Computer Algebra Independent Integration Tests

Summer 2023 edition

0-Independent-test-suites/1-Apostol-Problems

Nasser M. Abbasi

September 5, 2023 Compiled on September 5, 2023 at 10:11am

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [175]. This is test number [1].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (175)	0.00 (0)
Mathematica	100.00 (175)	0.00 (0)
Fricas	99.43 (174)	0.57 (1)
Maple	98.86 (173)	1.14 (2)
Giac	97.14 (170)	2.86 (5)
Mupad	96.57 (169)	3.43 (6)
Maxima	94.86 (166)	5.14 (9)
Sympy	94.29 (165)	5.71 (10)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

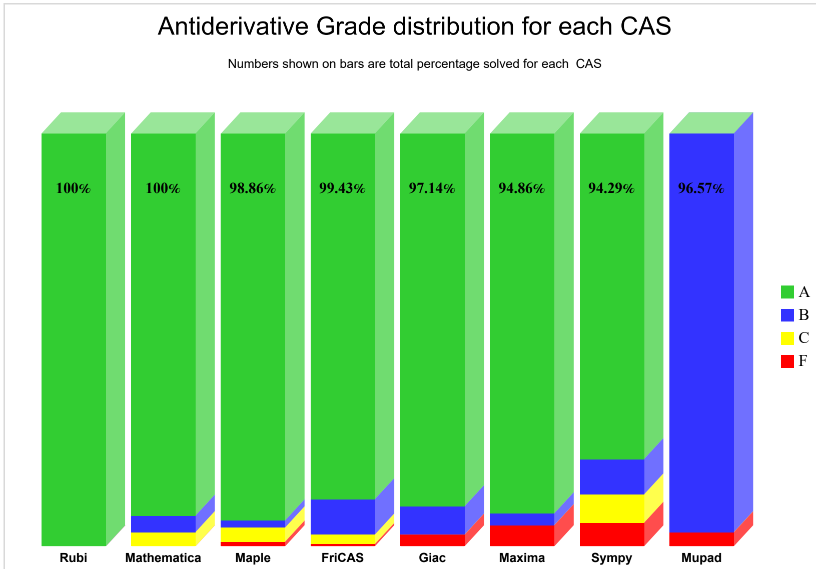
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

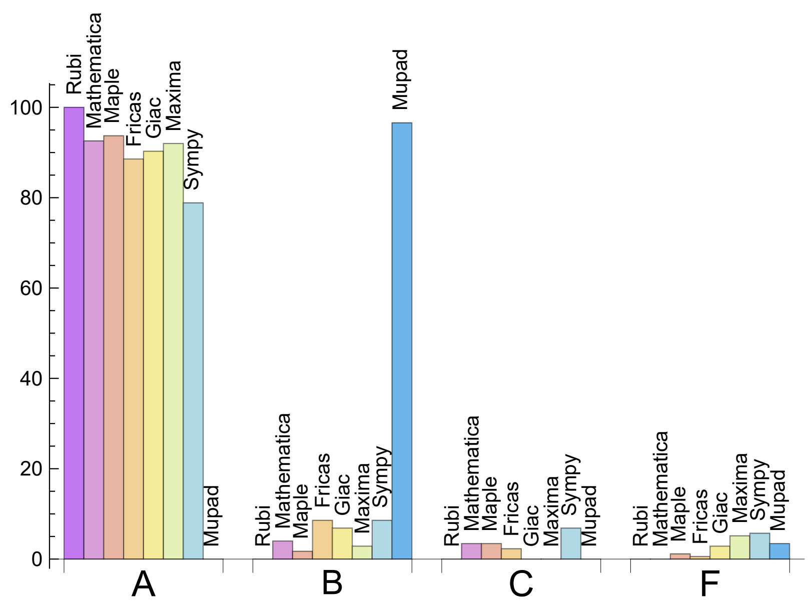
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	93.714	1.714	3.429	1.143
Mathematica	92.571	4.000	3.429	0.000
Maxima	92.000	2.857	0.000	5.143
Giac	90.286	6.857	0.000	2.857
Fricas	88.571	8.571	2.286	0.571
Sympy	78.857	8.571	6.857	5.714
Mupad	0.000	96.571	0.000	3.429

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	1	100.00	0.00	0.00
Maple	2	100.00	0.00	0.00
Giac	5	100.00	0.00	0.00
Mupad	6	0.00	100.00	0.00
Maxima	9	66.67	0.00	33.33
Sympy	10	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.01
Maple	0.09
Mathematica	0.14
Mupad	0.16
Maxima	0.21
Fricas	0.24
Giac	0.27
Sympy	1.44

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	17.84	0.90	15.00	0.82
Maxima	18.24	0.86	14.50	0.81
Giac	20.84	0.97	15.50	0.83
Fricas	21.74	1.04	17.00	0.85
Mathematica	21.81	1.04	18.00	1.00
Rubi	23.09	1.00	19.00	1.00
Mupad	31.83	1.11	14.00	0.79
Sympy	467.14	30.41	17.00	0.87

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

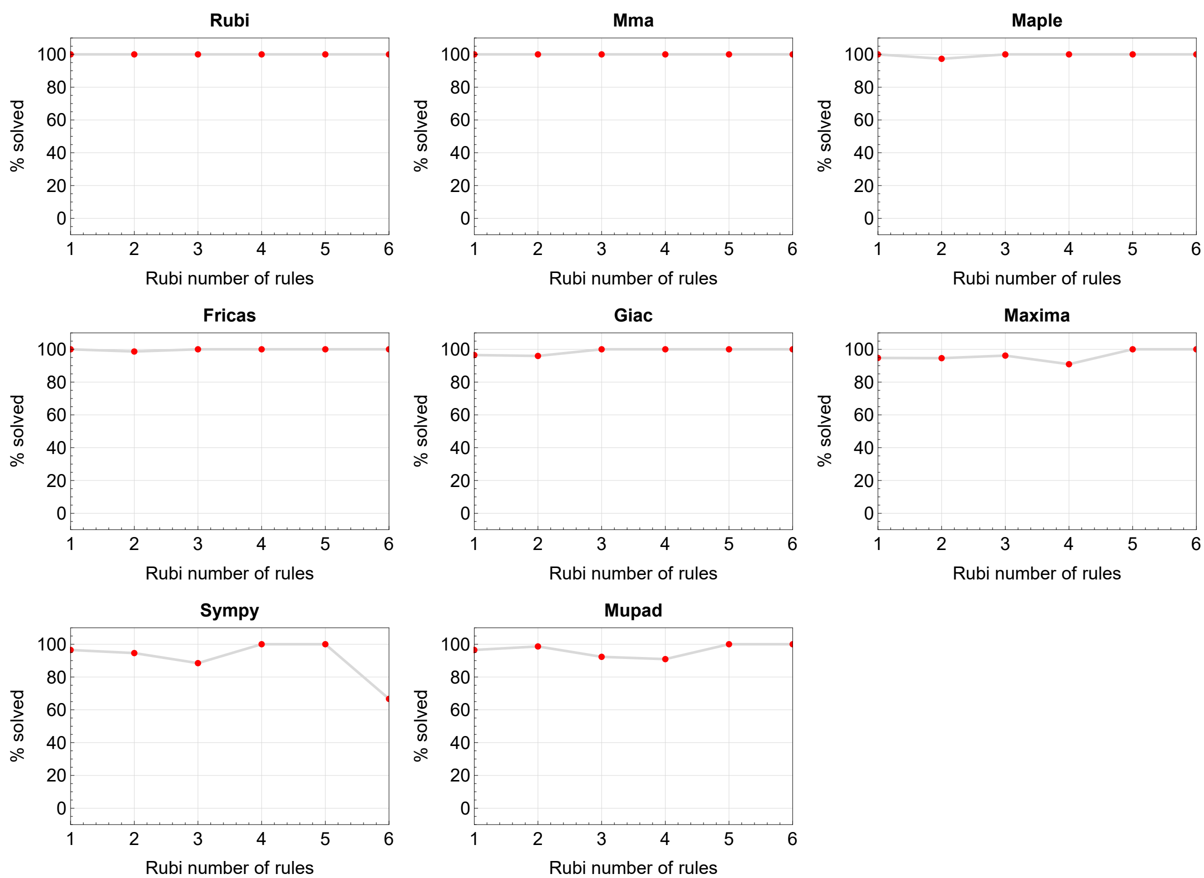


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

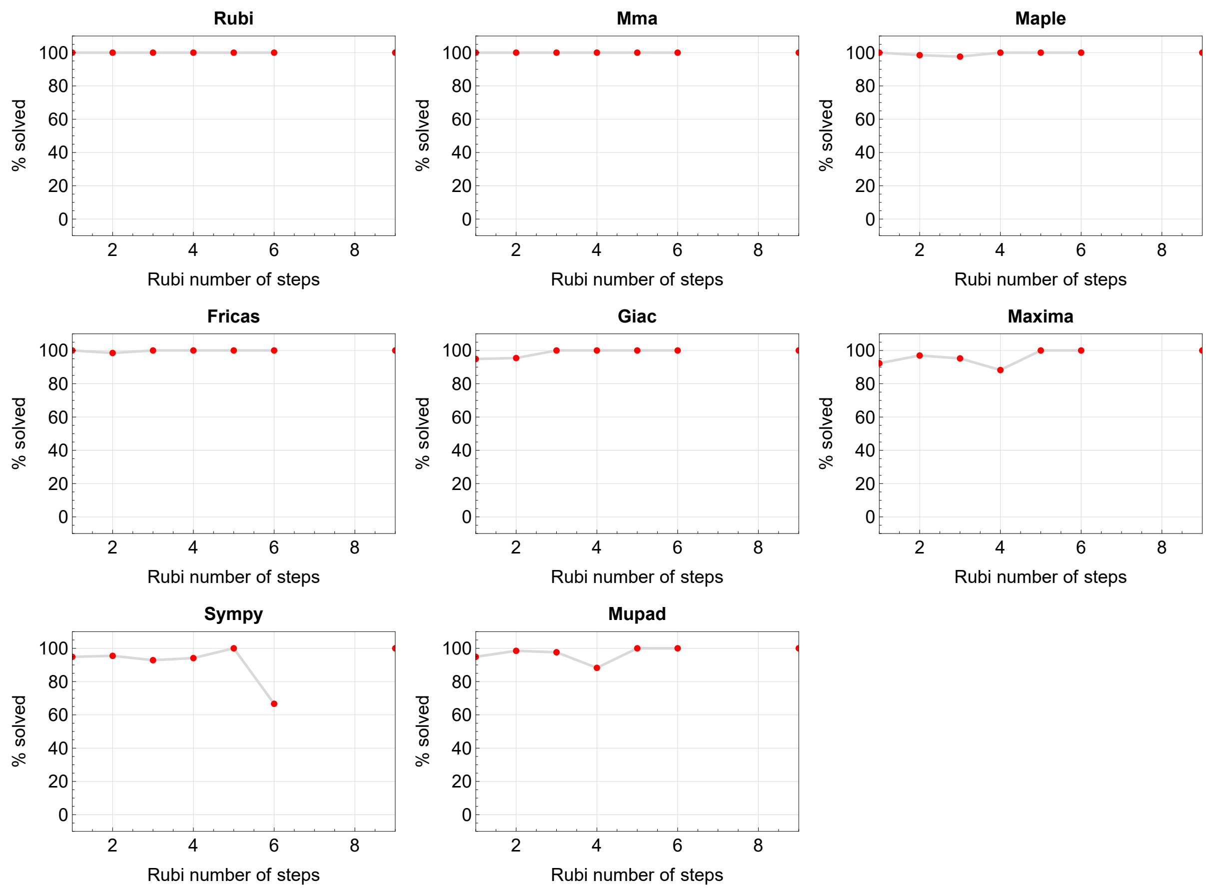


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

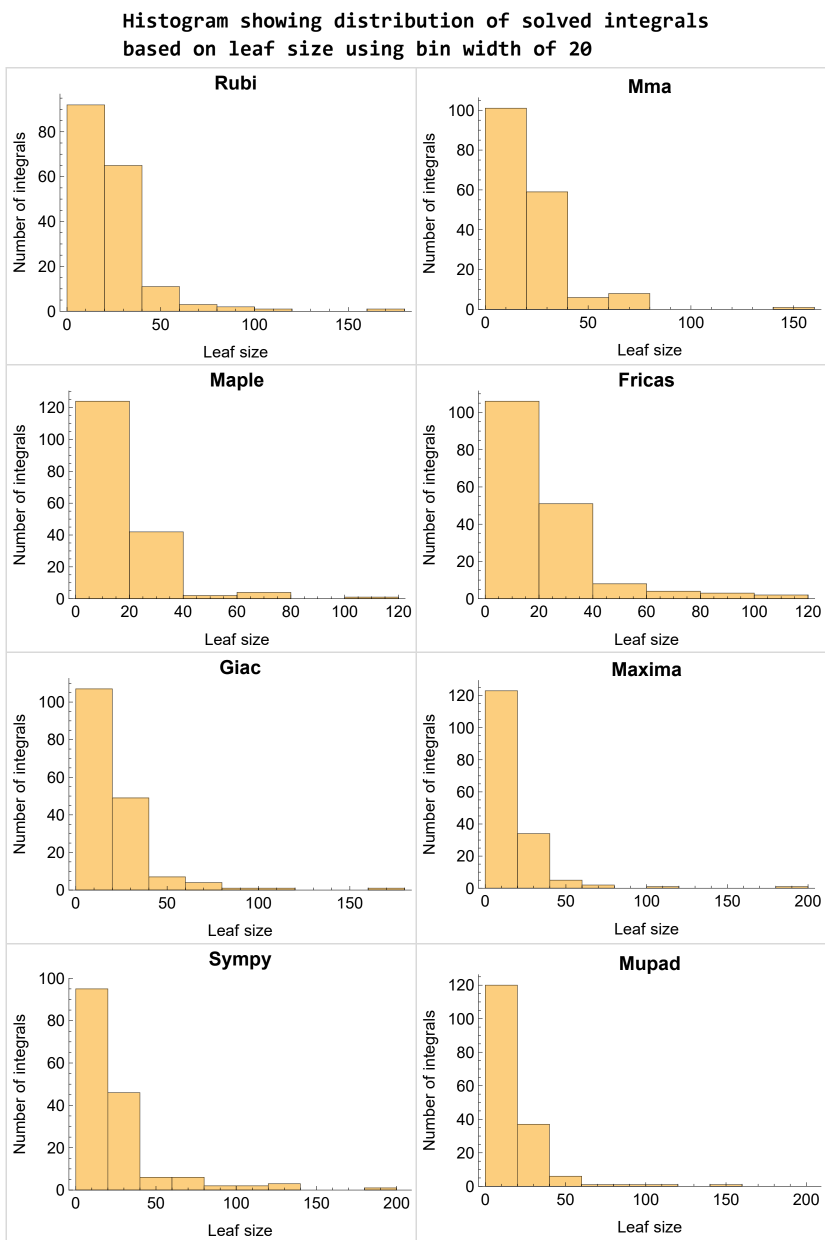


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

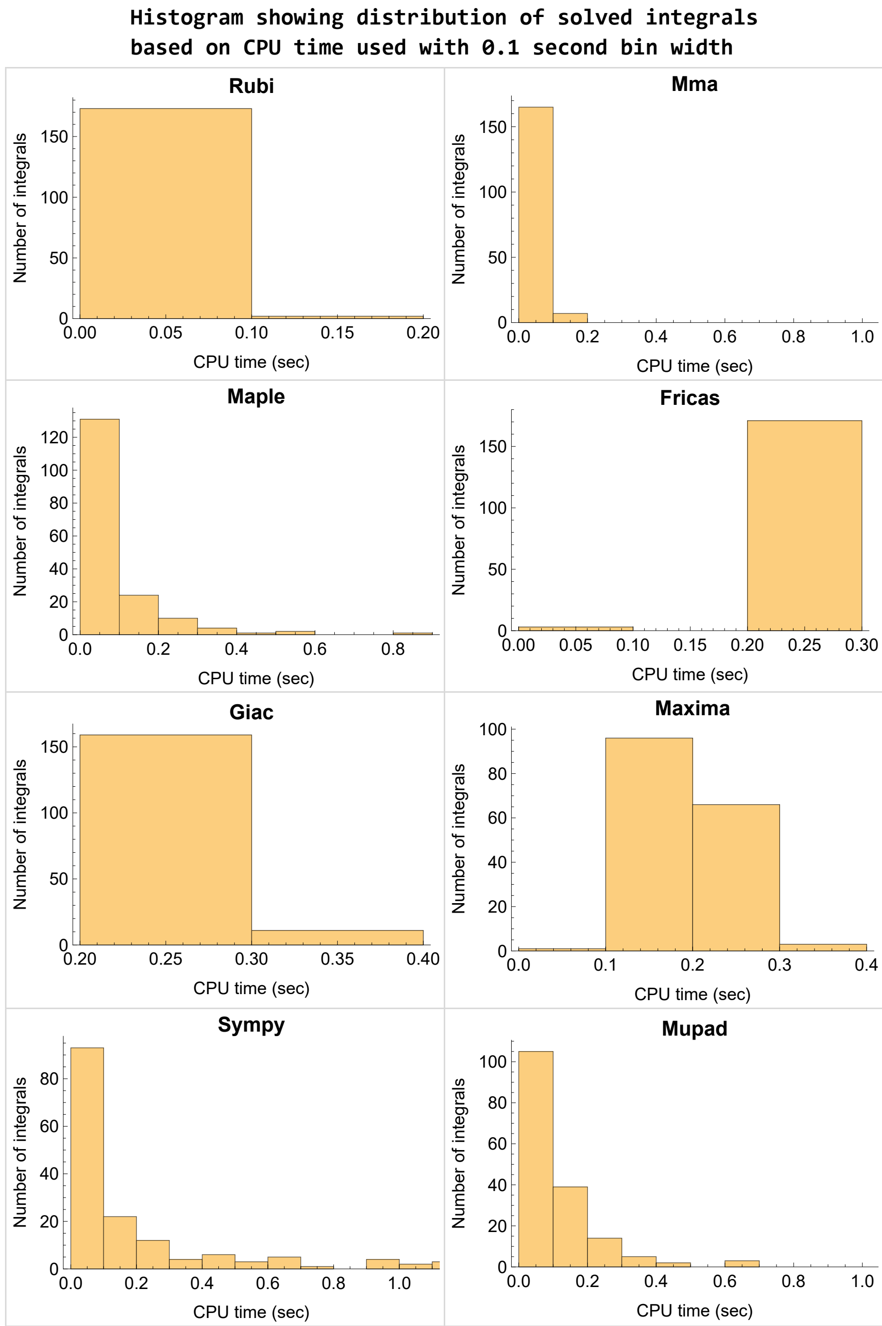


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

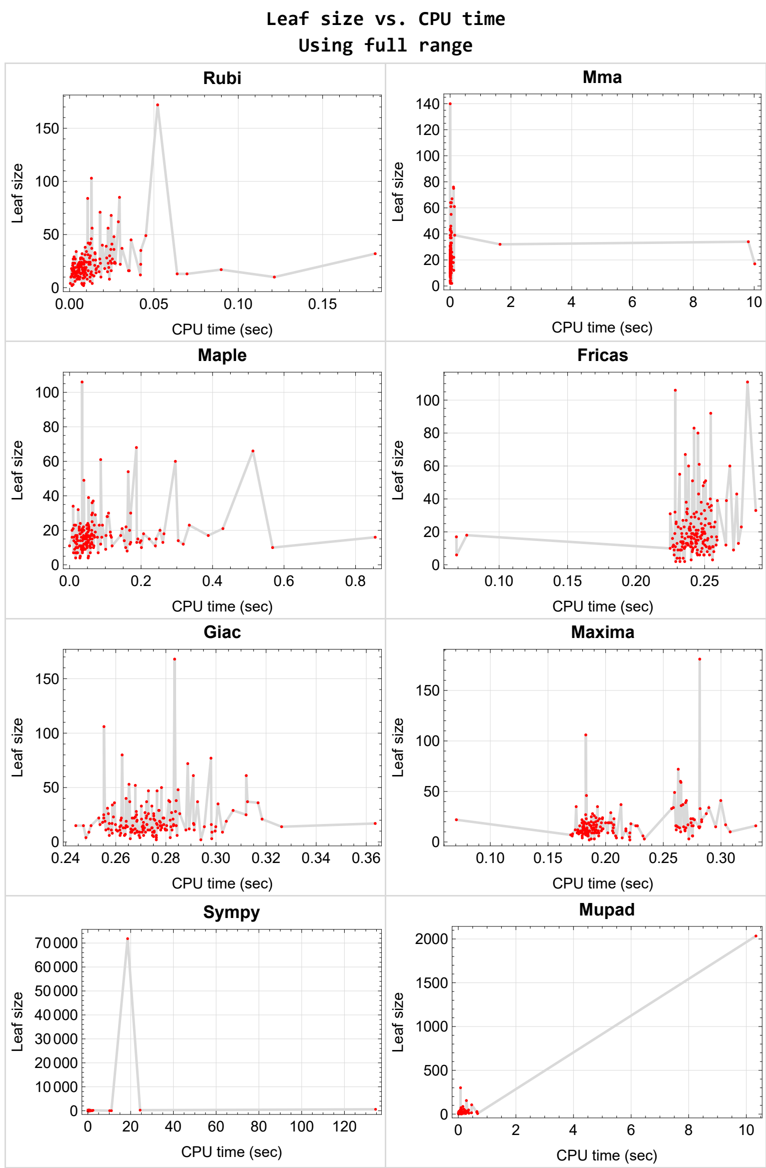


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

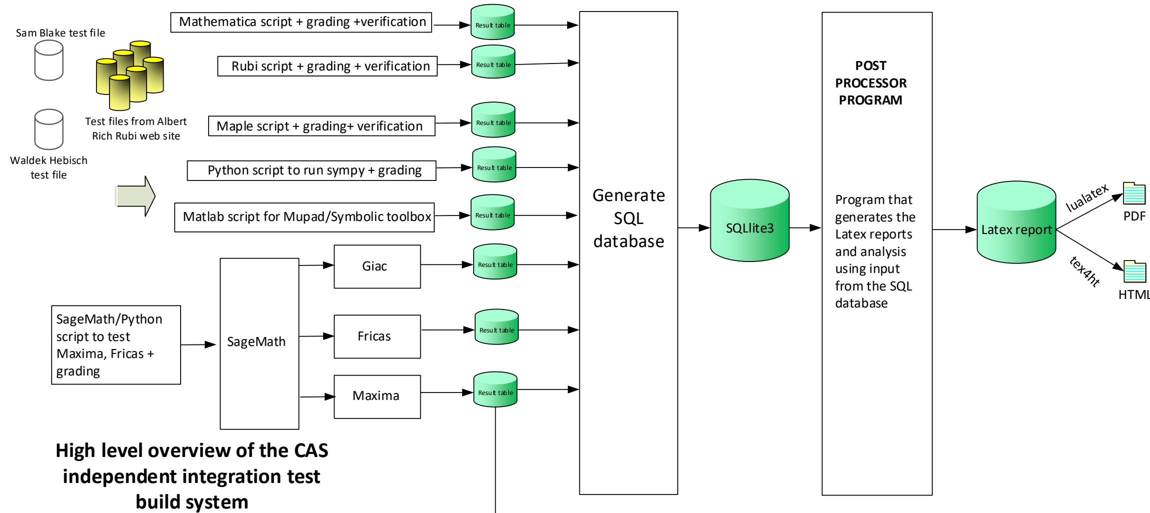
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

The following fields are present only in ***Rubi*** *Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade { 50, 51, 61, 83, 84, 88, 154 }

C grade { 41, 44, 45, 98, 113, 175 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 173, 174 }

B grade { 51, 158, 170 }

C grade { 35, 41, 131, 137, 147, 175 }

F normal fail { 19, 172 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 46, 47, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174 }

B grade { 16, 44, 45, 48, 50, 51, 61, 84, 88, 113, 114, 124, 131, 145, 146 }

C grade { 41, 137, 172, 175 }

F normal fail { 156 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173 }

B grade { 51, 83, 84, 113, 169 }

C grade { }

F normal fail { 19, 41, 98, 99, 174, 175 }

F(-1) timedout fail { }

F(-2) exception fail { 104, 105, 141 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 171, 173, 174 }

B grade { 44, 45, 51, 83, 84, 88, 105, 113, 136, 154, 155, 164 }

C grade { }

F normal fail { 41, 62, 156, 172, 175 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 175 }

C grade { }

F normal fail { }

F(-1) timedout fail { 98, 99, 104, 105, 165, 174 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 5, 6, 8, 10, 11, 12, 13, 14, 15, 16, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 43, 44, 45, 46, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 82, 84, 85, 86, 88, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 104, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 147, 148, 149, 152, 153, 154, 157, 158, 159, 160, 161, 166, 167, 168, 170, 171, 172, 175 }

B grade { 9, 17, 42, 47, 48, 50, 51, 62, 90, 101, 114, 141, 144, 145, 146 }

C grade { 4, 7, 39, 80, 81, 83, 87, 89, 105, 150, 156, 169 }

F normal fail { 19, 103, 151, 155, 162, 163, 164, 165, 173, 174 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	8	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.62	0.69	0.69
time (sec)	N/A	0.007	0.041	0.568	0.194	0.231	0.030	0.270	0.198

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	19	19	39	19	14
N.S.	1	1.00	0.67	0.56	0.70	0.70	1.44	0.70	0.52
time (sec)	N/A	0.006	0.014	0.191	0.182	0.233	0.600	0.256	0.052

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	21	18	22	22	48	22	19
N.S.	1	1.00	0.62	0.53	0.65	0.65	1.41	0.65	0.56
time (sec)	N/A	0.004	0.055	0.207	0.179	0.233	0.783	0.255	0.042

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	18	14	19	14	60	19	14
N.S.	1	1.00	0.67	0.52	0.70	0.52	2.22	0.70	0.52
time (sec)	N/A	0.003	0.007	0.200	0.205	0.240	0.625	0.264	0.038

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	22	22	12	12
N.S.	1	1.00	1.00	0.93	0.86	1.57	1.57	0.86	0.86
time (sec)	N/A	0.016	0.022	0.261	0.192	0.239	0.042	0.271	0.046

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	11	11	8	11	10
N.S.	1	1.00	1.15	0.85	0.85	0.85	0.62	0.85	0.77
time (sec)	N/A	0.064	0.053	0.240	0.179	0.236	0.020	0.267	0.103

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	13	15	17	92	15	12
N.S.	1	1.00	0.78	0.57	0.65	0.74	4.00	0.65	0.52
time (sec)	N/A	0.002	0.011	0.197	0.181	0.233	0.695	0.247	0.019

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	10	8	6	6
N.S.	1	1.00	1.00	0.88	0.75	1.25	1.00	0.75	0.75
time (sec)	N/A	0.010	0.014	0.036	0.185	0.225	0.041	0.271	0.260

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	18	29	12	12
N.S.	1	1.00	1.00	0.81	0.75	1.12	1.81	0.75	0.75
time (sec)	N/A	0.025	0.013	0.171	0.176	0.248	0.176	0.257	0.115

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.012	0.015	0.058	0.171	0.254	0.225	0.284	0.027

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	12	12	6	9
N.S.	1	1.00	1.00	0.92	0.83	1.00	1.00	0.50	0.75
time (sec)	N/A	0.042	0.118	0.119	0.185	0.250	0.294	0.275	0.117

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	10	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	1.00	0.80	0.80
time (sec)	N/A	0.019	0.021	0.101	0.177	0.252	0.124	0.259	0.104

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	9	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.78	1.00	1.00
time (sec)	N/A	0.008	0.012	0.157	0.190	0.271	1.484	0.275	0.130

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	10	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.67	0.73	0.73
time (sec)	N/A	0.003	0.015	0.318	0.206	0.240	0.098	0.258	0.216

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	13	15	15	34	15	12
N.S.	1	1.00	0.78	0.57	0.65	0.65	1.48	0.65	0.52
time (sec)	N/A	0.003	0.009	0.046	0.192	0.244	0.619	0.267	0.019

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	22	8	9	9
N.S.	1	1.00	1.00	0.91	0.82	2.00	0.73	0.82	0.82
time (sec)	N/A	0.001	0.024	0.033	0.180	0.249	0.421	0.275	0.048

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	27	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	1.80	0.73	0.73
time (sec)	N/A	0.003	0.009	0.087	0.180	0.229	0.096	0.274	0.115

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	12	11	15
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.80	0.73	1.00
time (sec)	N/A	0.023	0.044	0.169	0.182	0.251	0.126	0.278	0.149

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	23	0	15	47
N.S.	1	1.00	1.00	0.00	0.00	0.72	0.00	0.47	1.47
time (sec)	N/A	0.181	1.641	0.000	0.000	0.277	0.000	0.278	0.357

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.090	0.025	0.023	0.197	0.275	0.136	0.271	0.124

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	13	10	7	12	15	12	9
N.S.	1	1.00	0.81	0.62	0.44	0.75	0.94	0.75	0.56
time (sec)	N/A	0.006	0.005	0.058	0.170	0.238	0.589	0.266	0.107

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.006	0.010	0.040	0.185	0.247	0.070	0.262	0.049

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	15	17	15	15	17	15	15
N.S.	1	1.00	0.88	1.00	0.88	0.88	1.00	0.88	0.88
time (sec)	N/A	0.016	0.012	0.039	0.180	0.259	0.092	0.244	0.018

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	19	20	20	20	26	20	24
N.S.	1	1.00	0.83	0.87	0.87	0.87	1.13	0.87	1.04
time (sec)	N/A	0.025	0.012	0.040	0.187	0.249	0.140	0.276	0.018

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	20	23	21	21	26	21	23
N.S.	1	1.00	0.83	0.96	0.88	0.88	1.08	0.88	0.96
time (sec)	N/A	0.027	0.011	0.048	0.193	0.239	0.204	0.270	0.020

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.005	0.001	0.019	0.209	0.250	0.047	0.262	0.014

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	15	14	17	24	14	18
N.S.	1	1.00	0.78	0.65	0.61	0.74	1.04	0.61	0.78
time (sec)	N/A	0.008	0.002	0.043	0.180	0.247	0.107	0.282	0.068

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.004	0.006	0.031	0.184	0.254	0.022	0.300	0.036

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	11	11	8	11	10
N.S.	1	1.00	1.15	0.85	0.85	0.85	0.62	0.85	0.77
time (sec)	N/A	0.004	0.001	0.000	0.176	0.256	0.028	0.280	0.002

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	17	16	19	24	16	16
N.S.	1	1.00	0.92	0.71	0.67	0.79	1.00	0.67	0.67
time (sec)	N/A	0.009	0.006	0.063	0.187	0.245	0.020	0.272	0.039

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	17	17	17	17	17
N.S.	1	1.00	1.10	0.81	0.81	0.81	0.81	0.81	0.81
time (sec)	N/A	0.005	0.006	0.068	0.181	0.245	0.027	0.259	0.045

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	23	24	25	36	22	22
N.S.	1	1.00	0.88	0.68	0.71	0.74	1.06	0.65	0.65
time (sec)	N/A	0.010	0.006	0.083	0.197	0.247	0.020	0.267	0.043

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	36	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	1.44	0.76	0.76
time (sec)	N/A	0.009	0.010	0.033	0.190	0.250	0.099	0.268	0.106

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	31	23	23	23	39	23	25
N.S.	1	1.00	0.94	0.70	0.70	0.70	1.18	0.70	0.76
time (sec)	N/A	0.015	0.007	0.069	0.183	0.242	0.139	0.259	0.116

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	29	19	26	29	56	26	28
N.S.	1	1.00	0.71	0.46	0.63	0.71	1.37	0.63	0.68
time (sec)	N/A	0.025	0.025	0.072	0.190	0.240	0.179	0.257	0.067

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.005	0.005	0.030	0.177	0.248	0.018	0.266	0.027

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	9	10	8	9	9
N.S.	1	1.00	1.00	1.00	0.82	0.91	0.73	0.82	0.82
time (sec)	N/A	0.005	0.003	0.048	0.184	0.242	0.020	0.268	0.037

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	17	16	19	24	16	16
N.S.	1	1.00	0.92	0.71	0.67	0.79	1.00	0.67	0.67
time (sec)	N/A	0.011	0.005	0.059	0.187	0.249	0.017	0.279	0.031

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	61	54	60	60	180	50	37
N.S.	1	1.00	0.73	0.64	0.71	0.71	2.14	0.60	0.44
time (sec)	N/A	0.011	0.140	0.164	0.265	0.238	2.398	0.278	0.222

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	25	21	34	21	39	28	20
N.S.	1	1.00	0.66	0.55	0.89	0.55	1.03	0.74	0.53
time (sec)	N/A	0.009	0.013	0.036	0.259	0.240	0.211	0.284	0.031

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	34	17	0	17	31	0	301
N.S.	1	1.00	0.20	0.10	0.00	0.10	0.18	0.00	1.75
time (sec)	N/A	0.052	9.809	0.102	0.000	0.069	0.383	0.000	0.078

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	8	7	6	6	7	6	6
N.S.	1	1.00	1.33	1.17	1.00	1.00	1.17	1.00	1.00
time (sec)	N/A	0.003	0.004	0.011	0.275	0.249	0.029	0.257	0.069

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	12	12	19	12	12
N.S.	1	1.00	1.14	0.93	0.86	0.86	1.36	0.86	0.86
time (sec)	N/A	0.006	0.004	0.016	0.269	0.258	0.030	0.280	0.033

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	18	13	10	20	8	18	8
N.S.	1	1.00	2.25	1.62	1.25	2.50	1.00	2.25	1.00
time (sec)	N/A	0.004	0.007	0.017	0.308	0.248	0.026	0.278	0.074

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	22	11	16	48	19	34	10
N.S.	1	1.00	1.83	0.92	1.33	4.00	1.58	2.83	0.83
time (sec)	N/A	0.008	0.005	0.028	0.261	0.249	0.028	0.259	0.024

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	26	18	20	18	26	18	20
N.S.	1	1.00	1.18	0.82	0.91	0.82	1.18	0.82	0.91
time (sec)	N/A	0.009	0.055	0.062	0.181	0.244	0.071	0.271	0.112

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	22	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	1.69	0.69	0.69
time (sec)	N/A	0.002	0.001	0.023	0.192	0.245	0.075	0.270	0.027

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	51	58	9	9
N.S.	1	1.00	0.85	0.77	0.69	3.92	4.46	0.69	0.69
time (sec)	N/A	0.001	0.001	0.049	0.189	0.241	0.018	0.266	0.108

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	16	14	19	19	15	14	14
N.S.	1	1.00	0.89	0.78	1.06	1.06	0.83	0.78	0.78
time (sec)	N/A	0.001	0.003	0.030	0.180	0.228	0.043	0.262	0.042

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	43	10	9	31	31	9	31
N.S.	1	1.00	3.91	0.91	0.82	2.82	2.82	0.82	2.82
time (sec)	N/A	0.001	0.001	0.039	0.178	0.225	0.017	0.275	0.027

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	140	106	106	106	131	106	106
N.S.	1	1.00	2.50	1.89	1.89	1.89	2.34	1.89	1.89
time (sec)	N/A	0.023	0.001	0.035	0.183	0.229	0.040	0.255	0.465

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.007	0.008	0.053	0.214	0.230	0.179	0.270	0.101

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	46	49	37	37	60	37	41
N.S.	1	1.00	0.74	0.79	0.60	0.60	0.97	0.60	0.66
time (sec)	N/A	0.029	0.023	0.040	0.213	0.252	0.381	0.266	0.244

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.005	0.002	0.044	0.171	0.257	0.104	0.268	0.055

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	15	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.94	0.75	0.75
time (sec)	N/A	0.035	0.010	0.188	0.189	0.266	0.940	0.289	0.208

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	8	7	9	6
N.S.	1	1.00	1.00	0.70	0.80	0.80	0.70	0.90	0.60
time (sec)	N/A	0.001	0.000	0.021	0.180	0.241	0.022	0.266	0.075

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	12	15	15	15	12
N.S.	1	1.00	1.00	1.07	0.80	1.00	1.00	1.00	0.80
time (sec)	N/A	0.003	0.002	0.012	0.180	0.250	0.048	0.264	0.083

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.003	0.002	0.012	0.184	0.241	0.033	0.263	0.038

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	17	22	22	22	17
N.S.	1	1.00	1.00	0.82	0.61	0.79	0.79	0.79	0.61
time (sec)	N/A	0.007	0.002	0.013	0.190	0.239	0.055	0.272	0.034

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	5	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.25	1.00
time (sec)	N/A	0.000	0.000	0.030	0.180	0.232	0.020	0.275	0.016

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	7	4	3	11	3	4	3
N.S.	1	1.00	2.33	1.33	1.00	3.67	1.00	1.33	1.00
time (sec)	N/A	0.002	0.004	0.052	0.187	0.256	0.046	0.282	0.024

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	21	36	28	32	61	0	38
N.S.	1	1.00	0.75	1.29	1.00	1.14	2.18	0.00	1.36
time (sec)	N/A	0.007	0.005	0.063	0.189	0.244	0.348	0.000	0.219

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	17	22	26	22	17
N.S.	1	1.00	1.00	0.82	0.61	0.79	0.93	0.79	0.61
time (sec)	N/A	0.012	0.002	0.017	0.207	0.248	0.051	0.267	0.035

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	3	4	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	1.00	1.33	1.00
time (sec)	N/A	0.008	0.004	0.017	0.190	0.256	0.043	0.248	0.069

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.013	0.003	0.056	0.191	0.253	0.049	0.261	0.368

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	17	12	20	17	13
N.S.	1	1.00	0.70	0.78	0.74	0.52	0.87	0.74	0.57
time (sec)	N/A	0.027	0.019	0.047	0.190	0.227	1.148	0.257	0.180

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	23	31	37	31	23
N.S.	1	1.00	1.00	0.82	0.59	0.79	0.95	0.79	0.59
time (sec)	N/A	0.022	0.002	0.024	0.197	0.242	0.062	0.257	0.035

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.78	0.67	0.67	0.56	0.67	0.67
time (sec)	N/A	0.008	0.010	0.022	0.180	0.246	0.038	0.269	0.086

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	12	11	10	11	11
N.S.	1	1.00	1.00	0.86	0.86	0.79	0.71	0.79	0.79
time (sec)	N/A	0.009	0.003	0.048	0.174	0.256	0.068	0.256	0.117

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	7	7	7
N.S.	1	1.00	1.00	0.80	0.70	0.70	0.70	0.70	0.70
time (sec)	N/A	0.006	0.006	0.161	0.183	0.244	0.127	0.264	0.112

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	12	11	13	15	11	11
N.S.	1	1.00	0.74	0.63	0.58	0.68	0.79	0.58	0.58
time (sec)	N/A	0.006	0.012	0.063	0.176	0.240	0.138	0.260	0.020

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	12	10	9	13	15	9	9
N.S.	1	1.00	0.63	0.53	0.47	0.68	0.79	0.47	0.47
time (sec)	N/A	0.005	0.007	0.059	0.175	0.237	0.129	0.272	0.020

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	9	9	7	9	9
N.S.	1	1.00	1.00	1.00	0.90	0.90	0.70	0.90	0.90
time (sec)	N/A	0.005	0.009	0.021	0.202	0.241	0.040	0.269	0.049

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	7	7	6	6	5	6	6
N.S.	1	1.00	0.64	0.64	0.55	0.55	0.45	0.55	0.55
time (sec)	N/A	0.005	0.008	0.016	0.194	0.228	0.030	0.263	0.019

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	11	10	9	9	7	9	9
N.S.	1	1.00	0.69	0.62	0.56	0.56	0.44	0.56	0.56
time (sec)	N/A	0.005	0.009	0.019	0.221	0.232	0.029	0.249	0.023

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	12	12	11	11	10	11	11
N.S.	1	1.00	0.63	0.63	0.58	0.58	0.53	0.58	0.58
time (sec)	N/A	0.010	0.010	0.020	0.184	0.234	0.045	0.262	0.024

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	19	16	16	16	17	16	16
N.S.	1	1.00	0.59	0.50	0.50	0.50	0.53	0.50	0.50
time (sec)	N/A	0.015	0.012	0.021	0.178	0.242	0.038	0.298	0.077

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	16	16	11	11	20	11	11
N.S.	1	1.00	0.67	0.67	0.46	0.46	0.83	0.46	0.46
time (sec)	N/A	0.006	0.005	0.007	0.194	0.240	0.079	0.271	0.024

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	16	14	13	13	12	13	13
N.S.	1	1.00	0.62	0.54	0.50	0.50	0.46	0.50	0.50
time (sec)	N/A	0.014	0.016	0.023	0.182	0.244	0.039	0.264	0.109

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	28	28	27	31	139	36	27
N.S.	1	1.00	0.68	0.68	0.66	0.76	3.39	0.88	0.66
time (sec)	N/A	0.012	0.023	0.105	0.183	0.248	0.284	0.259	0.031

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	29	29	29	33	136	38	29
N.S.	1	1.00	0.69	0.69	0.69	0.79	3.24	0.90	0.69
time (sec)	N/A	0.011	0.022	0.072	0.204	0.250	0.275	0.284	0.020

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	21	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.40	0.87
time (sec)	N/A	0.003	0.003	0.036	0.198	0.237	0.071	0.283	0.109

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	64	22	35	33	17	37	21
N.S.	1	1.00	3.37	1.16	1.84	1.74	0.89	1.95	1.11
time (sec)	N/A	0.007	0.042	0.027	0.193	0.255	1.150	0.272	0.642

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	64	20	35	35	17	37	20
N.S.	1	1.00	3.76	1.18	2.06	2.06	1.00	2.18	1.18
time (sec)	N/A	0.008	0.024	0.007	0.174	0.253	1.180	0.282	0.205

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	23	22	23	22
N.S.	1	1.00	1.00	0.96	0.92	0.92	0.88	0.92	0.88
time (sec)	N/A	0.024	0.004	0.051	0.276	0.257	0.083	0.268	0.033

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	33	39	22	38	20
N.S.	1	1.00	1.00	0.95	1.50	1.77	1.00	1.73	0.91
time (sec)	N/A	0.011	0.003	0.010	0.257	0.266	1.097	0.281	0.025

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	6	23	19	28	14
N.S.	1	1.00	1.00	0.94	0.38	1.44	1.19	1.75	0.88
time (sec)	N/A	0.002	0.003	0.058	0.275	0.239	0.525	0.268	0.168

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	23	10	11	21	10	27	11
N.S.	1	1.00	2.30	1.00	1.10	2.10	1.00	2.70	1.10
time (sec)	N/A	0.009	0.050	0.201	0.269	0.244	0.262	0.282	0.091

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	20	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	2.00	1.00	1.00
time (sec)	N/A	0.121	0.002	0.068	0.265	0.235	0.050	0.275	0.040

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.010	0.009	0.056	0.282	0.236	0.057	0.250	0.105

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	16
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84
time (sec)	N/A	0.009	0.004	0.388	0.261	0.252	0.051	0.254	0.078

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	16	16	15	13	15	15	14
N.S.	1	1.00	0.76	0.76	0.71	0.62	0.71	0.71	0.67
time (sec)	N/A	0.006	0.003	0.042	0.268	0.249	0.123	0.271	0.026

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	30	34	33	24	32	33	24
N.S.	1	1.00	0.75	0.85	0.82	0.60	0.80	0.82	0.60
time (sec)	N/A	0.019	0.011	0.010	0.282	0.250	0.121	0.274	0.030

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	34	25	29	29	29
N.S.	1	1.00	0.74	0.86	0.97	0.71	0.83	0.83	0.83
time (sec)	N/A	0.042	0.007	0.064	0.289	0.254	0.137	0.277	0.126

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	18	17	16	14	19	16	16
N.S.	1	1.00	0.82	0.77	0.73	0.64	0.86	0.73	0.73
time (sec)	N/A	0.006	0.012	0.017	0.276	0.244	0.447	0.276	0.061

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.024	0.006	0.022	0.184	0.245	0.448	0.276	0.670

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	37	18	17	31	15	17	17
N.S.	1	1.00	1.61	0.78	0.74	1.35	0.65	0.74	0.74
time (sec)	N/A	0.002	0.034	0.264	0.304	0.237	0.106	0.276	0.077

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	37	16	0	15	31	24	0
N.S.	1	1.00	1.68	0.73	0.00	0.68	1.41	1.09	0.00
time (sec)	N/A	0.030	0.008	0.078	0.000	0.245	11.008	0.272	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	16	0	15	31	22	0
N.S.	1	1.00	1.00	0.80	0.00	0.75	1.55	1.10	0.00
time (sec)	N/A	0.017	0.005	0.041	0.000	0.252	10.172	0.279	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	12	15	17
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	0.89
time (sec)	N/A	0.002	0.006	0.049	0.281	0.245	0.041	0.283	0.029

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	3	15	3	3
N.S.	1	1.00	1.00	1.00	0.75	0.75	3.75	0.75	0.75
time (sec)	N/A	0.013	0.013	0.029	0.272	0.240	0.054	0.266	0.102

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	25	19	28	19	21	22
N.S.	1	1.00	1.00	0.93	0.70	1.04	0.70	0.78	0.81
time (sec)	N/A	0.018	0.013	0.063	0.202	0.251	1.997	0.274	0.137

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	67	61	49	38	0	36	49
N.S.	1	1.00	1.60	1.45	1.17	0.90	0.00	0.86	1.17
time (sec)	N/A	0.012	0.062	0.087	0.260	0.237	0.000	0.317	0.070

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	75	68	0	80	114	61	0
N.S.	1	1.00	1.06	0.96	0.00	1.13	1.61	0.86	0.00
time (sec)	N/A	0.018	0.114	0.187	0.000	0.245	1.077	0.291	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	55	28	0	43	71	61	0
N.S.	1	1.00	1.72	0.88	0.00	1.34	2.22	1.91	0.00
time (sec)	N/A	0.007	0.033	0.106	0.000	0.246	0.901	0.312	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	15	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.00	0.87
time (sec)	N/A	0.003	0.003	0.053	0.182	0.234	0.041	0.262	0.054

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	13	14	15	13
N.S.	1	1.00	1.00	0.74	0.68	0.68	0.74	0.79	0.68
time (sec)	N/A	0.004	0.003	0.052	0.181	0.230	0.047	0.278	0.040

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	19	21	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.83	0.91	0.83
time (sec)	N/A	0.023	0.003	0.062	0.200	0.241	0.056	0.275	0.046

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	20	19
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.87	0.83
time (sec)	N/A	0.027	0.005	0.030	0.192	0.259	0.069	0.269	0.205

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	19	18	26	19	24	18
N.S.	1	1.00	0.92	0.79	0.75	1.08	0.79	1.00	0.75
time (sec)	N/A	0.012	0.007	0.051	0.184	0.229	0.070	0.275	0.051

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	28	3	29	57
N.S.	1	1.00	1.00	1.04	1.00	1.00	0.11	1.04	2.04
time (sec)	N/A	0.022	0.021	0.062	0.287	0.246	0.064	0.307	0.192

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	61	37	36	51	14	37	53
N.S.	1	1.00	1.24	0.76	0.73	1.04	0.29	0.76	1.08
time (sec)	N/A	0.045	0.023	0.066	0.266	0.251	0.079	0.293	0.123

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	24	19	39	38	39	37	21
N.S.	1	1.00	1.14	0.90	1.86	1.81	1.86	1.76	1.00
time (sec)	N/A	0.007	0.018	0.114	0.269	0.247	0.239	0.313	0.339

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	14	55	48	14	87
N.S.	1	1.00	1.00	1.06	0.88	3.44	3.00	0.88	5.44
time (sec)	N/A	0.035	0.012	0.115	0.184	0.232	1.343	0.326	0.160

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	9	9	9	8	11	9
N.S.	1	1.00	0.82	0.82	0.82	0.82	0.73	1.00	0.82
time (sec)	N/A	0.003	0.003	0.041	0.182	0.233	0.036	0.282	0.050

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	20	22	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.87	0.96	0.83
time (sec)	N/A	0.007	0.004	0.054	0.185	0.244	0.052	0.262	0.124

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	21	20	27	22	22	18
N.S.	1	1.00	0.93	0.70	0.67	0.90	0.73	0.73	0.60
time (sec)	N/A	0.012	0.006	0.027	0.191	0.234	0.035	0.253	0.043

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	23	20	26	30
N.S.	1	1.00	1.00	0.89	0.85	0.85	0.74	0.96	1.11
time (sec)	N/A	0.025	0.004	0.030	0.207	0.235	0.062	0.275	0.095

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	24	16	20	32	17	16	15
N.S.	1	1.00	1.04	0.70	0.87	1.39	0.74	0.70	0.65
time (sec)	N/A	0.018	0.007	0.040	0.181	0.228	0.048	0.260	0.093

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	14	15	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	0.94	0.88
time (sec)	N/A	0.011	0.003	0.047	0.270	0.233	0.037	0.280	0.044

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	12	14	12	12
N.S.	1	1.00	1.00	0.72	0.67	0.67	0.78	0.67	0.67
time (sec)	N/A	0.009	0.005	0.049	0.261	0.238	0.063	0.268	0.039

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	13	11
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	1.18	1.00
time (sec)	N/A	0.004	0.002	0.040	0.196	0.227	0.039	0.277	0.104

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	24	32	19	29	20
N.S.	1	1.00	1.00	0.88	1.00	1.33	0.79	1.21	0.83
time (sec)	N/A	0.007	0.006	0.054	0.192	0.237	0.040	0.277	0.037

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	39	46	83	46	52	45
N.S.	1	1.00	0.96	0.85	1.00	1.80	1.00	1.13	0.98
time (sec)	N/A	0.013	0.012	0.053	0.183	0.242	0.087	0.268	0.097

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	16	8	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.60	0.80	1.10	1.00
time (sec)	N/A	0.003	0.003	0.033	0.185	0.227	0.027	0.291	0.030

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	17	13	10	16	13
N.S.	1	1.00	1.00	0.82	1.00	0.76	0.59	0.94	0.76
time (sec)	N/A	0.006	0.003	0.052	0.186	0.244	0.037	0.283	0.122

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	14	14	17	16	14
N.S.	1	1.00	1.00	0.75	0.70	0.70	0.85	0.80	0.70
time (sec)	N/A	0.006	0.003	0.063	0.188	0.234	0.045	0.291	0.054

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	12	13	12	16	8	13	12
N.S.	1	1.00	0.75	0.81	0.75	1.00	0.50	0.81	0.75
time (sec)	N/A	0.005	0.003	0.053	0.191	0.236	0.038	0.275	0.037

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	17	10	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.21	0.71	1.00	1.00
time (sec)	N/A	0.008	0.006	0.242	0.281	0.240	0.078	0.300	0.093

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	20	20	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.95	0.95	0.81
time (sec)	N/A	0.015	0.005	0.029	0.207	0.240	0.080	0.262	0.083

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	27	23	23	34	20	25	17
N.S.	1	1.00	1.29	1.10	1.10	1.62	0.95	1.19	0.81
time (sec)	N/A	0.002	0.005	0.053	0.195	0.237	0.054	0.255	0.102

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	17	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.77	0.77	0.73
time (sec)	N/A	0.008	0.003	0.052	0.275	0.240	0.037	0.291	0.172

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	18	8	14	10
N.S.	1	1.00	1.00	1.10	1.40	1.80	0.80	1.40	1.00
time (sec)	N/A	0.012	0.004	0.037	0.190	0.243	0.068	0.271	0.030

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	21	19	25	19	21	16
N.S.	1	1.00	1.00	0.68	0.61	0.81	0.61	0.68	0.52
time (sec)	N/A	0.009	0.002	0.049	0.196	0.229	0.051	0.318	0.045

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	15	17	24
N.S.	1	1.00	1.00	0.94	0.89	0.89	0.83	0.94	1.33
time (sec)	N/A	0.021	0.004	0.046	0.330	0.247	0.066	0.364	0.041

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	17	17	17	19	9
N.S.	1	1.00	1.92	0.77	1.31	1.31	1.31	1.46	0.69
time (sec)	N/A	0.002	0.003	0.067	0.271	0.255	0.066	0.277	0.029

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	64	22	72	61	73	72	33
N.S.	1	1.00	0.75	0.26	0.85	0.72	0.86	0.85	0.39
time (sec)	N/A	0.030	0.013	0.054	0.263	0.246	0.073	0.289	0.118

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	15	16	15	26	14	15	15
N.S.	1	1.00	0.65	0.70	0.65	1.13	0.61	0.65	0.65
time (sec)	N/A	0.005	0.006	0.117	0.295	0.258	0.041	0.284	0.089

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	11	11
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	1.00	1.00
time (sec)	N/A	0.005	0.005	0.042	0.185	0.243	0.045	0.280	0.062

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	23	20	23	36	39	47	21
N.S.	1	1.00	0.51	0.44	0.51	0.80	0.87	1.04	0.47
time (sec)	N/A	0.036	0.027	0.168	0.274	0.255	0.228	0.276	0.100

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	31	30	0	111	110	53	28
N.S.	1	1.00	0.84	0.81	0.00	3.00	2.97	1.43	0.76
time (sec)	N/A	0.031	0.024	0.109	0.000	0.281	1.449	0.265	0.339

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	20	16	37	50	36	35	15
N.S.	1	1.00	0.36	0.29	0.66	0.89	0.64	0.62	0.27
time (sec)	N/A	0.013	0.015	0.070	0.267	0.250	0.165	0.301	0.271

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	16	19	23	32	40	32
N.S.	1	1.00	0.65	0.52	0.61	0.74	1.03	1.29	1.03
time (sec)	N/A	0.011	0.013	0.090	0.274	0.255	0.133	0.264	0.224

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	18	17	14	33	248	48	26
N.S.	1	1.00	0.50	0.47	0.39	0.92	6.89	1.33	0.72
time (sec)	N/A	0.026	0.105	0.143	0.264	0.287	24.438	0.285	0.212

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	43	71839	26	15
N.S.	1	1.00	1.00	1.07	1.00	2.87	4789.27	1.73	1.00
time (sec)	N/A	0.018	0.042	0.855	0.262	0.273	18.580	0.286	0.460

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	39	602	13	29
N.S.	1	1.00	1.00	0.82	0.82	2.29	35.41	0.76	1.71
time (sec)	N/A	0.009	0.027	0.304	0.181	0.259	134.571	0.278	0.649

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	22	20	41	11	22	25	34
N.S.	1	1.00	0.73	0.67	1.37	0.37	0.73	0.83	1.13
time (sec)	N/A	0.024	0.053	0.253	0.300	0.253	0.147	0.312	0.319

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	41	23	22	29	24	22	22
N.S.	1	1.00	1.41	0.79	0.76	1.00	0.83	0.76	0.76
time (sec)	N/A	0.003	0.045	0.335	0.277	0.243	0.088	0.279	0.037

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	8	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.62	0.85	0.85
time (sec)	N/A	0.002	0.001	0.059	0.186	0.240	0.071	0.273	0.194

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	30	41	40	87	47	35
N.S.	1	1.00	0.89	0.81	1.11	1.08	2.35	1.27	0.95
time (sec)	N/A	0.013	0.033	0.171	0.270	0.254	0.910	0.273	0.224

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	42	22	25	25	0	26	21
N.S.	1	1.00	1.91	1.00	1.14	1.14	0.00	1.18	0.95
time (sec)	N/A	0.005	0.034	0.158	0.188	0.253	0.000	0.273	0.090

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	33	21	20	25	24	25	20
N.S.	1	1.00	1.22	0.78	0.74	0.93	0.89	0.93	0.74
time (sec)	N/A	0.002	0.019	0.147	0.283	0.240	0.099	0.268	0.087

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	33	21	22	27	26	27	23
N.S.	1	1.00	1.22	0.78	0.81	1.00	0.96	1.00	0.85
time (sec)	N/A	0.008	0.044	0.429	0.283	0.237	0.250	0.272	0.054

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	39	7	15	17	15	33	11
N.S.	1	1.00	2.79	0.50	1.07	1.21	1.07	2.36	0.79
time (sec)	N/A	0.002	0.022	0.080	0.195	0.238	0.214	0.270	0.203

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	76	60	59	92	0	168	73
N.S.	1	1.00	1.12	0.88	0.87	1.35	0.00	2.47	1.07
time (sec)	N/A	0.025	0.110	0.296	0.265	0.254	0.000	0.283	0.094

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	12	0	73	0	13
N.S.	1	1.00	1.00	1.00	0.92	0.00	5.62	0.00	1.00
time (sec)	N/A	0.013	0.003	0.062	0.190	0.000	0.990	0.000	0.033

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	15	2	2	15	2	2
N.S.	1	1.00	1.00	1.00	0.13	0.13	1.00	0.13	0.13
time (sec)	N/A	0.008	0.010	0.223	0.186	0.235	0.073	0.276	0.125

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	8	2	2	2	2	2
N.S.	1	1.00	1.00	4.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.008	0.009	0.036	0.221	0.229	0.387	0.294	0.008

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	4	4	3	4	4
N.S.	1	1.00	1.00	2.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.009	0.010	0.033	0.209	0.234	0.405	0.276	0.013

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	16	5	13	7	13	14
N.S.	1	1.00	1.00	1.45	0.45	1.18	0.64	1.18	1.27
time (sec)	N/A	0.014	0.011	0.033	0.221	0.240	0.599	0.274	0.025

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	9	13	10	18	9
N.S.	1	1.00	1.00	1.07	0.64	0.93	0.71	1.29	0.64
time (sec)	N/A	0.009	0.004	0.038	0.207	0.234	0.622	0.255	0.018

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	17	16	14	0	14	14
N.S.	1	1.00	1.00	1.13	1.07	0.93	0.00	0.93	0.93
time (sec)	N/A	0.013	0.029	0.068	0.228	0.234	0.000	0.265	0.029

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	10	0	10	10
N.S.	1	1.00	1.00	1.08	1.00	0.77	0.00	0.77	0.77
time (sec)	N/A	0.070	0.051	0.052	0.218	0.235	0.000	0.269	0.128

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	22	16	23	0	80	17
N.S.	1	1.00	1.00	1.16	0.84	1.21	0.00	4.21	0.89
time (sec)	N/A	0.023	0.040	0.039	0.226	0.231	0.000	0.263	0.125

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	19	0	16	0
N.S.	1	1.00	1.00	1.06	1.00	1.06	0.00	0.89	0.00
time (sec)	N/A	0.016	0.008	0.035	0.222	0.240	0.000	0.272	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	11	10	9	9	7	9	9
N.S.	1	1.00	0.69	0.62	0.56	0.56	0.44	0.56	0.56
time (sec)	N/A	0.009	0.009	0.018	0.188	0.228	0.031	0.303	0.019

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	16	15	14	14	12	14	14
N.S.	1	1.00	0.62	0.58	0.54	0.54	0.46	0.54	0.54
time (sec)	N/A	0.015	0.011	0.020	0.207	0.233	0.033	0.295	0.029

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	21	20	19	19	17	19	19
N.S.	1	1.00	0.58	0.56	0.53	0.53	0.47	0.53	0.53
time (sec)	N/A	0.024	0.012	0.027	0.203	0.237	0.039	0.304	0.022

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	39	66	181	60	360	77	2034
N.S.	1	1.00	0.81	1.38	3.77	1.25	7.50	1.60	42.38
time (sec)	N/A	0.026	0.150	0.513	0.282	0.268	0.409	0.298	10.335

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	9	3	2	2	3	2
N.S.	1	1.00	1.00	4.50	1.50	1.00	1.00	1.50	1.00
time (sec)	N/A	0.001	0.053	0.014	0.234	0.232	0.184	0.298	0.010

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	17	6	14	7	11	10
N.S.	1	1.00	1.00	1.70	0.60	1.40	0.70	1.10	1.00
time (sec)	N/A	0.003	0.004	0.021	0.232	0.249	0.172	0.288	0.035

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	22	18	24	0	22
N.S.	1	1.00	1.00	0.00	1.00	0.82	1.09	0.00	1.00
time (sec)	N/A	0.012	0.019	0.000	0.071	0.076	0.274	0.000	0.063

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	10	12	11	9	0	9	9
N.S.	1	1.00	0.83	1.00	0.92	0.75	0.00	0.75	0.75
time (sec)	N/A	0.010	0.038	0.043	0.218	0.233	0.000	0.298	0.015

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	20	0	20	0
N.S.	1	1.00	1.00	1.05	0.00	0.91	0.00	0.91	0.00
time (sec)	N/A	0.042	0.120	0.092	0.000	0.250	0.000	0.282	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	17	14	0	6	27	0	155
N.S.	1	1.00	0.17	0.14	0.00	0.06	0.26	0.00	1.50
time (sec)	N/A	0.013	10.020	0.153	0.000	0.069	0.411	0.000	0.281

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [83] had the largest ratio of [2]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	9	0.111
2	A	2	1	1.00	11	0.091
3	A	2	1	1.00	11	0.091
4	A	2	1	1.00	11	0.091
5	A	1	1	1.00	14	0.071
6	A	2	1	1.00	4	0.250
7	A	2	1	1.00	9	0.111
8	A	2	2	1.00	7	0.286
9	A	2	2	1.00	17	0.118
10	A	2	2	1.00	9	0.222
11	A	3	3	1.00	11	0.273
12	A	3	3	1.00	16	0.188
13	A	2	2	1.00	10	0.200
14	A	1	1	1.00	15	0.067
15	A	2	1	1.00	9	0.111
16	A	1	1	1.00	9	0.111
17	A	1	1	1.00	15	0.067
18	A	1	1	1.00	17	0.059
19	A	3	2	1.00	20	0.100
20	A	1	1	1.00	26	0.038
21	A	2	2	1.00	20	0.100
22	A	2	2	1.00	4	0.500
23	A	3	2	1.00	6	0.333
24	A	4	2	1.00	6	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	4	2	1.00	6	0.333
26	A	2	2	1.00	5	0.400
27	A	3	3	1.00	6	0.500
28	A	2	2	1.00	4	0.500
29	A	2	1	1.00	4	0.250
30	A	3	2	1.00	4	0.500
31	A	2	1	1.00	4	0.250
32	A	4	2	1.00	4	0.500
33	A	2	2	1.00	6	0.333
34	A	3	3	1.00	6	0.500
35	A	4	4	1.00	8	0.500
36	A	2	2	1.00	4	0.500
37	A	2	1	1.00	4	0.250
38	A	3	2	1.00	4	0.500
39	A	5	3	1.00	13	0.231
40	A	3	2	1.00	13	0.154
41	A	2	2	1.00	13	0.154
42	A	2	2	1.00	4	0.500
43	A	3	2	1.00	4	0.500
44	A	2	2	1.00	4	0.500
45	A	3	2	1.00	4	0.500
46	A	2	2	1.00	10	0.200
47	A	1	1	1.00	11	0.091
48	A	1	1	1.00	9	0.111
49	A	1	1	1.00	13	0.077
50	A	1	1	1.00	11	0.091
51	A	2	1	1.00	11	0.091
52	A	2	2	1.00	8	0.250
53	A	5	3	1.00	8	0.375
54	A	1	1	1.00	10	0.100
55	A	3	2	1.00	17	0.118
56	A	1	1	1.00	7	0.143
57	A	2	2	1.00	4	0.500
58	A	1	1	1.00	4	0.250
59	A	2	2	1.00	6	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	1	1	1.00	5	0.200
61	A	1	1	1.00	2	0.500
62	A	1	1	1.00	8	0.125
63	A	2	2	1.00	8	0.250
64	A	2	2	1.00	8	0.250
65	A	2	2	1.00	14	0.143
66	A	3	2	1.00	14	0.143
67	A	3	2	1.00	8	0.250
68	A	1	1	1.00	9	0.111
69	A	1	1	1.00	13	0.077
70	A	2	2	1.00	9	0.222
71	A	1	1	1.00	6	0.167
72	A	1	1	1.00	6	0.167
73	A	4	4	1.00	7	0.571
74	A	2	2	1.00	5	0.400
75	A	2	2	1.00	7	0.286
76	A	3	2	1.00	7	0.286
77	A	3	2	1.00	9	0.222
78	A	3	3	1.00	7	0.429
79	A	2	2	1.00	11	0.182
80	A	1	1	1.00	10	0.100
81	A	1	1	1.00	10	0.100
82	A	2	2	1.00	2	1.000
83	A	4	4	1.00	2	2.000
84	A	4	4	1.00	2	2.000
85	A	3	3	1.00	4	0.750
86	A	4	4	1.00	6	0.667
87	A	2	2	1.00	13	0.154
88	A	2	2	1.00	14	0.143
89	A	1	1	1.00	9	0.111
90	A	1	1	1.00	9	0.111
91	A	2	2	1.00	10	0.200
92	A	3	3	1.00	4	0.750
93	A	4	3	1.00	6	0.500
94	A	5	5	1.00	6	0.833

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	4	4	1.00	6	0.667
96	A	1	3	1.00	17	0.176
97	A	2	2	1.00	11	0.182
98	A	1	1	1.00	15	0.067
99	A	1	1	1.00	14	0.071
100	A	2	2	1.00	11	0.182
101	A	2	2	1.00	13	0.154
102	A	5	6	1.00	10	0.600
103	A	3	3	1.00	15	0.200
104	A	4	4	1.00	15	0.267
105	A	3	3	1.00	15	0.200
106	A	3	2	1.00	16	0.125
107	A	3	2	1.00	16	0.125
108	A	6	4	1.00	18	0.222
109	A	3	2	1.00	23	0.087
110	A	2	1	1.00	19	0.053
111	A	5	5	1.00	18	0.278
112	A	6	5	1.00	31	0.161
113	A	2	2	1.00	7	0.286
114	A	3	2	1.00	22	0.091
115	A	2	1	1.00	16	0.062
116	A	2	1	1.00	17	0.059
117	A	2	1	1.00	12	0.083
118	A	3	2	1.00	21	0.095
119	A	2	1	1.00	20	0.050
120	A	3	3	1.00	16	0.188
121	A	4	3	1.00	16	0.188
122	A	2	1	1.00	11	0.091
123	A	3	2	1.00	11	0.182
124	A	2	1	1.00	16	0.062
125	A	2	1	1.00	7	0.143
126	A	5	5	1.00	9	0.556
127	A	4	3	1.00	12	0.250
128	A	3	2	1.00	14	0.143
129	A	4	4	1.00	21	0.190

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	3	2	1.00	18	0.111
131	A	2	2	1.00	7	0.286
132	A	3	3	1.00	11	0.273
133	A	3	2	1.00	16	0.125
134	A	3	2	1.00	11	0.182
135	A	5	4	1.00	18	0.222
136	A	3	3	1.00	7	0.429
137	A	9	6	1.00	7	0.857
138	A	3	3	1.00	14	0.214
139	A	1	1	1.00	16	0.062
140	A	3	3	1.00	12	0.250
141	A	2	2	1.00	8	0.250
142	A	2	2	1.00	8	0.250
143	A	1	1	1.00	10	0.100
144	A	3	3	1.00	13	0.231
145	A	2	1	1.00	19	0.053
146	A	1	1	1.00	11	0.091
147	A	3	3	1.00	11	0.273
148	A	2	2	1.00	11	0.182
149	A	1	1	1.00	13	0.077
150	A	4	4	1.00	15	0.267
151	A	3	3	1.00	13	0.231
152	A	2	2	1.00	9	0.222
153	A	3	3	1.00	12	0.250
154	A	2	2	1.00	9	0.222
155	A	6	6	1.00	18	0.333
156	A	2	2	1.00	8	0.250
157	A	3	3	1.00	5	0.600
158	A	1	1	1.00	7	0.143
159	A	1	1	1.00	9	0.111
160	A	2	2	1.00	7	0.286
161	A	2	2	1.00	5	0.400
162	A	1	1	1.00	14	0.071
163	A	2	2	1.00	14	0.143
164	A	2	2	1.00	9	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	2	3	1.00	8	0.375
166	A	2	2	1.00	7	0.286
167	A	3	2	1.00	9	0.222
168	A	4	2	1.00	9	0.222
169	A	1	1	1.00	21	0.048
170	A	1	1	1.00	4	0.250
171	A	2	2	1.00	4	0.500
172	A	2	2	1.00	8	0.250
173	A	1	1	1.00	11	0.091
174	A	4	2	1.00	16	0.125
175	A	1	1	1.00	9	0.111

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sqrt{1+2x}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \sqrt{1+2x}\,dx = \frac{1}{3}(1+2x)^{3/2}$$

```
[Out] 1/3*(1+2*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$\int \sqrt{1+2x}\,dx = \frac{1}{3}(2x+1)^{3/2}$$

```
[In] Int[Sqrt[1 + 2*x],x]
```

```
[Out] (1 + 2*x)^(3/2)/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{3}(1+2x)^{3/2}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{1+2x}\,dx = \frac{1}{3}(1+2x)^{3/2}$$

```
[In] Integrate[Sqrt[1 + 2*x],x]
[Out] (1 + 2*x)^(3/2)/3
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{(1+2x)^{\frac{3}{2}}}{3}$	10
derivativedivides	$\frac{(1+2x)^{\frac{3}{2}}}{3}$	10
default	$\frac{(1+2x)^{\frac{3}{2}}}{3}$	10
risch	$\frac{(1+2x)^{\frac{3}{2}}}{3}$	10
pseudoelliptic	$\frac{(1+2x)^{\frac{3}{2}}}{3}$	10
trager	$\left(\frac{1}{3}+\frac{2x}{3}\right)\sqrt{1+2x}$	14
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,(2+4x)\sqrt{1+2x}}{3}}{4\sqrt{\pi}}$	29

```
[In] int((1+2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(1+2*x)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{1+2x}\,dx = \frac{1}{3}\,(2\,x+1)^{\frac{3}{2}}$$

```
[In] integrate((1+2*x)^(1/2),x, algorithm="fricas")
[Out] 1/3*(2*x + 1)^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \sqrt{1+2x}\, dx = \frac{(2x+1)^{\frac{3}{2}}}{3}$$

```
[In] integrate((1+2*x)**(1/2),x)
[Out] (2*x + 1)**(3/2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{1+2x}\, dx = \frac{1}{3}\,(2\,x+1)^{\frac{3}{2}}$$

```
[In] integrate((1+2*x)^(1/2),x, algorithm="maxima")
[Out] 1/3*(2*x + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{1+2x}\, dx = \frac{1}{3}\,(2\,x+1)^{\frac{3}{2}}$$

```
[In] integrate((1+2*x)^(1/2),x, algorithm="giac")
[Out] 1/3*(2*x + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{1+2x}\, dx = \frac{(2\,x+1)^{3/2}}{3}$$

```
[In] int((2*x + 1)^(1/2),x)
[Out] (2*x + 1)^(3/2)/3
```

3.2 $\int x\sqrt{1+3x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int x\sqrt{1+3x}\,dx = -\frac{2}{27}(1+3x)^{3/2}+\frac{2}{45}(1+3x)^{5/2}$$

```
[Out] -2/27*(1+3*x)^(3/2)+2/45*(1+3*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\int x\sqrt{1+3x}\,dx = \frac{2}{45}(3x+1)^{5/2}-\frac{2}{27}(3x+1)^{3/2}$$

```
[In] Int[x*Sqrt[1 + 3*x],x]
[Out] (-2*(1 + 3*x)^(3/2))/27 + (2*(1 + 3*x)^(5/2))/45
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int\left(-\frac{1}{3}\sqrt{1+3x}+\frac{1}{3}(1+3x)^{3/2}\right)dx\\ &= -\frac{2}{27}(1+3x)^{3/2}+\frac{2}{45}(1+3x)^{5/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int x\sqrt{1+3x}\,dx = \frac{2}{135}(1+3x)^{3/2}(-2+9x)$$

```
[In] Integrate[x*Sqrt[1 + 3*x],x]
[Out] (2*(1 + 3*x)^(3/2)*(-2 + 9*x))/135
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{2(1+3x)^{\frac{3}{2}}(9x-2)}{135}$	15
trager	$\left(\frac{2}{5}x^2+\frac{2}{45}x-\frac{4}{135}\right)\sqrt{1+3x}$	19
derivativedivides	$-\frac{2(1+3x)^{\frac{3}{2}}}{27}+\frac{2(1+3x)^{\frac{5}{2}}}{45}$	20
default	$-\frac{2(1+3x)^{\frac{3}{2}}}{27}+\frac{2(1+3x)^{\frac{5}{2}}}{45}$	20
risch	$\frac{2(27x^2+3x-2)\sqrt{1+3x}}{135}$	20
pseudoelliptic	$\frac{2(27x^2+3x-2)\sqrt{1+3x}}{135}$	20
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(1+3x)^{\frac{3}{2}}(-9x+2)}{15}}{18\sqrt{\pi}}$	29

```
[In] int(x*(1+3*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/135*(1+3*x)^(3/2)*(9*x-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x\sqrt{1+3x}\,dx = \frac{2}{135}\left(27\,x^2+3\,x-2\right)\sqrt{3\,x+1}$$

```
[In] integrate(x*(1+3*x)^(1/2),x, algorithm="fricas")
[Out] 2/135*(27*x^2 + 3*x - 2)*sqrt(3*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int x\sqrt{1+3x}\,dx = \frac{2x^2\sqrt{3x+1}}{5} + \frac{2x\sqrt{3x+1}}{45} - \frac{4\sqrt{3x+1}}{135}$$

```
[In] integrate(x*(1+3*x)**(1/2),x)
[Out] 2*x**2*sqrt(3*x + 1)/5 + 2*x*sqrt(3*x + 1)/45 - 4*sqrt(3*x + 1)/135
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x\sqrt{1+3x}\,dx = \frac{2}{45}\,(3\,x+1)^{\frac{5}{2}} - \frac{2}{27}\,(3\,x+1)^{\frac{3}{2}}$$

```
[In] integrate(x*(1+3*x)^(1/2),x, algorithm="maxima")
[Out] 2/45*(3*x + 1)^(5/2) - 2/27*(3*x + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x\sqrt{1+3x}\,dx = \frac{2}{45}\,(3\,x+1)^{\frac{5}{2}} - \frac{2}{27}\,(3\,x+1)^{\frac{3}{2}}$$

```
[In] integrate(x*(1+3*x)^(1/2),x, algorithm="giac")
[Out] 2/45*(3*x + 1)^(5/2) - 2/27*(3*x + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int x\sqrt{1+3x}\,dx = \frac{2\,(3\,x+1)^{3/2}\,(9\,x-2)}{135}$$

```
[In] int(x*(3*x + 1)^(1/2),x)
[Out] (2*(3*x + 1)^(3/2)*(9*x - 2))/135
```

3.3 $\int x^2\sqrt{1+x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x^2\sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2} - \frac{4}{5}(1+x)^{5/2} + \frac{2}{7}(1+x)^{7/2}$$

```
[Out] 2/3*(1+x)^(3/2)-4/5*(1+x)^(5/2)+2/7*(1+x)^(7/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\int x^2\sqrt{1+x}\,dx = \frac{2}{7}(x+1)^{7/2} - \frac{4}{5}(x+1)^{5/2} + \frac{2}{3}(x+1)^{3/2}$$

```
[In] Int[x^2*Sqrt[1 + x],x]
[Out] (2*(1 + x)^(3/2))/3 - (4*(1 + x)^(5/2))/5 + (2*(1 + x)^(7/2))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\sqrt{1+x} - 2(1+x)^{3/2} + (1+x)^{5/2}\right) dx \\ &= \frac{2}{3}(1+x)^{3/2} - \frac{4}{5}(1+x)^{5/2} + \frac{2}{7}(1+x)^{7/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

$$\int x^2\sqrt{1+x}\,dx = \frac{2}{105}(1+x)^{3/2}\left(8-12x+15x^2\right)$$

```
[In] Integrate[x^2*Sqrt[1 + x],x]
[Out] (2*(1 + x)^(3/2)*(8 - 12*x + 15*x^2))/105
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.53

method	result	size
gosper	$\frac{2(1+x)^{\frac{3}{2}}(15x^2-12x+8)}{105}$	18
trager	$\left(\frac{2}{7}x^3+\frac{2}{35}x^2-\frac{8}{105}x+\frac{16}{105}\right)\sqrt{1+x}$	22
derivativedivides	$\frac{2(1+x)^{\frac{3}{2}}}{3}-\frac{4(1+x)^{\frac{5}{2}}}{5}+\frac{2(1+x)^{\frac{7}{2}}}{7}$	23
default	$\frac{2(1+x)^{\frac{3}{2}}}{3}-\frac{4(1+x)^{\frac{5}{2}}}{5}+\frac{2(1+x)^{\frac{7}{2}}}{7}$	23
risch	$\frac{2\left(15x^3+3x^2-4x+8\right)\sqrt{1+x}}{105}$	23
meijerg	$-\frac{\frac{32\sqrt{\pi}}{105}-\frac{4\sqrt{\pi}\,(1+x)^{\frac{3}{2}}\left(15x^2-12x+8\right)}{105}}{2\sqrt{\pi}}$	32

```
[In] int(x^2*(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/105*(1+x)^(3/2)*(15*x^2-12*x+8)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int x^2\sqrt{1+x}\,dx = \frac{2}{105}\left(15\,x^3+3\,x^2-4\,x+8\right)\sqrt{x+1}$$

```
[In] integrate(x^2*(1+x)^(1/2),x, algorithm="fricas")
[Out] 2/105*(15*x^3 + 3*x^2 - 4*x + 8)*sqrt(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int x^2\sqrt{1+x}\,dx = \frac{2x^3\sqrt{x+1}}{7} + \frac{2x^2\sqrt{x+1}}{35} - \frac{8x\sqrt{x+1}}{105} + \frac{16\sqrt{x+1}}{105}$$

```
[In] integrate(x**2*(1+x)**(1/2),x)
[Out] 2*x**3*sqrt(x + 1)/7 + 2*x**2*sqrt(x + 1)/35 - 8*x*sqrt(x + 1)/105 + 16*sqr
t(x + 1)/105
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int x^2\sqrt{1+x}\,dx = \frac{2}{7}\,(x+1)^{\frac{7}{2}} - \frac{4}{5}\,(x+1)^{\frac{5}{2}} + \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

```
[In] integrate(x^2*(1+x)^(1/2),x, algorithm="maxima")
[Out] 2/7*(x + 1)^(7/2) - 4/5*(x + 1)^(5/2) + 2/3*(x + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int x^2\sqrt{1+x}\,dx = \frac{2}{7}\,(x+1)^{\frac{7}{2}} - \frac{4}{5}\,(x+1)^{\frac{5}{2}} + \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

```
[In] integrate(x^2*(1+x)^(1/2),x, algorithm="giac")
[Out] 2/7*(x + 1)^(7/2) - 4/5*(x + 1)^(5/2) + 2/3*(x + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.56

$$\int x^2\sqrt{1+x}\,dx = -\frac{2\,{\left(x+1\right)}^{3/2}\,\left(42\,x-15\,{\left(x+1\right)}^2+7\right)}{105}$$

```
[In] int(x^2*(x + 1)^(1/2),x)
[Out] -(2*(x + 1)^(3/2)*(42*x - 15*(x + 1)^2 + 7))/105
```

3.4 $\int \frac{x}{\sqrt{2-3x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \frac{x}{\sqrt{2-3x}}\,dx = -\frac{4}{9}\sqrt{2-3x}+\frac{2}{27}(2-3x)^{3/2}$$

```
[Out] 2/27*(2-3*x)^(3/2)-4/9*(2-3*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{45\}$

$$\int \frac{x}{\sqrt{2-3x}}\,dx = \frac{2}{27}(2-3x)^{3/2}-\frac{4}{9}\sqrt{2-3x}$$

```
[In] Int[x/Sqrt[2 - 3*x],x]
[Out] (-4*Sqrt[2 - 3*x])/9 + (2*(2 - 3*x)^(3/2))/27
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{2}{3\sqrt{2-3x}}-\frac{1}{3}\sqrt{2-3x}\right)dx \\ &= -\frac{4}{9}\sqrt{2-3x}+\frac{2}{27}(2-3x)^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{2-3x}}\,dx = -\frac{2}{27}\sqrt{2-3x}(4+3x)$$

```
[In] Integrate[x/Sqrt[2 - 3*x],x]
[Out] (-2*Sqrt[2 - 3*x]*(4 + 3*x))/27
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

method	result	size
trager	$\left(-\frac{2x}{9}-\frac{8}{27}\right)\sqrt{2-3x}$	14
gosper	$-\frac{2(3x+4)\sqrt{2-3x}}{27}$	15
pseudoelliptic	$-\frac{2(3x+4)\sqrt{2-3x}}{27}$	15
derivativedivides	$\frac{2(2-3x)^{\frac{3}{2}}}{27}-\frac{4\sqrt{2-3x}}{9}$	20
default	$\frac{2(2-3x)^{\frac{3}{2}}}{27}-\frac{4\sqrt{2-3x}}{9}$	20
risch	$\frac{2(-2+3x)(3x+4)}{27\sqrt{2-3x}}$	20
meijerg	$\frac{2\sqrt{2}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,(6x+8)\sqrt{1-\frac{3x}{2}}}{6}\right)}{9\sqrt{\pi}}$	32

```
[In] int(x/(2-3*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-2/9*x-8/27)*(2-3*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{x}{\sqrt{2-3x}}\,dx = -\frac{2}{27}\,(3\,x+4)\sqrt{-3\,x+2}$$

```
[In] integrate(x/(2-3*x)^(1/2),x, algorithm="fricas")
[Out] -2/27*(3*x + 4)*sqrt(-3*x + 2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \frac{x}{\sqrt{2-3x}}\,dx = \begin{cases} -\frac{2ix\sqrt{3x-2}}{9} - \frac{8i\sqrt{3x-2}}{27} & \text{for } |x| > \frac{2}{3} \\ -\frac{2x\sqrt{2-3x}}{9} - \frac{8\sqrt{2-3x}}{27} & \text{otherwise} \end{cases}$$

```
[In] integrate(x/(2-3*x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*x*sqrt(3*x - 2)/9 - 8*I*sqrt(3*x - 2)/27, Abs(x) > 2/3), (-
2*x*sqrt(2 - 3*x)/9 - 8*sqrt(2 - 3*x)/27, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x}{\sqrt{2-3x}}\,dx = \frac{2}{27}\,(-3\,x+2)^{\frac{3}{2}} - \frac{4}{9}\,\sqrt{-3\,x+2}$$

```
[In] integrate(x/(2-3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/27*(-3*x + 2)^(3/2) - 4/9*sqrt(-3*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x}{\sqrt{2-3x}}\,dx = \frac{2}{27}\,(-3\,x+2)^{\frac{3}{2}} - \frac{4}{9}\,\sqrt{-3\,x+2}$$

```
[In] integrate(x/(2-3*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/27*(-3*x + 2)^(3/2) - 4/9*sqrt(-3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{x}{\sqrt{2-3x}}\,dx = -\frac{2\sqrt{2-3\,x}\,(3\,x+4)}{27}$$

```
[In] int(x/(2 - 3*x)^(1/2),x)
[Out] -(2*(2 - 3*x)^(1/2)*(3*x + 4))/27
```

3.5 $\int \frac{1+x}{(2+2x+x^2)^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1+x}{(2+2x+x^2)^3} dx = -\frac{1}{4(2+2x+x^2)^2}$$

```
[Out] -1/4/(x^2+2*x+2)^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{643\}$

$$\int \frac{1+x}{(2+2x+x^2)^3} dx = -\frac{1}{4(x^2+2x+2)^2}$$

```
[In] Int[(1 + x)/(2 + 2*x + x^2)^3,x]
```

```
[Out] -1/4*1/(2 + 2*x + x^2)^2
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{4(2+2x+x^2)^2}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{(2+2x+x^2)^3}\,dx = -\frac{1}{4\,(2+2x+x^2)^2}$$

```
[In] Integrate[(1 + x)/(2 + 2*x + x^2)^3,x]
[Out] -1/4*1/(2 + 2*x + x^2)^2
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{1}{4(x^2+2x+2)^2}$	13
default	$-\frac{1}{4(x^2+2x+2)^2}$	13
norman	$-\frac{1}{4(x^2+2x+2)^2}$	13
risch	$-\frac{1}{4(x^2+2x+2)^2}$	13
parallelrisch	$-\frac{1}{4(x^2+2x+2)^2}$	13

```
[In] int((1+x)/(x^2+2*x+2)^3,x,method=_RETURNVERBOSE)
[Out] -1/4/(x^2+2*x+2)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{1+x}{(2+2x+x^2)^3}\,dx = -\frac{1}{4\,(x^4+4\,x^3+8\,x^2+8\,x+4)}$$

```
[In] integrate((1+x)/(x^2+2*x+2)^3,x, algorithm="fricas")
[Out] -1/4/(x^4 + 4*x^3 + 8*x^2 + 8*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{1+x}{(2+2x+x^2)^3}\,dx = -\frac{1}{4x^4+16x^3+32x^2+32x+16}$$

```
[In] integrate((1+x)/(x**2+2*x+2)**3,x)
[Out] -1/(4*x**4 + 16*x**3 + 32*x**2 + 32*x + 16)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x}{(2+2x+x^2)^3}\,dx = -\frac{1}{4\,(x^2+2\,x+2)^2}$$

```
[In] integrate((1+x)/(x^2+2*x+2)^3,x, algorithm="maxima")
[Out] -1/4/(x^2 + 2*x + 2)^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x}{(2+2x+x^2)^3}\,dx = -\frac{1}{4\,(x^2+2\,x+2)^2}$$

```
[In] integrate((1+x)/(x^2+2*x+2)^3,x, algorithm="giac")
[Out] -1/4/(x^2 + 2*x + 2)^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x}{(2+2x+x^2)^3}\,dx = -\frac{1}{4\,(x^2+2\,x+2)^2}$$

```
[In] int((x + 1)/(2*x + x^2 + 2)^3,x)
[Out] -1/(4*(2*x + x^2 + 2)^2)
```

3.6 $\int \sin^3(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 13

$$\int \sin^3(x)\,dx = -\cos(x) + \frac{\cos^3(x)}{3}$$

```
[Out] -cos(x)+1/3*cos(x)^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2713}

$$\int \sin^3(x)\,dx = \frac{\cos^3(x)}{3} - \cos(x)$$

```
[In] Int[Sin[x]^3,x]
```

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(x)\right)\\ &= -\cos(x) + \frac{\cos^3(x)}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sin^3(x)\,dx = -\frac{3\cos(x)}{4} + \frac{1}{12}\cos(3x)$$

```
[In] Integrate[Sin[x]^3,x]
[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{(2+\sin^2(x))\cos(x)}{3}$	11
risch	$-\frac{3\cos(x)}{4} + \frac{\cos(3x)}{12}$	12
parallelrisch	$-\frac{2}{3} - \frac{3\cos(x)}{4} + \frac{\cos(3x)}{12}$	13
norman	$\frac{-4(\tan^2(\frac{x}{2}))-\frac{4}{3}}{(1+\tan^2(\frac{x}{2}))^3}$	22

```
[In] int(sin(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/3*(2+sin(x)^2)*cos(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos(x)^3 - \cos(x)$$

```
[In] integrate(sin(x)^3,x, algorithm="fricas")
[Out] 1/3*cos(x)^3 - cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \sin^3(x)\,dx = \frac{\cos^3(x)}{3} - \cos(x)$$

```
[In] integrate(sin(x)**3,x)
[Out] cos(x)**3/3 - cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos(x)^3 - \cos(x)$$

```
[In] integrate(sin(x)^3,x, algorithm="maxima")
[Out] 1/3*cos(x)^3 - cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos(x)^3 - \cos(x)$$

```
[In] integrate(sin(x)^3,x, algorithm="giac")
[Out] 1/3*cos(x)^3 - cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \sin^3(x)\,dx = \frac{\cos(x)\,\left(\cos(x)^2 - 3\right)}{3}$$

```
[In] int(sin(x)^3,x)
[Out] (cos(x)*(cos(x)^2 - 3))/3
```

3.7 $\int \sqrt[3]{-1+z}z\,dz$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int \sqrt[3]{-1+z}z\,dz = \frac{3}{4}(-1+z)^{4/3}+\frac{3}{7}(-1+z)^{7/3}$$

```
[Out] 3/4*(-1+z)^(4/3)+3/7*(-1+z)^(7/3)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\int \sqrt[3]{-1+z}z\,dz = \frac{3}{7}(z-1)^{7/3}+\frac{3}{4}(z-1)^{4/3}$$

```
[In] Int[(-1 + z)^(1/3)*z,z]
[Out] (3*(-1 + z)^(4/3))/4 + (3*(-1 + z)^(7/3))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\sqrt[3]{-1+z}+(-1+z)^{4/3}\right)\,dz \\ &= \frac{3}{4}(-1+z)^{4/3}+\frac{3}{7}(-1+z)^{7/3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \sqrt[3]{-1+z}z\,dz = \frac{3}{28}(7+4(-1+z))(-1+z)^{4/3}$$

```
[In] Integrate[(-1 + z)^(1/3)*z,z]
[Out] (3*(7 + 4*(-1 + z))*(-1 + z)^(4/3))/28
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{3(-1+z)^{\frac{4}{3}}(4z+3)}{28}$	13
derivativedivides	$\frac{3(-1+z)^{\frac{4}{3}}}{4}+\frac{3(-1+z)^{\frac{7}{3}}}{7}$	16
default	$\frac{3(-1+z)^{\frac{4}{3}}}{4}+\frac{3(-1+z)^{\frac{7}{3}}}{7}$	16
trager	$\left(\frac{3}{7}z^2-\frac{3}{28}z-\frac{9}{28}\right)(-1+z)^{\frac{1}{3}}$	17
risch	$\frac{3(-1+z)^{\frac{1}{3}}(4z^2-z-3)}{28}$	18
meijerg	$\frac{\operatorname{signum}(-1+z)^{\frac{1}{3}}z^2{}_2F_1\left(-\frac{1}{3},2;3;z\right)}{2(-\operatorname{signum}(-1+z))^{\frac{1}{3}}}$	27

```
[In] int((-1+z)^(1/3)*z,z,method=_RETURNVERBOSE)
[Out] 3/28*(-1+z)^(4/3)*(4*z+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt[3]{-1+z}z\,dz = \frac{3}{28}\left(4z^2-z-3\right)(z-1)^{\frac{1}{3}}$$

```
[In] integrate((-1+z)^(1/3)*z,z, algorithm="fricas")
[Out] 3/28*(4*z^2 - z - 3)*(z - 1)^(1/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.00

$$\int \sqrt[3]{-1+z}z\,dz = \begin{cases} \frac{3z^2\sqrt[3]{z-1}}{7} - \frac{3z\sqrt[3]{z-1}}{28} - \frac{9\sqrt[3]{z-1}}{28} & \text{for } |z| > 1 \\ \frac{3z^2\sqrt[3]{1-z}e^{\frac{i\pi}{3}}}{7} - \frac{3z\sqrt[3]{1-z}e^{\frac{i\pi}{3}}}{28} - \frac{9\sqrt[3]{1-z}e^{\frac{i\pi}{3}}}{28} & \text{otherwise} \end{cases}$$

```
[In] integrate((-1+z)**(1/3)*z,z)
```

```
[Out] Piecewise((3*z**2*(z - 1)**(1/3)/7 - 3*z*(z - 1)**(1/3)/28 - 9*(z - 1)**(1/
3)/28, Abs(z) > 1), (3*z**2*(1 - z)**(1/3)*exp(I*pi/3)/7 - 3*z*(1 - z)**(1/
3)*exp(I*pi/3)/28 - 9*(1 - z)**(1/3)*exp(I*pi/3)/28, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \sqrt[3]{-1+z}z\,dz = \frac{3}{7}(z-1)^{\frac{7}{3}} + \frac{3}{4}(z-1)^{\frac{4}{3}}$$

```
[In] integrate((-1+z)^(1/3)*z,z, algorithm="maxima")
```

```
[Out] 3/7*(z - 1)^(7/3) + 3/4*(z - 1)^(4/3)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \sqrt[3]{-1+z}z\,dz = \frac{3}{7}(z-1)^{\frac{7}{3}} + \frac{3}{4}(z-1)^{\frac{4}{3}}$$

```
[In] integrate((-1+z)^(1/3)*z,z, algorithm="giac")
```

```
[Out] 3/7*(z - 1)^(7/3) + 3/4*(z - 1)^(4/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \sqrt[3]{-1+z}z\,dz=\frac{3\,\left(4\,z+3\right)\,{\left(z-1\right)}^{4/3}}{28}$$

```
[In] int(z*(z - 1)^(1/3),z)
[Out] (3*(4*z + 3)*(z - 1)^(4/3))/28
```

3.8 $\int \cot(x)\csc^2(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \cot(x)\csc^2(x)\,dx = -\frac{1}{2}\csc^2(x)$$

```
[Out] -1/2*csc(x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2686, 30}

$$\int \cot(x)\csc^2(x)\,dx = -\frac{1}{2}\csc^2(x)$$

```
[In] Int[Cot[x]*Csc[x]^2,x]
```

```
[Out] -1/2*Csc[x]^2
```

<u>Rule 30</u>

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

<u>Rule 2686</u>

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int x\,dx, x, \csc(x)\right)$$
$$= -\frac{1}{2}\csc^2(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cot(x)\csc^2(x)\,dx = -\frac{1}{2}\csc^2(x)$$

```
[In] Integrate[Cot[x]*Csc[x]^2,x]
[Out] -1/2*Csc[x]^2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{1}{2\sin(x)^2}$	7
default	$-\frac{1}{2\sin(x)^2}$	7
risch	$\frac{2\,e^{2ix}}{(e^{2ix}-1)^2}$	17
norman	$\frac{-\frac{1}{8}-\frac{\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}}{\tan\left(\frac{x}{2}\right)^2}$	18
parallelrisch	$\frac{-1-\left(\tan^4\left(\frac{x}{2}\right)\right)}{8\tan\left(\frac{x}{2}\right)^2}$	19

```
[In] int(cos(x)/sin(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/sin(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \cot(x)\csc^2(x)\,dx = \frac{1}{2\left(\cos\left(x\right)^2 - 1\right)}$$

```
[In] integrate(cos(x)/sin(x)^3,x, algorithm="fricas")
[Out] 1/2/(cos(x)^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cot(x)\csc^2(x)\,dx = -\frac{1}{2\sin^2\left(x\right)}$$

```
[In] integrate(cos(x)/sin(x)**3,x)
[Out] -1/(2*sin(x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cot(x)\csc^2(x)\,dx = -\frac{1}{2\,\sin\left(x\right)^2}$$

```
[In] integrate(cos(x)/sin(x)^3,x, algorithm="maxima")
[Out] -1/2/sin(x)^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cot(x)\csc^2(x)\,dx = -\frac{1}{2\,\sin\left(x\right)^2}$$

```
[In] integrate(cos(x)/sin(x)^3,x, algorithm="giac")
[Out] -1/2/sin(x)^2
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cot(x)\csc^2(x)\,dx = -\frac{\cot(x)^2}{2}$$

```
[In] int(cos(x)/sin(x)^3,x)
[Out] -cot(x)^2/2
```

3.9 $\int \cos(2x)\sqrt{4-\sin(2x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \cos(2x)\sqrt{4-\sin(2x)}\,dx = -\frac{1}{3}(4-\sin(2x))^{3/2}$$

```
[Out] -1/3*(4-sin(2*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2747, 32}

$$\int \cos(2x)\sqrt{4-\sin(2x)}\,dx = -\frac{1}{3}(4-\sin(2x))^{3/2}$$

```
[In] Int[Cos[2*x]*Sqrt[4 - Sin[2*x]],x]
```

```
[Out] -1/3*(4 - Sin[2*x])^(3/2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int \sqrt{4+x}\,dx, x, -\sin(2x)\right)\right)$$
$$= -\frac{1}{3}(4-\sin(2x))^{3/2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \cos(2x)\sqrt{4-\sin(2x)}\,dx = -\frac{1}{3}(4-\sin(2x))^{3/2}$$

```
[In] Integrate[Cos[2*x]*Sqrt[4 - Sin[2*x]],x]
[Out] -1/3*(4 - Sin[2*x])^(3/2)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{(4-\sin(2x))^{\frac{3}{2}}}{3}$	13
default	$-\frac{(4-\sin(2x))^{\frac{3}{2}}}{3}$	13

```
[In] int(cos(2*x)*(4-sin(2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(4-sin(2*x))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \cos(2x)\sqrt{4-\sin(2x)}\,dx = \frac{1}{3}\left(\sin\left(2\,x\right)-4\right)\sqrt{-\sin\left(2\,x\right)+4}$$

```
[In] integrate(cos(2*x)*(4-sin(2*x))^(1/2),x, algorithm="fricas")
[Out] 1/3*(sin(2*x) - 4)*sqrt(-sin(2*x) + 4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \cos(2x)\sqrt{4-\sin(2x)}\,dx = \frac{\sqrt{4-\sin{(2x)}}\sin{(2x)}}{3} - \frac{4\sqrt{4-\sin{(2x)}}}{3}$$

```
[In] integrate(cos(2*x)*(4-sin(2*x))**(1/2),x)
[Out] sqrt(4 - sin(2*x))*sin(2*x)/3 - 4*sqrt(4 - sin(2*x))/3
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \cos(2x)\sqrt{4-\sin(2x)}\,dx = -\frac{1}{3}\left(-\sin\left(2\,x\right)+4\right)^{\frac{3}{2}}$$

```
[In] integrate(cos(2*x)*(4-sin(2*x))^(1/2),x, algorithm="maxima")
[Out] -1/3*(-sin(2*x) + 4)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \cos(2x)\sqrt{4-\sin(2x)}\,dx = -\frac{1}{3}\left(-\sin\left(2\,x\right)+4\right)^{\frac{3}{2}}$$

```
[In] integrate(cos(2*x)*(4-sin(2*x))^(1/2),x, algorithm="giac")
[Out] -1/3*(-sin(2*x) + 4)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \cos(2x)\sqrt{4-\sin(2x)}\,dx=-\frac{(4-\sin(2\,x))^{3/2}}{3}$$

```
[In] int(cos(2*x)*(4 - sin(2*x))^(1/2),x)
[Out] -(4 - sin(2*x))^(3/2)/3
```

3.10 $\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 6

$$\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx = \frac{1}{3+\cos(x)}$$

```
[Out] 1/(3+cos(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2747, 32}

$$\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx = \frac{1}{\cos(x)+3}$$

```
[In] Int[Sin[x]/(3 + Cos[x])^2,x]
```

```
[Out] (3 + Cos[x])^(-1)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1}{(3+x)^2}\,dx, x, \cos(x)\right)\\ &= \frac{1}{3+\cos(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx = \frac{1}{3+\cos(x)}$$

```
[In] Integrate[Sin[x]/(3 + Cos[x])^2,x]
[Out] (3 + Cos[x])^(-1)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{1}{3+\cos(x)}$	7
default	$\frac{1}{3+\cos(x)}$	7
parallelrisch	$-\frac{1}{2\left(\tan^2\left(\frac{x}{2}\right)\right)+4}$	15
risch	$\frac{2\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}+6\,\mathrm{e}^{ix}+1}$	24
norman	$\frac{-\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{1}{2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(\tan^2\left(\frac{x}{2}\right)+2\right)}$	32

```
[In] int(sin(x)/(3+cos(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/(3+cos(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx = \frac{1}{\cos(x)+3}$$

```
[In] integrate(sin(x)/(3+cos(x))^2,x, algorithm="fricas")
[Out] 1/(cos(x) + 3)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx = \frac{1}{\cos(x)+3}$$

```
[In] integrate(sin(x)/(3+cos(x))**2,x)
[Out] 1/(cos(x) + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx = \frac{1}{\cos(x)+3}$$

```
[In] integrate(sin(x)/(3+cos(x))^2,x, algorithm="maxima")
[Out] 1/(cos(x) + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx = \frac{1}{\cos(x)+3}$$

```
[In] integrate(sin(x)/(3+cos(x))^2,x, algorithm="giac")
[Out] 1/(cos(x) + 3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx = \frac{1}{\cos(x)+3}$$

```
[In] int(sin(x)/(cos(x) + 3)^2,x)
[Out] 1/(cos(x) + 3)
```

3.11 $\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx = \frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

```
[Out] 2*cos(x)/(cos(x)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3286, 2645, 30}

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx = \frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

```
[In] Int[Sin[x]/Sqrt[Cos[x]^3],x]
```

```
[Out] (2*Cos[x])/Sqrt[Cos[x]^3]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cos^{\frac{3}{2}}(x)\int \frac{\sin(x)}{\cos^{\frac{3}{2}}(x)}\,dx}{\sqrt{\cos^3(x)}} \\
&= -\frac{\cos^{\frac{3}{2}}(x)\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, \cos(x)\right)}{\sqrt{\cos^3(x)}} \\
&= \frac{2\cos(x)}{\sqrt{\cos^3(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx = \frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

```
[In] Integrate[Sin[x]/Sqrt[Cos[x]^3],x]
[Out] (2*Cos[x])/Sqrt[Cos[x]^3]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{4\cos(x)}{\sqrt{\cos(3x)+3\cos(x)}}$	11
default	$\frac{4\cos(x)}{\sqrt{\cos(3x)+3\cos(x)}}$	11

```
[In] int(sin(x)/(cos(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*cos(x)/(cos(x)^3)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx = \frac{2\sqrt{\cos\left(x\right)^3}}{\cos\left(x\right)^2}$$

```
[In] integrate(sin(x)/(cos(x)^3)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(cos(x)^3)/cos(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx = \frac{2\cos\left(x\right)}{\sqrt{\cos^3\left(x\right)}}$$

```
[In] integrate(sin(x)/(cos(x)**3)**(1/2),x)
[Out] 2*cos(x)/sqrt(cos(x)**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx = \frac{2\,\cos\left(x\right)}{\sqrt{\cos\left(x\right)^3}}$$

```
[In] integrate(sin(x)/(cos(x)^3)^(1/2),x, algorithm="maxima")
[Out] 2*cos(x)/sqrt(cos(x)^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx = \frac{2}{\sqrt{\cos{(x)}}}$$

```
[In] integrate(sin(x)/(cos(x)^3)^(1/2),x, algorithm="giac")
[Out] 2/sqrt(cos(x))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx = \frac{2\,|\cos{(x)}|}{\cos{(x)}^{3/2}}$$

```
[In] int(sin(x)/(cos(x)^3)^(1/2),x)
[Out] (2*abs(cos(x)))/cos(x)^(3/2)
```

3.12 $\int \frac{\sin(\sqrt{1+x})}{\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 10

$$\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx = -2\cos\left(\sqrt{1+x}\right)$$

```
[Out] -2*cos((1+x)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3512, 15, 2718}

$$\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx = -2\cos\left(\sqrt{x+1}\right)$$

```
[In] Int[Sin[Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] -2*Cos[Sqrt[1 + x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{x\sin(x)}{\sqrt{x^2}}\,dx, x, \sqrt{1+x}\right)\\ &= 2\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{1+x}\right)\\ &= -2\cos\left(\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx = -2\cos\left(\sqrt{1+x}\right)$$

```
[In] Integrate[Sin[Sqrt[1 + x]]/Sqrt[1 + x],x]
[Out] -2*Cos[Sqrt[1 + x]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-2\cos\left(\sqrt{1+x}\right)$	9
default	$-2\cos\left(\sqrt{1+x}\right)$	9

```
[In] int(sin((1+x)^(1/2))/(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*cos((1+x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx = -2\,\cos\left(\sqrt{x+1}\right)$$

```
[In] integrate(sin((1+x)^(1/2))/(1+x)^(1/2),x, algorithm="fricas")
[Out] -2*cos(sqrt(x + 1))
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx = -2\cos\left(\sqrt{x+1}\right)$$

```
[In] integrate(sin((1+x)**(1/2))/(1+x)**(1/2),x)
[Out] -2*cos(sqrt(x + 1))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx = -2\,\cos\left(\sqrt{x+1}\right)$$

```
[In] integrate(sin((1+x)^(1/2))/(1+x)^(1/2),x, algorithm="maxima")
[Out] -2*cos(sqrt(x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx = -2\,\cos\left(\sqrt{x+1}\right)$$

```
[In] integrate(sin((1+x)^(1/2))/(1+x)^(1/2),x, algorithm="giac")
[Out] -2*cos(sqrt(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin \left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx = -2\,\cos\left(\sqrt{x+1}\right)$$

```
[In] int(sin((x + 1)^(1/2))/(x + 1)^(1/2),x)
[Out] -2*cos((x + 1)^(1/2))
```

3.13 $\int x^{-1+n} \sin(x^n)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 9

$$\int x^{-1+n} \sin(x^n)\, dx = -\frac{\cos(x^n)}{n}$$

```
[Out] -cos(x^n)/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3460, 2718}

$$\int x^{-1+n} \sin(x^n)\, dx = -\frac{\cos(x^n)}{n}$$

```
[In] Int[x^(-1 + n)*Sin[x^n],x]
```

```
[Out] -(Cos[x^n]/n)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sin(x)\,dx, x, x^n\right)}{n} \\
&= -\frac{\cos\left(x^n\right)}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x^{-1+n}\sin\left(x^n\right)\,dx = -\frac{\cos\left(x^n\right)}{n}$$

```
[In] Integrate[x^(-1 + n)*Sin[x^n],x]
[Out] -(Cos[x^n]/n)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$-\frac{\cos(x^n)}{n}$	10
risch	$-\frac{\cos(x^n)}{n}$	10
norman	$\frac{2\left(\tan^2\left(\frac{e^{n\ln(x)}}{2}\right)\right)}{n\left(1+\tan^2\left(\frac{e^{n\ln(x)}}{2}\right)\right)}$	30
meijerg	$\frac{\sqrt{\pi}\left(\frac{2^{1-\frac{-1+n}{n}-\frac{1}{n}}(-1)^{\frac{1}{2}-\frac{-1+n}{2n}-\frac{1}{2n}}}{\sqrt{\pi}\,\Gamma\left(3-\frac{-1+n}{n}-\frac{1}{n}\right)}-\frac{(-1)^{\frac{1}{2}-\frac{-1+n}{2n}-\frac{1}{2n}}2^{1-\frac{-1+n}{n}-\frac{1}{n}}\cos(x^n)}{\sqrt{\pi}\,\Gamma\left(3-\frac{-1+n}{n}-\frac{1}{n}\right)}\right)}{n}$	126

```
[In] int(x^(-1+n)*sin(x^n),x,method=_RETURNVERBOSE)
[Out] -cos(x^n)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x^{-1+n} \sin (x^n) \, dx = -\frac{\cos (x^n)}{n}$$

```
[In] integrate(x^(-1+n)*sin(x^n),x, algorithm="fricas")
[Out] -cos(x^n)/n
```

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int x^{-1+n} \sin (x^n) \, dx = -\frac{\cos (x^n)}{n}$$

```
[In] integrate(x**(-1+n)*sin(x**n),x)
[Out] -cos(x**n)/n
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x^{-1+n} \sin (x^n) \, dx = -\frac{\cos (x^n)}{n}$$

```
[In] integrate(x^(-1+n)*sin(x^n),x, algorithm="maxima")
[Out] -cos(x^n)/n
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x^{-1+n} \sin (x^n) \, dx = -\frac{\cos (x^n)}{n}$$

```
[In] integrate(x^(-1+n)*sin(x^n),x, algorithm="giac")
[Out] -cos(x^n)/n
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x^{-1+n}\sin\left(x^n\right)\,dx=-\frac{\cos\left(x^n\right)}{n}$$

```
[In] int(x^(n - 1)*sin(x^n),x)
[Out] -cos(x^n)/n
```

3.14 $\int \frac{x^5}{\sqrt{1-x^6}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{x^5}{\sqrt{1-x^6}} \, dx = -\frac{1}{3}\sqrt{1-x^6}$$

```
[Out] -1/3*(-x^6+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {267}

$$\int \frac{x^5}{\sqrt{1-x^6}} \, dx = -\frac{1}{3}\sqrt{1-x^6}$$

```
[In] Int[x^5/Sqrt[1 - x^6],x]
```

```
[Out] -1/3*Sqrt[1 - x^6]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{3}\sqrt{1-x^6}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\sqrt{1-x^6}}\,dx = -\frac{1}{3}\sqrt{1-x^6}$$

```
[In] Integrate[x^5/Sqrt[1 - x^6],x]
[Out] -1/3*Sqrt[1 - x^6]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\sqrt{-x^6+1}}{3}$	12
default	$-\frac{\sqrt{-x^6+1}}{3}$	12
trager	$-\frac{\sqrt{-x^6+1}}{3}$	12
pseudoelliptic	$-\frac{\sqrt{-x^6+1}}{3}$	12
risch	$\frac{x^6-1}{3\sqrt{-x^6+1}}$	17
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^6+1}}{6\sqrt{\pi}}$	26
gosper	$\frac{(-1+x)(1+x)\left(x^2+x+1\right)\left(x^2-x+1\right)}{3\sqrt{-x^6+1}}$	32

```
[In] int(x^5/(-x^6+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(-x^6+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{\sqrt{1-x^6}}\,dx = -\frac{1}{3}\,\sqrt{-x^6+1}$$

```
[In] integrate(x^5/(-x^6+1)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(-x^6 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x^5}{\sqrt{1-x^6}}\,dx = -\frac{\sqrt{1-x^6}}{3}$$

```
[In] integrate(x**5/(-x**6+1)**(1/2),x)
[Out] -sqrt(1 - x**6)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{\sqrt{1-x^6}}\,dx = -\frac{1}{3}\sqrt{-x^6+1}$$

```
[In] integrate(x^5/(-x^6+1)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(-x^6 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{\sqrt{1-x^6}}\,dx = -\frac{1}{3}\sqrt{-x^6+1}$$

```
[In] integrate(x^5/(-x^6+1)^(1/2),x, algorithm="giac")
[Out] -1/3*sqrt(-x^6 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{\sqrt{1-x^6}}\,dx = -\frac{\sqrt{1-x^6}}{3}$$

```
[In] int(x^5/(1 - x^6)^(1/2),x)
[Out] -(1 - x^6)^(1/2)/3
```

3.15 $\int t\sqrt[4]{1+t}\,dt$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int t\sqrt[4]{1+t}\,dt=-\frac{4}{5}(1+t)^{5/4}+\frac{4}{9}(1+t)^{9/4}$$

```
[Out] -4/5*(1+t)^(5/4)+4/9*(1+t)^(9/4)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\int t\sqrt[4]{1+t}\,dt=\frac{4}{9}(t+1)^{9/4}-\frac{4}{5}(t+1)^{5/4}$$

```
[In] Int[t*(1 + t)^(1/4),t]
[Out] (-4*(1 + t)^(5/4))/5 + (4*(1 + t)^(9/4))/9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(-\sqrt[4]{1+t}+(1+t)^{5/4}\right)\,dt\\&=-\frac{4}{5}(1+t)^{5/4}+\frac{4}{9}(1+t)^{9/4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int t\sqrt[4]{1+t}\,dt = \frac{4}{45}(1+t)^{5/4}(-9+5(1+t))$$

```
[In] Integrate[t*(1 + t)^(1/4),t]
[Out] (4*(1 + t)^(5/4)*(-9 + 5*(1 + t)))/45
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{4(1+t)^{\frac{5}{4}}(5t-4)}{45}$	13
meijerg	$\frac{t^2\,{}_2F_1\left(-\frac{1}{4},2;3;-t\right)}{2}$	15
derivativedivides	$-\frac{4(1+t)^{\frac{5}{4}}}{5}+\frac{4(1+t)^{\frac{9}{4}}}{9}$	16
default	$-\frac{4(1+t)^{\frac{5}{4}}}{5}+\frac{4(1+t)^{\frac{9}{4}}}{9}$	16
risch	$\frac{4(1+t)^{\frac{1}{4}}(5t^2+t-4)}{45}$	16
trager	$\left(\frac{4}{9}t^2+\frac{4}{45}t-\frac{16}{45}\right)(1+t)^{\frac{1}{4}}$	17

```
[In] int(t*(1+t)^(1/4),t,method=_RETURNVERBOSE)
[Out] 4/45*(1+t)^(5/4)*(5*t-4)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int t\sqrt[4]{1+t}\,dt = \frac{4}{45}\left(5\,t^2+t-4\right)(t+1)^{\frac{1}{4}}$$

```
[In] integrate(t*(1+t)^(1/4),t, algorithm="fricas")
[Out] 4/45*(5*t^2 + t - 4)*(t + 1)^(1/4)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int t\sqrt[4]{1+t}\,dt = \frac{4t^2\sqrt[4]{t+1}}{9} + \frac{4t\sqrt[4]{t+1}}{45} - \frac{16\sqrt[4]{t+1}}{45}$$

```
[In] integrate(t*(1+t)**(1/4),t)
[Out] 4*t**2*(t + 1)**(1/4)/9 + 4*t*(t + 1)**(1/4)/45 - 16*(t + 1)**(1/4)/45
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int t\sqrt[4]{1+t}\,dt = \frac{4}{9}\,(t+1)^{\frac{9}{4}} - \frac{4}{5}\,(t+1)^{\frac{5}{4}}$$

```
[In] integrate(t*(1+t)^(1/4),t, algorithm="maxima")
[Out] 4/9*(t + 1)^(9/4) - 4/5*(t + 1)^(5/4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int t\sqrt[4]{1+t}\,dt = \frac{4}{9}\,(t+1)^{\frac{9}{4}} - \frac{4}{5}\,(t+1)^{\frac{5}{4}}$$

```
[In] integrate(t*(1+t)^(1/4),t, algorithm="giac")
[Out] 4/9*(t + 1)^(9/4) - 4/5*(t + 1)^(5/4)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int t\sqrt[4]{1+t}\,dt = \frac{4\,(5\,t-4)\,\left(t+1\right)^{5/4}}{45}$$

```
[In] int(t*(t + 1)^(1/4),t)
[Out] (4*(5*t - 4)*(t + 1)^(5/4))/45
```

3.16 $\int \frac{1}{(1+x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \frac{1}{(1+x^2)^{3/2}} \, dx = \frac{x}{\sqrt{1+x^2}}$$

```
[Out] x/(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {197}

$$\int \frac{1}{(1+x^2)^{3/2}} \, dx = \frac{x}{\sqrt{x^2+1}}$$

```
[In] Int[(1 + x^2)^(-3/2),x]
```

```
[Out] x/Sqrt[1 + x^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{x}{\sqrt{1+x^2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)^{3/2}}\,dx = \frac{x}{\sqrt{1+x^2}}$$

```
[In] Integrate[(1 + x^2)^(-3/2),x]
[Out] x/Sqrt[1 + x^2]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
gosper	$\frac{x}{\sqrt{x^2+1}}$	10
default	$\frac{x}{\sqrt{x^2+1}}$	10
trager	$\frac{x}{\sqrt{x^2+1}}$	10
meijerg	$\frac{x}{\sqrt{x^2+1}}$	10
risch	$\frac{x}{\sqrt{x^2+1}}$	10
pseudoelliptic	$\frac{x}{\sqrt{x^2+1}}$	10

```
[In] int(1/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] x/(x^2+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(9) = 18$.

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{1}{(1+x^2)^{3/2}}\,dx = \frac{x^2+\sqrt{x^2+1}x+1}{x^2+1}$$

```
[In] integrate(1/(x^2+1)^(3/2),x, algorithm="fricas")
[Out] (x^2 + sqrt(x^2 + 1)*x + 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{(1+x^2)^{3/2}}\,dx = \frac{x}{\sqrt{x^2+1}}$$

```
[In] integrate(1/(x**2+1)**(3/2),x)
[Out] x/sqrt(x**2 + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(1+x^2)^{3/2}}\,dx = \frac{x}{\sqrt{x^2+1}}$$

```
[In] integrate(1/(x^2+1)^(3/2),x, algorithm="maxima")
[Out] x/sqrt(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(1+x^2)^{3/2}}\,dx = \frac{x}{\sqrt{x^2+1}}$$

```
[In] integrate(1/(x^2+1)^(3/2),x, algorithm="giac")
[Out] x/sqrt(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(1+x^2)^{3/2}}\,dx = \frac{x}{\sqrt{x^2+1}}$$

```
[In] int(1/(x^2 + 1)^(3/2),x)
[Out] x/(x^2 + 1)^(1/2)
```

3.17 $\int x^2(27+8x^3)^{2/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int x^2\left(27+8x^3\right)^{2/3}\,dx=\frac{1}{40}\left(27+8x^3\right)^{5/3}$$

```
[Out] 1/40*(8*x^3+27)^(5/3)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {267}

$$\int x^2\left(27+8x^3\right)^{2/3}\,dx=\frac{1}{40}\left(8x^3+27\right)^{5/3}$$

```
[In] Int[x^2*(27 + 8*x^3)^(2/3),x]
```

```
[Out] (27 + 8*x^3)^(5/3)/40
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{40}\left(27+8x^3\right)^{5/3}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x^2 \left(27+8x^3\right)^{2/3} \, dx = \frac{1}{40}\left(27+8x^3\right)^{5/3}$$

```
[In] Integrate[x^2*(27 + 8*x^3)^(2/3),x]
[Out] (27 + 8*x^3)^(5/3)/40
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{(8x^3+27)^{\frac{5}{3}}}{40}$	12
default	$\frac{(8x^3+27)^{\frac{5}{3}}}{40}$	12
risch	$\frac{(8x^3+27)^{\frac{5}{3}}}{40}$	12
pseudoelliptic	$\frac{(8x^3+27)^{\frac{5}{3}}}{40}$	12
meijerg	$3x^3{}_2F_1\left(-\frac{2}{3},1;2;-\frac{8x^3}{27}\right)$	17
trager	$\left(\frac{x^3}{5}+\frac{27}{40}\right)(8x^3+27)^{\frac{2}{3}}$	18
gosper	$\frac{(3+2x)(4x^2-6x+9)(8x^3+27)^{\frac{2}{3}}}{40}$	27

```
[In] int(x^2*(8*x^3+27)^(2/3),x,method=_RETURNVERBOSE)
[Out] 1/40*(8*x^3+27)^(5/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x^2 \left(27+8x^3\right)^{2/3} \, dx = \frac{1}{40}\left(8\,x^3+27\right)^{\frac{5}{3}}$$

```
[In] integrate(x^2*(8*x^3+27)^(2/3),x, algorithm="fricas")
[Out] 1/40*(8*x^3 + 27)^(5/3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(10) = 20.

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int x^2(27+8x^3)^{2/3}\,dx=\frac{x^3(8x^3+27)^{\frac{2}{3}}}{5}+\frac{27(8x^3+27)^{\frac{2}{3}}}{40}$$

```
[In] integrate(x**2*(8*x**3+27)**(2/3),x)
[Out] x**3*(8*x**3 + 27)**(2/3)/5 + 27*(8*x**3 + 27)**(2/3)/40
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x^2(27+8x^3)^{2/3}\,dx=\frac{1}{40}\,(8\,x^3+27)^{\frac{5}{3}}$$

```
[In] integrate(x^2*(8*x^3+27)^(2/3),x, algorithm="maxima")
[Out] 1/40*(8*x^3 + 27)^(5/3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x^2(27+8x^3)^{2/3}\,dx=\frac{1}{40}\,(8\,x^3+27)^{\frac{5}{3}}$$

```
[In] integrate(x^2*(8*x^3+27)^(2/3),x, algorithm="giac")
[Out] 1/40*(8*x^3 + 27)^(5/3)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x^2 (27 + 8x^3)^{2/3}\, dx = \frac{(8\,x^3 + 27)^{5/3}}{40}$$

```
[In] int(x^2*(8*x^3 + 27)^(2/3),x)
[Out] (8*x^3 + 27)^(5/3)/40
```

3.18 $\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}} \, dx = \frac{3}{2}(-\cos(x)+\sin(x))^{2/3}$$

```
[Out] 3/2*(-cos(x)+sin(x))^(2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3224}

$$\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}} \, dx = \frac{3}{2}(\sin(x)-\cos(x))^{2/3}$$

```
[In] Int[(Cos[x] + Sin[x])/(-Cos[x] + Sin[x])^(1/3),x]
```

```
[Out] (3*(-Cos[x] + Sin[x])^(2/3))/2
```

Rule 3224

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_.)*(
cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :
> Simp[(c*B - b*C)*((b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(b
^2 + c^2))), x] /; FreeQ[{b, c, d, e, B, C}, x] && NeQ[n, -1] && NeQ[b^2 +
c^2, 0] && EqQ[b*B + c*C, 0]
```

Rubi steps

$$\text{integral} = \frac{3}{2}(-\cos(x)+\sin(x))^{2/3}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}}\,dx=\frac{3}{2}(-\cos(x)+\sin(x))^{2/3}$$

```
[In] Integrate[(Cos[x] + Sin[x])/(-Cos[x] + Sin[x])^(1/3),x]
[Out] (3*(-Cos[x] + Sin[x])^(2/3))/2
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{3(-\cos(x)+\sin(x))^{\frac{2}{3}}}{2}$	12
default	$\frac{3(-\cos(x)+\sin(x))^{\frac{2}{3}}}{2}$	12
risch	$\frac{\left(-\frac{3}{2}-\frac{3i}{2}\right)\left((1+i)\left(-e^{4ix}+ie^{2ix}\right)\right)^{\frac{1}{3}}\left(e^{ix}-ie^{-ix}\right)}{\left(-8\cos(x)+8\sin(x)\right)^{\frac{1}{3}}\left((-1-i)\left(e^{4ix}-ie^{2ix}\right)\right)^{\frac{1}{3}}}$	72

```
[In] int((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/2*(-cos(x)+sin(x))^(2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}}\,dx=\frac{3}{2}\left(-\cos\left(x\right)+\sin\left(x\right)\right)^{\frac{2}{3}}$$

```
[In] integrate((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x, algorithm="fricas")
[Out] 3/2*(-cos(x) + sin(x))^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{\cos(x) + \sin(x)}{\sqrt[3]{-\cos(x) + \sin(x)}}\,dx = \frac{3(\sin{(x)} - \cos{(x)})^{\frac{2}{3}}}{2}$$

```
[In] integrate((cos(x)+sin(x))/(-cos(x)+sin(x))**(1/3),x)
[Out] 3*(sin(x) - cos(x))**(2/3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\cos(x) + \sin(x)}{\sqrt[3]{-\cos(x) + \sin(x)}}\,dx = \frac{3}{2}\,(-\cos{(x)} + \sin{(x)})^{\frac{2}{3}}$$

```
[In] integrate((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x, algorithm="maxima")
[Out] 3/2*(-cos(x) + sin(x))^(2/3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\cos(x) + \sin(x)}{\sqrt[3]{-\cos(x) + \sin(x)}}\,dx = \frac{3}{2}\,(-\cos{(x)} + \sin{(x)})^{\frac{2}{3}}$$

```
[In] integrate((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x, algorithm="giac")
[Out] 3/2*(-cos(x) + sin(x))^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x) + \sin(x)}{\sqrt[3]{-\cos(x) + \sin(x)}}\,dx = \frac{3\,2^{1/3}\,\left(-\cos\left(x + \frac{\pi}{4}\right)\right)^{2/3}}{2}$$

```
[In] int((cos(x) + sin(x))/(sin(x) - cos(x))^(1/3),x)
[Out] (3*2^(1/3)*(-cos(x + pi/4))^(2/3))/2
```

3.19 $\int \frac{x}{\sqrt{1+x^2+\left(1+x^2\right)^{3/2}}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 32

$$\int \frac{x}{\sqrt{1+x^2+\left(1+x^2\right)^{3/2}}}\,dx=\frac{2\sqrt{\left(1+x^2\right)\left(1+\sqrt{1+x^2}\right)}}{\sqrt{1+x^2}}$$

```
[Out] 2*((x^2+1)*(1+(x^2+1)^(1/2)))^(1/2)/(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6847, 1602}

$$\int \frac{x}{\sqrt{1+x^2+\left(1+x^2\right)^{3/2}}}\,dx=\frac{2\sqrt{\left(x^2+1\right)\left(\sqrt{x^2+1}+1\right)}}{\sqrt{x^2+1}}$$

```
[In] Int[x/Sqrt[1 + x^2 + (1 + x^2)^(3/2)],x]
```

```
[Out] (2*Sqrt[(1 + x^2)*(1 + Sqrt[1 + x^2])])/Sqrt[1 + x^2]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x+(1+x)^{3/2}}}\,dx, x, x^2\right)\\ &= \text{Subst}\left(\int \frac{x}{\sqrt{x^2(1+x)}}\,dx, x, \sqrt{1+x^2}\right)\\ &= \frac{2\sqrt{(1+x^2)\left(1+\sqrt{1+x^2}\right)}}{\sqrt{1+x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1+x^2+(1+x^2)^{3/2}}}\,dx = \frac{2\sqrt{(1+x^2)\left(1+\sqrt{1+x^2}\right)}}{\sqrt{1+x^2}}$$

```
[In] Integrate[x/Sqrt[1 + x^2 + (1 + x^2)^(3/2)],x]
[Out] (2*Sqrt[(1 + x^2)*(1 + Sqrt[1 + x^2])])/Sqrt[1 + x^2]
```

Maple [F]

$$\int \frac{x}{\sqrt{1+x^2+(x^2+1)^{\frac{3}{2}}}}dx$$

```
[In] int(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x)
[Out] int(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{1+x^2+(1+x^2)^{3/2}}}\,dx = \frac{2\sqrt{x^2+(x^2+1)^{\frac{3}{2}}+1}}{\sqrt{x^2+1}}$$

```
[In] integrate(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x^2 + (x^2 + 1)^(3/2) + 1)/sqrt(x^2 + 1)
```

Sympy [F]

$$\int \frac{x}{\sqrt{1+x^2+(1+x^2)^{3/2}}}\,dx = \int \frac{x}{\sqrt{(x^2+1)\left(\sqrt{x^2+1}+1\right)}}\,dx$$

```
[In] integrate(x/(1+x**2+(x**2+1)**(3/2))**(1/2),x)
[Out] Integral(x/sqrt((x**2 + 1)*(sqrt(x**2 + 1) + 1)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1+x^2+(1+x^2)^{3/2}}}\,dx = \int \frac{x}{\sqrt{x^2+(x^2+1)^{\frac{3}{2}}+1}}\,dx$$

```
[In] integrate(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(x^2 + (x^2 + 1)^(3/2) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.47

$$\int \frac{x}{\sqrt{1+x^2+(1+x^2)^{3/2}}}\,dx = 2\sqrt{\sqrt{x^2+1}+1}-2$$

```
[In] integrate(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(sqrt(x^2 + 1) + 1) - 2
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{x}{\sqrt{1+x^2+(1+x^2)^{3/2}}}\,dx = \frac{2\,(x^2+1)\,\sqrt{\sqrt{x^2+1}+1}}{\left(\sqrt{\sqrt{x^2+1}+1}+1\right)\,\sqrt{(x^2+1)^{3/2}+x^2+1}}$$

```
[In] int(x/((x^2 + 1)^(3/2) + x^2 + 1)^(1/2),x)
[Out] (2*(x^2 + 1)*((x^2 + 1)^(1/2) + 1)^(1/2))/((((x^2 + 1)^(1/2) + 1)^(1/2) + 1
)*((x^2 + 1)^(3/2) + x^2 + 1)^(1/2))
```

3.20 $\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 17

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}} \, dx = 2\sqrt{1+\sqrt{1+x^2}}$$

```
[Out] 2*(1+(x^2+1)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6818}

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}} \, dx = 2\sqrt{\sqrt{x^2+1}+1}$$

```
[In] Int[x/(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[1 + x^2]]),x]
```

```
[Out] 2*Sqrt[1 + Sqrt[1 + x^2]]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = 2\sqrt{1+\sqrt{1+x^2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx = 2\sqrt{1+\sqrt{1+x^2}}$$

```
[In] Integrate[x/(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[1 + x^2]]),x]
[Out] 2*Sqrt[1 + Sqrt[1 + x^2]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$2\sqrt{1+\sqrt{x^2+1}}$	14
default	$2\sqrt{1+\sqrt{x^2+1}}$	14

```
[In] int(x/(x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(1+(x^2+1)^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx = 2\sqrt{\sqrt{x^2+1}+1}$$

```
[In] integrate(x/(x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(sqrt(x^2 + 1) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx = 2\sqrt{\sqrt{x^2+1}+1}$$

```
[In] integrate(x/(x**2+1)**(1/2)/(1+(x**2+1)**(1/2))**(1/2),x)
[Out] 2*sqrt(sqrt(x**2 + 1) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx = 2\sqrt{\sqrt{x^2+1}+1}$$

```
[In] integrate(x/(x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(sqrt(x^2 + 1) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx = 2\sqrt{\sqrt{x^2+1}+1}$$

```
[In] integrate(x/(x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(sqrt(x^2 + 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx = 2\sqrt{\sqrt{x^2+1}+1}$$

```
[In] int(x/((x^2 + 1)^(1/2)*((x^2 + 1)^(1/2) + 1)^(1/2)),x)
[Out] 2*((x^2 + 1)^(1/2) + 1)^(1/2)
```

3.21 $\int \frac{\sqrt[5]{1-2x+x^2}}{1-x} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 16

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x} \, dx = -\frac{5}{2}\sqrt[5]{1-2x+x^2}$$

```
[Out] -5/2*(x^2-2*x+1)^(1/5)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {657, 643}

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x} \, dx = -\frac{5}{2}\sqrt[5]{x^2-2x+1}$$

```
[In] Int[(1 - 2*x + x^2)^(1/5)/(1 - x),x]
```

```
[Out] (-5*(1 - 2*x + x^2)^(1/5))/2
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
```

```
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1-x}{(1-2x+x^2)^{4/5}}\,dx \\ &= -\frac{5}{2}\sqrt[5]{1-2x+x^2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx = -\frac{5}{2}\sqrt[5]{(-1+x)^2}$$

```
[In] Integrate[(1 - 2*x + x^2)^(1/5)/(1 - x),x]
[Out] (-5*((-1 + x)^2)^(1/5))/2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{5\left(\left(-1+x\right)^{2}\right)^{\frac{1}{5}}}{2}$	10
pseudoelliptic	$-\frac{5\left(\left(-1+x\right)^{2}\right)^{\frac{1}{5}}}{2}$	10
gosper	$-\frac{5\left(x^{2}-2x+1\right)^{\frac{1}{5}}}{2}$	13
trager	$-\frac{5\left(x^{2}-2x+1\right)^{\frac{1}{5}}}{2}$	13
meijerg	$\frac{\operatorname{signum}\left(-1+x\right)^{\frac{2}{5}} x\, {}_{2}F_{1}\left(\frac{3}{5},1;2;x\right)}{\left(-\operatorname{signum}\left(-1+x\right)\right)^{\frac{2}{5}}}$	24

```
[In] int((x^2-2*x+1)^(1/5)/(1-x),x,method=_RETURNVERBOSE)
[Out] -5/2*((-1+x)^2)^(1/5)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx = -\frac{5}{2}\left(x^2-2\,x+1\right)^{\frac{1}{5}}$$

```
[In] integrate((x^2-2*x+1)^(1/5)/(1-x),x, algorithm="fricas")
[Out] -5/2*(x^2 - 2*x + 1)^(1/5)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx = -\frac{5\sqrt[5]{x^2-2x+1}}{2}$$

```
[In] integrate((x**2-2*x+1)**(1/5)/(1-x),x)
[Out] -5*(x**2 - 2*x + 1)**(1/5)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx = -\frac{5}{2}\,(x-1)^{\frac{2}{5}}$$

```
[In] integrate((x^2-2*x+1)^(1/5)/(1-x),x, algorithm="maxima")
[Out] -5/2*(x - 1)^(2/5)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx = -\frac{5}{2}\left(x^2-2\,x+1\right)^{\frac{1}{5}}$$

```
[In] integrate((x^2-2*x+1)^(1/5)/(1-x),x, algorithm="giac")
[Out] -5/2*(x^2 - 2*x + 1)^(1/5)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx = -\frac{5\left((x-1)^2\right)^{1/5}}{2}$$

```
[In] int(-(x^2 - 2*x + 1)^(1/5)/(x - 1),x)
[Out] -(5*((x - 1)^2)^(1/5))/2
```

3.22 $\int x\sin(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 8

$$\int x\sin(x)\,dx = -x\cos(x)+\sin(x)$$

```
[Out] -x*cos(x)+sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3377, 2717}

$$\int x\sin(x)\,dx = \sin(x)-x\cos(x)$$

```
[In] Int[x*Sin[x],x]
```

```
[Out] -(x*Cos[x]) + Sin[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -x\cos(x) + \int \cos(x)\,dx\\ &= -x\cos(x) + \sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = -x\cos(x) + \sin(x)$$

```
[In] Integrate[x*Sin[x],x]
[Out] -(x*Cos[x]) + Sin[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$-x\cos(x)+\sin(x)$	9
risch	$-x\cos(x)+\sin(x)$	9
parallelrisch	$-x\cos(x)+\sin(x)$	9
parts	$-x\cos(x)+\sin(x)$	9
meijerg	$2\sqrt{\pi}\left(-\frac{x\cos(x)}{2\sqrt{\pi}}+\frac{\sin(x)}{2\sqrt{\pi}}\right)$	22
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)-x+2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$	30

```
[In] int(x*sin(x),x,method=_RETURNVERBOSE)
[Out] -x*cos(x)+sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = -x\cos(x) + \sin(x)$$

```
[In] integrate(x*sin(x),x, algorithm="fricas")
[Out] -x*cos(x) + sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int x\sin(x)\,dx = -x\cos(x) + \sin(x)$$

```
[In] integrate(x*sin(x),x)
[Out] -x*cos(x) + sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = -x\cos(x) + \sin(x)$$

```
[In] integrate(x*sin(x),x, algorithm="maxima")
[Out] -x*cos(x) + sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = -x\cos(x) + \sin(x)$$

```
[In] integrate(x*sin(x),x, algorithm="giac")
[Out] -x*cos(x) + sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = \sin(x) - x\ \cos(x)$$

```
[In] int(x*sin(x),x)
[Out] sin(x) - x*cos(x)
```

3.23 $\int x^2 \sin(x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 17

$$\int x^2 \sin(x)\,dx = 2\cos(x) - x^2\cos(x) + 2x\sin(x)$$

```
[Out] 2*cos(x)-x^2*cos(x)+2*x*sin(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3377, 2718}

$$\int x^2 \sin(x)\,dx = x^2(-\cos(x)) + 2x\sin(x) + 2\cos(x)$$

```
[In] Int[x^2*Sin[x],x]
```

```
[Out] 2*Cos[x] - x^2*Cos[x] + 2*x*Sin[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -x^2\cos(x) + 2\int x\cos(x)\,dx\\ &= -x^2\cos(x) + 2x\sin(x) - 2\int \sin(x)\,dx\\ &= 2\cos(x) - x^2\cos(x) + 2x\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2\sin(x)\,dx = -\left(\left(-2+x^2\right)\cos(x)\right) + 2x\sin(x)$$

```
[In] Integrate[x^2*Sin[x],x]
[Out] -((-2 + x^2)*Cos[x]) + 2*x*Sin[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

method	result	size
risch	$(-x^2+2)\cos(x) + 2x\sin(x)$	17
default	$2\cos(x) - x^2\cos(x) + 2x\sin(x)$	18
parts	$2\cos(x) - x^2\cos(x) + 2x\sin(x)$	18
parallelrisch	$-x^2\cos(x) + 2x\sin(x) + 2\cos(x) + 2$	19
meijerg	$4\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}} + \frac{\left(-\frac{x^2}{2}+1\right)\cos(x)}{2\sqrt{\pi}} + \frac{x\sin(x)}{2\sqrt{\pi}}\right)$	34
norman	$\frac{x^2\left(\tan^2\left(\frac{x}{2}\right)\right)-x^2+4x\tan\left(\frac{x}{2}\right)+4}{1+\tan^2\left(\frac{x}{2}\right)}$	36

```
[In] int(x^2*sin(x),x,method=_RETURNVERBOSE)
[Out] (-x^2+2)*cos(x)+2*x*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x)\, dx = -\left(x^2 - 2\right) \cos\left(x\right) + 2\, x \sin\left(x\right)$$

```
[In] integrate(x^2*sin(x),x, algorithm="fricas")
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2 \sin(x)\, dx = -x^2 \cos\left(x\right) + 2x \sin\left(x\right) + 2 \cos\left(x\right)$$

```
[In] integrate(x**2*sin(x),x)
[Out] -x**2*cos(x) + 2*x*sin(x) + 2*cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x)\, dx = -\left(x^2 - 2\right) \cos\left(x\right) + 2\, x \sin\left(x\right)$$

```
[In] integrate(x^2*sin(x),x, algorithm="maxima")
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x)\, dx = -\left(x^2 - 2\right) \cos\left(x\right) + 2\, x \sin\left(x\right)$$

```
[In] integrate(x^2*sin(x),x, algorithm="giac")
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x)\, dx = 2\,x\,\sin(x) - \cos(x)\,\left(x^2 - 2\right)$$

```
[In] int(x^2*sin(x),x)
[Out] 2*x*sin(x) - cos(x)*(x^2 - 2)
```

3.24 $\int x^3 \cos(x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 23

$$\int x^3 \cos(x)\,dx = -6\cos(x) + 3x^2\cos(x) - 6x\sin(x) + x^3\sin(x)$$

```
[Out] -6*cos(x)+3*x^2*cos(x)-6*x*sin(x)+x^3*sin(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3377, 2718}

$$\int x^3 \cos(x)\,dx = x^3\sin(x) + 3x^2\cos(x) - 6x\sin(x) - 6\cos(x)$$

```
[In] Int[x^3*Cos[x],x]
```

```
[Out] -6*Cos[x] + 3*x^2*Cos[x] - 6*x*Sin[x] + x^3*Sin[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x^3\sin(x) - 3\int x^2\sin(x)\,dx\\ &= 3x^2\cos(x) + x^3\sin(x) - 6\int x\cos(x)\,dx\\ &= 3x^2\cos(x) - 6x\sin(x) + x^3\sin(x) + 6\int \sin(x)\,dx\\ &= -6\cos(x) + 3x^2\cos(x) - 6x\sin(x) + x^3\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^3\cos(x)\,dx = 3\left(-2+x^2\right)\cos(x) + x\left(-6+x^2\right)\sin(x)$$

```
[In] Integrate[x^3*Cos[x],x]
[Out] 3*(-2 + x^2)*Cos[x] + x*(-6 + x^2)*Sin[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
risch	$3(x^2-2)\cos(x) + x(x^2-6)\sin(x)$	20
parallelrisch	$(3x^2-6)\cos(x) - 6 + (x^3-6x)\sin(x)$	23
default	$-6\cos(x) + 3x^2\cos(x) - 6x\sin(x) + x^3\sin(x)$	24
parts	$-6\cos(x) + 3x^2\cos(x) - 6x\sin(x) + x^3\sin(x)$	24
meijerg	$8\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}} - \frac{\left(-\frac{3x^2}{2}+3\right)\cos(x)}{4\sqrt{\pi}} - \frac{x\left(-\frac{x^2}{2}+3\right)\sin(x)}{4\sqrt{\pi}}\right)$	41
norman	$\frac{3x^2-12x\tan\left(\frac{x}{2}\right)-3x^2\left(\tan^2\left(\frac{x}{2}\right)\right)+2x^3\tan\left(\frac{x}{2}\right)-12}{1+\tan^2\left(\frac{x}{2}\right)}$	46

```
[In] int(x^3*cos(x),x,method=_RETURNVERBOSE)
[Out] 3*(x^2-2)*cos(x)+x*(x^2-6)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^3 \cos(x) \, dx = 3 \left(x^2 - 2\right) \cos\left(x\right) + \left(x^3 - 6 \, x\right) \sin\left(x\right)$$

```
[In] integrate(x^3*cos(x),x, algorithm="fricas")
[Out] 3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int x^3 \cos(x) \, dx = x^3 \sin\left(x\right) + 3x^2 \cos\left(x\right) - 6x \sin\left(x\right) - 6 \cos\left(x\right)$$

```
[In] integrate(x**3*cos(x),x)
[Out] x**3*sin(x) + 3*x**2*cos(x) - 6*x*sin(x) - 6*cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^3 \cos(x) \, dx = 3 \left(x^2 - 2\right) \cos\left(x\right) + \left(x^3 - 6 \, x\right) \sin\left(x\right)$$

```
[In] integrate(x^3*cos(x),x, algorithm="maxima")
[Out] 3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^3 \cos(x) \, dx = 3 \left(x^2 - 2\right) \cos\left(x\right) + \left(x^3 - 6 \, x\right) \sin\left(x\right)$$

```
[In] integrate(x^3*cos(x),x, algorithm="giac")
[Out] 3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int x^3 \cos(x)\, dx = \cos(x)\ \left(3\,x^2 - 6\right) - \sin(x)\ \left(6\,x - x^3\right)$$

```
[In] int(x^3*cos(x),x)
[Out] cos(x)*(3*x^2 - 6) - sin(x)*(6*x - x^3)
```

3.25 $\int x^3 \sin(x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 24

$$\int x^3 \sin(x)\,dx = 6x\cos(x) - x^3\cos(x) - 6\sin(x) + 3x^2\sin(x)$$

```
[Out] 6*x*cos(x)-x^3*cos(x)-6*sin(x)+3*x^2*sin(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3377, 2717}

$$\int x^3 \sin(x)\,dx = x^3(-\cos(x)) + 3x^2\sin(x) - 6\sin(x) + 6x\cos(x)$$

```
[In] Int[x^3*Sin[x],x]
```

```
[Out] 6*x*Cos[x] - x^3*Cos[x] - 6*Sin[x] + 3*x^2*Sin[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -x^3\cos(x)+3\int x^2\cos(x)\,dx\\ &= -x^3\cos(x)+3x^2\sin(x)-6\int x\sin(x)\,dx\\ &= 6x\cos(x)-x^3\cos(x)+3x^2\sin(x)-6\int\cos(x)\,dx\\ &= 6x\cos(x)-x^3\cos(x)-6\sin(x)+3x^2\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x^3\sin(x)\,dx = -x\left(-6+x^2\right)\cos(x)+3\left(-2+x^2\right)\sin(x)$$

```
[In] Integrate[x^3*Sin[x],x]
[Out] -(x*(-6 + x^2)*Cos[x]) + 3*(-2 + x^2)*Sin[x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
risch	$(-x^3+6x)\cos(x)+3(x^2-2)\sin(x)$	23
default	$6x\cos(x)-x^3\cos(x)-6\sin(x)+3x^2\sin(x)$	25
parallelrisch	$6x\cos(x)-x^3\cos(x)-6\sin(x)+3x^2\sin(x)$	25
parts	$6x\cos(x)-x^3\cos(x)-6\sin(x)+3x^2\sin(x)$	25
meijerg	$8\sqrt{\pi}\left(\frac{x\left(-\frac{5x^2}{2}+15\right)\cos(x)}{20\sqrt{\pi}}-\frac{\left(-\frac{15x^2}{2}+15\right)\sin(x)}{20\sqrt{\pi}}\right)$	36
norman	$\frac{x^3\left(\tan^2\left(\frac{x}{2}\right)\right)+6x-x^3-6x\left(\tan^2\left(\frac{x}{2}\right)\right)+6x^2\tan\left(\frac{x}{2}\right)-12\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$	55

```
[In] int(x^3*sin(x),x,method=_RETURNVERBOSE)
[Out] (-x^3+6*x)*cos(x)+3*(x^2-2)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int x^3 \sin(x)\,dx = -\left(x^3 - 6\,x\right)\cos\left(x\right) + 3\,\left(x^2 - 2\right)\sin\left(x\right)$$

```
[In] integrate(x^3*sin(x),x, algorithm="fricas")
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^3 \sin(x)\,dx = -x^3\cos\left(x\right) + 3x^2\sin\left(x\right) + 6x\cos\left(x\right) - 6\sin\left(x\right)$$

```
[In] integrate(x**3*sin(x),x)
[Out] -x**3*cos(x) + 3*x**2*sin(x) + 6*x*cos(x) - 6*sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int x^3 \sin(x)\,dx = -\left(x^3 - 6\,x\right)\cos\left(x\right) + 3\,\left(x^2 - 2\right)\sin\left(x\right)$$

```
[In] integrate(x^3*sin(x),x, algorithm="maxima")
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int x^3 \sin(x)\,dx = -\left(x^3 - 6\,x\right)\cos\left(x\right) + 3\,\left(x^2 - 2\right)\sin\left(x\right)$$

```
[In] integrate(x^3*sin(x),x, algorithm="giac")
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int x^3 \sin(x) \, dx = \cos(x) \left(6x - x^3\right) + \sin(x) \left(3x^2 - 6\right)$$

```
[In] int(x^3*sin(x),x)
[Out] cos(x)*(6*x - x^3) + sin(x)*(3*x^2 - 6)
```

3.26 $\int \cos(x)\sin(x)\,dx$

Optimal result

Integrand size = 5, antiderivative size = 8

$$\int \cos(x)\sin(x)\,dx = \frac{\sin^2(x)}{2}$$

```
[Out] 1/2*sin(x)^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2644, 30}

$$\int \cos(x)\sin(x)\,dx = \frac{\sin^2(x)}{2}$$

```
[In] Int[Cos[x]*Sin[x],x]
```

```
[Out] Sin[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}(\int x\,dx, x, \sin(x)) \\
&= \frac{\sin^2(x)}{2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos^2(x)$$

```
[In] Integrate[Cos[x]*Sin[x],x]
[Out] -1/2*Cos[x]^2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sin^2(x))}{2}$	7
default	$\frac{(\sin^2(x))}{2}$	7
risch	$-\frac{\cos(2x)}{4}$	7
parallelrisch	$\frac{1}{4}-\frac{\cos(2x)}{4}$	9
norman	$\frac{2(\tan^2(\frac{x}{2}))}{(1+\tan^2(\frac{x}{2}))^2}$	19
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{4}$	19

```
[In] int(cos(x)*sin(x),x,method=_RETURNVERBOSE)
[Out] 1/2*sin(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos\left(x\right)^2$$

```
[In] integrate(cos(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*cos(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \cos(x)\sin(x)\,dx = \frac{\sin^2\left(x\right)}{2}$$

```
[In] integrate(cos(x)*sin(x),x)
[Out] sin(x)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos\left(x\right)^2$$

```
[In] integrate(cos(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*cos(x)^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos\left(x\right)^2$$

```
[In] integrate(cos(x)*sin(x),x, algorithm="giac")
[Out] -1/2*cos(x)^2
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = \frac{\sin(x)^2}{2}$$

```
[In] int(cos(x)*sin(x),x)
[Out] sin(x)^2/2
```

3.27 $\int x \cos(x) \sin(x) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 23

$$\int x \cos(x) \sin(x) \, dx = -\frac{x}{4} + \frac{1}{4} \cos(x) \sin(x) + \frac{1}{2} x \sin^2(x)$$

```
[Out] -1/4*x+1/4*cos(x)*sin(x)+1/2*x*sin(x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3524, 2715, 8}

$$\int x \cos(x) \sin(x) \, dx = -\frac{x}{4} + \frac{1}{2} x \sin^2(x) + \frac{1}{4} \sin(x) \cos(x)$$

```
[In] Int[x*Cos[x]*Sin[x],x]
```

```
[Out] -1/4*x + (Cos[x]*Sin[x])/4 + (x*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x\sin^2(x) - \frac{1}{2}\int \sin^2(x)\,dx\\ &= \frac{1}{4}\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x) - \frac{\int 1\,dx}{4}\\ &= -\frac{x}{4} + \frac{1}{4}\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int x\cos(x)\sin(x)\,dx = -\frac{1}{4}x\cos(2x) + \frac{1}{8}\sin(2x)$$

```
[In] Integrate[x*Cos[x]*Sin[x],x]
[Out] -1/4*(x*Cos[2*x]) + Sin[2*x]/8
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

method	result	size
risch	$-\frac{x\cos(2x)}{4}+\frac{\sin(2x)}{8}$	15
parallelrisch	$-\frac{x\cos(2x)}{4}+\frac{\sin(2x)}{8}$	15
default	$-\frac{(\cos^2(x))x}{2}+\frac{\cos(x)\sin(x)}{4}+\frac{x}{4}$	18
meijerg	$\frac{\sqrt{\pi}\left(-\frac{x\cos(2x)}{\sqrt{\pi}}+\frac{\sin(2x)}{2\sqrt{\pi}}\right)}{4}$	26
norman	$\frac{-\frac{x}{4}-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{\tan\left(\frac{x}{2}\right)}{2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	48

```
[In] int(x*cos(x)*sin(x),x,method=_RETURNVERBOSE)
[Out] -1/4*x*cos(2*x)+1/8*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int x\cos(x)\sin(x)\,dx = -\frac{1}{2}\,x\cos\left(x\right)^{2} + \frac{1}{4}\,\cos\left(x\right)\sin\left(x\right) + \frac{1}{4}\,x$$

```
[In] integrate(x*cos(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*x*cos(x)^2 + 1/4*cos(x)*sin(x) + 1/4*x
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int x\cos(x)\sin(x)\,dx = \frac{x\sin^{2}\left(x\right)}{4} - \frac{x\cos^{2}\left(x\right)}{4} + \frac{\sin\left(x\right)\cos\left(x\right)}{4}$$

```
[In] integrate(x*cos(x)*sin(x),x)
[Out] x*sin(x)**2/4 - x*cos(x)**2/4 + sin(x)*cos(x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int x\cos(x)\sin(x)\,dx = -\frac{1}{4}\,x\cos\left(2\,x\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

```
[In] integrate(x*cos(x)*sin(x),x, algorithm="maxima")
[Out] -1/4*x*cos(2*x) + 1/8*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int x\cos(x)\sin(x)\,dx = -\frac{1}{4}\,x\cos\left(2\,x\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

```
[In] integrate(x*cos(x)*sin(x),x, algorithm="giac")
[Out] -1/4*x*cos(2*x) + 1/8*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int x\cos(x)\sin(x)\,dx = \frac{\sin(2\,x)}{8} + \frac{x\,\left(2\sin\left(x\right)^2 - 1\right)}{4}$$

```
[In] int(x*cos(x)*sin(x),x)
[Out] sin(2*x)/8 + (x*(2*sin(x)^2 - 1))/4
```

3.28 $\int \sin^2(x)\, dx$

Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \sin^2(x)\, dx = \frac{x}{2} - \frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\int \sin^2(x)\, dx = \frac{x}{2} - \frac{1}{2}\sin(x)\cos(x)$$

```
[In] Int[Sin[x]^2,x]
```

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2}\cos(x)\sin(x) + \frac{\int 1\,dx}{2}\\ &= \frac{x}{2} - \frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{4}\sin(2x)$$

```
[In] Integrate[Sin[x]^2,x]
[Out] x/2 - Sin[2*x]/4
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
meijerg	$\frac{\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}}-\frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$	22
norman	$\frac{\tan^3\left(\frac{x}{2}\right)+x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}-\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

```
[In] int(sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = -\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

```
[In] integrate(sin(x)^2,x, algorithm="fricas")
[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

```
[In] integrate(sin(x)**2,x)
[Out] x/2 - sin(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{1}{2}x - \frac{1}{4}\sin(2x)$$

```
[In] integrate(sin(x)^2,x, algorithm="maxima")
[Out] 1/2*x - 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{1}{2}x - \frac{1}{4}\sin(2x)$$

```
[In] integrate(sin(x)^2,x, algorithm="giac")
[Out] 1/2*x - 1/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{\sin(2\,x)}{4}$$

```
[In] int(sin(x)^2,x)
[Out] x/2 - sin(2*x)/4
```

3.29 $\int \sin^3(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 13

$$\int \sin^3(x)\,dx = -\cos(x) + \frac{\cos^3(x)}{3}$$

```
[Out] -cos(x)+1/3*cos(x)^3
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2713}

$$\int \sin^3(x)\,dx = \frac{\cos^3(x)}{3} - \cos(x)$$

```
[In] Int[Sin[x]^3,x]
```

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(x)\right) \\ &= -\cos(x) + \frac{\cos^3(x)}{3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sin^3(x)\,dx = -\frac{3\cos(x)}{4} + \frac{1}{12}\cos(3x)$$

```
[In] Integrate[Sin[x]^3,x]
[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{(2+\sin^2(x))\cos(x)}{3}$	11
risch	$-\frac{3\cos(x)}{4}+\frac{\cos(3x)}{12}$	12
parallelrisch	$-\frac{2}{3}-\frac{3\cos(x)}{4}+\frac{\cos(3x)}{12}$	13
norman	$\frac{-4(\tan^2(\frac{x}{2}))-\frac{4}{3}}{(1+\tan^2(\frac{x}{2}))^3}$	22

```
[In] int(sin(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/3*(2+sin(x)^2)*cos(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos{(x)}^3 - \cos{(x)}$$

```
[In] integrate(sin(x)^3,x, algorithm="fricas")
[Out] 1/3*cos(x)^3 - cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \sin^3(x)\,dx = \frac{\cos^3{(x)}}{3} - \cos{(x)}$$

```
[In] integrate(sin(x)**3,x)
[Out] cos(x)**3/3 - cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos{(x)}^3 - \cos{(x)}$$

```
[In] integrate(sin(x)^3,x, algorithm="maxima")
[Out] 1/3*cos(x)^3 - cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos{(x)}^3 - \cos{(x)}$$

```
[In] integrate(sin(x)^3,x, algorithm="giac")
[Out] 1/3*cos(x)^3 - cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \sin^3(x)\,dx = \frac{\cos{(x)}\,\left(\cos{(x)}^2 - 3\right)}{3}$$

```
[In] int(sin(x)^3,x)
[Out] (cos(x)*(cos(x)^2 - 3))/3
```

3.30 $\int \sin^4(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 24

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{3}{8}\cos(x)\sin(x) - \frac{1}{4}\cos(x)\sin^3(x)$$

```
[Out] 3/8*x-3/8*cos(x)*sin(x)-1/4*cos(x)*sin(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{1}{4}\sin^3(x)\cos(x) - \frac{3}{8}\sin(x)\cos(x)$$

```
[In] Int[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{4}\cos(x)\sin^3(x) + \frac{3}{4}\int \sin^2(x)\,dx \\
&= -\frac{3}{8}\cos(x)\sin(x) - \frac{1}{4}\cos(x)\sin^3(x) + \frac{3\int 1\,dx}{8} \\
&= \frac{3x}{8} - \frac{3}{8}\cos(x)\sin(x) - \frac{1}{4}\cos(x)\sin^3(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)$$

```
[In] Integrate[Sin[x]^4,x]
[Out] (3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{3x}{8}+\frac{\sin(4x)}{32}-\frac{\sin(2x)}{4}$	17
parallelrisch	$\frac{3x}{8}+\frac{\sin(4x)}{32}-\frac{\sin(2x)}{4}$	17
default	$-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}$	18
norman	$\frac{\frac{3x}{8}-\frac{11\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}+\frac{11\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{3\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{9x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}-\frac{3\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	82

```
[In] int(sin(x)^4,x,method=_RETURNVERBOSE)
[Out] 3/8*x+1/32*sin(4*x)-1/4*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \sin^4(x)\,dx = \frac{1}{8}\left(2\,\cos\left(x\right)^3 - 5\,\cos\left(x\right)\right)\sin\left(x\right) + \frac{3}{8}\,x$$

```
[In] integrate(sin(x)^4,x, algorithm="fricas")
[Out] 1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{\sin^3\left(x\right)\cos\left(x\right)}{4} - \frac{3\sin\left(x\right)\cos\left(x\right)}{8}$$

```
[In] integrate(sin(x)**4,x)
[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3}{8}\,x + \frac{1}{32}\,\sin\left(4\,x\right) - \frac{1}{4}\,\sin\left(2\,x\right)$$

```
[In] integrate(sin(x)^4,x, algorithm="maxima")
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3}{8}\,x + \frac{1}{32}\,\sin\left(4\,x\right) - \frac{1}{4}\,\sin\left(2\,x\right)$$

```
[In] integrate(sin(x)^4,x, algorithm="giac")
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3\,x}{8} - \frac{\sin\left(2\,x\right)}{4} + \frac{\sin\left(4\,x\right)}{32}$$

```
[In] int(sin(x)^4,x)
[Out] (3*x)/8 - sin(2*x)/4 + sin(4*x)/32
```

3.31 $\int \sin^5(x)\, dx$

Optimal result

Integrand size = 4, antiderivative size = 21

$$\int \sin^5(x)\, dx = -\cos(x) + \frac{2\cos^3(x)}{3} - \frac{\cos^5(x)}{5}$$

```
[Out] -cos(x)+2/3*cos(x)^3-1/5*cos(x)^5
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{2713\}$

$$\int \sin^5(x)\, dx = -\frac{1}{5}\cos^5(x) + \frac{2\cos^3(x)}{3} - \cos(x)$$

```
[In] Int[Sin[x]^5,x]
```

```
[Out] -Cos[x] + (2*Cos[x]^3)/3 - Cos[x]^5/5
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int \left(1-2x^2+x^4\right)\, dx, x, \cos(x)\right) \\ &= -\cos(x) + \frac{2\cos^3(x)}{3} - \frac{\cos^5(x)}{5} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^5(x)\,dx = -\frac{5\cos(x)}{8} + \frac{5}{48}\cos(3x) - \frac{1}{80}\cos(5x)$$

[In] Integrate[Sin[x]^5,x]

[Out] (-5*Cos[x])/8 + (5*Cos[3*x])/48 - Cos[5*x]/80

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\left(\frac{8}{3}+\sin^4(x)+\frac{4\left(\sin^2(x)\right)}{3}\right)\cos(x)}{5}$	17
risch	$-\frac{5\cos(x)}{8}-\frac{\cos(5x)}{80}+\frac{5\cos(3x)}{48}$	18
parallelrisch	$\frac{8}{15}-\frac{5\cos(x)}{8}+\frac{5\cos(3x)}{48}-\frac{\cos(5x)}{80}$	19
norman	$\frac{-\frac{32\left(\tan^4\left(\frac{x}{2}\right)\right)}{3}-\frac{16\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}-\frac{16}{15}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^5}$	30

[In] int(sin(x)^5,x,method=_RETURNVERBOSE)

[Out] -1/5*(8/3+sin(x)^4+4/3*sin(x)^2)*cos(x)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin^5(x)\,dx = -\frac{1}{5}\cos(x)^5 + \frac{2}{3}\cos(x)^3 - \cos(x)$$

[In] integrate(sin(x)^5,x, algorithm="fricas")

[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin^5(x)\,dx = -\frac{\cos^5(x)}{5} + \frac{2\cos^3(x)}{3} - \cos(x)$$

```
[In] integrate(sin(x)**5,x)
[Out] -cos(x)**5/5 + 2*cos(x)**3/3 - cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin^5(x)\,dx = -\frac{1}{5}\cos(x)^5 + \frac{2}{3}\cos(x)^3 - \cos(x)$$

```
[In] integrate(sin(x)^5,x, algorithm="maxima")
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin^5(x)\,dx = -\frac{1}{5}\cos(x)^5 + \frac{2}{3}\cos(x)^3 - \cos(x)$$

```
[In] integrate(sin(x)^5,x, algorithm="giac")
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin^5(x)\,dx = -\frac{\cos(x)^5}{5} + \frac{2\cos(x)^3}{3} - \cos(x)$$

```
[In] int(sin(x)^5,x)
[Out] (2*cos(x)^3)/3 - cos(x) - cos(x)^5/5
```

3.32 $\int \sin^6(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 34

$$\int \sin^6(x)\,dx = \frac{5x}{16} - \frac{5}{16}\cos(x)\sin(x) - \frac{5}{24}\cos(x)\sin^3(x) - \frac{1}{6}\cos(x)\sin^5(x)$$

```
[Out] 5/16*x-5/16*cos(x)*sin(x)-5/24*cos(x)*sin(x)^3-1/6*cos(x)*sin(x)^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\int \sin^6(x)\,dx = \frac{5x}{16} - \frac{1}{6}\sin^5(x)\cos(x) - \frac{5}{24}\sin^3(x)\cos(x) - \frac{5}{16}\sin(x)\cos(x)$$

```
[In] Int[Sin[x]^6,x]
```

```
[Out] (5*x)/16 - (5*Cos[x]*Sin[x])/16 - (5*Cos[x]*Sin[x]^3)/24 - (Cos[x]*Sin[x]^5
)/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{6}\cos(x)\sin^5(x) + \frac{5}{6}\int \sin^4(x)\,dx \\
&= -\frac{5}{24}\cos(x)\sin^3(x) - \frac{1}{6}\cos(x)\sin^5(x) + \frac{5}{8}\int \sin^2(x)\,dx \\
&= -\frac{5}{16}\cos(x)\sin(x) - \frac{5}{24}\cos(x)\sin^3(x) - \frac{1}{6}\cos(x)\sin^5(x) + \frac{5\int 1\,dx}{16} \\
&= \frac{5x}{16} - \frac{5}{16}\cos(x)\sin(x) - \frac{5}{24}\cos(x)\sin^3(x) - \frac{1}{6}\cos(x)\sin^5(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sin^6(x)\,dx = \frac{5x}{16} - \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) - \frac{1}{192}\sin(6x)$$

```
[In] Integrate[Sin[x]^6,x]
[Out] (5*x)/16 - (15*Sin[2*x])/64 + (3*Sin[4*x])/64 - Sin[6*x]/192
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result
risch	$\frac{5x}{16}-\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}-\frac{15\sin(2x)}{64}$
parallelrisch	$\frac{5x}{16}-\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}-\frac{15\sin(2x)}{64}$
default	$-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}$
norman	$\frac{\frac{5x}{16}-\frac{85\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}-\frac{33\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{33\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{85\left(\tan^9\left(\frac{x}{2}\right)\right)}{24}+\frac{5\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{8}+\frac{15x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{75x\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{25x\left(\tan^6\left(\frac{x}{2}\right)\right.}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

```
[In] int(sin(x)^6,x,method=_RETURNVERBOSE)
[Out] 5/16*x-1/192*sin(6*x)+3/64*sin(4*x)-15/64*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \sin^6(x)\,dx = -\frac{1}{48}\left(8\cos(x)^5 - 26\cos(x)^3 + 33\cos(x)\right)\sin(x) + \frac{5}{16}\,x$$

```
[In] integrate(sin(x)^6,x, algorithm="fricas")
[Out] -1/48*(8*cos(x)^5 - 26*cos(x)^3 + 33*cos(x))*sin(x) + 5/16*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \sin^6(x)\,dx = \frac{5x}{16} - \frac{\sin^5(x)\cos(x)}{6} - \frac{5\sin^3(x)\cos(x)}{24} - \frac{5\sin(x)\cos(x)}{16}$$

```
[In] integrate(sin(x)**6,x)
[Out] 5*x/16 - sin(x)**5*cos(x)/6 - 5*sin(x)**3*cos(x)/24 - 5*sin(x)*cos(x)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \sin^6(x)\,dx = \frac{1}{48}\sin(2x)^3 + \frac{5}{16}\,x + \frac{3}{64}\sin(4x) - \frac{1}{4}\sin(2x)$$

```
[In] integrate(sin(x)^6,x, algorithm="maxima")
[Out] 1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) - 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \sin^6(x)\,dx = \frac{5}{16}\,x - \frac{1}{192}\sin(6x) + \frac{3}{64}\sin(4x) - \frac{15}{64}\sin(2x)$$

```
[In] integrate(sin(x)^6,x, algorithm="giac")
[Out] 5/16*x - 1/192*sin(6*x) + 3/64*sin(4*x) - 15/64*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \sin^6(x)\,dx = \frac{5\,x}{16} - \frac{15\,\sin\left(2\,x\right)}{64} + \frac{3\,\sin\left(4\,x\right)}{64} - \frac{\sin\left(6\,x\right)}{192}$$

```
[In] int(sin(x)^6,x)
[Out] (5*x)/16 - (15*sin(2*x))/64 + (3*sin(4*x))/64 - sin(6*x)/192
```

3.33 $\int x \sin^2(x)\, dx$

Optimal result

Integrand size = 6, antiderivative size = 25

$$\int x \sin^2(x)\, dx = \frac{x^2}{4} - \frac{1}{2} x \cos(x) \sin(x) + \frac{\sin^2(x)}{4}$$

```
[Out] 1/4*x^2-1/2*x*cos(x)*sin(x)+1/4*sin(x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3391, 30}

$$\int x \sin^2(x)\, dx = \frac{x^2}{4} + \frac{\sin^2(x)}{4} - \frac{1}{2} x \sin(x) \cos(x)$$

```
[In] Int[x*Sin[x]^2,x]
[Out] x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

]

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2}x\cos(x)\sin(x)+\frac{\sin^2(x)}{4}+\frac{\int x\,dx}{2}\\ &= \frac{x^2}{4}-\frac{1}{2}x\cos(x)\sin(x)+\frac{\sin^2(x)}{4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\sin^2(x)\,dx=\frac{x^2}{4}-\frac{1}{8}\cos(2x)-\frac{1}{4}x\sin(2x)$$

```
[In] Integrate[x*Sin[x]^2,x]
[Out] x^2/4 - Cos[2*x]/8 - (x*Sin[2*x])/4
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{x^2}{4}-\frac{\cos(2x)}{8}-\frac{x\sin(2x)}{4}$	20
default	$x\left(\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}\right)-\frac{x^2}{4}+\frac{(\sin^2(x))}{4}$	25
meijerg	$\frac{\sqrt{\pi}\left(\frac{2x^2+1}{2\sqrt{\pi}}-\frac{\cos(2x)}{2\sqrt{\pi}}-\frac{x\sin(2x)}{\sqrt{\pi}}\right)}{4}$	38
norman	$\frac{\tan^2(\frac{x}{2})+(\tan^3(\frac{x}{2}))x+\frac{x^2}{4}-x\tan(\frac{x}{2})+\frac{x^2(\tan^2(\frac{x}{2}))}{2}+\frac{x^2(\tan^4(\frac{x}{2}))}{4}}{(1+\tan^2(\frac{x}{2}))^2}$	61

```
[In] int(x*sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x^2-1/8*cos(2*x)-1/4*x*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\sin^2(x)\,dx = -\frac{1}{2}\,x\cos\left(x\right)\sin\left(x\right) + \frac{1}{4}\,x^2 - \frac{1}{4}\,\cos\left(x\right)^2$$

```
[In] integrate(x*sin(x)^2,x, algorithm="fricas")
[Out] -1/2*x*cos(x)*sin(x) + 1/4*x^2 - 1/4*cos(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int x\sin^2(x)\,dx = \frac{x^2\sin^2\left(x\right)}{4} + \frac{x^2\cos^2\left(x\right)}{4} - \frac{x\sin\left(x\right)\cos\left(x\right)}{2} - \frac{\cos^2\left(x\right)}{4}$$

```
[In] integrate(x*sin(x)**2,x)
[Out] x**2*sin(x)**2/4 + x**2*cos(x)**2/4 - x*sin(x)*cos(x)/2 - cos(x)**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\sin^2(x)\,dx = \frac{1}{4}\,x^2 - \frac{1}{4}\,x\sin\left(2\,x\right) - \frac{1}{8}\,\cos\left(2\,x\right)$$

```
[In] integrate(x*sin(x)^2,x, algorithm="maxima")
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\sin^2(x)\,dx = \frac{1}{4}\,x^2 - \frac{1}{4}\,x\sin\left(2\,x\right) - \frac{1}{8}\,\cos\left(2\,x\right)$$

```
[In] integrate(x*sin(x)^2,x, algorithm="giac")
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\sin^2(x)\,dx = \frac{\sin(x)^2}{4} - \frac{x\,\sin(2\,x)}{4} + \frac{x^2}{4}$$

```
[In] int(x*sin(x)^2,x)
[Out] sin(x)^2/4 - (x*sin(2*x))/4 + x^2/4
```

3.34 $\int x\sin^3(x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 33

$$\int x\sin^3(x)\,dx = -\frac{2}{3}x\cos(x)+\frac{2\sin(x)}{3}-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}$$

```
[Out] -2/3*x*cos(x)+2/3*sin(x)-1/3*x*cos(x)*sin(x)^2+1/9*sin(x)^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3391, 3377, 2717}

$$\int x\sin^3(x)\,dx = \frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}-\frac{2}{3}x\cos(x)-\frac{1}{3}x\sin^2(x)\cos(x)$$

```
[In] Int[x*Sin[x]^3,x]
```

```
[Out] (-2*x*Cos[x])/3 + (2*Sin[x])/3 - (x*Cos[x]*Sin[x]^2)/3 + Sin[x]^3/9
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{3}x\cos(x)\sin^2(x) + \frac{\sin^3(x)}{9} + \frac{2}{3}\int x\sin(x)\,dx \\ &= -\frac{2}{3}x\cos(x) - \frac{1}{3}x\cos(x)\sin^2(x) + \frac{\sin^3(x)}{9} + \frac{2}{3}\int \cos(x)\,dx \\ &= -\frac{2}{3}x\cos(x) + \frac{2\sin(x)}{3} - \frac{1}{3}x\cos(x)\sin^2(x) + \frac{\sin^3(x)}{9}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int x\sin^3(x)\,dx = -\frac{3}{4}x\cos(x) + \frac{1}{12}x\cos(3x) + \frac{3\sin(x)}{4} - \frac{1}{36}\sin(3x)$$

```
[In] Integrate[x*Sin[x]^3,x]
[Out] (-3*x*Cos[x])/4 + (x*Cos[3*x])/12 + (3*Sin[x])/4 - Sin[3*x]/36
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{x(2+\sin^2(x))\cos(x)}{3} + \frac{(\sin^3(x))}{9} + \frac{2\sin(x)}{3}$	23
risch	$-\frac{3x\cos(x)}{4} + \frac{3\sin(x)}{4} + \frac{x\cos(3x)}{12} - \frac{\sin(3x)}{36}$	24
parallelrisch	$-\frac{3x\cos(x)}{4} + \frac{3\sin(x)}{4} + \frac{x\cos(3x)}{12} - \frac{\sin(3x)}{36}$	24
norman	$\frac{-\frac{2x}{3}+\frac{32\left(\tan^3\left(\frac{x}{2}\right)\right)}{9}+\frac{4\left(\tan^5\left(\frac{x}{2}\right)\right)}{3}-2x\left(\tan^2\left(\frac{x}{2}\right)\right)+2x\left(\tan^4\left(\frac{x}{2}\right)\right)+\frac{2x\left(\tan^6\left(\frac{x}{2}\right)\right)}{3}+\frac{4\tan\left(\frac{x}{2}\right)}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$	65

```
[In] int(x*sin(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/3*x*(2+sin(x)^2)*cos(x)+1/9*sin(x)^3+2/3*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int x\sin^3(x)\,dx = \frac{1}{3}\,x\cos\left(x\right)^3 - x\cos\left(x\right) - \frac{1}{9}\,\left(\cos\left(x\right)^2 - 7\right)\sin\left(x\right)$$

```
[In] integrate(x*sin(x)^3,x, algorithm="fricas")
[Out] 1/3*x*cos(x)^3 - x*cos(x) - 1/9*(cos(x)^2 - 7)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int x\sin^3(x)\,dx = -x\sin^2\left(x\right)\cos\left(x\right) - \frac{2x\cos^3\left(x\right)}{3} + \frac{7\sin^3\left(x\right)}{9} + \frac{2\sin\left(x\right)\cos^2\left(x\right)}{3}$$

```
[In] integrate(x*sin(x)**3,x)
[Out] -x*sin(x)**2*cos(x) - 2*x*cos(x)**3/3 + 7*sin(x)**3/9 + 2*sin(x)*cos(x)**2/
3
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int x\sin^3(x)\,dx = \frac{1}{12}\,x\cos\left(3\,x\right) - \frac{3}{4}\,x\cos\left(x\right) - \frac{1}{36}\,\sin\left(3\,x\right) + \frac{3}{4}\,\sin\left(x\right)$$

```
[In] integrate(x*sin(x)^3,x, algorithm="maxima")
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int x\sin^3(x)\,dx = \frac{1}{12}\,x\cos\left(3\,x\right) - \frac{3}{4}\,x\cos\left(x\right) - \frac{1}{36}\,\sin\left(3\,x\right) + \frac{3}{4}\,\sin\left(x\right)$$

```
[In] integrate(x*sin(x)^3,x, algorithm="giac")
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int x\sin^3(x)\,dx = \frac{x\cos(x)^3}{3} - \frac{\sin(x)\ \cos(x)^2}{9} - x\ \cos(x) + \frac{7\ \sin(x)}{9}$$

```
[In] int(x*sin(x)^3,x)
[Out] (7*sin(x))/9 + (x*cos(x)^3)/3 - (cos(x)^2*sin(x))/9 - x*cos(x)
```

3.35 $\int x^2 \sin^2(x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int x^2 \sin^2(x) \, dx = -\frac{x}{4} + \frac{x^3}{6} + \frac{1}{4} \cos(x) \sin(x) - \frac{1}{2} x^2 \cos(x) \sin(x) + \frac{1}{2} x \sin^2(x)$$

```
[Out] -1/4*x+1/6*x^3+1/4*cos(x)*sin(x)-1/2*x^2*cos(x)*sin(x)+1/2*x*sin(x)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3392, 30, 2715, 8}

$$\int x^2 \sin^2(x) \, dx = \frac{x^3}{6} - \frac{1}{2} x^2 \sin(x) \cos(x) - \frac{x}{4} + \frac{1}{2} x \sin^2(x) + \frac{1}{4} \sin(x) \cos(x)$$

```
[In] Int[x^2*Sin[x]^2,x]
```

```
[Out] -1/4*x + x^3/6 + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{2}x^2\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x) + \frac{\int x^2\,dx}{2} - \frac{1}{2}\int \sin^2(x)\,dx \\
&= \frac{x^3}{6} + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x) - \frac{\int 1\,dx}{4} \\
&= -\frac{x}{4} + \frac{x^3}{6} + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int x^2\sin^2(x)\,dx = \frac{1}{24}\left(4x^3 - 6x\cos(2x) + \left(3 - 6x^2\right)\sin(2x)\right)$$

```
[In] Integrate[x^2*Sin[x]^2,x]
[Out] (4*x^3 - 6*x*Cos[2*x] + (3 - 6*x^2)*Sin[2*x])/24
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

method	result	size
meijerg	$\frac{x^{5} {}_{2}F_{3}\left(1,\frac{5}{2};\frac{3}{2},2,\frac{7}{2};-x^{2}\right)}{5}$	19
risch	$\frac{x^{3}}{6}-\frac{x\cos\left(2x\right)}{4}-\frac{\left(2x^{2}-1\right)\sin\left(2x\right)}{8}$	27
default	$x^{2}\left(\frac{x}{2}-\frac{\cos\left(x\right)\sin\left(x\right)}{2}\right)-\frac{\left(\cos^{2}\left(x\right)\right)x}{2}+\frac{\cos\left(x\right)\sin\left(x\right)}{4}+\frac{x}{4}-\frac{x^{3}}{3}$	37
norman	$\frac{x^{2}\left(\tan^{3}\left(\frac{x}{2}\right)\right)-\frac{x}{4}+\frac{x^{3}}{6}-\frac{\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\frac{x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{4}-x^{2}\tan\left(\frac{x}{2}\right)+\frac{x^{3}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{3}+\frac{x^{3}\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{6}+\frac{\tan\left(\frac{x}{2}\right)}{2}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	94

```
[In] int(x^2*sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/5*x^5*hypergeom([1,5/2],[3/2,2,7/2],-x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int x^2\sin^2(x)\,dx=\frac{1}{6}\,x^{3}-\frac{1}{2}\,x\cos\left(x\right)^{2}-\frac{1}{4}\,\left(2\,x^{2}-1\right)\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\,x$$

```
[In] integrate(x^2*sin(x)^2,x, algorithm="fricas")
[Out] 1/6*x^3 - 1/2*x*cos(x)^2 - 1/4*(2*x^2 - 1)*cos(x)*sin(x) + 1/4*x
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int x^2\sin^2(x)\,dx=\frac{x^{3}\sin^{2}\left(x\right)}{6}+\frac{x^{3}\cos^{2}\left(x\right)}{6}-\frac{x^{2}\sin\left(x\right)\cos\left(x\right)}{2}+\frac{x\sin^{2}\left(x\right)}{4}-\frac{x\cos^{2}\left(x\right)}{4}+\frac{\sin\left(x\right)\cos\left(x\right)}{4}$$

```
[In] integrate(x**2*sin(x)**2,x)
[Out] x**3*sin(x)**2/6 + x**3*cos(x)**2/6 - x**2*sin(x)*cos(x)/2 + x*sin(x)**2/4
- x*cos(x)**2/4 + sin(x)*cos(x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int x^2 \sin^2(x)\,dx = \frac{1}{6}\,x^3 - \frac{1}{4}\,x\cos\left(2\,x\right) - \frac{1}{8}\,\left(2\,x^2 - 1\right)\sin\left(2\,x\right)$$

```
[In] integrate(x^2*sin(x)^2,x, algorithm="maxima")
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int x^2 \sin^2(x)\,dx = \frac{1}{6}\,x^3 - \frac{1}{4}\,x\cos\left(2\,x\right) - \frac{1}{8}\,\left(2\,x^2 - 1\right)\sin\left(2\,x\right)$$

```
[In] integrate(x^2*sin(x)^2,x, algorithm="giac")
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int x^2 \sin^2(x)\,dx = \frac{\sin\left(2\,x\right)}{8} - \frac{x\,\cos\left(2\,x\right)}{4} - \frac{x^2\,\sin\left(2\,x\right)}{4} + \frac{x^3}{6}$$

```
[In] int(x^2*sin(x)^2,x)
[Out] sin(2*x)/8 - (x*cos(2*x))/4 - (x^2*sin(2*x))/4 + x^3/6
```

3.36 $\int \cos^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cos^2(x)\,dx=\frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\int \cos^2(x)\,dx=\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\cos(x)\sin(x) + \frac{\int 1\,dx}{2} \\ &= \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{4}\sin(2x)$$

```
[In] Integrate[Cos[x]^2,x]
[Out] x/2 + Sin[2*x]/4
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

```
[In] int(cos(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

```
[In] integrate(cos(x)**2,x)
[Out] x/2 + sin(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}x + \frac{1}{4}\sin(2x)$$

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
[Out] 1/2*x + 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}x + \frac{1}{4}\sin(2x)$$

```
[In] integrate(cos(x)^2,x, algorithm="giac")
[Out] 1/2*x + 1/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(2\,x)}{4}$$

```
[In] int(cos(x)^2,x)
[Out] x/2 + sin(2*x)/4
```

3.37 $\int \cos^3(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 11

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin^3(x)}{3}$$

```
[Out] sin(x)-1/3*sin(x)^3
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2713}

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin^3(x)}{3}$$

```
[In] Int[Cos[x]^3,x]
```

```
[Out] Sin[x] - Sin[x]^3/3
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(x)\right) \\ &= \sin(x) - \frac{\sin^3(x)}{3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin^3(x)}{3}$$

```
[In] Integrate[Cos[x]^3,x]
[Out] Sin[x] - Sin[x]^3/3
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{(2+\cos^2(x))\sin(x)}{3}$	11
risch	$\frac{3\sin(x)}{4} + \frac{\sin(3x)}{12}$	12
parallelrisch	$\frac{3\sin(x)}{4} + \frac{\sin(3x)}{12}$	12

```
[In] int(cos(x)^3,x,method=_RETURNVERBOSE)
[Out] 1/3*(2+cos(x)^2)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \cos^3(x)\,dx = \frac{1}{3}\left(\cos\left(x\right)^2 + 2\right)\sin\left(x\right)$$

```
[In] integrate(cos(x)^3,x, algorithm="fricas")
[Out] 1/3*(cos(x)^2 + 2)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \cos^3(x)\,dx = -\frac{\sin^3(x)}{3} + \sin(x)$$

```
[In] integrate(cos(x)**3,x)
[Out] -sin(x)**3/3 + sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = -\frac{1}{3}\sin(x)^3 + \sin(x)$$

```
[In] integrate(cos(x)^3,x, algorithm="maxima")
[Out] -1/3*sin(x)^3 + sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = -\frac{1}{3}\sin(x)^3 + \sin(x)$$

```
[In] integrate(cos(x)^3,x, algorithm="giac")
[Out] -1/3*sin(x)^3 + sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin(x)^3}{3}$$

```
[In] int(cos(x)^3,x)
[Out] sin(x) - sin(x)^3/3
```

3.38 $\int \cos^4(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 24

$$\int \cos^4(x)\,dx = \frac{3x}{8} + \frac{3}{8}\cos(x)\sin(x) + \frac{1}{4}\cos^3(x)\sin(x)$$

```
[Out] 3/8*x+3/8*cos(x)*sin(x)+1/4*cos(x)^3*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\int \cos^4(x)\,dx = \frac{3x}{8} + \frac{1}{4}\sin(x)\cos^3(x) + \frac{3}{8}\sin(x)\cos(x)$$

```
[In] Int[Cos[x]^4,x]
```

```
[Out] (3*x)/8 + (3*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\cos^3(x)\sin(x) + \frac{3}{4}\int \cos^2(x)\,dx \\
&= \frac{3}{8}\cos(x)\sin(x) + \frac{1}{4}\cos^3(x)\sin(x) + \frac{3\int 1\,dx}{8} \\
&= \frac{3x}{8} + \frac{3}{8}\cos(x)\sin(x) + \frac{1}{4}\cos^3(x)\sin(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \cos^4(x)\,dx = \frac{3x}{8} + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)$$

```
[In] Integrate[Cos[x]^4,x]
[Out] (3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{3x}{8}+\frac{\sin(4x)}{32}+\frac{\sin(2x)}{4}$	17
parallelrisch	$\frac{3x}{8}+\frac{\sin(4x)}{32}+\frac{\sin(2x)}{4}$	17
default	$\frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{4}+\frac{3x}{8}$	18
norman	$\frac{\frac{3x}{8}-\frac{3\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}+\frac{3\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}-\frac{5\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{9x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}+\frac{5\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	82

```
[In] int(cos(x)^4,x,method=_RETURNVERBOSE)
[Out] 3/8*x+1/32*sin(4*x)+1/4*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \cos^4(x)\,dx = \frac{1}{8}\left(2\,\cos\left(x\right)^3 + 3\,\cos\left(x\right)\right)\sin\left(x\right) + \frac{3}{8}\,x$$

```
[In] integrate(cos(x)^4,x, algorithm="fricas")
[Out] 1/8*(2*cos(x)^3 + 3*cos(x))*sin(x) + 3/8*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cos^4(x)\,dx = \frac{3x}{8} + \frac{\sin\left(x\right)\cos^3\left(x\right)}{4} + \frac{3\sin\left(x\right)\cos\left(x\right)}{8}$$

```
[In] integrate(cos(x)**4,x)
[Out] 3*x/8 + sin(x)*cos(x)**3/4 + 3*sin(x)*cos(x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\,dx = \frac{3}{8}\,x + \frac{1}{32}\,\sin\left(4\,x\right) + \frac{1}{4}\,\sin\left(2\,x\right)$$

```
[In] integrate(cos(x)^4,x, algorithm="maxima")
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\,dx = \frac{3}{8}\,x + \frac{1}{32}\,\sin\left(4\,x\right) + \frac{1}{4}\,\sin\left(2\,x\right)$$

```
[In] integrate(cos(x)^4,x, algorithm="giac")
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\,dx = \frac{3\,x}{8} + \frac{\sin\left(2\,x\right)}{4} + \frac{\sin\left(4\,x\right)}{32}$$

```
[In] int(cos(x)^4,x)
[Out] (3*x)/8 + sin(2*x)/4 + sin(4*x)/32
```

3.39 $\int (a^2-x^2)^{5/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int (a^2-x^2)^{5/2}\,dx=\frac{5}{16}a^4x\sqrt{a^2-x^2}+\frac{5}{24}a^2x(a^2-x^2)^{3/2}+\frac{1}{6}x(a^2-x^2)^{5/2}+\frac{5}{16}a^6\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[Out] 5/24*a^2*x*(a^2-x^2)^(3/2)+1/6*x*(a^2-x^2)^(5/2)+5/16*a^6*arctan(x/(a^2-x^2
)^(1/2))+5/16*a^4*x*(a^2-x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {201, 223, 209}

$$\int (a^2-x^2)^{5/2}\,dx=\frac{5}{24}a^2x(a^2-x^2)^{3/2}+\frac{1}{6}x(a^2-x^2)^{5/2}+\frac{5}{16}a^6\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)+\frac{5}{16}a^4x\sqrt{a^2-x^2}$$

```
[In] Int[(a^2 - x^2)^(5/2),x]
```

```
[Out] (5*a^4*x*Sqrt[a^2 - x^2])/16 + (5*a^2*x*(a^2 - x^2)^(3/2))/24 + (x*(a^2 - x
^2)^(5/2))/6 + (5*a^6*ArcTan[x/Sqrt[a^2 - x^2]])/16
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{1}{6}\left(5a^2\right)\int\left(a^2-x^2\right)^{3/2}\,dx\\
&= \frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{1}{8}\left(5a^4\right)\int\sqrt{a^2-x^2}\,dx\\
&= \frac{5}{16}a^4x\sqrt{a^2-x^2}+\frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{1}{16}\left(5a^6\right)\int\frac{1}{\sqrt{a^2-x^2}}\,dx\\
&= \frac{5}{16}a^4x\sqrt{a^2-x^2}+\frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}\\
&\quad+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{1}{16}\left(5a^6\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt{a^2-x^2}}\right)\\
&= \frac{5}{16}a^4x\sqrt{a^2-x^2}+\frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{5}{16}a^6\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int\left(a^2-x^2\right)^{5/2}\,dx=\frac{1}{48}\sqrt{a^2-x^2}\left(33a^4x-26a^2x^3+8x^5\right)+\frac{5}{16}a^6\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[In] Integrate[(a^2 - x^2)^(5/2),x]
[Out] (Sqrt[a^2 - x^2]*(33*a^4*x - 26*a^2*x^3 + 8*x^5))/48 + (5*a^6*ArcTan[x/Sqrt
[a^2 - x^2]])/16
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{x(33a^4-26a^2x^2+8x^4)\sqrt{a^2-x^2}}{48}+\frac{5a^6\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)}{16}$	54
pseudoelliptic	$-\frac{5\arctan\left(\frac{\sqrt{a^2-x^2}}{x}\right)a^6}{16}+\frac{11\sqrt{a^2-x^2}\,(a^4-\frac{26}{33}a^2x^2+\frac{8}{33}x^4)x}{16}$	54
default	$\frac{x(a^2-x^2)^{\frac{5}{2}}}{6}+\frac{5a^2\left(\frac{(a^2-x^2)^{\frac{3}{2}}x}{4}+\frac{3a^2\left(\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)}{2}\right)}{4}\right)}{6}$	75

```
[In] int((a^2-x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/48*x*(33*a^4-26*a^2*x^2+8*x^4)*(a^2-x^2)^(1/2)+5/16*a^6*arctan(x/(a^2-x^2
)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int (a^2-x^2)^{5/2}\,dx=-\frac{5}{8}\,a^6\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)+\frac{1}{48}\,(33\,a^4x-26\,a^2x^3+8\,x^5)\sqrt{a^2-x^2}$$

```
[In] integrate((a^2-x^2)^(5/2),x, algorithm="fricas")
[Out] -5/8*a^6*arctan(-(a - sqrt(a^2 - x^2))/x) + 1/48*(33*a^4*x - 26*a^2*x^3 + 8
*x^5)*sqrt(a^2 - x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.14

$$\int (a^2-x^2)^{5/2}\,dx=\begin{cases}-\frac{5ia^6\operatorname{acosh}\left(\frac{x}{a}\right)}{16}-\frac{11ia^5x}{16\sqrt{-1+\frac{x^2}{a^2}}}+\frac{59ia^3x^3}{48\sqrt{-1+\frac{x^2}{a^2}}}-\frac{17iax^5}{24\sqrt{-1+\frac{x^2}{a^2}}}+\frac{ix^7}{6a\sqrt{-1+\frac{x^2}{a^2}}} & \text{for}\ \left|\frac{x^2}{a^2}\right|>1\\ \frac{5a^6\operatorname{asin}\left(\frac{x}{a}\right)}{16}+\frac{11a^5x\sqrt{1-\frac{x^2}{a^2}}}{16}-\frac{13a^3x^3\sqrt{1-\frac{x^2}{a^2}}}{24}+\frac{ax^5\sqrt{1-\frac{x^2}{a^2}}}{6} & \text{otherwise}\end{cases}$$

```
[In] integrate((a**2-x**2)**(5/2),x)
```

```
[Out] Piecewise((-5*I*a**6*acosh(x/a)/16 - 11*I*a**5*x/(16*sqrt(-1 + x**2/a**2))
+ 59*I*a**3*x**3/(48*sqrt(-1 + x**2/a**2)) - 17*I*a*x**5/(24*sqrt(-1 + x**2
/a**2)) + I*x**7/(6*a*sqrt(-1 + x**2/a**2)), Abs(x**2/a**2) > 1), (5*a**6*a
sin(x/a)/16 + 11*a**5*x*sqrt(1 - x**2/a**2)/16 - 13*a**3*x**3*sqrt(1 - x**2
/a**2)/24 + a*x**5*sqrt(1 - x**2/a**2)/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int (a^2-x^2)^{5/2}\, dx = \frac{5}{16}\, a^6 \arcsin\left(\frac{x}{a}\right) + \frac{5}{16}\, \sqrt{a^2-x^2} a^4 x + \frac{5}{24}\, \left(a^2-x^2\right)^{\frac{3}{2}} a^2 x + \frac{1}{6}\, \left(a^2-x^2\right)^{\frac{5}{2}} x$$

```
[In] integrate((a^2-x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^6*arcsin(x/a) + 5/16*sqrt(a^2 - x^2)*a^4*x + 5/24*(a^2 - x^2)^(3/2)*
a^2*x + 1/6*(a^2 - x^2)^(5/2)*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.60

$$\int (a^2-x^2)^{5/2}\, dx = \frac{5}{16}\, a^6 \arcsin\left(\frac{x}{a}\right) \operatorname{sgn}(a) + \frac{1}{48}\, \left(33\, a^4 - 2\, \left(13\, a^2 - 4\, x^2\right) x^2\right) \sqrt{a^2-x^2} x$$

```
[In] integrate((a^2-x^2)^(5/2),x, algorithm="giac")
```

```
[Out] 5/16*a^6*arcsin(x/a)*sgn(a) + 1/48*(33*a^4 - 2*(13*a^2 - 4*x^2)*x^2)*sqrt(a
^2 - x^2)*x
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.44

$$\int (a^2-x^2)^{5/2}\, dx = \frac{x\, \left(a^2-x^2\right)^{5/2}\, {}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ \frac{x^2}{a^2}\right)}{\left(1-\frac{x^2}{a^2}\right)^{5/2}}$$

```
[In] int((a^2 - x^2)^(5/2),x)
```

```
[Out] (x*(a^2 - x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, x^2/a^2))/(1 - x^2/a^2)^(5
/2)
```

3.40 $\int \frac{x^5}{\sqrt{5+x^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{x^5}{\sqrt{5+x^2}} \, dx = 25\sqrt{5+x^2} - \frac{10}{3}\left(5+x^2\right)^{3/2} + \frac{1}{5}\left(5+x^2\right)^{5/2}$$

```
[Out] -10/3*(x^2+5)^(3/2)+1/5*(x^2+5)^(5/2)+25*(x^2+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\int \frac{x^5}{\sqrt{5+x^2}} \, dx = \frac{1}{5}\left(x^2+5\right)^{5/2} - \frac{10}{3}\left(x^2+5\right)^{3/2} + 25\sqrt{x^2+5}$$

```
[In] Int[x^5/Sqrt[5 + x^2],x]
```

```
[Out] 25*Sqrt[5 + x^2] - (10*(5 + x^2)^(3/2))/3 + (5 + x^2)^(5/2)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{5+x}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{25}{\sqrt{5+x}} - 10\sqrt{5+x} + (5+x)^{3/2}\right)\,dx, x, x^2\right)\\ &= 25\sqrt{5+x^2} - \frac{10}{3}\left(5+x^2\right)^{3/2} + \frac{1}{5}\left(5+x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{x^5}{\sqrt{5+x^2}}\,dx = \frac{1}{15}\sqrt{5+x^2}\left(200-20x^2+3x^4\right)$$

```
[In] Integrate[x^5/Sqrt[5 + x^2],x]
[Out] (Sqrt[5 + x^2]*(200 - 20*x^2 + 3*x^4))/15
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

method	result	size
trager	$\sqrt{x^2+5}\,\left(\frac{1}{5}x^4-\frac{4}{3}x^2+\frac{40}{3}\right)$	21
gosper	$\frac{\sqrt{x^2+5}\,\left(3x^4-20x^2+200\right)}{15}$	22
risch	$\frac{\sqrt{x^2+5}\,\left(3x^4-20x^2+200\right)}{15}$	22
pseudoelliptic	$\frac{\sqrt{x^2+5}\,\left(3x^4-20x^2+200\right)}{15}$	22
default	$\frac{x^4\sqrt{x^2+5}}{5}-\frac{4x^2\sqrt{x^2+5}}{3}+\frac{40\sqrt{x^2+5}}{3}$	35
meijerg	$\frac{25\sqrt{5}\left(-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\left(\frac{6}{25}x^4-\frac{8}{5}x^2+16\right)\sqrt{1+\frac{x^2}{5}}}{15}\right)}{2\sqrt{\pi}}$	41

```
[In] int(x^5/(x^2+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] (x^2+5)^(1/2)*(1/5*x^4-4/3*x^2+40/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

$$\int \frac{x^5}{\sqrt{5+x^2}}\,dx = \frac{1}{15}\left(3\,x^4-20\,x^2+200\right)\sqrt{x^2+5}$$

```
[In] integrate(x^5/(x^2+5)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*x^4 - 20*x^2 + 200)*sqrt(x^2 + 5)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{x^5}{\sqrt{5+x^2}}\,dx = \frac{x^4\sqrt{x^2+5}}{5} - \frac{4x^2\sqrt{x^2+5}}{3} + \frac{40\sqrt{x^2+5}}{3}$$

```
[In] integrate(x**5/(x**2+5)**(1/2),x)
[Out] x**4*sqrt(x**2 + 5)/5 - 4*x**2*sqrt(x**2 + 5)/3 + 40*sqrt(x**2 + 5)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{\sqrt{5+x^2}}\,dx = \frac{1}{5}\,\sqrt{x^2+5}x^4 - \frac{4}{3}\,\sqrt{x^2+5}x^2 + \frac{40}{3}\,\sqrt{x^2+5}$$

```
[In] integrate(x^5/(x^2+5)^(1/2),x, algorithm="maxima")
[Out] 1/5*sqrt(x^2 + 5)*x^4 - 4/3*sqrt(x^2 + 5)*x^2 + 40/3*sqrt(x^2 + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{\sqrt{5+x^2}}\,dx = \frac{1}{5}\left(x^2+5\right)^{\frac{5}{2}} - \frac{10}{3}\left(x^2+5\right)^{\frac{3}{2}} + 25\,\sqrt{x^2+5}$$

```
[In] integrate(x^5/(x^2+5)^(1/2),x, algorithm="giac")
[Out] 1/5*(x^2 + 5)^(5/2) - 10/3*(x^2 + 5)^(3/2) + 25*sqrt(x^2 + 5)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

$$\int \frac{x^5}{\sqrt{5+x^2}}\,dx = \sqrt{x^2+5}\,\left(\frac{x^4}{5}-\frac{4\,x^2}{3}+\frac{40}{3}\right)$$

```
[In] int(x^5/(x^2 + 5)^(1/2),x)
[Out] (x^2 + 5)^(1/2)*(x^4/5 - (4*x^2)/3 + 40/3)
```

3.41 $\int \frac{t^3}{\sqrt{4+t^3}}\,dt$

Optimal result

Integrand size = 13, antiderivative size = 172

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt = \frac{2}{5}t\sqrt{4+t^3} - \frac{8\ 2^{2/3}\sqrt{2+\sqrt{3}}\left(2^{2/3}+t\right)\sqrt{\frac{2\sqrt[3]{2}-2^{2/3}t+t^2}{\left(2^{2/3}\left(1+\sqrt{3}\right)+t\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1-\sqrt{3}\right)+t}{2^{2/3}\left(1+\sqrt{3}\right)+t}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{2^{2/3}+t}{\left(2^{2/3}\left(1+\sqrt{3}\right)+t\right)^2}}\sqrt{4+t^3}}$$

```
[Out] 2/5*t*(t^3+4)^(1/2)-8/15*2^(2/3)*(2^(2/3)+t)*EllipticF((t+2^(2/3)*(1-3^(1/2
)))/(t+2^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((2*2^
(1/3)-2^(2/3)*t+t^2)/(t+2^(2/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(t^3+4)^(1/2)
/((2^(2/3)+t)/(t+2^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {327, 224}

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt = \frac{2}{5}t\sqrt{t^3+4} - \frac{8\ 2^{2/3}\sqrt{2+\sqrt{3}}\left(t+2^{2/3}\right)\sqrt{\frac{t^2-2^{2/3}t+2\sqrt[3]{2}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{t+2^{2/3}\left(1-\sqrt{3}\right)}{t+2^{2/3}\left(1+\sqrt{3}\right)}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{t+2^{2/3}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{t^3+4}}$$

```
[In] Int[t^3/Sqrt[4 + t^3],t]
```

```
[Out] (2*t*Sqrt[4 + t^3])/5 - (8*2^(2/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) + t)*Sqrt[(2*
2^(1/3) - 2^(2/3)*t + t^2)/(2^(2/3)*(1 + Sqrt[3]) + t)^2]*EllipticF[ArcSin[
(2^(2/3)*(1 - Sqrt[3]) + t)/(2^(2/3)*(1 + Sqrt[3]) + t)], -7 - 4*Sqrt[3]])/
(5*3^(1/4)*Sqrt[(2^(2/3) + t)/(2^(2/3)*(1 + Sqrt[3]) + t)^2]*Sqrt[4 + t^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2}{5}t\sqrt{4+t^3} - \frac{8}{5}\int \frac{1}{\sqrt{4+t^3}}\,dt \\ &= \frac{2}{5}t\sqrt{4+t^3} \\ &\quad - \frac{8\ 2^{2/3}\sqrt{2+\sqrt{3}}\left(2^{2/3}+t\right)\sqrt{\frac{2\sqrt[3]{2}-2^{2/3}t+t^2}{\left(2^{2/3}\left(1+\sqrt{3}\right)+t\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1-\sqrt{3}\right)+t}{2^{2/3}\left(1+\sqrt{3}\right)+t}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{2^{2/3}+t}{\left(2^{2/3}\left(1+\sqrt{3}\right)+t\right)^2}}\sqrt{4+t^3}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.20

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt = \frac{2}{5}t\left(\sqrt{4+t^3} - 2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{t^3}{4}\right)\right)$$

```
[In] Integrate[t^3/Sqrt[4 + t^3],t]
```

```
[Out] (2*t*(Sqrt[4 + t^3] - 2*Hypergeometric2F1[1/3, 1/2, 4/3, -1/4*t^3]))/5
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.10

method	result
meijerg	$\frac{t^4\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};-\frac{t^3}{4}\right)}{8}$
default	$\frac{2t\sqrt{t^3+4}}{5}+\frac{8i\sqrt{3}\,2^{\frac{2}{3}}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\sqrt{\frac{2^{\frac{2}{3}}+t}{\frac{3\,2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}}}\sqrt{-i\left(t-\frac{2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\,F\left(\frac{\sqrt{6}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2}}{6}\right.}{15\sqrt{t^3+4}}$
risch	$\frac{2t\sqrt{t^3+4}}{5}+\frac{8i\sqrt{3}\,2^{\frac{2}{3}}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\sqrt{\frac{2^{\frac{2}{3}}+t}{\frac{3\,2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}}}\sqrt{-i\left(t-\frac{2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\,F\left(\frac{\sqrt{6}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2}}{6}\right.}{15\sqrt{t^3+4}}$
elliptic	$\frac{2t\sqrt{t^3+4}}{5}+\frac{8i\sqrt{3}\,2^{\frac{2}{3}}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\sqrt{\frac{2^{\frac{2}{3}}+t}{\frac{3\,2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}}}\sqrt{-i\left(t-\frac{2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\,F\left(\frac{\sqrt{6}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2}}{6}\right.}{15\sqrt{t^3+4}}$

```
[In] int(t^3/(t^3+4)^(1/2),t,method=_RETURNVERBOSE)
[Out] 1/8*t^4*hypergeom([1/2,4/3],[7/3],-1/4*t^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.10

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt = \frac{2}{5}\sqrt{t^3+4}t - \frac{16}{5}\,\mathrm{weierstrassPInverse}(0,-16,t)$$

```
[In] integrate(t^3/(t^3+4)^(1/2),t, algorithm="fricas")
[Out] 2/5*sqrt(t^3 + 4)*t - 16/5*weierstrassPInverse(0, -16, t)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.18

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt = \frac{t^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{t^3e^{i\pi}}{4}\right)}{6\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate(t**3/(t**3+4)**(1/2),t)
[Out] t**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), t**3*exp_polar(I*pi)/4)/(6*gamma(
7/3))
```

Maxima [F]

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt = \int \frac{t^3}{\sqrt{t^3+4}}\,dt$$

```
[In] integrate(t^3/(t^3+4)^(1/2),t, algorithm="maxima")
[Out] integrate(t^3/sqrt(t^3 + 4), t)
```

Giac [F]

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt = \int \frac{t^3}{\sqrt{t^3+4}}\,dt$$

```
[In] integrate(t^3/(t^3+4)^(1/2),t, algorithm="giac")
[Out] integrate(t^3/sqrt(t^3 + 4), t)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.75

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt = \frac{2\,t\,\sqrt{t^3+4}}{5} - \frac{16\sqrt{-\frac{t-2^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2^{2/3}+2^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\sqrt{-\frac{t+2^{2/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2^{2/3}-2^{2/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\sqrt{\frac{t+2^{2/3}}{2^{2/3}+2^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\left(2^{2/3}+2^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right.}{5\sqrt{t^3+\left(2^{2/3}+2^{2/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-2^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)t^2+\left(2\,2^{1/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-2\,2^{1/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\right.}}$$

```
[In] int(t^3/(t^3 + 4)^(1/2),t)

[Out] (2*t*(t^3 + 4)^(1/2))/5 - (16*(-(t - 2^(2/3)*((3^(1/2)*1i)/2 + 1/2))/(2^(2/
3) + 2^(2/3)*((3^(1/2)*1i)/2 + 1/2)))^(1/2)*(-(t + 2^(2/3)*((3^(1/2)*1i)/2
- 1/2))/(2^(2/3) - 2^(2/3)*((3^(1/2)*1i)/2 - 1/2)))^(1/2)*((t + 2^(2/3))/(2
^(2/3) + 2^(2/3)*((3^(1/2)*1i)/2 + 1/2)))^(1/2)*(2^(2/3) + 2^(2/3)*((3^(1/2
)*1i)/2 + 1/2))*ellipticF(asin(((t + 2^(2/3))/(2^(2/3) + 2^(2/3)*((3^(1/2)*
1i)/2 + 1/2)))^(1/2)), (2^(2/3) + 2^(2/3)*((3^(1/2)*1i)/2 + 1/2))/(2^(2/3)
- 2^(2/3)*((3^(1/2)*1i)/2 - 1/2))))/(5*(t^2*(2^(2/3) + 2^(2/3)*((3^(1/2)*1i
)/2 - 1/2) - 2^(2/3)*((3^(1/2)*1i)/2 + 1/2)) - 4*((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2) + t^3 - t*(2*2^(1/3)*((3^(1/2)*1i)/2 + 1/2) - 2*2^(1/3)
*((3^(1/2)*1i)/2 - 1/2) + 2*2^(1/3)*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2)))^(1/2))
```

3.42 $\int \tan^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 6

$$\int \tan^2(x)\,dx = -x + \tan(x)$$

```
[Out] -x+tan(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$\int \tan^2(x)\,dx = \tan(x) - x$$

```
[In] Int[Tan[x]^2,x]
```

```
[Out] -x + Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \tan(x) - \int 1\,dx \\
&= -x + \tan(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \tan^2(x)\,dx = -\arctan(\tan(x)) + \tan(x)$$

```
[In] Integrate[Tan[x]^2,x]
[Out] -ArcTan[Tan[x]] + Tan[x]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
norman	$-x + \tan(x)$	7
parallelrisch	$-x + \tan(x)$	7
derivativedivides	$\tan(x) - \arctan(\tan(x))$	9
default	$\tan(x) - \arctan(\tan(x))$	9
risch	$-x + \frac{2i}{e^{2ix}+1}$	17

```
[In] int(tan(x)^2,x,method=_RETURNVERBOSE)
[Out] -x+tan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx = -x + \tan(x)$$

```
[In] integrate(tan(x)^2,x, algorithm="fricas")
[Out] -x + tan(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \tan^2(x)\,dx = -x + \frac{\sin(x)}{\cos(x)}$$

```
[In] integrate(tan(x)**2,x)
[Out] -x + sin(x)/cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx = -x + \tan(x)$$

```
[In] integrate(tan(x)^2,x, algorithm="maxima")
[Out] -x + tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx = -x + \tan(x)$$

```
[In] integrate(tan(x)^2,x, algorithm="giac")
[Out] -x + tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx = \tan(x) - x$$

```
[In] int(tan(x)^2,x)
[Out] tan(x) - x
```

3.43 $\int \tan^4(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \tan^4(x)\,dx = x - \tan(x) + \frac{\tan^3(x)}{3}$$

```
[Out] x-tan(x)+1/3*tan(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$\int \tan^4(x)\,dx = x + \frac{\tan^3(x)}{3} - \tan(x)$$

```
[In] Int[Tan[x]^4,x]
```

```
[Out] x - Tan[x] + Tan[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\tan^3(x)}{3} - \int \tan^2(x)\,dx\\ &= -\tan(x) + \frac{\tan^3(x)}{3} + \int 1\,dx\\ &= x - \tan(x) + \frac{\tan^3(x)}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \tan^4(x)\,dx = \arctan(\tan(x)) - \tan(x) + \frac{\tan^3(x)}{3}$$

```
[In] Integrate[Tan[x]^4,x]
[Out] ArcTan[Tan[x]] - Tan[x] + Tan[x]^3/3
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
norman	$x-\tan \left(x \right)+\frac{\left(\tan^{3}\left(x \right)\right)}{3}$	13
parallelrisch	$x-\tan \left(x \right)+\frac{\left(\tan^{3}\left(x \right)\right)}{3}$	13
derivativedivides	$\frac{\left(\tan^{3}\left(x \right)\right)}{3}-\tan \left(x \right)+\arctan \left(\tan \left(x \right)\right)$	15
default	$\frac{\left(\tan^{3}\left(x \right)\right)}{3}-\tan \left(x \right)+\arctan \left(\tan \left(x \right)\right)$	15
risch	$x-\frac{4 i \left(3 \,{\mathrm e}^{4 i x}+3 \,{\mathrm e}^{2 i x}+2\right)}{3 \left({\mathrm e}^{2 i x}+1\right)^{3}}$	31

```
[In] int(tan(x)^4,x,method=_RETURNVERBOSE)
[Out] x-tan(x)+1/3*tan(x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\,dx = \frac{1}{3}\,\tan\left(x\right)^3 + x - \tan\left(x\right)$$

```
[In] integrate(tan(x)^4,x, algorithm="fricas")
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \tan^4(x)\,dx = x + \frac{\sin^3\left(x\right)}{3\cos^3\left(x\right)} - \frac{\sin\left(x\right)}{\cos\left(x\right)}$$

```
[In] integrate(tan(x)**4,x)
[Out] x + sin(x)**3/(3*cos(x)**3) - sin(x)/cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\,dx = \frac{1}{3}\,\tan\left(x\right)^3 + x - \tan\left(x\right)$$

```
[In] integrate(tan(x)^4,x, algorithm="maxima")
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\,dx = \frac{1}{3}\,\tan\left(x\right)^3 + x - \tan\left(x\right)$$

```
[In] integrate(tan(x)^4,x, algorithm="giac")
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\,dx = \frac{\tan(x)^3}{3} - \tan(x) + x$$

```
[In] int(tan(x)^4,x)
[Out] x - tan(x) + tan(x)^3/3
```

3.44 $\int \cot^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 8

$$\int \cot^2(x)\,dx = -x - \cot(x)$$

```
[Out] -x-cot(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$\int \cot^2(x)\,dx = -x - \cot(x)$$

```
[In] Int[Cot[x]^2,x]
```

```
[Out] -x - Cot[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\cot(x) - \int 1\,dx\\ &= -x - \cot(x)\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \cot^2(x)\,dx = -\cot(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(x)\right)$$

```
[In] Integrate[Cot[x]^2,x]
[Out] -(Cot[x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[x]^2])
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62

method	result	size
norman	$\frac{-1-x\tan(x)}{\tan(x)}$	13
parallelrisch	$\frac{-1-x\tan(x)}{\tan(x)}$	13
derivativedivides	$-\cot\left(x\right)+\frac{\pi}{2}-\operatorname{arccot}\left(\cot\left(x\right)\right)$	14
default	$-\cot\left(x\right)+\frac{\pi}{2}-\operatorname{arccot}\left(\cot\left(x\right)\right)$	14
risch	$-x-\frac{2i}{e^{2ix}-1}$	17

```
[In] int(cot(x)^2,x,method=_RETURNVERBOSE)
[Out] (-1-x*tan(x))/tan(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \cot^2(x)\,dx = -\frac{x\sin(2x) + \cos(2x) + 1}{\sin(2x)}$$

```
[In] integrate(cot(x)^2,x, algorithm="fricas")
[Out] -(x*sin(2*x) + cos(2*x) + 1)/sin(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cot^2(x)\,dx = -x - \frac{\cos(x)}{\sin(x)}$$

```
[In] integrate(cot(x)**2,x)
[Out] -x - cos(x)/sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \cot^2(x)\,dx = -x - \frac{1}{\tan(x)}$$

```
[In] integrate(cot(x)^2,x, algorithm="maxima")
[Out] -x - 1/tan(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(8) = 16$.

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \cot^2(x)\,dx = -x - \frac{1}{2\tan\left(\frac{1}{2}x\right)} + \frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

```
[In] integrate(cot(x)^2,x, algorithm="giac")
[Out] -x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cot^2(x)\,dx = -x - \cot(x)$$

```
[In] int(cot(x)^2,x)
```

```
[Out] - x - cot(x)
```

3.45 $\int \cot^4(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 12

$$\int \cot^4(x)\,dx = x + \cot(x) - \frac{\cot^3(x)}{3}$$

```
[Out] x+cot(x)-1/3*cot(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 8}

$$\int \cot^4(x)\,dx = x - \frac{1}{3}\cot^3(x) + \cot(x)$$

```
[In] Int[Cot[x]^4,x]
```

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{3}\cot^3(x) - \int \cot^2(x)\,dx\\ &= \cot(x) - \frac{\cot^3(x)}{3} + \int 1\,dx\\ &= x + \cot(x) - \frac{\cot^3(x)}{3}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \cot^4(x)\,dx = -\frac{1}{3}\cot^3(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(x)\right)$$

```
[In] Integrate[Cot[x]^4,x]
[Out] -1/3*(Cot[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[x]^2])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$x+\cot(x)-\frac{(\cot^3(x))}{3}$	11
derivativedivides	$-\frac{(\cot^3(x))}{3}+\cot(x)-\frac{\pi}{2}+\operatorname{arccot}(\cot(x))$	16
default	$-\frac{(\cot^3(x))}{3}+\cot(x)-\frac{\pi}{2}+\operatorname{arccot}(\cot(x))$	16
norman	$\frac{-\frac{1}{3}+\tan^2(x)+x(\tan^3(x))}{\tan(x)^3}$	18
risch	$x+\frac{4i(3\,e^{4ix}-3\,e^{2ix}+2)}{3(e^{2ix}-1)^3}$	31

```
[In] int(cot(x)^4,x,method=_RETURNVERBOSE)
[Out] x+cot(x)-1/3*cot(x)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.00

$$\int \cot^4(x)\,dx = \frac{4\cos(2x)^2 + 3(x\cos(2x) - x)\sin(2x) + 2\cos(2x) - 2}{3(\cos(2x) - 1)\sin(2x)}$$

```
[In] integrate(cot(x)^4,x, algorithm="fricas")
[Out] 1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*x
) - 1)*sin(2*x))
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \cot^4(x)\,dx = x + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3\sin^3(x)}$$

```
[In] integrate(cot(x)**4,x)
[Out] x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \cot^4(x)\,dx = x + \frac{3\tan(x)^2 - 1}{3\tan(x)^3}$$

```
[In] integrate(cot(x)^4,x, algorithm="maxima")
[Out] x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(10) = 20.

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.83

$$\int \cot^4(x)\,dx = \frac{1}{24}\tan\left(\frac{1}{2}x\right)^3 + x + \frac{15\tan\left(\frac{1}{2}x\right)^2 - 1}{24\tan\left(\frac{1}{2}x\right)^3} - \frac{5}{8}\tan\left(\frac{1}{2}x\right)$$

```
[In] integrate(cot(x)^4,x, algorithm="giac")
[Out] 1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(1
/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \cot^4(x)\,dx = -\frac{\cot(x)^3}{3} + \cot(x) + x$$

```
[In] int(cot(x)^4,x)
[Out] x + cot(x) - cot(x)^3/3
```

3.46 $\int (2+3x) \sin (5x) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 22

$$\int (2+3x) \sin (5x) \, dx = -\frac{1}{5}(2+3x) \cos (5x)+\frac{3}{25} \sin (5x)$$

```
[Out] -1/5*(2+3*x)*cos(5*x)+3/25*sin(5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3377, 2717}

$$\int (2+3x) \sin (5x) \, dx = \frac{3}{25} \sin (5x)-\frac{1}{5}(3x+2) \cos (5x)$$

```
[In] Int[(2 + 3*x)*Sin[5*x],x]
```

```
[Out] -1/5*((2 + 3*x)*Cos[5*x]) + (3*Sin[5*x])/25
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{5}(2+3x)\cos(5x)+\frac{3}{5}\int\cos(5x)\,dx\\ &= -\frac{1}{5}(2+3x)\cos(5x)+\frac{3}{25}\sin(5x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int(2+3x)\sin(5x)\,dx=-\frac{2}{5}\cos(5x)-\frac{3}{5}x\cos(5x)+\frac{3}{25}\sin(5x)$$

```
[In] Integrate[(2 + 3*x)*Sin[5*x],x]
[Out] (-2*Cos[5*x])/5 - (3*x*Cos[5*x])/5 + (3*Sin[5*x])/25
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

method	result	size
risch	$\left(-\frac{2}{5}-\frac{3x}{5}\right)\cos\left(5x\right)+\frac{3\sin(5x)}{25}$	18
derivativedivides	$-\frac{2\cos(5x)}{5}+\frac{3\sin(5x)}{25}-\frac{3x\cos(5x)}{5}$	21
default	$-\frac{2\cos(5x)}{5}+\frac{3\sin(5x)}{25}-\frac{3x\cos(5x)}{5}$	21
parts	$-\frac{2\cos(5x)}{5}+\frac{3\sin(5x)}{25}-\frac{3x\cos(5x)}{5}$	21
norman	$\frac{-\frac{3x}{5}+\frac{3x\left(\tan^2\left(\frac{5x}{2}\right)\right)}{5}+\frac{6\tan\left(\frac{5x}{2}\right)}{25}-\frac{4}{5}}{1+\tan^2\left(\frac{5x}{2}\right)}$	32
parallelrisch	$\frac{15x\left(\tan^2\left(\frac{5x}{2}\right)\right)-20-15x+6\tan\left(\frac{5x}{2}\right)}{25\left(\tan^2\left(\frac{5x}{2}\right)\right)+25}$	34
meijerg	$\frac{2\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(5x)}{\sqrt{\pi}}\right)}{5}+\frac{6\sqrt{\pi}\left(-\frac{5x\cos(5x)}{2\sqrt{\pi}}+\frac{\sin(5x)}{2\sqrt{\pi}}\right)}{25}$	45

```
[In] int((2+3*x)*sin(5*x),x,method=_RETURNVERBOSE)
[Out] (-2/5-3/5*x)*cos(5*x)+3/25*sin(5*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (2+3x)\sin(5x)\,dx = -\frac{1}{5}\,(3\,x+2)\cos{(5\,x)} + \frac{3}{25}\,\sin{(5\,x)}$$

```
[In] integrate((2+3*x)*sin(5*x),x, algorithm="fricas")
[Out] -1/5*(3*x + 2)*cos(5*x) + 3/25*sin(5*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (2+3x)\sin(5x)\,dx = -\frac{3x\cos{(5x)}}{5} + \frac{3\sin{(5x)}}{25} - \frac{2\cos{(5x)}}{5}$$

```
[In] integrate((2+3*x)*sin(5*x),x)
[Out] -3*x*cos(5*x)/5 + 3*sin(5*x)/25 - 2*cos(5*x)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int (2+3x)\sin(5x)\,dx = -\frac{3}{5}\,x\cos{(5\,x)} - \frac{2}{5}\,\cos{(5\,x)} + \frac{3}{25}\,\sin{(5\,x)}$$

```
[In] integrate((2+3*x)*sin(5*x),x, algorithm="maxima")
[Out] -3/5*x*cos(5*x) - 2/5*cos(5*x) + 3/25*sin(5*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (2+3x)\sin(5x)\,dx = -\frac{1}{5}\,(3\,x+2)\cos{(5\,x)} + \frac{3}{25}\,\sin{(5\,x)}$$

```
[In] integrate((2+3*x)*sin(5*x),x, algorithm="giac")
[Out] -1/5*(3*x + 2)*cos(5*x) + 3/25*sin(5*x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int (2+3x)\sin(5x)\,dx = \frac{3\,\sin\left(5\,x\right)}{25} - \frac{2\,\cos\left(5\,x\right)}{5} - \frac{3\,x\,\cos\left(5\,x\right)}{5}$$

```
[In] int(sin(5*x)*(3*x + 2),x)
[Out] (3*sin(5*x))/25 - (2*cos(5*x))/5 - (3*x*cos(5*x))/5
```

3.47 $\int x\sqrt{1+x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int x\sqrt{1+x^2}\,dx=\frac{1}{3}\left(1+x^2\right)^{3/2}$$

```
[Out] 1/3*(x^2+1)^(3/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {267}

$$\int x\sqrt{1+x^2}\,dx=\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[In] Int[x*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{3}\left(1+x^2\right)^{3/2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int x\sqrt{1+x^2}\,dx=\frac{1}{3}\left(1+x^2\right)^{3/2}$$

```
[In] Integrate[x*Sqrt[1 + x^2],x]
[Out] (1 + x^2)^(3/2)/3
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{\left(x^{2}+1\right)^{\frac{3}{2}}}{3}$	10
derivativedivides	$\frac{\left(x^{2}+1\right)^{\frac{3}{2}}}{3}$	10
default	$\frac{\left(x^{2}+1\right)^{\frac{3}{2}}}{3}$	10
risch	$\frac{\left(x^{2}+1\right)^{\frac{3}{2}}}{3}$	10
pseudoelliptic	$\frac{\left(x^{2}+1\right)^{\frac{3}{2}}}{3}$	10
trager	$\left(\frac{x^{2}}{3}+\frac{1}{3}\right)\sqrt{x^{2}+1}$	16
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2x^{2}+2\right)\sqrt{x^{2}+1}}{3}}{4\sqrt{\pi}}$	31

```
[In] int(x*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(x^2+1)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx=\frac{1}{3}\left(x^{2}+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*(x^2 + 1)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int x\sqrt{1+x^2}\,dx = \frac{x^2\sqrt{x^2+1}}{3} + \frac{\sqrt{x^2+1}}{3}$$

```
[In] integrate(x*(x**2+1)**(1/2),x)
[Out] x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/3*(x^2 + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/3*(x^2 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx=\frac{\left(x^2+1\right)^{3/2}}{3}$$

```
[In] int(x*(x^2 + 1)^(1/2),x)
[Out] (x^2 + 1)^(3/2)/3
```

3.48 $\int x(-1+x^2)^9\,dx$

Optimal result

Integrand size = 9, antiderivative size = 13

$$\int x\left(-1+x^2\right)^9\,dx=\frac{1}{20}\left(1-x^2\right)^{10}$$

```
[Out] 1/20*(-x^2+1)^10
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {267}

$$\int x\left(-1+x^2\right)^9\,dx=\frac{1}{20}\left(1-x^2\right)^{10}$$

```
[In] Int[x*(-1 + x^2)^9,x]
```

```
[Out] (1 - x^2)^10/20
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{20}\left(1-x^2\right)^{10}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int x\left(-1+x^2\right)^9\,dx=\frac{1}{20}\left(-1+x^2\right)^{10}$$

```
[In] Integrate[x*(-1 + x^2)^9,x]
[Out] (-1 + x^2)^10/20
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\left(x^{2}-1\right)^{10}}{20}$	10
gosper	$\frac{x^{2}\left(x^{18}-10x^{16}+45x^{14}-120x^{12}+210x^{10}-252x^{8}+210x^{6}-120x^{4}+45x^{2}-10\right)}{20}$	51
norman	$-\frac{1}{2}x^{2}+\frac{9}{4}x^{4}-6x^{6}+\frac{21}{2}x^{8}-\frac{63}{5}x^{10}+\frac{21}{2}x^{12}-6x^{14}+\frac{9}{4}x^{16}-\frac{1}{2}x^{18}+\frac{1}{20}x^{20}$	52
parallelrisch	$-\frac{1}{2}x^{2}+\frac{9}{4}x^{4}-6x^{6}+\frac{21}{2}x^{8}-\frac{63}{5}x^{10}+\frac{21}{2}x^{12}-6x^{14}+\frac{9}{4}x^{16}-\frac{1}{2}x^{18}+\frac{1}{20}x^{20}$	52
risch	$\frac{1}{20}x^{20}-\frac{1}{2}x^{18}+\frac{9}{4}x^{16}-6x^{14}+\frac{21}{2}x^{12}-\frac{63}{5}x^{10}+\frac{21}{2}x^{8}-6x^{6}+\frac{9}{4}x^{4}-\frac{1}{2}x^{2}+\frac{1}{20}$	53

```
[In] int(x*(x^2-1)^9,x,method=_RETURNVERBOSE)
[Out] 1/20*(x^2-1)^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(9) = 18$.

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.92

$$\int x\left(-1+x^2\right)^9\,dx=\frac{1}{20}\,x^{20}-\frac{1}{2}\,x^{18}+\frac{9}{4}\,x^{16}-6\,x^{14}+\frac{21}{2}\,x^{12}-\frac{63}{5}\,x^{10}+\frac{21}{2}\,x^{8}-6\,x^{6}+\frac{9}{4}\,x^{4}-\frac{1}{2}\,x^{2}$$

```
[In] integrate(x*(x^2-1)^9,x, algorithm="fricas")
[Out] 1/20*x^20 - 1/2*x^18 + 9/4*x^16 - 6*x^14 + 21/2*x^12 - 63/5*x^10 + 21/2*x^8
 - 6*x^6 + 9/4*x^4 - 1/2*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(7) = 14$.

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.46

$$\int x(-1+x^2)^9\,dx = \frac{x^{20}}{20} - \frac{x^{18}}{2} + \frac{9x^{16}}{4} - 6x^{14} + \frac{21x^{12}}{2} - \frac{63x^{10}}{5} + \frac{21x^8}{2} - 6x^6 + \frac{9x^4}{4} - \frac{x^2}{2}$$

```
[In] integrate(x*(x**2-1)**9,x)
[Out] x**20/20 - x**18/2 + 9*x**16/4 - 6*x**14 + 21*x**12/2 - 63*x**10/5 + 21*x**
8/2 - 6*x**6 + 9*x**4/4 - x**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x(-1+x^2)^9\,dx = \frac{1}{20}\,(x^2-1)^{10}$$

```
[In] integrate(x*(x^2-1)^9,x, algorithm="maxima")
[Out] 1/20*(x^2 - 1)^10
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x(-1+x^2)^9\,dx = \frac{1}{20}\,(x^2-1)^{10}$$

```
[In] integrate(x*(x^2-1)^9,x, algorithm="giac")
[Out] 1/20*(x^2 - 1)^10
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\left(-1+x^2\right)^9\,dx=\frac{\left(x^2-1\right)^{10}}{20}$$

```
[In] int(x*(x^2 - 1)^9,x)
[Out] (x^2 - 1)^10/20
```

3.49 $\int \frac{3+2x}{(7+6x)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{3+2x}{(7+6x)^3} \, dx = -\frac{(3+2x)^2}{8(7+6x)^2}$$

```
[Out] -1/8*(3+2*x)^2/(7+6*x)^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{37\}$

$$\int \frac{3+2x}{(7+6x)^3} \, dx = -\frac{(2x+3)^2}{8(6x+7)^2}$$

```
[In] Int[(3 + 2*x)/(7 + 6*x)^3,x]
```

```
[Out] -1/8*(3 + 2*x)^2/(7 + 6*x)^2
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\text{integral} = -\frac{(3+2x)^2}{8(7+6x)^2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{3+2x}{(7+6x)^3}\,dx = -\frac{4+3x}{9(7+6x)^2}$$

```
[In] Integrate[(3 + 2*x)/(7 + 6*x)^3,x]
```

```
[Out] -1/9*(4 + 3*x)/(7 + 6*x)^2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

method	result	size
norman	$\frac{-\frac{x}{3}-\frac{4}{9}}{(7+6x)^2}$	14
gosper	$-\frac{3x+4}{9(7+6x)^2}$	15
risch	$\frac{-\frac{x}{3}-\frac{4}{9}}{(7+6x)^2}$	15
parallelrisch	$\frac{-12x-16}{36(7+6x)^2}$	15
default	$-\frac{1}{18(7+6x)}-\frac{1}{18(7+6x)^2}$	20
meijerg	$\frac{3x\left(\frac{6x}{7}+2\right)}{686\left(1+\frac{6x}{7}\right)^2}+\frac{x^2}{343\left(1+\frac{6x}{7}\right)^2}$	29

```
[In] int((3+2*x)/(7+6*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(7+6*x)^2*(-1/3*x-4/9)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{3+2x}{(7+6x)^3}\,dx = -\frac{3\,x+4}{9\,(36\,x^2+84\,x+49)}$$

```
[In] integrate((3+2*x)/(7+6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/9*(3*x + 4)/(36*x^2 + 84*x + 49)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{3+2x}{(7+6x)^3}\,dx = \frac{-3x-4}{324x^2+756x+441}$$

```
[In] integrate((3+2*x)/(7+6*x)**3,x)
[Out] (-3*x - 4)/(324*x**2 + 756*x + 441)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{3+2x}{(7+6x)^3}\,dx = -\frac{3\,x+4}{9\,(36\,x^2+84\,x+49)}$$

```
[In] integrate((3+2*x)/(7+6*x)^3,x, algorithm="maxima")
[Out] -1/9*(3*x + 4)/(36*x^2 + 84*x + 49)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{3+2x}{(7+6x)^3}\,dx = -\frac{3\,x+4}{9\,(6\,x+7)^2}$$

```
[In] integrate((3+2*x)/(7+6*x)^3,x, algorithm="giac")
[Out] -1/9*(3*x + 4)/(6*x + 7)^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{3+2x}{(7+6x)^3}\,dx = -\frac{3\,x+4}{9\,(6\,x+7)^2}$$

```
[In] int((2*x + 3)/(6*x + 7)^3,x)
[Out] -(3*x + 4)/(9*(6*x + 7)^2)
```

3.50 $\int x^4(1+x^5)^5\,dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int x^4(1+x^5)^5\,dx=\frac{1}{30}(1+x^5)^6$$

```
[Out] 1/30*(x^5+1)^6
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {267}

$$\int x^4(1+x^5)^5\,dx=\frac{1}{30}(x^5+1)^6$$

```
[In] Int[x^4*(1 + x^5)^5,x]
```

```
[Out] (1 + x^5)^6/30
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{30}(1+x^5)^6$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. $2(11) = 22$.

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.91

$$\int x^4(1+x^5)^5\,dx = \frac{x^5}{5}+\frac{x^{10}}{2}+\frac{2x^{15}}{3}+\frac{x^{20}}{2}+\frac{x^{25}}{5}+\frac{x^{30}}{30}$$

```
[In] Integrate[x^4*(1 + x^5)^5,x]
[Out] x^5/5 + x^10/2 + (2*x^15)/3 + x^20/2 + x^25/5 + x^30/30
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{(x^5+1)^6}{30}$	10
gosper	$\frac{x^5(x^{25}+6x^{20}+15x^{15}+20x^{10}+15x^5+6)}{30}$	31
norman	$\frac{1}{5}x^{25}+\frac{1}{30}x^{30}+\frac{1}{2}x^{10}+\frac{2}{3}x^{15}+\frac{1}{2}x^{20}+\frac{1}{5}x^5$	32
parallelrisch	$\frac{1}{5}x^{25}+\frac{1}{30}x^{30}+\frac{1}{2}x^{10}+\frac{2}{3}x^{15}+\frac{1}{2}x^{20}+\frac{1}{5}x^5$	32
risch	$\frac{1}{30}x^{30}+\frac{1}{5}x^{25}+\frac{1}{2}x^{20}+\frac{2}{3}x^{15}+\frac{1}{2}x^{10}+\frac{1}{5}x^5+\frac{1}{30}$	33

```
[In] int(x^4*(x^5+1)^5,x,method=_RETURNVERBOSE)
[Out] 1/30*(x^5+1)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(9) = 18$.

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int x^4(1+x^5)^5\,dx = \frac{1}{30}x^{30}+\frac{1}{5}x^{25}+\frac{1}{2}x^{20}+\frac{2}{3}x^{15}+\frac{1}{2}x^{10}+\frac{1}{5}x^5$$

```
[In] integrate(x^4*(x^5+1)^5,x, algorithm="fricas")
[Out] 1/30*x^30 + 1/5*x^25 + 1/2*x^20 + 2/3*x^15 + 1/2*x^10 + 1/5*x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(7) = 14$.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int x^4(1+x^5)^5\,dx = \frac{x^{30}}{30} + \frac{x^{25}}{5} + \frac{x^{20}}{2} + \frac{2x^{15}}{3} + \frac{x^{10}}{2} + \frac{x^5}{5}$$

```
[In] integrate(x**4*(x**5+1)**5,x)
[Out] x**30/30 + x**25/5 + x**20/2 + 2*x**15/3 + x**10/2 + x**5/5
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int x^4(1+x^5)^5\,dx = \frac{1}{30}\,(x^5+1)^6$$

```
[In] integrate(x^4*(x^5+1)^5,x, algorithm="maxima")
[Out] 1/30*(x^5 + 1)^6
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int x^4(1+x^5)^5\,dx = \frac{1}{30}\,(x^5+1)^6$$

```
[In] integrate(x^4*(x^5+1)^5,x, algorithm="giac")
[Out] 1/30*(x^5 + 1)^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int x^4(1+x^5)^5\,dx = \frac{x^{30}}{30} + \frac{x^{25}}{5} + \frac{x^{20}}{2} + \frac{2\,x^{15}}{3} + \frac{x^{10}}{2} + \frac{x^5}{5}$$

```
[In] int(x^4*(x^5 + 1)^5,x)
[Out] x^5/5 + x^10/2 + (2*x^15)/3 + x^20/2 + x^25/5 + x^30/30
```

3.51 $\int (1-x)^{20}x^4\,dx$

Optimal result

Integrand size = 11, antiderivative size = 56

$$\int (1-x)^{20}x^4\,dx = -\frac{1}{21}(1-x)^{21}+\frac{2}{11}(1-x)^{22}-\frac{6}{23}(1-x)^{23}+\frac{1}{6}(1-x)^{24}-\frac{1}{25}(1-x)^{25}$$

```
[Out] -1/21*(1-x)^21+2/11*(1-x)^22-6/23*(1-x)^23+1/6*(1-x)^24-1/25*(1-x)^25
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\int (1-x)^{20}x^4\,dx = -\frac{1}{25}(1-x)^{25}+\frac{1}{6}(1-x)^{24}-\frac{6}{23}(1-x)^{23}+\frac{2}{11}(1-x)^{22}-\frac{1}{21}(1-x)^{21}$$

```
[In] Int[(1 - x)^20*x^4,x]
[Out] -1/21*(1 - x)^21 + (2*(1 - x)^22)/11 - (6*(1 - x)^23)/23 + (1 - x)^24/6 - (
1 - x)^25/25
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left((1-x)^{20}-4(1-x)^{21}+6(1-x)^{22}-4(1-x)^{23}+(1-x)^{24}\right)\,dx \\ &= -\frac{1}{21}(1-x)^{21}+\frac{2}{11}(1-x)^{22}-\frac{6}{23}(1-x)^{23}+\frac{1}{6}(1-x)^{24}-\frac{1}{25}(1-x)^{25} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(56) = 112.

Time = 0.00 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.50

$$\int (1-x)^{20}x^4\,dx = \frac{x^5}{5}-\frac{10x^6}{3}+\frac{190x^7}{7}-\frac{285x^8}{2}+\frac{1615x^9}{3}-\frac{7752x^{10}}{5}+\frac{38760x^{11}}{11}-6460x^{12}+9690x^{13}-\frac{83980x^{14}}{7}+\frac{184756x^{15}}{15}-\frac{20995x^{16}}{2}+7410x^{17}-\frac{12920x^{18}}{3}+2040x^{19}-\frac{3876x^{20}}{5}+\frac{1615x^{21}}{7}-\frac{570x^{22}}{11}+\frac{190x^{23}}{23}-\frac{5x^{24}}{6}+\frac{x^{25}}{25}$$

```
[In] Integrate[(1 - x)^20*x^4,x]
```

```
[Out] x^5/5 - (10*x^6)/3 + (190*x^7)/7 - (285*x^8)/2 + (1615*x^9)/3 - (7752*x^10)
/5 + (38760*x^11)/11 - 6460*x^12 + 9690*x^13 - (83980*x^14)/7 + (184756*x^1
5)/15 - (20995*x^16)/2 + 7410*x^17 - (12920*x^18)/3 + 2040*x^19 - (3876*x^2
0)/5 + (1615*x^21)/7 - (570*x^22)/11 + (190*x^23)/23 - (5*x^24)/6 + x^25/25
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(46) = 92.

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.89

method	result
gosper	$\frac{x^5(10626x^{20}-221375x^{19}+2194500x^{18}-13765500x^{17}+61289250x^{16}-205931880x^{15}+541926000x^{14}-1144066000x^{13}+1968466$
default	$\frac{1}{5}x^5-\frac{10}{3}x^6+\frac{190}{7}x^7-\frac{285}{2}x^8+\frac{1615}{3}x^9-\frac{7752}{5}x^{10}+\frac{38760}{11}x^{11}-6460x^{12}+9690x^{13}-\frac{83980}{7}x^{14}+$
norman	$\frac{1}{5}x^5-\frac{10}{3}x^6+\frac{190}{7}x^7-\frac{285}{2}x^8+\frac{1615}{3}x^9-\frac{7752}{5}x^{10}+\frac{38760}{11}x^{11}-6460x^{12}+9690x^{13}-\frac{83980}{7}x^{14}+$
risch	$\frac{1}{5}x^5-\frac{10}{3}x^6+\frac{190}{7}x^7-\frac{285}{2}x^8+\frac{1615}{3}x^9-\frac{7752}{5}x^{10}+\frac{38760}{11}x^{11}-6460x^{12}+9690x^{13}-\frac{83980}{7}x^{14}+$
parallelrisch	$\frac{1}{5}x^5-\frac{10}{3}x^6+\frac{190}{7}x^7-\frac{285}{2}x^8+\frac{1615}{3}x^9-\frac{7752}{5}x^{10}+\frac{38760}{11}x^{11}-6460x^{12}+9690x^{13}-\frac{83980}{7}x^{14}+$

```
[In] int((1-x)^20*x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/265650*x^5*(10626*x^20-221375*x^19+2194500*x^18-13765500*x^17+61289250*x^
16-205931880*x^15+541926000*x^14-1144066000*x^13+1968466500*x^12-2788660875
*x^11+3272028760*x^10-3187041000*x^9+2574148500*x^8-1716099000*x^7+93605400
0*x^6-411863760*x^5+143008250*x^4-37855125*x^3+7210500*x^2-885500*x+53130)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(36) = 72$.

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.89

$$\int (1-x)^{20}x^4\,dx = \frac{1}{25}\,x^{25} - \frac{5}{6}\,x^{24} + \frac{190}{23}\,x^{23} - \frac{570}{11}\,x^{22} + \frac{1615}{7}\,x^{21} - \frac{3876}{5}\,x^{20} + 2040\,x^{19} - \frac{12920}{3}\,x^{18} + 7410\,x^{17} - \frac{20995}{2}\,x^{16} + \frac{184756}{15}\,x^{15} - \frac{83980}{7}\,x^{14} + 9690\,x^{13} - 6460\,x^{12} + \frac{38760}{11}\,x^{11} - \frac{7752}{5}\,x^{10} + \frac{1615}{3}\,x^{9} - \frac{285}{2}\,x^{8} + \frac{190}{7}\,x^{7} - \frac{10}{3}\,x^{6} + \frac{1}{5}\,x^{5}$$

```
[In] integrate((1-x)^20*x^4,x, algorithm="fricas")
```

```
[Out] 1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 - 3876/5*x^2
0 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16 + 184756/15*x^15 -
83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/11*x^11 - 7752/5*x^10 + 1615/3
*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3*x^6 + 1/5*x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(36) = 72$.

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.34

$$\int (1-x)^{20}x^4\,dx = \frac{x^{25}}{25} - \frac{5x^{24}}{6} + \frac{190x^{23}}{23} - \frac{570x^{22}}{11} + \frac{1615x^{21}}{7} - \frac{3876x^{20}}{5} + 2040x^{19} - \frac{12920x^{18}}{3} + 7410x^{17} - \frac{20995x^{16}}{2} + \frac{184756x^{15}}{15} - \frac{83980x^{14}}{7} + 9690x^{13} - 6460x^{12} + \frac{38760x^{11}}{11} - \frac{7752x^{10}}{5} + \frac{1615x^{9}}{3} - \frac{285x^{8}}{2} + \frac{190x^{7}}{7} - \frac{10x^{6}}{3} + \frac{x^{5}}{5}$$

```
[In] integrate((1-x)**20*x**4,x)
```

```
[Out] x**25/25 - 5*x**24/6 + 190*x**23/23 - 570*x**22/11 + 1615*x**21/7 - 3876*x*
*20/5 + 2040*x**19 - 12920*x**18/3 + 7410*x**17 - 20995*x**16/2 + 184756*x*
*15/15 - 83980*x**14/7 + 9690*x**13 - 6460*x**12 + 38760*x**11/11 - 7752*x*
*10/5 + 1615*x**9/3 - 285*x**8/2 + 190*x**7/7 - 10*x**6/3 + x**5/5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(36) = 72.

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.89

$$\int (1-x)^{20}x^4\,dx = \frac{1}{25}\,x^{25} - \frac{5}{6}\,x^{24} + \frac{190}{23}\,x^{23} - \frac{570}{11}\,x^{22} + \frac{1615}{7}\,x^{21} - \frac{3876}{5}\,x^{20} + 2040\,x^{19} - \frac{12920}{3}\,x^{18} + 7410\,x^{17} - \frac{20995}{2}\,x^{16} + \frac{184756}{15}\,x^{15} - \frac{83980}{7}\,x^{14} + 9690\,x^{13} - 6460\,x^{12} + \frac{38760}{11}\,x^{11} - \frac{7752}{5}\,x^{10} + \frac{1615}{3}\,x^{9} - \frac{285}{2}\,x^{8} + \frac{190}{7}\,x^{7} - \frac{10}{3}\,x^{6} + \frac{1}{5}\,x^{5}$$

```
[In] integrate((1-x)^20*x^4,x, algorithm="maxima")
```

```
[Out] 1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 - 3876/5*x^2
0 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16 + 184756/15*x^15 -
83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/11*x^11 - 7752/5*x^10 + 1615/3
*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3*x^6 + 1/5*x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.89

$$\int (1-x)^{20}x^4\,dx = \frac{1}{25}\,x^{25} - \frac{5}{6}\,x^{24} + \frac{190}{23}\,x^{23} - \frac{570}{11}\,x^{22} + \frac{1615}{7}\,x^{21} - \frac{3876}{5}\,x^{20} + 2040\,x^{19} - \frac{12920}{3}\,x^{18} + 7410\,x^{17} - \frac{20995}{2}\,x^{16} + \frac{184756}{15}\,x^{15} - \frac{83980}{7}\,x^{14} + 9690\,x^{13} - 6460\,x^{12} + \frac{38760}{11}\,x^{11} - \frac{7752}{5}\,x^{10} + \frac{1615}{3}\,x^{9} - \frac{285}{2}\,x^{8} + \frac{190}{7}\,x^{7} - \frac{10}{3}\,x^{6} + \frac{1}{5}\,x^{5}$$

```
[In] integrate((1-x)^20*x^4,x, algorithm="giac")
```

```
[Out] 1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 - 3876/5*x^2
0 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16 + 184756/15*x^15 -
83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/11*x^11 - 7752/5*x^10 + 1615/3
*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3*x^6 + 1/5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.89

$$\int (1-x)^{20}x^4\,dx = \frac{x^{25}}{25}-\frac{5\,x^{24}}{6}+\frac{190\,x^{23}}{23}-\frac{570\,x^{22}}{11}+\frac{1615\,x^{21}}{7}-\frac{3876\,x^{20}}{5}+2040\,x^{19}-\frac{12920\,x^{18}}{3}+7410\,x^{17}-\frac{20995\,x^{16}}{2}+\frac{184756\,x^{15}}{15}-\frac{83980\,x^{14}}{7}+9690\,x^{13}-6460\,x^{12}+\frac{38760\,x^{11}}{11}-\frac{7752\,x^{10}}{5}+\frac{1615\,x^{9}}{3}-\frac{285\,x^{8}}{2}+\frac{190\,x^{7}}{7}-\frac{10\,x^{6}}{3}+\frac{x^{5}}{5}$$

```
[In] int(x^4*(x - 1)^20,x)
```

```
[Out] x^5/5 - (10*x^6)/3 + (190*x^7)/7 - (285*x^8)/2 + (1615*x^9)/3 - (7752*x^10)
/5 + (38760*x^11)/11 - 6460*x^12 + 9690*x^13 - (83980*x^14)/7 + (184756*x^1
5)/15 - (20995*x^16)/2 + 7410*x^17 - (12920*x^18)/3 + 2040*x^19 - (3876*x^2
0)/5 + (1615*x^21)/7 - (570*x^22)/11 + (190*x^23)/23 - (5*x^24)/6 + x^25/25
```

3.52 $\int \frac{\sin\left(\frac{1}{x}\right)}{x^2} dx$

Optimal result

Integrand size = 8, antiderivative size = 4

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2} dx = \cos\left(\frac{1}{x}\right)$$

```
[Out] cos(1/x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3460, 2718}

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2} dx = \cos\left(\frac{1}{x}\right)$$

```
[In] Int[Sin[x^(-1)]/x^2,x]
```

```
[Out] Cos[x^(-1)]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
```

```
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \sin(x)\,dx, x, \frac{1}{x}\right)$$
$$= \cos\left(\frac{1}{x}\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \cos\left(\frac{1}{x}\right)$$

```
[In] Integrate[Sin[x^(-1)]/x^2,x]
[Out] Cos[x^(-1)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\cos\left(\frac{1}{x}\right)$	5
default	$\cos\left(\frac{1}{x}\right)$	5
risch	$\cos\left(\frac{1}{x}\right)$	5
parallelrisch	$1+\cos\left(\frac{1}{x}\right)$	7
norman	$\frac{2}{1+\tan^2\left(\frac{1}{2x}\right)}$	15
meijerg	$-\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\frac{1}{x}\right)}{\sqrt{\pi}}\right)$	19

```
[In] int(sin(1/x)/x^2,x,method=_RETURNVERBOSE)
[Out] cos(1/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \cos\left(\frac{1}{x}\right)$$

```
[In] integrate(sin(1/x)/x^2,x, algorithm="fricas")
[Out] cos(1/x)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \cos\left(\frac{1}{x}\right)$$

```
[In] integrate(sin(1/x)/x**2,x)
[Out] cos(1/x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \cos\left(\frac{1}{x}\right)$$

```
[In] integrate(sin(1/x)/x^2,x, algorithm="maxima")
[Out] cos(1/x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \cos\left(\frac{1}{x}\right)$$

```
[In] integrate(sin(1/x)/x^2,x, algorithm="giac")
[Out] cos(1/x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \cos\left(\frac{1}{x}\right)$$

```
[In] int(sin(1/x)/x^2,x)
[Out] cos(1/x)
```

3.53 $\int \sin\left(\sqrt[4]{-1+x}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 62

$$\int \sin\left(\sqrt[4]{-1+x}\right)\,dx = 24\sqrt[4]{-1+x}\cos\left(\sqrt[4]{-1+x}\right) - 4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right) - 24\sin\left(\sqrt[4]{-1+x}\right) + 12\sqrt{-1+x}\sin\left(\sqrt[4]{-1+x}\right)$$

```
[Out] 24*(-1+x)^(1/4)*cos((-1+x)^(1/4))-4*(-1+x)^(3/4)*cos((-1+x)^(1/4))-24*sin((
-1+x)^(1/4))+12*sin((-1+x)^(1/4))*(-1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3442, 3377, 2717}

$$\int \sin\left(\sqrt[4]{-1+x}\right)\,dx = 12\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right) - 24\sin\left(\sqrt[4]{x-1}\right) - 4(x-1)^{3/4}\cos\left(\sqrt[4]{x-1}\right) + 24\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right)$$

```
[In] Int[Sin[(-1 + x)^(1/4)],x]
```

```
[Out] 24*(-1 + x)^(1/4)*Cos[(-1 + x)^(1/4)] - 4*(-1 + x)^(3/4)*Cos[(-1 + x)^(1/4)
] - 24*Sin[(-1 + x)^(1/4)] + 12*Sqrt[-1 + x]*Sin[(-1 + x)^(1/4)]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 4\text{Subst}\left(\int x^3\sin(x)\,dx, x, \sqrt[4]{-1+x}\right)\\
&= -4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right)+12\text{Subst}\left(\int x^2\cos(x)\,dx, x, \sqrt[4]{-1+x}\right)\\
&= -4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right)+12\sqrt{-1+x}\sin\left(\sqrt[4]{-1+x}\right)\\
&\quad -24\text{Subst}\left(\int x\sin(x)\,dx, x, \sqrt[4]{-1+x}\right)\\
&= 24\sqrt[4]{-1+x}\cos\left(\sqrt[4]{-1+x}\right)-4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right)\\
&\quad +12\sqrt{-1+x}\sin\left(\sqrt[4]{-1+x}\right)-24\text{Subst}\left(\int \cos(x)\,dx, x, \sqrt[4]{-1+x}\right)\\
&= 24\sqrt[4]{-1+x}\cos\left(\sqrt[4]{-1+x}\right)\\
&\quad -4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right)-24\sin\left(\sqrt[4]{-1+x}\right)+12\sqrt{-1+x}\sin\left(\sqrt[4]{-1+x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\begin{aligned}
\int \sin\left(\sqrt[4]{-1+x}\right)\,dx &= -4\left(-6+\sqrt{-1+x}\right)\sqrt[4]{-1+x}\cos\left(\sqrt[4]{-1+x}\right)\\
&\quad +12\left(-2+\sqrt{-1+x}\right)\sin\left(\sqrt[4]{-1+x}\right)
\end{aligned}$$

```
[In] Integrate[Sin[(-1 + x)^(1/4)],x]
[Out] -4*(-6 + Sqrt[-1 + x])*(-1 + x)^(1/4)*Cos[(-1 + x)^(1/4)] + 12*(-2 + Sqrt[-
1 + x])*Sin[(-1 + x)^(1/4)]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

method	result
derivativedivides	$24(-1+x)^{\frac{1}{4}} \cos\left((-1+x)^{\frac{1}{4}}\right) - 4(-1+x)^{\frac{3}{4}} \cos\left((-1+x)^{\frac{1}{4}}\right) - 24 \sin\left((-1+x)^{\frac{1}{4}}\right) + 1$
default	$24(-1+x)^{\frac{1}{4}} \cos\left((-1+x)^{\frac{1}{4}}\right) - 4(-1+x)^{\frac{3}{4}} \cos\left((-1+x)^{\frac{1}{4}}\right) - 24 \sin\left((-1+x)^{\frac{1}{4}}\right) + 1$

```
[In] int(sin((-1+x)^(1/4)),x,method=_RETURNVERBOSE)
```

```
[Out] 24*(-1+x)^(1/4)*cos((-1+x)^(1/4))-4*(-1+x)^(3/4)*cos((-1+x)^(1/4))-24*sin((
-1+x)^(1/4))+12*sin((-1+x)^(1/4))*(-1+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

$$\int \sin\left(\sqrt[4]{-1+x}\right)\,dx = -4\left((x-1)^{\frac{3}{4}} - 6\,(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right) + 12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

```
[In] integrate(sin((-1+x)^(1/4)),x, algorithm="fricas")
```

```
[Out] -4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqrt(x - 1) -
 2)*sin((x - 1)^(1/4))
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \sin\left(\sqrt[4]{-1+x}\right)\,dx = -4(x-1)^{\frac{3}{4}}\cos\left(\sqrt[4]{x-1}\right) + 24\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right) + 12\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right) - 24\sin\left(\sqrt[4]{x-1}\right)$$

```
[In] integrate(sin((-1+x)**(1/4)),x)
```

```
[Out] -4*(x - 1)**(3/4)*cos((x - 1)**(1/4)) + 24*(x - 1)**(1/4)*cos((x - 1)**(1/4
)) + 12*sqrt(x - 1)*sin((x - 1)**(1/4)) - 24*sin((x - 1)**(1/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

$$\int \sin\left(\sqrt[4]{-1+x}\right)\,dx = -4\left((x-1)^{\frac{3}{4}} - 6\,(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right) + 12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

```
[In] integrate(sin((-1+x)^(1/4)),x, algorithm="maxima")
[Out] -4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqrt(x - 1) -
 2)*sin((x - 1)^(1/4))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

$$\int \sin\left(\sqrt[4]{-1+x}\right)\,dx = -4\left((x-1)^{\frac{3}{4}} - 6\,(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right) + 12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

```
[In] integrate(sin((-1+x)^(1/4)),x, algorithm="giac")
[Out] -4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqrt(x - 1) -
 2)*sin((x - 1)^(1/4))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

$$\int \sin\left(\sqrt[4]{-1+x}\right)\,dx = 4\,\cos\left((x-1)^{1/4}\right)\,\left(6\,(x-1)^{1/4} - (x-1)^{3/4}\right) + 4\,\sin\left((x-1)^{1/4}\right)\,\left(3\,\sqrt{x-1}-6\right)$$

```
[In] int(sin((x - 1)^(1/4)),x)
[Out] 4*cos((x - 1)^(1/4))*(6*(x - 1)^(1/4) - (x - 1)^(3/4)) + 4*sin((x - 1)^(1/4
))*(3*(x - 1)^(1/2) - 6)
```

3.54 $\int x\cos(x^2)\sin(x^2)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x\cos\left(x^2\right)\sin\left(x^2\right)\,dx=\frac{1}{4}\sin^2\left(x^2\right)$$

```
[Out] 1/4*sin(x^2)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3522}

$$\int x\cos\left(x^2\right)\sin\left(x^2\right)\,dx=\frac{1}{4}\sin^2\left(x^2\right)$$

```
[In] Int[x*Cos[x^2]*Sin[x^2],x]
```

```
[Out] Sin[x^2]^2/4
```

Rule 3522

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{4}\sin^2\left(x^2\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x \cos \left(x^2\right) \sin \left(x^2\right) \, dx = -\frac{1}{4} \cos^2 \left(x^2\right)$$

```
[In] Integrate[x*Cos[x^2]*Sin[x^2],x]
[Out] -1/4*Cos[x^2]^2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\frac{(\cos^2(x^2))}{4}$	9
default	$-\frac{(\cos^2(x^2))}{4}$	9
risch	$-\frac{\cos(2x^2)}{8}$	9
parallelrisch	$-\frac{\cos(2x^2)}{8}+\frac{1}{8}$	11
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x^2)}{\sqrt{\pi}}\right)}{8}$	21
norman	$\frac{\tan^2\left(\frac{x^2}{2}\right)}{\left(1+\tan^2\left(\frac{x^2}{2}\right)\right)^2}$	22

```
[In] int(x*cos(x^2)*sin(x^2),x,method=_RETURNVERBOSE)
[Out] -1/4*cos(x^2)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x \cos \left(x^2\right) \sin \left(x^2\right) \, dx = -\frac{1}{4} \cos \left(x^2\right)^2$$

```
[In] integrate(x*cos(x^2)*sin(x^2),x, algorithm="fricas")
[Out] -1/4*cos(x^2)^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\cos\left(x^2\right)\sin\left(x^2\right)\,dx = -\frac{\cos^2\left(x^2\right)}{4}$$

```
[In] integrate(x*cos(x**2)*sin(x**2),x)
[Out] -cos(x**2)**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\cos\left(x^2\right)\sin\left(x^2\right)\,dx = -\frac{1}{4}\,\cos\left(x^2\right)^2$$

```
[In] integrate(x*cos(x^2)*sin(x^2),x, algorithm="maxima")
[Out] -1/4*cos(x^2)^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\cos\left(x^2\right)\sin\left(x^2\right)\,dx = -\frac{1}{4}\,\cos\left(x^2\right)^2$$

```
[In] integrate(x*cos(x^2)*sin(x^2),x, algorithm="giac")
[Out] -1/4*cos(x^2)^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\cos\left(x^2\right)\sin\left(x^2\right)\,dx = \frac{\sin\left(x^2\right)^2}{4}$$

```
[In] int(x*cos(x^2)*sin(x^2),x)
[Out] sin(x^2)^2/4
```

3.55 $\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx = -\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

```
[Out] -2/9*(4-3*sin(x)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {12, 267}

$$\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx = -\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

```
[In] Int[Sqrt[1 + 3*Cos[x]^2]*Sin[2*x],x]
```

```
[Out] (-2*(4 - 3*Sin[x]^2)^(3/2))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int 2x\sqrt{4-3x^2}\,dx, x, \sin(x)\right)\\ &= 2\text{Subst}\left(\int x\sqrt{4-3x^2}\,dx, x, \sin(x)\right)\\ &= -\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx = -\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

```
[In] Integrate[Sqrt[1 + 3*Cos[x]^2]*Sin[2*x],x]
[Out] (-2*(4 - 3*Sin[x]^2)^(3/2))/9
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{2\left(1+3\left(\cos^2(x)\right)\right)^{\frac{3}{2}}}{9}$	13
default	$-\frac{2\left(1+3\left(\cos^2(x)\right)\right)^{\frac{3}{2}}}{9}$	13

```
[In] int(sin(2*x)*(1+3*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/9*(1+3*cos(x)^2)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx = -\frac{2}{9}\left(3\,\cos\left(x\right)^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(sin(2*x)*(1+3*cos(x)^2)^(1/2),x, algorithm="fricas")
[Out] -2/9*(3*cos(x)^2 + 1)^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx = -\frac{2(3\cos^2\left(x\right)+1)^{\frac{3}{2}}}{9}$$

```
[In] integrate(sin(2*x)*(1+3*cos(x)**2)**(1/2),x)
[Out] -2*(3*cos(x)**2 + 1)**(3/2)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx = -\frac{2}{9}\left(3\,\cos\left(x\right)^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(sin(2*x)*(1+3*cos(x)^2)^(1/2),x, algorithm="maxima")
[Out] -2/9*(3*cos(x)^2 + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx = -\frac{2}{9}\left(3\,\cos\left(x\right)^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(sin(2*x)*(1+3*cos(x)^2)^(1/2),x, algorithm="giac")
[Out] -2/9*(3*cos(x)^2 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx = -\frac{2\left(3\cos\left(x\right)^2+1\right)^{3/2}}{9}$$

```
[In] int(sin(2*x)*(3*cos(x)^2 + 1)^(1/2),x)
[Out] -(2*(3*cos(x)^2 + 1)^(3/2))/9
```

3.56 $\int \frac{1}{2+3x} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{2+3x} \, dx = \frac{1}{3} \log(2+3x)$$

```
[Out] 1/3*ln(2+3*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{31\}$

$$\int \frac{1}{2+3x} \, dx = \frac{1}{3} \log(3x+2)$$

```
[In] Int[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{3} \log(2+3x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+3x}\,dx=\frac{1}{3}\log(2+3x)$$

```
[In] Integrate[(2 + 3*x)^(-1),x]
[Out] Log[2 + 3*x]/3
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
parallelrisch	$\frac{\ln\left(\frac{2}{3}+x\right)}{3}$	7
default	$\frac{\ln(2+3x)}{3}$	9
norman	$\frac{\ln(2+3x)}{3}$	9
meijerg	$\frac{\ln\left(1+\frac{3x}{2}\right)}{3}$	9
risch	$\frac{\ln(2+3x)}{3}$	9

```
[In] int(1/(2+3*x),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(2/3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{2+3x}\,dx=\frac{1}{3}\,\log\left(3\,x+2\right)$$

```
[In] integrate(1/(2+3*x),x, algorithm="fricas")
[Out] 1/3*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{2+3x}\,dx = \frac{\log{(3x+2)}}{3}$$

```
[In] integrate(1/(2+3*x),x)
[Out] log(3*x + 2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{2+3x}\,dx = \frac{1}{3}\,\log\left(3\,x+2\right)$$

```
[In] integrate(1/(2+3*x),x, algorithm="maxima")
[Out] 1/3*log(3*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{2+3x}\,dx = \frac{1}{3}\,\log\left(|3\,x+2|\right)$$

```
[In] integrate(1/(2+3*x),x, algorithm="giac")
[Out] 1/3*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{2+3x}\,dx = \frac{\ln\left(x+\frac{2}{3}\right)}{3}$$

```
[In] int(1/(3*x + 2),x)
[Out] log(x + 2/3)/3
```

3.57 $\int \log^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 15

$$\int \log^2(x)\,dx = 2x - 2x\log(x) + x\log^2(x)$$

```
[Out] 2*x-2*x*ln(x)+x*ln(x)^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2333, 2332}

$$\int \log^2(x)\,dx = 2x + x\log^2(x) - 2x\log(x)$$

```
[In] Int[Log[x]^2,x]
```

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = x\log^2(x) - 2\int \log(x)\,dx$$
$$= 2x - 2x\log(x) + x\log^2(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x)\,dx = 2x - 2x\log(x) + x\log^2(x)$$

```
[In] Integrate[Log[x]^2,x]
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$2x - 2x\ln(x) + x\ln(x)^2$	16
norman	$2x - 2x\ln(x) + x\ln(x)^2$	16
risch	$2x - 2x\ln(x) + x\ln(x)^2$	16
parallelrisch	$2x - 2x\ln(x) + x\ln(x)^2$	16

```
[In] int(ln(x)^2,x,method=_RETURNVERBOSE)
[Out] 2*x-2*x*ln(x)+x*ln(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x)\,dx = x\log(x)^2 - 2\,x\log(x) + 2\,x$$

```
[In] integrate(log(x)^2,x, algorithm="fricas")
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x)\,dx = x\log(x)^2 - 2x\log(x) + 2x$$

```
[In] integrate(ln(x)**2,x)
[Out] x*log(x)**2 - 2*x*log(x) + 2*x
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \log^2(x)\,dx = \left(\log(x)^2 - 2\,\log(x) + 2\right)x$$

```
[In] integrate(log(x)^2,x, algorithm="maxima")
[Out] (log(x)^2 - 2*log(x) + 2)*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x)\,dx = x\log(x)^2 - 2\,x\log(x) + 2\,x$$

```
[In] integrate(log(x)^2,x, algorithm="giac")
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \log^2(x)\,dx = x\left(\ln(x)^2 - 2\,\ln(x) + 2\right)$$

```
[In] int(log(x)^2,x)
[Out] x*(log(x)^2 - 2*log(x) + 2)
```

3.58 $\int x \log(x) \, dx$

Optimal result

Integrand size = 4, antiderivative size = 17

$$\int x \log(x) \, dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2341}

$$\int x \log(x) \, dx = \frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

```
[In] Int[x*Log[x],x]
```

```
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x\log(x)\,dx = -\frac{x^2}{4} + \frac{1}{2}x^2\log(x)$$

```
[In] Integrate[x*Log[x],x]
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
norman	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
risch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
parallelrisch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
parts	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14

```
[In] int(x*ln(x),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right) - \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(x),x, algorithm="fricas")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x\log(x)\,dx = \frac{x^2\log{(x)}}{2} - \frac{x^2}{4}$$

```
[In] integrate(x*ln(x),x)
[Out] x**2*log(x)/2 - x**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log{(x)} - \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(x),x, algorithm="maxima")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log{(x)} - \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(x),x, algorithm="giac")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int x\log(x)\,dx = \frac{x^2\,\left(\ln{(x)} - \frac{1}{2}\right)}{2}$$

```
[In] int(x*log(x),x)
[Out] (x^2*(log(x) - 1/2))/2
```

3.59 $\int x \log^2(x) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 28

$$\int x \log^2(x) \, dx = \frac{x^2}{4} - \frac{1}{2}x^2 \log(x) + \frac{1}{2}x^2 \log^2(x)$$

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2342, 2341}

$$\int x \log^2(x) \, dx = \frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

```
[In] Int[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\log^2(x) - \int x\log(x)\,dx\\ &= \frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x\log^2(x)\,dx = \frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)$$

```
[In] Integrate[x*Log[x]^2,x]
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^2}{4} - \frac{x^2\ln(x)}{2} + \frac{x^2\ln(x)^2}{2}$	23
norman	$\frac{x^2}{4} - \frac{x^2\ln(x)}{2} + \frac{x^2\ln(x)^2}{2}$	23
risch	$\frac{x^2}{4} - \frac{x^2\ln(x)}{2} + \frac{x^2\ln(x)^2}{2}$	23
parallelrisch	$\frac{x^2}{4} - \frac{x^2\ln(x)}{2} + \frac{x^2\ln(x)^2}{2}$	23
parts	$\frac{x^2}{4} - \frac{x^2\ln(x)}{2} + \frac{x^2\ln(x)^2}{2}$	23

```
[In] int(x*ln(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right)^2 - \frac{1}{2}\,x^2\log\left(x\right) + \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(x)^2,x, algorithm="fricas")
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx = \frac{x^2\log\left(x\right)^2}{2} - \frac{x^2\log\left(x\right)}{2} + \frac{x^2}{4}$$

```
[In] integrate(x*ln(x)**2,x)
[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x\log^2(x)\,dx = \frac{1}{4}\,\left(2\,\log\left(x\right)^2 - 2\,\log\left(x\right) + 1\right)x^2$$

```
[In] integrate(x*log(x)^2,x, algorithm="maxima")
[Out] 1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right)^2 - \frac{1}{2}\,x^2\log\left(x\right) + \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(x)^2,x, algorithm="giac")
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x\log^2(x)\,dx=\frac{x^2\,\left(2\ln\left(x\right)^2-2\,\ln\left(x\right)+1\right)}{4}$$

```
[In] int(x*log(x)^2,x)
[Out] (x^2*(2*log(x)^2 - 2*log(x) + 1))/4
```

3.60 $\int \frac{1}{1+t} \, dt$

Optimal result

Integrand size = 5, antiderivative size = 4

$$\int \frac{1}{1+t} \, dt = \log(1+t)$$

```
[Out] ln(1+t)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{31\}$

$$\int \frac{1}{1+t} \, dt = \log(t+1)$$

```
[In] Int[(1 + t)^(-1),t]
```

```
[Out] Log[1 + t]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\text{integral} = \log(1+t)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+t}\,dt = \log(1+t)$$

```
[In] Integrate[(1 + t)^(-1),t]
[Out] Log[1 + t]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$\ln(1+t)$	5
norman	$\ln(1+t)$	5
meijerg	$\ln(1+t)$	5
risch	$\ln(1+t)$	5
parallelrisch	$\ln(1+t)$	5

```
[In] int(1/(1+t),t,method=_RETURNVERBOSE)
[Out] ln(1+t)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+t}\,dt = \log(t+1)$$

```
[In] integrate(1/(1+t),t, algorithm="fricas")
[Out] log(t + 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+t}\,dt = \log\left(t+1\right)$$

```
[In] integrate(1/(1+t),t)
[Out] log(t + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+t}\,dt = \log\left(t+1\right)$$

```
[In] integrate(1/(1+t),t, algorithm="maxima")
[Out] log(t + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \frac{1}{1+t}\,dt = \log\left(|t+1|\right)$$

```
[In] integrate(1/(1+t),t, algorithm="giac")
[Out] log(abs(t + 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+t}\,dt = \ln\left(t+1\right)$$

```
[In] int(1/(t + 1),t)
[Out] log(t + 1)
```

3.61 $\int \cot(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \cot(x)\,dx = \log(\sin(x))$$

```
[Out] ln(sin(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{3556\}$

$$\int \cot(x)\,dx = \log(\sin(x))$$

```
[In] Int[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \log(\sin(x))$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \cot(x)\,dx = \log(\cos(x)) + \log(\tan(x))$$

```
[In] Integrate[Cot[x],x]
[Out] Log[Cos[x]] + Log[Tan[x]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\ln(\sin(x))$	4
default	$\ln(\sin(x))$	4
derivativedivides	$-\frac{\ln(\cot^2(x)+1)}{2}$	10
parallelrisch	$\ln\left(\frac{1}{\sqrt{\sec^2(x)}}\right) + \ln(\tan(x))$	12
norman	$-\frac{\ln(1+\tan^2(x))}{2} + \ln(\tan(x))$	14
risch	$-ix + \ln(e^{2ix} - 1)$	14

```
[In] int(cot(x),x,method=_RETURNVERBOSE)
[Out] ln(sin(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \cot(x)\,dx = \frac{1}{2}\log\left(-\frac{1}{2}\cos(2x) + \frac{1}{2}\right)$$

```
[In] integrate(cot(x),x, algorithm="fricas")
[Out] 1/2*log(-1/2*cos(2*x) + 1/2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \log\left(\sin\left(x\right)\right)$$

```
[In] integrate(cot(x),x)
[Out] log(sin(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \log\left(\sin\left(x\right)\right)$$

```
[In] integrate(cot(x),x, algorithm="maxima")
[Out] log(sin(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int \cot(x)\,dx = \log\left(\left|\sin\left(x\right)\right|\right)$$

```
[In] integrate(cot(x),x, algorithm="giac")
[Out] log(abs(sin(x)))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \ln\left(\sin\left(x\right)\right)$$

```
[In] int(cot(x),x)
[Out] log(sin(x))
```

3.62 $\int x^n \log(ax)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 28

$$\int x^n \log(ax)\, dx = -\frac{x^{1+n}}{(1+n)^2} + \frac{x^{1+n}\log(ax)}{1+n}$$

```
[Out] -x^(1+n)/(1+n)^2+x^(1+n)*ln(a*x)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2341}

$$\int x^n \log(ax)\, dx = \frac{x^{n+1}\log(ax)}{n+1} - \frac{x^{n+1}}{(n+1)^2}$$

```
[In] Int[x^n*Log[a*x],x]
```

```
[Out] -(x^(1 + n)/(1 + n)^2) + (x^(1 + n)*Log[a*x])/(1 + n)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{x^{1+n}}{(1+n)^2} + \frac{x^{1+n}\log(ax)}{1+n}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int x^n \log(ax)\,dx = \frac{x^{1+n}(-1+(1+n)\log(ax))}{(1+n)^2}$$

```
[In] Integrate[x^n*Log[a*x],x]
[Out] (x^(1 + n)*(-1 + (1 + n)*Log[a*x]))/(1 + n)^2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

method	result
norman	$\frac{x\ln(ax)e^{n\ln(x)}}{1+n}-\frac{x\,e^{n\ln(x)}}{n^2+2n+1}$
parallelrisch	$\frac{x\,x^n\ln(ax)n+x^n\ln(ax)x-x\,x^n}{n^2+2n+1}$
risch	$\frac{x\left(-i\pi\ \mathrm{csgn}(ia)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iax)n+i\pi\ \mathrm{csgn}(ia)\,\mathrm{csgn}(iax)^2n+i\pi\ \mathrm{csgn}(ix)\,\mathrm{csgn}(iax)^2n-i\pi\,\mathrm{csgn}(iax)^3n-i\pi\ \mathrm{csgn}(ia)\,\mathrm{csgn}(ix\right.}{2(1+n)}$

```
[In] int(x^n*ln(a*x),x,method=_RETURNVERBOSE)
[Out] 1/(1+n)*x*ln(a*x)*exp(n*ln(x))-1/(n^2+2*n+1)*x*exp(n*ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int x^n \log(ax)\,dx = \frac{((n+1)x\log{(a)}+(n+1)x\log{(x)}-x)x^n}{n^2+2\,n+1}$$

```
[In] integrate(x^n*log(a*x),x, algorithm="fricas")
[Out] ((n + 1)*x*log(a) + (n + 1)*x*log(x) - x)*x^n/(n^2 + 2*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(22) = 44$.

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.18

$$\int x^n \log(ax)\,dx = \begin{cases} \frac{nxx^n \log{(ax)}}{n^2+2n+1} + \frac{xx^n \log{(ax)}}{n^2+2n+1} - \frac{xx^n}{n^2+2n+1} & \text{for}\ n \neq -1 \\ \frac{\log{(ax)}^2}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**n*ln(a*x),x)
[Out] Piecewise((n*x*x**n*log(a*x)/(n**2 + 2*n + 1) + x*x**n*log(a*x)/(n**2 + 2*n
 + 1) - x*x**n/(n**2 + 2*n + 1), Ne(n, -1)), (log(a*x)**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x^n \log(ax)\,dx = \frac{x^{n+1}\log{(ax)}}{n+1} - \frac{x^{n+1}}{(n+1)^2}$$

```
[In] integrate(x^n*log(a*x),x, algorithm="maxima")
[Out] x^(n + 1)*log(a*x)/(n + 1) - x^(n + 1)/(n + 1)^2
```

Giac [F]

$$\int x^n \log(ax)\,dx = \int x^n \log{(ax)}\,dx$$

```
[In] integrate(x^n*log(a*x),x, algorithm="giac")
[Out] integrate(x^n*log(a*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int x^n \log(ax)\,dx = \begin{cases} \frac{\ln(a\,x)^2}{2} & \text{if } n = -1 \\ \frac{x^{n+1}\left(\ln(a\,x)-\frac{1}{n+1}\right)}{n+1} & \text{if } n \neq -1 \end{cases}$$

```
[In] int(x^n*log(a*x),x)
[Out] piecewise(n == -1, log(a*x)^2/2, n ~= -1, (x^(n + 1)*(log(a*x) - 1/(n + 1))
)/(n + 1))
```

3.63 $\int x^2 \log^2(x)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 28

$$\int x^2 \log^2(x)\, dx = \frac{2x^3}{27} - \frac{2}{9}x^3 \log(x) + \frac{1}{3}x^3 \log^2(x)$$

```
[Out] 2/27*x^3-2/9*x^3*ln(x)+1/3*x^3*ln(x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2342, 2341}

$$\int x^2 \log^2(x)\, dx = \frac{2x^3}{27} + \frac{1}{3}x^3 \log^2(x) - \frac{2}{9}x^3 \log(x)$$

```
[In] Int[x^2*Log[x]^2,x]
```

```
[Out] (2*x^3)/27 - (2*x^3*Log[x])/9 + (x^3*Log[x]^2)/3
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}x^3\log^2(x) - \frac{2}{3}\int x^2\log(x)\,dx\\ &= \frac{2x^3}{27} - \frac{2}{9}x^3\log(x) + \frac{1}{3}x^3\log^2(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x^2\log^2(x)\,dx = \frac{2x^3}{27} - \frac{2}{9}x^3\log(x) + \frac{1}{3}x^3\log^2(x)$$

```
[In] Integrate[x^2*Log[x]^2,x]
[Out] (2*x^3)/27 - (2*x^3*Log[x])/9 + (x^3*Log[x]^2)/3
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{2x^3}{27} - \frac{2x^3\ln(x)}{9} + \frac{x^3\ln(x)^2}{3}$	23
norman	$\frac{2x^3}{27} - \frac{2x^3\ln(x)}{9} + \frac{x^3\ln(x)^2}{3}$	23
risch	$\frac{2x^3}{27} - \frac{2x^3\ln(x)}{9} + \frac{x^3\ln(x)^2}{3}$	23
parallelrisch	$\frac{2x^3}{27} - \frac{2x^3\ln(x)}{9} + \frac{x^3\ln(x)^2}{3}$	23
parts	$\frac{2x^3}{27} - \frac{2x^3\ln(x)}{9} + \frac{x^3\ln(x)^2}{3}$	23

```
[In] int(x^2*ln(x)^2,x,method=_RETURNVERBOSE)
[Out] 2/27*x^3-2/9*x^3*ln(x)+1/3*x^3*ln(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x^2 \log^2(x)\, dx = \frac{1}{3}\, x^3 \log\left(x\right)^2 - \frac{2}{9}\, x^3 \log\left(x\right) + \frac{2}{27}\, x^3$$

```
[In] integrate(x^2*log(x)^2,x, algorithm="fricas")
[Out] 1/3*x^3*log(x)^2 - 2/9*x^3*log(x) + 2/27*x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int x^2 \log^2(x)\, dx = \frac{x^3 \log\left(x\right)^2}{3} - \frac{2x^3 \log\left(x\right)}{9} + \frac{2x^3}{27}$$

```
[In] integrate(x**2*ln(x)**2,x)
[Out] x**3*log(x)**2/3 - 2*x**3*log(x)/9 + 2*x**3/27
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x^2 \log^2(x)\, dx = \frac{1}{27}\left(9\, \log\left(x\right)^2 - 6\, \log\left(x\right) + 2\right)x^3$$

```
[In] integrate(x^2*log(x)^2,x, algorithm="maxima")
[Out] 1/27*(9*log(x)^2 - 6*log(x) + 2)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x^2 \log^2(x)\, dx = \frac{1}{3}\, x^3 \log\left(x\right)^2 - \frac{2}{9}\, x^3 \log\left(x\right) + \frac{2}{27}\, x^3$$

```
[In] integrate(x^2*log(x)^2,x, algorithm="giac")
[Out] 1/3*x^3*log(x)^2 - 2/9*x^3*log(x) + 2/27*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x^2 \log^2(x)\, dx = \frac{2\, x^3 \left(\frac{9 \ln(x)^2}{2} - 3\, \ln\left(x\right) + 1\right)}{27}$$

```
[In] int(x^2*log(x)^2,x)
[Out] (2*x^3*((9*log(x)^2)/2 - 3*log(x) + 1))/27
```

3.64 $\int \frac{1}{x \log(x)} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 3

$$\int \frac{1}{x \log(x)} \, dx = \log(\log(x))$$

```
[Out] ln(ln(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2339, 29}

$$\int \frac{1}{x \log(x)} \, dx = \log(\log(x))$$

```
[In] Int[1/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{x}\,dx, x, \log(x)\right)$$
$$= \log(\log(x))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log(\log(x))$$

```
[In] Integrate[1/(x*Log[x]),x]
[Out] Log[Log[x]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\ln(\ln(x))$	4
default	$\ln(\ln(x))$	4
norman	$\ln(\ln(x))$	4
risch	$\ln(\ln(x))$	4
parallelrisch	$\ln(\ln(x))$	4

```
[In] int(1/x/ln(x),x,method=_RETURNVERBOSE)
[Out] ln(ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/log(x),x, algorithm="fricas")
[Out] log(log(x))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/ln(x),x)
[Out] log(log(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/log(x),x, algorithm="maxima")
[Out] log(log(x))
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\log(x)}\,dx = \log\left(|\log\left(x\right)|\right)$$

```
[In] integrate(1/x/log(x),x, algorithm="giac")
[Out] log(abs(log(x)))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \ln\left(\ln\left(x\right)\right)$$

```
[In] int(1/(x*log(x)),x)
[Out] log(log(x))
```

3.65 $\int \frac{\log(1-t)}{1-t} \, dt$

Optimal result

Integrand size = 14, antiderivative size = 12

$$\int \frac{\log(1-t)}{1-t} \, dt = -\frac{1}{2} \log^2(1-t)$$

```
[Out] -1/2*ln(1-t)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2437, 2338}

$$\int \frac{\log(1-t)}{1-t} \, dt = -\frac{1}{2} \log^2(1-t)$$

```
[In] Int[Log[1 - t]/(1 - t),t]
[Out] -1/2*Log[1 - t]^2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{\log(t)}{t}\,dt, t, 1-t\right)\\ &= -\frac{1}{2}\log^2(1-t)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log(1-t)}{1-t}\,dt = -\frac{1}{2}\log^2(1-t)$$

```
[In] Integrate[Log[1 - t]/(1 - t),t]
[Out] -1/2*Log[1 - t]^2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{\ln(1-t)^2}{2}$	11
default	$-\frac{\ln(1-t)^2}{2}$	11
norman	$-\frac{\ln(1-t)^2}{2}$	11
risch	$-\frac{\ln(1-t)^2}{2}$	11
parts	$-\ln(-1+t)\ln(1-t)+\frac{\ln(-1+t)^2}{2}$	22

```
[In] int(ln(1-t)/(1-t),t,method=_RETURNVERBOSE)
[Out] -1/2*ln(1-t)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\log(1-t)}{1-t}\,dt = -\frac{1}{2}\,\log\left(-t+1\right)^2$$

```
[In] integrate(log(1-t)/(1-t),t, algorithm="fricas")
[Out] -1/2*log(-t + 1)^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\log(1-t)}{1-t}\,dt = -\frac{\log\left(1-t\right)^2}{2}$$

```
[In] integrate(ln(1-t)/(1-t),t)
[Out] -log(1 - t)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\log(1-t)}{1-t}\,dt = -\frac{1}{2}\,\log\left(-t+1\right)^2$$

```
[In] integrate(log(1-t)/(1-t),t, algorithm="maxima")
[Out] -1/2*log(-t + 1)^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\log(1-t)}{1-t}\,dt = -\frac{1}{2}\,\log\left(-t+1\right)^2$$

```
[In] integrate(log(1-t)/(1-t),t, algorithm="giac")
[Out] -1/2*log(-t + 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\log(1-t)}{1-t}\,dt = -\frac{\ln{(1-t)}^2}{2}$$

```
[In] int(-log(1 - t)/(t - 1),t)
[Out] -log(1 - t)^2/2
```

3.66 $\int \frac{\log(x)}{x\sqrt{1+\log(x)}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}} \, dx = -2\sqrt{1+\log(x)} + \frac{2}{3}(1+\log(x))^{3/2}$$

```
[Out] 2/3*(1+ln(x))^(3/2)-2*(1+ln(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2412, 45}

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}} \, dx = \frac{2}{3}(\log(x)+1)^{3/2} - 2\sqrt{\log(x)+1}$$

```
[In] Int[Log[x]/(x*Sqrt[1 + Log[x]]),x]
```

```
[Out] -2*Sqrt[1 + Log[x]] + (2*(1 + Log[x])^(3/2))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2412

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
```

```
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right)\,dx, x, \log(x)\right)\\ &= -2\sqrt{1+\log(x)}+\frac{2}{3}(1+\log(x))^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2}{3}(-2+\log(x))\sqrt{1+\log(x)}$$

```
[In] Integrate[Log[x]/(x*Sqrt[1 + Log[x]]),x]
[Out] (2*(-2 + Log[x])*Sqrt[1 + Log[x]])/3
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{2(1+\ln(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\ln(x)}$	18
default	$\frac{2(1+\ln(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\ln(x)}$	18

```
[In] int(ln(x)/x/(1+ln(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(1+ln(x))^(3/2)-2*(1+ln(x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2}{3}\,\sqrt{\log{(x)}+1}(\log{(x)}-2)$$

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(log(x) + 1)*(log(x) - 2)
```

Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2(\log{(x)}+1)^{\frac{3}{2}}}{3} - 2\sqrt{\log{(x)}+1}$$

```
[In] integrate(ln(x)/x/(1+ln(x))**(1/2),x)
[Out] 2*(log(x) + 1)**(3/2)/3 - 2*sqrt(log(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2}{3}\,(\log{(x)}+1)^{\frac{3}{2}} - 2\,\sqrt{\log{(x)}+1}$$

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="maxima")
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2}{3}\,(\log{(x)}+1)^{\frac{3}{2}} - 2\,\sqrt{\log{(x)}+1}$$

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="giac")
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \sqrt{\ln(x)+1}\,\left(\frac{2\,\ln(x)}{3}-\frac{4}{3}\right)$$

```
[In] int(log(x)/(x*(log(x) + 1)^(1/2)),x)
[Out] (log(x) + 1)^(1/2)*((2*log(x))/3 - 4/3)
```

3.67 $\int x^3 \log^3(x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 39

$$\int x^3 \log^3(x) \, dx = -\frac{3x^4}{128} + \frac{3}{32}x^4 \log(x) - \frac{3}{16}x^4 \log^2(x) + \frac{1}{4}x^4 \log^3(x)$$

```
[Out] -3/128*x^4+3/32*x^4*ln(x)-3/16*x^4*ln(x)^2+1/4*x^4*ln(x)^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2342, 2341}

$$\int x^3 \log^3(x) \, dx = -\frac{3x^4}{128} + \frac{1}{4}x^4 \log^3(x) - \frac{3}{16}x^4 \log^2(x) + \frac{3}{32}x^4 \log(x)$$

```
[In] Int[x^3*Log[x]^3,x]
```

```
[Out] (-3*x^4)/128 + (3*x^4*Log[x])/32 - (3*x^4*Log[x]^2)/16 + (x^4*Log[x]^3)/4
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\log^3(x) - \frac{3}{4}\int x^3\log^2(x)\,dx \\
&= -\frac{3}{16}x^4\log^2(x) + \frac{1}{4}x^4\log^3(x) + \frac{3}{8}\int x^3\log(x)\,dx \\
&= -\frac{3x^4}{128} + \frac{3}{32}x^4\log(x) - \frac{3}{16}x^4\log^2(x) + \frac{1}{4}x^4\log^3(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int x^3\log^3(x)\,dx = -\frac{3x^4}{128} + \frac{3}{32}x^4\log(x) - \frac{3}{16}x^4\log^2(x) + \frac{1}{4}x^4\log^3(x)$$

```
[In] Integrate[x^3*Log[x]^3,x]
[Out] (-3*x^4)/128 + (3*x^4*Log[x])/32 - (3*x^4*Log[x]^2)/16 + (x^4*Log[x]^3)/4
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{3x^4}{128}+\frac{3x^4\ln(x)}{32}-\frac{3x^4\ln(x)^2}{16}+\frac{x^4\ln(x)^3}{4}$	32
norman	$-\frac{3x^4}{128}+\frac{3x^4\ln(x)}{32}-\frac{3x^4\ln(x)^2}{16}+\frac{x^4\ln(x)^3}{4}$	32
risch	$-\frac{3x^4}{128}+\frac{3x^4\ln(x)}{32}-\frac{3x^4\ln(x)^2}{16}+\frac{x^4\ln(x)^3}{4}$	32
parallelrisch	$-\frac{3x^4}{128}+\frac{3x^4\ln(x)}{32}-\frac{3x^4\ln(x)^2}{16}+\frac{x^4\ln(x)^3}{4}$	32
parts	$-\frac{3x^4}{128}+\frac{3x^4\ln(x)}{32}-\frac{3x^4\ln(x)^2}{16}+\frac{x^4\ln(x)^3}{4}$	32

```
[In] int(x^3*ln(x)^3,x,method=_RETURNVERBOSE)
[Out] -3/128*x^4+3/32*x^4*ln(x)-3/16*x^4*ln(x)^2+1/4*x^4*ln(x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^3 \log^3(x)\,dx = \frac{1}{4}\,x^4 \log{(x)}^3 - \frac{3}{16}\,x^4 \log{(x)}^2 + \frac{3}{32}\,x^4 \log{(x)} - \frac{3}{128}\,x^4$$

```
[In] integrate(x^3*log(x)^3,x, algorithm="fricas")
[Out] 1/4*x^4*log(x)^3 - 3/16*x^4*log(x)^2 + 3/32*x^4*log(x) - 3/128*x^4
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int x^3 \log^3(x)\,dx = \frac{x^4 \log{(x)}^3}{4} - \frac{3x^4 \log{(x)}^2}{16} + \frac{3x^4 \log{(x)}}{32} - \frac{3x^4}{128}$$

```
[In] integrate(x**3*ln(x)**3,x)
[Out] x**4*log(x)**3/4 - 3*x**4*log(x)**2/16 + 3*x**4*log(x)/32 - 3*x**4/128
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.59

$$\int x^3 \log^3(x)\,dx = \frac{1}{128}\,\left(32\,\log{(x)}^3 - 24\,\log{(x)}^2 + 12\,\log{(x)} - 3\right)x^4$$

```
[In] integrate(x^3*log(x)^3,x, algorithm="maxima")
[Out] 1/128*(32*log(x)^3 - 24*log(x)^2 + 12*log(x) - 3)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^3 \log^3(x)\,dx = \frac{1}{4}\,x^4 \log{(x)}^3 - \frac{3}{16}\,x^4 \log{(x)}^2 + \frac{3}{32}\,x^4 \log{(x)} - \frac{3}{128}\,x^4$$

```
[In] integrate(x^3*log(x)^3,x, algorithm="giac")
[Out] 1/4*x^4*log(x)^3 - 3/16*x^4*log(x)^2 + 3/32*x^4*log(x) - 3/128*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.59

$$\int x^3 \log^3(x)\,dx = \frac{3\,x^4\,\left(\frac{32\ln(x)^3}{3}-8\ln\left(x\right)^2+4\,\ln\left(x\right)-1\right)}{128}$$

```
[In] int(x^3*log(x)^3,x)
[Out] (3*x^4*(4*log(x) - 8*log(x)^2 + (32*log(x)^3)/3 - 1))/128
```

3.68 $\int e^{x^3} x^2 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 9

$$\int e^{x^3} x^2 \, dx = \frac{e^{x^3}}{3}$$

```
[Out] 1/3*exp(x^3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{2240\}$

$$\int e^{x^3} x^2 \, dx = \frac{e^{x^3}}{3}$$

```
[In] Int[E^x^3*x^2,x]
```

```
[Out] E^x^3/3
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{e^{x^3}}{3}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int e^{x^3} x^2 \, dx = \frac{e^{x^3}}{3}$$

```
[In] Integrate[E^x^3*x^2,x]
[Out] E^x^3/3
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{\mathrm{e}^{x^3}}{3}$	7
derivativedivides	$\frac{\mathrm{e}^{x^3}}{3}$	7
default	$\frac{\mathrm{e}^{x^3}}{3}$	7
norman	$\frac{\mathrm{e}^{x^3}}{3}$	7
risch	$\frac{\mathrm{e}^{x^3}}{3}$	7
parallelrisch	$\frac{\mathrm{e}^{x^3}}{3}$	7
meijerg	$-\frac{1}{3}+\frac{\mathrm{e}^{x^3}}{3}$	9

```
[In] int(exp(x^3)*x^2,x,method=_RETURNVERBOSE)
[Out] 1/3*exp(x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^3} x^2 \, dx = \frac{1}{3} \, e^{\left(x^3\right)}$$

```
[In] integrate(exp(x^3)*x^2,x, algorithm="fricas")
[Out] 1/3*e^(x^3)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int e^{x^3}x^2\,dx=\frac{e^{x^3}}{3}$$

```
[In] integrate(exp(x**3)*x**2,x)
[Out] exp(x**3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^3}x^2\,dx=\frac{1}{3}\,e^{\left(x^3\right)}$$

```
[In] integrate(exp(x^3)*x^2,x, algorithm="maxima")
[Out] 1/3*e^(x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^3}x^2\,dx=\frac{1}{3}\,e^{\left(x^3\right)}$$

```
[In] integrate(exp(x^3)*x^2,x, algorithm="giac")
[Out] 1/3*e^(x^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^3}x^2\,dx=\frac{\mathrm{e}^{x^3}}{3}$$

```
[In] int(x^2*exp(x^3),x)
[Out] exp(x^3)/3
```

3.69 $\int \frac{2^{\sqrt{x}}}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}} \, dx = \frac{2^{1+\sqrt{x}}}{\log(2)}$$

```
[Out] 2^(1+x^(1/2))/ln(2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2240}

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}} \, dx = \frac{2^{\sqrt{x}+1}}{\log(2)}$$

```
[In] Int[2^Sqrt[x]/Sqrt[x],x]
```

```
[Out] 2^(1 + Sqrt[x])/Log[2]
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{2^{1+\sqrt{x}}}{\log(2)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx=\frac{2^{1+\sqrt{x}}}{\log(2)}$$

```
[In] Integrate[2^Sqrt[x]/Sqrt[x],x]
[Out] 2^(1 + Sqrt[x])/Log[2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2\,2^{\sqrt{x}}}{\ln(2)}$	12
default	$\frac{2\,2^{\sqrt{x}}}{\ln(2)}$	12
meijerg	$-\frac{2\left(1-\mathrm{e}^{\sqrt{x}\,\ln(2)}\right)}{\ln(2)}$	18

```
[In] int(2^(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*2^(x^(1/2))/ln(2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx=\frac{2\cdot 2^{\left(\sqrt{x}\right)}}{\log\left(2\right)}$$

```
[In] integrate(2^(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 2*2^sqrt(x)/log(2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2\cdot 2^{\sqrt{x}}}{\log{(2)}}$$

```
[In] integrate(2**(x**(1/2))/x**(1/2),x)
[Out] 2*2**(sqrt(x))/log(2)
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2^{\sqrt{x}+1}}{\log{(2)}}$$

```
[In] integrate(2^(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2^(sqrt(x) + 1)/log(2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2\cdot 2^{(\sqrt{x})}}{\log{(2)}}$$

```
[In] integrate(2^(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 2*2^sqrt(x)/log(2)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2\,2^{\sqrt{x}}}{\ln{(2)}}$$

```
[In] int(2^(x^(1/2))/x^(1/2),x)
[Out] (2*2^(x^(1/2)))/log(2)
```

3.70 $\int e^{2\sin(x)}\cos(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int e^{2\sin(x)}\cos(x)\,dx=\frac{1}{2}e^{2\sin(x)}$$

```
[Out] 1/2*exp(2*sin(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4419, 2225}

$$\int e^{2\sin(x)}\cos(x)\,dx=\frac{1}{2}e^{2\sin(x)}$$

```
[In] Int[E^(2*Sin[x])*Cos[x],x]
```

```
[Out] E^(2*Sin[x])/2
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int e^{2x}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}e^{2\sin(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int e^{2\sin(x)}\cos(x)\,dx = \frac{1}{2}e^{2\sin(x)}$$

```
[In] Integrate[E^(2*Sin[x])*Cos[x],x]
[Out] E^(2*Sin[x])/2
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{e^{2\sin(x)}}{2}$	8
default	$\frac{e^{2\sin(x)}}{2}$	8
risch	$\frac{e^{2\sin(x)}}{2}$	8
parallelrisch	$\frac{e^{2\sin(x)}}{2}$	8
norman	$\frac{\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)e^{\frac{4\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}}}{2}+\frac{e^{\frac{4\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}}}{2}}{1+\tan^2\left(\frac{x}{2}\right)}$	57

```
[In] int(exp(2*sin(x))*cos(x),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(2*sin(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2\sin(x)}\cos(x)\,dx = \frac{1}{2}\,e^{(2\,\sin(x))}$$

```
[In] integrate(exp(2*sin(x))*cos(x),x, algorithm="fricas")
[Out] 1/2*e^(2*sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2\sin(x)}\cos(x)\,dx = \frac{e^{2\sin{(x)}}}{2}$$

```
[In] integrate(exp(2*sin(x))*cos(x),x)
[Out] exp(2*sin(x))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2\sin(x)}\cos(x)\,dx = \frac{1}{2}\,e^{(2\,\sin(x))}$$

```
[In] integrate(exp(2*sin(x))*cos(x),x, algorithm="maxima")
[Out] 1/2*e^(2*sin(x))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2\sin(x)}\cos(x)\,dx = \frac{1}{2}\,e^{(2\,\sin(x))}$$

```
[In] integrate(exp(2*sin(x))*cos(x),x, algorithm="giac")
[Out] 1/2*e^(2*sin(x))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2\sin(x)}\cos(x)\,dx = \frac{\mathrm{e}^{2\ \sin(x)}}{2}$$

```
[In] int(exp(2*sin(x))*cos(x),x)
[Out] exp(2*sin(x))/2
```

3.71 $\int e^x \sin(x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 19

$$\int e^x \sin(x)\,dx = -\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)$$

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4517}

$$\int e^x \sin(x)\,dx = \frac{1}{2}e^x\sin(x) - \frac{1}{2}e^x\cos(x)$$

```
[In] Int[E^x*Sin[x],x]
```

```
[Out] -1/2*(E^x*Cos[x]) + (E^x*Sin[x])/2
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int e^x \sin(x)\, dx = \frac{1}{2}e^x(-\cos(x)+\sin(x))$$

```
[In] Integrate[E^x*Sin[x],x]
[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{e^x(-\cos(x)+\sin(x))}{2}$	12
default	$-\frac{e^x\cos(x)}{2}+\frac{e^x\sin(x)}{2}$	14
norman	$\frac{e^x\tan\left(\frac{x}{2}\right)+\frac{e^x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{e^x}{2}}{1+\tan^2\left(\frac{x}{2}\right)}$	34
risch	$-\frac{e^{(1+i)x}}{4}-\frac{i e^{(1+i)x}}{4}-\frac{e^{(1-i)x}}{4}+\frac{i e^{(1-i)x}}{4}$	36

```
[In] int(exp(x)*sin(x),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x)*(-cos(x)+sin(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sin(x)\, dx = -\frac{1}{2}\cos(x)e^x + \frac{1}{2}e^x\sin(x)$$

```
[In] integrate(exp(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int e^x \sin(x)\,dx = \frac{e^x \sin(x)}{2} - \frac{e^x \cos(x)}{2}$$

```
[In] integrate(exp(x)*sin(x),x)
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\,(\cos(x) - \sin(x))e^x$$

```
[In] integrate(exp(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\,(\cos(x) - \sin(x))e^x$$

```
[In] integrate(exp(x)*sin(x),x, algorithm="giac")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{\mathrm{e}^x\,(\cos(x) - \sin(x))}{2}$$

```
[In] int(exp(x)*sin(x),x)
[Out] -(exp(x)*(cos(x) - sin(x)))/2
```

3.72 $\int e^x \cos(x) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 19

$$\int e^x \cos(x) \, dx = \frac{1}{2} e^x \cos(x) + \frac{1}{2} e^x \sin(x)$$

```
[Out] 1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{4518\}$

$$\int e^x \cos(x) \, dx = \frac{1}{2} e^x \sin(x) + \frac{1}{2} e^x \cos(x)$$

```
[In] Int[E^x*Cos[x],x]
```

```
[Out] (E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2} e^x \cos(x) + \frac{1}{2} e^x \sin(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^x \cos(x) \, dx = \frac{1}{2} e^x (\cos(x) + \sin(x))$$

```
[In] Integrate[E^x*Cos[x],x]
[Out] (E^x*(Cos[x] + Sin[x]))/2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

method	result	size
parallelrisch	$\frac{e^x(\cos(x)+\sin(x))}{2}$	10
default	$\frac{e^x \cos(x)}{2} + \frac{e^x \sin(x)}{2}$	14
norman	$\frac{e^x \tan(\frac{x}{2}) - \frac{e^x(\tan^2(\frac{x}{2}))}{2} + \frac{e^x}{2}}{1+\tan^2(\frac{x}{2})}$	34
risch	$\frac{e^{(1+i)x}}{4} - \frac{i e^{(1+i)x}}{4} + \frac{e^{(1-i)x}}{4} + \frac{i e^{(1-i)x}}{4}$	36

```
[In] int(exp(x)*cos(x),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x)*(cos(x)+sin(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \cos(x) \, dx = \frac{1}{2} \cos(x) e^x + \frac{1}{2} e^x \sin(x)$$

```
[In] integrate(exp(x)*cos(x),x, algorithm="fricas")
[Out] 1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int e^x \cos(x)\,dx = \frac{e^x \sin(x)}{2} + \frac{e^x \cos(x)}{2}$$

```
[In] integrate(exp(x)*cos(x),x)
[Out] exp(x)*sin(x)/2 + exp(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.47

$$\int e^x \cos(x)\,dx = \frac{1}{2}(\cos(x) + \sin(x))e^x$$

```
[In] integrate(exp(x)*cos(x),x, algorithm="maxima")
[Out] 1/2*(cos(x) + sin(x))*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.47

$$\int e^x \cos(x)\,dx = \frac{1}{2}(\cos(x) + \sin(x))e^x$$

```
[In] integrate(exp(x)*cos(x),x, algorithm="giac")
[Out] 1/2*(cos(x) + sin(x))*e^x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.47

$$\int e^x \cos(x)\,dx = \frac{\mathrm{e}^x\,(\cos(x) + \sin(x))}{2}$$

```
[In] int(exp(x)*cos(x),x)
[Out] (exp(x)*(cos(x) + sin(x)))/2
```

3.73 $\int \frac{1}{1+e^x} dx$

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{1+e^x} dx = x - \log(1+e^x)$$

```
[Out] x-ln(1+exp(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2320, 36, 29, 31}

$$\int \frac{1}{1+e^x} dx = x - \log(e^x+1)$$

```
[In] Int[(1 + E^x)^(-1),x]
```

```
[Out] x - Log[1 + E^x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \frac{1}{x}\,dx, x, e^x\right) - \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, e^x\right)\\
&= x - \log\left(1+e^x\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+e^x}\,dx = -2\text{arctanh}(1+2e^x)$$

```
[In] Integrate[(1 + E^x)^(-1),x]
[Out] -2*ArcTanh[1 + 2*E^x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

method	result	size
norman	$x-\ln\left(1+{\mathrm e}^{x}\right)$	10
risch	$x-\ln\left(1+{\mathrm e}^{x}\right)$	10
parallelrisch	$x-\ln\left(1+{\mathrm e}^{x}\right)$	10
derivativedivides	$-\ln\left(1+{\mathrm e}^{x}\right)+\ln\left({\mathrm e}^{x}\right)$	12
default	$-\ln\left(1+{\mathrm e}^{x}\right)+\ln\left({\mathrm e}^{x}\right)$	12

```
[In] int(1/(1+exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x-ln(1+exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{1+e^x} \, dx = x - \log\left(e^x + 1\right)$$

```
[In] integrate(1/(1+exp(x)),x, algorithm="fricas")
[Out] x - log(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{1+e^x} \, dx = x - \log\left(e^x + 1\right)$$

```
[In] integrate(1/(1+exp(x)),x)
[Out] x - log(exp(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{1+e^x} \, dx = x - \log\left(e^x + 1\right)$$

```
[In] integrate(1/(1+exp(x)),x, algorithm="maxima")
[Out] x - log(e^x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{1+e^x}\,dx = x - \log\left(e^x + 1\right)$$

```
[In] integrate(1/(1+exp(x)),x, algorithm="giac")
[Out] x - log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{1+e^x}\,dx = x - \ln\left(\mathrm{e}^x + 1\right)$$

```
[In] int(1/(exp(x) + 1),x)
[Out] x - log(exp(x) + 1)
```

3.74 $\int e^x x \, dx$

Optimal result

Integrand size = 5, antiderivative size = 11

$$\int e^x x \, dx = -e^x + e^x x$$

```
[Out] -exp(x)+exp(x)*x
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2207, 2225}

$$\int e^x x \, dx = e^x x - e^x$$

```
[In] Int[E^x*x,x]
```

```
[Out] -E^x + E^x*x
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\text{integral} = e^x x - \int e^x \, dx$$
$$= -e^x + e^x x$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int e^x x \, dx = e^x(-1+x)$$

```
[In] Integrate[E^x*x,x]
[Out] E^x*(-1 + x)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

method	result	size
gosper	$(-1+x)\,\mathrm{e}^x$	7
risch	$(-1+x)\,\mathrm{e}^x$	7
default	$-\mathrm{e}^x+\mathrm{e}^x x$	10
norman	$-\mathrm{e}^x+\mathrm{e}^x x$	10
parallelrisch	$-\mathrm{e}^x+\mathrm{e}^x x$	10
parts	$-\mathrm{e}^x+\mathrm{e}^x x$	10
meijerg	$1-\frac{(-2x+2)\mathrm{e}^x}{2}$	12

```
[In] int(exp(x)*x,x,method=_RETURNVERBOSE)
[Out] (-1+x)*exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int e^x x\,dx = (x-1)e^x$$

```
[In] integrate(exp(x)*x,x, algorithm="fricas")
[Out] (x - 1)*e^x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.45

$$\int e^x x\,dx = (x-1)\,e^x$$

```
[In] integrate(exp(x)*x,x)
[Out] (x - 1)*exp(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int e^x x\,dx = (x-1)e^x$$

```
[In] integrate(exp(x)*x,x, algorithm="maxima")
[Out] (x - 1)*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int e^x x\,dx = (x-1)e^x$$

```
[In] integrate(exp(x)*x,x, algorithm="giac")
[Out] (x - 1)*e^x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int e^x x\,dx = \mathrm{e}^x\,(x-1)$$

```
[In] int(x*exp(x),x)
[Out] exp(x)*(x - 1)
```

3.75 $\int e^{-x}x\,dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int e^{-x}x\,dx = -e^{-x} - e^{-x}x$$

```
[Out] -1/exp(x)-x/exp(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2207, 2225}

$$\int e^{-x}x\,dx = -e^{-x}x - e^{-x}$$

```
[In] Int[x/E^x,x]
```

```
[Out] -E^(-x) - x/E^x
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -e^{-x}x + \int e^{-x}\,dx\\ &= -e^{-x} - e^{-x}x\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int e^{-x}x\,dx = e^{-x}(-1-x)$$

```
[In] Integrate[x/E^x,x]
[Out] (-1 - x)/E^x
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

method	result	size
gosper	$-(1+x)\,\mathrm{e}^{-x}$	10
norman	$(-1-x)\,\mathrm{e}^{-x}$	11
risch	$(-1-x)\,\mathrm{e}^{-x}$	11
parallelrisch	$(-1-x)\,\mathrm{e}^{-x}$	11
meijerg	$1-\frac{(2x+2)\mathrm{e}^{-x}}{2}$	14
default	$-\mathrm{e}^{-x}-x\,\mathrm{e}^{-x}$	15

```
[In] int(x/exp(x),x,method=_RETURNVERBOSE)
[Out] -(1+x)/exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-x}x\,dx = -(x+1)e^{(-x)}$$

```
[In] integrate(x/exp(x),x, algorithm="fricas")
[Out] -(x + 1)*e^(-x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int e^{-x}x\,dx = (-x-1)\,e^{-x}$$

```
[In] integrate(x/exp(x),x)
[Out] (-x - 1)*exp(-x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-x}x\,dx = -(x+1)e^{(-x)}$$

```
[In] integrate(x/exp(x),x, algorithm="maxima")
[Out] -(x + 1)*e^(-x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-x}x\,dx = -(x+1)e^{(-x)}$$

```
[In] integrate(x/exp(x),x, algorithm="giac")
[Out] -(x + 1)*e^(-x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-x}x\,dx = -\mathrm{e}^{-x}\,(x+1)$$

```
[In] int(x*exp(-x),x)
[Out] -exp(-x)*(x + 1)
```

3.76 $\int e^x x^2 \, dx$

Optimal result

Integrand size = 7, antiderivative size = 19

$$\int e^x x^2 \, dx = 2e^x - 2e^x x + e^x x^2$$

```
[Out] 2*exp(x)-2*exp(x)*x+exp(x)*x^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2207, 2225}

$$\int e^x x^2 \, dx = e^x x^2 - 2e^x x + 2e^x$$

```
[In] Int[E^x*x^2,x]
```

```
[Out] 2*E^x - 2*E^x*x + E^x*x^2
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= e^x x^2 - 2\int e^x x\,dx\\ &= -2e^x x + e^x x^2 + 2\int e^x\,dx\\ &= 2e^x - 2e^x x + e^x x^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^x x^2\,dx = e^x\left(2 - 2x + x^2\right)$$

```
[In] Integrate[E^x*x^2,x]
[Out] E^x*(2 - 2*x + x^2)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
gosper	$(x^2 - 2x + 2)\,\mathrm{e}^x$	12
risch	$(x^2 - 2x + 2)\,\mathrm{e}^x$	12
default	$2\,\mathrm{e}^x - 2\,\mathrm{e}^x x + \mathrm{e}^x x^2$	17
norman	$2\,\mathrm{e}^x - 2\,\mathrm{e}^x x + \mathrm{e}^x x^2$	17
meijerg	$-2 + \frac{(3x^2 - 6x + 6)\,\mathrm{e}^x}{3}$	17
parallelrisch	$2\,\mathrm{e}^x - 2\,\mathrm{e}^x x + \mathrm{e}^x x^2$	17
parts	$2\,\mathrm{e}^x - 2\,\mathrm{e}^x x + \mathrm{e}^x x^2$	17

```
[In] int(exp(x)*x^2,x,method=_RETURNVERBOSE)
[Out] (x^2-2*x+2)*exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x x^2 \, dx = (x^2 - 2\,x + 2)e^x$$

```
[In] integrate(exp(x)*x^2,x, algorithm="fricas")
[Out] (x^2 - 2*x + 2)*e^x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int e^x x^2 \, dx = (x^2 - 2x + 2)\,e^x$$

```
[In] integrate(exp(x)*x**2,x)
[Out] (x**2 - 2*x + 2)*exp(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x x^2 \, dx = (x^2 - 2\,x + 2)e^x$$

```
[In] integrate(exp(x)*x^2,x, algorithm="maxima")
[Out] (x^2 - 2*x + 2)*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x x^2 \, dx = (x^2 - 2\,x + 2)e^x$$

```
[In] integrate(exp(x)*x^2,x, algorithm="giac")
[Out] (x^2 - 2*x + 2)*e^x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x x^2\,dx = \mathrm{e}^x\left(x^2 - 2\,x + 2\right)$$

```
[In] int(x^2*exp(x),x)
[Out] exp(x)*(x^2 - 2*x + 2)
```

3.77 $\int e^{-2x}x^2\,dx$

Optimal result

Integrand size = 9, antiderivative size = 32

$$\int e^{-2x}x^2\,dx = -\frac{1}{4}e^{-2x} - \frac{1}{2}e^{-2x}x - \frac{1}{2}e^{-2x}x^2$$

```
[Out] -1/4/exp(2*x)-1/2*x/exp(2*x)-1/2*x^2/exp(2*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2207, 2225}

$$\int e^{-2x}x^2\,dx = -\frac{1}{2}e^{-2x}x^2 - \frac{1}{2}e^{-2x}x - \frac{e^{-2x}}{4}$$

```
[In] Int[x^2/E^(2*x),x]
```

```
[Out] -1/4*1/E^(2*x) - x/(2*E^(2*x)) - x^2/(2*E^(2*x))
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{2}e^{-2x}x^2 + \int e^{-2x}x\,dx \\
&= -\frac{1}{2}e^{-2x}x - \frac{1}{2}e^{-2x}x^2 + \frac{1}{2}\int e^{-2x}\,dx \\
&= -\frac{1}{4}e^{-2x} - \frac{1}{2}e^{-2x}x - \frac{1}{2}e^{-2x}x^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int e^{-2x}x^2\,dx = -\frac{1}{4}e^{-2x}\left(1+2x+2x^2\right)$$

```
[In] Integrate[x^2/E^(2*x),x]
[Out] -1/4*(1 + 2*x + 2*x^2)/E^(2*x)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

method	result	size
risch	$\left(-\frac{1}{2}x^2-\frac{1}{2}x-\frac{1}{4}\right){\mathrm e}^{-2x}$	16
norman	$\left(-\frac{1}{2}x^2-\frac{1}{2}x-\frac{1}{4}\right){\mathrm e}^{-2x}$	18
gosper	$-\frac{\left(2x^2+2x+1\right){\mathrm e}^{-2x}}{4}$	19
meijerg	$\frac{1}{4}-\frac{\left(12x^2+12x+6\right){\mathrm e}^{-2x}}{24}$	19
parallelrisch	$\frac{\left(-2x^2-2x-1\right){\mathrm e}^{-2x}}{4}$	19
derivativedivides	$-\frac{{\mathrm e}^{-2x}}{4}-\frac{x\,{\mathrm e}^{-2x}}{2}-\frac{x^2{\mathrm e}^{-2x}}{2}$	30
default	$-\frac{{\mathrm e}^{-2x}}{4}-\frac{x\,{\mathrm e}^{-2x}}{2}-\frac{x^2{\mathrm e}^{-2x}}{2}$	30

```
[In] int(x^2/exp(2*x),x,method=_RETURNVERBOSE)
[Out] (-1/2*x^2-1/2*x-1/4)*exp(-2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{-2x}x^2\,dx = -\frac{1}{4}\left(2\,x^2+2\,x+1\right)e^{(-2\,x)}$$

```
[In] integrate(x^2/exp(2*x),x, algorithm="fricas")
[Out] -1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int e^{-2x}x^2\,dx = \frac{(-2x^2-2x-1)\,e^{-2x}}{4}$$

```
[In] integrate(x**2/exp(2*x),x)
[Out] (-2*x**2 - 2*x - 1)*exp(-2*x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{-2x}x^2\,dx = -\frac{1}{4}\left(2\,x^2+2\,x+1\right)e^{(-2\,x)}$$

```
[In] integrate(x^2/exp(2*x),x, algorithm="maxima")
[Out] -1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{-2x}x^2\,dx = -\frac{1}{4}\left(2\,x^2+2\,x+1\right)e^{(-2\,x)}$$

```
[In] integrate(x^2/exp(2*x),x, algorithm="giac")
[Out] -1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{-2x} x^2 \, dx = -\frac{e^{-2x} (4x^2 + 4x + 2)}{8}$$

```
[In] int(x^2*exp(-2*x),x)
[Out] -(exp(-2*x)*(4*x + 4*x^2 + 2))/8
```

3.78 $\int e^{\sqrt{x}}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 24

$$\int e^{\sqrt{x}}\,dx = -2e^{\sqrt{x}} + 2e^{\sqrt{x}}\sqrt{x}$$

```
[Out] -2*exp(x^(1/2))+2*exp(x^(1/2))*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2238, 2207, 2225}

$$\int e^{\sqrt{x}}\,dx = 2e^{\sqrt{x}}\sqrt{x} - 2e^{\sqrt{x}}$$

```
[In] Int[E^Sqrt[x],x]
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2238

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Dist[k/d, Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (c
 + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Inte
gerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int e^x x\,dx, x, \sqrt{x}\right)\\ &= 2e^{\sqrt{x}}\sqrt{x} - 2\text{Subst}\left(\int e^x\,dx, x, \sqrt{x}\right)\\ &= -2e^{\sqrt{x}} + 2e^{\sqrt{x}}\sqrt{x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int e^{\sqrt{x}}\,dx = 2e^{\sqrt{x}}\left(-1+\sqrt{x}\right)$$

```
[In] Integrate[E^Sqrt[x],x]
[Out] 2*E^Sqrt[x]*(-1 + Sqrt[x])
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
meijerg	$2-\left(-2\sqrt{x}+2\right)\mathrm{e}^{\sqrt{x}}$	16
derivativedivides	$-2\,\mathrm{e}^{\sqrt{x}}+2\,\mathrm{e}^{\sqrt{x}}\sqrt{x}$	17
default	$-2\,\mathrm{e}^{\sqrt{x}}+2\,\mathrm{e}^{\sqrt{x}}\sqrt{x}$	17

```
[In] int(exp(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2-(-2*x^(1/2)+2)*exp(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int e^{\sqrt{x}}\,dx = 2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

```
[In] integrate(exp(x^(1/2)),x, algorithm="fricas")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int e^{\sqrt{x}}\,dx = 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}}$$

```
[In] integrate(exp(x**(1/2)),x)
[Out] 2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int e^{\sqrt{x}}\,dx = 2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

```
[In] integrate(exp(x^(1/2)),x, algorithm="maxima")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int e^{\sqrt{x}}\,dx = 2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

```
[In] integrate(exp(x^(1/2)),x, algorithm="giac")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int e^{\sqrt{x}}\,dx = 2\,\mathrm{e}^{\sqrt{x}}\,\left(\sqrt{x}-1\right)$$

```
[In] int(exp(x^(1/2)),x)
[Out] 2*exp(x^(1/2))*(x^(1/2) - 1)
```

3.79 $\int e^{-x^2} x^3 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int e^{-x^2} x^3 \, dx = -\frac{e^{-x^2}}{2} - \frac{1}{2} e^{-x^2} x^2$$

```
[Out] -1/2/exp(x^2)-1/2*x^2/exp(x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2243, 2240}

$$\int e^{-x^2} x^3 \, dx = -\frac{1}{2} e^{-x^2} x^2 - \frac{e^{-x^2}}{2}$$

```
[In] Int[x^3/E^x^2,x]
```

```
[Out] -1/2*1/E^x^2 - x^2/(2*E^x^2)
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
```

```
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2}e^{-x^2}x^2 + \int e^{-x^2}x\,dx\\ &= -\frac{e^{-x^2}}{2} - \frac{1}{2}e^{-x^2}x^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int e^{-x^2}x^3\,dx = -\frac{1}{2}e^{-x^2}\left(1+x^2\right)$$

```
[In] Integrate[x^3/E^x^2,x]
[Out] -1/2*(1 + x^2)/E^x^2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{(x^2+1)e^{-x^2}}{2}$	14
norman	$\left(-\frac{x^2}{2}-\frac{1}{2}\right)e^{-x^2}$	15
risch	$\left(-\frac{x^2}{2}-\frac{1}{2}\right)e^{-x^2}$	15
parallelrisch	$\frac{(-x^2-1)e^{-x^2}}{2}$	16
meijerg	$\frac{1}{2}-\frac{(2x^2+2)e^{-x^2}}{4}$	18
derivativedivides	$-\frac{e^{-x^2}}{2}-\frac{x^2e^{-x^2}}{2}$	21
default	$-\frac{e^{-x^2}}{2}-\frac{x^2e^{-x^2}}{2}$	21

```
[In] int(x^3/exp(x^2),x,method=_RETURNVERBOSE)
[Out] -1/2*(x^2+1)/exp(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int e^{-x^2} x^3 \, dx = -\frac{1}{2} \left(x^2 + 1\right) e^{\left(-x^2\right)}$$

```
[In] integrate(x^3/exp(x^2),x, algorithm="fricas")
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.46

$$\int e^{-x^2} x^3 \, dx = \frac{(-x^2 - 1)\, e^{-x^2}}{2}$$

```
[In] integrate(x**3/exp(x**2),x)
[Out] (-x**2 - 1)*exp(-x**2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int e^{-x^2} x^3 \, dx = -\frac{1}{2} \left(x^2 + 1\right) e^{\left(-x^2\right)}$$

```
[In] integrate(x^3/exp(x^2),x, algorithm="maxima")
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int e^{-x^2} x^3 \, dx = -\frac{1}{2} \left(x^2 + 1\right) e^{\left(-x^2\right)}$$

```
[In] integrate(x^3/exp(x^2),x, algorithm="giac")
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int e^{-x^2} x^3\,dx = -\frac{\mathrm{e}^{-x^2}\,(x^2+1)}{2}$$

```
[In] int(x^3*exp(-x^2),x)
[Out] -(exp(-x^2)*(x^2 + 1))/2
```

3.80 $\int e^{ax} \cos(bx) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int e^{ax} \cos(bx) \, dx = \frac{ae^{ax} \cos(bx)}{a^2 + b^2} + \frac{be^{ax} \sin(bx)}{a^2 + b^2}$$

```
[Out] a*exp(a*x)*cos(b*x)/(a^2+b^2)+b*exp(a*x)*sin(b*x)/(a^2+b^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4518}

$$\int e^{ax} \cos(bx) \, dx = \frac{be^{ax} \sin(bx)}{a^2 + b^2} + \frac{ae^{ax} \cos(bx)}{a^2 + b^2}$$

```
[In] Int[E^(a*x)*Cos[b*x],x]
```

```
[Out] (a*E^(a*x)*Cos[b*x])/(a^2 + b^2) + (b*E^(a*x)*Sin[b*x])/(a^2 + b^2)
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = \frac{ae^{ax} \cos(bx)}{a^2 + b^2} + \frac{be^{ax} \sin(bx)}{a^2 + b^2}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int e^{ax}\cos(bx)\,dx=\frac{e^{ax}(a\cos(bx)+b\sin(bx))}{a^2+b^2}$$

```
[In] Integrate[E^(a*x)*Cos[b*x],x]
[Out] (E^(a*x)*(a*Cos[b*x] + b*Sin[b*x]))/(a^2 + b^2)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{\mathrm{e}^{ax}(\cos(bx)a+b\sin(bx))}{a^2+b^2}$	28
default	$\frac{a\,\mathrm{e}^{ax}\cos(bx)}{a^2+b^2}+\frac{b\,\mathrm{e}^{ax}\sin(bx)}{a^2+b^2}$	40
risch	$\frac{\mathrm{e}^{x(ib+a)}}{2ib+2a}+\frac{\mathrm{e}^{x(-ib+a)}}{-2ib+2a}$	40
norman	$\frac{\frac{a\,\mathrm{e}^{ax}}{a^2+b^2}-\frac{a\,\mathrm{e}^{ax}\left(\tan^2\left(\frac{bx}{2}\right)\right)}{a^2+b^2}+\frac{2b\,\mathrm{e}^{ax}\tan\left(\frac{bx}{2}\right)}{a^2+b^2}}{1+\tan^2\left(\frac{bx}{2}\right)}$	73

```
[In] int(exp(a*x)*cos(b*x),x,method=_RETURNVERBOSE)
[Out] 1/(a^2+b^2)*exp(a*x)*(cos(b*x)*a+b*sin(b*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int e^{ax}\cos(bx)\,dx=\frac{a\cos(bx)\,e^{(ax)}+be^{(ax)}\sin(bx)}{a^2+b^2}$$

```
[In] integrate(exp(a*x)*cos(b*x),x, algorithm="fricas")
[Out] (a*cos(b*x)*e^(a*x) + b*e^(a*x)*sin(b*x))/(a^2 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.39

$$\int e^{ax}\cos(bx)\,dx = \begin{cases} x & \text{for } a=0 \wedge b=0 \\ \frac{ixe^{-ibx}\sin(bx)}{2} + \frac{xe^{-ibx}\cos(bx)}{2} + \frac{ie^{-ibx}\cos(bx)}{2b} & \text{for } a=-ib \\ -\frac{ixe^{ibx}\sin(bx)}{2} + \frac{xe^{ibx}\cos(bx)}{2} - \frac{ie^{ibx}\cos(bx)}{2b} & \text{for } a=ib \\ \frac{ae^{ax}\cos(bx)}{a^2+b^2} + \frac{be^{ax}\sin(bx)}{a^2+b^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(a*x)*cos(b*x),x)
```

```
[Out] Piecewise((x, Eq(a, 0) & Eq(b, 0)), (I*x*exp(-I*b*x)*sin(b*x)/2 + x*exp(-I*
b*x)*cos(b*x)/2 + I*exp(-I*b*x)*cos(b*x)/(2*b), Eq(a, -I*b)), (-I*x*exp(I*b
*x)*sin(b*x)/2 + x*exp(I*b*x)*cos(b*x)/2 - I*exp(I*b*x)*cos(b*x)/(2*b), Eq(
a, I*b)), (a*exp(a*x)*cos(b*x)/(a**2 + b**2) + b*exp(a*x)*sin(b*x)/(a**2 +
b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int e^{ax}\cos(bx)\,dx = \frac{(a\cos(bx) + b\sin(bx))e^{(ax)}}{a^2+b^2}$$

```
[In] integrate(exp(a*x)*cos(b*x),x, algorithm="maxima")
```

```
[Out] (a*cos(b*x) + b*sin(b*x))*e^(a*x)/(a^2 + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int e^{ax}\cos(bx)\,dx = \left(\frac{a\cos(bx)}{a^2+b^2} + \frac{b\sin(bx)}{a^2+b^2}\right)e^{(ax)}$$

```
[In] integrate(exp(a*x)*cos(b*x),x, algorithm="giac")
```

```
[Out] (a*cos(b*x)/(a^2 + b^2) + b*sin(b*x)/(a^2 + b^2))*e^(a*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int e^{ax} \cos(bx)\, dx = \frac{e^{a\,x}\,\left(a\,\cos\left(b\,x\right)+b\,\sin\left(b\,x\right)\right)}{a^2+b^2}$$

```
[In] int(exp(a*x)*cos(b*x),x)
[Out] (exp(a*x)*(a*cos(b*x) + b*sin(b*x)))/(a^2 + b^2)
```

3.81 $\int e^{ax} \sin(bx) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int e^{ax} \sin(bx) \, dx = -\frac{be^{ax} \cos(bx)}{a^2+b^2} + \frac{ae^{ax} \sin(bx)}{a^2+b^2}$$

```
[Out] -b*exp(a*x)*cos(b*x)/(a^2+b^2)+a*exp(a*x)*sin(b*x)/(a^2+b^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4517}

$$\int e^{ax} \sin(bx) \, dx = \frac{ae^{ax} \sin(bx)}{a^2+b^2} - \frac{be^{ax} \cos(bx)}{a^2+b^2}$$

```
[In] Int[E^(a*x)*Sin[b*x],x]
```

```
[Out] -((b*E^(a*x)*Cos[b*x])/(a^2 + b^2)) + (a*E^(a*x)*Sin[b*x])/(a^2 + b^2)
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{be^{ax} \cos(bx)}{a^2+b^2} + \frac{ae^{ax} \sin(bx)}{a^2+b^2}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int e^{ax} \sin(bx)\, dx = \frac{e^{ax}(-b\cos(bx) + a\sin(bx))}{a^2 + b^2}$$

```
[In] Integrate[E^(a*x)*Sin[b*x],x]
[Out] (E^(a*x)*(-(b*Cos[b*x]) + a*Sin[b*x]))/(a^2 + b^2)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$\frac{e^{ax}(\sin(bx)a-b\cos(bx))}{a^2+b^2}$	29
default	$-\frac{b\,e^{ax}\cos(bx)}{a^2+b^2}+\frac{a\,e^{ax}\sin(bx)}{a^2+b^2}$	41
risch	$-\frac{ie^{x(ib+a)}}{2(ib+a)}+\frac{ie^{x(-ib+a)}}{-2ib+2a}$	42
norman	$\frac{\frac{b\,e^{ax}\left(\tan^2\left(\frac{bx}{2}\right)\right)}{a^2+b^2}-\frac{b\,e^{ax}}{a^2+b^2}+\frac{2a\,e^{ax}\tan\left(\frac{bx}{2}\right)}{a^2+b^2}}{1+\tan^2\left(\frac{bx}{2}\right)}$	73

```
[In] int(exp(a*x)*sin(b*x),x,method=_RETURNVERBOSE)
[Out] 1/(a^2+b^2)*exp(a*x)*(sin(b*x)*a-b*cos(b*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int e^{ax} \sin(bx)\, dx = -\frac{b\cos(bx)\, e^{(ax)} - ae^{(ax)}\sin(bx)}{a^2 + b^2}$$

```
[In] integrate(exp(a*x)*sin(b*x),x, algorithm="fricas")
[Out] -(b*cos(b*x)*e^(a*x) - a*e^(a*x)*sin(b*x))/(a^2 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.24

$$\int e^{ax}\sin(bx)\,dx = \begin{cases} 0 & \text{for } a=0 \wedge b=0 \\ \frac{xe^{-ibx}\sin(bx)}{2} - \frac{ixe^{-ibx}\cos(bx)}{2} - \frac{e^{-ibx}\cos(bx)}{2b} & \text{for } a=-ib \\ \frac{xe^{ibx}\sin(bx)}{2} + \frac{ixe^{ibx}\cos(bx)}{2} - \frac{e^{ibx}\cos(bx)}{2b} & \text{for } a=ib \\ \frac{ae^{ax}\sin(bx)}{a^2+b^2} - \frac{be^{ax}\cos(bx)}{a^2+b^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(a*x)*sin(b*x),x)
[Out] Piecewise((0, Eq(a, 0) & Eq(b, 0)), (x*exp(-I*b*x)*sin(b*x)/2 - I*x*exp(-I*
b*x)*cos(b*x)/2 - exp(-I*b*x)*cos(b*x)/(2*b), Eq(a, -I*b)), (x*exp(I*b*x)*s
in(b*x)/2 + I*x*exp(I*b*x)*cos(b*x)/2 - exp(I*b*x)*cos(b*x)/(2*b), Eq(a, I*
b)), (a*exp(a*x)*sin(b*x)/(a**2 + b**2) - b*exp(a*x)*cos(b*x)/(a**2 + b**2)
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int e^{ax}\sin(bx)\,dx = -\frac{(b\cos(bx) - a\sin(bx))e^{(ax)}}{a^2+b^2}$$

```
[In] integrate(exp(a*x)*sin(b*x),x, algorithm="maxima")
[Out] -(b*cos(b*x) - a*sin(b*x))*e^(a*x)/(a^2 + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int e^{ax}\sin(bx)\,dx = -\left(\frac{b\cos(bx)}{a^2+b^2} - \frac{a\sin(bx)}{a^2+b^2}\right)e^{(ax)}$$

```
[In] integrate(exp(a*x)*sin(b*x),x, algorithm="giac")
[Out] -(b*cos(b*x)/(a^2 + b^2) - a*sin(b*x)/(a^2 + b^2))*e^(a*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int e^{ax} \sin(bx) \, dx = -\frac{e^{a\,x}\,\left(b\,\cos\left(b\,x\right)-a\,\sin\left(b\,x\right)\right)}{a^2+b^2}$$

```
[In] int(exp(a*x)*sin(b*x),x)
[Out] -(exp(a*x)*(b*cos(b*x) - a*sin(b*x)))/(a^2 + b^2)
```

3.82 $\int \cot^{-1}(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 15

$$\int \cot^{-1}(x)\,dx = x\cot^{-1}(x) + \frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] x*arccot(x)+1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4931, 266}

$$\int \cot^{-1}(x)\,dx = \frac{1}{2}\log\left(x^2+1\right) + x\cot^{-1}(x)$$

```
[In] Int[ArcCot[x],x]
```

```
[Out] x*ArcCot[x] + Log[1 + x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\cot^{-1}(x) + \int \frac{x}{1+x^2}\,dx\\ &= x\cot^{-1}(x) + \frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(x)\,dx = x\cot^{-1}(x) + \frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[ArcCot[x],x]
[Out] x*ArcCot[x] + Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
lookup	$x \operatorname{arccot}(x) + \frac{\ln(x^2+1)}{2}$	14
default	$x \operatorname{arccot}(x) + \frac{\ln(x^2+1)}{2}$	14
parallelrisch	$x \operatorname{arccot}(x) + \frac{\ln(x^2+1)}{2}$	14
parts	$x \operatorname{arccot}(x) + \frac{\ln(x^2+1)}{2}$	14
risch	$\frac{ix\ln(ix+1)}{2} - \frac{ix\ln(-ix+1)}{2} + \frac{\pi x}{2} + \frac{\ln(x^2+1)}{2}$	36

```
[In] int(arccot(x),x,method=_RETURNVERBOSE)
[Out] x*arccot(x)+1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot^{-1}(x)\,dx = x\,\mathrm{arccot}\,(x) + \frac{1}{2}\,\log\left(x^2 + 1\right)$$

```
[In] integrate(arccot(x),x, algorithm="fricas")
[Out] x*arccot(x) + 1/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \cot^{-1}(x)\,dx = x\,\mathrm{acot}\,(x) + \frac{\log\left(x^2 + 1\right)}{2}$$

```
[In] integrate(acot(x),x)
[Out] x*acot(x) + log(x**2 + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot^{-1}(x)\,dx = x\,\mathrm{arccot}\,(x) + \frac{1}{2}\,\log\left(x^2 + 1\right)$$

```
[In] integrate(arccot(x),x, algorithm="maxima")
[Out] x*arccot(x) + 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \cot^{-1}(x)\,dx = x\arctan\left(\frac{1}{x}\right) + \frac{1}{2}\,\log\left(\frac{1}{x^2} + 1\right) - \frac{1}{2}\,\log\left(\frac{1}{x^2}\right)$$

```
[In] integrate(arccot(x),x, algorithm="giac")
[Out] x*arctan(1/x) + 1/2*log(1/x^2 + 1) - 1/2*log(x^(-2))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot^{-1}(x)\,dx = \frac{\ln(x^2+1)}{2} + x\,\mathrm{acot}(x)$$

```
[In] int(acot(x),x)
[Out] log(x^2 + 1)/2 + x*acot(x)
```

3.83 $\int \sec^{-1}(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 19

$$\int \sec^{-1}(x)\,dx = x\sec^{-1}(x) - \operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] x*arcsec(x)-arctanh((1-1/x^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.000, Rules used = {5322, 272, 65, 212}

$$\int \sec^{-1}(x)\,dx = x\sec^{-1}(x) - \operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[In] Int[ArcSec[x],x]
```

```
[Out] x*ArcSec[x] - ArcTanh[Sqrt[1 - x^(-2)]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5322

```
Int[ArcSec[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSec[c*x], x] - Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\sec^{-1}(x) - \int \frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}\,dx \\
&= x\sec^{-1}(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x^2}\right) \\
&= x\sec^{-1}(x) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right) \\
&= x\sec^{-1}(x) - \operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. $2(19) = 38$.

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.37

$$\int \sec^{-1}(x)\,dx = x\sec^{-1}(x) - \frac{\sqrt{-1+x^2}\left(-\log\left(1-\frac{x}{\sqrt{-1+x^2}}\right)+\log\left(1+\frac{x}{\sqrt{-1+x^2}}\right)\right)}{2\sqrt{1-\frac{1}{x^2}}\,x}$$

```
[In] Integrate[ArcSec[x],x]
[Out] x*ArcSec[x] - (Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt[
-1 + x^2]]))/(2*Sqrt[1 - x^(-2)]*x)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

method	result	size
lookup	$x\,\mathrm{arcsec}\left(x\right)-\ln\left(x+x\sqrt{1-\frac{1}{x^{2}}}\right)$	22
default	$x\,\mathrm{arcsec}\left(x\right)-\ln\left(x+x\sqrt{1-\frac{1}{x^{2}}}\right)$	22
parts	$x\,\mathrm{arcsec}\left(x\right)-\frac{\sqrt{x^{2}-1}\,\ln\left(x+\sqrt{x^{2}-1}\right)}{\sqrt{\frac{x^{2}-1}{x^{2}}}\,x}$	39

```
[In] int(arcsec(x),x,method=_RETURNVERBOSE)
[Out] x*arcsec(x)-ln(x+x*(1-1/x^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \sec^{-1}(x)\,dx=(x-2)\,\mathrm{arcsec}\left(x\right)+4\,\arctan\left(-x+\sqrt{x^{2}-1}\right)+\log\left(-x+\sqrt{x^{2}-1}\right)$$

```
[In] integrate(arcsec(x),x, algorithm="fricas")
[Out] (x - 2)*arcsec(x) + 4*arctan(-x + sqrt(x^2 - 1)) + log(-x + sqrt(x^2 - 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \sec^{-1}(x)\,dx=x\,\mathrm{asec}\left(x\right)-\begin{cases}\mathrm{acosh}\left(x\right) & \text{for}\ \left|x^{2}\right|>1\\ -i\,\mathrm{asin}\left(x\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(asec(x),x)
[Out] x*asec(x) - Piecewise((acosh(x), Abs(x**2) > 1), (-I*asin(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \sec^{-1}(x)\,dx = x\,\mathrm{arcsec}\,(x) - \frac{1}{2}\log\left(\sqrt{-\frac{1}{x^2}+1}+1\right) + \frac{1}{2}\log\left(-\sqrt{-\frac{1}{x^2}+1}+1\right)$$

```
[In] integrate(arcsec(x),x, algorithm="maxima")
[Out] x*arcsec(x) - 1/2*log(sqrt(-1/x^2 + 1) + 1) + 1/2*log(-sqrt(-1/x^2 + 1) + 1
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \sec^{-1}(x)\,dx = x\arccos\left(\frac{1}{x}\right) - \frac{1}{2}\log\left(\sqrt{-\frac{1}{x^2}+1}+1\right) + \frac{1}{2}\log\left(-\sqrt{-\frac{1}{x^2}+1}+1\right)$$

```
[In] integrate(arcsec(x),x, algorithm="giac")
[Out] x*arccos(1/x) - 1/2*log(sqrt(-1/x^2 + 1) + 1) + 1/2*log(-sqrt(-1/x^2 + 1) +
 1)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sec^{-1}(x)\,dx = x\,\mathrm{acos}\left(\frac{1}{x}\right) - \ln\left(x+\sqrt{x^2-1}\right)\,\mathrm{sign}(x)$$

```
[In] int(acos(1/x),x)
[Out] x*acos(1/x) - log(x + (x^2 - 1)^(1/2))*sign(x)
```

3.84 $\int \csc^{-1}(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 17

$$\int \csc^{-1}(x)\,dx = x\csc^{-1}(x) + \operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] x*arccsc(x)+arctanh((1-1/x^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.000, Rules used = {5323, 272, 65, 212}

$$\int \csc^{-1}(x)\,dx = \operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right) + x\csc^{-1}(x)$$

```
[In] Int[ArcCsc[x],x]
```

```
[Out] x*ArcCsc[x] + ArcTanh[Sqrt[1 - x^(-2)]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5323

```
Int[ArcCsc[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsc[c*x], x] + Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\csc^{-1}(x)+\int \frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}\,dx \\
&= x\csc^{-1}(x)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,\frac{1}{x^2}\right) \\
&= x\csc^{-1}(x)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-\frac{1}{x^2}}\right) \\
&= x\csc^{-1}(x)+\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. $2(17) = 34$.

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.76

$$\int \csc^{-1}(x)\,dx = x\csc^{-1}(x)+\frac{\sqrt{-1+x^2}\left(-\log\left(1-\frac{x}{\sqrt{-1+x^2}}\right)+\log\left(1+\frac{x}{\sqrt{-1+x^2}}\right)\right)}{2\sqrt{1-\frac{1}{x^2}}\,x}$$

```
[In] Integrate[ArcCsc[x],x]
[Out] x*ArcCsc[x] + (Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt[
-1 + x^2]]))/(2*Sqrt[1 - x^(-2)]*x)
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

method	result	size
lookup	$x\ \mathrm{arccsc}\left(x\right)+\ln\left(x+x\sqrt{1-\frac{1}{x^{2}}}\right)$	20
default	$x\ \mathrm{arccsc}\left(x\right)+\ln\left(x+x\sqrt{1-\frac{1}{x^{2}}}\right)$	20
parts	$x\ \mathrm{arccsc}\left(x\right)+\frac{\sqrt{x^{2}-1}\,\ln\left(x+\sqrt{x^{2}-1}\right)}{\sqrt{\frac{x^{2}-1}{x^{2}}}\,x}$	38

```
[In] int(arccsc(x),x,method=_RETURNVERBOSE)
[Out] x*arccsc(x)+ln(x+x*(1-1/x^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(15) = 30.

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \csc^{-1}(x)\,dx = (x-2)\,\mathrm{arccsc}\left(x\right)-4\,\arctan\left(-x+\sqrt{x^{2}-1}\right)-\log\left(-x+\sqrt{x^{2}-1}\right)$$

```
[In] integrate(arccsc(x),x, algorithm="fricas")
[Out] (x - 2)*arccsc(x) - 4*arctan(-x + sqrt(x^2 - 1)) - log(-x + sqrt(x^2 - 1))
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc^{-1}(x)\,dx = x\,\mathrm{acsc}\left(x\right)+\begin{cases}\mathrm{acosh}\left(x\right) & \text{for}\ \left|x^{2}\right|>1\\ -i\,\mathrm{asin}\left(x\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(acsc(x),x)
[Out] x*acsc(x) + Piecewise((acosh(x), Abs(x**2) > 1), (-I*asin(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \csc^{-1}(x)\,dx = x\,\mathrm{arccsc}\,(x) + \frac{1}{2}\log\left(\sqrt{-\frac{1}{x^2}+1}+1\right) - \frac{1}{2}\log\left(-\sqrt{-\frac{1}{x^2}+1}+1\right)$$

```
[In] integrate(arccsc(x),x, algorithm="maxima")
[Out] x*arccsc(x) + 1/2*log(sqrt(-1/x^2 + 1) + 1) - 1/2*log(-sqrt(-1/x^2 + 1) + 1
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \csc^{-1}(x)\,dx = x\arcsin\left(\frac{1}{x}\right) + \frac{1}{2}\log\left(\sqrt{-\frac{1}{x^2}+1}+1\right) - \frac{1}{2}\log\left(-\sqrt{-\frac{1}{x^2}+1}+1\right)$$

```
[In] integrate(arccsc(x),x, algorithm="giac")
[Out] x*arcsin(1/x) + 1/2*log(sqrt(-1/x^2 + 1) + 1) - 1/2*log(-sqrt(-1/x^2 + 1) +
 1)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \csc^{-1}(x)\,dx = x\,\mathrm{asin}\left(\frac{1}{x}\right) + \ln\left(x+\sqrt{x^2-1}\right)\,\mathrm{sign}(x)$$

```
[In] int(asin(1/x),x)
[Out] x*asin(1/x) + log(x + (x^2 - 1)^(1/2))*sign(x)
```

3.85 $\int \arcsin(x)^2 \, dx$

Optimal result

Integrand size = 4, antiderivative size = 25

$$\int \arcsin(x)^2 \, dx = -2x + 2\sqrt{1-x^2} \arcsin(x) + x \arcsin(x)^2$$

```
[Out] -2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4715, 4767, 8}

$$\int \arcsin(x)^2 \, dx = 2\sqrt{1-x^2} \arcsin(x) + x \arcsin(x)^2 - 2x$$

```
[In] Int[ArcSin[x]^2,x]
```

```
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4767

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\arcsin(x)^2 - 2\int \frac{x\arcsin(x)}{\sqrt{1-x^2}}\,dx\\ &= 2\sqrt{1-x^2}\arcsin(x) + x\arcsin(x)^2 - 2\int 1\,dx\\ &= -2x + 2\sqrt{1-x^2}\arcsin(x) + x\arcsin(x)^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \arcsin(x)^2\,dx = -2x + 2\sqrt{1-x^2}\arcsin(x) + x\arcsin(x)^2$$

```
[In] Integrate[ArcSin[x]^2,x]
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
default	$-2x + x\arcsin(x)^2 + 2\arcsin(x)\sqrt{-x^2+1}$	24

```
[In] int(arcsin(x)^2,x,method=_RETURNVERBOSE)
[Out] -2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2\,dx = x\arcsin\left(x\right)^2 + 2\,\sqrt{-x^2+1}\arcsin\left(x\right) - 2\,x$$

```
[In] integrate(arcsin(x)^2,x, algorithm="fricas")
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \arcsin(x)^2\,dx = x\,\mathrm{asin}^2\left(x\right) - 2x + 2\sqrt{1-x^2}\,\mathrm{asin}\left(x\right)$$

```
[In] integrate(asin(x)**2,x)
[Out] x*asin(x)**2 - 2*x + 2*sqrt(1 - x**2)*asin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2\,dx = x\arcsin\left(x\right)^2 + 2\,\sqrt{-x^2+1}\arcsin\left(x\right) - 2\,x$$

```
[In] integrate(arcsin(x)^2,x, algorithm="maxima")
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2\,dx = x\arcsin\left(x\right)^2 + 2\,\sqrt{-x^2+1}\arcsin\left(x\right) - 2\,x$$

```
[In] integrate(arcsin(x)^2,x, algorithm="giac")
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \arcsin(x)^2\,dx = 2\,\mathrm{asin}(x)\,\sqrt{1-x^2} + x\,\left(\mathrm{asin}(x)^2 - 2\right)$$

```
[In] int(asin(x)^2,x)
[Out] 2*asin(x)*(1 - x^2)^(1/2) + x*(asin(x)^2 - 2)
```

3.86 $\int \frac{\arcsin(x)}{x^2} dx$

Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \frac{\arcsin(x)}{x^2} dx = -\frac{\arcsin(x)}{x} - \operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

```
[Out] -arcsin(x)/x-arctanh((-x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4723, 272, 65, 212}

$$\int \frac{\arcsin(x)}{x^2} dx = -\frac{\arcsin(x)}{x} - \operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

```
[In] Int[ArcSin[x]/x^2,x]
```

```
[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arcsin(x)}{x} + \int \frac{1}{x\sqrt{1-x^2}}\,dx \\
&= -\frac{\arcsin(x)}{x} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, x^2\right) \\
&= -\frac{\arcsin(x)}{x} - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right) \\
&= -\frac{\arcsin(x)}{x} - \operatorname{arctanh}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{\arcsin(x)}{x} - \operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

```
[In] Integrate[ArcSin[x]/x^2,x]
[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{\arcsin(x)}{x}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)$	21
parts	$-\frac{\arcsin(x)}{x}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)$	21

```
[In] int(arcsin(x)/x^2,x,method=_RETURNVERBOSE)
[Out] -arcsin(x)/x-arctanh(1/(-x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{x\log\left(\sqrt{-x^2+1}+1\right)-x\log\left(\sqrt{-x^2+1}-1\right)+2\arcsin\left(x\right)}{2\,x}$$

```
[In] integrate(arcsin(x)/x^2,x, algorithm="fricas")
[Out] -1/2*(x*log(sqrt(-x^2 + 1) + 1) - x*log(sqrt(-x^2 + 1) - 1) + 2*arcsin(x))/
x
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(x)}{x^2}\,dx = \begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for}\ \frac{1}{|x^2|}>1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases} - \frac{\operatorname{asin}\left(x\right)}{x}$$

```
[In] integrate(asin(x)/x**2,x)
[Out] Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) - asin(x)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{\arcsin(x)}{x} - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

```
[In] integrate(arcsin(x)/x^2,x, algorithm="maxima")
[Out] -arcsin(x)/x - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{\arcsin(x)}{x} - \frac{1}{2}\log\left(\sqrt{-x^2+1}+1\right) + \frac{1}{2}\log\left(-\sqrt{-x^2+1}+1\right)$$

```
[In] integrate(arcsin(x)/x^2,x, algorithm="giac")
[Out] -arcsin(x)/x - 1/2*log(sqrt(-x^2 + 1) + 1) + 1/2*log(-sqrt(-x^2 + 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\mathrm{atanh}\left(\frac{1}{\sqrt{1-x^2}}\right) - \frac{\mathrm{asin}(x)}{x}$$

```
[In] int(asin(x)/x^2,x)
[Out] - atanh(1/(1 - x^2)^(1/2)) - asin(x)/x
```

3.87 $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx=\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[Out] arctan(x/(a^2-x^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {223, 209}

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx=\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[In] Int[1/Sqrt[a^2 - x^2],x]
```

```
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{a^2-x^2}}\right)$$
$$= \arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[In] Integrate[1/Sqrt[a^2 - x^2],x]
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$	15
pseudoelliptic	$-\arctan\left(\frac{\sqrt{a^2-x^2}}{x}\right)$	19

```
[In] int(1/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan(x/(a^2-x^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = -2\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)$$

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="fricas")
[Out] -2*arctan(-(a - sqrt(a^2 - x^2))/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \begin{cases} -i\operatorname{acosh}\left(\frac{x}{a}\right) & \text{for}\ \left|\frac{x^2}{a^2}\right| > 1 \\ \operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a**2-x**2)**(1/2),x)
[Out] Piecewise((-I*acosh(x/a), Abs(x**2/a**2) > 1), (asin(x/a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.38

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right)$$

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="maxima")
[Out] arcsin(x/a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \frac{1}{2}\,a^2\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a) + \frac{1}{2}\,\sqrt{a^2-x^2}x$$

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="giac")
[Out] 1/2*a^2*arcsin(x/a)*sgn(a) + 1/2*sqrt(a^2 - x^2)*x
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx=\operatorname{atan}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[In] int(1/(a^2 - x^2)^(1/2),x)
[Out] atan(x/(a^2 - x^2)^(1/2))
```

3.88 $\int \frac{1}{\sqrt{1-2x-x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 10

$$\int \frac{1}{\sqrt{1-2x-x^2}}\,dx = \arcsin\left(\frac{1+x}{\sqrt{2}}\right)$$

```
[Out] arcsin(1/2*(1+x)*2^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {633, 222}

$$\int \frac{1}{\sqrt{1-2x-x^2}}\,dx = \arcsin\left(\frac{x+1}{\sqrt{2}}\right)$$

```
[In] Int[1/Sqrt[1 - 2*x - x^2],x]
```

```
[Out] ArcSin[(1 + x)/Sqrt[2]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{8}}}\,dx, x, -2-2x\right)}{2\sqrt{2}}$$
$$= \arcsin\left(\frac{1+x}{\sqrt{2}}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{1}{\sqrt{1-2x-x^2}}\,dx = 2\arctan\left(\frac{x}{-1+\sqrt{1-2x-x^2}}\right)$$

```
[In] Integrate[1/Sqrt[1 - 2*x - x^2],x]
[Out] 2*ArcTan[x/(-1 + Sqrt[1 - 2*x - x^2])]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

method	result	size
default	$\arcsin\left(\frac{(1+x)\sqrt{2}}{2}\right)$	10
trager	$\text{RootOf}\left(\_Z^2+1\right)\ln\left(-\text{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2-2x+1}-\text{RootOf}\left(\_Z^2+1\right)\right)$	39

```
[In] int(1/(-x^2-2*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(1/2*(1+x)*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \frac{1}{\sqrt{1-2x-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{-x^2-2\,x+1}-1}{x}\right)$$

```
[In] integrate(1/(-x^2-2*x+1)^(1/2),x, algorithm="fricas")
[Out] -2*arctan((sqrt(-x^2 - 2*x + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-2x-x^2}}\,dx = \operatorname{asin}\left(\frac{\sqrt{2}(x+1)}{2}\right)$$

```
[In] integrate(1/(-x**2-2*x+1)**(1/2),x)
[Out] asin(sqrt(2)*(x + 1)/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{1-2x-x^2}}\,dx = -\arcsin\left(-\frac{1}{2}\sqrt{2}(x+1)\right)$$

```
[In] integrate(1/(-x^2-2*x+1)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-1/2*sqrt(2)*(x + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(9) = 18.

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.70

$$\int \frac{1}{\sqrt{1-2x-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2-2\,x+1}(x+1)+\arcsin\left(\frac{1}{2}\sqrt{2}(x+1)\right)$$

```
[In] integrate(1/(-x^2-2*x+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 - 2*x + 1)*(x + 1) + arcsin(1/2*sqrt(2)*(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{1-2x-x^2}}\,dx = \operatorname{asin}\left(\frac{\sqrt{8}\,(2\,x+2)}{8}\right)$$

```
[In] int(1/(1 - x^2 - 2*x)^(1/2),x)
[Out] asin((8^(1/2)*(2*x + 2))/8)
```

3.89 $\int \frac{1}{a^2+x^2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{1}{a^2+x^2} \, dx = \frac{\arctan \left(\frac{x}{a}\right)}{a}$$

```
[Out] arctan(x/a)/a
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {209}

$$\int \frac{1}{a^2+x^2} \, dx = \frac{\arctan \left(\frac{x}{a}\right)}{a}$$

```
[In] Int[(a^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/a]/a
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\text{integral} = \frac{\arctan \left(\frac{x}{a}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+x^2}\,dx=\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

[In] Integrate[(a^2 + x^2)^(-1),x]

[Out] ArcTan[x/a]/a

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\arctan\left(\frac{x}{a}\right)}{a}$	11
risch	$\frac{\arctan\left(\frac{x}{a}\right)}{a}$	11
parallelrisch	$-\frac{i\ln(-ia+x)-i\ln(ia+x)}{2a}$	27

[In] int(1/(a^2+x^2),x,method=_RETURNVERBOSE)

[Out] arctan(x/a)/a

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+x^2}\,dx=\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

[In] integrate(1/(a^2+x^2),x, algorithm="fricas")

[Out] arctan(x/a)/a

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{1}{a^2+x^2}\,dx = \frac{-\frac{i\log\left(-ia+x\right)}{2}+\frac{i\log\left(ia+x\right)}{2}}{a}$$

```
[In] integrate(1/(a**2+x**2),x)
[Out] (-I*log(-I*a + x)/2 + I*log(I*a + x)/2)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+x^2}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

```
[In] integrate(1/(a^2+x^2),x, algorithm="maxima")
[Out] arctan(x/a)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+x^2}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

```
[In] integrate(1/(a^2+x^2),x, algorithm="giac")
[Out] arctan(x/a)/a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{\operatorname{atan}\left(\frac{x}{a}\right)}{a}$$

```
[In] int(1/(a^2 + x^2),x)
[Out] atan(x/a)/a
```

3.90 $\int \frac{1}{a+bx^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{1}{a+bx^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {211}

$$\int \frac{1}{a+bx^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[In] Int[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\text{integral}=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[In] Integrate[(a + b*x^2)^(-1),x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$-\frac{\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}$	41

```
[In] int(1/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{1}{a+bx^2}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

```
[In] integrate(1/(b*x^2+a),x, algorithm="fricas")
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{1}{a+bx^2}\,dx = -\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

```
[In] integrate(1/(b*x**2+a),x)
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

```
[In] integrate(1/(b*x^2+a),x, algorithm="maxima")
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

```
[In] integrate(1/(b*x^2+a),x, algorithm="giac")
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{a+bx^2}\,dx=\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[In] int(1/(a + b*x^2),x)
[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

3.91 $\int \frac{1}{2-x+x^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{1}{2-x+x^2} \, dx = -\frac{2 \arctan \left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] -2/7*arctan(1/7*(1-2*x)*7^(1/2))*7^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {632, 210}

$$\int \frac{1}{2-x+x^2} \, dx = -\frac{2 \arctan \left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[In] Int[(2 - x + x^2)^(-1),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, -1+2x\right)\right)$$
$$= -\frac{2\arctan\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2\arctan\left(\frac{-1+2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[In] Integrate[(2 - x + x^2)^(-1),x]
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{2\sqrt{7}\,\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$	17
risch	$\frac{2\sqrt{7}\,\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$	17

```
[In] int(1/(x^2-x+2),x,method=_RETURNVERBOSE)
[Out] 2/7*7^(1/2)*arctan(1/7*(2*x-1)*7^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2}{7}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x-1)\right)$$

```
[In] integrate(1/(x^2-x+2),x, algorithm="fricas")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x}{7}-\frac{\sqrt{7}}{7}\right)}{7}$$

```
[In] integrate(1/(x**2-x+2),x)
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x/7 - sqrt(7)/7)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2}{7}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x-1)\right)$$

```
[In] integrate(1/(x^2-x+2),x, algorithm="maxima")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2}{7}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x-1)\right)$$

```
[In] integrate(1/(x^2-x+2),x, algorithm="giac")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{\sqrt{7}\,(2\,x-1)}{7}\right)}{7}$$

```
[In] int(1/(x^2 - x + 2),x)
[Out] (2*7^(1/2)*atan((7^(1/2)*(2*x - 1))/7))/7
```

3.92 $\int x \arctan(x)\, dx$

Optimal result

Integrand size = 4, antiderivative size = 21

$$\int x \arctan(x)\, dx = -\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{1}{2}x^2 \arctan(x)$$

```
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {4946, 327, 209}

$$\int x \arctan(x)\, dx = \frac{1}{2}x^2 \arctan(x) + \frac{\arctan(x)}{2} - \frac{x}{2}$$

```
[In] Int[x*ArcTan[x],x]
```

```
[Out] -1/2*x + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\arctan(x) - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx\\ &= -\frac{x}{2} + \frac{1}{2}x^2\arctan(x) + \frac{1}{2}\int \frac{1}{1+x^2}\,dx\\ &= -\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{1}{2}x^2\arctan(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int x\arctan(x)\,dx = \frac{1}{2}\left(-x + \left(1+x^2\right)\arctan(x)\right)$$

```
[In] Integrate[x*ArcTan[x],x]
[Out] (-x + (1 + x^2)*ArcTan[x])/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2}+\frac{(3x^2+3)\arctan(x)}{6}$	16
parallelrisch	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
parts	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
risch	$-\frac{ix^2\ln(ix+1)}{4}+\frac{ix^2\ln(-ix+1)}{4}-\frac{x}{2}+\frac{\arctan(x)}{2}$	35

```
[In] int(x*arctan(x),x,method=_RETURNVERBOSE)
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x \arctan(x) \, dx = \frac{1}{2} \left(x^2 + 1\right) \arctan\left(x\right) - \frac{1}{2} \, x$$

```
[In] integrate(x*arctan(x),x, algorithm="fricas")
[Out] 1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \arctan(x) \, dx = \frac{x^2 \operatorname{atan}\left(x\right)}{2} - \frac{x}{2} + \frac{\operatorname{atan}\left(x\right)}{2}$$

```
[In] integrate(x*atan(x),x)
[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \arctan(x) \, dx = \frac{1}{2} \, x^2 \arctan\left(x\right) - \frac{1}{2} \, x + \frac{1}{2} \, \arctan\left(x\right)$$

```
[In] integrate(x*arctan(x),x, algorithm="maxima")
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x\arctan(x)\,dx = \frac{1}{2}\,x^2\arctan\left(x\right) - \frac{1}{2}\,x + \frac{1}{2}\,\arctan\left(x\right)$$

```
[In] integrate(x*arctan(x),x, algorithm="giac")
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int x\arctan(x)\,dx = \mathrm{atan}(x)\,\left(\frac{x^2}{2}+\frac{1}{2}\right) - \frac{x}{2}$$

```
[In] int(x*atan(x),x)
[Out] atan(x)*(x^2/2 + 1/2) - x/2
```

3.93 $\int x^2 \arccos(x)\, dx$

Optimal result

Integrand size = 6, antiderivative size = 40

$$\int x^2 \arccos(x)\, dx = -\frac{1}{3}\sqrt{1-x^2} + \frac{1}{9}\left(1-x^2\right)^{3/2} + \frac{1}{3}x^3 \arccos(x)$$

```
[Out] 1/9*(-x^2+1)^(3/2)+1/3*x^3*arccos(x)-1/3*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4724, 272, 45}

$$\int x^2 \arccos(x)\, dx = \frac{1}{3}x^3 \arccos(x) + \frac{1}{9}\left(1-x^2\right)^{3/2} - \frac{\sqrt{1-x^2}}{3}$$

```
[In] Int[x^2*ArcCos[x],x]
```

```
[Out] -1/3*Sqrt[1 - x^2] + (1 - x^2)^(3/2)/9 + (x^3*ArcCos[x])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arccos(x) + \frac{1}{3}\int \frac{x^3}{\sqrt{1-x^2}}\,dx \\
&= \frac{1}{3}x^3\arccos(x) + \frac{1}{6}\text{Subst}\left(\int \frac{x}{\sqrt{1-x}}\,dx, x, x^2\right) \\
&= \frac{1}{3}x^3\arccos(x) + \frac{1}{6}\text{Subst}\left(\int \left(\frac{1}{\sqrt{1-x}} - \sqrt{1-x}\right)\,dx, x, x^2\right) \\
&= -\frac{1}{3}\sqrt{1-x^2} + \frac{1}{9}\left(1-x^2\right)^{3/2} + \frac{1}{3}x^3\arccos(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int x^2\arccos(x)\,dx = -\frac{1}{9}\sqrt{1-x^2}\left(2+x^2\right) + \frac{1}{3}x^3\arccos(x)$$

```
[In] Integrate[x^2*ArcCos[x],x]
[Out] -1/9*(Sqrt[1 - x^2]*(2 + x^2)) + (x^3*ArcCos[x])/3
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{x^3\arccos(x)}{3} - \frac{x^2\sqrt{-x^2+1}}{9} - \frac{2\sqrt{-x^2+1}}{9}$	34
parts	$\frac{x^3\arccos(x)}{3} - \frac{x^2\sqrt{-x^2+1}}{9} - \frac{2\sqrt{-x^2+1}}{9}$	34

```
[In] int(x^2*arccos(x),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*arccos(x)-1/9*x^2*(-x^2+1)^(1/2)-2/9*(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int x^2 \arccos(x)\, dx = \frac{1}{3}\, x^3 \arccos\left(x\right) - \frac{1}{9}\, \left(x^2 + 2\right)\sqrt{-x^2 + 1}$$

```
[In] integrate(x^2*arccos(x),x, algorithm="fricas")
[Out] 1/3*x^3*arccos(x) - 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int x^2 \arccos(x)\, dx = \frac{x^3 \operatorname{acos}\left(x\right)}{3} - \frac{x^2\sqrt{1 - x^2}}{9} - \frac{2\sqrt{1 - x^2}}{9}$$

```
[In] integrate(x**2*acos(x),x)
[Out] x**3*acos(x)/3 - x**2*sqrt(1 - x**2)/9 - 2*sqrt(1 - x**2)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int x^2 \arccos(x)\, dx = \frac{1}{3}\, x^3 \arccos\left(x\right) - \frac{1}{9}\, \sqrt{-x^2 + 1}x^2 - \frac{2}{9}\, \sqrt{-x^2 + 1}$$

```
[In] integrate(x^2*arccos(x),x, algorithm="maxima")
[Out] 1/3*x^3*arccos(x) - 1/9*sqrt(-x^2 + 1)*x^2 - 2/9*sqrt(-x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int x^2 \arccos(x)\, dx = \frac{1}{3}\, x^3 \arccos\left(x\right) - \frac{1}{9}\, \sqrt{-x^2 + 1}x^2 - \frac{2}{9}\, \sqrt{-x^2 + 1}$$

```
[In] integrate(x^2*arccos(x),x, algorithm="giac")
[Out] 1/3*x^3*arccos(x) - 1/9*sqrt(-x^2 + 1)*x^2 - 2/9*sqrt(-x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int x^2 \arccos(x)\, dx = \frac{x^3 \operatorname{acos}(x)}{3} - \frac{\sqrt{1-x^2}\,(x^2+2)}{9}$$

```
[In] int(x^2*acos(x),x)
[Out] (x^3*acos(x))/3 - ((1 - x^2)^(1/2)*(x^2 + 2))/9
```

3.94 $\int x \arctan(x)^2 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 35

$$\int x \arctan(x)^2 \, dx = -x \arctan(x) + \frac{\arctan(x)^2}{2} + \frac{1}{2} x^2 \arctan(x)^2 + \frac{1}{2} \log\left(1 + x^2\right)$$

```
[Out] -x*arctan(x)+1/2*arctan(x)^2+1/2*x^2*arctan(x)^2+1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {4946, 5036, 4930, 266, 5004}

$$\int x \arctan(x)^2 \, dx = \frac{1}{2} x^2 \arctan(x)^2 + \frac{\arctan(x)^2}{2} - x \arctan(x) + \frac{1}{2} \log\left(x^2 + 1\right)$$

```
[In] Int[x*ArcTan[x]^2,x]
```

```
[Out] -(x*ArcTan[x]) + ArcTan[x]^2/2 + (x^2*ArcTan[x]^2)/2 + Log[1 + x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arctan(x)^2 - \int \frac{x^2\arctan(x)}{1+x^2}\,dx \\
&= \frac{1}{2}x^2\arctan(x)^2 - \int \arctan(x)\,dx + \int \frac{\arctan(x)}{1+x^2}\,dx \\
&= -x\arctan(x) + \frac{\arctan(x)^2}{2} + \frac{1}{2}x^2\arctan(x)^2 + \int \frac{x}{1+x^2}\,dx \\
&= -x\arctan(x) + \frac{\arctan(x)^2}{2} + \frac{1}{2}x^2\arctan(x)^2 + \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int x\arctan(x)^2\,dx = \frac{1}{2}\left(-2x\arctan(x) + \left(1+x^2\right)\arctan(x)^2 + \log\left(1+x^2\right)\right)$$

```
[In] Integrate[x*ArcTan[x]^2,x]
[Out] (-2*x*ArcTan[x] + (1 + x^2)*ArcTan[x]^2 + Log[1 + x^2])/2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result
default	$-x\arctan\left(x\right)+\frac{\arctan(x)^{2}}{2}+\frac{x^{2}\arctan(x)^{2}}{2}+\frac{\ln(x^{2}+1)}{2}$
parallelrisch	$-x\arctan\left(x\right)+\frac{\arctan(x)^{2}}{2}+\frac{x^{2}\arctan(x)^{2}}{2}+\frac{\ln(x^{2}+1)}{2}$
parts	$-x\arctan\left(x\right)+\frac{\arctan(x)^{2}}{2}+\frac{x^{2}\arctan(x)^{2}}{2}+\frac{\ln(x^{2}+1)}{2}$
risch	$-\frac{\left(\frac{x^{2}}{2}+\frac{1}{2}\right)\ln(ix+1)^{2}}{4}-\frac{\left(-x^{2}\ln(-ix+1)-2ix-\ln(-ix+1)\right)\ln(ix+1)}{4}-\frac{x^{2}\ln(-ix+1)^{2}}{8}-\frac{\ln(-ix+1)^{2}}{8}-\frac{ix\ln(-ix+1)}{2}$

```
[In] int(x*arctan(x)^2,x,method=_RETURNVERBOSE)
[Out] -x*arctan(x)+1/2*arctan(x)^2+1/2*x^2*arctan(x)^2+1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int x\arctan(x)^{2}\,dx=\frac{1}{2}\left(x^{2}+1\right)\arctan\left(x\right)^{2}-x\arctan\left(x\right)+\frac{1}{2}\log\left(x^{2}+1\right)$$

```
[In] integrate(x*arctan(x)^2,x, algorithm="fricas")
[Out] 1/2*(x^2 + 1)*arctan(x)^2 - x*arctan(x) + 1/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x\arctan(x)^{2}\,dx=\frac{x^{2}\operatorname{atan}^{2}\left(x\right)}{2}-x\operatorname{atan}\left(x\right)+\frac{\log\left(x^{2}+1\right)}{2}+\frac{\operatorname{atan}^{2}\left(x\right)}{2}$$

```
[In] integrate(x*atan(x)**2,x)
[Out] x**2*atan(x)**2/2 - x*atan(x) + log(x**2 + 1)/2 + atan(x)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int x\arctan(x)^2\,dx = \frac{1}{2}\,x^2\arctan\left(x\right)^2 - \left(x - \arctan\left(x\right)\right)\arctan\left(x\right) - \frac{1}{2}\,\arctan\left(x\right)^2 + \frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate(x*arctan(x)^2,x, algorithm="maxima")
[Out] 1/2*x^2*arctan(x)^2 - (x - arctan(x))*arctan(x) - 1/2*arctan(x)^2 + 1/2*log
(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x\arctan(x)^2\,dx = \frac{1}{2}\,x^2\arctan\left(x\right)^2 - x\arctan\left(x\right) + \frac{1}{2}\,\arctan\left(x\right)^2 + \frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate(x*arctan(x)^2,x, algorithm="giac")
[Out] 1/2*x^2*arctan(x)^2 - x*arctan(x) + 1/2*arctan(x)^2 + 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x\arctan(x)^2\,dx = \frac{\ln\left(x^2+1\right)}{2} + \frac{\mathrm{atan}(x)^2}{2} + \frac{x^2\,\mathrm{atan}(x)^2}{2} - x\,\mathrm{atan}(x)$$

```
[In] int(x*atan(x)^2,x)
[Out] log(x^2 + 1)/2 + atan(x)^2/2 + (x^2*atan(x)^2)/2 - x*atan(x)
```

3.95 $\int \arctan\left(\sqrt{x}\right) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \arctan\left(\sqrt{x}\right) \, dx = -\sqrt{x} + \arctan\left(\sqrt{x}\right) + x \arctan\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {4930, 52, 65, 209}

$$\int \arctan\left(\sqrt{x}\right) \, dx = x \arctan\left(\sqrt{x}\right) + \arctan\left(\sqrt{x}\right) - \sqrt{x}$$

```
[In] Int[ArcTan[Sqrt[x]],x]
```

```
[Out] -Sqrt[x] + ArcTan[Sqrt[x]] + x*ArcTan[Sqrt[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\arctan\left(\sqrt{x}\right)-\frac{1}{2}\int\frac{\sqrt{x}}{1+x}\,dx\\
&= -\sqrt{x}+x\arctan\left(\sqrt{x}\right)+\frac{1}{2}\int\frac{1}{\sqrt{x}(1+x)}\,dx\\
&= -\sqrt{x}+x\arctan\left(\sqrt{x}\right)+\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -\sqrt{x}+\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int\arctan\left(\sqrt{x}\right)\,dx=-\sqrt{x}+(1+x)\arctan\left(\sqrt{x}\right)$$

```
[In] Integrate[ArcTan[Sqrt[x]],x]
[Out] -Sqrt[x] + (1 + x)*ArcTan[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$	17
default	$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$	17
parts	$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$	17
meijerg	$-\sqrt{x}+\frac{(3x+3)\arctan\left(\sqrt{x}\right)}{3}$	18

```
[In] int(arctan(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \arctan\left(\sqrt{x}\right)\,dx=(x+1)\arctan\left(\sqrt{x}\right)-\sqrt{x}$$

```
[In] integrate(arctan(x^(1/2)),x, algorithm="fricas")
[Out] (x + 1)*arctan(sqrt(x)) - sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \arctan\left(\sqrt{x}\right)\,dx=-\sqrt{x}+x\,\mathrm{atan}\left(\sqrt{x}\right)+\mathrm{atan}\left(\sqrt{x}\right)$$

```
[In] integrate(atan(x**(1/2)),x)
[Out] -sqrt(x) + x*atan(sqrt(x)) + atan(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \arctan\left(\sqrt{x}\right)\,dx = x\arctan\left(\sqrt{x}\right) - \sqrt{x} + \arctan\left(\sqrt{x}\right)$$

```
[In] integrate(arctan(x^(1/2)),x, algorithm="maxima")
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \arctan\left(\sqrt{x}\right)\,dx = x\arctan\left(\sqrt{x}\right) - \sqrt{x} + \arctan\left(\sqrt{x}\right)$$

```
[In] integrate(arctan(x^(1/2)),x, algorithm="giac")
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \arctan\left(\sqrt{x}\right)\,dx = \mathrm{atan}\left(\sqrt{x}\right) + x\,\mathrm{atan}\left(\sqrt{x}\right) - \sqrt{x}$$

```
[In] int(atan(x^(1/2)),x)
[Out] atan(x^(1/2)) + x*atan(x^(1/2)) - x^(1/2)
```

3.96 $\int \frac{\arctan(\sqrt{x})}{\sqrt{x}(1+x)}\, dx$

Optimal result

Integrand size = 17, antiderivative size = 8

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\, dx = \arctan\left(\sqrt{x}\right)^2$$

```
[Out] arctan(x^(1/2))^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {65, 209, 6818}

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\, dx = \arctan\left(\sqrt{x}\right)^2$$

```
[In] Int[ArcTan[Sqrt[x]]/(Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \arctan\left(\sqrt{x}\right)^2$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)^2$$

```
[In] Integrate[ArcTan[Sqrt[x]]/(Sqrt[x]*(1 + x)),x]
[Out] ArcTan[Sqrt[x]]^2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)^2$	7
default	$\arctan\left(\sqrt{x}\right)^2$	7

```
[In] int(arctan(x^(1/2))/(1+x)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan(x^(1/2))^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)^2$$

```
[In] integrate(arctan(x^(1/2))/(1+x)/x^(1/2),x, algorithm="fricas")
[Out] arctan(sqrt(x))^2
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\,dx = \operatorname{atan}^2\left(\sqrt{x}\right)$$

```
[In] integrate(atan(x**(1/2))/(1+x)/x**(1/2),x)
[Out] atan(sqrt(x))**2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)^2$$

```
[In] integrate(arctan(x^(1/2))/(1+x)/x^(1/2),x, algorithm="maxima")
[Out] arctan(sqrt(x))^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)^2$$

```
[In] integrate(arctan(x^(1/2))/(1+x)/x^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(x))^2
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\,dx = \mathrm{atan}\left(\sqrt{x}\right)^2$$

```
[In] int(atan(x^(1/2))/(x^(1/2)*(x + 1)),x)
[Out] atan(x^(1/2))^2
```

3.97 $\int \sqrt{1-x^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \sqrt{1-x^2} \, dx = \frac{1}{2}x\sqrt{1-x^2} + \frac{\arcsin(x)}{2}$$

```
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {201, 222}

$$\int \sqrt{1-x^2} \, dx = \frac{\arcsin(x)}{2} + \frac{1}{2}\sqrt{1-x^2}x$$

```
[In] Int[Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= \frac{1}{2}x\sqrt{1-x^2} + \frac{\arcsin(x)}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}x\sqrt{1-x^2} - \arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

```
[In] Integrate[Sqrt[1 - x^2],x]
[Out] (x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\arcsin(x)}{2}+\frac{x\sqrt{-x^2+1}}{2}$	18
risch	$-\frac{x(x^2-1)}{2\sqrt{-x^2+1}}+\frac{\arcsin(x)}{2}$	23
pseudoelliptic	$\frac{x\sqrt{-x^2+1}}{2}-\frac{\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right)}{2}$	30
meijerg	$\frac{i\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\,\arcsin(x)\right)}{4\sqrt{\pi}}$	32
trager	$\frac{x\sqrt{-x^2+1}}{2}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	41

```
[In] int((-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\,\sqrt{-x^2+1}x - \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*x - arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \sqrt{1-x^2}\,dx = \frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}(x)}{2}$$

```
[In] integrate((-x**2+1)**(1/2),x)
[Out] x*sqrt(1 - x**2)/2 + asin(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\,\sqrt{-x^2+1}x + \frac{1}{2}\,\arcsin(x)$$

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\,\sqrt{-x^2+1}x + \frac{1}{2}\,\arcsin(x)$$

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{\operatorname{asin}(x)}{2} + \frac{x\,\sqrt{1-x^2}}{2}$$

```
[In] int((1 - x^2)^(1/2),x)
[Out] asin(x)/2 + (x*(1 - x^2)^(1/2))/2
```

3.98 $\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx = -\frac{e^{\arctan(x)}(1-x)}{2\sqrt{1+x^2}}$$

```
[Out] -1/2*exp(arctan(x))*(1-x)/(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{5185\}$

$$\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx = -\frac{(1-x)e^{\arctan(x)}}{2\sqrt{x^2+1}}$$

```
[In] Int[(E^ArcTan[x]*x)/(1 + x^2)^(3/2),x]
```

```
[Out] -1/2*(E^ArcTan[x]*(1 - x))/Sqrt[1 + x^2]
```

Rule 5185

```
Int[(E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[(-(1 - a*n*x))*(E^(n*ArcTan[a*x])/(d*(n^2 + 1)*Sqrt[c + d*x^2])
), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rubi steps

$$\text{integral} = -\frac{e^{\arctan(x)}(1-x)}{2\sqrt{1+x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx = \frac{1}{2}(1-ix)^{-\frac{1}{2}+\frac{i}{2}}(1+ix)^{-\frac{1}{2}-\frac{i}{2}}(-1+x)$$

```
[In] Integrate[(E^ArcTan[x]*x)/(1 + x^2)^(3/2),x]
[Out] (-1 + x)/(2*(1 - I*x)^(1/2 - I/2)*(1 + I*x)^(1/2 + I/2))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{(-1+x)\,\mathrm{e}^{\arctan(x)}}{2\sqrt{x^2+1}}$	16

```
[In] int(exp(arctan(x))*x/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(-1+x)*exp(arctan(x))/(x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx = \frac{(x-1)e^{\arctan(x)}}{2\sqrt{x^2+1}}$$

```
[In] integrate(exp(arctan(x))*x/(x^2+1)^(3/2),x, algorithm="fricas")
[Out] 1/2*(x - 1)*e^arctan(x)/sqrt(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 11.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx = \frac{xe^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}} - \frac{e^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}}$$

```
[In] integrate(exp(atan(x))*x/(x**2+1)**(3/2),x)
[Out] x*exp(atan(x))/(2*sqrt(x**2 + 1)) - exp(atan(x))/(2*sqrt(x**2 + 1))
```

Maxima [F]

$$\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx = \int \frac{xe^{\arctan(x)}}{(x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(arctan(x))*x/(x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(x*e^arctan(x)/(x^2 + 1)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx = \frac{1}{2}\left(\frac{x}{\sqrt{x^2+1}} - \frac{1}{\sqrt{x^2+1}}\right)e^{\arctan(x)}$$

```
[In] integrate(exp(arctan(x))*x/(x^2+1)^(3/2),x, algorithm="giac")
[Out] 1/2*(x/sqrt(x^2 + 1) - 1/sqrt(x^2 + 1))*e^arctan(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{e}^{\operatorname{atan}(x)}}{(x^2+1)^{3/2}}\,dx$$

```
[In] int((x*exp(atan(x)))/(x^2 + 1)^(3/2),x)
[Out] int((x*exp(atan(x)))/(x^2 + 1)^(3/2), x)
```

3.99 $\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 20

$$\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}}\,dx = \frac{e^{\arctan(x)}(1+x)}{2\sqrt{1+x^2}}$$

```
[Out] 1/2*exp(arctan(x))*(1+x)/(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5177}

$$\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}}\,dx = \frac{(x+1)e^{\arctan(x)}}{2\sqrt{x^2+1}}$$

```
[In] Int[E^ArcTan[x]/(1 + x^2)^(3/2),x]
```

```
[Out] (E^ArcTan[x]*(1 + x))/(2*Sqrt[1 + x^2])
```

Rule 5177

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rubi steps

$$\text{integral} = \frac{e^{\arctan(x)}(1+x)}{2\sqrt{1+x^2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}}\,dx = \frac{e^{\arctan(x)}(1+x)}{2\sqrt{1+x^2}}$$

```
[In] Integrate[E^ArcTan[x]/(1 + x^2)^(3/2),x]
[Out] (E^ArcTan[x]*(1 + x))/(2*Sqrt[1 + x^2])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{e^{\arctan(x)}(1+x)}{2\sqrt{x^2+1}}$	16

```
[In] int(exp(arctan(x))/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(arctan(x))*(1+x)/(x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}}\,dx = \frac{(x+1)e^{\arctan(x)}}{2\sqrt{x^2+1}}$$

```
[In] integrate(exp(arctan(x))/(x^2+1)^(3/2),x, algorithm="fricas")
[Out] 1/2*(x + 1)*e^arctan(x)/sqrt(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 10.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}}\,dx = \frac{xe^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}} + \frac{e^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}}$$

```
[In] integrate(exp(atan(x))/(x**2+1)**(3/2),x)
[Out] x*exp(atan(x))/(2*sqrt(x**2 + 1)) + exp(atan(x))/(2*sqrt(x**2 + 1))
```

Maxima [F]

$$\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}}\,dx = \int \frac{e^{\arctan(x)}}{(x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(arctan(x))/(x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(e^arctan(x)/(x^2 + 1)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}}\,dx = \frac{1}{2}\left(\frac{x}{\sqrt{x^2+1}} + \frac{1}{\sqrt{x^2+1}}\right)e^{\arctan(x)}$$

```
[In] integrate(exp(arctan(x))/(x^2+1)^(3/2),x, algorithm="giac")
[Out] 1/2*(x/sqrt(x^2 + 1) + 1/sqrt(x^2 + 1))*e^arctan(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}}\,dx = \int \frac{e^{\operatorname{atan}(x)}}{(x^2+1)^{3/2}}\,dx$$

```
[In] int(exp(atan(x))/(x^2 + 1)^(3/2),x)
[Out] int(exp(atan(x))/(x^2 + 1)^(3/2), x)
```

3.100 $\int \frac{x^2}{(1+x^2)^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{x^2}{(1+x^2)^2} \, dx = -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {294, 209}

$$\int \frac{x^2}{(1+x^2)^2} \, dx = \frac{\arctan(x)}{2} - \frac{x}{2(x^2+1)}$$

```
[In] Int[x^2/(1 + x^2)^2,x]
```

```
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{x}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx \\ &= -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+x^2)^2}\,dx = -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

```
[In] Integrate[x^2/(1 + x^2)^2,x]
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{x}{2(x^2+1)} + \frac{\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2(x^2+1)} + \frac{\arctan(x)}{2}$	16
risch	$-\frac{x}{2(x^2+1)} + \frac{\arctan(x)}{2}$	16
parallelrisch	$-\frac{i\ln(x-i)x^2 - i\ln(x+i)x^2 + i\ln(x-i) - i\ln(x+i) + 2x}{4(x^2+1)}$	52

```
[In] int(x^2/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(1+x^2)^2}\,dx = \frac{(x^2+1)\arctan(x) - x}{2\,(x^2+1)}$$

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{(1+x^2)^2}\,dx = -\frac{x}{2x^2+2} + \frac{\operatorname{atan}(x)}{2}$$

```
[In] integrate(x**2/(x**2+1)**2,x)
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(1+x^2)^2}\,dx = -\frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan(x)$$

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(1+x^2)^2}\,dx = -\frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan(x)$$

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(1+x^2)^2} \, dx = \frac{\operatorname{atan}(x)}{2} - \frac{x}{2\,(x^2+1)}$$

```
[In] int(x^2/(x^2 + 1)^2,x)
```

```
[Out] atan(x)/2 - x/(2*(x^2 + 1))
```

3.101 $\int \frac{e^x}{1+e^{2x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 4

$$\int \frac{e^x}{1+e^{2x}}\,dx=\arctan\left(e^x\right)$$

```
[Out] arctan(exp(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2281, 209}

$$\int \frac{e^x}{1+e^{2x}}\,dx=\arctan\left(e^x\right)$$

```
[In] Int[E^x/(1 + E^(2*x)),x]
```

```
[Out] ArcTan[E^x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
```

```
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\ &= \arctan(e^x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{1+e^{2x}}\,dx = \arctan(e^x)$$

```
[In] Integrate[E^x/(1 + E^(2*x)),x]
[Out] ArcTan[E^x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
default	$\arctan(e^x)$	4
risch	$\frac{i\ln(e^x+i)}{2}-\frac{i\ln(e^x-i)}{2}$	20

```
[In] int(exp(x)/(1+exp(2*x)),x,method=_RETURNVERBOSE)
[Out] arctan(exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{1+e^{2x}}\,dx = \arctan(e^x)$$

```
[In] integrate(exp(x)/(1+exp(2*x)),x, algorithm="fricas")
[Out] arctan(e^x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \frac{e^x}{1+e^{2x}}\,dx = \operatorname{RootSum}\left(4z^2+1,\left(i \mapsto i\log\left(2i+e^x\right)\right)\right)$$

```
[In] integrate(exp(x)/(1+exp(2*x)),x)
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{1+e^{2x}}\,dx = \arctan\left(e^x\right)$$

```
[In] integrate(exp(x)/(1+exp(2*x)),x, algorithm="maxima")
[Out] arctan(e^x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{1+e^{2x}}\,dx = \arctan\left(e^x\right)$$

```
[In] integrate(exp(x)/(1+exp(2*x)),x, algorithm="giac")
[Out] arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{1+e^{2x}}\,dx = \operatorname{atan}(\mathrm{e}^x)$$

```
[In] int(exp(x)/(exp(2*x) + 1),x)
[Out] atan(exp(x))
```

3.102 $\int e^{-x} \cot^{-1}(e^x) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{-x} \cot^{-1}(e^x) \, dx = -x - e^{-x} \cot^{-1}(e^x) + \frac{1}{2} \log(1 + e^{2x})$$

```
[Out] -x-arccot(exp(x))/exp(x)+1/2*ln(1+exp(2*x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2225, 5316, 2320, 36, 29, 31}

$$\int e^{-x} \cot^{-1}(e^x) \, dx = -x + \frac{1}{2} \log(e^{2x} + 1) - e^{-x} \cot^{-1}(e^x)$$

```
[In] Int[ArcCot[E^x]/E^x,x]
```

```
[Out] -x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -e^{-x}\cot^{-1}\left(e^x\right) - \int \frac{1}{1+e^{2x}}\,dx \\
&= -e^{-x}\cot^{-1}\left(e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, e^{2x}\right) \\
&= -e^{-x}\cot^{-1}\left(e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, e^{2x}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, e^{2x}\right) \\
&= -x - e^{-x}\cot^{-1}\left(e^x\right) + \frac{1}{2}\log\left(1+e^{2x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int e^{-x} \cot^{-1}\left(e^x\right) dx = -x - e^{-x} \cot^{-1}\left(e^x\right) + \frac{1}{2} \log\left(1 + e^{2x}\right)$$

```
[In] Integrate[ArcCot[E^x]/E^x,x]
[Out] -x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\operatorname{arccot}\left(\mathrm{e}^{x}\right) \mathrm{e}^{-x}+\frac{\ln\left(1+\mathrm{e}^{2 x}\right)}{2}-\ln\left(\mathrm{e}^{x}\right)$	25
default	$-\operatorname{arccot}\left(\mathrm{e}^{x}\right) \mathrm{e}^{-x}+\frac{\ln\left(1+\mathrm{e}^{2 x}\right)}{2}-\ln\left(\mathrm{e}^{x}\right)$	25
parallelrisch	$\frac{\left(\ln\left(1+\mathrm{e}^{2 x}\right) \mathrm{e}^{x}-2\,\mathrm{e}^{x} x-2\,\operatorname{arccot}\left(\mathrm{e}^{x}\right)\right) \mathrm{e}^{-x}}{2}$	28
risch	$-\frac{i \mathrm{e}^{-x} \ln\left(1+i \mathrm{e}^{x}\right)}{2}+\frac{\ln\left(1+\mathrm{e}^{2 x}\right)}{2}-x+\frac{i \mathrm{e}^{-x} \ln\left(1-i \mathrm{e}^{x}\right)}{2}-\frac{\mathrm{e}^{-x} \pi}{2}$	51

```
[In] int(arccot(exp(x))/exp(x),x,method=_RETURNVERBOSE)
[Out] -arccot(exp(x))/exp(x)+1/2*ln(1+exp(x)^2)-ln(exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int e^{-x} \cot^{-1}\left(e^x\right) dx = -\frac{1}{2} \left(2\, x e^{x} - e^{x} \log\left(e^{(2\,x)} + 1\right) + 2\, \operatorname{arccot}\left(e^{x}\right)\right) e^{(-x)}$$

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="fricas")
[Out] -1/2*(2*x*e^x - e^x*log(e^(2*x) + 1) + 2*arccot(e^x))*e^(-x)
```

Sympy [A] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-x}\cot^{-1}(e^x)\,dx = -x + \frac{\log(e^{2x}+1)}{2} - e^{-x}\operatorname{acot}(e^x)$$

```
[In] integrate(acot(exp(x))/exp(x),x)
[Out] -x + log(exp(2*x) + 1)/2 - exp(-x)*acot(exp(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-x}\cot^{-1}(e^x)\,dx = -\operatorname{arccot}(e^x)\,e^{(-x)} + \frac{1}{2}\log\left(e^{(-2x)}+1\right)$$

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="maxima")
[Out] -arccot(e^x)*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{-x}\cot^{-1}(e^x)\,dx = -\arctan\left(e^{(-x)}\right)e^{(-x)} + \frac{1}{2}\log\left(e^{(-2x)}+1\right)$$

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="giac")
[Out] -arctan(e^(-x))*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{-x}\cot^{-1}(e^x)\,dx = \frac{\ln(\mathrm{e}^{2x}+1)}{2} - x - \operatorname{acot}(\mathrm{e}^x)\,\mathrm{e}^{-x}$$

```
[In] int(acot(exp(x))*exp(-x),x)
[Out] log(exp(2*x) + 1)/2 - x - acot(exp(x))*exp(-x)
```

3.103 $\int \sqrt{\frac{a+x}{a-x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = -\left((a-x)\sqrt{\frac{a+x}{a-x}}\right) + 2a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)$$

```
[Out] 2*a*arctan(((a+x)/(a-x))^(1/2))-(a-x)*((a+x)/(a-x))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 294, 209}

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = 2a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right) - (a-x)\sqrt{\frac{a+x}{a-x}}$$

```
[In] Int[Sqrt[(a + x)/(a - x)],x]
[Out] -((a - x)*Sqrt[(a + x)/(a - x)]) + 2*a*ArcTan[Sqrt[(a + x)/(a - x)]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (4a)\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2}\,dx, x, \sqrt{\frac{a+x}{a-x}}\right) \\
&= -\left((a-x)\sqrt{\frac{a+x}{a-x}}\right) + (2a)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{a+x}{a-x}}\right) \\
&= -\left((a-x)\sqrt{\frac{a+x}{a-x}}\right) + 2a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.60

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = \frac{\sqrt{\frac{a+x}{a-x}}\left((-a+x)\sqrt{a+x} + 2a\sqrt{a-x}\arctan\left(\frac{\sqrt{a+x}}{\sqrt{a-x}}\right)\right)}{\sqrt{a+x}}$$

```
[In] Integrate[Sqrt[(a + x)/(a - x)],x]
[Out] (Sqrt[(a + x)/(a - x)]*((-a + x)*Sqrt[a + x] + 2*a*Sqrt[a - x]*ArcTan[Sqrt[
a + x]/Sqrt[a - x]]))/Sqrt[a + x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{\sqrt{\frac{a+x}{a-x}}\,(a-x)\left(\sqrt{a^{2}-x^{2}}-a\arctan\left(\frac{x}{\sqrt{a^{2}-x^{2}}}\right)\right)}{\sqrt{(a-x)(a+x)}}$	61
risch	$-\frac{(a-x)\sqrt{\frac{a+x}{a-x}}\,\sqrt{(a-x)(a+x)}}{\sqrt{-(-a+x)(a+x)}}+\frac{a\arctan\left(\frac{x}{\sqrt{a^{2}-x^{2}}}\right)\sqrt{\frac{a+x}{a-x}}\,\sqrt{(a-x)(a+x)}}{a+x}$	90

```
[In] int(((a+x)/(a-x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -((a+x)/(a-x))^(1/2)*(a-x)*((a^2-x^2)^(1/2)-a*arctan(x/(a^2-x^2)^(1/2)))/((
a-x)*(a+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int\sqrt{\frac{a+x}{a-x}}\,dx=2\,a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="fricas")
[Out] 2*a*arctan(sqrt((a + x)/(a - x))) - (a - x)*sqrt((a + x)/(a - x))
```

Sympy [F]

$$\int\sqrt{\frac{a+x}{a-x}}\,dx=\int\sqrt{\frac{a+x}{a-x}}\,dx$$

```
[In] integrate(((a+x)/(a-x))**(1/2),x)
[Out] Integral(sqrt((a + x)/(a - x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int\sqrt{\frac{a+x}{a-x}}\,dx=-2\,a\left(\frac{\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}-\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)\right)$$

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="maxima")
[Out] -2*a*(sqrt((a + x)/(a - x))/((a + x)/(a - x) + 1) - arctan(sqrt((a + x)/(a
- x))))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = a \arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a-x)\operatorname{sgn}(a) - \sqrt{a^2-x^2}\operatorname{sgn}(a-x)$$

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="giac")
```

```
[Out] a*arcsin(x/a)*sgn(a - x)*sgn(a) - sqrt(a^2 - x^2)*sgn(a - x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = 2\,a\,\mathrm{atan}\left(\sqrt{\frac{a+x}{a-x}}\right) - \frac{2\,a\,\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}$$

```
[In] int(((a + x)/(a - x))^(1/2),x)
```

```
[Out] 2*a*atan(((a + x)/(a - x))^(1/2)) - (2*a*((a + x)/(a - x))^(1/2))/((a + x)/
(a - x) + 1)
```

3.104 $\int \sqrt{(b-x)(-a+x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \sqrt{(b-x)(-a+x)}\,dx = -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2} - \frac{1}{8}(a-b)^2\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

```
[Out] -1/8*(a-b)^2*arctan(1/2*(a+b-2*x)/(-a*b+(a+b)*x-x^2)^(1/2))-1/4*(a+b-2*x)*(
-a*b+(a+b)*x-x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1976, 626, 635, 210}

$$\int \sqrt{(b-x)(-a+x)}\,dx = -\frac{1}{8}(a-b)^2\arctan\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right) - \frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}$$

```
[In] Int[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -1/4*((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2]) - ((a - b)^2*ArcTan[(a
+ b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \sqrt{-ab+(a+b)x-x^2}\,dx \\
&= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}+\frac{1}{8}(a-b)^2\int \frac{1}{\sqrt{-ab+(a+b)x-x^2}}\,dx \\
&= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2} \\
&\quad +\frac{1}{4}(a-b)^2\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,\frac{a+b-2x}{\sqrt{-ab+(a+b)x-x^2}}\right) \\
&= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}-\frac{1}{8}(a-b)^2\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \sqrt{(b-x)(-a+x)}\,dx = \frac{1}{4}\sqrt{(a-x)(-b+x)}\left(-a-b+2x+\frac{(a-b)^2\arctan\left(\frac{\sqrt{-a+x}}{\sqrt{b-x}}\right)}{\sqrt{b-x}\sqrt{-a+x}}\right)$$

```
[In] Integrate[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] (Sqrt[(a - x)*(-b + x)]*(-a - b + 2*x + ((a - b)^2*ArcTan[Sqrt[-a + x]/Sqrt
[b - x]])/(Sqrt[b - x]*Sqrt[-a + x])))/4
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{(a+b-2x)\sqrt{-ab+(a+b)x-x^2}}{4}-\frac{\left(4ab-(a+b)^2\right)\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)}{8}$	68
risch	$\frac{(b-x)(a-x)(a+b-2x)}{4\sqrt{-(-b+x)(-a+x)}}-\left(\frac{1}{4}ab-\frac{1}{8}b^2-\frac{1}{8}a^2\right)\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)$	78

```
[In] int(((b-x)*(-a+x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(a+b-2*x)*(-a*b+(a+b)*x-x^2)^(1/2)-1/8*(4*a*b-(a+b)^2)*arctan((x-1/2*b
-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \sqrt{(b-x)(-a+x)}\,dx &= -\frac{1}{8}\left(a^2-2\,ab+b^2\right)\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}(a+b-2\,x)}{2\,(ab-(a+b)x+x^2)}\right)\\ &\quad-\frac{1}{4}\sqrt{-ab+(a+b)x-x^2}(a+b-2\,x)\end{aligned}$$

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(a^2 - 2*a*b + b^2)*arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b -
2*x)/(a*b - (a + b)*x + x^2)) - 1/4*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2
*x)
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.61

$$\int \sqrt{(b-x)(-a+x)}\,dx = \left(-\frac{ab}{2} + \frac{\left(\frac{a}{4}+\frac{b}{4}\right)(a+b)}{2}\right)\left(\begin{cases} -i\log\left(a+b-2x+2i\sqrt{-ab-x^2+x\,(a+b)}\right) & \text{for } ab-\frac{(a+b)^2}{4}\neq 0 \\ \frac{\left(-\frac{a}{2}-\frac{b}{2}+x\right)\log\left(-\frac{a}{2}-\frac{b}{2}+x\right)}{\sqrt{-\left(-\frac{a}{2}-\frac{b}{2}+x\right)^2}} & \text{otherwise} \end{cases}\right) + \left(-\frac{a}{4}-\frac{b}{4}+\frac{x}{2}\right)\sqrt{-ab-x^2+x\,(a+b)}$$

```
[In] integrate(((b-x)*(-a+x))**(1/2),x)
```

```
[Out] (-a*b/2 + (a/4 + b/4)*(a + b)/2)*Piecewise((-I*log(a + b - 2*x + 2*I*sqrt(-
a*b - x**2 + x*(a + b))), Ne(a*b - (a + b)**2/4, 0)), ((-a/2 - b/2 + x)*log
(-a/2 - b/2 + x)/sqrt(-(-a/2 - b/2 + x)**2), True)) + (-a/4 - b/4 + x/2)*sq
rt(-a*b - x**2 + x*(a + b))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{(b-x)(-a+x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \sqrt{(b-x)(-a+x)}\,dx = \frac{1}{8}\left(a^2-2\,ab+b^2\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\mathrm{sgn}(-a+b) - \frac{1}{4}\sqrt{-ab+ax+bx-x^2}(a+b-2\,x)$$

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqr
t(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{(b-x)(-a+x)}\,dx = \int \sqrt{-\left(a-x\right)\,\left(b-x\right)}\,dx$$

```
[In] int((-(a - x)*(b - x))^(1/2),x)
```

```
[Out] int((-(a - x)*(b - x))^(1/2), x)
```

3.105 $\int \frac{1}{\sqrt{(b-x)(-a+x)}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}} \, dx = -\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

```
[Out] -arctan(1/2*(a+b-2*x)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1976, 635, 210}

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}} \, dx = -\arctan\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

```
[In] Int[1/Sqrt[(b - x)*(-a + x)],x]
[Out] -ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{-ab+(a+b)x-x^2}}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{a+b-2x}{\sqrt{-ab+(a+b)x-x^2}}\right)\\ &= -\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \frac{2\sqrt{b-x}\sqrt{-a+x}\arctan\left(\frac{\sqrt{-a+x}}{\sqrt{b-x}}\right)}{\sqrt{(a-x)(-b+x)}}$$

```
[In] Integrate[1/Sqrt[(b - x)*(-a + x)],x]
[Out] (2*Sqrt[b - x]*Sqrt[-a + x]*ArcTan[Sqrt[-a + x]/Sqrt[b - x]])/Sqrt[(a - x)*
(-b + x)]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

method	result	size
default	$\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)$	28

```
[In] int(1/((b-x)*(-a+x))^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = -\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}(a+b-2\,x)}{2\,(ab-(a+b)x+x^2)}\right)$$

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2*x)/(a*b - (a + b)*x +
x^2))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \begin{cases} -i\log\left(a+b-2x+2i\sqrt{-ab-x^2+x\,(a+b)}\right) & \text{for } ab-\frac{(a+b)^2}{4}\neq 0 \\ \frac{\left(-\frac{a}{2}-\frac{b}{2}+x\right)\log\left(-\frac{a}{2}-\frac{b}{2}+x\right)}{\sqrt{-\left(-\frac{a}{2}-\frac{b}{2}+x\right)^2}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Piecewise((-I*log(a + b - 2*x + 2*I*sqrt(-a*b - x**2 + x*(a + b))), Ne(a*b
- (a + b)**2/4, 0)), ((-a/2 - b/2 + x)*log(-a/2 - b/2 + x)/sqrt(-(-a/2 - b/
2 + x)**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(28) = 56$.

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \frac{1}{8}\left(a^2-2\,ab+b^2\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\operatorname{sgn}(-a+b) - \frac{1}{4}\sqrt{-ab+ax+bx-x^2}(a+b-2\,x)$$

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqr
t(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \int \frac{1}{\sqrt{-\left(a-x\right)\,\left(b-x\right)}}\,dx$$

```
[In] int(1/(-(a - x)*(b - x))^(1/2),x)
```

```
[Out] int(1/(-(a - x)*(b - x))^(1/2), x)
```

3.106 $\int \frac{3+5x}{-3+2x+x^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 15

$$\int \frac{3+5x}{-3+2x+x^2} \, dx = 2\log(1-x) + 3\log(3+x)$$

```
[Out] 2*ln(1-x)+3*ln(3+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {646, 31}

$$\int \frac{3+5x}{-3+2x+x^2} \, dx = 2\log(1-x) + 3\log(x+3)$$

```
[In] Int[(3 + 5*x)/(-3 + 2*x + x^2),x]
```

```
[Out] 2*Log[1 - x] + 3*Log[3 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{1}{-1+x}\,dx + 3\int \frac{1}{3+x}\,dx\\ &= 2\log(1-x) + 3\log(3+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{3+5x}{-3+2x+x^2}\,dx = 2\log(1-x) + 3\log(3+x)$$

```
[In] Integrate[(3 + 5*x)/(-3 + 2*x + x^2),x]
[Out] 2*Log[1 - x] + 3*Log[3 + x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$2\ln(-1+x)+3\ln(3+x)$	14
norman	$2\ln(-1+x)+3\ln(3+x)$	14
risch	$2\ln(-1+x)+3\ln(3+x)$	14
parallelrisch	$2\ln(-1+x)+3\ln(3+x)$	14

```
[In] int((3+5*x)/(x^2+2*x-3),x,method=_RETURNVERBOSE)
[Out] 2*ln(-1+x)+3*ln(3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{3+5x}{-3+2x+x^2}\,dx = 3\,\log\left(x+3\right)+2\,\log\left(x-1\right)$$

```
[In] integrate((3+5*x)/(x^2+2*x-3),x, algorithm="fricas")
[Out] 3*log(x + 3) + 2*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{3+5x}{-3+2x+x^2}\,dx = 2\log\left(x-1\right)+3\log\left(x+3\right)$$

```
[In] integrate((3+5*x)/(x**2+2*x-3),x)
[Out] 2*log(x - 1) + 3*log(x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{3+5x}{-3+2x+x^2}\,dx = 3\,\log\left(x+3\right)+2\,\log\left(x-1\right)$$

```
[In] integrate((3+5*x)/(x^2+2*x-3),x, algorithm="maxima")
[Out] 3*log(x + 3) + 2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{3+5x}{-3+2x+x^2}\,dx = 3\,\log\left(|x+3|\right)+2\,\log\left(|x-1|\right)$$

```
[In] integrate((3+5*x)/(x^2+2*x-3),x, algorithm="giac")
[Out] 3*log(abs(x + 3)) + 2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{3+5x}{-3+2x+x^2}\,dx = 2\,\ln\left(x-1\right)+3\,\ln\left(x+3\right)$$

```
[In] int((5*x + 3)/(2*x + x^2 - 3),x)
[Out] 2*log(x - 1) + 3*log(x + 3)
```

3.107 $\int \frac{5+2x}{-3+2x+x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 19

$$\int \frac{5+2x}{-3+2x+x^2} \, dx = \frac{7}{4}\log(1-x) + \frac{1}{4}\log(3+x)$$

```
[Out] 7/4*ln(1-x)+1/4*ln(3+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {646, 31}

$$\int \frac{5+2x}{-3+2x+x^2} \, dx = \frac{7}{4}\log(1-x) + \frac{1}{4}\log(x+3)$$

```
[In] Int[(5 + 2*x)/(-3 + 2*x + x^2),x]
```

```
[Out] (7*Log[1 - x])/4 + Log[3 + x]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\int \frac{1}{3+x}\,dx + \frac{7}{4}\int \frac{1}{-1+x}\,dx \\
&= \frac{7}{4}\log(1-x) + \frac{1}{4}\log(3+x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{5+2x}{-3+2x+x^2}\,dx = \frac{7}{4}\log(1-x) + \frac{1}{4}\log(3+x)$$

```
[In] Integrate[(5 + 2*x)/(-3 + 2*x + x^2),x]
[Out] (7*Log[1 - x])/4 + Log[3 + x]/4
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{7\ln(-1+x)}{4}+\frac{\ln(3+x)}{4}$	14
norman	$\frac{7\ln(-1+x)}{4}+\frac{\ln(3+x)}{4}$	14
risch	$\frac{7\ln(-1+x)}{4}+\frac{\ln(3+x)}{4}$	14
parallelrisch	$\frac{7\ln(-1+x)}{4}+\frac{\ln(3+x)}{4}$	14

```
[In] int((5+2*x)/(x^2+2*x-3),x,method=_RETURNVERBOSE)
[Out] 7/4*ln(-1+x)+1/4*ln(3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{5+2x}{-3+2x+x^2}\,dx = \frac{1}{4}\,\log\left(x+3\right) + \frac{7}{4}\,\log\left(x-1\right)$$

```
[In] integrate((5+2*x)/(x^2+2*x-3),x, algorithm="fricas")
[Out] 1/4*log(x + 3) + 7/4*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{5+2x}{-3+2x+x^2}\,dx = \frac{7\log\left(x-1\right)}{4} + \frac{\log\left(x+3\right)}{4}$$

```
[In] integrate((5+2*x)/(x**2+2*x-3),x)
[Out] 7*log(x - 1)/4 + log(x + 3)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{5+2x}{-3+2x+x^2}\,dx = \frac{1}{4}\,\log\left(x+3\right) + \frac{7}{4}\,\log\left(x-1\right)$$

```
[In] integrate((5+2*x)/(x^2+2*x-3),x, algorithm="maxima")
[Out] 1/4*log(x + 3) + 7/4*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{5+2x}{-3+2x+x^2}\,dx = \frac{1}{4}\,\log\left(|x+3|\right) + \frac{7}{4}\,\log\left(|x-1|\right)$$

```
[In] integrate((5+2*x)/(x^2+2*x-3),x, algorithm="giac")
[Out] 1/4*log(abs(x + 3)) + 7/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{5+2x}{-3+2x+x^2}\,dx = \frac{7\,\ln(x-1)}{4} + \frac{\ln(x+3)}{4}$$

```
[In] int((2*x + 5)/(2*x + x^2 - 3),x)
[Out] (7*log(x - 1))/4 + log(x + 3)/4
```

3.108 $\int \frac{3x+x^3}{-3-2x+x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 23

$$\int \frac{3x+x^3}{-3-2x+x^2}\,dx = 2x+\frac{x^2}{2}+9\log(3-x)+\log(1+x)$$

```
[Out] 2*x+1/2*x^2+9*ln(3-x)+ln(1+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1607, 1642, 646, 31}

$$\int \frac{3x+x^3}{-3-2x+x^2}\,dx = \frac{x^2}{2}+2x+9\log(3-x)+\log(x+1)$$

```
[In] Int[(3*x + x^3)/(-3 - 2*x + x^2),x]
```

```
[Out] 2*x + x^2/2 + 9*Log[3 - x] + Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(3+x^2)}{-3-2x+x^2}\,dx \\
&= \int \left(2+x+\frac{2(3+5x)}{-3-2x+x^2}\right)dx \\
&= 2x+\frac{x^2}{2}+2\int \frac{3+5x}{-3-2x+x^2}\,dx \\
&= 2x+\frac{x^2}{2}+9\int \frac{1}{-3+x}\,dx+\int \frac{1}{1+x}\,dx \\
&= 2x+\frac{x^2}{2}+9\log(3-x)+\log(1+x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{3x+x^3}{-3-2x+x^2}\,dx = 2x+\frac{x^2}{2}+9\log(3-x)+\log(1+x)$$

```
[In] Integrate[(3*x + x^3)/(-3 - 2*x + x^2),x]
[Out] 2*x + x^2/2 + 9*Log[3 - x] + Log[1 + x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$2x+\frac{x^2}{2}+\ln(1+x)+9\ln(-3+x)$	20
norman	$2x+\frac{x^2}{2}+\ln(1+x)+9\ln(-3+x)$	20
risch	$2x+\frac{x^2}{2}+\ln(1+x)+9\ln(-3+x)$	20
parallelrisch	$2x+\frac{x^2}{2}+\ln(1+x)+9\ln(-3+x)$	20

```
[In] int((x^3+3*x)/(x^2-2*x-3),x,method=_RETURNVERBOSE)
[Out] 2*x+1/2*x^2+ln(1+x)+9*ln(-3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{3x+x^3}{-3-2x+x^2}\,dx=\frac{1}{2}\,x^2+2\,x+\log(x+1)+9\,\log(x-3)$$

```
[In] integrate((x^3+3*x)/(x^2-2*x-3),x, algorithm="fricas")
[Out] 1/2*x^2 + 2*x + log(x + 1) + 9*log(x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{3x+x^3}{-3-2x+x^2}\,dx=\frac{x^2}{2}+2x+9\log(x-3)+\log(x+1)$$

```
[In] integrate((x**3+3*x)/(x**2-2*x-3),x)
[Out] x**2/2 + 2*x + 9*log(x - 3) + log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{3x + x^3}{-3 - 2x + x^2} \, dx = \frac{1}{2} \, x^2 + 2 \, x + \log\left(x + 1\right) + 9 \, \log\left(x - 3\right)$$

```
[In] integrate((x^3+3*x)/(x^2-2*x-3),x, algorithm="maxima")
[Out] 1/2*x^2 + 2*x + log(x + 1) + 9*log(x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{3x + x^3}{-3 - 2x + x^2} \, dx = \frac{1}{2} \, x^2 + 2 \, x + \log\left(|x + 1|\right) + 9 \, \log\left(|x - 3|\right)$$

```
[In] integrate((x^3+3*x)/(x^2-2*x-3),x, algorithm="giac")
[Out] 1/2*x^2 + 2*x + log(abs(x + 1)) + 9*log(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{3x + x^3}{-3 - 2x + x^2} \, dx = 2 \, x + \ln\left(x + 1\right) + 9 \, \ln\left(x - 3\right) + \frac{x^2}{2}$$

```
[In] int(-(3*x + x^3)/(2*x - x^2 + 3),x)
[Out] 2*x + log(x + 1) + 9*log(x - 3) + x^2/2
```

3.109 $\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx = 2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(2+x)$$

```
[Out] 2*ln(1-x)+1/2*ln(x)-1/2*ln(2+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1608, 1642}

$$\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx = 2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(x+2)$$

```
[In] Int[(-1 + 5*x + 2*x^2)/(-2*x + x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x]/2 - Log[2 + x]/2
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{-1+5x+2x^2}{x\,(-2+x+x^2)}\,dx \\
&= \int \left(\frac{2}{-1+x}+\frac{1}{2x}-\frac{1}{2(2+x)}\right)dx \\
&= 2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(2+x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx = 2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(2+x)$$

```
[In] Integrate[(-1 + 5*x + 2*x^2)/(-2*x + x^2 + x^3),x]
[Out] 2*Log[1 - x] + Log[x]/2 - Log[2 + x]/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$2\ln\left(-1+x\right)-\frac{\ln(2+x)}{2}+\frac{\ln(x)}{2}$	18
norman	$2\ln\left(-1+x\right)-\frac{\ln(2+x)}{2}+\frac{\ln(x)}{2}$	18
risch	$2\ln\left(-1+x\right)-\frac{\ln(2+x)}{2}+\frac{\ln(x)}{2}$	18
parallelrisch	$2\ln\left(-1+x\right)-\frac{\ln(2+x)}{2}+\frac{\ln(x)}{2}$	18

```
[In] int((2*x^2+5*x-1)/(x^3+x^2-2*x),x,method=_RETURNVERBOSE)
[Out] 2*ln(-1+x)-1/2*ln(2+x)+1/2*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx = -\frac{1}{2}\,\log\left(x+2\right) + 2\,\log\left(x-1\right) + \frac{1}{2}\,\log\left(x\right)$$

```
[In] integrate((2*x^2+5*x-1)/(x^3+x^2-2*x),x, algorithm="fricas")
[Out] -1/2*log(x + 2) + 2*log(x - 1) + 1/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx = \frac{\log\left(x\right)}{2} + 2\log\left(x-1\right) - \frac{\log\left(x+2\right)}{2}$$

```
[In] integrate((2*x**2+5*x-1)/(x**3+x**2-2*x),x)
[Out] log(x)/2 + 2*log(x - 1) - log(x + 2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx = -\frac{1}{2}\,\log\left(x+2\right) + 2\,\log\left(x-1\right) + \frac{1}{2}\,\log\left(x\right)$$

```
[In] integrate((2*x^2+5*x-1)/(x^3+x^2-2*x),x, algorithm="maxima")
[Out] -1/2*log(x + 2) + 2*log(x - 1) + 1/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx = -\frac{1}{2}\,\log\left(|x+2|\right) + 2\,\log\left(|x-1|\right) + \frac{1}{2}\,\log\left(|x|\right)$$

```
[In] integrate((2*x^2+5*x-1)/(x^3+x^2-2*x),x, algorithm="giac")
[Out] -1/2*log(abs(x + 2)) + 2*log(abs(x - 1)) + 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx = 2\,\ln\left(x-1\right)+\mathrm{atanh}\left(\frac{135}{11\,\left(11\,x-5\right)}+\frac{16}{11}\right)$$

```
[In] int((5*x + 2*x^2 - 1)/(x^2 - 2*x + x^3),x)
[Out] 2*log(x - 1) + atanh(135/(11*(11*x - 5)) + 16/11)
```

3.110 $\int \frac{3+2x+x^2}{(-1+x)(1+x)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \frac{3+2x+x^2}{(-1+x)(1+x)^2} \, dx = \frac{1}{1+x} + \frac{3}{2}\log(1-x) - \frac{1}{2}\log(1+x)$$

```
[Out] 1/(1+x)+3/2*ln(1-x)-1/2*ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {907}

$$\int \frac{3+2x+x^2}{(-1+x)(1+x)^2} \, dx = \frac{1}{x+1} + \frac{3}{2}\log(1-x) - \frac{1}{2}\log(x+1)$$

```
[In] Int[(3 + 2*x + x^2)/((-1 + x)*(1 + x)^2),x]
```

```
[Out] (1 + x)^(-1) + (3*Log[1 - x])/2 - Log[1 + x]/2
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{3}{2(-1+x)} - \frac{1}{(1+x)^2} - \frac{1}{2(1+x)}\right) dx \\ &= \frac{1}{1+x} + \frac{3}{2}\log(1-x) - \frac{1}{2}\log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx = \frac{1}{1+x} + \frac{3}{2}\log(-1+x) - \frac{1}{2}\log(1+x)$$

```
[In] Integrate[(3 + 2*x + x^2)/((-1 + x)*(1 + x)^2),x]
[Out] (1 + x)^(-1) + (3*Log[-1 + x])/2 - Log[1 + x]/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{3\ln(-1+x)}{2}+\frac{1}{1+x}-\frac{\ln(1+x)}{2}$	19
norman	$\frac{3\ln(-1+x)}{2}+\frac{1}{1+x}-\frac{\ln(1+x)}{2}$	19
risch	$\frac{3\ln(-1+x)}{2}+\frac{1}{1+x}-\frac{\ln(1+x)}{2}$	19
parallelrisch	$\frac{3\ln(-1+x)x-\ln(1+x)x+2+3\ln(-1+x)-\ln(1+x)}{2x+2}$	36

```
[In] int((x^2+2*x+3)/(-1+x)/(1+x)^2,x,method=_RETURNVERBOSE)
[Out] 3/2*ln(-1+x)+1/(1+x)-1/2*ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx = -\frac{(x+1)\log(x+1) - 3(x+1)\log(x-1) - 2}{2(x+1)}$$

```
[In] integrate((x^2+2*x+3)/(-1+x)/(1+x)^2,x, algorithm="fricas")
[Out] -1/2*((x + 1)*log(x + 1) - 3*(x + 1)*log(x - 1) - 2)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx = \frac{3\log{(x-1)}}{2} - \frac{\log{(x+1)}}{2} + \frac{1}{x+1}$$

```
[In] integrate((x**2+2*x+3)/(-1+x)/(1+x)**2,x)
[Out] 3*log(x - 1)/2 - log(x + 1)/2 + 1/(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx = \frac{1}{x+1} - \frac{1}{2}\log{(x+1)} + \frac{3}{2}\log{(x-1)}$$

```
[In] integrate((x^2+2*x+3)/(-1+x)/(1+x)^2,x, algorithm="maxima")
[Out] 1/(x + 1) - 1/2*log(x + 1) + 3/2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx = \frac{1}{x+1} + \log{(|x+1|)} + \frac{3}{2}\log{\left(\left|-\frac{2}{x+1}+1\right|\right)}$$

```
[In] integrate((x^2+2*x+3)/(-1+x)/(1+x)^2,x, algorithm="giac")
[Out] 1/(x + 1) + log(abs(x + 1)) + 3/2*log(abs(-2/(x + 1) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx = \frac{3\,\ln{(x-1)}}{2} - \frac{\ln{(x+1)}}{2} + \frac{1}{x+1}$$

```
[In] int((2*x + x^2 + 3)/((x - 1)*(x + 1)^2),x)
[Out] (3*log(x - 1))/2 - log(x + 1)/2 + 1/(x + 1)
```

3.111 $\int \frac{-2+2x+3x^2}{-1+x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{-2+2x+3x^2}{-1+x^3} dx = \frac{4\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log\left(1-x^3\right)$$

```
[Out] ln(-x^3+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1885, 1600, 632, 210, 266}

$$\int \frac{-2+2x+3x^2}{-1+x^3} dx = \frac{4\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} + \log\left(1-x^3\right)$$

```
[In] Int[(-2 + 2*x + 3*x^2)/(-1 + x^3),x]
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 - x^3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 3\int \frac{x^2}{-1+x^3}\,dx + \int \frac{-2+2x}{-1+x^3}\,dx\\ &= \log\left(1-x^3\right) + \int \frac{1}{\frac{1}{2}+\frac{x}{2}+\frac{x^2}{2}}\,dx\\ &= \log\left(1-x^3\right) - 2\text{Subst}\left(\int \frac{1}{-\frac{3}{4}-x^2}\,dx, x, \frac{1}{2}+x\right)\\ &= \frac{4\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log\left(1-x^3\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{-2+2x+3x^2}{-1+x^3}\,dx = \frac{4\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log\left(1-x^3\right)$$

```
[In] Integrate[(-2 + 2*x + 3*x^2)/(-1 + x^3),x]
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 - x^3]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result
default	$\ln(-1+x)+\ln(x^2+x+1)+\frac{4\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
risch	$\ln(4x^2+4x+4)+\frac{4\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\ln(-1+x)$
meijerg	$-\frac{2x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{1}{3}}}+\ln(-x^3+1)+\frac{2x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)\right.}{}\right.}{}$

```
[In] int((3*x^2+2*x-2)/(x^3-1),x,method=_RETURNVERBOSE)
[Out] ln(-1+x)+ln(x^2+x+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{-2+2x+3x^2}{-1+x^3}\,dx=\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\log\left(x^2+x+1\right)+\log(x-1)$$

```
[In] integrate((3*x^2+2*x-2)/(x^3-1),x, algorithm="fricas")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.11

$$\int \frac{-2+2x+3x^2}{-1+x^3}\,dx=\log(x-1)$$

```
[In] integrate((3*x**2+2*x-2)/(x**3-1),x)
[Out] log(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{-2+2x+3x^2}{-1+x^3}\,dx = \frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\log\left(x^2+x+1\right)+\log\left(x-1\right)$$

```
[In] integrate((3*x^2+2*x-2)/(x^3-1),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{-2+2x+3x^2}{-1+x^3}\,dx = \frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\log\left(x^2+x+1\right)+\log\left(|x-1|\right)$$

```
[In] integrate((3*x^2+2*x-2)/(x^3-1),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(abs(x -
1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.04

$$\int \frac{-2+2x+3x^2}{-1+x^3}\,dx = \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+\ln\left(x-1\right) - \frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,2\mathrm{i}}{3}+\frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,2\mathrm{i}}{3}$$

```
[In] int((2*x + 3*x^2 - 2)/(x^3 - 1),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2) + log(x + (3^(1/2)*1i)/2 + 1/2) + log(x - 1)
- (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*2i)/3 + (3^(1/2)*log(x + (3^(1/2)*
1i)/2 + 1/2)*2i)/3
```

3.112 $\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 49

$$\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2} \, dx = \frac{1}{2\left(2+x^2\right)} - \frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}} + \frac{1}{3}\log(1-x) + \frac{1}{3}\log\left(2+x^2\right)$$

```
[Out] 1/2/(x^2+2)+1/3*ln(1-x)+1/3*ln(x^2+2)-1/6*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1661, 1643, 649, 209, 266}

$$\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2} \, dx = -\frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}} + \frac{1}{2\left(x^2+2\right)} + \frac{1}{3}\log\left(x^2+2\right) + \frac{1}{3}\log(1-x)$$

```
[In] Int[(2 - x + 2*x^2 - x^3 + x^4)/((-1 + x)*(2 + x^2)^2),x]
```

```
[Out] 1/(2*(2 + x^2)) - ArcTan[x/Sqrt[2]]/(3*Sqrt[2]) + Log[1 - x]/3 + Log[2 + x^
2]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2\left(2+x^2\right)} - \frac{1}{4}\int \frac{-4+4x-4x^2}{(-1+x)\left(2+x^2\right)}\,dx \\
&= \frac{1}{2\left(2+x^2\right)} - \frac{1}{4}\int \left(-\frac{4}{3(-1+x)} - \frac{4(-1+2x)}{3\left(2+x^2\right)}\right)\,dx \\
&= \frac{1}{2\left(2+x^2\right)} + \frac{1}{3}\log(1-x) + \frac{1}{3}\int \frac{-1+2x}{2+x^2}\,dx \\
&= \frac{1}{2\left(2+x^2\right)} + \frac{1}{3}\log(1-x) - \frac{1}{3}\int \frac{1}{2+x^2}\,dx + \frac{2}{3}\int \frac{x}{2+x^2}\,dx \\
&= \frac{1}{2\left(2+x^2\right)} - \frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}} + \frac{1}{3}\log(1-x) + \frac{1}{3}\log\left(2+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2}\,dx = \frac{1}{2\left(3+2(-1+x)+(-1+x)^2\right)} - \frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}} + \frac{1}{3}\log\left(3+2(-1+x)+(-1+x)^2\right) + \frac{1}{3}\log(-1+x)$$

```
[In] Integrate[(2 - x + 2*x^2 - x^3 + x^4)/((-1 + x)*(2 + x^2)^2),x]
[Out] 1/(2*(3 + 2*(-1 + x) + (-1 + x)^2)) - ArcTan[x/Sqrt[2]]/(3*Sqrt[2]) + Log[3
 + 2*(-1 + x) + (-1 + x)^2]/3 + Log[-1 + x]/3
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\ln(-1+x)}{3}+\frac{1}{2x^2+4}+\frac{\ln(x^2+2)}{3}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{6}$	37
risch	$\frac{\ln(-1+x)}{3}+\frac{1}{2x^2+4}+\frac{\ln(x^2+2)}{3}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{6}$	37

```
[In] int((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*ln(-1+x)+1/2/(x^2+2)+1/3*ln(x^2+2)-1/6*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2}\,dx = -\frac{\sqrt{2}(x^2+2)\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 2\left(x^2+2\right)\log\left(x^2+2\right) - 2\left(x^2+2\right)\log\left(x-1\right) - 3}{6\left(x^2+2\right)}$$

```
[In] integrate((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x, algorithm="fricas")
[Out] -1/6*(sqrt(2)*(x^2 + 2)*arctan(1/2*sqrt(2)*x) - 2*(x^2 + 2)*log(x^2 + 2) -
2*(x^2 + 2)*log(x - 1) - 3)/(x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.29

$$\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2}\,dx=\frac{\log(x-1)}{3}+\frac{1}{2x^2+4}$$

```
[In] integrate((x**4-x**3+2*x**2-x+2)/(-1+x)/(x**2+2)**2,x)
[Out] log(x - 1)/3 + 1/(2*x**2 + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2}\,dx=-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{2\left(x^2+2\right)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log(x-1)$$

```
[In] integrate((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x, algorithm="maxima")
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/2/(x^2 + 2) + 1/3*log(x^2 + 2) + 1/3
*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2}\,dx=-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{2\left(x^2+2\right)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log(|x-1|)$$

```
[In] integrate((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x, algorithm="giac")
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/2/(x^2 + 2) + 1/3*log(x^2 + 2) + 1/3
*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\,(2+x^2)^2}\,dx = \frac{\ln(x-1)}{3} + \ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(\frac{1}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{12}\right) - \ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(-\frac{1}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{12}\right) + \frac{1}{2\,(x^2+2)}$$

```
[In] int((2*x^2 - x - x^3 + x^4 + 2)/((x^2 + 2)^2*(x - 1)),x)
[Out] log(x - 1)/3 + log(x - 2^(1/2)*1i)*((2^(1/2)*1i)/12 + 1/3) - log(x + 2^(1/2
)*1i)*((2^(1/2)*1i)/12 - 1/3) + 1/(2*(x^2 + 2))
```

3.113 $\int \frac{1}{\cos(x)+\sin(x)}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(1/2*(cos(x)-sin(x))*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3153, 212}

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[In] Int[(Cos[x] + Sin[x])^(-1),x]
```

```
[Out] -(ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \cos(x)-\sin(x)\right)$$
$$= -\frac{\operatorname{arctanh}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = (-1-i)(-1)^{3/4}\operatorname{arctanh}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$

```
[In] Integrate[(Cos[x] + Sin[x])^(-1),x]
[Out] (-1 - I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

method	result	size
default	$\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tan\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)$	19
risch	$\frac{\sqrt{2}\,\ln\left(e^{ix}-\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(e^{ix}+\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)}{2}$	48

```
[In] int(1/(cos(x)+sin(x)),x,method=_RETURNVERBOSE)
[Out] 2^(1/2)*arctanh(1/4*(2*tan(1/2*x)-2)*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}-\cos\left(x\right)\right)\sin\left(x\right)-2\sqrt{2}\cos\left(x\right)+3}{2\cos\left(x\right)\sin\left(x\right)+1}\right)$$

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((2*(sqrt(2) - cos(x))*sin(x) - 2*sqrt(2)*cos(x) + 3)/(2*cos
(x)*sin(x) + 1))
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = \frac{\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{2} - \frac{\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{2}$$

```
[In] integrate(1/(cos(x)+sin(x)),x)
[Out] sqrt(2)*log(tan(x/2) - 1 + sqrt(2))/2 - sqrt(2)*log(tan(x/2) - sqrt(2) - 1)
/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}+1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}-1}\right)$$

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="maxima")
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) + 1)/(sqrt(2) + sin(x)/(co
s(x) + 1) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{1}{\cos(x) + \sin(x)}\, dx = -\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 2\tan\left(\frac{1}{2}x\right) - 2\right|}{\left|2\sqrt{2} + 2\tan\left(\frac{1}{2}x\right) - 2\right|}\right)$$

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="giac")
[Out] -1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) - 2)/abs(2*sqrt(2) + 2*tan(1
/2*x) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos(x) + \sin(x)}\, dx = -\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}\tan\left(\frac{x}{2}\right)}{2}\right)$$

```
[In] int(1/(cos(x) + sin(x)),x)
[Out] -2^(1/2)*atanh(2^(1/2)/2 - (2^(1/2)*tan(x/2))/2)
```

3.114 $\int \frac{x}{4-x^2+\sqrt{4-x^2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{x}{4-x^2+\sqrt{4-x^2}} \, dx = -\log \left(1+\sqrt{4-x^2}\right)$$

```
[Out] -ln(1+(-x^2+4)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2186, 31}

$$\int \frac{x}{4-x^2+\sqrt{4-x^2}} \, dx = -\log \left(\sqrt{4-x^2}+1\right)$$

```
[In] Int[x/(4 - x^2 + Sqrt[4 - x^2]),x]
```

```
[Out] -Log[1 + Sqrt[4 - x^2]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{4+\sqrt{4-x}-x}\,dx, x, x^2\right) \\
&= -\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{4-x^2}\right) \\
&= -\log\left(1+\sqrt{4-x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$
\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx = -\log\left(1+\sqrt{4-x^2}\right)
$$

```
[In] Integrate[x/(4 - x^2 + Sqrt[4 - x^2]),x]
[Out] -Log[1 + Sqrt[4 - x^2]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result
trager	$-\ln\left(-1-\sqrt{-x^2+4}\right)$
default	$-\frac{\ln(x^2-3)}{2}+\frac{\sqrt{-(-2+x)^2-4x+8}-2\arcsin\left(\frac{x}{2}\right)}{2\left(2+\sqrt{3}\right)\left(-2+\sqrt{3}\right)}+\frac{\sqrt{-(2+x)^2+4x+8}+2\arcsin\left(\frac{x}{2}\right)}{2\left(2+\sqrt{3}\right)\left(-2+\sqrt{3}\right)}-\frac{\sqrt{-\left(x-\sqrt{3}\right)^2-2\sqrt{3}\left(x-\sqrt{3}\right)+1}-\sqrt{3}\,\text{ar}}{2\left(2+\right.}$

```
[In] int(x/(4-x^2+(-x^2+4)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -ln(-1-(-x^2+4)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(14) = 28$.

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.44

$$\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx = -\frac{1}{2}\log\left(x^2-3\right)+\frac{1}{2}\log\left(-\frac{x^2+3\sqrt{-x^2+4}-6}{x^2}\right) - \frac{1}{2}\log\left(-\frac{x^2+\sqrt{-x^2+4}-2}{x^2}\right)$$

```
[In] integrate(x/(4-x^2+(-x^2+4)^(1/2)),x, algorithm="fricas")
[Out] -1/2*log(x^2 - 3) + 1/2*log(-(x^2 + 3*sqrt(-x^2 + 4) - 6)/x^2) - 1/2*log(-(
x^2 + sqrt(-x^2 + 4) - 2)/x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(12) = 24$.

Time = 1.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 3.00

$$\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx = \frac{\log\left(2\sqrt{4-x^2}\right)}{2} - \frac{\log\left(2\sqrt{4-x^2}+2\right)}{2} - \frac{\log\left(2x^2-2\sqrt{4-x^2}-8\right)}{2}$$

```
[In] integrate(x/(4-x**2+(-x**2+4)**(1/2)),x)
[Out] log(2*sqrt(4 - x**2))/2 - log(2*sqrt(4 - x**2) + 2)/2 - log(2*x**2 - 2*sqrt
(4 - x**2) - 8)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx = -\log\left(\sqrt{-x^2+4}+1\right)$$

```
[In] integrate(x/(4-x^2+(-x^2+4)^(1/2)),x, algorithm="maxima")
[Out] -log(sqrt(-x^2 + 4) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx = -\log\left(\sqrt{-x^2+4}+1\right)$$

```
[In] integrate(x/(4-x^2+(-x^2+4)^(1/2)),x, algorithm="giac")
[Out] -log(sqrt(-x^2 + 4) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx = -\frac{\ln\left(x-\sqrt{3}\right)}{2} - \frac{\ln\left(\frac{\sqrt{3}\,x\,1\mathrm{i}+\sqrt{4-x^2}\,1\mathrm{i}+4\mathrm{i}}{x+\sqrt{3}}\right)}{2} - \frac{\ln\left(x+\sqrt{3}\right)}{2} - \frac{\ln\left(\frac{-\sqrt{3}\,x\,1\mathrm{i}+\sqrt{4-x^2}\,1\mathrm{i}+4\mathrm{i}}{x-\sqrt{3}}\right)}{2}$$

```
[In] int(x/((4 - x^2)^(1/2) - x^2 + 4),x)
[Out] - log(x - 3^(1/2))/2 - log((3^(1/2)*x*1i + (4 - x^2)^(1/2)*1i + 4i)/(x + 3^
(1/2)))/2 - log(x + 3^(1/2))/2 - log(((4 - x^2)^(1/2)*1i - 3^(1/2)*x*1i + 4
i)/(x - 3^(1/2)))/2
```

3.115 $\int \frac{3+2x}{(-2+x)(5+x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log(2-x)+\log(5+x)$$

```
[Out] ln(2-x)+ln(5+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {78}

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log(2-x)+\log(x+5)$$

```
[In] Int[(3 + 2*x)/((-2 + x)*(5 + x)),x]
[Out] Log[2 - x] + Log[5 + x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{-2+x}+\frac{1}{5+x}\right)\,dx \\ &= \log(2-x)+\log(5+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log(-2+x)+\log(5+x)$$

```
[In] Integrate[(3 + 2*x)/((-2 + x)*(5 + x)),x]
[Out] Log[-2 + x] + Log[5 + x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
default	$\ln\left(\left(-2+x\right)\left(5+x\right)\right)$	9
norman	$\ln\left(-2+x\right)+\ln\left(5+x\right)$	10
risch	$\ln\left(x^{2}+3x-10\right)$	10
parallelrisch	$\ln\left(-2+x\right)+\ln\left(5+x\right)$	10

```
[In] int((3+2*x)/(-2+x)/(5+x),x,method=_RETURNVERBOSE)
[Out] ln((-2+x)*(5+x))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log\left(x^2+3\,x-10\right)$$

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="fricas")
[Out] log(x^2 + 3*x - 10)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log\left(x^2+3x-10\right)$$

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x)
[Out] log(x**2 + 3*x - 10)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log(x+5) + \log(x-2)$$

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="maxima")
[Out] log(x + 5) + log(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log(|x+5|) + \log(|x-2|)$$

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="giac")
[Out] log(abs(x + 5)) + log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \ln\left(x^2+3\,x-10\right)$$

```
[In] int((2*x + 3)/((x - 2)*(x + 5)),x)
[Out] log(3*x + x^2 - 10)
```

3.116 $\int \frac{x}{(1+x)(2+x)(3+x)} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{x}{(1+x)(2+x)(3+x)} \, dx = -\frac{1}{2}\log(1+x) + 2\log(2+x) - \frac{3}{2}\log(3+x)$$

```
[Out] -1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {153}

$$\int \frac{x}{(1+x)(2+x)(3+x)} \, dx = -\frac{1}{2}\log(x+1) + 2\log(x+2) - \frac{3}{2}\log(x+3)$$

```
[In] Int[x/((1 + x)*(2 + x)*(3 + x)),x]
[Out] -1/2*Log[1 + x] + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\frac{1}{2(1+x)} + \frac{2}{2+x} - \frac{3}{2(3+x)}\right) dx \\ &= -\frac{1}{2}\log(1+x) + 2\log(2+x) - \frac{3}{2}\log(3+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{1}{2}\log(1+x)+2\log(2+x)-\frac{3}{2}\log(3+x)$$

```
[In] Integrate[x/((1 + x)*(2 + x)*(3 + x)),x]
[Out] -1/2*Log[1 + x] + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\ln(1+x)}{2}+2\ln\left(2+x\right)-\frac{3\ln(3+x)}{2}$	20
norman	$-\frac{\ln(1+x)}{2}+2\ln\left(2+x\right)-\frac{3\ln(3+x)}{2}$	20
risch	$-\frac{\ln(1+x)}{2}+2\ln\left(2+x\right)-\frac{3\ln(3+x)}{2}$	20
parallelrisch	$-\frac{\ln(1+x)}{2}+2\ln\left(2+x\right)-\frac{3\ln(3+x)}{2}$	20

```
[In] int(x/(1+x)/(2+x)/(3+x),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{3}{2}\,\log\left(x+3\right)+2\,\log\left(x+2\right)-\frac{1}{2}\,\log\left(x+1\right)$$

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="fricas")
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{\log(x+1)}{2} + 2\log(x+2) - \frac{3\log(x+3)}{2}$$

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x)
[Out] -log(x + 1)/2 + 2*log(x + 2) - 3*log(x + 3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{3}{2}\log(x+3) + 2\log(x+2) - \frac{1}{2}\log(x+1)$$

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="maxima")
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{3}{2}\log(|x+3|) + 2\log(|x+2|) - \frac{1}{2}\log(|x+1|)$$

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="giac")
[Out] -3/2*log(abs(x + 3)) + 2*log(abs(x + 2)) - 1/2*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = 2\ln(x+2) - \frac{\ln(x+1)}{2} - \frac{3\ln(x+3)}{2}$$

```
[In] int(x/((x + 1)*(x + 2)*(x + 3)),x)
[Out] 2*log(x + 2) - log(x + 1)/2 - (3*log(x + 3))/2
```

3.117 $\int \frac{x}{2-3x+x^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{x}{2-3x+x^3} dx = \frac{1}{3(1-x)} + \frac{2}{9}\log(1-x) - \frac{2}{9}\log(2+x)$$

```
[Out] 1/3/(1-x)+2/9*ln(1-x)-2/9*ln(2+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2099}

$$\int \frac{x}{2-3x+x^3} dx = \frac{1}{3(1-x)} + \frac{2}{9}\log(1-x) - \frac{2}{9}\log(x+2)$$

```
[In] Int[x/(2 - 3*x + x^3),x]
[Out] 1/(3*(1 - x)) + (2*Log[1 - x])/9 - (2*Log[2 + x])/9
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{3(-1+x)^2} + \frac{2}{9(-1+x)} - \frac{2}{9(2+x)}\right) dx \\ &= \frac{1}{3(1-x)} + \frac{2}{9}\log(1-x) - \frac{2}{9}\log(2+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{x}{2-3x+x^3}\,dx = -\frac{1}{3(-1+x)}+\frac{2}{9}\log(1-x)-\frac{2}{9}\log(2+x)$$

```
[In] Integrate[x/(2 - 3*x + x^3),x]
[Out] -1/3*1/(-1 + x) + (2*Log[1 - x])/9 - (2*Log[2 + x])/9
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{1}{3(-1+x)}+\frac{2\ln(-1+x)}{9}-\frac{2\ln(2+x)}{9}$	21
norman	$-\frac{1}{3(-1+x)}+\frac{2\ln(-1+x)}{9}-\frac{2\ln(2+x)}{9}$	21
risch	$-\frac{1}{3(-1+x)}+\frac{2\ln(-1+x)}{9}-\frac{2\ln(2+x)}{9}$	21
parallelrisch	$\frac{2\ln(-1+x)x-2\ln(2+x)x-3-2\ln(-1+x)+2\ln(2+x)}{-9+9x}$	36

```
[In] int(x/(x^3-3*x+2),x,method=_RETURNVERBOSE)
[Out] -1/3/(-1+x)+2/9*ln(-1+x)-2/9*ln(2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{x}{2-3x+x^3}\,dx = -\frac{2\,(x-1)\log\left(x+2\right)-2\,(x-1)\log\left(x-1\right)+3}{9\,(x-1)}$$

```
[In] integrate(x/(x^3-3*x+2),x, algorithm="fricas")
[Out] -1/9*(2*(x - 1)*log(x + 2) - 2*(x - 1)*log(x - 1) + 3)/(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x}{2-3x+x^3}\,dx = \frac{2\log{(x-1)}}{9} - \frac{2\log{(x+2)}}{9} - \frac{1}{3x-3}$$

```
[In] integrate(x/(x**3-3*x+2),x)
[Out] 2*log(x - 1)/9 - 2*log(x + 2)/9 - 1/(3*x - 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x}{2-3x+x^3}\,dx = -\frac{1}{3(x-1)} - \frac{2}{9}\log(x+2) + \frac{2}{9}\log(x-1)$$

```
[In] integrate(x/(x^3-3*x+2),x, algorithm="maxima")
[Out] -1/3/(x - 1) - 2/9*log(x + 2) + 2/9*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x}{2-3x+x^3}\,dx = -\frac{1}{3(x-1)} - \frac{2}{9}\log(|x+2|) + \frac{2}{9}\log(|x-1|)$$

```
[In] integrate(x/(x^3-3*x+2),x, algorithm="giac")
[Out] -1/3/(x - 1) - 2/9*log(abs(x + 2)) + 2/9*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{x}{2-3x+x^3}\,dx = -\frac{4\,\mathrm{atanh}\left(\frac{2x}{3}+\frac{1}{3}\right)}{9} - \frac{1}{3\,(x-1)}$$

```
[In] int(x/(x^3 - 3*x + 2),x)
[Out] - (4*atanh((2*x)/3 + 1/3))/9 - 1/(3*(x - 1))
```

3.118 $\int \frac{-6+2x+x^4}{-2x+x^2+x^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \frac{-6+2x+x^4}{-2x+x^2+x^3} \, dx = -x+\frac{x^2}{2}-\log(1-x)+3\log(x)+\log(2+x)$$

```
[Out] -x+1/2*x^2-ln(1-x)+3*ln(x)+ln(2+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1608, 1642}

$$\int \frac{-6+2x+x^4}{-2x+x^2+x^3} \, dx = \frac{x^2}{2}-x-\log(1-x)+3\log(x)+\log(x+2)$$

```
[In] Int[(-6 + 2*x + x^4)/(-2*x + x^2 + x^3),x]
```

```
[Out] -x + x^2/2 - Log[1 - x] + 3*Log[x] + Log[2 + x]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{-6+2x+x^4}{x\left(-2+x+x^2\right)}\,dx\\ &= \int \left(-1+\frac{1}{1-x}+\frac{3}{x}+x+\frac{1}{2+x}\right)dx\\ &= -x+\frac{x^2}{2}-\log(1-x)+3\log(x)+\log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{-6+2x+x^4}{-2x+x^2+x^3}\,dx=-x+\frac{x^2}{2}-\log(1-x)+3\log(x)+\log(2+x)$$

```
[In] Integrate[(-6 + 2*x + x^4)/(-2*x + x^2 + x^3),x]
[Out] -x + x^2/2 - Log[1 - x] + 3*Log[x] + Log[2 + x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-x+\frac{x^2}{2}-\ln\left(-1+x\right)+\ln\left(2+x\right)+3\ln\left(x\right)$	24
norman	$-x+\frac{x^2}{2}-\ln\left(-1+x\right)+\ln\left(2+x\right)+3\ln\left(x\right)$	24
risch	$-x+\frac{x^2}{2}-\ln\left(-1+x\right)+\ln\left(2+x\right)+3\ln\left(x\right)$	24
parallelrisch	$-x+\frac{x^2}{2}-\ln\left(-1+x\right)+\ln\left(2+x\right)+3\ln\left(x\right)$	24

```
[In] int((x^4+2*x-6)/(x^3+x^2-2*x),x,method=_RETURNVERBOSE)
[Out] -x+1/2*x^2-ln(-1+x)+ln(2+x)+3*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{-6+2x+x^4}{-2x+x^2+x^3}\,dx = \frac{1}{2}\,x^2 - x + \log\left(x+2\right) - \log\left(x-1\right) + 3\,\log\left(x\right)$$

```
[In] integrate((x^4+2*x-6)/(x^3+x^2-2*x),x, algorithm="fricas")
[Out] 1/2*x^2 - x + log(x + 2) - log(x - 1) + 3*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{-6+2x+x^4}{-2x+x^2+x^3}\,dx = \frac{x^2}{2} - x + 3\log\left(x\right) - \log\left(x-1\right) + \log\left(x+2\right)$$

```
[In] integrate((x**4+2*x-6)/(x**3+x**2-2*x),x)
[Out] x**2/2 - x + 3*log(x) - log(x - 1) + log(x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{-6+2x+x^4}{-2x+x^2+x^3}\,dx = \frac{1}{2}\,x^2 - x + \log\left(x+2\right) - \log\left(x-1\right) + 3\,\log\left(x\right)$$

```
[In] integrate((x^4+2*x-6)/(x^3+x^2-2*x),x, algorithm="maxima")
[Out] 1/2*x^2 - x + log(x + 2) - log(x - 1) + 3*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{-6+2x+x^4}{-2x+x^2+x^3}\,dx = \frac{1}{2}\,x^2 - x + \log\left(|x+2|\right) - \log\left(|x-1|\right) + 3\,\log\left(|x|\right)$$

```
[In] integrate((x^4+2*x-6)/(x^3+x^2-2*x),x, algorithm="giac")
[Out] 1/2*x^2 - x + log(abs(x + 2)) - log(abs(x - 1)) + 3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{-6+2x+x^4}{-2x+x^2+x^3}\,dx = 3\,\ln\left(x\right)-x+\frac{x^2}{2}+\mathrm{atan}\left(\frac{192\mathrm{i}}{7\,\left(28\,x-40\right)}+\frac{9}{7}\mathrm{i}\right)\,2\mathrm{i}$$

```
[In] int((2*x + x^4 - 6)/(x^2 - 2*x + x^3),x)
[Out] atan(192i/(7*(28*x - 40)) + 9i/7)*2i - x + 3*log(x) + x^2/2
```

3.119 $\int \frac{7+8x^3}{(1+x)(1+2x)^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \frac{7+8x^3}{(1+x)(1+2x)^3} \, dx = -\frac{3}{(1+2x)^2}+\frac{3}{1+2x}+\log(1+x)$$

```
[Out] -3/(1+2*x)^2+3/(1+2*x)+ln(1+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1634}

$$\int \frac{7+8x^3}{(1+x)(1+2x)^3} \, dx = \frac{3}{2x+1}-\frac{3}{(2x+1)^2}+\log(x+1)$$

```
[In] Int[(7 + 8*x^3)/((1 + x)*(1 + 2*x)^3),x]
```

```
[Out] -3/(1 + 2*x)^2 + 3/(1 + 2*x) + Log[1 + x]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{1+x}+\frac{12}{(1+2x)^3}-\frac{6}{(1+2x)^2}\right) \, dx \\ &= -\frac{3}{(1+2x)^2}+\frac{3}{1+2x}+\log(1+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{7+8x^3}{(1+x)(1+2x)^3}\,dx=\frac{6x+(1+2x)^2\log(1+x)}{(1+2x)^2}$$

```
[In] Integrate[(7 + 8*x^3)/((1 + x)*(1 + 2*x)^3),x]
[Out] (6*x + (1 + 2*x)^2*Log[1 + x])/(1 + 2*x)^2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
norman	$\frac{6x}{(1+2x)^2}+\ln(1+x)$	16
risch	$\frac{6x}{(1+2x)^2}+\ln(1+x)$	16
default	$-\frac{3}{(1+2x)^2}+\frac{3}{1+2x}+\ln(1+x)$	24
parallelrisch	$\frac{4\ln(1+x)x^2+4\ln(1+x)x+\ln(1+x)+6x}{(1+2x)^2}$	33

```
[In] int((8*x^3+7)/(1+x)/(1+2*x)^3,x,method=_RETURNVERBOSE)
[Out] 6*x/(1+2*x)^2+ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{7+8x^3}{(1+x)(1+2x)^3}\,dx=\frac{(4\,x^2+4\,x+1)\log(x+1)+6\,x}{4\,x^2+4\,x+1}$$

```
[In] integrate((8*x^3+7)/(1+x)/(1+2*x)^3,x, algorithm="fricas")
[Out] ((4*x^2 + 4*x + 1)*log(x + 1) + 6*x)/(4*x^2 + 4*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{7+8x^3}{(1+x)(1+2x)^3}\,dx = \frac{6x}{4x^2+4x+1} + \log{(x+1)}$$

```
[In] integrate((8*x**3+7)/(1+x)/(1+2*x)**3,x)
[Out] 6*x/(4*x**2 + 4*x + 1) + log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{7+8x^3}{(1+x)(1+2x)^3}\,dx = \frac{6\,x}{4\,x^2+4\,x+1} + \log{(x+1)}$$

```
[In] integrate((8*x^3+7)/(1+x)/(1+2*x)^3,x, algorithm="maxima")
[Out] 6*x/(4*x^2 + 4*x + 1) + log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{7+8x^3}{(1+x)(1+2x)^3}\,dx = \frac{6\,x}{(2\,x+1)^2} + \log{(|x+1|)}$$

```
[In] integrate((8*x^3+7)/(1+x)/(1+2*x)^3,x, algorithm="giac")
[Out] 6*x/(2*x + 1)^2 + log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{7+8x^3}{(1+x)(1+2x)^3}\,dx = \ln{(x+1)} + \frac{6\,x}{(2\,x+1)^2}$$

```
[In] int((8*x^3 + 7)/((2*x + 1)^3*(x + 1)),x)
[Out] log(x + 1) + (6*x)/(2*x + 1)^2
```

3.120 $\int \frac{1+x+4x^2}{-1+x^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1+x+4x^2}{-1+x^3} \, dx = 2\log(1-x)+\log\left(1+x+x^2\right)$$

```
[Out] 2*ln(1-x)+ln(x^2+x+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1889, 31, 642}

$$\int \frac{1+x+4x^2}{-1+x^3} \, dx = \log\left(x^2+x+1\right)+2\log(1-x)$$

```
[In] Int[(1 + x + 4*x^2)/(-1 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[1 + x + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1889

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Dist[q*((A + B*q
 + C*q^2)/(3*a)), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q -
 C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b
*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
 && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{3}\int \frac{-3-6x}{1+x+x^2}\,dx\right)-2\int \frac{1}{1-x}\,dx\\ &= 2\log(1-x)+\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1+x+4x^2}{-1+x^3}\,dx = 2\log(1-x)+\log\left(1+x+x^2\right)$$

```
[In] Integrate[(1 + x + 4*x^2)/(-1 + x^3),x]
[Out] 2*Log[1 - x] + Log[1 + x + x^2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$2\ln(-1+x)+\ln(x^2+x+1)$
norman	$2\ln(-1+x)+\ln(x^2+x+1)$
risch	$2\ln(-1+x)+\ln(x^2+x+1)$
parallelrisch	$2\ln(-1+x)+\ln(x^2+x+1)$
meijerg	$\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{1}{3}}}+\frac{4\ln(-x^3+1)}{3}+\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}\ldots\right.}{2}\ldots\right.}{3\ldots}$

```
[In] int((4*x^2+x+1)/(x^3-1),x,method=_RETURNVERBOSE)
[Out] 2*ln(-1+x)+ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x+4x^2}{-1+x^3}\,dx = \log\left(x^2+x+1\right)+2\,\log\left(x-1\right)$$

```
[In] integrate((4*x^2+x+1)/(x^3-1),x, algorithm="fricas")
[Out] log(x^2 + x + 1) + 2*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x+4x^2}{-1+x^3}\,dx = 2\log\left(x-1\right)+\log\left(x^2+x+1\right)$$

```
[In] integrate((4*x**2+x+1)/(x**3-1),x)
[Out] 2*log(x - 1) + log(x**2 + x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x+4x^2}{-1+x^3}\,dx = \log\left(x^2+x+1\right)+2\,\log\left(x-1\right)$$

```
[In] integrate((4*x^2+x+1)/(x^3-1),x, algorithm="maxima")
[Out] log(x^2 + x + 1) + 2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1+x+4x^2}{-1+x^3}\,dx = \log\left(x^2+x+1\right)+2\,\log\left(|x-1|\right)$$

```
[In] integrate((4*x^2+x+1)/(x^3-1),x, algorithm="giac")
[Out] log(x^2 + x + 1) + 2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x+4x^2}{-1+x^3}\,dx = \ln\left(x^2+x+1\right)+2\,\ln\left(x-1\right)$$

```
[In] int((x + 4*x^2 + 1)/(x^3 - 1),x)
[Out] log(x + x^2 + 1) + 2*log(x - 1)
```

3.121 $\int \frac{x^4}{4+5x^2+x^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 18

$$\int \frac{x^4}{4+5x^2+x^4} \, dx = x-\frac{8}{3}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{3}$$

```
[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1136, 1180, 209}

$$\int \frac{x^4}{4+5x^2+x^4} \, dx = -\frac{8}{3}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{3}+x$$

```
[In] Int[x^4/(4 + 5*x^2 + x^4),x]
[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
```

```
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x-\int \frac{4+5x^2}{4+5x^2+x^4}\,dx\\ &= x+\frac{1}{3}\int \frac{1}{1+x^2}\,dx-\frac{16}{3}\int \frac{1}{4+x^2}\,dx\\ &= x-\frac{8}{3}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{4+5x^2+x^4}\,dx=x+\frac{8}{3}\arctan\left(\frac{2}{x}\right)+\frac{\arctan(x)}{3}$$

```
[In] Integrate[x^4/(4 + 5*x^2 + x^4),x]
[Out] x + (8*ArcTan[2/x])/3 + ArcTan[x]/3
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

method	result	size
default	$x-\frac{8\arctan\left(\frac{x}{2}\right)}{3}+\frac{\arctan(x)}{3}$	13
risch	$x-\frac{8\arctan\left(\frac{x}{2}\right)}{3}+\frac{\arctan(x)}{3}$	13
parallelrisch	$x+\frac{i\ln(x+i)}{6}-\frac{i\ln(x-i)}{6}+\frac{4i\ln(x-2i)}{3}-\frac{4i\ln(x+2i)}{3}$	35

```
[In] int(x^4/(x^4+5*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="fricas")
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}(x)}{3}$$

```
[In] integrate(x**4/(x**4+5*x**2+4),x)
[Out] x - 8*atan(x/2)/3 + atan(x)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="maxima")
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8}{3}\arctan\left(\frac{1}{2}\,x\right) + \frac{1}{3}\arctan\left(x\right)$$

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="giac")
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}(x)}{3}$$

```
[In] int(x^4/(5*x^2 + x^4 + 4),x)
[Out] x - (8*atan(x/2))/3 + atan(x)/3
```

3.122 $\int \frac{2+x}{x+x^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{2+x}{x+x^2} dx = 2\log(x) - \log(1+x)$$

```
[Out] 2*ln(x)-ln(1+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {645}

$$\int \frac{2+x}{x+x^2} dx = 2\log(x) - \log(x+1)$$

```
[In] Int[(2 + x)/(x + x^2),x]
```

```
[Out] 2*Log[x] - Log[1 + x]
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{-1-x} + \frac{2}{x}\right) dx \\ &= 2\log(x) - \log(1+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{x+x^2}\,dx = 2\log(x) - \log(1+x)$$

```
[In] Integrate[(2 + x)/(x + x^2),x]
[Out] 2*Log[x] - Log[1 + x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$2\ln(x)-\ln(1+x)$	12
norman	$2\ln(x)-\ln(1+x)$	12
meijerg	$2\ln(x)-\ln(1+x)$	12
risch	$2\ln(x)-\ln(1+x)$	12
parallelrisch	$2\ln(x)-\ln(1+x)$	12

```
[In] int((2+x)/(x^2+x),x,method=_RETURNVERBOSE)
[Out] 2*ln(x)-ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{x+x^2}\,dx = -\log(x+1) + 2\log(x)$$

```
[In] integrate((2+x)/(x^2+x),x, algorithm="fricas")
[Out] -log(x + 1) + 2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{2+x}{x+x^2} \, dx = 2 \log(x) - \log(x+1)$$

```
[In] integrate((2+x)/(x**2+x),x)
[Out] 2*log(x) - log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{x+x^2} \, dx = -\log(x+1) + 2 \log(x)$$

```
[In] integrate((2+x)/(x^2+x),x, algorithm="maxima")
[Out] -log(x + 1) + 2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{2+x}{x+x^2} \, dx = -\log(|x+1|) + 2 \log(|x|)$$

```
[In] integrate((2+x)/(x^2+x),x, algorithm="giac")
[Out] -log(abs(x + 1)) + 2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{x+x^2} \, dx = 2 \ln(x) - \ln(x+1)$$

```
[In] int((x + 2)/(x + x^2),x)
[Out] 2*log(x) - log(x + 1)
```

3.123 $\int \frac{1}{x(1+x^2)^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{1}{x(1+x^2)^2} \, dx = \frac{1}{2(1+x^2)} + \log(x) - \frac{1}{2}\log(1+x^2)$$

```
[Out] 1/2/(x^2+1)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {272, 46}

$$\int \frac{1}{x(1+x^2)^2} \, dx = \frac{1}{2(x^2+1)} - \frac{1}{2}\log(x^2+1) + \log(x)$$

```
[In] Int[1/(x*(1 + x^2)^2),x]
```

```
[Out] 1/(2*(1 + x^2)) + Log[x] - Log[1 + x^2]/2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{1}{x}-\frac{1}{(1+x)^2}\right)\,dx, x, x^2\right) \\
&= \frac{1}{2(1+x^2)}+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+x^2\right)^2}\,dx = \frac{1}{2(1+x^2)}+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[1/(x*(1 + x^2)^2),x]
[Out] 1/(2*(1 + x^2)) + Log[x] - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{1}{2x^2+2}+\ln\left(x\right)-\frac{\ln\left(x^2+1\right)}{2}$	21
norman	$\frac{1}{2x^2+2}+\ln\left(x\right)-\frac{\ln\left(x^2+1\right)}{2}$	21
risch	$\frac{1}{2x^2+2}+\ln\left(x\right)-\frac{\ln\left(x^2+1\right)}{2}$	21
meijerg	$\frac{1}{2}+\ln\left(x\right)-\frac{x^2}{2x^2+2}-\frac{\ln\left(x^2+1\right)}{2}$	27
parallelrisch	$\frac{2x^2\ln(x)-\ln(x^2+1)x^2+1+2\ln(x)-\ln(x^2+1)}{2x^2+2}$	42

```
[In] int(1/x/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/2/(x^2+1)+ln(x)-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\left(1+x^2\right)^2}\,dx = -\frac{\left(x^2+1\right)\log\left(x^2+1\right)-2\left(x^2+1\right)\log\left(x\right)-1}{2\left(x^2+1\right)}$$

```
[In] integrate(1/x/(x^2+1)^2,x, algorithm="fricas")
[Out] -1/2*((x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x) - 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\left(1+x^2\right)^2}\,dx = \log\left(x\right)-\frac{\log\left(x^2+1\right)}{2}+\frac{1}{2x^2+2}$$

```
[In] integrate(1/x/(x**2+1)**2,x)
[Out] log(x) - log(x**2 + 1)/2 + 1/(2*x**2 + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+x^2\right)^2}\,dx = \frac{1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x^2\right)$$

```
[In] integrate(1/x/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/2/(x^2 + 1) - 1/2*log(x^2 + 1) + 1/2*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\left(1+x^2\right)^2}\,dx = \frac{x^2+2}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x^2\right)$$

```
[In] integrate(1/x/(x^2+1)^2,x, algorithm="giac")
[Out] 1/2*(x^2 + 2)/(x^2 + 1) - 1/2*log(x^2 + 1) + 1/2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(1+x^2\right)^2}\,dx = \ln\left(x\right) - \frac{\ln\left(x^2+1\right)}{2} + \frac{1}{2\,\left(x^2+1\right)}$$

```
[In] int(1/(x*(x^2 + 1)^2),x)
[Out] log(x) - log(x^2 + 1)/2 + 1/(2*(x^2 + 1))
```

3.124 $\int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx = \frac{1}{2+x} + \frac{1}{4(3+x)^2} + \frac{5}{4(3+x)} + \frac{1}{8}\log(1+x) + 2\log(2+x) - \frac{17}{8}\log(3+x)$$

```
[Out] 1/(2+x)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(2+x)-17/8*ln(3+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {90}

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx = \frac{1}{x+2} + \frac{5}{4(x+3)} + \frac{1}{4(x+3)^2} + \frac{1}{8}\log(x+1) + 2\log(x+2) - \frac{17}{8}\log(x+3)$$

```
[In] Int[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2*Log[2 + x
] - (17*Log[3 + x])/8
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{8(1+x)} - \frac{1}{(2+x)^2} + \frac{2}{2+x} - \frac{1}{2(3+x)^3} - \frac{5}{4(3+x)^2} - \frac{17}{8(3+x)}\right) dx \\ &= \frac{1}{2+x} + \frac{1}{4(3+x)^2} + \frac{5}{4(3+x)} + \frac{1}{8}\log(1+x) + 2\log(2+x) - \frac{17}{8}\log(3+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{1}{8}\left(\frac{8}{2+x} + \frac{2}{(3+x)^2} + \frac{10}{3+x} + \log(-1-x) + 16\log(2+x) - 17\log(3+x)\right)$$

```
[In] Integrate[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
[Out] (8/(2 + x) + 2/(3 + x)^2 + 10/(3 + x) + Log[-1 - x] + 16*Log[2 + x] - 17*Lo
g[3 + x])/8
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result
default	$\frac{1}{2+x}+\frac{1}{4(3+x)^2}+\frac{5}{4(3+x)}+\frac{\ln(1+x)}{8}+2\ln(2+x)-\frac{17\ln(3+x)}{8}$
norman	$\frac{\frac{9}{4}x^2+\frac{25}{2}x+17}{(2+x)(3+x)^2}+\frac{\ln(1+x)}{8}+2\ln(2+x)-\frac{17\ln(3+x)}{8}$
risch	$\frac{\frac{9}{4}x^2+\frac{25}{2}x+17}{(2+x)(3+x)^2}+\frac{\ln(1+x)}{8}+2\ln(2+x)-\frac{17\ln(3+x)}{8}$
parallelrisch	$\frac{\ln(1+x)x^3+16\ln(2+x)x^3-17\ln(3+x)x^3+136+8\ln(1+x)x^2+128\ln(2+x)x^2-136\ln(3+x)x^2+21\ln(1+x)x+336\ln(2+x)x-3}{8(2+x)(3+x)^2}$

```
[In] int(1/(1+x)/(2+x)^2/(3+x)^3,x,method=_RETURNVERBOSE)
[Out] 1/(2+x)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(2+x)-17/8*ln(3+x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(38) = 76$.

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{18\,x^2 - 17\,(x^3 + 8\,x^2 + 21\,x + 18)\log(x+3) + 16\,(x^3 + 8\,x^2 + 21\,x + 18)\log(x+2) + (x^3 + 8\,x^2 + 21\,x +}{8\,(x^3 + 8\,x^2 + 21\,x + 18)}$$

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(18*x^2 - 17*(x^3 + 8*x^2 + 21*x + 18)*log(x + 3) + 16*(x^3 + 8*x^2 + 2
1*x + 18)*log(x + 2) + (x^3 + 8*x^2 + 21*x + 18)*log(x + 1) + 100*x + 136)/
(x^3 + 8*x^2 + 21*x + 18)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{9x^2 + 50x + 68}{4x^3 + 32x^2 + 84x + 72} + \frac{\log(x+1)}{8} + 2\log(x+2) - \frac{17\log(x+3)}{8}$$

```
[In] integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)
```

```
[Out] (9*x**2 + 50*x + 68)/(4*x**3 + 32*x**2 + 84*x + 72) + log(x + 1)/8 + 2*log(
x + 2) - 17*log(x + 3)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{9\,x^2 + 50\,x + 68}{4\,(x^3 + 8\,x^2 + 21\,x + 18)} - \frac{17}{8}\,\log(x+3) + 2\,\log(x+2) + \frac{1}{8}\,\log(x+1)$$

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^2 + 50*x + 68)/(x^3 + 8*x^2 + 21*x + 18) - 17/8*log(x + 3) + 2*log
(x + 2) + 1/8*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{1}{x+2} - \frac{\frac{7}{x+2}+6}{4\left(\frac{1}{x+2}+1\right)^2} + \frac{1}{8}\log\left(\left|-\frac{1}{x+2}+1\right|\right) - \frac{17}{8}\log\left(\left|-\frac{1}{x+2}-1\right|\right)$$

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="giac")
```

```
[Out] 1/(x + 2) - 1/4*(7/(x + 2) + 6)/(1/(x + 2) + 1)^2 + 1/8*log(abs(-1/(x + 2)
+ 1)) - 17/8*log(abs(-1/(x + 2) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{\ln(x+1)}{8} + 2\,\ln(x+2) - \frac{17\,\ln(x+3)}{8} + \frac{\frac{9\,x^2}{4}+\frac{25\,x}{2}+17}{x^3+8\,x^2+21\,x+18}$$

```
[In] int(1/((x + 1)*(x + 2)^2*(x + 3)^3),x)
```

```
[Out] log(x + 1)/8 + 2*log(x + 2) - (17*log(x + 3))/8 + ((25*x)/2 + (9*x^2)/4 + 1
7)/(21*x + 8*x^2 + x^3 + 18)
```

3.125 $\int \frac{x}{(1+x)^2}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{x}{(1+x)^2}\,dx = \frac{1}{1+x} + \log(1+x)$$

```
[Out] 1/(1+x)+ln(1+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{45\}$

$$\int \frac{x}{(1+x)^2}\,dx = \frac{1}{x+1} + \log(x+1)$$

```
[In] Int[x/(1 + x)^2,x]
```

```
[Out] (1 + x)^(-1) + Log[1 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\frac{1}{(1+x)^2} + \frac{1}{1+x}\right) dx \\ &= \frac{1}{1+x} + \log(1+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)^2}\,dx = \frac{1}{1+x} + \log(1+x)$$

```
[In] Integrate[x/(1 + x)^2,x]
[Out] (1 + x)^(-1) + Log[1 + x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{1}{1+x}+\ln\left(1+x\right)$	11
norman	$\frac{1}{1+x}+\ln\left(1+x\right)$	11
risch	$\frac{1}{1+x}+\ln\left(1+x\right)$	11
meijerg	$-\frac{x}{1+x}+\ln\left(1+x\right)$	14
parallelrisch	$\frac{\ln(1+x)x+1+\ln(1+x)}{1+x}$	19

```
[In] int(x/(1+x)^2,x,method=_RETURNVERBOSE)
[Out] 1/(1+x)+ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{x}{(1+x)^2}\,dx = \frac{(x+1)\log\left(x+1\right)+1}{x+1}$$

```
[In] integrate(x/(1+x)^2,x, algorithm="fricas")
[Out] ((x + 1)*log(x + 1) + 1)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x)^2} \, dx = \log(x+1) + \frac{1}{x+1}$$

```
[In] integrate(x/(1+x)**2,x)
[Out] log(x + 1) + 1/(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)^2} \, dx = \frac{1}{x+1} + \log(x+1)$$

```
[In] integrate(x/(1+x)^2,x, algorithm="maxima")
[Out] 1/(x + 1) + log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x}{(1+x)^2} \, dx = \frac{1}{x+1} + \log(|x+1|)$$

```
[In] integrate(x/(1+x)^2,x, algorithm="giac")
[Out] 1/(x + 1) + log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)^2} \, dx = \ln(x+1) + \frac{1}{x+1}$$

```
[In] int(x/(x + 1)^2,x)
[Out] log(x + 1) + 1/(x + 1)
```

3.126 $\int \frac{1}{-x+x^3} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{1}{-x+x^3} \, dx = -\log(x) + \frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] -ln(x)+1/2*ln(-x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1607, 272, 36, 31, 29}

$$\int \frac{1}{-x+x^3} \, dx = \frac{1}{2}\log\left(1-x^2\right) - \log(x)$$

```
[In] Int[(-x + x^3)^(-1),x]
```

```
[Out] -Log[x] + Log[1 - x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x\left(-1+x^2\right)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)x}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) \\
&= -\log(x) + \frac{1}{2}\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x+x^3}\,dx = -\log(x) + \frac{1}{2}\log\left(1-x^2\right)$$

```
[In] Integrate[(-x + x^3)^(-1),x]
[Out] -Log[x] + Log[1 - x^2]/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
risch	$-\ln(x)+\frac{\ln(x^2-1)}{2}$	14
default	$\frac{\ln(-1+x)}{2}-\ln(x)+\frac{\ln(1+x)}{2}$	18
norman	$\frac{\ln(-1+x)}{2}-\ln(x)+\frac{\ln(1+x)}{2}$	18
parallelrisch	$\frac{\ln(-1+x)}{2}-\ln(x)+\frac{\ln(1+x)}{2}$	18
meijerg	$-\ln(x)-\frac{i\pi}{2}+\frac{\ln(-x^2+1)}{2}$	20

```
[In] int(1/(x^3-x),x,method=_RETURNVERBOSE)
[Out] -ln(x)+1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{-x+x^3}\,dx=\frac{1}{2}\log\left(x^2-1\right)-\log(x)$$

```
[In] integrate(1/(x^3-x),x, algorithm="fricas")
[Out] 1/2*log(x^2 - 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{1}{-x+x^3}\,dx=-\log(x)+\frac{\log(x^2-1)}{2}$$

```
[In] integrate(1/(x**3-x),x)
[Out] -log(x) + log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x+x^3}\,dx = \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1) - \log(x)$$

```
[In] integrate(1/(x^3-x),x, algorithm="maxima")
[Out] 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{-x+x^3}\,dx = -\frac{1}{2}\log(x^2) + \frac{1}{2}\log(|x^2-1|)$$

```
[In] integrate(1/(x^3-x),x, algorithm="giac")
[Out] -1/2*log(x^2) + 1/2*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{-x+x^3}\,dx = \frac{\ln(x^2-1)}{2} - \ln(x)$$

```
[In] int(-1/(x - x^3),x)
[Out] log(x^2 - 1)/2 - log(x)
```

3.127 $\int \frac{x^2}{-6+x+x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{x^2}{-6+x+x^2} \, dx = x + \frac{4}{5} \log(2-x) - \frac{9}{5} \log(3+x)$$

```
[Out] x+4/5*ln(2-x)-9/5*ln(3+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {717, 646, 31}

$$\int \frac{x^2}{-6+x+x^2} \, dx = x + \frac{4}{5} \log(2-x) - \frac{9}{5} \log(x+3)$$

```
[In] Int[x^2/(-6 + x + x^2),x]
```

```
[Out] x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, 
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x + \int \frac{6-x}{-6+x+x^2}\,dx\\ &= x + \frac{4}{5}\int \frac{1}{-2+x}\,dx - \frac{9}{5}\int \frac{1}{3+x}\,dx\\ &= x + \frac{4}{5}\log(2-x) - \frac{9}{5}\log(3+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{-6+x+x^2}\,dx = x + \frac{4}{5}\log(2-x) - \frac{9}{5}\log(3+x)$$

```
[In] Integrate[x^2/(-6 + x + x^2),x]
[Out] x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
default	$x-\frac{9\ln(3+x)}{5}+\frac{4\ln(-2+x)}{5}$	15
norman	$x-\frac{9\ln(3+x)}{5}+\frac{4\ln(-2+x)}{5}$	15
risch	$x-\frac{9\ln(3+x)}{5}+\frac{4\ln(-2+x)}{5}$	15
parallelrisch	$x-\frac{9\ln(3+x)}{5}+\frac{4\ln(-2+x)}{5}$	15

```
[In] int(x^2/(x^2+x-6),x,method=_RETURNVERBOSE)
[Out] x-9/5*ln(3+x)+4/5*ln(-2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{-6+x+x^2}\,dx = x - \frac{9}{5}\,\log\left(x+3\right) + \frac{4}{5}\,\log\left(x-2\right)$$

```
[In] integrate(x^2/(x^2+x-6),x, algorithm="fricas")
[Out] x - 9/5*log(x + 3) + 4/5*log(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{-6+x+x^2}\,dx = x + \frac{4\log\left(x-2\right)}{5} - \frac{9\log\left(x+3\right)}{5}$$

```
[In] integrate(x**2/(x**2+x-6),x)
[Out] x + 4*log(x - 2)/5 - 9*log(x + 3)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{-6+x+x^2}\,dx = x - \frac{9}{5}\,\log\left(x+3\right) + \frac{4}{5}\,\log\left(x-2\right)$$

```
[In] integrate(x^2/(x^2+x-6),x, algorithm="maxima")
[Out] x - 9/5*log(x + 3) + 4/5*log(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{-6+x+x^2}\,dx = x - \frac{9}{5}\,\log\left(|x+3|\right) + \frac{4}{5}\,\log\left(|x-2|\right)$$

```
[In] integrate(x^2/(x^2+x-6),x, algorithm="giac")
[Out] x - 9/5*log(abs(x + 3)) + 4/5*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{-6+x+x^2}\,dx = x + \frac{4\,\ln\left(x-2\right)}{5} - \frac{9\,\ln\left(x+3\right)}{5}$$

```
[In] int(x^2/(x + x^2 - 6),x)
[Out] x + (4*log(x - 2))/5 - (9*log(x + 3))/5
```

3.128 $\int \frac{2+x}{4-4x+x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{2+x}{4-4x+x^2}\,dx = \frac{4}{2-x} + \log(2-x)$$

```
[Out] 4/(2-x)+ln(2-x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {27, 45}

$$\int \frac{2+x}{4-4x+x^2}\,dx = \frac{4}{2-x} + \log(2-x)$$

```
[In] Int[(2 + x)/(4 - 4*x + x^2),x]
```

```
[Out] 4/(2 - x) + Log[2 - x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{2+x}{(-2+x)^2}\,dx \\ &= \int \left(\frac{4}{(-2+x)^2}+\frac{1}{-2+x}\right)dx \\ &= \frac{4}{2-x}+\log(2-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2+x}{4-4x+x^2}\,dx = -\frac{4}{-2+x}+\log(-2+x)$$

```
[In] Integrate[(2 + x)/(4 - 4*x + x^2),x]
[Out] -4/(-2 + x) + Log[-2 + x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{4}{-2+x}+\ln\left(-2+x\right)$	13
norman	$-\frac{4}{-2+x}+\ln\left(-2+x\right)$	13
risch	$-\frac{4}{-2+x}+\ln\left(-2+x\right)$	13
meijerg	$\frac{x}{1-\frac{x}{2}}+\ln\left(1-\frac{x}{2}\right)$	17
parallelrisch	$\frac{\ln(-2+x)x-4-2\ln(-2+x)}{-2+x}$	21

```
[In] int((2+x)/(x^2-4*x+4),x,method=_RETURNVERBOSE)
[Out] -4/(-2+x)+ln(-2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{4-4x+x^2}\,dx = \frac{(x-2)\log(x-2)-4}{x-2}$$

```
[In] integrate((2+x)/(x^2-4*x+4),x, algorithm="fricas")
[Out] ((x - 2)*log(x - 2) - 4)/(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{2+x}{4-4x+x^2}\,dx = \log(x-2) - \frac{4}{x-2}$$

```
[In] integrate((2+x)/(x**2-4*x+4),x)
[Out] log(x - 2) - 4/(x - 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2+x}{4-4x+x^2}\,dx = -\frac{4}{x-2} + \log(x-2)$$

```
[In] integrate((2+x)/(x^2-4*x+4),x, algorithm="maxima")
[Out] -4/(x - 2) + log(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{2+x}{4-4x+x^2}\,dx = -\frac{4}{x-2} + \log(|x-2|)$$

```
[In] integrate((2+x)/(x^2-4*x+4),x, algorithm="giac")
[Out] -4/(x - 2) + log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2+x}{4-4x+x^2} \, dx = \ln(x-2) - \frac{4}{x-2}$$

```
[In] int((x + 2)/(x^2 - 4*x + 4),x)
[Out] log(x - 2) - 4/(x - 2)
```

3.129 $\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 14

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = \frac{1}{2-x}+\arctan(2-x)$$

```
[Out] 1/(2-x)-arctan(-2+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {27, 707, 632, 210}

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = \arctan(2-x)+\frac{1}{2-x}$$

```
[In] Int[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
```

```
[Out] (2 - x)^(-1) + ArcTan[2 - x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 707

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[-2*b*d*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(d^2*(m
+ 1)*(b^2 - 4*a*c))), x] + Dist[b^2*((m + 2*p + 3)/(d^2*(m + 1)*(b^2 - 4*a*
c))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{(-2+x)^2\,(5-4x+x^2)}\,dx\\ &= \frac{1}{2-x} - \int \frac{1}{5-4x+x^2}\,dx\\ &= \frac{1}{2-x} + 2\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, -4+2x\right)\\ &= \frac{1}{2-x} + \arctan(2-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)\,(5-4x+x^2)}\,dx = -\frac{1}{-2+x} + \arctan(2-x)$$

```
[In] Integrate[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
[Out] -(-2 + x)^(-1) + ArcTan[2 - x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{1}{-2+x}-\arctan\left(-2+x\right)$	15
risch	$-\frac{1}{-2+x}-\arctan\left(-2+x\right)$	15
parallelrisch	$\frac{i\ln(x-2-i)x-i\ln(x-2+i)x-2i\ln(x-2-i)+2i\ln(x-2+i)-x}{-4+2x}$	50

```
[In] int(1/(x^2-4*x+4)/(x^2-4*x+5),x,method=_RETURNVERBOSE)
[Out] -1/(-2+x)-arctan(-2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{(x-2)\arctan(x-2)+1}{x-2}$$

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="fricas")
[Out] -((x - 2)*arctan(x - 2) + 1)/(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\operatorname{atan}(x-2)-\frac{1}{x-2}$$

```
[In] integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)
[Out] -atan(x - 2) - 1/(x - 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{1}{x-2} - \arctan(x-2)$$

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="maxima")
[Out] -1/(x - 2) - arctan(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{1}{x-2} - \arctan(x-2)$$

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="giac")
[Out] -1/(x - 2) - arctan(x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\mathrm{atan}(x-2) - \frac{1}{x-2}$$

```
[In] int(1/((x^2 - 4*x + 4)*(x^2 - 4*x + 5)),x)
[Out] - atan(x - 2) - 1/(x - 2)
```

3.130 $\int \frac{-3+x}{2x+3x^2+x^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{-3+x}{2x+3x^2+x^3} \, dx = -\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)$$

```
[Out] -3/2*ln(x)+4*ln(1+x)-5/2*ln(2+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1608, 814}

$$\int \frac{-3+x}{2x+3x^2+x^3} \, dx = -\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

```
[In] Int[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
```

```
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{-3+x}{x\left(2+3x+x^2\right)}\,dx\\ &= \int \left(-\frac{3}{2x}+\frac{4}{1+x}-\frac{5}{2(2+x)}\right)\,dx\\ &= -\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)$$

```
[In] Integrate[(-3 + x)/(2*x + 3*x^2 + x^3),x]
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{3\ln(x)}{2}+4\ln\left(1+x\right)-\frac{5\ln(2+x)}{2}$	18
norman	$-\frac{3\ln(x)}{2}+4\ln\left(1+x\right)-\frac{5\ln(2+x)}{2}$	18
risch	$-\frac{3\ln(x)}{2}+4\ln\left(1+x\right)-\frac{5\ln(2+x)}{2}$	18
parallelrisch	$-\frac{3\ln(x)}{2}+4\ln\left(1+x\right)-\frac{5\ln(2+x)}{2}$	18

```
[In] int((-3+x)/(x^3+3*x^2+2*x),x,method=_RETURNVERBOSE)
[Out] -3/2*ln(x)+4*ln(1+x)-5/2*ln(2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{5}{2}\,\log\left(x+2\right) + 4\,\log\left(x+1\right) - \frac{3}{2}\,\log\left(x\right)$$

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="fricas")
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{3\log\left(x\right)}{2} + 4\log\left(x+1\right) - \frac{5\log\left(x+2\right)}{2}$$

```
[In] integrate((-3+x)/(x**3+3*x**2+2*x),x)
[Out] -3*log(x)/2 + 4*log(x + 1) - 5*log(x + 2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{5}{2}\,\log\left(x+2\right) + 4\,\log\left(x+1\right) - \frac{3}{2}\,\log\left(x\right)$$

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="maxima")
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{5}{2}\,\log\left(|x+2|\right) + 4\,\log\left(|x+1|\right) - \frac{3}{2}\,\log\left(|x|\right)$$

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="giac")
[Out] -5/2*log(abs(x + 2)) + 4*log(abs(x + 1)) - 3/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = 4\,\ln\left(x+1\right)-\frac{5\,\ln\left(x+2\right)}{2}-\frac{3\,\ln\left(x\right)}{2}$$

```
[In] int((x - 3)/(2*x + 3*x^2 + x^3),x)
[Out] 4*log(x + 1) - (5*log(x + 2))/2 - (3*log(x))/2
```

3.131 $\int \frac{1}{(-1+x^2)^2} dx$

Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \frac{1}{(-1+x^2)^2} dx = \frac{x}{2(1-x^2)} + \frac{\operatorname{arctanh}(x)}{2}$$

```
[Out] 1/2*x/(-x^2+1)+1/2*arctanh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {205, 213}

$$\int \frac{1}{(-1+x^2)^2} dx = \frac{\operatorname{arctanh}(x)}{2} + \frac{x}{2(1-x^2)}$$

```
[In] Int[(-1 + x^2)^(-2),x]
[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x}{2(1-x^2)} - \frac{1}{2}\int \frac{1}{-1+x^2}\,dx \\ &= \frac{x}{2(1-x^2)} + \frac{\operatorname{arctanh}(x)}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{1}{(-1+x^2)^2}\,dx = \frac{1}{4}\left(-\frac{2x}{-1+x^2} - \log(1-x) + \log(1+x)\right)$$

```
[In] Integrate[(-1 + x^2)^(-2),x]
[Out] ((-2*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

method	result	size
meijerg	$-\frac{i\left(\frac{2ix}{-2x^2+2}+i\operatorname{arctanh}(x)\right)}{2}$	23
norman	$-\frac{x}{2(x^2-1)}-\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}$	24
risch	$-\frac{x}{2(x^2-1)}-\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}$	24
default	$-\frac{1}{4(-1+x)}-\frac{\ln(-1+x)}{4}-\frac{1}{4(1+x)}+\frac{\ln(1+x)}{4}$	28
parallelrisch	$-\frac{\ln(-1+x)x^2-\ln(1+x)x^2-\ln(-1+x)+\ln(1+x)+2x}{4(x^2-1)}$	41

```
[In] int(1/(x^2-1)^2,x,method=_RETURNVERBOSE)
[Out] -1/2*I*(2*I*x/(-2*x^2+2)+I*arctanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{1}{(-1+x^2)^2}\,dx = \frac{(x^2-1)\log(x+1) - (x^2-1)\log(x-1) - 2\,x}{4\,(x^2-1)}$$

```
[In] integrate(1/(x^2-1)^2,x, algorithm="fricas")
[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{(-1+x^2)^2}\,dx = -\frac{x}{2x^2-2} - \frac{\log(x-1)}{4} + \frac{\log(x+1)}{4}$$

```
[In] integrate(1/(x**2-1)**2,x)
[Out] -x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(-1+x^2)^2}\,dx = -\frac{x}{2\,(x^2-1)} + \frac{1}{4}\,\log(x+1) - \frac{1}{4}\,\log(x-1)$$

```
[In] integrate(1/(x^2-1)^2,x, algorithm="maxima")
[Out] -1/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-1+x^2)^2}\,dx = -\frac{x}{2\,(x^2-1)} + \frac{1}{4}\,\log(|x+1|) - \frac{1}{4}\,\log(|x-1|)$$

```
[In] integrate(1/(x^2-1)^2,x, algorithm="giac")
[Out] -1/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-1+x^2)^2}\,dx=\frac{\operatorname{atanh}(x)}{2}-\frac{x}{2\,(x^2-1)}$$

```
[In] int(1/(x^2 - 1)^2,x)
[Out] atanh(x)/2 - x/(2*(x^2 - 1))
```

3.132 $\int \frac{1+x}{-1+x^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{1+x}{-1+x^3} dx = \frac{2}{3}\log(1-x) - \frac{1}{3}\log\left(1+x+x^2\right)$$

```
[Out] 2/3*ln(1-x)-1/3*ln(x^2+x+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1875, 31, 642}

$$\int \frac{1+x}{-1+x^3} dx = \frac{2}{3}\log(1-x) - \frac{1}{3}\log\left(x^2+x+1\right)$$

```
[In] Int[(1 + x)/(-1 + x^3),x]
```

```
[Out] (2*Log[1 - x])/3 - Log[1 + x + x^2]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1875

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Dist[r*((B*r + A*s)/(3*a*
s)), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s*(B*
r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && N
eQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{3}\int \frac{-1-2x}{1+x+x^2}\,dx - \frac{2}{3}\int \frac{1}{1-x}\,dx \\ &= \frac{2}{3}\log(1-x) - \frac{1}{3}\log\left(1+x+x^2\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{-1+x^3}\,dx = \frac{2}{3}\log(1-x) - \frac{1}{3}\log\left(1+x+x^2\right)$$

[In] Integrate[(1 + x)/(-1 + x^3),x]

[Out] (2*Log[1 - x])/3 - Log[1 + x + x^2]/3

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result
default	$\frac{2\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{3}$
norman	$\frac{2\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{3}$
risch	$\frac{2\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{3}$
parallelrisch	$\frac{2\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{3}$
meijerg	$\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\mathrm{ar}\right.}{3(x^3)^{\frac{2}{3}}}$

[In] int((1+x)/(x^3-1),x,method=_RETURNVERBOSE)

[Out] 2/3*ln(-1+x)-1/3*ln(x^2+x+1)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{-1+x^3}\,dx = -\frac{1}{3}\,\log\left(x^2+x+1\right)+\frac{2}{3}\,\log\left(x-1\right)$$

```
[In] integrate((1+x)/(x^3-1),x, algorithm="fricas")
[Out] -1/3*log(x^2 + x + 1) + 2/3*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1+x}{-1+x^3}\,dx = \frac{2\log\left(x-1\right)}{3}-\frac{\log\left(x^2+x+1\right)}{3}$$

```
[In] integrate((1+x)/(x**3-1),x)
[Out] 2*log(x - 1)/3 - log(x**2 + x + 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{-1+x^3}\,dx = -\frac{1}{3}\,\log\left(x^2+x+1\right)+\frac{2}{3}\,\log\left(x-1\right)$$

```
[In] integrate((1+x)/(x^3-1),x, algorithm="maxima")
[Out] -1/3*log(x^2 + x + 1) + 2/3*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1+x}{-1+x^3}\,dx = -\frac{1}{3}\,\log\left(x^2+x+1\right)+\frac{2}{3}\,\log\left(|x-1|\right)$$

```
[In] integrate((1+x)/(x^3-1),x, algorithm="giac")
[Out] -1/3*log(x^2 + x + 1) + 2/3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{-1+x^3}\,dx = \frac{2\,\ln\left(x-1\right)}{3} - \frac{\ln\left(x^2+x+1\right)}{3}$$

```
[In] int((x + 1)/(x^3 - 1),x)
[Out] (2*log(x - 1))/3 - log(x + x^2 + 1)/3
```

3.133 $\int \frac{1+x^4}{x(1+x^2)^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 10

$$\int \frac{1+x^4}{x(1+x^2)^2} \, dx = \frac{1}{1+x^2} + \log(x)$$

```
[Out] 1/(x^2+1)+ln(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1266, 908}

$$\int \frac{1+x^4}{x(1+x^2)^2} \, dx = \frac{1}{x^2+1} + \log(x)$$

```
[In] Int[(1 + x^4)/(x*(1 + x^2)^2),x]
```

```
[Out] (1 + x^2)^(-1) + Log[x]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{x(1+x)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}-\frac{2}{(1+x)^2}\right)\,dx, x, x^2\right)\\ &= \frac{1}{1+x^2}+\log(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx = \frac{1}{1+x^2}+\log(x)$$

```
[In] Integrate[(1 + x^4)/(x*(1 + x^2)^2),x]
[Out] (1 + x^2)^(-1) + Log[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{1}{x^2+1}+\ln\left(x\right)$	11
norman	$\frac{1}{x^2+1}+\ln\left(x\right)$	11
risch	$\frac{1}{x^2+1}+\ln\left(x\right)$	11
parallelrisch	$\frac{x^2\ln(x)+1+\ln(x)}{x^2+1}$	19
meijerg	$-\frac{x^2}{2(x^2+1)}+\frac{1}{2}+\ln\left(x\right)-\frac{x^2}{2x^2+2}$	31

```
[In] int((x^4+1)/x/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/(x^2+1)+ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx = \frac{\left(x^2+1\right)\log\left(x\right)+1}{x^2+1}$$

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="fricas")
[Out] ((x^2 + 1)*log(x) + 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx = \log\left(x\right)+\frac{1}{x^2+1}$$

```
[In] integrate((x**4+1)/x/(x**2+1)**2,x)
[Out] log(x) + 1/(x**2 + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx = \frac{1}{x^2+1}+\frac{1}{2}\log\left(x^2\right)$$

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/(x^2 + 1) + 1/2*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx = \frac{1}{x^2+1}+\frac{1}{2}\log\left(x^2\right)$$

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="giac")
[Out] 1/(x^2 + 1) + 1/2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx=\ln\left(x\right)+\frac{1}{x^2+1}$$

```
[In] int((x^4 + 1)/(x*(x^2 + 1)^2),x)
```

```
[Out] log(x) + 1/(x^2 + 1)
```

3.134 $\int \frac{1}{-2x^3+x^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{1}{-2x^3+x^4}\,dx=\frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

```
[Out] 1/4/x^2+1/4/x+1/8*ln(2-x)-1/8*ln(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 46}

$$\int \frac{1}{-2x^3+x^4}\,dx=\frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

```
[In] Int[(-2*x^3 + x^4)^(-1),x]
```

```
[Out] 1/(4*x^2) + 1/(4*x) + Log[2 - x]/8 - Log[x]/8
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{(-2+x)x^3}\,dx \\ &= \int \left(\frac{1}{8(-2+x)} - \frac{1}{2x^3} - \frac{1}{4x^2} - \frac{1}{8x}\right) dx \\ &= \frac{1}{4x^2} + \frac{1}{4x} + \frac{1}{8}\log(2-x) - \frac{\log(x)}{8} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{-2x^3+x^4}\,dx = \frac{1}{4x^2} + \frac{1}{4x} + \frac{1}{8}\log(2-x) - \frac{\log(x)}{8}$$

```
[In] Integrate[(-2*x^3 + x^4)^(-1),x]
[Out] 1/(4*x^2) + 1/(4*x) + Log[2 - x]/8 - Log[x]/8
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

method	result	size
norman	$\frac{\frac{1}{4}+\frac{x}{4}}{x^2} - \frac{\ln(x)}{8} + \frac{\ln(-2+x)}{8}$	21
risch	$\frac{\frac{1}{4}+\frac{x}{4}}{x^2} - \frac{\ln(x)}{8} + \frac{\ln(-2+x)}{8}$	21
default	$\frac{1}{4x^2} + \frac{1}{4x} - \frac{\ln(x)}{8} + \frac{\ln(-2+x)}{8}$	22
parallelrisch	$-\frac{x^2\ln(x)-\ln(-2+x)x^2-2-2x}{8x^2}$	26
meijerg	$\frac{1}{4x^2} + \frac{1}{4x} - \frac{\ln(x)}{8} + \frac{\ln(2)}{8} - \frac{i\pi}{8} + \frac{\ln\left(1-\frac{x}{2}\right)}{8}$	32

```
[In] int(1/(x^4-2*x^3),x,method=_RETURNVERBOSE)
[Out] (1/4+1/4*x)/x^2-1/8*ln(x)+1/8*ln(-2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{-2x^3 + x^4}\,dx = \frac{x^2 \log(x-2) - x^2 \log(x) + 2\,x + 2}{8\,x^2}$$

```
[In] integrate(1/(x^4-2*x^3),x, algorithm="fricas")
[Out] 1/8*(x^2*log(x - 2) - x^2*log(x) + 2*x + 2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{1}{-2x^3 + x^4}\,dx = -\frac{\log(x)}{8} + \frac{\log(x-2)}{8} + \frac{x+1}{4x^2}$$

```
[In] integrate(1/(x**4-2*x**3),x)
[Out] -log(x)/8 + log(x - 2)/8 + (x + 1)/(4*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{1}{-2x^3 + x^4}\,dx = \frac{x+1}{4\,x^2} + \frac{1}{8}\,\log(x-2) - \frac{1}{8}\,\log(x)$$

```
[In] integrate(1/(x^4-2*x^3),x, algorithm="maxima")
[Out] 1/4*(x + 1)/x^2 + 1/8*log(x - 2) - 1/8*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{-2x^3 + x^4}\,dx = \frac{x+1}{4\,x^2} + \frac{1}{8}\,\log(|x-2|) - \frac{1}{8}\,\log(|x|)$$

```
[In] integrate(1/(x^4-2*x^3),x, algorithm="giac")
[Out] 1/4*(x + 1)/x^2 + 1/8*log(abs(x - 2)) - 1/8*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

$$\int \frac{1}{-2x^3+x^4}\,dx=\frac{\frac{x}{4}+\frac{1}{4}}{x^2}-\frac{\operatorname{atanh}(x-1)}{4}$$

```
[In] int(-1/(2*x^3 - x^4),x)
[Out] (x/4 + 1/4)/x^2 - atanh(x - 1)/4
```

3.135 $\int \frac{1-x^3}{x(1+x^2)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1-x^3}{x(1+x^2)} \, dx = -x + \arctan(x) + \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -x+arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1816, 649, 209, 266}

$$\int \frac{1-x^3}{x(1+x^2)} \, dx = \arctan(x) - \frac{1}{2}\log\left(x^2+1\right) - x + \log(x)$$

```
[In] Int[(1 - x^3)/(x*(1 + x^2)),x]
```

```
[Out] -x + ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-1+\frac{1}{x}+\frac{1-x}{1+x^2}\right) dx \\
&= -x+\log(x)+\int \frac{1-x}{1+x^2}\, dx \\
&= -x+\log(x)+\int \frac{1}{1+x^2}\, dx-\int \frac{x}{1+x^2}\, dx \\
&= -x+\arctan(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1-x^3}{x\left(1+x^2\right)}\, dx = -x+\arctan(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(1 - x^3)/(x*(1 + x^2)),x]
[Out] -x + ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$-x+\arctan(x)+\ln(x)-\frac{\ln(x^2+1)}{2}$	17
meijerg	$-x+\arctan(x)+\ln(x)-\frac{\ln(x^2+1)}{2}$	17
risch	$-x+\arctan(x)+\ln(x)-\frac{\ln(x^2+1)}{2}$	17
parallelrisch	$-x+\ln(x)-\frac{\ln(x-i)}{2}-\frac{i\ln(x-i)}{2}-\frac{\ln(x+i)}{2}+\frac{i\ln(x+i)}{2}$	37

```
[In] int((-x^3+1)/x/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -x+arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1-x^3}{x(1+x^2)}\,dx = -x+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

```
[In] integrate((-x^3+1)/x/(x^2+1),x, algorithm="fricas")
[Out] -x + arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1-x^3}{x(1+x^2)}\,dx = -x+\log(x)-\frac{\log(x^2+1)}{2}+\operatorname{atan}(x)$$

```
[In] integrate((-x**3+1)/x/(x**2+1),x)
[Out] -x + log(x) - log(x**2 + 1)/2 + atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1-x^3}{x(1+x^2)}\,dx = -x+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

```
[In] integrate((-x^3+1)/x/(x^2+1),x, algorithm="maxima")
[Out] -x + arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1-x^3}{x\left(1+x^2\right)}\,dx = -x + \arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right) + \log\left(|x|\right)$$

```
[In] integrate((-x^3+1)/x/(x^2+1),x, algorithm="giac")
[Out] -x + arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{1-x^3}{x\left(1+x^2\right)}\,dx = \ln\left(x\right) - x + \ln\left(x-\mathrm{i}\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

```
[In] int(-(x^3 - 1)/(x*(x^2 + 1)),x)
[Out] log(x) - log(x - 1i)*(1/2 + 1i/2) - log(x + 1i)*(1/2 - 1i/2) - x
```

3.136 $\int \frac{1}{-1+x^4} dx$

Optimal result

Integrand size = 7, antiderivative size = 13

$$\int \frac{1}{-1+x^4} dx = -\frac{\arctan(x)}{2} - \frac{\operatorname{arctanh}(x)}{2}$$

```
[Out] -1/2*arctan(x)-1/2*arctanh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {218, 212, 209}

$$\int \frac{1}{-1+x^4} dx = -\frac{\arctan(x)}{2} - \frac{\operatorname{arctanh}(x)}{2}$$

```
[In] Int[(-1 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[x] - ArcTanh[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b
, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int \frac{1}{1-x^2}\,dx\right) - \frac{1}{2}\int \frac{1}{1+x^2}\,dx \\ &= -\frac{\arctan(x)}{2} - \frac{\operatorname{arctanh}(x)}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{1}{-1+x^4}\,dx = -\frac{\arctan(x)}{2} + \frac{1}{4}\log(1-x) - \frac{1}{4}\log(1+x)$$

```
[In] Integrate[(-1 + x^4)^(-1),x]
[Out] -1/2*ArcTan[x] + Log[1 - x]/4 - Log[1 + x]/4
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\arctan(x)}{2}-\frac{\operatorname{arctanh}(x)}{2}$	10
risch	$-\frac{\arctan(x)}{2}-\frac{\ln(1+x)}{4}+\frac{\ln(-1+x)}{4}$	18
parallelrisch	$\frac{i\ln(x-i)}{4}-\frac{i\ln(x+i)}{4}-\frac{\ln(1+x)}{4}+\frac{\ln(-1+x)}{4}$	30
meijerg	$\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}$	38

```
[In] int(1/(x^4-1),x,method=_RETURNVERBOSE)
[Out] -1/2*arctan(x)-1/2*arctanh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{-1+x^4}\,dx = -\frac{1}{2}\,\arctan\left(x\right) - \frac{1}{4}\,\log\left(x+1\right) + \frac{1}{4}\,\log\left(x-1\right)$$

```
[In] integrate(1/(x^4-1),x, algorithm="fricas")
[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{-1+x^4}\,dx = \frac{\log\left(x-1\right)}{4} - \frac{\log\left(x+1\right)}{4} - \frac{\operatorname{atan}\left(x\right)}{2}$$

```
[In] integrate(1/(x**4-1),x)
[Out] log(x - 1)/4 - log(x + 1)/4 - atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{-1+x^4}\,dx = -\frac{1}{2}\,\arctan\left(x\right) - \frac{1}{4}\,\log\left(x+1\right) + \frac{1}{4}\,\log\left(x-1\right)$$

```
[In] integrate(1/(x^4-1),x, algorithm="maxima")
[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{1}{-1+x^4}\,dx = -\frac{1}{2}\,\arctan\left(x\right) - \frac{1}{4}\,\log\left(|x+1|\right) + \frac{1}{4}\,\log\left(|x-1|\right)$$

```
[In] integrate(1/(x^4-1),x, algorithm="giac")
[Out] -1/2*arctan(x) - 1/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{-1+x^4}\,dx=-\frac{\operatorname{atan}(x)}{2}-\frac{\operatorname{atanh}(x)}{2}$$

```
[In] int(1/(x^4 - 1),x)
[Out] - atan(x)/2 - atanh(x)/2
```

3.137 $\int \frac{1}{1+x^4} dx$

Optimal result

Integrand size = 7, antiderivative size = 85

$$\int \frac{1}{1+x^4} dx = -\frac{\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}x\right)}{2\sqrt{2}} -\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

```
[Out] 1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)-1/8*ln(1+x
^2-x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{1+x^4} dx = -\frac{\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\arctan\left(\sqrt{2}x+1\right)}{2\sqrt{2}} -\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}$$

```
[In] Int[(1 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) - Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx$$

$$= \frac{1}{4}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx - \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{4\sqrt{2}} - \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}$$

$$= -\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{2\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}x\right)}{2\sqrt{2}}$$

$$= -\frac{\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}x\right)}{2\sqrt{2}}-\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+x^4}\,dx = \frac{-2\arctan\left(1-\sqrt{2}x\right)+2\arctan\left(1+\sqrt{2}x\right)-\log\left(1-\sqrt{2}x+x^2\right)+\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

```
[In] Integrate[(1 + x^4)^(-1),x]
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqrt[2]*x + x
^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.26

method	result
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{8}$
meijerg	$-\frac{x\sqrt{2}\,\ln\left(1-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}\right)}{4\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}{2+\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}\right)}{4\left(x^4\right)^{\frac{1}{4}}}$

```
[In] int(1/(x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

$$\int \frac{1}{1+x^4} dx = \left(\frac{1}{8}i+\frac{1}{8}\right) \sqrt{2}\log\left(2x+(i+1)\sqrt{2}\right) - \left(\frac{1}{8}i-\frac{1}{8}\right) \sqrt{2}\log\left(2x-(i-1)\sqrt{2}\right) + \left(\frac{1}{8}i-\frac{1}{8}\right) \sqrt{2}\log\left(2x+(i-1)\sqrt{2}\right) - \left(\frac{1}{8}i+\frac{1}{8}\right) \sqrt{2}\log\left(2x-(i+1)\sqrt{2}\right)$$

```
[In] integrate(1/(x^4+1),x, algorithm="fricas")
[Out] (1/8*I + 1/8)*sqrt(2)*log(2*x + (I + 1)*sqrt(2)) - (1/8*I - 1/8)*sqrt(2)*lo
g(2*x - (I - 1)*sqrt(2)) + (1/8*I - 1/8)*sqrt(2)*log(2*x + (I - 1)*sqrt(2))
 - (1/8*I + 1/8)*sqrt(2)*log(2*x - (I + 1)*sqrt(2))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{1}{1+x^4} dx = -\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8} + \frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{4} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{4}$$

```
[In] integrate(1/(x**4+1),x)
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8
+ sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+x^4} dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

```
[In] integrate(1/(x^4+1),x, algorithm="maxima")
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+x^4}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

```
[In] integrate(1/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.39

$$\int \frac{1}{1+x^4}\,dx = \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

```
[In] int(1/(x^4 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 + 1i/4) + 2^(1/2)*atan(2^(1/2)*x*
(1/2 + 1i/2))*(1/4 - 1i/4)
```

3.138 $\int \frac{x^2}{(2+2x+x^2)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{x^2}{(2+2x+x^2)^2}\,dx = -\frac{x(2+x)}{2(2+2x+x^2)}+\arctan(1+x)$$

```
[Out] -1/2*x*(2+x)/(x^2+2*x+2)+arctan(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {736, 631, 210}

$$\int \frac{x^2}{(2+2x+x^2)^2}\,dx = \arctan(x+1)-\frac{x(x+2)}{2(x^2+2x+2)}$$

```
[In] Int[x^2/(2 + 2*x + x^2)^2,x]
```

```
[Out] -1/2*(x*(2 + x))/(2 + 2*x + x^2) + ArcTan[1 + x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c))), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(2+x)}{2\left(2+2x+x^2\right)}+\int \frac{1}{2+2x+x^2}\,dx \\
&= -\frac{x(2+x)}{2\left(2+2x+x^2\right)}-\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+x\right) \\
&= -\frac{x(2+x)}{2\left(2+2x+x^2\right)}+\arctan(1+x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{\left(2+2x+x^2\right)^2}\,dx=\frac{1}{2+2x+x^2}+\arctan(1+x)$$

```
[In] Integrate[x^2/(2 + 2*x + x^2)^2,x]
[Out] (2 + 2*x + x^2)^(-1) + ArcTan[1 + x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{1}{x^2+2x+2}+\arctan\left(1+x\right)$	16
risch	$\frac{1}{x^2+2x+2}+\arctan\left(1+x\right)$	16
parallelrisch	$-\frac{i\ln(x+1-i)x^2-i\ln(x+1+i)x^2+2i\ln(x+1-i)x-2i\ln(x+1+i)x-2+2i\ln(x+1-i)-2i\ln(x+1+i)}{2(x^2+2x+2)}$	77

```
[In] int(x^2/(x^2+2*x+2)^2,x,method=_RETURNVERBOSE)
[Out] 1/(x^2+2*x+2)+arctan(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{x^2}{(2+2x+x^2)^2}\,dx = \frac{(x^2+2\,x+2)\arctan\left(x+1\right)+1}{x^2+2\,x+2}$$

```
[In] integrate(x^2/(x^2+2*x+2)^2,x, algorithm="fricas")
[Out] ((x^2 + 2*x + 2)*arctan(x + 1) + 1)/(x^2 + 2*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{(2+2x+x^2)^2}\,dx = \operatorname{atan}\left(x+1\right) + \frac{1}{x^2+2x+2}$$

```
[In] integrate(x**2/(x**2+2*x+2)**2,x)
[Out] atan(x + 1) + 1/(x**2 + 2*x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{(2+2x+x^2)^2}\,dx = \frac{1}{x^2+2\,x+2} + \arctan\left(x+1\right)$$

```
[In] integrate(x^2/(x^2+2*x+2)^2,x, algorithm="maxima")
[Out] 1/(x^2 + 2*x + 2) + arctan(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{(2+2x+x^2)^2} \, dx = \frac{1}{x^2+2\,x+2} + \arctan\left(x+1\right)$$

```
[In] integrate(x^2/(x^2+2*x+2)^2,x, algorithm="giac")
[Out] 1/(x^2 + 2*x + 2) + arctan(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{(2+2x+x^2)^2} \, dx = \mathrm{atan}(x+1) + \frac{1}{x^2+2\,x+2}$$

```
[In] int(x^2/(2*x + x^2 + 2)^2,x)
[Out] atan(x + 1) + 1/(2*x + x^2 + 2)
```

3.139 $\int \frac{-1+4x^5}{(1+x+x^5)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{-1+4x^5}{(1+x+x^5)^2} dx = -\frac{x}{1+x+x^5}$$

```
[Out] -x/(x^5+x+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1602}

$$\int \frac{-1+4x^5}{(1+x+x^5)^2} dx = -\frac{x}{x^5+x+1}$$

```
[In] Int[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
```

```
[Out] -(x/(1 + x + x^5))
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{x}{1+x+x^5}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{1+x+x^5}$$

```
[In] Integrate[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
[Out] -(x/(1 + x + x^5))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{x}{x^5+x+1}$	12
norman	$-\frac{x}{x^5+x+1}$	12
risch	$-\frac{x}{x^5+x+1}$	12
parallelrisch	$-\frac{x}{x^5+x+1}$	12
default	$-\frac{-3x^2+5x-1}{7(x^3-x^2+1)}+\frac{-3x-1}{7x^2+7x+7}$	41

```
[In] int((4*x^5-1)/(x^5+x+1)^2,x,method=_RETURNVERBOSE)
[Out] -x/(x^5+x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="fricas")
[Out] -x/(x^5 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

```
[In] integrate((4*x**5-1)/(x**5+x+1)**2,x)
[Out] -x/(x**5 + x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="maxima")
[Out] -x/(x^5 + x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="giac")
[Out] -x/(x^5 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

```
[In] int((4*x^5 - 1)/(x + x^5 + 1)^2,x)
[Out] -x/(x + x^5 + 1)
```

3.140 $\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx=\frac{x}{2\sqrt{5}}+\frac{\arctan\left(\frac{2\cos(x)+\sin(x)}{5+2\sqrt{5}-\cos(x)+2\sin(x)}\right)}{\sqrt{5}}$$

```
[Out] 1/10*x*5^(1/2)+1/5*arctan((2*cos(x)+sin(x))/(5-cos(x)+2*sin(x)+2*5^(1/2)))*
5^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3203, 632, 210}

$$\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx=\frac{\arctan\left(\frac{\sin(x)+2\cos(x)}{2\sin(x)-\cos(x)+2\sqrt{5}+5}\right)}{\sqrt{5}}+\frac{x}{2\sqrt{5}}$$

```
[In] Int[(5 - Cos[x] + 2*Sin[x])^(-1),x]
```

```
[Out] x/(2*Sqrt[5]) + ArcTan[(2*Cos[x] + Sin[x])/(5 + 2*Sqrt[5] - Cos[x] + 2*Sin[
x])]/Sqrt[5]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{4+4x+6x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\left(4\text{Subst}\left(\int \frac{1}{-80-x^2}\,dx, x, 4+12\tan\left(\frac{x}{2}\right)\right)\right)\\ &= \frac{x}{2\sqrt{5}} + \frac{\arctan\left(\frac{2\cos(x)+\sin(x)}{5+2\sqrt{5}-\cos(x)+2\sin(x)}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.51

$$\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx = \frac{\arctan\left(\frac{1+3\tan\left(\frac{x}{2}\right)}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[In] Integrate[(5 - Cos[x] + 2*Sin[x])^(-1),x]
[Out] ArcTan[(1 + 3*Tan[x/2])/Sqrt[5]]/Sqrt[5]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.44

method	result	size
default	$\frac{\sqrt{5}\,\arctan\left(\frac{\left(6\tan\left(\frac{x}{2}\right)+2\right)\sqrt{5}}{10}\right)}{5}$	20
risch	$\frac{i\sqrt{5}\,\ln\left(\mathrm{e}^{ix}-1+2i+\frac{4i\sqrt{5}}{5}-\frac{2\sqrt{5}}{5}\right)}{10}-\frac{i\sqrt{5}\,\ln\left(\mathrm{e}^{ix}-1+2i-\frac{4i\sqrt{5}}{5}+\frac{2\sqrt{5}}{5}\right)}{10}$	56

```
[In] int(1/(5-cos(x)+2*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*5^(1/2)*arctan(1/10*(6*tan(1/2*x)+2)*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx = \frac{1}{10}\,\sqrt{5}\arctan\left(-\frac{\sqrt{5}\cos\left(x\right)-2\,\sqrt{5}\sin\left(x\right)-\sqrt{5}}{2\left(2\,\cos\left(x\right)+\sin\left(x\right)\right)}\right)$$

```
[In] integrate(1/(5-cos(x)+2*sin(x)),x, algorithm="fricas")
[Out] 1/10*sqrt(5)*arctan(-1/2*(sqrt(5)*cos(x) - 2*sqrt(5)*sin(x) - sqrt(5))/(2*c
os(x) + sin(x)))
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx = \frac{\sqrt{5}\left(\operatorname{atan}\left(\frac{3\sqrt{5}\tan\left(\frac{x}{2}\right)}{5}+\frac{\sqrt{5}}{5}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{5}$$

```
[In] integrate(1/(5-cos(x)+2*sin(x)),x)
[Out] sqrt(5)*(atan(3*sqrt(5)*tan(x/2)/5 + sqrt(5)/5) + pi*floor((x/2 - pi/2)/pi)
)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.51

$$\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx = \frac{1}{5}\,\sqrt{5}\arctan\left(\frac{1}{5}\,\sqrt{5}\left(\frac{3\,\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)\right)$$

```
[In] integrate(1/(5-cos(x)+2*sin(x)),x, algorithm="maxima")
[Out] 1/5*sqrt(5)*arctan(1/5*sqrt(5)*(3*sin(x)/(cos(x) + 1) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx = \frac{1}{10}\,\sqrt{5}\left(x+2\,\arctan\left(-\frac{\sqrt{5}\sin\left(x\right)-\cos\left(x\right)-3\,\sin\left(x\right)-1}{\sqrt{5}\cos\left(x\right)+\sqrt{5}-3\,\cos\left(x\right)+\sin\left(x\right)+3}\right)\right)$$

```
[In] integrate(1/(5-cos(x)+2*sin(x)),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5)*(x + 2*arctan(-(sqrt(5)*sin(x) - cos(x) - 3*sin(x) - 1)/(sqrt(
5)*cos(x) + sqrt(5) - 3*cos(x) + sin(x) + 3)))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.47

$$\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx = \frac{\sqrt{20}\,\mathrm{atan}\left(\frac{3\,\sqrt{20}\,\tan\left(\frac{x}{2}\right)}{10}+\frac{\sqrt{20}}{10}\right)}{10}$$

```
[In] int(1/(2*sin(x) - cos(x) + 5),x)
```

```
[Out] (20^(1/2)*atan((3*20^(1/2)*tan(x/2))/10 + 20^(1/2)/10))/10
```

3.141 $\int \frac{1}{1+a\cos(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \frac{1}{1+a\cos(x)}\,dx=\frac{2\arctan\left(\frac{\sqrt{1-a}\tan\left(\frac{x}{2}\right)}{\sqrt{1+a}}\right)}{\sqrt{1-a^2}}$$

```
[Out] 2*arctan((1-a)^(1/2)*tan(1/2*x)/(1+a)^(1/2))/(-a^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2738, 211}

$$\int \frac{1}{1+a\cos(x)}\,dx=\frac{2\arctan\left(\frac{\sqrt{1-a}\tan\left(\frac{x}{2}\right)}{\sqrt{a+1}}\right)}{\sqrt{1-a^2}}$$

```
[In] Int[(1 + a*Cos[x])^(-1),x]
```

```
[Out] (2*ArcTan[(Sqrt[1 - a]*Tan[x/2])/Sqrt[1 + a]])/Sqrt[1 - a^2]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{1}{1+a+(1-a)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= \frac{2\arctan\left(\frac{\sqrt{1-a}\tan\left(\frac{x}{2}\right)}{\sqrt{1+a}}\right)}{\sqrt{1-a^2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{1+a\cos(x)}\,dx = \frac{2\text{arctanh}\left(\frac{(-1+a)\tan\left(\frac{x}{2}\right)}{\sqrt{-1+a^2}}\right)}{\sqrt{-1+a^2}}$$

```
[In] Integrate[(1 + a*Cos[x])^(-1),x]
[Out] (2*ArcTanh[((-1 + a)*Tan[x/2])/Sqrt[-1 + a^2]])/Sqrt[-1 + a^2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{2\,\text{arctanh}\left(\frac{(a-1)\tan\left(\frac{x}{2}\right)}{\sqrt{(1+a)(a-1)}}\right)}{\sqrt{(1+a)(a-1)}}$	30
risch	$\frac{\ln\left(e^{ix}+\frac{i a^2+\sqrt{a^2-1}-i}{a\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}}-\frac{\ln\left(e^{ix}+\frac{-i a^2+\sqrt{a^2-1}+i}{a\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}}$	87

```
[In] int(1/(1+a*cos(x)),x,method=_RETURNVERBOSE)
[Out] 2/((1+a)*(a-1))^(1/2)*arctanh((a-1)*tan(1/2*x)/((1+a)*(a-1))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.00

$$\int \frac{1}{1+a\cos(x)}\,dx = \left[\frac{\log\left(-\frac{(a^2-2)\cos(x)^2-2\sqrt{a^2-1}(a+\cos(x))\sin(x)-2a^2-2a\cos(x)+1}{a^2\cos(x)^2+2a\cos(x)+1}\right)}{2\sqrt{a^2-1}},\ -\frac{\sqrt{-a^2+1}\arctan\left(\frac{\sqrt{-a^2+1}(a+\cos(x))}{(a^2-1)\sin(x)}\right)}{a^2-1}\right]$$

```
[In] integrate(1/(1+a*cos(x)),x, algorithm="fricas")
```

```
[Out] [1/2*log(-((a^2 - 2)*cos(x)^2 - 2*sqrt(a^2 - 1)*(a + cos(x))*sin(x) - 2*a^2
 - 2*a*cos(x) + 1)/(a^2*cos(x)^2 + 2*a*cos(x) + 1))/sqrt(a^2 - 1), -sqrt(-a
^2 + 1)*arctan(sqrt(-a^2 + 1)*(a + cos(x))/((a^2 - 1)*sin(x)))/(a^2 - 1)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(29) = 58.

Time = 1.45 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.97

$$\int \frac{1}{1+a\cos(x)}\,dx = \begin{cases} \tan\left(\frac{x}{2}\right) & \text{for } a=1 \\ -\frac{1}{\tan\left(\frac{x}{2}\right)} & \text{for } a=-1 \\ -\frac{\log\left(-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}} + \frac{\log\left(\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(1+a*cos(x)),x)
```

```
[Out] Piecewise((tan(x/2), Eq(a, 1)), (-1/tan(x/2), Eq(a, -1)), (-log(-sqrt(a/(a
- 1) + 1/(a - 1)) + tan(x/2))/(a*sqrt(a/(a - 1) + 1/(a - 1)) - sqrt(a/(a -
1) + 1/(a - 1))) + log(sqrt(a/(a - 1) + 1/(a - 1)) + tan(x/2))/(a*sqrt(a/(a
 - 1) + 1/(a - 1)) - sqrt(a/(a - 1) + 1/(a - 1))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{1 + a\cos(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(1+a*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a^2-1.0>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \frac{1}{1 + a\cos(x)}\,dx = -\frac{2\left(\pi\left\lfloor\frac{x}{2\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}(2a - 2) + \arctan\left(\frac{a\tan\left(\frac{1}{2}x\right) - \tan\left(\frac{1}{2}x\right)}{\sqrt{-a^2+1}}\right)\right)}{\sqrt{-a^2 + 1}}$$

```
[In] integrate(1/(1+a*cos(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a - 2) + arctan((a*tan(1/2*x) - tan(1/2*
x))/sqrt(-a^2 + 1)))/sqrt(-a^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{1}{1 + a\cos(x)}\,dx = \frac{2\,\operatorname{atanh}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{a-1}}{\sqrt{a+1}}\right)}{\sqrt{a - 1}\,\sqrt{a + 1}}$$

```
[In] int(1/(a*cos(x) + 1),x)
```

```
[Out] (2*atanh((tan(x/2)*(a - 1)^(1/2))/(a + 1)^(1/2)))/((a - 1)^(1/2)*(a + 1)^(1
/2))
```

3.142 $\int \frac{1}{1+2\cos(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 56

$$\int \frac{1}{1+2\cos(x)}\,dx=-\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}+\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}$$

```
[Out] -1/3*ln(-sin(1/2*x)+cos(1/2*x)*3^(1/2))*3^(1/2)+1/3*ln(sin(1/2*x)+cos(1/2*x
)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2738, 212}

$$\int \frac{1}{1+2\cos(x)}\,dx=\frac{\log\left(\sin\left(\frac{x}{2}\right)+\sqrt{3}\cos\left(\frac{x}{2}\right)\right)}{\sqrt{3}}-\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}$$

```
[In] Int[(1 + 2*Cos[x])^(-1),x]
```

```
[Out] -(Log[Sqrt[3]*Cos[x/2] - Sin[x/2]]/Sqrt[3]) + Log[Sqrt[3]*Cos[x/2] + Sin[x/
2]]/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{3-x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}+\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.36

$$\int \frac{1}{1+2\cos(x)}\,dx = \frac{2\text{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Integrate[(1 + 2*Cos[x])^(-1),x]
[Out] (2*ArcTanh[Tan[x/2]/Sqrt[3]])/Sqrt[3]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.29

method	result	size
default	$\frac{2\sqrt{3}\,\text{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{3}}{3}\right)}{3}$	16
risch	$\frac{\sqrt{3}\,\ln\left(e^{ix}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{3}-\frac{\sqrt{3}\,\ln\left(e^{ix}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{3}$	40

```
[In] int(1/(1+2*cos(x)),x,method=_RETURNVERBOSE)
[Out] 2/3*3^(1/2)*arctanh(1/3*tan(1/2*x)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{1}{1+2\cos(x)}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(-\frac{2\,\cos\left(x\right)^{2}-2\,\left(\sqrt{3}\cos\left(x\right)+2\,\sqrt{3}\right)\sin\left(x\right)-4\,\cos\left(x\right)-7}{4\,\cos\left(x\right)^{2}+4\,\cos\left(x\right)+1}\right)$$

```
[In] integrate(1/(1+2*cos(x)),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log(-(2*cos(x)^2 - 2*(sqrt(3)*cos(x) + 2*sqrt(3))*sin(x) - 4*co
s(x) - 7)/(4*cos(x)^2 + 4*cos(x) + 1))
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \frac{1}{1+2\cos(x)}\,dx = -\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{3}\right)}{3}+\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)+\sqrt{3}\right)}{3}$$

```
[In] integrate(1/(1+2*cos(x)),x)
[Out] -sqrt(3)*log(tan(x/2) - sqrt(3))/3 + sqrt(3)*log(tan(x/2) + sqrt(3))/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int \frac{1}{1+2\cos(x)}\,dx = -\frac{1}{3}\,\sqrt{3}\log\left(-\frac{\sqrt{3}-\frac{\sin(x)}{\cos(x)+1}}{\sqrt{3}+\frac{\sin(x)}{\cos(x)+1}}\right)$$

```
[In] integrate(1/(1+2*cos(x)),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*log(-(sqrt(3) - sin(x)/(cos(x) + 1))/(sqrt(3) + sin(x)/(cos(x)
 + 1)))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+2\cos(x)}\,dx = -\frac{1}{3}\,\sqrt{3}\log\left(\frac{\left|-2\,\sqrt{3}+2\,\tan\left(\frac{1}{2}\,x\right)\right|}{\left|2\,\sqrt{3}+2\,\tan\left(\frac{1}{2}\,x\right)\right|}\right)$$

```
[In] integrate(1/(1+2*cos(x)),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(1/2*x))/abs(2*sqrt(3) + 2*tan(1/2*x
)))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.27

$$\int \frac{1}{1+2\cos(x)}\,dx = \frac{2\,\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)}{3}$$

```
[In] int(1/(2*cos(x) + 1),x)
```

```
[Out] (2*3^(1/2)*atanh((3^(1/2)*tan(x/2))/3))/3
```

3.143 $\int \frac{1}{1+\frac{\cos(x)}{2}} dx$

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \frac{1}{1+\frac{\cos(x)}{2}} dx = \frac{2x}{\sqrt{3}} - \frac{4\arctan\left(\frac{\sin(x)}{2+\sqrt{3}+\cos(x)}\right)}{\sqrt{3}}$$

```
[Out] 2/3*x*3^(1/2)-4/3*arctan(sin(x)/(2+cos(x)+3^(1/2)))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2736}

$$\int \frac{1}{1+\frac{\cos(x)}{2}} dx = \frac{2x}{\sqrt{3}} - \frac{4\arctan\left(\frac{\sin(x)}{\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

```
[In] Int[(1 + Cos[x]/2)^(-1),x]
[Out] (2*x)/Sqrt[3] - (4*ArcTan[Sin[x]/(2 + Sqrt[3] + Cos[x])])/Sqrt[3]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\text{integral} = \frac{2x}{\sqrt{3}} - \frac{4\arctan\left(\frac{\sin(x)}{2+\sqrt{3}+\cos(x)}\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx = \frac{4\arctan\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Integrate[(1 + Cos[x]/2)^(-1),x]
[Out] (4*ArcTan[Tan[x/2]/Sqrt[3]])/Sqrt[3]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{4\sqrt{3}\,\arctan\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{3}}{3}\right)}{3}$	16
risch	$\frac{2i\sqrt{3}\,\ln\left(e^{ix}+\sqrt{3}+2\right)}{3}-\frac{2i\sqrt{3}\,\ln\left(e^{ix}-\sqrt{3}+2\right)}{3}$	38

```
[In] int(1/(1+1/2*cos(x)),x,method=_RETURNVERBOSE)
[Out] 4/3*3^(1/2)*arctan(1/3*tan(1/2*x)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx = -\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{2\,\sqrt{3}\cos\left(x\right)+\sqrt{3}}{3\,\sin\left(x\right)}\right)$$

```
[In] integrate(1/(1+1/2*cos(x)),x, algorithm="fricas")
[Out] -2/3*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(x) + sqrt(3))/sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx = \frac{4\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}$$

```
[In] integrate(1/(1+1/2*cos(x)),x)
[Out] 4*sqrt(3)*(atan(sqrt(3)*tan(x/2)/3) + pi*floor((x/2 - pi/2)/pi))/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx = \frac{4}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sin(x)}{3(\cos(x)+1)}\right)$$

```
[In] integrate(1/(1+1/2*cos(x)),x, algorithm="maxima")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx = \frac{2}{3}\sqrt{3}\left(x+2\arctan\left(-\frac{\sqrt{3}\sin(x)-\sin(x)}{\sqrt{3}\cos(x)+\sqrt{3}-\cos(x)+1}\right)\right)$$

```
[In] integrate(1/(1+1/2*cos(x)),x, algorithm="giac")
[Out] 2/3*sqrt(3)*(x + 2*arctan(-(sqrt(3)*sin(x) - sin(x))/(sqrt(3)*cos(x) + sqrt
(3) - cos(x) + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx = \frac{4\sqrt{3}\,\left(\frac{x}{2}-\mathrm{atan}\left(\tan\left(\frac{x}{2}\right)\right)\right)}{3}+\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)}{3}$$

```
[In] int(1/(cos(x)/2 + 1),x)
```

```
[Out] (4*3^(1/2)*(x/2 - atan(tan(x/2))))/3 + (4*3^(1/2)*atan((3^(1/2)*tan(x/2))/3
))/3
```

3.144 $\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx = x-\frac{x}{\sqrt{2}}-\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}}$$

```
[Out] x-1/2*x*2^(1/2)-1/2*arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3250, 3260, 209}

$$\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx = -\frac{\arctan\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}-\frac{x}{\sqrt{2}}+x$$

```
[In] Int[Sin[x]^2/(1 + Sin[x]^2),x]
```

```
[Out] x - x/Sqrt[2] - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3250

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Dist[(A*b - a*B)/b, Int[1/(a +
```

```
b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x - \int \frac{1}{1+\sin^2(x)}\,dx \\ &= x - \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(x)\right) \\ &= x - \frac{x}{\sqrt{2}} - \frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx = x - \frac{\arctan\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}}$$

```
[In] Integrate[Sin[x]^2/(1 + Sin[x]^2),x]
[Out] x - ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{\sqrt{2}\,\arctan\left(\tan(x)\sqrt{2}\right)}{2}+\arctan\left(\tan\left(x\right)\right)$	17
risch	$x-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}-3\right)}{4}+\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}-3\right)}{4}$	41

```
[In] int(sin(x)^2/(1+sin(x)^2),x,method=_RETURNVERBOSE)
[Out] -1/2*2^(1/2)*arctan(tan(x)*2^(1/2))+arctan(tan(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)+x$$

```
[In] integrate(sin(x)^2/(1+sin(x)^2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) +
x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(36) = 72.

Time = 24.44 (sec) , antiderivative size = 248, normalized size of antiderivative = 6.89

$$\begin{aligned}\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx &= \frac{31988856\sqrt{2}x}{31988856\sqrt{2}+45239074}+\frac{45239074x}{31988856\sqrt{2}+45239074} \\ &\quad-\frac{77227930\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}+45239074} \\ &\quad-\frac{54608393\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}+45239074} \\ &\quad-\frac{13250218\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}+45239074} \\ &\quad-\frac{9369319\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}+45239074}\end{aligned}$$

```
[In] integrate(sin(x)**2/(1+sin(x)**2),x)
[Out] 31988856*sqrt(2)*x/(31988856*sqrt(2) + 45239074) + 45239074*x/(31988856*sqr
t(2) + 45239074) - 77227930*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*s
qrt(2))) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2) + 45239074) - 54608
393*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*fl
oor((x/2 - pi/2)/pi))/(31988856*sqrt(2) + 45239074) - 13250218*sqrt(2*sqrt(
2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(3
1988856*sqrt(2) + 45239074) - 9369319*sqrt(2)*sqrt(2*sqrt(2) + 3)*(atan(tan
(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2) +
 45239074)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.39

$$\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx = -\frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}\tan\left(x\right)\right) + x$$

```
[In] integrate(sin(x)^2/(1+sin(x)^2),x, algorithm="maxima")
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*tan(x)) + x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx = -\frac{1}{2}\,\sqrt{2}\left(x + \arctan\left(-\frac{\sqrt{2}\sin\left(2\,x\right) - 2\,\sin\left(2\,x\right)}{\sqrt{2}\cos\left(2\,x\right) + \sqrt{2} - 2\,\cos\left(2\,x\right) + 2}\right)\right) + x$$

```
[In] integrate(sin(x)^2/(1+sin(x)^2),x, algorithm="giac")
[Out] -1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
 + sqrt(2) - 2*cos(2*x) + 2))) + x
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx = x - \frac{\sqrt{2}\,(x - \mathrm{atan}(\tan(x)))}{2} - \frac{\sqrt{2}\,\mathrm{atan}\big(\sqrt{2}\tan(x)\big)}{2}$$

```
[In] int(sin(x)^2/(sin(x)^2 + 1),x)
[Out] x - (2^(1/2)*(x - atan(tan(x))))/2 - (2^(1/2)*atan(2^(1/2)*tan(x)))/2
```

3.145 $\int \frac{1}{b^2 \cos^2(x)+a^2 \sin^2(x)} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 15

$$\int \frac{1}{b^2 \cos^2(x)+a^2 \sin^2(x)} \, dx=\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[Out] arctan(a*tan(x)/b)/a/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {211}

$$\int \frac{1}{b^2 \cos^2(x)+a^2 \sin^2(x)} \, dx=\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[In] Int[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{b^2+a^2x^2} \, dx,x,\tan(x)\right) \\ &= \frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx=\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[In] Integrate[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$	16
parallelrisch	$\frac{i\left(\ln\left(\frac{ia\sin(x)-b\cos(x)}{\cos(x)+1}\right)-\ln\left(\frac{-b\cos(x)-ia\sin(x)}{\cos(x)+1}\right)\right)}{2ab}$	53
risch	$\frac{i\ln\left(e^{2ix}-\frac{a+b}{a-b}\right)}{2ab}-\frac{i\ln\left(e^{2ix}-\frac{a-b}{a+b}\right)}{2ab}$	58

```
[In] int(1/(b^2*cos(x)^2+a^2*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] arctan(a*tan(x)/b)/a/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.87

$$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx=-\frac{\arctan\left(\frac{(a^2+b^2)\cos(x)^2-a^2}{2\,ab\cos(x)\sin(x)}\right)}{2\,ab}$$

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="fricas")
[Out] -1/2*arctan(1/2*((a^2 + b^2)*cos(x)^2 - a^2)/(a*b*cos(x)*sin(x)))/(a*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71839 vs. 2(10) = 20.

Time = 18.58 (sec) , antiderivative size = 71839, normalized size of antiderivative = 4789.27

$$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)
```

```
[Out] Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), ((tan(x/2)
/2 - 1/(2*tan(x/2)))/a**2, Eq(b, 0)), (-2*tan(x/2)/(b**2*(tan(x/2)**2 - 1))
, Eq(a, 0)), (x/(b**2*sin(x)**2 + b**2*cos(x)**2), Eq(a, b) | Eq(a, -b)), (
8192*a**15*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a
**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968
*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**1
0*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a
**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 8192*a**15*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqr
```

t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a
2 - b2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(
a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a
2 - b2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 8192*a**14*sqrt(a**2 - b**2)*sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqr
t(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqr

```
t(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b
**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 8192*a**14*sqrt(a**2 - b**2)
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*
sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a
**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 32768*a**13*b**2*sqrt(-2*a
**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
```

b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2
 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
2 + 2*a*sqrt(a2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*
a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 32768*a**13*b**2*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**1
3*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*s
qrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a
9*b8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
2 + 2*a*sqrt(a2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a*
*5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
2 + 2*a*sqrt(a2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*s
qrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*s

```
qrt(a**2 - b**2)/b**2 + 1)) + 2048*a**13*b**2*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14
*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a*
*10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a*
*6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b*
*14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)) - 2048*a**13*b**2*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(
x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(
a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b
**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
```

```
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a*
*2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
 + 1)) + 28672*a**12*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b
**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt
(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b
**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**1
0*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6
*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b
**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**1
4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqr
t(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b
**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
**2)/b**2 + 1)) - 28672*a**12*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192
*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**
2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b*
```

```
*4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33
280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*
b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt
(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5
824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b
**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a
**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1)) - 2048*a**12*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b
**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*
a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2
)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 337
92*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**
2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 240
8*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2
)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
```

```
*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**1
6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1)) + 2048*a**12*b**2*sqrt(a**2 - b**2)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a
**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2
 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqr
t(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**
2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqr
t(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2
 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2
*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 52736*a**11*b**4*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a*
*13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
```

b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792
*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*
a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1)) - 52736*a**11*b**4*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**
14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b
2)/b2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*
a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a
2/b2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*
a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*
b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1)) - 7168*a**11*b**4*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b

2)/b2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + t
an(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
2 + 1) + 45568*a11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6
*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a
2/b2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10
*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a
2/b2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
2 + 1) + 170*a3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt
(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a
2/b2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
2 + 1)) + 7168*a11*b**4*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8
192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b
2)*sqrt(-2*a2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
2 + 1) + 19968*a8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a
2 - b2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2

- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
2 + 1) + 728*a4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**
2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) -
 39424*a**10*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan
(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt
(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a
2/b2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a
2 - b2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1)) + 39424*a**10*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a
2 - b2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b
2*sqrt(a2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt

(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**1
0*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6
*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**1
4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqr
t(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1)) + 6144*a**10*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192
*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**
2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b*
*4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33
280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*
b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt
(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5
824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b
12*sqrt(-2*a2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -

b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a
2*b14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1)) - 6144*a**10*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a
13*b4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 3379
2*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2
)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408
*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1)) - 43520*a**9*b**6*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a*
*14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a
2/b2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280
*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**
2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**

```
8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824
*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**
2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**1
2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a
**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2
*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a
**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1)) + 43520*a**9*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + t
an(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6
*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a
**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10
*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a
**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt
(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a
**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1)) + 9728*a**9*b**6*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8
```

```
192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b
**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a
**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**
2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) -
 9728*a**9*b**6*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b*
*2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 266
24*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*
b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 1
9968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*
b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqr
```

t(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 7
28*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**
14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a
2/b2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 26880*a**8*
b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*
a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 4556
8*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b*
*2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 129
92*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a
2/b2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b*
*2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*
a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 268
80*a**8*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b*
*2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))
/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt

(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqr
t(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a*
*2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a*
*2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 -
b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
) - 6912*a**8*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + ta
n(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqr
t(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*
sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
2 + 1) + 12992*a7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*
sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(

a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1)) + 6912*a**8*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b*
*2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b*
*2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10
*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*
b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b*
*2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14
*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)) + 19392*a**7*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/
2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a*
*2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**
2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**
2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt
(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a
2/b2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b

2)/b2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt
(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a
2/b2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2
 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
2 + 2*a*sqrt(a2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1)) - 19392*a**7*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a
15*b2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568
*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**
2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 1299
2*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**
2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a
3*b14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*s
qrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 6400
*a**7*b**8*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*

```
sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a
**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968
*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**1
0*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a
**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 6400*a**7*b**8*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
```

```
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 9408*a**6*b**8*sqrt(a**2 -
b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**
2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**
2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*
b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8
*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b
**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 9408*a**6*b**8*sqrt(
a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b
**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*
a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**
```

6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
2 - b2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 1996
8*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**
2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**
10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
2 - b2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*
a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 3584*a**6*b**8
*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(
-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**1
5*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a
2 - b2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a*
*11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
2 + 2*a*sqrt(a2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
+ 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a
7*b10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
+ 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3
*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt

(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 3584*a*
*6*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(819
2*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**
2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45
568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 -
b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 1
2992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 -
b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 17
0*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 4
480*a**5*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqr
t(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b*
*2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
2 - b2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 266
24*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*
b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 1
9968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/

b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*
b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 7
28*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**
14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a
2/b2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 4480*a**5*b
10*sqrt(-2*a2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b
2 + 2*a*sqrt(a2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**
2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b*
*4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b
8*sqrt(a2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt
(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**
12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
2 + 1) - 2*a*b16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 2072*a**5*b**10*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt

(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqr
t(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a*
*2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**
2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**
2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a
2 - b2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a*
*2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a
2 - b2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 2072*a**5*b**10*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a*
*13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
2 + 2*a*sqrt(a2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792
*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*
a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*

sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1)) + 1568*a**4*b**10*sqrt(a**2 - b**2)*sqrt(-2*a
2/b2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
2 + 2*a*sqrt(a2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*
a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 1568*a**4*b**10*sqrt(a**2 - b**2)*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqr
t(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -

b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqr
t(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 840*a**4*b**10*sqrt(a**2 - b
2)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
2 - b2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b*
*2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b
4*sqrt(a2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt
(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*
b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b*
*12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
2 - b2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s

qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 840*a**4*b**10*sqrt(a
2 - b2)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a
12*b4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968
*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**1
0*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a
4*b12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 462*a**3*b**12*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*
sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a

```
**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 462*a**3*b**12*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a
**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 307
20*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a
**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b
**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 -
b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b
**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b
**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 292*a**3*b**12*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192
*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**
2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b*
```

```
*4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33
280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*
b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt
(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5
824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b
**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a
**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1)) + 292*a**3*b**12*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
 tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*
sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a
**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**
2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b*
*6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b*
*2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**
10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
```

```
/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1)) - 98*a**2*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14
*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a*
*10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a*
*6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b*
*14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)) + 98*a**2*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*
a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**
4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 332
80*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
```

```
**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b
**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 58
24*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b*
*12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a*
*2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
**2 - b**2)/b**2 + 1)) + 72*a**2*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a
**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**
13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b
**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*
a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b
**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a
**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a
**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1)) - 72*a**2*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2
```

/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 3072
0*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b*
*2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 3
3792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b
2)*sqrt(-2*a2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2
408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b*
*2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b
2)/b2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b*
*16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 14*a*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**1
4*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*s
qrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a
10*b6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b
2)/b2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a

6*b10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b
14*sqrt(a2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1)) + 14*a*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*log(sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2)
)/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
2 + 2*a*sqrt(a2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a
2 - b2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**
2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**
2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a
2 - b2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**
2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**
2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 -
b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
)) + 12*a*b**14*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b
2*sqrt(-2*a2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26

```
624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11
*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7
*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b*
*14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 12*a*b**14
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*
sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a
**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
```

```
 b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + b**14*sqrt(a**2 - b**2)*sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b
**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqr
t(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqr
t(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b
**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - b**14*sqrt(a**2 - b**2)*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a
**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
```

```
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(
a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b
**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a
**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - b**14*sqrt(a**2 - b**2)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**
2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a*
*2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**
2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**
2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**
2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + b**14*sqrt(a**2 - b**2)*sqrt(-2*a
**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*
```

```
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 3
0720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 -
 b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 -
 b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a
*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{b^2 \cos^2(x) + a^2 \sin^2(x)} \, dx = \frac{\arctan\left(\frac{a \tan(x)}{b}\right)}{ab}$$

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(a*tan(x)/b)/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{1}{b^2 \cos^2(x) + a^2 \sin^2(x)}\, dx = \frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor + \arctan\left(\frac{a \tan(x)}{b}\right)}{ab}$$

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")
[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)/b))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{b^2 \cos^2(x) + a^2 \sin^2(x)}\, dx = \frac{\operatorname{atan}\left(\frac{a \tan(x)}{b}\right)}{a\, b}$$

```
[In] int(1/(b^2*cos(x)^2 + a^2*sin(x)^2),x)
[Out] atan((a*tan(x))/b)/(a*b)
```

3.146 $\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx=\frac{\sin(x)}{b(b\cos(x)+a\sin(x))}$$

```
[Out] sin(x)/b/(b*cos(x)+a*sin(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3154}

$$\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx=\frac{\sin(x)}{b(a\sin(x)+b\cos(x))}$$

```
[In] Int[(b*Cos[x] + a*Sin[x])^(-2),x]
```

```
[Out] Sin[x]/(b*(b*Cos[x] + a*Sin[x]))
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral}=\frac{\sin(x)}{b(b\cos(x)+a\sin(x))}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx=\frac{\sin(x)}{b(b\cos(x)+a\sin(x))}$$

```
[In] Integrate[(b*Cos[x] + a*Sin[x])^(-2),x]
[Out] Sin[x]/(b*(b*Cos[x] + a*Sin[x]))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{1}{a(a\tan(x)+b)}$	14
parallelrisch	$\frac{\sin(x)}{b(b\cos(x)+a\sin(x))}$	18
norman	$\frac{-\frac{1}{a}+\frac{\tan^2(\frac{x}{2})}{a}}{-b(\tan^2(\frac{x}{2}))+2a\tan(\frac{x}{2})+b}$	38
risch	$-\frac{2i}{(a\,e^{2ix}+ib\,e^{2ix}-a+ib)(ib+a)}$	38

```
[In] int(1/(b*cos(x)+a*sin(x))^2,x,method=_RETURNVERBOSE)
[Out] -1/a/(a*tan(x)+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx=-\frac{a\cos(x)-b\sin(x)}{(a^2b+b^3)\cos(x)+(a^3+ab^2)\sin(x)}$$

```
[In] integrate(1/(b*cos(x)+a*sin(x))^2,x, algorithm="fricas")
[Out] -(a*cos(x) - b*sin(x))/((a^2*b + b^3)*cos(x) + (a^3 + a*b^2)*sin(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(14) = 28$.

Time = 134.57 (sec) , antiderivative size = 602, normalized size of antiderivative = 35.41

$$\int \frac{1}{(b\cos(x) + a\sin(x))^2}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} \\ \frac{x\tan^4\left(\frac{x}{2}\right)}{2b^2\sin^2(x)\tan^4\left(\frac{x}{2}\right)-4b^2\sin^2(x)\tan^2\left(\frac{x}{2}\right)+2b^2\sin^2(x)+8b^2\sin(x)\cos(x)\tan^3\left(\frac{x}{2}\right)-8b^2\sin(x)\cos(x)\tan\left(\frac{x}{2}\right)+8b^2\cos^2(x)\tan^2\left(\frac{x}{2}\right)} + \frac{}{2b^2\mathrm{s}} \\ \frac{\frac{\tan\left(\frac{x}{2}\right)}{2}-\frac{1}{2\tan\left(\frac{x}{2}\right)}}{a^2} \\ \frac{2\tan\left(\frac{x}{2}\right)}{2ab\tan\left(\frac{x}{2}\right)-b^2\tan^2\left(\frac{x}{2}\right)+b^2} \end{cases}$$

```
[In] integrate(1/(b*cos(x)+a*sin(x))**2,x)
```

```
[Out] Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), (x*tan(x/2
)**4/(2*b**2*sin(x)**2*tan(x/2)**4 - 4*b**2*sin(x)**2*tan(x/2)**2 + 2*b**2*
sin(x)**2 + 8*b**2*sin(x)*cos(x)*tan(x/2)**3 - 8*b**2*sin(x)*cos(x)*tan(x/2
) + 8*b**2*cos(x)**2*tan(x/2)**2) + 2*x*tan(x/2)**2/(2*b**2*sin(x)**2*tan(x
/2)**4 - 4*b**2*sin(x)**2*tan(x/2)**2 + 2*b**2*sin(x)**2 + 8*b**2*sin(x)*co
s(x)*tan(x/2)**3 - 8*b**2*sin(x)*cos(x)*tan(x/2) + 8*b**2*cos(x)**2*tan(x/2
)**2) + x/(2*b**2*sin(x)**2*tan(x/2)**4 - 4*b**2*sin(x)**2*tan(x/2)**2 + 2*
b**2*sin(x)**2 + 8*b**2*sin(x)*cos(x)*tan(x/2)**3 - 8*b**2*sin(x)*cos(x)*ta
n(x/2) + 8*b**2*cos(x)**2*tan(x/2)**2) + 2*tan(x/2)**3/(2*b**2*sin(x)**2*ta
n(x/2)**4 - 4*b**2*sin(x)**2*tan(x/2)**2 + 2*b**2*sin(x)**2 + 8*b**2*sin(x)
*cos(x)*tan(x/2)**3 - 8*b**2*sin(x)*cos(x)*tan(x/2) + 8*b**2*cos(x)**2*tan(
x/2)**2) - 2*tan(x/2)/(2*b**2*sin(x)**2*tan(x/2)**4 - 4*b**2*sin(x)**2*tan(
x/2)**2 + 2*b**2*sin(x)**2 + 8*b**2*sin(x)*cos(x)*tan(x/2)**3 - 8*b**2*sin(
x)*cos(x)*tan(x/2) + 8*b**2*cos(x)**2*tan(x/2)**2), Eq(a, b*(tan(x/2) - 1)*
(tan(x/2) + 1)/(2*tan(x/2)))), ((tan(x/2)/2 - 1/(2*tan(x/2)))/a**2, Eq(b, 0
)), (2*tan(x/2)/(2*a*b*tan(x/2) - b**2*tan(x/2)**2 + b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{(b\cos(x) + a\sin(x))^2}\,dx = -\frac{1}{a^2\tan(x) + ab}$$

```
[In] integrate(1/(b*cos(x)+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -1/(a^2*tan(x) + a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{(b\cos(x) + a\sin(x))^2}\,dx = -\frac{1}{(a\tan{(x)} + b)a}$$

```
[In] integrate(1/(b*cos(x)+a*sin(x))^2,x, algorithm="giac")
[Out] -1/((a*tan(x) + b)*a)
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{1}{(b\cos(x) + a\sin(x))^2}\,dx = \frac{2\tan\left(\frac{x}{2}\right)}{b\left(-b\tan\left(\frac{x}{2}\right)^2 + 2\,a\tan\left(\frac{x}{2}\right) + b\right)}$$

```
[In] int(1/(b*cos(x) + a*sin(x))^2,x)
[Out] (2*tan(x/2))/(b*(b + 2*a*tan(x/2) - b*tan(x/2)^2))
```

3.147 $\int \frac{\sin(x)}{1+\cos(x)+\sin(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)} dx = \frac{x}{2} - \frac{1}{2}\log(1+\cos(x)+\sin(x)) - \frac{1}{2}\log\left(1+\tan\left(\frac{x}{2}\right)\right)$$

```
[Out] 1/2*x-1/2*ln(1+cos(x)+sin(x))-1/2*ln(1+tan(1/2*x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3216, 3203, 31}

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)} dx = \frac{x}{2} - \frac{1}{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right) - \frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

```
[In] Int[Sin[x]/(1 + Cos[x] + Sin[x]),x]
```

```
[Out] x/2 - Log[1 + Cos[x] + Sin[x]]/2 - Log[1 + Tan[x/2]]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
```

```
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3216

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[b*C*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x}{2} - \frac{1}{2}\log(1+\cos(x)+\sin(x)) - \frac{1}{2}\int \frac{1}{1+\cos(x)+\sin(x)}\,dx \\
&= \frac{x}{2} - \frac{1}{2}\log(1+\cos(x)+\sin(x)) - \text{Subst}\left(\int \frac{1}{2+2x}\,dx, x, \tan\left(\frac{x}{2}\right)\right) \\
&= \frac{x}{2} - \frac{1}{2}\log(1+\cos(x)+\sin(x)) - \frac{1}{2}\log\left(1+\tan\left(\frac{x}{2}\right)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$
\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{x}{2} - \log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)
$$

```
[In] Integrate[Sin[x]/(1 + Cos[x] + Sin[x]),x]
[Out] x/2 - Log[Cos[x/2] + Sin[x/2]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{x}{2}+\frac{ix}{2}-\ln\left(i+e^{ix}\right)$	20
default	$\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{2}+\arctan\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	27
parallelrisch	$-\ln\left(-\frac{(\cot(x)-1-\csc(x))\sqrt{2}}{2}\right)+\ln\left(\sqrt{\frac{1}{\cos(x)+1}}\right)+\frac{x}{2}$	30
norman	$\frac{\frac{x}{2}+\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}}{1+\tan^2\left(\frac{x}{2}\right)}-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)+\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{2}$	46

```
[In] int(sin(x)/(1+cos(x)+sin(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/2*I*x-ln(I+exp(I*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.37

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{1}{2}\,x - \frac{1}{2}\,\log\left(\sin\left(x\right)+1\right)$$

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="fricas")
[Out] 1/2*x - 1/2*log(sin(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{x}{2} - \log\left(\tan\left(\frac{x}{2}\right)+1\right) + \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{2}$$

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x)
[Out] x/2 - log(tan(x/2) + 1) + log(tan(x/2)**2 + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) - \log\left(\frac{\sin(x)}{\cos(x)+1}+1\right) + \frac{1}{2}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) - log(sin(x)/(cos(x) + 1) + 1) + 1/2*log(sin(x)
^2/(cos(x) + 1)^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{1}{2}x + \frac{1}{2}\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right) - \log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)$$

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*log(tan(1/2*x)^2 + 1) - log(abs(tan(1/2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = -\ln\left(\tan\left(\frac{x}{2}\right)+1\right) + \ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right) + \ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

```
[In] int(sin(x)/(cos(x) + sin(x) + 1),x)
```

```
[Out] log(tan(x/2) - 1i)*(1/2 - 1i/2) - log(tan(x/2) + 1) + log(tan(x/2) + 1i)*(1
/2 + 1i/2)
```

3.148 $\int \sqrt{3-x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \sqrt{3-x^2}\,dx = \frac{1}{2}x\sqrt{3-x^2} + \frac{3}{2}\arcsin\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] 3/2*arcsin(1/3*x*3^(1/2))+1/2*x*(-x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {201, 222}

$$\int \sqrt{3-x^2}\,dx = \frac{3}{2}\arcsin\left(\frac{x}{\sqrt{3}}\right) + \frac{1}{2}\sqrt{3-x^2}x$$

```
[In] Int[Sqrt[3 - x^2],x]
```

```
[Out] (x*Sqrt[3 - x^2])/2 + (3*ArcSin[x/Sqrt[3]])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x\sqrt{3-x^2}+\frac{3}{2}\int \frac{1}{\sqrt{3-x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{3-x^2}+\frac{3}{2}\arcsin\left(\frac{x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \sqrt{3-x^2}\,dx = \frac{1}{2}x\sqrt{3-x^2}+3\arctan\left(\frac{-\sqrt{3}+x}{\sqrt{3-x^2}}\right)$$

```
[In] Integrate[Sqrt[3 - x^2],x]
[Out] (x*Sqrt[3 - x^2])/2 + 3*ArcTan[(-Sqrt[3] + x)/Sqrt[3 - x^2]]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{3\arcsin\left(\frac{x\sqrt{3}}{3}\right)}{2}+\frac{x\sqrt{-x^2+3}}{2}$	23
risch	$-\frac{x\left(x^2-3\right)}{2\sqrt{-x^2+3}}+\frac{3\arcsin\left(\frac{x\sqrt{3}}{3}\right)}{2}$	28
pseudoelliptic	$\frac{x\sqrt{-x^2+3}}{2}-\frac{3\arctan\left(\frac{\sqrt{-x^2+3}}{x}\right)}{2}$	30
meijerg	$\frac{3i\left(-\frac{2i\sqrt{\pi}\,x\sqrt{3}\,\sqrt{-\frac{x^2}{3}+1}}{3}-2i\sqrt{\pi}\,\arcsin\left(\frac{x\sqrt{3}}{3}\right)\right)}{4\sqrt{\pi}}$	40
trager	$\frac{x\sqrt{-x^2+3}}{2}+\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+3}+x\right)}{2}$	41

```
[In] int((-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 3/2*arcsin(1/3*x*3^(1/2))+1/2*x*(-x^2+3)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \sqrt{3-x^2}\,dx = \frac{1}{2}\,\sqrt{-x^2+3}x - \frac{3}{2}\,\arctan\left(\frac{\sqrt{-x^2+3}}{x}\right)$$

```
[In] integrate((-x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 3)*x - 3/2*arctan(sqrt(-x^2 + 3)/x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \sqrt{3-x^2}\,dx = \frac{x\sqrt{3-x^2}}{2} + \frac{3\,\mathrm{asin}\left(\frac{\sqrt{3}x}{3}\right)}{2}$$

```
[In] integrate((-x**2+3)**(1/2),x)
[Out] x*sqrt(3 - x**2)/2 + 3*asin(sqrt(3)*x/3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \sqrt{3-x^2}\,dx = \frac{1}{2}\,\sqrt{-x^2+3}x + \frac{3}{2}\,\arcsin\left(\frac{1}{3}\,\sqrt{3}x\right)$$

```
[In] integrate((-x^2+3)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 3)*x + 3/2*arcsin(1/3*sqrt(3)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \sqrt{3-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+3}x + \frac{3}{2}\arcsin\left(\frac{1}{3}\sqrt{3}x\right)$$

```
[In] integrate((-x^2+3)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 3)*x + 3/2*arcsin(1/3*sqrt(3)*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \sqrt{3-x^2}\,dx = \frac{3\,\mathrm{asin}\left(\frac{\sqrt{3}\,x}{3}\right)}{2} + \frac{x\,\sqrt{3-x^2}}{2}$$

```
[In] int((3 - x^2)^(1/2),x)
[Out] (3*asin((3^(1/2)*x)/3))/2 + (x*(3 - x^2)^(1/2))/2
```

3.149 $\int \frac{x}{\sqrt{3-x^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x}{\sqrt{3-x^2}} \, dx = -\sqrt{3-x^2}$$

```
[Out] -(-x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {267}

$$\int \frac{x}{\sqrt{3-x^2}} \, dx = -\sqrt{3-x^2}$$

```
[In] Int[x/Sqrt[3 - x^2],x]
```

```
[Out] -Sqrt[3 - x^2]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\sqrt{3-x^2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{3-x^2}}\,dx=-\sqrt{3-x^2}$$

```
[In] Integrate[x/Sqrt[3 - x^2],x]
[Out] -Sqrt[3 - x^2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\sqrt{-x^2+3}$	12
derivativedivides	$-\sqrt{-x^2+3}$	12
default	$-\sqrt{-x^2+3}$	12
trager	$-\sqrt{-x^2+3}$	12
pseudoelliptic	$-\sqrt{-x^2+3}$	12
risch	$\frac{x^2-3}{\sqrt{-x^2+3}}$	16
meijerg	$-\frac{\sqrt{3}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-\frac{x^2}{3}+1}\right)}{2\sqrt{\pi}}$	29

```
[In] int(x/(-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-x^2+3)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{3-x^2}}\,dx=-\sqrt{-x^2+3}$$

```
[In] integrate(x/(-x^2+3)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{x}{\sqrt{3-x^2}}\,dx = -\sqrt{3-x^2}$$

```
[In] integrate(x/(-x**2+3)**(1/2),x)
[Out] -sqrt(3 - x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{3-x^2}}\,dx = -\sqrt{-x^2+3}$$

```
[In] integrate(x/(-x^2+3)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{3-x^2}}\,dx = -\sqrt{-x^2+3}$$

```
[In] integrate(x/(-x^2+3)^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{3-x^2}}\,dx = -\sqrt{3-x^2}$$

```
[In] int(x/(3 - x^2)^(1/2),x)
[Out] -(3 - x^2)^(1/2)
```

3.150 $\int \frac{\sqrt{3-x^2}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{\sqrt{3-x^2}}{x} \, dx = \sqrt{3-x^2} - \sqrt{3}\text{arctanh}\left(\frac{\sqrt{3-x^2}}{\sqrt{3}}\right)$$

```
[Out] -arctanh(1/3*(-x^2+3)^(1/2)*3^(1/2))*3^(1/2)+(-x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {272, 52, 65, 212}

$$\int \frac{\sqrt{3-x^2}}{x} \, dx = \sqrt{3-x^2} - \sqrt{3}\text{arctanh}\left(\frac{\sqrt{3-x^2}}{\sqrt{3}}\right)$$

```
[In] Int[Sqrt[3 - x^2]/x,x]
```

```
[Out] Sqrt[3 - x^2] - Sqrt[3]*ArcTanh[Sqrt[3 - x^2]/Sqrt[3]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{3-x}}{x}\,dx, x, x^2\right)\\
&= \sqrt{3-x^2}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3-x}x}\,dx, x, x^2\right)\\
&= \sqrt{3-x^2}-3\text{Subst}\left(\int \frac{1}{3-x^2}\,dx, x, \sqrt{3-x^2}\right)\\
&= \sqrt{3-x^2}-\sqrt{3}\text{arctanh}\left(\frac{\sqrt{3-x^2}}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{3-x^2}}{x}\,dx = \sqrt{3-x^2}-\sqrt{3}\text{arctanh}\left(\sqrt{1-\frac{x^2}{3}}\right)$$

```
[In] Integrate[Sqrt[3 - x^2]/x,x]
[Out] Sqrt[3 - x^2] - Sqrt[3]*ArcTanh[Sqrt[1 - x^2/3]]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
default	$\sqrt{-x^{2}+3}-\sqrt{3}\, \operatorname{arctanh}\left(\frac{\sqrt{3}}{\sqrt{-x^{2}+3}}\right)$	30
pseudoelliptic	$-\operatorname{arctanh}\left(\frac{\sqrt{-x^{2}+3}\,\sqrt{3}}{3}\right) \sqrt{3}+\sqrt{-x^{2}+3}$	31
trager	$\sqrt{-x^{2}+3}+\operatorname{RootOf}\left(\_Z^{2}-3\right) \ln\left(\frac{\sqrt{-x^{2}+3}-\operatorname{RootOf}\left(\_Z^{2}-3\right)}{x}\right)$	41
meijerg	$-\frac{\sqrt{3}\left(-2(2-2\ln(2)+2\ln(x)-\ln(3)+i\pi)\sqrt{\pi}+4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{-\frac{x^{2}}{3}+1}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{x^{2}}{3}+1}}{2}\right)\right)}{4\sqrt{\pi}}$	71

```
[In] int((-x^2+3)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] (-x^2+3)^(1/2)-3^(1/2)*arctanh(3^(1/2)/(-x^2+3)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{3-x^{2}}}{x}\, dx = \frac{1}{2}\sqrt{3}\log\left(-\frac{x^{2}+2\sqrt{3}\sqrt{-x^{2}+3}-6}{x^{2}}\right)+\sqrt{-x^{2}+3}$$

```
[In] integrate((-x^2+3)^(1/2)/x,x, algorithm="fricas")
[Out] 1/2*sqrt(3)*log(-(x^2 + 2*sqrt(3)*sqrt(-x^2 + 3) - 6)/x^2) + sqrt(-x^2 + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{3-x^{2}}}{x}\, dx = \begin{cases} i\sqrt{x^{2}-3}-\sqrt{3}\log(x)+\frac{\sqrt{3}\log(x^{2})}{2}+\sqrt{3}i\operatorname{asin}\left(\frac{\sqrt{3}}{x}\right) & \text{for } |x^{2}|>3 \\ \sqrt{3-x^{2}}+\frac{\sqrt{3}\log(x^{2})}{2}-\sqrt{3}\log\left(\sqrt{1-\frac{x^{2}}{3}}+1\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((-x**2+3)**(1/2)/x,x)
[Out] Piecewise((I*sqrt(x**2 - 3) - sqrt(3)*log(x) + sqrt(3)*log(x**2)/2 + sqrt(3
)*I*asin(sqrt(3)/x), Abs(x**2) > 3), (sqrt(3 - x**2) + sqrt(3)*log(x**2)/2
- sqrt(3)*log(sqrt(1 - x**2/3) + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{3-x^2}}{x}\,dx = -\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{-x^2+3}}{|x|}+\frac{6}{|x|}\right)+\sqrt{-x^2+3}$$

```
[In] integrate((-x^2+3)^(1/2)/x,x, algorithm="maxima")
[Out] -sqrt(3)*log(2*sqrt(3)*sqrt(-x^2 + 3)/abs(x) + 6/abs(x)) + sqrt(-x^2 + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{3-x^2}}{x}\,dx = \frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt{3}-\sqrt{-x^2+3}}{\sqrt{3}+\sqrt{-x^2+3}}\right)+\sqrt{-x^2+3}$$

```
[In] integrate((-x^2+3)^(1/2)/x,x, algorithm="giac")
[Out] 1/2*sqrt(3)*log((sqrt(3) - sqrt(-x^2 + 3))/(sqrt(3) + sqrt(-x^2 + 3))) + sq
rt(-x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{3-x^2}}{x}\,dx = \sqrt{3}\,\ln\left(\sqrt{\frac{3}{x^2}-1}-\sqrt{3}\sqrt{\frac{1}{x^2}}\right)+\sqrt{3-x^2}$$

```
[In] int((3 - x^2)^(1/2)/x,x)
[Out] 3^(1/2)*log((3/x^2 - 1)^(1/2) - 3^(1/2)*(1/x^2)^(1/2)) + (3 - x^2)^(1/2)
```

3.151 $\int \frac{\sqrt{x+x^2}}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{\sqrt{x+x^2}}{x} \, dx = \sqrt{x+x^2} + \operatorname{arctanh}\left(\frac{x}{\sqrt{x+x^2}}\right)$$

```
[Out] arctanh(x/(x^2+x)^(1/2))+(x^2+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {678, 634, 212}

$$\int \frac{\sqrt{x+x^2}}{x} \, dx = \operatorname{arctanh}\left(\frac{x}{\sqrt{x^2+x}}\right) + \sqrt{x^2+x}$$

```
[In] Int[Sqrt[x + x^2]/x,x]
```

```
[Out] Sqrt[x + x^2] + ArcTanh[x/Sqrt[x + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \sqrt{x+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{x+x^2}}\,dx \\
&= \sqrt{x+x^2}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{x+x^2}}\right) \\
&= \sqrt{x+x^2}+\text{arctanh}\left(\frac{x}{\sqrt{x+x^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{x+x^2}}{x}\,dx=\sqrt{x(1+x)}\left(1-\frac{\log\left(-\sqrt{x}+\sqrt{1+x}\right)}{\sqrt{x}\sqrt{1+x}}\right)$$

```
[In] Integrate[Sqrt[x + x^2]/x,x]
[Out] Sqrt[x*(1 + x)]*(1 - Log[-Sqrt[x] + Sqrt[1 + x]]/(Sqrt[x]*Sqrt[1 + x]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

method	result	size
default	$\sqrt{x^2+x}+\frac{\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{2}$	22
trager	$\sqrt{x^2+x}+\frac{\ln\left(1+2x+2\sqrt{x^2+x}\right)}{2}$	26
risch	$\frac{x(1+x)}{\sqrt{x(1+x)}}+\frac{\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{2}$	27
meijerg	$-\frac{-2\sqrt{\pi}\,\sqrt{x}\,\sqrt{1+x}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{x}\right)}{2\sqrt{\pi}}$	29
pseudoelliptic	$\frac{\ln\left(\frac{\sqrt{x(1+x)}+x}{x}\right)}{2}-\frac{\ln\left(\frac{\sqrt{x(1+x)}-x}{x}\right)}{2}+\sqrt{x\,(1+x)}$	43

```
[In] int((x^2+x)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] (x^2+x)^(1/2)+1/2*ln(x+1/2+(x^2+x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{x+x^2}}{x}\,dx=\sqrt{x^2+x}-\frac{1}{2}\log\left(-2\,x+2\,\sqrt{x^2+x}-1\right)$$

```
[In] integrate((x^2+x)^(1/2)/x,x, algorithm="fricas")
[Out] sqrt(x^2 + x) - 1/2*log(-2*x + 2*sqrt(x^2 + x) - 1)
```

Sympy [F]

$$\int \frac{\sqrt{x+x^2}}{x}\,dx=\int \frac{\sqrt{x\,(x+1)}}{x}\,dx$$

```
[In] integrate((x**2+x)**(1/2)/x,x)
[Out] Integral(sqrt(x*(x + 1))/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{x+x^2}}{x}\,dx = \sqrt{x^2+x} + \frac{1}{2}\,\log\left(2\,x + 2\,\sqrt{x^2+x} + 1\right)$$

```
[In] integrate((x^2+x)^(1/2)/x,x, algorithm="maxima")
[Out] sqrt(x^2 + x) + 1/2*log(2*x + 2*sqrt(x^2 + x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{x+x^2}}{x}\,dx = \sqrt{x^2+x} - \frac{1}{2}\,\log\left(\left|-2\,x + 2\,\sqrt{x^2+x} - 1\right|\right)$$

```
[In] integrate((x^2+x)^(1/2)/x,x, algorithm="giac")
[Out] sqrt(x^2 + x) - 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{x+x^2}}{x}\,dx = \frac{\ln\left(x + \sqrt{x\,(x+1)} + \frac{1}{2}\right)}{2} + \sqrt{x^2+x}$$

```
[In] int((x + x^2)^(1/2)/x,x)
[Out] log(x + (x*(x + 1))^(1/2) + 1/2)/2 + (x + x^2)^(1/2)
```

3.152 $\int \sqrt{5+x^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 27

$$\int \sqrt{5+x^2}\,dx=\frac{1}{2}x\sqrt{5+x^2}+\frac{5}{2}\text{arcsinh}\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] 5/2*arcsinh(1/5*x*5^(1/2))+1/2*x*(x^2+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {201, 221}

$$\int \sqrt{5+x^2}\,dx=\frac{5}{2}\text{arcsinh}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^2+5}x$$

```
[In] Int[Sqrt[5 + x^2],x]
```

```
[Out] (x*Sqrt[5 + x^2])/2 + (5*ArcSinh[x/Sqrt[5]])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x\sqrt{5+x^2} + \frac{5}{2}\int \frac{1}{\sqrt{5+x^2}}\,dx$$
$$= \frac{1}{2}x\sqrt{5+x^2} + \frac{5}{2}\text{arcsinh}\left(\frac{x}{\sqrt{5}}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \sqrt{5+x^2}\,dx = \frac{1}{2}x\sqrt{5+x^2} - \frac{5}{2}\log\left(-x+\sqrt{5+x^2}\right)$$

```
[In] Integrate[Sqrt[5 + x^2],x]
[Out] (x*Sqrt[5 + x^2])/2 - (5*Log[-x + Sqrt[5 + x^2]])/2
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{5\ \text{arcsinh}\left(\frac{x\sqrt{5}}{5}\right)}{2}+\frac{x\sqrt{x^2+5}}{2}$	21
risch	$\frac{5\ \text{arcsinh}\left(\frac{x\sqrt{5}}{5}\right)}{2}+\frac{x\sqrt{x^2+5}}{2}$	21
trager	$\frac{x\sqrt{x^2+5}}{2}-\frac{5\ln\left(x-\sqrt{x^2+5}\right)}{2}$	26
meijerg	$-\frac{5\left(-\frac{2\sqrt{\pi}\,x\sqrt{5}\,\sqrt{1+\frac{x^2}{5}}}{5}-2\sqrt{\pi}\ \text{arcsinh}\left(\frac{x\sqrt{5}}{5}\right)\right)}{4\sqrt{\pi}}$	37
pseudoelliptic	$\frac{x\sqrt{x^2+5}}{2}+\frac{5\ln\left(\frac{\sqrt{x^2+5}+x}{x}\right)}{4}-\frac{5\ln\left(\frac{\sqrt{x^2+5}-x}{x}\right)}{4}$	46

```
[In] int((x^2+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] 5/2*arcsinh(1/5*x*5^(1/2))+1/2*x*(x^2+5)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sqrt{5+x^2}\, dx = \frac{1}{2}\sqrt{x^2+5}x - \frac{5}{2}\log\left(-x+\sqrt{x^2+5}\right)$$

```
[In] integrate((x^2+5)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(x^2 + 5)*x - 5/2*log(-x + sqrt(x^2 + 5))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \sqrt{5+x^2}\, dx = \frac{x\sqrt{x^2+5}}{2} + \frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}x}{5}\right)}{2}$$

```
[In] integrate((x**2+5)**(1/2),x)
[Out] x*sqrt(x**2 + 5)/2 + 5*asinh(sqrt(5)*x/5)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \sqrt{5+x^2}\, dx = \frac{1}{2}\sqrt{x^2+5}x + \frac{5}{2}\,\mathrm{arsinh}\left(\frac{1}{5}\sqrt{5}x\right)$$

```
[In] integrate((x^2+5)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^2 + 5)*x + 5/2*arcsinh(1/5*sqrt(5)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sqrt{5+x^2}\, dx = \frac{1}{2}\sqrt{x^2+5}x - \frac{5}{2}\log\left(-x+\sqrt{x^2+5}\right)$$

```
[In] integrate((x^2+5)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 + 5)*x - 5/2*log(-x + sqrt(x^2 + 5))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \sqrt{5+x^2}\, dx = \frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x}{5}\right)}{2} + \frac{x\,\sqrt{x^2+5}}{2}$$

```
[In] int((x^2 + 5)^(1/2),x)
[Out] (5*asinh((5^(1/2)*x)/5))/2 + (x*(x^2 + 5)^(1/2))/2
```

3.153 $\int \frac{x}{\sqrt{1+x+x^2}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{x}{\sqrt{1+x+x^2}} \, dx = \sqrt{1+x+x^2} - \frac{1}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

```
[Out] -1/2*arcsinh(1/3*(1+2*x)*3^(1/2))+(x^2+x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {654, 633, 221}

$$\int \frac{x}{\sqrt{1+x+x^2}} \, dx = \sqrt{x^2+x+1} - \frac{1}{2}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[In] Int[x/Sqrt[1 + x + x^2],x]
```

```
[Out] Sqrt[1 + x + x^2] - ArcSinh[(1 + 2*x)/Sqrt[3]]/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \sqrt{1+x+x^2}-\frac{1}{2}\int\frac{1}{\sqrt{1+x+x^2}}\,dx\\ &= \sqrt{1+x+x^2}-\frac{\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+2x\right)}{2\sqrt{3}}\\ &= \sqrt{1+x+x^2}-\frac{1}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int\frac{x}{\sqrt{1+x+x^2}}\,dx=\sqrt{1+x+x^2}+\frac{1}{2}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

```
[In] Integrate[x/Sqrt[1 + x + x^2],x]
[Out] Sqrt[1 + x + x^2] + Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]/2
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
default	$\sqrt{x^2+x+1}-\frac{\text{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{2}$	21
risch	$\sqrt{x^2+x+1}-\frac{\text{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{2}$	21
trager	$\sqrt{x^2+x+1}-\frac{\ln\left(2x+1+2\sqrt{x^2+x+1}\right)}{2}$	28

```
[In] int(x/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] (x^2+x+1)^(1/2)-1/2*arcsinh(2/3*3^(1/2)*(x+1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1+x+x^2}}\,dx = \sqrt{x^2+x+1} + \frac{1}{2}\log\left(-2\,x + 2\,\sqrt{x^2+x+1} - 1\right)$$

```
[In] integrate(x/(x^2+x+1)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^2 + x + 1) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x}{\sqrt{1+x+x^2}}\,dx = \sqrt{x^2+x+1} - \frac{\operatorname{asinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{2}$$

```
[In] integrate(x/(x**2+x+1)**(1/2),x)
[Out] sqrt(x**2 + x + 1) - asinh(2*sqrt(3)*(x + 1/2)/3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x}{\sqrt{1+x+x^2}}\,dx = \sqrt{x^2+x+1} - \frac{1}{2}\,\operatorname{arsinh}\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)$$

```
[In] integrate(x/(x^2+x+1)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 + x + 1) - 1/2*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1+x+x^2}}\,dx = \sqrt{x^2+x+1} + \frac{1}{2}\log\left(-2\,x + 2\,\sqrt{x^2+x+1} - 1\right)$$

```
[In] integrate(x/(x^2+x+1)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 + x + 1) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{1+x+x^2}}\,dx = \sqrt{x^2+x+1} - \frac{\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{2}$$

```
[In] int(x/(x + x^2 + 1)^(1/2),x)
[Out] (x + x^2 + 1)^(1/2) - log(x + (x + x^2 + 1)^(1/2) + 1/2)/2
```

3.154 $\int \frac{1}{\sqrt{x+x^2}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{\sqrt{x+x^2}}\,dx=2\text{arctanh}\left(\frac{x}{\sqrt{x+x^2}}\right)$$

```
[Out] 2*arctanh(x/(x^2+x)^(1/2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {634, 212}

$$\int \frac{1}{\sqrt{x+x^2}}\,dx=2\text{arctanh}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

```
[In] Int[1/Sqrt[x + x^2],x]
```

```
[Out] 2*ArcTanh[x/Sqrt[x + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{x+x^2}}\right)\\ &= 2\text{arctanh}\left(\frac{x}{\sqrt{x+x^2}}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(14) = 28.

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.79

$$\int \frac{1}{\sqrt{x+x^2}}\,dx = -\frac{2\sqrt{x}\sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)}{\sqrt{x(1+x)}}$$

```
[In] Integrate[1/Sqrt[x + x^2],x]
[Out] (-2*Sqrt[x]*Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]])/Sqrt[x*(1 + x)]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

method	result	size
meijerg	$2\,\text{arcsinh}\left(\sqrt{x}\right)$	7
default	$\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)$	12
pseudoelliptic	$2\,\text{arctanh}\left(\frac{\sqrt{x(1+x)}}{x}\right)$	15
trager	$\ln\left(1+2x+2\sqrt{x^2+x}\right)$	16

```
[In] int(1/(x^2+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arcsinh(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{x+x^2}}\,dx = -\log\left(-2\,x+2\,\sqrt{x^2+x}-1\right)$$

```
[In] integrate(1/(x^2+x)^(1/2),x, algorithm="fricas")
[Out] -log(-2*x + 2*sqrt(x^2 + x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{x+x^2}}\,dx = \log\left(2x+2\sqrt{x^2+x}+1\right)$$

```
[In] integrate(1/(x**2+x)**(1/2),x)
[Out] log(2*x + 2*sqrt(x**2 + x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{x+x^2}}\,dx = \log\left(2\,x+2\,\sqrt{x^2+x}+1\right)$$

```
[In] integrate(1/(x^2+x)^(1/2),x, algorithm="maxima")
[Out] log(2*x + 2*sqrt(x^2 + x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{1}{\sqrt{x+x^2}}\,dx = \frac{1}{4}\,\sqrt{x^2+x}(2\,x+1)+\frac{1}{8}\,\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)$$

```
[In] integrate(1/(x^2+x)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(x^2 + x)*(2*x + 1) + 1/8*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{x+x^2}}\,dx = \ln\left(x+\sqrt{x\,(x+1)}+\frac{1}{2}\right)$$

```
[In] int(1/(x + x^2)^(1/2),x)
[Out] log(x + (x*(x + 1))^(1/2) + 1/2)
```

3.155 $\int \frac{\sqrt{2-x-x^2}}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 68

$$\int \frac{\sqrt{2-x-x^2}}{x^2} dx = -\frac{\sqrt{2-x-x^2}}{x} + \arcsin\left(\frac{1}{3}(-1-2x)\right) + \frac{\operatorname{arctanh}\left(\frac{4-x}{2\sqrt{2}\sqrt{2-x-x^2}}\right)}{2\sqrt{2}}$$

```
[Out] -arcsin(1/3+2/3*x)+1/4*arctanh(1/4*(4-x)*2^(1/2)/(-x^2-x+2)^(1/2))*2^(1/2)-
(-x^2-x+2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {746, 857, 633, 222, 738, 212}

$$\int \frac{\sqrt{2-x-x^2}}{x^2} dx = \arcsin\left(\frac{1}{3}(-2x-1)\right) + \frac{\operatorname{arctanh}\left(\frac{4-x}{2\sqrt{2}\sqrt{-x^2-x+2}}\right)}{2\sqrt{2}} - \frac{\sqrt{-x^2-x+2}}{x}$$

```
[In] Int[Sqrt[2 - x - x^2]/x^2,x]
```

```
[Out] -(Sqrt[2 - x - x^2]/x) + ArcSin[(-1 - 2*x)/3] + ArcTanh[(4 - x)/(2*Sqrt[2]*
Sqrt[2 - x - x^2])]/(2*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{2-x-x^2}}{x}+\frac{1}{2}\int \frac{-1-2x}{x\sqrt{2-x-x^2}}\,dx \\
&= -\frac{\sqrt{2-x-x^2}}{x}-\frac{1}{2}\int \frac{1}{x\sqrt{2-x-x^2}}\,dx-\int \frac{1}{\sqrt{2-x-x^2}}\,dx \\
&= -\frac{\sqrt{2-x-x^2}}{x}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx,x,-1-2x\right) \\
&\quad +\text{Subst}\left(\int \frac{1}{8-x^2}\,dx,x,\frac{4-x}{\sqrt{2-x-x^2}}\right)
\end{aligned}$$

$$= -\frac{\sqrt{2-x-x^2}}{x} + \arcsin\left(\frac{1}{3}(-1-2x)\right) + \frac{\operatorname{arctanh}\left(\frac{4-x}{2\sqrt{2}\sqrt{2-x-x^2}}\right)}{2\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx = -\frac{\sqrt{2-x-x^2}}{x} + 2\arctan\left(\frac{\sqrt{2-x-x^2}}{2+x}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2-x-x^2}}{\sqrt{2}(-1+x)}\right)}{\sqrt{2}}$$

```
[In] Integrate[Sqrt[2 - x - x^2]/x^2,x]
[Out] -(Sqrt[2 - x - x^2]/x) + 2*ArcTan[Sqrt[2 - x - x^2]/(2 + x)] - ArcTanh[Sqrt
[2 - x - x^2]/(Sqrt[2]*(-1 + x))]/Sqrt[2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

method	result
risch	$\frac{x^2+x-2}{x\sqrt{-x^2-x+2}} - \arcsin\left(\frac{1}{3}+\frac{2x}{3}\right) + \frac{\operatorname{arctanh}\left(\frac{(4-x)\sqrt{2}}{4\sqrt{-x^2-x+2}}\right)\sqrt{2}}{4}$
default	$-\frac{(-x^2-x+2)^{\frac{3}{2}}}{2x} - \frac{\sqrt{-x^2-x+2}}{4} - \arcsin\left(\frac{1}{3}+\frac{2x}{3}\right) + \frac{\operatorname{arctanh}\left(\frac{(4-x)\sqrt{2}}{4\sqrt{-x^2-x+2}}\right)\sqrt{2}}{4} + \frac{(-2x-1)\sqrt{-x^2-x+2}}{4}$
trager	$-\frac{\sqrt{-x^2-x+2}}{x} + \frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{-x^2-x+2}+4\operatorname{RootOf}\left(\_Z^2-2\right)}{x}\right)}{4} - \operatorname{RootOf}\left(\_Z^2+1\right)$

```
[In] int((-x^2-x+2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] (x^2+x-2)/x/(-x^2-x+2)^(1/2)-arcsin(1/3+2/3*x)+1/4*arctanh(1/4*(4-x)*2^(1/2
)/(-x^2-x+2)^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx = \frac{\sqrt{2}x\log\left(-\frac{4\sqrt{2}\sqrt{-x^2-x+2}(x-4)+7\,x^2+16\,x-32}{x^2}\right)+8\,x\arctan\left(\frac{\sqrt{-x^2-x+2}(2\,x+1)}{2\,(x^2+x-2)}\right)-8\,\sqrt{-x^2-x+2}}{8\,x}$$

```
[In] integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*x*log(-(4*sqrt(2)*sqrt(-x^2 - x + 2)*(x - 4) + 7*x^2 + 16*x -
32)/x^2) + 8*x*arctan(1/2*sqrt(-x^2 - x + 2)*(2*x + 1)/(x^2 + x - 2)) - 8*s
qrt(-x^2 - x + 2))/x
```

Sympy [F]

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx = \int \frac{\sqrt{-(x-1)(x+2)}}{x^2}\,dx$$

```
[In] integrate((-x**2-x+2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 2))/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx = \frac{1}{4}\,\sqrt{2}\log\left(\frac{2\,\sqrt{2}\sqrt{-x^2-x+2}}{|x|}+\frac{4}{|x|}-1\right) - \frac{\sqrt{-x^2-x+2}}{x}+\arcsin\left(-\frac{2}{3}\,x-\frac{1}{3}\right)$$

```
[In] integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(2*sqrt(2)*sqrt(-x^2 - x + 2)/abs(x) + 4/abs(x) - 1) - sqrt(
-x^2 - x + 2)/x + arcsin(-2/3*x - 1/3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(52) = 104.

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.47

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx = -\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+6\right|}{\left|4\sqrt{2}+\frac{2\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+6\right|}\right) + \frac{6\left(\frac{3\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+1\right)}{\frac{6\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+\frac{\left(2\sqrt{-x^2-x+2}-3\right)^2}{(2x+1)^2}+1} - \arcsin\left(\frac{2}{3}x+\frac{1}{3}\right)$$

```
[In] integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(abs(-4*sqrt(2) + 2*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) +
6)/abs(4*sqrt(2) + 2*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) + 6)) + 6*(3*(2*s
qrt(-x^2 - x + 2) - 3)/(2*x + 1) + 1)/(6*(2*sqrt(-x^2 - x + 2) - 3)/(2*x +
1) + (2*sqrt(-x^2 - x + 2) - 3)^2/(2*x + 1)^2 + 1) - arcsin(2/3*x + 1/3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx = \frac{\sqrt{2}\,\ln\left(\frac{2}{x}+\frac{\sqrt{2}\sqrt{-x^2-x+2}}{x}-\frac{1}{2}\right)}{4} - \frac{\sqrt{-x^2-x+2}}{x} + \ln\left(x\,1\mathrm{i}+\sqrt{-x^2-x+2}+\frac{1}{2}\mathrm{i}\right)\,1\mathrm{i}$$

```
[In] int((2 - x^2 - x)^(1/2)/x^2,x)
```

```
[Out] log(x*1i + (2 - x^2 - x)^(1/2) + 1i/2)*1i - (2 - x^2 - x)^(1/2)/x + (2^(1/2
)*log(2/x + (2^(1/2)*(2 - x^2 - x)^(1/2))/x - 1/2))/4
```

3.156 $\int \frac{\log(t)}{1+t} \, dt$

Optimal result

Integrand size = 8, antiderivative size = 13

$$\int \frac{\log(t)}{1+t} \, dt = \log(t)\log(1+t) + \text{PolyLog}(2,-t)$$

```
[Out] ln(t)*ln(1+t)+polylog(2,-t)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2354, 2438}

$$\int \frac{\log(t)}{1+t} \, dt = \text{PolyLog}(2,-t) + \log(t)\log(t+1)$$

```
[In] Int[Log[t]/(1 + t),t]
```

```
[Out] Log[t]*Log[1 + t] + PolyLog[2, -t]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \log(t)\log(1+t) - \int \frac{\log(1+t)}{t}\,dt$$
$$= \log(t)\log(1+t) + \text{PolyLog}(2,-t)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log(t)}{1+t}\,dt = \log(t)\log(1+t) + \text{PolyLog}(2,-t)$$

```
[In] Integrate[Log[t]/(1 + t),t]
[Out] Log[t]*Log[1 + t] + PolyLog[2, -t]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
default	$\operatorname{dilog}(1+t)+\ln(t)\ln(1+t)$	13
risch	$\operatorname{dilog}(1+t)+\ln(t)\ln(1+t)$	13
parts	$\operatorname{dilog}(1+t)+\ln(t)\ln(1+t)$	13

```
[In] int(ln(t)/(1+t),t,method=_RETURNVERBOSE)
[Out] dilog(1+t)+ln(t)*ln(1+t)
```

Fricas [F]

$$\int \frac{\log(t)}{1+t}\,dt = \int \frac{\log(t)}{t+1}\,dt$$

```
[In] integrate(log(t)/(1+t),t, algorithm="fricas")
[Out] integral(log(t)/(t + 1), t)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 73, normalized size of antiderivative = 5.62

$$\int \frac{\log(t)}{1+t}\,dt = \begin{cases} -\operatorname{Li}_2(t+1) & \text{for } \frac{1}{|t+1|} < 1 \wedge |t+1| < 1 \\ i\pi \log(t+1) - \operatorname{Li}_2(t+1) & \text{for } |t+1| < 1 \\ -i\pi \log\left(\frac{1}{t+1}\right) - \operatorname{Li}_2(t+1) & \text{for } \frac{1}{|t+1|} < 1 \\ -i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| t+1\right) + i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| t+1\right) - \operatorname{Li}_2(t+1) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(t)/(1+t),t)
```

```
[Out] Piecewise((-polylog(2, t + 1), (Abs(t + 1) < 1) & (1/Abs(t + 1) < 1)), (I*p
i*log(t + 1) - polylog(2, t + 1), Abs(t + 1) < 1), (-I*pi*log(1/(t + 1)) -
polylog(2, t + 1), 1/Abs(t + 1) < 1), (-I*pi*meijerg(((), (1, 1)), ((0, 0),
 ()), t + 1) + I*pi*meijerg(((1, 1), ()), ((), (0, 0)), t + 1) - polylog(2,
 t + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{\log(t)}{1+t}\,dt = \log(t+1)\log(t) + \operatorname{Li}_2(-t)$$

```
[In] integrate(log(t)/(1+t),t, algorithm="maxima")
```

```
[Out] log(t + 1)*log(t) + dilog(-t)
```

Giac [F]

$$\int \frac{\log(t)}{1+t}\,dt = \int \frac{\log(t)}{t+1}\,dt$$

```
[In] integrate(log(t)/(1+t),t, algorithm="giac")
```

```
[Out] integrate(log(t)/(t + 1), t)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log(t)}{1+t} \, dt = \mathrm{polylog}(2,-t) + \ln(t+1) \, \ln(t)$$

```
[In] int(log(t)/(t + 1),t)
[Out] polylog(2, -t) + log(t + 1)*log(t)
```

3.157 $\int \log \left(e^{\cos (x)}\right) \, dx$

Optimal result

Integrand size = 5, antiderivative size = 15

$$\int \log \left(e^{\cos (x)}\right) \, dx = -x \cos (x) + x \log \left(e^{\cos (x)}\right) + \sin (x)$$

```
[Out] -x*cos(x)+x*ln(exp(cos(x)))+sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2628, 3377, 2717}

$$\int \log \left(e^{\cos (x)}\right) \, dx = \sin (x) - x \cos (x) + x \log \left(e^{\cos (x)}\right)$$

```
[In] Int[Log[E^Cos[x]],x]
```

```
[Out] -(x*Cos[x]) + x*Log[E^Cos[x]] + Sin[x]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log\left(e^{\cos(x)}\right)+\int x\sin(x)\,dx\\ &= -x\cos(x)+x\log\left(e^{\cos(x)}\right)+\int\cos(x)\,dx\\ &= -x\cos(x)+x\log\left(e^{\cos(x)}\right)+\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int\log\left(e^{\cos(x)}\right)\,dx=x\left(-\cos(x)+\log\left(e^{\cos(x)}\right)\right)+\sin(x)$$

```
[In] Integrate[Log[E^Cos[x]],x]
[Out] x*(-Cos[x] + Log[E^Cos[x]]) + Sin[x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

method	result	size
default	$-x\cos(x)+x\ln\left(\mathrm{e}^{\cos(x)}\right)+\sin(x)$	15
risch	$-x\cos(x)+x\ln\left(\mathrm{e}^{\cos(x)}\right)+\sin(x)$	15
parallelrisch	$-x\cos(x)+x\ln\left(\mathrm{e}^{\cos(x)}\right)+\sin(x)$	15
parts	$-x\cos(x)+x\ln\left(\mathrm{e}^{\cos(x)}\right)+\sin(x)$	15

```
[In] int(ln(exp(cos(x))),x,method=_RETURNVERBOSE)
[Out] -x*cos(x)+x*ln(exp(cos(x)))+sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.13

$$\int \log \left(e^{\cos(x)}\right) \, dx = \sin{(x)}$$

```
[In] integrate(log(exp(cos(x))),x, algorithm="fricas")
[Out] sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log \left(e^{\cos(x)}\right) \, dx = x \log \left(e^{\cos{(x)}}\right) - x \cos{(x)} + \sin{(x)}$$

```
[In] integrate(ln(exp(cos(x))),x)
[Out] x*log(exp(cos(x))) - x*cos(x) + sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.13

$$\int \log \left(e^{\cos(x)}\right) \, dx = \sin{(x)}$$

```
[In] integrate(log(exp(cos(x))),x, algorithm="maxima")
[Out] sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.13

$$\int \log \left(e^{\cos(x)}\right) \, dx = \sin{(x)}$$

```
[In] integrate(log(exp(cos(x))),x, algorithm="giac")
[Out] sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.13

$$\int \log \left(e^{\cos(x)}\right) \, dx = \sin \left(x\right)$$

```
[In] int(log(exp(cos(x))),x)
[Out] sin(x)
```

3.158 $\int \frac{e^t}{t} \, dt$

Optimal result

Integrand size = 7, antiderivative size = 2

$$\int \frac{e^t}{t} \, dt = \text{ExpIntegralEi}(t)$$

```
[Out] Ei(t)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{2209\}$

$$\int \frac{e^t}{t} \, dt = \text{ExpIntegralEi}(t)$$

```
[In] Int[E^t/t,t]
```

```
[Out] ExpIntegralEi[t]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$\text{integral} = \text{ExpIntegralEi}(t)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^t}{t} dt = \text{ExpIntegralEi}(t)$$

```
[In] Integrate[E^t/t,t]
[Out] ExpIntegralEi[t]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(2) = 4.

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 4.00

method	result	size
default	$-\text{Ei}_1(-t)$	8
risch	$-\text{Ei}_1(-t)$	8
meijerg	$\ln(t) + i\pi - \ln(-t) - \text{Ei}_1(-t)$	21

```
[In] int(exp(t)/t,t,method=_RETURNVERBOSE)
[Out] -Ei(1,-t)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^t}{t} dt = \text{Ei}(t)$$

```
[In] integrate(exp(t)/t,t, algorithm="fricas")
[Out] Ei(t)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^t}{t} \, dt = \text{Ei}\,(t)$$

```
[In] integrate(exp(t)/t,t)
[Out] Ei(t)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^t}{t} \, dt = \text{Ei}(t)$$

```
[In] integrate(exp(t)/t,t, algorithm="maxima")
[Out] Ei(t)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^t}{t} \, dt = \text{Ei}(t)$$

```
[In] integrate(exp(t)/t,t, algorithm="giac")
[Out] Ei(t)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^t}{t} \, dt = \text{ei}(t)$$

```
[In] int(exp(t)/t,t)
[Out] ei(t)
```

3.159 $\int \frac{e^{at}}{t} \, dt$

Optimal result

Integrand size = 9, antiderivative size = 4

$$\int \frac{e^{at}}{t} \, dt = \text{ExpIntegralEi}(at)$$

```
[Out] Ei(a*t)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2209}

$$\int \frac{e^{at}}{t} \, dt = \text{ExpIntegralEi}(at)$$

```
[In] Int[E^(a*t)/t,t]
```

```
[Out] ExpIntegralEi[a*t]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$\text{integral} = \text{ExpIntegralEi}(at)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{at}}{t}\,dt = \text{ExpIntegralEi}(at)$$

```
[In] Integrate[E^(a*t)/t,t]
[Out] ExpIntegralEi[a*t]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 2.25

method	result	size
derivativedivides	$-\operatorname{Ei}_1(-at)$	9
default	$-\operatorname{Ei}_1(-at)$	9
risch	$-\operatorname{Ei}_1(-at)$	9
meijerg	$\ln(t)+\ln(-a)-\ln(-at)-\operatorname{Ei}_1(-at)$	23

```
[In] int(exp(a*t)/t,t,method=_RETURNVERBOSE)
[Out] -Ei(1,-a*t)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{at}}{t}\,dt = \text{Ei}(at)$$

```
[In] integrate(exp(a*t)/t,t, algorithm="fricas")
[Out] Ei(a*t)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^{at}}{t}\,dt = \operatorname{Ei}(at)$$

```
[In] integrate(exp(a*t)/t,t)
[Out] Ei(a*t)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{at}}{t}\,dt = \operatorname{Ei}(at)$$

```
[In] integrate(exp(a*t)/t,t, algorithm="maxima")
[Out] Ei(a*t)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{at}}{t}\,dt = \operatorname{Ei}(at)$$

```
[In] integrate(exp(a*t)/t,t, algorithm="giac")
[Out] Ei(a*t)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{at}}{t}\,dt = \operatorname{ei}(a\,t)$$

```
[In] int(exp(a*t)/t,t)
[Out] ei(a*t)
```

3.160 $\int \frac{e^t}{t^2} \, dt$

Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \frac{e^t}{t^2} \, dt = -\frac{e^t}{t} + \text{ExpIntegralEi}(t)$$

```
[Out] -exp(t)/t+Ei(t)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2208, 2209}

$$\int \frac{e^t}{t^2} \, dt = \text{ExpIntegralEi}(t) - \frac{e^t}{t}$$

```
[In] Int[E^t/t^2,t]
```

```
[Out] -(E^t/t) + ExpIntegralEi[t]
```

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^t}{t}+\int \frac{e^t}{t}\,dt\\ &= -\frac{e^t}{t}+\text{ExpIntegralEi}(t)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{e^t}{t^2}\,dt=-\frac{e^t}{t}+\text{ExpIntegralEi}(t)$$

```
[In] Integrate[E^t/t^2,t]
[Out] -(E^t/t) + ExpIntegralEi[t]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{e^{t}}{t}-\mathrm{Ei}_{1}\left(-t\right)$	16
risch	$-\frac{e^{t}}{t}-\mathrm{Ei}_{1}\left(-t\right)$	16
meijerg	$-\frac{1}{t}-1+\ln\left(t\right)+i\pi+\frac{2+2t}{2t}-\frac{e^{t}}{t}-\ln\left(-t\right)-\mathrm{Ei}_{1}\left(-t\right)$	44

```
[In] int(exp(t)/t^2,t,method=_RETURNVERBOSE)
[Out] -exp(t)/t-Ei(1,-t)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{e^t}{t^2} dt = \frac{t\text{Ei}(t) - e^t}{t}$$

```
[In] integrate(exp(t)/t^2,t, algorithm="fricas")
[Out] (t*Ei(t) - e^t)/t
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{e^t}{t^2} dt = \text{Ei}(t) - \frac{e^t}{t}$$

```
[In] integrate(exp(t)/t**2,t)
[Out] Ei(t) - exp(t)/t
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.45

$$\int \frac{e^t}{t^2} dt = \Gamma(-1, -t)$$

```
[In] integrate(exp(t)/t^2,t, algorithm="maxima")
[Out] gamma(-1, -t)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{e^t}{t^2} dt = \frac{t\text{Ei}(t) - e^t}{t}$$

```
[In] integrate(exp(t)/t^2,t, algorithm="giac")
[Out] (t*Ei(t) - e^t)/t
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{e^t}{t^2} \, dt = -\frac{\mathrm{e}^t}{t} - \mathrm{expint}(-t)$$

```
[In] int(exp(t)/t^2,t)
[Out] - exp(t)/t - expint(-t)
```

3.161 $\int e^{\frac{1}{t}}\,dt$

Optimal result

Integrand size = 5, antiderivative size = 14

$$\int e^{\frac{1}{t}}\,dt = e^{\frac{1}{t}}t - \text{ExpIntegralEi}\left(\frac{1}{t}\right)$$

```
[Out] exp(1/t)*t-Ei(1/t)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2237, 2241}

$$\int e^{\frac{1}{t}}\,dt = e^{\frac{1}{t}}t - \text{ExpIntegralEi}\left(\frac{1}{t}\right)$$

```
[In] Int[E^t^(-1),t]
```

```
[Out] E^t^(-1)*t - ExpIntegralEi[t^(-1)]
```

Rule 2237

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Dist[b*n*Log[F], Int[(c + d*x)^n*F^(a
+ b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && I
LtQ[n, 0]
```

Rule 2241

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
```

```
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\text{integral} = e^{\frac{1}{t}}t + \int \frac{e^{\frac{1}{t}}}{t}\,dt$$
$$= e^{\frac{1}{t}}t - \text{ExpIntegralEi}\left(\frac{1}{t}\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int e^{\frac{1}{t}}\,dt = e^{\frac{1}{t}}t - \text{ExpIntegralEi}\left(\frac{1}{t}\right)$$

```
[In] Integrate[E^t^(-1),t]
[Out] E^t^(-1)*t - ExpIntegralEi[t^(-1)]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\mathrm{e}^{\frac{1}{t}}t + \mathrm{Ei}_1\left(-\frac{1}{t}\right)$	15
default	$\mathrm{e}^{\frac{1}{t}}t + \mathrm{Ei}_1\left(-\frac{1}{t}\right)$	15
risch	$\mathrm{e}^{\frac{1}{t}}t + \mathrm{Ei}_1\left(-\frac{1}{t}\right)$	15
meijerg	$t + 1 + \ln\left(t\right) - i\pi - \frac{t\left(2+\frac{2}{t}\right)}{2} + \mathrm{e}^{\frac{1}{t}}t + \ln\left(-\frac{1}{t}\right) + \mathrm{Ei}_1\left(-\frac{1}{t}\right)$	39

```
[In] int(exp(1/t),t,method=_RETURNVERBOSE)
[Out] exp(1/t)*t+Ei(1,-1/t)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int e^{\frac{1}{t}}\,dt = te^{\frac{1}{t}} - \mathrm{Ei}\left(\frac{1}{t}\right)$$

```
[In] integrate(exp(1/t),t, algorithm="fricas")
[Out] t*e^(1/t) - Ei(1/t)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int e^{\frac{1}{t}}\,dt = te^{\frac{1}{t}} - \mathrm{Ei}\left(\frac{1}{t}\right)$$

```
[In] integrate(exp(1/t),t)
[Out] t*exp(1/t) - Ei(1/t)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int e^{\frac{1}{t}}\,dt = -\Gamma\left(-1, -\frac{1}{t}\right)$$

```
[In] integrate(exp(1/t),t, algorithm="maxima")
[Out] -gamma(-1, -1/t)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int e^{\frac{1}{t}}\,dt = -t\left(\frac{\mathrm{Ei}\left(\frac{1}{t}\right)}{t} - e^{\frac{1}{t}}\right)$$

```
[In] integrate(exp(1/t),t, algorithm="giac")
[Out] -t*(Ei(1/t)/t - e^(1/t))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int e^{\frac{1}{t}}\,dt = t\,\mathrm{expint}\left(2,-\frac{1}{t}\right)$$

```
[In] int(exp(1/t),t)
[Out] t*expint(2, -1/t)
```

3.162 $\int \frac{e^{-t}}{-1-a+t} dt$

Optimal result

Integrand size = 14, antiderivative size = 15

$$\int \frac{e^{-t}}{-1-a+t} dt = e^{-1-a} \operatorname{ExpIntegralEi}(1+a-t)$$

```
[Out] exp(-1-a)*Ei(1+a-t)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2209}

$$\int \frac{e^{-t}}{-1-a+t} dt = e^{-a-1} \operatorname{ExpIntegralEi}(a-t+1)$$

```
[In] Int[1/(E^t*(-1 - a + t)),t]
```

```
[Out] E^(-1 - a)*ExpIntegralEi[1 + a - t]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$\text{integral} = e^{-1-a} \operatorname{ExpIntegralEi}(1+a-t)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{e^{-t}}{-1-a+t}\,dt = e^{-1-a}\,\text{ExpIntegralEi}(1+a-t)$$

```
[In] Integrate[1/(E^t*(-1 - a + t)),t]
[Out] E^(-1 - a)*ExpIntegralEi[1 + a - t]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

method	result	size
default	$-\mathrm{e}^{-1-a}\,\mathrm{Ei}_1\left(-1-a+t\right)$	17
risch	$-\mathrm{e}^{-1-a}\,\mathrm{Ei}_1\left(-1-a+t\right)$	17

```
[In] int(1/exp(t)/(-1-a+t),t,method=_RETURNVERBOSE)
[Out] -exp(-1-a)*Ei(1,-1-a+t)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{e^{-t}}{-1-a+t}\,dt = \text{Ei}(a-t+1)\,e^{(-a-1)}$$

```
[In] integrate(1/exp(t)/(-1-a+t),t, algorithm="fricas")
[Out] Ei(a - t + 1)*e^(-a - 1)
```

Sympy [F]

$$\int \frac{e^{-t}}{-1-a+t}\,dt = \int \frac{e^{-t}}{-a+t-1}\,dt$$

```
[In] integrate(1/exp(t)/(-1-a+t),t)
[Out] Integral(exp(-t)/(-a + t - 1), t)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{e^{-t}}{-1-a+t}\,dt = -e^{(-a-1)}E_1(-a+t-1)$$

```
[In] integrate(1/exp(t)/(-1-a+t),t, algorithm="maxima")
[Out] -e^(-a - 1)*exp_integral_e(1, -a + t - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{e^{-t}}{-1-a+t}\,dt = \mathrm{Ei}(a-t+1)\,e^{(-a-1)}$$

```
[In] integrate(1/exp(t)/(-1-a+t),t, algorithm="giac")
[Out] Ei(a - t + 1)*e^(-a - 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{e^{-t}}{-1-a+t}\,dt = \mathrm{e}^{-a-1}\,\mathrm{ei}(a-t+1)$$

```
[In] int(-exp(-t)/(a - t + 1),t)
[Out] exp(- a - 1)*ei(a - t + 1)
```

3.163 $\int \frac{e^{t^2}t}{1+t^2}\,dt$

Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{e^{t^2}t}{1+t^2}\,dt = \frac{\text{ExpIntegralEi}\left(1+t^2\right)}{2e}$$

```
[Out] 1/2*Ei(t^2+1)/exp(1)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6847, 2209}

$$\int \frac{e^{t^2}t}{1+t^2}\,dt = \frac{\text{ExpIntegralEi}\left(t^2+1\right)}{2e}$$

```
[In] Int[(E^t^2*t)/(1 + t^2),t]
```

```
[Out] ExpIntegralEi[1 + t^2]/(2*E)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
```

```
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{e^t}{1+t}\,dt, t, t^2\right)$$
$$= \frac{\text{ExpIntegralEi}\left(1+t^2\right)}{2e}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{t^2}t}{1+t^2}\,dt = \frac{\text{ExpIntegralEi}\left(1+t^2\right)}{2e}$$

```
[In] Integrate[(E^t^2*t)/(1 + t^2),t]
[Out] ExpIntegralEi[1 + t^2]/(2*E)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{e^{-1}\,\text{Ei}_1\left(-t^2-1\right)}{2}$	14
default	$-\frac{e^{-1}\,\text{Ei}_1\left(-t^2-1\right)}{2}$	14
risch	$-\frac{e^{-1}\,\text{Ei}_1\left(-t^2-1\right)}{2}$	14

```
[In] int(exp(t^2)*t/(t^2+1),t,method=_RETURNVERBOSE)
[Out] -1/2*exp(-1)*Ei(1,-t^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{e^{t^2}t}{1+t^2}\,dt = \frac{1}{2}\,\mathrm{Ei}\left(t^2+1\right)e^{(-1)}$$

```
[In] integrate(exp(t^2)*t/(t^2+1),t, algorithm="fricas")
[Out] 1/2*Ei(t^2 + 1)*e^(-1)
```

Sympy [F]

$$\int \frac{e^{t^2}t}{1+t^2}\,dt = \int \frac{te^{t^2}}{t^2+1}\,dt$$

```
[In] integrate(exp(t**2)*t/(t**2+1),t)
[Out] Integral(t*exp(t**2)/(t**2 + 1), t)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{t^2}t}{1+t^2}\,dt = -\frac{1}{2}\,e^{(-1)}E_1\left(-t^2-1\right)$$

```
[In] integrate(exp(t^2)*t/(t^2+1),t, algorithm="maxima")
[Out] -1/2*e^(-1)*exp_integral_e(1, -t^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{e^{t^2}t}{1+t^2}\,dt = \frac{1}{2}\,\mathrm{Ei}\left(t^2+1\right)e^{(-1)}$$

```
[In] integrate(exp(t^2)*t/(t^2+1),t, algorithm="giac")
[Out] 1/2*Ei(t^2 + 1)*e^(-1)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{e^{t^2} t}{1+t^2} \, dt = \frac{\mathrm{e}^{-1} \, \mathrm{ei}(t^2+1)}{2}$$

```
[In] int((t*exp(t^2))/(t^2 + 1),t)
[Out] (exp(-1)*ei(t^2 + 1))/2
```

3.164 $\int \frac{e^t}{(1+t)^2} dt$

Optimal result

Integrand size = 9, antiderivative size = 19

$$\int \frac{e^t}{(1+t)^2} dt = -\frac{e^t}{1+t} + \frac{\text{ExpIntegralEi}(1+t)}{e}$$

```
[Out] -exp(t)/(1+t)+Ei(1+t)/exp(1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2208, 2209}

$$\int \frac{e^t}{(1+t)^2} dt = \frac{\text{ExpIntegralEi}(t+1)}{e} - \frac{e^t}{t+1}$$

```
[In] Int[E^t/(1 + t)^2,t]
[Out] -(E^t/(1 + t)) + ExpIntegralEi[1 + t]/E
```

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$\text{integral} = -\frac{e^t}{1+t} + \int \frac{e^t}{1+t}\,dt$$
$$= -\frac{e^t}{1+t} + \frac{\text{ExpIntegralEi}(1+t)}{e}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^t}{(1+t)^2}\,dt = -\frac{e^t}{1+t} + \frac{\text{ExpIntegralEi}(1+t)}{e}$$

```
[In] Integrate[E^t/(1 + t)^2,t]
[Out] -(E^t/(1 + t)) + ExpIntegralEi[1 + t]/E
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

method	result	size
default	$-\frac{e^t}{1+t} - e^{-1}\,\text{Ei}_1(-1-t)$	22
risch	$-\frac{e^t}{1+t} - e^{-1}\,\text{Ei}_1(-1-t)$	22

```
[In] int(exp(t)/(1+t)^2,t,method=_RETURNVERBOSE)
[Out] -exp(t)/(1+t)-exp(-1)*Ei(1,-1-t)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{e^t}{(1+t)^2}\,dt = \frac{\left((t+1)\text{Ei}(t+1) - e^{(t+1)}\right)e^{(-1)}}{t+1}$$

```
[In] integrate(exp(t)/(1+t)^2,t, algorithm="fricas")
[Out] ((t + 1)*Ei(t + 1) - e^(t + 1))*e^(-1)/(t + 1)
```

Sympy [F]

$$\int \frac{e^t}{(1+t)^2}\,dt = \int \frac{e^t}{(t+1)^2}\,dt$$

```
[In] integrate(exp(t)/(1+t)**2,t)
[Out] Integral(exp(t)/(t + 1)**2, t)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{e^t}{(1+t)^2}\,dt = -\frac{e^{(-1)}E_2(-t-1)}{t+1}$$

```
[In] integrate(exp(t)/(1+t)^2,t, algorithm="maxima")
[Out] -e^(-1)*exp_integral_e(2, -t - 1)/(t + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 4.21

$$\int \frac{e^t}{(1+t)^2}\,dt = \frac{(t+1)\left(\frac{1}{t+1}-1\right)\mathrm{Ei}\left(-(t+1)\left(\frac{1}{t+1}-1\right)+1\right) - \mathrm{Ei}\left(-(t+1)\left(\frac{1}{t+1}-1\right)+1\right) + e^{\left(-(t+1)\left(\frac{1}{t+1}-1\right)+1\right)}}{(t+1)\left(\frac{1}{t+1}-1\right)e - e}$$

```
[In] integrate(exp(t)/(1+t)^2,t, algorithm="giac")
[Out] ((t + 1)*(1/(t + 1) - 1)*Ei(-(t + 1)*(1/(t + 1) - 1) + 1) - Ei(-(t + 1)*(1/
(t + 1) - 1) + 1) + e^(-(t + 1)*(1/(t + 1) - 1) + 1))/((t + 1)*(1/(t + 1) -
 1)*e - e)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{e^t}{(1+t)^2} \, dt = \mathrm{ei}(t+1) \ \mathrm{e}^{-1} - \frac{\mathrm{e}^t}{t+1}$$

```
[In] int(exp(t)/(t + 1)^2,t)
[Out] ei(t + 1)*exp(-1) - exp(t)/(t + 1)
```

3.165 $\int e^t \log(1+t)\,dt$

Optimal result

Integrand size = 8, antiderivative size = 18

$$\int e^t \log(1+t)\,dt = -\frac{\text{ExpIntegralEi}(1+t)}{e} + e^t \log(1+t)$$

```
[Out] -Ei(1+t)/exp(1)+exp(t)*ln(1+t)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2225, 2634, 2209}

$$\int e^t \log(1+t)\,dt = e^t \log(t+1) - \frac{\text{ExpIntegralEi}(t+1)}{e}$$

```
[In] Int[E^t*Log[1 + t],t]
```

```
[Out] -(ExpIntegralEi[1 + t]/E) + E^t*Log[1 + t]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= e^t \log(1+t) - \int \frac{e^t}{1+t}\,dt\\ &= -\frac{\text{ExpIntegralEi}(1+t)}{e} + e^t \log(1+t)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^t \log(1+t)\,dt = -\frac{\text{ExpIntegralEi}(1+t)}{e} + e^t \log(1+t)$$

```
[In] Integrate[E^t*Log[1 + t],t]
[Out] -(ExpIntegralEi[1 + t]/E) + E^t*Log[1 + t]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
risch	$\mathrm{e}^{t} \ln\left(1+t\right)+\mathrm{e}^{-1} \mathrm{Ei}_{1}\left(-1-t\right)$	19

```
[In] int(exp(t)*ln(1+t),t,method=_RETURNVERBOSE)
[Out] exp(t)*ln(1+t)+exp(-1)*Ei(1,-1-t)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int e^t \log(1+t)\,dt = \left(e^{(t+1)} \log\left(t+1\right) - \text{Ei}(t+1)\right)e^{(-1)}$$

```
[In] integrate(exp(t)*log(1+t),t, algorithm="fricas")
[Out] (e^(t + 1)*log(t + 1) - Ei(t + 1))*e^(-1)
```

Sympy [F]

$$\int e^t \log(1+t)\,dt = \int e^t \log{(t+1)}\,dt$$

```
[In] integrate(exp(t)*ln(1+t),t)
[Out] Integral(exp(t)*log(t + 1), t)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^t \log(1+t)\,dt = e^{(-1)}E_1(-t-1) + e^t \log{(t+1)}$$

```
[In] integrate(exp(t)*log(1+t),t, algorithm="maxima")
[Out] e^(-1)*exp_integral_e(1, -t - 1) + e^t*log(t + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int e^t \log(1+t)\,dt = -\mathrm{Ei}(t+1)\,e^{(-1)} + e^t \log{(t+1)}$$

```
[In] integrate(exp(t)*log(1+t),t, algorithm="giac")
[Out] -Ei(t + 1)*e^(-1) + e^t*log(t + 1)
```

Mupad [F(-1)]

Timed out.

$$\int e^t \log(1+t)\,dt = \int \ln{(t+1)}\,\mathrm{e}^t\,dt$$

```
[In] int(log(t + 1)*exp(t),t)
[Out] int(log(t + 1)*exp(t), t)
```

3.166 $\int e^{-t}t\,dt$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int e^{-t}t\,dt = -e^{-t} - e^{-t}t$$

```
[Out] -1/exp(t)-t/exp(t)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = $\{2207, 2225\}$

$$\int e^{-t}t\,dt = -e^{-t}t - e^{-t}$$

```
[In] Int[t/E^t,t]
```

```
[Out] -E^(-t) - t/E^t
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -e^{-t}t + \int e^{-t}\,dt \\ &= -e^{-t} - e^{-t}t \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int e^{-t}t\,dt = e^{-t}(-1-t)$$

```
[In] Integrate[t/E^t,t]
[Out] (-1 - t)/E^t
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

method	result	size
gosper	$-(1+t)\,\mathrm{e}^{-t}$	10
norman	$(-1-t)\,\mathrm{e}^{-t}$	11
risch	$(-1-t)\,\mathrm{e}^{-t}$	11
parallelrisch	$(-1-t)\,\mathrm{e}^{-t}$	11
meijerg	$1-\frac{(2+2t)\mathrm{e}^{-t}}{2}$	14
default	$-\mathrm{e}^{-t}-t\,\mathrm{e}^{-t}$	15

```
[In] int(t/exp(t),t,method=_RETURNVERBOSE)
[Out] -(1+t)/exp(t)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-t}t\,dt = -(t+1)e^{(-t)}$$

```
[In] integrate(t/exp(t),t, algorithm="fricas")
[Out] -(t + 1)*e^(-t)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int e^{-t}t\,dt = (-t-1)\,e^{-t}$$

```
[In] integrate(t/exp(t),t)
[Out] (-t - 1)*exp(-t)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-t}t\,dt = -(t+1)e^{(-t)}$$

```
[In] integrate(t/exp(t),t, algorithm="maxima")
[Out] -(t + 1)*e^(-t)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-t}t\,dt = -(t+1)e^{(-t)}$$

```
[In] integrate(t/exp(t),t, algorithm="giac")
[Out] -(t + 1)*e^(-t)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-t}t\,dt = -\mathrm{e}^{-t}\,(t+1)$$

```
[In] int(t*exp(-t),t)
[Out] -exp(-t)*(t + 1)
```

3.167 $\int e^{-t}t^2\,dt$

Optimal result

Integrand size = 9, antiderivative size = 26

$$\int e^{-t}t^2\,dt = -2e^{-t} - 2e^{-t}t - e^{-t}t^2$$

```
[Out] -2/exp(t)-2*t/exp(t)-t^2/exp(t)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2207, 2225}

$$\int e^{-t}t^2\,dt = -e^{-t}t^2 - 2e^{-t}t - 2e^{-t}$$

```
[In] Int[t^2/E^t,t]
```

```
[Out] -2/E^t - (2*t)/E^t - t^2/E^t
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -e^{-t}t^2 + 2\int e^{-t}t\,dt\\ &= -2e^{-t}t - e^{-t}t^2 + 2\int e^{-t}\,dt\\ &= -2e^{-t} - 2e^{-t}t - e^{-t}t^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int e^{-t}t^2\,dt = e^{-t}(-2-2t-t^2)$$

```
[In] Integrate[t^2/E^t,t]
[Out] (-2 - 2*t - t^2)/E^t
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

method	result	size
gosper	$-(t^2+2t+2)\,\mathrm{e}^{-t}$	15
norman	$(-t^2-2t-2)\,\mathrm{e}^{-t}$	16
risch	$(-t^2-2t-2)\,\mathrm{e}^{-t}$	16
parallelrisch	$(-t^2-2t-2)\,\mathrm{e}^{-t}$	16
meijerg	$2-\frac{(3t^2+6t+6)\mathrm{e}^{-t}}{3}$	19
default	$-2\,\mathrm{e}^{-t}-2t\,\mathrm{e}^{-t}-t^2\mathrm{e}^{-t}$	24

```
[In] int(t^2/exp(t),t,method=_RETURNVERBOSE)
[Out] -(t^2+2*t+2)/exp(t)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int e^{-t}t^2\,dt = -\left(t^2+2\,t+2\right)e^{(-t)}$$

```
[In] integrate(t^2/exp(t),t, algorithm="fricas")
[Out] -(t^2 + 2*t + 2)*e^(-t)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.46

$$\int e^{-t}t^2\,dt = \left(-t^2-2t-2\right)e^{-t}$$

```
[In] integrate(t**2/exp(t),t)
[Out] (-t**2 - 2*t - 2)*exp(-t)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int e^{-t}t^2\,dt = -\left(t^2+2\,t+2\right)e^{(-t)}$$

```
[In] integrate(t^2/exp(t),t, algorithm="maxima")
[Out] -(t^2 + 2*t + 2)*e^(-t)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int e^{-t}t^2\,dt = -\left(t^2+2\,t+2\right)e^{(-t)}$$

```
[In] integrate(t^2/exp(t),t, algorithm="giac")
[Out] -(t^2 + 2*t + 2)*e^(-t)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int e^{-t}t^2\,dt = -e^{-t}\left(t^2+2\,t+2\right)$$

```
[In] int(t^2*exp(-t),t)
[Out] -exp(-t)*(2*t + t^2 + 2)
```

3.168 $\int e^{-t}t^3\,dt$

Optimal result

Integrand size = 9, antiderivative size = 36

$$\int e^{-t}t^3\,dt = -6e^{-t} - 6e^{-t}t - 3e^{-t}t^2 - e^{-t}t^3$$

```
[Out] -6/exp(t)-6*t/exp(t)-3*t^2/exp(t)-t^3/exp(t)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2207, 2225}

$$\int e^{-t}t^3\,dt = -e^{-t}t^3 - 3e^{-t}t^2 - 6e^{-t}t - 6e^{-t}$$

```
[In] Int[t^3/E^t,t]
```

```
[Out] -6/E^t - (6*t)/E^t - (3*t^2)/E^t - t^3/E^t
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -e^{-t}t^3 + 3\int e^{-t}t^2\,dt \\
&= -3e^{-t}t^2 - e^{-t}t^3 + 6\int e^{-t}t\,dt \\
&= -6e^{-t}t - 3e^{-t}t^2 - e^{-t}t^3 + 6\int e^{-t}\,dt \\
&= -6e^{-t} - 6e^{-t}t - 3e^{-t}t^2 - e^{-t}t^3
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

$$\int e^{-t}t^3\,dt = e^{-t}\left(-6 - 6t - 3t^2 - t^3\right)$$

```
[In] Integrate[t^3/E^t,t]
[Out] (-6 - 6*t - 3*t^2 - t^3)/E^t
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\left(t^3+3t^2+6t+6\right)\mathrm{e}^{-t}$	20
norman	$\left(-t^3-3t^2-6t-6\right)\mathrm{e}^{-t}$	21
risch	$\left(-t^3-3t^2-6t-6\right)\mathrm{e}^{-t}$	21
parallelrisch	$\left(-t^3-3t^2-6t-6\right)\mathrm{e}^{-t}$	21
meijerg	$6-\frac{\left(4t^3+12t^2+24t+24\right)\mathrm{e}^{-t}}{4}$	24
default	$-6\,\mathrm{e}^{-t}-6t\,\mathrm{e}^{-t}-3t^2\mathrm{e}^{-t}-t^3\mathrm{e}^{-t}$	33

```
[In] int(t^3/exp(t),t,method=_RETURNVERBOSE)
[Out] -(t^3+3*t^2+6*t+6)/exp(t)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int e^{-t}t^3\,dt = -\left(t^3 + 3\,t^2 + 6\,t + 6\right)e^{(-t)}$$

```
[In] integrate(t^3/exp(t),t, algorithm="fricas")
[Out] -(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.47

$$\int e^{-t}t^3\,dt = \left(-t^3 - 3t^2 - 6t - 6\right)e^{-t}$$

```
[In] integrate(t**3/exp(t),t)
[Out] (-t**3 - 3*t**2 - 6*t - 6)*exp(-t)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int e^{-t}t^3\,dt = -\left(t^3 + 3\,t^2 + 6\,t + 6\right)e^{(-t)}$$

```
[In] integrate(t^3/exp(t),t, algorithm="maxima")
[Out] -(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int e^{-t}t^3\,dt = -\left(t^3 + 3\,t^2 + 6\,t + 6\right)e^{(-t)}$$

```
[In] integrate(t^3/exp(t),t, algorithm="giac")
[Out] -(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int e^{-t}t^3\,dt = -\mathrm{e}^{-t}\left(t^3+3\,t^2+6\,t+6\right)$$

```
[In] int(t^3*exp(-t),t)
[Out] -exp(-t)*(6*t + 3*t^2 + t^3 + 6)
```

3.169 $\int \frac{\mathbf{b1}\cos(x)+\mathbf{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \frac{\mathrm{b1}\cos(x)+\mathrm{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx=\frac{(a\mathrm{a1}+b\mathrm{b1})x}{a^2+b^2}-\frac{(\mathrm{a1}b-a\mathrm{b1})\log(b\cos(x)+a\sin(x))}{a^2+b^2}$$

```
[Out] (a*a1+b*b1)*x/(a^2+b^2)-(-a*b1+a1*b)*ln(b*cos(x)+a*sin(x))/(a^2+b^2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3212}

$$\int \frac{\mathrm{b1}\cos(x)+\mathrm{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx=\frac{x(a\mathrm{a1}+b\mathrm{b1})}{a^2+b^2}-\frac{(\mathrm{a1}b-a\mathrm{b1})\log(a\sin(x)+b\cos(x))}{a^2+b^2}$$

```
[In] Int[(b1*Cos[x] + a1*Sin[x])/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] ((a*a1 + b*b1)*x)/(a^2 + b^2) - ((a1*b - a*b1)*Log[b*Cos[x] + a*Sin[x]])/(a
^2 + b^2)
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\text{integral}=\frac{(a\mathrm{a1}+b\mathrm{b1})x}{a^2+b^2}-\frac{(\mathrm{a1}b-a\mathrm{b1})\log(b\cos(x)+a\sin(x))}{a^2+b^2}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{\mathrm{b1}\cos(x)+\mathrm{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx=\frac{(a\mathrm{a1}+b\mathrm{b1})x+(-\mathrm{a1}b+a\mathrm{b1})\log(b\cos(x)+a\sin(x))}{a^2+b^2}$$

```
[In] Integrate[(b1*Cos[x] + a1*Sin[x])/(b*Cos[x] + a*Sin[x]),x]
[Out] ((a*a1 + b*b1)*x + (-(a1*b) + a*b1)*Log[b*Cos[x] + a*Sin[x]])/(a^2 + b^2)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

method	result	si
default	$\frac{(a\,\mathrm{b1}-\mathrm{a1}\,b)\ln(a\tan(x)+b)}{a^2+b^2}+\frac{\frac{(-a\,\mathrm{b1}+\mathrm{a1}\,b)\ln\left(1+\tan^2(x)\right)}{2}+(a\,\mathrm{a1}+b\,\mathrm{b1})\arctan(\tan(x))}{a^2+b^2}$	6
parallelrisch	$\frac{-a\,\mathrm{b1}\ln\left(\frac{1}{\cos(x)+1}\right)+\mathrm{a1}\,b\ln\left(\frac{1}{\cos(x)+1}\right)+a\,\mathrm{b1}\ln\left(\frac{-b\cos(x)-a\sin(x)}{\cos(x)+1}\right)-\mathrm{a1}\,b\ln\left(\frac{-b\cos(x)-a\sin(x)}{\cos(x)+1}\right)+xa\,\mathrm{a1}+xb\,\mathrm{b1}}{a^2+b^2}$	8
norman	$\frac{\frac{(a\,\mathrm{a1}+b\,\mathrm{b1})x}{a^2+b^2}+\frac{(a\,\mathrm{a1}+b\,\mathrm{b1})x\left(\tan^2\left(\frac{x}{2}\right)\right)}{a^2+b^2}}{1+\tan^2\left(\frac{x}{2}\right)}+\frac{(a\,\mathrm{b1}-\mathrm{a1}\,b)\ln\left(-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2a\tan\left(\frac{x}{2}\right)+b\right)}{a^2+b^2}-\frac{(a\,\mathrm{b1}-\mathrm{a1}\,b)\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{a^2+b^2}$	1
risch	$\frac{ix\,\mathrm{b1}}{ib+a}+\frac{x\,\mathrm{a1}}{ib+a}-\frac{2ixa\,\mathrm{b1}}{a^2+b^2}+\frac{2ix\,\mathrm{a1}\,b}{a^2+b^2}+\frac{\ln\left(\mathrm{e}^{2ix}+\frac{ib-a}{ib+a}\right)a\,\mathrm{b1}}{a^2+b^2}-\frac{\ln\left(\mathrm{e}^{2ix}+\frac{ib-a}{ib+a}\right)\mathrm{a1}\,b}{a^2+b^2}$	1

```
[In] int((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x,method=_RETURNVERBOSE)
[Out] (a*b1-a1*b)/(a^2+b^2)*ln(a*tan(x)+b)+1/(a^2+b^2)*(1/2*(-a*b1+a1*b)*ln(1+tan
(x)^2)+(a*a1+b*b1)*arctan(tan(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{\mathrm{b1}\cos(x)+\mathrm{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx = \frac{2\,(aa_1+bb_1)x-(a_1b-ab_1)\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)-(a^2-b^2)\cos\left(x\right)^2+a^2\right)}{2\,(a^2+b^2)}$$

```
[In] integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x, algorithm="fricas")
[Out] 1/2*(2*(a*a1 + b*b1)*x - (a1*b - a*b1)*log(2*a*b*cos(x)*sin(x) - (a^2 - b^2
)*cos(x)^2 + a^2))/(a^2 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 360, normalized size of antiderivative = 7.50

$$\int \frac{\text{b1}\cos(x) + \text{a1}\sin(x)}{b\cos(x) + a\sin(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}(-a_1\log(\cos(x)) + b_1x) \\ \frac{a_1x+b_1\log(\sin(x))}{a} \\ \frac{a_1x\sin(x)}{-2ib\sin(x)+2b\cos(x)} + \frac{ia_1x\cos(x)}{-2ib\sin(x)+2b\cos(x)} - \frac{a_1\cos(x)}{-2ib\sin(x)+2b\cos(x)} - \frac{ib_1x\sin(x)}{-2ib\sin(x)+2b\cos(x)} + \frac{b_1x\cos(x)}{-2ib\sin(x)+2b\cos(x)} - \\ \frac{a_1x\sin(x)}{2ib\sin(x)+2b\cos(x)} - \frac{ia_1x\cos(x)}{2ib\sin(x)+2b\cos(x)} - \frac{a_1\cos(x)}{2ib\sin(x)+2b\cos(x)} + \frac{ib_1x\sin(x)}{2ib\sin(x)+2b\cos(x)} + \frac{b_1x\cos(x)}{2ib\sin(x)+2b\cos(x)} + \frac{ib_1\cos(x)}{2ib\sin(x)+2b\cos(x)} \\ \frac{aa_1x}{a^2+b^2} + \frac{ab_1\log\left(\frac{a\sin(x)}{b}+\cos(x)\right)}{a^2+b^2} - \frac{a_1b\log\left(\frac{a\sin(x)}{b}+\cos(x)\right)}{a^2+b^2} + \frac{bb_1x}{a^2+b^2} \end{cases}$$

```
[In] integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x)
```

```
[Out] Piecewise((zoo*(-a1*log(cos(x)) + b1*x), Eq(a, 0) & Eq(b, 0)), ((a1*x + b1*
log(sin(x)))/a, Eq(b, 0)), (a1*x*sin(x)/(-2*I*b*sin(x) + 2*b*cos(x)) + I*a1
*x*cos(x)/(-2*I*b*sin(x) + 2*b*cos(x)) - a1*cos(x)/(-2*I*b*sin(x) + 2*b*cos
(x)) - I*b1*x*sin(x)/(-2*I*b*sin(x) + 2*b*cos(x)) + b1*x*cos(x)/(-2*I*b*sin
(x) + 2*b*cos(x)) - I*b1*cos(x)/(-2*I*b*sin(x) + 2*b*cos(x)), Eq(a, -I*b)),
 (a1*x*sin(x)/(2*I*b*sin(x) + 2*b*cos(x)) - I*a1*x*cos(x)/(2*I*b*sin(x) + 2
*b*cos(x)) - a1*cos(x)/(2*I*b*sin(x) + 2*b*cos(x)) + I*b1*x*sin(x)/(2*I*b*s
in(x) + 2*b*cos(x)) + b1*x*cos(x)/(2*I*b*sin(x) + 2*b*cos(x)) + I*b1*cos(x)
/(2*I*b*sin(x) + 2*b*cos(x)), Eq(a, I*b)), (a*a1*x/(a**2 + b**2) + a*b1*log
(a*sin(x)/b + cos(x))/(a**2 + b**2) - a1*b*log(a*sin(x)/b + cos(x))/(a**2 +
 b**2) + b*b1*x/(a**2 + b**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(48) = 96.

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.77

$$\int \frac{\text{b1}\cos(x) + \text{a1}\sin(x)}{b\cos(x) + a\sin(x)}\,dx$$
$$= a_1\left(\frac{2\,a\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2+b^2} - \frac{b\log\left(-b - \frac{2\,a\sin(x)}{\cos(x)+1} + \frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^2+b^2} + \frac{b\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^2+b^2}\right)$$
$$+ b_1\left(\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2+b^2} + \frac{a\log\left(-b - \frac{2\,a\sin(x)}{\cos(x)+1} + \frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^2+b^2} - \frac{a\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^2+b^2}\right)$$

```
[In] integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x, algorithm="maxima")
```

```
[Out] a1*(2*a*arctan(sin(x)/(cos(x) + 1))/(a^2 + b^2) - b*log(-b - 2*a*sin(x)/(co
s(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(a^2 + b^2) + b*log(sin(x)^2/(cos(x)
 + 1)^2 + 1)/(a^2 + b^2)) + b1*(2*b*arctan(sin(x)/(cos(x) + 1))/(a^2 + b^2)
 + a*log(-b - 2*a*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(a^2 + b
^2) - a*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^2 + b^2))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.60

$$\int \frac{\mathrm{b1}\cos(x) + \mathrm{a1}\sin(x)}{b\cos(x) + a\sin(x)}\,dx = \frac{(aa_1 + bb_1)x}{a^2 + b^2} + \frac{(a_1 b - ab_1)\log\left(\tan(x)^2 + 1\right)}{2\,(a^2 + b^2)} - \frac{(aa_1 b - a^2 b_1)\log\left(|a\tan(x) + b|\right)}{a^3 + ab^2}$$

```
[In] integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x, algorithm="giac")
```

```
[Out] (a*a1 + b*b1)*x/(a^2 + b^2) + 1/2*(a1*b - a*b1)*log(tan(x)^2 + 1)/(a^2 + b^
2) - (a*a1*b - a^2*b1)*log(abs(a*tan(x) + b))/(a^3 + a*b^2)
```

Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 2034, normalized size of antiderivative = 42.38

$$\int \frac{\mathrm{b1}\cos(x) + \mathrm{a1}\sin(x)}{b\cos(x) + a\sin(x)}\,dx = \text{Too large to display}$$

```
[In] int((b1*cos(x) + a1*sin(x))/(b*cos(x) + a*sin(x)),x)
```

```
[Out] (2*atan((tan(x/2)*(((((a*a1 + b*b1)^3*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)^
3 + ((((a*a1 + b*b1)*(32*a^2*a1*b^2 - ((2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2
*b^3))/(2*(a^2 + b^2)) - 64*a1*b^4 + 128*a*b^3*b1 + 32*a^3*b*b1))/(a^2 + b^
2) - ((a*a1 + b*b1)*(2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^3))/(2*(a^2 + b^
2)^2))*(2*a*b1 - 2*a1*b))/(2*(a^2 + b^2)) - ((a*a1 + b*b1)*(32*b^3*b1^2 - (
(2*a*b1 - 2*a1*b)*(32*a^2*a1*b^2 - ((2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^
3))/(2*(a^2 + b^2)) - 64*a1*b^4 + 128*a*b^3*b1 + 32*a^3*b*b1))/(2*(a^2 + b^
2)) + 64*a^2*a1^2*b - 96*a^2*b*b1^2 + 192*a*a1*b^2*b1))/(a^2 + b^2))*(a^4*a
1^2 + 4*a1^2*b^4 - 4*a^4*b1^2 - b^4*b1^2 - 13*a^2*a1^2*b^2 + 13*a^2*b^2*b1^
2 - 18*a*a1*b^3*b1 + 18*a^3*a1*b*b1))/((a^2 + b^2)^2*(a^2*a1^2 + 4*a^2*b1^2
 + 4*a1^2*b^2 + b^2*b1^2 - 6*a*a1*b*b1)^2) - (((((a*a1 + b*b1)*(32*a^2*a1*
b^2 - ((2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^3))/(2*(a^2 + b^2)) - 64*a1*b
^4 + 128*a*b^3*b1 + 32*a^3*b*b1))/(a^2 + b^2) - ((a*a1 + b*b1)*(2*a*b1 - 2*
```

```
a1*b)*(96*a^4*b + 96*a^2*b^3))/(2*(a^2 + b^2)^2))*(a*a1 + b*b1))/(a^2 + b^2
) - 32*a1*b^2*b1^2 - 64*a1^3*b^2 + ((2*a*b1 - 2*a1*b)*(32*b^3*b1^2 - ((2*a*
b1 - 2*a1*b)*(32*a^2*a1*b^2 - ((2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^3))/(
2*(a^2 + b^2)) - 64*a1*b^4 + 128*a*b^3*b1 + 32*a^3*b*b1))/(2*(a^2 + b^2)) +
 64*a^2*a1^2*b - 96*a^2*b*b1^2 + 192*a*a1*b^2*b1))/(2*(a^2 + b^2)) + 32*a*b
*b1^3 - ((a*a1 + b*b1)^2*(2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^3))/(2*(a^2
 + b^2)^3) + 64*a*a1^2*b*b1)*(12*a*a1^2*b^3 - 6*a^3*a1^2*b - 6*a*b^3*b1^2 +
 12*a^3*b*b1^2 + 4*a^4*a1*b1 + 4*a1*b^4*b1 - 28*a^2*a1*b^2*b1))/((a^2 + b^2
)^2*(a^2*a1^2 + 4*a^2*b1^2 + 4*a1^2*b^2 + b^2*b1^2 - 6*a*a1*b*b1)^2))*(a^4
+ b^4 + 2*a^2*b^2))/(32*b^2*b1 + 32*a*a1*b) + ((a^4 + b^4 + 2*a^2*b^2)*(32*
a1^2*b^2*b1 + ((((a*a1 + b*b1)*(((2*a*b1 - 2*a1*b)*(96*a*b^4 + 96*a^3*b^2))
/(2*(a^2 + b^2)) - 32*b^4*b1 + 64*a^2*b^2*b1 - 64*a*a1*b^3 + 32*a^3*a1*b))/
(a^2 + b^2) + ((a*a1 + b*b1)*(2*a*b1 - 2*a1*b)*(96*a*b^4 + 96*a^3*b^2))/(2*
(a^2 + b^2)^2))*(a*a1 + b*b1))/(a^2 + b^2) - ((2*a*b1 - 2*a1*b)*(((2*a*b1 -
 2*a1*b)*(((2*a*b1 - 2*a1*b)*(96*a*b^4 + 96*a^3*b^2))/(2*(a^2 + b^2)) - 32*
b^4*b1 + 64*a^2*b^2*b1 - 64*a*a1*b^3 + 32*a^3*a1*b))/(2*(a^2 + b^2)) - 32*a
*a1^2*b^2 - 32*a*b^2*b1^2 + 64*a1*b^3*b1 + 64*a^2*a1*b*b1))/(2*(a^2 + b^2))
 + ((a*a1 + b*b1)^2*(2*a*b1 - 2*a1*b)*(96*a*b^4 + 96*a^3*b^2))/(2*(a^2 + b^
2)^3) - 32*a*a1*b*b1^2)*(12*a*a1^2*b^3 - 6*a^3*a1^2*b - 6*a*b^3*b1^2 + 12*a
^3*b*b1^2 + 4*a^4*a1*b1 + 4*a1*b^4*b1 - 28*a^2*a1*b^2*b1))/((32*b^2*b1 + 32
*a*a1*b)*(a^2 + b^2)^2*(a^2*a1^2 + 4*a^2*b1^2 + 4*a1^2*b^2 + b^2*b1^2 - 6*a
*a1*b*b1)^2) - ((a^4 + b^4 + 2*a^2*b^2)*(((((a*a1 + b*b1)*(((2*a*b1 - 2*a1*
b)*(96*a*b^4 + 96*a^3*b^2))/(2*(a^2 + b^2)) - 32*b^4*b1 + 64*a^2*b^2*b1 - 6
4*a*a1*b^3 + 32*a^3*a1*b))/(a^2 + b^2) + ((a*a1 + b*b1)*(2*a*b1 - 2*a1*b)*(
96*a*b^4 + 96*a^3*b^2))/(2*(a^2 + b^2)^2))*(2*a*b1 - 2*a1*b))/(2*(a^2 + b^2
)) - ((a*a1 + b*b1)^3*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^3 + ((a*a1 + b*b
1)*(((2*a*b1 - 2*a1*b)*(((2*a*b1 - 2*a1*b)*(96*a*b^4 + 96*a^3*b^2))/(2*(a^2
 + b^2)) - 32*b^4*b1 + 64*a^2*b^2*b1 - 64*a*a1*b^3 + 32*a^3*a1*b))/(2*(a^2
+ b^2)) - 32*a*a1^2*b^2 - 32*a*b^2*b1^2 + 64*a1*b^3*b1 + 64*a^2*a1*b*b1))/(
a^2 + b^2))*(a^4*a1^2 + 4*a1^2*b^4 - 4*a^4*b1^2 - b^4*b1^2 - 13*a^2*a1^2*b^
2 + 13*a^2*b^2*b1^2 - 18*a*a1*b^3*b1 + 18*a^3*a1*b*b1))/((32*b^2*b1 + 32*a*
a1*b)*(a^2 + b^2)^2*(a^2*a1^2 + 4*a^2*b1^2 + 4*a1^2*b^2 + b^2*b1^2 - 6*a*a1
*b*b1)^2))*(a*a1 + b*b1))/(a^2 + b^2) - (log(1/(cos(x) + 1))*(2*a*b1 - 2*a1
*b))/(2*(a^2 + b^2)) + (log(b + 2*a*tan(x/2) - b*tan(x/2)^2)*(a*b1 - a1*b))
/(a^2 + b^2)
```

3.170 $\int \frac{1}{\log(t)} dt$

Optimal result

Integrand size = 4, antiderivative size = 2

$$\int \frac{1}{\log(t)} dt = \text{LogIntegral}(t)$$

```
[Out] Li(t)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{2335\}$

$$\int \frac{1}{\log(t)} dt = \text{LogIntegral}(t)$$

```
[In] Int[Log[t]^(-1),t]
```

```
[Out] LogIntegral[t]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\text{integral} = \text{LogIntegral}(t)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(t)}\,dt = \text{LogIntegral}(t)$$

```
[In] Integrate[Log[t]^(-1),t]
[Out] LogIntegral[t]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. 2(2) = 4.

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 4.50

method	result	size
default	$-\operatorname{Ei}_{1}(-\ln(t))$	9
risch	$-\operatorname{Ei}_{1}(-\ln(t))$	9

```
[In] int(1/ln(t),t,method=_RETURNVERBOSE)
[Out] -Ei(1,-ln(t))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(t)}\,dt = \text{log\_integral}\,(t)$$

```
[In] integrate(1/log(t),t, algorithm="fricas")
[Out] log_integral(t)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(t)}\, dt = \mathrm{li}\,(t)$$

```
[In] integrate(1/ln(t),t)
[Out] li(t)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{\log(t)}\, dt = \mathrm{Ei}(\log\,(t))$$

```
[In] integrate(1/log(t),t, algorithm="maxima")
[Out] Ei(log(t))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{\log(t)}\, dt = \mathrm{Ei}(\log\,(t))$$

```
[In] integrate(1/log(t),t, algorithm="giac")
[Out] Ei(log(t))
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(t)}\, dt = \mathrm{logint}(t)$$

```
[In] int(1/log(t),t)
[Out] logint(t)
```

3.171 $\int \frac{1}{\log^2(t)}\,dt$

Optimal result

Integrand size = 4, antiderivative size = 10

$$\int \frac{1}{\log^2(t)}\,dt = -\frac{t}{\log(t)} + \text{LogIntegral}(t)$$

```
[Out] Li(t)-t/ln(t)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2334, 2335}

$$\int \frac{1}{\log^2(t)}\,dt = \text{LogIntegral}(t) - \frac{t}{\log(t)}$$

```
[In] Int[Log[t]^(-2),t]
```

```
[Out] -(t/Log[t]) + LogIntegral[t]
```

<u>Rule 2334</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

<u>Rule 2335</u>

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{t}{\log(t)} + \int \frac{1}{\log(t)}\,dt \\ &= -\frac{t}{\log(t)} + \text{LogIntegral}(t)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log^2(t)}\,dt = -\frac{t}{\log(t)} + \text{LogIntegral}(t)$$

```
[In] Integrate[Log[t]^(-2),t]
[Out] -(t/Log[t]) + LogIntegral[t]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

method	result	size
default	$-\frac{t}{\ln(t)} - \text{Ei}_1(-\ln(t))$	17
risch	$-\frac{t}{\ln(t)} - \text{Ei}_1(-\ln(t))$	17

```
[In] int(1/ln(t)^2,t,method=_RETURNVERBOSE)
[Out] -t/ln(t)-Ei(1,-ln(t))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1}{\log^2(t)}\,dt = \frac{\log(t)\,\text{log\_integral}(t) - t}{\log(t)}$$

```
[In] integrate(1/log(t)^2,t, algorithm="fricas")
[Out] (log(t)*log_integral(t) - t)/log(t)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{\log^2(t)}\,dt = -\frac{t}{\log{(t)}} + \operatorname{li}{(t)}$$

```
[In] integrate(1/ln(t)**2,t)
[Out] -t/log(t) + li(t)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\log^2(t)}\,dt = \Gamma(-1, -\log{(t)})$$

```
[In] integrate(1/log(t)^2,t, algorithm="maxima")
[Out] gamma(-1, -log(t))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1}{\log^2(t)}\,dt = -\frac{t}{\log{(t)}} + \operatorname{Ei}(\log{(t)})$$

```
[In] integrate(1/log(t)^2,t, algorithm="giac")
[Out] -t/log(t) + Ei(log(t))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log^2(t)}\,dt = \operatorname{logint}(t) - \frac{t}{\ln{(t)}}$$

```
[In] int(1/log(t)^2,t)
[Out] logint(t) - t/log(t)
```

3.172 $\int \log^{-1-n}(t)\,dt$

Optimal result

Integrand size = 8, antiderivative size = 22

$$\int \log^{-1-n}(t)\,dt = -\Gamma(-n,-\log(t))(-\log(t))^n \log^{-n}(t)$$

```
[Out] -GAMMA(-n,-ln(t))*(-ln(t))^n/(ln(t)^n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2336, 2212}

$$\int \log^{-1-n}(t)\,dt = (-\log(t))^n \log^{-n}(t)(-\Gamma(-n,-\log(t)))$$

```
[In] Int[Log[t]^(-1 - n),t]
[Out] -((Gamma[-n, -Log[t]]*(-Log[t])^n)/Log[t]^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
```

```
c, p}, x] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int e^t t^{-1-n}\,dt, t, \log(t)\right)\\ &= -\Gamma(-n,-\log(t))(-\log(t))^n \log^{-n}(t)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \log^{-1-n}(t)\,dt = -\Gamma(-n,-\log(t))(-\log(t))^n \log^{-n}(t)$$

```
[In] Integrate[Log[t]^(-1 - n),t]
[Out] -((Gamma[-n, -Log[t]]*(-Log[t])^n)/Log[t]^n)
```

Maple [F]

$$\int \ln(t)^{-1-n}\,dt$$

```
[In] int(ln(t)^(-1-n),t)
[Out] int(ln(t)^(-1-n),t)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \log^{-1-n}(t)\,dt = e^{(i\pi + i\pi n)}\Gamma(-n,-\log(t))$$

```
[In] integrate(log(t)^(-1-n),t, algorithm="fricas")
[Out] e^(I*pi + I*pi*n)*gamma(-n, -log(t))
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \log^{-1-n}(t)\,dt = (-\log(t))^{n+1}\log(t)^{-n-1}\Gamma(-n, -\log(t))$$

```
[In] integrate(ln(t)**(-1-n),t)
[Out] (-log(t))**(n + 1)*log(t)**(-n - 1)*uppergamma(-n, -log(t))
```

Maxima [A] (verification not implemented)

none

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \log^{-1-n}(t)\,dt = -(-\log(t))^n\log(t)^{-n}\,\Gamma(-n, -\log(t))$$

```
[In] integrate(log(t)^(-1-n),t, algorithm="maxima")
[Out] -(-log(t))^n*log(t)^(-n)*gamma(-n, -log(t))
```

Giac [F]

$$\int \log^{-1-n}(t)\,dt = \int \log(t)^{-n-1}\,dt$$

```
[In] integrate(log(t)^(-1-n),t, algorithm="giac")
[Out] integrate(log(t)^(-n - 1), t)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \log^{-1-n}(t)\,dt = -\frac{(-\ln(t))^n\,\Gamma(-n, -\ln(t))}{\ln(t)^n}$$

```
[In] int(1/log(t)^(n + 1),t)
[Out] -((-log(t))^n*igamma(-n, -log(t)))/log(t)^n
```

3.173 $\int \frac{e^{2t}}{-1+t}\,dt$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{e^{2t}}{-1+t}\,dt = e^2\,\text{ExpIntegralEi}(-2(1-t))$$

```
[Out] exp(2)*Ei(-2+2*t)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2209}

$$\int \frac{e^{2t}}{-1+t}\,dt = e^2\,\text{ExpIntegralEi}(-2(1-t))$$

```
[In] Int[E^(2*t)/(-1 + t),t]
```

```
[Out] E^2*ExpIntegralEi[-2*(1 - t)]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$\text{integral} = e^2\,\text{ExpIntegralEi}(-2(1-t))$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2t}}{-1+t}\,dt = e^2\,\mathrm{ExpIntegralEi}(2(-1+t))$$

```
[In] Integrate[E^(2*t)/(-1 + t),t]
[Out] E^2*ExpIntegralEi[2*(-1 + t)]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\mathrm{e}^{2}\,\mathrm{Ei}_{1}\left(-2t+2\right)$	12
default	$-\mathrm{e}^{2}\,\mathrm{Ei}_{1}\left(-2t+2\right)$	12
risch	$-\mathrm{e}^{2}\,\mathrm{Ei}_{1}\left(-2t+2\right)$	12

```
[In] int(exp(2*t)/(-1+t),t,method=_RETURNVERBOSE)
[Out] -exp(2)*Ei(1,-2*t+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{e^{2t}}{-1+t}\,dt = \mathrm{Ei}(2\,t-2)\,e^2$$

```
[In] integrate(exp(2*t)/(-1+t),t, algorithm="fricas")
[Out] Ei(2*t - 2)*e^2
```

Sympy [F]

$$\int \frac{e^{2t}}{-1+t}\,dt = \int \frac{e^{2t}}{t-1}\,dt$$

```
[In] integrate(exp(2*t)/(-1+t),t)
[Out] Integral(exp(2*t)/(t - 1), t)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{e^{2t}}{-1+t}\,dt = -e^2E_1(-2\,t+2)$$

```
[In] integrate(exp(2*t)/(-1+t),t, algorithm="maxima")
[Out] -e^2*exp_integral_e(1, -2*t + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{e^{2t}}{-1+t}\,dt = \mathrm{Ei}(2\,t-2)\,e^2$$

```
[In] integrate(exp(2*t)/(-1+t),t, algorithm="giac")
[Out] Ei(2*t - 2)*e^2
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{e^{2t}}{-1+t}\,dt = \mathrm{e}^2\,\mathrm{ei}(2\,t-2)$$

```
[In] int(exp(2*t)/(t - 1),t)
[Out] exp(2)*ei(2*t - 2)
```

3.174 $\int \frac{e^{2x}}{2-3x+x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{e^{2x}}{2-3x+x^2}\,dx = e^4\,\text{ExpIntegralEi}(-4+2x) - e^2\,\text{ExpIntegralEi}(-2+2x)$$

```
[Out] exp(4)*Ei(-4+2*x)-exp(2)*Ei(-2+2*x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2300, 2209}

$$\int \frac{e^{2x}}{2-3x+x^2}\,dx = e^4\,\text{ExpIntegralEi}(2x-4) - e^2\,\text{ExpIntegralEi}(2x-2)$$

```
[In] Int[E^(2*x)/(2 - 3*x + x^2),x]
```

```
[Out] E^4*ExpIntegralEi[-4 + 2*x] - E^2*ExpIntegralEi[-2 + 2*x]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2300

```
Int[(F_)^((g_.)*((d_.) + (e_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[F^(g*(d + e*x)^n), 1/(a + b*x + c*x^
```

```
2), x], x] /; FreeQ[{F, a, b, c, d, e, g, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{2e^{2x}}{4-2x}-\frac{2e^{2x}}{-2+2x}\right)dx\\ &= -\left(2\int \frac{e^{2x}}{4-2x}\,dx\right)-2\int \frac{e^{2x}}{-2+2x}\,dx\\ &= e^4\,\text{ExpIntegralEi}(-4+2x)-e^2\,\text{ExpIntegralEi}(-2+2x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{2-3x+x^2}\,dx = e^4\,\text{ExpIntegralEi}(-4+2x)-e^2\,\text{ExpIntegralEi}(-2+2x)$$

```
[In] Integrate[E^(2*x)/(2 - 3*x + x^2),x]
[Out] E^4*ExpIntegralEi[-4 + 2*x] - E^2*ExpIntegralEi[-2 + 2*x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-e^4\,\text{Ei}_1(-2x+4)+e^2\,\text{Ei}_1(-2x+2)$	23
default	$-e^4\,\text{Ei}_1(-2x+4)+e^2\,\text{Ei}_1(-2x+2)$	23
risch	$-e^4\,\text{Ei}_1(-2x+4)+e^2\,\text{Ei}_1(-2x+2)$	23

```
[In] int(exp(2*x)/(x^2-3*x+2),x,method=_RETURNVERBOSE)
[Out] -exp(4)*Ei(1,-2*x+4)+exp(2)*Ei(1,-2*x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{e^{2x}}{2-3x+x^2}\,dx = \mathrm{Ei}(2\,x-4)\,e^4-\mathrm{Ei}(2\,x-2)\,e^2$$

```
[In] integrate(exp(2*x)/(x^2-3*x+2),x, algorithm="fricas")
[Out] Ei(2*x - 4)*e^4 - Ei(2*x - 2)*e^2
```

Sympy [F]

$$\int \frac{e^{2x}}{2-3x+x^2}\,dx = \int \frac{e^{2x}}{(x-2)\,(x-1)}\,dx$$

```
[In] integrate(exp(2*x)/(x**2-3*x+2),x)
[Out] Integral(exp(2*x)/((x - 2)*(x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2x}}{2-3x+x^2}\,dx = \int \frac{e^{(2\,x)}}{x^2-3\,x+2}\,dx$$

```
[In] integrate(exp(2*x)/(x^2-3*x+2),x, algorithm="maxima")
[Out] integrate(e^(2*x)/(x^2 - 3*x + 2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{e^{2x}}{2-3x+x^2}\,dx = \mathrm{Ei}(2\,x-4)\,e^4-\mathrm{Ei}(2\,x-2)\,e^2$$

```
[In] integrate(exp(2*x)/(x^2-3*x+2),x, algorithm="giac")
[Out] Ei(2*x - 4)*e^4 - Ei(2*x - 2)*e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2x}}{2-3x+x^2}\,dx = \int \frac{e^{2\,x}}{x^2-3\,x+2}\,dx$$

```
[In] int(exp(2*x)/(x^2 - 3*x + 2),x)
[Out] int(exp(2*x)/(x^2 - 3*x + 2), x)
```

3.175 $\int \frac{1}{\sqrt{1+t^3}}\,dt$

Optimal result

Integrand size = 9, antiderivative size = 103

$$\int \frac{1}{\sqrt{1+t^3}}\,dt=\frac{2\sqrt{2+\sqrt{3}}(1+t)\sqrt{\frac{1-t+t^2}{\left(1+\sqrt{3}+t\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+t}{1+\sqrt{3}+t}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+t}{\left(1+\sqrt{3}+t\right)^2}}\sqrt{1+t^3}}$$

```
[Out] 2/3*(1+t)*EllipticF((1+t-3^(1/2))/(1+t+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((t^2-t+1)/(1+t+3^(1/2))^2)^(1/2)*3^(3/4)/(t^3+1)^(1/2)/((1+t
)/(1+t+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {224}

$$\int \frac{1}{\sqrt{1+t^3}}\,dt=\frac{2\sqrt{2+\sqrt{3}}(t+1)\sqrt{\frac{t^2-t+1}{\left(t+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{t-\sqrt{3}+1}{t+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{t+1}{\left(t+\sqrt{3}+1\right)^2}}\sqrt{t^3+1}}$$

```
[In] Int[1/Sqrt[1 + t^3],t]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*(1 + t)*Sqrt[(1 - t + t^2)/(1 + Sqrt[3] + t)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + t)/(1 + Sqrt[3] + t)], -7 - 4*Sqrt[3]])/(3^(1/4)*
Sqrt[(1 + t)/(1 + Sqrt[3] + t)^2]*Sqrt[1 + t^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{2+\sqrt{3}}(1+t)\sqrt{\frac{1-t+t^2}{\left(1+\sqrt{3}+t\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+t}{1+\sqrt{3}+t}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+t}{\left(1+\sqrt{3}+t\right)^2}}\sqrt{1+t^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.17

$$\int \frac{1}{\sqrt{1+t^3}}\,dt = t\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-t^3\right)$$

```
[In] Integrate[1/Sqrt[1 + t^3],t]
[Out] t*Hypergeometric2F1[1/3, 1/2, 4/3, -t^3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.14

method	result	size
meijerg	$t_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-t^3\right)$	14
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+t}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{t-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{t-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+t}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{t^3+1}}$	116
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+t}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{t-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{t-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+t}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{t^3+1}}$	116

```
[In] int(1/(t^3+1)^(1/2),t,method=_RETURNVERBOSE)
[Out] t*hypergeom([1/3,1/2],[4/3],-t^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.06

$$\int \frac{1}{\sqrt{1+t^3}} dt = 2 \,\mathrm{weierstrassPInverse}(0, -4, t)$$

```
[In] integrate(1/(t^3+1)^(1/2),t, algorithm="fricas")
[Out] 2*weierstrassPInverse(0, -4, t)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.26

$$\int \frac{1}{\sqrt{1+t^3}} dt = \frac{t\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{4}{3} \end{matrix}\middle| t^3 e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate(1/(t**3+1)**(1/2),t)
[Out] t*gamma(1/3)*hyper((1/3, 1/2), (4/3,), t**3*exp_polar(I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+t^3}} dt = \int \frac{1}{\sqrt{t^3+1}} dt$$

```
[In] integrate(1/(t^3+1)^(1/2),t, algorithm="maxima")
[Out] integrate(1/sqrt(t^3 + 1), t)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+t^3}} dt = \int \frac{1}{\sqrt{t^3+1}} dt$$

```
[In] integrate(1/(t^3+1)^(1/2),t, algorithm="giac")
[Out] integrate(1/sqrt(t^3 + 1), t)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{1+t^3}}\,dt = \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{t-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{t+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-t+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{t+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{t^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)t-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(1/(t^3 + 1)^(1/2),t)
```

```
[Out] ((3^(1/2)*1i + 3)*((t + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*((t + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - t + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((t + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(t^3 - t*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2))^(1/2)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                    end if
                else #result contains complex but optimal is not
                    if debug then
                            print("result contains complex but optimal is not");
                    fi;
                    return "C","Result contains complex when optimal does not.";
                fi;
            else # result do not contain complex
                 # this assumes optimal do not as well. No check is needed here.
                if debug then
                       print("result do not contain complex, this assumes optimal do not as well")
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    if debug then
                        print("leaf_count_result<=2*leaf_count_optimal");
                    fi;
                    return "A"," ";
                else
                    if debug then
                        print("leaf_count_result>2*leaf_count_optimal");
                    fi;
                    return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
                fi;
            fi;
        else #ExpnType(result) > ExpnType(optimal)
            if debug then
                print("ExpnType(result) > ExpnType(optimal)");
            fi;
            return "C",cat("Result contains higher order function than in optimal. Order ",
                           convert(ExpnType_result,string)," vs. order ",
                           convert(ExpnType_optimal,string),".");
        fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```