)/((pi*k*sgn(a) + pi
*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2) - (pi*k*s
gn(a) + pi*l*sgn(a) - pi*k - pi*l)*sin(-pi*k*x*sgn(a) - pi*l*x*sgn(a) + pi*
k*x + pi*l*x)/((pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a
)) + l*log(abs(a)))^2))*e^(2*(k*log(abs(a)) + l*log(abs(a)))*x) + 3*I*(I*e^
(I*pi*k*x*sgn(a) + I*pi*l*x*sgn(a) - I*pi*k*x - I*pi*l*x)/(I*pi*k*sgn(a) +
I*pi*l*sgn(a) - I*pi*k - I*pi*l + 2*k*log(abs(a)) + 2*l*log(abs(a))) - I*e^
(-I*pi*k*x*sgn(a) - I*pi*l*x*sgn(a) + I*pi*k*x + I*pi*l*x)/(-I*pi*k*sgn(a)
- I*pi*l*sgn(a) + I*pi*k + I*pi*l + 2*k*log(abs(a)) + 2*l*log(abs(a))))*e^(
2*(k*log(abs(a)) + l*log(abs(a)))*x)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \left(a^{kx}+a^{lx}\right)^4\,dx=\frac{3\,a^{2\,k\,x}\,a^{2\,l\,x}}{k\,\ln\left(a\right)+l\,\ln\left(a\right)}+\frac{4\,a^{k\,x}\,a^{3\,l\,x}}{k\,\ln\left(a\right)+3\,l\,\ln\left(a\right)}+\frac{4\,a^{3\,k\,x}\,a^{l\,x}}{3\,k\,\ln\left(a\right)+l\,\ln\left(a\right)}+\frac{a^{4\,k\,x}}{4\,k\,\ln\left(a\right)}+\frac{a^{4\,l\,x}}{4\,l\,\ln\left(a\right)}$$

```
[In] int((a^(k*x) + a^(l*x))^4,x)
```

```
[Out] (3*a^(2*k*x)*a^(2*l*x))/(k*log(a) + l*log(a)) + (4*a^(k*x)*a^(3*l*x))/(k*lo
g(a) + 3*l*log(a)) + (4*a^(3*k*x)*a^(l*x))/(3*k*log(a) + l*log(a)) + a^(4*k
*x)/(4*k*log(a)) + a^(4*l*x)/(4*l*log(a))
```

3.506 $\int (a^{kx} + a^{lx})^n \, dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int (a^{kx} + a^{lx})^n \, dx = \frac{(1 + a^{(k-l)x}) (a^{kx} + a^{lx})^n \text{Hypergeometric2F1}\left(1, 1 + \frac{kn}{k-l}, 1 + \frac{ln}{k-l}, -a^{(k-l)x}\right)}{ln \log(a)}$$

```
[Out] (1+a^((k-l)*x))*(a^(k*x)+a^(l*x))^n*hypergeom([1, 1+k*n/(k-l)],[1+l*n/(k-l)
],-a^((k-l)*x))/l/n/ln(a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2323, 2283}

$$\int (a^{kx} + a^{lx})^n \, dx = \frac{(a^{-(x(k-l))} + 1)^{-n} (a^{kx} + a^{lx})^n \text{Hypergeometric2F1}\left(-n, -\frac{kn}{k-l}, 1 - \frac{kn}{k-l}, -a^{-((k-l)x)}\right)}{kn \log(a)}$$

```
[In] Int[(a^(k*x) + a^(l*x))^n,x]
```

```
[Out] ((a^(k*x) + a^(l*x))^n*Hypergeometric2F1[-n, -((k*n)/(k - l)), 1 - (k*n)/(k
 - l), -a^(-((k - l)*x))])/((1 + a^(-((k - l)*x)))^n*k*n*Log[a])
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
```

```
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2323

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] && !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rubi steps

$$\text{integral} = \left(a^{-knx}\left(1+a^{-((k-l)x)}\right)^{-n}\left(a^{kx}+a^{lx}\right)^n\right)\int a^{knx}\left(1+a^{-((k-l)x)}\right)^n\,dx$$
$$=\frac{\left(1+a^{-((k-l)x)}\right)^{-n}\left(a^{kx}+a^{lx}\right)^n \text{Hypergeometric2F1}\left(-n,-\frac{kn}{k-l},1-\frac{kn}{k-l},-a^{-((k-l)x)}\right)}{kn\log(a)}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int\left(a^{kx}+a^{lx}\right)^n\,dx=\frac{\left(a^{kx}+a^{lx}\right)^n\left(1+a^{(-k+l)x}\right)\text{Hypergeometric2F1}\left(1,1+n+\frac{kn}{-k+l},1+\frac{kn}{-k+l},-a^{(-k+l)x}\right)}{kn\log(a)}$$

```
[In] Integrate[(a^(k*x) + a^(l*x))^n,x]
[Out] ((a^(k*x) + a^(l*x))^n*(1 + a^((-k + l)*x))*Hypergeometric2F1[1, 1 + n + (k
*n)/(-k + l), 1 + (k*n)/(-k + l), -a^((-k + l)*x)])/(k*n*Log[a])
```

Maple [F]

$$\int\left(a^{kx}+a^{lx}\right)^n\,dx$$

```
[In] int((a^(k*x)+a^(l*x))^n,x)
[Out] int((a^(k*x)+a^(l*x))^n,x)
```

Fricas [F]

$$\int \left(a^{kx} + a^{lx}\right)^n \, dx = \int \left(a^{kx} + a^{lx}\right)^n \, dx$$

```
[In] integrate((a^(k*x)+a^(l*x))^n,x, algorithm="fricas")
[Out] integral((a^(k*x) + a^(l*x))^n, x)
```

Sympy [F]

$$\int \left(a^{kx} + a^{lx}\right)^n \, dx = \int \left(a^{kx} + a^{lx}\right)^n \, dx$$

```
[In] integrate((a**(k*x)+a**(l*x))**n,x)
[Out] Integral((a**(k*x) + a**(l*x))**n, x)
```

Maxima [F]

$$\int \left(a^{kx} + a^{lx}\right)^n \, dx = \int \left(a^{kx} + a^{lx}\right)^n \, dx$$

```
[In] integrate((a^(k*x)+a^(l*x))^n,x, algorithm="maxima")
[Out] integrate((a^(k*x) + a^(l*x))^n, x)
```

Giac [F]

$$\int \left(a^{kx} + a^{lx}\right)^n \, dx = \int \left(a^{kx} + a^{lx}\right)^n \, dx$$

```
[In] integrate((a^(k*x)+a^(l*x))^n,x, algorithm="giac")
[Out] integrate((a^(k*x) + a^(l*x))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^{kx}+a^{lx}\right)^n\,dx=\int \left(a^{k\,x}+a^{l\,x}\right)^n\,dx$$

```
[In] int((a^(k*x) + a^(l*x))^n,x)
[Out] int((a^(k*x) + a^(l*x))^n, x)
```

3.507 $\int \left(a^{kx}-a^{lx}\right) dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \left(a^{kx}-a^{lx}\right) dx=\frac{a^{kx}}{k\log(a)}-\frac{a^{lx}}{l\log(a)}$$

```
[Out] a^(k*x)/k/ln(a)-a^(l*x)/l/ln(a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2225}

$$\int \left(a^{kx}-a^{lx}\right) dx=\frac{a^{kx}}{k\log(a)}-\frac{a^{lx}}{l\log(a)}$$

```
[In] Int[a^(k*x) - a^(l*x),x]
[Out] a^(k*x)/(k*Log[a]) - a^(l*x)/(l*Log[a])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int a^{kx}\,dx-\int a^{lx}\,dx\\&=\frac{a^{kx}}{k\log(a)}-\frac{a^{lx}}{l\log(a)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}-a^{lx}\right)\,dx=\frac{a^{kx}}{k\log(a)}-\frac{a^{lx}}{l\log(a)}$$

```
[In] Integrate[a^(k*x) - a^(l*x),x]
[Out] a^(k*x)/(k*Log[a]) - a^(l*x)/(l*Log[a])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{a^{kx}l-a^{lx}k}{\ln(a)kl}$	28
default	$\frac{a^{kx}}{k\ln(a)}-\frac{a^{lx}}{l\ln(a)}$	29
risch	$\frac{a^{kx}}{k\ln(a)}-\frac{a^{lx}}{l\ln(a)}$	29
parts	$\frac{a^{kx}}{k\ln(a)}-\frac{a^{lx}}{l\ln(a)}$	29
norman	$\frac{e^{kx\ln(a)}}{k\ln(a)}-\frac{e^{lx\ln(a)}}{l\ln(a)}$	31
meijerg	$-\frac{1-e^{kx\ln(a)}}{k\ln(a)}+\frac{1-e^{lx\ln(a)}}{l\ln(a)}$	39

```
[In] int(a^(k*x)-a^(l*x),x,method=_RETURNVERBOSE)
[Out] (a^(k*x)*l-a^(l*x)*k)/ln(a)/k/l
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}-a^{lx}\right)\,dx=-\frac{a^{lx}k-a^{kx}l}{kl\log{(a)}}$$

```
[In] integrate(a^(k*x)-a^(l*x),x, algorithm="fricas")
[Out] -(a^(l*x)*k - a^(k*x)*l)/(k*l*log(a))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \left(a^{kx}-a^{lx}\right)\,dx=\begin{cases}\frac{a^{kx}}{k\log\left(a\right)} & \text{for } k\log\left(a\right)\neq 0\\ x & \text{otherwise}\end{cases}-\begin{cases}\frac{a^{lx}}{l\log\left(a\right)} & \text{for } l\log\left(a\right)\neq 0\\ x & \text{otherwise}\end{cases}$$

```
[In] integrate(a**(k*x)-a**(l*x),x)
[Out] Piecewise((a**(k*x)/(k*log(a)), Ne(k*log(a), 0)), (x, True)) - Piecewise((a
**(l*x)/(l*log(a)), Ne(l*log(a), 0)), (x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}-a^{lx}\right)\,dx=\frac{a^{kx}}{k\log\left(a\right)}-\frac{a^{lx}}{l\log\left(a\right)}$$

```
[In] integrate(a^(k*x)-a^(l*x),x, algorithm="maxima")
[Out] a^(k*x)/(k*log(a)) - a^(l*x)/(l*log(a))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}-a^{lx}\right)\,dx=\frac{a^{kx}}{k\log\left(a\right)}-\frac{a^{lx}}{l\log\left(a\right)}$$

```
[In] integrate(a^(k*x)-a^(l*x),x, algorithm="giac")
[Out] a^(k*x)/(k*log(a)) - a^(l*x)/(l*log(a))
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \left(a^{kx}-a^{lx}\right)\,dx=\frac{a^{k\,x}\,l-a^{l\,x}\,k}{k\,l\,\ln \left(a\right)}$$

```
[In] int(a^(k*x) - a^(l*x),x)
[Out] (a^(k*x)*l - a^(l*x)*k)/(k*l*log(a))
```

3.508 $\int (a^{kx} - a^{lx})^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int (a^{kx} - a^{lx})^2 \, dx = \frac{a^{2kx}}{2k \log(a)} + \frac{a^{2lx}}{2l \log(a)} - \frac{2a^{(k+l)x}}{(k+l) \log(a)}$$

```
[Out] 1/2*a^(2*k*x)/k/ln(a)+1/2*a^(2*l*x)/l/ln(a)-2*a^((k+l)*x)/(k+l)/ln(a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6874, 2225}

$$\int (a^{kx} - a^{lx})^2 \, dx = -\frac{2a^{x(k+l)}}{\log(a)(k+l)} + \frac{a^{2kx}}{2k \log(a)} + \frac{a^{2lx}}{2l \log(a)}$$

```
[In] Int[(a^(k*x) - a^(l*x))^2,x]
```

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) - (2*a^((k + l)*x))/((k + l
)*Log[a])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \left(e^{kx}-e^{lx}\right)^2 \, dx, x, x\log(a)\right)}{\log(a)}\\ &= \frac{\text{Subst}\left(\int \left(e^{2kx}+e^{2lx}-2e^{(k+l)x}\right) \, dx, x, x\log(a)\right)}{\log(a)}\\ &= \frac{\text{Subst}\left(\int e^{2kx} \, dx, x, x\log(a)\right)}{\log(a)}+\frac{\text{Subst}\left(\int e^{2lx} \, dx, x, x\log(a)\right)}{\log(a)}\\ &\quad -\frac{2\text{Subst}\left(\int e^{(k+l)x} \, dx, x, x\log(a)\right)}{\log(a)}\\ &= \frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}-\frac{2a^{(k+l)x}}{(k+l)\log(a)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}-a^{lx}\right)^2 \, dx = \frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}-\frac{2a^{(k+l)x}}{(k+l)\log(a)}$$

```
[In] Integrate[(a^(k*x) - a^(l*x))^2,x]
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) - (2*a^((k + l)*x))/((k + l
)*Log[a])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

method	result	size
risch	$\frac{a^{2kx}}{2k\ln(a)}+\frac{a^{2lx}}{2l\ln(a)}-\frac{2a^{kx}a^{lx}}{\ln(a)(k+l)}$	55
norman	$\frac{e^{2kx\ln(a)}}{2k\ln(a)}+\frac{e^{2lx\ln(a)}}{2l\ln(a)}-\frac{2\,e^{kx\ln(a)}e^{lx\ln(a)}}{\ln(a)(k+l)}$	59
parallelrisch	$\frac{a^{2kx}lk+a^{2kx}l^2-4a^{kx}a^{lx}kl+a^{2lx}k^2+a^{2lx}kl}{2\ln(a)kl(k+l)}$	75
meijerg	$-\frac{1-e^{2kx\ln(a)}}{2k\ln(a)}+\frac{2-2\,e^{xl\ln(a)\left(1+\frac{k}{l}\right)}}{l\ln(a)\left(1+\frac{k}{l}\right)}-\frac{1-e^{2lx\ln(a)}}{2l\ln(a)}$	77

```
[In] int((a^(k*x)-a^(l*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/k/ln(a)*(a^(k*x))^2+1/2/l/ln(a)*(a^(l*x))^2-2/ln(a)/(k+l)*a^(k*x)*a^(l*
x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \left(a^{kx}-a^{lx}\right)^2\,dx=-\frac{4\,a^{kx}a^{lx}kl-(kl+l^2)a^{2\,kx}-(k^2+kl)a^{2\,lx}}{2\,(k^2l+kl^2)\log{(a)}}$$

```
[In] integrate((a^(k*x)-a^(l*x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^(k*x)*a^(l*x)*k*l - (k*l + l^2)*a^(2*k*x) - (k^2 + k*l)*a^(2*l*x)
)/((k^2*l + k*l^2)*log(a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(41) = 82.

Time = 0.44 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.68

$$\int \left(a^{kx}-a^{lx}\right)^2\,dx=\begin{cases}0\\ \frac{a^{2lx}}{2l\log{(a)}}-\frac{2a^{lx}}{l\log{(a)}}+x\\ \frac{a^{2lx}}{2l\log{(a)}}-2x-\frac{a^{-2lx}}{2l\log{(a)}}\\ \frac{a^{2kx}}{2k\log{(a)}}-\frac{2a^{kx}}{k\log{(a)}}+x\\ \frac{a^{2kx}kl}{2k^2l\log{(a)}+2kl^2\log{(a)}}+\frac{a^{2kx}l^2}{2k^2l\log{(a)}+2kl^2\log{(a)}}-\frac{4a^{kx}a^{lx}kl}{2k^2l\log{(a)}+2kl^2\log{(a)}}+\frac{a^{2lx}k^2}{2k^2l\log{(a)}+2kl^2\log{(a)}}+\frac{a^{2lx}kl}{2k^2l\log{(a)}+2kl^2\log{(a)}}\end{cases}$$

```
[In] integrate((a**(k*x)-a**(l*x))**2,x)
```

```
[Out] Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (a
**(2*l*x)/(2*l*log(a)) - 2*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(2*l*x)/
(2*l*log(a)) - 2*x - 1/(2*a**(2*l*x)*l*log(a)), Eq(k, -1)), (a**(2*k*x)/(2*
k*log(a)) - 2*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (a**(2*k*x)*k*l/(2*k**2*l
*log(a) + 2*k*l**2*log(a)) + a**(2*k*x)*l**2/(2*k**2*l*log(a) + 2*k*l**2*lo
g(a)) - 4*a**(k*x)*a**(l*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2
*l*x)*k**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*k*l/(2*k**2*l*l
og(a) + 2*k*l**2*log(a)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \left(a^{kx}-a^{lx}\right)^2\,dx=-\frac{2\,a^{kx+lx}}{(k+l)\log{(a)}}+\frac{a^{2\,kx}}{2\,k\log{(a)}}+\frac{a^{2\,lx}}{2\,l\log{(a)}}$$

```
[In] integrate((a^(k*x)-a^(l*x))^2,x, algorithm="maxima")
[Out] -2*a^(k*x + l*x)/((k + l)*log(a)) + 1/2*a^(2*k*x)/(k*log(a)) + 1/2*a^(2*l*x
)/(l*log(a))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 691, normalized size of antiderivative = 13.04

$$\int \left(a^{kx}-a^{lx}\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((a^(k*x)-a^(l*x))^2,x, algorithm="giac")
[Out] (2*k*cos(-pi*k*x*sgn(a) + pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2 + (pi*k*
sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-pi*k*x*sgn(a) + pi*k*x)/(4*k^
2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(2*k*x) + (2*l*cos(-pi*l*
x*sgn(a) + pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^
2) - (pi*l*sgn(a) - pi*l)*sin(-pi*l*x*sgn(a) + pi*l*x)/(4*l^2*log(abs(a))^2
 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(2*l*x) - 1/2*I*abs(a)^(2*k*x)*(-I*e^(I*
pi*k*x*sgn(a) - I*pi*k*x)/(I*pi*k*sgn(a) - I*pi*k + 2*k*log(abs(a))) + I*e^
(-I*pi*k*x*sgn(a) + I*pi*k*x)/(-I*pi*k*sgn(a) + I*pi*k + 2*k*log(abs(a))))
- 1/2*I*abs(a)^(2*l*x)*(-I*e^(I*pi*l*x*sgn(a) - I*pi*l*x)/(I*pi*l*sgn(a) -
I*pi*l + 2*l*log(abs(a))) + I*e^(-I*pi*l*x*sgn(a) + I*pi*l*x)/(-I*pi*l*sgn(
a) + I*pi*l + 2*l*log(abs(a)))) - 4*(2*(k*log(abs(a)) + l*log(abs(a)))*cos(
-1/2*pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*x)/((pi*k*sg
n(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2)
- (pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)*sin(-1/2*pi*k*x*sgn(a) - 1/2*pi
*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*x)/((pi*k*sgn(a) + pi*l*sgn(a) - pi*k -
 pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2))*e^((k*log(abs(a)) + l*log(
abs(a)))*x) + 2*I*(-I*e^(1/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l*x*sgn(a) - 1/2*I*
pi*k*x - 1/2*I*pi*l*x)/(I*pi*k*sgn(a) + I*pi*l*sgn(a) - I*pi*k - I*pi*l + 2
*k*log(abs(a)) + 2*l*log(abs(a))) + I*e^(-1/2*I*pi*k*x*sgn(a) - 1/2*I*pi*l*
x*sgn(a) + 1/2*I*pi*k*x + 1/2*I*pi*l*x)/(-I*pi*k*sgn(a) - I*pi*l*sgn(a) + I
*pi*k + I*pi*l + 2*k*log(abs(a)) + 2*l*log(abs(a))))*e^((k*log(abs(a)) + l*
log(abs(a)))*x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int \left(a^{kx} - a^{lx}\right)^2 \, dx = \frac{a^{2\,k\,x}}{2\,k\,\ln(a)} + \frac{\frac{a^{2\,l\,x}\,k^2}{2} - l\left(2\,a^{k\,x+l\,x}\,k - \frac{a^{2\,l\,x}\,k}{2}\right)}{k\,l\,\ln(a)\,(k+l)}$$

```
[In] int((a^(k*x) - a^(l*x))^2,x)
```

```
[Out] a^(2*k*x)/(2*k*log(a)) + ((a^(2*l*x)*k^2)/2 - l*(2*a^(k*x + l*x)*k - (a^(2*
l*x)*k)/2))/(k*l*log(a)*(k + l))
```

3.509 $\int \left(a^{kx}-a^{lx}\right)^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \left(a^{kx}-a^{lx}\right)^3 \, dx=\frac{a^{3kx}}{3k\log(a)}-\frac{a^{3lx}}{3l\log(a)}-\frac{3a^{(2k+l)x}}{(2k+l)\log(a)}+\frac{3a^{(k+2l)x}}{(k+2l)\log(a)}$$

```
[Out] 1/3*a^(3*k*x)/k/ln(a)-1/3*a^(3*l*x)/l/ln(a)-3*a^((2*k+l)*x)/(2*k+l)/ln(a)+3
*a^((k+2*l)*x)/(k+2*l)/ln(a)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6874, 2225}

$$\int \left(a^{kx}-a^{lx}\right)^3 \, dx=-\frac{3a^{x(2k+l)}}{\log(a)(2k+l)}+\frac{3a^{x(k+2l)}}{\log(a)(k+2l)}+\frac{a^{3kx}}{3k\log(a)}-\frac{a^{3lx}}{3l\log(a)}$$

```
[In] Int[(a^(k*x) - a^(l*x))^3,x]
```

```
[Out] a^(3*k*x)/(3*k*Log[a]) - a^(3*l*x)/(3*l*Log[a]) - (3*a^((2*k + l)*x))/((2*k
 + l)*Log[a]) + (3*a^((k + 2*l)*x))/((k + 2*l)*Log[a])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(e^{kx}-e^{lx}\right)^3 \, dx, x, x\log(a)\right)}{\log(a)} \\
&= \frac{\text{Subst}\left(\int \left(e^{3kx}-e^{3lx}-3e^{(2k+l)x}+3e^{(k+2l)x}\right) \, dx, x, x\log(a)\right)}{\log(a)} \\
&= \frac{\text{Subst}\left(\int e^{3kx} \, dx, x, x\log(a)\right)}{\log(a)} - \frac{\text{Subst}\left(\int e^{3lx} \, dx, x, x\log(a)\right)}{\log(a)} \\
&\quad - \frac{3\text{Subst}\left(\int e^{(2k+l)x} \, dx, x, x\log(a)\right)}{\log(a)} + \frac{3\text{Subst}\left(\int e^{(k+2l)x} \, dx, x, x\log(a)\right)}{\log(a)} \\
&= \frac{a^{3kx}}{3k\log(a)} - \frac{a^{3lx}}{3l\log(a)} - \frac{3a^{(2k+l)x}}{(2k+l)\log(a)} + \frac{3a^{(k+2l)x}}{(k+2l)\log(a)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$
\int \left(a^{kx}-a^{lx}\right)^3 \, dx = \frac{\frac{a^{3kx}}{k} - \frac{a^{3lx}}{l} - \frac{9a^{(2k+l)x}}{2k+l} + \frac{9a^{(k+2l)x}}{k+2l}}{3\log(a)}
$$

```
[In] Integrate[(a^(k*x) - a^(l*x))^3,x]
[Out] (a^(3*k*x)/k - a^(3*l*x)/l - (9*a^((2*k + l)*x))/(2*k + l) + (9*a^((k + 2*l
)*x))/(k + 2*l))/(3*Log[a])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06

method	result	s
risch	$\frac{a^{3kx}}{3k\ln(a)} - \frac{a^{3lx}}{3l\ln(a)} + \frac{3a^{kx}a^{2lx}}{\ln(a)(k+2l)} - \frac{3a^{2kx}a^{lx}}{\ln(a)(2k+l)}$	8
norman	$\frac{e^{3kx\ln(a)}}{3k\ln(a)} - \frac{e^{3lx\ln(a)}}{3l\ln(a)} + \frac{3e^{kx\ln(a)}e^{2lx\ln(a)}}{\ln(a)(k+2l)} - \frac{3e^{2kx\ln(a)}e^{lx\ln(a)}}{\ln(a)(2k+l)}$	9
meijerg	$-\frac{1-e^{3kx\ln(a)}}{3k\ln(a)} + \frac{3-3e^{xk\ln(a)\left(2+\frac{l}{k}\right)}}{k\ln(a)\left(2+\frac{l}{k}\right)} - \frac{3\left(1-e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{1}{1+\frac{k}{l}}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{1}{1+\frac{k}{l}}\right)} + \frac{1-e^{3lx\ln(a)}}{3l\ln(a)}$	1
parallelrisch	$\frac{2a^{3kx}k^2l+5a^{3kx}kl^2+2a^{3kx}l^3-9a^{2kx}a^{lx}k^2l-18a^{2kx}a^{lx}kl^2+18a^{kx}a^{2lx}k^2l+9a^{kx}a^{2lx}kl^2-2a^{3lx}k^3-5a^{3lx}k^2l-2a^{3lx}kl^2}{3\ln(a)kl(k+2l)(2k+l)}$	1

```
[In] int((a^(k*x)-a^(l*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/k/ln(a)*(a^(k*x))^3-1/3/l/ln(a)*(a^(l*x))^3+3/ln(a)/(k+2*l)*a^(k*x)*(a^
(l*x))^2-3/ln(a)/(2*k+l)*(a^(k*x))^2*a^(l*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.66

$$\int \left(a^{kx}-a^{lx}\right)^3\,dx = \frac{9\,(2\,k^2l+kl^2)a^{kx}a^{2\,lx}-9\,(k^2l+2\,kl^2)a^{2\,kx}a^{lx}+(2\,k^2l+5\,kl^2+2\,l^3)a^{3\,kx}-(2\,k^3+5\,k^2l+2\,kl^2)a^{3\,lx}}{3\,(2\,k^3l+5\,k^2l^2+2\,kl^3)\log(a)}$$

```
[In] integrate((a^(k*x)-a^(l*x))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(9*(2*k^2*l + k*l^2)*a^(k*x)*a^(2*l*x) - 9*(k^2*l + 2*k*l^2)*a^(2*k*x)*
a^(l*x) + (2*k^2*l + 5*k*l^2 + 2*l^3)*a^(3*k*x) - (2*k^3 + 5*k^2*l + 2*k*l^
2)*a^(3*l*x))/((2*k^3*l + 5*k^2*l^2 + 2*k*l^3)*log(a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. 2(63) = 126.

Time = 2.05 (sec) , antiderivative size = 663, normalized size of antiderivative = 8.39

$$\int \left(a^{kx}-a^{lx}\right)^3\,dx = \begin{cases} 0 \\ -\frac{a^{3lx}}{3l\log(a)}+\frac{3a^{2lx}}{2l\log(a)}-\frac{3a^{lx}}{l\log(a)}+x \\ -\frac{a^{3lx}}{3l\log(a)}+3x+\frac{a^{-3lx}}{l\log(a)}-\frac{a^{-6lx}}{6l\log(a)} \\ \frac{2a^{\frac{3lx}{2}}}{l\log(a)}-\frac{a^{3lx}}{3l\log(a)}-3x-\frac{2a^{-\frac{3lx}{2}}}{3l\log(a)} \\ \frac{a^{3kx}}{3k\log(a)}-\frac{3a^{2kx}}{2k\log(a)}+\frac{3a^{kx}}{k\log(a)}-x \\ \frac{2a^{3kx}k^2l}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)}+\frac{5a^{3kx}kl^2}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)}+\frac{2a^{3kx}l^3}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)}-\frac{}{6k^3l\log(a}\end{cases}$$

```
[In] integrate((a**(k*x)-a**(l*x))**3,x)
```

```
[Out] Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (-
a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(2*l*log(a)) - 3*a**(l*x)/(l*log(a))
 + x, Eq(k, 0)), (-a**(3*l*x)/(3*l*log(a)) + 3*x + 1/(a**(3*l*x)*l*log(a))
- 1/(6*a**(6*l*x)*l*log(a)), Eq(k, -2*l)), (2*a**(3*l*x/2)/(l*log(a)) - a**
```

```
(3*l*x)/(3*l*log(a)) - 3*x - 2/(3*a**(3*l*x/2)*l*log(a)), Eq(k, -l/2)), (a*
*(3*k*x)/(3*k*log(a)) - 3*a**(2*k*x)/(2*k*log(a)) + 3*a**(k*x)/(k*log(a)) -
 x, Eq(l, 0)), (2*a**(3*k*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a)
+ 6*k*l**3*log(a)) + 5*a**(3*k*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*lo
g(a) + 6*k*l**3*log(a)) + 2*a**(3*k*x)*l**3/(6*k**3*l*log(a) + 15*k**2*l**2
*log(a) + 6*k*l**3*log(a)) - 9*a**(2*k*x)*a**(l*x)*k**2*l/(6*k**3*l*log(a)
+ 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) - 18*a**(2*k*x)*a**(l*x)*k*l**2/(6
*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a**(k*x)*a**(2
*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 9*
a**(k*x)*a**(2*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**
3*log(a)) - 2*a**(3*l*x)*k**3/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*
l**3*log(a)) - 5*a**(3*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) +
 6*k*l**3*log(a)) - 2*a**(3*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log
(a) + 6*k*l**3*log(a)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int \left(a^{kx}-a^{lx}\right)^3\,dx=-\frac{3\,a^{2\,kx+lx}}{(2\,k+l)\log{(a)}}+\frac{3\,a^{kx+2\,lx}}{(k+2\,l)\log{(a)}}+\frac{a^{3\,kx}}{3\,k\log{(a)}}-\frac{a^{3\,lx}}{3\,l\log{(a)}}$$

```
[In] integrate((a^(k*x)-a^(l*x))^3,x, algorithm="maxima")
```

```
[Out] -3*a^(2*k*x + l*x)/((2*k + l)*log(a)) + 3*a^(k*x + 2*l*x)/((k + 2*l)*log(a)
) + 1/3*a^(3*k*x)/(k*log(a)) - 1/3*a^(3*l*x)/(l*log(a))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 1033, normalized size of antiderivative = 13.08

$$\int \left(a^{kx}-a^{lx}\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate((a^(k*x)-a^(l*x))^3,x, algorithm="giac")
```

```
[Out] 2/3*(2*k*cos(-3/2*pi*k*x*sgn(a) + 3/2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a)
)^2 + (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-3/2*pi*k*x*sgn(a)
 + 3/2*pi*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(3*k*
x) - 2/3*(2*l*cos(-3/2*pi*l*x*sgn(a) + 3/2*pi*l*x)*log(abs(a))/(4*l^2*log(a
bs(a))^2 + (pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-3/2*pi*l*x*s
gn(a) + 3/2*pi*l*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^
```

```
(3*l*x) + I*abs(a)^(3*k*x)*(I*e^(3/2*I*pi*k*x*sgn(a) - 3/2*I*pi*k*x)/(3*I*p
i*k*sgn(a) - 3*I*pi*k + 6*k*log(abs(a))) - I*e^(-3/2*I*pi*k*x*sgn(a) + 3/2*
I*pi*k*x)/(-3*I*pi*k*sgn(a) + 3*I*pi*k + 6*k*log(abs(a)))) + I*abs(a)^(3*l*
x)*(-I*e^(3/2*I*pi*l*x*sgn(a) - 3/2*I*pi*l*x)/(3*I*pi*l*sgn(a) - 3*I*pi*l +
 6*l*log(abs(a))) + I*e^(-3/2*I*pi*l*x*sgn(a) + 3/2*I*pi*l*x)/(-3*I*pi*l*sg
n(a) + 3*I*pi*l + 6*l*log(abs(a)))) - 6*(2*(2*k*log(abs(a)) + l*log(abs(a))
)*cos(-pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*l*x)/((2*pi*k*sg
n(a) + pi*l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a)) + l*log(abs(a)))
^2) - (2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)*sin(-pi*k*x*sgn(a) - 1/
2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*l*x)/((2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi
*k - pi*l)^2 + 4*(2*k*log(abs(a)) + l*log(abs(a)))^2))*e^((2*k*log(abs(a))
+ l*log(abs(a)))*x) + 3*I*(-I*e^(I*pi*k*x*sgn(a) + 1/2*I*pi*l*x*sgn(a) - I*
pi*k*x - 1/2*I*pi*l*x)/(2*I*pi*k*sgn(a) + I*pi*l*sgn(a) - 2*I*pi*k - I*pi*l
 + 4*k*log(abs(a)) + 2*l*log(abs(a))) + I*e^(-I*pi*k*x*sgn(a) - 1/2*I*pi*l*
x*sgn(a) + I*pi*k*x + 1/2*I*pi*l*x)/(-2*I*pi*k*sgn(a) - I*pi*l*sgn(a) + 2*I
*pi*k + I*pi*l + 4*k*log(abs(a)) + 2*l*log(abs(a))))*e^((2*k*log(abs(a)) +
l*log(abs(a)))*x) + 6*(2*(k*log(abs(a)) + 2*l*log(abs(a)))*cos(-1/2*pi*k*x*
sgn(a) - pi*l*x*sgn(a) + 1/2*pi*k*x + pi*l*x)/((pi*k*sgn(a) + 2*pi*l*sgn(a)
 - pi*k - 2*pi*l)^2 + 4*(k*log(abs(a)) + 2*l*log(abs(a)))^2) - (pi*k*sgn(a)
 + 2*pi*l*sgn(a) - pi*k - 2*pi*l)*sin(-1/2*pi*k*x*sgn(a) - pi*l*x*sgn(a) +
1/2*pi*k*x + pi*l*x)/((pi*k*sgn(a) + 2*pi*l*sgn(a) - pi*k - 2*pi*l)^2 + 4*(
k*log(abs(a)) + 2*l*log(abs(a)))^2))*e^((k*log(abs(a)) + 2*l*log(abs(a)))*x
) + 3*I*(I*e^(1/2*I*pi*k*x*sgn(a) + I*pi*l*x*sgn(a) - 1/2*I*pi*k*x - I*pi*l
*x)/(I*pi*k*sgn(a) + 2*I*pi*l*sgn(a) - I*pi*k - 2*I*pi*l + 2*k*log(abs(a))
+ 4*l*log(abs(a))) - I*e^(-1/2*I*pi*k*x*sgn(a) - I*pi*l*x*sgn(a) + 1/2*I*pi
*k*x + I*pi*l*x)/(-I*pi*k*sgn(a) - 2*I*pi*l*sgn(a) + I*pi*k + 2*I*pi*l + 2*
k*log(abs(a)) + 4*l*log(abs(a))))*e^((k*log(abs(a)) + 2*l*log(abs(a)))*x)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \left(a^{kx}-a^{lx}\right)^3\,dx=\frac{3\,a^{k\,x}\,a^{2\,l\,x}}{k\,\ln\left(a\right)+2\,l\,\ln\left(a\right)}-\frac{3\,a^{2\,k\,x}\,a^{l\,x}}{2\,k\,\ln\left(a\right)+l\,\ln\left(a\right)}+\frac{a^{3\,k\,x}}{3\,k\,\ln\left(a\right)}-\frac{a^{3\,l\,x}}{3\,l\,\ln\left(a\right)}$$

```
[In] int((a^(k*x) - a^(l*x))^3,x)
```

```
[Out] (3*a^(k*x)*a^(2*l*x))/(k*log(a) + 2*l*log(a)) - (3*a^(2*k*x)*a^(l*x))/(2*k*
log(a) + l*log(a)) + a^(3*k*x)/(3*k*log(a)) - a^(3*l*x)/(3*l*log(a))
```

3.510 $\int \left(a^{kx}-a^{lx}\right)^4 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 98

$$\int \left(a^{kx}-a^{lx}\right)^4 \, dx=\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}+\frac{3a^{2(k+l)x}}{(k+l)\log(a)}-\frac{4a^{(3k+l)x}}{(3k+l)\log(a)}-\frac{4a^{(k+3l)x}}{(k+3l)\log(a)}$$

```
[Out] 1/4*a^(4*k*x)/k/ln(a)+1/4*a^(4*l*x)/l/ln(a)+3*a^(2*(k+l)*x)/(k+l)/ln(a)-4*a
^((3*k+l)*x)/(3*k+l)/ln(a)-4*a^((k+3*l)*x)/(k+3*l)/ln(a)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6874, 2225}

$$\int \left(a^{kx}-a^{lx}\right)^4 \, dx=\frac{3a^{2x(k+l)}}{\log(a)(k+l)}-\frac{4a^{x(3k+l)}}{\log(a)(3k+l)}-\frac{4a^{x(k+3l)}}{\log(a)(k+3l)}+\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}$$

```
[In] Int[(a^(k*x) - a^(l*x))^4,x]
```

```
[Out] a^(4*k*x)/(4*k*Log[a]) + a^(4*l*x)/(4*l*Log[a]) + (3*a^(2*(k + l)*x))/((k +
 l)*Log[a]) - (4*a^((3*k + l)*x))/((3*k + l)*Log[a]) - (4*a^((k + 3*l)*x))/
((k + 3*l)*Log[a])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(e^{kx}-e^{lx}\right)^4 \, dx, x, x\log(a)\right)}{\log(a)}\\
&= \frac{\text{Subst}\left(\int \left(e^{4kx}+e^{4lx}+6e^{2(k+l)x}-4e^{(3k+l)x}-4e^{(k+3l)x}\right) \, dx, x, x\log(a)\right)}{\log(a)}\\
&= \frac{\text{Subst}\left(\int e^{4kx} \, dx, x, x\log(a)\right)}{\log(a)}+\frac{\text{Subst}\left(\int e^{4lx} \, dx, x, x\log(a)\right)}{\log(a)}\\
&\quad -\frac{4\text{Subst}\left(\int e^{(3k+l)x} \, dx, x, x\log(a)\right)}{\log(a)}-\frac{4\text{Subst}\left(\int e^{(k+3l)x} \, dx, x, x\log(a)\right)}{\log(a)}\\
&\quad +\frac{6\text{Subst}\left(\int e^{2(k+l)x} \, dx, x, x\log(a)\right)}{\log(a)}\\
&= \frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}+\frac{3a^{2(k+l)x}}{(k+l)\log(a)}-\frac{4a^{(3k+l)x}}{(3k+l)\log(a)}-\frac{4a^{(k+3l)x}}{(k+3l)\log(a)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \left(a^{kx}-a^{lx}\right)^4 \, dx = \frac{\frac{a^{4kx}}{k}+\frac{a^{4lx}}{l}+\frac{12a^{2(k+l)x}}{k+l}-\frac{16a^{(3k+l)x}}{3k+l}-\frac{16a^{(k+3l)x}}{k+3l}}{4\log(a)}$$

```
[In] Integrate[(a^(k*x) - a^(l*x))^4,x]
[Out] (a^(4*k*x)/k + a^(4*l*x)/l + (12*a^(2*(k + l)*x))/(k + l) - (16*a^((3*k + l
)*x))/(3*k + l) - (16*a^((k + 3*l)*x))/(k + 3*l))/(4*Log[a])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

method	result
risch	$\frac{a^{4kx}}{4k\ln(a)} - \frac{4a^{3kx}a^{lx}}{\ln(a)(3k+l)} + \frac{3a^{2kx}a^{2lx}}{\ln(a)(k+l)} - \frac{4a^{kx}a^{3lx}}{\ln(a)(k+3l)} + \frac{a^{4lx}}{4l\ln(a)}$
meijerg	$-\frac{1-e^{4kx\ln(a)}}{4k\ln(a)} + \frac{4-4\,e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2k}{l\left(1+\frac{k}{l}\right)}\right)}}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2k}{l\left(1+\frac{k}{l}\right)}\right)} - \frac{3\left(1-e^{2xl\ln(a)\left(1+\frac{k}{l}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)} + \frac{4-4\,e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2}{1+\frac{k}{l}}\right)}}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2}{1+\frac{k}{l}}\right)} - \frac{1-e^{4l}}{4l\ln}$
parallelrisch	$\frac{3a^{4kx}k^3l+13a^{4kx}k^2l^2+13a^{4kx}k\,l^3+3a^{4kx}l^4-16a^{3kx}a^{lx}k^3l-64a^{3kx}a^{lx}k^2l^2-48a^{3kx}a^{lx}k\,l^3+36a^{2kx}a^{2lx}k^3l+120a^{2kx}a^{2lx}k^2l}{4\ln(a)k(3k+l)(k+l)(}$

```
[In] int((a^(k*x)-a^(l*x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/ln(a)/k*(a^(k*x))^4-4*(a^(k*x))^3/ln(a)/(3*k+l)*a^(l*x)+3*(a^(k*x))^2/l
n(a)/(k+l)*(a^(l*x))^2-4*a^(k*x)/ln(a)/(k+3*l)*(a^(l*x))^3+1/4/ln(a)/l*(a^(
l*x))^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(94) = 188$.

Time = 0.25 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.11

$$\int \left(a^{kx} - a^{lx}\right)^4 \, dx = -\frac{16\,(3\,k^3l + 4\,k^2l^2 + kl^3)a^{kx}a^{3\,lx} - 12\,(3\,k^3l + 10\,k^2l^2 + 3\,kl^3)a^{2\,kx}a^{2\,lx} + 16\,(k^3l + 4\,k^2l^2 + 3\,kl^3)a^{3\,kx}a^{lx}}{4\,(3\,k^4l + 13\,k^3l^2 + 13\,k^2l^3 + 3\,kl^4}$$

```
[In] integrate((a^(k*x)-a^(l*x))^4,x, algorithm="fricas")
```

```
[Out] -1/4*(16*(3*k^3*l + 4*k^2*l^2 + k*l^3)*a^(k*x)*a^(3*l*x) - 12*(3*k^3*l + 10
*k^2*l^2 + 3*k*l^3)*a^(2*k*x)*a^(2*l*x) + 16*(k^3*l + 4*k^2*l^2 + 3*k*l^3)*
a^(3*k*x)*a^(l*x) - (3*k^3*l + 13*k^2*l^2 + 13*k*l^3 + 3*l^4)*a^(4*k*x) - (
3*k^4 + 13*k^3*l + 13*k^2*l^2 + 3*k*l^3)*a^(4*l*x))/((3*k^4*l + 13*k^3*l^2
+ 13*k^2*l^3 + 3*k*l^4)*log(a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1348 vs. $2(82) = 164$.

Time = 16.58 (sec) , antiderivative size = 1348, normalized size of antiderivative = 13.76

$$\int \left(a^{kx} - a^{lx}\right)^4 \, dx = \text{Too large to display}$$

```
[In] integrate((a**(k*x)-a**(l*x))**4,x)
```

```
[Out] Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (a
**(4*l*x)/(4*l*log(a)) - 4*a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(l*log(a)
```

```
) - 4*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(4*l*x)/(4*l*log(a)) - 4*x -
3/(2*a**(4*l*x)*l*log(a)) + 1/(2*a**(8*l*x)*l*log(a)) - 1/(12*a**(12*l*x)*l
*log(a)), Eq(k, -3*l)), (a**(4*l*x)/(4*l*log(a)) - 2*a**(2*l*x)/(l*log(a))
+ 6*x + 2/(a**(2*l*x)*l*log(a)) - 1/(4*a**(4*l*x)*l*log(a)), Eq(k, -l)), (-
3*a**(8*l*x/3)/(2*l*log(a)) + 9*a**(4*l*x/3)/(2*l*log(a)) + a**(4*l*x)/(4*l
*log(a)) - 4*x - 3/(4*a**(4*l*x/3)*l*log(a)), Eq(k, -l/3)), (a**(4*k*x)/(4*
k*log(a)) - 4*a**(3*k*x)/(3*k*log(a)) + 3*a**(2*k*x)/(k*log(a)) - 4*a**(k*x
)/(k*log(a)) + x, Eq(l, 0)), (3*a**(4*k*x)*k**3*l/(12*k**4*l*log(a) + 52*k*
*3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 13*a**(4*k*x)*k*
*2*l**2/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*
k*l**4*log(a)) + 13*a**(4*k*x)*k*l**3/(12*k**4*l*log(a) + 52*k**3*l**2*log(
a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 3*a**(4*k*x)*l**4/(12*k**4*l
*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) - 1
6*a**(3*k*x)*a**(l*x)*k**3*l/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k
**2*l**3*log(a) + 12*k*l**4*log(a)) - 64*a**(3*k*x)*a**(l*x)*k**2*l**2/(12*
k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a
)) - 48*a**(3*k*x)*a**(l*x)*k*l**3/(12*k**4*l*log(a) + 52*k**3*l**2*log(a)
+ 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 36*a**(2*k*x)*a**(2*l*x)*k**3*l
/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*
log(a)) + 120*a**(2*k*x)*a**(2*l*x)*k**2*l**2/(12*k**4*l*log(a) + 52*k**3*l
**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 36*a**(2*k*x)*a**(2*
l*x)*k*l**3/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) +
 12*k*l**4*log(a)) - 48*a**(k*x)*a**(3*l*x)*k**3*l/(12*k**4*l*log(a) + 52*k
**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) - 64*a**(k*x)*a**
(3*l*x)*k**2*l**2/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*lo
g(a) + 12*k*l**4*log(a)) - 16*a**(k*x)*a**(3*l*x)*k*l**3/(12*k**4*l*log(a)
+ 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 3*a**(4*l
*x)*k**4/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12
*k*l**4*log(a)) + 13*a**(4*l*x)*k**3*l/(12*k**4*l*log(a) + 52*k**3*l**2*log
(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 13*a**(4*l*x)*k**2*l**2/(12
*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(
a)) + 3*a**(4*l*x)*k*l**3/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2
*l**3*log(a) + 12*k*l**4*log(a)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \left(a^{kx}-a^{lx}\right)^4\,dx = -\frac{4\,a^{3\,kx+lx}}{(3\,k+l)\log(a)} - \frac{4\,a^{kx+3\,lx}}{(k+3\,l)\log(a)} + \frac{3\,a^{2\,kx+2\,lx}}{(k+l)\log(a)} + \frac{a^{4\,kx}}{4\,k\log(a)} + \frac{a^{4\,lx}}{4\,l\log(a)}$$

```
[In] integrate((a^(k*x)-a^(l*x))^4,x, algorithm="maxima")
```

```
[Out] -4*a^(3*k*x + l*x)/((3*k + l)*log(a)) - 4*a^(k*x + 3*l*x)/((k + 3*l)*log(a)
) + 3*a^(2*k*x + 2*l*x)/((k + l)*log(a)) + 1/4*a^(4*k*x)/(k*log(a)) + 1/4*a
^(4*l*x)/(l*log(a))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 1359, normalized size of antiderivative = 13.87

$$\int \left(a^{kx}-a^{lx}\right)^4\,dx = \text{Too large to display}$$

```
[In] integrate((a^(k*x)-a^(l*x))^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*k*cos(-2*pi*k*x*sgn(a) + 2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2
+ (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-2*pi*k*x*sgn(a) + 2*p
i*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(4*k*x) + 1/2
*(2*l*cos(-2*pi*l*x*sgn(a) + 2*pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2 + (
pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-2*pi*l*x*sgn(a) + 2*pi*l
*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(4*l*x) - 1/2*I*
abs(a)^(4*k*x)*(-I*e^(2*I*pi*k*x*sgn(a) - 2*I*pi*k*x)/(2*I*pi*k*sgn(a) - 2*
I*pi*k + 4*k*log(abs(a))) + I*e^(-2*I*pi*k*x*sgn(a) + 2*I*pi*k*x)/(-2*I*pi*
k*sgn(a) + 2*I*pi*k + 4*k*log(abs(a)))) - 1/2*I*abs(a)^(4*l*x)*(-I*e^(2*I*p
i*l*x*sgn(a) - 2*I*pi*l*x)/(2*I*pi*l*sgn(a) - 2*I*pi*l + 4*l*log(abs(a))) +
 I*e^(-2*I*pi*l*x*sgn(a) + 2*I*pi*l*x)/(-2*I*pi*l*sgn(a) + 2*I*pi*l + 4*l*l
og(abs(a)))) - 8*(2*(3*k*log(abs(a)) + l*log(abs(a)))*cos(-3/2*pi*k*x*sgn(a
) - 1/2*pi*l*x*sgn(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(a) + pi*l*sgn
(a) - 3*pi*k - pi*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^2) - (3*pi*k*s
gn(a) + pi*l*sgn(a) - 3*pi*k - pi*l)*sin(-3/2*pi*k*x*sgn(a) - 1/2*pi*l*x*sg
n(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(a) + pi*l*sgn(a) - 3*pi*k - pi
*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^2))*e^((3*k*log(abs(a)) + l*log
(abs(a)))*x) + 4*I*(-I*e^(3/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l*x*sgn(a) - 3/2*I
*pi*k*x - 1/2*I*pi*l*x)/(3*I*pi*k*sgn(a) + I*pi*l*sgn(a) - 3*I*pi*k - I*pi*
l + 6*k*log(abs(a)) + 2*l*log(abs(a))) + I*e^(-3/2*I*pi*k*x*sgn(a) - 1/2*I*
pi*l*x*sgn(a) + 3/2*I*pi*k*x + 1/2*I*pi*l*x)/(-3*I*pi*k*sgn(a) - I*pi*l*sgn
(a) + 3*I*pi*k + I*pi*l + 6*k*log(abs(a)) + 2*l*log(abs(a))))*e^((3*k*log(a
bs(a)) + l*log(abs(a)))*x) - 8*(2*(k*log(abs(a)) + 3*l*log(abs(a)))*cos(-1/
2*pi*k*x*sgn(a) - 3/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 3/2*pi*l*x)/((pi*k*sgn(a
) + 3*pi*l*sgn(a) - pi*k - 3*pi*l)^2 + 4*(k*log(abs(a)) + 3*l*log(abs(a)))^
2) - (pi*k*sgn(a) + 3*pi*l*sgn(a) - pi*k - 3*pi*l)*sin(-1/2*pi*k*x*sgn(a) -
 3/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 3/2*pi*l*x)/((pi*k*sgn(a) + 3*pi*l*sgn(a)
 - pi*k - 3*pi*l)^2 + 4*(k*log(abs(a)) + 3*l*log(abs(a)))^2))*e^((k*log(abs
(a)) + 3*l*log(abs(a)))*x) + 4*I*(-I*e^(1/2*I*pi*k*x*sgn(a) + 3/2*I*pi*l*x*
sgn(a) - 1/2*I*pi*k*x - 3/2*I*pi*l*x)/(I*pi*k*sgn(a) + 3*I*pi*l*sgn(a) - I*
pi*k - 3*I*pi*l + 2*k*log(abs(a)) + 6*l*log(abs(a))) + I*e^(-1/2*I*pi*k*x*s
```

```
gn(a) - 3/2*I*pi*l*x*sgn(a) + 1/2*I*pi*k*x + 3/2*I*pi*l*x)/(-I*pi*k*sgn(a)
- 3*I*pi*l*sgn(a) + I*pi*k + 3*I*pi*l + 2*k*log(abs(a)) + 6*l*log(abs(a))))
*e^((k*log(abs(a)) + 3*l*log(abs(a)))*x) + 6*(2*(k*log(abs(a)) + l*log(abs(
a)))*cos(-pi*k*x*sgn(a) - pi*l*x*sgn(a) + pi*k*x + pi*l*x)/((pi*k*sgn(a) +
pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2) - (pi*k
*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)*sin(-pi*k*x*sgn(a) - pi*l*x*sgn(a) + p
i*k*x + pi*l*x)/((pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs
(a)) + l*log(abs(a)))^2))*e^(2*(k*log(abs(a)) + l*log(abs(a)))*x) + 3*I*(I*
e^(I*pi*k*x*sgn(a) + I*pi*l*x*sgn(a) - I*pi*k*x - I*pi*l*x)/(I*pi*k*sgn(a)
+ I*pi*l*sgn(a) - I*pi*k - I*pi*l + 2*k*log(abs(a)) + 2*l*log(abs(a))) - I*
e^(-I*pi*k*x*sgn(a) - I*pi*l*x*sgn(a) + I*pi*k*x + I*pi*l*x)/(-I*pi*k*sgn(a
) - I*pi*l*sgn(a) + I*pi*k + I*pi*l + 2*k*log(abs(a)) + 2*l*log(abs(a))))*e
^(2*(k*log(abs(a)) + l*log(abs(a)))*x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \left(a^{kx}-a^{lx}\right)^4\,dx=\frac{3\,a^{2\,k\,x}\,a^{2\,l\,x}}{k\,\ln\left(a\right)+l\,\ln\left(a\right)}-\frac{4\,a^{k\,x}\,a^{3\,l\,x}}{k\,\ln\left(a\right)+3\,l\,\ln\left(a\right)}-\frac{4\,a^{3\,k\,x}\,a^{l\,x}}{3\,k\,\ln\left(a\right)+l\,\ln\left(a\right)}+\frac{a^{4\,k\,x}}{4\,k\,\ln\left(a\right)}+\frac{a^{4\,l\,x}}{4\,l\,\ln\left(a\right)}$$

```
[In] int((a^(k*x) - a^(l*x))^4,x)
```

```
[Out] (3*a^(2*k*x)*a^(2*l*x))/(k*log(a) + l*log(a)) - (4*a^(k*x)*a^(3*l*x))/(k*lo
g(a) + 3*l*log(a)) - (4*a^(3*k*x)*a^(l*x))/(3*k*log(a) + l*log(a)) + a^(4*k
*x)/(4*k*log(a)) + a^(4*l*x)/(4*l*log(a))
```

3.511 $\int \left(a^{kx}-a^{lx}\right)^n\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \left(a^{kx}-a^{lx}\right)^n\,dx = \frac{\left(1-a^{(k-l)x}\right)\left(a^{kx}-a^{lx}\right)^n \text{Hypergeometric2F1}\left(1,1+\frac{kn}{k-l},1+\frac{ln}{k-l},a^{(k-l)x}\right)}{ln\log(a)}$$

```
[Out] (1-a^((k-l)*x))*(a^(k*x)-a^(l*x))^n*hypergeom([1, 1+k*n/(k-l)],[1+l*n/(k-l)
],a^((k-l)*x))/l/n/ln(a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2323, 2283}

$$\int \left(a^{kx}-a^{lx}\right)^n\,dx = \frac{\left(1-a^{-(x(k-l))}\right)^{-n}\left(a^{kx}-a^{lx}\right)^n \text{Hypergeometric2F1}\left(-n,-\frac{kn}{k-l},1-\frac{kn}{k-l},a^{-((k-l)x)}\right)}{kn\log(a)}$$

```
[In] Int[(a^(k*x) - a^(l*x))^n,x]
```

```
[Out] ((a^(k*x) - a^(l*x))^n*Hypergeometric2F1[-n, -((k*n)/(k - l)), 1 - (k*n)/(k
 - l), a^(-((k - l)*x))])/((1 - a^(-((k - l)*x)))^n*k*n*Log[a])
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
```

```
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2323

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] && !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rubi steps

$$\text{integral} = \left(a^{-knx}\left(1-a^{-((k-l)x)}\right)^{-n}\left(a^{kx}-a^{lx}\right)^{n}\right)\int a^{knx}\left(1-a^{-((k-l)x)}\right)^{n}\,dx$$

$$=\frac{\left(1-a^{-((k-l)x)}\right)^{-n}\left(a^{kx}-a^{lx}\right)^{n}\text{Hypergeometric2F1}\left(-n,-\frac{kn}{k-l},1-\frac{kn}{k-l},a^{-((k-l)x)}\right)}{kn\log(a)}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \left(a^{kx}-a^{lx}\right)^{n}\,dx$$
$$=\frac{\left(a^{kx}-a^{lx}\right)^{n}\left(1-a^{(-k+l)x}\right)\text{Hypergeometric2F1}\left(1,1+n+\frac{kn}{-k+l},1+\frac{kn}{-k+l},a^{(-k+l)x}\right)}{kn\log(a)}$$

```
[In] Integrate[(a^(k*x) - a^(l*x))^n,x]
[Out] ((a^(k*x) - a^(l*x))^n*(1 - a^((-k + l)*x))*Hypergeometric2F1[1, 1 + n + (k
*n)/(-k + l), 1 + (k*n)/(-k + l), a^((-k + l)*x)])/(k*n*Log[a])
```

Maple [F]

$$\int \left(a^{kx}-a^{lx}\right)^{n}dx$$

```
[In] int((a^(k*x)-a^(l*x))^n,x)
[Out] int((a^(k*x)-a^(l*x))^n,x)
```

Fricas [F]

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx = \int \left(a^{kx} - a^{lx}\right)^n \, dx$$

```
[In] integrate((a^(k*x)-a^(l*x))^n,x, algorithm="fricas")
[Out] integral((a^(k*x) - a^(l*x))^n, x)
```

Sympy [F]

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx = \int \left(a^{kx} - a^{lx}\right)^n \, dx$$

```
[In] integrate((a**(k*x)-a**(l*x))**n,x)
[Out] Integral((a**(k*x) - a**(l*x))**n, x)
```

Maxima [F]

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx = \int \left(a^{kx} - a^{lx}\right)^n \, dx$$

```
[In] integrate((a^(k*x)-a^(l*x))^n,x, algorithm="maxima")
[Out] integrate((a^(k*x) - a^(l*x))^n, x)
```

Giac [F]

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx = \int \left(a^{kx} - a^{lx}\right)^n \, dx$$

```
[In] integrate((a^(k*x)-a^(l*x))^n,x, algorithm="giac")
[Out] integrate((a^(k*x) - a^(l*x))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx = \int \left(a^{k\,x} - a^{l\,x}\right)^n dx$$

```
[In] int((a^(k*x) - a^(l*x))^n,x)
[Out] int((a^(k*x) - a^(l*x))^n, x)
```

3.512 $\int (1+a^{mx})\,dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int (1+a^{mx})\,dx = x + \frac{a^{mx}}{m\log(a)}$$

```
[Out] x+a^(m*x)/m/ln(a)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2225}

$$\int (1+a^{mx})\,dx = \frac{a^{mx}}{m\log(a)} + x$$

```
[In] Int[1 + a^(m*x),x]
```

```
[Out] x + a^(m*x)/(m*Log[a])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x + \int a^{mx}\,dx\\ &= x + \frac{a^{mx}}{m\log(a)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1+a^{mx})\,dx = x+\frac{a^{mx}}{m\log(a)}$$

```
[In] Integrate[1 + a^(m*x),x]
[Out] x + a^(m*x)/(m*Log[a])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$x+\frac{a^{mx}}{m\ln(a)}$	16
risch	$x+\frac{a^{mx}}{m\ln(a)}$	16
parallelrisch	$x+\frac{a^{mx}}{m\ln(a)}$	16
parts	$x+\frac{a^{mx}}{m\ln(a)}$	16
norman	$x+\frac{\mathrm{e}^{mx\ln(a)}}{m\ln(a)}$	17
derivativedivides	$\frac{a^{mx}+\ln(a^{mx})}{m\ln(a)}$	21

```
[In] int(1+a^(m*x),x,method=_RETURNVERBOSE)
[Out] x+a^(m*x)/m/ln(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int (1+a^{mx})\,dx = \frac{mx\log(a)+a^{mx}}{m\log(a)}$$

```
[In] integrate(1+a^(m*x),x, algorithm="fricas")
[Out] (m*x*log(a) + a^(m*x))/(m*log(a))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1 + a^{mx})\ dx = x + \begin{cases} \frac{a^{mx}}{m\log{(a)}} & \text{for}\ m\log{(a)} \neq 0 \\ x & \text{otherwise} \end{cases}$$

```
[In] integrate(1+a**(m*x),x)
[Out] x + Piecewise((a**(m*x)/(m*log(a)), Ne(m*log(a), 0)), (x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1 + a^{mx})\ dx = x + \frac{a^{mx}}{m\log{(a)}}$$

```
[In] integrate(1+a^(m*x),x, algorithm="maxima")
[Out] x + a^(m*x)/(m*log(a))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1 + a^{mx})\ dx = x + \frac{a^{mx}}{m\log{(a)}}$$

```
[In] integrate(1+a^(m*x),x, algorithm="giac")
[Out] x + a^(m*x)/(m*log(a))
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1 + a^{mx})\ dx = x + \frac{a^{m\,x}}{m\,\ln{(a)}}$$

```
[In] int(a^(m*x) + 1,x)
[Out] x + a^(m*x)/(m*log(a))
```

3.513 $\int (1+a^{mx})^2\,dx$

Optimal result

Integrand size = 9, antiderivative size = 33

$$\int (1+a^{mx})^2\,dx = x+\frac{2a^{mx}}{m\log(a)}+\frac{a^{2mx}}{2m\log(a)}$$

```
[Out] x+2*a^(m*x)/m/ln(a)+1/2*a^(2*m*x)/m/ln(a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2320, 45}

$$\int (1+a^{mx})^2\,dx = \frac{2a^{mx}}{m\log(a)}+\frac{a^{2mx}}{2m\log(a)}+x$$

```
[In] Int[(1 + a^(m*x))^2,x]
```

```
[Out] x + (2*a^(m*x))/(m*Log[a]) + a^(2*m*x)/(2*m*Log[a])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x}\,dx, x, a^{mx}\right)}{m\log(a)} \\ &= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x}+x\right)\,dx, x, a^{mx}\right)}{m\log(a)} \\ &= x+\frac{2a^{mx}}{m\log(a)}+\frac{a^{2mx}}{2m\log(a)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int (1+a^{mx})^2\,dx = \frac{\frac{a^{mx}(4+a^{mx})}{2m}+\frac{\log(a^{mx})}{m}}{\log(a)}$$

```
[In] Integrate[(1 + a^(m*x))^2,x]
[Out] ((a^(m*x)*(4 + a^(m*x)))/(2*m) + Log[a^(m*x)]/m)/Log[a]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

method	result	size
parallelrisch	$\frac{2mx\ln(a)+a^{2mx}+4a^{mx}}{2\ln(a)m}$	31
derivativedivides	$\frac{\frac{a^{2mx}}{2}+2a^{mx}+\ln(a^{mx})}{m\ln(a)}$	32
default	$\frac{\frac{a^{2mx}}{2}+2a^{mx}+\ln(a^{mx})}{m\ln(a)}$	32
risch	$x+\frac{2a^{mx}}{m\ln(a)}+\frac{a^{2mx}}{2m\ln(a)}$	33
norman	$x+\frac{2\,\mathrm{e}^{mx\ln(a)}}{m\ln(a)}+\frac{\mathrm{e}^{2mx\ln(a)}}{2m\ln(a)}$	35

```
[In] int((1+a^(m*x))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*(2*m*x*ln(a)+(a^(m*x))^2+4*a^(m*x))/ln(a)/m
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int (1+a^{mx})^2 \, dx = \frac{2\,mx\log(a)+a^{2\,mx}+4\,a^{mx}}{2\,m\log(a)}$$

```
[In] integrate((1+a^(m*x))^2,x, algorithm="fricas")
[Out] 1/2*(2*m*x*log(a) + a^(2*m*x) + 4*a^(m*x))/(m*log(a))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int (1+a^{mx})^2 \, dx = x + \begin{cases} \frac{a^{2mx}m\log(a)+4a^{mx}m\log(a)}{2m^2\log(a)^2} & \text{for } m^2\log(a)^2 \neq 0 \\ 3x & \text{otherwise} \end{cases}$$

```
[In] integrate((1+a**(m*x))**2,x)
[Out] x + Piecewise(((a**(2*m*x)*m*log(a) + 4*a**(m*x)*m*log(a))/(2*m**2*log(a)**
2), Ne(m**2*log(a)**2, 0)), (3*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int (1+a^{mx})^2 \, dx = x + \frac{a^{2\,mx}}{2\,m\log(a)} + \frac{2\,a^{mx}}{m\log(a)}$$

```
[In] integrate((1+a^(m*x))^2,x, algorithm="maxima")
[Out] x + 1/2*a^(2*m*x)/(m*log(a)) + 2*a^(m*x)/(m*log(a))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int (1+a^{mx})^2\,dx=\frac{2\,mx\log(|a|)+a^{2\,mx}+4\,a^{mx}}{2\,m\log(a)}$$

```
[In] integrate((1+a^(m*x))^2,x, algorithm="giac")
[Out] 1/2*(2*m*x*log(abs(a)) + a^(2*m*x) + 4*a^(m*x))/(m*log(a))
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int (1+a^{mx})^2\,dx=x+\frac{2\,a^{m\,x}+\frac{a^{2\,m\,x}}{2}}{m\,\ln(a)}$$

```
[In] int((a^(m*x) + 1)^2,x)
[Out] x + (2*a^(m*x) + a^(2*m*x)/2)/(m*log(a))
```

3.514 $\int (1 + a^{mx})^3 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 50

$$\int (1 + a^{mx})^3 \, dx = x + \frac{3a^{mx}}{m \log(a)} + \frac{3a^{2mx}}{2m \log(a)} + \frac{a^{3mx}}{3m \log(a)}$$

```
[Out] x+3*a^(m*x)/m/ln(a)+3/2*a^(2*m*x)/m/ln(a)+1/3*a^(3*m*x)/m/ln(a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2320, 45}

$$\int (1 + a^{mx})^3 \, dx = \frac{3a^{mx}}{m \log(a)} + \frac{3a^{2mx}}{2m \log(a)} + \frac{a^{3mx}}{3m \log(a)} + x$$

```
[In] Int[(1 + a^(m*x))^3,x]
```

```
[Out] x + (3*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(2*m*Log[a]) + a^(3*m*x)/(3*m*Lo
g[a])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x}\,dx, x, a^{mx}\right)}{m\log(a)}\\ &= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x}+3x+x^2\right)\,dx, x, a^{mx}\right)}{m\log(a)}\\ &= x+\frac{3a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{2m\log(a)}+\frac{a^{3mx}}{3m\log(a)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int (1+a^{mx})^3\,dx = \frac{\frac{a^{mx}(18+9a^{mx}+2a^{2mx})}{6m}+\frac{\log(a^{mx})}{m}}{\log(a)}$$

[In] Integrate[(1 + a^(m*x))^3,x]

[Out] ((a^(m*x)*(18 + 9*a^(m*x) + 2*a^(2*m*x)))/(6*m) + Log[a^(m*x)]/m)/Log[a]

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{a^{3mx}}{3}+\frac{3a^{2mx}}{2}+3a^{mx}+\ln(a^{mx})}{m\ln(a)}$	41
default	$\frac{\frac{a^{3mx}}{3}+\frac{3a^{2mx}}{2}+3a^{mx}+\ln(a^{mx})}{m\ln(a)}$	41
parallelrisch	$\frac{2a^{3mx}+6mx\ln(a)+9a^{2mx}+18a^{mx}}{6\ln(a)m}$	42
risch	$x+\frac{3a^{mx}}{m\ln(a)}+\frac{3a^{2mx}}{2m\ln(a)}+\frac{a^{3mx}}{3m\ln(a)}$	49
norman	$x+\frac{3\,e^{mx\ln(a)}}{m\ln(a)}+\frac{3\,e^{2mx\ln(a)}}{2m\ln(a)}+\frac{e^{3mx\ln(a)}}{3m\ln(a)}$	52

[In] int((1+a^(m*x))^3,x,method=_RETURNVERBOSE)

[Out] 1/m/ln(a)*(1/3*(a^(m*x))^3+3/2*(a^(m*x))^2+3*a^(m*x)+ln(a^(m*x)))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int (1+a^{mx})^3 \, dx = \frac{6 \, mx \log (a) + 2 \, a^{3 \, mx} + 9 \, a^{2 \, mx} + 18 \, a^{mx}}{6 \, m \log (a)}$$

```
[In] integrate((1+a^(m*x))^3,x, algorithm="fricas")
[Out] 1/6*(6*m*x*log(a) + 2*a^(3*m*x) + 9*a^(2*m*x) + 18*a^(m*x))/(m*log(a))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int (1+a^{mx})^3 \, dx = x + \begin{cases} \frac{2a^{3mx}m^2 \log (a)^2 + 9a^{2mx}m^2 \log (a)^2 + 18a^{mx}m^2 \log (a)^2}{6m^3 \log (a)^3} & \text{for } m^3 \log (a)^3 \neq 0 \\ 7x & \text{otherwise} \end{cases}$$

```
[In] integrate((1+a**(m*x))**3,x)
[Out] x + Piecewise(((2*a**(3*m*x)*m**2*log(a)**2 + 9*a**(2*m*x)*m**2*log(a)**2 +
 18*a**(m*x)*m**2*log(a)**2)/(6*m**3*log(a)**3), Ne(m**3*log(a)**3, 0)), (7
*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int (1+a^{mx})^3 \, dx = x + \frac{a^{3 \, mx}}{3 \, m \log (a)} + \frac{3 \, a^{2 \, mx}}{2 \, m \log (a)} + \frac{3 \, a^{mx}}{m \log (a)}$$

```
[In] integrate((1+a^(m*x))^3,x, algorithm="maxima")
[Out] x + 1/3*a^(3*m*x)/(m*log(a)) + 3/2*a^(2*m*x)/(m*log(a)) + 3*a^(m*x)/(m*log(
a))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (1 + a^{mx})^3 \, dx = \frac{6\,mx \log(|a|) + 2\,a^{3\,mx} + 9\,a^{2\,mx} + 18\,a^{mx}}{6\,m \log(a)}$$

```
[In] integrate((1+a^(m*x))^3,x, algorithm="giac")
[Out] 1/6*(6*m*x*log(abs(a)) + 2*a^(3*m*x) + 9*a^(2*m*x) + 18*a^(m*x))/(m*log(a))
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int (1 + a^{mx})^3 \, dx = x + \frac{3\,a^{m\,x} + \frac{3\,a^{2\,m\,x}}{2} + \frac{a^{3\,m\,x}}{3}}{m\,\ln(a)}$$

```
[In] int((a^(m*x) + 1)^3,x)
[Out] x + (3*a^(m*x) + (3*a^(2*m*x))/2 + a^(3*m*x)/3)/(m*log(a))
```

3.515 $\int (1+a^{mx})^4 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 65

$$\int (1+a^{mx})^4 \, dx = x+\frac{4a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{m\log(a)}+\frac{4a^{3mx}}{3m\log(a)}+\frac{a^{4mx}}{4m\log(a)}$$

```
[Out] x+4*a^(m*x)/m/ln(a)+3*a^(2*m*x)/m/ln(a)+4/3*a^(3*m*x)/m/ln(a)+1/4*a^(4*m*x)
/m/ln(a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2320, 45}

$$\int (1+a^{mx})^4 \, dx = \frac{4a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{m\log(a)}+\frac{4a^{3mx}}{3m\log(a)}+\frac{a^{4mx}}{4m\log(a)}+x$$

```
[In] Int[(1 + a^(m*x))^4,x]
```

```
[Out] x + (4*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(m*Log[a]) + (4*a^(3*m*x))/(3*m*
Log[a]) + a^(4*m*x)/(4*m*Log[a])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x)^4}{x}\,dx, x, a^{mx}\right)}{m\log(a)}\\ &= \frac{\text{Subst}\left(\int \left(4+\frac{1}{x}+6x+4x^2+x^3\right)\,dx, x, a^{mx}\right)}{m\log(a)}\\ &= x+\frac{4a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{m\log(a)}+\frac{4a^{3mx}}{3m\log(a)}+\frac{a^{4mx}}{4m\log(a)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (1+a^{mx})^4\,dx = \frac{\frac{a^{mx}(48+36a^{mx}+16a^{2mx}+3a^{3mx})}{12m}+\frac{\log(a^{mx})}{m}}{\log(a)}$$

```
[In] Integrate[(1 + a^(m*x))^4,x]
[Out] ((a^(m*x)*(48 + 36*a^(m*x) + 16*a^(2*m*x) + 3*a^(3*m*x)))/(12*m) + Log[a^(m
*x)]/m)/Log[a]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\frac{a^{4mx}}{4}+\frac{4a^{3mx}}{3}+3a^{2mx}+4a^{mx}+\ln(a^{mx})}{m\ln(a)}$	50
default	$\frac{\frac{a^{4mx}}{4}+\frac{4a^{3mx}}{3}+3a^{2mx}+4a^{mx}+\ln(a^{mx})}{m\ln(a)}$	50
parallelrisch	$\frac{3a^{4mx}+16a^{3mx}+12mx\ln(a)+36a^{2mx}+48a^{mx}}{12\ln(a)m}$	51
risch	$x+\frac{4a^{mx}}{m\ln(a)}+\frac{3a^{2mx}}{m\ln(a)}+\frac{4a^{3mx}}{3m\ln(a)}+\frac{a^{4mx}}{4m\ln(a)}$	65
norman	$x+\frac{4\,e^{mx\ln(a)}}{m\ln(a)}+\frac{3\,e^{2mx\ln(a)}}{m\ln(a)}+\frac{4\,e^{3mx\ln(a)}}{3m\ln(a)}+\frac{e^{4mx\ln(a)}}{4m\ln(a)}$	69

```
[In] int((1+a^(m*x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/m/ln(a)*(1/4*(a^(m*x))^4+4/3*(a^(m*x))^3+3*(a^(m*x))^2+4*a^(m*x)+ln(a^(m*
x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int (1+a^{mx})^4 \, dx = \frac{12\,mx\log (a) + 3\,a^{4\,mx} + 16\,a^{3\,mx} + 36\,a^{2\,mx} + 48\,a^{mx}}{12\,m\log (a)}$$

```
[In] integrate((1+a^(m*x))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(12*m*x*log(a) + 3*a^(4*m*x) + 16*a^(3*m*x) + 36*a^(2*m*x) + 48*a^(m*x
))/(m*log(a))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int (1+a^{mx})^4 \, dx = x + \begin{cases} \frac{3a^{4mx}m^3\log (a)^3+16a^{3mx}m^3\log (a)^3+36a^{2mx}m^3\log (a)^3+48a^{mx}m^3\log (a)^3}{12m^4\log (a)^4} & \text{for } m^4\log (a)^4 \neq 0 \\ 15x & \text{otherwise} \end{cases}$$

```
[In] integrate((1+a**(m*x))**4,x)
```

```
[Out] x + Piecewise(((3*a**(4*m*x)*m**3*log(a)**3 + 16*a**(3*m*x)*m**3*log(a)**3
+ 36*a**(2*m*x)*m**3*log(a)**3 + 48*a**(m*x)*m**3*log(a)**3)/(12*m**4*log(a
)**4), Ne(m**4*log(a)**4, 0)), (15*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int (1+a^{mx})^4 \, dx = x + \frac{a^{4\,mx}}{4\,m\log (a)} + \frac{4\,a^{3\,mx}}{3\,m\log (a)} + \frac{3\,a^{2\,mx}}{m\log (a)} + \frac{4\,a^{mx}}{m\log (a)}$$

```
[In] integrate((1+a^(m*x))^4,x, algorithm="maxima")
```

```
[Out] x + 1/4*a^(4*m*x)/(m*log(a)) + 4/3*a^(3*m*x)/(m*log(a)) + 3*a^(2*m*x)/(m*lo
g(a)) + 4*a^(m*x)/(m*log(a))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int (1+a^{mx})^4 \, dx = \frac{12\,mx\log(|a|) + 3\,a^{4\,mx} + 16\,a^{3\,mx} + 36\,a^{2\,mx} + 48\,a^{mx}}{12\,m\log(a)}$$

```
[In] integrate((1+a^(m*x))^4,x, algorithm="giac")
[Out] 1/12*(12*m*x*log(abs(a)) + 3*a^(4*m*x) + 16*a^(3*m*x) + 36*a^(2*m*x) + 48*a
^(m*x))/(m*log(a))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int (1+a^{mx})^4 \, dx = x + \frac{4\,a^{m\,x} + 3\,a^{2\,m\,x} + \frac{4\,a^{3\,m\,x}}{3} + \frac{a^{4\,m\,x}}{4}}{m\,\ln(a)}$$

```
[In] int((a^(m*x) + 1)^4,x)
[Out] x + (4*a^(m*x) + 3*a^(2*m*x) + (4*a^(3*m*x))/3 + a^(4*m*x)/4)/(m*log(a))
```

3.516 $\int (1 + a^{mx})^n \, dx$

Optimal result

Integrand size = 9, antiderivative size = 40

$$\int (1 + a^{mx})^n \, dx = -\frac{(1 + a^{mx})^{1+n} \text{Hypergeometric2F1}(1, 1 + n, 2 + n, 1 + a^{mx})}{m(1 + n) \log(a)}$$

```
[Out] -(1+a^(m*x))^(1+n)*hypergeom([1, 1+n],[2+n],1+a^(m*x))/m/(1+n)/ln(a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2320, 67}

$$\int (1 + a^{mx})^n \, dx = -\frac{(a^{mx} + 1)^{n+1} \text{Hypergeometric2F1}(1, n + 1, n + 2, a^{mx} + 1)}{m(n + 1) \log(a)}$$

```
[In] Int[(1 + a^(m*x))^n,x]
```

```
[Out] -(((1 + a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + a^(m*x)])/(
m*(1 + n)*Log[a]))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x)^n}{x}\,dx, x, a^{mx}\right)}{m\log(a)}\\ &= -\frac{(1+a^{mx})^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+a^{mx}\right)}{m(1+n)\log(a)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int (1+a^{mx})^n\,dx = -\frac{(1+a^{mx})^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+a^{mx}\right)}{m(1+n)\log(a)}$$

```
[In] Integrate[(1 + a^(m*x))^n,x]
[Out] -(((1 + a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + a^(m*x)])/(
m*(1 + n)*Log[a]))
```

Maple [F]

$$\int (1+a^{mx})^n\,dx$$

```
[In] int((1+a^(m*x))^n,x)
[Out] int((1+a^(m*x))^n,x)
```

Fricas [F]

$$\int (1+a^{mx})^n\,dx = \int (a^{mx}+1)^n\,dx$$

```
[In] integrate((1+a^(m*x))^n,x, algorithm="fricas")
[Out] integral((a^(m*x) + 1)^n, x)
```

Sympy [F]

$$\int (1+a^{mx})^n \,dx = \int (a^{mx}+1)^n \,dx$$

```
[In] integrate((1+a**(m*x))**n,x)
[Out] Integral((a**(m*x) + 1)**n, x)
```

Maxima [F]

$$\int (1+a^{mx})^n \,dx = \int (a^{mx}+1)^n \,dx$$

```
[In] integrate((1+a^(m*x))^n,x, algorithm="maxima")
[Out] integrate((a^(m*x) + 1)^n, x)
```

Giac [F]

$$\int (1+a^{mx})^n \,dx = \int (a^{mx}+1)^n \,dx$$

```
[In] integrate((1+a^(m*x))^n,x, algorithm="giac")
[Out] integrate((a^(m*x) + 1)^n, x)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int (1+a^{mx})^n \,dx = \frac{(a^{m\,x}+1)^n\,{}_2\mathrm{F}_1\left(-n,-n;\ 1-n;\ -\frac{1}{a^{m\,x}}\right)}{m\,n\,\ln(a)\,\left(\frac{1}{a^{m\,x}}+1\right)^n}$$

```
[In] int((a^(m*x) + 1)^n,x)
[Out] ((a^(m*x) + 1)^n*hypergeom([-n, -n], 1 - n, -1/a^(m*x)))/(m*n*log(a)*(1/a^(
m*x) + 1)^n)
```

3.517 $\int (1 - a^{mx})\, dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int (1 - a^{mx})\, dx = x - \frac{a^{mx}}{m\log(a)}$$

```
[Out] x-a^(m*x)/m/ln(a)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2225}

$$\int (1 - a^{mx})\, dx = x - \frac{a^{mx}}{m\log(a)}$$

```
[In] Int[1 - a^(m*x),x]
```

```
[Out] x - a^(m*x)/(m*Log[a])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= x - \int a^{mx}\, dx \\ &= x - \frac{a^{mx}}{m\log(a)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (1 - a^{mx}) \, dx = x - \frac{a^{mx}}{m \log(a)}$$

```
[In] Integrate[1 - a^(m*x),x]
[Out] x - a^(m*x)/(m*Log[a])
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$x - \frac{a^{mx}}{m \ln(a)}$	17
risch	$x - \frac{a^{mx}}{m \ln(a)}$	17
parallelrisch	$x - \frac{a^{mx}}{m \ln(a)}$	17
parts	$x - \frac{a^{mx}}{m \ln(a)}$	17
norman	$x - \frac{e^{mx \ln(a)}}{m \ln(a)}$	18
derivativedivides	$\frac{-a^{mx} + \ln(a^{mx})}{m \ln(a)}$	23

```
[In] int(1-a^(m*x),x,method=_RETURNVERBOSE)
[Out] x-a^(m*x)/m/ln(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int (1 - a^{mx}) \, dx = \frac{mx \log(a) - a^{mx}}{m \log(a)}$$

```
[In] integrate(1-a^(m*x),x, algorithm="fricas")
[Out] (m*x*log(a) - a^(m*x))/(m*log(a))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int (1 - a^{mx})\, dx = x + \begin{cases} -\frac{a^{mx}}{m\log(a)} & \text{for } m\log(a) \neq 0 \\ -x & \text{otherwise} \end{cases}$$

```
[In] integrate(1-a**(m*x),x)
[Out] x + Piecewise((-a**(m*x)/(m*log(a)), Ne(m*log(a), 0)), (-x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (1 - a^{mx})\, dx = x - \frac{a^{mx}}{m\log(a)}$$

```
[In] integrate(1-a^(m*x),x, algorithm="maxima")
[Out] x - a^(m*x)/(m*log(a))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (1 - a^{mx})\, dx = x - \frac{a^{mx}}{m\log(a)}$$

```
[In] integrate(1-a^(m*x),x, algorithm="giac")
[Out] x - a^(m*x)/(m*log(a))
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (1 - a^{mx})\, dx = x - \frac{a^{m\,x}}{m\,\ln(a)}$$

```
[In] int(1 - a^(m*x),x)
[Out] x - a^(m*x)/(m*log(a))
```

3.518 $\int (1-a^{mx})^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int (1-a^{mx})^2 \, dx = x - \frac{2a^{mx}}{m \log(a)} + \frac{a^{2mx}}{2m \log(a)}$$

```
[Out] x-2*a^(m*x)/m/ln(a)+1/2*a^(2*m*x)/m/ln(a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2320, 45}

$$\int (1-a^{mx})^2 \, dx = -\frac{2a^{mx}}{m \log(a)} + \frac{a^{2mx}}{2m \log(a)} + x$$

```
[In] Int[(1 - a^(m*x))^2,x]
```

```
[Out] x - (2*a^(m*x))/(m*Log[a]) + a^(2*m*x)/(2*m*Log[a])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx, x, a^{mx}\right)}{m\log(a)}\\ &= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)\,dx, x, a^{mx}\right)}{m\log(a)}\\ &= x-\frac{2a^{mx}}{m\log(a)}+\frac{a^{2mx}}{2m\log(a)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int (1-a^{mx})^2\,dx = \frac{\frac{a^{mx}(-4+a^{mx})}{2m}+\frac{\log(a^{mx})}{m}}{\log(a)}$$

```
[In] Integrate[(1 - a^(m*x))^2,x]
[Out] ((a^(m*x)*(-4 + a^(m*x)))/(2*m) + Log[a^(m*x)]/m)/Log[a]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

method	result	size
parallelrisch	$\frac{2mx\ln(a)+a^{2mx}-4a^{mx}}{2\ln(a)m}$	31
derivativedivides	$\frac{\frac{a^{2mx}}{2}-2a^{mx}+\ln(a^{mx})}{m\ln(a)}$	32
default	$\frac{\frac{a^{2mx}}{2}-2a^{mx}+\ln(a^{mx})}{m\ln(a)}$	32
risch	$x-\frac{2a^{mx}}{m\ln(a)}+\frac{a^{2mx}}{2m\ln(a)}$	33
norman	$x-\frac{2\,\mathrm{e}^{mx\ln(a)}}{m\ln(a)}+\frac{\mathrm{e}^{2mx\ln(a)}}{2m\ln(a)}$	35

```
[In] int((1-a^(m*x))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*(2*m*x*ln(a)+(a^(m*x))^2-4*a^(m*x))/ln(a)/m
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int (1-a^{mx})^2 \, dx = \frac{2\,mx\log(a) + a^{2\,mx} - 4\,a^{mx}}{2\,m\log(a)}$$

```
[In] integrate((1-a^(m*x))^2,x, algorithm="fricas")
[Out] 1/2*(2*m*x*log(a) + a^(2*m*x) - 4*a^(m*x))/(m*log(a))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int (1-a^{mx})^2 \, dx = x + \begin{cases} \frac{a^{2mx}m\log(a) - 4a^{mx}m\log(a)}{2m^2\log(a)^2} & \text{for } m^2\log(a)^2 \neq 0 \\ -x & \text{otherwise} \end{cases}$$

```
[In] integrate((1-a**(m*x))**2,x)
[Out] x + Piecewise(((a**(2*m*x)*m*log(a) - 4*a**(m*x)*m*log(a))/(2*m**2*log(a)**
2), Ne(m**2*log(a)**2, 0)), (-x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int (1-a^{mx})^2 \, dx = x + \frac{a^{2\,mx}}{2\,m\log(a)} - \frac{2\,a^{mx}}{m\log(a)}$$

```
[In] integrate((1-a^(m*x))^2,x, algorithm="maxima")
[Out] x + 1/2*a^(2*m*x)/(m*log(a)) - 2*a^(m*x)/(m*log(a))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int (1-a^{mx})^2 \, dx = \frac{2\,mx\log(|a|) + a^{2\,mx} - 4\,a^{mx}}{2\,m\log(a)}$$

```
[In] integrate((1-a^(m*x))^2,x, algorithm="giac")
[Out] 1/2*(2*m*x*log(abs(a)) + a^(2*m*x) - 4*a^(m*x))/(m*log(a))
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int (1-a^{mx})^2 \, dx = x - \frac{2\,a^{m\,x} - \frac{a^{2\,m\,x}}{2}}{m\,\ln(a)}$$

```
[In] int((a^(m*x) - 1)^2,x)
[Out] x - (2*a^(m*x) - a^(2*m*x)/2)/(m*log(a))
```

3.519 $\int (1 - a^{mx})^3 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 50

$$\int (1 - a^{mx})^3 \, dx = x - \frac{3a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{2m\log(a)} - \frac{a^{3mx}}{3m\log(a)}$$

```
[Out] x-3*a^(m*x)/m/ln(a)+3/2*a^(2*m*x)/m/ln(a)-1/3*a^(3*m*x)/m/ln(a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2320, 45}

$$\int (1 - a^{mx})^3 \, dx = -\frac{3a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{2m\log(a)} - \frac{a^{3mx}}{3m\log(a)} + x$$

```
[In] Int[(1 - a^(m*x))^3,x]
```

```
[Out] x - (3*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(2*m*Log[a]) - a^(3*m*x)/(3*m*Lo
g[a])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x)^3}{x}\,dx, x, a^{mx}\right)}{m\log(a)}\\ &= \frac{\text{Subst}\left(\int \left(-3+\frac{1}{x}+3x-x^2\right)\,dx, x, a^{mx}\right)}{m\log(a)}\\ &= x-\frac{3a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{2m\log(a)}-\frac{a^{3mx}}{3m\log(a)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int (1-a^{mx})^3\,dx = \frac{-\frac{a^{mx}(18-9a^{mx}+2a^{2mx})}{6m}+\frac{\log(a^{mx})}{m}}{\log(a)}$$

```
[In] Integrate[(1 - a^(m*x))^3,x]
[Out] (-1/6*(a^(m*x)*(18 - 9*a^(m*x) + 2*a^(2*m*x)))/m + Log[a^(m*x)]/m)/Log[a]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{-\frac{a^{3mx}}{3}+\frac{3a^{2mx}}{2}-3a^{mx}+\ln(a^{mx})}{m\ln(a)}$	41
default	$\frac{-\frac{a^{3mx}}{3}+\frac{3a^{2mx}}{2}-3a^{mx}+\ln(a^{mx})}{m\ln(a)}$	41
parallelrisch	$\frac{6mx\ln(a)-2a^{3mx}+9a^{2mx}-18a^{mx}}{6\ln(a)m}$	42
risch	$x-\frac{3a^{mx}}{m\ln(a)}+\frac{3a^{2mx}}{2m\ln(a)}-\frac{a^{3mx}}{3m\ln(a)}$	49
norman	$x-\frac{3\,\mathrm{e}^{mx\ln(a)}}{m\ln(a)}+\frac{3\,\mathrm{e}^{2mx\ln(a)}}{2m\ln(a)}-\frac{\mathrm{e}^{3mx\ln(a)}}{3m\ln(a)}$	52

```
[In] int((1-a^(m*x))^3,x,method=_RETURNVERBOSE)
[Out] 1/m/ln(a)*(-1/3*(a^(m*x))^3+3/2*(a^(m*x))^2-3*a^(m*x)+ln(a^(m*x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int (1-a^{mx})^3 \, dx = \frac{6\,mx\log{(a)} - 2\,a^{3\,mx} + 9\,a^{2\,mx} - 18\,a^{mx}}{6\,m\log{(a)}}$$

```
[In] integrate((1-a^(m*x))^3,x, algorithm="fricas")
[Out] 1/6*(6*m*x*log(a) - 2*a^(3*m*x) + 9*a^(2*m*x) - 18*a^(m*x))/(m*log(a))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int (1-a^{mx})^3 \, dx = x + \begin{cases} \frac{-2a^{3mx}m^2\log{(a)}^2 + 9a^{2mx}m^2\log{(a)}^2 - 18a^{mx}m^2\log{(a)}^2}{6m^3\log{(a)}^3} & \text{for } m^3\log{(a)}^3 \neq 0 \\ -x & \text{otherwise} \end{cases}$$

```
[In] integrate((1-a**(m*x))**3,x)
[Out] x + Piecewise(((-2*a**(3*m*x)*m**2*log(a)**2 + 9*a**(2*m*x)*m**2*log(a)**2
- 18*a**(m*x)*m**2*log(a)**2)/(6*m**3*log(a)**3), Ne(m**3*log(a)**3, 0)), (
-x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int (1-a^{mx})^3 \, dx = x - \frac{a^{3\,mx}}{3\,m\log{(a)}} + \frac{3\,a^{2\,mx}}{2\,m\log{(a)}} - \frac{3\,a^{mx}}{m\log{(a)}}$$

```
[In] integrate((1-a^(m*x))^3,x, algorithm="maxima")
[Out] x - 1/3*a^(3*m*x)/(m*log(a)) + 3/2*a^(2*m*x)/(m*log(a)) - 3*a^(m*x)/(m*log(
a))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (1 - a^{mx})^3 \, dx = \frac{6\, mx \log(|a|) - 2\, a^{3\, mx} + 9\, a^{2\, mx} - 18\, a^{mx}}{6\, m \log(a)}$$

```
[In] integrate((1-a^(m*x))^3,x, algorithm="giac")
[Out] 1/6*(6*m*x*log(abs(a)) - 2*a^(3*m*x) + 9*a^(2*m*x) - 18*a^(m*x))/(m*log(a))
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int (1 - a^{mx})^3 \, dx = x - \frac{3\, a^{m\, x} - \frac{3\, a^{2\, m\, x}}{2} + \frac{a^{3\, m\, x}}{3}}{m\, \ln(a)}$$

```
[In] int(-(a^(m*x) - 1)^3,x)
[Out] x - (3*a^(m*x) - (3*a^(2*m*x))/2 + a^(3*m*x)/3)/(m*log(a))
```

3.520 $\int (1 - a^{mx})^4\, dx$

Optimal result

Integrand size = 11, antiderivative size = 65

$$\int (1 - a^{mx})^4\, dx = x - \frac{4a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{m\log(a)} - \frac{4a^{3mx}}{3m\log(a)} + \frac{a^{4mx}}{4m\log(a)}$$

```
[Out] x-4*a^(m*x)/m/ln(a)+3*a^(2*m*x)/m/ln(a)-4/3*a^(3*m*x)/m/ln(a)+1/4*a^(4*m*x)
/m/ln(a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2320, 45}

$$\int (1 - a^{mx})^4\, dx = -\frac{4a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{m\log(a)} - \frac{4a^{3mx}}{3m\log(a)} + \frac{a^{4mx}}{4m\log(a)} + x$$

```
[In] Int[(1 - a^(m*x))^4,x]
```

```
[Out] x - (4*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(m*Log[a]) - (4*a^(3*m*x))/(3*m*
Log[a]) + a^(4*m*x)/(4*m*Log[a])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x)^4}{x}\,dx, x, a^{mx}\right)}{m\log(a)}\\ &= \frac{\text{Subst}\left(\int \left(-4+\frac{1}{x}+6x-4x^2+x^3\right)\,dx, x, a^{mx}\right)}{m\log(a)}\\ &= x-\frac{4a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{m\log(a)}-\frac{4a^{3mx}}{3m\log(a)}+\frac{a^{4mx}}{4m\log(a)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (1-a^{mx})^4\,dx = \frac{\frac{a^{mx}(-48+36a^{mx}-16a^{2mx}+3a^{3mx})}{12m}+\frac{\log(a^{mx})}{m}}{\log(a)}$$

```
[In] Integrate[(1 - a^(m*x))^4,x]
```

```
[Out] ((a^(m*x)*(-48 + 36*a^(m*x) - 16*a^(2*m*x) + 3*a^(3*m*x)))/(12*m) + Log[a^(
m*x)]/m)/Log[a]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\frac{a^{4mx}}{4}-\frac{4a^{3mx}}{3}+3a^{2mx}-4a^{mx}+\ln(a^{mx})}{m\ln(a)}$	50
default	$\frac{\frac{a^{4mx}}{4}-\frac{4a^{3mx}}{3}+3a^{2mx}-4a^{mx}+\ln(a^{mx})}{m\ln(a)}$	50
parallelrisch	$\frac{3a^{4mx}-16a^{3mx}+12mx\ln(a)+36a^{2mx}-48a^{mx}}{12\ln(a)m}$	51
risch	$x-\frac{4a^{mx}}{m\ln(a)}+\frac{3a^{2mx}}{m\ln(a)}-\frac{4a^{3mx}}{3m\ln(a)}+\frac{a^{4mx}}{4m\ln(a)}$	65
norman	$x-\frac{4\,\mathrm{e}^{mx\ln(a)}}{m\ln(a)}+\frac{3\,\mathrm{e}^{2mx\ln(a)}}{m\ln(a)}-\frac{4\,\mathrm{e}^{3mx\ln(a)}}{3m\ln(a)}+\frac{\mathrm{e}^{4mx\ln(a)}}{4m\ln(a)}$	69

```
[In] int((1-a^(m*x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/m/ln(a)*(1/4*(a^(m*x))^4-4/3*(a^(m*x))^3+3*(a^(m*x))^2-4*a^(m*x)+ln(a^(m*
x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int (1-a^{mx})^4 \, dx = \frac{12\,mx\log(a) + 3\,a^{4\,mx} - 16\,a^{3\,mx} + 36\,a^{2\,mx} - 48\,a^{mx}}{12\,m\log(a)}$$

```
[In] integrate((1-a^(m*x))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(12*m*x*log(a) + 3*a^(4*m*x) - 16*a^(3*m*x) + 36*a^(2*m*x) - 48*a^(m*x
))/(m*log(a))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int (1-a^{mx})^4 \, dx = x + \begin{cases} \frac{3a^{4mx}m^3\log(a)^3 - 16a^{3mx}m^3\log(a)^3 + 36a^{2mx}m^3\log(a)^3 - 48a^{mx}m^3\log(a)^3}{12m^4\log(a)^4} & \text{for } m^4\log(a)^4 \neq 0 \\ -x & \text{otherwise} \end{cases}$$

```
[In] integrate((1-a**(m*x))**4,x)
```

```
[Out] x + Piecewise(((3*a**(4*m*x)*m**3*log(a)**3 - 16*a**(3*m*x)*m**3*log(a)**3
+ 36*a**(2*m*x)*m**3*log(a)**3 - 48*a**(m*x)*m**3*log(a)**3)/(12*m**4*log(a
)**4), Ne(m**4*log(a)**4, 0)), (-x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int (1-a^{mx})^4 \, dx = x + \frac{a^{4\,mx}}{4\,m\log(a)} - \frac{4\,a^{3\,mx}}{3\,m\log(a)} + \frac{3\,a^{2\,mx}}{m\log(a)} - \frac{4\,a^{mx}}{m\log(a)}$$

```
[In] integrate((1-a^(m*x))^4,x, algorithm="maxima")
```

```
[Out] x + 1/4*a^(4*m*x)/(m*log(a)) - 4/3*a^(3*m*x)/(m*log(a)) + 3*a^(2*m*x)/(m*lo
g(a)) - 4*a^(m*x)/(m*log(a))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int (1 - a^{mx})^4 \, dx = \frac{12\, mx \log(|a|) + 3\, a^{4\, mx} - 16\, a^{3\, mx} + 36\, a^{2\, mx} - 48\, a^{mx}}{12\, m \log(a)}$$

```
[In] integrate((1-a^(m*x))^4,x, algorithm="giac")
[Out] 1/12*(12*m*x*log(abs(a)) + 3*a^(4*m*x) - 16*a^(3*m*x) + 36*a^(2*m*x) - 48*a
^(m*x))/(m*log(a))
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int (1 - a^{mx})^4 \, dx = x - \frac{4\, a^{m\, x} - 3\, a^{2\, m\, x} + \frac{4\, a^{3\, m\, x}}{3} - \frac{a^{4\, m\, x}}{4}}{m\, \ln(a)}$$

```
[In] int((a^(m*x) - 1)^4,x)
[Out] x - (4*a^(m*x) - 3*a^(2*m*x) + (4*a^(3*m*x))/3 - a^(4*m*x)/4)/(m*log(a))
```

3.521 $\int (1-a^{mx})^n \, dx$

Optimal result

Integrand size = 11, antiderivative size = 44

$$\int (1-a^{mx})^n \, dx = -\frac{(1-a^{mx})^{1+n} \text{Hypergeometric2F1}(1,1+n,2+n,1-a^{mx})}{m(1+n)\log(a)}$$

```
[Out] -(1-a^(m*x))^(1+n)*hypergeom([1, 1+n],[2+n],1-a^(m*x))/m/(1+n)/ln(a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2320, 67}

$$\int (1-a^{mx})^n \, dx = -\frac{(1-a^{mx})^{n+1} \text{Hypergeometric2F1}(1,n+1,n+2,1-a^{mx})}{m(n+1)\log(a)}$$

```
[In] Int[(1 - a^(m*x))^n,x]
```

```
[Out] -(((1 - a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 - a^(m*x)])/(
m*(1 + n)*Log[a]))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1-x)^n}{x}\,dx, x, a^{mx}\right)}{m\log(a)}$$
$$= -\frac{(1-a^{mx})^{1+n}\,\text{Hypergeometric2F1}(1,1+n,2+n,1-a^{mx})}{m(1+n)\log(a)}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (1-a^{mx})^n\,dx = -\frac{(1-a^{mx})^{1+n}\,\text{Hypergeometric2F1}(1,1+n,2+n,1-a^{mx})}{m(1+n)\log(a)}$$

```
[In] Integrate[(1 - a^(m*x))^n,x]
[Out] -(((1 - a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 - a^(m*x)])/(
m*(1 + n)*Log[a]))
```

Maple [F]

$$\int (1-a^{mx})^n\,dx$$

```
[In] int((1-a^(m*x))^n,x)
[Out] int((1-a^(m*x))^n,x)
```

Fricas [F]

$$\int (1-a^{mx})^n\,dx = \int (-a^{mx}+1)^n\,dx$$

```
[In] integrate((1-a^(m*x))^n,x, algorithm="fricas")
[Out] integral((-a^(m*x) + 1)^n, x)
```

Sympy [F]

$$\int (1-a^{mx})^n \, dx = \int (1-a^{mx})^n \, dx$$

```
[In] integrate((1-a**(m*x))**n,x)
[Out] Integral((1 - a**(m*x))**n, x)
```

Maxima [F]

$$\int (1-a^{mx})^n \, dx = \int (-a^{mx}+1)^n \, dx$$

```
[In] integrate((1-a^(m*x))^n,x, algorithm="maxima")
[Out] integrate((-a^(m*x) + 1)^n, x)
```

Giac [F]

$$\int (1-a^{mx})^n \, dx = \int (-a^{mx}+1)^n \, dx$$

```
[In] integrate((1-a^(m*x))^n,x, algorithm="giac")
[Out] integrate((-a^(m*x) + 1)^n, x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int (1-a^{mx})^n \, dx = \frac{(1-a^{m\,x})^n \, {}_2\mathrm{F}_1\left(-n,-n;\ 1-n;\ \frac{1}{a^{m\,x}}\right)}{m\,n\,\ln(a)\,\left(1-\frac{1}{a^{m\,x}}\right)^n}$$

```
[In] int((1 - a^(m*x))^n,x)
[Out] ((1 - a^(m*x))^n*hypergeom([-n, -n], 1 - n, 1/a^(m*x)))/(m*n*log(a)*(1 - 1/
a^(m*x))^n)
```

3.522 $\int \frac{1}{b+ae^{nx}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{1}{b+ae^{nx}}\,dx=\frac{x}{b}-\frac{\log\left(b+ae^{nx}\right)}{bn}$$

```
[Out] x/b-ln(b+a*exp(n*x))/b/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2320, 36, 29, 31}

$$\int \frac{1}{b+ae^{nx}}\,dx=\frac{x}{b}-\frac{\log\left(ae^{nx}+b\right)}{bn}$$

```
[In] Int[(b + a*E^(n*x))^(-1),x]
```

```
[Out] x/b - Log[b + a*E^(n*x)]/(b*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(b+ax)}\,dx, x, e^{nx}\right)}{n}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, e^{nx}\right)}{bn} - \frac{a\text{Subst}\left(\int \frac{1}{b+ax}\,dx, x, e^{nx}\right)}{bn}\\ &= \frac{x}{b} - \frac{\log\left(b+ae^{nx}\right)}{bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{1}{b+ae^{nx}}\,dx = \frac{\log\left(e^{nx}\right)}{bn} - \frac{\log\left(b^2n+abe^{nx}n\right)}{bn}$$

```
[In] Integrate[(b + a*E^(n*x))^(-1),x]
[Out] Log[E^(n*x)]/(b*n) - Log[b^2*n + a*b*E^(n*x)*n]/(b*n)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
parallelrisch	$-\frac{-nx+\ln\left(b+a\,\mathrm{e}^{nx}\right)}{bn}$	23
norman	$\frac{x}{b}-\frac{\ln\left(b+a\,\mathrm{e}^{nx}\right)}{bn}$	24
risch	$\frac{x}{b}-\frac{\ln\left(\mathrm{e}^{nx}+\frac{b}{a}\right)}{bn}$	26
derivativedivides	$\frac{-\frac{\ln\left(b+a\,\mathrm{e}^{nx}\right)}{b}+\frac{\ln\left(\mathrm{e}^{nx}\right)}{b}}{n}$	29
default	$\frac{-\frac{\ln\left(b+a\,\mathrm{e}^{nx}\right)}{b}+\frac{\ln\left(\mathrm{e}^{nx}\right)}{b}}{n}$	29

```
[In] int(1/(b+a*exp(n*x)),x,method=_RETURNVERBOSE)
[Out] -(-n*x+ln(b+a*exp(n*x)))/b/n
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{b+ae^{nx}}\,dx = \frac{nx - \log\left(ae^{(nx)} + b\right)}{bn}$$

```
[In] integrate(1/(b+a*exp(n*x)),x, algorithm="fricas")
[Out] (n*x - log(a*e^(n*x) + b))/(b*n)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{b+ae^{nx}}\,dx = \frac{x}{b} - \frac{\log\left(e^{nx} + \frac{b}{a}\right)}{bn}$$

```
[In] integrate(1/(b+a*exp(n*x)),x)
[Out] x/b - log(exp(n*x) + b/a)/(b*n)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{b+ae^{nx}}\,dx = \frac{x}{b} - \frac{\log\left(ae^{(nx)} + b\right)}{bn}$$

```
[In] integrate(1/(b+a*exp(n*x)),x, algorithm="maxima")
[Out] x/b - log(a*e^(n*x) + b)/(b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{b+ae^{nx}}\,dx = \frac{\frac{nx}{b} - \frac{\log\left(\left|ae^{(nx)}+b\right|\right)}{b}}{n}$$

```
[In] integrate(1/(b+a*exp(n*x)),x, algorithm="giac")
[Out] (n*x/b - log(abs(a*e^(n*x) + b))/b)/n
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{b+ae^{nx}}\,dx = -\frac{\ln\left(b+a\,\mathrm{e}^{n\,x}\right)-n\,x}{b\,n}$$

```
[In] int(1/(b + a*exp(n*x)),x)
[Out] -(log(b + a*exp(n*x)) - n*x)/(b*n)
```

3.523 $\int \frac{e^x}{b+ae^{3x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 100

$$\int \frac{e^x}{b+ae^{3x}} \, dx = -\frac{\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}e^x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}} + \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}e^x\right)}{2\sqrt[3]{a}b^{2/3}} - \frac{\log\left(b+ae^{3x}\right)}{6\sqrt[3]{a}b^{2/3}}$$

```
[Out] 1/2*ln(b^(1/3)+a^(1/3)*exp(x))/a^(1/3)/b^(2/3)-1/6*ln(b+a*exp(3*x))/a^(1/3)
/b^(2/3)-1/3*arctan(1/3*(b^(1/3)-2*a^(1/3)*exp(x))/b^(1/3)*3^(1/2))/a^(1/3)
/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2281, 206, 31, 648, 631, 210, 642}

$$\int \frac{e^x}{b+ae^{3x}} \, dx = -\frac{\log\left(a^{2/3}e^{2x}-\sqrt[3]{a}\sqrt[3]{b}e^x+b^{2/3}\right)}{6\sqrt[3]{a}b^{2/3}} - \frac{\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}e^x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}} + \frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

```
[In] Int[E^x/(b + a*E^(3*x)),x]
```

```
[Out] -(ArcTan[(b^(1/3) - 2*a^(1/3)*E^x)/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(1/3)*b^(2
/3))) + Log[b^(1/3) + a^(1/3)*E^x]/(3*a^(1/3)*b^(2/3)) - Log[b^(2/3) - a^(1
/3)*b^(1/3)*E^x + a^(2/3)*E^(2*x)]/(6*a^(1/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{b+ax^3}\,dx, x, e^x\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx, x, e^x\right)}{3b^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, e^x\right)}{3b^{2/3}}$$

$$= \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}e^x\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, e^x\right)}{6\sqrt[3]{a}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, e^x\right)}{2\sqrt[3]{b}}$$

$$= \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}e^x\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}e^x+a^{2/3}e^{2x}\right)}{6\sqrt[3]{a}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}b^{2/3}}$$

$$= -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}} + \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}e^x\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}e^x+a^{2/3}e^{2x}\right)}{6\sqrt[3]{a}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int \frac{e^x}{b+ae^{3x}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{b}+\sqrt[3]{a}e^x\right) + \log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}e^x+a^{2/3}e^{2x}\right)}{6\sqrt[3]{a}b^{2/3}}$$

```
[In] Integrate[E^x/(b + a*E^(3*x)),x]
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*E^x)/b^(1/3))/Sqrt[3]] - 2*Log[b^(1/
3) + a^(1/3)*E^x] + Log[b^(2/3) - a^(1/3)*b^(1/3)*E^x + a^(2/3)*E^(2*x)])/(
a^(1/3)*b^(2/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.26

method	result	size
risch	$\sum_{\_R=\mathrm{RootOf}\left(27b^{2}a\_Z^{3}-1\right)}\_R\ln\left(3b\_R+\mathrm{e}^{x}\right)$	26
default	$\frac{\ln\left(\mathrm{e}^{x}+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\mathrm{e}^{2x}-\left(\frac{b}{a}\right)^{\frac{1}{3}}\mathrm{e}^{x}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\,\mathrm{e}^{x}}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$	95

```
[In] int(exp(x)/(b+a*exp(3*x)),x,method=_RETURNVERBOSE)
```

```
[Out] sum(_R*ln(3*b*_R+exp(x)),_R=RootOf(27*_Z^3*a*b^2-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.11

$$\int \frac{e^x}{b+ae^{3x}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,abe^{(3\,x)}-3\,(ab^2)^{\frac{1}{3}}be^x-b^2+3\sqrt{\frac{1}{3}}\left(2\,abe^{(2\,x)}+(ab^2)^{\frac{2}{3}}e^x-(ab^2)^{\frac{1}{3}}b\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}}{ae^{(3\,x)}+b}\right)-(ab^2)^{\frac{2}{3}}\log\left(abe^{(2}\right.}{6\,ab^2}\right.$$

```
[In] integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a*b^2)^(1/3)/a)*log((2*a*b*e^(3*x) - 3*(a*b^2)
^(1/3)*b*e^x - b^2 + 3*sqrt(1/3)*(2*a*b*e^(2*x) + (a*b^2)^(2/3)*e^x - (a*b^
2)^(1/3)*b)*sqrt(-(a*b^2)^(1/3)/a))/(a*e^(3*x) + b)) - (a*b^2)^(2/3)*log(a*
b*e^(2*x) - (a*b^2)^(2/3)*e^x + (a*b^2)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(a*b*
e^x + (a*b^2)^(2/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt((a*b^2)^(1/3)/a)*a
rctan(sqrt(1/3)*(2*(a*b^2)^(2/3)*e^x - (a*b^2)^(1/3)*b)*sqrt((a*b^2)^(1/3)/
a)/b^2) - (a*b^2)^(2/3)*log(a*b*e^(2*x) - (a*b^2)^(2/3)*e^x + (a*b^2)^(1/3)
*b) + 2*(a*b^2)^(2/3)*log(a*b*e^x + (a*b^2)^(2/3)))/(a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.22

$$\int \frac{e^x}{b+ae^{3x}}\,dx = \text{RootSum}\left(27z^3ab^2-1,\left(i\mapsto i\log\left(3ib+e^x\right)\right)\right)$$

```
[In] integrate(exp(x)/(b+a*exp(3*x)),x)
[Out] RootSum(27*_z**3*a*b**2 - 1, Lambda(_i, _i*log(3*_i*b + exp(x))))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{b+ae^{3x}}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\,e^x\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\log\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}}e^x+\left(\frac{b}{a}\right)^{\frac{2}{3}}+e^{(2\,x)}\right)}{6\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{\log\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}+e^x\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

```
[In] integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*((b/a)^(1/3) - 2*e^x)/(b/a)^(1/3))/(a*(b/a)
^(2/3)) - 1/6*log(-(b/a)^(1/3)*e^x + (b/a)^(2/3) + e^(2*x))/(a*(b/a)^(2/3))
 + 1/3*log((b/a)^(1/3) + e^x)/(a*(b/a)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \frac{e^x}{b+ae^{3x}}\,dx = -\frac{\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{b}{a}\right)^{\frac{1}{3}}+e^x\right|\right)}{3\,b} + \frac{\sqrt{3}(-a^2b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{b}{a}\right)^{\frac{1}{3}}+2\,e^x\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,ab} + \frac{(-a^2b)^{\frac{1}{3}}\log\left(\left(-\frac{b}{a}\right)^{\frac{1}{3}}e^x+\left(-\frac{b}{a}\right)^{\frac{2}{3}}+e^{(2\,x)}\right)}{6\,ab}$$

```
[In] integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="giac")
```

```
[Out] -1/3*(-b/a)^(1/3)*log(abs(-(-b/a)^(1/3) + e^x))/b + 1/3*sqrt(3)*(-a^2*b)^(1
/3)*arctan(1/3*sqrt(3)*((-b/a)^(1/3) + 2*e^x)/(-b/a)^(1/3))/(a*b) + 1/6*(-a
^2*b)^(1/3)*log((-b/a)^(1/3)*e^x + (-b/a)^(2/3) + e^(2*x))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{e^x}{b+ae^{3x}}\,dx = \frac{\ln\left(\frac{b^{1/3}}{a^{7/3}}+\frac{e^x}{a^2}\right)}{3\,a^{1/3}\,b^{2/3}} + \frac{\ln\left(\frac{e^x}{a^2}+\frac{b^{1/3}\left(-1+\sqrt{3}\,1i\right)}{2\,a^{7/3}}\right)\,\left(-1+\sqrt{3}\,1i\right)}{6\,a^{1/3}\,b^{2/3}} - \frac{\ln\left(\frac{e^x}{a^2}-\frac{b^{1/3}\left(1+\sqrt{3}\,1i\right)}{2\,a^{7/3}}\right)\,\left(1+\sqrt{3}\,1i\right)}{6\,a^{1/3}\,b^{2/3}}$$

```
[In] int(exp(x)/(b + a*exp(3*x)),x)
```

```
[Out] log(b^(1/3)/a^(7/3) + exp(x)/a^2)/(3*a^(1/3)*b^(2/3)) + (log(exp(x)/a^2 + (
b^(1/3)*(3^(1/2)*1i - 1))/(2*a^(7/3)))*(3^(1/2)*1i - 1))/(6*a^(1/3)*b^(2/3)
) - (log(exp(x)/a^2 - (b^(1/3)*(3^(1/2)*1i + 1))/(2*a^(7/3)))*(3^(1/2)*1i +
 1))/(6*a^(1/3)*b^(2/3))
```

3.524 $\int \frac{-1+e^x}{1+e^x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{-1+e^x}{1+e^x} \, dx = -x + 2\log(1+e^x)$$

[Out] -x+2*ln(1+exp(x))

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2320, 78}

$$\int \frac{-1+e^x}{1+e^x} \, dx = 2\log(e^x+1) - x$$

[In] Int[(-1 + E^x)/(1 + E^x),x]

[Out] -x + 2*Log[1 + E^x]

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{-1+x}{x(1+x)}\,dx, x, e^x\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{x}+\frac{2}{1+x}\right)\,dx, x, e^x\right)\\ &= -x+2\log\left(1+e^x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{-1+e^x}{1+e^x}\,dx = -\log\left(e^x\right)+2\log\left(1+e^x\right)$$

```
[In] Integrate[(-1 + E^x)/(1 + E^x),x]
[Out] -Log[E^x] + 2*Log[1 + E^x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

method	result	size
norman	$-x+2\ln\left(1+\mathrm{e}^{x}\right)$	12
risch	$-x+2\ln\left(1+\mathrm{e}^{x}\right)$	12
parallelrisch	$-x+2\ln\left(1+\mathrm{e}^{x}\right)$	12
derivativedivides	$2\ln\left(1+\mathrm{e}^{x}\right)-\ln\left(\mathrm{e}^{x}\right)$	14
default	$2\ln\left(1+\mathrm{e}^{x}\right)-\ln\left(\mathrm{e}^{x}\right)$	14

```
[In] int((-1+exp(x))/(1+exp(x)),x,method=_RETURNVERBOSE)
[Out] -x+2*ln(1+exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{-1+e^x}{1+e^x} \, dx = -x + 2 \log\left(e^x + 1\right)$$

```
[In] integrate((-1+exp(x))/(1+exp(x)),x, algorithm="fricas")
[Out] -x + 2*log(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{-1+e^x}{1+e^x} \, dx = -x + 2\log\left(e^x + 1\right)$$

```
[In] integrate((-1+exp(x))/(1+exp(x)),x)
[Out] -x + 2*log(exp(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{-1+e^x}{1+e^x} \, dx = -x + 2 \log\left(e^x + 1\right)$$

```
[In] integrate((-1+exp(x))/(1+exp(x)),x, algorithm="maxima")
[Out] -x + 2*log(e^x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{-1+e^x}{1+e^x} \, dx = -x + 2 \log\left(e^x + 1\right)$$

```
[In] integrate((-1+exp(x))/(1+exp(x)),x, algorithm="giac")
[Out] -x + 2*log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{-1+e^x}{1+e^x} \, dx = 2 \ln \left(\mathrm{e}^x+1\right)-x$$

```
[In] int((exp(x) - 1)/(exp(x) + 1),x)
[Out] 2*log(exp(x) + 1) - x
```

3.525 $\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx=-\frac{\arctan\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{1}{12}\log\left(1-2e^{2x}+3e^{4x}\right)$$

```
[Out] 1/12*ln(1-2*exp(2*x)+3*exp(4*x))-1/12*arctan(1/2*(1-3*exp(2*x))*2^(1/2))*2^
(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2320, 648, 632, 210, 642}

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx=\frac{1}{12}\log\left(-2e^{2x}+3e^{4x}+1\right)-\frac{\arctan\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)}{6\sqrt{2}}$$

```
[In] Int[E^(4*x)/(1 - 2*E^(2*x) + 3*E^(4*x)),x]
```

```
[Out] -1/6*ArcTan[(1 - 3*E^(2*x))/Sqrt[2]]/Sqrt[2] + Log[1 - 2*E^(2*x) + 3*E^(4*x
)]/12
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{1-2x+3x^2}\,dx, x, e^{2x}\right)\\
&= \frac{1}{12}\text{Subst}\left(\int \frac{-2+6x}{1-2x+3x^2}\,dx, x, e^{2x}\right)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx, x, e^{2x}\right)\\
&= \frac{1}{12}\log\left(1-2e^{2x}+3e^{4x}\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, -2+6e^{2x}\right)\\
&= -\frac{\arctan\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{1}{12}\log\left(1-2e^{2x}+3e^{4x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = \frac{1}{12}\left(-\sqrt{2}\arctan\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)+\log\left(1-2e^{2x}+3e^{4x}\right)\right)$$

```
[In] Integrate[E^(4*x)/(1 - 2*E^(2*x) + 3*E^(4*x)),x]
[Out] (-(Sqrt[2]*ArcTan[(1 - 3*E^(2*x))/Sqrt[2]]) + Log[1 - 2*E^(2*x) + 3*E^(4*x)
])/12
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\ln(1-2\,\mathrm{e}^{2x}+3\,\mathrm{e}^{4x})}{12}+\frac{\sqrt{2}\,\arctan\left(\frac{(6\,\mathrm{e}^{2x}-2)\sqrt{2}}{4}\right)}{12}$	38
risch	$\frac{\ln\left(\mathrm{e}^{2x}-\frac{1}{3}+\frac{i\sqrt{2}}{3}\right)}{12}+\frac{i\ln\left(\mathrm{e}^{2x}-\frac{1}{3}+\frac{i\sqrt{2}}{3}\right)\sqrt{2}}{24}+\frac{\ln\left(\mathrm{e}^{2x}-\frac{1}{3}-\frac{i\sqrt{2}}{3}\right)}{12}-\frac{i\ln\left(\mathrm{e}^{2x}-\frac{1}{3}-\frac{i\sqrt{2}}{3}\right)\sqrt{2}}{24}$	70

```
[In] int(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x,method=_RETURNVERBOSE)
[Out] 1/12*ln(1-2*exp(x)^2+3*exp(x)^4)+1/12*2^(1/2)*arctan(1/4*(6*exp(x)^2-2)*2^(
1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = \frac{1}{12}\sqrt{2}\arctan\left(\frac{3}{2}\sqrt{2}e^{(2x)}-\frac{1}{2}\sqrt{2}\right)+\frac{1}{12}\log\left(3\,e^{(4\,x)}-2\,e^{(2\,x)}+1\right)$$

```
[In] integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="fricas")
[Out] 1/12*sqrt(2)*arctan(3/2*sqrt(2)*e^(2*x) - 1/2*sqrt(2)) + 1/12*log(3*e^(4*x)
 - 2*e^(2*x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.47

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = \operatorname{RootSum}\left(96z^2-16z+1,\left(i\mapsto i\log\left(8i+e^{2x}-1\right)\right)\right)$$

```
[In] integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x)
[Out] RootSum(96*_z**2 - 16*_z + 1, Lambda(_i, _i*log(8*_i + exp(2*x) - 1)))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = \frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(3e^{(2x)}-1\right)\right)+\frac{1}{12}\log\left(3e^{(4x)}-2e^{(2x)}+1\right)$$

```
[In] integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="maxima")
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*e^(2*x) - 1)) + 1/12*log(3*e^(4*x) - 2*e
^(2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = \frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(3e^{(2x)}-1\right)\right)+\frac{1}{12}\log\left(3e^{(4x)}-2e^{(2x)}+1\right)$$

```
[In] integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="giac")
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*e^(2*x) - 1)) + 1/12*log(3*e^(4*x) - 2*e
^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = \frac{\ln\left(3\,\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1\right)}{12}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}}{2}-\frac{3\,\sqrt{2}\,\mathrm{e}^{2\,x}}{2}\right)}{12}$$

```
[In] int(exp(4*x)/(3*exp(4*x) - 2*exp(2*x) + 1),x)
[Out] log(3*exp(4*x) - 2*exp(2*x) + 1)/12 - (2^(1/2)*atan(2^(1/2)/2 - (3*2^(1/2)*
exp(2*x))/2))/12
```

3.526 $\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 39

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx = e^x+\frac{e^{2x}}{2}-\arctan\left(e^x\right)+\log\left(1-e^x\right)-\frac{1}{2}\log\left(1+e^{2x}\right)$$

```
[Out] exp(x)+1/2*exp(2*x)-arctan(exp(x))+ln(1-exp(x))-1/2*ln(1+exp(2*x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2320, 2099, 649, 209, 266}

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx = -\arctan\left(e^x\right)+e^x+\frac{e^{2x}}{2}+\log\left(1-e^x\right)-\frac{1}{2}\log\left(e^{2x}+1\right)$$

```
[In] Int[(E^x + E^(5*x))/(-1 + E^x - E^(2*x) + E^(3*x)),x]
```

```
[Out] E^x + E^(2*x)/2 - ArcTan[E^x] + Log[1 - E^x] - Log[1 + E^(2*x)]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{-1-x^4}{1-x+x^2-x^3}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \left(1+\frac{1}{-1+x}+x+\frac{-1-x}{1+x^2}\right)\,dx, x, e^x\right)\\
&= e^x+\frac{e^{2x}}{2}+\log\left(1-e^x\right)+\text{Subst}\left(\int \frac{-1-x}{1+x^2}\,dx, x, e^x\right)\\
&= e^x+\frac{e^{2x}}{2}+\log\left(1-e^x\right)-\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)-\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^x\right)\\
&= e^x+\frac{e^{2x}}{2}-\arctan\left(e^x\right)+\log\left(1-e^x\right)-\frac{1}{2}\log\left(1+e^{2x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx = e^x+\frac{e^{2x}}{2}-\arctan\left(e^x\right)+\log\left(-1+e^x\right)-\frac{1}{2}\log\left(1+e^{2x}\right)$$

```
[In] Integrate[(E^x + E^(5*x))/(-1 + E^x - E^(2*x) + E^(3*x)),x]
[Out] E^x + E^(2*x)/2 - ArcTan[E^x] + Log[-1 + E^x] - Log[1 + E^(2*x)]/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{\ln(1+{\mathrm e}^{2 x})}{2}-\arctan\left({\mathrm e}^{x}\right)+\ln\left(-1+{\mathrm e}^{x}\right)+\frac{{\mathrm e}^{2 x}}{2}+{\mathrm e}^{x}$	29
risch	$\frac{{\mathrm e}^{2 x}}{2}+{\mathrm e}^{x}-\frac{\ln({\mathrm e}^{x}-i)}{2}+\frac{i \ln({\mathrm e}^{x}-i)}{2}-\frac{\ln({\mathrm e}^{x}+i)}{2}-\frac{i \ln({\mathrm e}^{x}+i)}{2}+\ln\left(-1+{\mathrm e}^{x}\right)$	49

```
[In] int((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/2*ln(1+exp(x)^2)-arctan(exp(x))+ln(-1+exp(x))+1/2*exp(x)^2+exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx=-\arctan\left(e^x\right)+\frac{1}{2}\,e^{(2\,x)}+e^x-\frac{1}{2}\,\log\left(e^{(2\,x)}+1\right)+\log\left(e^x-1\right)$$

```
[In] integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="fri
cas")
```

```
[Out] -arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(e^x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx=\frac{e^{2x}}{2}+e^x+\log\left(e^x-1\right)+\operatorname{RootSum}\left(2z^2+2z+1,\left(i\mapsto i\log\left(\frac{4i^2}{5}-\frac{6i}{5}+e^x-\frac{3}{5}\right)\right)\right)$$

```
[In] integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x)
```

```
[Out] exp(2*x)/2 + exp(x) + log(exp(x) - 1) + RootSum(2*_z**2 + 2*_z + 1, Lambda(
_i, _i*log(4*_i**2/5 - 6*_i/5 + exp(x) - 3/5)))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{e^x + e^{5x}}{-1 + e^x - e^{2x} + e^{3x}}\,dx = -\arctan\left(e^x\right) + \frac{1}{2}\,e^{(2\,x)} + e^x - \frac{1}{2}\,\log\left(e^{(2\,x)} + 1\right) + \log\left(e^x - 1\right)$$

```
[In] integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="max
ima")
```

```
[Out] -arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{e^x + e^{5x}}{-1 + e^x - e^{2x} + e^{3x}}\,dx = -\arctan\left(e^x\right) + \frac{1}{2}\,e^{(2\,x)} + e^x - \frac{1}{2}\,\log\left(e^{(2\,x)} + 1\right) + \log\left(|e^x - 1|\right)$$

```
[In] integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="gia
c")
```

```
[Out] -arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{e^x + e^{5x}}{-1 + e^x - e^{2x} + e^{3x}}\,dx = \frac{\mathrm{e}^{2\,x}}{2} - \frac{\ln\left(\mathrm{e}^{2\,x} + 1\right)}{2} - \mathrm{atan}(\mathrm{e}^x) + \ln\left(\mathrm{e}^x - 1\right) + \mathrm{e}^x$$

```
[In] int(-(exp(5*x) + exp(x))/(exp(2*x) - exp(3*x) - exp(x) + 1),x)
```

```
[Out] exp(2*x)/2 - log(exp(2*x) + 1)/2 - atan(exp(x)) + log(exp(x) - 1) + exp(x)
```

3.527 $\int e^{nx}(a+be^{nx})^{r/s}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 30

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\frac{(a+be^{nx})^{\frac{r+s}{s}}s}{bn(r+s)}$$

```
[Out] (a+b*exp(n*x))^((r+s)/s)*s/b/n/(r+s)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2278, 32}

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\frac{s(a+be^{nx})^{\frac{r+s}{s}}}{bn(r+s)}$$

```
[In] Int[E^(n*x)*(a + b*E^(n*x))^(r/s),x]
```

```
[Out] ((a + b*E^(n*x))^((r + s)/s)*s)/(b*n*(r + s))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
```

```
e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (a+bx)^{r/s}\,dx, x, e^{nx}\right)}{n}$$
$$= \frac{(a+be^{nx})^{\frac{r+s}{s}}\,s}{bn(r+s)}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx = \frac{(a+be^{nx})^{1+\frac{r}{s}}\,s}{bnr+bns}$$

```
[In] Integrate[E^(n*x)*(a + b*E^(n*x))^(r/s),x]
[Out] ((a + b*E^(n*x))^(1 + r/s)*s)/(b*n*r + b*n*s)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{(a+b\,e^{nx})^{\frac{r}{s}+1}}{nb\left(\frac{r}{s}+1\right)}$	33
default	$\frac{(a+b\,e^{nx})^{\frac{r}{s}+1}}{nb\left(\frac{r}{s}+1\right)}$	33
risch	$\frac{s(a+b\,e^{nx})(a+b\,e^{nx})^{\frac{r}{s}}}{bn(r+s)}$	36
parallelrisch	$\frac{e^{nx}(a+b\,e^{nx})^{\frac{r}{s}}bs+(a+b\,e^{nx})^{\frac{r}{s}}as}{bn(r+s)}$	52
norman	$\frac{s\,e^{nx}e^{\frac{r\ln(a+b\,e^{nx})}{s}}}{n(r+s)}+\frac{as\,e^{\frac{r\ln(a+b\,e^{nx})}{s}}}{bn(r+s)}$	60

```
[In] int(exp(n*x)*(a+b*exp(n*x))^(r/s),x,method=_RETURNVERBOSE)
[Out] 1/n*(a+b*exp(n*x))^(r/s+1)/b/(r/s+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\frac{\left(bse^{(nx)}+as\right)\left(be^{(nx)}+a\right)^{\frac{r}{s}}}{bnr+bns}$$

```
[In] integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="fricas")
[Out] (b*s*e^(n*x) + a*s)*(b*e^(n*x) + a)^(r/s)/(b*n*r + b*n*s)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(20) = 40.

Time = 0.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.13

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\begin{cases}\frac{x}{a} & \text{for } b=0\wedge n=0\wedge r=-s\\ \frac{a^{\frac{r}{s}}e^{nx}}{n} & \text{for } b=0\\ x(a+b)^{\frac{r}{s}} & \text{for } n=0\\ \frac{\log\left(\frac{a}{b}+e^{nx}\right)}{bn} & \text{for } r=-s\\ \frac{as(a+be^{nx})^{\frac{r}{s}}}{bnr+bns}+\frac{bs(a+be^{nx})^{\frac{r}{s}}e^{nx}}{bnr+bns} & \text{otherwise}\end{cases}$$

```
[In] integrate(exp(n*x)*(a+b*exp(n*x))**(r/s),x)
[Out] Piecewise((x/a, Eq(b, 0) & Eq(n, 0) & Eq(r, -s)), (a**(r/s)*exp(n*x)/n, Eq(
b, 0)), (x*(a + b)**(r/s), Eq(n, 0)), (log(a/b + exp(n*x))/(b*n), Eq(r, -s)
), (a*s*(a + b*exp(n*x))**(r/s)/(b*n*r + b*n*s) + b*s*(a + b*exp(n*x))**(r/
s)*exp(n*x)/(b*n*r + b*n*s), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\frac{\left(be^{(nx)}+a\right)^{\frac{r}{s}+1}}{bn\left(\frac{r}{s}+1\right)}$$

```
[In] integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="maxima")
[Out] (b*e^(n*x) + a)^(r/s + 1)/(b*n*(r/s + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\frac{\left(be^{(nx)}+a\right)^{\frac{r}{s}+1}}{bn\left(\frac{r}{s}+1\right)}$$

```
[In] integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="giac")
[Out] (b*e^(n*x) + a)^(r/s + 1)/(b*n*(r/s + 1))
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\frac{s\,\left(a+b\,\mathrm{e}^{n\,x}\right)^{\frac{r}{s}+1}}{b\,n\,\left(r+s\right)}$$

```
[In] int(exp(n*x)*(a + b*exp(n*x))^(r/s),x)
[Out] (s*(a + b*exp(n*x))^(r/s + 1))/(b*n*(r + s))
```

3.528 $\int \sqrt[4]{1-2e^{x/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\sqrt[4]{1-2e^{x/3}} - 6\arctan\left(\sqrt[4]{1-2e^{x/3}}\right) - 6\text{arctanh}\left(\sqrt[4]{1-2e^{x/3}}\right)$$

```
[Out] 12*(1-2*exp(1/3*x))^(1/4)-6*arctan((1-2*exp(1/3*x))^(1/4))-6*arctanh((1-2*e
xp(1/3*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2320, 52, 65, 218, 212, 209}

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = -6\arctan\left(\sqrt[4]{1-2e^{x/3}}\right) - 6\text{arctanh}\left(\sqrt[4]{1-2e^{x/3}}\right) + 12\sqrt[4]{1-2e^{x/3}}$$

```
[In] Int[(1 - 2*E^(x/3))^(1/4),x]
```

```
[Out] 12*(1 - 2*E^(x/3))^(1/4) - 6*ArcTan[(1 - 2*E^(x/3))^(1/4)] - 6*ArcTanh[(1 -
 2*E^(x/3))^(1/4)]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{\sqrt[4]{1-2x}}{x}\,dx, x, e^{x/3}\right)\\
&= 12\sqrt[4]{1-2e^{x/3}} + 3\text{Subst}\left(\int \frac{1}{(1-2x)^{3/4}x}\,dx, x, e^{x/3}\right)\\
&= 12\sqrt[4]{1-2e^{x/3}} - 6\text{Subst}\left(\int \frac{1}{\frac{1}{2}-\frac{x^4}{2}}\,dx, x, \sqrt[4]{1-2e^{x/3}}\right)
\end{aligned}$$

$$= 12\sqrt[4]{1-2e^{x/3}} - 6\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1-2e^{x/3}}\right) - 6\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{1-2e^{x/3}}\right)$$

$$= 12\sqrt[4]{1-2e^{x/3}} - 6\arctan\left(\sqrt[4]{1-2e^{x/3}}\right) - 6\text{arctanh}\left(\sqrt[4]{1-2e^{x/3}}\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\sqrt[4]{1-2e^{x/3}} - 6\arctan\left(\sqrt[4]{1-2e^{x/3}}\right) - 6\text{arctanh}\left(\sqrt[4]{1-2e^{x/3}}\right)$$

```
[In] Integrate[(1 - 2*E^(x/3))^(1/4),x]
[Out] 12*(1 - 2*E^(x/3))^(1/4) - 6*ArcTan[(1 - 2*E^(x/3))^(1/4)] - 6*ArcTanh[(1 -
 2*E^(x/3))^(1/4)]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

method	result
derivativedivides	$12\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}+3\ln\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}-1\right)-3\ln\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}+1\right)-6\arctan\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)\right.$
default	$12\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}+3\ln\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}-1\right)-3\ln\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}+1\right)-6\arctan\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)\right.$

```
[In] int((1-2*exp(1/3*x))^(1/4),x,method=_RETURNVERBOSE)
[Out] 12*(1-2*exp(1/3*x))^(1/4)+3*ln((1-2*exp(1/3*x))^(1/4)-1)-3*ln((1-2*exp(1/3*
x))^(1/4)+1)-6*arctan((1-2*exp(1/3*x))^(1/4))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}} - 6\,\arctan\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}+1\right) + 3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}-1\right)$$

```
[In] integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="fricas")
[Out] 12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4)) - 3*log((-
2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log((-2*e^(1/3*x) + 1)^(1/4) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\sqrt[4]{1-2e^{\frac{x}{3}}} + 3\log\left(\sqrt[4]{1-2e^{\frac{x}{3}}}-1\right) - 3\log\left(\sqrt[4]{1-2e^{\frac{x}{3}}}+1\right) - 6\,\mathrm{atan}\left(\sqrt[4]{1-2e^{\frac{x}{3}}}\right)$$

```
[In] integrate((1-2*exp(1/3*x))**(1/4),x)
[Out] 12*(1 - 2*exp(x/3))**(1/4) + 3*log((1 - 2*exp(x/3))**(1/4) - 1) - 3*log((1
- 2*exp(x/3))**(1/4) + 1) - 6*atan((1 - 2*exp(x/3))**(1/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}} - 6\,\arctan\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}+1\right) + 3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}-1\right)$$

```
[In] integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="maxima")
[Out] 12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4)) - 3*log((-
2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log((-2*e^(1/3*x) + 1)^(1/4) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}} - 6\,\arctan\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}+1\right) + 3\,\log\left(\left|\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}-1\right|\right)$$

```
[In] integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="giac")
[Out] 12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4)) - 3*log((-
2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log(abs((-2*e^(1/3*x) + 1)^(1/4) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.61

$$\int \sqrt[4]{1-2e^{x/3}}\,dx=\frac{12\,{\left(2-4\,{\mathrm{e}}^{x/3}\right)}^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ \frac{{\mathrm{e}}^{-\frac{x}{3}}}{2}\right)}{{\left(2-{\mathrm{e}}^{-\frac{x}{3}}\right)}^{1/4}}$$

```
[In] int((1 - 2*exp(x/3))^(1/4),x)
```

```
[Out] (12*(2 - 4*exp(x/3))^(1/4)*hypergeom([-1/4, -1/4], 3/4, exp(-x/3)/2))/(2 -
exp(-x/3))^(1/4)
```

3.529 $\int (a+be^{nx})^{r/s}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int (a+be^{nx})^{r/s}\,dx=-\frac{(a+be^{nx})^{\frac{r+s}{s}}\,s\,\text{Hypergeometric2F1}\left(1,\frac{r+s}{s},2+\frac{r}{s},1+\frac{be^{nx}}{a}\right)}{an(r+s)}$$

```
[Out] -(a+b*exp(n*x))^((r+s)/s)*s*hypergeom([1, (r+s)/s],[2+r/s],1+b*exp(n*x)/a)/
a/n/(r+s)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2320, 67}

$$\int (a+be^{nx})^{r/s}\,dx=-\frac{s(a+be^{nx})^{\frac{r+s}{s}}\text{ Hypergeometric2F1}\left(1,\frac{r+s}{s},\frac{r}{s}+2,\frac{e^{nx}b}{a}+1\right)}{an(r+s)}$$

```
[In] Int[(a + b*E^(n*x))^(r/s),x]
```

```
[Out] -(((a + b*E^(n*x))^((r + s)/s)*s*Hypergeometric2F1[1, (r + s)/s, 2 + r/s, 1
 + (b*E^(n*x))/a])/(a*n*(r + s)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+bx)^{r/s}}{x}\,dx, x, e^{nx}\right)}{n}$$
$$= -\frac{(a+be^{nx})^{\frac{r+s}{s}}\,s\,\text{Hypergeometric2F1}\left(1, \frac{r+s}{s}, 2+\frac{r}{s}, 1+\frac{be^{nx}}{a}\right)}{an(r+s)}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int (a+be^{nx})^{r/s}\,dx = -\frac{(a+be^{nx})^{\frac{r+s}{s}}\,s\,\text{Hypergeometric2F1}\left(1, \frac{r+s}{s}, 2+\frac{r}{s}, 1+\frac{be^{nx}}{a}\right)}{an(r+s)}$$

```
[In] Integrate[(a + b*E^(n*x))^(r/s),x]
[Out] -(((a + b*E^(n*x))^((r + s)/s)*s*Hypergeometric2F1[1, (r + s)/s, 2 + r/s, 1
 + (b*E^(n*x))/a])/(a*n*(r + s)))
```

Maple [F]

$$\int (a+b\,e^{nx})^{\frac{r}{s}}\,dx$$

```
[In] int((a+b*exp(n*x))^(r/s),x)
[Out] int((a+b*exp(n*x))^(r/s),x)
```

Fricas [F]

$$\int (a+be^{nx})^{r/s}\,dx = \int \left(be^{(nx)}+a\right)^{\frac{r}{s}}\,dx$$

```
[In] integrate((a+b*exp(n*x))^(r/s),x, algorithm="fricas")
[Out] integral((b*e^(n*x) + a)^(r/s), x)
```

Sympy [F]

$$\int (a+be^{nx})^{r/s}\,dx=\int (a+be^{nx})^{\frac{r}{s}}\,dx$$

```
[In] integrate((a+b*exp(n*x))**(r/s),x)
[Out] Integral((a + b*exp(n*x))**(r/s), x)
```

Maxima [F]

$$\int (a+be^{nx})^{r/s}\,dx=\int \left(be^{(nx)}+a\right)^{\frac{r}{s}}\,dx$$

```
[In] integrate((a+b*exp(n*x))^(r/s),x, algorithm="maxima")
[Out] integrate((b*e^(n*x) + a)^(r/s), x)
```

Giac [F]

$$\int (a+be^{nx})^{r/s}\,dx=\int \left(be^{(nx)}+a\right)^{\frac{r}{s}}\,dx$$

```
[In] integrate((a+b*exp(n*x))^(r/s),x, algorithm="giac")
[Out] integrate((b*e^(n*x) + a)^(r/s), x)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int (a+be^{nx})^{r/s}\,dx=\frac{s\,(a+b\,e^{n\,x})^{r/s}\,{}_2\mathrm{F}_1\left(-\frac{r}{s},-\frac{r}{s};\ 1-\frac{r}{s};\ -\frac{a\,e^{-n\,x}}{b}\right)}{n\,r\,\left(\frac{a\,e^{-n\,x}}{b}+1\right)^{r/s}}$$

```
[In] int((a + b*exp(n*x))^(r/s),x)
[Out] (s*(a + b*exp(n*x))^(r/s)*hypergeom([-r/s, -r/s], 1 - r/s, -(a*exp(-n*x))/b
))/(n*r*((a*exp(-n*x))/b + 1)^(r/s))
```

3.530 $\int \frac{e^x}{\sqrt{a^2+e^{2x}}} dx$

Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}} dx = \operatorname{arctanh}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)$$

```
[Out] arctanh(exp(x)/(a^2+exp(2*x))^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2281, 223, 212}

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}} dx = \operatorname{arctanh}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)$$

```
[In] Int[E^x/Sqrt[a^2 + E^(2*x)],x]
```

```
[Out] ArcTanh[E^x/Sqrt[a^2 + E^(2*x)]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{a^2+x^2}}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{e^x}{\sqrt{a^2+e^{2x}}}\right)\\
&= \text{arctanh}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx = -\log\left(-e^x+\sqrt{a^2+e^{2x}}\right)$$

```
[In] Integrate[E^x/Sqrt[a^2 + E^(2*x)],x]
[Out] -Log[-E^x + Sqrt[a^2 + E^(2*x)]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\ln\left(\mathrm{e}^{x}+\sqrt{a^{2}+\mathrm{e}^{2x}}\right)$	15

```
[In] int(exp(x)/(a^2+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(exp(x)+(a^2+exp(x)^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx = -\log\left(\sqrt{a^2+e^{(2\,x)}}-e^x\right)$$

```
[In] integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="fricas")
[Out] -log(sqrt(a^2 + e^(2*x)) - e^x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(15) = 30.

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx = \begin{cases} \log\left(2\sqrt{a^2+e^{2x}}+2e^x\right) & \text{for } a^2 \neq 0 \\ \frac{e^x \log(e^x)}{\sqrt{e^{2x}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(x)/(a**2+exp(2*x))**(1/2),x)
[Out] Piecewise((log(2*sqrt(a**2 + exp(2*x)) + 2*exp(x)), Ne(a**2, 0)), (exp(x)*l
og(exp(x))/sqrt(exp(2*x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.39

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx = \operatorname{arsinh}\left(\frac{e^x}{a}\right)$$

```
[In] integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="maxima")
[Out] arcsinh(e^x/a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx = -\log\left(\sqrt{a^2+e^{(2x)}}-e^x\right)$$

```
[In] integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="giac")
[Out] -log(sqrt(a^2 + e^(2*x)) - e^x)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx = \ln\left(\mathrm{e}^x+\sqrt{a^2+\mathrm{e}^{2x}}\right)$$

```
[In] int(exp(x)/(exp(2*x) + a^2)^(1/2),x)
[Out] log(exp(x) + (exp(2*x) + a^2)^(1/2))
```

3.531 $\int \frac{e^x}{\sqrt{-a^2+e^{2x}}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}} \, dx = \operatorname{arctanh}\left(\frac{e^x}{\sqrt{-a^2+e^{2x}}}\right)$$

```
[Out] arctanh(exp(x)/(-a^2+exp(2*x))^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2281, 223, 212}

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}} \, dx = \operatorname{arctanh}\left(\frac{e^x}{\sqrt{e^{2x}-a^2}}\right)$$

```
[In] Int[E^x/Sqrt[-a^2 + E^(2*x)],x]
```

```
[Out] ArcTanh[E^x/Sqrt[-a^2 + E^(2*x)]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{-a^2+x^2}}\,dx, x, e^x\right)\\ &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{e^x}{\sqrt{-a^2+e^{2x}}}\right)\\ &= \operatorname{arctanh}\left(\frac{e^x}{\sqrt{-a^2+e^{2x}}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = -\log\left(-e^x+\sqrt{-a^2+e^{2x}}\right)$$

```
[In] Integrate[E^x/Sqrt[-a^2 + E^(2*x)],x]
[Out] -Log[-E^x + Sqrt[-a^2 + E^(2*x)]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
default	$\ln\left(\mathrm{e}^x+\sqrt{-a^2+\mathrm{e}^{2x}}\right)$	17

```
[In] int(exp(x)/(-a^2+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(exp(x)+(-a^2+exp(x)^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = -\log\left(\sqrt{-a^2+e^{(2\,x)}}-e^x\right)$$

```
[In] integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="fricas")
[Out] -log(sqrt(-a^2 + e^(2*x)) - e^x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = \begin{cases} \log\left(2\sqrt{-a^2+e^{2x}}+2e^x\right) & \text{for}\ a^2\neq 0 \\ \frac{e^x\log\left(e^x\right)}{\sqrt{e^{2x}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(x)/(-a**2+exp(2*x))**(1/2),x)
[Out] Piecewise((log(2*sqrt(-a**2 + exp(2*x)) + 2*exp(x)), Ne(a**2, 0)), (exp(x)*
log(exp(x))/sqrt(exp(2*x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = \log\left(2\,\sqrt{-a^2+e^{(2\,x)}}+2\,e^x\right)$$

```
[In] integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="maxima")
[Out] log(2*sqrt(-a^2 + e^(2*x)) + 2*e^x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = -\log\left(-\sqrt{-a^2+e^{(2x)}}+e^x\right)$$

```
[In] integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="giac")
[Out] -log(-sqrt(-a^2 + e^(2*x)) + e^x)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = \ln\left(\mathrm{e}^x+\sqrt{\mathrm{e}^{2x}-a^2}\right)$$

```
[In] int(exp(x)/(exp(2*x) - a^2)^(1/2),x)
[Out] log(exp(x) + (exp(2*x) - a^2)^(1/2))
```

3.532 $\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 40

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = \frac{2}{3}\text{arctanh}\left(\frac{2-5e^{3x/4}}{4\sqrt{-2+e^{3x/4}+e^{3x/2}}}\right)$$

```
[Out] 2/3*arctanh(1/4*(2-5*exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2320, 738, 212}

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = \frac{2}{3}\text{arctanh}\left(\frac{2-5e^{3x/4}}{4\sqrt{e^{3x/4}+e^{3x/2}-2}}\right)$$

```
[In] Int[E^((3*x)/4)/((-2 + E^((3*x)/4))*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)]),x
]
```

```
[Out] (2*ArcTanh[(2 - 5*E^((3*x)/4))/(4*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)])])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{4}{3}\text{Subst}\left(\int \frac{1}{(-2+x)\sqrt{-2+x+x^2}}\,dx, x, e^{3x/4}\right) \\ &= -\left(\frac{8}{3}\text{Subst}\left(\int \frac{1}{16-x^2}\,dx, x, \frac{-2+5e^{3x/4}}{\sqrt{-2+e^{3x/4}+e^{3x/2}}}\right)\right) \\ &= \frac{2}{3}\text{arctanh}\left(\frac{2-5e^{3x/4}}{4\sqrt{-2+e^{3x/4}+e^{3x/2}}}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = -\frac{4}{3}\text{arctanh}\left(1-\frac{1}{2}e^{3x/4}+\frac{1}{2}\sqrt{-2+e^{3x/4}+e^{3x/2}}\right)$$

```
[In] Integrate[E^((3*x)/4)/((-2 + E^((3*x)/4))*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/
2)]),x]
[Out] (-4*ArcTanh[1 - E^((3*x)/4)/2 + Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)]/2])/3
```

Maple [F]

$$\int \frac{e^{\frac{3x}{4}}}{\left(-2+e^{\frac{3x}{4}}\right)\sqrt{-2+e^{\frac{3x}{4}}+e^{\frac{3x}{2}}}}dx$$

```
[In] int(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x)
[Out] int(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = -\frac{2}{3}\log\left(\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}-e^{\left(\frac{3}{4}x\right)}+4\right)+\frac{2}{3}\log\left(\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}-e^{\left(\frac{3}{4}x\right)}\right)$$

```
[In] integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, al
gorithm="fricas")
[Out] -2/3*log(sqrt(e^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x) + 4) + 2/3*log(sqrt(e^
(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x))
```

Sympy [F]

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = \int \frac{e^{\frac{3x}{4}}}{\left(e^{\frac{3x}{4}}-2\right)\sqrt{e^{\frac{3x}{4}}+e^{\frac{3x}{2}}-2}}\,dx$$

```
[In] integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))**(1/2),x)
[Out] Integral(exp(3*x/4)/((exp(3*x/4) - 2)*sqrt(exp(3*x/4) + exp(3*x/2) - 2)), x
)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = -\frac{2}{3}\log\left(\frac{4\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}}{\left|e^{\left(\frac{3}{4}x\right)}-2\right|}+\frac{8}{\left|e^{\left(\frac{3}{4}x\right)}-2\right|}+5\right)$$

```
[In] integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, al
gorithm="maxima")
[Out] -2/3*log(4*sqrt(e^(3/2*x) + e^(3/4*x) - 2)/abs(e^(3/4*x) - 2) + 8/abs(e^(3/
4*x) - 2) + 5)
```

Giac [F]

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx=\int \frac{e^{\left(\frac{3}{4}\,x\right)}}{\sqrt{e^{\left(\frac{3}{2}\,x\right)}+e^{\left(\frac{3}{4}\,x\right)}-2}\left(e^{\left(\frac{3}{4}\,x\right)}-2\right)}\,dx$$

```
[In] integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, al
gorithm="giac")
```

```
[Out] integrate(e^(3/4*x)/(sqrt(e^(3/2*x) + e^(3/4*x) - 2)*(e^(3/4*x) - 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx=\int \frac{\mathrm{e}^{\frac{3\,x}{4}}}{\left(\mathrm{e}^{\frac{3\,x}{4}}-2\right)\,\sqrt{\mathrm{e}^{\frac{3\,x}{2}}+\mathrm{e}^{\frac{3\,x}{4}}-2}}\,dx$$

```
[In] int(exp((3*x)/4)/((exp((3*x)/4) - 2)*(exp((3*x)/2) + exp((3*x)/4) - 2)^(1/2
)),x)
```

```
[Out] int(exp((3*x)/4)/((exp((3*x)/4) - 2)*(exp((3*x)/2) + exp((3*x)/4) - 2)^(1/2
)), x)
```

3.533 $\int e^{-2x}(-3+e^{7x})^{2/3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\frac{1}{6}e^{-2x}\left(-3+e^{7x}\right)^{5/3}\text{Hypergeometric2F1}\left(1,\frac{29}{21},\frac{5}{7},\frac{e^{7x}}{3}\right)$$

```
[Out] 1/6*(-3+exp(7*x))^(5/3)*hypergeom([1, 29/21],[5/7],1/3*exp(7*x))/exp(2*x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2281, 342, 372, 371}

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=-\frac{3^{2/3}e^{-2x}\left(e^{7x}-3\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{2}{7},\frac{5}{7},\frac{e^{7x}}{3}\right)}{2\left(3-e^{7x}\right)^{2/3}}$$

```
[In] Int[(-3 + E^(7*x))^(2/3)/E^(2*x),x]
```

```
[Out] -1/2*(3^(2/3)*(-3 + E^(7*x))^(2/3)*Hypergeometric2F1[-2/3, -2/7, 5/7, E^(7*
x)/3])/(E^(2*x)*(3 - E^(7*x))^(2/3))
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \left(-3+\frac{1}{x^7}\right)^{2/3} x\,dx, x, e^{-x}\right)\\
&= \text{Subst}\left(\int \frac{(-3+x^7)^{2/3}}{x^3}\,dx, x, e^x\right)\\
&= \frac{(-3+e^{7x})^{2/3}\,\text{Subst}\left(\int \frac{\left(1-\frac{x^7}{3}\right)^{2/3}}{x^3}\,dx, x, e^x\right)}{\left(1-\frac{e^{7x}}{3}\right)^{2/3}}\\
&= -\frac{3^{2/3}e^{-2x}(-3+e^{7x})^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{2}{7},\frac{5}{7},\frac{e^{7x}}{3}\right)}{2\,(3-e^{7x})^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=-\frac{e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{2}{7},\frac{5}{7},\frac{e^{7x}}{3}\right)}{2\left(1-\frac{e^{7x}}{3}\right)^{2/3}}$$

```
[In] Integrate[(-3 + E^(7*x))^(2/3)/E^(2*x),x]
[Out] -1/2*((-3 + E^(7*x))^(2/3)*Hypergeometric2F1[-2/3, -2/7, 5/7, E^(7*x)/3])/(
E^(2*x)*(1 - E^(7*x)/3)^(2/3))
```

Maple [F]

$$\int \left(-3+\mathrm{e}^{7x}\right)^{\frac{2}{3}}\mathrm{e}^{-2x}dx$$

```
[In] int((-3+exp(7*x))^(2/3)/exp(2*x),x)
[Out] int((-3+exp(7*x))^(2/3)/exp(2*x),x)
```

Fricas [F]

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\int \left(e^{(7\,x)}-3\right)^{\frac{2}{3}}e^{(-2\,x)}\,dx$$

```
[In] integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="fricas")
[Out] integral((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

Sympy [F]

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\int \left(e^{7x}-3\right)^{\frac{2}{3}}e^{-2x}\,dx$$

```
[In] integrate((-3+exp(7*x))**(2/3)/exp(2*x),x)
[Out] Integral((exp(7*x) - 3)**(2/3)*exp(-2*x), x)
```

Maxima [F]

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\int\left(e^{(7\,x)}-3\right)^{\frac{2}{3}}e^{(-2\,x)}\,dx$$

```
[In] integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="maxima")
[Out] integrate((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

Giac [F]

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\int\left(e^{(7\,x)}-3\right)^{\frac{2}{3}}e^{(-2\,x)}\,dx$$

```
[In] integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="giac")
[Out] integrate((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\int \mathrm{e}^{-2\,x}\,\left(\mathrm{e}^{7\,x}-3\right)^{2/3}\,dx$$

```
[In] int(exp(-2*x)*(exp(7*x) - 3)^(2/3),x)
[Out] int(exp(-2*x)*(exp(7*x) - 3)^(2/3), x)
```

3.534 $\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{e^{2x}}{(3-e^{x/2})^{3/4}}\,dx = -216\sqrt[4]{3-e^{x/2}}+\frac{216}{5}\left(3-e^{x/2}\right)^{5/4}-8\left(3-e^{x/2}\right)^{9/4}+\frac{8}{13}\left(3-e^{x/2}\right)^{13/4}$$

```
[Out] -216*(3-exp(1/2*x))^(1/4)+216/5*(3-exp(1/2*x))^(5/4)-8*(3-exp(1/2*x))^(9/4)
+8/13*(3-exp(1/2*x))^(13/4)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2280, 45}

$$\int \frac{e^{2x}}{(3-e^{x/2})^{3/4}}\,dx = \frac{8}{13}\left(3-e^{x/2}\right)^{13/4}-8\left(3-e^{x/2}\right)^{9/4}+\frac{216}{5}\left(3-e^{x/2}\right)^{5/4}-216\sqrt[4]{3-e^{x/2}}$$

```
[In] Int[E^(2*x)/(3 - E^(x/2))^(3/4),x]
```

```
[Out] -216*(3 - E^(x/2))^(1/4) + (216*(3 - E^(x/2))^(5/4))/5 - 8*(3 - E^(x/2))^(9
/4) + (8*(3 - E^(x/2))^(13/4))/13
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2280

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Den
ominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{x^3}{(3-x)^{3/4}}\,dx, x, e^{x/2}\right)\\ &= 2\text{Subst}\left(\int \left(\frac{27}{(3-x)^{3/4}} - 27\sqrt[4]{3-x} + 9(3-x)^{5/4} - (3-x)^{9/4}\right) dx, x, e^{x/2}\right)\\ &= -216\sqrt[4]{3-e^{x/2}} + \frac{216}{5}\left(3-e^{x/2}\right)^{5/4} - 8\left(3-e^{x/2}\right)^{9/4} + \frac{8}{13}\left(3-e^{x/2}\right)^{13/4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx = -\frac{8}{65}\sqrt[4]{3-e^{x/2}}\left(1152+96e^{x/2}+20e^x+5e^{3x/2}\right)$$

```
[In] Integrate[E^(2*x)/(3 - E^(x/2))^(3/4),x]
[Out] (-8*(3 - E^(x/2))^(1/4)*(1152 + 96*E^(x/2) + 20*E^x + 5*E^((3*x)/2)))/65
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

method	result	size
risch	$\frac{8\left(5\,\mathrm{e}^{\frac{3x}{2}}+20\,\mathrm{e}^{x}+96\,\mathrm{e}^{\frac{x}{2}}+1152\right)\left(-3+\mathrm{e}^{\frac{x}{2}}\right)}{65\left(3-\mathrm{e}^{\frac{x}{2}}\right)^{\frac{3}{4}}}$	37

```
[In] int(exp(2*x)/(3-exp(1/2*x))^(3/4),x,method=_RETURNVERBOSE)
[Out] 8/65/(3-exp(1/2*x))^(3/4)*(5*exp(3/2*x)+20*exp(x)+96*exp(1/2*x)+1152)*(-3+e
xp(1/2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.41

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx=-\frac{8}{65}\left(5\,e^{\left(\frac{3}{2}\,x\right)}+96\,e^{\left(\frac{1}{2}\,x\right)}+20\,e^{x}+1152\right)\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{1}{4}}$$

```
[In] integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="fricas")
[Out] -8/65*(5*e^(3/2*x) + 96*e^(1/2*x) + 20*e^x + 1152)*(-e^(1/2*x) + 3)^(1/4)
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx=\frac{8\left(3-e^{\frac{x}{2}}\right)^{\frac{13}{4}}}{13}-8\left(3-e^{\frac{x}{2}}\right)^{\frac{9}{4}}+\frac{216\left(3-e^{\frac{x}{2}}\right)^{\frac{5}{4}}}{5}-216\sqrt[4]{3-e^{\frac{x}{2}}}$$

```
[In] integrate(exp(2*x)/(3-exp(1/2*x))**(3/4),x)
[Out] 8*(3 - exp(x/2))**(13/4)/13 - 8*(3 - exp(x/2))**(9/4) + 216*(3 - exp(x/2))*
*(5/4)/5 - 216*(3 - exp(x/2))**(1/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\begin{aligned}\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx&=\frac{8}{13}\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{13}{4}}-8\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{9}{4}}\\&\quad+\frac{216}{5}\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{5}{4}}-216\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{1}{4}}\end{aligned}$$

```
[In] integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="maxima")
[Out] 8/13*(-e^(1/2*x) + 3)^(13/4) - 8*(-e^(1/2*x) + 3)^(9/4) + 216/5*(-e^(1/2*x)
 + 3)^(5/4) - 216*(-e^(1/2*x) + 3)^(1/4)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx = -\frac{8}{13}\left(e^{\left(\frac{1}{2}x\right)}-3\right)^{3}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{1}{4}} - 8\left(e^{\left(\frac{1}{2}x\right)}-3\right)^{2}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{1}{4}} + \frac{216}{5}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{5}{4}} - 216\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{1}{4}}$$

```
[In] integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="giac")
```

```
[Out] -8/13*(e^(1/2*x) - 3)^3*(-e^(1/2*x) + 3)^(1/4) - 8*(e^(1/2*x) - 3)^2*(-e^(1
/2*x) + 3)^(1/4) + 216/5*(-e^(1/2*x) + 3)^(5/4) - 216*(-e^(1/2*x) + 3)^(1/4
)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.41

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx = -\left(3-\mathrm{e}^{x/2}\right)^{1/4}\left(\frac{768\,\mathrm{e}^{x/2}}{65}+\frac{8\,\mathrm{e}^{\frac{3\,x}{2}}}{13}+\frac{32\,\mathrm{e}^{x}}{13}+\frac{9216}{65}\right)$$

```
[In] int(exp(2*x)/(3 - exp(x/2))^(3/4),x)
```

```
[Out] -(3 - exp(x/2))^(1/4)*((768*exp(x/2))/65 + (8*exp((3*x)/2))/13 + (32*exp(x)
)/13 + 9216/65)
```

3.535 $\int e^{-x/2}x^3\,dx$

Optimal result

Integrand size = 11, antiderivative size = 44

$$\int e^{-x/2}x^3\,dx = -96e^{-x/2} - 48e^{-x/2}x - 12e^{-x/2}x^2 - 2e^{-x/2}x^3$$

```
[Out] -96/exp(1/2*x)-48*x/exp(1/2*x)-12*x^2/exp(1/2*x)-2*x^3/exp(1/2*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2207, 2225}

$$\int e^{-x/2}x^3\,dx = -2e^{-x/2}x^3 - 12e^{-x/2}x^2 - 48e^{-x/2}x - 96e^{-x/2}$$

```
[In] Int[x^3/E^(x/2),x]
```

```
[Out] -96/E^(x/2) - (48*x)/E^(x/2) - (12*x^2)/E^(x/2) - (2*x^3)/E^(x/2)
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -2e^{-x/2}x^3 + 6\int e^{-x/2}x^2\,dx\\ &= -12e^{-x/2}x^2 - 2e^{-x/2}x^3 + 24\int e^{-x/2}x\,dx\\ &= -48e^{-x/2}x - 12e^{-x/2}x^2 - 2e^{-x/2}x^3 + 48\int e^{-x/2}\,dx\\ &= -96e^{-x/2} - 48e^{-x/2}x - 12e^{-x/2}x^2 - 2e^{-x/2}x^3\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.52

$$\int e^{-x/2}x^3\,dx = e^{-x/2}\left(-96 - 48x - 12x^2 - 2x^3\right)$$

```
[In] Integrate[x^3/E^(x/2),x]
[Out] (-96 - 48*x - 12*x^2 - 2*x^3)/E^(x/2)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

method	result	size
risch	$\left(-2x^3-12x^2-48x-96\right)\mathrm{e}^{-\frac{x}{2}}$	21
gosper	$-2\left(x^3+6x^2+24x+48\right)\mathrm{e}^{-\frac{x}{2}}$	22
norman	$\left(-2x^3-12x^2-48x-96\right)\mathrm{e}^{-\frac{x}{2}}$	23
meijerg	$96-4\left(\frac{1}{2}x^3+3x^2+12x+24\right)\mathrm{e}^{-\frac{x}{2}}$	24
parallelrisch	$-\left(2x^3+12x^2+48x+96\right)\mathrm{e}^{-\frac{x}{2}}$	24
derivativedivides	$-96\,\mathrm{e}^{-\frac{x}{2}}-48x\,\mathrm{e}^{-\frac{x}{2}}-12x^2\mathrm{e}^{-\frac{x}{2}}-2x^3\mathrm{e}^{-\frac{x}{2}}$	41
default	$-96\,\mathrm{e}^{-\frac{x}{2}}-48x\,\mathrm{e}^{-\frac{x}{2}}-12x^2\mathrm{e}^{-\frac{x}{2}}-2x^3\mathrm{e}^{-\frac{x}{2}}$	41

```
[In] int(x^3/exp(1/2*x),x,method=_RETURNVERBOSE)
[Out] (-2*x^3-12*x^2-48*x-96)*exp(-1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.43

$$\int e^{-x/2}x^3\,dx = -2\left(x^3+6\,x^2+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

```
[In] integrate(x^3/exp(1/2*x),x, algorithm="fricas")
[Out] -2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.45

$$\int e^{-x/2}x^3\,dx = \left(-2x^3-12x^2-48x-96\right)e^{-\frac{x}{2}}$$

```
[In] integrate(x**3/exp(1/2*x),x)
[Out] (-2*x**3 - 12*x**2 - 48*x - 96)*exp(-x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.43

$$\int e^{-x/2}x^3\,dx = -2\left(x^3+6\,x^2+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

```
[In] integrate(x^3/exp(1/2*x),x, algorithm="maxima")
[Out] -2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.43

$$\int e^{-x/2}x^3\,dx = -2\left(x^3+6\,x^2+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

```
[In] integrate(x^3/exp(1/2*x),x, algorithm="giac")
[Out] -2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int e^{-x/2}x^3\,dx=-16\,\mathrm{e}^{-\frac{x}{2}}\,\left(\frac{x^3}{8}+\frac{3\,x^2}{4}+3\,x+6\right)$$

```
[In] int(x^3*exp(-x/2),x)
[Out] -16*exp(-x/2)*(3*x + (3*x^2)/4 + x^3/8 + 6)
```

3.536 $\int \frac{e^{-x/2}}{x^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{e^{-x/2}}{x^3} \, dx = -\frac{e^{-x/2}}{2x^2} + \frac{e^{-x/2}}{4x} + \frac{\text{ExpIntegralEi}\left(-\frac{x}{2}\right)}{8}$$

```
[Out] -1/2/exp(1/2*x)/x^2+1/4/exp(1/2*x)/x+1/8*Ei(-1/2*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2208, 2209}

$$\int \frac{e^{-x/2}}{x^3} \, dx = \frac{\text{ExpIntegralEi}\left(-\frac{x}{2}\right)}{8} - \frac{e^{-x/2}}{2x^2} + \frac{e^{-x/2}}{4x}$$

```
[In] Int[1/(E^(x/2)*x^3),x]
```

```
[Out] -1/2*1/(E^(x/2)*x^2) + 1/(4*E^(x/2)*x) + ExpIntegralEi[-1/2*x]/8
```

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{-x/2}}{2x^2} - \frac{1}{4}\int \frac{e^{-x/2}}{x^2}\,dx\\ &= -\frac{e^{-x/2}}{2x^2} + \frac{e^{-x/2}}{4x} + \frac{1}{8}\int \frac{e^{-x/2}}{x}\,dx\\ &= -\frac{e^{-x/2}}{2x^2} + \frac{e^{-x/2}}{4x} + \frac{\text{ExpIntegralEi}\left(-\frac{x}{2}\right)}{8}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int \frac{e^{-x/2}}{x^3}\,dx = \frac{1}{8}\left(\frac{2e^{-x/2}(-2+x)}{x^2} + \text{ExpIntegralEi}\left(-\frac{x}{2}\right)\right)$$

```
[In] Integrate[1/(E^(x/2)*x^3),x]
[Out] ((2*(-2 + x))/(E^(x/2)*x^2) + ExpIntegralEi[-1/2*x])/8
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{e^{-\frac{x}{2}}}{2x^2}+\frac{e^{-\frac{x}{2}}}{4x}-\frac{\text{Ei}_1\left(\frac{x}{2}\right)}{8}$	27
derivativedivides	$-\frac{e^{-\frac{x}{2}}}{2x^2}+\frac{e^{-\frac{x}{2}}}{4x}-\frac{\text{Ei}_1\left(\frac{x}{2}\right)}{8}$	31
default	$-\frac{e^{-\frac{x}{2}}}{2x^2}+\frac{e^{-\frac{x}{2}}}{4x}-\frac{\text{Ei}_1\left(\frac{x}{2}\right)}{8}$	31
meijerg	$-\frac{1}{2x^2}+\frac{1}{2x}-\frac{3}{16}+\frac{\ln(x)}{8}-\frac{\ln(2)}{8}+\frac{\frac{9}{4}x^2-6x+6}{12x^2}-\frac{\left(-\frac{3x}{2}+3\right)e^{-\frac{x}{2}}}{6x^2}-\frac{\ln\left(\frac{x}{2}\right)}{8}-\frac{\text{Ei}_1\left(\frac{x}{2}\right)}{8}$	63

```
[In] int(1/exp(1/2*x)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*exp(-1/2*x)/x^2+1/4*exp(-1/2*x)/x-1/8*Ei(1,1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.59

$$\int \frac{e^{-x/2}}{x^3}\,dx = \frac{x^2\mathrm{Ei}\left(-\frac{1}{2}\,x\right) + 2\,(x-2)e^{\left(-\frac{1}{2}\,x\right)}}{8\,x^2}$$

```
[In] integrate(1/exp(1/2*x)/x^3,x, algorithm="fricas")
[Out] 1/8*(x^2*Ei(-1/2*x) + 2*(x - 2)*e^(-1/2*x))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{e^{-x/2}}{x^3}\,dx = \frac{\mathrm{Ei}\left(\frac{xe^{i\pi}}{2}\right)}{8} + \frac{e^{-\frac{x}{2}}}{4x} - \frac{e^{-\frac{x}{2}}}{2x^2}$$

```
[In] integrate(1/exp(1/2*x)/x**3,x)
[Out] Ei(x*exp_polar(I*pi)/2)/8 + exp(-x/2)/(4*x) - exp(-x/2)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.18

$$\int \frac{e^{-x/2}}{x^3}\,dx = -\frac{1}{4}\,\Gamma\left(-2, \frac{1}{2}\,x\right)$$

```
[In] integrate(1/exp(1/2*x)/x^3,x, algorithm="maxima")
[Out] -1/4*gamma(-2, 1/2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{e^{-x/2}}{x^3}\,dx = \frac{x^2\mathrm{Ei}\left(-\frac{1}{2}\,x\right) + 2\,xe^{\left(-\frac{1}{2}\,x\right)} - 4\,e^{\left(-\frac{1}{2}\,x\right)}}{8\,x^2}$$

```
[In] integrate(1/exp(1/2*x)/x^3,x, algorithm="giac")
[Out] 1/8*(x^2*Ei(-1/2*x) + 2*x*e^(-1/2*x) - 4*e^(-1/2*x))/x^2
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.56

$$\int \frac{e^{-x/2}}{x^3}\,dx = \frac{\mathrm{e}^{-\frac{x}{2}}\left(\frac{1}{x}-\frac{2}{x^2}\right)}{4} - \frac{\mathrm{expint}\left(\frac{x}{2}\right)}{8}$$

```
[In] int(exp(-x/2)/x^3,x)
[Out] (exp(-x/2)*(1/x - 2/x^2))/4 - expint(x/2)/8
```

3.537 $\int a^{3x} x^2 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 44

$$\int a^{3x} x^2 \, dx = \frac{2a^{3x}}{27\log^3(a)} - \frac{2a^{3x}x}{9\log^2(a)} + \frac{a^{3x}x^2}{3\log(a)}$$

```
[Out] 2/27*a^(3*x)/ln(a)^3-2/9*a^(3*x)*x/ln(a)^2+1/3*a^(3*x)*x^2/ln(a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2207, 2225}

$$\int a^{3x} x^2 \, dx = \frac{x^2 a^{3x}}{3\log(a)} + \frac{2a^{3x}}{27\log^3(a)} - \frac{2xa^{3x}}{9\log^2(a)}$$

```
[In] Int[a^(3*x)*x^2,x]
```

```
[Out] (2*a^(3*x))/(27*Log[a]^3) - (2*a^(3*x)*x)/(9*Log[a]^2) + (a^(3*x)*x^2)/(3*L
og[a])
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{a^{3x}x^2}{3\log(a)} - \frac{2\int a^{3x}x\,dx}{3\log(a)} \\ &= -\frac{2a^{3x}x}{9\log^2(a)} + \frac{a^{3x}x^2}{3\log(a)} + \frac{2\int a^{3x}\,dx}{9\log^2(a)} \\ &= \frac{2a^{3x}}{27\log^3(a)} - \frac{2a^{3x}x}{9\log^2(a)} + \frac{a^{3x}x^2}{3\log(a)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int a^{3x}x^2\,dx = \frac{a^{3x}\left(2-6x\log(a)+9x^2\log^2(a)\right)}{27\log^3(a)}$$

```
[In] Integrate[a^(3*x)*x^2,x]
[Out] (a^(3*x)*(2 - 6*x*Log[a] + 9*x^2*Log[a]^2))/(27*Log[a]^3)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{\left(9x^2\ln(a)^2-6x\ln(a)+2\right)a^{3x}}{27\ln(a)^3}$	28
risch	$\frac{\left(9x^2\ln(a)^2-6x\ln(a)+2\right)a^{3x}}{27\ln(a)^3}$	28
meijerg	$-\frac{2-\frac{\left(27x^2\ln(a)^2-18x\ln(a)+6\right)e^{3x\ln(a)}}{3}}{27\ln(a)^3}$	33
parallelrisch	$\frac{9x^2a^{3x}\ln(a)^2-6x\,a^{3x}\ln(a)+2a^{3x}}{27\ln(a)^3}$	39
norman	$\frac{2\,e^{3x\ln(a)}}{27\ln(a)^3}-\frac{2x\,e^{3x\ln(a)}}{9\ln(a)^2}+\frac{x^2e^{3x\ln(a)}}{3\ln(a)}$	42

```
[In] int(a^(3*x)*x^2,x,method=_RETURNVERBOSE)
[Out] 1/27*(9*x^2*ln(a)^2-6*x*ln(a)+2)*a^(3*x)/ln(a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int a^{3x}x^2\,dx = \frac{\left(9\,x^2\log\left(a\right)^2 - 6\,x\log\left(a\right) + 2\right)a^{3\,x}}{27\,\log\left(a\right)^3}$$

```
[In] integrate(a^(3*x)*x^2,x, algorithm="fricas")
[Out] 1/27*(9*x^2*log(a)^2 - 6*x*log(a) + 2)*a^(3*x)/log(a)^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int a^{3x}x^2\,dx = \begin{cases} \frac{a^{3x}\left(9x^2\log\left(a\right)^2 - 6x\log\left(a\right) + 2\right)}{27\log\left(a\right)^3} & \text{for}\ \log\left(a\right)^3 \neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

```
[In] integrate(a**(3*x)*x**2,x)
[Out] Piecewise((a**(3*x)*(9*x**2*log(a)**2 - 6*x*log(a) + 2)/(27*log(a)**3), Ne(
log(a)**3, 0)), (x**3/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int a^{3x}x^2\,dx = \frac{\left(9\,x^2\log\left(a\right)^2 - 6\,x\log\left(a\right) + 2\right)a^{3\,x}}{27\,\log\left(a\right)^3}$$

```
[In] integrate(a^(3*x)*x^2,x, algorithm="maxima")
[Out] 1/27*(9*x^2*log(a)^2 - 6*x*log(a) + 2)*a^(3*x)/log(a)^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 826, normalized size of antiderivative = 18.77

$$\int a^{3x}x^2\,dx = \text{Too large to display}$$

```
[In] integrate(a^(3*x)*x^2,x, algorithm="giac")
```

```
[Out] -1/27*((6*(3*pi*x^2*log(abs(a))*sgn(a) - 3*pi*x^2*log(abs(a)) - pi*x*sgn(a)
 + pi*x)*(pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a))
^2)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a))^2)^
2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3)^2) -
 (9*pi^2*x^2*sgn(a) - 9*pi^2*x^2 + 18*x^2*log(abs(a))^2 - 12*x*log(abs(a))
+ 4)*(3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3)/((p
i^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a))^2)^2 + (3*
pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3)^2))*cos(-3/
2*pi*x*sgn(a) + 3/2*pi*x) - ((9*pi^2*x^2*sgn(a) - 9*pi^2*x^2 + 18*x^2*log(a
bs(a))^2 - 12*x*log(abs(a)) + 4)*(pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) -
 pi^3 + 3*pi*log(abs(a))^2)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^
3 + 3*pi*log(abs(a))^2)^2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a))
 + 2*log(abs(a))^3)^2) + 6*(3*pi*x^2*log(abs(a))*sgn(a) - 3*pi*x^2*log(abs(
a)) - pi*x*sgn(a) + pi*x)*(3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) +
 2*log(abs(a))^3)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*l
og(abs(a))^2)^2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(a
bs(a))^3)^2))*sin(-3/2*pi*x*sgn(a) + 3/2*pi*x))*abs(a)^(3*x) - 2*I*abs(a)^(
3*x)*((-9*I*pi^2*x^2*sgn(a) + 18*pi*x^2*log(abs(a))*sgn(a) + 9*I*pi^2*x^2 -
 18*pi*x^2*log(abs(a)) - 18*I*x^2*log(abs(a))^2 - 6*pi*x*sgn(a) + 6*pi*x +
12*I*x*log(abs(a)) - 4*I)*e^(3/2*I*pi*x*sgn(a) - 3/2*I*pi*x)/(-108*I*pi^3*s
gn(a) + 324*pi^2*log(abs(a))*sgn(a) + 324*I*pi*log(abs(a))^2*sgn(a) + 108*I
*pi^3 - 324*pi^2*log(abs(a)) - 324*I*pi*log(abs(a))^2 + 216*log(abs(a))^3)
- (-9*I*pi^2*x^2*sgn(a) - 18*pi*x^2*log(abs(a))*sgn(a) + 9*I*pi^2*x^2 + 18*
pi*x^2*log(abs(a)) - 18*I*x^2*log(abs(a))^2 + 6*pi*x*sgn(a) - 6*pi*x + 12*I
*x*log(abs(a)) - 4*I)*e^(-3/2*I*pi*x*sgn(a) + 3/2*I*pi*x)/(108*I*pi^3*sgn(a
) + 324*pi^2*log(abs(a))*sgn(a) - 324*I*pi*log(abs(a))^2*sgn(a) - 108*I*pi^
3 - 324*pi^2*log(abs(a)) + 324*I*pi*log(abs(a))^2 + 216*log(abs(a))^3))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int a^{3x}x^2\,dx=\frac{a^{3\,x}\,\left(9\,x^2\,\ln\left(a\right)^2-6\,x\,\ln\left(a\right)+2\right)}{27\,\ln\left(a\right)^3}$$

```
[In] int(a^(3*x)*x^2,x)
[Out] (a^(3*x)*(9*x^2*log(a)^2 - 6*x*log(a) + 2))/(27*log(a)^3)
```

3.538 $\int e^{x^2} x(1+x^2)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int e^{x^2} x\left(1+x^2\right)\,dx=\frac{1}{2}e^{x^2}x^2$$

```
[Out] 1/2*exp(x^2)*x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2258, 2240, 2243}

$$\int e^{x^2} x\left(1+x^2\right)\,dx=\frac{1}{2}e^{x^2}x^2$$

```
[In] Int[E^x^2*x*(1 + x^2),x]
```

```
[Out] (E^x^2*x^2)/2
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
```

```
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2258

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(e^{x^2}x + e^{x^2}x^3\right) dx \\ &= \int e^{x^2}x\,dx + \int e^{x^2}x^3\,dx \\ &= \frac{e^{x^2}}{2} + \frac{1}{2}e^{x^2}x^2 - \int e^{x^2}x\,dx \\ &= \frac{1}{2}e^{x^2}x^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int e^{x^2}x\left(1+x^2\right) dx = \frac{1}{2}e^{x^2}x^2$$

```
[In] Integrate[E^x^2*x*(1 + x^2),x]
[Out] (E^x^2*x^2)/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{e^{x^2} x^2}{2}$	10
derivativedivides	$\frac{e^{x^2} x^2}{2}$	10
default	$\frac{e^{x^2} x^2}{2}$	10
norman	$\frac{e^{x^2} x^2}{2}$	10
risch	$\frac{e^{x^2} x^2}{2}$	10
parallelrisch	$\frac{e^{x^2} x^2}{2}$	10
meijerg	$-\frac{(-2x^2+2)e^{x^2}}{4}+\frac{e^{x^2}}{2}$	21
parts	$\frac{\operatorname{erfi}(x)\sqrt{\pi}\,x^3}{2}+\frac{\operatorname{erfi}(x)\sqrt{\pi}\,x}{2}-\frac{\sqrt{\pi}\left(\operatorname{erfi}(x)x^3+\operatorname{erfi}(x)x-\frac{e^{x^2}x^2}{\sqrt{\pi}}\right)}{2}$	48

```
[In] int(exp(x^2)*x*(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x^2)*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int e^{x^2}x(1+x^2)\ dx=\frac{1}{2}\,x^2e^{(x^2)}$$

```
[In] integrate(exp(x^2)*x*(x^2+1),x, algorithm="fricas")
[Out] 1/2*x^2*e^(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int e^{x^2}x(1+x^2)\ dx=\frac{x^2e^{x^2}}{2}$$

```
[In] integrate(exp(x**2)*x*(x**2+1),x)
[Out] x**2*exp(x**2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int e^{x^2}x(1+x^2)\,dx = \frac{1}{2}\,(x^2-1)e^{(x^2)} + \frac{1}{2}\,e^{(x^2)}$$

```
[In] integrate(exp(x^2)*x*(x^2+1),x, algorithm="maxima")
[Out] 1/2*(x^2 - 1)*e^(x^2) + 1/2*e^(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int e^{x^2}x(1+x^2)\,dx = \frac{1}{2}\,(x^2-1)e^{(x^2)} + \frac{1}{2}\,e^{(x^2)}$$

```
[In] integrate(exp(x^2)*x*(x^2+1),x, algorithm="giac")
[Out] 1/2*(x^2 - 1)*e^(x^2) + 1/2*e^(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int e^{x^2}x(1+x^2)\,dx = \frac{x^2\,e^{x^2}}{2}$$

```
[In] int(x*exp(x^2)*(x^2 + 1),x)
[Out] (x^2*exp(x^2))/2
```

3.539 $\int \frac{x}{(e^{-x}+e^x)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{x}{(e^{-x}+e^x)^2} \, dx = \frac{x}{2} - \frac{x}{2(1+e^{2x})} - \frac{1}{4}\log\left(1+e^{2x}\right)$$

```
[Out] 1/2*x-1/2*x/(1+exp(2*x))-1/4*ln(1+exp(2*x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2321, 2222, 2320, 36, 29, 31}

$$\int \frac{x}{(e^{-x}+e^x)^2} \, dx = -\frac{x}{2(e^{2x}+1)} + \frac{x}{2} - \frac{1}{4}\log\left(e^{2x}+1\right)$$

```
[In] Int[x/(E^(-x) + E^x)^2,x]
[Out] x/2 - x/(2*(1 + E^(2*x))) - Log[1 + E^(2*x)]/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2321

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{e^{2x}x}{(1+e^{2x})^2}\,dx \\
&= -\frac{x}{2(1+e^{2x})}+\frac{1}{2}\int \frac{1}{1+e^{2x}}\,dx \\
&= -\frac{x}{2(1+e^{2x})}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,e^{2x}\right) \\
&= -\frac{x}{2(1+e^{2x})}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{x}\,dx,x,e^{2x}\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,e^{2x}\right) \\
&= \frac{x}{2}-\frac{x}{2(1+e^{2x})}-\frac{1}{4}\log\left(1+e^{2x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{x}{(e^{-x}+e^x)^2}\,dx = \frac{e^{2x}x}{2+2e^{2x}} - \frac{1}{4}\log\left(1+e^{2x}\right)$$

```
[In] Integrate[x/(E^(-x) + E^x)^2,x]
[Out] (E^(2*x)*x)/(2 + 2*E^(2*x)) - Log[1 + E^(2*x)]/4
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{x}{2}-\frac{x}{2(1+e^{2x})}-\frac{\ln(1+e^{2x})}{4}$	25
default	$-\frac{\ln(1+e^{2x})}{4}+\frac{e^{2x}x}{2+2\,e^{2x}}$	26
norman	$-\frac{\ln(1+e^{2x})}{4}+\frac{e^{2x}x}{2+2\,e^{2x}}$	26

```
[In] int(x/(exp(-x)+exp(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x-1/2*x/(1+exp(2*x))-1/4*ln(1+exp(2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{x}{(e^{-x}+e^x)^2}\,dx = \frac{2\,xe^{(2\,x)} - \left(e^{(2\,x)}+1\right)\log\left(e^{(2\,x)}+1\right)}{4\left(e^{(2\,x)}+1\right)}$$

```
[In] integrate(x/(exp(-x)+exp(x))^2,x, algorithm="fricas")
[Out] 1/4*(2*x*e^(2*x) - (e^(2*x) + 1)*log(e^(2*x) + 1))/(e^(2*x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{x}{(e^{-x} + e^{x})^2} \, dx = -\frac{x}{2} + \frac{x}{2 + 2e^{-2x}} - \frac{\log(1 + e^{-2x})}{4}$$

```
[In] integrate(x/(exp(-x)+exp(x))**2,x)
[Out] -x/2 + x/(2 + 2*exp(-2*x)) - log(1 + exp(-2*x))/4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x}{(e^{-x} + e^{x})^2} \, dx = \frac{x e^{(2x)}}{2(e^{(2x)} + 1)} - \frac{1}{4} \log\left(e^{(2x)} + 1\right)$$

```
[In] integrate(x/(exp(-x)+exp(x))^2,x, algorithm="maxima")
[Out] 1/2*x*e^(2*x)/(e^(2*x) + 1) - 1/4*log(e^(2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{x}{(e^{-x} + e^{x})^2} \, dx = \frac{2\,x e^{(2x)} - e^{(2x)} \log\left(e^{(2x)} + 1\right) - \log\left(e^{(2x)} + 1\right)}{4\,(e^{(2x)} + 1)}$$

```
[In] integrate(x/(exp(-x)+exp(x))^2,x, algorithm="giac")
[Out] 1/4*(2*x*e^(2*x) - e^(2*x)*log(e^(2*x) + 1) - log(e^(2*x) + 1))/(e^(2*x) +
1)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x}{(e^{-x}+e^x)^2}\,dx = \frac{x\,\mathrm{e}^{2\,x}}{2\,\left(\mathrm{e}^{2\,x}+1\right)} - \frac{\ln\left(\mathrm{e}^{2\,x}+1\right)}{4}$$

```
[In] int(x/(exp(-x) + exp(x))^2,x)
[Out] (x*exp(2*x))/(2*(exp(2*x) + 1)) - log(exp(2*x) + 1)/4
```

3.540 $\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 15

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}} \, dx = e^x\sqrt{1-x^2}$$

```
[Out] exp(x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{2326\}$

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}} \, dx = e^x\sqrt{1-x^2}$$

```
[In] Int[(E^x*(1 - x - x^2))/Sqrt[1 - x^2],x]
```

```
[Out] E^x*Sqrt[1 - x^2]
```

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\text{integral} = e^x\sqrt{1-x^2}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = e^x\sqrt{1-x^2}$$

```
[In] Integrate[(E^x*(1 - x - x^2))/Sqrt[1 - x^2],x]
[Out] E^x*Sqrt[1 - x^2]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

method	result	size
gosper	$-\frac{e^x(-1+x)(1+x)}{\sqrt{-x^2+1}}$	20

```
[In] int(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -exp(x)*(-1+x)*(1+x)/(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = \sqrt{-x^2+1}e^x$$

```
[In] integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] sqrt(-x^2 + 1)*e^x
```

Sympy [F]

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = -\int \left(-\frac{e^x}{\sqrt{1-x^2}}\right)dx - \int \frac{xe^x}{\sqrt{1-x^2}}\,dx - \int \frac{x^2e^x}{\sqrt{1-x^2}}\,dx$$

```
[In] integrate(exp(x)*(-x**2-x+1)/(-x**2+1)**(1/2),x)
[Out] -Integral(-exp(x)/sqrt(1 - x**2), x) - Integral(x*exp(x)/sqrt(1 - x**2), x)
 - Integral(x**2*exp(x)/sqrt(1 - x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = -\frac{(x^2-1)e^x}{\sqrt{x+1}\sqrt{-x+1}}$$

```
[In] integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -(x^2 - 1)*e^x/(sqrt(x + 1)*sqrt(-x + 1))
```

Giac [F]

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = \int -\frac{(x^2+x-1)e^x}{\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 + x - 1)*e^x/sqrt(-x^2 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = \mathrm{e}^x\,\sqrt{1-x^2}$$

```
[In] int(-(exp(x)*(x + x^2 - 1))/(1 - x^2)^(1/2),x)
[Out] exp(x)*(1 - x^2)^(1/2)
```

3.541 $\int e^{-3x}\cos(2x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{-3x}\cos(2x)\,dx = -\frac{3}{13}e^{-3x}\cos(2x)+\frac{2}{13}e^{-3x}\sin(2x)$$

```
[Out] -3/13*cos(2*x)/exp(3*x)+2/13*sin(2*x)/exp(3*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4518}

$$\int e^{-3x}\cos(2x)\,dx = \frac{2}{13}e^{-3x}\sin(2x)-\frac{3}{13}e^{-3x}\cos(2x)$$

```
[In] Int[Cos[2*x]/E^(3*x),x]
```

```
[Out] (-3*Cos[2*x])/(13*E^(3*x)) + (2*Sin[2*x])/(13*E^(3*x))
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{3}{13}e^{-3x}\cos(2x)+\frac{2}{13}e^{-3x}\sin(2x)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{-3x}\cos(2x)\,dx = \frac{1}{13}e^{-3x}(-3\cos(2x)+2\sin(2x))$$

```
[In] Integrate[Cos[2*x]/E^(3*x),x]
[Out] (-3*Cos[2*x] + 2*Sin[2*x])/(13*E^(3*x))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{e^{-3x}(-3\cos(2x)+2\sin(2x))}{13}$	20
default	$-\frac{3\,e^{-3x}\cos(2x)}{13}+\frac{2\,e^{-3x}\sin(2x)}{13}$	22
norman	$\frac{\left(-\frac{3}{13}+\frac{3\left(\tan^2(x)\right)}{13}+\frac{4\tan(x)}{13}\right)e^{-3x}}{1+\tan^2(x)}$	28
risch	$-\frac{3\,e^{(-3+2i)x}}{26}-\frac{i e^{(-3+2i)x}}{13}-\frac{3\,e^{(-3-2i)x}}{26}+\frac{i e^{(-3-2i)x}}{13}$	36

```
[In] int(cos(2*x)/exp(3*x),x,method=_RETURNVERBOSE)
[Out] 1/13*exp(-3*x)*(-3*cos(2*x)+2*sin(2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{-3x}\cos(2x)\,dx = -\frac{3}{13}\cos(2x)\,e^{(-3x)}+\frac{2}{13}\,e^{(-3x)}\sin(2x)$$

```
[In] integrate(cos(2*x)/exp(3*x),x, algorithm="fricas")
[Out] -3/13*cos(2*x)*e^(-3*x) + 2/13*e^(-3*x)*sin(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{-3x}\cos(2x)\,dx = \frac{2e^{-3x}\sin{(2x)}}{13} - \frac{3e^{-3x}\cos{(2x)}}{13}$$

```
[In] integrate(cos(2*x)/exp(3*x),x)
[Out] 2*exp(-3*x)*sin(2*x)/13 - 3*exp(-3*x)*cos(2*x)/13
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-3x}\cos(2x)\,dx = -\frac{1}{13}\,(3\,\cos{(2\,x)} - 2\,\sin{(2\,x)})e^{(-3\,x)}$$

```
[In] integrate(cos(2*x)/exp(3*x),x, algorithm="maxima")
[Out] -1/13*(3*cos(2*x) - 2*sin(2*x))*e^(-3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-3x}\cos(2x)\,dx = -\frac{1}{13}\,(3\,\cos{(2\,x)} - 2\,\sin{(2\,x)})e^{(-3\,x)}$$

```
[In] integrate(cos(2*x)/exp(3*x),x, algorithm="giac")
[Out] -1/13*(3*cos(2*x) - 2*sin(2*x))*e^(-3*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-3x}\cos(2x)\,dx = -\frac{\mathrm{e}^{-3\,x}\,(3\,\cos{(2\,x)} - 2\,\sin{(2\,x)})}{13}$$

```
[In] int(cos(2*x)*exp(-3*x),x)
[Out] -(exp(-3*x)*(3*cos(2*x) - 2*sin(2*x)))/13
```

3.542 $\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx = -\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}+\frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

```
[Out] -30/13*cos(1/2*x)/exp(x)^(1/3)+6/13*sin(1/2*x)/exp(x)^(1/3)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2319, 6874, 4518, 4517}

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx = \frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}-\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

```
[In] Int[(Cos[x/2] + Sin[x/2])/(E^x)^(1/3),x]
[Out] (-30*Cos[x/2])/(13*(E^x)^(1/3)) + (6*Sin[x/2])/(13*(E^x)^(1/3))
```

Rule 2319

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
```

```
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^{x/3}\int e^{-x/3}\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\,dx}{\sqrt[3]{e^x}}\\
&= \frac{\left(6e^{x/3}\right)\text{Subst}\left(\int e^{-2x}(\cos(3x)+\sin(3x))\,dx,x,\frac{x}{6}\right)}{\sqrt[3]{e^x}}\\
&= \frac{\left(6e^{x/3}\right)\text{Subst}\left(\int\left(e^{-2x}\cos(3x)+e^{-2x}\sin(3x)\right)\,dx,x,\frac{x}{6}\right)}{\sqrt[3]{e^x}}\\
&= \frac{\left(6e^{x/3}\right)\text{Subst}\left(\int e^{-2x}\cos(3x)\,dx,x,\frac{x}{6}\right)}{\sqrt[3]{e^x}}+\frac{\left(6e^{x/3}\right)\text{Subst}\left(\int e^{-2x}\sin(3x)\,dx,x,\frac{x}{6}\right)}{\sqrt[3]{e^x}}\\
&= -\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}+\frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int\frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx=\frac{6\left(-5\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{13\sqrt[3]{e^x}}$$

```
[In] Integrate[(Cos[x/2] + Sin[x/2])/(E^x)^(1/3),x]
[Out] (6*(-5*Cos[x/2] + Sin[x/2]))/(13*(E^x)^(1/3))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

method	result	size
parallelrisch	$\frac{-\frac{30\cos\left(\frac{x}{2}\right)}{13}+\frac{6\sin\left(\frac{x}{2}\right)}{13}}{\left(\mathrm{e}^{x}\right)^{\frac{1}{3}}}$	18
default	$-\frac{30\,\mathrm{e}^{-\frac{x}{3}}\cos\left(\frac{x}{2}\right)}{13}+\frac{6\,\mathrm{e}^{-\frac{x}{3}}\sin\left(\frac{x}{2}\right)}{13}$	22
parts	$-\frac{30\,\mathrm{e}^{-\frac{x}{3}}\cos\left(\frac{x}{2}\right)}{13}+\frac{6\,\mathrm{e}^{-\frac{x}{3}}\sin\left(\frac{x}{2}\right)}{13}$	22
risch	$\frac{\left(-\frac{15}{169}-\frac{3i}{169}\right)\left((25-5i)\cos\left(\frac{x}{2}\right)+(-5+i)\sin\left(\frac{x}{2}\right)\right)}{\left(\mathrm{e}^{x}\right)^{\frac{1}{3}}}$	26

```
[In] int((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x,method=_RETURNVERBOSE)
[Out] 6/13/exp(x)^(1/3)*(-5*cos(1/2*x)+sin(1/2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx = -\frac{30}{13}\cos\left(\frac{1}{2}\,x\right)e^{\left(-\frac{1}{3}\,x\right)}+\frac{6}{13}\,e^{\left(-\frac{1}{3}\,x\right)}\sin\left(\frac{1}{2}\,x\right)$$

```
[In] integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="fricas")
[Out] -30/13*cos(1/2*x)*e^(-1/3*x) + 6/13*e^(-1/3*x)*sin(1/2*x)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx = \frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}-\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

```
[In] integrate((cos(1/2*x)+sin(1/2*x))/exp(x)**(1/3),x)
[Out] 6*sin(x/2)/(13*exp(x)**(1/3)) - 30*cos(x/2)/(13*exp(x)**(1/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx = -\frac{6}{13}\left(3\cos\left(\frac{1}{2}x\right)+2\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)} - \frac{6}{13}\left(2\cos\left(\frac{1}{2}x\right)-3\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)}$$

[In] integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="maxima")

[Out] -6/13*(3*cos(1/2*x) + 2*sin(1/2*x))*e^(-1/3*x) - 6/13*(2*cos(1/2*x) - 3*sin(1/2*x))*e^(-1/3*x)

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx = -\frac{6}{13}\left(3\cos\left(\frac{1}{2}x\right)+2\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)} - \frac{6}{13}\left(2\cos\left(\frac{1}{2}x\right)-3\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)}$$

[In] integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="giac")

[Out] -6/13*(3*cos(1/2*x) + 2*sin(1/2*x))*e^(-1/3*x) - 6/13*(2*cos(1/2*x) - 3*sin(1/2*x))*e^(-1/3*x)

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.54

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx = -\frac{6\,\mathrm{e}^{-\frac{x}{3}}\left(5\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{13}$$

[In] int((cos(x/2) + sin(x/2))/exp(x)^(1/3),x)

[Out] -(6*exp(-x/3)*(5*cos(x/2) - sin(x/2)))/13

3.543 $\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = -\frac{4\cos\left(\frac{3x}{2}\right)\log(3)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}+\frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}$$

```
[Out] -4/3*cos(3/2*x)*ln(3)/(3^(3*x))^(1/4)/(4+ln(3)^2)+8/3*sin(3/2*x)/(3^(3*x))^
(1/4)/(4+ln(3)^2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2319, 4518}

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = \frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}-\frac{4\log(3)\cos\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}$$

```
[In] Int[Cos[(3*x)/2]/(3^(3*x))^(1/4),x]
```

```
[Out] (-4*Cos[(3*x)/2]*Log[3])/(3*(3^(3*x))^(1/4)*(4 + Log[3]^2)) + (8*Sin[(3*x)/
2])/(3*(3^(3*x))^(1/4)*(4 + Log[3]^2))
```

Rule 2319

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] && !IntegerQ[n]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3^{3x/4}\int 3^{-3x/4}\cos\left(\frac{3x}{2}\right)\,dx}{\sqrt[4]{3^{3x}}}\\ &= -\frac{4\cos\left(\frac{3x}{2}\right)\log(3)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}+\frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = -\frac{4\left(\cos\left(\frac{3x}{2}\right)\log(3)-2\sin\left(\frac{3x}{2}\right)\right)}{3\sqrt[4]{27^x}\left(4+\log^2(3)\right)}$$

```
[In] Integrate[Cos[(3*x)/2]/(3^(3*x))^(1/4),x]
[Out] (-4*(Cos[(3*x)/2]*Log[3] - 2*Sin[(3*x)/2]))/(3*(27^x)^(1/4)*(4 + Log[3]^2))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

method	result	size
parallelrisch	$-\frac{4\left(\cos\left(\frac{3x}{2}\right)\ln(3)-2\sin\left(\frac{3x}{2}\right)\right)}{\left(27^{x}\right)^{\frac{1}{4}}\left(3\ln(3)^{2}+12\right)}$	32
risch	$-\frac{2\left(2\cos\left(\frac{3x}{2}\right)\ln(3)-4\sin\left(\frac{3x}{2}\right)\right)}{3(2i+\ln(3))(-2i+\ln(3))\left(27^{x}\right)^{\frac{1}{4}}}$	37

```
[In] int(cos(3/2*x)/(3^(3*x))^(1/4),x,method=_RETURNVERBOSE)
[Out] -4*(cos(3/2*x)*ln(3)-2*sin(3/2*x))/(27^x)^(1/4)/(3*ln(3)^2+12)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = \frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\log(3)^2 + 12\sqrt[4]{3^{3x}}} - \frac{4\log(3)\cos\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\log(3)^2 + 12\sqrt[4]{3^{3x}}}$$

```
[In] integrate(cos(3/2*x)/(3**(3*x))**(1/4),x)
[Out] 8*sin(3*x/2)/(3*(3**(3*x))**(1/4)*log(3)**2 + 12*(3**(3*x))**(1/4)) - 4*log
(3)*cos(3*x/2)/(3*(3**(3*x))**(1/4)*log(3)**2 + 12*(3**(3*x))**(1/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.54

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = -\frac{4\left(\cos\left(\frac{3}{2}x\right)\log(3) - 2\sin\left(\frac{3}{2}x\right)\right)}{3\left(\log(3)^2 + 4\right)3^{\frac{3}{4}x}}$$

```
[In] integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="maxima")
[Out] -4/3*(cos(3/2*x)*log(3) - 2*sin(3/2*x))/((log(3)^2 + 4)*3^(3/4*x))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.68

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = -\frac{4\left(\frac{\cos\left(\frac{3}{2}\,x\right)\log(3)}{\log(3)^2+4} - \frac{2\,\sin\left(\frac{3}{2}\,x\right)}{\log(3)^2+4}\right)}{3\cdot 3^{\frac{3}{4}\,x}}$$

```
[In] integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="giac")
```

```
[Out] -4/3*(cos(3/2*x)*log(3)/(log(3)^2 + 4) - 2*sin(3/2*x)/(log(3)^2 + 4))/3^(3/
4*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.58

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = \frac{\frac{3\,\sin\left(\frac{3\,x}{2}\right)}{2} - \frac{3\,\cos\left(\frac{3\,x}{2}\right)\,\ln(3)}{4}}{3^{\frac{3\,x}{4}}\,\left(\frac{9\ln(3)^2}{16}+\frac{9}{4}\right)}$$

```
[In] int(cos((3*x)/2)/(3^(3*x))^(1/4),x)
```

```
[Out] ((3*sin((3*x)/2))/2 - (3*cos((3*x)/2)*log(3))/4)/(3^((3*x)/4)*((9*log(3)^2)
/16 + 9/4))
```

3.544 $\int e^{mx} \cos^2(x) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int e^{mx} \cos^2(x) \, dx = \frac{2e^{mx}}{m(4+m^2)} + \frac{e^{mx} m \cos^2(x)}{4+m^2} + \frac{2e^{mx} \cos(x) \sin(x)}{4+m^2}$$

```
[Out] 2*exp(m*x)/m/(m^2+4)+exp(m*x)*m*cos(x)^2/(m^2+4)+2*exp(m*x)*cos(x)*sin(x)/(
m^2+4)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4520, 2225}

$$\int e^{mx} \cos^2(x) \, dx = \frac{2e^{mx}}{m(m^2+4)} + \frac{me^{mx} \cos^2(x)}{m^2+4} + \frac{2e^{mx} \sin(x) \cos(x)}{m^2+4}$$

```
[In] Int[E^(m*x)*Cos[x]^2,x]
```

```
[Out] (2*E^(m*x))/(m*(4 + m^2)) + (E^(m*x)*m*Cos[x]^2)/(4 + m^2) + (2*E^(m*x)*Cos
[x]*Sin[x])/(4 + m^2)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4520

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
```

```
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[e*m*F^(c*(a + b*x))*Sin[d +
 e*x]*(Cos[d + e*x]^(m - 1)/(e^2*m^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{e^{mx}m\cos^2(x)}{4+m^2}+\frac{2e^{mx}\cos(x)\sin(x)}{4+m^2}+\frac{2\int e^{mx}\,dx}{4+m^2}\\ &= \frac{2e^{mx}}{m(4+m^2)}+\frac{e^{mx}m\cos^2(x)}{4+m^2}+\frac{2e^{mx}\cos(x)\sin(x)}{4+m^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.72

$$\int e^{mx}\cos^2(x)\,dx=\frac{e^{mx}(4+m^2+m^2\cos(2x)+2m\sin(2x))}{2m(4+m^2)}$$

```
[In] Integrate[E^(m*x)*Cos[x]^2,x]
[Out] (E^(m*x)*(4 + m^2 + m^2*Cos[2*x] + 2*m*Sin[2*x]))/(2*m*(4 + m^2))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$\frac{\mathrm{e}^{mx}(m^2\cos(2x)+2m\sin(2x)+m^2+4)}{2m(m^2+4)}$	37
risch	$\frac{\mathrm{e}^{mx}}{2m}+\frac{\mathrm{e}^{(2i+m)x}}{8i+4m}+\frac{\mathrm{e}^{x(m-2i)}}{4m-8i}$	41
default	$\frac{\mathrm{e}^{mx}}{2m}+\frac{m\,\mathrm{e}^{mx}\cos(2x)}{2m^2+8}+\frac{\mathrm{e}^{mx}\sin(2x)}{m^2+4}$	45
norman	$\frac{\frac{(m^2+2)\mathrm{e}^{mx}}{m(m^2+4)}+\frac{(m^2+2)\mathrm{e}^{mx}\left(\tan^4\left(\frac{x}{2}\right)\right)}{m(m^2+4)}+\frac{4\,\mathrm{e}^{mx}\tan\left(\frac{x}{2}\right)}{m^2+4}-\frac{4\,\mathrm{e}^{mx}\left(\tan^3\left(\frac{x}{2}\right)\right)}{m^2+4}-\frac{2(m^2-2)\mathrm{e}^{mx}\left(\tan^2\left(\frac{x}{2}\right)\right)}{m(m^2+4)}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	122

```
[In] int(exp(m*x)*cos(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*exp(m*x)*(m^2*cos(2*x)+2*m*sin(2*x)+m^2+4)/m/(m^2+4)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int e^{mx}\cos^2(x)\,dx = \frac{2\,m\cos(x)\,e^{(mx)}\sin(x) + \left(m^2\cos(x)^2 + 2\right)e^{(mx)}}{m^3 + 4\,m}$$

```
[In] integrate(exp(m*x)*cos(x)^2,x, algorithm="fricas")
[Out] (2*m*cos(x)*e^(m*x)*sin(x) + (m^2*cos(x)^2 + 2)*e^(m*x))/(m^3 + 4*m)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.91

$$\int e^{mx}\cos^2(x)\,dx$$
$$= \begin{cases} \frac{x\sin^2(x)}{2} + \frac{x\cos^2(x)}{2} + \frac{\sin(x)\cos(x)}{2} & \text{for } m = 0 \\ -\frac{xe^{-2ix}\sin^2(x)}{4} + \frac{ixe^{-2ix}\sin(x)\cos(x)}{2} + \frac{xe^{-2ix}\cos^2(x)}{4} - \frac{e^{-2ix}\sin(x)\cos(x)}{4} + \frac{ie^{-2ix}\cos^2(x)}{2} & \text{for } m = -2i \\ -\frac{xe^{2ix}\sin^2(x)}{4} - \frac{ixe^{2ix}\sin(x)\cos(x)}{2} + \frac{xe^{2ix}\cos^2(x)}{4} - \frac{e^{2ix}\sin(x)\cos(x)}{4} - \frac{ie^{2ix}\cos^2(x)}{2} & \text{for } m = 2i \\ \frac{m^2e^{mx}\cos^2(x)}{m^3+4m} + \frac{2me^{mx}\sin(x)\cos(x)}{m^3+4m} + \frac{2e^{mx}\sin^2(x)}{m^3+4m} + \frac{2e^{mx}\cos^2(x)}{m^3+4m} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(m*x)*cos(x)**2,x)
[Out] Piecewise((x*sin(x)**2/2 + x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, 0)), (-x*
exp(-2*I*x)*sin(x)**2/4 + I*x*exp(-2*I*x)*sin(x)*cos(x)/2 + x*exp(-2*I*x)*c
os(x)**2/4 - exp(-2*I*x)*sin(x)*cos(x)/4 + I*exp(-2*I*x)*cos(x)**2/2, Eq(m,
 -2*I)), (-x*exp(2*I*x)*sin(x)**2/4 - I*x*exp(2*I*x)*sin(x)*cos(x)/2 + x*ex
p(2*I*x)*cos(x)**2/4 - exp(2*I*x)*sin(x)*cos(x)/4 - I*exp(2*I*x)*cos(x)**2/
2, Eq(m, 2*I)), (m**2*exp(m*x)*cos(x)**2/(m**3 + 4*m) + 2*m*exp(m*x)*sin(x)
*cos(x)/(m**3 + 4*m) + 2*exp(m*x)*sin(x)**2/(m**3 + 4*m) + 2*exp(m*x)*cos(x
)**2/(m**3 + 4*m), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int e^{mx}\cos^2(x)\,dx = \frac{m^2\cos(2x)\,e^{(mx)} + 2\,me^{(mx)}\sin(2x) + (m^2+4)e^{(mx)}}{2\,(m^3+4\,m)}$$

```
[In] integrate(exp(m*x)*cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(m^2*cos(2*x)*e^(m*x) + 2*m*e^(m*x)*sin(2*x) + (m^2 + 4)*e^(m*x))/(m^3
+ 4*m)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int e^{mx}\cos^2(x)\,dx = \frac{1}{2}\left(\frac{m\cos(2x)}{m^2+4} + \frac{2\,\sin(2x)}{m^2+4}\right)e^{(mx)} + \frac{e^{(mx)}}{2\,m}$$

```
[In] integrate(exp(m*x)*cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(m*cos(2*x)/(m^2 + 4) + 2*sin(2*x)/(m^2 + 4))*e^(m*x) + 1/2*e^(m*x)/m
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int e^{mx}\cos^2(x)\,dx = \frac{\mathrm{e}^{m\,x}}{2\,m} + \frac{\mathrm{e}^{m\,x}\,(2\,\sin(2\,x) + m\,\cos(2\,x))}{2\,(m^2+4)}$$

```
[In] int(exp(m*x)*cos(x)^2,x)
```

```
[Out] exp(m*x)/(2*m) + (exp(m*x)*(2*sin(2*x) + m*cos(2*x)))/(2*(m^2 + 4))
```

3.545 $\int e^{mx}\sin^3(x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 82

$$\int e^{mx}\sin^3(x)\,dx=-\frac{6e^{mx}\cos(x)}{9+10m^2+m^4}+\frac{6e^{mx}m\sin(x)}{9+10m^2+m^4}-\frac{3e^{mx}\cos(x)\sin^2(x)}{9+m^2}+\frac{e^{mx}m\sin^3(x)}{9+m^2}$$

```
[Out] -6*exp(m*x)*cos(x)/(m^4+10*m^2+9)+6*exp(m*x)*m*sin(x)/(m^4+10*m^2+9)-3*exp(
m*x)*cos(x)*sin(x)^2/(m^2+9)+exp(m*x)*m*sin(x)^3/(m^2+9)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4519, 4517}

$$\int e^{mx}\sin^3(x)\,dx=\frac{me^{mx}\sin^3(x)}{m^2+9}-\frac{3e^{mx}\sin^2(x)\cos(x)}{m^2+9}+\frac{6me^{mx}\sin(x)}{m^4+10m^2+9}-\frac{6e^{mx}\cos(x)}{m^4+10m^2+9}$$

```
[In] Int[E^(m*x)*Sin[x]^3,x]
```

```
[Out] (-6*E^(m*x)*Cos[x])/(9 + 10*m^2 + m^4) + (6*E^(m*x)*m*Sin[x])/(9 + 10*m^2 +
 m^4) - (3*E^(m*x)*Cos[x]*Sin[x]^2)/(9 + m^2) + (E^(m*x)*m*Sin[x]^3)/(9 + m
^2)
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4519

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[e*n*F^(c*(a + b*x))*Cos[d +
 e*x]*(Sin[d + e*x]^(n - 1)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3e^{mx}\cos(x)\sin^2(x)}{9+m^2} + \frac{e^{mx}m\sin^3(x)}{9+m^2} + \frac{6\int e^{mx}\sin(x)\,dx}{9+m^2} \\
&= -\frac{6e^{mx}\cos(x)}{9+10m^2+m^4} + \frac{6e^{mx}m\sin(x)}{9+10m^2+m^4} - \frac{3e^{mx}\cos(x)\sin^2(x)}{9+m^2} + \frac{e^{mx}m\sin^3(x)}{9+m^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$
\int e^{mx}\sin^3(x)\,dx = \frac{e^{mx}(-3(9+m^2)\cos(x)+3(1+m^2)\cos(3x)-2m(-13-m^2+(1+m^2)\cos(2x))\sin(x))}{4(9+10m^2+m^4)}
$$

```
[In] Integrate[E^(m*x)*Sin[x]^3,x]
[Out] (E^(m*x)*(-3*(9 + m^2)*Cos[x] + 3*(1 + m^2)*Cos[3*x] - 2*m*(-13 - m^2 + (1
+ m^2)*Cos[2*x])*Sin[x]))/(4*(9 + 10*m^2 + m^4))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{3\,\mathrm{e}^{mx}\left(\left(m^2+1\right)\cos(3x)+\frac{\left(-m^3-m\right)\sin(3x)}{3}+\left(m^2+9\right)\left(m\sin(x)-\cos(x)\right)\right)}{4(m^4+10m^2+9)}$
risch	$\frac{i\mathrm{e}^{(3i+m)x}}{24i+8m}-\frac{3i\mathrm{e}^{x(m+i)}}{8(m+i)}+\frac{3i\mathrm{e}^{x(m-i)}}{8(m-i)}-\frac{i\mathrm{e}^{x(m-3i)}}{8(m-3i)}$
default	$-\frac{3\,\mathrm{e}^{mx}\cos(x)}{4(m^2+1)}+\frac{3m\,\mathrm{e}^{mx}\sin(x)}{4(m^2+1)}+\frac{3\,\mathrm{e}^{mx}\cos(3x)}{4(m^2+9)}-\frac{m\,\mathrm{e}^{mx}\sin(3x)}{4(m^2+9)}$
norman	$\frac{-\frac{6\,\mathrm{e}^{mx}}{m^4+10m^2+9}+\frac{6\,\mathrm{e}^{mx}\left(\tan^6\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9}+\frac{12m\,\mathrm{e}^{mx}\tan\left(\frac{x}{2}\right)}{m^4+10m^2+9}+\frac{12m\,\mathrm{e}^{mx}\left(\tan^5\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9}-\frac{6\left(2m^2+3\right)\mathrm{e}^{mx}\left(\tan^2\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9}+\frac{6\left(2m^2+3\right)\mathrm{e}^{mx}\left(\tan^4\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$

```
[In] int(exp(m*x)*sin(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*exp(m*x)*((m^2+1)*cos(3*x)+1/3*(-m^3-m)*sin(3*x)+(m^2+9)*(m*sin(x)-cos(
x)))/(m^4+10*m^2+9)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int e^{mx}\sin^3(x)\,dx = \frac{\left(m^3-(m^3+m)\cos(x)^2+7\,m\right)e^{(mx)}\sin(x)+3\left((m^2+1)\cos(x)^3-(m^2+3)\cos(x)\right)e^{(mx)}}{m^4+10\,m^2+9}$$

```
[In] integrate(exp(m*x)*sin(x)^3,x, algorithm="fricas")
```

```
[Out] ((m^3 - (m^3 + m)*cos(x)^2 + 7*m)*e^(m*x)*sin(x) + 3*((m^2 + 1)*cos(x)^3 -
(m^2 + 3)*cos(x))*e^(m*x))/(m^4 + 10*m^2 + 9)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 638, normalized size of antiderivative = 7.78

$$\int e^{mx}\sin^3(x)\,dx = \begin{cases} \frac{xe^{-3ix}\sin^3(x)}{8} - \frac{3ixe^{-3ix}\sin^2(x)\cos(x)}{8} - \frac{3xe^{-3ix}\sin(x)\cos^2(x)}{8} + \frac{ixe^{-3ix}\cos^3(x)}{8} + \frac{7ie^{-3ix}\sin^3(x)}{24} + \frac{ie^{-3ix}\sin(x)\cos^2(x)}{4} + \\ \frac{3xe^{-ix}\sin^3(x)}{8} - \frac{3ixe^{-ix}\sin^2(x)\cos(x)}{8} + \frac{3xe^{-ix}\sin(x)\cos^2(x)}{8} - \frac{3ixe^{-ix}\cos^3(x)}{8} + \frac{5ie^{-ix}\sin^3(x)}{8} + \frac{3ie^{-ix}\sin(x)\cos^2(x)}{4} + 3e \\ \frac{3xe^{ix}\sin^3(x)}{8} + \frac{3ixe^{ix}\sin^2(x)\cos(x)}{8} + \frac{3xe^{ix}\sin(x)\cos^2(x)}{8} + \frac{3ixe^{ix}\cos^3(x)}{8} - \frac{5ie^{ix}\sin^3(x)}{8} - \frac{3ie^{ix}\sin(x)\cos^2(x)}{4} + \frac{3e^{ix}\cos^3(x}{8} \\ \frac{xe^{3ix}\sin^3(x)}{8} + \frac{3ixe^{3ix}\sin^2(x)\cos(x)}{8} - \frac{3xe^{3ix}\sin(x)\cos^2(x)}{8} - \frac{ixe^{3ix}\cos^3(x)}{8} - \frac{7ie^{3ix}\sin^3(x)}{24} - \frac{ie^{3ix}\sin(x)\cos^2(x)}{4} + \frac{e^{3ix}\cos^3}{8} \\ \frac{m^3e^{mx}\sin^3(x)}{m^4+10m^2+9} - \frac{3m^2e^{mx}\sin^2(x)\cos(x)}{m^4+10m^2+9} + \frac{7me^{mx}\sin^3(x)}{m^4+10m^2+9} + \frac{6me^{mx}\sin(x)\cos^2(x)}{m^4+10m^2+9} - \frac{9e^{mx}\sin^2(x)\cos(x)}{m^4+10m^2+9} - \frac{6e^{mx}\cos^3(x)}{m^4+10m^2+9} \end{cases}$$

```
[In] integrate(exp(m*x)*sin(x)**3,x)
```

```
[Out] Piecewise((x*exp(-3*I*x)*sin(x)**3/8 - 3*I*x*exp(-3*I*x)*sin(x)**2*cos(x)/8
 - 3*x*exp(-3*I*x)*sin(x)*cos(x)**2/8 + I*x*exp(-3*I*x)*cos(x)**3/8 + 7*I*e
xp(-3*I*x)*sin(x)**3/24 + I*exp(-3*I*x)*sin(x)*cos(x)**2/4 + exp(-3*I*x)*co
s(x)**3/8, Eq(m, -3*I)), (3*x*exp(-I*x)*sin(x)**3/8 - 3*I*x*exp(-I*x)*sin(x
)**2*cos(x)/8 + 3*x*exp(-I*x)*sin(x)*cos(x)**2/8 - 3*I*x*exp(-I*x)*cos(x)**
3/8 + 5*I*exp(-I*x)*sin(x)**3/8 + 3*I*exp(-I*x)*sin(x)*cos(x)**2/4 + 3*exp(
-I*x)*cos(x)**3/8, Eq(m, -I)), (3*x*exp(I*x)*sin(x)**3/8 + 3*I*x*exp(I*x)*s
in(x)**2*cos(x)/8 + 3*x*exp(I*x)*sin(x)*cos(x)**2/8 + 3*I*x*exp(I*x)*cos(x)
```

```
**3/8 - 5*I*exp(I*x)*sin(x)**3/8 - 3*I*exp(I*x)*sin(x)*cos(x)**2/4 + 3*exp(
I*x)*cos(x)**3/8, Eq(m, I)), (x*exp(3*I*x)*sin(x)**3/8 + 3*I*x*exp(3*I*x)*s
in(x)**2*cos(x)/8 - 3*x*exp(3*I*x)*sin(x)*cos(x)**2/8 - I*x*exp(3*I*x)*cos(
x)**3/8 - 7*I*exp(3*I*x)*sin(x)**3/24 - I*exp(3*I*x)*sin(x)*cos(x)**2/4 + e
xp(3*I*x)*cos(x)**3/8, Eq(m, 3*I)), (m**3*exp(m*x)*sin(x)**3/(m**4 + 10*m**
2 + 9) - 3*m**2*exp(m*x)*sin(x)**2*cos(x)/(m**4 + 10*m**2 + 9) + 7*m*exp(m*
x)*sin(x)**3/(m**4 + 10*m**2 + 9) + 6*m*exp(m*x)*sin(x)*cos(x)**2/(m**4 + 1
0*m**2 + 9) - 9*exp(m*x)*sin(x)**2*cos(x)/(m**4 + 10*m**2 + 9) - 6*exp(m*x)
*cos(x)**3/(m**4 + 10*m**2 + 9), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int e^{mx}\sin^3(x)\,dx = \frac{3\,(m^2+1)\cos(3\,x)\,e^{(mx)} - 3\,(m^2+9)\cos(x)\,e^{(mx)} - (m^3+m)e^{(mx)}\sin(3\,x) + 3\,(m^3+9\,m)e^{(mx)}\sin(x)}{4\,(m^4+10\,m^2+9)}$$

```
[In] integrate(exp(m*x)*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*(m^2 + 1)*cos(3*x)*e^(m*x) - 3*(m^2 + 9)*cos(x)*e^(m*x) - (m^3 + m)*
e^(m*x)*sin(3*x) + 3*(m^3 + 9*m)*e^(m*x)*sin(x))/(m^4 + 10*m^2 + 9)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int e^{mx}\sin^3(x)\,dx = -\frac{1}{4}\left(\frac{m\sin(3\,x)}{m^2+9} - \frac{3\,\cos(3\,x)}{m^2+9}\right)e^{(mx)} + \frac{3}{4}\left(\frac{m\sin(x)}{m^2+1} - \frac{\cos(x)}{m^2+1}\right)e^{(mx)}$$

```
[In] integrate(exp(m*x)*sin(x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(m*sin(3*x)/(m^2 + 9) - 3*cos(3*x)/(m^2 + 9))*e^(m*x) + 3/4*(m*sin(x)/
(m^2 + 1) - cos(x)/(m^2 + 1))*e^(m*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

$$\int e^{mx} \sin^3(x) \, dx = -\frac{\mathrm{e}^{m\,x}\,\left(\frac{3\,\left(\cos(x)-m\,\sin(x)\right)}{m^2+1}-\frac{3\,\cos(3\,x)-m\,\sin(3\,x)}{m^2+9}\right)}{4}$$

```
[In] int(exp(m*x)*sin(x)^3,x)
```

```
[Out] -(exp(m*x)*((3*(cos(x) - m*sin(x)))/(m^2 + 1) - (3*cos(3*x) - m*sin(3*x))/(
m^2 + 9)))/4
```

3.546 $\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}} \, dx = -\frac{48\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}} - \frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}} + \frac{32\sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}} + \frac{4\cos^2\left(\frac{x}{3}\right)\sin\left(\frac{x}{3}\right)}{5\sqrt{e^x}}$$

```
[Out] -48/65*cos(1/3*x)/exp(x)^(1/2)-2/5*cos(1/3*x)^3/exp(x)^(1/2)+32/65*sin(1/3*
x)/exp(x)^(1/2)+4/5*cos(1/3*x)^2*sin(1/3*x)/exp(x)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2319, 4520, 4518}

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}} \, dx = \frac{32\sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}} - \frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}} - \frac{48\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}} + \frac{4\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)}{5\sqrt{e^x}}$$

```
[In] Int[Cos[x/3]^3/Sqrt[E^x],x]
```

```
[Out] (-48*Cos[x/3])/(65*Sqrt[E^x]) - (2*Cos[x/3]^3)/(5*Sqrt[E^x]) + (32*Sin[x/3]
)/(65*Sqrt[E^x]) + (4*Cos[x/3]^2*Sin[x/3])/(5*Sqrt[E^x])
```

Rule 2319

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
```

```
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4520

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[e*m*F^(c*(a + b*x))*Sin[d +
 e*x]*(Cos[d + e*x]^(m - 1)/(e^2*m^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^{x/2}\int e^{-x/2}\cos^3\left(\frac{x}{3}\right)\,dx}{\sqrt{e^x}}\\
&= -\frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}}+\frac{4\cos^2\left(\frac{x}{3}\right)\sin\left(\frac{x}{3}\right)}{5\sqrt{e^x}}+\frac{\left(8e^{x/2}\right)\int e^{-x/2}\cos\left(\frac{x}{3}\right)\,dx}{15\sqrt{e^x}}\\
&= -\frac{48\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}}-\frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}}+\frac{32\sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}}+\frac{4\cos^2\left(\frac{x}{3}\right)\sin\left(\frac{x}{3}\right)}{5\sqrt{e^x}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = \frac{-135\cos\left(\frac{x}{3}\right)-13\cos(x)+90\sin\left(\frac{x}{3}\right)+26\sin(x)}{130\sqrt{e^x}}$$

```
[In] Integrate[Cos[x/3]^3/Sqrt[E^x],x]
[Out] (-135*Cos[x/3] - 13*Cos[x] + 90*Sin[x/3] + 26*Sin[x])/(130*Sqrt[E^x])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.35

method	result	size
parallelrisch	$\frac{-13\cos(x)-135\cos\left(\frac{x}{3}\right)+26\sin(x)+90\sin\left(\frac{x}{3}\right)}{130\sqrt{e^x}}$	28
default	$-\frac{e^{-\frac{x}{2}}\cos(x)}{10}+\frac{e^{-\frac{x}{2}}\sin(x)}{5}-\frac{27\,e^{-\frac{x}{2}}\cos\left(\frac{x}{3}\right)}{26}+\frac{9\,e^{-\frac{x}{2}}\sin\left(\frac{x}{3}\right)}{13}$	38
risch	$\frac{\left(-\frac{1}{1300}-\frac{i}{650}\right)\left(-52i e^{-ix}+65\,e^{ix}-39\,e^{-ix}+(270-540i)\cos\left(\frac{x}{3}\right)+(-180+360i)\sin\left(\frac{x}{3}\right)\right)}{\sqrt{e^x}}$	48

```
[In] int(cos(1/3*x)^3/exp(x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/130*(-13*cos(x)-135*cos(1/3*x)+26*sin(x)+90*sin(1/3*x))/exp(x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.53

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = \frac{4}{65}\left(13\cos\left(\frac{1}{3}x\right)^2+8\right)e^{\left(-\frac{1}{2}x\right)}\sin\left(\frac{1}{3}x\right) - \frac{2}{65}\left(13\cos\left(\frac{1}{3}x\right)^3+24\cos\left(\frac{1}{3}x\right)\right)e^{\left(-\frac{1}{2}x\right)}$$

```
[In] integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="fricas")
[Out] 4/65*(13*cos(1/3*x)^2 + 8)*e^(-1/2*x)*sin(1/3*x) - 2/65*(13*cos(1/3*x)^3 +
24*cos(1/3*x))*e^(-1/2*x)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = \frac{32\sin^3\left(\frac{x}{3}\right)}{65\sqrt{e^x}} - \frac{48\sin^2\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}} + \frac{84\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)}{65\sqrt{e^x}} - \frac{74\cos^3\left(\frac{x}{3}\right)}{65\sqrt{e^x}}$$

```
[In] integrate(cos(1/3*x)**3/exp(x)**(1/2),x)
[Out] 32*sin(x/3)**3/(65*sqrt(exp(x))) - 48*sin(x/3)**2*cos(x/3)/(65*sqrt(exp(x))
) + 84*sin(x/3)*cos(x/3)**2/(65*sqrt(exp(x))) - 74*cos(x/3)**3/(65*sqrt(exp
(x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.34

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = -\frac{1}{130}\left(135\cos\left(\frac{1}{3}x\right)+13\cos\left(x\right)-90\sin\left(\frac{1}{3}x\right)-26\sin\left(x\right)\right)e^{\left(-\frac{1}{2}x\right)}$$

```
[In] integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="maxima")
[Out] -1/130*(135*cos(1/3*x) + 13*cos(x) - 90*sin(1/3*x) - 26*sin(x))*e^(-1/2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.42

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = -\frac{9}{26}\left(3\cos\left(\frac{1}{3}x\right) - 2\sin\left(\frac{1}{3}x\right)\right)e^{\left(-\frac{1}{2}x\right)} - \frac{1}{10}(\cos(x) - 2\sin(x))e^{\left(-\frac{1}{2}x\right)}$$

```
[In] integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="giac")
```

```
[Out] -9/26*(3*cos(1/3*x) - 2*sin(1/3*x))*e^(-1/2*x) - 1/10*(cos(x) - 2*sin(x))*e
^(-1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = -\frac{e^{-\frac{x}{2}}\left(\frac{8\cos\left(\frac{x}{3}\right)^3}{5} - \frac{16\sin\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)^2}{5} + \frac{192\cos\left(\frac{x}{3}\right)}{65} - \frac{128\sin\left(\frac{x}{3}\right)}{65}\right)}{4}$$

```
[In] int(cos(x/3)^3/exp(x)^(1/2),x)
```

```
[Out] -(exp(-x/2)*((192*cos(x/3))/65 - (128*sin(x/3))/65 - (16*cos(x/3)^2*sin(x/3
))/5 + (8*cos(x/3)^3)/5))/4
```

3.547 $\int e^{2x} \cos^2(x) \sin^2(x) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 36

$$\int e^{2x} \cos^2(x) \sin^2(x) \, dx = \frac{e^{2x}}{16} - \frac{1}{80} e^{2x} \cos(4x) - \frac{1}{40} e^{2x} \sin(4x)$$

```
[Out] 1/16*exp(2*x)-1/80*exp(2*x)*cos(4*x)-1/40*exp(2*x)*sin(4*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4557, 2225, 4518}

$$\int e^{2x} \cos^2(x) \sin^2(x) \, dx = \frac{e^{2x}}{16} - \frac{1}{40} e^{2x} \sin(4x) - \frac{1}{80} e^{2x} \cos(4x)$$

```
[In] Int[E^(2*x)*Cos[x]^2*Sin[x]^2,x]
```

```
[Out] E^(2*x)/16 - (E^(2*x)*Cos[4*x])/80 - (E^(2*x)*Sin[4*x])/40
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
```

```
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{e^{2x}}{8}-\frac{1}{8}e^{2x}\cos(4x)\right)dx\\ &= \frac{1}{8}\int e^{2x}\,dx-\frac{1}{8}\int e^{2x}\cos(4x)\,dx\\ &= \frac{e^{2x}}{16}-\frac{1}{80}e^{2x}\cos(4x)-\frac{1}{40}e^{2x}\sin(4x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx=-\frac{1}{80}e^{2x}(-5+\cos(4x)+2\sin(4x))$$

```
[In] Integrate[E^(2*x)*Cos[x]^2*Sin[x]^2,x]
[Out] -1/80*(E^(2*x)*(-5 + Cos[4*x] + 2*Sin[4*x]))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

method	result
parallelrisch	$-\frac{e^{2x}(-5+\cos(4x)+2\sin(4x))}{80}$
default	$\frac{e^{2x}}{16}-\frac{e^{2x}\cos(4x)}{80}-\frac{e^{2x}\sin(4x)}{40}$
risch	$\frac{e^{2x}}{16}-\frac{e^{(2+4i)x}}{160}+\frac{i e^{(2+4i)x}}{80}-\frac{e^{(2-4i)x}}{160}-\frac{i e^{(2-4i)x}}{80}$
norman	$\frac{-\frac{e^{2x}\tan\left(\frac{x}{2}\right)}{5}+\frac{3e^{2x}\left(\tan^2\left(\frac{x}{2}\right)\right)}{5}+\frac{7e^{2x}\left(\tan^3\left(\frac{x}{2}\right)\right)}{5}-\frac{e^{2x}\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}-\frac{7e^{2x}\left(\tan^5\left(\frac{x}{2}\right)\right)}{5}+\frac{3e^{2x}\left(\tan^6\left(\frac{x}{2}\right)\right)}{5}+\frac{e^{2x}\left(\tan^7\left(\frac{x}{2}\right)\right)}{5}+\frac{e^{2x}\left(\tan^8\left(\right.\right.}{20}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$

```
[In] int(exp(2*x)*cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)
[Out] -1/80*exp(2*x)*(-5+cos(4*x)+2*sin(4*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = -\frac{1}{10}\left(2\cos\left(x\right)^3 - \cos\left(x\right)\right)e^{(2x)}\sin\left(x\right) - \frac{1}{20}\left(2\cos\left(x\right)^4 - 2\cos\left(x\right)^2 - 1\right)e^{(2x)}$$

```
[In] integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="fricas")
[Out] -1/10*(2*cos(x)^3 - cos(x))*e^(2*x)*sin(x) - 1/20*(2*cos(x)^4 - 2*cos(x)^2
- 1)*e^(2*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(29) = 58.

Time = 0.52 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.94

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = \frac{e^{2x}\sin^4\left(x\right)}{20} + \frac{e^{2x}\sin^3\left(x\right)\cos\left(x\right)}{10} + \frac{e^{2x}\sin^2\left(x\right)\cos^2\left(x\right)}{5} - \frac{e^{2x}\sin\left(x\right)\cos^3\left(x\right)}{10} + \frac{e^{2x}\cos^4\left(x\right)}{20}$$

```
[In] integrate(exp(2*x)*cos(x)**2*sin(x)**2,x)
[Out] exp(2*x)*sin(x)**4/20 + exp(2*x)*sin(x)**3*cos(x)/10 + exp(2*x)*sin(x)**2*c
os(x)**2/5 - exp(2*x)*sin(x)*cos(x)**3/10 + exp(2*x)*cos(x)**4/20
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = -\frac{1}{80}\cos\left(4x\right)e^{(2x)} - \frac{1}{40}e^{(2x)}\sin\left(4x\right) + \frac{1}{16}e^{(2x)}$$

```
[In] integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="maxima")
[Out] -1/80*cos(4*x)*e^(2*x) - 1/40*e^(2*x)*sin(4*x) + 1/16*e^(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = -\frac{1}{80}\left(\cos(4x) + 2\sin(4x)\right)e^{(2x)} + \frac{1}{16}e^{(2x)}$$

```
[In] integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="giac")
[Out] -1/80*(cos(4*x) + 2*sin(4*x))*e^(2*x) + 1/16*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.50

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = -\frac{e^{2x}\left(\cos(4x) + 2\sin(4x) - 5\right)}{80}$$

```
[In] int(exp(2*x)*cos(x)^2*sin(x)^2,x)
[Out] -(exp(2*x)*(cos(4*x) + 2*sin(4*x) - 5))/80
```

3.548 $\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 36

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx=\frac{e^{3x}}{24}-\frac{1}{120}e^{3x}\cos(6x)-\frac{1}{60}e^{3x}\sin(6x)$$

```
[Out] 1/24*exp(3*x)-1/120*exp(3*x)*cos(6*x)-1/60*exp(3*x)*sin(6*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4557, 2225, 4518}

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx=\frac{e^{3x}}{24}-\frac{1}{60}e^{3x}\sin(6x)-\frac{1}{120}e^{3x}\cos(6x)$$

```
[In] Int[E^(3*x)*Cos[(3*x)/2]^2*Sin[(3*x)/2]^2,x]
```

```
[Out] E^(3*x)/24 - (E^(3*x)*Cos[6*x])/120 - (E^(3*x)*Sin[6*x])/60
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
```

```
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{e^{3x}}{8} - \frac{1}{8}e^{3x}\cos(6x)\right) dx \\ &= \frac{1}{8}\int e^{3x}\,dx - \frac{1}{8}\int e^{3x}\cos(6x)\,dx \\ &= \frac{e^{3x}}{24} - \frac{1}{120}e^{3x}\cos(6x) - \frac{1}{60}e^{3x}\sin(6x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx = -\frac{1}{120}e^{3x}(-5+\cos(6x)+2\sin(6x))$$

```
[In] Integrate[E^(3*x)*Cos[(3*x)/2]^2*Sin[(3*x)/2]^2,x]
[Out] -1/120*(E^(3*x)*(-5 + Cos[6*x] + 2*Sin[6*x]))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

method	result
parallelrisch	$-\frac{e^{3x}(-5+2\sin(6x)+\cos(6x))}{120}$
risch	$\frac{e^{3x}}{24}-\frac{e^{(3+6i)x}}{240}+\frac{i e^{(3+6i)x}}{120}-\frac{e^{(3-6i)x}}{240}-\frac{i e^{(3-6i)x}}{120}$
default	$-\frac{4(3\cos(x)+6\sin(x))e^{3x}(\cos^5(x))}{45}+\frac{2(3\cos(x)+4\sin(x))e^{3x}(\cos^3(x))}{15}-\frac{(3\cos(x)+2\sin(x))e^{3x}\cos(x)}{20}+\frac{e^{3x}}{20}$
norman	$\frac{-\frac{2e^{3x}\tan\left(\frac{3x}{4}\right)}{15}+\frac{2e^{3x}\left(\tan^2\left(\frac{3x}{4}\right)\right)}{5}+\frac{14e^{3x}\left(\tan^3\left(\frac{3x}{4}\right)\right)}{15}-\frac{e^{3x}\left(\tan^4\left(\frac{3x}{4}\right)\right)}{3}-\frac{14e^{3x}\left(\tan^5\left(\frac{3x}{4}\right)\right)}{15}+\frac{2e^{3x}\left(\tan^6\left(\frac{3x}{4}\right)\right)}{5}+\frac{2e^{3x}\left(\tan^7\left(\frac{3x}{4}\right)\right)}{15}}{\left(1+\tan^2\left(\frac{3x}{4}\right)\right)^4}$

```
[In] int(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*exp(3*x)*(-5+2*sin(6*x)+cos(6*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx = -\frac{1}{15}\left(2\,\cos\left(\frac{3}{2}\,x\right)^3 - \cos\left(\frac{3}{2}\,x\right)\right)e^{(3\,x)}\sin\left(\frac{3}{2}\,x\right) - \frac{1}{30}\left(2\,\cos\left(\frac{3}{2}\,x\right)^4 - 2\,\cos\left(\frac{3}{2}\,x\right)^2 - 1\right)e^{(3\,x)}$$

```
[In] integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(3/2*x)^3 - cos(3/2*x))*e^(3*x)*sin(3/2*x) - 1/30*(2*cos(3/2*x)
^4 - 2*cos(3/2*x)^2 - 1)*e^(3*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(29) = 58.

Time = 0.54 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.75

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx = \frac{e^{3x}\sin^4\left(\frac{3x}{2}\right)}{30} + \frac{e^{3x}\sin^3\left(\frac{3x}{2}\right)\cos\left(\frac{3x}{2}\right)}{15} + \frac{2e^{3x}\sin^2\left(\frac{3x}{2}\right)\cos^2\left(\frac{3x}{2}\right)}{15} - \frac{e^{3x}\sin\left(\frac{3x}{2}\right)\cos^3\left(\frac{3x}{2}\right)}{15} + \frac{e^{3x}\cos^4\left(\frac{3x}{2}\right)}{30}$$

```
[In] integrate(exp(3*x)*cos(3/2*x)**2*sin(3/2*x)**2,x)
```

```
[Out] exp(3*x)*sin(3*x/2)**4/30 + exp(3*x)*sin(3*x/2)**3*cos(3*x/2)/15 + 2*exp(3*
x)*sin(3*x/2)**2*cos(3*x/2)**2/15 - exp(3*x)*sin(3*x/2)*cos(3*x/2)**3/15 +
exp(3*x)*cos(3*x/2)**4/30
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx = -\frac{1}{120}\,\cos\left(6\,x\right)e^{(3\,x)} - \frac{1}{60}\,e^{(3\,x)}\sin\left(6\,x\right) + \frac{1}{24}\,e^{(3\,x)}$$

```
[In] integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="maxima")
[Out] -1/120*cos(6*x)*e^(3*x) - 1/60*e^(3*x)*sin(6*x) + 1/24*e^(3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx = -\frac{1}{120}\,(\cos\left(6\,x\right) + 2\,\sin\left(6\,x\right))e^{(3\,x)} + \frac{1}{24}\,e^{(3\,x)}$$

```
[In] integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="giac")
[Out] -1/120*(cos(6*x) + 2*sin(6*x))*e^(3*x) + 1/24*e^(3*x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.50

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx = -\frac{\mathrm{e}^{3\,x}\,\left(\cos\left(6\,x\right) + 2\,\sin\left(6\,x\right) - 5\right)}{120}$$

```
[In] int(cos((3*x)/2)^2*sin((3*x)/2)^2*exp(3*x),x)
[Out] -(exp(3*x)*(cos(6*x) + 2*sin(6*x) - 5))/120
```

3.549 $\int e^{mx} \tan^2(x)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 58

$$\int e^{mx} \tan^2(x)\, dx = -\frac{e^{mx}}{m} + \frac{4e^{(2i+m)x} \operatorname{Hypergeometric2F1}\left(2, 1-\frac{im}{2}, 2-\frac{im}{2}, -e^{2ix}\right)}{2i+m}$$

```
[Out] -exp(m*x)/m+4*exp((2*I+m)*x)*hypergeom([2, 1-1/2*I*m],[2-1/2*I*m],-exp(2*I*
x))/(2*I+m)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.47, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4527, 2225, 2283}

$$\int e^{mx} \tan^2(x)\, dx = \frac{4e^{mx} \operatorname{Hypergeometric2F1}\left(1, -\frac{im}{2}, 1-\frac{im}{2}, -e^{2ix}\right)}{m} - \frac{4e^{mx} \operatorname{Hypergeometric2F1}\left(2, -\frac{im}{2}, 1-\frac{im}{2}, -e^{2ix}\right)}{m} - \frac{e^{mx}}{m}$$

```
[In] Int[E^(m*x)*Tan[x]^2,x]
```

```
[Out] -(E^(m*x)/m) + (4*E^(m*x)*Hypergeometric2F1[1, (-1/2*I)*m, 1 - (I/2)*m, -E^
((2*I)*x)])/m - (4*E^(m*x)*Hypergeometric2F1[2, (-1/2*I)*m, 1 - (I/2)*m, -E
^((2*I)*x)])/m
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \left(e^{mx} + \frac{4e^{mx}}{(1+e^{2ix})^2} - \frac{4e^{mx}}{1+e^{2ix}}\right) dx \\
&= -\left(4\int \frac{e^{mx}}{(1+e^{2ix})^2}\,dx\right) + 4\int \frac{e^{mx}}{1+e^{2ix}}\,dx - \int e^{mx}\,dx \\
&= -\frac{e^{mx}}{m} + \frac{4e^{mx}\,\text{Hypergeometric2F1}\left(1,-\frac{im}{2},1-\frac{im}{2},-e^{2ix}\right)}{m} \\
&\quad - \frac{4e^{mx}\,\text{Hypergeometric2F1}\left(2,-\frac{im}{2},1-\frac{im}{2},-e^{2ix}\right)}{m}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.67

$$\int e^{mx}\tan^2(x)\,dx$$
$$= \frac{e^{mx}\left(-1 + \frac{ie^{2ix}m^2\,\text{Hypergeometric2F1}\left(1,1-\frac{im}{2},2-\frac{im}{2},-e^{2ix}\right)}{2i+m} - im\,\text{Hypergeometric2F1}\left(1,-\frac{im}{2},1-\frac{im}{2},-e^{2ix}\right) + m\,\text{ta}\right.}{m}$$

```
[In] Integrate[E^(m*x)*Tan[x]^2,x]
```

```
[Out] (E^(m*x)*(-1 + (I*E^((2*I)*x)*m^2*Hypergeometric2F1[1, 1 - (I/2)*m, 2 - (I/
2)*m, -E^((2*I)*x)])/(2*I + m) - I*m*Hypergeometric2F1[1, (-1/2*I)*m, 1 - (
I/2)*m, -E^((2*I)*x)] + m*Tan[x]))/m
```

Maple [F]

$$\int e^{mx} \left(\tan^2(x)\right) dx$$

[In] int(exp(m*x)*tan(x)^2,x)

[Out] int(exp(m*x)*tan(x)^2,x)

Fricas [F]

$$\int e^{mx} \tan^2(x)\, dx = \int e^{(mx)} \tan(x)^2\, dx$$

[In] integrate(exp(m*x)*tan(x)^2,x, algorithm="fricas")

[Out] integral(e^(m*x)*tan(x)^2, x)

Sympy [F]

$$\int e^{mx} \tan^2(x)\, dx = \int e^{mx} \tan^2(x)\, dx$$

[In] integrate(exp(m*x)*tan(x)**2,x)

[Out] Integral(exp(m*x)*tan(x)**2, x)

Maxima [F]

$$\int e^{mx} \tan^2(x)\, dx = \int e^{(mx)} \tan(x)^2\, dx$$

[In] integrate(exp(m*x)*tan(x)^2,x, algorithm="maxima")

[Out] -((m^4 + 20*m^2 + 64)*cos(4*x)^2*e^(m*x) - 4*(m^4 + 12*m^2 - 64)*cos(2*x)^2*e^(m*x) + (m^4 + 20*m^2 + 64)*e^(m*x)*sin(4*x)^2 - 4*(m^4 + 12*m^2 - 64)*e^(m*x)*sin(2*x)^2 - 16*(11*m^2 - 16)*cos(2*x)*e^(m*x) + 8*(5*m^3 - 16*m)*e^(m*x)*sin(2*x) + 2*(8*(m^2 + 16)*cos(2*x)*e^(m*x) + 4*(m^3 + 16*m)*e^(m*x)*sin(2*x) + (m^4 - 28*m^2 + 64)*e^(m*x))*cos(4*x) + (m^4 - 76*m^2 + 64)*e^(m*x) - 16*(m^6 + 20*m^4 + (m^6 + 20*m^4 + 64*m^2)*cos(4*x)^2 + 4*(m^6 + 20*m^4 + 64*m^2)*cos(2*x)^2 + (m^6 + 20*m^4 + 64*m^2)*sin(4*x)^2 + 4*(m^6 + 20*m^4 + 64*m^2)*sin(4*x)*sin(2*x) + 4*(m^6 + 20*m^4 + 64*m^2)*sin(2*x)^2 + 64*m^2 + 2*(m^6 + 20*m^4 + 64*m^2 + 2*(m^6 + 20*m^4 + 64*m^2)*cos(2*x))*cos(4

```
*x) + 4*(m^6 + 20*m^4 + 64*m^2)*cos(2*x))*integrate(-(6*m*cos(6*x)*e^(m*x)
+ 18*m*cos(4*x)*e^(m*x) + 18*m*cos(2*x)*e^(m*x) - (m^2 - 8)*e^(m*x)*sin(6*x
) - 3*(m^2 - 8)*e^(m*x)*sin(4*x) - 3*(m^2 - 8)*e^(m*x)*sin(2*x) + 6*m*e^(m*
x))/(m^4 + (m^4 + 20*m^2 + 64)*cos(6*x)^2 + 9*(m^4 + 20*m^2 + 64)*cos(4*x)^
2 + 9*(m^4 + 20*m^2 + 64)*cos(2*x)^2 + (m^4 + 20*m^2 + 64)*sin(6*x)^2 + 9*(
m^4 + 20*m^2 + 64)*sin(4*x)^2 + 18*(m^4 + 20*m^2 + 64)*sin(4*x)*sin(2*x) +
9*(m^4 + 20*m^2 + 64)*sin(2*x)^2 + 20*m^2 + 2*(m^4 + 20*m^2 + 3*(m^4 + 20*m
^2 + 64)*cos(4*x) + 3*(m^4 + 20*m^2 + 64)*cos(2*x) + 64)*cos(6*x) + 6*(m^4
+ 20*m^2 + 3*(m^4 + 20*m^2 + 64)*cos(2*x) + 64)*cos(4*x) + 6*(m^4 + 20*m^2
+ 64)*cos(2*x) + 6*((m^4 + 20*m^2 + 64)*sin(4*x) + (m^4 + 20*m^2 + 64)*sin(
2*x))*sin(6*x) + 64), x) - 8*((m^3 + 16*m)*cos(2*x)*e^(m*x) - 2*(m^2 + 16)*
e^(m*x)*sin(2*x) - 2*(m^3 - 8*m)*e^(m*x))*sin(4*x))/(m^5 + 20*m^3 + (m^5 +
20*m^3 + 64*m)*cos(4*x)^2 + 4*(m^5 + 20*m^3 + 64*m)*cos(2*x)^2 + (m^5 + 20*
m^3 + 64*m)*sin(4*x)^2 + 4*(m^5 + 20*m^3 + 64*m)*sin(4*x)*sin(2*x) + 4*(m^5
 + 20*m^3 + 64*m)*sin(2*x)^2 + 2*(m^5 + 20*m^3 + 2*(m^5 + 20*m^3 + 64*m)*co
s(2*x) + 64*m)*cos(4*x) + 4*(m^5 + 20*m^3 + 64*m)*cos(2*x) + 64*m)
```

Giac [F]

$$\int e^{mx}\tan^2(x)\,dx = \int e^{(mx)}\tan\left(x\right)^2\,dx$$

```
[In] integrate(exp(m*x)*tan(x)^2,x, algorithm="giac")
[Out] integrate(e^(m*x)*tan(x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{mx}\tan^2(x)\,dx = \int e^{m\,x}\,{\tan\left(x\right)}^2\,dx$$

```
[In] int(exp(m*x)*tan(x)^2,x)
[Out] int(exp(m*x)*tan(x)^2, x)
```

3.550 $\int e^{mx}\csc^2(x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 45

$$\int e^{mx}\csc^2(x)\,dx = -\frac{4e^{(2i+m)x}\,\text{Hypergeometric2F1}\left(2,1-\frac{im}{2},2-\frac{im}{2},e^{2ix}\right)}{2i+m}$$

```
[Out] -4*exp((2*I+m)*x)*hypergeom([2, 1-1/2*I*m],[2-1/2*I*m],exp(2*I*x))/(2*I+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4538}

$$\int e^{mx}\csc^2(x)\,dx = -\frac{4e^{(m+2i)x}\,\text{Hypergeometric2F1}\left(2,1-\frac{im}{2},2-\frac{im}{2},e^{2ix}\right)}{m+2i}$$

```
[In] Int[E^(m*x)*Csc[x]^2,x]
[Out] (-4*E^((2*I + m)*x)*Hypergeometric2F1[2, 1 - (I/2)*m, 2 - (I/2)*m, E^((2*I)
*x)])/(2*I + m)
```

Rule 4538

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rubi steps

$$\text{integral} = -\frac{4e^{(2i+m)x}\,\text{Hypergeometric2F1}\left(2,1-\frac{im}{2},2-\frac{im}{2},e^{2ix}\right)}{2i+m}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.00

$$\int e^{mx}\csc^2(x)\,dx$$
$$=\frac{e^{mx}\left(e^{2ix}m\,\text{Hypergeometric2F1}\left(1,1-\frac{im}{2},2-\frac{im}{2},e^{2ix}\right)+(2i+m)\left(-i\cot(x)+\text{Hypergeometric2F1}\left(1,-\right.\right.\right.}{-2+im}$$

```
[In] Integrate[E^(m*x)*Csc[x]^2,x]
[Out] (E^(m*x)*(E^((2*I)*x)*m*Hypergeometric2F1[1, 1 - (I/2)*m, 2 - (I/2)*m, E^((
2*I)*x)] + (2*I + m)*((-I)*Cot[x] + Hypergeometric2F1[1, (-1/2*I)*m, 1 - (I
/2)*m, E^((2*I)*x)])))/(-2 + I*m)
```

Maple [F]

$$\int \frac{e^{mx}}{\sin(x)^2}dx$$

```
[In] int(exp(m*x)/sin(x)^2,x)
[Out] int(exp(m*x)/sin(x)^2,x)
```

Fricas [F]

$$\int e^{mx}\csc^2(x)\,dx=\int \frac{e^{(mx)}}{\sin(x)^2}\,dx$$

```
[In] integrate(exp(m*x)/sin(x)^2,x, algorithm="fricas")
[Out] integral(-e^(m*x)/(cos(x)^2 - 1), x)
```

Sympy [F]

$$\int e^{mx}\csc^2(x)\,dx=\int \frac{e^{mx}}{\sin^2(x)}\,dx$$

```
[In] integrate(exp(m*x)/sin(x)**2,x)
[Out] Integral(exp(m*x)/sin(x)**2, x)
```

Maxima [F]

$$\int e^{mx}\csc^2(x)\,dx = \int \frac{e^{(mx)}}{\sin(x)^2}\,dx$$

```
[In] integrate(exp(m*x)/sin(x)^2,x, algorithm="maxima")
```

```
[Out] 4*(2*(m^3 + 16*m)*cos(2*x)^2*e^(m*x) + 2*(m^3 + 16*m)*e^(m*x)*sin(2*x)^2 -
(m^3 + 64*m)*cos(2*x)*e^(m*x) + 2*(5*m^2 - 16)*e^(m*x)*sin(2*x) - ((m^3 + 1
6*m)*cos(2*x)*e^(m*x) - 2*(m^2 + 16)*e^(m*x)*sin(2*x) - 24*m*e^(m*x))*cos(4
*x) + 24*m*e^(m*x) - 4*(m^5 + 20*m^3 + (m^5 + 20*m^3 + 64*m)*cos(4*x)^2 + 4
*(m^5 + 20*m^3 + 64*m)*cos(2*x)^2 + (m^5 + 20*m^3 + 64*m)*sin(4*x)^2 - 4*(m
^5 + 20*m^3 + 64*m)*sin(4*x)*sin(2*x) + 4*(m^5 + 20*m^3 + 64*m)*sin(2*x)^2
+ 2*(m^5 + 20*m^3 - 2*(m^5 + 20*m^3 + 64*m)*cos(2*x) + 64*m)*cos(4*x) - 4*(
m^5 + 20*m^3 + 64*m)*cos(2*x) + 64*m)*integrate(-(6*m*cos(6*x)*e^(m*x) - 18
*m*cos(4*x)*e^(m*x) + 18*m*cos(2*x)*e^(m*x) - (m^2 - 8)*e^(m*x)*sin(6*x) +
3*(m^2 - 8)*e^(m*x)*sin(4*x) - 3*(m^2 - 8)*e^(m*x)*sin(2*x) - 6*m*e^(m*x))/
(m^4 + (m^4 + 20*m^2 + 64)*cos(6*x)^2 + 9*(m^4 + 20*m^2 + 64)*cos(4*x)^2 +
9*(m^4 + 20*m^2 + 64)*cos(2*x)^2 + (m^4 + 20*m^2 + 64)*sin(6*x)^2 + 9*(m^4
+ 20*m^2 + 64)*sin(4*x)^2 - 18*(m^4 + 20*m^2 + 64)*sin(4*x)*sin(2*x) + 9*(m
^4 + 20*m^2 + 64)*sin(2*x)^2 + 20*m^2 - 2*(m^4 + 20*m^2 + 3*(m^4 + 20*m^2 +
 64)*cos(4*x) - 3*(m^4 + 20*m^2 + 64)*cos(2*x) + 64)*cos(6*x) + 6*(m^4 + 20
*m^2 - 3*(m^4 + 20*m^2 + 64)*cos(2*x) + 64)*cos(4*x) - 6*(m^4 + 20*m^2 + 64
)*cos(2*x) - 6*((m^4 + 20*m^2 + 64)*sin(4*x) - (m^4 + 20*m^2 + 64)*sin(2*x)
)*sin(6*x) + 64), x) - (2*(m^2 + 16)*cos(2*x)*e^(m*x) + (m^3 + 16*m)*e^(m*x
)*sin(2*x) + 4*(m^2 - 8)*e^(m*x))*sin(4*x))/(m^4 + (m^4 + 20*m^2 + 64)*cos(
4*x)^2 + 4*(m^4 + 20*m^2 + 64)*cos(2*x)^2 + (m^4 + 20*m^2 + 64)*sin(4*x)^2
- 4*(m^4 + 20*m^2 + 64)*sin(4*x)*sin(2*x) + 4*(m^4 + 20*m^2 + 64)*sin(2*x)^
2 + 20*m^2 + 2*(m^4 + 20*m^2 - 2*(m^4 + 20*m^2 + 64)*cos(2*x) + 64)*cos(4*x
) - 4*(m^4 + 20*m^2 + 64)*cos(2*x) + 64)
```

Giac [F]

$$\int e^{mx}\csc^2(x)\,dx = \int \frac{e^{(mx)}}{\sin(x)^2}\,dx$$

```
[In] integrate(exp(m*x)/sin(x)^2,x, algorithm="giac")
```

```
[Out] integrate(e^(m*x)/sin(x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{mx}\csc^2(x)\,dx=\int\frac{\mathrm{e}^{m\,x}}{\sin\left(x\right)^2}\,dx$$

```
[In] int(exp(m*x)/sin(x)^2,x)
[Out] int(exp(m*x)/sin(x)^2, x)
```

3.551 $\int e^{mx}\sec^3(x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 51

$$\int e^{mx}\sec^3(x)\,dx=\frac{8e^{(3i+m)x}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}(3-im),\frac{1}{2}(5-im),-e^{2ix}\right)}{3i+m}$$

```
[Out] 8*exp((3*I+m)*x)*hypergeom([3, 3/2-1/2*I*m],[5/2-1/2*I*m],-exp(2*I*x))/(3*I
+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4533, 4536}

$$\int e^{mx}\sec^3(x)\,dx=(-m+i)\left(-e^{(m+i)x}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-im),\frac{1}{2}(3-im),-e^{2ix}\right)-\frac{1}{2}me^{mx}\sec(x)+\frac{1}{2}e^{mx}\tan(x)\sec(x)$$

```
[In] Int[E^(m*x)*Sec[x]^3,x]
```

```
[Out] -(E^((I + m)*x)*(I - m)*Hypergeometric2F1[1, (1 - I*m)/2, (3 - I*m)/2, -E^(
(2*I)*x)]) - (E^(m*x)*m*Sec[x])/2 + (E^(m*x)*Sec[x]*Tan[x])/2
```

Rule 4533

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2)), Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x] + Simp[F^(c*(a + b*x
```

```
))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b,
c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && Ne
Q[n, 2]
```

Rule 4536

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2}e^{mx}m\sec(x)+\frac{1}{2}e^{mx}\sec(x)\tan(x)+\frac{1}{2}(1+m^2)\int e^{mx}\sec(x)\,dx\\ &= -e^{(i+m)x}(i-m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-im),\frac{1}{2}(3-im),-e^{2ix}\right)\\ &\quad-\frac{1}{2}e^{mx}m\sec(x)+\frac{1}{2}e^{mx}\sec(x)\tan(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int e^{mx}\sec^3(x)\,dx=\frac{1}{2}e^{mx}\left(2e^{ix}(-i+m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{im}{2},\frac{3}{2}-\frac{im}{2},-e^{2ix}\right)+\sec(x)(-m+\tan(x))\right)$$

```
[In] Integrate[E^(m*x)*Sec[x]^3,x]
[Out] (E^(m*x)*(2*E^(I*x)*(-I + m)*Hypergeometric2F1[1, 1/2 - (I/2)*m, 3/2 - (I/2
)*m, -E^((2*I)*x)] + Sec[x]*(-m + Tan[x])))/2
```

Maple [F]

$$\int \frac{e^{mx}}{\cos(x)^3}dx$$

```
[In] int(exp(m*x)/cos(x)^3,x)
[Out] int(exp(m*x)/cos(x)^3,x)
```

Fricas [F]

$$\int e^{mx} \sec^3(x)\,dx = \int \frac{e^{(mx)}}{\cos(x)^3}\,dx$$

```
[In] integrate(exp(m*x)/cos(x)^3,x, algorithm="fricas")
[Out] integral(e^(m*x)/cos(x)^3, x)
```

Sympy [F]

$$\int e^{mx} \sec^3(x)\,dx = \int \frac{e^{mx}}{\cos^3(x)}\,dx$$

```
[In] integrate(exp(m*x)/cos(x)**3,x)
[Out] Integral(exp(m*x)/cos(x)**3, x)
```

Maxima [F]

$$\int e^{mx} \sec^3(x)\,dx = \int \frac{e^{(mx)}}{\cos(x)^3}\,dx$$

```
[In] integrate(exp(m*x)/cos(x)^3,x, algorithm="maxima")
[Out] 8*(48*m*cos(x)*e^(m*x) + 6*(m^2 - 15)*e^(m*x)*sin(x) + ((m^3 + 25*m)*cos(3*
x)*e^(m*x) + 48*m*cos(x)*e^(m*x) - 3*(m^2 + 25)*e^(m*x)*sin(3*x) + 6*(m^2 -
 15)*e^(m*x)*sin(x))*cos(6*x) + 3*((m^3 + 25*m)*cos(3*x)*e^(m*x) + 48*m*cos
(x)*e^(m*x) - 3*(m^2 + 25)*e^(m*x)*sin(3*x) + 6*(m^2 - 15)*e^(m*x)*sin(x))*
cos(4*x) + (3*(m^3 + 25*m)*cos(2*x)*e^(m*x) + 9*(m^2 + 25)*e^(m*x)*sin(2*x)
 + (m^3 + 25*m)*e^(m*x))*cos(3*x) + 18*(8*m*cos(x)*e^(m*x) + (m^2 - 15)*e^(
m*x)*sin(x))*cos(2*x) - 6*(m^4 + (m^4 + 34*m^2 + 225)*cos(6*x)^2 + 9*(m^4 +
 34*m^2 + 225)*cos(4*x)^2 + 9*(m^4 + 34*m^2 + 225)*cos(2*x)^2 + (m^4 + 34*m
^2 + 225)*sin(6*x)^2 + 9*(m^4 + 34*m^2 + 225)*sin(4*x)^2 + 18*(m^4 + 34*m^2
 + 225)*sin(4*x)*sin(2*x) + 9*(m^4 + 34*m^2 + 225)*sin(2*x)^2 + 34*m^2 + 2*
(m^4 + 34*m^2 + 3*(m^4 + 34*m^2 + 225)*cos(4*x) + 3*(m^4 + 34*m^2 + 225)*co
s(2*x) + 225)*cos(6*x) + 6*(m^4 + 34*m^2 + 3*(m^4 + 34*m^2 + 225)*cos(2*x)
+ 225)*cos(4*x) + 6*(m^4 + 34*m^2 + 225)*cos(2*x) + 6*((m^4 + 34*m^2 + 225)
*sin(4*x) + (m^4 + 34*m^2 + 225)*sin(2*x))*sin(6*x) + 225)*integrate(((m^2
- 15)*cos(x)*e^(m*x) - 8*m*e^(m*x)*sin(x) + ((m^2 - 15)*cos(x)*e^(m*x) - 8*
m*e^(m*x)*sin(x))*cos(8*x) + 4*((m^2 - 15)*cos(x)*e^(m*x) - 8*m*e^(m*x)*sin
```

(x))*cos(6*x) + 6*((m^2 - 15)*cos(x)*e^(m*x) - 8*m*e^(m*x)*sin(x))*cos(4*x) + 4*((m^2 - 15)*cos(x)*e^(m*x) - 8*m*e^(m*x)*sin(x))*cos(2*x) + (8*m*cos(x)*e^(m*x) + (m^2 - 15)*e^(m*x)*sin(x))*sin(8*x) + 4*(8*m*cos(x)*e^(m*x) + (m^2 - 15)*e^(m*x)*sin(x))*sin(6*x) + 6*(8*m*cos(x)*e^(m*x) + (m^2 - 15)*e^(m*x)*sin(x))*sin(4*x) + 4*(8*m*cos(x)*e^(m*x) + (m^2 - 15)*e^(m*x)*sin(x))*sin(2*x))/(m^4 + (m^4 + 34*m^2 + 225)*cos(8*x)^2 + 16*(m^4 + 34*m^2 + 225)*cos(6*x)^2 + 36*(m^4 + 34*m^2 + 225)*cos(4*x)^2 + 16*(m^4 + 34*m^2 + 225)*cos(2*x)^2 + (m^4 + 34*m^2 + 225)*sin(8*x)^2 + 16*(m^4 + 34*m^2 + 225)*sin(6*x)^2 + 36*(m^4 + 34*m^2 + 225)*sin(4*x)^2 + 48*(m^4 + 34*m^2 + 225)*sin(4*x)*sin(2*x) + 16*(m^4 + 34*m^2 + 225)*sin(2*x)^2 + 34*m^2 + 2*(m^4 + 34*m^2 + 4*(m^4 + 34*m^2 + 225)*cos(6*x) + 6*(m^4 + 34*m^2 + 225)*cos(4*x) + 4*(m^4 + 34*m^2 + 225)*cos(2*x) + 225)*cos(8*x) + 8*(m^4 + 34*m^2 + 6*(m^4 + 34*m^2 + 225)*cos(4*x) + 4*(m^4 + 34*m^2 + 225)*cos(2*x) + 225)*cos(6*x) + 12*(m^4 + 34*m^2 + 4*(m^4 + 34*m^2 + 225)*cos(2*x) + 225)*cos(4*x) + 8*(m^4 + 34*m^2 + 225)*cos(2*x) + 4*(2*(m^4 + 34*m^2 + 225)*sin(6*x) + 3*(m^4 + 34*m^2 + 225)*sin(4*x) + 2*(m^4 + 34*m^2 + 225)*sin(2*x))*sin(8*x) + 16*(3*(m^4 + 34*m^2 + 225)*sin(4*x) + 2*(m^4 + 34*m^2 + 225)*sin(2*x))*sin(6*x) + 225), x) - 6*(m^5 + 34*m^3 + (m^5 + 34*m^3 + 225*m)*cos(6*x)^2 + 9*(m^5 + 34*m^3 + 225*m)*cos(4*x)^2 + 9*(m^5 + 34*m^3 + 225*m)*cos(2*x)^2 + (m^5 + 34*m^3 + 225*m)*sin(6*x)^2 + 9*(m^5 + 34*m^3 + 225*m)*sin(4*x)^2 + 18*(m^5 + 34*m^3 + 225*m)*sin(4*x)*sin(2*x) + 9*(m^5 + 34*m^3 + 225*m)*sin(2*x)^2 + 2*(m^5 + 34*m^3 + 3*(m^5 + 34*m^3 + 225*m)*cos(4*x) + 3*(m^5 + 34*m^3 + 225*m)*cos(2*x) + 225*m)*cos(6*x) + 6*(m^5 + 34*m^3 + 3*(m^5 + 34*m^3 + 225*m)*cos(2*x) + 225*m)*cos(4*x) + 6*(m^5 + 34*m^3 + 225*m)*cos(2*x) + 6*((m^5 + 34*m^3 + 225*m)*sin(4*x) + (m^5 + 34*m^3 + 225*m)*sin(2*x))*sin(6*x) + 225*m)*integrate((8*m*cos(x)*e^(m*x) + (m^2 - 15)*e^(m*x)*sin(x) + (8*m*cos(x)*e^(m*x) + (m^2 - 15)*e^(m*x)*sin(x))*cos(8*x) + 4*(8*m*cos(x)*e^(m*x) + (m^2 - 15)*e^(m*x)*sin(x))*cos(6*x) + 6*(8*m*cos(x)*e^(m*x) + (m^2 - 15)*e^(m*x)*sin(x))*cos(4*x) + 4*(8*m*cos(x)*e^(m*x) + (m^2 - 15)*e^(m*x)*sin(x))*cos(2*x) - ((m^2 - 15)*cos(x)*e^(m*x) - 8*m*e^(m*x)*sin(x))*sin(8*x) - 4*((m^2 - 15)*cos(x)*e^(m*x) - 8*m*e^(m*x)*sin(x))*sin(6*x) - 6*((m^2 - 15)*cos(x)*e^(m*x) - 8*m*e^(m*x)*sin(x))*sin(4*x) - 4*((m^2 - 15)*cos(x)*e^(m*x) - 8*m*e^(m*x)*sin(x))*sin(2*x))/(m^4 + (m^4 + 34*m^2 + 225)*cos(8*x)^2 + 16*(m^4 + 34*m^2 + 225)*cos(6*x)^2 + 36*(m^4 + 34*m^2 + 225)*cos(4*x)^2 + 16*(m^4 + 34*m^2 + 225)*cos(2*x)^2 + (m^4 + 34*m^2 + 225)*sin(8*x)^2 + 16*(m^4 + 34*m^2 + 225)*sin(6*x)^2 + 36*(m^4 + 34*m^2 + 225)*sin(4*x)^2 + 48*(m^4 + 34*m^2 + 225)*sin(4*x)*sin(2*x) + 16*(m^4 + 34*m^2 + 225)*sin(2*x)^2 + 34*m^2 + 2*(m^4 + 34*m^2 + 4*(m^4 + 34*m^2 + 225)*cos(6*x) + 6*(m^4 + 34*m^2 + 225)*cos(4*x) + 4*(m^4 + 34*m^2 + 225)*cos(2*x) + 225)*cos(8*x) + 8*(m^4 + 34*m^2 + 6*(m^4 + 34*m^2 + 225)*cos(4*x) + 4*(m^4 + 34*m^2 + 225)*cos(2*x) + 225)*cos(6*x) + 12*(m^4 + 34*m^2 + 4*(m^4 + 34*m^2 + 225)*cos(2*x) + 225)*cos(4*x) + 8*(m^4 + 34*m^2 + 225)*cos(2*x) + 4*(2*(m^4 + 34*m^2 + 225)*sin(6*x) + 3*(m^4 + 34*m^2 + 225)*sin(4*x) + 2*(m^4 + 34*m^2 + 225)*sin(2*x))*sin(8*x) + 16*(3*(m^4 + 34*m^2 + 225)*sin(4*x) + 2*(m^4 + 34*m^2 + 225)*sin(2*x))*sin(6*x) + 225), x) + (3*(m^2 + 25)*cos(3*x)*e^(m*x) - 6*(m^2 - 15)*cos

```
(x)*e^(m*x) + (m^3 + 25*m)*e^(m*x)*sin(3*x) + 48*m*e^(m*x)*sin(x))*sin(6*x)
 + 3*(3*(m^2 + 25)*cos(3*x)*e^(m*x) - 6*(m^2 - 15)*cos(x)*e^(m*x) + (m^3 +
25*m)*e^(m*x)*sin(3*x) + 48*m*e^(m*x)*sin(x))*sin(4*x) - 3*(3*(m^2 + 25)*co
s(2*x)*e^(m*x) - (m^3 + 25*m)*e^(m*x)*sin(2*x) + (m^2 + 25)*e^(m*x))*sin(3*
x) - 18*((m^2 - 15)*cos(x)*e^(m*x) - 8*m*e^(m*x)*sin(x))*sin(2*x))/(m^4 + (
m^4 + 34*m^2 + 225)*cos(6*x)^2 + 9*(m^4 + 34*m^2 + 225)*cos(4*x)^2 + 9*(m^4
 + 34*m^2 + 225)*cos(2*x)^2 + (m^4 + 34*m^2 + 225)*sin(6*x)^2 + 9*(m^4 + 34
*m^2 + 225)*sin(4*x)^2 + 18*(m^4 + 34*m^2 + 225)*sin(4*x)*sin(2*x) + 9*(m^4
 + 34*m^2 + 225)*sin(2*x)^2 + 34*m^2 + 2*(m^4 + 34*m^2 + 3*(m^4 + 34*m^2 +
225)*cos(4*x) + 3*(m^4 + 34*m^2 + 225)*cos(2*x) + 225)*cos(6*x) + 6*(m^4 +
34*m^2 + 3*(m^4 + 34*m^2 + 225)*cos(2*x) + 225)*cos(4*x) + 6*(m^4 + 34*m^2
+ 225)*cos(2*x) + 6*((m^4 + 34*m^2 + 225)*sin(4*x) + (m^4 + 34*m^2 + 225)*s
in(2*x))*sin(6*x) + 225)
```

Giac [F]

$$\int e^{mx}\sec^3(x)\,dx = \int \frac{e^{(mx)}}{\cos(x)^3}\,dx$$

```
[In] integrate(exp(m*x)/cos(x)^3,x, algorithm="giac")
[Out] integrate(e^(m*x)/cos(x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{mx}\sec^3(x)\,dx = \int \frac{e^{m\,x}}{\cos(x)^3}\,dx$$

```
[In] int(exp(m*x)/cos(x)^3,x)
[Out] int(exp(m*x)/cos(x)^3, x)
```

3.552 $\int \frac{e^x}{1+\cos(x)} dx$

Optimal result

Integrand size = 10, antiderivative size = 28

$$\int \frac{e^x}{1+\cos(x)} dx = (1-i)e^{(1+i)x} \text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{ix}\right)$$

```
[Out] (1-I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],-exp(I*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4542, 4536}

$$\int \frac{e^x}{1+\cos(x)} dx = (1-i)e^{(1+i)x} \text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{ix}\right)$$

```
[In] Int[E^x/(1 + Cos[x]),x]
[Out] (1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*x)]
```

Rule 4536

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 4542

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 + e*(x/2)]^
```

```
(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && ILt
Q[n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\int e^x \sec^2\left(\frac{x}{2}\right)\,dx \\ &= (1-i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{ix}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{1+\cos(x)}\,dx = (1-i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{ix}\right)$$

```
[In] Integrate[E^x/(1 + Cos[x]),x]
[Out] (1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*x)]
```

Maple [F]

$$\int \frac{e^x}{\cos(x)+1}dx$$

```
[In] int(exp(x)/(cos(x)+1),x)
[Out] int(exp(x)/(cos(x)+1),x)
```

Fricas [F]

$$\int \frac{e^x}{1+\cos(x)}\,dx = \int \frac{e^x}{\cos(x)+1}\,dx$$

```
[In] integrate(exp(x)/(1+cos(x)),x, algorithm="fricas")
[Out] integral(e^x/(cos(x) + 1), x)
```

Sympy [F]

$$\int \frac{e^x}{1 + \cos(x)} \, dx = \int \frac{e^x}{\cos(x) + 1} \, dx$$

```
[In] integrate(exp(x)/(1+cos(x)),x)
[Out] Integral(exp(x)/(cos(x) + 1), x)
```

Maxima [F]

$$\int \frac{e^x}{1 + \cos(x)} \, dx = \int \frac{e^x}{\cos(x) + 1} \, dx$$

```
[In] integrate(exp(x)/(1+cos(x)),x, algorithm="maxima")
[Out] -2*((cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 + s
in(x)^2 + 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x) +
 1)
```

Giac [F]

$$\int \frac{e^x}{1 + \cos(x)} \, dx = \int \frac{e^x}{\cos(x) + 1} \, dx$$

```
[In] integrate(exp(x)/(1+cos(x)),x, algorithm="giac")
[Out] integrate(e^x/(cos(x) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^x}{1 + \cos(x)} \, dx = \int \frac{e^x}{\cos(x) + 1} \, dx$$

```
[In] int(exp(x)/(cos(x) + 1),x)
[Out] int(exp(x)/(cos(x) + 1), x)
```

3.553 $\int \frac{e^x}{1-\cos(x)} dx$

Optimal result

Integrand size = 12, antiderivative size = 26

$$\int \frac{e^x}{1-\cos(x)} dx = (-1+i)e^{(1+i)x} \text{Hypergeometric2F1}\left(1-i,2,2-i,e^{ix}\right)$$

```
[Out] (-1+I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],exp(I*x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4543, 4538}

$$\int \frac{e^x}{1-\cos(x)} dx = (-1+i)e^{(1+i)x} \text{Hypergeometric2F1}\left(1-i,2,2-i,e^{ix}\right)$$

```
[In] Int[E^x/(1 - Cos[x]),x]
```

```
[Out] (-1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, E^(I*x)]
```

Rule 4538

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rule 4543

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Sin[d/2 + e*(x/2)]^
(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f + g, 0] && ILt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int e^x \csc^2\left(\frac{x}{2}\right)\,dx\\ &= (-1+i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,e^{ix}\right)\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 84 vs. $2(26) = 52$.

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.23

$$\int \frac{e^x}{1-\cos(x)}\,dx$$
$$= \frac{(1+i)e^x \sin\left(\frac{x}{2}\right)\left((1-i)\cos\left(\frac{x}{2}\right)+(1+i)\,\text{Hypergeometric2F1}\left(-i,1,1-i,e^{ix}\right)\sin\left(\frac{x}{2}\right)+e^{ix}\,\text{Hypergeomet}\right.}{-1+\cos(x)}$$

```
[In] Integrate[E^x/(1 - Cos[x]),x]
[Out] ((1 + I)*E^x*Sin[x/2]*((1 - I)*Cos[x/2] + (1 + I)*Hypergeometric2F1[-I, 1,
1 - I, E^(I*x)]*Sin[x/2] + E^(I*x)*Hypergeometric2F1[1, 1 - I, 2 - I, E^(I*
x)]*Sin[x/2]))/(-1 + Cos[x])
```

Maple [F]

$$\int \frac{e^x}{1-\cos(x)}dx$$

```
[In] int(exp(x)/(1-cos(x)),x)
[Out] int(exp(x)/(1-cos(x)),x)
```

Fricas [F]

$$\int \frac{e^x}{1-\cos(x)}\,dx = \int -\frac{e^x}{\cos(x)-1}\,dx$$

```
[In] integrate(exp(x)/(1-cos(x)),x, algorithm="fricas")
[Out] integral(-e^x/(cos(x) - 1), x)
```

Sympy [F]

$$\int \frac{e^x}{1-\cos(x)}\,dx = -\int \frac{e^x}{\cos(x)-1}\,dx$$

```
[In] integrate(exp(x)/(1-cos(x)),x)
[Out] -Integral(exp(x)/(cos(x) - 1), x)
```

Maxima [F]

$$\int \frac{e^x}{1-\cos(x)}\,dx = \int -\frac{e^x}{\cos(x)-1}\,dx$$

```
[In] integrate(exp(x)/(1-cos(x)),x, algorithm="maxima")
[Out] 2*((cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 - 2*cos(x) +
1)
```

Giac [F]

$$\int \frac{e^x}{1-\cos(x)}\,dx = \int -\frac{e^x}{\cos(x)-1}\,dx$$

```
[In] integrate(exp(x)/(1-cos(x)),x, algorithm="giac")
[Out] integrate(-e^x/(cos(x) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^x}{1-\cos(x)}\,dx = -\int \frac{e^x}{\cos(x)-1}\,dx$$

```
[In] int(-exp(x)/(cos(x) - 1),x)
[Out] -int(exp(x)/(cos(x) - 1), x)
```

3.554 $\int \frac{e^x}{1+\sin(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{e^x}{1+\sin(x)}\,dx = (-1+i)e^{(1-i)x}\,\text{Hypergeometric2F1}\left(1+i,2,2+i,-ie^{-ix}\right)$$

```
[Out] (-1+I)*exp((1-I)*x)*hypergeom([2, 1+I],[2+I],-I/exp(I*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4541, 4535}

$$\int \frac{e^x}{1+\sin(x)}\,dx = (-1+i)e^{(1-i)x}\,\text{Hypergeometric2F1}\left(1+i,2,2+i,-ie^{-ix}\right)$$

```
[In] Int[E^x/(1 + Sin[x]),x]
```

```
[Out] (-1 + I)*E^((1 - I)*x)*Hypergeometric2F1[1 + I, 2, 2 + I, (-I)/E^(I*x)]
```

Rule 4535

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n
_.), x_Symbol] :> Simp[2^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/
(I*e*n + b*c*Log[F]))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 +
n/2 - I*b*c*(Log[F]/(2*e)), (-E^(2*I*k*Pi))*E^(2*I*(d + e*x))], x] /; FreeQ
[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

Rule 4541

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 - f*(Pi/(4*
g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f
^2 - g^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int e^x \sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)\,dx\\ &= (-1+i)e^{(1-i)x}\,\text{Hypergeometric2F1}\left(1+i,2,2+i,-ie^{-ix}\right)\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 61 vs. 2(30) = 60.

Time = 0.61 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int \frac{e^x}{1+\sin(x)}\,dx = \frac{2e^x\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)} - (1-i)(1 - (1-i)\,\text{Hypergeometric2F1}(-i,1,1-i,i\cos(x)-\sin(x)))(\cosh(x) + \sinh(x))$$

```
[In] Integrate[E^x/(1 + Sin[x]),x]
[Out] (2*E^x*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - (1 - I)*(1 - (1 - I)*Hypergeometri
c2F1[-I, 1, 1 - I, I*Cos[x] - Sin[x]])*(Cosh[x] + Sinh[x])
```

Maple [F]

$$\int \frac{e^x}{\sin(x)+1}dx$$

```
[In] int(exp(x)/(sin(x)+1),x)
[Out] int(exp(x)/(sin(x)+1),x)
```

Fricas [F]

$$\int \frac{e^x}{1+\sin(x)}\,dx = \int \frac{e^x}{\sin(x)+1}\,dx$$

```
[In] integrate(exp(x)/(1+sin(x)),x, algorithm="fricas")
[Out] integral(e^x/(sin(x) + 1), x)
```

Sympy [F]

$$\int \frac{e^x}{1+\sin(x)}\,dx = \int \frac{e^x}{\sin(x)+1}\,dx$$

```
[In] integrate(exp(x)/(1+sin(x)),x)
[Out] Integral(exp(x)/(sin(x) + 1), x)
```

Maxima [F]

$$\int \frac{e^x}{1+\sin(x)}\,dx = \int \frac{e^x}{\sin(x)+1}\,dx$$

```
[In] integrate(exp(x)/(1+sin(x)),x, algorithm="maxima")
[Out] -2*(cos(x)*e^x - (cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(cos(x)*e^x/
(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 + 2*sin(x) +
 1)
```

Giac [F]

$$\int \frac{e^x}{1+\sin(x)}\,dx = \int \frac{e^x}{\sin(x)+1}\,dx$$

```
[In] integrate(exp(x)/(1+sin(x)),x, algorithm="giac")
[Out] integrate(e^x/(sin(x) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^x}{1+\sin(x)}\,dx = \int \frac{e^x}{\sin(x)+1}\,dx$$

```
[In] int(exp(x)/(sin(x) + 1),x)
[Out] int(exp(x)/(sin(x) + 1), x)
```

3.555 $\int \frac{e^x}{1-\sin(x)} dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{e^x}{1-\sin(x)}\,dx = (1+i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-ie^{ix}\right)$$

```
[Out] (1+I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],-I*exp(I*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4541, 4535}

$$\int \frac{e^x}{1-\sin(x)}\,dx = (1+i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-ie^{ix}\right)$$

```
[In] Int[E^x/(1 - Sin[x]),x]
```

```
[Out] (1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, (-I)*E^(I*x)]
```

Rule 4535

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n
_.), x_Symbol] :> Simp[2^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/
(I*e*n + b*c*Log[F]))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 +
n/2 - I*b*c*(Log[F]/(2*e)), (-E^(2*I*k*Pi))*E^(2*I*(d + e*x))], x] /; FreeQ
[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

Rule 4541

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 - f*(Pi/(4*
g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f
^2 - g^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int e^x \sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx\\ &= (1+i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-ie^{ix}\right)\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 61 vs. 2(30) = 60.

Time = 0.62 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int \frac{e^x}{1-\sin(x)}\,dx = \frac{2e^x\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)} + (1+i)(1 - (1+i)\,\text{Hypergeometric2F1}(-i,1,1-i,-i\cos(x)+\sin(x)))(\cosh(x) + \sinh(x))$$

```
[In] Integrate[E^x/(1 - Sin[x]),x]
[Out] (2*E^x*Sin[x/2])/(Cos[x/2] - Sin[x/2]) + (1 + I)*(1 - (1 + I)*Hypergeometri
c2F1[-I, 1, 1 - I, (-I)*Cos[x] + Sin[x]])*(Cosh[x] + Sinh[x])
```

Maple [F]

$$\int \frac{e^x}{-\sin(x)+1}dx$$

```
[In] int(exp(x)/(-sin(x)+1),x)
[Out] int(exp(x)/(-sin(x)+1),x)
```

Fricas [F]

$$\int \frac{e^x}{1-\sin(x)}\,dx = \int -\frac{e^x}{\sin(x)-1}\,dx$$

```
[In] integrate(exp(x)/(1-sin(x)),x, algorithm="fricas")
[Out] integral(-e^x/(sin(x) - 1), x)
```

Sympy [F]

$$\int \frac{e^x}{1-\sin(x)}\,dx = -\int \frac{e^x}{\sin(x)-1}\,dx$$

```
[In] integrate(exp(x)/(1-sin(x)),x)
[Out] -Integral(exp(x)/(sin(x) - 1), x)
```

Maxima [F]

$$\int \frac{e^x}{1-\sin(x)}\,dx = \int -\frac{e^x}{\sin(x)-1}\,dx$$

```
[In] integrate(exp(x)/(1-sin(x)),x, algorithm="maxima")
[Out] 2*(cos(x)*e^x - (cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(cos(x)*e^x/(
cos(x)^2 + sin(x)^2 - 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 - 2*sin(x) +
1)
```

Giac [F]

$$\int \frac{e^x}{1-\sin(x)}\,dx = \int -\frac{e^x}{\sin(x)-1}\,dx$$

```
[In] integrate(exp(x)/(1-sin(x)),x, algorithm="giac")
[Out] integrate(-e^x/(sin(x) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^x}{1-\sin(x)}\,dx = -\int \frac{e^x}{\sin(x)-1}\,dx$$

```
[In] int(-exp(x)/(sin(x) - 1),x)
[Out] -int(exp(x)/(sin(x) - 1), x)
```

3.556 $\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = -\frac{e^x\sin(x)}{1-\cos(x)}$$

```
[Out] -exp(x)*sin(x)/(1-cos(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2326}

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = -\frac{e^x\sin(x)}{1-\cos(x)}$$

```
[In] Int[(E^x*(1 - Sin[x]))/(1 - Cos[x]),x]
[Out] -((E^x*Sin[x])/(1 - Cos[x]))
```

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\text{integral} = -\frac{e^x\sin(x)}{1-\cos(x)}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = \frac{e^x\sin(x)}{-1+\cos(x)}$$

```
[In] Integrate[(E^x*(1 - Sin[x]))/(1 - Cos[x]),x]
[Out] (E^x*Sin[x])/(-1 + Cos[x])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

method	result	size
parallelrisch	$-\frac{\mathrm{e}^{x}}{\tan\left(\frac{x}{2}\right)}$	11
risch	$-i\mathrm{e}^{x}-\frac{2i\mathrm{e}^{x}}{\mathrm{e}^{ix}-1}$	21
norman	$\frac{-\mathrm{e}^{x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)-\mathrm{e}^{x}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)}$	33

```
[In] int(exp(x)*(-sin(x)+1)/(1-cos(x)),x,method=_RETURNVERBOSE)
[Out] -exp(x)/tan(1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = -\frac{(\cos(x)+1)e^x}{\sin(x)}$$

```
[In] integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="fricas")
[Out] -(cos(x) + 1)*e^x/sin(x)
```

Sympy [F]

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = \int \frac{(\sin(x)-1)\,e^x}{\cos(x)-1}\,dx$$

```
[In] integrate(exp(x)*(1-sin(x))/(1-cos(x)),x)
[Out] Integral((sin(x) - 1)*exp(x)/(cos(x) - 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = -\frac{2\,e^x\sin(x)}{\cos(x)^2+\sin(x)^2-2\,\cos(x)+1}$$

```
[In] integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="maxima")
[Out] -2*e^x*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = -\frac{e^x}{\tan\left(\frac{1}{2}\,x\right)}$$

```
[In] integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="giac")
[Out] -e^x/tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.53

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = -\cot\left(\frac{x}{2}\right)\,e^x$$

```
[In] int((exp(x)*(sin(x) - 1))/(cos(x) - 1),x)
[Out] -cot(x/2)*exp(x)
```

3.557 $\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx = (-2+2i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,e^{ix}\right)+\frac{e^x\sin(x)}{1-\cos(x)}$$

```
[Out] (-2+2*I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],exp(I*x))+exp(x)*sin(x)/(1-c
os(x))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4551, 4549, 4528, 2225, 2283, 2326}

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx = -4ie^x\,\text{Hypergeometric2F1}\left(-i,1,1-i,e^{ix}\right)+2ie^x-\frac{e^x\sin(x)}{1-\cos(x)}$$

```
[In] Int[(E^x*(1 + Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] (2*I)*E^x - (4*I)*E^x*Hypergeometric2F1[-I, 1, 1 - I, E^(I*x)] - (E^x*Sin[x
])/(1 - Cos[x])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
```

```
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rule 4528

```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(-I)^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d + e
*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e},
 x] && IntegerQ[n]
```

Rule 4549

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Dist[f^n, Int[F^(c*(a +
 b*x))*Cot[d/2 + e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] &
& EqQ[f + g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rule 4551

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*((h_) + (i_.)*Sin[(d_.) + (e_.)*(x_)
]))/(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_)), x_Symbol] :> Dist[2*i, Int[F^(c
*(a + b*x))*(Sin[d + e*x]/(f + g*Cos[d + e*x])), x], x] + Int[F^(c*(a + b*x
))*((h - i*Sin[d + e*x])/(f + g*Cos[d + e*x])), x] /; FreeQ[{F, a, b, c, d,
 e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h +
f*i, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\int \frac{e^x \sin(x)}{1-\cos(x)}\,dx + \int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx \\
&= -\frac{e^x \sin(x)}{1-\cos(x)} + 2\int e^x \cot\left(\frac{x}{2}\right)\,dx \\
&= -\frac{e^x \sin(x)}{1-\cos(x)} - 2i\int \left(-e^x - \frac{2e^x}{-1+e^{ix}}\right)\,dx \\
&= -\frac{e^x \sin(x)}{1-\cos(x)} + 2i\int e^x\,dx + 4i\int \frac{e^x}{-1+e^{ix}}\,dx \\
&= 2ie^x - 4ie^x\,\text{Hypergeometric2F1}\left(-i,1,1-i,e^{ix}\right) - \frac{e^x \sin(x)}{1-\cos(x)}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 100 vs. $2(41) = 82$.

Time = 0.62 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.44

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx$$

$$= \frac{2e^x \sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right) + 2i\,\text{Hypergeometric2F1}\left(-i,1,1-i,e^{ix}\right)\sin\left(\frac{x}{2}\right) + (1+i)e^{ix}\,\text{Hypergeometric2F1}\left(1,1\right.\right.}{(-1+\cos(x))\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}$$

```
[In] Integrate[(E^x*(1 + Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] (2*E^x*Sin[x/2]*(Cos[x/2] + (2*I)*Hypergeometric2F1[-I, 1, 1 - I, E^(I*x)]*
Sin[x/2] + (1 + I)*E^(I*x)*Hypergeometric2F1[1, 1 - I, 2 - I, E^(I*x)]*Sin[
x/2])*(1 + Sin[x]))/((-1 + Cos[x])*(Cos[x/2] + Sin[x/2])^2)
```

Maple [F]

$$\int \frac{e^x(\sin(x)+1)}{1-\cos(x)}dx$$

```
[In] int(exp(x)*(sin(x)+1)/(1-cos(x)),x)
```

```
[Out] int(exp(x)*(sin(x)+1)/(1-cos(x)),x)
```

Fricas [F]

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx = \int -\frac{(\sin(x)+1)e^x}{\cos(x)-1}\,dx$$

```
[In] integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] integral(-(e^x*sin(x) + e^x)/(cos(x) - 1), x)
```

Sympy [F]

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx = -\int \frac{e^x}{\cos(x)-1}\,dx - \int \frac{e^x\sin(x)}{\cos(x)-1}\,dx$$

```
[In] integrate(exp(x)*(1+sin(x))/(1-cos(x)),x)
[Out] -Integral(exp(x)/(cos(x) - 1), x) - Integral(exp(x)*sin(x)/(cos(x) - 1), x)
```

Maxima [F]

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx = \int -\frac{(\sin(x)+1)e^x}{\cos(x)-1}\,dx$$

```
[In] integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="maxima")
[Out] 2*(2*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 - 2*cos(x)
+ 1)
```

Giac [F]

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx = \int -\frac{(\sin(x)+1)e^x}{\cos(x)-1}\,dx$$

```
[In] integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="giac")
[Out] integrate(-(sin(x) + 1)*e^x/(cos(x) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx = \int -\frac{e^x(\sin(x)+1)}{\cos(x)-1}\,dx$$

```
[In] int(-(exp(x)*(sin(x) + 1))/(cos(x) - 1),x)
[Out] int(-(exp(x)*(sin(x) + 1))/(cos(x) - 1), x)
```

3.558 $\int \frac{e^x(1+\sin(x))}{1+\cos(x)} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 12

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)} \, dx = \frac{e^x \sin(x)}{1+\cos(x)}$$

```
[Out] exp(x)*sin(x)/(1+cos(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2326}

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)} \, dx = \frac{e^x \sin(x)}{\cos(x)+1}$$

```
[In] Int[(E^x*(1 + Sin[x]))/(1 + Cos[x]),x]
```

```
[Out] (E^x*Sin[x])/(1 + Cos[x])
```

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\text{integral} = \frac{e^x \sin(x)}{1+\cos(x)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \frac{e^x\sin(x)}{1+\cos(x)}$$

```
[In] Integrate[(E^x*(1 + Sin[x]))/(1 + Cos[x]),x]
[Out] (E^x*Sin[x])/(1 + Cos[x])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

method	result	size
norman	$e^x\tan\left(\frac{x}{2}\right)$	8
parallelrisch	$e^x\tan\left(\frac{x}{2}\right)$	8
risch	$-ie^x+\frac{2ie^x}{e^{ix}+1}$	21

```
[In] int(exp(x)*(sin(x)+1)/(cos(x)+1),x,method=_RETURNVERBOSE)
[Out] exp(x)*tan(1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \frac{e^x\sin\left(x\right)}{\cos\left(x\right)+1}$$

```
[In] integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="fricas")
[Out] e^x*sin(x)/(cos(x) + 1)
```

Sympy [F]

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \int \frac{(\sin(x)+1)\,e^x}{\cos(x)+1}\,dx$$

```
[In] integrate(exp(x)*(1+sin(x))/(1+cos(x)),x)
[Out] Integral((sin(x) + 1)*exp(x)/(cos(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \frac{2\,e^x\sin(x)}{\cos(x)^2+\sin(x)^2+2\,\cos(x)+1}$$

```
[In] integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="maxima")
[Out] 2*e^x*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = e^x\tan\left(\frac{1}{2}\,x\right)$$

```
[In] integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="giac")
[Out] e^x*tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)\,\mathrm{e}^x$$

```
[In] int((exp(x)*(sin(x) + 1))/(cos(x) + 1),x)
[Out] tan(x/2)*exp(x)
```

3.559 $\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = (2-2i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{ix}\right)-\frac{e^x\sin(x)}{1+\cos(x)}$$

```
[Out] (2-2*I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],-exp(I*x))-exp(x)*sin(x)/(1+c
os(x))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4551, 4548, 4527, 2225, 2283, 2326}

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = -4ie^x\,\text{Hypergeometric2F1}\left(-i,1,1-i,-e^{ix}\right)+2ie^x+\frac{e^x\sin(x)}{\cos(x)+1}$$

```
[In] Int[(E^x*(1 - Sin[x]))/(1 + Cos[x]),x]
```

```
[Out] (2*I)*E^x - (4*I)*E^x*Hypergeometric2F1[-I, 1, 1 - I, -E^(I*x)] + (E^x*Sin[
x])/(1 + Cos[x])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
```

```
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 4548

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Dist[f^n, Int[F^(c*(a +
 b*x))*Tan[d/2 + e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] &
& EqQ[f - g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rule 4551

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*((h_) + (i_.)*Sin[(d_.) + (e_.)*(x_)
]))/(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_)), x_Symbol] :> Dist[2*i, Int[F^(c
*(a + b*x))*(Sin[d + e*x]/(f + g*Cos[d + e*x])), x], x] + Int[F^(c*(a + b*x
))*((h - i*Sin[d + e*x])/(f + g*Cos[d + e*x])), x] /; FreeQ[{F, a, b, c, d,
 e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h +
f*i, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(2\int \frac{e^x \sin(x)}{1+\cos(x)}\,dx\right)+\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx\\
&= \frac{e^x \sin(x)}{1+\cos(x)}-2\int e^x \tan\left(\frac{x}{2}\right)\,dx\\
&= \frac{e^x \sin(x)}{1+\cos(x)}-2i\int \left(-e^x+\frac{2e^x}{1+e^{ix}}\right)\,dx\\
&= \frac{e^x \sin(x)}{1+\cos(x)}+2i\int e^x\,dx-4i\int \frac{e^x}{1+e^{ix}}\,dx\\
&= 2ie^x-4ie^x\,\text{Hypergeometric2F1}\left(-i,1,1-i,-e^{ix}\right)+\frac{e^x \sin(x)}{1+\cos(x)}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 87 vs. $2(42) = 84$.

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.07

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = -\frac{2e^x\cos\left(\frac{x}{2}\right)\left(2i\cos\left(\frac{x}{2}\right)\text{Hypergeometric2F1}\left(-i,1,1-i,-e^{ix}\right)-(1+i)e^{ix}\cos\left(\frac{x}{2}\right)\text{Hypergeometric2F1}\right.}{1+\cos(x)}$$

```
[In] Integrate[(E^x*(1 - Sin[x]))/(1 + Cos[x]),x]
[Out] (-2*E^x*Cos[x/2]*((2*I)*Cos[x/2]*Hypergeometric2F1[-I, 1, 1 - I, -E^(I*x)]
- (1 + I)*E^(I*x)*Cos[x/2]*Hypergeometric2F1[1, 1 - I, 2 - I, -E^(I*x)] - S
in[x/2]))/(1 + Cos[x])
```

Maple [F]

$$\int \frac{e^x(-\sin(x)+1)}{\cos(x)+1}dx$$

```
[In] int(exp(x)*(-sin(x)+1)/(cos(x)+1),x)
[Out] int(exp(x)*(-sin(x)+1)/(cos(x)+1),x)
```

Fricas [F]

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = \int -\frac{(\sin(x)-1)e^x}{\cos(x)+1}\,dx$$

```
[In] integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="fricas")
[Out] integral(-(e^x*sin(x) - e^x)/(cos(x) + 1), x)
```

Sympy [F]

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = -\int\left(-\frac{e^x}{\cos(x)+1}\right)dx - \int \frac{e^x\sin(x)}{\cos(x)+1}\,dx$$

```
[In] integrate(exp(x)*(1-sin(x))/(1+cos(x)),x)
[Out] -Integral(-exp(x)/(cos(x) + 1), x) - Integral(exp(x)*sin(x)/(cos(x) + 1), x
)
```

Maxima [F]

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = \int -\frac{(\sin(x)-1)e^x}{\cos(x)+1}\,dx$$

```
[In] integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] -2*(2*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 +
 sin(x)^2 + 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x)
 + 1)
```

Giac [F]

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = \int -\frac{(\sin(x)-1)e^x}{\cos(x)+1}\,dx$$

```
[In] integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="giac")
```

```
[Out] integrate(-(sin(x) - 1)*e^x/(cos(x) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = -\int \frac{e^x(\sin(x)-1)}{\cos(x)+1}\,dx$$

```
[In] int(-(exp(x)*(sin(x) - 1))/(cos(x) + 1),x)
```

```
[Out] -int((exp(x)*(sin(x) - 1))/(cos(x) + 1), x)
```

3.560 $\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = (2+2i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-ie^{ix}\right)-\frac{e^x\cos(x)}{1-\sin(x)}$$

```
[Out] (2+2*I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],-I*exp(I*x))-exp(x)*cos(x)/(1
-sin(x))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4550, 4547, 4527, 2225, 2283, 2326}

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = -4ie^x\,\text{Hypergeometric2F1}\left(-i,1,1-i,-ie^{ix}\right)+2ie^x+\frac{e^x\cos(x)}{1-\sin(x)}$$

```
[In] Int[(E^x*(1 - Cos[x]))/(1 - Sin[x]),x]
```

```
[Out] (2*I)*E^x - (4*I)*E^x*Hypergeometric2F1[-I, 1, 1 - I, (-I)*E^(I*x)] + (E^x*
Cos[x])/(1 - Sin[x])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
```

```
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 4547

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Dist[g^n, Int[F^(c*(a +
 b*x))*Tan[f*(Pi/(4*g)) - d/2 - e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d,
 e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rule 4550

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*(Cos[(d_.) + (e_.)*(x_)]*(i_.) + (h_
)))/((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Dist[2*i, Int[F^(c
*(a + b*x))*(Cos[d + e*x]/(f + g*Sin[d + e*x])), x], x] + Int[F^(c*(a + b*x
))*((h - i*Cos[d + e*x])/(f + g*Sin[d + e*x])), x] /; FreeQ[{F, a, b, c, d,
 e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h -
f*i, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(2\int \frac{e^x\cos(x)}{1-\sin(x)}\,dx\right)+\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx\\
&= \frac{e^x\cos(x)}{1-\sin(x)}-2\int e^x\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx\\
&= \frac{e^x\cos(x)}{1-\sin(x)}-2i\int \left(-e^x+\frac{2e^x}{1+e^{2i\left(\frac{\pi}{4}+\frac{x}{2}\right)}}\right)\,dx\\
&= \frac{e^x\cos(x)}{1-\sin(x)}+2i\int e^x\,dx-4i\int \frac{e^x}{1+e^{2i\left(\frac{\pi}{4}+\frac{x}{2}\right)}}\,dx\\
&= 2ie^x-4ie^x\,\text{Hypergeometric2F1}\left(-i,1,1-i,-ie^{ix}\right)+\frac{e^x\cos(x)}{1-\sin(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \frac{1}{2}(-1+\cos(x))\csc^2\left(\frac{x}{2}\right)\left(-\frac{e^x\left((1-2i)+(1+2i)\cot\left(\frac{x}{2}\right)\right)}{-1+\cot\left(\frac{x}{2}\right)} + 4i\,\text{Hypergeometric2F1}(-i,1,1-i,-i\cos(x)+\sin(x))(\cosh(x)+\sinh(x))\right)$$

```
[In] Integrate[(E^x*(1 - Cos[x]))/(1 - Sin[x]),x]
[Out] ((-1 + Cos[x])*Csc[x/2]^2*(-((E^x*((1 - 2*I) + (1 + 2*I)*Cot[x/2]))/(-1 + C
ot[x/2])) + (4*I)*Hypergeometric2F1[-I, 1, 1 - I, (-I)*Cos[x] + Sin[x]]*(Co
sh[x] + Sinh[x])))/2
```

Maple [F]

$$\int \frac{e^x(1-\cos(x))}{-\sin(x)+1}dx$$

```
[In] int(exp(x)*(1-cos(x))/(-sin(x)+1),x)
[Out] int(exp(x)*(1-cos(x))/(-sin(x)+1),x)
```

Fricas [F]

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \int \frac{(\cos(x)-1)e^x}{\sin(x)-1}\,dx$$

```
[In] integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="fricas")
[Out] integral((cos(x) - 1)*e^x/(sin(x) - 1), x)
```

Sympy [F]

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \int \frac{(\cos(x)-1)\,e^x}{\sin(x)-1}\,dx$$

```
[In] integrate(exp(x)*(1-cos(x))/(1-sin(x)),x)
[Out] Integral((cos(x) - 1)*exp(x)/(sin(x) - 1), x)
```

Maxima [F]

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \int \frac{(\cos(x)-1)e^x}{\sin(x)-1}\,dx$$

```
[In] integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="maxima")
```

```
[Out] 2*(cos(x)*e^x - 2*(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(cos(x)*e^x
/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 - 2*sin(x)
+ 1)
```

Giac [F]

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \int \frac{(\cos(x)-1)e^x}{\sin(x)-1}\,dx$$

```
[In] integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="giac")
```

```
[Out] integrate((cos(x) - 1)*e^x/(sin(x) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \int \frac{e^x\,(\cos(x)-1)}{\sin(x)-1}\,dx$$

```
[In] int((exp(x)*(cos(x) - 1))/(sin(x) - 1),x)
```

```
[Out] int((exp(x)*(cos(x) - 1))/(sin(x) - 1), x)
```

3.561 $\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = \frac{e^x\cos(x)}{1-\sin(x)}$$

```
[Out] exp(x)*cos(x)/(1-sin(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2326}

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = \frac{e^x\cos(x)}{1-\sin(x)}$$

```
[In] Int[(E^x*(1 + Cos[x]))/(1 - Sin[x]),x]
```

```
[Out] (E^x*Cos[x])/(1 - Sin[x])
```

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\text{integral} = \frac{e^x\cos(x)}{1-\sin(x)}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = -\frac{e^x\cos(x)}{-1+\sin(x)}$$

```
[In] Integrate[(E^x*(1 + Cos[x]))/(1 - Sin[x]),x]
[Out] -((E^x*Cos[x])/(-1 + Sin[x]))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

method	result	size
parallelrisch	$-\frac{\mathrm{e}^{x}\left(1+\tan\left(\frac{x}{2}\right)\right)}{\tan\left(\frac{x}{2}\right)-1}$	19
risch	$-i\mathrm{e}^{x}+\frac{2\,\mathrm{e}^{x}}{\mathrm{e}^{ix}-i}$	21
norman	$\frac{-\mathrm{e}^{x}\tan\left(\frac{x}{2}\right)-\mathrm{e}^{x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)-\mathrm{e}^{x}\left(\tan^{3}\left(\frac{x}{2}\right)\right)-\mathrm{e}^{x}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\left(\tan\left(\frac{x}{2}\right)-1\right)}$	53

```
[In] int(exp(x)*(cos(x)+1)/(-sin(x)+1),x,method=_RETURNVERBOSE)
[Out] -exp(x)*(1+tan(1/2*x))/(tan(1/2*x)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = \frac{(\cos(x)+1)e^x + e^x\sin(x)}{\cos(x)-\sin(x)+1}$$

```
[In] integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="fricas")
[Out] ((cos(x) + 1)*e^x + e^x*sin(x))/(cos(x) - sin(x) + 1)
```

Sympy [F]

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = -\int \frac{e^x}{\sin(x)-1}\,dx - \int \frac{e^x\cos(x)}{\sin(x)-1}\,dx$$

```
[In] integrate(exp(x)*(1+cos(x))/(1-sin(x)),x)
[Out] -Integral(exp(x)/(sin(x) - 1), x) - Integral(exp(x)*cos(x)/(sin(x) - 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = \frac{2\cos(x)e^x}{\cos(x)^2+\sin(x)^2-2\sin(x)+1}$$

```
[In] integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="maxima")
[Out] 2*cos(x)*e^x/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = -\frac{e^x\tan\left(\frac{1}{2}x\right)+e^x}{\tan\left(\frac{1}{2}x\right)-1}$$

```
[In] integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="giac")
[Out] -(e^x*tan(1/2*x) + e^x)/(tan(1/2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = -\frac{e^x\left(-1+e^{x\,1i}\,1i\right)}{e^{x\,1i}-i}$$

```
[In] int(-(exp(x)*(cos(x) + 1))/(sin(x) - 1),x)
[Out] -(exp(x)*(exp(x*1i)*1i - 1))/(exp(x*1i) - 1i)
```

3.562 $\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx=(-2-2i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,ie^{ix}\right)+\frac{e^x\cos(x)}{1+\sin(x)}$$

```
[Out] (-2-2*I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],I*exp(I*x))+exp(x)*cos(x)/(1
+sin(x))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4550, 4547, 4527, 2225, 2283, 2326}

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx=4ie^x\,\text{Hypergeometric2F1}\left(i,1,1+i,-ie^{-ix}\right)-2ie^x-\frac{e^x\cos(x)}{\sin(x)+1}$$

```
[In] Int[(E^x*(1 + Cos[x]))/(1 + Sin[x]),x]
```

```
[Out] (-2*I)*E^x + (4*I)*E^x*Hypergeometric2F1[I, 1, 1 + I, (-I)/E^(I*x)] - (E^x*
Cos[x])/(1 + Sin[x])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
```

```
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 4547

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Dist[g^n, Int[F^(c*(a +
 b*x))*Tan[f*(Pi/(4*g)) - d/2 - e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d,
 e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rule 4550

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*(Cos[(d_.) + (e_.)*(x_)]*(i_.) + (h_
)))/((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Dist[2*i, Int[F^(c
*(a + b*x))*(Cos[d + e*x]/(f + g*Sin[d + e*x])), x], x] + Int[F^(c*(a + b*x
))*((h - i*Cos[d + e*x])/(f + g*Sin[d + e*x])), x] /; FreeQ[{F, a, b, c, d,
 e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h -
f*i, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\int \frac{e^x\cos(x)}{1+\sin(x)}\,dx + \int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx \\
&= -\frac{e^x\cos(x)}{1+\sin(x)} + 2\int e^x\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\,dx \\
&= -\frac{e^x\cos(x)}{1+\sin(x)} + 2i\int\left(-e^x+\frac{2e^x}{1+e^{2i\left(\frac{\pi}{4}-\frac{x}{2}\right)}}\right)\,dx \\
&= -\frac{e^x\cos(x)}{1+\sin(x)} - 2i\int e^x\,dx + 4i\int\frac{e^x}{1+e^{2i\left(\frac{\pi}{4}-\frac{x}{2}\right)}}\,dx \\
&= -2ie^x + 4ie^x\,\text{Hypergeometric2F1}\left(i,1,1+i,-ie^{-ix}\right) - \frac{e^x\cos(x)}{1+\sin(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = \frac{1}{2}(1+\cos(x))\sec^2\left(\frac{x}{2}\right)\left(-4i\,\text{Hypergeometric2F1}(-i,1,1-i,i\cos(x)-\sin(x))(\cosh(x)+\sinh(x))+\frac{e^x\left((-1+2i)+(1+2i)\tan\left(\frac{x}{2}\right)\right)}{1+\tan\left(\frac{x}{2}\right)}\right)$$

```
[In] Integrate[(E^x*(1 + Cos[x]))/(1 + Sin[x]),x]
[Out] ((1 + Cos[x])*Sec[x/2]^2*((-4*I)*Hypergeometric2F1[-I, 1, 1 - I, I*Cos[x] -
 Sin[x]]*(Cosh[x] + Sinh[x]) + (E^x*((-1 + 2*I) + (1 + 2*I)*Tan[x/2]))/(1 +
 Tan[x/2])))/2
```

Maple [F]

$$\int \frac{e^x(\cos(x)+1)}{\sin(x)+1}dx$$

```
[In] int(exp(x)*(cos(x)+1)/(sin(x)+1),x)
[Out] int(exp(x)*(cos(x)+1)/(sin(x)+1),x)
```

Fricas [F]

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = \int \frac{(\cos(x)+1)e^x}{\sin(x)+1}\,dx$$

```
[In] integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="fricas")
[Out] integral((cos(x) + 1)*e^x/(sin(x) + 1), x)
```

Sympy [F]

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = \int \frac{(\cos(x)+1)\,e^x}{\sin(x)+1}\,dx$$

```
[In] integrate(exp(x)*(1+cos(x))/(1+sin(x)),x)
[Out] Integral((cos(x) + 1)*exp(x)/(sin(x) + 1), x)
```

Maxima [F]

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = \int \frac{(\cos(x)+1)e^x}{\sin(x)+1}\,dx$$

```
[In] integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="maxima")
```

```
[Out] -2*(cos(x)*e^x - 2*(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(cos(x)*e^
x/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 + 2*sin(x)
 + 1)
```

Giac [F]

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = \int \frac{(\cos(x)+1)e^x}{\sin(x)+1}\,dx$$

```
[In] integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="giac")
```

```
[Out] integrate((cos(x) + 1)*e^x/(sin(x) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = \int \frac{e^x\,(\cos(x)+1)}{\sin(x)+1}\,dx$$

```
[In] int((exp(x)*(cos(x) + 1))/(sin(x) + 1),x)
```

```
[Out] int((exp(x)*(cos(x) + 1))/(sin(x) + 1), x)
```

3.563 $\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 13

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = -\frac{e^x\cos(x)}{1+\sin(x)}$$

```
[Out] -exp(x)*cos(x)/(1+sin(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2326}

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = -\frac{e^x\cos(x)}{\sin(x)+1}$$

```
[In] Int[(E^x*(1 - Cos[x]))/(1 + Sin[x]),x]
```

```
[Out] -((E^x*Cos[x])/(1 + Sin[x]))
```

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\text{integral} = -\frac{e^x\cos(x)}{1+\sin(x)}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = -\frac{e^x\cos(x)}{1+\sin(x)}$$

```
[In] Integrate[(E^x*(1 - Cos[x]))/(1 + Sin[x]),x]
[Out] -((E^x*Cos[x])/(1 + Sin[x]))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

method	result	size
parallelrisch	$\frac{\mathrm{e}^{x}\left(\tan\left(\frac{x}{2}\right)-1\right)}{1+\tan\left(\frac{x}{2}\right)}$	18
risch	$-i\mathrm{e}^{x}-\frac{2\,\mathrm{e}^{x}}{i+\mathrm{e}^{ix}}$	21
norman	$\frac{\mathrm{e}^{x}\tan\left(\frac{x}{2}\right)+\mathrm{e}^{x}\left(\tan^{3}\left(\frac{x}{2}\right)\right)-\mathrm{e}^{x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)-\mathrm{e}^{x}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}$	51

```
[In] int(exp(x)*(1-cos(x))/(sin(x)+1),x,method=_RETURNVERBOSE)
[Out] exp(x)*(tan(1/2*x)-1)/(1+tan(1/2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = -\frac{(\cos(x)+1)e^x-e^x\sin(x)}{\cos(x)+\sin(x)+1}$$

```
[In] integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="fricas")
[Out] -((cos(x) + 1)*e^x - e^x*sin(x))/(cos(x) + sin(x) + 1)
```

Sympy [F]

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = -\int \left(-\frac{e^x}{\sin(x)+1}\right)dx - \int \frac{e^x\cos(x)}{\sin(x)+1}\,dx$$

```
[In] integrate(exp(x)*(1-cos(x))/(1+sin(x)),x)
[Out] -Integral(-exp(x)/(sin(x) + 1), x) - Integral(exp(x)*cos(x)/(sin(x) + 1), x
)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = -\frac{2\cos(x)e^x}{\cos(x)^2+\sin(x)^2+2\sin(x)+1}$$

```
[In] integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="maxima")
[Out] -2*cos(x)*e^x/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = \frac{e^x\tan\left(\frac{1}{2}x\right)-e^x}{\tan\left(\frac{1}{2}x\right)+1}$$

```
[In] integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="giac")
[Out] (e^x*tan(1/2*x) - e^x)/(tan(1/2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = -e^x\,1i - \frac{2\,e^x}{e^{x\,1i}+1i}$$

```
[In] int(-(exp(x)*(cos(x) - 1))/(sin(x) + 1),x)
[Out] - exp(x)*1i - (2*exp(x))/(exp(x*1i) + 1i)
```

3.564 $\int e^x x \cos(x) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 30

$$\int e^x x \cos(x) \, dx = \frac{1}{2} e^x x \cos(x) - \frac{1}{2} e^x \sin(x) + \frac{1}{2} e^x x \sin(x)$$

```
[Out] 1/2*exp(x)*x*cos(x)-1/2*exp(x)*sin(x)+1/2*exp(x)*x*sin(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4518, 4554, 4517}

$$\int e^x x \cos(x) \, dx = -\frac{1}{2} e^x \sin(x) + \frac{1}{2} e^x x \sin(x) + \frac{1}{2} e^x x \cos(x)$$

```
[In] Int[E^x*x*Cos[x],x]
```

```
[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
```

```
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}e^x x\cos(x) + \frac{1}{2}e^x x\sin(x) - \int \left(\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)\right)\,dx \\ &= \frac{1}{2}e^x x\cos(x) + \frac{1}{2}e^x x\sin(x) - \frac{1}{2}\int e^x\cos(x)\,dx - \frac{1}{2}\int e^x\sin(x)\,dx \\ &= \frac{1}{2}e^x x\cos(x) - \frac{1}{2}e^x\sin(x) + \frac{1}{2}e^x x\sin(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int e^x x\cos(x)\,dx = \frac{1}{2}e^x(x\cos(x) + (-1+x)\sin(x))$$

```
[In] Integrate[E^x*x*Cos[x],x]
[Out] (E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.53

method	result	size
parallelrisch	$\frac{e^x((-1+x)\sin(x)+x\cos(x))}{2}$	16
default	$\frac{e^x x\cos(x)}{2} - \left(-\frac{x}{2}+\frac{1}{2}\right)e^x\sin(x)$	20
risch	$\left(\frac{1}{8}-\frac{i}{8}\right)(-1+i+2x)\,e^{(1+i)x} + \left(\frac{1}{8}+\frac{i}{8}\right)(-1-i+2x)\,e^{(1-i)x}$	36
norman	$\frac{e^x x\tan\left(\frac{x}{2}\right)+\frac{e^x x}{2}-e^x\tan\left(\frac{x}{2}\right)-\frac{e^x x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}}{1+\tan^2\left(\frac{x}{2}\right)}$	45

```
[In] int(exp(x)*x*cos(x),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x)*((-1+x)*sin(x)+x*cos(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \cos(x) \, dx = \frac{1}{2} x \cos(x) e^x + \frac{1}{2} (x-1) e^x \sin(x)$$

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="fricas")
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int e^x x \cos(x) \, dx = \frac{xe^x \sin(x)}{2} + \frac{xe^x \cos(x)}{2} - \frac{e^x \sin(x)}{2}$$

```
[In] integrate(exp(x)*x*cos(x),x)
[Out] x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \cos(x) \, dx = \frac{1}{2} x \cos(x) e^x + \frac{1}{2} (x-1) e^x \sin(x)$$

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="maxima")
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.50

$$\int e^x x \cos(x) \, dx = \frac{1}{2} \left(x \cos\left(x\right) + \left(x - 1\right) \sin\left(x\right)\right) e^x$$

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="giac")
[Out] 1/2*(x*cos(x) + (x - 1)*sin(x))*e^x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \cos(x) \, dx = \frac{\mathrm{e}^x \left(x\ \cos\left(x\right) - \sin\left(x\right) + x\ \sin\left(x\right)\right)}{2}$$

```
[In] int(x*exp(x)*cos(x),x)
[Out] (exp(x)*(x*cos(x) - sin(x) + x*sin(x)))/2
```

3.565 $\int e^x x^2 \sin(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 50

$$\int e^x x^2 \sin(x)\,dx = -\frac{1}{2}e^x \cos(x) + e^x x \cos(x) - \frac{1}{2}e^x x^2 \cos(x) - \frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x^2 \sin(x)$$

```
[Out] -1/2*exp(x)*cos(x)+exp(x)*x*cos(x)-1/2*exp(x)*x^2*cos(x)-1/2*exp(x)*sin(x)+
1/2*exp(x)*x^2*sin(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4517, 4553, 14, 4518, 4554}

$$\int e^x x^2 \sin(x)\,dx = \frac{1}{2}e^x x^2 \sin(x) - \frac{1}{2}e^x x^2 \cos(x) - \frac{1}{2}e^x \sin(x) + e^x x \cos(x) - \frac{1}{2}e^x \cos(x)$$

```
[In] Int[E^x*x^2*Sin[x],x]
```

```
[Out] -1/2*(E^x*Cos[x]) + E^x*x*Cos[x] - (E^x*x^2*Cos[x])/2 - (E^x*Sin[x])/2 + (E
^x*x^2*Sin[x])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
```

```
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)-2\int x\left(-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)\right)\,dx\\
&= -\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)-2\int\left(-\frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)\right)\,dx\\
&= -\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)+\int e^x x\cos(x)\,dx-\int e^x x\sin(x)\,dx\\
&= e^x x\cos(x)-\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)\\
&\quad+\int\left(-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)\right)\,dx-\int\left(\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)\right)\,dx\\
&= e^x x\cos(x)-\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)-2\left(\frac{1}{2}\int e^x\cos(x)\,dx\right)\\
&= e^x x\cos(x)-\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)-2\left(\frac{1}{4}e^x\cos(x)+\frac{1}{4}e^x\sin(x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.50

$$\int e^x x^2 \sin(x)\,dx = \frac{1}{2}e^x\left(-(-1+x)^2\cos(x) + \left(-1+x^2\right)\sin(x)\right)$$

```
[In] Integrate[E^x*x^2*Sin[x],x]
[Out] (E^x*(-((-1 + x)^2*Cos[x]) + (-1 + x^2)*Sin[x]))/2
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.44

method	result	size
parallelrisch	$-\frac{e^x((-1+x)\cos(x)-\sin(x)(1+x))(-1+x)}{2}$	22
default	$\left(-\frac{1}{2}x^2+x-\frac{1}{2}\right)e^x\cos\left(x\right)+\left(\frac{x^2}{2}-\frac{1}{2}\right)e^x\sin\left(x\right)$	27
risch	$\left(-\frac{1}{4}-\frac{i}{4}\right)\left(x^2+ix-x-i\right)e^{(1+i)x}+\left(-\frac{1}{4}+\frac{i}{4}\right)\left(x^2-ix-x+i\right)e^{(1-i)x}$	48
norman	$\frac{e^x x+e^x x^2\tan\left(\frac{x}{2}\right)-\frac{e^x x^2}{2}-e^x\tan\left(\frac{x}{2}\right)+\frac{e^x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-e^x x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{e^x x^2\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{e^x}{2}}{1+\tan^2\left(\frac{x}{2}\right)}$	80

```
[In] int(exp(x)*x^2*sin(x),x,method=_RETURNVERBOSE)
[Out] -1/2*exp(x)*((-1+x)*cos(x)-sin(x)*(1+x))*(-1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int e^x x^2 \sin(x)\,dx = -\frac{1}{2}\left(x^2-2\,x+1\right)\cos\left(x\right)e^x + \frac{1}{2}\left(x^2-1\right)e^x\sin\left(x\right)$$

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="fricas")
[Out] -1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int e^x x^2 \sin(x)\,dx = \frac{x^2 e^x \sin{(x)}}{2} - \frac{x^2 e^x \cos{(x)}}{2} + x e^x \cos{(x)} - \frac{e^x \sin{(x)}}{2} - \frac{e^x \cos{(x)}}{2}$$

```
[In] integrate(exp(x)*x**2*sin(x),x)
[Out] x**2*exp(x)*sin(x)/2 - x**2*exp(x)*cos(x)/2 + x*exp(x)*cos(x) - exp(x)*sin(
x)/2 - exp(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int e^x x^2 \sin(x)\,dx = -\frac{1}{2}\left(x^2 - 2\,x + 1\right)\cos{(x)}\,e^x + \frac{1}{2}\left(x^2 - 1\right)e^x \sin{(x)}$$

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="maxima")
[Out] -1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.50

$$\int e^x x^2 \sin(x)\,dx = -\frac{1}{2}\left(\left(x^2 - 2\,x + 1\right)\cos{(x)} - \left(x^2 - 1\right)\sin{(x)}\right)e^x$$

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="giac")
[Out] -1/2*((x^2 - 2*x + 1)*cos(x) - (x^2 - 1)*sin(x))*e^x
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.42

$$\int e^x x^2 \sin(x)\,dx = \frac{e^x\,\left(x - 1\right)\,\left(\cos{(x)} + \sin{(x)} - x\,\cos{(x)} + x\,\sin{(x)}\right)}{2}$$

```
[In] int(x^2*exp(x)*sin(x),x)
[Out] (exp(x)*(x - 1)*(cos(x) + sin(x) - x*cos(x) + x*sin(x)))/2
```

3.566 $\int e^{-3x}x^2 \sin(x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 75

$$\int e^{-3x}x^2 \sin(x)\,dx = -\frac{13}{250}e^{-3x}\cos(x) - \frac{3}{25}e^{-3x}x\cos(x) - \frac{1}{10}e^{-3x}x^2\cos(x) - \frac{9}{250}e^{-3x}\sin(x) - \frac{4}{25}e^{-3x}x\sin(x) - \frac{3}{10}e^{-3x}x^2\sin(x)$$

```
[Out] -13/250*cos(x)/exp(3*x)-3/25*x*cos(x)/exp(3*x)-1/10*x^2*cos(x)/exp(3*x)-9/2
50*sin(x)/exp(3*x)-4/25*x*sin(x)/exp(3*x)-3/10*x^2*sin(x)/exp(3*x)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {4517, 4553, 14, 4518, 4554}

$$\int e^{-3x}x^2 \sin(x)\,dx = -\frac{3}{10}e^{-3x}x^2\sin(x) - \frac{1}{10}e^{-3x}x^2\cos(x) - \frac{4}{25}e^{-3x}x\sin(x) - \frac{9}{250}e^{-3x}\sin(x) - \frac{3}{25}e^{-3x}x\cos(x) - \frac{13}{250}e^{-3x}\cos(x)$$

```
[In] Int[(x^2*Sin[x])/E^(3*x),x]
```

```
[Out] (-13*Cos[x])/(250*E^(3*x)) - (3*x*Cos[x])/(25*E^(3*x)) - (x^2*Cos[x])/(10*E
^(3*x)) - (9*Sin[x])/(250*E^(3*x)) - (4*x*Sin[x])/(25*E^(3*x)) - (3*x^2*Sin
[x])/(10*E^(3*x))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{10}e^{-3x}x^2\cos(x)-\frac{3}{10}e^{-3x}x^2\sin(x)-2\int x\left(-\frac{1}{10}e^{-3x}\cos(x)-\frac{3}{10}e^{-3x}\sin(x)\right)\,dx\\
&= -\frac{1}{10}e^{-3x}x^2\cos(x)-\frac{3}{10}e^{-3x}x^2\sin(x)-2\int\left(-\frac{1}{10}e^{-3x}x\cos(x)-\frac{3}{10}e^{-3x}x\sin(x)\right)\,dx\\
&= -\frac{1}{10}e^{-3x}x^2\cos(x)-\frac{3}{10}e^{-3x}x^2\sin(x)+\frac{1}{5}\int e^{-3x}x\cos(x)\,dx+\frac{3}{5}\int e^{-3x}x\sin(x)\,dx\\
&= -\frac{3}{25}e^{-3x}x\cos(x)-\frac{1}{10}e^{-3x}x^2\cos(x)-\frac{4}{25}e^{-3x}x\sin(x)\\
&\quad-\frac{3}{10}e^{-3x}x^2\sin(x)-\frac{1}{5}\int\left(-\frac{3}{10}e^{-3x}\cos(x)+\frac{1}{10}e^{-3x}\sin(x)\right)\,dx\\
&\quad-\frac{3}{5}\int\left(-\frac{1}{10}e^{-3x}\cos(x)-\frac{3}{10}e^{-3x}\sin(x)\right)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3}{25}e^{-3x}x\cos(x) - \frac{1}{10}e^{-3x}x^2\cos(x) - \frac{4}{25}e^{-3x}x\sin(x) - \frac{3}{10}e^{-3x}x^2\sin(x) \\
&\quad - \frac{1}{50}\int e^{-3x}\sin(x)\,dx + 2\left(\frac{3}{50}\int e^{-3x}\cos(x)\,dx\right) + \frac{9}{50}\int e^{-3x}\sin(x)\,dx \\
&= -\frac{2}{125}e^{-3x}\cos(x) - \frac{3}{25}e^{-3x}x\cos(x) - \frac{1}{10}e^{-3x}x^2\cos(x) - \frac{6}{125}e^{-3x}\sin(x) \\
&\quad - \frac{4}{25}e^{-3x}x\sin(x) - \frac{3}{10}e^{-3x}x^2\sin(x) + 2\left(-\frac{9}{500}e^{-3x}\cos(x) + \frac{3}{500}e^{-3x}\sin(x)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.51

$$
\int e^{-3x}x^2\sin(x)\,dx = \frac{1}{250}e^{-3x}\left(-\left(\left(13+30x+25x^2\right)\cos(x)\right) - \left(9+40x+75x^2\right)\sin(x)\right)
$$

```
[In] Integrate[(x^2*Sin[x])/E^(3*x),x]
[Out] (-((13 + 30*x + 25*x^2)*Cos[x]) - (9 + 40*x + 75*x^2)*Sin[x])/(250*E^(3*x))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.41

method	result
parallelrisch	$-\frac{\mathrm{e}^{-3x}\left(\left(x^2+\frac{6}{5}x+\frac{13}{25}\right)\cos(x)+3\sin(x)\left(x^2+\frac{8}{15}x+\frac{3}{25}\right)\right)}{10}$
default	$\left(-\frac{1}{10}x^2-\frac{3}{25}x-\frac{13}{250}\right)\mathrm{e}^{-3x}\cos\left(x\right)+\left(-\frac{3}{10}x^2-\frac{4}{25}x-\frac{9}{250}\right)\mathrm{e}^{-3x}\sin\left(x\right)$
risch	$\left(-\frac{1}{500}+\frac{3i}{500}\right)\left(25x^2+5ix+15x+3i+4\right)\mathrm{e}^{(-3+i)x}+\left(-\frac{1}{500}-\frac{3i}{500}\right)\left(25x^2-5ix+15x-3i+\right.$
norman	$\frac{\left(-\frac{13}{250}-\frac{3x}{25}-\frac{x^2}{10}+\frac{13\left(\tan^2\left(\frac{x}{2}\right)\right)}{250}-\frac{8x\tan\left(\frac{x}{2}\right)}{25}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{25}-\frac{3x^2\tan\left(\frac{x}{2}\right)}{5}+\frac{x^2\left(\tan^2\left(\frac{x}{2}\right)\right)}{10}-\frac{9\tan\left(\frac{x}{2}\right)}{125}\right)\mathrm{e}^{-3x}}{1+\tan^2\left(\frac{x}{2}\right)}$

```
[In] int(x^2*sin(x)/exp(3*x),x,method=_RETURNVERBOSE)
[Out] -1/10*exp(-3*x)*((x^2+6/5*x+13/25)*cos(x)+3*sin(x)*(x^2+8/15*x+3/25))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

$$\int e^{-3x}x^2\sin(x)\,dx = -\frac{1}{250}\left(25\,x^2+30\,x+13\right)\cos\left(x\right)e^{\left(-3\,x\right)} - \frac{1}{250}\left(75\,x^2+40\,x+9\right)e^{\left(-3\,x\right)}\sin\left(x\right)$$

```
[In] integrate(x^2*sin(x)/exp(3*x),x, algorithm="fricas")
```

```
[Out] -1/250*(25*x^2 + 30*x + 13)*cos(x)*e^(-3*x) - 1/250*(75*x^2 + 40*x + 9)*e^(
-3*x)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int e^{-3x}x^2\sin(x)\,dx = -\frac{3x^2e^{-3x}\sin\left(x\right)}{10} - \frac{x^2e^{-3x}\cos\left(x\right)}{10} - \frac{4xe^{-3x}\sin\left(x\right)}{25} - \frac{3xe^{-3x}\cos\left(x\right)}{25} - \frac{9e^{-3x}\sin\left(x\right)}{250} - \frac{13e^{-3x}\cos\left(x\right)}{250}$$

```
[In] integrate(x**2*sin(x)/exp(3*x),x)
```

```
[Out] -3*x**2*exp(-3*x)*sin(x)/10 - x**2*exp(-3*x)*cos(x)/10 - 4*x*exp(-3*x)*sin(
x)/25 - 3*x*exp(-3*x)*cos(x)/25 - 9*exp(-3*x)*sin(x)/250 - 13*exp(-3*x)*cos
(x)/250
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.44

$$\int e^{-3x}x^2\sin(x)\,dx = -\frac{1}{250}\left(\left(25\,x^2+30\,x+13\right)\cos\left(x\right)+\left(75\,x^2+40\,x+9\right)\sin\left(x\right)\right)e^{\left(-3\,x\right)}$$

```
[In] integrate(x^2*sin(x)/exp(3*x),x, algorithm="maxima")
```

```
[Out] -1/250*((25*x^2 + 30*x + 13)*cos(x) + (75*x^2 + 40*x + 9)*sin(x))*e^(-3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.44

$$\int e^{-3x}x^2\sin(x)\,dx = -\frac{1}{250}\left(\left(25\,x^2+30\,x+13\right)\cos\left(x\right)+\left(75\,x^2+40\,x+9\right)\sin\left(x\right)\right)e^{\left(-3\,x\right)}$$

```
[In] integrate(x^2*sin(x)/exp(3*x),x, algorithm="giac")
[Out] -1/250*((25*x^2 + 30*x + 13)*cos(x) + (75*x^2 + 40*x + 9)*sin(x))*e^(-3*x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.52

$$\int e^{-3x}x^2\sin(x)\,dx$$
$$= -\frac{e^{-3\,x}\,\left(13\,\cos\left(x\right)+9\,\sin\left(x\right)+25\,x^2\,\cos\left(x\right)+75\,x^2\,\sin\left(x\right)+30\,x\,\cos\left(x\right)+40\,x\,\sin\left(x\right)\right)}{250}$$

```
[In] int(x^2*exp(-3*x)*sin(x),x)
[Out] -(exp(-3*x)*(13*cos(x) + 9*sin(x) + 25*x^2*cos(x) + 75*x^2*sin(x) + 30*x*co
s(x) + 40*x*sin(x)))/250
```

3.567 $\int e^{x/2}x^2\cos^3(x)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 187

$$\int e^{x/2}x^2\cos^3(x)\,dx=-\frac{132}{125}e^{x/2}\cos(x)$$
$$+\frac{18}{25}e^{x/2}x\cos(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)-\frac{428e^{x/2}\cos(3x)}{50653}+\frac{70e^{x/2}x\cos(3x)}{1369}-\frac{24}{125}e^{x/2}\sin(x)-$$

```
[Out] -132/125*exp(1/2*x)*cos(x)+18/25*exp(1/2*x)*x*cos(x)+48/185*exp(1/2*x)*x^2*
cos(x)+2/37*exp(1/2*x)*x^2*cos(x)^3-428/50653*exp(1/2*x)*cos(3*x)+70/1369*e
xp(1/2*x)*x*cos(3*x)-24/125*exp(1/2*x)*sin(x)-24/25*exp(1/2*x)*x*sin(x)+96/
185*exp(1/2*x)*x^2*sin(x)+12/37*exp(1/2*x)*x^2*cos(x)^2*sin(x)-792/50653*ex
p(1/2*x)*sin(3*x)-24/1369*exp(1/2*x)*x*sin(3*x)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.35, number of steps used = 31, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {4520, 4518, 4554, 14, 4517, 4557, 4553, 4558}

$$\int e^{x/2}x^2\cos^3(x)\,dx=\frac{96}{185}e^{x/2}x^2\sin(x)$$
$$+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{12}{37}e^{x/2}x^2\sin(x)\cos^2(x)-\frac{1218672e^{x/2}\sin(x)}{6331625}-\frac{32556e^{x/2}x\sin(x)}{34225}-\frac{81}{}$$

```
[In] Int[E^(x/2)*x^2*Cos[x]^3,x]
```

```
[Out] (-6687696*E^(x/2)*Cos[x])/6331625 + (24792*E^(x/2)*x*Cos[x])/34225 + (48*E^
(x/2)*x^2*Cos[x])/185 + (16*E^(x/2)*Cos[x]^3)/50653 - (8*E^(x/2)*x*Cos[x]^3
)/1369 + (2*E^(x/2)*x^2*Cos[x]^3)/37 - (432*E^(x/2)*Cos[3*x])/50653 + (72*E
^(x/2)*x*Cos[3*x])/1369 - (1218672*E^(x/2)*Sin[x])/6331625 - (32556*E^(x/2)
```

```
*x*Sin[x])/34225 + (96*E^(x/2)*x^2*Sin[x])/185 + (96*E^(x/2)*Cos[x]^2*Sin[x
])/50653 - (48*E^(x/2)*x*Cos[x]^2*Sin[x])/1369 + (12*E^(x/2)*x^2*Cos[x]^2*S
in[x])/37 - (816*E^(x/2)*Sin[3*x])/50653 - (12*E^(x/2)*x*Sin[3*x])/1369
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4520

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[e*m*F^(c*(a + b*x))*Sin[d +
 e*x]*(Cos[d + e*x]^(m - 1)/(e^2*m^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4558

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(x_)^(p
_.)*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[x^p*F^
(c*(a + b*x)), Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c,
d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{48}{185}e^{x/2}x^2\cos(x) + \frac{2}{37}e^{x/2}x^2\cos^3(x) + \frac{96}{185}e^{x/2}x^2\sin(x) + \frac{12}{37}e^{x/2}x^2\cos^2(x)\sin(x) - 2\int x\left(\frac{48}{185}e^{x/2}\cos(x) + \frac{2}{37}e^{x/2}\cos^3(x) + \frac{96}{185}e^{x/2}\sin(x) + \frac{12}{37}\right.$$

$$= \frac{48}{185}e^{x/2}x^2\cos(x) + \frac{2}{37}e^{x/2}x^2\cos^3(x) + \frac{96}{185}e^{x/2}x^2\sin(x) + \frac{12}{37}e^{x/2}x^2\cos^2(x)\sin(x) - 2\int\left(\frac{48}{185}e^{x/2}x\cos(x) + \frac{2}{37}e^{x/2}x\cos^3(x) + \frac{96}{185}e^{x/2}x\,s\right.$$

$$= \frac{48}{185}e^{x/2}x^2\cos(x) + \frac{2}{37}e^{x/2}x^2\cos^3(x) + \frac{96}{185}e^{x/2}x^2\sin(x) + \frac{12}{37}e^{x/2}x^2\cos^2(x)\sin(x) - \frac{4}{37}\int e^{x/2}x\cos^3(x)\,dx - \frac{96}{185}\int e^{x/2}x\cos(x)\,dx - \frac{24}{37}$$

$$= \frac{20352e^{x/2}x\cos(x)}{34225} + \frac{48}{185}e^{x/2}x^2\cos(x) - \frac{8e^{x/2}x\cos^3(x)}{1369} + \frac{2}{37}e^{x/2}x^2\cos^3(x) - \frac{30336e^{x/2}x\sin(x)}{34225} + \frac{96}{185}e^{x/2}x^2\sin(x) - \frac{48e^{x/2}x\cos^2(x)\sin(x)}{1369} + \frac{12}{37}e^{x/2}x^2\cos^2($$

$$= \frac{20352e^{x/2}x\cos(x)}{34225} + \frac{48}{185}e^{x/2}x^2\cos(x) - \frac{8e^{x/2}x\cos^3(x)}{1369} + \frac{2}{37}e^{x/2}x^2\cos^3(x) - \frac{30336e^{x/2}x\sin(x)}{34225} + \frac{96}{185}e^{x/2}x^2\sin(x) - \frac{48e^{x/2}x\cos^2(x)\sin(x)}{1369} + \frac{12}{37}e^{x/2}x^2\cos^2($$

$$= -\frac{48384e^{x/2}\cos(x)}{171125} + \frac{24792e^{x/2}x\cos(x)}{34225} + \frac{48}{185}e^{x/2}x^2\cos(x) + \frac{16e^{x/2}\cos^3(x)}{50653} - \frac{8e^{x/2}x\cos^3(x)}{1369} + \frac{2}{37}e^{x/2}x^2\cos^3(x) + \frac{72e^{x/2}x\cos(3x)}{1369} - \frac{77568e}{17}$$

$$= -\frac{1780608e^{x/2}\cos(x)}{6331625} + \frac{24792e^{x/2}x\cos(x)}{34225} + \frac{48}{185}e^{x/2}x^2\cos(x) + \frac{16e^{x/2}\cos^3(x)}{50653} - \frac{8e^{x/2}x\cos^3(x)}{1369} + \frac{2}{37}e^{x/2}x^2\cos^3(x) + \frac{72e^{x/2}x\cos(3x)}{1369} - \frac{28508}{6}$$

$$= -\frac{2482128e^{x/2}\cos(x)}{6331625} + \frac{24792e^{x/2}x\cos(x)}{34225} + \frac{48}{185}e^{x/2}x^2\cos(x) + \frac{16e^{x/2}\cos^3(x)}{50653} - \frac{8e^{x/2}x\cos^3(x)}{1369} + \frac{2}{37}e^{x/2}x^2\cos^3(x) - \frac{144e^{x/2}\cos(3x)}{50653} + \frac{72e^{x/2}}{1}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39

$$\int e^{x/2}x^2\cos^3(x)\,dx = \frac{e^{x/2}(151959(-88+60x+25x^2)\cos(x)+125(-856+5180x+1369x^2)\cos(3x)+30}{12663250}$$

```
[In] Integrate[E^(x/2)*x^2*Cos[x]^3,x]
```

```
[Out] (E^(x/2)*(151959*(-88 + 60*x + 25*x^2)*Cos[x] + 125*(-856 + 5180*x + 1369*x
^2)*Cos[3*x] + 303918*(-8 - 40*x + 25*x^2)*Sin[x] + 750*(-264 - 296*x + 136
9*x^2)*Sin[3*x]))/12663250
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.32

method	result
parallelrisch	$\frac{3\left(\frac{5\left(x^2+\frac{140}{37}x-\frac{856}{1369}\right)\cos(3x)}{111}+\frac{10\left(x^2-\frac{8}{37}x-\frac{264}{1369}\right)\sin(3x)}{37}+\left(x^2+\frac{12}{5}x-\frac{88}{25}\right)\cos(x)+2\sin(x)\left(x^2-\frac{8}{5}x-\frac{8}{25}\right)\right)e^{\frac{x}{2}}}{10}$
default	$\frac{\left(\frac{2}{37}x^2+\frac{280}{1369}x-\frac{1712}{50653}\right)e^{\frac{x}{2}}\cos(3x)}{4}-\frac{\left(-\frac{12}{37}x^2+\frac{96}{1369}x+\frac{3168}{50653}\right)e^{\frac{x}{2}}\sin(3x)}{4}+\frac{3\left(\frac{2}{5}x^2+\frac{24}{25}x-\frac{176}{125}\right)e^{\frac{x}{2}}\cos(x)}{4}-\frac{3\left(-\frac{4}{5}x^2+\frac{32}{25}x\right.}{}$
risch	$\left(\frac{1}{202612}-\frac{3i}{101306}\right)\left(1369x^2+888ix-148x-96i-280\right)e^{\left(\frac{1}{2}+3i\right)x}+\left(\frac{3}{500}-\frac{3i}{250}\right)\left(25x^2+40ix\right.$

```
[In] int(exp(1/2*x)*x^2*cos(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/10*(5/111*(x^2+140/37*x-856/1369)*cos(3*x)+10/37*(x^2-8/37*x-264/1369)*si
n(3*x)+(x^2+12/5*x-88/25)*cos(x)+2*sin(x)*(x^2-8/5*x-8/25))*exp(1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39

$$\int e^{x/2}x^2\cos^3(x)\,dx = \frac{12}{6331625}\left(125\left(1369\,x^2-296\,x-264\right)\cos\left(x\right)^2+273800\,x^2-497280\,x-93056\right)e^{\left(\frac{1}{2}\,x\right)} + \frac{2}{6331625}\left(125\left(1369\,x^2+5180\,x-856\right)\cos\left(x\right)^3+24\left(34225\,x^2+74740\,x-135952\right)\cos\left(x\right)\right)e^{\left(\frac{1}{2}\,x\right)}$$

```
[In] integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="fricas")
```

```
[Out] 12/6331625*(125*(1369*x^2 - 296*x - 264)*cos(x)^2 + 273800*x^2 - 497280*x -
 93056)*e^(1/2*x)*sin(x) + 2/6331625*(125*(1369*x^2 + 5180*x - 856)*cos(x)^
3 + 24*(34225*x^2 + 74740*x - 135952)*cos(x))*e^(1/2*x)
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.08

$$\int e^{x/2}x^2\cos^3(x)\,dx = \frac{96x^2e^{\frac{x}{2}}\sin^3(x)}{185} + \frac{48x^2e^{\frac{x}{2}}\sin^2(x)\cos(x)}{185} + \frac{156x^2e^{\frac{x}{2}}\sin(x)\cos^2(x)}{185} + \frac{58x^2e^{\frac{x}{2}}\cos^3(x)}{185} - \frac{32256xe^{\frac{x}{2}}\sin^3(x)}{34225} + \frac{19392xe^{\frac{x}{2}}\sin^2(x)\cos(x)}{34225} - \frac{34656xe^{\frac{x}{2}}\sin(x)\cos^2(x)}{34225} + \frac{26392xe^{\frac{x}{2}}\cos^3(x)}{34225} - \frac{1116672e^{\frac{x}{2}}\sin^3(x)}{6331625} - \frac{6525696e^{\frac{x}{2}}\sin^2(x)\cos(x)}{6331625} - \frac{1512672e^{\frac{x}{2}}\sin(x)\cos^2(x)}{6331625} - \frac{6739696e^{\frac{x}{2}}\cos^3(x)}{6331625}$$

```
[In] integrate(exp(1/2*x)*x**2*cos(x)**3,x)
```

```
[Out] 96*x**2*exp(x/2)*sin(x)**3/185 + 48*x**2*exp(x/2)*sin(x)**2*cos(x)/185 + 15
6*x**2*exp(x/2)*sin(x)*cos(x)**2/185 + 58*x**2*exp(x/2)*cos(x)**3/185 - 322
56*x*exp(x/2)*sin(x)**3/34225 + 19392*x*exp(x/2)*sin(x)**2*cos(x)/34225 - 3
4656*x*exp(x/2)*sin(x)*cos(x)**2/34225 + 26392*x*exp(x/2)*cos(x)**3/34225 -
 1116672*exp(x/2)*sin(x)**3/6331625 - 6525696*exp(x/2)*sin(x)**2*cos(x)/633
1625 - 1512672*exp(x/2)*sin(x)*cos(x)**2/6331625 - 6739696*exp(x/2)*cos(x)*
*3/6331625
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.41

$$\int e^{x/2}x^2\cos^3(x)\,dx = \frac{1}{101306}\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)e^{\left(\frac{1}{2}\,x\right)} + \frac{3}{250}\left(25\,x^2+60\,x-88\right)\cos\left(x\right)e^{\left(\frac{1}{2}\,x\right)} + \frac{3}{50653}\left(1369\,x^2-296\,x-264\right)e^{\left(\frac{1}{2}\,x\right)}\sin\left(3\,x\right) + \frac{3}{125}\left(25\,x^2-40\,x-8\right)e^{\left(\frac{1}{2}\,x\right)}\sin\left(x\right)$$

```
[In] integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="maxima")
[Out] 1/101306*(1369*x^2 + 5180*x - 856)*cos(3*x)*e^(1/2*x) + 3/250*(25*x^2 + 60*
x - 88)*cos(x)*e^(1/2*x) + 3/50653*(1369*x^2 - 296*x - 264)*e^(1/2*x)*sin(3
*x) + 3/125*(25*x^2 - 40*x - 8)*e^(1/2*x)*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.39

$$\int e^{x/2}x^2\cos^3(x)\,dx = \frac{1}{101306}\left(\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)+6\left(1369\,x^2-296\,x-264\right)\sin\left(3\,x\right)\right)e^{\left(\frac{1}{2}\,x\right)} + \frac{3}{250}\left(\left(25\,x^2+60\,x-88\right)\cos\left(x\right)+2\left(25\,x^2-40\,x-8\right)\sin\left(x\right)\right)e^{\left(\frac{1}{2}\,x\right)}$$

```
[In] integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="giac")
[Out] 1/101306*((1369*x^2 + 5180*x - 856)*cos(3*x) + 6*(1369*x^2 - 296*x - 264)*s
in(3*x))*e^(1/2*x) + 3/250*((25*x^2 + 60*x - 88)*cos(x) + 2*(25*x^2 - 40*x
- 8)*sin(x))*e^(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.44

$$\int e^{x/2}x^2\cos^3(x)\,dx = -\frac{e^{x/2}\left(107000\,\cos\left(3\,x\right)+198000\,\sin\left(3\,x\right)+13372392\,\cos\left(x\right)+2431344\,\sin\left(x\right)-647500\,x\,\cos\left(3\,x\right)-379\right.}{}$$

```
[In] int(x^2*exp(x/2)*cos(x)^3,x)
[Out] -(exp(x/2)*(107000*cos(3*x) + 198000*sin(3*x) + 13372392*cos(x) + 2431344*s
in(x) - 647500*x*cos(3*x) - 3798975*x^2*cos(x) + 222000*x*sin(3*x) - 759795
0*x^2*sin(x) - 171125*x^2*cos(3*x) - 1026750*x^2*sin(3*x) - 9117540*x*cos(x
) + 12156720*x*sin(x)))/12663250
```

3.568 $\int e^{2x}x^2\sin(4x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int e^{2x}x^2\sin(4x)\,dx=\frac{1}{250}e^{2x}\cos(4x)+\frac{2}{25}e^{2x}x\cos(4x)-\frac{1}{5}e^{2x}x^2\cos(4x)-\frac{11}{500}e^{2x}\sin(4x)+\frac{3}{50}e^{2x}x\sin(4x)+\frac{1}{10}e^{2x}x^2\sin(4x)$$

```
[Out] 1/250*exp(2*x)*cos(4*x)+2/25*exp(2*x)*x*cos(4*x)-1/5*exp(2*x)*x^2*cos(4*x)-
11/500*exp(2*x)*sin(4*x)+3/50*exp(2*x)*x*sin(4*x)+1/10*exp(2*x)*x^2*sin(4*x
)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {4517, 4553, 14, 4518, 4554}

$$\int e^{2x}x^2\sin(4x)\,dx=\frac{1}{10}e^{2x}x^2\sin(4x)-\frac{1}{5}e^{2x}x^2\cos(4x)+\frac{3}{50}e^{2x}x\sin(4x)-\frac{11}{500}e^{2x}\sin(4x)+\frac{2}{25}e^{2x}x\cos(4x)+\frac{1}{250}e^{2x}\cos(4x)$$

```
[In] Int[E^(2*x)*x^2*Sin[4*x],x]
```

```
[Out] (E^(2*x)*Cos[4*x])/250 + (2*E^(2*x)*x*Cos[4*x])/25 - (E^(2*x)*x^2*Cos[4*x])
/5 - (11*E^(2*x)*Sin[4*x])/500 + (3*E^(2*x)*x*Sin[4*x])/50 + (E^(2*x)*x^2*S
in[4*x])/10
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{5}e^{2x}x^2\cos(4x)+\frac{1}{10}e^{2x}x^2\sin(4x)-2\int x\left(-\frac{1}{5}e^{2x}\cos(4x)+\frac{1}{10}e^{2x}\sin(4x)\right)\,dx\\
&= -\frac{1}{5}e^{2x}x^2\cos(4x)+\frac{1}{10}e^{2x}x^2\sin(4x)-2\int\left(-\frac{1}{5}e^{2x}x\cos(4x)+\frac{1}{10}e^{2x}x\sin(4x)\right)\,dx\\
&= -\frac{1}{5}e^{2x}x^2\cos(4x)+\frac{1}{10}e^{2x}x^2\sin(4x)-\frac{1}{5}\int e^{2x}x\sin(4x)\,dx+\frac{2}{5}\int e^{2x}x\cos(4x)\,dx\\
&= \frac{2}{25}e^{2x}x\cos(4x)-\frac{1}{5}e^{2x}x^2\cos(4x)+\frac{3}{50}e^{2x}x\sin(4x)\\
&\quad+\frac{1}{10}e^{2x}x^2\sin(4x)+\frac{1}{5}\int\left(-\frac{1}{5}e^{2x}\cos(4x)+\frac{1}{10}e^{2x}\sin(4x)\right)\,dx\\
&\quad-\frac{2}{5}\int\left(\frac{1}{10}e^{2x}\cos(4x)+\frac{1}{5}e^{2x}\sin(4x)\right)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2}{25}e^{2x}x\cos(4x) - \frac{1}{5}e^{2x}x^2\cos(4x) + \frac{3}{50}e^{2x}x\sin(4x) + \frac{1}{10}e^{2x}x^2\sin(4x) \\
&\quad + \frac{1}{50}\int e^{2x}\sin(4x)\,dx - 2\left(\frac{1}{25}\int e^{2x}\cos(4x)\,dx\right) - \frac{2}{25}\int e^{2x}\sin(4x)\,dx \\
&= \frac{3}{250}e^{2x}\cos(4x) + \frac{2}{25}e^{2x}x\cos(4x) - \frac{1}{5}e^{2x}x^2\cos(4x) - \frac{3}{500}e^{2x}\sin(4x) \\
&\quad + \frac{3}{50}e^{2x}x\sin(4x) + \frac{1}{10}e^{2x}x^2\sin(4x) - 2\left(\frac{1}{250}e^{2x}\cos(4x) + \frac{1}{125}e^{2x}\sin(4x)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.46

$$\int e^{2x}x^2\sin(4x)\,dx = \frac{1}{500}e^{2x}\left((2+40x-100x^2)\cos(4x) + (-11+30x+50x^2)\sin(4x)\right)$$

```
[In] Integrate[E^(2*x)*x^2*Sin[4*x],x]
[Out] (E^(2*x)*((2 + 40*x - 100*x^2)*Cos[4*x] + (-11 + 30*x + 50*x^2)*Sin[4*x]))/
500
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.46

method	result
default	$\left(-\frac{1}{5}x^2+\frac{2}{25}x+\frac{1}{250}\right)\mathrm{e}^{2x}\cos(4x)+\left(\frac{1}{10}x^2+\frac{3}{50}x-\frac{11}{500}\right)\mathrm{e}^{2x}\sin(4x)$
parallelrisch	$\frac{(-100x^2+40x+2)\mathrm{e}^{2x}\cos(4x)}{500}+\frac{\mathrm{e}^{2x}\sin(4x)\left(x^2+\frac{3}{5}x-\frac{11}{50}\right)}{10}$
risch	$\left(-\frac{1}{500}-\frac{i}{1000}\right)(50x^2+20ix-10x-4i-3)\,\mathrm{e}^{(2+4i)x}+\left(-\frac{1}{500}+\frac{i}{1000}\right)(50x^2-20ix-10x+4$
norman	$\frac{\frac{2\,\mathrm{e}^{2x}x}{25}-\frac{\mathrm{e}^{2x}x^2}{5}-\frac{11\,\mathrm{e}^{2x}\tan(2x)}{250}-\frac{\mathrm{e}^{2x}\left(\tan^2(2x)\right)}{250}+\frac{3\,\mathrm{e}^{2x}x\tan(2x)}{25}-\frac{2\,\mathrm{e}^{2x}x\left(\tan^2(2x)\right)}{25}+\frac{\mathrm{e}^{2x}x^2\tan(2x)}{5}+\frac{\mathrm{e}^{2x}x^2\left(\tan^2(2x)\right)}{5}+\frac{\mathrm{e}^{2x}}{250}}{1+\tan^2(2x)}$

```
[In] int(exp(2*x)*x^2*sin(4*x),x,method=_RETURNVERBOSE)
[Out] (-1/5*x^2+2/25*x+1/250)*exp(2*x)*cos(4*x)+(1/10*x^2+3/50*x-11/500)*exp(2*x)
*sin(4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.47

$$\int e^{2x}x^2\sin(4x)\,dx = -\frac{1}{250}\left(50\,x^2-20\,x-1\right)\cos\left(4\,x\right)e^{(2\,x)} + \frac{1}{500}\left(50\,x^2+30\,x-11\right)e^{(2\,x)}\sin\left(4\,x\right)$$

```
[In] integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="fricas")
[Out] -1/250*(50*x^2 - 20*x - 1)*cos(4*x)*e^(2*x) + 1/500*(50*x^2 + 30*x - 11)*e^
(2*x)*sin(4*x)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int e^{2x}x^2\sin(4x)\,dx = \frac{x^2e^{2x}\sin\left(4x\right)}{10} - \frac{x^2e^{2x}\cos\left(4x\right)}{5} + \frac{3xe^{2x}\sin\left(4x\right)}{50} + \frac{2xe^{2x}\cos\left(4x\right)}{25} - \frac{11e^{2x}\sin\left(4x\right)}{500} + \frac{e^{2x}\cos\left(4x\right)}{250}$$

```
[In] integrate(exp(2*x)*x**2*sin(4*x),x)
[Out] x**2*exp(2*x)*sin(4*x)/10 - x**2*exp(2*x)*cos(4*x)/5 + 3*x*exp(2*x)*sin(4*x
)/50 + 2*x*exp(2*x)*cos(4*x)/25 - 11*exp(2*x)*sin(4*x)/500 + exp(2*x)*cos(4
*x)/250
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.47

$$\int e^{2x}x^2\sin(4x)\,dx = -\frac{1}{250}\left(50\,x^2-20\,x-1\right)\cos\left(4\,x\right)e^{(2\,x)} + \frac{1}{500}\left(50\,x^2+30\,x-11\right)e^{(2\,x)}\sin\left(4\,x\right)$$

```
[In] integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="maxima")
[Out] -1/250*(50*x^2 - 20*x - 1)*cos(4*x)*e^(2*x) + 1/500*(50*x^2 + 30*x - 11)*e^
(2*x)*sin(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.45

$$\int e^{2x}x^2\sin(4x)\,dx$$
$$= -\frac{1}{500}\left(2\left(50x^2-20x-1\right)\cos(4x)-\left(50x^2+30x-11\right)\sin(4x)\right)e^{(2x)}$$

```
[In] integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="giac")
[Out] -1/500*(2*(50*x^2 - 20*x - 1)*cos(4*x) - (50*x^2 + 30*x - 11)*sin(4*x))*e^(
2*x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int e^{2x}x^2\sin(4x)\,dx$$
$$= \frac{e^{2x}\left(2\cos(4x)-11\sin(4x)+40x\cos(4x)+30x\sin(4x)-100x^2\cos(4x)+50x^2\sin(4x)\right)}{500}$$

```
[In] int(x^2*sin(4*x)*exp(2*x),x)
[Out] (exp(2*x)*(2*cos(4*x) - 11*sin(4*x) + 40*x*cos(4*x) + 30*x*sin(4*x) - 100*x
^2*cos(4*x) + 50*x^2*sin(4*x)))/500
```

3.569 $\int e^{x/2} x^2 \cos(x) \sin^2(x) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 185

$$\int e^{x/2} x^2 \cos(x) \sin^2(x) \, dx = -\frac{44}{125} e^{x/2} \cos(x) + \frac{6}{25} e^{x/2} x \cos(x) + \frac{1}{10} e^{x/2} x^2 \cos(x) + \frac{428 e^{x/2} \cos(3x)}{50653} - \frac{70 e^{x/2} x \cos(3x)}{1369} - \frac{1}{74} e^{x/2} x^2 \cos(3x) - \frac{8}{125} e^{x/2} \sin(x) -$$

```
[Out] -44/125*exp(1/2*x)*cos(x)+6/25*exp(1/2*x)*x*cos(x)+1/10*exp(1/2*x)*x^2*cos(
x)+428/50653*exp(1/2*x)*cos(3*x)-70/1369*exp(1/2*x)*x*cos(3*x)-1/74*exp(1/2
*x)*x^2*cos(3*x)-8/125*exp(1/2*x)*sin(x)-8/25*exp(1/2*x)*x*sin(x)+1/5*exp(1
/2*x)*x^2*sin(x)+792/50653*exp(1/2*x)*sin(3*x)+24/1369*exp(1/2*x)*x*sin(3*x
)-3/37*exp(1/2*x)*x^2*sin(3*x)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4558, 4518, 4554, 14, 4517, 4553}

$$\int e^{x/2} x^2 \cos(x) \sin^2(x) \, dx = \frac{1}{5} e^{x/2} x^2 \sin(x) - \frac{3}{37} e^{x/2} x^2 \sin(3x) + \frac{1}{10} e^{x/2} x^2 \cos(x) - \frac{1}{74} e^{x/2} x^2 \cos(3x) - \frac{8}{25} e^{x/2} x \sin(x) + \frac{24 e^{x/2} x \sin(3x)}{1369} - \frac{8}{125} e^{x/2} \sin(x) +$$

```
[In] Int[E^(x/2)*x^2*Cos[x]*Sin[x]^2,x]
```

```
[Out] (-44*E^(x/2)*Cos[x])/125 + (6*E^(x/2)*x*Cos[x])/25 + (E^(x/2)*x^2*Cos[x])/1
0 + (428*E^(x/2)*Cos[3*x])/50653 - (70*E^(x/2)*x*Cos[3*x])/1369 - (E^(x/2)*
x^2*Cos[3*x])/74 - (8*E^(x/2)*Sin[x])/125 - (8*E^(x/2)*x*Sin[x])/25 + (E^(x
```

```
/2)*x^2*Sin[x])/5 + (792*E^(x/2)*Sin[3*x])/50653 + (24*E^(x/2)*x*Sin[3*x])/
1369 - (3*E^(x/2)*x^2*Sin[3*x])/37
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4558

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(x_)^(p
_.)*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[x^p*F^
(c*(a + b*x)), Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c,
d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{4}e^{x/2}x^2\cos(x) - \frac{1}{4}e^{x/2}x^2\cos(3x)\right)\,dx$$

$$= \frac{1}{4}\int e^{x/2}x^2\cos(x)\,dx - \frac{1}{4}\int e^{x/2}x^2\cos(3x)\,dx$$

$$= \frac{1}{10}e^{x/2}x^2\cos(x) - \frac{1}{74}e^{x/2}x^2\cos(3x) + \frac{1}{5}e^{x/2}x^2\sin(x) - \frac{3}{37}e^{x/2}x^2\sin(3x) - \frac{1}{2}\int x\left(\frac{2}{5}e^{x/2}\cos(x) + \frac{4}{5}e^{x/2}\sin(x)\right)\,dx + \frac{1}{2}\int x\left(\frac{2}{37}e^{x/2}\cos$$

$$= \frac{1}{10}e^{x/2}x^2\cos(x) - \frac{1}{74}e^{x/2}x^2\cos(3x) + \frac{1}{5}e^{x/2}x^2\sin(x) - \frac{3}{37}e^{x/2}x^2\sin(3x) - \frac{1}{2}\int\left(\frac{2}{5}e^{x/2}x\cos(x) + \frac{4}{5}e^{x/2}x\sin(x)\right)\,dx + \frac{1}{2}\int\left(\frac{2}{37}e^{x/2}x\,\mathrm{c}$$

$$= \frac{1}{10}e^{x/2}x^2\cos(x) - \frac{1}{74}e^{x/2}x^2\cos(3x) + \frac{1}{5}e^{x/2}x^2\sin(x) - \frac{3}{37}e^{x/2}x^2\sin(3x) + \frac{1}{37}\int e^{x/2}x\cos(3x)\,dx + \frac{6}{37}\int e^{x/2}x\sin(3x)\,dx - \frac{1}{5}\int e^{x/2}x\,\mathrm{c}$$

$$= \frac{6}{25}e^{x/2}x\cos(x) + \frac{1}{10}e^{x/2}x^2\cos(x) - \frac{70e^{x/2}x\cos(3x)}{1369} - \frac{1}{74}e^{x/2}x^2\cos(3x) - \frac{8}{25}e^{x/2}x\sin(x) + \frac{1}{5}e^{x/2}x^2\sin(x) + \frac{24e^{x/2}x\sin(3x)}{1369} - \frac{3}{37}e^{x/2}x^2\sin(3x) - \frac{1}{37}\int$$

$$= \frac{6}{25}e^{x/2}x\cos(x) + \frac{1}{10}e^{x/2}x^2\cos(x) - \frac{70e^{x/2}x\cos(3x)}{1369} - \frac{1}{74}e^{x/2}x^2\cos(3x) - \frac{8}{25}e^{x/2}x\sin(x) + \frac{1}{5}e^{x/2}x^2\sin(x) + \frac{24e^{x/2}x\sin(3x)}{1369} - \frac{3}{37}e^{x/2}x^2\sin(3x) - \frac{2\int e^{x}}{}$$

$$= -\frac{12}{125}e^{x/2}\cos(x) + \frac{6}{25}e^{x/2}x\cos(x) + \frac{1}{10}e^{x/2}x^2\cos(x) + \frac{140e^{x/2}\cos(3x)}{50653} - \frac{70e^{x/2}x\cos(3x)}{1369} - \frac{1}{74}e^{x/2}x^2\cos(3x) - \frac{24}{125}e^{x/2}\sin(x) - \frac{8}{25}e^{x/}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.41

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx = \frac{e^{x/2}(50653((-88+60x+25x^2)\cos(x)+2(-8-40x+25x^2)\sin(x))-125((-8}{12663250}$$

[In] Integrate[E^(x/2)*x^2*Cos[x]*Sin[x]^2,x]

[Out] (E^(x/2)*(50653*((-88 + 60*x + 25*x^2)*Cos[x] + 2*(-8 - 40*x + 25*x^2)*Sin[
x]) - 125*((-856 + 5180*x + 1369*x^2)*Cos[3*x] + 6*(-264 - 296*x + 1369*x^2
)*Sin[3*x])))/12663250

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.42

method	result
default	$\frac{(\frac{2}{5}x^2+\frac{24}{25}x-\frac{176}{125})e^{\frac{x}{2}}\cos(x)}{4}-\frac{(-\frac{4}{5}x^2+\frac{32}{25}x+\frac{32}{125})e^{\frac{x}{2}}\sin(x)}{4}-\frac{(\frac{2}{37}x^2+\frac{280}{1369}x-\frac{1712}{50653})e^{\frac{x}{2}}\cos(3x)}{4}+\frac{(-\frac{12}{37}x^2+\frac{96}{1369}x+\frac{3168}{50653})e^{\frac{x}{2}}\text{si}}{4}$
risch	$\left(-\frac{1}{202612}+\frac{3i}{101306}\right)(1369x^2+888ix-148x-96i-280)\,e^{(\frac{1}{2}+3i)x}+\left(\frac{1}{500}-\frac{i}{250}\right)(25x^2+40ix-20$

[In] int(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x,method=_RETURNVERBOSE)

[Out] 1/4*(2/5*x^2+24/25*x-176/125)*exp(1/2*x)*cos(x)-1/4*(-4/5*x^2+32/25*x+32/12
5)*exp(1/2*x)*sin(x)-1/4*(2/37*x^2+280/1369*x-1712/50653)*exp(1/2*x)*cos(3*
x)+1/4*(-12/37*x^2+96/1369*x+3168/50653)*exp(1/2*x)*sin(3*x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx =$$
$$-\frac{4}{6331625}\left(375\left(1369\,x^2-296\,x-264\right)\cos\left(x\right)^2-444925\,x^2+534280\,x+126056\right)e^{\left(\frac{1}{2}\,x\right)}\sin\left(x\right)$$
$$-\frac{2}{6331625}\left(125\left(1369\,x^2+5180\,x-856\right)\cos\left(x\right)^3-\left(444925\,x^2+1245420\,x-1194616\right)\cos\left(x\right)\right)e^{\left(\frac{1}{2}\,x\right)}$$

[In] integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="fricas")

[Out] -4/6331625*(375*(1369*x^2 - 296*x - 264)*cos(x)^2 - 444925*x^2 + 534280*x +
126056)*e^(1/2*x)*sin(x) - 2/6331625*(125*(1369*x^2 + 5180*x - 856)*cos(x)
^3 - (444925*x^2 + 1245420*x - 1194616)*cos(x))*e^(1/2*x)

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.09

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx = \frac{52x^2e^{\frac{x}{2}}\sin^3(x)}{185} + \frac{26x^2e^{\frac{x}{2}}\sin^2(x)\cos(x)}{185} - \frac{8x^2e^{\frac{x}{2}}\sin(x)\cos^2(x)}{185} + \frac{16x^2e^{\frac{x}{2}}\cos^3(x)}{185} - \frac{11552xe^{\frac{x}{2}}\sin^3(x)}{34225} + \frac{13464xe^{\frac{x}{2}}\sin^2(x)\cos(x)}{34225} - \frac{9152xe^{\frac{x}{2}}\sin(x)\cos^2(x)}{34225} + \frac{6464xe^{\frac{x}{2}}\cos^3(x)}{34225} - \frac{504224e^{\frac{x}{2}}\sin^3(x)}{6331625} - \frac{2389232e^{\frac{x}{2}}\sin^2(x)\cos(x)}{6331625} - \frac{108224e^{\frac{x}{2}}\sin(x)\cos^2(x)}{6331625} - \frac{2175232e^{\frac{x}{2}}\cos^3(x)}{6331625}$$

```
[In] integrate(exp(1/2*x)*x**2*cos(x)*sin(x)**2,x)
```

```
[Out] 52*x**2*exp(x/2)*sin(x)**3/185 + 26*x**2*exp(x/2)*sin(x)**2*cos(x)/185 - 8*
x**2*exp(x/2)*sin(x)*cos(x)**2/185 + 16*x**2*exp(x/2)*cos(x)**3/185 - 11552
*x*exp(x/2)*sin(x)**3/34225 + 13464*x*exp(x/2)*sin(x)**2*cos(x)/34225 - 915
2*x*exp(x/2)*sin(x)*cos(x)**2/34225 + 6464*x*exp(x/2)*cos(x)**3/34225 - 504
224*exp(x/2)*sin(x)**3/6331625 - 2389232*exp(x/2)*sin(x)**2*cos(x)/6331625
- 108224*exp(x/2)*sin(x)*cos(x)**2/6331625 - 2175232*exp(x/2)*cos(x)**3/633
1625
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.42

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx = -\frac{1}{101306}\left(1369\,x^2+5180\,x-856\right)\cos(3\,x)\,e^{\left(\frac{1}{2}\,x\right)} + \frac{1}{250}\left(25\,x^2+60\,x-88\right)\cos(x)\,e^{\left(\frac{1}{2}\,x\right)} - \frac{3}{50653}\left(1369\,x^2-296\,x-264\right)e^{\left(\frac{1}{2}\,x\right)}\sin(3\,x) + \frac{1}{125}\left(25\,x^2-40\,x-8\right)e^{\left(\frac{1}{2}\,x\right)}\sin(x)$$

```
[In] integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/101306*(1369*x^2 + 5180*x - 856)*cos(3*x)*e^(1/2*x) + 1/250*(25*x^2 + 60
*x - 88)*cos(x)*e^(1/2*x) - 3/50653*(1369*x^2 - 296*x - 264)*e^(1/2*x)*sin(
3*x) + 1/125*(25*x^2 - 40*x - 8)*e^(1/2*x)*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.39

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx = -\frac{1}{101306}\left(\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)+6\left(1369\,x^2-296\,x-264\right)\sin\left(3\,x\right)\right)e^{\left(\frac{1}{2}\,x\right)} + \frac{1}{250}\left(\left(25\,x^2+60\,x-88\right)\cos\left(x\right)+2\left(25\,x^2-40\,x-8\right)\sin\left(x\right)\right)e^{\left(\frac{1}{2}\,x\right)}$$

```
[In] integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/101306*((1369*x^2 + 5180*x - 856)*cos(3*x) + 6*(1369*x^2 - 296*x - 264)*
sin(3*x))*e^(1/2*x) + 1/250*((25*x^2 + 60*x - 88)*cos(x) + 2*(25*x^2 - 40*x
 - 8)*sin(x))*e^(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx = \frac{e^{x/2}\left(107000\,\cos\left(3\,x\right)+198000\,\sin\left(3\,x\right)-4457464\,\cos\left(x\right)-810448\,\sin\left(x\right)-6\right.}{}$$

```
[In] int(x^2*exp(x/2)*cos(x)*sin(x)^2,x)
```

```
[Out] (exp(x/2)*(107000*cos(3*x) + 198000*sin(3*x) - 4457464*cos(x) - 810448*sin(
x) - 647500*x*cos(3*x) + 1266325*x^2*cos(x) + 222000*x*sin(3*x) + 2532650*x
^2*sin(x) - 171125*x^2*cos(3*x) - 1026750*x^2*sin(3*x) + 3039180*x*cos(x) -
 4052240*x*sin(x)))/12663250
```

3.570 $\int \cosh(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 2

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[Out] sinh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{2717\}$

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[In] Int[Cosh[x],x]
```

```
[Out] Sinh[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \sinh(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[In] Integrate[Cosh[x],x]
[Out] Sinh[x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
lookup	$\sinh(x)$	3
default	$\sinh(x)$	3
meijerg	$\sinh(x)$	3
parallelrisch	$\sinh(x)$	3
risch	$-\frac{e^{-x}}{2}+\frac{e^{x}}{2}$	12

```
[In] int(cosh(x),x,method=_RETURNVERBOSE)
[Out] sinh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[In] integrate(cosh(x),x, algorithm="fricas")
[Out] sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[In] integrate(cosh(x),x)
[Out] sinh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[In] integrate(cosh(x),x, algorithm="maxima")
[Out] sinh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(2) = 4.

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 5.50

$$\int \cosh(x)\,dx = -\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(cosh(x),x, algorithm="giac")
[Out] -1/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[In] int(cosh(x),x)
[Out] sinh(x)
```

3.571 $\int \sinh(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 2

$$\int \sinh(x)\,dx = \cosh(x)$$

```
[Out] cosh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{2718\}$

$$\int \sinh(x)\,dx = \cosh(x)$$

```
[In] Int[Sinh[x],x]
```

```
[Out] Cosh[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\text{integral} = \cosh(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh(x)$$

```
[In] Integrate[Sinh[x],x]
[Out] Cosh[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
lookup	$\cosh\left(x\right)$	3
default	$\cosh\left(x\right)$	3
risch	$\frac{{\mathrm e}^{x}}{2}+\frac{{\mathrm e}^{-x}}{2}$	12
parallelrisch	$-\frac{2}{\tanh^{2}\left(\frac{x}{2}\right)-1}$	13
meijerg	$-\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh\left(x\right)}{\sqrt{\pi}}\right)$	17

```
[In] int(sinh(x),x,method=_RETURNVERBOSE)
[Out] cosh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh\left(x\right)$$

```
[In] integrate(sinh(x),x, algorithm="fricas")
[Out] cosh(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh(x)$$

```
[In] integrate(sinh(x),x)
[Out] cosh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh(x)$$

```
[In] integrate(sinh(x),x, algorithm="maxima")
[Out] cosh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(2) = 4$.

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 5.50

$$\int \sinh(x)\,dx = \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(sinh(x),x, algorithm="giac")
[Out] 1/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh(x)$$

```
[In] int(sinh(x),x)
[Out] cosh(x)
```

3.572 $\int \tanh(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

```
[Out] ln(cosh(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{3556\}$

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

```
[In] Int[Tanh[x],x]
```

```
[Out] Log[Cosh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \log(\cosh(x))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

```
[In] Integrate[Tanh[x],x]
[Out] Log[Cosh[x]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\ln(\cosh(x))$	4
derivativedivides	$\ln(\cosh(x))$	4
default	$\ln(\cosh(x))$	4
risch	$-x+\ln(1+e^{2x})$	12
parallelrisch	$-\ln(1-\tanh(x))-x$	14

```
[In] int(tanh(x),x,method=_RETURNVERBOSE)
[Out] ln(cosh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(3) = 6$.

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 6.00

$$\int \tanh(x)\,dx = -x + \log\left(\frac{2\cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

```
[In] integrate(tanh(x),x, algorithm="fricas")
[Out] -x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \tanh(x)\,dx = x - \log\left(\tanh\left(x\right) + 1\right)$$

```
[In] integrate(tanh(x),x)
[Out] x - log(tanh(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\,dx = \log\left(\cosh\left(x\right)\right)$$

```
[In] integrate(tanh(x),x, algorithm="maxima")
[Out] log(cosh(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \tanh(x)\,dx = -x + \log\left(e^{(2\,x)} + 1\right)$$

```
[In] integrate(tanh(x),x, algorithm="giac")
[Out] -x + log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\,dx = \ln\left(\cosh(x)\right)$$

```
[In] int(tanh(x),x)
[Out] log(cosh(x))
```

3.573 $\int \coth(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \coth(x)\,dx = \log(\sinh(x))$$

```
[Out] ln(sinh(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{3556\}$

$$\int \coth(x)\,dx = \log(\sinh(x))$$

```
[In] Int[Coth[x],x]
```

```
[Out] Log[Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \log(\sinh(x))$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \coth(x)\,dx = \log(\cosh(x)) + \log(\tanh(x))$$

```
[In] Integrate[Coth[x],x]
[Out] Log[Cosh[x]] + Log[Tanh[x]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\ln(\sinh(x))$	4
derivativedivides	$\ln(\sinh(x))$	4
default	$\ln(\sinh(x))$	4
risch	$-x+\ln(e^{2x}-1)$	12
parallelrisch	$\ln(\tanh(x))-\ln(1-\tanh(x))-x$	17

```
[In] int(coth(x),x,method=_RETURNVERBOSE)
[Out] ln(sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(3) = 6$.

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 6.00

$$\int \coth(x)\,dx = -x + \log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

```
[In] integrate(coth(x),x, algorithm="fricas")
[Out] -x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(3) = 6$.

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

$$\int \coth(x)\,dx = x - \log\left(\tanh\left(x\right) + 1\right) + \log\left(\tanh\left(x\right)\right)$$

```
[In] integrate(coth(x),x)
[Out] x - log(tanh(x) + 1) + log(tanh(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \coth(x)\,dx = \log\left(\sinh\left(x\right)\right)$$

```
[In] integrate(coth(x),x, algorithm="maxima")
[Out] log(sinh(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(3) = 6$.

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

$$\int \coth(x)\,dx = -x + \log\left(\left|e^{(2\,x)} - 1\right|\right)$$

```
[In] integrate(coth(x),x, algorithm="giac")
[Out] -x + log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \coth(x)\,dx = \ln\left(\sinh(x)\right)$$

```
[In] int(coth(x),x)
[Out] log(sinh(x))
```

3.574 $\int \operatorname{sech}(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \operatorname{sech}(x)\,dx = \arctan(\sinh(x))$$

```
[Out] arctan(sinh(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{3855\}$

$$\int \operatorname{sech}(x)\,dx = \arctan(\sinh(x))$$

```
[In] Int[Sech[x],x]
```

```
[Out] ArcTan[Sinh[x]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \arctan(\sinh(x))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \mathrm{sech}(x)\,dx = \arctan(\sinh(x))$$

```
[In] Integrate[Sech[x],x]
[Out] ArcTan[Sinh[x]]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\arctan(\sinh(x))$	4
default	$\arctan(\sinh(x))$	4
risch	$i\ln(\mathrm{e}^{x}+i)-i\ln(\mathrm{e}^{x}-i)$	20
parallelrisch	$-i\left(\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)\right)$	23

```
[In] int(sech(x),x,method=_RETURNVERBOSE)
[Out] arctan(sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. $2(3) = 6$.

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.67

$$\int \mathrm{sech}(x)\,dx = 2\arctan(\cosh(x)+\sinh(x))$$

```
[In] integrate(sech(x),x, algorithm="fricas")
[Out] 2*arctan(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \operatorname{sech}(x)\,dx = 2\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)$$

```
[In] integrate(sech(x),x)
[Out] 2*atan(tanh(x/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(x)\,dx = \arctan\left(\sinh\left(x\right)\right)$$

```
[In] integrate(sech(x),x, algorithm="maxima")
[Out] arctan(sinh(x))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(x)\,dx = 2\,\arctan\left(e^{x}\right)$$

```
[In] integrate(sech(x),x, algorithm="giac")
[Out] 2*arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(x)\,dx = 2\operatorname{atan}(\mathrm{e}^{x})$$

```
[In] int(1/cosh(x),x)
[Out] 2*atan(exp(x))
```

3.575 $\int \operatorname{csch}(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 5

$$\int \operatorname{csch}(x)\,dx = -\operatorname{arctanh}(\cosh(x))$$

```
[Out] -arctanh(cosh(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3855}

$$\int \operatorname{csch}(x)\,dx = -\operatorname{arctanh}(\cosh(x))$$

```
[In] Int[Csch[x],x]
```

```
[Out] -ArcTanh[Cosh[x]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\operatorname{arctanh}(\cosh(x))$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$.

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 3.40

$$\int \operatorname{csch}(x)\,dx = -\log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Csch[x],x]
[Out] -Log[Cosh[x/2]] + Log[Sinh[x/2]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
lookup	$\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	6
default	$\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	6
parallelrisch	$\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	6
risch	$\ln\left(-1+e^{x}\right)-\ln\left(1+e^{x}\right)$	14

```
[In] int(csch(x),x,method=_RETURNVERBOSE)
[Out] ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 3.40

$$\int \operatorname{csch}(x)\,dx = -\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) + \log\left(\cosh\left(x\right) + \sinh\left(x\right) - 1\right)$$

```
[In] integrate(csch(x),x, algorithm="fricas")
[Out] -log(cosh(x) + sinh(x) + 1) + log(cosh(x) + sinh(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(x)\,dx = \log\left(\tanh\left(\frac{x}{2}\right)\right)$$

```
[In] integrate(csch(x),x)
[Out] log(tanh(x/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(x)\,dx = \log\left(\tanh\left(\frac{1}{2}\,x\right)\right)$$

```
[In] integrate(csch(x),x, algorithm="maxima")
[Out] log(tanh(1/2*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(5) = 10.

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.80

$$\int \operatorname{csch}(x)\,dx = -\log\left(e^{x}+1\right) + \log\left(|e^{x}-1|\right)$$

```
[In] integrate(csch(x),x, algorithm="giac")
[Out] -log(e^x + 1) + log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(x)\,dx = \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

```
[In] int(1/sinh(x),x)
[Out] log(tanh(x/2))
```

3.576 $\int \cosh^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cosh^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\cosh(x)\sinh(x)$$

```
[Out] 1/2*x+1/2*cosh(x)*sinh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\int \cosh^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\sinh(x)\cosh(x)$$

```
[In] Int[Cosh[x]^2,x]
```

```
[Out] x/2 + (Cosh[x]*Sinh[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\cosh(x)\sinh(x) + \frac{\int 1\,dx}{2} \\ &= \frac{x}{2} + \frac{1}{2}\cosh(x)\sinh(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cosh^2(x)\,dx = \frac{x}{2} + \frac{1}{4}\sinh(2x)$$

```
[In] Integrate[Cosh[x]^2,x]
[Out] x/2 + Sinh[2*x]/4
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cosh(x)\sinh(x)}{2}$	11
parallelrisch	$\frac{x}{2}+\frac{\sinh(2x)}{4}$	11
risch	$\frac{x}{2}+\frac{e^{2x}}{8}-\frac{e^{-2x}}{8}$	17

```
[In] int(cosh(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/2*cosh(x)*sinh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cosh^2(x)\,dx = \frac{1}{2}\cosh(x)\sinh(x) + \frac{1}{2}x$$

```
[In] integrate(cosh(x)^2,x, algorithm="fricas")
[Out] 1/2*cosh(x)*sinh(x) + 1/2*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \cosh^2(x)\,dx = -\frac{x\sinh^2(x)}{2} + \frac{x\cosh^2(x)}{2} + \frac{\sinh(x)\cosh(x)}{2}$$

```
[In] integrate(cosh(x)**2,x)
[Out] -x*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \cosh^2(x)\,dx = \frac{1}{2}\,x + \frac{1}{8}\,e^{(2\,x)} - \frac{1}{8}\,e^{(-2\,x)}$$

```
[In] integrate(cosh(x)^2,x, algorithm="maxima")
[Out] 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \cosh^2(x)\,dx = -\frac{1}{8}\left(2\,e^{(2\,x)} + 1\right)e^{(-2\,x)} + \frac{1}{2}\,x + \frac{1}{8}\,e^{(2\,x)}$$

```
[In] integrate(cosh(x)^2,x, algorithm="giac")
[Out] -1/8*(2*e^(2*x) + 1)*e^(-2*x) + 1/2*x + 1/8*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cosh^2(x)\,dx = \frac{x}{2} + \frac{\sinh(2\,x)}{4}$$

```
[In] int(cosh(x)^2,x)
[Out] x/2 + sinh(2*x)/4
```

3.577 $\int \sinh^5(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 19

$$\int \sinh^5(x)\,dx = \cosh(x) - \frac{2\cosh^3(x)}{3} + \frac{\cosh^5(x)}{5}$$

```
[Out] cosh(x)-2/3*cosh(x)^3+1/5*cosh(x)^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{2713\}$

$$\int \sinh^5(x)\,dx = \frac{\cosh^5(x)}{5} - \frac{2\cosh^3(x)}{3} + \cosh(x)$$

```
[In] Int[Sinh[x]^5,x]
```

```
[Out] Cosh[x] - (2*Cosh[x]^3)/3 + Cosh[x]^5/5
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \cosh(x)\right)\\ &= \cosh(x) - \frac{2\cosh^3(x)}{3} + \frac{\cosh^5(x)}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \sinh^5(x)\,dx = \frac{5\cosh(x)}{8} - \frac{5}{48}\cosh(3x) + \frac{1}{80}\cosh(5x)$$

```
[In] Integrate[Sinh[x]^5,x]
```

```
[Out] (5*Cosh[x])/8 - (5*Cosh[3*x])/48 + Cosh[5*x]/80
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$\left(\frac{8}{15}+\frac{(\sinh^4(x))}{5}-\frac{4(\sinh^2(x))}{15}\right)\cosh(x)$	18
risch	$\frac{e^{5x}}{160}-\frac{5\,e^{3x}}{96}+\frac{5\,e^{x}}{16}+\frac{5\,e^{-x}}{16}-\frac{5\,e^{-3x}}{96}+\frac{e^{-5x}}{160}$	36
parallelrisch	$\frac{-18\cosh(7x)+90\cosh(5x)-162\cosh(3x)+90\cosh(x)+3\cosh(8x)-180\cosh(4x)+20\cosh(6x)+492\cosh(2x)-335}{480\cosh(3x)+7200\cosh(x)-2880\cosh(2x)-4800}$	70

```
[In] int(sinh(x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (8/15+1/5*sinh(x)^4-4/15*sinh(x)^2)*cosh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(15) = 30.

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \sinh^5(x)\,dx = \frac{1}{80}\cosh(x)^5 + \frac{1}{16}\cosh(x)\sinh(x)^4 - \frac{5}{48}\cosh(x)^3 + \frac{1}{16}\left(2\cosh(x)^3 - 5\cosh(x)\right)\sinh(x)^2 + \frac{5}{8}\cosh(x)$$

```
[In] integrate(sinh(x)^5,x, algorithm="fricas")
```

```
[Out] 1/80*cosh(x)^5 + 1/16*cosh(x)*sinh(x)^4 - 5/48*cosh(x)^3 + 1/16*(2*cosh(x)^
3 - 5*cosh(x))*sinh(x)^2 + 5/8*cosh(x)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \sinh^5(x)\,dx = \sinh^4(x)\cosh(x) - \frac{4\sinh^2(x)\cosh^3(x)}{3} + \frac{8\cosh^5(x)}{15}$$

```
[In] integrate(sinh(x)**5,x)
[Out] sinh(x)**4*cosh(x) - 4*sinh(x)**2*cosh(x)**3/3 + 8*cosh(x)**5/15
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(15) = 30.

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \sinh^5(x)\,dx = \frac{1}{160}\,e^{(5\,x)} - \frac{5}{96}\,e^{(3\,x)} + \frac{5}{16}\,e^{(-x)} - \frac{5}{96}\,e^{(-3\,x)} + \frac{1}{160}\,e^{(-5\,x)} + \frac{5}{16}\,e^{x}$$

```
[In] integrate(sinh(x)^5,x, algorithm="maxima")
[Out] 1/160*e^(5*x) - 5/96*e^(3*x) + 5/16*e^(-x) - 5/96*e^(-3*x) + 1/160*e^(-5*x)
 + 5/16*e^x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(15) = 30.

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \sinh^5(x)\,dx = \frac{1}{480}\left(150\,e^{(4\,x)} - 25\,e^{(2\,x)} + 3\right)e^{(-5\,x)} + \frac{1}{160}\,e^{(5\,x)} - \frac{5}{96}\,e^{(3\,x)} + \frac{5}{16}\,e^{x}$$

```
[In] integrate(sinh(x)^5,x, algorithm="giac")
[Out] 1/480*(150*e^(4*x) - 25*e^(2*x) + 3)*e^(-5*x) + 1/160*e^(5*x) - 5/96*e^(3*x
) + 5/16*e^x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \sinh^5(x)\, dx = \frac{\cosh(x)^5}{5} - \frac{2\cosh(x)^3}{3} + \cosh(x)$$

```
[In] int(sinh(x)^5,x)
[Out] cosh(x) - (2*cosh(x)^3)/3 + cosh(x)^5/5
```

3.578 $\int \tanh^4(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \tanh^4(x)\,dx = x - \tanh(x) - \frac{\tanh^3(x)}{3}$$

```
[Out] x-tanh(x)-1/3*tanh(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 8}

$$\int \tanh^4(x)\,dx = x - \frac{1}{3}\tanh^3(x) - \tanh(x)$$

```
[In] Int[Tanh[x]^4,x]
```

```
[Out] x - Tanh[x] - Tanh[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{3}\tanh^3(x) + \int \tanh^2(x)\,dx\\ &= -\tanh(x) - \frac{\tanh^3(x)}{3} + \int 1\,dx\\ &= x - \tanh(x) - \frac{\tanh^3(x)}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \tanh^4(x)\,dx = \operatorname{arctanh}(\tanh(x)) - \tanh(x) - \frac{\tanh^3(x)}{3}$$

```
[In] Integrate[Tanh[x]^4,x]
[Out] ArcTanh[Tanh[x]] - Tanh[x] - Tanh[x]^3/3
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$x-\tanh\left(x\right)-\frac{\left(\tanh^{3}\left(x\right)\right)}{3}$	13
derivativedivides	$-\frac{\left(\tanh^{3}\left(x\right)\right)}{3}-\tanh\left(x\right)-\frac{\ln\left(-1+\tanh\left(x\right)\right)}{2}+\frac{\ln\left(1+\tanh\left(x\right)\right)}{2}$	26
default	$-\frac{\left(\tanh^{3}\left(x\right)\right)}{3}-\tanh\left(x\right)-\frac{\ln\left(-1+\tanh\left(x\right)\right)}{2}+\frac{\ln\left(1+\tanh\left(x\right)\right)}{2}$	26
risch	$x+\frac{4\,{\mathrm e}^{4x}+4\,{\mathrm e}^{2x}+\frac{8}{3}}{\left(1+{\mathrm e}^{2x}\right)^{3}}$	27

```
[In] int(tanh(x)^4,x,method=_RETURNVERBOSE)
[Out] x-tanh(x)-1/3*tanh(x)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(12) = 24$.

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.86

$$\int \tanh^4(x)\,dx = \frac{(3x+4)\cosh(x)^3 + 3(3x+4)\cosh(x)\sinh(x)^2 - 12\cosh(x)^2\sinh(x) - 4\sinh(x)^3 + 3(3x+4)\cosh(x}{3\left(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + 3\cosh(x)\right)}$$

```
[In] integrate(tanh(x)^4,x, algorithm="fricas")
[Out] 1/3*((3*x + 4)*cosh(x)^3 + 3*(3*x + 4)*cosh(x)*sinh(x)^2 - 12*cosh(x)^2*sin
h(x) - 4*sinh(x)^3 + 3*(3*x + 4)*cosh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2
+ 3*cosh(x))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \tanh^4(x)\,dx = x - \frac{\tanh^3(x)}{3} - \tanh(x)$$

```
[In] integrate(tanh(x)**4,x)
[Out] x - tanh(x)**3/3 - tanh(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(12) = 24$.

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.71

$$\int \tanh^4(x)\,dx = x - \frac{4\left(3e^{(-2x)} + 3e^{(-4x)} + 2\right)}{3\left(3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1\right)}$$

```
[In] integrate(tanh(x)^4,x, algorithm="maxima")
[Out] x - 4/3*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) +
 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \tanh^4(x)\,dx = x + \frac{4\left(3\,e^{(4\,x)} + 3\,e^{(2\,x)} + 2\right)}{3\left(e^{(2\,x)} + 1\right)^3}$$

```
[In] integrate(tanh(x)^4,x, algorithm="giac")
[Out] x + 4/3*(3*e^(4*x) + 3*e^(2*x) + 2)/(e^(2*x) + 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tanh^4(x)\,dx = -\frac{\tanh(x)^3}{3} - \tanh(x) + x$$

```
[In] int(tanh(x)^4,x)
[Out] x - tanh(x) - tanh(x)^3/3
```

3.579 $\int \operatorname{csch}^3(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 16

$$\int \operatorname{csch}^3(x)\,dx = \frac{1}{2}\operatorname{arctanh}(\cosh(x)) - \frac{1}{2}\coth(x)\operatorname{csch}(x)$$

```
[Out] 1/2*arctanh(cosh(x))-1/2*coth(x)*csch(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3853, 3855}

$$\int \operatorname{csch}^3(x)\,dx = \frac{1}{2}\operatorname{arctanh}(\cosh(x)) - \frac{1}{2}\coth(x)\operatorname{csch}(x)$$

```
[In] Int[Csch[x]^3,x]
```

```
[Out] ArcTanh[Cosh[x]]/2 - (Coth[x]*Csch[x])/2
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2}\coth(x)\text{csch}(x) - \frac{1}{2}\int \text{csch}(x)\,dx \\ &= \frac{1}{2}\text{arctanh}(\cosh(x)) - \frac{1}{2}\coth(x)\text{csch}(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.94

$$\int \text{csch}^3(x)\,dx = -\frac{1}{8}\text{csch}^2\left(\frac{x}{2}\right) + \frac{1}{2}\log\left(\cosh\left(\frac{x}{2}\right)\right) - \frac{1}{2}\log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{1}{8}\text{sech}^2\left(\frac{x}{2}\right)$$

```
[In] Integrate[Csch[x]^3,x]
[Out] -1/8*Csch[x/2]^2 + Log[Cosh[x/2]]/2 - Log[Sinh[x/2]]/2 - Sech[x/2]^2/8
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

method	result	size
default	$-\frac{\coth(x)\,\text{csch}(x)}{2} + \text{arctanh}\left(e^{x}\right)$	11
parallelrisch	$\ln\left(\frac{1}{\sqrt{\coth(x)-\text{csch}(x)}}\right) - \frac{\coth(x)\,\text{csch}(x)}{2}$	18
risch	$-\frac{e^{x}\left(1+e^{2x}\right)}{\left(e^{2x}-1\right)^{2}} + \frac{\ln(1+e^{x})}{2} - \frac{\ln(-1+e^{x})}{2}$	34

```
[In] int(csch(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*coth(x)*csch(x)+arctanh(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(12) = 24$.

Time = 0.23 (sec) , antiderivative size = 211, normalized size of antiderivative = 13.19

$$\int \operatorname{csch}^3(x)\,dx = -\frac{2\cosh(x)^3 + 6\cosh(x)\sinh(x)^2 + 2\sinh(x)^3 - (\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\,(3\cos$$

```
[In] integrate(csch(x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(x)^3 + 6*cosh(x)*sinh(x)^2 + 2*sinh(x)^3 - (cosh(x)^4 + 4*cosh
(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4
*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) + 1) + (cosh(x)^4
 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh
(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2
*(3*cosh(x)^2 + 1)*sinh(x) + 2*cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 +
sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - co
sh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \operatorname{csch}^3(x)\,dx = \int \operatorname{csch}^3(x)\,dx$$

```
[In] integrate(csch(x)**3,x)
```

```
[Out] Integral(csch(x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(12) = 24$.

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.81

$$\int \operatorname{csch}^3(x)\,dx = \frac{e^{(-x)} + e^{(-3\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} + \frac{1}{2}\log\left(e^{(-x)} + 1\right) - \frac{1}{2}\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(csch(x)^3,x, algorithm="maxima")
```

```
[Out] (e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 1/2*log(e^(-x) + 1) - 1/2
*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.81

$$\int \operatorname{csch}^3(x)\, dx = -\frac{e^{(-x)} + e^x}{\left(e^{(-x)} + e^x\right)^2 - 4} + \frac{1}{4}\log\left(e^{(-x)} + e^x + 2\right) - \frac{1}{4}\log\left(e^{(-x)} + e^x - 2\right)$$

```
[In] integrate(csch(x)^3,x, algorithm="giac")
[Out] -(e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) + 1/4*log(e^(-x) + e^x + 2) - 1/4*lo
g(e^(-x) + e^x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^3(x)\, dx = -\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2} - \frac{\cosh(x)}{2\sinh(x)^2}$$

```
[In] int(1/sinh(x)^3,x)
[Out] - log(tanh(x/2))/2 - cosh(x)/(2*sinh(x)^2)
```

3.580 $\int \operatorname{sech}^5(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 26

$$\int \operatorname{sech}^5(x)\,dx=\frac{3}{8}\arctan(\sinh(x))+\frac{3}{8}\operatorname{sech}(x)\tanh(x)+\frac{1}{4}\operatorname{sech}^3(x)\tanh(x)$$

```
[Out] 3/8*arctan(sinh(x))+3/8*sech(x)*tanh(x)+1/4*sech(x)^3*tanh(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3853, 3855}

$$\int \operatorname{sech}^5(x)\,dx=\frac{3}{8}\arctan(\sinh(x))+\frac{1}{4}\tanh(x)\operatorname{sech}^3(x)+\frac{3}{8}\tanh(x)\operatorname{sech}(x)$$

```
[In] Int[Sech[x]^5,x]
```

```
[Out] (3*ArcTan[Sinh[x]])/8 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x])/4
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{sech}^3(x)\tanh(x) + \frac{3}{4}\int \text{sech}^3(x)\,dx \\ &= \frac{3}{8}\text{sech}(x)\tanh(x) + \frac{1}{4}\text{sech}^3(x)\tanh(x) + \frac{3}{8}\int \text{sech}(x)\,dx \\ &= \frac{3}{8}\arctan(\sinh(x)) + \frac{3}{8}\text{sech}(x)\tanh(x) + \frac{1}{4}\text{sech}^3(x)\tanh(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \text{sech}^5(x)\,dx = \frac{3}{8}\arctan(\sinh(x)) + \frac{3}{8}\text{sech}(x)\tanh(x) + \frac{1}{4}\text{sech}^3(x)\tanh(x)$$

```
[In] Integrate[Sech[x]^5,x]
[Out] (3*ArcTan[Sinh[x]])/8 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x])/4
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
default	$\left(\frac{\text{sech}(x)^3}{4}+\frac{3\,\text{sech}(x)}{8}\right)\tanh(x)+\frac{3\arctan(e^x)}{4}$	21
parallelrisch	$\frac{3i\ln(i+\coth(x)-\text{csch}(x))}{8}-\frac{3i\ln(-i+\coth(x)-\text{csch}(x))}{8}+\frac{3\,\text{sech}(x)\tanh(x)}{8}+\frac{\text{sech}(x)^3\tanh(x)}{4}$	42
risch	$\frac{e^x\left(3\,e^{6x}+11\,e^{4x}-11\,e^{2x}-3\right)}{4(1+e^{2x})^4}+\frac{3i\ln(e^x+i)}{8}-\frac{3i\ln(e^x-i)}{8}$	52

```
[In] int(1/cosh(x)^5,x,method=_RETURNVERBOSE)
[Out] (1/4*sech(x)^3+3/8*sech(x))*tanh(x)+3/4*arctan(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(20) = 40$.

Time = 0.24 (sec) , antiderivative size = 461, normalized size of antiderivative = 17.73

$$\int \operatorname{sech}^5(x)\,dx$$
$$= \frac{3\cosh(x)^7 + 21\cosh(x)\sinh(x)^6 + 3\sinh(x)^7 + (63\cosh(x)^2 + 11)\sinh(x)^5 + 11\cosh(x)^5 + 5\,(21\cos$$

```
[In] integrate(1/cosh(x)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(x)^7 + 21*cosh(x)*sinh(x)^6 + 3*sinh(x)^7 + (63*cosh(x)^2 + 11)
*sinh(x)^5 + 11*cosh(x)^5 + 5*(21*cosh(x)^3 + 11*cosh(x))*sinh(x)^4 + (105*
cosh(x)^4 + 110*cosh(x)^2 - 11)*sinh(x)^3 - 11*cosh(x)^3 + (63*cosh(x)^5 +
110*cosh(x)^3 - 33*cosh(x))*sinh(x)^2 + 3*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7
+ sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3
+ 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*
cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cos
h(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(
x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + s
inh(x)) + (21*cosh(x)^6 + 55*cosh(x)^4 - 33*cosh(x)^2 - 3)*sinh(x) - 3*cosh
(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sin
h(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x
)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(
x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 +
 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + co
sh(x))*sinh(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(27) = 54$.

Time = 1.17 (sec) , antiderivative size = 422, normalized size of antiderivative = 16.23

$$
\begin{aligned}
\int \operatorname{sech}^5(x)\,dx = {} & \frac{3\tanh^8\left(\frac{x}{2}\right)\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4} \\
& - \frac{5\tanh^7\left(\frac{x}{2}\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4} \\
& + \frac{12\tanh^6\left(\frac{x}{2}\right)\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4} \\
& + \frac{3\tanh^5\left(\frac{x}{2}\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4} \\
& + \frac{18\tanh^4\left(\frac{x}{2}\right)\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4} \\
& - \frac{3\tanh^3\left(\frac{x}{2}\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4} \\
& + \frac{12\tanh^2\left(\frac{x}{2}\right)\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4} \\
& + \frac{5\tanh\left(\frac{x}{2}\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4} \\
& + \frac{3\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4}
\end{aligned}
$$

```
[In] integrate(1/cosh(x)**5,x)
```

```
[Out] 3*tanh(x/2)**8*atan(tanh(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(
x/2)**4 + 16*tanh(x/2)**2 + 4) - 5*tanh(x/2)**7/(4*tanh(x/2)**8 + 16*tanh(x
/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 12*tanh(x/2)**6*atan(tanh
(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**
2 + 4) + 3*tanh(x/2)**5/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4
 + 16*tanh(x/2)**2 + 4) + 18*tanh(x/2)**4*atan(tanh(x/2))/(4*tanh(x/2)**8 +
 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) - 3*tanh(x/2)**3/
(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4)
+ 12*tanh(x/2)**2*atan(tanh(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*ta
nh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 5*tanh(x/2)/(4*tanh(x/2)**8 + 16*tanh(x
/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 3*atan(tanh(x/2))/(4*tanh
(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(20) = 40$.

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \operatorname{sech}^5(x)\,dx = \frac{3\,e^{(-x)} + 11\,e^{(-3\,x)} - 11\,e^{(-5\,x)} - 3\,e^{(-7\,x)}}{4\,(4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1)} - \frac{3}{4}\arctan\left(e^{(-x)}\right)$$

```
[In] integrate(1/cosh(x)^5,x, algorithm="maxima")
[Out] 1/4*(3*e^(-x) + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^(
-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) - 3/4*arctan(e^(-x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(20) = 40$.

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \operatorname{sech}^5(x)\,dx = \frac{3}{16}\,\pi - \frac{3\,\left(e^{(-x)} - e^{x}\right)^3 + 20\,e^{(-x)} - 20\,e^{x}}{4\,\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)^2} + \frac{3}{8}\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)$$

```
[In] integrate(1/cosh(x)^5,x, algorithm="giac")
[Out] 3/16*pi - 1/4*(3*(e^(-x) - e^x)^3 + 20*e^(-x) - 20*e^x)/((e^(-x) - e^x)^2 +
 4)^2 + 3/8*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \operatorname{sech}^5(x)\,dx = \frac{3\,\mathrm{atan}(\mathrm{e}^{x})}{4} + \frac{3\sinh(x)}{8\cosh(x)^2} + \frac{\sinh(x)}{4\cosh(x)^4}$$

```
[In] int(1/cosh(x)^5,x)
[Out] (3*atan(exp(x)))/4 + (3*sinh(x))/(8*cosh(x)^2) + sinh(x)/(4*cosh(x)^4)
```

3.581 $\int \sinh^4(x)\tanh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 18

$$\int \sinh^4(x)\tanh(x)\,dx = -\cosh^2(x) + \frac{\cosh^4(x)}{4} + \log(\cosh(x))$$

```
[Out] -cosh(x)^2+1/4*cosh(x)^4+ln(cosh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2670, 272, 45}

$$\int \sinh^4(x)\tanh(x)\,dx = \frac{\cosh^4(x)}{4} - \cosh^2(x) + \log(\cosh(x))$$

```
[In] Int[Sinh[x]^4*Tanh[x],x]
```

```
[Out] -Cosh[x]^2 + Cosh[x]^4/4 + Log[Cosh[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1-x^2)^2}{x}\,dx, x, \cosh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx, x, \cosh^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)\,dx, x, \cosh^2(x)\right)\\
&= -\cosh^2(x)+\frac{\cosh^4(x)}{4}+\log(\cosh(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \sinh^4(x)\tanh(x)\,dx = -\cosh^2(x)+\frac{\cosh^4(x)}{4}+\log(\cosh(x))$$

```
[In] Integrate[Sinh[x]^4*Tanh[x],x]
[Out] -Cosh[x]^2 + Cosh[x]^4/4 + Log[Cosh[x]]
```

Maple [A] (verified)

Time = 6.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(\sinh^4(x))}{4}-\frac{(\sinh^2(x))}{2}+\ln(\cosh(x))$	17
risch	$-x+\frac{e^{4x}}{64}-\frac{3\,e^{2x}}{16}-\frac{3\,e^{-2x}}{16}+\frac{e^{-4x}}{64}+\ln(1+e^{2x})$	36

```
[In] int(tanh(x)^5/sech(x)^4,x,method=_RETURNVERBOSE)
[Out] 1/4*sinh(x)^4-1/2*sinh(x)^2+ln(cosh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 257, normalized size of antiderivative = 14.28

$$\int \sinh^4(x)\tanh(x)\,dx = \frac{\cosh(x)^8 + 8\cosh(x)\sinh(x)^7 + \sinh(x)^8 + 4\left(7\cosh(x)^2 - 3\right)\sinh(x)^6 - 12\cosh(x)^6 + 8\left(7\cosh(x)^3\right.}{}$$

```
[In] integrate(tanh(x)^5/sech(x)^4,x, algorithm="fricas")
```

```
[Out] 1/64*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 3)*sin
h(x)^6 - 12*cosh(x)^6 + 8*(7*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 - 64*x*cosh(x
)^4 + 2*(35*cosh(x)^4 - 90*cosh(x)^2 - 32*x)*sinh(x)^4 + 8*(7*cosh(x)^5 - 3
0*cosh(x)^3 - 32*x*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 45*cosh(x)^4 - 96*
x*cosh(x)^2 - 3)*sinh(x)^2 - 12*cosh(x)^2 + 64*(cosh(x)^4 + 4*cosh(x)^3*sin
h(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)*log(2*cosh(
x)/(cosh(x) - sinh(x))) + 8*(cosh(x)^7 - 9*cosh(x)^5 - 32*x*cosh(x)^3 - 3*c
osh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)
^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
```

Sympy [F]

$$\int \sinh^4(x)\tanh(x)\,dx = \int \frac{\tanh^5(x)}{\operatorname{sech}^4(x)}\,dx$$

```
[In] integrate(tanh(x)**5/sech(x)**4,x)
```

```
[Out] Integral(tanh(x)**5/sech(x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

$$\int \sinh^4(x)\tanh(x)\,dx = -\frac{1}{64}\left(12\,e^{(-2\,x)} - 1\right)e^{(4\,x)} + x - \frac{3}{16}\,e^{(-2\,x)} + \frac{1}{64}\,e^{(-4\,x)} + \log\left(e^{(-2\,x)} + 1\right)$$

```
[In] integrate(tanh(x)^5/sech(x)^4,x, algorithm="maxima")
```

```
[Out] -1/64*(12*e^(-2*x) - 1)*e^(4*x) + x - 3/16*e^(-2*x) + 1/64*e^(-4*x) + log(e
^(-2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int \sinh^4(x)\tanh(x)\,dx = \frac{1}{64}\left(48\,e^{(4\,x)} - 12\,e^{(2\,x)} + 1\right)e^{(-4\,x)} - x + \frac{1}{64}\,e^{(4\,x)} - \frac{3}{16}\,e^{(2\,x)} + \log\left(e^{(2\,x)} + 1\right)$$

```
[In] integrate(tanh(x)^5/sech(x)^4,x, algorithm="giac")
[Out] 1/64*(48*e^(4*x) - 12*e^(2*x) + 1)*e^(-4*x) - x + 1/64*e^(4*x) - 3/16*e^(2*
x) + log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

$$\int \sinh^4(x)\tanh(x)\,dx = \ln\left(e^{2\,x} + 1\right) - x - \frac{3\,e^{-2\,x}}{16} - \frac{3\,e^{2\,x}}{16} + \frac{e^{-4\,x}}{64} + \frac{e^{4\,x}}{64}$$

```
[In] int(cosh(x)^4*tanh(x)^5,x)
[Out] log(exp(2*x) + 1) - x - (3*exp(-2*x))/16 - (3*exp(2*x))/16 + exp(-4*x)/64 +
 exp(4*x)/64
```

3.582 $\int \operatorname{sech}^{\frac{23}{4}}(x) \sinh^5(x) \, dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \operatorname{sech}^{\frac{23}{4}}(x) \sinh^5(x) \, dx = -\frac{4}{3} \operatorname{sech}^{\frac{3}{4}}(x) + \frac{8}{11} \operatorname{sech}^{\frac{11}{4}}(x) - \frac{4}{19} \operatorname{sech}^{\frac{19}{4}}(x)$$

```
[Out] -4/3*sech(x)^(3/4)+8/11*sech(x)^(11/4)-4/19*sech(x)^(19/4)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2702, 276}

$$\int \operatorname{sech}^{\frac{23}{4}}(x) \sinh^5(x) \, dx = -\frac{4}{19} \operatorname{sech}^{\frac{19}{4}}(x) + \frac{8}{11} \operatorname{sech}^{\frac{11}{4}}(x) - \frac{4}{3} \operatorname{sech}^{\frac{3}{4}}(x)$$

```
[In] Int[Sech[x]^(23/4)*Sinh[x]^5,x]
```

```
[Out] (-4*Sech[x]^(3/4))/3 + (8*Sech[x]^(11/4))/11 - (4*Sech[x]^(19/4))/19
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{(-1+x^2)^2}{\sqrt[4]{x}}\,dx, x, \text{sech}(x)\right)\\ &= -\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{x}} - 2x^{7/4} + x^{15/4}\right)\,dx, x, \text{sech}(x)\right)\\ &= -\frac{4}{3}\text{sech}^{\frac{3}{4}}(x) + \frac{8}{11}\text{sech}^{\frac{11}{4}}(x) - \frac{4}{19}\text{sech}^{\frac{19}{4}}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \text{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = \text{sech}^{\frac{3}{4}}(x)\left(-\frac{4}{3} + \frac{8\text{sech}^2(x)}{11} - \frac{4\text{sech}^4(x)}{19}\right)$$

```
[In] Integrate[Sech[x]^(23/4)*Sinh[x]^5,x]
[Out] Sech[x]^(3/4)*(-4/3 + (8*Sech[x]^2)/11 - (4*Sech[x]^4)/19)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$-\frac{4\,\text{sech}(x)^{\frac{3}{4}}}{3}+\frac{8\,\text{sech}(x)^{\frac{11}{4}}}{11}-\frac{4\,\text{sech}(x)^{\frac{19}{4}}}{19}$	20
default	$-\frac{4\,\text{sech}(x)^{\frac{3}{4}}}{3}+\frac{8\,\text{sech}(x)^{\frac{11}{4}}}{11}-\frac{4\,\text{sech}(x)^{\frac{19}{4}}}{19}$	20

```
[In] int(sech(x)^(3/4)*tanh(x)^5,x,method=_RETURNVERBOSE)
[Out] -4/3*sech(x)^(3/4)+8/11*sech(x)^(11/4)-4/19*sech(x)^(19/4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(19) = 38$.

Time = 0.24 (sec) , antiderivative size = 359, normalized size of antiderivative = 11.58

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx =$$

$$-\frac{4\cdot 2^{\frac{3}{4}}\left(209\,\cosh\left(x\right)^{8}+1672\,\cosh\left(x\right)\sinh\left(x\right)^{7}+209\,\sinh\left(x\right)^{8}+76\,\left(77\,\cosh\left(x\right)^{2}+5\right)\sinh\left(x\right)^{6}+380\,\mathrm{c}\right.}{627\,\left(\cosh\left(x\right)^{8}+8\,\cosh\left(x\right)\sinh\left(x\right)^{7}+\sinh\left(x\right)^{8}+4\right.}$$

```
[In] integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="fricas")
```

```
[Out] -4/627*2^(3/4)*(209*cosh(x)^8 + 1672*cosh(x)*sinh(x)^7 + 209*sinh(x)^8 + 76
*(77*cosh(x)^2 + 5)*sinh(x)^6 + 380*cosh(x)^6 + 152*(77*cosh(x)^3 + 15*cosh
(x))*sinh(x)^5 + 10*(1463*cosh(x)^4 + 570*cosh(x)^2 + 87)*sinh(x)^4 + 870*c
osh(x)^4 + 8*(1463*cosh(x)^5 + 950*cosh(x)^3 + 435*cosh(x))*sinh(x)^3 + 4*(
1463*cosh(x)^6 + 1425*cosh(x)^4 + 1305*cosh(x)^2 + 95)*sinh(x)^2 + 380*cosh
(x)^2 + 8*(209*cosh(x)^7 + 285*cosh(x)^5 + 435*cosh(x)^3 + 95*cosh(x))*sinh
(x) + 209)*((cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
+ 1))^(3/4)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 +
 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35
*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 1
0*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh
(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)
^3 + cosh(x))*sinh(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 39.54 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = -\frac{4\tanh^4\left(x\right)\operatorname{sech}^{\frac{3}{4}}\left(x\right)}{19}-\frac{64\tanh^2\left(x\right)\operatorname{sech}^{\frac{3}{4}}\left(x\right)}{209}-\frac{512\operatorname{sech}^{\frac{3}{4}}\left(x\right)}{627}$$

```
[In] integrate(sech(x)**(3/4)*tanh(x)**5,x)
```

```
[Out] -4*tanh(x)**4*sech(x)**(3/4)/19 - 64*tanh(x)**2*sech(x)**(3/4)/209 - 512*se
ch(x)**(3/4)/627
```

Maxima [F]

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = \int \operatorname{sech}(x)^{\frac{3}{4}}\tanh(x)^5\,dx$$

```
[In] integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="maxima")
[Out] integrate(sech(x)^(3/4)*tanh(x)^5, x)
```

Giac [F]

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = \int \operatorname{sech}(x)^{\frac{3}{4}}\tanh(x)^5\,dx$$

```
[In] integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="giac")
[Out] integrate(sech(x)^(3/4)*tanh(x)^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.87

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = \frac{32\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}\right)^{3/4}}{11\,(e^{2x}+1)} - \frac{1312\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}\right)^{3/4}}{209\,(e^{2x}+1)^2} + \frac{128\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}\right)^{3/4}}{19\,(e^{2x}+1)^3} - \frac{64\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}\right)^{3/4}}{19\,(e^{2x}+1)^4} - \frac{4\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}\right)^{3/4}}{3}$$

```
[In] int(tanh(x)^5*(1/cosh(x))^(3/4),x)
[Out] (32*(1/(exp(-x)/2 + exp(x)/2))^(3/4))/(11*(exp(2*x) + 1)) - (1312*(1/(exp(-
x)/2 + exp(x)/2))^(3/4))/(209*(exp(2*x) + 1)^2) + (128*(1/(exp(-x)/2 + exp(
x)/2))^(3/4))/(19*(exp(2*x) + 1)^3) - (64*(1/(exp(-x)/2 + exp(x)/2))^(3/4))
/(19*(exp(2*x) + 1)^4) - (4*(1/(exp(-x)/2 + exp(x)/2))^(3/4))/3
```

3.583 $\int \frac{1}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \frac{1}{a+b\cosh(x)}\,dx = \frac{2\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] 2*arctanh((a-b)*tanh(1/2*x)/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2738, 214}

$$\int \frac{1}{a+b\cosh(x)}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

```
[In] Int[(a + b*Cosh[x])^(-1),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)$$
$$= \frac{2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\cosh(x)}\,dx = -\frac{2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

```
[In] Integrate[(a + b*Cosh[x])^(-1),x]
[Out] (-2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{2\,\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}$	36
risch	$\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$	109

```
[In] int(1/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
[Out] 2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.27

$$\int \frac{1}{a+b\cosh(x)}\,dx = \left[\frac{\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)}{\sqrt{a^2-b^2}}, -\frac{2\,\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)}{a^2-b^2}\right]$$

```
[In] integrate(1/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*
cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*
cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b))/sqr
t(a^2 - b^2), -2*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*s
inh(x) + a)/(a^2 - b^2))/(a^2 - b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(31) = 62$.

Time = 1.83 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.07

$$\int \frac{1}{a+b\cosh(x)}\,dx = \begin{cases} \tilde{\infty}\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right) & \text{for } a=0\wedge b=0 \\ \frac{\tanh\left(\frac{x}{2}\right)}{b} & \text{for } a=b \\ -\frac{1}{b\tanh\left(\frac{x}{2}\right)} & \text{for } a=-b \\ -\frac{\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*atan(tanh(x/2)), Eq(a, 0) & Eq(b, 0)), (tanh(x/2)/b, Eq(a, b
)), (-1/(b*tanh(x/2)), Eq(a, -b)), (-log(-sqrt(a/(a - b) + b/(a - b)) + tan
h(x/2))/(a*sqrt(a/(a - b) + b/(a - b)) - b*sqrt(a/(a - b) + b/(a - b))) + l
og(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*sqrt(a/(a - b) + b/(a - b))
- b*sqrt(a/(a - b) + b/(a - b))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a + b\cosh(x)}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1}{a + b\cosh(x)}\, dx = \frac{2\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}}$$

```
[In] integrate(1/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{1}{a + b\cosh(x)}\, dx = \frac{2\,\mathrm{atan}\left(\frac{a}{\sqrt{b^{2}-a^{2}}}+\frac{b\,e^{x}}{\sqrt{b^{2}-a^{2}}}\right)}{\sqrt{b^{2}-a^{2}}}$$

```
[In] int(1/(a + b*cosh(x)),x)
```

```
[Out] (2*atan(a/(b^2 - a^2)^(1/2) + (b*exp(x))/(b^2 - a^2)^(1/2)))/(b^2 - a^2)^(1
/2)
```

3.584 $\int \frac{1}{(1+\cosh(x))^2} \, dx$

Optimal result

Integrand size = 6, antiderivative size = 25

$$\int \frac{1}{(1+\cosh(x))^2} \, dx = \frac{\sinh(x)}{3(1+\cosh(x))^2} + \frac{\sinh(x)}{3(1+\cosh(x))}$$

```
[Out] 1/3*sinh(x)/(1+cosh(x))^2+1/3*sinh(x)/(1+cosh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2729, 2727}

$$\int \frac{1}{(1+\cosh(x))^2} \, dx = \frac{\sinh(x)}{3(\cosh(x)+1)} + \frac{\sinh(x)}{3(\cosh(x)+1)^2}$$

```
[In] Int[(1 + Cosh[x])^(-2),x]
```

```
[Out] Sinh[x]/(3*(1 + Cosh[x])^2) + Sinh[x]/(3*(1 + Cosh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{\sinh(x)}{3(1+\cosh(x))^2} + \frac{1}{3}\int \frac{1}{1+\cosh(x)}\,dx$$
$$= \frac{\sinh(x)}{3(1+\cosh(x))^2} + \frac{\sinh(x)}{3(1+\cosh(x))}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{1}{(1+\cosh(x))^2}\,dx = \frac{(2+\cosh(x))\sinh(x)}{3(1+\cosh(x))^2}$$

```
[In] Integrate[(1 + Cosh[x])^(-2),x]
[Out] ((2 + Cosh[x])*Sinh[x])/(3*(1 + Cosh[x])^2)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

method	result	size
risch	$-\frac{2(1+3\,e^{x})}{3(1+e^{x})^{3}}$	15
default	$-\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{6}+\frac{\tanh\left(\frac{x}{2}\right)}{2}$	16
parallelrisch	$-\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{6}+\frac{\tanh\left(\frac{x}{2}\right)}{2}$	16

```
[In] int(1/(cosh(x)+1)^2,x,method=_RETURNVERBOSE)
[Out] -2/3*(1+3*exp(x))/(1+exp(x))^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(21) = 42$.

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \frac{1}{(1+\cosh(x))^2}\,dx = -\frac{2\,(3\,\cosh\left(x\right) + 3\,\sinh\left(x\right) + 1)}{3\,\left(\cosh\left(x\right)^{3} + 3\,(\cosh\left(x\right) + 1)\,\sinh\left(x\right)^{2} + \sinh\left(x\right)^{3} + 3\,\cosh\left(x\right)^{2} + 3\,\left(\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right) + 1\right)\,\sinh$$

```
[In] integrate(1/(1+cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(x) + 3*sinh(x) + 1)/(cosh(x)^3 + 3*(cosh(x) + 1)*sinh(x)^2 + s
inh(x)^3 + 3*cosh(x)^2 + 3*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x) + 3*cosh(x)
+ 1)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{1}{(1+\cosh(x))^2}\,dx = -\frac{\tanh^{3}\left(\frac{x}{2}\right)}{6} + \frac{\tanh\left(\frac{x}{2}\right)}{2}$$

```
[In] integrate(1/(1+cosh(x))**2,x)
```

```
[Out] -tanh(x/2)**3/6 + tanh(x/2)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(21) = 42$.

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{1}{(1+\cosh(x))^2}\,dx = \frac{2\,e^{(-x)}}{3\,e^{(-x)} + 3\,e^{(-2\,x)} + e^{(-3\,x)} + 1} + \frac{2}{3\,(3\,e^{(-x)} + 3\,e^{(-2\,x)} + e^{(-3\,x)} + 1)}$$

```
[In] integrate(1/(1+cosh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/3/(3*e^(-x) + 3*e^(-2*x
) + e^(-3*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{1}{(1+\cosh(x))^2} \, dx = -\frac{2\,(3\,e^{x}+1)}{3\,(e^{x}+1)^{3}}$$

```
[In] integrate(1/(1+cosh(x))^2,x, algorithm="giac")
[Out] -2/3*(3*e^x + 1)/(e^x + 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{1}{(1+\cosh(x))^2} \, dx = -\frac{2\,(3\,\mathrm{e}^{x}+1)}{3\,(\mathrm{e}^{x}+1)^{3}}$$

```
[In] int(1/(cosh(x) + 1)^2,x)
[Out] -(2*(3*exp(x) + 1))/(3*(exp(x) + 1)^3)
```

3.585 $\int \frac{1}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 39

$$\int \frac{1}{a+b\tanh(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[Out] a*x/(a^2-b^2)-b*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3565, 3611}

$$\int \frac{1}{a+b\tanh(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[In] Int[(a + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{ax}{a^2-b^2} - \frac{(ib)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^2-b^2}\\ &= \frac{ax}{a^2-b^2} - \frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{a+b\tanh(x)}\,dx = \frac{(-a+b)\log(1-\tanh(x))+(a+b)\log(1+\tanh(x))-2b\log(a+b\tanh(x))}{2(a-b)(a+b)}$$

```
[In] Integrate[(a + b*Tanh[x])^(-1),x]
[Out] ((-a + b)*Log[1 - Tanh[x]] + (a + b)*Log[1 + Tanh[x]] - 2*b*Log[a + b*Tanh[
x]])/(2*(a - b)*(a + b))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$-\frac{-\ln(1-\tanh(x))b+b\ln(a+b\tanh(x))-ax-bx}{a^2-b^2}$	42
derivativedivides	$\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{b\ln(a+b\tanh(x))}{(a+b)(a-b)}-\frac{\ln(-1+\tanh(x))}{2a+2b}$	55
default	$\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{b\ln(a+b\tanh(x))}{(a+b)(a-b)}-\frac{\ln(-1+\tanh(x))}{2a+2b}$	55
risch	$\frac{x}{a+b}+\frac{2bx}{a^2-b^2}-\frac{b\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^2-b^2}$	55

```
[In] int(1/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
[Out] -(-ln(1-tanh(x))*b+b*ln(a+b*tanh(x))-a*x-b*x)/(a^2-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+b\tanh(x)}\,dx = \frac{(a+b)x-b\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="fricas")
[Out] ((a + b)*x - b*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 - b
^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(29) = 58$.

Time = 0.22 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.74

$$\int \frac{1}{a+b\tanh(x)}\,dx = \begin{cases} \tilde{\infty}(x-\log(\tanh(x)+1)+\log(\tanh(x))) & \text{for } a=0 \wedge b=0 \\ \frac{x}{a} & \text{for } b=0 \\ -\frac{x\tanh(x)}{2b\tanh(x)-2b}+\frac{x}{2b\tanh(x)-2b}+\frac{1}{2b\tanh(x)-2b} & \text{for } a=-b \\ \frac{x\tanh(x)}{2b\tanh(x)+2b}+\frac{x}{2b\tanh(x)+2b}-\frac{1}{2b\tanh(x)+2b} & \text{for } a=b \\ \frac{ax}{a^2-b^2}-\frac{bx}{a^2-b^2}-\frac{b\log\left(\frac{a}{b}+\tanh(x)\right)}{a^2-b^2}+\frac{b\log(\tanh(x)+1)}{a^2-b^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a+b*tanh(x)),x)
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) & Eq(b, 0)),
 (x/a, Eq(b, 0)), (-x*tanh(x)/(2*b*tanh(x) - 2*b) + x/(2*b*tanh(x) - 2*b) +
 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b
*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (a*x/(a**2 - b**2) - b*
x/(a**2 - b**2) - b*log(a/b + tanh(x))/(a**2 - b**2) + b*log(tanh(x) + 1)/(
a**2 - b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{a+b\tanh(x)}\,dx = -\frac{b\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^2-b^2}+\frac{x}{a+b}$$

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="maxima")
[Out] -b*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+b\tanh(x)}\,dx = -\frac{b\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^2-b^2}+\frac{x}{a-b}$$

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="giac")
[Out] -b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) + x/(a - b)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{1}{a+b\tanh(x)}\,dx = \frac{a\,x-b\,\left(x-\ln\left(\tanh(x)+1\right)+\ln\left(a+b\tanh(x)\right)\right)}{a^2-b^2}$$

```
[In] int(1/(a + b*tanh(x)),x)
[Out] (a*x - b*(x - log(tanh(x) + 1) + log(a + b*tanh(x))))/(a^2 - b^2)
```

3.586 $\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

```
[Out] arctanh(a*tanh(x)/(a^2+b^2)^(1/2))/a/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3260, 214}

$$\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

```
[In] Int[(a^2 + b^2*Cosh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 + b^2]]/(a*Sqrt[a^2 + b^2])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{a^2-(a^2+b^2)\,x^2}\,dx, x, \coth(x)\right)\\ &= \frac{\operatorname{arctanh}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx = \frac{\operatorname{arctanh}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

```
[In] Integrate[(a^2 + b^2*Cosh[x]^2)^(-1),x]
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 + b^2]]/(a*Sqrt[a^2 + b^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(27) = 54.

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.16

method	result	size
default	$\frac{\ln\left(\sqrt{a^2+b^2}\,(\tanh^2(\frac{x}{2}))+2a\tanh(\frac{x}{2})+\sqrt{a^2+b^2}\right)}{2a\sqrt{a^2+b^2}}-\frac{\ln\left(\sqrt{a^2+b^2}\,(\tanh^2(\frac{x}{2}))-2a\tanh(\frac{x}{2})+\sqrt{a^2+b^2}\right)}{2a\sqrt{a^2+b^2}}$	98
risch	$\frac{\ln\left(e^{2x}+\frac{2a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}-2a^3-2b^2a}{b^2\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}}-\frac{\ln\left(e^{2x}+\frac{2a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}+2a^3+2b^2a}{b^2\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}}$	146

```
[In] int(1/(a^2+b^2*cosh(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/2/a/(a^2+b^2)^(1/2)*ln((a^2+b^2)^(1/2)*tanh(1/2*x)^2+2*a*tanh(1/2*x)+(a^2
+b^2)^(1/2))-1/2/a/(a^2+b^2)^(1/2)*ln((a^2+b^2)^(1/2)*tanh(1/2*x)^2-2*a*tan
h(1/2*x)+(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(27) = 54$.

Time = 0.25 (sec) , antiderivative size = 288, normalized size of antiderivative = 9.29

$$\int \frac{1}{a^2 + b^2 \cosh^2(x)}\, dx = \frac{\sqrt{a^2+b^2} \log \left(\frac{b^4 \cosh(x)^4 + 4\, b^4 \cosh(x) \sinh(x)^3 + b^4 \sinh(x)^4 + 8\, a^4 + 8\, a^2 b^2 + b^4 + 2\, (2\, a^2 b^2 + b^4) \cosh(x)^2 + 2\, \left(3\, b^4 \cosh(x)^2 + 2\, a^2 b^2 + b^4\right) \sinh}{b^2 \cosh(x)^4 + 4\, b^2 \cosh(x) \sinh(x)^3 + b^2 \sinh(x)^4 + 2\, (2\, a^2 + b^2) \cosh(x)^2 + 2\, \left(3\, b\right.} \right.}{2\, (a^3 + ab^2)}$$

```
[In] integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a^2 + b^2)*log((b^4*cosh(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh
(x)^4 + 8*a^4 + 8*a^2*b^2 + b^4 + 2*(2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(3*b^4*
cosh(x)^2 + 2*a^2*b^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 + (2*a^2*b^2 + b^
4)*cosh(x))*sinh(x) - 4*(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a*b^2*
sinh(x)^2 + 2*a^3 + a*b^2)*sqrt(a^2 + b^2))/(b^2*cosh(x)^4 + 4*b^2*cosh(x)*
sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a^2 + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2
+ 2*a^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + (2*a^2 + b^2)*cosh(x))*
sinh(x)))/(a^3 + a*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.58 (sec) , antiderivative size = 1129, normalized size of antiderivative = 36.42

$$\int \frac{1}{a^2 + b^2 \cosh^2(x)}\, dx = \text{Too large to display}$$

```
[In] integrate(1/(a**2+b**2*cosh(x)**2),x)
```

```
[Out] Piecewise((zoo*tanh(x/2)/(tanh(x/2)**2 + 1), Eq(a, 0) & Eq(b, 0)), (-tanh(x
/2)/(2*b**2) - 1/(2*b**2*tanh(x/2)), Eq(a, I*b) | Eq(a, -I*b)), (2*tanh(x/2
)/(b**2*(tanh(x/2)**2 + 1)), Eq(a, 0)), (-a*sqrt(a/(a - I*b) + I*b/(a - I*b
))*log(-sqrt(a/(a + I*b) - I*b/(a + I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(a -
I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqrt(a/(
a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))) + a*sqrt(a/(a
- I*b) + I*b/(a - I*b))*log(sqrt(a/(a + I*b) - I*b/(a + I*b)) + tanh(x/2))/
(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))
 + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I
*b))) - a*sqrt(a/(a + I*b) - I*b/(a + I*b))*log(-sqrt(a/(a - I*b) + I*b/(a
- I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a +
I*b) - I*b/(a + I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(
```

```
a + I*b) - I*b/(a + I*b))) + a*sqrt(a/(a + I*b) - I*b/(a + I*b))*log(sqrt(a
/(a - I*b) + I*b/(a - I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a
- I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b
/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))) + I*b*sqrt(a/(a - I*b) + I*b
/(a - I*b))*log(-sqrt(a/(a + I*b) - I*b/(a + I*b)) + tanh(x/2))/(2*a**3*sqr
t(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2
*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))) - I*b
*sqrt(a/(a - I*b) + I*b/(a - I*b))*log(sqrt(a/(a + I*b) - I*b/(a + I*b)) +
tanh(x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b
/(a + I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) -
 I*b/(a + I*b))) - I*b*sqrt(a/(a + I*b) - I*b/(a + I*b))*log(-sqrt(a/(a - I
*b) + I*b/(a - I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))
*sqrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I
*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))) + I*b*sqrt(a/(a + I*b) - I*b/(a + I
*b))*log(sqrt(a/(a - I*b) + I*b/(a - I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(a -
 I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqrt(a/
(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(27) = 54$.

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int \frac{1}{a^2 + b^2 \cosh^2(x)}\, dx = -\frac{\log\left(\frac{b^2 e^{(-2\,x)} + 2\,a^2 + b^2 - 2\,\sqrt{a^2 + b^2}a}{b^2 e^{(-2\,x)} + 2\,a^2 + b^2 + 2\,\sqrt{a^2 + b^2}a}\right)}{2\,\sqrt{a^2 + b^2}a}$$

```
[In] integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log((b^2*e^(-2*x) + 2*a^2 + b^2 - 2*sqrt(a^2 + b^2)*a)/(b^2*e^(-2*x) +
 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*a))/(sqrt(a^2 + b^2)*a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(27) = 54$.

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.55

$$\int \frac{1}{a^2 + b^2 \cosh^2(x)}\, dx = \frac{\log\left(\frac{b^2 e^{(2\,x)} + 2\,a^2 + b^2 - 2\,\sqrt{a^2 + b^2}|a|}{b^2 e^{(2\,x)} + 2\,a^2 + b^2 + 2\,\sqrt{a^2 + b^2}|a|}\right)}{2\,\sqrt{a^2 + b^2}|a|}$$

```
[In] integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*log((b^2*e^(2*x) + 2*a^2 + b^2 - 2*sqrt(a^2 + b^2)*abs(a))/(b^2*e^(2*x)
 + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*abs(a)))/(sqrt(a^2 + b^2)*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int \frac{1}{a^2 + b^2 \cosh^2(x)} \, dx = \frac{\operatorname{atan}\left(\frac{2\,a^2\,\left(-a^4-a^2\,b^2\right)^{3/2}+b^2\,\left(-a^4-a^2\,b^2\right)^{3/2}+b^2\,\mathrm{e}^{2\,x}\,\left(-a^4-a^2\,b^2\right)^{3/2}}{2\,a^8+4\,a^6\,b^2+2\,a^4\,b^4}\right)}{\sqrt{-a^4-a^2\,b^2}}$$

```
[In] int(1/(b^2*cosh(x)^2 + a^2),x)
```

```
[Out] atan((2*a^2*(- a^4 - a^2*b^2)^(3/2) + b^2*(- a^4 - a^2*b^2)^(3/2) + b^2*exp
(2*x)*(- a^4 - a^2*b^2)^(3/2))/(2*a^8 + 2*a^4*b^4 + 4*a^6*b^2))/(- a^4 - a^
2*b^2)^(1/2)
```

3.587 $\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

```
[Out] arctanh(a*tanh(x)/(a^2-b^2)^(1/2))/a/(a^2-b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3260, 214}

$$\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

```
[In] Int[(a^2 - b^2*Cosh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 - b^2]]/(a*Sqrt[a^2 - b^2])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{a^2-(a^2-b^2)\,x^2}\,dx, x, \coth(x)\right)$$
$$= \frac{\operatorname{arctanh}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx = \frac{\operatorname{arctanh}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

```
[In] Integrate[(a^2 - b^2*Cosh[x]^2)^(-1),x]
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 - b^2]]/(a*Sqrt[a^2 - b^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(31) = 62.

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.11

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}+\frac{\operatorname{arctanh}\left(\frac{(a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}$	74
risch	$\frac{\ln\left(e^{2x}-\frac{2a^2\sqrt{a^2-b^2}-b^2\sqrt{a^2-b^2}-2a^3+2b^2a}{b^2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,a}-\frac{\ln\left(e^{2x}-\frac{2a^2\sqrt{a^2-b^2}-b^2\sqrt{a^2-b^2}+2a^3-2b^2a}{b^2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,a}$	166

```
[In] int(1/(a^2-b^2*cosh(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+1/a/
((a+b)*(a-b))^(1/2)*arctanh((a+b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(31) = 62$.

Time = 0.27 (sec) , antiderivative size = 388, normalized size of antiderivative = 11.09

$$\int \frac{1}{a^2 - b^2 \cosh^2(x)}\,dx = \left[\frac{\sqrt{a^2-b^2}\log\left(\frac{b^4\cosh(x)^4+4\,b^4\cosh(x)\sinh(x)^3+b^4\sinh(x)^4+8\,a^4-8\,a^2b^2+b^4-2\,(2\,a^2b^2-b^4)\cosh(x)^2+2\left(3\,b^4\cosh(x)^2-2\,a^2b^2+b^4\right)\sinh}{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4-2\,(2\,a^2-b^2)\cosh(x)^2+2\left(3\,b\right.}\right.}{2\,(a^3-ab^2}\right.$$

```
[In] integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a^2 - b^2)*log((b^4*cosh(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sin
h(x)^4 + 8*a^4 - 8*a^2*b^2 + b^4 - 2*(2*a^2*b^2 - b^4)*cosh(x)^2 + 2*(3*b^4
*cosh(x)^2 - 2*a^2*b^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 - (2*a^2*b^2 - b
^4)*cosh(x))*sinh(x) + 4*(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a*b^2
*sinh(x)^2 - 2*a^3 + a*b^2)*sqrt(a^2 - b^2))/(b^2*cosh(x)^4 + 4*b^2*cosh(x)
*sinh(x)^3 + b^2*sinh(x)^4 - 2*(2*a^2 - b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2
 - 2*a^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 - (2*a^2 - b^2)*cosh(x))
*sinh(x)))/(a^3 - a*b^2), sqrt(-a^2 + b^2)*arctan(-1/2*(b^2*cosh(x)^2 + 2*b
^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - 2*a^2 + b^2)*sqrt(-a^2 + b^2)/(a^3 - a
*b^2))/(a^3 - a*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(27) = 54$.

Time = 14.55 (sec) , antiderivative size = 874, normalized size of antiderivative = 24.97

$$\int \frac{1}{a^2 - b^2 \cosh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a**2-b**2*cosh(x)**2),x)
```

```
[Out] Piecewise((zoo*tanh(x/2)/(tanh(x/2)**2 + 1), Eq(a, 0) & Eq(b, 0)), (tanh(x/
2)/(2*b**2) + 1/(2*b**2*tanh(x/2)), Eq(a, b) | Eq(a, -b)), (-2*tanh(x/2)/(b
**2*(tanh(x/2)**2 + 1)), Eq(a, 0)), (-a*sqrt(a/(a - b) + b/(a - b))*log(-sq
rt(a/(a + b) - b/(a + b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*
sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(
a + b) - b/(a + b))) + a*sqrt(a/(a - b) + b/(a - b))*log(sqrt(a/(a + b) - b
/(a + b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) -
 b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a +
b))) - a*sqrt(a/(a + b) - b/(a + b))*log(-sqrt(a/(a - b) + b/(a - b)) + tan
```

```
h(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2
*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) + a*sqrt(a
/(a + b) - b/(a + b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(2*a**3*
sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(
a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) + b*sqrt(a/(a - b) + b/(a
- b))*log(-sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b)
+ b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a -
 b))*sqrt(a/(a + b) - b/(a + b))) - b*sqrt(a/(a - b) + b/(a - b))*log(sqrt(
a/(a + b) - b/(a + b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqr
t(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a +
 b) - b/(a + b))) - b*sqrt(a/(a + b) - b/(a + b))*log(-sqrt(a/(a - b) + b/(
a - b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b
/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)
)) + b*sqrt(a/(a + b) - b/(a + b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x
/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*
b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a^2 - b^2 \cosh^2(x)} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{1}{a^2 - b^2 \cosh^2(x)} dx = -\frac{\arctan\left(\frac{b^2 e^{(2x)} - 2a^2 + b^2}{2\sqrt{-a^2 + b^2}a}\right)}{\sqrt{-a^2 + b^2}a}$$

```
[In] integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="giac")
```

```
[Out] -arctan(1/2*(b^2*e^(2*x) - 2*a^2 + b^2)/(sqrt(-a^2 + b^2)*a))/(sqrt(-a^2 +
b^2)*a)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.03

$$\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx=-\frac{\operatorname{atan}\left(\frac{b^2\,{\left(a^2\,b^2-a^4\right)}^{3/2}-2\,a^2\,{\left(a^2\,b^2-a^4\right)}^{3/2}+b^2\,{\mathrm{e}}^{2\,x}\,{\left(a^2\,b^2-a^4\right)}^{3/2}}{2\,a^8-4\,a^6\,b^2+2\,a^4\,b^4}\right)}{\sqrt{a^2\,b^2-a^4}}$$

```
[In] int(-1/(b^2*cosh(x)^2 - a^2),x)
```

```
[Out] -atan((b^2*(a^2*b^2 - a^4)^(3/2) - 2*a^2*(a^2*b^2 - a^4)^(3/2) + b^2*exp(2*
x)*(a^2*b^2 - a^4)^(3/2))/(2*a^8 + 2*a^4*b^4 - 4*a^6*b^2))/(a^2*b^2 - a^4)^
(1/2)
```

3.588 $\int \frac{1}{1-\sinh^4(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}} + \frac{\tanh(x)}{2}$$

```
[Out] 1/4*arctanh(2^(1/2)*tanh(x))*2^(1/2)+1/2*tanh(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3288, 396, 212}

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}} + \frac{\tanh(x)}{2}$$

```
[In] Int[(1 - Sinh[x]^4)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(2*Sqrt[2]) + Tanh[x]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1-x^2}{1-2x^2}\,dx, x, \tanh(x)\right)\\ &= \frac{\tanh(x)}{2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right)\\ &= \frac{\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \frac{1}{4}\left(\sqrt{2}\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right)+2\tanh(x)\right)$$

```
[In] Integrate[(1 - Sinh[x]^4)^(-1),x]
[Out] (Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + 2*Tanh[x])/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(17) = 34.

Time = 0.45 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

method	result	size
risch	$-\frac{1}{1+{\mathrm e}^{2 x}}+\frac{\sqrt{2}\, \ln\left({\mathrm e}^{2 x}-3+2 \sqrt{2}\right)}{8}-\frac{\sqrt{2}\, \ln\left({\mathrm e}^{2 x}-3-2 \sqrt{2}\right)}{8}$	46
default	$\frac{\tanh\left(\frac{x}{2}\right)}{\tanh^{2}\left(\frac{x}{2}\right)+1}+\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\left(2 \tanh\left(\frac{x}{2}\right)-2\right) \sqrt{2}}{4}\right)}{4}+\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\left(2 \tanh\left(\frac{x}{2}\right)+2\right) \sqrt{2}}{4}\right)}{4}$	55

```
[In] int(1/(1-sinh(x)^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(1+exp(2*x))+1/8*2^(1/2)*ln(exp(2*x)-3+2*2^(1/2))-1/8*2^(1/2)*ln(exp(2*x
)-3-2*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.52

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \frac{\left(\sqrt{2}\cosh\left(x\right)^2 + 2\sqrt{2}\cosh\left(x\right)\sinh\left(x\right) + \sqrt{2}\sinh\left(x\right)^2 + \sqrt{2}\right)\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)}{\cosh(x)^2+\sinh(x)^2-}\right.}{8\left(\cosh\left(x\right)^2 + 2\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^2 + 1\right)}$$

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="fricas")
```

```
[Out] 1/8*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + s
qrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(
x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 -
3)) - 8)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 908 vs. $2(20) = 40$.

Time = 2.48 (sec) , antiderivative size = 908, normalized size of antiderivative = 36.32

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(1-sinh(x)**4),x)
```

```
[Out] 3064704*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/
2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 2167073*sqrt
(2)*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**
2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 3064704*log(tanh
(x/2) - 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2
 + 12258816*sqrt(2) + 17336584) + 2167073*sqrt(2)*log(tanh(x/2) - 1 + sqrt(
2))/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(
2) + 17336584) + 3064704*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(1225881
6*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 1733658
4) + 2167073*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(12258816*sq
rt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) +
 3064704*log(tanh(x/2) + 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 1733
```

```
6584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 2167073*sqrt(2)*log(tanh
(x/2) + 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2
 + 12258816*sqrt(2) + 17336584) - 2167073*sqrt(2)*log(tanh(x/2) - sqrt(2) -
 1)*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 1
2258816*sqrt(2) + 17336584) - 3064704*log(tanh(x/2) - sqrt(2) - 1)*tanh(x/2
)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt
(2) + 17336584) - 2167073*sqrt(2)*log(tanh(x/2) - sqrt(2) - 1)/(12258816*sq
rt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) -
 3064704*log(tanh(x/2) - sqrt(2) - 1)/(12258816*sqrt(2)*tanh(x/2)**2 + 1733
6584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) - 2167073*sqrt(2)*log(tanh
(x/2) - sqrt(2) + 1)*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584
*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) - 3064704*log(tanh(x/2) - sqrt
(2) + 1)*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**
2 + 12258816*sqrt(2) + 17336584) - 2167073*sqrt(2)*log(tanh(x/2) - sqrt(2)
+ 1)/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt
(2) + 17336584) - 3064704*log(tanh(x/2) - sqrt(2) + 1)/(12258816*sqrt(2)*ta
nh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 1225881
6*sqrt(2)*tanh(x/2)/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2
+ 12258816*sqrt(2) + 17336584) + 17336584*tanh(x/2)/(12258816*sqrt(2)*tanh(
x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.76

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right) - \frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right) + \frac{1}{e^{(-2x)}+1}$$

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/8*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) + 1/(e^(-2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \frac{1}{1-\sinh^4(x)}\,dx = -\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right) - \frac{1}{e^{(2x)}+1}$$

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6)) - 1/(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \frac{\sqrt{2}\,\ln\left(2\,\mathrm{e}^{2\,x}+\frac{\sqrt{2}\,\left(12\,\mathrm{e}^{2\,x}-4\right)}{8}\right)}{8} - \frac{\sqrt{2}\,\ln\left(2\,\mathrm{e}^{2\,x}-\frac{\sqrt{2}\,\left(12\,\mathrm{e}^{2\,x}-4\right)}{8}\right)}{8} - \frac{1}{\mathrm{e}^{2\,x}+1}$$

```
[In] int(-1/(sinh(x)^4 - 1),x)
```

```
[Out] (2^(1/2)*log(2*exp(2*x) + (2^(1/2)*(12*exp(2*x) - 4))/8))/8 - (2^(1/2)*log(
2*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/8))/8 - 1/(exp(2*x) + 1)
```

3.589 $\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = -\frac{4\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3(1+\tanh(x))}$$

```
[Out] -4/9*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)-1/3/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2099, 632, 210}

$$\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = -\frac{4\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3(\tanh(x)+1)}$$

```
[In] Int[(Cosh[x]^3 - Sinh[x]^3)/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] (-4*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(3*(1 + Tanh[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1+x+x^2}{1+x+x^3+x^4}\,dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{3(1+x)^2}+\frac{2}{3\,(1-x+x^2)}\right)dx, x, \tanh(x)\right)\\
&= -\frac{1}{3(1+\tanh(x))}+\frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tanh(x)\right)\\
&= -\frac{1}{3(1+\tanh(x))}-\frac{4}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tanh(x)\right)\\
&= -\frac{4\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3(1+\tanh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx=\frac{1}{18}\left(8\sqrt{3}\arctan\left(\frac{-1+2\tanh(x)}{\sqrt{3}}\right)-3\cosh(2x)+3\sinh(2x)\right)$$

```
[In] Integrate[(Cosh[x]^3 - Sinh[x]^3)/(Cosh[x]^3 + Sinh[x]^3),x]
[Out] (8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]] - 3*Cosh[2*x] + 3*Sinh[2*x])/18
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{e^{-2x}}{6}+\frac{2i\sqrt{3}\,\ln\left(e^{2x}+i\sqrt{3}\right)}{9}-\frac{2i\sqrt{3}\,\ln\left(e^{2x}-i\sqrt{3}\right)}{9}$
default	$\frac{2i\sqrt{3}\,\ln\left(\tanh^2\left(\frac{x}{2}\right)+\left(-1-i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)+1\right)}{9}-\frac{2i\sqrt{3}\,\ln\left(\tanh^2\left(\frac{x}{2}\right)+\left(-1+i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)+1\right)}{9}-\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)\right.}$

```
[In] int((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x,method=_RETURNVERBOSE)
[Out] -1/6*exp(-2*x)+2/9*I*3^(1/2)*ln(exp(2*x)+I*3^(1/2))-2/9*I*3^(1/2)*ln(exp(2*
x)-I*3^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = -\frac{8\left(\sqrt{3}\cosh\left(x\right)^2+2\sqrt{3}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{3}\sinh\left(x\right)^2\right)\arctan\left(-\frac{\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)}{3\left(\cosh(x)-\sinh(x)\right)}\right)+3}{18\left(\cosh\left(x\right)^2+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2\right)}$$

```
[In] integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="fricas"
)
[Out] -1/18*(8*(sqrt(3)*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2
)*arctan(-1/3*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) + 3)
/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(34) = 68.

Time = 0.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.09

$$\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = \frac{4\sqrt{3}\sinh\left(x\right)\operatorname{atan}\left(\frac{2\sqrt{3}\sinh\left(x\right)}{3\cosh\left(x\right)}-\frac{\sqrt{3}}{3}\right)}{9\sinh\left(x\right)+9\cosh\left(x\right)}+\frac{3\sinh\left(x\right)}{9\sinh\left(x\right)+9\cosh\left(x\right)} + \frac{4\sqrt{3}\cosh\left(x\right)\operatorname{atan}\left(\frac{2\sqrt{3}\sinh\left(x\right)}{3\cosh\left(x\right)}-\frac{\sqrt{3}}{3}\right)}{9\sinh\left(x\right)+9\cosh\left(x\right)}$$

```
[In] integrate((cosh(x)**3-sinh(x)**3)/(cosh(x)**3+sinh(x)**3),x)
```

```
[Out] 4*sqrt(3)*sinh(x)*atan(2*sqrt(3)*sinh(x)/(3*cosh(x)) - sqrt(3)/3)/(9*sinh(x
) + 9*cosh(x)) + 3*sinh(x)/(9*sinh(x) + 9*cosh(x)) + 4*sqrt(3)*cosh(x)*atan
(2*sqrt(3)*sinh(x)/(3*cosh(x)) - sqrt(3)/3)/(9*sinh(x) + 9*cosh(x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(26) = 52$.

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.12

$$\int \frac{\cosh^3(x) - \sinh^3(x)}{\cosh^3(x) + \sinh^3(x)}\,dx = \frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)} + 3^{\frac{1}{4}}\sqrt{2}\right)\right) - \frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)} - 3^{\frac{1}{4}}\sqrt{2}\right)\right) - \frac{1}{6}e^{(-2x)}$$

```
[In] integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="maxima"
)
```

```
[Out] 4/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2))
) - 4/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sqrt
(2))) - 1/6*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \frac{\cosh^3(x) - \sinh^3(x)}{\cosh^3(x) + \sinh^3(x)}\,dx = \frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2x)}\right) - \frac{1}{6}e^{(-2x)}$$

```
[In] integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="giac")
```

```
[Out] 4/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) - 1/6*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \frac{\cosh^3(x) - \sinh^3(x)}{\cosh^3(x) + \sinh^3(x)}\,dx = \frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\mathrm{e}^{2x}}{3}\right)}{9} - \frac{\mathrm{e}^{-2x}}{6}$$

```
[In] int((cosh(x)^3 - sinh(x)^3)/(cosh(x)^3 + sinh(x)^3),x)
```

```
[Out] (4*3^(1/2)*atan((3^(1/2)*exp(2*x))/3))/9 - exp(-2*x)/6
```

3.590 $\int \cosh(x)\cosh(2x)\cosh(3x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)$$

```
[Out] 1/4*x+1/8*sinh(2*x)+1/16*sinh(4*x)+1/24*sinh(6*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4440, 2717}

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)$$

```
[In] Int[Cosh[x]*Cosh[2*x]*Cosh[3*x],x]
```

```
[Out] x/4 + Sinh[2*x]/8 + Sinh[4*x]/16 + Sinh[6*x]/24
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4440

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
```

```
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{4}+\frac{1}{4}\cosh(2x)+\frac{1}{4}\cosh(4x)+\frac{1}{4}\cosh(6x)\right)\,dx\\ &= \frac{x}{4}+\frac{1}{4}\int \cosh(2x)\,dx+\frac{1}{4}\int \cosh(4x)\,dx+\frac{1}{4}\int \cosh(6x)\,dx\\ &= \frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)$$

```
[In] Integrate[Cosh[x]*Cosh[2*x]*Cosh[3*x],x]
[Out] x/4 + Sinh[2*x]/8 + Sinh[4*x]/16 + Sinh[6*x]/24
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result
default	$\frac{x}{4}+\frac{\sinh(2x)}{8}+\frac{\sinh(4x)}{16}+\frac{\sinh(6x)}{24}$
risch	$\frac{x}{4}+\frac{e^{6x}}{48}+\frac{e^{4x}}{32}+\frac{e^{2x}}{16}-\frac{e^{-2x}}{16}-\frac{e^{-4x}}{32}-\frac{e^{-6x}}{48}$
parallelrisch	$\frac{(-24+24\cosh(x))\ln(1-\coth(x)+\operatorname{csch}(x))+(-24\cosh(x)+24)\ln(\coth(x)-\operatorname{csch}(x)+1)+48x\cosh(x)-48x-4\sinh(6x)+2\sinh(}{96\cosh(x)-96}$

```
[In] int(cosh(x)*cosh(2*x)*cosh(3*x),x,method=_RETURNVERBOSE)
[Out] 1/4*x+1/8*sinh(2*x)+1/16*sinh(4*x)+1/24*sinh(6*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{1}{4}\cosh(x)\sinh(x)^5 + \frac{1}{12}\left(10\cosh(x)^3 + 3\cosh(x)\right)\sinh(x)^3 + \frac{1}{4}\left(\cosh(x)^5 + \cosh(x)^3 + \cosh(x)\right)\sinh(x) + \frac{1}{4}x$$

```
[In] integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="fricas")
```

```
[Out] 1/4*cosh(x)*sinh(x)^5 + 1/12*(10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 1/4*(co
sh(x)^5 + cosh(x)^3 + cosh(x))*sinh(x) + 1/4*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(22) = 44.

Time = 0.94 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.87

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{x\sinh(x)\sinh(2x)\cosh(3x)}{4} - \frac{x\sinh(x)\sinh(3x)\cosh(2x)}{4} - \frac{x\sinh(2x)\sinh(3x)\cosh(x)}{4} + \frac{x\cosh(x)\cosh(2x)\cosh(3x)}{4} - \frac{3\sinh(x)\sinh(2x)\sinh(3x)}{8} + \frac{\sinh(x)\cosh(2x)\cosh(3x)}{3} + \frac{5\sinh(2x)\cosh(x)\cosh(3x)}{24}$$

```
[In] integrate(cosh(x)*cosh(2*x)*cosh(3*x),x)
```

```
[Out] x*sinh(x)*sinh(2*x)*cosh(3*x)/4 - x*sinh(x)*sinh(3*x)*cosh(2*x)/4 - x*sinh(
2*x)*sinh(3*x)*cosh(x)/4 + x*cosh(x)*cosh(2*x)*cosh(3*x)/4 - 3*sinh(x)*sinh
(2*x)*sinh(3*x)/8 + sinh(x)*cosh(2*x)*cosh(3*x)/3 + 5*sinh(2*x)*cosh(x)*cos
h(3*x)/24
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{1}{96}\left(3\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 2\right)e^{(6\,x)} + \frac{1}{4}\,x - \frac{1}{16}\,e^{(-2\,x)} - \frac{1}{32}\,e^{(-4\,x)} - \frac{1}{48}\,e^{(-6\,x)}$$

```
[In] integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="maxima")
```

```
[Out] 1/96*(3*e^(-2*x) + 6*e^(-4*x) + 2)*e^(6*x) + 1/4*x - 1/16*e^(-2*x) - 1/32*e
^(-4*x) - 1/48*e^(-6*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(22) = 44$.

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = -\frac{1}{96}\left(22\,e^{(6\,x)} + 6\,e^{(4\,x)} + 3\,e^{(2\,x)} + 2\right)e^{(-6\,x)} + \frac{1}{4}\,x + \frac{1}{48}\,e^{(6\,x)} + \frac{1}{32}\,e^{(4\,x)} + \frac{1}{16}\,e^{(2\,x)}$$

```
[In] integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="giac")
```

```
[Out] -1/96*(22*e^(6*x) + 6*e^(4*x) + 3*e^(2*x) + 2)*e^(-6*x) + 1/4*x + 1/48*e^(6
*x) + 1/32*e^(4*x) + 1/16*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{x}{4} - \frac{\mathrm{e}^{-2\,x}}{16} + \frac{\mathrm{e}^{2\,x}}{16} - \frac{\mathrm{e}^{-4\,x}}{32} + \frac{\mathrm{e}^{4\,x}}{32} - \frac{\mathrm{e}^{-6\,x}}{48} + \frac{\mathrm{e}^{6\,x}}{48}$$

```
[In] int(cosh(2*x)*cosh(3*x)*cosh(x),x)
```

```
[Out] x/4 - exp(-2*x)/16 + exp(2*x)/16 - exp(-4*x)/32 + exp(4*x)/32 - exp(-6*x)/4
8 + exp(6*x)/48
```

3.591 $\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx = -\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)$$

```
[Out] -1/4*x+1/8*sinh(2*x)-1/12*sinh(3*x)+1/20*sinh(5*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4440, 2717}

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx = -\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)$$

```
[In] Int[Cosh[(3*x)/2]*Sinh[x]*Sinh[(5*x)/2],x]
```

```
[Out] -1/4*x + Sinh[2*x]/8 - Sinh[3*x]/12 + Sinh[5*x]/20
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4440

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
```

```
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\int \left(\frac{1}{4}-\frac{1}{4}\cosh(2x)+\frac{1}{4}\cosh(3x)-\frac{1}{4}\cosh(5x)\right)\,dx \\ &= -\frac{x}{4}+\frac{1}{4}\int \cosh(2x)\,dx-\frac{1}{4}\int \cosh(3x)\,dx+\frac{1}{4}\int \cosh(5x)\,dx \\ &= -\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx = -\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)$$

```
[In] Integrate[Cosh[(3*x)/2]*Sinh[x]*Sinh[(5*x)/2],x]
[Out] -1/4*x + Sinh[2*x]/8 - Sinh[3*x]/12 + Sinh[5*x]/20
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result
default	$-\frac{x}{4}+\frac{\sinh(2x)}{8}-\frac{\sinh(3x)}{12}+\frac{\sinh(5x)}{20}$
risch	$-\frac{x}{4}+\frac{e^{5x}}{40}-\frac{e^{3x}}{24}+\frac{e^{2x}}{16}-\frac{e^{-2x}}{16}+\frac{e^{-3x}}{24}-\frac{e^{-5x}}{40}$
parallelrisch	$\frac{\left(-40\cosh\left(\frac{x}{2}\right)+40\right)\ln\left(1-\tanh\left(\frac{3x}{4}\right)\right)+\left(40\cosh\left(\frac{x}{2}\right)-40\right)\ln\left(\tanh\left(\frac{3x}{4}\right)+1\right)-120x\cosh\left(\frac{x}{2}\right)+120x+6\sinh\left(\frac{11x}{2}\right)-30\sinh(2x)+20}{-240+240\cosh\left(\frac{x}{2}\right)}$

```
[In] int(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x,method=_RETURNVERBOSE)
[Out] -1/4*x+1/8*sinh(2*x)-1/12*sinh(3*x)+1/20*sinh(5*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(22) = 44.

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.70

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx$$
$$= 6\cosh\left(\frac{1}{2}x\right)^3\sinh\left(\frac{1}{2}x\right)^7 + \frac{1}{2}\cosh\left(\frac{1}{2}x\right)\sinh\left(\frac{1}{2}x\right)^9$$
$$+ \frac{1}{10}\left(126\cosh\left(\frac{1}{2}x\right)^5 - 5\cosh\left(\frac{1}{2}x\right)\right)\sinh\left(\frac{1}{2}x\right)^5$$
$$+ \frac{1}{6}\left(36\cosh\left(\frac{1}{2}x\right)^7 - 10\cosh\left(\frac{1}{2}x\right)^3 + 3\cosh\left(\frac{1}{2}x\right)\right)\sinh\left(\frac{1}{2}x\right)^3$$
$$+ \frac{1}{2}\left(\cosh\left(\frac{1}{2}x\right)^9 - \cosh\left(\frac{1}{2}x\right)^5 + \cosh\left(\frac{1}{2}x\right)^3\right)\sinh\left(\frac{1}{2}x\right) - \frac{1}{4}x$$

```
[In] integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="fricas")
```

```
[Out] 6*cosh(1/2*x)^3*sinh(1/2*x)^7 + 1/2*cosh(1/2*x)*sinh(1/2*x)^9 + 1/10*(126*c
osh(1/2*x)^5 - 5*cosh(1/2*x))*sinh(1/2*x)^5 + 1/6*(36*cosh(1/2*x)^7 - 10*co
sh(1/2*x)^3 + 3*cosh(1/2*x))*sinh(1/2*x)^3 + 1/2*(cosh(1/2*x)^9 - cosh(1/2*
x)^5 + cosh(1/2*x)^3)*sinh(1/2*x) - 1/4*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(22) = 44.

Time = 0.94 (sec) , antiderivative size = 139, normalized size of antiderivative = 4.63

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx = -\frac{x\sinh(x)\sinh\left(\frac{3x}{2}\right)\cosh\left(\frac{5x}{2}\right)}{4} + \frac{x\sinh(x)\sinh\left(\frac{5x}{2}\right)\cosh\left(\frac{3x}{2}\right)}{4} + \frac{x\sinh\left(\frac{3x}{2}\right)\sinh\left(\frac{5x}{2}\right)\cosh(x)}{4} - \frac{x\cosh(x)\cosh\left(\frac{3x}{2}\right)\cosh\left(\frac{5x}{2}\right)}{4} - \frac{3\sinh(x)\sinh\left(\frac{3x}{2}\right)\sinh\left(\frac{5x}{2}\right)}{20} + \frac{5\sinh(x)\cosh\left(\frac{3x}{2}\right)\cosh\left(\frac{5x}{2}\right)}{12} - \frac{\sinh\left(\frac{5x}{2}\right)\cosh(x)\cosh\left(\frac{3x}{2}\right)}{15}$$

```
[In] integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x)
```

```
[Out] -x*sinh(x)*sinh(3*x/2)*cosh(5*x/2)/4 + x*sinh(x)*sinh(5*x/2)*cosh(3*x/2)/4
+ x*sinh(3*x/2)*sinh(5*x/2)*cosh(x)/4 - x*cosh(x)*cosh(3*x/2)*cosh(5*x/2)/4
 - 3*sinh(x)*sinh(3*x/2)*sinh(5*x/2)/20 + 5*sinh(x)*cosh(3*x/2)*cosh(5*x/2)
/12 - sinh(5*x/2)*cosh(x)*cosh(3*x/2)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx = -\frac{1}{240}\left(10\,e^{(-2\,x)} - 15\,e^{(-3\,x)} - 6\right)e^{(5\,x)} - \frac{1}{4}\,x - \frac{1}{16}\,e^{(-2\,x)} + \frac{1}{24}\,e^{(-3\,x)} - \frac{1}{40}\,e^{(-5\,x)}$$

```
[In] integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="maxima")
```

```
[Out] -1/240*(10*e^(-2*x) - 15*e^(-3*x) - 6)*e^(5*x) - 1/4*x - 1/16*e^(-2*x) + 1/
24*e^(-3*x) - 1/40*e^(-5*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(22) = 44.

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx = \frac{1}{240}\left(137\,e^{(5\,x)} - 15\,e^{(3\,x)} + 10\,e^{(2\,x)} - 6\right)e^{(-5\,x)} - \frac{1}{4}\,x + \frac{1}{40}\,e^{(5\,x)} - \frac{1}{24}\,e^{(3\,x)} + \frac{1}{16}\,e^{(2\,x)}$$

```
[In] integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="giac")
```

```
[Out] 1/240*(137*e^(5*x) - 15*e^(3*x) + 10*e^(2*x) - 6)*e^(-5*x) - 1/4*x + 1/40*e
^(5*x) - 1/24*e^(3*x) + 1/16*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx = \frac{e^{2x}}{16} - \frac{e^{-2x}}{16} - \frac{x}{4} + \frac{e^{-3x}}{24} - \frac{e^{3x}}{24} - \frac{e^{-5x}}{40} + \frac{e^{5x}}{40}$$

```
[In] int(cosh((3*x)/2)*sinh((5*x)/2)*sinh(x),x)
```

```
[Out] exp(2*x)/16 - exp(-2*x)/16 - x/4 + exp(-3*x)/24 - exp(3*x)/24 - exp(-5*x)/4
0 + exp(5*x)/40
```

3.592 $\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}(\sinh^2(x)+\sinh(2x))} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 69

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\,(\sinh^2(x)+\sinh(2x))} \, dx = \sqrt{2}\arctan\left(\operatorname{sech}(x)\sqrt{\cosh(x)\sinh(x)}\right) + \frac{1}{6}\arctan\left(\frac{\sinh(x)}{\sqrt{\sinh(2x)}}\right) - \frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\operatorname{sech}(x)\sqrt{\cosh(x)\sinh(x)}\right) + \frac{\cosh(x)}{\sqrt{\sinh(2x)}}$$

```
[Out] 1/6*arctan(sinh(x)/sinh(2*x)^(1/2))+arctan(sech(x)*(cosh(x)*sinh(x))^(1/2))
*2^(1/2)-1/3*arctanh(sech(x)*(cosh(x)*sinh(x))^(1/2))*2^(1/2)+cosh(x)/sinh(
2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.48, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used

$= \{4475, 6857, 213, 209\}$

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\left(\sinh^2(x)+\sinh(2x)\right)}\,dx = \frac{2\sinh(x)\arctan\left(\sqrt{\tanh(x)}\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}} + \frac{\sinh(x)\arctan\left(\frac{\sqrt{\tanh(x)}}{\sqrt{2}}\right)}{3\sqrt{2}\sqrt{\sinh(2x)}\sqrt{\tanh(x)}} - \frac{2\sinh(x)\text{arctanh}\left(\sqrt{\tanh(x)}\right)}{3\sqrt{\sinh(2x)}\sqrt{\tanh(x)}} + \frac{\cosh(x)}{\sqrt{\sinh(2x)}}$$

```
[In] Int[(Cosh[x]*(-Cosh[2*x] + Tanh[x]))/(Sqrt[Sinh[2*x]]*(Sinh[x]^2 + Sinh[2*x
])),x]
```

```
[Out] Cosh[x]/Sqrt[Sinh[2*x]] + (2*ArcTan[Sqrt[Tanh[x]]]*Sinh[x])/(Sqrt[Sinh[2*x]
]*Sqrt[Tanh[x]]) + (ArcTan[Sqrt[Tanh[x]]/Sqrt[2]]*Sinh[x])/(3*Sqrt[2]*Sqrt[
Sinh[2*x]]*Sqrt[Tanh[x]]) - (2*ArcTanh[Sqrt[Tanh[x]]]*Sinh[x])/(3*Sqrt[Sinh
[2*x]]*Sqrt[Tanh[x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 4475

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[u*(Sin
[v/2]^(2*m)/(c*Tan[v/2])^m), x]}, Dist[(c*Sin[v])^m*((c*Tan[v/2])^m/Sin[v/2
]^(2*m)), Int[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\sinh(x)\int\frac{-\cosh(2x)+\tanh(x)}{\left(\sinh^2(x)+\sinh(2x)\right)\sqrt{\tanh(x)}}\,dx}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$= \frac{\sinh(x)\text{Subst}\left(\int\frac{-1+x-x^2-x^3}{x^{3/2}(2+x)(1-x^2)}\,dx,x,\tanh(x)\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$= \frac{(2\sinh(x))\text{Subst}\left(\int\frac{1-x^2+x^4+x^6}{x^2(2+x^2)(-1+x^4)}\,dx,x,\sqrt{\tanh(x)}\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$= \frac{(2\sinh(x))\text{Subst}\left(\int\left(-\frac{1}{2x^2}+\frac{1}{3(-1+x^2)}+\frac{1}{1+x^2}+\frac{1}{6(2+x^2)}\right)\,dx,x,\sqrt{\tanh(x)}\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$= \frac{\cosh(x)}{\sqrt{\sinh(2x)}}+\frac{\sinh(x)\text{Subst}\left(\int\frac{1}{2+x^2}\,dx,x,\sqrt{\tanh(x)}\right)}{3\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$
$$+\frac{(2\sinh(x))\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\sqrt{\tanh(x)}\right)}{3\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$
$$+\frac{(2\sinh(x))\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{\tanh(x)}\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$= \frac{\cosh(x)}{\sqrt{\sinh(2x)}}+\frac{2\arctan\left(\sqrt{\tanh(x)}\right)\sinh(x)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$
$$+\frac{\arctan\left(\frac{\sqrt{\tanh(x)}}{\sqrt{2}}\right)\sinh(x)}{3\sqrt{2}\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}-\frac{2\text{arctanh}\left(\sqrt{\tanh(x)}\right)\sinh(x)}{3\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(69) = 138.

Time = 18.50 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.32

$$\int\frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\left(\sinh^2(x)+\sinh(2x)\right)}\,dx$$
$$= \frac{\sqrt{\sinh(2x)}\left(6\sqrt{2}\arctan\left(\frac{\sqrt{\tanh\left(\frac{x}{2}\right)}}{\sqrt{\frac{\cosh(x)}{1+\cosh(x)}}}\right)+\arctan\left(\frac{\sqrt{\tanh\left(\frac{x}{2}\right)}}{\sqrt{1+\tanh^2\left(\frac{x}{2}\right)}}\right)-2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\tanh\left(\frac{x}{2}\right)}}{\sqrt{\frac{\cosh(x)}{1+\cosh(x)}}}\right)+\frac{3\sqrt{\cosh(x)}\text{secl}}{\sqrt{\tanh(\frac{x}{2}}}\right.}{6(1+\cosh(x))\sqrt{\tanh\left(\frac{x}{2}\right)}\sqrt{1+\tanh^2\left(\frac{x}{2}\right)}}$$

```
[In] Integrate[(Cosh[x]*(-Cosh[2*x] + Tanh[x]))/(Sqrt[Sinh[2*x]]*(Sinh[x]^2 + Si
nh[2*x])),x]
```

```
[Out] (Sqrt[Sinh[2*x]]*(6*Sqrt[2]*ArcTan[Sqrt[Tanh[x/2]]/Sqrt[Cosh[x]/(1 + Cosh[x
])]] + ArcTan[Sqrt[Tanh[x/2]]/Sqrt[1 + Tanh[x/2]^2]] - 2*Sqrt[2]*ArcTanh[Sq
rt[Tanh[x/2]]/Sqrt[Cosh[x]/(1 + Cosh[x])]] + (3*Sqrt[Cosh[x]*Sech[x/2]^2])/
Sqrt[Tanh[x/2]]))/(6*(1 + Cosh[x])*Sqrt[Tanh[x/2]]*Sqrt[1 + Tanh[x/2]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(53) = 106$.

Time = 1.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.42

method	result
default	$\frac{\sqrt{\frac{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)\tanh\left(\frac{x}{2}\right)}{\left(\tanh^2\left(\frac{x}{2}\right)-1\right)^2}}\left(\tanh^2\left(\frac{x}{2}\right)-1\right)\left(2\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{\tanh^3\left(\frac{x}{2}\right)+\tanh\left(\frac{x}{2}\right)}\,\sqrt{2}}{2\tanh\left(\frac{x}{2}\right)}\right)\tanh\left(\frac{x}{2}\right)+6\sqrt{2}\,\arctan\left(\frac{\sqrt{\tanh^3\left(\frac{x}{2}\right)+\tanh\left(\frac{x}{2}\right)}\,\sqrt{2}}{2\tanh\left(\frac{x}{2}\right)}\right.\right.}{6\sqrt{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)\tanh\left(\frac{x}{2}\right)}\,\tanh\left(\frac{x}{2}\right)}$

```
[In] int(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 1/6*((tanh(1/2*x)^2+1)*tanh(1/2*x)/(tanh(1/2*x)^2-1)^2)^(1/2)*(tanh(1/2*x)^
2-1)*(2*2^(1/2)*arctanh(1/2/tanh(1/2*x)*(tanh(1/2*x)^3+tanh(1/2*x))^(1/2)*2
^(1/2))*tanh(1/2*x)+6*2^(1/2)*arctan(1/2/tanh(1/2*x)*(tanh(1/2*x)^3+tanh(1/
2*x))^(1/2)*2^(1/2))*tanh(1/2*x)+arctan(1/tanh(1/2*x)*(tanh(1/2*x)^3+tanh(1
/2*x))^(1/2))*tanh(1/2*x)-3*(tanh(1/2*x)^3+tanh(1/2*x))^(1/2))/((tanh(1/2*x
)^2+1)*tanh(1/2*x))^(1/2)/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(53) = 106$.

Time = 0.26 (sec) , antiderivative size = 376, normalized size of antiderivative = 5.45

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\left(\sinh^2(x)+\sinh(2x)\right)}\,dx =$$

$$-\frac{\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\arctan\left(\frac{\left(\sqrt{2}\cosh(x)^2+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^2+3\sqrt{2}\right)\sqrt{\frac{\cosh}{}}}{2\left(\cosh(x)^4+4\cosh(x)^3\sinh(x)+6\cosh(x)^2\sinh(x)^2+4\cos\right.}\right.}{}$$

```
[In] integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2
),x, algorithm="fricas")
```

```
[Out] -1/12*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*arctan(1/2*(sqrt(2)*
cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + 3*sqrt(2))*sqrt
(cosh(x)*sinh(x)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(cosh(x)^4 +
4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)
^4 - 1)) + 6*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(
x)^2 - sqrt(2))*arctan(2*sqrt(cosh(x)*sinh(x)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 +
4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)) - (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh
(x)*sinh(x) + sqrt(2)*sinh(x)^2 - sqrt(2))*log(2*cosh(x)^4 + 8*cosh(x)^3*si
nh(x) + 12*cosh(x)^2*sinh(x)^2 + 8*cosh(x)*sinh(x)^3 + 2*sinh(x)^4 - 4*(cos
h(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(cosh(x)*sinh(x)/(cosh(x)^2 - 2
*cosh(x)*sinh(x) + sinh(x)^2)) - 1) - 12*sqrt(2)*sqrt(cosh(x)*sinh(x)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 - 1)
```

Sympy [F]

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\,(\sinh^2(x)+\sinh(2x))}\,dx = -\int \frac{\cosh{(x)}\cosh{(2x)}}{\sinh^2{(x)}\sqrt{\sinh{(2x)}}+\sinh^{\frac{3}{2}}{(2x)}}\,dx - \int \left(-\frac{\cosh{(x)}\tanh{(x)}}{\sinh^2{(x)}\sqrt{\sinh{(2x)}}+\sinh^{\frac{3}{2}}{(2x)}}\right)dx$$

```
[In] integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)**2+sinh(2*x))/sinh(2*x)**(1
/2),x)
```

```
[Out] -Integral(cosh(x)*cosh(2*x)/(sinh(x)**2*sqrt(sinh(2*x)) + sinh(2*x)**(3/2))
, x) - Integral(-cosh(x)*tanh(x)/(sinh(x)**2*sqrt(sinh(2*x)) + sinh(2*x)**(
3/2)), x)
```

Maxima [F]

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\,(\sinh^2(x)+\sinh(2x))}\,dx = \int -\frac{(\cosh{(2\,x)}-\tanh{(x)})\cosh{(x)}}{\left(\sinh{(x)}^2+\sinh{(2\,x)}\right)\sqrt{\sinh{(2\,x)}}}\,dx$$

```
[In] integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2
),x, algorithm="maxima")
```

```
[Out] -integrate((cosh(2*x) - tanh(x))*cosh(x)/((sinh(x)^2 + sinh(2*x))*sqrt(sinh
(2*x))), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\left(\sinh^2(x)+\sinh(2x)\right)}\,dx = \sqrt{2}\arctan\left(\sqrt{e^{(4x)}-1}-e^{(2x)}\right) + \frac{1}{6}\sqrt{2}\log\left(-\sqrt{e^{(4x)}-1}+e^{(2x)}\right) + \frac{\sqrt{2}}{\sqrt{e^{(4x)}-1}-e^{(2x)}+1} + \frac{1}{6}\arctan\left(\frac{1}{4}\sqrt{2}\left(3\sqrt{e^{(4x)}-1}-3e^{(2x)}-1\right)\right)$$

```
[In] integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2
),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(sqrt(e^(4*x) - 1) - e^(2*x)) + 1/6*sqrt(2)*log(-sqrt(e^(4*x)
 - 1) + e^(2*x)) + sqrt(2)/(sqrt(e^(4*x) - 1) - e^(2*x) + 1) + 1/6*arctan(1
/4*sqrt(2)*(3*sqrt(e^(4*x) - 1) - 3*e^(2*x) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\left(\sinh^2(x)+\sinh(2x)\right)}\,dx = -\int \frac{\cosh(x)\,\left(\cosh(2\,x)-\tanh(x)\right)}{\sqrt{\sinh\left(2\,x\right)}\,\left({\sinh\left(x\right)}^2+\sinh\left(2\,x\right)\right)}\,dx$$

```
[In] int(-(cosh(x)*(cosh(2*x) - tanh(x)))/(sinh(2*x)^(1/2)*(sinh(2*x) + sinh(x)^
2)),x)
```

```
[Out] -int((cosh(x)*(cosh(2*x) - tanh(x)))/(sinh(2*x)^(1/2)*(sinh(2*x) + sinh(x)^
2)), x)
```

3.593 $\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = -\frac{\cosh(x)}{27\left(-9+4\cosh^2(x)\right)^{3/2}}+\frac{2\cosh(x)}{243\sqrt{-9+4\cosh^2(x)}}$$

```
[Out] -1/27*cosh(x)/(-9+4*cosh(x)^2)^(3/2)+2/243*cosh(x)/(-9+4*cosh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3269, 198, 197}

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = \frac{2\cosh(x)}{243\sqrt{4\cosh^2(x)-9}}-\frac{\cosh(x)}{27\left(4\cosh^2(x)-9\right)^{3/2}}$$

```
[In] Int[Sinh[x]/(-9 + 4*Cosh[x]^2)^(5/2),x]
```

```
[Out] -1/27*Cosh[x]/(-9 + 4*Cosh[x]^2)^(3/2) + (2*Cosh[x])/(243*Sqrt[-9 + 4*Cosh[
x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{(-9+4x^2)^{5/2}}\,dx, x, \cosh(x)\right)\\ &= -\frac{\cosh(x)}{27\left(-9+4\cosh^2(x)\right)^{3/2}} - \frac{2}{27}\text{Subst}\left(\int \frac{1}{(-9+4x^2)^{3/2}}\,dx, x, \cosh(x)\right)\\ &= -\frac{\cosh(x)}{27\left(-9+4\cosh^2(x)\right)^{3/2}} + \frac{2\cosh(x)}{243\sqrt{-9+4\cosh^2(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = \frac{\cosh(x)(-23+4\cosh(2x))}{243(-7+2\cosh(2x))^{3/2}}$$

```
[In] Integrate[Sinh[x]/(-9 + 4*Cosh[x]^2)^(5/2),x]
[Out] (Cosh[x]*(-23 + 4*Cosh[2*x]))/(243*(-7 + 2*Cosh[2*x])^(3/2))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{\cosh(x)}{27(-9+4(\cosh^2(x)))^{\frac{3}{2}}}+\frac{2\cosh(x)}{243\sqrt{-9+4(\cosh^2(x))}}$	30
default	$-\frac{\cosh(x)}{27(-9+4(\cosh^2(x)))^{\frac{3}{2}}}+\frac{2\cosh(x)}{243\sqrt{-9+4(\cosh^2(x))}}$	30

```
[In] int(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/27*cosh(x)/(-9+4*cosh(x)^2)^(3/2)+2/243*cosh(x)/(-9+4*cosh(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 474, normalized size of antiderivative = 12.81

$$\int \frac{\sinh(x)}{(-9+4\cosh^2(x))^{5/2}}\,dx = \frac{2\cosh(x)^8 + 16\cosh(x)\sinh(x)^7 + 2\sinh(x)^8 + 28\left(2\cosh(x)^2 - 1\right)\sinh(x}{}$$

```
[In] integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="fricas")
[Out] 1/486*(2*cosh(x)^8 + 16*cosh(x)*sinh(x)^7 + 2*sinh(x)^8 + 28*(2*cosh(x)^2 -
 1)*sinh(x)^6 - 28*cosh(x)^6 + 56*(2*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(
70*cosh(x)^4 - 210*cosh(x)^2 + 51)*sinh(x)^4 + 102*cosh(x)^4 + 8*(14*cosh(x
)^5 - 70*cosh(x)^3 + 51*cosh(x))*sinh(x)^3 + 4*(14*cosh(x)^6 - 105*cosh(x)^
4 + 153*cosh(x)^2 - 7)*sinh(x)^2 - 28*cosh(x)^2 + 8*(2*cosh(x)^7 - 21*cosh(
x)^5 + 51*cosh(x)^3 - 7*cosh(x))*sinh(x) + (2*cosh(x)^6 + 12*cosh(x)*sinh(x
)^5 + 2*sinh(x)^6 + 3*(10*cosh(x)^2 - 7)*sinh(x)^4 - 21*cosh(x)^4 + 4*(10*c
osh(x)^3 - 21*cosh(x))*sinh(x)^3 + 3*(10*cosh(x)^4 - 42*cosh(x)^2 - 7)*sinh
(x)^2 - 21*cosh(x)^2 + 6*(2*cosh(x)^5 - 14*cosh(x)^3 - 7*cosh(x))*sinh(x) +
 2)*sqrt((2*cosh(x)^2 + 2*sinh(x)^2 - 7)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2)) + 2)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 14*(2*cosh(x
)^2 - 1)*sinh(x)^6 - 14*cosh(x)^6 + 28*(2*cosh(x)^3 - 3*cosh(x))*sinh(x)^5
+ (70*cosh(x)^4 - 210*cosh(x)^2 + 51)*sinh(x)^4 + 51*cosh(x)^4 + 4*(14*cosh
(x)^5 - 70*cosh(x)^3 + 51*cosh(x))*sinh(x)^3 + 2*(14*cosh(x)^6 - 105*cosh(x
)^4 + 153*cosh(x)^2 - 7)*sinh(x)^2 - 14*cosh(x)^2 + 4*(2*cosh(x)^7 - 21*cos
h(x)^5 + 51*cosh(x)^3 - 7*cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)/(-9+4*cosh(x)**2)**(5/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(29) = 58.

Time = 0.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.38

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = -\frac{1855\,e^{(-2\,x)}-8485\,e^{(-4\,x)}+5285\,e^{(-6\,x)}-980\,e^{(-8\,x)}+56\,e^{(-10\,x)}-106}{12150\,\left(3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}\left(-3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}} + \frac{980\,e^{(-2\,x)}-5285\,e^{(-4\,x)}+8485\,e^{(-6\,x)}-1855\,e^{(-8\,x)}+106\,e^{(-10\,x)}-56}{12150\,\left(3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}\left(-3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}}$$

```
[In] integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="maxima")
[Out] -1/12150*(1855*e^(-2*x) - 8485*e^(-4*x) + 5285*e^(-6*x) - 980*e^(-8*x) + 56
*e^(-10*x) - 106)/((3*e^(-x) + e^(-2*x) + 1)^(5/2)*(-3*e^(-x) + e^(-2*x) +
1)^(5/2)) + 1/12150*(980*e^(-2*x) - 5285*e^(-4*x) + 8485*e^(-6*x) - 1855*e^
(-8*x) + 106*e^(-10*x) - 56)/((3*e^(-x) + e^(-2*x) + 1)^(5/2)*(-3*e^(-x) +
e^(-2*x) + 1)^(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = \frac{\left(\left(2\,e^{(2\,x)}-21\right)e^{(2\,x)}-21\right)e^{(2\,x)}+2}{486\,\left(e^{(4\,x)}-7\,e^{(2\,x)}+1\right)^{\frac{3}{2}}}$$

```
[In] integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="giac")
[Out] 1/486*(((2*e^(2*x) - 21)*e^(2*x) - 21)*e^(2*x) + 2)/(e^(4*x) - 7*e^(2*x) +
1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = -\frac{e^x\,\sqrt{4\left(\frac{e^{-x}}{2}+\frac{e^x}{2}\right)^2-9}\,\left(21\,e^{2\,x}+21\,e^{4\,x}-2\,e^{6\,x}-2\right)}{486\left(e^{4\,x}-7\,e^{2\,x}+1\right)^2}$$

```
[In] int(sinh(x)/(4*cosh(x)^2 - 9)^(5/2),x)
```

```
[Out] -(exp(x)*(4*(exp(-x)/2 + exp(x)/2)^2 - 9)^(1/2)*(21*exp(2*x) + 21*exp(4*x)
- 2*exp(6*x) - 2))/(486*(exp(4*x) - 7*exp(2*x) + 1)^2)
```

3.594 $\int \frac{\sinh^2(x)\sinh(2x)}{(1-\sinh^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \frac{\sinh^2(x)\sinh(2x)}{(1-\sinh^2(x))^{3/2}}\,dx = \frac{2}{\sqrt{1-\sinh^2(x)}} + 2\sqrt{1-\sinh^2(x)}$$

```
[Out] 2/(1-sinh(x)^2)^(1/2)+2*(1-sinh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 272, 45}

$$\int \frac{\sinh^2(x)\sinh(2x)}{(1-\sinh^2(x))^{3/2}}\,dx = 2\sqrt{1-\sinh^2(x)} + \frac{2}{\sqrt{1-\sinh^2(x)}}$$

```
[In] Int[(Sinh[x]^2*Sinh[2*x])/(1 - Sinh[x]^2)^(3/2),x]
[Out] 2/Sqrt[1 - Sinh[x]^2] + 2*Sqrt[1 - Sinh[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= i\text{Subst}\left(\int -\frac{2ix^3}{\left(1-x^2\right)^{3/2}}\,dx, x, \sinh(x)\right) \\
&= 2\text{Subst}\left(\int \frac{x^3}{\left(1-x^2\right)^{3/2}}\,dx, x, \sinh(x)\right) \\
&= \text{Subst}\left(\int \frac{x}{(1-x)^{3/2}}\,dx, x, \sinh^2(x)\right) \\
&= \text{Subst}\left(\int \left(\frac{1}{(1-x)^{3/2}} - \frac{1}{\sqrt{1-x}}\right)\,dx, x, \sinh^2(x)\right) \\
&= \frac{2}{\sqrt{1-\sinh^2(x)}} + 2\sqrt{1-\sinh^2(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$
\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx = \frac{5-\cosh(2x)}{\sqrt{1-\sinh^2(x)}}
$$

```
[In] Integrate[(Sinh[x]^2*Sinh[2*x])/(1 - Sinh[x]^2)^(3/2),x]
[Out] (5 - Cosh[2*x])/Sqrt[1 - Sinh[x]^2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$-\frac{2\left(\sinh^{2}(x)\right)}{\sqrt{1-\left(\sinh^{2}(x)\right)}}+\frac{4}{\sqrt{1-\left(\sinh^{2}(x)\right)}}$	30
default	$-\frac{2\left(\sinh^{2}(x)\right)}{\sqrt{1-\left(\sinh^{2}(x)\right)}}+\frac{4}{\sqrt{1-\left(\sinh^{2}(x)\right)}}$	30

```
[In] int(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*sinh(x)^2/(1-sinh(x)^2)^(1/2)+4/(1-sinh(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 5.55

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx = \frac{\sqrt{2}\left(\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\,\left(3\,\cosh\left(x\right)^{2}-5\right)\sinh\left(x\right.\right.}{\cosh\left(x\right)^{5}+5\,\cosh\left(x\right)\sinh\left(x\right)^{4}+\sinh\left(x\right)^{5}+2\,\left(5\,\cosh\left(x\right)^{2}-3\right)\sinh\left(x\right)^{3}-6\,\cosh}$$

```
[In] integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 5)*
sinh(x)^2 - 10*cosh(x)^2 + 4*(cosh(x)^3 - 5*cosh(x))*sinh(x) + 1)*sqrt(-(co
sh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(cosh
(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 + 2*(5*cosh(x)^2 - 3)*sinh(x)^3 - 6
*cosh(x)^3 + 2*(5*cosh(x)^3 - 9*cosh(x))*sinh(x)^2 + (5*cosh(x)^4 - 18*cosh
(x)^2 + 1)*sinh(x) + cosh(x))
```

Sympy [F]

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx = \int \frac{\sinh^{2}\left(x\right)\sinh\left(2x\right)}{\left(-\left(\sinh\left(x\right)-1\right)\left(\sinh\left(x\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sinh(x)**2*sinh(2*x)/(1-sinh(x)**2)**(3/2),x)
[Out] Integral(sinh(x)**2*sinh(2*x)/(-(sinh(x) - 1)*(sinh(x) + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(25) = 50.

Time = 0.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 6.10

$$\int \frac{\sinh^2(x)\sinh(2x)}{(1-\sinh^2(x))^{3/2}}\,dx = -\frac{16\,e^{(-x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}} + \frac{62\,e^{(-3\,x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}} - \frac{16\,e^{(-5\,x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}} + \frac{e^{(-7\,x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}} + \frac{e^{x}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}}$$

```
[In] integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="maxima")
[Out] -16*e^(-x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x) + 1)^(3/2)
) + 62*e^(-3*x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x) + 1)^
(3/2)) - 16*e^(-5*x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x)
+ 1)^(3/2)) + e^(-7*x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x
) + 1)^(3/2)) + e^x/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x) +
 1)^(3/2))
```

Giac [F]

$$\int \frac{\sinh^2(x)\sinh(2x)}{(1-\sinh^2(x))^{3/2}}\,dx = \int \frac{\sinh\left(2\,x\right)\sinh\left(x\right)^2}{\left(-\sinh\left(x\right)^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="giac")
[Out] integrate(sinh(2*x)*sinh(x)^2/(-sinh(x)^2 + 1)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx = \frac{2\sqrt{1-\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^2}\,\left(e^{4\,x}-10\,e^{2\,x}+1\right)}{e^{4\,x}-6\,e^{2\,x}+1}$$

```
[In] int((sinh(2*x)*sinh(x)^2)/(1 - sinh(x)^2)^(3/2),x)
```

```
[Out] (2*(1 - (exp(-x)/2 - exp(x)/2)^2)^(1/2)*(exp(4*x) - 10*exp(2*x) + 1))/(exp(
4*x) - 6*exp(2*x) + 1)
```

3.595 $\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = \frac{\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

```
[Out] 1/2*arcsinh(sinh(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4441, 221}

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = \frac{\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

```
[In] Int[Cosh[x]/Sqrt[Cosh[2*x]],x]
```

```
[Out] ArcSinh[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
```

```
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\sqrt{1+2x^2}}\,dx, x, \sinh(x)\right)$$
$$= \frac{\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = \frac{\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

```
[In] Integrate[Cosh[x]/Sqrt[Cosh[2*x]],x]
[Out] ArcSinh[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(12) = 24.

Time = 0.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 4.20

method	result	size
default	$\frac{\sqrt{(2(\cosh^2(x))-1)(\sinh^2(x))}\,\ln\left(\sqrt{2}\,(\sinh^2(x))+\frac{\sqrt{2}}{4}+\sqrt{2(\sinh^4(x))+\sinh^2(x)}\right)\sqrt{2}}{4\sinh(x)\sqrt{2(\cosh^2(x))-1}}$	63

```
[In] int(cosh(x)/cosh(2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*((2*cosh(x)^2-1)*sinh(x)^2)^(1/2)*ln(2^(1/2)*sinh(x)^2+1/4*2^(1/2)+(2*s
inh(x)^4+sinh(x)^2)^(1/2))*2^(1/2)/sinh(x)/(2*cosh(x)^2-1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(12) = 24.

Time = 0.26 (sec) , antiderivative size = 482, normalized size of antiderivative = 32.13

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$$
$$= \frac{1}{8}\sqrt{2}\log\left(-\frac{\cosh(x)^8 + 8\cosh(x)\sinh(x)^7 + \sinh(x)^8 + \left(28\cosh(x)^2 - 3\right)\sinh(x)^6 - 3\cosh(x)^6 + 2\right.$$
$$+\frac{1}{8}\sqrt{2}\log\left(\frac{\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + \left(6\cosh(x)^2 + 1\right)\sinh(x)^2 + \sqrt{2}\left(\cosh(x)^2 + \right.}{\cosh(x}\right.$$

```
[In] integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)
^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 +
5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x)^
5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 + 30
*cosh(x)^2 - 4)*sinh(x)^2 + sqrt(2)*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh
(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*co
sh(x))*sinh(x)^3 + (15*cosh(x)^4 - 18*cosh(x)^2 + 4)*sinh(x)^2 + 4*cosh(x)^
2 + 2*(3*cosh(x)^5 - 6*cosh(x)^3 + 4*cosh(x))*sinh(x) - 4)*sqrt((cosh(x)^2
+ sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*cosh(x)^2 + 2
*(4*cosh(x)^7 - 9*cosh(x)^5 + 10*cosh(x)^3 - 4*cosh(x))*sinh(x) + 4)/(cosh(
x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^
3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/8*sqrt(2
)*log((cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh
(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt((cosh(
x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + cosh(x)^2
+ 2*(2*cosh(x)^3 + cosh(x))*sinh(x) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2))
```

Sympy [F]

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = \int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$$

```
[In] integrate(cosh(x)/cosh(2*x)**(1/2),x)
```

```
[Out] Integral(cosh(x)/sqrt(cosh(2*x)), x)
```

Maxima [F]

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = \int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$$

```
[In] integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="maxima")
[Out] integrate(cosh(x)/sqrt(cosh(2*x)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(12) = 24$.

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.87

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = -\frac{1}{4}\sqrt{2}\left(\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)-\log\left(-\sqrt{e^{(4x)}+1}+e^{(2x)}+1\right)\right)$$

```
[In] integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="giac")
[Out] -1/4*sqrt(2)*(log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1)
- e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = \int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$$

```
[In] int(cosh(x)/cosh(2*x)^(1/2),x)
[Out] int(cosh(x)/cosh(2*x)^(1/2), x)
```

3.596 $\int x\tanh^2(x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 16

$$\int x\tanh^2(x)\,dx = \frac{x^2}{2} + \log(\cosh(x)) - x\tanh(x)$$

```
[Out] 1/2*x^2+ln(cosh(x))-x*tanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3801, 3556, 30}

$$\int x\tanh^2(x)\,dx = \frac{x^2}{2} - x\tanh(x) + \log(\cosh(x))$$

```
[In] Int[x*Tanh[x]^2,x]
```

```
[Out] x^2/2 + Log[Cosh[x]] - x*Tanh[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -x\tanh(x) + \int x\,dx + \int \tanh(x)\,dx \\ &= \frac{x^2}{2} + \log(\cosh(x)) - x\tanh(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x\tanh^2(x)\,dx = \frac{x^2}{2} + \log(\cosh(x)) - x\tanh(x)$$

```
[In] Integrate[x*Tanh[x]^2,x]
[Out] x^2/2 + Log[Cosh[x]] - x*Tanh[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

method	result	size
parallelrisch	$\frac{x^2}{2} - x\tanh(x) - x - \ln(1-\tanh(x))$	24
risch	$\frac{x^2}{2} - 2x + \frac{2x}{1+e^{2x}} + \ln(1+e^{2x})$	28

```
[In] int(x*tanh(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-x*tanh(x)-x-ln(1-tanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 5.81

$$\int x\tanh^2(x)\,dx$$
$$= \frac{(x^2-4\,x)\cosh(x)^2+2\,(x^2-4\,x)\cosh(x)\sinh(x)+(x^2-4\,x)\sinh(x)^2+x^2+2\left(\cosh(x)^2+2\,\cosh(x\right.}{2\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2+1\right)}$$

```
[In] integrate(x*tanh(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 4*x)*cosh(x)^2 + 2*(x^2 - 4*x)*cosh(x)*sinh(x) + (x^2 - 4*x)*si
nh(x)^2 + x^2 + 2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(2*cos
h(x)/(cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int x\tanh^2(x)\,dx = \frac{x^2}{2} - x\tanh(x) + x - \log(\tanh(x)+1)$$

```
[In] integrate(x*tanh(x)**2,x)
```

```
[Out] x**2/2 - x*tanh(x) + x - log(tanh(x) + 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(14) = 28$.

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int x\tanh^2(x)\,dx = -\frac{xe^{(2\,x)}}{e^{(2\,x)}+1} + \frac{x^2+(x^2-2\,x)e^{(2\,x)}}{2\,(e^{(2\,x)}+1)} + \log\left(e^{(2\,x)}+1\right)$$

```
[In] integrate(x*tanh(x)^2,x, algorithm="maxima")
```

```
[Out] -x*e^(2*x)/(e^(2*x) + 1) + 1/2*(x^2 + (x^2 - 2*x)*e^(2*x))/(e^(2*x) + 1) +
log(e^(2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(14) = 28$.

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int x\tanh^2(x)\,dx = \frac{x^2e^{(2x)} + x^2 - 4\,xe^{(2x)} + 2\,e^{(2x)}\log\left(e^{(2x)} + 1\right) + 2\,\log\left(e^{(2x)} + 1\right)}{2\left(e^{(2x)} + 1\right)}$$

```
[In] integrate(x*tanh(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e^(2*x) + x^2 - 4*x*e^(2*x) + 2*e^(2*x)*log(e^(2*x) + 1) + 2*log(e
^(2*x) + 1))/(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x\tanh^2(x)\,dx = \ln\left(e^{2x} + 1\right) - x - x\tanh(x) + \frac{x^2}{2}$$

```
[In] int(x*tanh(x)^2,x)
```

```
[Out] log(exp(2*x) + 1) - x - x*tanh(x) + x^2/2
```

3.597 $\int x \coth^2(x) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 16

$$\int x \coth^2(x) \, dx = \frac{x^2}{2} - x \coth(x) + \log(\sinh(x))$$

```
[Out] 1/2*x^2-x*coth(x)+ln(sinh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3801, 3556, 30}

$$\int x \coth^2(x) \, dx = \frac{x^2}{2} - x \coth(x) + \log(\sinh(x))$$

```
[In] Int[x*Coth[x]^2,x]
```

```
[Out] x^2/2 - x*Coth[x] + Log[Sinh[x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -x\coth(x)+\int x\,dx+\int \coth(x)\,dx\\ &= \frac{x^2}{2}-x\coth(x)+\log(\sinh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x\coth^2(x)\,dx=\frac{x^2}{2}-x\coth(x)+\log(\sinh(x))$$

```
[In] Integrate[x*Coth[x]^2,x]
[Out] x^2/2 - x*Coth[x] + Log[Sinh[x]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

method	result	size
risch	$\frac{x^2}{2}-2x-\frac{2x}{e^{2x}-1}+\ln\left(e^{2x}-1\right)$	28
parallelrisch	$\frac{-2\ln(1-\tanh(x))\tanh(x)+2\ln(\tanh(x))\tanh(x)+x(-2+(-2+x)\tanh(x))}{2\tanh(x)}$	36

```
[In] int(x*coth(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-2*x-2*x/(exp(2*x)-1)+ln(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 5.94

$$\int x\coth^2(x)\,dx$$
$$= \frac{(x^2-4\,x)\cosh(x)^2+2\,(x^2-4\,x)\cosh(x)\sinh(x)+(x^2-4\,x)\sinh(x)^2-x^2+2\,\left(\cosh(x)^2+2\,\cosh(x\right.}{2\,\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2-1\right)}$$

```
[In] integrate(x*coth(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 4*x)*cosh(x)^2 + 2*(x^2 - 4*x)*cosh(x)*sinh(x) + (x^2 - 4*x)*si
nh(x)^2 - x^2 + 2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(2*sin
h(x)/(cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int x\coth^2(x)\,dx = \frac{x^2}{2}+x-\frac{x}{\tanh(x)}-\log(\tanh(x)+1)+\log(\tanh(x))$$

```
[In] integrate(x*coth(x)**2,x)
```

```
[Out] x**2/2 + x - x/tanh(x) - log(tanh(x) + 1) + log(tanh(x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

$$\int x\coth^2(x)\,dx = -\frac{xe^{(2\,x)}}{e^{(2\,x)}-1}-\frac{x^2-(x^2-2\,x)e^{(2\,x)}}{2\,(e^{(2\,x)}-1)}+\log(e^x+1)+\log(e^x-1)$$

```
[In] integrate(x*coth(x)^2,x, algorithm="maxima")
```

```
[Out] -x*e^(2*x)/(e^(2*x) - 1) - 1/2*(x^2 - (x^2 - 2*x)*e^(2*x))/(e^(2*x) - 1) +
log(e^x + 1) + log(e^x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

$$\int x\coth^2(x)\,dx = \frac{x^2e^{(2\,x)} - x^2 - 4\,xe^{(2\,x)} + 2\,e^{(2\,x)}\log\left(e^{(2\,x)} - 1\right) - 2\,\log\left(e^{(2\,x)} - 1\right)}{2\left(e^{(2\,x)} - 1\right)}$$

```
[In] integrate(x*coth(x)^2,x, algorithm="giac")
[Out] 1/2*(x^2*e^(2*x) - x^2 - 4*x*e^(2*x) + 2*e^(2*x)*log(e^(2*x) - 1) - 2*log(e
^(2*x) - 1))/(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int x\coth^2(x)\,dx = \ln\left(e^{2\,x} - 1\right) - 2\,x - \frac{2\,x}{e^{2\,x} - 1} + \frac{x^2}{2}$$

```
[In] int(x*coth(x)^2,x)
[Out] log(exp(2*x) - 1) - 2*x - (2*x)/(exp(2*x) - 1) + x^2/2
```

3.598 $\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 20

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = -e^x + \frac{e^{2x}}{2} + e^x x$$

```
[Out] -exp(x)+1/2*exp(2*x)+exp(x)*x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5767, 6874, 2207, 2225, 2320, 12, 14}

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = e^x x - e^x + \frac{e^{2x}}{2}$$

```
[In] Int[(x + Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

```
[Out] -E^x + E^(2*x)/2 + E^x*x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5767

```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a/b)*v))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int e^x(x+\cosh(x)+\sinh(x))\,dx \\
&= \int \left(e^x x+e^x\cosh(x)+e^x\sinh(x)\right)\,dx \\
&= \int e^x x\,dx+\int e^x\cosh(x)\,dx+\int e^x\sinh(x)\,dx \\
&= e^x x-\int e^x\,dx+\text{Subst}\left(\int \frac{-1+x^2}{2x}\,dx,x,e^x\right)+\text{Subst}\left(\int \frac{1+x^2}{2x}\,dx,x,e^x\right) \\
&= -e^x+e^x x+\frac{1}{2}\text{Subst}\left(\int \frac{-1+x^2}{x}\,dx,x,e^x\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{x}\,dx,x,e^x\right)
\end{aligned}
$$

$$= -e^x + e^x x + \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x} + x\right) dx, x, e^x\right) + \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x} + x\right) dx, x, e^x\right)$$

$$= -e^x + \frac{e^{2x}}{2} + e^x x$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{x + \cosh(x) + \sinh(x)}{\cosh(x) - \sinh(x)} dx = \frac{1}{2}\cosh(2x) + (-1 + x)\sinh(x) + \cosh(x)(-1 + x + \sinh(x))$$

```
[In] Integrate[(x + Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
[Out] Cosh[2*x]/2 + (-1 + x)*Sinh[x] + Cosh[x]*(-1 + x + Sinh[x])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

method	result	size
risch	$(-1 + x)\,\mathrm{e}^{x} + \frac{\mathrm{e}^{2x}}{2}$	14
default	$-\cosh(x) + x\sinh(x) + x\cosh(x) - \sinh(x) + \cosh(x)\sinh(x) + \cosh^2(x)$	27

```
[In] int((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x,method=_RETURNVERBOSE)
[Out] (-1+x)*exp(x)+1/2*exp(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x + \cosh(x) + \sinh(x)}{\cosh(x) - \sinh(x)} dx = \frac{2x + \cosh(x) + \sinh(x) - 2}{2(\cosh(x) - \sinh(x))}$$

```
[In] integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="fricas")
[Out] 1/2*(2*x + cosh(x) + sinh(x) - 2)/(cosh(x) - sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = \frac{x}{-\sinh{(x)}+\cosh{(x)}}+\frac{\sinh{(x)}}{-\sinh{(x)}+\cosh{(x)}} - \frac{1}{-\sinh{(x)}+\cosh{(x)}}$$

```
[In] integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)
[Out] x/(-sinh(x) + cosh(x)) + sinh(x)/(-sinh(x) + cosh(x)) - 1/(-sinh(x) + cosh(
x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = (x-1)e^{x}+\frac{1}{2}\,e^{(2\,x)}$$

```
[In] integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="maxima")
[Out] (x - 1)*e^x + 1/2*e^(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.55

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = \frac{1}{2}\,(2\,x+e^{x}-2)e^{x}$$

```
[In] integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="giac")
[Out] 1/2*(2*x + e^x - 2)*e^x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = e^x \left(x+\frac{e^{-x}}{2}+\frac{e^x}{2}-1\right)$$

```
[In] int((x + cosh(x) + sinh(x))/(cosh(x) - sinh(x)),x)
[Out] exp(x)*(x + exp(-x)/2 + exp(x)/2 - 1)
```

3.599 $\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)} dx = x-(1-x)\tanh\left(\frac{x}{2}\right)$$

```
[Out] x-(1-x)*tanh(1/2*x)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6874, 3399, 4269, 3556}

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)} dx = x-(1-x)\tanh\left(\frac{x}{2}\right)$$

```
[In] Int[(x + Cosh[x] + Sinh[x])/(1 + Cosh[x]),x]
[Out] x - (1 - x)*Tanh[x/2]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x+\cosh(x)}{1+\cosh(x)}+\tanh\left(\frac{x}{2}\right)\right)\,dx\\
&= \int \frac{x+\cosh(x)}{1+\cosh(x)}\,dx+\int \tanh\left(\frac{x}{2}\right)\,dx\\
&= 2\log\left(\cosh\left(\frac{x}{2}\right)\right)+\int \left(1+\frac{-1+x}{1+\cosh(x)}\right)\,dx\\
&= x+2\log\left(\cosh\left(\frac{x}{2}\right)\right)+\int \frac{-1+x}{1+\cosh(x)}\,dx\\
&= x+2\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{1}{2}\int (-1+x)\text{sech}^2\left(\frac{x}{2}\right)\,dx\\
&= x+2\log\left(\cosh\left(\frac{x}{2}\right)\right)-(1-x)\tanh\left(\frac{x}{2}\right)-\int \tanh\left(\frac{x}{2}\right)\,dx\\
&= x-(1-x)\tanh\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx=\frac{\left(-1+x+x\coth\left(\frac{x}{2}\right)\right)\sinh(x)}{1+\cosh(x)}$$

```
[In] Integrate[(x + Cosh[x] + Sinh[x])/(1 + Cosh[x]),x]
[Out] ((-1 + x + x*Coth[x/2])*Sinh[x])/(1 + Cosh[x])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
risch	$2x-\frac{2(-1+x)}{1+e^{x}}$	16

```
[In] int((x+cosh(x)+sinh(x))/(cosh(x)+1),x,method=_RETURNVERBOSE)
[Out] 2*x-2*(-1+x)/(1+exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx = \frac{2\left(x\cosh\left(x\right)+x\sinh\left(x\right)+1\right)}{\cosh\left(x\right)+\sinh\left(x\right)+1}$$

```
[In] integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="fricas")
[Out] 2*(x*cosh(x) + x*sinh(x) + 1)/(cosh(x) + sinh(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx = x\tanh\left(\frac{x}{2}\right)+x-\tanh\left(\frac{x}{2}\right)$$

```
[In] integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x)
[Out] x*tanh(x/2) + x - tanh(x/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(10) = 20.

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx = x+\frac{2\,xe^{x}}{e^{x}+1}-\frac{2}{e^{(-x)}+1}+\log\left(\cosh\left(x\right)+1\right)-2\,\log\left(e^{x}+1\right)$$

```
[In] integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="maxima")
[Out] x + 2*x*e^x/(e^x + 1) - 2/(e^(-x) + 1) + log(cosh(x) + 1) - 2*log(e^x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx = \frac{2\,(xe^x+1)}{e^x+1}$$

```
[In] integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="giac")
[Out] 2*(x*e^x + 1)/(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx = 2\,x-\frac{2\,x-2}{\mathrm{e}^x+1}$$

```
[In] int((x + cosh(x) + sinh(x))/(cosh(x) + 1),x)
[Out] 2*x - (2*x - 2)/(exp(x) + 1)
```

3.600 $\int e^{2x}\operatorname{csch}^4(x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 20

$$\int e^{2x}\operatorname{csch}^4(x)\,dx=\frac{8e^{6x}}{3\,(1-e^{2x})^3}$$

```
[Out] 8/3*exp(6*x)/(1-exp(2*x))^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2320, 12, 270}

$$\int e^{2x}\operatorname{csch}^4(x)\,dx=\frac{8e^{6x}}{3\,(1-e^{2x})^3}$$

```
[In] Int[E^(2*x)*Csch[x]^4,x]
```

```
[Out] (8*E^(6*x))/(3*(1 - E^(2*x))^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{16x^5}{(1-x^2)^4}\,dx, x, e^x\right)\\ &= 16\text{Subst}\left(\int \frac{x^5}{(1-x^2)^4}\,dx, x, e^x\right)\\ &= \frac{8e^{6x}}{3(1-e^{2x})^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int e^{2x}\text{csch}^4(x)\,dx = \frac{8e^{6x}}{3(1-e^{2x})^3}$$

```
[In] Integrate[E^(2*x)*Csch[x]^4,x]
[Out] (8*E^(6*x))/(3*(1 - E^(2*x))^3)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
parallelrisch	$-\frac{e^{2x}\,\text{csch}(x)^2(\coth(x)+1)}{3}$	15
default	$-\frac{1}{3\tanh(x)^3}-\frac{1}{\tanh(x)^2}-\frac{1}{\tanh(x)}$	20
risch	$-\frac{8(3\,e^{4x}-3\,e^{2x}+1)}{3(e^{2x}-1)^3}$	25

```
[In] int(exp(2*x)/sinh(x)^4,x,method=_RETURNVERBOSE)
[Out] -1/3*exp(2*x)*csch(x)^2*(coth(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.75

$$\int e^{2x}\operatorname{csch}^4(x)\,dx =$$
$$-\frac{8\left(4\cosh(x)^2+4\cosh(x)\sinh(x)+4\sinh(x)^2-3\right)}{3\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2-2\right)\sinh(x)^2-4\cosh(x)^2+4\left(\cosh(x)\right.\right.}$$

```
[In] integrate(exp(2*x)/sinh(x)^4,x, algorithm="fricas")
[Out] -8/3*(4*cosh(x)^2 + 4*cosh(x)*sinh(x) + 4*sinh(x)^2 - 3)/(cosh(x)^4 + 4*cos
h(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 2)*sinh(x)^2 - 4*cosh(x)^2 +
4*(cosh(x)^3 - cosh(x))*sinh(x) + 3)
```

Sympy [F]

$$\int e^{2x}\operatorname{csch}^4(x)\,dx = \int \frac{e^{2x}}{\sinh^4(x)}\,dx$$

```
[In] integrate(exp(2*x)/sinh(x)**4,x)
[Out] Integral(exp(2*x)/sinh(x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int e^{2x}\operatorname{csch}^4(x)\,dx = \frac{8}{3\left(3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1\right)}$$

```
[In] integrate(exp(2*x)/sinh(x)^4,x, algorithm="maxima")
[Out] 8/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int e^{2x}\text{csch}^4(x)\,dx = -\frac{8\left(3\,e^{(4\,x)} - 3\,e^{(2\,x)} + 1\right)}{3\left(e^{(2\,x)} - 1\right)^3}$$

```
[In] integrate(exp(2*x)/sinh(x)^4,x, algorithm="giac")
[Out] -8/3*(3*e^(4*x) - 3*e^(2*x) + 1)/(e^(2*x) - 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int e^{2x}\text{csch}^4(x)\,dx = -\frac{8\left(3\,\mathrm{e}^{4\,x} - 3\,\mathrm{e}^{2\,x} + 1\right)}{3\left(\mathrm{e}^{2\,x} - 1\right)^3}$$

```
[In] int(exp(2*x)/sinh(x)^4,x)
[Out] -(8*(3*exp(4*x) - 3*exp(2*x) + 1))/(3*(exp(2*x) - 1)^3)
```

3.601 $\int e^{-2x}\mathbf{sech}^4(x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 13

$$\int e^{-2x}\text{sech}^4(x)\,dx=-\frac{8}{3\left(1+e^{2x}\right)^3}$$

```
[Out] -8/3/(1+exp(2*x))^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2320, 12, 267}

$$\int e^{-2x}\text{sech}^4(x)\,dx=-\frac{8}{3\left(e^{2x}+1\right)^3}$$

```
[In] Int[Sech[x]^4/E^(2*x),x]
```

```
[Out] -8/(3*(1 + E^(2*x))^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{16x}{(1+x^2)^4}\,dx, x, e^x\right)\\ &= 16\text{Subst}\left(\int \frac{x}{(1+x^2)^4}\,dx, x, e^x\right)\\ &= -\frac{8}{3(1+e^{2x})^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{-2x}\text{sech}^4(x)\,dx = -\frac{8}{3(1+e^{2x})^3}$$

```
[In] Integrate[Sech[x]^4/E^(2*x),x]
[Out] -8/(3*(1 + E^(2*x))^3)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{(-1+\tanh(x))^3}{3}$	9
risch	$-\frac{8}{3(1+e^{2x})^3}$	11
parallelrisch	$\frac{\text{sech}(x)^2(-1+\tanh(x))e^{-2x}}{3}$	15

```
[In] int(1/exp(2*x)/cosh(x)^4,x,method=_RETURNVERBOSE)
[Out] 1/3*(-1+tanh(x))^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 7.85

$$\int e^{-2x}\text{sech}^4(x)\,dx =$$

$$-\frac{}{3\left(\cosh(x)^6+6\cosh(x)\sinh(x)^5+\sinh(x)^6+3\left(5\cosh(x)^2+1\right)\sinh(x)^4+3\cosh(x)^4+4\left(5\cosh(x\right.\right.}$$

```
[In] integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="fricas")
```

```
[Out] -8/3/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sin
h(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)
^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3
+ cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int e^{-2x}\text{sech}^4(x)\,dx = \int \frac{e^{-2x}}{\cosh^4(x)}\,dx$$

```
[In] integrate(1/exp(2*x)/cosh(x)**4,x)
```

```
[Out] Integral(exp(-2*x)/cosh(x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(10) = 20$.

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 5.77

$$\int e^{-2x}\text{sech}^4(x)\,dx = \frac{8\,e^{(-2\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1} + \frac{8\,e^{(-4\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1} + \frac{8}{3\,(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1)}$$

```
[In] integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="maxima")
```

```
[Out] 8*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 8*e^(-4*x)/(3*e^(-2*x
) + 3*e^(-4*x) + e^(-6*x) + 1) + 8/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) +
1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{-2x}\text{sech}^4(x)\,dx = -\frac{8}{3\,{\left(e^{\left(2\,x\right)} + 1\right)}^{3}}$$

```
[In] integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="giac")
[Out] -8/3/(e^(2*x) + 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int e^{-2x}\text{sech}^4(x)\,dx = -\frac{\mathrm{e}^{-3\,x}}{3\,{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)}^{3}}$$

```
[In] int(exp(-2*x)/cosh(x)^4,x)
[Out] -exp(-3*x)/(3*(exp(-x)/2 + exp(x)/2)^3)
```

3.602 $\int \frac{e^x}{\cosh(x)-\sinh(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 9

$$\int \frac{e^x}{\cosh(x)-\sinh(x)} \, dx = \frac{e^{2x}}{2}$$

```
[Out] 1/2*exp(2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2320, 30}

$$\int \frac{e^x}{\cosh(x)-\sinh(x)} \, dx = \frac{e^{2x}}{2}$$

```
[In] Int[E^x/(Cosh[x] - Sinh[x]),x]
```

```
[Out] E^(2*x)/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int x\,dx, x, e^x\right)$$
$$= \frac{e^{2x}}{2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx = \frac{e^{2x}}{2}$$

```
[In] Integrate[E^x/(Cosh[x] - Sinh[x]),x]
[Out] E^(2*x)/2
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{e^{2x}}{2}$	7
gosper	$\frac{e^{x}}{2\cosh(x)-2\sinh(x)}$	14
default	$\frac{2}{\tanh(\frac{x}{2})-1}+\frac{2}{\left(\tanh(\frac{x}{2})-1\right)^{2}}$	22

```
[In] int(exp(x)/(cosh(x)-sinh(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.78

$$\int \frac{e^x}{\cosh(x) - \sinh(x)}\, dx = \frac{\cosh(x) + \sinh(x)}{2(\cosh(x) - \sinh(x))}$$

```
[In] integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="fricas")
[Out] 1/2*(cosh(x) + sinh(x))/(cosh(x) - sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(5) = 10.

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{e^x}{\cosh(x) - \sinh(x)}\, dx = \frac{e^x}{-2\sinh(x) + 2\cosh(x)}$$

```
[In] integrate(exp(x)/(cosh(x)-sinh(x)),x)
[Out] exp(x)/(-2*sinh(x) + 2*cosh(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^x}{\cosh(x) - \sinh(x)}\, dx = \frac{1}{2}\, e^{(2x)}$$

```
[In] integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="maxima")
[Out] 1/2*e^(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^x}{\cosh(x) - \sinh(x)}\, dx = \frac{1}{2}\, e^{(2\,x)}$$

```
[In] integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="giac")
[Out] 1/2*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^x}{\cosh(x) - \sinh(x)}\, dx = \frac{\mathrm{e}^{2\,x}}{2}$$

```
[In] int(exp(x)/(cosh(x) - sinh(x)),x)
[Out] exp(2*x)/2
```

3.603 $\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx = \frac{e^{(-1+m)x}}{-1+m}$$

```
[Out] exp((-1+m)*x)/(-1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5767, 2259, 2225}

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx = -\frac{e^{-((1-m)x)}}{1-m}$$

```
[In] Int[E^(m*x)/(Cosh[x] + Sinh[x]),x]
[Out] -(1/(E^((1 - m)*x)*(1 - m)))
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2259

```
Int[(u_.)*(F_)^((a_.) + (b_.)*(v_)), x_Symbol] :> Int[u*F^(a + b*NormalizeP
owerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && Powe
rOfLinearQ[v, x] &&  !PowerOfLinearMatchQ[v, x]
```

Rule 5767

```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a/b)*v))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int e^{-x+mx}\,dx\\ &= \int e^{-((1-m)x)}\,dx\\ &= -\frac{e^{-((1-m)x)}}{1-m}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx = \frac{e^{mx}(\cosh(x)-\sinh(x))}{-1+m}$$

```
[In] Integrate[E^(m*x)/(Cosh[x] + Sinh[x]),x]
[Out] (E^(m*x)*(Cosh[x] - Sinh[x]))/(-1 + m)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{\mathrm{e}^{(-1+m)x}}{-1+m}$	13
gosper	$\frac{\mathrm{e}^{mx}}{(-1+m)(\cosh(x)+\sinh(x))}$	18
default	$\frac{\sinh((-1+m)x)}{-1+m}+\frac{\cosh((-1+m)x)}{-1+m}$	26

```
[In] int(exp(m*x)/(cosh(x)+sinh(x)),x,method=_RETURNVERBOSE)
[Out] exp((-1+m)*x)/(-1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{e^{mx}}{\cosh(x) + \sinh(x)} \, dx = \frac{\cosh(mx) + \sinh(mx)}{(m-1)\cosh(x) + (m-1)\sinh(x)}$$

```
[In] integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="fricas")
[Out] (cosh(m*x) + sinh(m*x))/((m - 1)*cosh(x) + (m - 1)*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.46

$$\int \frac{e^{mx}}{\cosh(x) + \sinh(x)} \, dx = \begin{cases} \frac{e^{mx}}{m\sinh(x) + m\cosh(x) - \sinh(x) - \cosh(x)} & \text{for } m \neq 1 \\ \frac{xe^{x}}{\sinh(x) + \cosh(x)} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(m*x)/(cosh(x)+sinh(x)),x)
[Out] Piecewise((exp(m*x)/(m*sinh(x) + m*cosh(x) - sinh(x) - cosh(x)), Ne(m, 1)),
 (x*exp(x)/(sinh(x) + cosh(x)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{mx}}{\cosh(x) + \sinh(x)} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-m>0)', see `assume?` for more deta
ils)Is
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{e^{mx}}{\cosh(x) + \sinh(x)}\, dx = \frac{e^{(mx)}}{me^x - e^x}$$

```
[In] integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="giac")
[Out] e^(m*x)/(m*e^x - e^x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{e^{mx}}{\cosh(x) + \sinh(x)}\, dx = \frac{\mathrm{e}^{m\,x-x}}{m-1}$$

```
[In] int(exp(m*x)/(cosh(x) + sinh(x)),x)
[Out] exp(m*x - x)/(m - 1)
```

3.604 $\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 1

$$\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx = x$$

```
[Out] x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2320, 29}

$$\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx = x$$

```
[In] Int[E^x/(Cosh[x] + Sinh[x]),x]
```

```
[Out] x
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, e^x\right)\\ &= x\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx = x$$

```
[In] Integrate[E^x/(Cosh[x] + Sinh[x]),x]
[Out] x
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
default	x	2

```
[In] int(exp(x)/(cosh(x)+sinh(x)),x,method=_RETURNVERBOSE)
[Out] x
```

Fricas [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx = x$$

```
[In] integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="fricas")
[Out] x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(0) = 0$.

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 10.00

$$\int \frac{e^x}{\cosh(x) + \sinh(x)}\, dx = \frac{xe^x}{\sinh{(x)} + \cosh{(x)}}$$

```
[In] integrate(exp(x)/(cosh(x)+sinh(x)),x)
[Out] x*exp(x)/(sinh(x) + cosh(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x) + \sinh(x)}\, dx = x$$

```
[In] integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="maxima")
[Out] x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x) + \sinh(x)}\, dx = x$$

```
[In] integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="giac")
[Out] x
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x) + \sinh(x)} \, dx = x$$

```
[In] int(exp(x)/(cosh(x) + sinh(x)),x)
[Out] x
```

3.605 $\int \frac{e^x}{1-\cosh(x)} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{e^x}{1-\cosh(x)} \, dx = -\frac{2}{1-e^x} - 2\log\left(1-e^x\right)$$

```
[Out] -2/(1-exp(x))-2*ln(1-exp(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2320, 12, 45}

$$\int \frac{e^x}{1-\cosh(x)} \, dx = -\frac{2}{1-e^x} - 2\log\left(1-e^x\right)$$

```
[In] Int[E^x/(1 - Cosh[x]),x]
[Out] -2/(1 - E^x) - 2*Log[1 - E^x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int -\frac{2x}{(1-x)^2}\,dx, x, e^x\right) \\
&= -\left(2\text{Subst}\left(\int \frac{x}{(1-x)^2}\,dx, x, e^x\right)\right) \\
&= -\left(2\text{Subst}\left(\int \left(\frac{1}{(-1+x)^2}+\frac{1}{-1+x}\right)\,dx, x, e^x\right)\right) \\
&= -\frac{2}{1-e^x}-2\log\left(1-e^x\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{e^x}{1-\cosh(x)}\,dx = \frac{4\left(\frac{1}{1-e^x}+\log\left(1-e^x\right)\right)\sinh^2\left(\frac{x}{2}\right)}{1-\cosh(x)}$$

```
[In] Integrate[E^x/(1 - Cosh[x]),x]
[Out] (4*((1 - E^x)^(-1) + Log[1 - E^x])*Sinh[x/2]^2)/(1 - Cosh[x])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{2}{-1+e^x}-2\ln\left(-1+e^x\right)$	17
default	$2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)}-2\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	24

```
[In] int(exp(x)/(1-cosh(x)),x,method=_RETURNVERBOSE)
[Out] 2/(-1+exp(x))-2*ln(-1+exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{e^x}{1-\cosh(x)}\,dx = -\frac{2\,((\cosh(x)+\sinh(x)-1)\log(\cosh(x)+\sinh(x)-1)-1)}{\cosh(x)+\sinh(x)-1}$$

```
[In] integrate(exp(x)/(1-cosh(x)),x, algorithm="fricas")
[Out] -2*((cosh(x) + sinh(x) - 1)*log(cosh(x) + sinh(x) - 1) - 1)/(cosh(x) + sinh
(x) - 1)
```

Sympy [F]

$$\int \frac{e^x}{1-\cosh(x)}\,dx = -\int \frac{e^x}{\cosh(x)-1}\,dx$$

```
[In] integrate(exp(x)/(1-cosh(x)),x)
[Out] -Integral(exp(x)/(cosh(x) - 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{e^x}{1-\cosh(x)}\,dx = \frac{2}{e^x-1} - 2\,\log(e^x-1)$$

```
[In] integrate(exp(x)/(1-cosh(x)),x, algorithm="maxima")
[Out] 2/(e^x - 1) - 2*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{e^x}{1-\cosh(x)}\,dx = \frac{2}{e^x-1} - 2\,\log(|e^x-1|)$$

```
[In] integrate(exp(x)/(1-cosh(x)),x, algorithm="giac")
[Out] 2/(e^x - 1) - 2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{e^x}{1-\cosh(x)}\,dx = \frac{2}{\mathrm{e}^x-1}-2\,\ln\left(\mathrm{e}^x-1\right)$$

```
[In] int(-exp(x)/(cosh(x) - 1),x)
[Out] 2/(exp(x) - 1) - 2*log(exp(x) - 1)
```

3.606 $\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)} dx$

Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)} dx = e^x + \frac{2}{1+e^x}$$

```
[Out] exp(x)+2/(1+exp(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2320, 697}

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)} dx = e^x + \frac{2}{e^x+1}$$

```
[In] Int[(E^x*(1 + Sinh[x]))/(1 + Cosh[x]),x]
```

```
[Out] E^x + 2/(1 + E^x)
```

Rule 697

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{-1+2x+x^2}{(1+x)^2}\,dx, x, e^x\right) \\ &= \text{Subst}\left(\int \left(1-\frac{2}{(1+x)^2}\right)\,dx, x, e^x\right) \\ &= e^x+\frac{2}{1+e^x} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = \frac{2+e^x+e^{2x}}{1+e^x}$$

```
[In] Integrate[(E^x*(1 + Sinh[x]))/(1 + Cosh[x]),x]
[Out] (2 + E^x + E^(2*x))/(1 + E^x)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
risch	$e^x+\frac{2}{1+e^x}$	12
default	$-\tanh\left(\frac{x}{2}\right)-\frac{2}{\tanh\left(\frac{x}{2}\right)-1}$	18

```
[In] int(exp(x)*(1+sinh(x))/(cosh(x)+1),x,method=_RETURNVERBOSE)
[Out] exp(x)+2/(1+exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = \frac{3\,\cosh\left(x\right) - \sinh\left(x\right) + 1}{\cosh\left(x\right) - \sinh\left(x\right) + 1}$$

```
[In] integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="fricas")
[Out] (3*cosh(x) - sinh(x) + 1)/(cosh(x) - sinh(x) + 1)
```

Sympy [F]

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = \int \frac{\left(\sinh\left(x\right)+1\right)e^{x}}{\cosh\left(x\right)+1}\,dx$$

```
[In] integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x)
[Out] Integral((sinh(x) + 1)*exp(x)/(cosh(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = \frac{2}{e^{x}+1} + e^{x}$$

```
[In] integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="maxima")
[Out] 2/(e^x + 1) + e^x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = \frac{2}{e^{x}+1} + e^{x}$$

```
[In] integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="giac")
[Out] 2/(e^x + 1) + e^x
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = e^x + \frac{2}{e^x+1}$$

```
[In] int((exp(x)*(sinh(x) + 1))/(cosh(x) + 1),x)
[Out] exp(x) + 2/(exp(x) + 1)
```

3.607 $\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = e^x - \frac{2}{1-e^x}$$

```
[Out] exp(x)-2/(1-exp(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2320, 697}

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = e^x - \frac{2}{1-e^x}$$

```
[In] Int[(E^x*(1 - Sinh[x]))/(1 - Cosh[x]),x]
```

```
[Out] E^x - 2/(1 - E^x)
```

Rule 697

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{-1-2x+x^2}{(1-x)^2}\,dx, x, e^x\right)\\ &= \text{Subst}\left(\int \left(1-\frac{2}{(-1+x)^2}\right)\,dx, x, e^x\right)\\ &= e^x-\frac{2}{1-e^x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = \frac{2-e^x+e^{2x}}{-1+e^x}$$

```
[In] Integrate[(E^x*(1 - Sinh[x]))/(1 - Cosh[x]),x]
[Out] (2 - E^x + E^(2*x))/(-1 + E^x)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
risch	$\mathrm{e}^{x}+\frac{2}{-1+\mathrm{e}^{x}}$	12
default	$-\frac{2}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{\tanh\left(\frac{x}{2}\right)}$	18

```
[In] int(exp(x)*(1-sinh(x))/(1-cosh(x)),x,method=_RETURNVERBOSE)
[Out] exp(x)+2/(-1+exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = -\frac{3\,\cosh(x)-\sinh(x)-1}{\cosh(x)-\sinh(x)-1}$$

```
[In] integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="fricas")
[Out] -(3*cosh(x) - sinh(x) - 1)/(cosh(x) - sinh(x) - 1)
```

Sympy [F]

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = \int \frac{(\sinh(x)-1)\,e^x}{\cosh(x)-1}\,dx$$

```
[In] integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x)
[Out] Integral((sinh(x) - 1)*exp(x)/(cosh(x) - 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = \frac{2}{e^x-1}+e^x$$

```
[In] integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="maxima")
[Out] 2/(e^x - 1) + e^x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = \frac{2}{e^x-1}+e^x$$

```
[In] integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="giac")
[Out] 2/(e^x - 1) + e^x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = e^x + \frac{2}{e^x-1}$$

```
[In] int((exp(x)*(sinh(x) - 1))/(cosh(x) - 1),x)
[Out] exp(x) + 2/(exp(x) - 1)
```

3.608 $\int x^m \log(x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 26

$$\int x^m \log(x)\,dx = -\frac{x^{1+m}}{(1+m)^2} + \frac{x^{1+m}\log(x)}{1+m}$$

```
[Out] -x^(1+m)/(1+m)^2+x^(1+m)*ln(x)/(1+m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2341}

$$\int x^m \log(x)\,dx = \frac{x^{m+1}\log(x)}{m+1} - \frac{x^{m+1}}{(m+1)^2}$$

```
[In] Int[x^m*Log[x],x]
```

```
[Out] -(x^(1 + m)/(1 + m)^2) + (x^(1 + m)*Log[x])/(1 + m)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{x^{1+m}}{(1+m)^2} + \frac{x^{1+m}\log(x)}{1+m}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int x^m \log(x)\,dx = \frac{x^{1+m}(-1+(1+m)\log(x))}{(1+m)^2}$$

```
[In] Integrate[x^m*Log[x],x]
[Out] (x^(1 + m)*(-1 + (1 + m)*Log[x]))/(1 + m)^2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{x(m\ln(x)+\ln(x)-1)x^m}{(1+m)^2}$	19
norman	$\frac{x\ln(x)e^{m\ln(x)}}{1+m}-\frac{x\,e^{m\ln(x)}}{m^2+2m+1}$	34
parallelrisch	$\frac{x\,x^m\ln(x)m+x^m\ln(x)x-x\,x^m}{m^2+2m+1}$	34

```
[In] int(x^m*ln(x),x,method=_RETURNVERBOSE)
[Out] x*(m*ln(x)+ln(x)-1)/(1+m)^2*x^m
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int x^m \log(x)\,dx = \frac{((m+1)x\log(x)-x)x^m}{m^2+2m+1}$$

```
[In] integrate(x^m*log(x),x, algorithm="fricas")
[Out] ((m + 1)*x*log(x) - x)*x^m/(m^2 + 2*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(20) = 40$.

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.15

$$\int x^m \log(x)\,dx = \begin{cases} \frac{mxx^m \log(x)}{m^2+2m+1} + \frac{xx^m \log(x)}{m^2+2m+1} - \frac{xx^m}{m^2+2m+1} & \text{for } m \neq -1 \\ \frac{\log(x)^2}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**m*ln(x),x)
[Out] Piecewise((m*x*x**m*log(x)/(m**2 + 2*m + 1) + x*x**m*log(x)/(m**2 + 2*m + 1
) - x*x**m/(m**2 + 2*m + 1), Ne(m, -1)), (log(x)**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^m \log(x)\,dx = \frac{x^{m+1}\log(x)}{m+1} - \frac{x^{m+1}}{(m+1)^2}$$

```
[In] integrate(x^m*log(x),x, algorithm="maxima")
[Out] x^(m + 1)*log(x)/(m + 1) - x^(m + 1)/(m + 1)^2
```

Giac [F]

$$\int x^m \log(x)\,dx = \int x^m \log(x)\,dx$$

```
[In] integrate(x^m*log(x),x, algorithm="giac")
[Out] integrate(x^m*log(x), x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int x^m \log(x)\,dx = \begin{cases} \frac{\ln(x)^2}{2} & \text{if } m = -1 \\ \frac{x^{m+1}\,(\ln(x)\,(m+1)-1)}{(m+1)^2} & \text{if } m \neq -1 \end{cases}$$

```
[In] int(x^m*log(x),x)
```

```
[Out] piecewise(m == -1, log(x)^2/2, m ~= -1, (x^(m + 1)*(log(x)*(m + 1) - 1))/(m
 + 1)^2)
```

3.609 $\int x^m \log^2(x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 42

$$\int x^m \log^2(x) \, dx = \frac{2x^{1+m}}{(1+m)^3} - \frac{2x^{1+m}\log(x)}{(1+m)^2} + \frac{x^{1+m}\log^2(x)}{1+m}$$

```
[Out] 2*x^(1+m)/(1+m)^3-2*x^(1+m)*ln(x)/(1+m)^2+x^(1+m)*ln(x)^2/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2342, 2341}

$$\int x^m \log^2(x) \, dx = \frac{2x^{m+1}}{(m+1)^3} + \frac{x^{m+1}\log^2(x)}{m+1} - \frac{2x^{m+1}\log(x)}{(m+1)^2}$$

```
[In] Int[x^m*Log[x]^2,x]
```

```
[Out] (2*x^(1 + m))/(1 + m)^3 - (2*x^(1 + m)*Log[x])/(1 + m)^2 + (x^(1 + m)*Log[x
]^2)/(1 + m)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{x^{1+m}\log^2(x)}{1+m} - \frac{2\int x^m \log(x)\,dx}{1+m}$$
$$= \frac{2x^{1+m}}{(1+m)^3} - \frac{2x^{1+m}\log(x)}{(1+m)^2} + \frac{x^{1+m}\log^2(x)}{1+m}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int x^m \log^2(x)\,dx = \frac{x^{1+m}\left(2-2(1+m)\log(x)+(1+m)^2\log^2(x)\right)}{(1+m)^3}$$

```
[In] Integrate[x^m*Log[x]^2,x]
[Out] (x^(1 + m)*(2 - 2*(1 + m)*Log[x] + (1 + m)^2*Log[x]^2))/(1 + m)^3
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{x\left(m^2\ln(x)^2+2m\ln(x)^2-2m\ln(x)+\ln(x)^2-2\ln(x)+2\right)x^m}{(1+m)^3}$	41
norman	$\frac{x\ln(x)^2 e^{m\ln(x)}}{1+m}+\frac{2x\,e^{m\ln(x)}}{m^3+3m^2+3m+1}-\frac{2x\ln(x)e^{m\ln(x)}}{m^2+2m+1}$	61
parallelrisch	$\frac{x\,x^m\ln(x)^2m^2+2x\,x^m\ln(x)^2m+x^m\ln(x)^2x-2x\,x^m\ln(x)m-2x^m\ln(x)x+2x\,x^m}{m^3+3m^2+3m+1}$	73

```
[In] int(x^m*ln(x)^2,x,method=_RETURNVERBOSE)
[Out] x*(m^2*ln(x)^2+2*m*ln(x)^2-2*m*ln(x)+ln(x)^2-2*ln(x)+2)/(1+m)^3*x^m
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int x^m \log^2(x)\,dx = \frac{\left((m^2+2\,m+1)x\log{(x)}^2-2\,(m+1)x\log{(x)}+2\,x\right)x^m}{m^3+3\,m^2+3\,m+1}$$

```
[In] integrate(x^m*log(x)^2,x, algorithm="fricas")
```

```
[Out] ((m^2 + 2*m + 1)*x*log(x)^2 - 2*(m + 1)*x*log(x) + 2*x)*x^m/(m^3 + 3*m^2 +
3*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(39) = 78.

Time = 0.36 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.69

$$\int x^m \log^2(x)\,dx$$
$$= \begin{cases} \frac{m^2xx^m\log{(x)}^2}{m^3+3m^2+3m+1} + \frac{2mxx^m\log{(x)}^2}{m^3+3m^2+3m+1} - \frac{2mxx^m\log{(x)}}{m^3+3m^2+3m+1} + \frac{xx^m\log{(x)}^2}{m^3+3m^2+3m+1} - \frac{2xx^m\log{(x)}}{m^3+3m^2+3m+1} + \frac{2xx^m}{m^3+3m^2+3m+1} & \text{for } m \neq \\ \frac{\log{(x)}^3}{3} & \text{otherwis} \end{cases}$$

```
[In] integrate(x**m*ln(x)**2,x)
```

```
[Out] Piecewise((m**2*x*x**m*log(x)**2/(m**3 + 3*m**2 + 3*m + 1) + 2*m*x*x**m*log
(x)**2/(m**3 + 3*m**2 + 3*m + 1) - 2*m*x*x**m*log(x)/(m**3 + 3*m**2 + 3*m +
 1) + x*x**m*log(x)**2/(m**3 + 3*m**2 + 3*m + 1) - 2*x*x**m*log(x)/(m**3 +
3*m**2 + 3*m + 1) + 2*x*x**m/(m**3 + 3*m**2 + 3*m + 1), Ne(m, -1)), (log(x)
**3/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int x^m \log^2(x)\,dx = \frac{x^{m+1}\log{(x)}^2}{m+1} - \frac{2\,x^{m+1}\log{(x)}}{(m+1)^2} + \frac{2\,x^{m+1}}{(m+1)^3}$$

```
[In] integrate(x^m*log(x)^2,x, algorithm="maxima")
```

```
[Out] x^(m + 1)*log(x)^2/(m + 1) - 2*x^(m + 1)*log(x)/(m + 1)^2 + 2*x^(m + 1)/(m
+ 1)^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.00

$$\int x^m \log^2(x)\,dx = -\frac{2\,mxx^m\log(x)}{(m^2+2\,m+1)(m+1)} + \frac{x^{m+1}\log(x)^2}{m+1} - \frac{2\,xx^m\log(x)}{(m^2+2\,m+1)(m+1)} + \frac{2\,xx^m}{(m^2+2\,m+1)(m+1)}$$

```
[In] integrate(x^m*log(x)^2,x, algorithm="giac")
```

```
[Out] -2*m*x*x^m*log(x)/((m^2 + 2*m + 1)*(m + 1)) + x^(m + 1)*log(x)^2/(m + 1) -
2*x*x^m*log(x)/((m^2 + 2*m + 1)*(m + 1)) + 2*x*x^m/((m^2 + 2*m + 1)*(m + 1)
)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int x^m \log^2(x)\,dx = \begin{cases} \frac{\ln(x)^3}{3} & \text{if } m=-1 \\ \frac{x^{m+1}\left(\ln(x)^2\,(m+1)^2-2\,\ln(x)\,(m+1)+2\right)}{(m+1)^3} & \text{if } m\neq -1 \end{cases}$$

```
[In] int(x^m*log(x)^2,x)
```

```
[Out] piecewise(m == -1, log(x)^3/3, m ~= -1, (x^(m + 1)*(- 2*log(x)*(m + 1) + lo
g(x)^2*(m + 1)^2 + 2))/(m + 1)^3)
```

3.610 $\int \frac{\log^2(x)}{x^{5/2}} dx$

Optimal result

Integrand size = 10, antiderivative size = 34

$$\int \frac{\log^2(x)}{x^{5/2}} dx = -\frac{16}{27x^{3/2}} - \frac{8\log(x)}{9x^{3/2}} - \frac{2\log^2(x)}{3x^{3/2}}$$

```
[Out] -16/27/x^(3/2)-8/9*ln(x)/x^(3/2)-2/3*ln(x)^2/x^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2342, 2341}

$$\int \frac{\log^2(x)}{x^{5/2}} dx = -\frac{16}{27x^{3/2}} - \frac{2\log^2(x)}{3x^{3/2}} - \frac{8\log(x)}{9x^{3/2}}$$

```
[In] Int[Log[x]^2/x^(5/2),x]
```

```
[Out] -16/(27*x^(3/2)) - (8*Log[x])/(9*x^(3/2)) - (2*Log[x]^2)/(3*x^(3/2))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\log^2(x)}{3x^{3/2}} + \frac{4}{3}\int \frac{\log(x)}{x^{5/2}}\,dx$$
$$= -\frac{16}{27x^{3/2}} - \frac{8\log(x)}{9x^{3/2}} - \frac{2\log^2(x)}{3x^{3/2}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{2(8+12\log(x)+9\log^2(x))}{27x^{3/2}}$$

```
[In] Integrate[Log[x]^2/x^(5/2),x]
[Out] (-2*(8 + 12*Log[x] + 9*Log[x]^2))/(27*x^(3/2))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$-\frac{16}{27x^{\frac{3}{2}}}-\frac{8\ln(x)}{9x^{\frac{3}{2}}}-\frac{2\ln(x)^2}{3x^{\frac{3}{2}}}$	23
default	$-\frac{16}{27x^{\frac{3}{2}}}-\frac{8\ln(x)}{9x^{\frac{3}{2}}}-\frac{2\ln(x)^2}{3x^{\frac{3}{2}}}$	23
risch	$-\frac{16}{27x^{\frac{3}{2}}}-\frac{8\ln(x)}{9x^{\frac{3}{2}}}-\frac{2\ln(x)^2}{3x^{\frac{3}{2}}}$	23

```
[In] int(ln(x)^2/x^(5/2),x,method=_RETURNVERBOSE)
[Out] -16/27/x^(3/2)-8/9*ln(x)/x^(3/2)-2/3*ln(x)^2/x^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.50

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{2\left(9\,\log\left(x\right)^2+12\,\log\left(x\right)+8\right)}{27\,x^{\frac{3}{2}}}$$

```
[In] integrate(log(x)^2/x^(5/2),x, algorithm="fricas")
[Out] -2/27*(9*log(x)^2 + 12*log(x) + 8)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{2\log\left(x\right)^2}{3x^{\frac{3}{2}}}-\frac{8\log\left(x\right)}{9x^{\frac{3}{2}}}-\frac{16}{27x^{\frac{3}{2}}}$$

```
[In] integrate(ln(x)**2/x**(5/2),x)
[Out] -2*log(x)**2/(3*x**(3/2)) - 8*log(x)/(9*x**(3/2)) - 16/(27*x**(3/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{2\,\log\left(x\right)^2}{3\,x^{\frac{3}{2}}}-\frac{8\,\log\left(x\right)}{9\,x^{\frac{3}{2}}}-\frac{16}{27\,x^{\frac{3}{2}}}$$

```
[In] integrate(log(x)^2/x^(5/2),x, algorithm="maxima")
[Out] -2/3*log(x)^2/x^(3/2) - 8/9*log(x)/x^(3/2) - 16/27/x^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{2\,\log\left(x\right)^2}{3\,x^{\frac{3}{2}}}-\frac{8\,\log\left(x\right)}{9\,x^{\frac{3}{2}}}-\frac{16}{27\,x^{\frac{3}{2}}}$$

```
[In] integrate(log(x)^2/x^(5/2),x, algorithm="giac")
[Out] -2/3*log(x)^2/x^(3/2) - 8/9*log(x)/x^(3/2) - 16/27/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.50

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{18\ln(x)^2 + 24\,\ln(x) + 16}{27\,x^{3/2}}$$

```
[In] int(log(x)^2/x^(5/2),x)
[Out] -(24*log(x) + 18*log(x)^2 + 16)/(27*x^(3/2))
```

3.611 $\int (a+bx)\log(x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 28

$$\int (a+bx)\log(x)\,dx=-ax-\frac{bx^2}{4}+ax\log(x)+\frac{1}{2}bx^2\log(x)$$

```
[Out] -a*x-1/4*b*x^2+a*x*ln(x)+1/2*b*x^2*ln(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2350}

$$\int (a+bx)\log(x)\,dx=-ax+ax\log(x)-\frac{bx^2}{4}+\frac{1}{2}bx^2\log(x)$$

```
[In] Int[(a + b*x)*Log[x],x]
```

```
[Out] -(a*x) - (b*x^2)/4 + a*x*Log[x] + (b*x^2*Log[x])/2
```

Rule 2350

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u
, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=ax\log(x)+\frac{1}{2}bx^2\log(x)-\int\left(a+\frac{bx}{2}\right)\,dx\\&=-ax-\frac{bx^2}{4}+ax\log(x)+\frac{1}{2}bx^2\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a+bx)\log(x)\,dx = -ax - \frac{bx^2}{4} + ax\log(x) + \frac{1}{2}bx^2\log(x)$$

```
[In] Integrate[(a + b*x)*Log[x],x]
[Out] -(a*x) - (b*x^2)/4 + a*x*Log[x] + (b*x^2*Log[x])/2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
norman	$-ax-\frac{x^2b}{4}+ax\ln(x)+\frac{b\,x^2\ln(x)}{2}$	25
risch	$\left(\frac{1}{2}x^2b+ax\right)\ln(x)-\frac{x^2b}{4}-ax$	25
parallelrisch	$-ax-\frac{x^2b}{4}+ax\ln(x)+\frac{b\,x^2\ln(x)}{2}$	25
parts	$-ax-\frac{x^2b}{4}+ax\ln(x)+\frac{b\,x^2\ln(x)}{2}$	25
default	$b\left(-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}\right)+a(-x+x\ln(x))$	27

```
[In] int((b*x+a)*ln(x),x,method=_RETURNVERBOSE)
[Out] -a*x-1/4*x^2*b+a*x*ln(x)+1/2*b*x^2*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a+bx)\log(x)\,dx = -\frac{1}{4}\,bx^2 - ax + \frac{1}{2}\,\left(bx^2+2\,ax\right)\log(x)$$

```
[In] integrate((b*x+a)*log(x),x, algorithm="fricas")
[Out] -1/4*b*x^2 - a*x + 1/2*(b*x^2 + 2*a*x)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int (a+bx)\log(x)\,dx = -ax - \frac{bx^2}{4} + \left(ax + \frac{bx^2}{2}\right)\log(x)$$

```
[In] integrate((b*x+a)*ln(x),x)
[Out] -a*x - b*x**2/4 + (a*x + b*x**2/2)*log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a+bx)\log(x)\,dx = -\frac{1}{4}\,bx^2 - ax + \frac{1}{2}\,(bx^2 + 2\,ax)\log(x)$$

```
[In] integrate((b*x+a)*log(x),x, algorithm="maxima")
[Out] -1/4*b*x^2 - a*x + 1/2*(b*x^2 + 2*a*x)*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx)\log(x)\,dx = \frac{1}{2}\,bx^2\log(x) - \frac{1}{4}\,bx^2 + ax\log(x) - ax$$

```
[In] integrate((b*x+a)*log(x),x, algorithm="giac")
[Out] 1/2*b*x^2*log(x) - 1/4*b*x^2 + a*x*log(x) - a*x
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int (a+bx)\log(x)\,dx = -\frac{x\,(4\,a + b\,x - 4\,a\,\ln(x) - 2\,b\,x\,\ln(x))}{4}$$

```
[In] int(log(x)*(a + b*x),x)
[Out] -(x*(4*a + b*x - 4*a*log(x) - 2*b*x*log(x)))/4
```

3.612 $\int (a+bx)^3 \log(x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 67

$$\int (a+bx)^3\log(x)\,dx = -a^3x-\frac{3}{4}a^2bx^2-\frac{1}{3}ab^2x^3-\frac{b^3x^4}{16}-\frac{a^4\log(x)}{4b}+\frac{(a+bx)^4\log(x)}{4b}$$

```
[Out] -a^3*x-3/4*a^2*b*x^2-1/3*a*b^2*x^3-1/16*b^3*x^4-1/4*a^4*ln(x)/b+1/4*(b*x+a)
^4*ln(x)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {32, 2350, 12, 45}

$$\int (a+bx)^3\log(x)\,dx = -\frac{a^4\log(x)}{4b}-a^3x-\frac{3}{4}a^2bx^2-\frac{1}{3}ab^2x^3+\frac{\log(x)(a+bx)^4}{4b}-\frac{b^3x^4}{16}$$

```
[In] Int[(a + b*x)^3*Log[x],x]
```

```
[Out] -(a^3*x) - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - (b^3*x^4)/16 - (a^4*Log[x])/(4
*b) + ((a + b*x)^4*Log[x])/(4*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2350

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u
, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+bx)^4\log(x)}{4b} - \int \frac{(a+bx)^4}{4bx}\,dx\\
&= \frac{(a+bx)^4\log(x)}{4b} - \frac{\int \frac{(a+bx)^4}{x}\,dx}{4b}\\
&= \frac{(a+bx)^4\log(x)}{4b} - \frac{\int \left(4a^3b+\frac{a^4}{x}+6a^2b^2x+4ab^3x^2+b^4x^3\right)dx}{4b}\\
&= -a^3x - \frac{3}{4}a^2bx^2 - \frac{1}{3}ab^2x^3 - \frac{b^3x^4}{16} - \frac{a^4\log(x)}{4b} + \frac{(a+bx)^4\log(x)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

$$\begin{aligned}
\int (a+bx)^3\log(x)\,dx = &-a^3x - \frac{3}{4}a^2bx^2 - \frac{1}{3}ab^2x^3 - \frac{b^3x^4}{16} + a^3x\log(x)\\
&+ \frac{3}{2}a^2bx^2\log(x) + ab^2x^3\log(x) + \frac{1}{4}b^3x^4\log(x)
\end{aligned}$$

```
[In] Integrate[(a + b*x)^3*Log[x],x]
[Out] -(a^3*x) - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - (b^3*x^4)/16 + a^3*x*Log[x] +
(3*a^2*b*x^2*Log[x])/2 + a*b^2*x^3*Log[x] + (b^3*x^4*Log[x])/4
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

method	result
risch	$-a^{3}x-\frac{3a^{2}b\,x^{2}}{4}-\frac{a\,b^{2}x^{3}}{3}-\frac{b^{3}x^{4}}{16}-\frac{a^{4}\ln(x)}{4b}+\frac{(bx+a)^{4}\ln(x)}{4b}$
default	$b^{3}\left(-\frac{x^{4}}{16}+\frac{x^{4}\ln(x)}{4}\right)+3b^{2}a\left(-\frac{x^{3}}{9}+\frac{x^{3}\ln(x)}{3}\right)+3a^{2}b\left(-\frac{x^{2}}{4}+\frac{x^{2}\ln(x)}{2}\right)+a^{3}(-x+x\ln(x))$
norman	$a^{3}x\ln(x)+a\,b^{2}x^{3}\ln(x)-a^{3}x-\frac{b^{3}x^{4}}{16}-\frac{a\,b^{2}x^{3}}{3}-\frac{3a^{2}b\,x^{2}}{4}+\frac{b^{3}x^{4}\ln(x)}{4}+\frac{3a^{2}b\,x^{2}\ln(x)}{2}$
parallelrisch	$a^{3}x\ln(x)+a\,b^{2}x^{3}\ln(x)-a^{3}x-\frac{b^{3}x^{4}}{16}-\frac{a\,b^{2}x^{3}}{3}-\frac{3a^{2}b\,x^{2}}{4}+\frac{b^{3}x^{4}\ln(x)}{4}+\frac{3a^{2}b\,x^{2}\ln(x)}{2}$
parts	$\frac{b^{3}x^{4}\ln(x)}{4}+a\,b^{2}x^{3}\ln(x)+\frac{3a^{2}b\,x^{2}\ln(x)}{2}+a^{3}x\ln(x)+\frac{a^{4}\ln(x)}{4b}-\frac{\frac{b^{4}x^{4}}{4}+\frac{4a\,b^{3}x^{3}}{3}+3a^{2}b^{2}x^{2}+4a^{3}bx+a^{4}\ln(x)}{4b}$

```
[In] int((b*x+a)^3*ln(x),x,method=_RETURNVERBOSE)
[Out] -a^3*x-3/4*a^2*b*x^2-1/3*a*b^2*x^3-1/16*b^3*x^4-1/4*a^4*ln(x)/b+1/4*(b*x+a)
^4*ln(x)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int (a+bx)^3\log(x)\,dx=-\frac{1}{16}\,b^3x^4-\frac{1}{3}\,ab^2x^3-\frac{3}{4}\,a^2bx^2-a^3x+\frac{1}{4}\,\left(b^3x^4+4\,ab^2x^3+6\,a^2bx^2+4\,a^3x\right)\log\left(x\right)$$

```
[In] integrate((b*x+a)^3*log(x),x, algorithm="fricas")
[Out] -1/16*b^3*x^4 - 1/3*a*b^2*x^3 - 3/4*a^2*b*x^2 - a^3*x + 1/4*(b^3*x^4 + 4*a*
b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int (a+bx)^3\log(x)\,dx=-a^3x-\frac{3a^2bx^2}{4}-\frac{ab^2x^3}{3}-\frac{b^3x^4}{16}+\left(a^3x+\frac{3a^2bx^2}{2}+ab^2x^3+\frac{b^3x^4}{4}\right)\log\left(x\right)$$

```
[In] integrate((b*x+a)**3*ln(x),x)
[Out] -a**3*x - 3*a**2*b*x**2/4 - a*b**2*x**3/3 - b**3*x**4/16 + (a**3*x + 3*a**2
*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4)*log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int (a+bx)^3 \log(x)\, dx = -\frac{1}{16}\, b^3x^4 - \frac{1}{3}\, ab^2x^3 - \frac{3}{4}\, a^2bx^2 - a^3x + \frac{1}{4}\, \left(b^3x^4 + 4\, ab^2x^3 + 6\, a^2bx^2 + 4\, a^3x\right) \log\left(x\right)$$

```
[In] integrate((b*x+a)^3*log(x),x, algorithm="maxima")
```

```
[Out] -1/16*b^3*x^4 - 1/3*a*b^2*x^3 - 3/4*a^2*b*x^2 - a^3*x + 1/4*(b^3*x^4 + 4*a*
b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int (a+bx)^3 \log(x)\, dx = \frac{1}{4}\, b^3x^4 \log\left(x\right) - \frac{1}{16}\, b^3x^4 + ab^2x^3 \log\left(x\right) - \frac{1}{3}\, ab^2x^3 + \frac{3}{2}\, a^2bx^2 \log\left(x\right) - \frac{3}{4}\, a^2bx^2 + a^3x \log\left(x\right) - a^3x$$

```
[In] integrate((b*x+a)^3*log(x),x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*log(x) - 1/16*b^3*x^4 + a*b^2*x^3*log(x) - 1/3*a*b^2*x^3 + 3/2*
a^2*b*x^2*log(x) - 3/4*a^2*b*x^2 + a^3*x*log(x) - a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int (a+bx)^3 \log(x)\, dx = a^3\, x\, \ln\left(x\right) - \frac{b^3\, x^4}{16} - \frac{3\, a^2\, b\, x^2}{4} - \frac{a\, b^2\, x^3}{3} - a^3\, x + \frac{b^3\, x^4\, \ln\left(x\right)}{4} + \frac{3\, a^2\, b\, x^2\, \ln\left(x\right)}{2} + a\, b^2\, x^3\, \ln\left(x\right)$$

```
[In] int(log(x)*(a + b*x)^3,x)
```

```
[Out] a^3*x*log(x) - (b^3*x^4)/16 - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - a^3*x + (b^
3*x^4*log(x))/4 + (3*a^2*b*x^2*log(x))/2 + a*b^2*x^3*log(x)
```

3.613 $\int \left(-1-8 \log ^2(x)+3 \log ^3(x)\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \left(-1-8 \log ^2(x)+3 \log ^3(x)\right) \, dx=-35 x+34 x \log (x)-17 x \log ^2(x)+3 x \log ^3(x)$$

```
[Out] -35*x+34*x*ln(x)-17*x*ln(x)^2+3*x*ln(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2333, 2332}

$$\int \left(-1-8 \log ^2(x)+3 \log ^3(x)\right) \, dx=-35 x+3 x \log ^3(x)-17 x \log ^2(x)+34 x \log (x)$$

```
[In] Int[-1 - 8*Log[x]^2 + 3*Log[x]^3,x]
```

```
[Out] -35*x + 34*x*Log[x] - 17*x*Log[x]^2 + 3*x*Log[x]^3
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -x+3\int \log^3(x)\,dx-8\int \log^2(x)\,dx\\
&= -x-8x\log^2(x)+3x\log^3(x)-9\int \log^2(x)\,dx+16\int \log(x)\,dx\\
&= -17x+16x\log(x)-17x\log^2(x)+3x\log^3(x)+18\int \log(x)\,dx\\
&= -35x+34x\log(x)-17x\log^2(x)+3x\log^3(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$
\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx=-35x+34x\log(x)-17x\log^2(x)+3x\log^3(x)
$$

```
[In] Integrate[-1 - 8*Log[x]^2 + 3*Log[x]^3,x]
[Out] -35*x + 34*x*Log[x] - 17*x*Log[x]^2 + 3*x*Log[x]^3
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
default	$-35x+34x\ln(x)-17x\ln(x)^2+3x\ln(x)^3$	24
norman	$-35x+34x\ln(x)-17x\ln(x)^2+3x\ln(x)^3$	24
risch	$-35x+34x\ln(x)-17x\ln(x)^2+3x\ln(x)^3$	24
parallelrisch	$-35x+34x\ln(x)-17x\ln(x)^2+3x\ln(x)^3$	24
parts	$-35x+34x\ln(x)-17x\ln(x)^2+3x\ln(x)^3$	24

```
[In] int(-1-8*ln(x)^2+3*ln(x)^3,x,method=_RETURNVERBOSE)
[Out] -35*x+34*x*ln(x)-17*x*ln(x)^2+3*x*ln(x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx=3\,x\log\left(x\right)^3-17\,x\log\left(x\right)^2+34\,x\log\left(x\right)-35\,x$$

```
[In] integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="fricas")
[Out] 3*x*log(x)^3 - 17*x*log(x)^2 + 34*x*log(x) - 35*x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx=3x\log\left(x\right)^3-17x\log\left(x\right)^2+34x\log\left(x\right)-35x$$

```
[In] integrate(-1-8*ln(x)**2+3*ln(x)**3,x)
[Out] 3*x*log(x)**3 - 17*x*log(x)**2 + 34*x*log(x) - 35*x
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx=3\left(\log\left(x\right)^3-3\,\log\left(x\right)^2+6\,\log\left(x\right)-6\right)x-8\left(\log\left(x\right)^2-2\,\log\left(x\right)+2\right)x-x$$

```
[In] integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="maxima")
[Out] 3*(log(x)^3 - 3*log(x)^2 + 6*log(x) - 6)*x - 8*(log(x)^2 - 2*log(x) + 2)*x
- x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx=3\,x\log\left(x\right)^3-17\,x\log\left(x\right)^2+34\,x\log\left(x\right)-35\,x$$

```
[In] integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="giac")
[Out] 3*x*log(x)^3 - 17*x*log(x)^2 + 34*x*log(x) - 35*x
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx=x\left(3\ln\left(x\right)^3-17\ln\left(x\right)^2+34\,\ln\left(x\right)-35\right)$$

```
[In] int(3*log(x)^3 - 8*log(x)^2 - 1,x)
[Out] x*(34*log(x) - 17*log(x)^2 + 3*log(x)^3 - 35)
```

3.614 $\int (1+x^4) (1-2\log(x)+\log^3(x)) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 60

$$\int (1+x^4) (1-2\log(x)+\log^3(x)) \, dx = -3x+\frac{169x^5}{625}+4x\log(x)-\frac{44}{125}x^5\log(x)-3x\log^2(x) -\frac{3}{25}x^5\log^2(x)+x\log^3(x)+\frac{1}{5}x^5\log^3(x)$$

```
[Out] -3*x+169/625*x^5+4*x*ln(x)-44/125*x^5*ln(x)-3*x*ln(x)^2-3/25*x^5*ln(x)^2+x*
ln(x)^3+1/5*x^5*ln(x)^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6874, 2350, 12, 2367, 2333, 2332, 2342, 2341}

$$\int (1+x^4) (1-2\log(x)+\log^3(x)) \, dx = \frac{169x^5}{625}+\frac{1}{5}x^5\log^3(x)-\frac{3}{25}x^5\log^2(x)-\frac{44}{125}x^5\log(x) -3x+x\log^3(x)-3x\log^2(x)+4x\log(x)$$

```
[In] Int[(1 + x^4)*(1 - 2*Log[x] + Log[x]^3),x]
```

```
[Out] -3*x + (169*x^5)/625 + 4*x*Log[x] - (44*x^5*Log[x])/125 - 3*x*Log[x]^2 - (3
*x^5*Log[x]^2)/25 + x*Log[x]^3 + (x^5*Log[x]^3)/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2350

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u
, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IGtQ[q, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(1+x^4-2\left(1+x^4\right)\log(x)+\left(1+x^4\right)\log^3(x)\right)\,dx$$

$$= x+\frac{x^5}{5}-2\int \left(1+x^4\right)\log(x)\,dx+\int \left(1+x^4\right)\log^3(x)\,dx$$

$$= x+\frac{x^5}{5}-2x\log(x)-\frac{2}{5}x^5\log(x)+2\int \frac{1}{5}\left(5+x^4\right)\,dx+\int \left(\log^3(x)+x^4\log^3(x)\right)\,dx$$

$$= x+\frac{x^5}{5}-2x\log(x)-\frac{2}{5}x^5\log(x)+\frac{2}{5}\int \left(5+x^4\right)\,dx+\int \log^3(x)\,dx+\int x^4\log^3(x)\,dx$$

$$= 3x+\frac{7x^5}{25}-2x\log(x)-\frac{2}{5}x^5\log(x)+x\log^3(x) + \frac{1}{5}x^5\log^3(x)-\frac{3}{5}\int x^4\log^2(x)\,dx-3\int \log^2(x)\,dx$$

$$= 3x+\frac{7x^5}{25}-2x\log(x)-\frac{2}{5}x^5\log(x)-3x\log^2(x)-\frac{3}{25}x^5\log^2(x) + x\log^3(x)+\frac{1}{5}x^5\log^3(x)+\frac{6}{25}\int x^4\log(x)\,dx+6\int \log(x)\,dx$$

$$= -3x+\frac{169x^5}{625}+4x\log(x)-\frac{44}{125}x^5\log(x)-3x\log^2(x)-\frac{3}{25}x^5\log^2(x)+x\log^3(x)+\frac{1}{5}x^5\log^3(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \left(1+x^4\right)\left(1-2\log(x)+\log^3(x)\right)\,dx = -3x+\frac{169x^5}{625}+4x\log(x)-\frac{44}{125}x^5\log(x)-3x\log^2(x) - \frac{3}{25}x^5\log^2(x)+x\log^3(x)+\frac{1}{5}x^5\log^3(x)$$

```
[In] Integrate[(1 + x^4)*(1 - 2*Log[x] + Log[x]^3),x]
[Out] -3*x + (169*x^5)/625 + 4*x*Log[x] - (44*x^5*Log[x])/125 - 3*x*Log[x]^2 - (3
*x^5*Log[x]^2)/25 + x*Log[x]^3 + (x^5*Log[x]^3)/5
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

method	result	size
risch	$\left(\frac{1}{5}x^{5}+x\right)\ln\left(x\right)^{3}+\left(-\frac{3}{25}x^{5}-3x\right)\ln\left(x\right)^{2}+\left(-\frac{44}{125}x^{5}+4x\right)\ln\left(x\right)+\frac{169x^{5}}{625}-3x$	48
default	$-3x+\frac{169x^{5}}{625}+4x\ln\left(x\right)-\frac{44x^{5}\ln(x)}{125}-3x\ln\left(x\right)^{2}-\frac{3x^{5}\ln(x)^{2}}{25}+x\ln\left(x\right)^{3}+\frac{x^{5}\ln(x)^{3}}{5}$	53
norman	$-3x+\frac{169x^{5}}{625}+4x\ln\left(x\right)-\frac{44x^{5}\ln(x)}{125}-3x\ln\left(x\right)^{2}-\frac{3x^{5}\ln(x)^{2}}{25}+x\ln\left(x\right)^{3}+\frac{x^{5}\ln(x)^{3}}{5}$	53
parallelrisch	$-3x+\frac{169x^{5}}{625}+4x\ln\left(x\right)-\frac{44x^{5}\ln(x)}{125}-3x\ln\left(x\right)^{2}-\frac{3x^{5}\ln(x)^{2}}{25}+x\ln\left(x\right)^{3}+\frac{x^{5}\ln(x)^{3}}{5}$	53
parts	$-3x+\frac{169x^{5}}{625}+4x\ln\left(x\right)-\frac{44x^{5}\ln(x)}{125}-3x\ln\left(x\right)^{2}-\frac{3x^{5}\ln(x)^{2}}{25}+x\ln\left(x\right)^{3}+\frac{x^{5}\ln(x)^{3}}{5}$	53

```
[In] int((x^4+1)*(1-2*ln(x)+ln(x)^3),x,method=_RETURNVERBOSE)
[Out] (1/5*x^5+x)*ln(x)^3+(-3/25*x^5-3*x)*ln(x)^2+(-44/125*x^5+4*x)*ln(x)+169/625
*x^5-3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int\left(1+x^{4}\right)\left(1-2\log(x)+\log^{3}(x)\right)\,dx=\frac{169}{625}\,x^{5}+\frac{1}{5}\left(x^{5}+5\,x\right)\log\left(x\right)^{3}-\frac{3}{25}\left(x^{5}+25\,x\right)\log\left(x\right)^{2}-\frac{4}{125}\left(11\,x^{5}-125\,x\right)\log\left(x\right)-3\,x$$

```
[In] integrate((x^4+1)*(1-2*log(x)+log(x)^3),x, algorithm="fricas")
[Out] 169/625*x^5 + 1/5*(x^5 + 5*x)*log(x)^3 - 3/25*(x^5 + 25*x)*log(x)^2 - 4/125
*(11*x^5 - 125*x)*log(x) - 3*x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int\left(1+x^{4}\right)\left(1-2\log(x)+\log^{3}(x)\right)\,dx=\frac{169x^{5}}{625}-3x+\left(-\frac{44x^{5}}{125}+4x\right)\log\left(x\right)+\left(-\frac{3x^{5}}{25}-3x\right)\log\left(x\right)^{2}+\left(\frac{x^{5}}{5}+x\right)\log\left(x\right)^{3}$$

```
[In] integrate((x**4+1)*(1-2*ln(x)+ln(x)**3),x)
```

```
[Out] 169*x**5/625 - 3*x + (-44*x**5/125 + 4*x)*log(x) + (-3*x**5/25 - 3*x)*log(x
)**2 + (x**5/5 + x)*log(x)**3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int (1+x^4) (1-2\log(x)+\log^3(x)) \, dx = \frac{1}{625} \left(125 \log(x)^3 - 75 \log(x)^2 + 30 \log(x) - 6\right)x^5 - \frac{2}{25} x^5 (5 \log(x) - 1) + \frac{1}{5} x^5 + \left(\log(x)^3 - 3 \log(x)^2 + 6 \log(x) - 6\right)x - 2 x(\log(x) - 1) + x$$

```
[In] integrate((x^4+1)*(1-2*log(x)+log(x)^3),x, algorithm="maxima")
```

```
[Out] 1/625*(125*log(x)^3 - 75*log(x)^2 + 30*log(x) - 6)*x^5 - 2/25*x^5*(5*log(x)
 - 1) + 1/5*x^5 + (log(x)^3 - 3*log(x)^2 + 6*log(x) - 6)*x - 2*x*(log(x) -
1) + x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int (1+x^4) (1-2\log(x)+\log^3(x)) \, dx = \frac{1}{5} x^5 \log(x)^3 - \frac{3}{25} x^5 \log(x)^2 - \frac{44}{125} x^5 \log(x) + \frac{169}{625} x^5 + x \log(x)^3 - 3 x \log(x)^2 + 4 x \log(x) - 3 x$$

```
[In] integrate((x^4+1)*(1-2*log(x)+log(x)^3),x, algorithm="giac")
```

```
[Out] 1/5*x^5*log(x)^3 - 3/25*x^5*log(x)^2 - 44/125*x^5*log(x) + 169/625*x^5 + x*
log(x)^3 - 3*x*log(x)^2 + 4*x*log(x) - 3*x
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \left(1+x^4\right) \left(1-2 \log(x)+\log^3(x)\right) \, dx$$
$$= \frac{x \left(125 x^4 \ln(x)^3-75 x^4 \ln(x)^2-220 x^4 \ln(x)+169 x^4+625 \ln(x)^3-1875 \ln(x)^2+2500 \ln(x)-1875\right)}{625}$$

```
[In] int((x^4 + 1)*(log(x)^3 - 2*log(x) + 1),x)
```

```
[Out] (x*(2500*log(x) - 220*x^4*log(x) - 1875*log(x)^2 + 625*log(x)^3 - 75*x^4*lo
g(x)^2 + 125*x^4*log(x)^3 + 169*x^4 - 1875))/625
```

3.615 $\int \frac{1}{x^3\log^4(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \frac{1}{x^3\log^4(x)}\,dx=-\frac{4}{3}\text{ExpIntegralEi}(-2\log(x))-\frac{1}{3x^2\log^3(x)}+\frac{1}{3x^2\log^2(x)}-\frac{2}{3x^2\log(x)}$$

```
[Out] -4/3*Ei(-2*ln(x))-1/3/x^2/ln(x)^3+1/3/x^2/ln(x)^2-2/3/x^2/ln(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2343, 2346, 2209}

$$\int \frac{1}{x^3\log^4(x)}\,dx=-\frac{4}{3}\text{ExpIntegralEi}(-2\log(x))-\frac{1}{3x^2\log^3(x)}+\frac{1}{3x^2\log^2(x)}-\frac{2}{3x^2\log(x)}$$

```
[In] Int[1/(x^3*Log[x]^4),x]
```

```
[Out] (-4*ExpIntegralEi[-2*Log[x]])/3 - 1/(3*x^2*Log[x]^3) + 1/(3*x^2*Log[x]^2) -
 2/(3*x^2*Log[x])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
```

```
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{3x^2\log^3(x)} - \frac{2}{3}\int \frac{1}{x^3\log^3(x)}\,dx \\
&= -\frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} + \frac{2}{3}\int \frac{1}{x^3\log^2(x)}\,dx \\
&= -\frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} - \frac{2}{3x^2\log(x)} - \frac{4}{3}\int \frac{1}{x^3\log(x)}\,dx \\
&= -\frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} - \frac{2}{3x^2\log(x)} - \frac{4}{3}\text{Subst}\left(\int \frac{e^{-2x}}{x}\,dx, x, \log(x)\right) \\
&= -\frac{4}{3}\,\text{ExpIntegralEi}(-2\log(x)) - \frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} - \frac{2}{3x^2\log(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\log^4(x)}\,dx = -\frac{4}{3}\,\text{ExpIntegralEi}(-2\log(x)) - \frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} - \frac{2}{3x^2\log(x)}$$

```
[In] Integrate[1/(x^3*Log[x]^4),x]
[Out] (-4*ExpIntegralEi[-2*Log[x]])/3 - 1/(3*x^2*Log[x]^3) + 1/(3*x^2*Log[x]^2) -
 2/(3*x^2*Log[x])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{2\ln(x)^2-\ln(x)+1}{3x^2\ln(x)^3}+\frac{4\,\mathrm{Ei}_1(2\ln(x))}{3}$	31
default	$-\frac{1}{3x^2\ln(x)^3}+\frac{1}{3x^2\ln(x)^2}-\frac{2}{3x^2\ln(x)}+\frac{4\,\mathrm{Ei}_1(2\ln(x))}{3}$	37

```
[In] int(1/x^3/ln(x)^4,x,method=_RETURNVERBOSE)
[Out] -1/3*(2*ln(x)^2-ln(x)+1)/x^2/ln(x)^3+4/3*Ei(1,2*ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^3\log^4(x)}\,dx = -\frac{4\,x^2\log(x)^3\,\text{log\_integral}\left(\frac{1}{x^2}\right)+2\,\log(x)^2-\log(x)+1}{3\,x^2\log(x)^3}$$

```
[In] integrate(1/x^3/log(x)^4,x, algorithm="fricas")
[Out] -1/3*(4*x^2*log(x)^3*log_integral(x^(-2)) + 2*log(x)^2 - log(x) + 1)/(x^2*l
og(x)^3)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3\log^4(x)}\,dx = -\frac{4\,\mathrm{Ei}\left(-2\log(x)\right)}{3}+\frac{-2\log(x)^2+\log(x)-1}{3x^2\log(x)^3}$$

```
[In] integrate(1/x**3/ln(x)**4,x)
[Out] -4*Ei(-2*log(x))/3 + (-2*log(x)**2 + log(x) - 1)/(3*x**2*log(x)**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^3 \log^4(x)} \, dx = -8 \Gamma(-3, 2 \log(x))$$

```
[In] integrate(1/x^3/log(x)^4,x, algorithm="maxima")
[Out] -8*gamma(-3, 2*log(x))
```

Giac [F]

$$\int \frac{1}{x^3 \log^4(x)} \, dx = \int \frac{1}{x^3 \log(x)^4} \, dx$$

```
[In] integrate(1/x^3/log(x)^4,x, algorithm="giac")
[Out] integrate(1/(x^3*log(x)^4), x)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^3 \log^4(x)} \, dx = -\frac{4 \,\mathrm{ei}(-2 \ln(x))}{3} - \frac{\frac{2 \ln(x)^2}{3} - \frac{\ln(x)}{3} + \frac{1}{3}}{x^2 \ln(x)^3}$$

```
[In] int(1/(x^3*log(x)^4),x)
[Out] - (4*ei(-2*log(x)))/3 - ((2*log(x)^2)/3 - log(x)/3 + 1/3)/(x^2*log(x)^3)
```

3.616 $\int \frac{\log(x)}{a+bx} dx$

Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{\log(x)}{a+bx} dx = \frac{\log(x)\log\left(1+\frac{bx}{a}\right)}{b} + \frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}$$

```
[Out] ln(x)*ln(1+b*x/a)/b+polylog(2,-b*x/a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2354, 2438}

$$\int \frac{\log(x)}{a+bx} dx = \frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b} + \frac{\log(x)\log\left(\frac{bx}{a}+1\right)}{b}$$

```
[In] Int[Log[x]/(a + b*x),x]
[Out] (Log[x]*Log[1 + (b*x)/a])/b + PolyLog[2, -((b*x)/a)]/b
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\log(x)\log\left(1+\frac{bx}{a}\right)}{b} - \frac{\int \frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx}{b}\\ &= \frac{\log(x)\log\left(1+\frac{bx}{a}\right)}{b} + \frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\log(x)}{a+bx}\,dx = \frac{\log(x)\log\left(\frac{a+bx}{a}\right)}{b} + \frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}$$

```
[In] Integrate[Log[x]/(a + b*x),x]
[Out] (Log[x]*Log[(a + b*x)/a])/b + PolyLog[2, -((b*x)/a)]/b
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\text{dilog}\left(\frac{bx+a}{a}\right)}{b}+\frac{\ln(x)\ln\left(\frac{bx+a}{a}\right)}{b}$	32
risch	$\frac{\text{dilog}\left(\frac{bx+a}{a}\right)}{b}+\frac{\ln(x)\ln\left(\frac{bx+a}{a}\right)}{b}$	32
parts	$\frac{\ln(bx+a)\ln(x)}{b}-\frac{\text{dilog}\left(-\frac{bx}{a}\right)+\ln(bx+a)\ln\left(-\frac{bx}{a}\right)}{b}$	43

```
[In] int(ln(x)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] dilog((b*x+a)/a)/b+ln(x)*ln((b*x+a)/a)/b
```

Fricas [F]

$$\int \frac{\log(x)}{a+bx}\,dx = \int \frac{\log(x)}{bx+a}\,dx$$

```
[In] integrate(log(x)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(log(x)/(b*x + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.78 (sec) , antiderivative size = 177, normalized size of antiderivative = 6.10

$$\int \frac{\log(x)}{a+bx}\,dx = \begin{cases} -\frac{\operatorname{Li}_{2}\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} \\ \frac{\log\left(\frac{a}{b}\right)\log\left(\frac{a}{b}+x\right)}{b} + \frac{i\pi\log\left(\frac{a}{b}+x\right)}{b} - \frac{\operatorname{Li}_{2}\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} \\ -\frac{\log\left(\frac{a}{b}\right)\log\left(\frac{1}{\frac{a}{b}+x}\right)}{b} - \frac{i\pi\log\left(\frac{1}{\frac{a}{b}+x}\right)}{b} - \frac{\operatorname{Li}_{2}\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} \\ -\frac{G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| \frac{a}{b}+x\right)\log\left(\frac{a}{b}\right)}{b} - \frac{i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| \frac{a}{b}+x\right)}{b} + \frac{G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| \frac{a}{b}+x\right)\log\left(\frac{a}{b}\right)}{b} + \frac{i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| \frac{a}{b}+x\right)}{b} \end{cases}$$

```
[In] integrate(ln(x)/(b*x+a),x)
```

```
[Out] Piecewise((-polylog(2, b*(a/b + x)/a)/b, (Abs(a/b + x) < 1) & (1/Abs(a/b +
x) < 1)), (log(a/b)*log(a/b + x)/b + I*pi*log(a/b + x)/b - polylog(2, b*(a/
b + x)/a)/b, Abs(a/b + x) < 1), (-log(a/b)*log(1/(a/b + x))/b - I*pi*log(1/
(a/b + x))/b - polylog(2, b*(a/b + x)/a)/b, 1/Abs(a/b + x) < 1), (-meijerg(
((), (1, 1)), ((0, 0), ()), a/b + x)*log(a/b)/b - I*pi*meijerg(((), (1, 1))
, ((0, 0), ()), a/b + x)/b + meijerg(((1, 1), ()), ((), (0, 0)), a/b + x)*l
og(a/b)/b + I*pi*meijerg(((1, 1), ()), ((), (0, 0)), a/b + x)/b - polylog(2
, b*(a/b + x)/a)/b, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{\log(x)}{a+bx}\,dx = \frac{\log\left(\frac{bx}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)}{b}$$

```
[In] integrate(log(x)/(b*x+a),x, algorithm="maxima")
[Out] (log(b*x/a + 1)*log(x) + dilog(-b*x/a))/b
```

Giac [F]

$$\int \frac{\log(x)}{a+bx}\,dx = \int \frac{\log(x)}{bx+a}\,dx$$

```
[In] integrate(log(x)/(b*x+a),x, algorithm="giac")
[Out] integrate(log(x)/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)}{a+bx}\,dx = \int \frac{\ln(x)}{a+b\,x}\,dx$$

```
[In] int(log(x)/(a + b*x),x)
[Out] int(log(x)/(a + b*x), x)
```

3.617 $\int \frac{\log(x)}{(a+bx)^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{\log(x)}{(a+bx)^2} dx = \frac{x\log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab}$$

```
[Out] x*ln(x)/a/(b*x+a)-ln(b*x+a)/a/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2351, 31}

$$\int \frac{\log(x)}{(a+bx)^2} dx = \frac{x\log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab}$$

```
[In] Int[Log[x]/(a + b*x)^2,x]
```

```
[Out] (x*Log[x])/(a*(a + b*x)) - Log[a + b*x]/(a*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\log(x)}{a(a+bx)} - \frac{\int \frac{1}{a+bx}\,dx}{a}\\ &= \frac{x\log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = \frac{\frac{x\log(x)}{a+bx} - \frac{\log(a+bx)}{b}}{a}$$

```
[In] Integrate[Log[x]/(a + b*x)^2,x]
[Out] ((x*Log[x])/(a + b*x) - Log[a + b*x]/b)/a
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{x\ln(x)}{a(bx+a)} - \frac{\ln(bx+a)}{ab}$	30
norman	$\frac{x\ln(x)}{a(bx+a)} - \frac{\ln(bx+a)}{ab}$	30
parts	$-\frac{\ln(x)}{b(bx+a)} + \frac{\frac{\ln(x)}{a} - \frac{\ln(bx+a)}{a}}{b}$	38
parallelrisch	$\frac{-b\ln(bx+a)x+bx\ln(x)-\ln(bx+a)a}{ab(bx+a)}$	40
risch	$-\frac{\ln(x)}{b(bx+a)} - \frac{\ln(bx+a)}{ab} + \frac{\ln(-x)}{ab}$	41

```
[In] int(ln(x)/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] x*ln(x)/a/(b*x+a)-ln(b*x+a)/a/b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = \frac{bx\log(x) - (bx+a)\log(bx+a)}{ab^2x + a^2b}$$

```
[In] integrate(log(x)/(b*x+a)^2,x, algorithm="fricas")
[Out] (b*x*log(x) - (b*x + a)*log(b*x + a))/(a*b^2*x + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = -\frac{\log(x)}{ab+b^2x} + \frac{\log(x) - \log\left(\frac{a}{b}+x\right)}{ab}$$

```
[In] integrate(ln(x)/(b*x+a)**2,x)
[Out] -log(x)/(a*b + b**2*x) + (log(x) - log(a/b + x))/(a*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = -\frac{\frac{\log(bx+a)}{a} - \frac{\log(x)}{a}}{b} - \frac{\log(x)}{(bx+a)b}$$

```
[In] integrate(log(x)/(b*x+a)^2,x, algorithm="maxima")
[Out] -(log(b*x + a)/a - log(x)/a)/b - log(x)/((b*x + a)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(29) = 58$.

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 4.76

$$\int \frac{\log(x)}{(a+bx)^2}\,dx$$

$$= b^2\left(\frac{\log\left(\frac{(bx+a)^2|b|\left|\frac{a}{bx+a}-1\right|}{b^2|bx+a|}\right)}{ab^3} + \frac{\log\left(-\frac{a+\frac{(bx+a)b\left(\frac{a}{bx+a}-1\right)-ab}{b}}{b}\right)}{\left((bx+a)\left(\frac{a}{bx+a}-1\right)-a\right)b^3} - \frac{\log\left(\left|-(bx+a)\left(\frac{a}{bx+a}-1\right)+a\right|\right)}{ab^3}\right)$$

```
[In] integrate(log(x)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] b^2*(log((b*x + a)^2*abs(b)*abs(a/(b*x + a) - 1)/(b^2*abs(b*x + a)))/(a*b^3
) + log(-(a + ((b*x + a)*b*(a/(b*x + a) - 1) - a*b)/b)/b)/(((b*x + a)*(a/(b
*x + a) - 1) - a)*b^3) - log(abs(-(b*x + a)*(a/(b*x + a) - 1) + a))/(a*b^3)
)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = \frac{x^2\,\ln(x)}{a\,\left(b\,x^2+a\,x\right)} - \frac{\ln(a+b\,x)}{a\,b}$$

```
[In] int(log(x)/(a + b*x)^2,x)
```

```
[Out] (x^2*log(x))/(a*(a*x + b*x^2)) - log(a + b*x)/(a*b)
```

3.618 $\int \frac{\log^n(x)}{x}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \frac{\log^n(x)}{x}\,dx = \frac{\log^{1+n}(x)}{1+n}$$

```
[Out] ln(x)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2339, 30}

$$\int \frac{\log^n(x)}{x}\,dx = \frac{\log^{n+1}(x)}{n+1}$$

```
[In] Int[Log[x]^n/x,x]
```

```
[Out] Log[x]^(1 + n)/(1 + n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^n\,dx, x, \log(x)\right)\\ &= \frac{\log^{1+n}(x)}{1+n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(x)}{x}\,dx = \frac{\log^{1+n}(x)}{1+n}$$

```
[In] Integrate[Log[x]^n/x,x]
[Out] Log[x]^(1 + n)/(1 + n)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\ln(x)^{1+n}}{1+n}$	13
default	$\frac{\ln(x)^{1+n}}{1+n}$	13
risch	$\frac{\ln(x)\ln(x)^{n}}{1+n}$	13
norman	$\frac{\ln(x)\mathrm{e}^{n\ln(\ln(x))}}{1+n}$	15

```
[In] int(ln(x)^n/x,x,method=_RETURNVERBOSE)
[Out] ln(x)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(x)}{x}\,dx = \frac{\log(x)^n\log(x)}{n+1}$$

```
[In] integrate(log(x)^n/x,x, algorithm="fricas")
[Out] log(x)^n*log(x)/(n + 1)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\log^n(x)}{x}\,dx = \begin{cases} \frac{\log(x)^{n+1}}{n+1} & \text{for}\ n \neq -1 \\ \log(\log(x)) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(x)**n/x,x)
[Out] Piecewise((log(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(log(x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(x)}{x}\,dx = \frac{\log(x)^{n+1}}{n+1}$$

```
[In] integrate(log(x)^n/x,x, algorithm="maxima")
[Out] log(x)^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(x)}{x}\,dx = \frac{\log(x)^{n+1}}{n+1}$$

```
[In] integrate(log(x)^n/x,x, algorithm="giac")
[Out] log(x)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\log^n(x)}{x} \, dx = \begin{cases} \ln(\ln(x)) & \text{if } n = -1 \\ \frac{\ln(x)^{n+1}}{n+1} & \text{if } n \neq -1 \end{cases}$$

```
[In] int(log(x)^n/x,x)
[Out] piecewise(n == -1, log(log(x)), n ~= -1, log(x)^(n + 1)/(n + 1))
```

3.619 $\int \frac{(a+b\log(x))^n}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(a+b\log(x))^{1+n}}{b(1+n)}$$

```
[Out] (a+b*ln(x))^(1+n)/b/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2339, 30}

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(a+b\log(x))^{n+1}}{b(n+1)}$$

```
[In] Int[(a + b*Log[x])^n/x,x]
```

```
[Out] (a + b*Log[x])^(1 + n)/(b*(1 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int x^n\,dx, x, a+b\log(x)\right)}{b} \\
&= \frac{(a+b\log(x))^{1+n}}{b(1+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$
\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(a+b\log(x))^{1+n}}{b(1+n)}
$$

```
[In] Integrate[(a + b*Log[x])^n/x,x]
[Out] (a + b*Log[x])^(1 + n)/(b*(1 + n))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{(a+b\ln(x))^{1+n}}{b(1+n)}$	20
default	$\frac{(a+b\ln(x))^{1+n}}{b(1+n)}$	20
risch	$\frac{(a+b\ln(x))(a+b\ln(x))^{n}}{b(1+n)}$	24
parallelrisch	$\frac{\ln(x)(a+b\ln(x))^{n}b+(a+b\ln(x))^{n}a}{b(1+n)}$	33
norman	$\frac{\ln(x)\,\mathrm{e}^{n\ln(a+b\ln(x))}}{1+n}+\frac{a\,\mathrm{e}^{n\ln(a+b\ln(x))}}{b(1+n)}$	40

```
[In] int((a+b*ln(x))^n/x,x,method=_RETURNVERBOSE)
[Out] (a+b*ln(x))^(1+n)/b/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(b\log{(x)}+a)(b\log{(x)}+a)^n}{bn+b}$$

```
[In] integrate((a+b*log(x))^n/x,x, algorithm="fricas")
[Out] (b*log(x) + a)*(b*log(x) + a)^n/(b*n + b)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{(a+b\log(x))^n}{x}\,dx = -\begin{cases} -a^n\log{(x)} & \text{for } b=0 \\ -\frac{\begin{cases} \frac{(a+b\log{(x)})^{n+1}}{n+1} & \text{for } n\neq -1 \\ \log{(a+b\log{(x)})} & \text{otherwise} \end{cases}}{b} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*ln(x))**n/x,x)
[Out] -Piecewise((-a**n*log(x), Eq(b, 0)), (-Piecewise(((a + b*log(x))**(n + 1)/(
n + 1), Ne(n, -1)), (log(a + b*log(x)), True))/b, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(b\log{(x)}+a)^{n+1}}{b(n+1)}$$

```
[In] integrate((a+b*log(x))^n/x,x, algorithm="maxima")
[Out] (b*log(x) + a)^(n + 1)/(b*(n + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(b\log{(x)}+a)^{n+1}}{b(n+1)}$$

```
[In] integrate((a+b*log(x))^n/x,x, algorithm="giac")
[Out] (b*log(x) + a)^(n + 1)/(b*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(a+b\,\ln{(x)})^{n+1}}{b\,(n+1)}$$

```
[In] int((a + b*log(x))^n/x,x)
[Out] (a + b*log(x))^(n + 1)/(b*(n + 1))
```

3.620 $\int \frac{1}{x(a+b\log(x))}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 11

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log(a+b\log(x))}{b}$$

```
[Out] ln(a+b*ln(x))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2339, 29}

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log(a+b\log(x))}{b}$$

```
[In] Int[1/(x*(a + b*Log[x])),x]
```

```
[Out] Log[a + b*Log[x]]/b
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x} \, dx, x, a + b\log(x)\right)}{b}$$
$$= \frac{\log(a + b\log(x))}{b}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a + b\log(x))} \, dx = \frac{\log(a + b\log(x))}{b}$$

```
[In] Integrate[1/(x*(a + b*Log[x])),x]
[Out] Log[a + b*Log[x]]/b
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\ln(x))}{b}$	12
default	$\frac{\ln(a+b\ln(x))}{b}$	12
norman	$\frac{\ln(a+b\ln(x))}{b}$	12
parallelrisch	$\frac{\ln(a+b\ln(x))}{b}$	12
risch	$\frac{\ln\left(\ln(x)+\frac{a}{b}\right)}{b}$	14

```
[In] int(1/x/(a+b*ln(x)),x,method=_RETURNVERBOSE)
[Out] ln(a+b*ln(x))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log\left(b\log\left(x\right)+a\right)}{b}$$

```
[In] integrate(1/x/(a+b*log(x)),x, algorithm="fricas")
[Out] log(b*log(x) + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log\left(\frac{a}{b}+\log\left(x\right)\right)}{b}$$

```
[In] integrate(1/x/(a+b*ln(x)),x)
[Out] log(a/b + log(x))/b
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log\left(b\log\left(x\right)+a\right)}{b}$$

```
[In] integrate(1/x/(a+b*log(x)),x, algorithm="maxima")
[Out] log(b*log(x) + a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(11) = 22.

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log\left(\frac{1}{4}\,\pi^2b^2(\mathrm{sgn}(x)-1)^2+\left(b\log\left(|x|\right)+a\right)^2\right)}{2\,b}$$

```
[In] integrate(1/x/(a+b*log(x)),x, algorithm="giac")
[Out] 1/2*log(1/4*pi^2*b^2*(sgn(x) - 1)^2 + (b*log(abs(x)) + a)^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\log(x))}\,dx=\frac{\ln\left(a+b\,\ln\left(x\right)\right)}{b}$$

```
[In] int(1/(x*(a + b*log(x))),x)
[Out] log(a + b*log(x))/b
```

3.621 $\int \frac{(a+b\log(x))^{-n}}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{(a+b\log(x))^{-n}}{x} dx = \frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

```
[Out] (a+b*ln(x))^(1-n)/b/(1-n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2339, 30}

$$\int \frac{(a+b\log(x))^{-n}}{x} dx = \frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

```
[In] Int[1/(x*(a + b*Log[x])^n),x]
```

```
[Out] (a + b*Log[x])^(1 - n)/(b*(1 - n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^{-n}\,dx, x, a+b\log(x)\right)}{b}\\ &= \frac{(a+b\log(x))^{1-n}}{b(1-n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = \frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

```
[In] Integrate[1/(x*(a + b*Log[x])^n),x]
[Out] (a + b*Log[x])^(1 - n)/(b*(1 - n))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{(a+b\ln(x))^{1-n}}{b(1-n)}$	24
default	$\frac{(a+b\ln(x))^{1-n}}{b(1-n)}$	24
risch	$-\frac{(a+b\ln(x))(a+b\ln(x))^{-n}}{b(-1+n)}$	27
parallelrisch	$\frac{(-\ln(x)bn-an)(a+b\ln(x))^{-n}}{nb(-1+n)}$	34
norman	$\left(-\frac{\ln(x)}{-1+n}-\frac{a}{b(-1+n)}\right)\mathrm{e}^{-n\ln(a+b\ln(x))}$	35

```
[In] int(1/x/((a+b*ln(x))^n),x,method=_RETURNVERBOSE)
[Out] (a+b*ln(x))^(1-n)/b/(1-n)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = -\frac{b\log(x)+a}{(bn-b)(b\log(x)+a)^n}$$

```
[In] integrate(1/x/((a+b*log(x))^n),x, algorithm="fricas")
[Out] -(b*log(x) + a)/((b*n - b)*(b*log(x) + a)^n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(14) = 28.

Time = 5.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = \begin{cases} \frac{\log(x)}{a} & \text{for } b=0 \wedge n=1 \\ a^{-n}\log(x) & \text{for } b=0 \\ \frac{\log\left(\frac{a}{b}+\log(x)\right)}{b} & \text{for } n=1 \\ -\frac{a}{bn(a+b\log(x))^n-b(a+b\log(x))^n} - \frac{b\log(x)}{bn(a+b\log(x))^n-b(a+b\log(x))^n} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/((a+b*ln(x))**n),x)
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 1)), (log(x)/a**n, Eq(b, 0)), (log(a/
b + log(x))/b, Eq(n, 1)), (-a/(b*n*(a + b*log(x))**n - b*(a + b*log(x))**n)
 - b*log(x)/(b*n*(a + b*log(x))**n - b*(a + b*log(x))**n), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x/((a+b*log(x))^n),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-n>0)', see `assume?` for more deta
ils)Is
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = -\frac{(b\log(x)+a)^{-n+1}}{b(n-1)}$$

```
[In] integrate(1/x/((a+b*log(x))^n),x, algorithm="giac")
[Out] -(b*log(x) + a)^(-n + 1)/(b*(n - 1))
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = -\frac{(a+b\,\ln(x))^{1-n}}{b\,(n-1)}$$

```
[In] int(1/(x*(a + b*log(x))^n),x)
[Out] -(a + b*log(x))^(1 - n)/(b*(n - 1))
```

3.622 $\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = \operatorname{arctanh}\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

```
[Out] arctanh(ln(x)/(a^2+ln(x)^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {223, 212}

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = \operatorname{arctanh}\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

```
[In] Int[1/(x*Sqrt[a^2 + Log[x]^2]),x]
```

```
[Out] ArcTanh[Log[x]/Sqrt[a^2 + Log[x]^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{a^2+x^2}}\,dx, x, \log(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)\\
&= \text{arctanh}\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. $2(16) = 32$.

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.88

$$
\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = -\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)+\frac{1}{2}\log\left(1+\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)
$$

```
[In] Integrate[1/(x*Sqrt[a^2 + Log[x]^2]),x]
[Out] -1/2*Log[1 - Log[x]/Sqrt[a^2 + Log[x]^2]] + Log[1 + Log[x]/Sqrt[a^2 + Log[x
]^2]]/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\ln\left(\ln(x)+\sqrt{a^2+\ln(x)^2}\right)$	15
default	$\ln\left(\ln(x)+\sqrt{a^2+\ln(x)^2}\right)$	15

```
[In] int(1/x/(a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(ln(x)+(a^2+ln(x)^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = -\log\left(\sqrt{a^2+\log{(x)}^2}-\log{(x)}\right)$$

```
[In] integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="fricas")
[Out] -log(sqrt(a^2 + log(x)^2) - log(x))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{a^2+\log{(x)}^2}}\,dx$$

```
[In] integrate(1/x/(a**2+ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(a**2 + log(x)**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = \operatorname{arsinh}\left(\frac{\log{(x)}}{a}\right)$$

```
[In] integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="maxima")
[Out] arcsinh(log(x)/a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = -\log\left(\sqrt{a^2+\log(x)^2}-\log(x)\right)$$

```
[In] integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="giac")
[Out] -log(sqrt(a^2 + log(x)^2) - log(x))
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = \ln\left(\ln(x)+\sqrt{a^2+\ln(x)^2}\right)$$

```
[In] int(1/(x*(log(x)^2 + a^2)^(1/2)),x)
[Out] log(log(x) + (log(x)^2 + a^2)^(1/2))
```

3.623 $\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = \operatorname{arctanh}\left(\frac{\log(x)}{\sqrt{-a^2+\log^2(x)}}\right)$$

```
[Out] arctanh(ln(x)/(-a^2+ln(x)^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {223, 212}

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = \operatorname{arctanh}\left(\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}\right)$$

```
[In] Int[1/(x*Sqrt[-a^2 + Log[x]^2]),x]
```

```
[Out] ArcTanh[Log[x]/Sqrt[-a^2 + Log[x]^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{-a^2+x^2}}\,dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\log(x)}{\sqrt{-a^2+\log^2(x)}}\right)\\ &= \text{arctanh}\left(\frac{\log(x)}{\sqrt{-a^2+\log^2(x)}}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = -\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{-a^2+\log^2(x)}}\right)+\frac{1}{2}\log\left(1+\frac{\log(x)}{\sqrt{-a^2+\log^2(x)}}\right)$$

```
[In] Integrate[1/(x*Sqrt[-a^2 + Log[x]^2]),x]
[Out] -1/2*Log[1 - Log[x]/Sqrt[-a^2 + Log[x]^2]] + Log[1 + Log[x]/Sqrt[-a^2 + Log
[x]^2]]/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\ln\left(\ln(x)+\sqrt{-a^2+\ln(x)^2}\right)$	17
default	$\ln\left(\ln(x)+\sqrt{-a^2+\ln(x)^2}\right)$	17

```
[In] int(1/x/(-a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(ln(x)+(-a^2+ln(x)^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = -\log\left(\sqrt{-a^2+\log(x)^2}-\log(x)\right)$$

```
[In] integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="fricas")
[Out] -log(sqrt(-a^2 + log(x)^2) - log(x))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{-(a-\log(x))(a+\log(x))}}\,dx$$

```
[In] integrate(1/x/(-a**2+ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(-(a - log(x))*(a + log(x)))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = \log\left(2\sqrt{-a^2+\log(x)^2}+2\log(x)\right)$$

```
[In] integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="maxima")
[Out] log(2*sqrt(-a^2 + log(x)^2) + 2*log(x))
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = \ln\left(\ln(x)+\sqrt{\ln(x)^2-a^2}\right)$$

```
[In] int(1/(x*(log(x)^2 - a^2)^(1/2)),x)
[Out] log(log(x) + (log(x)^2 - a^2)^(1/2))
```

3.624 $\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = \arctan\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

```
[Out] arctan(ln(x)/(a^2-ln(x)^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {223, 209}

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = \arctan\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

```
[In] Int[1/(x*Sqrt[a^2 - Log[x]^2]),x]
```

```
[Out] ArcTan[Log[x]/Sqrt[a^2 - Log[x]^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{a^2-x^2}}\,dx, x, \log(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)\\
&= \arctan\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = \arctan\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

```
[In] Integrate[1/(x*Sqrt[a^2 - Log[x]^2]),x]
[Out] ArcTan[Log[x]/Sqrt[a^2 - Log[x]^2]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\arctan\left(\frac{\ln(x)}{\sqrt{a^2-\ln(x)^2}}\right)$	17
default	$\arctan\left(\frac{\ln(x)}{\sqrt{a^2-\ln(x)^2}}\right)$	17

```
[In] int(1/x/(a^2-ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan(ln(x)/(a^2-ln(x)^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = -2\arctan\left(-\frac{a-\sqrt{a^2-\log\left(x\right)^2}}{\log\left(x\right)}\right)$$

```
[In] integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="fricas")
[Out] -2*arctan(-(a - sqrt(a^2 - log(x)^2))/log(x))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{\left(a-\log\left(x\right)\right)\left(a+\log\left(x\right)\right)}}\,dx$$

```
[In] integrate(1/x/(a**2-ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt((a - log(x))*(a + log(x)))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.39

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = \arcsin\left(\frac{\log\left(x\right)}{a}\right)$$

```
[In] integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="maxima")
[Out] arcsin(log(x)/a)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = \arcsin\left(\frac{\log(x)}{a}\right)\mathrm{sgn}(a)$$

```
[In] integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="giac")
[Out] arcsin(log(x)/a)*sgn(a)
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = \mathrm{atan}\left(\frac{\ln(x)}{\sqrt{a^2-\ln(x)^2}}\right)$$

```
[In] int(1/(x*(a^2 - log(x)^2)^(1/2)),x)
[Out] atan(log(x)/(a^2 - log(x)^2)^(1/2))
```

3.625 $\int \frac{1}{x \log(x)\sqrt{a^2+\log^2(x)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 22

$$\int \frac{1}{x \log(x)\sqrt{a^2+\log^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

```
[Out] -arctanh((a^2+ln(x)^2)^(1/2)/a)/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {272, 65, 213}

$$\int \frac{1}{x \log(x)\sqrt{a^2+\log^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

```
[In] Int[1/(x*Log[x]*Sqrt[a^2 + Log[x]^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[a^2 + Log[x]^2]/a]/a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x\sqrt{a^2+x^2}}\,dx, x, \log(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a^2+x}}\,dx, x, \log^2(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{-a^2+x^2}\,dx, x, \sqrt{a^2+\log^2(x)}\right)\\
&= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

```
[In] Integrate[1/(x*Log[x]*Sqrt[a^2 + Log[x]^2]),x]
[Out] -(ArcTanh[Sqrt[a^2 + Log[x]^2]/a]/a)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2+\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$	37
default	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2+\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$	37

```
[In] int(1/x/ln(x)/(a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2+ln(x)^2)^(1/2))/ln(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(20) = 40.

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = -\frac{\log\left(a+\sqrt{a^2+\log\left(x\right)^2}-\log\left(x\right)\right)-\log\left(-a+\sqrt{a^2+\log\left(x\right)^2}-\log\left(x\right)\right)}{a}$$

```
[In] integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="fricas")
[Out] -(log(a + sqrt(a^2 + log(x)^2) - log(x)) - log(-a + sqrt(a^2 + log(x)^2) -
log(x)))/a
```

Sympy [F]

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{a^2+\log\left(x\right)^2}\log\left(x\right)}\,dx$$

```
[In] integrate(1/x/ln(x)/(a**2+ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(a**2 + log(x)**2)*log(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = -\frac{\operatorname{arsinh}\left(\frac{a}{|\log(x)|}\right)}{a}$$

```
[In] integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="maxima")
[Out] -arcsinh(a/abs(log(x)))/a
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{a^2+\ln(x)^2}}{\sqrt{-a^2}}\right)}{\sqrt{-a^2}}$$

```
[In] int(1/(x*log(x)*(log(x)^2 + a^2)^(1/2)),x)
[Out] atan((log(x)^2 + a^2)^(1/2)/(-a^2)^(1/2))/(-a^2)^(1/2)
```

3.626 $\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 24

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

```
[Out] -arctanh((a^2-ln(x)^2)^(1/2)/a)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {272, 65, 212}

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

```
[In] Int[1/(x*Log[x]*Sqrt[a^2 - Log[x]^2]),x]
[Out] -(ArcTanh[Sqrt[a^2 - Log[x]^2]/a]/a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{x\sqrt{a^2-x^2}}\,dx, x, \log(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a^2-xx}}\,dx, x, \log^2(x)\right)\\ &= -\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, \sqrt{a^2-\log^2(x)}\right)\\ &= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

```
[In] Integrate[1/(x*Log[x]*Sqrt[a^2 - Log[x]^2]),x]
[Out] -(ArcTanh[Sqrt[a^2 - Log[x]^2]/a]/a)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2-\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$	39
default	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2-\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$	39

```
[In] int(1/x/ln(x)/(a^2-ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2-ln(x)^2)^(1/2))/ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx=\frac{\log\left(-\frac{a-\sqrt{a^2-\log(x)^2}}{\log(x)}\right)}{a}$$

```
[In] integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="fricas")
[Out] log(-(a - sqrt(a^2 - log(x)^2))/log(x))/a
```

Sympy [F]

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx=\int \frac{1}{x\sqrt{(a-\log{(x)})\,(a+\log{(x)})}\log{(x)}}\,dx$$

```
[In] integrate(1/x/ln(x)/(a**2-ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt((a - log(x))*(a + log(x)))*log(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = -\frac{\log\left(\frac{2\,a^2}{|\log(x)|}+\frac{2\,\sqrt{a^2-\log(x)^2}a}{|\log(x)|}\right)}{a}$$

```
[In] integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="maxima")
[Out] -log(2*a^2/abs(log(x)) + 2*sqrt(a^2 - log(x)^2)*a/abs(log(x)))/a
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{a^2-\ln(x)^2}}{a}\right)}{a}$$

```
[In] int(1/(x*log(x)*(a^2 - log(x)^2)^(1/2)),x)
[Out] -atanh((a^2 - log(x)^2)^(1/2)/a)/a
```

3.627 $\int \frac{1}{x \log(x)\sqrt{-a^2+\log^2(x)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 23

$$\int \frac{1}{x \log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \frac{\arctan\left(\frac{\sqrt{-a^2+\log^2(x)}}{a}\right)}{a}$$

```
[Out] arctan((-a^2+ln(x)^2)^(1/2)/a)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {272, 65, 209}

$$\int \frac{1}{x \log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \frac{\arctan\left(\frac{\sqrt{\log^2(x)-a^2}}{a}\right)}{a}$$

```
[In] Int[1/(x*Log[x]*Sqrt[-a^2 + Log[x]^2]),x]
```

```
[Out] ArcTan[Sqrt[-a^2 + Log[x]^2]/a]/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx, x, \log(x)\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{-a^2+x}}\,dx, x, \log^2(x)\right) \\
&= \text{Subst}\left(\int \frac{1}{a^2+x^2}\,dx, x, \sqrt{-a^2+\log^2(x)}\right) \\
&= \frac{\arctan\left(\frac{\sqrt{-a^2+\log^2(x)}}{a}\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$
\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \frac{\arctan\left(\frac{\sqrt{-a^2+\log^2(x)}}{a}\right)}{a}
$$

```
[In] Integrate[1/(x*Log[x]*Sqrt[-a^2 + Log[x]^2]),x]
[Out] ArcTan[Sqrt[-a^2 + Log[x]^2]/a]/a
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{-2a^{2}+2\sqrt{-a^{2}}\,\sqrt{-a^{2}+\ln(x)^{2}}}{\ln(x)}\right)}{\sqrt{-a^{2}}}$	43
default	$-\frac{\ln\left(\frac{-2a^{2}+2\sqrt{-a^{2}}\,\sqrt{-a^{2}+\ln(x)^{2}}}{\ln(x)}\right)}{\sqrt{-a^{2}}}$	43

```
[In] int(1/x/ln(x)/(-a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/(-a^2)^(1/2)*ln((-2*a^2+2*(-a^2)^(1/2)*(-a^2+ln(x)^2)^(1/2))/ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{-a^2+\log(x)^2}-\log(x)}{a}\right)}{a}$$

```
[In] integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="fricas")
[Out] 2*arctan((sqrt(-a^2 + log(x)^2) - log(x))/a)/a
```

Sympy [F]

$$\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{-(a-\log(x))(a+\log(x))}\log(x)}\,dx$$

```
[In] integrate(1/x/ln(x)/(-a**2+ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(-(a - log(x))*(a + log(x)))*log(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx = -\frac{\arcsin\left(\frac{a}{|\log(x)|}\right)}{a}$$

```
[In] integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="maxima")
[Out] -arcsin(a/abs(log(x)))/a
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \frac{\arctan\left(\frac{\sqrt{-a^2+\log(x)^2}}{a}\right)}{a}$$

```
[In] integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(-a^2 + log(x)^2)/a)/a
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{\ln(x)^2-a^2}}{\sqrt{a^2}}\right)}{\sqrt{a^2}}$$

```
[In] int(1/(x*log(x)*(log(x)^2 - a^2)^(1/2)),x)
[Out] atan((log(x)^2 - a^2)^(1/2)/(a^2)^(1/2))/(a^2)^(1/2)
```

3.628 $\int \frac{\log(\log(x))}{x} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \frac{\log(\log(x))}{x} \, dx = -\log(x) + \log(x)\log(\log(x))$$

```
[Out] -ln(x)+ln(x)*ln(ln(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{2601\}$

$$\int \frac{\log(\log(x))}{x} \, dx = \log(x)\log(\log(x)) - \log(x)$$

```
[In] Int[Log[Log[x]]/x,x]
```

```
[Out] -Log[x] + Log[x]*Log[Log[x]]
```

Rule 2601

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\text{integral} = -\log(x) + \log(x)\log(\log(x))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x} \, dx = -\log(x) + \log(x)\log(\log(x))$$

```
[In] Integrate[Log[Log[x]]/x,x]
[Out] -Log[x] + Log[x]*Log[Log[x]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\ln(x)+\ln(x)\ln(\ln(x))$	12
default	$-\ln(x)+\ln(x)\ln(\ln(x))$	12
norman	$-\ln(x)+\ln(x)\ln(\ln(x))$	12
risch	$-\ln(x)+\ln(x)\ln(\ln(x))$	12

```
[In] int(ln(ln(x))/x,x,method=_RETURNVERBOSE)
[Out] -ln(x)+ln(x)*ln(ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x} \, dx = \log(x)\log(\log(x)) - \log(x)$$

```
[In] integrate(log(log(x))/x,x, algorithm="fricas")
[Out] log(x)*log(log(x)) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\log(\log(x))}{x}\,dx = \log\left(x\right)\log\left(\log\left(x\right)\right) - \log\left(x\right)$$

```
[In] integrate(ln(ln(x))/x,x)
[Out] log(x)*log(log(x)) - log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x}\,dx = \log\left(x\right)\log\left(\log\left(x\right)\right) - \log\left(x\right)$$

```
[In] integrate(log(log(x))/x,x, algorithm="maxima")
[Out] log(x)*log(log(x)) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x}\,dx = \log\left(x\right)\log\left(\log\left(x\right)\right) - \log\left(x\right)$$

```
[In] integrate(log(log(x))/x,x, algorithm="giac")
[Out] log(x)*log(log(x)) - log(x)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{\log(\log(x))}{x}\,dx = \ln\left(x\right)\,\left(\ln\left(\ln\left(x\right)\right) - 1\right)$$

```
[In] int(log(log(x))/x,x)
[Out] log(x)*(log(log(x)) - 1)
```

3.629 $\int \frac{\log^2(\log(x))}{x} dx$

Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \frac{\log^2(\log(x))}{x} dx = 2\log(x) - 2\log(x)\log(\log(x)) + \log(x)\log^2(\log(x))$$

```
[Out] 2*ln(x)-2*ln(x)*ln(ln(x))+ln(x)*ln(ln(x))^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2333, 2332}

$$\int \frac{\log^2(\log(x))}{x} dx = \log(x)\log^2(\log(x)) - 2\log(x)\log(\log(x)) + 2\log(x)$$

```
[In] Int[Log[Log[x]]^2/x,x]
[Out] 2*Log[x] - 2*Log[x]*Log[Log[x]] + Log[x]*Log[Log[x]]^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \log^2(x)\,dx, x, \log(x)\right) \\ &= \log(x)\log^2(\log(x)) - 2\text{Subst}(\int \log(x)\,dx, x, \log(x)) \\ &= 2\log(x) - 2\log(x)\log(\log(x)) + \log(x)\log^2(\log(x)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(\log(x))}{x}\,dx = 2\log(x) - 2\log(x)\log(\log(x)) + \log(x)\log^2(\log(x))$$

```
[In] Integrate[Log[Log[x]]^2/x,x]
[Out] 2*Log[x] - 2*Log[x]*Log[Log[x]] + Log[x]*Log[Log[x]]^2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$2\ln(x) - 2\ln(x)\ln(\ln(x)) + \ln(x)\ln(\ln(x))^2$	21
default	$2\ln(x) - 2\ln(x)\ln(\ln(x)) + \ln(x)\ln(\ln(x))^2$	21
norman	$2\ln(x) - 2\ln(x)\ln(\ln(x)) + \ln(x)\ln(\ln(x))^2$	21
risch	$2\ln(x) - 2\ln(x)\ln(\ln(x)) + \ln(x)\ln(\ln(x))^2$	21

```
[In] int(ln(ln(x))^2/x,x,method=_RETURNVERBOSE)
[Out] 2*ln(x)-2*ln(x)*ln(ln(x))+ln(x)*ln(ln(x))^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(\log(x))}{x}\,dx = \log(x)\log(\log(x))^2 - 2\,\log(x)\log(\log(x)) + 2\,\log(x)$$

```
[In] integrate(log(log(x))^2/x,x, algorithm="fricas")
[Out] log(x)*log(log(x))^2 - 2*log(x)*log(log(x)) + 2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\log^2(\log(x))}{x}\,dx = \log(x)\log(\log(x))^2 - 2\log(x)\log(\log(x)) + 2\log(x)$$

```
[In] integrate(ln(ln(x))**2/x,x)
[Out] log(x)*log(log(x))**2 - 2*log(x)*log(log(x)) + 2*log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{\log^2(\log(x))}{x}\,dx = \left(\log(\log(x))^2 - 2\,\log(\log(x)) + 2\right)\log(x)$$

```
[In] integrate(log(log(x))^2/x,x, algorithm="maxima")
[Out] (log(log(x))^2 - 2*log(log(x)) + 2)*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(\log(x))}{x}\,dx = \log(x)\log(\log(x))^2 - 2\,\log(x)\log(\log(x)) + 2\,\log(x)$$

```
[In] integrate(log(log(x))^2/x,x, algorithm="giac")
[Out] log(x)*log(log(x))^2 - 2*log(x)*log(log(x)) + 2*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{\log^2(\log(x))}{x}\,dx = \ln(x)\ \left(\ln(\ln(x))^2 - 2\ \ln(\ln(x)) + 2\right)$$

```
[In] int(log(log(x))^2/x,x)
[Out] log(x)*(log(log(x))^2 - 2*log(log(x)) + 2)
```

3.630 $\int \frac{\log^3(\log(x))}{x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 29

$$\int \frac{\log^3(\log(x))}{x} \, dx = -6\log(x) + 6\log(x)\log(\log(x)) - 3\log(x)\log^2(\log(x)) + \log(x)\log^3(\log(x))$$

```
[Out] -6*ln(x)+6*ln(x)*ln(ln(x))-3*ln(x)*ln(ln(x))^2+ln(x)*ln(ln(x))^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2333, 2332}

$$\int \frac{\log^3(\log(x))}{x} \, dx = \log(x)\log^3(\log(x)) - 3\log(x)\log^2(\log(x)) + 6\log(x)\log(\log(x)) - 6\log(x)$$

```
[In] Int[Log[Log[x]]^3/x,x]
```

```
[Out] -6*Log[x] + 6*Log[x]*Log[Log[x]] - 3*Log[x]*Log[Log[x]]^2 + Log[x]*Log[Log[
x]]^3
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
```

```
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \log^3(x)\,dx, x, \log(x)\right)\\
&= \log(x)\log^3(\log(x)) - 3\text{Subst}\left(\int \log^2(x)\,dx, x, \log(x)\right)\\
&= -3\log(x)\log^2(\log(x)) + \log(x)\log^3(\log(x)) + 6\text{Subst}(\int \log(x)\,dx, x, \log(x))\\
&= -6\log(x) + 6\log(x)\log(\log(x)) - 3\log(x)\log^2(\log(x)) + \log(x)\log^3(\log(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\log^3(\log(x))}{x}\,dx = -6\log(x) + 6\log(x)\log(\log(x)) - 3\log(x)\log^2(\log(x)) + \log(x)\log^3(\log(x))$$

```
[In] Integrate[Log[Log[x]]^3/x,x]
[Out] -6*Log[x] + 6*Log[x]*Log[Log[x]] - 3*Log[x]*Log[Log[x]]^2 + Log[x]*Log[Log[
x]]^3
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)\ln(\ln(x))^2+\ln(x)\ln(\ln(x))^3$	30
default	$-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)\ln(\ln(x))^2+\ln(x)\ln(\ln(x))^3$	30
norman	$-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)\ln(\ln(x))^2+\ln(x)\ln(\ln(x))^3$	30
risch	$-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)\ln(\ln(x))^2+\ln(x)\ln(\ln(x))^3$	30

```
[In] int(ln(ln(x))^3/x,x,method=_RETURNVERBOSE)
[Out] -6*ln(x)+6*ln(x)*ln(ln(x))-3*ln(x)*ln(ln(x))^2+ln(x)*ln(ln(x))^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\log^3(\log(x))}{x}\,dx = \log(x)\log(\log(x))^3 - 3\log(x)\log(\log(x))^2 + 6\log(x)\log(\log(x)) - 6\log(x)$$

```
[In] integrate(log(log(x))^3/x,x, algorithm="fricas")
[Out] log(x)*log(log(x))^3 - 3*log(x)*log(log(x))^2 + 6*log(x)*log(log(x)) - 6*lo
g(x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{\log^3(\log(x))}{x}\,dx = \log(x)\log(\log(x))^3 - 3\log(x)\log(\log(x))^2 + 6\log(x)\log(\log(x)) - 6\log(x)$$

```
[In] integrate(ln(ln(x))**3/x,x)
[Out] log(x)*log(log(x))**3 - 3*log(x)*log(log(x))**2 + 6*log(x)*log(log(x)) - 6*
log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{\log^3(\log(x))}{x}\,dx = \left(\log(\log(x))^3 - 3\log(\log(x))^2 + 6\log(\log(x)) - 6\right)\log(x)$$

```
[In] integrate(log(log(x))^3/x,x, algorithm="maxima")
[Out] (log(log(x))^3 - 3*log(log(x))^2 + 6*log(log(x)) - 6)*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\log^3(\log(x))}{x} \, dx = \log(x) \log(\log(x))^3 - 3 \log(x) \log(\log(x))^2 + 6 \log(x) \log(\log(x)) - 6 \log(x)$$

```
[In] integrate(log(log(x))^3/x,x, algorithm="giac")
[Out] log(x)*log(log(x))^3 - 3*log(x)*log(log(x))^2 + 6*log(x)*log(log(x)) - 6*lo
g(x)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\log^3(\log(x))}{x} \, dx = \ln(x) \ln(\ln(x))^3 - 3 \ln(x) \ln(\ln(x))^2 + 6 \ln(x) \ln(\ln(x)) - 6 \ln(x)$$

```
[In] int(log(log(x))^3/x,x)
[Out] 6*log(log(x))*log(x) - 6*log(x) - 3*log(log(x))^2*log(x) + log(log(x))^3*lo
g(x)
```

3.631 $\int \frac{\log^4(\log(x))}{x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \frac{\log^4(\log(x))}{x} \, dx = 24\log(x) - 24\log(x)\log(\log(x)) + 12\log(x)\log^2(\log(x)) - 4\log(x)\log^3(\log(x)) + \log(x)\log^4(\log(x))$$

```
[Out] 24*ln(x)-24*ln(x)*ln(ln(x))+12*ln(x)*ln(ln(x))^2-4*ln(x)*ln(ln(x))^3+ln(x)*
ln(ln(x))^4
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2333, 2332}

$$\int \frac{\log^4(\log(x))}{x} \, dx = \log(x)\log^4(\log(x)) - 4\log(x)\log^3(\log(x)) + 12\log(x)\log^2(\log(x)) - 24\log(x)\log(\log(x)) + 24\log(x)$$

```
[In] Int[Log[Log[x]]^4/x,x]
```

```
[Out] 24*Log[x] - 24*Log[x]*Log[Log[x]] + 12*Log[x]*Log[Log[x]]^2 - 4*Log[x]*Log[
Log[x]]^3 + Log[x]*Log[Log[x]]^4
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \log^4(x)\,dx, x, \log(x)\right)\\
&= \log(x)\log^4(\log(x)) - 4\text{Subst}\left(\int \log^3(x)\,dx, x, \log(x)\right)\\
&= -4\log(x)\log^3(\log(x)) + \log(x)\log^4(\log(x)) + 12\text{Subst}\left(\int \log^2(x)\,dx, x, \log(x)\right)\\
&= 12\log(x)\log^2(\log(x)) - 4\log(x)\log^3(\log(x))\\
&\quad + \log(x)\log^4(\log(x)) - 24\text{Subst}(\int \log(x)\,dx, x, \log(x))\\
&= 24\log(x) - 24\log(x)\log(\log(x)) + 12\log(x)\log^2(\log(x))\\
&\quad - 4\log(x)\log^3(\log(x)) + \log(x)\log^4(\log(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\log^4(\log(x))}{x}\,dx = 24\log(x) - 24\log(x)\log(\log(x)) + 12\log(x)\log^2(\log(x)) - 4\log(x)\log^3(\log(x)) + \log(x)\log^4(\log(x))$$

```
[In] Integrate[Log[Log[x]]^4/x,x]
```

```
[Out] 24*Log[x] - 24*Log[x]*Log[Log[x]] + 12*Log[x]*Log[Log[x]]^2 - 4*Log[x]*Log[
Log[x]]^3 + Log[x]*Log[Log[x]]^4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

method	result
derivativedivides	$24\ln(x)-24\ln(x)\ln(\ln(x))+12\ln(x)\ln(\ln(x))^2-4\ln(x)\ln(\ln(x))^3+\ln(x)\ln(\ln$
default	$24\ln(x)-24\ln(x)\ln(\ln(x))+12\ln(x)\ln(\ln(x))^2-4\ln(x)\ln(\ln(x))^3+\ln(x)\ln(\ln$
norman	$24\ln(x)-24\ln(x)\ln(\ln(x))+12\ln(x)\ln(\ln(x))^2-4\ln(x)\ln(\ln(x))^3+\ln(x)\ln(\ln$
risch	$24\ln(x)-24\ln(x)\ln(\ln(x))+12\ln(x)\ln(\ln(x))^2-4\ln(x)\ln(\ln(x))^3+\ln(x)\ln(\ln$

```
[In] int(ln(ln(x))^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] 24*ln(x)-24*ln(x)*ln(ln(x))+12*ln(x)*ln(ln(x))^2-4*ln(x)*ln(ln(x))^3+ln(x)*
ln(ln(x))^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\log^4(\log(x))}{x}\,dx = \log(x)\log(\log(x))^4 - 4\log(x)\log(\log(x))^3 + 12\log(x)\log(\log(x))^2 - 24\log(x)\log(\log(x)) + 24\log(x)$$

```
[In] integrate(log(log(x))^4/x,x, algorithm="fricas")
```

```
[Out] log(x)*log(log(x))^4 - 4*log(x)*log(log(x))^3 + 12*log(x)*log(log(x))^2 - 2
4*log(x)*log(log(x)) + 24*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{\log^4(\log(x))}{x}\,dx = \log(x)\log(\log(x))^4 - 4\log(x)\log(\log(x))^3 + 12\log(x)\log(\log(x))^2 - 24\log(x)\log(\log(x)) + 24\log(x)$$

```
[In] integrate(ln(ln(x))**4/x,x)
```

```
[Out] log(x)*log(log(x))**4 - 4*log(x)*log(log(x))**3 + 12*log(x)*log(log(x))**2
- 24*log(x)*log(log(x)) + 24*log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{\log^4(\log(x))}{x}\,dx = \left(\log\left(\log\left(x\right)\right)^4 - 4\,\log\left(\log\left(x\right)\right)^3 + 12\,\log\left(\log\left(x\right)\right)^2 - 24\,\log\left(\log\left(x\right)\right) + 24\right)\log\left(x\right)$$

```
[In] integrate(log(log(x))^4/x,x, algorithm="maxima")
```

```
[Out] (log(log(x))^4 - 4*log(log(x))^3 + 12*log(log(x))^2 - 24*log(log(x)) + 24)*
log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\log^4(\log(x))}{x}\,dx = \log\left(x\right)\log\left(\log\left(x\right)\right)^4 - 4\,\log\left(x\right)\log\left(\log\left(x\right)\right)^3 + 12\,\log\left(x\right)\log\left(\log\left(x\right)\right)^2 - 24\,\log\left(x\right)\log\left(\log\left(x\right)\right) + 24\,\log\left(x\right)$$

```
[In] integrate(log(log(x))^4/x,x, algorithm="giac")
```

```
[Out] log(x)*log(log(x))^4 - 4*log(x)*log(log(x))^3 + 12*log(x)*log(log(x))^2 - 2
4*log(x)*log(log(x)) + 24*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\log^4(\log(x))}{x}\,dx = \ln\left(x\right)\,\ln\left(\ln\left(x\right)\right)^4 - 4\,\ln\left(x\right)\,\ln\left(\ln\left(x\right)\right)^3 + 12\,\ln\left(x\right)\,\ln\left(\ln\left(x\right)\right)^2 - 24\,\ln\left(x\right)\,\ln\left(\ln\left(x\right)\right) + 24\,\ln\left(x\right)$$

```
[In] int(log(log(x))^4/x,x)
```

```
[Out] 24*log(x) - 24*log(log(x))*log(x) + 12*log(log(x))^2*log(x) - 4*log(log(x))
^3*log(x) + log(log(x))^4*log(x)
```

3.632 $\int \frac{\log^n(\log(x))}{x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{\log^n(\log(x))}{x} \, dx = \Gamma(1+n,-\log(\log(x)))(-\log(\log(x)))^{-n}\log^n(\log(x))$$

```
[Out] GAMMA(1+n,-ln(ln(x)))*ln(ln(x))^n/((-ln(ln(x)))^n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2336, 2212}

$$\int \frac{\log^n(\log(x))}{x} \, dx = (-\log(\log(x)))^{-n}\log^n(\log(x))\Gamma(n+1,-\log(\log(x)))$$

```
[In] Int[Log[Log[x]]^n/x,x]
```

```
[Out] (Gamma[1 + n, -Log[Log[x]]]*Log[Log[x]]^n)/(-Log[Log[x]])^n
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \log^n(x)\,dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int e^x x^n\,dx, x, \log(\log(x))\right)\\ &= \Gamma(1+n, -\log(\log(x)))(-\log(\log(x)))^{-n}\log^n(\log(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(\log(x))}{x}\,dx = \Gamma(1+n, -\log(\log(x)))(-\log(\log(x)))^{-n}\log^n(\log(x))$$

```
[In] Integrate[Log[Log[x]]^n/x,x]
[Out] (Gamma[1 + n, -Log[Log[x]]]*Log[Log[x]]^n)/(-Log[Log[x]])^n
```

Maple [F]

$$\int \frac{\ln(\ln(x))^n}{x}dx$$

```
[In] int(ln(ln(x))^n/x,x)
[Out] int(ln(ln(x))^n/x,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{\log^n(\log(x))}{x}\,dx = e^{(-i\pi n)}\Gamma(n+1, -\log(\log(x)))$$

```
[In] integrate(log(log(x))^n/x,x, algorithm="fricas")
[Out] e^(-I*pi*n)*gamma(n + 1, -log(log(x)))
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(\log(x))}{x} \, dx = (-\log{(\log{(x)})})^{-n} \log{(\log{(x)})}^n \Gamma(n+1, -\log{(\log{(x)})})$$

```
[In] integrate(ln(ln(x))**n/x,x)
[Out] log(log(x))**n*uppergamma(n + 1, -log(log(x)))/(-log(log(x)))**n
```

Maxima [A] (verification not implemented)

none

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{\log^n(\log(x))}{x} \, dx = -(-\log{(\log{(x)})})^{-n-1} \log{(\log{(x)})}^{n+1} \Gamma(n+1, -\log{(\log{(x)})})$$

```
[In] integrate(log(log(x))^n/x,x, algorithm="maxima")
[Out] -(-log(log(x)))^(-n - 1)*log(log(x))^(n + 1)*gamma(n + 1, -log(log(x)))
```

Giac [F]

$$\int \frac{\log^n(\log(x))}{x} \, dx = \int \frac{\log{(\log{(x)})}^n}{x} \, dx$$

```
[In] integrate(log(log(x))^n/x,x, algorithm="giac")
[Out] integrate(log(log(x))^n/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(\log(x))}{x} \, dx = \frac{\ln{(\ln{(x)})}^n \, \Gamma(n+1, -\ln{(\ln{(x)})})}{(-\ln{(\ln{(x)})})^n}$$

```
[In] int(log(log(x))^n/x,x)
[Out] (log(log(x))^n*igamma(n + 1, -log(log(x))))/(-log(log(x)))^n
```

3.633 $\int \frac{\cot(x)}{\log(\sin(x))}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 4

$$\int \frac{\cot(x)}{\log(\sin(x))}\,dx = \log(\log(\sin(x)))$$

```
[Out] ln(ln(sin(x)))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4423, 2339, 29}

$$\int \frac{\cot(x)}{\log(\sin(x))}\,dx = \log(\log(\sin(x)))$$

```
[In] Int[Cot[x]/Log[Sin[x]],x]
```

```
[Out] Log[Log[Sin[x]]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 4423

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{x \log(x)}\,dx, x, \sin(x)\right) \\ &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, \log(\sin(x))\right) \\ &= \log(\log(\sin(x))) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\log(\sin(x))}\,dx = \log(\log(\sin(x)))$$

```
[In] Integrate[Cot[x]/Log[Sin[x]],x]
[Out] Log[Log[Sin[x]]]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\ln\left(\ln\left(\sin\left(x\right)\right)\right)$
default	$\ln\left(\ln\left(\sin\left(x\right)\right)\right)$
risch	$\ln\left(-\frac{i\pi\ \text{csgn}\left(i\left(\mathrm{e}^{2ix}-1\right)\right)\text{csgn}\left(i\mathrm{e}^{-ix}\right)\text{csgn}(\sin(x))}{2}-\frac{i\pi\ \text{csgn}\left(i\left(\mathrm{e}^{2ix}-1\right)\right)\text{csgn}(\sin(x))^{2}}{2}-\frac{i\pi\ \text{csgn}\left(i\mathrm{e}^{-ix}\right)\text{csgn}(\sin(x$

```
[In] int(cot(x)/ln(sin(x)),x,method=_RETURNVERBOSE)
[Out] ln(ln(sin(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\log(\sin(x))} \, dx = \log(\log(\sin(x)))$$

```
[In] integrate(cot(x)/log(sin(x)),x, algorithm="fricas")
[Out] log(log(sin(x)))
```

Sympy [F]

$$\int \frac{\cot(x)}{\log(\sin(x))} \, dx = \int \frac{\cot(x)}{\log(\sin(x))} \, dx$$

```
[In] integrate(cot(x)/ln(sin(x)),x)
[Out] Integral(cot(x)/log(sin(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\log(\sin(x))} \, dx = \log(\log(\sin(x)))$$

```
[In] integrate(cot(x)/log(sin(x)),x, algorithm="maxima")
[Out] log(log(sin(x)))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \frac{\cot(x)}{\log(\sin(x))} \, dx = \log(|\log(\sin(x))|)$$

```
[In] integrate(cot(x)/log(sin(x)),x, algorithm="giac")
[Out] log(abs(log(sin(x))))
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\log(\sin(x))}\,dx = \ln\left(\ln\left(\sin\left(x\right)\right)\right)$$

```
[In] int(cot(x)/log(sin(x)),x)
[Out] log(log(sin(x)))
```

3.634 $\int (\cos(x)+\sec(x))\tan(x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 7

$$\int (\cos(x)+\sec(x))\tan(x)\,dx=-\cos(x)+\sec(x)$$

```
[Out] -cos(x)+sec(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {4321}

$$\int (\cos(x)+\sec(x))\tan(x)\,dx=\sec(x)-\cos(x)$$

```
[In] Int[(Cos[x] + Sec[x])*Tan[x],x]
```

```
[Out] -Cos[x] + Sec[x]
```

Rule 4321

```
Int[(u_)*((A_.) + cos[(a_.) + (b_.)*(x_)]*(B_.) + (C_.)*sec[(a_.) + (b_.)*(
x_)]), x_Symbol] :> Int[ActivateTrig[u]*((C + A*Cos[a + b*x] + B*Cos[a + b*
x]^2)/Cos[a + b*x]), x] /; FreeQ[{a, b, A, B, C}, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(1+\cos^2(x)\right)\sec(x)\tan(x)\,dx\\&=-\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx,x,\cos(x)\right)\\&=-\cos(x)+\sec(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\cos(x)+\sec(x))\tan(x)\,dx=-\cos(x)+\sec(x)$$

```
[In] Integrate[(Cos[x] + Sec[x])*Tan[x],x]
[Out] -Cos[x] + Sec[x]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.43

method	result	size
default	$\frac{1}{\cos(x)}-\cos\left(x\right)$	10
parts	$\frac{1}{\cos(x)}-\cos\left(x\right)$	10
risch	$-\frac{{\mathrm e}^{3 i x}-\cos(x)-3 i \sin(x)}{2\left({\mathrm e}^{2 i x}+1\right)}$	27

```
[In] int((1/cos(x)+cos(x))*tan(x),x,method=_RETURNVERBOSE)
[Out] 1/cos(x)-cos(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int (\cos(x)+\sec(x))\tan(x)\,dx=-\frac{\cos\left(x\right)^{2}-1}{\cos\left(x\right)}$$

```
[In] integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="fricas")
[Out] -(cos(x)^2 - 1)/cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\cos(x) + \sec(x)) \tan(x)\, dx = -\cos{(x)} + \frac{1}{\cos{(x)}}$$

```
[In] integrate((1/cos(x)+cos(x))*tan(x),x)
[Out] -cos(x) + 1/cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int (\cos(x) + \sec(x)) \tan(x)\, dx = \frac{1}{\cos{(x)}} - \cos{(x)}$$

```
[In] integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="maxima")
[Out] 1/cos(x) - cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int (\cos(x) + \sec(x)) \tan(x)\, dx = \frac{1}{\cos{(x)}} - \cos{(x)}$$

```
[In] integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="giac")
[Out] 1/cos(x) - cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int (\cos(x) + \sec(x)) \tan(x)\, dx = \frac{1}{\cos{(x)}} - \cos{(x)}$$

```
[In] int(tan(x)*(cos(x) + 1/cos(x)),x)
[Out] 1/cos(x) - cos(x)
```

3.635 $\int \log(\cosh(x))\sinh(x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \log(\cosh(x))\sinh(x)\,dx = -\cosh(x) + \cosh(x)\log(\cosh(x))$$

```
[Out] -cosh(x)+cosh(x)*ln(cosh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2718, 2634}

$$\int \log(\cosh(x))\sinh(x)\,dx = \cosh(x)\log(\cosh(x)) - \cosh(x)$$

```
[In] Int[Log[Cosh[x]]*Sinh[x],x]
[Out] -Cosh[x] + Cosh[x]*Log[Cosh[x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\text{integral} = \cosh(x)\log(\cosh(x)) - \int \sinh(x)\,dx$$
$$= -\cosh(x) + \cosh(x)\log(\cosh(x))$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \log(\cosh(x))\sinh(x)\,dx = -\cosh(x) + \cosh(x)\log(\cosh(x))$$

```
[In] Integrate[Log[Cosh[x]]*Sinh[x],x]
[Out] -Cosh[x] + Cosh[x]*Log[Cosh[x]]
```

Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result
derivativedivides	$-\cosh(x) + \cosh(x)\ln(\cosh(x))$
default	$-\cosh(x) + \cosh(x)\ln(\cosh(x))$
risch	$-\frac{(1+e^{2x})e^{-x}\ln(e^{x})}{2} - \frac{\left(-i\pi\,\mathrm{csgn}\left(i\left(1+e^{2x}\right)\right)\mathrm{csgn}\left(ie^{-x}\left(1+e^{2x}\right)\right)^{2} - i\pi\,\mathrm{csgn}\left(i\left(1+e^{2x}\right)\right)\mathrm{csgn}\left(ie^{-x}\left(1+e^{2x}\right)\right)^{2}e^{2x} + i\right.}{}$

```
[In] int(ln(cosh(x))*sinh(x),x,method=_RETURNVERBOSE)
[Out] -cosh(x)+cosh(x)*ln(cosh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(11) = 22.

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 4.18

$$\int \log(\cosh(x))\sinh(x)\,dx = -\frac{\cosh(x)^2 - \left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\log(\cosh(x)) + 2\cosh(x)\sinh(x) + \sinh}{2\left(\cosh(x) + \sinh(x)\right)}$$

```
[In] integrate(log(cosh(x))*sinh(x),x, algorithm="fricas")
[Out] -1/2*(cosh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(cosh(
x)) + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \log(\cosh(x))\sinh(x)\,dx = \log\left(\cosh\left(x\right)\right)\cosh\left(x\right) - \cosh\left(x\right)$$

```
[In] integrate(ln(cosh(x))*sinh(x),x)
[Out] log(cosh(x))*cosh(x) - cosh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \log(\cosh(x))\sinh(x)\,dx = \cosh\left(x\right)\log\left(\cosh\left(x\right)\right) - \cosh\left(x\right)$$

```
[In] integrate(log(cosh(x))*sinh(x),x, algorithm="maxima")
[Out] cosh(x)*log(cosh(x)) - cosh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(11) = 22.

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.45

$$\int \log(\cosh(x))\sinh(x)\,dx = \frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}\log\left(\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}\right) - \frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}$$

```
[In] integrate(log(cosh(x))*sinh(x),x, algorithm="giac")
[Out] 1/2*(e^(2*x) + 1)*e^(-x)*log(1/2*(e^(2*x) + 1)*e^(-x)) - 1/2*(e^(2*x) + 1)*
e^(-x)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \log(\cosh(x))\sinh(x)\,dx = \cosh(x)\ \left(\ln\left(\cosh(x)\right)-1\right)$$

```
[In] int(log(cosh(x))*sinh(x),x)
[Out] cosh(x)*(log(cosh(x)) - 1)
```

3.636 $\int \log(\cosh(x)) \tanh(x) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \log(\cosh(x)) \tanh(x) \, dx = \frac{1}{2} \log^2(\cosh(x))$$

```
[Out] 1/2*ln(cosh(x))^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556, 4426, 2338}

$$\int \log(\cosh(x)) \tanh(x) \, dx = \frac{1}{2} \log^2(\cosh(x))$$

```
[In] Int[Log[Cosh[x]]*Tanh[x],x]
```

```
[Out] Log[Cosh[x]]^2/2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4426

```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Cosh[c*(a
+ b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \cosh(x)\right)$$
$$= \frac{1}{2}\log^2(\cosh(x))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \log(\cosh(x))\tanh(x)\,dx = \frac{1}{2}\log^2(\cosh(x))$$

```
[In] Integrate[Log[Cosh[x]]*Tanh[x],x]
[Out] Log[Cosh[x]]^2/2
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\ln(\cosh(x))^2}{2}$
default	$\frac{\ln(\cosh(x))^2}{2}$
risch	$(x-\ln(1+e^{2x}))\ln(e^x)+\frac{\ln(1+e^{2x})^2}{2}+\frac{i\ln(1+e^{2x})\pi\ \mathrm{csgn}(i(1+e^{2x}))\ \mathrm{csgn}(ie^{-x}(1+e^{2x}))^2}{2}+\frac{i\pi\ \mathrm{csgn}(ie^{-}$

```
[In] int(ln(cosh(x))*tanh(x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(cosh(x))^2
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cosh(x))\tanh(x)\,dx = \frac{1}{2}\log\left(\cosh\left(x\right)\right)^2$$

```
[In] integrate(log(cosh(x))*tanh(x),x, algorithm="fricas")
[Out] 1/2*log(cosh(x))^2
```

Sympy [A] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cosh(x))\tanh(x)\,dx = \frac{\log\left(\cosh\left(x\right)\right)^2}{2}$$

```
[In] integrate(ln(cosh(x))*tanh(x),x)
[Out] log(cosh(x))**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cosh(x))\tanh(x)\,dx = \frac{1}{2}\log\left(\cosh\left(x\right)\right)^2$$

```
[In] integrate(log(cosh(x))*tanh(x),x, algorithm="maxima")
[Out] 1/2*log(cosh(x))^2
```

Giac [F]

$$\int \log(\cosh(x))\tanh(x)\,dx = \int \log\left(\cosh\left(x\right)\right)\tanh\left(x\right)\,dx$$

```
[In] integrate(log(cosh(x))*tanh(x),x, algorithm="giac")
[Out] integrate(log(cosh(x))*tanh(x), x)
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.78

$$\int \log(\cosh(x))\tanh(x)\,dx = \frac{\ln\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^2}{2}$$

```
[In] int(log(cosh(x))*tanh(x),x)
[Out] log(exp(-x)/2 + exp(x)/2)^2/2
```

3.637 $\int \log \left(x-\sqrt{1+x^2}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \log \left(x-\sqrt{1+x^2}\right) \, dx=\sqrt{1+x^2}+x \log \left(x-\sqrt{1+x^2}\right)$$

```
[Out] x*ln(x-(x^2+1)^(1/2))+(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2614, 267}

$$\int \log \left(x-\sqrt{1+x^2}\right) \, dx=\sqrt{x^2+1}+x \log \left(x-\sqrt{x^2+1}\right)$$

```
[In] Int[Log[x - Sqrt[1 + x^2]],x]
[Out] Sqrt[1 + x^2] + x*Log[x - Sqrt[1 + x^2]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2614

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Dist[a*c*f^2, Int[x/(d*e*(a
 + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
```

```
f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rubi steps

$$\text{integral} = x\log\left(x-\sqrt{1+x^2}\right)+\int\frac{x}{\sqrt{1+x^2}}\,dx$$
$$=\sqrt{1+x^2}+x\log\left(x-\sqrt{1+x^2}\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int\log\left(x-\sqrt{1+x^2}\right)\,dx=\sqrt{1+x^2}+x\log\left(x-\sqrt{1+x^2}\right)$$

```
[In] Integrate[Log[x - Sqrt[1 + x^2]],x]
[Out] Sqrt[1 + x^2] + x*Log[x - Sqrt[1 + x^2]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$x\ln\left(-\sqrt{x^2+1}+x\right)+\sqrt{x^2+1}$	23
parts	$x\ln\left(-\sqrt{x^2+1}+x\right)-\frac{x^2\sqrt{x^2+1}}{3}+\frac{2\sqrt{x^2+1}}{3}+\frac{(x^2+1)^{\frac{3}{2}}}{3}$	46

```
[In] int(ln(-(x^2+1)^(1/2)+x),x,method=_RETURNVERBOSE)
[Out] x*ln(-(x^2+1)^(1/2)+x)+(x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int\log\left(x-\sqrt{1+x^2}\right)\,dx=x\log\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

```
[In] integrate(log(x-(x^2+1)^(1/2)),x, algorithm="fricas")
[Out] x*log(x - sqrt(x^2 + 1)) + sqrt(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \log\left(x-\sqrt{1+x^2}\right)\,dx = x\log\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

```
[In] integrate(ln(x-(x**2+1)**(1/2)),x)
[Out] x*log(x - sqrt(x**2 + 1)) + sqrt(x**2 + 1)
```

Maxima [F]

$$\int \log\left(x-\sqrt{1+x^2}\right)\,dx = \int \log\left(x-\sqrt{x^2+1}\right)\,dx$$

```
[In] integrate(log(x-(x^2+1)^(1/2)),x, algorithm="maxima")
[Out] x*log(x - sqrt(x^2 + 1)) - x + arctan(x) + integrate(-x/(x^3 - (x^2 + 1)^(3
/2) + x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log\left(x-\sqrt{1+x^2}\right)\,dx = x\log\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

```
[In] integrate(log(x-(x^2+1)^(1/2)),x, algorithm="giac")
[Out] x*log(x - sqrt(x^2 + 1)) + sqrt(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log\left(x-\sqrt{1+x^2}\right)\,dx = x\,\ln\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

```
[In] int(log(x - (x^2 + 1)^(1/2)),x)
[Out] x*log(x - (x^2 + 1)^(1/2)) + (x^2 + 1)^(1/2)
```

3.638 $\int \frac{\log(-1+x)}{x^3} dx$

Optimal result

Integrand size = 8, antiderivative size = 35

$$\int \frac{\log(-1+x)}{x^3} dx = \frac{1}{2x} + \frac{1}{2}\log(1-x) - \frac{\log(-1+x)}{2x^2} - \frac{\log(x)}{2}$$

```
[Out] 1/2/x+1/2*ln(1-x)-1/2*ln(-1+x)/x^2-1/2*ln(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2442, 46}

$$\int \frac{\log(-1+x)}{x^3} dx = -\frac{\log(x-1)}{2x^2} + \frac{1}{2x} + \frac{1}{2}\log(1-x) - \frac{\log(x)}{2}$$

```
[In] Int[Log[-1 + x]/x^3,x]
```

```
[Out] 1/(2*x) + Log[1 - x]/2 - Log[-1 + x]/(2*x^2) - Log[x]/2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log(-1+x)}{2x^2}+\frac{1}{2}\int \frac{1}{(-1+x)x^2}\,dx \\
&= -\frac{\log(-1+x)}{2x^2}+\frac{1}{2}\int \left(\frac{1}{-1+x}-\frac{1}{x^2}-\frac{1}{x}\right)dx \\
&= \frac{1}{2x}+\frac{1}{2}\log(1-x)-\frac{\log(-1+x)}{2x^2}-\frac{\log(x)}{2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\log(-1+x)}{x^3}\,dx = \frac{1}{2}\left(\frac{1}{x}+\log(1-x)-\frac{\log(-1+x)}{x^2}-\log(x)\right)$$

```
[In] Integrate[Log[-1 + x]/x^3,x]
[Out] (x^(-1) + Log[1 - x] - Log[-1 + x]/x^2 - Log[x])/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{\ln(x)}{2}+\frac{1}{2x}+\frac{\ln(-1+x)(-1+x)(1+x)}{2x^2}$	26
default	$-\frac{\ln(x)}{2}+\frac{1}{2x}+\frac{\ln(-1+x)(-1+x)(1+x)}{2x^2}$	26
parts	$-\frac{\ln(-1+x)}{2x^2}+\frac{\ln(-1+x)}{2}+\frac{1}{2x}-\frac{\ln(x)}{2}$	26
norman	$\frac{\frac{x}{2}+\frac{x^2\ln(-1+x)}{2}-\frac{\ln(-1+x)}{2}}{x^2}-\frac{\ln(x)}{2}$	29
risch	$-\frac{\ln(-1+x)}{2x^2}+\frac{\ln(-1+x)x-x\ln(x)+1}{2x}$	29
parallelrisch	$-\frac{x^2\ln(x)-x^2\ln(-1+x)-x+\ln(-1+x)}{2x^2}$	29

```
[In] int(ln(-1+x)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x)+1/2/x+1/2*ln(-1+x)*(-1+x)*(1+x)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{\log(-1+x)}{x^3} dx = -\frac{x^2 \log(x) - (x^2-1)\log(x-1) - x}{2x^2}$$

```
[In] integrate(log(-1+x)/x^3,x, algorithm="fricas")
[Out] -1/2*(x^2*log(x) - (x^2 - 1)*log(x - 1) - x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{\log(-1+x)}{x^3} dx = -\frac{\log(x)}{2} + \frac{\log(x-1)}{2} + \frac{1}{2x} - \frac{\log(x-1)}{2x^2}$$

```
[In] integrate(ln(-1+x)/x**3,x)
[Out] -log(x)/2 + log(x - 1)/2 + 1/(2*x) - log(x - 1)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \frac{\log(-1+x)}{x^3} dx = \frac{1}{2x} - \frac{\log(x-1)}{2x^2} + \frac{1}{2}\log(x-1) - \frac{1}{2}\log(x)$$

```
[In] integrate(log(-1+x)/x^3,x, algorithm="maxima")
[Out] 1/2/x - 1/2*log(x - 1)/x^2 + 1/2*log(x - 1) - 1/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\log(-1+x)}{x^3} dx = \frac{1}{2x} - \frac{\log(x-1)}{2x^2} + \frac{1}{2}\log(|x-1|) - \frac{1}{2}\log(|x|)$$

```
[In] integrate(log(-1+x)/x^3,x, algorithm="giac")
[Out] 1/2/x - 1/2*log(x - 1)/x^2 + 1/2*log(abs(x - 1)) - 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \frac{\log(-1+x)}{x^3}\,dx = \frac{x-\ln\left(x-1\right)+x^2\,\ln\left(1-\frac{1}{x}\right)}{2\,x^2}$$

```
[In] int(log(x - 1)/x^3,x)
[Out] (x - log(x - 1) + x^2*log(1 - 1/x))/(2*x^2)
```

3.639 $\int (-e^{-x}+e^{x})\log(1+e^{2x})\,dx$

Optimal result

Integrand size = 20, antiderivative size = 32

$$\int (-e^{-x}+e^{x})\log\left(1+e^{2x}\right)\,dx=-2e^{x}+e^{-x}\log\left(1+e^{2x}\right)+e^{x}\log\left(1+e^{2x}\right)$$

```
[Out] -2*exp(x)+ln(1+exp(2*x))/exp(x)+exp(x)*ln(1+exp(2*x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2320, 2526, 2498, 327, 209, 2505}

$$\int (-e^{-x}+e^{x})\log\left(1+e^{2x}\right)\,dx=-2e^{x}+e^{-x}\log\left(e^{2x}+1\right)+e^{x}\log\left(e^{2x}+1\right)$$

```
[In] Int[(-E^(-x) + E^x)*Log[1 + E^(2*x)],x]
```

```
[Out] -2*E^x + Log[1 + E^(2*x)]/E^x + E^x*Log[1 + E^(2*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(-1+x^2)\log(1+x^2)}{x^2}\,dx, x, e^x\right) \\
&= \text{Subst}\left(\int \left(\log(1+x^2) - \frac{\log(1+x^2)}{x^2}\right)\,dx, x, e^x\right) \\
&= \text{Subst}\left(\int \log(1+x^2)\,dx, x, e^x\right) - \text{Subst}\left(\int \frac{\log(1+x^2)}{x^2}\,dx, x, e^x\right) \\
&= e^{-x}\log(1+e^{2x}) + e^x\log(1+e^{2x}) \\
&\quad - 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right) - 2\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, e^x\right)
\end{aligned}
$$

$$= -2e^x - 2\arctan(e^x) + e^{-x}\log(1+e^{2x}) + e^x\log(1+e^{2x}) + 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)$$

$$= -2e^x + e^{-x}\log(1+e^{2x}) + e^x\log(1+e^{2x})$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int (-e^{-x}+e^x)\log(1+e^{2x})\,dx = -2e^x + (e^{-x}+e^x)\log(1+e^{2x})$$

```
[In] Integrate[(-E^(-x) + E^x)*Log[1 + E^(2*x)],x]
[Out] -2*E^x + (E^(-x) + E^x)*Log[1 + E^(2*x)]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

method	result	size
risch	$(1+e^{2x})e^{-x}\ln(1+e^{2x}) - 2e^x$	24
norman	$(e^{2x}\ln(1+e^{2x}) - 2e^{2x} + \ln(1+e^{2x}))e^{-x}$	32

```
[In] int((-1/exp(x)+exp(x))*ln(1+exp(2*x)),x,method=_RETURNVERBOSE)
[Out] (1+exp(2*x))*exp(-x)*ln(1+exp(2*x))-2*exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int (-e^{-x}+e^x)\log(1+e^{2x})\,dx = \left((e^{(2x)}+1)\log(e^{(2x)}+1) - 2e^{(2x)}\right)e^{(-x)}$$

```
[In] integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="fricas")
[Out] ((e^(2*x) + 1)*log(e^(2*x) + 1) - 2*e^(2*x))*e^(-x)
```

Sympy [A] (verification not implemented)

Time = 81.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \left(-e^{-x}+e^{x}\right) \log \left(1+e^{2x}\right) \, dx = e^{x} \log \left(e^{2x}+1\right) - 2e^{x} + e^{-x} \log \left(e^{2x}+1\right)$$

```
[In] integrate((-1/exp(x)+exp(x))*ln(1+exp(2*x)),x)
[Out] exp(x)*log(exp(2*x) + 1) - 2*exp(x) + exp(-x)*log(exp(2*x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \left(-e^{-x}+e^{x}\right) \log \left(1+e^{2x}\right) \, dx = \left(e^{(-x)}+e^{x}\right) \log \left(e^{(2x)}+1\right) - 2e^{x}$$

```
[In] integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="maxima")
[Out] (e^(-x) + e^x)*log(e^(2*x) + 1) - 2*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \left(-e^{-x}+e^{x}\right) \log \left(1+e^{2x}\right) \, dx = \left(e^{(-x)}+e^{x}\right) \log \left(e^{(2x)}+1\right) - 2e^{x}$$

```
[In] integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="giac")
[Out] (e^(-x) + e^x)*log(e^(2*x) + 1) - 2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(-e^{-x}+e^{x}\right) \log \left(1+e^{2x}\right) \, dx = 2 \ln \left(e^{2x}+1\right) \cosh(x) - \frac{e^{2x}+1}{\cosh(x)}$$

```
[In] int(-log(exp(2*x) + 1)*(exp(-x) - exp(x)),x)
[Out] 2*log(exp(2*x) + 1)*cosh(x) - (exp(2*x) + 1)/cosh(x)
```

3.640 $\int e^{3x/2}\log(-1+e^x)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int e^{3x/2}\log(-1+e^x)\,dx=-\frac{4e^{x/2}}{3}-\frac{4}{9}e^{3x/2}+\frac{4}{3}\operatorname{arctanh}\left(e^{x/2}\right)+\frac{2}{3}e^{3x/2}\log(-1+e^x)$$

```
[Out] -4/3*exp(1/2*x)-4/9*exp(3/2*x)+4/3*arctanh(exp(1/2*x))+2/3*exp(3/2*x)*ln(-1
+exp(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2225, 2634, 12, 2280, 308, 213}

$$\int e^{3x/2}\log(-1+e^x)\,dx=\frac{4}{3}\operatorname{arctanh}\left(e^{x/2}\right)-\frac{4e^{x/2}}{3}-\frac{4}{9}e^{3x/2}+\frac{2}{3}e^{3x/2}\log(e^x-1)$$

```
[In] Int[E^((3*x)/2)*Log[-1 + E^x],x]
```

```
[Out] (-4*E^(x/2))/3 - (4*E^((3*x)/2))/9 + (4*ArcTanh[E^(x/2)])/3 + (2*E^((3*x)/2
)*Log[-1 + E^x])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2280

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Den
ominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{3}e^{3x/2}\log\left(-1+e^x\right)-\int \frac{2e^{5x/2}}{3\left(-1+e^x\right)}\,dx\\
&= \frac{2}{3}e^{3x/2}\log\left(-1+e^x\right)-\frac{2}{3}\int \frac{e^{5x/2}}{-1+e^x}\,dx\\
&= \frac{2}{3}e^{3x/2}\log\left(-1+e^x\right)-\frac{4}{3}\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx,x,e^{x/2}\right)\\
&= \frac{2}{3}e^{3x/2}\log\left(-1+e^x\right)-\frac{4}{3}\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx,x,e^{x/2}\right)\\
&= -\frac{4e^{x/2}}{3}-\frac{4}{9}e^{3x/2}+\frac{2}{3}e^{3x/2}\log\left(-1+e^x\right)-\frac{4}{3}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,e^{x/2}\right)\\
&= -\frac{4e^{x/2}}{3}-\frac{4}{9}e^{3x/2}+\frac{4}{3}\text{arctanh}\left(e^{x/2}\right)+\frac{2}{3}e^{3x/2}\log\left(-1+e^x\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int e^{3x/2}\log\left(-1+e^x\right)\,dx=\frac{2}{9}\left(6\text{arctanh}\left(e^{x/2}\right)+e^{x/2}\left(-2(3+e^x)+3e^x\log\left(-1+e^x\right)\right)\right)$$

```
[In] Integrate[E^((3*x)/2)*Log[-1 + E^x],x]
[Out] (2*(6*ArcTanh[E^(x/2)] + E^(x/2)*(-2*(3 + E^x) + 3*E^x*Log[-1 + E^x])))/9
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{2\,\mathrm{e}^{\frac{3x}{2}}\ln\left(-1+\mathrm{e}^{x}\right)}{3}-\frac{4\,\mathrm{e}^{\frac{3x}{2}}}{9}-\frac{4\,\mathrm{e}^{\frac{x}{2}}}{3}+\frac{2\ln\left(\mathrm{e}^{\frac{x}{2}}+1\right)}{3}-\frac{2\ln\left(-1+\mathrm{e}^{\frac{x}{2}}\right)}{3}$	43

```
[In] int(exp(3/2*x)*ln(-1+exp(x)),x,method=_RETURNVERBOSE)
[Out] 2/3*exp(3/2*x)*ln(-1+exp(x))-4/9*exp(3/2*x)-4/3*exp(1/2*x)+2/3*ln(exp(1/2*x
)+1)-2/3*ln(-1+exp(1/2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int e^{3x/2}\log\left(-1+e^x\right)\,dx=\frac{2}{3}\,e^{\left(\frac{3}{2}\,x\right)}\log\left(e^x-1\right)-\frac{4}{9}\,e^{\left(\frac{3}{2}\,x\right)}-\frac{4}{3}\,e^{\left(\frac{1}{2}\,x\right)}+\frac{2}{3}\,\log\left(e^{\left(\frac{1}{2}\,x\right)}+1\right)-\frac{2}{3}\,\log\left(e^{\left(\frac{1}{2}\,x\right)}-1\right)$$

```
[In] integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="fricas")
[Out] 2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*log(e^(1/2
*x) + 1) - 2/3*log(e^(1/2*x) - 1)
```

Sympy [F(-1)]

Timed out.

$$\int e^{3x/2}\log\left(-1+e^x\right)\,dx=\text{Timed out}$$

```
[In] integrate(exp(3/2*x)*ln(-1+exp(x)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int e^{3x/2}\log\left(-1+e^x\right)\,dx=\frac{2}{3}\,e^{\left(\frac{3}{2}\,x\right)}\log\left(e^x-1\right)-\frac{4}{9}\,e^{\left(\frac{3}{2}\,x\right)}-\frac{4}{3}\,e^{\left(\frac{1}{2}\,x\right)}+\frac{2}{3}\,\log\left(e^{\left(\frac{1}{2}\,x\right)}+1\right)-\frac{2}{3}\,\log\left(e^{\left(\frac{1}{2}\,x\right)}-1\right)$$

```
[In] integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="maxima")
[Out] 2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*log(e^(1/2
*x) + 1) - 2/3*log(e^(1/2*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int e^{3x/2}\log\left(-1+e^x\right)\,dx=\frac{2}{3}\,e^{\left(\frac{3}{2}\,x\right)}\log\left(e^x-1\right)-\frac{4}{9}\,e^{\left(\frac{3}{2}\,x\right)}-\frac{4}{3}\,e^{\left(\frac{1}{2}\,x\right)}+\frac{2}{3}\,\log\left(e^{\left(\frac{1}{2}\,x\right)}+1\right)-\frac{2}{3}\,\log\left(\left|e^{\left(\frac{1}{2}\,x\right)}-1\right|\right)$$

```
[In] integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="giac")
[Out] 2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*log(e^(1/2
*x) + 1) - 2/3*log(abs(e^(1/2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int e^{3x/2}\log\left(-1+e^x\right)\,dx=\frac{4\,\mathrm{atanh}\left(\sqrt{\mathrm{e}^x}\right)}{3}-\frac{4\,\mathrm{e}^{\frac{3\,x}{2}}}{9}-\frac{4\,\mathrm{e}^{x/2}}{3}+\frac{2\,\mathrm{e}^{\frac{3\,x}{2}}\,\ln\left(\mathrm{e}^x-1\right)}{3}$$

```
[In] int(exp((3*x)/2)*log(exp(x) - 1),x)
```

```
[Out] (4*atanh(exp(x)^(1/2)))/3 - (4*exp((3*x)/2))/9 - (4*exp(x/2))/3 + (2*exp((3
*x)/2)*log(exp(x) - 1))/3
```

3.641 $\int \cos^3(x) \log(\sin(x)) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 30

$$\int \cos^3(x) \log(\sin(x)) \, dx = -\sin(x) + \log(\sin(x)) \sin(x) + \frac{\sin^3(x)}{9} - \frac{1}{3} \log(\sin(x)) \sin^3(x)$$

```
[Out] -sin(x)+ln(sin(x))*sin(x)+1/9*sin(x)^3-1/3*ln(sin(x))*sin(x)^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2713, 2634, 12, 4441}

$$\int \cos^3(x) \log(\sin(x)) \, dx = \frac{\sin^3(x)}{9} - \sin(x) - \frac{1}{3} \sin^3(x) \log(\sin(x)) + \sin(x) \log(\sin(x))$$

```
[In] Int[Cos[x]^3*Log[Sin[x]],x]
[Out] -Sin[x] + Log[Sin[x]]*Sin[x] + Sin[x]^3/9 - (Log[Sin[x]]*Sin[x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \log(\sin(x))\sin(x) - \frac{1}{3}\log(\sin(x))\sin^3(x) - \int \frac{1}{6}\cos(x)(5+\cos(2x))\,dx \\
&= \log(\sin(x))\sin(x) - \frac{1}{3}\log(\sin(x))\sin^3(x) - \frac{1}{6}\int \cos(x)(5+\cos(2x))\,dx \\
&= \log(\sin(x))\sin(x) - \frac{1}{3}\log(\sin(x))\sin^3(x) - \frac{1}{6}\text{Subst}\left(\int \left(6-2x^2\right)\,dx, x, \sin(x)\right) \\
&= -\sin(x) + \log(\sin(x))\sin(x) + \frac{\sin^3(x)}{9} - \frac{1}{3}\log(\sin(x))\sin^3(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \cos^3(x)\log(\sin(x))\,dx = -\sin(x) + \log(\sin(x))\sin(x) + \frac{\sin^3(x)}{9} - \frac{1}{3}\log(\sin(x))\sin^3(x)$$

```
[In] Integrate[Cos[x]^3*Log[Sin[x]],x]
[Out] -Sin[x] + Log[Sin[x]]*Sin[x] + Sin[x]^3/9 - (Log[Sin[x]]*Sin[x]^3)/3
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

method	result	size
parallelrisch	$\frac{(3\ln(\sin(x))-1)\sin(3x)}{36}+\frac{3\ln(\sin(x))\sin(x)}{4}-\frac{11\sin(x)}{12}$	26
derivativedivides	$-\sin(x)+\ln(\sin(x))\sin(x)+\frac{(\sin^3(x))}{9}-\frac{\ln(\sin(x))(\sin^3(x))}{3}$	27
default	$-\sin(x)+\ln(\sin(x))\sin(x)+\frac{(\sin^3(x))}{9}-\frac{\ln(\sin(x))(\sin^3(x))}{3}$	27
risch	Expression too large to display	577

```
[In] int(cos(x)^3*ln(sin(x)),x,method=_RETURNVERBOSE)
[Out] 1/36*(3*ln(sin(x))-1)*sin(3*x)+3/4*ln(sin(x))*sin(x)-11/12*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \cos^3(x)\log(\sin(x))\,dx = \frac{1}{3}\left(\cos(x)^2+2\right)\log(\sin(x))\sin(x)-\frac{1}{9}\left(\cos(x)^2+8\right)\sin(x)$$

```
[In] integrate(cos(x)^3*log(sin(x)),x, algorithm="fricas")
[Out] 1/3*(cos(x)^2 + 2)*log(sin(x))*sin(x) - 1/9*(cos(x)^2 + 8)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \cos^3(x)\log(\sin(x))\,dx = \frac{2\log(\sin(x))\sin^3(x)}{3}+\log(\sin(x))\sin(x)\cos^2(x)-\frac{8\sin^3(x)}{9}-\sin(x)\cos^2(x)$$

```
[In] integrate(cos(x)**3*ln(sin(x)),x)
[Out] 2*log(sin(x))*sin(x)**3/3 + log(sin(x))*sin(x)*cos(x)**2 - 8*sin(x)**3/9 -
sin(x)*cos(x)**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \cos^3(x)\log(\sin(x))\,dx = \frac{1}{9}\sin(x)^3 - \frac{1}{3}\left(\sin(x)^3 - 3\sin(x)\right)\log(\sin(x)) - \sin(x)$$

```
[In] integrate(cos(x)^3*log(sin(x)),x, algorithm="maxima")
[Out] 1/9*sin(x)^3 - 1/3*(sin(x)^3 - 3*sin(x))*log(sin(x)) - sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \cos^3(x)\log(\sin(x))\,dx = -\frac{1}{3}\log(\sin(x))\sin(x)^3 + \frac{1}{9}\sin(x)^3 + \log(\sin(x))\sin(x) - \sin(x)$$

```
[In] integrate(cos(x)^3*log(sin(x)),x, algorithm="giac")
[Out] -1/3*log(sin(x))*sin(x)^3 + 1/9*sin(x)^3 + log(sin(x))*sin(x) - sin(x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(x)\log(\sin(x))\,dx = \int \ln(\sin(x))\,\cos(x)^3\,dx$$

```
[In] int(log(sin(x))*cos(x)^3,x)
[Out] int(log(sin(x))*cos(x)^3, x)
```

3.642 $\int \log(\tan(x)) \sec^4(x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 30

$$\int \log(\tan(x)) \sec^4(x) \, dx = -\tan(x) + \log(\tan(x)) \tan(x) - \frac{\tan^3(x)}{9} + \frac{1}{3} \log(\tan(x)) \tan^3(x)$$

```
[Out] -tan(x)+ln(tan(x))*tan(x)-1/9*tan(x)^3+1/3*ln(tan(x))*tan(x)^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3852, 2634, 12}

$$\int \log(\tan(x)) \sec^4(x) \, dx = -\frac{\tan^3(x)}{9} - \tan(x) + \frac{1}{3} \tan^3(x) \log(\tan(x)) + \tan(x) \log(\tan(x))$$

```
[In] Int[Log[Tan[x]]*Sec[x]^4,x]
```

```
[Out] -Tan[x] + Log[Tan[x]]*Tan[x] - Tan[x]^3/9 + (Log[Tan[x]]*Tan[x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \log(\tan(x))\tan(x) + \frac{1}{3}\log(\tan(x))\tan^3(x) - \int \frac{1}{3}(2+\cos(2x))\sec^4(x)\,dx \\
&= \log(\tan(x))\tan(x) + \frac{1}{3}\log(\tan(x))\tan^3(x) - \frac{1}{3}\int (2+\cos(2x))\sec^4(x)\,dx \\
&= \log(\tan(x))\tan(x) + \frac{1}{3}\log(\tan(x))\tan^3(x) - \frac{1}{3}\text{Subst}\left(\int \left(3+x^2\right)\,dx, x, \tan(x)\right) \\
&= -\tan(x) + \log(\tan(x))\tan(x) - \frac{\tan^3(x)}{9} + \frac{1}{3}\log(\tan(x))\tan^3(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \log(\tan(x))\sec^4(x)\,dx = \frac{1}{9}\left(-8+(-1+6\log(\tan(x))+3\cos(2x)\log(\tan(x)))\sec^2(x)\right)\tan(x)$$

```
[In] Integrate[Log[Tan[x]]*Sec[x]^4,x]
[Out] ((-8 + (-1 + 6*Log[Tan[x]] + 3*Cos[2*x]*Log[Tan[x]])*Sec[x]^2)*Tan[x])/9
```

Maple [A] (verified)

Time = 10.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\tan(x)+\ln(\tan(x))\tan(x)-\frac{(\tan^3(x))}{9}+\frac{\ln(\tan(x))(\tan^3(x))}{3}$	27
default	$-\tan(x)+\ln(\tan(x))\tan(x)-\frac{(\tan^3(x))}{9}+\frac{\ln(\tan(x))(\tan^3(x))}{3}$	27
risch	Expression too large to display	782

```
[In] int(ln(tan(x))/cos(x)^4,x,method=_RETURNVERBOSE)
[Out] -tan(x)+ln(tan(x))*tan(x)-1/9*tan(x)^3+1/3*ln(tan(x))*tan(x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int \log(\tan(x)) \sec^4(x) \, dx = \frac{3 \left(2 \cos(x)^2 + 1\right) \log\left(\frac{\sin(x)}{\cos(x)}\right) \sin(x) - \left(8 \cos(x)^2 + 1\right) \sin(x)}{9 \cos(x)^3}$$

```
[In] integrate(log(tan(x))/cos(x)^4,x, algorithm="fricas")
[Out] 1/9*(3*(2*cos(x)^2 + 1)*log(sin(x)/cos(x))*sin(x) - (8*cos(x)^2 + 1)*sin(x)
)/cos(x)^3
```

Sympy [A] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \log(\tan(x)) \sec^4(x) \, dx = \frac{\log(\tan(x)) \tan^3(x)}{3} + \log(\tan(x)) \tan(x) - \frac{\sin^3(x)}{9 \cos^3(x)} + \frac{\sin(x)}{3 \cos(x)} - \frac{4 \tan(x)}{3}$$

```
[In] integrate(ln(tan(x))/cos(x)**4,x)
[Out] log(tan(x))*tan(x)**3/3 + log(tan(x))*tan(x) - sin(x)**3/(9*cos(x)**3) + si
n(x)/(3*cos(x)) - 4*tan(x)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \log(\tan(x)) \sec^4(x) \, dx = -\frac{1}{9} \tan(x)^3 + \frac{1}{3} \left(\tan(x)^3 + 3 \tan(x)\right) \log(\tan(x)) - \tan(x)$$

```
[In] integrate(log(tan(x))/cos(x)^4,x, algorithm="maxima")
[Out] -1/9*tan(x)^3 + 1/3*(tan(x)^3 + 3*tan(x))*log(tan(x)) - tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \log(\tan(x))\sec^4(x)\,dx = \frac{1}{3}\log\left(\tan\left(x\right)\right)\tan\left(x\right)^{3}-\frac{1}{9}\tan\left(x\right)^{3}+\log\left(\tan\left(x\right)\right)\tan\left(x\right)-\tan\left(x\right)$$

```
[In] integrate(log(tan(x))/cos(x)^4,x, algorithm="giac")
[Out] 1/3*log(tan(x))*tan(x)^3 - 1/9*tan(x)^3 + log(tan(x))*tan(x) - tan(x)
```

Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.93

$$\int \log(\tan(x))\sec^4(x)\,dx = \frac{\ln\left(-\frac{8\,{\mathrm{e}}^{x\,2\mathrm{i}}}{3}-\frac{8}{3}\right)\,2\mathrm{i}}{3}-\frac{\ln\left(\frac{8}{3}-\frac{8\,{\mathrm{e}}^{x\,2\mathrm{i}}}{3}\right)\,2\mathrm{i}}{3}+\frac{8\mathrm{i}}{9\,\left(3\,{\mathrm{e}}^{x\,2\mathrm{i}}+3\,{\mathrm{e}}^{x\,4\mathrm{i}}+{\mathrm{e}}^{x\,6\mathrm{i}}+1\right)}-\frac{4\mathrm{i}}{3\,\left(2\,{\mathrm{e}}^{x\,2\mathrm{i}}+{\mathrm{e}}^{x\,4\mathrm{i}}+1\right)}-\frac{4\mathrm{i}}{3\,\left({\mathrm{e}}^{x\,2\mathrm{i}}+1\right)}+\frac{\ln\left(-\frac{{\mathrm{e}}^{x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}}{{\mathrm{e}}^{x\,2\mathrm{i}}+1}\right)\,\left({\mathrm{e}}^{x\,2\mathrm{i}}\,4\mathrm{i}+\frac{4}{3}\mathrm{i}\right)}{3\,{\mathrm{e}}^{x\,2\mathrm{i}}+3\,{\mathrm{e}}^{x\,4\mathrm{i}}+{\mathrm{e}}^{x\,6\mathrm{i}}+1}$$

```
[In] int(log(tan(x))/cos(x)^4,x)
[Out] (log(- (8*exp(x*2i))/3 - 8/3)*2i)/3 - (log(8/3 - (8*exp(x*2i))/3)*2i)/3 + 8
i/(9*(3*exp(x*2i) + 3*exp(x*4i) + exp(x*6i) + 1)) - 4i/(3*(2*exp(x*2i) + ex
p(x*4i) + 1)) - 4i/(3*(exp(x*2i) + 1)) + (log(-(exp(x*2i)*1i - 1i)/(exp(x*2
i) + 1))*(exp(x*2i)*4i + 4i/3))/(3*exp(x*2i) + 3*exp(x*4i) + exp(x*6i) + 1)
```

3.643 $\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 28

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = -\frac{x}{2}+\frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)}+\tan\left(\frac{x}{2}\right)$$

```
[Out] -1/2*x+ln(cos(1/2*x))*sin(x)/(1+cos(x))+tan(1/2*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2727, 2634, 12, 3554, 8}

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = -\frac{x}{2}+\tan\left(\frac{x}{2}\right)+\frac{\sin(x)\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1}$$

```
[In] Int[Log[Cos[x/2]]/(1 + Cos[x]),x]
[Out] -1/2*x + (Log[Cos[x/2]]*Sin[x])/(1 + Cos[x]) + Tan[x/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)} - \int -\frac{1}{2}\tan^2\left(\frac{x}{2}\right)\,dx \\
&= \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)} + \frac{1}{2}\int \tan^2\left(\frac{x}{2}\right)\,dx \\
&= \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)} + \tan\left(\frac{x}{2}\right) - \frac{\int 1\,dx}{2} \\
&= -\frac{x}{2} + \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)} + \tan\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = -\frac{\left(x\cot\left(\frac{x}{2}\right) - 2\left(1+\log\left(\cos\left(\frac{x}{2}\right)\right)\right)\right)\sin(x)}{2(1+\cos(x))}$$

```
[In] Integrate[Log[Cos[x/2]]/(1 + Cos[x]),x]
[Out] -1/2*((x*Cot[x/2] - 2*(1 + Log[Cos[x/2]]))*Sin[x])/(1 + Cos[x])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 164, normalized size of antiderivative = 5.86

method	result
risch	$-\frac{2i\ln\left(e^{\frac{ix}{2}}\right)}{e^{ix}+1}+\frac{\pi\ \mathrm{csgn}\left(i\left(e^{ix}+1\right)\right)\mathrm{csgn}\left(ie^{-\frac{ix}{2}}\right)\mathrm{csgn}\left(i\cos\left(\frac{x}{2}\right)\right)-\pi\ \mathrm{csgn}\left(i\left(e^{ix}+1\right)\right)\mathrm{csgn}\left(i\cos\left(\frac{x}{2}\right)\right)^{2}-\pi\ \mathrm{csgn}\left(ie^{-\frac{ix}{2}}\right)\mathrm{csgn}\left(i\cos\left(\frac{x}{2}\right.\right.}{e^{ix}+1}$

```
[In] int(ln(cos(1/2*x))/(cos(x)+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/(exp(I*x)+1)*ln(exp(1/2*I*x))+(Pi*csgn(I*(exp(I*x)+1))*csgn(I*exp(-1/2
*I*x))*csgn(I*cos(1/2*x))-Pi*csgn(I*(exp(I*x)+1))*csgn(I*cos(1/2*x))^2-Pi*c
sgn(I*exp(-1/2*I*x))*csgn(I*cos(1/2*x))^2+Pi*csgn(I*cos(1/2*x))^3-I*ln(exp(
I*x)+1)*exp(I*x)-x*exp(I*x)-2*I*ln(2)+I*ln(exp(I*x)+1)+2*I-x)/(exp(I*x)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx=-\frac{x\cos\left(\frac{1}{2}x\right)-2\log\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right)-2\sin\left(\frac{1}{2}x\right)}{2\cos\left(\frac{1}{2}x\right)}$$

```
[In] integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(x*cos(1/2*x) - 2*log(cos(1/2*x))*sin(1/2*x) - 2*sin(1/2*x))/cos(1/2*x
)
```

Sympy [F]

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx=\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1}\,dx$$

```
[In] integrate(ln(cos(1/2*x))/(1+cos(x)),x)
```

```
[Out] Integral(log(cos(x/2))/(cos(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(22) = 44$.

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = \frac{\log\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(x\right)}{\cos\left(x\right)+1} - \frac{x\cos\left(x\right)^2 + x\sin\left(x\right)^2 + 2\,x\cos\left(x\right) + x - 4\,\sin\left(x\right)}{2\,\left(\cos\left(x\right)^2 + \sin\left(x\right)^2 + 2\,\cos\left(x\right) + 1\right)}$$

```
[In] integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="maxima")
[Out] log(cos(1/2*x))*sin(x)/(cos(x) + 1) - 1/2*(x*cos(x)^2 + x*sin(x)^2 + 2*x*co
s(x) + x - 4*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = -\frac{1}{2}\,x - \frac{2\,\log\left(\cos\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)}{\left(x^2+1\right)\left(\frac{x^2-1}{x^2+1}-1\right)} + \tan\left(\frac{1}{2}\,x\right)$$

```
[In] integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="giac")
[Out] -1/2*x - 2*log(cos(1/2*x))*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
 + tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right) - x + \tan\left(\frac{x}{2}\right)\,\ln\left(\cos\left(\frac{x}{2}\right)\right) + \ln\left(\cos\left(\frac{x}{2}\right)\right)\,1\mathrm{i} - \ln\left(\cos\left(x\right) + 1 + \sin\left(x\right)\,1\mathrm{i}\right)\,1\mathrm{i}$$

```
[In] int(log(cos(x/2))/(cos(x) + 1),x)
[Out] tan(x/2) - x + log(cos(x/2))*1i - log(cos(x) + sin(x)*1i + 1)*1i + tan(x/2)
*log(cos(x/2))
```

3.644 $\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{2x}{3} - \frac{\sin(x)}{9(1+\cos(x))^2} + \frac{8\sin(x)}{9(1+\cos(x))} - \frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2} + \frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))}$$

```
[Out] -2/3*x-1/9*sin(x)/(1+cos(x))^2+8/9*sin(x)/(1+cos(x))-1/3*ln(sin(x))*sin(x)/
(1+cos(x))^2+2/3*ln(sin(x))*sin(x)/(1+cos(x))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {2829, 2727, 2634, 12, 3047, 3098, 2814}

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{2x}{3} + \frac{8\sin(x)}{9(\cos(x)+1)} - \frac{\sin(x)}{9(\cos(x)+1)^2} + \frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)} - \frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

```
[In] Int[(Cos[x]*Log[Sin[x]])/(1 + Cos[x])^2,x]
```

```
[Out] (-2*x)/3 - Sin[x]/(9*(1 + Cos[x])^2) + (8*Sin[x])/(9*(1 + Cos[x])) - (Log[S
in[x]]*Sin[x])/(3*(1 + Cos[x])^2) + (2*Log[Sin[x]]*Sin[x])/(3*(1 + Cos[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2} + \frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))} - \int \frac{\cos(x)(1+2\cos(x))}{3(1+\cos(x))^2}\,dx$$

$$= -\frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2} + \frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))} - \frac{1}{3}\int \frac{\cos(x)(1+2\cos(x))}{(1+\cos(x))^2}\,dx$$
$$= -\frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2} + \frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))} - \frac{1}{3}\int \frac{\cos(x)+2\cos^2(x)}{(1+\cos(x))^2}\,dx$$
$$= -\frac{\sin(x)}{9(1+\cos(x))^2} - \frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2} + \frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))} + \frac{1}{9}\int \frac{2-6\cos(x)}{1+\cos(x)}\,dx$$
$$= -\frac{2x}{3} - \frac{\sin(x)}{9(1+\cos(x))^2} - \frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2} + \frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))} + \frac{8}{9}\int \frac{1}{1+\cos(x)}\,dx$$
$$= -\frac{2x}{3} - \frac{\sin(x)}{9(1+\cos(x))^2} + \frac{8\sin(x)}{9(1+\cos(x))} - \frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2} + \frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{1}{18}\sec^3\left(\frac{x}{2}\right)\left(9x\cos\left(\frac{x}{2}\right) + 3x\cos\left(\frac{3x}{2}\right) - (7+3\log(\sin(x))+\cos(x)(8+6\log(\sin(x))))\sin\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[(Cos[x]*Log[Sin[x]])/(1 + Cos[x])^2,x]
```

```
[Out] -1/18*(Sec[x/2]^3*(9*x*Cos[x/2] + 3*x*Cos[(3*x)/2] - (7 + 3*Log[Sin[x]] + C
os[x]*(8 + 6*Log[Sin[x]]))*Sin[x/2]))
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

method	result
parallelrisch	$\frac{-6x\cos(2x)+(6\ln(\sin(x))+8)\sin(2x)-24x\cos(x)+6\ln(\sin(x))\sin(x)-18x+14\sin(x)}{9\cos(2x)+27+36\cos(x)}$
default	$\frac{6\ln\left(\frac{\sin(x)}{2}\right)\cos(x)\sin(x)+12\arctan(-\csc(x)+\cot(x))\left(\cos^2(x)\right)+6\ln(2)\cos(x)\sin(x)+3\ln\left(\frac{\sin(x)}{2}\right)\sin(x)+24\arctan(-\csc(}{9(\cos(x)+1)^2}$
risch	Expression too large to display

```
[In] int(cos(x)*ln(sin(x))/(cos(x)+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-6*x*cos(2*x)+(6*ln(sin(x))+8)*sin(2*x)-24*x*cos(x)+6*ln(sin(x))*sin(x)-18
*x+14*sin(x))/(9*cos(2*x)+27+36*cos(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{6\,x\cos\left(x\right)^2-3\left(2\,\cos\left(x\right)+1\right)\log\left(\sin\left(x\right)\right)\sin\left(x\right)+12\,x\cos\left(x\right)-\left(8\,\cos\left(x\right)+7\right)\sin\left(x\right)+6\,x}{9\left(\cos\left(x\right)^2+2\,\cos\left(x\right)+1\right)}$$

```
[In] integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="fricas")
[Out] -1/9*(6*x*cos(x)^2 - 3*(2*cos(x) + 1)*log(sin(x))*sin(x) + 12*x*cos(x) - (8
*cos(x) + 7)*sin(x) + 6*x)/(cos(x)^2 + 2*cos(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.47

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{2x}{3} - \frac{\log\left(\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan^3\left(\frac{x}{2}\right)}{6} + \frac{\log\left(\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan\left(\frac{x}{2}\right)}{2} - \frac{\log\left(2\right)\tan^3\left(\frac{x}{2}\right)}{6} - \frac{\tan^3\left(\frac{x}{2}\right)}{18} + \frac{\log\left(2\right)\tan\left(\frac{x}{2}\right)}{2} + \frac{5\tan\left(\frac{x}{2}\right)}{6}$$

```
[In] integrate(cos(x)*ln(sin(x))/(1+cos(x))**2,x)
[Out] -2*x/3 - log(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**3/6 + log(tan(x/2)/(tan(
x/2)**2 + 1))*tan(x/2)/2 - log(2)*tan(x/2)**3/6 - tan(x/2)**3/18 + log(2)*t
an(x/2)/2 + 5*tan(x/2)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = \frac{1}{6}\left(\frac{3\,\sin\left(x\right)}{\cos\left(x\right)+1} - \frac{\sin\left(x\right)^3}{\left(\cos\left(x\right)+1\right)^3}\right)\log\left(\frac{2\,\sin\left(x\right)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)\left(\cos\left(x\right)+1\right)}\right) + \frac{5\,\sin\left(x\right)}{6\left(\cos\left(x\right)+1\right)} - \frac{\sin\left(x\right)^3}{18\left(\cos\left(x\right)+1\right)^3} - \frac{4}{3}\,\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

```
[In] integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sin(x)/(cos(x) + 1) - sin(x)^3/(cos(x) + 1)^3)*log(2*sin(x)/((sin(x)
^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1))) + 5/6*sin(x)/(cos(x) + 1) - 1/18*sin(
x)^3/(cos(x) + 1)^3 - 4/3*arctan(sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{1}{18}\tan\left(\frac{1}{2}x\right)^3 - \frac{1}{6}\left(\tan\left(\frac{1}{2}x\right)^3 - 3\tan\left(\frac{1}{2}x\right)\right)\log\left(\sin\left(x\right)\right) - \frac{2}{3}x + \frac{5}{6}\tan\left(\frac{1}{2}x\right)$$

```
[In] integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="giac")
```

```
[Out] -1/18*tan(1/2*x)^3 - 1/6*(tan(1/2*x)^3 - 3*tan(1/2*x))*log(sin(x)) - 2/3*x
+ 5/6*tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.73

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = \frac{\frac{4\sin(2x)}{9} - \frac{\ln\left(-2\sin(x)^2+\sin(2x)\,1\mathrm{i}\right)7\mathrm{i}}{3} - \frac{14x}{3} + \frac{\ln(\sin(x))\,7\mathrm{i}}{3} + \frac{7\sin(x)}{9} + \frac{\sin(2x)\ln(\sin(x))}{3} - \frac{\sin(x)^2\,8\mathrm{i}}{9} + \sin\left(\frac{x}{2}\right)^2\left(\frac{16x}{3} + \right.}{}$$

```
[In] int((log(sin(x))*cos(x))/(cos(x) + 1)^2,x)
```

```
[Out] ((4*sin(2*x))/9 - (log(sin(2*x)*1i - 2*sin(x)^2)*7i)/3 - (14*x)/3 + (log(si
n(x))*7i)/3 + (7*sin(x))/9 + (sin(2*x)*log(sin(x)))/3 - (sin(x)^2*8i)/9 + s
in(x/2)^2*((16*x)/3 + (log(sin(2*x)*1i - 2*sin(x)^2)*8i)/3 - (log(sin(x))*8
i)/3 - 32i/9) + (log(sin(2*x)*1i - 2*sin(x)^2)*(2*sin(x)^2 - 1)*1i)/3 + (lo
g(sin(x))*sin(x))/3 - (log(sin(x))*(2*sin(x)^2 - 1)*1i)/3 + (2*x*(2*sin(x)^
2 - 1))/3 + 32i/9)/(2*sin(x/2)^2 - 2)^2
```

3.645 $\int \frac{\arccos(x)^2}{x^5} dx$

Optimal result

Integrand size = 8, antiderivative size = 65

$$\int \frac{\arccos(x)^2}{x^5} dx = -\frac{1}{12x^2} + \frac{\sqrt{1-x^2}\arccos(x)}{6x^3} + \frac{\sqrt{1-x^2}\arccos(x)}{3x} - \frac{\arccos(x)^2}{4x^4} + \frac{\log(x)}{3}$$

```
[Out] -1/12/x^2-1/4*arccos(x)^2/x^4+1/3*ln(x)+1/6*arccos(x)*(-x^2+1)^(1/2)/x^3+1/
3*arccos(x)*(-x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {4724, 4790, 4772, 29, 30}

$$\int \frac{\arccos(x)^2}{x^5} dx = -\frac{\arccos(x)^2}{4x^4} + \frac{\sqrt{1-x^2}\arccos(x)}{3x} + \frac{\sqrt{1-x^2}\arccos(x)}{6x^3} - \frac{1}{12x^2} + \frac{\log(x)}{3}$$

```
[In] Int[ArcCos[x]^2/x^5,x]
```

```
[Out] -1/12*1/x^2 + (Sqrt[1 - x^2]*ArcCos[x])/(6*x^3) + (Sqrt[1 - x^2]*ArcCos[x])
/(3*x) - ArcCos[x]^2/(4*x^4) + Log[x]/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4772

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*Ar
cCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2
*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 4790

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c
*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1
- c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arccos(x)^2}{4x^4} - \frac{1}{2}\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx \\
&= \frac{\sqrt{1-x^2}\arccos(x)}{6x^3} - \frac{\arccos(x)^2}{4x^4} + \frac{1}{6}\int \frac{1}{x^3}\,dx - \frac{1}{3}\int \frac{\arccos(x)}{x^2\sqrt{1-x^2}}\,dx \\
&= -\frac{1}{12x^2} + \frac{\sqrt{1-x^2}\arccos(x)}{6x^3} + \frac{\sqrt{1-x^2}\arccos(x)}{3x} - \frac{\arccos(x)^2}{4x^4} + \frac{1}{3}\int \frac{1}{x}\,dx \\
&= -\frac{1}{12x^2} + \frac{\sqrt{1-x^2}\arccos(x)}{6x^3} + \frac{\sqrt{1-x^2}\arccos(x)}{3x} - \frac{\arccos(x)^2}{4x^4} + \frac{\log(x)}{3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{\arccos(x)^2}{x^5}\,dx = -\frac{1}{12x^2} + \frac{\sqrt{1-x^2}(1+2x^2)\arccos(x)}{6x^3} - \frac{\arccos(x)^2}{4x^4} + \frac{\log(x)}{3}$$

```
[In] Integrate[ArcCos[x]^2/x^5,x]
```

```
[Out] -1/12*1/x^2 + (Sqrt[1 - x^2]*(1 + 2*x^2)*ArcCos[x])/(6*x^3) - ArcCos[x]^2/(
4*x^4) + Log[x]/3
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{1}{12x^2}-\frac{\arccos(x)^2}{4x^4}+\frac{\ln(x)}{3}+\frac{\arccos(x)\sqrt{-x^2+1}}{6x^3}+\frac{\arccos(x)\sqrt{-x^2+1}}{3x}$	52

```
[In] int(arccos(x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/x^2-1/4*arccos(x)^2/x^4+1/3*ln(x)+1/6*arccos(x)*(-x^2+1)^(1/2)/x^3+1/
3*arccos(x)*(-x^2+1)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int \frac{\arccos(x)^2}{x^5}\,dx = \frac{4\,x^4\log(x) + 2\,(2\,x^3+x)\sqrt{-x^2+1}\arccos(x) - x^2 - 3\,\arccos(x)^2}{12\,x^4}$$

```
[In] integrate(arccos(x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^4*log(x) + 2*(2*x^3 + x)*sqrt(-x^2 + 1)*arccos(x) - x^2 - 3*arcco
s(x)^2)/x^4
```

Sympy [F]

$$\int \frac{\arccos(x)^2}{x^5}\,dx = \int \frac{\operatorname{acos}^2(x)}{x^5}\,dx$$

```
[In] integrate(acos(x)**2/x**5,x)
```

```
[Out] Integral(acos(x)**2/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{\arccos(x)^2}{x^5}\,dx = \frac{1}{6}\left(\frac{2\sqrt{-x^2+1}}{x} + \frac{\sqrt{-x^2+1}}{x^3}\right)\arccos(x) - \frac{1}{12\,x^2} - \frac{\arccos(x)^2}{4\,x^4} + \frac{1}{3}\log(x)$$

```
[In] integrate(arccos(x)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*(2*sqrt(-x^2 + 1)/x + sqrt(-x^2 + 1)/x^3)*arccos(x) - 1/12/x^2 - 1/4*ar
ccos(x)^2/x^4 + 1/3*log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(51) = 102$.

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.60

$$\int \frac{\arccos(x)^2}{x^5}\,dx = -\frac{1}{48}\left(\frac{x^3\left(\frac{9\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{\left(\sqrt{-x^2+1}-1\right)^3} - \frac{9\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^3}{x^3}\right)\arccos(x) - \frac{2\,x^2+1}{12\,x^2} - \frac{\arccos(x)^2}{4\,x^4} + \frac{1}{6}\log\left(x^2\right)$$

```
[In] integrate(arccos(x)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(x^3*(9*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1)^3 - 9*(s
qrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^3/x^3)*arccos(x) - 1/12*(2*x^2
+ 1)/x^2 - 1/4*arccos(x)^2/x^4 + 1/6*log(x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(x)^2}{x^5}\,dx = \int \frac{\operatorname{acos}(x)^2}{x^5}\,dx$$

```
[In] int(acos(x)^2/x^5,x)
[Out] int(acos(x)^2/x^5, x)
```

3.646 $\int x^2 \arcsin(x)^2 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 61

$$\int x^2 \arcsin(x)^2 \, dx = -\frac{4x}{9} - \frac{2x^3}{27} + \frac{4}{9}\sqrt{1-x^2}\arcsin(x) + \frac{2}{9}x^2\sqrt{1-x^2}\arcsin(x) + \frac{1}{3}x^3\arcsin(x)^2$$

```
[Out] -4/9*x-2/27*x^3+1/3*x^3*arcsin(x)^2+4/9*arcsin(x)*(-x^2+1)^(1/2)+2/9*x^2*ar
csin(x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4723, 4795, 4767, 8, 30}

$$\int x^2 \arcsin(x)^2 \, dx = \frac{1}{3}x^3\arcsin(x)^2 + \frac{2}{9}\sqrt{1-x^2}x^2\arcsin(x) + \frac{4}{9}\sqrt{1-x^2}\arcsin(x) - \frac{2x^3}{27} - \frac{4x}{9}$$

```
[In] Int[x^2*ArcSin[x]^2,x]
```

```
[Out] (-4*x)/9 - (2*x^3)/27 + (4*Sqrt[1 - x^2]*ArcSin[x])/9 + (2*x^2*Sqrt[1 - x^2
]*ArcSin[x])/9 + (x^3*ArcSin[x]^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4767

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4795

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arcsin(x)^2-\frac{2}{3}\int\frac{x^3\arcsin(x)}{\sqrt{1-x^2}}\,dx\\
&= \frac{2}{9}x^2\sqrt{1-x^2}\arcsin(x)+\frac{1}{3}x^3\arcsin(x)^2-\frac{2\int x^2\,dx}{9}-\frac{4}{9}\int\frac{x\arcsin(x)}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2x^3}{27}+\frac{4}{9}\sqrt{1-x^2}\arcsin(x)+\frac{2}{9}x^2\sqrt{1-x^2}\arcsin(x)+\frac{1}{3}x^3\arcsin(x)^2-\frac{4\int 1\,dx}{9}\\
&= -\frac{4x}{9}-\frac{2x^3}{27}+\frac{4}{9}\sqrt{1-x^2}\arcsin(x)+\frac{2}{9}x^2\sqrt{1-x^2}\arcsin(x)+\frac{1}{3}x^3\arcsin(x)^2
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int x^2 \arcsin(x)^2\,dx = \frac{1}{27}\left(-2x\left(6+x^2\right)+6\sqrt{1-x^2}\left(2+x^2\right)\arcsin(x)+9x^3\arcsin(x)^2\right)$$

```
[In] Integrate[x^2*ArcSin[x]^2,x]
[Out] (-2*x*(6 + x^2) + 6*Sqrt[1 - x^2]*(2 + x^2)*ArcSin[x] + 9*x^3*ArcSin[x]^2)/
27
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{x^3\arcsin(x)^2}{3}+\frac{2\arcsin(x)(x^2+2)\sqrt{-x^2+1}}{9}-\frac{2x^3}{27}-\frac{4x}{9}$	37

```
[In] int(x^2*arcsin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*arcsin(x)^2+2/9*arcsin(x)*(x^2+2)*(-x^2+1)^(1/2)-2/27*x^3-4/9*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int x^2 \arcsin(x)^2\,dx = \frac{1}{3}\,x^3\arcsin\left(x\right)^2-\frac{2}{27}\,x^3+\frac{2}{9}\,\left(x^2+2\right)\sqrt{-x^2+1}\arcsin\left(x\right)-\frac{4}{9}\,x$$

```
[In] integrate(x^2*arcsin(x)^2,x, algorithm="fricas")
[Out] 1/3*x^3*arcsin(x)^2 - 2/27*x^3 + 2/9*(x^2 + 2)*sqrt(-x^2 + 1)*arcsin(x) - 4
/9*x
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int x^2 \arcsin(x)^2 \, dx = \frac{x^3 \operatorname{asin}^2(x)}{3} - \frac{2x^3}{27} + \frac{2x^2\sqrt{1-x^2}\operatorname{asin}(x)}{9} - \frac{4x}{9} + \frac{4\sqrt{1-x^2}\operatorname{asin}(x)}{9}$$

```
[In] integrate(x**2*asin(x)**2,x)
[Out] x**3*asin(x)**2/3 - 2*x**3/27 + 2*x**2*sqrt(1 - x**2)*asin(x)/9 - 4*x/9 + 4
*sqrt(1 - x**2)*asin(x)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int x^2 \arcsin(x)^2 \, dx = \frac{1}{3}\,x^3 \arcsin(x)^2 - \frac{2}{27}\,x^3 + \frac{2}{9}\left(\sqrt{-x^2+1}x^2 + 2\,\sqrt{-x^2+1}\right)\arcsin(x) - \frac{4}{9}\,x$$

```
[In] integrate(x^2*arcsin(x)^2,x, algorithm="maxima")
[Out] 1/3*x^3*arcsin(x)^2 - 2/27*x^3 + 2/9*(sqrt(-x^2 + 1)*x^2 + 2*sqrt(-x^2 + 1)
)*arcsin(x) - 4/9*x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int x^2 \arcsin(x)^2 \, dx = \frac{1}{3}\,(x^2-1)x\arcsin(x)^2 + \frac{1}{3}\,x\arcsin(x)^2 - \frac{2}{9}\,(-x^2+1)^{\frac{3}{2}}\arcsin(x) - \frac{2}{27}\,(x^2-1)x + \frac{2}{3}\,\sqrt{-x^2+1}\arcsin(x) - \frac{14}{27}\,x$$

```
[In] integrate(x^2*arcsin(x)^2,x, algorithm="giac")
[Out] 1/3*(x^2 - 1)*x*arcsin(x)^2 + 1/3*x*arcsin(x)^2 - 2/9*(-x^2 + 1)^(3/2)*arcs
in(x) - 2/27*(x^2 - 1)*x + 2/3*sqrt(-x^2 + 1)*arcsin(x) - 14/27*x
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arcsin(x)^2 \, dx = \int x^2 \operatorname{asin}(x)^2 \, dx$$

```
[In] int(x^2*asin(x)^2,x)
[Out] int(x^2*asin(x)^2, x)
```

3.647 $\int x^3 \arctan(x)^2 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 53

$$\int x^3 \arctan(x)^2 \, dx = \frac{x^2}{12} + \frac{1}{2}x \arctan(x) - \frac{1}{6}x^3 \arctan(x) - \frac{\arctan(x)^2}{4} + \frac{1}{4}x^4 \arctan(x)^2 - \frac{1}{3}\log\left(1+x^2\right)$$

```
[Out] 1/12*x^2+1/2*x*arctan(x)-1/6*x^3*arctan(x)-1/4*arctan(x)^2+1/4*x^4*arctan(x
)^2-1/3*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {4946, 5036, 272, 45, 4930, 266, 5004}

$$\int x^3 \arctan(x)^2 \, dx = \frac{1}{4}x^4 \arctan(x)^2 - \frac{1}{6}x^3 \arctan(x) + \frac{1}{2}x \arctan(x) - \frac{\arctan(x)^2}{4} + \frac{x^2}{12} - \frac{1}{3}\log\left(x^2+1\right)$$

```
[In] Int[x^3*ArcTan[x]^2,x]
```

```
[Out] x^2/12 + (x*ArcTan[x])/2 - (x^3*ArcTan[x])/6 - ArcTan[x]^2/4 + (x^4*ArcTan[
x]^2)/4 - Log[1 + x^2]/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{4}x^4\arctan(x)^2 - \frac{1}{2}\int \frac{x^4\arctan(x)}{1+x^2}\,dx$$

$$= \frac{1}{4}x^4 \arctan(x)^2 - \frac{1}{2}\int x^2 \arctan(x)\,dx + \frac{1}{2}\int \frac{x^2 \arctan(x)}{1+x^2}\,dx$$

$$= -\frac{1}{6}x^3 \arctan(x) + \frac{1}{4}x^4 \arctan(x)^2 + \frac{1}{6}\int \frac{x^3}{1+x^2}\,dx + \frac{1}{2}\int \arctan(x)\,dx - \frac{1}{2}\int \frac{\arctan(x)}{1+x^2}\,dx$$

$$= \frac{1}{2}x\arctan(x) - \frac{1}{6}x^3\arctan(x) - \frac{\arctan(x)^2}{4} + \frac{1}{4}x^4\arctan(x)^2 + \frac{1}{12}\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, x^2\right) - \frac{1}{2}\int \frac{x}{1+x^2}\,dx$$

$$= \frac{1}{2}x\arctan(x) - \frac{1}{6}x^3\arctan(x) - \frac{\arctan(x)^2}{4} + \frac{1}{4}x^4\arctan(x)^2 - \frac{1}{4}\log\left(1+x^2\right) + \frac{1}{12}\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, x^2\right)$$

$$= \frac{x^2}{12} + \frac{1}{2}x\arctan(x) - \frac{1}{6}x^3\arctan(x) - \frac{\arctan(x)^2}{4} + \frac{1}{4}x^4\arctan(x)^2 - \frac{1}{3}\log\left(1+x^2\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int x^3 \arctan(x)^2\,dx = \frac{1}{12}\left(x^2 - 2x\left(-3+x^2\right)\arctan(x) + 3\left(-1+x^4\right)\arctan(x)^2 - 4\log\left(1+x^2\right)\right)$$

```
[In] Integrate[x^3*ArcTan[x]^2,x]
[Out] (x^2 - 2*x*(-3 + x^2)*ArcTan[x] + 3*(-1 + x^4)*ArcTan[x]^2 - 4*Log[1 + x^2]
)/12
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

method	result
default	$\frac{x^2}{12} + \frac{x\arctan(x)}{2} - \frac{x^3\arctan(x)}{6} - \frac{\arctan(x)^2}{4} + \frac{x^4\arctan(x)^2}{4} - \frac{\ln(x^2+1)}{3}$
parts	$\frac{x^2}{12} + \frac{x\arctan(x)}{2} - \frac{x^3\arctan(x)}{6} - \frac{\arctan(x)^2}{4} + \frac{x^4\arctan(x)^2}{4} - \frac{\ln(x^2+1)}{3}$
parallelrisch	$\frac{x^4\arctan(x)^2}{4} - \frac{x^3\arctan(x)}{6} + \frac{x^2}{12} + \frac{x\arctan(x)}{2} - \frac{\arctan(x)^2}{4} - \frac{\ln(x^2+1)}{3} - \frac{1}{12}$
risch	$-\frac{\left(\frac{x^4}{4}-\frac{1}{4}\right)\ln(ix+1)^2}{4} - \frac{\left(-\frac{x^4\ln(-ix+1)}{2} - \frac{ix^3}{3} + ix + \frac{\ln(-ix+1)}{2}\right)\ln(ix+1)}{4} - \frac{x^4\ln(-ix+1)^2}{16} + \frac{\ln(-ix+1)^2}{16} - \frac{ix^3\ln(-ix}{12}$

```
[In] int(x^3*arctan(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*x^2+1/2*x*arctan(x)-1/6*x^3*arctan(x)-1/4*arctan(x)^2+1/4*x^4*arctan(x
)^2-1/3*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int x^3 \arctan(x)^2 \, dx = \frac{1}{4} \left(x^4 - 1\right) \arctan\left(x\right)^2 + \frac{1}{12} x^2 - \frac{1}{6} \left(x^3 - 3\,x\right) \arctan\left(x\right) - \frac{1}{3} \log\left(x^2 + 1\right)$$

```
[In] integrate(x^3*arctan(x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x^4 - 1)*arctan(x)^2 + 1/12*x^2 - 1/6*(x^3 - 3*x)*arctan(x) - 1/3*log(
x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int x^3 \arctan(x)^2 \, dx = \frac{x^4 \operatorname{atan}^2\left(x\right)}{4} - \frac{x^3 \operatorname{atan}\left(x\right)}{6} + \frac{x^2}{12} + \frac{x \operatorname{atan}\left(x\right)}{2} - \frac{\log\left(x^2 + 1\right)}{3} - \frac{\operatorname{atan}^2\left(x\right)}{4}$$

```
[In] integrate(x**3*atan(x)**2,x)
```

```
[Out] x**4*atan(x)**2/4 - x**3*atan(x)/6 + x**2/12 + x*atan(x)/2 - log(x**2 + 1)/
3 - atan(x)**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\begin{aligned}\int x^3 \arctan(x)^2 \, dx = {} & \frac{1}{4} x^4 \arctan\left(x\right)^2 + \frac{1}{12} x^2 - \frac{1}{6} \left(x^3 - 3\,x + 3 \arctan\left(x\right)\right) \arctan\left(x\right) \\ & + \frac{1}{4} \arctan\left(x\right)^2 - \frac{1}{3} \log\left(x^2 + 1\right)\end{aligned}$$

```
[In] integrate(x^3*arctan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan(x)^2 + 1/12*x^2 - 1/6*(x^3 - 3*x + 3*arctan(x))*arctan(x) +
1/4*arctan(x)^2 - 1/3*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^3 \arctan(x)^2 \, dx = \frac{1}{4} \, x^4 \arctan\left(x\right)^2 - \frac{1}{6} \, x^3 \arctan\left(x\right) + \frac{1}{12} \, x^2 + \frac{1}{2} \, x \arctan\left(x\right) - \frac{1}{4} \, \arctan\left(x\right)^2 - \frac{1}{3} \, \log\left(x^2 + 1\right)$$

```
[In] integrate(x^3*arctan(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^4*arctan(x)^2 - 1/6*x^3*arctan(x) + 1/12*x^2 + 1/2*x*arctan(x) - 1/4*
arctan(x)^2 - 1/3*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^3 \arctan(x)^2 \, dx = \frac{x^4 \operatorname{atan}(x)^2}{4} - \frac{x^3 \operatorname{atan}(x)}{6} - \frac{\operatorname{atan}(x)^2}{4} - \frac{\ln\left(x^2+1\right)}{3} + \frac{x \operatorname{atan}(x)}{2} + \frac{x^2}{12}$$

```
[In] int(x^3*atan(x)^2,x)
```

```
[Out] (x^4*atan(x)^2)/4 - (x^3*atan(x))/6 - atan(x)^2/4 - log(x^2 + 1)/3 + (x*ata
n(x))/2 + x^2/12
```

3.648 $\int \frac{\arctan(x)^2}{x^5}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 61

$$\int \frac{\arctan(x)^2}{x^5}\,dx = -\frac{1}{12x^2}-\frac{\arctan(x)}{6x^3}+\frac{\arctan(x)}{2x}+\frac{\arctan(x)^2}{4}-\frac{\arctan(x)^2}{4x^4}-\frac{2\log(x)}{3}+\frac{1}{3}\log\left(1+x^2\right)$$

```
[Out] -1/12/x^2-1/6*arctan(x)/x^3+1/2*arctan(x)/x+1/4*arctan(x)^2-1/4*arctan(x)^2
/x^4-2/3*ln(x)+1/3*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4946, 5038, 272, 46, 36, 29, 31, 5004}

$$\int \frac{\arctan(x)^2}{x^5}\,dx = -\frac{\arctan(x)^2}{4x^4}-\frac{\arctan(x)}{6x^3}+\frac{\arctan(x)^2}{4}+\frac{\arctan(x)}{2x}-\frac{1}{12x^2}+\frac{1}{3}\log\left(x^2+1\right)-\frac{2\log(x)}{3}$$

```
[In] Int[ArcTan[x]^2/x^5,x]
```

```
[Out] -1/12*1/x^2 - ArcTan[x]/(6*x^3) + ArcTan[x]/(2*x) + ArcTan[x]^2/4 - ArcTan[
x]^2/(4*x^4) - (2*Log[x])/3 + Log[1 + x^2]/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\arctan(x)^2}{4x^4}+\frac{1}{2}\int\frac{\arctan(x)}{x^4\left(1+x^2\right)}\,dx\\
&= -\frac{\arctan(x)^2}{4x^4}+\frac{1}{2}\int\frac{\arctan(x)}{x^4}\,dx-\frac{1}{2}\int\frac{\arctan(x)}{x^2\left(1+x^2\right)}\,dx\\
&= -\frac{\arctan(x)}{6x^3}-\frac{\arctan(x)^2}{4x^4}+\frac{1}{6}\int\frac{1}{x^3\left(1+x^2\right)}\,dx-\frac{1}{2}\int\frac{\arctan(x)}{x^2}\,dx+\frac{1}{2}\int\frac{\arctan(x)}{1+x^2}\,dx\\
&= -\frac{\arctan(x)}{6x^3}+\frac{\arctan(x)}{2x}+\frac{\arctan(x)^2}{4}-\frac{\arctan(x)^2}{4x^4}\\
&\quad+\frac{1}{12}\text{Subst}\left(\int\frac{1}{x^2(1+x)}\,dx,x,x^2\right)-\frac{1}{2}\int\frac{1}{x\left(1+x^2\right)}\,dx\\
&= -\frac{\arctan(x)}{6x^3}+\frac{\arctan(x)}{2x}+\frac{\arctan(x)^2}{4}-\frac{\arctan(x)^2}{4x^4}\\
&\quad+\frac{1}{12}\text{Subst}\left(\int\left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)\,dx,x,x^2\right)-\frac{1}{4}\text{Subst}\left(\int\frac{1}{x(1+x)}\,dx,x,x^2\right)\\
&= -\frac{1}{12x^2}-\frac{\arctan(x)}{6x^3}+\frac{\arctan(x)}{2x}+\frac{\arctan(x)^2}{4}-\frac{\arctan(x)^2}{4x^4}-\frac{\log(x)}{6}\\
&\quad+\frac{1}{12}\log\left(1+x^2\right)-\frac{1}{4}\text{Subst}\left(\int\frac{1}{x}\,dx,x,x^2\right)+\frac{1}{4}\text{Subst}\left(\int\frac{1}{1+x}\,dx,x,x^2\right)\\
&= -\frac{1}{12x^2}-\frac{\arctan(x)}{6x^3}+\frac{\arctan(x)}{2x}+\frac{\arctan(x)^2}{4}-\frac{\arctan(x)^2}{4x^4}-\frac{2\log(x)}{3}+\frac{1}{3}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$
\int\frac{\arctan(x)^2}{x^5}\,dx=-\frac{1}{12x^2}+\frac{\left(-1+3x^2\right)\arctan(x)}{6x^3}+\frac{\left(-1+x^4\right)\arctan(x)^2}{4x^4}-\frac{2\log(x)}{3}+\frac{1}{3}\log\left(1+x^2\right)
$$

```
[In] Integrate[ArcTan[x]^2/x^5,x]
```

```
[Out] -1/12*1/x^2 + ((-1 + 3*x^2)*ArcTan[x])/(6*x^3) + ((-1 + x^4)*ArcTan[x]^2)/(
4*x^4) - (2*Log[x])/3 + Log[1 + x^2]/3
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

method	result
default	$-\frac{1}{12x^{2}}-\frac{\arctan(x)}{6x^{3}}+\frac{\arctan(x)}{2x}+\frac{\arctan(x)^{2}}{4}-\frac{\arctan(x)^{2}}{4x^{4}}-\frac{2\ln(x)}{3}+\frac{\ln(x^{2}+1)}{3}$
parts	$-\frac{1}{12x^{2}}-\frac{\arctan(x)}{6x^{3}}+\frac{\arctan(x)}{2x}+\frac{\arctan(x)^{2}}{4}-\frac{\arctan(x)^{2}}{4x^{4}}-\frac{2\ln(x)}{3}+\frac{\ln(x^{2}+1)}{3}$
parallelrisch	$-\frac{-3x^{4}\arctan(x)^{2}+8x^{4}\ln(x)-4\ln(x^{2}+1)x^{4}-6x^{3}\arctan(x)+x^{2}+2x\arctan(x)+3\arctan(x)^{2}}{12x^{4}}$
risch	$-\frac{(x^{4}-1)\ln(ix+1)^{2}}{16x^{4}}+\frac{(3x^{4}\ln(-ix+1)-6ix^{3}+2ix-3\ln(-ix+1))\ln(ix+1)}{24x^{4}}-\frac{3x^{4}\ln(-ix+1)^{2}+32x^{4}\ln(x)-16\ln(x^{2}+1)x^{4}-}{}$

[In] int(arctan(x)^2/x^5,x,method=_RETURNVERBOSE)

[Out] -1/12/x^2-1/6/x^3*arctan(x)+1/2/x*arctan(x)+1/4*arctan(x)^2-1/4*arctan(x)^2
/x^4-2/3*ln(x)+1/3*ln(x^2+1)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{\arctan(x)^2}{x^5}\,dx = \frac{4\,x^{4}\log\left(x^{2}+1\right)-8\,x^{4}\log\left(x\right)+3\,\left(x^{4}-1\right)\arctan\left(x\right)^{2}-x^{2}+2\,\left(3\,x^{3}-x\right)\arctan\left(x\right)}{12\,x^{4}}$$

[In] integrate(arctan(x)^2/x^5,x, algorithm="fricas")

[Out] 1/12*(4*x^4*log(x^2 + 1) - 8*x^4*log(x) + 3*(x^4 - 1)*arctan(x)^2 - x^2 + 2
*(3*x^3 - x)*arctan(x))/x^4

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{\arctan(x)^2}{x^5}\,dx = -\frac{2\log\left(x\right)}{3}+\frac{\log\left(x^{2}+1\right)}{3}+\frac{\operatorname{atan}^{2}\left(x\right)}{4}+\frac{\operatorname{atan}\left(x\right)}{2x}-\frac{1}{12x^{2}}-\frac{\operatorname{atan}\left(x\right)}{6x^{3}}-\frac{\operatorname{atan}^{2}\left(x\right)}{4x^{4}}$$

[In] integrate(atan(x)**2/x**5,x)

[Out] -2*log(x)/3 + log(x**2 + 1)/3 + atan(x)**2/4 + atan(x)/(2*x) - 1/(12*x**2)
- atan(x)/(6*x**3) - atan(x)**2/(4*x**4)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(x)^2}{x^5}\,dx = \frac{1}{6}\left(\frac{3\,x^2-1}{x^3}+3\,\arctan\left(x\right)\right)\arctan\left(x\right) - \frac{3\,x^2\arctan\left(x\right)^2-4\,x^2\log\left(x^2+1\right)+8\,x^2\log\left(x\right)+1}{12\,x^2} - \frac{\arctan\left(x\right)^2}{4\,x^4}$$

```
[In] integrate(arctan(x)^2/x^5,x, algorithm="maxima")
[Out] 1/6*((3*x^2 - 1)/x^3 + 3*arctan(x))*arctan(x) - 1/12*(3*x^2*arctan(x)^2 - 4
*x^2*log(x^2 + 1) + 8*x^2*log(x) + 1)/x^2 - 1/4*arctan(x)^2/x^4
```

Giac [F]

$$\int \frac{\arctan(x)^2}{x^5}\,dx = \int \frac{\arctan\left(x\right)^2}{x^5}\,dx$$

```
[In] integrate(arctan(x)^2/x^5,x, algorithm="giac")
[Out] integrate(arctan(x)^2/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{\arctan(x)^2}{x^5}\,dx = \frac{\ln\left(x^2+1\right)}{3} - \frac{2\,\ln\left(x\right)}{3} - \mathrm{atan}(x)^2\left(\frac{1}{4\,x^4}-\frac{1}{4}\right) - \frac{1}{12\,x^2} + \frac{\mathrm{atan}(x)\,\left(\frac{x^2}{2}-\frac{1}{6}\right)}{x^3}$$

```
[In] int(atan(x)^2/x^5,x)
[Out] log(x^2 + 1)/3 - (2*log(x))/3 - atan(x)^2*(1/(4*x^4) - 1/4) - 1/(12*x^2) +
(atan(x)*(x^2/2 - 1/6))/x^3
```

3.649 $\int x^3 \csc^{-1}(x)^2 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 63

$$\int x^3 \csc^{-1}(x)^2 \, dx = \frac{x^2}{12} + \frac{1}{3}\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x) + \frac{1}{6}\sqrt{1-\frac{1}{x^2}}x^3\csc^{-1}(x) + \frac{1}{4}x^4\csc^{-1}(x)^2 + \frac{\log(x)}{3}$$

```
[Out] 1/12*x^2+1/4*x^4*arccsc(x)^2+1/3*ln(x)+1/3*x*arccsc(x)*(1-1/x^2)^(1/2)+1/6*
x^3*arccsc(x)*(1-1/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5331, 3843, 4270, 4269, 3556}

$$\int x^3 \csc^{-1}(x)^2 \, dx = \frac{1}{4}x^4\csc^{-1}(x)^2 + \frac{x^2}{12} + \frac{1}{3}\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x) + \frac{1}{6}\sqrt{1-\frac{1}{x^2}}x^3\csc^{-1}(x) + \frac{\log(x)}{3}$$

```
[In] Int[x^3*ArcCsc[x]^2,x]
```

```
[Out] x^2/12 + (Sqrt[1 - x^(-2)]*x*ArcCsc[x])/3 + (Sqrt[1 - x^(-2)]*x^3*ArcCsc[x]
)/6 + (x^4*ArcCsc[x]^2)/4 + Log[x]/3
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3843

```
Int[Cot[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)
*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^p/(b*n*p)),
 x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csc[a + b*x^n]^p, x], x] /; F
reeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^2\cot(x)\csc^4(x)\,dx, x, \csc^{-1}(x)\right)\\
&= \frac{1}{4}x^4\csc^{-1}(x)^2 - \frac{1}{2}\text{Subst}\left(\int x\csc^4(x)\,dx, x, \csc^{-1}(x)\right)\\
&= \frac{x^2}{12} + \frac{1}{6}\sqrt{1-\frac{1}{x^2}}x^3\csc^{-1}(x) + \frac{1}{4}x^4\csc^{-1}(x)^2 - \frac{1}{3}\text{Subst}\left(\int x\csc^2(x)\,dx, x, \csc^{-1}(x)\right)\\
&= \frac{x^2}{12} + \frac{1}{3}\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x) + \frac{1}{6}\sqrt{1-\frac{1}{x^2}}x^3\csc^{-1}(x)\\
&\quad + \frac{1}{4}x^4\csc^{-1}(x)^2 - \frac{1}{3}\text{Subst}\left(\int\cot(x)\,dx, x, \csc^{-1}(x)\right)\\
&= \frac{x^2}{12} + \frac{1}{3}\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x) + \frac{1}{6}\sqrt{1-\frac{1}{x^2}}x^3\csc^{-1}(x) + \frac{1}{4}x^4\csc^{-1}(x)^2 + \frac{\log(x)}{3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int x^3 \csc^{-1}(x)^2\,dx = \frac{1}{12}\left(x^2+2\sqrt{1-\frac{1}{x^2}}x(2+x^2)\csc^{-1}(x)+3x^4\csc^{-1}(x)^2+4\log(x)\right)$$

```
[In] Integrate[x^3*ArcCsc[x]^2,x]
[Out] (x^2 + 2*Sqrt[1 - x^(-2)]*x*(2 + x^2)*ArcCsc[x] + 3*x^4*ArcCsc[x]^2 + 4*Log
[x])/12
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{x^4\,\mathrm{arccsc}(x)^2}{4}+\frac{x^3\,\mathrm{arccsc}(x)\sqrt{\frac{x^2-1}{x^2}}}{6}+\frac{x^2}{12}+\frac{\mathrm{arccsc}(x)\sqrt{\frac{x^2-1}{x^2}}\,x}{3}-\frac{\ln\left(\frac{1}{x}\right)}{3}$	56

```
[In] int(x^3*arccsc(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x^4*arccsc(x)^2+1/6*x^3*arccsc(x)*((x^2-1)/x^2)^(1/2)+1/12*x^2+1/3*arcc
sc(x)*((x^2-1)/x^2)^(1/2)*x-1/3*ln(1/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

$$\int x^3 \csc^{-1}(x)^2\,dx = \frac{1}{4}\,x^4\,\mathrm{arccsc}\left(x\right)^2+\frac{1}{6}\,(x^2+2)\sqrt{x^2-1}\,\mathrm{arccsc}\left(x\right)+\frac{1}{12}\,x^2+\frac{1}{3}\,\log\left(x\right)$$

```
[In] integrate(x^3*arccsc(x)^2,x, algorithm="fricas")
[Out] 1/4*x^4*arccsc(x)^2 + 1/6*(x^2 + 2)*sqrt(x^2 - 1)*arccsc(x) + 1/12*x^2 + 1/
3*log(x)
```

Sympy [F]

$$\int x^3 \csc^{-1}(x)^2 \, dx = \int x^3 \operatorname{acsc}^2(x) \, dx$$

```
[In] integrate(x**3*acsc(x)**2,x)
```

```
[Out] Integral(x**3*acsc(x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.51

$$\int x^3 \csc^{-1}(x)^2 \, dx = \frac{1}{4} x^4 \operatorname{arccsc}(x)^2 + \frac{2 x^4 \arctan\left(1, \sqrt{x+1}\sqrt{x-1}\right) + 2 x^2 \arctan\left(1, \sqrt{x+1}\sqrt{x-1}\right) + \left(x^2 + 2 \log\left(x^2\right)\right)\sqrt{x+1}\sqrt{x-1} - 4}{12 \sqrt{x+1}\sqrt{x-1}}$$

```
[In] integrate(x^3*arccsc(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccsc(x)^2 + 1/12*(2*x^4*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + 2*x
^2*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + (x^2 + 2*log(x^2))*sqrt(x + 1)*sqr
t(x - 1) - 4*arctan2(1, sqrt(x + 1)*sqrt(x - 1)))/(sqrt(x + 1)*sqrt(x - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(49) = 98.

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.68

$$\int x^3 \csc^{-1}(x)^2 \, dx = \frac{1}{4} x^4 \arcsin\left(\frac{1}{x}\right)^2 + \frac{1}{12} x^2 \left(\frac{2}{x^2} + 1\right) + \frac{1}{48} \left(x^3 \left(\sqrt{-\frac{1}{x^2} + 1} - 1\right)^3 + 9 x \left(\sqrt{-\frac{1}{x^2} + 1} - 1\right) - \frac{9 x^2 \left(\sqrt{-\frac{1}{x^2} + 1} - 1\right)^2 + 1}{x^3 \left(\sqrt{-\frac{1}{x^2} + 1} - 1\right)^3} \right) \arcsin\left(\frac{1}{x}\right) - \frac{1}{6} \log\left(\frac{1}{x^2}\right)$$

```
[In] integrate(x^3*arccsc(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^4*arcsin(1/x)^2 + 1/12*x^2*(2/x^2 + 1) + 1/48*(x^3*(sqrt(-1/x^2 + 1)
- 1)^3 + 9*x*(sqrt(-1/x^2 + 1) - 1) - (9*x^2*(sqrt(-1/x^2 + 1) - 1)^2 + 1)/
(x^3*(sqrt(-1/x^2 + 1) - 1)^3))*arcsin(1/x) - 1/6*log(x^(-2))
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \csc^{-1}(x)^2 \, dx = \int x^3 \operatorname{asin}\left(\frac{1}{x}\right)^2 \, dx$$

```
[In] int(x^3*asin(1/x)^2,x)
[Out] int(x^3*asin(1/x)^2, x)
```

3.650 $\int \frac{\sec^{-1}(x)^4}{x^5} dx$

Optimal result

Integrand size = 8, antiderivative size = 148

$$\int \frac{\sec^{-1}(x)^4}{x^5} dx = -\frac{3}{128x^4} - \frac{45}{128x^2} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{32x^3} - \frac{45\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{64x} - \frac{45}{128}\sec^{-1}(x)^2 + \frac{3\sec^{-1}(x)^2}{16x^4} + \frac{9\sec^{-1}(x)^2}{16x^2} + \frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3} + \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{8x} + \frac{3}{32}\sec^{-1}(x)^4 - \frac{\sec^{-1}(x)^4}{4x^4}$$

```
[Out] -3/128/x^4-45/128/x^2-45/128*arcsec(x)^2+3/16*arcsec(x)^2/x^4+9/16*arcsec(x
)^2/x^2+3/32*arcsec(x)^4-1/4*arcsec(x)^4/x^4-3/32*arcsec(x)*(1-1/x^2)^(1/2)
/x^3-45/64*arcsec(x)*(1-1/x^2)^(1/2)/x+1/4*arcsec(x)^3*(1-1/x^2)^(1/2)/x^3+
3/8*arcsec(x)^3*(1-1/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5330, 3525, 3392, 30, 3391}

$$\int \frac{\sec^{-1}(x)^4}{x^5} dx = -\frac{3}{128x^4} - \frac{\sec^{-1}(x)^4}{4x^4} + \frac{3\sec^{-1}(x)^2}{16x^4} - \frac{45}{128x^2} + \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{8x} + \frac{9\sec^{-1}(x)^2}{16x^2} - \frac{45\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{64x} + \frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{32x^3} + \frac{3}{32}\sec^{-1}(x)^4 - \frac{45}{128}\sec^{-1}(x)^2$$

```
[In] Int[ArcSec[x]^4/x^5,x]
```

```
[Out] -3/(128*x^4) - 45/(128*x^2) - (3*Sqrt[1 - x^(-2)]*ArcSec[x])/(32*x^3) - (45
*Sqrt[1 - x^(-2)]*ArcSec[x])/(64*x) - (45*ArcSec[x]^2)/128 + (3*ArcSec[x]^2
)/(16*x^4) + (9*ArcSec[x]^2)/(16*x^2) + (Sqrt[1 - x^(-2)]*ArcSec[x]^3)/(4*x
^3) + (3*Sqrt[1 - x^(-2)]*ArcSec[x]^3)/(8*x) + (3*ArcSec[x]^4)/32 - ArcSec[
x]^4/(4*x^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3525

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p
 + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int x^4\cos^3(x)\sin(x)\,dx, x, \sec^{-1}(x)\right)$$

$$= -\frac{\sec^{-1}(x)^4}{4x^4} + \text{Subst}\left(\int x^3\cos^4(x)\,dx, x, \sec^{-1}(x)\right)$$

$$= \frac{3\sec^{-1}(x)^2}{16x^4} + \frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3} - \frac{\sec^{-1}(x)^4}{4x^4} - \frac{3}{8}\text{Subst}\left(\int x\cos^4(x)\,dx, x, \sec^{-1}(x)\right) + \frac{3}{4}\text{Subst}\left(\int x^3\cos^2(x)\,dx, x, \sec^{-1}(x)\right)$$

$$= -\frac{3}{128x^4} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{32x^3} + \frac{3\sec^{-1}(x)^2}{16x^4} + \frac{9\sec^{-1}(x)^2}{16x^2} + \frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3} + \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{8x} - \frac{\sec^{-1}(x)^4}{4x^4} - \frac{9}{32}\text{Subst}\left(\int x\cos^2(x)\,dx, x, \sec^{-1}(x)\right) + \frac{3}{8}\text{Subst}\left(\int x^3\,dx, x, \sec^{-1}(x)\right) - \frac{9}{8}\text{Subst}\left(\int x\cos^2(x)\,dx, x, \sec^{-1}(x)\right)$$

$$= -\frac{3}{128x^4} - \frac{45}{128x^2} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{32x^3} - \frac{45\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{64x} + \frac{3\sec^{-1}(x)^2}{16x^4} + \frac{9\sec^{-1}(x)^2}{16x^2} + \frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3} + \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{8x} + \frac{3}{32}\sec^{-1}(x)^4 - \frac{\sec^{-1}(x)^4}{4x^4} - \frac{9}{64}\text{Subst}\left(\int x\,dx, x, \sec^{-1}(x)\right) - \frac{9}{16}\text{Subst}\left(\int x\,dx, x, \sec^{-1}(x)\right)$$

$$= -\frac{3}{128x^4} - \frac{45}{128x^2} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{32x^3} - \frac{45\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{64x} - \frac{45}{128}\sec^{-1}(x)^2 + \frac{3\sec^{-1}(x)^2}{16x^4} + \frac{9\sec^{-1}(x)^2}{16x^2} + \frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3} + \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{8x} + \frac{3}{32}\sec^{-1}(x)^4 - \frac{\sec^{-1}(x)^4}{4x^4}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.62

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx$$

$$= \frac{-3 - 45x^2 - 6\sqrt{1-\frac{1}{x^2}}x(2+15x^2)\sec^{-1}(x) + (24+72x^2-45x^4)\sec^{-1}(x)^2 + 16\sqrt{1-\frac{1}{x^2}}x(2+3x^2)\sec}{128x^4}$$

```
[In] Integrate[ArcSec[x]^4/x^5,x]
```

```
[Out] (-3 - 45*x^2 - 6*Sqrt[1 - x^(-2)]*x*(2 + 15*x^2)*ArcSec[x] + (24 + 72*x^2 -
 45*x^4)*ArcSec[x]^2 + 16*Sqrt[1 - x^(-2)]*x*(2 + 3*x^2)*ArcSec[x]^3 + 4*(-
8 + 3*x^4)*ArcSec[x]^4)/(128*x^4)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.18

method	result
default	$-\frac{\operatorname{arcsec}(x)^4}{4x^4}+\frac{\operatorname{arcsec}(x)^3\left(3\operatorname{arcsec}(x)x^3+3x^2\sqrt{\frac{x^2-1}{x^2}}+2\sqrt{\frac{x^2-1}{x^2}}\right)}{8x^3}+\frac{3\operatorname{arcsec}(x)^2}{16x^4}-\frac{3\operatorname{arcsec}(x)\left(3\operatorname{arcsec}(x)x^3+3x^2\sqrt{\frac{x^2-1}{x^2}}+2\right.}{64x^3}$

```
[In] int(arcsec(x)^4/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arcsec(x)^4/x^4+1/8*arcsec(x)^3*(3*arcsec(x)*x^3+3*x^2*((x^2-1)/x^2)^(
1/2)+2*((x^2-1)/x^2)^(1/2))/x^3+3/16*arcsec(x)^2/x^4-3/64*arcsec(x)*(3*arcs
ec(x)*x^3+3*x^2*((x^2-1)/x^2)^(1/2)+2*((x^2-1)/x^2)^(1/2))/x^3+45/128*arcse
c(x)^2-3/512*(3*x^2+2)^2/x^4+9/16*arcsec(x)^2/x^2-9/16*arcsec(x)*(x*arcsec(
x)+((x^2-1)/x^2)^(1/2))/x+9/32-9/32/x^2-9/32*arcsec(x)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx$$
$$=\frac{4\left(3x^4-8\right)\operatorname{arcsec}(x)^4-3\left(15x^4-24x^2-8\right)\operatorname{arcsec}(x)^2-45x^2+2\left(8\left(3x^2+2\right)\operatorname{arcsec}(x)^3-3\left(15x^2+\right.\right.}{128x^4}$$

```
[In] integrate(arcsec(x)^4/x^5,x, algorithm="fricas")
```

```
[Out] 1/128*(4*(3*x^4 - 8)*arcsec(x)^4 - 3*(15*x^4 - 24*x^2 - 8)*arcsec(x)^2 - 45
*x^2 + 2*(8*(3*x^2 + 2)*arcsec(x)^3 - 3*(15*x^2 + 2)*arcsec(x))*sqrt(x^2 -
1) - 3)/x^4
```

Sympy [F]

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx=\int \frac{\operatorname{asec}^4(x)}{x^5}\,dx$$

```
[In] integrate(asec(x)**4/x**5,x)
```

```
[Out] Integral(asec(x)**4/x**5, x)
```

Maxima [F]

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx = \int \frac{\operatorname{arcsec}(x)^4}{x^5}\,dx$$

```
[In] integrate(arcsec(x)^4/x^5,x, algorithm="maxima")
```

```
[Out] 1/64*(64*x^4*integrate(1/8*(12*(x^2 - 1)*log(x^2)^2*log(x)^2 - 16*(x^2 - 1)
*log(x^2)*log(x)^3 + 8*(x^2 - 1)*log(x)^4 + (x^2 - 4*(x^2 - 1)*log(x) - 1)*
log(x^2)^3 - 12*(4*(x^2 - 1)*log(x)^2 + (x^2 - 4*(x^2 - 1)*log(x) - 1)*log(
x^2))*arctan(sqrt(x + 1)*sqrt(x - 1))^2 + 2*(4*arctan(sqrt(x + 1)*sqrt(x -
1))^3 - 3*arctan(sqrt(x + 1)*sqrt(x - 1))*log(x^2)^2)*sqrt(x + 1)*sqrt(x -
1))/(x^7 - x^5), x) - 16*arctan(sqrt(x + 1)*sqrt(x - 1))^4 + 24*arctan(sqrt
(x + 1)*sqrt(x - 1))^2*log(x^2)^2 - log(x^2)^4)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{\sec^{-1}(x)^4}{x^5}\,dx = {} & \frac{3}{32}\arccos\left(\frac{1}{x}\right)^4 + \frac{3\sqrt{-\frac{1}{x^2}+1}\arccos\left(\frac{1}{x}\right)^3}{8\,x} - \frac{45}{128}\arccos\left(\frac{1}{x}\right)^2 \\ & - \frac{45\sqrt{-\frac{1}{x^2}+1}\arccos\left(\frac{1}{x}\right)}{64\,x} + \frac{\sqrt{-\frac{1}{x^2}+1}\arccos\left(\frac{1}{x}\right)^3}{4\,x^3} \\ & + \frac{9\arccos\left(\frac{1}{x}\right)^2}{16\,x^2} - \frac{\arccos\left(\frac{1}{x}\right)^4}{4\,x^4} - \frac{3\sqrt{-\frac{1}{x^2}+1}\arccos\left(\frac{1}{x}\right)}{32\,x^3} \\ & - \frac{45}{128\,x^2} + \frac{3\arccos\left(\frac{1}{x}\right)^2}{16\,x^4} - \frac{3}{128\,x^4} + \frac{189}{1024}\end{aligned}$$

```
[In] integrate(arcsec(x)^4/x^5,x, algorithm="giac")
```

```
[Out] 3/32*arccos(1/x)^4 + 3/8*sqrt(-1/x^2 + 1)*arccos(1/x)^3/x - 45/128*arccos(1
/x)^2 - 45/64*sqrt(-1/x^2 + 1)*arccos(1/x)/x + 1/4*sqrt(-1/x^2 + 1)*arccos(
1/x)^3/x^3 + 9/16*arccos(1/x)^2/x^2 - 1/4*arccos(1/x)^4/x^4 - 3/32*sqrt(-1/
x^2 + 1)*arccos(1/x)/x^3 - 45/128/x^2 + 3/16*arccos(1/x)^2/x^4 - 3/128/x^4
+ 189/1024
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx = \int \frac{\mathrm{acos}\left(\frac{1}{x}\right)^4}{x^5}\,dx$$

```
[In] int(acos(1/x)^4/x^5,x)
[Out] int(acos(1/x)^4/x^5, x)
```

3.651 $\int \sqrt{1-x^2} \arcsin(x) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \sqrt{1-x^2} \arcsin(x) \, dx = -\frac{x^2}{4} + \frac{1}{2} x \sqrt{1-x^2} \arcsin(x) + \frac{\arcsin(x)^2}{4}$$

```
[Out] -1/4*x^2+1/4*arcsin(x)^2+1/2*x*arcsin(x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4741, 4737, 30}

$$\int \sqrt{1-x^2} \arcsin(x) \, dx = \frac{1}{2} \sqrt{1-x^2} x \arcsin(x) + \frac{\arcsin(x)^2}{4} - \frac{x^2}{4}$$

```
[In] Int[Sqrt[1 - x^2]*ArcSin[x],x]
```

```
[Out] -1/4*x^2 + (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4737

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
```

```
+ e, 0] && NeQ[n, -1]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcSin[c*x])^n/Sqrt[1
 - c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}x\sqrt{1-x^2}\arcsin(x) - \frac{\int x\,dx}{2} + \frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}\,dx \\ &= -\frac{x^2}{4} + \frac{1}{2}x\sqrt{1-x^2}\arcsin(x) + \frac{\arcsin(x)^2}{4} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2}\arcsin(x)\,dx = \frac{1}{4}\left(-x^2 + 2x\sqrt{1-x^2}\arcsin(x) + \arcsin(x)^2\right)$$

```
[In] Integrate[Sqrt[1 - x^2]*ArcSin[x],x]
[Out] (-x^2 + 2*x*Sqrt[1 - x^2]*ArcSin[x] + ArcSin[x]^2)/4
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\arcsin(x)\left(x\sqrt{-x^2+1}+\arcsin(x)\right)}{2} - \frac{\arcsin(x)^2}{4} - \frac{x^2}{4}$	31

```
[In] int(arcsin(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsin(x)*(x*(-x^2+1)^(1/2)+arcsin(x))-1/4*arcsin(x)^2-1/4*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \sqrt{1-x^2} \arcsin(x)\, dx = \frac{1}{2} \sqrt{-x^2+1} x \arcsin{(x)} - \frac{1}{4} x^2 + \frac{1}{4} \arcsin{(x)}^2$$

```
[In] integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*x*arcsin(x) - 1/4*x^2 + 1/4*arcsin(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \sqrt{1-x^2} \arcsin(x)\, dx = -\frac{x^2}{4} + \left(\frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}{(x)}}{2}\right) \operatorname{asin}{(x)} - \frac{\operatorname{asin}^2{(x)}}{4}$$

```
[In] integrate(asin(x)*(-x**2+1)**(1/2),x)
[Out] -x**2/4 + (x*sqrt(1 - x**2)/2 + asin(x)/2)*asin(x) - asin(x)**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2} \arcsin(x)\, dx = -\frac{1}{4} x^2 + \frac{1}{2} \left(\sqrt{-x^2+1} x + \arcsin{(x)}\right) \arcsin{(x)} - \frac{1}{4} \arcsin{(x)}^2$$

```
[In] integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arcsin(x) - 1/4*arcsin(x)^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \sqrt{1-x^2} \arcsin(x)\, dx = \frac{1}{2} \sqrt{-x^2+1} x \arcsin{(x)} - \frac{1}{4} x^2 + \frac{1}{4} \arcsin{(x)}^2 + \frac{1}{8}$$

```
[In] integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*x*arcsin(x) - 1/4*x^2 + 1/4*arcsin(x)^2 + 1/8
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-x^2} \arcsin(x) \, dx = \int \text{asin}(x) \sqrt{1-x^2} \, dx$$

```
[In] int(asin(x)*(1 - x^2)^(1/2),x)
[Out] int(asin(x)*(1 - x^2)^(1/2), x)
```

3.652 $\int \sqrt{1-x^2} \arccos(x) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \sqrt{1-x^2} \arccos(x) \, dx = \frac{x^2}{4} + \frac{1}{2} x \sqrt{1-x^2} \arccos(x) - \frac{\arccos(x)^2}{4}$$

```
[Out] 1/4*x^2-1/4*arccos(x)^2+1/2*x*arccos(x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4742, 4738, 30}

$$\int \sqrt{1-x^2} \arccos(x) \, dx = \frac{1}{2} \sqrt{1-x^2} x \arccos(x) - \frac{\arccos(x)^2}{4} + \frac{x^2}{4}$$

```
[In] Int[Sqrt[1 - x^2]*ArcCos[x],x]
```

```
[Out] x^2/4 + (x*Sqrt[1 - x^2]*ArcCos[x])/2 - ArcCos[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
```

```
2*d + e, 0] && NeQ[n, -1]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcCos[c*x])^n/Sqrt[1
 - c^2*x^2], x], x] + Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x\sqrt{1-x^2}\arccos(x) + \frac{\int x\,dx}{2} + \frac{1}{2}\int \frac{\arccos(x)}{\sqrt{1-x^2}}\,dx \\ &= \frac{x^2}{4} + \frac{1}{2}x\sqrt{1-x^2}\arccos(x) - \frac{\arccos(x)^2}{4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{4}\left(x^2 + 2x\sqrt{1-x^2}\arccos(x) - \arccos(x)^2\right)$$

```
[In] Integrate[Sqrt[1 - x^2]*ArcCos[x],x]
[Out] (x^2 + 2*x*Sqrt[1 - x^2]*ArcCos[x] - ArcCos[x]^2)/4
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\arccos(x)\left(-x\sqrt{-x^2+1}+\arccos(x)\right)}{2}+\frac{\arccos(x)^2}{4}+\frac{x^2}{4}-\frac{1}{4}$	33

```
[In] int(arccos(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*arccos(x)*(-x*(-x^2+1)^(1/2)+arccos(x))+1/4*arccos(x)^2+1/4*x^2-1/4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{2}\,\sqrt{-x^2+1}x\arccos\left(x\right) + \frac{1}{4}\,x^2 - \frac{1}{4}\,\arccos\left(x\right)^2$$

```
[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{x^2}{4} + \left(\frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}\left(x\right)}{2}\right)\operatorname{acos}\left(x\right) + \frac{\operatorname{asin}^2\left(x\right)}{4}$$

```
[In] integrate(acos(x)*(-x**2+1)**(1/2),x)
[Out] x**2/4 + (x*sqrt(1 - x**2)/2 + asin(x)/2)*acos(x) + asin(x)**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{4}\,x^2 + \frac{1}{2}\,\left(\sqrt{-x^2+1}x + \arcsin\left(x\right)\right)\arccos\left(x\right) + \frac{1}{4}\,\arcsin\left(x\right)^2$$

```
[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arccos(x) + 1/4*arcsin(x)^2
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{2}\,\sqrt{-x^2+1}x\arccos\left(x\right) + \frac{1}{4}\,x^2 - \frac{1}{4}\,\arccos\left(x\right)^2 - \frac{1}{8}$$

```
[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2 - 1/8
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-x^2} \arccos(x) \, dx = \int \operatorname{acos}(x) \sqrt{1-x^2} \, dx$$

```
[In] int(acos(x)*(1 - x^2)^(1/2),x)
[Out] int(acos(x)*(1 - x^2)^(1/2), x)
```

3.653 $\int x\sqrt{1-x^2}\arccos(x)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int x\sqrt{1-x^2}\arccos(x)\,dx = -\frac{x}{3}+\frac{x^3}{9}-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)$$

```
[Out] -1/3*x+1/9*x^3-1/3*(-x^2+1)^(3/2)*arccos(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4768}

$$\int x\sqrt{1-x^2}\arccos(x)\,dx = -\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)+\frac{x^3}{9}-\frac{x}{3}$$

```
[In] Int[x*Sqrt[1 - x^2]*ArcCos[x],x]
[Out] -1/3*x + x^3/9 - ((1 - x^2)^(3/2)*ArcCos[x])/3
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)-\frac{1}{3}\int\left(1-x^2\right)\,dx \\ &= -\frac{x}{3}+\frac{x^3}{9}-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x\sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{9}\left(-3x+x^3-3\left(1-x^2\right)^{3/2}\arccos(x)\right)$$

```
[In] Integrate[x*Sqrt[1 - x^2]*ArcCos[x],x]
[Out] (-3*x + x^3 - 3*(1 - x^2)^(3/2)*ArcCos[x])/9
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{(x^2-1)\sqrt{-x^2+1}\,\arccos(x)}{3}+\frac{(x^2-3)x}{9}$	28

```
[In] int(x*arccos(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(x^2-1)*(-x^2+1)^(1/2)*arccos(x)+1/9*(x^2-3)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int x\sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{9}\,x^3+\frac{1}{3}\,\left(x^2-1\right)\sqrt{-x^2+1}\arccos\left(x\right)-\frac{1}{3}\,x$$

```
[In] integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/9*x^3 + 1/3*(x^2 - 1)*sqrt(-x^2 + 1)*arccos(x) - 1/3*x
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int x\sqrt{1-x^2}\arccos(x)\,dx = \frac{x^3}{9}+\frac{x^2\sqrt{1-x^2}\operatorname{acos}\left(x\right)}{3}-\frac{x}{3}-\frac{\sqrt{1-x^2}\operatorname{acos}\left(x\right)}{3}$$

```
[In] integrate(x*acos(x)*(-x**2+1)**(1/2),x)
[Out] x**3/9 + x**2*sqrt(1 - x**2)*acos(x)/3 - x/3 - sqrt(1 - x**2)*acos(x)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int x\sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{9}\,x^3 - \frac{1}{3}\,\left(-x^2+1\right)^{\frac{3}{2}}\arccos\left(x\right) - \frac{1}{3}\,x$$

```
[In] integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/9*x^3 - 1/3*(-x^2 + 1)^(3/2)*arccos(x) - 1/3*x
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int x\sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{9}\,x^3 - \frac{1}{3}\,\left(-x^2+1\right)^{\frac{3}{2}}\arccos\left(x\right) - \frac{1}{3}\,x$$

```
[In] integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/9*x^3 - 1/3*(-x^2 + 1)^(3/2)*arccos(x) - 1/3*x
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{1-x^2}\arccos(x)\,dx = \int x\,\mathrm{acos}(x)\,\sqrt{1-x^2}\,dx$$

```
[In] int(x*acos(x)*(1 - x^2)^(1/2),x)
[Out] int(x*acos(x)*(1 - x^2)^(1/2), x)
```

3.654 $\int (1-x^2)^{3/2} \arcsin(x) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 59

$$\int (1-x^2)^{3/2} \arcsin(x) \, dx = -\frac{5x^2}{16} + \frac{x^4}{16} + \frac{3}{8}x\sqrt{1-x^2}\arcsin(x) + \frac{1}{4}x(1-x^2)^{3/2}\arcsin(x) + \frac{3\arcsin(x)^2}{16}$$

```
[Out] -5/16*x^2+1/16*x^4+1/4*x*(-x^2+1)^(3/2)*arcsin(x)+3/16*arcsin(x)^2+3/8*x*ar
csin(x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4743, 4741, 4737, 30, 14}

$$\int (1-x^2)^{3/2} \arcsin(x) \, dx = \frac{1}{4}(1-x^2)^{3/2} x \arcsin(x) + \frac{3}{8}\sqrt{1-x^2}x\arcsin(x) + \frac{3\arcsin(x)^2}{16} + \frac{x^4}{16} - \frac{5x^2}{16}$$

```
[In] Int[(1 - x^2)^(3/2)*ArcSin[x],x]
```

```
[Out] (-5*x^2)/16 + x^4/16 + (3*x*Sqrt[1 - x^2]*ArcSin[x])/8 + (x*(1 - x^2)^(3/2)
*ArcSin[x])/4 + (3*ArcSin[x]^2)/16
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4737

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcSin[c*x])^n/Sqrt[1
 - c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4743

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (D
ist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x
] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[x*(1 -
c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x(1-x^2)^{3/2}\arcsin(x) - \frac{1}{4}\int x(1-x^2)\,dx + \frac{3}{4}\int \sqrt{1-x^2}\arcsin(x)\,dx \\
&= \frac{3}{8}x\sqrt{1-x^2}\arcsin(x) \\
&\quad + \frac{1}{4}x(1-x^2)^{3/2}\arcsin(x) - \frac{1}{4}\int (x-x^3)\,dx - \frac{3\int x\,dx}{8} + \frac{3}{8}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}\,dx \\
&= -\frac{5x^2}{16} + \frac{x^4}{16} + \frac{3}{8}x\sqrt{1-x^2}\arcsin(x) + \frac{1}{4}x(1-x^2)^{3/2}\arcsin(x) + \frac{3\arcsin(x)^2}{16}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int (1-x^2)^{3/2} \arcsin(x)\,dx = \frac{1}{16}\left(-5x^2+x^4-2x\sqrt{1-x^2}(-5+2x^2)\arcsin(x)+3\arcsin(x)^2\right)$$

```
[In] Integrate[(1 - x^2)^(3/2)*ArcSin[x],x]
[Out] (-5*x^2 + x^4 - 2*x*Sqrt[1 - x^2]*(-5 + 2*x^2)*ArcSin[x] + 3*ArcSin[x]^2)/1
6
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\arcsin(x)\left(-2\sqrt{-x^2+1}\,x^3+5x\sqrt{-x^2+1}+3\arcsin(x)\right)}{8}-\frac{3\arcsin(x)^2}{16}+\frac{(2x^2-5)^2}{64}$	54

```
[In] int((-x^2+1)^(3/2)*arcsin(x),x,method=_RETURNVERBOSE)
[Out] 1/8*arcsin(x)*(-2*(-x^2+1)^(1/2)*x^3+5*x*(-x^2+1)^(1/2)+3*arcsin(x))-3/16*a
rcsin(x)^2+1/64*(2*x^2-5)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int (1-x^2)^{3/2} \arcsin(x)\,dx = \frac{1}{16}\,x^4 - \frac{1}{8}\left(2\,x^3-5\,x\right)\sqrt{-x^2+1}\arcsin\left(x\right) - \frac{5}{16}\,x^2 + \frac{3}{16}\,\arcsin\left(x\right)^2$$

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="fricas")
[Out] 1/16*x^4 - 1/8*(2*x^3 - 5*x)*sqrt(-x^2 + 1)*arcsin(x) - 5/16*x^2 + 3/16*arc
sin(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \left(1-x^2\right)^{3/2} \arcsin(x)\,dx = \frac{x^4}{16} - \frac{x^3\sqrt{1-x^2}\,\mathrm{asin}\,(x)}{4} - \frac{5x^2}{16} + \frac{5x\sqrt{1-x^2}\,\mathrm{asin}\,(x)}{8} + \frac{3\,\mathrm{asin}^2\,(x)}{16}$$

```
[In] integrate((-x**2+1)**(3/2)*asin(x),x)
```

```
[Out] x**4/16 - x**3*sqrt(1 - x**2)*asin(x)/4 - 5*x**2/16 + 5*x*sqrt(1 - x**2)*as
in(x)/8 + 3*asin(x)**2/16
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \left(1-x^2\right)^{3/2} \arcsin(x)\,dx = \frac{1}{16}\,x^4 - \frac{5}{16}\,x^2 + \frac{1}{8}\left(2\left(-x^2+1\right)^{\frac{3}{2}}x + 3\sqrt{-x^2+1}x + 3\arcsin\,(x)\right)\arcsin\,(x) - \frac{3}{16}\arcsin\,(x)^2$$

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="maxima")
```

```
[Out] 1/16*x^4 - 5/16*x^2 + 1/8*(2*(-x^2 + 1)^(3/2)*x + 3*sqrt(-x^2 + 1)*x + 3*ar
csin(x))*arcsin(x) - 3/16*arcsin(x)^2
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \left(1-x^2\right)^{3/2} \arcsin(x)\,dx = \frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x\arcsin\,(x) + \frac{3}{8}\sqrt{-x^2+1}x\arcsin\,(x) + \frac{1}{16}\left(x^2-1\right)^2 - \frac{3}{16}\,x^2 + \frac{3}{16}\arcsin\,(x)^2 + \frac{9}{128}$$

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="giac")
```

```
[Out] 1/4*(-x^2 + 1)^(3/2)*x*arcsin(x) + 3/8*sqrt(-x^2 + 1)*x*arcsin(x) + 1/16*(x
^2 - 1)^2 - 3/16*x^2 + 3/16*arcsin(x)^2 + 9/128
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-x^2\right)^{3/2}\arcsin(x)\,dx=\int \operatorname{asin}(x)\,\left(1-x^2\right)^{3/2}\,dx$$

```
[In] int(asin(x)*(1 - x^2)^(3/2),x)
[Out] int(asin(x)*(1 - x^2)^(3/2), x)
```

3.655 $\int x(1-x^2)^{3/2} \arcsin(x)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int x(1-x^2)^{3/2} \arcsin(x)\,dx = \frac{x}{5} - \frac{2x^3}{15} + \frac{x^5}{25} - \frac{1}{5}(1-x^2)^{5/2} \arcsin(x)$$

```
[Out] 1/5*x-2/15*x^3+1/25*x^5-1/5*(-x^2+1)^(5/2)*arcsin(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4767, 200}

$$\int x(1-x^2)^{3/2} \arcsin(x)\,dx = -\frac{1}{5}(1-x^2)^{5/2} \arcsin(x) + \frac{x^5}{25} - \frac{2x^3}{15} + \frac{x}{5}$$

```
[In] Int[x*(1 - x^2)^(3/2)*ArcSin[x],x]
```

```
[Out] x/5 - (2*x^3)/15 + x^5/25 - ((1 - x^2)^(5/2)*ArcSin[x])/5
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4767

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
```

```
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{5}\left(1-x^2\right)^{5/2}\arcsin(x)+\frac{1}{5}\int\left(1-x^2\right)^2\,dx\\ &= -\frac{1}{5}\left(1-x^2\right)^{5/2}\arcsin(x)+\frac{1}{5}\int\left(1-2x^2+x^4\right)\,dx\\ &= \frac{x}{5}-\frac{2x^3}{15}+\frac{x^5}{25}-\frac{1}{5}\left(1-x^2\right)^{5/2}\arcsin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int x\left(1-x^2\right)^{3/2}\arcsin(x)\,dx=\frac{1}{5}\left(x-\frac{2x^3}{3}+\frac{x^5}{5}-\left(1-x^2\right)^{5/2}\arcsin(x)\right)$$

```
[In] Integrate[x*(1 - x^2)^(3/2)*ArcSin[x],x]
[Out] (x - (2*x^3)/3 + x^5/5 - (1 - x^2)^(5/2)*ArcSin[x])/5
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\left(x^2-1\right)^2\sqrt{-x^2+1}\,\arcsin(x)}{5}+\frac{\left(3x^4-10x^2+15\right)x}{75}$	37

```
[In] int(x*(-x^2+1)^(3/2)*arcsin(x),x,method=_RETURNVERBOSE)
[Out] -1/5*(x^2-1)^2*(-x^2+1)^(1/2)*arcsin(x)+1/75*(3*x^4-10*x^2+15)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x(1-x^2)^{3/2}\arcsin(x)\,dx = \frac{1}{25}\,x^5 - \frac{2}{15}\,x^3 - \frac{1}{5}\,(x^4 - 2\,x^2 + 1)\sqrt{-x^2+1}\arcsin(x) + \frac{1}{5}\,x$$

```
[In] integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="fricas")
[Out] 1/25*x^5 - 2/15*x^3 - 1/5*(x^4 - 2*x^2 + 1)*sqrt(-x^2 + 1)*arcsin(x) + 1/5*
x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(27) = 54.

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.70

$$\int x(1-x^2)^{3/2}\arcsin(x)\,dx = \frac{x^5}{25} - \frac{x^4\sqrt{1-x^2}\operatorname{asin}(x)}{5} - \frac{2x^3}{15} + \frac{2x^2\sqrt{1-x^2}\operatorname{asin}(x)}{5} + \frac{x}{5} - \frac{\sqrt{1-x^2}\operatorname{asin}(x)}{5}$$

```
[In] integrate(x*(-x**2+1)**(3/2)*asin(x),x)
[Out] x**5/25 - x**4*sqrt(1 - x**2)*asin(x)/5 - 2*x**3/15 + 2*x**2*sqrt(1 - x**2)
*asin(x)/5 + x/5 - sqrt(1 - x**2)*asin(x)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int x(1-x^2)^{3/2}\arcsin(x)\,dx = \frac{1}{25}\,x^5 - \frac{1}{5}\,(-x^2+1)^{\frac{5}{2}}\arcsin(x) - \frac{2}{15}\,x^3 + \frac{1}{5}\,x$$

```
[In] integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="maxima")
[Out] 1/25*x^5 - 1/5*(-x^2 + 1)^(5/2)*arcsin(x) - 2/15*x^3 + 1/5*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x(1-x^2)^{3/2}\arcsin(x)\,dx = \frac{1}{25}\,x^5 - \frac{1}{5}\,(x^2-1)^2\sqrt{-x^2+1}\arcsin{(x)} - \frac{2}{15}\,x^3 + \frac{1}{5}\,x$$

```
[In] integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="giac")
[Out] 1/25*x^5 - 1/5*(x^2 - 1)^2*sqrt(-x^2 + 1)*arcsin(x) - 2/15*x^3 + 1/5*x
```

Mupad [F(-1)]

Timed out.

$$\int x(1-x^2)^{3/2}\arcsin(x)\,dx = \int x\,\mathrm{asin}(x)\,\left(1-x^2\right)^{3/2}\,dx$$

```
[In] int(x*asin(x)*(1 - x^2)^(3/2),x)
[Out] int(x*asin(x)*(1 - x^2)^(3/2), x)
```

3.656 $\int x^3(1-x^2)^{3/2}\arccos(x)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx = -\frac{2x}{35}-\frac{x^3}{105}+\frac{8x^5}{175}-\frac{x^7}{49} -\frac{1}{5}(1-x^2)^{5/2}\arccos(x)+\frac{1}{7}(1-x^2)^{7/2}\arccos(x)$$

```
[Out] -2/35*x-1/105*x^3+8/175*x^5-1/49*x^7-1/5*(-x^2+1)^(5/2)*arccos(x)+1/7*(-x^2
+1)^(7/2)*arccos(x)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {272, 45, 4780, 12, 380}

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx = \frac{1}{7}(1-x^2)^{7/2}\arccos(x) -\frac{1}{5}(1-x^2)^{5/2}\arccos(x)-\frac{x^7}{49}+\frac{8x^5}{175}-\frac{x^3}{105}-\frac{2x}{35}$$

```
[In] Int[x^3*(1 - x^2)^(3/2)*ArcCos[x],x]
```

```
[Out] (-2*x)/35 - x^3/105 + (8*x^5)/175 - x^7/49 - ((1 - x^2)^(5/2)*ArcCos[x])/5
+ ((1 - x^2)^(7/2)*ArcCos[x])/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 4780

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos[
c*x], u, x] + Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[Simplif
yIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && E
qQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2
, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{5}\left(1-x^2\right)^{5/2}\arccos(x)+\frac{1}{7}\left(1-x^2\right)^{7/2}\arccos(x)+\int \frac{1}{35}\left(-2-5x^2\right)\left(1-x^2\right)^2\,dx\\
&= -\frac{1}{5}\left(1-x^2\right)^{5/2}\arccos(x)+\frac{1}{7}\left(1-x^2\right)^{7/2}\arccos(x)+\frac{1}{35}\int \left(-2-5x^2\right)\left(1-x^2\right)^2\,dx\\
&= -\frac{1}{5}\left(1-x^2\right)^{5/2}\arccos(x)+\frac{1}{7}\left(1-x^2\right)^{7/2}\arccos(x)+\frac{1}{35}\int \left(-2-x^2+8x^4-5x^6\right)dx\\
&= -\frac{2x}{35}-\frac{x^3}{105}+\frac{8x^5}{175}-\frac{x^7}{49}-\frac{1}{5}\left(1-x^2\right)^{5/2}\arccos(x)+\frac{1}{7}\left(1-x^2\right)^{7/2}\arccos(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx = -\frac{x(210+35x^2-168x^4+75x^6)}{3675} - \frac{1}{35}(1-x^2)^{5/2}(2+5x^2)\arccos(x)$$

```
[In] Integrate[x^3*(1 - x^2)^(3/2)*ArcCos[x],x]
```

```
[Out] -1/3675*(x*(210 + 35*x^2 - 168*x^4 + 75*x^6)) - ((1 - x^2)^(5/2)*(2 + 5*x^2
)*ArcCos[x])/35
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{(x^2-1)^2\sqrt{-x^2+1}\,\arccos(x)}{5}-\frac{(3x^4-10x^2+15)x}{75}-\frac{(x^2-1)^3\sqrt{-x^2+1}\,\arccos(x)}{7}-\frac{(5x^6-21x^4+35x^2-35)x}{245}$	77

```
[In] int(x^3*(-x^2+1)^(3/2)*arccos(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(x^2-1)^2*(-x^2+1)^(1/2)*arccos(x)-1/75*(3*x^4-10*x^2+15)*x-1/7*(x^2-1
)^3*(-x^2+1)^(1/2)*arccos(x)-1/245*(5*x^6-21*x^4+35*x^2-35)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx = -\frac{1}{49}\,x^7+\frac{8}{175}\,x^5-\frac{1}{105}\,x^3 - \frac{1}{35}\left(5\,x^6-8\,x^4+x^2+2\right)\sqrt{-x^2+1}\arccos\left(x\right)-\frac{2}{35}\,x$$

```
[In] integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="fricas")
```

```
[Out] -1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*x^6 - 8*x^4 + x^2 + 2)*sqrt(-x^
2 + 1)*arccos(x) - 2/35*x
```

Sympy [A] (verification not implemented)

Time = 6.92 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx = -\frac{x^7}{49} - \frac{x^6\sqrt{1-x^2}\operatorname{acos}(x)}{7} + \frac{8x^5}{175} + \frac{8x^4\sqrt{1-x^2}\operatorname{acos}(x)}{35} - \frac{x^3}{105} - \frac{x^2\sqrt{1-x^2}\operatorname{acos}(x)}{35} - \frac{2x}{35} - \frac{2\sqrt{1-x^2}\operatorname{acos}(x)}{35}$$

```
[In] integrate(x**3*(-x**2+1)**(3/2)*acos(x),x)
```

```
[Out] -x**7/49 - x**6*sqrt(1 - x**2)*acos(x)/7 + 8*x**5/175 + 8*x**4*sqrt(1 - x**
2)*acos(x)/35 - x**3/105 - x**2*sqrt(1 - x**2)*acos(x)/35 - 2*x/35 - 2*sqrt
(1 - x**2)*acos(x)/35
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx = -\frac{1}{49}x^7 + \frac{8}{175}x^5 - \frac{1}{105}x^3 - \frac{1}{35}\left(5\left(-x^2+1\right)^{\frac{5}{2}}x^2 + 2\left(-x^2+1\right)^{\frac{5}{2}}\right)\arccos(x) - \frac{2}{35}x$$

```
[In] integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="maxima")
```

```
[Out] -1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*(-x^2 + 1)^(5/2)*x^2 + 2*(-x^2
+ 1)^(5/2))*arccos(x) - 2/35*x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx = -\frac{1}{49}x^7 + \frac{8}{175}x^5 - \frac{1}{105}x^3 - \frac{1}{35}\left(5\left(x^2-1\right)^3\sqrt{-x^2+1} + 7\left(x^2-1\right)^2\sqrt{-x^2+1}\right)\arccos(x) - \frac{2}{35}x$$

```
[In] integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="giac")
```

```
[Out] -1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*(x^2 - 1)^3*sqrt(-x^2 + 1) + 7*
(x^2 - 1)^2*sqrt(-x^2 + 1))*arccos(x) - 2/35*x
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (1-x^2)^{3/2} \arccos(x)\, dx = \int x^3 \operatorname{acos}(x) \left(1-x^2\right)^{3/2} dx$$

```
[In] int(x^3*acos(x)*(1 - x^2)^(3/2),x)
[Out] int(x^3*acos(x)*(1 - x^2)^(3/2), x)
```

3.657 $\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx = \frac{4x}{3} - \frac{x^3}{9} + \sqrt{1-x^2}\arccos(x) + \frac{1}{3}(1-x^2)^{3/2}\arccos(x) + 2i\arccos(x)\arctan\left(e^{i\arccos(x)}\right) - i\,\text{PolyLog}\left(2,-ie^{i\arccos(x)}\right) + i\,\text{PolyLog}\left(2,ie^{i\arccos(x)}\right)$$

```
[Out] 4/3*x-1/9*x^3+1/3*(-x^2+1)^(3/2)*arccos(x)+2*I*arccos(x)*arctan(x+I*(-x^2+1
)^(1/2))-I*polylog(2,-I*(x+I*(-x^2+1)^(1/2)))+I*polylog(2,I*(x+I*(-x^2+1)^(
1/2)))+arccos(x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4788, 4784, 4804, 4266, 2317, 2438, 8}

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx = 2i\arccos(x)\arctan\left(e^{i\arccos(x)}\right) - i\,\text{PolyLog}\left(2,-ie^{i\arccos(x)}\right) + i\,\text{PolyLog}\left(2,ie^{i\arccos(x)}\right) + \frac{1}{3}(1-x^2)^{3/2}\arccos(x) + \sqrt{1-x^2}\arccos(x) - \frac{x^3}{9} + \frac{4x}{3}$$

```
[In] Int[((1 - x^2)^(3/2)*ArcCos[x])/x,x]
```

```
[Out] (4*x)/3 - x^3/9 + Sqrt[1 - x^2]*ArcCos[x] + ((1 - x^2)^(3/2)*ArcCos[x])/3 +
 (2*I)*ArcCos[x]*ArcTan[E^(I*ArcCos[x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[x]
)] + I*PolyLog[2, I*E^(I*ArcCos[x])]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4784

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]], Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]
 + Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(f
*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 4788

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x)^
m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c*(n/(f*(m + 2
*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 4804

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
```

```
d + e*x^2]], Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)+\frac{1}{3}\int\left(1-x^2\right)\,dx+\int\frac{\sqrt{1-x^2}\arccos(x)}{x}\,dx\\
&= \frac{x}{3}-\frac{x^3}{9}+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)+\int 1\,dx+\int\frac{\arccos(x)}{x\sqrt{1-x^2}}\,dx\\
&= \frac{4x}{3}-\frac{x^3}{9}+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)-\text{Subst}\left(\int x\sec(x)\,dx,x,\arccos(x)\right)\\
&= \frac{4x}{3}-\frac{x^3}{9}+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)+2i\arccos(x)\arctan\left(e^{i\arccos(x)}\right)\\
&\quad+\text{Subst}\left(\int\log\left(1-ie^{ix}\right)\,dx,x,\arccos(x)\right)\\
&\quad-\text{Subst}\left(\int\log\left(1+ie^{ix}\right)\,dx,x,\arccos(x)\right)\\
&= \frac{4x}{3}-\frac{x^3}{9}+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)+2i\arccos(x)\arctan\left(e^{i\arccos(x)}\right)\\
&\quad-i\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{i\arccos(x)}\right)\\
&\quad+i\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{i\arccos(x)}\right)\\
&= \frac{4x}{3}-\frac{x^3}{9}+\sqrt{1-x^2}\arccos(x)\\
&\quad+\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)+2i\arccos(x)\arctan\left(e^{i\arccos(x)}\right)\\
&\quad-i\,\text{PolyLog}\left(2,-ie^{i\arccos(x)}\right)+i\,\text{PolyLog}\left(2,ie^{i\arccos(x)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25

$$
\begin{aligned}
\int\frac{\left(1-x^2\right)^{3/2}\arccos(x)}{x}\,dx &= x+\sqrt{1-x^2}\arccos(x)\\
&\quad+\frac{1}{36}\left(9x+12\left(1-x^2\right)^{3/2}\arccos(x)-\cos(3\arccos(x))\right)-\arccos(x)\log\left(1-ie^{i\arccos(x)}\right)\\
&\quad+\arccos(x)\log\left(1+ie^{i\arccos(x)}\right)-i\,\text{PolyLog}\left(2,-ie^{i\arccos(x)}\right)+i\,\text{PolyLog}\left(2,ie^{i\arccos(x)}\right)
\end{aligned}
$$

```
[In] Integrate[((1 - x^2)^(3/2)*ArcCos[x])/x,x]
```

```
[Out] x + Sqrt[1 - x^2]*ArcCos[x] + (9*x + 12*(1 - x^2)^(3/2)*ArcCos[x] - Cos[3*A
rcCos[x]])/36 - ArcCos[x]*Log[1 - I*E^(I*ArcCos[x])] + ArcCos[x]*Log[1 + I*
E^(I*ArcCos[x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[x])] + I*PolyLog[2, I*E^(I
*ArcCos[x])]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.63

method	result
default	$-\frac{5\left(-\sqrt{-x^2+1}+ix\right)(\arccos(x)+i)}{8}+\frac{5\left(ix+\sqrt{-x^2+1}\right)(\arccos(x)-i)}{8}+\arccos(x)\ln\left(1+i\left(i\sqrt{-x^2+1}+x\right)\right)-$

```
[In] int((-x^2+1)^(3/2)*arccos(x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -5/8*(-(-x^2+1)^(1/2)+I*x)*(arccos(x)+I)+5/8*(I*x+(-x^2+1)^(1/2))*(arccos(x
)-I)+arccos(x)*ln(1+I*(I*(-x^2+1)^(1/2)+x))-arccos(x)*ln(1-I*(I*(-x^2+1)^(1
/2)+x))-I*dilog(1+I*(I*(-x^2+1)^(1/2)+x))+I*dilog(1-I*(I*(-x^2+1)^(1/2)+x))
-1/36*cos(3*arccos(x))-1/12*arccos(x)*sin(3*arccos(x))
```

Fricas [F]

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx=\int \frac{(-x^2+1)^{\frac{3}{2}}\arccos(x)}{x}\,dx$$

```
[In] integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="fricas")
```

```
[Out] integral(-(x^2 - 1)*sqrt(-x^2 + 1)*arccos(x)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx=\text{Timed out}$$

```
[In] integrate((-x**2+1)**(3/2)*acos(x)/x,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx = \int \frac{(-x^2+1)^{\frac{3}{2}}\arccos{(x)}}{x}\,dx$$

```
[In] integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="maxima")
[Out] integrate((-x^2 + 1)^(3/2)*arccos(x)/x, x)
```

Giac [F]

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx = \int \frac{(-x^2+1)^{\frac{3}{2}}\arccos{(x)}}{x}\,dx$$

```
[In] integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="giac")
[Out] integrate((-x^2 + 1)^(3/2)*arccos(x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx = \int \frac{\mathrm{acos}(x)\,\left(1-x^2\right)^{3/2}}{x}\,dx$$

```
[In] int((acos(x)*(1 - x^2)^(3/2))/x,x)
[Out] int((acos(x)*(1 - x^2)^(3/2))/x, x)
```

3.658 $\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx=-\frac{1}{20x^4}+\frac{1}{5x^2}-\frac{(1-x^2)^{5/2}\arcsin(x)}{5x^5}+\frac{\log(x)}{5}$$

```
[Out] -1/20/x^4+1/5/x^2-1/5*(-x^2+1)^(5/2)*arcsin(x)/x^5+1/5*ln(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4771, 272, 45}

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx=-\frac{(1-x^2)^{5/2}\arcsin(x)}{5x^5}-\frac{1}{20x^4}+\frac{1}{5x^2}+\frac{\log(x)}{5}$$

```
[In] Int[((1 - x^2)^(3/2)*ArcSin[x])/x^6,x]
[Out] -1/20*1/x^4 + 1/(5*x^2) - ((1 - x^2)^(5/2)*ArcSin[x])/(5*x^5) + Log[x]/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4771

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*Ar
cSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2
*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(1-x^2)^{5/2}\arcsin(x)}{5x^5}+\frac{1}{5}\int\frac{(1-x^2)^2}{x^5}\,dx\\
&= -\frac{(1-x^2)^{5/2}\arcsin(x)}{5x^5}+\frac{1}{10}\text{Subst}\left(\int\frac{(1-x)^2}{x^3}\,dx,x,x^2\right)\\
&= -\frac{(1-x^2)^{5/2}\arcsin(x)}{5x^5}+\frac{1}{10}\text{Subst}\left(\int\left(\frac{1}{x^3}-\frac{2}{x^2}+\frac{1}{x}\right)\,dx,x,x^2\right)\\
&= -\frac{1}{20x^4}+\frac{1}{5x^2}-\frac{(1-x^2)^{5/2}\arcsin(x)}{5x^5}+\frac{\log(x)}{5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int\frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx=-\frac{x-4x^3+4(1-x^2)^{5/2}\arcsin(x)-4x^5\log(x)}{20x^5}$$

```
[In] Integrate[((1 - x^2)^(3/2)*ArcSin[x])/x^6,x]
[Out] -1/20*(x - 4*x^3 + 4*(1 - x^2)^(5/2)*ArcSin[x] - 4*x^5*Log[x])/x^5
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 201, normalized size of antiderivative = 4.90

method	result
default	$-\frac{2i\arcsin(x)}{5}+\frac{\left(-x^{4}\sqrt{-x^{2}+1}+ix^{5}+2x^{2}\sqrt{-x^{2}+1}-\sqrt{-x^{2}+1}\right)\left(20\arcsin(x)x^{8}-4ix^{8}-4\sqrt{-x^{2}+1}\,x^{7}-40\arcsin(x)x^{6}+ix^{6}+9\sqrt{-}\right.}{20(5x^{8}-10x^{6}+10x^{4}-5x^{2}+1)x^{5}}$

```
[In] int((-x^2+1)^(3/2)*arcsin(x)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*I*arcsin(x)+1/20*(-x^4*(-x^2+1)^(1/2)+I*x^5+2*x^2*(-x^2+1)^(1/2)-(-x^2
+1)^(1/2))*(20*arcsin(x)*x^8-4*I*x^8-4*(-x^2+1)^(1/2)*x^7-40*arcsin(x)*x^6+
I*x^6+9*(-x^2+1)^(1/2)*x^5+40*arcsin(x)*x^4-6*(-x^2+1)^(1/2)*x^3-20*x^2*arc
sin(x)+x*(-x^2+1)^(1/2)+4*arcsin(x))/(5*x^8-10*x^6+10*x^4-5*x^2+1)/x^5+1/5*
ln((I*x+(-x^2+1)^(1/2))^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx=\frac{4\,x^5\log(x)+4\,x^3-4\,(x^4-2\,x^2+1)\sqrt{-x^2+1}\arcsin(x)-x}{20\,x^5}$$

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(4*x^5*log(x) + 4*x^3 - 4*(x^4 - 2*x^2 + 1)*sqrt(-x^2 + 1)*arcsin(x) -
 x)/x^5
```

Sympy [F]

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx=\int \frac{(-(x-1)\,(x+1))^{\frac{3}{2}}\operatorname{asin}(x)}{x^6}\,dx$$

```
[In] integrate((-x**2+1)**(3/2)*asin(x)/x**6,x)
```

```
[Out] Integral((-(x - 1)*(x + 1))**(3/2)*asin(x)/x**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = -\frac{(-x^2+1)^{\frac{5}{2}}\arcsin(x)}{5\,x^5} + \frac{4\,x^2-1}{20\,x^4} + \frac{1}{10}\log\left(x^2\right)$$

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="maxima")
[Out] -1/5*(-x^2 + 1)^(5/2)*arcsin(x)/x^5 + 1/20*(4*x^2 - 1)/x^4 + 1/10*log(x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(31) = 62$.

Time = 0.34 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.29

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = -\frac{1}{160}\left(\frac{x^5\left(\frac{5\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - \frac{10\left(\sqrt{-x^2+1}-1\right)^4}{x^4} - 1\right)}{\left(\sqrt{-x^2+1}-1\right)^5} + \frac{10\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{5\left(\sqrt{-x^2+1}-1\right)^3}{x^3} + \frac{\left(\sqrt{-x^2+1}-1\right)^5}{x^5}\right)\arcsin(x) - \frac{3\,x^4-4\,x^2+1}{20\,x^4} + \frac{1}{10}\log\left(x^2\right)$$

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="giac")
[Out] -1/160*(x^5*(5*(sqrt(-x^2 + 1) - 1)^2/x^2 - 10*(sqrt(-x^2 + 1) - 1)^4/x^4 -
 1)/(sqrt(-x^2 + 1) - 1)^5 + 10*(sqrt(-x^2 + 1) - 1)/x - 5*(sqrt(-x^2 + 1)
- 1)^3/x^3 + (sqrt(-x^2 + 1) - 1)^5/x^5)*arcsin(x) - 1/20*(3*x^4 - 4*x^2 +
1)/x^4 + 1/10*log(x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = \int \frac{\mathrm{asin}(x)\,\left(1-x^2\right)^{3/2}}{x^6}\,dx$$

```
[In] int((asin(x)*(1 - x^2)^(3/2))/x^6,x)
[Out] int((asin(x)*(1 - x^2)^(3/2))/x^6, x)
```

3.659 $\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 34

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}} dx = \frac{x^2}{4} - \frac{1}{2}x\sqrt{1-x^2}\arcsin(x) + \frac{\arcsin(x)^2}{4}$$

```
[Out] 1/4*x^2+1/4*arcsin(x)^2-1/2*x*arcsin(x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4795, 4737, 30}

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}} dx = -\frac{1}{2}\sqrt{1-x^2}x\arcsin(x) + \frac{\arcsin(x)^2}{4} + \frac{x^2}{4}$$

```
[In] Int[(x^2*ArcSin[x])/Sqrt[1 - x^2],x]
[Out] x^2/4 - (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4737

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
```

```
+ e, 0] && NeQ[n, -1]
```

Rule 4795

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)+\frac{\int x\,dx}{2}+\frac{1}{2}\int\frac{\arcsin(x)}{\sqrt{1-x^2}}\,dx\\ &= \frac{x^2}{4}-\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)+\frac{\arcsin(x)^2}{4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int\frac{x^2\arcsin(x)}{\sqrt{1-x^2}}\,dx=\frac{1}{4}\left(x^2-2x\sqrt{1-x^2}\arcsin(x)+\arcsin(x)^2\right)$$

```
[In] Integrate[(x^2*ArcSin[x])/Sqrt[1 - x^2],x]
[Out] (x^2 - 2*x*Sqrt[1 - x^2]*ArcSin[x] + ArcSin[x]^2)/4
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\arcsin(x)\left(-x\sqrt{-x^2+1}+\arcsin(x)\right)}{2}-\frac{\arcsin(x)^2}{4}+\frac{x^2}{4}$	32

```
[In] int(x^2*arcsin(x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsin(x)*(-x*(-x^2+1)^(1/2)+arcsin(x))-1/4*arcsin(x)^2+1/4*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\,\sqrt{-x^2+1}x\arcsin\left(x\right) + \frac{1}{4}\,x^2 + \frac{1}{4}\,\arcsin\left(x\right)^2$$

```
[In] integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-x^2 + 1)*x*arcsin(x) + 1/4*x^2 + 1/4*arcsin(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \frac{x^2}{4} - \frac{x\sqrt{1-x^2}\operatorname{asin}\left(x\right)}{2} + \frac{\operatorname{asin}^2\left(x\right)}{4}$$

```
[In] integrate(x**2*asin(x)/(-x**2+1)**(1/2),x)
[Out] x**2/4 - x*sqrt(1 - x**2)*asin(x)/2 + asin(x)**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \frac{1}{4}\,x^2 - \frac{1}{2}\left(\sqrt{-x^2+1}x - \arcsin\left(x\right)\right)\arcsin\left(x\right) - \frac{1}{4}\,\arcsin\left(x\right)^2$$

```
[In] integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/4*x^2 - 1/2*(sqrt(-x^2 + 1)*x - arcsin(x))*arcsin(x) - 1/4*arcsin(x)^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\,\sqrt{-x^2+1}x\arcsin\left(x\right) + \frac{1}{4}\,x^2 + \frac{1}{4}\,\arcsin\left(x\right)^2 - \frac{1}{8}$$

```
[In] integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^2 + 1)*x*arcsin(x) + 1/4*x^2 + 1/4*arcsin(x)^2 - 1/8
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\, dx = \int \frac{x^2 \operatorname{asin}(x)}{\sqrt{1-x^2}}\, dx$$

```
[In] int((x^2*asin(x))/(1 - x^2)^(1/2),x)
[Out] int((x^2*asin(x))/(1 - x^2)^(1/2), x)
```

3.660 $\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}} dx = \frac{3x^2}{16} + \frac{x^4}{16} - \frac{3}{8}x\sqrt{1-x^2}\arcsin(x) - \frac{1}{4}x^3\sqrt{1-x^2}\arcsin(x) + \frac{3\arcsin(x)^2}{16}$$

```
[Out] 3/16*x^2+1/16*x^4+3/16*arcsin(x)^2-3/8*x*arcsin(x)*(-x^2+1)^(1/2)-1/4*x^3*a
rcsin(x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4795, 4737, 30}

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}} dx = -\frac{3}{8}\sqrt{1-x^2}x\arcsin(x) - \frac{1}{4}\sqrt{1-x^2}x^3\arcsin(x) + \frac{3\arcsin(x)^2}{16} + \frac{x^4}{16} + \frac{3x^2}{16}$$

```
[In] Int[(x^4*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] (3*x^2)/16 + x^4/16 - (3*x*Sqrt[1 - x^2]*ArcSin[x])/8 - (x^3*Sqrt[1 - x^2]*
ArcSin[x])/4 + (3*ArcSin[x]^2)/16
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4737

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
```

```
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

Rule 4795

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{4}x^3\sqrt{1-x^2}\arcsin(x)+\frac{\int x^3\,dx}{4}+\frac{3}{4}\int\frac{x^2\arcsin(x)}{\sqrt{1-x^2}}\,dx\\
&= \frac{x^4}{16}-\frac{3}{8}x\sqrt{1-x^2}\arcsin(x)-\frac{1}{4}x^3\sqrt{1-x^2}\arcsin(x)+\frac{3\int x\,dx}{8}+\frac{3}{8}\int\frac{\arcsin(x)}{\sqrt{1-x^2}}\,dx\\
&= \frac{3x^2}{16}+\frac{x^4}{16}-\frac{3}{8}x\sqrt{1-x^2}\arcsin(x)-\frac{1}{4}x^3\sqrt{1-x^2}\arcsin(x)+\frac{3\arcsin(x)^2}{16}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int\frac{x^4\arcsin(x)}{\sqrt{1-x^2}}\,dx=\frac{1}{16}\left(x^2(3+x^2)-2x\sqrt{1-x^2}(3+2x^2)\arcsin(x)+3\arcsin(x)^2\right)$$

```
[In] Integrate[(x^4*ArcSin[x])/Sqrt[1 - x^2],x]
[Out] (x^2*(3 + x^2) - 2*x*Sqrt[1 - x^2]*(3 + 2*x^2)*ArcSin[x] + 3*ArcSin[x]^2)/1
6
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\arcsin(x)\left(-2\sqrt{-x^2+1}\,x^3-3x\sqrt{-x^2+1}+3\arcsin(x)\right)}{8}-\frac{3\arcsin(x)^2}{16}+\frac{(2x^2+3)^2}{64}$	54

```
[In] int(x^4*arcsin(x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*arcsin(x)*(-2*(-x^2+1)^(1/2)*x^3-3*x*(-x^2+1)^(1/2)+3*arcsin(x))-3/16*a
rcsin(x)^2+1/64*(2*x^2+3)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \frac{1}{16}\,x^4 - \frac{1}{8}\left(2\,x^3 + 3\,x\right)\sqrt{-x^2+1}\arcsin\left(x\right) + \frac{3}{16}\,x^2 + \frac{3}{16}\,\arcsin\left(x\right)^2$$

```
[In] integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*x^4 - 1/8*(2*x^3 + 3*x)*sqrt(-x^2 + 1)*arcsin(x) + 3/16*x^2 + 3/16*arc
sin(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \frac{x^4}{16} - \frac{x^3\sqrt{1-x^2}\operatorname{asin}\left(x\right)}{4} + \frac{3x^2}{16} - \frac{3x\sqrt{1-x^2}\operatorname{asin}\left(x\right)}{8} + \frac{3\operatorname{asin}^2\left(x\right)}{16}$$

```
[In] integrate(x**4*asin(x)/(-x**2+1)**(1/2),x)
```

```
[Out] x**4/16 - x**3*sqrt(1 - x**2)*asin(x)/4 + 3*x**2/16 - 3*x*sqrt(1 - x**2)*as
in(x)/8 + 3*asin(x)**2/16
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}\,dx = {} & \frac{1}{16}\,x^4 + \frac{3}{16}\,x^2 \\ & - \frac{1}{8}\left(2\,\sqrt{-x^2+1}x^3 + 3\,\sqrt{-x^2+1}x - 3\,\arcsin\left(x\right)\right)\arcsin\left(x\right) \\ & - \frac{3}{16}\,\arcsin\left(x\right)^2\end{aligned}$$

```
[In] integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*x^4 + 3/16*x^2 - 1/8*(2*sqrt(-x^2 + 1)*x^3 + 3*sqrt(-x^2 + 1)*x - 3*ar
csin(x))*arcsin(x) - 3/16*arcsin(x)^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}} x \arcsin\left(x\right) - \frac{5}{8}\sqrt{-x^2+1}x \arcsin\left(x\right) + \frac{1}{16}\left(x^2-1\right)^2 + \frac{5}{16}x^2 + \frac{3}{16}\arcsin\left(x\right)^2 - \frac{23}{128}$$

```
[In] integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/4*(-x^2 + 1)^(3/2)*x*arcsin(x) - 5/8*sqrt(-x^2 + 1)*x*arcsin(x) + 1/16*(x
^2 - 1)^2 + 5/16*x^2 + 3/16*arcsin(x)^2 - 23/128
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \int \frac{x^4 \operatorname{asin}(x)}{\sqrt{1-x^2}}\,dx$$

```
[In] int((x^4*asin(x))/(1 - x^2)^(1/2),x)
[Out] int((x^4*asin(x))/(1 - x^2)^(1/2), x)
```

3.661 $\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}} dx = \frac{\arcsin(x)}{\sqrt{1-x^2}} - \operatorname{arctanh}(x)$$

```
[Out] -arctanh(x)+arcsin(x)/(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4767, 212}

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}} dx = \frac{\arcsin(x)}{\sqrt{1-x^2}} - \operatorname{arctanh}(x)$$

```
[In] Int[(x*ArcSin[x])/(1 - x^2)^(3/2),x]
```

```
[Out] ArcSin[x]/Sqrt[1 - x^2] - ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4767

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
```

```
1))), x] + Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\arcsin(x)}{\sqrt{1-x^2}} - \int \frac{1}{1-x^2}\,dx\\ &= \frac{\arcsin(x)}{\sqrt{1-x^2}} - \operatorname{arctanh}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x\arcsin(x)}{(1-x^2)^{3/2}}\,dx = \frac{\arcsin(x)}{\sqrt{1-x^2}} - \operatorname{arctanh}(x)$$

```
[In] Integrate[(x*ArcSin[x])/(1 - x^2)^(3/2),x]
[Out] ArcSin[x]/Sqrt[1 - x^2] - ArcTanh[x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(17) = 34.

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

method	result	size
default	$-\frac{\sqrt{-x^2+1}\,\arcsin(x)}{x^2-1} - \ln\left(\frac{1}{\sqrt{-x^2+1}} + \frac{x}{\sqrt{-x^2+1}}\right)$	46

```
[In] int(x*arcsin(x)/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)-ln(1/(-x^2+1)^(1/2)+1/(-x^2+1)^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(17) = 34.

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}}\,dx = -\frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)+2\sqrt{-x^2+1}\arcsin(x)}{2(x^2-1)}$$

```
[In] integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="fricas")
[Out] -1/2*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) + 2*sqrt(-x^2 + 1)*arcsin
(x))/(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}}\,dx = \frac{\log(x-1)}{2} - \frac{\log(x+1)}{2} + \frac{\operatorname{asin}(x)}{\sqrt{1-x^2}}$$

```
[In] integrate(x*asin(x)/(-x**2+1)**(3/2),x)
[Out] log(x - 1)/2 - log(x + 1)/2 + asin(x)/sqrt(1 - x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}}\,dx = \frac{\arcsin(x)}{\sqrt{-x^2+1}} - \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1)$$

```
[In] integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="maxima")
[Out] arcsin(x)/sqrt(-x^2 + 1) - 1/2*log(x + 1) + 1/2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{x\arcsin(x)}{(1-x^2)^{3/2}}\,dx = \frac{\arcsin(x)}{\sqrt{-x^2+1}} - \frac{1}{2}\log(|x+1|) + \frac{1}{2}\log(|x-1|)$$

```
[In] integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="giac")
[Out] arcsin(x)/sqrt(-x^2 + 1) - 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\arcsin(x)}{(1-x^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{asin}(x)}{(1-x^2)^{3/2}}\,dx$$

```
[In] int((x*asin(x))/(1 - x^2)^(3/2),x)
[Out] int((x*asin(x))/(1 - x^2)^(3/2), x)
```

3.662 $\int \frac{x \arccos(x)}{(1-x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}} \, dx = \frac{\arccos(x)}{\sqrt{1-x^2}} + \operatorname{arctanh}(x)$$

```
[Out] arctanh(x)+arccos(x)/(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4768, 212}

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}} \, dx = \frac{\arccos(x)}{\sqrt{1-x^2}} + \operatorname{arctanh}(x)$$

```
[In] Int[(x*ArcCos[x])/(1 - x^2)^(3/2),x]
```

```
[Out] ArcCos[x]/Sqrt[1 - x^2] + ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
```

```
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\arccos(x)}{\sqrt{1-x^2}} + \int \frac{1}{1-x^2}\,dx \\ &= \frac{\arccos(x)}{\sqrt{1-x^2}} + \operatorname{arctanh}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int \frac{x\arccos(x)}{(1-x^2)^{3/2}}\,dx = \frac{1}{2}\left(\frac{2\arccos(x)}{\sqrt{1-x^2}} - \log(1-x) + \log(1+x)\right)$$

```
[In] Integrate[(x*ArcCos[x])/(1 - x^2)^(3/2),x]
[Out] ((2*ArcCos[x])/Sqrt[1 - x^2] - Log[1 - x] + Log[1 + x])/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(15) = 30.

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

method	result	size
default	$-\frac{\sqrt{-x^2+1}\,\arccos(x)}{x^2-1} - \ln\left(-\frac{x}{\sqrt{-x^2+1}} + \frac{1}{\sqrt{-x^2+1}}\right)$	47

```
[In] int(x*arccos(x)/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -(-x^2+1)^(1/2)/(x^2-1)*arccos(x)-ln(-1/(-x^2+1)^(1/2)*x+1/(-x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.59

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}}\,dx = \frac{(x^2-1)\log(x+1) - (x^2-1)\log(x-1) - 2\sqrt{-x^2+1}\arccos(x)}{2(x^2-1)}$$

```
[In] integrate(x*arccos(x)/(-x^2+1)^(3/2),x, algorithm="fricas")
[Out] 1/2*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*sqrt(-x^2 + 1)*arccos(
x))/(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}}\,dx = -\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2} + \frac{\operatorname{acos}(x)}{\sqrt{1-x^2}}$$

```
[In] integrate(x*acos(x)/(-x**2+1)**(3/2),x)
[Out] -log(x - 1)/2 + log(x + 1)/2 + acos(x)/sqrt(1 - x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}}\,dx = \frac{\arccos(x)}{\sqrt{-x^2+1}} + \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

```
[In] integrate(x*arccos(x)/(-x^2+1)^(3/2),x, algorithm="maxima")
[Out] arccos(x)/sqrt(-x^2 + 1) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}}\,dx = \frac{\arccos(x)}{\sqrt{-x^2+1}} + \frac{1}{2}\log(|x+1|) - \frac{1}{2}\log(|x-1|)$$

```
[In] integrate(x*arccos(x)/(-x^2+1)^(3/2),x, algorithm="giac")
[Out] arccos(x)/sqrt(-x^2 + 1) + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{acos}(x)}{(1-x^2)^{3/2}}\,dx$$

```
[In] int((x*acos(x))/(1 - x^2)^(3/2),x)
[Out] int((x*acos(x))/(1 - x^2)^(3/2), x)
```

3.663 $\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = -\frac{1}{6(1-x^2)}+\frac{x\arcsin(x)}{3(1-x^2)^{3/2}}+\frac{2x\arcsin(x)}{3\sqrt{1-x^2}}+\frac{1}{3}\log\left(1-x^2\right)$$

```
[Out] -1/6/(-x^2+1)+1/3*x*arcsin(x)/(-x^2+1)^(3/2)+1/3*ln(-x^2+1)+2/3*x*arcsin(x)
/(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4747, 4745, 266, 267}

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = \frac{2x\arcsin(x)}{3\sqrt{1-x^2}}+\frac{x\arcsin(x)}{3(1-x^2)^{3/2}}-\frac{1}{6(1-x^2)}+\frac{1}{3}\log\left(1-x^2\right)$$

```
[In] Int[ArcSin[x]/(1 - x^2)^(5/2),x]
```

```
[Out] -1/6*1/(1 - x^2) + (x*ArcSin[x])/(3*(1 - x^2)^(3/2)) + (2*x*ArcSin[x])/(3*S
qrt[1 - x^2]) + Log[1 - x^2]/3
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4745

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcSin[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSin[c*x]
)^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0]
```

Rule 4747

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p + 1
))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sin[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*
x^2)^p], Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\arcsin(x)}{3(1-x^2)^{3/2}} - \frac{1}{3}\int \frac{x}{(1-x^2)^2}\,dx + \frac{2}{3}\int \frac{\arcsin(x)}{(1-x^2)^{3/2}}\,dx \\
&= -\frac{1}{6(1-x^2)} + \frac{x\arcsin(x)}{3(1-x^2)^{3/2}} + \frac{2x\arcsin(x)}{3\sqrt{1-x^2}} - \frac{2}{3}\int \frac{x}{1-x^2}\,dx \\
&= -\frac{1}{6(1-x^2)} + \frac{x\arcsin(x)}{3(1-x^2)^{3/2}} + \frac{2x\arcsin(x)}{3\sqrt{1-x^2}} + \frac{1}{3}\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = \frac{1}{6}\left(\frac{1}{-1+x^2} - \frac{2x(-3+2x^2)\arcsin(x)}{(1-x^2)^{3/2}} + 2\log\left(1-x^2\right)\right)$$

```
[In] Integrate[ArcSin[x]/(1 - x^2)^(5/2),x]
[Out] ((-1 + x^2)^(-1) - (2*x*(-3 + 2*x^2)*ArcSin[x])/(1 - x^2)^(3/2) + 2*Log[1 -
 x^2])/6
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{1}{6x^2-6}+\frac{\sqrt{-x^2+1}\,\arcsin(x)x}{3(x^2-1)^2}+\frac{\ln(-x^2+1)}{3}-\frac{2\sqrt{-x^2+1}\,\arcsin(x)x}{3(x^2-1)}$	63

```
[In] int(arcsin(x)/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(x^2-1)+1/3*(-x^2+1)^(1/2)/(x^2-1)^2*arcsin(x)*x+1/3*ln(-x^2+1)-2/3*(-x
^2+1)^(1/2)/(x^2-1)*arcsin(x)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = -\frac{2\,(2\,x^3-3\,x)\sqrt{-x^2+1}\arcsin(x)-x^2-2\,(x^4-2\,x^2+1)\log(x^2-1)+1}{6\,(x^4-2\,x^2+1)}$$

```
[In] integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(2*x^3 - 3*x)*sqrt(-x^2 + 1)*arcsin(x) - x^2 - 2*(x^4 - 2*x^2 + 1)*
log(x^2 - 1) + 1)/(x^4 - 2*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 12.59 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.26

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = \left(\begin{cases} \frac{x^3}{3(1-x^2)^{\frac{3}{2}}}+\frac{x}{\sqrt{1-x^2}} & \text{for}\: x>-1\wedge x<1 \end{cases}\right)\operatorname{asin}(x) - \begin{cases} \text{NaN} & \text{for}\: x<-1 \\ -\frac{2x^2\log(1-x^2)}{6x^2-6}-\frac{x^2}{6x^2-6}+\frac{2\log(1-x^2)}{6x^2-6} & \text{for}\: x<1 \\ \text{NaN} & \text{otherwise} \end{cases}$$

```
[In] integrate(asin(x)/(-x**2+1)**(5/2),x)
```

```
[Out] Piecewise((x**3/(3*(1 - x**2)**(3/2)) + x/sqrt(1 - x**2), (x > -1) & (x < 1
)))*asin(x) - Piecewise((nan, x < -1), (-2*x**2*log(1 - x**2)/(6*x**2 - 6)
- x**2/(6*x**2 - 6) + 2*log(1 - x**2)/(6*x**2 - 6), x < 1), (nan, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{2\,x}{\sqrt{-x^2+1}} + \frac{x}{(-x^2+1)^{\frac{3}{2}}}\right)\arcsin(x) + \frac{1}{6\,(x^2-1)} + \frac{1}{3}\log\left(-3\,x^2+3\right)$$

```
[In] integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="maxima")
[Out] 1/3*(2*x/sqrt(-x^2 + 1) + x/(-x^2 + 1)^(3/2))*arcsin(x) + 1/6/(x^2 - 1) + 1
/3*log(-3*x^2 + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = -\frac{(2\,x^2-3)\sqrt{-x^2+1}x\arcsin(x)}{3\,(x^2-1)^2} - \frac{2\,x^2-3}{6\,(x^2-1)} + \frac{1}{3}\log\left(|x^2-1|\right)$$

```
[In] integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="giac")
[Out] -1/3*(2*x^2 - 3)*sqrt(-x^2 + 1)*x*arcsin(x)/(x^2 - 1)^2 - 1/6*(2*x^2 - 3)/(
x^2 - 1) + 1/3*log(abs(x^2 - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = \int \frac{\mathrm{asin}(x)}{(1-x^2)^{5/2}}\,dx$$

```
[In] int(asin(x)/(1 - x^2)^(5/2),x)
[Out] int(asin(x)/(1 - x^2)^(5/2), x)
```

3.664 $\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}} \, dx = -x + \frac{\arcsin(x)}{\sqrt{1-x^2}} + \sqrt{1-x^2} \arcsin(x) - \operatorname{arctanh}(x)$$

```
[Out] -x-arctanh(x)+arcsin(x)/(-x^2+1)^(1/2)+arcsin(x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {272, 45, 4779, 396, 212}

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}} \, dx = \sqrt{1-x^2} \arcsin(x) + \frac{\arcsin(x)}{\sqrt{1-x^2}} - \operatorname{arctanh}(x) - x$$

```
[In] Int[(x^3*ArcSin[x])/(1 - x^2)^(3/2),x]
```

```
[Out] -x + ArcSin[x]/Sqrt[1 - x^2] + Sqrt[1 - x^2]*ArcSin[x] - ArcTanh[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4779

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin[
c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[Simplif
yIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && E
qQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2
, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\arcsin(x)}{\sqrt{1-x^2}} + \sqrt{1-x^2}\arcsin(x) - \int \frac{2-x^2}{1-x^2}\,dx \\
&= -x + \frac{\arcsin(x)}{\sqrt{1-x^2}} + \sqrt{1-x^2}\arcsin(x) - \int \frac{1}{1-x^2}\,dx \\
&= -x + \frac{\arcsin(x)}{\sqrt{1-x^2}} + \sqrt{1-x^2}\arcsin(x) - \operatorname{arctanh}(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}}\,dx = \frac{1}{2}\left(-2x - \frac{2(-2+x^2)\arcsin(x)}{\sqrt{1-x^2}} + \log(1-x) - \log(1+x)\right)$$

```
[In] Integrate[(x^3*ArcSin[x])/(1 - x^2)^(3/2),x]
[Out] (-2*x - (2*(-2 + x^2)*ArcSin[x])/Sqrt[1 - x^2] + Log[1 - x] - Log[1 + x])/2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.83

method	result
default	$\frac{(\arcsin(x)+i)\left(ix+\sqrt{-x^2+1}\right)}{2}-\frac{\left(-\sqrt{-x^2+1}+ix\right)(\arcsin(x)-i)}{2}-\frac{\sqrt{-x^2+1}\,\arcsin(x)}{x^2-1}-\ln\left(ix+\sqrt{-x^2+1}+i\right)+\mathrm{l}$

```
[In] int(x^3*arcsin(x)/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(arcsin(x)+I)*(I*x+(-x^2+1)^(1/2))-1/2*(-(-x^2+1)^(1/2)+I*x)*(arcsin(x)
-I)-(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)-ln(I*x+(-x^2+1)^(1/2)+I)+ln(I*x+(-x^2+
1)^(1/2)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}}\,dx = -\frac{2\,x^3-2\,(x^2-2)\sqrt{-x^2+1}\arcsin(x)+(x^2-1)\log(x+1)-(x^2-1)\log(x-1)-2\,x}{2\,(x^2-1)}$$

```
[In] integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="fricas")
[Out] -1/2*(2*x^3 - 2*(x^2 - 2)*sqrt(-x^2 + 1)*arcsin(x) + (x^2 - 1)*log(x + 1) -
 (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}}\,dx = -x-\left(-\sqrt{1-x^2}-\frac{1}{\sqrt{1-x^2}}\right)\operatorname{asin}(x)+\frac{\log(x-1)}{2}-\frac{\log(x+1)}{2}$$

```
[In] integrate(x**3*asin(x)/(-x**2+1)**(3/2),x)
[Out] -x - (-sqrt(1 - x**2) - 1/sqrt(1 - x**2))*asin(x) + log(x - 1)/2 - log(x +
1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}}\,dx = -\left(\frac{x^2}{\sqrt{-x^2+1}} - \frac{2}{\sqrt{-x^2+1}}\right)\arcsin(x) - x - \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1)$$

```
[In] integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="maxima")
[Out] -(x^2/sqrt(-x^2 + 1) - 2/sqrt(-x^2 + 1))*arcsin(x) - x - 1/2*log(x + 1) + 1
/2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}}\,dx = \left(\sqrt{-x^2+1} + \frac{1}{\sqrt{-x^2+1}}\right)\arcsin(x) - x - \frac{1}{2}\log(|x+1|) + \frac{1}{2}\log(|x-1|)$$

```
[In] integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="giac")
[Out] (sqrt(-x^2 + 1) + 1/sqrt(-x^2 + 1))*arcsin(x) - x - 1/2*log(abs(x + 1)) + 1
/2*log(abs(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}}\,dx = \int \frac{x^3 \operatorname{asin}(x)}{(1-x^2)^{3/2}}\,dx$$

```
[In] int((x^3*asin(x))/(1 - x^2)^(3/2),x)
[Out] int((x^3*asin(x))/(1 - x^2)^(3/2), x)
```

3.665 $\int \frac{\arcsin(x)}{x(1-x^2)^{3/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int \frac{\arcsin(x)}{x(1-x^2)^{3/2}} dx = \frac{\arcsin(x)}{\sqrt{1-x^2}} - 2\arcsin(x)\text{arctanh}\left(e^{i\arcsin(x)}\right) - \text{arctanh}(x) + i\,\text{PolyLog}\left(2,-e^{i\arcsin(x)}\right) - i\,\text{PolyLog}\left(2,e^{i\arcsin(x)}\right)$$

```
[Out] -2*arcsin(x)*arctanh(I*x+(-x^2+1)^(1/2))-arctanh(x)+I*polylog(2,-I*x-(-x^2+
1)^(1/2))-I*polylog(2,I*x+(-x^2+1)^(1/2))+arcsin(x)/(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4793, 4803, 4268, 2317, 2438, 212}

$$\int \frac{\arcsin(x)}{x(1-x^2)^{3/2}} dx = -2\arcsin(x)\text{arctanh}\left(e^{i\arcsin(x)}\right) + i\,\text{PolyLog}\left(2,-e^{i\arcsin(x)}\right) - i\,\text{PolyLog}\left(2,e^{i\arcsin(x)}\right) + \frac{\arcsin(x)}{\sqrt{1-x^2}} - \text{arctanh}(x)$$

```
[In] Int[ArcSin[x]/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] ArcSin[x]/Sqrt[1 - x^2] - 2*ArcSin[x]*ArcTanh[E^(I*ArcSin[x])] - ArcTanh[x]
 + I*PolyLog[2, -E^(I*ArcSin[x])] - I*PolyLog[2, E^(I*ArcSin[x])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4793

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Dist[b*c*
(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1
 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 4803

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]], Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\arcsin(x)}{\sqrt{1-x^2}} - \int \frac{1}{1-x^2}\,dx + \int \frac{\arcsin(x)}{x\sqrt{1-x^2}}\,dx \\ &= \frac{\arcsin(x)}{\sqrt{1-x^2}} - \operatorname{arctanh}(x) + \operatorname{Subst}\left(\int x\csc(x)\,dx, x, \arcsin(x)\right)\end{aligned}$$

$$= \frac{\arcsin(x)}{\sqrt{1-x^2}} - 2\arcsin(x)\text{arctanh}\left(e^{i\arcsin(x)}\right) - \text{arctanh}(x) - \text{Subst}\left(\int \log\left(1-e^{ix}\right)\,dx, x, \arcsin(x)\right) + \text{Subst}\left(\int \log\left(1+e^{ix}\right)\,dx, x, \arcsin(x)\right)$$

$$= \frac{\arcsin(x)}{\sqrt{1-x^2}} - 2\arcsin(x)\text{arctanh}\left(e^{i\arcsin(x)}\right) - \text{arctanh}(x) + i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{i\arcsin(x)}\right) - i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{i\arcsin(x)}\right)$$

$$= \frac{\arcsin(x)}{\sqrt{1-x^2}} - 2\arcsin(x)\text{arctanh}\left(e^{i\arcsin(x)}\right) - \text{arctanh}(x) + i\,\text{PolyLog}\left(2, -e^{i\arcsin(x)}\right) - i\,\text{PolyLog}\left(2, e^{i\arcsin(x)}\right)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.81

$$\int \frac{\arcsin(x)}{x\left(1-x^2\right)^{3/2}}\,dx = \frac{\arcsin(x)}{\sqrt{1-x^2}} + \arcsin(x)\log\left(1-e^{i\arcsin(x)}\right) - \arcsin(x)\log\left(1+e^{i\arcsin(x)}\right) + \log\left(\cos\left(\frac{\arcsin(x)}{2}\right) - \sin\left(\frac{\arcsin(x)}{2}\right)\right) - \log\left(\cos\left(\frac{\arcsin(x)}{2}\right) + \sin\left(\frac{\arcsin(x)}{2}\right)\right) + i\,\text{PolyLog}\left(2, -e^{i\arcsin(x)}\right) - i\,\text{PolyLog}\left(2, e^{i\arcsin(x)}\right)$$

```
[In] Integrate[ArcSin[x]/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] ArcSin[x]/Sqrt[1 - x^2] + ArcSin[x]*Log[1 - E^(I*ArcSin[x])] - ArcSin[x]*Lo
g[1 + E^(I*ArcSin[x])] + Log[Cos[ArcSin[x]/2] - Sin[ArcSin[x]/2]] - Log[Cos
[ArcSin[x]/2] + Sin[ArcSin[x]/2]] + I*PolyLog[2, -E^(I*ArcSin[x])] - I*Poly
Log[2, E^(I*ArcSin[x])]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 429 vs. 2(76) = 152.

Time = 0.69 (sec) , antiderivative size = 430, normalized size of antiderivative = 6.94

method	result
default	$-\frac{\sqrt{-x^2+1}\,\arcsin(x)}{x^2-1}-\frac{i\left(\ln\left(ix+\sqrt{-x^2+1}+1\right)-\ln\left(ix+\sqrt{-x^2+1}-1\right)-2\arctan\left(ix+\sqrt{-x^2+1}\right)\right)}{2}+\frac{i\left(i\arcsin(x)\ln\left(ix+\sqrt{-x^2+1}\right.\right.}{}$

```
[In] int(arcsin(x)/x/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)-1/2*I*(ln(I*x+(-x^2+1)^(1/2)+1)-ln(I*x+(-
x^2+1)^(1/2)-1)-2*arctan(I*x+(-x^2+1)^(1/2)))+1/2*I*(I*arcsin(x)*ln(I*x+(-x
^2+1)^(1/2)+1)+arcsin(x)*ln(1+I*(I*x+(-x^2+1)^(1/2)))-arcsin(x)*ln(1-I*(I*x
+(-x^2+1)^(1/2)))-I*dilog(1+I*(I*x+(-x^2+1)^(1/2)))+I*dilog(1-I*(I*x+(-x^2+
1)^(1/2)))+dilog(I*x+(-x^2+1)^(1/2)+1)+dilog(I*x+(-x^2+1)^(1/2)))+1/2*I*(I*
arcsin(x)*ln(I*x+(-x^2+1)^(1/2)+1)-arcsin(x)*ln(1+I*(I*x+(-x^2+1)^(1/2)))+a
rcsin(x)*ln(1-I*(I*x+(-x^2+1)^(1/2)))+I*dilog(1+I*(I*x+(-x^2+1)^(1/2)))-I*d
ilog(1-I*(I*x+(-x^2+1)^(1/2)))+dilog(I*x+(-x^2+1)^(1/2)+1)+dilog(I*x+(-x^2+
1)^(1/2)))+1/2*I*(ln(I*x+(-x^2+1)^(1/2)+1)-ln(I*x+(-x^2+1)^(1/2)-1)+2*arcta
n(I*x+(-x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\arcsin(x)}{x\left(1-x^2\right)^{3/2}}\,dx=\int \frac{\arcsin\left(x\right)}{\left(-x^2+1\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)*arcsin(x)/(x^5 - 2*x^3 + x), x)
```

Sympy [F]

$$\int \frac{\arcsin(x)}{x\left(1-x^2\right)^{3/2}}\,dx=\int \frac{\operatorname{asin}\left(x\right)}{x\left(-\left(x-1\right)\left(x+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(asin(x)/x/(-x**2+1)**(3/2),x)
```

```
[Out] Integral(asin(x)/(x*(-(x - 1)*(x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{\arcsin(x)}{x\,(1-x^2)^{3/2}}\,dx = \int \frac{\arcsin\left(x\right)}{\left(-x^2+1\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(arcsin(x)/((-x^2 + 1)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{\arcsin(x)}{x\,(1-x^2)^{3/2}}\,dx = \int \frac{\arcsin\left(x\right)}{\left(-x^2+1\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(arcsin(x)/((-x^2 + 1)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(x)}{x\,(1-x^2)^{3/2}}\,dx = \int \frac{\mathrm{asin}(x)}{x\,(1-x^2)^{3/2}}\,dx$$

```
[In] int(asin(x)/(x*(1 - x^2)^(3/2)),x)
[Out] int(asin(x)/(x*(1 - x^2)^(3/2)), x)
```

3.666 $\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = \frac{1}{6x^2} - \frac{\sqrt{1-x^2}\arccos(x)}{3x^3} - \frac{2\sqrt{1-x^2}\arccos(x)}{3x} - \frac{2\log(x)}{3}$$

```
[Out] 1/6/x^2-2/3*ln(x)-1/3*arccos(x)*(-x^2+1)^(1/2)/x^3-2/3*arccos(x)*(-x^2+1)^(
1/2)/x
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {4790, 4772, 29, 30}

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = -\frac{2\sqrt{1-x^2}\arccos(x)}{3x} - \frac{\sqrt{1-x^2}\arccos(x)}{3x^3} + \frac{1}{6x^2} - \frac{2\log(x)}{3}$$

```
[In] Int[ArcCos[x]/(x^4*Sqrt[1 - x^2]),x]
```

```
[Out] 1/(6*x^2) - (Sqrt[1 - x^2]*ArcCos[x])/(3*x^3) - (2*Sqrt[1 - x^2]*ArcCos[x])
/(3*x) - (2*Log[x])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4772

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*Ar
cCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2
*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 4790

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c
*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1
- c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{1-x^2}\arccos(x)}{3x^3} - \frac{1}{3}\int \frac{1}{x^3}\,dx + \frac{2}{3}\int \frac{\arccos(x)}{x^2\sqrt{1-x^2}}\,dx \\
&= \frac{1}{6x^2} - \frac{\sqrt{1-x^2}\arccos(x)}{3x^3} - \frac{2\sqrt{1-x^2}\arccos(x)}{3x} - \frac{2}{3}\int \frac{1}{x}\,dx \\
&= \frac{1}{6x^2} - \frac{\sqrt{1-x^2}\arccos(x)}{3x^3} - \frac{2\sqrt{1-x^2}\arccos(x)}{3x} - \frac{2\log(x)}{3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = \frac{x-2\sqrt{1-x^2}(1+2x^2)\arccos(x)-4x^3\log(x)}{6x^3}$$

```
[In] Integrate[ArcCos[x]/(x^4*Sqrt[1 - x^2]),x]
[Out] (x - 2*Sqrt[1 - x^2]*(1 + 2*x^2)*ArcCos[x] - 4*x^3*Log[x])/(6*x^3)
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{1}{6x^{2}}-\frac{2\ln\left(x\right)}{3}-\frac{\arccos\left(x\right)\sqrt{-x^{2}+1}}{3x^{3}}-\frac{2\arccos\left(x\right)\sqrt{-x^{2}+1}}{3x}$	43

```
[In] int(arccos(x)/x^4/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6/x^2-2/3*ln(x)-1/3*arccos(x)*(-x^2+1)^(1/2)/x^3-2/3*arccos(x)*(-x^2+1)^(
1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.67

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = -\frac{4\,x^{3}\log\left(x\right)+2\left(2\,x^{2}+1\right)\sqrt{-x^{2}+1}\arccos\left(x\right)-x}{6\,x^{3}}$$

```
[In] integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/6*(4*x^3*log(x) + 2*(2*x^2 + 1)*sqrt(-x^2 + 1)*arccos(x) - x)/x^3
```

Sympy [A] (verification not implemented)

Time = 5.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = \left(\begin{cases} -\frac{\sqrt{1-x^{2}}}{x}-\frac{\left(1-x^{2}\right)^{\frac{3}{2}}}{3x^{3}} & \text{for}\: x > -1 \wedge x < 1 \end{cases}\right)\operatorname{acos}\left(x\right) + \begin{cases} \text{NaN} & \text{for}\: x < -1 \\ -\frac{2\log\left(x\right)}{3}+\frac{1}{6x^{2}} & \text{for}\: x < 1 \\ \text{NaN} & \text{otherwise} \end{cases}$$

```
[In] integrate(acos(x)/x**4/(-x**2+1)**(1/2),x)
[Out] Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
1)))*acos(x) + Piecewise((nan, x < -1), (-2*log(x)/3 + 1/(6*x**2), x < 1),
(nan, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = -\frac{1}{3}\left(\frac{2\sqrt{-x^2+1}}{x} + \frac{\sqrt{-x^2+1}}{x^3}\right)\arccos(x) + \frac{1}{6x^2} - \frac{2}{3}\log(x)$$

```
[In] integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/3*(2*sqrt(-x^2 + 1)/x + sqrt(-x^2 + 1)/x^3)*arccos(x) + 1/6/x^2 - 2/3*lo
g(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(42) = 84.

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.76

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = \frac{1}{24}\left(\frac{x^3\left(\frac{9\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{\left(\sqrt{-x^2+1}-1\right)^3} - \frac{9\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^3}{x^3}\right)\arccos(x) + \frac{2x^2+1}{6x^2} - \frac{1}{3}\log\left(x^2\right)$$

```
[In] integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/24*(x^3*(9*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1)^3 - 9*(sq
rt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^3/x^3)*arccos(x) + 1/6*(2*x^2 +
1)/x^2 - 1/3*log(x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = \int \frac{\mathrm{acos}(x)}{x^4\sqrt{1-x^2}}\,dx$$

```
[In] int(acos(x)/(x^4*(1 - x^2)^(1/2)),x)
[Out] int(acos(x)/(x^4*(1 - x^2)^(1/2)), x)
```

3.667 $\int x\sqrt{1-x^2}\arccos(x)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = \frac{4\sqrt{1-x^2}}{9}+\frac{2}{27}\left(1-x^2\right)^{3/2} -\frac{2}{3}x\arccos(x)+\frac{2}{9}x^3\arccos(x)-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)^2$$

```
[Out] 2/27*(-x^2+1)^(3/2)-2/3*x*arccos(x)+2/9*x^3*arccos(x)-1/3*(-x^2+1)^(3/2)*ar
ccos(x)^2+4/9*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4768, 4740, 455, 45}

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = \frac{2}{9}x^3\arccos(x)-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)^2-\frac{2}{3}x\arccos(x) +\frac{2}{27}\left(1-x^2\right)^{3/2}+\frac{4\sqrt{1-x^2}}{9}$$

```
[In] Int[x*Sqrt[1 - x^2]*ArcCos[x]^2,x]
```

```
[Out] (4*Sqrt[1 - x^2])/9 + (2*(1 - x^2)^(3/2))/27 - (2*x*ArcCos[x])/3 + (2*x^3*A
rcCos[x])/9 - ((1 - x^2)^(3/2)*ArcCos[x]^2)/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4740

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCos[c*x], u, x] +
 Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)^2-\frac{2}{3}\int\left(1-x^2\right)\arccos(x)\,dx\\
&= -\frac{2}{3}x\arccos(x)+\frac{2}{9}x^3\arccos(x)-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)^2-\frac{2}{3}\int\frac{x\left(1-\frac{x^2}{3}\right)}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2}{3}x\arccos(x)+\frac{2}{9}x^3\arccos(x)-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)^2-\frac{1}{3}\mathrm{Subst}\left(\int\frac{1-\frac{x}{3}}{\sqrt{1-x}}\,dx,x,x^2\right)\\
&= -\frac{2}{3}x\arccos(x)+\frac{2}{9}x^3\arccos(x)\\
&\quad-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)^2-\frac{1}{3}\mathrm{Subst}\left(\int\left(\frac{2}{3\sqrt{1-x}}+\frac{\sqrt{1-x}}{3}\right)\,dx,x,x^2\right)\\
&= \frac{4\sqrt{1-x^2}}{9}+\frac{2}{27}\left(1-x^2\right)^{3/2}-\frac{2}{3}x\arccos(x)+\frac{2}{9}x^3\arccos(x)-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)^2
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = \frac{1}{27}\Big(-2\sqrt{1-x^2}\left(-7+x^2\right)+6x\left(-3+x^2\right)\arccos(x) - 9\left(1-x^2\right)^{3/2}\arccos(x)^2\Big)$$

```
[In] Integrate[x*Sqrt[1 - x^2]*ArcCos[x]^2,x]
```

```
[Out] (-2*Sqrt[1 - x^2]*(-7 + x^2) + 6*x*(-3 + x^2)*ArcCos[x] - 9*(1 - x^2)^(3/2)
*ArcCos[x]^2)/27
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{(x^2-1)\sqrt{-x^2+1}\,\arccos(x)^2}{3}+\frac{2\arccos(x)(x^2-3)x}{9}-\frac{2(x^2-1)\sqrt{-x^2+1}}{27}+\frac{4\sqrt{-x^2+1}}{9}$	59

```
[In] int(x*arccos(x)^2*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(x^2-1)*(-x^2+1)^(1/2)*arccos(x)^2+2/9*arccos(x)*(x^2-3)*x-2/27*(x^2-1)
*(-x^2+1)^(1/2)+4/9*(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.62

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = \frac{2}{9}\left(x^3-3\,x\right)\arccos\left(x\right) + \frac{1}{27}\left(9\left(x^2-1\right)\arccos\left(x\right)^2-2\,x^2+14\right)\sqrt{-x^2+1}$$

```
[In] integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(x^3 - 3*x)*arccos(x) + 1/27*(9*(x^2 - 1)*arccos(x)^2 - 2*x^2 + 14)*sqr
t(-x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = \frac{2x^3\operatorname{acos}(x)}{9} + \frac{x^2\sqrt{1-x^2}\operatorname{acos}^2(x)}{3} - \frac{2x^2\sqrt{1-x^2}}{27} - \frac{2x\operatorname{acos}(x)}{3} - \frac{\sqrt{1-x^2}\operatorname{acos}^2(x)}{3} + \frac{14\sqrt{1-x^2}}{27}$$

```
[In] integrate(x*acos(x)**2*(-x**2+1)**(1/2),x)
[Out] 2*x**3*acos(x)/9 + x**2*sqrt(1 - x**2)*acos(x)**2/3 - 2*x**2*sqrt(1 - x**2)
/27 - 2*x*acos(x)/3 - sqrt(1 - x**2)*acos(x)**2/3 + 14*sqrt(1 - x**2)/27
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = -\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos(x)^2 - \frac{2}{27}\sqrt{-x^2+1}x^2 + \frac{2}{9}\left(x^3-3x\right)\arccos(x) + \frac{14}{27}\sqrt{-x^2+1}$$

```
[In] integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/3*(-x^2 + 1)^(3/2)*arccos(x)^2 - 2/27*sqrt(-x^2 + 1)*x^2 + 2/9*(x^3 - 3*
x)*arccos(x) + 14/27*sqrt(-x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = \frac{2}{9}x^3\arccos(x) - \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos(x)^2 - \frac{2}{27}\sqrt{-x^2+1}x^2 - \frac{2}{3}x\arccos(x) + \frac{14}{27}\sqrt{-x^2+1}$$

```
[In] integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 2/9*x^3*arccos(x) - 1/3*(-x^2 + 1)^(3/2)*arccos(x)^2 - 2/27*sqrt(-x^2 + 1)*
x^2 - 2/3*x*arccos(x) + 14/27*sqrt(-x^2 + 1)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx=\int x\,\mathrm{acos}(x)^2\,\sqrt{1-x^2}\,dx$$

```
[In] int(x*acos(x)^2*(1 - x^2)^(1/2),x)
[Out] int(x*acos(x)^2*(1 - x^2)^(1/2), x)
```

3.668 $\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = -\frac{3x^2}{8} + \frac{3}{4}x\sqrt{1-x^2}\arcsin(x) - \frac{3\arcsin(x)^2}{8} + \frac{3}{4}x^2\arcsin(x)^2 - \frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3 + \frac{\arcsin(x)^4}{8}$$

```
[Out] -3/8*x^2-3/8*arcsin(x)^2+3/4*x^2*arcsin(x)^2+1/8*arcsin(x)^4+3/4*x*arcsin(x
)*(-x^2+1)^(1/2)-1/2*x*arcsin(x)^3*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {4795, 4737, 4723, 30}

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3 + \frac{3}{4}x^2\arcsin(x)^2 + \frac{3}{4}x\sqrt{1-x^2}\arcsin(x) + \frac{\arcsin(x)^4}{8} - \frac{3\arcsin(x)^2}{8} - \frac{3x^2}{8}$$

```
[In] Int[(x^2*ArcSin[x]^3)/Sqrt[1 - x^2],x]
```

```
[Out] (-3*x^2)/8 + (3*x*Sqrt[1 - x^2]*ArcSin[x])/4 - (3*ArcSin[x]^2)/8 + (3*x^2*A
rcSin[x]^2)/4 - (x*Sqrt[1 - x^2]*ArcSin[x]^3)/2 + ArcSin[x]^4/8
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4737

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

Rule 4795

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3 + \frac{1}{2}\int \frac{\arcsin(x)^3}{\sqrt{1-x^2}}\,dx + \frac{3}{2}\int x\arcsin(x)^2\,dx \\
&= \frac{3}{4}x^2\arcsin(x)^2 - \frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3 + \frac{\arcsin(x)^4}{8} - \frac{3}{2}\int \frac{x^2\arcsin(x)}{\sqrt{1-x^2}}\,dx \\
&= \frac{3}{4}x\sqrt{1-x^2}\arcsin(x) + \frac{3}{4}x^2\arcsin(x)^2 - \frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3 \\
&\quad + \frac{\arcsin(x)^4}{8} - \frac{3\int x\,dx}{4} - \frac{3}{4}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}\,dx \\
&= -\frac{3x^2}{8} + \frac{3}{4}x\sqrt{1-x^2}\arcsin(x) - \frac{3\arcsin(x)^2}{8} \\
&\quad + \frac{3}{4}x^2\arcsin(x)^2 - \frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3 + \frac{\arcsin(x)^4}{8}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = \frac{1}{8}\Big(-3x^2+6x\sqrt{1-x^2}\arcsin(x)+\left(-3+6x^2\right)\arcsin(x)^2 - 4x\sqrt{1-x^2}\arcsin(x)^3+\arcsin(x)^4\Big)$$

```
[In] Integrate[(x^2*ArcSin[x]^3)/Sqrt[1 - x^2],x]
```

```
[Out] (-3*x^2 + 6*x*Sqrt[1 - x^2]*ArcSin[x] + (-3 + 6*x^2)*ArcSin[x]^2 - 4*x*Sqrt
[1 - x^2]*ArcSin[x]^3 + ArcSin[x]^4)/8
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

method	result
default	$\frac{\arcsin(x)^3\left(-x\sqrt{-x^2+1}+\arcsin(x)\right)}{2}+\frac{3\arcsin(x)^2(x^2-1)}{4}+\frac{3\arcsin(x)\left(x\sqrt{-x^2+1}+\arcsin(x)\right)}{4}-\frac{3\arcsin(x)^2}{8}-\frac{3x^2}{8}-\frac{3}{}$

```
[In] int(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsin(x)^3*(-x*(-x^2+1)^(1/2)+arcsin(x))+3/4*arcsin(x)^2*(x^2-1)+3/4*a
rcsin(x)*(x*(-x^2+1)^(1/2)+arcsin(x))-3/8*arcsin(x)^2-3/8*x^2-3/8*arcsin(x)
^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = \frac{1}{8}\arcsin\left(x\right)^4+\frac{3}{8}\left(2\,x^2-1\right)\arcsin\left(x\right)^2-\frac{3}{8}\,x^2 - \frac{1}{4}\left(2\,x\arcsin\left(x\right)^3-3\,x\arcsin\left(x\right)\right)\sqrt{-x^2+1}$$

```
[In] integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*arcsin(x)^4 + 3/8*(2*x^2 - 1)*arcsin(x)^2 - 3/8*x^2 - 1/4*(2*x*arcsin(x
)^3 - 3*x*arcsin(x))*sqrt(-x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = \frac{3x^2 \operatorname{asin}^2(x)}{4} - \frac{3x^2}{8} - \frac{x\sqrt{1-x^2}\operatorname{asin}^3(x)}{2} + \frac{3x\sqrt{1-x^2}\operatorname{asin}(x)}{4} + \frac{\operatorname{asin}^4(x)}{8} - \frac{3\operatorname{asin}^2(x)}{8}$$

```
[In] integrate(x**2*asin(x)**3/(-x**2+1)**(1/2),x)
[Out] 3*x**2*asin(x)**2/4 - 3*x**2/8 - x*sqrt(1 - x**2)*asin(x)**3/2 + 3*x*sqrt(1
 - x**2)*asin(x)/4 + asin(x)**4/8 - 3*asin(x)**2/8
```

Maxima [F]

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = \int \frac{x^2 \arcsin(x)^3}{\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2*arcsin(x)^3/sqrt(-x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+1}x \arcsin(x)^3 + \frac{1}{8}\arcsin(x)^4 + \frac{3}{4}(x^2-1)\arcsin(x)^2 + \frac{3}{4}\sqrt{-x^2+1}x\arcsin(x) - \frac{3}{8}x^2 + \frac{3}{8}\arcsin(x)^2 + \frac{3}{16}$$

```
[In] integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^2 + 1)*x*arcsin(x)^3 + 1/8*arcsin(x)^4 + 3/4*(x^2 - 1)*arcsin(
x)^2 + 3/4*sqrt(-x^2 + 1)*x*arcsin(x) - 3/8*x^2 + 3/8*arcsin(x)^2 + 3/16
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\, dx = \int \frac{x^2 \operatorname{asin}(x)^3}{\sqrt{1-x^2}}\, dx$$

```
[In] int((x^2*asin(x)^3)/(1 - x^2)^(1/2),x)
[Out] int((x^2*asin(x)^3)/(1 - x^2)^(1/2), x)
```

3.669 $\int \frac{x\arctan(x)}{(1+x^2)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{x\arctan(x)}{(1+x^2)^2}\,dx = \frac{x}{4(1+x^2)} + \frac{\arctan(x)}{4} - \frac{\arctan(x)}{2(1+x^2)}$$

```
[Out] 1/4*x/(x^2+1)+1/4*arctan(x)-1/2*arctan(x)/(x^2+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5050, 205, 209}

$$\int \frac{x\arctan(x)}{(1+x^2)^2}\,dx = -\frac{\arctan(x)}{2(x^2+1)} + \frac{\arctan(x)}{4} + \frac{x}{4(x^2+1)}$$

```
[In] Int[(x*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] x/(4*(1 + x^2)) + ArcTan[x]/4 - ArcTan[x]/(2*(1 + x^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\arctan(x)}{2(1+x^2)}+\frac{1}{2}\int\frac{1}{(1+x^2)^2}\,dx\\ &= \frac{x}{4(1+x^2)}-\frac{\arctan(x)}{2(1+x^2)}+\frac{1}{4}\int\frac{1}{1+x^2}\,dx\\ &= \frac{x}{4(1+x^2)}+\frac{\arctan(x)}{4}-\frac{\arctan(x)}{2(1+x^2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int\frac{x\arctan(x)}{(1+x^2)^2}\,dx=\frac{x+(-1+x^2)\arctan(x)}{4(1+x^2)}$$

```
[In] Integrate[(x*ArcTan[x])/(1 + x^2)^2,x]
[Out] (x + (-1 + x^2)*ArcTan[x])/(4*(1 + x^2))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$\frac{x^2\arctan(x)+x-\arctan(x)}{4x^2+4}$	22
default	$\frac{x}{4x^2+4}+\frac{\arctan(x)}{4}-\frac{\arctan(x)}{2(x^2+1)}$	27
parts	$\frac{x}{4x^2+4}+\frac{\arctan(x)}{4}-\frac{\arctan(x)}{2(x^2+1)}$	27
risch	$\frac{i\ln(ix+1)}{4x^2+4}-\frac{i(2\ln(-ix+1)-\ln(x+i)x^2-\ln(x+i)+x^2\ln(x-i)+\ln(x-i)+2ix)}{8(x-i)(x+i)}$	79

```
[In] int(x*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*(x^2*arctan(x)+x-arctan(x))/(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \frac{x\arctan(x)}{(1+x^2)^2}\,dx = \frac{(x^2-1)\arctan(x)+x}{4\,(x^2+1)}$$

```
[In] integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/4*((x^2 - 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{x\arctan(x)}{(1+x^2)^2}\,dx = \frac{x^2\operatorname{atan}(x)}{4x^2+4} + \frac{x}{4x^2+4} - \frac{\operatorname{atan}(x)}{4x^2+4}$$

```
[In] integrate(x*atan(x)/(x**2+1)**2,x)
[Out] x**2*atan(x)/(4*x**2 + 4) + x/(4*x**2 + 4) - atan(x)/(4*x**2 + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x\arctan(x)}{(1+x^2)^2}\,dx = \frac{x}{4\,(x^2+1)} - \frac{\arctan(x)}{2\,(x^2+1)} + \frac{1}{4}\arctan(x)$$

```
[In] integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/4*x/(x^2 + 1) - 1/2*arctan(x)/(x^2 + 1) + 1/4*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x \arctan(x)}{(1+x^2)^2} \, dx = \frac{x}{4(x^2+1)} - \frac{\arctan(x)}{2(x^2+1)} + \frac{1}{4}\arctan(x)$$

```
[In] integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="giac")
[Out] 1/4*x/(x^2 + 1) - 1/2*arctan(x)/(x^2 + 1) + 1/4*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{x \arctan(x)}{(1+x^2)^2} \, dx = \frac{\mathrm{atan}(x)}{4} + \frac{\frac{x}{4} - \frac{\mathrm{atan}(x)}{2}}{x^2+1}$$

```
[In] int((x*atan(x))/(x^2 + 1)^2,x)
[Out] atan(x)/4 + (x/4 - atan(x)/2)/(x^2 + 1)
```

3.670 $\int \frac{x \arctan(x)}{(1+x^2)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 44

$$\int \frac{x \arctan(x)}{(1+x^2)^3} dx = \frac{x}{16(1+x^2)^2} + \frac{3x}{32(1+x^2)} + \frac{3\arctan(x)}{32} - \frac{\arctan(x)}{4(1+x^2)^2}$$

```
[Out] 1/16*x/(x^2+1)^2+3/32*x/(x^2+1)+3/32*arctan(x)-1/4*arctan(x)/(x^2+1)^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5050, 205, 209}

$$\int \frac{x \arctan(x)}{(1+x^2)^3} dx = -\frac{\arctan(x)}{4(x^2+1)^2} + \frac{3\arctan(x)}{32} + \frac{3x}{32(x^2+1)} + \frac{x}{16(x^2+1)^2}$$

```
[In] Int[(x*ArcTan[x])/(1 + x^2)^3,x]
```

```
[Out] x/(16*(1 + x^2)^2) + (3*x)/(32*(1 + x^2)) + (3*ArcTan[x])/32 - ArcTan[x]/(4
*(1 + x^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\arctan(x)}{4(1+x^2)^2}+\frac{1}{4}\int\frac{1}{(1+x^2)^3}\,dx\\&=\frac{x}{16(1+x^2)^2}-\frac{\arctan(x)}{4(1+x^2)^2}+\frac{3}{16}\int\frac{1}{(1+x^2)^2}\,dx\\&=\frac{x}{16(1+x^2)^2}+\frac{3x}{32(1+x^2)}-\frac{\arctan(x)}{4(1+x^2)^2}+\frac{3}{32}\int\frac{1}{1+x^2}\,dx\\&=\frac{x}{16(1+x^2)^2}+\frac{3x}{32(1+x^2)}+\frac{3\arctan(x)}{32}-\frac{\arctan(x)}{4(1+x^2)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int\frac{x\arctan(x)}{(1+x^2)^3}\,dx=\frac{x(5+3x^2)+(-5+6x^2+3x^4)\arctan(x)}{32(1+x^2)^2}$$

```
[In] Integrate[(x*ArcTan[x])/(1 + x^2)^3,x]
[Out] (x*(5 + 3*x^2) + (-5 + 6*x^2 + 3*x^4)*ArcTan[x])/(32*(1 + x^2)^2)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{x}{16(x^2+1)^2}+\frac{3x}{32(x^2+1)}+\frac{3\arctan(x)}{32}-\frac{\arctan(x)}{4(x^2+1)^2}$	37
parallelrisch	$\frac{3\arctan(x)x^4+3x^3+6x^2\arctan(x)+5x-5\arctan(x)}{32(x^2+1)^2}$	37
parts	$\frac{x}{16(x^2+1)^2}+\frac{3x}{32(x^2+1)}+\frac{3\arctan(x)}{32}-\frac{\arctan(x)}{4(x^2+1)^2}$	37
risch	$\frac{i\ln(ix+1)}{8(x^2+1)^2}-\frac{i(8\ln(-ix+1)+3\ln(x-i)x^4+6x^2\ln(x-i)+3\ln(x-i)-3\ln(x+i)x^4-6\ln(x+i)x^2-3\ln(x+i)+6ix^3+10ix)}{64(x+i)^2(x-i)^2}$	108

```
[In] int(x*arctan(x)/(x^2+1)^3,x,method=_RETURNVERBOSE)
[Out] 1/16*x/(x^2+1)^2+3/32*x/(x^2+1)+3/32*arctan(x)-1/4*arctan(x)/(x^2+1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{x\arctan(x)}{(1+x^2)^3}\,dx=\frac{3\,x^3+(3\,x^4+6\,x^2-5)\arctan{(x)}+5\,x}{32\,(x^4+2\,x^2+1)}$$

```
[In] integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="fricas")
[Out] 1/32*(3*x^3 + (3*x^4 + 6*x^2 - 5)*arctan(x) + 5*x)/(x^4 + 2*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(37) = 74.

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.00

$$\int \frac{x\arctan(x)}{(1+x^2)^3}\,dx=\frac{3x^4\operatorname{atan}{(x)}}{32x^4+64x^2+32}+\frac{3x^3}{32x^4+64x^2+32}+\frac{6x^2\operatorname{atan}{(x)}}{32x^4+64x^2+32}+\frac{5x}{32x^4+64x^2+32}-\frac{5\operatorname{atan}{(x)}}{32x^4+64x^2+32}$$

```
[In] integrate(x*atan(x)/(x**2+1)**3,x)
[Out] 3*x**4*atan(x)/(32*x**4 + 64*x**2 + 32) + 3*x**3/(32*x**4 + 64*x**2 + 32) +
 6*x**2*atan(x)/(32*x**4 + 64*x**2 + 32) + 5*x/(32*x**4 + 64*x**2 + 32) - 5
*atan(x)/(32*x**4 + 64*x**2 + 32)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{x \arctan(x)}{(1+x^2)^3} \, dx = \frac{3\,x^3+5\,x}{32\,(x^4+2\,x^2+1)} - \frac{\arctan(x)}{4\,(x^2+1)^2} + \frac{3}{32}\,\arctan(x)$$

```
[In] integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(3*x^3 + 5*x)/(x^4 + 2*x^2 + 1) - 1/4*arctan(x)/(x^2 + 1)^2 + 3/32*arc
tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{x \arctan(x)}{(1+x^2)^3} \, dx = \frac{3\,x^3+5\,x}{32\,(x^2+1)^2} - \frac{\arctan(x)}{4\,(x^2+1)^2} + \frac{3}{32}\,\arctan(x)$$

```
[In] integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/32*(3*x^3 + 5*x)/(x^2 + 1)^2 - 1/4*arctan(x)/(x^2 + 1)^2 + 3/32*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{x \arctan(x)}{(1+x^2)^3} \, dx = \frac{3\,\mathrm{atan}(x)}{32} + \frac{\frac{5\,x}{32} - \frac{\mathrm{atan}(x)}{4} + \frac{3\,x^3}{32}}{(x^2+1)^2}$$

```
[In] int((x*atan(x))/(x^2 + 1)^3,x)
```

```
[Out] (3*atan(x))/32 + ((5*x)/32 - atan(x)/4 + (3*x^3)/32)/(x^2 + 1)^2
```

3.671 $\int \frac{x^2 \arctan(x)}{1+x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{1}{2} \log \left(1+x^2\right)$$

```
[Out] x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5036, 4930, 266, 5004}

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = -\frac{1}{2} \arctan(x)^2 + x \arctan(x) - \frac{1}{2} \log \left(x^2+1\right)$$

```
[In] Int[(x^2*ArcTan[x])/(1 + x^2),x]
```

```
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \arctan(x)\,dx - \int \frac{\arctan(x)}{1+x^2}\,dx \\
&= x\arctan(x) - \frac{\arctan(x)^2}{2} - \int \frac{x}{1+x^2}\,dx \\
&= x\arctan(x) - \frac{\arctan(x)^2}{2} - \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2\arctan(x)}{1+x^2}\,dx = x\arctan(x) - \frac{\arctan(x)^2}{2} - \frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(x^2*ArcTan[x])/(1 + x^2),x]
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{\ln(x^2+1)}{2}$	20
parallelrisch	$x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{\ln(x^2+1)}{2}$	20
parts	$x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{\ln(x^2+1)}{2}$	20
risch	$\frac{\ln(ix+1)^2}{8} + \frac{i\left(-x+\frac{i\ln(-ix+1)}{2}\right)\ln(ix+1)}{2} + \frac{\ln(-ix+1)^2}{8} + \frac{ix\ln(-ix+1)}{2} - \frac{\ln(x^2+1)}{2}$	67

```
[In] int(x^2*arctan(x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log\left(x^2+1\right)$$

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="fricas")
[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \operatorname{atan}(x) - \frac{\log(x^2+1)}{2} - \frac{\operatorname{atan}^2(x)}{2}$$

```
[In] integrate(x**2*atan(x)/(x**2+1),x)
[Out] x*atan(x) - log(x**2 + 1)/2 - atan(x)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = (x - \arctan(x)) \arctan(x) + \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2+1)$$

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="maxima")
[Out] (x - arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2+1)$$

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="giac")
[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = -\frac{\mathrm{atan}(x)^2}{2} + x \,\mathrm{atan}(x) - \frac{\ln(x^2+1)}{2}$$

```
[In] int((x^2*atan(x))/(x^2 + 1),x)
[Out] x*atan(x) - atan(x)^2/2 - log(x^2 + 1)/2
```

3.672 $\int \frac{x^3 \arctan(x)}{1+x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{x^3 \arctan(x)}{1+x^2} dx = -\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{1}{2}x^2 \arctan(x) + \frac{1}{2}i \arctan(x)^2 + \arctan(x) \log\left(\frac{2}{1+ix}\right) + \frac{1}{2}i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+ix}\right)$$

```
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)+1/2*I*arctan(x)^2+arctan(x)*ln(2/(1+
I*x))+1/2*I*polylog(2,1-2/(1+I*x))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5036, 4946, 327, 209, 5040, 4964, 2449, 2352}

$$\int \frac{x^3 \arctan(x)}{1+x^2} dx = \frac{1}{2}x^2 \arctan(x) + \frac{1}{2}i \arctan(x)^2 + \frac{\arctan(x)}{2} + \arctan(x) \log\left(\frac{2}{1+ix}\right) + \frac{1}{2}i \operatorname{PolyLog}\left(2, 1 - \frac{2}{ix+1}\right) - \frac{x}{2}$$

```
[In] Int[(x^3*ArcTan[x])/(1 + x^2),x]
```

```
[Out] -1/2*x + ArcTan[x]/2 + (x^2*ArcTan[x])/2 + (I/2)*ArcTan[x]^2 + ArcTan[x]*Lo
g[2/(1 + I*x)] + (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
```

```
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x\arctan(x)\,dx - \int \frac{x\arctan(x)}{1+x^2}\,dx \\
&= \frac{1}{2}x^2\arctan(x) + \frac{1}{2}i\arctan(x)^2 - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx + \int \frac{\arctan(x)}{i-x}\,dx \\
&= -\frac{x}{2} + \frac{1}{2}x^2\arctan(x) + \frac{1}{2}i\arctan(x)^2 \\
&\quad + \arctan(x)\log\left(\frac{2}{1+ix}\right) + \frac{1}{2}\int \frac{1}{1+x^2}\,dx - \int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx \\
&= -\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{1}{2}x^2\arctan(x) + \frac{1}{2}i\arctan(x)^2 \\
&\quad + \arctan(x)\log\left(\frac{2}{1+ix}\right) + i\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+ix}\right) \\
&= -\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{1}{2}x^2\arctan(x) + \frac{1}{2}i\arctan(x)^2 \\
&\quad + \arctan(x)\log\left(\frac{2}{1+ix}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2, 1-\frac{2}{1+ix}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{x^3\arctan(x)}{1+x^2}\,dx = \frac{1}{2}\left(-x + i\arctan(x)^2 + \arctan(x)\left(1+x^2+2\log\left(-\frac{2i}{-i+x}\right)\right) + i\,\text{PolyLog}\left(2, \frac{i+x}{-i+x}\right)\right)$$

```
[In] Integrate[(x^3*ArcTan[x])/(1 + x^2),x]
[Out] (-x + I*ArcTan[x]^2 + ArcTan[x]*(1 + x^2 + 2*Log[(-2*I)/(-I + x)]) + I*Poly
Log[2, (I + x)/(-I + x)])/2
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(53) = 106.

Time = 0.44 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.88

method	result
default	$\frac{x^{2}\arctan\left(x\right)}{2}-\frac{\arctan\left(x\right)\ln\left(x^{2}+1\right)}{2}-\frac{x}{2}+\frac{\arctan\left(x\right)}{2}-\frac{i\left(\ln\left(x-i\right)\ln\left(x^{2}+1\right)-\frac{\ln\left(x-i\right)^{2}}{2}-\operatorname{dilog}\left(-\frac{i\left(x+i\right)}{2}\right)-\ln\left(x-i\right)\ln\left(-\frac{i\left(x+i\right)}{2}\right.\right.}{4}$
parts	$\frac{x^{2}\arctan\left(x\right)}{2}-\frac{\arctan\left(x\right)\ln\left(x^{2}+1\right)}{2}-\frac{x}{2}+\frac{\arctan\left(x\right)}{2}-\frac{i\left(\ln\left(x-i\right)\ln\left(x^{2}+1\right)-\frac{\ln\left(x-i\right)^{2}}{2}-\operatorname{dilog}\left(-\frac{i\left(x+i\right)}{2}\right)-\ln\left(x-i\right)\ln\left(-\frac{i\left(x+i\right)}{2}\right.\right.}{4}$
risch	$-\frac{i\ln\left(ix+1\right)}{4}-\frac{i\ln\left(-ix+1\right)^{2}}{8}+\frac{i\operatorname{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4}-\frac{x}{2}+\frac{i\ln\left(\frac{1}{2}-\frac{ix}{2}\right)\ln\left(ix+1\right)}{4}-\frac{i\ln\left(\frac{1}{2}+\frac{ix}{2}\right)\ln\left(-ix+1\right)}{4}+\frac{i\ln\left(ix+1\right)^{2}}{8}+\frac{ix^{2}}{}$

```
[In] int(x^3*arctan(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arctan(x)-1/2*arctan(x)*ln(x^2+1)-1/2*x+1/2*arctan(x)-1/4*I*(ln(x-I
)*ln(x^2+1)-1/2*ln(x-I)^2-dilog(-1/2*I*(x+I))-ln(x-I)*ln(-1/2*I*(x+I)))+1/4
*I*(ln(x+I)*ln(x^2+1)-1/2*ln(x+I)^2-dilog(1/2*I*(x-I))-ln(x+I)*ln(1/2*I*(x-
I)))
```

Fricas [F]

$$\int \frac{x^3 \arctan(x)}{1+x^2}\, dx = \int \frac{x^{3} \arctan\left(x\right)}{x^{2} + 1}\, dx$$

```
[In] integrate(x^3*arctan(x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(x)/(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{x^3 \arctan(x)}{1+x^2}\, dx = \int \frac{x^{3} \operatorname{atan}{\left(x \right)}}{x^{2} + 1}\, dx$$

```
[In] integrate(x**3*atan(x)/(x**2+1),x)
```

```
[Out] Integral(x**3*atan(x)/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{x^3 \arctan(x)}{1+x^2} \, dx = \int \frac{x^3 \arctan{(x)}}{x^2 + 1} \, dx$$

```
[In] integrate(x^3*arctan(x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(x)/(x^2 + 1), x)
```

Giac [F]

$$\int \frac{x^3 \arctan(x)}{1+x^2} \, dx = \int \frac{x^3 \arctan{(x)}}{x^2 + 1} \, dx$$

```
[In] integrate(x^3*arctan(x)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(x)/(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(x)}{1+x^2} \, dx = \int \frac{x^3 \operatorname{atan}(x)}{x^2 + 1} \, dx$$

```
[In] int((x^3*atan(x))/(x^2 + 1),x)
```

```
[Out] int((x^3*atan(x))/(x^2 + 1), x)
```

3.673 $\int \frac{x^2 \arctan(x)}{(1+x^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{x^2 \arctan(x)}{(1+x^2)^2} dx = -\frac{1}{4(1+x^2)} - \frac{x \arctan(x)}{2(1+x^2)} + \frac{\arctan(x)^2}{4}$$

```
[Out] -1/4/(x^2+1)-1/2*x*arctan(x)/(x^2+1)+1/4*arctan(x)^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5054, 5004}

$$\int \frac{x^2 \arctan(x)}{(1+x^2)^2} dx = -\frac{x \arctan(x)}{2(x^2+1)} + \frac{\arctan(x)^2}{4} - \frac{1}{4(x^2+1)}$$

```
[In] Int[(x^2*ArcTan[x])/(1 + x^2)^2,x]
[Out] -1/4*1/(1 + x^2) - (x*ArcTan[x])/(2*(1 + x^2)) + ArcTan[x]^2/4
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5054

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (-Dis
```

```
t[1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]
+ Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{4(1+x^2)} - \frac{x\arctan(x)}{2(1+x^2)} + \frac{1}{2}\int \frac{\arctan(x)}{1+x^2}\,dx \\ &= -\frac{1}{4(1+x^2)} - \frac{x\arctan(x)}{2(1+x^2)} + \frac{\arctan(x)^2}{4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{x^2\arctan(x)}{(1+x^2)^2}\,dx = \frac{-1-2x\arctan(x)+(1+x^2)\arctan(x)^2}{4(1+x^2)}$$

```
[In] Integrate[(x^2*ArcTan[x])/(1 + x^2)^2,x]
[Out] (-1 - 2*x*ArcTan[x] + (1 + x^2)*ArcTan[x]^2)/(4*(1 + x^2))
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{1}{4(x^2+1)}-\frac{x\arctan(x)}{2(x^2+1)}+\frac{\arctan(x)^2}{4}$	29
parts	$-\frac{1}{4(x^2+1)}-\frac{x\arctan(x)}{2(x^2+1)}+\frac{\arctan(x)^2}{4}$	29
risch	$-\frac{\ln(ix+1)^2}{16}+\frac{(x^2\ln(-ix+1)+\ln(-ix+1)+2ix)\ln(ix+1)}{8x^2+8}-\frac{x^2\ln(-ix+1)^2+\ln(-ix+1)^2+4ix\ln(-ix+1)+4}{16(x+i)(x-i)}$	101

```
[In] int(x^2*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] -1/4/(x^2+1)-1/2*x*arctan(x)/(x^2+1)+1/4*arctan(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{x^2 \arctan(x)}{(1+x^2)^2}\, dx = \frac{(x^2+1)\arctan(x)^2 - 2\,x\arctan(x) - 1}{4\,(x^2+1)}$$

```
[In] integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/4*((x^2 + 1)*arctan(x)^2 - 2*x*arctan(x) - 1)/(x^2 + 1)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(x)}{(1+x^2)^2}\, dx = \text{Exception raised: RecursionError}$$

```
[In] integrate(x**2*atan(x)/(x**2+1)**2,x)
[Out] Exception raised: RecursionError >> maximum recursion depth exceeded
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

$$\int \frac{x^2 \arctan(x)}{(1+x^2)^2}\, dx = -\frac{1}{2}\left(\frac{x}{x^2+1} - \arctan(x)\right)\arctan(x) - \frac{(x^2+1)\arctan(x)^2 + 1}{4\,(x^2+1)}$$

```
[In] integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2*(x/(x^2 + 1) - arctan(x))*arctan(x) - 1/4*((x^2 + 1)*arctan(x)^2 + 1)/
(x^2 + 1)
```

Giac [F]

$$\int \frac{x^2 \arctan(x)}{(1+x^2)^2} \, dx = \int \frac{x^2 \arctan(x)}{(x^2+1)^2} \, dx$$

[In] integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="giac")

[Out] integrate(x^2*arctan(x)/(x^2 + 1)^2, x)

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{x^2 \arctan(x)}{(1+x^2)^2} \, dx = \frac{\operatorname{atan}(x)^2}{4} - \frac{\frac{x \operatorname{atan}(x)}{2} + \frac{1}{4}}{x^2+1}$$

[In] int((x^2*atan(x))/(x^2 + 1)^2,x)

[Out] atan(x)^2/4 - ((x*atan(x))/2 + 1/4)/(x^2 + 1)

3.674 $\int \frac{x^3 \arctan(x)}{(1+x^2)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 79

$$\int \frac{x^3 \arctan(x)}{(1+x^2)^2} \, dx = -\frac{x}{4(1+x^2)} - \frac{\arctan(x)}{4} + \frac{\arctan(x)}{2(1+x^2)} - \frac{1}{2} i \arctan(x)^2 - \arctan(x) \log\left(\frac{2}{1+ix}\right) - \frac{1}{2} i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+ix}\right)$$

```
[Out] -1/4*x/(x^2+1)-1/4*arctan(x)+1/2*arctan(x)/(x^2+1)-1/2*I*arctan(x)^2-arctan
(x)*ln(2/(1+I*x))-1/2*I*polylog(2,1-2/(1+I*x))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {5084, 5040, 4964, 2449, 2352, 5050, 205, 209}

$$\int \frac{x^3 \arctan(x)}{(1+x^2)^2} \, dx = \frac{\arctan(x)}{2(x^2+1)} - \frac{1}{2} i \arctan(x)^2 - \frac{\arctan(x)}{4} - \arctan(x) \log\left(\frac{2}{1+ix}\right) - \frac{1}{2} i \operatorname{PolyLog}\left(2, 1 - \frac{2}{ix+1}\right) - \frac{x}{4(x^2+1)}$$

```
[In] Int[(x^3*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] -1/4*x/(1 + x^2) - ArcTan[x]/4 + ArcTan[x]/(2*(1 + x^2)) - (I/2)*ArcTan[x]^
2 - ArcTan[x]*Log[2/(1 + I*x)] - (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
```

```
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{x\arctan(x)}{(1+x^2)^2}\,dx+\int \frac{x\arctan(x)}{1+x^2}\,dx\\
&= \frac{\arctan(x)}{2(1+x^2)}-\frac{1}{2}i\arctan(x)^2-\frac{1}{2}\int \frac{1}{(1+x^2)^2}\,dx-\int \frac{\arctan(x)}{i-x}\,dx\\
&= -\frac{x}{4(1+x^2)}+\frac{\arctan(x)}{2(1+x^2)}-\frac{1}{2}i\arctan(x)^2\\
&\quad -\arctan(x)\log\left(\frac{2}{1+ix}\right)-\frac{1}{4}\int \frac{1}{1+x^2}\,dx+\int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx\\
&= -\frac{x}{4(1+x^2)}-\frac{\arctan(x)}{4}+\frac{\arctan(x)}{2(1+x^2)}-\frac{1}{2}i\arctan(x)^2\\
&\quad -\arctan(x)\log\left(\frac{2}{1+ix}\right)-i\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+ix}\right)\\
&= -\frac{x}{4(1+x^2)}-\frac{\arctan(x)}{4}+\frac{\arctan(x)}{2(1+x^2)}-\frac{1}{2}i\arctan(x)^2\\
&\quad -\arctan(x)\log\left(\frac{2}{1+ix}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

$$\begin{aligned}
\int \frac{x^3\arctan(x)}{(1+x^2)^2}\,dx &= \frac{1}{2}i\arctan(x)^2+\frac{1}{4}\arctan(x)\cos(2\arctan(x))\\
&\quad -\arctan(x)\log\left(1+e^{2i\arctan(x)}\right)\\
&\quad +\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arctan(x)}\right)-\frac{1}{8}\sin(2\arctan(x))
\end{aligned}$$

```
[In] Integrate[(x^3*ArcTan[x])/(1 + x^2)^2,x]
[Out] (I/2)*ArcTan[x]^2 + (ArcTan[x]*Cos[2*ArcTan[x]])/4 - ArcTan[x]*Log[1 + E^((
2*I)*ArcTan[x])] + (I/2)*PolyLog[2, -E^((2*I)*ArcTan[x])] - Sin[2*ArcTan[x]
]/8
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(65) = 130$.

Time = 0.35 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.73

method	result
default	$\frac{\arctan(x)}{2x^2+2}+\frac{\arctan(x)\ln(x^2+1)}{2}-\frac{x}{4(x^2+1)}-\frac{\arctan(x)}{4}+\frac{i\left(\ln(x-i)\ln(x^2+1)-\frac{\ln(x-i)^2}{2}-\operatorname{dilog}\left(-\frac{i(x+i)}{2}\right)-\ln(x-i)\ln\left(-\frac{i(x-}{2}\right.\right.}{4}$
parts	$\frac{\arctan(x)}{2x^2+2}+\frac{\arctan(x)\ln(x^2+1)}{2}-\frac{x}{4(x^2+1)}-\frac{\arctan(x)}{4}+\frac{i\left(\ln(x-i)\ln(x^2+1)-\frac{\ln(x-i)^2}{2}-\operatorname{dilog}\left(-\frac{i(x+i)}{2}\right)-\ln(x-i)\ln\left(-\frac{i(x-}{2}\right.\right.}{4}$
risch	$\frac{i\ln(-ix+1)^2}{8}+\frac{i\operatorname{dilog}\left(\frac{1}{2}+\frac{ix}{2}\right)}{4}+\frac{i\ln(ix+1)}{16ix-16}+\frac{i}{-8ix+8}+\frac{i\ln\left(\frac{1}{2}+\frac{ix}{2}\right)\ln(-ix+1)}{4}-\frac{i\operatorname{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4}-\frac{i}{8(ix+1)}-\frac{\arctan(x)}{8}$

```
[In] int(x^3*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(x)/(x^2+1)+1/2*arctan(x)*ln(x^2+1)-1/4*x/(x^2+1)-1/4*arctan(x)+1
/4*I*(ln(x-I)*ln(x^2+1)-1/2*ln(x-I)^2-dilog(-1/2*I*(x+I))-ln(x-I)*ln(-1/2*I
*(x+I)))-1/4*I*(ln(x+I)*ln(x^2+1)-1/2*ln(x+I)^2-dilog(1/2*I*(x-I))-ln(x+I)*
ln(1/2*I*(x-I)))
```

Fricas [F]

$$\int \frac{x^3\arctan(x)}{(1+x^2)^2}\,dx=\int \frac{x^3\arctan(x)}{(x^2+1)^2}\,dx$$

```
[In] integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(x)/(x^4 + 2*x^2 + 1), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^3\arctan(x)}{(1+x^2)^2}\,dx=\text{Exception raised: RecursionError}$$

```
[In] integrate(x**3*atan(x)/(x**2+1)**2,x)
```

```
[Out] Exception raised: RecursionError >> maximum recursion depth exceeded while
calling a Python object
```

Maxima [F]

$$\int \frac{x^3 \arctan(x)}{(1+x^2)^2}\,dx = \int \frac{x^3 \arctan(x)}{(x^2+1)^2}\,dx$$

```
[In] integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
[Out] integrate(x^3*arctan(x)/(x^2 + 1)^2, x)
```

Giac [F]

$$\int \frac{x^3 \arctan(x)}{(1+x^2)^2}\,dx = \int \frac{x^3 \arctan(x)}{(x^2+1)^2}\,dx$$

```
[In] integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="giac")
[Out] integrate(x^3*arctan(x)/(x^2 + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(x)}{(1+x^2)^2}\,dx = \int \frac{x^3 \operatorname{atan}(x)}{(x^2+1)^2}\,dx$$

```
[In] int((x^3*atan(x))/(x^2 + 1)^2,x)
[Out] int((x^3*atan(x))/(x^2 + 1)^2, x)
```

3.675 $\int \frac{x^5 \arctan(x)}{(1+x^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 89

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2} dx = -\frac{x}{2} + \frac{x}{4(1+x^2)} + \frac{3\arctan(x)}{4} + \frac{1}{2}x^2 \arctan(x) - \frac{\arctan(x)}{2(1+x^2)} + i\arctan(x)^2 + 2\arctan(x)\log\left(\frac{2}{1+ix}\right) + i\,\text{PolyLog}\left(2, 1 - \frac{2}{1+ix}\right)$$

```
[Out] -1/2*x+1/4*x/(x^2+1)+3/4*arctan(x)+1/2*x^2*arctan(x)-1/2*arctan(x)/(x^2+1)+
I*arctan(x)^2+2*arctan(x)*ln(2/(1+I*x))+I*polylog(2,1-2/(1+I*x))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {5084, 5036, 4946, 327, 209, 5040, 4964, 2449, 2352, 5050, 205}

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2} dx = \frac{1}{2}x^2 \arctan(x) - \frac{\arctan(x)}{2(x^2+1)} + i\arctan(x)^2 + \frac{3\arctan(x)}{4} + 2\arctan(x)\log\left(\frac{2}{1+ix}\right) + i\,\text{PolyLog}\left(2, 1 - \frac{2}{ix+1}\right) + \frac{x}{4(x^2+1)} - \frac{x}{2}$$

```
[In] Int[(x^5*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] -1/2*x + x/(4*(1 + x^2)) + (3*ArcTan[x])/4 + (x^2*ArcTan[x])/2 - ArcTan[x]/
(2*(1 + x^2)) + I*ArcTan[x]^2 + 2*ArcTan[x]*Log[2/(1 + I*x)] + I*PolyLog[2,
 1 - 2/(1 + I*x)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
```

```
x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{x^3\arctan(x)}{(1+x^2)^2}\,dx+\int \frac{x^3\arctan(x)}{1+x^2}\,dx\\
&= \int x\arctan(x)\,dx+\int \frac{x\arctan(x)}{(1+x^2)^2}\,dx-2\int \frac{x\arctan(x)}{1+x^2}\,dx\\
&= \frac{1}{2}x^2\arctan(x)-\frac{\arctan(x)}{2(1+x^2)}+\frac{1}{2}\int \frac{1}{(1+x^2)^2}\,dx\\
&\quad -\frac{1}{2}\int \frac{x^2}{1+x^2}\,dx-2\left(-\frac{1}{2}i\arctan(x)^2-\int \frac{\arctan(x)}{i-x}\,dx\right)
\end{aligned}$$

$$= -\frac{x}{2} + \frac{x}{4(1+x^2)} + \frac{1}{2}x^2\arctan(x) - \frac{\arctan(x)}{2(1+x^2)} + \frac{1}{4}\int\frac{1}{1+x^2}\,dx + \frac{1}{2}\int\frac{1}{1+x^2}\,dx - 2\left(-\frac{1}{2}i\arctan(x)^2 - \arctan(x)\log\left(\frac{2}{1+ix}\right) + \int\frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx\right)$$

$$= -\frac{x}{2} + \frac{x}{4(1+x^2)} + \frac{3\arctan(x)}{4} + \frac{1}{2}x^2\arctan(x) - \frac{\arctan(x)}{2(1+x^2)} - 2\left(-\frac{1}{2}i\arctan(x)^2 - \arctan(x)\log\left(\frac{2}{1+ix}\right) - i\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+ix}\right)\right)$$

$$= -\frac{x}{2} + \frac{x}{4(1+x^2)} + \frac{3\arctan(x)}{4} + \frac{1}{2}x^2\arctan(x) - \frac{\arctan(x)}{2(1+x^2)} - 2\left(-\frac{1}{2}i\arctan(x)^2 - \arctan(x)\log\left(\frac{2}{1+ix}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2, 1-\frac{2}{1+ix}\right)\right)$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.79

$$\int\frac{x^5\arctan(x)}{(1+x^2)^2}\,dx = \frac{1}{8}\big(-4x + 4(1+x^2)\arctan(x) - 8i\arctan(x)^2 - 2\arctan(x)\cos(2\arctan(x)) + 16\arctan(x)\log\left(1+e^{2i\arctan(x)}\right) - 8i\,\text{PolyLog}\left(2, -e^{2i\arctan(x)}\right) + \sin(2\arctan(x))\big)$$

```
[In] Integrate[(x^5*ArcTan[x])/(1 + x^2)^2,x]
[Out] (-4*x + 4*(1 + x^2)*ArcTan[x] - (8*I)*ArcTan[x]^2 - 2*ArcTan[x]*Cos[2*ArcTa
n[x]] + 16*ArcTan[x]*Log[1 + E^((2*I)*ArcTan[x])] - (8*I)*PolyLog[2, -E^((2
*I)*ArcTan[x])] + Sin[2*ArcTan[x]])/8
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.65

method	result
default	$\frac{x^2\arctan(x)}{2} - \arctan(x)\ln(x^2+1) - \frac{\arctan(x)}{2(x^2+1)} - \frac{x}{2} + \frac{x}{4x^2+4} + \frac{3\arctan(x)}{4} - \frac{i\left(\ln(x-i)\ln(x^2+1) - \frac{\ln(x-i)^2}{2} - \text{d}\right.}{}$
parts	$\frac{x^2\arctan(x)}{2} - \arctan(x)\ln(x^2+1) - \frac{\arctan(x)}{2(x^2+1)} - \frac{x}{2} + \frac{x}{4x^2+4} + \frac{3\arctan(x)}{4} - \frac{i\left(\ln(x-i)\ln(x^2+1) - \frac{\ln(x-i)^2}{2} - \text{d}\right.}{}$
risch	$-\frac{x}{2} - \frac{i\ln(ix+1)}{4} - \frac{i\ln(-ix+1)^2}{4} + \frac{i\ln\left(\frac{1}{2}-\frac{ix}{2}\right)\ln(ix+1)}{2} + \frac{\arctan(x)}{8} + \frac{\ln(-ix+1)x}{-16ix-16} - \frac{i}{8(-ix+1)} + \frac{i\,\text{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{2} -$

```
[In] int(x^5*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arctan(x)-arctan(x)*ln(x^2+1)-1/2*arctan(x)/(x^2+1)-1/2*x+1/4*x/(x^
2+1)+3/4*arctan(x)-1/2*I*(ln(x-I)*ln(x^2+1)-1/2*ln(x-I)^2-dilog(-1/2*I*(x+I
))-ln(x-I)*ln(-1/2*I*(x+I)))+1/2*I*(ln(x+I)*ln(x^2+1)-1/2*ln(x+I)^2-dilog(1
/2*I*(x-I))-ln(x+I)*ln(1/2*I*(x-I)))
```

Fricas [F]

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2}\,dx = \int \frac{x^5 \arctan(x)}{(x^2+1)^2}\,dx$$

```
[In] integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(x^5*arctan(x)/(x^4 + 2*x^2 + 1), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2}\,dx = \text{Exception raised: RecursionError}$$

```
[In] integrate(x**5*atan(x)/(x**2+1)**2,x)
```

```
[Out] Exception raised: RecursionError >> maximum recursion depth exceeded in com
parison
```

Maxima [F]

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2}\,dx = \int \frac{x^5 \arctan(x)}{(x^2+1)^2}\,dx$$

```
[In] integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^5*arctan(x)/(x^2 + 1)^2, x)
```

Giac [F]

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2}\,dx = \int \frac{x^5 \arctan{(x)}}{(x^2+1)^2}\,dx$$

```
[In] integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="giac")
[Out] integrate(x^5*arctan(x)/(x^2 + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2}\,dx = \int \frac{x^5 \operatorname{atan}(x)}{(x^2+1)^2}\,dx$$

```
[In] int((x^5*atan(x))/(x^2 + 1)^2,x)
[Out] int((x^5*atan(x))/(x^2 + 1)^2, x)
```

3.676 $\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = -\frac{\arctan(x)}{x} + x\arctan(x) + \log(x) - \log\left(1+x^2\right)$$

```
[Out] -arctan(x)/x+x*arctan(x)+ln(x)-ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {5070, 4946, 272, 36, 29, 31, 4930, 266}

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = x\arctan(x) - \frac{\arctan(x)}{x} - \log\left(x^2+1\right) + \log(x)$$

```
[In] Int[((1 + x^2)*ArcTan[x])/x^2,x]
```

```
[Out] -(ArcTan[x]/x) + x*ArcTan[x] + Log[x] - Log[1 + x^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \arctan(x)\,dx + \int \frac{\arctan(x)}{x^2}\,dx\\ &= -\frac{\arctan(x)}{x} + x\arctan(x) + \int \frac{1}{x\,(1+x^2)}\,dx - \int \frac{x}{1+x^2}\,dx\end{aligned}$$

$$= -\frac{\arctan(x)}{x} + x\arctan(x) - \frac{1}{2}\log\left(1+x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^2\right)$$

$$= -\frac{\arctan(x)}{x} + x\arctan(x) - \frac{1}{2}\log\left(1+x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right)$$

$$= -\frac{\arctan(x)}{x} + x\arctan(x) + \log(x) - \log\left(1+x^2\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = -\frac{\arctan(x)}{x} + x\arctan(x) + \log(x) - \log\left(1+x^2\right)$$

```
[In] Integrate[((1 + x^2)*ArcTan[x])/x^2,x]
[Out] -(ArcTan[x]/x) + x*ArcTan[x] + Log[x] - Log[1 + x^2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
default	$-\frac{\arctan(x)}{x}+x\arctan\left(x\right)+\ln\left(x\right)-\ln\left(x^{2}+1\right)$	23
parts	$-\frac{\arctan(x)}{x}+x\arctan\left(x\right)+\ln\left(x\right)-\ln\left(x^{2}+1\right)$	23
parallelrisch	$\frac{x^{2}\arctan(x)+x\ln(x)-x\ln(x^{2}+1)-\arctan(x)}{x}$	29
meijerg	$\ln\left(x\right)-\frac{\arctan\left(\sqrt{x^{2}}\right)}{\sqrt{x^{2}}}-\ln\left(x^{2}+1\right)+\frac{x^{2}\arctan\left(\sqrt{x^{2}}\right)}{\sqrt{x^{2}}}$	40
risch	$-\frac{i(x^{2}-1)\ln(ix+1)}{2x}+\frac{i(-2i\ln(x)x+2i\ln(x^{2}+1)x+x^{2}\ln(-ix+1)-\ln(-ix+1))}{2x}$	63

```
[In] int((x^2+1)*arctan(x)/x^2,x,method=_RETURNVERBOSE)
[Out] -1/x*arctan(x)+x*arctan(x)+ln(x)-ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = \frac{(x^2-1)\arctan(x) - x\log(x^2+1) + x\log(x)}{x}$$

```
[In] integrate((x^2+1)*arctan(x)/x^2,x, algorithm="fricas")
[Out] ((x^2 - 1)*arctan(x) - x*log(x^2 + 1) + x*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = x\,\mathrm{atan}(x) + \log(x) - \log(x^2+1) - \frac{\mathrm{atan}(x)}{x}$$

```
[In] integrate((x**2+1)*atan(x)/x**2,x)
[Out] x*atan(x) + log(x) - log(x**2 + 1) - atan(x)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = \left(x - \frac{1}{x}\right)\arctan(x) - \log(x^2+1) + \log(x)$$

```
[In] integrate((x^2+1)*arctan(x)/x^2,x, algorithm="maxima")
[Out] (x - 1/x)*arctan(x) - log(x^2 + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = \left(x - \frac{1}{x}\right)\arctan(x) - \log(x^2+1) + \frac{1}{2}\log(x^2)$$

```
[In] integrate((x^2+1)*arctan(x)/x^2,x, algorithm="giac")
[Out] (x - 1/x)*arctan(x) - log(x^2 + 1) + 1/2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = \ln(x) - \ln(x^2+1) - \frac{\operatorname{atan}(x)}{x} + x\operatorname{atan}(x)$$

```
[In] int((atan(x)*(x^2 + 1))/x^2,x)
[Out] log(x) - log(x^2 + 1) - atan(x)/x + x*atan(x)
```

3.677 $\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{1}{12x^3}-\frac{1}{4x}-\frac{(1+x^2)^2\arctan(x)}{4x^4}$$

```
[Out] -1/12/x^3-1/4/x-1/4*(x^2+1)^2*arctan(x)/x^4
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5064, 14}

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{(x^2+1)^2\arctan(x)}{4x^4}-\frac{1}{12x^3}-\frac{1}{4x}$$

```
[In] Int[((1 + x^2)*ArcTan[x])/x^5,x]
```

```
[Out] -1/12*1/x^3 - 1/(4*x) - ((1 + x^2)^2*ArcTan[x])/(4*x^4)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
```

```
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(1+x^2)^2\arctan(x)}{4x^4}+\frac{1}{4}\int\frac{1+x^2}{x^4}\,dx\\
&= -\frac{(1+x^2)^2\arctan(x)}{4x^4}+\frac{1}{4}\int\left(\frac{1}{x^4}+\frac{1}{x^2}\right)\,dx\\
&= -\frac{1}{12x^3}-\frac{1}{4x}-\frac{(1+x^2)^2\arctan(x)}{4x^4}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$
\int\frac{(1+x^2)\arctan(x)}{x^5}\,dx=-\frac{\arctan(x)}{4x^4}-\frac{\arctan(x)}{2x^2}-\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-x^2\right)}{12x^3}-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-x^2\right)}{2x}
$$

```
[In] Integrate[((1 + x^2)*ArcTan[x])/x^5,x]
[Out] -1/4*ArcTan[x]/x^4 - ArcTan[x]/(2*x^2) - Hypergeometric2F1[-3/2, 1, -1/2, -
x^2]/(12*x^3) - Hypergeometric2F1[-1/2, 1, 1/2, -x^2]/(2*x)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\arctan(x)}{2x^2}-\frac{\arctan(x)}{4x^4}-\frac{\arctan(x)}{4}-\frac{1}{12x^3}-\frac{1}{4x}$	30
parts	$-\frac{\arctan(x)}{2x^2}-\frac{\arctan(x)}{4x^4}-\frac{\arctan(x)}{4}-\frac{1}{12x^3}-\frac{1}{4x}$	30
parallelrisch	$-\frac{3\arctan(x)x^4+3x^3+6x^2\arctan(x)+x+3\arctan(x)}{12x^4}$	31
meijerg	$-\frac{1}{12x^3}-\frac{1}{4x}-\frac{2\left(-\frac{3x^4}{8}+\frac{3}{8}\right)\arctan\left(\sqrt{x^2}\right)}{3x^3\sqrt{x^2}}-\frac{(x^2+1)\arctan(x)}{2x^2}$	47
risch	$\frac{i(2x^2+1)\ln(ix+1)}{8x^4}+\frac{i(3\ln(x-i)x^4-3\ln(x+i)x^4+6ix^3-6x^2\ln(-ix+1)+2ix-3\ln(-ix+1))}{24x^4}$	80

```
[In] int((x^2+1)*arctan(x)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/2*arctan(x)/x^2-1/4*arctan(x)/x^4-1/4*arctan(x)-1/12/x^3-1/4/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{3\,x^3+3\,(x^4+2\,x^2+1)\arctan{(x)}+x}{12\,x^4}$$

```
[In] integrate((x^2+1)*arctan(x)/x^5,x, algorithm="fricas")
[Out] -1/12*(3*x^3 + 3*(x^4 + 2*x^2 + 1)*arctan(x) + x)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{\operatorname{atan}{(x)}}{4} - \frac{1}{4x} - \frac{\operatorname{atan}{(x)}}{2x^2} - \frac{1}{12x^3} - \frac{\operatorname{atan}{(x)}}{4x^4}$$

```
[In] integrate((x**2+1)*atan(x)/x**5,x)
[Out] -atan(x)/4 - 1/(4*x) - atan(x)/(2*x**2) - 1/(12*x**3) - atan(x)/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{3\,x^2+1}{12\,x^3} - \frac{(2\,x^2+1)\arctan{(x)}}{4\,x^4} - \frac{1}{4}\,\arctan{(x)}$$

```
[In] integrate((x^2+1)*arctan(x)/x^5,x, algorithm="maxima")
[Out] -1/12*(3*x^2 + 1)/x^3 - 1/4*(2*x^2 + 1)*arctan(x)/x^4 - 1/4*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{3\,x^2+1}{12\,x^3} - \frac{(2\,x^2+1)\arctan(x)}{4\,x^4} - \frac{1}{4}\arctan(x)$$

```
[In] integrate((x^2+1)*arctan(x)/x^5,x, algorithm="giac")
[Out] -1/12*(3*x^2 + 1)/x^3 - 1/4*(2*x^2 + 1)*arctan(x)/x^4 - 1/4*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{\operatorname{atan}(x)}{4} - \frac{\frac{x}{12}+\frac{\operatorname{atan}(x)}{4}+\frac{x^2\operatorname{atan}(x)}{2}+\frac{x^3}{4}}{x^4}$$

```
[In] int((atan(x)*(x^2 + 1))/x^5,x)
[Out] - atan(x)/4 - (x/12 + atan(x)/4 + (x^2*atan(x))/2 + x^3/4)/x^4
```

3.678 $\int \frac{(1+x^2)^2 \arctan(x)}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5} dx = -\frac{1}{12x^3} - \frac{3}{4x} - \frac{3\arctan(x)}{4} - \frac{\arctan(x)}{4x^4} - \frac{\arctan(x)}{x^2} + \frac{1}{2} i \operatorname{PolyLog}(2,-ix) - \frac{1}{2} i \operatorname{PolyLog}(2,ix)$$

```
[Out] -1/12/x^3-3/4/x-3/4*arctan(x)-1/4*arctan(x)/x^4-arctan(x)/x^2+1/2*I*polylog
(2,-I*x)-1/2*I*polylog(2,I*x)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5068, 4946, 331, 209, 4940, 2438}

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5} dx = -\frac{\arctan(x)}{4x^4} - \frac{\arctan(x)}{x^2} - \frac{3\arctan(x)}{4} + \frac{1}{2} i \operatorname{PolyLog}(2,-ix) - \frac{1}{2} i \operatorname{PolyLog}(2,ix) - \frac{1}{12x^3} - \frac{3}{4x}$$

```
[In] Int[((1 + x^2)^2*ArcTan[x])/x^5,x]
```

```
[Out] -1/12*1/x^3 - 3/(4*x) - (3*ArcTan[x])/4 - ArcTan[x]/(4*x^4) - ArcTan[x]/x^2
 + (I/2)*PolyLog[2, (-I)*x] - (I/2)*PolyLog[2, I*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\arctan(x)}{x^5}+\frac{2\arctan(x)}{x^3}+\frac{\arctan(x)}{x}\right)dx\\
&= 2\int \frac{\arctan(x)}{x^3}\,dx+\int \frac{\arctan(x)}{x^5}\,dx+\int \frac{\arctan(x)}{x}\,dx\\
&= -\frac{\arctan(x)}{4x^4}-\frac{\arctan(x)}{x^2}+\frac{1}{2}i\int \frac{\log(1-ix)}{x}\,dx\\
&\quad -\frac{1}{2}i\int \frac{\log(1+ix)}{x}\,dx+\frac{1}{4}\int \frac{1}{x^4\,(1+x^2)}\,dx+\int \frac{1}{x^2\,(1+x^2)}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{12x^3} - \frac{1}{x} - \frac{\arctan(x)}{4x^4} - \frac{\arctan(x)}{x^2} + \frac{1}{2} i \operatorname{PolyLog}(2,-ix) \\
&\quad - \frac{1}{2} i \operatorname{PolyLog}(2,ix) - \frac{1}{4}\int \frac{1}{x^2(1+x^2)}\,dx - \int \frac{1}{1+x^2}\,dx \\
&= -\frac{1}{12x^3} - \frac{3}{4x} - \arctan(x) - \frac{\arctan(x)}{4x^4} - \frac{\arctan(x)}{x^2} \\
&\quad + \frac{1}{2} i \operatorname{PolyLog}(2,-ix) - \frac{1}{2} i \operatorname{PolyLog}(2,ix) + \frac{1}{4}\int \frac{1}{1+x^2}\,dx \\
&= -\frac{1}{12x^3} - \frac{3}{4x} - \frac{3\arctan(x)}{4} - \frac{\arctan(x)}{4x^4} - \frac{\arctan(x)}{x^2} \\
&\quad + \frac{1}{2} i \operatorname{PolyLog}(2,-ix) - \frac{1}{2} i \operatorname{PolyLog}(2,ix)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.29

$$
\begin{aligned}
\int \frac{(1+x^2)^2 \arctan(x)}{x^5}\,dx &= -\frac{\arctan(x)}{4x^4} - \frac{\arctan(x)}{x^2} \\
&\quad - \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-x^2\right)}{12x^3} \\
&\quad - \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-x^2\right)}{x} \\
&\quad + \frac{1}{2} i \operatorname{PolyLog}(2,-ix) - \frac{1}{2} i \operatorname{PolyLog}(2,ix)
\end{aligned}
$$

```
[In] Integrate[((1 + x^2)^2*ArcTan[x])/x^5,x]
```

```
[Out] -1/4*ArcTan[x]/x^4 - ArcTan[x]/x^2 - Hypergeometric2F1[-3/2, 1, -1/2, -x^2]
/(12*x^3) - Hypergeometric2F1[-1/2, 1, 1/2, -x^2]/x + (I/2)*PolyLog[2, (-I)
*x] - (I/2)*PolyLog[2, I*x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.25

method	result
default	$\arctan(x)\ln(x)-\frac{\arctan(x)}{x^2}-\frac{\arctan(x)}{4x^4}+\frac{i\ln(x)\ln(ix+1)}{2}-\frac{i\ln(x)\ln(-ix+1)}{2}+\frac{i\,\mathrm{dilog}(ix+1)}{2}-\frac{i\,\mathrm{dilog}(-ix+1)}{2}-$
parts	$\arctan(x)\ln(x)-\frac{\arctan(x)}{x^2}-\frac{\arctan(x)}{4x^4}+\frac{i\ln(x)\ln(ix+1)}{2}-\frac{i\ln(x)\ln(-ix+1)}{2}+\frac{i\,\mathrm{dilog}(ix+1)}{2}-\frac{i\,\mathrm{dilog}(-ix+1)}{2}-$
meijerg	$-\frac{1}{12x^3}-\frac{3}{4x}-\frac{2\left(-\frac{3x^4}{8}+\frac{3}{8}\right)\arctan\left(\sqrt{x^2}\right)}{3x^3\sqrt{x^2}}-\frac{ix\,\mathrm{Li}_2\left(i\sqrt{x^2}\right)}{2\sqrt{x^2}}+\frac{ix\,\mathrm{Li}_2\left(-i\sqrt{x^2}\right)}{2\sqrt{x^2}}-\frac{(x^2+1)\arctan(x)}{x^2}$
risch	$-\frac{3}{4x}-\frac{1}{12x^3}+\frac{3i\ln(-ix)}{8}-\frac{3\arctan(x)}{4}-\frac{i\ln(-ix+1)}{8x^4}-\frac{i\,\mathrm{dilog}(-ix+1)}{2}-\frac{i\ln(-ix+1)}{2x^2}-\frac{3i\ln(ix)}{8}+\frac{i\ln(ix+1)}{8x^4}+\frac{i\,\mathrm{di}}{}$

[In] int((x^2+1)^2*arctan(x)/x^5,x,method=_RETURNVERBOSE)

[Out] arctan(x)*ln(x)-arctan(x)/x^2-1/4*arctan(x)/x^4+1/2*I*ln(x)*ln(1+I*x)-1/2*I*ln(x)*ln(1-I*x)+1/2*I*dilog(1+I*x)-1/2*I*dilog(1-I*x)-1/12/x^3-3/4/x-3/4*arctan(x)

Fricas [F]

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5}\,dx = \int \frac{(x^2+1)^2 \arctan(x)}{x^5}\,dx$$

[In] integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="fricas")

[Out] integral((x^4 + 2*x^2 + 1)*arctan(x)/x^5, x)

Sympy [F]

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5}\,dx = \int \frac{(x^2+1)^2 \operatorname{atan}(x)}{x^5}\,dx$$

[In] integrate((x**2+1)**2*atan(x)/x**5,x)

[Out] Integral((x**2 + 1)**2*atan(x)/x**5, x)

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5}\,dx = -\frac{3\pi x^4\log(x^2+1)-12x^4\arctan(x)\log(x)+6i\,x^4\mathrm{Li}_2(i\,x+1)-6i\,x^4\mathrm{Li}_2(-i\,x+1)+9x^3+3(3x^4+4}{12x^4}$$

```
[In] integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*pi*x^4*log(x^2 + 1) - 12*x^4*arctan(x)*log(x) + 6*I*x^4*dilog(I*x
+ 1) - 6*I*x^4*dilog(-I*x + 1) + 9*x^3 + 3*(3*x^4 + 4*x^2 + 1)*arctan(x) +
x)/x^4
```

Giac [F]

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5} \, dx = \int \frac{(x^2+1)^2 \arctan(x)}{x^5} \, dx$$

```
[In] integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^2*arctan(x)/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5} \, dx = \frac{x^2-\frac{1}{3}}{4\,x^3} - \frac{\mathrm{atan}(x)}{x^2} - \frac{\mathrm{atan}(x)}{4\,x^4} - \frac{3\,\mathrm{atan}(x)}{4} - \frac{1}{x} - \frac{\mathrm{Li}_2(1-x\,1\mathrm{i})\,\,1\mathrm{i}}{2} + \frac{\mathrm{polylog}(2,-x\,1\mathrm{i})\,\,1\mathrm{i}}{2}$$

```
[In] int((atan(x)*(x^2 + 1)^2)/x^5,x)
```

```
[Out] (polylog(2, -x*1i)*1i)/2 - (3*atan(x))/4 - atan(x)/x^2 - atan(x)/(4*x^4) -
(dilog(1 - x*1i)*1i)/2 + (x^2 - 1/3)/(4*x^3) - 1/x
```

3.679 $\int \frac{\arctan(x)}{x^2(1+x^2)} dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{\arctan(x)}{x^2(1+x^2)} dx = -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} + \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -arctan(x)/x-1/2*arctan(x)^2+ln(x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5038, 4946, 272, 36, 29, 31, 5004}

$$\int \frac{\arctan(x)}{x^2(1+x^2)} dx = -\frac{1}{2}\arctan(x)^2 - \frac{\arctan(x)}{x} - \frac{1}{2}\log\left(x^2+1\right) + \log(x)$$

```
[In] Int[ArcTan[x]/(x^2*(1 + x^2)),x]
```

```
[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 + Log[x] - Log[1 + x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\arctan(x)}{x^2}\,dx - \int \frac{\arctan(x)}{1+x^2}\,dx \\
&= -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} + \int \frac{1}{x\,(1+x^2)}\,dx \\
&= -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^2\right) \\
&= -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) \\
&= -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} + \log(x) - \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx = -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} + \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[ArcTan[x]/(x^2*(1 + x^2)),x]
[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 + Log[x] - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\arctan(x)}{x}-\frac{\arctan(x)^2}{2}+\ln(x)-\frac{\ln(x^2+1)}{2}$	25
parts	$-\frac{\arctan(x)}{x}-\frac{\arctan(x)^2}{2}+\ln(x)-\frac{\ln(x^2+1)}{2}$	25
parallelrisch	$\frac{-x\arctan(x)^2+2x\ln(x)-x\ln(x^2+1)-2\arctan(x)}{2x}$	32
risch	$\frac{\ln(ix+1)^2}{8}-\frac{(\ln(-ix+1)x-2i)\ln(ix+1)}{4x}-\frac{-x\ln(-ix+1)^2-8x\ln(x)+4x\ln(x^2+1)+4i\ln(-ix+1)}{8x}$	79

```
[In] int(arctan(x)/x^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/x*arctan(x)-1/2*arctan(x)^2+ln(x)-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx = -\frac{x\arctan(x)^2 + x\log(x^2+1) - 2\,x\log(x) + 2\,\arctan(x)}{2\,x}$$

```
[In] integrate(arctan(x)/x^2/(x^2+1),x, algorithm="fricas")
[Out] -1/2*(x*arctan(x)^2 + x*log(x^2 + 1) - 2*x*log(x) + 2*arctan(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(x)}{x^2 (1+x^2)} \, dx = \log(x) - \frac{\log(x^2+1)}{2} - \frac{\operatorname{atan}^2(x)}{2} - \frac{\operatorname{atan}(x)}{x}$$

```
[In] integrate(atan(x)/x**2/(x**2+1),x)
[Out] log(x) - log(x**2 + 1)/2 - atan(x)**2/2 - atan(x)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\arctan(x)}{x^2 (1+x^2)} \, dx = -\left(\frac{1}{x} + \arctan(x)\right)\arctan(x) + \frac{1}{2}\arctan(x)^2 - \frac{1}{2}\log\left(x^2+1\right) + \log(x)$$

```
[In] integrate(arctan(x)/x^2/(x^2+1),x, algorithm="maxima")
[Out] -(1/x + arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1) + log(x)
```

Giac [F]

$$\int \frac{\arctan(x)}{x^2 (1+x^2)} \, dx = \int \frac{\arctan(x)}{(x^2+1)x^2} \, dx$$

```
[In] integrate(arctan(x)/x^2/(x^2+1),x, algorithm="giac")
[Out] integrate(arctan(x)/((x^2 + 1)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\arctan(x)}{x^2 (1+x^2)} \, dx = \ln(x) - \frac{\ln(x^2+1)}{2} - \frac{\operatorname{atan}(x)}{x} - \frac{\operatorname{atan}(x)^2}{2}$$

```
[In] int(atan(x)/(x^2*(x^2 + 1)),x)
[Out] log(x) - log(x^2 + 1)/2 - atan(x)/x - atan(x)^2/2
```

3.680 $\int \frac{\arctan(x)^2}{x^3} dx$

Optimal result

Integrand size = 8, antiderivative size = 39

$$\int \frac{\arctan(x)^2}{x^3} dx = -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} - \frac{\arctan(x)^2}{2x^2} + \log(x) - \frac{1}{2}\log(1+x^2)$$

```
[Out] -arctan(x)/x-1/2*arctan(x)^2-1/2*arctan(x)^2/x^2+ln(x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {4946, 5038, 272, 36, 29, 31, 5004}

$$\int \frac{\arctan(x)^2}{x^3} dx = -\frac{\arctan(x)^2}{2x^2} - \frac{\arctan(x)^2}{2} - \frac{\arctan(x)}{x} - \frac{1}{2}\log(x^2+1) + \log(x)$$

```
[In] Int[ArcTan[x]^2/x^3,x]
```

```
[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 - ArcTan[x]^2/(2*x^2) + Log[x] - Log[1 + x^2
]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arctan(x)^2}{2x^2} + \int \frac{\arctan(x)}{x^2(1+x^2)}\,dx \\
&= -\frac{\arctan(x)^2}{2x^2} + \int \frac{\arctan(x)}{x^2}\,dx - \int \frac{\arctan(x)}{1+x^2}\,dx \\
&= -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} - \frac{\arctan(x)^2}{2x^2} + \int \frac{1}{x(1+x^2)}\,dx \\
&= -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} - \frac{\arctan(x)^2}{2x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^2\right)
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} - \frac{\arctan(x)^2}{2x^2} \\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) \\
&= -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} - \frac{\arctan(x)^2}{2x^2} + \log(x) - \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$
\int \frac{\arctan(x)^2}{x^3}\,dx = -\frac{\arctan(x)}{x} + \frac{(-1-x^2)\arctan(x)^2}{2x^2} + \log(x) - \frac{1}{2}\log\left(1+x^2\right)
$$

```
[In] Integrate[ArcTan[x]^2/x^3,x]
[Out] -(ArcTan[x]/x) + ((-1 - x^2)*ArcTan[x]^2)/(2*x^2) + Log[x] - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result
default	$-\frac{\arctan(x)}{x}-\frac{\arctan(x)^2}{2}-\frac{\arctan(x)^2}{2x^2}+\ln(x)-\frac{\ln(x^2+1)}{2}$
parts	$-\frac{\arctan(x)}{x}-\frac{\arctan(x)^2}{2}-\frac{\arctan(x)^2}{2x^2}+\ln(x)-\frac{\ln(x^2+1)}{2}$
parallelrisch	$\frac{-x^2\arctan(x)^2+2x^2\ln(x)-\ln(x^2+1)x^2-2x\arctan(x)-\arctan(x)^2}{2x^2}$
risch	$\frac{(x^2+1)\ln(ix+1)^2}{8x^2}-\frac{(x^2\ln(-ix+1)-2ix+\ln(-ix+1))\ln(ix+1)}{4x^2}+\frac{x^2\ln(-ix+1)^2-4ix\ln(-ix+1)+8x^2\ln(x)-4\ln(x^2+1)x^2}{8x^2}$

```
[In] int(arctan(x)^2/x^3,x,method=_RETURNVERBOSE)
[Out] -1/x*arctan(x)-1/2*arctan(x)^2-1/2*arctan(x)^2/x^2+ln(x)-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{\arctan(x)^2}{x^3}\,dx = -\frac{(x^2+1)\arctan(x)^2 + x^2\log(x^2+1) - 2\,x^2\log(x) + 2\,x\arctan(x)}{2\,x^2}$$

```
[In] integrate(arctan(x)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 + 1)*arctan(x)^2 + x^2*log(x^2 + 1) - 2*x^2*log(x) + 2*x*arctan(
x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{\arctan(x)^2}{x^3}\,dx = \log(x) - \frac{\log(x^2+1)}{2} - \frac{\operatorname{atan}^2(x)}{2} - \frac{\operatorname{atan}(x)}{x} - \frac{\operatorname{atan}^2(x)}{2x^2}$$

```
[In] integrate(atan(x)**2/x**3,x)
```

```
[Out] log(x) - log(x**2 + 1)/2 - atan(x)**2/2 - atan(x)/x - atan(x)**2/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(x)^2}{x^3}\,dx = -\left(\frac{1}{x} + \arctan(x)\right)\arctan(x) + \frac{1}{2}\arctan(x)^2 - \frac{\arctan(x)^2}{2\,x^2} - \frac{1}{2}\log\left(x^2+1\right) + \log(x)$$

```
[In] integrate(arctan(x)^2/x^3,x, algorithm="maxima")
```

```
[Out] -(1/x + arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*arctan(x)^2/x^2 - 1/2*
log(x^2 + 1) + log(x)
```

Giac [F]

$$\int \frac{\arctan(x)^2}{x^3}\,dx = \int \frac{\arctan(x)^2}{x^3}\,dx$$

```
[In] integrate(arctan(x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)^2/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(x)^2}{x^3}\,dx = \ln(x) - \frac{\ln(x^2+1)}{2} - \frac{\operatorname{atan}(x)}{x} - \operatorname{atan}(x)^2\left(\frac{1}{2\,x^2} + \frac{1}{2}\right)$$

```
[In] int(atan(x)^2/x^3,x)
```

```
[Out] log(x) - log(x^2 + 1)/2 - atan(x)/x - atan(x)^2*(1/(2*x^2) + 1/2)
```

3.681 $\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = -\frac{1}{12x^2}-\frac{\arctan(x)}{6x^3}-\frac{\arctan(x)}{2x}-\frac{(1+x^2)^2\arctan(x)^2}{4x^4}+\frac{\log(x)}{3}-\frac{1}{6}\log\left(1+x^2\right)$$

```
[Out] -1/12/x^2-1/6*arctan(x)/x^3-1/2*arctan(x)/x-1/4*(x^2+1)^2*arctan(x)^2/x^4+1
/3*ln(x)-1/6*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5064, 5070, 4946, 272, 46, 36, 29, 31}

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = -\frac{\arctan(x)}{6x^3}-\frac{(x^2+1)^2\arctan(x)^2}{4x^4}-\frac{\arctan(x)}{2x}-\frac{1}{12x^2}-\frac{1}{6}\log\left(x^2+1\right)+\frac{\log(x)}{3}$$

```
[In] Int[((1 + x^2)*ArcTan[x]^2)/x^5,x]
```

```
[Out] -1/12*1/x^2 - ArcTan[x]/(6*x^3) - ArcTan[x]/(2*x) - ((1 + x^2)^2*ArcTan[x]^
2)/(4*x^4) + Log[x]/3 - Log[1 + x^2]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
```

```
EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
IntegerQ[q]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(1+x^2)^2\arctan(x)^2}{4x^4}+\frac{1}{2}\int\frac{(1+x^2)\arctan(x)}{x^4}\,dx\\
&= -\frac{(1+x^2)^2\arctan(x)^2}{4x^4}+\frac{1}{2}\int\frac{\arctan(x)}{x^4}\,dx+\frac{1}{2}\int\frac{\arctan(x)}{x^2}\,dx\\
&= -\frac{\arctan(x)}{6x^3}-\frac{\arctan(x)}{2x}-\frac{(1+x^2)^2\arctan(x)^2}{4x^4}+\frac{1}{6}\int\frac{1}{x^3(1+x^2)}\,dx+\frac{1}{2}\int\frac{1}{x(1+x^2)}\,dx\\
&= -\frac{\arctan(x)}{6x^3}-\frac{\arctan(x)}{2x}-\frac{(1+x^2)^2\arctan(x)^2}{4x^4}\\
&\quad+\frac{1}{12}\text{Subst}\left(\int\frac{1}{x^2(1+x)}\,dx,x,x^2\right)+\frac{1}{4}\text{Subst}\left(\int\frac{1}{x(1+x)}\,dx,x,x^2\right)\\
&= -\frac{\arctan(x)}{6x^3}-\frac{\arctan(x)}{2x}-\frac{(1+x^2)^2\arctan(x)^2}{4x^4}\\
&\quad+\frac{1}{12}\text{Subst}\left(\int\left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx,x,x^2\right)\\
&\quad+\frac{1}{4}\text{Subst}\left(\int\frac{1}{x}\,dx,x,x^2\right)-\frac{1}{4}\text{Subst}\left(\int\frac{1}{1+x}\,dx,x,x^2\right)\\
&= -\frac{1}{12x^2}-\frac{\arctan(x)}{6x^3}-\frac{\arctan(x)}{2x}-\frac{(1+x^2)^2\arctan(x)^2}{4x^4}+\frac{\log(x)}{3}-\frac{1}{6}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$
\int\frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = \frac{-2(x+3x^3)\arctan(x)-3(1+x^2)^2\arctan(x)^2+x^2(-1+4x^2\log(x)-2x^2\log\left(1+x^2\right))}{12x^4}
$$

```
[In] Integrate[((1 + x^2)*ArcTan[x]^2)/x^5,x]
[Out] (-2*(x + 3*x^3)*ArcTan[x] - 3*(1 + x^2)^2*ArcTan[x]^2 + x^2*(-1 + 4*x^2*Log
[x] - 2*x^2*Log[1 + x^2]))/(12*x^4)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

method	result
default	$-\frac{\arctan(x)^2}{2x^2}-\frac{\arctan(x)^2}{4x^4}-\frac{\arctan(x)^2}{4}-\frac{\arctan(x)}{6x^3}-\frac{\arctan(x)}{2x}-\frac{1}{12x^2}+\frac{\ln(x)}{3}-\frac{\ln(x^2+1)}{6}$
parts	$-\frac{\arctan(x)^2}{2x^2}-\frac{\arctan(x)^2}{4x^4}-\frac{\arctan(x)^2}{4}-\frac{\arctan(x)}{6x^3}-\frac{\arctan(x)}{2x}-\frac{1}{12x^2}+\frac{\ln(x)}{3}-\frac{\ln(x^2+1)}{6}$
parallelrisch	$\frac{-3x^4\arctan(x)^2+4x^4\ln(x)-2\ln(x^2+1)x^4-6x^3\arctan(x)-6x^2\arctan(x)^2-x^2-2x\arctan(x)-3\arctan(x)^2}{12x^4}$
risch	$\frac{(x^4+2x^2+1)\ln(ix+1)^2}{16x^4}-\frac{(3x^4\ln(-ix+1)-6ix^3+6x^2\ln(-ix+1)-2ix+3\ln(-ix+1))\ln(ix+1)}{24x^4}+\frac{3x^4\ln(-ix+1)^2-12ix^3\ln}{}$

[In] int((x^2+1)*arctan(x)^2/x^5,x,method=_RETURNVERBOSE)

[Out] -1/2*arctan(x)^2/x^2-1/4*arctan(x)^2/x^4-1/4*arctan(x)^2-1/6/x^3*arctan(x)-1/2/x*arctan(x)-1/12/x^2+1/3*ln(x)-1/6*ln(x^2+1)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = -\frac{2\,x^4\log\left(x^2+1\right)-4\,x^4\log\left(x\right)+3\,\left(x^4+2\,x^2+1\right)\arctan\left(x\right)^2+x^2+2\,\left(3\,x^3+x\right)\arctan\left(x\right)}{12\,x^4}$$

[In] integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="fricas")

[Out] -1/12*(2*x^4*log(x^2 + 1) - 4*x^4*log(x) + 3*(x^4 + 2*x^2 + 1)*arctan(x)^2 + x^2 + 2*(3*x^3 + x)*arctan(x))/x^4

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = \frac{\log\left(x\right)}{3}-\frac{\log\left(x^2+1\right)}{6}-\frac{\operatorname{atan}^2\left(x\right)}{4}-\frac{\operatorname{atan}\left(x\right)}{2x}-\frac{\operatorname{atan}^2\left(x\right)}{2x^2}-\frac{1}{12x^2}-\frac{\operatorname{atan}\left(x\right)}{6x^3}-\frac{\operatorname{atan}^2\left(x\right)}{4x^4}$$

[In] integrate((x**2+1)*atan(x)**2/x**5,x)

[Out] log(x)/3 - log(x**2 + 1)/6 - atan(x)**2/4 - atan(x)/(2*x) - atan(x)**2/(2*x**2) - 1/(12*x**2) - atan(x)/(6*x**3) - atan(x)**2/(4*x**4)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = -\frac{1}{6}\left(\frac{3\,x^2+1}{x^3}+3\,\arctan\left(x\right)\right)\arctan\left(x\right) + \frac{3\,x^2\arctan\left(x\right)^2-2\,x^2\log\left(x^2+1\right)+4\,x^2\log\left(x\right)-1}{12\,x^2} - \frac{\left(2\,x^2+1\right)\arctan\left(x\right)^2}{4\,x^4}$$

```
[In] integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/6*((3*x^2 + 1)/x^3 + 3*arctan(x))*arctan(x) + 1/12*(3*x^2*arctan(x)^2 -
2*x^2*log(x^2 + 1) + 4*x^2*log(x) - 1)/x^2 - 1/4*(2*x^2 + 1)*arctan(x)^2/x^
4
```

Giac [F]

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = \int \frac{\left(x^2+1\right)\arctan\left(x\right)^2}{x^5}\,dx$$

```
[In] integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)*arctan(x)^2/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = \frac{\ln\left(x\right)}{3} - \frac{\ln\left(x^2+1\right)}{6} - \mathrm{atan}(x)^2\left(\frac{\frac{x^2}{2}+\frac{1}{4}}{x^4}+\frac{1}{4}\right) - \frac{1}{12\,x^2} - \frac{\mathrm{atan}(x)\,\left(\frac{x^2}{2}+\frac{1}{6}\right)}{x^3}$$

```
[In] int((atan(x)^2*(x^2 + 1))/x^5,x)
```

```
[Out] log(x)/3 - log(x^2 + 1)/6 - atan(x)^2*((x^2/2 + 1/4)/x^4 + 1/4) - 1/(12*x^2
) - (atan(x)*(x^2/2 + 1/6))/x^3
```

3.682 $\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3} \, dx = -\frac{1}{32(1+x^2)^2} + \frac{5}{32(1+x^2)} + \frac{x^3 \arctan(x)}{8(1+x^2)^2} + \frac{3x \arctan(x)}{16(1+x^2)} - \frac{3\arctan(x)^2}{32} + \frac{x^4 \arctan(x)^2}{4(1+x^2)^2}$$

```
[Out] -1/32/(x^2+1)^2+5/32/(x^2+1)+1/8*x^3*arctan(x)/(x^2+1)^2+3/16*x*arctan(x)/(
x^2+1)-3/32*arctan(x)^2+1/4*x^4*arctan(x)^2/(x^2+1)^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5064, 5058, 5054, 5004}

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3} \, dx = \frac{3x \arctan(x)}{16(x^2+1)} + \frac{x^4 \arctan(x)^2}{4(x^2+1)^2} + \frac{x^3 \arctan(x)}{8(x^2+1)^2} - \frac{3\arctan(x)^2}{32} + \frac{3}{32(x^2+1)} - \frac{x^4}{32(x^2+1)^2}$$

```
[In] Int[(x^3*ArcTan[x]^2)/(1 + x^2)^3,x]
```

```
[Out] -1/32*x^4/(1 + x^2)^2 + 3/(32*(1 + x^2)) + (x^3*ArcTan[x])/(8*(1 + x^2)^2)
+ (3*x*ArcTan[x])/(16*(1 + x^2)) - (3*ArcTan[x]^2)/32 + (x^4*ArcTan[x]^2)/(
4*(1 + x^2)^2)
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5054

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (-Dis
t[1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]
+ Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rule 5058

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
b*ArcTan[c*x]), x], x] - Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*A
rcTan[c*x])/(c^2*d*m)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^4\arctan(x)^2}{4\left(1+x^2\right)^2}-\frac{1}{2}\int\frac{x^4\arctan(x)}{\left(1+x^2\right)^3}\,dx\\
&= -\frac{x^4}{32\left(1+x^2\right)^2}+\frac{x^3\arctan(x)}{8\left(1+x^2\right)^2}+\frac{x^4\arctan(x)^2}{4\left(1+x^2\right)^2}-\frac{3}{8}\int\frac{x^2\arctan(x)}{\left(1+x^2\right)^2}\,dx\\
&= -\frac{x^4}{32\left(1+x^2\right)^2}+\frac{3}{32\left(1+x^2\right)}+\frac{x^3\arctan(x)}{8\left(1+x^2\right)^2}\\
&\quad+\frac{3x\arctan(x)}{16\left(1+x^2\right)}+\frac{x^4\arctan(x)^2}{4\left(1+x^2\right)^2}-\frac{3}{16}\int\frac{\arctan(x)}{1+x^2}\,dx\\
&= -\frac{x^4}{32\left(1+x^2\right)^2}+\frac{3}{32\left(1+x^2\right)}+\frac{x^3\arctan(x)}{8\left(1+x^2\right)^2}+\frac{3x\arctan(x)}{16\left(1+x^2\right)}-\frac{3\arctan(x)^2}{32}+\frac{x^4\arctan(x)^2}{4\left(1+x^2\right)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3} \, dx = \frac{4+5x^2+2x(3+5x^2)\arctan(x)+(-3-6x^2+5x^4)\arctan(x)^2}{32\left(1+x^2\right)^2}$$

```
[In] Integrate[(x^3*ArcTan[x]^2)/(1 + x^2)^3,x]
[Out] (4 + 5*x^2 + 2*x*(3 + 5*x^2)*ArcTan[x] + (-3 - 6*x^2 + 5*x^4)*ArcTan[x]^2)/
(32*(1 + x^2)^2)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

method	result
default	$\frac{\arctan(x)^2}{4(x^2+1)^2}-\frac{\arctan(x)^2}{2(x^2+1)}+\frac{5x^3\arctan(x)}{16(x^2+1)^2}+\frac{3x\arctan(x)}{16(x^2+1)^2}+\frac{5\arctan(x)^2}{32}-\frac{1}{32(x^2+1)^2}+\frac{5}{32(x^2+1)}$
parts	$\frac{\arctan(x)^2}{4(x^2+1)^2}-\frac{\arctan(x)^2}{2(x^2+1)}+\frac{5x^3\arctan(x)}{16(x^2+1)^2}+\frac{3x\arctan(x)}{16(x^2+1)^2}+\frac{5\arctan(x)^2}{32}-\frac{1}{32(x^2+1)^2}+\frac{5}{32(x^2+1)}$
risch	$-\frac{(5x^4-6x^2-3)\ln(ix+1)^2}{128(x^2+1)^2}+\frac{(-6x^2\ln(-ix+1)-3\ln(-ix+1)+5x^4\ln(-ix+1)-10ix^3-6ix)\ln(ix+1)}{64(x+i)^2(x-i)^2}-\frac{5x^4\ln(-ix+1)^2-6x^2\ln(-}{}$

```
[In] int(x^3*arctan(x)^2/(x^2+1)^3,x,method=_RETURNVERBOSE)
[Out] 1/4*arctan(x)^2/(x^2+1)^2-1/2*arctan(x)^2/(x^2+1)+5/16*x^3*arctan(x)/(x^2+1
)^2+3/16*x*arctan(x)/(x^2+1)^2+5/32*arctan(x)^2-1/32/(x^2+1)^2+5/32/(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3} \, dx = \frac{(5\,x^4-6\,x^2-3)\arctan\left(x\right)^2+5\,x^2+2\,(5\,x^3+3\,x)\arctan\left(x\right)+4}{32\,(x^4+2\,x^2+1)}$$

```
[In] integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="fricas")
[Out] 1/32*((5*x^4 - 6*x^2 - 3)*arctan(x)^2 + 5*x^2 + 2*(5*x^3 + 3*x)*arctan(x) +
 4)/(x^4 + 2*x^2 + 1)
```

Sympy [F]

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3}\,dx = \int \frac{x^3 \operatorname{atan}^2(x)}{(x^2+1)^3}\,dx$$

```
[In] integrate(x**3*atan(x)**2/(x**2+1)**3,x)
[Out] Integral(x**3*atan(x)**2/(x**2 + 1)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3}\,dx = \frac{1}{16}\left(\frac{5\,x^3+3\,x}{x^4+2\,x^2+1}+5\,\arctan(x)\right)\arctan(x) - \frac{(2\,x^2+1)\arctan(x)^2}{4\,(x^4+2\,x^2+1)} - \frac{5\,(x^4+2\,x^2+1)\arctan(x)^2-5\,x^2-4}{32\,(x^4+2\,x^2+1)}$$

```
[In] integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="maxima")
[Out] 1/16*((5*x^3 + 3*x)/(x^4 + 2*x^2 + 1) + 5*arctan(x))*arctan(x) - 1/4*(2*x^2
 + 1)*arctan(x)^2/(x^4 + 2*x^2 + 1) - 1/32*(5*(x^4 + 2*x^2 + 1)*arctan(x)^2
 - 5*x^2 - 4)/(x^4 + 2*x^2 + 1)
```

Giac [F]

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3}\,dx = \int \frac{x^3 \arctan(x)^2}{(x^2+1)^3}\,dx$$

```
[In] integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="giac")
[Out] integrate(x^3*arctan(x)^2/(x^2 + 1)^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.71

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3}\,dx$$
$$= -\frac{-5\,x^4\,\mathrm{atan}(x)^2 + 4\,x^4 - 10\,x^3\,\mathrm{atan}(x) + 6\,x^2\,\mathrm{atan}(x)^2 + 3\,x^2 - 6\,x\,\mathrm{atan}(x) + 3\,\mathrm{atan}(x)^2}{32\,(x^2+1)^2}$$

```
[In] int((x^3*atan(x)^2)/(x^2 + 1)^3,x)
[Out] -(3*atan(x)^2 - 10*x^3*atan(x) + 6*x^2*atan(x)^2 - 5*x^4*atan(x)^2 - 6*x*at
an(x) + 3*x^2 + 4*x^4)/(32*(x^2 + 1)^2)
```

3.683 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 107

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx=-\frac{1}{\sqrt{x^2}}-\frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x}-\frac{2i\sqrt{x^2}\sec^{-1}(x)\arctan\left(e^{i\sec^{-1}(x)}\right)}{x}+\frac{i\sqrt{x^2}\,\mathrm{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{x}-\frac{i\sqrt{x^2}\,\mathrm{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{x}$$

```
[Out] -(x^2)^(1/2)/x^2-2*I*arcsec(x)*arctan(1/x+I*(1-1/x^2)^(1/2))*(x^2)^(1/2)/x+
I*polylog(2,-I*(1/x+I*(1-1/x^2)^(1/2)))*(x^2)^(1/2)/x-I*polylog(2,I*(1/x+I*
(1-1/x^2)^(1/2)))*(x^2)^(1/2)/x-arcsec(x)*(x^2-1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5350, 4784, 4804, 4266, 2317, 2438, 8}

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx=-\frac{2i\sqrt{x^2}\sec^{-1}(x)\arctan\left(e^{i\sec^{-1}(x)}\right)}{x}+\frac{i\sqrt{x^2}\,\mathrm{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{x}-\frac{i\sqrt{x^2}\,\mathrm{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{x}-\frac{1}{\sqrt{x^2}}-\frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x}$$

```
[In] Int[(Sqrt[-1 + x^2]*ArcSec[x])/x^2,x]
```

```
[Out] -(1/Sqrt[x^2]) - (Sqrt[1 - x^(-2)]*Sqrt[x^2]*ArcSec[x])/x - ((2*I)*Sqrt[x^2
]*ArcSec[x]*ArcTan[E^(I*ArcSec[x])])/x + (I*Sqrt[x^2]*PolyLog[2, (-I)*E^(I*
ArcSec[x])])/x - (I*Sqrt[x^2]*PolyLog[2, I*E^(I*ArcSec[x])])/x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4784

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]], Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]
 + Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(f
*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 4804

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]], Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5350

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Dist[-Sqrt[x^2]/x, Subst[Int[(e + d*x^2)^p*((a + b*A
```

```
rcCos[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{x^2}\text{Subst}\left(\int \frac{\sqrt{1-x^2}\arccos(x)}{x}\,dx, x, \frac{1}{x}\right)}{x}$$

$$= -\frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x} - \frac{\sqrt{x^2}\text{Subst}\left(\int 1\,dx, x, \frac{1}{x}\right)}{x} - \frac{\sqrt{x^2}\text{Subst}\left(\int \frac{\arccos(x)}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)}{x}$$

$$= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x} + \frac{\sqrt{x^2}\text{Subst}\left(\int x\sec(x)\,dx, x, \sec^{-1}(x)\right)}{x}$$

$$= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x} - \frac{2i\sqrt{x^2}\sec^{-1}(x)\arctan\left(e^{i\sec^{-1}(x)}\right)}{x} - \frac{\sqrt{x^2}\text{Subst}\left(\int \log\left(1-ie^{ix}\right)\,dx, x, \sec^{-1}(x)\right)}{x} + \frac{\sqrt{x^2}\text{Subst}\left(\int \log\left(1+ie^{ix}\right)\,dx, x, \sec^{-1}(x)\right)}{x}$$

$$= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x} - \frac{2i\sqrt{x^2}\sec^{-1}(x)\arctan\left(e^{i\sec^{-1}(x)}\right)}{x} + \frac{\left(i\sqrt{x^2}\right)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{i\sec^{-1}(x)}\right)}{x} - \frac{\left(i\sqrt{x^2}\right)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{i\sec^{-1}(x)}\right)}{x}$$

$$= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x} - \frac{2i\sqrt{x^2}\sec^{-1}(x)\arctan\left(e^{i\sec^{-1}(x)}\right)}{x} + \frac{i\sqrt{x^2}\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{x} - \frac{i\sqrt{x^2}\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{x}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = -\frac{\sqrt{1-\frac{1}{x^2}}\left(1+\sqrt{1-\frac{1}{x^2}}x\sec^{-1}(x)-x\sec^{-1}(x)\log\left(1-ie^{i\sec^{-1}(x)}\right)+x\sec^{-1}(x)\log\left(1+ie^{i\sec^{-1}(x)}\right)-i\right.}{\sqrt{-1+x^2}}$$

[In] Integrate[(Sqrt[-1 + x^2]*ArcSec[x])/x^2,x]

[Out] -((Sqrt[1 - x^(-2)]*(1 + Sqrt[1 - x^(-2)]*x*ArcSec[x] - x*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])] + x*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])] - I*x*PolyLog[2, (-I)*E^(I*ArcSec[x])] + I*x*PolyLog[2, I*E^(I*ArcSec[x])]))/Sqrt[-1 + x^2])

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.81

method	result
default	$-\frac{\left(\sqrt{\frac{x^2-1}{x^2}}\,x-i\right)\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)(\operatorname{arcsec}(x)+i)}{2x}-\frac{\left(\sqrt{\frac{x^2-1}{x^2}}\,x+i\right)\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)(\operatorname{arcsec}(x)-i)}{2x}-\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(a\right.$

[In] int(arcsec(x)*(x^2-1)^(1/2)/x^2,x,method=_RETURNVERBOSE)

[Out] -1/2*(((x^2-1)/x^2)^(1/2)*x-I)/x*csgn(x*(1-1/x^2)^(1/2))*(arcsec(x)+I)-1/2*(((x^2-1)/x^2)^(1/2)*x+I)/x*csgn(x*(1-1/x^2)^(1/2))*(arcsec(x)-I)-csgn(x*(1-1/x^2)^(1/2))*(arcsec(x)*ln(1+I*(1/x+I*(1-1/x^2)^(1/2)))-arcsec(x)*ln(1-I*(1/x+I*(1-1/x^2)^(1/2)))-I*dilog(1+I*(1/x+I*(1-1/x^2)^(1/2)))+I*dilog(1-I*(1/x+I*(1-1/x^2)^(1/2))))

Fricas [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = \int \frac{\sqrt{x^2-1}\operatorname{arcsec}(x)}{x^2}\,dx$$

[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="fricas")

[Out] integral(sqrt(x^2 - 1)*arcsec(x)/x^2, x)

Sympy [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = \int \frac{\sqrt{(x-1)(x+1)}\operatorname{asec}(x)}{x^2}\,dx$$

```
[In] integrate(asec(x)*(x**2-1)**(1/2)/x**2,x)
[Out] Integral(sqrt((x - 1)*(x + 1))*asec(x)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = \int \frac{\sqrt{x^2-1}\operatorname{arcsec}(x)}{x^2}\,dx$$

```
[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(x^2 - 1)*arcsec(x)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = \int \frac{\sqrt{x^2-1}\operatorname{arcsec}(x)}{x^2}\,dx$$

```
[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(x^2 - 1)*arcsec(x)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = \int \frac{\operatorname{acos}\left(\frac{1}{x}\right)\sqrt{x^2-1}}{x^2}\,dx$$

```
[In] int((acos(1/x)*(x^2 - 1)^(1/2))/x^2,x)
[Out] int((acos(1/x)*(x^2 - 1)^(1/2))/x^2, x)
```

3.684 $\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 106

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \frac{3+2x^4}{12x\sqrt{x^2}} - \frac{5\sqrt{-1+x^2}\csc^{-1}(x)}{2x^2} - \frac{5(-1+x^2)^{3/2}\csc^{-1}(x)}{3x^2} + \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{3x^2} - \frac{5x\csc^{-1}(x)^2}{4\sqrt{x^2}} - \frac{7x\log(x)}{3\sqrt{x^2}}$$

```
[Out] -5/3*(x^2-1)^(3/2)*arccsc(x)/x^2+1/3*(x^2-1)^(5/2)*arccsc(x)/x^2+1/12*(2*x^
4+3)/x/(x^2)^(1/2)-5/4*x*arccsc(x)^2/(x^2)^(1/2)-7/3*x*ln(x)/(x^2)^(1/2)-5/
2*arccsc(x)*(x^2-1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.25, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5351, 4785, 4741, 4737, 30, 14, 272, 45}

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \frac{x\sqrt{x^2}}{6} - \frac{7\sqrt{x^2}\log(x)}{3x} + \frac{1}{3}(x^2)^{3/2}\left(1-\frac{1}{x^2}\right)^{5/2}\csc^{-1}(x) - \frac{5}{3}\sqrt{x^2}\left(1-\frac{1}{x^2}\right)^{3/2}\csc^{-1}(x) - \frac{5\sqrt{1-\frac{1}{x^2}}\csc^{-1}(x)}{2\sqrt{x^2}} - \frac{5\sqrt{x^2}\csc^{-1}(x)^2}{4x} + \frac{\sqrt{x^2}}{4x^3}$$

```
[In] Int[((-1 + x^2)^(5/2)*ArcCsc[x])/x^3,x]
```

```
[Out] Sqrt[x^2]/(4*x^3) + (x*Sqrt[x^2])/6 - (5*Sqrt[1 - x^(-2)]*ArcCsc[x])/(2*Sqr
t[x^2]) - (5*(1 - x^(-2))^(3/2)*Sqrt[x^2]*ArcCsc[x])/3 + ((1 - x^(-2))^(5/2
)*(x^2)^(3/2)*ArcCsc[x])/3 - (5*Sqrt[x^2]*ArcCsc[x]^2)/(4*x) - (7*Sqrt[x^2]
*Log[x])/(3*x)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4737

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcSin[c*x])^n/Sqrt[1
 - c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4785

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Dist[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(
```

```
p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5351

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Dist[-Sqrt[x^2]/x, Subst[Int[(e + d*x^2)^p*((a + b*A
rcSin[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{x^2}\text{Subst}\left(\int \frac{(1-x^2)^{5/2}\arcsin(x)}{x^4}\,dx, x, \frac{1}{x}\right)}{x}$$

$$= \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}(x^2)^{3/2}\csc^{-1}(x) - \frac{\sqrt{x^2}\text{Subst}\left(\int \frac{(1-x^2)^2}{x^3}\,dx, x, \frac{1}{x}\right)}{3x} + \frac{\left(5\sqrt{x^2}\right)\text{Subst}\left(\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^2}\,dx, x, \frac{1}{x}\right)}{3x}$$

$$= -\frac{5}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\csc^{-1}(x) + \frac{1}{3}\left(1 -\frac{1}{x^2}\right)^{5/2}(x^2)^{3/2}\csc^{-1}(x) - \frac{\sqrt{x^2}\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, \frac{1}{x^2}\right)}{6x} + \frac{\left(5\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, \frac{1}{x}\right)}{3x} - \frac{(5\sqrt{}}{}$$

$$= -\frac{5\sqrt{1-\frac{1}{x^2}}\csc^{-1}(x)}{2\sqrt{x^2}} - \frac{5}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\csc^{-1}(x) + \frac{1}{3}\left(1 -\frac{1}{x^2}\right)^{5/2}(x^2)^{3/2}\csc^{-1}(x) - \frac{\sqrt{x^2}\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)dx, x, \frac{1}{x^2}\right)}{6x} + \frac{\left(5\sqrt{x^2}\right)\text{Subst}\left(\int \left(\frac{1}{x}-x\right)dx, x\right.}{3x}$$

$$= \frac{\sqrt{x^2}}{4x^3} + \frac{x\sqrt{x^2}}{6} - \frac{5\sqrt{1-\frac{1}{x^2}}\csc^{-1}(x)}{2\sqrt{x^2}} - \frac{5}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\csc^{-1}(x) + \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}(x^2)^{3/2}\csc^{-1}(x) - \frac{5\sqrt{x^2}\csc^{-1}(x)^2}{4x} - \frac{7\sqrt{x^2}\log(x)}{3x}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.81

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \frac{\sqrt{-1+x^2}\left(4x^2-30\csc^{-1}(x)^2-3\cos\left(2\csc^{-1}(x)\right)+48\log\left(\frac{1}{x}\right)-8\log(x)+\right.}{24\sqrt{1-\frac{1}{x^2}}x}$$

[In] Integrate[((-1 + x^2)^(5/2)*ArcCsc[x])/x^3,x]

[Out] (Sqrt[-1 + x^2]*(4*x^2 - 30*ArcCsc[x]^2 - 3*Cos[2*ArcCsc[x]] + 48*Log[x^(-1)] - 8*Log[x] + ArcCsc[x]*(8*Sqrt[1 - x^(-2)]*x*(-7 + x^2) - 6*Sin[2*ArcCsc[x]])))/(24*Sqrt[1 - x^(-2)]*x)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.50 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.58

method	result
default	$-\frac{5\,\mathrm{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\mathrm{arccsc}(x)^2}{4}+\frac{\left(-2\sqrt{\frac{x^2-1}{x^2}}\,x+ix^2-2i\right)\mathrm{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)(2\,\mathrm{arccsc}(x)+i)}{16x^2}-\frac{\left(2\sqrt{\frac{x^2-1}{x^2}}\,x+ix^2-2i\right)\mathrm{csgn}\left(x\right.}{16x^2}$

[In] int((x^2-1)^(5/2)*arccsc(x)/x^3,x,method=_RETURNVERBOSE)

[Out] -5/4*csgn(x*(1-1/x^2)^(1/2))*arccsc(x)^2+1/16*(-2*((x^2-1)/x^2)^(1/2)*x+I*x^2-2*I)/x^2*csgn(x*(1-1/x^2)^(1/2))*(2*arccsc(x)+I)-1/16*(2*((x^2-1)/x^2)^(1/2)*x+I*x^2-2*I)*csgn(x*(1-1/x^2)^(1/2))*(-I+2*arccsc(x))/x^2-14/3*I*csgn(x*(1-1/x^2)^(1/2))*arccsc(x)+1/6*(((x^2-1)/x^2)^(1/2)*x^3-7*((x^2-1)/x^2)^(1/2)*x+7*I)*csgn(x*(1-1/x^2)^(1/2))*(2*arccsc(x)*x^4+((x^2-1)/x^2)^(1/2)*x^3-30*arccsc(x)*x^2-7*((x^2-1)/x^2)^(1/2)*x+126*arccsc(x)-7*I)/(x^4-15*x^2+63)+7/3*csgn(x*(1-1/x^2)^(1/2))*ln((I/x+(1-1/x^2)^(1/2))^2-1)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.48

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \frac{2\,x^4-15\,x^2\,\mathrm{arccsc}\,(x)^2-28\,x^2\log{(x)}+2\,(2\,x^4-14\,x^2-3)\sqrt{x^2-1}\,\mathrm{arccsc}\,(}{12\,x^2}$$

[In] integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="fricas")

[Out] 1/12*(2*x^4 - 15*x^2*arccsc(x)^2 - 28*x^2*log(x) + 2*(2*x^4 - 14*x^2 - 3)*sqrt(x^2 - 1)*arccsc(x) + 3)/x^2

Sympy [F(-1)]

Timed out.

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((x**2-1)**(5/2)*acsc(x)/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \int \frac{(x^2-1)^{\frac{5}{2}}\operatorname{arccsc}(x)}{x^3}\,dx$$

```
[In] integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="maxima")
[Out] integrate((x^2 - 1)^(5/2)*arccsc(x)/x^3, x)
```

Giac [F]

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \int \frac{(x^2-1)^{\frac{5}{2}}\operatorname{arccsc}(x)}{x^3}\,dx$$

```
[In] integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="giac")
[Out] integrate((x^2 - 1)^(5/2)*arccsc(x)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \int \frac{\operatorname{asin}\left(\frac{1}{x}\right)\,\left(x^2-1\right)^{5/2}}{x^3}\,dx$$

```
[In] int((asin(1/x)*(x^2 - 1)^(5/2))/x^3,x)
[Out] int((asin(1/x)*(x^2 - 1)^(5/2))/x^3, x)
```

3.685 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = -\frac{1}{9(x^2)^{3/2}}+\frac{1}{3\sqrt{x^2}}+\frac{(-1+x^2)^{3/2}\sec^{-1}(x)}{3x^3}$$

```
[Out] 1/3*(x^2-1)^(3/2)*arcsec(x)/x^3+1/3/(x^2)^(1/2)-1/9/x^2/(x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {270, 5346, 12, 14}

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \frac{1}{3\sqrt{x^2}}-\frac{1}{9(x^2)^{3/2}}+\frac{(x^2-1)^{3/2}\sec^{-1}(x)}{3x^3}$$

```
[In] Int[(Sqrt[-1 + x^2]*ArcSec[x])/x^4,x]
[Out] -1/9*1/(x^2)^(3/2) + 1/(3*Sqrt[x^2]) + ((-1 + x^2)^(3/2)*ArcSec[x])/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(-1+x^2)^{3/2}\sec^{-1}(x)}{3x^3} - \frac{x\int \frac{-1+x^2}{3x^4}\,dx}{\sqrt{x^2}} \\
&= \frac{(-1+x^2)^{3/2}\sec^{-1}(x)}{3x^3} - \frac{x\int \frac{-1+x^2}{x^4}\,dx}{3\sqrt{x^2}} \\
&= \frac{(-1+x^2)^{3/2}\sec^{-1}(x)}{3x^3} - \frac{x\int \left(-\frac{1}{x^4}+\frac{1}{x^2}\right)\,dx}{3\sqrt{x^2}} \\
&= -\frac{1}{9(x^2)^{3/2}} + \frac{1}{3\sqrt{x^2}} + \frac{(-1+x^2)^{3/2}\sec^{-1}(x)}{3x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \frac{\sqrt{1-\frac{1}{x^2}}x(-1+3x^2)+3(-1+x^2)^2\sec^{-1}(x)}{9x^3\sqrt{-1+x^2}}$$

```
[In] Integrate[(Sqrt[-1 + x^2]*ArcSec[x])/x^4,x]
[Out] (Sqrt[1 - x^(-2)]*x*(-1 + 3*x^2) + 3*(-1 + x^2)^2*ArcSec[x])/(9*x^3*Sqrt[-1
 + x^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.07

method	result	size
default	$\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\sqrt{\frac{x^2-1}{x^2}}\left(3\operatorname{arcsec}(x)x^4+3\sqrt{\frac{x^2-1}{x^2}}\,x^3-6\operatorname{arcsec}(x)x^2-\sqrt{\frac{x^2-1}{x^2}}\,x+3\operatorname{arcsec}(x)\right)}{9(x^2-1)x^2}$	85

[In] int(arcsec(x)*(x^2-1)^(1/2)/x^4,x,method=_RETURNVERBOSE)

[Out] 1/9*csgn(x*(1-1/x^2)^(1/2))*((x^2-1)/x^2)^(1/2)/(x^2-1)/x^2*(3*arcsec(x)*x^4+3*((x^2-1)/x^2)^(1/2)*x^3-6*arcsec(x)*x^2-((x^2-1)/x^2)^(1/2)*x+3*arcsec(x))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \frac{3\,(x^2-1)^{\frac{3}{2}}\operatorname{arcsec}(x)+3\,x^2-1}{9\,x^3}$$

[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="fricas")

[Out] 1/9*(3*(x^2 - 1)^(3/2)*arcsec(x) + 3*x^2 - 1)/x^3

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \text{Timed out}$$

[In] integrate(asec(x)*(x**2-1)**(1/2)/x**4,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \frac{(x^2-1)^{\frac{3}{2}}\operatorname{arcsec}(x)}{3\,x^3} + \frac{3\,x^2-1}{9\,x^3}$$

```
[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="maxima")
[Out] 1/3*(x^2 - 1)^(3/2)*arcsec(x)/x^3 + 1/9*(3*x^2 - 1)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(32) = 64.

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = -\frac{2\arctan\left(-x+\sqrt{x^2-1}\right)}{3\operatorname{sgn}(x)} + \frac{2\left(3\left(x-\sqrt{x^2-1}\right)^4+1\right)\arccos\left(\frac{1}{x}\right)}{3\left(\left(x-\sqrt{x^2-1}\right)^2+1\right)^3} + \frac{3\,x^2-1}{9\,x^3\operatorname{sgn}(x)}$$

```
[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="giac")
[Out] -2/3*arctan(-x + sqrt(x^2 - 1))/sgn(x) + 2/3*(3*(x - sqrt(x^2 - 1))^4 + 1)*
arccos(1/x)/((x - sqrt(x^2 - 1))^2 + 1)^3 + 1/9*(3*x^2 - 1)/(x^3*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \int \frac{\operatorname{acos}\left(\frac{1}{x}\right)\sqrt{x^2-1}}{x^4}\,dx$$

```
[In] int((acos(1/x)*(x^2 - 1)^(1/2))/x^4,x)
[Out] int((acos(1/x)*(x^2 - 1)^(1/2))/x^4, x)
```

3.686 $\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx=\frac{\sqrt{x^2}}{6(1-x^2)}+\frac{5}{6}\coth^{-1}\left(\sqrt{x^2}\right)-\frac{x\sec^{-1}(x)}{3(-1+x^2)^{3/2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{-1+x^2}}$$

```
[Out] 5/6*arccoth((x^2)^(1/2))-1/3*x*arcsec(x)/(x^2-1)^(3/2)+1/6*(x^2)^(1/2)/(-x^
2+1)+2/3*x*arcsec(x)/(x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {198, 197, 5336, 12, 393, 212}

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx=\frac{5x\text{arctanh}(x)}{6\sqrt{x^2}}+\frac{\sqrt{x^2}}{6(1-x^2)}+\frac{2x\sec^{-1}(x)}{3\sqrt{x^2-1}}-\frac{x\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

```
[In] Int[ArcSec[x]/(-1 + x^2)^(5/2),x]
```

```
[Out] Sqrt[x^2]/(6*(1 - x^2)) - (x*ArcSec[x])/(3*(-1 + x^2)^(3/2)) + (2*x*ArcSec[
x])/(3*Sqrt[-1 + x^2]) + (5*x*ArcTanh[x])/(6*Sqrt[x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 5336

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x]
- Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{-1+x^2}}-\frac{x\int\frac{-3+2x^2}{3\left(1-x^2\right)^2}\,dx}{\sqrt{x^2}}\\
&= -\frac{x\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{-1+x^2}}-\frac{x\int\frac{-3+2x^2}{\left(1-x^2\right)^2}\,dx}{3\sqrt{x^2}}\\
&= \frac{\sqrt{x^2}}{6\left(1-x^2\right)}-\frac{x\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{-1+x^2}}+\frac{(5x)\int\frac{1}{1-x^2}\,dx}{6\sqrt{x^2}}\\
&= \frac{\sqrt{x^2}}{6\left(1-x^2\right)}-\frac{x\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{-1+x^2}}+\frac{5x\text{arctanh}(x)}{6\sqrt{x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \frac{4x(-3+2x^2)\sec^{-1}(x)+\sqrt{1-\frac{1}{x^2}}x(-2x-5(-1+x^2)\log(1-x)+5(-1+x^2)\log(1$$

[In] Integrate[ArcSec[x]/(-1 + x^2)^(5/2),x]

[Out] (4*x*(-3 + 2*x^2)*ArcSec[x] + Sqrt[1 - x^(-2)]*x*(-2*x - 5*(-1 + x^2)*Log[1 - x] + 5*(-1 + x^2)*Log[1 + x]))/(12*(-1 + x^2)^(3/2))

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.95

method	result
default	$\frac{\sqrt{\frac{x^2-1}{x^2}}\,x^2\,\mathrm{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(4\,\mathrm{arcsec}(x)x^2-\sqrt{\frac{x^2-1}{x^2}}\,x-6\,\mathrm{arcsec}(x)\right)}{6(x^2-1)^2}+\frac{5\,\mathrm{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\ln\left(\frac{1}{x}+i\sqrt{1-\frac{1}{x^2}}+1\right)}{6}-\frac{5\,\mathrm{csgn}\left(x\sqrt{1\right.}$

[In] int(arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/6*((x^2-1)/x^2)^(1/2)*x^2/(x^2-1)^2*csgn(x*(1-1/x^2)^(1/2))*(4*arcsec(x)*x^2-((x^2-1)/x^2)^(1/2)*x-6*arcsec(x))+5/6*csgn(x*(1-1/x^2)^(1/2))*ln(1/x+I*(1-1/x^2)^(1/2)+1)-5/6*csgn(x*(1-1/x^2)^(1/2))*ln(1/x+I*(1-1/x^2)^(1/2)-1)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx =$$
$$-\frac{2\,x^3-4\,(2\,x^3-3\,x)\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)-5\,(x^4-2\,x^2+1)\log\,(x+1)+5\,(x^4-2\,x^2+1)\log\,(x-1)-2\,x}{12\,(x^4-2\,x^2+1)}$$

[In] integrate(arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")

[Out] -1/12*(2*x^3 - 4*(2*x^3 - 3*x)*sqrt(x^2 - 1)*arcsec(x) - 5*(x^4 - 2*x^2 + 1)*log(x + 1) + 5*(x^4 - 2*x^2 + 1)*log(x - 1) - 2*x)/(x^4 - 2*x^2 + 1)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(asec(x)/(x**2-1)**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{2\,x}{\sqrt{x^2-1}} - \frac{x}{(x^2-1)^{\frac{3}{2}}}\right)\operatorname{arcsec}(x) - \frac{x}{6\,(x^2-1)} + \frac{5}{12}\log(x+1) - \frac{5}{12}\log(x-1)$$

```
[In] integrate(arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
[Out] 1/3*(2*x/sqrt(x^2 - 1) - x/(x^2 - 1)^(3/2))*arcsec(x) - 1/6*x/(x^2 - 1) + 5
/12*log(x + 1) - 5/12*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \frac{(2\,x^2-3)x\arccos\left(\frac{1}{x}\right)}{3\,(x^2-1)^{\frac{3}{2}}} + \frac{5\,\log(|x+1|)}{12\,\mathrm{sgn}(x)} - \frac{5\,\log(|x-1|)}{12\,\mathrm{sgn}(x)} - \frac{x}{6\,(x^2-1)\mathrm{sgn}(x)}$$

```
[In] integrate(arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
[Out] 1/3*(2*x^2 - 3)*x*arccos(1/x)/(x^2 - 1)^(3/2) + 5/12*log(abs(x + 1))/sgn(x)
 - 5/12*log(abs(x - 1))/sgn(x) - 1/6*x/((x^2 - 1)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{\mathrm{acos}\left(\frac{1}{x}\right)}{(x^2-1)^{5/2}}\,dx$$

```
[In] int(acos(1/x)/(x^2 - 1)^(5/2),x)
[Out] int(acos(1/x)/(x^2 - 1)^(5/2), x)
```

3.687 $\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \frac{\sqrt{x^2}}{6(1-x^2)} - \frac{1}{6}\coth^{-1}\left(\sqrt{x^2}\right) - \frac{x^3 \sec^{-1}(x)}{3(-1+x^2)^{3/2}}$$

```
[Out] -1/6*arccoth((x^2)^(1/2))-1/3*x^3*arcsec(x)/(x^2-1)^(3/2)+1/6*(x^2)^(1/2)/(
-x^2+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 5346, 12, 294, 213}

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = -\frac{x\text{arctanh}(x)}{6\sqrt{x^2}} + \frac{\sqrt{x^2}}{6(1-x^2)} - \frac{x^3 \sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

```
[In] Int[(x^2*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] Sqrt[x^2]/(6*(1 - x^2)) - (x^3*ArcSec[x])/(3*(-1 + x^2)^(3/2)) - (x*ArcTanh
[x])/(6*Sqrt[x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^3\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}-\frac{x\int -\frac{x^2}{3(-1+x^2)^2}\,dx}{\sqrt{x^2}}\\
&= -\frac{x^3\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{x\int \frac{x^2}{(-1+x^2)^2}\,dx}{3\sqrt{x^2}}\\
&= \frac{\sqrt{x^2}}{6\left(1-x^2\right)}-\frac{x^3\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{x\int \frac{1}{-1+x^2}\,dx}{6\sqrt{x^2}}\\
&= \frac{\sqrt{x^2}}{6\left(1-x^2\right)}-\frac{x^3\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}-\frac{x\operatorname{arctanh}(x)}{6\sqrt{x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{x^2\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \frac{-4x^3\sec^{-1}(x)+\sqrt{1-\frac{1}{x^2}}x(-2x+(-1+x^2)\log(1-x)-(-1+x^2)\log(1+x))}{12(-1+x^2)^{3/2}}$$

```
[In] Integrate[(x^2*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] (-4*x^3*ArcSec[x] + Sqrt[1 - x^(-2)]*x*(-2*x + (-1 + x^2)*Log[1 - x] - (-1
+ x^2)*Log[1 + x]))/(12*(-1 + x^2)^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.49 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.63

method	result
default	$-\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(2\,\mathrm{arcsec}(x)x^4\sqrt{\frac{x^2-1}{x^2}}-\ln\left(\frac{1}{\sqrt{1-\frac{1}{x^2}}}-\frac{1}{x\sqrt{1-\frac{1}{x^2}}}\right)x^4+2\ln\left(\frac{1}{\sqrt{1-\frac{1}{x^2}}}-\frac{1}{x\sqrt{1-\frac{1}{x^2}}}\right)x^2+x^3-\ln\left(\frac{1}{\sqrt{1-\frac{1}{x^2}}}-\frac{1}{x\sqrt{1-\frac{1}{x^2}}}\right.\right.}{6(x^2-1)^2}$

```
[In] int(x^2*arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*csgn(x*(1-1/x^2)^(1/2))*(2*arcsec(x)*x^4*((x^2-1)/x^2)^(1/2)-ln(1/(1-1
/x^2)^(1/2)-1/x/(1-1/x^2)^(1/2))*x^4+2*ln(1/(1-1/x^2)^(1/2)-1/x/(1-1/x^2)^(
1/2))*x^2+x^3-ln(1/(1-1/x^2)^(1/2)-1/x/(1-1/x^2)^(1/2))-x)/(x^2-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

$$\int \frac{x^2\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = -\frac{4\sqrt{x^2-1}x^3\,\mathrm{arcsec}\,(x)+2\,x^3+(x^4-2\,x^2+1)\log(x+1)-(x^4-2\,x^2+1)\log(x-1)-2\,x}{12\,(x^4-2\,x^2+1)}$$

```
[In] integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(x^2 - 1)*x^3*arcsec(x) + 2*x^3 + (x^4 - 2*x^2 + 1)*log(x + 1)
 - (x^4 - 2*x^2 + 1)*log(x - 1) - 2*x)/(x^4 - 2*x^2 + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \text{Timed out}$$

```
[In] integrate(x**2*asec(x)/(x**2-1)**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = -\frac{1}{3}\left(\frac{x}{\sqrt{x^2-1}} + \frac{x}{(x^2-1)^{\frac{3}{2}}}\right)\operatorname{arcsec}(x) - \frac{x}{6\,(x^2-1)} - \frac{1}{12}\log(x+1) + \frac{1}{12}\log(x-1)$$

```
[In] integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
[Out] -1/3*(x/sqrt(x^2 - 1) + x/(x^2 - 1)^(3/2))*arcsec(x) - 1/6*x/(x^2 - 1) - 1/
12*log(x + 1) + 1/12*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = -\frac{x^3 \arccos\left(\frac{1}{x}\right)}{3\,(x^2-1)^{\frac{3}{2}}} - \frac{\log(|x+1|)}{12\operatorname{sgn}(x)} + \frac{\log(|x-1|)}{12\operatorname{sgn}(x)} - \frac{x}{6\,(x^2-1)\operatorname{sgn}(x)}$$

```
[In] integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
[Out] -1/3*x^3*arccos(1/x)/(x^2 - 1)^(3/2) - 1/12*log(abs(x + 1))/sgn(x) + 1/12*l
og(abs(x - 1))/sgn(x) - 1/6*x/((x^2 - 1)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{x^2 \operatorname{acos}\left(\frac{1}{x}\right)}{(x^2-1)^{5/2}}\,dx$$

```
[In] int((x^2*acos(1/x))/(x^2 - 1)^(5/2),x)
[Out] int((x^2*acos(1/x))/(x^2 - 1)^(5/2), x)
```

3.688 $\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 82

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \frac{x}{6\sqrt{x^2}\,(1-x^2)} - \frac{\sec^{-1}(x)}{3\,(-1+x^2)^{3/2}} - \frac{\sec^{-1}(x)}{\sqrt{-1+x^2}} - \frac{2x\log(x)}{3\sqrt{x^2}} + \frac{x\log(-1+x^2)}{3\sqrt{x^2}}$$

```
[Out] -1/3*arcsec(x)/(x^2-1)^(3/2)+1/6*x/(-x^2+1)/(x^2)^(1/2)-2/3*x*ln(x)/(x^2)^(
1/2)+1/3*x*ln(x^2-1)/(x^2)^(1/2)-arcsec(x)/(x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {272, 45, 5346, 12, 457, 78}

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \frac{x}{6\sqrt{x^2}\,(1-x^2)} - \frac{2x\log(x)}{3\sqrt{x^2}} + \frac{x\log(1-x^2)}{3\sqrt{x^2}} - \frac{\sec^{-1}(x)}{\sqrt{x^2-1}} - \frac{\sec^{-1}(x)}{3\,(x^2-1)^{3/2}}$$

```
[In] Int[(x^3*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] x/(6*Sqrt[x^2]*(1 - x^2)) - ArcSec[x]/(3*(-1 + x^2)^(3/2)) - ArcSec[x]/Sqrt
[-1 + x^2] - (2*x*Log[x])/(3*Sqrt[x^2]) + (x*Log[1 - x^2])/(3*Sqrt[x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}} - \frac{\sec^{-1}(x)}{\sqrt{-1+x^2}} - \frac{x\int \frac{2-3x^2}{3x\left(1-x^2\right)^2}\,dx}{\sqrt{x^2}} \\
&= -\frac{\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}} - \frac{\sec^{-1}(x)}{\sqrt{-1+x^2}} - \frac{x\int \frac{2-3x^2}{x\left(1-x^2\right)^2}\,dx}{3\sqrt{x^2}}
\end{aligned}$$

$$= -\frac{\sec^{-1}(x)}{3(-1+x^2)^{3/2}} - \frac{\sec^{-1}(x)}{\sqrt{-1+x^2}} - \frac{x\text{Subst}\left(\int \frac{2-3x}{(1-x)^2x}\,dx, x, x^2\right)}{6\sqrt{x^2}}$$

$$= -\frac{\sec^{-1}(x)}{3(-1+x^2)^{3/2}} - \frac{\sec^{-1}(x)}{\sqrt{-1+x^2}} - \frac{x\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2} - \frac{2}{-1+x} + \frac{2}{x}\right)\,dx, x, x^2\right)}{6\sqrt{x^2}}$$

$$= \frac{x}{6\sqrt{x^2}\,(1-x^2)} - \frac{\sec^{-1}(x)}{3(-1+x^2)^{3/2}} - \frac{\sec^{-1}(x)}{\sqrt{-1+x^2}} - \frac{2x\log(x)}{3\sqrt{x^2}} + \frac{x\log(1-x^2)}{3\sqrt{x^2}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{x^3\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \frac{-2(-2+3x^2)\sec^{-1}(x) + \sqrt{1-\frac{1}{x^2}}x(-1+2(-1+x^2)\log(-1+x) - 4(-1+x^2)\log(}{6(-1+x^2)^{3/2}}$$

```
[In] Integrate[(x^3*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] (-2*(-2 + 3*x^2)*ArcSec[x] + Sqrt[1 - x^(-2)]*x*(-1 + 2*(-1 + x^2)*Log[-1 +
 x] - 4*(-1 + x^2)*Log[x] - 2*Log[1 + x] + 2*x^2*Log[1 + x]))/(6*(-1 + x^2)
^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.23

method	result	size
default	$-\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(6x^3\operatorname{arcsec}(x)\sqrt{\frac{x^2-1}{x^2}}-2\ln\left(1-\frac{1}{x^2}\right)x^4+x^4-4\operatorname{arcsec}(x)x\sqrt{\frac{x^2-1}{x^2}}+4\ln\left(1-\frac{1}{x^2}\right)x^2-x^2-2\ln\left(1-\frac{1}{x^2}\right)\right)}{6(x^2-1)^2}$	101

```
[In] int(x^3*arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*csgn(x*(1-1/x^2)^(1/2))*(6*x^3*arcsec(x)*((x^2-1)/x^2)^(1/2)-2*ln(1-1/
x^2)*x^4+x^4-4*arcsec(x)*x*((x^2-1)/x^2)^(1/2)+4*ln(1-1/x^2)*x^2-x^2-2*ln(1
-1/x^2))/(x^2-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = -\frac{2\,(3\,x^2-2)\sqrt{x^2-1}\,\text{arcsec}\,(x)+x^2-2\,(x^4-2\,x^2+1)\log\,(x^2-1)+4\,(x^4-2\,x^2+1)\log\,(x)-1}{6\,(x^4-2\,x^2+1)}$$

```
[In] integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
[Out] -1/6*(2*(3*x^2 - 2)*sqrt(x^2 - 1)*arcsec(x) + x^2 - 2*(x^4 - 2*x^2 + 1)*log
(x^2 - 1) + 4*(x^4 - 2*x^2 + 1)*log(x) - 1)/(x^4 - 2*x^2 + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \text{Timed out}$$

```
[In] integrate(x**3*asec(x)/(x**2-1)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \int \frac{x^3\,\text{arcsec}\,(x)}{(x^2-1)^{\frac{5}{2}}} \, dx$$

```
[In] integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
[Out] integrate(x^3*arcsec(x)/(x^2 - 1)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = -\frac{(3x^2-2)\arccos\left(\frac{1}{x}\right)}{3(x^2-1)^{\frac{3}{2}}} - \frac{\log(x^2)}{3\,\mathrm{sgn}(x)} + \frac{\log(|x^2-1|)}{3\,\mathrm{sgn}(x)} - \frac{2x^2-1}{6(x^2-1)\mathrm{sgn}(x)}$$

```
[In] integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
[Out] -1/3*(3*x^2 - 2)*arccos(1/x)/(x^2 - 1)^(3/2) - 1/3*log(x^2)/sgn(x) + 1/3*lo
g(abs(x^2 - 1))/sgn(x) - 1/6*(2*x^2 - 1)/((x^2 - 1)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{x^3 \operatorname{acos}\left(\frac{1}{x}\right)}{(x^2-1)^{5/2}}\,dx$$

```
[In] int((x^3*acos(1/x))/(x^2 - 1)^(5/2),x)
[Out] int((x^3*acos(1/x))/(x^2 - 1)^(5/2), x)
```

3.689 $\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 175

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \frac{\sqrt{x^2}(2-3x^2)}{6(-1+x^2)} - \frac{13}{6}\coth^{-1}\left(\sqrt{x^2}\right) - \frac{5x^3\sec^{-1}(x)}{6(-1+x^2)^{3/2}} + \frac{x^5\sec^{-1}(x)}{2(-1+x^2)^{3/2}} - \frac{5x\sec^{-1}(x)}{2\sqrt{-1+x^2}} - \frac{5i\sqrt{x^2}\sec^{-1}(x)\arctan\left(e^{i\sec^{-1}(x)}\right)}{x} + \frac{5i\sqrt{x^2}\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{2x} - \frac{5i\sqrt{x^2}\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{2x}$$

```
[Out] -13/6*arccoth((x^2)^(1/2))-5/6*x^3*arcsec(x)/(x^2-1)^(3/2)+1/2*x^5*arcsec(x
)/(x^2-1)^(3/2)+1/6*(-3*x^2+2)*(x^2)^(1/2)/(x^2-1)-5*I*arcsec(x)*arctan(1/x
+I*(1-1/x^2)^(1/2))*(x^2)^(1/2)/x+5/2*I*polylog(2,-I*(1/x+I*(1-1/x^2)^(1/2)
))*(x^2)^(1/2)/x-5/2*I*polylog(2,I*(1/x+I*(1-1/x^2)^(1/2)))*(x^2)^(1/2)/x-5
/2*x*arcsec(x)/(x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.33, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules

used = {5350, 4790, 4794, 4804, 4266, 2317, 2438, 212, 205, 296, 331}

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = -\frac{5i\sqrt{x^2}\sec^{-1}(x)\arctan\left(e^{i\sec^{-1}(x)}\right)}{x} + \frac{5i\sqrt{x^2}\,\mathrm{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{2x} - \frac{5i\sqrt{x^2}\,\mathrm{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{2x} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{3\sqrt{x^2}}{4} - \frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} + \frac{x\sqrt{x^2}\sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}\,x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{13\sqrt{x^2}\coth^{-1}(x)}{6x}$$

```
[In] Int[(x^6*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] -5/(12*(1 - x^(-2))*Sqrt[x^2]) - (3*Sqrt[x^2])/4 + Sqrt[x^2]/(4*(1 - x^(-2)
)) - (13*Sqrt[x^2]*ArcCoth[x])/(6*x) - (5*Sqrt[x^2]*ArcSec[x])/(6*(1 - x^(-
2))^(3/2)*x) - (5*Sqrt[x^2]*ArcSec[x])/(2*Sqrt[1 - x^(-2)]*x) + (x*Sqrt[x^2
]*ArcSec[x])/(2*(1 - x^(-2))^(3/2)) - ((5*I)*Sqrt[x^2]*ArcSec[x]*ArcTan[E^(
I*ArcSec[x])])/x + (((5*I)/2)*Sqrt[x^2]*PolyLog[2, (-I)*E^(I*ArcSec[x])])/x
 - (((5*I)/2)*Sqrt[x^2]*PolyLog[2, I*E^(I*ArcSec[x])])/x
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4790

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c
*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1
- c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 4794

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Dist[b*c*
(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1
 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Rule 4804

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]], Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5350

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Dist[-Sqrt[x^2]/x, Subst[Int[(e + d*x^2)^p*((a + b*A
rcCos[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{x^2}\text{Subst}\left(\int \frac{\arccos(x)}{x^3(1-x^2)^{5/2}}\,dx, x, \frac{1}{x}\right)}{x} \\
&= \frac{x\sqrt{x^2}\sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}} + \frac{\sqrt{x^2}\text{Subst}\left(\int \frac{1}{x^2(1-x^2)^2}\,dx, x, \frac{1}{x}\right)}{2x} - \frac{\left(5\sqrt{x^2}\right)\text{Subst}\left(\int \frac{\arccos(x)}{x(1-x^2)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2x} \\
&= \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} + \frac{x\sqrt{x^2}\sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}} + \frac{\left(3\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{x^2(1-x^2)}\,dx, x, \frac{1}{x}\right)}{4x} \\
&\quad - \frac{\left(5\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx, x, \frac{1}{x}\right)}{6x} - \frac{\left(5\sqrt{x^2}\right)\text{Subst}\left(\int \frac{\arccos(x)}{x(1-x^2)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2x} \\
&= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} \\
&\quad - \frac{5\sqrt{x^2}\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}\,x} + \frac{x\sqrt{x^2}\sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}} \\
&\quad - \frac{\left(5\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{1}{x}\right)}{12x} + \frac{\left(3\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{1}{x}\right)}{4x} \\
&\quad - \frac{\left(5\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{1}{x}\right)}{2x} - \frac{\left(5\sqrt{x^2}\right)\text{Subst}\left(\int \frac{\arccos(x)}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)}{2x} \\
&= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{13\sqrt{x^2}\coth^{-1}(x)}{6x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} \\
&\quad - \frac{5\sqrt{x^2}\sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}\,x} + \frac{x\sqrt{x^2}\sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}} + \frac{\left(5\sqrt{x^2}\right)\text{Subst}\left(\int x\sec(x)\,dx, x, \sec^{-1}(x)\right)}{2x}
\end{aligned}
$$

$$= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{13\sqrt{x^2}\coth^{-1}(x)}{6x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}x} + \frac{x\sqrt{x^2}\sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{5i\sqrt{x^2}\sec^{-1}(x)\arctan\left(e^{i\sec^{-1}(x)}\right)}{x} - \frac{\left(5\sqrt{x^2}\right)\text{Subst}\left(\int\log\left(1-ie^{ix}\right)dx, x, \sec^{-1}(x)\right)}{2x} + \frac{\left(5\sqrt{x^2}\right)\text{Subst}\left(\int\log\left(1+ie^{ix}\right)dx, x, \sec^{-1}(x)\right)}{2x}$$

$$= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{13\sqrt{x^2}\coth^{-1}(x)}{6x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}x} + \frac{x\sqrt{x^2}\sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{5i\sqrt{x^2}\sec^{-1}(x)\arctan\left(e^{i\sec^{-1}(x)}\right)}{x} + \frac{\left(5i\sqrt{x^2}\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}dx, x, e^{i\sec^{-1}(x)}\right)}{2x} - \frac{\left(5i\sqrt{x^2}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}dx, x, e^{i\sec^{-1}(x)}\right)}{2x}$$

$$= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{13\sqrt{x^2}\coth^{-1}(x)}{6x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}x} + \frac{x\sqrt{x^2}\sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{5i\sqrt{x^2}\sec^{-1}(x)\arctan\left(e^{i\sec^{-1}(x)}\right)}{x} + \frac{5i\sqrt{x^2}\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{2x} - \frac{5i\sqrt{x^2}\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{2x}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 383 vs. $2(175) = 350$.

Time = 1.25 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.19

$$\int \frac{x^6\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx =$$

$$-\frac{x^5\left(22\sec^{-1}(x) + 40\sec^{-1}(x)\cos\left(2\sec^{-1}(x)\right) - 30\sec^{-1}(x)\cos\left(4\sec^{-1}(x)\right) - 30\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)\log\left(1-\right.\right.}{}$$

[In] Integrate[(x^6*ArcSec[x])/(-1 + x^2)^(5/2),x]

[Out] -1/96*(x^5*(22*ArcSec[x] + 40*ArcSec[x]*Cos[2*ArcSec[x]] - 30*ArcSec[x]*Cos[4*ArcSec[x]] - 30*Sqrt[1 - x^(-2)]*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])] + 30*Sqrt[1 - x^(-2)]*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])] + 26*Sqrt[1 - x^(-2)]*Log[Cos[ArcSec[x]/2]] - 26*Sqrt[1 - x^(-2)]*Log[Sin[ArcSec[x]/2]] + 16*Sin[2*ArcSec[x]] - (60*I)*Sqrt[1 - x^(-2)]*PolyLog[2, (-I)*E^(I*ArcSec[x])]*Sin[2*ArcSec[x]]^2 + (60*I)*Sqrt[1 - x^(-2)]*PolyLog[2, I*E^(I*ArcSec[x])]*Sin[2*ArcSec[x]]^2 - 15*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])]*Sin[3*ArcSec[x]] + 15*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])]*Sin[3*ArcSec[x]] + 13*Log[Cos[ArcSec[x]/2]]*Sin[3*ArcSec[x]] - 13*Log[Sin[ArcSec[x]/2]]*Sin[3*ArcSec[x]] - 4*Sin[4*ArcSec[x]] + 15*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])]*Sin[5*ArcSec[x]] - 15*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])]*Sin[5*ArcSec[x]] - 13*Log[Cos[ArcSec[x]/2]]*Sin[5*ArcSec[x]] + 13*Log[Sin[ArcSec[x]/2]]*Sin[5*ArcSec[x]]))/(-1 + x^2)^(3/2)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.83 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.41

method	result
default	$\frac{\sqrt{\frac{x^2-1}{x^2}}\, x^2 \operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(3 \operatorname{arcsec}(x) x^4-3\sqrt{\frac{x^2-1}{x^2}}\, x^3-20 \operatorname{arcsec}(x) x^2+2\sqrt{\frac{x^2-1}{x^2}}\, x+15 \operatorname{arcsec}(x)\right)}{6\left(x^2-1\right)^2}+\frac{i \operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(15 i \operatorname{a}\right.}{}$

[In] int(x^6*arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/6*((x^2-1)/x^2)^(1/2)*x^2/(x^2-1)^2*csgn(x*(1-1/x^2)^(1/2))*(3*arcsec(x)*x^4-3*((x^2-1)/x^2)^(1/2)*x^3-20*arcsec(x)*x^2+2*((x^2-1)/x^2)^(1/2)*x+15*arcsec(x))+1/6*I*csgn(x*(1-1/x^2)^(1/2))*(15*I*arcsec(x)*ln(1+I*(1/x+I*(1-1/x^2)^(1/2)))-15*I*arcsec(x)*ln(1-I*(1/x+I*(1-1/x^2)^(1/2)))+13*I*ln(1/x+I*(1-1/x^2)^(1/2)+1)-13*I*ln(1/x+I*(1-1/x^2)^(1/2)-1)+15*dilog(1+I*(1/x+I*(1-1/x^2)^(1/2)))-15*dilog(1-I*(1/x+I*(1-1/x^2)^(1/2))))

Fricas [F]

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\, dx = \int \frac{x^6 \operatorname{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}}\, dx$$

[In] integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(x^2 - 1)*x^6*arcsec(x)/(x^6 - 3*x^4 + 3*x^2 - 1), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**6*asec(x)/(x**2-1)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{x^6 \operatorname{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
[Out] integrate(x^6*arcsec(x)/(x^2 - 1)^(5/2), x)
```

Giac [F]

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{x^6 \operatorname{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
[Out] integrate(x^6*arcsec(x)/(x^2 - 1)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{x^6 \operatorname{acos}\left(\frac{1}{x}\right)}{(x^2-1)^{5/2}}\,dx$$

```
[In] int((x^6*acos(1/x))/(x^2 - 1)^(5/2),x)
[Out] int((x^6*acos(1/x))/(x^2 - 1)^(5/2), x)
```

3.690 $\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \frac{1}{\sqrt{x^2}} + \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x}$$

```
[Out] 1/(x^2)^(1/2)+arcsec(x)*(x^2-1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {270, 5346, 30}

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \frac{1}{\sqrt{x^2}} + \frac{\sqrt{x^2-1}\sec^{-1}(x)}{x}$$

```
[In] Int[ArcSec[x]/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] 1/Sqrt[x^2] + (Sqrt[-1 + x^2]*ArcSec[x])/x
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x} - \frac{x\int \frac{1}{x^2}\,dx}{\sqrt{x^2}}\\ &= \frac{1}{\sqrt{x^2}} + \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-\frac{1}{x^2}}x + (-1+x^2)\sec^{-1}(x)}{x\sqrt{-1+x^2}}$$

```
[In] Integrate[ArcSec[x]/(x^2*Sqrt[-1 + x^2]),x]
[Out] (Sqrt[1 - x^(-2)]*x + (-1 + x^2)*ArcSec[x])/(x*Sqrt[-1 + x^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.43

method	result	size
default	$\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\sqrt{\frac{x^2-1}{x^2}}\left(\operatorname{arcsec}(x)x^2-\operatorname{arcsec}(x)+\sqrt{\frac{x^2-1}{x^2}}\,x\right)}{x^2-1}$	56

```
[In] int(arcsec(x)/x^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] csgn(x*(1-1/x^2)^(1/2))*((x^2-1)/x^2)^(1/2)/(x^2-1)*(arcsec(x)*x^2-arcsec(x
)+((x^2-1)/x^2)^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \frac{\sqrt{x^2-1}\,\text{arcsec}\,(x)+1}{x}$$

```
[In] integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="fricas")
[Out] (sqrt(x^2 - 1)*arcsec(x) + 1)/x
```

Sympy [F]

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{\text{asec}\,(x)}{x^2\sqrt{(x-1)\,(x+1)}}\,dx$$

```
[In] integrate(asec(x)/x**2/(x**2-1)**(1/2),x)
[Out] Integral(asec(x)/(x**2*sqrt((x - 1)*(x + 1))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \frac{\sqrt{x^2-1}\,\text{arcsec}\,(x)}{x}+\frac{1}{x}$$

```
[In] integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 - 1)*arcsec(x)/x + 1/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(19) = 38.

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \frac{2\,\arccos\left(\frac{1}{x}\right)}{\left(x-\sqrt{x^2-1}\right)^2+1}-\frac{2\,\arctan\left(-x+\sqrt{x^2-1}\right)}{\text{sgn}\,(x)}+\frac{1}{x\text{sgn}\,(x)}$$

```
[In] integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="giac")
[Out] 2*arccos(1/x)/((x - sqrt(x^2 - 1))^2 + 1) - 2*arctan(-x + sqrt(x^2 - 1))/sg
n(x) + 1/(x*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{\mathrm{acos}\left(\frac{1}{x}\right)}{x^2\,\sqrt{x^2-1}}\,dx$$

```
[In] int(acos(1/x)/(x^2*(x^2 - 1)^(1/2)),x)
[Out] int(acos(1/x)/(x^2*(x^2 - 1)^(1/2)), x)
```

3.691 $\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}} dx = -\frac{1}{\sqrt{x^2}}+\frac{\sqrt{x^2}}{6(-1+x^2)}-\frac{11}{6}\coth^{-1}\left(\sqrt{x^2}\right)+\frac{(3-12x^2+8x^4)\csc^{-1}(x)}{3x(-1+x^2)^{3/2}}$$

```
[Out] -11/6*arccoth((x^2)^(1/2))+1/3*(8*x^4-12*x^2+3)*arccsc(x)/x/(x^2-1)^(3/2)-1
/(x^2)^(1/2)+1/6*(x^2)^(1/2)/(x^2-1)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {277, 198, 197, 5347, 12, 1273, 464, 212}

$$\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}} dx = -\frac{11x\text{arctanh}(x)}{6\sqrt{x^2}}-\frac{1}{\sqrt{x^2}} -\frac{\sqrt{x^2}}{6(1-x^2)}+\frac{8x\csc^{-1}(x)}{3\sqrt{x^2-1}}-\frac{4x\csc^{-1}(x)}{3(x^2-1)^{3/2}}+\frac{\csc^{-1}(x)}{x(x^2-1)^{3/2}}$$

```
[In] Int[ArcCsc[x]/(x^2*(-1 + x^2)^(5/2)),x]
```

```
[Out] -(1/Sqrt[x^2]) - Sqrt[x^2]/(6*(1 - x^2)) + ArcCsc[x]/(x*(-1 + x^2)^(3/2)) -
 (4*x*ArcCsc[x])/(3*(-1 + x^2)^(3/2)) + (8*x*ArcCsc[x])/(3*Sqrt[-1 + x^2])
- (11*x*ArcTanh[x])/(6*Sqrt[x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\csc^{-1}(x)}{x\left(-1+x^2\right)^{3/2}}-\frac{4x\csc^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{-1+x^2}}+\frac{x\int\frac{3-12x^2+8x^4}{3x^2\left(1-x^2\right)^2}\,dx}{\sqrt{x^2}}\\
&= \frac{\csc^{-1}(x)}{x\left(-1+x^2\right)^{3/2}}-\frac{4x\csc^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{-1+x^2}}+\frac{x\int\frac{3-12x^2+8x^4}{x^2\left(1-x^2\right)^2}\,dx}{3\sqrt{x^2}}\\
&= -\frac{\sqrt{x^2}}{6\left(1-x^2\right)}+\frac{\csc^{-1}(x)}{x\left(-1+x^2\right)^{3/2}}-\frac{4x\csc^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{-1+x^2}}-\frac{x\int\frac{-6+17x^2}{x^2\left(1-x^2\right)}\,dx}{6\sqrt{x^2}}\\
&= -\frac{1}{\sqrt{x^2}}-\frac{\sqrt{x^2}}{6\left(1-x^2\right)}+\frac{\csc^{-1}(x)}{x\left(-1+x^2\right)^{3/2}}-\frac{4x\csc^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{-1+x^2}}-\frac{(11x)\int\frac{1}{1-x^2}\,dx}{6\sqrt{x^2}}\\
&= -\frac{1}{\sqrt{x^2}}-\frac{\sqrt{x^2}}{6\left(1-x^2\right)}+\frac{\csc^{-1}(x)}{x\left(-1+x^2\right)^{3/2}}-\frac{4x\csc^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{-1+x^2}}-\frac{11x\,\text{arctanh}(x)}{6\sqrt{x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int\frac{\csc^{-1}(x)}{x^2\left(-1+x^2\right)^{5/2}}\,dx=\frac{4(3-12x^2+8x^4)\csc^{-1}(x)+\sqrt{1-\frac{1}{x^2}}x(12-10x^2+11x(-1+x^2)\log(1-x)-1}{12x\left(-1+x^2\right)^{3/2}}$$

```
[In] Integrate[ArcCsc[x]/(x^2*(-1 + x^2)^(5/2)),x]
```

```
[Out] (4*(3 - 12*x^2 + 8*x^4)*ArcCsc[x] + Sqrt[1 - x^(-2)]*x*(12 - 10*x^2 + 11*x*
(-1 + x^2)*Log[1 - x] - 11*x*(-1 + x^2)*Log[1 + x]))/(12*x*(-1 + x^2)^(3/2)
)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.72 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.91

method	result
default	$\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)(\operatorname{arccsc}(x)+i)\left(\sqrt{\frac{x^2-1}{x^2}}\,x+i\right)}{2x}+\frac{\left(\sqrt{\frac{x^2-1}{x^2}}\,x-i\right)\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)(\operatorname{arccsc}(x)-i)}{2x}+\frac{\sqrt{\frac{x^2-1}{x^2}}\,x^2\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(1\right.}{}$

```
[In] int(arccsc(x)/x^2/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/2*csgn(x*(1-1/x^2)^(1/2))*(arccsc(x)+I)*(((x^2-1)/x^2)^(1/2)*x+I)/x+1/2*(
((x^2-1)/x^2)^(1/2)*x-I)/x*csgn(x*(1-1/x^2)^(1/2))*(arccsc(x)-I)+1/6*((x^2-
1)/x^2)^(1/2)*x^2/(x^2-1)^2*csgn(x*(1-1/x^2)^(1/2))*(10*arccsc(x)*x^2+((x^2
-1)/x^2)^(1/2)*x-12*arccsc(x))-11/6*csgn(x*(1-1/x^2)^(1/2))*ln(I/x+(1-1/x^2
)^(1/2)+I)+11/6*csgn(x*(1-1/x^2)^(1/2))*ln(I/x+(1-1/x^2)^(1/2)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\int \frac{\csc^{-1}(x)}{x^2\left(-1+x^2\right)^{5/2}}\,dx = -\frac{10\,x^4-4\,(8\,x^4-12\,x^2+3)\sqrt{x^2-1}\operatorname{arccsc}(x)-22\,x^2+11\,(x^5-2\,x^3+x)\log(x+1)-11\,(x^5-2\,x^3+x)}{12\,(x^5-2\,x^3+x)}$$

```
[In] integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="fricas")
[Out] -1/12*(10*x^4 - 4*(8*x^4 - 12*x^2 + 3)*sqrt(x^2 - 1)*arccsc(x) - 22*x^2 + 1
1*(x^5 - 2*x^3 + x)*log(x + 1) - 11*(x^5 - 2*x^3 + x)*log(x - 1) + 12)/(x^5
 - 2*x^3 + x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(x)}{x^2\left(-1+x^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(acsc(x)/x**2/(x**2-1)**(5/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(56) = 112.

Time = 0.55 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.76

$$\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}}\,dx = \frac{32\,x^4 \arctan\left(1, \sqrt{x+1}\sqrt{x-1}\right) - (x^3-x)\sqrt{x+1}\sqrt{x-1}\left(\frac{2\,(5\,x^2-6)}{x^3-x} + 11\,\log\left(x+\right.\right.}{12\,(x^3-x)}$$

```
[In] integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(32*x^4*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) - (x^3 - x)*sqrt(x + 1)*sq
rt(x - 1)*(2*(5*x^2 - 6)/(x^3 - x) + 11*log(x + 1) - 11*log(x - 1)) - 48*x^
2*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + 12*arctan2(1, sqrt(x + 1)*sqrt(x -
1)))/((x^3 - x)*sqrt(x + 1)*sqrt(x - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.50

$$\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{(5\,x^2-6)x}{(x^2-1)^{\frac{3}{2}}} + \frac{6}{\left(x-\sqrt{x^2-1}\right)^2+1}\right)\arcsin\left(\frac{1}{x}\right) + \frac{2\,\arctan\left(-x+\sqrt{x^2-1}\right)}{\mathrm{sgn}\,(x)} - \frac{11\,\log\left(|x+1|\right)}{12\,\mathrm{sgn}\,(x)} + \frac{11\,\log\left(|x-1|\right)}{12\,\mathrm{sgn}\,(x)} - \frac{5\,x^2-6}{6\,(x^3-x)\mathrm{sgn}\,(x)}$$

```
[In] integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((5*x^2 - 6)*x/(x^2 - 1)^(3/2) + 6/((x - sqrt(x^2 - 1))^2 + 1))*arcsin(
1/x) + 2*arctan(-x + sqrt(x^2 - 1))/sgn(x) - 11/12*log(abs(x + 1))/sgn(x) +
 11/12*log(abs(x - 1))/sgn(x) - 1/6*(5*x^2 - 6)/((x^3 - x)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}}\,dx = \int \frac{\mathrm{asin}\left(\frac{1}{x}\right)}{x^2\,(x^2-1)^{5/2}}\,dx$$

```
[In] int(asin(1/x)/(x^2*(x^2 - 1)^(5/2)),x)
```

```
[Out] int(asin(1/x)/(x^2*(x^2 - 1)^(5/2)), x)
```

3.692 $\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \frac{24\sqrt{-1+x^2}}{x}+\frac{24\csc^{-1}(x)}{\sqrt{x^2}}-\frac{12\sqrt{-1+x^2}\csc^{-1}(x)^2}{x}-\frac{4\csc^{-1}(x)^3}{\sqrt{x^2}}+\frac{\sqrt{-1+x^2}\csc^{-1}(x)^4}{x}$$

```
[Out] 24*arccsc(x)/(x^2)^(1/2)-4*arccsc(x)^3/(x^2)^(1/2)+24*(x^2-1)^(1/2)/x-12*ar
ccsc(x)^2*(x^2-1)^(1/2)/x+arccsc(x)^4*(x^2-1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5351, 4767, 4715, 267}

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \frac{24\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}}{x}+\frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x}-\frac{4\csc^{-1}(x)^3}{\sqrt{x^2}}-\frac{12\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^2}{x}+\frac{24\csc^{-1}(x)}{\sqrt{x^2}}$$

```
[In] Int[ArcCsc[x]^4/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (24*Sqrt[1 - x^(-2)]*Sqrt[x^2])/x + (24*ArcCsc[x])/Sqrt[x^2] - (12*Sqrt[1 -
 x^(-2)]*Sqrt[x^2]*ArcCsc[x]^2)/x - (4*ArcCsc[x]^3)/Sqrt[x^2] + (Sqrt[1 - x
^(-2)]*Sqrt[x^2]*ArcCsc[x]^4)/x
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4767

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5351

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Dist[-Sqrt[x^2]/x, Subst[Int[(e + d*x^2)^p*((a + b*A
rcSin[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{x \arcsin(x)^4}{\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)}{x} \\
&= \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x} - \frac{\left(4\sqrt{x^2}\right)\text{Subst}\left(\int \arcsin(x)^3\,dx, x, \frac{1}{x}\right)}{x} \\
&= -\frac{4\csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x} + \frac{\left(12\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x \arcsin(x)^2}{\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)}{x} \\
&= -\frac{12\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^2}{x} - \frac{4\csc^{-1}(x)^3}{\sqrt{x^2}} \\
&\quad + \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x} + \frac{\left(24\sqrt{x^2}\right)\text{Subst}\left(\int \arcsin(x)\,dx, x, \frac{1}{x}\right)}{x}
\end{aligned}$$

$$= \frac{24\csc^{-1}(x)}{\sqrt{x^2}} - \frac{12\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^2}{x} - \frac{4\csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x} - \frac{\left(24\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)}{x}$$

$$= \frac{24\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}}{x} + \frac{24\csc^{-1}(x)}{\sqrt{x^2}} - \frac{12\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^2}{x} - \frac{4\csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \frac{24(-1+x^2) + 24\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x) - 12(-1+x^2)\csc^{-1}(x)^2 - 4\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x)^3 + (-1+x^2)\csc^{-1}(x}{x\sqrt{-1+x^2}}$$

```
[In] Integrate[ArcCsc[x]^4/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (24*(-1 + x^2) + 24*Sqrt[1 - x^(-2)]*x*ArcCsc[x] - 12*(-1 + x^2)*ArcCsc[x]^
2 - 4*Sqrt[1 - x^(-2)]*x*ArcCsc[x]^3 + (-1 + x^2)*ArcCsc[x]^4)/(x*Sqrt[-1 +
 x^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

method	result
default	$\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\sqrt{\frac{x^2-1}{x^2}}\left(\operatorname{arccsc}(x)^4x^2-\operatorname{arccsc}(x)^4-12\operatorname{arccsc}(x)^2x^2+12\operatorname{arccsc}(x)^2-4\sqrt{\frac{x^2-1}{x^2}}\operatorname{arccsc}(x)^3x+24x^2-24+24\operatorname{arccsc}(x)\right.}{x^2-1}$

```
[In] int(arccsc(x)^4/x^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(x*(1-1/x^2)^(1/2))*((x^2-1)/x^2)^(1/2)/(x^2-1)*(arccsc(x)^4*x^2-arccsc
(x)^4-12*arccsc(x)^2*x^2+12*arccsc(x)^2-4*((x^2-1)/x^2)^(1/2)*arccsc(x)^3*x
+24*x^2-24+24*arccsc(x)*((x^2-1)/x^2)^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.50

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = -\frac{4\,\mathrm{arccsc}\,(x)^3 - \left(\mathrm{arccsc}\,(x)^4 - 12\,\mathrm{arccsc}\,(x)^2 + 24\right)\sqrt{x^2-1} - 24\,\mathrm{arccsc}\,(x)}{x}$$

```
[In] integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(4*arccsc(x)^3 - (arccsc(x)^4 - 12*arccsc(x)^2 + 24)*sqrt(x^2 - 1) - 24*ar
ccsc(x))/x
```

Sympy [F]

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{\mathrm{acsc}^4\,(x)}{x^2\sqrt{(x-1)\,(x+1)}}\,dx$$

```
[In] integrate(acsc(x)**4/x**2/(x**2-1)**(1/2),x)
```

```
[Out] Integral(acsc(x)**4/(x**2*sqrt((x - 1)*(x + 1))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \frac{\sqrt{x^2-1}\,\mathrm{arccsc}\,(x)^4}{x} - 12\sqrt{-\frac{1}{x^2}+1}\,\mathrm{arccsc}\,(x)^2 - \frac{4\,\mathrm{arccsc}\,(x)^3}{x} + 24\sqrt{-\frac{1}{x^2}+1} + \frac{24\,\mathrm{arccsc}\,(x)}{x}$$

```
[In] integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 1)*arccsc(x)^4/x - 12*sqrt(-1/x^2 + 1)*arccsc(x)^2 - 4*arccsc(x)
^3/x + 24*sqrt(-1/x^2 + 1) + 24*arccsc(x)/x
```

Giac [F]

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{\operatorname{arccsc}(x)^4}{\sqrt{x^2-1}x^2}\,dx$$

```
[In] integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(arccsc(x)^4/(sqrt(x^2 - 1)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{\operatorname{asin}\left(\frac{1}{x}\right)^4}{x^2\,\sqrt{x^2-1}}\,dx$$

```
[In] int(asin(1/x)^4/(x^2*(x^2 - 1)^(1/2)),x)
[Out] int(asin(1/x)^4/(x^2*(x^2 - 1)^(1/2)), x)
```

3.693 $\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 133

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx = \frac{\sqrt{-1+x^2}(-2+17x^2)}{64x^4}-\frac{3\sec^{-1}(x)}{8x\sqrt{x^2}}+\frac{9x\sec^{-1}(x)}{64\sqrt{x^2}}+\frac{(-1+x^2)^2\sec^{-1}(x)}{8x^3\sqrt{x^2}}-\frac{3\sqrt{-1+x^2}\sec^{-1}(x)^2}{8x^2}-\frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{4x^4}+\frac{x\sec^{-1}(x)^3}{8\sqrt{x^2}}$$

```
[Out] -1/4*(x^2-1)^(3/2)*arcsec(x)^2/x^4-3/8*arcsec(x)/x/(x^2)^(1/2)+9/64*x*arcse
c(x)/(x^2)^(1/2)+1/8*(x^2-1)^2*arcsec(x)/x^3/(x^2)^(1/2)+1/8*x*arcsec(x)^3/
(x^2)^(1/2)+1/64*(17*x^2-2)*(x^2-1)^(1/2)/x^4-3/8*arcsec(x)^2*(x^2-1)^(1/2)
/x^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {5350, 4744, 4742, 4738, 4724, 327, 222, 4768, 201}

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx = \frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{32\sqrt{x^2}}+\frac{15\sqrt{1-\frac{1}{x^2}}}{64\sqrt{x^2}}-\frac{9\sqrt{x^2}\csc^{-1}(x)}{64x}+\frac{\sqrt{x^2}\sec^{-1}(x)^3}{8x}-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sec^{-1}(x)^2}{4\sqrt{x^2}}-\frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^2}{8\sqrt{x^2}}+\frac{\left(1-\frac{1}{x^2}\right)^2\sqrt{x^2}\sec^{-1}(x)}{8x}-\frac{3\sqrt{x^2}\sec^{-1}(x)}{8x^3}$$

```
[In] Int[((-1 + x^2)^(3/2)*ArcSec[x]^2)/x^5,x]
```

```
[Out] (15*Sqrt[1 - x^(-2)])/(64*Sqrt[x^2]) + (1 - x^(-2))^(3/2)/(32*Sqrt[x^2]) -
(9*Sqrt[x^2]*ArcCsc[x])/(64*x) - (3*Sqrt[x^2]*ArcSec[x])/(8*x^3) + ((1 - x^
(-2))^2*Sqrt[x^2]*ArcSec[x])/(8*x) - (3*Sqrt[1 - x^(-2)]*ArcSec[x]^2)/(8*Sq
rt[x^2]) - ((1 - x^(-2))^(3/2)*ArcSec[x]^2)/(4*Sqrt[x^2]) + (Sqrt[x^2]*ArcS
ec[x]^3)/(8*x)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcCos[c*x])^n/Sqrt[1
 - c^2*x^2], x], x] + Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
```

```
], Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (D
ist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[x*(1 -
c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5350

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Dist[-Sqrt[x^2]/x, Subst[Int[(e + d*x^2)^p*((a + b*A
rcCos[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{x^2}\text{Subst}\left(\int (1-x^2)^{3/2}\arccos(x)^2\,dx,x,\frac{1}{x}\right)}{x}$$

$$= -\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sec^{-1}(x)^2}{4\sqrt{x^2}} - \frac{\sqrt{x^2}\text{Subst}\left(\int x(1-x^2)\arccos(x)\,dx,x,\frac{1}{x}\right)}{2x} - \frac{\left(3\sqrt{x^2}\right)\text{Subst}\left(\int \sqrt{1-x^2}\arccos(x)^2\,dx,x,\frac{1}{x}\right)}{4x}$$

$$= \frac{\left(1-\frac{1}{x^2}\right)^2\sqrt{x^2}\sec^{-1}(x)}{8x} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^2}{8\sqrt{x^2}} - \frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sec^{-1}(x)^2}{4\sqrt{x^2}} + \frac{\sqrt{x^2}\text{Subst}\left(\int (1-x^2)^{3/2}\,dx,x,\frac{1}{x}\right)}{8x} - \frac{\left(3\sqrt{x^2}\right)\text{Subst}\left(\int \frac{\arccos(x)^2}{\sqrt{1-x^2}}\,dx,x,\frac{1}{x}\right)}{8x} - \frac{\left(3\sqrt{x^2}\right)\text{Subst}\left(\int x\arccos(x)\,dx,x,\frac{1}{x}\right)}{4x}$$

$$= \frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{32\sqrt{x^2}} - \frac{3\sqrt{x^2}\sec^{-1}(x)}{8x^3} + \frac{\left(1-\frac{1}{x^2}\right)^2\sqrt{x^2}\sec^{-1}(x)}{8x} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^2}{8\sqrt{x^2}} - \frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sec^{-1}(x)^2}{4\sqrt{x^2}} + \frac{\sqrt{x^2}\sec^{-1}(x)^3}{8x} + \frac{\left(3\sqrt{x^2}\right)\text{Subst}\left(\int\sqrt{1-x^2}\,dx,x,\frac{1}{x}\right)}{32x} - \frac{\left(3\sqrt{x^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-x^2}}\,dx,x,\frac{1}{x}\right)}{8x}$$

$$= \frac{15\sqrt{1-\frac{1}{x^2}}}{64\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{32\sqrt{x^2}} - \frac{3\sqrt{x^2}\sec^{-1}(x)}{8x^3} + \frac{\left(1-\frac{1}{x^2}\right)^2\sqrt{x^2}\sec^{-1}(x)}{8x} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^2}{8\sqrt{x^2}} - \frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sec^{-1}(x)^2}{4\sqrt{x^2}} + \frac{\sqrt{x^2}\sec^{-1}(x)^3}{8x} + \frac{\left(3\sqrt{x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}}\,dx,x,\frac{1}{x}\right)}{64x} - \frac{\left(3\sqrt{x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}}\,dx,x,\frac{1}{x}\right)}{16x}$$

$$= \frac{15\sqrt{1-\frac{1}{x^2}}}{64\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{32\sqrt{x^2}} - \frac{9\sqrt{x^2}\csc^{-1}(x)}{64x} - \frac{3\sqrt{x^2}\sec^{-1}(x)}{8x^3} + \frac{\left(1-\frac{1}{x^2}\right)^2\sqrt{x^2}\sec^{-1}(x)}{8x} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^2}{8\sqrt{x^2}} - \frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sec^{-1}(x)^2}{4\sqrt{x^2}} + \frac{\sqrt{x^2}\sec^{-1}(x)^3}{8x}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.63

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx = \frac{\sqrt{-1+x^2}(32\sec^{-1}(x)^3+4\sec^{-1}(x)\left(-16\cos\left(2\sec^{-1}(x)\right)+\cos\left(4\sec^{-1}(x)\right)\right)}{}$$

```
[In] Integrate[((-1 + x^2)^(3/2)*ArcSec[x]^2)/x^5,x]
```

```
[Out] (Sqrt[-1 + x^2]*(32*ArcSec[x]^3 + 4*ArcSec[x]*(-16*Cos[2*ArcSec[x]] + Cos[4
*ArcSec[x]]) + 32*Sin[2*ArcSec[x]] - Sin[4*ArcSec[x]] + 8*ArcSec[x]^2*(-8*S
in[2*ArcSec[x]] + Sin[4*ArcSec[x]])))/(256*Sqrt[1 - x^(-2)]*x)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

method	result
default	$\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(8\,\mathrm{arcsec}(x)^3x^4-40\,\mathrm{arcsec}(x)^2\sqrt{\frac{x^2-1}{x^2}}\,x^3+17\;\mathrm{arcsec}(x)x^4+16\,\mathrm{arcsec}(x)^2\sqrt{\frac{x^2-1}{x^2}}\,x+17\sqrt{\frac{x^2-1}{x^2}}\,x^3-40\;\mathrm{arcsec}(x)x^2}{64x^4}$

```
[In] int((x^2-1)^(3/2)*arcsec(x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*csgn(x*(1-1/x^2)^(1/2))*(8*arcsec(x)^3*x^4-40*arcsec(x)^2*((x^2-1)/x^2
)^(1/2)*x^3+17*arcsec(x)*x^4+16*arcsec(x)^2*((x^2-1)/x^2)^(1/2)*x+17*((x^2-
1)/x^2)^(1/2)*x^3-40*arcsec(x)*x^2-2*((x^2-1)/x^2)^(1/2)*x+8*arcsec(x))/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.44

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx = \frac{8\,x^4\,\mathrm{arcsec}\,(x)^3+(17\,x^4-40\,x^2+8)\,\mathrm{arcsec}\,(x)-\left(8\,(5\,x^2-2)\,\mathrm{arcsec}\,(x)^2-\right.}{64\,x^4}$$

```
[In] integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/64*(8*x^4*arcsec(x)^3 + (17*x^4 - 40*x^2 + 8)*arcsec(x) - (8*(5*x^2 - 2)*
arcsec(x)^2 - 17*x^2 + 2)*sqrt(x^2 - 1))/x^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate((x**2-1)**(3/2)*asec(x)**2/x**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(-1+x^2)^{3/2} \sec^{-1}(x)^2}{x^5} \, dx = \int \frac{(x^2-1)^{\frac{3}{2}} \operatorname{arcsec}(x)^2}{x^5} \, dx$$

```
[In] integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="maxima")
[Out] integrate((x^2 - 1)^(3/2)*arcsec(x)^2/x^5, x)
```

Giac [F]

$$\int \frac{(-1+x^2)^{3/2} \sec^{-1}(x)^2}{x^5} \, dx = \int \frac{(x^2-1)^{\frac{3}{2}} \operatorname{arcsec}(x)^2}{x^5} \, dx$$

```
[In] integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="giac")
[Out] integrate((x^2 - 1)^(3/2)*arcsec(x)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-1+x^2)^{3/2} \sec^{-1}(x)^2}{x^5} \, dx = \int \frac{\operatorname{acos}\left(\frac{1}{x}\right)^2 (x^2-1)^{3/2}}{x^5} \, dx$$

```
[In] int((acos(1/x)^2*(x^2 - 1)^(3/2))/x^5,x)
[Out] int((acos(1/x)^2*(x^2 - 1)^(3/2))/x^5, x)
```

3.694 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 110

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = \frac{2(1-21x^2)}{27(x^2)^{3/2}} - \frac{4\sqrt{-1+x^2}\sec^{-1}(x)}{3x} - \frac{2(-1+x^2)^{3/2}\sec^{-1}(x)}{9x^3} + \frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}} + \frac{(-1+x^2)\sec^{-1}(x)^2}{3(x^2)^{3/2}} + \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^3}{3x^3}$$

```
[Out] -2/9*(x^2-1)^(3/2)*arcsec(x)/x^3+1/3*(x^2-1)^(3/2)*arcsec(x)^3/x^3+2/27*(-2
1*x^2+1)/x^2/(x^2)^(1/2)+2/3*arcsec(x)^2/(x^2)^(1/2)+1/3*(x^2-1)*arcsec(x)^
2/x^2/(x^2)^(1/2)-4/3*arcsec(x)*(x^2-1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5350, 4768, 4744, 4716, 8}

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = -\frac{14}{9\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)^3}{3x} + \frac{\left(1-\frac{1}{x^2}\right)\sec^{-1}(x)^2}{3\sqrt{x^2}} + \frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}} - \frac{2\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)}{9x} - \frac{4\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{3x} + \frac{2\sqrt{x^2}}{27x^4}$$

```
[In] Int[(Sqrt[-1 + x^2]*ArcSec[x]^3)/x^4,x]
```

```
[Out] -14/(9*Sqrt[x^2]) + (2*Sqrt[x^2])/(27*x^4) - (4*Sqrt[1 - x^(-2)]*Sqrt[x^2]*
ArcSec[x])/(3*x) - (2*(1 - x^(-2))^(3/2)*Sqrt[x^2]*ArcSec[x])/(9*x) + (2*Ar
cSec[x]^2)/(3*Sqrt[x^2]) + ((1 - x^(-2))*ArcSec[x]^2)/(3*Sqrt[x^2]) + ((1 -
 x^(-2))^(3/2)*Sqrt[x^2]*ArcSec[x]^3)/(3*x)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (D
ist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[x*(1 -
c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5350

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Dist[-Sqrt[x^2]/x, Subst[Int[(e + d*x^2)^p*((a + b*A
rcCos[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{x^2}\text{Subst}\left(\int x\sqrt{1-x^2}\arccos(x)^3\,dx, x, \frac{1}{x}\right)}{x}$$

$$= \frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)^3}{3x} + \frac{\sqrt{x^2}\text{Subst}\left(\int \left(1-x^2\right)\arccos(x)^2\,dx, x, \frac{1}{x}\right)}{x}$$

$$
\begin{aligned}
&= \frac{\left(1-\frac{1}{x^2}\right)\sec^{-1}(x)^2}{3\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)^3}{3x} \\
&\quad + \frac{\left(2\sqrt{x^2}\right)\operatorname{Subst}\left(\int x\sqrt{1-x^2}\arccos(x)\,dx, x, \frac{1}{x}\right)}{3x} \\
&\quad + \frac{\left(2\sqrt{x^2}\right)\operatorname{Subst}\left(\int \arccos(x)^2\,dx, x, \frac{1}{x}\right)}{3x}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)}{9x} + \frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)\sec^{-1}(x)^2}{3\sqrt{x^2}} \\
&\quad + \frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)^3}{3x} - \frac{\left(2\sqrt{x^2}\right)\operatorname{Subst}\left(\int (1-x^2)\,dx, x, \frac{1}{x}\right)}{9x} \\
&\quad + \frac{\left(4\sqrt{x^2}\right)\operatorname{Subst}\left(\int \frac{x\arccos(x)}{\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)}{3x}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2}{9\sqrt{x^2}} + \frac{2\sqrt{x^2}}{27x^4} - \frac{4\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{3x} - \frac{2\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)}{9x} + \frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}} \\
&\quad + \frac{\left(1-\frac{1}{x^2}\right)\sec^{-1}(x)^2}{3\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)^3}{3x} - \frac{\left(4\sqrt{x^2}\right)\operatorname{Subst}\left(\int 1\,dx, x, \frac{1}{x}\right)}{3x}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{14}{9\sqrt{x^2}} + \frac{2\sqrt{x^2}}{27x^4} - \frac{4\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{3x} - \frac{2\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)}{9x} \\
&\quad + \frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)\sec^{-1}(x)^2}{3\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)^3}{3x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$
\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx
$$

$$
= \frac{2\sqrt{1-\frac{1}{x^2}}x(1-21x^2) - 6(1-8x^2+7x^4)\sec^{-1}(x) + 9\sqrt{1-\frac{1}{x^2}}x(-1+3x^2)\sec^{-1}(x)^2 + 9(-1+x^2)^2\sec}{27x^3\sqrt{-1+x^2}}
$$

```
[In] Integrate[(Sqrt[-1 + x^2]*ArcSec[x]^3)/x^4,x]
```

```
[Out] (2*Sqrt[1 - x^(-2)]*x*(1 - 21*x^2) - 6*(1 - 8*x^2 + 7*x^4)*ArcSec[x] + 9*Sq
rt[1 - x^(-2)]*x*(-1 + 3*x^2)*ArcSec[x]^2 + 9*(-1 + x^2)^2*ArcSec[x]^3)/(27
*x^3*Sqrt[-1 + x^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.54 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.34

method	result
default	$\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\sqrt{\frac{x^2-1}{x^2}}\left(9\,\mathrm{arcsec}(x)^3x^4+27\,\mathrm{arcsec}(x)^2\sqrt{\frac{x^2-1}{x^2}}\,x^3-18x^2\,\mathrm{arcsec}(x)^3-42\,\mathrm{arcsec}(x)x^4-9\,\mathrm{arcsec}(x)^2\sqrt{\frac{x^2-1}{x^2}}\,x-42\sqrt{\frac{x^2-1}{x^2}}\right.}{27(x^2-1)x^2}$

```
[In] int(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*csgn(x*(1-1/x^2)^(1/2))*((x^2-1)/x^2)^(1/2)/(x^2-1)/x^2*(9*arcsec(x)^3
*x^4+27*arcsec(x)^2*((x^2-1)/x^2)^(1/2)*x^3-18*x^2*arcsec(x)^3-42*arcsec(x)
*x^4-9*arcsec(x)^2*((x^2-1)/x^2)^(1/2)*x-42*((x^2-1)/x^2)^(1/2)*x^3+9*arcse
c(x)^3+48*arcsec(x)*x^2+2*((x^2-1)/x^2)^(1/2)*x-6*arcsec(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = \frac{9\,(3\,x^2-1)\,\mathrm{arcsec}\,(x)^2-42\,x^2+3\,\left(3\,(x^2-1)\,\mathrm{arcsec}\,(x)^3-2\,(7\,x^2-1)\,\mathrm{arcsec}\,(x)\right)\sqrt{x^2-1}+2}{27\,x^3}$$

```
[In] integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(9*(3*x^2 - 1)*arcsec(x)^2 - 42*x^2 + 3*(3*(x^2 - 1)*arcsec(x)^3 - 2*(
7*x^2 - 1)*arcsec(x))*sqrt(x^2 - 1) + 2)/x^3
```

Sympy [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = \int \frac{\sqrt{(x-1)\,(x+1)}\,\mathrm{asec}^3\,(x)}{x^4}\,dx$$

```
[In] integrate(asec(x)**3*(x**2-1)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1))*asec(x)**3/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = \frac{(x^2-1)^{\frac{3}{2}}\operatorname{arcsec}(x)^3}{3\,x^3} + \frac{(3\,x^2-1)\operatorname{arcsec}(x)^2}{3\,x^3} - \frac{2\left((21\,x^2-1)\sqrt{x+1}\sqrt{x-1}+3\,(7\,x^4-8\,x^2+1)\arctan\left(\sqrt{x+1}\sqrt{x-1}\right)\right)}{27\,\sqrt{x+1}\sqrt{x-1}x^3}$$

```
[In] integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="maxima")
[Out] 1/3*(x^2 - 1)^(3/2)*arcsec(x)^3/x^3 + 1/3*(3*x^2 - 1)*arcsec(x)^2/x^3 - 2/2
7*((21*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1) + 3*(7*x^4 - 8*x^2 + 1)*arctan(sqrt
(x + 1)*sqrt(x - 1)))/(sqrt(x + 1)*sqrt(x - 1)*x^3)
```

Giac [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = \int \frac{\sqrt{x^2-1}\operatorname{arcsec}(x)^3}{x^4}\,dx$$

```
[In] integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(x^2 - 1)*arcsec(x)^3/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = \int \frac{\operatorname{acos}\left(\frac{1}{x}\right)^3\sqrt{x^2-1}}{x^4}\,dx$$

```
[In] int((acos(1/x)^3*(x^2 - 1)^(1/2))/x^4,x)
[Out] int((acos(1/x)^3*(x^2 - 1)^(1/2))/x^4, x)
```

3.695 $\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right) dx = -\frac{\sqrt{2}a\sqrt{\frac{-a+x}{a+x}}}{\sqrt{\frac{a}{a+x}}} + (a+x)\arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)$$

```
[Out] (a+x)*arcsin(((-a+x)/(a+x))^(1/2))-a*2^(1/2)*((-a+x)/(a+x))^(1/2)/(a/(a+x))
^(1/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 125 vs. 2(55) = 110.

Time = 0.52 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.27, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4924, 12, 1973, 1972, 21, 393, 222}

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right) dx = \frac{a^2\sqrt{\frac{a+x}{a}}\sqrt{\frac{x}{a}+1}\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)}{a+x} + x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right) - \sqrt{2}a\sqrt{\frac{a}{a+x}}\sqrt{-\frac{a-x}{a+x}}\sqrt{\frac{a+x}{a}}\sqrt{\frac{x}{a}+1}$$

```
[In] Int[ArcSin[Sqrt[(-a + x)/(a + x)]],x]
```

```
[Out] -(Sqrt[2]*a*Sqrt[a/(a + x)]*Sqrt[-((a - x)/(a + x))]*Sqrt[(a + x)/a]*Sqrt[1
 + x/a]) + x*ArcSin[Sqrt[-((a - x)/(a + x))]] + (a^2*Sqrt[(a + x)/a]*Sqrt[1
 + x/a]*ArcSin[Sqrt[-((a - x)/(a + x))]])/(a + x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rule 4924

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\text{integral} = x \arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right) - \int \frac{x\left(\frac{a}{a+x}\right)^{3/2}}{\sqrt{2}a\sqrt{\frac{-a+x}{a+x}}}\,dx$$

$$= x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{\int \frac{x\left(\frac{a}{a+x}\right)^{3/2}}{\sqrt{\frac{-a+x}{a+x}}}\,dx}{\sqrt{2}a}$$

$$= x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{\left(\sqrt{\frac{a}{a+x}}\sqrt{1+\frac{x}{a}}\right)\int \frac{x}{\sqrt{\frac{-a+x}{a+x}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx}{\sqrt{2}a}$$

$$= x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right) - \left(a\sqrt{\frac{a}{a+x}}\sqrt{1+\frac{x}{a}}\right)\text{Subst}\left(\int \frac{1+x^2}{\sqrt{\frac{1}{1-x^2}}\left(-1+x^2\right)^2}\,dx, x, \sqrt{\frac{-a+x}{a+x}}\right)$$

$$= x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{\left(a^2\sqrt{\frac{a+x}{a}}\sqrt{1+\frac{x}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}\left(1+x^2\right)}{\left(-1+x^2\right)^2}\,dx, x, \sqrt{\frac{-a+x}{a+x}}\right)}{a+x}$$

$$= x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{\left(a^2\sqrt{\frac{a+x}{a}}\sqrt{1+\frac{x}{a}}\right)\text{Subst}\left(\int \frac{1+x^2}{\left(1-x^2\right)^{3/2}}\,dx, x, \sqrt{\frac{-a+x}{a+x}}\right)}{a+x}$$

$$= -\frac{\sqrt{2}a^2\sqrt{-\frac{a-x}{a+x}}\sqrt{\frac{a+x}{a}}\sqrt{1+\frac{x}{a}}}{\sqrt{\frac{a}{a+x}}(a+x)} + x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right) + \frac{\left(a^2\sqrt{\frac{a+x}{a}}\sqrt{1+\frac{x}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \sqrt{\frac{-a+x}{a+x}}\right)}{a+x}$$

$$= -\frac{\sqrt{2}a^2\sqrt{-\frac{a-x}{a+x}}\sqrt{\frac{a+x}{a}}\sqrt{1+\frac{x}{a}}}{\sqrt{\frac{a}{a+x}}(a+x)} + x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right) + \frac{a^2\sqrt{\frac{a+x}{a}}\sqrt{1+\frac{x}{a}}\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)}{a+x}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.80

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = x\arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right) + \frac{\sqrt{\frac{a}{a+x}}\left(2a-2x+\sqrt{2}\sqrt{a}\sqrt{-a+x}\arctan\left(\frac{\sqrt{-a+x}}{\sqrt{2}\sqrt{a}}\right)\right)}{\sqrt{2}\sqrt{\frac{-a+x}{a+x}}}$$

```
[In] Integrate[ArcSin[Sqrt[(-a + x)/(a + x)]],x]
[Out] x*ArcSin[Sqrt[(-a + x)/(a + x)]] + (Sqrt[a/(a + x)]*(2*a - 2*x + Sqrt[2]*Sq
rt[a]*Sqrt[-a + x]*ArcTan[Sqrt[-a + x]/(Sqrt[2]*Sqrt[a])]))/(Sqrt[2]*Sqrt[(
-a + x)/(a + x)])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.56

method	result	size
default	$x \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{\sqrt{-a+x}\,\sqrt{2}\,\sqrt{\frac{a}{a+x}}\,\left(-2\sqrt{-a+x}+\sqrt{a}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-a+x}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{2\sqrt{-\frac{a-x}{a+x}}}$	86
parts	$x \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{\sqrt{-a+x}\,\sqrt{2}\,\sqrt{\frac{a}{a+x}}\,\left(-2\sqrt{-a+x}+\sqrt{a}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-a+x}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{2\sqrt{-\frac{a-x}{a+x}}}$	86

```
[In] int(arcsin(((-a+x)/(a+x))^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*arcsin(((-a+x)/(a+x))^(1/2))+1/2/(-(a-x)/(a+x))^(1/2)*(-a+x)^(1/2)*2^(1/2
)*(a/(a+x))^(1/2)*(-2*(-a+x)^(1/2)+a^(1/2)*2^(1/2)*arctan(1/2*(-a+x)^(1/2)*
2^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = -\sqrt{2}(a+x)\sqrt{-\frac{a-x}{a+x}}\sqrt{\frac{a}{a+x}}+(a+x)\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

```
[In] integrate(arcsin(((-a+x)/(a+x))^(1/2)),x, algorithm="fricas")
[Out] -sqrt(2)*(a + x)*sqrt(-(a - x)/(a + x))*sqrt(a/(a + x)) + (a + x)*arcsin(sq
rt(-(a - x)/(a + x)))
```

Sympy [F]

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = \int \operatorname{asin}\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx$$

```
[In] integrate(asin(((-a+x)/(a+x))**(1/2)),x)
[Out] Integral(asin(sqrt((-a + x)/(a + x))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(49) = 98$.

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.87

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = a\left(\frac{2\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)}{\frac{a-x}{a+x}+1} + \frac{\sqrt{\frac{a-x}{a+x}+1}}{\sqrt{-\frac{a-x}{a+x}}+1} + \frac{\sqrt{\frac{a-x}{a+x}+1}}{\sqrt{-\frac{a-x}{a+x}}-1}\right)$$

```
[In] integrate(arcsin(((-a+x)/(a+x))^(1/2)),x, algorithm="maxima")
[Out] a*(2*arcsin(sqrt(-(a - x)/(a + x)))/((a - x)/(a + x) + 1) + sqrt((a - x)/(a
 + x) + 1)/(sqrt(-(a - x)/(a + x)) + 1) + sqrt((a - x)/(a + x) + 1)/(sqrt(-
(a - x)/(a + x)) - 1))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.71

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = x\arcsin\left(\frac{\sqrt{-a^2+x^2}\operatorname{sgn}(a+x)}{a+x}\right) + \frac{\sqrt{2}\left(\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt{-a^2+ax}}{2a}\right) - \sqrt{2}\left(a\arctan\left(\frac{i|a|}{a}\right) - 2i|a|\right) - 2\sqrt{-a^2+ax}\right)a}{2|a|}$$

```
[In] integrate(arcsin(((-a+x)/(a+x))^(1/2)),x, algorithm="giac")
[Out] x*arcsin(sqrt(-a^2 + x^2)*sgn(a + x)/(a + x)) + 1/2*sqrt(2)*(sqrt(2)*a*arct
an(1/2*sqrt(2)*sqrt(-a^2 + a*x)/a) - sqrt(2)*(a*arctan(I*abs(a)/a) - 2*I*ab
s(a)) - 2*sqrt(-a^2 + a*x))*a/abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = \int \operatorname{asin}\left(\sqrt{-\frac{a-x}{a+x}}\right)\,dx$$

```
[In] int(asin((-(a - x)/(a + x))^(1/2)),x)
[Out] int(asin((-(a - x)/(a + x))^(1/2)), x)
```

3.696 $\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right) - a\text{arctanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

```
[Out] x*arctan(((-a+x)/(a+x))^(1/2))-a*arctanh(((-a+x)/(a+x))^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5311, 12, 1983, 214}

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right) - a\text{arctanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

```
[In] Int[ArcTan[Sqrt[(-a + x)/(a + x)]],x]
```

```
[Out] x*ArcTan[Sqrt[-((a - x)/(a + x))]] - a*ArcTanh[Sqrt[-((a - x)/(a + x))]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1983

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n),
Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(
b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1/
n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-\int\frac{a}{2\sqrt{\frac{-a+x}{a+x}}(a+x)}\,dx\\
&= x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-\frac{1}{2}a\int\frac{1}{\sqrt{\frac{-a+x}{a+x}}(a+x)}\,dx\\
&= x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-\left(2a^2\right)\text{Subst}\left(\int\frac{1}{2a-2ax^2}\,dx,x,\sqrt{\frac{-a+x}{a+x}}\right)\\
&= x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\text{arctanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$
\int\arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx=x\arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)-\frac{a\sqrt{-a+x}\,\text{arctanh}\left(\frac{\sqrt{a+x}}{\sqrt{-a+x}}\right)}{\sqrt{\frac{-a+x}{a+x}}\sqrt{a+x}}
$$

```
[In] Integrate[ArcTan[Sqrt[(-a + x)/(a + x)]],x]
```

```
[Out] x*ArcTan[Sqrt[(-a + x)/(a + x)]] - (a*Sqrt[-a + x]*ArcTanh[Sqrt[a + x]/Sqrt
[-a + x]])/(Sqrt[(-a + x)/(a + x)]*Sqrt[a + x])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

method	result	size
default	$x \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{(a-x)a\ln\left(x+\sqrt{-a^2+x^2}\right)}{2\sqrt{-\frac{a-x}{a+x}}\,\sqrt{-(a-x)(a+x)}}$	66
parts	$x \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{(a-x)a\ln\left(x+\sqrt{-a^2+x^2}\right)}{2\sqrt{-\frac{a-x}{a+x}}\,\sqrt{-(a-x)(a+x)}}$	66

```
[In] int(arctan(((-a+x)/(a+x))^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*arctan(((-a+x)/(a+x))^(1/2))+1/2*(a-x)*a*ln(x+(-a^2+x^2)^(1/2))/(-(a-x)/(
a+x))^(1/2)/(-(a-x)*(a+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-\frac{1}{2}\,a\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+\frac{1}{2}\,a\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)$$

```
[In] integrate(arctan(((-a+x)/(a+x))^(1/2)),x, algorithm="fricas")
[Out] x*arctan(sqrt(-(a - x)/(a + x))) - 1/2*a*log(sqrt(-(a - x)/(a + x)) + 1) +
1/2*a*log(sqrt(-(a - x)/(a + x)) - 1)
```

Sympy [F]

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = \int \operatorname{atan}\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx$$

```
[In] integrate(atan(((-a+x)/(a+x))**(1/2)),x)
[Out] Integral(atan(sqrt((-a + x)/(a + x))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(36) = 72$.

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.22

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx$$
$$=\frac{1}{2}\,a\left(\frac{4\,\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)}{\frac{a-x}{a+x}+1}-2\,\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)\right)$$

```
[In] integrate(arctan(((-a+x)/(a+x))^(1/2)),x, algorithm="maxima")
[Out] 1/2*a*(4*arctan(sqrt(-(a - x)/(a + x)))/((a - x)/(a + x) + 1) - 2*arctan(sq
rt(-(a - x)/(a + x))) - log(sqrt(-(a - x)/(a + x)) + 1) + log(sqrt(-(a - x)
/(a + x)) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx=\frac{1}{2}\,a\log\left(\left|-x+\sqrt{-a^2+x^2}\right|\right)\operatorname{sgn}(a+x)+x\arctan\left(\frac{\sqrt{-a^2+x^2}\operatorname{sgn}(a+x)}{a+x}\right)$$

```
[In] integrate(arctan(((-a+x)/(a+x))^(1/2)),x, algorithm="giac")
[Out] 1/2*a*log(abs(-x + sqrt(-a^2 + x^2)))*sgn(a + x) + x*arctan(sqrt(-a^2 + x^2
)*sgn(a + x)/(a + x))
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx=x\,\mathrm{atan}\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\,\mathrm{atanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

```
[In] int(atan((-(a - x)/(a + x))^(1/2)),x)
[Out] x*atan((-(a - x)/(a + x))^(1/2)) - a*atanh((-(a - x)/(a + x))^(1/2))
```

3.697 $\int \frac{\arctan(x)}{(1+x)^3} dx$

Optimal result

Integrand size = 8, antiderivative size = 39

$$\int \frac{\arctan(x)}{(1+x)^3} dx = -\frac{1}{4(1+x)} - \frac{\arctan(x)}{2(1+x)^2} + \frac{1}{4}\log(1+x) - \frac{1}{8}\log\left(1+x^2\right)$$

```
[Out] -1/4/(1+x)-1/2*arctan(x)/(1+x)^2+1/4*ln(1+x)-1/8*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4972, 724, 815, 266}

$$\int \frac{\arctan(x)}{(1+x)^3} dx = -\frac{\arctan(x)}{2(x+1)^2} - \frac{1}{8}\log\left(x^2+1\right) - \frac{1}{4(x+1)} + \frac{1}{4}\log(x+1)$$

```
[In] Int[ArcTan[x]/(1 + x)^3,x]
```

```
[Out] -1/4*1/(1 + x) - ArcTan[x]/(2*(1 + x)^2) + Log[1 + x]/4 - Log[1 + x^2]/8
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
```

```
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arctan(x)}{2(1+x)^2}+\frac{1}{2}\int\frac{1}{(1+x)^2\,(1+x^2)}\,dx\\
&= -\frac{1}{4(1+x)}-\frac{\arctan(x)}{2(1+x)^2}+\frac{1}{4}\int\frac{1-x}{(1+x)\,(1+x^2)}\,dx\\
&= -\frac{1}{4(1+x)}-\frac{\arctan(x)}{2(1+x)^2}+\frac{1}{4}\int\left(\frac{1}{1+x}-\frac{x}{1+x^2}\right)dx\\
&= -\frac{1}{4(1+x)}-\frac{\arctan(x)}{2(1+x)^2}+\frac{1}{4}\log(1+x)-\frac{1}{4}\int\frac{x}{1+x^2}\,dx\\
&= -\frac{1}{4(1+x)}-\frac{\arctan(x)}{2(1+x)^2}+\frac{1}{4}\log(1+x)-\frac{1}{8}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int\frac{\arctan(x)}{(1+x)^3}\,dx=\frac{1}{8}\left(-\frac{2}{1+x}-\frac{4\arctan(x)}{(1+x)^2}+2\log(1+x)-\log\left(1+x^2\right)\right)$$

```
[In] Integrate[ArcTan[x]/(1 + x)^3,x]
[Out] (-2/(1 + x) - (4*ArcTan[x])/(1 + x)^2 + 2*Log[1 + x] - Log[1 + x^2])/8
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result
default	$-\frac{1}{4(1+x)}-\frac{\arctan(x)}{2(1+x)^{2}}+\frac{\ln(1+x)}{4}-\frac{\ln(x^{2}+1)}{8}$
parts	$-\frac{1}{4(1+x)}-\frac{\arctan(x)}{2(1+x)^{2}}+\frac{\ln(1+x)}{4}-\frac{\ln(x^{2}+1)}{8}$
parallelrisch	$\frac{2\ln(1+x)x^{2}-\ln(x^{2}+1)x^{2}-2+4\ln(1+x)x-2x\ln(x^{2}+1)+2\ln(1+x)-\ln(x^{2}+1)-2x-4\arctan(x)}{8(1+x)^{2}}$
risch	$\frac{i\ln(ix+1)}{4(1+x)^{2}}-\frac{i(2i\ln(1+x)x^{2}-i\ln(x^{2}+1)x^{2}+4i\ln(1+x)x-2i\ln(x^{2}+1)x+2i\ln(1+x)-i\ln(x^{2}+1)-2ix-2i+2\ln(-ix+1))}{8(1+x)^{2}}$

```
[In] int(arctan(x)/(1+x)^3,x,method=_RETURNVERBOSE)
[Out] -1/4/(1+x)-1/2*arctan(x)/(1+x)^2+1/4*ln(1+x)-1/8*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \frac{\arctan(x)}{(1+x)^3}\,dx = -\frac{(x^2+2x+1)\log(x^2+1)-2(x^2+2x+1)\log(x+1)+2x+4\arctan(x)+2}{8(x^2+2x+1)}$$

```
[In] integrate(arctan(x)/(1+x)^3,x, algorithm="fricas")
[Out] -1/8*((x^2 + 2*x + 1)*log(x^2 + 1) - 2*(x^2 + 2*x + 1)*log(x + 1) + 2*x + 4
*arctan(x) + 2)/(x^2 + 2*x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(31) = 62.

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.92

$$\int \frac{\arctan(x)}{(1+x)^3}\,dx = \frac{2x^2\log(x+1)}{8x^2+16x+8}-\frac{x^2\log(x^2+1)}{8x^2+16x+8}+\frac{4x\log(x+1)}{8x^2+16x+8}-\frac{2x\log(x^2+1)}{8x^2+16x+8}-\frac{2x}{8x^2+16x+8}+\frac{2\log(x+1)}{8x^2+16x+8}-\frac{\log(x^2+1)}{8x^2+16x+8}-\frac{4\operatorname{atan}(x)}{8x^2+16x+8}-\frac{2}{8x^2+16x+8}$$

```
[In] integrate(atan(x)/(1+x)**3,x)
```

```
[Out] 2*x**2*log(x + 1)/(8*x**2 + 16*x + 8) - x**2*log(x**2 + 1)/(8*x**2 + 16*x +
 8) + 4*x*log(x + 1)/(8*x**2 + 16*x + 8) - 2*x*log(x**2 + 1)/(8*x**2 + 16*x
 + 8) - 2*x/(8*x**2 + 16*x + 8) + 2*log(x + 1)/(8*x**2 + 16*x + 8) - log(x*
*2 + 1)/(8*x**2 + 16*x + 8) - 4*atan(x)/(8*x**2 + 16*x + 8) - 2/(8*x**2 + 1
6*x + 8)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(x)}{(1+x)^3}\,dx = -\frac{1}{4\,(x+1)} - \frac{\arctan(x)}{2\,(x+1)^2} - \frac{1}{8}\,\log\left(x^2+1\right) + \frac{1}{4}\,\log\left(x+1\right)$$

```
[In] integrate(arctan(x)/(1+x)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(x + 1) - 1/2*arctan(x)/(x + 1)^2 - 1/8*log(x^2 + 1) + 1/4*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{\arctan(x)}{(1+x)^3}\,dx = -\frac{1}{4\,(x+1)} - \frac{\arctan(x)}{2\,(x+1)^2} - \frac{1}{8}\,\log\left(x^2+1\right) + \frac{1}{4}\,\log\left(|x+1|\right)$$

```
[In] integrate(arctan(x)/(1+x)^3,x, algorithm="giac")
```

```
[Out] -1/4/(x + 1) - 1/2*arctan(x)/(x + 1)^2 - 1/8*log(x^2 + 1) + 1/4*log(abs(x +
 1))
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(x)}{(1+x)^3}\,dx = \frac{\ln\left(x+1\right)}{4} - \frac{\ln\left(x^2+1\right)}{8} - \frac{\frac{x}{4}+\frac{\mathrm{atan}(x)}{2}+\frac{1}{4}}{{\left(x+1\right)}^2}$$

```
[In] int(atan(x)/(x + 1)^3,x)
```

```
[Out] log(x + 1)/4 - log(x^2 + 1)/8 - (x/4 + atan(x)/2 + 1/4)/(x + 1)^2
```

3.698 $\int -\frac{\arctan(a-x)}{a+x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 122

$$\int -\frac{\arctan(a-x)}{a+x}\,dx = \arctan(a-x)\log\left(\frac{2}{1-i(a-x)}\right) - \arctan(a-x)\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a-x)}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2,1+\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right)$$

```
[Out] arctan(a-x)*ln(2/(1-I*(a-x)))-arctan(a-x)*ln(-2*(a+x)/(I-2*a)/(1-I*(a-x)))-
1/2*I*polylog(2,1-2/(1-I*(a-x)))+1/2*I*polylog(2,1+2*(a+x)/(I-2*a)/(1-I*(a-
x)))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5155, 4966, 2449, 2352, 2497}

$$\int -\frac{\arctan(a-x)}{a+x}\,dx = \arctan(a-x)\log\left(\frac{2}{1-i(a-x)}\right) - \arctan(a-x)\log\left(-\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a-x)}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2,\frac{2(a+x)}{(i-2a)(1-i(a-x))}+1\right)$$

```
[In] Int[-(ArcTan[a - x]/(a + x)),x]
```

```
[Out] ArcTan[a - x]*Log[2/(1 - I*(a - x))] - ArcTan[a - x]*Log[(-2*(a + x))/((I -
 2*a)*(1 - I*(a - x)))] - (I/2)*PolyLog[2, 1 - 2/(1 - I*(a - x))] + (I/2)*P
olyLog[2, 1 + (2*(a + x))/((I - 2*a)*(1 - I*(a - x)))]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\arctan(x)}{2a-x}\,dx, x, a-x\right)$$

$$= \arctan(a-x)\log\left(\frac{2}{1-i(a-x)}\right)-\arctan(a-x)\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right) - \text{Subst}\left(\int\frac{\log\left(\frac{2}{1-ix}\right)}{1+x^2}\,dx,x,a-x\right)+\text{Subst}\left(\int\frac{\log\left(\frac{2(2a-x)}{(-i+2a)(1-ix)}\right)}{1+x^2}\,dx,x,a-x\right)$$

$$= \arctan(a-x)\log\left(\frac{2}{1-i(a-x)}\right)-\arctan(a-x)\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2,1+\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right)-i\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-i(a-x)}\right)$$

$$= \arctan(a-x)\log\left(\frac{2}{1-i(a-x)}\right)-\arctan(a-x)\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a-x)}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1+\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.86

$$\int-\frac{\arctan(a-x)}{a+x}\,dx = -\frac{1}{2}i\left(-\log(1+i(a-x))\log\left(\frac{a+x}{-i+2a}\right) + \log(1-ia+ix)\log\left(\frac{a+x}{i+2a}\right)+\text{PolyLog}\left(2,\frac{i+a-x}{i+2a}\right) - \text{PolyLog}\left(2,\frac{i-a+x}{i-2a}\right)\right)$$

```
[In] Integrate[-(ArcTan[a - x]/(a + x)),x]
```

```
[Out] (-1/2*I)*(-(Log[1 + I*(a - x)]*Log[(a + x)/(-I + 2*a)]) + Log[1 - I*a + I*x
]*Log[(a + x)/(I + 2*a)] + PolyLog[2, (I + a - x)/(I + 2*a)] - PolyLog[2, (
I - a + x)/(I - 2*a)])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\ln(a+x)\arctan(a-x)+\frac{i\ln(a+x)\ln\left(\frac{i+a-x}{2a+i}\right)}{2}-\frac{i\ln(a+x)\ln\left(\frac{i-a+x}{i-2a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{i+a-x}{2a+i}\right)}{2}-\frac{i\,\mathrm{dilog}(}{}$
default	$-\ln(a+x)\arctan(a-x)+\frac{i\ln(a+x)\ln\left(\frac{i+a-x}{2a+i}\right)}{2}-\frac{i\ln(a+x)\ln\left(\frac{i-a+x}{i-2a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{i+a-x}{2a+i}\right)}{2}-\frac{i\,\mathrm{dilog}(}{}$
parts	$-\ln(a+x)\arctan(a-x)+\frac{i\ln(a+x)\ln\left(\frac{i+a-x}{2a+i}\right)}{2}-\frac{i\ln(a+x)\ln\left(\frac{i-a+x}{i-2a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{i+a-x}{2a+i}\right)}{2}-\frac{i\,\mathrm{dilog}(}{}$
risch	$-\frac{i\,\mathrm{dilog}\left(\frac{ia+ix}{2ia-1}\right)}{2}-\frac{i\ln(-ia+ix+1)\ln\left(\frac{ia+ix}{2ia-1}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{-ia-ix}{-2ia-1}\right)}{2}+\frac{i\ln(ia-ix+1)\ln\left(\frac{-ia-ix}{-2ia-1}\right)}{2}$

```
[In] int(-arctan(a-x)/(a+x),x,method=_RETURNVERBOSE)
[Out] -ln(a+x)*arctan(a-x)+1/2*I*ln(a+x)*ln((I+a-x)/(2*a+I))-1/2*I*ln(a+x)*ln((I-
a+x)/(I-2*a))+1/2*I*dilog((I+a-x)/(2*a+I))-1/2*I*dilog((I-a+x)/(I-2*a))
```

Fricas [F]

$$\int -\frac{\arctan(a-x)}{a+x}\,dx=\int -\frac{\arctan(a-x)}{a+x}\,dx$$

```
[In] integrate(-arctan(a-x)/(a+x),x, algorithm="fricas")
[Out] integral(arctan(-a + x)/(a + x), x)
```

Sympy [F]

$$\int -\frac{\arctan(a-x)}{a+x}\,dx=-\int \frac{\operatorname{atan}(a-x)}{a+x}\,dx$$

```
[In] integrate(-atan(a-x)/(a+x),x)
[Out] -Integral(atan(a - x)/(a + x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.97

$$\begin{aligned}\int -\frac{\arctan(a-x)}{a+x}\,dx=&-\frac{1}{2}\arctan\left(\frac{a+x}{4\,a^2+1},\frac{2\,(a^2+ax)}{4\,a^2+1}\right)\log\left(a^2-2\,ax+x^2+1\right)\\&+\frac{1}{2}\arctan(-a+x)\log\left(\frac{a^2+2\,ax+x^2}{4\,a^2+1}\right)\\&-\frac{1}{2}i\,\mathrm{Li}_2\left(-\frac{-i\,a+i\,x+1}{2i\,a-1}\right)+\frac{1}{2}i\,\mathrm{Li}_2\left(-\frac{-i\,a+i\,x-1}{2i\,a+1}\right)\end{aligned}$$

```
[In] integrate(-arctan(a-x)/(a+x),x, algorithm="maxima")
```

```
[Out] -1/2*arctan2((a + x)/(4*a^2 + 1), 2*(a^2 + a*x)/(4*a^2 + 1))*log(a^2 - 2*a*
x + x^2 + 1) + 1/2*arctan(-a + x)*log((a^2 + 2*a*x + x^2)/(4*a^2 + 1)) - 1/
2*I*dilog(-(-I*a + I*x + 1)/(2*I*a - 1)) + 1/2*I*dilog(-(-I*a + I*x - 1)/(2
*I*a + 1))
```

Giac [F]

$$\int -\frac{\arctan(a-x)}{a+x}\,dx = \int -\frac{\arctan\left(a-x\right)}{a+x}\,dx$$

```
[In] integrate(-arctan(a-x)/(a+x),x, algorithm="giac")
```

```
[Out] integrate(-arctan(a - x)/(a + x), x)
```

Mupad [F(-1)]

Timed out.

$$\int -\frac{\arctan(a-x)}{a+x}\,dx = -\int \frac{\operatorname{atan}(a-x)}{a+x}\,dx$$

```
[In] int(-atan(a - x)/(a + x),x)
```

```
[Out] -int(atan(a - x)/(a + x), x)
```

3.699 $\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 28

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = -\frac{\sqrt{x^2}\arcsin\left(\sqrt{1-x^2}\right)^2}{2x}$$

```
[Out] -1/2*arcsin((-x^2+1)^(1/2))^2*(x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4918, 4737}

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = -\frac{\sqrt{x^2}\arcsin\left(\sqrt{1-x^2}\right)^2}{2x}$$

```
[In] Int[ArcSin[Sqrt[1 - x^2]]/Sqrt[1 - x^2],x]
```

```
[Out] -1/2*(Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2)/x
```

Rule 4737

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

Rule 4918

```
Int[ArcSin[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
:> Dist[Sqrt[(-b)*x^2]/(b*x), Subst[Int[ArcSin[x]^n/Sqrt[1 - x^2], x], x, S
qrt[1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{\arcsin(x)}{\sqrt{1-x^2}}\,dx, x, \sqrt{1-x^2}\right)}{x}\\ &= -\frac{\sqrt{x^2}\arcsin\left(\sqrt{1-x^2}\right)^2}{2x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = -\frac{\sqrt{x^2}\arcsin\left(\sqrt{1-x^2}\right)^2}{2x}$$

```
[In] Integrate[ArcSin[Sqrt[1 - x^2]]/Sqrt[1 - x^2],x]
[Out] -1/2*(Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2)/x
```

Maple [F]

$$\int \frac{\arcsin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}dx$$

```
[In] int(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)
[Out] int(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.50

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\arcsin\left(\sqrt{-x^2+1}\right)^2$$

```
[In] integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*arcsin(sqrt(-x^2 + 1))^2
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = \frac{x\,\mathrm{asin}^2\,(x)}{2\sqrt{x^2}} + \mathrm{asin}\,(x)\,\mathrm{asin}\left(\sqrt{1-x^2}\right)$$

```
[In] integrate(asin((-x**2+1)**(1/2))/(-x**2+1)**(1/2),x)
[Out] x*asin(x)**2/(2*sqrt(x**2)) + asin(x)*asin(sqrt(1 - x**2))
```

Maxima [F]

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = \int \frac{\arcsin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsin(sqrt(-x^2 + 1))/sqrt(-x^2 + 1), x)
```

Giac [F]

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = \int \frac{\arcsin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsin(sqrt(-x^2 + 1))/sqrt(-x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = \int \frac{\mathrm{asin}\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx$$

```
[In] int(asin((1 - x^2)^(1/2))/(1 - x^2)^(1/2),x)
[Out] int(asin((1 - x^2)^(1/2))/(1 - x^2)^(1/2), x)
```

3.700 $\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} \, dx = \sqrt{1+x^2} \arctan\left(\sqrt{1+x^2}\right) - \frac{1}{2} \log\left(2+x^2\right)$$

```
[Out] -1/2*ln(x^2+2)+arctan((x^2+1)^(1/2))*(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {267, 5315, 266}

$$\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} \, dx = \sqrt{x^2+1} \arctan\left(\sqrt{x^2+1}\right) - \frac{1}{2} \log\left(x^2+2\right)$$

```
[In] Int[(x*ArcTan[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

```
[Out] Sqrt[1 + x^2]*ArcTan[Sqrt[1 + x^2]] - Log[2 + x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \sqrt{1+x^2}\arctan\left(\sqrt{1+x^2}\right) - \int \frac{x}{2+x^2}\,dx\\ &= \sqrt{1+x^2}\arctan\left(\sqrt{1+x^2}\right) - \frac{1}{2}\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x\arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \sqrt{1+x^2}\arctan\left(\sqrt{1+x^2}\right) - \frac{1}{2}\log\left(2+x^2\right)$$

```
[In] Integrate[(x*ArcTan[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
[Out] Sqrt[1 + x^2]*ArcTan[Sqrt[1 + x^2]] - Log[2 + x^2]/2
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{\ln(x^2+2)}{2}+\arctan\left(\sqrt{x^2+1}\right)\sqrt{x^2+1}$	26
default	$-\frac{\ln(x^2+2)}{2}+\arctan\left(\sqrt{x^2+1}\right)\sqrt{x^2+1}$	26

```
[In] int(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x^2+2)+arctan((x^2+1)^(1/2))*(x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right) - \frac{1}{2}\log\left(x^2+2\right)$$

```
[In] integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \sqrt{x^2+1}\,\mathrm{atan}\left(\sqrt{x^2+1}\right) - \frac{\log\left(x^2+2\right)}{2}$$

```
[In] integrate(x*atan((x**2+1)**(1/2))/(x**2+1)**(1/2),x)
[Out] sqrt(x**2 + 1)*atan(sqrt(x**2 + 1)) - log(x**2 + 2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right) - \frac{1}{2}\log\left(x^2+2\right)$$

```
[In] integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right) - \frac{1}{2}\log\left(x^2+2\right)$$

```
[In] integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \operatorname{atan}\left(\sqrt{x^2+1}\right)\,\sqrt{x^2+1} - \frac{\ln\left(x^2+2\right)}{2}$$

```
[In] int((x*atan((x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2),x)
[Out] atan((x^2 + 1)^(1/2))*(x^2 + 1)^(1/2) - log(x^2 + 2)/2
```

3.701 $\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx=-\frac{\sqrt{1+x}}{3(1-x)}+\frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[Out] 2/3*arcsin(x)/(1-x)^(3/2)-1/6*arctanh(1/2*2^(1/2)*(1+x)^(1/2))*2^(1/2)-1/3*
(1+x)^(1/2)/(1-x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4827, 641, 44, 65, 212}

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx=\frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{3\sqrt{2}}-\frac{\sqrt{x+1}}{3(1-x)}$$

```
[In] Int[ArcSin[x]/(1 - x)^(5/2),x]
```

```
[Out] -1/3*Sqrt[1 + x]/(1 - x) + (2*ArcSin[x])/(3*(1 - x)^(3/2)) - ArcTanh[Sqrt[1
 + x]/Sqrt[2]]/(3*Sqrt[2])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4827

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{2}{3}\int\frac{1}{(1-x)^{3/2}\sqrt{1-x^2}}\,dx\\
&= \frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{2}{3}\int\frac{1}{(1-x)^2\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{3(1-x)}+\frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{1}{6}\int\frac{1}{(1-x)\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{3(1-x)}+\frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{1}{3}\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+x}\right)\\
&= -\frac{\sqrt{1+x}}{3(1-x)}+\frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)}{3\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \frac{1}{6}\left(-\frac{2\left(\sqrt{1-x^2}-2\arcsin(x)\right)}{(1-x)^{3/2}}-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-x^2}}{\sqrt{2-2x}}\right)\right)$$

[In] Integrate[ArcSin[x]/(1 - x)^(5/2),x]

[Out] ((-2*(Sqrt[1 - x^2] - 2*ArcSin[x]))/(1 - x)^(3/2) - Sqrt[2]*ArcTanh[Sqrt[1 - x^2]/Sqrt[2 - 2*x]])/6

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$\frac{2\arcsin(x)}{3(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}\left(\sqrt{2}\ \text{arctanh}\left(\frac{\sqrt{2}}{\sqrt{1+x}}\right)(1-x)+2\sqrt{1+x}\right)}{6\sqrt{1-x}\,\sqrt{-(1-x)^2+2-2x}}$	70
default	$\frac{2\arcsin(x)}{3(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}\left(\sqrt{2}\ \text{arctanh}\left(\frac{\sqrt{2}}{\sqrt{1+x}}\right)(1-x)+2\sqrt{1+x}\right)}{6\sqrt{1-x}\,\sqrt{-(1-x)^2+2-2x}}$	70

[In] int(arcsin(x)/(1-x)^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/3*arcsin(x)/(1-x)^(3/2)-1/6/(1-x)^(1/2)*(1+x)^(1/2)*(2^(1/2)*arctanh(2^(1/2)/(1+x)^(1/2))*(1-x)+2*(1+x)^(1/2))/(-(1-x)^2+2-2*x)^(1/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(40) = 80.

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \frac{\sqrt{2}(x^2-2\,x+1)\log\left(-\frac{x^2+2\,\sqrt{2}\sqrt{-x^2+1}\sqrt{-x+1}+2\,x-3}{x^2-2\,x+1}\right)-4\,\sqrt{-x+1}\left(\sqrt{-x^2+1}-2\,\arcsin(\right.}{12\,(x^2-2\,x+1)}$$

[In] integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="fricas")

[Out] 1/12*(sqrt(2)*(x^2 - 2*x + 1)*log(-(x^2 + 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(-x + 1) + 2*x - 3)/(x^2 - 2*x + 1)) - 4*sqrt(-x + 1)*(sqrt(-x^2 + 1) - 2*arcsin(x)))/(x^2 - 2*x + 1)

Sympy [F]

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \int \frac{\operatorname{asin}(x)}{(1-x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(asin(x)/(1-x)**(5/2),x)
[Out] Integral(asin(x)/(1 - x)**(5/2), x)
```

Maxima [F]

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \int \frac{\arcsin(x)}{(-x+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="maxima")
[Out] -2/3*(3*(x - 1)*sqrt(-x + 1)*integrate(1/3*sqrt(x + 1)*x^2/(x^5 - x^4 - x^3
 + x^2 + (x^3 - x^2 - x + 1)*e^(log(x + 1) + log(-x + 1))), x) + arctan2(x,
 sqrt(x + 1)*sqrt(-x + 1)))/((x - 1)*sqrt(-x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{x+1}}{\sqrt{2}+\sqrt{x+1}}\right) + \frac{\sqrt{x+1}}{3\,(x-1)} - \frac{2\arcsin(x)}{3\,(x-1)\sqrt{-x+1}}$$

```
[In] integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="giac")
[Out] 1/12*sqrt(2)*log((sqrt(2) - sqrt(x + 1))/(sqrt(2) + sqrt(x + 1))) + 1/3*sqr
t(x + 1)/(x - 1) - 2/3*arcsin(x)/((x - 1)*sqrt(-x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \int \frac{\operatorname{asin}(x)}{(1-x)^{5/2}}\,dx$$

```
[In] int(asin(x)/(1 - x)^(5/2),x)
[Out] int(asin(x)/(1 - x)^(5/2), x)
```

3.702 $\int (-1+x)^{5/2} \csc^{-1}(x) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 82

$$\int (-1+x)^{5/2} \csc^{-1}(x) \, dx = \frac{4x\sqrt{-1+x^2}(83-19x+3x^2)}{105\sqrt{-1+x}\sqrt{x^2}} + \frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x) + \frac{4x\text{arctanh}\left(\frac{\sqrt{-1+x^2}}{\sqrt{-1+x}}\right)}{7\sqrt{x^2}}$$

```
[Out] 2/7*(-1+x)^(7/2)*arccsc(x)+4/7*x*arctanh((x^2-1)^(1/2)/(-1+x)^(1/2))/(x^2)^
(1/2)+4/105*x*(3*x^2-19*x+83)*(x^2-1)^(1/2)/(-1+x)^(1/2)/(x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.71, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5335, 1588, 906, 90, 65, 213}

$$\int (-1+x)^{5/2} \csc^{-1}(x) \, dx = \frac{4\sqrt{x+1}\sqrt{x-1}\text{arctanh}\left(\sqrt{x+1}\right)}{7\sqrt{1-\frac{1}{x^2}}x} + \frac{4(x+1)^3\sqrt{x-1}}{35\sqrt{1-\frac{1}{x^2}}x} - \frac{20(x+1)^2\sqrt{x-1}}{21\sqrt{1-\frac{1}{x^2}}x} + \frac{4(x+1)\sqrt{x-1}}{\sqrt{1-\frac{1}{x^2}}x} + \frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

```
[In] Int[(-1 + x)^(5/2)*ArcCsc[x],x]
```

```
[Out] (4*Sqrt[-1 + x]*(1 + x))/(Sqrt[1 - x^(-2)]*x) - (20*Sqrt[-1 + x]*(1 + x)^2)
/(21*Sqrt[1 - x^(-2)]*x) + (4*Sqrt[-1 + x]*(1 + x)^3)/(35*Sqrt[1 - x^(-2)]*
x) + (2*(-1 + x)^(7/2)*ArcCsc[x])/7 + (4*Sqrt[-1 + x]*Sqrt[1 + x]*ArcTanh[S
qrt[1 + x]])/(7*Sqrt[1 - x^(-2)]*x)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 906

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*
x)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)+\frac{2}{7}\int\frac{(-1+x)^{7/2}}{\sqrt{1-\frac{1}{x^2}}x^2}\,dx$$

$$=\frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)+\frac{\left(2\sqrt{-1+x^2}\right)\int\frac{(-1+x)^{7/2}}{x\sqrt{-1+x^2}}\,dx}{7\sqrt{1-\frac{1}{x^2}}x}$$

$$=\frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)+\frac{\left(2\sqrt{-1+x}\sqrt{1+x}\right)\int\frac{(-1+x)^3}{x\sqrt{1+x}}\,dx}{7\sqrt{1-\frac{1}{x^2}}x}$$

$$=\frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)+\frac{\left(2\sqrt{-1+x}\sqrt{1+x}\right)\int\left(\frac{7}{\sqrt{1+x}}-\frac{1}{x\sqrt{1+x}}-5\sqrt{1+x}+(1+x)^{3/2}\right)\,dx}{7\sqrt{1-\frac{1}{x^2}}x}$$

$$=\frac{4\sqrt{-1+x}(1+x)}{\sqrt{1-\frac{1}{x^2}}x}-\frac{20\sqrt{-1+x}(1+x)^2}{21\sqrt{1-\frac{1}{x^2}}x}+\frac{4\sqrt{-1+x}(1+x)^3}{35\sqrt{1-\frac{1}{x^2}}x}+\frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)-\frac{\left(2\sqrt{-1+x}\sqrt{1+x}\right)\int\frac{1}{x\sqrt{1+x}}\,dx}{7\sqrt{1-\frac{1}{x^2}}x}$$

$$=\frac{4\sqrt{-1+x}(1+x)}{\sqrt{1-\frac{1}{x^2}}x}-\frac{20\sqrt{-1+x}(1+x)^2}{21\sqrt{1-\frac{1}{x^2}}x}+\frac{4\sqrt{-1+x}(1+x)^3}{35\sqrt{1-\frac{1}{x^2}}x}+\frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)-\frac{\left(4\sqrt{-1+x}\sqrt{1+x}\right)\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\sqrt{1+x}\right)}{7\sqrt{1-\frac{1}{x^2}}x}$$

$$=\frac{4\sqrt{-1+x}(1+x)}{\sqrt{1-\frac{1}{x^2}}x}-\frac{20\sqrt{-1+x}(1+x)^2}{21\sqrt{1-\frac{1}{x^2}}x}+\frac{4\sqrt{-1+x}(1+x)^3}{35\sqrt{1-\frac{1}{x^2}}x}+\frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)+\frac{4\sqrt{-1+x}\sqrt{1+x}\,\text{arctanh}\left(\sqrt{1+x}\right)}{7\sqrt{1-\frac{1}{x^2}}x}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int (-1+x)^{5/2} \csc^{-1}(x)\,dx = \frac{4\sqrt{1-\frac{1}{x^2}}x(83-19x+3x^2)}{105\sqrt{-1+x}} + \frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x) + \frac{4}{7}\text{arctanh}\left(\frac{\sqrt{1-\frac{1}{x^2}}x}{\sqrt{-1+x}}\right)$$

```
[In] Integrate[(-1 + x)^(5/2)*ArcCsc[x],x]
[Out] (4*Sqrt[1 - x^(-2)]*x*(83 - 19*x + 3*x^2))/(105*Sqrt[-1 + x]) + (2*(-1 + x)
^(7/2)*ArcCsc[x])/7 + (4*ArcTanh[(Sqrt[1 - x^(-2)]*x)/Sqrt[-1 + x]])/7
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{2(-1+x)^{\frac{7}{2}}\,\text{arccsc}(x)}{7}+\frac{4\sqrt{-1+x}\,\sqrt{1+x}\,\left(3(-1+x)^2\sqrt{1+x}-13(-1+x)\sqrt{1+x}+15\,\text{arctanh}\left(\sqrt{1+x}\right)+67\sqrt{1+x}\right)}{105\sqrt{\frac{(-1+x)(1+x)}{x^2}}\,x}$	76
default	$\frac{2(-1+x)^{\frac{7}{2}}\,\text{arccsc}(x)}{7}+\frac{4\sqrt{-1+x}\,\sqrt{1+x}\,\left(3(-1+x)^2\sqrt{1+x}-13(-1+x)\sqrt{1+x}+15\,\text{arctanh}\left(\sqrt{1+x}\right)+67\sqrt{1+x}\right)}{105\sqrt{\frac{(-1+x)(1+x)}{x^2}}\,x}$	76

```
[In] int((-1+x)^(5/2)*arccsc(x),x,method=_RETURNVERBOSE)
[Out] 2/7*(-1+x)^(7/2)*arccsc(x)+4/105*(-1+x)^(1/2)*(1+x)^(1/2)*(3*(-1+x)^2*(1+x)
^(1/2)-13*(-1+x)*(1+x)^(1/2)+15*arctanh((1+x)^(1/2))+67*(1+x)^(1/2))/((-1+x
)*(1+x)/x^2)^(1/2)/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(62) = 124$.

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.52

$$\int (-1+x)^{5/2}\csc^{-1}(x)\,dx = \frac{2\left(15\,(x^4-4\,x^3+6\,x^2-4\,x+1)\sqrt{x-1}\,\text{arccsc}\,(x)+2\,(3\,x^2-19\,x+83)\sqrt{x^2-1}\sqrt{x-}\right.}{105\,(x-1)}$$

```
[In] integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="fricas")
```

```
[Out] 2/105*(15*(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)*sqrt(x - 1)*arccsc(x) + 2*(3*x^2
- 19*x + 83)*sqrt(x^2 - 1)*sqrt(x - 1) + 15*(x - 1)*log((x^2 + sqrt(x^2 - 1
)*sqrt(x - 1) - 1)/(x^2 - 1)) - 15*(x - 1)*log(-(x^2 - sqrt(x^2 - 1)*sqrt(x
 - 1) - 1)/(x^2 - 1)))/(x - 1)
```

Sympy [F(-1)]

Timed out.

$$\int (-1+x)^{5/2}\csc^{-1}(x)\,dx = \text{Timed out}$$

```
[In] integrate((-1+x)**(5/2)*acsc(x),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.78 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.41

$$\int (-1+x)^{5/2}\csc^{-1}(x)\,dx = \frac{4}{35}\,(x+1)^{\frac{5}{2}} - \frac{20}{21}\,(x+1)^{\frac{3}{2}} + \frac{2}{7}\left(x^3\arctan\left(1, \sqrt{x+1}\sqrt{x-1}\right) - 3\,x^2\arctan\left(1, \sqrt{x+1}\sqrt{x-1}\right) + 3\,x\arctan\left(1, \sqrt{x+1}\sqrt{x-1}\right) - \text{arc}\right. + 4\sqrt{x+1} + \frac{2}{7}\,\log\left(\sqrt{x+1}+1\right) - \frac{2}{7}\,\log\left(\sqrt{x+1}-1\right)$$

```
[In] integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="maxima")
```

```
[Out] 4/35*(x + 1)^(5/2) - 20/21*(x + 1)^(3/2) + 2/7*(x^3*arctan2(1, sqrt(x + 1)*
sqrt(x - 1)) - 3*x^2*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + 3*x*arctan2(1, s
qrt(x + 1)*sqrt(x - 1)) - arctan2(1, sqrt(x + 1)*sqrt(x - 1)))*sqrt(x - 1)
+ 4*sqrt(x + 1) + 2/7*log(sqrt(x + 1) + 1) - 2/7*log(sqrt(x + 1) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(62) = 124.

Time = 0.44 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.78

$$\int (-1+x)^{5/2} \csc^{-1}(x)\, dx = \frac{2}{35}\left(5(x-1)^{\frac{7}{2}} + 21(x-1)^{\frac{5}{2}} + 35(x-1)^{\frac{3}{2}} + 35\sqrt{x-1}\right)\arcsin\left(\frac{1}{x}\right) - \frac{2}{5}\left(3(x-1)^{\frac{5}{2}} + 10(x-1)^{\frac{3}{2}} + 15\sqrt{x-1}\right)\arcsin\left(\frac{1}{x}\right) + 2\left((x-1)^{\frac{3}{2}} + 3\sqrt{x-1}\right)\arcsin\left(\frac{1}{x}\right) - 2\sqrt{x-1}\arcsin\left(\frac{1}{x}\right) + \frac{4\left(3(x+1)^{\frac{5}{2}} - 4(x+1)^{\frac{3}{2}} + 21\sqrt{x+1}\right)}{105\,\mathrm{sgn}\left((x-1)^{\frac{3}{2}} + \sqrt{x-1}\right)} - \frac{4\left((x+1)^{\frac{3}{2}} + \sqrt{x+1}\right)}{5\,\mathrm{sgn}\left((x-1)^{\frac{3}{2}} + \sqrt{x-1}\right)} + \frac{2\log\left(\sqrt{x+1}+1\right)}{7\,\mathrm{sgn}\left((x-1)^{\frac{3}{2}} + \sqrt{x-1}\right)} - \frac{2\log\left(\sqrt{x+1}-1\right)}{7\,\mathrm{sgn}\left((x-1)^{\frac{3}{2}} + \sqrt{x-1}\right)} + \frac{4\sqrt{x+1}}{\mathrm{sgn}\left((x-1)^{\frac{3}{2}} + \sqrt{x-1}\right)}$$

```
[In] integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="giac")
```

```
[Out] 2/35*(5*(x - 1)^(7/2) + 21*(x - 1)^(5/2) + 35*(x - 1)^(3/2) + 35*sqrt(x - 1
))*arcsin(1/x) - 2/5*(3*(x - 1)^(5/2) + 10*(x - 1)^(3/2) + 15*sqrt(x - 1))*
arcsin(1/x) + 2*((x - 1)^(3/2) + 3*sqrt(x - 1))*arcsin(1/x) - 2*sqrt(x - 1)
*arcsin(1/x) + 4/105*(3*(x + 1)^(5/2) - 4*(x + 1)^(3/2) + 21*sqrt(x + 1))/s
gn((x - 1)^(3/2) + sqrt(x - 1)) - 4/5*((x + 1)^(3/2) + sqrt(x + 1))/sgn((x
- 1)^(3/2) + sqrt(x - 1)) + 2/7*log(sqrt(x + 1) + 1)/sgn((x - 1)^(3/2) + sq
rt(x - 1)) - 2/7*log(sqrt(x + 1) - 1)/sgn((x - 1)^(3/2) + sqrt(x - 1)) + 4*
sqrt(x + 1)/sgn((x - 1)^(3/2) + sqrt(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int (-1+x)^{5/2} \csc^{-1}(x)\, dx = \int \mathrm{asin}\left(\frac{1}{x}\right)\,(x-1)^{5/2}\, dx$$

```
[In] int(asin(1/x)*(x - 1)^(5/2),x)
```

```
[Out] int(asin(1/x)*(x - 1)^(5/2), x)
```

3.703 $\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 49

$$\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx = -\frac{2}{3}\arcsin\left(\frac{\cosh(x)}{\sqrt{2}}\right)+\frac{1}{6}\text{sech}(x)\sqrt{1-\sinh^2(x)} + \arcsin(\sinh(x))\tanh(x)-\frac{1}{3}\arcsin(\sinh(x))\tanh^3(x)$$

```
[Out] -2/3*arcsin(1/2*cosh(x)*2^(1/2))+1/6*sech(x)*(1-sinh(x)^2)^(1/2)+arcsin(sin
h(x))*tanh(x)-1/3*arcsin(sinh(x))*tanh(x)^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3852, 4928, 12, 4442, 462, 222}

$$\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx = -\frac{2}{3}\arcsin\left(\frac{\cosh(x)}{\sqrt{2}}\right)-\frac{1}{3}\tanh^3(x)\arcsin(\sinh(x)) + \tanh(x)\arcsin(\sinh(x))+\frac{1}{6}\sqrt{2-\cosh^2(x)}\text{sech}(x)$$

```
[In] Int[ArcSin[Sinh[x]]*Sech[x]^4,x]
```

```
[Out] (-2*ArcSin[Cosh[x]/Sqrt[2]])/3 + (Sqrt[2 - Cosh[x]^2]*Sech[x])/6 + ArcSin[S
inh[x]]*Tanh[x] - (ArcSin[Sinh[x]]*Tanh[x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4442

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 4928

```
Int[((a_.) + ArcSin[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcSin[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/Sqr
t[1 - u^2]), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
eQ[{c, d, m}, x]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \arcsin(\sinh(x))\tanh(x) - \frac{1}{3}\arcsin(\sinh(x))\tanh^3(x) - \int \frac{(2+\cosh(2x))\text{sech}(x)\tanh(x)}{3\sqrt{1-\sinh^2(x)}}\,dx \\ &= \arcsin(\sinh(x))\tanh(x) - \frac{1}{3}\arcsin(\sinh(x))\tanh^3(x) - \frac{1}{3}\int \frac{(2+\cosh(2x))\text{sech}(x)\tanh(x)}{\sqrt{1-\sinh^2(x)}}\,dx\end{aligned}$$

$$= \arcsin(\sinh(x))\tanh(x) - \frac{1}{3}\arcsin(\sinh(x))\tanh^3(x) - \frac{1}{3}\text{Subst}\left(\int \frac{1+2x^2}{x^2\sqrt{2-x^2}}\,dx, x, \cosh(x)\right)$$

$$= \frac{1}{6}\sqrt{2-\cosh^2(x)}\text{sech}(x) + \arcsin(\sinh(x))\tanh(x) - \frac{1}{3}\arcsin(\sinh(x))\tanh^3(x) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}}\,dx, x, \cosh(x)\right)$$

$$= -\frac{2}{3}\arcsin\left(\frac{\cosh(x)}{\sqrt{2}}\right) + \frac{1}{6}\sqrt{2-\cosh^2(x)}\text{sech}(x) + \arcsin(\sinh(x))\tanh(x) - \frac{1}{3}\arcsin(\sinh(x))\tanh^3(x)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

$$\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx = \frac{1}{12}\left(8i\log\left(i\sqrt{2}\cosh(x) + \sqrt{3-\cosh(2x)}\right) + \sqrt{6-2\cosh(2x)}\text{sech}(x) + 4\arcsin(\sinh(x))(2+\cosh(2x))\text{sech}^2(x)\tanh(x)\right)$$

```
[In] Integrate[ArcSin[Sinh[x]]*Sech[x]^4,x]
```

```
[Out] ((8*I)*Log[I*Sqrt[2]*Cosh[x] + Sqrt[3 - Cosh[2*x]]] + Sqrt[6 - 2*Cosh[2*x]]
*Sech[x] + 4*ArcSin[Sinh[x]]*(2 + Cosh[2*x])*Sech[x]^2*Tanh[x])/12
```

Maple [F]

$$\int \arcsin(\sinh(x))\,\text{sech}(x)^4\,dx$$

```
[In] int(arcsin(sinh(x))*sech(x)^4,x)
```

```
[Out] int(arcsin(sinh(x))*sech(x)^4,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. 2(40) = 80.

Time = 0.28 (sec) , antiderivative size = 519, normalized size of antiderivative = 10.59

$$\int \arcsin(\sinh(x))\mathrm{sech}^4(x)\,dx = \text{Too large to display}$$

```
[In] integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
+ 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(-(
cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4
*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)
^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 +
 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + co
sh(x))*sinh(x) + 1)*arctan(sqrt(2)*(3*cosh(x)^2 + 6*cosh(x)*sinh(x) + 3*sin
h(x)^2 - 1)*sqrt(-(cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 6*(cosh(x)^2
 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x))*sinh(x) + 1)) + 8
*(3*cosh(x)^2 + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 + 1)*arctan(sqrt(2)*(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-(cosh(x)^2 + sinh(x)^2 - 3)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(cosh(x)^4 + 4*cosh(x)*sinh(x)
^3 + sinh(x)^4 + 6*(cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 4*(cosh(x)^3 -
 3*cosh(x))*sinh(x) + 1)))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3
*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*si
nh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(co
sh(x)^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \arcsin(\sinh(x))\mathrm{sech}^4(x)\,dx = \text{Timed out}$$

```
[In] integrate(asin(sinh(x))*sech(x)**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx = \int \arcsin\left(\sinh\left(x\right)\right)\text{sech}\left(x\right)^4\,dx$$

```
[In] integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(4*(3*e^(2*x) + 1)*arctan2(e^(2*x) - 1, sqrt(e^(2*x) + 2*e^x - 1)*sqrt
(-e^(2*x) + 2*e^x + 1)) + 3*(e^(6*x) + 3*e^(4*x) + 3*e^(2*x) + 1)*integrate
(16/3*(3*e^(4*x) + e^(2*x))*e^(1/2*log(e^(2*x) + 2*e^x - 1) + 1/2*log(-e^(2
*x) + 2*e^x + 1))/((e^(8*x) - 4*e^(6*x) - 10*e^(4*x) - 4*e^(2*x) + 1)*e^(lo
g(e^(2*x) + 2*e^x - 1) + log(-e^(2*x) + 2*e^x + 1)) + e^(12*x) - 6*e^(10*x)
 - e^(8*x) + 12*e^(6*x) - e^(4*x) - 6*e^(2*x) + 1), x))/(e^(6*x) + 3*e^(4*x
) + 3*e^(2*x) + 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 218, normalized size of antiderivative = 4.45

$$\begin{aligned}
&\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx \\
&= -\frac{16\left(-8i\sqrt{2}\arctan\left(-i\right) - 3\sqrt{2} + 32\arctan\left(-i\right) - 3i\right)}{96i\sqrt{2} - 384} \\
&\quad + \frac{\sqrt{2} + \frac{2\sqrt{2} - \sqrt{-e^{(4x)} + 6e^{(2x)} - 1}}{e^{(2x)} - 3}}{6\left(\frac{\sqrt{2}\left(2\sqrt{2} - \sqrt{-e^{(4x)} + 6e^{(2x)} - 1}\right)}{e^{(2x)} - 3} + \frac{\left(2\sqrt{2} - \sqrt{-e^{(4x)} + 6e^{(2x)} - 1}\right)^2}{\left(e^{(2x)} - 3\right)^2} + 1\right)} \\
&\quad - \frac{4\left(3e^{(2x)} + 1\right)\arcsin\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)}{3\left(e^{(2x)} + 1\right)^3} \\
&\quad - \frac{4}{3}\arctan\left(-2\sqrt{2} - \frac{3\left(2\sqrt{2} - \sqrt{-e^{(4x)} + 6e^{(2x)} - 1}\right)}{e^{(2x)} - 3}\right)
\end{aligned}$$

```
[In] integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="giac")
```

```
[Out] -16*(-8*I*sqrt(2)*arctan(-I) - 3*sqrt(2) + 32*arctan(-I) - 3*I)/(96*I*sqrt(
2) - 384) + 1/6*(sqrt(2) + (2*sqrt(2) - sqrt(-e^(4*x) + 6*e^(2*x) - 1))/(e^
(2*x) - 3))/(sqrt(2)*(2*sqrt(2) - sqrt(-e^(4*x) + 6*e^(2*x) - 1))/(e^(2*x)
- 3) + (2*sqrt(2) - sqrt(-e^(4*x) + 6*e^(2*x) - 1))^2/(e^(2*x) - 3)^2 + 1)
- 4/3*(3*e^(2*x) + 1)*arcsin(1/2*(e^(2*x) - 1)*e^(-x))/(e^(2*x) + 1)^3 - 4/
3*arctan(-2*sqrt(2) - 3*(2*sqrt(2) - sqrt(-e^(4*x) + 6*e^(2*x) - 1))/(e^(2*
x) - 3))
```

Mupad [F(-1)]

Timed out.

$$\int \arcsin(\sinh(x))\operatorname{sech}^4(x)\,dx = \int \frac{\operatorname{asin}(\sinh(x))}{\cosh(x)^4}\,dx$$

```
[In] int(asin(sinh(x))/cosh(x)^4,x)
[Out] int(asin(sinh(x))/cosh(x)^4, x)
```

3.704 $\int \cot^{-1}(\cosh(x)) \coth(x)\mathbf{csch}^3(x) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \cot^{-1}(\cosh(x)) \coth(x)\text{csch}^3(x) \, dx = \frac{\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{6\sqrt{2}} + \frac{\coth(x)}{6} - \frac{1}{3}\cot^{-1}(\cosh(x))\text{csch}^3(x)$$

```
[Out] 1/6*coth(x)-1/3*arccot(cosh(x))*csch(x)^3+1/12*arctanh(1/2*2^(1/2)*tanh(x))
*2^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2686, 30, 5316, 12, 464, 212}

$$\int \cot^{-1}(\cosh(x)) \coth(x)\text{csch}^3(x) \, dx = \frac{\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{6\sqrt{2}} + \frac{\coth(x)}{6} - \frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

```
[In] Int[ArcCot[Cosh[x]]*Coth[x]*Csch[x]^3,x]
```

```
[Out] ArcTanh[Tanh[x]/Sqrt[2]]/(6*Sqrt[2]) + Coth[x]/6 - (ArcCot[Cosh[x]]*Csch[x]
^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{3}\cot^{-1}(\cosh(x))\operatorname{csch}^3(x)+\int\frac{2\operatorname{csch}^2(x)}{3(-3-\cosh(2x))}\,dx\\ &= -\frac{1}{3}\cot^{-1}(\cosh(x))\operatorname{csch}^3(x)+\frac{2}{3}\int\frac{\operatorname{csch}^2(x)}{-3-\cosh(2x)}\,dx\end{aligned}$$

$$= -\frac{1}{3}\cot^{-1}(\cosh(x))\text{csch}^3(x) - \frac{2}{3}\text{Subst}\left(\int \frac{1-x^2}{2x^2(2-x^2)}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{3}\cot^{-1}(\cosh(x))\text{csch}^3(x) - \frac{1}{3}\text{Subst}\left(\int \frac{1-x^2}{x^2(2-x^2)}\,dx, x, \tanh(x)\right)$$

$$= \frac{\coth(x)}{6} - \frac{1}{3}\cot^{-1}(\cosh(x))\text{csch}^3(x) + \frac{1}{6}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \tanh(x)\right)$$

$$= \frac{\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{6\sqrt{2}} + \frac{\coth(x)}{6} - \frac{1}{3}\cot^{-1}(\cosh(x))\text{csch}^3(x)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 4.00

$$\int \cot^{-1}(\cosh(x))\coth(x)\text{csch}^3(x)\,dx = \frac{1}{48}\text{csch}^3(x)\Big(-16\cot^{-1}(\cosh(x)) - 2\cosh(x) + 2\cosh(3x) - 3i\sqrt{2}\arctan\left(1 - i\sqrt{2}\tanh\left(\frac{x}{2}\right)\right)\sinh(x) + 3i\sqrt{2}\arctan\left(1 + i\sqrt{2}\tanh\left(\frac{x}{2}\right)\right)\sinh(x) + i\sqrt{2}\arctan\left(1 - i\sqrt{2}\tanh\left(\frac{x}{2}\right)\right)\sinh(3x) - i\sqrt{2}\arctan\left(1 + i\sqrt{2}\tanh\left(\frac{x}{2}\right)\right)\sinh(3x)\Big)$$

```
[In] Integrate[ArcCot[Cosh[x]]*Coth[x]*Csch[x]^3,x]
```

```
[Out] (Csch[x]^3*(-16*ArcCot[Cosh[x]] - 2*Cosh[x] + 2*Cosh[3*x] - (3*I)*Sqrt[2]*A
rcTan[1 - I*Sqrt[2]*Tanh[x/2]]*Sinh[x] + (3*I)*Sqrt[2]*ArcTan[1 + I*Sqrt[2]
*Tanh[x/2]]*Sinh[x] + I*Sqrt[2]*ArcTan[1 - I*Sqrt[2]*Tanh[x/2]]*Sinh[3*x] -
 I*Sqrt[2]*ArcTan[1 + I*Sqrt[2]*Tanh[x/2]]*Sinh[3*x]))/48
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.37 (sec) , antiderivative size = 850, normalized size of antiderivative = 23.61

Expression too large to display

```
[In] int(arccot(cosh(x))*cosh(x)/sinh(x)^4,x)
```

```
[Out] 4/3*I*exp(3*x)/(exp(2*x)-1)^3*ln(exp(2*x)+1+2*I*exp(x))-1/24*(-8-16*Pi*csgn
(-I*exp(2*x)+2*exp(x)-I)*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))^2*exp(3*x)
```

```
-16*Pi*csgn(I*exp(2*x)+I+2*exp(x))*csgn(I*exp(-x)*(2*I*exp(x)-exp(2*x)-1))^
2*exp(3*x)-16*Pi*csgn(I*exp(-x))*csgn(I*exp(2*x)+I+2*exp(x))*csgn(I*exp(-x)
*(2*I*exp(x)-exp(2*x)-1))*exp(3*x)+16*exp(2*x)+16*Pi*csgn(I*exp(-x)*(2*I*ex
p(x)-exp(2*x)-1))*csgn(exp(-x)*(2*I*exp(x)-exp(2*x)-1))*exp(3*x)-16*Pi*csgn
(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))*csgn(exp(-x)*(exp(2*x)+1+2*I*exp(x)))*e
xp(3*x)-8*exp(4*x)+16*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp
(x)))^2*exp(3*x)+16*Pi*csgn(I*exp(-x)*(2*I*exp(x)-exp(2*x)-1))*csgn(exp(-x)
*(2*I*exp(x)-exp(2*x)-1))^2*exp(3*x)+16*Pi*csgn(I*exp(-x)*(exp(2*x)+1+2*I*e
xp(x)))*csgn(exp(-x)*(exp(2*x)+1+2*I*exp(x)))^2*exp(3*x)+16*Pi*csgn(I*exp(-
x))*csgn(-I*exp(2*x)+2*exp(x)-I)*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))*ex
p(3*x)-2^(1/2)*ln(exp(2*x)+(1+2^(1/2))^2)-16*Pi*csgn(I*exp(-x))*csgn(I*exp(
-x)*(2*I*exp(x)-exp(2*x)-1))^2*exp(3*x)+2^(1/2)*ln(exp(2*x)+(2^(1/2)-1)^2)+
16*Pi*csgn(exp(-x)*(exp(2*x)+1+2*I*exp(x)))^2*exp(3*x)-16*Pi*csgn(exp(-x)*(
exp(2*x)+1+2*I*exp(x)))^3*exp(3*x)+16*Pi*csgn(exp(-x)*(2*I*exp(x)-exp(2*x)-
1))^3*exp(3*x)+16*Pi*csgn(exp(-x)*(2*I*exp(x)-exp(2*x)-1))^2*exp(3*x)-ln(ex
p(2*x)+(2^(1/2)-1)^2)*2^(1/2)*exp(6*x)+32*I*exp(3*x)*ln(exp(2*x)+1-2*I*exp(
x))-16*Pi*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))^3*exp(3*x)+ln(exp(2*x)+(1
+2^(1/2))^2)*2^(1/2)*exp(6*x)+3*ln(exp(2*x)+(2^(1/2)-1)^2)*2^(1/2)*exp(4*x)
-3*ln(exp(2*x)+(1+2^(1/2))^2)*2^(1/2)*exp(4*x)-3*ln(exp(2*x)+(2^(1/2)-1)^2)
*2^(1/2)*exp(2*x)+3*ln(exp(2*x)+(1+2^(1/2))^2)*2^(1/2)*exp(2*x)-16*Pi*csgn(
I*exp(-x)*(2*I*exp(x)-exp(2*x)-1))^3*exp(3*x))/(exp(2*x)-1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(27) = 54$.

Time = 0.26 (sec) , antiderivative size = 423, normalized size of antiderivative = 11.75

$$\int \cot^{-1}(\cosh(x))\coth(x)\text{csch}^3(x)\,dx$$

$$= \frac{8\cosh(x)^4 + 32\cosh(x)\sinh(x)^3 + 8\sinh(x)^4 + 16\left(3\cosh(x)^2 - 1\right)\sinh(x)^2 - 64\left(\cosh(x)^3 + 3\cosh(\ldots\right.}{\ldots}$$

```
[In] integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(8*cosh(x)^4 + 32*cosh(x)*sinh(x)^3 + 8*sinh(x)^4 + 16*(3*cosh(x)^2 -
1)*sinh(x)^2 - 64*(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 +
sinh(x)^3)*arctan(2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2 + 1)) - 16*cosh(x)^2 + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*sinh(
x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4 - 3*
sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x)^3 +
 3*(5*sqrt(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 3*sqrt
(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 - 2*sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x
))*sinh(x) - sqrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)
```

```
*cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(cosh(x)^2
+ sinh(x)^2 + 3)) + 32*(cosh(x)^3 - cosh(x))*sinh(x) + 8)/(cosh(x)^6 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 +
 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*
sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*sinh(x) - 1
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(34) = 68$.

Time = 78.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 5.94

$$\int \cot^{-1}(\cosh(x))\coth(x)\operatorname{csch}^3(x)\,dx = -\frac{\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{24} + \frac{\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{24} - \frac{\tanh^3\left(\frac{x}{2}\right)\operatorname{acot}\left(\frac{\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1}+\frac{1}{\tanh^2\left(\frac{x}{2}\right)-1}\right)}{24} + \frac{\tanh\left(\frac{x}{2}\right)\operatorname{acot}\left(\frac{\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1}+\frac{1}{\tanh^2\left(\frac{x}{2}\right)-1}\right)}{8} + \frac{\tanh\left(\frac{x}{2}\right)}{12} - \frac{\operatorname{acot}\left(\frac{\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1}+\frac{1}{\tanh^2\left(\frac{x}{2}\right)-1}\right)}{8\tanh\left(\frac{x}{2}\right)} + \frac{1}{12\tanh\left(\frac{x}{2}\right)} + \frac{\operatorname{acot}\left(\frac{\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1}+\frac{1}{\tanh^2\left(\frac{x}{2}\right)-1}\right)}{24\tanh^3\left(\frac{x}{2}\right)}$$

```
[In] integrate(acot(cosh(x))*cosh(x)/sinh(x)**4,x)
```

```
[Out] -sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)/24 + sqrt(2)*log(4*t
anh(x/2)**2 + 4*sqrt(2)*tanh(x/2) + 4)/24 - tanh(x/2)**3*acot(tanh(x/2)**2/
(tanh(x/2)**2 - 1) + 1/(tanh(x/2)**2 - 1))/24 + tanh(x/2)*acot(tanh(x/2)**2
/(tanh(x/2)**2 - 1) + 1/(tanh(x/2)**2 - 1))/8 + tanh(x/2)/12 - acot(tanh(x/
2)**2/(tanh(x/2)**2 - 1) + 1/(tanh(x/2)**2 - 1))/(8*tanh(x/2)) + 1/(12*tanh
(x/2)) + acot(tanh(x/2)**2/(tanh(x/2)**2 - 1) + 1/(tanh(x/2)**2 - 1))/(24*t
anh(x/2)**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \cot^{-1}(\cosh(x))\coth(x)\text{csch}^3(x)\,dx = -\frac{1}{24}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right) - \frac{1}{3\left(e^{(-2x)}-1\right)} - \frac{\operatorname{arccot}\left(\cosh\left(x\right)\right)}{3\,\sinh\left(x\right)^{3}}$$

```
[In] integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) -
 1/3/(e^(-2*x) - 1) - 1/3*arccot(cosh(x))/sinh(x)^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(27) = 54.

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.94

$$\int \cot^{-1}(\cosh(x))\coth(x)\text{csch}^3(x)\,dx = \frac{1}{24}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right) + \frac{1}{3\left(e^{(2x)}-1\right)} + \frac{8\,\arctan\left(\frac{2}{e^{(-x)}+e^{x}}\right)}{3\left(e^{(-x)}-e^{x}\right)^{3}}$$

```
[In] integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="giac")
```

```
[Out] 1/24*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + 1/
3/(e^(2*x) - 1) + 8/3*arctan(2/(e^(-x) + e^x))/(e^(-x) - e^x)^3
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.86

$$\int \cot^{-1}(\cosh(x))\coth(x)\text{csch}^3(x)\,dx = \frac{\sqrt{2}\,\ln\left(-\frac{2\,e^{2\,x}}{3}-\frac{\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{24}\right)}{24} - \frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{24}-\frac{2\,e^{2\,x}}{3}\right)}{24} + \frac{1}{3\,\left(e^{2\,x}-1\right)} - \frac{8\,e^{3\,x}\,\mathrm{acot}\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)}{3\,\left(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1\right)}$$

```
[In] int((acot(cosh(x))*cosh(x))/sinh(x)^4,x)
```

```
[Out] (2^(1/2)*log(- (2*exp(2*x))/3 - (2^(1/2)*(12*exp(2*x) + 4))/24))/24 - (2^(1
/2)*log((2^(1/2)*(12*exp(2*x) + 4))/24 - (2*exp(2*x))/3))/24 + 1/(3*(exp(2*
x) - 1)) - (8*exp(3*x)*acot(exp(-x)/2 + exp(x)/2))/(3*(3*exp(2*x) - 3*exp(4
*x) + exp(6*x) - 1))
```

3.705 $\int e^x \arcsin(\tanh(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 28

$$\int e^x \arcsin(\tanh(x))\,dx = e^x \arcsin(\tanh(x)) - \cosh(x)\log\left(1+e^{2x}\right)\sqrt{\operatorname{sech}^2(x)}$$

```
[Out] exp(x)*arcsin(tanh(x))-cosh(x)*ln(1+exp(2*x))*(sech(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2225, 4928, 6852, 2320, 12, 266}

$$\int e^x \arcsin(\tanh(x))\,dx = e^x \arcsin(\tanh(x)) - \log\left(e^{2x}+1\right)\cosh(x)\sqrt{\operatorname{sech}^2(x)}$$

```
[In] Int[E^x*ArcSin[Tanh[x]],x]
```

```
[Out] E^x*ArcSin[Tanh[x]] - Cosh[x]*Log[1 + E^(2*x)]*Sqrt[Sech[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 4928

```
Int[((a_.) + ArcSin[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcSin[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/Sqr
t[1 - u^2]), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
eQ[{c, d, m}, x]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= e^x \arcsin(\tanh(x)) - \int e^x \sqrt{\operatorname{sech}^2(x)}\, dx \\
&= e^x \arcsin(\tanh(x)) - \left(\cosh(x)\sqrt{\operatorname{sech}^2(x)}\right) \int e^x \operatorname{sech}(x)\, dx \\
&= e^x \arcsin(\tanh(x)) - \left(\cosh(x)\sqrt{\operatorname{sech}^2(x)}\right) \operatorname{Subst}\left(\int \frac{2x}{1+x^2}\, dx, x, e^x\right) \\
&= e^x \arcsin(\tanh(x)) - \left(2\cosh(x)\sqrt{\operatorname{sech}^2(x)}\right) \operatorname{Subst}\left(\int \frac{x}{1+x^2}\, dx, x, e^x\right) \\
&= e^x \arcsin(\tanh(x)) - \cosh(x) \log\left(1+e^{2x}\right) \sqrt{\operatorname{sech}^2(x)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.89

$$\int e^x \arcsin(\tanh(x))\,dx = -e^{-x}\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\left(1+e^{2x}\right)\log\left(1+e^{2x}\right) - ie^x\log\left(\frac{-ie^{-x}+ie^x+2(e^{-x}+e^x)\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}}{e^{-x}+e^x}\right)$$

```
[In] Integrate[E^x*ArcSin[Tanh[x]],x]
[Out] -((Sqrt[E^(2*x)/(1 + E^(2*x))^2]*(1 + E^(2*x))*Log[1 + E^(2*x)])/E^x) - I*E
^x*Log[((-I)/E^x + I*E^x + 2*(E^(-x) + E^x)*Sqrt[E^(2*x)/(1 + E^(2*x))^2])/
(E^(-x) + E^x)]
```

Maple [F]

$$\int \mathrm{e}^x \arcsin\left(\tanh\left(x\right)\right)dx$$

```
[In] int(exp(x)*arcsin(tanh(x)),x)
[Out] int(exp(x)*arcsin(tanh(x)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int e^x \arcsin(\tanh(x))\,dx = (\cosh(x)+\sinh(x))\arctan(\sinh(x)) - \log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

```
[In] integrate(exp(x)*arcsin(tanh(x)),x, algorithm="fricas")
[Out] (cosh(x) + sinh(x))*arctan(sinh(x)) - log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int e^x \arcsin(\tanh(x))\,dx = \int e^x \operatorname{asin}(\tanh(x))\,dx$$

```
[In] integrate(exp(x)*asin(tanh(x)),x)
[Out] Integral(exp(x)*asin(tanh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int e^x \arcsin(\tanh(x))\,dx = \arcsin(\tanh(x))\,e^x - \log\left(e^{(2x)} + 1\right)$$

```
[In] integrate(exp(x)*arcsin(tanh(x)),x, algorithm="maxima")
[Out] arcsin(tanh(x))*e^x - log(e^(2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int e^x \arcsin(\tanh(x))\,dx = \arcsin\left(\frac{e^{(2x)} - 1}{e^{(2x)} + 1}\right)e^x - \log\left(e^{(2x)} + 1\right)$$

```
[In] integrate(exp(x)*arcsin(tanh(x)),x, algorithm="giac")
[Out] arcsin((e^(2*x) - 1)/(e^(2*x) + 1))*e^x - log(e^(2*x) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int e^x \arcsin(\tanh(x))\,dx = \int \operatorname{asin}(\tanh(x))\,\mathrm{e}^x\,dx$$

```
[In] int(asin(tanh(x))*exp(x),x)
[Out] int(asin(tanh(x))*exp(x), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                 end if
             else #result contains complex but optimal is not
                 if debug then
                         print("result contains complex but optimal is not");
                 fi;
                 return "C","Result contains complex when optimal does not.";
             fi;
         else # result do not contain complex
              # this assumes optimal do not as well. No check is needed here.
             if debug then
                    print("result do not contain complex, this assumes optimal do not as well")
             fi;
             if leaf_count_result<=2*leaf_count_optimal then
                 if debug then
                     print("leaf_count_result<=2*leaf_count_optimal");
                 fi;
                 return "A"," ";
             else
                 if debug then
                     print("leaf_count_result>2*leaf_count_optimal");
                 fi;
                 return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
             fi;
         fi;
     else #ExpnType(result) > ExpnType(optimal)
         if debug then
             print("ExpnType(result) > ExpnType(optimal)");
         fi;
         return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
     fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:  #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```