Computer Algebra Independent Integration Tests

Summer 2023 edition

0-Independent-test-suites/11-Welz-Problems

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [116]. This is test number [11].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	97.41 (113)	2.59 (3)
Mathematica	87.93 (102)	12.07 (14)
Fricas	77.59 (90)	22.41 (26)
Maple	75.86 (88)	24.14 (28)
Mupad	31.90 (37)	68.10 (79)
Giac	31.03 (36)	68.97 (80)
Sympy	25.00 (29)	75.00 (87)
Maxima	17.24 (20)	82.76 (96)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

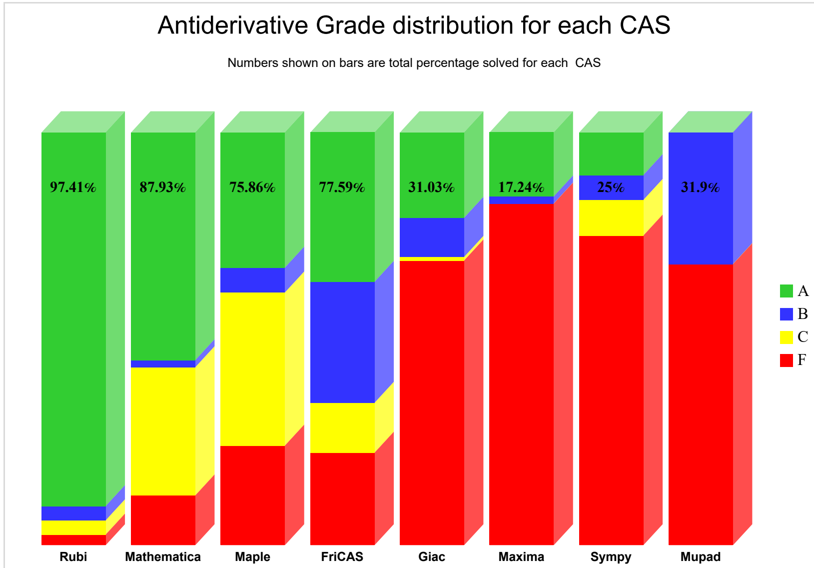
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

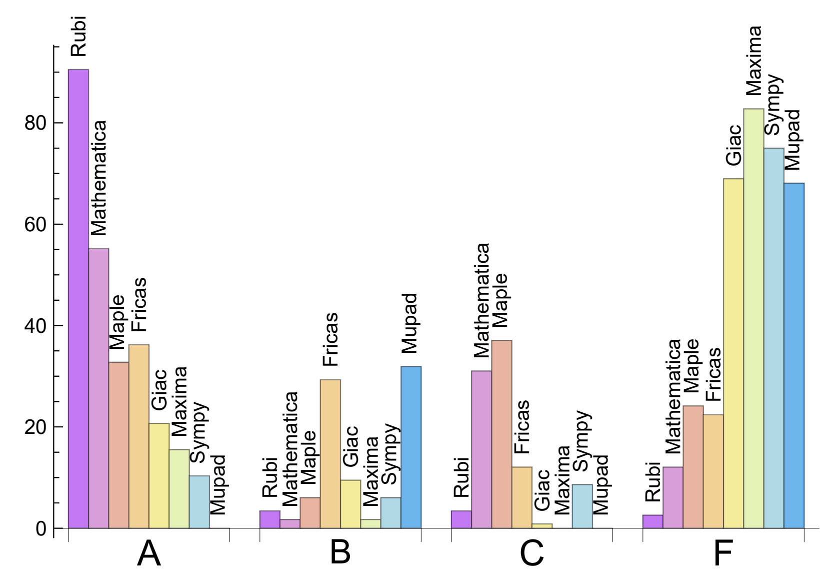
System	% A grade	% B grade	% C grade	% F grade
Rubi	90.517	3.448	3.448	2.586
Mathematica	55.172	1.724	31.034	12.069
Fricas	36.207	29.310	12.069	22.414
Maple	32.759	6.034	37.069	24.138
Giac	20.690	9.483	0.862	68.966
Maxima	15.517	1.724	0.000	82.759
Sympy	10.345	6.034	8.621	75.000
Mupad	0.000	31.897	0.000	68.103

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	3	100.00	0.00	0.00
Mathematica	14	100.00	0.00	0.00
Fricas	26	42.31	26.92	30.77
Maple	28	100.00	0.00	0.00
Mupad	79	0.00	100.00	0.00
Giac	80	98.75	1.25	0.00
Sympy	87	90.80	8.05	1.15
Maxima	96	98.96	0.00	1.04

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.18
Maxima	0.25
Sympy	0.72
Mupad	0.92
Giac	1.46
Fricas	2.66
Mathematica	3.80
Maple	4.56

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	81.80	1.16	62.00	1.05
Mathematica	132.28	1.16	83.50	1.07
Mupad	151.70	1.71	76.00	1.09
Rubi	162.17	1.23	88.00	1.00
Sympy	208.41	5.09	37.00	0.82
Giac	247.89	1.54	69.50	1.11
Maple	336.27	3.08	111.00	1.18
Fricas	459.56	3.39	194.50	1.75

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

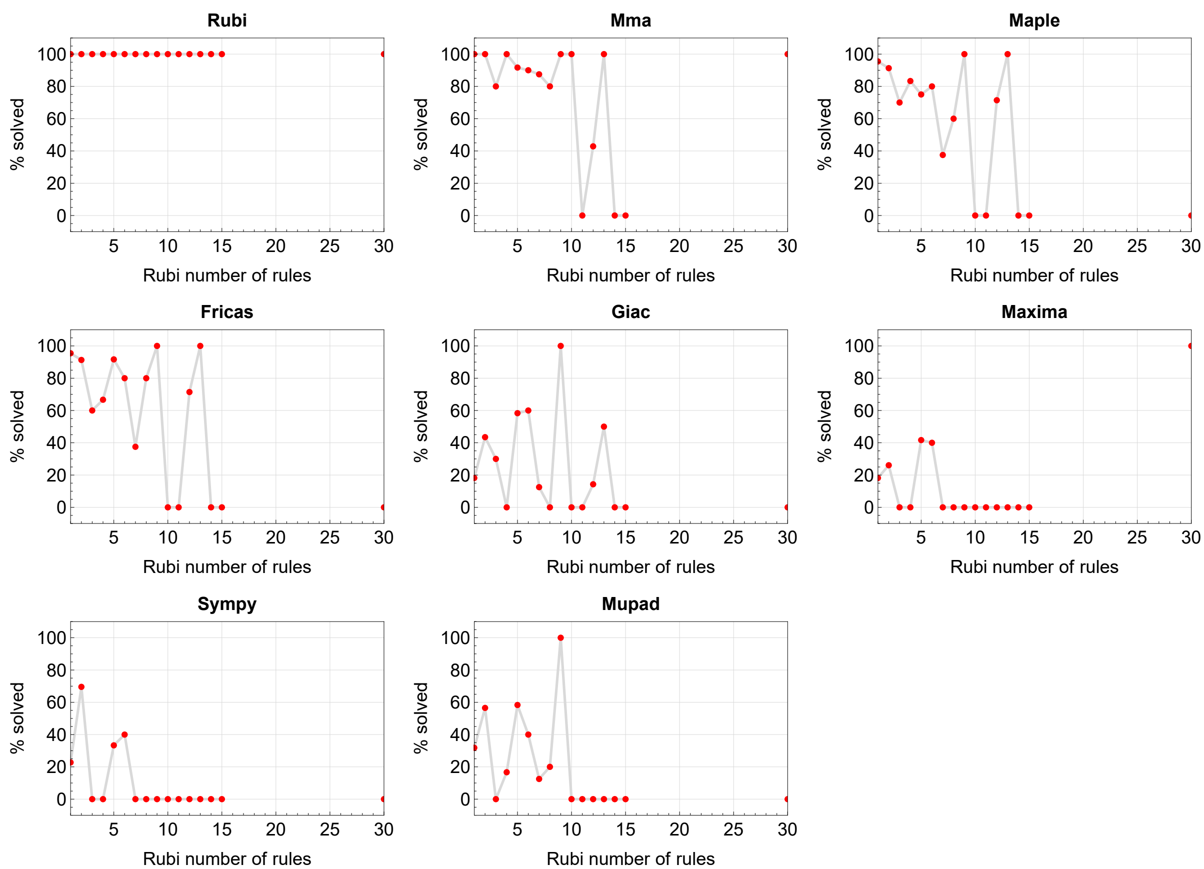


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

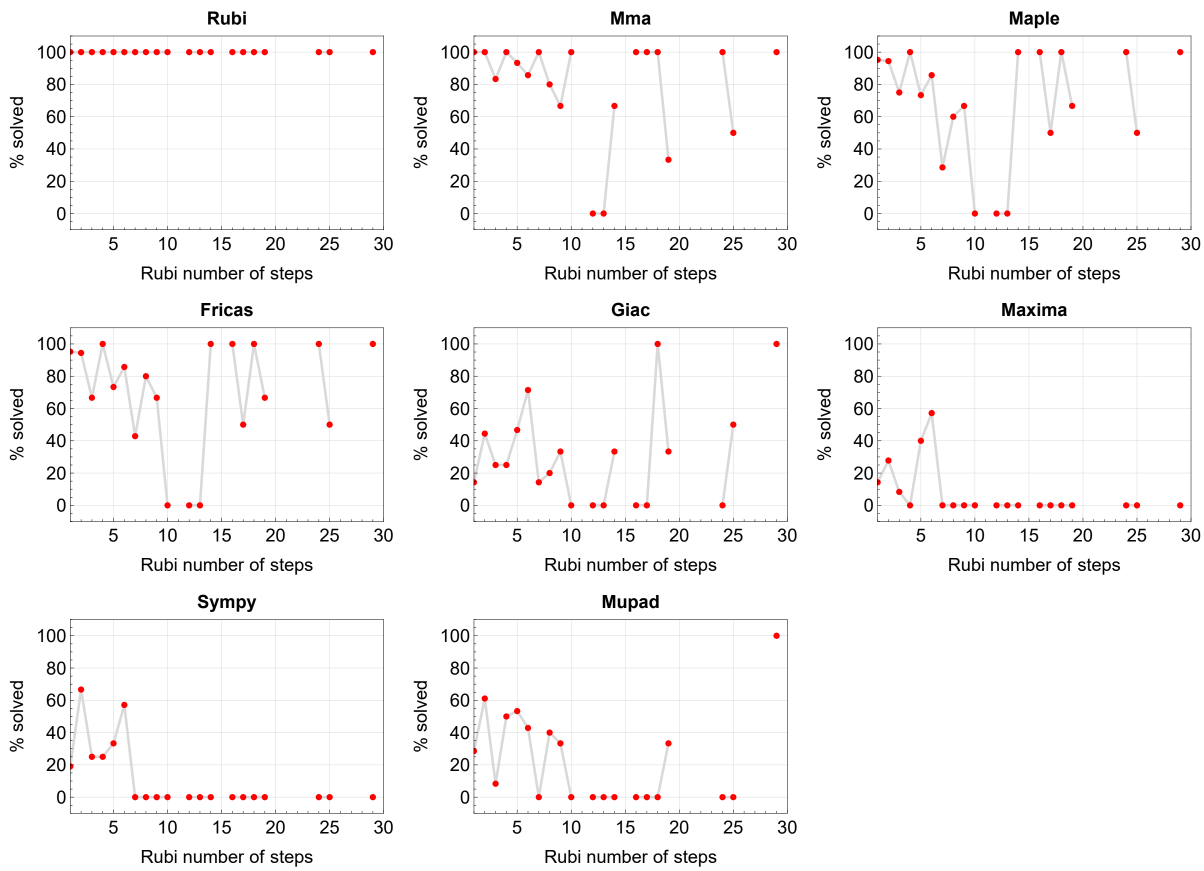


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

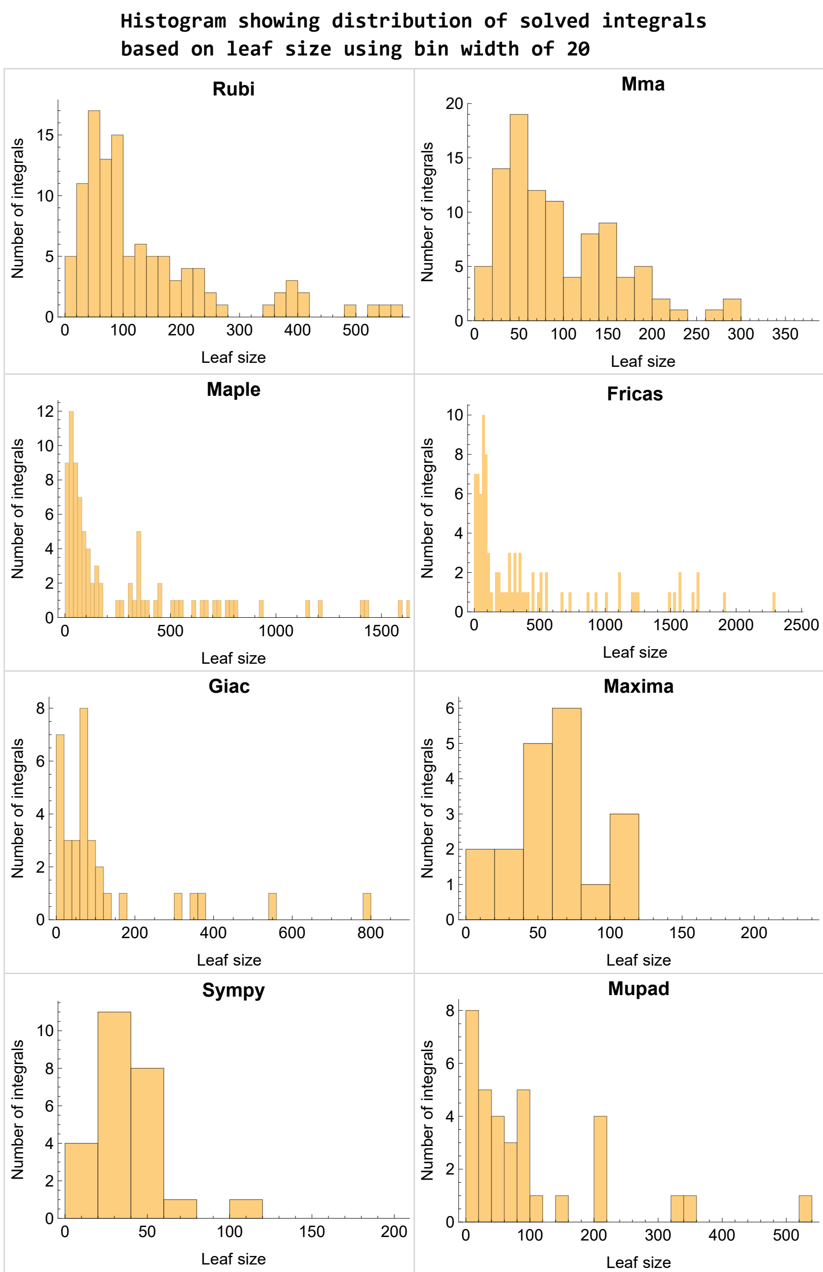


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

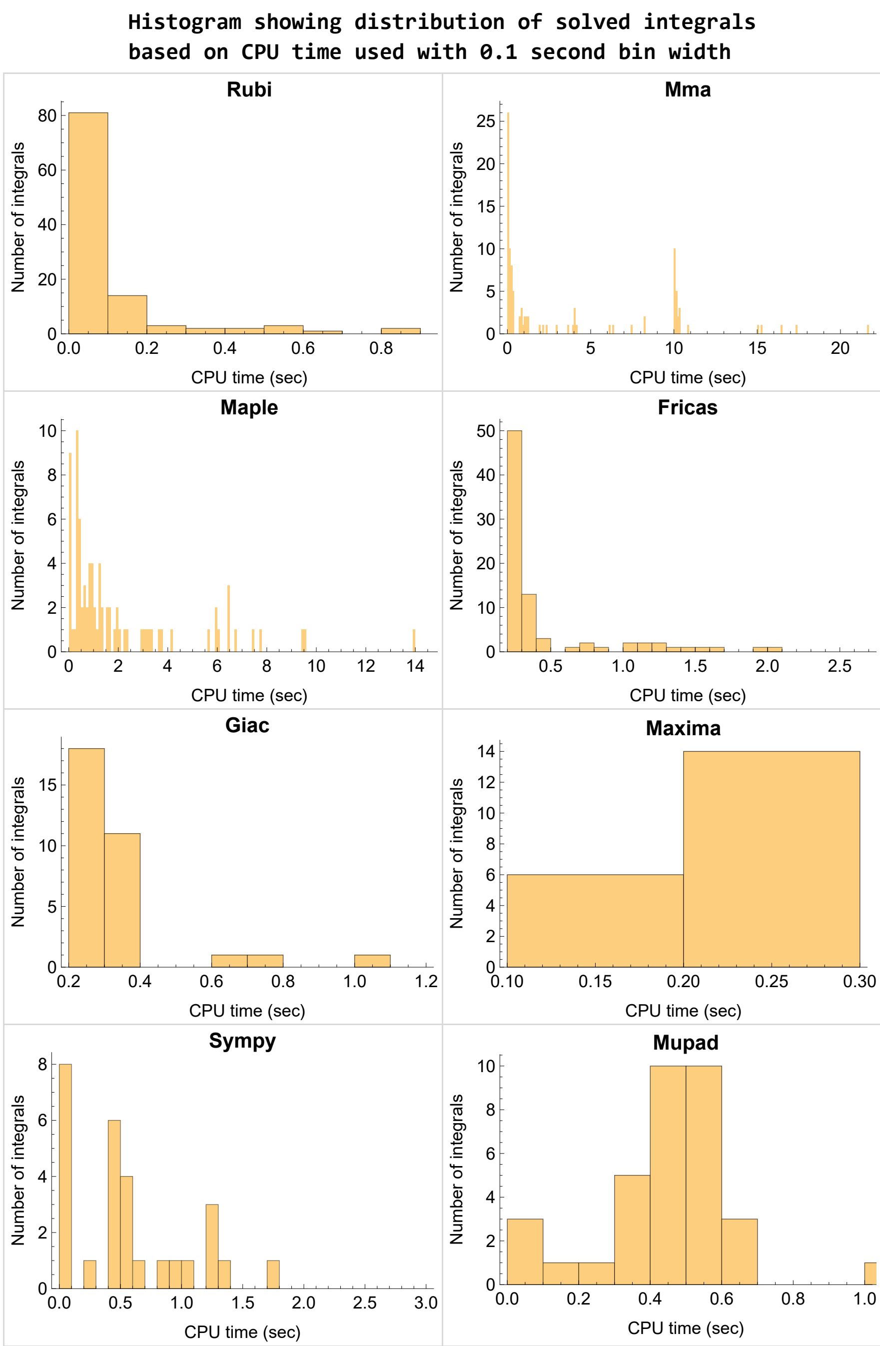


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

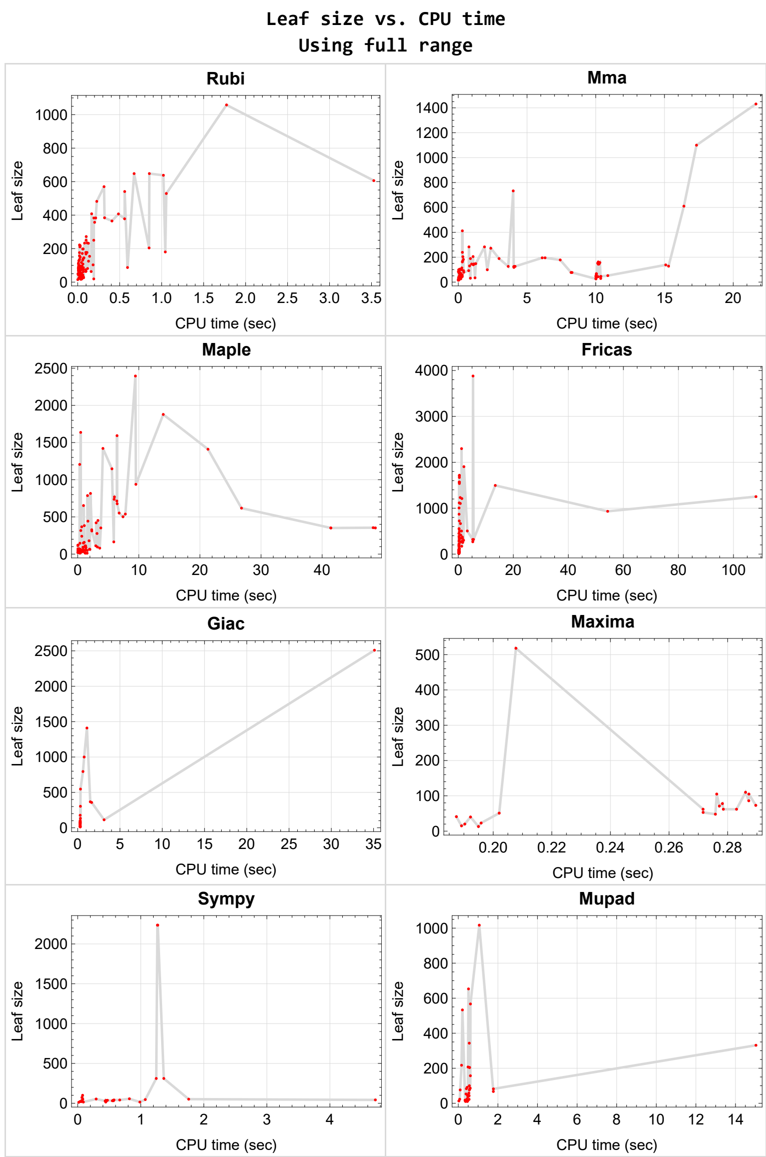


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{82\}$

Mathematica $\{53, 54, 69, 70, 71, 72, 77, 78, 79, 80, 115\}$

Maple $\{37, 38, 39, 52, 59, 74, 76, 77, 100, 101, 102, 116\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

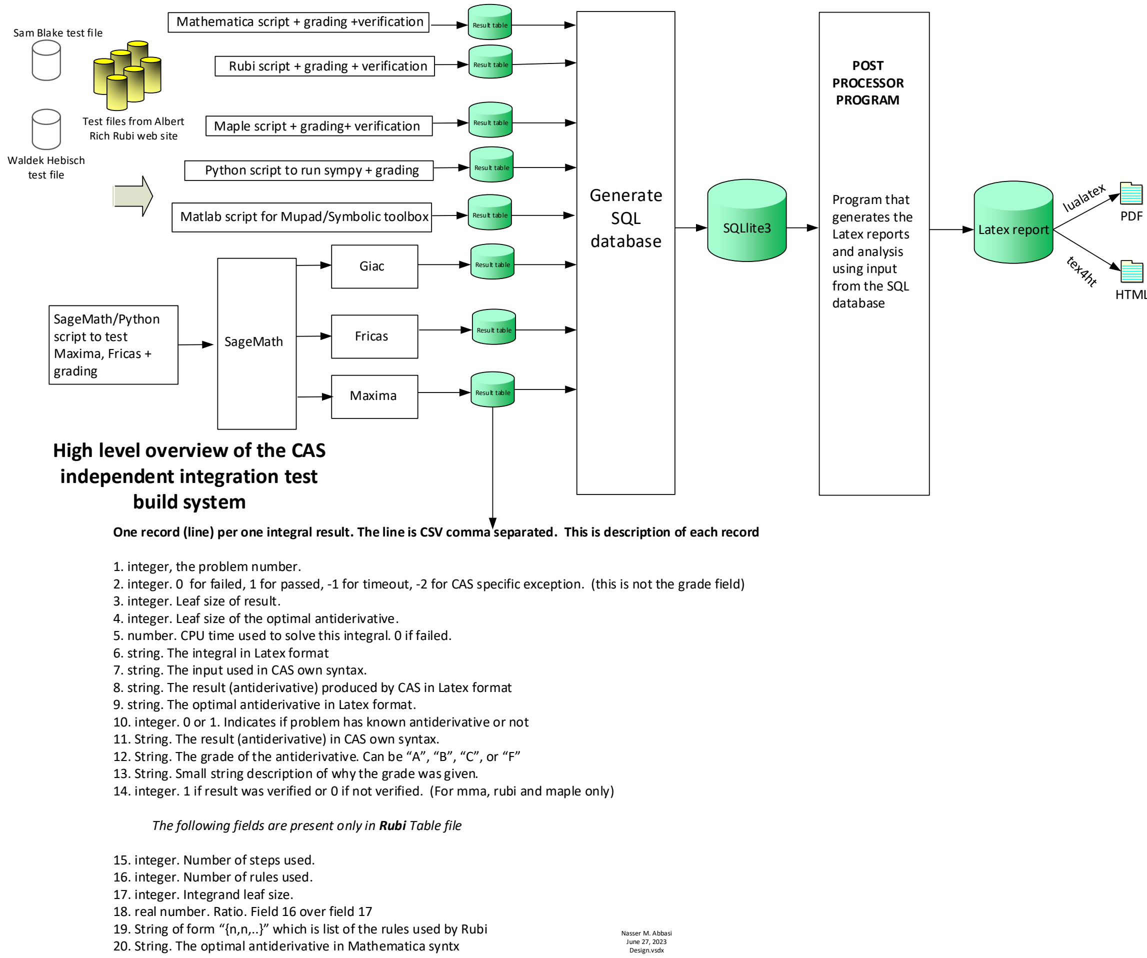
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

B grade { 10, 100, 101, 102 }

C grade { 2, 52, 82, 83 }

F normal fail { 43, 44, 45 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 41, 42, 43, 44, 52, 55, 56, 57, 62, 66, 67, 81, 82, 83, 84, 85, 90, 91, 92, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 113, 116 }

B grade { 12, 13 }

C grade { 2, 15, 24, 40, 47, 48, 49, 50, 51, 53, 54, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 86, 87, 88, 89, 94, 96, 106, 114, 115 }

F normal fail { 38, 45, 46, 58, 59, 60, 61, 93, 95, 108, 109, 110, 111, 112 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 4, 6, 7, 8, 11, 16, 19, 20, 21, 22, 23, 25, 26, 30, 32, 33, 36, 41, 44, 47, 48, 49, 50, 51, 62, 63, 66, 67, 81, 94, 96, 97, 99, 103, 104, 105, 113 }

B grade { 5, 9, 10, 17, 24, 31, 85 }

C grade { 2, 3, 15, 28, 34, 35, 37, 38, 39, 40, 52, 55, 56, 57, 59, 64, 65, 68, 73, 74, 75, 76, 77, 78, 79, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 100, 101, 102, 106, 107, 110, 116 }

F normal fail { 12, 13, 14, 18, 27, 29, 42, 43, 45, 46, 53, 54, 58, 60, 61, 69, 70, 71, 72, 80, 95, 98, 108, 109, 111, 112, 114, 115 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 3, 5, 6, 8, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 30, 31, 32, 33, 34, 36, 40, 52, 56, 57, 62, 63, 64, 65, 66, 67, 81, 83, 99, 106, 107, 113, 116 }

B grade { 4, 7, 9, 10, 12, 13, 14, 35, 37, 39, 41, 42, 43, 45, 55, 74, 76, 78, 80, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 100, 101, 102 }

C grade { 24, 47, 48, 49, 50, 51, 59, 68, 73, 75, 77, 79, 82, 110 }

F normal fail { 2, 29, 61, 103, 104, 105, 109, 111, 112, 114, 115 }

F(-1) timedout fail { 44, 69, 70, 71, 72, 95, 108 }

F(-2) exception fail { 38, 46, 53, 54, 58, 60, 84, 92 }

Maxima

A grade { 1, 6, 16, 21, 22, 23, 29, 32, 33, 34, 35, 36, 56, 57, 62, 96, 99, 107 }

B grade { 2, 106 }

C grade { }

F normal fail { 3, 4, 5, 7, 8, 9, 10, 12, 13, 14, 15, 17, 18, 19, 20, 24, 25, 26, 27, 28, 30, 31, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

F(-1) timedout fail { }

F(-2) exception fail { 11 }

Giac

A grade { 1, 6, 8, 16, 19, 20, 21, 22, 23, 25, 26, 30, 33, 34, 36, 41, 49, 57, 62, 63, 64, 96, 99, 107 }

B grade { 2, 3, 4, 5, 7, 9, 10, 11, 24, 47, 48 }

C grade { 50 }

F normal fail { 12, 13, 14, 15, 17, 18, 27, 28, 29, 31, 32, 35, 37, 38, 39, 40, 42, 43, 44, 45, 46, 52, 53, 54, 55, 56, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

F(-1) timedout fail { 51 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 5, 8, 16, 19, 20, 21, 22, 23, 25, 26, 30, 33, 34, 35, 36, 41, 47, 48, 49, 57, 62, 63, 64, 73, 74, 75, 81, 82, 84, 85, 96, 99, 106, 107 }

C grade { }

F normal fail { }

F(-1) timedout fail { 4, 6, 7, 9, 10, 11, 12, 13, 14, 15, 17, 18, 24, 27, 28, 29, 31, 32, 37, 38, 39, 40, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 76, 77, 78, 79, 80, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 14, 15, 21, 22, 23, 25, 26, 62, 63, 64 }

B grade { 8, 16, 17, 19, 20, 30, 31 }

C grade { 27, 28, 33, 34, 35, 36, 56, 57, 106, 107 }

F normal fail { 3, 4, 5, 6, 7, 9, 12, 13, 18, 24, 29, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 53, 54, 55, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

F(-1) timedout fail { 10, 32, 47, 48, 49, 51, 52 }

F(-2) exception fail { 11 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	0.87	0.87
time (sec)	N/A	0.001	0.002	0.315	0.195	0.242	0.017	0.300	0.029

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	C	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	52	37	34	41	0	42	41	43
N.S.	1	3.47	2.47	2.27	2.73	0.00	2.80	2.73	2.87
time (sec)	N/A	0.045	0.019	0.342	0.187	0.000	4.721	0.283	0.547

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	60	73	0	100	0	177	204
N.S.	1	1.00	0.73	0.89	0.00	1.22	0.00	2.16	2.49
time (sec)	N/A	0.057	0.188	0.201	0.000	0.247	0.000	0.292	0.568

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	71	52	0	80	0	94	0
N.S.	1	1.00	1.65	1.21	0.00	1.86	0.00	2.19	0.00
time (sec)	N/A	0.008	0.074	0.460	0.000	0.243	0.000	0.299	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	115	0	105	0	133	82
N.S.	1	1.00	0.99	1.55	0.00	1.42	0.00	1.80	1.11
time (sec)	N/A	0.041	0.065	0.024	0.000	0.240	0.000	0.323	1.773

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	70	65	53	89	0	84	0
N.S.	1	1.00	1.09	1.02	0.83	1.39	0.00	1.31	0.00
time (sec)	N/A	0.019	0.186	0.424	0.272	0.243	0.000	0.286	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	55	37	0	83	0	101	0
N.S.	1	1.00	1.15	0.77	0.00	1.73	0.00	2.10	0.00
time (sec)	N/A	0.009	0.104	0.416	0.000	0.238	0.000	0.298	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	26	21	0	18	53	20	21
N.S.	1	1.00	0.87	0.70	0.00	0.60	1.77	0.67	0.70
time (sec)	N/A	0.058	0.181	0.329	0.000	0.235	0.292	0.281	0.438

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	365	195	1206	0	450	0	367	0
N.S.	1	1.66	0.89	5.48	0.00	2.05	0.00	1.67	0.00
time (sec)	N/A	0.408	6.105	0.324	0.000	0.245	0.000	1.489	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	541	195	1637	0	450	0	358	0
N.S.	1	2.46	0.89	7.44	0.00	2.05	0.00	1.63	0.00
time (sec)	N/A	0.560	6.314	0.485	0.000	0.250	0.000	1.649	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	126	146	0	161	0	547	0
N.S.	1	1.00	0.91	1.06	0.00	1.17	0.00	3.96	0.00
time (sec)	N/A	0.064	3.633	0.366	0.000	0.252	0.000	0.331	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	272	0	0	502	0	0	0
N.S.	1	1.00	2.18	0.00	0.00	4.02	0.00	0.00	0.00
time (sec)	N/A	0.129	2.366	0.000	0.000	1.171	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	205	0	0	504	0	0	0
N.S.	1	1.00	2.53	0.00	0.00	6.22	0.00	0.00	0.00
time (sec)	N/A	0.118	1.074	0.000	0.000	3.271	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	44	0	0	60	15	0	0
N.S.	1	1.00	1.42	0.00	0.00	1.94	0.48	0.00	0.00
time (sec)	N/A	0.039	0.134	0.000	0.000	0.368	0.986	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	57	22	0	29	15	0	0
N.S.	1	1.00	1.73	0.67	0.00	0.88	0.45	0.00	0.00
time (sec)	N/A	0.058	0.158	0.358	0.000	0.385	0.444	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	28	56	15	15
N.S.	1	1.00	1.00	0.84	0.79	1.47	2.95	0.79	0.79
time (sec)	N/A	0.192	0.009	0.411	0.189	0.240	0.820	0.308	0.476

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	43	120	0	32	2236	0	0
N.S.	1	1.00	0.83	2.31	0.00	0.62	43.00	0.00	0.00
time (sec)	N/A	0.018	0.144	0.040	0.000	0.248	1.264	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	50	0	0	33	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.021	0.140	0.000	0.000	0.255	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	0	15	311	15	15
N.S.	1	1.00	1.00	0.94	0.00	0.88	18.29	0.88	0.88
time (sec)	N/A	0.040	0.022	0.332	0.000	0.240	1.365	0.303	0.343

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	18	41	18	18
N.S.	1	1.00	1.00	0.95	0.00	0.90	2.05	0.90	0.90
time (sec)	N/A	0.039	0.023	0.356	0.000	0.245	0.666	0.286	0.342

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	39	43	40	52	36	47	58
N.S.	1	1.00	0.93	1.02	0.95	1.24	0.86	1.12	1.38
time (sec)	N/A	0.023	0.040	0.056	0.192	0.239	0.068	0.280	0.534

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	20	20	19	20	19	24
N.S.	1	1.00	1.00	0.91	0.91	0.86	0.91	0.86	1.09
time (sec)	N/A	0.018	0.022	0.046	0.190	0.254	0.055	0.292	0.436

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	49	50	51	58	51	74	47
N.S.	1	1.00	0.79	0.81	0.82	0.94	0.82	1.19	0.76
time (sec)	N/A	0.066	0.071	0.104	0.202	0.267	0.081	0.294	0.454

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	239	158	0	285	0	304	0
N.S.	1	1.00	2.78	1.84	0.00	3.31	0.00	3.53	0.00
time (sec)	N/A	0.068	0.299	1.065	0.000	0.256	0.000	0.322	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	0	15	15	15	15
N.S.	1	1.00	1.00	0.84	0.00	0.79	0.79	0.79	0.79
time (sec)	N/A	0.050	0.048	0.435	0.000	0.243	0.090	0.292	0.484

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	27	22	22
N.S.	1	1.00	1.00	0.88	0.00	0.85	1.04	0.85	0.85
time (sec)	N/A	0.071	0.068	0.343	0.000	0.242	0.454	0.294	0.546

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	56	0	0	198	46	0	0
N.S.	1	1.00	0.89	0.00	0.00	3.14	0.73	0.00	0.00
time (sec)	N/A	0.161	0.105	0.000	0.000	0.258	1.072	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	25	0	216	51	0	0
N.S.	1	1.00	1.00	0.30	0.00	2.63	0.62	0.00	0.00
time (sec)	N/A	0.053	0.013	0.023	0.000	0.261	1.760	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	606	606	412	0	518	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.85	0.00	0.00	0.00	0.00
time (sec)	N/A	3.527	0.295	0.000	0.208	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	0	15	311	15	15
N.S.	1	1.00	1.00	0.94	0.00	0.88	18.29	0.88	0.88
time (sec)	N/A	0.037	0.043	0.367	0.000	0.245	1.246	0.305	0.355

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	46	120	0	32	2236	0	0
N.S.	1	1.00	0.88	2.31	0.00	0.62	43.00	0.00	0.00
time (sec)	N/A	0.018	0.209	0.036	0.000	0.259	1.270	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	44	48	48	0	0	0
N.S.	1	1.00	0.97	1.29	1.41	1.41	0.00	0.00	0.00
time (sec)	N/A	0.031	0.196	0.723	0.276	0.246	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	77	63	62	64	36	64	86
N.S.	1	1.00	1.33	1.09	1.07	1.10	0.62	1.10	1.48
time (sec)	N/A	0.027	0.069	1.300	0.283	0.242	0.447	0.287	0.603

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	77	48	62	64	37	64	76
N.S.	1	1.00	1.33	0.83	1.07	1.10	0.64	1.10	1.31
time (sec)	N/A	0.029	0.054	1.234	0.279	0.244	0.472	0.298	0.557

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	86	12	78	82	29	0	10
N.S.	1	1.00	1.76	0.24	1.59	1.67	0.59	0.00	0.20
time (sec)	N/A	0.004	0.043	1.540	0.278	0.241	0.436	0.000	0.367

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	79	63	62	64	32	63	80
N.S.	1	1.00	1.44	1.15	1.13	1.16	0.58	1.15	1.45
time (sec)	N/A	0.025	0.056	1.997	0.272	0.245	0.453	0.289	0.575

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	97	97	148	769	0	301	0	0	0
N.S.	1	1.00	1.53	7.93	0.00	3.10	0.00	0.00	0.00
time (sec)	N/A	0.028	1.271	6.006	0.000	1.929	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	145	145	0	1878	0	0	0	0	0
N.S.	1	1.00	0.00	12.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.074	0.000	13.997	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	110	176	162	1593	0	277	0	0	0
N.S.	1	1.60	1.47	14.48	0.00	2.52	0.00	0.00	0.00
time (sec)	N/A	0.021	0.294	6.433	0.000	1.226	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	131	85	653	0	120	0	0	0
N.S.	1	1.62	1.05	8.06	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	0.060	0.015	0.950	0.000	0.404	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	117	130	92	0	415	0	67	37
N.S.	1	1.77	1.97	1.39	0.00	6.29	0.00	1.02	0.56
time (sec)	N/A	0.044	0.818	0.849	0.000	0.898	0.000	0.319	0.483

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	145	145	0	0	665	0	0	0
N.S.	1	1.84	1.84	0.00	0.00	8.42	0.00	0.00	0.00
time (sec)	N/A	0.073	0.968	0.000	0.000	0.783	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	B	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	0	189	0	0	1496	0	0	0
N.S.	1	0.00	1.60	0.00	0.00	12.68	0.00	0.00	0.00
time (sec)	N/A	0.000	2.966	0.000	0.000	13.441	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	0	128	111	0	0	0	0	0
N.S.	1	0.00	1.15	1.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	15.299	1.215	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	F	F	F	B	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	176	0	0	0	0	932	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	5.30	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	54.282	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	493	570	0	0	0	0	0	0	0
N.S.	1	1.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	178	277	0	555	0	795	343
N.S.	1	1.00	0.44	0.68	0.00	1.36	0.00	1.95	0.84
time (sec)	N/A	0.485	7.416	3.160	0.000	0.272	0.000	0.623	0.554

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	648	648	610	352	0	1005	0	1000	567
N.S.	1	1.00	0.94	0.54	0.00	1.55	0.00	1.54	0.88
time (sec)	N/A	0.854	16.413	3.783	0.000	0.284	0.000	0.765	0.614

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1058	1058	1100	502	0	1905	0	1410	1017
N.S.	1	1.00	1.04	0.47	0.00	1.80	0.00	1.33	0.96
time (sec)	N/A	1.773	17.330	7.409	0.000	2.036	0.000	1.091	1.060

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	733	452	0	869	0	2509	0
N.S.	1	1.00	1.94	1.20	0.00	2.30	0.00	6.64	0.00
time (sec)	N/A	0.559	3.996	3.310	0.000	0.285	0.000	35.108	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	638	638	1431	552	0	1569	0	0	0
N.S.	1	1.00	2.24	0.87	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	1.021	21.655	6.780	0.000	0.322	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	66	204	99	787	0	546	0	0	0
N.S.	1	3.09	1.50	11.92	0.00	8.27	0.00	0.00	0.00
time (sec)	N/A	0.851	2.110	1.605	0.000	0.297	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	198	198	145	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.065	10.240	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	198	198	153	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	10.270	0.000	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	168	2395	0	171	0	0	0
N.S.	1	1.00	1.73	24.69	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	0.012	0.312	9.443	0.000	1.244	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	99	15	105	96	32	0	0
N.S.	1	1.00	1.36	0.21	1.44	1.32	0.44	0.00	0.00
time (sec)	N/A	0.009	0.240	1.189	0.276	0.257	0.568	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	88	49	71	73	37	72	83
N.S.	1	1.00	1.31	0.73	1.06	1.09	0.55	1.07	1.24
time (sec)	N/A	0.028	0.074	1.553	0.277	0.245	0.561	0.295	0.407

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	482	482	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	280	408	0	1410	0	3880	0	0	0
N.S.	1	1.46	0.00	5.04	0.00	13.86	0.00	0.00	0.00
time (sec)	N/A	0.166	0.000	21.318	0.000	5.355	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	232	232	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.128	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	168	168	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.099	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	23	23	22	23	23
N.S.	1	1.00	1.00	0.88	0.92	0.92	0.88	0.92	0.92
time (sec)	N/A	0.014	0.009	0.022	0.196	0.264	0.038	0.304	0.071

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	86	52	0	43	73	40	53
N.S.	1	1.00	1.46	0.88	0.00	0.73	1.24	0.68	0.90
time (sec)	N/A	0.020	0.025	0.046	0.000	0.249	0.067	0.311	0.389

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	99	70	0	66	100	64	76
N.S.	1	1.00	1.27	0.90	0.00	0.85	1.28	0.82	0.97
time (sec)	N/A	0.048	0.027	0.036	0.000	0.247	0.078	0.302	0.096

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	57	55	0	56	0	0	0
N.S.	1	1.00	1.16	1.12	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.005	0.289	0.904	0.000	0.280	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	44	62	0	61	0	0	0
N.S.	1	1.00	0.83	1.17	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.009	0.291	0.862	0.000	0.285	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	79	89	0	359	0	0	0
N.S.	1	1.00	1.05	1.19	0.00	4.79	0.00	0.00	0.00
time (sec)	N/A	0.065	0.399	1.347	0.000	0.313	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	92	149	0	229	0	0	0
N.S.	1	1.00	0.54	0.87	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.055	0.380	0.925	0.000	0.328	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	157	0	0	0	0	0	0
N.S.	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	10.301	0.000	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	162	0	0	0	0	0	0
N.S.	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	10.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	144	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	10.184	0.000	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	152	0	0	0	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.023	10.138	0.000	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	28	164	0	1116	0	0	653
N.S.	1	1.00	0.22	1.29	0.00	8.79	0.00	0.00	5.14
time (sec)	N/A	0.013	10.018	5.903	0.000	0.338	0.000	0.000	0.515

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	157	157	54	240	0	1669	0	0	331
N.S.	1	1.00	0.34	1.53	0.00	10.63	0.00	0.00	2.11
time (sec)	N/A	0.021	10.025	0.665	0.000	0.391	0.000	0.000	15.033

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	48	421	0	723	0	0	533
N.S.	1	1.00	0.65	5.69	0.00	9.77	0.00	0.00	7.20
time (sec)	N/A	0.093	10.017	3.015	0.000	0.348	0.000	0.000	0.208

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	103	103	32	383	0	497	0	0	0
N.S.	1	1.00	0.31	3.72	0.00	4.83	0.00	0.00	0.00
time (sec)	N/A	0.183	10.022	1.086	0.000	0.423	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	81	81	126	538	0	1210	0	0	0
N.S.	1	1.00	1.56	6.64	0.00	14.94	0.00	0.00	0.00
time (sec)	N/A	0.013	4.134	7.796	0.000	1.301	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	126	443	0	345	0	0	0
N.S.	1	1.00	1.56	5.47	0.00	4.26	0.00	0.00	0.00
time (sec)	N/A	0.007	4.070	1.669	0.000	1.009	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	118	938	0	1232	0	0	0
N.S.	1	1.00	1.04	8.30	0.00	10.90	0.00	0.00	0.00
time (sec)	N/A	0.009	4.047	9.519	0.000	0.681	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	1103	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	10.12	0.00	0.00	0.00
time (sec)	N/A	0.008	4.032	0.000	0.000	0.739	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	29	33	0	85	0	0	217
N.S.	1	1.00	0.33	0.38	0.00	0.98	0.00	0.00	2.49
time (sec)	N/A	0.594	10.362	0.759	0.000	0.253	0.000	0.000	0.163

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	F	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	529	46	317	0	70	0	0	207
N.S.	1	7.45	0.65	4.46	0.00	0.99	0.00	0.00	2.92
time (sec)	N/A	1.056	10.372	0.506	0.000	0.264	0.000	0.000	0.490

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	180	52	367	0	63	0	0	0
N.S.	1	3.91	1.13	7.98	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	1.044	10.879	0.655	0.000	0.276	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	34	111	0	0	0	0	67
N.S.	1	1.00	1.06	3.47	0.00	0.00	0.00	0.00	2.09
time (sec)	N/A	0.068	1.217	2.957	0.000	0.000	0.000	0.000	1.770

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	31	42	0	44	0	0	204
N.S.	1	1.00	1.35	1.83	0.00	1.91	0.00	0.00	8.87
time (sec)	N/A	0.039	0.879	0.817	0.000	0.268	0.000	0.000	0.567

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	47	355	0	1569	0	0	0
N.S.	1	1.00	0.22	1.63	0.00	7.20	0.00	0.00	0.00
time (sec)	N/A	0.030	10.046	48.349	0.000	0.426	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	50	352	0	1702	0	0	0
N.S.	1	1.00	0.24	1.68	0.00	8.10	0.00	0.00	0.00
time (sec)	N/A	0.027	10.051	48.707	0.000	0.393	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	65	351	0	1718	0	0	0
N.S.	1	1.00	0.29	1.58	0.00	7.74	0.00	0.00	0.00
time (sec)	N/A	0.022	10.053	41.432	0.000	0.396	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	68	352	0	1538	0	0	0
N.S.	1	1.00	0.32	1.64	0.00	7.19	0.00	0.00	0.00
time (sec)	N/A	0.029	10.045	41.404	0.000	0.391	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	77	327	0	323	0	0	0
N.S.	1	1.00	1.18	5.03	0.00	4.97	0.00	0.00	0.00
time (sec)	N/A	0.099	8.275	2.295	0.000	0.360	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	77	311	0	112	0	0	0
N.S.	1	1.00	1.22	4.94	0.00	1.78	0.00	0.00	0.00
time (sec)	N/A	0.097	8.204	2.308	0.000	0.380	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	92	816	0	0	0	0	0
N.S.	1	1.00	1.74	15.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.757	2.089	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	0	1421	0	267	0	0	0
N.S.	1	1.00	0.00	13.16	0.00	2.47	0.00	0.00	0.00
time (sec)	N/A	0.063	0.000	4.127	0.000	1.044	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	189	111	0	1252	0	0	0
N.S.	1	1.00	1.93	1.13	0.00	12.78	0.00	0.00	0.00
time (sec)	N/A	0.022	0.882	1.293	0.000	108.172	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.142	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	182	92	110	387	0	113	157
N.S.	1	1.00	1.90	0.96	1.15	4.03	0.00	1.18	1.64
time (sec)	N/A	0.051	0.370	0.911	0.286	0.263	0.000	3.121	0.611

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	114	95	0	253	0	0	0
N.S.	1	1.00	1.30	1.08	0.00	2.88	0.00	0.00	0.00
time (sec)	N/A	0.008	0.258	3.221	0.000	1.607	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	233	233	283	0	0	373	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.082	0.775	0.000	0.000	1.485	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	104	80	86	90	0	87	100
N.S.	1	1.00	1.27	0.98	1.05	1.10	0.00	1.06	1.22
time (sec)	N/A	0.041	0.097	3.605	0.287	0.245	0.000	0.315	0.558

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	135	383	145	677	0	318	0	0	0
N.S.	1	2.84	1.07	5.01	0.00	2.36	0.00	0.00	0.00
time (sec)	N/A	0.193	1.163	6.457	0.000	5.537	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	135	383	145	737	0	318	0	0	0
N.S.	1	2.84	1.07	5.46	0.00	2.36	0.00	0.00	0.00
time (sec)	N/A	0.215	1.122	5.957	0.000	5.248	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	119	357	139	714	0	268	0	0	0
N.S.	1	3.00	1.17	6.00	0.00	2.25	0.00	0.00	0.00
time (sec)	N/A	0.202	1.088	6.401	0.000	5.265	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	0	0	0	0	0
N.S.	1	1.00	1.00	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	10.181	0.862	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	0	0	0	0	0
N.S.	1	1.00	1.00	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	10.141	0.606	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	43	34	0	0	0	0	0
N.S.	1	1.00	1.10	0.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.023	10.017	0.534	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	101	12	105	94	31	0	10
N.S.	1	1.00	1.51	0.18	1.57	1.40	0.46	0.00	0.15
time (sec)	N/A	0.009	0.082	1.276	0.287	0.250	0.550	0.000	0.457

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	90	66	73	75	41	74	91
N.S.	1	1.00	1.29	0.94	1.04	1.07	0.59	1.06	1.30
time (sec)	N/A	0.033	0.050	1.934	0.290	0.249	0.576	0.295	0.449

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	384	384	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	234	234	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.113	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	199	250	0	618	0	2298	0	0	0
N.S.	1	1.26	0.00	3.11	0.00	11.55	0.00	0.00	0.00
time (sec)	N/A	0.192	0.000	26.803	0.000	1.184	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	177	177	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.097	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	177	177	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.110	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	204	179	0	191	0	0	0
N.S.	1	1.00	1.55	1.36	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.019	0.331	1.854	0.000	0.225	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	250	250	26	0	0	0	0	0	0
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	10.012	0.000	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	383	648	138	0	0	0	0	0	0
N.S.	1	1.69	0.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.672	15.090	0.000	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	272	272	283	1147	0	341	0	0	0
N.S.	1	1.00	1.04	4.22	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.102	1.905	5.600	0.000	1.540	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [29] had the largest ratio of [2.14299999999999979]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	10	0.100
2	C	5	2	3.47	37	0.054
3	A	9	6	1.00	15	0.400
4	A	3	3	1.00	19	0.158
5	A	8	7	1.00	17	0.412
6	A	6	6	1.00	17	0.353
7	A	3	3	1.00	17	0.176
8	A	4	2	1.00	23	0.087
9	A	18	12	1.66	27	0.444
10	B	25	13	2.46	39	0.333
11	A	7	3	1.00	45	0.067
12	A	7	4	1.00	32	0.125
13	A	5	3	1.00	32	0.094
14	A	2	2	1.00	27	0.074
15	A	2	2	1.00	29	0.069
16	A	2	1	1.00	30	0.033
17	A	3	2	1.00	13	0.154
18	A	3	2	1.00	15	0.133
19	A	2	2	1.00	23	0.087
20	A	2	2	1.00	25	0.080
21	A	3	2	1.00	11	0.182
22	A	2	2	1.00	18	0.111
23	A	6	6	1.00	20	0.300
24	A	6	5	1.00	31	0.161

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	2	2	1.00	29	0.069
26	A	2	2	1.00	35	0.057
27	A	5	5	1.00	32	0.156
28	A	6	6	1.00	21	0.286
29	A	359	30	1.00	14	2.143
30	A	2	2	1.00	23	0.087
31	A	3	2	1.00	13	0.154
32	A	2	2	1.00	33	0.061
33	A	5	5	1.00	15	0.333
34	A	5	5	1.00	15	0.333
35	A	1	1	1.00	11	0.091
36	A	5	5	1.00	15	0.333
37	A	1	1	1.00	17	0.059
38	A	3	3	1.00	18	0.167
39	A	2	2	1.60	16	0.125
40	A	5	5	1.62	17	0.294
41	A	5	5	1.77	13	0.385
42	A	5	5	1.84	16	0.312
43	F	0	0	N/A	0.000	N/A
44	F	0	0	N/A	0.000	N/A
45	F	0	0	N/A	0.000	N/A
46	A	19	14	1.16	32	0.438
47	A	19	9	1.00	20	0.450
48	A	29	9	1.00	20	0.450
49	A	49	9	1.00	20	0.450
50	A	14	6	1.00	23	0.261
51	A	24	6	1.00	23	0.261
52	C	9	8	3.09	48	0.167
53	A	7	7	1.00	24	0.292
54	A	7	7	1.00	24	0.292
55	A	1	1	1.00	18	0.056
56	A	2	2	1.00	13	0.154
57	A	6	6	1.00	15	0.400
58	A	25	15	1.00	17	0.882
59	A	19	12	1.46	22	0.546

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	12	11	1.00	17	0.647
61	A	9	8	1.00	21	0.381
62	A	1	1	1.00	38	0.026
63	A	1	1	1.00	33	0.030
64	A	2	2	1.00	38	0.053
65	A	1	1	1.00	19	0.053
66	A	4	4	1.00	19	0.210
67	A	4	4	1.00	24	0.167
68	A	1	1	1.00	24	0.042
69	A	7	7	1.00	24	0.292
70	A	7	7	1.00	24	0.292
71	A	3	3	1.00	26	0.115
72	A	3	3	1.00	22	0.136
73	A	1	1	1.00	22	0.045
74	A	1	1	1.00	23	0.043
75	A	8	8	1.00	18	0.444
76	A	8	7	1.00	23	0.304
77	A	1	1	1.00	21	0.048
78	A	1	1	1.00	19	0.053
79	A	1	1	1.00	19	0.053
80	A	1	1	1.00	19	0.053
81	A	4	4	1.00	34	0.118
82	C	5	5	7.45	40	0.125
83	C	7	7	3.91	51	0.137
84	A	2	2	1.00	29	0.069
85	A	2	2	1.00	18	0.111
86	A	1	1	1.00	25	0.040
87	A	1	1	1.00	25	0.040
88	A	1	1	1.00	25	0.040
89	A	1	1	1.00	25	0.040
90	A	2	2	1.00	40	0.050
91	A	2	2	1.00	40	0.050
92	A	1	1	1.00	18	0.056
93	A	3	3	1.00	15	0.200
94	A	1	1	1.00	21	0.048

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	8	7	1.00	23	0.304
96	A	5	5	1.00	24	0.208
97	A	1	1	1.00	19	0.053
98	A	8	8	1.00	20	0.400
99	A	5	5	1.00	22	0.227
100	B	16	12	2.84	25	0.480
101	B	17	13	2.84	24	0.542
102	B	16	12	3.00	23	0.522
103	A	5	4	1.00	20	0.200
104	A	5	4	1.00	25	0.160
105	A	3	3	1.00	19	0.158
106	A	2	2	1.00	11	0.182
107	A	6	6	1.00	15	0.400
108	A	13	12	1.00	19	0.632
109	A	13	12	1.00	22	0.546
110	A	14	12	1.26	27	0.444
111	A	5	5	1.00	17	0.294
112	A	6	6	1.00	27	0.222
113	A	3	3	1.00	19	0.158
114	A	10	10	1.00	20	0.500
115	A	17	6	1.69	24	0.250
116	A	14	8	1.00	19	0.421

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{1}{\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{1-ax}}{a}$$

```
[Out] -2*(-a*x+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {32}

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{1-ax}}{a}$$

```
[In] Int[1/Sqrt[1 - a*x],x]
```

```
[Out] (-2*Sqrt[1 - a*x])/a
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{1-ax}}{a}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{1-ax}}{a}$$

```
[In] Integrate[1/Sqrt[1 - a*x],x]
[Out] (-2*Sqrt[1 - a*x])/a
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{2\sqrt{-ax+1}}{a}$	14
derivativedivides	$-\frac{2\sqrt{-ax+1}}{a}$	14
default	$-\frac{2\sqrt{-ax+1}}{a}$	14
trager	$-\frac{2\sqrt{-ax+1}}{a}$	14
pseudoelliptic	$-\frac{2\sqrt{-ax+1}}{a}$	14
risch	$\frac{2ax-2}{a\sqrt{-ax+1}}$	19
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-ax+1}}{\sqrt{\pi}\,a}$	28

```
[In] int(1/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(-a*x+1)^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\,\sqrt{-ax+1}}{a}$$

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-a*x + 1)/a
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{-ax+1}}{a}$$

```
[In] integrate(1/(-a*x+1)**(1/2),x)
[Out] -2*sqrt(-a*x + 1)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{-ax+1}}{a}$$

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(-a*x + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{-ax+1}}{a}$$

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(-a*x + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{1-a\,x}}{a}$$

```
[In] int(1/(1 - a*x)^(1/2),x)
[Out] -(2*(1 - a*x)^(1/2))/a
```

3.2 $\int \frac{-2\log(-\sqrt{-1+ax})+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 15

$$\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx=-\frac{2\sqrt{1-ax}}{a}$$

```
[Out] -2*(-a*x+1)^(1/2)/a
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2 in optimal.

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 3.47, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {12, 2332}

$$\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx=\frac{\sqrt{ax-1}\log(ax-1)}{\pi a}-\frac{2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

```
[In] Int[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-1 + a*x]),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Log[-Sqrt[-1 + a*x]])/(a*Pi) + (Sqrt[-1 + a*x]*Log[-1 + a*x])/(a*Pi)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{\sqrt{-1+ax}}\,dx}{2\pi} \\
&= \frac{\text{Subst}\left(\int \left(-2\log(-x)+\log\left(x^2\right)\right)\,dx, x, \sqrt{-1+ax}\right)}{a\pi} \\
&= \frac{\text{Subst}\left(\int \log\left(x^2\right)\,dx, x, \sqrt{-1+ax}\right)}{a\pi} - \frac{2\text{Subst}\left(\int \log(-x)\,dx, x, \sqrt{-1+ax}\right)}{a\pi} \\
&= -\frac{2\sqrt{-1+ax}\log\left(-\sqrt{-1+ax}\right)}{a\pi} + \frac{\sqrt{-1+ax}\log(-1+ax)}{a\pi}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2 in optimal.

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$
\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx = \frac{\sqrt{-1+ax}\left(-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)\right)}{a\pi}
$$

```
[In] Integrate[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-1 + a*x]),x
]
[Out] (Sqrt[-1 + a*x]*(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x]))/(a*Pi)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

method	result	size
gosper	$\frac{\sqrt{ax-1}\,(\ln(ax-1)-2\ln(-\sqrt{ax-1}))}{a\pi}$	34
derivativedivides	$\frac{-2\ln(-\sqrt{ax-1})\sqrt{ax-1}+\sqrt{ax-1}\,\ln(ax-1)}{\pi a}$	42
default	$\frac{-2\ln(-\sqrt{ax-1})\sqrt{ax-1}+\sqrt{ax-1}\,\ln(ax-1)}{\pi a}$	42
meijerg	$\frac{i\sqrt{-\operatorname{signum}(ax-1)}\,(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-ax+1})}{\sqrt{\pi}\,\sqrt{\operatorname{signum}(ax-1)}\,a}$	47
parts	$\frac{-2\ln(-\sqrt{ax-1})\sqrt{ax-1}+2\sqrt{ax-1}}{a\pi}+\frac{\sqrt{ax-1}\,\ln(ax-1)-2\sqrt{ax-1}}{\pi a}$	68

```
[In] int(1/2*(ln(a*x-1)-2*ln(-(a*x-1)^(1/2)))/Pi/(a*x-1)^(1/2),x,method=_RETURNV
ERBOSE)
[Out] (a*x-1)^(1/2)*(ln(a*x-1)-2*ln(-(a*x-1)^(1/2)))/a/Pi
```

Fricas [F]

$$\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx = \int \frac{\log\left(ax-1\right)-2\,\log\left(-\sqrt{ax-1}\right)}{2\,\pi\sqrt{ax-1}}\,dx$$

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="fricas")
[Out] 0
```

Sympy [A] (verification not implemented)

Time = 4.72 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.80

$$\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx = \frac{\begin{cases} \frac{-2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)+\sqrt{ax-1}\log\left(ax-1\right)}{a} & \text{for } a\neq 0 \\ \pi x & \text{otherwise} \end{cases}}{\pi}$$

```
[In] integrate(1/2*(ln(a*x-1)-2*ln(-(a*x-1)**(1/2)))/pi/(a*x-1)**(1/2),x)
[Out] Piecewise(((-2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)) + sqrt(a*x - 1)*log(a*x -
1))/a, Ne(a, 0)), (pi*x, True))/pi
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(13) = 26$.

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.73

$$\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx = \frac{\sqrt{ax-1}\log\left(ax-1\right)-2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="maxima")
[Out] (sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)))/(pi*a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(13) = 26$.

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.73

$$\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx = \frac{\sqrt{ax-1}\log\left(ax-1\right)-2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="giac")
[Out] (sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)))/(pi*a)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.87

$$\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx = -\frac{2\,\ln\left(-\sqrt{a\,x-1}\right)\,\sqrt{a\,x-1}-\ln\left(a\,x-1\right)\,\sqrt{a\,x-1}}{\Pi\,a}$$

```
[In] int((log(a*x - 1)/2 - log(-(a*x - 1)^(1/2)))/(Pi*(a*x - 1)^(1/2)),x)
[Out] -(2*log(-(a*x - 1)^(1/2))*(a*x - 1)^(1/2) - log(a*x - 1)*(a*x - 1)^(1/2))/(
Pi*a)
```

3.3 $\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 82

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2} dx = \frac{4x}{3(1-3x^2)} - \frac{2\sqrt{1+x^2}}{3(1-3x^2)} - \frac{\operatorname{arctanh}\left(\sqrt{3}x\right)}{3\sqrt{3}} + \frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{3}\sqrt{1+x^2}\right)}{3\sqrt{3}}$$

```
[Out] 4/3*x/(-3*x^2+1)-1/9*arctanh(x*3^(1/2))*3^(1/2)+1/9*arctanh(1/2*3^(1/2)*(x^
2+1)^(1/2))*3^(1/2)-2/3*(x^2+1)^(1/2)/(-3*x^2+1)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6874, 205, 213, 455, 43, 65}

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2} dx = \frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right)}{3\sqrt{3}} - \frac{\operatorname{arctanh}\left(\sqrt{3}x\right)}{3\sqrt{3}} + \frac{4x}{3(1-3x^2)} - \frac{2\sqrt{x^2+1}}{3(1-3x^2)}$$

```
[In] Int[(2*x + Sqrt[1 + x^2])^(-2),x]
```

```
[Out] (4*x)/(3*(1 - 3*x^2)) - (2*Sqrt[1 + x^2])/(3*(1 - 3*x^2)) - ArcTanh[Sqrt[3]
*x]/(3*Sqrt[3]) + ArcTanh[(Sqrt[3]*Sqrt[1 + x^2])/2]/(3*Sqrt[3])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{8}{3\left(-1+3x^2\right)^2}-\frac{4x\sqrt{1+x^2}}{\left(-1+3x^2\right)^2}+\frac{5}{3\left(-1+3x^2\right)}\right) dx \\
&= \frac{5}{3}\int \frac{1}{-1+3x^2}\,dx+\frac{8}{3}\int \frac{1}{\left(-1+3x^2\right)^2}\,dx-4\int \frac{x\sqrt{1+x^2}}{\left(-1+3x^2\right)^2}\,dx \\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{5\operatorname{arctanh}\left(\sqrt{3}x\right)}{3\sqrt{3}}-\frac{4}{3}\int \frac{1}{-1+3x^2}\,dx-2\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{(-1+3x)^2}\,dx,x,x^2\right) \\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\operatorname{arctanh}\left(\sqrt{3}x\right)}{3\sqrt{3}}-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x}(-1+3x)}\,dx,x,x^2\right)
\end{aligned}
$$

$$= \frac{4x}{3(1-3x^2)} - \frac{2\sqrt{1+x^2}}{3(1-3x^2)} - \frac{\operatorname{arctanh}(\sqrt{3}x)}{3\sqrt{3}} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{-4+3x^2}\,dx, x, \sqrt{1+x^2}\right)$$

$$= \frac{4x}{3(1-3x^2)} - \frac{2\sqrt{1+x^2}}{3(1-3x^2)} - \frac{\operatorname{arctanh}(\sqrt{3}x)}{3\sqrt{3}} + \frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{3}\sqrt{1+x^2}\right)}{3\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx = \frac{6\left(-2x+\sqrt{1+x^2}\right) - 2\sqrt{3}(-1+3x^2)\operatorname{arctanh}\left(\frac{x-\sqrt{1+x^2}}{\sqrt{3}}\right)}{-9+27x^2}$$

```
[In] Integrate[(2*x + Sqrt[1 + x^2])^(-2),x]
[Out] (6*(-2*x + Sqrt[1 + x^2]) - 2*Sqrt[3]*(-1 + 3*x^2)*ArcTanh[(x - Sqrt[1 + x^
2])/Sqrt[3]])/(-9 + 27*x^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

method	result
trager	$-\frac{4x}{3(3x^2-1)}+\frac{2\sqrt{x^2+1}}{3(3x^2-1)}+\frac{\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{3\sqrt{x^2+1}+2\,\mathrm{RootOf}\left(\_Z^2-3\right)}{\mathrm{RootOf}\left(\_Z^2-3\right)x+1}\right)}{9}$
default	$-\frac{x}{2(3x^2-1)}-\frac{\operatorname{arctanh}\left(x\sqrt{3}\right)\sqrt{3}}{9}-\frac{5x}{18\left(x^2-\frac{1}{3}\right)}-\sqrt{3}\left(-\frac{\left(\left(x-\frac{\sqrt{3}}{3}\right)^2+\frac{2\sqrt{3}\left(x-\frac{\sqrt{3}}{3}\right)}{3}+\frac{4}{3}\right)^{\frac{3}{2}}}{12\left(x-\frac{\sqrt{3}}{3}\right)}+\frac{\sqrt{3}\left(\frac{\sqrt{9\left(x-\frac{\sqrt{3}}{3}\right)^2+6\sqrt{3}\left(x-\right.}}{3}\right.}{}\right.$

[In] int(1/(2*x+(x^2+1)^(1/2))^2,x,method=_RETURNVERBOSE)

[Out] -4/3*x/(3*x^2-1)+2/3/(3*x^2-1)*(x^2+1)^(1/2)+1/9*RootOf(_Z^2-3)*ln(-(3*(x^2+1)^(1/2)+2*RootOf(_Z^2-3))/(RootOf(_Z^2-3)*x+1))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx = \frac{\sqrt{3}(3\,x^2-1)\log\left(\frac{3\,x^2-2\,\sqrt{3}x+1}{3\,x^2-1}\right)+\sqrt{3}(3\,x^2-1)\log\left(\frac{3\,x^2+4\,\sqrt{3}\sqrt{x^2+1}+7}{3\,x^2-1}\right)-24\,x+12\,\sqrt{x^2+1}}{18\,(3\,x^2-1)}$$

[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="fricas")

[Out] 1/18*(sqrt(3)*(3*x^2 - 1)*log((3*x^2 - 2*sqrt(3)*x + 1)/(3*x^2 - 1)) + sqrt(3)*(3*x^2 - 1)*log((3*x^2 + 4*sqrt(3)*sqrt(x^2 + 1) + 7)/(3*x^2 - 1)) - 24*x + 12*sqrt(x^2 + 1))/(3*x^2 - 1)

Sympy [F]

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx=\int \frac{1}{\left(2x+\sqrt{x^2+1}\right)^2}\,dx$$

```
[In] integrate(1/(2*x+(x**2+1)**(1/2))**2,x)
[Out] Integral((2*x + sqrt(x**2 + 1))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx=\int \frac{1}{\left(2\,x+\sqrt{x^2+1}\right)^2}\,dx$$

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="maxima")
[Out] integrate((2*x + sqrt(x^2 + 1))^(-2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(60) = 120$.

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.16

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx=\frac{1}{18}\,\sqrt{3}\log\left(\frac{\left|6\,x-2\,\sqrt{3}\right|}{\left|6\,x+2\,\sqrt{3}\right|}\right)-\frac{1}{18}\,\sqrt{3}\log\left(-\frac{\left|-6\,x-8\,\sqrt{3}+6\,\sqrt{x^2+1}-\frac{6}{x-\sqrt{x^2+1}}\right|}{2\left(3\,x-4\,\sqrt{3}-3\,\sqrt{x^2+1}+\frac{3}{x-\sqrt{x^2+1}}\right)}\right)-\frac{4\left(x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}\right)}{3\left(3\left(x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}\right)^2-16\right)}-\frac{4\,x}{3\,(3\,x^2-1)}$$

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="giac")
[Out] 1/18*sqrt(3)*log(abs(6*x - 2*sqrt(3))/abs(6*x + 2*sqrt(3))) - 1/18*sqrt(3)*
log(-1/2*abs(-6*x - 8*sqrt(3) + 6*sqrt(x^2 + 1) - 6/(x - sqrt(x^2 + 1)))/(3
*x - 4*sqrt(3) - 3*sqrt(x^2 + 1) + 3/(x - sqrt(x^2 + 1)))) - 4/3*(x - sqrt(
x^2 + 1) + 1/(x - sqrt(x^2 + 1)))/(3*(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 +
 1)))^2 - 16) - 4/3*x/(3*x^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.49

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx = \frac{\sqrt{3}\left(\ln\left(x-\frac{\sqrt{3}}{3}\right)-\ln\left(x+\sqrt{3}+2\sqrt{x^2+1}\right)\right)}{18} - \frac{4\,x}{9\,\left(x^2-\frac{1}{3}\right)} + \frac{\sqrt{3}\left(\ln\left(x+\frac{\sqrt{3}}{3}\right)-\ln\left(x-\sqrt{3}-2\sqrt{x^2+1}\right)\right)}{18} - \frac{\sqrt{3}\left(6\,\ln\left(x-\frac{\sqrt{3}}{3}\right)-6\,\ln\left(x+\sqrt{3}+2\sqrt{x^2+1}\right)\right)}{54} - \frac{\sqrt{3}\left(6\,\ln\left(x+\frac{\sqrt{3}}{3}\right)-6\,\ln\left(x-\sqrt{3}-2\sqrt{x^2+1}\right)\right)}{54} + \frac{\sqrt{3}\sqrt{x^2+1}}{9\left(x-\frac{\sqrt{3}}{3}\right)} - \frac{\sqrt{3}\sqrt{x^2+1}}{9\left(x+\frac{\sqrt{3}}{3}\right)} + \frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{9}$$

```
[In] int(1/(2*x + (x^2 + 1)^(1/2))^2,x)
```

```
[Out] (3^(1/2)*(log(x - 3^(1/2)/3) - log(x + 3^(1/2) + 2*(x^2 + 1)^(1/2))))/18 +
(3^(1/2)*atan(3^(1/2)*x*1i)*1i)/9 - (4*x)/(9*(x^2 - 1/3)) + (3^(1/2)*(log(x
 + 3^(1/2)/3) - log(x - 3^(1/2) - 2*(x^2 + 1)^(1/2))))/18 - (3^(1/2)*(6*log
(x - 3^(1/2)/3) - 6*log(x + 3^(1/2) + 2*(x^2 + 1)^(1/2))))/54 - (3^(1/2)*(6
*log(x + 3^(1/2)/3) - 6*log(x - 3^(1/2) - 2*(x^2 + 1)^(1/2))))/54 + (3^(1/2
)*(x^2 + 1)^(1/2))/(9*(x - 3^(1/2)/3)) - (3^(1/2)*(x^2 + 1)^(1/2))/(9*(x +
3^(1/2)/3))
```

3.4 $\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)^2} \, dx = \frac{3x\sqrt{-1+x^2}}{8\left(4-3x^2\right)} + \frac{5}{16}\operatorname{arctanh}\left(\frac{x}{2\sqrt{-1+x^2}}\right)$$

```
[Out] 5/16*arctanh(1/2*x/(x^2-1)^(1/2))+3/8*x*(x^2-1)^(1/2)/(-3*x^2+4)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 213}

$$\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)^2} \, dx = \frac{5}{16}\operatorname{arctanh}\left(\frac{x}{2\sqrt{x^2-1}}\right) + \frac{3\sqrt{x^2-1}x}{8\left(4-3x^2\right)}$$

```
[In] Int[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
[Out] (3*x*Sqrt[-1 + x^2])/(8*(4 - 3*x^2)) + (5*ArcTanh[x/(2*Sqrt[-1 + x^2])])/16
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3x\sqrt{-1+x^2}}{8(4-3x^2)} - \frac{5}{8}\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)}\,dx \\
&= \frac{3x\sqrt{-1+x^2}}{8(4-3x^2)} - \frac{5}{8}\text{Subst}\left(\int \frac{1}{-4+x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right) \\
&= \frac{3x\sqrt{-1+x^2}}{8(4-3x^2)} + \frac{5}{16}\text{arctanh}\left(\frac{x}{2\sqrt{-1+x^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)^2}\,dx = -\frac{3x\sqrt{-1+x^2}}{8(-4+3x^2)} + \frac{5}{32}\log\left(2-x^2+x\sqrt{-1+x^2}\right) - \frac{5}{32}\log\left(2-3x^2+3x\sqrt{-1+x^2}\right)$$

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

```
[Out] (-3*x*Sqrt[-1 + x^2])/(8*(-4 + 3*x^2)) + (5*Log[2 - x^2 + x*Sqrt[-1 + x^2]]
)/32 - (5*Log[2 - 3*x^2 + 3*x*Sqrt[-1 + x^2]])/32
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

method	result
trager	$-\frac{3x\sqrt{x^2-1}}{8(3x^2-4)}+\frac{5\ln\left(-\frac{4\sqrt{x^2-1}\,x+5x^2-4}{3x^2-4}\right)}{32}$
pseudoelliptic	$\frac{(-15x^2+20)\ln\left(\frac{2\sqrt{x^2-1}-x}{x}\right)+15\ln\left(\frac{x+2\sqrt{x^2-1}}{x}\right)x^2-12\sqrt{x^2-1}\,x-20\ln\left(\frac{x+2\sqrt{x^2-1}}{x}\right)}{96x^2-128}$
risch	$-\frac{3x\sqrt{x^2-1}}{8(3x^2-4)}-\frac{5\,\mathrm{arctanh}\left(\frac{3\left(\frac{2}{3}-\frac{4\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{2\sqrt{9\left(x+\frac{2\sqrt{3}}{3}\right)^2-12\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)+3}}\right)}{32}+\frac{5\,\mathrm{arctanh}\left(\frac{3\left(\frac{2}{3}+\frac{4\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{2\sqrt{9\left(x-\frac{2\sqrt{3}}{3}\right)^2+12\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)+3}}\right)}{32}$
default	$-\frac{\sqrt{\left(x+\frac{2\sqrt{3}}{3}\right)^2-\frac{4\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)}{3}+\frac{1}{3}}}{16\left(x+\frac{2\sqrt{3}}{3}\right)}-\frac{5\,\mathrm{arctanh}\left(\frac{3\left(\frac{2}{3}-\frac{4\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{2\sqrt{9\left(x+\frac{2\sqrt{3}}{3}\right)^2-12\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)+3}}\right)}{32}-\frac{\sqrt{\left(x-\frac{2\sqrt{3}}{3}\right)^2+\frac{4\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)}{3}}}{16\left(x-\frac{2\sqrt{3}}{3}\right)}$

```
[In] int(1/(3*x^2-4)^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*x/(3*x^2-4)*(x^2-1)^(1/2)+5/32*ln(-(4*(x^2-1)^(1/2)*x+5*x^2-4)/(3*x^2-
4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(33) = 66$.

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.86

$$\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)^2}\,dx = -\frac{12\,x^2+5\,(3\,x^2-4)\log\left(3\,x^2-3\,\sqrt{x^2-1}x-2\right)-5\,(3\,x^2-4)\log\left(x^2-\sqrt{x^2-1}x-2\right)+12\,\sqrt{x^2-1}x}{32\,(3\,x^2-4)}$$

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*(12*x^2 + 5*(3*x^2 - 4)*log(3*x^2 - 3*sqrt(x^2 - 1)*x - 2) - 5*(3*x^2
 - 4)*log(x^2 - sqrt(x^2 - 1)*x - 2) + 12*sqrt(x^2 - 1)*x - 16)/(3*x^2 - 4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)^2}\,dx = \int \frac{1}{\sqrt{(x-1)\,(x+1)}\,(3x^2-4)^2}\,dx$$

```
[In] integrate(1/(3*x**2-4)**2/(x**2-1)**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(3*x**2 - 4)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)^2}\,dx = \int \frac{1}{(3\,x^2-4)^2\sqrt{x^2-1}}\,dx$$

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 - 4)^2*sqrt(x^2 - 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(33) = 66$.

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.19

$$\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)^2}\,dx = \frac{5\left(x-\sqrt{x^2-1}\right)^2-3}{4\left(3\left(x-\sqrt{x^2-1}\right)^4-10\left(x-\sqrt{x^2-1}\right)^2+3\right)} - \frac{5}{32}\log\left(\left|3\left(x-\sqrt{x^2-1}\right)^2-1\right|\right) + \frac{5}{32}\log\left(\left|\left(x-\sqrt{x^2-1}\right)^2-3\right|\right)$$

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="giac")
[Out] 1/4*(5*(x - sqrt(x^2 - 1))^2 - 3)/(3*(x - sqrt(x^2 - 1))^4 - 10*(x - sqrt(x
^2 - 1))^2 + 3) - 5/32*log(abs(3*(x - sqrt(x^2 - 1))^2 - 1)) + 5/32*log(abs
((x - sqrt(x^2 - 1))^2 - 3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)^2}\,dx = \int \frac{1}{\sqrt{x^2-1}\,(3\,x^2-4)^2}\,dx$$

```
[In] int(1/((x^2 - 1)^(1/2)*(3*x^2 - 4)^2),x)
[Out] int(1/((x^2 - 1)^(1/2)*(3*x^2 - 4)^2), x)
```

3.5 $\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx = \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)}-\frac{8\operatorname{arcsinh}\left(\sqrt{x}\right)}{9}+\frac{10}{9}\operatorname{arctanh}\left(\frac{2\sqrt{x}}{\sqrt{1+x}}\right)+\frac{5}{9}\log(1-3x)$$

```
[Out] 8/9/(1-3*x)-8/9*arcsinh(x^(1/2))+10/9*arctanh(2*x^(1/2)/(1+x)^(1/2))+5/9*ln
(1-3*x)-4/3*x^(1/2)*(1+x)^(1/2)/(1-3*x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6874, 99, 163, 56, 221, 95, 213}

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx = -\frac{8}{9}\operatorname{arcsinh}\left(\sqrt{x}\right)+\frac{10}{9}\operatorname{arctanh}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)-\frac{4\sqrt{x}\sqrt{x+1}}{3(1-3x)}+\frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)$$

```
[In] Int[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] 8/(9*(1 - 3*x)) - (4*Sqrt[x]*Sqrt[1 + x])/(3*(1 - 3*x)) - (8*ArcSinh[Sqrt[x
]])/9 + (10*ArcTanh[(2*Sqrt[x])/Sqrt[1 + x]])/9 + (5*Log[1 - 3*x])/9
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{8}{3(-1+3x)^2} - \frac{4\sqrt{x}\sqrt{1+x}}{(-1+3x)^2} + \frac{5}{3(-1+3x)}\right) dx$$

$$= \frac{8}{9(1-3x)} + \frac{5}{9}\log(1-3x) - 4\int \frac{\sqrt{x}\sqrt{1+x}}{(-1+3x)^2}\,dx$$

$$= \frac{8}{9(1-3x)} - \frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)} + \frac{5}{9}\log(1-3x) - \frac{4}{3}\int \frac{\frac{1}{2}+x}{\sqrt{x}\sqrt{1+x}(-1+3x)}\,dx$$

$$= \frac{8}{9(1-3x)} - \frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)} + \frac{5}{9}\log(1-3x) - \frac{4}{9}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx - \frac{10}{9}\int \frac{1}{\sqrt{x}\sqrt{1+x}(-1+3x)}\,dx$$

$$= \frac{8}{9(1-3x)} - \frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)} + \frac{5}{9}\log(1-3x) - \frac{8}{9}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right) - \frac{20}{9}\text{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{1+x}}\right)$$

$$= \frac{8}{9(1-3x)} - \frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)} - \frac{8\text{arcsinh}(\sqrt{x})}{9} + \frac{10}{9}\text{arctanh}\left(\frac{2\sqrt{x}}{\sqrt{1+x}}\right) + \frac{5}{9}\log(1-3x)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx = \frac{2\left(-4+6\sqrt{x}\sqrt{1+x}+(1-3x)\log\left(-\sqrt{x}+\sqrt{1+x}\right)+5(-1+3x)\log\left(1-x+\sqrt{x}\sqrt{1+x}\right)\right)}{-9+27x}$$

```
[In] Integrate[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
[Out] (2*(-4 + 6*Sqrt[x]*Sqrt[1 + x] + (1 - 3*x)*Log[-Sqrt[x] + Sqrt[1 + x]] + 5*
(-1 + 3*x)*Log[1 - x + Sqrt[x]*Sqrt[1 + x]]))/(-9 + 27*x)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(54) = 108.

Time = 0.02 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.55

method	result
default	$-\frac{8}{9(-1+3x)}+\frac{5\ln(-1+3x)}{9}-\frac{\sqrt{x}\,\sqrt{1+x}\,\left(12\ln\left(\frac{1}{2}+x+\sqrt{x(1+x)}\right)x-15\,\text{arctanh}\left(\frac{5x+1}{4\sqrt{x(1+x)}}\right)x-4\ln\left(\frac{1}{2}+x+\sqrt{x(1+x)}\right)+5\,\text{arcta}\cdots\right.}{9\sqrt{x(1+x)}\,(-1+3x)}$

```
[In] int(1/(2*x^(1/2)+(1+x)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -8/9/(-1+3*x)+5/9*ln(-1+3*x)-1/9*x^(1/2)*(1+x)^(1/2)*(12*ln(1/2+x+(x*(1+x))
^(1/2))*x-15*arctanh(1/4*(5*x+1)/(x*(1+x))^(1/2))*x-4*ln(1/2+x+(x*(1+x))^(1
/2))+5*arctanh(1/4*(5*x+1)/(x*(1+x))^(1/2))-12*(x*(1+x))^(1/2))/(x*(1+x))^(
1/2)/(-1+3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx = -\frac{5\,(3\,x-1)\log\left(3\,\sqrt{x+1}\sqrt{x}-3\,x-1\right)-4\,(3\,x-1)\log\left(2\,\sqrt{x+1}\sqrt{x}-2\,x-1\right)-5\,(3\,x-1)\log\left(\sqrt{x}\right.}{9\,(3\,x-1)}$$

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/9*(5*(3*x - 1)*log(3*sqrt(x + 1)*sqrt(x) - 3*x - 1) - 4*(3*x - 1)*log(2*
sqrt(x + 1)*sqrt(x) - 2*x - 1) - 5*(3*x - 1)*log(sqrt(x + 1)*sqrt(x) - x +
1) - 5*(3*x - 1)*log(3*x - 1) - 12*sqrt(x + 1)*sqrt(x) - 12*x + 12)/(3*x -
1)
```

Sympy [F]

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx = \int \frac{1}{\left(2\sqrt{x}+\sqrt{x+1}\right)^2}\,dx$$

```
[In] integrate(1/(2*x**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Integral((2*sqrt(x) + sqrt(x + 1))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx = \int \frac{1}{\left(\sqrt{x+1}+2\,\sqrt{x}\right)^2}\,dx$$

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(x + 1) + 2*sqrt(x))^(-2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(54) = 108$.

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.80

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx = -\frac{8\left(5\left(\sqrt{x+1}-\sqrt{x}\right)^2-3\right)}{9\left(3\left(\sqrt{x+1}-\sqrt{x}\right)^4-10\left(\sqrt{x+1}-\sqrt{x}\right)^2+3\right)} - \frac{5x+1}{3(3x-1)} + \frac{4}{9}\log\left(\left(\sqrt{x+1}-\sqrt{x}\right)^2\right) - \frac{5}{9}\log\left(\left|3\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right|\right) + \frac{5}{9}\log\left(\left|\left(\sqrt{x+1}-\sqrt{x}\right)^2-3\right|\right) + \frac{5}{9}\log\left(|3x-1|\right)$$

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -8/9*(5*(sqrt(x + 1) - sqrt(x))^2 - 3)/(3*(sqrt(x + 1) - sqrt(x))^4 - 10*(s
qrt(x + 1) - sqrt(x))^2 + 3) - 1/3*(5*x + 1)/(3*x - 1) + 4/9*log((sqrt(x +
1) - sqrt(x))^2) - 5/9*log(abs(3*(sqrt(x + 1) - sqrt(x))^2 - 1)) + 5/9*log(
abs((sqrt(x + 1) - sqrt(x))^2 - 3)) + 5/9*log(abs(3*x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx = \frac{10\,\mathrm{atanh}\left(\frac{2662400\sqrt{x}}{81\left(\frac{665600\,x}{81\left(\sqrt{x+1}-1\right)^2}+\frac{665600}{81}\right)\left(\sqrt{x+1}-1\right)}\right)}{9} + \frac{5\,\ln\left(x-\frac{1}{3}\right)}{9} - \frac{16\,\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)}{9} - \frac{8}{27\left(x-\frac{1}{3}\right)} + \frac{4\sqrt{x}\sqrt{x+1}}{3\,(3x-1)}$$

```
[In] int(1/((x + 1)^(1/2) + 2*x^(1/2))^2,x)
```

```
[Out] (10*atanh((2662400*x^(1/2))/(81*((665600*x)/(81*((x + 1)^(1/2) - 1)^2) + 66
5600/81)*((x + 1)^(1/2) - 1))))/9 + (5*log(x - 1/3))/9 - (16*atanh(x^(1/2)/
((x + 1)^(1/2) - 1)))/9 - 8/(27*(x - 1/3)) + (4*x^(1/2)*(x + 1)^(1/2))/(3*(
3*x - 1))
```

3.6 $\int \frac{\sqrt{-1+x^2}}{(-i+x)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 64

$$\int \frac{\sqrt{-1+x^2}}{(-i+x)^2} \, dx = \frac{\sqrt{-1+x^2}}{i-x} - \frac{i \arctan\left(\frac{1-ix}{\sqrt{2}\sqrt{-1+x^2}}\right)}{\sqrt{2}} + \operatorname{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

```
[Out] arctanh(x/(x^2-1)^(1/2))-1/2*I*arctan(1/2*(1-I*x)*2^(1/2)/(x^2-1)^(1/2))*2^
(1/2)+(x^2-1)^(1/2)/(I-x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {747, 858, 223, 212, 739, 210}

$$\int \frac{\sqrt{-1+x^2}}{(-i+x)^2} \, dx = -\frac{i \arctan\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}} + \operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right) + \frac{\sqrt{x^2-1}}{-x+i}$$

```
[In] Int[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

```
[Out] Sqrt[-1 + x^2]/(I - x) - (I*ArcTan[(1 - I*x)/(Sqrt[2]*Sqrt[-1 + x^2])])/Sqr
t[2] + ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{-1+x^2}}{i-x}+\int \frac{x}{(-i+x)\sqrt{-1+x^2}}\,dx \\
&= \frac{\sqrt{-1+x^2}}{i-x}+i\int \frac{1}{(-i+x)\sqrt{-1+x^2}}\,dx+\int \frac{1}{\sqrt{-1+x^2}}\,dx \\
&= \frac{\sqrt{-1+x^2}}{i-x}-i\,\text{Subst}\left(\int \frac{1}{-2-x^2}\,dx,x,\frac{-1+ix}{\sqrt{-1+x^2}}\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right) \\
&= \frac{\sqrt{-1+x^2}}{i-x}-\frac{i\arctan\left(\frac{1-ix}{\sqrt{2}\sqrt{-1+x^2}}\right)}{\sqrt{2}}+\operatorname{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx = \frac{\sqrt{-1+x^2}}{i-x} - \sqrt{2}\text{arctanh}\left(\frac{1+ix-i\sqrt{-1+x^2}}{\sqrt{2}}\right) - \log\left(-x+\sqrt{-1+x^2}\right)$$

```
[In] Integrate[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

```
[Out] Sqrt[-1 + x^2]/(I - x) - Sqrt[2]*ArcTanh[(1 + I*x - I*Sqrt[-1 + x^2])/Sqrt[
2]] - Log[-x + Sqrt[-1 + x^2]]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{\sqrt{x^2-1}}{x-i} + \ln\left(x+\sqrt{x^2-1}\right) + \frac{i\sqrt{2}\,\arctan\left(\frac{(-4+2i(x-i))\sqrt{2}}{4\sqrt{(x-i)^2+2i(x-i)-2}}\right)}{2}$
default	$\frac{\left((x-i)^2+2i(x-i)-2\right)^{\frac{3}{2}}}{2x-2i} - \frac{i\left(\sqrt{(x-i)^2+2i(x-i)-2}+i\ln\left(x+\sqrt{(x-i)^2+2i(x-i)-2}\right)-\sqrt{2}\,\arctan\left(\frac{(-4+2i(x-i))\sqrt{2}}{4\sqrt{(x-i)^2+2i(x-i)-2}}\right)\right)}{2} - x$

```
[In] int((x^2-1)^(1/2)/(x-I)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(x^2-1)^(1/2)/(x-I)+ln(x+(x^2-1)^(1/2))+1/2*I*2^(1/2)*arctan(1/4*(-4+2*I*(
x-I))*2^(1/2)/((x-I)^2+2*I*(x-I)-2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx = -\frac{\sqrt{2}(x-i)\log\left(-x+i\sqrt{2}+\sqrt{x^2-1}+i\right) - \sqrt{2}(x-i)\log\left(-x-i\sqrt{2}+\sqrt{x^2-1}+i\right) + 2\,(x-i)\log\left(\right.}{2\,(x-i)}$$

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(x - I)*log(-x + I*sqrt(2) + sqrt(x^2 - 1) + I) - sqrt(2)*(x
- I)*log(-x - I*sqrt(2) + sqrt(x^2 - 1) + I) + 2*(x - I)*log(-x + sqrt(x^2
- 1)) + 2*x + 2*sqrt(x^2 - 1) - 2*I)/(x - I)
```

Sympy [F]

$$\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx = \int \frac{\sqrt{(x-1)(x+1)}}{(x-i)^2}\,dx$$

```
[In] integrate((x**2-1)**(1/2)/(-I+x)**2,x)
[Out] Integral(sqrt((x - 1)*(x + 1))/(x - I)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx = \frac{1}{2}i\sqrt{2}\arcsin\left(\frac{i\,x}{|x-i|} - \frac{1}{|x-i|}\right) - \frac{\sqrt{x^2-1}}{x-i} + \log\left(2\,x + 2\sqrt{x^2-1}\right)$$

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="maxima")
[Out] 1/2*I*sqrt(2)*arcsin(I*x/abs(x - I) - 1/abs(x - I)) - sqrt(x^2 - 1)/(x - I)
 + log(2*x + 2*sqrt(x^2 - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx = i\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2-1}-i\right)\right) + \frac{2\left(i\,x - i\sqrt{x^2-1}-1\right)}{\left(x-\sqrt{x^2-1}\right)^2 - 2i\,x + 2i\sqrt{x^2-1}+1} - \log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="giac")
[Out] I*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 - 1) - I)) + 2*(I*x - I*sqrt(x^
2 - 1) - 1)/((x - sqrt(x^2 - 1))^2 - 2*I*x + 2*I*sqrt(x^2 - 1) + 1) - log(a
bs(-x + sqrt(x^2 - 1)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx = \int \frac{\sqrt{x^2-1}}{(x-\mathrm{i})^2}\,dx$$

```
[In] int((x^2 - 1)^(1/2)/(x - 1i)^2,x)
[Out] int((x^2 - 1)^(1/2)/(x - 1i)^2, x)
```

3.7 $\int \frac{1}{\sqrt{-1+x^2}(1+x^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{1}{\sqrt{-1+x^2}\left(1+x^2\right)^2}\,dx=-\frac{x\sqrt{-1+x^2}}{4\left(1+x^2\right)}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)}{4\sqrt{2}}$$

```
[Out] 3/8*arctanh(x*2^(1/2)/(x^2-1)^(1/2))*2^(1/2)-1/4*x*(x^2-1)^(1/2)/(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {390, 385, 212}

$$\int \frac{1}{\sqrt{-1+x^2}\left(1+x^2\right)^2}\,dx=\frac{3\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2-1}}{4\left(x^2+1\right)}$$

```
[In] Int[1/(Sqrt[-1 + x^2]*(1 + x^2)^2),x]
```

```
[Out] -1/4*(x*Sqrt[-1 + x^2])/(1 + x^2) + (3*ArcTanh[(Sqrt[2]*x)/Sqrt[-1 + x^2]])
/(4*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x\sqrt{-1+x^2}}{4(1+x^2)}+\frac{3}{4}\int\frac{1}{\sqrt{-1+x^2}\,(1+x^2)}\,dx\\ &= -\frac{x\sqrt{-1+x^2}}{4(1+x^2)}+\frac{3}{4}\text{Subst}\left(\int\frac{1}{1-2x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right)\\ &= -\frac{x\sqrt{-1+x^2}}{4(1+x^2)}+\frac{3\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int\frac{1}{\sqrt{-1+x^2}\,(1+x^2)^2}\,dx=\frac{1}{8}\left(-\frac{2x\sqrt{-1+x^2}}{1+x^2}+3\sqrt{2}\text{arctanh}\left(\frac{1+x^2-x\sqrt{-1+x^2}}{\sqrt{2}}\right)\right)$$

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(1 + x^2)^2),x]
[Out] ((-2*x*Sqrt[-1 + x^2])/(1 + x^2) + 3*Sqrt[2]*ArcTanh[(1 + x^2 - x*Sqrt[-1 +
 x^2])/Sqrt[2]])/8
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{3\,\mathrm{arctanh}\left(\frac{x\sqrt{2}}{\sqrt{x^2-1}}\right)\sqrt{2}}{8}-\frac{x\sqrt{x^2-1}}{4(x^2+1)}$	37
default	$-\frac{x}{8\sqrt{x^2-1}\left(\frac{x^2}{x^2-1}-\frac{1}{2}\right)}+\frac{3\,\mathrm{arctanh}\left(\frac{x\sqrt{2}}{\sqrt{x^2-1}}\right)\sqrt{2}}{8}$	45
pseudoelliptic	$\frac{(3x^2+3)\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x^2-1}}{2x}\right)-2\sqrt{x^2-1}\,x}{8x^2+8}$	49
trager	$-\frac{x\sqrt{x^2-1}}{4(x^2+1)}+\frac{3\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2-2\right)x^2+4\sqrt{x^2-1}\,x-\mathrm{RootOf}\left(\_Z^2-2\right)}{x^2+1}\right)}{16}$	66

```
[In] int(1/(x^2+1)^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*arctanh(x*2^(1/2)/(x^2-1)^(1/2))*2^(1/2)-1/4*x*(x^2-1)^(1/2)/(x^2+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(36) = 72.

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)^2}\,dx = \frac{3\,\sqrt{2}(x^2+1)\log\left(\frac{9\,x^2+2\,\sqrt{2}(3\,x^2-1)+2\,\sqrt{x^2-1}\left(3\,\sqrt{2}x+4\,x\right)-3}{x^2+1}\right)-4\,x^2-4\,\sqrt{x^2-1}x-4}{16\,(x^2+1)}$$

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(x^2 + 1)*log((9*x^2 + 2*sqrt(2)*(3*x^2 - 1) + 2*sqrt(x^2 -
 1)*(3*sqrt(2)*x + 4*x) - 3)/(x^2 + 1)) - 4*x^2 - 4*sqrt(x^2 - 1)*x - 4)/(x
^2 + 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)^2}\,dx = \int \frac{1}{\sqrt{(x-1)\,(x+1)}\,(x^2+1)^2}\,dx$$

```
[In] integrate(1/(x**2+1)**2/(x**2-1)**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(x**2 + 1)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)^2}\,dx = \int \frac{1}{(x^2+1)^2\sqrt{x^2-1}}\,dx$$

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 1)^2*sqrt(x^2 - 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(36) = 72.

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.10

$$\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)^2}\,dx = -\frac{3}{16}\sqrt{2}\log\left(\frac{\left(x-\sqrt{x^2-1}\right)^2-2\sqrt{2}+3}{\left(x-\sqrt{x^2-1}\right)^2+2\sqrt{2}+3}\right) - \frac{3\left(x-\sqrt{x^2-1}\right)^2+1}{2\left(\left(x-\sqrt{x^2-1}\right)^4+6\left(x-\sqrt{x^2-1}\right)^2+1\right)}$$

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="giac")
[Out] -3/16*sqrt(2)*log(((x - sqrt(x^2 - 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 -
1))^2 + 2*sqrt(2) + 3)) - 1/2*(3*(x - sqrt(x^2 - 1))^2 + 1)/((x - sqrt(x^2
- 1))^4 + 6*(x - sqrt(x^2 - 1))^2 + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)^2}\,dx = \int \frac{1}{\sqrt{x^2-1}\,(x^2+1)^2}\,dx$$

```
[In] int(1/((x^2 - 1)^(1/2)*(x^2 + 1)^2),x)
[Out] int(1/((x^2 - 1)^(1/2)*(x^2 + 1)^2), x)
```

3.8 $\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 30

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx=2\sqrt{-1+x}+\frac{4}{3}(-1+x)^{3/2}-\frac{4x^{3/2}}{3}$$

```
[Out] 4/3*(-1+x)^(3/2)-4/3*x^(3/2)+2*(-1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6821, 45}

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx=-\frac{4x^{3/2}}{3}+\frac{4}{3}(x-1)^{3/2}+2\sqrt{x-1}$$

```
[In] Int[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]),x]
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 - (4*x^(3/2))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
```

```
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{\sqrt{-1+x}} - 2\sqrt{x} + \frac{2x}{\sqrt{-1+x}}\right) dx \\
&= -2\sqrt{-1+x} - \frac{4x^{3/2}}{3} + 2\int \frac{x}{\sqrt{-1+x}}\, dx \\
&= -2\sqrt{-1+x} - \frac{4x^{3/2}}{3} + 2\int \left(\frac{1}{\sqrt{-1+x}} + \sqrt{-1+x}\right) dx \\
&= 2\sqrt{-1+x} + \frac{4}{3}(-1+x)^{3/2} - \frac{4x^{3/2}}{3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2 \sqrt{-1+x}}\, dx = -\frac{4x^{3/2}}{3} + \frac{2}{3}\sqrt{-1+x}(1+2x)$$

```
[In] Integrate[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]),x]
[Out] (-4*x^(3/2))/3 + (2*Sqrt[-1 + x]*(1 + 2*x))/3
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{4(-1+x)^{\frac{3}{2}}}{3} - \frac{4x^{\frac{3}{2}}}{3} + 2\sqrt{-1+x}$	21

```
[In] int(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x,method=_RETURNVERBOSE)
[Out] 4/3*(-1+x)^(3/2)-4/3*x^(3/2)+2*(-1+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2 \sqrt{-1+x}}\,dx = \frac{2}{3}\,(2\,x+1)\sqrt{x-1}-\frac{4}{3}\,x^{\frac{3}{2}}$$

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="fricas")
[Out] 2/3*(2*x + 1)*sqrt(x - 1) - 4/3*x^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(26) = 52.

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2 \sqrt{-1+x}}\,dx = -\frac{4\sqrt{x}}{6\sqrt{x}\sqrt{x-1}+6x-3}-\frac{2\sqrt{x-1}}{6\sqrt{x}\sqrt{x-1}+6x-3}$$

```
[In] integrate(1/(-1+x)**(1/2)/((-1+x)**(1/2)+x**(1/2))**2,x)
[Out] -4*sqrt(x)/(6*sqrt(x)*sqrt(x - 1) + 6*x - 3) - 2*sqrt(x - 1)/(6*sqrt(x)*sqr
t(x - 1) + 6*x - 3)
```

Maxima [F]

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2 \sqrt{-1+x}}\,dx = \int \frac{1}{\sqrt{x-1}\left(\sqrt{x-1}+\sqrt{x}\right)^2}\,dx$$

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="maxima")
[Out] integrate(1/(sqrt(x - 1)*(sqrt(x - 1) + sqrt(x))^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2 \sqrt{-1+x}}\,dx = \frac{4}{3}\,(x-1)^{\frac{3}{2}}-\frac{4}{3}\,x^{\frac{3}{2}}+2\,\sqrt{x-1}$$

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="giac")
[Out] 4/3*(x - 1)^(3/2) - 4/3*x^(3/2) + 2*sqrt(x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx=\frac{4\,x\,\sqrt{x-1}}{3}+\frac{2\,\sqrt{x-1}}{3}-\frac{4\,x^{3/2}}{3}$$

```
[In] int(1/(((x - 1)^(1/2) + x^(1/2))^2*(x - 1)^(1/2)),x)
[Out] (4*x*(x - 1)^(1/2))/3 + (2*(x - 1)^(1/2))/3 - (4*x^(3/2))/3
```

3.9 $\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 220

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx = \frac{2-4x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25}\sqrt{-110+50\sqrt{5}}\arctan\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}\sqrt{x}\right) - \frac{1}{50}\sqrt{-110+50\sqrt{5}}\arctan\left(\frac{\sqrt{-2+2\sqrt{5}}\sqrt{-1+x^2}}{2-\left(1-\sqrt{5}\right)x}\right) - \frac{1}{25}\sqrt{110+50\sqrt{5}}\text{arctanh}\left(\frac{1}{2}\sqrt{-2+2\sqrt{5}}\sqrt{x}\right) - \frac{1}{50}\sqrt{110+50\sqrt{5}}\text{arctanh}\left(\frac{\sqrt{2+2\sqrt{5}}\sqrt{-1+x^2}}{2-x-\sqrt{5}x}\right)$$

```
[Out] 1/5*(2-4*x)/(x^(1/2)+(x^2-1)^(1/2))-1/50*arctan((x^2-1)^(1/2)*(-2+2*5^(1/2)
)^(1/2)/(2-x*(-5^(1/2)+1)))*(-110+50*5^(1/2))^(1/2)+1/25*arctan(1/2*x^(1/2)
*(2+2*5^(1/2))^(1/2))*(-110+50*5^(1/2))^(1/2)-1/25*arctanh(1/2*x^(1/2)*(-2+
2*5^(1/2))^(1/2))*(110+50*5^(1/2))^(1/2)-1/50*arctanh((x^2-1)^(1/2)*(2+2*5^
(1/2))^(1/2)/(2-x-x*5^(1/2)))*(110+50*5^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.66, number of steps used = 18, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6874, 750, 840, 1180, 213, 209, 1032, 1048, 739, 212, 210, 999}

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx = -\frac{2}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\arctan\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right) + \sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\arctan\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right) + \frac{1}{5}\sqrt{\frac{2}{5}\left(5\sqrt{5}-11\right)}\arctan\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x}\right) - \frac{2}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right) + \sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\operatorname{arctanh}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right) - \frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right) - \frac{2\sqrt{x^2-1}(1-2x)}{5\left(-x^2+x+1\right)} + \frac{2\sqrt{x}(1-2x)}{5\left(-x^2+x+1\right)}$$

```
[In] Int[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - (2*(1 - 2*x)*Sqrt[-1 + x^2])/(5*(
1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(-1 + Sqrt[5])
]*Sqrt[x]])/5 + Sqrt[2/(5*(-1 + Sqrt[5]))]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sq
rt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])] - (2*Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTan[
(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(
2*(11 + 5*Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]])/5 + Sqrt[2/(
5*(1 + Sqrt[5]))]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt
[-1 + x^2])] - (2*Sqrt[(2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sq
rt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 750

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 999

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[1/((b - q + 2*c*x)
```

```
*Sqrt[d + f*x^2]), x], x] - Dist[2*(c/q), Int[1/((b + q + 2*c*x)*Sqrt[d + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

integral

$$= \int \left(-\frac{2\sqrt{x}}{(-1-x+x^2)^2}+\frac{2x}{\sqrt{-1+x^2}\,(-1-x+x^2)^2}+\frac{1}{\sqrt{-1+x^2}\,(-1-x+x^2)}\right) dx$$

$$= -\left(2\int \frac{\sqrt{x}}{(-1-x+x^2)^2}\,dx\right)+2\int \frac{x}{\sqrt{-1+x^2}\,(-1-x+x^2)^2}\,dx + \int \frac{1}{\sqrt{-1+x^2}\,(-1-x+x^2)}\,dx$$

$$= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)}-\frac{2}{5}\int \frac{-\frac{1}{2}-x}{\sqrt{x}\,(-1-x+x^2)}\,dx + \frac{2}{5}\int \frac{-3-x}{\sqrt{-1+x^2}\,(-1-x+x^2)}\,dx+\frac{2\int \frac{1}{\left(-1-\sqrt{5}+2x\right)\sqrt{-1+x^2}}\,dx}{\sqrt{5}}-\frac{2\int \frac{1}{\left(-1+\sqrt{5}+2x\right)\sqrt{-1+x^2}}\,dx}{\sqrt{5}}$$

$$= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)}-\frac{4}{5}\text{Subst}\left(\int \frac{-\frac{1}{2}-x^2}{-1-x^2+x^4}\,dx,x,\sqrt{x}\right) - \frac{2\text{Subst}\left(\int \frac{1}{-4+\left(-1-\sqrt{5}\right)^2-x^2}\,dx,x,\frac{-2-\left(-1-\sqrt{5}\right)x}{\sqrt{-1+x^2}}\right)}{\sqrt{5}} + \frac{2\text{Subst}\left(\int \frac{1}{-4+\left(-1+\sqrt{5}\right)^2-x^2}\,dx,x,\frac{-2-\left(-1+\sqrt{5}\right)x}{\sqrt{-1+x^2}}\right)}{\sqrt{5}} - \frac{1}{25}\left(2\left(5-7\sqrt{5}\right)\right)\int \frac{1}{\left(-1+\sqrt{5}+2x\right)\sqrt{-1+x^2}}\,dx - \frac{1}{25}\left(2\left(5+7\sqrt{5}\right)\right)\int \frac{1}{\left(-1-\sqrt{5}+2x\right)\sqrt{-1+x^2}}\,dx$$

$$
\begin{aligned}
&= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)} - \frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)} \\
&\quad + \sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}} \arctan\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right) \\
&\quad + \sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}} \operatorname{arctanh}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right) \\
&\quad + \frac{1}{25}\left(2\left(5-7\sqrt{5}\right)\right) \operatorname{Subst}\left(\int \frac{1}{-4+\left(-1+\sqrt{5}\right)^2-x^2}\,dx, x, \frac{-2-\left(-1+\sqrt{5}\right)x}{\sqrt{-1+x^2}}\right) \\
&\quad + \frac{1}{25}\left(2\left(5-2\sqrt{5}\right)\right) \operatorname{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, \sqrt{x}\right) \\
&\quad + \frac{1}{25}\left(2\left(5+2\sqrt{5}\right)\right) \operatorname{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, \sqrt{x}\right) \\
&\quad + \frac{1}{25}\left(2\left(5+7\sqrt{5}\right)\right) \operatorname{Subst}\left(\int \frac{1}{-4+\left(-1-\sqrt{5}\right)^2-x^2}\,dx, x, \frac{-2-\left(-1-\sqrt{5}\right)x}{\sqrt{-1+x^2}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)} - \frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)} + \frac{1}{5}\sqrt{\frac{2}{5}\left(-11+5\sqrt{5}\right)} \arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{x}\right) \\
&\quad + \sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}} \arctan\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right) \\
&\quad - \frac{2}{5}\sqrt{\frac{1}{5}\left(-2+5\sqrt{5}\right)} \arctan\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right) \\
&\quad - \frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)} \operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right) \\
&\quad + \sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}} \operatorname{arctanh}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right) \\
&\quad - \frac{2}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)} \operatorname{arctanh}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 6.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx = \frac{1}{25}\left(-\frac{10(-1+2x)\left(-\sqrt{x}+\sqrt{-1+x^2}\right)}{-1-x+x^2}\right.$$
$$+\sqrt{-110+50\sqrt{5}}\arctan\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{x}\right)$$
$$-\sqrt{-110+50\sqrt{5}}\arctan\left(\frac{\sqrt{-2+\sqrt{5}}\sqrt{-1+x^2}}{1+x}\right)$$
$$-\sqrt{110+50\sqrt{5}}\text{arctanh}\left(\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\sqrt{x}\right)$$
$$\left.+\sqrt{110+50\sqrt{5}}\text{arctanh}\left(\frac{\sqrt{2+\sqrt{5}}\sqrt{-1+x^2}}{1+x}\right)\right)$$

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] ((-10*(-1 + 2*x)*(-Sqrt[x] + Sqrt[-1 + x^2]))/(-1 - x + x^2) + Sqrt[-110 +
50*Sqrt[5]]*ArcTan[Sqrt[(1 + Sqrt[5])/2]*Sqrt[x]] - Sqrt[-110 + 50*Sqrt[5]]
*ArcTan[(Sqrt[-2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1 + x)] - Sqrt[110 + 50*Sqrt[5
]]*ArcTanh[Sqrt[(-1 + Sqrt[5])/2]*Sqrt[x]] + Sqrt[110 + 50*Sqrt[5]]*ArcTanh
[(Sqrt[2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1 + x)])/25
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1205 vs. 2(158) = 316.

Time = 0.32 (sec) , antiderivative size = 1206, normalized size of antiderivative = 5.48

method	result	size
default	Expression too large to display	1206

```
[In] int(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -6/25*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*
5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*
(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))-6/25*5^(1/2)/(2+2*5^(1/2))^(1/2)*ar
ctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*
(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))
+(2/5+2/5*5^(1/2))*(-1/4/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1
/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)+1/4*(5^(1
/2)+1)/(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+
```

```
1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(
1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)))+(2/5-2/5*5^(1/2))*(-1/4/(1
/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+
1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)-1/4*(-5^(1/2)+1)/(1/2-1/2*5^(1/2))/
(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/
(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)
-1/2)+2-2*5^(1/2))^(1/2)))+1/5*(5^(1/2)+1)^2*(-1/4/(1/2+1/2*5^(1/2))/(x-1/2
*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/
2*5^(1/2))^(1/2)+1/4*(5^(1/2)+1)/(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arct
anh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x
-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)))+
1/5*(5^(1/2)-1)^2*(-1/4/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/
2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)-1/4*(-5^(
1/2)+1)/(1/2-1/2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2
)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(
-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)))+1/5/(1/2+1/2*5^(1/2))/
(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+
1/2+1/2*5^(1/2))^(1/2)-1/5*(5^(1/2)+1)/(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2
)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)
/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1
/2))+1/5/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(
1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)+1/5*(-5^(1/2)+1)/(1/2-1/
2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(
1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+
1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))+2/5*x^(1/2)/(x+1/2*5^(1/2)-1/2)-8/25*(
-5/2+5^(1/2))/(-2+2*5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))+2
/5*x^(1/2)/(x-1/2*5^(1/2)-1/2)-8/25*(5/2+5^(1/2))/(2+2*5^(1/2))^(1/2)*arcta
nh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(153) = 306$.

Time = 0.24 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx = -\frac{\sqrt{5}(x^2-x-1)\sqrt{10\sqrt{5}+22}\log\left(\sqrt{10\sqrt{5}+22}(\sqrt{5}-3)-4x+2\sqrt{5}+4\sqrt{x^2-1}+2\right)-\sqrt{5}(x^2-x}{}$$

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/50*(sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(5) + 22
)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^2 - 1) + 2) - sqrt(5)*(x^2 - x
```

```
 - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) + 4*sqr
t(x)) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sqrt(5) +
22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^2 - 1) + 2) + sqrt(5)*(x^2 -
 x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) + 4*
sqrt(x)) + sqrt(5)*(x^2 - x - 1)*sqrt(-10*sqrt(5) + 22)*log((sqrt(5) + 3)*s
qrt(-10*sqrt(5) + 22) - 4*x - 2*sqrt(5) + 4*sqrt(x^2 - 1) + 2) - sqrt(5)*(x
^2 - x - 1)*sqrt(-10*sqrt(5) + 22)*log((sqrt(5) + 3)*sqrt(-10*sqrt(5) + 22)
 + 4*sqrt(x)) - sqrt(5)*(x^2 - x - 1)*sqrt(-10*sqrt(5) + 22)*log(-(sqrt(5)
+ 3)*sqrt(-10*sqrt(5) + 22) - 4*x - 2*sqrt(5) + 4*sqrt(x^2 - 1) + 2) + sqrt
(5)*(x^2 - x - 1)*sqrt(-10*sqrt(5) + 22)*log(-(sqrt(5) + 3)*sqrt(-10*sqrt(5
) + 22) + 4*sqrt(x)) + 40*x^2 + 20*sqrt(x^2 - 1)*(2*x - 1) - 20*(2*x - 1)*s
qrt(x) - 40*x - 40)/(x^2 - x - 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx=\int \frac{1}{\sqrt{(x-1)(x+1)}\left(\sqrt{x}+\sqrt{x^2-1}\right)^2}\,dx$$

```
[In] integrate(1/(x**2-1)**(1/2)/(x**(1/2)+(x**2-1)**(1/2))**2,x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(sqrt(x) + sqrt(x**2 - 1))**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx=\int \frac{1}{\sqrt{x^2-1}\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}\,dx$$

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(153) = 306$.

Time = 1.49 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$$
$$= \frac{2}{5}\sqrt{\frac{1}{10}}\sqrt{5\sqrt{5}-11}\arctan\left(\frac{2x+\sqrt{5}-2\sqrt{x^2-1}-1}{\sqrt{2\sqrt{5}-2}}\right)$$
$$+\frac{1}{5}\sqrt{\frac{1}{10}}\sqrt{5\sqrt{5}+11}\log\left(\left|-153040x+22956\sqrt{5}\sqrt{50\sqrt{5}+110}+76520\sqrt{5}+153040\sqrt{x^2-1}-38260\right.\right.$$
$$-\frac{1}{5}\sqrt{\frac{1}{10}}\sqrt{5\sqrt{5}+11}\log\left(\left|-153040x-22956\sqrt{5}\sqrt{50\sqrt{5}+110}+76520\sqrt{5}+153040\sqrt{x^2-1}+38260\right.\right.$$
$$+\frac{1}{25}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{\sqrt{x}}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{50}\sqrt{50\sqrt{5}+110}\log\left(\sqrt{x}+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)$$
$$+\frac{1}{50}\sqrt{50\sqrt{5}+110}\log\left(\left|\sqrt{x}-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)$$
$$+\frac{4\left(\left(x-\sqrt{x^2-1}\right)^3+2\left(x-\sqrt{x^2-1}\right)^2+3x-3\sqrt{x^2-1}-2\right)}{5\left(\left(x-\sqrt{x^2-1}\right)^4-2\left(x-\sqrt{x^2-1}\right)^3-2\left(x-\sqrt{x^2-1}\right)^2-2x+2\sqrt{x^2-1}+1\right)}$$
$$+\frac{2\left(2x^{\frac{3}{2}}-\sqrt{x}\right)}{5\left(x^2-x-1\right)}$$

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2/5*sqrt(1/10)*sqrt(5*sqrt(5) - 11)*arctan((2*x + sqrt(5) - 2*sqrt(x^2 - 1)
 - 1)/sqrt(2*sqrt(5) - 2)) + 1/5*sqrt(1/10)*sqrt(5*sqrt(5) + 11)*log(abs(-1
53040*x + 22956*sqrt(5)*sqrt(50*sqrt(5) + 110) + 76520*sqrt(5) + 153040*sqr
t(x^2 - 1) - 38260*sqrt(50*sqrt(5) + 110) + 76520)) - 1/5*sqrt(1/10)*sqrt(5
*sqrt(5) + 11)*log(abs(-153040*x - 22956*sqrt(5)*sqrt(50*sqrt(5) + 110) + 7
6520*sqrt(5) + 153040*sqrt(x^2 - 1) + 38260*sqrt(50*sqrt(5) + 110) + 76520)
) + 1/25*sqrt(50*sqrt(5) - 110)*arctan(sqrt(x)/sqrt(1/2*sqrt(5) - 1/2)) - 1
/50*sqrt(50*sqrt(5) + 110)*log(sqrt(x) + sqrt(1/2*sqrt(5) + 1/2)) + 1/50*sq
rt(50*sqrt(5) + 110)*log(abs(sqrt(x) - sqrt(1/2*sqrt(5) + 1/2))) + 4/5*((x
- sqrt(x^2 - 1))^3 + 2*(x - sqrt(x^2 - 1))^2 + 3*x - 3*sqrt(x^2 - 1) - 2)/(
(x - sqrt(x^2 - 1))^4 - 2*(x - sqrt(x^2 - 1))^3 - 2*(x - sqrt(x^2 - 1))^2 -
 2*x + 2*sqrt(x^2 - 1) + 1) + 2/5*(2*x^(3/2) - sqrt(x))/(x^2 - x - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx = \int \frac{1}{\sqrt{x^2-1}\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}\,dx$$

```
[In] int(1/((x^2 - 1)^(1/2)*((x^2 - 1)^(1/2) + x^(1/2))^2),x)
[Out] int(1/((x^2 - 1)^(1/2)*((x^2 - 1)^(1/2) + x^(1/2))^2), x)
```

3.10 $\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 220

$$\begin{aligned}\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx &= \frac{2-4x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)} \\ &+\frac{1}{25}\sqrt{-110+50\sqrt{5}}\arctan\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}\sqrt{x}\right) \\ &-\frac{1}{50}\sqrt{-110+50\sqrt{5}}\arctan\left(\frac{\sqrt{-2+2\sqrt{5}}\sqrt{-1+x^2}}{2-\left(1-\sqrt{5}\right)x}\right) \\ &-\frac{1}{25}\sqrt{110+50\sqrt{5}}\operatorname{arctanh}\left(\frac{1}{2}\sqrt{-2+2\sqrt{5}}\sqrt{x}\right) \\ &-\frac{1}{50}\sqrt{110+50\sqrt{5}}\operatorname{arctanh}\left(\frac{\sqrt{2+2\sqrt{5}}\sqrt{-1+x^2}}{2-x-\sqrt{5}x}\right)\end{aligned}$$

```
[Out] 1/5*(2-4*x)/(x^(1/2)+(x^2-1)^(1/2))-1/50*arctan((x^2-1)^(1/2)*(-2+2*5^(1/2)
)^(1/2)/(2-x*(-5^(1/2)+1)))*(-110+50*5^(1/2))^(1/2)+1/25*arctan(1/2*x^(1/2)
*(2+2*5^(1/2))^(1/2))*(-110+50*5^(1/2))^(1/2)-1/25*arctanh(1/2*x^(1/2)*(-2+
2*5^(1/2))^(1/2))*(110+50*5^(1/2))^(1/2)-1/50*arctanh((x^2-1)^(1/2)*(2+2*5^
(1/2))^(1/2)/(2-x-x*5^(1/2)))*(110+50*5^(1/2))^(1/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 541 vs. $2(220) = 440$.

Time = 0.56 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.46, number of steps used = 25, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 750, 840, 1180, 213, 209, 989, 1048, 739, 212, 210, 1032, 1079}

$$\begin{aligned}\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx = {} & \frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\arctan\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)\\
&-\frac{1}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\arctan\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)\\
&-\frac{1}{5}\sqrt{\frac{1}{10}\left(5\sqrt{5}-11\right)}\arctan\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)\\
&+\frac{1}{5}\sqrt{\frac{2}{5}\left(5\sqrt{5}-11\right)}\arctan\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x}\right)\\
&+\frac{1}{5}\sqrt{\frac{1}{10}\left(11+5\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)\\
&-\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)\\
&-\frac{1}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\operatorname{arctanh}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)\\
&-\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)\\
&-\frac{\sqrt{x^2-1}(1-2x)}{5\left(-x^2+x+1\right)}+\frac{2\sqrt{x}(1-2x)}{5\left(-x^2+x+1\right)}\\
&-\frac{(3-x)\sqrt{x^2-1}}{5\left(-x^2+x+1\right)}+\frac{(x+2)\sqrt{x^2-1}}{5\left(-x^2+x+1\right)}\end{aligned}$$

```
[In] Int[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - ((1 - 2*x)*Sqrt[-1 + x^2])/(5*(1
+ x - x^2)) - ((3 - x)*Sqrt[-1 + x^2])/(5*(1 + x - x^2)) + ((2 + x)*Sqrt[-1
 + x^2])/(5*(1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(
-1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-11 + 5*Sqrt[5])/10]*ArcTan[(2 - (1 - S
qrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(-2 + 5*Sqrt
```

```
[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2]
)])/5 + (Sqrt[(2 + 5*Sqrt[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 +
 Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(2*(11 + 5*Sqrt[5]))/5]*ArcTanh[Sqrt
[2/(1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 +
 Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(2 + 5*Sqrt
[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2]
)])/5 + (Sqrt[(11 + 5*Sqrt[5])/10]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1
 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 750

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
```

```
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 989

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f
))*x)*(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
```

```
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1079

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2
)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((A*c - a*C)*(
(-b)*(c*d + a*f)) + (A*b)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(2*a*f)) + C*(b^2*d - 2*a*(c*d - a*f)))*x), x] + Dist[1/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) +
(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) -
c*d*(p + 2)) - (2*f*((A*c - a*C)*((-b)*(c*d + a*f)) + (A*b)*(2*c^2*d + b^2*
f - c*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*
C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f)
 - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d
, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*
d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{2\sqrt{x}}{(-1-x+x^2)^2} - \frac{1}{\sqrt{-1+x^2}\,(-1-x+x^2)^2} + \frac{x}{\sqrt{-1+x^2}\,(-1-x+x^2)^2} + \frac{x^2}{\sqrt{-1+x^2}\,(-1-x+x^2)^2}\right) dx$$

$$= -\left(2\int \frac{\sqrt{x}}{(-1-x+x^2)^2}\,dx\right) - \int \frac{1}{\sqrt{-1+x^2}\,(-1-x+x^2)^2}\,dx + \int \frac{x}{\sqrt{-1+x^2}\,(-1-x+x^2)^2}\,dx + \int \frac{x^2}{\sqrt{-1+x^2}\,(-1-x+x^2)^2}\,dx$$

$$
\begin{aligned}
&= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)} - \frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)} - \frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)} + \frac{(2+x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)} \\
&\quad + \frac{1}{5}\int \frac{1-3x}{\sqrt{-1+x^2}\left(-1-x+x^2\right)}\,dx + \frac{1}{5}\int \frac{-3-x}{\sqrt{-1+x^2}\left(-1-x+x^2\right)}\,dx \\
&\quad + \frac{1}{5}\int \frac{1+2x}{\sqrt{-1+x^2}\left(-1-x+x^2\right)}\,dx - \frac{2}{5}\int \frac{-\frac{1}{2}-x}{\sqrt{x}\left(-1-x+x^2\right)}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)} - \frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)} - \frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)} \\
&\quad + \frac{(2+x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)} - \frac{4}{5}\mathrm{Subst}\left(\int \frac{-\frac{1}{2}-x^2}{-1-x^2+x^4}\,dx, x, \sqrt{x}\right) \\
&\quad + \frac{1}{25}\left(2\left(5-2\sqrt{5}\right)\right)\int \frac{1}{\left(-1+\sqrt{5}+2x\right)\sqrt{-1+x^2}}\,dx \\
&\quad + \frac{1}{25}\left(-15+\sqrt{5}\right)\int \frac{1}{\left(-1+\sqrt{5}+2x\right)\sqrt{-1+x^2}}\,dx \\
&\quad - \frac{1}{25}\left(15+\sqrt{5}\right)\int \frac{1}{\left(-1-\sqrt{5}+2x\right)\sqrt{-1+x^2}}\,dx \\
&\quad + \frac{1}{25}\left(2\left(5+2\sqrt{5}\right)\right)\int \frac{1}{\left(-1-\sqrt{5}+2x\right)\sqrt{-1+x^2}}\,dx \\
&\quad + \frac{1}{25}\left(-5+7\sqrt{5}\right)\int \frac{1}{\left(-1+\sqrt{5}+2x\right)\sqrt{-1+x^2}}\,dx \\
&\quad - \frac{1}{25}\left(5+7\sqrt{5}\right)\int \frac{1}{\left(-1-\sqrt{5}+2x\right)\sqrt{-1+x^2}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)} - \frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)} - \frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)} + \frac{(2+x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)} \\
&\quad + \frac{1}{25}\left(5-7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-4+\left(-1+\sqrt{5}\right)^2-x^2}\,dx, x, \frac{-2-\left(-1+\sqrt{5}\right)x}{\sqrt{-1+x^2}}\right) \\
&\quad - \frac{1}{25}\left(2\left(5-2\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-4+\left(-1+\sqrt{5}\right)^2-x^2}\,dx, x, \frac{-2-\left(-1+\sqrt{5}\right)x}{\sqrt{-1+x^2}}\right) \\
&\quad + \frac{1}{25}\left(2\left(5-2\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, \sqrt{x}\right) \\
&\quad + \frac{1}{25}\left(15-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-4+\left(-1+\sqrt{5}\right)^2-x^2}\,dx, x, \frac{-2-\left(-1+\sqrt{5}\right)x}{\sqrt{-1+x^2}}\right) \\
&\quad + \frac{1}{25}\left(15+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-4+\left(-1-\sqrt{5}\right)^2-x^2}\,dx, x, \frac{-2-\left(-1-\sqrt{5}\right)x}{\sqrt{-1+x^2}}\right) \\
&\quad - \frac{1}{25}\left(2\left(5+2\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-4+\left(-1-\sqrt{5}\right)^2-x^2}\,dx, x, \frac{-2-\left(-1-\sqrt{5}\right)x}{\sqrt{-1+x^2}}\right) \\
&\quad + \frac{1}{25}\left(2\left(5+2\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, \sqrt{x}\right) \\
&\quad + \frac{1}{25}\left(5+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-4+\left(-1-\sqrt{5}\right)^2-x^2}\,dx, x, \frac{-2-\left(-1-\sqrt{5}\right)x}{\sqrt{-1+x^2}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} - \frac{(3-x)\sqrt{-1+x^2}}{5(1+x-x^2)} \\
&\quad + \frac{(2+x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{1}{5}\sqrt{\frac{2}{5}\left(-11+5\sqrt{5}\right)} \arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{x}\right) \\
&\quad - \frac{1}{5}\sqrt{\frac{1}{10}\left(-11+5\sqrt{5}\right)} \arctan\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right) \\
&\quad - \frac{1}{5}\sqrt{\frac{1}{5}\left(-2+5\sqrt{5}\right)} \arctan\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right) \\
&\quad + \frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)} \arctan\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right) \\
&\quad - \frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)} \operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right) \\
&\quad - \frac{1}{5}\sqrt{\frac{1}{5}\left(-2+5\sqrt{5}\right)} \operatorname{arctanh}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right) \\
&\quad - \frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)} \operatorname{arctanh}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right) \\
&\quad + \frac{1}{5}\sqrt{\frac{1}{10}\left(11+5\sqrt{5}\right)} \operatorname{arctanh}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 6.31 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.89

$$
\begin{aligned}
\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{(1+x-x^2)^2\sqrt{-1+x^2}}\,dx &= \frac{1}{25}\left(-\frac{10(-1+2x)\left(-\sqrt{x}+\sqrt{-1+x^2}\right)}{-1-x+x^2}\right. \\
&\quad + \sqrt{-110+50\sqrt{5}} \arctan\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{x}\right) \\
&\quad - \sqrt{-110+50\sqrt{5}} \arctan\left(\frac{\sqrt{-2+\sqrt{5}}\sqrt{-1+x^2}}{1+x}\right) \\
&\quad - \sqrt{110+50\sqrt{5}} \operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\sqrt{x}\right) \\
&\quad \left. + \sqrt{110+50\sqrt{5}} \operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{5}}\sqrt{-1+x^2}}{1+x}\right)\right)
\end{aligned}
$$

```
[In] Integrate[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]),x]
```

```
[Out] ((-10*(-1 + 2*x)*(-Sqrt[x] + Sqrt[-1 + x^2]))/(-1 - x + x^2) + Sqrt[-110 +
50*Sqrt[5]]*ArcTan[Sqrt[(1 + Sqrt[5])/2]*Sqrt[x]] - Sqrt[-110 + 50*Sqrt[5]]
*ArcTan[(Sqrt[-2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1 + x)] - Sqrt[110 + 50*Sqrt[5
]]*ArcTanh[Sqrt[(-1 + Sqrt[5])/2]*Sqrt[x]] + Sqrt[110 + 50*Sqrt[5]]*ArcTanh
[(Sqrt[2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1 + x)])/25
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1636 vs. $2(158) = 316$.

Time = 0.48 (sec) , antiderivative size = 1637, normalized size of antiderivative = 7.44

method	result	size
default	Expression too large to display	1637

```
[In] int((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/25*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5
^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(
x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))+2/25*5^(1/2)/(2+2*5^(1/2))^(1/2)*arc
tanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(
x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))+
(2/5+2/5*5^(1/2))*(-1/4/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/
2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)+1/4*(5^(1/
2)+1)/(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1
)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1
/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)))+(2/5-2/5*5^(1/2))*(-1/4/(1/
2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1
/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)-1/4*(-5^(1/2)+1)/(1/2-1/2*5^(1/2))/(
-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(
-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-
1/2)+2-2*5^(1/2))^(1/2)))+2/5*x^(1/2)/(x+1/2*5^(1/2)-1/2)-8/25*(-5/2+5^(1/2
))/(-2+2*5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))+2/5*x^(1/2)/
(x-1/2*5^(1/2)-1/2)-8/25*(5/2+5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*x^(1/2
)/(2+2*5^(1/2))^(1/2))-1/5/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^
(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(3/2)+1/10*(5
^(1/2)+1)/(1/2+1/2*5^(1/2))*(1/2*(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-
1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)+1/2*(5^(1/2)+1)*ln(x+((x-1/2*5^(1/2)-1/
2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2))-2*(1/2+1/2*5^(
1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1
/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1
/2)-1/2)+2+2*5^(1/2))^(1/2)))+2/5/(1/2+1/2*5^(1/2))*(1/2*x*((x-1/2*5^(1/2)-
1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)+1/8*(2+2*5^(1
```

```
/2)-(5^(1/2)+1)^2)*ln(x+((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1
/2)+1/2+1/2*5^(1/2))^(1/2)))-1/5/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+
1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(3/2)+
1/10*(-5^(1/2)+1)/(1/2-1/2*5^(1/2))*(1/2*(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/
2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)+1/2*(-5^(1/2)+1)*ln(x+((x+1/2*
5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))+2*(
1/2-1/2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1
/2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+
1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)))+2/5/(1/2-1/2*5^(1/2))*(1/2*x*((
x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2
)+1/8*(2-2*5^(1/2)-(-5^(1/2)+1)^2)*ln(x+((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)
*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)))-4/25*5^(1/2)*(1/4*(4*(x-1/2*5
^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)+1/4*(5^(
1/2)+1)*ln(x+((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2
*5^(1/2))^(1/2))-(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)
+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2
)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)))+4/25*5^(1/2)*(1/
4*(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^
(1/2)+1/4*(-5^(1/2)+1)*ln(x+((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1
/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))+(1/2-1/2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arc
tan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*
(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(153) = 306$.

Time = 0.25 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.05

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{(1+x-x^2)^2\sqrt{-1+x^2}}\,dx = -\frac{\sqrt{5}(x^2-x-1)\sqrt{10\sqrt{5}+22}\log\left(\sqrt{10\sqrt{5}+22}(\sqrt{5}-3)-4x+2\sqrt{5}+4\sqrt{x^2-1}+2\right)-\sqrt{5}(x^2-x}{}$$

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/50*(sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(5) + 22
)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^2 - 1) + 2) - sqrt(5)*(x^2 - x
 - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) + 4*sqr
t(x)) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sqrt(5) +
22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^2 - 1) + 2) + sqrt(5)*(x^2 -
 x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) + 4*
```

```
sqrt(x)) + sqrt(5)*(x^2 - x - 1)*sqrt(-10*sqrt(5) + 22)*log((sqrt(5) + 3)*s
qrt(-10*sqrt(5) + 22) - 4*x - 2*sqrt(5) + 4*sqrt(x^2 - 1) + 2) - sqrt(5)*(x
^2 - x - 1)*sqrt(-10*sqrt(5) + 22)*log((sqrt(5) + 3)*sqrt(-10*sqrt(5) + 22)
 + 4*sqrt(x)) - sqrt(5)*(x^2 - x - 1)*sqrt(-10*sqrt(5) + 22)*log(-(sqrt(5)
+ 3)*sqrt(-10*sqrt(5) + 22) - 4*x - 2*sqrt(5) + 4*sqrt(x^2 - 1) + 2) + sqrt
(5)*(x^2 - x - 1)*sqrt(-10*sqrt(5) + 22)*log(-(sqrt(5) + 3)*sqrt(-10*sqrt(5
) + 22) + 4*sqrt(x)) + 40*x^2 + 20*sqrt(x^2 - 1)*(2*x - 1) - 20*(2*x - 1)*s
qrt(x) - 40*x - 40)/(x^2 - x - 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{(1+x-x^2)^2\sqrt{-1+x^2}}\,dx=\text{Timed out}$$

```
[In] integrate((x**(1/2)-(x**2-1)**(1/2))**2/(-x**2+x+1)**2/(x**2-1)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{(1+x-x^2)^2\sqrt{-1+x^2}}\,dx=\int \frac{\left(\sqrt{x^2-1}-\sqrt{x}\right)^2}{\left(x^2-x-1\right)^2\sqrt{x^2-1}}\,dx$$

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="maxima")
[Out] -2/5*(x^(5/2) - 3*x^(3/2))/(x^2 - x - 1) + integrate(1/5*(x^(3/2) + sqrt(x)
)/(x^2 - x - 1), x) + integrate((x^2 + x - 1)/((x^4 - 2*x^3 - x^2 + 2*x + 1
)*sqrt(x + 1)*sqrt(x - 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(153) = 306$.

Time = 1.65 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.63

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx = \frac{2}{5}\sqrt{\frac{1}{2}\sqrt{5}-\frac{11}{10}}\arctan\left(\frac{2x+\sqrt{5}-2\sqrt{x^2-1}-1}{\sqrt{2\sqrt{5}-2}}\right)$$
$$+\frac{1}{25}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{\sqrt{x}}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{50}\sqrt{50\sqrt{5}+110}\log\left(\sqrt{x}+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)$$
$$-\frac{1}{5}\sqrt{\frac{1}{2}\sqrt{5}+\frac{11}{10}}\log\left(\left|-520x-78\sqrt{5}\sqrt{50\sqrt{5}+110}+260\sqrt{5}+520\sqrt{x^2-1}+130\sqrt{50\sqrt{5}+110}+\right.\right.$$
$$+\frac{1}{5}\sqrt{\frac{1}{2}\sqrt{5}+\frac{11}{10}}\log\left(\left|-1040x+156\sqrt{5}\sqrt{50\sqrt{5}+110}+520\sqrt{5}+1040\sqrt{x^2-1}-260\sqrt{50\sqrt{5}+110}\right.\right.$$
$$+\frac{1}{50}\sqrt{50\sqrt{5}+110}\log\left(\left|\sqrt{x}-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)$$
$$+\frac{4\left(\left(x-\sqrt{x^2-1}\right)^3+2\left(x-\sqrt{x^2-1}\right)^2+3x-3\sqrt{x^2-1}-2\right)}{5\left(\left(x-\sqrt{x^2-1}\right)^4-2\left(x-\sqrt{x^2-1}\right)^3-2\left(x-\sqrt{x^2-1}\right)^2-2x+2\sqrt{x^2-1}+1\right)}$$
$$+\frac{2\left(2x^{\frac{3}{2}}-\sqrt{x}\right)}{5\left(x^2-x-1\right)}$$

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="giac")
```

```
[Out] 2/5*sqrt(1/2*sqrt(5) - 11/10)*arctan((2*x + sqrt(5) - 2*sqrt(x^2 - 1) - 1)/
sqrt(2*sqrt(5) - 2)) + 1/25*sqrt(50*sqrt(5) - 110)*arctan(sqrt(x)/sqrt(1/2*
sqrt(5) - 1/2)) - 1/50*sqrt(50*sqrt(5) + 110)*log(sqrt(x) + sqrt(1/2*sqrt(5
) + 1/2)) - 1/5*sqrt(1/2*sqrt(5) + 11/10)*log(abs(-520*x - 78*sqrt(5)*sqrt(
50*sqrt(5) + 110) + 260*sqrt(5) + 520*sqrt(x^2 - 1) + 130*sqrt(50*sqrt(5) +
 110) + 260)) + 1/5*sqrt(1/2*sqrt(5) + 11/10)*log(abs(-1040*x + 156*sqrt(5)
*sqrt(50*sqrt(5) + 110) + 520*sqrt(5) + 1040*sqrt(x^2 - 1) - 260*sqrt(50*sq
rt(5) + 110) + 520)) + 1/50*sqrt(50*sqrt(5) + 110)*log(abs(sqrt(x) - sqrt(1
/2*sqrt(5) + 1/2))) + 4/5*((x - sqrt(x^2 - 1))^3 + 2*(x - sqrt(x^2 - 1))^2
+ 3*x - 3*sqrt(x^2 - 1) - 2)/((x - sqrt(x^2 - 1))^4 - 2*(x - sqrt(x^2 - 1))
^3 - 2*(x - sqrt(x^2 - 1))^2 - 2*x + 2*sqrt(x^2 - 1) + 1) + 2/5*(2*x^(3/2)
- sqrt(x))/(x^2 - x - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx=\int \frac{\left(\sqrt{x^2-1}-\sqrt{x}\right)^2}{\sqrt{x^2-1}\left(-x^2+x+1\right)^2}\,dx$$

```
[In] int(((x^2 - 1)^(1/2) - x^(1/2))^2/((x^2 - 1)^(1/2)*(x - x^2 + 1)^2),x)
[Out] int(((x^2 - 1)^(1/2) - x^(1/2))^2/((x^2 - 1)^(1/2)*(x - x^2 + 1)^2), x)
```

3.11 $\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right)\,dx$

Optimal result

Integrand size = 45, antiderivative size = 138

$$\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right)\,dx = -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)}+\frac{\operatorname{arctanh}\left(\frac{i+x}{\sqrt{1-i}\sqrt{-i+x^2}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\operatorname{arctanh}\left(\frac{i-x}{\sqrt{1+i}\sqrt{i+x^2}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

```
[Out] 1/2*arctanh((I+x)/(1-I)^(1/2)/(-I+x^2)^(1/2))/(1-I)^(3/2)*2^(1/2)-1/2*arcta
nh((I-x)/(1+I)^(1/2)/(I+x^2)^(1/2))/(1+I)^(3/2)*2^(1/2)-(1/4+1/4*I)*(-I+x^2
)^(1/2)/(1+x)*2^(1/2)+(-1/4+1/4*I)*(I+x^2)^(1/2)/(1+x)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {745, 739, 212}

$$\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right)\,dx = \frac{\operatorname{arctanh}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\operatorname{arctanh}\left(\frac{-x+i}{\sqrt{1+i}\sqrt{x^2+i}}\right)}{(1+i)^{3/2}\sqrt{2}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^2-i}}{\sqrt{2}(x+1)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{x^2+i}}{\sqrt{2}(x+1)}$$

```
[In] Int[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x)^2*Sqrt[I + x^
2]),x]
```

```
[Out] ((-1/2 - I/2)*Sqrt[-I + x^2])/(Sqrt[2]*(1 + x)) - ((1/2 - I/2)*Sqrt[I + x^2
])/(Sqrt[2]*(1 + x)) + ArcTanh[(I + x)/(Sqrt[1 - I]*Sqrt[-I + x^2])]/((1 -
```

```
I)^(3/2)*Sqrt[2]) - ArcTanh[(I - x)/(Sqrt[1 + I]*Sqrt[I + x^2])]/((1 + I)^(
3/2)*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{(1+x)^2\sqrt{-i+x^2}}\,dx}{\sqrt{2}} + \frac{\int \frac{1}{(1+x)^2\sqrt{i+x^2}}\,dx}{\sqrt{2}} \\
&= -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)} - \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)} + \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{i+x^2}}\,dx}{\sqrt{2}} + \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{-i+x^2}}\,dx}{\sqrt{2}} \\
&= -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)} - \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)} + -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{(1-i)-x^2}\,dx, x, \frac{-i-x}{\sqrt{-i+x^2}}\right)}{\sqrt{2}} \\
&\quad + -\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{(1+i)-x^2}\,dx, x, \frac{i-x}{\sqrt{i+x^2}}\right)}{\sqrt{2}} \\
&= -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)} - \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)} + \frac{\operatorname{arctanh}\left(\frac{i+x}{\sqrt{1-i}\sqrt{-i+x^2}}\right)}{(1-i)^{3/2}\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{i-x}{\sqrt{1+i}\sqrt{i+x^2}}\right)}{(1+i)^{3/2}\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.63 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int\left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right)dx=$$
$$-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(i\sqrt{-i+x^2}+\sqrt{i+x^2}+\frac{2(1+x)\arctan\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\left(1+x-\sqrt{-i+x^2}\right)\right)}{\sqrt{1-i}}+(1+i)^{3/2}(1+x)\arctan\left(\sqrt{-\frac{1}{2}\ldots}\right.\right.}{\sqrt{2}(1+x)}$$

```
[In] Integrate[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x)^2*Sqrt[
I + x^2]),x]
```

```
[Out] ((-1/2 + I/2)*(I*Sqrt[-I + x^2] + Sqrt[I + x^2] + (2*(1 + x)*ArcTan[Sqrt[-1
/2 - I/2]*(1 + x - Sqrt[-I + x^2])])/Sqrt[1 - I] + (1 + I)^(3/2)*(1 + x)*Ar
cTan[Sqrt[-1/2 + I/2]*(1 + x - Sqrt[I + x^2])]))/(Sqrt[2]*(1 + x))
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.06

method	result
default	$\frac{\sqrt{2}\left(\frac{\left(-\frac{1}{2}-\frac{i}{2}\right)\sqrt{(1+x)^2-2x-1-i}}{1+x}+\frac{\left(-\frac{1}{2}-\frac{i}{2}\right)\ln\left(\frac{-2i-2x+2\sqrt{1-i}\sqrt{(1+x)^2-2x-1-i}}{1+x}\right)}{\sqrt{1-i}}\right)}{2}+\frac{\sqrt{2}\left(\frac{\left(-\frac{1}{2}+\frac{i}{2}\right)\sqrt{(1+x)^2-2x-1+i}}{1+x}+\frac{\left(-\frac{1}{2}+\frac{i}{2}\right)\ldots}{\ldots}\right.}{2}$

```
[In] int(1/2/(1+x)^2*2^(1/2)/(x^2-I)^(1/2)+1/2/(1+x)^2*2^(1/2)/(x^2+I)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*((-1/2-1/2*I)/(1+x)*((1+x)^2-2*x-1-I)^(1/2)-(1/2+1/2*I)/(1-I)^(
1/2)*ln((-2*I-2*x+2*(1-I)^(1/2)*((1+x)^2-2*x-1-I)^(1/2))/(1+x)))+1/2*2^(1/2
)*((-1/2+1/2*I)/(1+x)*((1+x)^2-2*x-1+I)^(1/2)+(-1/2+1/2*I)/(1+I)^(1/2)*ln((
2*I-2*x+2*(1+I)^(1/2)*((1+x)^2-2*x-1+I)^(1/2))/(1+x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.17

$$\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right) dx$$
$$= \frac{\sqrt{-\frac{1}{2}i+\frac{1}{2}}(-(i-1)\,x-i+1)\log\left(\sqrt{2}\sqrt{-\frac{1}{2}i+\frac{1}{2}}-x+\sqrt{x^2-i}-1\right)+\sqrt{-\frac{1}{2}i+\frac{1}{2}}((i-1)\,x+i-1)\log}{}$$

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-1/2*I + 1/2)*(-(I - 1)*x - I + 1)*log(sqrt(2)*sqrt(-1/2*I + 1/2) - x
 + sqrt(x^2 - I) - 1) + sqrt(-1/2*I + 1/2)*((I - 1)*x + I - 1)*log(-sqrt(2)
*sqrt(-1/2*I + 1/2) - x + sqrt(x^2 - I) - 1) + sqrt(-1/2*I - 1/2)*(-(I + 1)
*x - I - 1)*log(I*sqrt(2)*sqrt(-1/2*I - 1/2) - x + sqrt(x^2 + I) - 1) + sqr
t(-1/2*I - 1/2)*((I + 1)*x + I + 1)*log(-I*sqrt(2)*sqrt(-1/2*I - 1/2) - x +
 sqrt(x^2 + I) - 1) + sqrt(2)*(-(I + 1)*x - I - 1) - sqrt(2)*sqrt(x^2 + I)
- I*sqrt(2)*sqrt(x^2 - I))/((2*I + 2)*x + 2*I + 2)
```

Sympy [F(-2)]

Exception generated.

$$\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right) dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/2/(1+x)**2*2**(1/2)/(-I+x**2)**(1/2)+1/2/(1+x)**2*2**(1/2)/(I+x
**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

Maxima [F(-2)]

Exception generated.

$$\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right) dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  1which is not of the expected type LIST
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(82) = 164.

Time = 0.33 (sec) , antiderivative size = 547, normalized size of antiderivative = 3.96

$$\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right)\,dx$$
$$=\sqrt{2}\left(\frac{-(i-1)\sqrt{2\,x^2+2\,\sqrt{x^4+1}}\left(\frac{i}{x^2+\sqrt{x^4+1}}+1\right)+(2i-2)\,x+2i+2}{\left(\sqrt{2\,x^2+2\,\sqrt{x^4+1}}\left(\frac{i}{x^2+\sqrt{x^4+1}}+1\right)-2\,x\right)^2-4\,\sqrt{2\,x^2+2\,\sqrt{x^4+1}}\left(\frac{i}{x^2+\sqrt{x^4+1}}+1\right)+8\,x-4i}-\cdots\right.$$
$$+\sqrt{2}\left(\frac{(i+1)\sqrt{2\,x^2+2\,\sqrt{x^4+1}}\left(-\frac{i}{x^2+\sqrt{x^4+1}}+1\right)-(2i+2)\,x-2i+2}{\left(\sqrt{2\,x^2+2\,\sqrt{x^4+1}}\left(-\frac{i}{x^2+\sqrt{x^4+1}}+1\right)-2\,x\right)^2-4\,\sqrt{2\,x^2+2\,\sqrt{x^4+1}}\left(-\frac{i}{x^2+\sqrt{x^4+1}}+1\right)+8\,x+\cdots}\right.$$

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*((-(I - 1)*sqrt(2*x^2 + 2*sqrt(x^4 + 1))*(I/(x^2 + sqrt(x^4 + 1)) +
 1) + (2*I - 2)*x + 2*I + 2)/((sqrt(2*x^2 + 2*sqrt(x^4 + 1))*(I/(x^2 + sqrt
(x^4 + 1)) + 1) - 2*x)^2 - 4*sqrt(2*x^2 + 2*sqrt(x^4 + 1))*(I/(x^2 + sqrt(x
^4 + 1)) + 1) + 8*x - 4*I) - (I - 1)*arctan((sqrt(2)*(sqrt(2*x^2 + 2*sqrt(x
^4 + 1))*(I/(x^2 + sqrt(x^4 + 1)) + 1) - 2*x) - sqrt(2*x^2 + 2*sqrt(x^4 + 1
))*(I/(x^2 + sqrt(x^4 + 1)) + 1) + 2*x - 2*sqrt(2) + 2)/(sqrt(2)*sqrt(2*sqr
t(2) - 2) - (I + 1)*sqrt(2*sqrt(2) - 2)))/(sqrt(2*sqrt(2) - 2)*(-I/(sqrt(2)
 - 1) + 1))) + sqrt(2)*(((I + 1)*sqrt(2*x^2 + 2*sqrt(x^4 + 1))*(-I/(x^2 + s
qrt(x^4 + 1)) + 1) - (2*I + 2)*x - 2*I + 2)/((sqrt(2*x^2 + 2*sqrt(x^4 + 1))
*(-I/(x^2 + sqrt(x^4 + 1)) + 1) - 2*x)^2 - 4*sqrt(2*x^2 + 2*sqrt(x^4 + 1))*
(-I/(x^2 + sqrt(x^4 + 1)) + 1) + 8*x + 4*I) + (I + 1)*arctan((sqrt(2)*(sqrt
(2*x^2 + 2*sqrt(x^4 + 1))*(-I/(x^2 + sqrt(x^4 + 1)) + 1) - 2*x) - sqrt(2*x^
2 + 2*sqrt(x^4 + 1))*(-I/(x^2 + sqrt(x^4 + 1)) + 1) + 2*x - 2*sqrt(2) + 2)/
(sqrt(2)*sqrt(2*sqrt(2) - 2) + (I - 1)*sqrt(2*sqrt(2) - 2)))/(sqrt(2*sqrt(2
) - 2)*(I/(sqrt(2) - 1) + 1)))
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right)\,dx$$
$$=\int \frac{\sqrt{2}}{2\,\sqrt{x^2-\mathrm{i}}\,{\left(x+1\right)}^2}+\frac{\sqrt{2}}{2\,\sqrt{x^2+1\mathrm{i}}\,{\left(x+1\right)}^2}\,dx$$

```
[In] int(2^(1/2)/(2*(x^2 - 1i)^(1/2)*(x + 1)^2) + 2^(1/2)/(2*(x^2 + 1i)^(1/2)*(x
 + 1)^2),x)
[Out] int(2^(1/2)/(2*(x^2 - 1i)^(1/2)*(x + 1)^2) + 2^(1/2)/(2*(x^2 + 1i)^(1/2)*(x
 + 1)^2), x)
```

3.12 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 125

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx = -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}-\frac{1}{4}(1-i)^{3/2}\text{arctanh}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\text{arctanh}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

```
[Out] -1/4*(1-I)^(3/2)*arctanh((1+I*x)/(1-I)^(1/2)/(1-I*x^2)^(1/2))-1/4*(1+I)^(3/
2)*arctanh((1-I*x)/(1+I)^(1/2)/(1+I*x^2)^(1/2))-1/2*(1-I*x^2)^(1/2)/(1+x)-1
/2*(1+I*x^2)^(1/2)/(1+x)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2158, 745, 739, 212}

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx = -\frac{1}{4}(1-i)^{3/2}\text{arctanh}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\text{arctanh}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)-\frac{\sqrt{1-ix^2}}{2(x+1)}-\frac{\sqrt{1+ix^2}}{2(x+1)}$$

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

```
[Out] -1/2*Sqrt[1 - I*x^2]/(1 + x) - Sqrt[1 + I*x^2]/(2*(1 + x)) - ((1 - I)^(3/2)
*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/4 - ((1 + I)^(3/2)*ArcTa
nh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2158

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)^2\sqrt{1-ix^2}}\,dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+x)^2\sqrt{1+ix^2}}\,dx \\
&= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}-\frac{1}{2}i\int \frac{1}{(1+x)\sqrt{1-ix^2}}\,dx+\frac{1}{2}i\int \frac{1}{(1+x)\sqrt{1+ix^2}}\,dx \\
&= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}+\frac{1}{2}i\text{Subst}\left(\int \frac{1}{(1-i)-x^2}\,dx,x,\frac{1+ix}{\sqrt{1-ix^2}}\right) \\
&\quad -\frac{1}{2}i\text{Subst}\left(\int \frac{1}{(1+i)-x^2}\,dx,x,\frac{1-ix}{\sqrt{1+ix^2}}\right) \\
&= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)} \\
&\quad -\frac{1}{4}(1-i)^{3/2}\text{arctanh}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\text{arctanh}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 272 vs. 2(125) = 250.

Time = 2.37 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx = \frac{1}{2}\left(\frac{-1-2x^4-\sqrt{1+x^4}-x^2\left(1+2\sqrt{1+x^4}\right)}{(1+x)\left(x^2+\sqrt{1+x^4}\right)^{3/2}} + \frac{\arctan\left(\sqrt{1+\sqrt{2}}\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{-1+\sqrt{2}}} - \sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right) - \frac{\operatorname{arctanh}\left(\sqrt{-1+\sqrt{2}}\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{1+\sqrt{2}}} + \sqrt{-1+\sqrt{2}}\operatorname{arctanh}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)$$

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

```
[Out] ((-1 - 2*x^4 - Sqrt[1 + x^4] - x^2*(1 + 2*Sqrt[1 + x^4]))/((1 + x)*(x^2 + S
qrt[1 + x^4])^(3/2)) + ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]]/
Sqrt[-1 + Sqrt[2]] - Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])]*x*Sqr
t[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])] - ArcTanh[Sqrt[-1 + Sqrt
[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[1 + Sqrt[2]] + Sqrt[-1 + Sqrt[2]]*ArcT
anh[(Sqrt[2*(1 + Sqrt[2])]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 +
 x^4])])/2
```

Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(1+x)^2\sqrt{x^4+1}}\,dx$$

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(81) = 162$.

Time = 1.17 (sec) , antiderivative size = 502, normalized size of antiderivative = 4.02

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx = -\frac{(x+1)\sqrt{-\sqrt{2}-1}\log\left(-\frac{\sqrt{2}(x^2+1)\sqrt{-\sqrt{2}-1}+\left(2x^3+\sqrt{2}(x^3-x^2-x-1)-\sqrt{x^4+1}\left(\sqrt{2}(x-1)+2x\right)-2\right)\sqrt{x^2+\sqrt{x^4+1}}-2\sqrt{x^4+1}\sqrt{\cdots}}{x^2+2x+1}\right.}{\cdots}$$

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/8*((x + 1)*sqrt(-sqrt(2) - 1)*log(-(sqrt(2)*(x^2 + 1)*sqrt(-sqrt(2) - 1)
 + (2*x^3 + sqrt(2)*(x^3 - x^2 - x - 1) - sqrt(x^4 + 1)*(sqrt(2)*(x - 1) +
2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - 2*sqrt(x^4 + 1)*sqrt(-sqrt(2) - 1))/(
x^2 + 2*x + 1)) - (x + 1)*sqrt(-sqrt(2) - 1)*log((sqrt(2)*(x^2 + 1)*sqrt(-s
qrt(2) - 1) - (2*x^3 + sqrt(2)*(x^3 - x^2 - x - 1) - sqrt(x^4 + 1)*(sqrt(2)
*(x - 1) + 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - 2*sqrt(x^4 + 1)*sqrt(-sqrt
(2) - 1))/(x^2 + 2*x + 1)) - (x + 1)*sqrt(sqrt(2) - 1)*log(-((2*x^3 - sqrt(
2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^
2 + sqrt(x^4 + 1)) + (sqrt(2)*(x^2 + 1) + 2*sqrt(x^4 + 1))*sqrt(sqrt(2) - 1
))/(x^2 + 2*x + 1)) + (x + 1)*sqrt(sqrt(2) - 1)*log(-((2*x^3 - sqrt(2)*(x^3
 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqr
t(x^4 + 1)) - (sqrt(2)*(x^2 + 1) + 2*sqrt(x^4 + 1))*sqrt(sqrt(2) - 1))/(x^2
 + 2*x + 1)) - 4*sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1))/(x +
1)
```

Sympy [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)^2\sqrt{x^4+1}}\,dx$$

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)**2/(x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)**2*sqrt(x**4 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)^2}\,dx$$

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)^2}\,dx$$

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}\,(x+1)^2}\,dx$$

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)^2),x)
```

```
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)^2), x)
```

3.13 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 81

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = -\frac{1}{2}\sqrt{1-i}\operatorname{arctanh}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) - \frac{1}{2}\sqrt{1+i}\operatorname{arctanh}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

```
[Out] -1/2*arctanh((1+I*x)/(1-I)^(1/2)/(1-I*x^2)^(1/2))*(1-I)^(1/2)-1/2*arctanh((
1-I*x)/(1+I)^(1/2)/(1+I*x^2)^(1/2))*(1+I)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2158, 739, 212}

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = -\frac{1}{2}\sqrt{1-i}\operatorname{arctanh}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) - \frac{1}{2}\sqrt{1+i}\operatorname{arctanh}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] -1/2*(Sqrt[1 - I]*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])]) - (Sqrt
[1 + I]*ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2158

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{1-ix^2}}\,dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{1+ix^2}}\,dx \\
&= \left(-\frac{1}{2}-\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{(1+i)-x^2}\,dx,x,\frac{1-ix}{\sqrt{1+ix^2}}\right) \\
&\quad+\left(-\frac{1}{2}+\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{(1-i)-x^2}\,dx,x,\frac{1+ix}{\sqrt{1-ix^2}}\right) \\
&= -\frac{1}{2}\sqrt{1-i}\,\text{arctanh}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\,\text{arctanh}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 205 vs. $2(81) = 162$.

Time = 1.07 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.53

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$
$$=\frac{\sqrt{-1+\sqrt{2}}\left(\arctan\left(\sqrt{1+\sqrt{2}}\sqrt{x^2+\sqrt{1+x^4}}\right)-\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)-\sqrt{1+\sqrt{2}}\text{arcta}}{\sqrt{2}}$$

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

[Out] (Sqrt[-1 + Sqrt[2]]*(ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]] - ArcTan[(Sqrt[2*(-1 + Sqrt[2])]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])]) - Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[(Sqrt[2*(1 + Sqrt[2])]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])])/Sqrt[2]

Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(1+x)\sqrt{x^4+1}}dx$$

[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)

[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. 2(49) = 98.

Time = 3.27 (sec) , antiderivative size = 504, normalized size of antiderivative = 6.22

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$
$$= \frac{1}{8}\sqrt{-2\sqrt{2}+2}\log\left(-\frac{\sqrt{x^4+1}(\sqrt{2}+2)\sqrt{-2\sqrt{2}+2}+\left(2x^3+\sqrt{2}(x^3-x^2-x-1)-\sqrt{x^4+1}(\sqrt{2}(x-\right.}{x^2+2x+1}\right.$$
$$-\frac{1}{8}\sqrt{-2\sqrt{2}+2}\log\left(\frac{\sqrt{x^4+1}(\sqrt{2}+2)\sqrt{-2\sqrt{2}+2}-\left(2x^3+\sqrt{2}(x^3-x^2-x-1)-\sqrt{x^4+1}(\sqrt{2}(x-\right.}{x^2+2x+1}\right.$$
$$-\frac{1}{8}\sqrt{2\sqrt{2}+2}\log\left(-\frac{\left(2x^3-\sqrt{2}(x^3-x^2-x-1)+\sqrt{x^4+1}(\sqrt{2}(x-1)-2x)-2\right)\sqrt{x^2+\sqrt{x^4+1}}+}{x^2+2x+1}\right.$$
$$+\frac{1}{8}\sqrt{2\sqrt{2}+2}\log\left(-\frac{\left(2x^3-\sqrt{2}(x^3-x^2-x-1)+\sqrt{x^4+1}(\sqrt{2}(x-1)-2x)-2\right)\sqrt{x^2+\sqrt{x^4+1}}-}{x^2+2x+1}\right.$$

[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="fricas")

[Out] 1/8*sqrt(-2*sqrt(2) + 2)*log(-(sqrt(x^4 + 1)*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 2) + (2*x^3 + sqrt(2)*(x^3 - x^2 - x - 1) - sqrt(x^4 + 1)*(sqrt(2)*(x - 1) + 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - (x^2 + sqrt(2)*(x^2 + 1) + 1)*sqrt(-2*sqrt(2) + 2))/(x^2 + 2*x + 1)) - 1/8*sqrt(-2*sqrt(2) + 2)*log((sqrt(x^4 + 1)*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 2) - (2*x^3 + sqrt(2)*(x^3 - x^2 - x - 1) - sqrt(x^4 + 1)*(sqrt(2)*(x - 1) + 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - (x^2 + sqrt(2)*(x^2 + 1) + 1)*sqrt(-2*sqrt(2) + 2))/(x^2 + 2*x + 1)) -

```
1/8*sqrt(2*sqrt(2) + 2)*log(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x
^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) + (x^2 - sqr
t(2)*(x^2 + 1) + sqrt(x^4 + 1)*(sqrt(2) - 2) + 1)*sqrt(2*sqrt(2) + 2))/(x^2
 + 2*x + 1)) + 1/8*sqrt(2*sqrt(2) + 2)*log(-((2*x^3 - sqrt(2)*(x^3 - x^2 -
x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1
)) - (x^2 - sqrt(2)*(x^2 + 1) + sqrt(x^4 + 1)*(sqrt(2) - 2) + 1)*sqrt(2*sqr
t(2) + 2))/(x^2 + 2*x + 1))
```

Sympy [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)\sqrt{x^4+1}}\,dx$$

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)/(x**4+1)**(1/2),x)
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)*sqrt(x**4 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)}\,dx$$

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="maxim
a")
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)}\,dx$$

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="giac"
)
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}\,(x+1)}\,dx$$

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)),x)
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)), x)
```

3.14 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}} dx$

Optimal result

Integrand size = 27, antiderivative size = 31

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2157, 212}

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2157

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
```

```
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{\log\left(x^2+\sqrt{1+x^4}+\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
[Out] Log[x^2 + Sqrt[1 + x^4] + Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}dx$$

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(24) = 48.

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{1}{4}\sqrt{2}\log\Big(4\,x^4+4\,\sqrt{x^4+1}x^2 + 2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\Big)$$

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt
(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix} 1,1 & \frac{1}{2} \\ \frac{1}{4},\frac{3}{4} & 0 \end{matrix}\middle|\, x^4\right)}{4\sqrt{\pi}}$$

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
[Out] meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}\,dx$$

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2),x)
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2), x)
```

3.15 $\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 33

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(x*2^(1/2)/(-x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2157, 209}

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

```
[In] Int[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2157

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
```

```
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)$$
$$= \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = -\frac{i\log\left(ix^2-i\sqrt{1+x^4}+\sqrt{2}x\sqrt{-x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

```
[In] Integrate[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
[Out] ((-I)*Log[I*x^2 - I*Sqrt[1 + x^4] + Sqrt[2]*x*Sqrt[-x^2 + Sqrt[1 + x^4]]])/
Sqrt[2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

method	result	size
meijerg	$-\frac{\sqrt{2}\,{}_3F_2\left(\frac{1}{2},\frac{3}{4},\frac{5}{4};\frac{3}{2},\frac{3}{2};-\frac{1}{x^4}\right)}{4x^2}$	22

```
[In] int((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*2^(1/2)/x^2*hypergeom([1/2,3/4,5/4],[3/2,3/2],-1/x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = -\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+\sqrt{x^4+1}}}{2\,x}\right)$$

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + sqrt(x^4 + 1))/x)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix} \frac{1}{2}, 1 & 1 \\ \frac{1}{4}, \frac{3}{4} & 0 \end{matrix} \middle| {x^{4}} \right)}{4\sqrt{\pi}}$$

```
[In] integrate((-x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
[Out] meijerg(((1/2, 1), (1,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}-x^2}}{\sqrt{x^4+1}}\,dx$$

```
[In] int(((x^4 + 1)^(1/2) - x^2)^(1/2)/(x^4 + 1)^(1/2),x)
[Out] int(((x^4 + 1)^(1/2) - x^2)^(1/2)/(x^4 + 1)^(1/2), x)
```

3.16 $\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}} dx$

Optimal result

Integrand size = 30, antiderivative size = 19

$$\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}} dx = -\frac{2}{\sqrt{-1+x}} - \frac{2}{\sqrt{1+x}}$$

```
[Out] -2/(-1+x)^(1/2)-2/(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {6820}

$$\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}} dx = -\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

```
[In] Int[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{(-1+x)^{3/2}} + \frac{1}{(1+x)^{3/2}}\right) dx \\ &= -\frac{2}{\sqrt{-1+x}} - \frac{2}{\sqrt{1+x}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx = -\frac{2}{\sqrt{-1+x}}-\frac{2}{\sqrt{1+x}}$$

```
[In] Integrate[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x
]
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{2}{\sqrt{-1+x}}-\frac{2}{\sqrt{1+x}}$	16
meijerg	$\frac{2\sqrt{\pi}-\frac{2\sqrt{\pi}}{\sqrt{1+x}}}{\sqrt{\pi}}-\frac{2(-\operatorname{signum}(-1+x))^{\frac{3}{2}}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{1-x}}\right)}{\sqrt{\pi}\operatorname{signum}(-1+x)^{\frac{3}{2}}}$	56

```
[In] int(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x,method=_RETURNVER
BOSE)
[Out] -2/(-1+x)^(1/2)-2/(1+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx = -\frac{2\left((x+1)\sqrt{x-1}+\sqrt{x+1}(x-1)\right)}{x^2-1}$$

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"fricas")
[Out] -2*((x + 1)*sqrt(x - 1) + sqrt(x + 1)*(x - 1))/(x^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(17) = 34$.

Time = 0.82 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.95

$$\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx = -\frac{2x\sqrt{x-1}}{x^2-1}-\frac{2x\sqrt{x+1}}{x^2-1}-\frac{2\sqrt{x-1}}{x^2-1}+\frac{2\sqrt{x+1}}{x^2-1}$$

```
[In] integrate(((-1+x)**(3/2)+(1+x)**(3/2))/(-1+x)**(3/2)/(1+x)**(3/2),x)
[Out] -2*x*sqrt(x - 1)/(x**2 - 1) - 2*x*sqrt(x + 1)/(x**2 - 1) - 2*sqrt(x - 1)/(x
**2 - 1) + 2*sqrt(x + 1)/(x**2 - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx = -\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"maxima")
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx = -\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"giac")
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx=-\frac{2}{\sqrt{x-1}}-\frac{2}{\sqrt{x+1}}$$

```
[In] int(((x - 1)^(3/2) + (x + 1)^(3/2))/((x - 1)^(3/2)*(x + 1)^(3/2)),x)
[Out] - 2/(x - 1)^(1/2) - 2/(x + 1)^(1/2)
```

3.17 $\int \left(x+\sqrt{a+x^2}\right)^b \, dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \left(x+\sqrt{a+x^2}\right)^b \, dx = -\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)}+\frac{\left(x+\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}$$

```
[Out] -1/2*a*(x+(x^2+a)^(1/2))^(-1+b)/(1-b)+1/2*(x+(x^2+a)^(1/2))^(1+b)/(1+b)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2142, 14}

$$\int \left(x+\sqrt{a+x^2}\right)^b \, dx = \frac{\left(\sqrt{a+x^2}+x\right)^{b+1}}{2(b+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{b-1}}{2(1-b)}$$

```
[In] Int[(x + Sqrt[a + x^2])^b,x]
```

```
[Out] -1/2*(a*(x + Sqrt[a + x^2])^(-1 + b))/(1 - b) + (x + Sqrt[a + x^2])^(1 + b)
/(2*(1 + b))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^{-2+b}(a+x^2)\,dx, x, x+\sqrt{a+x^2}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(ax^{-2+b}+x^b\right)\,dx, x, x+\sqrt{a+x^2}\right)\\ &= -\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)}+\frac{\left(x+\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \left(x+\sqrt{a+x^2}\right)^b\,dx = \frac{\left(x+\sqrt{a+x^2}\right)^{-1+b}\left(ab+(-1+b)x\left(x+\sqrt{a+x^2}\right)\right)}{-1+b^2}$$

```
[In] Integrate[(x + Sqrt[a + x^2])^b,x]
[Out] ((x + Sqrt[a + x^2])^(-1 + b)*(a*b + (-1 + b)*x*(x + Sqrt[a + x^2])))/(-1 +
 b^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(44) = 88.

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.31

method	result	size
meijerg	$\frac{a^{\frac{b}{2}+\frac{1}{2}}b\left(\frac{8\sqrt{\pi}\,x^{1+b}a^{-\frac{b}{2}-\frac{1}{2}}\left(\frac{ab}{x^2}+b-1\right)\left(\sqrt{1+\frac{a}{x^2}}+1\right)^{-1+b}}{(1+b)b(2b-2)}+\frac{4\sqrt{\pi}\,x^{1+b}a^{-\frac{b}{2}-\frac{1}{2}}\sqrt{1+\frac{a}{x^2}}\left(\sqrt{1+\frac{a}{x^2}}+1\right)^{-1+b}}{(1+b)b}\right)}{4\sqrt{\pi}}$	120

```
[In] int((x+(x^2+a)^(1/2))^b,x,method=_RETURNVERBOSE)
[Out] 1/4*a^(1/2*b+1/2)/Pi^(1/2)*b*(8*Pi^(1/2)/(1+b)/b*x^(1+b)*a^(-1/2*b-1/2)*(a*
b/x^2+b-1)/(2*b-2)*((1+1/x^2*a)^(1/2)+1)^(-1+b)+4*Pi^(1/2)/(1+b)/b*x^(1+b)*
a^(-1/2*b-1/2)*(1+1/x^2*a)^(1/2)*((1+1/x^2*a)^(1/2)+1)^(-1+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \left(x+\sqrt{a+x^2}\right)^b\,dx=\frac{\left(\sqrt{x^2+a}b-x\right)\left(x+\sqrt{x^2+a}\right)^b}{b^2-1}$$

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="fricas")
[Out] (sqrt(x^2 + a)*b - x)*(x + sqrt(x^2 + a))^b/(b^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2236 vs. 2(37) = 74.

Time = 1.26 (sec) , antiderivative size = 2236, normalized size of antiderivative = 43.00

$$\int \left(x+\sqrt{a+x^2}\right)^b\,dx=\text{Too large to display}$$

```
[In] integrate((x+(x**2+a)**(1/2))**b,x)
[Out] Piecewise((2*a**(9/2)*a**(b/2 + 1/2)*b*cosh(b*asinh(x/sqrt(a)) + asinh(x/sq
rt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1
 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b
/2)) - 2*a**(9/2)*a**(b/2 + 1/2)*b*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1
- b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) -
2*a**(7/2)*x**2*gamma(1 - b/2)) - 2*a**(7/2)*a**(b/2 + 1/2)*b*x**2*sqrt(a/x
**2 + 1)*sinh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(
9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2
*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) + 4*a**(7/2)*a**(b/2 + 1/
2)*b*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**
(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**
2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) - 2*a**(7/2)*a**(b/2 + 1
/2)*b*x**2*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamm
a(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1
- b/2)) - 2*a**(7/2)*a**(b/2 + 1/2)*x**2*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sq
rt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) -
 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/
2)*x**2*gamma(1 - b/2)) + 2*a**(7/2)*a**(b/2 + 1/2)*x**2*cosh(b*asinh(x/sqr
t(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) -
2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2
)*x**2*gamma(1 - b/2)) - 2*a**(5/2)*a**(b/2 + 1/2)*b*x**4*sqrt(a/x**2 + 1)*
sinh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2
```

```
*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1
- b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) + 2*a**(5/2)*a**(b/2 + 1/2)*b*x**4
*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**
2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1
 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) - 2*a**(5/2)*a**(b/2 + 1/2)*x**4*
sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)
/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b
**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) + 2*a**(5/2)*a**(
b/2 + 1/2)*x**4*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/
(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b*
*2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) - a**4*a**(b/2 + 1
/2)*b**2*x*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(9/2
)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*ga
mma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) + a**4*a**(b/2 + 1/2)*b*x*co
sh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(
9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2
*gamma(1 - b/2)) - a**3*a**(b/2 + 1/2)*b**2*x**3*sqrt(a/x**2 + 1)*sinh(b*as
inh(x/sqrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*ga
mma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(
1 - b/2)) + a**3*a**(b/2 + 1/2)*b*x**3*cosh(b*asinh(x/sqrt(a)))*gamma(-b/2)
/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b
**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)), Abs(x**2/a) > 1)
, (-a**(5/2)*a**(b/2 + 1/2)*b**2*sqrt(1 + x**2/a)*sinh(b*asinh(x/sqrt(a)))*
gamma(-b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)) +
2*a**(5/2)*a**(b/2 + 1/2)*b*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gam
ma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)) +
2*a**(3/2)*a**(b/2 + 1/2)*b*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a))
)*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2
)) + 2*a**(3/2)*a**(b/2 + 1/2)*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(
a)))*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 -
b/2)) - 2*a**2*a**(b/2 + 1/2)*b*x*sqrt(1 + x**2/a)*sinh(b*asinh(x/sqrt(a))
+ asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(
5/2)*gamma(1 - b/2)) + a**2*a**(b/2 + 1/2)*b*x*cosh(b*asinh(x/sqrt(a)))*gam
ma(-b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)) - 2*a
**2*a**(b/2 + 1/2)*x*sqrt(1 + x**2/a)*sinh(b*asinh(x/sqrt(a)) + asinh(x/sqr
t(a)))*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1
- b/2)), True))
```

Maxima [F]

$$\int \left(x+\sqrt{a+x^2}\right)^b\,dx=\int \left(x+\sqrt{x^2+a}\right)^b\,dx$$

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 + a))^b, x)
```

Giac [F]

$$\int \left(x+\sqrt{a+x^2}\right)^b\,dx=\int \left(x+\sqrt{x^2+a}\right)^b\,dx$$

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="giac")
[Out] integrate((x + sqrt(x^2 + a))^b, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(x+\sqrt{a+x^2}\right)^b\,dx=\int \left(x+\sqrt{x^2+a}\right)^b\,dx$$

```
[In] int((x + (a + x^2)^(1/2))^b,x)
[Out] int((x + (a + x^2)^(1/2))^b, x)
```

3.18 $\int \left(x-\sqrt{a+x^2}\right)^b \, dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \left(x-\sqrt{a+x^2}\right)^b \, dx=-\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)}+\frac{\left(x-\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}$$

```
[Out] -1/2*a*(x-(x^2+a)^(1/2))^(-1+b)/(1-b)+1/2*(x-(x^2+a)^(1/2))^(1+b)/(1+b)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2142, 14}

$$\int \left(x-\sqrt{a+x^2}\right)^b \, dx=\frac{\left(x-\sqrt{a+x^2}\right)^{b+1}}{2(b+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{b-1}}{2(1-b)}$$

```
[In] Int[(x - Sqrt[a + x^2])^b,x]
```

```
[Out] -1/2*(a*(x - Sqrt[a + x^2])^(-1 + b))/(1 - b) + (x - Sqrt[a + x^2])^(1 + b)
/(2*(1 + b))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^{-2+b}(a+x^2)\,dx, x, x-\sqrt{a+x^2}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int (ax^{-2+b}+x^b)\,dx, x, x-\sqrt{a+x^2}\right)\\ &= -\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)}+\frac{\left(x-\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \left(x-\sqrt{a+x^2}\right)^b\,dx = \frac{1}{2}\left(x-\sqrt{a+x^2}\right)^{-1+b}\left(\frac{a}{-1+b}+\frac{\left(x-\sqrt{a+x^2}\right)^2}{1+b}\right)$$

```
[In] Integrate[(x - Sqrt[a + x^2])^b,x]
[Out] ((x - Sqrt[a + x^2])^(-1 + b)*(a/(-1 + b) + (x - Sqrt[a + x^2])^2/(1 + b)))
/2
```

Maple [F]

$$\int \left(x-\sqrt{x^2+a}\right)^b dx$$

```
[In] int((x-(x^2+a)^(1/2))^b,x)
[Out] int((x-(x^2+a)^(1/2))^b,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.59

$$\int \left(x-\sqrt{a+x^2}\right)^b dx = -\frac{\left(\sqrt{x^2+a}b+x\right)\left(x-\sqrt{x^2+a}\right)^b}{b^2-1}$$

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="fricas")
[Out] -(sqrt(x^2 + a)*b + x)*(x - sqrt(x^2 + a))^b/(b^2 - 1)
```

Sympy [F]

$$\int \left(x-\sqrt{a+x^2}\right)^b dx = \int \left(x-\sqrt{a+x^2}\right)^b dx$$

```
[In] integrate((x-(x**2+a)**(1/2))**b,x)
[Out] Integral((x - sqrt(a + x**2))**b, x)
```

Maxima [F]

$$\int \left(x-\sqrt{a+x^2}\right)^b dx = \int \left(x-\sqrt{x^2+a}\right)^b dx$$

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^b, x)
```

Giac [F]

$$\int \left(x-\sqrt{a+x^2}\right)^b dx = \int \left(x-\sqrt{x^2+a}\right)^b dx$$

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^b, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(x-\sqrt{a+x^2}\right)^b\,dx=\int \left(x-\sqrt{x^2+a}\right)^b\,dx$$

```
[In] int((x - (a + x^2)^(1/2))^b,x)
[Out] int((x - (a + x^2)^(1/2))^b, x)
```

3.19 $\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 17

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}} \, dx=\frac{\left(x+\sqrt{a+x^2}\right)^b}{b}$$

```
[Out] (x+(x^2+a)^(1/2))^b/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2147, 30}

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}} \, dx=\frac{\left(\sqrt{a+x^2}+x\right)^b}{b}$$

```
[In] Int[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
[Out] (x + Sqrt[a + x^2])^b/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
```

```
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^{-1+b}\,dx, x, x+\sqrt{a+x^2}\right)\\ &= \frac{\left(x+\sqrt{a+x^2}\right)^b}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = \frac{\left(x+\sqrt{a+x^2}\right)^b}{b}$$

```
[In] Integrate[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
[Out] (x + Sqrt[a + x^2])^b/b
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$	16
default	$\frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$	16

```
[In] int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] (x+(x^2+a)^(1/2))^b/b
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = \frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$$

```
[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="fricas")
[Out] (x + sqrt(x^2 + a))^b/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(12) = 24$.

Time = 1.37 (sec) , antiderivative size = 311, normalized size of antiderivative = 18.29

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = \begin{cases} \frac{\sqrt{a}a^{\frac{b}{2}}\sinh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{bx\sqrt{\frac{a}{x^2}+1}} - \frac{2a^{\frac{b}{2}}\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{b}{2}\right)}{b^2\Gamma\left(-\frac{b}{2}\right)} + \frac{a^{\frac{b}{2}}x\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}b} + \cdots \\ \frac{a^{\frac{b}{2}}\sinh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{b\sqrt{1+\frac{x^2}{a}}} - \frac{2a^{\frac{b}{2}}\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{b}{2}\right)}{b^2\Gamma\left(-\frac{b}{2}\right)} + \frac{a^{\frac{b}{2}}x^2\sinh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{ab\sqrt{1+\frac{x^2}{a}}} + \cdots \end{cases}$$

```
[In] integrate((x+(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)
[Out] Piecewise((sqrt(a)*a**(b/2)*sinh(b*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(b*
x*sqrt(a/x**2 + 1)) - 2*a**(b/2)*cosh(b*asinh(x/sqrt(a)))*gamma(1 - b/2)/(b
**2*gamma(-b/2)) + a**(b/2)*x*cosh(b*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(
sqrt(a)*b) + a**(b/2)*x*sinh(b*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(sqrt(a
)*b*sqrt(a/x**2 + 1)), Abs(x**2/a) > 1), (a**(b/2)*sinh(b*asinh(x/sqrt(a))
- asinh(x/sqrt(a)))/(b*sqrt(1 + x**2/a)) - 2*a**(b/2)*cosh(b*asinh(x/sqrt(a
)))*gamma(1 - b/2)/(b**2*gamma(-b/2)) + a**(b/2)*x**2*sinh(b*asinh(x/sqrt(a
)) - asinh(x/sqrt(a)))/(a*b*sqrt(1 + x**2/a)) + a**(b/2)*x*cosh(b*asinh(x/s
qrt(a)) - asinh(x/sqrt(a)))/(sqrt(a)*b), True))
```

Maxima [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

```
[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = \frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$$

```
[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="giac")
[Out] (x + sqrt(x^2 + a))^b/b
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = \frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$$

```
[In] int((x + (a + x^2)^(1/2))^b/(a + x^2)^(1/2),x)
[Out] (x + (a + x^2)^(1/2))^b/b
```

3.20 $\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 20

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}} \, dx = -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

```
[Out] -(x-(x^2+a)^(1/2))^b/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2147, 30}

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}} \, dx = -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

```
[In] Int[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] -((x - Sqrt[a + x^2])^b/b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
```

```
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int x^{-1+b}\,dx, x, x-\sqrt{a+x^2}\right)\\ &= -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

```
[In] Integrate[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
[Out] -((x - Sqrt[a + x^2])^b/b)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$	19
default	$-\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$	19

```
[In] int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(x-(x^2+a)^(1/2))^b/b
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = -\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="fricas")
[Out] -(x - sqrt(x^2 + a))^b/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(14) = 28.

Time = 0.67 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = \begin{cases} -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b} & \text{for } b \neq 0 \\ \begin{cases} \log\left(2x+2\sqrt{a+x^2}\right) & \text{for } a \neq 0 \\ \frac{x\log(x)}{\sqrt{x^2}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

```
[In] integrate((x-(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)
[Out] Piecewise((-(x - sqrt(a + x**2))**b/b, Ne(b, 0)), (Piecewise((log(2*x + 2*s
qrt(a + x**2)), Ne(a, 0)), (x*log(x)/sqrt(x**2), True)), True))
```

Maxima [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = \int \frac{\left(x-\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = -\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="giac")
[Out] -(x - sqrt(x^2 + a))^b/b
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = -\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

```
[In] int((x - (a + x^2)^(1/2))^b/(a + x^2)^(1/2),x)
[Out] -(x - (a + x^2)^(1/2))^b/b
```

3.21 $\int \frac{1}{(a+be^{px})^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{1}{(a+be^{px})^2} \, dx = \frac{1}{a(a+be^{px})\, p} + \frac{x}{a^2} - \frac{\log(a+be^{px})}{a^2p}$$

```
[Out] 1/a/(a+b*exp(p*x))/p+x/a^2-ln(a+b*exp(p*x))/a^2/p
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2320, 46}

$$\int \frac{1}{(a+be^{px})^2} \, dx = -\frac{\log(a+be^{px})}{a^2p} + \frac{x}{a^2} + \frac{1}{ap(a+be^{px})}$$

```
[In] Int[(a + b*E^(p*x))^(-2),x]
[Out] 1/(a*(a + b*E^(p*x))*p) + x/a^2 - Log[a + b*E^(p*x)]/(a^2*p)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, e^{px}\right)}{p}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)\,dx, x, e^{px}\right)}{p}\\ &= \frac{1}{a(a+be^{px})\,p}+\frac{x}{a^2}-\frac{\log(a+be^{px})}{a^2p}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{1}{(a+be^{px})^2}\,dx = \frac{\frac{a}{a+be^{px}}+\log(e^{px})-\log(a+be^{px})}{a^2p}$$

```
[In] Integrate[(a + b*E^(p*x))^(-2),x]
[Out] (a/(a + b*E^(p*x)) + Log[E^(p*x)] - Log[a + b*E^(p*x)])/(a^2*p)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{-\frac{\ln(a+b\,e^{px})}{a^2}+\frac{1}{a(a+b\,e^{px})}+\frac{\ln(e^{px})}{a^2}}{p}$	43
default	$\frac{-\frac{\ln(a+b\,e^{px})}{a^2}+\frac{1}{a(a+b\,e^{px})}+\frac{\ln(e^{px})}{a^2}}{p}$	43
risch	$\frac{x}{a^2}+\frac{1}{a(a+b\,e^{px})p}-\frac{\ln\left(e^{px}+\frac{a}{b}\right)}{a^2p}$	43
norman	$\frac{\frac{x}{a}+\frac{bx\,e^{px}}{a^2}-\frac{b\,e^{px}}{a^2p}}{a+b\,e^{px}}-\frac{\ln(a+b\,e^{px})}{a^2p}$	59
parallelrisch	$-\frac{-b^2e^{px}xp+\ln(a+b\,e^{px})e^{px}b^2-xabp+\ln(a+b\,e^{px})ab-ab}{a^2bp(a+b\,e^{px})}$	73

```
[In] int(1/(a+b*exp(p*x))^2,x,method=_RETURNVERBOSE)
[Out] 1/p*(-1/a^2*ln(a+b*exp(p*x))+1/a/(a+b*exp(p*x))+1/a^2*ln(exp(p*x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+be^{px}\right)^2}\,dx = \frac{bpxe^{(px)} + apx - \left(be^{(px)} + a\right)\log\left(be^{(px)} + a\right) + a}{a^2bpe^{(px)} + a^3p}$$

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="fricas")
[Out] (b*p*x*e^(p*x) + a*p*x - (b*e^(p*x) + a)*log(b*e^(p*x) + a) + a)/(a^2*b*p*e
^(p*x) + a^3*p)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+be^{px}\right)^2}\,dx = \frac{1}{a^2p + abpe^{px}} + \frac{x}{a^2} - \frac{\log\left(\frac{a}{b} + e^{px}\right)}{a^2p}$$

```
[In] integrate(1/(a+b*exp(p*x))**2,x)
[Out] 1/(a**2*p + a*b*p*exp(p*x)) + x/a**2 - log(a/b + exp(p*x))/(a**2*p)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+be^{px}\right)^2}\,dx = \frac{x}{a^2} + \frac{1}{(abe^{(px)} + a^2)p} - \frac{\log\left(be^{(px)} + a\right)}{a^2p}$$

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="maxima")
[Out] x/a^2 + 1/((a*b*e^(p*x) + a^2)*p) - log(b*e^(p*x) + a)/(a^2*p)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+be^{px})^2}\,dx = \frac{b\left(\frac{\log\left(\left|-\frac{a}{be^{(px)}+a}+1\right|\right)}{a^2b} + \frac{1}{(be^{(px)}+a)ab}\right)}{p}$$

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="giac")
[Out] b*(log(abs(-a/(b*e^(p*x) + a) + 1))/(a^2*b) + 1/((b*e^(p*x) + a)*a*b))/p
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{1}{(a+be^{px})^2}\,dx = \frac{\frac{x}{a} + \frac{b\,x\,e^{p\,x}}{a^2} - \frac{b\,e^{p\,x}}{a^2\,p}}{a+b\,e^{p\,x}} - \frac{\ln\left(a+b\,e^{p\,x}\right)}{a^2\,p}$$

```
[In] int(1/(a + b*exp(p*x))^2,x)
[Out] (x/a + (b*x*exp(p*x))/a^2 - (b*exp(p*x))/(a^2*p))/(a + b*exp(p*x)) - log(a
+ b*exp(p*x))/(a^2*p)
```

3.22 $\int \frac{1}{(be^{-px}+ae^{px})^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 22

$$\int \frac{1}{(be^{-px}+ae^{px})^2} \, dx = -\frac{1}{2a(b+ae^{2px})\,p}$$

```
[Out] -1/2/a/(b+a*exp(2*p*x))/p
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2320, 267}

$$\int \frac{1}{(be^{-px}+ae^{px})^2} \, dx = -\frac{1}{2ap(ae^{2px}+b)}$$

```
[In] Int[(b/E^(p*x) + a*E^(p*x))^(-2),x]
```

```
[Out] -1/2*1/(a*(b + a*E^(2*p*x))*p)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{(b+ax^2)^2}\,dx, x, e^{px}\right)}{p}\\ &= -\frac{1}{2a\left(b+ae^{2px}\right)p}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(be^{-px}+ae^{px}\right)^2}\,dx = -\frac{1}{2a\left(b+ae^{2px}\right)p}$$

```
[In] Integrate[(b/E^(p*x) + a*E^(p*x))^(-2),x]
[Out] -1/2*1/(a*(b + a*E^(2*p*x))*p)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{1}{2a(b+a\,\mathrm{e}^{2px})p}$	20
derivativedivides	$-\frac{1}{2a(b+a\,\mathrm{e}^{2px})p}$	21
default	$-\frac{1}{2a(b+a\,\mathrm{e}^{2px})p}$	21
norman	$-\frac{1}{2a(b+a\,\mathrm{e}^{2px})p}$	21
parallelrisch	$-\frac{1}{2a(b+a\,\mathrm{e}^{2px})p}$	21

```
[In] int(1/(b/exp(p*x)+a*exp(p*x))^2,x,method=_RETURNVERBOSE)
[Out] -1/2/a/(b+a*exp(2*p*x))/p
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{(be^{-px} + ae^{px})^2}\,dx = -\frac{1}{2\,(a^2pe^{(2\,px)} + abp)}$$

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="fricas")
[Out] -1/2/(a^2*p*e^(2*p*x) + a*b*p)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(be^{-px} + ae^{px})^2}\,dx = \frac{1}{2abp + 2b^2pe^{-2px}}$$

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))**2,x)
[Out] 1/(2*a*b*p + 2*b**2*p*exp(-2*p*x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(be^{-px} + ae^{px})^2}\,dx = \frac{1}{2\,(b^2e^{(-2\,px)} + ab)p}$$

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="maxima")
[Out] 1/2/((b^2*e^(-2*p*x) + a*b)*p)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{(be^{-px} + ae^{px})^2}\,dx = -\frac{1}{2\,(ae^{(2\,px)} + b)ap}$$

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="giac")
[Out] -1/2/((a*e^(2*p*x) + b)*a*p)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(be^{-px}+ae^{px})^2}\,dx=\frac{\mathrm{e}^{2\,p\,x}}{2\,b\,p\,\left(b+a\,\mathrm{e}^{2\,p\,x}\right)}$$

```
[In] int(1/(a*exp(p*x) + b*exp(-p*x))^2,x)
[Out] exp(2*p*x)/(2*b*p*(b + a*exp(2*p*x)))
```

3.23 $\int \frac{x}{(be^{-px}+ae^{px})^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{x}{(be^{-px}+ae^{px})^2} \, dx = \frac{x}{2abp} - \frac{x}{2a(b+ae^{2px})p} - \frac{\log(b+ae^{2px})}{4abp^2}$$

```
[Out] 1/2*x/a/b/p-1/2*x/a/(b+a*exp(2*p*x))/p-1/4*ln(b+a*exp(2*p*x))/a/b/p^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2321, 2222, 2320, 36, 29, 31}

$$\int \frac{x}{(be^{-px}+ae^{px})^2} \, dx = -\frac{\log(ae^{2px}+b)}{4abp^2} + \frac{x}{2abp} - \frac{x}{2ap(ae^{2px}+b)}$$

```
[In] Int[x/(b/E^(p*x) + a*E^(p*x))^2,x]
```

```
[Out] x/(2*a*b*p) - x/(2*a*(b + a*E^(2*p*x))*p) - Log[b + a*E^(2p*x)]/(4*a*b*p^2
)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2321

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{e^{2px}x}{\left(b+ae^{2px}\right)^2}\,dx \\
&= -\frac{x}{2a\left(b+ae^{2px}\right)p}+\frac{\int \frac{1}{b+ae^{2px}}\,dx}{2ap} \\
&= -\frac{x}{2a\left(b+ae^{2px}\right)p}+\frac{\text{Subst}\left(\int \frac{1}{x(b+ax)}\,dx,x,e^{2px}\right)}{4ap^2} \\
&= -\frac{x}{2a\left(b+ae^{2px}\right)p}-\frac{\text{Subst}\left(\int \frac{1}{b+ax}\,dx,x,e^{2px}\right)}{4bp^2}+\frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,e^{2px}\right)}{4abp^2} \\
&= \frac{x}{2abp}-\frac{x}{2a\left(b+ae^{2px}\right)p}-\frac{\log\left(b+ae^{2px}\right)}{4abp^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx = \frac{\frac{2e^{2px}px}{b+ae^{2px}} - \frac{\log(b+ae^{2px})}{a}}{4bp^2}$$

[In] Integrate[x/(b/E^(p*x) + a*E^(p*x))^2,x]

[Out] ((2*E^(2*p*x)*p*x)/(b + a*E^(2*p*x)) - Log[b + a*E^(2*p*x)]/a)/(4*b*p^2)

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(b+a\,\mathrm{e}^{2px}\right)}{4ba}+\frac{px\,\mathrm{e}^{2px}}{2b\left(b+a\,\mathrm{e}^{2px}\right)}}{p^2}$	50
default	$\frac{-\frac{\ln\left(b+a\,\mathrm{e}^{2px}\right)}{4ba}+\frac{px\,\mathrm{e}^{2px}}{2b\left(b+a\,\mathrm{e}^{2px}\right)}}{p^2}$	50
norman	$\frac{x\,\mathrm{e}^{2px}}{2pb(b+a\,\mathrm{e}^{2px})}-\frac{\ln(b+a\,\mathrm{e}^{2px})}{4ab\,p^2}$	51
risch	$\frac{x}{2abp}-\frac{x}{2a(b+a\,\mathrm{e}^{2px})p}-\frac{\ln\left(\mathrm{e}^{2px}+\frac{b}{a}\right)}{4ab\,p^2}$	57
parallelrisch	$-\frac{-2\,\mathrm{e}^{2px}apx+\ln(b+a\,\mathrm{e}^{2px})\mathrm{e}^{2px}a+\ln(b+a\,\mathrm{e}^{2px})b}{4ab\,p^2(b+a\,\mathrm{e}^{2px})}$	68

[In] int(x/(b/exp(p*x)+a*exp(p*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/p^2*(-1/4/b/a*ln(a*exp(p*x)^2+b)+1/2*p*x*exp(p*x)^2/b/(a*exp(p*x)^2+b))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx = \frac{2\,apxe^{(2\,px)} - \left(ae^{(2\,px)}+b\right)\log\left(ae^{(2\,px)}+b\right)}{4\left(a^2bp^2e^{(2\,px)}+ab^2p^2\right)}$$

[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="fricas")

[Out] 1/4*(2*a*p*x*e^(2*p*x) - (a*e^(2*p*x) + b)*log(a*e^(2*p*x) + b))/(a^2*b*p^2
*e^(2*p*x) + a*b^2*p^2)

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx = \frac{x}{2abp+2b^2pe^{-2px}} - \frac{x}{2abp} - \frac{\log\left(\frac{a}{b}+e^{-2px}\right)}{4abp^2}$$

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))**2,x)
[Out] x/(2*a*b*p + 2*b**2*p*exp(-2*p*x)) - x/(2*a*b*p) - log(a/b + exp(-2*p*x))/(
4*a*b*p**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx = \frac{xe^{(2\,px)}}{2\,(abpe^{(2\,px)}+b^2p)} - \frac{\log\left(\frac{ae^{(2\,px)}+b}{a}\right)}{4\,abp^2}$$

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="maxima")
[Out] 1/2*x*e^(2*p*x)/(a*b*p*e^(2*p*x) + b^2*p) - 1/4*log((a*e^(2*p*x) + b)/a)/(a
*b*p^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

$$\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx = \frac{2\,apxe^{(2\,px)} - ae^{(2\,px)}\log\left(-ae^{(2\,px)}-b\right) - b\log\left(-ae^{(2\,px)}-b\right)}{4\,(a^2bp^2e^{(2\,px)}+ab^2p^2)}$$

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="giac")
[Out] 1/4*(2*a*p*x*e^(2*p*x) - a*e^(2*p*x)*log(-a*e^(2*p*x) - b) - b*log(-a*e^(2*
p*x) - b))/(a^2*b*p^2*e^(2*p*x) + a*b^2*p^2)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \frac{x}{(be^{-px} + ae^{px})^2}\,dx = \frac{x\,\mathrm{e}^{2\,p\,x}}{2\,b\,p\,\left(b + a\,\mathrm{e}^{2\,p\,x}\right)} - \frac{\ln\left(b + a\,\mathrm{e}^{2\,p\,x}\right)}{4\,a\,b\,p^2}$$

```
[In] int(x/(a*exp(p*x) + b*exp(-p*x))^2,x)
[Out] (x*exp(2*p*x))/(2*b*p*(b + a*exp(2*p*x))) - log(b + a*exp(2*p*x))/(4*a*b*p^
2)
```

3.24 $\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\left(1+x+x^2\right)^2} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 86

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\left(1+x+x^2\right)^2} \, dx = \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2}+\sqrt{2}\arctan\left(\frac{\sqrt{2}(1+x)}{\sqrt{1-x+x^2}}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{1-x+x^2}}\right)}{\sqrt{6}}$$

```
[Out] arctan((1+x)*2^(1/2)/(x^2-x+1)^(1/2))*2^(1/2)-1/6*arctanh(1/3*(1-x)*6^(1/2)
/(x^2-x+1)^(1/2))*6^(1/2)+(1+x)*(x^2-x+1)^(1/2)/(x^2+x+1)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1074, 1049, 1043, 212, 210}

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\left(1+x+x^2\right)^2} \, dx = \sqrt{2}\arctan\left(\frac{\sqrt{2}(x+1)}{\sqrt{x^2-x+1}}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{x^2-x+1}}\right)}{\sqrt{6}}+\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1}$$

```
[In] Int[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2),x]
```

```
[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + Sqrt[2]*ArcTan[(Sqrt[2]*(1 + x)
)/Sqrt[1 - x + x^2]] - ArcTanh[(Sqrt[2/3]*(1 - x))/Sqrt[1 - x + x^2]]/Sqrt[
6]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
```

```
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \frac{1}{12}\int \frac{18-6x}{\sqrt{1-x+x^2}\,(1+x+x^2)}\,dx \\
&= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \frac{1}{48}\int \frac{24+24x}{\sqrt{1-x+x^2}\,(1+x+x^2)}\,dx \\
&\quad - \frac{1}{48}\int \frac{-48+48x}{\sqrt{1-x+x^2}\,(1+x+x^2)}\,dx \\
&= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + 24\text{Subst}\left(\int \frac{1}{1728-2x^2}\,dx, x, \frac{-24+24x}{\sqrt{1-x+x^2}}\right) \\
&\quad + 288\text{Subst}\left(\int \frac{1}{-20736-2x^2}\,dx, x, \frac{-144-144x}{\sqrt{1-x+x^2}}\right) \\
&= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \sqrt{2}\arctan\left(\frac{\sqrt{2}(1+x)}{\sqrt{1-x+x^2}}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{1-x+x^2}}\right)}{\sqrt{6}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.78

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\,(1+x+x^2)^2}\,dx = \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} - \text{RootSum}\Big[3+6\#1+\#1^2-2\#1^3 + \#1^4\&, \frac{19\log\left(-x+\sqrt{1-x+x^2}-\#1\right)+6\log\left(-x+\sqrt{1-x+x^2}-\#1\right)\#1}{3+\#1-3\#1^2+2\#1^3}\&\Big] - \frac{1}{2}\text{RootSum}\Big[3+6\#1+\#1^2-2\#1^3 + \#1^4\&, \frac{-36\log\left(-x+\sqrt{1-x+x^2}-\#1\right)-6\log\left(-x+\sqrt{1-x+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{1-x\right.}}{3+\#1-3\#1^2+2\#1^3}$$

```
[In] Integrate[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2),x]
```

```
[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) - RootSum[3 + 6*#1 + #1^2 - 2*#1^
3 + #1^4 & , (19*Log[-x + Sqrt[1 - x + x^2] - #1] + 6*Log[-x + Sqrt[1 - x +
 x^2] - #1]*#1)/(3 + #1 - 3*#1^2 + 2*#1^3) & ] - RootSum[3 + 6*#1 + #1^2 -
2*#1^3 + #1^4 & , (-36*Log[-x + Sqrt[1 - x + x^2] - #1] - 6*Log[-x + Sqrt[1
 - x + x^2] - #1]*#1 + Log[-x + Sqrt[1 - x + x^2] - #1]*#1^2)/(3 + #1 - 3*#
1^2 + 2*#1^3) & ]/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(71) = 142$.

Time = 1.06 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.84

method	result
risch	$\frac{(1+x)\sqrt{x^2-x+1}}{x^2+x+1}+\frac{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\left(6\sqrt{2}\arctan\left(\frac{2\sqrt{2}\,(1+x)}{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,(1-x)}\right)-\sqrt{6}\operatorname{arctanh}\left(\frac{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,\sqrt{6}}{4}\right)\right)}{6\sqrt{\frac{\frac{(1+x)^2}{(1-x)^2}+3}{\left(\frac{1+x}{1-x}+1\right)^2}}\left(\frac{1+x}{1-x}+1\right)}$
default	$\frac{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\left(3\sqrt{2}\arctan\left(\frac{2\sqrt{2}\,(1+x)}{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,(1-x)}\right)-\sqrt{6}\operatorname{arctanh}\left(\frac{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,\sqrt{6}}{4}\right)\right)}{2\sqrt{\frac{\frac{(1+x)^2}{(1-x)^2}+3}{\left(\frac{1+x}{1-x}+1\right)^2}}\left(\frac{1+x}{1-x}+1\right)}-\frac{9\sqrt{2}\arctan\left(\frac{2\sqrt{2}\,(1+x)}{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,(1-x)}\right)\sqrt{\frac{(1+x)^2}{(1-x)^2}+3\,(1}}{(1-x)^2}$
trager	$\frac{(1+x)\sqrt{x^2-x+1}}{x^2+x+1}-\operatorname{RootOf}\left(576\_Z^4+528\_Z^2+169\right)\ln\left(-\frac{-3456x\operatorname{RootOf}\left(576\_Z^4+528\_Z^2+169\right)^5-6312x\operatorname{R}}{}\right.$

```
[In] int((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1+x)*(x^2-x+1)^(1/2)/(x^2+x+1)+1/6*((1+x)^2/(1-x)^2+3)^(1/2)*(6*2^(1/2)*ar
ctan(2*2^(1/2)/((1+x)^2/(1-x)^2+3)^(1/2)*(1+x)/(1-x))-6^(1/2)*arctanh(1/4*(
(1+x)^2/(1-x)^2+3)^(1/2)*6^(1/2)))/(((1+x)^2/(1-x)^2+3)/((1+x)/(1-x)+1)^2)^
(1/2)/((1+x)/(1-x)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.31

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\,(1+x+x^2)^2}\,dx$$
$$=\frac{\sqrt{6}(x^2+x+1)\sqrt{4i\sqrt{3}-11}\log\left(\sqrt{6}\left(5i\sqrt{3}-9\right)\sqrt{4i\sqrt{3}-11}-78\,x-39i\sqrt{3}+78\sqrt{x^2-x+1}-39\right)-}{}$$

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(6)*(x^2 + x + 1)*sqrt(4*I*sqrt(3) - 11)*log(sqrt(6)*(5*I*sqrt(3)
 - 9)*sqrt(4*I*sqrt(3) - 11) - 78*x - 39*I*sqrt(3) + 78*sqrt(x^2 - x + 1) -
 39) - sqrt(6)*(x^2 + x + 1)*sqrt(-4*I*sqrt(3) - 11)*log(sqrt(6)*(5*I*sqrt(
3) + 9)*sqrt(-4*I*sqrt(3) - 11) - 78*x + 39*I*sqrt(3) + 78*sqrt(x^2 - x + 1
) - 39) - sqrt(6)*(x^2 + x + 1)*sqrt(4*I*sqrt(3) - 11)*log(sqrt(6)*sqrt(4*I
*sqrt(3) - 11)*(-5*I*sqrt(3) + 9) - 78*x - 39*I*sqrt(3) + 78*sqrt(x^2 - x +
```

```
1) - 39) + sqrt(6)*(x^2 + x + 1)*sqrt(-4*I*sqrt(3) - 11)*log(sqrt(6)*sqrt(
-4*I*sqrt(3) - 11)*(-5*I*sqrt(3) - 9) - 78*x + 39*I*sqrt(3) + 78*sqrt(x^2 -
x + 1) - 39) + 12*x^2 + 12*sqrt(x^2 - x + 1)*(x + 1) + 12*x + 12)/(x^2 + x
+ 1)
```

Sympy [F]

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\,(1+x+x^2)^2}\,dx = \int \frac{3x^2-x+1}{\sqrt{x^2-x+1}\,(x^2+x+1)^2}\,dx$$

```
[In] integrate((3*x**2-x+1)/(x**2+x+1)**2/(x**2-x+1)**(1/2),x)
[Out] Integral((3*x**2 - x + 1)/(sqrt(x**2 - x + 1)*(x**2 + x + 1)**2), x)
```

Maxima [F]

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\,(1+x+x^2)^2}\,dx = \int \frac{3\,x^2-x+1}{(x^2+x+1)^2\sqrt{x^2-x+1}}\,dx$$

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(69) = 138$.

Time = 0.32 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.53

$$\begin{aligned}\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\,(1+x+x^2)^2}\,dx &= -\frac{1}{3}\sqrt{6}\sqrt{3}\arctan\left(-\frac{2\,x+\sqrt{6}-2\,\sqrt{x^2-x+1}+1}{\sqrt{3}+\sqrt{2}}\right) \\ &+\frac{1}{3}\sqrt{6}\sqrt{3}\arctan\left(-\frac{2\,x-\sqrt{6}-2\,\sqrt{x^2-x+1}+1}{\sqrt{3}-\sqrt{2}}\right) \\ &+\frac{1}{12}\sqrt{6}\log\left(4\left(\sqrt{6}\sqrt{3}+3\,\sqrt{3}\right)^2+36\left(2\,x+\sqrt{6}-2\,\sqrt{x^2-x+1}+1\right)^2\right) \\ &-\frac{1}{12}\sqrt{6}\log\left(4\left(\sqrt{6}\sqrt{3}-3\,\sqrt{3}\right)^2+36\left(2\,x-\sqrt{6}-2\,\sqrt{x^2-x+1}+1\right)^2\right) \\ &+\frac{\left(x-\sqrt{x^2-x+1}\right)^3+4\left(x-\sqrt{x^2-x+1}\right)^2-10\,x+10\,\sqrt{x^2-x+1}+5}{\left(x-\sqrt{x^2-x+1}\right)^4+2\left(x-\sqrt{x^2-x+1}\right)^3+\left(x-\sqrt{x^2-x+1}\right)^2-6\,x+6\,\sqrt{x^2-x+1}+3}\end{aligned}$$

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(6)*sqrt(3)*arctan(-(2*x + sqrt(6) - 2*sqrt(x^2 - x + 1) + 1)/(sqr
t(3) + sqrt(2))) + 1/3*sqrt(6)*sqrt(3)*arctan(-(2*x - sqrt(6) - 2*sqrt(x^2
- x + 1) + 1)/(sqrt(3) - sqrt(2))) + 1/12*sqrt(6)*log(4*(sqrt(6)*sqrt(3) +
3*sqrt(3))^2 + 36*(2*x + sqrt(6) - 2*sqrt(x^2 - x + 1) + 1)^2) - 1/12*sqrt(
6)*log(4*(sqrt(6)*sqrt(3) - 3*sqrt(3))^2 + 36*(2*x - sqrt(6) - 2*sqrt(x^2 -
 x + 1) + 1)^2) + ((x - sqrt(x^2 - x + 1))^3 + 4*(x - sqrt(x^2 - x + 1))^2
- 10*x + 10*sqrt(x^2 - x + 1) + 5)/((x - sqrt(x^2 - x + 1))^4 + 2*(x - sqrt
(x^2 - x + 1))^3 + (x - sqrt(x^2 - x + 1))^2 - 6*x + 6*sqrt(x^2 - x + 1) +
3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\,(1+x+x^2)^2}\,dx = \int \frac{3\,x^2-x+1}{\sqrt{x^2-x+1}\,(x^2+x+1)^2}\,dx$$

```
[In] int((3*x^2 - x + 1)/((x^2 - x + 1)^(1/2)*(x + x^2 + 1)^2),x)
[Out] int((3*x^2 - x + 1)/((x^2 - x + 1)^(1/2)*(x + x^2 + 1)^2), x)
```

3.25 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 19

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}} \, dx = 2\sqrt{x+\sqrt{a^2+x^2}}$$

```
[Out] 2*(x+(a^2+x^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2147, 30}

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}} \, dx = 2\sqrt{\sqrt{a^2+x^2}+x}$$

```
[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
```

```
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, x+\sqrt{a^2+x^2}\right)\\ &= 2\sqrt{x+\sqrt{a^2+x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx = 2\sqrt{x+\sqrt{a^2+x^2}}$$

```
[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$2\sqrt{x+\sqrt{a^2+x^2}}$	16
default	$2\sqrt{x+\sqrt{a^2+x^2}}$	16

```
[In] int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(x+(a^2+x^2)^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx = 2\sqrt{x+\sqrt{a^2+x^2}}$$

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x + sqrt(a^2 + x^2))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx = 2\sqrt{x+\sqrt{a^2+x^2}}$$

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/(a**2+x**2)**(1/2),x)
[Out] 2*sqrt(x + sqrt(a**2 + x**2))
```

Maxima [F]

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx = \int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx$$

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx = 2\sqrt{x+\sqrt{a^2+x^2}}$$

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x + sqrt(a^2 + x^2))
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx = 2\sqrt{x+\sqrt{a^2+x^2}}$$

```
[In] int((x + (a^2 + x^2)^(1/2))^(1/2)/(a^2 + x^2)^(1/2),x)
[Out] 2*(x + (a^2 + x^2)^(1/2))^(1/2)
```

3.26 $\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 26

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx = \frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}$$

```
[Out] 2*(b*x+(b^2*x^2+a)^(1/2))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2147, 30}

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx = \frac{2\sqrt{\sqrt{a+b^2x^2}+bx}}{b}$$

```
[In] Int[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]
```

```
[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
```

```
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, bx+\sqrt{a+b^2x^2}\right)}{b}\\ &= \frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx = \frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}$$

```
[In] Integrate[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]
[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{2\sqrt{bx+\sqrt{x^2b^2+a}}}{b}$	23
default	$\frac{2\sqrt{bx+\sqrt{x^2b^2+a}}}{b}$	23

```
[In] int((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x,method=_RETURNVERBOSE
)
[Out] 2*(b*x+(b^2*x^2+a)^(1/2))^(1/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx = \frac{2\sqrt{bx+\sqrt{b^2x^2+a}}}{b}$$

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2*sqrt(b*x + sqrt(b^2*x^2 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx = \begin{cases} \frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b} & \text{for } b\neq 0 \\ \frac{x}{\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

```
[In] integrate((b*x+(b**2*x**2+a)**(1/2))**(1/2)/(b**2*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(b*x + sqrt(a + b**2*x**2))/b, Ne(b, 0)), (x/a**(1/4), Tru
e))
```

Maxima [F]

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx = \int \frac{\sqrt{bx+\sqrt{b^2x^2+a}}}{\sqrt{b^2x^2+a}}\,dx$$

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx=\frac{2\,\sqrt{bx+\sqrt{b^2x^2+a}}}{b}$$

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] 2*sqrt(b*x + sqrt(b^2*x^2 + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx=\frac{2\,\sqrt{\sqrt{b^2\,x^2+a}+b\,x}}{b}$$

```
[In] int(((a + b^2*x^2)^(1/2) + b*x)^(1/2)/(a + b^2*x^2)^(1/2),x)
```

```
[Out] (2*((a + b^2*x^2)^(1/2) + b*x)^(1/2))/b
```

3.27 $\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 63

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2\text{arctanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -2*arctan((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))/a^(3/2)-2*arctanh((x+(a^2+x^2)
^(1/2))^(1/2)/a^(1/2))/a^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2145, 335, 218, 212, 209}

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2\text{arctanh}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[In] Int[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]
```

```
[Out] (-2*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2) - (2*ArcTanh[Sqrt[x
+ Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2145

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + p + 1)*e^(p + 1)*f^(2*
m)))*(i/c)^m, Subst[Int[x^(n - 2*m - p - 2)*((-a)*f^2 + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-a^2+x^2)}\,dx, x, x+\sqrt{a^2+x^2}\right)\\
&= 4\text{Subst}\left(\int \frac{1}{-a^2+x^4}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)\\
&= -\frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)}{a} - \frac{2\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)}{a}\\
&= -\frac{2\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2\text{arctanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx=-\frac{2\left(\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)\right)}{a^{3/2}}$$

```
[In] Integrate[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]
```

```
[Out] (-2*(ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] + ArcTanh[Sqrt[x + Sqrt[a^2
+ x^2]]/Sqrt[a]]))/a^(3/2)
```

Maple [F]

$$\int \frac{1}{x\sqrt{a^2+x^2}\,\sqrt{x+\sqrt{a^2+x^2}}}dx$$

```
[In] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(47) = 94$.

Time = 0.26 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.14

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx$$
$$=\left[-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)-\sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)}{a^2},\frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\right.}{}\right.}{}\right.$$

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-(2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) - sqrt(a)*log((a^2 +
 sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + s
qrt(a^2 + x^2)))/x))/a^2, (2*sqrt(-a)*arctan(sqrt(-a)*sqrt(x + sqrt(a^2 + x
^2))/a) - sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a - (sqrt(-a)*(a + x) - sqrt
(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x))/a^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx = -\frac{\Gamma^2\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_3F_2\left(\begin{matrix}\frac{3}{4},\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4}\end{matrix}\middle|\frac{a^2e^{i\pi}}{x^2}\right)}{\pi x^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(1/x/(a**2+x**2)**(1/2)/(x+(a**2+x**2)**(1/2))**(1/2),x)
[Out] -gamma(3/4)**2*gamma(5/4)*hyper((3/4, 3/4, 5/4), (3/2, 7/4), a**2*exp_polar
(I*pi)/x**2)/(pi*x**(3/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx = \int \frac{1}{\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}\,x}\,dx$$

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="maxim
a")
[Out] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx = \int \frac{1}{\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}\,x}\,dx$$

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="giac"
)
[Out] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx = \int \frac{1}{x\,\sqrt{x+\sqrt{a^2+x^2}}\,\sqrt{a^2+x^2}}\,dx$$

```
[In] int(1/(x*(x + (a^2 + x^2)^(1/2))^(1/2)*(a^2 + x^2)^(1/2)),x)
[Out] int(1/(x*(x + (a^2 + x^2)^(1/2))^(1/2)*(a^2 + x^2)^(1/2)), x)
```

3.28 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} \, dx = 2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

```
[Out] -2*arctan((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))*a^(1/2)-2*arctanh((x+(a^2+x^2)
^(1/2))^(1/2)/a^(1/2))*a^(1/2)+2*(x+(a^2+x^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2144, 470, 335, 218, 212, 209}

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} \, dx = -2\sqrt{a}\arctan\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right) - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right) + 2\sqrt{\sqrt{a^2+x^2}+x}$$

```
[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2144

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{a^2+x^2}{\sqrt{x}\,(-a^2+x^2)}\,dx, x, x+\sqrt{a^2+x^2}\right)\\ &= 2\sqrt{x+\sqrt{a^2+x^2}}+(2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-a^2+x^2)}\,dx, x, x+\sqrt{a^2+x^2}\right)\end{aligned}$$

$$= 2\sqrt{x+\sqrt{a^2+x^2}} + (4a^2)\,\text{Subst}\left(\int \frac{1}{-a^2+x^4}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)$$
$$= 2\sqrt{x+\sqrt{a^2+x^2}} - (2a)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)$$
$$- (2a)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)$$
$$= 2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = 2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

```
[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.30

method	result	size
meijerg	$2\sqrt{2}\,\sqrt{x}\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},\frac{1}{4};\frac{1}{2},\frac{3}{4};-\frac{a^2}{x^2}\right)$	25

```
[In] int((x+(a^2+x^2)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 2*2^(1/2)*x^(1/2)*hypergeom([-1/4,-1/4,1/4],[1/2,3/4],-a^2/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \left[-2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) + \sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right) + 2\sqrt{x+\sqrt{a^2+x^2}}, 2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{x+\sqrt{a^2+x^2}}}{a}\right) + \sqrt{-a}\log\left(-\frac{a^2-\sqrt{a^2+x^2}a+\left(\sqrt{-a}(a+x)-\sqrt{a^2+x^2}\sqrt{-a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right) + 2\sqrt{x+\sqrt{a^2+x^2}}\right]$$

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*log((a^2 +
sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + sq
rt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2)), 2*sqrt(-a)*arctan(sqrt(-a
)*sqrt(x + sqrt(a^2 + x^2))/a) + sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a + (
sqrt(-a)*(a + x) - sqrt(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x)
+ 2*sqrt(x + sqrt(a^2 + x^2))]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \frac{\sqrt{x}\Gamma^2\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_3F_2\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{3}{4}\end{matrix}\middle|\frac{a^2e^{i\pi}}{x^2}\right)}{8\pi\Gamma\left(\frac{3}{4}\right)}$$

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)
```

```
[Out] sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2, 3/4), a**2
*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))
```

Maxima [F]

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

Giac [F]

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

```
[In] int((x + (a^2 + x^2)^(1/2))^(1/2)/x,x)
[Out] int((x + (a^2 + x^2)^(1/2))^(1/2)/x, x)
```

3.29 $\int x^3 \log^3(2+x) \log(3+x)\, dx$

Optimal result

Integrand size = 14, antiderivative size = 606

$$\int x^3\log^3(2+x)\log(3+x)\,dx = -\frac{302177x}{1152}+\frac{8029x^2}{2304}-\frac{763x^3}{3456}+\frac{3x^4}{256}+\frac{377}{64}(2+x)^2 -\frac{71}{216}(2+x)^3+\frac{3}{256}(2+x)^4+\frac{2069}{144}\log(2+x) -\frac{187}{64}x^2\log(2+x)+\frac{83}{288}x^3\log(2+x) -\frac{3}{128}x^4\log(2+x)+\frac{6733}{32}(2+x)\log(2+x) -\frac{377}{32}(2+x)^2\log(2+x)+\frac{71}{72}(2+x)^3\log(2+x) -\frac{3}{64}(2+x)^4\log(2+x)-\frac{43}{12}\log^2(2+x)-\frac{17}{48}x^3\log^2(2+x) +\frac{3}{64}x^4\log^2(2+x)-\frac{1251}{16}(2+x)\log^2(2+x) +\frac{273}{32}(2+x)^2\log^2(2+x)-\frac{3}{4}(2+x)^3\log^2(2+x) +\frac{3}{64}(2+x)^4\log^2(2+x)+\frac{65}{4}(2+x)\log^3(2+x) -\frac{33}{8}(2+x)^2\log^3(2+x)+\frac{3}{4}(2+x)^3\log^3(2+x) -\frac{1}{16}(2+x)^4\log^3(2+x)+\frac{3891}{128}\log(3+x) -\frac{115}{48}x^2\log(3+x)+\frac{37}{144}x^3\log(3+x)-\frac{3}{128}x^4\log(3+x) +\frac{415}{12}(3+x)\log(3+x)-\frac{4083}{32}\log(2+x)\log(3+x) -25x\log(2+x)\log(3+x)+\frac{13}{4}x^2\log(2+x)\log(3+x) -\frac{7}{12}x^3\log(2+x)\log(3+x)+\frac{3}{32}x^4\log(2+x)\log(3+x) +\frac{963}{16}\log^2(2+x)\log(3+x)+6x\log^2(2+x)\log(3+x) -\frac{3}{2}x^2\log^2(2+x)\log(3+x)+\frac{1}{2}x^3\log^2(2+x)\log(3+x) -\frac{3}{16}x^4\log^2(2+x)\log(3+x)-\frac{81}{4}\log^3(2+x)\log(3+x) +\frac{1}{4}x^4\log^3(2+x)\log(3+x)-\frac{5609\,\text{PolyLog}(2,-2-x)}{96} +\frac{563}{8}\log(2+x)\,\text{PolyLog}(2,-2-x) -\frac{195}{4}\log^2(2+x)\,\text{PolyLog}(2,-2-x) -\frac{563\,\text{PolyLog}(3,-2-x)}{8} +\frac{195}{2}\log(2+x)\,\text{PolyLog}(3,-2-x) -\frac{195\,\text{PolyLog}(4,-2-x)}{2}$$

```
[Out] -302177/1152*x-25*x*ln(2+x)*ln(3+x)+13/4*x^2*ln(2+x)*ln(3+x)-7/12*x^3*ln(2+
x)*ln(3+x)+3/32*x^4*ln(2+x)*ln(3+x)+6*x*ln(2+x)^2*ln(3+x)-3/2*x^2*ln(2+x)^2
*ln(3+x)+1/2*x^3*ln(2+x)^2*ln(3+x)-3/16*x^4*ln(2+x)^2*ln(3+x)+1/4*x^4*ln(2+
x)^3*ln(3+x)-43/12*ln(2+x)^2-5609/96*polylog(2,-2-x)-563/8*polylog(3,-2-x)-
195/2*polylog(4,-2-x)+377/64*(2+x)^2-71/216*(2+x)^3+3/256*(2+x)^4+3891/128*
ln(3+x)+2069/144*ln(2+x)+3/256*x^4-763/3456*x^3+8029/2304*x^2+963/16*ln(2+x
)^2*ln(3+x)-81/4*ln(2+x)^3*ln(3+x)+563/8*ln(2+x)*polylog(2,-2-x)-195/4*ln(2
+x)^2*polylog(2,-2-x)+195/2*ln(2+x)*polylog(3,-2-x)-187/64*x^2*ln(2+x)+83/2
88*x^3*ln(2+x)-3/128*x^4*ln(2+x)+6733/32*(2+x)*ln(2+x)-377/32*(2+x)^2*ln(2+
x)+71/72*(2+x)^3*ln(2+x)-3/64*(2+x)^4*ln(2+x)-17/48*x^3*ln(2+x)^2+3/64*x^4*
ln(2+x)^2-1251/16*(2+x)*ln(2+x)^2+273/32*(2+x)^2*ln(2+x)^2-3/4*(2+x)^3*ln(2
+x)^2+3/64*(2+x)^4*ln(2+x)^2+65/4*(2+x)*ln(2+x)^3-33/8*(2+x)^2*ln(2+x)^3+3/
4*(2+x)^3*ln(2+x)^3-1/16*(2+x)^4*ln(2+x)^3-115/48*x^2*ln(3+x)+37/144*x^3*ln
(3+x)-3/128*x^4*ln(3+x)+415/12*(3+x)*ln(3+x)-4083/32*ln(2+x)*ln(3+x)
```

Rubi [A] (verified)

Time = 3.53 (sec) , antiderivative size = 606, normalized size of antiderivative = 1.00, number of steps used = 359, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.143, Rules used = {2489, 2463, 2436, 2333, 2332, 2448, 2437, 2342, 2341, 2443, 2481, 2421, 2430, 6724, 2458, 2388, 2339, 30, 2367, 2356, 45, 2372, 2338, 6874, 2479, 2440, 2438, 2441, 2442, 2445}

$$\int x^3 \log^3(2+x)\log(3+x)\,dx = -\frac{5609\,\mathrm{PolyLog}(2,-x-2)}{96} - \frac{563\,\mathrm{PolyLog}(3,-x-2)}{8} - \frac{195\,\mathrm{PolyLog}(4,-x-2)}{2} - \frac{195}{4}\mathrm{PolyLog}(2,-x-2)\log^2(x+2) + \frac{563}{8}\mathrm{PolyLog}(2,-x-2)\log(x+2) + \frac{195}{2}\mathrm{PolyLog}(3,-x-2)\log(x+2) + \frac{3x^4}{256} + \frac{1}{4}x^4\log^3(x+2)\log(x+3) + \frac{3}{64}x^4\log^2(x+2) - \frac{3}{16}x^4\log^2(x+2)\log(x+3) - \frac{3}{128}x^4\log(x+2) + \frac{3}{32}x^4\log(x+2)\log(x+3) - \frac{3}{128}x^4\log(x+3) - \frac{763x^3}{3456} - \frac{17}{48}x^3\log^2(x+2) + \frac{1}{2}x^3\log^2(x+2)\log(x+3) + \frac{83}{288}x^3\log(x+2) - \frac{7}{12}x^3\log(x+2)\log(x+3) + \frac{37}{144}x^3\log(x+3) + \frac{8029x^2}{2304} - \frac{3}{2}x^2\log^2(x+2)\log(x+3) - \frac{187}{64}x^2\log(x+2) + \frac{13}{4}x^2\log(x+2)\log(x+3) - \frac{115}{48}x^2\log(x+3) - \frac{302177x}{1152} + \frac{3}{256}(x+2)^4 - \frac{71}{216}(x+2)^3 + \frac{377}{64}(x+2)^2 - \frac{1}{16}(x+2)^4\log^3(x+2) + \frac{3}{4}(x+2)^3\log^3(x+2) - \frac{33}{8}(x+2)^2\log^3(x+2) + \frac{65}{4}(x+2)\log^3(x+2) - \frac{81}{4}\log^3(x+2)\log(x+3) + 6x\log^2(x+2)\log(x+3) + \frac{3}{64}(x+2)^4\log^2(x+2) - \frac{3}{4}(x+2)^3\log^2(x+2) + \frac{273}{32}(x+2)^2\log^2(x+2) - \frac{1251}{16}(x+2)\log^2(x+2) - \frac{43}{12}\log^2(x+2) + \frac{963}{16}\log^2(x+2)\log(x+3) - 25x\log(x+2)\log(x+3) - \frac{3}{64}(x+2)^4\log(x+2) + \frac{71}{72}(x+2)^3\log(x+2) - \frac{377}{32}(x+2)^2\log(x+2) + \frac{6733}{32}(x+2)\log(x+2) + \frac{2069}{144}\log(x+2) + \frac{415}{12}(x+3)\log(x+3) - \frac{4083}{32}\log(x+2)\log(x+3) + \frac{3891}{128}\log(x+3)$$

```
[In] Int[x^3*Log[2 + x]^3*Log[3 + x],x]
```

```
[Out] (-302177*x)/1152 + (8029*x^2)/2304 - (763*x^3)/3456 + (3*x^4)/256 + (377*(2
 + x)^2)/64 - (71*(2 + x)^3)/216 + (3*(2 + x)^4)/256 + (2069*Log[2 + x])/14
4 - (187*x^2*Log[2 + x])/64 + (83*x^3*Log[2 + x])/288 - (3*x^4*Log[2 + x])/
128 + (6733*(2 + x)*Log[2 + x])/32 - (377*(2 + x)^2*Log[2 + x])/32 + (71*(2
 + x)^3*Log[2 + x])/72 - (3*(2 + x)^4*Log[2 + x])/64 - (43*Log[2 + x]^2)/12
 - (17*x^3*Log[2 + x]^2)/48 + (3*x^4*Log[2 + x]^2)/64 - (1251*(2 + x)*Log[2
 + x]^2)/16 + (273*(2 + x)^2*Log[2 + x]^2)/32 - (3*(2 + x)^3*Log[2 + x]^2)/
4 + (3*(2 + x)^4*Log[2 + x]^2)/64 + (65*(2 + x)*Log[2 + x]^3)/4 - (33*(2 +
x)^2*Log[2 + x]^3)/8 + (3*(2 + x)^3*Log[2 + x]^3)/4 - ((2 + x)^4*Log[2 + x]
^3)/16 + (3891*Log[3 + x])/128 - (115*x^2*Log[3 + x])/48 + (37*x^3*Log[3 +
x])/144 - (3*x^4*Log[3 + x])/128 + (415*(3 + x)*Log[3 + x])/12 - (4083*Log[
2 + x]*Log[3 + x])/32 - 25*x*Log[2 + x]*Log[3 + x] + (13*x^2*Log[2 + x]*Log
[3 + x])/4 - (7*x^3*Log[2 + x]*Log[3 + x])/12 + (3*x^4*Log[2 + x]*Log[3 + x
])/32 + (963*Log[2 + x]^2*Log[3 + x])/16 + 6*x*Log[2 + x]^2*Log[3 + x] - (3
*x^2*Log[2 + x]^2*Log[3 + x])/2 + (x^3*Log[2 + x]^2*Log[3 + x])/2 - (3*x^4*
Log[2 + x]^2*Log[3 + x])/16 - (81*Log[2 + x]^3*Log[3 + x])/4 + (x^4*Log[2 +
 x]^3*Log[3 + x])/4 - (5609*PolyLog[2, -2 - x])/96 + (563*Log[2 + x]*PolyLo
g[2, -2 - x])/8 - (195*Log[2 + x]^2*PolyLog[2, -2 - x])/4 - (563*PolyLog[3,
 -2 - x])/8 + (195*Log[2 + x]*PolyLog[3, -2 - x])/2 - (195*PolyLog[4, -2 -
x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2479

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[x*((a +
 b*Log[c*(d + e*x)^n])^p/(i + j*x)), x], x] - Dist[b*e*n*p, Int[x*(a + b*Lo
g[c*(d + e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2489

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Dist[g*j*(m/(r + 1)), Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
+ j*x)), x], x] - Dist[b*e*n*(p/(r + 1)), Int[x^(r + 1)*(a + b*Log[c*(d + e
*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int \frac{x^4\log^3(2+x)}{3+x}\,dx \\
&\quad - \frac{3}{4}\int \frac{x^4\log^2(2+x)\log(3+x)}{2+x}\,dx \\
&= \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int \bigg(-27\log^3(2+x) + 9x\log^3(2+x) - 3x^2\log^3(2+x) \\
&\quad + x^3\log^3(2+x) + \frac{81\log^3(2+x)}{3+x}\bigg)\,dx - \frac{3}{4}\int \bigg(-8\log^2(2+x)\log(3+x) \\
&\quad + 4x\log^2(2+x)\log(3+x) - 2x^2\log^2(2+x)\log(3+x) \\
&\quad + x^3\log^2(2+x)\log(3+x) + \frac{16\log^2(2+x)\log(3+x)}{2+x}\bigg)\,dx \\
&= \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int x^3\log^3(2+x)\,dx + \frac{3}{4}\int x^2\log^3(2+x)\,dx \\
&\quad - \frac{3}{4}\int x^3\log^2(2+x)\log(3+x)\,dx + \frac{3}{2}\int x^2\log^2(2+x)\log(3+x)\,dx \\
&\quad - \frac{9}{4}\int x\log^3(2+x)\,dx - 3\int x\log^2(2+x)\log(3+x)\,dx \\
&\quad + 6\int \log^2(2+x)\log(3+x)\,dx + \frac{27}{4}\int \log^3(2+x)\,dx \\
&\quad - 12\int \frac{\log^2(2+x)\log(3+x)}{2+x}\,dx - \frac{81}{4}\int \frac{\log^3(2+x)}{3+x}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= 6x\log^2(2+x)\log(3+x) - \frac{3}{2}x^2\log^2(2+x)\log(3+x) + \frac{1}{2}x^3\log^2(2+x)\log(3+x) \\
&\quad - \frac{3}{16}x^4\log^2(2+x)\log(3+x) - \frac{81}{4}\log^3(2+x)\log(3+x) + \frac{1}{4}x^4\log^3(2+x)\log(3+x) \\
&\quad + \frac{3}{16}\int \frac{x^4\log^2(2+x)}{3+x}\,dx - \frac{1}{4}\int \big(-8\log^3(2+x) + 12(2+x)\log^3(2+x) \\
&\qquad\qquad - 6(2+x)^2\log^3(2+x) + (2+x)^3\log^3(2+x)\big)\,dx \\
&\quad + \frac{3}{8}\int \frac{x^4\log(2+x)\log(3+x)}{2+x}\,dx - \frac{1}{2}\int \frac{x^3\log^2(2+x)}{3+x}\,dx \\
&\quad + \frac{3}{4}\int \big(4\log^3(2+x) - 4(2+x)\log^3(2+x) + (2+x)^2\log^3(2+x)\big)\,dx \\
&\quad + \frac{3}{2}\int \frac{x^2\log^2(2+x)}{3+x}\,dx - \frac{9}{4}\int \big(-2\log^3(2+x) + (2+x)\log^3(2+x)\big)\,dx \\
&\quad + 3\int \frac{x^2\log(2+x)\log(3+x)}{2+x}\,dx - 6\int \frac{x\log^2(2+x)}{3+x}\,dx \\
&\quad + \frac{27}{4}\text{Subst}\left(\int \log^3(x)\,dx, x, 2+x\right) - 12\int \frac{x\log(2+x)\log(3+x)}{2+x}\,dx \\
&\quad - 12\text{Subst}\left(\int \frac{\log^2(x)\log(1+x)}{x}\,dx, x, 2+x\right) \\
&\quad + \frac{243}{4}\int \frac{\log^2(2+x)\log(3+x)}{2+x}\,dx - \int \frac{x^3\log(2+x)\log(3+x)}{2+x}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{27}{4}(2+x)\log^3(2+x) + 6x\log^2(2+x)\log(3+x) - \frac{3}{2}x^2\log^2(2+x)\log(3+x) \\
&\quad + \frac{1}{2}x^3\log^2(2+x)\log(3+x) - \frac{3}{16}x^4\log^2(2+x)\log(3+x) - \frac{81}{4}\log^3(2+x)\log(3+x) \\
&\quad + \frac{1}{4}x^4\log^3(2+x)\log(3+x) + 12\log^2(2+x)\,\mathrm{PolyLog}(2,-2-x) \\
&\quad + \frac{3}{16}\int\bigg(-27\log^2(2+x) + 9x\log^2(2+x) - 3x^2\log^2(2+x) + x^3\log^2(2+x) \\
&\qquad\qquad + \frac{81\log^2(2+x)}{3+x}\bigg)\,dx - \frac{1}{4}\int(2+x)^3\log^3(2+x)\,dx \\
&\quad + \frac{3}{8}\int\bigg(-8\log(2+x)\log(3+x) + 4x\log(2+x)\log(3+x) - 2x^2\log(2+x)\log(3+x) \\
&\qquad\qquad + x^3\log(2+x)\log(3+x) + \frac{16\log(2+x)\log(3+x)}{2+x}\bigg)\,dx \\
&\quad - \frac{1}{2}\int\bigg(9\log^2(2+x) - 3x\log^2(2+x) + x^2\log^2(2+x) - \frac{27\log^2(2+x)}{3+x}\bigg)\,dx \\
&\quad + \frac{3}{4}\int(2+x)^2\log^3(2+x)\,dx + \frac{3}{2}\int(2+x)^2\log^3(2+x)\,dx \\
&\quad + \frac{3}{2}\int\bigg(-3\log^2(2+x) + x\log^2(2+x) + \frac{9\log^2(2+x)}{3+x}\bigg)\,dx \\
&\quad + 2\int\log^3(2+x)\,dx - \frac{9}{4}\int(2+x)\log^3(2+x)\,dx + 3\int\log^3(2+x)\,dx \\
&\quad - 2\bigg(3\int(2+x)\log^3(2+x)\,dx\bigg) + 3\int\bigg(-2\log(2+x)\log(3+x) \\
&\qquad\qquad + x\log(2+x)\log(3+x) + \frac{4\log(2+x)\log(3+x)}{2+x}\bigg)\,dx \\
&\quad + \frac{9}{2}\int\log^3(2+x)\,dx - 6\int\bigg(\log^2(2+x) - \frac{3\log^2(2+x)}{3+x}\bigg)\,dx \\
&\quad - 12\int\bigg(\log(2+x)\log(3+x) - \frac{2\log(2+x)\log(3+x)}{2+x}\bigg)\,dx \\
&\quad - \frac{81}{4}\mathrm{Subst}\bigg(\int\log^2(x)\,dx, x, 2+x\bigg) \\
&\quad - 24\mathrm{Subst}\bigg(\int\frac{\log(x)\,\mathrm{PolyLog}(2,-x)}{x}\,dx, x, 2+x\bigg) \\
&\quad + \frac{243}{4}\mathrm{Subst}\bigg(\int\frac{\log^2(x)\log(1+x)}{x}\,dx, x, 2+x\bigg) - \int\bigg(4\log(2+x)\log(3+x) \\
&\quad - 2x\log(2+x)\log(3+x) + x^2\log(2+x)\log(3+x) - \frac{8\log(2+x)\log(3+x)}{2+x}\bigg)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{81}{4}(2+x)\log^2(2+x) + \frac{27}{4}(2+x)\log^3(2+x) + 6x\log^2(2+x)\log(3+x) \\
&\quad - \frac{3}{2}x^2\log^2(2+x)\log(3+x) + \frac{1}{2}x^3\log^2(2+x)\log(3+x) \\
&\quad - \frac{3}{16}x^4\log^2(2+x)\log(3+x) - \frac{81}{4}\log^3(2+x)\log(3+x) + \frac{1}{4}x^4\log^3(2+x)\log(3+x) \\
&\quad - \frac{195}{4}\log^2(2+x)\,\text{PolyLog}(2,-2-x) - 24\log(2+x)\,\text{PolyLog}(3,-2-x) \\
&\quad + \frac{3}{16}\int x^3\log^2(2+x)\,dx - \frac{1}{4}\text{Subst}\left(\int x^3\log^3(x)\,dx, x, 2+x\right) \\
&\quad + \frac{3}{8}\int x^3\log(2+x)\log(3+x)\,dx - \frac{1}{2}\int x^2\log^2(2+x)\,dx - \frac{9}{16}\int x^2\log^2(2+x)\,dx \\
&\quad - \frac{3}{4}\int x^2\log(2+x)\log(3+x)\,dx + \frac{3}{4}\text{Subst}\left(\int x^2\log^3(x)\,dx, x, 2+x\right) \\
&\quad + 2\left(\frac{3}{2}\int x\log^2(2+x)\,dx\right) + \frac{3}{2}\int x\log(2+x)\log(3+x)\,dx \\
&\quad + \frac{3}{2}\text{Subst}\left(\int x^2\log^3(x)\,dx, x, 2+x\right) + \frac{27}{16}\int x\log^2(2+x)\,dx \\
&\quad + 2\int x\log(2+x)\log(3+x)\,dx + 2\text{Subst}\left(\int \log^3(x)\,dx, x, 2+x\right) \\
&\quad - \frac{9}{4}\text{Subst}\left(\int x\log^3(x)\,dx, x, 2+x\right) - 3\int \log(2+x)\log(3+x)\,dx \\
&\quad + 3\int x\log(2+x)\log(3+x)\,dx + 3\text{Subst}\left(\int \log^3(x)\,dx, x, 2+x\right) \\
&\quad - 2\left(3\text{Subst}\left(\int x\log^3(x)\,dx, x, 2+x\right)\right) - 4\int \log(2+x)\log(3+x)\,dx \\
&\quad - 2\left(\frac{9}{2}\int \log^2(2+x)\,dx\right) + \frac{9}{2}\text{Subst}\left(\int \log^3(x)\,dx, x, 2+x\right) - \frac{81}{16}\int \log^2(2+x)\,dx \\
&\quad - 6\int \log^2(2+x)\,dx - 6\int \log(2+x)\log(3+x)\,dx + 6\int \frac{\log(2+x)\log(3+x)}{2+x}\,dx \\
&\quad + 8\int \frac{\log(2+x)\log(3+x)}{2+x}\,dx - 12\int \log(2+x)\log(3+x)\,dx \\
&\quad + 12\int \frac{\log(2+x)\log(3+x)}{2+x}\,dx + 2\left(\frac{27}{2}\int \frac{\log^2(2+x)}{3+x}\,dx\right) \\
&\quad + \frac{243}{16}\int \frac{\log^2(2+x)}{3+x}\,dx + 18\int \frac{\log^2(2+x)}{3+x}\,dx + 24\int \frac{\log(2+x)\log(3+x)}{2+x}\,dx \\
&\quad + 24\text{Subst}\left(\int \frac{\text{PolyLog}(3,-x)}{x}\,dx, x, 2+x\right) + \frac{81}{2}\text{Subst}(\int \log(x)\,dx, x, 2+x) \\
&\quad + \frac{243}{2}\text{Subst}\left(\int \frac{\log(x)\,\text{PolyLog}(2,-x)}{x}\,dx, x, 2+x\right) - \int x^2\log(2+x)\log(3+x)\,dx
\end{aligned}
$$

$= \text{Too large to display}$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.68

$$\int x^3 \log^3(2+x)\log(3+x)\,dx$$
$$= \frac{-195984 - 558290x + 17705x^2 - 1050x^3 + 54x^4 + 910528\log(2+x) + 400008x\log(2+x) - 22836x^2\log}{}$$

```
[In] Integrate[x^3*Log[2 + x]^3*Log[3 + x],x]
```

```
[Out] (-195984 - 558290*x + 17705*x^2 - 1050*x^3 + 54*x^4 + 910528*Log[2 + x] + 4
00008*x*Log[2 + x] - 22836*x^2*Log[2 + x] + 2072*x^3*Log[2 + x] - 162*x^4*L
og[2 + x] - 302016*Log[2 + x]^2 - 118800*x*Log[2 + x]^2 + 11880*x^2*Log[2 +
 x]^2 - 1680*x^3*Log[2 + x]^2 + 216*x^4*Log[2 + x]^2 + 48384*Log[2 + x]^3 +
 15552*x*Log[2 + x]^3 - 2592*x^2*Log[2 + x]^3 + 576*x^3*Log[2 + x]^3 - 144*
x^4*Log[2 + x]^3 + 309078*Log[3 + x] + 79680*x*Log[3 + x] - 5520*x^2*Log[3
+ x] + 592*x^3*Log[3 + x] - 54*x^4*Log[3 + x] - 293976*Log[2 + x]*Log[3 + x
] - 57600*x*Log[2 + x]*Log[3 + x] + 7488*x^2*Log[2 + x]*Log[3 + x] - 1344*x
^3*Log[2 + x]*Log[3 + x] + 216*x^4*Log[2 + x]*Log[3 + x] + 138672*Log[2 + x
]^2*Log[3 + x] + 13824*x*Log[2 + x]^2*Log[3 + x] - 3456*x^2*Log[2 + x]^2*Lo
g[3 + x] + 1152*x^3*Log[2 + x]^2*Log[3 + x] - 432*x^4*Log[2 + x]^2*Log[3 +
x] - 46656*Log[2 + x]^3*Log[3 + x] + 576*x^4*Log[2 + x]^3*Log[3 + x] - 24*(
5609 - 6756*Log[2 + x] + 4680*Log[2 + x]^2)*PolyLog[2, -2 - x] + 288*(-563
+ 780*Log[2 + x])*PolyLog[3, -2 - x] - 224640*PolyLog[4, -2 - x])/2304
```

Maple [F]

$$\int x^3 \ln(2+x)^3 \ln(3+x)\,dx$$

```
[In] int(x^3*ln(2+x)^3*ln(3+x),x)
```

```
[Out] int(x^3*ln(2+x)^3*ln(3+x),x)
```

Fricas [F]

$$\int x^3 \log^3(2+x)\log(3+x)\,dx = \int x^3 \log(x+3)\log(x+2)^3\,dx$$

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(x + 3)*log(x + 2)^3, x)
```

Sympy [F]

$$\int x^3 \log^3(2+x)\log(3+x)\,dx = \left(\frac{x^4 \log{(x+2)}^3}{4} - \frac{3x^4\log{(x+2)}^2}{16} + \frac{3x^4\log{(x+2)}}{32} - \frac{3x^4}{128} + \frac{x^3\log{(x+2)}^2}{2} - \frac{7x^3\log{(x+2)}}{12} + \frac{37x^3}{144} - \frac{3x^2\log{(x+2)}^2}{2} + \frac{13x^2\log{(x+2)}}{4} - \frac{115x^2}{48} + 6x\log{(x+2)}^2 - 25x\log{(x+2)} + \frac{415x}{12} - 4\log{(x+2)}^3 + 25\log{(x+2)}^2 - \frac{415\log{(x+2)}}{6} + \frac{10955281}{240000}\right)\log{(x+3)} - \frac{\int \frac{24900000x}{x+3}\,dx + \int\left(-\frac{1725000x^2}{x+3}\right)dx + \int \frac{185000x^3}{x+3}\,dx + \int\left(-\frac{16875x^4}{x+3}\right)dx + \int\left(-\frac{49800000\log{(x+2)}}{x+3}\right)dx + \int \frac{1}{}$$

```
[In] integrate(x**3*ln(2+x)**3*ln(3+x),x)
```

```
[Out] (x**4*log(x + 2)**3/4 - 3*x**4*log(x + 2)**2/16 + 3*x**4*log(x + 2)/32 - 3*
x**4/128 + x**3*log(x + 2)**2/2 - 7*x**3*log(x + 2)/12 + 37*x**3/144 - 3*x*
*2*log(x + 2)**2/2 + 13*x**2*log(x + 2)/4 - 115*x**2/48 + 6*x*log(x + 2)**2
 - 25*x*log(x + 2) + 415*x/12 - 4*log(x + 2)**3 + 25*log(x + 2)**2 - 415*lo
g(x + 2)/6 + 10955281/240000)*log(x + 3) - (Integral(24900000*x/(x + 3), x)
 + Integral(-1725000*x**2/(x + 3), x) + Integral(185000*x**3/(x + 3), x) +
Integral(-16875*x**4/(x + 3), x) + Integral(-49800000*log(x + 2)/(x + 3), x
) + Integral(18000000*log(x + 2)**2/(x + 3), x) + Integral(-2880000*log(x +
 2)**3/(x + 3), x) + Integral(-18000000*x*log(x + 2)/(x + 3), x) + Integral
(4320000*x*log(x + 2)**2/(x + 3), x) + Integral(2340000*x**2*log(x + 2)/(x
+ 3), x) + Integral(-1080000*x**2*log(x + 2)**2/(x + 3), x) + Integral(-420
000*x**3*log(x + 2)/(x + 3), x) + Integral(360000*x**3*log(x + 2)**2/(x + 3
), x) + Integral(67500*x**4*log(x + 2)/(x + 3), x) + Integral(-135000*x**4*
log(x + 2)**2/(x + 3), x) + Integral(180000*x**4*log(x + 2)**3/(x + 3), x)
+ Integral(32865843/(x + 3), x))/720000
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 518, normalized size of antiderivative = 0.85

$$\int x^3 \log^3(2+x)\log(3+x)\,dx$$

$$= \frac{3}{128}x^4 + \frac{1}{16}\left(4x^4\log(x+3) - x^4 + 4x^3 - 18x^2 + 108x - 324\log(x+3)\right)\log(x+2)^3$$
$$- \frac{65}{4}\log(x+3)\log(x+2)^3 + \frac{195}{4}\log(x+3)\log(x+2)^2\log(-x-2) - \frac{175}{384}x^3$$
$$+ \frac{1}{96}\left(9x^4 - 70x^3 + 495x^2 - 6\left(3x^4 - 8x^3 + 24x^2 - 96x\right)\log(x+3) + 4680\log(x+3)\log(-x-2) - \cdots\right.$$
$$\left. +2)^2 + \frac{195}{4}\mathrm{Li}_2(x+3)\log(x+2)^2 - \frac{195}{4}\mathrm{Li}_2(-x-2)\log(x+2)^2 + \frac{563}{16}\log(x+3)\log(x+2)^2\right.$$
$$+ 21\log(x+2)^3 + \frac{17705}{2304}x^2 + \frac{1}{8}\left(780\log(x+2)^2 - 563\log(x+2)\right)\mathrm{Li}_2(-x-2)$$
$$- \frac{1}{1152}\left(27x^4 - 296x^3 - 18720\log(x+2)^3 + 2760x^2 + 40536\log(x+2)^2 - 39840x - 67308\log(x+2\cdots\right.$$
$$\left. +3)\right.$$
$$- \frac{1}{1152}\left(81x^4 - 1036x^3 + 56160\log(x+3)\log(x+2)^2 + 112320\log(x+3)\log(x+2)\log(-x-2) + 1\cdots\right.$$
$$\left. +2) + \frac{563}{8}\mathrm{Li}_2(-x-2)\log(x+2)\right.$$
$$- \frac{5609}{96}\log(x+3)\log(x+2) + \frac{1573}{12}\log(x+2)^2 - \frac{279145}{1152}x - \frac{5609}{96}\mathrm{Li}_2(-x-2)$$
$$+ \frac{17171}{128}\log(x+3) + \frac{14227}{36}\log(x+2) - \frac{195}{2}\mathrm{Li}_4(-x-2) - \frac{563}{8}\mathrm{Li}_3(-x-2)$$

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="maxima")
```

```
[Out] 3/128*x^4 + 1/16*(4*x^4*log(x + 3) - x^4 + 4*x^3 - 18*x^2 + 108*x - 324*log
(x + 3))*log(x + 2)^3 - 65/4*log(x + 3)*log(x + 2)^3 + 195/4*log(x + 3)*log
(x + 2)^2*log(-x - 2) - 175/384*x^3 + 1/96*(9*x^4 - 70*x^3 + 495*x^2 - 6*(3
*x^4 - 8*x^3 + 24*x^2 - 96*x)*log(x + 3) + 4680*log(x + 3)*log(-x - 2) - 49
50*x + 4680*dilog(x + 3) + 5778*log(x + 3) + 6048*log(x + 2))*log(x + 2)^2
+ 195/4*dilog(x + 3)*log(x + 2)^2 - 195/4*dilog(-x - 2)*log(x + 2)^2 + 563/
16*log(x + 3)*log(x + 2)^2 + 21*log(x + 2)^3 + 17705/2304*x^2 + 1/8*(780*lo
g(x + 2)^2 - 563*log(x + 2))*dilog(-x - 2) - 1/1152*(27*x^4 - 296*x^3 - 187
20*log(x + 2)^3 + 2760*x^2 + 40536*log(x + 2)^2 - 39840*x - 67308*log(x + 2
))*log(x + 3) - 1/1152*(81*x^4 - 1036*x^3 + 56160*log(x + 3)*log(x + 2)^2 +
 112320*log(x + 3)*log(x + 2)*log(-x - 2) + 11418*x^2 - 12*(9*x^4 - 56*x^3
+ 312*x^2 + 4680*log(x + 2)^2 - 2400*x - 6756*log(x + 2))*log(x + 3) + 1123
20*dilog(x + 3)*log(x + 2) + 112320*dilog(-x - 2)*log(x + 2) - 81072*log(x
+ 3)*log(x + 2) + 72576*log(x + 2)^2 - 200004*x - 81072*dilog(-x - 2) + 146
988*log(x + 3) + 302016*log(x + 2) - 112320*polylog(3, -x - 2))*log(x + 2)
```

```
+ 563/8*dilog(-x - 2)*log(x + 2) - 5609/96*log(x + 3)*log(x + 2) + 1573/12*
log(x + 2)^2 - 279145/1152*x - 5609/96*dilog(-x - 2) + 17171/128*log(x + 3)
 + 14227/36*log(x + 2) - 195/2*polylog(4, -x - 2) - 563/8*polylog(3, -x - 2
)
```

Giac [F]

$$\int x^3 \log^3(2+x) \log(3+x) \, dx = \int x^3 \log(x+3) \log(x+2)^3 \, dx$$

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="giac")
[Out] integrate(x^3*log(x + 3)*log(x + 2)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \log^3(2+x) \log(3+x) \, dx = \int x^3 \ln(x+2)^3 \ln(x+3) \, dx$$

```
[In] int(x^3*log(x + 2)^3*log(x + 3),x)
[Out] int(x^3*log(x + 2)^3*log(x + 3), x)
```

3.30 $\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 17

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}} dx = \frac{\left(x+\sqrt{b+x^2}\right)^a}{a}$$

```
[Out] (x+(x^2+b)^(1/2))^a/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2147, 30}

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}} dx = \frac{\left(\sqrt{b+x^2}+x\right)^a}{a}$$

```
[In] Int[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]
```

```
[Out] (x + Sqrt[b + x^2])^a/a
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
```

```
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^{-1+a}\,dx, x, x+\sqrt{b+x^2}\right)\\ &= \frac{\left(x+\sqrt{b+x^2}\right)^a}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx = \frac{\left(x+\sqrt{b+x^2}\right)^a}{a}$$

```
[In] Integrate[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]
[Out] (x + Sqrt[b + x^2])^a/a
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$	16
default	$\frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$	16

```
[In] int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x,method=_RETURNVERBOSE)
[Out] (x+(x^2+b)^(1/2))^a/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx=\frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$$

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="fricas")
[Out] (x + sqrt(x^2 + b))^a/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(12) = 24$.

Time = 1.25 (sec) , antiderivative size = 311, normalized size of antiderivative = 18.29

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx = \begin{cases} \frac{\sqrt{b}b^{\frac{a}{2}}\sinh\left(a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ax\sqrt{\frac{b}{x^2}+1}}+\frac{b^{\frac{a}{2}}x\cosh\left(a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}}+\frac{b^{\frac{a}{2}}x\sinh\left(a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}\sqrt{\frac{b}{x^2}+1}}-\frac{2b^{\frac{a}{2}}\cosh\left(\right.}{} \\ \frac{b^{\frac{a}{2}}\sinh\left(a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{1+\frac{x^2}{b}}}+\frac{b^{\frac{a}{2}}x^2\sinh\left(a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ab\sqrt{1+\frac{x^2}{b}}}+\frac{b^{\frac{a}{2}}x\cosh\left(a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}}-\frac{2b^{\frac{a}{2}}\cosh\left(a\right.}{} \end{cases}$$

```
[In] integrate((x+(x**2+b)**(1/2))**a/(x**2+b)**(1/2),x)
[Out] Piecewise((sqrt(b)*b**(a/2)*sinh(a*asinh(x/sqrt(b)) - asinh(x/sqrt(b)))/(a*
x*sqrt(b/x**2 + 1)) + b**(a/2)*x*cosh(a*asinh(x/sqrt(b)) - asinh(x/sqrt(b))
)/(a*sqrt(b)) + b**(a/2)*x*sinh(a*asinh(x/sqrt(b)) - asinh(x/sqrt(b)))/(a*s
qrt(b)*sqrt(b/x**2 + 1)) - 2*b**(a/2)*cosh(a*asinh(x/sqrt(b)))*gamma(1 - a/
2)/(a**2*gamma(-a/2)), Abs(x**2/b) > 1), (b**(a/2)*sinh(a*asinh(x/sqrt(b))
- asinh(x/sqrt(b)))/(a*sqrt(1 + x**2/b)) + b**(a/2)*x**2*sinh(a*asinh(x/sqr
t(b)) - asinh(x/sqrt(b)))/(a*b*sqrt(1 + x**2/b)) + b**(a/2)*x*cosh(a*asinh(
x/sqrt(b)) - asinh(x/sqrt(b)))/(a*sqrt(b)) - 2*b**(a/2)*cosh(a*asinh(x/sqrt
(b)))*gamma(1 - a/2)/(a**2*gamma(-a/2)), True))
```

Maxima [F]

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx = \int \frac{\left(x+\sqrt{x^2+b}\right)^a}{\sqrt{x^2+b}}\,dx$$

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx = \frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$$

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="giac")
[Out] (x + sqrt(x^2 + b))^a/a
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx = \frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$$

```
[In] int((x + (b + x^2)^(1/2))^a/(b + x^2)^(1/2),x)
[Out] (x + (b + x^2)^(1/2))^a/a
```

3.31 $\int \left(x+\sqrt{b+x^2}\right)^a\,dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \left(x+\sqrt{b+x^2}\right)^a\,dx=-\frac{b\left(x+\sqrt{b+x^2}\right)^{-1+a}}{2(1-a)}+\frac{\left(x+\sqrt{b+x^2}\right)^{1+a}}{2(1+a)}$$

```
[Out] -1/2*b*(x+(x^2+b)^(1/2))^(-1+a)/(1-a)+1/2*(x+(x^2+b)^(1/2))^(1+a)/(1+a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2142, 14}

$$\int \left(x+\sqrt{b+x^2}\right)^a\,dx=\frac{\left(\sqrt{b+x^2}+x\right)^{a+1}}{2(a+1)}-\frac{b\left(\sqrt{b+x^2}+x\right)^{a-1}}{2(1-a)}$$

```
[In] Int[(x + Sqrt[b + x^2])^a,x]
```

```
[Out] -1/2*(b*(x + Sqrt[b + x^2])^(-1 + a))/(1 - a) + (x + Sqrt[b + x^2])^(1 + a)
/(2*(1 + a))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
```

```
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^{-2+a}\left(b+x^2\right)\,dx, x, x+\sqrt{b+x^2}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(bx^{-2+a}+x^a\right)\,dx, x, x+\sqrt{b+x^2}\right)\\ &= -\frac{b\left(x+\sqrt{b+x^2}\right)^{-1+a}}{2(1-a)}+\frac{\left(x+\sqrt{b+x^2}\right)^{1+a}}{2(1+a)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \left(x+\sqrt{b+x^2}\right)^a\,dx = \frac{1}{2}\left(x+\sqrt{b+x^2}\right)^{-1+a}\left(\frac{b}{-1+a}+\frac{\left(x+\sqrt{b+x^2}\right)^2}{1+a}\right)$$

```
[In] Integrate[(x + Sqrt[b + x^2])^a,x]
[Out] ((x + Sqrt[b + x^2])^(-1 + a)*(b/(-1 + a) + (x + Sqrt[b + x^2])^2/(1 + a)))
/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(44) = 88.

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.31

method	result	size
meijerg	$\frac{b^{\frac{a}{2}+\frac{1}{2}} a \left(\frac{8\sqrt{\pi}\, x^{1+a} b^{-\frac{a}{2}-\frac{1}{2}} \left(\frac{ab}{x^2}+a-1\right)\left(\sqrt{1+\frac{b}{x^2}}+1\right)^{a-1}}{(1+a)a(2a-2)}+\frac{4\sqrt{\pi}\, x^{1+a} b^{-\frac{a}{2}-\frac{1}{2}} \sqrt{1+\frac{b}{x^2}}\left(\sqrt{1+\frac{b}{x^2}}+1\right)^{a-1}}{(1+a)a}\right)}{4\sqrt{\pi}}$	120

```
[In] int((x+(x^2+b)^(1/2))^a,x,method=_RETURNVERBOSE)
[Out] 1/4*b^(1/2*a+1/2)/Pi^(1/2)*a*(8*Pi^(1/2)/(1+a)/a*x^(1+a)*b^(-1/2*a-1/2)*(a*
b/x^2+a-1)/(2*a-2)*((1+1/x^2*b)^(1/2)+1)^(a-1)+4*Pi^(1/2)/(1+a)/a*x^(1+a)*b
^(-1/2*a-1/2)*(1+1/x^2*b)^(1/2)*((1+1/x^2*b)^(1/2)+1)^(a-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \left(x+\sqrt{b+x^2}\right)^a \, dx = \frac{\left(\sqrt{x^2+b}a-x\right)\left(x+\sqrt{x^2+b}\right)^a}{a^2-1}$$

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="fricas")
[Out] (sqrt(x^2 + b)*a - x)*(x + sqrt(x^2 + b))^a/(a^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2236 vs. 2(37) = 74.

Time = 1.27 (sec) , antiderivative size = 2236, normalized size of antiderivative = 43.00

$$\int \left(x+\sqrt{b+x^2}\right)^a \, dx = \text{Too large to display}$$

```
[In] integrate((x+(x**2+b)**(1/2))**a,x)
[Out] Piecewise((-a**2*b**4*b**(a/2 + 1/2)*x*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt
(b)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*ga
mma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2))
- a**2*b**3*b**(a/2 + 1/2)*x**3*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)))*g
amma(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 -
 a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) + 2*a*b
**(9/2)*b**(a/2 + 1/2)*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1
- a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2
) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) - 2*a*b**(9
/2)*b**(a/2 + 1/2)*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*
b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*
gamma(1 - a/2)) - 2*a*b**(7/2)*b**(a/2 + 1/2)*x**2*sqrt(b/x**2 + 1)*sinh(a*
asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(
1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2)
- 2*b**(7/2)*x**2*gamma(1 - a/2)) + 4*a*b**(7/2)*b**(a/2 + 1/2)*x**2*cosh(a
*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma
(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2)
 - 2*b**(7/2)*x**2*gamma(1 - a/2)) - 2*a*b**(7/2)*b**(a/2 + 1/2)*x**2*gamma
(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 -
a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) - 2*a*b*
*(5/2)*b**(a/2 + 1/2)*x**4*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)) + asinh
(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/
```

```
2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(
1 - a/2)) + 2*a*b**(5/2)*b**(a/2 + 1/2)*x**4*cosh(a*asinh(x/sqrt(b)) + asin
h(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7
/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma
(1 - a/2)) + a*b**4*b**(a/2 + 1/2)*x*cosh(a*asinh(x/sqrt(b)))*gamma(-a/2)/(
2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b*
*(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) + a*b**3*b**(a/2 +
1/2)*x**3*cosh(a*asinh(x/sqrt(b)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a
/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b
**(7/2)*x**2*gamma(1 - a/2)) - 2*b**(7/2)*b**(a/2 + 1/2)*x**2*sqrt(b/x**2 +
 1)*sinh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(
9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamm
a(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) + 2*b**(7/2)*b**(a/2 + 1/2)*x*
*2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9
/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma
(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) - 2*b**(5/2)*b**(a/2 + 1/2)*x**
4*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/
2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) -
2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) + 2*b**(5/2)*b*
*(a/2 + 1/2)*x**4*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2
)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2
*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)), Abs(x**2/b) > 1
), (-a**2*b**(5/2)*b**(a/2 + 1/2)*sqrt(1 + x**2/b)*sinh(a*asinh(x/sqrt(b)))
*gamma(-a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)) +
 2*a*b**(5/2)*b**(a/2 + 1/2)*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*ga
mma(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)) +
 2*a*b**(3/2)*b**(a/2 + 1/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)
))*gamma(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/
2)) - 2*a*b**2*b**(a/2 + 1/2)*x*sqrt(1 + x**2/b)*sinh(a*asinh(x/sqrt(b)) +
asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/
2)*gamma(1 - a/2)) + a*b**2*b**(a/2 + 1/2)*x*cosh(a*asinh(x/sqrt(b)))*gamma
(-a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)) + 2*b**
(3/2)*b**(a/2 + 1/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma
(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)) - 2*
b**2*b**(a/2 + 1/2)*x*sqrt(1 + x**2/b)*sinh(a*asinh(x/sqrt(b)) + asinh(x/sq
rt(b)))*gamma(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1
 - a/2)), True))
```

Maxima [F]

$$\int \left(x + \sqrt{b + x^2}\right)^a \, dx = \int \left(x + \sqrt{x^2 + b}\right)^a \, dx$$

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 + b))^a, x)
```

Giac [F]

$$\int \left(x + \sqrt{b + x^2}\right)^a \, dx = \int \left(x + \sqrt{x^2 + b}\right)^a \, dx$$

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="giac")
[Out] integrate((x + sqrt(x^2 + b))^a, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(x + \sqrt{b + x^2}\right)^a \, dx = \int \left(x + \sqrt{x^2 + b}\right)^a \, dx$$

```
[In] int((x + (b + x^2)^(1/2))^a,x)
[Out] int((x + (b + x^2)^(1/2))^a, x)
```

3.32 $\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 34

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right) dx = \frac{x^{1+a}\left(6+3x^a+2x^{2a}\right)^{1+\frac{1}{a}}}{6(1+a)}$$

```
[Out] 1/6*x^(1+a)*(6+3*x^a+2*x^(2*a))^(1+1/a)/(1+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {1608, 1761}

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right) dx = \frac{x^{a+1}\left(2x^{2a}+3x^a+6\right)^{\frac{1}{a}+1}}{6(a+1)}$$

```
[In] Int[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)),x]
```

```
[Out] (x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6*(1 + a))
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1761

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.)*
((d_) + (e_.)*(x_)^(n_.) + (f_.)*(x_)^(n2_.)), x_Symbol] :> Simp[d*(g*x)^(m
 + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*g*(m + 1))), x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[a*e*(m + 1) - b*d*(m + n*
(p + 1) + 1), 0] && EqQ[a*f*(m + 1) - c*d*(m + 2*n*(p + 1) + 1), 0] && NeQ[
m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^a\left(1+x^a+x^{2a}\right)\left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}}\,dx\\ &= \frac{x^{1+a}(6+3x^a+2x^{2a})^{1+\frac{1}{a}}}{6(1+a)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}}\left(x^a+x^{2a}+x^{3a}\right)\,dx = \frac{x^{1+a}(6+3x^a+2x^{2a})^{1+\frac{1}{a}}}{6+6a}$$

```
[In] Integrate[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)),x]
[Out] (x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6 + 6*a)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

method	result	size
risch	$\frac{x\,x^a\left(6+3x^a+2x^{2a}\right)\left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}}}{6+6a}$	44

```
[In] int((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x,method=_RETURNVERBOSE
)
[Out] 1/6*x*x^a*(6+3*x^a+2*(x^a)^2)/(1+a)*(6+3*x^a+2*(x^a)^2)^(1/a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right) \, dx = \frac{(2\,xx^{3\,a}+3\,xx^{2\,a}+6\,xx^a)(2\,x^{2\,a}+3\,x^a+6)^{\left(\frac{1}{a}\right)}}{6\,(a+1)}$$

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="fri
cas")
```

```
[Out] 1/6*(2*x*x^(3*a) + 3*x*x^(2*a) + 6*x*x^a)*(2*x^(2*a) + 3*x^a + 6)^(1/a)/(a
+ 1)
```

Sympy [F(-1)]

Timed out.

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right) \, dx = \text{Timed out}$$

```
[In] integrate((6+3*x**a+2*x**(2*a))**(1/a)*(x**a+x**(2*a)+x**(3*a)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right) \, dx = \frac{(2\,xx^{3\,a}+3\,xx^{2\,a}+6\,xx^a)(2\,x^{2\,a}+3\,x^a+6)^{\left(\frac{1}{a}\right)}}{6\,(a+1)}$$

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="max
ima")
```

```
[Out] 1/6*(2*x*x^(3*a) + 3*x*x^(2*a) + 6*x*x^a)*(2*x^(2*a) + 3*x^a + 6)^(1/a)/(a
+ 1)
```

Giac [F]

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}}\left(x^a+x^{2a}+x^{3a}\right)\,dx=\int \left(2\,x^{2\,a}+3\,x^a+6\right)^{\left(\frac{1}{a}\right)}\left(x^{3\,a}+x^{2\,a}+x^a\right)dx$$

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="gia
c")
[Out] integrate((2*x^(2*a) + 3*x^a + 6)^(1/a)*(x^(3*a) + x^(2*a) + x^a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}}\left(x^a+x^{2a}+x^{3a}\right)\,dx=\int \left(x^a+x^{2\,a}+x^{3\,a}\right)\,\left(3\,x^a+2\,x^{2\,a}+6\right)^{1/a}dx$$

```
[In] int((x^a + x^(2*a) + x^(3*a))*(3*x^a + 2*x^(2*a) + 6)^(1/a),x)
[Out] int((x^a + x^(2*a) + x^(3*a))*(3*x^a + 2*x^(2*a) + 6)^(1/a), x)
```

3.33 $\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx=\frac{1}{2}\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)$$

```
[Out] -1/2*ln(x)+3/4*ln(1-(-x^2+1)^(1/3))+1/2*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(
1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {272, 57, 632, 210, 31}

$$\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx=\frac{1}{2}\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log(x)}{2}$$

```
[In] Int[1/(x*(1 - x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3*Log[1 -
 (1 - x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}\,dx, x, x^2\right)\\
&= -\frac{\log(x)}{2} - \frac{3}{4}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^2}\right) + \frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right)\\
&= -\frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^2}\right)\\
&= \frac{1}{2}\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right) - \frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx = \frac{1}{4}\left(2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)+2\log\left(-1+\sqrt[3]{1-x^2}\right)-\log\left(1+\sqrt[3]{1-x^2}+\left(1-x^2\right)^{2/3}\right)\right)$$

```
[In] Integrate[1/(x*(1 - x^2)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] + 2*Log[-1 + (1 - x^2)^(
1/3)] - Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/4
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

method	result
pseudoelliptic	$-\frac{\ln\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)}{4}+\frac{\arctan\left(\frac{\left(1+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2}+\frac{\ln\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)}{2}$
meijerg	$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}+\frac{2\pi\sqrt{3}\,x^2\,{}_3F_2\left(1,1,\frac{4}{3};2,2;x^2\right)}{9\Gamma\left(\frac{2}{3}\right)}\right)}{4\pi}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)^2 x^2+15\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)\left(-x^2+1\right)^{\frac{2}{3}}-\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)}{}\right.}{}$

```
[In] int(1/x/(-x^2+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln((-x^2+1)^(2/3)+(-x^2+1)^(1/3)+1)+1/2*arctan(1/3*(1+2*(-x^2+1)^(1/3)
)*3^(1/2))*3^(1/2)+1/2*ln((-x^2+1)^(1/3)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx = \frac{1}{2}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}(-x^2+1)^{\frac{1}{3}} + \frac{1}{3}\,\sqrt{3}\right) - \frac{1}{4}\,\log\left((-x^2+1)^{\frac{2}{3}} + (-x^2+1)^{\frac{1}{3}} + 1\right) + \frac{1}{2}\,\log\left((-x^2+1)^{\frac{1}{3}} - 1\right)$$

```
[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="fricas")
[Out] 1/2*sqrt(3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/4*log((-
x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx = -\frac{e^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{1}{x^2}\right)}{2x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate(1/x/(-x**2+1)**(1/3),x)
[Out] -exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**(-2))/(2*x**(2/3)*gam
ma(4/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx = \frac{1}{2}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,(-x^2+1)^{\frac{1}{3}} + 1\right)\right) - \frac{1}{4}\,\log\left((-x^2+1)^{\frac{2}{3}} + (-x^2+1)^{\frac{1}{3}} + 1\right) + \frac{1}{2}\,\log\left((-x^2+1)^{\frac{1}{3}} - 1\right)$$

```
[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="maxima")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right) + \frac{1}{2}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

```
[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log(-(-x^2 + 1)^(1/3) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.48

$$\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx = \frac{\ln\left(\frac{9\left(1-x^2\right)^{1/3}}{4}-\frac{9}{4}\right)}{2} + \ln\left(\frac{9\left(1-x^2\right)^{1/3}}{4}-9\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)^2\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right) - \ln\left(\frac{9\left(1-x^2\right)^{1/3}}{4}-9\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)^2\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

```
[In] int(1/(x*(1 - x^2)^(1/3)),x)
```

```
[Out] log((9*(1 - x^2)^(1/3))/4 - 9/4)/2 + log((9*(1 - x^2)^(1/3))/4 - 9*((3^(1/2
)*1i)/4 - 1/4)^2)*((3^(1/2)*1i)/4 - 1/4) - log((9*(1 - x^2)^(1/3))/4 - 9*((
3^(1/2)*1i)/4 + 1/4)^2)*((3^(1/2)*1i)/4 + 1/4)
```

3.34 $\int \frac{1}{x(1-x^2)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \frac{1}{x\left(1-x^2\right)^{2/3}}\,dx = -\frac{1}{2}\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)$$

```
[Out] -1/2*ln(x)+3/4*ln(1-(-x^2+1)^(1/3))-1/2*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(
1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {272, 59, 632, 210, 31}

$$\int \frac{1}{x\left(1-x^2\right)^{2/3}}\,dx = -\frac{1}{2}\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log(x)}{2}$$

```
[In] Int[1/(x*(1 - x^2)^(2/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]) - Log[x]/2 + (3*Log[
1 - (1 - x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^2\right)\\
&= -\frac{\log(x)}{2} - \frac{3}{4}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^2}\right) - \frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right)\\
&= -\frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right) + \frac{3}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^2}\right)\\
&= -\frac{1}{2}\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right) - \frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\left(1-x^2\right)^{2/3}}\,dx = \frac{1}{4}\left(-2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right) + 2\log\left(-1+\sqrt[3]{1-x^2}\right) - \log\left(1+\sqrt[3]{1-x^2}+\left(1-x^2\right)^{2/3}\right)\right)$$

```
[In] Integrate[1/(x*(1 - x^2)^(2/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] + 2*Log[-1 + (1 - x^2)^
(1/3)] - Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

method	result
meijerg	$\frac{\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+\frac{2\Gamma\left(\frac{2}{3}\right)x^2\,{}_3F_2\left(1,1,\frac{5}{3};2,2;x^2\right)}{3}}{2\Gamma\left(\frac{2}{3}\right)}$
pseudoelliptic	$-\frac{\ln\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)}{4}-\frac{\arctan\left(\frac{\left(1+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2}+\frac{\ln\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)}{2}$
trager	$\frac{\ln\left(\frac{-4\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^2+15\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(-x^2+1\right)^{\frac{2}{3}}+9\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2-9\left(-x^2+1\right)^{\frac{2}{3}}+9\,\mathrm{RootOf}}{x^2}\right.}{2}$

```
[In] int(1/x/(-x^2+1)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/GAMMA(2/3)*((1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x)+I*Pi)*GAMMA(2/3)+2/3*GAMM
A(2/3)*x^2*hypergeom([1,1,5/3],[2,2],x^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(1-x^2\right)^{2/3}}\,dx = -\frac{1}{2}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}(-x^2+1)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right) - \frac{1}{4}\,\log\left((-x^2+1)^{\frac{2}{3}}+(-x^2+1)^{\frac{1}{3}}+1\right)+\frac{1}{2}\,\log\left((-x^2+1)^{\frac{1}{3}}-1\right)$$

```
[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/4*log((
-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.64

$$\int \frac{1}{x\left(1-x^2\right)^{2/3}}\,dx = -\frac{e^{-\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{1}{x^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate(1/x/(-x**2+1)**(2/3),x)
```

```
[Out] -exp(-2*I*pi/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), x**(-2))/(2*x**(4/3)*g
amma(5/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(1-x^2\right)^{2/3}}\,dx = -\frac{1}{2}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{4}\,\log\left((-x^2+1)^{\frac{2}{3}}+(-x^2+1)^{\frac{1}{3}}+1\right)+\frac{1}{2}\,\log\left((-x^2+1)^{\frac{1}{3}}-1\right)$$

```
[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(1-x^{2}\right)^{2/3}}\,dx = -\frac{1}{2}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(-x^{2}+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{4}\,\log\left(\left(-x^{2}+1\right)^{\frac{2}{3}}+\left(-x^{2}+1\right)^{\frac{1}{3}}+1\right) + \frac{1}{2}\,\log\left(-\left(-x^{2}+1\right)^{\frac{1}{3}}+1\right)$$

```
[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log(-(-x^2 + 1)^(1/3) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31

$$\int \frac{1}{x\left(1-x^{2}\right)^{2/3}}\,dx = \frac{\ln\left(\frac{9\left(1-x^{2}\right)^{1/3}}{4}-\frac{9}{4}\right)}{2} + \ln\left(\frac{9\left(1-x^{2}\right)^{1/3}}{2}+\frac{9}{4}-\frac{\sqrt{3}\,9\mathrm{i}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right) - \ln\left(\frac{9\left(1-x^{2}\right)^{1/3}}{2}+\frac{9}{4}+\frac{\sqrt{3}\,9\mathrm{i}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

```
[In] int(1/(x*(1 - x^2)^(2/3)),x)
```

```
[Out] log((9*(1 - x^2)^(1/3))/4 - 9/4)/2 + log((9*(1 - x^2)^(1/3))/2 - (3^(1/2)*9
i)/4 + 9/4)*((3^(1/2)*1i)/4 - 1/4) - log((3^(1/2)*9i)/4 + (9*(1 - x^2)^(1/3
))/2 + 9/4)*((3^(1/2)*1i)/4 + 1/4)
```

3.35 $\int \frac{1}{\sqrt[3]{1-x^3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx=-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

```
[Out] 1/2*ln(x+(-x^3+1)^(1/3))-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(
1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {245}

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx=\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Int[(1 - x^3)^(-1/3),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[x + (1 - x^3)^
(1/3)]/2
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\text{integral} = -\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx = \frac{\arctan\left(\frac{-1+\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{6}\log\left(1+\frac{x^2}{(1-x^3)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right)$$

```
[In] Integrate[(1 - x^3)^(-1/3),x]
```

```
[Out] ArcTan[(-1 + (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 + x^2/(1 - x^3
)^(2/3) - x/(1 - x^3)^(1/3)]/6 + Log[1 + x/(1 - x^3)^(1/3)]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.24

method	result
meijerg	$x{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};x^3\right)$
pseudoelliptic	$\frac{\ln\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)}{3}-\frac{\ln\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}-x\left(-x^3+1\right)^{\frac{1}{3}}+x^2}{x^2}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(-2\left(-x^3+1\right)^{\frac{1}{3}}+x\right)\sqrt{3}}{3x}\right)}{3}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3+3\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x-3\operatorname{RootOf}\left(\_Z^2+\_Z\right.\right.}{3}$

```
[In] int(1/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] x*hypergeom([1/3,1/3],[4/3],x^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(40) = 80.

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3x}\right)+\frac{1}{3}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right) - \frac{1}{6}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/3*
log((x + (-x^3 + 1)^(1/3))/x) - 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 +
 1)^(2/3))/x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate(1/(-x**3+1)**(1/3),x)
[Out] x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3
))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(-x^3+1)^{\frac{1}{3}}}{x}-1\right)\right) + \frac{1}{3}\log\left(\frac{(-x^3+1)^{\frac{1}{3}}}{x}+1\right)-\frac{1}{6}\log\left(-\frac{(-x^3+1)^{\frac{1}{3}}}{x}+\frac{(-x^3+1)^{\frac{2}{3}}}{x^2}+1\right)$$

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) + 1/3*log((-x^3
 + 1)^(1/3)/x + 1) - 1/6*log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1
)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(-1/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.20

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx = x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ x^3\right)$$

```
[In] int(1/(1 - x^3)^(1/3),x)
```

```
[Out] x*hypergeom([1/3, 1/3], 4/3, x^3)
```

3.36 $\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx=\frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

```
[Out] -1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(
1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {272, 57, 632, 210, 31}

$$\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx=\frac{\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log(x)}{2}$$

```
[In] Int[1/(x*(1 - x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x
^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-xx}}\,dx, x, x^3\right)\\
&= -\frac{\log(x)}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)\\
&= -\frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right)\\
&= \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

$$\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx = \frac{\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(-1+\sqrt[3]{1-x^3}\right) - \frac{1}{6}\log\left(1+\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)$$

```
[In] Integrate[1/(x*(1 - x^3)^(1/3)),x]
```

```
[Out] ArcTan[1/Sqrt[3] + (2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[-1 + (1 - x^3
)^(1/3)]/3 - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$-\frac{\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)}{6}+\frac{\arctan\left(\frac{\left(1+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)}{3}$
meijerg	$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}+\frac{2\pi\sqrt{3}\,x^3\,{}_3F_2\left(1,1,\frac{4}{3};2,2;x^3\right)}{9\Gamma\left(\frac{2}{3}\right)}\right)}{6\pi}$
trager	$\text{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(\frac{-1438\,\text{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3-9855\,\text{RootOf}\left(\_Z^2+\_Z+1\right)x^3+5502\,\text{RootOf}\left(\_Z^2+\_Z+\right.}{}\right.$

```
[In] int(1/x/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*ln((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)+1/3*ln((-x^3+1)^(1/3)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}(-x^3+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right) - \frac{1}{6}\log\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left((-x^3+1)^{\frac{1}{3}}-1\right)$$

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 1/6*log((-
x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.58

$$\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx = -\frac{e^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate(1/x/(-x**3+1)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**(-3))/(3*x*gamma(4/3)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(-x^3+1)^{\frac{1}{3}}+1\right)\right) - \frac{1}{6}\log\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left((-x^3+1)^{\frac{1}{3}}-1\right)$$

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 +
1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 +
1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx = \frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3} + \ln\left(\left(1-x^3\right)^{1/3}-9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(\left(1-x^3\right)^{1/3}-9\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

```
[In] int(1/(x*(1 - x^3)^(1/3)),x)
```

```
[Out] log((1 - x^3)^(1/3) - 1)/3 + log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 - 1/6)
^2)*((3^(1/2)*1i)/6 - 1/6) - log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 + 1/6)
^2)*((3^(1/2)*1i)/6 + 1/6)
```

3.37 $\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 97

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}} + \frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] -1/8*ln((1-x)*(1+x)^2)*2^(2/3)+3/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-
1/4*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2174}

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}} + \frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}} - \frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

```
[In] Int[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(1
/3) - Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + (3*Log[-1 + x + 2^(2/3)*(1 - x^3
)^(1/3)])/(4*2^(1/3))
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}} + \frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.53

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)+2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x+2\sqrt[3]{1-x^3}\right)-\log\left(2^{2/3}-2\ 2^{2/3}x+2^{2/3}x^2-\right.}{4\sqrt[3]{2}}$$

```
[In] Integrate[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1 - x^3
)^(1/3))] + 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] - Log[2^(2/3) -
 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3) + 4*(1 - x^3)^(2/
3)])/(4*2^(1/3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.01 (sec) , antiderivative size = 769, normalized size of antiderivative = 7.93

method	result	size
trager	Expression too large to display	769

```
[In] int(1/(1+x)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*ln(-(4*(-x^3+1)^(2/
3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2-10*
RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-2*
RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x-9*(-
x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^
3-4)*x+2*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x+9*(-x^3+1)^(1/3)*RootOf(RootOf(_
Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)-2*(-x^3+1)^(1/3)*RootOf
(_Z^3-4)^2-35*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-7*Roo
tOf(_Z^3-4)*x^2+26*(-x^3+1)^(2/3)-30*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)*x-6*RootOf(_Z^3-4)*x-35*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_
Z^3-4)+4*_Z^2)-7*RootOf(_Z^3-4))/(1+x)^2)+1/4*RootOf(_Z^3-4)*ln((8*(-x^3+1)
^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2
-8*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x
-10*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x+
26*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootO
f(_Z^3-4)*x+4*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x-26*(-x^3+1)^(1/3)*RootOf(Ro
otOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)-4*(-x^3+1)^(1/3)*
RootOf(_Z^3-4)^2+28*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2
+35*RootOf(_Z^3-4)*x^2-36*(-x^3+1)^(2/3)+8*RootOf(RootOf(_Z^3-4)^2+2*_Z*Roo
tOf(_Z^3-4)+4*_Z^2)*x+10*RootOf(_Z^3-4)*x+28*RootOf(RootOf(_Z^3-4)^2+2*_Z*R
ootOf(_Z^3-4)+4*_Z^2)+35*RootOf(_Z^3-4))/(1+x)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(71) = 142.

Time = 1.93 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.10

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$
$$= \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{1}{6}}\left(2^{\frac{5}{6}}(13\,x^6+2\,x^5+19\,x^4-4\,x^3+19\,x^2+2\,x+13)-4\sqrt{2}(5\,x^5-5\,x^4+6\,x^3-6\right.}{6\,(3\,x^6-18\,x^5-3\,x^4-28\,x^3}\right.$$
$$-\frac{1}{24}$$
$$\cdot 2^{\frac{2}{3}}\log\left(\frac{4\cdot 2^{\frac{2}{3}}(-x^3+1)^{\frac{2}{3}}(x^2+1)+2^{\frac{1}{3}}(5\,x^4+6\,x^2+5)-2\,(3\,x^3-x^2+x-3)(-x^3+1)^{\frac{1}{3}}}{x^4+4\,x^3+6\,x^2+4\,x+1}\right)$$
$$+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}(x^2+2\,x+1)-2\cdot 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}(x-1)-4\,(-x^3+1)^{\frac{2}{3}}}{x^2+2\,x+1}\right)$$

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(13*x^6 + 2*x^5 +
19*x^4 - 4*x^3 + 19*x^2 + 2*x + 13) - 4*sqrt(2)*(5*x^5 - 5*x^4 + 6*x^3 - 6*
x^2 + 5*x - 5)*(-x^3 + 1)^(1/3) + 16*2^(1/6)*(x^4 + 2*x^3 + 2*x^2 + 2*x + 1
)*(-x^3 + 1)^(2/3))/(3*x^6 - 18*x^5 - 3*x^4 - 28*x^3 - 3*x^2 - 18*x + 3)) -
 1/24*2^(2/3)*log((4*2^(2/3)*(-x^3 + 1)^(2/3)*(x^2 + 1) + 2^(1/3)*(5*x^4 +
6*x^2 + 5) - 2*(3*x^3 - x^2 + x - 3)*(-x^3 + 1)^(1/3))/(x^4 + 4*x^3 + 6*x^2
 + 4*x + 1)) + 1/12*2^(2/3)*log((2^(2/3)*(x^2 + 2*x + 1) - 2*2^(1/3)*(-x^3
+ 1)^(1/3)*(x - 1) - 4*(-x^3 + 1)^(2/3))/(x^2 + 2*x + 1))
```

Sympy [F]

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)}\,dx$$

```
[In] integrate(1/(1+x)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{(-x^3+1)^{\frac{1}{3}}(x+1)}\,dx$$

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{(-x^3+1)^{\frac{1}{3}}(x+1)}\,dx$$

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="giac")
[Out] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{(1-x^3)^{1/3}\,(x+1)}\,dx$$

```
[In] int(1/((1 - x^3)^(1/3)*(x + 1)),x)
[Out] int(1/((1 - x^3)^(1/3)*(x + 1)), x)
```

3.38 $\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 145

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}} + \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right) - \frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/8*ln((1-x)*(1+x)^2)*2^(2/3)+1/2*ln(x+(-x^3+1)^(1/3))-3/8*ln(-1+x+2^(2/3)*
(-x^3+1)^(1/3))*2^(2/3)-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1
/2)+1/4*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3
)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used

$= \{2177, 245, 2174\}$

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right) - \frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}} + \frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

```
[In] Int[x/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3
)) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[(1 - x)*(1 +
 x)^2]/(4*2^(1/3)) + Log[x + (1 - x^3)^(1/3)]/2 - (3*Log[-1 + x + 2^(2/3)*(
1 - x^3)^(1/3)])/(4*2^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt[3]{1-x^3}}\,dx - \int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

$$= \frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}} + \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right) - \frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [F]

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

```
[In] Integrate[x/((1 + x)*(1 - x^3)^(1/3)),x]
[Out] Integrate[x/((1 + x)*(1 - x^3)^(1/3)), x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 14.00 (sec) , antiderivative size = 1878, normalized size of antiderivative = 12.95

method	result	size
trager	Expression too large to display	1878

```
[In] int(x/(1+x)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
[Out] 1/2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*ln(-(8*RootOf(RootO
f(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)+12*
RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x-4*
RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x+8*(-
x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^
3+4)*x-9*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x-8*(-x^3+1)^(1/3)*RootOf(RootOf(_
Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+9*(-x^3+1)^(1/3)*RootOf
(_Z^3+4)^2+42*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2-14*Ro
otOf(_Z^3+4)*x^2+36*(-x^3+1)^(2/3)+12*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_
Z^3+4)+4*_Z^2)*x-4*RootOf(_Z^3+4)*x+42*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(
_Z^3+4)+4*_Z^2)-14*RootOf(_Z^3+4))/(1+x)^2)-1/4*ln((8*RootOf(RootOf(_Z^3+4)
```

```
^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)-12*RootOf(Ro
otOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x-10*RootOf(R
ootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x+8*(-x^3+1)^(
1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x+1
3*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x-8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^
2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)-13*(-x^3+1)^(1/3)*RootOf(_Z^3+
4)^2+42*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2+35*RootOf(_
Z^3+4)*x^2-52*(-x^3+1)^(2/3)+36*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)
+4*_Z^2)*x+30*RootOf(_Z^3+4)*x+42*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+
4)+4*_Z^2)+35*RootOf(_Z^3+4))/(1+x)^2)*RootOf(_Z^3+4)-1/2*ln((8*RootOf(Root
Of(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)-12
*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x-1
0*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x+8*
(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_
Z^3+4)*x+13*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x-8*(-x^3+1)^(1/3)*RootOf(RootO
f(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)-13*(-x^3+1)^(1/3)*Ro
otOf(_Z^3+4)^2+42*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2+3
5*RootOf(_Z^3+4)*x^2-52*(-x^3+1)^(2/3)+36*RootOf(RootOf(_Z^3+4)^2+2*_Z*Root
Of(_Z^3+4)+4*_Z^2)*x+30*RootOf(_Z^3+4)*x+42*RootOf(RootOf(_Z^3+4)^2+2*_Z*Ro
otOf(_Z^3+4)+4*_Z^2)+35*RootOf(_Z^3+4))/(1+x)^2)*RootOf(RootOf(_Z^3+4)^2+2*
_Z*RootOf(_Z^3+4)+4*_Z^2)-1/3*ln(RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4
)+4*_Z^2)^2*RootOf(_Z^3+4)^4*x^3+6*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3
+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)*x-6*RootOf(RootOf(_Z^3+4)^2+2*_
Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(1/3)*x^2+4*RootOf(RootO
f(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*x^3-2*RootOf(_Z^3+
4)^2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)+4*x^3-4)+1/6*ln(Ro
otOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^4*x^3+6*
RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+
1)^(2/3)*x-6*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^
3+4)^2*(-x^3+1)^(1/3)*x^2+4*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_
Z^2)*RootOf(_Z^3+4)^2*x^3-2*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*_Z*R
ootOf(_Z^3+4)+4*_Z^2)+4*x^3-4)*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*_
Z*RootOf(_Z^3+4)+4*_Z^2)-1/6*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*_Z*
RootOf(_Z^3+4)+4*_Z^2)*ln(RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^
2)^2*RootOf(_Z^3+4)^4*x^3-6*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_
Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)*x+6*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootO
f(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(1/3)*x^2-8*RootOf(RootOf(_Z^3+
4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*x^3+12*x*(-x^3+1)^(2/3)-1
2*x^2*(-x^3+1)^(1/3)+2*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf
(_Z^3+4)+4*_Z^2)+16*x^3-8)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

Sympy [F]

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{x}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)}\,dx$$

```
[In] integrate(x/(1+x)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{x}{(-x^3+1)^{\frac{1}{3}}(x+1)}\,dx$$

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

Giac [F]

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{x}{(-x^3+1)^{\frac{1}{3}}(x+1)}\,dx$$

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{x}{\left(1-x^3\right)^{1/3}\,(x+1)}\,dx$$

```
[In] int(x/((1 - x^3)^(1/3)*(x + 1)),x)
[Out] int(x/((1 - x^3)^(1/3)*(x + 1)), x)
```

3.39 $\int \frac{1}{x \sqrt[3]{2-3x+x^2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 110

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx=-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-x)}{\sqrt{3}\sqrt[3]{2-3x+x^2}}\right)}{2\sqrt[3]{2}}-\frac{\log(2-x)}{4\sqrt[3]{2}}-\frac{\log(x)}{2\sqrt[3]{2}}+\frac{3\log\left(2-x-2^{2/3}\sqrt[3]{2-3x+x^2}\right)}{4\sqrt[3]{2}}$$

```
[Out] -1/8*ln(2-x)*2^(2/3)-1/4*ln(x)*2^(2/3)+3/8*ln(2-x-2^(2/3)*(x^2-3*x+2)^(1/3)
)*2^(2/3)+1/4*arctan(-1/3*3^(1/2)-1/3*2^(1/3)*(2-x)/(x^2-3*x+2)^(1/3)*3^(1/
2))*3^(1/2)*2^(2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.60, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {769, 124}

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx=-\frac{\sqrt{3}\sqrt[3]{x-2}\sqrt[3]{x-1}\arctan\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}(x-2)^{2/3}}{\sqrt{3}\sqrt[3]{x-1}}\right)}{2\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}+\frac{3\sqrt[3]{x-2}\sqrt[3]{x-1}\log\left(-\frac{(x-2)^{2/3}}{\sqrt[3]{2}}-\sqrt[3]{2}\sqrt[3]{x-1}\right)}{4\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}-\frac{\sqrt[3]{x-2}\sqrt[3]{x-1}\log(x)}{2\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}$$

```
[In] Int[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*(-2 + x)^(1/3)*(-1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2^(1/3)*(-2
 + x)^(2/3))/(Sqrt[3]*(-1 + x)^(1/3))])/(2^(1/3)*(2 - 3*x + x^2)^(1/3)) + (
3*(-2 + x)^(1/3)*(-1 + x)^(1/3)*Log[-((-2 + x)^(2/3)/2^(1/3)) - 2^(1/3)*(-1
 + x)^(1/3)])/(4*2^(1/3)*(2 - 3*x + x^2)^(1/3)) - ((-2 + x)^(1/3)*(-1 + x)^
(1/3)*Log[x])/(2*2^(1/3)*(2 - 3*x + x^2)^(1/3))
```

Rule 124

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)*((e_.) + (f_.)*(x_))
^(1/3)), x_Symbol] :> With[{q = Rt[b*((b*e - a*f)/(b*c - a*d)^2), 3]}, Simp
[-Log[a + b*x]/(2*q*(b*c - a*d)), x] + (-Simp[Sqrt[3]*(ArcTan[1/Sqrt[3] + 2
*q*((c + d*x)^(2/3)/(Sqrt[3]*(e + f*x)^(1/3)))]/(2*q*(b*c - a*d))), x] + Si
mp[3*(Log[q*(c + d*x)^(2/3) - (e + f*x)^(1/3)]/(4*q*(b*c - a*d))), x])] /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - b*c*f - a*d*f, 0]
```

Rule 769

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(b + q + 2*c*x)^(1/3)*((b - q
+ 2*c*x)^(1/3)/(a + b*x + c*x^2)^(1/3)), Int[1/((d + e*x)*(b + q + 2*c*x)^(
1/3)*(b - q + 2*c*x)^(1/3)), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^
2 - 4*a*c, 0] && EqQ[c^2*d^2 - b*c*d*e - 2*b^2*e^2 + 9*a*c*e^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt[3]{-4+2x}\sqrt[3]{-2+2x}\right)\int \frac{1}{x\sqrt[3]{-4+2x}\sqrt[3]{-2+2x}}\,dx}{\sqrt[3]{2-3x+x^2}}$$

$$= -\frac{\sqrt{3}\sqrt[3]{-2+x}\sqrt[3]{-1+x}\arctan\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}(-2+x)^{2/3}}{\sqrt{3}\sqrt[3]{-1+x}}\right)}{2\sqrt[3]{2}\sqrt[3]{2-3x+x^2}} + \frac{3\sqrt[3]{-2+x}\sqrt[3]{-1+x}\log\left(-\frac{(-2+x)^{2/3}}{\sqrt[3]{2}}-\sqrt[3]{2}\sqrt[3]{-1+x}\right)}{4\sqrt[3]{2}\sqrt[3]{2-3x+x^2}} - \frac{\sqrt[3]{-2+x}\sqrt[3]{-1+x}\log(x)}{2\sqrt[3]{2}\sqrt[3]{2-3x+x^2}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.47

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{2-3x+x^2}}{2\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{2-3x+x^2}}\right)+2\log\left(-2\sqrt[3]{2}+\sqrt[3]{2}x+2\sqrt[3]{2-3x+x^2}\right)-\log\left(4\,2^{2/3}-4\,2^{2/}\right.}{4\sqrt[3]{2}}$$

```
[In] Integrate[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*(2 - 3*x + x^2)^(1/3))/(2*2^(1/3) - 2^(1/3)*x +
(2 - 3*x + x^2)^(1/3))] + 2*Log[-2*2^(1/3) + 2^(1/3)*x + 2*(2 - 3*x + x^2)^
(1/3)] - Log[4*2^(2/3) - 4*2^(2/3)*x + 2^(2/3)*x^2 - 2*2^(1/3)*(-2 + x)*(2
- 3*x + x^2)^(1/3) + 4*(2 - 3*x + x^2)^(2/3)])/(4*2^(1/3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.43 (sec) , antiderivative size = 1593, normalized size of antiderivative = 14.48

method	result	size
trager	Expression too large to display	1593

```
[In] int(1/x/(x^2-3*x+2)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*RootOf(_Z^3-4)*ln(-(-12*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_
Z^2)*RootOf(_Z^3-4)^3*x^2+112*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4
*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+216*(x^2-3*x+2)^(2/3)*RootOf(RootOf(_Z^3-4)^2
+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2+54*RootOf(RootOf(_Z^3-4)^2+2*
_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x-504*RootOf(RootOf(_Z^3-4)^2+2*
_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-237*(x^2-3*x+2)^(1/3)*RootOf
(_Z^3-4)^2*x-258*(x^2-3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3
-4)+4*_Z^2)*RootOf(_Z^3-4)*x-54*RootOf(_Z^3-4)^3*RootOf(RootOf(_Z^3-4)^2+2*
_Z*RootOf(_Z^3-4)+4*_Z^2)+504*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4
*_Z^2)^2*RootOf(_Z^3-4)^2+474*(x^2-3*x+2)^(1/3)*RootOf(_Z^3-4)^2+516*(x^2-3
*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3
-4)-3*RootOf(_Z^3-4)*x^2+28*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_
Z^2)*x^2-516*(x^2-3*x+2)^(2/3)-72*RootOf(_Z^3-4)*x+672*RootOf(RootOf(_Z^3-4
)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+72*RootOf(_Z^3-4)-672*RootOf(RootOf(_Z^3-
4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/x^2)-1/4*ln((12*RootOf(RootOf(_Z^3-4)^2+2
*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x^2+136*RootOf(RootOf(_Z^3-4)^2
+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+216*(x^2-3*x+2)^(2/3)*R
ootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2-54*Root
```

```
Of(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x-612*Root
Of(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x+129*(x
^2-3*x+2)^(1/3)*RootOf(_Z^3-4)^2*x+474*(x^2-3*x+2)^(1/3)*RootOf(RootOf(_Z^3
-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x+54*RootOf(_Z^3-4)^3*Root
Of(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+612*RootOf(RootOf(_Z^3-4)^2
+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2-258*(x^2-3*x+2)^(1/3)*RootO
f(_Z^3-4)^2-948*(x^2-3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-
4)+4*_Z^2)*RootOf(_Z^3-4)+21*RootOf(_Z^3-4)*x^2+238*RootOf(RootOf(_Z^3-4)^2
+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2+948*(x^2-3*x+2)^(2/3)-180*RootOf(_Z^3-4)*x
-2040*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+180*RootOf(_Z^3
-4)+2040*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/x^2)*RootOf(_
Z^3-4)-1/2*ln((12*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootO
f(_Z^3-4)^3*x^2+136*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*R
ootOf(_Z^3-4)^2*x^2+216*(x^2-3*x+2)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*Root
Of(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2-54*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(
_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x-612*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(
_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x+129*(x^2-3*x+2)^(1/3)*RootOf(_Z^3-4)^2
*x+474*(x^2-3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2
)*RootOf(_Z^3-4)*x+54*RootOf(_Z^3-4)^3*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(
_Z^3-4)+4*_Z^2)+612*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*R
ootOf(_Z^3-4)^2-258*(x^2-3*x+2)^(1/3)*RootOf(_Z^3-4)^2-948*(x^2-3*x+2)^(1/3
)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)+21*Roo
tOf(_Z^3-4)*x^2+238*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2
+948*(x^2-3*x+2)^(2/3)-180*RootOf(_Z^3-4)*x-2040*RootOf(RootOf(_Z^3-4)^2+2*
_Z*RootOf(_Z^3-4)+4*_Z^2)*x+180*RootOf(_Z^3-4)+2040*RootOf(RootOf(_Z^3-4)^2
+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/x^2)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3
-4)+4*_Z^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(81) = 162$.

Time = 1.23 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.52

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx =$$

$$-\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{1}{6}}\left(2^{\frac{5}{6}}(x^6+36\,x^5-612\,x^4+2880\,x^3-5760\,x^2+5184\,x-1728)+12\,\sqrt{2}(x^5-38\right.}{6\,(x^6-108\,x^5+972\,x^4-}\right.$$

$$+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}x^2+6\cdot 2^{\frac{1}{3}}(x^2-3\,x+2)^{\frac{1}{3}}(x-2)+12\,(x^2-3\,x+2)^{\frac{2}{3}}}{x^2}\right)-\frac{1}{24}$$

$$\cdot 2^{\frac{2}{3}}\log\left(\frac{12\cdot 2^{\frac{2}{3}}(x^2-3\,x+2)^{\frac{2}{3}}(x^2-6\,x+6)+2^{\frac{1}{3}}(x^4-36\,x^3+180\,x^2-288\,x+144)-6\,(x^3-14\,x^2+3}{x^4}\right.$$

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^6 + 36*x^5 - 6
12*x^4 + 2880*x^3 - 5760*x^2 + 5184*x - 1728) + 12*sqrt(2)*(x^5 - 38*x^4 +
252*x^3 - 648*x^2 + 720*x - 288)*(x^2 - 3*x + 2)^(1/3) + 48*2^(1/6)*(x^4 -
6*x^3 + 6*x^2)*(x^2 - 3*x + 2)^(2/3))/(x^6 - 108*x^5 + 972*x^4 - 3456*x^3 +
 6048*x^2 - 5184*x + 1728)) + 1/12*2^(2/3)*log((2^(2/3)*x^2 + 6*2^(1/3)*(x^
2 - 3*x + 2)^(1/3)*(x - 2) + 12*(x^2 - 3*x + 2)^(2/3))/x^2) - 1/24*2^(2/3)*
log((12*2^(2/3)*(x^2 - 3*x + 2)^(2/3)*(x^2 - 6*x + 6) + 2^(1/3)*(x^4 - 36*x
^3 + 180*x^2 - 288*x + 144) - 6*(x^3 - 14*x^2 + 36*x - 24)*(x^2 - 3*x + 2)^
(1/3))/x^4)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx = \int \frac{1}{x\sqrt[3]{(x-2)\,(x-1)}}\,dx$$

```
[In] integrate(1/x/(x**2-3*x+2)**(1/3),x)
```

```
[Out] Integral(1/(x*((x - 2)*(x - 1))**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx = \int \frac{1}{(x^2-3\,x+2)^{\frac{1}{3}}x}\,dx$$

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx = \int \frac{1}{(x^2-3\,x+2)^{\frac{1}{3}}x}\,dx$$

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx = \int \frac{1}{x\,(x^2-3\,x+2)^{1/3}}\,dx$$

```
[In] int(1/(x*(x^2 - 3*x + 2)^(1/3)),x)
[Out] int(1/(x*(x^2 - 3*x + 2)^(1/3)), x)
```

3.40 $\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2(-1+x)}{\sqrt{3}\sqrt[3]{-5+7x-3x^2+x^3}}\right) + \frac{1}{4}\log(1-x) - \frac{3}{4}\log\left(1-x+\sqrt[3]{-5+7x-3x^2+x^3}\right)$$

```
[Out] 1/4*ln(1-x)-3/4*ln(1-x+(x^3-3*x^2+7*x-5)^(1/3))+1/2*arctan(1/3*3^(1/2)+2/3*
(-1+x)/(x^3-3*x^2+7*x-5)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.62, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2092, 2036, 335, 281, 245}

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx = \frac{\sqrt{3}\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}\arctan\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{(x-1)^2+4}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{(x-1)^3+4(x-1)}} - \frac{3\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}\log\left((x-1)^{2/3}-\sqrt[3]{(x-1)^2+4}\right)}{4\sqrt[3]{(x-1)^3+4(x-1)}}$$

```
[In] Int[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]
```

```
[Out] (Sqrt[3]*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*ArcTan[(1 + (2*(-1 + x)^(2/3
))/(4 + (-1 + x)^2)^(1/3))/Sqrt[3]])/(2*(4*(-1 + x) + (-1 + x)^3)^(1/3)) -
```

```
(3*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*Log[-(4 + (-1 + x)^2)^(1/3) + (-1
+ x)^(2/3)])/(4*(4*(-1 + x) + (-1 + x)^3)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\sqrt[3]{4x+x^3}}\,dx, x, -1+x\right)$$

$$= \frac{\left(\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\sqrt[3]{4+x^2}}\,dx, x, -1+x\right)}{\sqrt[3]{4(-1+x)+(-1+x)^3}}$$

$$= \frac{\left(3\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{4+x^6}}\,dx, x, \sqrt[3]{-1+x}\right)}{\sqrt[3]{4(-1+x)+(-1+x)^3}}$$

$$= \frac{\left(3\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{4+x^3}}\,dx, x, (-1+x)^{2/3}\right)}{2\sqrt[3]{4(-1+x)+(-1+x)^3}}$$

$$= \frac{\sqrt{3}\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\arctan\left(\frac{1+\frac{2(-1+x)^{2/3}}{\sqrt[3]{4+(-1+x)^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{-4(1-x)+(-1+x)^3}} - \frac{3\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\log\left(\sqrt[3]{4+(-1+x)^2}-(-1+x)^{2/3}\right)}{4\sqrt[3]{-4(1-x)+(-1+x)^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx = \frac{3\sqrt[3]{(2-i)+ix}\sqrt[3]{i(-1+x)}((-1+2i)+x)\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{1}{4}i((-1+2i)+x),-\frac{1}{2}i((-1+2i)+x\right.}{4\sqrt[3]{-5+7x-3x^2+x^3}}$$

```
[In] Integrate[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]
```

```
[Out] (3*((2 - I) + I*x)^(1/3)*(I*(-1 + x))^(1/3)*((-1 + 2*I) + x)*AppellF1[2/3,
1/3, 1/3, 5/3, (-1/4*I)*((-1 + 2*I) + x), (-1/2*I)*((-1 + 2*I) + x)])/(4*(-
5 + 7*x - 3*x^2 + x^3)^(1/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.95 (sec) , antiderivative size = 653, normalized size of antiderivative = 8.06

method	result
trager	$\frac{\text{RootOf}\left(\_Z^2-\_Z+1\right)\ln\left(-304\,\text{RootOf}\left(\_Z^2-\_Z+1\right)^2 x^2+624\,\text{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3-3x^2+7x-5\right)^{\frac{2}{3}}+624\,\text{RootOf}\left(\_\right.\right.}{}$

```
[In] int(1/(x^3-3*x^2+7*x-5)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*RootOf(_Z^2-_Z+1)*ln(-304*RootOf(_Z^2-_Z+1)^2*x^2+624*RootOf(_Z^2-_Z+1)
*(x^3-3*x^2+7*x-5)^(2/3)+624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)*x+60
8*RootOf(_Z^2-_Z+1)^2*x+928*RootOf(_Z^2-_Z+1)*x^2+51*(x^3-3*x^2+7*x-5)^(2/3
)-624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)+51*(x^3-3*x^2+7*x-5)^(1/3)*
```

```
x-1856*RootOf(_Z^2-_Z+1)*x-253*x^2-51*(x^3-3*x^2+7*x-5)^(1/3)+2356*RootOf(_
Z^2-_Z+1)+506*x-713)-1/2*ln(-304*RootOf(_Z^2-_Z+1)^2*x^2-624*RootOf(_Z^2-_Z
+1)*(x^3-3*x^2+7*x-5)^(2/3)-624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)*x
+608*RootOf(_Z^2-_Z+1)^2*x-320*RootOf(_Z^2-_Z+1)*x^2+675*(x^3-3*x^2+7*x-5)^
(2/3)+624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)+675*(x^3-3*x^2+7*x-5)^(
1/3)*x+640*RootOf(_Z^2-_Z+1)*x+371*x^2-675*(x^3-3*x^2+7*x-5)^(1/3)-2356*Roo
tOf(_Z^2-_Z+1)-742*x+1643)*RootOf(_Z^2-_Z+1)+1/2*ln(-304*RootOf(_Z^2-_Z+1)^
2*x^2-624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(2/3)-624*RootOf(_Z^2-_Z+1)*(
x^3-3*x^2+7*x-5)^(1/3)*x+608*RootOf(_Z^2-_Z+1)^2*x-320*RootOf(_Z^2-_Z+1)*x^
2+675*(x^3-3*x^2+7*x-5)^(2/3)+624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)
+675*(x^3-3*x^2+7*x-5)^(1/3)*x+640*RootOf(_Z^2-_Z+1)*x+371*x^2-675*(x^3-3*x
^2+7*x-5)^(1/3)-2356*RootOf(_Z^2-_Z+1)-742*x+1643)
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx = -\frac{1}{2}\sqrt{3}\arctan\left(\frac{22791076\sqrt{3}(x^3-3\,x^2+7\,x-5)^{\frac{1}{3}}(x-1)+\sqrt{3}(20389537\,x^2-40779074\,x+53222437)}{7204617\,x^2-14409234\,x-20666867}\right. - \frac{1}{4}\log\left(3\left(x^3-3\,x^2+7\,x-5\right)^{\frac{1}{3}}(x-1)-3\left(x^3-3\,x^2+7\,x-5\right)^{\frac{2}{3}}+4\right)$$

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan((22791076*sqrt(3)*(x^3 - 3*x^2 + 7*x - 5)^(1/3)*(x - 1)
 + sqrt(3)*(20389537*x^2 - 40779074*x + 53222437) + 17987998*sqrt(3)*(x^3 -
 3*x^2 + 7*x - 5)^(2/3))/(7204617*x^2 - 14409234*x - 20666867)) - 1/4*log(3
*(x^3 - 3*x^2 + 7*x - 5)^(1/3)*(x - 1) - 3*(x^3 - 3*x^2 + 7*x - 5)^(2/3) +
4)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx = \int \frac{1}{\sqrt[3]{x^3-3x^2+7x-5}}\,dx$$

```
[In] integrate(1/(x**3-3*x**2+7*x-5)**(1/3),x)
```

```
[Out] Integral((x**3 - 3*x**2 + 7*x - 5)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx=\int \frac{1}{\left(x^3-3\,x^2+7\,x-5\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="maxima")
[Out] integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx=\int \frac{1}{\left(x^3-3\,x^2+7\,x-5\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="giac")
[Out] integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx=\int \frac{1}{\left(x^3-3\,x^2+7\,x-5\right)^{1/3}}\,dx$$

```
[In] int(1/(7*x - 3*x^2 + x^3 - 5)^(1/3),x)
[Out] int(1/(7*x - 3*x^2 + x^3 - 5)^(1/3), x)
```

3.41 $\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx=\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2x}{\sqrt{3}\sqrt[3]{x\left(-q+x^2\right)}}\right)+\frac{\log(x)}{4}-\frac{3}{4}\log\left(-x+\sqrt[3]{x\left(-q+x^2\right)}\right)$$

```
[Out] 1/4*ln(x)-3/4*ln(-x+(x*(x^2-q))^(1/3))+1/2*arctan(1/3*3^(1/2)+2/3*x/(x*(x^2
-q))^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.77, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2004, 2036, 335, 281, 245}

$$\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx=\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{x^2-q}\arctan\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-q}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{x^3-qx}}-\frac{3\sqrt[3]{x}\sqrt[3]{x^2-q}\log\left(x^{2/3}-\sqrt[3]{x^2-q}\right)}{4\sqrt[3]{x^3-qx}}$$

```
[In] Int[(x*(-q + x^2))^(-1/3),x]
```

```
[Out] (Sqrt[3]*x^(1/3)*(-q + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3))/(-q + x^2)^(1/3))
/Sqrt[3]])/(2*(-(q*x) + x^3)^(1/3)) - (3*x^(1/3)*(-q + x^2)^(1/3)*Log[x^(2/
3) - (-q + x^2)^(1/3)])/(4*(-(q*x) + x^3)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt[3]{-qx+x^3}}\,dx$$

$$= \frac{\left(\sqrt[3]{x}\sqrt[3]{-q+x^2}\right)\int \frac{1}{\sqrt[3]{x}\sqrt[3]{-q+x^2}}\,dx}{\sqrt[3]{-qx+x^3}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-q+x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-q+x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-qx+x^3}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-q+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-q+x^3}}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{-qx+x^3}}$$

$$= \frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{-q+x^2}\arctan\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-q+x^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{-qx+x^3}} - \frac{3\sqrt[3]{x}\sqrt[3]{-q+x^2}\log\left(x^{2/3}-\sqrt[3]{-q+x^2}\right)}{4\sqrt[3]{-qx+x^3}}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.97

$$\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx$$
$$= \frac{\sqrt[3]{x}\sqrt[3]{-q+x^2}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}x^{2/3}}{x^{2/3}+2\sqrt[3]{-q+x^2}}\right)-2\log\left(-x^{2/3}+\sqrt[3]{-q+x^2}\right)+\log\left(x^{4/3}+x^{2/3}\sqrt[3]{-q+x\right.\right.}{4\sqrt[3]{-qx+x^3}}$$

```
[In] Integrate[(x*(-q + x^2))^(-1/3),x]
```

```
[Out] (x^(1/3)*(-q + x^2)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*x^(2/3))/(x^(2/3) + 2*
(-q + x^2)^(1/3))] - 2*Log[-x^(2/3) + (-q + x^2)^(1/3)] + Log[x^(4/3) + x^(
2/3)*(-q + x^2)^(1/3) + (-q + x^2)^(2/3)]))/(4*(-(q*x) + x^3)^(1/3))
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

method	result	si
pseudoelliptic	$\frac{\ln\left(\frac{\left(-x\left(-x^2+q\right)\right)^{\frac{2}{3}}+\left(-x\left(-x^2+q\right)\right)^{\frac{1}{3}}x+x^2}{x^2}\right)}{4}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(x+2\left(-x\left(-x^2+q\right)\right)^{\frac{1}{3}}\right)\sqrt{3}}{3x}\right)}{2}-\frac{\ln\left(\frac{\left(-x\left(-x^2+q\right)\right)^{\frac{1}{3}}-x}{x}\right)}{2}$	9

```
[In] int(1/(x*(x^2-q))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(((-x*(-x^2+q))^(2/3)+(-x*(-x^2+q))^(1/3)*x+x^2)/x^2)-1/2*3^(1/2)*arc
tan(1/3*(x+2*(-x*(-x^2+q))^(1/3))*3^(1/2)/x)-1/2*ln(((-x*(-x^2+q))^(1/3)-x)
/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(52) = 104.

Time = 0.90 (sec) , antiderivative size = 415, normalized size of antiderivative = 6.29

$$\int \frac{1}{\sqrt[3]{x(-q+x^2)}}\,dx$$
$$=\frac{1}{2}\sqrt{3}\arctan\left(\frac{4\sqrt{3}(q^{12}-15q^{10}+90q^8-351q^6+810q^4-1215q^2+729)(x^3-qx)^{\frac{1}{3}}x-2\sqrt{3}(q^{12}+6q^{1}}{}\right.$$
$$-\frac{1}{4}\log\left(-3\left(x^3-qx\right)^{\frac{1}{3}}x+q+3\left(x^3-qx\right)^{\frac{2}{3}}\right)$$

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan((4*sqrt(3)*(q^12 - 15*q^10 + 90*q^8 - 351*q^6 + 810*q^4
- 1215*q^2 + 729)*(x^3 - q*x)^(1/3)*x - 2*sqrt(3)*(q^12 + 6*q^11 - 15*q^10
- 54*q^9 + 90*q^8 + 270*q^7 - 351*q^6 - 810*q^5 + 810*q^4 + 1458*q^3 - 1215
*q^2 - 1458*q + 729)*(x^3 - q*x)^(2/3) - sqrt(3)*(q^13 + 10*q^12 - 15*q^11
- 282*q^10 + 90*q^9 + 2178*q^8 - 351*q^7 - 6534*q^6 + 810*q^5 + 7614*q^4 -
1215*q^3 - (q^12 - 6*q^11 - 15*q^10 + 54*q^9 + 90*q^8 - 270*q^7 - 351*q^6 +
 810*q^5 + 810*q^4 - 1458*q^3 - 1215*q^2 + 1458*q + 729)*x^2 - 2430*q^2 + 7
29*q))/(q^13 + 18*q^12 + 81*q^11 - 162*q^10 - 1350*q^9 + 810*q^8 + 6561*q^7
 - 2430*q^6 - 12150*q^5 + 4374*q^4 + 6561*q^3 - 9*(q^12 + 2*q^11 - 15*q^10
- 18*q^9 + 90*q^8 + 90*q^7 - 351*q^6 - 270*q^5 + 810*q^4 + 486*q^3 - 1215*q
^2 - 486*q + 729)*x^2 - 4374*q^2 + 729*q)) - 1/4*log(-3*(x^3 - q*x)^(1/3)*x
 + q + 3*(x^3 - q*x)^(2/3))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{x(-q+x^2)}}\,dx=\int \frac{1}{\sqrt[3]{x(-q+x^2)}}\,dx$$

```
[In] integrate(1/(x*(x**2-q))**(1/3),x)
```

```
[Out] Integral((x*(-q + x**2))**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{x\left(-q+x^{2}\right)}}\,dx = \int \frac{1}{\left((x^{2}-q)x\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x^2 - q)*x)^(-1/3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt[3]{x\left(-q+x^{2}\right)}}\,dx = -\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{q}{x^{2}}+1\right)^{\frac{1}{3}}+1\right)\right) + \frac{1}{4}\log\left(\left(-\frac{q}{x^{2}}+1\right)^{\frac{2}{3}}+\left(-\frac{q}{x^{2}}+1\right)^{\frac{1}{3}}+1\right) - \frac{1}{2}\log\left(\left|\left(-\frac{q}{x^{2}}+1\right)^{\frac{1}{3}}-1\right|\right)$$

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-q/x^2 + 1)^(1/3) + 1)) + 1/4*log((-q/x
^2 + 1)^(2/3) + (-q/x^2 + 1)^(1/3) + 1) - 1/2*log(abs((-q/x^2 + 1)^(1/3) -
1))
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt[3]{x\left(-q+x^{2}\right)}}\,dx = \frac{3\,x\,{\left(1-\frac{x^{2}}{q}\right)}^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ \frac{x^{2}}{q}\right)}{2\,{\left(x^{3}-q\,x\right)}^{1/3}}$$

```
[In] int(1/(-x*(q - x^2))^(1/3),x)
```

```
[Out] (3*x*(1 - x^2/q)^(1/3)*hypergeom([1/3, 1/3], 4/3, x^2/q))/(2*(x^3 - q*x)^(1
/3))
```

3.42 $\int \frac{1}{\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \frac{1}{\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\,dx=\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2(-1+x)}{\sqrt{3}\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\right)+\frac{1}{4}\log(1-x)-\frac{3}{4}\log\left(1-x+\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}\right)$$

```
[Out] 1/4*ln(1-x)-3/4*ln(1-x+((-1+x)*(x^2+q-2*x))^(1/3))+1/2*arctan(1/3*3^(1/2)+2
/3*(-1+x)/((-1+x)*(x^2+q-2*x))^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.84, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2092, 2036, 335, 281, 245}

$$\int \frac{1}{\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\,dx=\frac{\sqrt{3}\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1}\arctan\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{q+(x-1)^2-1}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{(x-1)^3-(1-q)(x-1)}}-\frac{3\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1}\log\left((x-1)^{2/3}-\sqrt[3]{q+(x-1)^2-1}\right)}{4\sqrt[3]{(x-1)^3-(1-q)(x-1)}}$$

```
[In] Int[((-1 + x)*(q - 2*x + x^2))^(-1/3),x]
```

```
[Out] (Sqrt[3]*(-1 + q + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*ArcTan[(1 + (2*(-1 + x)
^(2/3))/(-1 + q + (-1 + x)^2)^(1/3))/Sqrt[3]])/(2*(-((1 - q)*(-1 + x)) + (-
1 + x)^3)^(1/3)) - (3*(-1 + q + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*Log[-(-1 +
 q + (-1 + x)^2)^(1/3) + (-1 + x)^(2/3)])/(4*(-((1 - q)*(-1 + x)) + (-1 + x
)^3)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\sqrt[3]{-((1-q)x)+x^3}}\,dx, x, -1+x\right)$$

$$= \frac{\left(\sqrt[3]{-1+q+(-1+x)^2}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\sqrt[3]{-1+q+x^2}}\,dx, x, -1+x\right)}{\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}$$

$$= \frac{\left(3\sqrt[3]{-1+q+(-1+x)^2}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-1+q+x^6}}\,dx, x, \sqrt[3]{-1+x}\right)}{\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}$$

$$= \frac{\left(3\sqrt[3]{-1+q+(-1+x)^2}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+q+x^3}}\,dx, x, (-1+x)^{2/3}\right)}{2\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}$$

$$= \frac{\sqrt{3}\sqrt[3]{-1+q+(-1+x)^2}\sqrt[3]{-1+x}\arctan\left(\frac{1+\frac{2(-1+x)^{2/3}}{\sqrt[3]{q-(2-x)x}}}{\sqrt{3}}\right)}{2\sqrt[3]{(1-q)(1-x)+(-1+x)^3}} - \frac{3\sqrt[3]{-1+q+(-1+x)^2}\sqrt[3]{-1+x}\log\left((-1+x)^{2/3}-\sqrt[3]{q-(2-x)x}\right)}{4\sqrt[3]{(1-q)(1-x)+(-1+x)^3}}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sqrt[3]{(-1+x)\,(q-2x+x^2)}}\,dx = \frac{\sqrt[3]{-1+x}\sqrt[3]{q+(-2+x)x}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}(-1+x)^{2/3}}{(-1+x)^{2/3}+2\sqrt[3]{q+(-2+x)x}}\right)-2\log\left(-(-1+x)^{2/3}+\sqrt[3]{q+(\cdots}\right.\right.}{4\sqrt[3]{(-1+x)(q+(-2+\cdots}}$$

```
[In] Integrate[((-1 + x)*(q - 2*x + x^2))^(-1/3),x]
```

```
[Out] ((-1 + x)^(1/3)*(q + (-2 + x)*x)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*(-1 + x)^
(2/3))/((-1 + x)^(2/3) + 2*(q + (-2 + x)*x)^(1/3))] - 2*Log[-(-1 + x)^(2/3)
 + (q + (-2 + x)*x)^(1/3)] + Log[(-1 + x)^(4/3) + (-1 + x)^(2/3)*(q + (-2 +
 x)*x)^(1/3) + (q + (-2 + x)*x)^(2/3)]))/(4*((-1 + x)*(q + (-2 + x)*x))^(1/
3))
```

Maple [F]

$$\int \frac{1}{\left((-1+x)\,(x^2+q-2x)\right)^{\frac{1}{3}}}dx$$

```
[In] int(1/((-1+x)*(x^2+q-2*x))^(1/3),x)
```

```
[Out] int(1/((-1+x)*(x^2+q-2*x))^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(65) = 130$.

Time = 0.78 (sec) , antiderivative size = 665, normalized size of antiderivative = 8.42

$$\int \frac{1}{\sqrt[3]{(-1+x)(q-2x+x^2)}}\,dx$$
$$= \frac{1}{2}\sqrt{3}\arctan\left(\frac{2\sqrt{3}(q^{12}-18q^{11}+117q^{10}-346q^9+414q^8-18q^7+69q^6-774q^5-234q^4+1058q^3+6\cdots}{\cdots}\right.$$
$$-\frac{1}{4}\log\left(3\left(x^3+(q+2)x-3x^2-q\right)^{\frac{1}{3}}(x-1)+q-3\left(x^3+(q+2)x-3x^2-q\right)^{\frac{2}{3}}-1\right)$$

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan((2*sqrt(3)*(q^12 - 18*q^11 + 117*q^10 - 346*q^9 + 414*q^
8 - 18*q^7 + 69*q^6 - 774*q^5 - 234*q^4 + 1058*q^3 + 621*q^2 + 378*q - 539)
*(x^3 + (q + 2)*x - 3*x^2 - q)^(2/3) + 4*sqrt(3)*(q^12 - 12*q^11 + 51*q^10
- 70*q^9 - 90*q^8 + 288*q^7 - 57*q^6 + 54*q^5 - 810*q^4 + 320*q^3 + 291*q^2
 - (q^12 - 12*q^11 + 51*q^10 - 70*q^9 - 90*q^8 + 288*q^7 - 57*q^6 + 54*q^5
- 810*q^4 + 320*q^3 + 291*q^2 + 714*q + 49)*x + 714*q + 49)*(x^3 + (q + 2)*
x - 3*x^2 - q)^(1/3) - sqrt(3)*(q^13 - 22*q^12 + 177*q^11 - 514*q^10 - 434*
q^9 + 5346*q^8 - 8247*q^7 - 4542*q^6 + 19638*q^5 - 8050*q^4 - 10343*q^3 + (
q^12 - 6*q^11 - 15*q^10 + 206*q^9 - 594*q^8 + 594*q^7 - 183*q^6 + 882*q^5 -
 1386*q^4 - 418*q^3 - 39*q^2 + 1050*q + 637)*x^2 + 6186*q^2 - 2*(q^12 - 6*q
^11 - 15*q^10 + 206*q^9 - 594*q^8 + 594*q^7 - 183*q^6 + 882*q^5 - 1386*q^4
- 418*q^3 - 39*q^2 + 1050*q + 637)*x + 1501*q + 32))/(q^13 - 22*q^12 + 249*
q^11 - 1546*q^10 + 4702*q^9 - 4230*q^8 - 10623*q^7 + 25338*q^6 - 3546*q^5 -
 31306*q^4 + 18817*q^3 + 9*(q^12 - 14*q^11 + 73*q^10 - 162*q^9 + 78*q^8 + 1
86*q^7 - 15*q^6 - 222*q^5 - 618*q^4 + 566*q^3 + 401*q^2 + 602*q - 147)*x^2
+ 9714*q^2 - 18*(q^12 - 14*q^11 + 73*q^10 - 162*q^9 + 78*q^8 + 186*q^7 - 15
*q^6 - 222*q^5 - 618*q^4 + 566*q^3 + 401*q^2 + 602*q - 147)*x - 995*q + 8))
 - 1/4*log(3*(x^3 + (q + 2)*x - 3*x^2 - q)^(1/3)*(x - 1) + q - 3*(x^3 + (q
+ 2)*x - 3*x^2 - q)^(2/3) - 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)(q-2x+x^2)}}\,dx = \int \frac{1}{\sqrt[3]{(x-1)(q+x^2-2x)}}\,dx$$

```
[In] integrate(1/((-1+x)*(x**2+q-2*x))**(1/3),x)
```

```
[Out] Integral(((x - 1)*(q + x**2 - 2*x))**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)\,(q-2x+x^2)}}\,dx = \int \frac{1}{\left((x^2+q-2\,x)(x-1)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="maxima")
[Out] integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)\,(q-2x+x^2)}}\,dx = \int \frac{1}{\left((x^2+q-2\,x)(x-1)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="giac")
[Out] integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{(-1+x)\,(q-2x+x^2)}}\,dx = \int \frac{1}{\left((x-1)\,\left(x^2-2\,x+q\right)\right)^{1/3}}\,dx$$

```
[In] int(1/((x - 1)*(q - 2*x + x^2))^(1/3),x)
[Out] int(1/((x - 1)*(q - 2*x + x^2))^(1/3), x)
```

3.43 $\int \frac{1}{x\sqrt[3]{(-1+x)(q-2qx+x^2)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 118

$$\int \frac{1}{x\sqrt[3]{(-1+x)(q-2qx+x^2)}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{q}(-1+x)}{\sqrt{3}\sqrt[3]{(-1+x)(q-2qx+x^2)}}\right)}{2\sqrt[3]{q}} + \frac{\log(1-x)}{4\sqrt[3]{q}} + \frac{\log(x)}{2\sqrt[3]{q}} - \frac{3\log\left(-\sqrt[3]{q}(-1+x)+\sqrt[3]{(-1+x)(q-2qx+x^2)}\right)}{4\sqrt[3]{q}}$$

```
[Out] 1/4*ln(1-x)/q^(1/3)+1/2*ln(x)/q^(1/3)-3/4*ln(-q^(1/3)*(-1+x)+((-1+x)*(-2*q*
x+x^2+q))^(1/3))/q^(1/3)+1/2*arctan(1/3*3^(1/2)+2/3*q^(1/3)*(-1+x)/((-1+x)*
(-2*q*x+x^2+q))^(1/3)*3^(1/2))*3^(1/2)/q^(1/3)
```

Rubi [F]

$$\int \frac{1}{x\sqrt[3]{(-1+x)(q-2qx+x^2)}}\,dx = \int \frac{1}{x\sqrt[3]{(-1+x)(q-2qx+x^2)}}\,dx$$

```
[In] Int[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

```
[Out] ((-1 - 2*q - (1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[
-((-1 + q)^3*q)])^(2/3))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[-((-1 +
 q)^3*q)])^(1/3) + 3*x)^(1/3)*(-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(
1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q -
15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (3*(1 - 5*q + 4*q^2 +
```

```
(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*((-1 - 2*q
)/3 + x))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3) +
9*((-1 - 2*q)/3 + x)^2)^(1/3)*Defer[Subst][Defer[Int][1/(((1 + 2*q)/3 + x)*
(-1/3*(1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)
^3*q])^(2/3))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3
) + x)^(1/3)*((-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(1 + 6*q - 15*q^2
 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q - 15*q^2 + 8*q^3 +
 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))/9 + ((1 - 5*q + 4*q^2 + (1 + 6*q - 15*
q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*x)/(3*(1 + 6*q - 15*q^2 +
 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3)) + x^2)^(1/3)), x], x, (-1 - 2*
q)/3 + x])/(3*(-q + 3*q*x + (-1 - 2*q)*x^2 + x^3)^(1/3))
```

Rubi steps

integral

$$= \text{Subst}\left(\int \frac{1}{\left(\frac{1}{3}(1+2q)+x\right)\sqrt[3]{-\frac{2}{27}(1-q)^2(1+8q)-\frac{1}{3}(1-4q)(1-q)x+x^3}}\,dx, x, \frac{1}{3}(-1-2q)+x\right)$$

$$= \frac{\left(\sqrt[3]{-1-2q-\frac{1-5q+4q^2+\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q}\right)^{2/3}}{\sqrt[3]{1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q}}}+3x}\sqrt[3]{-1+5q-4q^2+\cdots}\right.}{\cdots}$$

Mathematica [A] (verified)

Time = 2.97 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.60

$$\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx$$
$$= \frac{\sqrt[3]{-1+x}\sqrt[3]{q-2qx+x^2}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{q}(-1+x)^{2/3}}{\sqrt[3]{q}(-1+x)^{2/3}+2\sqrt[3]{q-2qx+x^2}}\right)-2\log\left(-\sqrt[3]{q}(-1+x)^{2/3}+\sqrt[3]{q\cdots}\right.\right.}{4\sqrt[3]{q}\sqrt[3]{(-1+x)(q-\cdots}}$$

```
[In] Integrate[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

```
[Out] ((-1 + x)^(1/3)*(q - 2*q*x + x^2)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*q^(1/3)*
(-1 + x)^(2/3))/(q^(1/3)*(-1 + x)^(2/3) + 2*(q - 2*q*x + x^2)^(1/3))] - 2*L
og[-(q^(1/3)*(-1 + x)^(2/3)) + (q - 2*q*x + x^2)^(1/3)] + Log[q^(2/3)*(-1 +
 x)^(4/3) + q^(1/3)*(-1 + x)^(2/3)*(q - 2*q*x + x^2)^(1/3) + (q - 2*q*x + x
^2)^(2/3)]))/(4*q^(1/3)*((-1 + x)*(q - 2*q*x + x^2))^(1/3))
```

Maple [F]

$$\int \frac{1}{x((-1+x)\,(-2qx+x^2+q))^{\frac{1}{3}}}dx$$

```
[In] int(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)
```

```
[Out] int(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(100) = 200$.

Time = 13.44 (sec) , antiderivative size = 1496, normalized size of antiderivative = 12.68

$$\int \frac{1}{x\sqrt[3]{(-1+x)\,(q-2qx+x^2)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(3)*q*sqrt((-q)^(1/3)/q)*log(-((q^3 - 30*q^2 - 51*q - 1)*x^6 + 5
4*(q^3 + 6*q^2 + 2*q)*x^5 - 27*(17*q^3 + 26*q^2 + 2*q)*x^4 + 486*q^3*x + 54
0*(2*q^3 + q^2)*x^3 - 81*q^3 - 135*(8*q^3 + q^2)*x^2 + 9*((2*q^2 - q - 1)*x
^4 - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^
(2/3)*(-q)^(1/3) + 9*((q^2 + 7*q + 1)*x^5 - (19*q^2 + 25*q + 1)*x^4 + 9*(7*
q^2 + 3*q)*x^3 + 45*q^2*x - 9*(9*q^2 + q)*x^2 - 9*q^2)*(-(2*q + 1)*x^2 + x^
3 + 3*q*x - q)^(1/3)*(-q)^(2/3) + sqrt(3)*(3*((4*q^2 + 13*q + 1)*x^4 - 6*(7
*q^2 + 5*q)*x^3 - 72*q^2*x + 3*(31*q^2 + 5*q)*x^2 + 18*q^2)*(-(2*q + 1)*x^2
 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^3 - 5*q^2 - 5*q)*x^5 + 5*(q^3
+ 7*q^2 + q)*x^4 - 45*q^3*x - 45*(q^3 + q^2)*x^3 + 9*q^3 + 15*(5*q^3 + q^2)
*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) + ((q^3 + 24*q^2 + 3*q - 1)*
x^6 - 54*(q^3 + 2*q^2)*x^5 + 81*(3*q^3 + 2*q^2)*x^4 - 162*q^3*x - 108*(4*q^
3 + q^2)*x^3 + 27*q^3 + 27*(14*q^3 + q^2)*x^2)*(-q)^(1/3))*sqrt((-q)^(1/3)/
q))/x^6) - 2*(-q)^(2/3)*log(((-q)^(2/3)*(q - 1)*x^2 + 3*(-(2*q + 1)*x^2 + x
^3 + 3*q*x - q)^(1/3)*(q*x - q)*(-q)^(1/3) + 3*(-(2*q + 1)*x^2 + x^3 + 3*q*
x - q)^(2/3)*q)/x^2) + (-q)^(2/3)*log((3*((2*q + 1)*x^2 - 6*q*x + 3*q)*(-(2
*q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^2 + 2*q)*x^3 + 9*q^
2*x - (7*q^2 + 2*q)*x^2 - 3*q^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) -
```

```
((q^2 + 7*q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36*q^2*x + 9*(5*q^2 + q)*x^2 + 9
*q^2)*(-q)^(1/3))/x^4))/q, 1/12*(2*sqrt(3)*q*sqrt(-(-q)^(1/3)/q)*arctan(1/3
*sqrt(3)*(6*((2*q^2 - q - 1)*x^4 - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2*
q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) - 6*((q^3 + 7*q^2 + q)*x^5 -
 (19*q^3 + 25*q^2 + q)*x^4 + 45*q^3*x + 9*(7*q^3 + 3*q^2)*x^3 - 9*q^3 - 9*(
9*q^3 + q^2)*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) - ((q^3 - 12*q^2
 - 15*q - 1)*x^6 + 18*(q^3 + 6*q^2 + 2*q)*x^5 - 9*(17*q^3 + 26*q^2 + 2*q)*x
^4 + 162*q^3*x + 180*(2*q^3 + q^2)*x^3 - 27*q^3 - 45*(8*q^3 + q^2)*x^2)*(-q
)^(1/3))*sqrt(-(-q)^(1/3)/q)/((q^3 + 24*q^2 + 3*q - 1)*x^6 - 54*(q^3 + 2*q^
2)*x^5 + 81*(3*q^3 + 2*q^2)*x^4 - 162*q^3*x - 108*(4*q^3 + q^2)*x^3 + 27*q^
3 + 27*(14*q^3 + q^2)*x^2)) - 2*(-q)^(2/3)*log(((-q)^(2/3)*(q - 1)*x^2 + 3*
(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3)*(q*x - q)*(-q)^(1/3) + 3*(-(2*q +
1)*x^2 + x^3 + 3*q*x - q)^(2/3)*q)/x^2) + (-q)^(2/3)*log((3*((2*q + 1)*x^2
- 6*q*x + 3*q)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^
2 + 2*q)*x^3 + 9*q^2*x - (7*q^2 + 2*q)*x^2 - 3*q^2)*(-(2*q + 1)*x^2 + x^3 +
 3*q*x - q)^(1/3) - ((q^2 + 7*q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36*q^2*x + 9*
(5*q^2 + q)*x^2 + 9*q^2)*(-q)^(1/3))/x^4))/q]
```

Sympy [F]

$$\int \frac{1}{x\sqrt[3]{(-1+x)\,(q-2qx+x^2)}}\,dx = \int \frac{1}{x\sqrt[3]{(x-1)\,(-2qx+q+x^2)}}\,dx$$

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x**2+q))**(1/3),x)
[Out] Integral(1/(x*((x - 1)*(-2*q*x + q + x**2))**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[3]{(-1+x)\,(q-2qx+x^2)}}\,dx = \int \frac{1}{(-(2\,qx-x^2-q)(x-1))^{\frac{1}{3}}\,x}\,dx$$

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="maxima")
[Out] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx=\int \frac{1}{\left(-(2\,qx-x^2-q)(x-1)\right)^{\frac{1}{3}}\,x}\,dx$$

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="giac")
[Out] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx=\int \frac{1}{x\left((x-1)\,\left(x^2-2\,q\,x+q\right)\right)^{1/3}}\,dx$$

```
[In] int(1/(x*((x - 1)*(q - 2*q*x + x^2))^(1/3)),x)
[Out] int(1/(x*((x - 1)*(q - 2*q*x + x^2))^(1/3)), x)
```

3.44 $\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 111

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{k}x}{\sqrt[3]{(1-x)x(1-kx)}}}{\sqrt{3}}\right)}{\sqrt[3]{k}} + \frac{\log(x)}{2\sqrt[3]{k}} + \frac{\log(1-(1+k)x)}{2\sqrt[3]{k}} - \frac{3\log\left(-\sqrt[3]{k}x+\sqrt[3]{(1-x)x(1-kx)}\right)}{2\sqrt[3]{k}}$$

```
[Out] 1/2*ln(x)/k^(1/3)+1/2*ln(1-(1+k)*x)/k^(1/3)-3/2*ln(-k^(1/3)*x+((1-x)*x*(-k*
x+1))^(1/3))/k^(1/3)+arctan(1/3*(1+2*k^(1/3)*x/((1-x)*x*(-k*x+1))^(1/3))*3^
(1/2))*3^(1/2)/k^(1/3)
```

Rubi [F]

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx = \int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx$$

```
[In] Int[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
```

```
[Out] (3*(1 - x)^(1/3)*x*(1 - k*x)^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, x, k*x])/(2
*((1 - x)*x*(1 - k*x))^(1/3)) + ((1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defe
r[Int][1/((1 - x)^(1/3)*x^(1/3)*(1 + (-1 - k)*x)*(1 - k*x)^(1/3)), x])/((1
- x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\text{integral}=\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int\frac{2-(1+k)x}{\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}(1-(1+k)x)}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$=\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int\frac{1}{\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}+\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int\frac{1}{\sqrt[3]{1-x}\sqrt[3]{x}(1+(-1-k)x)\sqrt[3]{1-kx}}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$=\frac{3\sqrt[3]{1-x}x\sqrt[3]{1-kx}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},x,kx\right)}{2\sqrt[3]{(1-x)x(1-kx)}}+\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int\frac{1}{\sqrt[3]{1-x}\sqrt[3]{x}(1+(-1-k)x)\sqrt[3]{1-kx}}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}$$

Mathematica [A] (verified)

Time = 15.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

$$\int\frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{k}x}{\sqrt[3]{k}x+2\sqrt[3]{(-1+x)x(-1+kx)}}\right)-2\log\left(-\sqrt[3]{k}x+\sqrt[3]{(-1+x)x(-1+kx)}\right)+\log\left(k^{2/3}x^2+\right.}{2\sqrt[3]{k}}$$

```
[In] Integrate[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*k^(1/3)*x)/(k^(1/3)*x + 2*((-1 + x)*x*(-1 + k*x)
)^(1/3))] - 2*Log[-(k^(1/3)*x) + ((-1 + x)*x*(-1 + k*x))^(1/3)] + Log[k^(2/
3)*x^2 + k^(1/3)*x*((-1 + x)*x*(-1 + k*x))^(1/3) + ((-1 + x)*x*(-1 + k*x))^
(2/3)])/(2*k^(1/3))
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(k^{\frac{1}{3}}x+2((-1+x)x(kx-1))^{\frac{1}{3}}\right)}{3k^{\frac{1}{3}}x}\right)-\frac{\ln\left(\frac{k^{\frac{2}{3}}x^{2}+k^{\frac{1}{3}}((-1+x)x(kx-1))^{\frac{1}{3}}x+((-1+x)x(kx-1))^{\frac{2}{3}}}{x^{2}}\right)}{2}+\ln\left(\frac{-k^{\frac{1}{3}}x+((-1\cdots}{}\right.}{k^{\frac{1}{3}}}$

```
[In] int((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x,method=_RETURNVERBOS
E)
[Out] -(3^(1/2)*arctan(1/3*3^(1/2)*(k^(1/3)*x+2*((-1+x)*x*(k*x-1))^(1/3))/k^(1/3)
/x)-1/2*ln((k^(2/3)*x^2+k^(1/3)*((-1+x)*x*(k*x-1))^(1/3)*x+((-1+x)*x*(k*x-1
))^(2/3))/x^2)+ln((-k^(1/3)*x+((-1+x)*x*(k*x-1))^(1/3))/x))/k^(1/3)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx = \text{Timed out}$$

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="fr
icas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx = \int \frac{kx+x-2}{\sqrt[3]{x\,(x-1)\,(kx-1)}\,(kx+x-1)}\,dx$$

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(1-(1+k)*x),x)
[Out] Integral((k*x + x - 2)/((x*(x - 1)*(k*x - 1))**(1/3)*(k*x + x - 1)), x)
```

Maxima [F]

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx = \int \frac{(k+1)x-2}{((kx-1)(x-1)x)^{\frac{1}{3}}\,((k+1)x-1)}\,dx$$

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="ma
xima")
```

```
[Out] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x)
```

Giac [F]

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx = \int \frac{(k+1)x-2}{((kx-1)(x-1)x)^{\frac{1}{3}}\,((k+1)x-1)}\,dx$$

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="gi
ac")
```

```
[Out] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx = \int \frac{x\,(k+1)-2}{(x\,(k+1)-1)\,\left(x\,(k\,x-1)\,(x-1)\right)^{1/3}}\,dx$$

```
[In] int((x*(k + 1) - 2)/((x*(k + 1) - 1)*(x*(k*x - 1)*(x - 1))^(1/3)),x)
```

```
[Out] int((x*(k + 1) - 2)/((x*(k + 1) - 1)*(x*(k*x - 1)*(x - 1))^(1/3)), x)
```

3.45 $\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 176

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-kx)}{\sqrt[3]{1-k}\sqrt[3]{(1-x)x(1-kx)}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt[3]{1-k}}+\frac{\log(1-(2-k)x)}{2^{2/3}\sqrt[3]{1-k}}+\frac{\log(1-kx)}{2\,2^{2/3}\sqrt[3]{1-k}}-\frac{3\log\left(-1+kx+2^{2/3}\sqrt[3]{1-k}\sqrt[3]{(1-x)x(1-kx)}\right)}{2\,2^{2/3}\sqrt[3]{1-k}}$$

```
[Out] 1/2*ln(1-(2-k)*x)*2^(1/3)/(1-k)^(1/3)+1/4*ln(-k*x+1)*2^(1/3)/(1-k)^(1/3)-3/
4*ln(-1+k*x+2^(2/3)*(1-k)^(1/3)*((1-x)*x*(-k*x+1))^(1/3))*2^(1/3)/(1-k)^(1/
3)-1/2*arctan(1/3*(1+2^(1/3)*(-k*x+1)/(1-k)^(1/3)/((1-x)*x*(-k*x+1))^(1/3))
*3^(1/2))*3^(1/2)*2^(1/3)/(1-k)^(1/3)
```

Rubi [F]

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx = \int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$$

```
[In] Int[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

```
[Out] ((1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Int][(1 - k*x)^(1/3)/((1 - x)^
(2/3)*x^(2/3)*(1 + (-2 + k)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\text{integral} = \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{\sqrt[3]{1-kx}}{(1-x)^{2/3}x^{2/3}(1+(-2+k)x)}\,dx}{((1-x)x(1-kx))^{2/3}}$$

Mathematica [F]

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx = \int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$$

```
[In] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
[Out] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)), x]
```

Maple [F]

$$\int \frac{-kx+1}{(1+(-2+k)\,x)\,((1-x)\,x\,(-kx+1))^{\frac{2}{3}}}dx$$

```
[In] int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)
[Out] int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(132) = 264$.

Time = 54.28 (sec) , antiderivative size = 932, normalized size of antiderivative = 5.30

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx = \text{Too large to display}$$

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="fric
as")
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/3*(24*sqrt(3)*2^(1/3)*((k^5 - 3*k^4 - 4*k^3 +
22*k^2 - 24*k + 8)*x^4 - 2*(k^4 - 10*k^3 + 27*k^2 - 26*k + 8)*x^3 - 6*(k^3
- 4*k^2 + 4*k - 1)*x^2 - 2*(k^2 - 1)*x + k - 1)*(k*x^3 - (k + 1)*x^2 + x)^(
2/3)/(k - 1)^(1/3) - 6*sqrt(3)*2^(2/3)*((k^6 + 27*k^5 - 40*k^4 - 20*k^3 + 4
8*k^2 - 16*k)*x^5 - (33*k^5 + 55*k^4 - 220*k^3 + 132*k^2 + 16*k - 16)*x^4 +
 2*(55*k^4 - 55*k^3 - 66*k^2 + 82*k - 16)*x^3 - 2*(55*k^3 - 99*k^2 + 38*k +
 6)*x^2 + (33*k^2 - 61*k + 28)*x - k + 1)*(k*x^3 - (k + 1)*x^2 + x)^(1/3)/(
k - 1)^(2/3) + sqrt(3)*((k^6 - 48*k^5 - 192*k^4 + 416*k^3 - 48*k^2 - 192*k
+ 64)*x^6 + 6*(7*k^5 + 104*k^4 - 80*k^3 - 176*k^2 + 176*k - 32)*x^5 - 3*(13
```

```
9*k^4 + 256*k^3 - 768*k^2 + 352*k + 16)*x^4 + 4*(203*k^3 - 192*k^2 - 120*k
+ 104)*x^3 - 3*(139*k^2 - 208*k + 64)*x^2 + 6*(7*k - 8)*x + 1))/((k^6 + 96*
k^5 - 48*k^4 - 160*k^3 + 240*k^2 - 192*k + 64)*x^6 - 6*(17*k^5 + 64*k^4 - 1
12*k^3 + 80*k^2 - 80*k + 32)*x^5 + 3*(149*k^4 + 32*k^3 - 96*k^2 - 160*k + 8
0)*x^4 - 4*(157*k^3 - 24*k^2 - 168*k + 40)*x^3 + 3*(149*k^2 - 128*k - 16)*x
^2 - 6*(17*k - 16)*x + 1))/(k - 1)^(1/3) - 1/12*2^(1/3)*log((12*2^(2/3)*(k*
x^3 - (k + 1)*x^2 + x)^(2/3)*((k^3 + k^2 - 4*k + 2)*x^2 - 2*(2*k^2 - 3*k +
1)*x + k - 1)/(k - 1)^(2/3) + 6*((k^3 + 8*k^2 - 8*k)*x^3 - (11*k^2 - 8)*x^2
 + (11*k - 8)*x - 1)*(k*x^3 - (k + 1)*x^2 + x)^(1/3) + 2^(1/3)*((k^4 + 28*k
^3 - 12*k^2 - 32*k + 16)*x^4 - 4*(8*k^3 + 15*k^2 - 30*k + 8)*x^3 + 6*(13*k^
2 - 10*k - 2)*x^2 - 4*(8*k - 7)*x + 1)/(k - 1)^(1/3))/((k^4 - 8*k^3 + 24*k^
2 - 32*k + 16)*x^4 + 4*(k^3 - 6*k^2 + 12*k - 8)*x^3 + 6*(k^2 - 4*k + 4)*x^2
 + 4*(k - 2)*x + 1))/(k - 1)^(1/3) + 1/6*2^(1/3)*log((6*2^(1/3)*(k*x^3 - (k
 + 1)*x^2 + x)^(1/3)*(k*x - 1)/(k - 1)^(1/3) - 2^(2/3)*((k^2 - 4*k + 4)*x^2
 + 2*(k - 2)*x + 1)/(k - 1)^(2/3) - 12*(k*x^3 - (k + 1)*x^2 + x)^(2/3))/((k
^2 - 4*k + 4)*x^2 + 2*(k - 2)*x + 1))/(k - 1)^(1/3)
```

Sympy [F]

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx =$$
$$-\int \frac{kx}{kx\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}-2x\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}+\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}}\,dx$$
$$-\int\left(-\frac{1}{kx\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}-2x\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}+\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}}\right)dx$$

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))**(2/3),x)
```

```
[Out] -Integral(k*x/(k*x*(k*x**3 - k*x**2 - x**2 + x)**(2/3) - 2*x*(k*x**3 - k*x*
*2 - x**2 + x)**(2/3) + (k*x**3 - k*x**2 - x**2 + x)**(2/3)), x) - Integral
(-1/(k*x*(k*x**3 - k*x**2 - x**2 + x)**(2/3) - 2*x*(k*x**3 - k*x**2 - x**2
+ x)**(2/3) + (k*x**3 - k*x**2 - x**2 + x)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx = \int -\frac{kx-1}{((kx-1)(x-1)x)^{\frac{2}{3}}\,((k-2)x+1)}\,dx$$

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="maxi
ma")
```

```
[Out] -integrate((k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)
```

Giac [F]

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx = \int -\frac{kx-1}{((kx-1)(x-1)x)^{\frac{2}{3}}\,((k-2)x+1)}\,dx$$

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="giac
")
```

```
[Out] integrate(-(k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx = -\int \frac{k\,x-1}{(x\,(k-2)+1)\,(x\,(k\,x-1)\,(x-1))^{2/3}}\,dx$$

```
[In] int(-(k*x - 1)/((x*(k - 2) + 1)*(x*(k*x - 1)*(x - 1))^(2/3)),x)
```

```
[Out] -int((k*x - 1)/((x*(k - 2) + 1)*(x*(k*x - 1)*(x - 1))^(2/3)), x)
```

3.46 $\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 493

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \frac{(a+b)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{(a+b)\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{c\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{(a-c)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{(b+c)\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{(a+b)\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{(a-c)\log(1+x^3)}{6\sqrt[3]{2}} - \frac{(b+c)\log(1+x^3)}{6\sqrt[3]{2}} + \frac{(a+b)\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{(a+b)\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{(b+c)\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{(a-c)\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{1}{2}c\log\left(x+\sqrt[3]{1-x^3}\right) - \frac{(a+b)\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/24*(a+b)*ln((1-x)*(1+x)^2)*2^(2/3)-1/12*(a-c)*ln(x^3+1)*2^(2/3)-1/12*(b+c
)*ln(x^3+1)*2^(2/3)+1/12*(a+b)*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*
(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*(a+b)*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*
2^(2/3)+1/4*(b+c)*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/4*(a-c)*ln(-2^(1/3)*
x-(-x^3+1)^(1/3))*2^(2/3)+1/2*c*ln(x+(-x^3+1)^(1/3))-1/8*(a+b)*ln(-1+x+2^(2
```

```
/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*(a+b)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1
)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*(a+b)*arctan(1/3*(1+2^(1/3)*(1-x)/(-
x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/3*c*arctan(1/3*(1-2*x/(-x^3+1)^(1/
3))*3^(1/2))*3^(1/2)-1/6*(a-c)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^
(1/2))*2^(2/3)*3^(1/2)+1/6*(b+c)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1
/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.16, number of steps used = 19, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules

used = {2183, 384, 502, 2174, 206, 31, 648, 631, 210, 642, 455, 57, 494, 245}

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \frac{(a+b)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{(a+b)\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{a\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{(a+b)\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}} - \frac{(a+b)\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}} - \frac{(a+b)\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}} + \frac{(a+b)\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}} - \frac{a\log\left(x^3+1\right)}{6\sqrt[3]{2}} + \frac{a\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}} + \frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)(b+c)}{\sqrt[3]{2}\sqrt{3}} - \frac{c\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{(b+c)\log\left(x^3+1\right)}{6\sqrt[3]{2}} + \frac{(b+c)\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{c\log\left(x^3+1\right)}{6\sqrt[3]{2}} - \frac{c\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}} + \frac{1}{2}c\log\left(\sqrt[3]{1-x^3}+x\right)$$

```
[In] Int[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] ((a + b)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3
)*Sqrt[3]) + ((a + b)*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3
```

```
]])/(2*2^(1/3)*Sqrt[3]) - (c*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/S
qrt[3] - (a*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*S
qrt[3]) + (c*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*
Sqrt[3]) + ((b + c)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3)
*Sqrt[3]) + ((a + b)*Log[(1 - x)*(1 + x)^2])/(12*2^(1/3)) - (a*Log[1 + x^3]
)/(6*2^(1/3)) + (c*Log[1 + x^3])/(6*2^(1/3)) - ((b + c)*Log[1 + x^3])/(6*2^
(1/3)) + ((a + b)*Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1
 - x))/(1 - x^3)^(1/3)])/(6*2^(1/3)) - ((a + b)*Log[1 + (2^(1/3)*(1 - x))/(
1 - x^3)^(1/3)])/(3*2^(1/3)) + ((b + c)*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*
2^(1/3)) + (a*Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)])/(2*2^(1/3)) - (c*Log[-(2
^(1/3)*x) - (1 - x^3)^(1/3)])/(2*2^(1/3)) + (c*Log[x + (1 - x^3)^(1/3)])/2
- ((a + b)*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a}{\sqrt[3]{1-x^3}\,(1+x^3)}+\frac{(a+b)x}{\sqrt[3]{1-x^3}\,(1+x^3)}+\frac{(b+c)x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}+\frac{cx^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\right)dx\\
&= a\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx+(a+b)\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\
&\quad +c\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx+(b+c)\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\
&= -\frac{a\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{a\log\left(1+x^3\right)}{6\sqrt[3]{2}}\\
&\quad +\frac{a\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{1}{3}(-a-b)\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx\\
&\quad +(-a-b)\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)+c\int \frac{1}{\sqrt[3]{1-x^3}}\,dx\\
&\quad -c\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx+\frac{1}{3}(b+c)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx,x,x^3\right)
\end{aligned}$$

$$
\begin{aligned}
= & \frac{(a+b)\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{c\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} \\
& - \frac{a\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} \\
& + \frac{(a+b)\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{a\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{c\log\left(1+x^3\right)}{6\sqrt[3]{2}} \\
& - \frac{(b+c)\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{a\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{c\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} \\
& + \frac{1}{2}c\log\left(x+\sqrt[3]{1-x^3}\right) - \frac{(a+b)\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} \\
& + \frac{1}{3}(-a-b)\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) \\
& \qquad + \frac{1}{3}(-a-b)\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) \\
& + \frac{1}{2}(b+c)\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{(b+c)\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}
$$

$$= \frac{(a+b)\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{c\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$- \frac{a\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

$$+ \frac{(a+b)\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{a\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{c\log\left(1+x^3\right)}{6\sqrt[3]{2}}$$

$$- \frac{(b+c)\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{(a+b)\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{(b+c)\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

$$+ \frac{a\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{c\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

$$+ \frac{1}{2}c\log\left(x+\sqrt[3]{1-x^3}\right) - \frac{(a+b)\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

$$+ \frac{1}{2}(-a-b)\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$+ \frac{(a+b)\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}$$

$$- \frac{(b+c)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}$$

$$= \frac{(a+b)\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{c\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$- \frac{a\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

$$+ \frac{(b+c)\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{(a+b)\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{a\log\left(1+x^3\right)}{6\sqrt[3]{2}}$$

$$+ \frac{c\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{(b+c)\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{(a+b)\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}$$

$$- \frac{(a+b)\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{(b+c)\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

$$+ \frac{a\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{c\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

$$+ \frac{1}{2}c\log\left(x+\sqrt[3]{1-x^3}\right) - \frac{(a+b)\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2 \ldots}{\ldots}\right.}{\sqrt[3]{2}}$$

$$
= \frac{(a+b)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{(a+b)\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}
- \frac{c\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{a\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}
+ \frac{(b+c)\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{(a+b)\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{a\log\left(1+x^3\right)}{6\sqrt[3]{2}}
+ \frac{c\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{(b+c)\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{(a+b)\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}
- \frac{(a+b)\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{(b+c)\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
+ \frac{a\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{c\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
+ \frac{1}{2}c\log\left(x+\sqrt[3]{1-x^3}\right) - \frac{(a+b)\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}
$$

Mathematica [F]

$$
\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx
$$

```
[In] Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
[Out] Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)), x]
```

Maple [F]

$$\int \frac{c\,x^{2}+bx+a}{\left(x^{2}-x+1\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}dx$$

[In] int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)

[Out] int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)

Fricas [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)

Sympy [F]

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \frac{a+bx+cx^2}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x^2-x+1)}\,dx$$

[In] integrate((c*x**2+b*x+a)/(x**2-x+1)/(-x**3+1)**(1/3),x)

[Out] Integral((a + b*x + c*x**2)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1
)), x)

Maxima [F]

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \frac{cx^2+bx+a}{(-x^3+1)^{\frac{1}{3}}(x^2-x+1)}\,dx$$

[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)

Giac [F]

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \frac{cx^2+bx+a}{(-x^3+1)^{\frac{1}{3}}(x^2-x+1)}\,dx$$

```
[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \frac{c\,x^2+b\,x+a}{(1-x^3)^{1/3}\,(x^2-x+1)}\,dx$$

```
[In] int((a + b*x + c*x^2)/((1 - x^3)^(1/3)*(x^2 - x + 1)),x)
[Out] int((a + b*x + c*x^2)/((1 - x^3)^(1/3)*(x^2 - x + 1)), x)
```

3.47 $\int \frac{1}{(3-2x)^{11/2}(1+x+2x^2)^5} dx$

Optimal result

Integrand size = 20, antiderivative size = 407

$$\begin{aligned}\int \frac{1}{(3-2x)^{11/2}(1+x+2x^2)^5} dx &= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} \\ &- \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2x)^{3/2}} - \frac{38225}{240945152\sqrt{3-2x}} \\ &+ \frac{x}{28(3-2x)^{9/2}(1+x+2x^2)^4} + \frac{23+73x}{1176(3-2x)^{9/2}(1+x+2x^2)^3} \\ &+ \frac{1387+3049x}{32928(3-2x)^{9/2}(1+x+2x^2)^2} + \frac{5(3049+4377x)}{153664(3-2x)^{9/2}(1+x+2x^2)} \\ &+ \frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\arctan\left(\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right)}{3373232128} \\ &- \frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\arctan\left(\frac{\sqrt{7+2\sqrt{14}}+2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right)}{3373232128} \\ &+ \frac{5\sqrt{\frac{1}{2}\left(-149046503977+40815066112\sqrt{14}\right)}\log\left(3+\sqrt{14}-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right)}{6746464256} \\ &- \frac{5\sqrt{\frac{1}{2}\left(-149046503977+40815066112\sqrt{14}\right)}\log\left(3+\sqrt{14}+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right)}{6746464256}\end{aligned}$$

```
[Out] -19255/395136/(3-2*x)^(9/2)-462025/30118144/(3-2*x)^(7/2)-38491/8605184/(3-
2*x)^(5/2)-141045/120472576/(3-2*x)^(3/2)+1/28*x/(3-2*x)^(9/2)/(2*x^2+x+1)^
4+1/1176*(23+73*x)/(3-2*x)^(9/2)/(2*x^2+x+1)^3+1/32928*(1387+3049*x)/(3-2*x
)^(9/2)/(2*x^2+x+1)^2+5/153664*(3049+4377*x)/(3-2*x)^(9/2)/(2*x^2+x+1)-3822
5/240945152/(3-2*x)^(1/2)+5/13492928512*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+
2*14^(1/2))^(1/2))*(-298093007954+81630132224*14^(1/2))^(1/2)-5/13492928512
*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(-298093007954+81630
```

```
132224*14^(1/2))^(1/2)+5/6746464256*arctan((-2*(3-2*x)^(1/2)+(7+2*14^(1/2))
^(1/2))/(-7+2*14^(1/2))^(1/2))*(298093007954+81630132224*14^(1/2))^(1/2)-5/
6746464256*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1
/2))*(298093007954+81630132224*14^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {754, 836, 842, 840, 1183, 648, 632, 210, 642}

$$\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx = \frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\arctan\left(\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{2\sqrt{14}-7}}\right)}{3373232128} - \frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\arctan\left(\frac{2\sqrt{3-2x}+\sqrt{7+2\sqrt{14}}}{\sqrt{2\sqrt{14}-7}}\right)}{3373232128} + \frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4} + \frac{5(4377x+3049)}{153664(3-2x)^{9/2}\left(2x^2+x+1\right)} + \frac{3049x+1387}{32928(3-2x)^{9/2}\left(2x^2+x+1\right)^2} + \frac{73x+23}{1176(3-2x)^{9/2}\left(2x^2+x+1\right)^3} - \frac{38225}{240945152\sqrt{3-2x}} - \frac{141045}{120472576(3-2x)^{3/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{19255}{395136(3-2x)^{9/2}} + \frac{5\sqrt{\frac{1}{2}\left(40815066112\sqrt{14}-149046503977\right)}\log\left(-2x-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3\right)}{6746464256} - \frac{5\sqrt{\frac{1}{2}\left(40815066112\sqrt{14}-149046503977\right)}\log\left(-2x+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3\right)}{6746464256}$$

```
[In] Int[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] -19255/(395136*(3 - 2*x)^(9/2)) - 462025/(30118144*(3 - 2*x)^(7/2)) - 38491
/(8605184*(3 - 2*x)^(5/2)) - 141045/(120472576*(3 - 2*x)^(3/2)) - 38225/(24
0945152*Sqrt[3 - 2*x]) + x/(28*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^4) + (23 + 7
3*x)/(1176*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^3) + (1387 + 3049*x)/(32928*(3 -
 2*x)^(9/2)*(1 + x + 2*x^2)^2) + (5*(3049 + 4377*x))/(153664*(3 - 2*x)^(9/2
)*(1 + x + 2*x^2)) + (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTa
n[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/33732321
28 - (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTan[(Sqrt[7 + 2*Sq
rt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/3373232128 + (5*Sqrt[(-1
49046503977 + 40815066112*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[7 + 2*Sqrt[1
4]]*Sqrt[3 - 2*x] - 2*x])/6746464256 - (5*Sqrt[(-149046503977 + 40815066112
*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])
/6746464256
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
```

```
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 842

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)
^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4}+\frac{1}{784}\int\frac{700-644x}{(3-2x)^{11/2}\left(1+x+2x^2\right)^4}\,dx\\
&=\frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4}+\frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3}+\frac{\int\frac{325752-543704x}{(3-2x)^{11/2}\left(1+x+2x^2\right)^3}\,dx}{460992}\\
&=\frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4}+\frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3}\\
&\quad+\frac{1387+3049x}{32928(3-2x)^{9/2}\left(1+x+2x^2\right)^2}+\frac{\int\frac{54660480-250993680x}{(3-2x)^{11/2}\left(1+x+2x^2\right)^2}\,dx}{180708864}
\end{aligned}$$

$$= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{1387+3049x}{32928(3-2x)^{9/2}\left(1+x+2x^2\right)^2} + \frac{5(3049+4377x)}{153664(3-2x)^{9/2}\left(1+x+2x^2\right)} + \frac{\int \frac{-25503229920-55488454560x}{(3-2x)^{11/2}(1+x+2x^2)}\,dx}{35418937344}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{1387+3049x}{32928(3-2x)^{9/2}\left(1+x+2x^2\right)^2} + \frac{5(3049+4377x)}{153664(3-2x)^{9/2}\left(1+x+2x^2\right)} + \frac{\int \frac{-93048930240-434943647040x}{(3-2x)^{9/2}(1+x+2x^2)}\,dx}{991730245632}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{1387+3049x}{32928(3-2x)^{9/2}\left(1+x+2x^2\right)^2} + \frac{5(3049+4377x)}{153664(3-2x)^{9/2}\left(1+x+2x^2\right)} + \frac{\int \frac{125495852160-2981857603200x}{(3-2x)^{7/2}(1+x+2x^2)}\,dx}{27768446877696}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{1387+3049x}{32928(3-2x)^{9/2}\left(1+x+2x^2\right)^2} + \frac{5(3049+4377x)}{153664(3-2x)^{9/2}\left(1+x+2x^2\right)} + \frac{\int \frac{6967682023680-17389162210560x}{(3-2x)^{5/2}(1+x+2x^2)}\,dx}{777516512575488}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2x)^{3/2}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{1387+3049x}{32928(3-2x)^{9/2}\left(1+x+2x^2\right)^2} + \frac{5(3049+4377x)}{153664(3-2x)^{9/2}\left(1+x+2x^2\right)} + \frac{\int \frac{90519780610560-76464245168640x}{(3-2x)^{3/2}(1+x+2x^2)}\,dx}{21770462352113664}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2x)^{3/2}} - \frac{38225}{240945152\sqrt{3-2x}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{1387+3049x}{32928(3-2x)^{9/2}\left(1+x+2x^2\right)^2} + \frac{5(3049+4377x)}{153664(3-2x)^{9/2}\left(1+x+2x^2\right)} + \frac{\int \frac{877086735221760-96706348569600x}{\sqrt{3-2x}(1+x+2x^2)}\,dx}{609572945859182592}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2x)^{3/2}} - \frac{38225}{240945152\sqrt{3-2x}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{1387+3049x}{32928(3-2x)^{9/2}\left(1+x+2x^2\right)^2} + \frac{5(3049+4377x)}{153664(3-2x)^{9/2}\left(1+x+2x^2\right)} + \frac{\text{Subst}\left(\int \frac{-1464054424734720-96706348569600x^2}{28-14x^2+2x^4}\,dx, x, \sqrt{3-2x}\right)}{304786472929591296}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2x)^{3/2}} - \frac{38225}{240945152\sqrt{3-2x}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{1387+3049x}{32928(3-2x)^{9/2}\left(1+x+2x^2\right)^2} + \frac{5(3049+4377x)}{153664(3-2x)^{9/2}\left(1+x+2x^2\right)} + \frac{\text{Subst}\left(\int \frac{-1464054424734720\sqrt{7+2\sqrt{14}}-\left(-1464054424734720+96706348569600\sqrt{14}\right)x}{\sqrt{14}-\sqrt{7+2\sqrt{14}}x+x^2}\,dx, x, \sqrt{3-2x}\right)}{1219145891718365184\sqrt{14\left(7+2\sqrt{14}\right)}} + \frac{\text{Subst}\left(\int \frac{-1464054424734720\sqrt{7+2\sqrt{14}}+\left(-1464054424734720+96706348569600\sqrt{14}\right)x}{\sqrt{14}+\sqrt{7+2\sqrt{14}}x+x^2}\,dx, x, \sqrt{3-2x}\right)}{1219145891718365184\sqrt{14\left(7+2\sqrt{14}\right)}}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2x)^{3/2}} - \frac{38225}{240945152\sqrt{3-2x}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{1387+3049x}{32928(3-2x)^{9/2}\left(1+x+2x^2\right)^2} + \frac{5(3049+4377x)}{153664(3-2x)^{9/2}\left(1+x+2x^2\right)} - \frac{\left(5\left(107030+115739\sqrt{14}\right)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{14}-\sqrt{7+2\sqrt{14}}x+x^2}\,dx, x, \sqrt{3-2x}\right)}{13492928512} - \frac{\left(5\left(107030+115739\sqrt{14}\right)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{14}+\sqrt{7+2\sqrt{14}}x+x^2}\,dx, x, \sqrt{3-2x}\right)}{13492928512} + \frac{\left(5\sqrt{\frac{1}{2}\left(-149046503977+40815066112\sqrt{14}\right)}\right)\operatorname{Subst}\left(\int \frac{-\sqrt{7+2\sqrt{14}}+2x}{\sqrt{14}-\sqrt{7+2\sqrt{14}}x+x^2}\,dx, x, \sqrt{3-2x}\right)}{6746464256} - \frac{\left(5\sqrt{\frac{1}{2}\left(-149046503977+40815066112\sqrt{14}\right)}\right)\operatorname{Subst}\left(\int \frac{\sqrt{7+2\sqrt{14}}+2x}{\sqrt{14}+\sqrt{7+2\sqrt{14}}x+x^2}\,dx, x, \sqrt{3-2x}\right)}{6746464256}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2x)^{3/2}} - \frac{38225}{240945152\sqrt{3-2x}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{1387+3049x}{32928(3-2x)^{9/2}\left(1+x+2x^2\right)^2} + \frac{5(3049+4377x)}{153664(3-2x)^{9/2}\left(1+x+2x^2\right)} + \frac{5\sqrt{\frac{1}{2}\left(-149046503977+40815066112\sqrt{14}\right)}\log\left(3+\sqrt{14}-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right)}{6746464256} - \frac{5\sqrt{\frac{1}{2}\left(-149046503977+40815066112\sqrt{14}\right)}\log\left(3+\sqrt{14}+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right)}{6746464256} + \frac{\left(5\left(107030+115739\sqrt{14}\right)\right)\operatorname{Subst}\left(\int \frac{1}{7-2\sqrt{14}-x^2}\,dx, x, -\sqrt{7+2\sqrt{14}}+2\sqrt{3-2x}\right)}{6746464256} + \frac{\left(5\left(107030+115739\sqrt{14}\right)\right)\operatorname{Subst}\left(\int \frac{1}{7-2\sqrt{14}-x^2}\,dx, x, \sqrt{7+2\sqrt{14}}+2\sqrt{3-2x}\right)}{6746464256}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{141045}{120472576(3-2x)^{3/2}} - \frac{38225}{240945152\sqrt{3-2x}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{1387+3049x}{32928(3-2x)^{9/2}\left(1+x+2x^2\right)^2} + \frac{5(3049+4377x)}{153664(3-2x)^{9/2}\left(1+x+2x^2\right)} + \frac{5\sqrt{298093007954+81630132224\sqrt{14}}\arctan\left(\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right)}{6746464256} - \frac{5\sqrt{298093007954+81630132224\sqrt{14}}\arctan\left(\frac{\sqrt{7+2\sqrt{14}}+2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right)}{6746464256} + \frac{5\sqrt{\frac{1}{2}\left(-149046503977+40815066112\sqrt{14}\right)}\log\left(3+\sqrt{14}-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right)}{6746464256} - \frac{5\sqrt{\frac{1}{2}\left(-149046503977+40815066112\sqrt{14}\right)}\log\left(3+\sqrt{14}+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right)}{6746464256}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.42 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.44

$$\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx = \frac{-\frac{14(40289347-429812744x+135202154x^2-1073855156x^3+1627773523x^4-1470758860x^5+288\ldots}{(3-2\ldots}}{}$$

```
[In] Integrate[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] ((-14*(40289347 - 429812744*x + 135202154*x^2 - 1073855156*x^3 + 1627773523
*x^4 - 1470758860*x^5 + 2888625656*x^6 - 3106712560*x^7 + 2343370048*x^8 -
2443779648*x^9 + 1873554048*x^10 - 677249280*x^11 + 88070400*x^12))/((3 - 2
*x)^(9/2)*(1 + x + 2*x^2)^4) - 45*Sqrt[149046503977 + (12577271771*I)*Sqrt[
7]]*ArcTan[(Sqrt[-1 - I/Sqrt[7]]*Sqrt[3 - 2*x])/2] - 45*Sqrt[149046503977 -
 (12577271771*I)*Sqrt[7]]*ArcTan[(Sqrt[-1 + I/Sqrt[7]]*Sqrt[3 - 2*x])/2])/3
0359089152
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{5\left(\left(x-\frac{3}{2}\right)^{4}\left(x^{2}+\frac{1}{2}x+\frac{1}{2}\right)^{4}\left(\frac{\sqrt{7+2\sqrt{14}}\,\left(146319\sqrt{14}-569986\right)\left(\ln\left(3-2x+\sqrt{14}-\sqrt{3-2x}\,\sqrt{7+2\sqrt{14}}\right)-\ln\left(3-2x+\sqrt{14}+\sqrt{3-2x}\,\sqrt{7+2\right.\right.}{2}\right.\right.}{}$
derivativedivides	$\frac{\frac{567651623\sqrt{3-2x}}{32}-\frac{6194606411(3-2x)^{\frac{3}{2}}}{192}+\frac{9801432515(3-2x)^{\frac{5}{2}}}{384}-\frac{8763772549(3-2x)^{\frac{7}{2}}}{768}+\frac{149630663(3-2x)^{\frac{9}{2}}}{48}-\frac{200063633(3-2x)^{\frac{11}{2}}}{384}+}{6588344\left((3-2x)^{2}-7+14x\right)^{4}}$
default	$\frac{\frac{567651623\sqrt{3-2x}}{32}-\frac{6194606411(3-2x)^{\frac{3}{2}}}{192}+\frac{9801432515(3-2x)^{\frac{5}{2}}}{384}-\frac{8763772549(3-2x)^{\frac{7}{2}}}{768}+\frac{149630663(3-2x)^{\frac{9}{2}}}{48}-\frac{200063633(3-2x)^{\frac{11}{2}}}{384}+}{6588344\left((3-2x)^{2}-7+14x\right)^{4}}$
trager	Expression too large to display
risch	$-\frac{88070400x^{12}-677249280x^{11}+1873554048x^{10}-2443779648x^{9}+2343370048x^{8}-3106712560x^{7}+2888625656x^{6}-147075}{2168506368(2x-3)^{4}\sqrt{3-2x}\,(2x^{2}+x+1)}$

```
[In] int(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -5/26353376/(3-2*x)^(9/2)*((x-3/2)^4*(x^2+1/2*x+1/2)^4*(1/2*(7+2*14^(1/2))^
(1/2)*(146319*14^(1/2)-569986)*(ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/
2))^(1/2))-ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2)))*(-7+2*14^
(1/2))^(1/2)+(115739*14^(1/2)+107030)*(arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2
))^(1/2))/(-7+2*14^(1/2))^(1/2))+arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/
2))/(-7+2*14^(1/2))^(1/2))))*(3-2*x)^(1/2)+214060*(-7+2*14^(1/2))^(1/2)*(16
26349/76450*x^10+x^12-24405799/2001600*x^3+1627773523/88070400*x^4+36107820
7/11008800*x^6+67601077/44035200*x^2-53726593/11008800*x+36615157/1376100*x
^8-73537943/4403520*x^5+40289347/88070400-38833907/1100880*x^7-4242673/1529
00*x^9-58789/7645*x^11))/(-7+2*14^(1/2))^(1/2)/(2*x^2+x+1)^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.36

$$\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^{2}\right)^{5}}\,dx = \frac{9\left(512\,x^{13}-2816\,x^{12}+5632\,x^{11}-5888\,x^{10}+6848\,x^{9}-8992\,x^{8}+6112\,x\right.}{}$$

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/60718178304*(9*(512*x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x^9 -
 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*x^2
```

```
- 162*x - 243)*sqrt(314431794275*I*sqrt(7) - 3726162599425)*log(sqrt(314431
794275*I*sqrt(7) - 3726162599425)*(146319*I*sqrt(7) - 115739) + 40815066112
0*sqrt(-2*x + 3)) - 9*(512*x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10 + 6848*
x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242
*x^2 - 162*x - 243)*sqrt(314431794275*I*sqrt(7) - 3726162599425)*log(sqrt(3
14431794275*I*sqrt(7) - 3726162599425)*(-146319*I*sqrt(7) + 115739) + 40815
0661120*sqrt(-2*x + 3)) - 9*(512*x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10 +
 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3
- 1242*x^2 - 162*x - 243)*sqrt(-314431794275*I*sqrt(7) - 3726162599425)*log
((146319*I*sqrt(7) + 115739)*sqrt(-314431794275*I*sqrt(7) - 3726162599425)
+ 408150661120*sqrt(-2*x + 3)) + 9*(512*x^13 - 2816*x^12 + 5632*x^11 - 5888
*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4 + 9
36*x^3 - 1242*x^2 - 162*x - 243)*sqrt(-314431794275*I*sqrt(7) - 37261625994
25)*log((-146319*I*sqrt(7) - 115739)*sqrt(-314431794275*I*sqrt(7) - 3726162
599425) + 408150661120*sqrt(-2*x + 3)) + 28*(88070400*x^12 - 677249280*x^11
 + 1873554048*x^10 - 2443779648*x^9 + 2343370048*x^8 - 3106712560*x^7 + 288
8625656*x^6 - 1470758860*x^5 + 1627773523*x^4 - 1073855156*x^3 + 135202154*
x^2 - 429812744*x + 40289347)*sqrt(-2*x + 3))/(512*x^13 - 2816*x^12 + 5632*
x^11 - 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1
707*x^4 + 936*x^3 - 1242*x^2 - 162*x - 243)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx = \text{Timed out}$$

```
[In] integrate(1/(3-2*x)**(11/2)/(2*x**2+x+1)**5,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx = \int \frac{1}{\left(2\,x^2+x+1\right)^5\left(-2\,x+3\right)^{\frac{11}{2}}}\,dx$$

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="maxima")
[Out] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. 2(298) = 596.

Time = 0.62 (sec) , antiderivative size = 795, normalized size of antiderivative = 1.95

$$\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="giac")
[Out] -5/1511207993344*sqrt(7)*(22935*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-2*sqr
t(14) + 8) + 7645*14^(3/4)*sqrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) + 5
3515*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 160545*14^(3/4)*sqrt(2*
sqrt(14) + 8)*(sqrt(14) - 4) + 925912*14^(1/4)*sqrt(7)*sqrt(-2*sqrt(14) + 8
) + 6481384*14^(1/4)*sqrt(2*sqrt(14) + 8))*arctan(1/28*14^(3/4)*(14^(1/4)*s
qrt(1/2)*sqrt(sqrt(14) + 4) + 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2))
- 5/1511207993344*sqrt(7)*(22935*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-2*sq
rt(14) + 8) + 7645*14^(3/4)*sqrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) +
53515*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 160545*14^(3/4)*sqrt(2
*sqrt(14) + 8)*(sqrt(14) - 4) + 925912*14^(1/4)*sqrt(7)*sqrt(-2*sqrt(14) +
8) + 6481384*14^(1/4)*sqrt(2*sqrt(14) + 8))*arctan(-1/28*14^(3/4)*(14^(1/4)
*sqrt(1/2)*sqrt(sqrt(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)
) - 5/3022415986688*sqrt(7)*(7645*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sq
rt(14) + 4) + 22935*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) -
160545*14^(3/4)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) - 53515*14^(3/4)*(sqrt
(14) - 4)*sqrt(-2*sqrt(14) + 8) + 925912*14^(1/4)*sqrt(7)*sqrt(2*sqrt(14) +
 8) - 6481384*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(14^(1/4)*sqrt(1/2)*sqrt(-
2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 5/3022415986688*sqrt(7)
*(7645*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 22935*14^(3/4
)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) - 160545*14^(3/4)*(sqrt(14) +
 4)*sqrt(-2*sqrt(14) + 8) - 53515*14^(3/4)*(sqrt(14) - 4)*sqrt(-2*sqrt(14)
+ 8) + 925912*14^(1/4)*sqrt(7)*sqrt(2*sqrt(14) + 8) - 6481384*14^(1/4)*sqrt
(-2*sqrt(14) + 8))*log(-14^(1/4)*sqrt(1/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4
) - 2*x + sqrt(14) + 3) + 1/5059848192*(1578405*(2*x - 3)^7*sqrt(-2*x + 3)
+ 37939930*(2*x - 3)^6*sqrt(-2*x + 3) + 400127266*(2*x - 3)^5*sqrt(-2*x + 3
) + 2394090608*(2*x - 3)^4*sqrt(-2*x + 3) + 8763772549*(2*x - 3)^3*sqrt(-2*
x + 3) + 19602865030*(2*x - 3)^2*sqrt(-2*x + 3) - 24778425644*(-2*x + 3)^(3
/2) + 13623638952*sqrt(-2*x + 3))/((2*x - 3)^2 + 14*x - 7)^4 + 1/59295096*(
9090*(2*x - 3)^4 - 3885*(2*x - 3)^3 + 2394*(2*x - 3)^2 - 2520*x + 4172)/((2
*x - 3)^4*sqrt(-2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.84

$$\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx =$$

$$-\frac{\frac{272\,x}{441}-\frac{164\,(2\,x-3)^2}{441}+\frac{1966\,(2\,x-3)^3}{3087}-\frac{9091\,(2\,x-3)^4}{3087}-\frac{32070727\,(2\,x-3)^5}{5531904}-\frac{41014777\,(2\,x-3)^6}{11063808}-\frac{141921511\,(2\,x-3)^7}{154893312}+\frac{2326265\ldots}{3097\ldots}}{38416\,(3-2\,x)^{9/2}-76832\,(3-2\,x)^{11/2}+68600\,(3-2\,x)^{13/2}-35672\,(3-2\,x)^{15/2}+11809\,(3-2\,x)^{17/\ldots}}$$

$$+\frac{\operatorname{atan}\left(\frac{\sqrt{3-2\,x}\,\sqrt{-149046503977+\sqrt{7}\,12577271771\mathrm{i}}\,1572158971375\mathrm{i}}{391663056253676053933850624\left(-\frac{230036728532618625}{2797593258954828956670361 6}+\frac{\sqrt{7}\,181960107187971125\mathrm{i}}{19583152812683802696692531 2}\right)}-\frac{1572158971375\,\sqrt{7}\,\sqrt{3-\ldots}}{391663056253676053933850624\left(-\frac{\ldots}{279\ldots}\right.}\right.}{3373232128}$$

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{3-2\,x}\,\sqrt{-149046503977-\sqrt{7}\,12577271771\mathrm{i}}\,1572158971375\mathrm{i}}{391663056253676053933850624\left(\frac{230036728532618625}{27975932589548289566703616}+\frac{\sqrt{7}\,181960107187971125\mathrm{i}}{195831528126838026966925312}\right)}+\frac{1572158971375\,\sqrt{7}\,\sqrt{3-2\ldots}}{391663056253676053933850624\left(\frac{23\ldots}{27975\ldots}\right.}\right.}{3373232128}$$

```
[In] int(1/((3 - 2*x)^(11/2)*(x + 2*x^2 + 1)^5),x)
```

```
[Out] (atan(((3 - 2*x)^(1/2)*(7^(1/2)*12577271771i - 149046503977)^(1/2)*15721589
71375i)/(391663056253676053933850624*((7^(1/2)*181960107187971125i)/1958315
2812683802696692531 2 - 230036728532618625/27975932589548289566703616)) - (1
572158971375*7^(1/2)*(3 - 2*x)^(1/2)*(7^(1/2)*12577271771i - 149046503977)^
(1/2))/(391663056253676053933850624*((7^(1/2)*181960107187971125i)/19583152
812683802696692531 2 - 230036728532618625/27975932589548289566703616)))*(7^(
1/2)*12577271771i - 149046503977)^(1/2)*5i)/3373232128 - ((272*x)/441 - (16
4*(2*x - 3)^2)/441 + (1966*(2*x - 3)^3)/3087 - (9091*(2*x - 3)^4)/3087 - (3
2070727*(2*x - 3)^5)/5531904 - (41014777*(2*x - 3)^6)/11063808 - (141921511
*(2*x - 3)^7)/154893312 + (23262655*(2*x - 3)^8)/309786624 + (1571659*(2*x
- 3)^9)/15059072 + (468427*(2*x - 3)^10)/17210368 + (394105*(2*x - 3)^11)/1
20472576 + (38225*(2*x - 3)^12)/240945152 - 520/441)/(38416*(3 - 2*x)^(9/2)
 - 76832*(3 - 2*x)^(11/2) + 68600*(3 - 2*x)^(13/2) - 35672*(3 - 2*x)^(15/2)
 + 11809*(3 - 2*x)^(17/2) - 2548*(3 - 2*x)^(19/2) + 350*(3 - 2*x)^(21/2) -
28*(3 - 2*x)^(23/2) + (3 - 2*x)^(25/2)) - (atan(((3 - 2*x)^(1/2)*(- 7^(1/2)
*12577271771i - 149046503977)^(1/2)*1572158971375i)/(391663056253676053933 8
50624*((7^(1/2)*181960107187971125i)/195831528126838026966925312 + 23003672
8532618625/27975932589548289566703616)) + (1572158971375*7^(1/2)*(3 - 2*x)^
(1/2)*(- 7^(1/2)*12577271771i - 149046503977)^(1/2))/(391663056253676053933
850624*((7^(1/2)*181960107187971125i)/195831528126838026966925312 + 2300367
28532618625/27975932589548289566703616)))*(- 7^(1/2)*12577271771i - 1490465
03977)^(1/2)*5i)/3373232128
```

3.48 $\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 648

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{3029508823715}{1555033025150976(3-2x)^{15/2}} - \frac{13515743021825}{13476952884641792(3-2x)^{13/2}} - \frac{5846828446875}{14513641568075776(3-2x)^{11/2}} - \frac{37283626871975}{261245548225363968(3-2x)^{9/2}} - \frac{132355162272575}{2844673747342852096(3-2x)^{7/2}} - \frac{11557581705725}{812763927812243456(3-2x)^{5/2}} - \frac{46601678385075}{11378694989371408384(3-2x)^{3/2}} - \frac{24229218097975}{227573899787428167768\sqrt{3-2x}} + \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} + \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3} + \frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}\left(1+x+2x^2\right)^2} + \frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(1+x+2x^2\right)} + \frac{11275\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(9756589235+2148932869\sqrt{14}\right)\arctan\left(\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right)}{318603459702399434752} - \frac{11275\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(9756589235+2148932869\sqrt{14}\right)\arctan\left(\frac{\sqrt{7+2\sqrt{14}}+2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right)}{318603459702399434752} + \frac{11275\left(9756589235-2148932869\sqrt{14}\right)\sqrt{\frac{1}{2}\left(-7+2\sqrt{14}\right)}\log\left(3+\sqrt{14}-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right)}{637206919404798869504} - \frac{11275\left(9756589235-2148932869\sqrt{14}\right)\sqrt{\frac{1}{2}\left(-7+2\sqrt{14}\right)}\log\left(3+\sqrt{14}+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right)}{637206919404798869504}$$

```
[Out] 4718120139975/351733660450816/(3-2*x)^(19/2)-815900548375/629418129227776/(
3-2*x)^(17/2)-3029508823715/1555033025150976/(3-2*x)^(15/2)-13515743021825/
13476952884641792/(3-2*x)^(13/2)-5846828446875/14513641568075776/(3-2*x)^(1
1/2)-37283626871975/261245548225363968/(3-2*x)^(9/2)-132355162272575/284467
3747342852096/(3-2*x)^(7/2)-11557581705725/812763927812243456/(3-2*x)^(5/2)
-46601678385075/11378694989371408384/(3-2*x)^(3/2)+1/63*x/(3-2*x)^(19/2)/(2
*x^2+x+1)^9+1/7056*(53+173*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^8+1/691488*(8477+2
1409*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^7+5/6453888*(21409+47471*x)/(3-2*x)^(19/
```

```
2)/(2*x^2+x+1)^6+41/90354432*(47471+92875*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^5+4
1/5059848192*(3436375+5677637*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^4+451/101196963
84*(811091+998691*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^3+451/283351498752*(2896203
9+14627273*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^2+11275/3966920982528*(14627273-35
058731*x)/(3-2*x)^(19/2)/(2*x^2+x+1)-24229218097975/22757389978742816768/(3
-2*x)^(1/2)+11275/1274413838809597739008*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7
+2*14^(1/2))^(1/2))*(9756589235-2148932869*14^(1/2))*(-14+4*14^(1/2))^(1/2)
-11275/1274413838809597739008*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2)
)^(1/2))*(9756589235-2148932869*14^(1/2))*(-14+4*14^(1/2))^(1/2)+11275/6372
06919404798869504*arctan((-2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(
1/2))^(1/2))*(9756589235+2148932869*14^(1/2))*(14+4*14^(1/2))^(1/2)-11275/6
37206919404798869504*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14
^(1/2))^(1/2))*(9756589235+2148932869*14^(1/2))*(14+4*14^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used

$= \{754, 836, 842, 840, 1183, 648, 632, 210, 642\}$

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \frac{11275\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(9756589235+2148932869\sqrt{14}\right)\arctan\left(\frac{\sqrt{7+2\sqrt{\cdots}}}{\sqrt{2\cdots}}\right.}{318603459702399434752}$$
$$-\frac{11275\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(9756589235+2148932869\sqrt{14}\right)\arctan\left(\frac{2\sqrt{3-2x}+\sqrt{7+2\sqrt{14}}}{\sqrt{2\sqrt{14}-7}}\right)}{318603459702399434752}$$
$$+\frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(2x^2+x+1\right)}$$
$$+\frac{451(14627273x+28962039)}{283351498752(3-2x)^{19/2}\left(2x^2+x+1\right)^2}+\frac{451(998691x+811091)}{10119696384(3-2x)^{19/2}\left(2x^2+x+1\right)^3}$$
$$+\frac{41(5677637x+3436375)}{5059848192(3-2x)^{19/2}\left(2x^2+x+1\right)^4}+\frac{41(92875x+47471)}{90354432(3-2x)^{19/2}\left(2x^2+x+1\right)^5}$$
$$+\frac{5(47471x+21409)}{6453888(3-2x)^{19/2}\left(2x^2+x+1\right)^6}+\frac{21409x+8477}{691488(3-2x)^{19/2}\left(2x^2+x+1\right)^7}$$
$$+\frac{173x+53}{7056(3-2x)^{19/2}\left(2x^2+x+1\right)^8}+\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$
$$-\frac{24229218097975}{227573899787428167 68\sqrt{3-2x}}-\frac{46601678385075}{113786949893714083 84(3-2x)^{3/2}}$$
$$-\frac{11557581705725}{812763927812243456(3-2x)^{5/2}}-\frac{132355162272575}{2844673747342852096(3-2x)^{7/2}}$$
$$-\frac{37283626871975}{261245548225363968(3-2x)^{9/2}}-\frac{5846828446875}{14513641568075776(3-2x)^{11/2}}$$
$$-\frac{13515743021825}{13476952884641792(3-2x)^{13/2}}-\frac{3029508823715}{1555033025150976(3-2x)^{15/2}}$$
$$-\frac{815900548375}{629418129227776(3-2x)^{17/2}}+\frac{4718120139975}{351733660450816(3-2x)^{19/2}}$$
$$+\frac{11275\left(9756589235-2148932869\sqrt{14}\right)\sqrt{\frac{1}{2}\left(2\sqrt{14}-7\right)}\log\left(-2x-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3\right)}{637206919404798869504}$$
$$-\frac{11275\left(9756589235-2148932869\sqrt{14}\right)\sqrt{\frac{1}{2}\left(2\sqrt{14}-7\right)}\log\left(-2x+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3\right)}{637206919404798869504}$$

```
[In] Int[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] 4718120139975/(351733660450816*(3 - 2*x)^(19/2)) - 815900548375/(6294181292
27776*(3 - 2*x)^(17/2)) - 3029508823715/(1555033025150976*(3 - 2*x)^(15/2))
 - 13515743021825/(13476952884641792*(3 - 2*x)^(13/2)) - 5846828446875/(145
13641568075776*(3 - 2*x)^(11/2)) - 37283626871975/(261245548225363968*(3 -
2*x)^(9/2)) - 132355162272575/(2844673747342852096*(3 - 2*x)^(7/2)) - 11557
581705725/(812763927812243456*(3 - 2*x)^(5/2)) - 46601678385075/(1137869498
9371408384*(3 - 2*x)^(3/2)) - 24229218097975/(22757389978742816768*Sqrt[3 -
 2*x]) + x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + (53 + 173*x)/(7056*(3
```

```
- 2*x)^(19/2)*(1 + x + 2*x^2)^8) + (8477 + 21409*x)/(691488*(3 - 2*x)^(19/2
)*(1 + x + 2*x^2)^7) + (5*(21409 + 47471*x))/(6453888*(3 - 2*x)^(19/2)*(1 +
 x + 2*x^2)^6) + (41*(47471 + 92875*x))/(90354432*(3 - 2*x)^(19/2)*(1 + x +
 2*x^2)^5) + (41*(3436375 + 5677637*x))/(5059848192*(3 - 2*x)^(19/2)*(1 + x
 + 2*x^2)^4) + (451*(811091 + 998691*x))/(10119696384*(3 - 2*x)^(19/2)*(1 +
 x + 2*x^2)^3) + (451*(28962039 + 14627273*x))/(283351498752*(3 - 2*x)^(19/
2)*(1 + x + 2*x^2)^2) + (11275*(14627273 - 35058731*x))/(3966920982528*(3 -
 2*x)^(19/2)*(1 + x + 2*x^2)) + (11275*Sqrt[(7 + 2*Sqrt[14])/2]*(9756589235
 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqr
t[-7 + 2*Sqrt[14]]])/318603459702399434752 - (11275*Sqrt[(7 + 2*Sqrt[14])/2
]*(9756589235 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[
3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/318603459702399434752 + (11275*(975658923
5 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[
7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504 - (11275*(97565
89235 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + S
qrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
```

```
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 842

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)
^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{\int \frac{1680-1484x}{(3-2x)^{21/2}(1+x+2x^2)^9}\,dx}{1764}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{\int \frac{2534672-3322984x}{(3-2x)^{21/2}(1+x+2x^2)^8}\,dx}{2765952}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} + \frac{\int \frac{3218135760-5287166640x}{(3-2x)^{21/2}(1+x+2x^2)^7}\,dx}{3794886144}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} + \frac{\int \frac{3218122918080-6729253503840x}{(3-2x)^{21/2}(1+x+2x^2)^6}\,dx}{4462786105344}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} + \frac{\int \frac{2223971291223360-6819728658120000x}{(3-2x)^{21/2}(1+x+2x^2)^5}\,dx}{4373530383237120}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} + \frac{\int \frac{602017891719552000-520566411314182464 0x}{(3-2x)^{21/2}(1+x+2x^2)^4}\,dx}{342884782045790208 0}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} + \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3} + \frac{\int \frac{-6440138511651578764 80-2602338158011857027840x}{(3-2x)^{21/2}(1+x+2x^2)^3}\,dx}{20161625184292464230 40}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} + \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3} + \frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}\left(1+x+2x^2\right)^2} + \frac{\int \frac{-781280013553524600192000-460008659488539446208000x}{(3-2x)^{21/2}(1+x+2x^2)^2}\,dx}{7903357072242645978316 80}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} + \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3} + \frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}\left(1+x+2x^2\right)^2} + \frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(1+x+2x^2\right)} + \frac{\int \frac{-20986566494194624791283 2000+32415010207984186772774 4000x}{(3-2x)^{21/2}(1+x+2x^2)}\,dx}{1549057986159558611750092 80}$$

$$
\begin{aligned}
&= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} + \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} \\
&\quad + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} \\
&\quad + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} \\
&\quad + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} \\
&\quad + \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3} \\
&\quad + \frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}\left(1+x+2x^2\right)^2} \\
&\quad + \frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(1+x+2x^2\right)} \\
&\quad + \frac{\int \frac{-2327225523695253718758144000+1105437952711266214715136000x}{(3-2x)^{19/2}(1+x+2x^2)}\,dx}{4337362361246764112900259840} \\
&= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} \\
&\quad + \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} \\
&\quad + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} \\
&\quad + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} \\
&\quad + \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3} \\
&\quad + \frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}\left(1+x+2x^2\right)^2} \\
&\quad + \frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(1+x+2x^2\right)} \\
&\quad + \frac{\int \frac{-2082868009498456217949542400 0-2676274378513417586741760000x}{(3-2x)^{17/2}(1+x+2x^2)}\,dx}{1214461461149093951612072755 20}
\end{aligned}
$$

$$
\begin{aligned}
=& \frac{4718120139975}{351733660450816(3-2x)^{19/2}}-\frac{815900548375}{629418129227776(3-2x)^{17/2}} \\
&-\frac{3029508823715}{1555033025150976(3-2x)^{15/2}}+\frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} \\
&+\frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8}+\frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} \\
&+\frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6}+\frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} \\
&+\frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} \\
&+\frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3} \\
&+\frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}\left(1+x+2x^2\right)^2} \\
&+\frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(1+x+2x^2\right)} \\
&+\frac{\int \frac{-1612768920028496622624798720000-99372366651018754238432256000x}{(3-2x)^{15/2}(1+x+2x^2)}\,dx}{34004920912174630645138037145600} \\
=& \frac{4718120139975}{351733660450816(3-2x)^{19/2}}-\frac{815900548375}{629418129227776(3-2x)^{17/2}} \\
&-\frac{3029508823715}{1555033025150976(3-2x)^{15/2}}-\frac{13515743021825}{13476952884641792(3-2x)^{13/2}} \\
&+\frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9}+\frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} \\
&+\frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7}+\frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} \\
&+\frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5}+\frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} \\
&+\frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3} \\
&+\frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}\left(1+x+2x^2\right)^2} \\
&+\frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(1+x+2x^2\right)} \\
&+\frac{\int \frac{-10914704027207597896229744640000-124134176791751117448051302400x}{(3-2x)^{13/2}(1+x+2x^2)}\,dx}{952137785540889658063865040076800}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} \\
&\quad - \frac{3029508823715}{1555033025150976(3-2x)^{15/2}} - \frac{13515743021825}{13476952884641792(3-2x)^{13/2}} \\
&\quad - \frac{5846828446875}{14513641568075776(3-2x)^{11/2}} + \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} \\
&\quad + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} \\
&\quad + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} \\
&\quad + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} \\
&\quad + \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3} \\
&\quad + \frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}\left(1+x+2x^2\right)^2} \\
&\quad + \frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(1+x+2x^2\right)} \\
&\quad + \frac{\int \frac{-6249079685931055968022769664000-1181393221838810620537497600000x}{(3-2x)^{11/2}(1+x+2x^2)}\,dx}{2665985799514491042578822112215040} \\
&= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} \\
&\quad - \frac{3029508823715}{1555033025150976(3-2x)^{15/2}} - \frac{13515743021825}{13476952884641792(3-2x)^{13/2}} \\
&\quad - \frac{5846828446875}{14513641568075776(3-2x)^{11/2}} - \frac{37283626871975}{261245548225363968(3-2x)^{9/2}} \\
&\quad + \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} \\
&\quad + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} \\
&\quad + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} \\
&\quad + \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3} \\
&\quad + \frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}\left(1+x+2x^2\right)^2} \\
&\quad + \frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(1+x+2x^2\right)} \\
&\quad + \frac{\int \frac{-263647730506722353334322053120000-95879912054052861104340934656000x}{(3-2x)^{9/2}(1+x+2x^2)}\,dx}{746476023864057491922070191420211120}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} \\
&\quad - \frac{3029508823715}{1555033025150976(3-2x)^{15/2}} - \frac{13515743021825}{13476952884641792(3-2x)^{13/2}} \\
&\quad - \frac{5846828446875}{14513641568075776(3-2x)^{11/2}} - \frac{37283626871975}{261245548225363968(3-2x)^{9/2}} \\
&\quad - \frac{132355162272575}{2844673747342852096(3-2x)^{7/2}} + \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} \\
&\quad + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} \\
&\quad + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} \\
&\quad + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} \\
&\quad + \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3} \\
&\quad + \frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}\left(1+x+2x^2\right)^2} \\
&\quad + \frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(1+x+2x^2\right)} \\
&\quad + \frac{\int \frac{-1915836029727216045877577318400 0-680738564527006107959774429184000x}{(3-2x)^{7/2}(1+x+2x^2)}\,dx}{20901328668193609773817965359765 91360}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} \\
&\quad - \frac{3029508823715}{1555033025150976(3-2x)^{15/2}} - \frac{13515743021825}{13476952884641792(3-2x)^{13/2}} \\
&\quad - \frac{5846828446875}{14513641568075776(3-2x)^{11/2}} - \frac{37283626871975}{261245548225363968(3-2x)^{9/2}} \\
&\quad - \frac{132355162272575}{2844673747342852096(3-2x)^{7/2}} - \frac{11557581705725}{812763927812243456(3-2x)^{5/2}} \\
&\quad + \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} \\
&\quad + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} \\
&\quad + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} \\
&\quad + \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3} \\
&\quad + \frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}\left(1+x+2x^2\right)^2} \\
&\quad + \frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(1+x+2x^2\right)} \\
&\quad + \frac{\int \frac{120821024667583493224934267289600 0-416106482835112528959374966784000 0x}{(3-2x)^{5/2}(1+x+2x^2)}\,dx}{585237202709421073666903030073445580 80}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} \\
&\quad - \frac{3029508823715}{1555033025150976(3-2x)^{15/2}} - \frac{13515743021825}{13476952884641792(3-2x)^{13/2}} \\
&\quad - \frac{5846828446875}{14513641568075776(3-2x)^{11/2}} - \frac{37283626871975}{261245548225363968(3-2x)^{9/2}} \\
&\quad - \frac{132355162272575}{2844673747342852096(3-2x)^{7/2}} - \frac{11557581705725}{812763927812243456(3-2x)^{5/2}} \\
&\quad - \frac{46601678385075}{11378694989371408384(3-2x)^{3/2}} + \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} \\
&\quad + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} \\
&\quad + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} \\
&\quad + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} \\
&\quad + \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3} \\
&\quad + \frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}\left(1+x+2x^2\right)^2} \\
&\quad + \frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(1+x+2x^2\right)} \\
&\quad + \frac{\int \frac{1798781163010893003718224071884800 0-2013354798340341200856512731545600 0x}{(3-2x)^{3/2}(1+x+2x^2)}\,dx}{16386641675863790062673284842056476 26240}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} \\
&- \frac{3029508823715}{1555033025150976(3-2x)^{15/2}} - \frac{13515743021825}{13476952884641792(3-2x)^{13/2}} \\
&- \frac{5846828446875}{14513641568075776(3-2x)^{11/2}} - \frac{37283626871975}{261245548225363968(3-2x)^{9/2}} \\
&- \frac{132355162272575}{2844673747342852096(3-2x)^{7/2}} - \frac{11557581705725}{812763927812243456(3-2x)^{5/2}} \\
&- \frac{46601678385075}{11378694989371408384(3-2x)^{3/2}} - \frac{24229218097975}{22757389978742816768\sqrt{3-2x}} \\
&+ \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} \\
&+ \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} \\
&+ \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} \\
&+ \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3} \\
&+ \frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}\left(1+x+2x^2\right)^2} \\
&+ \frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(1+x+2x^2\right)} \\
&+ \frac{\int \frac{1841695890076782643145881803816960000-48850041379984751902661801017344000x}{\sqrt{3-2x}\left(1+x+2x^2\right)}\,dx}{4588259669241861217548519755775813353472 0}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} \\
&\quad - \frac{3029508823715}{1555033025150976(3-2x)^{15/2}} - \frac{13515743021825}{13476952884641792(3-2x)^{13/2}} \\
&\quad - \frac{5846828446875}{14513641568075776(3-2x)^{11/2}} - \frac{37283626871975}{261245548225363968(3-2x)^{9/2}} \\
&\quad - \frac{132355162272575}{2844673747342852096(3-2x)^{7/2}} - \frac{11557581705725}{812763927812243456(3-2x)^{5/2}} \\
&\quad - \frac{46601678385075}{11378694989371408384(3-2x)^{3/2}} - \frac{24229218097975}{22757389978742816768\sqrt{3-2x}} \\
&\quad + \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} \\
&\quad + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} \\
&\quad + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} \\
&\quad + \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3} \\
&\quad + \frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}\left(1+x+2x^2\right)^2} \\
&\quad + \frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(1+x+2x^2\right)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-2217890538754022729211909577113600000-488500413799847519026618010173440000x^2}{28-14x^2+2x^4}\,dx, x, \sqrt{3-2x}\right)}{22941298346209306087742598778879066767360}
\end{aligned}
$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}}$$

$$- \frac{3029508823715}{1555033025150976(3-2x)^{15/2}} - \frac{13515743021825}{13476952884641792(3-2x)^{13/2}}$$

$$- \frac{5846828446875}{14513641568075776(3-2x)^{11/2}} - \frac{37283626871975}{261245548225363968(3-2x)^{9/2}}$$

$$- \frac{132355162272575}{2844673747342852096(3-2x)^{7/2}} - \frac{11557581705725}{812763927812243456(3-2x)^{5/2}}$$

$$- \frac{46601678385075}{11378694989371408384(3-2x)^{3/2}} - \frac{24229218097975}{22757389978742816768\sqrt{3-2x}}$$

$$+ \frac{x}{63(3-2x)^{19/2}(1+x+2x^2)^9} + \frac{53+173x}{7056(3-2x)^{19/2}(1+x+2x^2)^8}$$

$$+ \frac{8477+21409x}{691488(3-2x)^{19/2}(1+x+2x^2)^7} + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}(1+x+2x^2)^6}$$

$$+ \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}(1+x+2x^2)^5} + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}(1+x+2x^2)^4}$$

$$+ \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}(1+x+2x^2)^3}$$

$$+ \frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}(1+x+2x^2)^2}$$

$$+ \frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}(1+x+2x^2)}$$

$$+ \frac{\mathrm{Subst}\Big(\int \frac{-22178905387540227292119095771136 0000\sqrt{7+2\sqrt{14}} - (-221789053875402272921190957711360000+4885004137998}{\sqrt{14}-\sqrt{7+2\sqrt{14}}x+x^2}}{917651933848372243509703951155162670694 40\sqrt{14 (7+2}}$$

$$+ \frac{\mathrm{Subst}\Big(\int \frac{-221789053875402272921190957711360000\sqrt{7+2\sqrt{14}} + (-221789053875402272921190957711360000+4885004137998}{\sqrt{14}+\sqrt{7+2\sqrt{14}}x+x^2}}{91765193384837224350970395115516267069440\sqrt{14 (7+2}}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{3029508823715}{1555033025150976(3-2x)^{15/2}} - \frac{13515743021825}{13476952884641792(3-2x)^{13/2}} - \frac{5846828446875}{14513641568075776(3-2x)^{11/2}} - \frac{37283626871975}{261245548225363968(3-2x)^{9/2}} - \frac{132355162272575}{2844673747342852096(3-2x)^{7/2}} - \frac{11557581705725}{812763927812243456(3-2x)^{5/2}} - \frac{46601678385075}{11378694989371408384(3-2x)^{3/2}} - \frac{24229218097975}{22757389978742816768\sqrt{3-2x}} + \frac{x}{63(3-2x)^{19/2}(1+x+2x^2)^9} + \frac{53+173x}{7056(3-2x)^{19/2}(1+x+2x^2)^8} + \frac{8477+21409x}{691488(3-2x)^{19/2}(1+x+2x^2)^7} + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}(1+x+2x^2)^6} + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}(1+x+2x^2)^5} + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}(1+x+2x^2)^4} + \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}(1+x+2x^2)^3} + \frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}(1+x+2x^2)^2} + \frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}(1+x+2x^2)} + \frac{\left(11275\left(9756589235-2148932869\sqrt{14}\right)\right)\operatorname{Subst}\left(\int \frac{-\sqrt{7+2\sqrt{14}}+2x}{\sqrt{14}-\sqrt{7+2\sqrt{14}}x+x^2}\,dx, x, \sqrt{3-2x}\right)}{910295599149712670 72\sqrt{14\left(7+2\sqrt{14}\right)}} - \frac{\left(11275\left(9756589235-2148932869\sqrt{14}\right)\right)\operatorname{Subst}\left(\int \frac{\sqrt{7+2\sqrt{14}}+2x}{\sqrt{14}+\sqrt{7+2\sqrt{14}}x+x^2}\,dx, x, \sqrt{3-2x}\right)}{91029559914971267072\sqrt{14\left(7+2\sqrt{14}\right)}} - \frac{\left(11275\left(30085060166+9756589235\sqrt{14}\right)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{14}-\sqrt{7+2\sqrt{14}}x+x^2}\,dx, x, \sqrt{3-2x}\right)}{1274413838809597739008} - \frac{\left(11275\left(30085060166+9756589235\sqrt{14}\right)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{14}+\sqrt{7+2\sqrt{14}}x+x^2}\,dx, x, \sqrt{3-2x}\right)}{1274413838809597739008}$$

$$
\begin{aligned}
&= \frac{4718120139975}{351733660450816(3-2x)^{19/2}}-\frac{815900548375}{629418129227776(3-2x)^{17/2}}\\
&\quad-\frac{3029508823715}{1555033025150976(3-2x)^{15/2}}-\frac{13515743021825}{13476952884641792(3-2x)^{13/2}}\\
&\quad-\frac{5846828446875}{14513641568075776(3-2x)^{11/2}}-\frac{37283626871975}{261245548225363968(3-2x)^{9/2}}\\
&\quad-\frac{132355162272575}{2844673747342852096(3-2x)^{7/2}}-\frac{11557581705725}{812763927812243456(3-2x)^{5/2}}\\
&\quad-\frac{46601678385075}{11378694989371408384(3-2x)^{3/2}}-\frac{24229218097975}{22757389978742816768\sqrt{3-2x}}\\
&\quad+\frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9}+\frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8}\\
&\quad+\frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7}+\frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6}\\
&\quad+\frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5}+\frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4}\\
&\quad+\frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3}\\
&\quad+\frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}\left(1+x+2x^2\right)^2}\\
&\quad+\frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(1+x+2x^2\right)}\\
&\quad+\frac{11275\left(9756589235-2148932869\sqrt{14}\right)\log\left(3+\sqrt{14}-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right)}{910295599149712670 72\sqrt{14\left(7+2\sqrt{14}\right)}}\\
&\quad-\frac{11275\left(9756589235-2148932869\sqrt{14}\right)\log\left(3+\sqrt{14}+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right)}{91029559914971267072\sqrt{14\left(7+2\sqrt{14}\right)}}\\
&\quad+\frac{\left(11275\left(30085060166+9756589235\sqrt{14}\right)\right)\operatorname{Subst}\left(\int\frac{1}{7-2\sqrt{14}-x^2}\,dx,x,-\sqrt{7+2\sqrt{14}}+2\sqrt{3-2x}\right)}{637206919404798869504}\\
&\quad+\frac{\left(11275\left(30085060166+9756589235\sqrt{14}\right)\right)\operatorname{Subst}\left(\int\frac{1}{7-2\sqrt{14}-x^2}\,dx,x,\sqrt{7+2\sqrt{14}}+2\sqrt{3-2x}\right)}{637206919404798869504}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} \\
&\quad - \frac{3029508823715}{1555033025150976(3-2x)^{15/2}} - \frac{13515743021825}{13476952884641792(3-2x)^{13/2}} \\
&\quad - \frac{5846828446875}{14513641568075776(3-2x)^{11/2}} - \frac{37283626871975}{261245548225363968(3-2x)^{9/2}} \\
&\quad - \frac{132355162272575}{2844673747342852096(3-2x)^{7/2}} - \frac{11557581705725}{812763927812243456(3-2x)^{5/2}} \\
&\quad - \frac{46601678385075}{11378694989371408384(3-2x)^{3/2}} - \frac{24229218097975}{22757389978742816768\sqrt{3-2x}} \\
&\quad + \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} \\
&\quad + \frac{8477+21409x}{691488(3-2x)^{19/2}\left(1+x+2x^2\right)^7} + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}\left(1+x+2x^2\right)^6} \\
&\quad + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}\left(1+x+2x^2\right)^5} + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}\left(1+x+2x^2\right)^4} \\
&\quad + \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}\left(1+x+2x^2\right)^3} \\
&\quad + \frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}\left(1+x+2x^2\right)^2} \\
&\quad + \frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(1+x+2x^2\right)} \\
&\quad + \frac{11275\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(9756589235+2148932869\sqrt{14}\right)\arctan\left(\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right)}{318603459702399434752} \\
&\quad - \frac{11275\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(9756589235+2148932869\sqrt{14}\right)\arctan\left(\frac{\sqrt{7+2\sqrt{14}}+2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right)}{318603459702399434752} \\
&\quad + \frac{11275\left(9756589235-2148932869\sqrt{14}\right)\log\left(3+\sqrt{14}-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right)}{910295599149712670 72\sqrt{14\left(7+2\sqrt{14}\right)}} \\
&\quad - \frac{11275\left(9756589235-2148932869\sqrt{14}\right)\log\left(3+\sqrt{14}+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right)}{91029559914971267072\sqrt{14\left(7+2\sqrt{14}\right)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.41 (sec) , antiderivative size = 610, normalized size of antiderivative = 0.94

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{\frac{20776+67816x}{1568(3-2x)^{19/2}(1+x+2x^2)^8} + \frac{\frac{46521776+117492592x}{1372(3-2x)^{19/2}\left(1+x+2x^2\right)^7} + \frac{\frac{74020332960+164128134240x}{1176(3-2x)^{19/2}\left(1+x+2x^2\right)^6} + \frac{\frac{94209549053760+184316990760000x}{980(3-2x)^{19/2}\left(1+x+2x^2\right)^5} + \frac{1}{980}\left(\frac{954762012136800\cdots}{784(3-2x\cdots}\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

```
[In] Integrate[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + ((20776 + 67816*x)/(1568*(3 - 2
*x)^(19/2)*(1 + x + 2*x^2)^8) + ((46521776 + 117492592*x)/(1372*(3 - 2*x)^(
19/2)*(1 + x + 2*x^2)^7) + ((74020332960 + 164128134240*x)/(1176*(3 - 2*x)^
(19/2)*(1 + x + 2*x^2)^6) + ((94209549053760 + 184316990760000*x)/(980*(3 -
 2*x)^(19/2)*(1 + x + 2*x^2)^5) + ((95476201213680000 + 157747397367934080*
x)/(784*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^4) + ((72879297583985544960 + 8973
5798552133000960*x)/(588*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^3) + ((3643273421
2165998389760 + 18400346379541577848320*x)/(392*(3 - 2*x)^(19/2)*(1 + x + 2
*x^2)^2) + ((6440121232839552246912000 - 1543571914665913655846400 0*x)/(196
*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)) + (394799268825452219541120000/(19*(3 - 2
*x)^(19/2)) + (-90802166413848096693024 0000/(17*(3 - 2*x)^(17/2)) + (-19105
520493023248582746201600/(3 - 2*x)^(15/2) + (-268495574355372394658843107 20
000/(13*(3 - 2*x)^(13/2)) + (-15099442385859879653927412 0000000/(3 - 2*x)^(
11/2) + (-823771811358751413978497661984 0000/(3 - 2*x)^(9/2) + (-3383893120
365604664600440728470400 00/(3 - 2*x)^(7/2) + (-101353055285765105508363945 1
5648960000/(3 - 2*x)^(5/2) + (-2043343757384956488128059567910736 00000/(3 -
 2*x)^(3/2) + (-223099486651988979682856103640622880 0000/Sqrt[3 - 2*x] + ((
Sqrt[(7 - I*Sqrt[7])/2]*(-31233928131278457155599854509687203200000 - (7175
059724092334984605434771301389120 0000*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 -
 2*x])/Sqrt[7 - I*Sqrt[7]]])/(-14 + (2*I)*Sqrt[7]) + (Sqrt[(7 + I*Sqrt[7])/
2]*(-31233928131278457155599854509687203200000 + (717505972409233498460543 4
771301389120 0000*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 + I*Sqr
t[7]]])/(-14 - (2*I)*Sqrt[7]))/7)/42)/70)/98)/126)/154)/182)/210)/238)/266)
/196)/392)/588)/784)/980)/1176)/1372)/1568)/1764
```

Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.54

method	result
pseudoelliptic	$\frac{11275\left(x-\frac{3}{2}\right)^{9}\left(\sqrt{7+2\sqrt{14}}\left(18352320711\sqrt{14}-69111417106\right)\left(\ln\left(3-2x+\sqrt{14}-\sqrt{3-2x}\,\sqrt{7+2\sqrt{14}}\right)-\ln\left(3-2x+\sqrt{14}+\sqrt{3-2x}\,\sqrt{7+2\sqrt{14}}\right.\right.\right.}{48615}$
derivativedivides	$\frac{9364999706478908741137(3-2x)^{\frac{5}{2}}}{2048}-\frac{23851905772903279054347(3-2x)^{\frac{7}{2}}}{4096}+\frac{19298361379538354104131 7(3-2x)^{\frac{9}{2}}}{36864}-\frac{5775842147534844}{163}$
default	$\frac{9364999706478908741137(3-2x)^{\frac{5}{2}}}{2048}-\frac{23851905772903279054347(3-2x)^{\frac{7}{2}}}{4096}+\frac{192983613795383541041317(3-2x)^{\frac{9}{2}}}{36864}-\frac{5775842147534844}{163}$
trager	Expression too large to display
risch	$-\frac{2400312049377144274944 00x^{27}-26219489415962370637824 00x^{26}+12365045055896811105484800x^{25}-3396989006}{}$

[In] int(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x,method=_RETURNVERBOSE)

[Out] 11275/4861502986181632*((x-3/2)^9*((7+2*14^(1/2))^(1/2)*(18352320711*14^(1/2)-69111417106)*(ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))-ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2)))*(-7+2*14^(1/2))^(1/2)+2*(9756589235*14^(1/2)+30085060166)*(arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))+arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))))*(x^2+1/2*x+1/2)^9*(3-2*x)^(1/2)+120340240664*(-7+2*14^(1/2))^(1/2)*(-9651082208977600419673/2667013388196826972 16*x^6+22855828001615591421921/2667013388196826972 1600*x+18366515291389111126934 63/1200156024688572137472 00*x^5-13499839368250736834249 3/4000520082295240458240 0*x^4+996043194154916251217/175461407118212300800*x^3+847930065890931816713/18184182192251092992 00*x^2-465471892878599/515743888560*x^20-38014370445393391293/31762259291597120*x^18-29920886744155956452 3/254098074332776960*x^16-20254438577741909746663/8131138378648862720 0*x^10-11290018748073034054 53/2032784594662215680*x^12-10928359993529274103333/11434413344974963200*x^14+193096157908388472533/152458844599666176*x^17+271924352600651293/257184285761920*x^19+3324327068969447/4916758404272*x^21-63047885074067/141829569354*x^22+21065437682057/77361583284*x^23-1216492052933/8595731476*x^24+112774755927521146576673/650491070291909017600*x^9+121988968955425435853 63/286981354540548096 00*x^11+219321255455557633732 43/3049176891993323520 0*x^13+297725881275469254209863/66675334704920674304 00*x^7-22151710773233080936621 1/19514732108757270528 00*x^8+442803288917/8595731476*x^25+14420378290318520107 1/133735828596198400*x^15-23473582374/2148932869*x^26+x^27-5427130111302619721 93/2667013388196826972 1600))/(3-2*x)^(19/2)/(-7+2*14^(1/2))^(1/2)/(2*x^2+x+1)^9

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1005, normalized size of antiderivative = 1.55

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="fricas")
```

```
[Out] 1/240806866912267540774256640*(37791*(524288*x^28 - 5505024*x^27 + 24772608*
x^26 - 64684032*x^25 + 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 3
86777088*x^21 + 449261568*x^20 - 515594240*x^19 + 540503040*x^18 - 49658112
0*x^17 + 467712000*x^16 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13
+ 186495624*x^12 - 105219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065
*x^8 - 20036484*x^7 + 5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3
+ 826686*x^2 + 137781*x + 59049)*sqrt(388244949319992410911898l875*I*sqrt(7
) - 2914994336158615430697241206625)*log(sqrt(38824494931999241091189818 75*I
*sqrt(7) - 2914994336158615430697241206625)*(18352320711*I*sqrt(7) - 9756589
235) + 138279123449649741430784000*sqrt(-2*x + 3)) - 37791*(524288*x^28 - 55
05024*x^27 + 24772608*x^26 - 64684032*x^25 + 119734272*x^24 - 194052096*x^2
3 + 295206912*x^22 - 386777088*x^21 + 449261568*x^20 - 515594240*x^19 + 540
503040*x^18 - 496581120*x^17 + 467712000*x^16 - 411828480*x^15 + 303534720*
x^14 - 248434368*x^13 + 186495624*x^12 - 105219828*x^11 + 83621482*x^10 - 4
9793667*x^9 + 19105065*x^8 - 20036484*x^7 + 5497632*x^6 - 2235114*x^5 + 327
6126*x^4 + 734832*x^3 + 826686*x^2 + 137781*x + 59049)*sqrt(388244949319992
4109118981875*I*sqrt(7) - 2914994336158615430697241206625)*log(sqrt(38824494
9319992410911898l875*I*sqrt(7) - 2914994336158615430697241206625)*(-18352320
711*I*sqrt(7) + 9756589235) + 138279123449649741430784000*sqrt(-2*x + 3)) -
37791*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 64684032*x^25 + 1197342
72*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^21 + 449261568*x^20
 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 467712000*x^16 - 4118
28480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624*x^12 - 105219828*x
^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 20036484*x^7 + 5497632*
x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 + 137781*x + 5904
9)*sqrt(-3882449493199924109118981875*I*sqrt(7) - 291499433615861543069724l
206625)*log((18352320711*I*sqrt(7) + 9756589235)*sqrt(-3882449493199924109l1
8981875*I*sqrt(7) - 2914994336158615430697241206625) + 138279123449649741430
78400*sqrt(-2*x + 3)) + 37791*(524288*x^28 - 5505024*x^27 + 24772608*x^26 -
 64684032*x^25 + 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 3867770
88*x^21 + 449261568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17
 + 467712000*x^16 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 + 1864
95624*x^12 - 105219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 -
 20036484*x^7 + 5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 8266
86*x^2 + 137781*x + 59049)*sqrt(-3882449493199924109118981875*I*sqrt(7) - 2
```

```
91499433615861543069724120625)*log((-18352320711*I*sqrt(7) - 9756589235)*sq
rt(-3882449493199924109118981875*I*sqrt(7) - 291499433615861543069724120625
) + 13827912344964974143078400*sqrt(-2*x + 3)) + 28*(2400312049377144274944
00*x^27 - 2621948941596237063782400*x^26 + 12365045055896811105484800*x^25
- 33969890064381284111155200*x^24 + 65360120291258796757811200*x^23 - 10670
1725825102321939251200*x^22 + 162290307223249502039654400*x^21 - 2166342283
26470609547509760*x^20 + 253788172995391086570485760*x^19 - 287279159180291
305208156160*x^18 + 304010591010966811155955200*x^17 - 28264466453999482703
1006720*x^16 + 258819256815163249845447936*x^15 - 2294081329841665219771663
36*x^14 + 172649692294614969274168896*x^13 - 133312541377246386115890240*x^
12 + 102031573634317834547976132*x^11 - 59791102681494117572149176*x^10 + 4
1613884937255303086792337*x^9 - 27246604251076689552043953*x^8 + 1071813172
5916893151555068*x^7 - 8685973988079840377705700*x^6 + 36733030582778222253
86926*x^5 - 809990362095044210054958*x^4 + 1362587089603925431664856*x^3 +
111926768697602999806116*x^2 + 205702452014540322797289*x - 488441710017235
7749737)*sqrt(-2*x + 3))/(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 6468
4032*x^25 + 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^
21 + 449261568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 46
7712000*x^16 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624
*x^12 - 105219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 2003
6484*x^7 + 5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^
2 + 137781*x + 59049)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(3-2*x)**(21/2)/(2*x**2+x+1)**10,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \int \frac{1}{(2\,x^2+x+1)^{10}(-2\,x+3)^{\frac{21}{2}}}\,dx$$

```
[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="maxima")
[Out] integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1000 vs. $2(491) = 982$.

Time = 0.76 (sec) , antiderivative size = 1000, normalized size of antiderivative = 1.54

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="giac")
```

```
[Out] -11275/1427343499466749467688 96*sqrt(7)*(6446798607*14^(3/4)*sqrt(7)*(sqrt(
14) + 4)*sqrt(-2*sqrt(14) + 8) + 2148932869*14^(3/4)*sqrt(7)*(sqrt(14) - 4)
*sqrt(-2*sqrt(14) + 8) - 15042530083*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14
) + 4) - 45127590249*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 7805271
3880*14^(1/4)*sqrt(7)*sqrt(-2*sqrt(14) + 8) - 546368997160*14^(1/4)*sqrt(2*
sqrt(14) + 8))*arctan(1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4)
+ 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)) - 11275/14273434994667494676
8896*sqrt(7)*(6446798607*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) +
 8) + 2148932869*14^(3/4)*sqrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) - 15
042530083*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) - 45127590249*14^(3/
4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 78052713880*14^(1/4)*sqrt(7)*sqrt(
-2*sqrt(14) + 8) - 546368997160*14^(1/4)*sqrt(2*sqrt(14) + 8))*arctan(-1/28
*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(-
1/8*sqrt(14) + 1/2)) - 11275/285468699893349893537792*sqrt(7)*(2148932869*1
4^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 6446798607*14^(3/4)*s
qrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 45127590249*14^(3/4)*(sqrt(14)
 + 4)*sqrt(-2*sqrt(14) + 8) + 15042530083*14^(3/4)*(sqrt(14) - 4)*sqrt(-2*s
qrt(14) + 8) + 78052713880*14^(1/4)*sqrt(7)*sqrt(2*sqrt(14) + 8) + 54636899
7160*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(14^(1/4)*sqrt(1/2)*sqrt(-2*x + 3)*
sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 11275/285468699893349893537792*s
qrt(7)*(2148932869*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 6
446798607*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 4512759024
9*14^(3/4)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) + 15042530083*14^(3/4)*(sqr
t(14) - 4)*sqrt(-2*sqrt(14) + 8) + 78052713880*14^(1/4)*sqrt(7)*sqrt(2*sqrt
(14) + 8) + 546368997160*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(-14^(1/4)*sqrt
(1/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 1/204816509
808685350912*(232787883652335*(2*x - 3)^17*sqrt(-2*x + 3) + 138201066680105
55*(2*x - 3)^16*sqrt(-2*x + 3) + 389618236717151904*(2*x - 3)^15*sqrt(-2*x
+ 3) + 6925854690067471092*(2*x - 3)^14*sqrt(-2*x + 3) + 869247176222685156
82*(2*x - 3)^13*sqrt(-2*x + 3) + 817308030405306394458*(2*x - 3)^12*sqrt(-2
*x + 3) + 5960699611609964201316*(2*x - 3)^11*sqrt(-2*x + 3) + 344385392534
55396724476*(2*x - 3)^10*sqrt(-2*x + 3) + 159569809573892673649239*(2*x - 3
)^9*sqrt(-2*x + 3) + 596312099501239401271299*(2*x - 3)^8*sqrt(-2*x + 3) +
1797250621001927736488676*(2*x - 3)^7*sqrt(-2*x + 3) + 4343978582610098069б
31672*(2*x - 3)^6*sqrt(-2*x + 3) + 8317212692450176764092592*(2*x - 3)^5*sq
```

```
rt(-2*x + 3) + 12350951282904546626644288*(2*x - 3)^4*sqrt(-2*x + 3) + 1373
869772519228873530387 2*(2*x - 3)^3*sqrt(-2*x + 3) + 1078847966186370286978 9
824*(2*x - 3)^2*sqrt(-2*x + 3) - 5340653236079401357791744*(-2*x + 3)^(3/2)
 + 1255138952440667471476992*sqrt(-2*x + 3))/((2*x - 3)^2 + 14*x - 7)^9 + 1
/3280733202692679552*(235862511885*(2*x - 3)^9 - 107316677325*(2*x - 3)^8 +
 80348352084*(2*x - 3)^7 - 64554208290*(2*x - 3)^6 + 49954696792*(2*x - 3)^
5 - 35035280280*(2*x - 3)^4 + 21058773120*(2*x - 3)^3 - 10093321056*(2*x -
3)^2 + 6831901440*x - 10859127552)/((2*x - 3)^9*sqrt(-2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 567, normalized size of antiderivative = 0.88

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \text{Too large to display}$$

```
[In] int(1/((3 - 2*x)^(21/2)*(x + 2*x^2 + 1)^10),x)
```

```
[Out] ((184192*(2*x - 3)^2)/47481 - (18944*x)/2261 - (15552*(2*x - 3)^3)/4199 + (
5666272*(2*x - 3)^4)/1440257 - (63490768*(2*x - 3)^5)/12962313 + (533495672
*(2*x - 3)^6)/70572593 - (1111521492*(2*x - 3)^7)/70572593 + (78007323158*(
2*x - 3)^8)/1482024453 - (250239440467*(2*x - 3)^9)/494008151 + (1118693654
785651073*(2*x - 3)^10)/453254454575104 + (16243004501522493 01*(2*x - 3)^11
)/97125954551808 + (35048653520674948897*(2*x - 3)^12)/906508909150208 + (9
552751196743757791 5*(2*x - 3)^13)/1813017818300416 + (56406629997314156105 4
7*(2*x - 3)^14)/114220122552926208 + (173714228876444750014 9*(2*x - 3)^15)/
50764498912411648 + (1297121066722909760105 5*(2*x - 3)^16)/7107029847737630
72 + (32723441206946795665235*(2*x - 3)^17)/4264217908642578432 + (10264579
7034777710681325*(2*x - 3)^18)/39799367147330732032 + (14609317874302006653
15*(2*x - 3)^19)/20947035340700385 28 + (687618468821894139745*(2*x - 3)^20)
/452825616923964 2112 + (39968995676603847725*(2*x - 3)^21)/1509418723079880
704 + (5940132943613849875*(2*x - 3)^22)/1625527855624486912 + (57179785036
20010375*(2*x - 3)^23)/14629750700620382208 + (178056995818325525*(2*x - 3)
^24)/5689347494685704192 + (179665281323275*(2*x - 3)^25)/1015954909765304 3
2 + (1433237383402275*(2*x - 3)^26)/227573899787428167 68 + (24229218097975*
(2*x - 3)^27)/22757389978742816768 + 37120/2261)/(20661046784*(3 - 2*x)^(19
/2) - 92974710528*(3 - 2*x)^(21/2) + 199231522560*(3 - 2*x)^(23/2) - 270069
397248*(3 - 2*x)^(25/2) + 259475340096*(3 - 2*x)^(27/2) - 187609683744*(3 -
 2*x)^(29/2) + 105782451264*(3 - 2*x)^(31/2) - 47554666992*(3 - 2*x)^(33/2)
 + 17278167438*(3 - 2*x)^(35/2) - 5111496103*(3 - 2*x)^(37/2) + 1234154817*
(3 - 2*x)^(39/2) - 242625852*(3 - 2*x)^(41/2) + 38550456*(3 - 2*x)^(43/2) -
 4883634*(3 - 2*x)^(45/2) + 482454*(3 - 2*x)^(47/2) - 35868*(3 - 2*x)^(49/2
) + 1890*(3 - 2*x)^(51/2) - 63*(3 - 2*x)^(53/2) + (3 - 2*x)^(55/2)) - (atan
(((- 7^(1/2)*30540258843957888971i - 229300295369923682239 3)^(1/2)*(3 - 2*x
)^(1/2)*43774618035829144330316520640625i)/(330008698047761583560870082619 2
```

6380643009360058915812383l296*((7^(1/2)*42709096709460747387242744942497717
8671875i)/165004349023880791780435041309631903215046800294579061915648 + 80
33658291950613455506761069384011754843 75/23572049860554398825776434472804 55
760214954289922558027366 4)) + (43774618035829144330316520640625*7^(1/2)*(-
7^(1/2)*30540258843957888971i - 22930029536992368223 93)^(1/2)*(3 - 2*x)^(1/
2))/(33000869804776158356087008261926380643009360058915812383l296*((7^(1/2)
*42709096709460747387242744942497717867187 5i)/165004349023880791780435041 30
963190321504680029457906191564 8 + 803365829195061345550676106938401175484 37
5/2357204986055439882577643447280455760214954289922558027366 4)))*(- 7^(1/2)
*30540258843957888971i - 22930029536992368223 93)^(1/2)*11275i)/318603459702
399434752 + (atan(((7^(1/2)*30540258843957888971i - 22930029536992368223 93)
^(1/2)*(3 - 2*x)^(1/2)*43774618035829144330316520640625i)/(3300086980477615
8356087008261926380643009360058915812383l296*((7^(1/2)*42709096709460747387
242744942497717867187 5i)/16500434902388079178043504130963190321504680029457
9061915648 - 80336582919506134555067610693840117548437 5/235720498605543988 2
57764344728045576021495428992255802736 64)) - (4377461803582914433031652064 0
625*7^(1/2)*(7^(1/2)*30540258843957888971i - 22930029536992368223 93)^(1/2)*
(3 - 2*x)^(1/2))/(3300086980477615835608700826192638064300936005891581238 31
296*((7^(1/2)*42709096709460747387242744942497717867187 5i)/1650043490238807
9178043504130963190321504680029457906191564 8 - 8033658291950613455506761069
3840117548437 5/23572049860554398825776434472804557602149542899225580273664)
))*(7^(1/2)*30540258843957888971i - 22930029536992368223 93)^(1/2)*11275i)/3
18603459702399434752

3.49 $\int \frac{1}{(3-2x)^{41/2}(1+x+2x^2)^{20}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 1058

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx = -\frac{1305695962836335553428578425}{106924014357253562723941220352(3-2x)^{39/2}}$$
$$-\frac{39481943432914017403219964 15}{202881463139404195937734623232(3-2x)^{37/2}}$$
$$-\frac{3046882292626202227364808 11}{5373617131800435459972430561 28(3-2x)^{35/2}}$$
$$+\frac{212431584675656745565386292 5}{168885109856585114456276389068 8(3-2x)^{33/2}}$$
$$+\frac{4765751507451411879609592953 5}{666328524343253997036581389598 72(3-2x)^{31/2}}$$
$$+\frac{3491161999397471406217275198 5}{124667917457770102671360389021696(3-2x)^{29/2}}$$
$$+\frac{14906630980879476084301740482 5}{1624981820656451683095663001731072(3-2x)^{27/2}}$$
$$+\frac{15848613964169066543734380171}{6018451187616487715169122228633 60(3-2x)^{25/2}}$$
$$+\frac{111551682229707742323768911 45}{168516633253261656024735422401740 8(3-2x)^{23/2}}$$
$$+\frac{1401181849809102027247495637 5}{1011099799519569936148412534410444 8(3-2x)^{21/2}}$$
$$+\frac{1734413681498043786619358697 05}{8965084889073520100515924471772610 56(3-2x)^{19/2}}$$
$$-\frac{227240908234699051527135195 45}{160427834857105096535548122126457241 6(3-2x)^{17/2}}$$
$$-\frac{1011902744127796186785732752 45}{396351121411671414970177713488894361 6(3-2x)^{15/2}}$$
$$-\frac{4605031904169582830874393371 35}{343504305223448559640820685023708446 72(3-2x)^{13/2}}$$
$$-\frac{2211619588790911794826342607 495}{406920484649315986036049119181931544576(3-2x)^{11/2}}$$
$$-\frac{1434014675507772476279404370 25}{739855426635119974610998398512602808 32(3-2x)^{9/2}}$$
$$-\frac{4611053278117143010907562317 585}{725058318102417575118778430542350752153 6(3-2x)^{7/2}}$$
$$-\frac{4059653724406305107209268902 27}{207159519457833592891079551583528786329 6(3-2x)^{5/2}}$$
$$-\frac{4986681479187781853417316522 775}{870069981722901090142534116650820902584 32(3-2x)^{3/2}}$$
$$-\frac{927027754781476746208047620505}{58004665448193406009502274443388060172288\sqrt{3-2x}}$$
$$+\frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}}+\frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}}$$
$$+\frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}}+\frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}}$$
$$184959785+429411497x \qquad 41652915209+92630823167x$$

[Out] 115/3248261265098830736532127368829731369648128*ln(3-2*x+14^(1/2)-(3-2*x)^(
1/2)*(7+2*14^(1/2))^(1/2))*(30297118912219360725028693061-80611109111432760
53983022787*14^(1/2))*(-14+4*14^(1/2))^(1/2)-115/32482612650988307365321273
68829731369648128*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(30
297118912219360725028693061-8061110911143276053983022787*14^(1/2))*(-14+4*1
4^(1/2))^(1/2)+115/1624130632549415368266063684414865684824064*arctan((-2*(
3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(3029711891221936
0725028693061+8061110911143276053983022787*14^(1/2))*(14+4*14^(1/2))^(1/2)-
115/1624130632549415368266063684414865684824064*arctan((2*(3-2*x)^(1/2)+(7+
2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(30297118912219360725028693061+80
61110911143276053983022787*14^(1/2))*(14+4*14^(1/2))^(1/2)+23/1576711628592
227945545728*(919498192874055581221+908287136092467468517*x)/(3-2*x)^(39/2)
/(2*x^2+x+1)^7+115/10187982830903626725064704*(908287136092467468517+298281
884944522225747*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^6+23/203759656618072534501294
08*(2599313568802265110081-10426142448623187379187*x)/(3-2*x)^(39/2)/(2*x^2
+x+1)^5-23/20018492580021161284337664*(10426142448623187379187+275137234631
94262383705*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^4-115/76434244396444433994743808*
(26513224428169016478843+30673415406553789342019*x)/(3-2*x)^(39/2)/(2*x^2+x
+1)^3-115/125891696652967303050166272*(88411609113007981044643-571226953624
5152162963*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^2+115/195831528126838026966925312*
(28561347681225760814815+965934812839019490346107*x)/(3-2*x)^(39/2)/(2*x^2+
x+1)+1/133*x/(3-2*x)^(39/2)/(2*x^2+x+1)^19+1/33516*(113+373*x)/(3-2*x)^(39/
2)/(2*x^2+x+1)^18+1/7976808*(40657+107329*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^17+
5/595601664*(751303+1831285*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^16+1/25015269888*
(184959785+429411497*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^15+1/4902992898048*(4165
2915209+92630823167*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^14+1/297448235814912*(287
1555518177+6100156355517*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^13+1/713875765955788
8*(77559130805859+156274047129113*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^12+5/109936
8679571914752*(2656658801194921+5020880176134289*x)/(3-2*x)^(39/2)/(2*x^2+x
+1)^11+1/3420258114223734784*(45187921585208601+78752911037377255*x)/(3-2*x
)^(39/2)/(2*x^2+x+1)^10+1/430952522392190582784*(6063974149878048635+947717
2618423641847*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^9+1/48266682507925345271808*(69
1833601144925854831+919498192874055581221*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^8-1
4340146755077247627940437025/73985542663511997461099839851260280832/(3-2*x
)^(9/2)-4611053278117143010907562317585/72505831810241757511877843054235075
21536/(3-2*x)^(7/2)-405965372440630510720926890227/207159519457833592891079
5515835287863296/(3-2*x)^(5/2)-4986681479187781853417316522775/870069981722
90109014253411665082090258432/(3-2*x)^(3/2)-927027754781476746208047620505/
58004665448193406009502274443388060172288/(3-2*x)^(1/2)+1734413681498043786
61935869705/896508488907352010051592447177261056/(3-2*x)^(19/2)-22724090823
469905152713519545/1604278348571050965355481221264572416/(3-2*x)^(17/2)-101
190274412779618678573275245/3963511214116714149701777134888943616/(3-2*x)^(
15/2)-460503190416958283087439337135/34350430522344855964082068502370844672
/(3-2*x)^(13/2)-2211619588790911794826342607495/406920484649315986036049119
181931544576/(3-2*x)^(11/2)-3948194343291401740321996415/202881463139404195

```
937734623232/(3-2*x)^(37/2)-304688229262620222736480811/5373617131800435459
97243056128/(3-2*x)^(35/2)+2124315846756567455653862925/1688851098565851144
562763890688/(3-2*x)^(33/2)+4765751507451411879609592953 5/66632852434325399
703658138959872/(3-2*x)^(31/2)+3491161999397471406217275198 5/12466791745777
0102671360389021696/(3-2*x)^(29/2)+1490663098087947608430174048 25/162498182
0656451683095663001731072/(3-2*x)^(27/2)+1584861396416906654373438017 1/6018
45118761648771516912222863360/(3-2*x)^(25/2)+1115516822297077423237689114 5/
1685166332532616560247354224017408/(3-2*x)^(23/2)+1401181849809102027247495
6375/10110997995195699361484125344104448/(3-2*x)^(21/2)-1305695962836335553
4285785425/1069240143572535627239412203 52/(3-2*x)^(39/2)
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 1058, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used

$= \{754, 836, 842, 840, 1183, 648, 632, 210, 642\}$

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx = \frac{23(25993135688022651100\,81-10426142448623187379187x)}{203759656618072534501294\,08(3-2x)^{39/2}\left(2x^2+x+1\right)^5}$$

$$+\frac{115\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(302971189122193607250286930\,61+806111091114327605398302278\,7\sqrt{14}\right)\arctan\left(\right.}{8120653162747076841330318422074328424120\,32}$$

$$-\frac{115\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(30297118912219360725028693061+8061110911143276053983022787\sqrt{14}\right)\arctan\left(\right.}{812065316274707684133031842207432842412032}$$

$$+\frac{115\left(30297118912219360725028693061-8061110911143276053983022787\sqrt{14}\right)\sqrt{\frac{1}{2}\left(-7+2\sqrt{14}\right)}\log\left(-\right.}{1624130632549415368266063684414865684824064}$$

$$-\frac{115\left(30297118912219360725028693061-8061110911143276053983022787\sqrt{14}\right)\sqrt{\frac{1}{2}\left(-7+2\sqrt{14}\right)}\log\left(-\right.}{1624130632549415368266063684414865684824064}$$

$$-\frac{9270277547814767462080476205\,05}{580046654481934060095022744433880601722\,88\sqrt{3-2x}}+\frac{115(965934812839019490346107x+285613476812257608148\,15)}{195831528126838026966925312(3-2x)^{39/2}\left(2x^2+x+1\right)}$$

$$-\frac{498668147918778185341731652277\,5}{870069981722901090142534116650820902584\,32(3-2x)^{3/2}}-\frac{115(884116091130079810446\,43-571226953624515216296\,3x)}{125891696652967303050166272(3-2x)^{39/2}\left(2x^2+x+1\right)^2}$$

$$-\frac{405965372440630510720926890227}{20715951945783359289107955158352878632\,96(3-2x)^{5/2}}-\frac{115(306734154065537893420\,19x+265132244281690164788\,43)}{764342443964444339947438\,08(3-2x)^{39/2}\left(2x^2+x+1\right)^3}$$

$$-\frac{461105327811714301090756231758\,5}{725058318102417575118778430542350752153\,6(3-2x)^{7/2}}-\frac{23(275137234631942623837\,05x+10426142448623187379187)}{200184925800211612843376\,64(3-2x)^{39/2}\left(2x^2+x+1\right)^4}$$

$$-\frac{143401467550777247627940437025}{7398554266351199746109983985126028083\,2(3-2x)^{9/2}}-\frac{221161958879091179482634260749\,5}{406920484649315986036049119181931544576(3-2x)^{11/2}}$$

$$+\frac{115(298281884944522225747x+908287136092467468517)}{101879828309036267250647\,04(3-2x)^{39/2}\left(2x^2+x+1\right)^6}-\frac{460503190416958283087439337135}{343504305223448559640820685023708446\,72(3-2x)^{13/2}}$$

$$+\frac{23(908287136092467468517x+919498192874055581221)}{157671162859222794554572\,8(3-2x)^{39/2}\left(2x^2+x+1\right)^7}-\frac{101190274412779618678573275245}{39635112141167141497017771348889436\,16(3-2x)^{15/2}}$$

$$+\frac{919498192874055581221x+691833601144925854831}{482666825079253452718\,08(3-2x)^{39/2}\left(2x^2+x+1\right)^8}-\frac{227240908234699051527135195\,45}{160427834857105096535548122126457241\,6(3-2x)^{17/2}}$$

$$+\frac{9477172618423641847x+6063974149878048635}{}$$

[In] Int[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]

[Out] -13056959628363355534285785425/(106924014357253562723941220352*(3 - 2*x)^(39/2)) - 3948194343291401740321996415/(202881463139404195937734623232*(3 - 2*x)^(37/2)) - 304688229262620222736480811/(537361713180043545997243056128*(3 - 2*x)^(35/2)) + 2124315846756567455653862925/(1688851098565851144562763890688*(3 - 2*x)^(33/2)) + 47657515074514118796095929535/(66632852434325399703658138959872*(3 - 2*x)^(31/2)) + 34911619993974714062172751985/(124667917457770102671360389021696*(3 - 2*x)^(29/2)) + 149066309808794760843017404825/(1624981820656451683095663001731072*(3 - 2*x)^(27/2)) + 15848613964169066543734380171/(601845118761648771516912222863360*(3 - 2*x)^(25/2)) + 11155168222970774232376891145/(1685166332532616560247354224017408*(3 - 2*x)^(23/2)) + 14011818498091020272474956375/(10110997995195699361484125344104448*(3 - 2*x)^(21/2)) + 173441368149804378661935869705/(896508488907352010051592447177261056*(3 - 2*x)^(19/2)) - 22724090823469905152713519545/(1604278348571050965355481221264572416*(3 - 2*x)^(17/2)) - 101190274412779618678573275245/(3963511214116714149701777134888943616*(3 - 2*x)^(15/2)) - 460503190416958283087439337135/(34350430522344855964082068502370844672*(3 - 2*x)^(13/2)) - 2211619588790911794826342607495/(406920484649315986036049119181931544576*(3 - 2*x)^(11/2)) - 143401467550777247627940437025/(73985542663511997461099839851260280832*(3 - 2*x)^(9/2)) - 4611053278117143010907562317585/(7250583181024175751187784305423507521536*(3 - 2*x)^(7/2)) - 405965372440630510720926890227/(2071595194578335928910795515835287863296*(3 - 2*x)^(5/2)) - 4986681479187781853417316522775/(87006998172290109014253411665082090258432*(3 - 2*x)^(3/2)) - 927027754781476746208047620505/(58004665448193406009502274443388060172288*Sqrt[3 - 2*x]) + x/(133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + (113 + 373*x)/(33516*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^18) + (40657 + 107329*x)/(7976808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^17) + (5*(751303 + 1831285*x))/(595601664*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + (184959785 + 429411497*x)/(25015269888*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + (41652915209 + 92630823167*x)/(4902992898048*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^14) + (2871555518177 + 6100156355517*x)/(297448235814912*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^13) + (77559130805859 + 156274047129113*x)/(7138757659557888*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^12) + (5*(2656658801194921 + 5020880176134289*x))/(1099368679571914752*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^11) + (45187921585208601 + 78752911037377255*x)/(3420258114223734784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^10) + (6063974149878048635 + 9477172618423641847*x)/(430952522392190582784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^9) + (691833601144925854831 + 919498192874055581221*x)/(48266682507925345271808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^8) + (23*(919498192874055581221 + 908287136092467468517*x))/(1576711628592227945545728*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^7) + (115*(908287136092467468517 + 298281884944522225747*x))/(10187982830903626725064704*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^6) + (23*(2599313568802265110081 - 10426142448623187379187*x))/(20375965661807253450129408*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^5) - (23*(10426142448623187379187 + 27513723463194262383705*x))/(20018492580021161284337664*

```
(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^4) - (115*(265132244281690164788843 + 30673
415406553789342019*x))/(764342443964444339947438808*(3 - 2*x)^(39/2)*(1 + x
+ 2*x^2)^3) - (115*(884116091130079810446443 - 571226953624515216296 3*x))/(1
2589169665296730305016627 2*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^2) + (115*(2856
134768122576081481 5 + 96593481283901949034610 7*x))/(1958315281268380269669 2
5312*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) + (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302
971189122193607250286930 61 + 806111091114327605398302278 7*Sqrt[14])*ArcTan[
(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/8120653162
74707684133031842207432842412032 - (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302971189
122193607250286930 61 + 806111091114327605398302278 7*Sqrt[14])*ArcTan[(Sqrt[
7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/8120653162747076
841330318422074328424120 32 + (115*(302971189122193607250286930 61 - 80611109
111432760539830227 87*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] -
 Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/16241306325494153682660636844 14
865684824064 - (115*(302971189122193607250286930 61 - 806111091114327605398 3
022787*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sq
rt[14]]*Sqrt[3 - 2*x] - 2*x])/162413063254941536826606368441486568 4824064
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
```

```
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 842

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)
^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}}+\frac{\int \frac{3640-3164x}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{19}}\,dx}{3724}\\
&= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}}\\
&\quad+\frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}}+\frac{\int \frac{13067712-15937544x}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{18}}\,dx}{13138272}\\
&= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}}+\frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}}\\
&\quad+\frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}}+\frac{\int \frac{44452059120-61847262960x}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{17}}\,dx}{43776722304}\\
&= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}}+\frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}}\\
&\quad+\frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}}\\
&\quad+\frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}}+\frac{\int \frac{140862854522880-213162453016800x}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{16}}\,dx}{137283801145344}\\
&= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}}+\frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}}\\
&\quad+\frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}}+\frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}}\\
&\quad+\frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}}\\
&\quad+\frac{\int \frac{411257135544050880-672058124080956480x}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{15}}\,dx}{403614375367311360}\\
&= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}}+\frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}}\\
&\quad+\frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}}+\frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}}\\
&\quad+\frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}}\\
&\quad+\frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}}\\
&\quad+\frac{\int \frac{1093943980547175340800-1945933258510113828480x}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{14}}\,dx}{1107517846007902371840}
\end{aligned}
$$

$$= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} + \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} + \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} + \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} + \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} + \frac{\int \frac{261374877121081525893504 0-5150741239627162779559680x}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{13}}\,dx}{2821955471628135243448320}$$

$$= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} + \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} + \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} + \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} + \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} + \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} + \frac{\int \frac{5495319948977447333781657600-12350104150638064023941414400x}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{12}}\,dx}{66372392692693740925904486 40}$$

$$
\begin{aligned}
&= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} \\
&\quad + \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} \\
&\quad + \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} \\
&\quad + \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} \\
&\quad + \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} \\
&\quad + \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} \\
&\quad + \frac{5(2656658801194921+5020880176134289x)}{1099368679571914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}} \\
&\quad + \frac{\int \frac{98301898113008748085789763328000-26468403751104415156166363366400x}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{11}}\,dx}{1430988786454477054362500726784 0}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} \\
&\quad + \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} \\
&\quad + \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} \\
&\quad + \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} \\
&\quad + \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} \\
&\quad + \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} \\
&\quad + \frac{5(2656658801194921+5020880176134289x)}{1099368679571914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}} \\
&\quad + \frac{45187921585208601+78752911037377255x}{34202581142237347 84(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}} \\
&\quad + \frac{\int \frac{1399474425581489939465821055477760 0-49726828476855245625643612682496000x}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{10}}\,dx}{28047380214507750265505014244966400}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} \\
&\quad + \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} \\
&\quad + \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} \\
&\quad + \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} \\
&\quad + \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} \\
&\quad + \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} \\
&\quad + \frac{5(2656658801194921+5020880176134289x)}{1099368679571914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}} \\
&\quad + \frac{45187921585208601+78752911037377255x}{3420258114223734784(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}} \\
&\quad + \frac{606397414987804863 5+9477172618423641847x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}} \\
&\quad + \frac{\int \frac{130685363696294294016406350687022080000-7942607596223171539208915453547632640 0x}{(3-2x)^{41/2}(1+x+2x^2)^9}\,dx}{494755786983916714683508451281207296 00}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} \\
&\quad + \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} \\
&\quad + \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} \\
&\quad + \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} \\
&\quad + \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} \\
&\quad + \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} \\
&\quad + \frac{5(2656658801194921+5020880176134289x)}{1099368679571914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}} \\
&\quad + \frac{45187921585208601+78752911037377255x}{3420258114223734784(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}} \\
&\quad + \frac{6063974149878048635+9477172618423641847x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}} \\
&\quad + \frac{691833601144925854831+919498192874055581221x}{48266682507925345271808(3-2x)^{39/2}\left(1+x+2x^2\right)^{8}} \\
&\quad + \frac{\int \frac{-621663169952319046771764500985957580800-101974001645639894672727830234002797158400x}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{8}}\,dx}{77577707399078140862374125160893304012800}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} \\
&\quad + \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} \\
&\quad + \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} \\
&\quad + \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} \\
&\quad + \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} \\
&\quad + \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} \\
&\quad + \frac{5(2656658801194921+5020880176134289x)}{1099368679571914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}} \\
&\quad + \frac{45187921585208601+78752911037377255x}{3420258114223734784(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}} \\
&\quad + \frac{6063974149878048635+9477172618423641847x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}} \\
&\quad + \frac{69183360114492585483\,1+919498192874055581221x}{4826668250792534527180\,8(3-2x)^{39/2}\left(1+x+2x^2\right)^{8}} \\
&\quad + \frac{23(919498192874055581221+908287136092467468517x)}{157671162859222794554572\,8(3-2x)^{39/2}\left(1+x+2x^2\right)^{7}} \\
&\quad + \frac{\int \frac{-3078105864671671596602401476606673572864000\,0-916649145282190834870577141211481026170880\,00x}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{7}}\,dx}{1064366145515352092631772997207456131055616\,00}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} \\
&\quad + \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} \\
&\quad + \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} \\
&\quad + \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} \\
&\quad + \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} \\
&\quad + \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} \\
&\quad + \frac{5(2656658801194921+5020880176134289x)}{1099368679571914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}} \\
&\quad + \frac{45187921585208601+787529110373772 55x}{34202581142237347 84(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}} \\
&\quad + \frac{606397414987804863 5+94771726184236418 47x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}} \\
&\quad + \frac{691833601144925854831+919498192874055581221x}{4826668250792534527180 8(3-2x)^{39/2}\left(1+x+2x^2\right)^{8}} \\
&\quad + \frac{23(919498192874055581221+908287136092467468517x)}{1576711628592227945545728(3-2x)^{39/2}\left(1+x+2x^2\right)^{7}} \\
&\quad + \frac{115(908287136092467468517+2982818849445222257 47x)}{1018798283090362672506470 4(3-2x)^{39/2}\left(1+x+2x^2\right)^{6}} \\
&\quad + \frac{\int \frac{-644123273236962658865496160692506378950164480 00-257077888385069464279782706390184950104760320 00x}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{6}}\,dx}{125169458712605406093496504471596841012140441600}
\end{aligned}
$$

$$= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}}$$
$$+ \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}}$$
$$+ \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}}$$
$$+ \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}}$$
$$+ \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}}$$
$$+ \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}}$$
$$+ \frac{5(2656658801194921+5020880176134289x)}{1099368679571914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}}$$
$$+ \frac{45187921585208601+78752911037377255x}{342025811422373 4784(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}}$$
$$+ \frac{6063974149878048635+9477172618423641847x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}}$$
$$+ \frac{691833601144925854831+919498192874055581221x}{48266682507925345271808(3-2x)^{39/2}\left(1+x+2x^2\right)^{8}}$$
$$+ \frac{23(919498192874055581221+908287136092467468517x)}{1576711628592227945545728(3-2x)^{39/2}\left(1+x+2x^2\right)^{7}}$$
$$+ \frac{115(908287136092467468517+298281884944522225747x)}{101879828309036267250647 04(3-2x)^{39/2}\left(1+x+2x^2\right)^{6}}$$
$$+ \frac{23(2599313568802265110081-1042614244862318737 9187x)}{20375965661807253450129408(3-2x)^{39/2}\left(1+x+2x^2\right)^{5}}$$
$$+ \frac{\int \frac{-6743098946298747998412424229959756266789467750 4000+82287258915490697105406527276587259062086377472000x}{(3-2x)^{41/2}(1+x+2x^2)^5}\,dx}{122666069538353297971626574382164904191897632768000}$$

$$
\begin{aligned}
&= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} \\
&\quad + \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} \\
&\quad + \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} \\
&\quad + \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} \\
&\quad + \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} \\
&\quad + \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} \\
&\quad + \frac{5(2656658801194921+5020880176134289x)}{1099368679571914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}} \\
&\quad + \frac{45187921585208601+78752911037377255x}{3420258114223734784(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}} \\
&\quad + \frac{6063974149878048635+9477172618423641847x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}} \\
&\quad + \frac{69183360114492585483 1+919498192874055581221x}{4826668250792534527180 8(3-2x)^{39/2}\left(1+x+2x^2\right)^{8}} \\
&\quad + \frac{23(919498192874055581221+9082871360924674685 17x)}{1576711628592227945545728(3-2x)^{39/2}\left(1+x+2x^2\right)^{7}} \\
&\quad + \frac{115(908287136092467468517+2982818849445222257 47x)}{10187982830903626725064704(3-2x)^{39/2}\left(1+x+2x^2\right)^{6}} \\
&\quad + \frac{23(2599313568802265110081-1042614244862318737918 7x)}{20375965661807253450129408(3-2x)^{39/2}\left(1+x+2x^2\right)^{5}} \\
&\quad - \frac{23(10426142448623187379187+27513723463194262383705x)}{200184925800211612843376 64(3-2x)^{39/2}\left(1+x+2x^2\right)^{4}} \\
&\quad + \frac{\int \frac{-12641043879585143204616228153551645061449995714560000+16112473447836751754865201881183052922322379350016000}{(3-2x)^{41/2}\left(1+x+2x^2\right)^4}}{96170198518068985609755234315617284886447744090112000}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} \\
&\quad + \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} \\
&\quad + \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} \\
&\quad + \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} \\
&\quad + \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} \\
&\quad + \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} \\
&\quad + \frac{5(2656658801194921+5020880176134289x)}{1099368679571914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}} \\
&\quad + \frac{451879215852208601+787529110373772 55x}{3420258114223734784(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}} \\
&\quad + \frac{6063974149878048635+9477172618423641847x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}} \\
&\quad + \frac{691833601144925854831+919498192874055581221x}{48266682507925345271808(3-2x)^{39/2}\left(1+x+2x^2\right)^{8}} \\
&\quad + \frac{23(919498192874055581221+908287136092467468517x)}{1576711628592227945545728(3-2x)^{39/2}\left(1+x+2x^2\right)^{7}} \\
&\quad + \frac{115(908287136092467468517+2982818849445222225747x)}{101879828309036267250647 04(3-2x)^{39/2}\left(1+x+2x^2\right)^{6}} \\
&\quad + \frac{23(2599313568802265110081-10426142448623187379187x)}{203759656618072534501294 08(3-2x)^{39/2}\left(1+x+2x^2\right)^{5}} \\
&\quad - \frac{23(10426142448623187379187+27513723463194262383705x)}{20018492580021161284337664(3-2x)^{39/2}\left(1+x+2x^2\right)^{4}} \\
&\quad - \frac{115(2651322442816901647884 3+30673415406553789342019x)}{76434244396444339947438 08(3-2x)^{39/2}\left(1+x+2x^2\right)^{3}} \\
&\quad + \frac{\int \frac{680685412136948809323506537016756753901585947569356800 00+12787508005495093351510874993158860007144091646148608}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{3}}}{565480767286245635385360777775829635132312735249858560 00}
\end{aligned}
$$

$$= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}}$$

$$+ \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}}$$

$$+ \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}}$$

$$+ \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}}$$

$$+ \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}}$$

$$+ \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}}$$

$$+ \frac{5(2656658801194921+5020880176134289x)}{1099368679571914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}}$$

$$+ \frac{4518792158520860l+78752911037377255x}{342025811422373478 4(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}}$$

$$+ \frac{6063974149878048635+94771726184236418 47x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}}$$

$$+ \frac{691833601144925854831+919498192874055581221x}{4826668250792534527180 8(3-2x)^{39/2}\left(1+x+2x^2\right)^{8}}$$

$$+ \frac{23(919498192874055581221+908287136092467468517x)}{1576711628592227945545728(3-2x)^{39/2}\left(1+x+2x^2\right)^{7}}$$

$$+ \frac{115(908287136092467468517+2982818849445222225747x)}{1018798283090362672506470 4(3-2x)^{39/2}\left(1+x+2x^2\right)^{6}}$$

$$+ \frac{23(2599313568802265110081-1042614244862318737918 7x)}{203759656618072534501294 08(3-2x)^{39/2}\left(1+x+2x^2\right)^{5}}$$

$$- \frac{23(10426142448623187379187+27513723463194262383705x)}{200184925800211612843376 64(3-2x)^{39/2}\left(1+x+2x^2\right)^{4}}$$

$$- \frac{115(26513224428169016478843+30673415406553789342019x)}{7643424439644443399474380 8(3-2x)^{39/2}\left(1+x+2x^2\right)^{3}}$$

$$- \frac{115(8841160911300798104464 3-57122695362451521629 63x)}{125891696652967303050166272(3-2x)^{39/2}\left(1+x+2x^2\right)^{2}}$$

$$+ \frac{\int \frac{854141510415132938187788113056367624561750690817251737600 00-520506149463646186024331120721053294659205202300}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{2}}}{2216684607762082890710614248881252169718665922179445555 2000}$$

$$
\begin{aligned}
&= \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} \\
&+ \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} \\
&+ \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} \\
&+ \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} \\
&+ \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} \\
&+ \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} \\
&+ \frac{5(2656658801194921+5020880176134289x)}{1099368679571914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}} \\
&+ \frac{451879215852086 01+787529110373772 55x}{3420258114223734784(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}} \\
&+ \frac{6063974149878048635+9477172618423641847x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}} \\
&+ \frac{691833601144925854831+919498192874055581221x}{48266682507925345271808(3-2x)^{39/2}\left(1+x+2x^2\right)^{8}} \\
&+ \frac{23(919498192874055581221+908287136092467468517x)}{1576711628592227945545728(3-2x)^{39/2}\left(1+x+2x^2\right)^{7}} \\
&+ \frac{115(908287136092467468517+2982818849445222257 47x)}{101879828309036267250647 04(3-2x)^{39/2}\left(1+x+2x^2\right)^{6}} \\
&+ \frac{23(2599313568802265110081-10426142448623187379187x)}{203759656618072534501294 08(3-2x)^{39/2}\left(1+x+2x^2\right)^{5}} \\
&- \frac{23(10426142448623187379187+27513723463194262383705x)}{2001849258002116128433766 4(3-2x)^{39/2}\left(1+x+2x^2\right)^{4}} \\
&- \frac{115(26513224428169016478843+30673415406553789342019x)}{764342443964444339947438 08(3-2x)^{39/2}\left(1+x+2x^2\right)^{3}} \\
&- \frac{115(88411609113007981044643-57122695362451521629 63x)}{125891696652967303050166272(3-2x)^{39/2}\left(1+x+2x^2\right)^{2}} \\
&+ \frac{115(28561347681225760814815+96593481283901949034 6107x)}{1958315281268380269669253 12(3-2x)^{39/2}\left(1+x+2x^2\right)} \\
&+ \frac{\int \frac{672442134057798073201037894709436640625234733259373084672 0000-1010431506935785904117377360118098492110328171 67\cdots}{(3-2x)^{41/2}\left(1+x+2x^2\right)}}{434470183121368246579280392780725425264858520747171328819 2000}
\end{aligned}
$$

$$
\begin{aligned}
= & -\frac{1305695962836335553428578542 5}{106924014357253562723941220352(3-2x)^{39/2}} \\
& + \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} \\
& + \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} \\
& + \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} \\
& + \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} \\
& + \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} \\
& + \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} \\
& + \frac{5(2656658801194921+5020880176134289x)}{1099368679571914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}} \\
& + \frac{45187921585208601+787529110373772 55x}{3420258114223734784(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}} \\
& + \frac{6063974149878048635+9477172618423641847x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}} \\
& + \frac{691833601144925854831+919498192874055581221x}{48266682507925345271808(3-2x)^{39/2}\left(1+x+2x^2\right)^{8}} \\
& + \frac{23(919498192874055581221+908287136092467468517x)}{1576711628592227945545728(3-2x)^{39/2}\left(1+x+2x^2\right)^{7}} \\
& + \frac{115(908287136092467468517+298281884944522225747x)}{1018798283090362672506470 4(3-2x)^{39/2}\left(1+x+2x^2\right)^{6}} \\
& + \frac{23(259931356880226511008 1-1042614244862318737918 7x)}{203759656618072534501294 08(3-2x)^{39/2}\left(1+x+2x^2\right)^{5}} \\
& - \frac{23(10426142448623187379187+27513723463194262383705x)}{200184925800211612843376 64(3-2x)^{39/2}\left(1+x+2x^2\right)^{4}} \\
& - \frac{115(26513224428169016478843+30673415406553789342019x)}{764342443964444339947438 08(3-2x)^{39/2}\left(1+x+2x^2\right)^{3}} \\
& - \frac{115(88411609113007981044643-57122695362451521629 63x)}{125891696652967303050166272(3-2x)^{39/2}\left(1+x+2x^2\right)^{2}} \\
& + \frac{115(285613476812257608148 15+96593481283901949034610 7x)}{195831528126838026966925312(3-2x)^{39/2}\left(1+x+2x^2\right)} \\
& + \frac{\int \frac{2558816721117810266795585036003746296720844129947979756339200 00-5793612187991596195423849002824816296411875 1}{(3-2x)^{39/2}\left(1+x+2x^2\right)}}{1216516512739831090421985099786031190741603858092079720693760 00}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{130569596283633555342857854 25}{106924014357253562723941220352(3-2x)^{39/2}} \\
&- \frac{3948194343291401740321996415}{20288146313940419593773462323 2(3-2x)^{37/2}} \\
&+ \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} \\
&+ \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} \\
&+ \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} \\
&+ \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} \\
&+ \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} \\
&+ \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} \\
&+ \frac{5(2656658801194921+5020880176134289x)}{109936867957191475 2(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}} \\
&+ \frac{45187921585208601+78752911037377255x}{3420258114223734784(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}} \\
&+ \frac{606397414987804863 5+94771726184236418 47x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}} \\
&+ \frac{6918336011449258548 31+919498192874055581221x}{4826668250792534527180 8(3-2x)^{39/2}\left(1+x+2x^2\right)^{8}} \\
&+ \frac{23(919498192874055581221+908287136092467468517x)}{1576711628592227945545728(3-2x)^{39/2}\left(1+x+2x^2\right)^{7}} \\
&+ \frac{115(908287136092467468517+2982818849445222257 47x)}{101879828309036267250647 04(3-2x)^{39/2}\left(1+x+2x^2\right)^{6}} \\
&+ \frac{23(2599313568802265110081-1042614244862318737918 7x)}{203759656618072534501294 08(3-2x)^{39/2}\left(1+x+2x^2\right)^{5}} \\
&- \frac{23(10426142448623187379187+27513723463194262383705x)}{200184925800211612843376 64(3-2x)^{39/2}\left(1+x+2x^2\right)^{4}} \\
&- \frac{115(26513224428169016478843+30673415406553789342019x)}{764342443964444339947438 08(3-2x)^{39/2}\left(1+x+2x^2\right)^{3}} \\
&- \frac{115(88411609113007981044643-57122695362451521629 63x)}{125891696652967303050166272(3-2x)^{39/2}\left(1+x+2x^2\right)^{2}} \\
&+ \frac{115(285613476812257608148 15+96593481283901949034610 7x)}{1958315281268380269669253 12(3-2x)^{39/2}\left(1+x+2x^2\right)} \\
&+ \frac{\int \frac{32057758144925674525212378293679602966590503313019227314585600 00-245264062434783361053607538729339125915878743\cdots}{(3-2x)^{37/2}(1+x+2x^2)}}{3406246235671527053181558279400887334076490802657823217942528000\cdots}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1305695962836335553428578425}{1069240143572535627239412203352(3-2x)^{39/2}} \\
&\quad - \frac{394819434329140174032199 6415}{202881463139404195937734623232(3-2x)^{37/2}} \\
&\quad - \frac{304688229262620222736480811}{537361713180043545997243056128(3-2x)^{35/2}} \\
&\quad + \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} \\
&\quad + \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} \\
&\quad + \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} \\
&\quad + \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} \\
&\quad + \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} \\
&\quad + \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} \\
&\quad + \frac{5(2656658801194921+5020880176134289x)}{109936867957 1914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}} \\
&\quad + \frac{45187921585208601+78752911037377255x}{342025811422373478 4(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}} \\
&\quad + \frac{606397414987804863 5+9477172618423641847x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}} \\
&\quad + \frac{691833601144925854831+919498192874055581221x}{4826668250792534527 1808(3-2x)^{39/2}\left(1+x+2x^2\right)^{8}} \\
&\quad + \frac{23(919498192874055581221+908287136092467468517x)}{15767116285922279455457 28(3-2x)^{39/2}\left(1+x+2x^2\right)^{7}} \\
&\quad + \frac{115(908287136092467468517+2982818849445222225747x)}{10187982830903626725064704(3-2x)^{39/2}\left(1+x+2x^2\right)^{6}} \\
&\quad + \frac{23(25993135688022651100 81-10426142448623187379187x)}{20375965661807253450129408(3-2x)^{39/2}\left(1+x+2x^2\right)^{5}} \\
&\quad - \frac{23(10426142448623187379187+27513723463194262383705x)}{200184925800211612843376 64(3-2x)^{39/2}\left(1+x+2x^2\right)^{4}} \\
&\quad - \frac{115(26513224428169016478843+30673415406553789342019x)}{76434244396444433994743808(3-2x)^{39/2}\left(1+x+2x^2\right)^{3}} \\
&\quad - \frac{115(88411609113007981044643-57122695362451521629 63x)}{125891696652967303050166272(3-2x)^{39/2}\left(1+x+2x^2\right)^{2}} \\
&\quad + \frac{115(28561347681225760814815+96593481283901949034610 7x)}{195831528126838026966925312(3-2x)^{39/2}\left(1+x+2x^2\right)} \\
&\quad + \frac{\int \frac{305514877646362068412420534095304648915899775105182316049203200 00-1892740488116731853131501006288506368 31652}{(3-2x)^{35/2}\left(1+x+2x^2\right)}}{953748945988027574890836318232248453541417424744190501023907840}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{130569596283633555342857854 25}{106924014357253562723941220352(3-2x)^{39/2}} \\
&\quad - \frac{39481943432914017403219964 15}{202881463139404195937734623232(3-2x)^{37/2}} \\
&\quad - \frac{30468822926262022273648081 1}{537361713180043545997243056128(3-2x)^{35/2}} \\
&\quad + \frac{21243158467565674556538629 25}{1688851098565851144562763890688(3-2x)^{33/2}} \\
&\quad + \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} \\
&\quad + \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} \\
&\quad + \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} \\
&\quad + \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} \\
&\quad + \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} \\
&\quad + \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} \\
&\quad + \frac{5(2656658801194921+5020880176134289x)}{109936867957 1914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}} \\
&\quad + \frac{45187921585208601+78752911037377255x}{342025811422373 4784(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}} \\
&\quad + \frac{606397414987804863 5+947717261842364184 7x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}} \\
&\quad + \frac{691833601144925854831+919498192874055581221x}{4826668250792534527180 8(3-2x)^{39/2}\left(1+x+2x^2\right)^{8}} \\
&\quad + \frac{23(919498192874055581221+908287136092467468517x)}{1576711628592227945545728(3-2x)^{39/2}\left(1+x+2x^2\right)^{7}} \\
&\quad + \frac{115(908287136092467468517+2982818849445222257 47x)}{1018798283090362672506470 4(3-2x)^{39/2}\left(1+x+2x^2\right)^{6}} \\
&\quad + \frac{23(2599313568802265110081-1042614244862318737918 7x)}{2037596566180725345012940 8(3-2x)^{39/2}\left(1+x+2x^2\right)^{5}} \\
&\quad - \frac{23(10426142448623187379187+2751372346319426238370 5x)}{2001849258002116128433766 4(3-2x)^{39/2}\left(1+x+2x^2\right)^{4}} \\
&\quad - \frac{115(2651322442816901647884 3+3067341540655378934201 9x)}{764342443964444339947438 08(3-2x)^{39/2}\left(1+x+2x^2\right)^{3}} \\
&\quad - \frac{115(8841160911300798104464 3-5712269536245152162963x)}{125891696652967303050166272(3-2x)^{39/2}\left(1+x+2x^2\right)^{2}} \\
&\quad + \frac{115(28561347681225760814815+96593481283901949034610 7x)}{195831528126838026966925312(3-2x)^{39/2}\left(1+x+2x^2\right)} \\
&\quad + \frac{\int \frac{2481973830933231184361994292888207318693528665943475695072051200 00+1108495081298444362461792076003908213564607\cdots}{(3-2x)^{33/2}\left(1+x+2x^2\right)}}{2670497048766477209694341691050295669915968789283733402866941952\cdots}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1305695962836335553428578542 5}{1069240143572535627239412203 52(3-2x)^{39/2}} \\
&\quad - \frac{394819434329140174032199641 5}{2028814631394041959377346232 32(3-2x)^{37/2}} \\
&\quad - \frac{3046882292626202227364808 11}{5373617131800435459972430561 28(3-2x)^{35/2}} \\
&\quad + \frac{2124315846756567455653862925}{168885109856585114456276389068 8(3-2x)^{33/2}} \\
&\quad + \frac{4765751507451411879609592953 5}{666328524343253997036581389598 72(3-2x)^{31/2}} \\
&\quad + \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} \\
&\quad + \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} \\
&\quad + \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} \\
&\quad + \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} \\
&\quad + \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} \\
&\quad + \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} \\
&\quad + \frac{5(2656658801194921+5020880176134289x)}{1099368679571914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}} \\
&\quad + \frac{45187921585208601+787529110373772 55x}{3420258114223734784(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}} \\
&\quad + \frac{6063974149878048635+9477172618423641847x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}} \\
&\quad + \frac{691833601144925854831+919498192874055581221x}{4826668250792534527180 8(3-2x)^{39/2}\left(1+x+2x^2\right)^{8}} \\
&\quad + \frac{23(919498192874055581221+908287136092467468517x)}{1576711628592227945545728(3-2x)^{39/2}\left(1+x+2x^2\right)^{7}} \\
&\quad + \frac{115(908287136092467468517+2982818849445222225747x)}{10187982830903626725064704(3-2x)^{39/2}\left(1+x+2x^2\right)^{6}} \\
&\quad + \frac{23(2599313568802265110081-10426142448623187379187x)}{20375965661807253450129408(3-2x)^{39/2}\left(1+x+2x^2\right)^{5}} \\
&\quad - \frac{23(10426142448623187379187+27513723463194262383705x)}{20018492580021161284337664(3-2x)^{39/2}\left(1+x+2x^2\right)^{4}} \\
&\quad - \frac{115(26513224428169016478843+30673415406553789342019x)}{76434244396444433994743808(3-2x)^{39/2}\left(1+x+2x^2\right)^{3}} \\
&\quad - \frac{115(88411609113007981044643-57122695362451521629 63x)}{125891696652967303050166272(3-2x)^{39/2}\left(1+x+2x^2\right)^{2}} \\
&\quad + \frac{115(28561347681225760814815+96593481283901949034610 7x)}{195831528126838026966925312(3-2x)^{39/2}\left(1+x+2x^2\right)} \\
&\quad + \frac{\int \frac{17638800484868960749972370191097842122419013917318450032253337600 00+1657886581152359091221872962757627855616}{(3-2x)^{31/2}\left(1+x+2x^2\right)}}{747739173654613618714415673494082787576471260999445352802743746}
\end{aligned}
$$

$$
\begin{aligned}
= &-\frac{130569596283633555342857854 25}{10692401435725356272394122035 2(3-2x)^{39/2}}\\
&-\frac{394819434329140174032199641 5}{20288146313940419593773462323 2(3-2x)^{37/2}}\\
&-\frac{304688229262620222736480811}{53736171318004354599724305612 8(3-2x)^{35/2}}\\
&+\frac{212431584675656745565386292 5}{16888510985658511445627638906 88(3-2x)^{33/2}}\\
&+\frac{476575150745141187960959295 35}{66632852434325399703658138959 872(3-2x)^{31/2}}\\
&+\frac{349116199939747140621727519 85}{12466791745777010267136038902 1696(3-2x)^{29/2}}\\
&+\frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}}+\frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}}\\
&+\frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}}+\frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}}\\
&+\frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}}\\
&+\frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}}\\
&+\frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}}\\
&+\frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}}\\
&+\frac{5(2656658801194921+5020880176134289x)}{1099368679571914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}}\\
&+\frac{45187921585208601+78752911037377255x}{342025811422373478 4(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}}\\
&+\frac{606397414987804863 5+947717261842364184 7x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}}\\
&+\frac{691833601144925854831+919498192874055581221x}{4826668250792534527180 8(3-2x)^{39/2}\left(1+x+2x^2\right)^{8}}\\
&+\frac{23(919498192874055581221+908287136092467468517x)}{157671162859222794554572 8(3-2x)^{39/2}\left(1+x+2x^2\right)^{7}}\\
&+\frac{115(908287136092467468517+2982818849445222257 47x)}{1018798283090362672506470 4(3-2x)^{39/2}\left(1+x+2x^2\right)^{6}}\\
&+\frac{23(2599313568802265110081-1042614244862318737918 7x)}{2037596566180725345012940 8(3-2x)^{39/2}\left(1+x+2x^2\right)^{5}}\\
&-\frac{23(10426142448623187379187+2751372346319426238370 5x)}{2001849258002116128433766 4(3-2x)^{39/2}\left(1+x+2x^2\right)^{4}}\\
&-\frac{115(2651322442816901647884 3+3067341540655378934201 9x)}{7643424439644443399474380 8(3-2x)^{39/2}\left(1+x+2x^2\right)^{3}}\\
&-\frac{115(8841160911300798104464 3-5712269536245152162963x)}{1258916966529673030501662 72(3-2x)^{39/2}\left(1+x+2x^2\right)^{2}}\\
&+\frac{115(2856134768122576081481 5+9659348128390194903461 07x)}{1958315281268380269669253 12(3-2x)^{39/2}\left(1+x+2x^2\right)}
\end{aligned}
$$

$$
\begin{aligned}
= & -\frac{130569596283633555342857854 25}{1069240143572535627239412203 52(3-2x)^{39/2}} \\
& -\frac{394819434329140174032199641 5}{202881463139404195937734623232(3-2x)^{37/2}} \\
& -\frac{304688229262620222736480811}{537361713180043545997243056128(3-2x)^{35/2}} \\
& +\frac{212431584675656745565386292 5}{1688851098565851144562763890688(3-2x)^{33/2}} \\
& +\frac{476575150745141187960959295 35}{66632852434325399703658138959872(3-2x)^{31/2}} \\
& +\frac{349116199939747140621727519 85}{124667917457770102671360389021696(3-2x)^{29/2}} \\
& +\frac{149066309808794760843017404825}{1624981820656451683095663001731072(3-2x)^{27/2}} \\
& +\frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}}+\frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} \\
& +\frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}}+\frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} \\
& +\frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} \\
& +\frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} \\
& +\frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} \\
& +\frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} \\
& +\frac{5(2656658801194921+5020880176134289x)}{1099368679571914752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}} \\
& +\frac{45187921585208601+78752911037377255x}{3420258114223734784(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}} \\
& +\frac{6063974149878048635+9477172618423641847x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}} \\
& +\frac{691833601144925854831+919498192874055581221x}{48266682507925345271808(3-2x)^{39/2}\left(1+x+2x^2\right)^{8}} \\
& +\frac{23(919498192874055581221+908287136092467468517x)}{157671162859222794554 5728(3-2x)^{39/2}\left(1+x+2x^2\right)^{7}} \\
& +\frac{115(908287136092467468517+298281884944522225747x)}{10187982830903626725064704(3-2x)^{39/2}\left(1+x+2x^2\right)^{6}} \\
& +\frac{23(259931356880226511008 1-1042614244862318737918 7x)}{203759656618072534501294 08(3-2x)^{39/2}\left(1+x+2x^2\right)^{5}} \\
& -\frac{23(1042614244862318737918 7+2751372346319426238370 5x)}{200184925800216128433766 4(3-2x)^{39/2}\left(1+x+2x^2\right)^{4}} \\
& -\frac{115(265132244281690164788 43+306734154065537893420 19x)}{764342443964444339947438 08(3-2x)^{39/2}\left(1+x+2x^2\right)^{3}} \\
& -\frac{115(884116091130079810446 43-571226953624515216296 3x)}{125891696652967303050166272(3-2x)^{39/2}\left(1+x+2x^2\right)^{2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1305695962836335553428578542 5}{1069240143572535627239412203 52(3-2x)^{39/2}} \\
&\quad - \frac{39481943432914017403219964 15}{2028814631394041959377346232 32(3-2x)^{37/2}} \\
&\quad - \frac{3046882292626202222736480811}{537361713180043545997243056128(3-2x)^{35/2}} \\
&\quad + \frac{2124315846756567455653862925}{1688851098565851144562763890688(3-2x)^{33/2}} \\
&\quad + \frac{47657515074514118796095929535}{666328524343253997036581389 59872(3-2x)^{31/2}} \\
&\quad + \frac{34911619993974714062172751985}{124667917457770102671360389021696(3-2x)^{29/2}} \\
&\quad + \frac{149066309808794760843017404825}{1624981820656451683095663001731072(3-2x)^{27/2}} \\
&\quad + \frac{15848613964169066543734380171}{601845118761648771516912222863360(3-2x)^{25/2}} \\
&\quad + \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{113+373x}{33516(3-2x)^{39/2}\left(1+x+2x^2\right)^{18}} \\
&\quad + \frac{40657+107329x}{7976808(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{5(751303+1831285x)}{595601664(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} \\
&\quad + \frac{184959785+429411497x}{25015269888(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} \\
&\quad + \frac{41652915209+92630823167x}{4902992898048(3-2x)^{39/2}\left(1+x+2x^2\right)^{14}} \\
&\quad + \frac{2871555518177+6100156355517x}{297448235814912(3-2x)^{39/2}\left(1+x+2x^2\right)^{13}} \\
&\quad + \frac{77559130805859+156274047129113x}{7138757659557888(3-2x)^{39/2}\left(1+x+2x^2\right)^{12}} \\
&\quad + \frac{5(2656658801194921+5020880176134289x)}{109936867957191 4752(3-2x)^{39/2}\left(1+x+2x^2\right)^{11}} \\
&\quad + \frac{451879215852086 01+78752911037377255x}{3420258114223734784(3-2x)^{39/2}\left(1+x+2x^2\right)^{10}} \\
&\quad + \frac{606397414987804 8635+94771726184236418 47x}{430952522392190582784(3-2x)^{39/2}\left(1+x+2x^2\right)^{9}} \\
&\quad + \frac{691833601144925854831+919498192874055581221x}{48266682507925345271808(3-2x)^{39/2}\left(1+x+2x^2\right)^{8}} \\
&\quad + \frac{23(919498192874055581221+908287136092467468517x)}{157671162859222794554 5728(3-2x)^{39/2}\left(1+x+2x^2\right)^{7}} \\
&\quad + \frac{115(908287136092467468517+2982818849445222257 47x)}{101879828309036267250647 04(3-2x)^{39/2}\left(1+x+2x^2\right)^{6}} \\
&\quad + \frac{23(259931356880226511008 1-104261424486231873791 87x)}{203759656618072534501294 08(3-2x)^{39/2}\left(1+x+2x^2\right)^{5}} \\
&\quad - \frac{23(104261424486231873791 87+275137234631942623837 05x)}{200184925800211612843376 64(3-2x)^{39/2}\left(1+x+2x^2\right)^{4}} \\
&\quad - \frac{115(265132244281690164788 43+306734154065537893420 19x)}{764342443964444339947438 08(3-2x)^{39/2}\left(1+x+2x^2\right)^{3}}
\end{aligned}
$$

$= \text{Too large to display}$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 17.33 (sec) , antiderivative size = 1100, normalized size of antiderivative = 1.04

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx = \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}}$$

$$+ \frac{44296+146216x}{3528(3-2x)^{39/2}(1+x+2x^2)^{18}} + \frac{\frac{223125616+589021552x}{3332(3-2x)^{39/2}\left(1+x+2x^2\right)^{17}} + \frac{\frac{865861681440+2110519336800x}{3136(3-2x)^{39/2}\left(1+x+2x^2\right)^{16}} + \frac{\frac{2984274342235200+6928434268875840x}{2940(3-2x)^{39/2}\left(1+x+2x^2\right)^{15}} + \frac{94088137371\ldots}{2744\ldots}}{\ldots}}{\ldots}}{\ldots}$$

```
[In] Integrate[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]
```

```
[Out] x/(133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + ((44296 + 146216*x)/(3528*(3
- 2*x)^(39/2)*(1 + x + 2*x^2)^18) + ((223125616 + 589021552*x)/(3332*(3 - 2
*x)^(39/2)*(1 + x + 2*x^2)^17) + ((865861681440 + 2110519336800*x)/(3136*(3
 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + ((2984274342235200 + 6928434268875840*
x)/(2940*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + ((9408813737133390720 + 209
24013532366815360*x)/(2744*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^14) + ((2724306
5619141593598720 + 57873497074462503141120*x)/(2548*(3 - 2*x)^(39/2)*(1 + x
 + 2*x^2)^13) + ((72110377354780278913835520 + 145295342948683106164016640*
x)/(2352*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^12) + ((172901458108932896335179801600 + 326770416680301421681066214400*x)/(2156*(3 - 2*x)^(39/2)*(1 + x + 2
*x^2)^11) + ((370557652515461812186329087129600 + 645802967231886306826540424448000*x)/(1960*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^10) + ((696175598675973438759010577554944000 + 1088028437838790621809440473088716800*x)/(1764*(3 -
2*x)^(39/2)*(1 + x + 2*x^2)^9) + ((1111965063471244015489248163496668569600
 + 1477884081820868038735185945420330393600*x)/(1568*(3 - 2*x)^(39/2)*(1 +
x + 2*x^2)^8) + ((1427636023038958525418189623276039160217600 + 14102294542802935921085802172484323479552 00*x)/(1372*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
7) + ((1283308803395067168818807997696073436639232000 + 4214391612869991217
```

7013558424620483623731200O*x)/(1176*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^6) + (
(359909043739097249991695788946258930146664448000 - 14436361213243981948316
934609927589309137960960000*x)/(980*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^5) + ((
-115202162481686975947569138187222162686920928460800O - 3040089329780519199
031170166260953381570260254720000*x)/(784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
4) + ((-2255746282697145245681128263365627409125133109002240000 - 260969551
132552925541038265166507347084573298900992000O*x)/(588*(3 - 2*x)^(39/2)*(1
+ x + 2*x^2)^3) + ((-1790251120769313069211522499042240401000172830460805120
0000 + 1156680332141435968942958046046785099242678227333939200000*x)/(392*(3
 - 2*x)^(39/2)*(1 + x + 2*x^2)^2) + ((7287086092491046604340635690094746125
2288728322038169600000 + 246446709008728269296921307345877681002519066261O3
43034880000*x)/(196*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) + (-53055056666589708
7493026465460148012491929957574880460800000/(3 - 2*x)^(39/2) + (-1708089006
2422412644804810732936117697712983887858137533644800000/(37*(3 - 2*x)^(37/2)
) + (-69674095008990920001753978369242721670427118803840269792051200O/(3 -
2*x)^(35/2) + (757366667762147355602446006474261151597409525795681824661504
000000/(3 - 2*x)^(33/2) + (616772664905423340350737254793402194192083509401
0816556282758758400000/(31*(3 - 2*x)^(31/2)) + (980445504127015992472138196
645778610361943940861637274650890661068800000/(29*(3 - 2*x)^(29/2)) + (4496
4233234365801798259356678072391756466292408034159102502223134720000000/(3 -
2*x)^(27/2) + (487904184130260773926886832047572655461484781443782543411352
841560457216000/(3 - 2*x)^(25/2) + (429268867215238023064148871550918822599
025420880676981706228025454182400000000/(3 - 2*x)^(23/2) + (2893692593980364
723231826294558630623656919099359688069727689450554368000000000/(3 - 2*x)^(
21/2) + (118767476492930264374166633243140666046068763101817907661320807641
190359040000000/(3 - 2*x)^(19/2) + (-23130641371662285970537372414163682847
225169124231597674893328104378032537600000000/(3 - 2*x)^(17/2) + (-992239519
653790860422623948957964852355985846800936213338418761762097950023680000000
/(3 - 2*x)^(15/2) + (-10941518315154632243157241587901809625083601209973176
69014678416546026147551232000000000/(3 - 2*x)^(13/2) + (-8073268485314233063
8403379340954315600692165352258493007480189439306347456219136000000000/(3 -
2*x)^(11/2) + (-44337987226211231305207361494572283981715203938096393248399
66665118399975472138240000000000/(3 - 2*x)^(9/2) + (-18330190892216697744173
70679014370008735856157613617875417454472757811732535979192320000000O/(3 -
2*x)^(7/2) + (-553541210002735957048844214716028245499086746401723523324780
66055766166841372505894912000000000/(3 - 2*x)^(5/2) + (-113323856633918397403
43974428370683887566771471384841151672642393999283182139266339840000000000/
(3 - 2*x)^(3/2) + (-1327220262908131487403839635355234271426655189754352930
64356777236410088640362467513344000000000/Sqrt[3 - 2*x] + ((Sqrt[(7 - I*Sqr
t[7])/2]*(-1858108368071384082365375489497327979997317265656094102900994881
309741240965074545186816000000000 - (38534140062781031467679876224014966993
36335555921865837542016885265897482833115690092544000000000*I)*Sqrt[7])*Arc
Tanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 - I*Sqrt[7]]])/(-14 + (2*I)*Sqrt[7]) +
(Sqrt[(7 + I*Sqrt[7])/2]*(-185810836807138408236537548949732797999731726565
60941029009948813097412409650745451868160000000000 + (3853414006278103146767

```
98762240149669933633555592186583754201688526589748283311569009254400000000
*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 + I*Sqrt[7]]])/(-14 - (
2*I)*Sqrt[7]))/7)/42)/70)/98)/126)/154)/182)/210)/238)/266)/294)/322)/350)/
378)/406)/434)/462)/490)/518)/546)/196)/392)/588)/784)/980)/1176)/1372)/156
8)/1764)/1960)/2156)/2352)/2548)/2744)/2940)/3136)/3332)/3528)/3724
```

Maple [A] (verified)

Time = 7.41 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.47

method	result	size
pseudoelliptic	Expression too large to display	502
trager	Expression too large to display	733
risch	Expression too large to display	761
derivativedivides	Expression too large to display	820
default	Expression too large to display	820

```
[In] int(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x,method=_RETURNVERBOSE)
```

```
[Out] 115/5908552821163231304184823545856/(3-2*x)^(39/2)/(-7+2*14^(1/2))^(1/2)*((
x-3/2)^19*(1/2*(7+2*14^(1/2))^(1/2)*(62541562556792464940960784209*14^(1/2)
-234044028404883307655877091262)*(ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(
1/2))^(1/2))-ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2)))*(-7+2*1
4^(1/2))^(1/2)+(302971189122193607250286930610*14^(1/2)+112855552756005864755762319018)*(arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^
(1/2))+arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))
))*(x^2+1/2*x+1/2)^19*(3-2*x)^(1/2)+225711105512011729511524638036*(-7+2*14
^(1/2))^(1/2)*(440996520277951008903098744562486494852026613994907/44191133
0168408577552269171810580697101813940224000*x^6+351285717820454846300261178
0168757024408387405317 1/4517315819499287681645418200730380459262986944512 00*
x+2105745625521655913347866299366467066542716176806 29/188221492479136986735
2257583637658524692911226880 0*x^5+3283584831830973024106759848201028935360 8
1023906041/1129328954874821920411354550182595114815746736128 00*x^4+26422837
2907554078899652568589725089315378041327 65/1806926327799715072658167280292 1
521837051947778048*x^3+10258861744182705485679996525882139164879327899743/4
1066507449993524378594710915730731447845335859200*x^2-339692530351150840696
3020210127759103831979726486117490 49/2202191462005902744802141372856060473 8
90706135449600*x^20-522812506810746872215164433461641835721700045394393837 8
3/9272385103182748399166911043604465153224025833472 00*x^18-2515882948258187
0810753820521494677060478585099733446 1/1487548412275307229812873429455261 78
92872768716800*x^16-2382588328555401515212229149049548745208186041061239 11/
6775973729248931522468127301095570688894480416768 00*x^10-827017337294263956
0376990410516488461130104109549459/5943836604604325896901866053592605867451
298611200*x^12-3542038068347974323734210174480917607561286860807670 53/77269
```

87585985623665972425869670387627686688194560 0*x^14+1832128020262488605142 88
501929804345458085916920873773/5783192372047452224012626430522535438601 2635
13600*x^17+16088554099632166662170276395247605557702557286910897246 99/17617
531696047221958417130982848483791125649083596800*x^19+330991985104115172193
83291043269897730264414699250122 81/1355194745849786304493625460219114137778
8960833536*x^21-41071938883859099778530822150921198736931709708724611 9213/1
101095731002951372401070686428030236945353067724800*x^22+625318899982989425
38387740068578546377843212221911556 9133/110109573100295137240107068642803 02
36945353067724800*x^23-274067096576512309634246914644855503028036706067 4320
773/33299266058556997149225928016976720875363500032 00*x^24+2347963612040894
483703653776453936934180545909453415/27103894916995726089872509204382282755
577921667072*x^9+27439447440933045346986900471010627083603249499481 99/71326
039255251910762822392643111270409415583334 40*x^11+637360073442058123717571 3
799612806445667343432127466 9/2575662528661874555324141956556795875895562731
5200*x^13+410939330411662540158083263982470040269451485756393 97/10163960593
873397283702190951643356033341720625152 00*x^7-50284576958027788162848722514
4/2418333273342982816194906836 1*x^56-12928739610639609510023998542972187988
5913594602271/42976577563946711558994464911811230584954421248 00*x^8+9607924
61248230371780904754517979393899188617856449406597/825821798252213529300803
014821022677709014800793600*x^25-1883534998025385823080659025665371558289 23
16535850547/4353395950637934006519921425971146875574681600*x^30+14196593640
43018931112335188299367292249732216540418143/1390857765477412259875036656 54
066977298360387502080*x^15+97069094157269730492383696446991/483666654668596
563238981367220*x^55-26245938369736234526012047920253051775801543552378884 7
/162819755175909607511987976108245796078275788800*x^26+739179522404372504 68
20643823179286910083882501053857487/34409241593842230387533458950875944904 5
4228336640*x^27-58175485784798347597186133724915 1/483666654668596563238981 3
67220*x^54+109306897181322181669945457975855 1/21496295763048736143954727432
0*x^53-116896892587430274288859944498569 35/709377760180608292750506005256*x
^52+538283761872548702012635418152217439 9/122958811764638770743421040911040
*x^51-11131922352242500061613487752310013471 3/11066293058817489366907893681
99360*x^50+9731082213994105693982824418221611994 93/470317454999743298093585
4814847280*x^49-1258933056799485296542965303408408804684 7/32494660527254991
504647724175308480*x^48+7346222764586479584756115243682891167195 1/109981927
93840150970803845105489024 0*x^47-705751653232795796568704403045357490685791
7/65769192907164102805406993730824363 52*x^46+106569782717850749209997459468
9936795606132653/657691929071641028054069937308243635200*x^45-6835238006675
34227787826206727872897796022179 1/295961368082238462624331471788709635840 0*
x^44+15412073198854459007499596713002894165623265 37/49362356143119571048774
1922753275596800*x^43-171373486311427568922354420956457358202869163931 59/42
571083184949180463883838902087994019225 60*x^42+42029782210469591904656064 39
27269118982162390144 7/847185735023864287838484356260457592422400*x^41-41540
750768017179675819160419113877614943832368997/70951805308248634106473064836
81332336537600*x^40+55803824490321264266984942630037522606224294206455979/8
40069374849663827820641087667869748646051840 0*x^39-36948096352450654613819 2

```
808431464318687931642138663/50913295445434171383069156828355742342184960*x^
38+14197448176524133949804186032458558891454559001223/185969201361373363843
18802095696933945344 00*x^37-22870997009642099691726659303114887864408733274
70459/29476118415777678169145301321679640303370240 0*x^36+16191679600845810 4
277098014234073439700351452727531557/212228052593599282817846169516093410 18
426572800*x^35-2366011645461650212967693020996896818678912962565347/3265046
962978450504889941069478360156681011 20*x^34+3331303172078907621464497191742
22348466351299094658 1/49936012374964537133610863415551390631591936 0*x^33-84
34629516124572789973054877156920000889504501777677/141485368395732855211897
4463440622734561771520*x^32+583189064340477381734933810124991377592806643 87
653599/1131882947165862841695179570752498187649417216 0*x^31+152088307038562
76782109872871420891815553883141484488061/430115519923027879844168236885949
3113067785420800*x^29-6004843304687499790014904282586413326057205713308656 1
63/2150577599615139399220841184429746556533892710400*x^28+x^57+121784713558
23914289546890500249914864097822800 31/451731581949928768164541820073038045 9
26298694451200))/(2*x^2+x+1)^19
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 1905, normalized size of antiderivative = 1.80

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="fricas")
```

```
[Out] 1/336864077912586356135291702496114974019074478550548480*(207411935445*(549
755813888*x^58 - 11269994184704*x^57 + 107064944754688*x^56 - 6306386380062
72*x^55 + 2618521301286912*x^54 - 8342252417974272*x^53 + 21849572376576000
*x^52 - 49684091485814784*x^51 + 101394501297242112*x^50 - 1885833123636183
04*x^49 + 323261995581177856*x^48 - 517079841212727296*x^47 + 7781178962608
12800*x^46 - 1105641165387988992*x^45 + 1491287028233404416*x^44 - 19199296
63119949824*x^43 + 2363050939901804544*x^42 - 2786274020645928960*x^41 + 31
61145685194047488*x^40 - 3453753931369283584*x^39 + 3634098467102523392*x^3
8 - 3697893960325791744*x^37 + 3640651752731836416*x^36 - 34617982122476175
36*x^35 + 3194540251789393920*x^34 - 2861544579495297024*x^33 + 24776329382
17930752*x^32 - 2088430257127768064*x^31 + 1712761005459316736*x^30 - 13554
47485390974976*x^29 + 1048940886155151360*x^28 - 790511024135089152*x^27 +
571750925528393856*x^26 - 408374103192240192*x^25 + 282845069599813728*x^24
 - 186113897194906128*x^23 + 123982890381352520*x^22 - 78116367732251996*x^
21 + 46488580159296898*x^20 - 29591055660829971*x^19 + 16200795673453545*x^
18 - 8941894120163277*x^17 + 5578893209169441*x^16 - 2296849711499532*x^15
+ 1448289882400788*x^14 - 756896247319212*x^13 + 182213447974992*x^12 - 240
```

797810407770*x^11 + 25549234281774*x^10 - 26500281727302*x^9 + 255207013325
82*x^8 + 9965507230260*x^7 + 10389354811164*x^6 + 3755740313808*x^5 + 18206
18017974*x^4 + 463742325333*x^3 + 139858796529*x^2 + 19758444939*x + 348678
4401)*sqrt(10810494129824083725997607827275078715898246902554284468987 5*I*s
qrt(7) - 35007088396631542305692106207357215633834159066833652223179822 5)*l
og(sqrt(108104941298240837259976078272750787158982469025542844689875*I*sqrt
(7) - 350070883966315423056921062073572156338341590668336522231798225)*(625
415625556792464940960784209*I*sqrt(7) - 3029711891221936072502869 3061) + 162
71375727626037643406407392186742716391685886146967607 70560*sqrt(-2*x + 3))
- 207411935445*(549755813888*x^58 - 11269994184704*x^57 + 107064944754688*x
^56 - 630638638006272*x^55 + 2618521301286912*x^54 - 8342252417974272*x^53
+ 21849572376576000*x^52 - 49684091485814784*x^51 + 101394501297242112*x^50
 - 188583312363618304*x^49 + 323261995581177856*x^48 - 517079841212727296*x
^47 + 778117896260812800*x^46 - 1105641165387988992*x^45 + 1491287028233404
416*x^44 - 1919929663119949824*x^43 + 2363050939901804544*x^42 - 2786274020
645928960*x^41 + 3161145685194047488*x^40 - 3453753931369283584*x^39 + 3634
098467102523392*x^38 - 3697893960325791744*x^37 + 3640651752731836416*x^36
- 3461798212247617536*x^35 + 3194540251789393920*x^34 - 2861544579495297024
*x^33 + 2477632938217930752*x^32 - 2088430257127768064*x^31 + 1712761005459
316736*x^30 - 1355447485390974976*x^29 + 1048940886155151360*x^28 - 7905110
24135089152*x^27 + 571750925528393856*x^26 - 408374103192240192*x^25 + 2828
45069599813728*x^24 - 186113897194906128*x^23 + 123982890381352520*x^22 - 7
8116367732251996*x^21 + 46488580159296898*x^20 - 29591055660829971*x^19 + 1
6200795673453545*x^18 - 8941894120163277*x^17 + 5578893209169441*x^16 - 229
6849711499532*x^15 + 1448289882400788*x^14 - 756896247319212*x^13 + 1822134
47974992*x^12 - 240797810407770*x^11 + 25549234281774*x^10 - 26500281727302
*x^9 + 25520701332582*x^8 + 9965507230260*x^7 + 10389354811164*x^6 + 375574
0313808*x^5 + 1820618017974*x^4 + 463742325333*x^3 + 139858796529*x^2 + 197
58444939*x + 3486784401)*sqrt(108104941298240837259976078272750787158982469
025542844689875*I*sqrt(7) - 350070883966315423056921062073572156338341590 66
8336522231798225)*log(sqrt(108104941298240837259976078272750787158982469025
542844689875*I*sqrt(7) - 350070883966315423056921062073572156338341590668 33
6522231798225)*(-625415625556792464940960784209*I*sqrt(7) + 3029711891221936
0725028693061) + 1627137572762603764340640739218674271639168588614696760770
560*sqrt(-2*x + 3)) - 207411935445*(549755813888*x^58 - 11269994184704*x^57
 + 107064944754688*x^56 - 630638638006272*x^55 + 2618521301286912*x^54 - 83
42252417974272*x^53 + 21849572376576000*x^52 - 49684091485814784*x^51 + 101
394501297242112*x^50 - 188583312363618304*x^49 + 323261995581177856*x^48 -
517079841212727296*x^47 + 778117896260812800*x^46 - 1105641165387988992*x^4
5 + 1491287028233404416*x^44 - 1919929663119949824*x^43 + 23630509399018045
44*x^42 - 2786274020645928960*x^41 + 3161145685194047488*x^40 - 34537539313
69283584*x^39 + 3634098467102523392*x^38 - 3697893960325791744*x^37 + 36406
51752731836416*x^36 - 3461798212247617536*x^35 + 3194540251789393920*x^34 -
 2861544579495297024*x^33 + 2477632938217930752*x^32 - 2088430257127768064*
x^31 + 1712761005459316736*x^30 - 1355447485390974976*x^29 + 10489408861551

51360*x^28 - 790511024135089152*x^27 + 571750925528393856*x^26 - 4083741031
92240192*x^25 + 282845069599813728*x^24 - 186113897194906128*x^23 + 1239828
90381352520*x^22 - 78116367732251996*x^21 + 46488580159296898*x^20 - 295910
55660829971*x^19 + 16200795673453545*x^18 - 8941894120163277*x^17 + 5578893
209169441*x^16 - 2296849711499532*x^15 + 1448289882400788*x^14 - 7568962473
19212*x^13 + 182213447974992*x^12 - 240797810407770*x^11 + 25549234281774*x
^10 - 26500281727302*x^9 + 25520701332582*x^8 + 9965507230260*x^7 + 1038935
4811164*x^6 + 3755740313808*x^5 + 1820618017974*x^4 + 463742325333*x^3 + 13
9858796529*x^2 + 19758444939*x + 3486784401)*sqrt(-108104941298240837259976
078272750787158982469025542844689875*I*sqrt(7) - 350070883966315423056921062
073572156338341590668336522231798225)*log((6254156255679246940960784209*I
*sqrt(7) + 30297118912219360725028693061)*sqrt(-108104941298240837259976078
272750787158982469025542844689875*I*sqrt(7) - 35007088396631542305692106207
3572156338341590668336522231798225) + 1627137572762603764340640739218674271
639168588614696760770560*sqrt(-2*x + 3)) + 207411935445*(549755813888*x^58
- 11269994184704*x^57 + 107064944754688*x^56 - 630638638006272*x^55 + 26185
21301286912*x^54 - 8342252417974272*x^53 + 21849572376576000*x^52 - 4968409
1485814784*x^51 + 101394501297242112*x^50 - 188583312363618304*x^49 + 32326
1995581177856*x^48 - 517079841212727296*x^47 + 778117896260812800*x^46 - 11
05641165387988992*x^45 + 1491287028233404416*x^44 - 1919929663119949824*x^4
3 + 2363050939901804544*x^42 - 2786274020645928960*x^41 + 31611456851940474
88*x^40 - 3453753931369283584*x^39 + 3634098467102523392*x^38 - 36978939603
25791744*x^37 + 3640651752731836416*x^36 - 3461798212247617536*x^35 + 31945
40251789393920*x^34 - 2861544579495297024*x^33 + 2477632938217930752*x^32 -
2088430257127768064*x^31 + 1712761005459316736*x^30 - 1355447485390974976*
x^29 + 1048940886155151360*x^28 - 790511024135089152*x^27 + 571750925528393
856*x^26 - 408374103192240192*x^25 + 282845069599813728*x^24 - 186113897194
906128*x^23 + 123982890381352520*x^22 - 78116367732251996*x^21 + 4648858015
9296898*x^20 - 29591055660829971*x^19 + 16200795673453545*x^18 - 8941894120
163277*x^17 + 5578893209169441*x^16 - 2296849711499532*x^15 + 1448289882400
788*x^14 - 756896247319212*x^13 + 182213447974992*x^12 - 240797810407770*x^
11 + 25549234281774*x^10 - 26500281727302*x^9 + 25520701332582*x^8 + 996550
7230260*x^7 + 10389354811164*x^6 + 3755740313808*x^5 + 1820618017974*x^4 +
463742325333*x^3 + 139858796529*x^2 + 19758444939*x + 3486784401)*sqrt(-108
104941298240837259976078272750787158982469025542844689875*I*sqrt(7) - 35007
0883966315423056921062073572156338341590668336522231798225)*log((-625415625
5679246940960784209*I*sqrt(7) - 30297118912219360725028693061)*sqrt(-10810
4941298240837259976078272750787158982469025542844689875*I*sqrt(7) - 3500708
83966315423056921062073572156338341590668336522231798225) + 162713757276260
3764340640739218674271639168588614696760770560*sqrt(-2*x + 3)) + 28*(528525
95088141665875251392948545451373376947250790400*x^57 - 10989677950662733151
62856093421299059440183747910041600*x^56 + 10607209489316853390896228799650
834948444579920210821120*x^55 - 6357116755023475399401410440007422334658088
0315719352320*x^54 + 268751102085050752152483783816672599931031121283482910
720*x^53 - 870946973219521114804962921504691759517713269107195904000*x^52 +

```
 231375802193244831242532164933608498102950607249760845824O*x^51 - 53166040
4716026729045985632329296934574488676816107077632O*x^50 + 109354424880090 47
```

```
876986188529240039099445267636 27)*sqrt(-2*x + 3))/(549755813888*x^58 - 1126
9994184704*x^57 + 107064944754688*x^56 - 630638638006272*x^55 + 26185213012
86912*x^54 - 8342252417974272*x^53 + 21849572376576000*x^52 - 4968409148581
4784*x^51 + 101394501297242112*x^50 - 188583312363618304*x^49 + 32326199558
1177856*x^48 - 517079841212727296*x^47 + 778117896260812800*x^46 - 11056411
65387988992*x^45 + 1491287028233404416*x^44 - 1919929663119949824*x^43 + 23
63050939901804544*x^42 - 2786274020645928960*x^41 + 3161145685194047488*x^4
0 - 3453753931369283584*x^39 + 3634098467102523392*x^38 - 36978939603257917
44*x^37 + 3640651752731836416*x^36 - 3461798212247617536*x^35 + 31945402517
89393920*x^34 - 2861544579495297024*x^33 + 2477632938217930752*x^32 - 20884
30257127768064*x^31 + 1712761005459316736*x^30 - 1355447485390974976*x^29 +
 1048940886155151360*x^28 - 790511024135089152*x^27 + 571750925528393856*x^
26 - 408374103192240192*x^25 + 282845069599813728*x^24 - 186113897194906128
*x^23 + 123982890381352520*x^22 - 78116367732251996*x^21 + 4648858015929689
8*x^20 - 29591055660829971*x^19 + 16200795673453545*x^18 - 8941894120163277
*x^17 + 5578893209169441*x^16 - 2296849711499532*x^15 + 1448289882400788*x^
14 - 756896247319212*x^13 + 182213447974992*x^12 - 240797810407770*x^11 + 2
5549234281774*x^10 - 26500281727302*x^9 + 25520701332582*x^8 + 996550723026
0*x^7 + 10389354811164*x^6 + 3755740313808*x^5 + 1820618017974*x^4 + 463742
325333*x^3 + 139858796529*x^2 + 19758444939*x + 3486784401)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(3-2*x)**(41/2)/(2*x**2+x+1)**20,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx=\int \frac{1}{\left(2\,x^2+x+1\right)^{20}(-2\,x+3)^{\frac{41}{2}}}\,dx$$

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="maxima")
[Out] integrate(1/((2*x^2 + x + 1)^20*(-2*x + 3)^(41/2)), x)
```

Giac [A] (verification not implemented)

none

Time = 1.09 (sec) , antiderivative size = 1410, normalized size of antiderivative = 1.33

$$\int \frac{1}{(3-2x)^{41/2} \left(1+x+2x^2\right)^{20}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="giac")
```

```
[Out] -115/36380526169106904249159826530892991340059 0336*sqrt(7)*(241833327334298
28161949068361*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) + 8061
110911143276053983022787*14^(3/4)*sqrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) +
 8) - 56427776378002932377881159509*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14)
 + 4) - 16928332913400879713364347852 7*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(
14) - 4) + 24237695129775488580022954448 8*14^(1/4)*sqrt(7)*sqrt(-2*sqrt(14)
 + 8) - 16966386590842842006016068114 16*14^(1/4)*sqrt(2*sqrt(14) + 8))*arct
an(1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4) + 2*sqrt(-2*x + 3))
/sqrt(-1/8*sqrt(14) + 1/2)) - 115/36380526169106904249159826530892991340059
0336*sqrt(7)*(24183332733429828161949068361*14^(3/4)*sqrt(7)*(sqrt(14) + 4)
*sqrt(-2*sqrt(14) + 8) + 8061110911143276053983022787*14^(3/4)*sqrt(7)*(sqr
t(14) - 4)*sqrt(-2*sqrt(14) + 8) - 56427776378002932377881159509*14^(3/4)*s
qrt(2*sqrt(14) + 8)*(sqrt(14) + 4) - 169283329134008797133643478527*14^(3/4
)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 242376951297754885800229544488*14^(
1/4)*sqrt(7)*sqrt(-2*sqrt(14) + 8) - 169663865908428420060160681141 6*14^(1/
4)*sqrt(2*sqrt(14) + 8))*arctan(-1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqr
t(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)) - 115/72761052338
213808498319653061785982680118 0672*sqrt(7)*(8061110911143276053983022787*14
^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 241833327334298281619 4
9068361*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 169283329134
008797133643478527*14^(3/4)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) + 56427776
378002932377881159509*14^(3/4)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) + 24237
6951297754885800229544488*14^(1/4)*sqrt(7)*sqrt(2*sqrt(14) + 8) + 169663865
908428420060160681141 6*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(14^(1/4)*sqrt(1/
2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 115/7276105233
8213808498319653061785982680118 0672*sqrt(7)*(8061110911143276053983022787*1
4^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 241833327334298281619
49068361*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 16928332913
4008797133643478527*14^(3/4)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) + 5642777
6378002932377881159509*14^(3/4)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) + 2423
76951297754885800229544488*14^(1/4)*sqrt(7)*sqrt(2*sqrt(14) + 8) + 16966386
5908428420060160681141 6*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(-14^(1/4)*sqrt(
1/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 1/2411259743
14794470715561049883908600016802938 88*(38591279629413862313248614614480980 5
*(2*x - 3)^37*sqrt(-2*x + 3) + 4994416662656937088431754278268478521 5*(2*x
```

- 3)^36*sqrt(-2*x + 3) + 315710432519019081879041701576867210025l*(2*x - 3)
^35*sqrt(-2*x + 3) + 129862663539742829727010168448772257537793*(2*x - 3)^3
4*sqrt(-2*x + 3) + 3907056032933059027385185682832433217956200*(2*x - 3)^33
*sqrt(-2*x + 3) + 9162634230824006291365946903167694132884768 8*(2*x - 3)^32
*sqrt(-2*x + 3) + 174305183978371665445857016880893373017462 7004*(2*x - 3)^
31*sqrt(-2*x + 3) + 2763854450762272912509362183729143783091746 2708*(2*x -
3)^30*sqrt(-2*x + 3) + 37249851007044541162953738829085171370508 0145718*(2*
x - 3)^29*sqrt(-2*x + 3) + 43299535169306873423374720142726663639696515873 1
4*(2*x - 3)^28*sqrt(-2*x + 3) + 43899444560112308623605331157143896725828 41
5934650*(2*x - 3)^27*sqrt(-2*x + 3) + 391609357365773780316151578457972453 6
48367489837454*(2*x - 3)^26*sqrt(-2*x + 3) + 309503170175884957504062693739
9363198202032753884252*(2*x - 3)^25*sqrt(-2*x + 3) + 217907196222246813794 1
65678259100933686683346767977 80*(2*x - 3)^24*sqrt(-2*x + 3) + 1372614029241
98725794062163116053277099106968046586092*(2*x - 3)^23*sqrt(-2*x + 3) + 776
17118305565254538487138855317391269135216850095 1876*(2*x - 3)^22*sqrt(-2*x
+ 3) + 395009552637699460788078433865593460380216799543 3166405*(2*x - 3)^21
*sqrt(-2*x + 3) + 181258038168328615978327668733398821189240151833 38007655*
(2*x - 3)^20*sqrt(-2*x + 3) + 750834145086940501444266399776850855400388047
54309758915*(2*x - 3)^19*sqrt(-2*x + 3) + 280932652073348343517776090631271
8952356113432842758 20345*(2*x - 3)^18*sqrt(-2*x + 3) + 949449516366891514 86
66417793095975364784904899879544625 80*(2*x - 3)^17*sqrt(-2*x + 3) + 2896666
9537605702496505134563936009837035495096544691179 00*(2*x - 3)^16*sqrt(-2*x
+ 3) + 79682836929579885676507951291082957044837682603798 20818752*(2*x - 3)
^15*sqrt(-2*x + 3) + 1972749457881227765860600971283186162692222652326643 57
34336*(2*x - 3)^14*sqrt(-2*x + 3) + 438441033794236958424800303207601166664
91172278035172870400*(2*x - 3)^13*sqrt(-2*x + 3) + 871807724494537191124097
15850861698835279004734515297162496*(2*x - 3)^12*sqrt(-2*x + 3) + 154427451
620079851403012035013949923367197814895239131529728*(2*x - 3)^11*sqrt(-2*x
+ 3) + 242351725944359254347670713000225450988365795247877220072960*(2*x -
3)^10*sqrt(-2*x + 3) + 334646091432259174045261099248092390902126268663782 6
08549888*(2*x - 3)^9*sqrt(-2*x + 3) + 403034519668261986708991686890381317 8
41470126237337802123264*(2*x - 3)^8*sqrt(-2*x + 3) + 418646794645473329714 8
96095169087072615863373434634780753920*(2*x - 3)^7*sqrt(-2*x + 3) + 3696217
15112196031007775193340564258755874521674193323966464*(2*x - 3)^6*sqrt(-2*x
+ 3) + 272008032423513780299697431707644217391623176190273099661312*(2*x -
3)^5*sqrt(-2*x + 3) + 162377109720555022535973021706211388170650620411678 7
44248320*(2*x - 3)^4*sqrt(-2*x + 3) + 755566667488842917662208922971666033 7
604020075527569480 0896*(2*x - 3)^3*sqrt(-2*x + 3) + 257152174791471563114 80
45127160359569651927811226569755 8528*(2*x - 3)^2*sqrt(-2*x + 3) - 569505889
84884579140566167635220880459306245787692525158 40*(-2*x + 3)^(3/2) + 616047
39327042324976730399736940635285540412723029737 4720*sqrt(-2*x + 3))/((2*x -
3)^2 + 14*x - 7)^19 + 1/43768013439874312895399492130064309616640*(9918560
554799127296649333 75*(2*x - 3)^19 - 465215115289202563341931875*(2*x - 3)^1
8 + 376870004361848629670138100*(2*x - 3)^17 - 347816399209073565143694750*
(2*x - 3)^16 + 333480450533749292133360000*(2*x - 3)^15 - 31977824826109400

```
5065228000*(2*x - 3)^14 + 300292311231869293365336000*(2*x - 3)^13 - 272225
522279980529558298000*(2*x - 3)^12 + 235508819476507302437712000*(2*x - 3)^
11 - 192403914635036320216640640*(2*x - 3)^10 + 146870291549367152461094400
*(2*x - 3)^9 - 103544963718981484751251200*(2*x - 3)^8 + 665207702174834449
75816704*(2*x - 3)^7 - 38308222816032989365145600*(2*x - 3)^6 + 19364536310
461049463275520*(2*x - 3)^5 - 8351885944887834417868800*(2*x - 3)^4 + 29503
96963171184804659200*(2*x - 3)^3 - 800398003403553957642240*(2*x - 3)^2 + 2
96499732880545408614400*x - 458814330239510651535360)/((2*x - 3)^19*sqrt(-2
*x + 3))
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 1017, normalized size of antiderivative = 0.96

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx = \text{Too large to display}$$

```
[In] int(1/((3 - 2*x)^(41/2)*(x + 2*x^2 + 1)^20),x)
```

```
[Out] ((64356352*(2*x - 3)^2)/38073 - (5767168*x)/1443 - (7517962240*(2*x - 3)^3)
/5444439 + (1357449428992*(2*x - 3)^4)/1181443263 - (34130408095744*(2*x -
3)^5)/34261854627 + (1965832636456960*(2*x - 3)^6)/2158496841501 - (9552588
571922432*(2*x - 3)^7)/10792484207505 + (69571472879183872*(2*x - 3)^8)/755
47389452535 - (5204838729946112*(2*x - 3)^9)/5036492630169 + (3250820527817
55904*(2*x - 3)^10)/257635969158645 - (461538785202937088*(2*x - 3)^11)/272
428464995505 + (1772667874456220326 4*(2*x - 3)^12)/6992330601551295 - (1432
471149647610304*(2*x - 3)^13)/332968123883395 + (2043463601243388704*(2*x -
 3)^14)/241114848329355 - (969727684773439768 16*(2*x - 3)^15)/4840844262612
435 + (108338706701225459276 56*(2*x - 3)^16)/181389282075536535 - (44340157
049832305729324*(2*x - 3)^17)/181389282075536535 + (691509778132186261807 28
2*(2*x - 3)^18)/423241658176251915 - (1357735833153708223970 3407*(2*x - 3)^
19)/423241658176251915 + (509495943858959939640753039465067 2614981*(2*x - 3
)^20)/20359461697924305362362564 6080 + (474753402737241482257498862608 84632
526403*(2*x - 3)^21)/203594616979243053623625646080 + (5473624067276673458 6
817623075460075234149 9*(2*x - 3)^22)/518240843219891409223774371840 + (1363
2173991688467418032505314434961676475 59*(2*x - 3)^23)/438511482724523500112
424468480 + (4003570481422480713899753107522400202013 88159*(2*x - 3)^24)/59
8568173918974577653459399475 20 + (16780353218671061875177851217431652450855
3291*(2*x - 3)^25)/1496420434797436444133648498688 0 + (511087710606983153 19
12486309354814419579941506 7*(2*x - 3)^26)/335198177394625763485937263706112
0 + (3939870831872067350820038893812216643460900 53*(2*x - 3)^27)/2280259710
167522200584607236096 0 + (194509919512254900809288150922829785396777195281*
(2*x - 3)^28)/11688962083504898418996786631802880 + (399049412174158598496 7
811280954752578787283887167 7*(2*x - 3)^29)/288717363462570990949220629805 53
11360 + (2982029082982520685654165296540313515739996589545 19*(2*x - 3)^30)/
```

2978347538877048117160381233783738240 + (1727837071783712649879020657943557255298602982441l*(2*x - 3)^31)/2707588671706407379236710212530339840 + (13658990914062315748322961696111086760908746919545 7*(2*x - 3)^32)/37906241403889703309313942975424757760 + (1212444851028213221312106677792572151674677283084 7*(2*x - 3)^33)/668933671833347705458481346625142784 0 + (52681032254640039242845987567705145658956828241 29*(2*x - 3)^34)/64586699349426675009784405881048268 80 + (617176100928620262663130059020160395102877327114 13*(2*x - 3)^35)/18730142811333735752837477055039979520 + (23627912036802812325678333491117787914105632657738 7*(2*x - 3)^36)/197226802936437285876032243873341243 3920 + (10069182899664488193697417735778758301092236673480 01*(2*x - 3)^37)/25639484381736847163884191703534361640960 + (83431525141223413404125137068400689545183372518688 59*(2*x - 3)^38)/717905562688631720588757367698962125946880 + (66901645261129343103617051181305776742494483919549 23*(2*x - 3)^39)/215371668806589516176627210309688637784064 0 + (3055810652078339448493840154331404088748812305746 13*(2*x - 3)^40)/40745991395841259817199742491022174 7159040 + (731867339371195846981841457176808134814103613309*(2*x - 3)^41)/44775814720704681117801914825299093094 40 + (98156536112115492322904146290693532441307132676 41*(2*x - 3)^42)/30559493546880944862899806868266631036 9280 + (11199801517259481678687287141859390404145132617*(2*x - 3)^43)/19715802288310287008322456044042987765 76 + (136564747272427838170630719416707180545547422 1*(2*x - 3)^44)/151422305976050793637586261153308213248 0 + (403050116590493478621865418191675419450056550 1*(2*x - 3)^45)/31462190241690553789142923150742928752640 + (1428009628445556490988667295522054915842433631*(2*x - 3)^46)/88094132676733550609600184822080200507392 + (160089053926633694221849846408842457682603621*(2*x - 3)^47)/8809413267673355060960018482208020050739 2 + (210019981409672089241582716785447580068246038 9*(2*x - 3)^48)/11716519646005562231076824581336666667483136 + (731521029491460764762993572365861799703833*(2*x - 3)^49)/474353022105488349436308687503508771962 88 + (145278250114246808817452879440670605483477*(2*x - 3)^50)/12695919121058658764324732518476264190771 2 + (3054176246891199033401768204622054595917*(2*x - 3)^51)/423197304035288625477491083949208806359 04 + (4322624121559696023583903787645234779 3*(2*x - 3)^52)/1139377357018084760900937533709408324812 8 + (16757214169421265746492710756597657 5*(2*x - 3)^53)/103579759728916796445539775791764393164 8 + (935756145095208333386444273642906999*(2*x - 3)^54)/174013996344580218028506823330164180516864 + (3250015519725523200399609528788299*(2*x - 3)^55)/248591423349400311469295461900234543595 52 + (35991071119943365803017636753594 5*(2*x - 3)^56)/174013996344580218028506823330164180516864 + (927027754781476746208047620505*(2*x - 3)^57)/58004665448193406009502274443388060172288 + 79953920/10101)/(5976303958948914397184*(3 - 2*x)^(39/2) - 56774887610014686773248*(3 - 2*x)^(41/2) + 263597692475068188590080*(3 - 2*x)^(43/2) - 796876101097706139353088*(3 - 2*x)^(45/2) + 1763207861643670399942656*(3 - 2*x)^(47/2) - 3043249843014358669590528*(3 - 2*x)^(49/2) + 4264137522753475514499072*(3 - 2*x)^(51/2) - 4984324075408572529754112*(3 - 2*x)^(53/2) + 4956568063057422401458176*(3 - 2*x)^(55/2) - 4255315771373708518529024*(3 - 2*x)^(57/2) + 3189779613484873345291264*(3 - 2*x)^(59/2) - 21062355

39086912777861632*(3 - 2*x)^(61/2) + 1233708448609783150169088*(3 - 2*x)^(6
3/2) - 644615788666077029453568*(3 - 2*x)^(65/2) + 301787157080763250721664
*(3 - 2*x)^(67/2) - 127037834354660188150464*(3 - 2*x)^(69/2) + 48214067552
985728953272*(3 - 2*x)^(71/2) - 16530947936007918636468*(3 - 2*x)^(73/2) +
5127550624086495626518*(3 - 2*x)^(75/2) - 1440010379792375040419*(3 - 2*x)^
(77/2) + 366253616006178259037*(3 - 2*x)^(79/2) - 84341571102081217533*(3 -
2*x)^(81/2) + 17570724326889842913*(3 - 2*x)^(83/2) - 3306899061710229804*
(3 - 2*x)^(85/2) + 561126236614140036*(3 - 2*x)^(87/2) - 85611621840452988*
(3 - 2*x)^(89/2) + 11703514272799272*(3 - 2*x)^(91/2) - 1427192816292922*(3
- 2*x)^(93/2) + 154386157043846*(3 - 2*x)^(95/2) - 14711313018374*(3 - 2*x
)^(97/2) + 1223975378934*(3 - 2*x)^(99/2) - 87916389372*(3 - 2*x)^(101/2) +
5372380188*(3 - 2*x)^(103/2) - 273870408*(3 - 2*x)^(105/2) + 11333994*(3 -
2*x)^(107/2) - 365883*(3 - 2*x)^(109/2) + 8645*(3 - 2*x)^(111/2) - 133*(3
- 2*x)^(113/2) + (3 - 2*x)^(115/2)) - (atan(((3 - 2*x)^(1/2)*(- 7^(1/2)*817
4286676615564254062463385463197516747256637092086555i - 2647038820161175221
6024276905374076093636415173409188826601)^(1/2)*124320682492976962848972490
01366340523282983937937427139335625i)/(546445444973747744833043391094451536
038531013369836763902595689946460970498681963431302609552363376731445542355
7204931772416*((7^(1/2)*376655850073799072335964720186587398406296145585988
886284558062903152597529420137587598125i)/273222722486873872416521695547225
768019265506684918381951297844973230485249340981715651304776181688365722771
1778602465886208 + 77752097412376525349979023523894633857634059343371363891
6736753944556393731049211251145625/3903181749812483891664595650674653828846
650095498834027875683499617578360704871167366447211088309833796039588255146
37983744)) + (1243206824929769628489724900136634052328298393793742713933562
5*7^(1/2)*(3 - 2*x)^(1/2)*(- 7^(1/2)*81742866766155642540624633854631975167
47256637092086555i - 264703882016117522160242769053740760936364151734091888
26601)^(1/2))/(546445444973747744833043391094451536038531013369836763902595
6899464609704986819634313026095523633767314455423557204931772416*((7^(1/2)*
376655850073799072335964720186587398406296145585988886284558062903152597529
420137587598125i)/273222722486873872416521695547225768019265506684918381951
2978449732304852493409817156513047761816883657227711778602465886208 + 77752
097412376525349979023523894633857634059343371363891673675394455639373104921
1251145625/3903181749812483891664595650674653828846650095498834027875683499
61757836070487116736644721108830983379603958825514637983744)))*(- 7^(1/2)*8
174286676615564254062463385463197516747256637092086555i - 26470388201611752
216024276905374076093636415173409188826601)^(1/2)*115i)/8120653162747076841
33031842207432842412032 + (atan(((3 - 2*x)^(1/2)*(7^(1/2)*81742866766155642
54062463385463197516747256637092086555i - 264703882016117522160242769053740
76093636415173409188826601)^(1/2)*12432068249297696284897249001366340523282
983937937427139335625i)/(54644544497374774483304339109445153603853101336983
67639025956899464609704986819634313026095523633767314455423557204931772416*
((7^(1/2)*37665585007379907233596472018658739840629614558598888628455806290
3152597529420137587598125i)/27322272248687387241652169554722576801926550668
491838195129784497323048524934098171565130477618168836572277117786024658862

08 - 7775209741237652534997902352389463385763405934337136389167367539445563
93731049211251145625/390318174981248389166459565067465382884665009549883402
787568349961757836070487116736644721108830983379603958825514637983744)) - (
12432068249297696284897249001366340523282983937937427139335625*7^(1/2)*(3 -
 2*x)^(1/2)*(7^(1/2)*817428667661556425406246338546319751674725663709208655
5i - 26470388201611752216024276905374076093636415173409188826601)^(1/2))/(5
464454449737477448330433910944515360385310133698367639025956899464609704986
819634313026095523633767314455423557204931772416*((7^(1/2)*3766558500737990
72335964720186587398406296145585988886284558062903152597529420137587598125i
)/2732227224868738724165216955472257680192655066849183819512978449732304852
493409817156513047761816883657227711778602465886208 - 777520974123765253499
790235238946338576340593433713638916736753944556393731049211251145625/39031
817498124838916645956506746538288466500954988340278756834996175783607048711
6736644721108830983379603958825514637983744)))*(7^(1/2)*8174286676615564254
062463385463197516747256637092086555i - 26470388201611752216024276905374076
093636415173409188826601)^(1/2)*115i)/8120653162747076841330318422074328424
12032

3.50 $\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 378

$$\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx = -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}}$$
$$-\frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}}-\frac{30316369-15043110x}{6860000000(3-2x+x^2)^{5/2}}$$
$$-\frac{63043297-29625922x}{41160000000(3-2x+x^2)^{3/2}}-\frac{31(7434109-3088870x)}{411600000000\sqrt{3-2x+x^2}}$$
$$-\frac{1-10x}{280(3-2x+x^2)^{9/2}(1+x+2x^2)^4}+\frac{28+67x}{1050(3-2x+x^2)^{9/2}(1+x+2x^2)^3}$$
$$+\frac{5485+8878x}{117600(3-2x+x^2)^{9/2}(1+x+2x^2)^2}+\frac{3(8822+8233x)}{343000(3-2x+x^2)^{9/2}(1+x+2x^2)}$$
$$+\frac{\sqrt{\frac{1}{70}\left(151363871237318045+110320475741093888\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{5}{7\left(151363871237318045+110320475741093888\sqrt{2}\right)}}\left(30810816\cdots\right.}{\sqrt{3-2x+\cdots}}\right.}{137200000000}$$
$$-\frac{\sqrt{\frac{1}{70}\left(-151363871237318045+110320475741093888\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{5}{7\left(-151363871237318045+110320475741093888\sqrt{2}\right)}}\left(308\cdots\right.}{\sqrt{3-\cdots}}\right.}{137200000000}$$

```
[Out] 1/123480000*(-3450497+2004270*x)/(x^2-2*x+3)^(9/2)+1/411600000*(-4878869+25
78034*x)/(x^2-2*x+3)^(7/2)+1/6860000000*(-30316369+15043110*x)/(x^2-2*x+3)^
(5/2)+1/41160000000*(-63043297+29625922*x)/(x^2-2*x+3)^(3/2)+1/280*(-1+10*x
)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)^4+1/1050*(28+67*x)/(x^2-2*x+3)^(9/2)/(2*x^2
+x+1)^3+1/117600*(5485+8878*x)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)^2+3/343000*(88
22+8233*x)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)-31/411600000000*(7434109-3088870*x
```

```
)/(x^2-2*x+3)^(1/2)-1/9604000000000*arctanh(1/7*(308108167+x*(932587773-620
347970*2^(1/2))-312239803*2^(1/2))*35^(1/2)/(-151363871237318045+1103204757
41093888*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(-1059547098661226315O+772243330
1876572160*2^(1/2))^(1/2)+1/9604000000000*arctan(1/7*(308108167+312239803*2
^(1/2)+x*(932587773+620347970*2^(1/2)))*35^(1/2)/(151363871237318045+110320
475741093888*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(1059547098661226315O+772243
3301876572160*2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {988, 1074, 1049, 1043, 212, 210}

$$\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx = \frac{\sqrt{\frac{1}{70}\left(151363871237318045+110320475741093888\sqrt{2}\right)}\arctan\left(\cdots\right)}{\cdots} - \frac{\sqrt{\frac{1}{70}\left(110320475741093888\sqrt{2}-151363871237318045\right)}\text{arctanh}\left(\frac{\sqrt{\frac{5}{7\left(110320475741093888\sqrt{2}-151363871237318045\right)}}\left(\left(9325\cdots\right.\right.}{\sqrt{x^2-\cdots}}\right)}{137200000000} - \frac{63043297-29625922x}{41160000000\left(x^2-2x+3\right)^{3/2}} - \frac{31(7434109-3088870x)}{411600000000\sqrt{x^2-2x+3}} + \frac{3(8233x+8822)}{343000\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)} + \frac{8878x+5485}{117600\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^2} - \frac{30316369-15043110x}{6860000000\left(x^2-2x+3\right)^{5/2}} + \frac{67x+28}{1050\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^3} - \frac{4878869-2578034x}{411600000\left(x^2-2x+3\right)^{7/2}} - \frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4} - \frac{3450497-2004270x}{123480000\left(x^2-2x+3\right)^{9/2}}$$

```
[In] Int[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5),x]

[Out] -1/123480000*(3450497 - 2004270*x)/(3 - 2*x + x^2)^(9/2) - (4878869 - 25780
34*x)/(411600000*(3 - 2*x + x^2)^(7/2)) - (30316369 - 15043110*x)/(68600000
00*(3 - 2*x + x^2)^(5/2)) - (63043297 - 29625922*x)/(41160000000*(3 - 2*x +
 x^2)^(3/2)) - (31*(7434109 - 3088870*x))/(411600000000*Sqrt[3 - 2*x + x^2]
) - (1 - 10*x)/(280*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^4) + (28 + 67*x)/
(1050*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^3) + (5485 + 8878*x)/(117600*(3
 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^2) + (3*(8822 + 8233*x))/(343000*(3 - 2
*x + x^2)^(9/2)*(1 + x + 2*x^2)) + (Sqrt[(151363871237318045 + 110320475741
```

```
093888*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(151363871237318045 + 110320475741093
888*Sqrt[2]))]*(308108167 + 312239803*Sqrt[2] + (932587773 + 620347970*Sqrt
[2])*x))/Sqrt[3 - 2*x + x^2]])/137200000000 - (Sqrt[(-151363871237318045 +
110320475741093888*Sqrt[2])/70]*ArcTanh[(Sqrt[5/(7*(-151363871237318045 + 1
10320475741093888*Sqrt[2]))]*(308108167 - 312239803*Sqrt[2] + (932587773 -
620347970*Sqrt[2])*x))/Sqrt[3 - 2*x + x^2]])/137200000000
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} - \frac{\int \frac{-1235+1335x-800x^2}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^4}\,dx}{1400} \\ &= -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} \\ &\quad + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^3} - \frac{\int \frac{-1015350+1334900x-1313200x^2}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^3}\,dx}{1470000}\end{aligned}$$

$$= -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^3} + \frac{5485+8878x}{117600\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^2} - \frac{\int \frac{-333716250+619001250x-932190000x^2}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^2}\,dx}{1029000000}$$

$$= -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^3} + \frac{5485+8878x}{117600\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^2} + \frac{3(8822+8233x)}{343000\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)} - \frac{\int \frac{127736962500-7441875000x-259339500000x^2}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)}\,dx}{360150000000}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^3} + \frac{5485+8878x}{117600\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^2} + \frac{3(8822+8233x)}{343000\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)} - \frac{\int \frac{32819267250000+52489111500000x-168358680000000x^2}{\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)}\,dx}{648270000000000}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^3} + \frac{5485+8878x}{117600\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^2} + \frac{3(8822+8233x)}{343000\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)} - \frac{\int \frac{-4101557985000000+36919386630000000x-68214779640000000x^2}{\left(3-2x+x^2\right)^{7/2}\left(1+x+2x^2\right)}\,dx}{907578000000000000}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{30316369-15043110x}{6860000000\left(3-2x+x^2\right)^{5/2}} - \frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^3} + \frac{5485+8878x}{117600\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^2} + \frac{3(8822+8233x)}{343000\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)} - \frac{\int \frac{-606174190650000000+1224570784500000000x-1592162762400000000x^2}{\left(3-2x+x^2\right)^{5/2}\left(1+x+2x^2\right)}\,dx}{90757800000000000000}$$

$$
\begin{aligned}
&= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} \\
&\quad - \frac{30316369-15043110x}{6860000000\left(3-2x+x^2\right)^{5/2}} - \frac{63043297-29625922x}{41160000000\left(3-2x+x^2\right)^{3/2}} \\
&\quad - \frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^3} \\
&\quad + \frac{5485+8878x}{117600\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^2} + \frac{3(8822+8233x)}{343000\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)} \\
&\quad - \frac{\int \frac{-16544602525500000000000+18687693313800000000000x-15678037922400000000000x^2}{\left(3-2x+x^2\right)^{3/2}\left(1+x+2x^2\right)}\,dx}{5445468000000000000000000} \\
&= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} \\
&\quad - \frac{30316369-15043110x}{6860000000\left(3-2x+x^2\right)^{5/2}} - \frac{63043297-29625922x}{41160000000\left(3-2x+x^2\right)^{3/2}} \\
&\quad - \frac{31(7434109-3088870x)}{411600000000\sqrt{3-2x+x^2}} - \frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} \\
&\quad + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^3} + \frac{5485+8878x}{117600\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^2} \\
&\quad + \frac{3(8822+8233x)}{343000\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)} \\
&\quad - \frac{\int \frac{-105286925935800000000000+71284514842800000000000x}{\sqrt{3-2x+x^2}\left(1+x+2x^2\right)}\,dx}{108909360000000000000000000} \\
&= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} \\
&\quad - \frac{30316369-15043110x}{6860000000\left(3-2x+x^2\right)^{5/2}} - \frac{63043297-29625922x}{41160000000\left(3-2x+x^2\right)^{3/2}} \\
&\quad - \frac{31(7434109-3088870x)}{411600000000\sqrt{3-2x+x^2}} - \frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} \\
&\quad + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^3} + \frac{5485+8878x}{117600\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^2} \\
&\quad + \frac{3(8822+8233x)}{343000\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)} \\
&\quad - \frac{\int \frac{3969000000000000\left(222438197-132636591\sqrt{2}\right)-3969000000000000\left(42834985-89801606\sqrt{2}\right)x}{\sqrt{3-2x+x^2}\left(1+x+2x^2\right)}\,dx}{108909360000000000000000000\sqrt{2}} \\
&\quad + \frac{\int \frac{3969000000000000\left(222438197+132636591\sqrt{2}\right)-3969000000000000\left(42834985+89801606\sqrt{2}\right)x}{\sqrt{3-2x+x^2}\left(1+x+2x^2\right)}\,dx}{108909360000000000000000000\sqrt{2}}
\end{aligned}
$$

$$= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{30316369-15043110x}{6860000000(3-2x+x^2)^{5/2}} - \frac{63043297-29625922x}{41160000000(3-2x+x^2)^{3/2}} - \frac{31(7434109-3088870x)}{411600000000\sqrt{3-2x+x^2}} - \frac{1-10x}{280(3-2x+x^2)^{9/2}(1+x+2x^2)^4} + \frac{28+67x}{1050(3-2x+x^2)^{9/2}(1+x+2x^2)^3} + \frac{5485+8878x}{117600(3-2x+x^2)^{9/2}(1+x+2x^2)^2} + \frac{3(8822+8233x)}{343000(3-2x+x^2)^{9/2}(1+x+2x^2)} - \frac{1}{7}\left(101250\left(220640951482187776-151363871237318045\sqrt{2}\right)\right)\text{Subst}\left(\int \frac{}{-11027072700000000000\ldots}\right.$$

$$= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{30316369-15043110x}{6860000000(3-2x+x^2)^{5/2}} - \frac{63043297-29625922x}{41160000000(3-2x+x^2)^{3/2}} - \frac{31(7434109-3088870x)}{411600000000\sqrt{3-2x+x^2}} - \frac{1-10x}{280(3-2x+x^2)^{9/2}(1+x+2x^2)^4} + \frac{28+67x}{1050(3-2x+x^2)^{9/2}(1+x+2x^2)^3} + \frac{5485+8878x}{117600(3-2x+x^2)^{9/2}(1+x+2x^2)^2} + \frac{3(8822+8233x)}{343000(3-2x+x^2)^{9/2}(1+x+2x^2)} + \frac{\sqrt{\frac{1}{70}\left(151363871237318045+110320475741093888\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{5}{7\left(151363871237318045+110320475741093888\ldots\right.}}}{}\right.}{137200000000} - \frac{\sqrt{\frac{1}{70}\left(-151363871237318045+110320475741093888\sqrt{2}\right)}\text{arctanh}\left(\frac{\sqrt{\frac{5}{7\left(-151363871237318045+11032047574110\ldots\right.}}}{}\right.}{137200000000}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 4.00 (sec) , antiderivative size = 733, normalized size of antiderivative = 1.94

$$\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx = \frac{-53205422447+261702502714x-266966654968x^2+1002897791524x^3-1409335257371x^4+2\ldots}{}$$

```
[In] Integrate[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] ((-53205422447 + 261702502714*x - 266966654968*x^2 + 1002897791524*x^3 - 14
09335257371*x^4 + 2503427226914*x^5 - 3359813871472*x^6 + 4591320676952*x^7
 - 5134334619701*x^8 + 5380603084494*x^9 - 4915797913008*x^10 + 39996561325
32*x^11 - 2679143870481*x^12 + 1459208021718*x^13 - 606785954952*x^14 + 188
603773872*x^15 - 38639385552*x^16 + 4596238560*x^17)/((3 - 2*x + x^2)^(9/2)
*(1 + x + 2*x^2)^4) - 49392*RootSum[14 + 7*#1 - 5*#1^2 - #1^3 + #1^4 & , (-
6014*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 10727*Log[-x + Sqrt[3 - 2*x + x^2
] - #1]*#1 + 3229*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1^2)/(7 - 10*#1 - 3*#
1^2 + 4*#1^3) & ] - 56448*RootSum[14 + 7*#1 - 5*#1^2 - #1^3 + #1^4 & , (737
81*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 60407*Log[-x + Sqrt[3 - 2*x + x^2]
- #1]*#1 + 13104*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1^2)/(7 - 10*#1 - 3*#1
^2 + 4*#1^3) & ] - 504*RootSum[14 + 7*#1 - 5*#1^2 - #1^3 + #1^4 & , (275935
046*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 208696097*Log[-x + Sqrt[3 - 2*x +
x^2] - #1]*#1 + 50007219*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1^2)/(7 - 10*#
1 - 3*#1^2 + 4*#1^3) & ] + 1440*RootSum[14 + 7*#1 - 5*#1^2 - #1^3 + #1^4 &
, (3276009822*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 2447831621*Log[-x + Sqrt
[3 - 2*x + x^2] - #1]*#1 + 590084719*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1^
2)/(7 - 10*#1 - 3*#1^2 + 4*#1^3) & ] - 18*RootSum[14 + 7*#1 - 5*#1^2 - #1^3
 + #1^4 & , (254137663854*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 189631531133
*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1 + 45801521671*Log[-x + Sqrt[3 - 2*x
+ x^2] - #1]*#1^2)/(7 - 10*#1 - 3*#1^2 + 4*#1^3) & ])/1234800000000
```

Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.20

method	result
risch	$\frac{4596238560x^{17}-38639385552x^{16}+188603773872x^{15}-606785954952x^{14}+1459208021718x^{13}-2679143870481x^{12}+3999656132532x}{}$
trager	Expression too large to display
default	Expression too large to display

```
[In] int(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1234800000000*(4596238560*x^17-38639385552*x^16+188603773872*x^15-6067859
54952*x^14+1459208021718*x^13-2679143870481*x^12+3999656132532*x^11-4915797
913008*x^10+5380603084494*x^9-5134334619701*x^8+4591320676952*x^7-335981387
1472*x^6+2503427226914*x^5-1409335257371*x^4+1002897791524*x^3-266966654968
*x^2+261702502714*x-53205422447)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)^4+1/26891200
```

```
0000000*4^(1/2)*((2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+1)^(1/2)*2^(1/2)*(96257226
25*(-6050+4280*2^(1/2))^(1/2)*arctan(1/49*(-6050+4280*2^(1/2))^(1/2)/((2^(1
/2)-1+x)^2/(2^(1/2)+1-x)^2+1)^(1/2)*(40*2^(1/2)+57)*(2^(1/2)-1+x)/(2^(1/2)+
1-x))*(-350+280*2^(1/2))^(1/2)*2^(1/2)+13664181884*(-6050+4280*2^(1/2))^(1/
2)*arctan(1/49*(-6050+4280*2^(1/2))^(1/2)/((2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+
1)^(1/2)*(40*2^(1/2)+57)*(2^(1/2)-1+x)/(2^(1/2)+1-x))*(-350+280*2^(1/2))^(1
/2)+456968008770*arctanh(7*((2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+1)^(1/2)/(-350+
280*2^(1/2))^(1/2))*2^(1/2)-607941010600*arctanh(7*((2^(1/2)-1+x)^2/(2^(1/2
)+1-x)^2+1)^(1/2)/(-350+280*2^(1/2))^(1/2)))/(((2^(1/2)-1+x)^2/(2^(1/2)+1-x
)^2+1)/((2^(1/2)-1+x)/(2^(1/2)+1-x)+1)^2)^(1/2)/((2^(1/2)-1+x)/(2^(1/2)+1-x
)+1)/(-350+280*2^(1/2))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 869, normalized size of antiderivative = 2.30

$$\int \frac{1}{(3-2x+x^2)^{11/2}\,(1+x+2x^2)^5}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/86436000000000*(321736699200*x^18 - 2573893593600*x^17 + 12386862919200*x
^16 - 39091008952800*x^15 + 93484619661300*x^14 - 169997628439800*x^13 + 25
5076876834500*x^12 - 312647637447600*x^11 + 349667466399300*x^10 - 33175075
3962600*x^9 + 310817759970900*x^8 - 219424428854400*x^7 + 184898059321500*x
^6 - 88356941017800*x^5 + 88618352085900*x^4 - 7962983305200*x^3 - 9*sqrt(3
5)*(16*x^18 - 128*x^17 + 616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13 + 126
85*x^12 - 15548*x^11 + 17389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7 + 919
5*x^6 - 4394*x^5 + 4407*x^4 - 396*x^3 + 1647*x^2 + 162*x + 243)*sqrt(142938
20940408247*I*sqrt(7) - 151363871237318045)*log(sqrt(35)*sqrt(1429382094040
8247*I*sqrt(7) - 151363871237318045)*(932587773*I*sqrt(7) + 299844895) - 30
88973320750628864*x + 772243330187657216*I*sqrt(7) + 3088973320750628864*sq
rt(x^2 - 2*x + 3) - 772243330187657216) + 9*sqrt(35)*(16*x^18 - 128*x^17 +
616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 17
389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*x
^4 - 396*x^3 + 1647*x^2 + 162*x + 243)*sqrt(14293820940408247*I*sqrt(7) - 1
51363871237318045)*log(sqrt(35)*sqrt(14293820940408247*I*sqrt(7) - 15136387
1237318045)*(-932587773*I*sqrt(7) - 299844895) - 3088973320750628864*x + 77
2243330187657216*I*sqrt(7) + 3088973320750628864*sqrt(x^2 - 2*x + 3) - 7722
43330187657216) + 9*sqrt(35)*(16*x^18 - 128*x^17 + 616*x^16 - 1944*x^15 + 4
649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 17389*x^10 - 16498*x^9 + 1
5457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*x^3 + 1647*x^2
+ 162*x + 243)*sqrt(-14293820940408247*I*sqrt(7) - 151363871237318045)*log(
```

```
sqrt(35)*(932587773*I*sqrt(7) - 299844895)*sqrt(-14293820940408247*I*sqrt(7
) - 151363871237318045) - 3088973320750628864*x - 772243330187657216*I*sqrt
(7) + 3088973320750628864*sqrt(x^2 - 2*x + 3) - 772243330187657216) - 9*sqr
t(35)*(16*x^18 - 128*x^17 + 616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13 +
12685*x^12 - 15548*x^11 + 17389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7 +
9195*x^6 - 4394*x^5 + 4407*x^4 - 396*x^3 + 1647*x^2 + 162*x + 243)*sqrt(-14
293820940408247*I*sqrt(7) - 151363871237318045)*log(sqrt(35)*(-932587773*I*
sqrt(7) + 299844895)*sqrt(-14293820940408247*I*sqrt(7) - 151363871237318045
) - 3088973320750628864*x - 772243330187657216*I*sqrt(7) + 3088973320750628
864*sqrt(x^2 - 2*x + 3) - 772243330187657216) + 33118771473900*x^2 + 70*(45
96238560*x^17 - 38639385552*x^16 + 188603773872*x^15 - 606785954952*x^14 +
1459208021718*x^13 - 2679143870481*x^12 + 3999656132532*x^11 - 491579791300
8*x^10 + 5380603084494*x^9 - 5134334619701*x^8 + 4591320676952*x^7 - 335981
3871472*x^6 + 2503427226914*x^5 - 1409335257371*x^4 + 1002897791524*x^3 - 2
66966654968*x^2 + 261702502714*x - 53205422447)*sqrt(x^2 - 2*x + 3) + 32575
84079400*x + 4886376119100)/(16*x^18 - 128*x^17 + 616*x^16 - 1944*x^15 + 46
49*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 17389*x^10 - 16498*x^9 + 15
457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*x^3 + 1647*x^2 +
 162*x + 243)
```

Sympy [F]

$$\int \frac{1}{(3-2x+x^2)^{11/2}\,(1+x+2x^2)^5}\,dx = \int \frac{1}{(x^2-2x+3)^{\frac{11}{2}}\,(2x^2+x+1)^5}\,dx$$

```
[In] integrate(1/(x**2-2*x+3)**(11/2)/(2*x**2+x+1)**5,x)
```

```
[Out] Integral(1/((x**2 - 2*x + 3)**(11/2)*(2*x**2 + x + 1)**5), x)
```

Maxima [F]

$$\int \frac{1}{(3-2x+x^2)^{11/2}\,(1+x+2x^2)^5}\,dx = \int \frac{1}{(2\,x^2+x+1)^5(x^2-2\,x+3)^{\frac{11}{2}}}\,dx$$

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 35.11 (sec) , antiderivative size = 2509, normalized size of antiderivative = 6.64

$$\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="giac")
```

```
[Out] 1/19208000000000*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*l
og(3136*(247430153598830145135914226638091465128017779071251327216101236181
293485559300330785024470114864584026604284622700*sqrt(7)*sqrt(2)*sqrt(77224
33301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2)
- 151363871237318045)^2 + 1443342562659842513292832988722200213246770377915
632742093923877724211999095918596245976075670043406821858326965750*sqrt(7)*
(110320475741093888*sqrt(2) - 151363871237318045)^3 + 288668512531968502658
566597744440042649354075583126548418784775544842399819183719249195215134008
6813643716653931500*sqrt(2)*(110320475741093888*sqrt(2) - 15136387123731804
5)^3 + 20619179466569178761326185553174288760668148255937610601341769681774
4571299416942320853725095720486688836903852250*sqrt(7722433301876572160*sqr
t(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 1513638712373180
45)^3 - 1049138541122969625735220807296230414367334042656225923210840932892
59251415027575686933144355006438004151420024881229481000*sqrt(7)*sqrt(2)*(1
10320475741093888*sqrt(2) - 151363871237318045)^2 - 10491385411229696257352
208072962304143673340426562259232108409328925925141502757568693314435500643
800415142002488122948100*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 1059547
0986612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 - 209827
708224593925147044161459246082873466808531245184642168186578518502830055151
37386628871001287600830284004976245896200*sqrt(2)*sqrt(7722433301876572160*
sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 1513638712373
18045)^2 - 1223994964643464563357757608512268816761889716432263577079314421
70802459984198838301422001747507511004843323362361434394500*(11032047574109
3888*sqrt(2) - 151363871237318045)^3 + 724503996256958016683144110303659048
527226579099834730629600402015843465282345034154512602754539838193840458016
13291562600472200700*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 105
95470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045) + 6339
409967248382662472854538412359683674181009662984901543522122388718802293934
79427155097805557896986440201529880194683449362574125*sqrt(7)*(110320475741
093888*sqrt(2) - 151363871237318045)^2 + 1267881993449676532082187318351088
361508312490869111205095341459358991548431951565218821053012281909331172952
868319415989224917443750*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237
318045)^2 + 126788199344967653538125603300215696332050217937699740680224518
030900924464663471430273419380295298646483255439984720301061537907975*sqrt(
7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqr
```

t(2) - 151363871237318045)^2 - 21264528220686985082784156444749824400286141
322404339508073021441899306648522000622634130083780322911432744853704088500
185306368048860002880*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 1513638
71237318045) - 354408803678116418845746257791902502469699160932438355128709
627960208195159816911628734559796548815662417323599879097891861163466576081
8080*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(1103
20475741093888*sqrt(2) - 151363871237318045) - 7088176073562328374916566029
889536476564991204751185360922127716049447858985212646112610216597117728735
334198568887900615291459333783931760*sqrt(2)*sqrt(7722433301876572160*sqrt(
2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045
) - 62021540643670373420405091577929394267973114014403593823713156536262791
860899401705831007948594881042345033377486353135919026969012248710900*(1103
20475741093888*sqrt(2) - 151363871237318045)^2 + 32899118880973852710380417
819638428242533122398715783525424459521797423442868355883980260844124987688
79366524177397299680463206375263895127106986700*sqrt(7)*sqrt(2)*sqrt(772243
3301876572160*sqrt(2) - 10595470986612263150) + 575734580417042425329673296
871504559468760385065959616403976214712902997702584706455877479060569410162
15068838775424495834136105150442722719839966690*sqrt(7)*(110320475741093888
*sqrt(2) - 151363871237318045) + 115146916083408484993484259748605110296131
995582491774181190935500623259371276266538520099917051160383062730419039816
825148360727220332176154717624880*sqrt(2)*(110320475741093888*sqrt(2) - 151
363871237318045) + 19191152680568080928847909459028587443440938343434315974
646074253344559590752575102628626876766807523046716011108365196910710221166
349350116972946360*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
*(110320475741093888*sqrt(2) - 151363871237318045) - 4219083468134411357204
411829911575030935087599894152681923125521189411158761429808471398986722740
73729706037742726523339689588219472775435857599079592655320*sqrt(7)*sqrt(2)
 - 210954173406720568670297857045559135287403242478401319837374548657411401
676856640700014506909797905726420309651469993318236639490549988690120558780
166576108*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
- 4219083468134411371380763977036231747195786343738197611526042097212408620
850995875571065161946256176959320357812676471048958085730443394260704501501
50098280*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) -
 92058995214015314954086068104250082609842263587571418594372338376217809267
922249720111767473806366273424910555828722901509465001692379638157996477568
878185171912773*sqrt(7) - 8145397671270700392375835131235891700241076006749
872956875216413467916040501467073084588856054007249498459026692980650865954
42158392477166657536193932267555915756504095096650*sqrt(2) - 92058995214015
311115855531990633917126372266976678439922113048655699796659517564449092929
141850974541546613899009112709389970185858812421123596605881876967985540965
*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) + 115328821454793
719911555111506838054918092483693711591974348115163386041674344938146595482
926045968304684846611487161656745020914533707787852282815214568718566601574
0023320320)^2 + 3136*(34124314806601555041367954040995026009203193083759390
91863756572751913121135591082568720686452963161513000*sqrt(7)*sqrt(2)*sqrt(

7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqr
t(2) - 151363871237318045)^2 + 19905850303850907107464639857247098505368529
298859644702538580007719493206624281314984204004308951775492500*sqrt(7)*(11
0320475741093888*sqrt(2) - 151363871237318045)^3 + 398117006077018142149292
797144941970107370585977192894050771600154389864132485626299684080086179035
50985000*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 2843
692900550129586780662836749585500766932756979949243219797143959927600946325
902140600572044135967927500*sqrt(7722433301876572160*sqrt(2) - 105954709866
12263150)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 76858147264
002002271476515530447061472149684267202070486664776785256332938718528406617
898269719011792142 50*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 15136387
1237318045)^2 + 76858147264002002271476515530447061472149684267202070486664
776785256332938718528406617898269719011792142 5*sqrt(7)*sqrt(772243330187657
2160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 15136387
1237318045)^2 + 15371629452800400454295303106089412294429936853440414097332
955357051266587743705681323579653943802358428 50*sqrt(2)*sqrt(77224333018765
72160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 1513638
71237318045)^2 + 8966783847466900265005593478552157171750796497840241556777
557291613238842850494980772088131467218042416625*(110320475741093888*sqrt(2
) - 151363871237318045)^3 + 97651082935133698531842609272064332805391755699
900630343142766909193115243498547681531918595187516033980577557159475001400
0*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*
(110320475741093888*sqrt(2) - 151363871237318045) + 85444697568241986442857
715157066658147170813177379891611604541603939915295645000738134636846495148
700981051126045418060425 00*sqrt(7)*(110320475741093888*sqrt(2) - 1513638712
37318045)^2 + 1708893951364839728288415723031307245587283696197680732078704
28068280845726894058597070721674063779359150090811208116762300 00*sqrt(2)*(1
10320475741093888*sqrt(2) - 151363871237318045)^2 + 17088939513648397328383
243639115145844363442349970175333057966918507272464206160163065913782547592
3701646182338269871630 7000*sqrt(7722433301876572160*sqrt(2) - 1059547098661
2263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 - 161566291032
164298591825052515595385305284590856268393753077809569128347680816594314529
2528342610931516231547450242131454340*sqrt(7)*sqrt(2)*(110320475741093888*s
qrt(2) - 151363871237318045) - 26927715172027383040078920360978657724993896
167132685456256065962700766147439126263654812280972255786894984252003836063
5590*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(1103
20475741093888*sqrt(2) - 151363871237318045) - 5385543034405476609479748781
986245873979284291149338262149302416677335524471916239116797369312899317505
0520555382721992288 0*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986
612263150)*(110320475741093888*sqrt(2) - 151363871237318045) - 471235015510
479202304702721570436483686833835069606278309401504563239251902428980452635
330631964978193453411263345326870620 0*(110320475741093888*sqrt(2) - 1513638
71237318045)^2 + 4193859254705817563786545204110979417249615218275883808083
699006997908745917921159286375728564093032299349286334300105037477765458575
2870920*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 1059547098661226

3150) + 733925369573518077568688728124769337212428798249901052299318450769533849414676062708934630055035134045830109366690499889371007117918795898700*sqrt(7)*(110320475741093888*sqrt(2) - 151363871237318045) + 146785073914703615416086662689820169008141192682491749201237517428277642423230140157620641963615809777688763882024614874428500622232316501400*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045) + 2446417898578393603288255436181495977780897934081649365296599297616538778164584609859198885592495876512187088988513142432922439383397920921 00*sqrt(772243330187657 2160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045) - 1292077753312694019345893430149910326331364817398263414897431400481314306688204378121592147185828297550084259097863598421441509496331599911 50*sqrt(7)*sqrt(2) + 758762916008240003409819101375410993692930390503123728189868562072536874773198417845474059096092571231597614031305995383233253822344649520984786771597244159013334688*x - 646038876656347010288437347578438205687950441405951033530044590248134676986715874118750732031211145579039572078252198592335248430544666615 75*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) - 12920777533126940204230020370310056506867752862435766623805367556854879262268547580075255076534844025016463869994363709342246652349944171058 2*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) - 36018223914820167914680328062277541844579926085292035680717885777055540822317688033834995139282160837267303073663773328283377336607108359807277238605193066 0*sqrt(7) - 256653251552859302745755374084882225122579558135538546802966417819334438408482678055766092810745610958399639440237969215404840097857114692088861260268773750000*sqrt(2) - 758762916008240003409819101375410993692930390503123728189868562072536874773198417845474059096092571231597614031305995383233253822344649520984786771597244159013334688*sqrt(x^2 - 2*x + 3) - 36018223914820167425396748346598823921159161032736404931012591722796526817745688546882730709582846841129178670301992545389090569566407753301601299449048450 80*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) + 189691081878191176772044460297494206938487074845844976108591897448936648822154611593038166610328900989958412425584079869742930256624900190943273280656848430181850997)^2) - 1/19208000000000*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*log(3136*(2474301532945128077011663304566412660865899546342905585182468026346391921433693240822436073126340210807888656257 00*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 + 14433425608846580449234702609970740521717747353666949246897730153687286208363210571464210426570317896379350494832 50*sqrt(7)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 288668512176931608984694052199414810434354947073338984937954603073745724167264211429284208531406357927587009896650 0*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 2061917944120940064176386087138677217388249621952421320985390021955326601194744367352030060938616842339907213547 50*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 1049138549006054635980882612252778717922436689548063193763378141360297457464252692057521154195454324912741482216699450010 00*sqrt(7)*sqrt(2)*(

110320475741093888*sqrt(2) - 151363871237318045)^2 + 10491385490060546359808826122527787179224366895480631937633781413602974574642526920575211541954543249127414822166994500100*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 + 2098277098012109271961765224505557435844873379096126387526756282720594914928505384115042308390908649825482964433398900020 0*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 + 12239949738403970753110297142949085042428428044727403927239411649203470337082948074004413465613633790648650625861493583450 0*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 72450399556079180248329015146999235208892617070047340278059551131392900835267393877060925502058563859855540246511032336003698229700*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045) + 6339409961156928288224132378329953594189192690738560013436207699616249390969017139885443790988079276553600259078650068119581303478 75*sqrt(7)*(110320475741093888*sqrt(2) - 151363871237318045)^2 + 12678819922313856572324428868418027060025613207199765592095916155327656139967254235860234379586669818403140396280066451559348179862 50*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045)^2 + 12678819922313856605315115974353068086847790601418601070307910106566397275733494587145459998702579696035621371297935928997863596622 5*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 + 21264527927126900961164377132668373311820329495714368356539982937044757067834357827140293364137684307991178111246209024321545469979809839680*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045) + 35440879878544835015208327526337165880148748188307087674038005051117505114076550006018390461315393241510718124111630271982879190286307 82880*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045) + 7088175975708967001043306364303519583685211328608505691145816839655761606123161160517397220902597330097341426354794006943734919549262613360*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045) + 62021539787453461399014070555129747821363280759028253853850789286729673222028079002566886678131418912687891373331690882456544841615974534900*(110320475741093888*sqrt(2) - 151363871237318045)^2 + 32899118847630152211015411144055054747532701564810653878870517591440238490285735651535789555031009928566367922372193172392300964981792236163142097 00*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) + 57573457983352766659078567726413066650683718699283969000978205977352244855554840995810704156013886991645243042141434050503060360394862431867078254790*sqrt(7)*(110320475741093888*sqrt(2) - 151363871237318045) + 11514691596670553324570673589674695312373275176428664785053988930084036831493297504145438133911660943041218690890125397893446827788841283652727703008 0*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045) + 19191152661117588970884988724230066021393869653903156391556097575709573739753723383708361939864243785854901671586306846977795770326107328538508811760*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150

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)*(110320475741093888*sqrt(2) - 151363871237318045) + 421908326717543641924
3588206898612547738784116489303793819018092992027507823310015083748035178706
62815101635538037057318579767492941175495676148000511482108 0*sqrt(7)*sqrt(2
) + 2109541633587718217722566664711669711256358377483192354207771689878568670
89453384049969627290987030043155180365618048585416083969716512306115822804
6039881708*sqrt(7)*sqrt(772243330187657216 0*sqrt(2) - 1059547098661226315 0)
 + 42190832671754364334199401583165465162529632769025946098011971581201199 1
659696962885724904699692807901543935675281424621198266774916716624735720821
529103720*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
- 92058996408756032091676158104201528390543378064521592176251540724536532 23
47939602047491610965392490295654216539646083128661631912861645825999759027 8
2992478976911083*sqrt(7) + 81453972796152704669483783306214586458459063445 2
974565084319757683318515854195479301757363227685529077360827463499277712322
1082185639753988444491134597154930322061732374698 50*sqrt(2) - 9205899640875
602825344562588068375449965750892067890031749508881924444728233783716057581
267806102568826874405209859338973827430110551686881735838388090152238069483
5*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) - 1153288160734 8
687037747868321346713994558911627676219472353978719356293894710618931600085
322372043380298052663139565536675674564328460228800898840894611032414487006
75305025280)^2 + 3136*(34124314806601555041367954040995026009203193083759 39
09186375657275191312113559108256872068645296316151300 0*sqrt(7)*sqrt(2)*sqrt
(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sq
rt(2) - 151363871237318045)^2 + 1990585030385090710746463985724709850536852
929885964470253858000771949320662428131498420400430895177549250 0*sqrt(7)*(1
10320475741093888*sqrt(2) - 151363871237318045)^3 + 398117006077018142149 29
27971449419701073705859771928940507716001543898641324856262996840800861790 3
550985000*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 284
369290055012958678066283674958550076693275697994924321979714395992760094632
5902140600572044135967927500*sqrt(7722433301876572160*sqrt(2) - 10595470986
612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 7685814726
400200227147651553044706147214968426720207048666477678525633293871852840661
78982697190117921425 0*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 1513638
71237318045)^2 + 768581472640020022714765155304470614721496842672020704866 6
4776785256332938718528406617898269719011792142 5*sqrt(7)*sqrt(77224333018765
72160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 1513638
71237318045)^2 + 153716294528004004542953031060894122944299368534404140973 3
2955357051266587743705681323579653943802358428 50*sqrt(2)*sqrt(7722433301876
572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363
871237318045)^2 + 8966783847466900265005593478552157171750796497840241556 77
755729161323884285049498077208813146721804241662 5*(110320475741093888*sqrt(
2) - 151363871237318045)^3 + 976510829351336985318426092720643328053917556 9
99006303431427669091931152434985476815319185951875160339805775571594750014 0
00*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
*(110320475741093888*sqrt(2) - 151363871237318045) + 854446975682419864428 5
7715157066658147170813177379891611604541603939915295645000738134636846495 14
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870098105112604541806042500*sqrt(7)*(110320475741093888*sqrt(2) - 151363871
237318045)^2 + 170889395136483972828841572303130724558728369619768073207870
4280682808457268940585970707216740637793591500908112081167623 0000*sqrt(2)*(
110320475741093888*sqrt(2) - 151363871237318045)^2 + 1708893951364839732838
324363911514584436344234997017533305796691850727246420616016306591378254759
23701646182338269871630 7000*sqrt(7722433301876572160*sqrt(2) - 105954709866
12263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 - 16156629103
216429859182505251559538530528459085626839375307780956912834768081659431452
925283426109315162315474502421314543 40*sqrt(7)*sqrt(2)*(110320475741093888*
sqrt(2) - 151363871237318045) - 2692771517202738304007892036097865772499389
616713268545625606596270076614743912626365481228097225578689498425200383606
35590*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110
320475741093888*sqrt(2) - 151363871237318045) - 538554303440547660947974878
198624587397928429114933826214930241667733552447191623911679736931289931750
5052055538272199228 80*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 1059547098
6612263150)*(110320475741093888*sqrt(2) - 151363871237318045) - 47123501551
047920230470272157043648368683383506960627830940150456323925190242898045263
533063196497819345341126334532687062 00*(110320475741093888*sqrt(2) - 151363
871237318045)^2 + 419385925470581756378654520411097941724961521827588380808
369900699790874591792115928637572856409303229934928633430010503747776545857
52870920*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 105954709866122
63150) + 733925369573518077568688728124769337212428798249901052299318450769
533849414676062708934630055035134045830109366690499889371007117918795898700
sqrt(7)(110320475741093888*sqrt(2) - 151363871237318045) + 14678507391470
361541608666268982016900814119268249174920123751742827764242323014015762064
1963615809777688763882024614874428500622223231650140 0*sqrt(2)*(110320475741
093888*sqrt(2) - 151363871237318045) + 244641789857839360328825543618149597
778089793408164936529659929761653877816458460985919888559249587651218708898
8513142432922439383397920921 00*sqrt(7722433301876572160*sqrt(2) - 105954709
86612263150)*(110320475741093888*sqrt(2) - 151363871237318045) - 1292077753
312694019345893430149910326331364817398263414897431400481314306688204378121
5921471858282975500842590978635984214415094963315999115 0*sqrt(7)*sqrt(2) -
758762916008240003409819101375410993692930390503123728189868562072536874773
198417845474059096092571231597614031305995383233253822344649520984786771597
244159013334688*x - 6460388766563470102884373475784382056879504414059510335
300445902481346769867158741187507320312111455790395720782521985923352484305
4466661575*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
- 129207775331269402042300203703100565068677528624357666238053675568548792
622685475800752550765348440250164638699943637093422466523499441710582*sqrt(
2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) - 3601822391482
016791468032806227754184457992608529203568071788577705554082231768803383499
513928216083726730307366377332828337773366071083598072772386051930660*sqrt(
7) - 2566532515528593027457553740848822251225795581355385468029664178193344
384084826780557660928107456109583996394402379692154048400978571146920888612
60268773750000*sqrt(2) + 75876291600824000340981910137541099369293039050312

3728189868562072536874773198417845474059096092571231597614031305995383233253822344649520984786771597244159013334688*sqrt(x^2 - 2*x + 3) - 360182239148201674253967483465988239211591610327364049310125917227965268177456885468827307095828468411291786703019925453890905695664077533016012994490484508O*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) - 18969037612592882493286509039021128990797812040571688798634238358733178856444459732969886293771738462584039459006891782187369665454742456954912010514177364932481634 7)^2)
+ 41672947348129/28000000000*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*arctan(-75876291600824000340981910137541099369293039050312372818986856207253687477319841784547405909609257123159761403130599538323325382234464952098478677159724415901333468 8*(4*x - 4*sqrt(x^2 - 2*x + 3) - I*sqrt(20*sqrt(2) - 25) + 1)/(240456903400223659024212076421857594876596481118805954534597897163550520465792352560762361627662375147331666 3*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 10595470986612263150))^7 - 14273993756775096948778514384982726726086177451105114601508039903300578423813275603664373381633528980156655785698126 46*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 10595470986612263150))^6 + 103500570893851144987969410044573593528466831057025915469516609398945671350435784286059468190138006696033871436116351184730996386*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 10595470986612263150))^5 - 50629829096873773899573990740257783361032907165973215049765624930757608699456368874374542973258344666404687384663029077837715816904194414*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 10595470986612263150))^4 + 10966372960324617518376520492953903536419826007364881536262517747994521927583758739165722209949618191560706809147945162088690245763393164652919228 47*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 10595470986612263150))^3 - 42190834681344113420154631008494428482305699904760055166613689392979753170802688007006183501301022019174774824416270657812179967072764778464276943820525414 6*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 10595470986612263150))^2 - 368235980856061293812100449423314733558612942895100398043329245333136823181580024721476354281502066792356860508683478554304309475540771033672165453813191045495483200*sqrt(7) - 736471961712122587624200898846629467117225885790200796086658490666273646363160049442952708563004133584713721017366957108608618951081542067344330907626382090990966400*sqrt(2) - 368235980856061293812100449423314733558612942895100398043329245333136823181580024721476354281502066792356860508683478554304309475540771033672165453813191045495483200*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) + 142835210520388728827903853693232289300402954893648600418526215611670264583041444569915814770291499043879486110165753955023507391431740254479476659166189278113070264611396886))/(110320475741093888*sqrt(2) - 151363871237318045) - 41672947348129/28000000000*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*arctan(-75876291600824000340981910137541099369293039050312372818986856207253687477319841784547405909609257123159761403130599538323325382234464952098478677159724415901333468 8*(4*x - 4*sqrt(x^2 - 2*x + 3) - I*sqrt(20*sqrt(2) - 25) + 1)/(
240456903104482806317945899374772853339737565242264877082844317429192606553

3229582917819313047949670367238733*(sqrt(7) + 2*sqrt(2) + sqrt(772243330187
6572160*sqrt(2) - 1059547098661226315O))^7 + 142739938640279542310324164932
350845975841726469124244049439202906162919382891522728914442747680860532345
7798934284966*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 105
9547098661226315O))^6 + 103500570794398828673704559093149103664574830042270
418676217772397611737399109743666577656693380270736279743910695330805730930
506*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 1059547098661
2263150))^5 + 506298283979211926576221344309055178192100417667234568432139O
261731100045776430693261660418696870154376935391839699513483958286559484145
4*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 105954709866122
63150))^4 + 109663729492100506852548516508189371952187779338119894553749726
838300179564207424160536321027108218235029918594318831383181052429437744237
8547095677*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 105954
70986612263150))^3 + 421908326717543640405463968752225473761982330345651759
435786313555664940163096802031617835780482986046113584519114737539776217075
8509647257854327953473930 26*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160
*sqrt(2) - 10595470986612263150))^2 - 368235985635024162362460774967963334O
042438977933035786955504105077991958445671969042859007463098310599443798853
87481474312223533832259969580105714665875528461138380*sqrt(7) - 73647197127
004832472492154993592666800848779558660715739110082101559839168913439380857
180149261966211988875977077496294862444706766451993916021142933175105692227
6760*sqrt(2) - 36823598563502416236246077496796333400424389779330357869555O
410507799195844567196904285900746309831059944379885387481474312223533832259
9695801057146658755284611383 80*sqrt(7722433301876572160*sqrt(2) - 105954709
86612263150) - 142835208193613592915330036722260451533458578946048957790307
351258262277239751604641916384506889981140070748629723613145062430716033621
3112668130748323345181748400248288944 86))/(110320475741093888*sqrt(2) - 151
363871237318045) + 1/205800000000*(108121281*(x - sqrt(x^2 - 2*x + 3))^15 +
135317265*(x - sqrt(x^2 - 2*x + 3))^14 - 2309618731*(x - sqrt(x^2 - 2*x +
3))^13 - 4089866767*(x - sqrt(x^2 - 2*x + 3))^12 + 23951599406*(x - sqrt(x^
2 - 2*x + 3))^11 + 45641347654*(x - sqrt(x^2 - 2*x + 3))^10 - 149568395690*
(x - sqrt(x^2 - 2*x + 3))^9 - 288215430978*(x - sqrt(x^2 - 2*x + 3))^8 + 66
0704292769*(x - sqrt(x^2 - 2*x + 3))^7 + 1062639157153*(x - sqrt(x^2 - 2*x
+ 3))^6 - 2094971437979*(x - sqrt(x^2 - 2*x + 3))^5 - 2301192104575*(x - sq
rt(x^2 - 2*x + 3))^4 + 4977175786352*(x - sqrt(x^2 - 2*x + 3))^3 + 13029940
04424*(x - sqrt(x^2 - 2*x + 3))^2 - 6052879270032*x + 6052879270032*sqrt(x^
2 - 2*x + 3) + 2841437414928)/((x - sqrt(x^2 - 2*x + 3))^4 + (x - sqrt(x^2
- 2*x + 3))^3 - 5*(x - sqrt(x^2 - 2*x + 3))^2 - 7*x + 7*sqrt(x^2 - 2*x + 3)
+ 14)^4 + 1/3150000000*(3*((((((((29420*x - 332589)*x + 1860912)*x - 67437
44)*x + 17167416)*x - 31960026)*x + 43362368)*x - 42014736)*x + 26516604)*x
- 27199867)/(x^2 - 2*x + 3)^(9/2)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x+x^2)^{11/2}\left(1+x+2x^2\right)^5}\,dx=\int \frac{1}{\left(2\,x^2+x+1\right)^5\,\left(x^2-2\,x+3\right)^{11/2}}\,dx$$

```
[In] int(1/((x + 2*x^2 + 1)^5*(x^2 - 2*x + 3)^(11/2)),x)
[Out] int(1/((x + 2*x^2 + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

3.51 $\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 638

$$\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx = \frac{37358055634422583-14024622879097678x}{18401244792000000\,(3-2x+x^2)^{19/2}}$$
$$+\frac{476849951294984711-125181871472148210x}{104273720488000000\,(3-2x+x^2)^{17/2}}$$
$$+\frac{7851758375483333511+19421649962045842 34x}{1564105807320000000 0\,(3-2x+x^2)^{15/2}}$$
$$-\frac{11(7502325106308201089-7813986379726516886x)}{406667509903200000000\,(3-2x+x^2)^{13/2}}$$
$$-\frac{3(6905326851529635 9011-44840736195018286006x)}{11470109253680000000000\,(3-2x+x^2)^{11/2}}$$
$$-\frac{838519439380295335657-466189390555853643870x}{93846348439200000000000\,(3-2x+x^2)^{9/2}}$$
$$-\frac{1117646664729238460189-568839749685437871554x}{312821161464000000000000\,(3-2x+x^2)^{7/2}}$$
$$-\frac{655140551156544930 1689-312729855998330930 1910x}{5213686024400000000000000\,(3-2x+x^2)^{5/2}}$$
$$-\frac{4179039782398459850819-1886993445589652402694x}{10427372048800000000000000\,(3-2x+x^2)^{3/2}}$$
$$-\frac{12105495874518671061833-5117656435043679338190x}{104273720488000000000000000\sqrt{3-2x+x^2}}$$
$$-\frac{1-10x}{630\,(3-2x+x^2)^{19/2}(1+x+2x^2)^9}+\frac{887+2218x}{88200\,(3-2x+x^2)^{19/2}(1+x+2x^2)^8}$$
$$+\frac{14453+29371x}{1080450\,(3-2x+x^2)^{19/2}(1+x+2x^2)^7}+\frac{8837931+17459234x}{605052000\,(3-2x+x^2)^{19/2}(1+x+2x^2)^6}$$
$$+\frac{447940041+813432205x}{26471025000\,(3-2x+x^2)^{19/2}(1+x+2x^2)^5}$$
$$+\frac{592729157441+911061463974x}{29647548000000\,(3-2x+x^2)^{19/2}(1+x+2x^2)^4}$$
$$+\frac{277010166219+310705340015x}{12353145000000\,(3-2x+x^2)^{19/2}(1+x+2x^2)^3}$$
$$+\frac{5488221294349+1384103301166x}{276710448000000\,(3-2x+x^2)^{19/2}(1+x+2x^2)^2}$$
$$-\frac{37857197792117+146548895467025x}{2421216420000000\,(3-2x+x^2)^{19/2}(1+x+2x^2)}$$

$$+\frac{\sqrt{\frac{1}{70}\left(81042225921274689605478944797800854846405+573059225230017071260263638786665003089 92\right.}}{}$$

$$-\frac{\sqrt{\frac{1}{70}\left(-81042225921274689605478944797800854846405+57305922523001707126026363878666500308 9\right.}}{}$$

```
[Out] 1/1840124479200000000*(37358055634422583-14024622879097678*x)/(x^2-2*x+3)^(
19/2)+1/10427372048800000000*(476849951294984711-125181871472148210*x)/(x^
2-2*x+3)^(17/2)+1/156410580732000000000*(7851758375483333511+194216499620
4584234*x)/(x^2-2*x+3)^(15/2)-11/4066675099032000000000*(7502325106308201
089-7813986379726516886*x)/(x^2-2*x+3)^(13/2)-3/11470109253680000000000*(
69053268515296359011-44840736195018286006*x)/(x^2-2*x+3)^(11/2)+1/938463484
392000000000000*(-838519439380295335657+466189390555853643870*x)/(x^2-2*x+
3)^(9/2)+1/3128211614640000000000000*(-1117646664729238460189+568839749685
437871554*x)/(x^2-2*x+3)^(7/2)+1/5213686024400000000000000*(-655140551156
5449301689+3127298559983309301910*x)/(x^2-2*x+3)^(5/2)+1/104273720488000000
0000000000*(-417903978239845985081 9+18869934455896524 02694*x)/(x^2-2*x+3)^(
3/2)+1/630*(-1+10*x)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1)^9+1/88200*(887+2218*x)/
(x^2-2*x+3)^(19/2)/(2*x^2+x+1)^8+1/1080450*(14453+29371*x)/(x^2-2*x+3)^(19/
2)/(2*x^2+x+1)^7+1/605052000*(8837931+17459234*x)/(x^2-2*x+3)^(19/2)/(2*x^2
+x+1)^6+1/26471025000*(447940041+813432205*x)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1
)^5+1/29647548000000*(592729157441+911061463974*x)/(x^2-2*x+3)^(19/2)/(2*x^
2+x+1)^4+1/12353145000000*(277010166219+310705340015*x)/(x^2-2*x+3)^(19/2)/
(2*x^2+x+1)^3+1/276710448000000*(5488221294349+1384103301166*x)/(x^2-2*x+3)
^(19/2)/(2*x^2+x+1)^2+1/2421216420000000*(-37857197792117-146548895467025*x
)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1)+1/104273720488000000000000000*(-12105495
874518671061833+5117656435043679338190*x)/(x^2-2*x+3)^(1/2)-1/2259801992000
000000000000000*arctanh(1/7*(2729445895232483817 49+x*(656826642296538601431
-464885615909893491590*2^(1/2))-191941026386645109841*2^(1/2))*35^(1/2)/(-8
1042225921274689605478944797800854846405+5730592252300170712602636387866650
0308992*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(-5672955814489228272383526135846
059839248350+4011414576610119498821845471506655021629440*2^(1/2))^(1/2)+1/2
2598019920000000000000000*arctan(1/7*(272944589523248381749+1919410263866
45109841*2^(1/2)+x*(656826642296538601431+464885615909893491590*2^(1/2)))*3
5^(1/2)/(81042225921274689605478944797800854846405+57305922523001707126026 3
63878666500308992*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(567295581448922827238 3
526135846059839248350+4011414576610119498821845471506655021629440*2^(1/2))^
(1/2)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

$= \{988, 1074, 1049, 1043, 212, 210\}$

$$\int \frac{1}{(3-2x+x^2)^{21/2}\,(1+x+2x^2)^{10}}\,dx = \frac{\sqrt{\frac{1}{70}\left(81042225921274689605478944797800854846405 + 5730592\right.}}{}$$

$$-\frac{\sqrt{\frac{1}{70}\left(5730592252300170712602636387866650030899 2\sqrt{2} - 8104222592127468960547894479780085484 6\right.}}{}$$

$$-\frac{12105495874518671061833 - 51176564350436793381 90x}{104273720488000000000000000\sqrt{x^2-2x+3}}$$
$$-\frac{146548895467025x + 37857197792117}{2421216420000000\,(x^2-2x+3)^{19/2}\,(2x^2+x+1)}$$
$$-\frac{41790397823984598508 19 - 18869934455896524026 94x}{104273720488000000000000000\,(x^2-2x+3)^{3/2}}$$
$$+\frac{1384103301166x + 5488221294349}{276710448000000\,(x^2-2x+3)^{19/2}\,(2x^2+x+1)^2}$$
$$-\frac{6551405511565449301689 - 3127298559983309301910x}{52136860244000000000000000\,(x^2-2x+3)^{5/2}}$$
$$+\frac{310705340015x + 277010166219}{12353145000000\,(x^2-2x+3)^{19/2}\,(2x^2+x+1)^3}$$
$$-\frac{1117646664729238460189 - 568839749685437871554x}{3128211614640000000000000\,(x^2-2x+3)^{7/2}}$$
$$+\frac{911061463974x + 592729157441}{29647548000000\,(x^2-2x+3)^{19/2}\,(2x^2+x+1)^4}$$
$$-\frac{838519439380295335657 - 466189390555853643870x}{938463484392000000000000\,(x^2-2x+3)^{9/2}}$$
$$+\frac{813432205x + 447940041}{26471025000\,(x^2-2x+3)^{19/2}\,(2x^2+x+1)^5}$$
$$-\frac{3(690532685152963590 11 - 448407361950182860 06x)}{114701092536800000000000\,(x^2-2x+3)^{11/2}}$$
$$+\frac{17459234x + 8837931}{605052000\,(x^2-2x+3)^{19/2}\,(2x^2+x+1)^6}$$
$$-\frac{11(750232510630820108 9 - 781398637972651688 6x)}{406667509903200000000000\,(x^2-2x+3)^{13/2}}$$
$$+\frac{29371x + 14453}{1080450\,(x^2-2x+3)^{19/2}\,(2x^2+x+1)^7}$$
$$+\frac{1942164996204584234x + 7851758375483333511}{156410580732000000000000\,(x^2-2x+3)^{15/2}}$$
$$+\frac{2218x + 887}{88200\,(x^2-2x+3)^{19/2}\,(2x^2+x+1)^8} + \frac{476849951294984711 - 1251818714721482 10x}{10427372048800000000 0\,(x^2-2x+3)^{17/2}}$$
$$-\frac{1-10x}{630\,(x^2-2x+3)^{19/2}\,(2x^2+x+1)^9} + \frac{37358055634422583 - 14024622879097678x}{1840124479200000000\,(x^2-2x+3)^{19/2}}$$

```
[In] Int[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] (37358055634422583 - 14024622879097678*x)/(1840124479200000000*(3 - 2*x + x
^2)^(19/2)) + (476849951294984711 - 125181871472148210*x)/(1042737204880000
00000*(3 - 2*x + x^2)^(17/2)) + (7851758375483333511 + 1942164996204584234*
x)/(156410580732000000000000*(3 - 2*x + x^2)^(15/2)) - (11*(7502325106308201
089 - 781398637972651688 6*x))/(406667509903200000000000*(3 - 2*x + x^2)^(13
/2)) - (3*(69053268515296359011 - 44840736195018286006*x))/(114701092536800
0000000000*(3 - 2*x + x^2)^(11/2)) - (838519439380295335657 - 4661893905558
53643870*x)/(938463484392000000000000*(3 - 2*x + x^2)^(9/2)) - (1117646664
729238460189 - 568839749685437871554*x)/(31282116146400000000000000*(3 - 2*
x + x^2)^(7/2)) - (6551405511565449301689 - 3127298559983309301910*x)/(5213
686024400000000000000000*(3 - 2*x + x^2)^(5/2)) - (41790397823984598508 19 -
1886993445589652402694*x)/(1042737204880000000000000000*(3 - 2*x + x^2)^(3/
2)) - (12105495874518671061833 - 5117656435043679338190*x)/(104273720488000
00000000000000*Sqrt[3 - 2*x + x^2]) - (1 - 10*x)/(630*(3 - 2*x + x^2)^(19/2
)*(1 + x + 2*x^2)^9) + (887 + 2218*x)/(88200*(3 - 2*x + x^2)^(19/2)*(1 + x
+ 2*x^2)^8) + (14453 + 29371*x)/(1080450*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*
x^2)^7) + (8837931 + 17459234*x)/(605052000*(3 - 2*x + x^2)^(19/2)*(1 + x +
 2*x^2)^6) + (447940041 + 813432205*x)/(26471025000*(3 - 2*x + x^2)^(19/2)*
(1 + x + 2*x^2)^5) + (592729157441 + 911061463974*x)/(29647548000000*(3 - 2
*x + x^2)^(19/2)*(1 + x + 2*x^2)^4) + (277010166219 + 310705340015*x)/(1235
3145000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^3) + (5488221294349 + 138
4103301166*x)/(276710448000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^2) -
(37857197792117 + 146548895467025*x)/(2421216420000000*(3 - 2*x + x^2)^(19/
2)*(1 + x + 2*x^2)) + (Sqrt[(810422259212746896054789447978008548464 05 + 57
305922523001707126026363878666500308992*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(810
422259212746896054789447978008548464 05 + 573059225230017071260263638786665 0
0308992*Sqrt[2]))]*(272944589523248381749 + 191941026386645109841*Sqrt[2] +
 (656826642296538601431 + 464885615909893491590*Sqrt[2])*x))/Sqrt[3 - 2*x +
 x^2]])/3228288560000000000000000 - (Sqrt[(-8104222592127468960547894479 78
00854846405 + 57305922523001707126026363878666500308992*Sqrt[2])/70]*ArcTan
h[(Sqrt[5/(7*(-81042225921274689605478944797800854846405 + 5730592252300170
7126026363878666500308992*Sqrt[2]))]*(272944589523248381749 - 1919410263866
45109841*Sqrt[2] + (656826642296538601431 - 464885615909893491590*Sqrt[2])*
x))/Sqrt[3 - 2*x + x^2]])/3228288560000000000000000
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
```

```
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = -\frac{1-10x}{630(3-2x+x^2)^{19/2}(1+x+2x^2)^9} - \frac{\int \frac{-2960+3060x-1800x^2}{(3-2x+x^2)^{21/2}(1+x+2x^2)^9}\,dx}{3150}$$

$$= -\frac{1-10x}{630(3-2x+x^2)^{19/2}(1+x+2x^2)^9} + \frac{887+2218x}{88200(3-2x+x^2)^{19/2}(1+x+2x^2)^8} - \frac{\int \frac{-8066100+8650900x-7541200x^2}{(3-2x+x^2)^{21/2}(1+x+2x^2)^8}\,dx}{8820000}$$

$$= -\frac{1-10x}{630(3-2x+x^2)^{19/2}(1+x+2x^2)^9} + \frac{887+2218x}{88200(3-2x+x^2)^{19/2}(1+x+2x^2)^8} + \frac{14453+29371x}{1080450(3-2x+x^2)^{19/2}(1+x+2x^2)^7} - \frac{\int \frac{-18577805000+18950890000x-18797440000x^2}{(3-2x+x^2)^{21/2}(1+x+2x^2)^7}\,dx}{21609000000}$$

$$= -\frac{1-10x}{630(3-2x+x^2)^{19/2}(1+x+2x^2)^9} + \frac{887+2218x}{88200(3-2x+x^2)^{19/2}(1+x+2x^2)^8} + \frac{14453+29371x}{1080450(3-2x+x^2)^{19/2}(1+x+2x^2)^7} + \frac{8837931+17459234x}{605052000(3-2x+x^2)^{19/2}(1+x+2x^2)^6} - \frac{\int \frac{-34422218025000+37067282625000x-39283276500000x^2}{(3-2x+x^2)^{21/2}(1+x+2x^2)^6}\,dx}{45378900000000}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^8} + \frac{14453+29371x}{1080450\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^7} + \frac{8837931+17459234x}{605052000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^6} + \frac{447940041+813432205x}{26471025000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^5} - \frac{\int \frac{-4754271120675000+5742093272550000x-6832830522000000x^2}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^5}\,dx}{7941307500000000}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^8} + \frac{14453+29371x}{1080450\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^7} + \frac{8837931+17459234x}{605052000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^6} + \frac{447940041+813432205x}{26471025000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^5} + \frac{592729157441+911061463974x}{29647548000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^4} - \frac{\int \frac{-407512138369162500000+595629899556862500000x-888284927374650000000x^2}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^4}\,dx}{1111783050000000000000}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^8} + \frac{14453+29371x}{1080450\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^7} + \frac{8837931+17459234x}{605052000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^6} + \frac{447940041+813432205x}{26471025000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^5} + \frac{592729157441+911061463974x}{29647548000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^4} + \frac{277010166219+310705340015x}{12353145000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^3} - \frac{\int \frac{-78028174316413125000000+241093948565846250000000x-704679711154020000000000x^2}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^3}\,dx}{116737220250000000000000}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^8} + \frac{14453+29371x}{1080450\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^7} + \frac{8837931+17459234x}{605052000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^6} + \frac{447940041+813432205x}{26471025000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^5} + \frac{592729157441+911061463974x}{29647548000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^4} + \frac{277010166219+310705340015x}{12353145000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^3} + \frac{5488221294349+1384103301166x}{276710448000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^2} - \frac{\int \frac{1683388137906454218750000-266497099134459046875000x-899234613476285625000000x^2}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^2}\,dx}{81716054175000000000000000}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^8} + \frac{14453+29371x}{1080450\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^7} + \frac{8837931+17459234x}{605052000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^6} + \frac{447940041+813432205x}{26471025000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^5} + \frac{592729157441+911061463974x}{29647548000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^4} + \frac{277010166219+310705340015x}{12353145000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^3} + \frac{5488221294349+1384103301166x}{276710448000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^2} - \frac{37857197792117+146548895467025x}{2421216420000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)} - \frac{\int \frac{-6456478383150221671875000000-2782788866833398215625000000x+3462217655408465625000000000x^2}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)}\,dx}{2860061896125000000000000000}$$

$$
\begin{aligned}
&= \frac{37358055634422583 - 14024622879097678x}{18401244792000000000\left(3-2x+x^2\right)^{19/2}} \\
&\quad - \frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^8} \\
&\quad + \frac{14453+29371x}{1080450\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^7} \\
&\quad + \frac{8837931+17459234x}{605052000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^6} \\
&\quad + \frac{447940041+813432205x}{26471025000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^5} \\
&\quad + \frac{592729157441+911061463974x}{29647548000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^4} \\
&\quad + \frac{277010166219+310705340015x}{12353145000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^3} \\
&\quad + \frac{5488221294349+1384103301166x}{276710448000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^2} \\
&\quad - \frac{37857197792117+146548895467025x}{2421216420000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)} \\
&\quad - \frac{\int \frac{496771244496421006218750000000-3745904594189161473562500000000x+2981985439668143784750000000000x^2}{\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)}\,dx}{108682352052750000000000000000}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{37358055634422583 - 14024622879097678x}{18401244792000000000\left(3-2x+x^2\right)^{19/2}} \\
&+ \frac{476849951294984711 - 125181871472148210x}{1042737204880000000000\left(3-2x+x^2\right)^{17/2}} \\
&- \frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^8} \\
&+ \frac{14453+29371x}{1080450\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^7} \\
&+ \frac{8837931+17459234x}{605052000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^6} \\
&+ \frac{447940041+813432205x}{26471025000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^5} \\
&+ \frac{592729157441+911061463974x}{29647548000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^4} \\
&+ \frac{277010166219+310705340015x}{12353145000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^3} \\
&+ \frac{5488221294349+1384103301166x}{276710448000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^2} \\
&- \frac{37857197792117+146548895467025x}{2421216420000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)} \\
&- \frac{\int \frac{1476220393170575791239375000000000000-352737956551832094072375000000000000x+14195624224941607014000000000000000x}{\left(3-2x+x^2\right)^{17/2}\left(1+x+2x^2\right)}}{36951999697935000000000000000000000}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{37358055634422583 - 14024622879097678x}{18401244792000000000\left(3-2x+x^2\right)^{19/2}} \\
&\quad + \frac{476849951294984711 - 1251818714721482 10x}{1042737204880000000000\left(3-2x+x^2\right)^{17/2}} \\
&\quad + \frac{7851758375483333511 + 19421649962045842 34x}{15641058073200000000000\left(3-2x+x^2\right)^{15/2}} \\
&\quad - \frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^8} \\
&\quad + \frac{14453+29371x}{1080450\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^7} \\
&\quad + \frac{8837931+17459234x}{605052000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^6} \\
&\quad + \frac{447940041+813432205x}{26471025000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^5} \\
&\quad + \frac{592729157441+911061463974x}{29647548000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^4} \\
&\quad + \frac{277010166219+310705340015x}{12353145000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^3} \\
&\quad + \frac{5488221294349+1384103301166x}{276710448000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^2} \\
&\quad - \frac{37857197792117+146548895467025x}{2421216420000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)} \\
&\quad - \frac{\int \frac{17682321750123664132939125000000000000-22751442786535433811492750000000000000x-385422643496799741237300000\cdots}{\left(3-2x+x^2\right)^{15/2}\left(1+x+2x^2\right)}}{110855999093805000000000000000000000000}
\end{aligned}
$$

$$
\begin{aligned}
= & \frac{37358055634422583 - 14024622879097678x}{18401244792000000000\left(3 - 2x + x^2\right)^{19/2}} \\
& + \frac{476849951294984711 - 125181871472148210x}{1042737204880000000000\left(3 - 2x + x^2\right)^{17/2}} \\
& + \frac{785175837548333511 + 1942164996204584234x}{156410580732000000000000\left(3 - 2x + x^2\right)^{15/2}} \\
& - \frac{11(7502325106308201089 - 7813986379726516886x)}{40666750990320000000000000\left(3 - 2x + x^2\right)^{13/2}} \\
& - \frac{1 - 10x}{630\left(3 - 2x + x^2\right)^{19/2}\left(1 + x + 2x^2\right)^9} + \frac{887 + 2218x}{88200\left(3 - 2x + x^2\right)^{19/2}\left(1 + x + 2x^2\right)^8} \\
& + \frac{14453 + 29371x}{1080450\left(3 - 2x + x^2\right)^{19/2}\left(1 + x + 2x^2\right)^7} \\
& + \frac{8837931 + 17459234x}{605052000\left(3 - 2x + x^2\right)^{19/2}\left(1 + x + 2x^2\right)^6} \\
& + \frac{447940041 + 813432205x}{26471025000\left(3 - 2x + x^2\right)^{19/2}\left(1 + x + 2x^2\right)^5} \\
& + \frac{592729157441 + 911061463974x}{29647548000000\left(3 - 2x + x^2\right)^{19/2}\left(1 + x + 2x^2\right)^4} \\
& + \frac{277010166219 + 310705340015x}{12353145000000\left(3 - 2x + x^2\right)^{19/2}\left(1 + x + 2x^2\right)^3} \\
& + \frac{5488221294349 + 1384103301166x}{276710448000000\left(3 - 2x + x^2\right)^{19/2}\left(1 + x + 2x^2\right)^2} \\
& - \frac{37857197792117 + 146548895467025x}{2421216420000000\left(3 - 2x + x^2\right)^{19/2}\left(1 + x + 2x^2\right)} \\
& - \frac{\int \frac{1339931000513420726920031250000000000000000 - 79421201503039044511112250000000000000000x - 14620749915106285745394600}{(3-2x+x^2)^{13/2}(1+x+2x^2)}}{288225597643893000000000000000000000000000000}
\end{aligned}
$$

$$
\begin{aligned}
= & \frac{37358055634422583-14024622879097678x}{18401244792000000000\left(3-2x+x^2\right)^{19/2}} \\
& +\frac{476849951294984711-125181871472148210x}{104273720488000000000\left(3-2x+x^2\right)^{17/2}} \\
& +\frac{7851758375483333511+1942164996204584234x}{1564105807320000000000\left(3-2x+x^2\right)^{15/2}} \\
& -\frac{11(7502325106308201089-7813986379726516886x)}{40666750990320000000000\left(3-2x+x^2\right)^{13/2}} \\
& -\frac{3(69053268515296359011-44840736195018286006x)}{114701092536800000000000\left(3-2x+x^2\right)^{11/2}} \\
& -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9}+\frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^8} \\
& +\frac{14453+29371x}{1080450\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^7} \\
& +\frac{8837931+17459234x}{605052000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^6} \\
& +\frac{447940041+813432205x}{26471025000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^5} \\
& +\frac{592729157441+911061463974x}{29647548000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^4} \\
& +\frac{277010166219+310705340015x}{12353145000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^3} \\
& +\frac{5488221294349+1384103301166x}{276710448000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^2} \\
& -\frac{37857197792117+146548895467025x}{2421216420000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)} \\
& -\frac{\int \frac{637065202167552345252340275000000000000000000+1397569502668803779552737500000000000000000x-148734479922065903\ldots}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)}}{6340963148165646000000000000000000000000000000}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{37358055634422583 - 14024622879097678x}{18401244792000000000\left(3-2x+x^2\right)^{19/2}} \\
&\quad + \frac{476849951294984711 - 125181871472148210x}{1042737204880000000000\left(3-2x+x^2\right)^{17/2}} \\
&\quad + \frac{7851758375483333511 + 1942164996204584234x}{15641058073200000000000\left(3-2x+x^2\right)^{15/2}} \\
&\quad - \frac{11(7502325106308201089 - 7813986379726516886x)}{406667509903200000000000\left(3-2x+x^2\right)^{13/2}} \\
&\quad - \frac{3(69053268515296359011 - 44840736195018286006x)}{1147010925368000000000000\left(3-2x+x^2\right)^{11/2}} \\
&\quad - \frac{838519439380295335657 - 466189390555853643870x}{9384634843920000000000000\left(3-2x+x^2\right)^{9/2}} \\
&\quad - \frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^8} \\
&\quad + \frac{14453+29371x}{1080450\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^7} \\
&\quad + \frac{8837931+17459234x}{605052000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^6} \\
&\quad + \frac{447940041+813432205x}{26471025000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^5} \\
&\quad + \frac{592729157441+911061463974x}{29647548000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^4} \\
&\quad + \frac{277010166219+310705340015x}{12353145000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^3} \\
&\quad + \frac{5488221294349+1384103301166x}{276710448000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^2} \\
&\quad - \frac{37857197792117+146548895467025x}{2421216420000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)} \\
&\quad - \frac{\int \frac{13299179827424839360251589350000000000000000000+361510878887923002334625529000000000000000000000x-90717844741582}{(3-2x+x^2)^{9/2}(1+x+2x^2)}}{114137336666981628000000000000000000000000000000}
\end{aligned}
$$

$$
\begin{aligned}
= &\frac{37358055634422583 - 14024622879097678x}{18401244792000000\left(3-2x+x^2\right)^{19/2}} \\
&+ \frac{476849951294984711 - 125181871472148210x}{1042737204880000000\left(3-2x+x^2\right)^{17/2}} \\
&+ \frac{7851758375483333511 + 1942164996204584234x}{156410580732000000000\left(3-2x+x^2\right)^{15/2}} \\
&- \frac{11(7502325106308201089 - 7813986379726516886x)}{406667509903200000000\left(3-2x+x^2\right)^{13/2}} \\
&- \frac{3(69053268515296359011 - 44840736195018286006x)}{1147010925368000000000\left(3-2x+x^2\right)^{11/2}} \\
&- \frac{838519439380295335657 - 466189390555853643870x}{93846348439200000000000\left(3-2x+x^2\right)^{9/2}} \\
&- \frac{1117646664729238460189 - 568839749685437871554x}{312821161464000000000000\left(3-2x+x^2\right)^{7/2}} \\
&- \frac{1 - 10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887 + 2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^8} \\
&+ \frac{14453 + 29371x}{1080450\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^7} \\
&+ \frac{8837931 + 17459234x}{605052000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^6} \\
&+ \frac{447940041 + 813432205x}{26471025000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^5} \\
&+ \frac{592729157441 + 911061463974x}{29647548000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^4} \\
&+ \frac{277010166219 + 310705340015x}{12353145000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^3} \\
&+ \frac{5488221294349 + 1384103301166x}{276710448000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^2} \\
&- \frac{37857197792117 + 146548895467025x}{2421216420000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)} \\
&- \frac{\int \frac{-42091507044156141050332163310000000000000000000000+21812919296859155366923319298000000000000000000000x-348683}{\left(3-2x+x^2\right)^{7/2}\left(1+x+2x^2\right)}}{15979227133377427920000000000000000000000000000000}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{37358055634422583 - 14024622879097678x}{18401244792000000\left(3-2x+x^2\right)^{19/2}} \\
&+ \frac{476849951294984711 - 1251818714721482 10x}{104273720488000000\left(3-2x+x^2\right)^{17/2}} \\
&+ \frac{7851758375483333511 + 19421649962045842 34x}{1564105807320000000\left(3-2x+x^2\right)^{15/2}} \\
&- \frac{11(75023251063082010 89 - 78139863797265168 86x)}{40666750990320000000\left(3-2x+x^2\right)^{13/2}} \\
&- \frac{3(690532685152963590 11 - 448407361950182860 06x)}{114701092536800000000\left(3-2x+x^2\right)^{11/2}} \\
&- \frac{838519439380295335657 - 466189390555853643870x}{938463484392000000000\left(3-2x+x^2\right)^{9/2}} \\
&- \frac{1117646664729238460189 - 568839749685437871554x}{3128211614640000000000\left(3-2x+x^2\right)^{7/2}} \\
&- \frac{6551405511565449301689 - 3127298559983309301910x}{52136860244000000000000\left(3-2x+x^2\right)^{5/2}} \\
&- \frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^8} \\
&+ \frac{14453+29371x}{1080450\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^7} \\
&+ \frac{8837931+17459234x}{605052000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^6} \\
&+ \frac{447940041+813432205x}{26471025000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^5} \\
&+ \frac{592729157441+911061463974x}{29647548000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^4} \\
&+ \frac{277010166219+310705340015x}{12353145000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^3} \\
&+ \frac{5488221294349+1384103301166x}{276710448000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^2} \\
&- \frac{37857197792117+146548895467025x}{2421216420000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)} \\
&- \frac{\int \frac{-37911018602208199974074247699000000000000000000000000+6660519437543409120771215127000000000000000000000000x-766\cdots}{\left(3-2x+x^2\right)^{5/2}\left(1+x+2x^2\right)}}{15979227133377427920000000000000000000000000000000\cdots}
\end{aligned}
$$

$$
\begin{aligned}
= & \frac{37358055634422583 - 14024622879097678x}{18401244792000000\,(3-2x+x^2)^{19/2}} \\
& + \frac{476849951294984711 - 125181871472148210x}{104273720488000000\,(3-2x+x^2)^{17/2}} \\
& + \frac{7851758375483333511 + 19421649962045842234x}{1564105807320000000000\,(3-2x+x^2)^{15/2}} \\
& - \frac{11(7502325106308201089 - 7813986379726516886x)}{40666750990320000000000\,(3-2x+x^2)^{13/2}} \\
& - \frac{3(69053268515296359011 - 44840736195018286006x)}{1147010925368000000000000\,(3-2x+x^2)^{11/2}} \\
& - \frac{83851943938029533565 7 - 46618939055585364387 0x}{93846348439200000000000000\,(3-2x+x^2)^{9/2}} \\
& - \frac{1117646664729238460189 - 568839749685437871554x}{312821161464000000000000000\,(3-2x+x^2)^{7/2}} \\
& - \frac{6551405511565449301689 - 3127298559983309301910x}{5213686024400000000000000000\,(3-2x+x^2)^{5/2}} \\
& - \frac{4179039782398459850819 - 1886993445589652402694x}{104273720488000000000000000000\,(3-2x+x^2)^{3/2}} \\
& - \frac{1-10x}{630\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)^9} + \frac{887+2218x}{88200\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)^8} \\
& + \frac{14453+29371x}{1080450\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)^7} \\
& + \frac{8837931+17459234x}{605052000\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)^6} \\
& + \frac{447940041+813432205x}{26471025000\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)^5} \\
& + \frac{592729157441+911061463974x}{29647548000000\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)^4} \\
& + \frac{277010166219+310705340015x}{12353145000000\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)^3} \\
& + \frac{5488221294349+1384103301166x}{276710448000000\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)^2} \\
& - \frac{37857197792117+146548895467025x}{2421216420000000\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)} \\
& - \frac{\int \frac{-968964529889621435020898777730000000000000000000000000+917462499378458365778779481148000000000000000000000}{(3-2x+x^2)^{3/2}(1+x+2x^2)}}{9587536280026456752000000000000000000000000000}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{37358055634422583 - 14024622879097678x}{18401244792000000\,(3-2x+x^2)^{19/2}} \\
&\quad + \frac{476849951294984711 - 125181871472148210x}{104273720488000000\,(3-2x+x^2)^{17/2}} \\
&\quad + \frac{7851758375483333511 + 1942164996204584234x}{1564105807320000000\,(3-2x+x^2)^{15/2}} \\
&\quad - \frac{11(7502325106308201089 - 7813986379726516886x)}{40666750990320000000\,(3-2x+x^2)^{13/2}} \\
&\quad - \frac{3(690532685152963590 11 - 44840736195018286006x)}{114701092536800000000\,(3-2x+x^2)^{11/2}} \\
&\quad - \frac{838519439380295335657 - 466189390555853643870x}{938463484392000000000\,(3-2x+x^2)^{9/2}} \\
&\quad - \frac{1117646664729238460189 - 568839749685437871554x}{3128211614640000000000\,(3-2x+x^2)^{7/2}} \\
&\quad - \frac{6551405511565449301689 - 3127298559983309301910x}{52136860244000000000000\,(3-2x+x^2)^{5/2}} \\
&\quad - \frac{4179039782398459850819 - 1886993445589652402694x}{104273720488000000000000\,(3-2x+x^2)^{3/2}} \\
&\quad - \frac{12105495874518671061833 - 5117656435043679338190x}{104273720488000000000000\sqrt{3-2x+x^2}} \\
&\quad - \frac{1-10x}{630\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)^9} + \frac{887+2218x}{88200\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)^8} \\
&\quad + \frac{14453+29371x}{1080450\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)^7} \\
&\quad + \frac{8837931+17459234x}{605052000\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)^6} \\
&\quad + \frac{447940041+813432205x}{26471025000\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)^5} \\
&\quad + \frac{592729157441+911061463974x}{29647548000000\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)^4} \\
&\quad + \frac{277010166219+310705340015x}{12353145000000\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)^3} \\
&\quad + \frac{5488221294349+1384103301166x}{276710448000000\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)^2} \\
&\quad - \frac{37857197792117+146548895467025x}{2421216420000000\,(3-2x+x^2)^{19/2}\,(1+x+2x^2)} \\
&\quad - \frac{\int \frac{-62607023873175003518521377832680000000000000+257001145950963770269315050448800000000000000000000000000}{\sqrt{3-2x+x^2}(1+x+2x^2)}}{19175072560052913504000000000000000000000000000}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{37358055634422583 - 14024622879097678x}{1840124479200000000\left(3-2x+x^2\right)^{19/2}} \\
&+ \frac{476849951294984711 - 125181871472148210x}{10427372048800000000\left(3-2x+x^2\right)^{17/2}} \\
&+ \frac{785175837548333511 + 1942164996204584234x}{156410580732000000000\left(3-2x+x^2\right)^{15/2}} \\
&- \frac{11(7502325106308201089 - 7813986379726516886x)}{4066675099032000000000\left(3-2x+x^2\right)^{13/2}} \\
&- \frac{3(69053268515296359011 - 44840736195018286006x)}{11470109253680000000000\left(3-2x+x^2\right)^{11/2}} \\
&- \frac{838519439380295335657 - 466189390555853643870x}{938463484392000000000000\left(3-2x+x^2\right)^{9/2}} \\
&- \frac{1117646664729238460189 - 568839749685437871554x}{3128211614640000000000000\left(3-2x+x^2\right)^{7/2}} \\
&- \frac{655140551156544930168 9 - 3127298559983309301910x}{52136860244000000000000000\left(3-2x+x^2\right)^{5/2}} \\
&- \frac{4179039782398459850819 - 1886993445589652402694x}{104273720488000000000000000\left(3-2x+x^2\right)^{3/2}} \\
&- \frac{12105495874518671061833 - 5117656435043679338190x}{1042737204880000000000000000\sqrt{3-2x+x^2}} \\
&- \frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^8} \\
&+ \frac{14453+29371x}{1080450\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^7} \\
&+ \frac{8837931+17459234x}{605052000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^6} \\
&+ \frac{447940041+813432205x}{26471025000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^5} \\
&+ \frac{592729157441+911061463974x}{29647548000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^4} \\
&+ \frac{277010166219+310705340015x}{12353145000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^3} \\
&+ \frac{5488221294349+1384103301166x}{276710448000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^2} \\
&- \frac{37857197792117+146548895467025x}{2421216420000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)} \\
&- \frac{\int \frac{29698510842000000000000000000000000\left(14867267072426126015 9-10540431506187741047 7\sqrt{2}\right)-29698510842000000000 \cdots}{\sqrt{3-2x+x^2}\left(1+x+2x^2\right)}}{191750725600529135040000000000000000000000000000 \cdots} \\
&+ \frac{\int \frac{29698510842000000000000000000000000\left(14867267072426126015 9+10540431506187741047 7\sqrt{2}\right)-29698510842000000000 \cdots}{\sqrt{3-2x+x^2}\left(1+x+2x^2\right)}}{191750725600529135040000000000000000000000000000 \cdots}
\end{aligned}
$$

= Too large to display

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.66 (sec) , antiderivative size = 1431, normalized size of antiderivative = 2.24

$$\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx = \sqrt{3-2x+x^2}\left(\frac{1-x}{11875000000\,(3-2x+x^2)^{10}} + \frac{265-113x}{403750000000\,(3-2x+x^2)^9} + \frac{82361-4841x}{60562500000000\,(3-2x+x^2)^8} + \frac{1062937+1642511x}{1574625000000000\,(3-2x+x^2)^7} + \frac{7(-678331+833371x)}{2220625000000000\,(3-2x+x^2)^6} + \frac{7(-73161291+43964675x)}{90843750000000000\,(3-2x+x^2)^5} + \frac{-1340879383+430593031x}{181687500000000000\,(3-2x+x^2)^4} - \frac{11(1626125723+112950205x)}{3028125000000000000\,(3-2x+x^2)^3} - \frac{11(3311570647+15286717673x)}{36337500000000000000\,(3-2x+x^2)^2} - \frac{11(-411521923277+484788625685x)}{363375000000000000000\,(3-2x+x^2)} + \frac{251943+221770x}{6300000000000\,(1+x+2x^2)^9} - \frac{73(-888423+1604678x)}{882000000000000\,(1+x+2x^2)^8} + \frac{-2596903794-4965311863x}{10804500000000000\,(1+x+2x^2)^7} + \frac{-539608494637-334647150510x}{1210104000000000000\,(1+x+2x^2)^6} + \frac{-40800462989458+56711874696335x}{26471025000000000000\,(1+x+2x^2)^5} + \frac{42018358198215561+129196597088670934x}{296475480000000000000000\,(1+x+2x^2)^4} + \frac{62819559864314747+169630389653846945x}{370594350000000000000000\,(1+x+2x^2)^3} + \frac{1082422109196374795+4797048907791526114x}{8301313440000000000000000\,(1+x+2x^2)^2} + \frac{65571203144429922747+367152793968978953465x}{363182463000000000000000000\,(1+x+2x^2)}\right) + \frac{(232442807954946745795i+2163417783119192\cdots}{\cdots}$$

$$- \frac{i(-232442807954946745795i+216341778311919248 41\sqrt{7})\arctan\left(\frac{\cdots}{1350637388604350168995865589487332591135\cdots}\right.}{\cdots}$$

$$- \frac{(-232442807954946745795i+21634177831191924841\sqrt{7})\log\left(\left(-i+\sqrt{7}-4ix\right)^2\left(i+\sqrt{7}+4ix\right)^2\right)}{322828856000000000000000000\sqrt{70\left(5+i\sqrt{7}\right)}}$$

$$+ \frac{i(232442807954946745795i+21634177831191924841\sqrt{7})\log\left(\left(-i+\sqrt{7}-4ix\right)^2\left(i+\sqrt{7}+4ix\right)^2\right)}{322828856000000000000000000\sqrt{70\left(-5+i\sqrt{7}\right)}}$$

$$- \frac{i(232442807954946745795i+21634177831191924841\sqrt{7})\log\left((1+x+2x^2)\left(-13i+15\sqrt{7}+22ix-1\cdots\right.\right.}{322828856000000000000000\cdots}$$

$$+ \frac{(-232442807954946745795i+21634177831191924841\sqrt{7})\log\left((1+x+2x^2)\left(-163i+15\sqrt{7}+122ix-\cdots\right.\right.}{3228288560000000000000\cdots}$$

```
[In] Integrate[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]

[Out] Sqrt[3 - 2*x + x^2]*((1 - x)/(11875000000*(3 - 2*x + x^2)^10) + (265 - 113*
x)/(403750000000*(3 - 2*x + x^2)^9) + (82361 - 4841*x)/(60562500000000*(3 -
 2*x + x^2)^8) + (1062937 + 1642511*x)/(1574625000000000*(3 - 2*x + x^2)^7)
 + (7*(-678331 + 833371*x))/(2220625000000000*(3 - 2*x + x^2)^6) + (7*(-731
61291 + 43964675*x))/(90843750000000000*(3 - 2*x + x^2)^5) + (-1340879383 +
 430593031*x)/(181687500000000000*(3 - 2*x + x^2)^4) - (11*(1626125723 + 11
2950205*x))/(3028125000000000000*(3 - 2*x + x^2)^3) - (11*(3311570647 + 152
86717673*x))/(36337500000000000000*(3 - 2*x + x^2)^2) - (11*(-411521923277
+ 484788625685*x))/(363375000000000000000*(3 - 2*x + x^2)) + (251943 + 2217
70*x)/(6300000000000*(1 + x + 2*x^2)^9) - (73*(-888423 + 1604678*x))/(88200
0000000000*(1 + x + 2*x^2)^8) + (-2596903794 - 4965311863*x)/(1080450000000
0000*(1 + x + 2*x^2)^7) + (-539608494637 - 334647150510*x)/(121010400000000
0000*(1 + x + 2*x^2)^6) + (-40800462989458 + 56711874696335*x)/(26471025000
0000000000*(1 + x + 2*x^2)^5) + (42018358198215561 + 129196597088670934*x)/
(2964754800000000000000*(1 + x + 2*x^2)^4) + (62819559864314747 + 1696303
89653846945*x)/(37059435000000000000000*(1 + x + 2*x^2)^3) + (108242210919
6374795 + 4797048907791526114*x)/(830131344000000000000000*(1 + x + 2*x^2)
^2) + (65571203144429922747 + 367152793968978953465*x)/(3631824630000000000
00000000*(1 + x + 2*x^2))) + ((232442807954946745795*I + 216341778311919248
41*Sqrt[7])*ArcTan[(-135063738860435016899586558948733259113515 + (18863089
4626466690216855285995045889396405*I)*Sqrt[7] - 150624136187268800855926877
6761430483700000*x - (105711500937472192718115651350352447938680*I)*Sqrt[7]
*x + 491153540508443587025809789813541985707360*x^2 - (46076406417713999339
9975100872663310399420*I)*Sqrt[7]*x^2 - 18008498514724668919944874526497767
8818020*x^3 + (197868296377913870863837680953446009396860*I)*Sqrt[7]*x^3 -
176004816500761880926774485599831047775825*x^4 - (2073428332284595771635570
43035558264835165*I)*Sqrt[7]*x^4 + (186244248199755548159585682605666126004
224*I)*Sqrt[10*(-5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] + (1146118450460034142
52052727757333000617984*I)*Sqrt[10*(-5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2]
+ (300856093245758962411638410362999126622208*I)*Sqrt[10*(-5 + I*Sqrt[7])]*
x^2*Sqrt[3 - 2*x + x^2] - (143264806307504267815065909696666250772480*I)*Sq
rt[10*(-5 + I*Sqrt[7])]*x^3*Sqrt[3 - 2*x + x^2])/(2368773290838836979864678
493023884746594823*I + 423642940259238735473942663180025956729505*Sqrt[7] +
 (1890613486065620301760074218556745311646936*I)*x + 6150574559311228258394
328777942059796320*Sqrt[7]*x + (2511300259855822962340893027852239157667820
*I)*x^2 - 2027867550801106189867763431094227596320*Sqrt[7]*x^2 - (313421774
6230760357128318797499380812303788*I)*x^3 + 6343043160272004327919286696836
9397935660*Sqrt[7]*x^3 + (944749064886626467328385369190460703669697*I)*x^4
 + 16381317765107264789462917221030750634835*Sqrt[7]*x^4)])/(16141442800000
000000000000*Sqrt[70*(-5 + I*Sqrt[7])]) - ((I/16141442800000000000000000)*(
-232442807954946745795*I + 21634177831191924841*Sqrt[7])*ArcTan[(35*(436249
4290663946676585186218212607628595*I + 121040840074068210135412189480007416
20843*Sqrt[7] - (40919031596617332707196094500783237405000*I)*x + 175730701
```

```
694606521668409393655487422752*Sqrt[7]*x + (2648728832926512757773396585336
4310310620*I)*x^2 - 57939072880031605424793240888406502752*Sqrt[7]*x^2 - (1
5238894149752825683924814021007863070620*I)*x^3 + 1812298045792001236548367
627667697083876*Sqrt[7]*x^3 - (795837271959975808913244203765619963595*I)*x
^4 + 468037650431636136841797634886592875281*Sqrt[7]*x^4))/(135063738860435
016899586558948733259113515 + (188630894626466690216855285995045889396405*I
)*Sqrt[7] + 1506241361872688008559268776761430483700000*x - (10571150093747
2192718115651350352447938680*I)*Sqrt[7]*x - 4911535405084435870258097898135
41985707360*x^2 - (460764064177139993399975100872663310399420*I)*Sqrt[7]*x^
2 + 180084985147246689199448745264977678818020*x^3 + (197868296377913870863
837680953446009396860*I)*Sqrt[7]*x^3 + 176004816500761880926774485599831047
775825*x^4 - (207342833228459577163557043035558264835165*I)*Sqrt[7]*x^4 - (
14326480630750426781506590969666625077248*I)*Sqrt[70*(5 + I*Sqrt[7])]*Sqrt[
3 - 2*x + x^2] - (14326480630750426781506590969666625077248*I)*Sqrt[70*(5 +
 I*Sqrt[7])]*x^2*Sqrt[3 - 2*x + x^2] + (28652961261500853563013181939333250
154496*I)*Sqrt[70*(5 + I*Sqrt[7])]*x^3*Sqrt[3 - 2*x + x^2])])/Sqrt[70*(5 +
I*Sqrt[7])] - ((-232442807954946745795*I + 21634177831191924841*Sqrt[7])*Lo
g[(-I + Sqrt[7] - (4*I)*x)^2*(I + Sqrt[7] + (4*I)*x)^2])/(3228288560000000
000000000*Sqrt[70*(5 + I*Sqrt[7])]) + ((I/3228288560000000000000000)*(2324
42807954946745795*I + 21634177831191924841*Sqrt[7])*Log[(-I + Sqrt[7] - (4*
I)*x)^2*(I + Sqrt[7] + (4*I)*x)^2])/Sqrt[70*(-5 + I*Sqrt[7])] - ((I/3228288
5600000000000000000)*(232442807954946745795*I + 21634177831191924841*Sqrt[7
])*Log[(1 + x + 2*x^2)*(-13*I + 15*Sqrt[7] + (22*I)*x - 10*Sqrt[7]*x + (9*I
)*x^2 + 5*Sqrt[7]*x^2 + I*Sqrt[70*(-5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] - I
*Sqrt[70*(-5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2])])/Sqrt[70*(-5 + I*Sqrt[7]
)] + ((-232442807954946745795*I + 21634177831191924841*Sqrt[7])*Log[(1 + x
+ 2*x^2)*(-163*I + 15*Sqrt[7] + (122*I)*x - 10*Sqrt[7]*x - (41*I)*x^2 + 5*S
qrt[7]*x^2 - (13*I)*Sqrt[10*(5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] + (5*I)*Sq
rt[10*(5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2])])/(3228288560000000000000000
*Sqrt[70*(5 + I*Sqrt[7])])
```

Maple [A] (verified)

Time = 6.78 (sec) , antiderivative size = 552, normalized size of antiderivative = 0.87

method	result
risch	$\frac{33722490019334222378242713600x^{37}-535022053996400313947961477120x^{36}+4691493940829897017294945758720x^{35}-2847499220912\ldots}{}$
trager	Expression too large to display
default	Expression too large to display

```
[In] int(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13420027826805600000000000000000*(33722490019334222378242713600*x^37-53502
205399640031394796147712*x^36+469149394082989701729494575872*x^35-284749922
0912667753383035299072*x^34+13254252261100740556512388253568*x^33-497700800
58525077628064229832576*x^32+156010734937008739388220889457760*x^31-4175163
98850754397130111919794336*x^30+971538171913365251873706873353652*x^29-1993
653213575521837888601204380228*x^28+3655553471852957606257345414140031*x^27
-6054769996581738503753686155104785*x^26+9155494158513869230271529746307221
*x^25-12740106677685048178693605103009787*x^24+1644277020247007631319721593
6814318*x^23-19772569734288744720189854470201506*x^22+222864376176219099216
09206629636086*x^21-23584986647560742443188031208946882*x^20+23579397211179
175240196614296051673*x^19-22218747553941794885903840542461607*x^18+1991229
5454080246583636391613811979*x^17-16801760806053390242995145349148613*x^16+
13613407965006475288139078599341572*x^15-1027930565073317866922363402096207
6*x^14+7606288378303449524327938977040824*x^13-5069838234992751929471190426
115248*x^12+3507425970596197680016078213030977*x^11-19748144830613444052758
51094534735*x^10+1357002388430055881833293557852283*x^9-5669690107591694616
15951049236597*x^8+458426000073846882432457044306894*x^7-947045576652534893
32536549937026*x^6+135183920426913231415208872303230*x^5-102309531890177463
8403186272874*x^4+29398041153524973343917601742151*x^3+19339571955700627087
81629134823*x^2+3397462350398947848063583843461*x-8003871087155531686134536
9643)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1)^9+1/63274455776000000000000000000*4^(1
/2)*((2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+1)^(1/2)*2^(1/2)*(70032181387618409398
75*(-6050+4280*2^(1/2))^(1/2)*arctan(1/49*(-6050+4280*2^(1/2))^(1/2)/((2^(1
/2)-1+x)^2/(2^(1/2)+1-x)^2+1)^(1/2)*(40*2^(1/2)+57)*(2^(1/2)-1+x)/(2^(1/2)+
1-x))*(-350+280*2^(1/2))^(1/2)*2^(1/2)+9903469297471243727348*(-6050+4280*2
^(1/2))^(1/2)*arctan(1/49*(-6050+4280*2^(1/2))^(1/2)/((2^(1/2)-1+x)^2/(2^(1
/2)+1-x)^2+1)^(1/2)*(40*2^(1/2)+57)*(2^(1/2)-1+x)/(2^(1/2)+1-x))*(-350+280*
2^(1/2))^(1/2)+321845054725303914701190*arctanh(7*((2^(1/2)-1+x)^2/(2^(1/2)
+1-x)^2+1)^(1/2)/(-350+280*2^(1/2))^(1/2))*2^(1/2)-455587903591695621758200
*arctanh(7*((2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+1)^(1/2)/(-350+280*2^(1/2))^(1/
2)))/(((2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+1)/((2^(1/2)-1+x)/(2^(1/2)+1-x)+1)^2
```

```
)^(1/2)/((2^(1/2)-1+x)/(2^(1/2)+1-x)+1)/(-350+280*2^(1/2))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1569, normalized size of antiderivative = 2.46

$$\int \frac{1}{(3-2x+x^2)^{21/2}\,(1+x+2x^2)^{10}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="fricas")
```

```
[Out] 1/93940194787639200000000000000000*(2360574301353395566476989952000*x^38 -
3658890167097763128039334425600*x^37 + 31513666923067830812467815859200*x^3
6 - 188019743102797956869892249676800*x^35 + 861226026670019454984548821612
800*x^34 - 3183721313824708059229806829449600*x^33 + 9831378864634155690152
529676838400*x^32 - 25933999577342884900069590431438400*x^31 + 595378010536
69957636238873995743000*x^30 - 120659917686431634864896285011569300*x^29 +
218815205755728685314344512920641100*x^28 - 358981964316724903316209908944
483400*x^27 + 538611677703407694971607759062726400*x^26 - 744747058599416616
000621120456199500*x^25 + 956690308445988798962145796617987300*x^24 - 11462
15696378789191186353021849349200*x^23 + 12893735409422786378759267697290562
00*x^22 - 1362598128377218516278181645663204500*x^21 + 13632710921486602471
73548652450625900*x^20 - 1285053072164246491655277964217182200*x^19 + 11560
90273753138114015372080442309200*x^18 - 97666203123362882057380739721863550
0*x^17 + 798237355988012151640630610578068900*x^16 - 6023785757897600295628
40059112791200*x^15 + 453947813134211818985370625408991400*x^14 - 299561768
273477509253114104689745500*x^13 + 216090200276716466450059917698391300*x^1
2 - 116372548125131610054621102465641400*x^11 + 886982879899635151006076604
42952800*x^10 - 31524301955764963385813894907485700*x^9 + 33341076472331463
305896468245703500*x^8 - 3040034262620530630502524237160400*x^7 + 115994388
73255147841572220445070200*x^6 + 1565914164733200701041734359348100*x^5 + 3
098553913860351419382646288652100*x^4 + 611039268402008773233284624446200*x
^3 + 415701*sqrt(35)*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 186
7968*x^34 - 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 -
 261706983*x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 16153
29345*x^25 + 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 29554258
95*x^21 + 2956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*
x^17 + 1731347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13
+ 468691803*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 7231558
5*x^8 - 6593724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^
3 + 1023516*x^2 + 137781*x + 59049)*sqrt(1165762470424230935049788475756180
29777*I*sqrt(7) - 81042225921274689605478944797800854846405)*log(sqrt(35)*s
qrt(116576247042423093504978847575618029777*I*sqrt(7) - 8104222592127468960
```

5478944797800854846405)*(656826642296538601431*I*sqrt(7) - 4349517157964549
25565) - 16045658306440477995287381886026620086 51776*x - 401141457661011949
88218454715066550216 2944*I*sqrt(7) + 1604565830644047799528738188602662008 6
51776*sqrt(x^2 - 2*x + 3) - 401141457661011949882184547150665502162944) - 4
15701*sqrt(35)*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x
^34 - 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 26170
6983*x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329345*
x^25 + 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*x^2
1 + 2956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^17 +
 1731347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 4686
91803*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8
 - 6593724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 10
23516*x^2 + 137781*x + 59049)*sqrt(11657624704242309350497884757561802977 7*
I*sqrt(7) - 8104222592127468960547894479780085 4846405)*log(sqrt(35)*sqrt(11
65762470424230935049788475756180297 77*I*sqrt(7) - 810422259212746896054789 4
4797800854846405)*(-656826642296538601431*I*sqrt(7) + 434951715796454925565
) - 16045658306440477995287381886026620086 51776*x - 401141457661011949882 18
454715066550216 2944*I*sqrt(7) + 16045658306440477995287381886026620086 51776
*sqrt(x^2 - 2*x + 3) - 401141457661011949882184547150665502162944) - 415701
sqrt(35)(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 -
 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*
x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25
 + 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 2
956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731
347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803
*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 659
3724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516
*x^2 + 137781*x + 59049)*sqrt(-11657624704242309350497884757561802977 7*I*sq
rt(7) - 8104222592127468960547894479780085 4846405)*log(sqrt(35)*(6568266422
96538601431*I*sqrt(7) + 434951715796454925565)*sqrt(-116576247042423093504 9
78847575618029777*I*sqrt(7) - 8104222592127468960547894479780085 4846405) -
16045658306440477995287381886026620086 51776*x + 401141457661011949882184547
150665502162944*I*sqrt(7) + 16045658306440477995287381886026620086 51776*sqr
t(x^2 - 2*x + 3) - 401141457661011949882184547150665502162944) + 415701*sqr
t(35)*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 - 690
5376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*x^29
 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25 + 20
75026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 29568
85529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 17313478
59*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803*x^1
2 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 6593724
*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516*x^2
 + 137781*x + 59049)*sqrt(-11657624704242309350497884757561802977 7*I*sqrt(7
) - 8104222592127468960547894479780085 4846405)*log(sqrt(35)*(-6568266422965
38601431*I*sqrt(7) - 434951715796454925565)*sqrt(-116576247042423093504978 8

```
47575618029777*I*sqrt(7) - 810422259212746896054789447978008548464O5) - 160
4565830644047799528738188602662008651776*x + 401141457661011949882184547150
665502162944*I*sqrt(7) + 1604565830644047799528738188602662008651776*sqrt(x
^2 - 2*x + 3) - 401141457661011949882184547150665502162944) + 4718917122312
5430012075446244360O*x^2 + 70*(3372249001933422237824271360*x^37 - 53502205
39964003139479614771 2*x^36 + 469149394082989701729494575872*x^35 - 28474992
2091266775338303529907 2*x^34 + 13254252261100740556512388253568*x^33 - 4977
0080058525077628064229832576*x^32 + 156010734937008739388220889457760*x^31
- 417516398850754397130111919794336*x^30 + 971538171913365251873706873353 65
2*x^29 - 19936532135755218378886012043802 28*x^28 + 365555347185295760625734
541414003 1*x^27 - 6054769996581738503753686155104785*x^26 + 915549415851386
92302715297463072 21*x^25 - 12740106677685048178693605103009787*x^24 + 16442
770202470076313197215936814318*x^23 - 19772569734288744720189854470201506*x
^22 + 22286437617621909921609206629636086*x^21 - 235849866475607424431880 31
208946882*x^20 + 23579397211179175240196614296051673*x^19 - 222187475539417
9488590384054246160 7*x^18 + 19912295454080246583636391613811979*x^17 - 1680
17608060533902429951453491486 13*x^16 + 13613407965006475288139078599341572*
x^15 - 10279305650733178669223634020962076*x^14 + 760628837830344952432793 8
977040824*x^13 - 5069838234992751929471190426115248*x^12 + 350742597059619 7
680016078213030977*x^11 - 1974814483061344405275851094534735*x^10 + 1357002
388430055881833293557852283*x^9 - 566969010759169461615951049236597*x^8 + 4
584260000738468824324570443068 94*x^7 - 947045576652534893325365499370 26*x^6
 + 135183920426913231415208872303230*x^5 - 10230953189017746384031862728 74*
x^4 + 2939804115352497334391760174215 1*x^3 + 193395719557006270878162913482
3*x^2 + 3397462350398947848063583843461*x - 800387108715553168613453696 43)*
sqrt(x^2 - 2*x + 3) + 6352388433882269424702463917510 0*x + 272245218594954 4
0391581988217900)/(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 186796
8*x^34 - 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 26
1706983*x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 16153293
45*x^25 + 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*
x^21 + 2956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^1
7 + 1731347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 4
68691803*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x
^8 - 6593724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 +
 1023516*x^2 + 137781*x + 59049)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(x**2-2*x+3)**(21/2)/(2*x**2+x+1)**10,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx = \int \frac{1}{(2\,x^2+x+1)^{10}(x^2-2\,x+3)^{\frac{21}{2}}}\,dx$$

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="maxima")
[Out] integrate(1/((2*x^2 + x + 1)^10*(x^2 - 2*x + 3)^(21/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx = \int \frac{1}{(2\,x^2+x+1)^{10}\,(x^2-2\,x+3)^{21/2}}\,dx$$

```
[In] int(1/((x + 2*x^2 + 1)^10*(x^2 - 2*x + 3)^(21/2)),x)
[Out] int(1/((x + 2*x^2 + 1)^10*(x^2 - 2*x + 3)^(21/2)), x)
```

3.52 $\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)(1+x^2)}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 66

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)(1+x^2)}}\,dx = -\sqrt{2}\sqrt{a+\sqrt{1+a^2}}\arctan\left(\frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}(-a+x)}{\sqrt{(-a+x)(1+x^2)}}\right)$$

```
[Out] -arctan((-a+x)*2^(1/2)*(-a+(a^2+1)^(1/2))^(1/2)/((-a+x)*(x^2+1))^(1/2))*2^(
1/2)*(a+(a^2+1)^(1/2))^(1/2)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.85 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6851, 6874, 733, 430, 946, 174, 552, 551}

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)(1+x^2)}}\,dx = \frac{4\sqrt{a^2+1}\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}\,\text{EllipticPi}\left(\frac{2}{1-i\left(a-\sqrt{a^2+1}\right)},\arcsin\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right),\frac{2}{1-ia}\right)}{\left(1-i\left(a-\sqrt{a^2+1}\right)\right)\sqrt{-\left((x^2+1)(a-x)\right)}} + \frac{2i\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right),\frac{2}{1-ia}\right)}{\sqrt{-\left((x^2+1)(a-x)\right)}}$$

```
[In] Int[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)*Sqrt[(-a + x)*(1 + x
^2)]),x]
```

```
[Out] ((2*I)*Sqrt[(a - x)/(I + a)]*Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[1 - I*x]/S
qrt[2]], 2/(1 - I*a)])/Sqrt[-((a - x)*(1 + x^2))] + (4*Sqrt[1 + a^2]*Sqrt[(
a - x)/(I + a)]*Sqrt[1 + x^2]*EllipticPi[2/(1 - I*(a - Sqrt[1 + a^2])), Arc
Sin[Sqrt[1 - I*x]/Sqrt[2]], 2/(1 - I*a)])/((1 - I*(a - Sqrt[1 + a^2]))*Sqrt
[-((a - x)*(1 + x^2))])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{-a+x}\sqrt{1+x^2}\right)\int \frac{-a-\sqrt{1+a^2}+x}{\sqrt{-a+x}\left(-a+\sqrt{1+a^2}+x\right)\sqrt{1+x^2}}\,dx}{\sqrt{(-a+x)\left(1+x^2\right)}}$$

$$= \frac{\left(\sqrt{-a+x}\sqrt{1+x^2}\right)\int \left(\frac{1}{\sqrt{-a+x}\sqrt{1+x^2}} - \frac{2\sqrt{1+a^2}}{\sqrt{-a+x}\left(-a+\sqrt{1+a^2}+x\right)\sqrt{1+x^2}}\right)dx}{\sqrt{(-a+x)\left(1+x^2\right)}}$$

$$= \frac{\left(\sqrt{-a+x}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-a+x}\sqrt{1+x^2}}\,dx}{\sqrt{(-a+x)\left(1+x^2\right)}} - \frac{\left(2\sqrt{1+a^2}\sqrt{-a+x}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-a+x}\left(-a+\sqrt{1+a^2}+x\right)\sqrt{1+x^2}}\,dx}{\sqrt{(-a+x)\left(1+x^2\right)}}$$

$$= -\frac{\left(2\sqrt{1+a^2}\sqrt{-a+x}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1-ix}\sqrt{1+ix}\sqrt{-a+x}\left(-a+\sqrt{1+a^2}+x\right)}\,dx}{\sqrt{(-a+x)\left(1+x^2\right)}} + \frac{\left(2i\sqrt{\frac{-a+x}{-i-a}}\sqrt{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2ix^2}{-i-a}}}\,dx, x, \frac{\sqrt{1-ix}}{\sqrt{2}}\right)}{\sqrt{(-a+x)\left(1+x^2\right)}}$$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right),\frac{2}{1-ia}\right)}{\sqrt{-((a-x)(1+x^2))}} + \frac{\left(4\sqrt{1+a^2}\sqrt{-a+x}\sqrt{1+x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(1-i\left(a-\sqrt{1+a^2}\right)-x^2\right)\sqrt{-i-a+ix^2}}\,dx,x,\sqrt{1-ix}\right)}{\sqrt{(-a+x)(1+x^2)}}$$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right),\frac{2}{1-ia}\right)}{\sqrt{-((a-x)(1+x^2))}} + \frac{\left(4\sqrt{1+a^2}\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(1-i\left(a-\sqrt{1+a^2}\right)-x^2\right)\sqrt{1+\frac{ix^2}{-i-a}}}\,dx,x,\sqrt{1-ix}\right)}{\sqrt{(-a+x)(1+x^2)}}$$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right),\frac{2}{1-ia}\right)}{\sqrt{-((a-x)(1+x^2))}} + \frac{4\sqrt{1+a^2}\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}\,\text{EllipticPi}\left(\frac{2}{1-i\left(a-\sqrt{1+a^2}\right)},\arcsin\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right),\frac{2}{1-ia}\right)}{\left(1-i\left(a-\sqrt{1+a^2}\right)\right)\sqrt{-((a-x)(1+x^2))}}$$

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.50

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)(1+x^2)}}\,dx = -\frac{\sqrt{2}\sqrt{-a+x}\sqrt{1+x^2}\arctan\left(\frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\sqrt{-a+x}}{\sqrt{1+x^2}}\right)}{\sqrt{-a+\sqrt{1+a^2}}\sqrt{(-a+x)(1+x^2)}}$$

```
[In] Integrate[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)*Sqrt[(-a + x)*
(1 + x^2)]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[-a + x]*Sqrt[1 + x^2]*ArcTan[(Sqrt[2]*Sqrt[-a + Sqrt[1 + a^
2]]*Sqrt[-a + x])/Sqrt[1 + x^2]])/(Sqrt[-a + Sqrt[1 + a^2]]*Sqrt[(-a + x)*(
1 + x^2)]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.60 (sec) , antiderivative size = 787, normalized size of antiderivative = 11.92

method	result
default	$\frac{2i\sqrt{-i(x+i)}\,\sqrt{\frac{-a+x}{-i-a}}\,\sqrt{i(x-i)}\,F\left(\frac{\sqrt{2}\,\sqrt{-i(x+i)}}{2},\sqrt{2}\,\sqrt{-\frac{i}{-i-a}}\right)}{\sqrt{-a\,x^2+x^3-a+x}} - \frac{2\sqrt{a^2+1}\,(2ax-x^2+1)\sqrt{-(a-x)(x^2+1)(a^2+1)}\left(-\frac{i\sqrt{a^2+1}\sqrt{-i}}{}\right.}{}$
elliptic	Expression too large to display

```
[In] int((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),x,meth
od=_RETURNVERBOSE)
```

```
[Out] 2*I*(-I*(x+I))^(1/2)*((-a+x)/(-I-a))^(1/2)*(I*(x-I))^(1/2)/(-a*x^2+x^3-a+x)
^(1/2)*EllipticF(1/2*2^(1/2)*(-I*(x+I))^(1/2),2^(1/2)*(-I/(-I-a))^(1/2))-2*
(a^2+1)^(1/2)*(2*a*x-x^2+1)*(-(a-x)*(x^2+1)*(a^2+1))^(1/2)/(-a+x+(a^2+1)^(1
/2))/((-(a-x)*(x^2+1))^(1/2)*a^2+(-(a-x)*(x^2+1)*(a^2+1))^(1/2)*a-(-(a-x)*(
x^2+1)*(a^2+1))^(1/2)*x+(-(a-x)*(x^2+1))^(1/2))*(-I*(a^2+1)^(1/2)*(1-I*x)^(
1/2)*(-1/(-I-a)*a+1/(-I-a)*x)^(1/2)*(1+I*x)^(1/2)/(-a^3*x^2+a^2*x^3-a^3+a^2
*x-a*x^2+x^3-a+x)^(1/2)/(-I-a-(a^2+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+
I))^(1/2),-2*I/(-I-a-(a^2+1)^(1/2)),2^(1/2)*(-I/(-I-a))^(1/2))+I*(a^2+1)^(1
/2)*(1-I*x)^(1/2)*(-1/(-I-a)*a+1/(-I-a)*x)^(1/2)*(1+I*x)^(1/2)/(-a^3*x^2+a^
2*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^(1/2)/(-I-a+(a^2+1)^(1/2))*EllipticPi(1/2*2^
(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a+(a^2+1)^(1/2)),2^(1/2)*(-I/(-I-a))^(1/2))
+I*(1-I*x)^(1/2)*(-1/(-I-a)*a+1/(-I-a)*x)^(1/2)*(1+I*x)^(1/2)/(-a*x^2+x^3-a
+x)^(1/2)/(-I-a-(a^2+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I))^(1/2),-2*I
/(-I-a-(a^2+1)^(1/2)),2^(1/2)*(-I/(-I-a))^(1/2))+I*(1-I*x)^(1/2)*(-1/(-I-a)
*a+1/(-I-a)*x)^(1/2)*(1+I*x)^(1/2)/(-a*x^2+x^3-a+x)^(1/2)/(-I-a+(a^2+1)^(1/
2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a+(a^2+1)^(1/2)),2^(1/
2)*(-I/(-I-a))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 546, normalized size of antiderivative = 8.27

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\,(1+x^2)}}\,dx$$
$$= \left[\frac{1}{4}\sqrt{-2\,a-2\,\sqrt{a^2+1}}\log\left(-\frac{8\,ax^7+x^8+4\,(2\,a^2+15)x^6-8\,(4\,a^3+15\,a)x^5+2\,(8\,a^4+80\,a^2+67)x^4 -}{}\right.\right.$$
$$-\frac{1}{2}\sqrt{2\,a+2\,\sqrt{a^2+1}}\arctan\left(-\frac{\sqrt{-ax^2+x^3-a+x}\,(2\,a^2-2\,ax-x^2-2\,\sqrt{a^2+1}(a-x)-1)\sqrt{2\,a+}}{4\,(ax^2-x^3+a-x)}\right.$$

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(-2*a - 2*sqrt(a^2 + 1))*log(-(8*a*x^7 + x^8 + 4*(2*a^2 + 15)*x^6
- 8*(4*a^3 + 15*a)*x^5 + 2*(8*a^4 + 80*a^2 + 67)*x^4 + 64*a^4 - 8*(20*a^3 +
 37*a)*x^3 + 4*(16*a^4 + 74*a^2 + 15)*x^2 + 48*a^2 - 4*(a*x^6 + 2*(2*a^2 +
3)*x^5 - (4*a^3 - a)*x^4 - 8*a^3 - (4*a^3 + 29*a)*x^2 + 20*x^3 + 2*(10*a^2
+ 3)*x - (4*a*x^5 + x^6 - (4*a^2 - 15)*x^4 - 16*a*x^3 + (4*a^2 + 15)*x^2 +
8*a^2 - 20*a*x + 1)*sqrt(a^2 + 1) - 5*a)*sqrt(-a*x^2 + x^3 - a + x)*sqrt(-2
*a - 2*sqrt(a^2 + 1)) - 8*(24*a^3 + 13*a)*x + 16*(a*x^6 - x^7 + 15*a*x^4 -
7*x^5 - (12*a^2 + 7)*x^3 + 4*a^3 + (4*a^3 + 15*a)*x^2 - (12*a^2 + 1)*x + a)
*sqrt(a^2 + 1) + 1)/(8*a*x^7 - x^8 - 4*(6*a^2 - 1)*x^6 + 8*(4*a^3 - 3*a)*x^
5 - 2*(8*a^4 - 24*a^2 + 3)*x^4 - 8*(4*a^3 - 3*a)*x^3 - 4*(6*a^2 - 1)*x^2 -
8*a*x - 1)), -1/2*sqrt(2*a + 2*sqrt(a^2 + 1))*arctan(-1/4*sqrt(-a*x^2 + x^3
 - a + x)*(2*a^2 - 2*a*x - x^2 - 2*sqrt(a^2 + 1)*(a - x) - 1)*sqrt(2*a + 2*
sqrt(a^2 + 1))/(a*x^2 - x^3 + a - x))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx = \text{Timed out}$$

```
[In] integrate((-a+x-(a**2+1)**(1/2))/(-a+x+(a**2+1)**(1/2))/((-a+x)*(x**2+1))**
(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx$$
$$= \int \frac{a-x+\sqrt{a^2+1}}{\sqrt{-(x^2+1)(a-x)}\left(a-x-\sqrt{a^2+1}\right)}\,dx$$

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="maxima")
```

```
[Out] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a
^2 + 1))), x)
```

Giac [F]

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)(1+x^2)}}\,dx$$
$$=\int \frac{a-x+\sqrt{a^2+1}}{\sqrt{-(x^2+1)(a-x)}\left(a-x-\sqrt{a^2+1}\right)}\,dx$$

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="giac")
```

```
[Out] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a
^2 + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)(1+x^2)}}\,dx$$
$$=\int -\frac{a-x+\sqrt{a^2+1}}{\sqrt{-\left(x^2+1\right)\,(a-x)}\,\left(x-a+\sqrt{a^2+1}\right)}\,dx$$

```
[In] int(-(a - x + (a^2 + 1)^(1/2))/((-(x^2 + 1)*(a - x))^(1/2)*(x - a + (a^2 +
1)^(1/2))),x)
```

```
[Out] int(-(a - x + (a^2 + 1)^(1/2))/((-(x^2 + 1)*(a - x))^(1/2)*(x - a + (a^2 +
1)^(1/2))), x)
```

3.53 $\int \frac{a+bx}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 198

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{a\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\sqrt{3}b\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}}+\frac{a\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{a\text{arctanh}(x)}{6\ 2^{2/3}}+\frac{a\text{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}-\frac{b\log\left(3+x^2\right)}{4\ 2^{2/3}}+\frac{3b\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}$$

```
[Out] -1/12*a*arctanh(x)*2^(1/3)+1/4*a*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1
/3)-1/8*b*ln(x^2+3)*2^(1/3)+3/8*b*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)+1/12*a
*arctan(3^(1/2)/x)*2^(1/3)*3^(1/2)+1/12*a*arctan((1-2^(1/3)*(-x^2+1)^(1/3))
*3^(1/2)/x)*2^(1/3)*3^(1/2)+1/4*b*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*
3^(1/2)*2^(1/3)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1024, 402, 455, 57, 631, 210, 31}

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{a\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{a\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{a\text{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}} - \frac{a\text{arctanh}(x)}{6\ 2^{2/3}} + \frac{\sqrt{3}b\arctan\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}} - \frac{b\log\left(x^2+3\right)}{4\ 2^{2/3}} + \frac{3b\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}$$

```
[In] Int[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (a*ArcTan[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) + (Sqrt[3]*b*ArcTan[(1 + (2 - 2*x
^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) + (a*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2
)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTanh[x])/(6*2^(2/3)) + (a*ArcTanh
[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(2*2^(2/3)) - (b*Log[3 + x^2])/(4*2^(2/3
)) + (3*b*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\text{integral} = a\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx + b\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= \frac{a\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{a\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\text{arctanh}(x)}{6\ 2^{2/3}} + \frac{a\text{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}} + \frac{1}{2}b\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(3+x)}\,dx, x, x^2\right)$$

$$= \frac{a \arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{a \arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\text{arctanh}(x)}{6\ 2^{2/3}} + \frac{a\text{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}} - \frac{b\log(3+x^2)}{4\ 2^{2/3}} + \frac{1}{4}(3b)\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right) - \frac{(3b)\text{Subst}\left(\int \frac{1}{2^{2/3}-x}\,dx, x, \sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}$$

$$= \frac{a \arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{a \arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\text{arctanh}(x)}{6\ 2^{2/3}} + \frac{a\text{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}} - \frac{b\log(3+x^2)}{4\ 2^{2/3}} + \frac{3b\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}} - \frac{(3b)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\sqrt[3]{2-2x^2}\right)}{2\ 2^{2/3}}$$

$$= \frac{a \arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\sqrt{3}b \arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}} + \frac{a \arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\text{arctanh}(x)}{6\ 2^{2/3}} + \frac{a\text{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}} - \frac{b\log(3+x^2)}{4\ 2^{2/3}} + \frac{3b\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.73

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{1}{6}bx^2\,\text{AppellF1}\left(1,\frac{1}{3},1,2,x^2,-\frac{x^2}{3}\right) - \frac{9ax\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)\left(-9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)+2x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right)-\text{AppellF1}\left(\frac{3}{2}\right.\right.\right.}$$

```
[In] Integrate[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/3, 1, 2, x^2, -1/3*x^2])/6 - (9*a*x*AppellF1[1/2, 1/3,
 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)*(-9*AppellF1[1/2, 1/3,
1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2] -
 AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

Maple [F]

$$\int \frac{bx+a}{(-x^2+1)^{\frac{1}{3}}(x^2+3)}dx$$

```
[In] int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
[Out] int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

Sympy [F]

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{a+bx}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

```
[In] integrate((b*x+a)/(-x**2+1)**(1/3)/(x**2+3),x)
[Out] Integral((a + b*x)/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{bx+a}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
[Out] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{bx+a}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
[Out] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{a+b\,x}{(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

```
[In] int((a + b*x)/((1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] int((a + b*x)/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.54 $\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 198

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\frac{a\arctan(x)}{6\ 2^{2/3}}+\frac{a\arctan\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{\sqrt{3}b\arctan\left(\frac{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}} - \frac{a\text{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{b\log(3-x^2)}{4\ 2^{2/3}} - \frac{3b\log\left(2^{2/3}-\sqrt[3]{1+x^2}\right)}{4\ 2^{2/3}}$$

```
[Out] -1/12*a*arctan(x)*2^(1/3)+1/4*a*arctan(x/(1+2^(1/3)*(x^2+1)^(1/3)))*2^(1/3)
+1/8*b*ln(-x^2+3)*2^(1/3)-3/8*b*ln(2^(2/3)-(x^2+1)^(1/3))*2^(1/3)-1/12*a*ar
ctanh(3^(1/2)/x)*2^(1/3)*3^(1/2)-1/12*a*arctanh((1-2^(1/3)*(x^2+1)^(1/3))*3
^(1/2)/x)*2^(1/3)*3^(1/2)-1/4*b*arctan(1/3*(1+2^(1/3)*(x^2+1)^(1/3))*3^(1/2
))*3^(1/2)*2^(1/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1024, 401, 455, 57, 631, 210, 31}

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \frac{a\arctan\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}} - \frac{a\arctan(x)}{6\ 2^{2/3}} - \frac{a\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\text{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\sqrt{3}b\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}} + \frac{b\log(3-x^2)}{4\ 2^{2/3}} - \frac{3b\log\left(2^{2/3}-\sqrt[3]{x^2+1}\right)}{4\ 2^{2/3}}$$

```
[In] Int[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -1/6*(a*ArcTan[x])/2^(2/3) + (a*ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))])/(2
*2^(2/3)) - (Sqrt[3]*b*ArcTan[(1 + 2^(1/3)*(1 + x^2)^(1/3))/Sqrt[3]])/(2*2^
(2/3)) - (a*ArcTanh[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTanh[(Sqrt[3]*(
1 - 2^(1/3)*(1 + x^2)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) + (b*Log[3 - x^2])/(4
*2^(2/3)) - (3*b*Log[2^(2/3) - (1 + x^2)^(1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\text{integral} = a\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx + b\int \frac{x}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$$

$$= -\frac{a\arctan(x)}{6\ 2^{2/3}} + \frac{a\arctan\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{a\text{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{1}{2}b\text{Subst}\left(\int \frac{1}{(3-x)\sqrt[3]{1+x}}\,dx, x, x^2\right)$$

$$= -\frac{a\arctan(x)}{6\ 2^{2/3}} + \frac{a\arctan\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{a\text{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{b\log(3-x^2)}{4\ 2^{2/3}} - \frac{1}{4}(3b)\text{Subst}\left(\int\frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \sqrt[3]{1+x^2}\right) + \frac{(3b)\text{Subst}\left(\int\frac{1}{2^{2/3}-x}\,dx, x, \sqrt[3]{1+x^2}\right)}{4\ 2^{2/3}}$$

$$= -\frac{a\arctan(x)}{6\ 2^{2/3}} + \frac{a\arctan\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{a\text{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{b\log(3-x^2)}{4\ 2^{2/3}} - \frac{3b\log\left(2^{2/3}-\sqrt[3]{1+x^2}\right)}{4\ 2^{2/3}} + \frac{(3b)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1+\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{2\ 2^{2/3}}$$

$$= -\frac{a\arctan(x)}{6\ 2^{2/3}} + \frac{a\arctan\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{\sqrt{3}b\arctan\left(\frac{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}} - \frac{a\text{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{b\log(3-x^2)}{4\ 2^{2/3}} - \frac{3b\log\left(2^{2/3}-\sqrt[3]{1+x^2}\right)}{4\ 2^{2/3}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.77

$$\int\frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \frac{1}{6}bx^2\,\text{AppellF1}\left(1,\frac{1}{3},1,2,-x^2,\frac{x^2}{3}\right) - \frac{9ax\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right)}{(-3+x^2)\sqrt[3]{1+x^2}\left(9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right)+2x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-x^2,\frac{x^2}{3}\right)-\text{AppellF1}\,(\right.\right.}$$

```
[In] Integrate[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/3, 1, 2, -x^2, x^2/3])/6 - (9*a*x*AppellF1[1/2, 1/3, 1
, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*(9*AppellF1[1/2, 1/3, 1, 3
/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -x^2, x^2/3] - AppellF
1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

Maple [F]

$$\int \frac{bx + a}{(-x^2 + 3)(x^2 + 1)^{\frac{1}{3}}} dx$$

```
[In] int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
```

```
[Out] int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{a + bx}{(3 - x^2)\sqrt[3]{1 + x^2}} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

Sympy [F]

$$\int \frac{a + bx}{(3 - x^2)\sqrt[3]{1 + x^2}} dx = -\int \frac{a}{x^2\sqrt[3]{x^2 + 1} - 3\sqrt[3]{x^2 + 1}} dx - \int \frac{bx}{x^2\sqrt[3]{x^2 + 1} - 3\sqrt[3]{x^2 + 1}} dx$$

```
[In] integrate((b*x+a)/(-x**2+3)/(x**2+1)**(1/3),x)
```

```
[Out] -Integral(a/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x) - Integral(b
*x/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

Maxima [F]

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \int -\frac{bx+a}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
[Out] -integrate((b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

Giac [F]

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \int -\frac{bx+a}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
[Out] integrate(-(b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \int -\frac{a+b\,x}{(x^2+1)^{1/3}\,(x^2-3)}\,dx$$

```
[In] int(-(a + b*x)/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
[Out] int(-(a + b*x)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.55 $\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 97

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx = -\frac{\arctan\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{4-6x+3x^2}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\ 2^{2/3}}+\frac{\log\left(6-3x-3\sqrt[3]{2}\sqrt[3]{4-6x+3x^2}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/4*ln(x)*2^(1/3)+1/4*ln(6-3*x-3*2^(1/3)*(3*x^2-6*x+4)^(1/3))*2^(1/3)+1/6*
arctan(-1/3*3^(1/2)-1/3*2^(2/3)*(2-x)/(3*x^2-6*x+4)^(1/3)*3^(1/2))*2^(1/3)*
3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {764}

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx = -\frac{\arctan\left(\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{3x^2-6x+4}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}-3x+6\right)}{2\ 2^{2/3}}-\frac{\log(x)}{2\ 2^{2/3}}$$

```
[In] Int[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2^(2/3)*(2 - x))/(Sqrt[3]*(4 - 6*x + 3*x^2)^(1/3))]/(
2^(2/3)*Sqrt[3])) - Log[x]/(2*2^(2/3)) + Log[6 - 3*x - 3*2^(1/3)*(4 - 6*x +
 3*x^2)^(1/3)]/(2*2^(2/3))
```

Rule 764

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[3*c*e^2*(2*c*d - b*e), 3]}, Simp[(-Sqrt[3])*c*e*(ArcT
an[1/Sqrt[3] + 2*((c*d - b*e - c*e*x)/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3)))]
/q^2), x] + (-Simp[3*c*e*(Log[d + e*x]/(2*q^2)), x] + Simp[3*c*e*(Log[c*d -
 b*e - c*e*x - q*(a + b*x + c*x^2)^(1/3)]/(2*q^2)), x])] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*
c*e^2, 0] && PosQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\text{integral} = -\frac{\arctan\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{4-6x+3x^2}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\ 2^{2/3}}+\frac{\log\left(6-3x-3\sqrt[3]{2}\sqrt[3]{4-6x+3x^2}\right)}{2\ 2^{2/3}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.73

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{2\ 2^{2/3}-2^{2/3}x+\sqrt[3]{4-6x+3x^2}}{\sqrt{3}\sqrt[3]{4-6x+3x^2}}\right)-2\log\left(-22^{2/3}+2^{2/3}x+2\sqrt[3]{4-6x+3x^2}\right)+\log\left(-4\sqrt[3]{2}\right.}{6\ 2^{2/3}}$$

```
[In] Integrate[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(2*2^(2/3) - 2^(2/3)*x + (4 - 6*x + 3*x^2)^(1/3))/(S
qrt[3]*(4 - 6*x + 3*x^2)^(1/3))] - 2*Log[-2*2^(2/3) + 2^(2/3)*x + 2*(4 - 6*
x + 3*x^2)^(1/3)] + Log[-4*2^(1/3) + 4*2^(1/3)*x - 2^(1/3)*x^2 + 2^(2/3)*(-
2 + x)*(4 - 6*x + 3*x^2)^(1/3) - 2*(4 - 6*x + 3*x^2)^(2/3)])/2^(2/3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 9.44 (sec) , antiderivative size = 2395, normalized size of antiderivative = 24.69

method	result	size
trager	Expression too large to display	2395

```
[In] int(1/x/(3*x^2-6*x+4)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*ln(-(600*RootOf(Roo
tOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+30*RootOf(RootOf(_Z^3-2)^2+2*_Z
*RootOf(_Z^3-2)+4*_Z^2)*x^3-300*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)
+4*_Z^2)*x^2-400*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-160*Ro
otOf(_Z^3-2)+12*RootOf(_Z^3-2)*x^3-120*RootOf(_Z^3-2)*x^2+240*RootOf(_Z^3-2
)*x+20*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)
^2*x^3+8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)
^3*x^3-60*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3
-2)^2*x^2-24*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^
3-2)^3*x^2+120*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf
(_Z^3-2)^2*x+48*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(
_Z^3-2)^3*x-80*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf
(_Z^3-2)^2-32*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z
^3-2)^3+30*(3*x^2-6*x+4)^(2/3)*x-60*(3*x^2-6*x+4)^(1/3)*RootOf(_Z^3-2)^2+48
*(3*x^2-6*x+4)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*Ro
otOf(_Z^3-2)^2*x-48*(3*x^2-6*x+4)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf
(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2+192*(3*x^2-6*x+4)^(1/3)*RootOf(RootOf(_
Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-60*(3*x^2-6*x+4)^(2/3
)-96*(3*x^2-6*x+4)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2
)*RootOf(_Z^3-2)^2-15*(3*x^2-6*x+4)^(1/3)*RootOf(_Z^3-2)^2*x^2+60*(3*x^2-6*
x+4)^(1/3)*RootOf(_Z^3-2)^2*x-192*(3*x^2-6*x+4)^(1/3)*RootOf(RootOf(_Z^3-2)
^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2))/x^3)-1/6*ln((480*RootOf(Root
Of(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+10*RootOf(RootOf(_Z^3-2)^2+2*_Z*
RootOf(_Z^3-2)+4*_Z^2)*x^3-240*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+
4*_Z^2)*x^2-320*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-32*Root
Of(_Z^3-2)+RootOf(_Z^3-2)*x^3-24*RootOf(_Z^3-2)*x^2+48*RootOf(_Z^3-2)*x-20*
RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-
2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3+
60*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x
^2+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x
^2-120*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)
^2*x-12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^
3*x+80*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)
^2+8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3+1
8*(3*x^2-6*x+4)^(2/3)*x-36*(3*x^2-6*x+4)^(1/3)*RootOf(_Z^3-2)^2+48*(3*x^2-6
*x+4)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3
-2)^2*x-48*(3*x^2-6*x+4)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+
4*_Z^2)*RootOf(_Z^3-2)*x^2+192*(3*x^2-6*x+4)^(1/3)*RootOf(RootOf(_Z^3-2)^2+
2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-36*(3*x^2-6*x+4)^(2/3)-96*(3*x
^2-6*x+4)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(
_Z^3-2)^2-9*(3*x^2-6*x+4)^(1/3)*RootOf(_Z^3-2)^2*x^2+36*(3*x^2-6*x+4)^(1/3)
*RootOf(_Z^3-2)^2*x-192*(3*x^2-6*x+4)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*Ro
otOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2))/x^3)*RootOf(_Z^3-2)-1/3*ln((480*RootOf
(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+10*RootOf(RootOf(_Z^3-2)^2+
2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^3-240*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^
```

```
3-2)+4*_Z^2)*x^2-320*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-32
*RootOf(_Z^3-2)+RootOf(_Z^3-2)*x^3-24*RootOf(_Z^3-2)*x^2+48*RootOf(_Z^3-2)*
x-20*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2
*x^3-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3
*x^3+60*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2
)^2*x^2+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2
)^3*x^2-120*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z
^3-2)^2*x-12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^
3-2)^3*x+80*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z
^3-2)^2+8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2
)^3+18*(3*x^2-6*x+4)^(2/3)*x-36*(3*x^2-6*x+4)^(1/3)*RootOf(_Z^3-2)^2+48*(3*
x^2-6*x+4)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf
(_Z^3-2)^2*x-48*(3*x^2-6*x+4)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^
3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2+192*(3*x^2-6*x+4)^(1/3)*RootOf(RootOf(_Z^3-
2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-36*(3*x^2-6*x+4)^(2/3)-96
*(3*x^2-6*x+4)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*Ro
otOf(_Z^3-2)^2-9*(3*x^2-6*x+4)^(1/3)*RootOf(_Z^3-2)^2*x^2+36*(3*x^2-6*x+4)^
(1/3)*RootOf(_Z^3-2)^2*x-192*(3*x^2-6*x+4)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*
_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2))/x^3)*RootOf(RootOf(_Z^3-2)^2+2*_Z
*RootOf(_Z^3-2)+4*_Z^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(74) = 148$.

Time = 1.24 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.76

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx = -\frac{1}{6} \cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}x^3+2\cdot 4^{\frac{2}{3}}(3\,x^2-6\,x+4)^{\frac{2}{3}}(x-2)+4\,(3\,x^2-6\,x+4)^{\frac{1}{3}}(x^2-4\,x+4)\right)}{6\,(x^3-12\,x^2+24\,x-16)}\right) + \frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{1}{3}}(x-2)+2\,(3\,x^2-6\,x+4)^{\frac{1}{3}}}{x}\right) - \frac{1}{24} \cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}(3\,x^2-6\,x+4)^{\frac{2}{3}}+4^{\frac{1}{3}}(x^2-4\,x+4)-2\,(3\,x^2-6\,x+4)^{\frac{1}{3}}(x-2)}{x^2}\right)$$

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3)*x^3 + 2*4^(2/3)*(3
*x^2 - 6*x + 4)^(2/3)*(x - 2) + 4*(3*x^2 - 6*x + 4)^(1/3)*(x^2 - 4*x + 4))/
(x^3 - 12*x^2 + 24*x - 16)) + 1/12*4^(2/3)*log((4^(1/3)*(x - 2) + 2*(3*x^2
- 6*x + 4)^(1/3))/x) - 1/24*4^(2/3)*log((4^(2/3)*(3*x^2 - 6*x + 4)^(2/3) +
4^(1/3)*(x^2 - 4*x + 4) - 2*(3*x^2 - 6*x + 4)^(1/3)*(x - 2))/x^2)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx = \int \frac{1}{x\sqrt[3]{3x^2-6x+4}}\,dx$$

```
[In] integrate(1/x/(3*x**2-6*x+4)**(1/3),x)
[Out] Integral(1/(x*(3*x**2 - 6*x + 4)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx = \int \frac{1}{(3\,x^2-6\,x+4)^{\frac{1}{3}}x}\,dx$$

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx = \int \frac{1}{(3\,x^2-6\,x+4)^{\frac{1}{3}}x}\,dx$$

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="giac")
[Out] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx = \int \frac{1}{x\,(3\,x^2-6\,x+4)^{1/3}}\,dx$$

```
[In] int(1/(x*(3*x^2 - 6*x + 4)^(1/3)),x)
[Out] int(1/(x*(3*x^2 - 6*x + 4)^(1/3)), x)
```

3.56 $\int x\sqrt[3]{1-x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 73

$$\int x\sqrt[3]{1-x^3}\,dx=\frac{1}{3}x^2\sqrt[3]{1-x^3}-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(-x-\sqrt[3]{1-x^3}\right)$$

```
[Out] 1/3*x^2*(-x^3+1)^(1/3)-1/6*ln(-x-(-x^3+1)^(1/3))-1/9*arctan(1/3*(1-2*x/(-x^
3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {285, 337}

$$\int x\sqrt[3]{1-x^3}\,dx=-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{1}{3}\sqrt[3]{1-x^3}x^2$$

```
[In] Int[x*(1 - x^3)^(1/3),x]
```

```
[Out] (x^2*(1 - x^3)^(1/3))/3 - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(3*Sq
rt[3]) - Log[-x - (1 - x^3)^(1/3)]/6
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}x^2\sqrt[3]{1-x^3}+\frac{1}{3}\int\frac{x}{(1-x^3)^{2/3}}\,dx\\ &= \frac{1}{3}x^2\sqrt[3]{1-x^3}-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(-x-\sqrt[3]{1-x^3}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int x\sqrt[3]{1-x^3}\,dx=\frac{1}{18}\left(6x^2\sqrt[3]{1-x^3}-2\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)-2\log\left(x+\sqrt[3]{1-x^3}\right)+\log\left(x^2-x\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)\right)$$

```
[In] Integrate[x*(1 - x^3)^(1/3),x]
[Out] (6*x^2*(1 - x^3)^(1/3) - 2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3
))] - 2*Log[x + (1 - x^3)^(1/3)] + Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^
(2/3)])/18
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.21

method	result
meijerg	$\frac{x^{2} {}_{2}F_{1}\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^{3}\right)}{2}$
risch	$-\frac{x^{2}\left(x^{3}-1\right)}{3\left(-x^{3}+1\right)^{\frac{2}{3}}}+\frac{\left(x^{3}-1\right)^{\frac{2}{3}}\left(-\operatorname{signum}\left(x^{3}-1\right)\right)^{\frac{2}{3}} x^{2} {}_{2}F_{1}\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};x^{3}\right)}{6 \operatorname{signum}\left(x^{3}-1\right)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{2}{3}}}$
pseudoelliptic	$\frac{6x^{2}\left(-x^{3}+1\right)^{\frac{1}{3}}+2\sqrt{3}\,\arctan\left(\frac{\left(-2\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)\sqrt{3}}{3x}\right)-2\ln\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)+\ln\left(\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}-x\left(-x^{3}+1\right)^{\frac{1}{3}}+x^{2}}{x^{2}}\right)}{18\left(x+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\left(\left(-x^{3}+1\right)^{\frac{2}{3}}-x\left(-x^{3}+1\right)^{\frac{1}{3}}+x^{2}\right)}$
trager	$\frac{x^{2}\left(-x^{3}+1\right)^{\frac{1}{3}}}{3}-\frac{\ln\left(-2\operatorname{RootOf}\left(\_Z^{2}-\_Z+1\right)^{2}x^{3}+3\operatorname{RootOf}\left(\_Z^{2}-\_Z+1\right)\left(-x^{3}+1\right)^{\frac{2}{3}}x-\operatorname{RootOf}\left(\_Z^{2}-\_Z+1\right)\right.}{9}$

```
[In] int(x*(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*hypergeom([-1/3,2/3],[5/3],x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32

$$\int x\sqrt[3]{1-x^3}\,dx = \frac{1}{3}\left(-x^{3}+1\right)^{\frac{1}{3}}x^{2}-\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^{3}+1)^{\frac{1}{3}}}{3\,x}\right) -\frac{1}{9}\log\left(\frac{x+(-x^{3}+1)^{\frac{1}{3}}}{x}\right)+\frac{1}{18}\log\left(\frac{x^{2}-(-x^{3}+1)^{\frac{1}{3}}x+(-x^{3}+1)^{\frac{2}{3}}}{x^{2}}\right)$$

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*(-x^3 + 1)^(1/3)*x^2 - 1/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(
-x^3 + 1)^(1/3))/x) - 1/9*log((x + (-x^3 + 1)^(1/3))/x) + 1/18*log((x^2 - (
-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.44

$$\int x\sqrt[3]{1-x^3}\,dx = \frac{x^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate(x*(-x**3+1)**(1/3),x)
[Out] x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(5/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int x\sqrt[3]{1-x^3}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\,(-x^3+1)^{\frac{1}{3}}}{x}-1\right)\right)-\frac{(-x^3+1)^{\frac{1}{3}}}{3\,x\left(\frac{x^3-1}{x^3}-1\right)} -\frac{1}{9}\log\left(\frac{(-x^3+1)^{\frac{1}{3}}}{x}+1\right)+\frac{1}{18}\log\left(-\frac{(-x^3+1)^{\frac{1}{3}}}{x}+\frac{(-x^3+1)^{\frac{2}{3}}}{x^2}+1\right)$$

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="maxima")
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*(-x^3 + 1
)^(1/3)/(x*((x^3 - 1)/x^3 - 1)) - 1/9*log((-x^3 + 1)^(1/3)/x + 1) + 1/18*lo
g(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)
```

Giac [F]

$$\int x\sqrt[3]{1-x^3}\,dx = \int (-x^3+1)^{\frac{1}{3}}x\,dx$$

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(1/3)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt[3]{1-x^3}\,dx = \int x\,(1-x^3)^{1/3}\,dx$$

```
[In] int(x*(1 - x^3)^(1/3),x)
[Out] int(x*(1 - x^3)^(1/3), x)
```

3.57 $\int \frac{\sqrt[3]{1-x^3}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \frac{\sqrt[3]{1-x^3}}{x}\,dx = \sqrt[3]{1-x^3} - \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

```
[Out] (-x^3+1)^(1/3)-1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))-1/3*arctan(1/3*(1+2*(-x^3
+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {272, 52, 59, 632, 210, 31}

$$\int \frac{\sqrt[3]{1-x^3}}{x}\,dx = -\frac{\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} + \sqrt[3]{1-x^3} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log(x)}{2}$$

```
[In] Int[(1 - x^3)^(1/3)/x,x]
```

```
[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/
2 + Log[1 - (1 - x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{1-x}}{x}\,dx, x, x^3\right) \\
&= \sqrt[3]{1-x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^3\right) \\
&= \sqrt[3]{1-x^3} - \frac{\log(x)}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) \\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)
\end{aligned}$$

$$= \sqrt[3]{1-x^3} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right)$$

$$= \sqrt[3]{1-x^3} - \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt[3]{1-x^3}}{x}\,dx = \sqrt[3]{1-x^3} - \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(-1+\sqrt[3]{1-x^3}\right) - \frac{1}{6}\log\left(1+\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)$$

```
[In] Integrate[(1 - x^3)^(1/3)/x,x]
[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[-1
+ (1 - x^3)^(1/3)]/3 - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

method	result
meijerg	$-\frac{-3\left(3+\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+\Gamma\left(\frac{2}{3}\right)x^3\,{}_3F_2\left(\frac{2}{3},1,1;2,2;x^3\right)}{9\Gamma\left(\frac{2}{3}\right)}$
pseudoelliptic	$\left(-x^3+1\right)^{\frac{1}{3}} - \frac{\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)}{6} - \frac{\arctan\left(\frac{\left(1+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3} + \frac{\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)}{3}$
trager	$\left(-x^3+1\right)^{\frac{1}{3}} + \frac{\ln\left(-\frac{1438\,\text{RootOf}\left(\_Z^2+\_Z+1\right)^2 x^3-6979\,\text{RootOf}\left(\_Z^2+\_Z+1\right)x^3+5502\,\text{RootOf}\left(\_Z^2+\_Z+1\right)\left(-x\right.}{}\right.}{}$

```
[In] int((-x^3+1)^(1/3)/x,x,method=_RETURNVERBOSE)
[Out] -1/9/GAMMA(2/3)*(-3*(3+1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*GAMMA(2/3)+GA
MMA(2/3)*x^3*hypergeom([2/3,1,1],[2,2],x^3))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt[3]{1-x^3}}{x}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}(-x^3+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+(-x^3+1)^{\frac{1}{3}} -\frac{1}{6}\log\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left((-x^3+1)^{\frac{1}{3}}-1\right)$$

```
[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + (-x^3 + 1
)^(1/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3
+ 1)^(1/3) - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt[3]{1-x^3}}{x}\,dx = -\frac{xe^{\frac{i\pi}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3\Gamma\left(\frac{2}{3}\right)}$$

```
[In] integrate((-x**3+1)**(1/3)/x,x)
```

```
[Out] -x*exp(I*pi/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), x**(-3))/(3*gamma(2/
3))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt[3]{1-x^3}}{x}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(-x^3+1)^{\frac{1}{3}}+1\right)\right)+(-x^3+1)^{\frac{1}{3}} -\frac{1}{6}\log\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left((-x^3+1)^{\frac{1}{3}}-1\right)$$

```
[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/3
) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(
1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{1-x^3}}{x}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\Big(2\left(-x^3+1\right)^{\frac{1}{3}}+1\Big)\right)+\left(-x^3+1\right)^{\frac{1}{3}} - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

```
[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/3
) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 +
1)^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt[3]{1-x^3}}{x}\,dx = \frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3}+\ln\left(3\left(1-x^3\right)^{1/3}+\frac{3}{2}-\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(3\left(1-x^3\right)^{1/3}+\frac{3}{2}+\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\left(1-x^3\right)^{1/3}$$

```
[In] int((1 - x^3)^(1/3)/x,x)
```

```
[Out] log((1 - x^3)^(1/3) - 1)/3 + log(3*(1 - x^3)^(1/3) - (3^(1/2)*3i)/2 + 3/2)*
((3^(1/2)*1i)/6 - 1/6) - log((3^(1/2)*3i)/2 + 3*(1 - x^3)^(1/3) + 3/2)*((3^
(1/2)*1i)/6 + 1/6) + (1 - x^3)^(1/3)
```

3.58 $\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 482

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx = \sqrt[3]{1-x^3}+\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\sqrt[3]{2}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\sqrt[3]{2}\log\left(1+x^3\right)+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)-\frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2^{2/3}}-\frac{1}{2}\log\left(-x-\sqrt[3]{\right.$$

```
[Out] (-x^3+1)^(1/3)-1/3*2^(1/3)*ln(x^3+1)+1/6*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*
2^(1/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3
))*2^(1/3)+1/3*2^(1/3)*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))-1/12*ln(2*2^(1/3)
+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/2*ln(2^(1/3
)-(-x^3+1)^(1/3))*2^(1/3)-1/2*ln(-x-(-x^3+1)^(1/3))+1/2*ln(-2^(1/3)*x-(-x^3
+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))
*2^(1/3)*3^(1/2)-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/3
*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/3*2^(
1/3)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.882, Rules used = {2181, 420, 493, 298, 31, 648, 631, 210, 642, 495, 337, 503, 455, 52, 59}

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx = \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\sqrt[3]{2}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} + \sqrt[3]{1-x^3} - \frac{1}{3}\sqrt[3]{2}\log\left(x^3+1\right) + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right) - \frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2^{2/3}} - \frac{1}{2}\log\left(-\sqrt[3]{1-\right.$$

```
[In] Int[(1 - x^3)^(1/3)/(1 + x),x]
```

```
[Out] (1 - x^3)^(1/3) + (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]])/Sqrt[3] + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]
]/(2^(2/3)*Sqrt[3]) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] +
 (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - (2
^(1/3)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - (2^(1/3)*Lo
g[1 + x^3])/3 + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[1
+ (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/
(3*2^(2/3)) + (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - Log[
2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/
(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/2^(2/3) - Log[-x - (1 - x^3)^(
1/3)]/2 + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/2^(2/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
```

```
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sqrt[3]{1-x^3}}{1+x^3} - \frac{x\sqrt[3]{1-x^3}}{1+x^3} + \frac{x^2\sqrt[3]{1-x^3}}{1+x^3} \right) dx \\
&= \int \frac{\sqrt[3]{1-x^3}}{1+x^3}\, dx - \int \frac{x\sqrt[3]{1-x^3}}{1+x^3}\, dx + \int \frac{x^2\sqrt[3]{1-x^3}}{1+x^3}\, dx \\
&= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{1-x}}{1+x}\, dx, x, x^3 \right) - 2\int \frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\, dx \\
&\quad - 9\text{Subst}\left(\int \frac{x}{\left(4-x^3\right)\left(1+2x^3\right)}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}} \right) + \int \frac{x}{\left(1-x^3\right)^{2/3}}\, dx \\
&= \sqrt[3]{1-x^3} - \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} \\
&\quad - \frac{\log\left(1+x^3\right)}{3\ 2^{2/3}} - \frac{1}{2}\log\left(-x-\sqrt[3]{1-x^3}\right) + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2^{2/3}} \\
&\quad + \frac{2}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\, dx, x, x^3 \right) - 2\text{Subst}\left(\int \frac{x}{1+2x^3}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}} \right) - \text{Subst}\left(\int \frac{x}{4-x} \right. \\
&= \sqrt[3]{1-x^3} - \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} \\
&\quad - \frac{1}{3}\sqrt[3]{2}\log\left(1+x^3\right) - \frac{1}{2}\log\left(-x-\sqrt[3]{1-x^3}\right) + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2^{2/3}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-x}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}} \right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{2^{2/3}-x}{2\sqrt[3]{2}+2^{2/3}x+x^2}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}} \right)}{3\ 2^{2/3}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\, dx, x, \sqrt[3]{1-x^3} \right)}{2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\, dx, x, \sqrt[3]{1-x^3} \right)}{\sqrt[3]{2}} \\
&\quad + \frac{1}{3}2^{2/3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}} \right) - \frac{1}{3}2^{2/3}\text{Subst}\left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}} \right)
\end{aligned}$$

$$= \sqrt[3]{1-x^3} - \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\sqrt[3]{2}\log\left(1+x^3\right)$$

$$+ \frac{\log\left(2^{2/3} - \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\log\left(1 + \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2^{2/3}}$$

$$-\frac{1}{2}\log\left(-x-\sqrt[3]{1-x^3}\right) + \frac{\log\left(-\sqrt[3]{2}x - \sqrt[3]{1-x^3}\right)}{2^{2/3}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x}}\right.$$

$$= \sqrt[3]{1-x^3} - \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$+ \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\sqrt[3]{2}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$-\frac{1}{3}\sqrt[3]{2}\log\left(1+x^3\right) + \frac{\log\left(2^{2/3} - \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\log\left(1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}$$

$$+\frac{1}{3}\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(2\sqrt[3]{2} + \frac{(1-x)^2}{(1-x^3)^{2/3}} + \frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x^3}\right)}{2^{2/3}} - \frac{1}{2}\log$$

$$= \sqrt[3]{1-x^3} + \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\sqrt[3]{2}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\sqrt[3]{2}\log\left(1+x^3\right) + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2^{2/3}} - \frac{1}{2}\log\left(\right.$$

Mathematica [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx = \int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$$

```
[In] Integrate[(1 - x^3)^(1/3)/(1 + x),x]
[Out] Integrate[(1 - x^3)^(1/3)/(1 + x), x]
```

Maple [F]

$$\int \frac{(-x^3+1)^{\frac{1}{3}}}{1+x}dx$$

```
[In] int((-x^3+1)^(1/3)/(1+x),x)
[Out] int((-x^3+1)^(1/3)/(1+x),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

Sympy [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx = \int \frac{\sqrt[3]{-(x-1)\,(x^2+x+1)}}{x+1}\,dx$$

```
[In] integrate((-x**3+1)**(1/3)/(1+x),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx = \int \frac{(-x^3+1)^{\frac{1}{3}}}{x+1}\,dx$$

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 1), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx = \int \frac{(-x^3+1)^{\frac{1}{3}}}{x+1}\,dx$$

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx = \int \frac{(1-x^3)^{1/3}}{x+1}\,dx$$

```
[In] int((1 - x^3)^(1/3)/(x + 1),x)
[Out] int((1 - x^3)^(1/3)/(x + 1), x)
```

3.59 $\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 280

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}(-1+x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log\left(-3(-1+x)\left(1-x+x^2\right)\right)}{2\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} + \frac{3\log\left(-\sqrt[3]{2}(-1+x)+\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} + \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/4*ln(-3*(-1+x)*(x^2-x+1))*2^(1/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)
+3/4*ln(-2^(1/3)*(-1+x)+(-x^3+1)^(1/3))*2^(1/3)+1/2*ln(x+(-x^3+1)^(1/3))-1/
4*ln(2^(1/3)*x+(-x^3+1)^(1/3))*2^(1/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)-1/6*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^
(1/2))*3^(1/2)-1/6*2^(1/3)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3
^(1/2)+1/2*arctan(1/3*(1+2*2^(1/3)*(-1+x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*
2^(1/3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.46, number of steps used = 19, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {2183, 420, 493, 298, 31, 648, 631, 210, 642, 495, 337, 503}

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx = \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log(x^3+1)}{3\ 2^{2/3}} + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right) - \frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}} + \frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right) - \frac{\log\left(-\sqrt[3]{1-x\right.}}{2^{2/3}}$$

```
[In] Int[(1 - x^3)^(1/3)/(1 - x + x^2),x]
```

```
[Out] (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]
) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - (2^(1/3)*ArcTan[(
1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + Log[1 + x^3]/(3*2^(2
/3)) + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[1 + (2^(2/3
)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3
)) + (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - Log[2*2^(1/3)
 + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(2/3
)) + Log[-x - (1 - x^3)^(1/3)]/2 - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/2^(2
/3)
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\sqrt[3]{1-x^3}}{1+x^3}+\frac{x\sqrt[3]{1-x^3}}{1+x^3}\right) dx \\
&= \int \frac{\sqrt[3]{1-x^3}}{1+x^3}\, dx+\int \frac{x\sqrt[3]{1-x^3}}{1+x^3}\, dx \\
&= 2\int \frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\, dx \\
&\quad -9\text{Subst}\left(\int \frac{x}{\left(4-x^3\right)\left(1+2x^3\right)}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)-\int \frac{x}{\left(1-x^3\right)^{2/3}}\, dx \\
&= \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log\left(1+x^3\right)}{3\ 2^{2/3}} \\
&\quad +\frac{1}{2}\log\left(-x-\sqrt[3]{1-x^3}\right)-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2^{2/3}} \\
&\quad -2\text{Subst}\left(\int \frac{x}{1+2x^3}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)-\text{Subst}\left(\int \frac{x}{4-x^3}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)
\end{aligned}
$$

$$= \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log(1+x^3)}{3\ 2^{2/3}} + \frac{1}{2}\log\left(-x-\sqrt[3]{1-x^3}\right) - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{2^{2/3}-x}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{1}{3}2^{2/3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}2^{2/3}\text{Subst}\left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log(1+x^3)}{3\ 2^{2/3}} + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) + \frac{1}{2}\log\left(-x-\sqrt[3]{1-x^3}\right) - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2^{2/3}} + \frac{1}{2}\text{Subst}\left(\int \frac{}{2\sqrt[}}\right.$$

$$= \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log(1+x^3)}{3\ 2^{2/3}} + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{1}{2}\log\left(-x-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\right.}{}$$

$$= \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log(1+x^3)}{3\ 2^{2/3}} + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{1}{2}\log\left(-x-\sqrt[3]{1-x^3}\right) - \frac{\log(-}{}$$

Mathematica [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx = \int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx$$

```
[In] Integrate[(1 - x^3)^(1/3)/(1 - x + x^2),x]
[Out] Integrate[(1 - x^3)^(1/3)/(1 - x + x^2), x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 21.32 (sec) , antiderivative size = 1410, normalized size of antiderivative = 5.04

method	result	size
trager	Expression too large to display	1410

```
[In] int((-x^3+1)^(1/3)/(x^2-x+1),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(RootOf(_Z^2+_Z+1)^2*x^3-3*RootOf(_Z^2+_Z+1)*(-x^3+1)^(2/3)*x+3*RootO
f(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x^2-2*RootOf(_Z^2+_Z+1)*x^3+x^3+RootOf(_Z^2+_Z+
1)-1)*RootOf(_Z^2+_Z+1)-1/3*RootOf(_Z^2+_Z+1)*ln(RootOf(_Z^2+_Z+1)^2*x^3+3*
RootOf(_Z^2+_Z+1)*(-x^3+1)^(2/3)*x-3*RootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x^2+4
*RootOf(_Z^2+_Z+1)*x^3+3*x*(-x^3+1)^(2/3)-3*x^2*(-x^3+1)^(1/3)+4*x^3-RootOf
(_Z^2+_Z+1)-2)-1/3*ln(RootOf(_Z^2+_Z+1)^2*x^3+3*RootOf(_Z^2+_Z+1)*(-x^3+1)^
(2/3)*x-3*RootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x^2+4*RootOf(_Z^2+_Z+1)*x^3+3*x*
```

```
(-x^3+1)^(2/3)-3*x^2*(-x^3+1)^(1/3)+4*x^3-RootOf(_Z^2+_Z+1)-2)-1/18*ln(-(12
*(-x^3+1)^(1/3)*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)*RootOf(_Z^2+_Z+1)*x^
3+RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)^2*x^4-36*(-x^3+1)^(1/3)*RootOf(_Z^
3-324*RootOf(_Z^2+_Z+1)-162)*RootOf(_Z^2+_Z+1)*x^2+6*(-x^3+1)^(1/3)*RootOf(
_Z^3-324*RootOf(_Z^2+_Z+1)-162)*x^3+2*x^3*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)
-162)^2+108*(-x^3+1)^(2/3)*x^2+12*(-x^3+1)^(1/3)*RootOf(_Z^3-324*RootOf(_Z^
2+_Z+1)-162)*RootOf(_Z^2+_Z+1)*x-18*(-x^3+1)^(1/3)*RootOf(_Z^3-324*RootOf(_
Z^2+_Z+1)-162)*x^2-RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)^2*x^2-108*x*(-x^3
+1)^(2/3)+6*(-x^3+1)^(1/3)*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)*x-2*RootO
f(_Z^3-324*RootOf(_Z^2+_Z+1)-162)^2*x+RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162
)^2)/(x^2-x+1)^2)*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)*RootOf(_Z^2+_Z+1)-
1/18*ln(-(12*(-x^3+1)^(1/3)*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)*RootOf(_
Z^2+_Z+1)*x^3+RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)^2*x^4-36*(-x^3+1)^(1/3
)*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)*RootOf(_Z^2+_Z+1)*x^2+6*(-x^3+1)^(
1/3)*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)*x^3+2*x^3*RootOf(_Z^3-324*RootO
f(_Z^2+_Z+1)-162)^2+108*(-x^3+1)^(2/3)*x^2+12*(-x^3+1)^(1/3)*RootOf(_Z^3-32
4*RootOf(_Z^2+_Z+1)-162)*RootOf(_Z^2+_Z+1)*x-18*(-x^3+1)^(1/3)*RootOf(_Z^3-
324*RootOf(_Z^2+_Z+1)-162)*x^2-RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)^2*x^2
-108*x*(-x^3+1)^(2/3)+6*(-x^3+1)^(1/3)*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-16
2)*x-2*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)^2*x+RootOf(_Z^3-324*RootOf(_Z
^2+_Z+1)-162)^2)/(x^2-x+1)^2)*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)+1/18*R
ootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)*ln(-(-5*RootOf(_Z^2+_Z+1)*RootOf(_Z^3
-324*RootOf(_Z^2+_Z+1)-162)^2*x^4+18*(-x^3+1)^(1/3)*RootOf(_Z^3-324*RootOf(
_Z^2+_Z+1)-162)*RootOf(_Z^2+_Z+1)*x^3+2*RootOf(_Z^2+_Z+1)*RootOf(_Z^3-324*R
ootOf(_Z^2+_Z+1)-162)^2*x^3-6*(-x^3+1)^(1/3)*RootOf(_Z^3-324*RootOf(_Z^2+_Z
+1)-162)*RootOf(_Z^2+_Z+1)*x^2-18*(-x^3+1)^(1/3)*RootOf(_Z^3-324*RootOf(_Z^
2+_Z+1)-162)*x^3+RootOf(_Z^2+_Z+1)*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)^2
*x^2+216*(-x^3+1)^(2/3)*x^2-6*(-x^3+1)^(1/3)*RootOf(_Z^3-324*RootOf(_Z^2+_Z
+1)-162)*RootOf(_Z^2+_Z+1)*x+6*(-x^3+1)^(1/3)*RootOf(_Z^3-324*RootOf(_Z^2+_
Z+1)-162)*x^2+2*RootOf(_Z^2+_Z+1)*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)^2*
x-108*x*(-x^3+1)^(2/3)+6*(-x^3+1)^(1/3)*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-1
62)*x-RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)^2*RootOf(_Z^2+_Z+1))/(x^2-x+1)
^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.35 (sec) , antiderivative size = 3880, normalized size of antiderivative = 13.86

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx = \text{Too large to display}$$

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(3)*(sqrt(-3)*(-4)^(1/6) + (-4)^(1/6))*log(3*(6*sqrt(-3)*(-4)^(1/3
)*(26655*x^12 - 185476*x^11 + 155872*x^10 + 361508*x^9 - 363117*x^8 - 87612
*x^7 - 197936*x^6 + 492924*x^5 - 182367*x^4 - 39436*x^3 + 18254*x^2 + 3652*
x - 1278) + 48*(11866*x^10 - 16425*x^9 - 125794*x^8 + 251931*x^7 - 71187*x^
6 - 79049*x^5 - 2745*x^4 + 52032*x^3 - 20629*x^2 - sqrt(3)*(3104*I*x^10 - 4
3815*I*x^9 + 84520*I*x^8 + 11329*I*x^7 - 92013*I*x^6 + 5291*I*x^5 + 53855*I
*x^4 - 20262*I*x^3 - 2009*I*x^2 + 1278*I*x) + 2008*x)*(-x^3 + 1)^(2/3) + sq
rt(3)*(sqrt(-3)*(-4)^(5/6)*(31397*x^12 + 113940*x^11 - 831396*x^10 + 973364
*x^9 - 140709*x^8 + 407484*x^7 - 1009896*x^6 + 313212*x^5 + 248121*x^4 - 75
940*x^3 - 48198*x^2 + 19716*x - 2008) - (-4)^(5/6)*(31397*x^12 + 113940*x^1
1 - 831396*x^10 + 973364*x^9 - 140709*x^8 + 407484*x^7 - 1009896*x^6 + 3132
12*x^5 + 248121*x^4 - 75940*x^3 - 48198*x^2 + 19716*x - 2008)) - 6*(-4)^(1/
3)*(26655*x^12 - 185476*x^11 + 155872*x^10 + 361508*x^9 - 363117*x^8 - 8761
2*x^7 - 197936*x^6 + 492924*x^5 - 182367*x^4 - 39436*x^3 + 18254*x^2 + 3652
*x - 1278) - 6*(-x^3 + 1)^(1/3)*(sqrt(-3)*(-4)^(2/3)*(1459*x^11 + 94937*x^1
0 - 314364*x^9 + 204807*x^8 + 73586*x^7 + 103515*x^6 - 263973*x^5 + 67714*x
^4 + 54774*x^3 - 25376*x^2 + 2008*x) + (-4)^(2/3)*(1459*x^11 + 94937*x^10 -
 314364*x^9 + 204807*x^8 + 73586*x^7 + 103515*x^6 - 263973*x^5 + 67714*x^4
+ 54774*x^3 - 25376*x^2 + 2008*x) + 2*sqrt(3)*(sqrt(-3)*(-4)^(1/6)*(12049*x
^11 - 48557*x^10 - 31048*x^9 + 203745*x^8 - 117748*x^7 - 29753*x^6 - 67923*
x^5 + 127612*x^4 - 49654*x^3 + 1642*x^2 + 1278*x) + (-4)^(1/6)*(12049*x^11
- 48557*x^10 - 31048*x^9 + 203745*x^8 - 117748*x^7 - 29753*x^6 - 67923*x^5
+ 127612*x^4 - 49654*x^3 + 1642*x^2 + 1278*x))))/(x^12 - 6*x^11 + 21*x^10 -
 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2 -
 6*x + 1)) - 1/72*sqrt(3)*(sqrt(-3)*(-4)^(1/6) - (-4)^(1/6))*log(-3*(6*sqrt
(-3)*(-4)^(1/3)*(26655*x^12 - 185476*x^11 + 155872*x^10 + 361508*x^9 - 3631
17*x^8 - 87612*x^7 - 197936*x^6 + 492924*x^5 - 182367*x^4 - 39436*x^3 + 182
54*x^2 + 3652*x - 1278) - 48*(11866*x^10 - 16425*x^9 - 125794*x^8 + 251931*
x^7 - 71187*x^6 - 79049*x^5 - 2745*x^4 + 52032*x^3 - 20629*x^2 - sqrt(3)*(3
104*I*x^10 - 43815*I*x^9 + 84520*I*x^8 + 11329*I*x^7 - 92013*I*x^6 + 5291*I
*x^5 + 53855*I*x^4 - 20262*I*x^3 - 2009*I*x^2 + 1278*I*x) + 2008*x)*(-x^3 +
 1)^(2/3) + sqrt(3)*(sqrt(-3)*(-4)^(5/6)*(31397*x^12 + 113940*x^11 - 831396
*x^10 + 973364*x^9 - 140709*x^8 + 407484*x^7 - 1009896*x^6 + 313212*x^5 + 2
48121*x^4 - 75940*x^3 - 48198*x^2 + 19716*x - 2008) + (-4)^(5/6)*(31397*x^1
2 + 113940*x^11 - 831396*x^10 + 973364*x^9 - 140709*x^8 + 407484*x^7 - 1009
896*x^6 + 313212*x^5 + 248121*x^4 - 75940*x^3 - 48198*x^2 + 19716*x - 2008)
) + 6*(-4)^(1/3)*(26655*x^12 - 185476*x^11 + 155872*x^10 + 361508*x^9 - 363
117*x^8 - 87612*x^7 - 197936*x^6 + 492924*x^5 - 182367*x^4 - 39436*x^3 + 18
254*x^2 + 3652*x - 1278) - 6*(-x^3 + 1)^(1/3)*(sqrt(-3)*(-4)^(2/3)*(1459*x^
11 + 94937*x^10 - 314364*x^9 + 204807*x^8 + 73586*x^7 + 103515*x^6 - 263973
*x^5 + 67714*x^4 + 54774*x^3 - 25376*x^2 + 2008*x) - (-4)^(2/3)*(1459*x^11
+ 94937*x^10 - 314364*x^9 + 204807*x^8 + 73586*x^7 + 103515*x^6 - 263973*x^
5 + 67714*x^4 + 54774*x^3 - 25376*x^2 + 2008*x) + 2*sqrt(3)*(sqrt(-3)*(-4)^
(1/6)*(12049*x^11 - 48557*x^10 - 31048*x^9 + 203745*x^8 - 117748*x^7 - 2975
3*x^6 - 67923*x^5 + 127612*x^4 - 49654*x^3 + 1642*x^2 + 1278*x) - (-4)^(1/6
```

)*(12049*x^11 - 48557*x^10 - 31048*x^9 + 203745*x^8 - 117748*x^7 - 29753*x^
6 - 67923*x^5 + 127612*x^4 - 49654*x^3 + 1642*x^2 + 1278*x))))/(x^12 - 6*x^
11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*
x^3 + 21*x^2 - 6*x + 1)) - 1/72*sqrt(3)*(sqrt(-3)*(-4)^(1/6) + (-4)^(1/6))*
log(3*(6*sqrt(-3)*(-4)^(1/3)*(26655*x^12 - 185476*x^11 + 155872*x^10 + 3615
08*x^9 - 363117*x^8 - 87612*x^7 - 197936*x^6 + 492924*x^5 - 182367*x^4 - 39
436*x^3 + 18254*x^2 + 3652*x - 1278) + 48*(11866*x^10 - 16425*x^9 - 125794*
x^8 + 251931*x^7 - 71187*x^6 - 79049*x^5 - 2745*x^4 + 52032*x^3 - 20629*x^2
 - sqrt(3)*(-3104*I*x^10 + 43815*I*x^9 - 84520*I*x^8 - 11329*I*x^7 + 92013*
I*x^6 - 5291*I*x^5 - 53855*I*x^4 + 20262*I*x^3 + 2009*I*x^2 - 1278*I*x) + 2
008*x)*(-x^3 + 1)^(2/3) - sqrt(3)*(sqrt(-3)*(-4)^(5/6)*(31397*x^12 + 113940
*x^11 - 831396*x^10 + 973364*x^9 - 140709*x^8 + 407484*x^7 - 1009896*x^6 +
313212*x^5 + 248121*x^4 - 75940*x^3 - 48198*x^2 + 19716*x - 2008) - (-4)^(5
/6)*(31397*x^12 + 113940*x^11 - 831396*x^10 + 973364*x^9 - 140709*x^8 + 407
484*x^7 - 1009896*x^6 + 313212*x^5 + 248121*x^4 - 75940*x^3 - 48198*x^2 + 1
9716*x - 2008)) - 6*(-4)^(1/3)*(26655*x^12 - 185476*x^11 + 155872*x^10 + 36
1508*x^9 - 363117*x^8 - 87612*x^7 - 197936*x^6 + 492924*x^5 - 182367*x^4 -
39436*x^3 + 18254*x^2 + 3652*x - 1278) - 6*(-x^3 + 1)^(1/3)*(sqrt(-3)*(-4)^
(2/3)*(1459*x^11 + 94937*x^10 - 314364*x^9 + 204807*x^8 + 73586*x^7 + 10351
5*x^6 - 263973*x^5 + 67714*x^4 + 54774*x^3 - 25376*x^2 + 2008*x) + (-4)^(2/
3)*(1459*x^11 + 94937*x^10 - 314364*x^9 + 204807*x^8 + 73586*x^7 + 103515*x
^6 - 263973*x^5 + 67714*x^4 + 54774*x^3 - 25376*x^2 + 2008*x) - 2*sqrt(3)*(
sqrt(-3)*(-4)^(1/6)*(12049*x^11 - 48557*x^10 - 31048*x^9 + 203745*x^8 - 117
748*x^7 - 29753*x^6 - 67923*x^5 + 127612*x^4 - 49654*x^3 + 1642*x^2 + 1278*
x) + (-4)^(1/6)*(12049*x^11 - 48557*x^10 - 31048*x^9 + 203745*x^8 - 117748*
x^7 - 29753*x^6 - 67923*x^5 + 127612*x^4 - 49654*x^3 + 1642*x^2 + 1278*x)))
)/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*x^5
+ 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) + 1/72*sqrt(3)*(sqrt(-3)*(-4)^(1/6)
- (-4)^(1/6))*log(-3*(6*sqrt(-3)*(-4)^(1/3)*(26655*x^12 - 185476*x^11 + 155
872*x^10 + 361508*x^9 - 363117*x^8 - 87612*x^7 - 197936*x^6 + 492924*x^5 -
182367*x^4 - 39436*x^3 + 18254*x^2 + 3652*x - 1278) - 48*(11866*x^10 - 1642
5*x^9 - 125794*x^8 + 251931*x^7 - 71187*x^6 - 79049*x^5 - 2745*x^4 + 52032*
x^3 - 20629*x^2 - sqrt(3)*(-3104*I*x^10 + 43815*I*x^9 - 84520*I*x^8 - 11329
*I*x^7 + 92013*I*x^6 - 5291*I*x^5 - 53855*I*x^4 + 20262*I*x^3 + 2009*I*x^2
- 1278*I*x) + 2008*x)*(-x^3 + 1)^(2/3) - sqrt(3)*(sqrt(-3)*(-4)^(5/6)*(3139
7*x^12 + 113940*x^11 - 831396*x^10 + 973364*x^9 - 140709*x^8 + 407484*x^7 -
 1009896*x^6 + 313212*x^5 + 248121*x^4 - 75940*x^3 - 48198*x^2 + 19716*x -
2008) + (-4)^(5/6)*(31397*x^12 + 113940*x^11 - 831396*x^10 + 973364*x^9 - 1
40709*x^8 + 407484*x^7 - 1009896*x^6 + 313212*x^5 + 248121*x^4 - 75940*x^3
- 48198*x^2 + 19716*x - 2008)) + 6*(-4)^(1/3)*(26655*x^12 - 185476*x^11 + 1
55872*x^10 + 361508*x^9 - 363117*x^8 - 87612*x^7 - 197936*x^6 + 492924*x^5
- 182367*x^4 - 39436*x^3 + 18254*x^2 + 3652*x - 1278) - 6*(-x^3 + 1)^(1/3)*
(sqrt(-3)*(-4)^(2/3)*(1459*x^11 + 94937*x^10 - 314364*x^9 + 204807*x^8 + 73
586*x^7 + 103515*x^6 - 263973*x^5 + 67714*x^4 + 54774*x^3 - 25376*x^2 + 200
8*x) - (-4)^(2/3)*(1459*x^11 + 94937*x^10 - 314364*x^9 + 204807*x^8 + 73586

```
*x^7 + 103515*x^6 - 263973*x^5 + 67714*x^4 + 54774*x^3 - 25376*x^2 + 2008*x
) - 2*sqrt(3)*(sqrt(-3)*(-4)^(1/6)*(12049*x^11 - 48557*x^10 - 31048*x^9 + 2
03745*x^8 - 117748*x^7 - 29753*x^6 - 67923*x^5 + 127612*x^4 - 49654*x^3 + 1
642*x^2 + 1278*x) - (-4)^(1/6)*(12049*x^11 - 48557*x^10 - 31048*x^9 + 20374
5*x^8 - 117748*x^7 - 29753*x^6 - 67923*x^5 + 127612*x^4 - 49654*x^3 + 1642*
x^2 + 1278*x))))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 141
*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) - 1/36*sqrt(3)*(-4)^(
1/6)*log(3*(sqrt(3)*(-4)^(5/6)*(31397*x^12 + 113940*x^11 - 831396*x^10 + 97
3364*x^9 - 140709*x^8 + 407484*x^7 - 1009896*x^6 + 313212*x^5 + 248121*x^4
- 75940*x^3 - 48198*x^2 + 19716*x - 2008) + 24*(11866*x^10 - 16425*x^9 - 12
5794*x^8 + 251931*x^7 - 71187*x^6 - 79049*x^5 - 2745*x^4 + 52032*x^3 - 2062
9*x^2 - sqrt(3)*(3104*I*x^10 - 43815*I*x^9 + 84520*I*x^8 + 11329*I*x^7 - 92
013*I*x^6 + 5291*I*x^5 + 53855*I*x^4 - 20262*I*x^3 - 2009*I*x^2 + 1278*I*x)
 + 2008*x)*(-x^3 + 1)^(2/3) + 6*(-4)^(1/3)*(26655*x^12 - 185476*x^11 + 1558
72*x^10 + 361508*x^9 - 363117*x^8 - 87612*x^7 - 197936*x^6 + 492924*x^5 - 1
82367*x^4 - 39436*x^3 + 18254*x^2 + 3652*x - 1278) + 6*(-x^3 + 1)^(1/3)*(2*
sqrt(3)*(-4)^(1/6)*(12049*x^11 - 48557*x^10 - 31048*x^9 + 203745*x^8 - 1177
48*x^7 - 29753*x^6 - 67923*x^5 + 127612*x^4 - 49654*x^3 + 1642*x^2 + 1278*x
) + (-4)^(2/3)*(1459*x^11 + 94937*x^10 - 314364*x^9 + 204807*x^8 + 73586*x^
7 + 103515*x^6 - 263973*x^5 + 67714*x^4 + 54774*x^3 - 25376*x^2 + 2008*x)))
/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*x^5 +
 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) + 1/36*sqrt(3)*(-4)^(1/6)*log(-3*(sqr
t(3)*(-4)^(5/6)*(31397*x^12 + 113940*x^11 - 831396*x^10 + 973364*x^9 - 1407
09*x^8 + 407484*x^7 - 1009896*x^6 + 313212*x^5 + 248121*x^4 - 75940*x^3 - 4
8198*x^2 + 19716*x - 2008) - 24*(11866*x^10 - 16425*x^9 - 125794*x^8 + 2519
31*x^7 - 71187*x^6 - 79049*x^5 - 2745*x^4 + 52032*x^3 - 20629*x^2 - sqrt(3)
*(-3104*I*x^10 + 43815*I*x^9 - 84520*I*x^8 - 11329*I*x^7 + 92013*I*x^6 - 52
91*I*x^5 - 53855*I*x^4 + 20262*I*x^3 + 2009*I*x^2 - 1278*I*x) + 2008*x)*(-x
^3 + 1)^(2/3) - 6*(-4)^(1/3)*(26655*x^12 - 185476*x^11 + 155872*x^10 + 3615
08*x^9 - 363117*x^8 - 87612*x^7 - 197936*x^6 + 492924*x^5 - 182367*x^4 - 39
436*x^3 + 18254*x^2 + 3652*x - 1278) + 6*(-x^3 + 1)^(1/3)*(2*sqrt(3)*(-4)^(
1/6)*(12049*x^11 - 48557*x^10 - 31048*x^9 + 203745*x^8 - 117748*x^7 - 29753
*x^6 - 67923*x^5 + 127612*x^4 - 49654*x^3 + 1642*x^2 + 1278*x) - (-4)^(2/3)
*(1459*x^11 + 94937*x^10 - 314364*x^9 + 204807*x^8 + 73586*x^7 + 103515*x^6
 - 263973*x^5 + 67714*x^4 + 54774*x^3 - 25376*x^2 + 2008*x)))/(x^12 - 6*x^1
1 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*x
^3 + 21*x^2 - 6*x + 1)) + 1/3*sqrt(3)*arctan((4*sqrt(3)*(-x^3 + 1)^(1/3)*x^
2 + 2*sqrt(3)*(-x^3 + 1)^(2/3)*x - sqrt(3)*(x^3 - 1))/(9*x^3 - 1)) + 1/6*lo
g(3*(-x^3 + 1)^(1/3)*x^2 + 3*(-x^3 + 1)^(2/3)*x + 1)
```

Sympy [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx = \int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{x^2-x+1}\,dx$$

```
[In] integrate((-x**3+1)**(1/3)/(x**2-x+1),x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x**2 - x + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx = \int \frac{(-x^3+1)^{\frac{1}{3}}}{x^2-x+1}\,dx$$

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx = \int \frac{(-x^3+1)^{\frac{1}{3}}}{x^2-x+1}\,dx$$

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx = \int \frac{(1-x^3)^{1/3}}{x^2-x+1}\,dx$$

```
[In] int((1 - x^3)^(1/3)/(x^2 - x + 1),x)
[Out] int((1 - x^3)^(1/3)/(x^2 - x + 1), x)
```

3.60 $\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 232

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx$$

$$= \sqrt[3]{1-x^3}+\frac{1}{2}x\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-\frac{x^3}{8}\right)-\frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$
$$+\sqrt[6]{3}\arctan\left(\frac{1-\frac{3^{2/3}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)-\sqrt[6]{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-x^3}}{3\sqrt[6]{3}}\right)-\frac{\log(8+x^3)}{\sqrt[3]{3}}$$
$$+\frac{1}{2}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{1-x^3}\right)-\log\left(-x-\sqrt[3]{1-x^3}\right)+\frac{1}{2}3^{2/3}\log\left(-\frac{1}{2}3^{2/3}x-\sqrt[3]{1-x^3}\right)$$

```
[Out] (-x^3+1)^(1/3)+1/2*x*AppellF1(1/3,-1/3,1,4/3,x^3,-1/8*x^3)-3^(1/6)*arctan(2
/9*(-x^3+1)^(1/3)*3^(5/6)+1/3*3^(1/2))+3^(1/6)*arctan(1/3*(1-3^(2/3)*x/(-x^
3+1)^(1/3))*3^(1/2))-1/3*ln(x^3+8)*3^(2/3)+1/2*3^(2/3)*ln(3^(2/3)-(-x^3+1)^
(1/3))-ln(-x-(-x^3+1)^(1/3))+1/2*3^(2/3)*ln(-1/2*3^(2/3)*x-(-x^3+1)^(1/3))-
2/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {2181, 440, 495, 337, 503, 455, 52, 59, 631, 210, 31}

$$\int \frac{\sqrt[3]{1-x^3}}{2+x} \, dx$$

$$= \frac{1}{2}x \operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-\frac{x^3}{8}\right) - \frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \sqrt[6]{3}\arctan\left(\frac{1-\frac{3^{2/3}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right) - \sqrt[6]{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}}{3\sqrt[6]{3}}+\frac{1}{\sqrt{3}}\right) + \sqrt[3]{1-x^3} - \frac{\log\left(x^3+8\right)}{\sqrt[3]{3}} + \frac{1}{2}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{1-x^3}\right) - \log\left(-\sqrt[3]{1-x^3}-x\right) + \frac{1}{2}3^{2/3}\log\left(-\sqrt[3]{1-x^3}-\frac{1}{2}3^{2/3}x\right)$$

```
[In] Int[(1 - x^3)^(1/3)/(2 + x),x]
```

```
[Out] (1 - x^3)^(1/3) + (x*AppellF1[1/3, -1/3, 1, 4/3, x^3, -1/8*x^3])/2 - (2*Arc
Tan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + 3^(1/6)*ArcTan[(1 - (3^
(2/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - 3^(1/6)*ArcTan[1/Sqrt[3] + (2*(1 - x^3
)^(1/3))/(3*3^(1/6))] - Log[8 + x^3]/3^(1/3) + (3^(2/3)*Log[3^(2/3) - (1 -
x^3)^(1/3)])/2 - Log[-x - (1 - x^3)^(1/3)] + (3^(2/3)*Log[-1/2*(3^(2/3)*x)
- (1 - x^3)^(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\text{integral} = \int \left(\frac{4\sqrt[3]{1-x^3}}{8+x^3} - \frac{2x\sqrt[3]{1-x^3}}{8+x^3} + \frac{x^2\sqrt[3]{1-x^3}}{8+x^3}\right) dx$$

$$= -\left(2\int \frac{x\sqrt[3]{1-x^3}}{8+x^3}\,dx\right) + 4\int \frac{\sqrt[3]{1-x^3}}{8+x^3}\,dx + \int \frac{x^2\sqrt[3]{1-x^3}}{8+x^3}\,dx$$

$$= \frac{1}{2}x\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-\frac{x^3}{8}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{1-x}}{8+x}\,dx, x, x^3\right) + 2\int \frac{x}{(1-x^3)^{2/3}}\,dx - 18\int \frac{x}{(1-x^3)^{2/3}(8+x^3)}\,dx$$

$$= \sqrt[3]{1-x^3} + \frac{1}{2}x\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-\frac{x^3}{8}\right) - \frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \sqrt[6]{3}\arctan\left(\frac{1-\frac{3^{2/3}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right) - \frac{\log(8+x^3)}{2\sqrt[3]{3}} - \log\left(-x-\sqrt[3]{1-x^3}\right) + \frac{1}{2}3^{2/3}\log\left(-\frac{1}{2}3^{2/3}x-\sqrt[3]{1-x^3}\right) + 3\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(8+x)}\,dx, x, x^3\right)$$

$$= \sqrt[3]{1-x^3} + \frac{1}{2}x\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-\frac{x^3}{8}\right) - \frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \sqrt[6]{3}\arctan\left(\frac{1-\frac{3^{2/3}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right) - \frac{\log(8+x^3)}{\sqrt[3]{3}} - \log\left(-x-\sqrt[3]{1-x^3}\right) + \frac{1}{2}3^{2/3}\log\left(-\frac{1}{2}3^{2/3}x-\sqrt[3]{1-x^3}\right) - \frac{1}{2}\left(3\sqrt[3]{3}\right)\text{Subst}\left(\int \frac{1}{3\sqrt[3]{3}+3^{2/3}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{1}{2}3^{2/}$$

$$= \sqrt[3]{1-x^3} + \frac{1}{2}x\,\text{AppellF1}\left(\frac{1}{3}, -\frac{1}{3}, 1, \frac{4}{3}, x^3, -\frac{x^3}{8}\right) - \frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \sqrt[6]{3}\arctan\left(\frac{1-\frac{3^{2/3}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right) - \frac{\log(8+x^3)}{\sqrt[3]{3}} + \frac{1}{2}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{1-x^3}\right) - \log\left(-x-\sqrt[3]{1-x^3}\right) + \frac{1}{2}3^{2/3}\log\left(-\frac{1}{2}3^{2/3}x-\sqrt[3]{1-x^3}\right) + 3^{2/3}\text{Subst}\Big($$

$$= \sqrt[3]{1-x^3} + \frac{1}{2}x\,\text{AppellF1}\left(\frac{1}{3}, -\frac{1}{3}, 1, \frac{4}{3}, x^3, -\frac{x^3}{8}\right) - \frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \sqrt[6]{3}\arctan\left(\frac{1-\frac{3^{2/3}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right) - \sqrt[6]{3}\arctan\left(\frac{3+2\sqrt[3]{3}\sqrt[3]{1-x^3}}{3\sqrt{3}}\right) - \frac{\log(8+x^3)}{\sqrt[3]{3}} + \frac{1}{2}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{1-x^3}\right) - \log\left(-x-\sqrt[3]{1-x^3}\right) + \frac{1}{2}3^{2/3}\log\left(-\frac{1}{2}3^{2/3}x-\sqrt[3]{1-x^3}\right)$$

Mathematica [F]

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx = \int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx$$

```
[In] Integrate[(1 - x^3)^(1/3)/(2 + x),x]
[Out] Integrate[(1 - x^3)^(1/3)/(2 + x), x]
```

Maple [F]

$$\int \frac{(-x^3+1)^{\frac{1}{3}}}{2+x}dx$$

```
[In] int((-x^3+1)^(1/3)/(2+x),x)
[Out] int((-x^3+1)^(1/3)/(2+x),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

Sympy [F]

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx = \int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{x+2}\,dx$$

```
[In] integrate((-x**3+1)**(1/3)/(2+x),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x + 2), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx = \int \frac{(-x^3+1)^{\frac{1}{3}}}{x+2}\,dx$$

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 2), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx = \int \frac{(-x^3+1)^{\frac{1}{3}}}{x+2}\,dx$$

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx = \int \frac{\left(1-x^3\right)^{1/3}}{x+2}\,dx$$

```
[In] int((1 - x^3)^(1/3)/(x + 2),x)
[Out] int((1 - x^3)^(1/3)/(x + 2), x)
```

3.61 $\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 168

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx = -\frac{x^2\operatorname{AppellF1}\left(\frac{2}{3},1,\frac{1}{3},\frac{5}{3},x^3,-\frac{x^3}{2}\right)}{2\sqrt[3]{2}} + \frac{2\arctan\left(\frac{1+\frac{2\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3^{5/6}} + \frac{\arctan\left(\frac{\sqrt[3]{3}+2\sqrt[3]{2+x^3}}{3^{5/6}}\right)}{3^{5/6}} + \frac{\log(1-x^3)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}-\sqrt[3]{2+x^3}\right)}{2\sqrt[3]{3}} - \frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{2+x^3}\right)}{\sqrt[3]{3}}$$

```
[Out] -1/4*x^2*AppellF1(2/3,1,1/3,5/3,x^3,-1/2*x^3)*2^(2/3)+1/3*arctan(1/3*(3^(1/
3)+2*(x^3+2)^(1/3))*3^(1/6))*3^(1/6)+2/3*arctan(1/3*(1+2*3^(1/3)*x/(x^3+2)^
(1/3))*3^(1/2))*3^(1/6)+1/18*ln(-x^3+1)*3^(2/3)+1/6*ln(3^(1/3)-(x^3+2)^(1/3
))*3^(2/3)-1/3*ln(3^(1/3)*x-(x^3+2)^(1/3))*3^(2/3)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used

$= \{2183, 384, 524, 455, 57, 631, 210, 31\}$

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx = -\frac{x^2\,\text{AppellF1}\left(\frac{2}{3},1,\frac{1}{3},\frac{5}{3},x^3,-\frac{x^3}{2}\right)}{2\sqrt[3]{2}} + \frac{2\arctan\left(\frac{\frac{2\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}} + \frac{\arctan\left(\frac{2\sqrt[3]{x^3+2}+\sqrt[3]{3}}{3^{5/6}}\right)}{3^{5/6}} + \frac{\log(1-x^3)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}-\sqrt[3]{x^3+2}\right)}{2\sqrt[3]{3}} - \frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{x^3+2}\right)}{\sqrt[3]{3}}$$

```
[In] Int[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)),x]
```

```
[Out] -1/2*(x^2*AppellF1[2/3, 1, 1/3, 5/3, x^3, -1/2*x^3])/2^(1/3) + (2*ArcTan[(1
 + (2*3^(1/3)*x)/(2 + x^3)^(1/3))/Sqrt[3]])/3^(5/6) + ArcTan[(3^(1/3) + 2*(
2 + x^3)^(1/3))/3^(5/6)]/3^(5/6) + Log[1 - x^3]/(6*3^(1/3)) + Log[3^(1/3) -
 (2 + x^3)^(1/3)]/(2*3^(1/3)) - Log[3^(1/3)*x - (2 + x^3)^(1/3)]/3^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{2}{(1-x^3)\sqrt[3]{2+x^3}} - \frac{x}{(1-x^3)\sqrt[3]{2+x^3}} - \frac{x^2}{(1-x^3)\sqrt[3]{2+x^3}}\right) dx \\
&= 2\int \frac{1}{(1-x^3)\sqrt[3]{2+x^3}}\, dx - \int \frac{x}{(1-x^3)\sqrt[3]{2+x^3}}\, dx - \int \frac{x^2}{(1-x^3)\sqrt[3]{2+x^3}}\, dx \\
&= -\frac{x^2 \operatorname{AppellF1}\left(\frac{2}{3},1,\frac{1}{3},\frac{5}{3},x^3,-\frac{x^3}{2}\right)}{2\sqrt[3]{2}} + \frac{2\arctan\left(\frac{1+\frac{2\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3^{5/6}} + \frac{\log(1-x^3)}{3\sqrt[3]{3}} \\
&\quad - \frac{\log\left(\sqrt[3]{3}x - \sqrt[3]{2+x^3}\right)}{\sqrt[3]{3}} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt[3]{2+x}}\, dx, x, x^3\right)
\end{aligned}
$$

$$= -\frac{x^2 \operatorname{AppellF1}\left(\frac{2}{3},1,\frac{1}{3},\frac{5}{3},x^3,-\frac{x^3}{2}\right)}{2\sqrt[3]{2}}+\frac{2\arctan\left(\frac{1+\frac{2\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3^{5/6}}+\frac{\log(1-x^3)}{6\sqrt[3]{3}}-\frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{2+x^3}\right)}{\sqrt[3]{3}}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{3^{2/3}+\sqrt[3]{3}x+x^2}\,dx,x,\sqrt[3]{2+x^3}\right)-\frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{3}-x}\,dx,x,\sqrt[3]{2+x^3}\right)}{2\sqrt[3]{3}}$$

$$= -\frac{x^2 \operatorname{AppellF1}\left(\frac{2}{3},1,\frac{1}{3},\frac{5}{3},x^3,-\frac{x^3}{2}\right)}{2\sqrt[3]{2}}+\frac{2\arctan\left(\frac{1+\frac{2\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3^{5/6}}+\frac{\log(1-x^3)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}-\sqrt[3]{2+x^3}\right)}{2\sqrt[3]{3}}-\frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{2+x^3}\right)}{\sqrt[3]{3}}-\frac{\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{2+x^3}}{\sqrt[3]{3}}\right)}{\sqrt[3]{3}}$$

$$= -\frac{x^2 \operatorname{AppellF1}\left(\frac{2}{3},1,\frac{1}{3},\frac{5}{3},x^3,-\frac{x^3}{2}\right)}{2\sqrt[3]{2}}+\frac{2\arctan\left(\frac{1+\frac{2\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3^{5/6}}+\frac{\arctan\left(\frac{1}{3}\left(\sqrt{3}+2\sqrt[6]{3}\sqrt[3]{2+x^3}\right)\right)}{3^{5/6}}+\frac{\log(1-x^3)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}-\sqrt[3]{2+x^3}\right)}{2\sqrt[3]{3}}-\frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{2+x^3}\right)}{\sqrt[3]{3}}$$

Mathematica [F]

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx=\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx$$

```
[In] Integrate[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)),x]
[Out] Integrate[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)), x]
```

Maple [F]

$$\int \frac{2+x}{(x^2+x+1)(x^3+2)^{\frac{1}{3}}}dx$$

[In] int((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x)

[Out] int((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x)

Fricas [F]

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx = \int \frac{x+2}{(x^3+2)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="fricas")

[Out] integral((x^3 + 2)^(2/3)*(x + 2)/(x^5 + x^4 + x^3 + 2*x^2 + 2*x + 2), x)

Sympy [F]

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx = \int \frac{x+2}{\sqrt[3]{x^3+2}\,(x^2+x+1)}\,dx$$

[In] integrate((2+x)/(x**2+x+1)/(x**3+2)**(1/3),x)

[Out] Integral((x + 2)/((x**3 + 2)**(1/3)*(x**2 + x + 1)), x)

Maxima [F]

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx = \int \frac{x+2}{(x^3+2)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="maxima")

[Out] integrate((x + 2)/((x^3 + 2)^(1/3)*(x^2 + x + 1)), x)

Giac [F]

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx = \int \frac{x+2}{(x^3+2)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="giac")
[Out] integrate((x + 2)/((x^3 + 2)^(1/3)*(x^2 + x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx = \int \frac{x+2}{(x^3+2)^{1/3}\,(x^2+x+1)}\,dx$$

```
[In] int((x + 2)/((x^3 + 2)^(1/3)*(x + x^2 + 1)),x)
[Out] int((x + 2)/((x^3 + 2)^(1/3)*(x + x^2 + 1)), x)
```

3.62 $\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 25

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{1}{8}\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

```
[Out] 1/8*ln(320*x^4+80*x^3-12*x^2+24*x+9)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1601}

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{1}{8}\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

```
[In] Int[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\text{integral} = \frac{1}{8}\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx=\frac{1}{8}\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

```
[In] Integrate[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^
4),x]
```

```
[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$\frac{\ln\left(x^4+\frac{1}{4}x^3-\frac{3}{80}x^2+\frac{3}{40}x+\frac{9}{320}\right)}{8}$	22
default	$\frac{\ln\left(320x^4+80x^3-12x^2+24x+9\right)}{8}$	24
norman	$\frac{\ln\left(320x^4+80x^3-12x^2+24x+9\right)}{8}$	24
risch	$\frac{\ln\left(320x^4+80x^3-12x^2+24x+9\right)}{8}$	24

```
[In] int((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/8*ln(x^4+1/4*x^3-3/80*x^2+3/40*x+9/320)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx=\frac{1}{8}\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="fricas")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{\log(320x^4+80x^3-12x^2+24x+9)}{8}$$

```
[In] integrate((160*x**3+30*x**2-3*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

```
[Out] log(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{1}{8}\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="maxima")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{1}{8}\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="giac")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{\ln\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)}{8}$$

```
[In] int((30*x^2 - 3*x + 160*x^3 + 3)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9),x)
[Out] log(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9)/8
```

3.63 $\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 59

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx = -\frac{\arctan\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}+\frac{\arctan\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right)}{2\sqrt{11}}$$

```
[Out] -1/22*arctan(1/55*(7-40*x)*11^(1/2))*11^(1/2)+1/22*arctan(1/66*(800*x^3-40*
x^2+30*x+57)*11^(1/2))*11^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {2115}

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{\arctan\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}}-\frac{\arctan\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}$$

```
[In] Int[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] -1/2*ArcTan[(7 - 40*x)/(5*Sqrt[11])]/Sqrt[11] + ArcTan[(57 + 30*x - 40*x^2
+ 800*x^3)/(6*Sqrt[11])]/(2*Sqrt[11])
```

Rule 2115

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 +
 (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-C)*(2*e*(B*d - 4
*A*e) + C*(d^2 - 4*c*e)), 2]}, Simp[2*(C^2/q)*ArcTan[(C*d - B*e + 2*C*e*x)/
q], x] - Simp[2*(C^2/q)*ArcTan[C*((4*B*c*C - 3*B^2*d - 4*A*C*d + 12*A*B*e +
 4*C*(2*c*C - B*d + 2*A*e)*x + 4*C*(2*C*d - B*e)*x^2 + 8*C^2*e*x^3)/(q*(B^2
 - 4*A*C)))], x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2*d + 2*C*
```

```
(b*C + A*d) - 2*B*(c*C + 2*A*e), 0] && EqQ[2*B^2*c*C - 8*a*C^3 - B^3*d - 4*
A*B*C*d + 4*A*(B^2 + 2*A*C)*e, 0] && NegQ[C*(2*e*(B*d - 4*A*e) + C*(d^2 - 4
*c*e))]
```

Rubi steps

$$\text{integral} = -\frac{\arctan\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}} + \frac{\arctan\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right)}{2\sqrt{11}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{1}{8}\text{RootSum}\left[9+24\#1-12\#1^2+80\#1^3 + 320\#1^4\&, \frac{3\log(x-\#1)+12\log(x-\#1)\#1+20\log(x-\#1)\#1^2}{3-3\#1+30\#1^2+160\#1^3}\&\right]$$

```
[In] Integrate[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (3*Log[x - #1] + 12*Lo
g[x - #1]*#1 + 20*Log[x - #1]*#1^2)/(3 - 3*#1 + 30*#1^2 + 160*#1^3) & ]/8
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{\sqrt{11}\,\arctan\left(-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}+\frac{400\sqrt{11}\,x^3}{33}\right)}{22}+\frac{\sqrt{11}\,\arctan\left(\frac{(40x-7)\sqrt{11}}{55}\right)}{22}$	52
default	$\frac{i\sqrt{11}\,\ln\left(80x^2+\left(10i\sqrt{11}+10\right)x+3i\sqrt{11}-9\right)}{44}-\frac{i\sqrt{11}\,\ln\left(80x^2+\left(-10i\sqrt{11}+10\right)x-3i\sqrt{11}-9\right)}{44}$	62

```
[In] int((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x,method=_RETURNVERBOSE)
```

```
[Out] 1/22*11^(1/2)*arctan(-20/33*11^(1/2)*x^2+5/11*11^(1/2)*x+19/22*11^(1/2)+400
/33*11^(1/2)*x^3)+1/22*11^(1/2)*arctan(1/55*(40*x-7)*11^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{1}{22}\sqrt{11}\arctan\left(\frac{1}{66}\sqrt{11}(800\,x^3-40\,x^2+30\,x+57)\right) + \frac{1}{22}\sqrt{11}\arctan\left(\frac{1}{55}\sqrt{11}(40\,x-7)\right)$$

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="fric
as")
[Out] 1/22*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) + 1/22*s
qrt(11)*arctan(1/55*sqrt(11)*(40*x - 7))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{\sqrt{11}\cdot\left(2\operatorname{atan}\left(\frac{8\sqrt{11}x}{11}-\frac{7\sqrt{11}}{55}\right)+2\operatorname{atan}\left(\frac{400\sqrt{11}x^3}{33}-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}\right)\right)}{44}$$

```
[In] integrate((20*x**2+12*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
[Out] sqrt(11)*(2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) + 2*atan(400*sqrt(11)*x**
3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22))/44
```

Maxima [F]

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx = \int \frac{20\,x^2+12\,x+3}{320\,x^4+80\,x^3-12\,x^2+24\,x+9}\,dx$$

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="maxi
ma")
[Out] integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx$$
$$=\frac{1}{22}\sqrt{11}\left(\arctan\left(\frac{1}{66}\sqrt{11}(800\,x^3-40\,x^2+30\,x+57)\right)-\arctan\left(-\frac{1}{55}\sqrt{11}(40\,x-7)\right)\right)$$

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="giac
")
```

```
[Out] 1/22*sqrt(11)*(arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - arcta
n(-1/55*sqrt(11)*(40*x - 7)))
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx=\frac{\sqrt{11}\,\mathrm{atan}\left(\frac{8\sqrt{11}\,x}{11}-\frac{7\sqrt{11}}{55}\right)}{22}+\frac{\sqrt{11}\,\mathrm{atan}\left(\frac{400\sqrt{11}\,x^3}{33}-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}\right)}{22}$$

```
[In] int((12*x + 20*x^2 + 3)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9),x)
```

```
[Out] (11^(1/2)*atan((8*11^(1/2)*x)/11 - (7*11^(1/2))/55))/22 + (11^(1/2)*atan((5
*11^(1/2)*x)/11 + (19*11^(1/2))/22 - (20*11^(1/2)*x^2)/33 + (400*11^(1/2)*x
^3)/33))/22
```

3.64 $\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4} dx$

Optimal result

Integrand size = 38, antiderivative size = 78

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4} dx = 2\sqrt{11}\arctan\left(\frac{7-40x}{5\sqrt{11}}\right) - 2\sqrt{11}\arctan\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right) + 2\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

```
[Out] 2*ln(320*x^4+80*x^3-12*x^2+24*x+9)+2*arctan(1/55*(7-40*x)*11^(1/2))*11^(1/2
)-2*arctan(1/66*(800*x^3-40*x^2+30*x+57)*11^(1/2))*11^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2125, 2115}

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4} dx = -2\sqrt{11}\arctan\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right) + 2\sqrt{11}\arctan\left(\frac{7-40x}{5\sqrt{11}}\right) + 2\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

```
[In] Int[(-84 - 576*x - 400*x^2 + 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^
4),x]
```

```
[Out] 2*Sqrt[11]*ArcTan[(7 - 40*x)/(5*Sqrt[11])] - 2*Sqrt[11]*ArcTan[(57 + 30*x -
 40*x^2 + 800*x^3)/(6*Sqrt[11])] + 2*Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x
^4]
```

Rule 2115

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 +
 (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-C)*(2*e*(B*d - 4
*A*e) + C*(d^2 - 4*c*e)), 2]}, Simp[2*(C^2/q)*ArcTan[(C*d - B*e + 2*C*e*x)/
q], x] - Simp[2*(C^2/q)*ArcTan[C*((4*B*c*C - 3*B^2*d - 4*A*C*d + 12*A*B*e +
 4*C*(2*c*C - B*d + 2*A*e)*x + 4*C*(2*C*d - B*e)*x^2 + 8*C^2*e*x^3)/(q*(B^2
 - 4*A*C)))], x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2*d + 2*C*
(b*C + A*d) - 2*B*(c*C + 2*A*e), 0] && EqQ[2*B^2*c*C - 8*a*C^3 - B^3*d - 4*
A*B*C*d + 4*A*(B^2 + 2*A*C)*e, 0] && NegQ[C*(2*e*(B*d - 4*A*e) + C*(d^2 - 4
*c*e))]
```

Rule 2125

```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Dist[1/(n*Coeff[Qn,
x, n]), Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x]
/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\log\left(9+24x-12x^2+80x^3+320x^4\right)+\frac{\int\frac{-168960-675840x-1126400x^2}{9+24x-12x^2+80x^3+320x^4}\,dx}{1280}\\ &= 2\sqrt{11}\arctan\left(\frac{7-40x}{5\sqrt{11}}\right)-2\sqrt{11}\arctan\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right)\\ &\quad+2\log\left(9+24x-12x^2+80x^3+320x^4\right)\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

$$\int\frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx=\frac{1}{2}\text{RootSum}\Big[9+24\#1-12\#1^2+80\#1^3+320\#1^4\&,\frac{-21\log(x-\#1)-144\log(x-\#1)\#1-100\log(x-\#1)\#1^2+640\log(x-\#1)\#1^3}{3-3\#1+30\#1^2+160\#1^3}\&\Big]$$

```
[In] Integrate[(-84 - 576*x - 400*x^2 + 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 +
320*x^4),x]
```

```
[Out] RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (-21*Log[x - #1] - 144
*Log[x - #1]*#1 - 100*Log[x - #1]*#1^2 + 640*Log[x - #1]*#1^3)/(3 - 3*#1 +
30*#1^2 + 160*#1^3) & ]/2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

method	result
default	$4\left(\frac{i\sqrt{11}}{4}+\frac{1}{2}\right)\ln\left(80x^{2}+\left(-10i\sqrt{11}+10\right)x-3i\sqrt{11}-9\right)+4\left(\frac{1}{2}-\frac{i\sqrt{11}}{4}\right)\ln\left(80x^{2}+\left(10i\sqrt{11}+\right.\right.$
risch	$2\ln\left(6400x^{4}+1600x^{3}-240x^{2}+480x+180\right)-2\sqrt{11}\,\arctan\left(-\frac{20\sqrt{11}\,x^{2}}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}+\right.$

```
[In] int((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x,method=_RE
TURNVERBOSE)
```

```
[Out] 4*(1/4*I*11^(1/2)+1/2)*ln(80*x^2+(-10*I*11^(1/2)+10)*x-3*I*11^(1/2)-9)+4*(1
/2-1/4*I*11^(1/2))*ln(80*x^2+(10*I*11^(1/2)+10)*x+3*I*11^(1/2)-9)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$$
$$=-2\sqrt{11}\arctan\left(\frac{1}{66}\sqrt{11}(800\,x^3-40\,x^2+30\,x+57)\right)$$
$$-2\sqrt{11}\arctan\left(\frac{1}{55}\sqrt{11}(40\,x-7)\right)+2\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="fricas")
```

```
[Out] -2*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - 2*sqrt(1
1)*arctan(1/55*sqrt(11)*(40*x - 7)) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 24*
x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.28

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \sqrt{11}\left(-2\,\mathrm{atan}\left(\frac{8\sqrt{11}x}{11}-\frac{7\sqrt{11}}{55}\right) - 2\,\mathrm{atan}\left(\frac{400\sqrt{11}x^3}{33}-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}\right)\right) + 2\log\left(x^4+\frac{x^3}{4}-\frac{3x^2}{80}+\frac{3x}{40}+\frac{9}{320}\right)$$

```
[In] integrate((2560*x**3-400*x**2-576*x-84)/(320*x**4+80*x**3-12*x**2+24*x+9),x
)
```

```
[Out] sqrt(11)*(-2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) - 2*atan(400*sqrt(11)*x*
*3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22)) + 2*log(x*
*4 + x**3/4 - 3*x**2/80 + 3*x/40 + 9/320)
```

Maxima [F]

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \int \frac{4\,(640\,x^3-100\,x^2-144\,x-21)}{320\,x^4+80\,x^3-12\,x^2+24\,x+9}\,dx$$

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="maxima")
```

```
[Out] 4*integrate((640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 + 2
4*x + 9), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = -2\sqrt{11}\left(\arctan\left(\frac{1}{66}\sqrt{11}(800\,x^3-40\,x^2+30\,x+57)\right)-\arctan\left(-\frac{1}{55}\sqrt{11}(40\,x-7)\right)\right) + 2\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="giac")
```

```
[Out] -2*sqrt(11)*(arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - arctan(
-1/55*sqrt(11)*(40*x - 7))) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = 2\,\ln\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right) - 2\,\sqrt{11}\,\mathrm{atan}\left(\frac{8\,\sqrt{11}\,x}{11}-\frac{7\,\sqrt{11}}{55}\right) - 2\,\sqrt{11}\,\mathrm{atan}\left(\frac{400\,\sqrt{11}\,x^3}{33}-\frac{20\,\sqrt{11}\,x^2}{33}+\frac{5\,\sqrt{11}\,x}{11}+\frac{19\,\sqrt{11}}{22}\right)$$

```
[In] int(-(576*x + 400*x^2 - 2560*x^3 + 84)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 +
9),x)
```

```
[Out] 2*log(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9) - 2*11^(1/2)*atan((8*11^(1/2)*x
)/11 - (7*11^(1/2))/55) - 2*11^(1/2)*atan((5*11^(1/2)*x)/11 + (19*11^(1/2))
/22 - (20*11^(1/2)*x^2)/33 + (400*11^(1/2)*x^3)/33)
```

3.65 $\int \frac{\sqrt{1-x^4}}{1+x^4} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{\sqrt{1-x^4}}{1+x^4} \, dx = \frac{1}{2}\arctan\left(\frac{x(1+x^2)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\text{arctanh}\left(\frac{x(1-x^2)}{\sqrt{1-x^4}}\right)$$

```
[Out] 1/2*arctan(x*(x^2+1)/(-x^4+1)^(1/2))+1/2*arctanh(x*(-x^2+1)/(-x^4+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {414}

$$\int \frac{\sqrt{1-x^4}}{1+x^4} \, dx = \frac{1}{2}\arctan\left(\frac{x(x^2+1)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\text{arctanh}\left(\frac{x(1-x^2)}{\sqrt{1-x^4}}\right)$$

```
[In] Int[Sqrt[1 - x^4]/(1 + x^4),x]
```

```
[Out] ArcTan[(x*(1 + x^2))/Sqrt[1 - x^4]]/2 + ArcTanh[(x*(1 - x^2))/Sqrt[1 - x^4]
]/2
```

Rule 414

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[(-a)*b, 4]}, Simp[(a/(2*c*q))*ArcTan[q*x*((a + q^2*x^2)/(a*Sqrt[a + b*x
^4]))], x] + Simp[(a/(2*c*q))*ArcTanh[q*x*((a - q^2*x^2)/(a*Sqrt[a + b*x^4]
))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\arctan\left(\frac{x(1+x^2)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\text{arctanh}\left(\frac{x(1-x^2)}{\sqrt{1-x^4}}\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{1-x^4}}{1+x^4}\,dx = \left(\frac{1}{4}-\frac{i}{4}\right)\arctan\left(\frac{(1+i)x}{\sqrt{1-x^4}}\right)-\left(\frac{1}{4}+\frac{i}{4}\right)\arctan\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{1-x^4}}{x}\right)$$

```
[In] Integrate[Sqrt[1 - x^4]/(1 + x^4),x]
```

```
[Out] (1/4 - I/4)*ArcTan[((1 + I)*x)/Sqrt[1 - x^4]] - (1/4 + I/4)*ArcTan[((1/2 + I/2)*Sqrt[1 - x^4])/x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\left(\frac{1}{4}-\frac{i}{4}\right)\left(\ln\left(\frac{(1+i)\sqrt{-x^4+1}+2ix}{x^2+i}\right)-\arctan\left(\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{-x^4+1}}{x}\right)+\ln(2)\right)$
default	$-\frac{\ln\left(\frac{1+\frac{-x^4+1}{2x^2}-\frac{\sqrt{-x^4+1}}{x}}{1+\frac{-x^4+1}{2x^2}+\frac{\sqrt{-x^4+1}}{x}}\right)}{8}-\frac{\arctan\left(1+\frac{\sqrt{-x^4+1}}{x}\right)}{4}-\frac{\arctan\left(-1+\frac{\sqrt{-x^4+1}}{x}\right)}{4}$
elliptic	$-\frac{\ln\left(\frac{1+\frac{-x^4+1}{2x^2}-\frac{\sqrt{-x^4+1}}{x}}{1+\frac{-x^4+1}{2x^2}+\frac{\sqrt{-x^4+1}}{x}}\right)}{8}-\frac{\arctan\left(1+\frac{\sqrt{-x^4+1}}{x}\right)}{4}-\frac{\arctan\left(-1+\frac{\sqrt{-x^4+1}}{x}\right)}{4}$
trager	$\operatorname{RootOf}\left(8\_Z^2+4\_Z+1\right)\ln\left(-\frac{-4\operatorname{RootOf}\left(8\_Z^2+4\_Z+1\right)x+\sqrt{-x^4+1}-2x}{4\operatorname{RootOf}\left(8\_Z^2+4\_Z+1\right)x^2+x^2-1}\right)-\frac{\ln\left(-\frac{4\operatorname{RootOf}\left(8\_Z^2 \ldots\right.\right.}{4\operatorname{RootOf}\left(8\_Z \ldots\right.}\right.}{}$

```
[In] int((-x^4+1)^(1/2)/(x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] (1/4-1/4*I)*(ln(((1+I)*(-x^4+1)^(1/2)+2*I*x)/(x^2+I))-arctan((1/2-1/2*I)*(-x^4+1)^(1/2)/x)+ln(2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{1-x^4}}{1+x^4}\,dx = -\frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}x}{x^2-1}\right)+\frac{1}{4}\log\left(-\frac{x^4-2\,x^2-2\,\sqrt{-x^4+1}x-1}{x^4+1}\right)$$

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="fricas")
[Out] -1/2*arctan(sqrt(-x^4 + 1)*x/(x^2 - 1)) + 1/4*log(-(x^4 - 2*x^2 - 2*sqrt(-x
^4 + 1)*x - 1)/(x^4 + 1))
```

Sympy [F]

$$\int \frac{\sqrt{1-x^4}}{1+x^4}\,dx = \int \frac{\sqrt{-(x-1)(x+1)(x^2+1)}}{x^4+1}\,dx$$

```
[In] integrate((-x**4+1)**(1/2)/(x**4+1),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/(x**4 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^4}}{1+x^4}\,dx = \int \frac{\sqrt{-x^4+1}}{x^4+1}\,dx$$

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x^4}}{1+x^4}\,dx = \int \frac{\sqrt{-x^4+1}}{x^4+1}\,dx$$

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^4}}{1+x^4}\,dx = \int \frac{\sqrt{1-x^4}}{x^4+1}\,dx$$

```
[In] int((1 - x^4)^(1/2)/(x^4 + 1),x)
[Out] int((1 - x^4)^(1/2)/(x^4 + 1), x)
```

3.66 $\int \frac{\sqrt{1+x^4}}{1-x^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \frac{\sqrt{1+x^4}}{1-x^4} dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)+1/4*arctanh(x*2^(1/2)/(x^4+1)^(
1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {413, 218, 212, 209}

$$\int \frac{\sqrt{1+x^4}}{1-x^4} dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

```
[In] Int[Sqrt[1 + x^4]/(1 - x^4),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ArcTanh[(Sqrt[2]*x)/Sqrt[1
+ x^4]]/(2*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 413

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[a/c,
 Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{1-4x^4}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right) + \operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}$$

```
[In] Integrate[Sqrt[1 + x^4]/(1 - x^4),x]
[Out] (ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]] + ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]])/(2
*Sqrt[2])
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{\sqrt{2}\left(2\arctan\left(\frac{x\sqrt{2}}{\sqrt{x^4+1}}\right)-\operatorname{arctanh}\left(\frac{\left(x^2+x+1\right)\sqrt{2}}{\sqrt{x^4+1}}\right)+\operatorname{arctanh}\left(\frac{\left(x^2-x+1\right)\sqrt{2}}{\sqrt{x^4+1}}\right)\right)}{8}$	62
pseudoelliptic	$\frac{\sqrt{2}\left(2\arctan\left(\frac{x\sqrt{2}}{\sqrt{x^4+1}}\right)-\operatorname{arctanh}\left(\frac{\left(x^2+x+1\right)\sqrt{2}}{\sqrt{x^4+1}}\right)+\operatorname{arctanh}\left(\frac{\left(x^2-x+1\right)\sqrt{2}}{\sqrt{x^4+1}}\right)\right)}{8}$	62
elliptic	$\frac{\left(-\frac{\ln\left(-1+\frac{\sqrt{2}\sqrt{x^4+1}}{2x}\right)}{4}+\frac{\ln\left(1+\frac{\sqrt{2}\sqrt{x^4+1}}{2x}\right)}{4}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{x^4+1}}{2x}\right)}{2}\right)\sqrt{2}}{2}$	65
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+2\right)x+\sqrt{x^4+1}}{x^2+1}\right)}{4}+\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x+\sqrt{x^4+1}}{(-1+x)(1+x)}\right)}{4}$	72

```
[In] int((x^4+1)^(1/2)/(-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*2^(1/2)*(2*arctan(x*2^(1/2)/(x^4+1)^(1/2))-arctanh((x^2+x+1)*2^(1/2)/(x
^4+1)^(1/2))+arctanh((x^2-x+1)*2^(1/2)/(x^4+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx = \frac{1}{4}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)+\frac{1}{8}\,\sqrt{2}\log\left(\frac{x^4+2\,\sqrt{2}\sqrt{x^4+1}x+2\,x^2+1}{x^4-2\,x^2+1}\right)$$

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) + 1/8*sqrt(2)*log((x^4 + 2*sqrt
(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1))
```

Sympy [F]

$$\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx = -\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

```
[In] integrate((x**4+1)**(1/2)/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(x**4 + 1)/(x**4 - 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx = \int -\frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="maxima")
[Out] -integrate(sqrt(x^4 + 1)/(x^4 - 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx = \int -\frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="giac")
[Out] integrate(-sqrt(x^4 + 1)/(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx = -\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

```
[In] int(-(x^4 + 1)^(1/2)/(x^4 - 1),x)
[Out] -int((x^4 + 1)^(1/2)/(x^4 - 1), x)
```

3.67 $\int \frac{\sqrt{1+px^2+x^4}}{1-x^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4} dx = \frac{1}{4}\sqrt{2-p}\arctan\left(\frac{\sqrt{2-p}x}{\sqrt{1+px^2+x^4}}\right) + \frac{1}{4}\sqrt{2+p}\text{arctanh}\left(\frac{\sqrt{2+p}x}{\sqrt{1+px^2+x^4}}\right)$$

```
[Out] 1/4*arctan(x*(2-p)^(1/2)/(x^4+p*x^2+1)^(1/2))*(2-p)^(1/2)+1/4*arctanh(x*(2+
p)^(1/2)/(x^4+p*x^2+1)^(1/2))*(2+p)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2096, 1107, 211, 214}

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4} dx = \frac{1}{4}\sqrt{2-p}\arctan\left(\frac{\sqrt{2-p}x}{\sqrt{px^2+x^4+1}}\right) + \frac{1}{4}\sqrt{p+2}\text{arctanh}\left(\frac{\sqrt{p+2}x}{\sqrt{px^2+x^4+1}}\right)$$

```
[In] Int[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]
```

```
[Out] (Sqrt[2 - p]*ArcTan[(Sqrt[2 - p]*x)/Sqrt[1 + p*x^2 + x^4]])/4 + (Sqrt[2 + p
]*ArcTanh[(Sqrt[2 + p]*x)/Sqrt[1 + p*x^2 + x^4]])/4
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 2096

```
Int[Sqrt[v_]/((d_) + (e_.)*(x_)^4), x_Symbol] :> With[{a = Coeff[v, x, 0],
b = Coeff[v, x, 2], c = Coeff[v, x, 4]}, Dist[a/d, Subst[Int[1/(1 - 2*b*x^2
 + (b^2 - 4*a*c)*x^4), x], x, x/Sqrt[v]], x] /; EqQ[c*d + a*e, 0] && PosQ[a
*c]] /; FreeQ[{d, e}, x] && PolyQ[v, x^2, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{1-2px^2+(-4+p^2)\,x^4}\,dx, x, \frac{x}{\sqrt{1+px^2+x^4}}\right)\\
&= \frac{1}{4}(-4+p^2)\,\text{Subst}\left(\int \frac{1}{-2-p+(-4+p^2)\,x^2}\,dx, x, \frac{x}{\sqrt{1+px^2+x^4}}\right)\\
&\quad -\frac{1}{4}(-4+p^2)\,\text{Subst}\left(\int \frac{1}{2-p+(-4+p^2)\,x^2}\,dx, x, \frac{x}{\sqrt{1+px^2+x^4}}\right)\\
&= \frac{1}{4}\sqrt{2-p}\arctan\left(\frac{\sqrt{2-p}x}{\sqrt{1+px^2+x^4}}\right)+\frac{1}{4}\sqrt{2+p}\text{arctanh}\left(\frac{\sqrt{2+p}x}{\sqrt{1+px^2+x^4}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\begin{aligned}
\int \frac{\sqrt{1+px^2+x^4}}{1-x^4}\,dx = &-\frac{1}{4}\sqrt{-2-p}\arctan\left(\frac{\sqrt{-2-p}x}{\sqrt{1+px^2+x^4}}\right)\\
&+\frac{1}{4}\sqrt{2-p}\arctan\left(\frac{\sqrt{2-p}x}{\sqrt{1+px^2+x^4}}\right)
\end{aligned}$$

```
[In] Integrate[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]
```

```
[Out] -1/4*(Sqrt[-2 - p]*ArcTan[(Sqrt[-2 - p]*x)/Sqrt[1 + p*x^2 + x^4]]) + (Sqrt[
2 - p]*ArcTan[(Sqrt[2 - p]*x)/Sqrt[1 + p*x^2 + x^4]])/4
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{\left(\frac{4\left(\frac{1}{4}+\frac{p}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{x^4+p\,x^2+1}\,\sqrt{2}}{x\sqrt{4+2p}}\right)}{\sqrt{4+2p}}+\frac{4\left(\frac{1}{4}-\frac{p}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{x^4+p\,x^2+1}\,\sqrt{2}}{x\sqrt{2p-4}}\right)}{\sqrt{2p-4}}\right)\sqrt{2}}{2}$
pseudoelliptic	$\frac{\left(\ln\left(\frac{\sqrt{2+p}\,\sqrt{x^4+p\,x^2+1}-2-2x^2+x(p-2)}{(1+x)^2}\right)+\ln\left(\frac{\sqrt{2+p}\,\sqrt{x^4+p\,x^2+1}+2+2x^2+x(p-2)}{(-1+x)^2}\right)+2\ln(2)\right)\sqrt{2+p}}{8}-\frac{\left(\ln\left(\frac{\sqrt{p-2}\,\sqrt{x^4+p\,x^2}}{x^2+1}\right.\right.}{}$
default	$-\frac{\left(-\ln\left(\frac{\sqrt{2+p}\,\sqrt{x^4+p\,x^2+1}-2-2x^2+x(p-2)}{(1+x)^2}\right)-\ln\left(\frac{\sqrt{2+p}\,\sqrt{x^4+p\,x^2+1}+2+2x^2+x(p-2)}{(-1+x)^2}\right)-2\ln(2)\right)\sqrt{2+p}}{8}-\frac{\left(\ln\left(\sqrt{p-2}\,\sqrt{x^4}\right.\right.}{}$

```
[In] int((x^4+p*x^2+1)^(1/2)/(-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(4*(1/4+1/8*p)/(4+2*p)^(1/2)*arctanh((x^4+p*x^2+1)^(1/2)*2^(1/2)/x/(4+2
*p)^(1/2))+4*(1/4-1/8*p)/(2*p-4)^(1/2)*arctanh((x^4+p*x^2+1)^(1/2)*2^(1/2)/
x/(2*p-4)^(1/2)))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 359, normalized size of antiderivative = 4.79

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4}\,dx = \left[\frac{1}{8}\sqrt{p-2}\log\left(\frac{x^4+2\,(p-1)x^2-2\sqrt{x^4+px^2+1}\sqrt{p-2}x+1}{x^4+2\,x^2+1}\right)\right.$$
$$+\frac{1}{8}\sqrt{p+2}\log\left(\frac{x^4+2\,(p+1)x^2+2\sqrt{x^4+px^2+1}\sqrt{p+2}x+1}{x^4-2\,x^2+1}\right),\frac{1}{4}\sqrt{-p+2}\arctan\left(\frac{\sqrt{-p+2}x}{\sqrt{x^4+px^2+1}}\right.$$
$$+\frac{1}{8}\sqrt{p+2}\log\left(\frac{x^4+2\,(p+1)x^2+2\sqrt{x^4+px^2+1}\sqrt{p+2}x+1}{x^4-2\,x^2+1}\right),$$
$$-\frac{1}{4}\sqrt{-p-2}\arctan\left(\frac{\sqrt{x^4+px^2+1}\sqrt{-p-2}}{(p+2)x}\right)$$
$$+\frac{1}{8}\sqrt{p-2}\log\left(\frac{x^4+2\,(p-1)x^2-2\sqrt{x^4+px^2+1}\sqrt{p-2}x+1}{x^4+2\,x^2+1}\right),\frac{1}{4}\sqrt{-p+2}\arctan\left(\frac{\sqrt{-p+2}x}{\sqrt{x^4+px^2+1}}\right.$$
$$\left.-\frac{1}{4}\sqrt{-p-2}\arctan\left(\frac{\sqrt{x^4+px^2+1}\sqrt{-p-2}}{(p+2)x}\right)\right]$$

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p
- 2)*x + 1)/(x^4 + 2*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x^4 + 2*(p + 1)*x^2 +
 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(x^4 - 2*x^2 + 1)), 1/4*sqrt(-p
```

```
 + 2)*arctan(sqrt(-p + 2)*x/sqrt(x^4 + p*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x
^4 + 2*(p + 1)*x^2 + 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(x^4 - 2*x^
2 + 1)), -1/4*sqrt(-p - 2)*arctan(sqrt(x^4 + p*x^2 + 1)*sqrt(-p - 2)/((p +
2)*x)) + 1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 + 1)
*sqrt(p - 2)*x + 1)/(x^4 + 2*x^2 + 1)), 1/4*sqrt(-p + 2)*arctan(sqrt(-p + 2
)*x/sqrt(x^4 + p*x^2 + 1)) - 1/4*sqrt(-p - 2)*arctan(sqrt(x^4 + p*x^2 + 1)*
sqrt(-p - 2)/((p + 2)*x))]
```

Sympy [F]

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4}\,dx = -\int \frac{\sqrt{px^2+x^4+1}}{x^4-1}\,dx$$

```
[In] integrate((x**4+p*x**2+1)**(1/2)/(-x**4+1),x)
[Out] -Integral(sqrt(p*x**2 + x**4 + 1)/(x**4 - 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4}\,dx = \int -\frac{\sqrt{x^4+px^2+1}}{x^4-1}\,dx$$

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="maxima")
[Out] -integrate(sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4}\,dx = \int -\frac{\sqrt{x^4+px^2+1}}{x^4-1}\,dx$$

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="giac")
[Out] integrate(-sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4}\,dx=-\int \frac{\sqrt{x^4+p\,x^2+1}}{x^4-1}\,dx$$

```
[In] int(-(p*x^2 + x^4 + 1)^(1/2)/(x^4 - 1),x)
[Out] -int((p*x^2 + x^4 + 1)^(1/2)/(x^4 - 1), x)
```

3.68 $\int \frac{\sqrt{1+px^2-x^4}}{1+x^4} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 171

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4} \, dx = -\frac{\sqrt{p+\sqrt{4+p^2}} \arctan\left(\frac{\sqrt{p+\sqrt{4+p^2}}x\left(p-\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\sqrt{1+px^2-x^4}}\right)}{2\sqrt{2}} + \frac{\sqrt{-p+\sqrt{4+p^2}}\text{arctanh}\left(\frac{\sqrt{-p+\sqrt{4+p^2}}x\left(p+\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\sqrt{1+px^2-x^4}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctanh(1/4*x*(p-2*x^2+(p^2+4)^(1/2))*(-p+(p^2+4)^(1/2))^(1/2)*2^(1/2)/
(-x^4+p*x^2+1)^(1/2))*(-p+(p^2+4)^(1/2))^(1/2)*2^(1/2)-1/4*arctan(1/4*x*(p-
2*x^2-(p^2+4)^(1/2))*(p+(p^2+4)^(1/2))^(1/2)*2^(1/2)/(-x^4+p*x^2+1)^(1/2))*
(p+(p^2+4)^(1/2))^(1/2)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2097}

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4} \, dx = \frac{\sqrt{\sqrt{p^2+4}-p}\text{arctanh}\left(\frac{\sqrt{\sqrt{p^2+4}-p}x\left(\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}} - \frac{\sqrt{\sqrt{p^2+4}+p}\arctan\left(\frac{\sqrt{\sqrt{p^2+4}+p}x\left(-\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}$$

```
[In] Int[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]
```

```
[Out] -1/2*(Sqrt[p + Sqrt[4 + p^2]]*ArcTan[(Sqrt[p + Sqrt[4 + p^2]]*x*(p - Sqrt[4
 + p^2] - 2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/Sqrt[2] + (Sqrt[-p +
Sqrt[4 + p^2]]*ArcTanh[(Sqrt[-p + Sqrt[4 + p^2]]*x*(p + Sqrt[4 + p^2] - 2*x
^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sqrt[2])
```

Rule 2097

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^4), x_Symbo
l] :> With[{q = Sqrt[b^2 - 4*a*c]}, Simp[(-a)*(Sqrt[b + q]/(2*Sqrt[2]*Rt[(-
a)*c, 2]*d))*ArcTan[Sqrt[b + q]*x*((b - q + 2*c*x^2)/(2*Sqrt[2]*Rt[(-a)*c,
2]*Sqrt[a + b*x^2 + c*x^4]))], x] + Simp[a*(Sqrt[-b + q]/(2*Sqrt[2]*Rt[(-a)
*c, 2]*d))*ArcTanh[Sqrt[-b + q]*x*((b + q + 2*c*x^2)/(2*Sqrt[2]*Rt[(-a)*c,
2]*Sqrt[a + b*x^2 + c*x^4]))], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d
+ a*e, 0] && NegQ[a*c]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{p+\sqrt{4+p^2}}\arctan\left(\frac{\sqrt{p+\sqrt{4+p^2}}x\left(p-\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\sqrt{1+px^2-x^4}}\right)}{2\sqrt{2}} + \frac{\sqrt{-p+\sqrt{4+p^2}}\text{arctanh}\left(\frac{\sqrt{-p+\sqrt{4+p^2}}x\left(p+\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\sqrt{1+px^2-x^4}}\right)}{2\sqrt{2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx = \frac{1}{4}i\left(\sqrt{-2i-p}\arctan\left(\frac{\sqrt{-2i-p}x}{\sqrt{1+px^2-x^4}}\right) - \sqrt{2i-p}\arctan\left(\frac{\sqrt{2i-p}x}{\sqrt{1+px^2-x^4}}\right)\right)$$

```
[In] Integrate[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]
```

```
[Out] (I/4)*(Sqrt[-2*I - p]*ArcTan[(Sqrt[-2*I - p]*x)/Sqrt[1 + p*x^2 - x^4]] - Sq
rt[2*I - p]*ArcTan[(Sqrt[2*I - p]*x)/Sqrt[1 + p*x^2 - x^4]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{i\sqrt{2i+p}\,\sqrt{2i-p}\,\ln(2)+i\sqrt{2i+p}\,\sqrt{2i-p}\,\ln\left(\frac{\sqrt{2i+p}\,\sqrt{-x^4+p\,x^2+1}+2x\left(i+\frac{p}{2}\right)}{x^2+i}\right)+ip\arctan\left(\frac{\sqrt{-x^4+p\,x^2+1}}{x\sqrt{2i-p}}\right)+2\arctan\left(\frac{\sqrt{-x\dots}}{x\dots}\right.\cdots}{4\sqrt{2i-p}}$
default	$\frac{\left(-\frac{\sqrt{p+\sqrt{p^2+4}}\,\sqrt{p^2+4}\,\ln\left(\frac{\sqrt{-x^4+p\,x^2+1}\,\sqrt{2}\,\sqrt{p+\sqrt{p^2+4}}}{x}-\frac{-x^4+p\,x^2+1}{x^2}-\sqrt{p^2+4}\right)}{16}+\frac{\sqrt{p^2+4}\,\left(p+\sqrt{p^2+4}\right)\arctan\left(\frac{2\sqrt{p+\sqrt{p^2+4}}-\dots}{2\sqrt{-p\dots}}\right.}{8\sqrt{-p+\sqrt{p^2+4}}}\cdots\right.}{\cdots}$
elliptic	$\frac{\left(-\frac{\sqrt{p+\sqrt{p^2+4}}\,\sqrt{p^2+4}\,\ln\left(\frac{\sqrt{-x^4+p\,x^2+1}\,\sqrt{2}\,\sqrt{p+\sqrt{p^2+4}}}{x}-\frac{-x^4+p\,x^2+1}{x^2}-\sqrt{p^2+4}\right)}{16}+\frac{\sqrt{p^2+4}\,\left(p+\sqrt{p^2+4}\right)\arctan\left(\frac{2\sqrt{p+\sqrt{p^2+4}}-\dots}{2\sqrt{-p\dots}}\right.}{8\sqrt{-p+\sqrt{p^2+4}}}\cdots\right.}{\cdots}$

```
[In] int((-x^4+p*x^2+1)^(1/2)/(x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(I*(2*I+p)^(1/2)*(2*I-p)^(1/2)*ln(2)+I*(2*I+p)^(1/2)*(2*I-p)^(1/2)*ln(
((2*I+p)^(1/2)*(-x^4+p*x^2+1)^(1/2)+2*x*(I+1/2*p))/(x^2+I))+I*p*arctan((-x^
4+p*x^2+1)^(1/2)/x/(2*I-p)^(1/2))+2*arctan((-x^4+p*x^2+1)^(1/2)/x/(2*I-p)^(
1/2)))/(2*I-p)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.34

$$\begin{aligned}
\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx
&= \frac{1}{8}\,\sqrt{-p+2i}\log\left(-\frac{\sqrt{-x^4+px^2+1}(x^2+i)-(i\,x^3-x)\sqrt{-p+2i}}{x^4+1}\right)\\
&\quad-\frac{1}{8}\,\sqrt{-p+2i}\log\left(-\frac{\sqrt{-x^4+px^2+1}(x^2+i)-(-i\,x^3+x)\sqrt{-p+2i}}{x^4+1}\right)\\
&\quad-\frac{1}{8}\,\sqrt{-p-2i}\log\left(-\frac{\sqrt{-x^4+px^2+1}(x^2-i)-(i\,x^3+x)\sqrt{-p-2i}}{x^4+1}\right)\\
&\quad+\frac{1}{8}\,\sqrt{-p-2i}\log\left(-\frac{\sqrt{-x^4+px^2+1}(x^2-i)-(-i\,x^3-x)\sqrt{-p-2i}}{x^4+1}\right)
\end{aligned}$$

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(-p + 2*I)*log(-(sqrt(-x^4 + p*x^2 + 1)*(x^2 + I) - (I*x^3 - x)*sqr
t(-p + 2*I))/(x^4 + 1)) - 1/8*sqrt(-p + 2*I)*log(-(sqrt(-x^4 + p*x^2 + 1)*(
x^2 + I) - (-I*x^3 + x)*sqrt(-p + 2*I))/(x^4 + 1)) - 1/8*sqrt(-p - 2*I)*log
(-(sqrt(-x^4 + p*x^2 + 1)*(x^2 - I) - (I*x^3 + x)*sqrt(-p - 2*I))/(x^4 + 1)
) + 1/8*sqrt(-p - 2*I)*log(-(sqrt(-x^4 + p*x^2 + 1)*(x^2 - I) - (-I*x^3 - x
)*sqrt(-p - 2*I))/(x^4 + 1))
```

Sympy [F]

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx=\int \frac{\sqrt{px^2-x^4+1}}{x^4+1}\,dx$$

```
[In] integrate((-x**4+p*x**2+1)**(1/2)/(x**4+1),x)
```

```
[Out] Integral(sqrt(p*x**2 - x**4 + 1)/(x**4 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx=\int \frac{\sqrt{-x^4+px^2+1}}{x^4+1}\,dx$$

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx=\int \frac{\sqrt{-x^4+px^2+1}}{x^4+1}\,dx$$

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx = \int \frac{\sqrt{-x^4+p\,x^2+1}}{x^4+1}\,dx$$

```
[In] int((p*x^2 - x^4 + 1)^(1/2)/(x^4 + 1),x)
[Out] int((p*x^2 - x^4 + 1)^(1/2)/(x^4 + 1), x)
```

3.69 $\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \frac{a\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} - b\arctan\left(\sqrt[4]{-1+x^2}\right) + \frac{a\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + b\text{arctanh}\left(\sqrt[4]{-1+x^2}\right)$$

```
[Out] -b*arctan((x^2-1)^(1/4))+b*arctanh((x^2-1)^(1/4))+1/4*a*arctan(1/2*x/(x^2-1
)^(1/4)*2^(1/2))*2^(1/2)+1/4*a*arctanh(1/2*x/(x^2-1)^(1/4)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1024, 407, 455, 65, 304, 209, 212}

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \frac{a\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}} + \frac{a\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}} - b\arctan\left(\sqrt[4]{x^2-1}\right) + b\text{arctanh}\left(\sqrt[4]{x^2-1}\right)$$

```
[In] Int[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTan[(-1 + x^2)^
(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTanh[
(-1 + x^2)^(1/4)]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
```

```
, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= a\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx + b\int \frac{x}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx \\
&= \frac{a\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} \\
&\quad + \frac{1}{2}b\text{Subst}\left(\int \frac{1}{(2-x)\sqrt[4]{-1+x}}\,dx, x, x^2\right) \\
&= \frac{a\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} \\
&\quad + (2b)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt[4]{-1+x^2}\right) \\
&= \frac{a\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} \\
&\quad + b\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+x^2}\right) - b\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{-1+x^2}\right) \\
&= \frac{a\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} - b\arctan\left(\sqrt[4]{-1+x^2}\right) \\
&\quad + \frac{a\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + b\text{arctanh}\left(\sqrt[4]{-1+x^2}\right)
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.96

$$
\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx
$$

$$
= \frac{x\left(bx\sqrt[4]{1-x^2}(-2+x^2)\,\text{AppellF1}\left(1,\frac{1}{4},1,2,x^2,\frac{x^2}{2}\right) - \frac{24a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},x^2,\frac{x^2}{2}\right)}{6\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},x^2,\frac{x^2}{2}\right)+x^2\left(2\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},2,\frac{5}{2},x^2,\frac{x^2}{2}\right)+\text{Appe}\right.}\right.}{4(-2+x^2)\sqrt[4]{-1+x^2}}
$$

```
[In] Integrate[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (x*(b*x*(1 - x^2)^(1/4)*(-2 + x^2)*AppellF1[1, 1/4, 1, 2, x^2, x^2/2] - (24
*a*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2])/(6*AppellF1[1/2, 1/4, 1, 3/2, x^
2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/2, 5
/4, 1, 5/2, x^2, x^2/2]))))/(4*(-2 + x^2)*(-1 + x^2)^(1/4))
```

Maple [F]

$$\int \frac{bx+a}{\left(-x^{2}+2\right)\left(x^{2}-1\right)^{\frac{1}{4}}}dx$$

```
[In] int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
[Out] int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = -\int \frac{a}{x^2\sqrt[4]{x^2-1}-2\sqrt[4]{x^2-1}}\,dx - \int \frac{bx}{x^2\sqrt[4]{x^2-1}-2\sqrt[4]{x^2-1}}\,dx$$

```
[In] integrate((b*x+a)/(-x**2+2)/(x**2-1)**(1/4),x)
[Out] -Integral(a/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x) - Integral(b
*x/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x)
```

Maxima [F]

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \int -\frac{bx+a}{\left(x^{2}-1\right)^{\frac{1}{4}}\left(x^{2}-2\right)}\,dx$$

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="maxima")
[Out] -integrate((b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

Giac [F]

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx=\int -\frac{bx+a}{(x^2-1)^{\frac{1}{4}}(x^2-2)}\,dx$$

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="giac")
[Out] integrate(-(b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx=\int -\frac{a+b\,x}{(x^2-1)^{1/4}\,(x^2-2)}\,dx$$

```
[In] int(-(a + b*x)/((x^2 - 1)^(1/4)*(x^2 - 2)),x)
[Out] int(-(a + b*x)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

3.70 $\int \frac{a+bx}{\sqrt[4]{-1-x^2}(2+x^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx = \frac{a\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + b\arctan\left(\sqrt[4]{-1-x^2}\right) + \frac{a\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - b\text{arctanh}\left(\sqrt[4]{-1-x^2}\right)$$

```
[Out] b*arctan((-x^2-1)^(1/4))-b*arctanh((-x^2-1)^(1/4))+1/4*a*arctan(1/2*x/(-x^2
-1)^(1/4)*2^(1/2))*2^(1/2)+1/4*a*arctanh(1/2*x/(-x^2-1)^(1/4)*2^(1/2))*2^(1
/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1024, 407, 455, 65, 304, 209, 212}

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx = \frac{a\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}} + \frac{a\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}} + b\arctan\left(\sqrt[4]{-x^2-1}\right) - b\text{arctanh}\left(\sqrt[4]{-x^2-1}\right)$$

```
[In] Int[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)),x]
```

[Out] (a*ArcTan[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTan[(-1 - x^2)^(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTanh[(-1 - x^2)^(1/4)]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a\int \frac{1}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx + b\int \frac{x}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx \\
&= \frac{a\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} \\
&\quad + \frac{1}{2}b\text{Subst}\left(\int \frac{1}{\sqrt[4]{-1-x}(2+x)}\,dx, x, x^2\right) \\
&= \frac{a\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} \\
&\quad - (2b)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt[4]{-1-x^2}\right) \\
&= \frac{a\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} \\
&\quad - b\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1-x^2}\right) + b\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{-1-x^2}\right) \\
&= \frac{a\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + b\arctan\left(\sqrt[4]{-1-x^2}\right) \\
&\quad + \frac{a\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - b\text{arctanh}\left(\sqrt[4]{-1-x^2}\right)
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.84

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx$$

$$= \frac{x\left(bx\sqrt[4]{1+x^2}\,\text{AppellF1}\left(1,\frac{1}{4},1,2,-x^2,-\frac{x^2}{2}\right) - \frac{24a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-x^2,-\frac{x^2}{2}\right)}{(2+x^2)\left(-6\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-x^2,-\frac{x^2}{2}\right)+x^2\left(2\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-x^2,-\right.\right.\right.}\right.}{4\sqrt[4]{-1-x^2}}$$

```
[In] Integrate[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (x*(b*x*(1 + x^2)^(1/4)*AppellF1[1, 1/4, 1, 2, -x^2, -1/2*x^2] - (24*a*Appe
llF1[1/2, 1/4, 1, 3/2, -x^2, -1/2*x^2])/((2 + x^2)*(-6*AppellF1[1/2, 1/4, 1
, 3/2, -x^2, -1/2*x^2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -1/2*x^2]
+ AppellF1[3/2, 5/4, 1, 5/2, -x^2, -1/2*x^2])))))/(4*(-1 - x^2)^(1/4))
```

Maple [F]

$$\int \frac{bx+a}{\left(-x^{2}-1\right)^{\frac{1}{4}} \left(x^{2}+2\right)}dx$$

```
[In] int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
```

```
[Out] int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx = \int \frac{a+bx}{\sqrt[4]{-x^2-1}\,(x^2+2)}\,dx$$

```
[In] integrate((b*x+a)/(-x**2-1)**(1/4)/(x**2+2),x)
```

```
[Out] Integral((a + b*x)/((-x**2 - 1)**(1/4)*(x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx = \int \frac{bx+a}{(x^2+2)(-x^2-1)^{\frac{1}{4}}}\,dx$$

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)
```

Giac [F]

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx = \int \frac{bx+a}{(x^2+2)(-x^2-1)^{\frac{1}{4}}}\,dx$$

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="giac")
[Out] integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx = \int \frac{a+b\,x}{(-x^2-1)^{1/4}\,(x^2+2)}\,dx$$

```
[In] int((a + b*x)/((- x^2 - 1)^(1/4)*(x^2 + 2)),x)
[Out] int((a + b*x)/((- x^2 - 1)^(1/4)*(x^2 + 2)), x)
```

3.71 $\int \frac{a+bx}{\sqrt[4]{1-x^2}(2-x^2)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 149

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = \frac{b\arctan\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{1}{2}a\arctan\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right) + \frac{b\operatorname{arctanh}\left(\frac{1+\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{1}{2}a\operatorname{arctanh}\left(\frac{1+\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)$$

```
[Out] 1/2*a*arctan((1-(-x^2+1)^(1/2))/x/(-x^2+1)^(1/4))+1/2*a*arctanh((1+(-x^2+1)
^(1/2))/x/(-x^2+1)^(1/4))+1/2*b*arctan(1/2*(1-(-x^2+1)^(1/2))/(-x^2+1)^(1/4
)*2^(1/2))*2^(1/2)+1/2*b*arctanh(1/2*(1+(-x^2+1)^(1/2))/(-x^2+1)^(1/4)*2^(1
/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1024, 406, 450}

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = \frac{1}{2}a\arctan\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right) + \frac{1}{2}a\operatorname{arctanh}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right) + \frac{b\arctan\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{b\operatorname{arctanh}\left(\frac{\sqrt{1-x^2}+1}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}$$

```
[In] Int[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)),x]
```

```
[Out] (b*ArcTan[(1 - Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcT
an[(1 - Sqrt[1 - x^2])/(x*(1 - x^2)^(1/4))])/2 + (b*ArcTanh[(1 + Sqrt[1 - x
^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcTanh[(1 + Sqrt[1 - x^2])/(
x*(1 - x^2)^(1/4))])/2
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 450

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
Simp[(-(Sqrt[2]*Rt[a, 4]*d)^(-1))*ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sq
rt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x] - Simp[(1/(Sqrt[2]*Rt[a, 4]*d))*ArcT
anh[(Rt[a, 4]^2 + Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= a\int \frac{1}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx + b\int \frac{x}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx \\
&= \frac{b\arctan\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{1}{2}a\arctan\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right) \\
&\quad + \frac{b\text{arctanh}\left(\frac{1+\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{1}{2}a\text{arctanh}\left(\frac{1+\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.97

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = \frac{1}{4}bx^2\,\text{AppellF1}\left(1,\frac{1}{4},1,2,x^2,\frac{x^2}{2}\right) - \frac{6ax\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},x^2,\frac{x^2}{2}\right)}{\sqrt[4]{1-x^2}\,(-2+x^2)\left(6\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},x^2,\frac{x^2}{2}\right)+x^2\left(2\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},2,\frac{5}{2},x^2,\frac{x^2}{2}\right)+\text{AppellF1}\left(\frac{3}{2},\frac{5}{4},1\right.\right.\right.}$$

```
[In] Integrate[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)),x]
[Out] (b*x^2*AppellF1[1, 1/4, 1, 2, x^2, x^2/2])/4 - (6*a*x*AppellF1[1/2, 1/4, 1,
 3/2, x^2, x^2/2])/((1 - x^2)^(1/4)*(-2 + x^2)*(6*AppellF1[1/2, 1/4, 1, 3/2
, x^2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/
2, 5/4, 1, 5/2, x^2, x^2/2])))
```

Maple [F]

$$\int \frac{bx+a}{\left(-x^2+1\right)^{\frac{1}{4}}\left(-x^2+2\right)}dx$$

```
[In] int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
[Out] int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = -\int \frac{a}{x^2\sqrt[4]{1-x^2}-2\sqrt[4]{1-x^2}}\,dx - \int \frac{bx}{x^2\sqrt[4]{1-x^2}-2\sqrt[4]{1-x^2}}\,dx$$

```
[In] integrate((b*x+a)/(-x**2+1)**(1/4)/(-x**2+2),x)
[Out] -Integral(a/(x**2*(1 - x**2)**(1/4) - 2*(1 - x**2)**(1/4)), x) - Integral(b
*x/(x**2*(1 - x**2)**(1/4) - 2*(1 - x**2)**(1/4)), x)
```

Maxima [F]

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = \int -\frac{bx+a}{(x^2-2)(-x^2+1)^{\frac{1}{4}}}\,dx$$

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="maxima")
[Out] -integrate((b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)
```

Giac [F]

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = \int -\frac{bx+a}{(x^2-2)(-x^2+1)^{\frac{1}{4}}}\,dx$$

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="giac")
[Out] integrate(-(b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = \int -\frac{a+b\,x}{(1-x^2)^{1/4}\,(x^2-2)}\,dx$$

```
[In] int(-(a + b*x)/((1 - x^2)^(1/4)*(x^2 - 2)),x)
[Out] int(-(a + b*x)/((1 - x^2)^(1/4)*(x^2 - 2)), x)
```

3.72 $\int \frac{a+bx}{\sqrt[4]{1+x^2}(2+x^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = -\frac{b\arctan\left(\frac{1-\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right)}{\sqrt{2}} - \frac{1}{2}a\arctan\left(\frac{1+\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right) - \frac{1}{2}a\text{arctanh}\left(\frac{1-\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right) - \frac{b\text{arctanh}\left(\frac{1+\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*a*arctan((1+(x^2+1)^(1/2))/x/(x^2+1)^(1/4))-1/2*a*arctanh((1-(x^2+1)^(
1/2))/x/(x^2+1)^(1/4))-1/2*b*arctan(1/2*(1-(x^2+1)^(1/2))/(x^2+1)^(1/4)*2^(
1/2))*2^(1/2)-1/2*b*arctanh(1/2*(1+(x^2+1)^(1/2))/(x^2+1)^(1/4)*2^(1/2))*2^
(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1024, 406, 450}

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = -\frac{1}{2}a\arctan\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right) - \frac{1}{2}a\text{arctanh}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right) - \frac{b\arctan\left(\frac{1-\sqrt{x^2+1}}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}} - \frac{b\text{arctanh}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}$$

```
[In] Int[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] -((b*ArcTan[(1 - Sqrt[1 + x^2])/(Sqrt[2]*(1 + x^2)^(1/4))])/Sqrt[2]) - (a*A
rcTan[(1 + Sqrt[1 + x^2])/(x*(1 + x^2)^(1/4))])/2 - (a*ArcTanh[(1 - Sqrt[1
+ x^2])/(x*(1 + x^2)^(1/4))])/2 - (b*ArcTanh[(1 + Sqrt[1 + x^2])/(Sqrt[2]*(
1 + x^2)^(1/4))])/Sqrt[2]
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 450

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
Simp[(-(Sqrt[2]*Rt[a, 4]*d)^(-1))*ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sq
rt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x] - Simp[(1/(Sqrt[2]*Rt[a, 4]*d))*ArcT
anh[(Rt[a, 4]^2 + Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a\int \frac{1}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx + b\int \frac{x}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx\\
&= -\frac{b\arctan\left(\frac{1-\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right)}{\sqrt{2}} - \frac{1}{2}a\arctan\left(\frac{1+\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right)\\
&\quad - \frac{1}{2}a\text{arctanh}\left(\frac{1-\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right) - \frac{b\text{arctanh}\left(\frac{1+\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = \frac{1}{4}bx^2\,\text{AppellF1}\left(1,\frac{1}{4},1,2,-x^2,-\frac{x^2}{2}\right) - \frac{6ax\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-x^2,-\frac{x^2}{2}\right)}{\sqrt[4]{1+x^2}\,(2+x^2)\left(-6\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-x^2,-\frac{x^2}{2}\right)+x^2\left(2\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-x^2,-\frac{x^2}{2}\right)+\text{AppellF1}\right.\right.}$$

```
[In] Integrate[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/4, 1, 2, -x^2, -1/2*x^2])/4 - (6*a*x*AppellF1[1/2, 1/4
, 1, 3/2, -x^2, -1/2*x^2])/((1 + x^2)^(1/4)*(2 + x^2)*(-6*AppellF1[1/2, 1/4
, 1, 3/2, -x^2, -1/2*x^2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -1/2*x^
2] + AppellF1[3/2, 5/4, 1, 5/2, -x^2, -1/2*x^2])))
```

Maple [F]

$$\int \frac{bx+a}{(x^2+1)^{\frac{1}{4}}\,(x^2+2)}dx$$

```
[In] int((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
```

```
[Out] int((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = \int \frac{a+bx}{\sqrt[4]{x^2+1}\,(x^2+2)}\,dx$$

```
[In] integrate((b*x+a)/(x**2+1)**(1/4)/(x**2+2),x)
[Out] Integral((a + b*x)/((x**2 + 1)**(1/4)*(x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = \int \frac{bx+a}{(x^2+2)(x^2+1)^{\frac{1}{4}}}\,dx$$

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="maxima")
[Out] integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

Giac [F]

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = \int \frac{bx+a}{(x^2+2)(x^2+1)^{\frac{1}{4}}}\,dx$$

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="giac")
[Out] integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = \int \frac{a+b\,x}{(x^2+1)^{1/4}\,(x^2+2)}\,dx$$

```
[In] int((a + b*x)/((x^2 + 1)^(1/4)*(x^2 + 2)),x)
[Out] int((a + b*x)/((x^2 + 1)^(1/4)*(x^2 + 2)), x)
```

3.73 $\int \frac{x}{\sqrt{1-x^3}(4-x^3)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\text{arctanh}\left(\frac{1+\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\text{arctanh}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

```
[Out] -1/6*arctanh((1+2^(1/3)*x)/(-x^3+1)^(1/2))*2^(1/3)+1/18*arctanh((-x^3+1)^(1
/2))*2^(1/3)-1/18*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*2^(1/3)*3^(1
/2)+1/18*arctan(1/3*(-x^3+1)^(1/2)*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {497}

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\text{arctanh}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\text{arctanh}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

```
[In] Int[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] -1/3*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(2^(2/3)*Sqrt[3]) + Ar
cTan[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)/S
qrt[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\text{integral} = -\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{1+\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\operatorname{arctanh}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.22

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \frac{1}{8}x^2\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{4}\right)$$

```
[In] Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/8
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.90 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.29

method	result
default	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^3-4\right)}\frac{\_\alpha^2\sqrt{2}\sqrt{i\left(-i\sqrt{3}+2x+1\right)}\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\sqrt{-\frac{i\left(i\sqrt{3}+2x+1\right)}{2}}\left(-2\_\alpha^2+\_\alpha+1+i\sqrt{3}\,(1-\_\alpha)\right)\Pi\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\cdots\right.}{2\sqrt{-x^3+1}}\cdots\right.}{36}$
elliptic	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^3-4\right)}\frac{\_\alpha^2\sqrt{2}\sqrt{i\left(-i\sqrt{3}+2x+1\right)}\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\sqrt{-\frac{i\left(i\sqrt{3}+2x+1\right)}{2}}\left(-2\_\alpha^2+\_\alpha+1+i\sqrt{3}\,(1-\_\alpha)\right)\Pi\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\cdots\right.}{2\sqrt{-x^3+1}}\cdots\right.}{36}$
trager	Expression too large to display

```
[In] int(x/(-x^3+4)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*I*2^(1/2)*sum(_alpha^2*(1/2*I*(-I*3^(1/2)+2*x+1))^(1/2)*((-1+x)/(I*3^(
1/2)-3))^(1/2)*(-1/2*I*(I*3^(1/2)+2*x+1))^(1/2)/(-x^3+1)^(1/2)*(-2*_alpha^2
+_alpha+1+I*3^(1/2)*(1-_alpha))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2),1/2*_alpha-1/3*I*_alpha^2*3^(1/2)-1/2+1/6*I*_alpha*3^(1
/2)+1/6*I*3^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^
3-4))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 1116, normalized size of antiderivative = 8.79

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15552*432^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log(-(72*x^9 + 4752*x^6 - 5184
*x^3 + 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*x^2 + sqrt(-3)*(x^8 + 7*x^5
- 8*x^2)) - 864*2^(1/3)*(-1)^(2/3)*(x^7 + x^4 - sqrt(-3)*(x^7 + x^4 - 2*x)
- 2*x) + sqrt(-x^3 + 1)*(432^(5/6)*(-1)^(1/6)*(x^7 + 16*x^4 - sqrt(-3)*(x^7
 + 16*x^4 - 8*x) - 8*x) + 648*432^(1/6)*(-1)^(5/6)*(sqrt(-3)*x^5 + x^5) + 1
44*sqrt(3)*(5*I*x^6 + 20*I*x^3 - 16*I)) + 2304)/(x^9 - 12*x^6 + 48*x^3 - 64
)) + 1/15552*432^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log(-(72*x^9 + 4752*x^6 -
5184*x^3 + 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*x^2 + sqrt(-3)*(x^8 + 7*
x^5 - 8*x^2)) - 864*2^(1/3)*(-1)^(2/3)*(x^7 + x^4 - sqrt(-3)*(x^7 + x^4 - 2
*x) - 2*x) - sqrt(-x^3 + 1)*(432^(5/6)*(-1)^(1/6)*(x^7 + 16*x^4 - sqrt(-3)*
```

```
(x^7 + 16*x^4 - 8*x) - 8*x) + 648*432^(1/6)*(-1)^(5/6)*(sqrt(-3)*x^5 + x^5)
 - 144*sqrt(3)*(-5*I*x^6 - 20*I*x^3 + 16*I)) + 2304)/(x^9 - 12*x^6 + 48*x^3
 - 64)) + 1/15552*432^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*log(-(72*x^9 + 4752*x
^6 - 5184*x^3 + 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*x^2 - sqrt(-3)*(x^8
 + 7*x^5 - 8*x^2)) - 864*2^(1/3)*(-1)^(2/3)*(x^7 + x^4 + sqrt(-3)*(x^7 + x^
4 - 2*x) - 2*x) + sqrt(-x^3 + 1)*(432^(5/6)*(-1)^(1/6)*(x^7 + 16*x^4 + sqrt
(-3)*(x^7 + 16*x^4 - 8*x) - 8*x) - 648*432^(1/6)*(-1)^(5/6)*(sqrt(-3)*x^5 -
 x^5) + 144*sqrt(3)*(5*I*x^6 + 20*I*x^3 - 16*I)) + 2304)/(x^9 - 12*x^6 + 48
*x^3 - 64)) - 1/15552*432^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*log(-(72*x^9 + 47
52*x^6 - 5184*x^3 + 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*x^2 - sqrt(-3)*
(x^8 + 7*x^5 - 8*x^2)) - 864*2^(1/3)*(-1)^(2/3)*(x^7 + x^4 + sqrt(-3)*(x^7
+ x^4 - 2*x) - 2*x) - sqrt(-x^3 + 1)*(432^(5/6)*(-1)^(1/6)*(x^7 + 16*x^4 +
sqrt(-3)*(x^7 + 16*x^4 - 8*x) - 8*x) - 648*432^(1/6)*(-1)^(5/6)*(sqrt(-3)*x
^5 - x^5) - 144*sqrt(3)*(-5*I*x^6 - 20*I*x^3 + 16*I)) + 2304)/(x^9 - 12*x^6
 + 48*x^3 - 64)) - 1/7776*432^(5/6)*(-1)^(1/6)*log(-(36*x^9 + 2376*x^6 - 25
92*x^3 - 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*x^2) + 864*2^(1/3)*(-1)^(2
/3)*(x^7 + x^4 - 2*x) - (648*432^(1/6)*(-1)^(5/6)*x^5 + 432^(5/6)*(-1)^(1/6
)*(x^7 + 16*x^4 - 8*x) - 72*sqrt(3)*(5*I*x^6 + 20*I*x^3 - 16*I))*sqrt(-x^3
+ 1) + 1152)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/7776*432^(5/6)*(-1)^(1/6)*lo
g(-(36*x^9 + 2376*x^6 - 2592*x^3 - 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*
x^2) + 864*2^(1/3)*(-1)^(2/3)*(x^7 + x^4 - 2*x) + (648*432^(1/6)*(-1)^(5/6)
*x^5 + 432^(5/6)*(-1)^(1/6)*(x^7 + 16*x^4 - 8*x) + 72*sqrt(3)*(-5*I*x^6 - 2
0*I*x^3 + 16*I))*sqrt(-x^3 + 1) + 1152)/(x^9 - 12*x^6 + 48*x^3 - 64))
```

Sympy [F]

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = -\int \frac{x}{x^3\sqrt{1-x^3}-4\sqrt{1-x^3}}\,dx$$

```
[In] integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)
[Out] -Integral(x/(x**3*sqrt(1 - x**3) - 4*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \int -\frac{x}{(x^3-4)\sqrt{-x^3+1}}\,dx$$

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \int -\frac{x}{(x^3-4)\sqrt{-x^3+1}}\,dx$$

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 653, normalized size of antiderivative = 5.14

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \text{Too large to display}$$

```
[In] int(-x/((1 - x^3)^(1/2)*(x^3 - 4)),x)
[Out] - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1
/2)*1i)/2 + 3/2)/(2^(2/3) - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2)*(2^
(2/3) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1
i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(2/
3)*((3^(1/2)*1i)/2 + 1/2) + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 + 1
/2)*(1 - x^3)^(1/2)*(2^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 1)*(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(
-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i
)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(2/3)*((3^(1/2)*1i)/2 - 1/2) -
1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 - 1/2)*(1 - x^3)^(1/2)*(2^(2/3)
*((3^(1/2)*1i)/2 - 1/2) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.74 $\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}} dx$

Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = -\frac{\arctan\left(\frac{1+\sqrt[3]{2}\sqrt[3]{d}x}{\sqrt{-1+dx^3}}\right)}{3\ 2^{2/3}d^{2/3}} - \frac{\arctan\left(\sqrt{-1+dx^3}\right)}{9\ 2^{2/3}d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{-1+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{-1+dx^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}$$

```
[Out] -1/6*arctan((1+2^(1/3)*d^(1/3)*x)/(d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)-1/18*arc
tan((d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)-1/18*arctanh((1-2^(1/3)*d^(1/3)*x)*3^(
1/2)/(d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)-1/18*arctanh(1/3*(d*x^3-1)^(1
/2)*3^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {498}

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{d}x+1}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}d^{2/3}} - \frac{\arctan\left(\sqrt{dx^3-1}\right)}{9\ 2^{2/3}d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{dx^3-1}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}$$

```
[In] Int[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]
```

```
[Out] -1/3*ArcTan[(1 + 2^(1/3)*d^(1/3)*x)/Sqrt[-1 + d*x^3]]/(2^(2/3)*d^(2/3)) - A
rcTan[Sqrt[-1 + d*x^3]]/(9*2^(2/3)*d^(2/3)) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)
*d^(1/3)*x))/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3)) - ArcTanh[Sqrt[-
1 + d*x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3))
```

Rule 498

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(-q)*(ArcTan[Sqrt[c + d*x^3]/Rt[-c, 2]]/(9*2^(2/3
)*b*Rt[-c, 2])), x] + (-Simp[q*(ArcTan[Rt[-c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c
+ d*x^3])]/(3*2^(2/3)*b*Rt[-c, 2])), x] - Simp[q*(ArcTanh[Sqrt[c + d*x^3]/(
Sqrt[3]*Rt[-c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[-c, 2])), x] - Simp[q*(ArcTanh[
Sqrt[3]*Rt[-c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b
*Rt[-c, 2])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*
b*c - a*d, 0] && NegQ[c]
```

Rubi steps

$$\text{integral} = -\frac{\arctan\left(\frac{1+\sqrt[3]{2}\sqrt[3]{d}x}{\sqrt{-1+dx^3}}\right)}{3\ 2^{2/3}d^{2/3}} - \frac{\arctan\left(\sqrt{-1+dx^3}\right)}{9\ 2^{2/3}d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{-1+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{-1+dx^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.34

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = \frac{x^2\sqrt{1-dx^3}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},dx^3,\frac{dx^3}{4}\right)}{8\sqrt{-1+dx^3}}$$

```
[In] Integrate[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 - d*x^3]*AppellF1[2/3, 1/2, 1, 5/3, d*x^3, (d*x^3)/4])/(8*Sqrt[
-1 + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.53

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-4\right)}\frac{\sqrt{-\frac{i\left(2x+\frac{1}{d^{\frac{1}{3}}}+\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}{2}}\sqrt{\frac{x-\frac{1}{d^{\frac{1}{3}}}}{-\frac{3}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}}}\sqrt{2}\sqrt{i\left(2x+\frac{1}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}\left(-2\_\alpha^{2}d+i\sqrt{3}\_\alpha d^{\frac{2}{3}}-i\sqrt{3}d^{\frac{1}{3}}+\_\alpha d^{\frac{2}{3}}\cdots\right.}{2\_\alpha\cdots}\right)}{9\cdots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-4\right)}\frac{\sqrt{-\frac{i\left(2x+\frac{1}{d^{\frac{1}{3}}}+\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}{2}}\sqrt{\frac{x-\frac{1}{d^{\frac{1}{3}}}}{-\frac{3}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}}}\sqrt{2}\sqrt{i\left(2x+\frac{1}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}\left(-2\_\alpha^{2}d+i\sqrt{3}\_\alpha d^{\frac{2}{3}}-i\sqrt{3}d^{\frac{1}{3}}+\_\alpha d^{\frac{2}{3}}\cdots\right.}{2\_\alpha\cdots}\right)}{9\cdots}$

```
[In] int(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*I*2^(1/2)*sum(1/_alpha/d^(4/3)*(-1/2*I*(2*x+1/d^(1/3)+I*3^(1/2)/d^(1/3
))*d^(1/3))^(1/2)*((x-1/d^(1/3))/(-3/d^(1/3)-I*3^(1/2)/d^(1/3)))^(1/2)*(1/2
*I*(2*x+1/d^(1/3)-I*3^(1/2)/d^(1/3))*d^(1/3))^(1/2)/(d*x^3-1)^(1/2)*(-2*_al
pha^2*d+I*3^(1/2)*_alpha*d^(2/3)-I*3^(1/2)*d^(1/3)+_alpha*d^(2/3)+d^(1/3))*
EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/d^(1/3)+1/2*I*3^(1/2)/d^(1/3))*3^(1/2)*d^
(1/3))^(1/2),1/3*I*3^(1/2)*_alpha^2*d^(2/3)-1/6*I*3^(1/2)*_alpha*d^(1/3)-1/
6*I*3^(1/2)+1/2*_alpha*d^(1/3)-1/2,(-I*3^(1/2)/d^(1/3)/(-3/2/d^(1/3)-1/2*I*
3^(1/2)/d^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1669 vs. $2(110) = 220$.

Time = 0.39 (sec) , antiderivative size = 1669, normalized size of antiderivative = 10.63

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/36*(1/432)^(1/6)*(sqrt(-3) + 1)*(d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6
 - 72*d*x^3 - 24*(1/2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x + sqrt(-3)*(d^5*x
```

```
^7 + d^4*x^4 - 2*d^3*x))*(d^(-4))^(2/3) - 6*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^
5 - 8*d^2*x^2 - sqrt(-3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2))*(d^(-4))^(1/3)
+ 6*sqrt(d*x^3 - 1)*(648*(1/432)^(5/6)*(sqrt(-3)*d^5*x^5 - d^5*x^5)*(d^(-4)
)^(5/6) + sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4)) - (1/432
)^(1/6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x + sqrt(-3)*(d^3*x^7 + 16*d^2*x^4 - 8*
d*x))*(d^(-4))^(1/6)) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) + 1/36*
(1/432)^(1/6)*(sqrt(-3) + 1)*(d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*
d*x^3 - 24*(1/2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x + sqrt(-3)*(d^5*x^7 + d
^4*x^4 - 2*d^3*x))*(d^(-4))^(2/3) - 6*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*
d^2*x^2 - sqrt(-3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2))*(d^(-4))^(1/3) - 6*sq
rt(d*x^3 - 1)*(648*(1/432)^(5/6)*(sqrt(-3)*d^5*x^5 - d^5*x^5)*(d^(-4))^(5/6
) + sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4)) - (1/432)^(1/6
)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x + sqrt(-3)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x))*
(d^(-4))^(1/6)) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) - 1/36*(1/432
)^(1/6)*(sqrt(-3) - 1)*(d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3
- 24*(1/2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x - sqrt(-3)*(d^5*x^7 + d^4*x^4
 - 2*d^3*x))*(d^(-4))^(2/3) - 6*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^
2 + sqrt(-3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2))*(d^(-4))^(1/3) + 6*sqrt(d*x
^3 - 1)*(648*(1/432)^(5/6)*(sqrt(-3)*d^5*x^5 + d^5*x^5)*(d^(-4))^(5/6) - sq
rt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4)) + (1/432)^(1/6)*(d^3
*x^7 + 16*d^2*x^4 - 8*d*x - sqrt(-3)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x))*(d^(-4
))^(1/6)) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) + 1/36*(1/432)^(1/6
)*(sqrt(-3) - 1)*(d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 - 24*(
1/2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x - sqrt(-3)*(d^5*x^7 + d^4*x^4 - 2*d
^3*x))*(d^(-4))^(2/3) - 6*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2 + sq
rt(-3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2))*(d^(-4))^(1/3) - 6*sqrt(d*x^3 - 1
)*(648*(1/432)^(5/6)*(sqrt(-3)*d^5*x^5 + d^5*x^5)*(d^(-4))^(5/6) - sqrt(1/3
)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4)) + (1/432)^(1/6)*(d^3*x^7 +
 16*d^2*x^4 - 8*d*x - sqrt(-3)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x))*(d^(-4))^(1/
6)) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) + 1/18*(1/432)^(1/6)*(d^(
-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 + 48*(1/2)^(2/3)*(d^5*x^7 +
 d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 -
8*d^2*x^2)*(d^(-4))^(1/3) + 6*(1296*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 +
sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4)) + 2*(1/432)^(1/6)*
(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^3*x
^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) - 1/18*(1/432)^(1/6)*(d^(-4))^(1/6)*log((
d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 + 48*(1/2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3
*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-
4))^(1/3) - 6*(1296*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 + sqrt(1/3)*(5*d^4
*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4)) + 2*(1/432)^(1/6)*(d^3*x^7 + 16*d^
2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6
+ 48*d*x^3 - 64))
```

Sympy [F]

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = -\int \frac{x}{dx^3\sqrt{dx^3-1}-4\sqrt{dx^3-1}}\,dx$$

```
[In] integrate(x/(-d*x**3+4)/(d*x**3-1)**(1/2),x)
[Out] -Integral(x/(d*x**3*sqrt(d*x**3 - 1) - 4*sqrt(d*x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = \int -\frac{x}{\sqrt{dx^3-1}(dx^3-4)}\,dx$$

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

Giac [F]

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = \int -\frac{x}{\sqrt{dx^3-1}(dx^3-4)}\,dx$$

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

Mupad [B] (verification not implemented)

Time = 15.03 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.11

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = \frac{\sqrt{3}\,314928^{1/3}\ln\left(\frac{\left(54\sqrt{d\,x^3-1}+54\sqrt{3}-54\,2^{1/3}\sqrt{3}\,d^{1/3}\,x\right)\left(\sqrt{d\,x^3-1}-\sqrt{3}+2^{1/3}\sqrt{3}\,d^{1/3}\,x\right)^3}{\left(2^{2/3}-d^{1/3}\,x\right)^6}\right)}{2916\,d^{2/3}} + \frac{\sqrt{3}\,314928^{1/3}\ln\left(\frac{\left(2\sqrt{d\,x^3-1}+2\sqrt{3}+2^{1/3}\sqrt{3}\,d^{1/3}\,x+2^{1/3}\,d^{1/3}\,x\,3\mathrm{i}\right)^3\left(108\sqrt{3}-108\sqrt{d\,x^3-1}+54\,2^{1/3}\sqrt{3}\,d^{1/3}\,x+2^{1/3}\,d^{1/3}\,x\,162\mathrm{i}\right)}{\left(2^{2/3}+2\,d^{1/3}\,x-2^{2/3}\sqrt{3}\,1\mathrm{i}\right)^6}\right)}{2916\,d^{2/3}} + \frac{\sqrt{3}\,314928^{1/3}\ln\left(\frac{\left(2\sqrt{d\,x^3-1}-2\sqrt{3}-2^{1/3}\sqrt{3}\,d^{1/3}\,x+2^{1/3}\,d^{1/3}\,x\,3\mathrm{i}\right)^3\left(108\sqrt{d\,x^3-1}+108\sqrt{3}+54\,2^{1/3}\sqrt{3}\,d^{1/3}\,x-2^{1/3}\,d^{1/3}\,x\,162\mathrm{i}\right)}{\left(2^{2/3}+2\,d^{1/3}\,x+2^{2/3}\sqrt{3}\,1\mathrm{i}\right)^6}\right)}{2916\,d^{2/3}}$$

```
[In] int(-x/((d*x^3 - 1)^(1/2)*(d*x^3 - 4)),x)
```

```
[Out] (3^(1/2)*314928^(1/3)*log(((54*(d*x^3 - 1)^(1/2) + 54*3^(1/2) - 54*2^(1/3)*
3^(1/2)*d^(1/3)*x)*((d*x^3 - 1)^(1/2) - 3^(1/2) + 2^(1/3)*3^(1/2)*d^(1/3)*x
)^3)/(2^(2/3) - d^(1/3)*x)^6))/(2916*d^(2/3)) + (3^(1/2)*314928^(1/3)*log((
(2*(d*x^3 - 1)^(1/2) + 2*3^(1/2) + 2^(1/3)*d^(1/3)*x*3i + 2^(1/3)*3^(1/2)*d
^(1/3)*x)^3*(108*3^(1/2) - 108*(d*x^3 - 1)^(1/2) + 2^(1/3)*d^(1/3)*x*162i +
 54*2^(1/3)*3^(1/2)*d^(1/3)*x))/(2^(2/3) - 2^(2/3)*3^(1/2)*1i + 2*d^(1/3)*x
)^6)*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(2916*d^(2/3)) + (3^(1/2)*314928^(1/3)*l
og(((2*(d*x^3 - 1)^(1/2) - 2*3^(1/2) + 2^(1/3)*d^(1/3)*x*3i - 2^(1/3)*3^(1/
2)*d^(1/3)*x)^3*(108*(d*x^3 - 1)^(1/2) + 108*3^(1/2) - 2^(1/3)*d^(1/3)*x*16
2i + 54*2^(1/3)*3^(1/2)*d^(1/3)*x))/(2^(2/3)*3^(1/2)*1i + 2^(2/3) + 2*d^(1/
3)*x)^6)*((3^(1/2)*1i)/2 + 1/2)^(1/2)*1i)/(2916*d^(2/3))
```

3.75 $\int \frac{x}{\sqrt{-1+x^3}(8+x^3)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 74

$$\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)} \, dx = \frac{1}{18}\arctan\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) + \frac{1}{18}\arctan\left(\frac{1}{3}\sqrt{-1+x^3}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}}$$

```
[Out] 1/18*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))+1/18*arctan(1/3*(x^3-1)^(1/2))-1/18*
arctanh((1-x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {499, 455, 65, 210, 2163, 209, 2170, 212}

$$\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)} \, dx = \frac{1}{18}\arctan\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right) + \frac{1}{18}\arctan\left(\frac{\sqrt{x^3-1}}{3}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}(1-x)}{\sqrt{x^3-1}}\right)}{6\sqrt{3}}$$

```
[In] Int[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]
```

```
[Out] ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])]/18 + ArcTan[Sqrt[-1 + x^3]/3]/18 - Arc
Tanh[(Sqrt[3]*(1 - x))/Sqrt[-1 + x^3]]/(6*Sqrt[3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{12}\int\frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx\right) \\
&\quad -\frac{1}{12}\int\frac{-2-2x+x^2}{(4-2x+x^2)\sqrt{-1+x^3}}\,dx-\frac{1}{4}\int\frac{x^2}{(-8-x^3)\sqrt{-1+x^3}}\,dx \\
&= -\left(\frac{1}{12}\text{Subst}\left(\int\frac{1}{(-8-x)\sqrt{-1+x}}\,dx,x,x^3\right)\right) \\
&\quad +\frac{1}{6}\text{Subst}\left(\int\frac{1}{9+x^2}\,dx,x,\frac{(1-x)^2}{\sqrt{-1+x^3}}\right)-\frac{1}{3}\text{Subst}\left(\int\frac{1}{2-6x^2}\,dx,x,\frac{1-x}{\sqrt{-1+x^3}}\right) \\
&= \frac{1}{18}\arctan\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{3}(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}}-\frac{1}{6}\text{Subst}\left(\int\frac{1}{-9-x^2}\,dx,x,\sqrt{-1+x^3}\right) \\
&= \frac{1}{18}\arctan\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)+\frac{1}{18}\arctan\left(\frac{1}{3}\sqrt{-1+x^3}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{3}(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

$$\int\frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx=\frac{x^2\sqrt{1-x^3}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,-\frac{x^3}{8}\right)}{16\sqrt{-1+x^3}}$$

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]
[Out] (x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, -1/8*x^3])/(16*Sqrt[-1 +
 x^3])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 3.02 (sec) , antiderivative size = 421, normalized size of antiderivative = 5.69

method	result
default	$-\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{i\sqrt{3}}{6}+\frac{1}{2},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{9\sqrt{x^3-1}}+\frac{i\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\cdots}}{}$
trager	$324\,\mathrm{RootOf}\left(104976\_Z^4-324\_Z^2+1\right)^3\ln\left(\frac{-629856\,\mathrm{RootOf}\left(104976\_Z^4-324\_Z^2+1\right)^5x^2+2519424x\,\mathrm{RootOf}(\cdots}{}\right.$
elliptic	$\frac{i\sqrt{-\frac{1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}+\frac{x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}+\frac{1}{3-i\sqrt{3}}-\frac{i\sqrt{3}}{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\sqrt{\frac{x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}+\frac{1}{i\sqrt{3}+3}+\frac{i\sqrt{3}}{i\sqrt{3}+3}}\sqrt{3}\,\Pi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{i\left(1+i\sqrt{3}\right)\sqrt{3}}{6}+\frac{i\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\cdots}{\frac{3}{2}-\cdots}}\right.}{6\sqrt{x^3-1}}$

```
[In] int(x/(x^3+8)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/
6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+1/9*I*(1/2
-1/2*I*3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((
x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2
+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I
*3^(1/2)))^(1/2),1/6*I*(1+I*3^(1/2))*3^(1/2)+1/3*I*3^(1/2),((3/2+1/2*I*3^(1
/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/9*I*(1/2+1/2*I*3^(1/2))*(-3/2-1/2*I*3^(1
/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*
3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(
1/2)*3^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/6*I*(1-I*3^(1
/2))*3^(1/2)-2/3*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 723, normalized size of antiderivative = 9.77

$$\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx =$$

$$-\frac{1}{216}\sqrt{2}\sqrt{i\sqrt{3}+1}\log\left(\frac{2x^6+6x^5-150x^4+176x^3+12x^2-3\sqrt{x^3-1}\big(\sqrt{3}\sqrt{2}(7i\,x^3-12i\,x^2-12i}{x^6-}\right.$$

$$+\frac{1}{216}\sqrt{2}\sqrt{i\sqrt{3}+1}\log\left(\frac{2x^6+6x^5-150x^4+176x^3+12x^2-3\sqrt{x^3-1}\big(\sqrt{3}\sqrt{2}(-7i\,x^3+12i\,x^2+12}{x^6}\right.$$

$$+\frac{1}{216}\sqrt{2}\sqrt{-i\sqrt{3}+1}\log\left(\frac{2x^6+6x^5-150x^4+176x^3+12x^2-3\sqrt{x^3-1}\big(\sqrt{3}\sqrt{2}(7i\,x^3-12i\,x^2-12}{x^6}\right.$$

$$-\frac{1}{216}\sqrt{2}\sqrt{-i\sqrt{3}+1}\log\left(\frac{2x^6+6x^5-150x^4+176x^3+12x^2-3\sqrt{x^3-1}\big(\sqrt{3}\sqrt{2}(-7i\,x^3+12i\,x^2+}{x}\right.$$

$$+\frac{1}{54}\arctan\left(\frac{(x^3-12x^2-6x-10)\sqrt{x^3-1}}{6(x^4-x^3-x+1)}\right)$$

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/216*sqrt(2)*sqrt(I*sqrt(3) + 1)*log((2*x^6 + 6*x^5 - 150*x^4 + 176*x^3 +
 12*x^2 - 3*sqrt(x^3 - 1)*(sqrt(3)*sqrt(2)*(7*I*x^3 - 12*I*x^2 - 12*I*x + 8
*I) + sqrt(2)*(2*x^4 - 5*x^3 - 24*x^2 + 28*x + 8))*sqrt(I*sqrt(3) + 1) - 18
*sqrt(3)*(-I*x^5 + I*x^4 + 6*I*x^3 - 2*I*x^2 - 4*I*x) + 168*x - 160)/(x^6 -
 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)) + 1/216*sqrt(2)*sqrt(I*sqrt
(3) + 1)*log((2*x^6 + 6*x^5 - 150*x^4 + 176*x^3 + 12*x^2 - 3*sqrt(x^3 - 1)*
(sqrt(3)*sqrt(2)*(-7*I*x^3 + 12*I*x^2 + 12*I*x - 8*I) - sqrt(2)*(2*x^4 - 5*
x^3 - 24*x^2 + 28*x + 8))*sqrt(I*sqrt(3) + 1) - 18*sqrt(3)*(-I*x^5 + I*x^4
+ 6*I*x^3 - 2*I*x^2 - 4*I*x) + 168*x - 160)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3
+ 96*x^2 - 96*x + 64)) + 1/216*sqrt(2)*sqrt(-I*sqrt(3) + 1)*log((2*x^6 + 6*
x^5 - 150*x^4 + 176*x^3 + 12*x^2 - 3*sqrt(x^3 - 1)*(sqrt(3)*sqrt(2)*(7*I*x^
3 - 12*I*x^2 - 12*I*x + 8*I) - sqrt(2)*(2*x^4 - 5*x^3 - 24*x^2 + 28*x + 8))
*sqrt(-I*sqrt(3) + 1) - 18*sqrt(3)*(I*x^5 - I*x^4 - 6*I*x^3 + 2*I*x^2 + 4*I
*x) + 168*x - 160)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)) -
1/216*sqrt(2)*sqrt(-I*sqrt(3) + 1)*log((2*x^6 + 6*x^5 - 150*x^4 + 176*x^3 +
 12*x^2 - 3*sqrt(x^3 - 1)*(sqrt(3)*sqrt(2)*(-7*I*x^3 + 12*I*x^2 + 12*I*x -
8*I) + sqrt(2)*(2*x^4 - 5*x^3 - 24*x^2 + 28*x + 8))*sqrt(-I*sqrt(3) + 1) -
18*sqrt(3)*(I*x^5 - I*x^4 - 6*I*x^3 + 2*I*x^2 + 4*I*x) + 168*x - 160)/(x^6
- 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)) + 1/54*arctan(1/6*(x^3 - 1
2*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x + 1))
```

Sympy [F]

$$\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx = \int \frac{x}{\sqrt{(x-1)\,(x^2+x+1)}\,(x+2)\,(x^2-2x+4)}\,dx$$

```
[In] integrate(x/(x**3+8)/(x**3-1)**(1/2),x)
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)*(x**2 - 2*x + 4)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx = \int \frac{x}{(x^3+8)\sqrt{x^3-1}}\,dx$$

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx = \int \frac{x}{(x^3+8)\sqrt{x^3-1}}\,dx$$

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 533, normalized size of antiderivative = 7.20

$$\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx$$

$$= \frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{9\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

$$-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{3};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{9\left(-1+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

$$-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{3};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{9\left(1+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(x/((x^3 - 1)^(1/2)*(x^3 + 8)),x)
```

```
[Out] (((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2)
)^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x -
 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
 - 3/2)))/(9*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (3^(1/2)*((3^(1/2)
*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((
x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)*ellipticPi(-(3^(1/2)*((3^(1/2)*1i)/2 + 3/2)*1i)/3, asi
n((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/
2)*1i)/2 - 3/2))*2i)/(9*(3^(1/2)*1i - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)
^(1/2)) - (3^(1/2)*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*((3^(1
/2)*1i)/2 + 3/2)*1i)/3, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3
^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*2i)/(9*(3^(1/2)*1i + 1)*(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.76 $\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{d}x\right)}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}d^{2/3}} + \frac{\operatorname{arctanh}\left(\frac{\left(1+\sqrt[3]{d}x\right)^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{1}{3}\sqrt{1+dx^3}\right)}{18d^{2/3}}$$

```
[Out] 1/18*arctanh(1/3*(1+d^(1/3)*x)^2/(d*x^3+1)^(1/2))/d^(2/3)-1/18*arctanh(1/3*
(d*x^3+1)^(1/2))/d^(2/3)-1/18*arctan((1+d^(1/3)*x)*3^(1/2)/(d*x^3+1)^(1/2))
/d^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{d}x+1\right)}{\sqrt{dx^3+1}}\right)}{6\sqrt{3}d^{2/3}} + \frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{d}x+1\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

```
[In] Int[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
```

```
[Out] -1/6*ArcTan[(Sqrt[3]*(1 + d^(1/3)*x))/Sqrt[1 + d*x^3]]/(Sqrt[3]*d^(2/3)) +
ArcTanh[(1 + d^(1/3)*x)^2/(3*Sqrt[1 + d*x^3])]/(18*d^(2/3)) - ArcTanh[Sqrt[
1 + d*x^3]/3]/(18*d^(2/3))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{2d^{2/3}-2dx-d^{4/3}x^2}{\left(4+2\sqrt[3]{d}x+d^{2/3}x^2\right)\sqrt{1+dx^3}}\,dx}{12d} + \frac{\int \frac{1+\sqrt[3]{d}x}{\left(2-\sqrt[3]{d}x\right)\sqrt{1+dx^3}}\,dx}{12\sqrt[3]{d}} \\
&\quad -\frac{1}{4}\sqrt[3]{d}\int \frac{x^2}{(8-dx^3)\sqrt{1+dx^3}}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{\left(1+\sqrt[3]{d}x\right)^2}{\sqrt{1+dx^3}}\right)}{6d^{2/3}} - \frac{1}{12}\sqrt[3]{d}\text{Subst}\left(\int \frac{1}{(8-dx)\sqrt{1+dx}}\,dx, x, x^3\right) \\
&\quad + \frac{1}{3}d^{4/3}\text{Subst}\left(\int \frac{1}{-2d^2-6d^2x^2}\,dx, x, \frac{1+\sqrt[3]{d}x}{\sqrt{1+dx^3}}\right) \\
&= -\frac{\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{d}x\right)}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}d^{2/3}} + \frac{\text{arctanh}\left(\frac{\left(1+\sqrt[3]{d}x\right)^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \sqrt{1+dx^3}\right)}{6d^{2/3}} \\
&= -\frac{\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{d}x\right)}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}d^{2/3}} + \frac{\text{arctanh}\left(\frac{\left(1+\sqrt[3]{d}x\right)^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}} - \frac{\text{arctanh}\left(\frac{1}{3}\sqrt{1+dx^3}\right)}{18d^{2/3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.31

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx = \frac{1}{16}x^2 \operatorname{AppellF1}\left(\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -dx^3, \frac{dx^3}{8}\right)$$

```
[In] Integrate[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, -(d*x^3), (d*x^3)/8])/16
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.09 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.72

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(d\_Z^3-8\right)} \frac{\left(-d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}+\left(-d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}+\left(-\cdots\right.}{d}\right.}{2\left(-d^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(d\_Z^3-8\right)} \frac{\left(-d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}+\left(-d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}+\left(-\cdots\right.}{d}\right.}{2\left(-d^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}\right.}{\cdots}$

```
[In] int(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*I/d^3*2^(1/2)*sum(1/_alpha*(-d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^(1/2)*(d*(x-1/d*(-d^2)^(1/3))/(-
3*(-d^2)^(1/3)+I*3^(1/2)*(-d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)
*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^(1/2)/(d*x^3+1)^(1/2)*(I*(-d^2)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2)^(2/3)+2*_alpha^2*d^2-(-d^2)^(1/3)*_
alpha*d-(-d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-d^2)^(1/3))*3^(1/2)*d/(-d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2)^
(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2)^(2/3)*3^(1/2)*_alpha-3*(-d^2)^(2/3)*_alph
```

```
a+I*3^(1/2)*d-3*d),(I*3^(1/2)/d*(-d^2)^(1/3)/(-3/2/d*(-d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(73) = 146$.

Time = 0.42 (sec) , antiderivative size = 497, normalized size of antiderivative = 4.83

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx = \frac{2\sqrt{3}(d^2)^{\frac{1}{6}}d\arctan\left(-\frac{\left(9\sqrt{3}d^3x^5-\sqrt{3}(d^2x^6-40\,dx^3-32)(d^2)^{\frac{2}{3}}+3\sqrt{3}(5\,d^2x^4+8\,dx)(d^2)^{\frac{1}{3}}\right)\sqrt{dx^3+1}(d^2)^{\frac{1}{6}}}{9\,(d^4x^7-7\,d^3x^4-8\,d^2x)}\right)+2\,(d^2)^{\frac{2}{3}}\log\left(\frac{d^4x^9\cdots}{\cdots}\right)\cdots}{\cdots}$$

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/108*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(-1/9*(9*sqrt(3)*d^3*x^5 - sqrt(3)*(d^
2*x^6 - 40*d*x^3 - 32)*(d^2)^(2/3) + 3*sqrt(3)*(5*d^2*x^4 + 8*d*x)*(d^2)^(1
/3))*sqrt(d*x^3 + 1)*(d^2)^(1/6)/(d^4*x^7 - 7*d^3*x^4 - 8*d^2*x)) + 2*(d^2)
^(2/3)*log((d^4*x^9 + 318*d^3*x^6 + 1200*d^2*x^3 + 18*(5*d^2*x^7 + 64*d*x^4
 + 32*x)*(d^2)^(2/3) + 6*(7*d^3*x^6 + 152*d^2*x^3 + (d^2*x^7 + 80*d*x^4 + 1
60*x)*(d^2)^(2/3) + 6*(5*d^2*x^5 + 32*d*x^2)*(d^2)^(1/3) + 64*d)*sqrt(d*x^3
 + 1) + 18*(d^3*x^8 + 38*d^2*x^5 + 64*d*x^2)*(d^2)^(1/3) + 640*d)/(d^3*x^9
- 24*d^2*x^6 + 192*d*x^3 - 512)) - (d^2)^(2/3)*log((d^4*x^9 - 276*d^3*x^6 -
 1608*d^2*x^3 - 18*(d^2*x^7 - 52*d*x^4 - 80*x)*(d^2)^(2/3) - 6*(4*d^3*x^6 +
 164*d^2*x^3 + (d^2*x^7 - 28*d*x^4 - 272*x)*(d^2)^(2/3) - 24*(d^2*x^5 + d*x
^2)*(d^2)^(1/3) + 160*d)*sqrt(d*x^3 + 1) + 18*(d^3*x^8 + 20*d^2*x^5 - 8*d*x
^2)*(d^2)^(1/3) - 1088*d)/(d^3*x^9 - 24*d^2*x^6 + 192*d*x^3 - 512)))/d^2
```

Sympy [F]

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx = -\int \frac{x}{dx^3\sqrt{dx^3+1}-8\sqrt{dx^3+1}}\,dx$$

```
[In] integrate(x/(-d*x**3+8)/(d*x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(d*x**3*sqrt(d*x**3 + 1) - 8*sqrt(d*x**3 + 1)), x)
```

Maxima [F]

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx = \int -\frac{x}{\sqrt{dx^3+1}(dx^3-8)}\,dx$$

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)
```

Giac [F]

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx = \int -\frac{x}{\sqrt{dx^3+1}(dx^3-8)}\,dx$$

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(-x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx = -\int \frac{x}{\sqrt{d\,x^3+1}\,(d\,x^3-8)}\,dx$$

```
[In] int(-x/((d*x^3 + 1)^(1/2)*(d*x^3 - 8)),x)
[Out] -int(x/((d*x^3 + 1)^(1/2)*(d*x^3 - 8)), x)
```

3.77 $\int \frac{1}{\sqrt[3]{1-3x^2}(3-x^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx = \frac{1}{4}\arctan\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}$$

```
[Out] 1/4*arctan((1-(-3*x^2+1)^(1/3))/x)+1/12*arctanh(1/3*x*3^(1/2))*3^(1/2)-1/12
*arctanh(1/9*(1-(-3*x^2+1)^(1/3))^2/x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {404}

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx = \frac{1}{4}\arctan\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)-\frac{\operatorname{arctanh}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[In] Int[1/((1 - 3*x^2)^(1/3)*(3 - x^2)),x]
```

```
[Out] ArcTan[(1 - (1 - 3*x^2)^(1/3))/x]/4 + ArcTanh[x/Sqrt[3]]/(4*Sqrt[3]) - ArcT
anh[(1 - (1 - 3*x^2)^(1/3))^2/(3*Sqrt[3]*x)]/(4*Sqrt[3])
```

Rule 404

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\arctan\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)+\frac{\text{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{\text{arctanh}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx = -\frac{9x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},3x^2,\frac{x^2}{3}\right)}{\sqrt[3]{1-3x^2}\,(-3+x^2)\left(9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},3x^2,\frac{x^2}{3}\right)+2x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},3x^2,\frac{x^2}{3}\right)+3\,\text{AppellF1}\right.\right.}$$

```
[In] Integrate[1/((1 - 3*x^2)^(1/3)*(3 - x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3])/((1 - 3*x^2)^(1/3)*(-3 + x^
2)*(9*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, 3*x^2, x^2/3] + 3*AppellF1[3/2, 4/3, 1, 5/2, 3*x^2, x^2/3])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 7.80 (sec) , antiderivative size = 538, normalized size of antiderivative = 6.64

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{8\left(-3x^2+1\right)^{\frac{1}{3}}\operatorname{RootOf}\left(\_Z^2-3\right)^2\operatorname{RootOf}\left(4\_Z\operatorname{RootOf}\left(\_Z^2-3\right)+48\_Z^2+1\right)x+192\left(-3x^2+1\right)^{\frac{1}{3}}\operatorname{RootOf}\left(\_Z\right.}{}\right.}{}$

```
[In] int(1/(-3*x^2+1)^(1/3)/(-x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*RootOf(_Z^2-3)*ln((8*(-3*x^2+1)^(1/3)*RootOf(_Z^2-3)^2*RootOf(4*_Z*Ro
otOf(_Z^2-3)+48*_Z^2+1)*x+192*(-3*x^2+1)^(1/3)*RootOf(_Z^2-3)*RootOf(4*_Z*R
ootOf(_Z^2-3)+48*_Z^2+1)^2*x-16*RootOf(_Z^2-3)^2*RootOf(4*_Z*RootOf(_Z^2-3)
+48*_Z^2+1)*x-384*RootOf(_Z^2-3)*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)^2*x+
12*RootOf(_Z^2-3)*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)*x^2+24*(-3*x^2+1)^(
1/3)*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)*RootOf(_Z^2-3)+6*(-3*x^2+1)^(2/3
)+12*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)*RootOf(_Z^2-3)-4*RootOf(_Z^2-3)*
x-96*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)*x+3*x^2+3)/(x^2-3))-1/12*ln((2*(
-3*x^2+1)^(1/3)*RootOf(_Z^2-3)*x+48*(-3*x^2+1)^(1/3)*RootOf(4*_Z*RootOf(_Z^
2-3)+48*_Z^2+1)*x+6*(-3*x^2+1)^(2/3)+4*RootOf(_Z^2-3)*x+96*RootOf(4*_Z*Root
Of(_Z^2-3)+48*_Z^2+1)*x-3*x^2+6*(-3*x^2+1)^(1/3)-3)/(x^2-3))*RootOf(_Z^2-3)
-ln((2*(-3*x^2+1)^(1/3)*RootOf(_Z^2-3)*x+48*(-3*x^2+1)^(1/3)*RootOf(4*_Z*Ro
otOf(_Z^2-3)+48*_Z^2+1)*x+6*(-3*x^2+1)^(2/3)+4*RootOf(_Z^2-3)*x+96*RootOf(4
*_Z*RootOf(_Z^2-3)+48*_Z^2+1)*x-3*x^2+6*(-3*x^2+1)^(1/3)-3)/(x^2-3))*RootOf
(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 1210, normalized size of antiderivative = 14.94

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="fricas")
```

```
[Out] 1/144*sqrt(6)*sqrt(-I*sqrt(3) - 1)*log(-(42*x^5 + 276*x^3 + sqrt(6)*(x^6 +
105*x^4 + 63*x^2 - 9)*sqrt(-I*sqrt(3) - 1) - 3*(40*x^3 + sqrt(6)*(x^4 + 12*
x^2 - sqrt(3)*(-I*x^4 - 12*I*x^2 - 3*I) + 3)*sqrt(-I*sqrt(3) - 1) + 72*x)*(
-3*x^2 + 1)^(2/3) + 6*sqrt(3)*(-7*I*x^5 - 46*I*x^3 + 9*I*x) - 3*(2*x^5 + 52
*x^3 - sqrt(6)*(5*x^4 + 18*x^2 + sqrt(3)*(-5*I*x^4 - 18*I*x^2 + 3*I) - 3)*s
qrt(-I*sqrt(3) - 1) - 2*sqrt(3)*(-I*x^5 - 26*I*x^3 - 9*I*x) + 18*x)*(-3*x^2
 + 1)^(1/3) - 54*x)/(x^6 - 9*x^4 + 27*x^2 - 27)) - 1/144*sqrt(6)*sqrt(-I*sq
rt(3) - 1)*log(-(42*x^5 + 276*x^3 - sqrt(6)*(x^6 + 105*x^4 + 63*x^2 - 9)*sq
rt(-I*sqrt(3) - 1) - 3*(40*x^3 - sqrt(6)*(x^4 + 12*x^2 + sqrt(3)*(I*x^4 + 1
2*I*x^2 + 3*I) + 3)*sqrt(-I*sqrt(3) - 1) + 72*x)*(-3*x^2 + 1)^(2/3) + 6*sqr
```

```
t(3)*(-7*I*x^5 - 46*I*x^3 + 9*I*x) - 3*(2*x^5 + 52*x^3 + sqrt(6)*(5*x^4 + 1
8*x^2 - sqrt(3)*(5*I*x^4 + 18*I*x^2 - 3*I) - 3)*sqrt(-I*sqrt(3) - 1) - 2*sq
rt(3)*(-I*x^5 - 26*I*x^3 - 9*I*x) + 18*x)*(-3*x^2 + 1)^(1/3) - 54*x)/(x^6 -
 9*x^4 + 27*x^2 - 27)) + 1/144*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(-(42*x^5 + 2
76*x^3 - 24*(5*x^3 + 9*x)*(-3*x^2 + 1)^(2/3) + 6*sqrt(3)*(7*I*x^5 + 46*I*x^
3 - 9*I*x) - (3*sqrt(6)*(x^4 + 12*x^2 - sqrt(3)*(I*x^4 + 12*I*x^2 + 3*I) +
3)*(-3*x^2 + 1)^(2/3) - 3*sqrt(6)*(5*x^4 + 18*x^2 + sqrt(3)*(5*I*x^4 + 18*I
*x^2 - 3*I) - 3)*(-3*x^2 + 1)^(1/3) - sqrt(6)*(x^6 + 105*x^4 + 63*x^2 - 9))
*sqrt(I*sqrt(3) - 1) - 6*(x^5 + 26*x^3 - sqrt(3)*(I*x^5 + 26*I*x^3 + 9*I*x)
 + 9*x)*(-3*x^2 + 1)^(1/3) - 54*x)/(x^6 - 9*x^4 + 27*x^2 - 27)) - 1/144*sqr
t(6)*sqrt(I*sqrt(3) - 1)*log(-(42*x^5 + 276*x^3 - 24*(5*x^3 + 9*x)*(-3*x^2
+ 1)^(2/3) + 6*sqrt(3)*(7*I*x^5 + 46*I*x^3 - 9*I*x) + (3*sqrt(6)*(x^4 + 12*
x^2 + sqrt(3)*(-I*x^4 - 12*I*x^2 - 3*I) + 3)*(-3*x^2 + 1)^(2/3) - 3*sqrt(6)
*(5*x^4 + 18*x^2 - sqrt(3)*(-5*I*x^4 - 18*I*x^2 + 3*I) - 3)*(-3*x^2 + 1)^(1
/3) - sqrt(6)*(x^6 + 105*x^4 + 63*x^2 - 9))*sqrt(I*sqrt(3) - 1) - 6*(x^5 +
26*x^3 - sqrt(3)*(I*x^5 + 26*I*x^3 + 9*I*x) + 9*x)*(-3*x^2 + 1)^(1/3) - 54*
x)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/72*sqrt(3)*log(-(x^12 + 2598*x^10 + 551
43*x^8 + 114228*x^6 - 22113*x^4 - 7290*x^2 + 8*(3*x^10 + 576*x^8 + 5598*x^6
 + 5832*x^4 - 729*x^2 - sqrt(3)*(41*x^9 + 1368*x^7 + 4482*x^5 + 864*x^3 - 2
43*x))*(-3*x^2 + 1)^(2/3) - 4*sqrt(3)*(25*x^11 + 2359*x^9 + 15426*x^7 + 696
6*x^5 - 4347*x^3 + 243*x) - 4*(84*x^10 + 4536*x^8 + 20880*x^6 + 5832*x^4 -
2916*x^2 - sqrt(3)*(x^11 + 521*x^9 + 7362*x^7 + 10746*x^5 - 1971*x^3 - 243*
x))*(-3*x^2 + 1)^(1/3) + 729)/(x^12 - 18*x^10 + 135*x^8 - 540*x^6 + 1215*x^
4 - 1458*x^2 + 729))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx = -\int \frac{1}{x^2\sqrt[3]{1-3x^2} - 3\sqrt[3]{1-3x^2}}\,dx$$

```
[In] integrate(1/(-3*x**2+1)**(1/3)/(-x**2+3),x)
[Out] -Integral(1/(x**2*(1 - 3*x**2)**(1/3) - 3*(1 - 3*x**2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx = \int -\frac{1}{(x^2-3)(-3\,x^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="maxima")
[Out] -integrate(1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx = \int -\frac{1}{(x^2-3)(-3\,x^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="giac")
[Out] integrate(-1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx = -\int \frac{1}{(x^2-3)\,\left(1-3\,x^2\right)^{1/3}}\,dx$$

```
[In] int(-1/((x^2 - 3)*(1 - 3*x^2)^(1/3)),x)
[Out] -int(1/((x^2 - 3)*(1 - 3*x^2)^(1/3)), x)
```

3.78 $\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx = \frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\arctan\left(\frac{\left(1-\sqrt[3]{1+3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}} - \frac{1}{4}\operatorname{arctanh}\left(\frac{1-\sqrt[3]{1+3x^2}}{x}\right)$$

```
[Out] -1/4*arctanh((1-(3*x^2+1)^(1/3))/x)+1/12*arctan(1/3*x*3^(1/2))*3^(1/2)+1/12
*arctan(1/9*(1-(3*x^2+1)^(1/3))^2/x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {403}

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx = \frac{\arctan\left(\frac{\left(1-\sqrt[3]{3x^2+1}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}} + \frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\operatorname{arctanh}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right)$$

```
[In] Int[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] ArcTan[x/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 - (1 + 3*x^2)^(1/3))^2/(3*Sqrt[3]
*x)]/(4*Sqrt[3]) - ArcTanh[(1 - (1 + 3*x^2)^(1/3))/x]/4
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\arctan\left(\frac{\left(1-\sqrt[3]{1+3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}} - \frac{1}{4}\text{arctanh}\left(\frac{1-\sqrt[3]{1+3x^2}}{x}\right)$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx = -\frac{9x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-3x^2,-\frac{x^2}{3}\right)}{(3+x^2)\sqrt[3]{1+3x^2}\left(-9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-3x^2,-\frac{x^2}{3}\right)+2x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-3x^2,-\frac{x^2}{3}\right)+3\,\text{App}\right.\right.}$$

```
[In] Integrate[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -1/3*x^2])/((3 + x^2)*(1 + 3*x^2)^
(1/3)*(-9*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/
2, 1/3, 2, 5/2, -3*x^2, -1/3*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/2, -3*x^2, -1
/3*x^2])))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.67 (sec) , antiderivative size = 443, normalized size of antiderivative = 5.47

method	result
trager	$-\frac{\ln\left(-\frac{12\,\mathrm{RootOf}\left(48\_Z^2+12\_Z+1\right)\left(3x^2+1\right)^{\frac{1}{3}}x-6\,\mathrm{RootOf}\left(48\_Z^2+12\_Z+1\right)x^2+\left(3x^2+1\right)^{\frac{2}{3}}+12\,\mathrm{RootOf}\left(48\_Z^2+12\_Z+1\right)\left(3x^2\cdots\right.}{x^2+3}\right.}{4}$

```
[In] int(1/(x^2+3)/(3*x^2+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(-(12*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)*x-6*RootOf(48*_Z^2+12*
_Z+1)*x^2+(3*x^2+1)^(2/3)+12*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)+(3*x^2
+1)^(1/3)*x-24*RootOf(48*_Z^2+12*_Z+1)*x-x^2+(3*x^2+1)^(1/3)+6*RootOf(48*_Z
^2+12*_Z+1)-4*x+1)/(x^2+3))-ln(-(12*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)
*x-6*RootOf(48*_Z^2+12*_Z+1)*x^2+(3*x^2+1)^(2/3)+12*RootOf(48*_Z^2+12*_Z+1)
*(3*x^2+1)^(1/3)+(3*x^2+1)^(1/3)*x-24*RootOf(48*_Z^2+12*_Z+1)*x-x^2+(3*x^2+
1)^(1/3)+6*RootOf(48*_Z^2+12*_Z+1)-4*x+1)/(x^2+3))*RootOf(48*_Z^2+12*_Z+1)+
RootOf(48*_Z^2+12*_Z+1)*ln(-(-24*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)*x+
12*RootOf(48*_Z^2+12*_Z+1)*x^2+2*(3*x^2+1)^(2/3)-24*RootOf(48*_Z^2+12*_Z+1)
*(3*x^2+1)^(1/3)-4*(3*x^2+1)^(1/3)*x+48*RootOf(48*_Z^2+12*_Z+1)*x+x^2-4*(3*
x^2+1)^(1/3)-12*RootOf(48*_Z^2+12*_Z+1)+4*x-1)/(x^2+3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(59) = 118.

Time = 1.01 (sec) , antiderivative size = 345, normalized size of antiderivative = 4.26

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx$$

$$= \frac{1}{36}\sqrt{3}\arctan\left(\frac{4\sqrt{3}(3x^4-10x^3-36x^2+18x+9)(3x^2+1)^{\frac{2}{3}}-4\sqrt{3}(x^5+15x^4-26x^3-54x^2+9x\cdots}{x^6+126x^5-225x^4-828x^3-81\cdots}\right.$$

$$-\frac{1}{36}\sqrt{3}\arctan\left(\frac{2\left(2\sqrt{3}(23x^3+9x)(3x^2+1)^{\frac{2}{3}}+\sqrt{3}(x^5-80x^3-9x)(3x^2+1)^{\frac{1}{3}}+\sqrt{3}(11x^5+10x^3\cdots\right.}{x^6-657x^4-189x^2-27}\right.$$

$$+\frac{1}{24}\log\left(\frac{x^6+108x^5+549x^4+360x^3+99x^2+6(3x^4+32x^3+42x^2+3)(3x^2+1)^{\frac{2}{3}}+6(x^5+27x\cdots}{x^6+9x^4+27x^2+27}\right.$$

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(3)*arctan((4*sqrt(3)*(3*x^4 - 10*x^3 - 36*x^2 + 18*x + 9)*(3*x^2
+ 1)^(2/3) - 4*sqrt(3)*(x^5 + 15*x^4 - 26*x^3 - 54*x^2 + 9*x - 9)*(3*x^2 +
1)^(1/3) + sqrt(3)*(x^6 - 2*x^5 - 105*x^4 - 28*x^3 + 63*x^2 + 126*x + 9))/(
x^6 + 126*x^5 - 225*x^4 - 828*x^3 - 81*x^2 - 162*x + 81)) - 1/36*sqrt(3)*ar
ctan(2*(2*sqrt(3)*(23*x^3 + 9*x)*(3*x^2 + 1)^(2/3) + sqrt(3)*(x^5 - 80*x^3
- 9*x)*(3*x^2 + 1)^(1/3) + sqrt(3)*(11*x^5 + 10*x^3 - 9*x))/(x^6 - 657*x^4
- 189*x^2 - 27)) + 1/24*log((x^6 + 108*x^5 + 549*x^4 + 360*x^3 + 99*x^2 + 6
*(3*x^4 + 32*x^3 + 42*x^2 + 3)*(3*x^2 + 1)^(2/3) + 6*(x^5 + 27*x^4 + 70*x^3
 + 18*x^2 + 9*x + 3)*(3*x^2 + 1)^(1/3) + 108*x - 9)/(x^6 + 9*x^4 + 27*x^2 +
 27))
```

Sympy [F]

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx = \int \frac{1}{(x^2+3)\sqrt[3]{3x^2+1}}\,dx$$

```
[In] integrate(1/(x**2+3)/(3*x**2+1)**(1/3),x)
[Out] Integral(1/((x**2 + 3)*(3*x**2 + 1)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx = \int \frac{1}{(3x^2+1)^{\frac{1}{3}}(x^2+3)}\,dx$$

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

Giac [F]

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx = \int \frac{1}{(3x^2+1)^{\frac{1}{3}}(x^2+3)}\,dx$$

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="giac")
[Out] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx = \int \frac{1}{(x^2+3)\,(3\,x^2+1)^{1/3}}\,dx$$

```
[In] int(1/((x^2 + 3)*(3*x^2 + 1)^(1/3)),x)
[Out] int(1/((x^2 + 3)*(3*x^2 + 1)^(1/3)), x)
```

3.79 $\int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)} dx$

Optimal result

Integrand size = 19, antiderivative size = 113

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}} + \frac{\operatorname{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/12*arctanh(x)*2^(1/3)+1/4*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)+
1/12*arctan(3^(1/2)/x)*2^(1/3)*3^(1/2)+1/12*arctan((1-2^(1/3)*(-x^2+1)^(1/3
))*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {402}

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}} - \frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}}$$

```
[In] Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1
+ 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}} + \frac{\operatorname{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{9x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)\left(-9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)+2x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right)-\text{AppellF1}\left(\right.\right.\right.}$$

```
[In] Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)
*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, x^2, -1/3*x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 9.52 (sec) , antiderivative size = 938, normalized size of antiderivative = 8.30

method	result	size
trager	Expression too large to display	938

```
[In] int(1/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/432*ln((RootOf(_Z^6+108)^4*x^6-72*RootOf(_Z^6+108)^4*x^5-36*RootOf(_Z^6+1
08)^2*(-x^2+1)^(1/3)*x^5+225*RootOf(_Z^6+108)^4*x^4+648*RootOf(_Z^6+108)^2*
(-x^2+1)^(1/3)*x^4+72*RootOf(_Z^6+108)^4*x^3+648*(-x^2+1)^(2/3)*x^4-864*Roo
tOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^3-189*x^2*RootOf(_Z^6+108)^4-4536*(-x^2+1)
^(2/3)*x^3-648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^2+1944*(-x^2+1)^(2/3)*x^
2+324*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x+27*RootOf(_Z^6+108)^4+1944*(-x^2+
1)^(2/3)*x)/(x^2+3)^3)*RootOf(_Z^6+108)^4+1/72*ln((RootOf(_Z^6+108)^4*x^6-7
2*RootOf(_Z^6+108)^4*x^5-36*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5+225*RootO
f(_Z^6+108)^4*x^4+648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^4+72*RootOf(_Z^6+
108)^4*x^3+648*(-x^2+1)^(2/3)*x^4-864*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^3
-189*x^2*RootOf(_Z^6+108)^4-4536*(-x^2+1)^(2/3)*x^3-648*RootOf(_Z^6+108)^2*
(-x^2+1)^(1/3)*x^2+1944*(-x^2+1)^(2/3)*x^2+324*RootOf(_Z^6+108)^2*(-x^2+1)^
(1/3)*x+27*RootOf(_Z^6+108)^4+1944*(-x^2+1)^(2/3)*x)/(x^2+3)^3)*RootOf(_Z^6
+108)-1/36*RootOf(_Z^6+108)*ln((486*RootOf(_Z^6+108)-27*RootOf(_Z^6+108)^4-
72*RootOf(_Z^6+108)^4*x^5+189*x^2*RootOf(_Z^6+108)^4-225*RootOf(_Z^6+108)^4
*x^4+1296*RootOf(_Z^6+108)*x^5-3402*RootOf(_Z^6+108)*x^2-1296*RootOf(_Z^6+1
08)*x^3+72*RootOf(_Z^6+108)^4*x^3+4050*RootOf(_Z^6+108)*x^4-RootOf(_Z^6+108
)^4*x^6+18*RootOf(_Z^6+108)*x^6+1296*(-x^2+1)^(2/3)*x^4+9072*(-x^2+1)^(2/3)
*x^3+3888*(-x^2+1)^(2/3)*x^2-3888*(-x^2+1)^(2/3)*x-36*RootOf(_Z^6+108)^2*(-
x^2+1)^(1/3)*x^5-648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^4-864*RootOf(_Z^6+
108)^2*(-x^2+1)^(1/3)*x^3+648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^2+324*Roo
tOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x+6*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^5+10
8*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^4+144*RootOf(_Z^6+108)^5*(-x^2+1)^(1/
3)*x^3-108*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^2-54*RootOf(_Z^6+108)^5*(-x^
2+1)^(1/3)*x)/(x^2+3)^3)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 1232, normalized size of antiderivative = 10.90

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] 1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*log((432^(5/6)*(-1)^(1/6)*(x^6
- 69*x^4 + 63*x^2 + sqrt(-3)*(x^6 - 69*x^4 + 63*x^2 - 27) - 27) + 432*2^(1/
3)*(-1)^(2/3)*(5*x^5 - 30*x^3 + sqrt(-3)*(5*x^5 - 30*x^3 + 9*x) + 9*x) + 17
28*(9*x^3 - sqrt(3)*(I*x^4 - 9*I*x^2) - 9*x)*(-x^2 + 1)^(2/3) - 432*(2^(2/3
)*(-1)^(1/3)*(x^5 - 18*x^3 - sqrt(-3)*(x^5 - 18*x^3 + 9*x) + 9*x) + 4*432^(
1/6)*(-1)^(5/6)*(x^4 - 3*x^2 - sqrt(-3)*(x^4 - 3*x^2)))*(-x^2 + 1)^(1/3))/(
x^6 + 9*x^4 + 27*x^2 + 27)) - 1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*l
og(-(432^(5/6)*(-1)^(1/6)*(x^6 - 69*x^4 + 63*x^2 + sqrt(-3)*(x^6 - 69*x^4 +
 63*x^2 - 27) - 27) - 432*2^(1/3)*(-1)^(2/3)*(5*x^5 - 30*x^3 + sqrt(-3)*(5*
x^5 - 30*x^3 + 9*x) + 9*x) - 1728*(9*x^3 - sqrt(3)*(-I*x^4 + 9*I*x^2) - 9*x
)*(-x^2 + 1)^(2/3) + 432*(2^(2/3)*(-1)^(1/3)*(x^5 - 18*x^3 - sqrt(-3)*(x^5
- 18*x^3 + 9*x) + 9*x) - 4*432^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2 - sqrt(-3)*(x^
4 - 3*x^2)))*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/10368*432^(
5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log((432^(5/6)*(-1)^(1/6)*(x^6 - 69*x^4 + 63
*x^2 - sqrt(-3)*(x^6 - 69*x^4 + 63*x^2 - 27) - 27) + 432*2^(1/3)*(-1)^(2/3)
*(5*x^5 - 30*x^3 - sqrt(-3)*(5*x^5 - 30*x^3 + 9*x) + 9*x) + 1728*(9*x^3 - s
qrt(3)*(I*x^4 - 9*I*x^2) - 9*x)*(-x^2 + 1)^(2/3) - 432*(2^(2/3)*(-1)^(1/3)*
(x^5 - 18*x^3 + sqrt(-3)*(x^5 - 18*x^3 + 9*x) + 9*x) + 4*432^(1/6)*(-1)^(5/
6)*(x^4 - 3*x^2 + sqrt(-3)*(x^4 - 3*x^2)))*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 +
 27*x^2 + 27)) + 1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log(-(432^(5/6
)*(-1)^(1/6)*(x^6 - 69*x^4 + 63*x^2 - sqrt(-3)*(x^6 - 69*x^4 + 63*x^2 - 27)
 - 27) - 432*2^(1/3)*(-1)^(2/3)*(5*x^5 - 30*x^3 - sqrt(-3)*(5*x^5 - 30*x^3
+ 9*x) + 9*x) - 1728*(9*x^3 - sqrt(3)*(-I*x^4 + 9*I*x^2) - 9*x)*(-x^2 + 1)^
(2/3) + 432*(2^(2/3)*(-1)^(1/3)*(x^5 - 18*x^3 + sqrt(-3)*(x^5 - 18*x^3 + 9*
x) + 9*x) - 4*432^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2 + sqrt(-3)*(x^4 - 3*x^2)))*
(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/5184*432^(5/6)*(-1)^(1/6
)*log(-(432^(5/6)*(-1)^(1/6)*(x^6 - 69*x^4 + 63*x^2 - 27) + 432*2^(1/3)*(-1
)^(2/3)*(5*x^5 - 30*x^3 + 9*x) - 864*(9*x^3 - sqrt(3)*(I*x^4 - 9*I*x^2) - 9
*x)*(-x^2 + 1)^(2/3) - 432*(2^(2/3)*(-1)^(1/3)*(x^5 - 18*x^3 + 9*x) + 4*432
^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2))*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 +
27)) + 1/5184*432^(5/6)*(-1)^(1/6)*log((432^(5/6)*(-1)^(1/6)*(x^6 - 69*x^4
+ 63*x^2 - 27) - 432*2^(1/3)*(-1)^(2/3)*(5*x^5 - 30*x^3 + 9*x) + 864*(9*x^3
 - sqrt(3)*(-I*x^4 + 9*I*x^2) - 9*x)*(-x^2 + 1)^(2/3) + 432*(2^(2/3)*(-1)^(
1/3)*(x^5 - 18*x^3 + 9*x) - 4*432^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2))*(-x^2 + 1
)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{\sqrt[3]{-(x-1)\,(x+1)}\,(x^2+3)}\,dx$$

```
[In] integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

```
[In] int(1/((1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] int(1/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.80 $\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\frac{\arctan(x)}{6\ 2^{2/3}} + \frac{\arctan\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

```
[Out] -1/12*arctan(x)*2^(1/3)+1/4*arctan(x/(1+2^(1/3)*(x^2+1)^(1/3)))*2^(1/3)-1/1
2*arctanh(3^(1/2)/x)*2^(1/3)*3^(1/2)-1/12*arctanh((1-2^(1/3)*(x^2+1)^(1/3))
*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {401}

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \frac{\arctan\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}} - \frac{\arctan(x)}{6\ 2^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

```
[In] Int[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -1/6*ArcTan[x]/2^(2/3) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3)
) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*
(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\text{integral} = -\frac{\arctan(x)}{6\ 2^{2/3}} + \frac{\arctan\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\frac{9x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right)}{(-3+x^2)\sqrt[3]{1+x^2}\left(9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right)+2x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-x^2,\frac{x^2}{3}\right)-\text{AppellF1}\left(\frac{3}{2},\right.\right.\right.}$$

```
[In] Integrate[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*
(9*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5
/2, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

Maple [F]

$$\int \frac{1}{\left(-x^{2}+3\right) \left(x^{2}+1\right)^{\frac{1}{3}}}dx$$

[In] int(1/(-x^2+3)/(x^2+1)^(1/3),x)

[Out] int(1/(-x^2+3)/(x^2+1)^(1/3),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1103 vs. 2(77) = 154.

Time = 0.74 (sec) , antiderivative size = 1103, normalized size of antiderivative = 10.12

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \text{Too large to display}$$

[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")

```
[Out] -1/10368*432^(5/6)*(sqrt(-3) + 1)*log((432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + s
qrt(-3)*(x^6 + 69*x^4 + 63*x^2 + 27) + 27) - 1728*(9*x^3 + sqrt(3)*(x^4 + 9
*x^2) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x^5 + 30*x^3 + sqrt(-3)*(5*x^
5 + 30*x^3 + 9*x) + 9*x) + 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 - sqr
t(-3)*(x^5 + 18*x^3 + 9*x) + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2 - sqrt(-3)*(x^
4 + 3*x^2))))/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/10368*432^(5/6)*(sqrt(-3) +
1)*log(-(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + sqrt(-3)*(x^6 + 69*x^4 + 63*x^2
 + 27) + 27) + 1728*(9*x^3 - sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) -
 432*2^(1/3)*(5*x^5 + 30*x^3 + sqrt(-3)*(5*x^5 + 30*x^3 + 9*x) + 9*x) - 432
*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 - sqrt(-3)*(x^5 + 18*x^3 + 9*x) + 9
*x) - 4*432^(1/6)*(x^4 + 3*x^2 - sqrt(-3)*(x^4 + 3*x^2))))/(x^6 - 9*x^4 + 2
7*x^2 - 27)) + 1/10368*432^(5/6)*(sqrt(-3) - 1)*log((432^(5/6)*(x^6 + 69*x^
4 + 63*x^2 - sqrt(-3)*(x^6 + 69*x^4 + 63*x^2 + 27) + 27) - 1728*(9*x^3 + sq
rt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x^5 + 30*x^3 -
sqrt(-3)*(5*x^5 + 30*x^3 + 9*x) + 9*x) + 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5
+ 18*x^3 + sqrt(-3)*(x^5 + 18*x^3 + 9*x) + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2
+ sqrt(-3)*(x^4 + 3*x^2))))/(x^6 - 9*x^4 + 27*x^2 - 27)) - 1/10368*432^(5/6
)*(sqrt(-3) - 1)*log(-(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 - sqrt(-3)*(x^6 + 6
9*x^4 + 63*x^2 + 27) + 27) + 1728*(9*x^3 - sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^
2 + 1)^(2/3) - 432*2^(1/3)*(5*x^5 + 30*x^3 - sqrt(-3)*(5*x^5 + 30*x^3 + 9*x
) + 9*x) - 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 + sqrt(-3)*(x^5 + 18*
x^3 + 9*x) + 9*x) - 4*432^(1/6)*(x^4 + 3*x^2 + sqrt(-3)*(x^4 + 3*x^2))))/(x
^6 - 9*x^4 + 27*x^2 - 27)) + 1/5184*432^(5/6)*log(-(432^(5/6)*(x^6 + 69*x^4
 + 63*x^2 + 27) + 864*(9*x^3 + sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3)
 + 432*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) + 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 +
 18*x^3 + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 - 9*x^4 + 27*x^2 - 27)) -
```

```
1/5184*432^(5/6)*log((432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + 27) - 864*(9*x^3
- sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) - 432*2^(1/3)*(5*x^5 + 30*x^
3 + 9*x) - 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 + 9*x) - 4*432^(1/6)*
(x^4 + 3*x^2)))/(x^6 - 9*x^4 + 27*x^2 - 27))
```

Sympy [F]

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\int \frac{1}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

```
[In] integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)
[Out] -Integral(1/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \int -\frac{1}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
[Out] -integrate(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

Giac [F]

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \int -\frac{1}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
[Out] integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\int \frac{1}{(x^2+1)^{1/3}\,(x^2-3)}\,dx$$

```
[In] int(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
[Out] -int(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.81 $\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)x^2+x^3}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 87

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}\,dx = -\frac{2\sqrt{x}\sqrt{a^2-(1+a^2)\,x+x^2}\arctan\left(\frac{(1-a)\sqrt{x}}{\sqrt{a^2-(1+a^2)x+x^2}}\right)}{(1-a)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}$$

```
[Out] -2*arctan((1-a)*x^(1/2)/(a^2-(a^2+1)*x+x^2)^(1/2))*x^(1/2)*(a^2-(a^2+1)*x+x
^2)^(1/2)/(1-a)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2081, 6865, 1712, 211}

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}\,dx = -\frac{2\sqrt{x}\sqrt{-(a^2+1)\,x+a^2+x^2}\arctan\left(\frac{(1-a)\sqrt{x}}{\sqrt{-(a^2+1)x+a^2+x^2}}\right)}{(1-a)\sqrt{-(a^2+1)\,x^2+a^2x+x^3}}$$

```
[In] Int[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a^2 - (1 + a^2)*x + x^2]*ArcTan[((1 - a)*Sqrt[x])/Sqrt[a^2
 - (1 + a^2)*x + x^2]])/((1 - a)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{x}\sqrt{a^2-(1+a^2)\,x+x^2}\right)\int\frac{a+x}{\sqrt{x}(-a+x)\sqrt{a^2-(1+a^2)x+x^2}}\,dx}{\sqrt{a^2x-(1+a^2)\,x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{a^2-(1+a^2)\,x+x^2}\right)\text{Subst}\left(\int\frac{a+x^2}{(-a+x^2)\sqrt{a^2+(-1-a^2)x^2+x^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{a^2x-(1+a^2)\,x^2+x^3}}$$

$$= \frac{\left(2a\sqrt{x}\sqrt{a^2-(1+a^2)\,x+x^2}\right)\text{Subst}\left(\int\frac{1}{-a-(-2a^2-a(-1-a^2))x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a^2-(1+a^2)x+x^2}}\right)}{\sqrt{a^2x-(1+a^2)\,x^2+x^3}}$$

$$= -\frac{2\sqrt{x}\sqrt{a^2-(1+a^2)\,x+x^2}\arctan\left(\frac{(1-a)\sqrt{x}}{\sqrt{a^2-(1+a^2)x+x^2}}\right)}{(1-a)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}$$

Mathematica [A] (verified)

Time = 10.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.33

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}\,dx=-\frac{2\arctan\left(\frac{(-1+a)x}{\sqrt{(-1+x)x(-a^2+x)}}\right)}{-1+a}$$

```
[In] Integrate[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]
[Out] (-2*ArcTan[((-1 + a)*x)/Sqrt[(-1 + x)*x*(-a^2 + x)]])/(-1 + a)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.38

method	result
default	$\frac{2\arctan\left(\frac{\sqrt{-\left(a^2-x\right)x(-1+x)}}{x(a-1)}\right)}{a-1}$
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{-\left(a^2-x\right)x(-1+x)}}{x(a-1)}\right)}{a-1}$
elliptic	$-\frac{2a^2\sqrt{-\frac{-a^2+x}{a^2}}\,\sqrt{\frac{-1+x}{a^2-1}}\,\sqrt{\frac{x}{a^2}}\,F\left(\sqrt{-\frac{-a^2+x}{a^2}},\sqrt{\frac{a^2}{a^2-1}}\right)}{\sqrt{-a^2x^2+a^2x+x^3-x^2}}-\frac{4a^3\sqrt{-\frac{-a^2+x}{a^2}}\,\sqrt{\frac{-1+x}{a^2-1}}\,\sqrt{\frac{x}{a^2}}\,\Pi\left(\sqrt{-\frac{-a^2+x}{a^2}},\frac{a^2}{a^2-a},\sqrt{\frac{a^2}{a^2-1}}\right)}{\sqrt{-a^2x^2+a^2x+x^3-x^2}\,(a^2-a)}$

```
[In] int((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arctan((-(a^2-x)*x*(-1+x))^(1/2)/x/(a-1))/(a-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}\,dx=\frac{\arctan\left(\frac{\sqrt{a^2x-(a^2+1)x^2+x^3}\,(a^2-2\,(a^2-a+1)x+x^2)}{2\,((a-1)x^3-(a^3-a^2+a-1)x^2+(a^3-a^2)x)}\right)}{a-1}$$

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="fricas")
[Out] arctan(1/2*sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a^2 - 2*(a^2 - a + 1)*x + x^2
)/((a - 1)*x^3 - (a^3 - a^2 + a - 1)*x^2 + (a^3 - a^2)*x))/(a - 1)
```

Sympy [F]

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}\,dx=\int \frac{a+x}{\sqrt{x\,(-a^2+x)\,(x-1)}\,(-a+x)}\,dx$$

```
[In] integrate((a+x)/(-a+x)/(a**2*x-(a**2+1)*x**2+x**3)**(1/2),x)
[Out] Integral((a + x)/(sqrt(x*(-a**2 + x)*(x - 1))*(-a + x)), x)
```

Maxima [F]

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}\,dx=\int -\frac{a+x}{\sqrt{a^2x-(a^2+1)x^2+x^3}(a-x)}\,dx$$

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="maxima")
[Out] -integrate((a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x)
```

Giac [F]

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}\,dx=\int -\frac{a+x}{\sqrt{a^2x-(a^2+1)x^2+x^3}(a-x)}\,dx$$

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="giac")
[Out] integrate(-(a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.49

$$\begin{aligned}&\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}\,dx\\&=\frac{4\,a\,(a^2-1)\,\sqrt{\frac{x}{a^2}}\,\sqrt{\frac{x-1}{a^2-1}}\,\sqrt{-\frac{x-a^2}{a^2-1}}\,\Pi\left(-\frac{a^2-1}{a-a^2};\operatorname{asin}\left(\sqrt{-\frac{x-a^2}{a^2-1}}\right)\middle|\frac{a^2-1}{a^2}\right)}{(a-a^2)\,\sqrt{a^2\,x-x^2\,(a^2+1)+x^3}}\\&\quad-\frac{2\,(a^2-1)\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{-\frac{x-a^2}{a^2-1}}\right)\middle|\frac{a^2-1}{a^2}\right)\,\sqrt{\frac{x}{a^2}}\,\sqrt{\frac{x-1}{a^2-1}}\,\sqrt{-\frac{x-a^2}{a^2-1}}}{\sqrt{a^2\,x-x^2\,(a^2+1)+x^3}}\end{aligned}$$

```
[In] int(-(a + x)/((a - x)*(a^2*x - x^2*(a^2 + 1) + x^3)^(1/2)),x)
```

```
[Out] (4*a*(a^2 - 1)*(x/a^2)^(1/2)*((x - 1)/(a^2 - 1))^(1/2)*(-(x - a^2)/(a^2 - 1
))^(1/2)*ellipticPi(-(a^2 - 1)/(a - a^2), asin((-(x - a^2)/(a^2 - 1))^(1/2)
), (a^2 - 1)/a^2))/((a - a^2)*(a^2*x - x^2*(a^2 + 1) + x^3)^(1/2)) - (2*(a^
2 - 1)*ellipticF(asin((-(x - a^2)/(a^2 - 1))^(1/2)), (a^2 - 1)/a^2)*(x/a^2)
^(1/2)*((x - 1)/(a^2 - 1))^(1/2)*(-(x - a^2)/(a^2 - 1))^(1/2))/(a^2*x - x^2
*(a^2 + 1) + x^3)^(1/2)
```

3.82 $\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)x^2+x^3}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 71

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+\left(-1-2a+a^2\right)x^2+x^3}}\,dx = \frac{\log\left(\frac{-a^2+2(-a+a^2)x+x^2-2a\sqrt{-((-2a+a^2)x)+(-1-2a+a^2)x^2+x^3}}{a^2-2ax+x^2}\right)}{a}$$

```
[Out] 0
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.06 (sec) , antiderivative size = 529, normalized size of antiderivative = 7.45, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used

= {2081, 6865, 1722, 1117, 1720}

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}\,dx$$

$$= \frac{2(1-a)\sqrt{x}\sqrt{-\left(-a^2+2a+1\right)x+(2-a)a+x^2}\arctan\left(\frac{\sqrt{-a^2+2a-1}\sqrt{x}}{\sqrt{-\left(-a^2+2a+1\right)x+(2-a)a+x^2}}\right)}{a\sqrt{-a^2+2a-1}\sqrt{-\left(-a^2+2a+1\right)x^2+(2-a)ax+x^3}}$$

$$+ \frac{((2-a)a)^{3/4}\sqrt{x}\left(\frac{x}{\sqrt{(2-a)a}}+1\right)\sqrt{\frac{-\left(-a^2+2a+1\right)x+(2-a)a+x^2}{(2-a)a\left(\frac{x}{\sqrt{(2-a)a}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{x}}{\sqrt[4]{(2-a)a}}\right),\frac{1}{4}\left(\frac{-a^2+2a+}{\sqrt{(2-a)c}}\right.\right.}{a\sqrt{-\left(-a^2+2a+1\right)x^2+(2-a)ax+x^3}}$$

$$+ \frac{(2-a)\left(1-\sqrt{(2-a)a}\right)\sqrt{x}\left(\frac{x}{\sqrt{(2-a)a}}+1\right)\sqrt{\frac{-\left(-a^2+2a+1\right)x+(2-a)a+x^2}{(2-a)a\left(\frac{x}{\sqrt{(2-a)a}}+1\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{2-a}+\sqrt{a}\right)^2}{4\sqrt{(2-a)a}},2\arctan\left(\right.\right.}{((2-a)a)^{3/4}\sqrt{-\left(-a^2+2a+1\right)x^2+(2-a)ax+x^3}}$$

```
[In] Int[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 + x^3]),
x]
```

```
[Out] (2*(1 - a)*Sqrt[x]*Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2]*ArcTan[(Sqrt[-
1 + 2*a - a^2]*Sqrt[x])/Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2]])/(a*Sqrt
[-1 + 2*a - a^2]*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2)*x^2 + x^3]) + (((2 - a)
*a)^(3/4)*Sqrt[x]*(1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2)
*x + x^2)/((2 - a)*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticF[2*ArcTan[Sqrt[x]
/((2 - a)*a)^(1/4)], (2 + (1 + 2*a - a^2)/Sqrt[(2 - a)*a])/4])/(a*Sqrt[(2 -
 a)*a*x - (1 + 2*a - a^2)*x^2 + x^3]) + ((2 - a)*(1 - Sqrt[(2 - a)*a])*Sqrt
[x]*(1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2)*x + x^2)/((2
- a)*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticPi[(Sqrt[2 - a] + Sqrt[a])^2/(4*
Sqrt[(2 - a)*a]), 2*ArcTan[Sqrt[x]/((2 - a)*a)^(1/4)], (2 + (1 + 2*a - a^2)
/Sqrt[(2 - a)*a])/4])/(((2 - a)*a)^(3/4)*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2)
*x^2 + x^3])
```

<u>Rule 1117</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

<u>Rule 1720</u>

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
```

```
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{x}\sqrt{(2-a)a+(-1-2a+a^2)\,x+x^2}\right)\int \frac{-2+a+x}{\sqrt{x}(-a+x)\sqrt{(2-a)a+(-1-2a+a^2)x+x^2}}\,dx}{\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{(2-a)a+(-1-2a+a^2)\,x+x^2}\right)\text{Subst}\left(\int \frac{-2+a+x^2}{(-a+x^2)\sqrt{(2-a)a+(-1-2a+a^2)x^2+x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}$$

$$= \frac{\left(2\sqrt{(2-a)a}\sqrt{x}\sqrt{(2-a)a+(-1-2a+a^2)\,x+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{(2-a)a+(-1-2a+a^2)x^2+x^4}}\,dx, x, \sqrt{x}\right)}{a\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}$$

$$+ \frac{\left(2\left(1-\frac{\sqrt{a}}{\sqrt{2-a}}\right)(2-2a)(2-a)a\sqrt{x}\sqrt{(2-a)a+(-1-2a+a^2)\,x+x^2}\right)\text{Subst}\left(\int \frac{1+\cdots}{(-a+x^2)\sqrt{(2-a)a\cdots}}\right.}{(-((2-a)a)+a^2)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}$$

$$= \frac{2(1-a)\sqrt{x}\sqrt{(2-a)a-(1+2a-a^2)\,x+x^2}\arctan\left(\frac{\sqrt{-1+2a-a^2}\sqrt{x}}{\sqrt{(2-a)a-(1+2a-a^2)x+x^2}}\right)}{a\sqrt{-1+2a-a^2}\sqrt{(2-a)ax-(1+2a-a^2)\,x^2+x^3}}$$
$$+ \frac{((2-a)a)^{3/4}\sqrt{x}\left(1+\frac{x}{\sqrt{(2-a)a}}\right)\sqrt{\frac{(2-a)a-(1+2a-a^2)x+x^2}{(2-a)a\left(1+\frac{x}{\sqrt{(2-a)a}}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{x}}{\sqrt[4]{(2-a)a}}\right),\frac{1}{4}\left(2 \right.\right.}{a\sqrt{(2-a)ax-(1+2a-a^2)\,x^2+x^3}}$$
$$+ \frac{\sqrt[4]{(2-a)a}\left(1-\sqrt{(2-a)a}\right)\sqrt{x}\left(1+\frac{x}{\sqrt{(2-a)a}}\right)\sqrt{\frac{(2-a)a-(1+2a-a^2)x+x^2}{(2-a)a\left(1+\frac{x}{\sqrt{(2-a)a}}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{2-a}+\sqrt{a}\right)^2}{4\sqrt{(2-a)a}},2\right.}{a\sqrt{(2-a)ax-(1+2a-a^2)\,x^2+x^3}}$$

Mathematica [A] (verified)

Time = 10.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.65

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{(2a-a^2)x+(-1-2a+a^2)x^2+x^3}}{a(-1+x)}\right)}{a}$$

```
[In] Integrate[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 +
x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[(2*a - a^2)*x + (-1 - 2*a + a^2)*x^2 + x^3]/(a*(-1 + x))])
/a
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 1.

Time = 0.51 (sec) , antiderivative size = 317, normalized size of antiderivative = 4.46

method	result
default	$\frac{2(a^2-2a)\sqrt{\frac{a^2-2a+x}{a^2-2a}}\sqrt{\frac{-1+x}{-a^2+2a-1}}\sqrt{\frac{x}{-a^2+2a}}F\left(\sqrt{\frac{a^2-2a+x}{a^2-2a}},\sqrt{\frac{-a^2+2a}{-a^2+2a-1}}\right)}{\sqrt{a^2x^2-a^2x-2a\,x^2+x^3+2ax-x^2}} - \frac{2(-2a+2)(a^2-2a)\sqrt{\frac{a^2-2a+x}{a^2-2a}}\sqrt{\frac{-1+x}{-a^2+2a-1}}\sqrt{-}}{\sqrt{a^2x^2-a^2x-2a\,x^2+x^3}}$
elliptic	$\frac{2(a^2-2a)\sqrt{\frac{a^2-2a+x}{a^2-2a}}\sqrt{\frac{-1+x}{-a^2+2a-1}}\sqrt{\frac{x}{-a^2+2a}}F\left(\sqrt{\frac{a^2-2a+x}{a^2-2a}},\sqrt{\frac{-a^2+2a}{-a^2+2a-1}}\right)}{\sqrt{a^2x^2-a^2x-2a\,x^2+x^3+2ax-x^2}} + \frac{2(2a-2)(a^2-2a)\sqrt{\frac{a^2-2a+x}{a^2-2a}}\sqrt{\frac{-1+x}{-a^2+2a-1}}\sqrt{\frac{x}{-a^2}}}{\sqrt{a^2x^2-a^2x-2a\,x^2+x^3}}$

```
[In] int((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 2*(a^2-2*a)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((-1+x)/(-a^2+2*a-1))^(1/2)*(x/(-
a^2+2*a))^(1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(1/2)*EllipticF(((a^2
```

```
-2*a+x)/(a^2-2*a))^(1/2),((-a^2+2*a)/(-a^2+2*a-1))^(1/2))-2*(-2*a+2)*(a^2-2
*a)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((-1+x)/(-a^2+2*a-1))^(1/2)*(x/(-a^2+2*a)
)^(1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(1/2)/(-a^2+a)*EllipticPi(((a
^2-2*a+x)/(a^2-2*a))^(1/2),(-a^2+2*a)/(-a^2+a),((-a^2+2*a)/(-a^2+2*a-1))^(1
/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}\,dx = \frac{\log\left(-\frac{a^2-2\,(a^2-a)x-x^2+2\,\sqrt{(a^2-2\,a-1)x^2+x^3-(a^2-2\,a)x}a}{a^2-2\,ax+x^2}\right)}{a}$$

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="fricas")
[Out] log(-(a^2 - 2*(a^2 - a)*x - x^2 + 2*sqrt((a^2 - 2*a - 1)*x^2 + x^3 - (a^2 -
 2*a)*x)*a)/(a^2 - 2*a*x + x^2))/a
```

Sympy [F]

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}\,dx = \int \frac{a+x-2}{\sqrt{x\,(x-1)\,(a^2-2a+x)}\,(-a+x)}\,dx$$

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a**2-2*a-1)*x**2+x**3)**(1/2),x)
[Out] Integral((a + x - 2)/(sqrt(x*(x - 1)*(a**2 - 2*a + x))*(-a + x)), x)
```

Maxima [F]

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}\,dx = \int -\frac{a+x-2}{\sqrt{-(a-2)ax+(a^2-2\,a-1)x^2+x^3}(a-x)}\,dx$$

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="maxima")
```

```
[Out] -integrate((a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a -
 x)), x)
```

Giac [F]

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}\,dx = \int -\frac{a+x-2}{\sqrt{-(a-2)ax+(a^2-2\,a-1)x^2+x^3}(a-x)}\,dx$$

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(-(a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a -
 x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.92

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}\,dx = \frac{2\sqrt{\frac{x}{2\,a-a^2}}\,\sqrt{-\frac{x-1}{a^2-2\,a+1}}\,(a-1)^2\,\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a+1}}\,\left(a\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a+1}}\right)\middle|-\frac{a^2-2\,a+1}{2\,a-a^2}\right)-2\,\Pi\left(-\frac{a^2-2\,a+1}{a-a^2};\mathrm{asin}\left(\sqrt{\frac{a^2}{a^2}}\right.\right.\right.}{a\,\sqrt{x^3+(a^2-2\,a-1)\,x^2+(2\,a-a^2)\,x}}$$

```
[In] int(-(a + x - 2)/((a - x)*(x^3 - x^2*(2*a - a^2 + 1) - a*x*(a - 2))^(1/2)),
x)
```

```
[Out] (2*(x/(2*a - a^2))^(1/2)*(-(x - 1)/(a^2 - 2*a + 1))^(1/2)*(a - 1)^2*((x - 2
*a + a^2)/(a^2 - 2*a + 1))^(1/2)*(a*ellipticF(asin(((x - 2*a + a^2)/(a^2 -
2*a + 1))^(1/2)), -(a^2 - 2*a + 1)/(2*a - a^2)) - 2*ellipticPi(-(a^2 - 2*a
+ 1)/(a - a^2), asin(((x - 2*a + a^2)/(a^2 - 2*a + 1))^(1/2)), -(a^2 - 2*a
+ 1)/(2*a - a^2))))/(a*(x*(2*a - a^2) - x^2*(2*a - a^2 + 1) + x^3)^(1/2))
```

3.83 $\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)x^2+(-1+2a)x^3}}\,dx$

Optimal result

Integrand size = 51, antiderivative size = 46

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx = \log\left(\frac{-a^2+2ax+x^2-2\left(x+\sqrt{(1-x)x\,(a^2+x-2ax)}\right)}{(a-x)^2}\right)$$

```
[Out] ln((-a^2+2*a*x+x^2-2*x-2*((1-x)*x*(a^2-2*a*x+x))^(1/2))/(a-x)^2)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.91, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.137, Rules used = {2081, 6865, 1724, 1118, 430, 1234, 551}

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx = \frac{4(1-a)\sqrt{1-x}\sqrt{x}\sqrt{\frac{(1-2a)x}{a^2}+1}\,\text{EllipticPi}\left(\frac{1}{a},\arcsin\left(\sqrt{x}\right),-\frac{1-2a}{a^2}\right)}{\sqrt{(-a^2-2a+1)\,x^2+a^2x-((1-2a)x^3)}} - \frac{2(1-2a)\sqrt{1-x}\sqrt{x}\sqrt{\frac{(1-2a)x}{a^2}+1}\,\text{EllipticF}\left(\arcsin\left(\sqrt{x}\right),-\frac{1-2a}{a^2}\right)}{\sqrt{(-a^2-2a+1)\,x^2+a^2x-((1-2a)x^3)}}$$

```
[In] Int[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 + (-1 +
 2*a)*x^3]),x]
```

```
[Out] (-2*(1 - 2*a)*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*EllipticF[Arc
Sin[Sqrt[x]], -((1 - 2*a)/a^2)])/Sqrt[a^2*x + (1 - 2*a - a^2)*x^2 - (1 - 2*
a)*x^3] + (4*(1 - a)*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*Ellipt
icPi[a^(-1), ArcSin[Sqrt[x]], -((1 - 2*a)/a^2)])/Sqrt[a^2*x + (1 - 2*a - a^
2)*x^2 - (1 - 2*a)*x^3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1118

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2
*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 1234

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/((d + e*x^2)*Sq
rt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1724

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + b*x^2 + c*x^4], x],
 x] + Dist[(e*A - d*B)/e, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

integral

$$=\frac{\left(\sqrt{x}\sqrt{a^2-\left(-1+2a+a^2\right)x+(-1+2a)x^2}\right)\int\frac{-a+(-1+2a)x}{\sqrt{x}(-a+x)\sqrt{a^2-\left(-1+2a+a^2\right)x+(-1+2a)x^2}}\,dx}{\sqrt{a^2x-\left(-1+2a+a^2\right)x^2+(-1+2a)x^3}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{a^2-\left(-1+2a+a^2\right)x+(-1+2a)x^2}\right)\text{Subst}\left(\int\frac{-a+(-1+2a)x^2}{\left(-a+x^2\right)\sqrt{a^2+\left(1-2a-a^2\right)x^2+(-1+2a)x^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{a^2x-\left(-1+2a+a^2\right)x^2+(-1+2a)x^3}}$$

$$=-\frac{\left(4(1-a)a\sqrt{x}\sqrt{a^2-\left(-1+2a+a^2\right)x+(-1+2a)x^2}\right)\text{Subst}\left(\int\frac{1}{\left(-a+x^2\right)\sqrt{a^2+\left(1-2a-a^2\right)x^2+(-1+2a)x^4}}\,dx,x,\ldots\right.}{\sqrt{a^2x-\left(-1+2a+a^2\right)x^2+(-1+2a)x^3}}+\frac{\left(2(-1+2a)\sqrt{x}\sqrt{a^2-\left(-1+2a+a^2\right)x+(-1+2a)x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+\left(1-2a-a^2\right)x^2+(-1+2a)x^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{a^2x-\left(-1+2a+a^2\right)x^2+(-1+2a)x^3}}$$

$$=-\frac{\left(4(1-a)a\sqrt{1-x}\sqrt{x}\sqrt{1+\frac{(1-2a)x}{a^2}}\sqrt{a^2-\left(-1+2a+a^2\right)x+(-1+2a)x^2}\right)\text{Subst}\left(\int\frac{}{\left(-a+x^2\right)\sqrt{1+\frac{2(-\ldots}{1-(-1+\ldots}}}\right.}{\sqrt{a^2+\left(1-2a-a^2\right)x+(-1+2a)x^2}\sqrt{a^2x-\left(-1+2a+a^2\right)x^2+(-1\ldots}}+\frac{\left(2(-1+2a)\sqrt{1-x}\sqrt{x}\sqrt{1+\frac{(1-2a)x}{a^2}}\sqrt{a^2-\left(-1+2a+a^2\right)x+(-1+2a)x^2}\right)\text{Subst}\left(\int\frac{}{\sqrt{1+\frac{2(-1+2a)x^2\ldots}{1-(-1+a)^2-2a\ldots}}}\right.}{\sqrt{a^2+\left(1-2a-a^2\right)x+(-1+2a)x^2}\sqrt{a^2x-\left(-1+2a+a^2\right)x^2+(-1+2\ldots}}$$

$$=-\frac{2(1-2a)\sqrt{1-x}\sqrt{x}\sqrt{1+\frac{(1-2a)x}{a^2}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{x}\right),-\frac{1-2a}{a^2}\right)}{\sqrt{a^2x+\left(1-2a-a^2\right)x^2-(1-2a)x^3}}+\frac{4(1-a)\sqrt{1-x}\sqrt{x}\sqrt{1+\frac{(1-2a)x}{a^2}}\,\text{EllipticPi}\left(\frac{1}{a},\arcsin\left(\sqrt{x}\right),-\frac{1-2a}{a^2}\right)}{\sqrt{a^2x+\left(1-2a-a^2\right)x^2-(1-2a)x^3}}$$

Mathematica [A] (verified)

Time = 10.88 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx = -2\text{arctanh}\left(\frac{\sqrt{a^2x+(1-2a-a^2)\,x^2+(-1+2a)x^3}}{-a^2+(-1+2a)x}\right)$$

```
[In] Integrate[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 +
 (-1 + 2*a)*x^3]),x]
```

```
[Out] -2*ArcTanh[Sqrt[a^2*x + (1 - 2*a - a^2)*x^2 + (-1 + 2*a)*x^3]/(-a^2 + (-1 +
 2*a)*x)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 367, normalized size of antiderivative = 7.98

method	result
elliptic	$-\frac{2a^2\sqrt{-\frac{\left(x-\frac{a^2}{-1+2a}\right)(-1+2a)}{a^2}}\sqrt{\frac{-1+x}{\frac{a^2}{-1+2a}-1}}\sqrt{\frac{x(-1+2a)}{a^2}}\,F\left(\sqrt{-\frac{\left(x-\frac{a^2}{-1+2a}\right)(-1+2a)}{a^2}},\sqrt{\frac{a^2}{(-1+2a)\left(\frac{a^2}{-1+2a}-1\right)}}\right)}{\sqrt{-a^2x^2+2a\,x^3+a^2x-2a\,x^2-x^3+x^2}}-\frac{4a^3(a-1)\sqrt{-\cdots}}{\cdots}$
default	$\frac{2a^2\sqrt{-\frac{\left(x-\frac{a^2}{-1+2a}\right)(-1+2a)}{a^2}}\sqrt{\frac{-1+x}{\frac{a^2}{-1+2a}-1}}\sqrt{\frac{x(-1+2a)}{a^2}}\,F\left(\sqrt{-\frac{\left(x-\frac{a^2}{-1+2a}\right)(-1+2a)}{a^2}},\sqrt{\frac{a^2}{(-1+2a)\left(\frac{a^2}{-1+2a}-1\right)}}\right)}{(-1+2a)\sqrt{-a^2x^2+2a\,x^3+a^2x-2a\,x^2-x^3+x^2}}-\frac{4a^3\sqrt{-\frac{\left(x-\frac{a^2}{-1+\cdots}\right)\cdots}{\cdots}}}{\cdots}$

```
[In] int((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2*a^2*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^2/(-1+2*a)-1))^(1/
2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^(1/2)*El
lipticF((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),(a^2/(-1+2*a)/(a^2/(-1+2*a)-
1))^(1/2))-4*a^3*(a-1)/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((-1
+x)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-
2*a*x^2-x^3+x^2)^(1/2)/(a^2/(-1+2*a)-a)*EllipticPi((-(x-a^2/(-1+2*a))/a^2*(
-1+2*a))^(1/2),a^2/(-1+2*a)/(a^2/(-1+2*a)-a),(a^2/(-1+2*a)/(a^2/(-1+2*a)-1)
)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx = \log\left(-\frac{a^2-2\,(a-1)x-x^2+2\,\sqrt{(2\,a-1)x^3+a^2x-(a^2+2\,a-1)x^2}}{a^2-2\,ax+x^2}\right)$$

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="fricas")
```

```
[Out] log(-(a^2 - 2*(a - 1)*x - x^2 + 2*sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a -
 1)*x^2))/(a^2 - 2*a*x + x^2))
```

Sympy [F]

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx = \int \frac{2ax-a-x}{\sqrt{x\,(x-1)\,(-a^2+2ax-x)}\,(-a+x)}\,dx$$

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a**2*x-(a**2+2*a-1)*x**2+(-1+2*a)*x**3)**
(1/2),x)
```

```
[Out] Integral((2*a*x - a - x)/(sqrt(x*(x - 1)*(-a**2 + 2*a*x - x))*(-a + x)), x)
```

Maxima [F]

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx = \int -\frac{(2\,a-1)x-a}{\sqrt{(2\,a-1)x^3+a^2x-(a^2+2\,a-1)x^2}(a-x)}\,dx$$

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="maxima")
```

```
[Out] -integrate(((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*
x^2)*(a - x)), x)
```

Giac [F]

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx = \int -\frac{(2\,a-1)x-a}{\sqrt{(2\,a-1)x^3+a^2x-(a^2+2\,a-1)x^2}(a-x)}\,dx$$

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate(-((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*
x^2)*(a - x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx = \text{Hanged}$$

```
[In] int((a - x*(2*a - 1))/((a - x)*(x^3*(2*a - 1) - x^2*(2*a + a^2 - 1) + a^2*x
)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

3.84 $\int \frac{1-\sqrt[3]{2}x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 32

$$\int \frac{1-\sqrt[3]{2}x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}$$

```
[Out] 2/3*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2162, 209}

$$\int \frac{1-\sqrt[3]{2}x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

```
[In] Int[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1+\sqrt[3]{2}x}{\sqrt{1+x^3}}\right)$$
$$= \frac{2\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1-\sqrt[3]{2}x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{1+x^3}}{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}\right)}{\sqrt{3}}$$

```
[In] Integrate[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
[Out] (-2*ArcTan[Sqrt[1 + x^3]/(Sqrt[3]*(1 + 2^(1/3)*x))])/Sqrt[3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.96 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.47

method	result
trager	$-\frac{2^{\frac{1}{3}} \mathrm{RootOf}\left(\_Z^{2}+6\,2^{\frac{1}{3}}\right) \ln\left(\frac{12\sqrt{x^{3}+1}\,x+3\,2^{\frac{2}{3}} x^{2} \mathrm{RootOf}\left(\_Z^{2}+6\,2^{\frac{1}{3}}\right)-\mathrm{RootOf}\left(\_Z^{2}+6\,2^{\frac{1}{3}}\right) x^{3}+6\sqrt{x^{3}+1}\,2^{\frac{2}{3}}+6\,\mathrm{RootOf}\left(\_Z^{2}+6\,2^{\frac{1}{3}}\right)}{\left(2^{\frac{1}{3}} x+2\right)^{3}}\right)}{6}$
default	$-\frac{2\,2^{\frac{1}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+}{-\frac{3}{2}+}}}{\sqrt{x^{3}+1}\,(}$
elliptic	$-\frac{2\,2^{\frac{1}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+}{-\frac{3}{2}+}}}{\sqrt{x^{3}+1}\,(}$

```
[In] int((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*2^(1/3)*RootOf(_Z^2+6*2^(1/3))*ln((12*(x^3+1)^(1/2)*x+3*2^(2/3)*x^2*Ro
otOf(_Z^2+6*2^(1/3))-RootOf(_Z^2+6*2^(1/3))*x^3+6*(x^3+1)^(1/2)*2^(2/3)+6*R
ootOf(_Z^2+6*2^(1/3))*2^(1/3)*x+2*RootOf(_Z^2+6*2^(1/3)))/(2^(1/3)*x+2)^3)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt[3]{2}x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]

$$\int \frac{1-\sqrt[3]{2}x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = -\int \frac{\sqrt[3]{2}x}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx - \int \left(-\frac{1}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\right)dx$$

```
[In] integrate((1-2**(1/3)*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2**(1/3)*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x) - Int
egral(-1/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt[3]{2}x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=\int -\frac{2^{\frac{1}{3}}x-1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Giac [F]

$$\int \frac{1-\sqrt[3]{2}x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=\int -\frac{2^{\frac{1}{3}}x-1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.09

$$\int \frac{1-\sqrt[3]{2}x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=\frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}\,1\mathrm{i}+\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)\,{\left(\sqrt{3}\,1\mathrm{i}-\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)}^3}{{\left(x+2^{2/3}\right)}^6}\right)\,1\mathrm{i}}{3}$$

```
[In] int(-(2^(1/3)*x - 1)/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
[Out] (3^(1/2)*log(((3^(1/2)*1i + (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)*(3^(1/2
)*1i - (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)^3)/(x + 2^(2/3))^6)*1i)/3
```

3.85 $\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 23

$$\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx = -\frac{2}{3}\text{arctanh}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)$$

```
[Out] -2/3*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2163, 212}

$$\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx = -\frac{2}{3}\text{arctanh}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

```
[In] Int[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(2\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1+x)^2}{\sqrt{1+x^3}}\right)\right)\\ &= -\frac{2}{3}\text{arctanh}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx = -\frac{2}{3}\text{arctanh}\left(\frac{\frac{1}{3}+\frac{2x}{3}+\frac{x^2}{3}}{\sqrt{1+x^3}}\right)$$

```
[In] Integrate[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
[Out] (-2*ArcTanh[(1/3 + (2*x)/3 + x^2/3)/Sqrt[1 + x^3]])/3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(17) = 34.

Time = 0.82 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

method	result
trager	$-\frac{\ln\left(\frac{x^3+6\sqrt{x^3+1}\,x+12x^2+6\sqrt{x^3+1}-6x+10}{(-2+x)^3}\right)}{3}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3+1}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3+1}}$

```
[In] int((1+x)/(-2+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*ln((x^3+6*(x^3+1)^(1/2)*x+12*x^2+6*(x^3+1)^(1/2)-6*x+10)/(-2+x)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx = \frac{1}{3}\log\left(\frac{x^3+12\,x^2-6\,\sqrt{x^3+1}(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right)$$

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*log((x^3 + 12*x^2 - 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
12*x - 8))
```

Sympy [F]

$$\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{(x+1)\,(x^2-x+1)}\,(x-2)}\,dx$$

```
[In] integrate((1+x)/(-2+x)/(x**3+1)**(1/2),x)
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - 2)), x)
```

Maxima [F]

$$\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Giac [F]

$$\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 204, normalized size of antiderivative = 8.87

$$\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx$$
$$= \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int((x + 1)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
+ 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(((x
+ 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/
2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.86 $\int \frac{x}{\sqrt{1+x^3}(10+6\sqrt{3}+x^3)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 218

$$\int \frac{x}{\sqrt{1+x^3}\,(10+6\sqrt{3}+x^3)}\,dx = -\frac{(2-\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}3^{3/4}} - \frac{(2-\sqrt{3})\arctan\left(\frac{(1-\sqrt{3})\sqrt{1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} - \frac{(2-\sqrt{3})\,\text{arctanh}\left(\frac{\sqrt[4]{3}(1+\sqrt{3}-2x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{(2-\sqrt{3})\,\text{arctanh}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}}$$

```
[Out] -1/12*arctan(1/2*3^(1/4)*(1+x)*(1+3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2
))*3^(1/4)*2^(1/2)-1/18*arctan(1/6*(1-3^(1/2))*(x^3+1)^(1/2)*3^(1/4)*2^(1/2
))*(2-3^(1/2))*3^(1/4)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*(1+x)*(1-3^(1/2))*2
^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*
(1-2*x+3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{500\}$

$$\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}} - \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(-2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}$$

```
[In] Int[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] -1/2*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1
+ x^3])])/(Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[1 +
 x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTanh[(3^
(1/4)*(1 + Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 + x^3])])/(3*Sqrt[2]*3^(1/4)) -
((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^
3])])/(6*Sqrt[2]*3^(1/4))
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\text{integral} = -\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}3^{3/4}} - \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} - \frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}-2x\right)}{\sqrt{2}\sqrt{1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.22

$$\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx = \frac{x^2 \,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-x^3,-\frac{x^3}{10+6\sqrt{3}}\right)}{20+12\sqrt{3}}$$

```
[In] Integrate[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, -(x^3/(10 + 6*Sqrt[3]))])/(20 + 12*Sq
rt[3])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 48.35 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.63

method	result
default	$\frac{2\left(-1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\Pi\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\left(12+6\sqrt{3}\right)\sqrt{x^3+1}} - \frac{\sqrt{2}\left(\underset{\_\alpha=\text{RootOf}(\_Z}{}\right.}{}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{\left(-1-\sqrt{3}\right)^2}{3}+\frac{2\left(-1-\sqrt{3}\right)^2\sqrt{3}}{9}-\frac{2}{3}-\frac{\sqrt{3}}{9}-\frac{2\left(-1-\sqrt{3}\right)\sqrt{3}}{9}\right)\Pi\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{i\left(-1-\sqrt{3}\right)^2}{3}-\right.}{3\left(-1-\sqrt{3}\right)\sqrt{x^3+}}$
trager	Expression too large to display

```
[In] int(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-1-3^(1/2))/(12+6*3^(1/2))*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/
2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I
*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((1+
x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I
*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-_alpha*3^(1/2)+_a
lpha-2)/(-1+2*_alpha-3^(1/2))*(3-I*3^(1/2))*((1+x)/(3-I*3^(1/2)))^(1/2)*((-
I*3^(1/2)+2*x-1)/(-3-I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-3))^(1
/2)/(x^3+1)^(1/2)*(-1+2*_alpha-_alpha*3^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I
*3^(1/2)))^(1/2),-1/2*I*_alpha+1/3*I*_alpha*3^(1/2)+1/2*_alpha*3^(1/2)-_alp
ha-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_al
pha=RootOf(_Z^2+(-1-3^(1/2))*_Z+2*3^(1/2)+4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1569 vs. $2(148) = 296$.

Time = 0.43 (sec) , antiderivative size = 1569, normalized size of antiderivative = 7.20

$$\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(14*sqrt(3) - 24)*arctan(1/12*(3*x^2 + sqrt(3)*(x^2 - 10*x - 8) -
18*x - 12)*sqrt(14*sqrt(3) - 24)/sqrt(x^3 + 1)) + 1/72*sqrt(7*sqrt(3) + 3*s
qrt(56*sqrt(3) - 97) - 12)*log((x^8 - x^7 - 11*x^6 - 16*x^5 - 20*x^4 + 32*x
^3 - 44*x^2 + 2*sqrt(3)*(x^7 - 8*x^6 - 7*x^4 - 16*x^3 - 8*x - 8) + 3*(26*x^
7 + 12*x^6 - 48*x^5 - 98*x^4 - 96*x^3 - 48*x^2 + sqrt(3)*(15*x^7 + 7*x^6 -
28*x^5 - 56*x^4 - 56*x^3 - 28*x^2 - 8*x) - 16*x)*sqrt(56*sqrt(3) - 97) + ((
336*x^5 + 33*x^4 - 132*x^3 - 474*x^2 + sqrt(3)*(194*x^5 + 19*x^4 - 76*x^3 -
 274*x^2 - 152*x - 76) - 264*x - 132)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) - 97) +
 (5*x^6 - 6*x^5 - 33*x^4 - 44*x^3 - 42*x^2 + sqrt(3)*(3*x^6 - 4*x^5 - 17*x^
4 - 28*x^3 - 22*x^2 - 8*x - 4) - 24*x - 4)*sqrt(x^3 + 1))*sqrt(7*sqrt(3) +
3*sqrt(56*sqrt(3) - 97) - 12) + 8*x + 16)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 +
28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)) - 1/72*sqrt(7*sqrt(3) + 3*sqrt(56*sq
rt(3) - 97) - 12)*log((x^8 - x^7 - 11*x^6 - 16*x^5 - 20*x^4 + 32*x^3 - 44*x
^2 + 2*sqrt(3)*(x^7 - 8*x^6 - 7*x^4 - 16*x^3 - 8*x - 8) + 3*(26*x^7 + 12*x^
6 - 48*x^5 - 98*x^4 - 96*x^3 - 48*x^2 + sqrt(3)*(15*x^7 + 7*x^6 - 28*x^5 -
56*x^4 - 56*x^3 - 28*x^2 - 8*x) - 16*x)*sqrt(56*sqrt(3) - 97) - ((336*x^5 +
 33*x^4 - 132*x^3 - 474*x^2 + sqrt(3)*(194*x^5 + 19*x^4 - 76*x^3 - 274*x^2
- 152*x - 76) - 264*x - 132)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) - 97) + (5*x^6 -
 6*x^5 - 33*x^4 - 44*x^3 - 42*x^2 + sqrt(3)*(3*x^6 - 4*x^5 - 17*x^4 - 28*x^
3 - 22*x^2 - 8*x - 4) - 24*x - 4)*sqrt(x^3 + 1))*sqrt(7*sqrt(3) + 3*sqrt(56
*sqrt(3) - 97) - 12) + 8*x + 16)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 +
32*x^3 + 64*x^2 + 32*x + 16)) - 1/72*sqrt(7*sqrt(3) - 3*sqrt(56*sqrt(3) - 9
```

```
7) - 12)*log((x^8 - x^7 - 11*x^6 - 16*x^5 - 20*x^4 + 32*x^3 - 44*x^2 + 2*sq
rt(3)*(x^7 - 8*x^6 - 7*x^4 - 16*x^3 - 8*x - 8) - 3*(26*x^7 + 12*x^6 - 48*x^
5 - 98*x^4 - 96*x^3 - 48*x^2 + sqrt(3)*(15*x^7 + 7*x^6 - 28*x^5 - 56*x^4 -
56*x^3 - 28*x^2 - 8*x) - 16*x)*sqrt(56*sqrt(3) - 97) + ((336*x^5 + 33*x^4 -
 132*x^3 - 474*x^2 + sqrt(3)*(194*x^5 + 19*x^4 - 76*x^3 - 274*x^2 - 152*x -
 76) - 264*x - 132)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) - 97) - (5*x^6 - 6*x^5 -
33*x^4 - 44*x^3 - 42*x^2 + sqrt(3)*(3*x^6 - 4*x^5 - 17*x^4 - 28*x^3 - 22*x^
2 - 8*x - 4) - 24*x - 4)*sqrt(x^3 + 1))*sqrt(7*sqrt(3) - 3*sqrt(56*sqrt(3)
- 97) - 12) + 8*x + 16)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 +
64*x^2 + 32*x + 16)) + 1/72*sqrt(7*sqrt(3) - 3*sqrt(56*sqrt(3) - 97) - 12)*
log((x^8 - x^7 - 11*x^6 - 16*x^5 - 20*x^4 + 32*x^3 - 44*x^2 + 2*sqrt(3)*(x^
7 - 8*x^6 - 7*x^4 - 16*x^3 - 8*x - 8) - 3*(26*x^7 + 12*x^6 - 48*x^5 - 98*x^
4 - 96*x^3 - 48*x^2 + sqrt(3)*(15*x^7 + 7*x^6 - 28*x^5 - 56*x^4 - 56*x^3 -
28*x^2 - 8*x) - 16*x)*sqrt(56*sqrt(3) - 97) - ((336*x^5 + 33*x^4 - 132*x^3
- 474*x^2 + sqrt(3)*(194*x^5 + 19*x^4 - 76*x^3 - 274*x^2 - 152*x - 76) - 26
4*x - 132)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) - 97) - (5*x^6 - 6*x^5 - 33*x^4 -
44*x^3 - 42*x^2 + sqrt(3)*(3*x^6 - 4*x^5 - 17*x^4 - 28*x^3 - 22*x^2 - 8*x -
 4) - 24*x - 4)*sqrt(x^3 + 1))*sqrt(7*sqrt(3) - 3*sqrt(56*sqrt(3) - 97) - 1
2) + 8*x + 16)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 +
32*x + 16))
```

Sympy [F]

$$\int \frac{x}{\sqrt{1+x^3}\,(10+6\sqrt{3}+x^3)}\,dx = \int \frac{x}{\sqrt{(x+1)\,(x^2-x+1)}\,(x^3+10+6\sqrt{3})}\,dx$$

[In] integrate(x/(10+x**3+6*3**(1/2))/(x**3+1)**(1/2),x)

[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 + 10 + 6*sqrt(3))), x)

Maxima [F]

$$\int \frac{x}{\sqrt{1+x^3}\,(10+6\sqrt{3}+x^3)}\,dx = \int \frac{x}{(x^3+6\,\sqrt{3}+10)\sqrt{x^3+1}}\,dx$$

[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)

Giac [F]

$$\int \frac{x}{\sqrt{1+x^3}\,(10+6\sqrt{3}+x^3)}\,dx = \int \frac{x}{(x^3+6\sqrt{3}+10)\sqrt{x^3+1}}\,dx$$

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+x^3}\,(10+6\sqrt{3}+x^3)}\,dx = \int \frac{x}{\sqrt{x^3+1}\,(x^3+6\sqrt{3}+10)}\,dx$$

```
[In] int(x/((x^3 + 1)^(1/2)*(6*3^(1/2) + x^3 + 10)),x)
[Out] int(x/((x^3 + 1)^(1/2)*(6*3^(1/2) + x^3 + 10)), x)
```

3.87 $\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 210

$$\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)} \, dx = -\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}-2x\right)}{\sqrt{2}\sqrt{1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}3^{3/4}} + \frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

```
[Out] -1/18*arctan(1/2*3^(1/4)*(1-2*x-3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2+3^(1/2))
*3^(3/4)*2^(1/2)-1/36*arctan(1/2*3^(1/4)*(1+x)*(1+3^(1/2))*2^(1/2)/(x^3+1)^
(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*(1+x)*(1-3^(1/2
))*2^(1/2)/(x^3+1)^(1/2))*(2+3^(1/2))*3^(1/4)*2^(1/2)+1/18*arctanh(1/6*(1+3
^(1/2))*(x^3+1)^(1/2)*3^(1/4)*2^(1/2))*(2+3^(1/2))*3^(1/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {500}

$$\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(-2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}} + \frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

```
[In] Int[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] -1/3*((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 +
x^3])])/(Sqrt[2]*3^(1/4)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1
 + x))/(Sqrt[2]*Sqrt[1 + x^3])])/(6*Sqrt[2]*3^(1/4)) + ((2 + Sqrt[3])*ArcTa
nh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3])])/(2*Sqrt[2]*3^(
3/4)) + ((2 + Sqrt[3])*ArcTanh[((1 + Sqrt[3])*Sqrt[1 + x^3])/(Sqrt[2]*3^(3/
4))])/(3*Sqrt[2]*3^(3/4))
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\text{integral} = -\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}(1-\sqrt{3}-2x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{(2+\sqrt{3})\operatorname{arctanh}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}3^{3/4}} + \frac{(2+\sqrt{3})\operatorname{arctanh}\left(\frac{(1+\sqrt{3})\sqrt{1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.24

$$\int \frac{x}{\sqrt{1+x^3}\,(10-6\sqrt{3}+x^3)}\,dx = -\frac{x^2 \operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-x^3,\frac{1}{4}(5+3\sqrt{3})\,x^3\right)}{4\,(-5+3\sqrt{3})}$$

```
[In] Integrate[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]
[Out] -1/4*(x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, ((5 + 3*Sqrt[3])*x^3)/4])/(-5 +
3*Sqrt[3])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 48.71 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.68

method	result
default	$\frac{2(\sqrt{3}-1)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\Pi\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3(6\sqrt{3}-12)\sqrt{x^3+1}} - \frac{\sqrt{2}\left(\_\alpha=\text{RootOf}(\_Z\right.}{}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{2(\sqrt{3}-1)^2\sqrt{3}}{9}-\frac{(\sqrt{3}-1)^2}{3}+\frac{2(\sqrt{3}-1)\sqrt{3}}{9}+\frac{\sqrt{3}}{9}-\frac{2}{3}\right)\Pi\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{i(\sqrt{3}-1)^2\sqrt{3}}{6}-\right.}{3(\sqrt{3}-1)\sqrt{x^3+1}}$
trager	Expression too large to display

```
[In] int(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(3^(1/2)-1)/(6*3^(1/2)-12)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2
)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*
3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((1+x
)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I
*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-_alpha*3^(1/2)-_a
lpha+2)/(1-2*_alpha-3^(1/2))*(3-I*3^(1/2))*((1+x)/(3-I*3^(1/2)))^(1/2)*((-I
*3^(1/2)+2*x-1)/(-3-I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-3))^(1/
2)/(x^3+1)^(1/2)*(-1+2*_alpha+_alpha*3^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*
3^(1/2)))^(1/2),1/3*I*_alpha*3^(1/2)+1/2*I*_alpha-1/2*_alpha*3^(1/2)-_alpha
-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alph
a=RootOf(_Z^2+(3^(1/2)-1)*_Z-2*3^(1/2)+4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1702 vs. $2(146) = 292$.

Time = 0.39 (sec) , antiderivative size = 1702, normalized size of antiderivative = 8.10

$$\int \frac{x}{\sqrt{1+x^3}\,(10-6\sqrt{3}+x^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(-7*sqrt(3) + 3*sqrt(-56*sqrt(3) - 97) - 12)*log((x^8 - x^7 - 11*x
^6 - 16*x^5 - 20*x^4 + 32*x^3 - 44*x^2 + (5*x^6 - 6*x^5 - 33*x^4 - 44*x^3 -
 42*x^2 - sqrt(3)*(3*x^6 - 4*x^5 - 17*x^4 - 28*x^3 - 22*x^2 - 8*x - 4) + (3
36*x^5 + 33*x^4 - 132*x^3 - 474*x^2 - sqrt(3)*(194*x^5 + 19*x^4 - 76*x^3 -
274*x^2 - 152*x - 76) - 264*x - 132)*sqrt(-56*sqrt(3) - 97) - 24*x - 4)*sqr
t(x^3 + 1)*sqrt(-7*sqrt(3) + 3*sqrt(-56*sqrt(3) - 97) - 12) - 2*sqrt(3)*(x^
7 - 8*x^6 - 7*x^4 - 16*x^3 - 8*x - 8) + 3*(26*x^7 + 12*x^6 - 48*x^5 - 98*x^
4 - 96*x^3 - 48*x^2 - sqrt(3)*(15*x^7 + 7*x^6 - 28*x^5 - 56*x^4 - 56*x^3 -
28*x^2 - 8*x) - 16*x)*sqrt(-56*sqrt(3) - 97) + 8*x + 16)/(x^8 - 4*x^7 + 16*
x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)) - 1/72*sqrt(-7*sqrt(3
) + 3*sqrt(-56*sqrt(3) - 97) - 12)*log((x^8 - x^7 - 11*x^6 - 16*x^5 - 20*x^
4 + 32*x^3 - 44*x^2 - (5*x^6 - 6*x^5 - 33*x^4 - 44*x^3 - 42*x^2 - sqrt(3)*(
3*x^6 - 4*x^5 - 17*x^4 - 28*x^3 - 22*x^2 - 8*x - 4) + (336*x^5 + 33*x^4 - 1
32*x^3 - 474*x^2 - sqrt(3)*(194*x^5 + 19*x^4 - 76*x^3 - 274*x^2 - 152*x - 7
6) - 264*x - 132)*sqrt(-56*sqrt(3) - 97) - 24*x - 4)*sqrt(x^3 + 1)*sqrt(-7*
sqrt(3) + 3*sqrt(-56*sqrt(3) - 97) - 12) - 2*sqrt(3)*(x^7 - 8*x^6 - 7*x^4 -
 16*x^3 - 8*x - 8) + 3*(26*x^7 + 12*x^6 - 48*x^5 - 98*x^4 - 96*x^3 - 48*x^2
 - sqrt(3)*(15*x^7 + 7*x^6 - 28*x^5 - 56*x^4 - 56*x^3 - 28*x^2 - 8*x) - 16*
x)*sqrt(-56*sqrt(3) - 97) + 8*x + 16)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x
^4 + 32*x^3 + 64*x^2 + 32*x + 16)) + 1/72*sqrt(-7*sqrt(3) - 3*sqrt(-56*sqrt
(3) - 97) - 12)*log((x^8 - x^7 - 11*x^6 - 16*x^5 - 20*x^4 + 32*x^3 - 44*x^2
 + (5*x^6 - 6*x^5 - 33*x^4 - 44*x^3 - 42*x^2 - sqrt(3)*(3*x^6 - 4*x^5 - 17*
```

```
x^4 - 28*x^3 - 22*x^2 - 8*x - 4) - (336*x^5 + 33*x^4 - 132*x^3 - 474*x^2 -
sqrt(3)*(194*x^5 + 19*x^4 - 76*x^3 - 274*x^2 - 152*x - 76) - 264*x - 132)*s
qrt(-56*sqrt(3) - 97) - 24*x - 4)*sqrt(x^3 + 1)*sqrt(-7*sqrt(3) - 3*sqrt(-5
6*sqrt(3) - 97) - 12) - 2*sqrt(3)*(x^7 - 8*x^6 - 7*x^4 - 16*x^3 - 8*x - 8)
- 3*(26*x^7 + 12*x^6 - 48*x^5 - 98*x^4 - 96*x^3 - 48*x^2 - sqrt(3)*(15*x^7
+ 7*x^6 - 28*x^5 - 56*x^4 - 56*x^3 - 28*x^2 - 8*x) - 16*x)*sqrt(-56*sqrt(3)
 - 97) + 8*x + 16)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^
2 + 32*x + 16)) - 1/72*sqrt(-7*sqrt(3) - 3*sqrt(-56*sqrt(3) - 97) - 12)*log
((x^8 - x^7 - 11*x^6 - 16*x^5 - 20*x^4 + 32*x^3 - 44*x^2 - (5*x^6 - 6*x^5 -
 33*x^4 - 44*x^3 - 42*x^2 - sqrt(3)*(3*x^6 - 4*x^5 - 17*x^4 - 28*x^3 - 22*x
^2 - 8*x - 4) - (336*x^5 + 33*x^4 - 132*x^3 - 474*x^2 - sqrt(3)*(194*x^5 +
19*x^4 - 76*x^3 - 274*x^2 - 152*x - 76) - 264*x - 132)*sqrt(-56*sqrt(3) - 9
7) - 24*x - 4)*sqrt(x^3 + 1)*sqrt(-7*sqrt(3) - 3*sqrt(-56*sqrt(3) - 97) - 1
2) - 2*sqrt(3)*(x^7 - 8*x^6 - 7*x^4 - 16*x^3 - 8*x - 8) - 3*(26*x^7 + 12*x^
6 - 48*x^5 - 98*x^4 - 96*x^3 - 48*x^2 - sqrt(3)*(15*x^7 + 7*x^6 - 28*x^5 -
56*x^4 - 56*x^3 - 28*x^2 - 8*x) - 16*x)*sqrt(-56*sqrt(3) - 97) + 8*x + 16)/
(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)) + 1
/72*sqrt(14*sqrt(3) + 24)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 +
224*x^3 + 64*x^2 + 2*(5*x^6 - 54*x^5 + 96*x^4 - 56*x^3 - 36*x^2 - 3*sqrt(3)
*(x^6 - 10*x^5 + 20*x^4 - 8*x^3 - 4*x^2 + 8*x) + 24*x - 16)*sqrt(x^3 + 1)*s
qrt(14*sqrt(3) + 24) + 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*
x^2 + 4*x + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*
x^3 + 64*x^2 - 64*x + 16))
```

Sympy [F]

$$\int \frac{x}{\sqrt{1+x^3}\,(10-6\sqrt{3}+x^3)}\,dx = \int \frac{x}{\sqrt{(x+1)\,(x^2-x+1)}\,(x^3-6\sqrt{3}+10)}\,dx$$

```
[In] integrate(x/(10+x**3-6*3**(1/2))/(x**3+1)**(1/2),x)
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 - 6*sqrt(3) + 10)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1+x^3}\,(10-6\sqrt{3}+x^3)}\,dx = \int \frac{x}{(x^3-6\sqrt{3}+10)\sqrt{x^3+1}}\,dx$$

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx = \int \frac{x}{\left(x^3-6\sqrt{3}+10\right)\sqrt{x^3+1}}\,dx$$

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx = \int \frac{x}{\sqrt{x^3+1}\,\left(x^3-6\sqrt{3}+10\right)}\,dx$$

```
[In] int(x/((x^3 + 1)^(1/2)*(x^3 - 6*3^(1/2) + 10)),x)
[Out] int(x/((x^3 + 1)^(1/2)*(x^3 - 6*3^(1/2) + 10)), x)
```

3.88 $\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 222

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx = \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}+2x\right)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} + \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{2\sqrt{2}3^{3/4}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

```
[Out] 1/36*arctan(1/2*3^(1/4)*(1-x)*(1-3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2-3^(1/2)
)*3^(3/4)*2^(1/2)+1/18*arctan(1/2*3^(1/4)*(1+2*x+3^(1/2))*2^(1/2)/(x^3-1)^(
1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*(1-x)*(1+3^(1/2)
)*2^(1/2)/(x^3-1)^(1/2))*(2-3^(1/2))*3^(1/4)*2^(1/2)-1/18*arctanh(1/6*(1-3^
(1/2))*(x^3-1)^(1/2)*3^(1/4)*2^(1/2))*(2-3^(1/2))*3^(1/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {501}

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx = \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}} + \frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}} - \frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

```
[In] Int[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^
3])])/(6*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3] + 2
*x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(3*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan
h[(3^(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(2*Sqrt[2]*3^(
3/4)) - ((2 - Sqrt[3])*ArcTanh[((1 - Sqrt[3])*Sqrt[-1 + x^3])/(Sqrt[2]*3^(3
/4))])/(3*Sqrt[2]*3^(3/4))
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\text{integral} = \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}+2x\right)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} + \frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{2\sqrt{2}3^{3/4}} - \frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.29

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx = -\frac{x^2\sqrt{1-x^3}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{10+6\sqrt{3}}\right)}{\left(20+12\sqrt{3}\right)\sqrt{-1+x^3}}$$

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]
[Out] -((x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/(10 + 6*Sqrt[3])])
/((20 + 12*Sqrt[3])*Sqrt[-1 + x^3]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 41.43 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.58

method	result
default	$\frac{2\left(-1-\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\Pi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\left(12+6\sqrt{3}\right)\sqrt{x^3-1}} - \frac{\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}} \cdots\right.}{\cdots}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\frac{\left(1+\sqrt{3}\right)^2}{3}-\frac{2\left(1+\sqrt{3}\right)^2\sqrt{3}}{9}+\frac{2}{3}+\frac{\sqrt{3}}{9}-\frac{2\left(1+\sqrt{3}\right)\sqrt{3}}{9}\right)\Pi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{i\left(1+\sqrt{3}\right)^2}{3}+\cdots\right.}{3\left(1+\sqrt{3}\right)\sqrt{x^3-1}\cdots}$
trager	Expression too large to display

```
[In] int(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-1-3^(1/2))/(12+6*3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^
(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2
*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((-
1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2
*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-_alpha*3^(1/2)+_
alpha+2)/(-1-2*_alpha-3^(1/2))*(-3-I*3^(1/2))*((-1+x)/(-3-I*3^(1/2)))^(1/2)
*((-I*3^(1/2)+2*x+1)/(3-I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x+1)/(I*3^(1/2)+3))
^(1/2)/(x^3-1)^(1/2)*(1+2*_alpha-_alpha*3^(1/2))*EllipticPi(((-1+x)/(-3/2-1
/2*I*3^(1/2)))^(1/2),-1/2*I*_alpha+1/3*I*_alpha*3^(1/2)-1/2*_alpha*3^(1/2)+
_alpha+1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_
alpha=RootOf(_Z^2+(1+3^(1/2))*_Z+2*3^(1/2)+4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1718 vs. $2(146) = 292$.

Time = 0.40 (sec) , antiderivative size = 1718, normalized size of antiderivative = 7.74

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/72*sqrt(-7*sqrt(3) + 3*sqrt(56*sqrt(3) - 97) + 12)*log((x^8 + x^7 - 11*x
^6 + 16*x^5 - 20*x^4 - 32*x^3 - 44*x^2 - 2*sqrt(3)*(x^7 + 8*x^6 + 7*x^4 - 1
6*x^3 - 8*x + 8) + 3*(26*x^7 - 12*x^6 - 48*x^5 + 98*x^4 - 96*x^3 + 48*x^2 +
 sqrt(3)*(15*x^7 - 7*x^6 - 28*x^5 + 56*x^4 - 56*x^3 + 28*x^2 - 8*x) - 16*x)
*sqrt(56*sqrt(3) - 97) + ((336*x^5 - 33*x^4 - 132*x^3 + 474*x^2 + sqrt(3)*(
194*x^5 - 19*x^4 - 76*x^3 + 274*x^2 - 152*x + 76) - 264*x + 132)*sqrt(x^3 -
 1)*sqrt(56*sqrt(3) - 97) + (5*x^6 + 6*x^5 - 33*x^4 + 44*x^3 - 42*x^2 + sqr
t(3)*(3*x^6 + 4*x^5 - 17*x^4 + 28*x^3 - 22*x^2 + 8*x - 4) + 24*x - 4)*sqrt(
x^3 - 1))*sqrt(-7*sqrt(3) + 3*sqrt(56*sqrt(3) - 97) + 12) - 8*x + 16)/(x^8
+ 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) + 1/72*s
qrt(-7*sqrt(3) + 3*sqrt(56*sqrt(3) - 97) + 12)*log((x^8 + x^7 - 11*x^6 + 16
*x^5 - 20*x^4 - 32*x^3 - 44*x^2 - 2*sqrt(3)*(x^7 + 8*x^6 + 7*x^4 - 16*x^3 -
 8*x + 8) + 3*(26*x^7 - 12*x^6 - 48*x^5 + 98*x^4 - 96*x^3 + 48*x^2 + sqrt(3
)*(15*x^7 - 7*x^6 - 28*x^5 + 56*x^4 - 56*x^3 + 28*x^2 - 8*x) - 16*x)*sqrt(5
6*sqrt(3) - 97) - ((336*x^5 - 33*x^4 - 132*x^3 + 474*x^2 + sqrt(3)*(194*x^5
 - 19*x^4 - 76*x^3 + 274*x^2 - 152*x + 76) - 264*x + 132)*sqrt(x^3 - 1)*sqr
t(56*sqrt(3) - 97) + (5*x^6 + 6*x^5 - 33*x^4 + 44*x^3 - 42*x^2 + sqrt(3)*(3
*x^6 + 4*x^5 - 17*x^4 + 28*x^3 - 22*x^2 + 8*x - 4) + 24*x - 4)*sqrt(x^3 - 1
))*sqrt(-7*sqrt(3) + 3*sqrt(56*sqrt(3) - 97) + 12) - 8*x + 16)/(x^8 + 4*x^7
 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) + 1/72*sqrt(-7*
sqrt(3) - 3*sqrt(56*sqrt(3) - 97) + 12)*log((x^8 + x^7 - 11*x^6 + 16*x^5 -
20*x^4 - 32*x^3 - 44*x^2 - 2*sqrt(3)*(x^7 + 8*x^6 + 7*x^4 - 16*x^3 - 8*x +
```

```
8) - 3*(26*x^7 - 12*x^6 - 48*x^5 + 98*x^4 - 96*x^3 + 48*x^2 + sqrt(3)*(15*x
^7 - 7*x^6 - 28*x^5 + 56*x^4 - 56*x^3 + 28*x^2 - 8*x) - 16*x)*sqrt(56*sqrt(
3) - 97) + ((336*x^5 - 33*x^4 - 132*x^3 + 474*x^2 + sqrt(3)*(194*x^5 - 19*x
^4 - 76*x^3 + 274*x^2 - 152*x + 76) - 264*x + 132)*sqrt(x^3 - 1)*sqrt(56*sq
rt(3) - 97) - (5*x^6 + 6*x^5 - 33*x^4 + 44*x^3 - 42*x^2 + sqrt(3)*(3*x^6 +
4*x^5 - 17*x^4 + 28*x^3 - 22*x^2 + 8*x - 4) + 24*x - 4)*sqrt(x^3 - 1))*sqrt
(-7*sqrt(3) - 3*sqrt(56*sqrt(3) - 97) + 12) - 8*x + 16)/(x^8 + 4*x^7 + 16*x
^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 1/72*sqrt(-7*sqrt(3)
 - 3*sqrt(56*sqrt(3) - 97) + 12)*log((x^8 + x^7 - 11*x^6 + 16*x^5 - 20*x^4
- 32*x^3 - 44*x^2 - 2*sqrt(3)*(x^7 + 8*x^6 + 7*x^4 - 16*x^3 - 8*x + 8) - 3*
(26*x^7 - 12*x^6 - 48*x^5 + 98*x^4 - 96*x^3 + 48*x^2 + sqrt(3)*(15*x^7 - 7*
x^6 - 28*x^5 + 56*x^4 - 56*x^3 + 28*x^2 - 8*x) - 16*x)*sqrt(56*sqrt(3) - 97
) - ((336*x^5 - 33*x^4 - 132*x^3 + 474*x^2 + sqrt(3)*(194*x^5 - 19*x^4 - 76
*x^3 + 274*x^2 - 152*x + 76) - 264*x + 132)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) -
 97) - (5*x^6 + 6*x^5 - 33*x^4 + 44*x^3 - 42*x^2 + sqrt(3)*(3*x^6 + 4*x^5 -
 17*x^4 + 28*x^3 - 22*x^2 + 8*x - 4) + 24*x - 4)*sqrt(x^3 - 1))*sqrt(-7*sqr
t(3) - 3*sqrt(56*sqrt(3) - 97) + 12) - 8*x + 16)/(x^8 + 4*x^7 + 16*x^6 + 16
*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) + 1/72*sqrt(14*sqrt(3) - 24)*
log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*x^4 - 224*x^3 + 64*x^2 - 2*(5*x^
6 + 54*x^5 + 96*x^4 + 56*x^3 - 36*x^2 + 3*sqrt(3)*(x^6 + 10*x^5 + 20*x^4 +
8*x^3 - 4*x^2 - 8*x) - 24*x - 16)*sqrt(x^3 - 1)*sqrt(14*sqrt(3) - 24) + 16*
sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*x - 4) - 128*x + 1
12)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16))
```

Sympy [F]

$$\int \frac{x}{\sqrt{-1+x^3}\,\left(-10-6\sqrt{3}+x^3\right)}\,dx = \int \frac{x}{\sqrt{(x-1)\,(x^2+x+1)}\,\left(x^3-6\sqrt{3}-10\right)}\,dx$$

```
[In] integrate(x/(-10+x**3-6*3**(1/2))/(x**3-1)**(1/2),x)
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 6*sqrt(3) - 10)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{-1+x^3}\,\left(-10-6\sqrt{3}+x^3\right)}\,dx = \int \frac{x}{\left(x^3-6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx = \int \frac{x}{\left(x^3-6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx = \int -\frac{x}{\sqrt{x^3-1}\left(-x^3+6\sqrt{3}+10\right)}\,dx$$

```
[In] int(-x/((x^3 - 1)^(1/2)*(6*3^(1/2) - x^3 + 10)),x)
[Out] int(-x/((x^3 - 1)^(1/2)*(6*3^(1/2) - x^3 + 10)), x)
```

3.89 $\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{2\sqrt{2}3^{3/4}} + \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} + \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}+2x\right)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}}$$

```
[Out] -1/12*arctan(1/2*3^(1/4)*(1-x)*(1-3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2+3^(1/2
))*3^(1/4)*2^(1/2)+1/18*arctan(1/6*(1+3^(1/2))*(x^3-1)^(1/2)*3^(1/4)*2^(1/2
))*(2+3^(1/2))*3^(1/4)*2^(1/2)+1/18*arctanh(1/2*3^(1/4)*(1+2*x-3^(1/2))*2^(
1/2)/(x^3-1)^(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)+1/36*arctanh(1/2*3^(1/4)*(1
-x)*(1+3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {501}

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}} + \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} + \frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}$$

```
[In] Int[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] -1/2*((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1
 + x^3])])/(Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-1
 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTanh[(
3^(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(6*Sqrt[2]*3^(1/4
)) + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3] + 2*x))/(Sqrt[2]*Sqrt[-1
+ x^3])])/(3*Sqrt[2]*3^(1/4))
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\text{integral} = -\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}(1-\sqrt{3})(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{2\sqrt{2}3^{3/4}} + \frac{(2+\sqrt{3})\arctan\left(\frac{(1+\sqrt{3})\sqrt{-1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} + \frac{(2+\sqrt{3})\operatorname{arctanh}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{(2+\sqrt{3})\operatorname{arctanh}\left(\frac{\sqrt[4]{3}(1-\sqrt{3}+2x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.32

$$\int \frac{x}{\sqrt{-1+x^3}\,(-10+6\sqrt{3}+x^3)}\,dx = \frac{x^2\sqrt{1-x^3}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,-\frac{x^3}{-10+6\sqrt{3}}\right)}{4\,(-5+3\sqrt{3})\,\sqrt{-1+x^3}}$$

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)),x]
[Out] (x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, -(x^3/(-10 + 6*Sqrt[3]))
])/(4*(-5 + 3*Sqrt[3])*Sqrt[-1 + x^3])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 41.40 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.64

method	result
default	$\frac{2\left(\sqrt{3}-1\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\Pi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\left(6\sqrt{3}-12\right)\sqrt{x^3-1}} - \frac{\sqrt{2}\left(\underset{\_\alpha=\text{RootOf}(\_Z}{}\right.}{}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\frac{2\left(1-\sqrt{3}\right)^2\sqrt{3}}{9}+\frac{\left(1-\sqrt{3}\right)^2}{3}+\frac{2\left(1-\sqrt{3}\right)\sqrt{3}}{9}+\frac{2}{3}-\frac{\sqrt{3}}{9}\right)\Pi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{i\left(1-\sqrt{3}\right)^2\sqrt{3}}{6}+\right.}{3\left(1-\sqrt{3}\right)\sqrt{x^3-1}}$
trager	Expression too large to display

```
[In] int(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(3^(1/2)-1)/(6*3^(1/2)-12)*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(
1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*
I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((-1
+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-_alpha*3^(1/2)-_al
pha-2)/(-3^(1/2)+2*_alpha+1)*(-3-I*3^(1/2))*((-1+x)/(-3-I*3^(1/2)))^(1/2)*(
(-I*3^(1/2)+2*x+1)/(3-I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x+1)/(I*3^(1/2)+3))^(
1/2)/(x^3-1)^(1/2)*(1+2*_alpha+_alpha*3^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2
*I*3^(1/2)))^(1/2),1/3*I*_alpha*3^(1/2)+1/2*I*_alpha+1/2*_alpha*3^(1/2)+_al
pha+1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alp
ha=RootOf(_Z^2+(1-3^(1/2))*_Z-2*3^(1/2)+4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1538 vs. $2(148) = 296$.

Time = 0.39 (sec) , antiderivative size = 1538, normalized size of antiderivative = 7.19

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(14*sqrt(3) + 24)*arctan(-1/12*(3*x^2 - sqrt(3)*(x^2 + 10*x - 8)
+ 18*x - 12)*sqrt(14*sqrt(3) + 24)/sqrt(x^3 - 1)) - 1/72*sqrt(7*sqrt(3) + 3
*sqrt(-56*sqrt(3) - 97) + 12)*log((x^8 + x^7 - 11*x^6 + 16*x^5 - 20*x^4 - 3
2*x^3 - 44*x^2 + (5*x^6 + 6*x^5 - 33*x^4 + 44*x^3 - 42*x^2 - sqrt(3)*(3*x^6
 + 4*x^5 - 17*x^4 + 28*x^3 - 22*x^2 + 8*x - 4) + (336*x^5 - 33*x^4 - 132*x^
3 + 474*x^2 - sqrt(3)*(194*x^5 - 19*x^4 - 76*x^3 + 274*x^2 - 152*x + 76) -
264*x + 132)*sqrt(-56*sqrt(3) - 97) + 24*x - 4)*sqrt(x^3 - 1)*sqrt(7*sqrt(3
) + 3*sqrt(-56*sqrt(3) - 97) + 12) + 2*sqrt(3)*(x^7 + 8*x^6 + 7*x^4 - 16*x^
3 - 8*x + 8) + 3*(26*x^7 - 12*x^6 - 48*x^5 + 98*x^4 - 96*x^3 + 48*x^2 - sqr
t(3)*(15*x^7 - 7*x^6 - 28*x^5 + 56*x^4 - 56*x^3 + 28*x^2 - 8*x) - 16*x)*sqr
t(-56*sqrt(3) - 97) - 8*x + 16)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 3
2*x^3 + 64*x^2 - 32*x + 16)) + 1/72*sqrt(7*sqrt(3) + 3*sqrt(-56*sqrt(3) - 9
7) + 12)*log((x^8 + x^7 - 11*x^6 + 16*x^5 - 20*x^4 - 32*x^3 - 44*x^2 - (5*x
^6 + 6*x^5 - 33*x^4 + 44*x^3 - 42*x^2 - sqrt(3)*(3*x^6 + 4*x^5 - 17*x^4 + 2
8*x^3 - 22*x^2 + 8*x - 4) + (336*x^5 - 33*x^4 - 132*x^3 + 474*x^2 - sqrt(3)
*(194*x^5 - 19*x^4 - 76*x^3 + 274*x^2 - 152*x + 76) - 264*x + 132)*sqrt(-56
*sqrt(3) - 97) + 24*x - 4)*sqrt(x^3 - 1)*sqrt(7*sqrt(3) + 3*sqrt(-56*sqrt(3
) - 97) + 12) + 2*sqrt(3)*(x^7 + 8*x^6 + 7*x^4 - 16*x^3 - 8*x + 8) + 3*(26*
x^7 - 12*x^6 - 48*x^5 + 98*x^4 - 96*x^3 + 48*x^2 - sqrt(3)*(15*x^7 - 7*x^6
- 28*x^5 + 56*x^4 - 56*x^3 + 28*x^2 - 8*x) - 16*x)*sqrt(-56*sqrt(3) - 97) -
 8*x + 16)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x
 + 16)) - 1/72*sqrt(7*sqrt(3) - 3*sqrt(-56*sqrt(3) - 97) + 12)*log((x^8 + x
```

```
^7 - 11*x^6 + 16*x^5 - 20*x^4 - 32*x^3 - 44*x^2 + (5*x^6 + 6*x^5 - 33*x^4 +
 44*x^3 - 42*x^2 - sqrt(3)*(3*x^6 + 4*x^5 - 17*x^4 + 28*x^3 - 22*x^2 + 8*x
- 4) - (336*x^5 - 33*x^4 - 132*x^3 + 474*x^2 - sqrt(3)*(194*x^5 - 19*x^4 -
76*x^3 + 274*x^2 - 152*x + 76) - 264*x + 132)*sqrt(-56*sqrt(3) - 97) + 24*x
 - 4)*sqrt(x^3 - 1)*sqrt(7*sqrt(3) - 3*sqrt(-56*sqrt(3) - 97) + 12) + 2*sqr
t(3)*(x^7 + 8*x^6 + 7*x^4 - 16*x^3 - 8*x + 8) - 3*(26*x^7 - 12*x^6 - 48*x^5
 + 98*x^4 - 96*x^3 + 48*x^2 - sqrt(3)*(15*x^7 - 7*x^6 - 28*x^5 + 56*x^4 - 5
6*x^3 + 28*x^2 - 8*x) - 16*x)*sqrt(-56*sqrt(3) - 97) - 8*x + 16)/(x^8 + 4*x
^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) + 1/72*sqrt(7
*sqrt(3) - 3*sqrt(-56*sqrt(3) - 97) + 12)*log((x^8 + x^7 - 11*x^6 + 16*x^5
- 20*x^4 - 32*x^3 - 44*x^2 - (5*x^6 + 6*x^5 - 33*x^4 + 44*x^3 - 42*x^2 - sq
rt(3)*(3*x^6 + 4*x^5 - 17*x^4 + 28*x^3 - 22*x^2 + 8*x - 4) - (336*x^5 - 33*
x^4 - 132*x^3 + 474*x^2 - sqrt(3)*(194*x^5 - 19*x^4 - 76*x^3 + 274*x^2 - 15
2*x + 76) - 264*x + 132)*sqrt(-56*sqrt(3) - 97) + 24*x - 4)*sqrt(x^3 - 1)*s
qrt(7*sqrt(3) - 3*sqrt(-56*sqrt(3) - 97) + 12) + 2*sqrt(3)*(x^7 + 8*x^6 + 7
*x^4 - 16*x^3 - 8*x + 8) - 3*(26*x^7 - 12*x^6 - 48*x^5 + 98*x^4 - 96*x^3 +
48*x^2 - sqrt(3)*(15*x^7 - 7*x^6 - 28*x^5 + 56*x^4 - 56*x^3 + 28*x^2 - 8*x)
 - 16*x)*sqrt(-56*sqrt(3) - 97) - 8*x + 16)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5
+ 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16))
```

Sympy [F]

$$\int \frac{x}{\sqrt{-1+x^3}\,(-10+6\sqrt{3}+x^3)}\,dx = \int \frac{x}{\sqrt{(x-1)\,(x^2+x+1)}\,(x^3-10+6\sqrt{3})}\,dx$$

```
[In] integrate(x/(-10+x**3+6*3**(1/2))/(x**3-1)**(1/2),x)
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 10 + 6*sqrt(3))), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{-1+x^3}\,(-10+6\sqrt{3}+x^3)}\,dx = \int \frac{x}{(x^3+6\sqrt{3}-10)\sqrt{x^3-1}}\,dx$$

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{-1+x^3}\,(-10+6\sqrt{3}+x^3)}\,dx = \int \frac{x}{(x^3+6\sqrt{3}-10)\sqrt{x^3-1}}\,dx$$

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-1+x^3}\,(-10+6\sqrt{3}+x^3)}\,dx = \int \frac{x}{\sqrt{x^3-1}\,(x^3+6\sqrt{3}-10)}\,dx$$

```
[In] int(x/((x^3 - 1)^(1/2)*(6*3^(1/2) + x^3 - 10)),x)
[Out] int(x/((x^3 - 1)^(1/2)*(6*3^(1/2) + x^3 - 10)), x)
```

3.90 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 65

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \frac{1}{3}\sqrt{-3+2\sqrt{3}}\text{arctanh}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-4+4\sqrt{3}x^2+x^4}}\right)$$

```
[Out] 1/3*arctanh((1+x-3^(1/2))^2/(-9+6*3^(1/2))^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/
2))*(-3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1754, 213}

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \frac{1}{3}\sqrt{2\sqrt{3}-3}\text{arctanh}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)$$

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3])]
*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1754

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Dist[(-A^2)*((B*d + A*e)/e), Subst[Int[1/(6*A^3*B
*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /;
 FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^
4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && E
qQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\text{integral} = -\left(\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{3\left(1-\sqrt{3}\right)^4+6\left(1-\sqrt{3}\right)^3\left(1+\sqrt{3}\right)+4x^2}\,dx, x, \frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{-4+4\sqrt{3}x^2+x^4}}\right)\right)$$

$$= \frac{1}{3}\sqrt{-3+2\sqrt{3}}\text{arctanh}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-4+4\sqrt{3}x^2+x^4}}\right)$$

Mathematica [A] (verified)

Time = 8.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \frac{1}{3}\sqrt{-3+2\sqrt{3}}\text{arctanh}\left(\frac{\sqrt{9+6\sqrt{3}}\sqrt{-4+4\sqrt{3}x^2+x^4}}{2+\left(-2-2\sqrt{3}\right)x+\left(2+\sqrt{3}\right)x^2}\right)$$

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^
4]),x]

[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(Sqrt[9 + 6*Sqrt[3]]*Sqrt[-4 + 4*Sqrt[3]*x^2
+ x^4])/(2 + (-2 - 2*Sqrt[3])*x + (2 + Sqrt[3])*x^2)])/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 2.30 (sec) , antiderivative size = 327, normalized size of antiderivative = 5.03

method	result
elliptic	$\frac{\sqrt{1-\left(-1+\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(\frac{\sqrt{3}}{2}+1\right)x^2}\,F\left(x\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right), i\sqrt{1+4\sqrt{3}\left(\frac{\sqrt{3}}{2}+1\right)}\right)}{\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{-4+x^4+4\sqrt{3}\,x^2}}-2\sqrt{3}\left(-\frac{\operatorname{arctanh}\left(\frac{4\left(-1-\sqrt{3}\right)^2\sqrt{3}-8+4\sqrt{3}\,x^2+2x}{2\sqrt{\left(-1-\sqrt{3}\right)^4+4\left(-1-\sqrt{3}\right)^2\sqrt{3}-4}}\right.}{2\sqrt{\left(-1-\sqrt{3}\right)^4+4\left(-1-\sqrt{3}\right)^2}}\right.$

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/(1/2*I*3^(1/2)-1/2*I)*(1-(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1/2*3^(1/2)+1)*x
^2)^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2*I*3^(1/2)-1/2*I),I*
(1+4*3^(1/2)*(1/2*3^(1/2)+1))^(1/2))-2*3^(1/2)*(-1/2/((-1-3^(1/2))^4+4*(-1-
3^(1/2))^2*3^(1/2)-4)^(1/2)*arctanh(1/2*(4*(-1-3^(1/2))^2*3^(1/2)-8+4*3^(1/
2)*x^2+2*x^2*(-1-3^(1/2))^2)/((-1-3^(1/2))^4+4*(-1-3^(1/2))^2*3^(1/2)-4)^(1
/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/2))-1/(-1+1/2*3^(1/2))^(1/2)/(-1-3^(1/2))*(1-
(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1/2*3^(1/2)+1)*x^2)^(1/2)/(-4+x^4+4*3^(1/2)
*x^2)^(1/2)*EllipticPi((-1+1/2*3^(1/2))^(1/2)*x,1/(-1+1/2*3^(1/2))/(-1-3^(1
/2))^2,(1/2*3^(1/2)+1)^(1/2)/(-1+1/2*3^(1/2))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(47) = 94$.

Time = 0.36 (sec) , antiderivative size = 323, normalized size of antiderivative = 4.97

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx$$
$$=\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{37\,x^{12}-204\,x^{11}+804\,x^{10}-2408\,x^9+3708\,x^8-5472\,x^7+6432\,x^6+10944\,x^5+14\right.}{}$$

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(37*x^12 - 204*x^11 + 804*x^10 - 2408*x^9 + 3
708*x^8 - 5472*x^7 + 6432*x^6 + 10944*x^5 + 14832*x^4 + 19264*x^3 + 12864*x
^2 + (54*x^10 - 300*x^9 + 1026*x^8 - 2232*x^7 + 3024*x^6 - 3024*x^5 - 1008*
x^4 - 2016*x^3 - 2592*x^2 + sqrt(3)*(31*x^10 - 176*x^9 + 576*x^8 - 1320*x^7
 + 1848*x^6 - 1008*x^5 + 1344*x^4 + 1632*x^3 + 1008*x^2 + 832*x + 256) - 11
52*x - 480)*sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(
```

```
7*x^12 - 40*x^11 + 160*x^10 - 400*x^9 + 924*x^8 - 960*x^7 - 1920*x^5 - 3696
*x^4 - 3200*x^3 - 2560*x^2 - 1280*x - 448) + 6528*x + 2368)/(x^12 + 12*x^11
 + 48*x^10 + 40*x^9 - 180*x^8 - 288*x^7 + 384*x^6 + 576*x^5 - 720*x^4 - 320
*x^3 + 768*x^2 - 384*x + 64))
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x-\sqrt{3}+1}{\left(x+1+\sqrt{3}\right)\sqrt{x^4+4\sqrt{3}x^2-4}}\,dx$$

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),x)
[Out] Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(3)*x**2 -
4)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\,\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\,\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="giac")
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\,\sqrt{x^4+4\,\sqrt{3}\,x^2-4}}\,dx$$

```
[In] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),x
)

[Out] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),
x)
```

3.91 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 63

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = -\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}x^2+x^4}}\right)$$

```
[Out] -1/3*arctan((1+x+3^(1/2))^2/(9+6*3^(1/2))^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2
))*(3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1754, 209}

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = -\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)$$

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] -1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3]
)]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4])])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1754

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Dist[(-A^2)*((B*d + A*e)/e), Subst[Int[1/(6*A^3*B
*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /;
 FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^
4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && E
qQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

integral =

$$-\left(\left(4\left(2+\sqrt{3}\right)\right)\text{Subst}\left(\int\frac{1}{6\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)^3+3\left(1+\sqrt{3}\right)^4+4x^2}dx,x,\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{-4-4\sqrt{3}x^2+x^4}}\right)\right)$$

$$=-\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}x^2+x^4}}\right)$$

Mathematica [A] (verified)

Time = 8.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int\frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$$
$$=-\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\sqrt{-9+6\sqrt{3}}\sqrt{-4-4\sqrt{3}x^2+x^4}}{-2+\left(2-2\sqrt{3}\right)x+\left(-2+\sqrt{3}\right)x^2}\right)$$

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^
4]),x]

[Out] -1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(Sqrt[-9 + 6*Sqrt[3]]*Sqrt[-4 - 4*Sqrt[3]*
x^2 + x^4])/(-2 + (2 - 2*Sqrt[3])*x + (-2 + Sqrt[3])*x^2)])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 2.31 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.94

method	result
elliptic	$\frac{\sqrt{1-\left(-\frac{\sqrt{3}}{2}-1\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{3}}{2}\right)x^2}\,F\left(x\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right),i\sqrt{1-4\sqrt{3}\left(1-\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{-4+x^4-4\sqrt{3}\,x^2}}+2\sqrt{3}\left(-\frac{\operatorname{arctanh}\left(\frac{-4\left(\sqrt{3}-1\right)^2\sqrt{3}-8-4\sqrt{3}\,x^2+2}{2\sqrt{\left(\sqrt{3}-1\right)^4-4\left(\sqrt{3}-1\right)^2\sqrt{3}-4}\right.}{2\sqrt{\left(\sqrt{3}-1\right)^4-4\left(\sqrt{3}-1\right)^2\sqrt{}}}\right.$

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/(1/2*I+1/2*I*3^(1/2))*(1-(-1/2*3^(1/2)-1)*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x
^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2*I+1/2*I*3^(1/2)),I*
(1-4*3^(1/2)*(1-1/2*3^(1/2)))^(1/2))+2*3^(1/2)*(-1/2/((3^(1/2)-1)^4-4*(3^(1
/2)-1)^2*3^(1/2)-4)^(1/2)*arctanh(1/2*(-4*(3^(1/2)-1)^2*3^(1/2)-8-4*3^(1/2)
*x^2+2*x^2*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4*(3^(1/2)-1)^2*3^(1/2)-4)^(1/2)/(
-4+x^4-4*3^(1/2)*x^2)^(1/2))-1/(-1/2*3^(1/2)-1)^(1/2)/(3^(1/2)-1)*(1-(-1/2*
3^(1/2)-1)*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^
(1/2)*EllipticPi((-1/2*3^(1/2)-1)^(1/2)*x,1/(-1/2*3^(1/2)-1)/(3^(1/2)-1)^2,
(1-1/2*3^(1/2))^(1/2)/(-1/2*3^(1/2)-1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(45) = 90.

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$$
$$=\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(9\,x^4-30\,x^3+18\,x^2-2\,\sqrt{3}(2\,x^4-10\,x^3+3\,x^2-10\,x+2)+24\right)\sqrt{x^4-4\,\sqrt{3}x}}{11\,x^6-42\,x^5+66\,x^4-176\,x^3-132\,x^2-168\,x-88}\right.$$

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*sqrt(2*sqrt(3) + 3)*arctan(-(9*x^4 - 30*x^3 + 18*x^2 - 2*sqrt(3)*(2*x^4
 - 10*x^3 + 3*x^2 - 10*x + 2) + 24)*sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*sqrt(2*sq
rt(3) + 3)/(11*x^6 - 42*x^5 + 66*x^4 - 176*x^3 - 132*x^2 - 168*x - 88))
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x+1+\sqrt{3}}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}\,dx$$

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2),x)
[Out] Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(3)*x**2 -
4)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="giac")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}\,x^2-4}\,\left(x-\sqrt{3}+1\right)}\,dx$$

```
[In] int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),x
)
[Out] int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),
x)
```

3.92 $\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx=\sqrt{3}\arctan\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)+\log(1+x)-\frac{3}{2}\log\left(2+x-\sqrt[3]{2+x^3}\right)$$

```
[Out] ln(1+x)-3/2*ln(2+x-(x^3+2)^(1/3))+arctan(1/3*(1+2*(2+x)/(x^3+2)^(1/3))*3^(1
/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2176}

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx=\sqrt{3}\arctan\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)-\frac{3}{2}\log\left(-\sqrt[3]{x^3+2}+x+2\right)+\log(x+1)$$

```
[In] Int[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]] + Log[1 + x] - (3
*Log[2 + x - (2 + x^3)^(1/3)])/2
```

Rule 2176

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[Sqrt[3]*f*(ArcTan[(1 + 2*Rt[b, 3]*((2*c + d*x)/(d*(a + b*
x^3)^(1/3))))/Sqrt[3]]/(Rt[b, 3]*d)), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
```

```
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\text{integral} = \sqrt{3}\arctan\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)+\log(1+x)-\frac{3}{2}\log\left(2+x-\sqrt[3]{2+x^3}\right)$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.74

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx = -\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{2+x^3}}{4+2x+\sqrt[3]{2+x^3}}\right)-\log\left(-2-x+\sqrt[3]{2+x^3}\right)+\frac{1}{2}\log\left(4+4x+x^2+(2+x)\sqrt[3]{2+x^3}+\left(2+x^3\right)^{2/3}\right)$$

```
[In] Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*(2 + x^3)^(1/3))/(4 + 2*x + (2 + x^3)^(1/3))]) -
Log[-2 - x + (2 + x^3)^(1/3)] + Log[4 + 4*x + x^2 + (2 + x)*(2 + x^3)^(1/3)
 + (2 + x^3)^(2/3)]/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.09 (sec) , antiderivative size = 816, normalized size of antiderivative = 15.40

method	result	size
trager	Expression too large to display	816

```
[In] int((-1+x)/(1+x)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] RootOf(_Z^2-_Z+1)*ln(-(1239*RootOf(_Z^2-_Z+1)^2*x^3+4504*RootOf(_Z^2-_Z+1)*
(x^3+2)^(2/3)*x+4504*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x^2-2478*RootOf(_Z^2-_
Z+1)^2*x^2+3265*RootOf(_Z^2-_Z+1)*x^3+9008*RootOf(_Z^2-_Z+1)*(x^3+2)^(2/3)+
335*x*(x^3+2)^(2/3)+18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x+335*(x^3+2)^(1/
3)*x^2-4956*RootOf(_Z^2-_Z+1)^2*x+10816*RootOf(_Z^2-_Z+1)*x^2+1574*x^3+670*
(x^3+2)^(2/3)+18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)+1340*x*(x^3+2)^(1/3)+21
632*RootOf(_Z^2-_Z+1)*x+7870*x^2+1340*(x^3+2)^(1/3)+17346*RootOf(_Z^2-_Z+1)
+15740*x+11018)/(1+x)^2)-ln((-1239*RootOf(_Z^2-_Z+1)^2*x^3+4504*RootOf(_Z^2
-_Z+1)*(x^3+2)^(2/3)*x+4504*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x^2+2478*RootOf
```

```
(_Z^2-_Z+1)^2*x^2+5743*RootOf(_Z^2-_Z+1)*x^3+9008*RootOf(_Z^2-_Z+1)*(x^3+2)
^(2/3)-4839*x*(x^3+2)^(2/3)+18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x-4839*(x
^3+2)^(1/3)*x^2+4956*RootOf(_Z^2-_Z+1)^2*x+5860*RootOf(_Z^2-_Z+1)*x^2-6078*
x^3-9678*(x^3+2)^(2/3)+18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)-19356*x*(x^3+2
)^(1/3)+11720*RootOf(_Z^2-_Z+1)*x-16208*x^2-19356*(x^3+2)^(1/3)+17346*RootO
f(_Z^2-_Z+1)-32416*x-28364)/(1+x)^2)*RootOf(_Z^2-_Z+1)+ln((-1239*RootOf(_Z^
2-_Z+1)^2*x^3+4504*RootOf(_Z^2-_Z+1)*(x^3+2)^(2/3)*x+4504*RootOf(_Z^2-_Z+1)
*(x^3+2)^(1/3)*x^2+2478*RootOf(_Z^2-_Z+1)^2*x^2+5743*RootOf(_Z^2-_Z+1)*x^3+
9008*RootOf(_Z^2-_Z+1)*(x^3+2)^(2/3)-4839*x*(x^3+2)^(2/3)+18016*RootOf(_Z^2
-_Z+1)*(x^3+2)^(1/3)*x-4839*(x^3+2)^(1/3)*x^2+4956*RootOf(_Z^2-_Z+1)^2*x+58
60*RootOf(_Z^2-_Z+1)*x^2-6078*x^3-9678*(x^3+2)^(2/3)+18016*RootOf(_Z^2-_Z+1
)*(x^3+2)^(1/3)-19356*x*(x^3+2)^(1/3)+11720*RootOf(_Z^2-_Z+1)*x-16208*x^2-1
9356*(x^3+2)^(1/3)+17346*RootOf(_Z^2-_Z+1)-32416*x-28364)/(1+x)^2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

Sympy [F]

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{x-1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

```
[In] integrate((-1+x)/(1+x)/(x**3+2)**(1/3),x)
[Out] Integral((x - 1)/((x + 1)*(x**3 + 2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{x-1}{(x^3+2)^{\frac{1}{3}}(x+1)}\,dx$$

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")
[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

Giac [F]

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{x-1}{(x^3+2)^{\frac{1}{3}}(x+1)}\,dx$$

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="giac")
[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{x-1}{(x^3+2)^{1/3}\,(x+1)}\,dx$$

```
[In] int((x - 1)/((x^3 + 2)^(1/3)*(x + 1)),x)
[Out] int((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

3.93 $\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \frac{\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt{3}\arctan\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right) - \frac{1}{2}\log(1+x) + \frac{3}{4}\log\left(2+x-\sqrt[3]{2+x^3}\right) - \frac{1}{4}\log\left(-x+\sqrt[3]{2+x^3}\right)$$

```
[Out] -1/2*ln(1+x)+3/4*ln(2+x-(x^3+2)^(1/3))-1/4*ln(-x+(x^3+2)^(1/3))+1/6*arctan(
1/3*(1+2*x/(x^3+2)^(1/3))*3^(1/2))*3^(1/2)-1/2*arctan(1/3*(1+2*(2+x)/(x^3+2
)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2175, 245, 2176}

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \frac{\arctan\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt{3}\arctan\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right) + \frac{3}{4}\log\left(-\sqrt[3]{x^3+2}+x+2\right) - \frac{1}{4}\log\left(\sqrt[3]{x^3+2}-x\right) - \frac{1}{2}\log(x+1)$$

```
[In] Int[1/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - (Sqrt[3]*ArcTan[(
1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]])/2 - Log[1 + x]/2 + (3*Log[2 + x
- (2 + x^3)^(1/3)])/4 - Log[-x + (2 + x^3)^(1/3)]/4
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2175

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Dist[
1/(2*c), Int[1/(a + b*x^3)^(1/3), x], x] + Dist[1/(2*c), Int[(c - d*x)/((c
+ d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[2*b*c^3
- a*d^3, 0]
```

Rule 2176

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[Sqrt[3]*f*(ArcTan[(1 + 2*Rt[b, 3]*((2*c + d*x)/(d*(a + b*
x^3)^(1/3))))/Sqrt[3]]/(Rt[b, 3]*d)), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{\sqrt[3]{2+x^3}}\,dx + \frac{1}{2}\int \frac{1-x}{(1+x)\sqrt[3]{2+x^3}}\,dx \\
&= \frac{\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt{3}\arctan\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right) \\
&\quad - \frac{1}{2}\log(1+x) + \frac{3}{4}\log\left(2+x-\sqrt[3]{2+x^3}\right) - \frac{1}{4}\log\left(-x+\sqrt[3]{2+x^3}\right)
\end{aligned}
$$

Mathematica [F]

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

```
[In] Integrate[1/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((1 + x)*(2 + x^3)^(1/3)), x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.13 (sec) , antiderivative size = 1421, normalized size of antiderivative = 13.16

method	result	size
trager	Expression too large to display	1421

```
[In] int(1/(1+x)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*ln((-4550781346817636-68457312523761*x^6-6728375859478224*x-62589542878
8672*x^5+4993190285176576*RootOf(_Z^2+_Z+1)^2*x^3+8816461926585488*RootOf(_
Z^2+_Z+1)*x^3+1055101552116528*RootOf(_Z^2+_Z+1)*x^2-2128312852753752O*Root
Of(_Z^2+_Z+1)*x+9094739448000192*RootOf(_Z^2+_Z+1)^2*x^2+5884831407529536*R
ootOf(_Z^2+_Z+1)^2*x-234710785795752*x^4+2151515536461060*x^3-4694215715915
04*x^2+1326316169500028*RootOf(_Z^2+_Z+1)^2*x^6+4346750471470680*RootOf(_Z^
2+_Z+1)^2*x^5+4547369724000096*RootOf(_Z^2+_Z+1)^2*x^4+153868976350327*Root
Of(_Z^2+_Z+1)*x^6-868588603920114*RootOf(_Z^2+_Z+1)*x^5-15559137585059152*R
ootOf(_Z^2+_Z+1)+16295099853018372*x*(x^3+2)^(2/3)-9125490357912936*(x^3+2)
^(1/3)+527550776058264*RootOf(_Z^2+_Z+1)*x^4-14648813469281292*x*(x^3+2)^(1
/3)+10107087250606332*(x^3+2)^(2/3)-4682817420507954*(x^3+2)^(1/3)*x^2-9282
01890361806*(x^3+2)^(2/3)*x^4-540325086981687*(x^3+2)^(1/3)*x^5-19595373240
97146*(x^3+2)^(2/3)*x^3-480288966205944*(x^3+2)^(1/3)*x^4+5569211342170836*
(x^3+2)^(2/3)*x^2+1200722415514860*(x^3+2)^(1/3)*x^3+7115580883942020*RootO
f(_Z^2+_Z+1)*(x^3+2)^(2/3)+3372637147591320*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)
+36303984101745*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x^4+288449229728205*(x^3+
2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^5-1306943427662820*RootOf(_Z^2+_Z+1)^2*(x^3+
2)^(2/3)*x^3-601904942144643*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^4+1286927332
633530*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^4-580773949503594*RootOf(_Z^2+_Z
+1)*(x^3+2)^(1/3)*x^5-3775614346581480*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x^
2-3235955176589922*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^3+1420057746354240*(x^
3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^3-3304587786698814*RootOf(_Z^2+_Z+1)*(x^3+
2)^(1/3)*x^4-2613886855325640*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x-144211397
2435308*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^2-310637632014990*(x^3+2)^(1/3)*R
ootOf(_Z^2+_Z+1)^2*x^2-4286079775383252*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)*x^3
```

```
+7759251414704196*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x-798782482324260*(x^3+2)
^(1/3)*RootOf(_Z^2+_Z+1)^2*x+2571201069229632*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/
3)*x^2+7575270505902288*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)*x)/(1+x)^6)+1/6*Roo
tOf(_Z^2+_Z+1)*ln((-15559137585059152-1560371964117680*x^6-2888917236423590
4*x-6486689165117784*x^5-1460422667173568*RootOf(_Z^2+_Z+1)^2*x^3-430209741
5889084*RootOf(_Z^2+_Z+1)*x^3-12224216591943552*RootOf(_Z^2+_Z+1)*x^2-14334
419696176608*RootOf(_Z^2+_Z+1)*x-2660055572351856*RootOf(_Z^2+_Z+1)^2*x^2-1
721212429168848*RootOf(_Z^2+_Z+1)^2*x-5349846734117760*x^4+2362848974235344
*x^3-10699693468235520*x^2-387924770967979*RootOf(_Z^2+_Z+1)^2*x^6-12713500
89726990*RootOf(_Z^2+_Z+1)^2*x^5-1330027786175928*RootOf(_Z^2+_Z+1)^2*x^4-1
782698252991768*RootOf(_Z^2+_Z+1)*x^6-6243995989986342*RootOf(_Z^2+_Z+1)*x^
5-4550781346817636*RootOf(_Z^2+_Z+1)+5921961582988536*x*(x^3+2)^(2/3)+91254
90357912936*(x^3+2)^(1/3)-6112108295971776*RootOf(_Z^2+_Z+1)*x^4+1681011381
7208040*x*(x^3+2)^(1/3)+2991506366664312*(x^3+2)^(2/3)+5523323111368356*(x^
3+2)^(1/3)*x^2-289992964115418*(x^3+2)^(2/3)*x^4-240144483102972*(x^3+2)^(1
/3)*x^5-30525575170044*(x^3+2)^(2/3)*x^3-3001806038787150*(x^3+2)^(1/3)*x^4
+3235710968024664*(x^3+2)^(2/3)*x^2-5043034145162412*(x^3+2)^(1/3)*x^3-7115
580883942020*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)+12498127505504256*RootOf(_Z^2+
_Z+1)*(x^3+2)^(1/3)+36303984101745*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x^4-10
68918799812864*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^5-1306943427662820*RootO
f(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x^3+674512910348133*RootOf(_Z^2+_Z+1)*(x^3+2)^
(2/3)*x^4-4769022337626624*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^4-1109367662
334771*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)*x^5-3775614346581480*RootOf(_Z^2+_Z+
1)^2*(x^3+2)^(2/3)*x^2+622068321264282*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^3-
5262369476001792*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^3-7593321158119494*Roo
tOf(_Z^2+_Z+1)*(x^3+2)^(1/3)*x^4-2613886855325640*RootOf(_Z^2+_Z+1)^2*(x^3+
2)^(2/3)*x-6109114720727652*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^2+11511433228
75392*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^2-10749171666899904*RootOf(_Z^2+_
Z+1)*(x^3+2)^(1/3)*x^3-12987025125355476*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x+
2960082830251008*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x+8405161812612978*RootO
f(_Z^2+_Z+1)*(x^3+2)^(1/3)*x^2+25184166805434588*RootOf(_Z^2+_Z+1)*(x^3+2)^
(1/3)*x)/(1+x)^6)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(84) = 168$.

Time = 1.04 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.47

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$$
$$= \frac{1}{6}\sqrt{3}\arctan\left(\frac{13910019318573948542\sqrt{3}(7114781247\,x^4 + 13663058416\,x^3 - 46178206896\,x^2 - 1268425\right.$$
$$+ \frac{1}{12}\log\left(\frac{22\,x^6 + 6\,x^5 - 48\,x^4 + 44\,x^3 + 24\,x^2 + 3\,(7\,x^4 - 2\,x^3 - 32\,x^2 - 20\,x + 4)(x^3+2)^{\frac{2}{3}} + 3\,(7\,x^5 -}{x^6 + 6\,x^5 + 15\,x^4 + 20\,x^3 + 15\,x^2 + 6\,x + 1}\right.$$

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*(13910019318573948542*sqrt(3)*(7114781247*x^4 + 1366
3058416*x^3 - 46178206896*x^2 - 126842559344*x - 77084338088)*(x^3 + 2)^(2/
3) - 27820038637147897084*sqrt(3)*(1625757424*x^5 + 16302821713*x^4 + 26102
613730*x^3 - 26431113242*x^2 - 80188343316*x - 42779182428)*(x^3 + 2)^(1/3)
 + sqrt(3)*(932925708335594356631323018 85*x^6 + 382151535711085278859235047
618*x^5 + 673924074224408772959625384792*x^4 + 889426563183087468015580290 0
48*x^3 + 888876515195959220955879945824*x^2 + 351260598258508240019971964 88
0*x - 476740009955972110578168843 04))/(7890543481456472174570846488 3*x^6 +
33774670583645822286334793445 0*x^5 + 1559895277605858789433607097 6*x^4 - 89
54305253151001086847879648 24*x^3 + 3616678622404770288695333753 52*x^2 + 254
18023010116325106459720903 36*x + 1554815286823334092314485968880)) + 1/12*l
og((22*x^6 + 6*x^5 - 48*x^4 + 44*x^3 + 24*x^2 + 3*(7*x^4 - 2*x^3 - 32*x^2 -
 20*x + 4)*(x^3 + 2)^(2/3) + 3*(7*x^5 - 16*x^3 + 34*x^2 + 76*x + 32)*(x^3 +
 2)^(1/3) - 192*x - 140)/(x^6 + 6*x^5 + 15*x^4 + 20*x^3 + 15*x^2 + 6*x + 1)
)
```

Sympy [F]

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

```
[In] integrate(1/(1+x)/(x**3+2)**(1/3),x)
```

```
[Out] Integral(1/((x + 1)*(x**3 + 2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x^3+2)^{\frac{1}{3}}(x+1)}\,dx$$

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x^3+2)^{\frac{1}{3}}(x+1)}\,dx$$

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="giac")
[Out] integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x^3+2)^{1/3}\,(x+1)}\,dx$$

```
[In] int(1/((x^3 + 2)^(1/3)*(x + 1)),x)
[Out] int(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

3.94 $\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

```
[Out] 1/6*ln(-x^3+1)/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)*x-(b*x^3+a)^(1/3))/(a+b)^(1/3
)+1/3*arctan(1/3*(1+2*(a+b)^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/(a+b)^(1/3)*3
^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {384}

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = \frac{\arctan\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log\left(x\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

```
[In] Int[1/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3)) + Log[1 - x^3]/(6*(a + b)^(1/3)) - Log[(a + b)^(1/3)*x - (a + b*x^
3)^(1/3)]/(2*(a + b)^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\text{integral} = \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}x - \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.93

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$$
$$= \frac{-2\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{3\sqrt[3]{a+b}x}{\sqrt{3}\sqrt[3]{a+b}x-(3i+\sqrt{3})\sqrt[3]{a+bx^3}}\right) + (1+i\sqrt{3})\left(2\log\left(2\sqrt[3]{a+b}x + (1+i\sqrt{3})\sqrt[3]{a-\right.\right.}{12\sqrt[3]{a+b}}$$

```
[In] Integrate[1/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] (-2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(3*(a + b)^(1/3)*x)/(Sqrt[3]*(a + b)^(1
/3)*x - (3*I + Sqrt[3])*(a + b*x^3)^(1/3))] + (1 + I*Sqrt[3])*(2*Log[2*(a +
 b)^(1/3)*x + (1 + I*Sqrt[3])*(a + b*x^3)^(1/3)] - Log[(-((a + b)^(1/3)*x)
+ (a + b*x^3)^(1/3))*((2*I)*(a + b)^(1/3)*x + (I + Sqrt[3])*(a + b*x^3)^(1/
3))]))/(12*(a + b)^(1/3))
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left((a+b)^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3(a+b)^{\frac{1}{3}} x}\right)+\ln\left(\frac{-(a+b)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{(a+b)^{\frac{2}{3}} x^{2}+(a+b)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right.}{2}}{3(a+b)^{\frac{1}{3}}}$

```
[In] int(1/(-x^3+1)/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(a+b)^(1/3)*(3^(1/2)*arctan(1/3*3^(1/2)*((a+b)^(1/3)*x+2*(b*x^3+a)^(1/
3))/(a+b)^(1/3)/x)+ln((-(a+b)^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln(((a+b)^(2/
3)*x^2+(a+b)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(78) = 156$.

Time = 108.17 (sec) , antiderivative size = 1252, normalized size of antiderivative = 12.78

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(a + b)*sqrt((-a - b)^(1/3)/(a + b))*log(-((a^3 - 27*a^2
*b - 108*a*b^2 - 81*b^3)*x^9 - 3*(10*a^3 + 54*a^2*b + 45*a*b^2)*x^6 - 3*(17
*a^3 + 18*a^2*b)*x^3 - a^3 + 9*((2*a^2 + 3*a*b)*x^7 - (a^2 + 3*a*b)*x^4 - a
^2*x)*(b*x^3 + a)^(2/3)*(-a - b)^(1/3) + 9*((a^2 + 9*a*b + 9*b^2)*x^8 + (7*
a^2 + 9*a*b)*x^5 + a^2*x^2)*(b*x^3 + a)^(1/3)*(-a - b)^(2/3) + 3*sqrt(1/3)*
(3*((4*a^2 + 21*a*b + 18*b^2)*x^7 + (13*a^2 + 15*a*b)*x^4 + a^2*x)*(b*x^3 +
 a)^(2/3)*(-a - b)^(2/3) + 3*((a^3 - 2*a^2*b - 12*a*b^2 - 9*b^3)*x^8 - 5*(a
^3 + 4*a^2*b + 3*a*b^2)*x^5 - 5*(a^3 + a^2*b)*x^2)*(b*x^3 + a)^(1/3) + ((a^
3 + 27*a^2*b + 54*a*b^2 + 27*b^3)*x^9 + 3*(8*a^3 + 18*a^2*b + 9*a*b^2)*x^6
+ 3*a^3*x^3 - a^3)*(-a - b)^(1/3))*sqrt((-a - b)^(1/3)/(a + b)))/(x^9 - 3*x
^6 + 3*x^3 - 1)) - 2*(-a - b)^(2/3)*log(-(3*(b*x^3 + a)^(1/3)*(a + b)*(-a -
 b)^(1/3)*x^2 + 3*(b*x^3 + a)^(2/3)*(a + b)*x + (a*x^3 - a)*(-a - b)^(2/3))
/(x^3 - 1)) + (-a - b)^(2/3)*log((3*((2*a + 3*b)*x^4 + a*x)*(b*x^3 + a)^(2/
3)*(-a - b)^(2/3) + 3*((a^2 + 4*a*b + 3*b^2)*x^5 + 2*(a^2 + a*b)*x^2)*(b*x^
3 + a)^(1/3) - ((a^2 + 9*a*b + 9*b^2)*x^6 + (7*a^2 + 9*a*b)*x^3 + a^2)*(-a
- b)^(1/3))/(x^6 - 2*x^3 + 1)))/(a + b), 1/18*(6*sqrt(1/3)*(a + b)*sqrt(-(-
a - b)^(1/3)/(a + b))*arctan(sqrt(1/3)*(6*((2*a^2 + 3*a*b)*x^7 - (a^2 + 3*a
*b)*x^4 - a^2*x)*(b*x^3 + a)^(2/3)*(-a - b)^(2/3) - 6*((a^3 + 10*a^2*b + 18
*a*b^2 + 9*b^3)*x^8 + (7*a^3 + 16*a^2*b + 9*a*b^2)*x^5 + (a^3 + a^2*b)*x^2)
```

```
*(b*x^3 + a)^(1/3) - ((a^3 - 9*a^2*b - 36*a*b^2 - 27*b^3)*x^9 - 3*(4*a^3 +
18*a^2*b + 15*a*b^2)*x^6 - 3*(5*a^3 + 6*a^2*b)*x^3 - a^3)*(-a - b)^(1/3))*s
qrt(-(-a - b)^(1/3)/(a + b))/((a^3 + 27*a^2*b + 54*a*b^2 + 27*b^3)*x^9 + 3*
(8*a^3 + 18*a^2*b + 9*a*b^2)*x^6 + 3*a^3*x^3 - a^3)) - 2*(-a - b)^(2/3)*log
(-(3*(b*x^3 + a)^(1/3)*(a + b)*(-a - b)^(1/3)*x^2 + 3*(b*x^3 + a)^(2/3)*(a
+ b)*x + (a*x^3 - a)*(-a - b)^(2/3))/(x^3 - 1)) + (-a - b)^(2/3)*log((3*((2
*a + 3*b)*x^4 + a*x)*(b*x^3 + a)^(2/3)*(-a - b)^(2/3) + 3*((a^2 + 4*a*b + 3
*b^2)*x^5 + 2*(a^2 + a*b)*x^2)*(b*x^3 + a)^(1/3) - ((a^2 + 9*a*b + 9*b^2)*x
^6 + (7*a^2 + 9*a*b)*x^3 + a^2)*(-a - b)^(1/3))/(x^6 - 2*x^3 + 1)))/(a + b)
]
```

Sympy [F]

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = -\int \frac{1}{x^3\sqrt[3]{a+bx^3}-\sqrt[3]{a+bx^3}}\,dx$$

```
[In] integrate(1/(-x**3+1)/(b*x**3+a)**(1/3),x)
[Out] -Integral(1/(x**3*(a + b*x**3)**(1/3) - (a + b*x**3)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{1}{3}}(x^3-1)}\,dx$$

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] -integrate(1/((b*x^3 + a)^(1/3)*(x^3 - 1)), x)
```

Giac [F]

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{1}{3}}(x^3-1)}\,dx$$

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate(-1/((b*x^3 + a)^(1/3)*(x^3 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = -\int \frac{1}{(x^3-1)\,(b\,x^3+a)^{1/3}}\,dx$$

```
[In] int(-1/((x^3 - 1)*(a + b*x^3)^(1/3)),x)
[Out] -int(1/((x^3 - 1)*(a + b*x^3)^(1/3)), x)
```

3.95 $\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 154

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx = \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

```
[Out] 1/2*ln((a+b)^(1/3)-(b*x^3+a)^(1/3))/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)*x-(b*x^3
+a)^(1/3))/(a+b)^(1/3)+1/3*arctan(1/3*(1+2*(a+b)^(1/3)*x/(b*x^3+a)^(1/3))*3
^(1/2))/(a+b)^(1/3)*3^(1/2)+1/3*arctan(1/3*(1+2*(b*x^3+a)^(1/3)/(a+b)^(1/3)
)*3^(1/2))/(a+b)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2183, 384, 455, 57, 631, 210, 31}

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx = \frac{\arctan\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}} - \frac{\log\left(x\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

```
[In] Int[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3)) + ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]]/(Sqrt[
3]*(a + b)^(1/3)) + Log[(a + b)^(1/3) - (a + b*x^3)^(1/3)]/(2*(a + b)^(1/3)
) - Log[(a + b)^(1/3)*x - (a + b*x^3)^(1/3)]/(2*(a + b)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{(1-x^3)\sqrt[3]{a+bx^3}} - \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\right) dx \\
&= \int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\, dx - \int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\, dx \\
&= \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}x - \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}} \\
&\quad - \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt[3]{a+bx}}\, dx, x, x^3\right) \\
&= \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}x - \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}} \\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+b)^{2/3} + \sqrt[3]{a+b}x + x^2}\, dx, x, \sqrt[3]{a+bx^3}\right) \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+b}-x}\, dx, x, \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}} \\
&= \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log\left(\sqrt[3]{a+b} - \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}} \\
&\quad - \frac{\log\left(\sqrt[3]{a+b}x - \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\, dx, x, 1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}\right)}{\sqrt[3]{a+b}}
\end{aligned}$$

$$= \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

Mathematica [F]

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx = \int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$$

```
[In] Integrate[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)),x]
[Out] Integrate[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)), x]
```

Maple [F]

$$\int \frac{1+x}{(x^2+x+1)\,(b\,x^3+a)^{\frac{1}{3}}}dx$$

```
[In] int((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x)
[Out] int((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx = \int \frac{x+1}{\sqrt[3]{a+bx^3}\,(x^2+x+1)}\,dx$$

```
[In] integrate((1+x)/(x**2+x+1)/(b*x**3+a)**(1/3),x)
[Out] Integral((x + 1)/((a + b*x**3)**(1/3)*(x**2 + x + 1)), x)
```

Maxima [F]

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx = \int \frac{x+1}{(bx^3+a)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] integrate((x + 1)/((b*x^3 + a)^(1/3)*(x^2 + x + 1)), x)
```

Giac [F]

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx = \int \frac{x+1}{(bx^3+a)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((x + 1)/((b*x^3 + a)^(1/3)*(x^2 + x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx = \int \frac{x+1}{(b\,x^3+a)^{1/3}\,(x^2+x+1)}\,dx$$

```
[In] int((x + 1)/((a + b*x^3)^(1/3)*(x + x^2 + 1)),x)
[Out] int((x + 1)/((a + b*x^3)^(1/3)*(x + x^2 + 1)), x)
```

3.96 $\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

```
[Out] 1/6*ln(-x^3+1)/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)-(b*x^3+a)^(1/3))/(a+b)^(1/3)-
1/3*arctan(1/3*(1+2*(b*x^3+a)^(1/3)/(a+b)^(1/3))*3^(1/2))/(a+b)^(1/3)*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {455, 57, 631, 210, 31}

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = -\frac{\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

```
[In] Int[x^2/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3))) + Log[1 - x^3]/(6*(a + b)^(1/3)) - Log[(a + b)^(1/3) - (a + b*x^3
)^(1/3)]/(2*(a + b)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt[3]{a+bx}}\,dx, x, x^3\right)$$

$$= \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+b)^{2/3}+\sqrt[3]{a+b}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+b}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

$$= \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}\right)}{\sqrt[3]{a+b}}$$

$$= -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.90

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{1+\frac{(-1-i\sqrt{3})\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right) - i(-i+\sqrt{3})\left(\log\left(\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)\left(2\sqrt[3]{a+b}+\sqrt[3]{}\right.\right.\right.}{12\sqrt[3]{a+b}}$$

```
[In] Integrate[x^2/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(1 + ((-1 - I*Sqrt[3])*(a + b*x^3)^(1/3)
)/(a + b)^(1/3))/Sqrt[3]] - I*(-I + Sqrt[3])*(Log[((a + b)^(1/3) - (a + b*x
^3)^(1/3))*(2*(a + b)^(1/3) + (a + b*x^3)^(1/3) - I*Sqrt[3]*(a + b*x^3)^(1/
3))] - 2*Log[2*(a + b)^(1/3) + (1 + I*Sqrt[3])*(a + b*x^3)^(1/3)]))/(12*(a
+ b)^(1/3))
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

method	result	size
pseudoelliptic	$-\frac{\arctan\left(\frac{\left((a+b)^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3(a+b)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-(a+b)^{\frac{1}{3}}\right)-\frac{\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+(a+b)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+(a+b)^{\frac{2}{3}}\right)}{2}}{3(a+b)^{\frac{1}{3}}}$	92

```
[In] int(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(a+b)^(1/3)*(arctan(1/3*((a+b)^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/(a+b)^
(1/3))*3^(1/2)+ln((b*x^3+a)^(1/3)-(a+b)^(1/3))-1/2*ln((b*x^3+a)^(2/3)+(a+b)
^(1/3)*(b*x^3+a)^(1/3)+(a+b)^(2/3)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(75) = 150.

Time = 0.26 (sec) , antiderivative size = 387, normalized size of antiderivative = 4.03

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(a+b)\sqrt{\frac{(-a-b)^{\frac{1}{3}}}{a+b}}\log\left(\frac{2\,bx^3+3\sqrt{\frac{1}{3}}\left(\left(bx^3+a\right)^{\frac{1}{3}}(a+b)-(a+b)(-a-b)^{\frac{1}{3}}-2\left(bx^3+a\right)^{\frac{2}{3}}(-a-b)^{\frac{2}{3}}\right)\sqrt{\frac{(-a-b)^{\frac{1}{3}}}{a+b}}+3\,a-3\left(bx^3+a\right)^{\frac{1}{3}}(-}{x^3-1}\right.}{}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}(a+b)\sqrt{-\frac{(-a-b)^{\frac{1}{3}}}{a+b}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\left(bx^3+a\right)^{\frac{1}{3}}-(-a-b)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a-b)^{\frac{1}{3}}}{a+b}}\right)-(-a-b)^{\frac{2}{3}}\log\left(\left(bx^3+\right.\right.}{6\,(a+b)}$$

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(a + b)*sqrt((-a - b)^(1/3)/(a + b))*log((2*b*x^3 + 3*sqr
t(1/3)*((b*x^3 + a)^(1/3)*(a + b) - (a + b)*(-a - b)^(1/3) - 2*(b*x^3 + a)^
(2/3)*(-a - b)^(2/3))*sqrt((-a - b)^(1/3)/(a + b)) + 3*a - 3*(b*x^3 + a)^(1
/3)*(-a - b)^(2/3) + b)/(x^3 - 1)) + (-a - b)^(2/3)*log((b*x^3 + a)^(2/3) -
 (b*x^3 + a)^(1/3)*(-a - b)^(1/3) + (-a - b)^(2/3)) - 2*(-a - b)^(2/3)*log(
(b*x^3 + a)^(1/3) + (-a - b)^(1/3)))/(a + b), -1/6*(6*sqrt(1/3)*(a + b)*sqr
```

```
t(-(-a - b)^(1/3)/(a + b))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) - (-a - b)
^(1/3))*sqrt(-(-a - b)^(1/3)/(a + b))) - (-a - b)^(2/3)*log((b*x^3 + a)^(2/
3) - (b*x^3 + a)^(1/3)*(-a - b)^(1/3) + (-a - b)^(2/3)) + 2*(-a - b)^(2/3)*
log((b*x^3 + a)^(1/3) + (-a - b)^(1/3)))/(a + b)]
```

Sympy [F]

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = -\int \frac{x^2}{x^3\sqrt[3]{a+bx^3}-\sqrt[3]{a+bx^3}}\,dx$$

```
[In] integrate(x**2/(-x**3+1)/(b*x**3+a)**(1/3),x)
[Out] -Integral(x**2/(x**3*(a + b*x**3)**(1/3) - (a + b*x**3)**(1/3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = -\frac{\frac{2\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+(a+b)^{\frac{1}{3}}\right)}{3(a+b)^{\frac{1}{3}}}\right)}{(a+b)^{\frac{1}{3}}}-\frac{b\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}(a+b)^{\frac{1}{3}}+(a+b)^{\frac{2}{3}}\right)}{(a+b)^{\frac{1}{3}}}+\frac{2b\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-(a+b)^{\frac{1}{3}}\right)}{(a+b)^{\frac{1}{3}}}}{6b}$$

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] -1/6*(2*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (a + b)^(1/3))/
(a + b)^(1/3))/(a + b)^(1/3) - b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*
(a + b)^(1/3) + (a + b)^(2/3))/(a + b)^(1/3) + 2*b*log((b*x^3 + a)^(1/3) -
(a + b)^(1/3))/(a + b)^(1/3))/b
```

Giac [A] (verification not implemented)

none

Time = 3.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = -\frac{(a+b)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+(a+b)^{\frac{1}{3}}\right)}{3\,(a+b)^{\frac{1}{3}}}\right)}{\sqrt{3}a+\sqrt{3}b} + \frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}(a+b)^{\frac{1}{3}}+(a+b)^{\frac{2}{3}}\right)}{6\,(a+b)^{\frac{1}{3}}} - \frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-(a+b)^{\frac{1}{3}}\right|\right)}{3\,(a+b)^{\frac{1}{3}}}$$

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] -(a + b)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (a + b)^(1/3))/(a
+ b)^(1/3))/(sqrt(3)*a + sqrt(3)*b) + 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*(a + b)^(1/3) + (a + b)^(2/3))/(a + b)^(1/3) - 1/3*log(abs((b*x^3
+ a)^(1/3) - (a + b)^(1/3)))/(a + b)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.64

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = \frac{\ln\left((b\,x^3+a)^{1/3}-\frac{9\,a+9\,b}{9\,(-a-b)^{2/3}}\right)}{3\,(-a-b)^{1/3}} + \frac{\ln\left((b\,x^3+a)^{1/3}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,(9\,a+9\,b)}{36\,(-a-b)^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-a-b)^{1/3}} - \frac{\ln\left((b\,x^3+a)^{1/3}-\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,(9\,a+9\,b)}{36\,(-a-b)^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-a-b)^{1/3}}$$

```
[In] int(-x^2/((x^3 - 1)*(a + b*x^3)^(1/3)),x)
```

```
[Out] log((a + b*x^3)^(1/3) - (9*a + 9*b)/(9*(- a - b)^(2/3)))/(3*(- a - b)^(1/3)
) + (log((a + b*x^3)^(1/3) - ((3^(1/2)*1i - 1)^2*(9*a + 9*b))/(36*(- a - b)
^(2/3)))*(3^(1/2)*1i - 1))/(6*(- a - b)^(1/3)) - (log((a + b*x^3)^(1/3) - (
(3^(1/2)*1i + 1)^2*(9*a + 9*b))/(36*(- a - b)^(2/3)))*(3^(1/2)*1i + 1))/(6*
(- a - b)^(1/3))
```

3.97 $\int \frac{1}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 88

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/12*ln(x^3+1)*2^(2/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*arcta
n(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {384}

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}$$

```
[In] Int[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) -
Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
```

```
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 2\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + \log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}(1-x^3)^{2/3}\right)}{6\sqrt[3]{2}}$$

```
[In] Integrate[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*Log[2
*x + 2^(2/3)*(1 - x^3)^(1/3)] + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^
(1/3)*(1 - x^3)^(2/3)])/2^(1/3)
```

Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

method	result	size
pseudoelliptic	$\frac{2^{\frac{2}{3}}\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)+\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}\right)}{6}$	95
trager	Expression too large to display	780

```
[In] int(1/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*2^(2/3)*(3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)+ln((
2^(1/3)*x+(-x^3+1)^(1/3))/x)-1/2*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(
-x^3+1)^(2/3))/x^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(67) = 134$.

Time = 1.61 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.88

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx =$$

$$-\frac{1}{18}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-\sqrt{6}2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)+12\sqrt{\cdots}\right.}{6\,(109x^9-105x^6+3x^3+1)}\right.$$

$$+\frac{1}{18}\cdot 2^{\frac{2}{3}}\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}x^2+2^{\frac{2}{3}}(x^3+1)+6\,(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\right)-\frac{1}{36}$$

$$\cdot 2^{\frac{2}{3}}\log\left(\frac{3\cdot 2^{\frac{2}{3}}(5x^4-x)(-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(19x^6-16x^3+1)-12\,(2x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right)$$

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7 + 4*x^4
- x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1) + 1
2*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*
x^3 + 1)) + 1/18*2^(2/3)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3
 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/36*2^(2/3)*log((3*2^(2/3)*(5*x
^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)
*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

```
[In] integrate(1/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

```
[In] int(1/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.98 $\int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 233

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/24*ln((1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(
1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2
^(2/3)-1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2^(1
/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {502, 2174, 206, 31, 648, 631, 210, 642}

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{6\sqrt[3]{2}} - \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} + x - 1\right)}{4\sqrt[3]{2}} + \frac{\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}$$

```
[In] Int[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[
3]) + Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1
- x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^
(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x^3
)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{3}\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx\right) - \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

$$-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

$$-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$+\frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}$$

$$= \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}$$

$$+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

$$-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

$$= \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

$$+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}$$

$$-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.21

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$
$$= \frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{}\right.}{}$$

```
[In] Integrate[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1 - x^
3)^(1/3))] - 4*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2^(
1/3)*x + (1 - x^3)^(1/3))] - 4*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/3)]
- 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] + 2*Log[2^(2/3) - 2*2^(2/
3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)] + Log[2^
(2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3) + 4*(1 - x
^3)^(2/3)])/(12*2^(1/3))
```

Maple [F]

$$\int \frac{x}{(-x^3+1)^{\frac{1}{3}}\,(x^3+1)}dx$$

```
[In] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(171) = 342.

Time = 1.48 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.60

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx =$$

$$-\frac{1}{36}\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{2^{\frac{1}{6}}\left(24\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\,(x^{14}-2\,x^{11}-6\,x^8-2\,x^5+x^2)(-x^3+1)^{\frac{2}{3}}+12\sqrt{6}(-1)^{\frac{1}{3}}\,(x\right.}{6\,(x^{18}-1}\right.$$

$$-\frac{1}{72}$$

$$\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{12\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(x^8-4\,x^5+x^2)(-x^3+1)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(x^{12}-32\,x^9+78\,x^6-32\,x^3+1)-}{x^{12}+4\,x^9+6\,x^6+4\,x^3+1}\right.$$

$$+\frac{1}{36}$$

$$\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{12\,(-x^3+1)^{\frac{2}{3}}x^2-6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(x^4-x)(-x^3+1)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(x^6+2\,x^3+1)}{x^6+2\,x^3+1}\right)$$

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(24*sqrt(6)*2^(2/3)*(-1
)^(2/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) + 12*sqrt(6)
*(-1)^(1/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(
1/3) + sqrt(6)*2^(1/3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*
x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) -
 1/72*2^(2/3)*(-1)^(1/3)*log(-(12*2^(2/3)*(-1)^(1/3)*(x^8 - 4*x^5 + x^2)*(-
x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) -
 6*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4
*x^3 + 1)) + 1/36*2^(2/3)*(-1)^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^2 - 6*2^(1
/3)*(-1)^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3)*(x^6 + 2*x^3
 + 1))/(x^6 + 2*x^3 + 1))
```

Sympy [F]

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

```
[In] integrate(x/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

```
[In] int(x/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(x/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.99 $\int \frac{x^2}{\sqrt[3]{1-x^3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/12*ln(x^3+1)*2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1
/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {455, 57, 631, 210, 31}

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[In] Int[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x
^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right) \\
&= -\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} \\
&\quad +\frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)-\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} \\
&= -\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}} \\
&= \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+2\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3)^{2/3}\right)}{6\sqrt[3]{2}}$$

```
[In] Integrate[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
[Out] (2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 2*Log[-2 + 2^(2/
3)*(1 - x^3)^(1/3)] - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(
2/3)])/(6*2^(1/3))
```

Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

method	result	size
pseudoelliptic	$\frac{2^{\frac{2}{3}}\left(2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}+2\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)-\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)\right)}{12}$	80
trager	Expression too large to display	655

```
[In] int(x^2/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/12*2^(2/3)*(2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+2*ln
((-x^3+1)^(1/3)-2^(1/3))-ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)$$

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 +
 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 +
 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Sympy [F]

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^2}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

```
[In] integrate(x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6} + \frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} - \frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

```
[In] int(x^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2^(2/3)*log((1 - x^3)^(1/3) -
 (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x
^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

3.100 $\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

```
[Out] 1/4*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/
3)-1/2*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*arctan(1/3*(1-2*2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 383 vs. 2(135) = 270.

Time = 0.19 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.84, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules

used = {2183, 384, 502, 2174, 206, 31, 648, 631, 210, 642, 455, 57}

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$

$$= \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

$$+ \frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(x^3+1)}{3\sqrt[3]{2}} + \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}}$$

$$-\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-\right.}{2\sqrt[3]{2}}$$

```
[In] Int[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]
) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) +
 ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[(1 -
 x)*(1 + x)^2]/(6*2^(1/3)) - Log[1 + x^3]/(3*2^(1/3)) + Log[1 + (2^(2/3)*(1
 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) -
 (2^(2/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 + Log[2^(1/3) - (1
- x^3)^(1/3)]/(2*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))
 - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}+\frac{2x}{\sqrt[3]{1-x^3}\,(1+x^3)}+\frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\right)dx \\
&= 2\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx+\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx+\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx \\
&= -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} \\
&\quad +\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx,x,x^3\right) \\
&\quad -\frac{2}{3}\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx-2\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)
\end{aligned}
$$

$$= \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

$$+ \frac{\log\left((1-x)(1+x)^2\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+x^3\right)}{3\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

$$- \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)$$

$$-\frac{2}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{2}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Sub}}{}$$

$$= \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{6\sqrt[3]{2}}$$

$$- \frac{\log\left(1+x^3\right)}{3\sqrt[3]{2}} - \frac{1}{3}2^{2/3}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

$$+ \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

$$+ \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}$$

$$- \text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

$$+ \frac{\log\left((1-x)(1+x)^2\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+x^3\right)}{3\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

$$-\frac{1}{3}2^{2/3}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{\log\left(-1+x+\right.}{2\sqrt[3]{}}$$

$$= \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+x^3\right)}{3\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{1}{3}2^{2/3}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{\log\left(-1+x+2\cdots\right.}{2\sqrt[3]{2}}$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right) - 2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right) + \log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2\cdots\right.}{2\sqrt[3]{2}}$$

```
[In] Integrate[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2^(1/3)*x + (1
 - x^3)^(1/3))] - 2*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/3)] + Log[2^(2/
3) - 2*2^(2/3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/
3)])/(2*2^(1/3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.46 (sec) , antiderivative size = 677, normalized size of antiderivative = 5.01

method	result	size
trager	Expression too large to display	677

```
[In] int((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*RootOf(_Z^3+4)*ln((RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*
RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)-RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)
+4*_Z^2)*RootOf(_Z^3+4)^3*x-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4
*_Z^2)^2*RootOf(_Z^3+4)^2*x+(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x+2*(-x^3+1)^(1
/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x-(-
x^3+1)^(1/3)*RootOf(_Z^3+4)^2-2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z
*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+RootOf(_Z^3+4)*x^2+2*RootOf(RootOf(_
Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2-RootOf(_Z^3+4)*x-2*RootOf(RootOf(_
Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+RootOf(_Z^3+4)+2*RootOf(RootOf(_Z^3+
4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2-x+1))+RootOf(RootOf(_Z^3+4)^2+2*_Z*R
ootOf(_Z^3+4)+4*_Z^2)*ln(-(RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z
^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)-RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^
3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+
4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x+(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x-(-x^3+1)^
(1/3)*RootOf(_Z^3+4)^2-RootOf(_Z^3+4)*x^2-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*Ro
otOf(_Z^3+4)+4*_Z^2)*x^2-2*(-x^3+1)^(2/3)+3*RootOf(_Z^3+4)*x+6*RootOf(RootO
f(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x-RootOf(_Z^3+4)-2*RootOf(RootOf(_Z
^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2-x+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(101) = 202$.

Time = 5.54 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.36

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$
$$= \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{1}{6}}\left(4\cdot 2^{\frac{1}{6}}(-1)^{\frac{2}{3}}\left(x^4-4x^3+5x^2-4x+1\right)(-x^3+1)^{\frac{2}{3}}-4\sqrt{2}(-1)^{\frac{1}{3}}\left(x^5-x^4\right.\right.}{6\left(3x^6-9x^5+6x^4\right.}\right.$$
$$-\frac{1}{12}$$
$$\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{2}{3}}(x^2-3x+1)+2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^4-3x^2+1\right)+4\left(-x^3+1\right)^{\frac{1}{3}}(x^2-x)}{x^4-2x^3+3x^2-2x+1}\right)$$
$$+\frac{1}{6}$$
$$\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}(x-1)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^2-x+1\right)-2\left(-x^3+1\right)^{\frac{2}{3}}}{x^2-x+1}\right)$$

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(4*2^(1/6)*(-1)^(
2/3)*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) - 4*sqrt(2)*(-1)^(1/3
)*(x^5 - x^4 - 3*x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 2^(5/6)*(x^6 - 7*x
```

```
^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x
^2 - 9*x + 3)) - 1/12*2^(2/3)*(-1)^(1/3)*log(-(2^(2/3)*(-1)^(1/3)*(-x^3 + 1
)^(2/3)*(x^2 - 3*x + 1) + 2^(1/3)*(-1)^(2/3)*(x^4 - 3*x^2 + 1) + 4*(-x^3 +
1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/6*2^(2/3)*(-1)^(1/
3)*log(-(2*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) + 2^(2/3)*(-1)^(1/3)
*(x^2 - x + 1) - 2*(-x^3 + 1)^(2/3))/(x^2 - x + 1))
```

Sympy [F]

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \frac{x+1}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x^2-x+1)}\,dx$$

```
[In] integrate((1+x)/(x**2-x+1)/(-x**3+1)**(1/3),x)
[Out] Integral((x + 1)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \frac{x+1}{(-x^3+1)^{\frac{1}{3}}(x^2-x+1)}\,dx$$

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

Giac [F]

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \frac{x+1}{(-x^3+1)^{\frac{1}{3}}(x^2-x+1)}\,dx$$

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \frac{x+1}{(1-x^3)^{1/3}\,(x^2-x+1)}\,dx$$

```
[In] int((x + 1)/((1 - x^3)^(1/3)*(x^2 - x + 1)),x)
[Out] int((x + 1)/((1 - x^3)^(1/3)*(x^2 - x + 1)), x)
```

3.101 $\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 135

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

```
[Out] 1/4*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/
3)-1/2*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*arctan(1/3*(1-2*2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 383 vs. 2(135) = 270.

Time = 0.21 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.84, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules

used = {1600, 2183, 384, 502, 2174, 206, 31, 648, 631, 210, 642, 455, 57}

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$
$$= \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$
$$+ \frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(x^3+1\right)}{3\sqrt[3]{2}} + \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}}$$
$$-\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x -\right.}{2\sqrt[3]{2}}$$

```
[In] Int[(1 + x)^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]
) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) +
 ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[(1 -
 x)*(1 + x)^2]/(6*2^(1/3)) - Log[1 + x^3]/(3*2^(1/3)) + Log[1 + (2^(2/3)*(1
 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) -
 (2^(2/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 + Log[2^(1/3) - (1
- x^3)^(1/3)]/(2*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))
 - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx \\
&= \int \left(\frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}+\frac{2x}{\sqrt[3]{1-x^3}\,(1+x^3)}+\frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\right)dx \\
&= 2\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx+\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx+\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx
\end{aligned}
$$

$$= -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$
$$+ \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)$$
$$- \frac{2}{3}\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx - 2\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$
$$+ \frac{\log((1-x)(1+x)^2)}{6\sqrt[3]{2}} - \frac{\log(1+x^3)}{3\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$
$$- \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)$$
$$-\frac{2}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{2}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}}{}$$

$$= \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log((1-x)(1+x)^2)}{6\sqrt[3]{2}}$$
$$- \frac{\log(1+x^3)}{3\sqrt[3]{2}} - \frac{1}{3}2^{2/3}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$
$$+ \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$
$$+ \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}$$
$$- \text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+x^3\right)}{3\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{1}{3}2^{2/3}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{\log\left(-1+x+\right.}{2\sqrt[3]{}}$$

$$= \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+x^3\right)}{3\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{1}{3}2^{2/3}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{\log\left(-1+x+\right.}{2\sqrt[3]{}}$$

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right) - 2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right) + \log\left(2^{2/3}-2\ 2^{2/3}x+2^{2/3}x^{2}\right.}{2\sqrt[3]{2}}$$

```
[In] Integrate[(1 + x)^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2^(1/3)*x + (1
 - x^3)^(1/3))] - 2*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/3)] + Log[2^(2/
3) - 2*2^(2/3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/
3)])/(2*2^(1/3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.96 (sec) , antiderivative size = 737, normalized size of antiderivative = 5.46

method	result	size
trager	Expression too large to display	737

```
[In] int((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*RootOf(_Z^3+4)*ln(-(RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)
*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3
+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x+2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2
*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x-2*(-x^3+1)^(1/3)*RootOf(RootOf(
_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+2*RootOf(RootOf(_Z^3+4
)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2-2*(-x^3+1)^(2/3)-6*RootOf(RootOf(_Z^3+4
)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^
3+4)+4*_Z^2))/(x^2-x+1))-1/2*ln((RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4
)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*Ro
otOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x
+(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3
+4)+4*_Z^2)*x^2+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x-2*R
ootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2-x+1))*RootOf(_Z^3+
4)-ln((RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2
*(-x^3+1)^(2/3)+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*Roo
tOf(_Z^3+4)^2*x-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x+(-x^3+1)^(1/3)*RootOf(_Z^
3+4)^2-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2+2*RootOf(R
ootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x-2*RootOf(RootOf(_Z^3+4)^2+2*_
Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2-x+1))*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z
^3+4)+4*_Z^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(101) = 202$.

Time = 5.25 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.36

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$
$$=\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{1}{6}}\left(4\cdot 2^{\frac{1}{6}}(-1)^{\frac{2}{3}}\,(x^4-4\,x^3+5\,x^2-4\,x+1)(-x^3+1)^{\frac{2}{3}}-4\,\sqrt{2}(-1)^{\frac{1}{3}}\,(x^5-x^4}{6\,(3\,x^6-9\,x^5+6\,x^4}\right.$$
$$-\frac{1}{12}$$
$$\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(-x^3+1)^{\frac{2}{3}}(x^2-3\,x+1)+2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(x^4-3\,x^2+1)+4\,(-x^3+1)^{\frac{1}{3}}(x^2-x)}{x^4-2\,x^3+3\,x^2-2\,x+1}\right)$$
$$+\frac{1}{6}$$
$$\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(-x^3+1)^{\frac{1}{3}}(x-1)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(x^2-x+1)-2\,(-x^3+1)^{\frac{2}{3}}}{x^2-x+1}\right)$$

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(4*2^(1/6)*(-1)^(
2/3)*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) - 4*sqrt(2)*(-1)^(1/3
)*(x^5 - x^4 - 3*x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 2^(5/6)*(x^6 - 7*x
^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x
^2 - 9*x + 3)) - 1/12*2^(2/3)*(-1)^(1/3)*log(-(2^(2/3)*(-1)^(1/3)*(-x^3 + 1
)^(2/3)*(x^2 - 3*x + 1) + 2^(1/3)*(-1)^(2/3)*(x^4 - 3*x^2 + 1) + 4*(-x^3 +
1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/6*2^(2/3)*(-1)^(1/
3)*log(-(2*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) + 2^(2/3)*(-1)^(1/3)
*(x^2 - x + 1) - 2*(-x^3 + 1)^(2/3))/(x^2 - x + 1))
```

Sympy [F]

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\int \frac{x+1}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x^2-x+1)}\,dx$$

```
[In] integrate((1+x)**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral((x + 1)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{(x+1)^2}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate((x + 1)^2/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{(x+1)^2}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate((x + 1)^2/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{(x+1)^2}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

```
[In] int((x + 1)^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int((x + 1)^2/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.102 $\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1+x)^2}{(1+x^3)^{2/3}}-\frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}\right)}{2\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}\right)}{\sqrt[3]{2}}$$

```
[Out] -1/4*ln(1+2^(2/3)*(1+x)^2/(x^3+1)^(2/3)-2^(1/3)*(1+x)/(x^3+1)^(1/3))*2^(2/3
)+1/2*ln(1+2^(1/3)*(1+x)/(x^3+1)^(1/3))*2^(2/3)-1/2*arctan(1/3*(1-2*2^(1/3)
*(1+x)/(x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 357 vs. 2(119) = 238.

Time = 0.20 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules

used = {2183, 384, 502, 2174, 206, 31, 648, 631, 210, 642, 455, 57}

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx$$

$$= \frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$
$$- \frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(1-x^3)}{3\sqrt[3]{2}} - \frac{\log\left(\frac{2^{2/3}(x+1)^2}{(x^3+1)^{2/3}} - \frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1\right)}{3\sqrt[3]{2}}$$
$$+\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{x^3+1}\right)}{2\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}x-\sqrt[3]{x^3+1}\right)}{2\sqrt[3]{2}} + \frac{\log\left(-2^{2/3}\sqrt[3]{x^3+1}+x+\right.}{2\sqrt[3]{2}}$$

```
[In] Int[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*2^(1/3)*x)/(1 + x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - (2^
(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 + x))/(1 + x^3)^(1/3))/Sqrt[3]])/Sqrt[3] -
ArcTan[(1 + (2^(1/3)*(1 + x))/(1 + x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) -
 ArcTan[(1 + 2^(2/3)*(1 + x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[(1 -
 x)^2*(1 + x)]/(6*2^(1/3)) + Log[1 - x^3]/(3*2^(1/3)) - Log[1 + (2^(2/3)*(1
 + x)^2)/(1 + x^3)^(2/3) - (2^(1/3)*(1 + x))/(1 + x^3)^(1/3)]/(3*2^(1/3)) +
 (2^(2/3)*Log[1 + (2^(1/3)*(1 + x))/(1 + x^3)^(1/3)])/3 - Log[2^(1/3) - (1
+ x^3)^(1/3)]/(2*2^(1/3)) - Log[2^(1/3)*x - (1 + x^3)^(1/3)]/(2*2^(1/3)) +
Log[1 + x - 2^(2/3)*(1 + x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{(1-x^3)\sqrt[3]{1+x^3}} - \frac{2x}{(1-x^3)\sqrt[3]{1+x^3}} + \frac{x^2}{(1-x^3)\sqrt[3]{1+x^3}}\right) dx$$

$$= -\left(2\int \frac{x}{(1-x^3)\sqrt[3]{1+x^3}}\,dx\right) + \int \frac{1}{(1-x^3)\sqrt[3]{1+x^3}}\,dx + \int \frac{x^2}{(1-x^3)\sqrt[3]{1+x^3}}\,dx$$

$$= \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{2}x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(1-x^3)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}x - \sqrt[3]{1+x^3}\right)}{2\sqrt[3]{2}} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt[3]{1+x}}\,dx, x, x^3\right) - \frac{2}{3}\int \frac{1}{(1-x)\sqrt[3]{1+x^3}}\,dx + 2\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{1+x}{\sqrt[3]{1+x^3}}\right)$$

$$=\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{2}x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

$$-\frac{\log\left((1-x)^2(1+x)\right)}{6\sqrt[3]{2}}+\frac{\log\left(1-x^3\right)}{3\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}x-\sqrt[3]{1+x^3}\right)}{2\sqrt[3]{2}}$$

$$+\frac{\log\left(1+x-2^{2/3}\sqrt[3]{1+x^3}\right)}{2\sqrt[3]{2}}-\frac{1}{2}\text{Subst}\left(\int\frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx,x,\sqrt[3]{1+x^3}\right)$$

$$+\frac{2}{3}\text{Subst}\left(\int\frac{1}{1+\sqrt[3]{2}x}\,dx,x,\frac{1+x}{\sqrt[3]{1+x^3}}\right)$$

$$+\frac{2}{3}\text{Subst}\left(\int\frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1+x}{\sqrt[3]{1+x^3}}\right)+\frac{\text{Subst}\left(\int\frac{1}{\sqrt[3]{2}-x}\,dx,x,\sqrt[3]{1+x^3}\right)}{2\sqrt[3]{2}}$$

$$=\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{2}x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

$$-\frac{\log\left((1-x)^2(1+x)\right)}{6\sqrt[3]{2}}+\frac{\log\left(1-x^3\right)}{3\sqrt[3]{2}}+\frac{1}{3}2^{2/3}\log\left(1+\frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}\right)$$

$$-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1+x^3}\right)}{2\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}x-\sqrt[3]{1+x^3}\right)}{2\sqrt[3]{2}}$$

$$+\frac{\log\left(1+x-2^{2/3}\sqrt[3]{1+x^3}\right)}{2\sqrt[3]{2}}-\frac{\text{Subst}\left(\int\frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1+x}{\sqrt[3]{1+x^3}}\right)}{3\sqrt[3]{2}}$$

$$+\frac{\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2^{2/3}\sqrt[3]{1+x^3}\right)}{\sqrt[3]{2}}+\text{Subst}\left(\int\frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1+x}{\sqrt[3]{1+x^3}}\right)$$

$$=\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{2}x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1+x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

$$-\frac{\log\left((1-x)^2(1+x)\right)}{6\sqrt[3]{2}}+\frac{\log\left(1-x^3\right)}{3\sqrt[3]{2}}-\frac{\log\left(1+\frac{2^{2/3}(1+x)^2}{(1+x^3)^{2/3}}-\frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}\right)}{3\sqrt[3]{2}}$$

$$+\frac{1}{3}2^{2/3}\log\left(1+\frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1+x^3}\right)}{2\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}x-\sqrt[3]{1+x^3}\right)}{2\sqrt[3]{2}}+\frac{\log\left(1+x-2^{2/3}\right.}{2\sqrt[3]{2}}$$

$$
\begin{aligned}
&= \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{2}x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} \\
&- \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1+x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} \\
&- \frac{\log\left((1-x)^2(1+x)\right)}{6\sqrt[3]{2}} + \frac{\log\left(1-x^3\right)}{3\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1+x)^2}{(1+x^3)^{2/3}} - \frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}\right)}{3\sqrt[3]{2}} \\
&+ \frac{1}{3}2^{2/3}\log\left(1+\frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1+x^3}\right)}{2\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}x-\sqrt[3]{1+x^3}\right)}{2\sqrt[3]{2}} + \frac{\log\left(1+x-2^{2/3}\sqrt[3]{\right.}{2\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.17

$$
\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1+x^3}}{-2\sqrt[3]{2}-2\sqrt[3]{2}x+\sqrt[3]{1+x^3}}\right) + 2\log\left(\sqrt[3]{2}+\sqrt[3]{2}x+\sqrt[3]{1+x^3}\right) - \log\left(2^{2/3}+2\ 2^{2/3}x+2^{2/3}x^2-\sqrt[3]{\right.}{2\sqrt[3]{2}}
$$

```
[In] Integrate[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 + x^3)^(1/3))/(-2*2^(1/3) - 2*2^(1/3)*x + (1
+ x^3)^(1/3))] + 2*Log[2^(1/3) + 2^(1/3)*x + (1 + x^3)^(1/3)] - Log[2^(2/3)
 + 2*2^(2/3)*x + 2^(2/3)*x^2 - 2^(1/3)*(1 + x)*(1 + x^3)^(1/3) + (1 + x^3)^
(2/3)])/(2*2^(1/3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.40 (sec) , antiderivative size = 714, normalized size of antiderivative = 6.00

method	result	size
trager	Expression too large to display	714

```
[In] int((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(-((x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)
*RootOf(_Z^3-4)^2+2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*R
ootOf(_Z^3-4)^2*x-(x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x-(x^3+1)^(1/3)*RootOf(_Z^
3-4)^2-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-2*RootOf(R
ootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-2*RootOf(RootOf(_Z^3-4)^2+2*_
Z*RootOf(_Z^3-4)+4*_Z^2))/(x^2+x+1))*RootOf(_Z^3-4)-ln(-((x^3+1)^(2/3)*Root
Of(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2+2*RootOf(R
ootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-(x^3+1)^(1
/3)*RootOf(_Z^3-4)^2*x-(x^3+1)^(1/3)*RootOf(_Z^3-4)^2-2*RootOf(RootOf(_Z^3-
4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(
_Z^3-4)+4*_Z^2)*x-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(x
^2+x+1))*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+1/2*RootOf(_Z^
3-4)*ln(((x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*
RootOf(_Z^3-4)^2+2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*Ro
otOf(_Z^3-4)^2*x+2*(x^3+1)^(1/3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+2*_
Z*RootOf(_Z^3-4)+4*_Z^2)*x+2*(x^3+1)^(1/3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^
3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)*x^2+2*(x^3+1)^(2/3)+6*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^
3-4)+4*_Z^2)*x+2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(x^2+
x+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(93) = 186$.

Time = 5.27 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.25

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx$$
$$= \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{1}{6}}\left(2^{\frac{5}{6}}(x^6+7x^5+10x^4+7x^3+10x^2+7x+1)-4\sqrt{2}(x^5+x^4-3x^3-3x^2+x}{6(3x^6+9x^5+6x^4+x^3+6x^2+}\right.$$
$$-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}(x^3+1)^{\frac{2}{3}}(x^2+3x+1)-2^{\frac{1}{3}}(x^4-3x^2+1)-4(x^3+1)^{\frac{1}{3}}(x^2+x)}{x^4+2x^3+3x^2+2x+1}\right)$$
$$+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}(x^2+x+1)+2\cdot 2^{\frac{1}{3}}(x^3+1)^{\frac{1}{3}}(x+1)+2(x^3+1)^{\frac{2}{3}}}{x^2+x+1}\right)$$

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^6 + 7*x^5 + 10*x
^4 + 7*x^3 + 10*x^2 + 7*x + 1) - 4*sqrt(2)*(x^5 + x^4 - 3*x^3 - 3*x^2 + x +
 1)*(x^3 + 1)^(1/3) + 4*2^(1/6)*(x^4 + 4*x^3 + 5*x^2 + 4*x + 1)*(x^3 + 1)^(
2/3))/(3*x^6 + 9*x^5 + 6*x^4 + x^3 + 6*x^2 + 9*x + 3)) - 1/12*2^(2/3)*log((
2^(2/3)*(x^3 + 1)^(2/3)*(x^2 + 3*x + 1) - 2^(1/3)*(x^4 - 3*x^2 + 1) - 4*(x^
```

```
3 + 1)^(1/3)*(x^2 + x))/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)) + 1/6*2^(2/3)*log(
(2^(2/3)*(x^2 + x + 1) + 2*2^(1/3)*(x^3 + 1)^(1/3)*(x + 1) + 2*(x^3 + 1)^(2
/3))/(x^2 + x + 1))
```

Sympy [F]

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx = -\int \frac{x}{x^2\sqrt[3]{x^3+1}+x\sqrt[3]{x^3+1}+\sqrt[3]{x^3+1}}\,dx - \int \left(-\frac{1}{x^2\sqrt[3]{x^3+1}+x\sqrt[3]{x^3+1}+\sqrt[3]{x^3+1}}\right)dx$$

```
[In] integrate((1-x)/(x**2+x+1)/(x**3+1)**(1/3),x)
[Out] -Integral(x/(x**2*(x**3 + 1)**(1/3) + x*(x**3 + 1)**(1/3) + (x**3 + 1)**(1/
3)), x) - Integral(-1/(x**2*(x**3 + 1)**(1/3) + x*(x**3 + 1)**(1/3) + (x**3
 + 1)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx = \int -\frac{x-1}{(x^3+1)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="maxima")
[Out] -integrate((x - 1)/((x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

Giac [F]

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx = \int -\frac{x-1}{(x^3+1)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="giac")
[Out] integrate(-(x - 1)/((x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx = -\int \frac{x-1}{(x^3+1)^{1/3}\,(x^2+x+1)}\,dx$$

```
[In] int(-(x - 1)/((x^3 + 1)^(1/3)*(x + x^2 + 1)),x)
[Out] -int((x - 1)/((x^3 + 1)^(1/3)*(x + x^2 + 1)), x)
```

3.103 $\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2} dx = \frac{1}{\sqrt[3]{1-x^3}} + \frac{x}{\sqrt[3]{1-x^3}} - x^2 \text{Hypergeometric2F1}\left(\frac{2}{3}, \frac{4}{3}, \frac{5}{3}, x^3\right)$$

```
[Out] 1/(-x^3+1)^(1/3)+x/(-x^3+1)^(1/3)-x^2*hypergeom([2/3, 4/3],[5/3],x^3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2183, 197, 371, 267}

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2} dx = x^2 \left(-\text{Hypergeometric2F1}\left(\frac{2}{3}, \frac{4}{3}, \frac{5}{3}, x^3\right)\right) + \frac{x}{\sqrt[3]{1-x^3}} + \frac{1}{\sqrt[3]{1-x^3}}$$

```
[In] Int[(1 - x^3)^(2/3)/(1 + x + x^2)^2,x]
```

```
[Out] (1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3,
 x^3]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{(1-x^3)^{4/3}} - \frac{2x}{(1-x^3)^{4/3}} + \frac{x^2}{(1-x^3)^{4/3}}\right) dx \\ &= -\left(2\int \frac{x}{(1-x^3)^{4/3}}\, dx\right) + \int \frac{1}{(1-x^3)^{4/3}}\, dx + \int \frac{x^2}{(1-x^3)^{4/3}}\, dx \\ &= \frac{1}{\sqrt[3]{1-x^3}} + \frac{x}{\sqrt[3]{1-x^3}} - x^2\, \text{Hypergeometric2F1}\left(\frac{2}{3}, \frac{4}{3}, \frac{5}{3}, x^3\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 10.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2}\, dx = \frac{(1+2x)\,(1-x^3)^{2/3}}{1+x+x^2} + x^2\, \text{Hypergeometric2F1}\left(\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right)$$

```
[In] Integrate[(1 - x^3)^(2/3)/(1 + x + x^2)^2,x]
[Out] ((1 + 2*x)*(1 - x^3)^(2/3))/(1 + x + x^2) + x^2*Hypergeometric2F1[1/3, 2/3,
 5/3, x^3]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{(-1+x)(1+2x)}{(-x^3+1)^{\frac{1}{3}}}+x^2{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$	34

```
[In] int((-x^3+1)^(2/3)/(x^2+x+1)^2,x,method=_RETURNVERBOSE)
[Out] -(-1+x)*(1+2*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

Fricas [F]

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2}\,dx=\int \frac{(-x^3+1)^{\frac{2}{3}}}{(x^2+x+1)^2}\,dx$$

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

Sympy [F]

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2}\,dx=\int \frac{(-(x-1)(x^2+x+1))^{\frac{2}{3}}}{(x^2+x+1)^2}\,dx$$

```
[In] integrate((-x**3+1)**(2/3)/(x**2+x+1)**2,x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x**2 + x + 1)**2, x)
```

Maxima [F]

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2}\,dx=\int \frac{(-x^3+1)^{\frac{2}{3}}}{(x^2+x+1)^2}\,dx$$

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 + x + 1)^2, x)
```

Giac [F]

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{(x^2+x+1)^2}\,dx$$

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 + x + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2}\,dx = \int \frac{(1-x^3)^{2/3}}{(x^2+x+1)^2}\,dx$$

```
[In] int((1 - x^3)^(2/3)/(x + x^2 + 1)^2,x)
[Out] int((1 - x^3)^(2/3)/(x + x^2 + 1)^2, x)
```

3.104 $\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 43

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx=\frac{1}{\sqrt[3]{1-x^3}}+\frac{x}{\sqrt[3]{1-x^3}}-x^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},x^3\right)$$

```
[Out] 1/(-x^3+1)^(1/3)+x/(-x^3+1)^(1/3)-x^2*hypergeom([2/3, 4/3],[5/3],x^3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2183, 197, 371, 267}

$$\begin{aligned}&\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx\\&=x^2\left(-\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},x^3\right)\right)+\frac{x}{\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}}\end{aligned}$$

```
[In] Int[(1 - x)/((1 + x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] (1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3,
 x^3]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{(1-x^3)^{4/3}}-\frac{2x}{(1-x^3)^{4/3}}+\frac{x^2}{(1-x^3)^{4/3}}\right)dx \\
&= -\left(2\int \frac{x}{(1-x^3)^{4/3}}\,dx\right)+\int \frac{1}{(1-x^3)^{4/3}}\,dx+\int \frac{x^2}{(1-x^3)^{4/3}}\,dx \\
&= \frac{1}{\sqrt[3]{1-x^3}}+\frac{x}{\sqrt[3]{1-x^3}}-x^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},x^3\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx=\frac{(1+2x)\,(1-x^3)^{2/3}}{1+x+x^2}+x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

```
[In] Integrate[(1 - x)/((1 + x + x^2)*(1 - x^3)^(1/3)),x]
[Out] ((1 + 2*x)*(1 - x^3)^(2/3))/(1 + x + x^2) + x^2*Hypergeometric2F1[1/3, 2/3,
 5/3, x^3]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{(-1+x)(1+2x)}{(-x^3+1)^{\frac{1}{3}}}+x^2{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$	34

```
[In] int((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
[Out] -(-1+x)*(1+2*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

Fricas [F]

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx=\int -\frac{x-1}{(-x^3+1)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

Sympy [F]

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx=-\int \frac{x}{x^2\sqrt[3]{1-x^3}+x\sqrt[3]{1-x^3}+\sqrt[3]{1-x^3}}\,dx-\int \left(-\frac{1}{x^2\sqrt[3]{1-x^3}+x\sqrt[3]{1-x^3}+\sqrt[3]{1-x^3}}\right)dx$$

```
[In] integrate((1-x)/(x**2+x+1)/(-x**3+1)**(1/3),x)
[Out] -Integral(x/(x**2*(1 - x**3)**(1/3) + x*(1 - x**3)**(1/3) + (1 - x**3)**(1/
3)), x) - Integral(-1/(x**2*(1 - x**3)**(1/3) + x*(1 - x**3)**(1/3) + (1 -
x**3)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx = \int -\frac{x-1}{(-x^3+1)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
[Out] -integrate((x - 1)/((-x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

Giac [F]

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx = \int -\frac{x-1}{(-x^3+1)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
[Out] integrate(-(x - 1)/((-x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx = -\int \frac{x-1}{(1-x^3)^{1/3}\,(x^2+x+1)}\,dx$$

```
[In] int(-(x - 1)/((1 - x^3)^(1/3)*(x + x^2 + 1)),x)
[Out] -int((x - 1)/((1 - x^3)^(1/3)*(x + x^2 + 1)), x)
```

3.105 $\int \frac{(1-x)^2}{(1-x^3)^{4/3}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}} \, dx = \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} + x^2 \operatorname{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

```
[Out] (1+(1-2*x)*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3, 2/3],[5/3],x^3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1868, 12, 371}

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}} \, dx = x^2 \operatorname{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{(1-2x)x+1}{\sqrt[3]{1-x^3}}$$

```
[In] Int[(1 - x)^2/(1 - x^3)^(4/3),x]
```

```
[Out] (1 + (1 - 2*x)*x)/(1 - x^3)^(1/3) + x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^
3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} - \int -\frac{2x}{\sqrt[3]{1-x^3}}\,dx \\
&= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} + 2\int \frac{x}{\sqrt[3]{1-x^3}}\,dx \\
&= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} + x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}}\,dx = \frac{1}{\sqrt[3]{1-x^3}} + \frac{x}{\sqrt[3]{1-x^3}} - x^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},x^3\right)$$

```
[In] Integrate[(1 - x)^2/(1 - x^3)^(4/3),x]
[Out] (1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3,
 x^3]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
risch	$-\frac{(-1+x)(1+2x)}{(-x^3+1)^{\frac{1}{3}}}+x^2{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$	34
meijerg	$\frac{x}{(-x^3+1)^{\frac{1}{3}}}-x^2{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)+\frac{x^3{}_2F_1\left(1,\frac{4}{3};2;x^3\right)}{3}$	41

```
[In] int((1-x)^2/(-x^3+1)^(4/3),x,method=_RETURNVERBOSE)
[Out] -(-1+x)*(1+2*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

Fricas [F]

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}}\,dx = \int \frac{(x-1)^2}{(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

Sympy [F]

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}}\,dx = \int \frac{(x-1)^2}{(-(x-1)(x^2+x+1))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((1-x)**2/(-x**3+1)**(4/3),x)
[Out] Integral((x - 1)**2/(-(x - 1)*(x**2 + x + 1))**(4/3), x)
```

Maxima [F]

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}}\,dx = \int \frac{(x-1)^2}{(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="maxima")
[Out] x/(-x^3 + 1)^(1/3) - integrate((x^2 - 2*x)/((x^3 - 1)*(x^2 + x + 1)^(1/3)*(
-x + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}}\,dx = \int \frac{(x-1)^2}{(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="giac")
[Out] integrate((x - 1)^2/(-x^3 + 1)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}}\,dx = \int \frac{(x-1)^2}{(1-x^3)^{4/3}}\,dx$$

```
[In] int((x - 1)^2/(1 - x^3)^(4/3),x)
[Out] int((x - 1)^2/(1 - x^3)^(4/3), x)
```

3.106 $\int \left(1-x^3\right)^{2/3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 67

$$\int \left(1-x^3\right)^{2/3} \, dx=\frac{1}{3}x\left(1-x^3\right)^{2/3}-\frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(x+\sqrt[3]{1-x^3}\right)$$

```
[Out] 1/3*x*(-x^3+1)^(2/3)+1/3*ln(x+(-x^3+1)^(1/3))-2/9*arctan(1/3*(1-2*x/(-x^3+1
)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {201, 245}

$$\int \left(1-x^3\right)^{2/3} \, dx=-\frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\left(1-x^3\right)^{2/3}x+\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)$$

```
[In] Int[(1 - x^3)^(2/3),x]
```

```
[Out] (x*(1 - x^3)^(2/3))/3 - (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*
Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/3
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{3}x(1-x^3)^{2/3} + \frac{2}{3}\int \frac{1}{\sqrt[3]{1-x^3}}\,dx$$

$$= \frac{1}{3}x(1-x^3)^{2/3} - \frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3}\log\left(x+\sqrt[3]{1-x^3}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.51

$$\int (1-x^3)^{2/3}\,dx = \frac{3(-1+x)\,(1-x^3)^{2/3}\,\text{AppellF1}\left(\frac{5}{3},-\frac{2}{3},-\frac{2}{3},\frac{8}{3},-\frac{-1+x}{1-(-1)^{2/3}},-\frac{-1+x}{1+\sqrt[3]{-1}}\right)}{5\left(1+\frac{-1+x}{1+\sqrt[3]{-1}}\right)^{2/3}\left(1+\frac{-1+x}{1-(-1)^{2/3}}\right)^{2/3}}$$

```
[In] Integrate[(1 - x^3)^(2/3),x]
[Out] (3*(-1 + x)*(1 - x^3)^(2/3)*AppellF1[5/3, -2/3, -2/3, 8/3, -((-1 + x)/(1 -
(-1)^(2/3))), -((-1 + x)/(1 + (-1)^(1/3)))])/(5*(1 + (-1 + x)/(1 + (-1)^(1/
3)))^(2/3)*(1 + (-1 + x)/(1 - (-1)^(2/3)))^(2/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.18

method	result
meijerg	$x{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};x^3\right)$
risch	$-\frac{x(x^3-1)}{3(-x^3+1)^{\frac{1}{3}}}+\frac{2x{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};x^3\right)}{3}$
pseudoelliptic	$\frac{3x(-x^3+1)^{\frac{2}{3}}+2\sqrt{3}\arctan\left(\frac{\left(-2\left(-x^3+1\right)^{\frac{1}{3}}+x\right)\sqrt{3}}{3x}\right)+2\ln\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-\ln\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}-x\left(-x^3+1\right)^{\frac{1}{3}}+x^2}{x^2}\right)}{9\left(x+(-x^3+1)^{\frac{1}{3}}\right)\left((-x^3+1)^{\frac{2}{3}}-x(-x^3+1)^{\frac{1}{3}}+x^2\right)}$
trager	$\frac{x\left(-x^3+1\right)^{\frac{2}{3}}}{3}+\frac{2\ln\left(-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3+3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)(-x^3+1)^{\frac{2}{3}}x-5\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)}{9}$

```
[In] int((-x^3+1)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] x*hypergeom([-2/3,1/3],[4/3],x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \left(1-x^3\right)^{2/3}\,dx=\frac{1}{3}\left(-x^3+1\right)^{\frac{2}{3}}x-\frac{2}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3\,x}\right)+\frac{2}{9}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right)-\frac{1}{9}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3*(-x^3 + 1)^(2/3)*x - 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x
^3 + 1)^(1/3))/x) + 2/9*log((x + (-x^3 + 1)^(1/3))/x) - 1/9*log((x^2 - (-x^
3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

$$\int \left(1-x^3\right)^{2/3}\,dx=\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((-x**3+1)**(2/3),x)
[Out] x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/
3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(52) = 104.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.57

$$\int \left(1-x^3\right)^{2/3}\,dx=-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right)-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{3\,x^2\left(\frac{x^3-1}{x^3}-1\right)}+\frac{2}{9}\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right)-\frac{1}{9}\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)$$

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="maxima")
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*(-x^3 + 1
)^(2/3)/(x^2*((x^3 - 1)/x^3 - 1)) + 2/9*log((-x^3 + 1)^(1/3)/x + 1) - 1/9*l
og(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)
```

Giac [F]

$$\int \left(1-x^3\right)^{2/3}\,dx=\int \left(-x^3+1\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.15

$$\int \left(1-x^3\right)^{2/3}\,dx=x\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{1}{3};\ \frac{4}{3};\ x^3\right)$$

```
[In] int((1 - x^3)^(2/3),x)
[Out] x*hypergeom([-2/3, 1/3], 4/3, x^3)
```

3.107 $\int \frac{(1-x^3)^{2/3}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{(1-x^3)^{2/3}}{x} \, dx = \frac{1}{2}(1-x^3)^{2/3} + \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

```
[Out] 1/2*(-x^3+1)^(2/3)-1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))+1/3*arctan(1/3*(1+2*(
-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {272, 52, 57, 632, 210, 31}

$$\int \frac{(1-x^3)^{2/3}}{x} \, dx = \frac{\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}(1-x^3)^{2/3} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log(x)}{2}$$

```
[In] Int[(1 - x^3)^(2/3)/x,x]
```

```
[Out] (1 - x^3)^(2/3)/2 + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x
]/2 + Log[1 - (1 - x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(1-x)^{2/3}}{x}\,dx, x, x^3\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}\,dx, x, x^3\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{\log(x)}{2}\\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)
\end{aligned}$$

$$= \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right)$$

$$= \frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

$$\int \frac{\left(1-x^3\right)^{2/3}}{x}\,dx = \frac{1}{6}\left(3\left(1-x^3\right)^{2/3} + 2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 2\log\left(-1+\sqrt[3]{1-x^3}\right) - \log\left(1+\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)\right)$$

```
[In] Integrate[(1 - x^3)^(2/3)/x,x]
[Out] (3*(1 - x^3)^(2/3) + 2*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] + 2*
Log[-1 + (1 - x^3)^(1/3)] - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)])/6
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.93 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

method	result
meijerg	$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{\left(\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}+\frac{2\pi\sqrt{3}\,x^3\,{}_3F_2\left(\frac{1}{3},1,1;2,2;x^3\right)}{3\Gamma\left(\frac{2}{3}\right)}\right)}{9\pi}$
pseudoelliptic	$\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{2}-\frac{\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)}{6}+\frac{\arctan\left(\frac{\left(1+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)}{3}$
trager	$\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{2}+\frac{\ln\left(\frac{-211\,\text{RootOf}\left(\_Z^2+\_Z+1\right)^2 x^3-3126\,\text{RootOf}\left(\_Z^2+\_Z+1\right)x^3+5502\,\text{RootOf}\left(\_Z^2+\_Z+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}{}\right.}{}$

```
[In] int((-x^3+1)^(2/3)/x,x,method=_RETURNVERBOSE)
[Out] -1/9/Pi*3^(1/2)*GAMMA(2/3)*(-(3/2-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*Pi
*3^(1/2)/GAMMA(2/3)+2/3*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1/3,1,1],[2,2]
,x^3))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

$$\int \frac{(1-x^3)^{2/3}}{x}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}(-x^3+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}} - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

```
[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 1/2*(-x^3
+ 1)^(2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x
^3 + 1)^(1/3) - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int \frac{(1-x^3)^{2/3}}{x}\,dx = -\frac{x^2 e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\right|\frac{1}{x^3}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

```
[In] integrate((-x**3+1)**(2/3)/x,x)
```

```
[Out] -x**2*exp(2*I*pi/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), x**(-3))/(3*gam
ma(1/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{(1-x^3)^{2/3}}{x}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}} - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

```
[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2*(-x^3 + 1)^(
2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1
)^(1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int \frac{(1-x^3)^{2/3}}{x}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}} -\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

```
[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2*(-x^3 + 1)^(
2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3
 + 1)^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.30

$$\int \frac{(1-x^3)^{2/3}}{x}\,dx = \frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3} +\ln\left(\left(1-x^3\right)^{1/3}-9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(\left(1-x^3\right)^{1/3}-9\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

```
[In] int((1 - x^3)^(2/3)/x,x)
```

```
[Out] log((1 - x^3)^(1/3) - 1)/3 + log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 - 1/6)
^2)*((3^(1/2)*1i)/6 - 1/6) - log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 + 1/6)
^2)*((3^(1/2)*1i)/6 + 1/6) + (1 - x^3)^(2/3)/2
```

3.108 $\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 384

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx = \frac{(1-x^3)^{2/3}}{2b} - \frac{(a^3+b^3)\,x^2\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-\frac{b^3x^3}{a^3}\right)}{2a^2b^2} + \frac{a^2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} - \frac{(a^3+b^3)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a^3+b^3}x}{a\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} + \frac{(a^3+b^3)^{2/3}\arctan\left(\frac{1+\frac{2b\sqrt[3]{1-x^3}}{\sqrt[3]{a^3+b^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} + \frac{ax^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)}{2b^2} - \frac{(a^3+b^3)^{2/3}\log(a^3+b^3x^3)}{3b^3} + \frac{(a^3+b^3)^{2/3}\log\left(-\frac{\sqrt[3]{a^3+b^3}x}{a}-\sqrt[3]{1-x^3}\right)}{2b^3} - \frac{a^2\log\left(x+\sqrt[3]{1-x^3}\right)}{2b^3} + \frac{(a^3+b^3)^{2/3}\log\left(\sqrt[3]{a^3+b^3}-b\sqrt[3]{1-x^3}\right)}{2b^3}$$

```
[Out] 1/2*(-x^3+1)^(2/3)/b-1/2*(a^3+b^3)*x^2*AppellF1(2/3,1/3,1,5/3,x^3,-b^3*x^3/
a^3)/a^2/b^2+1/2*a*x^2*hypergeom([1/3, 2/3],[5/3],x^3)/b^2-1/3*(a^3+b^3)^(2
/3)*ln(b^3*x^3+a^3)/b^3+1/2*(a^3+b^3)^(2/3)*ln(-(a^3+b^3)^(1/3)*x/a-(-x^3+1
)^(1/3))/b^3-1/2*a^2*ln(x+(-x^3+1)^(1/3))/b^3+1/2*(a^3+b^3)^(2/3)*ln((a^3+b
^3)^(1/3)-b*(-x^3+1)^(1/3))/b^3+1/3*a^2*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3
^(1/2))/b^3*3^(1/2)-1/3*(a^3+b^3)^(2/3)*arctan(1/3*(1-2*(a^3+b^3)^(1/3)*x/a
/(-x^3+1)^(1/3))*3^(1/2))/b^3*3^(1/2)+1/3*(a^3+b^3)^(2/3)*arctan(1/3*(1+2*b
*(-x^3+1)^(1/3)/(a^3+b^3)^(1/3))*3^(1/2))/b^3*3^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {2178, 2177, 245, 2181, 384, 524, 455, 57, 631, 210, 31, 371}

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx = -\frac{(a^3+b^3)^{2/3}\arctan\left(\frac{1-\frac{2x\sqrt[3]{a^3+b^3}}{a\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} + \frac{(a^3+b^3)^{2/3}\arctan\left(\frac{\frac{2b\sqrt[3]{1-x^3}}{\sqrt[3]{a^3+b^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^3} - \frac{(a^3+b^3)^{2/3}\log(a^3+b^3x^3)}{3b^3} + \frac{(a^3+b^3)^{2/3}\log\left(-\frac{x\sqrt[3]{a^3+b^3}}{a}-\sqrt[3]{1-x^3}\right)}{2b^3} + \frac{(a^3+b^3)^{2/3}\log\left(\sqrt[3]{a^3+b^3}-b\sqrt[3]{1-x^3}\right)}{2b^3} + \frac{a^2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} - \frac{a^2\log\left(\sqrt[3]{1-x^3}+x\right)}{2b^3} - \frac{x^2(a^3+b^3)\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-\frac{b^3x^3}{a^3}\right)}{2a^2b^2} + \frac{ax^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)}{2b^2} + \frac{(1-x^3)^{2/3}}{2b}$$

```
[In] Int[(1 - x^3)^(2/3)/(a + b*x),x]
```

```
[Out] (1 - x^3)^(2/3)/(2*b) - ((a^3 + b^3)*x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -(
(b^3*x^3)/a^3)])/(2*a^2*b^2) + (a^2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt
[3]])/(Sqrt[3]*b^3) - ((a^3 + b^3)^(2/3)*ArcTan[(1 - (2*(a^3 + b^3)^(1/3)*x
)/(a*(1 - x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^3) + ((a^3 + b^3)^(2/3)*ArcTan[
(1 + (2*b*(1 - x^3)^(1/3))/(a^3 + b^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^3) + (a*
x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/(2*b^2) - ((a^3 + b^3)^(2/3)*Log
[a^3 + b^3*x^3])/(3*b^3) + ((a^3 + b^3)^(2/3)*Log[-(((a^3 + b^3)^(1/3)*x)/a
) - (1 - x^3)^(1/3)])/(2*b^3) - (a^2*Log[x + (1 - x^3)^(1/3)])/(2*b^3) + ((
a^3 + b^3)^(2/3)*Log[(a^3 + b^3)^(1/3) - b*(1 - x^3)^(1/3)])/(2*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rule 2178

```
Int[((a_) + (b_.)*(x_)^3)^(2/3)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(a +
 b*x^3)^(2/3)/(2*d), x] + (Dist[1/d^2, Int[(a*d^2 + b*c^2*x)/((c + d*x)*(a
+ b*x^3)^(1/3)), x], x] - Dist[b*(c/d^2), Int[x/(a + b*x^3)^(1/3), x], x])
/; FreeQ[{a, b, c, d}, x]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\text{integral} = \frac{(1-x^3)^{2/3}}{2b} + \frac{\int \frac{b^2-a^2x}{(a+bx)\sqrt[3]{1-x^3}}\,dx}{b^2} + \frac{a\int \frac{x}{\sqrt[3]{1-x^3}}\,dx}{b^2}$$

$$= \frac{(1-x^3)^{2/3}}{2b} + \frac{ax^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)}{2b^2} - \frac{a^2\int \frac{1}{\sqrt[3]{1-x^3}}\,dx}{b^3} + \frac{(a^3+b^3)\int \frac{1}{(a+bx)\sqrt[3]{1-x^3}}\,dx}{b^3}$$

$$= \frac{(1-x^3)^{2/3}}{2b} + \frac{a^2 \arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} + \frac{ax^2 \text{Hypergeometric2F1}\left(\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right)}{2b^2} - \frac{a^2 \log\left(x+\sqrt[3]{1-x^3}\right)}{2b^3} + \frac{(a^3+b^3)\int\left(\frac{a^2}{\sqrt[3]{1-x^3}(a^3+b^3x^3)} - \frac{abx}{\sqrt[3]{1-x^3}(a^3+b^3x^3)} + \frac{b^2x^2}{\sqrt[3]{1-x^3}(a^3+b^3x^3)}\right)dx}{b^3}$$

$$= \frac{(1-x^3)^{2/3}}{2b} + \frac{a^2 \arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} + \frac{ax^2 \text{Hypergeometric2F1}\left(\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right)}{2b^2} - \frac{a^2 \log\left(x+\sqrt[3]{1-x^3}\right)}{2b^3} + \frac{(a^2(a^3+b^3))\int \frac{1}{\sqrt[3]{1-x^3}(a^3+b^3x^3)}dx}{b^3} - \frac{(a(a^3+b^3))\int \frac{x}{\sqrt[3]{1-x^3}(a^3+b^3x^3)}dx}{b^2} + \frac{(a^3+b^3)\int \frac{x^2}{\sqrt[3]{1-x^3}(a^3+b^3x^3)}dx}{b}$$

$$= \frac{(1-x^3)^{2/3}}{2b} - \frac{(a^3+b^3)\,x^2 \text{AppellF1}\left(\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, x^3, -\frac{b^3x^3}{a^3}\right)}{2a^2b^2} + \frac{a^2 \arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} - \frac{(a^3+b^3)^{2/3} \arctan\left(\frac{1-\frac{2\sqrt[3]{a^3+b^3}x}{a\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} + \frac{ax^2 \text{Hypergeometric2F1}\left(\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right)}{2b^2} - \frac{(a^3+b^3)^{2/3}\log(a^3+b^3x^3)}{6b^3} + \frac{(a^3+b^3)^{2/3}\log\left(-\frac{\sqrt[3]{a^3+b^3}x}{a} - \sqrt[3]{1-x^3}\right)}{2b^3} - \frac{a^2 \log\left(x+\sqrt[3]{1-x^3}\right)}{2b^3} + \frac{(a^3+b^3)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(a^3+b^3x)}dx, x, x^3\right)}{3b}$$

$$= \frac{(1-x^3)^{2/3}}{2b} - \frac{(a^3+b^3)\,x^2\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-\frac{b^3x^3}{a^3}\right)}{2a^2b^2} + \frac{a^2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} - \frac{(a^3+b^3)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a^3+b^3}x}{a\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} + \frac{ax^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)}{2b^2} - \frac{(a^3+b^3)^{2/3}\log\left(a^3+b^3x^3\right)}{3b^3} + \frac{(a^3+b^3)^{2/3}\log\left(-\frac{\sqrt[3]{a^3+b^3}x}{a}-\sqrt[3]{1-x^3}\right)}{2b^3} - \frac{a^2\log\left(x+\sqrt[3]{1-x^3}\right)}{2b^3} - \frac{(a^3+b^3)^{2/3}\,\text{Subst}\left(\int\frac{1}{\frac{\sqrt[3]{a^3+b^3}}{b}-x}\,dx,x,\sqrt[3]{1-x^3}\right)}{2b^3} + \frac{(a^3+b^3)\,\text{Subst}\left(\int\frac{1}{\frac{(a^3+b^3)^{2/3}}{b^2}+\frac{\sqrt[3]{a^3+b^3}x}{b}+x^2}\,dx,x,\sqrt[3]{1-x^3}\right)}{2b^4}$$

$$= \frac{(1-x^3)^{2/3}}{2b} - \frac{(a^3+b^3)\,x^2\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-\frac{b^3x^3}{a^3}\right)}{2a^2b^2} + \frac{a^2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} - \frac{(a^3+b^3)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a^3+b^3}x}{a\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} + \frac{ax^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)}{2b^2} - \frac{(a^3+b^3)^{2/3}\log\left(a^3+b^3x^3\right)}{3b^3} + \frac{(a^3+b^3)^{2/3}\log\left(-\frac{\sqrt[3]{a^3+b^3}x}{a}-\sqrt[3]{1-x^3}\right)}{2b^3} - \frac{a^2\log\left(x+\sqrt[3]{1-x^3}\right)}{2b^3} + \frac{(a^3+b^3)^{2/3}\log\left(\sqrt[3]{a^3+b^3}-b\sqrt[3]{1-x^3}\right)}{2b^3} - \frac{(a^3+b^3)^{2/3}\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2b\sqrt[3]{1-x^3}}{\sqrt[3]{a^3+b^3}}\right)}{b^3}$$

$$= \frac{(1-x^3)^{2/3}}{2b} - \frac{(a^3+b^3)\,x^2\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-\frac{b^3x^3}{a^3}\right)}{2a^2b^2}$$
$$+ \frac{a^2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} - \frac{(a^3+b^3)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a^3+b^3}x}{a\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3}$$
$$+ \frac{(a^3+b^3)^{2/3}\arctan\left(\frac{1+\frac{2b\sqrt[3]{1-x^3}}{\sqrt[3]{a^3+b^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} + \frac{ax^2\,\mathrm{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)}{2b^2}$$
$$- \frac{(a^3+b^3)^{2/3}\log(a^3+b^3x^3)}{3b^3} + \frac{(a^3+b^3)^{2/3}\log\left(-\frac{\sqrt[3]{a^3+b^3}x}{a}-\sqrt[3]{1-x^3}\right)}{2b^3}$$
$$- \frac{a^2\log\left(x+\sqrt[3]{1-x^3}\right)}{2b^3} + \frac{(a^3+b^3)^{2/3}\log\left(\sqrt[3]{a^3+b^3}-b\sqrt[3]{1-x^3}\right)}{2b^3}$$

Mathematica [F]

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx = \int \frac{(1-x^3)^{2/3}}{a+bx}\,dx$$

```
[In] Integrate[(1 - x^3)^(2/3)/(a + b*x),x]
[Out] Integrate[(1 - x^3)^(2/3)/(a + b*x), x]
```

Maple [F]

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{bx+a}dx$$

```
[In] int((-x^3+1)^(2/3)/(b*x+a),x)
[Out] int((-x^3+1)^(2/3)/(b*x+a),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx = \text{Timed out}$$

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx = \int \frac{(-(x-1)(x^2+x+1))^{\frac{2}{3}}}{a+bx}\,dx$$

```
[In] integrate((-x**3+1)**(2/3)/(b*x+a),x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(a + b*x), x)
```

Maxima [F]

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{bx+a}\,dx$$

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)/(b*x + a), x)
```

Giac [F]

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{bx+a}\,dx$$

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx = \int \frac{(1-x^3)^{2/3}}{a+b\,x}\,dx$$

```
[In] int((1 - x^3)^(2/3)/(a + b*x),x)
[Out] int((1 - x^3)^(2/3)/(a + b*x), x)
```

3.109 $\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 234

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2} \, dx = -\frac{(1-x^3)^{2/3}}{3(1+x^3)} + \frac{x(1-x^3)^{2/3}}{3(1+x^3)} + \frac{2x^2(1-x^3)^{2/3}}{3(1+x^3)} - \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2^{2/3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}}$$

```
[Out] -1/3*(-x^3+1)^(2/3)/(x^3+1)+1/3*x*(-x^3+1)^(2/3)/(x^3+1)+2/3*x^2*(-x^3+1)^(
2/3)/(x^3+1)+1/3*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/6*ln(2^(1/3)-(-x^3+1
)^(1/3))*2^(2/3)+1/6*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/9*arctan(1/3*(
1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/9*arctan(1/3*(1+2^
(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {2183, 386, 384, 480, 21, 371, 455, 43, 57, 631, 210, 31}

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = -\frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2^{2/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{(1-x^3)^{2/3}x}{3(x^3+1)} - \frac{(1-x^3)^{2/3}}{3(x^3+1)} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{1-\right.}{3\sqrt[3]{}}$$

```
[In] Int[(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]
```

```
[Out] -1/3*(1 - x^3)^(2/3)/(1 + x^3) + (x*(1 - x^3)^(2/3))/(3*(1 + x^3)) + (2*x^2
*(1 - x^3)^(2/3))/(3*(1 + x^3)) - (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x
^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) - (2^(2/3)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(
1/3))/Sqrt[3]])/(3*Sqrt[3]) + (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/3
 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(3*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3
)^(1/3)]/(3*2^(1/3))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{(1-x^3)^{2/3}}{(1+x^3)^2}+\frac{2x(1-x^3)^{2/3}}{(1+x^3)^2}+\frac{x^2(1-x^3)^{2/3}}{(1+x^3)^2}\right) dx \\
&= 2\int \frac{x(1-x^3)^{2/3}}{(1+x^3)^2}\,dx+\int \frac{(1-x^3)^{2/3}}{(1+x^3)^2}\,dx+\int \frac{x^2(1-x^3)^{2/3}}{(1+x^3)^2}\,dx \\
&= \frac{x(1-x^3)^{2/3}}{3(1+x^3)}+\frac{2x^2(1-x^3)^{2/3}}{3(1+x^3)} \\
&\quad +\frac{1}{3}\text{Subst}\left(\int \frac{(1-x)^{2/3}}{(1+x)^2}\,dx,x,x^3\right)+\frac{2}{3}\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx-\frac{2}{3}\int \frac{x(-1-x^3)}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx \\
&= -\frac{(1-x^3)^{2/3}}{3(1+x^3)}+\frac{x(1-x^3)^{2/3}}{3(1+x^3)}+\frac{2x^2(1-x^3)^{2/3}}{3(1+x^3)}-\frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\log(1+x^3)}{9\sqrt[3]{2}} \\
&\quad +\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}}-\frac{2}{9}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx,x,x^3\right)+\frac{2}{3}\int \frac{x}{\sqrt[3]{1-x^3}}\,dx
\end{aligned}
$$

$$= -\frac{(1-x^3)^{2/3}}{3(1+x^3)} + \frac{x(1-x^3)^{2/3}}{3(1+x^3)} + \frac{2x^2(1-x^3)^{2/3}}{3(1+x^3)} - \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\right.$$

$$= -\frac{(1-x^3)^{2/3}}{3(1+x^3)} + \frac{x(1-x^3)^{2/3}}{3(1+x^3)} + \frac{2x^2(1-x^3)^{2/3}}{3(1+x^3)} - \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}} + \frac{1}{3}2^{2/3}$$

$$= -\frac{(1-x^3)^{2/3}}{3(1+x^3)} + \frac{x(1-x^3)^{2/3}}{3(1+x^3)} + \frac{2x^2(1-x^3)^{2/3}}{3(1+x^3)} - \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2^{2/3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}}$$

Mathematica [F]

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = \int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx$$

```
[In] Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]
[Out] Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2, x]
```

Maple [F]

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{(x^2-x+1)^2}dx$$

```
[In] int((-x^3+1)^(2/3)/(x^2-x+1)^2,x)
[Out] int((-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

Fricas [F]

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{(x^2-x+1)^2}\,dx$$

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1), x)
```

Sympy [F]

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = \int \frac{(-(x-1)(x^2+x+1))^{\frac{2}{3}}}{(x^2-x+1)^2}\,dx$$

```
[In] integrate((-x**3+1)**(2/3)/(x**2-x+1)**2,x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x**2 - x + 1)**2, x)
```

Maxima [F]

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{(x^2-x+1)^2}\,dx$$

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 - x + 1)^2, x)
```

Giac [F]

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{(x^2-x+1)^2}\,dx$$

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 - x + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = \int \frac{(1-x^3)^{2/3}}{(x^2-x+1)^2}\,dx$$

```
[In] int((1 - x^3)^(2/3)/(x^2 - x + 1)^2,x)
[Out] int((1 - x^3)^(2/3)/(x^2 - x + 1)^2, x)
```

3.110 $\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 199

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx = \frac{\left(1-x^3\right)^{2/3}}{1-x+x^2} - \frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2^{2/3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}} + \log\left(x+\sqrt[3]{1-x^3}\right)$$

```
[Out] (-x^3+1)^(2/3)/(x^2-x+1)+1/2*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(-2^(
1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+ln(x+(-x^3+1)^(1/3))-2/3*arctan(1/3*(1-2*x/(
-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/3*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3
))*3^(1/2))*2^(2/3)*3^(1/2)+1/3*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/
2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2183, 386, 384, 455, 43, 57, 631, 210, 31, 478, 544, 245}

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx=-\frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2^{2/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\left(1-x^3\right)^{2/3}x}{x^3+1}+\frac{\left(1-x^3\right)^{2/3}}{x^3+1}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}-\frac{2}{3}2^{2/3}\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}+\log\left(\sqrt[3]{1-x^3}+x\right)$$

```
[In] Int[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2,x]
```

```
[Out] (1 - x^3)^(2/3)/(1 + x^3) + (x*(1 - x^3)^(2/3))/(1 + x^3) - (2*ArcTan[(1 -
(2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*
x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + (2^(2/3)*ArcTan[(1 + 2^(2/3)*(1 - x
^3)^(1/3))/Sqrt[3]])/Sqrt[3] + Log[2^(1/3) - (1 - x^3)^(1/3)]/2^(1/3) + Log
[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(3*2^(1/3)) - (2*2^(2/3)*Log[-(2^(1/3)*x)
- (1 - x^3)^(1/3)])/3 + Log[x + (1 - x^3)^(1/3)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{(1-x^3)^{2/3}}{(1+x^3)^2}-\frac{3x^2(1-x^3)^{2/3}}{(1+x^3)^2}-\frac{2x^3(1-x^3)^{2/3}}{(1+x^3)^2}\right)dx \\
&= -\left(2\int \frac{x^3(1-x^3)^{2/3}}{(1+x^3)^2}\,dx\right)-3\int \frac{x^2(1-x^3)^{2/3}}{(1+x^3)^2}\,dx+\int \frac{(1-x^3)^{2/3}}{(1+x^3)^2}\,dx \\
&= \frac{x(1-x^3)^{2/3}}{1+x^3}+\frac{2}{3}\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx \\
&\quad -\frac{2}{3}\int \frac{1-3x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx-\text{Subst}\left(\int \frac{(1-x)^{2/3}}{(1+x)^2}\,dx,x,x^3\right) \\
&= \frac{(1-x^3)^{2/3}}{1+x^3}+\frac{x(1-x^3)^{2/3}}{1+x^3}-\frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} \\
&\quad -\frac{\log(1+x^3)}{9\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}} \\
&\quad +\frac{2}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx,x,x^3\right)+2\int \frac{1}{\sqrt[3]{1-x^3}}\,dx-\frac{8}{3}\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx
\end{aligned}
$$

$$= \frac{(1-x^3)^{2/3}}{1+x^3} + \frac{x(1-x^3)^{2/3}}{1+x^3} - \frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}} - \frac{2}{3}2^{2/3}\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right) + \log\left(x+\sqrt[3]{1-x^3}\right) - \frac{\text{Subst}\left(\int\frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}} + \text{Subst}\Big(\int$$

$$= \frac{(1-x^3)^{2/3}}{1+x^3} + \frac{x(1-x^3)^{2/3}}{1+x^3} - \frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}} - \frac{2}{3}2^{2/3}\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right) + \log\left(x+\sqrt[3]{1-x^3}\right) - 2^{2/3}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x}\right.$$

$$= \frac{(1-x^3)^{2/3}}{1+x^3} + \frac{x(1-x^3)^{2/3}}{1+x^3} - \frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2^{2/3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}} - \frac{2}{3}2^{2/3}\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right) + \log\left(x+\sqrt[3]{1-x^3}\right)$$

Mathematica [F]

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx = \int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$$

```
[In] Integrate[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2,x]
[Out] Integrate[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 26.80 (sec) , antiderivative size = 618, normalized size of antiderivative = 3.11

method	result
trager	$\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^2-x+1}+\frac{2\ln\left(-\operatorname{RootOf}\left(\_Z^6+432\right)^6 x^3-36\operatorname{RootOf}\left(\_Z^6+432\right)^3\left(-x^3+1\right)^{\frac{2}{3}}x+36\operatorname{RootOf}\left(\_Z^6+432\right)^3 x^3-24\operatorname{RootOf}\left(\_\right.\right.}{3}$
risch	Expression too large to display

```
[In] int((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
[Out] (-x^3+1)^(2/3)/(x^2-x+1)+2/3*ln(-RootOf(_Z^6+432)^6*x^3-36*RootOf(_Z^6+432)
^3*(-x^3+1)^(2/3)*x+36*RootOf(_Z^6+432)^3*x^3-24*RootOf(_Z^6+432)^3+432*x*(
-x^3+1)^(2/3)+864*x^2*(-x^3+1)^(1/3))+1/3*RootOf(_Z^6+432)*ln((RootOf(_Z^6+
432)^5*(-x^3+1)^(1/3)+RootOf(_Z^6+432)^4*x^2-RootOf(_Z^6+432)^4*x-RootOf(_Z
^6+432)^4+12*RootOf(_Z^6+432)^2*(-x^3+1)^(1/3)+36*RootOf(_Z^6+432)*x^2-36*R
ootOf(_Z^6+432)*x+144*(-x^3+1)^(2/3)-36*RootOf(_Z^6+432))/(x^2-x+1))+1/72*l
n(-(RootOf(_Z^6+432)^4*x^2+RootOf(_Z^6+432)^4*x-RootOf(_Z^6+432)^4+12*RootO
f(_Z^6+432)^2*(-x^3+1)^(1/3)*x+72*(-x^3+1)^(2/3))/(x^2-x+1))*RootOf(_Z^6+43
2)^4-1/6*ln(-(RootOf(_Z^6+432)^4*x^2+RootOf(_Z^6+432)^4*x-RootOf(_Z^6+432)^
4+12*RootOf(_Z^6+432)^2*(-x^3+1)^(1/3)*x+72*(-x^3+1)^(2/3))/(x^2-x+1))*Root
Of(_Z^6+432)-1/36*ln(RootOf(_Z^6+432)^6*x^3+72*RootOf(_Z^6+432)^3*(-x^3+1)^
(2/3)*x+72*RootOf(_Z^6+432)^3*(-x^3+1)^(1/3)*x^2-24*RootOf(_Z^6+432)^3+864*
x*(-x^3+1)^(2/3)-864*x^2*(-x^3+1)^(1/3)-1296*x^3+864)*RootOf(_Z^6+432)^3-1/
3*ln(RootOf(_Z^6+432)^6*x^3+72*RootOf(_Z^6+432)^3*(-x^3+1)^(2/3)*x+72*RootO
f(_Z^6+432)^3*(-x^3+1)^(1/3)*x^2-24*RootOf(_Z^6+432)^3+864*x*(-x^3+1)^(2/3)
-864*x^2*(-x^3+1)^(1/3)-1296*x^3+864)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 2298, normalized size of antiderivative = 11.55

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(2*sqrt(3)*(-16)^(1/6)*(x^2 - x + 1)*log(9*(4*sqrt(3)*(-16)^(1/6)*(131
*x^6 - 48*x^5 - 381*x^4 - 152*x^3 + 267*x^2 + 276*x - 112) + 12*2^(2/3)*(15
*x^6 - 200*x^5 + 5*x^4 + 216*x^3 + 157*x^2 - 124*x - 42) + 24*(31*x^4 + 107
*x^3 - 243*x^2 - sqrt(3)*(-23*I*x^4 + 85*I*x^3 + 57*I*x^2 - 104*I*x + 4*I)
- 26*x + 50)*(-x^3 + 1)^(2/3) + 3*(-x^3 + 1)^(1/3)*(sqrt(3)*(-16)^(5/6)*(4*
x^5 + 69*x^4 - 58*x^3 - 77*x^2 + 12*x + 23) - 8*(-2)^(1/3)*(50*x^5 - 93*x^4
 - 88*x^3 - 7*x^2 + 150*x - 31)))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*
x + 1)) - 2*sqrt(3)*(-16)^(1/6)*(x^2 - x + 1)*log(-9*(4*sqrt(3)*(-16)^(1/6)
*(131*x^6 - 48*x^5 - 381*x^4 - 152*x^3 + 267*x^2 + 276*x - 112) - 12*2^(2/3
)*(15*x^6 - 200*x^5 + 5*x^4 + 216*x^3 + 157*x^2 - 124*x - 42) - 24*(31*x^4
+ 107*x^3 - 243*x^2 - sqrt(3)*(23*I*x^4 - 85*I*x^3 - 57*I*x^2 + 104*I*x - 4
*I) - 26*x + 50)*(-x^3 + 1)^(2/3) + 3*(-x^3 + 1)^(1/3)*(sqrt(3)*(-16)^(5/6)
*(4*x^5 + 69*x^4 - 58*x^3 - 77*x^2 + 12*x + 23) + 8*(-2)^(1/3)*(50*x^5 - 93
*x^4 - 88*x^3 - 7*x^2 + 150*x - 31)))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2
- 3*x + 1)) - 24*sqrt(3)*(x^2 - x + 1)*arctan((4*sqrt(3)*(-x^3 + 1)^(1/3)*x
^2 + 2*sqrt(3)*(-x^3 + 1)^(2/3)*x - sqrt(3)*(x^3 - 1))/(9*x^3 - 1)) + sqrt(
3)*(sqrt(-3)*(-16)^(1/6)*(x^2 - x + 1) + (-16)^(1/6)*(x^2 - x + 1))*log(-9*
(12*2^(2/3)*sqrt(-3)*(15*x^6 - 200*x^5 + 5*x^4 + 216*x^3 + 157*x^2 - 124*x
- 42) + 12*2^(2/3)*(15*x^6 - 200*x^5 + 5*x^4 + 216*x^3 + 157*x^2 - 124*x -
42) - 48*(31*x^4 + 107*x^3 - 243*x^2 - sqrt(3)*(23*I*x^4 - 85*I*x^3 - 57*I*
x^2 + 104*I*x - 4*I) - 26*x + 50)*(-x^3 + 1)^(2/3) - 4*sqrt(3)*(sqrt(-3)*(-
16)^(1/6)*(131*x^6 - 48*x^5 - 381*x^4 - 152*x^3 + 267*x^2 + 276*x - 112) +
(-16)^(1/6)*(131*x^6 - 48*x^5 - 381*x^4 - 152*x^3 + 267*x^2 + 276*x - 112))
 + 3*(-x^3 + 1)^(1/3)*(8*(-2)^(1/3)*sqrt(-3)*(50*x^5 - 93*x^4 - 88*x^3 - 7*
x^2 + 150*x - 31) + sqrt(3)*(sqrt(-3)*(-16)^(5/6)*(4*x^5 + 69*x^4 - 58*x^3
- 77*x^2 + 12*x + 23) - (-16)^(5/6)*(4*x^5 + 69*x^4 - 58*x^3 - 77*x^2 + 12*
x + 23)) - 8*(-2)^(1/3)*(50*x^5 - 93*x^4 - 88*x^3 - 7*x^2 + 150*x - 31)))/(
x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) - sqrt(3)*(sqrt(-3)*(-16)^(
1/6)*(x^2 - x + 1) + (-16)^(1/6)*(x^2 - x + 1))*log(-9*(12*2^(2/3)*sqrt(-3)
*(15*x^6 - 200*x^5 + 5*x^4 + 216*x^3 + 157*x^2 - 124*x - 42) + 12*2^(2/3)*(
15*x^6 - 200*x^5 + 5*x^4 + 216*x^3 + 157*x^2 - 124*x - 42) - 48*(31*x^4 + 1
07*x^3 - 243*x^2 - sqrt(3)*(-23*I*x^4 + 85*I*x^3 + 57*I*x^2 - 104*I*x + 4*I
) - 26*x + 50)*(-x^3 + 1)^(2/3) + 4*sqrt(3)*(sqrt(-3)*(-16)^(1/6)*(131*x^6
- 48*x^5 - 381*x^4 - 152*x^3 + 267*x^2 + 276*x - 112) + (-16)^(1/6)*(131*x^
6 - 48*x^5 - 381*x^4 - 152*x^3 + 267*x^2 + 276*x - 112)) + 3*(-x^3 + 1)^(1/
```

```
3)*(8*(-2)^(1/3)*sqrt(-3)*(50*x^5 - 93*x^4 - 88*x^3 - 7*x^2 + 150*x - 31) -
 sqrt(3)*(sqrt(-3)*(-16)^(5/6)*(4*x^5 + 69*x^4 - 58*x^3 - 77*x^2 + 12*x + 2
3) - (-16)^(5/6)*(4*x^5 + 69*x^4 - 58*x^3 - 77*x^2 + 12*x + 23)) - 8*(-2)^(
1/3)*(50*x^5 - 93*x^4 - 88*x^3 - 7*x^2 + 150*x - 31)))/(x^6 - 3*x^5 + 6*x^4
 - 7*x^3 + 6*x^2 - 3*x + 1)) - sqrt(3)*(sqrt(-3)*(-16)^(1/6)*(x^2 - x + 1)
- (-16)^(1/6)*(x^2 - x + 1))*log(9*(12*2^(2/3)*sqrt(-3)*(15*x^6 - 200*x^5 +
 5*x^4 + 216*x^3 + 157*x^2 - 124*x - 42) - 12*2^(2/3)*(15*x^6 - 200*x^5 + 5
*x^4 + 216*x^3 + 157*x^2 - 124*x - 42) + 48*(31*x^4 + 107*x^3 - 243*x^2 - s
qrt(3)*(23*I*x^4 - 85*I*x^3 - 57*I*x^2 + 104*I*x - 4*I) - 26*x + 50)*(-x^3
+ 1)^(2/3) - 4*sqrt(3)*(sqrt(-3)*(-16)^(1/6)*(131*x^6 - 48*x^5 - 381*x^4 -
152*x^3 + 267*x^2 + 276*x - 112) - (-16)^(1/6)*(131*x^6 - 48*x^5 - 381*x^4
- 152*x^3 + 267*x^2 + 276*x - 112)) + 3*(-x^3 + 1)^(1/3)*(8*(-2)^(1/3)*sqrt
(-3)*(50*x^5 - 93*x^4 - 88*x^3 - 7*x^2 + 150*x - 31) + sqrt(3)*(sqrt(-3)*(-
16)^(5/6)*(4*x^5 + 69*x^4 - 58*x^3 - 77*x^2 + 12*x + 23) + (-16)^(5/6)*(4*x
^5 + 69*x^4 - 58*x^3 - 77*x^2 + 12*x + 23)) + 8*(-2)^(1/3)*(50*x^5 - 93*x^4
 - 88*x^3 - 7*x^2 + 150*x - 31)))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*
x + 1)) + sqrt(3)*(sqrt(-3)*(-16)^(1/6)*(x^2 - x + 1) - (-16)^(1/6)*(x^2 -
x + 1))*log(9*(12*2^(2/3)*sqrt(-3)*(15*x^6 - 200*x^5 + 5*x^4 + 216*x^3 + 15
7*x^2 - 124*x - 42) - 12*2^(2/3)*(15*x^6 - 200*x^5 + 5*x^4 + 216*x^3 + 157*
x^2 - 124*x - 42) + 48*(31*x^4 + 107*x^3 - 243*x^2 - sqrt(3)*(-23*I*x^4 + 8
5*I*x^3 + 57*I*x^2 - 104*I*x + 4*I) - 26*x + 50)*(-x^3 + 1)^(2/3) + 4*sqrt(
3)*(sqrt(-3)*(-16)^(1/6)*(131*x^6 - 48*x^5 - 381*x^4 - 152*x^3 + 267*x^2 +
276*x - 112) - (-16)^(1/6)*(131*x^6 - 48*x^5 - 381*x^4 - 152*x^3 + 267*x^2
+ 276*x - 112)) + 3*(-x^3 + 1)^(1/3)*(8*(-2)^(1/3)*sqrt(-3)*(50*x^5 - 93*x^
4 - 88*x^3 - 7*x^2 + 150*x - 31) - sqrt(3)*(sqrt(-3)*(-16)^(5/6)*(4*x^5 + 6
9*x^4 - 58*x^3 - 77*x^2 + 12*x + 23) + (-16)^(5/6)*(4*x^5 + 69*x^4 - 58*x^3
 - 77*x^2 + 12*x + 23)) + 8*(-2)^(1/3)*(50*x^5 - 93*x^4 - 88*x^3 - 7*x^2 +
150*x - 31)))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 12*(x^2 -
x + 1)*log(3*(-x^3 + 1)^(1/3)*x^2 + 3*(-x^3 + 1)^(2/3)*x + 1) + 36*(-x^3 +
1)^(2/3))/(x^2 - x + 1)
```

Sympy [F]

$$\int \frac{(1-2x)\,(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = -\int \left(-\frac{(1-x^3)^{\frac{2}{3}}}{x^4-2x^3+3x^2-2x+1}\right)dx - \int \frac{2x(1-x^3)^{\frac{2}{3}}}{x^4-2x^3+3x^2-2x+1}\,dx$$

```
[In] integrate((1-2*x)*(-x**3+1)**(2/3)/(x**2-x+1)**2,x)
```

```
[Out] -Integral(-(1 - x**3)**(2/3)/(x**4 - 2*x**3 + 3*x**2 - 2*x + 1), x) - Integ
ral(2*x*(1 - x**3)**(2/3)/(x**4 - 2*x**3 + 3*x**2 - 2*x + 1), x)
```

Maxima [F]

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx = \int -\frac{\left(-x^3+1\right)^{\frac{2}{3}}(2\,x-1)}{\left(x^2-x+1\right)^2}\,dx$$

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="maxima")
[Out] -integrate((-x^3 + 1)^(2/3)*(2*x - 1)/(x^2 - x + 1)^2, x)
```

Giac [F]

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx = \int -\frac{\left(-x^3+1\right)^{\frac{2}{3}}(2\,x-1)}{\left(x^2-x+1\right)^2}\,dx$$

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="giac")
[Out] integrate(-(-x^3 + 1)^(2/3)*(2*x - 1)/(x^2 - x + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx = -\int \frac{(2\,x-1)\,\left(1-x^3\right)^{2/3}}{\left(x^2-x+1\right)^2}\,dx$$

```
[In] int(-((2*x - 1)*(1 - x^3)^(2/3))/(x^2 - x + 1)^2,x)
[Out] -int(((2*x - 1)*(1 - x^3)^(2/3))/(x^2 - x + 1)^2, x)
```

3.111 $\int \frac{(1-x^3)^{2/3}}{1+x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 177

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx = \frac{1}{2}(1-x^3)^{2/3} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}} + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{2\sqrt[3]{2}} - \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right) + \frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] 1/2*(-x^3+1)^(2/3)+1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/4*ln((1-x)*(1+
x)^2)*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+3/4*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*
2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/2*arctan(1
/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used

$= \{2178, 2177, 245, 2174, 371\}$

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}} + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$
$$+ \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{1}{2}(1-x^3)^{2/3}$$
$$-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right) + \frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}} - \frac{\log\left((1-x)(x+1)^2\right)}{2\sqrt[3]{2}}$$

```
[In] Int[(1 - x^3)^(2/3)/(1 + x),x]
```

```
[Out] (1 - x^3)^(2/3)/2 - (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]])/2^(1/3) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] +
(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x)^2]/(2*2
^(1/3)) - Log[x + (1 - x^3)^(1/3)]/2 + (3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1
/3)])/(2*2^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
```

```
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rule 2178

```
Int[((a_) + (b_.)*(x_)^3)^(2/3)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(a +
 b*x^3)^(2/3)/(2*d), x] + (Dist[1/d^2, Int[(a*d^2 + b*c^2*x)/((c + d*x)*(a
+ b*x^3)^(1/3)), x], x] - Dist[b*(c/d^2), Int[x/(a + b*x^3)^(1/3), x], x])
/; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\left(1-x^3\right)^{2/3}+\int \frac{x}{\sqrt[3]{1-x^3}}\,dx+\int \frac{1-x}{(1+x)\sqrt[3]{1-x^3}}\,dx \\
&= \frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) \\
&\quad +2\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx-\int \frac{1}{\sqrt[3]{1-x^3}}\,dx \\
&= \frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} \\
&\quad +\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)-\frac{\log\left((1-x)(1+x)^2\right)}{2\sqrt[3]{2}} \\
&\quad -\frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)+\frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}$$

Mathematica [F]

$$\int \frac{\left(1-x^3\right)^{2/3}}{1+x}\,dx=\int \frac{\left(1-x^3\right)^{2/3}}{1+x}\,dx$$

```
[In] Integrate[(1 - x^3)^(2/3)/(1 + x),x]
[Out] Integrate[(1 - x^3)^(2/3)/(1 + x), x]
```

Maple [F]

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{1+x}dx$$

```
[In] int((-x^3+1)^(2/3)/(1+x),x)
[Out] int((-x^3+1)^(2/3)/(1+x),x)
```

Fricas [F]

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{x+1}\,dx$$

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x + 1), x)
```

Sympy [F]

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx = \int \frac{(-(x-1)(x^2+x+1))^{\frac{2}{3}}}{x+1}\,dx$$

```
[In] integrate((-x**3+1)**(2/3)/(1+x),x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x + 1), x)
```

Maxima [F]

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{x+1}\,dx$$

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)/(x + 1), x)
```

Giac [F]

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{x+1}\,dx$$

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)/(x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx = \int \frac{(1-x^3)^{2/3}}{x+1}\,dx$$

```
[In] int((1 - x^3)^(2/3)/(x + 1),x)
[Out] int((1 - x^3)^(2/3)/(x + 1), x)
```

3.112 $\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 177

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx = \frac{1}{2}(1-x^3)^{2/3} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}} + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log((1-x)(1+x)^2)}{2\sqrt[3]{2}} - \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right) + \frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] 1/2*(-x^3+1)^(2/3)+1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/4*ln((1-x)*(1+
x)^2)*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+3/4*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*
2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/2*arctan(1
/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used

$= \{1600, 2178, 2177, 245, 2174, 371\}$

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}} + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{1}{2}(1-x^3)^{2/3} - \frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right) + \frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}} - \frac{\log\left((1-x)(x+1)^2\right)}{2\sqrt[3]{2}}$$

```
[In] Int[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] (1 - x^3)^(2/3)/2 - (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]])/2^(1/3) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] +
(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x)^2]/(2*2
^(1/3)) - Log[x + (1 - x^3)^(1/3)]/2 + (3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1
/3)])/(2*2^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
```

```
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rule 2178

```
Int[((a_) + (b_.)*(x_)^3)^(2/3)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(a +
 b*x^3)^(2/3)/(2*d), x] + (Dist[1/d^2, Int[(a*d^2 + b*c^2*x)/((c + d*x)*(a
+ b*x^3)^(1/3)), x], x] - Dist[b*(c/d^2), Int[x/(a + b*x^3)^(1/3), x], x])
/; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1-x^3)^{2/3}}{1+x}\,dx \\
&= \frac{1}{2}(1-x^3)^{2/3} + \int \frac{x}{\sqrt[3]{1-x^3}}\,dx + \int \frac{1-x}{(1+x)\sqrt[3]{1-x^3}}\,dx \\
&= \frac{1}{2}(1-x^3)^{2/3} + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) \\
&\quad + 2\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx - \int \frac{1}{\sqrt[3]{1-x^3}}\,dx \\
&= \frac{1}{2}(1-x^3)^{2/3} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}} + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} \\
&\quad + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{2\sqrt[3]{2}} \\
&\quad - \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right) + \frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}$$

Mathematica [F]

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx$$

```
[In] Integrate[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3),x]
[Out] Integrate[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3), x]
```

Maple [F]

$$\int \frac{(x^2-x+1)(-x^3+1)^{\frac{2}{3}}}{x^3+1}dx$$

```
[In] int((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}(x^2-x+1)}{x^3+1}\,dx$$

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x + 1), x)
```

Sympy [F]

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-(x-1)(x^2+x+1))^{\frac{2}{3}}}{x+1}\,dx$$

```
[In] integrate((x**2-x+1)*(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x + 1), x)
```

Maxima [F]

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}(x^2-x+1)}{x^3+1}\,dx$$

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)*(x^2 - x + 1)/(x^3 + 1), x)
```

Giac [F]

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}(x^2-x+1)}{x^3+1}\,dx$$

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)*(x^2 - x + 1)/(x^3 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(1-x^3)^{2/3}(x^2-x+1)}{x^3+1}\,dx$$

```
[In] int(((1 - x^3)^(2/3)*(x^2 - x + 1))/(x^3 + 1),x)
[Out] int(((1 - x^3)^(2/3)*(x^2 - x + 1))/(x^3 + 1), x)
```

3.113 $\int \frac{(1-x^3)^{2/3}}{1+x^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 132

$$\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx = \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(1+x^3)}{3\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}} - \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

```
[Out] -1/6*ln(x^3+1)*2^(2/3)+1/2*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(x+(
-x^3+1)^(1/3))+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/3*a
rctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {399, 245, 384}

$$\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx = \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x^3+1)}{3\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{\sqrt[3]{2}} - \frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

```
[In] Int[(1 - x^3)^(2/3)/(1 + x^3),x]
```

```
[Out] ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - (2^(2/3)*ArcTan[(1 -
(2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - Log[1 + x^3]/(3*2^(1/3))
 + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/2^(1/3) - Log[x + (1 - x^3)^(1/3)]/2
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = 2\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx - \int \frac{1}{\sqrt[3]{1-x^3}}\,dx$$

$$= \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(1+x^3)}{3\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}} - \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.55

$$\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx = \frac{1}{6}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right) - 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 2\log\left(x+\sqrt[3]{1-x^3}\right) + 2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + \log\left(x^2-x\sqrt[3]{1\right.\right.$$

```
[In] Integrate[(1 - x^3)^(2/3)/(1 + x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3))] - 2*2^(2/3)*Sqrt[3]*
ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*Log[x + (1 - x^3)^(1/
3)] + 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + Log[x^2 - x*(1 - x^3)^
(1/3) + (1 - x^3)^(2/3)] - 2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) -
 2^(1/3)*(1 - x^3)^(2/3)])/6
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$-\frac{\ln\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)}{3}+\frac{2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)}{3}-\frac{2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)}{6}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\right.}{}$

```
[In] int((-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*ln((x+(-x^3+1)^(1/3))/x)+1/3*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)-
1/6*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)+1
/3*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)+1/6*ln
(((-x^3+1)^(2/3)-x*(-x^3+1)^(1/3)+x^2)/x^2)-1/3*3^(1/2)*arctan(1/3*(-2*(-x^
3+1)^(1/3)+x)*3^(1/2)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.45

$$\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx = -\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3\,x}\right) + \frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3\,x}\right) + \frac{1}{3}\cdot 4^{\frac{1}{3}}\log\left(\frac{4^{\frac{2}{3}}x+2\,(-x^3+1)^{\frac{1}{3}}}{x}\right) - \frac{1}{6}\cdot 4^{\frac{1}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}x^2-4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}x+2\,(-x^3+1)^{\frac{2}{3}}}{x^2}\right) - \frac{1}{3}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right) + \frac{1}{6}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*4^(1/3)*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(-x^3 + 1)^(1
/3))/x) + 1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/
x) + 1/3*4^(1/3)*log((4^(2/3)*x + 2*(-x^3 + 1)^(1/3))/x) - 1/6*4^(1/3)*log(
(2*4^(1/3)*x^2 - 4^(2/3)*(-x^3 + 1)^(1/3)*x + 2*(-x^3 + 1)^(2/3))/x^2) - 1/
3*log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3
 + 1)^(2/3))/x^2)
```

Sympy [F]

$$\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{\left(-(x-1)\,(x^2+x+1)\right)^{\frac{2}{3}}}{(x+1)\,(x^2-x+1)}\,dx$$

```
[In] integrate((-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/((x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{x^3+1}\,dx$$

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)/(x^3 + 1), x)
```

Giac [F]

$$\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{x^3+1}\,dx$$

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)/(x^3 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(1-x^3)^{2/3}}{x^3+1}\,dx$$

```
[In] int((1 - x^3)^(2/3)/(x^3 + 1),x)
[Out] int((1 - x^3)^(2/3)/(x^3 + 1), x)
```

3.114 $\int \frac{x\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 250

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx = \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{\log\left((1-x)(1+x)^2\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{1}{3}2^{2/3}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/12*ln((1-x)*(1+x)^2)*2^(2/3)+1/6
*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-
1/3*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/4*ln(-1+x+2^(2/3)*(-x^3+1)
^(1/3))*2^(2/3)+1/3*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*
2^(2/3)*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^
(1/2)*2^(2/3)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {495, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx = \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}} - \frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}+\frac{\log\left((1-x)(x+1)^2\right)}{6\sqrt[3]{2}}$$

```
[In] Int[(x*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]
) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 + Log[(1 - x)*(1 + x)^2]/
(6*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x
))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - (2^(2/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x
^3)^(1/3)])/3 - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
```

```
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx - \int \frac{x}{\sqrt[3]{1-x^3}}\,dx \\
&= -\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) \\
&\quad -\frac{2}{3}\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx - 2\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) \\
&= \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) \\
&\quad + \frac{\log\left((1-x)(1+x)^2\right)}{6\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} \\
&\quad -\frac{2}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) \\
&\quad -\frac{2}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) \\
&= \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) \\
&\quad + \frac{\log\left((1-x)(1+x)^2\right)}{6\sqrt[3]{2}} - \frac{1}{3}2^{2/3}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) \\
&\quad - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} \\
&\quad - \text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)
\end{aligned}$$

$$= \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{\log\left((1-x)(1+x)^2\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{1}{3}2^{2/3}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - 2^{2/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.$$

$$= \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{\log\left((1-x)(1+x)^2\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{1}{3}2^{2/3}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.10

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx = \frac{1}{2}x^2\,\text{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},x^3,-x^3\right)$$

```
[In] Integrate[(x*(1 - x^3)^(2/3))/(1 + x^3),x]
[Out] (x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3])/2
```

Maple [F]

$$\int \frac{x(-x^3+1)^{\frac{2}{3}}}{x^3+1}dx$$

[In] int(x*(-x^3+1)^(2/3)/(x^3+1),x)

[Out] int(x*(-x^3+1)^(2/3)/(x^3+1),x)

Fricas [F]

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\,dx$$

[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")

[Out] integral((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)

Sympy [F]

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{x(-(x-1)(x^2+x+1))^{\frac{2}{3}}}{(x+1)(x^2-x+1)}\,dx$$

[In] integrate(x*(-x**3+1)**(2/3)/(x**3+1),x)

[Out] Integral(x*(-(x - 1)*(x**2 + x + 1))**(2/3)/((x + 1)*(x**2 - x + 1)), x)

Maxima [F]

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\,dx$$

[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)

Giac [F]

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\,dx$$

```
[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{x\,(1-x^3)^{2/3}}{x^3+1}\,dx$$

```
[In] int((x*(1 - x^3)^(2/3))/(x^3 + 1),x)
[Out] int((x*(1 - x^3)^(2/3))/(x^3 + 1), x)
```

3.115 $\int \frac{(1-x)(1-x^3)^{2/3}}{1+x^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 383

$$\int \frac{(1-x)(1-x^3)^{2/3}}{1+x^3} dx = -\frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+x^3\right)}{3\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-\ldots}{\sqrt[3]{1-\ldots}}\right.}{3\sqrt[3]{2}}$$

```
[Out] 1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/12*ln((1-x)*(1+x)^2)*2^(2/3)-1/6*
ln(x^3+1)*2^(2/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x
^3+1)^(1/3))*2^(2/3)+1/3*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*ln(
-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+1/4*ln(-1+x+2^(
2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/3*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/
3))*3^(1/2))*2^(2/3)*3^(1/2)-1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)*2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3
^(1/2)-1/3*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.69, number of steps used = 17, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6857, 2178, 2177, 245, 2174, 371}

$$\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx = -\frac{2^{2/3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\left(1+(-1)^{2/3}\right)\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\left(1-\sqrt[3]{-1}\right)\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\left(1-\sqrt[3]{-1}\right)\arctan\left(\frac{1-\frac{\sqrt[3]{2}\left(x+\sqrt[3]{-1}\right)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\left(1+(-1)^{2/3}\right)\arctan\left(\frac{\frac{(-1)^{2/3}\sqrt[3]{2}\left(\sqrt[3]{-1}x+1\right)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{1}{6}\left(1+(-1)^{2/3}\right)x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{1}{6}\left(1-\sqrt[3]{-1}\right)x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) +$$

```
[In] Int[((1 - x)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] -((2^(2/3)*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
) + (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) + ((1 - (-1
)^(1/3))*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) + ((1 + (
-1)^(2/3))*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) - ((1 -
 (-1)^(1/3))*ArcTan[(1 - (2^(1/3)*((-1)^(1/3) + x))/(1 - x^3)^(1/3))/Sqrt[3
]])/(2^(1/3)*Sqrt[3]) - ((1 + (-1)^(2/3))*ArcTan[(1 + ((-1)^(2/3)*2^(1/3)*(
1 + (-1)^(1/3)*x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]) + (x^2*Hype
rgeometric2F1[1/3, 2/3, 5/3, x^3])/3 + ((1 - (-1)^(1/3))*x^2*Hypergeometric
2F1[1/3, 2/3, 5/3, x^3])/6 + ((1 + (-1)^(2/3))*x^2*Hypergeometric2F1[1/3, 2
/3, 5/3, x^3])/6 - Log[-((1 - x)*(1 + x)^2)]/(3*2^(1/3)) - ((1 + (-1)^(2/3)
)*Log[-((-1)^(2/3)*((-1)^(2/3) + x)^2*(1 + (-1)^(1/3)*x))])/(6*2^(1/3)) - (
(1 - (-1)^(1/3))*Log[(-1)^(2/3)*((-1)^(1/3) + x)*(1 + (-1)^(2/3)*x)^2])/(6*
2^(1/3)) - Log[x + (1 - x^3)^(1/3)]/3 - ((1 - (-1)^(1/3))*Log[x + (1 - x^3)
```

```
^(1/3)])/6 - ((1 + (-1)^(2/3))*Log[x + (1 - x^3)^(1/3)])/6 + ((1 - (-1)^(1/
3))*Log[1 - (-1)^(2/3)*x - (-2)^(2/3)*(1 - x^3)^(1/3)])/(2*2^(1/3)) + Log[1
 - x - 2^(2/3)*(1 - x^3)^(1/3)]/2^(1/3) + ((1 + (-1)^(2/3))*Log[1 + (-1)^(1
/3)*x + (-1)^(1/3)*2^(2/3)*(1 - x^3)^(1/3)])/(2*2^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rule 2178

```
Int[((a_) + (b_.)*(x_)^3)^(2/3)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(a +
 b*x^3)^(2/3)/(2*d), x] + (Dist[1/d^2, Int[(a*d^2 + b*c^2*x)/((c + d*x)*(a
+ b*x^3)^(1/3)), x], x] - Dist[b*(c/d^2), Int[x/(a + b*x^3)^(1/3), x], x])
/; FreeQ[{a, b, c, d}, x]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{2(1-x^3)^{2/3}}{3(-1-x)} + \frac{\left(-1-(-1)^{2/3}\right)(1-x^3)^{2/3}}{3\left(-1+\sqrt[3]{-1}x\right)} + \frac{\left(-1+\sqrt[3]{-1}\right)(1-x^3)^{2/3}}{3\left(-1-(-1)^{2/3}x\right)}\right)dx$$

$$= -\left(\frac{2}{3}\int \frac{(1-x^3)^{2/3}}{-1-x}\,dx\right) + \frac{1}{3}\left(-1+\sqrt[3]{-1}\right)\int \frac{(1-x^3)^{2/3}}{-1-(-1)^{2/3}x}\,dx + \frac{1}{3}\left(-1-(-1)^{2/3}\right)\int \frac{(1-x^3)^{2/3}}{-1+\sqrt[3]{-1}x}\,dx$$

$$= -\left(\frac{2}{3}\int \frac{1-x}{(-1-x)\sqrt[3]{1-x^3}}\,dx\right) + \frac{2}{3}\int \frac{x}{\sqrt[3]{1-x^3}}\,dx + \frac{1}{3}\left(1-\sqrt[3]{-1}\right)\int \frac{x}{\sqrt[3]{1-x^3}}\,dx + \frac{1}{3}\left(-1+\sqrt[3]{-1}\right)\int \frac{(-1)^{2/3}-x}{\left(-1+\sqrt[3]{-1}x\right)\sqrt[3]{1-x^3}}\,dx + \frac{1}{3}\left((-1)^{2/3}\left(-1+\sqrt[3]{-1}\right)\right)\int \frac{-\sqrt[3]{-1}-x}{\left(-1-(-1)^{2/3}x\right)\sqrt[3]{1-x^3}}\,dx + \frac{1}{3}\left(1+(-1)^{2/3}\right)\int \frac{x}{\sqrt[3]{1-x^3}}\,dx$$

$$= \frac{1}{3}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{1}{6}\left(1-\sqrt[3]{-1}\right)x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{1}{6}\left(1+(-1)^{2/3}\right)x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{2}{3}\int \frac{1}{\sqrt[3]{1-x^3}}\,dx - \frac{4}{3}\int \frac{1}{(-1-x)\sqrt[3]{1-x^3}}\,dx - \frac{1}{3}\left(2\left(1-\sqrt[3]{-1}\right)\right)\int \frac{1}{\left(-1-(-1)^{2/3}x\right)\sqrt[3]{1-x^3}}\,dx + \frac{1}{3}\left(-1+\sqrt[3]{-1}\right)\int \frac{1}{\sqrt[3]{1-x^3}}\,dx + \frac{1}{3}\left((-1)^{2/3}\left(-1+\sqrt[3]{-1}\right)\right)\int \frac{1}{\sqrt[3]{1-x^3}}\,dx + \frac{1}{3}\left(2(-1)^{2/3}\left(-1+\sqrt[3]{-1}\right)\right)\int \frac{1}{\left(-1+\sqrt[3]{-1}x\right)\sqrt[3]{1-x^3}}\,d$$

$$= -\frac{2^{2/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

$$+ \frac{\left(1-\sqrt[3]{-1}\right)\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

$$+ \frac{1}{18}\left(3i+\sqrt{3}\right)\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right) - \frac{\left(1-\sqrt[3]{-1}\right)\arctan\left(\frac{1-\frac{\sqrt[3]{2}\left(\sqrt[3]{-1}+x\right)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

$$- \frac{(-1)^{2/3}\left(1-\sqrt[3]{-1}\right)\arctan\left(\frac{1+\frac{(-1)^{2/3}\sqrt[3]{2}\left(1+\sqrt[3]{-1}x\right)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

$$+ \frac{1}{3}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{1}{6}\left(1-\sqrt[3]{-1}\right)x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{1}{6}(1$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 15.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.36

$$\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx = -\frac{1}{2}x^2\,\text{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},x^3,-x^3\right)$$

$$- \frac{4x\left(1-x^3\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right)}{\left(1+x^3\right)\left(-4\,\text{AppellF1}\left(\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right)+x^3\left(3\,\text{AppellF1}\left(\frac{4}{3},-\frac{2}{3},2,\frac{7}{3},x^3,-x^3\right)+2\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3}\right.\right.\right.}$$

```
[In] Integrate[((1 - x)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] -1/2*(x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3]) - (4*x*(1 - x^3)^(2/3)*Ap
pellF1[1/3, -2/3, 1, 4/3, x^3, -x^3])/((1 + x^3)*(-4*AppellF1[1/3, -2/3, 1,
 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -2/3, 2, 7/3, x^3, -x^3] + 2*Appell
F1[4/3, 1/3, 1, 7/3, x^3, -x^3])))
```

Maple [F]

$$\int \frac{(1-x)\left(-x^3+1\right)^{\frac{2}{3}}}{x^3+1}dx$$

```
[In] int((1-x)*(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int((1-x)*(-x^3+1)^(2/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx = \int -\frac{\left(-x^3+1\right)^{\frac{2}{3}}(x-1)}{x^3+1}\,dx$$

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] integral(-(-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

Sympy [F]

$$\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx = -\int \left(-\frac{\left(1-x^3\right)^{\frac{2}{3}}}{x^3+1}\right)dx - \int \frac{x\left(1-x^3\right)^{\frac{2}{3}}}{x^3+1}\,dx$$

```
[In] integrate((1-x)*(-x**3+1)**(2/3)/(x**3+1),x)
[Out] -Integral(-(1 - x**3)**(2/3)/(x**3 + 1), x) - Integral(x*(1 - x**3)**(2/3)/
(x**3 + 1), x)
```

Maxima [F]

$$\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx = \int -\frac{\left(-x^3+1\right)^{\frac{2}{3}}(x-1)}{x^3+1}\,dx$$

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] -integrate((-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

Giac [F]

$$\int \frac{(1-x)(1-x^3)^{2/3}}{1+x^3}\,dx = \int -\frac{(-x^3+1)^{\frac{2}{3}}(x-1)}{x^3+1}\,dx$$

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(-(-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)(1-x^3)^{2/3}}{1+x^3}\,dx = -\int \frac{(1-x^3)^{2/3}(x-1)}{x^3+1}\,dx$$

```
[In] int(-((1 - x^3)^(2/3)*(x - 1))/(x^3 + 1),x)
[Out] -int(((1 - x^3)^(2/3)*(x - 1))/(x^3 + 1), x)
```

3.116 $\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 272

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx = \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)-\frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}$$

```
[Out] 1/6*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*2^(1/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^
3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*ln(1+2^(1/3)*(
1-x)/(-x^3+1)^(1/3))-1/12*ln(2*2^(1/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)
/(-x^3+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)
^(1/3))*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^
(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {420, 493, 298, 31, 648, 631, 210, 642}

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx = \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}}$$

```
[In] Int[(1 - x^3)^(1/3)/(1 + x^3),x]
```

```
[Out] (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]
) + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[1 + (2^(2/3)*(
1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3))
+ (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - Log[2*2^(1/3) +
(1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(9\text{Subst}\left(\int \frac{x}{(4-x^3)(1+2x^3)}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)\right)\\
&= -\left(2\text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)\right) - \text{Subst}\left(\int \frac{x}{4-x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{2^{2/3}-x}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}\\
&\quad + \frac{1}{3}2^{2/3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}2^{2/3}\text{Subst}\left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)\end{aligned}$$

$$= \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)$$

$$+\frac{1}{2}\text{Subst}\left(\int\frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)-\frac{\text{Subst}\left(\int\frac{2^{2/3}+2x}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}-\text{Sub}$$

$$= \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}$$

$$+\frac{1}{3}\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)-\frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}-\frac{\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{\sqrt[3]{2}\ldots}{\sqrt[3]{1}\ldots}\right.}{2^{2/3}}$$

$$= \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

$$+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}$$

$$+\frac{1}{3}\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)-\frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}$$

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.04

$$\int\frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx =$$

$$-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)+4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{1}\ldots\right.}{\ldots}$$

```
[In] Integrate[(1 - x^3)^(1/3)/(1 + x^3),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1
- x^3)^(1/3))] + 4*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2
*2^(1/3)*x + (1 - x^3)^(1/3))] - 4*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/
3)] - 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] + 2*Log[2^(2/3) - 2*2
^(2/3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)] + Lo
g[2^(2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3) + 4*(1
 - x^3)^(2/3)])/2^(2/3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.60 (sec) , antiderivative size = 1147, normalized size of antiderivative = 4.22

method	result	size
trager	Expression too large to display	1147

```
[In] int((-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*RootOf(_Z^3-2)*ln((6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2
)*RootOf(_Z^3-2)^4*x^3-18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^
2)^2*RootOf(_Z^3-2)^3*x^3+RootOf(_Z^3-2)^2*x^6-3*RootOf(RootOf(_Z^3-2)^2+3*
_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)*x^6+18*(-x^3+1)^(2/3)*RootOf(_Z^3-
2)^2*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^2-6*(-x^3+1)^(1/
3)*RootOf(_Z^3-2)*x^4-2*RootOf(_Z^3-2)^2*x^3+6*RootOf(RootOf(_Z^3-2)^2+3*_Z
*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)*x^3+12*(-x^3+1)^(2/3)*x^2+6*(-x^3+1)
^(1/3)*RootOf(_Z^3-2)*x+RootOf(_Z^3-2)^2-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*Roo
tOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2))/(1+x)^2/(x^2-x+1)^2)-1/6*ln(-(-6*RootOf
(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^4*x^3+18*RootO
f(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_Z^3-2)^3*x^3+RootO
f(_Z^3-2)^2*x^6-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootO
f(_Z^3-2)*x^6+18*(-x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+3*
_Z*RootOf(_Z^3-2)+9*_Z^2)*x^2-6*(-x^3+1)^(1/3)*RootOf(_Z^3-2)*x^4-18*(-x^3+
1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^4-6*RootOf(_
Z^3-2)^2*x^3+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(
_Z^3-2)*x^3+6*(-x^3+1)^(1/3)*RootOf(_Z^3-2)*x+18*(-x^3+1)^(1/3)*RootOf(Root
Of(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x+RootOf(_Z^3-2)^2-3*RootOf(RootOf
(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2))/(1+x)^2/(x^2-x+1)^2)
*RootOf(_Z^3-2)-1/2*ln(-(-6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_
Z^2)*RootOf(_Z^3-2)^4*x^3+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*
_Z^2)^2*RootOf(_Z^3-2)^3*x^3+RootOf(_Z^3-2)^2*x^6-3*RootOf(RootOf(_Z^3-2)^2
+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)*x^6+18*(-x^3+1)^(2/3)*RootOf(_Z
^3-2)^2*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^2-6*(-x^3+1)^
(1/3)*RootOf(_Z^3-2)*x^4-18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*Roo
tOf(_Z^3-2)+9*_Z^2)*x^4-6*RootOf(_Z^3-2)^2*x^3+18*RootOf(RootOf(_Z^3-2)^2+3
*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)*x^3+6*(-x^3+1)^(1/3)*RootOf(_Z^3-
2)*x+18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*
x+RootOf(_Z^3-2)^2-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*Ro
otOf(_Z^3-2))/(1+x)^2/(x^2-x+1)^2)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3
-2)+9*_Z^2)
```

Fricas [A] (verification not implemented)

none

Time = 1.54 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx$$
$$= \frac{1}{18}\sqrt{3}2^{\frac{1}{3}}\arctan\left(-\frac{6\sqrt{3}2^{\frac{2}{3}}(x^{16}-33x^{13}+110x^{10}-110x^7+33x^4-x)(-x^3+1)^{\frac{1}{3}}-24\sqrt{3}2^{\frac{1}{3}}(x^{14}-2x^{1}}{3(x^{18}-102x^{15}+447x^{12}}\right.$$
$$+\frac{1}{18}\cdot 2^{\frac{1}{3}}\log\left(-\frac{12(-x^3+1)^{\frac{2}{3}}x^2+2^{\frac{2}{3}}(x^6+2x^3+1)-6\cdot 2^{\frac{1}{3}}(x^4-x)(-x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right)-\frac{1}{36}$$
$$\cdot 2^{\frac{1}{3}}\log\left(\frac{12\cdot 2^{\frac{2}{3}}(x^8-4x^5+x^2)(-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(x^{12}-32x^9+78x^6-32x^3+1)+6(x^{10}-11x^7+11x^4}{x^{12}+4x^9+6x^6+4x^3+1}\right.$$

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*2^(1/3)*arctan(-1/3*(6*sqrt(3)*2^(2/3)*(x^16 - 33*x^13 + 110*x
^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) - 24*sqrt(3)*2^(1/3)*(x^14 - 2
*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - sqrt(3)*(x^18 + 42*x^15 - 4
17*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 62
8*x^9 + 447*x^6 - 102*x^3 + 1)) + 1/18*2^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^
2 + 2^(2/3)*(x^6 + 2*x^3 + 1) - 6*2^(1/3)*(x^4 - x)*(-x^3 + 1)^(1/3))/(x^6
+ 2*x^3 + 1)) - 1/36*2^(1/3)*log((12*2^(2/3)*(x^8 - 4*x^5 + x^2)*(-x^3 + 1)
^(2/3) + 2^(1/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 6*(x^10 - 11*x^7 +
 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1))
```

Sympy [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx = \int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate((-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/((x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{1}{3}}}{x^3+1}\,dx$$

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(1/3)/(x^3 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{1}{3}}}{x^3+1}\,dx$$

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(1/3)/(x^3 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx = \int \frac{(1-x^3)^{1/3}}{x^3+1}\,dx$$

```
[In] int((1 - x^3)^(1/3)/(x^3 + 1),x)
[Out] int((1 - x^3)^(1/3)/(x^3 + 1), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well")
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_count
        fi;
    fi;
else #ExpnType(result) > ExpnType(optimal)
    if debug then
        print("ExpnType(result) > ExpnType(optimal)");
    fi;
    return "C",cat("Result contains higher order function than in optimal. Order ",
                  convert(ExpnType_result,string)," vs. order ",
                  convert(ExpnType_optimal,string),".");
fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation ="none"
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```