Computer Algebra
Independent Integration Tests

Summer 2023 edition

0-Independent-test-suites/8-Moses-Problems

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [113]. This is test number [8].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (113)	0.00 (0)
Mathematica	100.00 (113)	0.00 (0)
Maple	100.00 (113)	0.00 (0)
Fricas	99.12 (112)	0.88 (1)
Giac	98.23 (111)	1.77 (2)
Maxima	98.23 (111)	1.77 (2)
Sympy	94.69 (107)	5.31 (6)
Mupad	93.81 (106)	6.19 (7)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

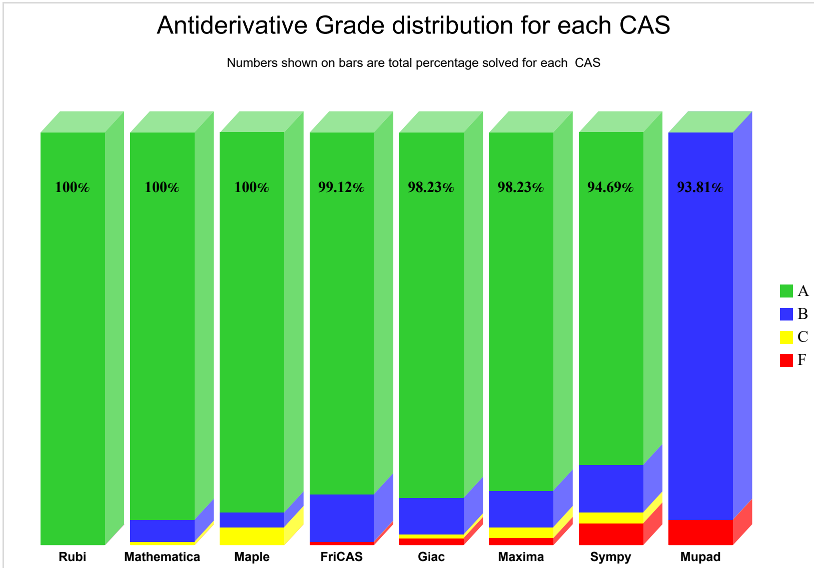
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

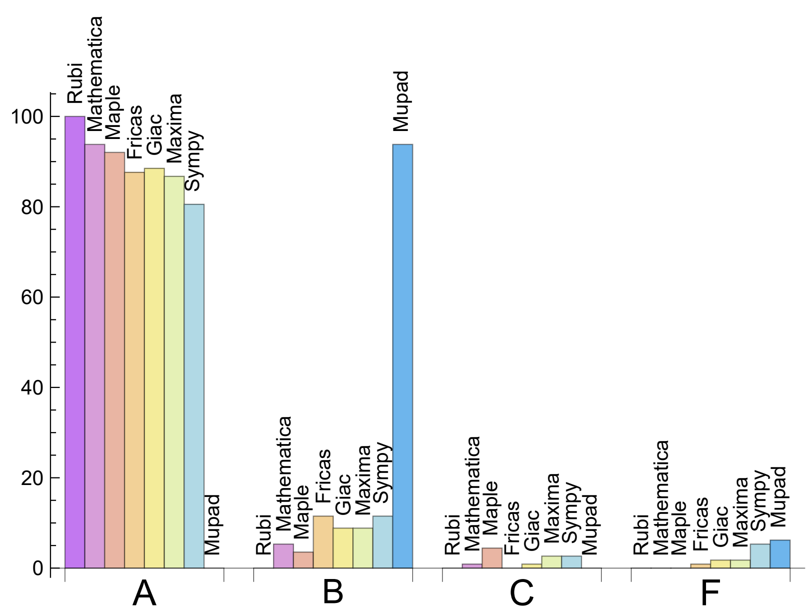
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	93.805	5.310	0.885	0.000
Maple	92.035	3.540	4.425	0.000
Giac	88.496	8.850	0.885	1.770
Fricas	87.611	11.504	0.000	0.885
Maxima	86.726	8.850	2.655	1.770
Sympy	80.531	11.504	2.655	5.310
Mupad	0.000	93.805	0.000	6.195

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Fricas	1	100.00	0.00	0.00
Giac	2	100.00	0.00	0.00
Maxima	2	100.00	0.00	0.00
Sympy	6	100.00	0.00	0.00
Mupad	7	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.02
Mathematica	0.05
Mupad	0.11
Maple	0.13
Maxima	0.22
Fricas	0.24
Giac	0.28
Sympy	1.29

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	20.01	1.08	13.00	0.88
Rubi	20.14	1.01	16.00	1.00
Giac	23.58	1.15	14.00	0.85
Maxima	23.66	1.16	14.00	0.88
Mathematica	24.39	1.13	16.00	1.00
Fricas	24.87	1.16	14.00	0.95
Mupad	28.25	1.65	12.00	0.83
Sympy	30.84	1.68	15.00	0.83

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

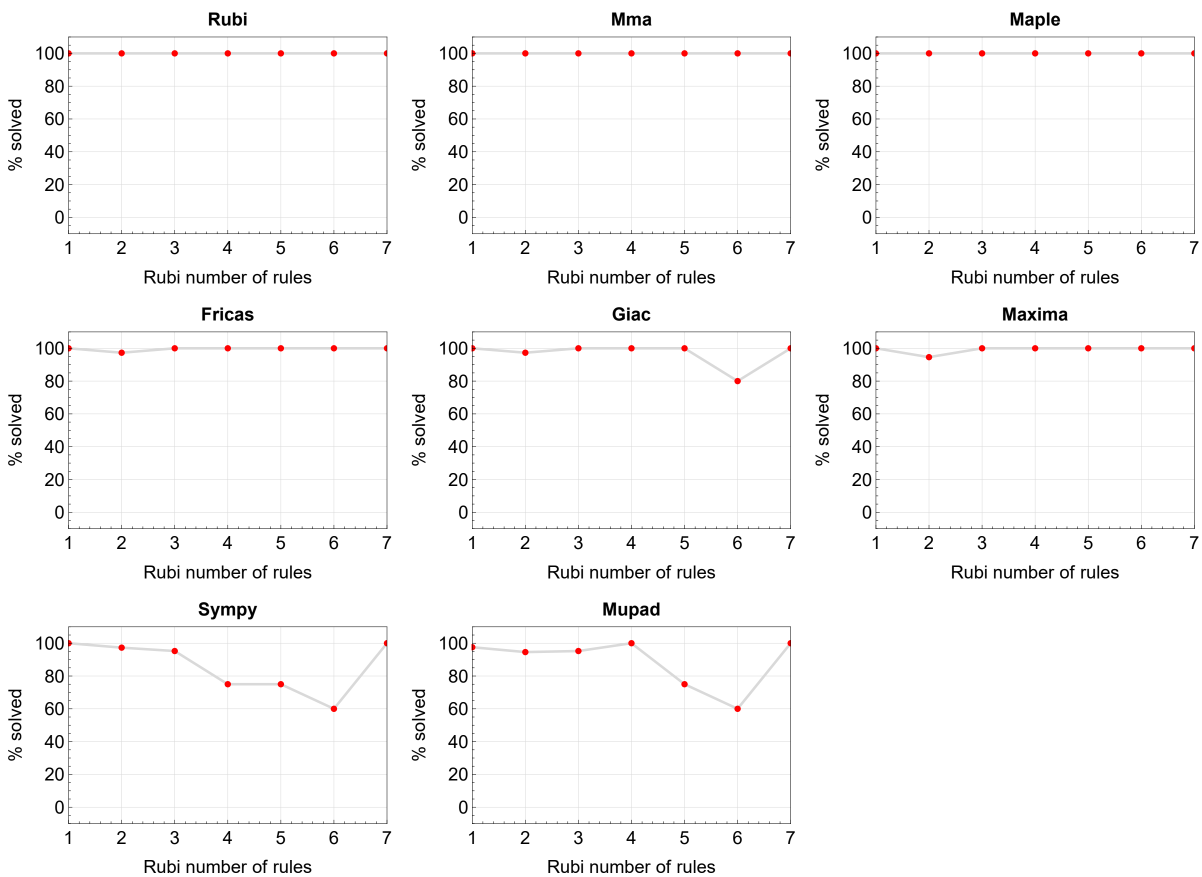


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

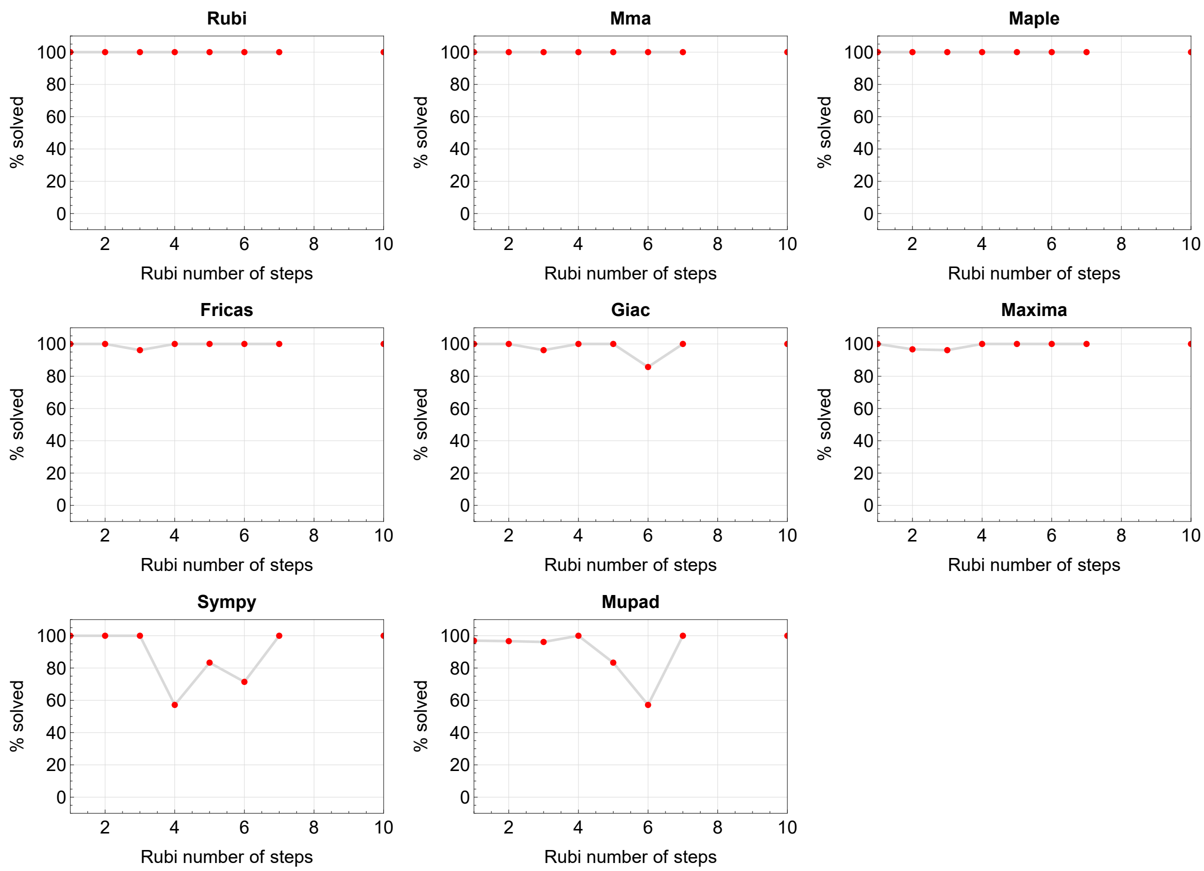


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

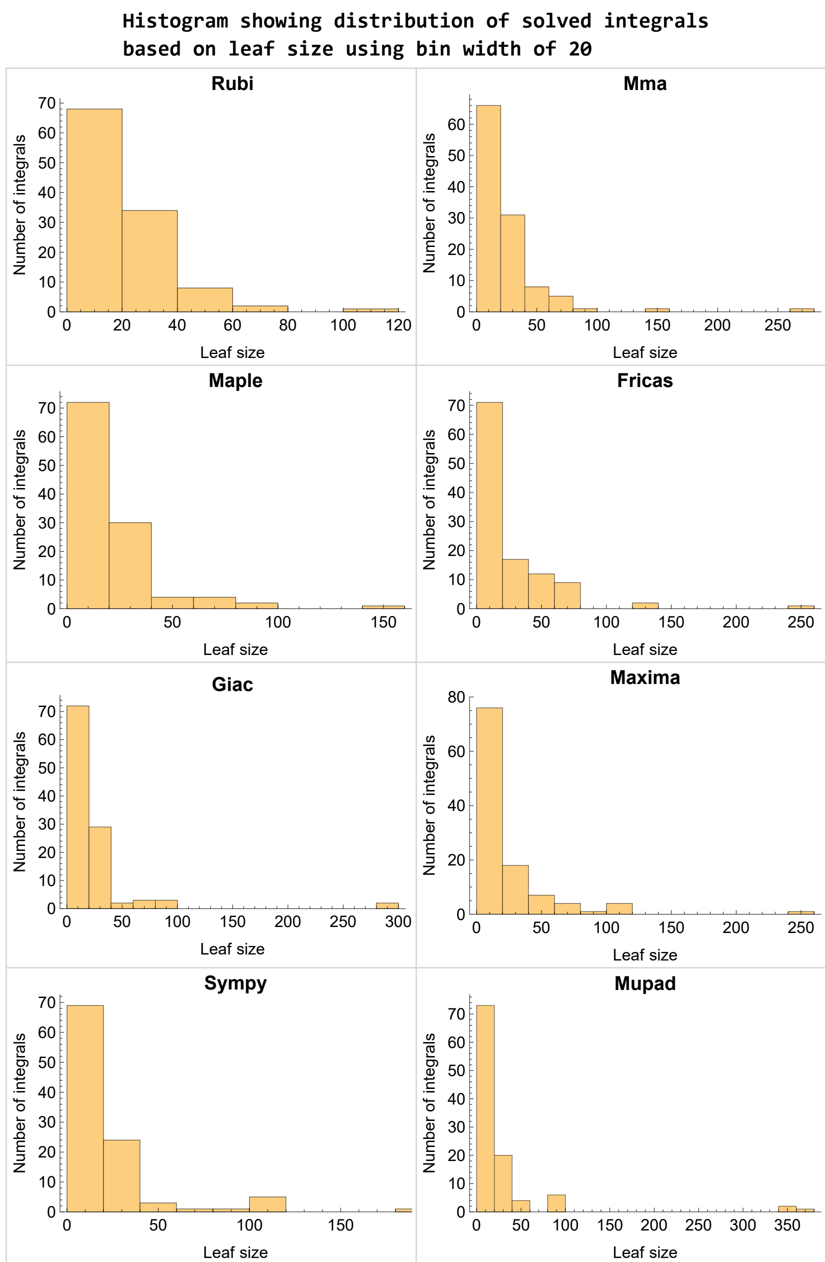


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

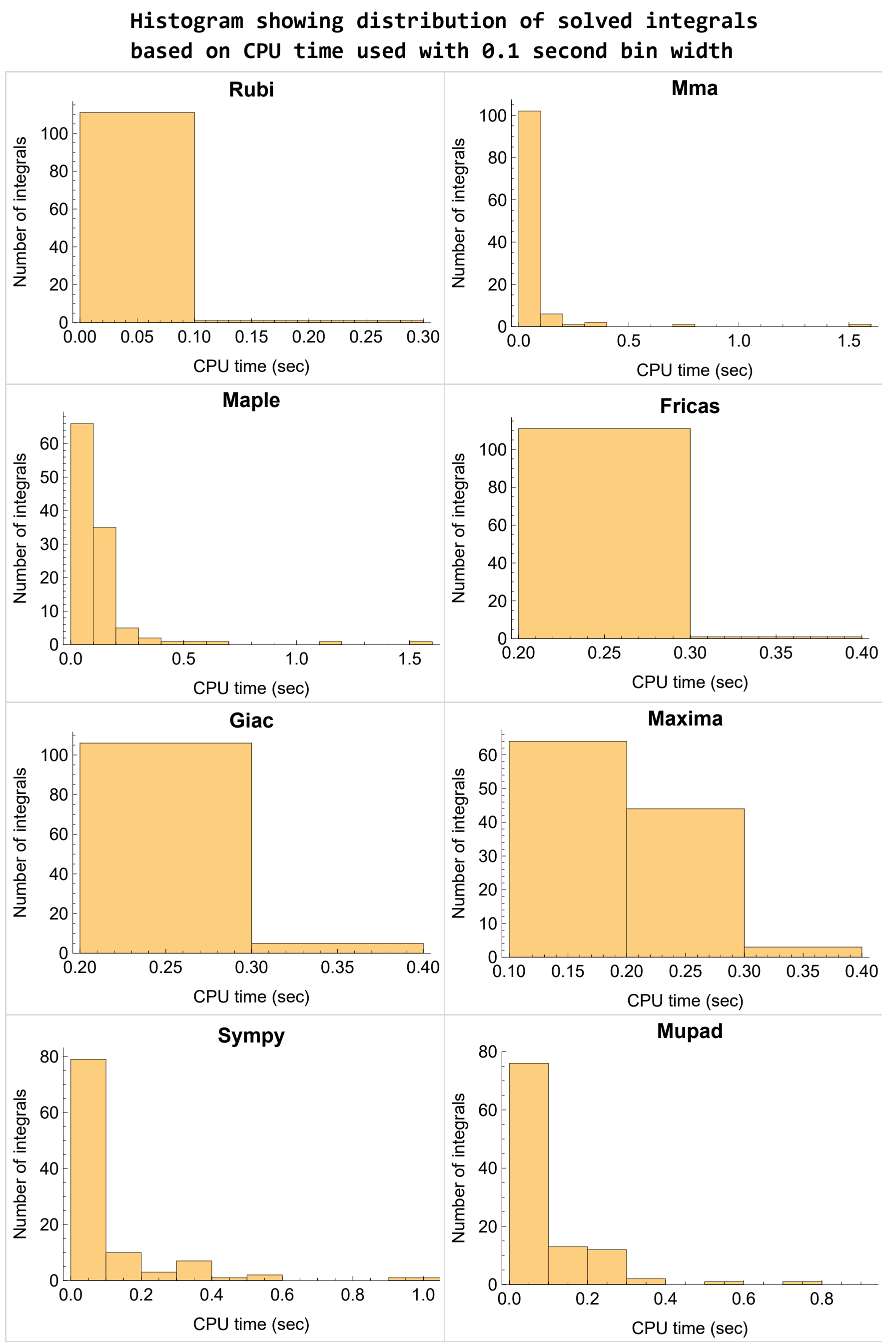


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

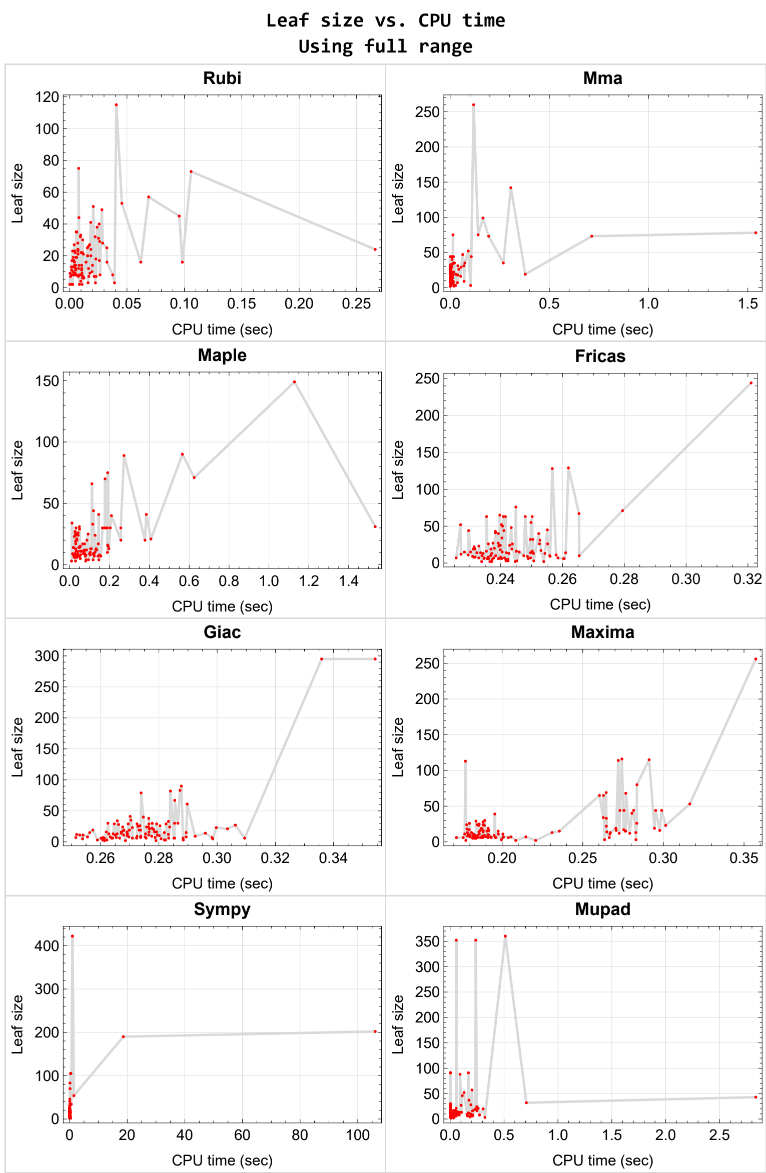


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
 # 1.7 is a fudge factor since it is low side from actual leaf count
 leafCount = round(1.7*count_ops(anti))

 except Exception as ee:
      leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

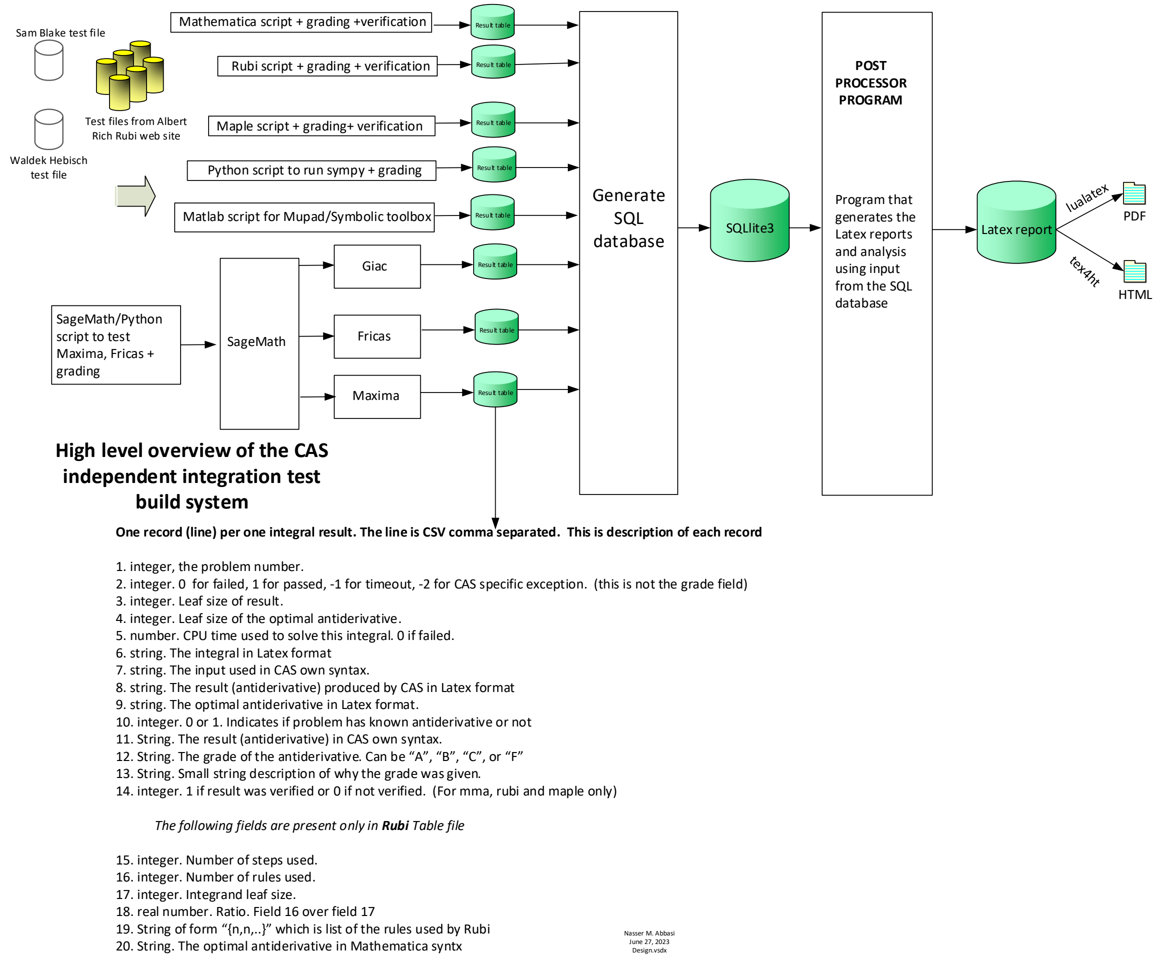
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 22
2.2 Detailed conclusion table per each integral for all CAS systems 25
2.3 Detailed conclusion table specific for Rubi results 48

2.1 List of integrals sorted by grade for each CAS

Rubi 22
Mma 22
Maple 23
Fricas 23
Maxima 23
Giac 24
Mupad 24
Sympy 24

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { 31, 42, 70, 71, 72, 84 }

C grade { 1 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { 36, 48, 57, 70 }

C grade { 32, 42, 68, 71, 72 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 38, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { 1, 9, 37, 39, 40, 42, 69, 70, 73, 74, 83, 84, 100 }

C grade { }

F normal fail { 32 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 39, 41, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { 36, 38, 40, 42, 68, 69, 70, 71, 72, 87 }

C grade { 10, 11, 47 }

F normal fail { 32, 67 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 44, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { 1, 25, 36, 40, 43, 45, 69, 70, 71, 72 }

C grade { 47 }

F normal fail { 32, 42 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 39, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

C grade { }

F normal fail { }

F(-1) timedout fail { 10, 11, 32, 38, 40, 42, 69 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 41, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 70, 74, 75, 76, 77, 78, 79, 80, 81, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { 7, 18, 37, 39, 50, 51, 56, 66, 73, 82, 83, 84, 100 }

C grade { 38, 71, 72 }

F normal fail { 35, 36, 40, 42, 69, 87 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	22	11	16	48	19	34	10
N.S.	1	1.00	1.83	0.92	1.33	4.00	1.58	2.83	0.83
time (sec)	N/A	0.011	0.001	0.058	0.271	0.243	0.025	0.266	0.002

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	19	14	15	12
N.S.	1	1.00	1.00	0.92	1.15	1.46	1.08	1.15	0.92
time (sec)	N/A	0.004	0.006	0.136	0.277	0.231	0.045	0.256	0.170

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	11	11	14	15	11	10
N.S.	1	1.00	0.74	0.58	0.58	0.74	0.79	0.58	0.53
time (sec)	N/A	0.003	0.012	0.073	0.198	0.242	0.074	0.277	0.030

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.003	0.001	0.030	0.208	0.245	0.032	0.261	0.031

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.78	0.67	0.67	0.56	0.67	0.67
time (sec)	N/A	0.005	0.001	0.032	0.204	0.235	0.029	0.260	0.019

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.009	0.007	0.155	0.193	0.261	0.020	0.288	0.024

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	22	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	1.69	0.69	0.69
time (sec)	N/A	0.001	0.002	0.103	0.195	0.231	0.068	0.292	0.029

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	12	11	13	15	11	11
N.S.	1	1.00	0.74	0.63	0.58	0.68	0.79	0.58	0.58
time (sec)	N/A	0.006	0.010	0.085	0.201	0.237	0.098	0.280	0.018

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	14	8	6	6
N.S.	1	1.00	1.00	0.88	0.75	1.75	1.00	0.75	0.75
time (sec)	N/A	0.010	0.007	0.157	0.196	0.232	0.022	0.285	0.054

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	15	3	3	3	0
N.S.	1	1.00	1.00	1.00	3.75	0.75	0.75	0.75	0.00
time (sec)	N/A	0.010	0.018	0.147	0.235	0.242	0.281	0.282	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	13	2	2	2	0
N.S.	1	1.00	1.00	1.50	6.50	1.00	1.00	1.00	0.00
time (sec)	N/A	0.011	0.019	0.081	0.231	0.254	0.389	0.279	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	7	7	5	7	7
N.S.	1	1.00	1.00	1.00	0.88	0.88	0.62	0.88	0.88
time (sec)	N/A	0.005	0.010	0.104	0.193	0.241	0.032	0.298	0.042

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.00	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.017	0.036	0.043	0.201	0.236	0.035	0.309	0.191

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	19	19	20	19	21
N.S.	1	1.00	0.93	0.75	0.68	0.68	0.71	0.68	0.75
time (sec)	N/A	0.029	0.044	0.036	0.191	0.237	0.039	0.273	0.056

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.68	0.73	0.73
time (sec)	N/A	0.005	0.013	0.031	0.188	0.235	0.037	0.279	0.046

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.006	0.001	0.049	0.188	0.260	0.034	0.283	0.017

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.78	0.67	0.67	0.56	0.67	0.67
time (sec)	N/A	0.005	0.000	0.033	0.191	0.249	0.029	0.272	0.002

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	22	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	1.69	0.69	0.69
time (sec)	N/A	0.002	0.001	0.104	0.180	0.256	0.070	0.272	0.002

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	5	5	5
N.S.	1	1.00	1.00	1.00	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.013	0.013	0.029	0.182	0.242	0.030	0.299	0.034

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56
time (sec)	N/A	0.000	0.001	0.037	0.183	0.237	0.032	0.281	0.042

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.003	0.008	0.076	0.184	0.240	0.045	0.263	0.067

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	7	7	7	7	7
N.S.	1	1.00	1.00	1.00	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.004	0.001	0.032	0.190	0.226	0.046	0.251	0.017

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	19	8	7	7	8	7	7
N.S.	1	1.00	1.90	0.80	0.70	0.70	0.80	0.70	0.70
time (sec)	N/A	0.019	0.029	0.150	0.201	0.259	0.285	0.261	0.209

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	13	15	15	34	15	12
N.S.	1	1.00	0.78	0.57	0.65	0.65	1.48	0.65	0.52
time (sec)	N/A	0.003	0.016	0.109	0.191	0.230	0.555	0.289	0.032

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	17	17	17	19	9
N.S.	1	1.00	1.92	0.77	1.31	1.31	1.31	1.46	0.69
time (sec)	N/A	0.002	0.004	0.127	0.275	0.253	0.063	0.268	0.157

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	14	13	13	15	13	13
N.S.	1	1.00	1.00	0.78	0.72	0.72	0.83	0.72	0.72
time (sec)	N/A	0.023	0.038	0.049	0.283	0.251	0.062	0.264	0.090

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	20	19	76	22	19	19
N.S.	1	1.00	1.00	0.65	0.61	2.45	0.71	0.61	0.61
time (sec)	N/A	0.025	0.039	0.087	0.294	0.245	0.078	0.270	0.235

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	11	8	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.38	1.00	1.00	1.00
time (sec)	N/A	0.027	0.016	0.041	0.182	0.258	0.040	0.289	0.211

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	15	14	14	10	13	12
N.S.	1	1.00	1.00	1.25	1.17	1.17	0.83	1.08	1.00
time (sec)	N/A	0.005	0.004	0.054	0.186	0.233	0.033	0.266	0.066

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.016	0.001	0.144	0.186	0.245	0.122	0.267	0.207

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	260	41	40	40	46	40	52
N.S.	1	1.00	5.31	0.84	0.82	0.82	0.94	0.82	1.06
time (sec)	N/A	0.028	0.118	0.146	0.280	0.239	0.078	0.275	0.129

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	142	41	0	0	54	0	0
N.S.	1	1.00	1.23	0.36	0.00	0.00	0.47	0.00	0.00
time (sec)	N/A	0.041	0.306	0.385	0.000	0.000	1.501	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.011	0.018	0.075	0.192	0.250	0.096	0.277	0.248

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	13	15	15	34	15	12
N.S.	1	1.00	0.78	0.57	0.65	0.65	1.48	0.65	0.52
time (sec)	N/A	0.005	0.002	0.105	0.198	0.228	0.559	0.282	0.002

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	33	25	24	24	0	24	24
N.S.	1	1.00	1.03	0.78	0.75	0.75	0.00	0.75	0.75
time (sec)	N/A	0.009	0.006	0.093	0.188	0.240	0.000	0.284	0.002

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	75	75	80	55	0	61	57
N.S.	1	1.00	1.70	1.70	1.82	1.25	0.00	1.39	1.30
time (sec)	N/A	0.008	0.141	0.191	0.284	0.250	0.000	0.290	0.203

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	44	30	44	63	105	29	91
N.S.	1	1.00	1.26	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.006	0.106	0.257	0.273	0.250	0.398	0.279	0.169

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	52	44	113	44	190	90	0
N.S.	1	1.00	0.69	0.59	1.51	0.59	2.53	1.20	0.00
time (sec)	N/A	0.008	0.091	0.120	0.177	0.241	18.654	0.288	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	44	30	44	63	105	29	91
N.S.	1	1.00	1.26	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.006	0.005	0.165	0.276	0.235	0.391	0.272	0.002

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	78	90	114	129	0	295	0
N.S.	1	1.00	1.53	1.76	2.24	2.53	0.00	5.78	0.00
time (sec)	N/A	0.021	1.540	0.566	0.272	0.262	0.000	0.336	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.004	0.002	0.061	0.191	0.256	0.018	0.269	0.029

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	57	99	149	116	244	0	0	0
N.S.	1	1.16	2.02	3.04	2.37	4.98	0.00	0.00	0.00
time (sec)	N/A	0.069	0.166	1.128	0.274	0.321	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	9	9	3	30	4
N.S.	1	1.00	0.67	0.56	1.00	1.00	0.33	3.33	0.44
time (sec)	N/A	0.011	0.003	0.054	0.189	0.249	0.098	0.285	0.184

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	7	7	6	6	5	6	6
N.S.	1	1.00	0.64	0.64	0.55	0.55	0.45	0.55	0.55
time (sec)	N/A	0.005	0.001	0.026	0.185	0.236	0.028	0.267	0.016

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	8	8	5	30	8
N.S.	1	1.00	1.00	1.00	0.89	0.89	0.56	3.33	0.89
time (sec)	N/A	0.018	0.070	0.100	0.188	0.232	0.033	0.287	0.084

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.00	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.023	0.002	0.040	0.187	0.239	0.032	0.285	0.162

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	7	8	9	7
N.S.	1	1.00	1.00	0.73	0.82	0.64	0.73	0.82	0.64
time (sec)	N/A	0.002	0.001	0.018	0.185	0.235	0.081	0.274	0.018

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	8	2	2	2	2	2
N.S.	1	1.00	1.00	4.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.009	0.001	0.033	0.221	0.237	0.366	0.269	0.008

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	40	33	34	34	41	35	46
N.S.	1	1.00	0.98	0.80	0.83	0.83	1.00	0.85	1.12
time (sec)	N/A	0.018	0.010	0.116	0.263	0.240	0.061	0.270	0.112

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	73	75	66	65	65	83	67	88
N.S.	1	1.55	1.60	1.40	1.38	1.38	1.77	1.43	1.87
time (sec)	N/A	0.106	0.014	0.112	0.260	0.240	0.127	0.285	0.092

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	30	39	27	70	41	21
N.S.	1	1.00	1.00	1.43	1.86	1.29	3.33	1.95	1.00
time (sec)	N/A	0.007	0.005	0.189	0.195	0.237	0.151	0.270	0.260

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.003	0.000	0.019	0.179	0.243	0.031	0.275	0.035

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	29	34	33	24	32	38	24
N.S.	1	1.00	0.72	0.85	0.82	0.60	0.80	0.95	0.60
time (sec)	N/A	0.026	0.009	0.012	0.265	0.243	0.093	0.277	0.002

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	5	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	0.71	1.00	1.00
time (sec)	N/A	0.001	0.001	0.122	0.206	0.238	0.026	0.265	0.019

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.62	1.00	1.00
time (sec)	N/A	0.038	0.016	0.119	0.179	0.231	0.035	0.275	0.272

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	15	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.023	0.028	0.095	0.264	0.242	0.069	0.259	0.323

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	9	3	2	2	3	2
N.S.	1	1.00	1.00	4.50	1.50	1.00	1.00	1.50	1.00
time (sec)	N/A	0.002	0.001	0.015	0.200	0.237	0.173	0.261	0.009

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	15	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	1.07	1.00	1.00
time (sec)	N/A	0.020	0.006	0.194	0.188	0.261	0.069	0.267	0.061

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	13	13	11	14	8	11	15
N.S.	1	1.00	0.76	0.76	0.65	0.82	0.47	0.65	0.88
time (sec)	N/A	0.026	0.006	0.030	0.186	0.250	0.044	0.268	0.057

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	29	25	24	24	26	24	28
N.S.	1	1.00	0.76	0.66	0.63	0.63	0.68	0.63	0.74
time (sec)	N/A	0.024	0.055	0.037	0.189	0.240	0.042	0.276	0.193

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	7	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.008	0.003	0.082	0.215	0.233	0.070	0.279	0.019

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.010	0.006	0.066	0.192	0.245	0.107	0.279	0.002

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	7	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.012	0.002	0.055	0.180	0.250	0.061	0.274	0.002

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	17	22	22	22	17
N.S.	1	1.00	1.00	0.82	0.61	0.79	0.79	0.79	0.61
time (sec)	N/A	0.007	0.001	0.024	0.180	0.231	0.040	0.278	0.031

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	15	8	9	9
N.S.	1	1.00	1.00	0.91	0.82	1.36	0.73	0.82	0.82
time (sec)	N/A	0.011	0.004	0.190	0.183	0.240	0.121	0.278	0.033

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	15	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.016	0.003	0.083	0.283	0.245	0.091	0.282	0.002

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	0	3	3	3	43
N.S.	1	1.00	1.00	1.33	0.00	1.00	1.00	1.00	14.33
time (sec)	N/A	0.039	0.103	0.118	0.000	0.251	0.133	0.288	2.835

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	20	53	15	12	21	13
N.S.	1	1.00	1.00	1.25	3.31	0.94	0.75	1.31	0.81
time (sec)	N/A	0.033	0.053	0.257	0.316	0.246	0.055	0.304	0.029

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	73	89	115	128	0	295	0
N.S.	1	1.00	1.38	1.68	2.17	2.42	0.00	5.57	0.00
time (sec)	N/A	0.046	0.194	0.273	0.291	0.257	0.000	0.354	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	35	71	69	67	202	83	360
N.S.	1	1.00	2.19	4.44	4.31	4.19	12.62	5.19	22.50
time (sec)	N/A	0.062	0.268	0.625	0.265	0.265	106.059	0.287	0.512

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	35	70	68	26	422	82	352
N.S.	1	1.00	2.19	4.38	4.25	1.62	26.38	5.12	22.00
time (sec)	N/A	0.098	0.022	0.178	0.277	0.255	0.991	0.284	0.238

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	35	40	65	26	422	79	352
N.S.	1	1.00	2.19	2.50	4.06	1.62	26.38	4.94	22.00
time (sec)	N/A	0.014	0.016	0.210	0.263	0.253	1.009	0.274	0.057

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	44	30	44	63	105	29	91
N.S.	1	1.00	1.26	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.006	0.016	0.204	0.282	0.241	0.382	0.277	0.002

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	12	26	19	12	12
N.S.	1	1.00	1.14	0.93	0.86	1.86	1.36	0.86	0.86
time (sec)	N/A	0.008	0.007	0.197	0.279	0.244	0.022	0.283	0.073

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	8	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.62	0.85	0.85
time (sec)	N/A	0.004	0.005	0.132	0.267	0.229	0.032	0.265	0.028

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.00	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.021	0.002	0.039	0.188	0.240	0.032	0.276	0.002

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	19	24	23	23	20	30	24
N.S.	1	1.00	0.79	1.00	0.96	0.96	0.83	1.25	1.00
time (sec)	N/A	0.266	0.379	0.128	0.301	0.239	0.048	0.278	0.247

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	8	8	7	8	8
N.S.	1	1.00	1.00	1.00	0.89	0.89	0.78	0.89	0.89
time (sec)	N/A	0.004	0.015	0.026	0.182	0.236	0.029	0.255	0.033

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	24	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.96	0.76	0.76	0.76	0.76
time (sec)	N/A	0.032	0.004	0.037	0.185	0.244	0.039	0.257	0.228

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.002	0.004	0.124	0.266	0.236	0.038	0.271	0.054

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.004	0.006	0.133	0.273	0.227	0.040	0.252	0.021

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	16	20	36	16	10
N.S.	1	1.00	1.00	0.79	1.14	1.43	2.57	1.14	0.71
time (sec)	N/A	0.011	0.003	0.055	0.298	0.238	0.178	0.262	0.190

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	44	30	44	63	105	29	91
N.S.	1	1.00	1.26	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.006	0.001	0.176	0.295	0.242	0.391	0.269	0.002

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	35	16	15	45	34	21	37
N.S.	1	1.00	2.06	0.94	0.88	2.65	2.00	1.24	2.18
time (sec)	N/A	0.004	0.074	0.191	0.276	0.255	0.180	0.277	0.175

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	12	11	13	15	11	11
N.S.	1	1.00	0.74	0.63	0.58	0.68	0.79	0.58	0.58
time (sec)	N/A	0.006	0.007	0.074	0.177	0.254	0.088	0.253	0.002

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	3	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.50	1.00
time (sec)	N/A	0.000	0.000	0.010	0.178	0.234	0.033	0.262	0.007

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	73	31	256	71	0	33	32
N.S.	1	1.00	1.62	0.69	5.69	1.58	0.00	0.73	0.71
time (sec)	N/A	0.096	0.713	1.533	0.357	0.279	0.000	0.283	0.707

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	16	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	1.14	0.71
time (sec)	N/A	0.005	0.002	0.054	0.179	0.237	0.023	0.269	0.003

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	14	11	11	10	14	11	10
N.S.	1	1.00	0.82	0.65	0.65	0.59	0.82	0.65	0.59
time (sec)	N/A	0.002	0.013	0.040	0.182	0.234	0.077	0.254	0.026

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	31	20	19	27	19	27	27
N.S.	1	1.00	1.35	0.87	0.83	1.17	0.83	1.17	1.17
time (sec)	N/A	0.007	0.069	0.378	0.271	0.239	0.255	0.306	0.101

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	10	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	1.25	0.62	0.75	0.75
time (sec)	N/A	0.009	0.002	0.079	0.176	0.265	0.034	0.264	0.027

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	5	5	5
N.S.	1	1.00	1.00	1.00	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.011	0.001	0.023	0.184	0.242	0.032	0.254	0.003

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.017	0.002	0.033	0.191	0.238	0.037	0.280	0.047

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	10	10	7	8	6
N.S.	1	1.00	0.67	0.75	0.83	0.83	0.58	0.67	0.50
time (sec)	N/A	0.008	0.006	0.069	0.179	0.247	0.163	0.270	0.208

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.009	0.002	0.142	0.172	0.239	0.021	0.261	0.003

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.003	0.000	0.019	0.181	0.234	0.038	0.269	0.003

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.006	0.001	0.036	0.196	0.236	0.032	0.264	0.003

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	47	21	22	32	15	20	20
N.S.	1	1.00	1.96	0.88	0.92	1.33	0.62	0.83	0.83
time (sec)	N/A	0.007	0.063	0.408	0.265	0.250	0.348	0.278	0.306

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	14	13	19	15	13	13
N.S.	1	1.00	1.00	0.70	0.65	0.95	0.75	0.65	0.65
time (sec)	N/A	0.019	0.006	0.046	0.267	0.232	0.049	0.263	0.106

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	44	30	44	63	105	29	91
N.S.	1	1.00	1.26	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.006	0.013	0.180	0.299	0.248	0.403	0.282	0.002

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	15	14	14	24	14	14
N.S.	1	1.00	0.90	0.75	0.70	0.70	1.20	0.70	0.70
time (sec)	N/A	0.016	0.035	0.053	0.265	0.250	0.051	0.296	0.073

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	26	32	31	26	26
N.S.	1	1.00	1.00	0.82	0.79	0.97	0.94	0.79	0.79
time (sec)	N/A	0.010	0.011	0.028	0.283	0.250	0.054	0.280	0.002

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	19	31	15	19	19
N.S.	1	1.00	1.00	0.95	0.90	1.48	0.71	0.90	0.90
time (sec)	N/A	0.012	0.000	0.035	0.187	0.253	0.016	0.274	0.003

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	24	40	19	24	24
N.S.	1	1.00	1.00	0.96	0.92	1.54	0.73	0.92	0.92
time (sec)	N/A	0.017	0.000	0.039	0.184	0.252	0.018	0.268	0.002

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	25	43	22	25	25
N.S.	1	1.00	1.00	0.96	0.93	1.59	0.81	0.93	0.93
time (sec)	N/A	0.017	0.000	0.043	0.186	0.238	0.017	0.273	0.002

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	30	52	26	30	30
N.S.	1	1.00	1.00	0.97	0.94	1.62	0.81	0.94	0.94
time (sec)	N/A	0.022	0.000	0.049	0.189	0.240	0.018	0.262	0.003

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	25	24	23	41	20	23	23
N.S.	1	1.00	1.04	1.00	0.96	1.71	0.83	0.96	0.96
time (sec)	N/A	0.019	0.000	0.043	0.189	0.238	0.015	0.300	0.003

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	30	29	28	50	24	28	28
N.S.	1	1.00	1.03	1.00	0.97	1.72	0.83	0.97	0.97
time (sec)	N/A	0.026	0.000	0.050	0.188	0.241	0.017	0.266	0.003

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	18	18	17	18	18
N.S.	1	1.00	1.00	1.00	0.95	0.95	0.89	0.95	0.95
time (sec)	N/A	0.006	0.000	0.030	0.178	0.240	0.012	0.268	0.003

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	24	23	23	20	23	23
N.S.	1	1.00	1.00	1.00	0.96	0.96	0.83	0.96	0.96
time (sec)	N/A	0.007	0.000	0.029	0.185	0.233	0.012	0.271	0.003

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	24	44	24	24	24
N.S.	1	1.00	1.00	1.00	0.96	1.76	0.96	0.96	0.96
time (sec)	N/A	0.016	0.000	0.030	0.178	0.230	0.012	0.276	0.003

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	30	29	52	27	29	29
N.S.	1	1.00	1.00	1.00	0.97	1.73	0.90	0.97	0.97
time (sec)	N/A	0.011	0.000	0.032	0.184	0.227	0.013	0.265	0.003

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	28	27	26	26	24	26	26
N.S.	1	1.00	1.04	1.00	0.96	0.96	0.89	0.96	0.96
time (sec)	N/A	0.004	0.000	0.027	0.180	0.236	0.012	0.273	0.002

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [50] had the largest ratio of [.8570999999999999973]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	4	0.500
2	A	3	2	1.00	11	0.182
3	A	2	1	1.00	11	0.091
4	A	1	1	1.00	2	0.500
5	A	1	1	1.00	7	0.143
6	A	2	2	1.00	7	0.286
7	A	1	1	1.00	11	0.091
8	A	1	1	1.00	6	0.167
9	A	2	2	1.00	7	0.286
10	A	2	2	1.00	4	0.500
11	A	1	1	1.00	6	0.167
12	A	3	2	1.00	6	0.333
13	A	4	2	1.00	16	0.125
14	A	5	3	1.00	7	0.429
15	A	3	1	1.00	14	0.071
16	A	2	2	1.00	5	0.400
17	A	1	1	1.00	7	0.143
18	A	1	1	1.00	11	0.091
19	A	2	2	1.00	11	0.182
20	A	1	1	1.00	5	0.200
21	A	1	1	1.00	6	0.167
22	A	2	2	1.00	9	0.222
23	A	3	3	1.00	14	0.214
24	A	2	1	1.00	9	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	3	1.00	7	0.429
26	A	2	2	1.00	15	0.133
27	A	2	2	1.00	17	0.118
28	A	3	3	1.00	13	0.231
29	A	1	1	1.00	5	0.200
30	A	3	3	1.00	8	0.375
31	A	7	7	1.00	11	0.636
32	A	3	2	1.00	12	0.167
33	A	3	3	1.00	6	0.500
34	A	2	1	1.00	9	0.111
35	A	4	3	1.00	13	0.231
36	A	4	4	1.00	15	0.267
37	A	3	2	1.00	15	0.133
38	A	6	3	1.00	13	0.231
39	A	3	2	1.00	15	0.133
40	A	5	5	1.00	29	0.172
41	A	2	2	1.00	4	0.500
42	A	6	6	1.16	19	0.316
43	A	1	1	1.00	6	0.167
44	A	2	2	1.00	5	0.400
45	A	1	1	1.00	10	0.100
46	A	5	3	1.00	13	0.231
47	A	1	1	1.00	5	0.200
48	A	1	1	1.00	7	0.143
49	A	6	6	1.00	9	0.667
50	A	10	6	1.55	7	0.857
51	A	1	1	1.00	27	0.037
52	A	1	1	1.00	4	0.250
53	A	4	3	1.00	6	0.500
54	A	2	2	1.00	10	0.200
55	A	7	4	1.00	9	0.444
56	A	2	1	1.00	12	0.083
57	A	1	1	1.00	4	0.250
58	A	6	4	1.00	7	0.571
59	A	4	3	1.00	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	6	3	1.00	9	0.333
61	A	2	2	1.00	4	0.500
62	A	3	3	1.00	6	0.500
63	A	2	2	1.00	4	0.500
64	A	2	2	1.00	6	0.333
65	A	2	1	1.00	9	0.111
66	A	2	1	1.00	12	0.083
67	A	2	2	1.00	20	0.100
68	A	2	2	1.00	10	0.200
69	A	6	6	1.00	30	0.200
70	A	5	5	1.00	39	0.128
71	A	6	5	1.00	48	0.104
72	A	4	4	1.00	31	0.129
73	A	3	2	1.00	15	0.133
74	A	3	2	1.00	4	0.500
75	A	3	2	1.00	11	0.182
76	A	5	3	1.00	13	0.231
77	A	10	6	1.00	33	0.182
78	A	1	1	1.00	7	0.143
79	A	5	5	1.00	8	0.625
80	A	2	2	1.00	9	0.222
81	A	3	3	1.00	11	0.273
82	A	3	3	1.00	8	0.375
83	A	3	2	1.00	15	0.133
84	A	2	2	1.00	16	0.125
85	A	1	1	1.00	6	0.167
86	A	1	1	1.00	3	0.333
87	A	4	2	1.00	17	0.118
88	A	2	2	1.00	4	0.500
89	A	2	1	1.00	11	0.091
90	A	3	3	1.00	14	0.214
91	A	2	2	1.00	7	0.286
92	A	2	2	1.00	11	0.182
93	A	3	2	1.00	13	0.154
94	A	1	1	1.00	8	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	2	2	1.00	7	0.286
96	A	1	1	1.00	4	0.250
97	A	2	2	1.00	5	0.400
98	A	3	3	1.00	17	0.176
99	A	3	3	1.00	16	0.188
100	A	3	2	1.00	15	0.133
101	A	3	3	1.00	15	0.200
102	A	3	3	1.00	8	0.375
103	A	1	1	1.00	20	0.050
104	A	1	1	1.00	25	0.040
105	A	1	1	1.00	26	0.038
106	A	1	1	1.00	31	0.032
107	A	1	1	1.00	24	0.042
108	A	1	1	1.00	29	0.034
109	A	1	1	1.00	18	0.056
110	A	1	1	1.00	23	0.043
111	A	1	1	1.00	24	0.042
112	A	1	1	1.00	29	0.034
113	A	1	1	1.00	27	0.037

CHAPTER 3

LISTING OF INTEGRALS

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3.92 $\int \frac{e^x}{1+e^x}\,dx$ 409
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3.1 $\int \cot^4(x)\,dx$

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Mathematica [C] (verified) 58
Maple [A] (verified) 58
Fricas [B] (verification not implemented) 59
Sympy [A] (verification not implemented) 59
Maxima [A] (verification not implemented) 59
Giac [B] (verification not implemented) 60
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Optimal result

Integrand size = 4, antiderivative size = 12

$$\int \cot^4(x)\,dx = x + \cot(x) - \frac{\cot^3(x)}{3}$$

```
[Out] x+cot(x)-1/3*cot(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 8}

$$\int \cot^4(x)\,dx = x - \frac{1}{3}\cot^3(x) + \cot(x)$$

```
[In] Int[Cot[x]^4,x]
```

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{3}\cot^3(x) - \int \cot^2(x)\,dx\\ &= \cot(x) - \frac{\cot^3(x)}{3} + \int 1\,dx\\ &= x + \cot(x) - \frac{\cot^3(x)}{3}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \cot^4(x)\,dx = -\frac{1}{3}\cot^3(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(x)\right)$$

```
[In] Integrate[Cot[x]^4,x]
[Out] -1/3*(Cot[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[x]^2])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$x+\cot\left(x\right)-\frac{\left(\cot^{3}\left(x\right)\right)}{3}$	11
derivativedivides	$-\frac{\left(\cot^{3}\left(x\right)\right)}{3}+\cot\left(x\right)-\frac{\pi}{2}+\operatorname{arccot}\left(\cot\left(x\right)\right)$	16
default	$-\frac{\left(\cot^{3}\left(x\right)\right)}{3}+\cot\left(x\right)-\frac{\pi}{2}+\operatorname{arccot}\left(\cot\left(x\right)\right)$	16
norman	$\frac{-\frac{1}{3}+\tan^{2}\left(x\right)+x\left(\tan^{3}\left(x\right)\right)}{\tan\left(x\right)^{3}}$	18
risch	$x+\frac{4i\left(3\,{\mathrm e}^{4ix}-3\,{\mathrm e}^{2ix}+2\right)}{3\left({\mathrm e}^{2ix}-1\right)^{3}}$	31

```
[In] int(cot(x)^4,x,method=_RETURNVERBOSE)
[Out] x+cot(x)-1/3*cot(x)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(10) = 20$.

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.00

$$\int \cot^4(x)\,dx = \frac{4\cos(2x)^2 + 3(x\cos(2x) - x)\sin(2x) + 2\cos(2x) - 2}{3(\cos(2x) - 1)\sin(2x)}$$

```
[In] integrate(cot(x)^4,x, algorithm="fricas")
[Out] 1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*x
) - 1)*sin(2*x))
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \cot^4(x)\,dx = x + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3\sin^3(x)}$$

```
[In] integrate(cot(x)**4,x)
[Out] x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \cot^4(x)\,dx = x + \frac{3\tan(x)^2 - 1}{3\tan(x)^3}$$

```
[In] integrate(cot(x)^4,x, algorithm="maxima")
[Out] x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(10) = 20.

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.83

$$\int \cot^4(x)\,dx = \frac{1}{24}\tan\left(\frac{1}{2}x\right)^3 + x + \frac{15\tan\left(\frac{1}{2}x\right)^2 - 1}{24\tan\left(\frac{1}{2}x\right)^3} - \frac{5}{8}\tan\left(\frac{1}{2}x\right)$$

```
[In] integrate(cot(x)^4,x, algorithm="giac")
[Out] 1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(1
/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \cot^4(x)\,dx = -\frac{\cot(x)^3}{3} + \cot(x) + x$$

```
[In] int(cot(x)^4,x)
[Out] x + cot(x) - cot(x)^3/3
```

3.2 $\int \frac{1}{x^4(1+x^2)}\,dx$

Optimal result 61
Rubi [A] (verified) 61
Mathematica [A] (verified) 62
Maple [A] (verified) 62
Fricas [A] (verification not implemented) 63
Sympy [A] (verification not implemented) 63
Maxima [A] (verification not implemented) 63
Giac [A] (verification not implemented) 63
Mupad [B] (verification not implemented) 64

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{1}{x^4\left(1+x^2\right)}\,dx=-\frac{1}{3x^3}+\frac{1}{x}+\arctan(x)$$

```
[Out] -1/3/x^3+1/x+arctan(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {331, 209}

$$\int \frac{1}{x^4\left(1+x^2\right)}\,dx=\arctan(x)-\frac{1}{3x^3}+\frac{1}{x}$$

```
[In] Int[1/(x^4*(1 + x^2)),x]
```

```
[Out] -1/3*1/x^3 + x^(-1) + ArcTan[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{1}{3x^3} - \int \frac{1}{x^2(1+x^2)}\,dx \\ &= -\frac{1}{3x^3} + \frac{1}{x} + \int \frac{1}{1+x^2}\,dx \\ &= -\frac{1}{3x^3} + \frac{1}{x} + \arctan(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(1+x^2)}\,dx = -\frac{1}{3x^3} + \frac{1}{x} + \arctan(x)$$

```
[In] Integrate[1/(x^4*(1 + x^2)),x]
[Out] -1/3*1/x^3 + x^(-1) + ArcTan[x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{1}{3x^3}+\frac{1}{x}+\arctan\left(x\right)$	12
meijerg	$-\frac{1}{3x^3}+\frac{1}{x}+\arctan\left(x\right)$	12
risch	$\frac{x^2-\frac{1}{3}}{x^3}+\arctan\left(x\right)$	13
parallelrisch	$-\frac{3i\ln(x-i)x^3-3i\ln(x+i)x^3+2-6x^2}{6x^3}$	35

```
[In] int(1/x^4/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/3/x^3+1/x+arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^4(1+x^2)}\,dx = \frac{3\,x^3\arctan\left(x\right)+3\,x^2-1}{3\,x^3}$$

```
[In] integrate(1/x^4/(x^2+1),x, algorithm="fricas")
[Out] 1/3*(3*x^3*arctan(x) + 3*x^2 - 1)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4(1+x^2)}\,dx = \operatorname{atan}\left(x\right)+\frac{3x^2-1}{3x^3}$$

```
[In] integrate(1/x**4/(x**2+1),x)
[Out] atan(x) + (3*x**2 - 1)/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^4(1+x^2)}\,dx = \frac{3\,x^2-1}{3\,x^3}+\arctan\left(x\right)$$

```
[In] integrate(1/x^4/(x^2+1),x, algorithm="maxima")
[Out] 1/3*(3*x^2 - 1)/x^3 + arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^4(1+x^2)}\,dx = \frac{3\,x^2-1}{3\,x^3}+\arctan\left(x\right)$$

```
[In] integrate(1/x^4/(x^2+1),x, algorithm="giac")
[Out] 1/3*(3*x^2 - 1)/x^3 + arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (1+x^2)} \, dx = \mathrm{atan}(x) + \frac{x^2 - \frac{1}{3}}{x^3}$$

```
[In] int(1/(x^4*(x^2 + 1)),x)
[Out] atan(x) + (x^2 - 1/3)/x^3
```

3.3 $\int \frac{x+x^2}{\sqrt{x}}\,dx$

Optimal result 65
Rubi [A] (verified) 65
Mathematica [A] (verified) 66
Maple [A] (verified) 66
Fricas [A] (verification not implemented) 66
Sympy [A] (verification not implemented) 67
Maxima [A] (verification not implemented) 67
Giac [A] (verification not implemented) 67
Mupad [B] (verification not implemented) 67

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{x+x^2}{\sqrt{x}}\,dx = \frac{2x^{3/2}}{3}+\frac{2x^{5/2}}{5}$$

```
[Out] 2/3*x^(3/2)+2/5*x^(5/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\int \frac{x+x^2}{\sqrt{x}}\,dx = \frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{3}$$

```
[In] Int[(x + x^2)/Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3 + (2*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\sqrt{x}+x^{3/2}\right)\,dx\\ &= \frac{2x^{3/2}}{3}+\frac{2x^{5/2}}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{x+x^2}{\sqrt{x}}\,dx=\frac{2}{15}x^{3/2}(5+3x)$$

```
[In] Integrate[(x + x^2)/Sqrt[x],x]
[Out] (2*x^(3/2)*(5 + 3*x))/15
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}(5+3x)}{15}$	11
trager	$\frac{2x^{\frac{3}{2}}(5+3x)}{15}$	11
risch	$\frac{2x^{\frac{3}{2}}(5+3x)}{15}$	11
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3}+\frac{2x^{\frac{5}{2}}}{5}$	12
default	$\frac{2x^{\frac{3}{2}}}{3}+\frac{2x^{\frac{5}{2}}}{5}$	12

```
[In] int((x^2+x)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15*x^(3/2)*(5+3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{x+x^2}{\sqrt{x}}\,dx=\frac{2}{15}\left(3\,x^2+5\,x\right)\sqrt{x}$$

```
[In] integrate((x^2+x)/x^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*x^2 + 5*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x + x^2}{\sqrt{x}} \, dx = \frac{2x^{\frac{5}{2}}}{5} + \frac{2x^{\frac{3}{2}}}{3}$$

```
[In] integrate((x**2+x)/x**(1/2),x)
[Out] 2*x**(5/2)/5 + 2*x**(3/2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{x + x^2}{\sqrt{x}} \, dx = \frac{2}{5} x^{\frac{5}{2}} + \frac{2}{3} x^{\frac{3}{2}}$$

```
[In] integrate((x^2+x)/x^(1/2),x, algorithm="maxima")
[Out] 2/5*x^(5/2) + 2/3*x^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{x + x^2}{\sqrt{x}} \, dx = \frac{2}{5} x^{\frac{5}{2}} + \frac{2}{3} x^{\frac{3}{2}}$$

```
[In] integrate((x^2+x)/x^(1/2),x, algorithm="giac")
[Out] 2/5*x^(5/2) + 2/3*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int \frac{x + x^2}{\sqrt{x}} \, dx = \frac{2 \, x^{3/2} \, (3 \, x + 5)}{15}$$

```
[In] int((x + x^2)/x^(1/2),x)
[Out] (2*x^(3/2)*(3*x + 5))/15
```

3.4 $\int \cos(x)\,dx$

Optimal result 68
Rubi [A] (verified) 68
Mathematica [A] (verified) 69
Maple [A] (verified) 69
Fricas [A] (verification not implemented) 69
Sympy [A] (verification not implemented) 70
Maxima [A] (verification not implemented) 70
Giac [A] (verification not implemented) 70
Mupad [B] (verification not implemented) 70

Optimal result

Integrand size = 2, antiderivative size = 2

$$\int \cos(x)\,dx = \sin(x)$$

```
[Out] sin(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{2717\}$

$$\int \cos(x)\,dx = \sin(x)$$

```
[In] Int[Cos[x],x]
```

```
[Out] Sin[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \sin(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx=\sin(x)$$

```
[In] Integrate[Cos[x],x]
```

```
[Out] Sin[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
lookup	$\sin\left(x\right)$	3
default	$\sin\left(x\right)$	3
meijerg	$\sin\left(x\right)$	3
risch	$\sin\left(x\right)$	3
parallelrisch	$\sin\left(x\right)$	3
norman	$\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}$	17

```
[In] int(cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx=\sin\left(x\right)$$

```
[In] integrate(cos(x),x, algorithm="fricas")
```

```
[Out] sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

```
[In] integrate(cos(x),x)
[Out] sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

```
[In] integrate(cos(x),x, algorithm="maxima")
[Out] sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

```
[In] integrate(cos(x),x, algorithm="giac")
[Out] sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

```
[In] int(cos(x),x)
[Out] sin(x)
```

3.5 $\int e^{x^2} x \, dx$

Optimal result 71
Rubi [A] (verified) 71
Mathematica [A] (verified) 72
Maple [A] (verified) 72
Fricas [A] (verification not implemented) 72
Sympy [A] (verification not implemented) 73
Maxima [A] (verification not implemented) 73
Giac [A] (verification not implemented) 73
Mupad [B] (verification not implemented) 73

Optimal result

Integrand size = 7, antiderivative size = 9

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

```
[Out] 1/2*exp(x^2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2240}

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

```
[In] Int[E^x^2*x,x]
```

```
[Out] E^x^2/2
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{e^{x^2}}{2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

```
[In] Integrate[E^x^2*x,x]
[Out] E^x^2/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{e^{x^2}}{2}$	7
derivativedivides	$\frac{e^{x^2}}{2}$	7
default	$\frac{e^{x^2}}{2}$	7
norman	$\frac{e^{x^2}}{2}$	7
risch	$\frac{e^{x^2}}{2}$	7
parallelrisch	$\frac{e^{x^2}}{2}$	7
meijerg	$-\frac{1}{2}+\frac{e^{x^2}}{2}$	9
parts	$\frac{\operatorname{erfi}(x)\sqrt{\pi}\, x}{2}-\frac{\sqrt{\pi}\left(x \operatorname{erfi}(x)-\frac{e^{x^2}}{\sqrt{\pi}}\right)}{2}$	29

```
[In] int(exp(x^2)*x,x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x \, dx = \frac{1}{2} e^{\left(x^2\right)}$$

```
[In] integrate(exp(x^2)*x,x, algorithm="fricas")
[Out] 1/2*e^(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int e^{x^2}x\,dx=\frac{e^{x^2}}{2}$$

```
[In] integrate(exp(x**2)*x,x)
[Out] exp(x**2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2}x\,dx=\frac{1}{2}\,e^{\left(x^2\right)}$$

```
[In] integrate(exp(x^2)*x,x, algorithm="maxima")
[Out] 1/2*e^(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2}x\,dx=\frac{1}{2}\,e^{\left(x^2\right)}$$

```
[In] integrate(exp(x^2)*x,x, algorithm="giac")
[Out] 1/2*e^(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2}x\,dx=\frac{\mathrm{e}^{x^2}}{2}$$

```
[In] int(x*exp(x^2),x)
[Out] exp(x^2)/2
```

3.6 $\int \sec^2(x)\tan(x)\,dx$

Optimal result 74
Rubi [A] (verified) 74
Mathematica [A] (verified) 75
Maple [A] (verified) 75
Fricas [A] (verification not implemented) 75
Sympy [A] (verification not implemented) 76
Maxima [A] (verification not implemented) 76
Giac [A] (verification not implemented) 76
Mupad [B] (verification not implemented) 76

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \sec^2(x)\tan(x)\,dx = \frac{\sec^2(x)}{2}$$

```
[Out] 1/2*sec(x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2686, 30}

$$\int \sec^2(x)\tan(x)\,dx = \frac{\sec^2(x)}{2}$$

```
[In] Int[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int x\,dx, x, \sec(x)\right)$$
$$= \frac{\sec^2(x)}{2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\tan(x)\,dx = \frac{\sec^2(x)}{2}$$

```
[In] Integrate[Sec[x]^2*Tan[x],x]
[Out] Sec[x]^2/2
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sec^2(x))}{2}$	7
default	$\frac{(\sec^2(x))}{2}$	7
risch	$\frac{2\,e^{2ix}}{(e^{2ix}+1)^2}$	17

```
[In] int(sec(x)^2*tan(x),x,method=_RETURNVERBOSE)
[Out] 1/2*sec(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2\cos(x)^2}$$

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="fricas")
[Out] 1/2/cos(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2\cos^2(x)}$$

```
[In] integrate(sec(x)**2*tan(x),x)
[Out] 1/(2*cos(x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2}\tan(x)^2$$

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="maxima")
[Out] 1/2*tan(x)^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2\cos(x)^2}$$

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="giac")
[Out] 1/2/cos(x)^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{\tan(x)^2}{2}$$

```
[In] int(tan(x)/cos(x)^2,x)
[Out] tan(x)^2/2
```

3.7 $\int x\sqrt{1+x^2}\,dx$

Optimal result 77
Rubi [A] (verified) 77
Mathematica [A] (verified) 78
Maple [A] (verified) 78
Fricas [A] (verification not implemented) 78
Sympy [B] (verification not implemented) 79
Maxima [A] (verification not implemented) 79
Giac [A] (verification not implemented) 79
Mupad [B] (verification not implemented) 80

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(1+x^2\right)^{3/2}$$

```
[Out] 1/3*(x^2+1)^(3/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {267}

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[In] Int[x*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\left(1+x^2\right)^{3/2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(1+x^2\right)^{3/2}$$

```
[In] Integrate[x*Sqrt[1 + x^2],x]
[Out] (1 + x^2)^(3/2)/3
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$	10
derivativedivides	$\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$	10
default	$\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$	10
risch	$\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$	10
pseudoelliptic	$\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$	10
trager	$\left(\frac{x^2}{3}+\frac{1}{3}\right)\sqrt{x^2+1}$	16
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2x^2+2\right)\sqrt{x^2+1}}{3}}{4\sqrt{\pi}}$	31

```
[In] int(x*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(x^2+1)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*(x^2 + 1)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int x\sqrt{1+x^2}\,dx = \frac{x^2\sqrt{x^2+1}}{3} + \frac{\sqrt{x^2+1}}{3}$$

```
[In] integrate(x*(x**2+1)**(1/2),x)
[Out] x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\,\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/3*(x^2 + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\,\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/3*(x^2 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{(x^2+1)^{3/2}}{3}$$

```
[In] int(x*(x^2 + 1)^(1/2),x)
[Out] (x^2 + 1)^(3/2)/3
```

3.8 $\int e^x \sin(x)\,dx$

Optimal result 81
Rubi [A] (verified) 81
Mathematica [A] (verified) 82
Maple [A] (verified) 82
Fricas [A] (verification not implemented) 82
Sympy [A] (verification not implemented) 83
Maxima [A] (verification not implemented) 83
Giac [A] (verification not implemented) 83
Mupad [B] (verification not implemented) 83

Optimal result

Integrand size = 6, antiderivative size = 19

$$\int e^x \sin(x)\,dx = -\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)$$

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{4517\}$

$$\int e^x \sin(x)\,dx = \frac{1}{2}e^x\sin(x) - \frac{1}{2}e^x\cos(x)$$

```
[In] Int[E^x*Sin[x],x]
```

```
[Out] -1/2*(E^x*Cos[x]) + (E^x*Sin[x])/2
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int e^x \sin(x)\,dx = \frac{1}{2}e^x(-\cos(x)+\sin(x))$$

```
[In] Integrate[E^x*Sin[x],x]
[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{e^x(-\cos(x)+\sin(x))}{2}$	12
default	$-\frac{e^x\cos(x)}{2}+\frac{e^x\sin(x)}{2}$	14
norman	$\frac{e^x\tan\left(\frac{x}{2}\right)+\frac{e^x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{e^x}{2}}{1+\tan^2\left(\frac{x}{2}\right)}$	34
risch	$-\frac{e^{(1+i)x}}{4}-\frac{ie^{(1+i)x}}{4}-\frac{e^{(1-i)x}}{4}+\frac{ie^{(1-i)x}}{4}$	36

```
[In] int(exp(x)*sin(x),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x)*(-cos(x)+sin(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\cos(x)e^x + \frac{1}{2}e^x\sin(x)$$

```
[In] integrate(exp(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int e^x \sin(x)\,dx = \frac{e^x \sin{(x)}}{2} - \frac{e^x \cos{(x)}}{2}$$

```
[In] integrate(exp(x)*sin(x),x)
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\left(\cos{(x)} - \sin{(x)}\right)e^x$$

```
[In] integrate(exp(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\left(\cos{(x)} - \sin{(x)}\right)e^x$$

```
[In] integrate(exp(x)*sin(x),x, algorithm="giac")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{\mathrm{e}^x\left(\cos{(x)} - \sin{(x)}\right)}{2}$$

```
[In] int(exp(x)*sin(x),x)
[Out] -(exp(x)*(cos(x) - sin(x)))/2
```

3.9 $\int \cot(x)\csc^3(x)\,dx$

Optimal result 84
Rubi [A] (verified) 84
Mathematica [A] (verified) 85
Maple [A] (verified) 85
Fricas [B] (verification not implemented) 86
Sympy [A] (verification not implemented) 86
Maxima [A] (verification not implemented) 86
Giac [A] (verification not implemented) 86
Mupad [B] (verification not implemented) 87

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \cot(x)\csc^3(x)\,dx = -\frac{1}{3}\csc^3(x)$$

```
[Out] -1/3*csc(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2686, 30}

$$\int \cot(x)\csc^3(x)\,dx = -\frac{1}{3}\csc^3(x)$$

```
[In] Int[Cot[x]*Csc[x]^3,x]
[Out] -1/3*Csc[x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int x^2\,dx, x, \csc(x)\right)$$
$$= -\frac{1}{3}\csc^3(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cot(x)\csc^3(x)\,dx = -\frac{1}{3}\csc^3(x)$$

```
[In] Integrate[Cot[x]*Csc[x]^3,x]
[Out] -1/3*Csc[x]^3
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{(\csc^3(x))}{3}$	7
default	$-\frac{(\csc^3(x))}{3}$	7
parallelrisch	$-\frac{(\csc^3(x))}{3}$	7
risch	$\frac{8ie^{3ix}}{3(e^{2ix}-1)^3}$	18
norman	$\frac{-\frac{1}{24}-\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}-\frac{\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}-\frac{\left(\tan^6\left(\frac{x}{2}\right)\right)}{24}}{\tan\left(\frac{x}{2}\right)^3}$	34

```
[In] int(cos(x)*csc(x)^2/sin(x)^2,x,method=_RETURNVERBOSE)
[Out] -1/3*csc(x)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \cot(x)\csc^3(x)\,dx = \frac{1}{3\left(\cos\left(x\right)^2 - 1\right)\sin\left(x\right)}$$

```
[In] integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="fricas")
[Out] 1/3/((cos(x)^2 - 1)*sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cot(x)\csc^3(x)\,dx = -\frac{1}{3\sin^3\left(x\right)}$$

```
[In] integrate(cos(x)*csc(x)**2/sin(x)**2,x)
[Out] -1/(3*sin(x)**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cot(x)\csc^3(x)\,dx = -\frac{1}{3\,\sin\left(x\right)^3}$$

```
[In] integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="maxima")
[Out] -1/3/sin(x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cot(x)\csc^3(x)\,dx = -\frac{1}{3\,\sin\left(x\right)^3}$$

```
[In] integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="giac")
[Out] -1/3/sin(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cot(x)\csc^3(x)\,dx = -\frac{1}{3\sin(x)^3}$$

```
[In] int(cos(x)/sin(x)^4,x)
[Out] -1/(3*sin(x)^3)
```

3.10 $\int \sin(e^x) \, dx$

Optimal result 88
Rubi [A] (verified) 88
Mathematica [A] (verified) 89
Maple [A] (verified) 89
Fricas [A] (verification not implemented) 89
Sympy [A] (verification not implemented) 90
Maxima [C] (verification not implemented) 90
Giac [A] (verification not implemented) 90
Mupad [F(-1)] 90

Optimal result

Integrand size = 4, antiderivative size = 4

$$\int \sin(e^x) \, dx = \text{Si}(e^x)$$

```
[Out] Si(exp(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2320, 3380}

$$\int \sin(e^x) \, dx = \text{Si}(e^x)$$

```
[In] Int[Sin[E^x],x]
```

```
[Out] SinIntegral[E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, e^x\right)\\ &= \text{Si}(e^x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin\left(e^x\right)\,dx = \text{Si}(e^x)$$

```
[In] Integrate[Sin[E^x],x]
[Out] SinIntegral[E^x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\text{Si}\left(e^x\right)$	4
default	$\text{Si}\left(e^x\right)$	4
risch	$-\frac{\pi\ \text{csgn}(e^x)}{2} + \text{Si}\left(e^x\right)$	11

```
[In] int(sin(exp(x)),x,method=_RETURNVERBOSE)
[Out] Si(exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \sin\left(e^x\right)\,dx = \text{Si}\left(e^x\right)$$

```
[In] integrate(sin(exp(x)),x, algorithm="fricas")
[Out] sin_integral(e^x)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \sin\left(e^{x}\right)\,dx = \operatorname{Si}\left(e^{x}\right)$$

```
[In] integrate(sin(exp(x)),x)
[Out] Si(exp(x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \sin\left(e^{x}\right)\,dx = -\frac{1}{2}i\,\mathrm{Ei}(i\,e^{x}) + \frac{1}{2}i\,\mathrm{Ei}(-i\,e^{x})$$

```
[In] integrate(sin(exp(x)),x, algorithm="maxima")
[Out] -1/2*I*Ei(I*e^x) + 1/2*I*Ei(-I*e^x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \sin\left(e^{x}\right)\,dx = \operatorname{Si}\left(e^{x}\right)$$

```
[In] integrate(sin(exp(x)),x, algorithm="giac")
[Out] sin_integral(e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin\left(e^{x}\right)\,dx = \mathrm{sinint}(\mathrm{e}^{x})$$

```
[In] int(sin(exp(x)),x)
[Out] sinint(exp(x))
```

3.11 $\int \frac{\sin(y)}{y} \, dy$

Optimal result 91
Rubi [A] (verified) 91
Mathematica [A] (verified) 92
Maple [A] (verified) 92
Fricas [A] (verification not implemented) 92
Sympy [A] (verification not implemented) 93
Maxima [C] (verification not implemented) 93
Giac [A] (verification not implemented) 93
Mupad [F(-1)] 93

Optimal result

Integrand size = 6, antiderivative size = 2

$$\int \frac{\sin(y)}{y} \, dy = \mathrm{Si}(y)$$

```
[Out] Si(y)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3380}

$$\int \frac{\sin(y)}{y} \, dy = \mathrm{Si}(y)$$

```
[In] Int[Sin[y]/y,y]
```

```
[Out] SinIntegral[y]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\text{integral} = \mathrm{Si}(y)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(y)}{y} \, dy = \text{Si}(y)$$

```
[In] Integrate[Sin[y]/y,y]
[Out] SinIntegral[y]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\text{Si}\left(y\right)$	3
meijerg	$\text{Si}\left(y\right)$	3
risch	$-\frac{\pi \,\text{csgn}(y)}{2}+\text{Si}\left(y\right)$	9

```
[In] int(sin(y)/y,y,method=_RETURNVERBOSE)
[Out] Si(y)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(y)}{y} \, dy = \text{Si}\left(y\right)$$

```
[In] integrate(sin(y)/y,y, algorithm="fricas")
[Out] sin_integral(y)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(y)}{y}\,dy = \mathrm{Si}\,(y)$$

```
[In] integrate(sin(y)/y,y)
[Out] Si(y)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{\sin(y)}{y}\,dy = -\frac{1}{2}i\,\mathrm{Ei}(i\,y) + \frac{1}{2}i\,\mathrm{Ei}(-i\,y)$$

```
[In] integrate(sin(y)/y,y, algorithm="maxima")
[Out] -1/2*I*Ei(I*y) + 1/2*I*Ei(-I*y)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(y)}{y}\,dy = \mathrm{Si}\,(y)$$

```
[In] integrate(sin(y)/y,y, algorithm="giac")
[Out] sin_integral(y)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(y)}{y}\,dy = \mathrm{sinint}(y)$$

```
[In] int(sin(y)/y,y)
[Out] sinint(y)
```

3.12 $\int (e^x+\sin(x))\,dx$

Optimal result 94
Rubi [A] (verified) 94
Mathematica [A] (verified) 95
Maple [A] (verified) 95
Fricas [A] (verification not implemented) 96
Sympy [A] (verification not implemented) 96
Maxima [A] (verification not implemented) 96
Giac [A] (verification not implemented) 96
Mupad [B] (verification not implemented) 97

Optimal result

Integrand size = 6, antiderivative size = 8

$$\int (e^x+\sin(x))\,dx=e^x-\cos(x)$$

```
[Out] exp(x)-cos(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {2225, 2718}

$$\int (e^x+\sin(x))\,dx=e^x-\cos(x)$$

```
[In] Int[E^x + Sin[x],x]
```

```
[Out] E^x - Cos[x]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int e^x\,dx + \int \sin(x)\,dx \\ &= e^x - \cos(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int (e^x + \sin(x))\,dx = e^x - \cos(x)$$

```
[In] Integrate[E^x + Sin[x],x]
[Out] E^x - Cos[x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

method	result	size
default	$\mathrm{e}^{x}-\cos\left(x\right)$	8
risch	$\mathrm{e}^{x}-\cos\left(x\right)$	8
parts	$\mathrm{e}^{x}-\cos\left(x\right)$	8
parallelrisch	$-\cos\left(x\right)+\mathrm{e}^{x}-1$	9
meijerg	$-1+\mathrm{e}^{x}+\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(x)}{\sqrt{\pi}}\right)$	20
norman	$\frac{\mathrm{e}^{x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)-2+\mathrm{e}^{x}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	25

```
[In] int(exp(x)+sin(x),x,method=_RETURNVERBOSE)
[Out] exp(x)-cos(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int (e^x + \sin(x))\, dx = -\cos(x) + e^x$$

```
[In] integrate(exp(x)+sin(x),x, algorithm="fricas")
[Out] -cos(x) + e^x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int (e^x + \sin(x))\, dx = e^x - \cos(x)$$

```
[In] integrate(exp(x)+sin(x),x)
[Out] exp(x) - cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int (e^x + \sin(x))\, dx = -\cos(x) + e^x$$

```
[In] integrate(exp(x)+sin(x),x, algorithm="maxima")
[Out] -cos(x) + e^x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int (e^x + \sin(x))\, dx = -\cos(x) + e^x$$

```
[In] integrate(exp(x)+sin(x),x, algorithm="giac")
[Out] -cos(x) + e^x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int (e^x + \sin(x))\ dx = e^x - \cos(x)$$

```
[In] int(exp(x) + sin(x),x)
[Out] exp(x) - cos(x)
```

3.13 $\int \left(e^{x^2}+2e^{x^2}x^2\right) dx$

Optimal result 98
Rubi [A] (verified) 98
Mathematica [A] (verified) 99
Maple [A] (verified) 99
Fricas [A] (verification not implemented) 100
Sympy [A] (verification not implemented) 100
Maxima [A] (verification not implemented) 100
Giac [A] (verification not implemented) 100
Mupad [B] (verification not implemented) 101

Optimal result

Integrand size = 16, antiderivative size = 7

$$\int \left(e^{x^2}+2e^{x^2}x^2\right) dx = e^{x^2}x$$

```
[Out] exp(x^2)*x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2235, 2243}

$$\int \left(e^{x^2}+2e^{x^2}x^2\right) dx = e^{x^2}x$$

```
[In] Int[E^x^2 + 2*E^x^2*x^2,x]
```

```
[Out] E^x^2*x
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
```

```
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int e^{x^2}x^2\,dx+\int e^{x^2}\,dx\\ &= e^{x^2}x+\frac{1}{2}\sqrt{\pi}\text{erfi}(x)-\int e^{x^2}\,dx\\ &= e^{x^2}x\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \left(e^{x^2}+2e^{x^2}x^2\right)\,dx=e^{x^2}x$$

```
[In] Integrate[E^x^2 + 2*E^x^2*x^2,x]
[Out] E^x^2*x
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

method	result	size
gosper	$\mathrm{e}^{x^2}x$	7
default	$\mathrm{e}^{x^2}x$	7
norman	$\mathrm{e}^{x^2}x$	7
risch	$\mathrm{e}^{x^2}x$	7
parallelrisch	$\mathrm{e}^{x^2}x$	7
meijerg	$i\left(-ix\,\mathrm{e}^{x^2}+\frac{i\,\mathrm{erfi}(x)\sqrt{\pi}}{2}\right)+\frac{\mathrm{erfi}(x)\sqrt{\pi}}{2}$	29
parts	$\mathrm{erfi}\left(x\right)\sqrt{\pi}\,x^2+\frac{\mathrm{erfi}(x)\sqrt{\pi}}{2}-2\sqrt{\pi}\left(\frac{x^2\,\mathrm{erfi}(x)}{2}-\frac{\frac{\mathrm{e}^{x^2}x}{2}-\frac{\mathrm{erfi}(x)\sqrt{\pi}}{4}}{\sqrt{\pi}}\right)$	51

```
[In] int(2*exp(x^2)*x^2+exp(x^2),x,method=_RETURNVERBOSE)
[Out] exp(x^2)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \left(e^{x^2} + 2e^{x^2}x^2\right) dx = xe^{(x^2)}$$

```
[In] integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="fricas")
[Out] x*e^(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \left(e^{x^2} + 2e^{x^2}x^2\right) dx = xe^{x^2}$$

```
[In] integrate(exp(x**2)+2*exp(x**2)*x**2,x)
[Out] x*exp(x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \left(e^{x^2} + 2e^{x^2}x^2\right) dx = xe^{(x^2)}$$

```
[In] integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="maxima")
[Out] x*e^(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \left(e^{x^2} + 2e^{x^2}x^2\right) dx = xe^{(x^2)}$$

```
[In] integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="giac")
[Out] x*e^(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \left(e^{x^2} + 2e^{x^2}x^2\right) dx = x\,\mathrm{e}^{x^2}$$

```
[In] int(exp(x^2) + 2*x^2*exp(x^2),x)
[Out] x*exp(x^2)
```

3.14 $\int (e^x + x)^2 \, dx$

Optimal result 102
Rubi [A] (verified) 102
Mathematica [A] (verified) 103
Maple [A] (verified) 103
Fricas [A] (verification not implemented) 104
Sympy [A] (verification not implemented) 104
Maxima [A] (verification not implemented) 104
Giac [A] (verification not implemented) 104
Mupad [B] (verification not implemented) 105

Optimal result

Integrand size = 7, antiderivative size = 28

$$\int (e^x + x)^2 \, dx = -2e^x + \frac{e^{2x}}{2} + 2e^x x + \frac{x^3}{3}$$

```
[Out] -2*exp(x)+1/2*exp(2*x)+2*exp(x)*x+1/3*x^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6874, 2225, 2207}

$$\int (e^x + x)^2 \, dx = \frac{x^3}{3} + 2e^x x - 2e^x + \frac{e^{2x}}{2}$$

```
[In] Int[(E^x + x)^2,x]
```

```
[Out] -2*E^x + E^(2*x)/2 + 2*E^x*x + x^3/3
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(e^{2x} + 2e^x x + x^2\right) \, dx \\ &= \frac{x^3}{3} + 2\int e^x x \, dx + \int e^{2x} \, dx \\ &= \frac{e^{2x}}{2} + 2e^x x + \frac{x^3}{3} - 2\int e^x \, dx \\ &= -2e^x + \frac{e^{2x}}{2} + 2e^x x + \frac{x^3}{3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (e^x + x)^2 \, dx = \frac{e^{2x}}{2} + \frac{x^3}{3} + e^x(-2 + 2x)$$

```
[In] Integrate[(E^x + x)^2,x]
[Out] E^(2*x)/2 + x^3/3 + E^x*(-2 + 2*x)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{x^3}{3} + (-2 + 2x)\,\mathrm{e}^x + \frac{\mathrm{e}^{2x}}{2}$	21
default	$-2\,\mathrm{e}^x + \frac{\mathrm{e}^{2x}}{2} + 2\,\mathrm{e}^x x + \frac{x^3}{3}$	22
norman	$-2\,\mathrm{e}^x + \frac{\mathrm{e}^{2x}}{2} + 2\,\mathrm{e}^x x + \frac{x^3}{3}$	22
parallelrisch	$-2\,\mathrm{e}^x + \frac{\mathrm{e}^{2x}}{2} + 2\,\mathrm{e}^x x + \frac{x^3}{3}$	22
parts	$-2\,\mathrm{e}^x + \frac{\mathrm{e}^{2x}}{2} + 2\,\mathrm{e}^x x + \frac{x^3}{3}$	22

```
[In] int((exp(x)+x)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*x^3+(-2+2*x)*exp(x)+1/2*exp(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int (e^x + x)^2 \, dx = \frac{1}{3}\,x^3 + 2\,(x-1)e^x + \frac{1}{2}\,e^{(2\,x)}$$

```
[In] integrate((exp(x)+x)^2,x, algorithm="fricas")
[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int (e^x + x)^2 \, dx = \frac{x^3}{3} + \frac{(4x-4)\,e^x}{2} + \frac{e^{2x}}{2}$$

```
[In] integrate((exp(x)+x)**2,x)
[Out] x**3/3 + (4*x - 4)*exp(x)/2 + exp(2*x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int (e^x + x)^2 \, dx = \frac{1}{3}\,x^3 + 2\,(x-1)e^x + \frac{1}{2}\,e^{(2\,x)}$$

```
[In] integrate((exp(x)+x)^2,x, algorithm="maxima")
[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int (e^x + x)^2 \, dx = \frac{1}{3}\,x^3 + 2\,(x-1)e^x + \frac{1}{2}\,e^{(2\,x)}$$

```
[In] integrate((exp(x)+x)^2,x, algorithm="giac")
[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int (e^x + x)^2 \, dx = \frac{e^{2x}}{2} - 2e^x + 2x\,e^x + \frac{x^3}{3}$$

```
[In] int((x + exp(x))^2,x)
[Out] exp(2*x)/2 - 2*exp(x) + 2*x*exp(x) + x^3/3
```

3.15 $\int (2e^x + e^{2x} + x^2) \, dx$

Optimal result 106
Rubi [A] (verified) 106
Mathematica [A] (verified) 107
Maple [A] (verified) 107
Fricas [A] (verification not implemented) 107
Sympy [A] (verification not implemented) 108
Maxima [A] (verification not implemented) 108
Giac [A] (verification not implemented) 108
Mupad [B] (verification not implemented) 108

Optimal result

Integrand size = 14, antiderivative size = 22

$$\int (2e^x + e^{2x} + x^2) \, dx = 2e^x + \frac{e^{2x}}{2} + \frac{x^3}{3}$$

```
[Out] 2*exp(x)+1/2*exp(2*x)+1/3*x^3
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2225}

$$\int (2e^x + e^{2x} + x^2) \, dx = \frac{x^3}{3} + 2e^x + \frac{e^{2x}}{2}$$

```
[In] Int[2*E^x + E^(2*x) + x^2,x]
```

```
[Out] 2*E^x + E^(2*x)/2 + x^3/3
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{x^3}{3} + 2\int e^x \, dx + \int e^{2x} \, dx \\ &= 2e^x + \frac{e^{2x}}{2} + \frac{x^3}{3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (2e^x + e^{2x} + x^2) \, dx = 2e^x + \frac{e^{2x}}{2} + \frac{x^3}{3}$$

```
[In] Integrate[2*E^x + E^(2*x) + x^2,x]
[Out] 2*E^x + E^(2*x)/2 + x^3/3
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
default	$2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+\frac{x^{3}}{3}$	17
norman	$2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+\frac{x^{3}}{3}$	17
risch	$2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+\frac{x^{3}}{3}$	17
parallelrisch	$2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+\frac{x^{3}}{3}$	17
parts	$2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+\frac{x^{3}}{3}$	17

```
[In] int(2*exp(x)+exp(2*x)+x^2,x,method=_RETURNVERBOSE)
[Out] 2*exp(x)+1/2*exp(2*x)+1/3*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int (2e^x + e^{2x} + x^2) \, dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,e^{(2\,x)} + 2\,e^x$$

```
[In] integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \left(2e^x + e^{2x} + x^2\right) \, dx = \frac{x^3}{3} + \frac{e^{2x}}{2} + 2e^x$$

```
[In] integrate(2*exp(x)+exp(2*x)+x**2,x)
[Out] x**3/3 + exp(2*x)/2 + 2*exp(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \left(2e^x + e^{2x} + x^2\right) \, dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,e^{(2\,x)} + 2\,e^x$$

```
[In] integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="maxima")
[Out] 1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \left(2e^x + e^{2x} + x^2\right) \, dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,e^{(2\,x)} + 2\,e^x$$

```
[In] integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="giac")
[Out] 1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \left(2e^x + e^{2x} + x^2\right) \, dx = \frac{\mathrm{e}^{2\,x}}{2} + 2\,\mathrm{e}^x + \frac{x^3}{3}$$

```
[In] int(exp(2*x) + 2*exp(x) + x^2,x)
[Out] exp(2*x)/2 + 2*exp(x) + x^3/3
```

3.16 $\int \cos(x)\sin(x)\,dx$

Optimal result 109
Rubi [A] (verified) 109
Mathematica [A] (verified) 110
Maple [A] (verified) 110
Fricas [A] (verification not implemented) 111
Sympy [A] (verification not implemented) 111
Maxima [A] (verification not implemented) 111
Giac [A] (verification not implemented) 111
Mupad [B] (verification not implemented) 112

Optimal result

Integrand size = 5, antiderivative size = 8

$$\int \cos(x)\sin(x)\,dx = \frac{\sin^2(x)}{2}$$

```
[Out] 1/2*sin(x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2644, 30}

$$\int \cos(x)\sin(x)\,dx = \frac{\sin^2(x)}{2}$$

```
[In] Int[Cos[x]*Sin[x],x]
```

```
[Out] Sin[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int x\,dx, x, \sin(x)\right)$$
$$= \frac{\sin^2(x)}{2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos^2(x)$$

```
[In] Integrate[Cos[x]*Sin[x],x]
[Out] -1/2*Cos[x]^2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sin^2(x))}{2}$	7
default	$\frac{(\sin^2(x))}{2}$	7
risch	$-\frac{\cos(2x)}{4}$	7
parallelrisch	$\frac{1}{4}-\frac{\cos(2x)}{4}$	9
norman	$\frac{2(\tan^2(\frac{x}{2}))}{(1+\tan^2(\frac{x}{2}))^2}$	19
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{4}$	19

```
[In] int(cos(x)*sin(x),x,method=_RETURNVERBOSE)
[Out] 1/2*sin(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos{(x)}^2$$

```
[In] integrate(cos(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*cos(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \cos(x)\sin(x)\,dx = \frac{\sin^2{(x)}}{2}$$

```
[In] integrate(cos(x)*sin(x),x)
[Out] sin(x)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos{(x)}^2$$

```
[In] integrate(cos(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*cos(x)^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos{(x)}^2$$

```
[In] integrate(cos(x)*sin(x),x, algorithm="giac")
[Out] -1/2*cos(x)^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = \frac{\sin{(x)}^2}{2}$$

```
[In] int(cos(x)*sin(x),x)
[Out] sin(x)^2/2
```

3.17 $\int e^{x^2} x \, dx$

Optimal result 113
Rubi [A] (verified) 113
Mathematica [A] (verified) 114
Maple [A] (verified) 114
Fricas [A] (verification not implemented) 114
Sympy [A] (verification not implemented) 115
Maxima [A] (verification not implemented) 115
Giac [A] (verification not implemented) 115
Mupad [B] (verification not implemented) 115

Optimal result

Integrand size = 7, antiderivative size = 9

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

```
[Out] 1/2*exp(x^2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2240}

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

```
[In] Int[E^x^2*x,x]
```

```
[Out] E^x^2/2
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{e^{x^2}}{2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int e^{x^2} x\,dx = \frac{e^{x^2}}{2}$$

```
[In] Integrate[E^x^2*x,x]
[Out] E^x^2/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{e^{x^2}}{2}$	7
derivativedivides	$\frac{e^{x^2}}{2}$	7
default	$\frac{e^{x^2}}{2}$	7
norman	$\frac{e^{x^2}}{2}$	7
risch	$\frac{e^{x^2}}{2}$	7
parallelrisch	$\frac{e^{x^2}}{2}$	7
meijerg	$-\frac{1}{2}+\frac{e^{x^2}}{2}$	9
parts	$\frac{\operatorname{erfi}(x)\sqrt{\pi}\,x}{2}-\frac{\sqrt{\pi}\left(x\operatorname{erfi}(x)-\frac{e^{x^2}}{\sqrt{\pi}}\right)}{2}$	29

```
[In] int(exp(x^2)*x,x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x\,dx = \frac{1}{2}\,e^{(x^2)}$$

```
[In] integrate(exp(x^2)*x,x, algorithm="fricas")
[Out] 1/2*e^(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int e^{x^2} x\,dx = \frac{e^{x^2}}{2}$$

```
[In] integrate(exp(x**2)*x,x)
[Out] exp(x**2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x\,dx = \frac{1}{2}\,e^{\left(x^2\right)}$$

```
[In] integrate(exp(x^2)*x,x, algorithm="maxima")
[Out] 1/2*e^(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x\,dx = \frac{1}{2}\,e^{\left(x^2\right)}$$

```
[In] integrate(exp(x^2)*x,x, algorithm="giac")
[Out] 1/2*e^(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x\,dx = \frac{\mathrm{e}^{x^2}}{2}$$

```
[In] int(x*exp(x^2),x)
[Out] exp(x^2)/2
```

3.18 $\int x\sqrt{1+x^2}\,dx$

Optimal result 116
Rubi [A] (verified) 116
Mathematica [A] (verified) 117
Maple [A] (verified) 117
Fricas [A] (verification not implemented) 117
Sympy [B] (verification not implemented) 118
Maxima [A] (verification not implemented) 118
Giac [A] (verification not implemented) 118
Mupad [B] (verification not implemented) 119

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(1+x^2\right)^{3/2}$$

```
[Out] 1/3*(x^2+1)^(3/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {267}

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[In] Int[x*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\left(1+x^2\right)^{3/2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(1+x^2\right)^{3/2}$$

```
[In] Integrate[x*Sqrt[1 + x^2],x]
[Out] (1 + x^2)^(3/2)/3
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$	10
derivativedivides	$\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$	10
default	$\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$	10
risch	$\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$	10
pseudoelliptic	$\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$	10
trager	$\left(\frac{x^2}{3}+\frac{1}{3}\right)\sqrt{x^2+1}$	16
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2x^2+2\right)\sqrt{x^2+1}}{3}}{4\sqrt{\pi}}$	31

```
[In] int(x*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(x^2+1)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*(x^2 + 1)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int x\sqrt{1+x^2}\,dx = \frac{x^2\sqrt{x^2+1}}{3} + \frac{\sqrt{x^2+1}}{3}$$

```
[In] integrate(x*(x**2+1)**(1/2),x)
[Out] x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/3*(x^2 + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/3*(x^2 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{\left(x^2+1\right)^{3/2}}{3}$$

```
[In] int(x*(x^2 + 1)^(1/2),x)
[Out] (x^2 + 1)^(3/2)/3
```

3.19 $\int \frac{e^x}{1+e^x} dx$

Optimal result 120
Rubi [A] (verified) 120
Mathematica [A] (verified) 121
Maple [A] (verified) 121
Fricas [A] (verification not implemented) 121
Sympy [A] (verification not implemented) 122
Maxima [A] (verification not implemented) 122
Giac [A] (verification not implemented) 122
Mupad [B] (verification not implemented) 122

Optimal result

Integrand size = 11, antiderivative size = 6

$$\int \frac{e^x}{1+e^x} dx = \log(1+e^x)$$

```
[Out] ln(1+exp(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2278, 31}

$$\int \frac{e^x}{1+e^x} dx = \log(e^x+1)$$

```
[In] Int[E^x/(1 + E^x),x]
```

```
[Out] Log[1 + E^x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2278

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
 e, n, p}, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, e^x\right)$$
$$= \log\left(1+e^x\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{1+e^x}\,dx = \log\left(1+e^x\right)$$

```
[In] Integrate[E^x/(1 + E^x),x]
[Out] Log[1 + E^x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\ln\left(1+\mathrm{e}^{x}\right)$	6
default	$\ln\left(1+\mathrm{e}^{x}\right)$	6
norman	$\ln\left(1+\mathrm{e}^{x}\right)$	6
risch	$\ln\left(1+\mathrm{e}^{x}\right)$	6
parallelrisch	$\ln\left(1+\mathrm{e}^{x}\right)$	6

```
[In] int(1/(1+exp(x))*exp(x),x,method=_RETURNVERBOSE)
[Out] ln(1+exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x}\,dx = \log\left(e^x+1\right)$$

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="fricas")
[Out] log(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x} \, dx = \log \left(e^x + 1\right)$$

```
[In] integrate(exp(x)/(1+exp(x)),x)
[Out] log(exp(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x} \, dx = \log \left(e^x + 1\right)$$

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="maxima")
[Out] log(e^x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x} \, dx = \log \left(e^x + 1\right)$$

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="giac")
[Out] log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x} \, dx = \ln \left(\mathrm{e}^x + 1\right)$$

```
[In] int(exp(x)/(exp(x) + 1),x)
[Out] log(exp(x) + 1)
```

3.20 $\int x^{3/2}\,dx$

Optimal result 123
Rubi [A] (verified) 123
Mathematica [A] (verified) 124
Maple [A] (verified) 124
Fricas [A] (verification not implemented) 124
Sympy [A] (verification not implemented) 125
Maxima [A] (verification not implemented) 125
Giac [A] (verification not implemented) 125
Mupad [B] (verification not implemented) 125

Optimal result

Integrand size = 5, antiderivative size = 9

$$\int x^{3/2}\,dx=\frac{2x^{5/2}}{5}$$

```
[Out] 2/5*x^(5/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\int x^{3/2}\,dx=\frac{2x^{5/2}}{5}$$

```
[In] Int[x^(3/2),x]
```

```
[Out] (2*x^(5/2))/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{2x^{5/2}}{5}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x^{3/2}\,dx = \frac{2x^{5/2}}{5}$$

```
[In] Integrate[x^(3/2),x]
[Out] (2*x^(5/2))/5
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}}{5}$	6
derivativedivides	$\frac{2x^{\frac{5}{2}}}{5}$	6
default	$\frac{2x^{\frac{5}{2}}}{5}$	6
trager	$\frac{2x^{\frac{5}{2}}}{5}$	6
risch	$\frac{2x^{\frac{5}{2}}}{5}$	6

```
[In] int(x^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/5*x^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int x^{3/2}\,dx = \frac{2}{5}\,x^{\frac{5}{2}}$$

```
[In] integrate(x^(3/2),x, algorithm="fricas")
[Out] 2/5*x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int x^{3/2}\,dx = \frac{2x^{\frac{5}{2}}}{5}$$

```
[In] integrate(x**(3/2),x)
[Out] 2*x**(5/2)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int x^{3/2}\,dx = \frac{2}{5}\,x^{\frac{5}{2}}$$

```
[In] integrate(x^(3/2),x, algorithm="maxima")
[Out] 2/5*x^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int x^{3/2}\,dx = \frac{2}{5}\,x^{\frac{5}{2}}$$

```
[In] integrate(x^(3/2),x, algorithm="giac")
[Out] 2/5*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int x^{3/2}\,dx = \frac{2\,x^{5/2}}{5}$$

```
[In] int(x^(3/2),x)
[Out] (2*x^(5/2))/5
```

3.21 $\int \cos(3+2x)\,dx$

Optimal result 126
Rubi [A] (verified) 126
Mathematica [A] (verified) 127
Maple [A] (verified) 127
Fricas [A] (verification not implemented) 127
Sympy [A] (verification not implemented) 128
Maxima [A] (verification not implemented) 128
Giac [A] (verification not implemented) 128
Mupad [B] (verification not implemented) 128

Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \cos(3+2x)\,dx = \frac{1}{2}\sin(3+2x)$$

```
[Out] 1/2*sin(3+2*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2717}

$$\int \cos(3+2x)\,dx = \frac{1}{2}\sin(2x+3)$$

```
[In] Int[Cos[3 + 2*x],x]
```

```
[Out] Sin[3 + 2*x]/2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\sin(3+2x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(3+2x)\,dx = \frac{1}{2}\sin(3+2x)$$

```
[In] Integrate[Cos[3 + 2*x],x]
[Out] Sin[3 + 2*x]/2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\sin(3+2x)}{2}$	9
default	$\frac{\sin(3+2x)}{2}$	9
risch	$\frac{\sin(3+2x)}{2}$	9
parallelrisch	$\frac{\sin(3+2x)}{2}$	9
norman	$\frac{\tan\left(\frac{3}{2}+x\right)}{1+\tan^2\left(\frac{3}{2}+x\right)}$	16
meijerg	$\frac{\cos(3)\sin(2x)}{2}-\frac{\sin(3)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{2}$	30

```
[In] int(cos(3+2*x),x,method=_RETURNVERBOSE)
[Out] 1/2*sin(3+2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \cos(3+2x)\,dx = \frac{1}{2}\sin\left(2\,x+3\right)$$

```
[In] integrate(cos(3+2*x),x, algorithm="fricas")
[Out] 1/2*sin(2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \cos(3+2x)\,dx = \frac{\sin{(2x+3)}}{2}$$

```
[In] integrate(cos(3+2*x),x)
[Out] sin(2*x + 3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \cos(3+2x)\,dx = \frac{1}{2}\,\sin\left(2\,x+3\right)$$

```
[In] integrate(cos(3+2*x),x, algorithm="maxima")
[Out] 1/2*sin(2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \cos(3+2x)\,dx = \frac{1}{2}\,\sin\left(2\,x+3\right)$$

```
[In] integrate(cos(3+2*x),x, algorithm="giac")
[Out] 1/2*sin(2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \cos(3+2x)\,dx = \frac{\sin\left(2\,x+3\right)}{2}$$

```
[In] int(cos(2*x + 3),x)
[Out] sin(2*x + 3)/2
```

3.22 $\int 2e^{2x}yz\,dx$

Optimal result 129
Rubi [A] (verified) 129
Mathematica [A] (verified) 130
Maple [A] (verified) 130
Fricas [A] (verification not implemented) 131
Sympy [A] (verification not implemented) 131
Maxima [A] (verification not implemented) 131
Giac [A] (verification not implemented) 131
Mupad [B] (verification not implemented) 132

Optimal result

Integrand size = 9, antiderivative size = 8

$$\int 2e^{2x}yz\,dx = e^{2x}yz$$

```
[Out] exp(2*x)*y*z
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {12, 2225}

$$\int 2e^{2x}yz\,dx = e^{2x}yz$$

```
[In] Int[2*E^(2*x)*y*z,x]
```

```
[Out] E^(2*x)*y*z
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (2yz)\int e^{2x}\,dx\\ &= e^{2x}yz\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int 2e^{2x}yz\,dx = e^{2x}yz$$

```
[In] Integrate[2*E^(2*x)*y*z,x]
[Out] E^(2*x)*y*z
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

method	result	size
gosper	$e^{2x}yz$	8
derivativedivides	$e^{2x}yz$	8
default	$e^{2x}yz$	8
norman	$e^{2x}yz$	8
risch	$e^{2x}yz$	8
parallelrisch	$e^{2x}yz$	8
parts	$e^{2x}yz$	8
meijerg	$-yz(1-e^{2x})$	13

```
[In] int(2*exp(2*x)*y*z,x,method=_RETURNVERBOSE)
[Out] exp(2*x)*y*z
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int 2e^{2x}yz\,dx = yze^{(2\,x)}$$

```
[In] integrate(2*exp(2*x)*y*z,x, algorithm="fricas")
[Out] y*z*e^(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int 2e^{2x}yz\,dx = yze^{2x}$$

```
[In] integrate(2*exp(2*x)*y*z,x)
[Out] y*z*exp(2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int 2e^{2x}yz\,dx = yze^{(2\,x)}$$

```
[In] integrate(2*exp(2*x)*y*z,x, algorithm="maxima")
[Out] y*z*e^(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int 2e^{2x}yz\,dx = yze^{(2\,x)}$$

```
[In] integrate(2*exp(2*x)*y*z,x, algorithm="giac")
[Out] y*z*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int 2e^{2x}yz\,dx = y\,z\,\mathrm{e}^{2\,x}$$

```
[In] int(2*y*z*exp(2*x),x)
[Out] y*z*exp(2*x)
```

3.23 $\int e^x \cos^2(e^x) \sin(e^x) \, dx$

Optimal result 133
Rubi [A] (verified) 133
Mathematica [A] (verified) 134
Maple [A] (verified) 134
Fricas [A] (verification not implemented) 135
Sympy [A] (verification not implemented) 135
Maxima [A] (verification not implemented) 135
Giac [A] (verification not implemented) 136
Mupad [B] (verification not implemented) 136

Optimal result

Integrand size = 14, antiderivative size = 10

$$\int e^x \cos^2(e^x) \sin(e^x) \, dx = -\frac{1}{3} \cos^3(e^x)$$

```
[Out] -1/3*cos(exp(x))^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2320, 2645, 30}

$$\int e^x \cos^2(e^x) \sin(e^x) \, dx = -\frac{1}{3} \cos^3(e^x)$$

```
[In] Int[E^x*Cos[E^x]^2*Sin[E^x],x]
```

```
[Out] -1/3*Cos[E^x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \cos^2(x)\sin(x)\,dx, x, e^x\right)\\
&= -\text{Subst}\left(\int x^2\,dx, x, \cos\left(e^x\right)\right)\\
&= -\frac{1}{3}\cos^3\left(e^x\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int e^x \cos^2\left(e^x\right)\sin\left(e^x\right)\,dx = -\frac{1}{4}\cos\left(e^x\right) - \frac{1}{12}\cos\left(3e^x\right)$$

```
[In] Integrate[E^x*Cos[E^x]^2*Sin[E^x],x]
[Out] -1/4*Cos[E^x] - Cos[3*E^x]/12
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\left(\cos^3\left(e^x\right)\right)}{3}$	8
default	$-\frac{\left(\cos^3\left(e^x\right)\right)}{3}$	8
risch	$-\frac{\cos\left(e^x\right)}{4}-\frac{\cos\left(3\,e^x\right)}{12}$	14
parallelrisch	$\frac{1}{3}-\frac{\cos\left(e^x\right)}{4}-\frac{\cos\left(3\,e^x\right)}{12}$	15
norman	$\frac{-2\left(\tan^4\left(\frac{e^x}{2}\right)\right)-\frac{2}{3}}{\left(1+\tan^2\left(\frac{e^x}{2}\right)\right)^3}$	24

```
[In] int(exp(x)*cos(exp(x))^2*sin(exp(x)),x,method=_RETURNVERBOSE)
[Out] -1/3*cos(exp(x))^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{x}\cos^{2}\left(e^{x}\right)\sin\left(e^{x}\right)\,dx=-\frac{1}{3}\,\cos\left(e^{x}\right)^{3}$$

```
[In] integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="fricas")
[Out] -1/3*cos(e^x)^3
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int e^{x}\cos^{2}\left(e^{x}\right)\sin\left(e^{x}\right)\,dx=-\frac{\cos^{3}\left(e^{x}\right)}{3}$$

```
[In] integrate(exp(x)*cos(exp(x))**2*sin(exp(x)),x)
[Out] -cos(exp(x))**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{x}\cos^{2}\left(e^{x}\right)\sin\left(e^{x}\right)\,dx=-\frac{1}{3}\,\cos\left(e^{x}\right)^{3}$$

```
[In] integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="maxima")
[Out] -1/3*cos(e^x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^x \cos^2(e^x) \sin(e^x) \, dx = -\frac{1}{3} \cos(e^x)^3$$

```
[In] integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="giac")
[Out] -1/3*cos(e^x)^3
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^x \cos^2(e^x) \sin(e^x) \, dx = -\frac{\cos(e^x)^3}{3}$$

```
[In] int(cos(exp(x))^2*sin(exp(x))*exp(x),x)
[Out] -cos(exp(x))^3/3
```

3.24 $\int x\sqrt{1+x}\,dx$

Optimal result 137
Rubi [A] (verified) 137
Mathematica [A] (verified) 138
Maple [A] (verified) 138
Fricas [A] (verification not implemented) 138
Sympy [A] (verification not implemented) 139
Maxima [A] (verification not implemented) 139
Giac [A] (verification not implemented) 139
Mupad [B] (verification not implemented) 139

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x\sqrt{1+x}\,dx = -\frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2}$$

```
[Out] -2/3*(1+x)^(3/2)+2/5*(1+x)^(5/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{45\}$

$$\int x\sqrt{1+x}\,dx = \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2}$$

```
[In] Int[x*Sqrt[1 + x],x]
```

```
[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\sqrt{1+x} + (1+x)^{3/2}\right) dx \\ &= -\frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int x\sqrt{1+x}\,dx = \frac{2}{15}(1+x)^{3/2}(-5+3(1+x))$$

```
[In] Integrate[x*Sqrt[1 + x],x]
[Out] (2*(1 + x)^(3/2)*(-5 + 3*(1 + x)))/15
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{2(1+x)^{\frac{3}{2}}(-2+3x)}{15}$	13
derivativedivides	$-\frac{2(1+x)^{\frac{3}{2}}}{3}+\frac{2(1+x)^{\frac{5}{2}}}{5}$	16
default	$-\frac{2(1+x)^{\frac{3}{2}}}{3}+\frac{2(1+x)^{\frac{5}{2}}}{5}$	16
risch	$\frac{2(3x^2+x-2)\sqrt{1+x}}{15}$	16
trager	$\left(\frac{2}{5}x^2+\frac{2}{15}x-\frac{4}{15}\right)\sqrt{1+x}$	17
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(1+x)^{\frac{3}{2}}(2-3x)}{15}}{2\sqrt{\pi}}$	27

```
[In] int(x*(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(1+x)^(3/2)*(-2+3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int x\sqrt{1+x}\,dx = \frac{2}{15}\left(3\,x^2+x-2\right)\sqrt{x+1}$$

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int x\sqrt{1+x}\,dx = \frac{2x^2\sqrt{x+1}}{5} + \frac{2x\sqrt{x+1}}{15} - \frac{4\sqrt{x+1}}{15}$$

```
[In] integrate(x*(1+x)**(1/2),x)
[Out] 2*x**2*sqrt(x + 1)/5 + 2*x*sqrt(x + 1)/15 - 4*sqrt(x + 1)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int x\sqrt{1+x}\,dx = \frac{2}{5}\,(x+1)^{\frac{5}{2}} - \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="maxima")
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int x\sqrt{1+x}\,dx = \frac{2}{5}\,(x+1)^{\frac{5}{2}} - \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="giac")
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int x\sqrt{1+x}\,dx = \frac{2\,(3\,x-2)\,\left(x+1\right)^{3/2}}{15}$$

```
[In] int(x*(x + 1)^(1/2),x)
[Out] (2*(3*x - 2)*(x + 1)^(3/2))/15
```

3.25 $\int \frac{1}{-1+x^4} \, dx$

Optimal result 140
Rubi [A] (verified) 140
Mathematica [A] (verified) 141
Maple [A] (verified) 141
Fricas [A] (verification not implemented) 142
Sympy [A] (verification not implemented) 142
Maxima [A] (verification not implemented) 142
Giac [B] (verification not implemented) 142
Mupad [B] (verification not implemented) 143

Optimal result

Integrand size = 7, antiderivative size = 13

$$\int \frac{1}{-1+x^4} \, dx = -\frac{\arctan(x)}{2} - \frac{\text{arctanh}(x)}{2}$$

```
[Out] -1/2*arctan(x)-1/2*arctanh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {218, 212, 209}

$$\int \frac{1}{-1+x^4} \, dx = -\frac{\arctan(x)}{2} - \frac{\text{arctanh}(x)}{2}$$

```
[In] Int[(-1 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[x] - ArcTanh[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int \frac{1}{1-x^2}\,dx\right) - \frac{1}{2}\int \frac{1}{1+x^2}\,dx\\ &= -\frac{\arctan(x)}{2} - \frac{\operatorname{arctanh}(x)}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{1}{-1+x^4}\,dx = -\frac{\arctan(x)}{2} + \frac{1}{4}\log(1-x) - \frac{1}{4}\log(1+x)$$

```
[In] Integrate[(-1 + x^4)^(-1),x]
[Out] -1/2*ArcTan[x] + Log[1 - x]/4 - Log[1 + x]/4
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\arctan(x)}{2}-\frac{\operatorname{arctanh}(x)}{2}$	10
risch	$-\frac{\arctan(x)}{2}-\frac{\ln(1+x)}{4}+\frac{\ln(-1+x)}{4}$	18
parallelrisch	$\frac{i\ln(x-i)}{4}-\frac{i\ln(x+i)}{4}-\frac{\ln(1+x)}{4}+\frac{\ln(-1+x)}{4}$	30
meijerg	$\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}$	38

```
[In] int(1/(x^4-1),x,method=_RETURNVERBOSE)
[Out] -1/2*arctan(x)-1/2*arctanh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{-1+x^4} \, dx = -\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x+1) + \frac{1}{4} \log(x-1)$$

```
[In] integrate(1/(x^4-1),x, algorithm="fricas")
[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{-1+x^4} \, dx = \frac{\log(x-1)}{4} - \frac{\log(x+1)}{4} - \frac{\operatorname{atan}(x)}{2}$$

```
[In] integrate(1/(x**4-1),x)
[Out] log(x - 1)/4 - log(x + 1)/4 - atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{-1+x^4} \, dx = -\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x+1) + \frac{1}{4} \log(x-1)$$

```
[In] integrate(1/(x^4-1),x, algorithm="maxima")
[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{1}{-1+x^4} \, dx = -\frac{1}{2} \arctan(x) - \frac{1}{4} \log(|x+1|) + \frac{1}{4} \log(|x-1|)$$

```
[In] integrate(1/(x^4-1),x, algorithm="giac")
[Out] -1/2*arctan(x) - 1/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{-1+x^4}\,dx=-\frac{\operatorname{atan}(x)}{2}-\frac{\operatorname{atanh}(x)}{2}$$

```
[In] int(1/(x^4 - 1),x)
[Out] - atan(x)/2 - atanh(x)/2
```

3.26 $\int \frac{e^x}{2+3e^{2x}}\,dx$

Optimal result 144
Rubi [A] (verified) 144
Mathematica [A] (verified) 145
Maple [A] (verified) 145
Fricas [A] (verification not implemented) 146
Sympy [A] (verification not implemented) 146
Maxima [A] (verification not implemented) 146
Giac [A] (verification not implemented) 146
Mupad [B] (verification not implemented) 147

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{e^x}{2+3e^{2x}}\,dx = \frac{\arctan\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

```
[Out] 1/6*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2281, 209}

$$\int \frac{e^x}{2+3e^{2x}}\,dx = \frac{\arctan\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

```
[In] Int[E^x/(2 + 3*E^(2*x)),x]
```

```
[Out] ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]
```

<u>Rule 209</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

<u>Rule 2281</u>

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
```

```
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, e^x\right)\\ &= \frac{\arctan\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{2+3e^{2x}}\,dx = \frac{\arctan\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

```
[In] Integrate[E^x/(2 + 3*E^(2*x)),x]
[Out] ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\arctan\left(\frac{e^x\sqrt{6}}{2}\right)\sqrt{6}}{6}$	14
risch	$\frac{i\sqrt{6}\,\ln\left(e^x+\frac{i\sqrt{6}}{3}\right)}{12}-\frac{i\sqrt{6}\,\ln\left(e^x-\frac{i\sqrt{6}}{3}\right)}{12}$	34

```
[In] int(exp(x)/(2+3*exp(2*x)),x,method=_RETURNVERBOSE)
[Out] 1/6*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{e^x}{2+3e^{2x}}\,dx = \frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^x\right)$$

```
[In] integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="fricas")
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{2+3e^{2x}}\,dx = \operatorname{RootSum}\left(24z^2+1,\left(i\mapsto i\log\left(4i+e^x\right)\right)\right)$$

```
[In] integrate(exp(x)/(2+3*exp(2*x)),x)
[Out] RootSum(24*_z**2 + 1, Lambda(_i, _i*log(4*_i + exp(x))))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{e^x}{2+3e^{2x}}\,dx = \frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^x\right)$$

```
[In] integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="maxima")
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{e^x}{2+3e^{2x}}\,dx = \frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^x\right)$$

```
[In] integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="giac")
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{e^x}{2+3e^{2x}}\,dx = \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,\mathrm{e}^x}{2}\right)}{6}$$

```
[In] int(exp(x)/(3*exp(2*x) + 2),x)
[Out] (6^(1/2)*atan((6^(1/2)*exp(x))/2))/6
```

3.27 $\int \frac{e^{2x}}{A+Be^{4x}}\,dx$

Optimal result 148
Rubi [A] (verified) 148
Mathematica [A] (verified) 149
Maple [A] (verified) 149
Fricas [A] (verification not implemented) 150
Sympy [A] (verification not implemented) 150
Maxima [A] (verification not implemented) 150
Giac [A] (verification not implemented) 151
Mupad [B] (verification not implemented) 151

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{e^{2x}}{A+Be^{4x}}\,dx=\frac{\arctan\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

```
[Out] 1/2*arctan(exp(2*x)*B^(1/2)/A^(1/2))/A^(1/2)/B^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2281, 211}

$$\int \frac{e^{2x}}{A+Be^{4x}}\,dx=\frac{\arctan\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

```
[In] Int[E^(2*x)/(A + B*E^(4*x)),x]
```

```
[Out] ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
```

```
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{A+Bx^2}\,dx, x, e^{2x}\right) \\ &= \frac{\arctan\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{A+Be^{4x}}\,dx = \frac{\arctan\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

```
[In] Integrate[E^(2*x)/(A + B*E^(4*x)),x]
[Out] ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\arctan\left(\frac{B\,e^{2x}}{\sqrt{AB}}\right)}{2\sqrt{AB}}$	20
risch	$-\frac{\ln\left(e^{2x}-\frac{A}{\sqrt{-AB}}\right)}{4\sqrt{-AB}}+\frac{\ln\left(e^{2x}+\frac{A}{\sqrt{-AB}}\right)}{4\sqrt{-AB}}$	47

```
[In] int(exp(2*x)/(A+B*exp(4*x)),x,method=_RETURNVERBOSE)
[Out] 1/2/(A*B)^(1/2)*arctan(B*exp(x)^2/(A*B)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int \frac{e^{2x}}{A+Be^{4x}}\,dx = \left[-\frac{\sqrt{-AB}\log\left(\frac{Be^{(4x)}-2\sqrt{-AB}e^{(2x)}-A}{Be^{(4x)}+A}\right)}{4AB}, -\frac{\sqrt{AB}\arctan\left(\frac{\sqrt{AB}e^{(-2x)}}{B}\right)}{2AB}\right]$$

```
[In] integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="fricas")
[Out] [-1/4*sqrt(-A*B)*log((B*e^(4*x) - 2*sqrt(-A*B)*e^(2*x) - A)/(B*e^(4*x) + A)
)/(A*B), -1/2*sqrt(A*B)*arctan(sqrt(A*B)*e^(-2*x)/B)/(A*B)]
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{e^{2x}}{A+Be^{4x}}\,dx = \operatorname{RootSum}\left(16z^2AB+1, \left(i\mapsto i\log\left(4iA+e^{2x}\right)\right)\right)$$

```
[In] integrate(exp(2*x)/(A+B*exp(4*x)),x)
[Out] RootSum(16*_z**2*A*B + 1, Lambda(_i, _i*log(4*_i*A + exp(2*x))))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{e^{2x}}{A+Be^{4x}}\,dx = \frac{\arctan\left(\frac{Be^{(2x)}}{\sqrt{AB}}\right)}{2\sqrt{AB}}$$

```
[In] integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="maxima")
[Out] 1/2*arctan(B*e^(2*x)/sqrt(A*B))/sqrt(A*B)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{e^{2x}}{A+Be^{4x}}\,dx = \frac{\arctan\left(\frac{Be^{(2\,x)}}{\sqrt{AB}}\right)}{2\,\sqrt{AB}}$$

```
[In] integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="giac")
[Out] 1/2*arctan(B*e^(2*x)/sqrt(A*B))/sqrt(A*B)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{e^{2x}}{A+Be^{4x}}\,dx = \frac{\mathrm{atan}\left(\frac{B\,\mathrm{e}^{2\,x}}{\sqrt{A\,B}}\right)}{2\,\sqrt{A\,B}}$$

```
[In] int(exp(2*x)/(A + B*exp(4*x)),x)
[Out] atan((B*exp(2*x))/(A*B)^(1/2))/(2*(A*B)^(1/2))
```

3.28 $\int \frac{e^{1+x}}{1+e^x} dx$

Optimal result 152
Rubi [A] (verified) 152
Mathematica [A] (verified) 153
Maple [A] (verified) 153
Fricas [A] (verification not implemented) 154
Sympy [A] (verification not implemented) 154
Maxima [A] (verification not implemented) 154
Giac [A] (verification not implemented) 154
Mupad [B] (verification not implemented) 155

Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{e^{1+x}}{1+e^x} dx = e \log (1+e^x)$$

```
[Out] exp(1)*ln(1+exp(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2279, 2278, 31}

$$\int \frac{e^{1+x}}{1+e^x} dx = e \log (e^x+1)$$

```
[In] Int[E^(1 + x)/(1 + E^x),x]
```

```
[Out] E*Log[1 + E^x]
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 2278</u>

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
 e, n, p}, x]
```

Rule 2279

```
Int[((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.)*((G_)^((
h_.)*((f_.) + (g_.)*(x_))))^(m_.), x_Symbol] :> Dist[(G^(h*(f + g*x)))^m/(F
^(e*(c + d*x)))^n, Int[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p, x
], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d*e*n*Log
[F], g*h*m*Log[G]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= e\int \frac{e^x}{1+e^x}\,dx \\
&= e\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, e^x\right) \\
&= e\log\left(1+e^x\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{e^{1+x}}{1+e^x}\,dx = e\log\left(1+e^x\right)$$

```
[In] Integrate[E^(1 + x)/(1 + E^x),x]
[Out] E*Log[1 + E^x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$\mathrm{e}\ln\left(1+\mathrm{e}^{x}\right)$	9
norman	$\mathrm{e}\ln\left(1+\mathrm{e}^{x}\right)$	9
risch	$\mathrm{e}\ln\left(1+\mathrm{e}^{x}\right)$	9

```
[In] int(exp(1+x)/(1+exp(x)),x,method=_RETURNVERBOSE)
[Out] exp(1)*ln(1+exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int \frac{e^{1+x}}{1+e^x}\,dx = e\log\left(e+e^{(x+1)}\right)$$

```
[In] integrate(exp(1+x)/(1+exp(x)),x, algorithm="fricas")
[Out] e*log(e + e^(x + 1))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{e^{1+x}}{1+e^x}\,dx = e\log\left(e^x+1\right)$$

```
[In] integrate(exp(1+x)/(1+exp(x)),x)
[Out] E*log(exp(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{e^{1+x}}{1+e^x}\,dx = e\log\left(e^x+1\right)$$

```
[In] integrate(exp(1+x)/(1+exp(x)),x, algorithm="maxima")
[Out] e*log(e^x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{e^{1+x}}{1+e^x}\,dx = e\log\left(e^x+1\right)$$

```
[In] integrate(exp(1+x)/(1+exp(x)),x, algorithm="giac")
[Out] e*log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{e^{1+x}}{1+e^x}\,dx = \mathrm{e}\,\ln\left(\mathrm{e}^x+1\right)$$

```
[In] int(exp(x + 1)/(exp(x) + 1),x)
[Out] exp(1)*log(exp(x) + 1)
```

3.29 $\int (10e)^x \, dx$

Optimal result 156
Rubi [A] (verified) 156
Mathematica [A] (verified) 157
Maple [A] (verified) 157
Fricas [A] (verification not implemented) 157
Sympy [A] (verification not implemented) 158
Maxima [A] (verification not implemented) 158
Giac [A] (verification not implemented) 158
Mupad [B] (verification not implemented) 158

Optimal result

Integrand size = 5, antiderivative size = 12

$$\int (10e)^x \, dx = \frac{(10e)^x}{1+\log(10)}$$

```
[Out] (10*exp(1))^x/(1+ln(10))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2225}

$$\int (10e)^x \, dx = \frac{(10e)^x}{1+\log(10)}$$

```
[In] Int[(10*E)^x,x]
```

```
[Out] (10*E)^x/(1 + Log[10])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\text{integral} = \frac{(10e)^x}{1+\log(10)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (10e)^x \, dx = \frac{(10e)^x}{\log(10e)}$$

```
[In] Integrate[(10*E)^x,x]
[Out] (10*E)^x/Log[10*E]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

method	result	size
gosper	$\frac{(10\,\mathrm{e})^x}{\ln(10\,\mathrm{e})}$	15
parallelrisch	$\frac{(10\,\mathrm{e})^x}{\ln(10\,\mathrm{e})}$	15
norman	$\frac{\mathrm{e}^{x\ln(10\,\mathrm{e})}}{1+\ln(10)}$	16
risch	$\frac{5^x 2^x \mathrm{e}^x}{1+\ln(2)+\ln(5)}$	18
meijerg	$-\frac{1-\mathrm{e}^{x(1+\ln(10))}}{1+\ln(10)}$	20

```
[In] int((10*exp(1))^x,x,method=_RETURNVERBOSE)
[Out] 1/ln(10*exp(1))*(10*exp(1))^x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (10e)^x \, dx = \frac{e^{(x\log(10)+x)}}{\log(10)+1}$$

```
[In] integrate((10*exp(1))^x,x, algorithm="fricas")
[Out] e^(x*log(10) + x)/(log(10) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (10e)^x \, dx = \frac{(10e)^x}{1 + \log(10)}$$

```
[In] integrate((10*exp(1))**x,x)
[Out] (10*E)**x/(1 + log(10))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (10e)^x \, dx = \frac{(10\,e)^x}{\log(10\,e)}$$

```
[In] integrate((10*exp(1))^x,x, algorithm="maxima")
[Out] (10*e)^x/log(10*e)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int (10e)^x \, dx = \frac{(10\,e)^x}{\log(10) + 1}$$

```
[In] integrate((10*exp(1))^x,x, algorithm="giac")
[Out] (10*e)^x/(log(10) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (10e)^x \, dx = \frac{10^x\,\mathrm{e}^x}{\ln(10) + 1}$$

```
[In] int((10*exp(1))^x,x)
[Out] (10^x*exp(x))/(log(10) + 1)
```

3.30 $\int x^3 \sin(x^2)\, dx$

Optimal result 159
Rubi [A] (verified) 159
Mathematica [A] (verified) 160
Maple [A] (verified) 161
Fricas [A] (verification not implemented) 161
Sympy [A] (verification not implemented) 161
Maxima [A] (verification not implemented) 162
Giac [A] (verification not implemented) 162
Mupad [B] (verification not implemented) 162

Optimal result

Integrand size = 8, antiderivative size = 20

$$\int x^3 \sin(x^2)\, dx = -\frac{1}{2}x^2\cos(x^2) + \frac{\sin(x^2)}{2}$$

```
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3460, 3377, 2717}

$$\int x^3 \sin(x^2)\, dx = \frac{\sin(x^2)}{2} - \frac{1}{2}x^2\cos(x^2)$$

```
[In] Int[x^3*Sin[x^2],x]
```

```
[Out] -1/2*(x^2*Cos[x^2]) + Sin[x^2]/2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x\sin(x)\,dx, x, x^2\right)\\ &= -\frac{1}{2}x^2\cos\left(x^2\right) + \frac{1}{2}\text{Subst}\left(\int \cos(x)\,dx, x, x^2\right)\\ &= -\frac{1}{2}x^2\cos\left(x^2\right) + \frac{\sin\left(x^2\right)}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^3\sin\left(x^2\right)\,dx = -\frac{1}{2}x^2\cos\left(x^2\right) + \frac{\sin\left(x^2\right)}{2}$$

```
[In] Integrate[x^3*Sin[x^2],x]
[Out] -1/2*(x^2*Cos[x^2]) + Sin[x^2]/2
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{x^{2} \cos \left(x^{2}\right)}{2}+\frac{\sin \left(x^{2}\right)}{2}$	17
default	$-\frac{x^{2} \cos \left(x^{2}\right)}{2}+\frac{\sin \left(x^{2}\right)}{2}$	17
risch	$-\frac{x^{2} \cos \left(x^{2}\right)}{2}+\frac{\sin \left(x^{2}\right)}{2}$	17
parallelrisch	$-\frac{x^{2} \cos \left(x^{2}\right)}{2}+\frac{\sin \left(x^{2}\right)}{2}$	17
meijerg	$\sqrt{\pi}\, \left(-\frac{x^{2} \cos \left(x^{2}\right)}{2 \sqrt{\pi}}+\frac{\sin \left(x^{2}\right)}{2 \sqrt{\pi}}\right)$	27
norman	$\frac{-\frac{x^{2}}{2}+\frac{x^{2} \left(\tan^{2}\left(\frac{x^{2}}{2}\right)\right)}{2}+\tan \left(\frac{x^{2}}{2}\right)}{1+\tan^{2}\left(\frac{x^{2}}{2}\right)}$	39
parts	$\frac{\sqrt{2}\, \sqrt{\pi}\, \mathrm{S}\left(\frac{\sqrt{2}\, x}{\sqrt{\pi}}\right) x^{3}}{2}-\frac{3 \pi^{2} \left(\frac{2\, \mathrm{S}\left(\frac{\sqrt{2}\, x}{\sqrt{\pi}}\right) \sqrt{2}\, x^{3}}{3 \pi^{\frac{3}{2}}}+\frac{2 x^{2} \cos \left(x^{2}\right)}{3 \pi^{2}}-\frac{2 \sin \left(x^{2}\right)}{3 \pi^{2}}\right)}{4}$	69

```
[In] int(x^3*sin(x^2),x,method=_RETURNVERBOSE)
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin \left(x^2\right) \, dx = -\frac{1}{2} \, x^{2} \cos \left(x^{2}\right) + \frac{1}{2} \, \sin \left(x^{2}\right)$$

```
[In] integrate(x^3*sin(x^2),x, algorithm="fricas")
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int x^3 \sin \left(x^2\right) \, dx = - \frac{x^{2} \cos {\left(x^{2} \right)}}{2} + \frac{\sin {\left(x^{2} \right)}}{2}$$

```
[In] integrate(x**3*sin(x**2),x)
[Out] -x**2*cos(x**2)/2 + sin(x**2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin \left(x^2\right) \, dx = -\frac{1}{2} \, x^2 \cos \left(x^2\right) + \frac{1}{2} \, \sin \left(x^2\right)$$

```
[In] integrate(x^3*sin(x^2),x, algorithm="maxima")
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin \left(x^2\right) \, dx = -\frac{1}{2} \, x^2 \cos \left(x^2\right) + \frac{1}{2} \, \sin \left(x^2\right)$$

```
[In] integrate(x^3*sin(x^2),x, algorithm="giac")
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin \left(x^2\right) \, dx = \frac{\sin \left(x^2\right)}{2} - \frac{x^2\,\cos \left(x^2\right)}{2}$$

```
[In] int(x^3*sin(x^2),x)
[Out] sin(x^2)/2 - (x^2*cos(x^2))/2
```

3.31 $\int \frac{x^7}{1+x^{12}}\,dx$

Optimal result 163
Rubi [A] (verified) 163
Mathematica [B] (verified) 165
Maple [A] (verified) 165
Fricas [A] (verification not implemented) 166
Sympy [A] (verification not implemented) 166
Maxima [A] (verification not implemented) 167
Giac [A] (verification not implemented) 167
Mupad [B] (verification not implemented) 167

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{x^7}{1+x^{12}}\,dx=-\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\log\left(1-x^4+x^8\right)$$

```
[Out] -1/12*ln(x^4+1)+1/24*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1
/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {281, 298, 31, 648, 632, 210, 642}

$$\int \frac{x^7}{1+x^{12}}\,dx=-\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{12}\log\left(x^4+1\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)$$

```
[In] Int[x^7/(1 + x^12),x]
```

```
[Out] -1/4*ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3] - Log[1 + x^4]/12 + Log[1 - x^4 +
x^8]/24
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, x^4\right)\\
&= -\left(\frac{1}{12}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^4\right)\right)+\frac{1}{12}\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, x^4\right)\\
&= -\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)
\end{aligned}$$

$$= -\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\log\left(1-x^4+x^8\right)-\frac{1}{4}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,-1+2x^4\right)$$

$$= -\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\log\left(1-x^4+x^8\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 260 vs. 2(49) = 98.

Time = 0.12 (sec) , antiderivative size = 260, normalized size of antiderivative = 5.31

$$\int\frac{x^7}{1+x^{12}}\,dx=\frac{1}{24}\left(2\sqrt{3}\arctan\left(\frac{1+\sqrt{3}-2\sqrt{2}x}{1-\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1-\sqrt{3}+2\sqrt{2}x}{1+\sqrt{3}}\right)\right.$$
$$+2\sqrt{3}\arctan\left(\frac{-1+\sqrt{3}+2\sqrt{2}x}{1+\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+\sqrt{3}+2\sqrt{2}x}{-1+\sqrt{3}}\right)$$
$$-2\log\left(1-\sqrt{2}x+x^2\right)-2\log\left(1+\sqrt{2}x+x^2\right)$$
$$+\log\left(2+\sqrt{2}x-\sqrt{6}x+2x^2\right)+\log\left(2+\sqrt{2}\left(-1+\sqrt{3}\right)x+2x^2\right)$$
$$\left.+\log\left(2-\left(\sqrt{2}+\sqrt{6}\right)x+2x^2\right)+\log\left(2+\left(\sqrt{2}+\sqrt{6}\right)x+2x^2\right)\right)$$

```
[In] Integrate[x^7/(1 + x^12),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + Sqrt[3] - 2*Sqrt[2]*x)/(1 - Sqrt[3])] - 2*Sqrt[3]*Ar
cTan[(1 - Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] + 2*Sqrt[3]*ArcTan[(-1 + Sq
rt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] - 2*Sqrt[3]*ArcTan[(1 + Sqrt[3] + 2*Sqr
t[2]*x)/(-1 + Sqrt[3])] - 2*Log[1 - Sqrt[2]*x + x^2] - 2*Log[1 + Sqrt[2]*x
+ x^2] + Log[2 + Sqrt[2]*x - Sqrt[6]*x + 2*x^2] + Log[2 + Sqrt[2]*(-1 + Sqr
t[3])*x + 2*x^2] + Log[2 - (Sqrt[2] + Sqrt[6])*x + 2*x^2] + Log[2 + (Sqrt[2
] + Sqrt[6])*x + 2*x^2])/24
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln(x^8-x^4+1)}{24}+\frac{\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln(x^4+1)}{12}$	41
risch	$\frac{\ln(4x^8-4x^4+4)}{24}+\frac{\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln(x^4+1)}{12}$	43
meijerg	$-\frac{x^8\ln\left(1+(x^{12})^{\frac{1}{3}}\right)}{12(x^{12})^{\frac{2}{3}}}+\frac{x^8\ln\left(1-(x^{12})^{\frac{1}{3}}+(x^{12})^{\frac{2}{3}}\right)}{24(x^{12})^{\frac{2}{3}}}+\frac{x^8\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^{12})^{\frac{1}{3}}}{2-(x^{12})^{\frac{1}{3}}}\right)}{12(x^{12})^{\frac{2}{3}}}$	80

```
[In] int(x^7/(x^12+1),x,method=_RETURNVERBOSE)
[Out] 1/24*ln(x^8-x^4+1)+1/12*arctan(1/3*(2*x^4-1)*3^(1/2))*3^(1/2)-1/12*ln(x^4+1
)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+x^{12}}\,dx=\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

```
[In] integrate(x^7/(x^12+1),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{x^7}{1+x^{12}}\,dx=-\frac{\log\left(x^4+1\right)}{12}+\frac{\log\left(x^8-x^4+1\right)}{24}+\frac{\sqrt{3}\,\text{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

```
[In] integrate(x**7/(x**12+1),x)
[Out] -log(x**4 + 1)/12 + log(x**8 - x**4 + 1)/24 + sqrt(3)*atan(2*sqrt(3)*x**4/3
 - sqrt(3)/3)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+x^{12}}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right) + \frac{1}{24}\log\left(x^8-x^4+1\right) - \frac{1}{12}\log\left(x^4+1\right)$$

```
[In] integrate(x^7/(x^12+1),x, algorithm="maxima")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+x^{12}}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right) + \frac{1}{24}\log\left(x^8-x^4+1\right) - \frac{1}{12}\log\left(x^4+1\right)$$

```
[In] integrate(x^7/(x^12+1),x, algorithm="giac")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{x^7}{1+x^{12}}\,dx = -\frac{\ln\left(x^4+1\right)}{12} - \ln\left(x^4-\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(-\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right) + \ln\left(x^4+\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right)$$

```
[In] int(x^7/(x^12 + 1),x)
[Out] log((3^(1/2)*1i)/2 + x^4 - 1/2)*((3^(1/2)*1i)/24 + 1/24) - log(x^4 - (3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/24 - 1/24) - log(x^4 + 1)/12
```

3.32 $\int x^{3a}\sin\left(x^{2a}\right)\,dx$

Optimal result 168
Rubi [A] (verified) 168
Mathematica [A] (verified) 169
Maple [C] (verified) 169
Fricas [F] 170
Sympy [A] (verification not implemented) 170
Maxima [F] 170
Giac [F] 171
Mupad [F(-1)] 171

Optimal result

Integrand size = 12, antiderivative size = 115

$$\int x^{3a}\sin\left(x^{2a}\right)\,dx=\frac{ix^{1+3a}\left(-ix^{2a}\right)^{-\frac{1+3a}{2a}}\Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right),-ix^{2a}\right)}{4a}-\frac{ix^{1+3a}\left(ix^{2a}\right)^{-\frac{1+3a}{2a}}\Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right),ix^{2a}\right)}{4a}$$

```
[Out] 1/4*I*x^(1+3*a)*GAMMA(3/2+1/2/a,-I*x^(2*a))/a/((-I*x^(2*a))^(1/2*(1+3*a)/a)
)-1/4*I*x^(1+3*a)*GAMMA(3/2+1/2/a,I*x^(2*a))/a/((I*x^(2*a))^(1/2*(1+3*a)/a)
)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3504, 2250}

$$\int x^{3a}\sin\left(x^{2a}\right)\,dx=\frac{ix^{3a+1}\left(-ix^{2a}\right)^{-\frac{3a+1}{2a}}\Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right),-ix^{2a}\right)}{4a}-\frac{ix^{3a+1}\left(ix^{2a}\right)^{-\frac{3a+1}{2a}}\Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right),ix^{2a}\right)}{4a}$$

```
[In] Int[x^(3*a)*Sin[x^(2*a)],x]
```

```
[Out] ((I/4)*x^(1 + 3*a)*Gamma[(3 + a^(-1))/2, (-I)*x^(2*a)])/(a*((-I)*x^(2*a))^(
(1 + 3*a)/(2*a))) - ((I/4)*x^(1 + 3*a)*Gamma[(3 + a^(-1))/2, I*x^(2*a)])/(a
*(I*x^(2*a))^((1 + 3*a)/(2*a)))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}i\int e^{-ix^{2a}}x^{3a}\,dx - \frac{1}{2}i\int e^{ix^{2a}}x^{3a}\,dx\\ &= \frac{ix^{1+3a}\left(-ix^{2a}\right)^{-\frac{1+3a}{2a}}\Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right),-ix^{2a}\right)}{4a} - \frac{ix^{1+3a}\left(ix^{2a}\right)^{-\frac{1+3a}{2a}}\Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right),ix^{2a}\right)}{4a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.23

$$\int x^{3a}\sin\left(x^{2a}\right)\,dx = -\frac{x^{1+a}\left(x^{4a}\right)^{-\frac{1+a}{2a}}\left(4a\left(x^{4a}\right)^{\frac{1+a}{2a}}\cos\left(x^{2a}\right)+(1+a)\left(ix^{2a}\right)^{\frac{1+a}{2a}}\Gamma\left(\frac{1+a}{2a},-ix^{2a}\right)+(1+a)\left(-ix^{2a}\right)^{\frac{1+a}{2a}}\Gamma\left(\frac{1+a}{2a},ix^{2a}\right.\right.}{8a^2}$$

```
[In] Integrate[x^(3*a)*Sin[x^(2*a)],x]
```

```
[Out] -1/8*(x^(1 + a)*(4*a*(x^(4*a))^((1 + a)/(2*a))*Cos[x^(2*a)] + (1 + a)*(I*x^
(2*a))^((1 + a)/(2*a))*Gamma[(1 + a)/(2*a), (-I)*x^(2*a)] + (1 + a)*((-I)*x
^(2*a))^((1 + a)/(2*a))*Gamma[(1 + a)/(2*a), I*x^(2*a)]))/(a^2*(x^(4*a))^((
1 + a)/(2*a)))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.36

method	result	size
meijerg	$\frac{x^{5a+1}{}_1F_2\left(\frac{5}{4}+\frac{1}{4a};\frac{3}{2},\frac{9}{4}+\frac{1}{4a};-\frac{x^{4a}}{4}\right)}{5a+1}$	41

```
[In] int(x^(3*a)*sin(x^(2*a)),x,method=_RETURNVERBOSE)
[Out] 1/(5*a+1)*x^(5*a+1)*hypergeom([5/4+1/4/a],[3/2,9/4+1/4/a],-1/4*x^(4*a))
```

Fricas [F]

$$\int x^{3a} \sin \left(x^{2a}\right) \, dx = \int x^{3\,a} \sin \left(x^{2\,a}\right) \, dx$$

```
[In] integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="fricas")
[Out] integral(x^(3*a)*sin(x^(2*a)), x)
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int x^{3a} \sin \left(x^{2a}\right) \, dx = \frac{x^{5a+1}\Gamma\left(\frac{5}{4}+\frac{1}{4a}\right) {}_1F_2\left(\begin{matrix} \frac{5}{4}+\frac{1}{4a} \\ \frac{3}{2}, \frac{9}{4}+\frac{1}{4a} \end{matrix}\middle| -\frac{x^{4a}}{4}\right)}{4a\Gamma\left(\frac{9}{4}+\frac{1}{4a}\right)}$$

```
[In] integrate(x**(3*a)*sin(x**(2*a)),x)
[Out] x**(5*a + 1)*gamma(5/4 + 1/(4*a))*hyper((5/4 + 1/(4*a),), (3/2, 9/4 + 1/(4*
a)), -x**(4*a)/4)/(4*a*gamma(9/4 + 1/(4*a)))
```

Maxima [F]

$$\int x^{3a} \sin \left(x^{2a}\right) \, dx = \int x^{3\,a} \sin \left(x^{2\,a}\right) \, dx$$

```
[In] integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="maxima")
[Out] -1/2*(x*x^a*cos(x^(2*a)) - (a + 1)*integrate(x^a*cos(x^(2*a)), x))/a
```

Giac [F]

$$\int x^{3a} \sin\left(x^{2a}\right) \, dx = \int x^{3\,a} \sin\left(x^{2\,a}\right) \, dx$$

```
[In] integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="giac")
[Out] integrate(x^(3*a)*sin(x^(2*a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3a} \sin\left(x^{2a}\right) \, dx = \int x^{3\,a} \sin\left(x^{2\,a}\right) \, dx$$

```
[In] int(x^(3*a)*sin(x^(2*a)),x)
[Out] int(x^(3*a)*sin(x^(2*a)), x)
```

3.33 $\int \cos\left(\sqrt{x}\right)\,dx$

Optimal result 172
Rubi [A] (verified) 172
Mathematica [A] (verified) 173
Maple [A] (verified) 173
Fricas [A] (verification not implemented) 174
Sympy [A] (verification not implemented) 174
Maxima [A] (verification not implemented) 174
Giac [A] (verification not implemented) 174
Mupad [B] (verification not implemented) 175

Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\sin\left(\sqrt{x}\right)$$

```
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3443, 3377, 2718}

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

```
[In] Int[Cos[Sqrt[x]],x]
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3443

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int x\cos(x)\,dx, x, \sqrt{x}\right)\\ &= 2\sqrt{x}\sin\left(\sqrt{x}\right) - 2\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{x}\right)\\ &= 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \cos\left(\sqrt{x}\right)\, dx = 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\sin\left(\sqrt{x}\right)$$

```
[In] Integrate[Cos[Sqrt[x]],x]
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$2\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)\sqrt{x}$	17
default	$2\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)\sqrt{x}$	17
meijerg	$4\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}} + \frac{\cos(\sqrt{x})}{2\sqrt{\pi}} + \frac{\sqrt{x}\sin(\sqrt{x})}{2\sqrt{\pi}}\right)$	33

```
[In] int(cos(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\,\sqrt{x}\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x**(1/2)),x)
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\,\sqrt{x}\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\,\sqrt{x}\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\, dx = 2\,\cos\left(\sqrt{x}\right) + 2\,\sqrt{x}\,\sin\left(\sqrt{x}\right)$$

```
[In] int(cos(x^(1/2)),x)
[Out] 2*cos(x^(1/2)) + 2*x^(1/2)*sin(x^(1/2))
```

3.34 $\int x\sqrt{1+x}\,dx$

Optimal result 176
Rubi [A] (verified) 176
Mathematica [A] (verified) 177
Maple [A] (verified) 177
Fricas [A] (verification not implemented) 177
Sympy [A] (verification not implemented) 178
Maxima [A] (verification not implemented) 178
Giac [A] (verification not implemented) 178
Mupad [B] (verification not implemented) 178

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x\sqrt{1+x}\,dx = -\frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2}$$

```
[Out] -2/3*(1+x)^(3/2)+2/5*(1+x)^(5/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = $\{45\}$

$$\int x\sqrt{1+x}\,dx = \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2}$$

```
[In] Int[x*Sqrt[1 + x],x]
[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\sqrt{1+x} + (1+x)^{3/2}\right) dx \\ &= -\frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int x\sqrt{1+x}\,dx = \frac{2}{15}(1+x)^{3/2}(-5+3(1+x))$$

```
[In] Integrate[x*Sqrt[1 + x],x]
[Out] (2*(1 + x)^(3/2)*(-5 + 3*(1 + x)))/15
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{2(1+x)^{\frac{3}{2}}(-2+3x)}{15}$	13
derivativedivides	$-\frac{2(1+x)^{\frac{3}{2}}}{3}+\frac{2(1+x)^{\frac{5}{2}}}{5}$	16
default	$-\frac{2(1+x)^{\frac{3}{2}}}{3}+\frac{2(1+x)^{\frac{5}{2}}}{5}$	16
risch	$\frac{2(3x^2+x-2)\sqrt{1+x}}{15}$	16
trager	$\left(\frac{2}{5}x^2+\frac{2}{15}x-\frac{4}{15}\right)\sqrt{1+x}$	17
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(1+x)^{\frac{3}{2}}(2-3x)}{15}}{2\sqrt{\pi}}$	27

```
[In] int(x*(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(1+x)^(3/2)*(-2+3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int x\sqrt{1+x}\,dx = \frac{2}{15}\left(3\,x^2+x-2\right)\sqrt{x+1}$$

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int x\sqrt{1+x}\,dx = \frac{2x^2\sqrt{x+1}}{5} + \frac{2x\sqrt{x+1}}{15} - \frac{4\sqrt{x+1}}{15}$$

```
[In] integrate(x*(1+x)**(1/2),x)
[Out] 2*x**2*sqrt(x + 1)/5 + 2*x*sqrt(x + 1)/15 - 4*sqrt(x + 1)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int x\sqrt{1+x}\,dx = \frac{2}{5}\,(x+1)^{\frac{5}{2}} - \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="maxima")
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int x\sqrt{1+x}\,dx = \frac{2}{5}\,(x+1)^{\frac{5}{2}} - \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="giac")
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int x\sqrt{1+x}\,dx = \frac{2\,(3\,x-2)\,\left(x+1\right)^{3/2}}{15}$$

```
[In] int(x*(x + 1)^(1/2),x)
[Out] (2*(3*x - 2)*(x + 1)^(3/2))/15
```

3.35 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$

Optimal result 179
Rubi [A] (verified) 179
Mathematica [A] (verified) 180
Maple [A] (verified) 180
Fricas [A] (verification not implemented) 181
Sympy [F] 181
Maxima [A] (verification not implemented) 181
Giac [A] (verification not implemented) 181
Mupad [B] (verification not implemented) 182

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)$$

```
[Out] 6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1607, 272, 45}

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

```
[In] Int[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\left(1+\sqrt[6]{x}\right)\sqrt[3]{x}}\,dx\\ &= 6\text{Subst}\left(\int \frac{x^3}{1+x}\,dx, x, \sqrt[6]{x}\right)\\ &= 6\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2\right)\,dx, x, \sqrt[6]{x}\right)\\ &= 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = \left(6-3\sqrt[6]{x}+2\sqrt[3]{x}\right)\sqrt[6]{x}-6\log\left(1+\sqrt[6]{x}\right)$$

```
[In] Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
[Out] (6 - 3*x^(1/6) + 2*x^(1/3))*x^(1/6) - 6*Log[1 + x^(1/6)]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result
derivativedivides	$6x^{\frac{1}{6}}-3x^{\frac{1}{3}}-6\ln\left(1+x^{\frac{1}{6}}\right)+2\sqrt{x}$
meijerg	$\frac{x^{\frac{1}{6}}\left(4x^{\frac{1}{3}}-6x^{\frac{1}{6}}+12\right)}{2}-6\ln\left(1+x^{\frac{1}{6}}\right)$
default	$-\ln\left(x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+2\ln\left(x^{\frac{1}{6}}-1\right)-2\ln\left(1+x^{\frac{1}{6}}\right)+\ln\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)+2\sqrt{x}+\ln($

```
[In] int(1/(x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\log\left(x^{\frac{1}{6}} + 1\right)$$

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = \int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$$

```
[In] integrate(1/(x**(1/3)+x**(1/2)),x)
[Out] Integral(1/(x**(1/3) + sqrt(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\log\left(x^{\frac{1}{6}} + 1\right)$$

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\log\left(x^{\frac{1}{6}} + 1\right)$$

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 6\,\ln\left(x^{1/6}+1\right) - 3\,x^{1/3} + 6\,x^{1/6}$$

```
[In] int(1/(x^(1/2) + x^(1/3)),x)
[Out] 2*x^(1/2) - 6*log(x^(1/6) + 1) - 3*x^(1/3) + 6*x^(1/6)
```

3.36 $\int \sqrt{\frac{1+x}{3+2x}}\,dx$

Optimal result 183
Rubi [A] (verified) 183
Mathematica [A] (verified) 184
Maple [B] (verified) 185
Fricas [A] (verification not implemented) 185
Sympy [F] 185
Maxima [B] (verification not implemented) 186
Giac [B] (verification not implemented) 186
Mupad [B] (verification not implemented) 186

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \sqrt{\frac{1+x}{3+2x}}\,dx = \frac{1}{2}\sqrt{1+x}\sqrt{3+2x}-\frac{\operatorname{arcsinh}\left(\sqrt{2}\sqrt{1+x}\right)}{2\sqrt{2}}$$

```
[Out] -1/4*arcsinh(2^(1/2)*(1+x)^(1/2))*2^(1/2)+1/2*(1+x)^(1/2)*(3+2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1978, 52, 56, 221}

$$\int \sqrt{\frac{1+x}{3+2x}}\,dx = \frac{1}{2}\sqrt{x+1}\sqrt{2x+3}-\frac{\operatorname{arcsinh}\left(\sqrt{2}\sqrt{x+1}\right)}{2\sqrt{2}}$$

```
[In] Int[Sqrt[(1 + x)/(3 + 2*x)],x]
```

```
[Out] (Sqrt[1 + x]*Sqrt[3 + 2*x])/2 - ArcSinh[Sqrt[2]*Sqrt[1 + x]]/(2*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1+x}}{\sqrt{3+2x}}\,dx \\
&= \frac{1}{2}\sqrt{1+x}\sqrt{3+2x} - \frac{1}{4}\int \frac{1}{\sqrt{1+x}\sqrt{3+2x}}\,dx \\
&= \frac{1}{2}\sqrt{1+x}\sqrt{3+2x} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+2x^2}}\,dx, x, \sqrt{1+x}\right) \\
&= \frac{1}{2}\sqrt{1+x}\sqrt{3+2x} - \frac{\text{arcsinh}\left(\sqrt{2}\sqrt{1+x}\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int \sqrt{\frac{1+x}{3+2x}}\,dx = \frac{\sqrt{\frac{1+x}{3+2x}}\left(2\sqrt{1+x}(3+2x) - \sqrt{6+4x}\,\text{arctanh}\left(\frac{\sqrt{3+2x}}{\sqrt{2}\sqrt{1+x}}\right)\right)}{4\sqrt{1+x}}$$

```
[In] Integrate[Sqrt[(1 + x)/(3 + 2*x)],x]
```

```
[Out] (Sqrt[(1 + x)/(3 + 2*x)]*(2*Sqrt[1 + x]*(3 + 2*x) - Sqrt[6 + 4*x]*ArcTanh[S
qrt[3 + 2*x]/(Sqrt[2]*Sqrt[1 + x])]))/(4*Sqrt[1 + x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(30) = 60$.

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

method	result
default	$-\frac{\sqrt{\frac{1+x}{3+2x}}\,(3+2x)\left(\ln\left(\frac{5\sqrt{2}}{4}+x\sqrt{2}+\sqrt{2x^2+5x+3}\right)\sqrt{2}-4\sqrt{2x^2+5x+3}\right)}{8\sqrt{(3+2x)(1+x)}}$
risch	$\frac{(3+2x)\sqrt{\frac{1+x}{3+2x}}}{2}-\frac{\ln\left(\frac{\left(\frac{5}{2}+2x\right)\sqrt{2}}{2}+\sqrt{2x^2+5x+3}\right)\sqrt{2}\,\sqrt{\frac{1+x}{3+2x}}\,\sqrt{(3+2x)(1+x)}}{8(1+x)}$
trager	$3\left(\frac{1}{2}+\frac{x}{3}\right)\sqrt{-\frac{-1-x}{3+2x}}-\frac{\text{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\text{RootOf}\left(\_Z^2-2\right)x+8\sqrt{-\frac{-1-x}{3+2x}}\,x+5\,\text{RootOf}\left(\_Z^2-2\right)+12\sqrt{-\frac{-1-x}{3+2x}}\right)}{8}$

```
[In] int(((1+x)/(3+2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/8*((1+x)/(3+2*x))^(1/2)*(3+2*x)*(ln(5/4*2^(1/2)+x*2^(1/2)+(2*x^2+5*x+3)^
(1/2))*2^(1/2)-4*(2*x^2+5*x+3)^(1/2))/((3+2*x)*(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \sqrt{\frac{1+x}{3+2x}}\,dx = \frac{1}{2}\,(2\,x+3)\sqrt{\frac{x+1}{2\,x+3}}+\frac{1}{8}\,\sqrt{2}\log\left(2\,\sqrt{2}(2\,x+3)\sqrt{\frac{x+1}{2\,x+3}}-4\,x-5\right)$$

```
[In] integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="fricas")
[Out] 1/2*(2*x + 3)*sqrt((x + 1)/(2*x + 3)) + 1/8*sqrt(2)*log(2*sqrt(2)*(2*x + 3)
*sqrt((x + 1)/(2*x + 3)) - 4*x - 5)
```

Sympy [F]

$$\int \sqrt{\frac{1+x}{3+2x}}\,dx = \int \sqrt{\frac{x+1}{2x+3}}\,dx$$

```
[In] integrate(((1+x)/(3+2*x))**(1/2),x)
[Out] Integral(sqrt((x + 1)/(2*x + 3)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.82

$$\int \sqrt{\frac{1+x}{3+2x}}\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\sqrt{\frac{x+1}{2x+3}}}{\sqrt{2}+2\sqrt{\frac{x+1}{2x+3}}}\right) - \frac{\sqrt{\frac{x+1}{2x+3}}}{2\left(\frac{2(x+1)}{2x+3}-1\right)}$$

```
[In] integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(-(sqrt(2) - 2*sqrt((x + 1)/(2*x + 3)))/(sqrt(2) + 2*sqrt((x
 + 1)/(2*x + 3)))) - 1/2*sqrt((x + 1)/(2*x + 3))/(2*(x + 1)/(2*x + 3) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(30) = 60$.

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \sqrt{\frac{1+x}{3+2x}}\,dx = \frac{1}{8}\sqrt{2}\log\left(\left|-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2+5x+3}\right)-5\right|\right)\operatorname{sgn}(2x+3) + \frac{1}{2}\sqrt{2x^2+5x+3}\operatorname{sgn}(2x+3)$$

```
[In] integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 + 5*x + 3)) - 5))*sg
n(2*x + 3) + 1/2*sqrt(2*x^2 + 5*x + 3)*sgn(2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \sqrt{\frac{1+x}{3+2x}}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\sqrt{\frac{x+1}{2x+3}}\right)}{4} - \frac{\sqrt{\frac{x+1}{2x+3}}}{2\left(\frac{2x+2}{2x+3}-1\right)}$$

```
[In] int(((x + 1)/(2*x + 3))^(1/2),x)
```

```
[Out] - (2^(1/2)*atanh(2^(1/2)*((x + 1)/(2*x + 3))^(1/2)))/4 - ((x + 1)/(2*x + 3)
)^(1/2)/(2*((2*x + 2)/(2*x + 3) - 1))
```

3.37 $\int \frac{x^4}{(1-x^2)^{5/2}}\,dx$

Optimal result 187
Rubi [A] (verified) 187
Mathematica [A] (verified) 188
Maple [A] (verified) 188
Fricas [B] (verification not implemented) 189
Sympy [B] (verification not implemented) 189
Maxima [A] (verification not implemented) 189
Giac [A] (verification not implemented) 190
Mupad [B] (verification not implemented) 190

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \arcsin(x)$$

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {294, 222}

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \arcsin(x) - \frac{x}{\sqrt{1-x^2}} + \frac{x^3}{3(1-x^2)^{3/2}}$$

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\int \frac{x^2}{\left(1-x^2\right)^{3/2}}\,dx\\ &= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\arcsin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx=\frac{x(-3+4x^2)}{3\left(1-x^2\right)^{3/2}}+2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x^{3}}{3\left(-x^{2}+1\right)^{\frac{3}{2}}}+\arcsin\left(x\right)-\frac{x}{\sqrt{-x^{2}+1}}$	30
risch	$-\frac{x\left(4x^{2}-3\right)}{3\left(x^{2}-1\right)\sqrt{-x^{2}+1}}+\arcsin\left(x\right)$	30
meijerg	$-\frac{2i\left(-\frac{i\sqrt{\pi}\,x\left(-20x^{2}+15\right)}{10\left(-x^{2}+1\right)^{\frac{3}{2}}}+\frac{3i\sqrt{\pi}\,\arcsin\left(x\right)}{2}\right)}{3\sqrt{\pi}}$	39
pseudoelliptic	$\frac{\left(x^{2}-1\right)\sqrt{-x^{2}+1}\,\arctan\left(\frac{\sqrt{-x^{2}+1}}{x}\right)+\frac{4x^{3}}{3}-x}{\left(-x^{2}+1\right)^{\frac{3}{2}}}$	49
trager	$\frac{\left(4x^{2}-3\right)x\sqrt{-x^{2}+1}}{3\left(x^{2}-1\right)^{2}}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)$	54

```
[In] int(x^4/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-1/(-x^2+1)^(1/2)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = -\frac{6\,(x^4-2\,x^2+1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-(4\,x^3-3\,x)\sqrt{-x^2+1}}{3\,(x^4-2\,x^2+1)}$$

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(26) = 52$.

Time = 0.40 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.00

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3} - \frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3}-\frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3}+\frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{1}{3}\,x\left(\frac{3\,x^2}{(-x^2+1)^{\frac{3}{2}}}-\frac{2}{(-x^2+1)^{\frac{3}{2}}}\right)-\frac{x}{3\,\sqrt{-x^2+1}}+\arcsin(x)$$

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{(4\,x^2-3)\sqrt{-x^2+1}x}{3\,(x^2-1)^2} + \arcsin{(x)}$$

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.60

$$\begin{aligned}\int \frac{x^4}{(1-x^2)^{5/2}}\,dx &= \operatorname{asin}(x) + \frac{3\sqrt{1-x^2}}{4\,(x-1)} + \frac{3\sqrt{1-x^2}}{4\,(x+1)}\\ &\quad - \sqrt{1-x^2}\left(\frac{1}{12\,(x-1)} - \frac{1}{12\,(x-1)^2}\right) - \sqrt{1-x^2}\left(\frac{1}{12\,(x+1)} + \frac{1}{12\,(x+1)^2}\right)\end{aligned}$$

```
[In] int(x^4/(1 - x^2)^(5/2),x)
[Out] asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1))
 - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*(1
/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

3.38 $\int \sqrt{x}(1+x)^{5/2}\,dx$

Optimal result 191
Rubi [A] (verified) 191
Mathematica [A] (verified) 192
Maple [A] (verified) 193
Fricas [A] (verification not implemented) 193
Sympy [C] (verification not implemented) 193
Maxima [B] (verification not implemented) 194
Giac [A] (verification not implemented) 194
Mupad [F(-1)] 195

Optimal result

Integrand size = 13, antiderivative size = 75

$$\int \sqrt{x}(1+x)^{5/2}\,dx = \frac{5}{64}\sqrt{x}\sqrt{1+x}+\frac{5}{32}x^{3/2}\sqrt{1+x} + \frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}-\frac{5\text{arcsinh}\left(\sqrt{x}\right)}{64}$$

```
[Out] 5/24*x^(3/2)*(1+x)^(3/2)+1/4*x^(3/2)*(1+x)^(5/2)-5/64*arcsinh(x^(1/2))+5/32
*x^(3/2)*(1+x)^(1/2)+5/64*x^(1/2)*(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {52, 56, 221}

$$\int \sqrt{x}(1+x)^{5/2}\,dx = -\frac{5\text{arcsinh}\left(\sqrt{x}\right)}{64} + \frac{1}{4}x^{3/2}(x+1)^{5/2}+\frac{5}{24}x^{3/2}(x+1)^{3/2}+\frac{5}{32}x^{3/2}\sqrt{x+1}+\frac{5}{64}\sqrt{x}\sqrt{x+1}$$

```
[In] Int[Sqrt[x]*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[x]*Sqrt[1 + x])/64 + (5*x^(3/2)*Sqrt[1 + x])/32 + (5*x^(3/2)*(1 + x
)^(3/2))/24 + (x^(3/2)*(1 + x)^(5/2))/4 - (5*ArcSinh[Sqrt[x]])/64
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^{3/2}(1+x)^{5/2}+\frac{5}{8}\int \sqrt{x}(1+x)^{3/2}\,dx \\
&= \frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}+\frac{5}{16}\int \sqrt{x}\sqrt{1+x}\,dx \\
&= \frac{5}{32}x^{3/2}\sqrt{1+x}+\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}+\frac{5}{64}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx \\
&= \frac{5}{64}\sqrt{x}\sqrt{1+x}+\frac{5}{32}x^{3/2}\sqrt{1+x}+\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}-\frac{5}{128}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx \\
&= \frac{5}{64}\sqrt{x}\sqrt{1+x}+\frac{5}{32}x^{3/2}\sqrt{1+x} \\
&\quad +\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}-\frac{5}{64}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{x}\right) \\
&= \frac{5}{64}\sqrt{x}\sqrt{1+x}+\frac{5}{32}x^{3/2}\sqrt{1+x}+\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}-\frac{5\text{arcsinh}\left(\sqrt{x}\right)}{64}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(1+x)^{5/2}\,dx=\frac{1}{192}\sqrt{x}\sqrt{1+x}\left(15+118x+136x^2+48x^3\right)+\frac{5}{64}\log\left(-\sqrt{x}+\sqrt{1+x}\right)$$

```
[In] Integrate[Sqrt[x]*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(15 + 118*x + 136*x^2 + 48*x^3))/192 + (5*Log[-Sqrt[x]
 + Sqrt[1 + x]])/64
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59

method	result	size
meijerg	$-\frac{15\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\left(48x^{3}+136x^{2}+118x+15\right)\sqrt{1+x}}{360}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{x}\right)}{24}\right)}{8\sqrt{\pi}}$	44
risch	$\frac{\left(48x^{3}+136x^{2}+118x+15\right)\sqrt{x}\,\sqrt{1+x}}{192}-\frac{5\sqrt{x(1+x)}\,\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)}{128\sqrt{1+x}\,\sqrt{x}}$	55
default	$\frac{\sqrt{x}\,(1+x)^{\frac{7}{2}}}{4}-\frac{\sqrt{x}\,(1+x)^{\frac{5}{2}}}{24}-\frac{5\sqrt{x}\,(1+x)^{\frac{3}{2}}}{96}-\frac{5\sqrt{x}\,\sqrt{1+x}}{64}-\frac{5\sqrt{x(1+x)}\,\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)}{128\sqrt{1+x}\,\sqrt{x}}$	70

```
[In] int(x^(1/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
[Out] -15/8/Pi^(1/2)*(-1/360*Pi^(1/2)*x^(1/2)*(48*x^3+136*x^2+118*x+15)*(1+x)^(1/
2)+1/24*Pi^(1/2)*arcsinh(x^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59

$$\int \sqrt{x}(1+x)^{5/2}\,dx = \frac{1}{192}\left(48\,x^{3}+136\,x^{2}+118\,x+15\right)\sqrt{x+1}\sqrt{x} + \frac{5}{128}\log\left(2\sqrt{x+1}\sqrt{x}-2\,x-1\right)$$

```
[In] integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="fricas")
[Out] 1/192*(48*x^3 + 136*x^2 + 118*x + 15)*sqrt(x + 1)*sqrt(x) + 5/128*log(2*sqr
t(x + 1)*sqrt(x) - 2*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.65 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.53

$$\int \sqrt{x}(1+x)^{5/2}\,dx = \begin{cases} -\frac{5\operatorname{acosh}\left(\sqrt{x+1}\right)}{64}+\frac{(x+1)^{\frac{9}{2}}}{4\sqrt{x}}-\frac{7(x+1)^{\frac{7}{2}}}{24\sqrt{x}}-\frac{(x+1)^{\frac{5}{2}}}{96\sqrt{x}}-\frac{5(x+1)^{\frac{3}{2}}}{192\sqrt{x}}+\frac{5\sqrt{x+1}}{64\sqrt{x}} & \text{for } |x+1|>1 \\ \frac{5i\operatorname{asin}\left(\sqrt{x+1}\right)}{64}-\frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{-x}}+\frac{7i(x+1)^{\frac{7}{2}}}{24\sqrt{-x}}+\frac{i(x+1)^{\frac{5}{2}}}{96\sqrt{-x}}+\frac{5i(x+1)^{\frac{3}{2}}}{192\sqrt{-x}}-\frac{5i\sqrt{x+1}}{64\sqrt{-x}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(1/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-5*acosh(sqrt(x + 1))/64 + (x + 1)**(9/2)/(4*sqrt(x)) - 7*(x + 1
)**(7/2)/(24*sqrt(x)) - (x + 1)**(5/2)/(96*sqrt(x)) - 5*(x + 1)**(3/2)/(192
*sqrt(x)) + 5*sqrt(x + 1)/(64*sqrt(x)), Abs(x + 1) > 1), (5*I*asin(sqrt(x +
 1))/64 - I*(x + 1)**(9/2)/(4*sqrt(-x)) + 7*I*(x + 1)**(7/2)/(24*sqrt(-x))
+ I*(x + 1)**(5/2)/(96*sqrt(-x)) + 5*I*(x + 1)**(3/2)/(192*sqrt(-x)) - 5*I*
sqrt(x + 1)/(64*sqrt(-x)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(47) = 94$.

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.51

$$\int \sqrt{x}(1+x)^{5/2}\,dx = \frac{\frac{15\,(x+1)^{\frac{7}{2}}}{x^{\frac{7}{2}}} + \frac{73\,(x+1)^{\frac{5}{2}}}{x^{\frac{5}{2}}} - \frac{55\,(x+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}} + \frac{15\,\sqrt{x+1}}{\sqrt{x}}}{192\left(\frac{(x+1)^4}{x^4} - \frac{4\,(x+1)^3}{x^3} + \frac{6\,(x+1)^2}{x^2} - \frac{4\,(x+1)}{x} + 1\right)} - \frac{5}{128}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}} + 1\right) + \frac{5}{128}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}} - 1\right)$$

```
[In] integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/192*(15*(x + 1)^(7/2)/x^(7/2) + 73*(x + 1)^(5/2)/x^(5/2) - 55*(x + 1)^(3/
2)/x^(3/2) + 15*sqrt(x + 1)/sqrt(x))/((x + 1)^4/x^4 - 4*(x + 1)^3/x^3 + 6*(
x + 1)^2/x^2 - 4*(x + 1)/x + 1) - 5/128*log(sqrt(x + 1)/sqrt(x) + 1) + 5/12
8*log(sqrt(x + 1)/sqrt(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int \sqrt{x}(1+x)^{5/2}\,dx = \frac{1}{192}\,(2\,(4\,(6\,x-19)(x+1)+163)(x+1)-279)\sqrt{x+1}\sqrt{x} + \frac{1}{8}\,(2\,(4\,x-9)(x+1)+33)\sqrt{x+1}\sqrt{x} + \frac{3}{4}\,(2\,x-3)\sqrt{x+1}\sqrt{x} + \sqrt{x+1}\sqrt{x} + \frac{5}{64}\,\log\left(\sqrt{x+1}-\sqrt{x}\right)$$

```
[In] integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/192*(2*(4*(6*x - 19)*(x + 1) + 163)*(x + 1) - 279)*sqrt(x + 1)*sqrt(x) +
1/8*(2*(4*x - 9)*(x + 1) + 33)*sqrt(x + 1)*sqrt(x) + 3/4*(2*x - 3)*sqrt(x +
 1)*sqrt(x) + sqrt(x + 1)*sqrt(x) + 5/64*log(sqrt(x + 1) - sqrt(x))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(1+x)^{5/2}\,dx = \int \sqrt{x}\,(x+1)^{5/2}\,dx$$

```
[In] int(x^(1/2)*(x + 1)^(5/2),x)
[Out] int(x^(1/2)*(x + 1)^(5/2), x)
```

3.39 $\int \frac{x^4}{(1-x^2)^{5/2}} dx$

Optimal result 196
Rubi [A] (verified) 196
Mathematica [A] (verified) 197
Maple [A] (verified) 197
Fricas [B] (verification not implemented) 198
Sympy [B] (verification not implemented) 198
Maxima [A] (verification not implemented) 198
Giac [A] (verification not implemented) 199
Mupad [B] (verification not implemented) 199

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{x^4}{(1-x^2)^{5/2}} dx = \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \arcsin(x)$$

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {294, 222}

$$\int \frac{x^4}{(1-x^2)^{5/2}} dx = \arcsin(x) - \frac{x}{\sqrt{1-x^2}} + \frac{x^3}{3(1-x^2)^{3/2}}$$

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^3}{3(1-x^2)^{3/2}} - \int \frac{x^2}{(1-x^2)^{3/2}}\,dx\\ &= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \arcsin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{x(-3+4x^2)}{3(1-x^2)^{3/2}} + 2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x^3}{3(-x^2+1)^{\frac{3}{2}}} + \arcsin(x) - \frac{x}{\sqrt{-x^2+1}}$	30
risch	$-\frac{x(4x^2-3)}{3(x^2-1)\sqrt{-x^2+1}} + \arcsin(x)$	30
meijerg	$-\frac{2i\left(-\frac{i\sqrt{\pi}\,x(-20x^2+15)}{10(-x^2+1)^{\frac{3}{2}}} + \frac{3i\sqrt{\pi}\arcsin(x)}{2}\right)}{3\sqrt{\pi}}$	39
pseudoelliptic	$\frac{(x^2-1)\sqrt{-x^2+1}\,\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right) + \frac{4x^3}{3} - x}{(-x^2+1)^{\frac{3}{2}}}$	49
trager	$\frac{(4x^2-3)x\sqrt{-x^2+1}}{3(x^2-1)^2} + \operatorname{RootOf}(\_Z^2+1)\ln\left(\operatorname{RootOf}(\_Z^2+1)\sqrt{-x^2+1}+x\right)$	54

```
[In] int(x^4/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-1/(-x^2+1)^(1/2)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = -\frac{6\,(x^4-2\,x^2+1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-(4\,x^3-3\,x)\sqrt{-x^2+1}}{3\,(x^4-2\,x^2+1)}$$

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(26) = 52$.

Time = 0.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.00

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3} - \frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3}-\frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3}+\frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{1}{3}\,x\left(\frac{3\,x^2}{(-x^2+1)^{\frac{3}{2}}}-\frac{2}{(-x^2+1)^{\frac{3}{2}}}\right)-\frac{x}{3\,\sqrt{-x^2+1}}+\arcsin(x)$$

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{(4\,x^2-3)\sqrt{-x^2+1}x}{3\,(x^2-1)^2} + \arcsin{(x)}$$

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.60

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \mathrm{asin}(x) + \frac{3\sqrt{1-x^2}}{4\ (x-1)} + \frac{3\sqrt{1-x^2}}{4\ (x+1)} - \sqrt{1-x^2}\left(\frac{1}{12\ (x-1)} - \frac{1}{12\,(x-1)^2}\right) - \sqrt{1-x^2}\left(\frac{1}{12\ (x+1)} + \frac{1}{12\,(x+1)^2}\right)$$

```
[In] int(x^4/(1 - x^2)^(5/2),x)
[Out] asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1))
 - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*(1
/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

3.40 $\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2} dy$

Optimal result 200
Rubi [A] (verified) 200
Mathematica [A] (verified) 201
Maple [A] (verified) 202
Fricas [B] (verification not implemented) 202
Sympy [F] 203
Maxima [B] (verification not implemented) 203
Giac [B] (verification not implemented) 203
Mupad [F(-1)] 204

Optimal result

Integrand size = 29, antiderivative size = 51

$$\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2} dy = B \arctan\left(\frac{By}{\sqrt{A^2+B^2-B^2y^2}}\right) + A\operatorname{arctanh}\left(\frac{Ay}{\sqrt{A^2+B^2-B^2y^2}}\right)$$

```
[Out] B*arctan(B*y/(-B^2*y^2+A^2+B^2)^(1/2))+A*arctanh(A*y/(-B^2*y^2+A^2+B^2)^(1/
2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {399, 223, 209, 385, 212}

$$\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2} dy = B \arctan\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right) + A\operatorname{arctanh}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

```
[In] Int[Sqrt[A^2 + B^2 - B^2*y^2]/(1 - y^2),y]
```

```
[Out] B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]] + A*ArcTanh[(A*y)/Sqrt[A^2 + B^2
- B^2*y^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= A^2\int \frac{1}{(1-y^2)\sqrt{A^2+B^2-B^2y^2}}\,dy + B^2\int \frac{1}{\sqrt{A^2+B^2-B^2y^2}}\,dy \\
&= A^2\text{Subst}\left(\int \frac{1}{1-A^2y^2}\,dy, y, \frac{y}{\sqrt{A^2+B^2-B^2y^2}}\right) \\
&\quad + B^2\text{Subst}\left(\int \frac{1}{1+B^2y^2}\,dy, y, \frac{y}{\sqrt{A^2+B^2-B^2y^2}}\right) \\
&= B\arctan\left(\frac{By}{\sqrt{A^2+B^2-B^2y^2}}\right) + A\text{arctanh}\left(\frac{Ay}{\sqrt{A^2+B^2-B^2y^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy = -2B\arctan\left(\frac{-\frac{\sqrt{A^2+B^2}}{B}+\frac{\sqrt{A^2+B^2-B^2y^2}}{B}}{y}\right) + A\text{arctanh}\left(\frac{\sqrt{A^2+B^2-B^2y^2}}{Ay}\right)$$

```
[In] Integrate[Sqrt[A^2 + B^2 - B^2*y^2]/(1 - y^2),y]
[Out] -2*B*ArcTan[(-(Sqrt[A^2 + B^2]/B) + Sqrt[A^2 + B^2 - B^2*y^2]/B)/y] + A*Arc
Tanh[Sqrt[A^2 + B^2 - B^2*y^2]/(A*y)]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.76

method	result
pseudoelliptic	$\frac{A\ln\left(\frac{Ay+\sqrt{-B^2y^2+A^2+B^2}}{y}\right)}{2}-\frac{A\ln\left(\frac{Ay-\sqrt{-B^2y^2+A^2+B^2}}{y}\right)}{2}-B\arctan\left(\frac{\sqrt{-B^2y^2+A^2+B^2}}{By}\right)$
default	$-\frac{\sqrt{-B^2(y-1)^2-2B^2(y-1)+A^2}}{2}+\frac{B^2\arctan\left(\frac{\sqrt{B^2}\,y}{\sqrt{-B^2(y-1)^2-2B^2(y-1)+A^2}}\right)}{2\sqrt{B^2}}+\frac{A^2\ln\left(\frac{2A^2-2B^2(y-1)+2\sqrt{A^2}\,\sqrt{-B^2(y-1}}{y-1}\right.}{2\sqrt{A^2}}$

```
[In] int((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y,method=_RETURNVERBOSE)
```

```
[Out] 1/2*A*ln((A*y+(-B^2*y^2+A^2+B^2)^(1/2))/y)-1/2*A*ln((A*y-(-B^2*y^2+A^2+B^2)
^(1/2))/y)-B*arctan(1/B/y*(-B^2*y^2+A^2+B^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(47) = 94$.

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.53

$$\begin{aligned}
&\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy \\
&= -B\arctan\left(\frac{\sqrt{-B^2y^2+A^2+B^2}}{By}\right) \\
&\quad+\frac{1}{4}\,A\log\left(-\frac{(A^2-B^2)y^2+2\sqrt{-B^2y^2+A^2+B^2}Ay+A^2+B^2}{y^2}\right) \\
&\quad-\frac{1}{4}\,A\log\left(-\frac{(A^2-B^2)y^2-2\sqrt{-B^2y^2+A^2+B^2}Ay+A^2+B^2}{y^2}\right)
\end{aligned}$$

```
[In] integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="fricas")
```

```
[Out] -B*arctan(sqrt(-B^2*y^2 + A^2 + B^2)/(B*y)) + 1/4*A*log(-((A^2 - B^2)*y^2 +
 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2) - 1/4*A*log(-((A^2 - B^
2)*y^2 - 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2)
```

Sympy [F]

$$\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy = -\int \frac{\sqrt{A^2-B^2y^2+B^2}}{y^2-1}\,dy$$

```
[In] integrate((-B**2*y**2+A**2+B**2)**(1/2)/(-y**2+1),y)
[Out] -Integral(sqrt(A**2 - B**2*y**2 + B**2)/(y**2 - 1), y)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(47) = 94$.

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.24

$$\begin{aligned}\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy &= B\arcsin\left(\frac{B^2y}{\sqrt{A^2B^2+B^4}}\right) \\ &\quad - \frac{1}{2}A\log\left(B^2+\frac{A^2}{y+1}+\frac{\sqrt{-B^2y^2+A^2+B^2}A}{y+1}\right) \\ &\quad + \frac{1}{2}A\log\left(-B^2+\frac{2A^2}{|2y-2|}+\frac{2\sqrt{-B^2y^2+A^2+B^2}A}{|2y-2|}\right)\end{aligned}$$

```
[In] integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="maxima")
[Out] B*arcsin(B^2*y/sqrt(A^2*B^2 + B^4)) - 1/2*A*log(B^2 + A^2/(y + 1) + sqrt(-B
^2*y^2 + A^2 + B^2)*A/(y + 1)) + 1/2*A*log(-B^2 + 2*A^2/abs(2*y - 2) + 2*sq
rt(-B^2*y^2 + A^2 + B^2)*A/abs(2*y - 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(47) = 94$.

Time = 0.34 (sec) , antiderivative size = 295, normalized size of antiderivative = 5.78

$$\begin{aligned}&\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy \\ &= -\frac{\left(\pi\,\mathrm{sgn}(y)-2\arctan\left(-\frac{B^2y\left(\frac{\left(\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|\right)^2}{B^4y^2}-1\right)}{2\left(\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|\right)}\right)\right)B^2}{2|B|} \\ &\quad + \frac{AB\log\left(\left|-\left(\frac{B^2y}{\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|}-\frac{\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|}{B^2y}\right)B+2A\right|\right)}{2|B|} \\ &\quad - \frac{AB\log\left(\left|-\left(\frac{B^2y}{\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|}-\frac{\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|}{B^2y}\right)B-2A\right|\right)}{2|B|}\end{aligned}$$

```
[In] integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="giac")
```

```
[Out] -1/2*(pi*sgn(y) - 2*arctan(-1/2*B^2*y*((sqrt(A^2 + B^2)*B + sqrt(-B^2*y^2 +
 A^2 + B^2)*abs(B))^2/(B^4*y^2) - 1)/(sqrt(A^2 + B^2)*B + sqrt(-B^2*y^2 + A
^2 + B^2)*abs(B))))*B^2/abs(B) + 1/2*A*B*log(abs(-(B^2*y/(sqrt(A^2 + B^2)*B
 + sqrt(-B^2*y^2 + A^2 + B^2)*abs(B)) - (sqrt(A^2 + B^2)*B + sqrt(-B^2*y^2
+ A^2 + B^2)*abs(B))/(B^2*y))*B + 2*A))/abs(B) - 1/2*A*B*log(abs(-(B^2*y/(s
qrt(A^2 + B^2)*B + sqrt(-B^2*y^2 + A^2 + B^2)*abs(B)) - (sqrt(A^2 + B^2)*B
+ sqrt(-B^2*y^2 + A^2 + B^2)*abs(B))/(B^2*y))*B - 2*A))/abs(B)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy$$
$$= \begin{cases} -\int \frac{\sqrt{-B^2\,y^2}}{y^2-1}\,dy & \text{if } A^2+B^2=0 \\ -\ln\left(2\,y\,\sqrt{-B^2}+2\,\sqrt{A^2-B^2\,y^2+B^2}\right)\,\sqrt{-B^2}-\operatorname{atan}\left(\frac{y\,\sqrt{A^2}\,1\mathrm{i}}{\sqrt{A^2-B^2\,y^2+B^2}}\right)\,\sqrt{A^2}\,1\mathrm{i} & \text{if } A^2+B^2\neq 0 \end{cases}$$

```
[In] int(-(A^2 + B^2 - B^2*y^2)^(1/2)/(y^2 - 1),y)
```

```
[Out] piecewise(A^2 + B^2 == 0, -int((-B^2*y^2)^(1/2)/(y^2 - 1), y), A^2 + B^2 ~=
 0, - atan((y*(A^2)^(1/2)*1i)/(A^2 + B^2 - B^2*y^2)^(1/2))*(A^2)^(1/2)*1i -
 log(2*y*(-B^2)^(1/2) + 2*(A^2 + B^2 - B^2*y^2)^(1/2))*(-B^2)^(1/2))
```

3.41 $\int \sin^2(x)\,dx$

Optimal result 205
Rubi [A] (verified) 205
Mathematica [A] (verified) 206
Maple [A] (verified) 206
Fricas [A] (verification not implemented) 207
Sympy [A] (verification not implemented) 207
Maxima [A] (verification not implemented) 207
Giac [A] (verification not implemented) 207
Mupad [B] (verification not implemented) 208

Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{2}\sin(x)\cos(x)$$

```
[In] Int[Sin[x]^2,x]
```

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\text{integral} = -\frac{1}{2}\cos(x)\sin(x) + \frac{\int 1\,dx}{2}$$
$$= \frac{x}{2} - \frac{1}{2}\cos(x)\sin(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{4}\sin(2x)$$

```
[In] Integrate[Sin[x]^2,x]
[Out] x/2 - Sin[2*x]/4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
meijerg	$\frac{\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}}-\frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$	22
norman	$\frac{\tan^3\left(\frac{x}{2}\right)+x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}-\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

```
[In] int(sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = -\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

```
[In] integrate(sin(x)^2,x, algorithm="fricas")
[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

```
[In] integrate(sin(x)**2,x)
[Out] x/2 - sin(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{1}{2}x - \frac{1}{4}\sin(2x)$$

```
[In] integrate(sin(x)^2,x, algorithm="maxima")
[Out] 1/2*x - 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{1}{2}x - \frac{1}{4}\sin(2x)$$

```
[In] integrate(sin(x)^2,x, algorithm="giac")
[Out] 1/2*x - 1/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{\sin(2\,x)}{4}$$

```
[In] int(sin(x)^2,x)
[Out] x/2 - sin(2*x)/4
```

3.42 $\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx$

Optimal result . . . 209
Rubi [A] (verified) . . . 209
Mathematica [B] (verified) . . . 211
Maple [C] (verified) . . . 211
Fricas [B] (verification not implemented) . . . 212
Sympy [F] . . . 212
Maxima [B] (verification not implemented) . . . 213
Giac [F] . . . 213
Mupad [F(-1)] . . . 213

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx = -B\arctan\left(\frac{B\cos(x)}{\sqrt{A^2+B^2\sin^2(x)}}\right) - A\text{arctanh}\left(\frac{A\cos(x)}{\sqrt{A^2+B^2\sin^2(x)}}\right)$$

```
[Out] -B*arctan(B*cos(x)/(A^2+B^2*sin(x)^2)^(1/2))-A*arctanh(A*cos(x)/(A^2+B^2*si
n(x)^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3265, 399, 223, 209, 385, 212}

$$\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx = -B\arctan\left(\frac{B\cos(x)}{\sqrt{A^2-B^2\cos^2(x)+B^2}}\right) - A\text{arctanh}\left(\frac{A\cos(x)}{\sqrt{A^2-B^2\cos^2(x)+B^2}}\right)$$

```
[In] Int[Csc[x]*Sqrt[A^2 + B^2*Sin[x]^2],x]
```

```
[Out] -(B*ArcTan[(B*Cos[x])/Sqrt[A^2 + B^2 - B^2*Cos[x]^2]]) - A*ArcTanh[(A*Cos[x
])/Sqrt[A^2 + B^2 - B^2*Cos[x]^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt{A^2+B^2-B^2x^2}}{1-x^2}\,dx, x, \cos(x)\right)\\ &= -\left(A^2\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{A^2+B^2-B^2x^2}}\,dx, x, \cos(x)\right)\right)\\ &\quad - B^2\text{Subst}\left(\int \frac{1}{\sqrt{A^2+B^2-B^2x^2}}\,dx, x, \cos(x)\right)\end{aligned}$$

$$= -\left(A^2\text{Subst}\left(\int \frac{1}{1-A^2x^2}\,dx, x, \frac{\cos(x)}{\sqrt{A^2+B^2-B^2\cos^2(x)}}\right)\right) - B^2\text{Subst}\left(\int \frac{1}{1+B^2x^2}\,dx, x, \frac{\cos(x)}{\sqrt{A^2+B^2-B^2\cos^2(x)}}\right)$$

$$= -B\arctan\left(\frac{B\cos(x)}{\sqrt{A^2+B^2-B^2\cos^2(x)}}\right) - A\text{arctanh}\left(\frac{A\cos(x)}{\sqrt{A^2+B^2-B^2\cos^2(x)}}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 99 vs. 2(49) = 98.

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.02

$$\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx = -\sqrt{A^2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{A^2}\cos(x)}{\sqrt{2A^2+B^2-B^2\cos(2x)}}\right) + \sqrt{-B^2}\log\left(\sqrt{2}\sqrt{-B^2}\cos(x) + \sqrt{2A^2+B^2-B^2\cos(2x)}\right)$$

```
[In] Integrate[Csc[x]*Sqrt[A^2 + B^2*Sin[x]^2],x]
```

```
[Out] -(Sqrt[A^2]*ArcTanh[(Sqrt[2]*Sqrt[A^2]*Cos[x])/Sqrt[2*A^2 + B^2 - B^2*Cos[2
*x]]]) + Sqrt[-B^2]*Log[Sqrt[2]*Sqrt[-B^2]*Cos[x] + Sqrt[2*A^2 + B^2 - B^2*
Cos[2*x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.04

method	result
default	$-\frac{\sqrt{\left(A^2+B^2\left(\sin^2(x)\right)\right)\left(\cos^2(x)\right)}\left(A\ \text{csgn}(A)\ln\left(-\frac{A^2\left(\sin^2(x)\right)-B^2\left(\sin^2(x)\right)-2\ \text{csgn}(A)A\sqrt{\left(A^2+B^2\left(\sin^2(x)\right)\right)\left(\cos^2(x)\right)}-2A^2}{\sin(x)^2}\right)-B\ \text{c}}{2\cos(x)\sqrt{A^2+B^2\left(\sin^2(x)\right)}}$

```
[In] int((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((A^2+B^2*sin(x)^2)*cos(x)^2)^(1/2)*(A*csgn(A)*ln(-(A^2*sin(x)^2-B^2*s
in(x)^2-2*csgn(A)*A*((A^2+B^2*sin(x)^2)*cos(x)^2)^(1/2)-2*A^2)/sin(x)^2)-B*
csgn(B)*arctan(1/2*csgn(B)/B*(2*B^2*sin(x)^2+A^2-B^2)/((A^2+B^2*sin(x)^2)*c
os(x)^2)^(1/2)))/cos(x)/(A^2+B^2*sin(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(45) = 90$.

Time = 0.32 (sec) , antiderivative size = 244, normalized size of antiderivative = 4.98

$$\int \csc(x)\sqrt{A^2 + B^2\sin^2(x)}\,dx$$
$$= \frac{1}{2}B\arctan\left(-\frac{(A^4 + 2A^2B^2 + B^4)\cos(x)\sin(x) - 2\left(2B^3\cos(x)^3 - (A^2B + B^3)\cos(x)\right)\sqrt{-B^2\cos(x)^2}}{4B^4\cos(x)^4 + A^4 + 2A^2B^2 + B^4 - (A^4 + 6A^2B^2 + 5B^4)\cos(x)^2}\right.$$
$$- \frac{1}{2}B\arctan\left(\frac{\sin(x)}{\cos(x)}\right) - \frac{1}{2}A\log\left(-B^2\cos(x)^2 + AB\cos(x)\sin(x) + A^2 + B^2\right.$$
$$\left. + \sqrt{-B^2\cos(x)^2 + A^2 + B^2}(A\cos(x) + B\sin(x))\right) + \frac{1}{2}A\log\left(-B^2\cos(x)^2\right.$$
$$\left. - AB\cos(x)\sin(x) + A^2 + B^2 - \sqrt{-B^2\cos(x)^2 + A^2 + B^2}(A\cos(x) - B\sin(x))\right)$$

```
[In] integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="fricas")
```

```
[Out] 1/2*B*arctan(-((A^4 + 2*A^2*B^2 + B^4)*cos(x)*sin(x) - 2*(2*B^3*cos(x)^3 -
(A^2*B + B^3)*cos(x))*sqrt(-B^2*cos(x)^2 + A^2 + B^2))/(4*B^4*cos(x)^4 + A^
4 + 2*A^2*B^2 + B^4 - (A^4 + 6*A^2*B^2 + 5*B^4)*cos(x)^2)) - 1/2*B*arctan(s
in(x)/cos(x)) - 1/2*A*log(-B^2*cos(x)^2 + A*B*cos(x)*sin(x) + A^2 + B^2 + s
qrt(-B^2*cos(x)^2 + A^2 + B^2)*(A*cos(x) + B*sin(x))) + 1/2*A*log(-B^2*cos(
x)^2 - A*B*cos(x)*sin(x) + A^2 + B^2 - sqrt(-B^2*cos(x)^2 + A^2 + B^2)*(A*c
os(x) - B*sin(x)))
```

Sympy [F]

$$\int \csc(x)\sqrt{A^2 + B^2\sin^2(x)}\,dx = \int \frac{\sqrt{A^2 + B^2\sin^2(x)}}{\sin(x)}\,dx$$

```
[In] integrate((A**2+B**2*sin(x)**2)**(1/2)/sin(x),x)
```

```
[Out] Integral(sqrt(A**2 + B**2*sin(x)**2)/sin(x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.37

$$\int \csc(x)\sqrt{A^2 + B^2\sin^2(x)}\,dx = -B\arcsin\left(\frac{B^2\cos(x)}{\sqrt{A^2B^2 + B^4}}\right) - \frac{1}{2}A\log\left(B^2 - \frac{A^2}{\cos(x) - 1} - \frac{\sqrt{-B^2\cos(x)^2 + A^2 + B^2}A}{\cos(x) - 1}\right) + \frac{1}{2}A\log\left(-B^2 + \frac{A^2}{\cos(x) + 1} + \frac{\sqrt{-B^2\cos(x)^2 + A^2 + B^2}A}{\cos(x) + 1}\right)$$

```
[In] integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="maxima")
[Out] -B*arcsin(B^2*cos(x)/sqrt(A^2*B^2 + B^4)) - 1/2*A*log(B^2 - A^2/(cos(x) - 1
) - sqrt(-B^2*cos(x)^2 + A^2 + B^2)*A/(cos(x) - 1)) + 1/2*A*log(-B^2 + A^2/
(cos(x) + 1) + sqrt(-B^2*cos(x)^2 + A^2 + B^2)*A/(cos(x) + 1))
```

Giac [F]

$$\int \csc(x)\sqrt{A^2 + B^2\sin^2(x)}\,dx = \int \frac{\sqrt{B^2\sin(x)^2 + A^2}}{\sin(x)}\,dx$$

```
[In] integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="giac")
[Out] integrate(sqrt(B^2*sin(x)^2 + A^2)/sin(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(x)\sqrt{A^2 + B^2\sin^2(x)}\,dx = \int \frac{\sqrt{A^2 + B^2\sin(x)^2}}{\sin(x)}\,dx$$

```
[In] int((B^2*sin(x)^2 + A^2)^(1/2)/sin(x),x)
[Out] int((B^2*sin(x)^2 + A^2)^(1/2)/sin(x), x)
```

3.43 $\int \frac{1}{1+\cos(x)}\,dx$

Optimal result 214
Rubi [A] (verified) 214
Mathematica [A] (verified) 215
Maple [A] (verified) 215
Fricas [A] (verification not implemented) 215
Sympy [A] (verification not implemented) 216
Maxima [A] (verification not implemented) 216
Giac [B] (verification not implemented) 216
Mupad [B] (verification not implemented) 216

Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{1+\cos(x)}$$

```
[Out] sin(x)/(1+cos(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2727}

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{\cos(x)+1}$$

```
[In] Int[(1 + Cos[x])^(-1),x]
```

```
[Out] Sin[x]/(1 + Cos[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\sin(x)}{1+\cos(x)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

```
[In] Integrate[(1 + Cos[x])^(-1),x]
[Out] Tan[x/2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

method	result	size
default	$\tan\left(\frac{x}{2}\right)$	5
norman	$\tan\left(\frac{x}{2}\right)$	5
parallelrisch	$\tan\left(\frac{x}{2}\right)$	5
risch	$\frac{2i}{e^{ix}+1}$	13

```
[In] int(1/(cos(x)+1),x,method=_RETURNVERBOSE)
[Out] tan(1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{\cos(x)+1}$$

```
[In] integrate(1/(1+cos(x)),x, algorithm="fricas")
[Out] sin(x)/(cos(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.33

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

```
[In] integrate(1/(1+cos(x)),x)
[Out] tan(x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{\cos(x)+1}$$

```
[In] integrate(1/(1+cos(x)),x, algorithm="maxima")
[Out] sin(x)/(cos(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(9) = 18$.

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 3.33

$$\int \frac{1}{1+\cos(x)}\,dx = -\frac{2\tan\left(\frac{1}{2}x\right)}{(x^2+1)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

```
[In] integrate(1/(1+cos(x)),x, algorithm="giac")
[Out] -2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

```
[In] int(1/(cos(x) + 1),x)
[Out] tan(x/2)
```

3.44 $\int e^x x \, dx$

Optimal result 217
Rubi [A] (verified) 217
Mathematica [A] (verified) 218
Maple [A] (verified) 218
Fricas [A] (verification not implemented) 219
Sympy [A] (verification not implemented) 219
Maxima [A] (verification not implemented) 219
Giac [A] (verification not implemented) 219
Mupad [B] (verification not implemented) 220

Optimal result

Integrand size = 5, antiderivative size = 11

$$\int e^x x \, dx = -e^x + e^x x$$

```
[Out] -exp(x)+exp(x)*x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2207, 2225}

$$\int e^x x \, dx = e^x x - e^x$$

```
[In] Int[E^x*x,x]
```

```
[Out] -E^x + E^x*x
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= e^x x - \int e^x\,dx\\ &= -e^x + e^x x\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int e^x x\,dx = e^x(-1+x)$$

```
[In] Integrate[E^x*x,x]
[Out] E^x*(-1 + x)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

method	result	size
gosper	$(-1+x)\,\mathrm{e}^{x}$	7
risch	$(-1+x)\,\mathrm{e}^{x}$	7
default	$-\mathrm{e}^{x}+\mathrm{e}^{x}x$	10
norman	$-\mathrm{e}^{x}+\mathrm{e}^{x}x$	10
parallelrisch	$-\mathrm{e}^{x}+\mathrm{e}^{x}x$	10
parts	$-\mathrm{e}^{x}+\mathrm{e}^{x}x$	10
meijerg	$1-\frac{(-2x+2)\mathrm{e}^{x}}{2}$	12

```
[In] int(exp(x)*x,x,method=_RETURNVERBOSE)
[Out] (-1+x)*exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int e^x x\,dx = (x-1)e^x$$

```
[In] integrate(exp(x)*x,x, algorithm="fricas")
[Out] (x - 1)*e^x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.45

$$\int e^x x\,dx = (x-1)\,e^x$$

```
[In] integrate(exp(x)*x,x)
[Out] (x - 1)*exp(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int e^x x\,dx = (x-1)e^x$$

```
[In] integrate(exp(x)*x,x, algorithm="maxima")
[Out] (x - 1)*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int e^x x\,dx = (x-1)e^x$$

```
[In] integrate(exp(x)*x,x, algorithm="giac")
[Out] (x - 1)*e^x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int e^x x\,dx = \mathrm{e}^x\,(x-1)$$

```
[In] int(x*exp(x),x)
[Out] exp(x)*(x - 1)
```

3.45 $\int \frac{e^x x}{(1+x)^2} dx$

Optimal result 221
Rubi [A] (verified) 221
Mathematica [A] (verified) 222
Maple [A] (verified) 222
Fricas [A] (verification not implemented) 222
Sympy [A] (verification not implemented) 223
Maxima [A] (verification not implemented) 223
Giac [B] (verification not implemented) 223
Mupad [B] (verification not implemented) 223

Optimal result

Integrand size = 10, antiderivative size = 9

$$\int \frac{e^x x}{(1+x)^2} dx = \frac{e^x}{1+x}$$

```
[Out] exp(x)/(1+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2228}

$$\int \frac{e^x x}{(1+x)^2} dx = \frac{e^x}{x+1}$$

```
[In] Int[(E^x*x)/(1 + x)^2,x]
```

```
[Out] E^x/(1 + x)
```

Rule 2228

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> With[{b = Coefficient[v
, x, 1], d = Coefficient[u, x, 0], e = Coefficient[u, x, 1], f = Coefficien
t[w, x, 0], g = Coefficient[w, x, 1]}, Simp[g*u^(m + 1)*(F^(c*v)/(b*c*e*Log
[F])), x] /; EqQ[e*g*(m + 1) - b*c*(e*f - d*g)*Log[F], 0]] /; FreeQ[{F, c,
m}, x] && LinearQ[{u, v, w}, x]
```

Rubi steps

$$\text{integral} = \frac{e^x}{1+x}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{e^x x}{(1+x)^2} \, dx = \frac{e^x}{1+x}$$

[In] Integrate[(E^x*x)/(1 + x)^2,x]

[Out] E^x/(1 + x)

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{e^x}{1+x}$	9
default	$\frac{e^x}{1+x}$	9
norman	$\frac{e^x}{1+x}$	9
risch	$\frac{e^x}{1+x}$	9
parallelrisch	$\frac{e^x}{1+x}$	9

[In] int(exp(x)*x/(1+x)^2,x,method=_RETURNVERBOSE)

[Out] exp(x)/(1+x)

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{e^x x}{(1+x)^2} \, dx = \frac{e^x}{x+1}$$

[In] integrate(exp(x)*x/(1+x)^2,x, algorithm="fricas")

[Out] e^x/(x + 1)

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int \frac{e^x x}{(1+x)^2}\, dx = \frac{e^x}{x+1}$$

```
[In] integrate(exp(x)*x/(1+x)**2,x)
[Out] exp(x)/(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{e^x x}{(1+x)^2}\, dx = \frac{e^x}{x+1}$$

```
[In] integrate(exp(x)*x/(1+x)^2,x, algorithm="maxima")
[Out] e^x/(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(8) = 16$.

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 3.33

$$\int \frac{e^x x}{(1+x)^2}\, dx = -\frac{e^{\left(-(x+1)\left(\frac{1}{x+1}-1\right)\right)}}{(x+1)\left(\frac{1}{x+1}-1\right)-1}$$

```
[In] integrate(exp(x)*x/(1+x)^2,x, algorithm="giac")
[Out] -e^(-(x + 1)*(1/(x + 1) - 1))/((x + 1)*(1/(x + 1) - 1) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{e^x x}{(1+x)^2}\, dx = \frac{\mathrm{e}^x}{x+1}$$

```
[In] int((x*exp(x))/(x + 1)^2,x)
[Out] exp(x)/(x + 1)
```

3.46 $\int e^{x^2}(1+2x^2)\,dx$

Optimal result 224
Rubi [A] (verified) 224
Mathematica [A] (verified) 225
Maple [A] (verified) 225
Fricas [A] (verification not implemented) 226
Sympy [A] (verification not implemented) 226
Maxima [A] (verification not implemented) 227
Giac [A] (verification not implemented) 227
Mupad [B] (verification not implemented) 227

Optimal result

Integrand size = 13, antiderivative size = 7

$$\int e^{x^2}\left(1+2x^2\right)\,dx=e^{x^2}x$$

```
[Out] exp(x^2)*x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2258, 2235, 2243}

$$\int e^{x^2}\left(1+2x^2\right)\,dx=e^{x^2}x$$

```
[In] Int[E^x^2*(1 + 2*x^2),x]
```

```
[Out] E^x^2*x
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
```

```
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2258

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(e^{x^2} + 2e^{x^2}x^2\right) dx \\
&= 2\int e^{x^2}x^2\,dx + \int e^{x^2}\,dx \\
&= e^{x^2}x + \frac{1}{2}\sqrt{\pi}\,\text{erfi}(x) - \int e^{x^2}\,dx \\
&= e^{x^2}x
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int e^{x^2}\left(1+2x^2\right)\,dx = e^{x^2}x$$

```
[In] Integrate[E^x^2*(1 + 2*x^2),x]
[Out] E^x^2*x
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

method	result	size
gosper	$e^{x^2}x$	7
default	$e^{x^2}x$	7
norman	$e^{x^2}x$	7
risch	$e^{x^2}x$	7
parallelrisch	$e^{x^2}x$	7
meijerg	$i\left(-ix\,e^{x^2}+\frac{i\,\mathrm{erfi}(x)\sqrt{\pi}}{2}\right)+\frac{\mathrm{erfi}(x)\sqrt{\pi}}{2}$	29
parts	$\mathrm{erfi}\left(x\right)\sqrt{\pi}\,x^{2}+\frac{\mathrm{erfi}(x)\sqrt{\pi}}{2}-2\sqrt{\pi}\left(\frac{x^{2}\,\mathrm{erfi}(x)}{2}-\frac{\frac{e^{x^2}x}{2}-\frac{\mathrm{erfi}(x)\sqrt{\pi}}{4}}{\sqrt{\pi}}\right)$	51

```
[In] int(exp(x^2)*(2*x^2+1),x,method=_RETURNVERBOSE)
[Out] exp(x^2)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int e^{x^2}\left(1+2x^2\right)\,dx=xe^{(x^2)}$$

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="fricas")
[Out] x*e^(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int e^{x^2}\left(1+2x^2\right)\,dx=xe^{x^2}$$

```
[In] integrate(exp(x**2)*(2*x**2+1),x)
[Out] x*exp(x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int e^{x^2}\left(1+2x^2\right)\,dx=xe^{(x^2)}$$

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="maxima")
[Out] x*e^(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int e^{x^2}\left(1+2x^2\right)\,dx=xe^{(x^2)}$$

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="giac")
[Out] x*e^(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int e^{x^2}\left(1+2x^2\right)\,dx=x\,\mathrm{e}^{x^2}$$

```
[In] int(exp(x^2)*(2*x^2 + 1),x)
[Out] x*exp(x^2)
```

3.47 $\int e^{x^2} dx$

Optimal result 228
Rubi [A] (verified) 228
Mathematica [A] (verified) 229
Maple [A] (verified) 229
Fricas [A] (verification not implemented) 229
Sympy [A] (verification not implemented) 230
Maxima [C] (verification not implemented) 230
Giac [C] (verification not implemented) 230
Mupad [B] (verification not implemented) 230

Optimal result

Integrand size = 5, antiderivative size = 11

$$\int e^{x^2} dx = \frac{1}{2}\sqrt{\pi}\text{erfi}(x)$$

```
[Out] 1/2*erfi(x)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{2235\}$

$$\int e^{x^2} dx = \frac{1}{2}\sqrt{\pi}\text{erfi}(x)$$

```
[In] Int[E^x^2,x]
```

```
[Out] (Sqrt[Pi]*Erfi[x])/2
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\sqrt{\pi}\text{erfi}(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{x^2}\,dx=\frac{1}{2}\sqrt{\pi}\text{erfi}(x)$$

```
[In] Integrate[E^x^2,x]
[Out] (Sqrt[Pi]*Erfi[x])/2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\text{erfi}(x)\sqrt{\pi}}{2}$	8
meijerg	$\frac{\text{erfi}(x)\sqrt{\pi}}{2}$	8
risch	$\frac{\text{erfi}(x)\sqrt{\pi}}{2}$	8

```
[In] int(exp(x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*erfi(x)*Pi^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int e^{x^2}\,dx=\frac{1}{2}\,\sqrt{\pi}\,\text{erfi}\left(x\right)$$

```
[In] integrate(exp(x^2),x, algorithm="fricas")
[Out] 1/2*sqrt(pi)*erfi(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{x^2}\,dx = \frac{\sqrt{\pi}\,\mathrm{erfi}\,(x)}{2}$$

```
[In] integrate(exp(x**2),x)
[Out] sqrt(pi)*erfi(x)/2
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int e^{x^2}\,dx = -\frac{1}{2}i\,\sqrt{\pi}\,\mathrm{erf}\,(i\,x)$$

```
[In] integrate(exp(x^2),x, algorithm="maxima")
[Out] -1/2*I*sqrt(pi)*erf(I*x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int e^{x^2}\,dx = \frac{1}{2}i\,\sqrt{\pi}\,\mathrm{erf}\,(-i\,x)$$

```
[In] integrate(exp(x^2),x, algorithm="giac")
[Out] 1/2*I*sqrt(pi)*erf(-I*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int e^{x^2}\,dx = \frac{\sqrt{\pi}\,\mathrm{erfi}(x)}{2}$$

```
[In] int(exp(x^2),x)
[Out] (pi^(1/2)*erfi(x))/2
```

3.48 $\int \frac{e^x}{x} dx$

Optimal result 231
Rubi [A] (verified) 231
Mathematica [A] (verified) 232
Maple [B] (verified) 232
Fricas [A] (verification not implemented) 232
Sympy [A] (verification not implemented) 233
Maxima [A] (verification not implemented) 233
Giac [A] (verification not implemented) 233
Mupad [B] (verification not implemented) 233

Optimal result

Integrand size = 7, antiderivative size = 2

$$\int \frac{e^x}{x} dx = \text{ExpIntegralEi}(x)$$

```
[Out] Ei(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2209}

$$\int \frac{e^x}{x} dx = \text{ExpIntegralEi}(x)$$

```
[In] Int[E^x/x,x]
```

```
[Out] ExpIntegralEi[x]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$\text{integral} = \text{ExpIntegralEi}(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{x}\, dx = \text{ExpIntegralEi}(x)$$

```
[In] Integrate[E^x/x,x]
[Out] ExpIntegralEi[x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(2) = 4.

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 4.00

method	result	size
default	$-\operatorname{Ei}_1(-x)$	8
risch	$-\operatorname{Ei}_1(-x)$	8
meijerg	$\ln(x) + i\pi - \ln(-x) - \operatorname{Ei}_1(-x)$	21

```
[In] int(exp(x)/x,x,method=_RETURNVERBOSE)
[Out] -Ei(1,-x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{x}\, dx = \text{Ei}(x)$$

```
[In] integrate(exp(x)/x,x, algorithm="fricas")
[Out] Ei(x)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{x}\,dx = \mathrm{Ei}\,(x)$$

```
[In] integrate(exp(x)/x,x)
[Out] Ei(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{x}\,dx = \mathrm{Ei}(x)$$

```
[In] integrate(exp(x)/x,x, algorithm="maxima")
[Out] Ei(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{x}\,dx = \mathrm{Ei}(x)$$

```
[In] integrate(exp(x)/x,x, algorithm="giac")
[Out] Ei(x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{x}\,dx = \mathrm{ei}(x)$$

```
[In] int(exp(x)/x,x)
[Out] ei(x)
```

3.49 $\int \frac{x}{1+x^3} \, dx$

Optimal result 234
Rubi [A] (verified) 234
Mathematica [A] (verified) 236
Maple [A] (verified) 236
Fricas [A] (verification not implemented) 236
Sympy [A] (verification not implemented) 237
Maxima [A] (verification not implemented) 237
Giac [A] (verification not implemented) 237
Mupad [B] (verification not implemented) 238

Optimal result

Integrand size = 9, antiderivative size = 41

$$\int \frac{x}{1+x^3} \, dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right)$$

```
[Out] -1/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {298, 31, 648, 632, 210, 642}

$$\int \frac{x}{1+x^3} \, dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log(x+1)$$

```
[In] Int[x/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{3}\int \frac{1}{1+x}\,dx\right) + \frac{1}{3}\int \frac{1+x}{1-x+x^2}\,dx \\
&= -\frac{1}{3}\log(1+x) + \frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{2}\int \frac{1}{1-x+x^2}\,dx \\
&= -\frac{1}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) \\
&= \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{x}{1+x^3}\,dx = \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right)$$

```
[In] Integrate[x/(1 + x^3),x]
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	33
default	$\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{\ln(1+x)}{3}$	35
meijerg	$-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}+\frac{x^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{2}{3}}}$	80

```
[In] int(x/(x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log\left(x+1\right)$$

```
[In] integrate(x/(x^3+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x^3}\,dx = -\frac{\log(x+1)}{3} + \frac{\log(x^2-x+1)}{6} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate(x/(x**3+1),x)
```

```
[Out] -log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/
3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log(x+1)$$

```
[In] integrate(x/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log(|x+1|)$$

```
[In] integrate(x/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{x}{1+x^3}\,dx = -\frac{\ln(x+1)}{3} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6} + \frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6} + \frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

```
[In] int(x/(x^3 + 1),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6) - log(x - (3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x + 1)/3
```

3.50 $\int \frac{1}{-1+x^6}\,dx$

Optimal result 239
Rubi [A] (verified) 239
Mathematica [A] (verified) 241
Maple [A] (verified) 241
Fricas [A] (verification not implemented) 242
Sympy [B] (verification not implemented) 242
Maxima [A] (verification not implemented) 242
Giac [A] (verification not implemented) 243
Mupad [B] (verification not implemented) 243

Optimal result

Integrand size = 7, antiderivative size = 47

$$\int \frac{1}{-1+x^6}\,dx=-\frac{\arctan\left(\frac{\sqrt{3}x}{1-x^2}\right)}{2\sqrt{3}}-\frac{\operatorname{arctanh}(x)}{3}-\frac{1}{6}\operatorname{arctanh}\left(\frac{x}{1+x^2}\right)$$

```
[Out] -1/3*arctanh(x)-1/6*arctanh(x/(x^2+1))-1/6*arctan(x*3^(1/2)/(-x^2+1))*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {216, 648, 632, 210, 642, 212}

$$\int \frac{1}{-1+x^6}\,dx=\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\operatorname{arctanh}(x)}{3}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{12}\log\left(x^2+x+1\right)$$

```
[In] Int[(-1 + x^6)^(-1),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[3
]) - ArcTanh[x]/3 + Log[1 - x + x^2]/12 - Log[1 + x + x^2]/12
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx\right)-\frac{1}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx-\frac{1}{3}\int \frac{1}{1-x^2}\,dx\\
&= -\frac{\operatorname{arctanh}(x)}{3}+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{12}\int \frac{1+2x}{1+x+x^2}\,dx\\
&\quad -\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx\\
&= -\frac{\operatorname{arctanh}(x)}{3}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{12}\log\left(1+x+x^2\right)\\
&\quad +\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)
\end{aligned}$$

$$= \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\operatorname{arctanh}(x)}{3} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{1}{12}\log\left(1+x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int \frac{1}{-1+x^6}\,dx = \frac{1}{12}\left(-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) + 2\log(1-x) - 2\log(1+x) + \log\left(1-x+x^2\right) - \log\left(1+x+x^2\right)\right)$$

```
[In] Integrate[(-1 + x^6)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]
] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])/12
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

method	result
default	$\frac{\ln(-1+x)}{6}-\frac{\ln(x^2+x+1)}{12}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(1+x)}{6}$
risch	$\frac{\ln(4x^2-4x+4)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(1+x)}{6}-\frac{\ln(x^2+x+1)}{12}-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{6}+\frac{\ln(-1+x)}{6}$
meijerg	$\frac{x\left(\ln\left(1-(x^6)^{\frac{1}{6}}\right)-\ln\left(1+(x^6)^{\frac{1}{6}}\right)+\frac{\ln\left(1-(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{2-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6}{2+(x^6)}\right.\right.}{6(x^6)^{\frac{1}{6}}}$

```
[In] int(1/(x^6-1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*ln(-1+x)-1/12*ln(x^2+x+1)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/12*
ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/6*ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{1}{-1+x^6}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{1}{12}\log\left(x^2+x+1\right) + \frac{1}{12}\log\left(x^2-x+1\right) - \frac{1}{6}\log\left(x+1\right) + \frac{1}{6}\log\left(x-1\right)$$

```
[In] integrate(1/(x^6-1),x, algorithm="fricas")
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1
) + 1/6*log(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(36) = 72.

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.77

$$\int \frac{1}{-1+x^6}\,dx = \frac{\log\left(x-1\right)}{6} - \frac{\log\left(x+1\right)}{6} + \frac{\log\left(x^2-x+1\right)}{12} - \frac{\log\left(x^2+x+1\right)}{12} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

```
[In] integrate(1/(x**6-1),x)
[Out] log(x - 1)/6 - log(x + 1)/6 + log(x**2 - x + 1)/12 - log(x**2 + x + 1)/12 -
 sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + s
qrt(3)/3)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{1}{-1+x^6}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{1}{12}\log\left(x^2+x+1\right) + \frac{1}{12}\log\left(x^2-x+1\right) - \frac{1}{6}\log\left(x+1\right) + \frac{1}{6}\log\left(x-1\right)$$

```
[In] integrate(1/(x^6-1),x, algorithm="maxima")
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1
) + 1/6*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{1}{-1+x^6}\,dx = -\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{12}\,\log\left(x^2+x+1\right) + \frac{1}{12}\,\log\left(x^2-x+1\right) - \frac{1}{6}\,\log\left(|x+1|\right) + \frac{1}{6}\,\log\left(|x-1|\right)$$

```
[In] integrate(1/(x^6-1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x
 + 1)) + 1/6*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.87

$$\int \frac{1}{-1+x^6}\,dx = -\frac{\mathrm{atanh}(x)}{3} - \mathrm{atan}\left(\frac{x\,1\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}} + \frac{\sqrt{3}\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{6}\mathrm{i}\right) - \mathrm{atan}\left(\frac{x\,1\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}} - \frac{\sqrt{3}\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{6}\mathrm{i}\right)$$

```
[In] int(1/(x^6 - 1),x)
```

```
[Out] - atanh(x)/3 - atan((x*1i)/(3^(1/2)*1i + 1) + (3^(1/2)*x)/(3^(1/2)*1i + 1))
*(3^(1/2)/6 + 1i/6) - atan((x*1i)/(3^(1/2)*1i - 1) - (3^(1/2)*x)/(3^(1/2)*1
i - 1))*(3^(1/2)/6 - 1i/6)
```

3.51 $\int \frac{1}{A^4-A^2B^2+(-A^2+B^2)x^2}\,dx$

Optimal result 244
Rubi [A] (verified) 244
Mathematica [A] (verified) 245
Maple [A] (verified) 245
Fricas [A] (verification not implemented) 245
Sympy [B] (verification not implemented) 246
Maxima [A] (verification not implemented) 246
Giac [A] (verification not implemented) 246
Mupad [B] (verification not implemented) 247

Optimal result

Integrand size = 27, antiderivative size = 21

$$\int \frac{1}{A^4-A^2B^2+(-A^2+B^2)\,x^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{x}{A}\right)}{A\,(A^2-B^2)}$$

```
[Out] arctanh(x/A)/A/(A^2-B^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {214}

$$\int \frac{1}{A^4-A^2B^2+(-A^2+B^2)\,x^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{x}{A}\right)}{A\,(A^2-B^2)}$$

```
[In] Int[(A^4 - A^2*B^2 + (-A^2 + B^2)*x^2)^(-1),x]
```

```
[Out] ArcTanh[x/A]/(A*(A^2 - B^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\text{integral}=\frac{\operatorname{arctanh}\left(\frac{x}{A}\right)}{A\,(A^2-B^2)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{A^4 - A^2B^2 + (-A^2 + B^2)\,x^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{x}{A}\right)}{A\,(A^2 - B^2)}$$

```
[In] Integrate[(A^4 - A^2*B^2 + (-A^2 + B^2)*x^2)^(-1),x]
[Out] ArcTanh[x/A]/(A*(A^2 - B^2))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

method	result	size
parallelrisch	$\frac{-\ln(A-x)+\ln(A+x)}{2A(A^2-B^2)}$	30
default	$\frac{-\frac{\ln(A-x)}{2A}+\frac{\ln(A+x)}{2A}}{A^2-B^2}$	34
norman	$-\frac{\ln(A-x)}{2A(A^2-B^2)}+\frac{\ln(A+x)}{2A(A^2-B^2)}$	44
risch	$-\frac{\ln(A-x)}{2A(A^2-B^2)}+\frac{\ln(A+x)}{2A(A^2-B^2)}$	44

```
[In] int(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*(-ln(A-x)+ln(A+x))/A/(A^2-B^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{1}{A^4 - A^2B^2 + (-A^2 + B^2)\,x^2}\,dx = \frac{\log(A+x) - \log(-A+x)}{2\,(A^3 - AB^2)}$$

```
[In] integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="fricas")
[Out] 1/2*(log(A + x) - log(-A + x))/(A^3 - A*B^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(12) = 24$.

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.33

$$\int \frac{1}{A^4 - A^2B^2 + (-A^2 + B^2)\,x^2}\,dx = -\frac{\log\left(-\frac{A^3}{(A-B)(A+B)} + \frac{AB^2}{(A-B)(A+B)} + x\right)}{2A\,(A-B)\,(A+B)} + \frac{\log\left(\frac{A^3}{(A-B)(A+B)} - \frac{AB^2}{(A-B)(A+B)} + x\right)}{2A\,(A-B)\,(A+B)}$$

```
[In] integrate(1/(A**4-A**2*B**2+(-A**2+B**2)*x**2),x)
```

```
[Out] -log(-A**3/((A - B)*(A + B)) + A*B**2/((A - B)*(A + B)) + x)/(2*A*(A - B)*(
A + B)) + log(A**3/((A - B)*(A + B)) - A*B**2/((A - B)*(A + B)) + x)/(2*A*(
A - B)*(A + B))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{1}{A^4 - A^2B^2 + (-A^2 + B^2)\,x^2}\,dx = \frac{\log\left(A + x\right)}{2\left(A^3 - AB^2\right)} - \frac{\log\left(-A + x\right)}{2\left(A^3 - AB^2\right)}$$

```
[In] integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(A + x)/(A^3 - A*B^2) - 1/2*log(-A + x)/(A^3 - A*B^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \frac{1}{A^4 - A^2B^2 + (-A^2 + B^2)\,x^2}\,dx = \frac{\log\left(|A + x|\right)}{2\left(A^3 - AB^2\right)} - \frac{\log\left(|-A + x|\right)}{2\left(A^3 - AB^2\right)}$$

```
[In] integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(A + x))/(A^3 - A*B^2) - 1/2*log(abs(-A + x))/(A^3 - A*B^2)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{A^4 - A^2B^2 + (-A^2 + B^2)\,x^2}\,dx = -\frac{\operatorname{atanh}\left(\frac{x}{A}\right)}{A\,B^2 - A^3}$$

```
[In] int(-1/(x^2*(A^2 - B^2) - A^4 + A^2*B^2),x)
[Out] -atanh(x/A)/(A*B^2 - A^3)
```

3.52 $\int x \log(x)\, dx$

Optimal result 248
Rubi [A] (verified) 248
Mathematica [A] (verified) 249
Maple [A] (verified) 249
Fricas [A] (verification not implemented) 249
Sympy [A] (verification not implemented) 250
Maxima [A] (verification not implemented) 250
Giac [A] (verification not implemented) 250
Mupad [B] (verification not implemented) 250

Optimal result

Integrand size = 4, antiderivative size = 17

$$\int x \log(x)\, dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2341}

$$\int x \log(x)\, dx = \frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

```
[In] Int[x*Log[x],x]
```

```
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x \log(x)\,dx = -\frac{x^2}{4} + \frac{1}{2}x^2\log(x)$$

```
[In] Integrate[x*Log[x],x]
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
norman	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
risch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
parallelrisch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
parts	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14

```
[In] int(x*ln(x),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x \log(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right) - \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(x),x, algorithm="fricas")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x\log(x)\,dx = \frac{x^2\log{(x)}}{2} - \frac{x^2}{4}$$

```
[In] integrate(x*ln(x),x)
[Out] x**2*log(x)/2 - x**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log{(x)} - \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(x),x, algorithm="maxima")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log{(x)} - \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(x),x, algorithm="giac")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int x\log(x)\,dx = \frac{x^2\,\left(\ln{(x)} - \frac{1}{2}\right)}{2}$$

```
[In] int(x*log(x),x)
[Out] (x^2*(log(x) - 1/2))/2
```

3.53 $\int x^2 \arcsin(x)\,dx$

Optimal result 251
Rubi [A] (verified) 251
Mathematica [A] (verified) 252
Maple [A] (verified) 252
Fricas [A] (verification not implemented) 253
Sympy [A] (verification not implemented) 253
Maxima [A] (verification not implemented) 253
Giac [A] (verification not implemented) 253
Mupad [B] (verification not implemented) 254

Optimal result

Integrand size = 6, antiderivative size = 40

$$\int x^2 \arcsin(x)\,dx = \frac{\sqrt{1-x^2}}{3} - \frac{1}{9}\left(1-x^2\right)^{3/2} + \frac{1}{3}x^3 \arcsin(x)$$

```
[Out] -1/9*(-x^2+1)^(3/2)+1/3*x^3*arcsin(x)+1/3*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4723, 272, 45}

$$\int x^2 \arcsin(x)\,dx = \frac{1}{3}x^3 \arcsin(x) - \frac{1}{9}\left(1-x^2\right)^{3/2} + \frac{\sqrt{1-x^2}}{3}$$

```
[In] Int[x^2*ArcSin[x],x]
```

```
[Out] Sqrt[1 - x^2]/3 - (1 - x^2)^(3/2)/9 + (x^3*ArcSin[x])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arcsin(x) - \frac{1}{3}\int \frac{x^3}{\sqrt{1-x^2}}\,dx \\
&= \frac{1}{3}x^3\arcsin(x) - \frac{1}{6}\text{Subst}\left(\int \frac{x}{\sqrt{1-x}}\,dx, x, x^2\right) \\
&= \frac{1}{3}x^3\arcsin(x) - \frac{1}{6}\text{Subst}\left(\int \left(\frac{1}{\sqrt{1-x}} - \sqrt{1-x}\right)\,dx, x, x^2\right) \\
&= \frac{\sqrt{1-x^2}}{3} - \frac{1}{9}\left(1-x^2\right)^{3/2} + \frac{1}{3}x^3\arcsin(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int x^2\arcsin(x)\,dx = \frac{1}{9}\left(\sqrt{1-x^2}\left(2+x^2\right) + 3x^3\arcsin(x)\right)$$

```
[In] Integrate[x^2*ArcSin[x],x]
[Out] (Sqrt[1 - x^2]*(2 + x^2) + 3*x^3*ArcSin[x])/9
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{x^3\arcsin(x)}{3} + \frac{x^2\sqrt{-x^2+1}}{9} + \frac{2\sqrt{-x^2+1}}{9}$	34
parts	$\frac{x^3\arcsin(x)}{3} + \frac{x^2\sqrt{-x^2+1}}{9} + \frac{2\sqrt{-x^2+1}}{9}$	34

```
[In] int(x^2*arcsin(x),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*arcsin(x)+1/9*x^2*(-x^2+1)^(1/2)+2/9*(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int x^2 \arcsin(x)\,dx = \frac{1}{3}\,x^3 \arcsin\left(x\right) + \frac{1}{9}\,\left(x^2 + 2\right)\sqrt{-x^2 + 1}$$

```
[In] integrate(x^2*arcsin(x),x, algorithm="fricas")
[Out] 1/3*x^3*arcsin(x) + 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int x^2 \arcsin(x)\,dx = \frac{x^3 \operatorname{asin}\left(x\right)}{3} + \frac{x^2\sqrt{1 - x^2}}{9} + \frac{2\sqrt{1 - x^2}}{9}$$

```
[In] integrate(x**2*asin(x),x)
[Out] x**3*asin(x)/3 + x**2*sqrt(1 - x**2)/9 + 2*sqrt(1 - x**2)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int x^2 \arcsin(x)\,dx = \frac{1}{3}\,x^3 \arcsin\left(x\right) + \frac{1}{9}\,\sqrt{-x^2 + 1}x^2 + \frac{2}{9}\,\sqrt{-x^2 + 1}$$

```
[In] integrate(x^2*arcsin(x),x, algorithm="maxima")
[Out] 1/3*x^3*arcsin(x) + 1/9*sqrt(-x^2 + 1)*x^2 + 2/9*sqrt(-x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int x^2 \arcsin(x)\,dx = \frac{1}{3}\,\left(x^2 - 1\right)x \arcsin\left(x\right) + \frac{1}{3}\,x \arcsin\left(x\right) - \frac{1}{9}\,\left(-x^2 + 1\right)^{\frac{3}{2}} + \frac{1}{3}\,\sqrt{-x^2 + 1}$$

```
[In] integrate(x^2*arcsin(x),x, algorithm="giac")
[Out] 1/3*(x^2 - 1)*x*arcsin(x) + 1/3*x*arcsin(x) - 1/9*(-x^2 + 1)^(3/2) + 1/3*sq
rt(-x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int x^2 \arcsin(x)\,dx = \frac{x^3\,\mathrm{asin}(x)}{3} + \frac{\sqrt{1-x^2}\,(x^2+2)}{9}$$

```
[In] int(x^2*asin(x),x)
[Out] (x^3*asin(x))/3 + ((1 - x^2)^(1/2)*(x^2 + 2))/9
```

3.54 $\int \frac{1}{1+2x+x^2}\,dx$

Optimal result 255
Rubi [A] (verified) 255
Mathematica [A] (verified) 256
Maple [A] (verified) 256
Fricas [A] (verification not implemented) 257
Sympy [A] (verification not implemented) 257
Maxima [A] (verification not implemented) 257
Giac [A] (verification not implemented) 257
Mupad [B] (verification not implemented) 258

Optimal result

Integrand size = 10, antiderivative size = 7

$$\int \frac{1}{1+2x+x^2}\,dx=-\frac{1}{1+x}$$

```
[Out] -1/(1+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {27, 32}

$$\int \frac{1}{1+2x+x^2}\,dx=-\frac{1}{x+1}$$

```
[In] Int[(1 + 2*x + x^2)^(-1),x]
```

```
[Out] -(1 + x)^(-1)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{(1+x)^2} \, dx \\ &= -\frac{1}{1+x} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+2x+x^2} \, dx = -\frac{1}{1+x}$$

```
[In] Integrate[(1 + 2*x + x^2)^(-1),x]
[Out] -(1 + x)^(-1)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
gosper	$-\frac{1}{1+x}$	8
default	$-\frac{1}{1+x}$	8
norman	$-\frac{1}{1+x}$	8
meijerg	$\frac{x}{1+x}$	8
risch	$-\frac{1}{1+x}$	8
parallelrisch	$-\frac{1}{1+x}$	8

```
[In] int(1/(x^2+2*x+1),x,method=_RETURNVERBOSE)
[Out] -1/(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+2x+x^2}\,dx = -\frac{1}{x+1}$$

```
[In] integrate(1/(x^2+2*x+1),x, algorithm="fricas")
[Out] -1/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{1+2x+x^2}\,dx = -\frac{1}{x+1}$$

```
[In] integrate(1/(x**2+2*x+1),x)
[Out] -1/(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+2x+x^2}\,dx = -\frac{1}{x+1}$$

```
[In] integrate(1/(x^2+2*x+1),x, algorithm="maxima")
[Out] -1/(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+2x+x^2}\,dx = -\frac{1}{x+1}$$

```
[In] integrate(1/(x^2+2*x+1),x, algorithm="giac")
[Out] -1/(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+2x+x^2}\,dx=-\frac{1}{x+1}$$

```
[In] int(1/(2*x + x^2 + 1),x)
[Out] -1/(x + 1)
```

3.55 $\int \frac{\log(x)}{(1+\log(x))^2}\,dx$

Optimal result 259
Rubi [A] (verified) 259
Mathematica [A] (verified) 260
Maple [A] (verified) 261
Fricas [A] (verification not implemented) 261
Sympy [A] (verification not implemented) 261
Maxima [A] (verification not implemented) 262
Giac [A] (verification not implemented) 262
Mupad [B] (verification not implemented) 262

Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \frac{\log(x)}{(1+\log(x))^2}\,dx = \frac{x}{1+\log(x)}$$

```
[Out] x/(1+ln(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2407, 2334, 2336, 2209}

$$\int \frac{\log(x)}{(1+\log(x))^2}\,dx = \frac{x}{\log(x)+1}$$

```
[In] Int[Log[x]/(1 + Log[x])^2,x]
```

```
[Out] x/(1 + Log[x])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
```

```
gerQ[2*p]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2407

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(Log[(c_.)*(x_)^(n_.)]*(e_.
) + (d_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*x^n])^p*(d +
 e*Log[c*x^n])^q, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IntegerQ[p] &&
IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{(1+\log(x))^2}+\frac{1}{1+\log(x)}\right) dx \\
&= -\int \frac{1}{(1+\log(x))^2}\, dx+\int \frac{1}{1+\log(x)}\, dx \\
&= \frac{x}{1+\log(x)}-\int \frac{1}{1+\log(x)}\, dx+\text{Subst}\left(\int \frac{e^x}{1+x}\, dx,x,\log(x)\right) \\
&= \frac{\text{ExpIntegralEi}(1+\log(x))}{e}+\frac{x}{1+\log(x)}-\text{Subst}\left(\int \frac{e^x}{1+x}\, dx,x,\log(x)\right) \\
&= \frac{x}{1+\log(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{(1+\log(x))^2}\, dx=\frac{x}{1+\log(x)}$$

```
[In] Integrate[Log[x]/(1 + Log[x])^2,x]
[Out] x/(1 + Log[x])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{x}{1+\ln(x)}$	9
norman	$\frac{x}{1+\ln(x)}$	9
risch	$\frac{x}{1+\ln(x)}$	9
parallelrisch	$\frac{x}{1+\ln(x)}$	9

```
[In] int(ln(x)/(1+ln(x))^2,x,method=_RETURNVERBOSE)
[Out] x/(1+ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{(1+\log(x))^2}\,dx = \frac{x}{\log(x)+1}$$

```
[In] integrate(log(x)/(1+log(x))^2,x, algorithm="fricas")
[Out] x/(log(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{\log(x)}{(1+\log(x))^2}\,dx = \frac{x}{\log(x)+1}$$

```
[In] integrate(ln(x)/(1+ln(x))**2,x)
[Out] x/(log(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{(1+\log(x))^2}\,dx = \frac{x}{\log(x)+1}$$

```
[In] integrate(log(x)/(1+log(x))^2,x, algorithm="maxima")
[Out] x/(log(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{(1+\log(x))^2}\,dx = \frac{x}{\log(x)+1}$$

```
[In] integrate(log(x)/(1+log(x))^2,x, algorithm="giac")
[Out] x/(log(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{(1+\log(x))^2}\,dx = \frac{x}{\ln(x)+1}$$

```
[In] int(log(x)/(log(x) + 1)^2,x)
[Out] x/(log(x) + 1)
```

3.56 $\int \frac{1}{x(1+\log^2(x))} dx$

Optimal result 263
Rubi [A] (verified) 263
Mathematica [A] (verified) 264
Maple [A] (verified) 264
Fricas [A] (verification not implemented) 264
Sympy [B] (verification not implemented) 265
Maxima [A] (verification not implemented) 265
Giac [A] (verification not implemented) 265
Mupad [B] (verification not implemented) 266

Optimal result

Integrand size = 12, antiderivative size = 3

$$\int \frac{1}{x\left(1+\log^2(x)\right)} dx = \arctan(\log(x))$$

```
[Out] arctan(ln(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {209}

$$\int \frac{1}{x\left(1+\log^2(x)\right)} dx = \arctan(\log(x))$$

```
[In] Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \log(x)\right) \\ &= \arctan(\log(x)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx=\arctan(\log(x))$$

```
[In] Integrate[1/(x*(1 + Log[x]^2)),x]
[Out] ArcTan[Log[x]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arctan\left(\ln\left(x\right)\right)$	4
default	$\arctan\left(\ln\left(x\right)\right)$	4
risch	$\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$	20
parallelrisch	$\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$	20

```
[In] int(1/x/(1+ln(x)^2),x,method=_RETURNVERBOSE)
[Out] arctan(ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx=\arctan\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
[Out] arctan(log(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \operatorname{RootSum}\left(4z^2+1,\left(i\mapsto i\log\left(2i+\log\left(x\right)\right)\right)\right)$$

```
[In] integrate(1/x/(1+ln(x)**2),x)
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
[Out] arctan(log(x))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="giac")
[Out] arctan(log(x))
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \operatorname{atan}(\ln(x))$$

```
[In] int(1/(x*(log(x)^2 + 1)),x)
[Out] atan(log(x))
```

3.57 $\int \frac{1}{\log(x)} \, dx$

Optimal result 267
Rubi [A] (verified) 267
Mathematica [A] (verified) 268
Maple [B] (verified) 268
Fricas [A] (verification not implemented) 268
Sympy [A] (verification not implemented) 269
Maxima [A] (verification not implemented) 269
Giac [A] (verification not implemented) 269
Mupad [B] (verification not implemented) 269

Optimal result

Integrand size = 4, antiderivative size = 2

$$\int \frac{1}{\log(x)} \, dx = \text{LogIntegral}(x)$$

```
[Out] Li(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2335}

$$\int \frac{1}{\log(x)} \, dx = \text{LogIntegral}(x)$$

```
[In] Int[Log[x]^(-1),x]
```

```
[Out] LogIntegral[x]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\text{integral} = \text{LogIntegral}(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(x)}\,dx = \text{LogIntegral}(x)$$

```
[In] Integrate[Log[x]^(-1),x]
```

```
[Out] LogIntegral[x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. 2(2) = 4.

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 4.50

method	result	size
default	$-\operatorname{Ei}_1(-\ln(x))$	9
risch	$-\operatorname{Ei}_1(-\ln(x))$	9

```
[In] int(1/ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] -Ei(1,-ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(x)}\,dx = \text{log\_integral}\,(x)$$

```
[In] integrate(1/log(x),x, algorithm="fricas")
```

```
[Out] log_integral(x)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(x)}\,dx = \mathrm{li}\,(x)$$

```
[In] integrate(1/ln(x),x)
[Out] li(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{\log(x)}\,dx = \mathrm{Ei}(\log{(x)})$$

```
[In] integrate(1/log(x),x, algorithm="maxima")
[Out] Ei(log(x))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{\log(x)}\,dx = \mathrm{Ei}(\log{(x)})$$

```
[In] integrate(1/log(x),x, algorithm="giac")
[Out] Ei(log(x))
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(x)}\,dx = \mathrm{logint}(x)$$

```
[In] int(1/log(x),x)
[Out] logint(x)
```

3.58 $\int x(\cos(x)+\sin(x))\,dx$

Optimal result 270
Rubi [A] (verified) 270
Mathematica [A] (verified) 271
Maple [A] (verified) 271
Fricas [A] (verification not implemented) 272
Sympy [A] (verification not implemented) 272
Maxima [A] (verification not implemented) 273
Giac [A] (verification not implemented) 273
Mupad [B] (verification not implemented) 273

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int x(\cos(x)+\sin(x))\,dx=\cos(x)-x\cos(x)+\sin(x)+x\sin(x)$$

```
[Out] cos(x)-x*cos(x)+sin(x)+x*sin(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}=0.571$, Rules used = {14, 3377, 2718, 2717}

$$\int x(\cos(x)+\sin(x))\,dx=x\sin(x)+\sin(x)-x\cos(x)+\cos(x)$$

```
[In] Int[x*(Cos[x] + Sin[x]),x]
```

```
[Out] Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (x\cos(x) + x\sin(x))\,dx \\
&= \int x\cos(x)\,dx + \int x\sin(x)\,dx \\
&= -x\cos(x) + x\sin(x) + \int \cos(x)\,dx - \int \sin(x)\,dx \\
&= \cos(x) - x\cos(x) + \sin(x) + x\sin(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x(\cos(x) + \sin(x))\,dx = \cos(x) - x\cos(x) + \sin(x) + x\sin(x)$$

```
[In] Integrate[x*(Cos[x] + Sin[x]),x]
[Out] Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$\cos(x) - x\cos(x) + \sin(x) + x\sin(x)$	15
parts	$\cos(x) - x\cos(x) + \sin(x) + x\sin(x)$	15
risch	$(1-x)\cos(x) + (1+x)\sin(x)$	16
parallelrisch	$(1-x)\cos(x) + 1 + (1+x)\sin(x)$	17
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right) - x + 2x\tan\left(\frac{x}{2}\right) + 2\tan\left(\frac{x}{2}\right) + 2}{1+\tan^2\left(\frac{x}{2}\right)}$	38
meijerg	$2\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}} + \frac{\cos(x)}{2\sqrt{\pi}} + \frac{x\sin(x)}{2\sqrt{\pi}}\right) + 2\sqrt{\pi}\left(-\frac{x\cos(x)}{2\sqrt{\pi}} + \frac{\sin(x)}{2\sqrt{\pi}}\right)$	49

```
[In] int(x*(cos(x)+sin(x)),x,method=_RETURNVERBOSE)
[Out] cos(x)-x*cos(x)+sin(x)+x*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x(\cos(x)+\sin(x))\,dx = -(x-1)\cos(x) + (x+1)\sin(x)$$

```
[In] integrate(x*(cos(x)+sin(x)),x, algorithm="fricas")
[Out] -(x - 1)*cos(x) + (x + 1)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int x(\cos(x)+\sin(x))\,dx = x\sin(x) - x\cos(x) + \sin(x) + \cos(x)$$

```
[In] integrate(x*(cos(x)+sin(x)),x)
[Out] x*sin(x) - x*cos(x) + sin(x) + cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x(\cos(x) + \sin(x))\,dx = -x\cos(x) + x\sin(x) + \cos(x) + \sin(x)$$

```
[In] integrate(x*(cos(x)+sin(x)),x, algorithm="maxima")
[Out] -x*cos(x) + x*sin(x) + cos(x) + sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x(\cos(x) + \sin(x))\,dx = -x\cos(x) + x\sin(x) + \cos(x) + \sin(x)$$

```
[In] integrate(x*(cos(x)+sin(x)),x, algorithm="giac")
[Out] -x*cos(x) + x*sin(x) + cos(x) + sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x(\cos(x) + \sin(x))\,dx = \cos(x) + \sin(x) - x\,\cos(x) + x\,\sin(x)$$

```
[In] int(x*(cos(x) + sin(x)),x)
[Out] cos(x) + sin(x) - x*cos(x) + x*sin(x)
```

3.59 $\int e^{-x}(e^x + x)\, dx$

Optimal result 274
Rubi [A] (verified) 274
Mathematica [A] (verified) 275
Maple [A] (verified) 275
Fricas [A] (verification not implemented) 276
Sympy [A] (verification not implemented) 276
Maxima [A] (verification not implemented) 276
Giac [A] (verification not implemented) 276
Mupad [B] (verification not implemented) 277

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int e^{-x}(e^x + x)\, dx = -e^{-x} + x - e^{-x}x$$

```
[Out] -1/exp(x)+x-x/exp(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6874, 2207, 2225}

$$\int e^{-x}(e^x + x)\, dx = -e^{-x}x + x - e^{-x}$$

```
[In] Int[(E^x + x)/E^x,x]
```

```
[Out] -E^(-x) + x - x/E^x
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(1+e^{-x}x\right)\,dx \\ &= x+\int e^{-x}x\,dx \\ &= x-e^{-x}x+\int e^{-x}\,dx \\ &= -e^{-x}+x-e^{-x}x \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int e^{-x}(e^x+x)\,dx = e^{-x}(-1-x)+x$$

```
[In] Integrate[(E^x + x)/E^x,x]
[Out] (-1 - x)/E^x + x
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
risch	$x+(-1-x)\,\mathrm{e}^{-x}$	13
norman	$(-1+\mathrm{e}^{x}x-x)\,\mathrm{e}^{-x}$	15
parallelrisch	$(-1+\mathrm{e}^{x}x-x)\,\mathrm{e}^{-x}$	15
default	$-x\,\mathrm{e}^{-x}-\mathrm{e}^{-x}+x$	16
parts	$-x\,\mathrm{e}^{-x}-\mathrm{e}^{-x}+x$	16

```
[In] int((exp(x)+x)/exp(x),x,method=_RETURNVERBOSE)
[Out] x+(-1-x)*exp(-x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int e^{-x}(e^x+x)\,dx=(xe^x-x-1)e^{(-x)}$$

```
[In] integrate((exp(x)+x)/exp(x),x, algorithm="fricas")
[Out] (x*e^x - x - 1)*e^(-x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.47

$$\int e^{-x}(e^x+x)\,dx=x+(-x-1)\,e^{-x}$$

```
[In] integrate((exp(x)+x)/exp(x),x)
[Out] x + (-x - 1)*exp(-x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int e^{-x}(e^x+x)\,dx=-(x+1)e^{(-x)}+x$$

```
[In] integrate((exp(x)+x)/exp(x),x, algorithm="maxima")
[Out] -(x + 1)*e^(-x) + x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int e^{-x}(e^x+x)\,dx=-(x+1)e^{(-x)}+x$$

```
[In] integrate((exp(x)+x)/exp(x),x, algorithm="giac")
[Out] -(x + 1)*e^(-x) + x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int e^{-x}(e^x + x)\,dx = x - \mathrm{e}^{-x} - x\,\mathrm{e}^{-x}$$

```
[In] int(exp(-x)*(x + exp(x)),x)
[Out] x - exp(-x) - x*exp(-x)
```

3.60 $\int (1+e^x)^2 x \, dx$

Optimal result 278
Rubi [A] (verified) 278
Mathematica [A] (verified) 279
Maple [A] (verified) 279
Fricas [A] (verification not implemented) 280
Sympy [A] (verification not implemented) 280
Maxima [A] (verification not implemented) 280
Giac [A] (verification not implemented) 281
Mupad [B] (verification not implemented) 281

Optimal result

Integrand size = 9, antiderivative size = 38

$$\int (1+e^x)^2 x \, dx = -2e^x - \frac{e^{2x}}{4} + 2e^x x + \frac{1}{2}e^{2x} x + \frac{x^2}{2}$$

```
[Out] -2*exp(x)-1/4*exp(2*x)+2*exp(x)*x+1/2*exp(2*x)*x+1/2*x^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2214, 2207, 2225}

$$\int (1+e^x)^2 x \, dx = \frac{x^2}{2} + 2e^x x + \frac{1}{2}e^{2x} x - 2e^x - \frac{e^{2x}}{4}$$

```
[In] Int[(1 + E^x)^2*x,x]
```

```
[Out] -2*E^x - E^(2*x)/4 + 2*E^x*x + (E^(2*x)*x)/2 + x^2/2
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
```

```
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(x+2e^x x+e^{2x}x\right)\,dx\\
&= \frac{x^2}{2}+2\int e^x x\,dx+\int e^{2x}x\,dx\\
&= 2e^x x+\frac{1}{2}e^{2x}x+\frac{x^2}{2}-\frac{1}{2}\int e^{2x}\,dx-2\int e^x\,dx\\
&= -2e^x-\frac{e^{2x}}{4}+2e^x x+\frac{1}{2}e^{2x}x+\frac{x^2}{2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int (1+e^x)^2\,x\,dx=\frac{1}{4}\left(8e^x(-1+x)+2x^2+e^{2x}(-1+2x)\right)$$

```
[In] Integrate[(1 + E^x)^2*x,x]
[Out] (8*E^x*(-1 + x) + 2*x^2 + E^(2*x)*(-1 + 2*x))/4
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{x^2}{2}+\left(-\frac{1}{4}+\frac{x}{2}\right)\mathrm{e}^{2x}+\left(-2+2x\right)\mathrm{e}^{x}$	25
default	$-2\,\mathrm{e}^{x}-\frac{\mathrm{e}^{2x}}{4}+2\,\mathrm{e}^{x}x+\frac{\mathrm{e}^{2x}x}{2}+\frac{x^2}{2}$	29
norman	$-2\,\mathrm{e}^{x}-\frac{\mathrm{e}^{2x}}{4}+2\,\mathrm{e}^{x}x+\frac{\mathrm{e}^{2x}x}{2}+\frac{x^2}{2}$	29
parallelrisch	$-2\,\mathrm{e}^{x}-\frac{\mathrm{e}^{2x}}{4}+2\,\mathrm{e}^{x}x+\frac{\mathrm{e}^{2x}x}{2}+\frac{x^2}{2}$	29
parts	$-2\,\mathrm{e}^{x}-\frac{\mathrm{e}^{2x}}{4}+2\,\mathrm{e}^{x}x+\frac{\mathrm{e}^{2x}x}{2}+\frac{x^2}{2}$	29

```
[In] int((1+exp(x))^2*x,x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+(-1/4+1/2*x)*exp(x)^2+(-2+2*x)*exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int (1+e^x)^2\,x\,dx = \frac{1}{2}\,x^2 + \frac{1}{4}\,(2\,x-1)e^{(2\,x)} + 2\,(x-1)e^x$$

```
[In] integrate((1+exp(x))^2*x,x, algorithm="fricas")
[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int (1+e^x)^2\,x\,dx = \frac{x^2}{2} + \frac{(2x-1)\,e^{2x}}{4} + \frac{(8x-8)\,e^x}{4}$$

```
[In] integrate((1+exp(x))**2*x,x)
[Out] x**2/2 + (2*x - 1)*exp(2*x)/4 + (8*x - 8)*exp(x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int (1+e^x)^2\,x\,dx = \frac{1}{2}\,x^2 + \frac{1}{4}\,(2\,x-1)e^{(2\,x)} + 2\,(x-1)e^x$$

```
[In] integrate((1+exp(x))^2*x,x, algorithm="maxima")
[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int (1+e^x)^2 x\,dx = \frac{1}{2}\,x^2 + \frac{1}{4}\,(2\,x-1)e^{(2\,x)} + 2\,(x-1)e^x$$

```
[In] integrate((1+exp(x))^2*x,x, algorithm="giac")
[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int (1+e^x)^2 x\,dx = \frac{x\,\mathrm{e}^{2\,x}}{2} - 2\,\mathrm{e}^x - \frac{\mathrm{e}^{2\,x}}{4} + 2\,x\,\mathrm{e}^x + \frac{x^2}{2}$$

```
[In] int(x*(exp(x) + 1)^2,x)
[Out] (x*exp(2*x))/2 - 2*exp(x) - exp(2*x)/4 + 2*x*exp(x) + x^2/2
```

3.61 $\int x \cos(x) \, dx$

Optimal result 282
Rubi [A] (verified) 282
Mathematica [A] (verified) 283
Maple [A] (verified) 283
Fricas [A] (verification not implemented) 284
Sympy [A] (verification not implemented) 284
Maxima [A] (verification not implemented) 284
Giac [A] (verification not implemented) 284
Mupad [B] (verification not implemented) 285

Optimal result

Integrand size = 4, antiderivative size = 7

$$\int x \cos(x) \, dx = \cos(x) + x \sin(x)$$

```
[Out] cos(x)+x*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3377, 2718}

$$\int x \cos(x) \, dx = x \sin(x) + \cos(x)$$

```
[In] Int[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = x\sin(x) - \int \sin(x)\,dx$$
$$= \cos(x) + x\sin(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = \cos(x) + x\sin(x)$$

```
[In] Integrate[x*Cos[x],x]
[Out] Cos[x] + x*Sin[x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$\cos(x) + x\sin(x)$	8
risch	$\cos(x) + x\sin(x)$	8
parts	$\cos(x) + x\sin(x)$	8
parallelrisch	$x\sin(x) + \cos(x) + 1$	9
norman	$\frac{2x\tan\left(\frac{x}{2}\right)+2}{1+\tan^2\left(\frac{x}{2}\right)}$	21
meijerg	$2\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}} + \frac{\cos(x)}{2\sqrt{\pi}} + \frac{x\sin(x)}{2\sqrt{\pi}}\right)$	27

```
[In] int(x*cos(x),x,method=_RETURNVERBOSE)
[Out] cos(x)+x*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin{(x)} + \cos{(x)}$$

```
[In] integrate(x*cos(x),x, algorithm="fricas")
[Out] x*sin(x) + cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin{(x)} + \cos{(x)}$$

```
[In] integrate(x*cos(x),x)
[Out] x*sin(x) + cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin{(x)} + \cos{(x)}$$

```
[In] integrate(x*cos(x),x, algorithm="maxima")
[Out] x*sin(x) + cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin{(x)} + \cos{(x)}$$

```
[In] integrate(x*cos(x),x, algorithm="giac")
[Out] x*sin(x) + cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = \cos(x) + x\ \sin(x)$$

```
[In] int(x*cos(x),x)
[Out] cos(x) + x*sin(x)
```

3.62 $\int \cos\left(\sqrt{x}\right)\, dx$

Optimal result 286
Rubi [A] (verified) 286
Mathematica [A] (verified) 287
Maple [A] (verified) 287
Fricas [A] (verification not implemented) 288
Sympy [A] (verification not implemented) 288
Maxima [A] (verification not implemented) 288
Giac [A] (verification not implemented) 288
Mupad [B] (verification not implemented) 289

Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \cos\left(\sqrt{x}\right)\, dx = 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\sin\left(\sqrt{x}\right)$$

```
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3443, 3377, 2718}

$$\int \cos\left(\sqrt{x}\right)\, dx = 2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

```
[In] Int[Cos[Sqrt[x]],x]
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3443

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int x\cos(x)\,dx, x, \sqrt{x}\right)$$
$$= 2\sqrt{x}\sin\left(\sqrt{x}\right) - 2\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{x}\right)$$
$$= 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\sin\left(\sqrt{x}\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\sin\left(\sqrt{x}\right)$$

```
[In] Integrate[Cos[Sqrt[x]],x]
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$2\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\sqrt{x}$	17
default	$2\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\sqrt{x}$	17
meijerg	$4\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(\sqrt{x})}{2\sqrt{\pi}}+\frac{\sqrt{x}\,\sin(\sqrt{x})}{2\sqrt{\pi}}\right)$	33

```
[In] int(cos(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x**(1/2)),x)
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\,\cos\left(\sqrt{x}\right) + 2\,\sqrt{x}\,\sin\left(\sqrt{x}\right)$$

```
[In] int(cos(x^(1/2)),x)
[Out] 2*cos(x^(1/2)) + 2*x^(1/2)*sin(x^(1/2))
```

3.63 $\int x\cos(x)\,dx$

Optimal result 290
Rubi [A] (verified) 290
Mathematica [A] (verified) 291
Maple [A] (verified) 291
Fricas [A] (verification not implemented) 292
Sympy [A] (verification not implemented) 292
Maxima [A] (verification not implemented) 292
Giac [A] (verification not implemented) 292
Mupad [B] (verification not implemented) 293

Optimal result

Integrand size = 4, antiderivative size = 7

$$\int x\cos(x)\,dx = \cos(x) + x\sin(x)$$

```
[Out] cos(x)+x*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3377, 2718}

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

```
[In] Int[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\sin(x) - \int \sin(x)\,dx\\ &= \cos(x) + x\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = \cos(x) + x\sin(x)$$

```
[In] Integrate[x*Cos[x],x]
[Out] Cos[x] + x*Sin[x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$\cos(x) + x\sin(x)$	8
risch	$\cos(x) + x\sin(x)$	8
parts	$\cos(x) + x\sin(x)$	8
parallelrisch	$x\sin(x) + \cos(x) + 1$	9
norman	$\frac{2x\tan(\frac{x}{2})+2}{1+\tan^2(\frac{x}{2})}$	21
meijerg	$2\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}} + \frac{\cos(x)}{2\sqrt{\pi}} + \frac{x\sin(x)}{2\sqrt{\pi}}\right)$	27

```
[In] int(x*cos(x),x,method=_RETURNVERBOSE)
[Out] cos(x)+x*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin{(x)} + \cos{(x)}$$

```
[In] integrate(x*cos(x),x, algorithm="fricas")
[Out] x*sin(x) + cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin{(x)} + \cos{(x)}$$

```
[In] integrate(x*cos(x),x)
[Out] x*sin(x) + cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin{(x)} + \cos{(x)}$$

```
[In] integrate(x*cos(x),x, algorithm="maxima")
[Out] x*sin(x) + cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin{(x)} + \cos{(x)}$$

```
[In] integrate(x*cos(x),x, algorithm="giac")
[Out] x*sin(x) + cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = \cos(x) + x\sin(x)$$

```
[In] int(x*cos(x),x)
[Out] cos(x) + x*sin(x)
```

3.64 $\int x \log^2(x) \, dx$

Optimal result 294
Rubi [A] (verified) 294
Mathematica [A] (verified) 295
Maple [A] (verified) 295
Fricas [A] (verification not implemented) 296
Sympy [A] (verification not implemented) 296
Maxima [A] (verification not implemented) 296
Giac [A] (verification not implemented) 296
Mupad [B] (verification not implemented) 297

Optimal result

Integrand size = 6, antiderivative size = 28

$$\int x \log^2(x) \, dx = \frac{x^2}{4} - \frac{1}{2} x^2 \log(x) + \frac{1}{2} x^2 \log^2(x)$$

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2342, 2341}

$$\int x \log^2(x) \, dx = \frac{x^2}{4} + \frac{1}{2} x^2 \log^2(x) - \frac{1}{2} x^2 \log(x)$$

```
[In] Int[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}x^2\log^2(x) - \int x\log(x)\,dx \\ &= \frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x\log^2(x)\,dx = \frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)$$

```
[In] Integrate[x*Log[x]^2,x]
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
norman	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
risch	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
parallelrisch	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
parts	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23

```
[In] int(x*ln(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x \log^2(x) \, dx = \frac{1}{2} \, x^2 \log\left(x\right)^2 - \frac{1}{2} \, x^2 \log\left(x\right) + \frac{1}{4} \, x^2$$

```
[In] integrate(x*log(x)^2,x, algorithm="fricas")
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x \log^2(x) \, dx = \frac{x^2 \log\left(x\right)^2}{2} - \frac{x^2 \log\left(x\right)}{2} + \frac{x^2}{4}$$

```
[In] integrate(x*ln(x)**2,x)
[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x \log^2(x) \, dx = \frac{1}{4} \, {\left(2 \, \log\left(x\right)^2 - 2 \, \log\left(x\right) + 1\right)} x^2$$

```
[In] integrate(x*log(x)^2,x, algorithm="maxima")
[Out] 1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x \log^2(x) \, dx = \frac{1}{2} \, x^2 \log\left(x\right)^2 - \frac{1}{2} \, x^2 \log\left(x\right) + \frac{1}{4} \, x^2$$

```
[In] integrate(x*log(x)^2,x, algorithm="giac")
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x \log^2(x) \, dx = \frac{x^2 \left(2 \ln(x)^2 - 2 \ln(x) + 1\right)}{4}$$

```
[In] int(x*log(x)^2,x)
[Out] (x^2*(2*log(x)^2 - 2*log(x) + 1))/4
```

3.65 $\int \cos(x) \left(1+\sin^3(x)\right) \, dx$

Optimal result . . . 298
Rubi [A] (verified) . . . 298
Mathematica [A] (verified) . . . 299
Maple [A] (verified) . . . 299
Fricas [A] (verification not implemented) . . . 299
Sympy [A] (verification not implemented) . . . 300
Maxima [A] (verification not implemented) . . . 300
Giac [A] (verification not implemented) . . . 300
Mupad [B] (verification not implemented) . . . 300

Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \cos(x) \left(1+\sin^3(x)\right) \, dx = \sin(x)+\frac{\sin^4(x)}{4}$$

```
[Out] sin(x)+1/4*sin(x)^4
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3302}

$$\int \cos(x) \left(1+\sin^3(x)\right) \, dx = \frac{\sin^4(x)}{4}+\sin(x)$$

```
[In] Int[Cos[x]*(1 + Sin[x]^3),x]
```

```
[Out] Sin[x] + Sin[x]^4/4
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \left(1+x^3\right) \, dx,x,\sin(x)\right) \\ &= \sin(x)+\frac{\sin^4(x)}{4} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos(x) \left(1+\sin^3(x)\right) \, dx = \sin(x) + \frac{\sin^4(x)}{4}$$

```
[In] Integrate[Cos[x]*(1 + Sin[x]^3),x]
[Out] Sin[x] + Sin[x]^4/4
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\sin\left(x\right)+\frac{\left(\sin^{4}\left(x\right)\right)}{4}$	10
default	$\sin\left(x\right)+\frac{\left(\sin^{4}\left(x\right)\right)}{4}$	10
risch	$\sin\left(x\right)+\frac{\cos\left(4x\right)}{32}-\frac{\cos\left(2x\right)}{8}$	16
parallelrisch	$-\frac{\cos\left(2x\right)}{8}+\frac{\cos\left(4x\right)}{32}+\frac{3}{32}+\sin\left(x\right)$	17

```
[In] int(cos(x)*(1+sin(x)^3),x,method=_RETURNVERBOSE)
[Out] sin(x)+1/4*sin(x)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \cos(x) \left(1+\sin^3(x)\right) \, dx = \frac{1}{4} \, \cos\left(x\right)^{4} - \frac{1}{2} \, \cos\left(x\right)^{2} + \sin\left(x\right)$$

```
[In] integrate(cos(x)*(1+sin(x)^3),x, algorithm="fricas")
[Out] 1/4*cos(x)^4 - 1/2*cos(x)^2 + sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \cos(x)\left(1+\sin^3(x)\right)\,dx=\frac{\sin^4\left(x\right)}{4}+\sin\left(x\right)$$

```
[In] integrate(cos(x)*(1+sin(x)**3),x)
[Out] sin(x)**4/4 + sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos(x)\left(1+\sin^3(x)\right)\,dx=\frac{1}{4}\sin\left(x\right)^4+\sin\left(x\right)$$

```
[In] integrate(cos(x)*(1+sin(x)^3),x, algorithm="maxima")
[Out] 1/4*sin(x)^4 + sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos(x)\left(1+\sin^3(x)\right)\,dx=\frac{1}{4}\sin\left(x\right)^4+\sin\left(x\right)$$

```
[In] integrate(cos(x)*(1+sin(x)^3),x, algorithm="giac")
[Out] 1/4*sin(x)^4 + sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos(x)\left(1+\sin^3(x)\right)\,dx=\frac{\sin\left(x\right)^4}{4}+\sin\left(x\right)$$

```
[In] int(cos(x)*(sin(x)^3 + 1),x)
[Out] sin(x) + sin(x)^4/4
```

3.66 $\int \frac{1}{x(1+\log^2(x))} \, dx$

Optimal result 301
Rubi [A] (verified) 301
Mathematica [A] (verified) 302
Maple [A] (verified) 302
Fricas [A] (verification not implemented) 302
Sympy [B] (verification not implemented) 303
Maxima [A] (verification not implemented) 303
Giac [A] (verification not implemented) 303
Mupad [B] (verification not implemented) 304

Optimal result

Integrand size = 12, antiderivative size = 3

$$\int \frac{1}{x\left(1+\log^2(x)\right)} \, dx = \arctan(\log(x))$$

```
[Out] arctan(ln(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {209}

$$\int \frac{1}{x\left(1+\log^2(x)\right)} \, dx = \arctan(\log(x))$$

```
[In] Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \log(x)\right) \\ &= \arctan(\log(x)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan(\log(x))$$

```
[In] Integrate[1/(x*(1 + Log[x]^2)),x]
[Out] ArcTan[Log[x]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arctan\left(\ln\left(x\right)\right)$	4
default	$\arctan\left(\ln\left(x\right)\right)$	4
risch	$\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$	20
parallelrisch	$\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$	20

```
[In] int(1/x/(1+ln(x)^2),x,method=_RETURNVERBOSE)
[Out] arctan(ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
[Out] arctan(log(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \operatorname{RootSum}\left(4z^2+1, \left(i \mapsto i\log\left(2i+\log\left(x\right)\right)\right)\right)$$

```
[In] integrate(1/x/(1+ln(x)**2),x)
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
[Out] arctan(log(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="giac")
[Out] arctan(log(x))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \operatorname{atan}(\ln{(x)})$$

```
[In] int(1/(x*(log(x)^2 + 1)),x)
[Out] atan(log(x))
```

3.67 $\int \frac{1}{\sqrt{1-x^2}(1+\arcsin(x)^2)} \, dx$

Optimal result 305
Rubi [A] (verified) 305
Mathematica [A] (verified) 306
Maple [A] (verified) 306
Fricas [A] (verification not implemented) 306
Sympy [A] (verification not implemented) 307
Maxima [F] 307
Giac [A] (verification not implemented) 307
Mupad [B] (verification not implemented) 307

Optimal result

Integrand size = 20, antiderivative size = 3

$$\int \frac{1}{\sqrt{1-x^2}\,(1+\arcsin(x)^2)} \, dx = \arctan(\arcsin(x))$$

```
[Out] arctan(arcsin(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6828, 209}

$$\int \frac{1}{\sqrt{1-x^2}\,(1+\arcsin(x)^2)} \, dx = \arctan(\arcsin(x))$$

```
[In] Int[1/(Sqrt[1 - x^2]*(1 + ArcSin[x]^2)),x]
```

```
[Out] ArcTan[ArcSin[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6828

```
Int[(u_.)*((a_.) + (b_.)*(y_)^(n_))^(p_), x_Symbol] :> With[{q = Derivative
Divides[y, u, x]}, Dist[q, Subst[Int[(a + b*x^n)^p, x], x, y], x] /;  !Fals
eQ[q]] /; FreeQ[{a, b, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \arcsin(x)\right)\\ &= \arctan(\arcsin(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\,(1+\arcsin(x)^2)}\,dx = \arctan(\arcsin(x))$$

```
[In] Integrate[1/(Sqrt[1 - x^2]*(1 + ArcSin[x]^2)),x]
[Out] ArcTan[ArcSin[x]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arctan(\arcsin(x))$	4
default	$\arctan(\arcsin(x))$	4

```
[In] int(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan(arcsin(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\,(1+\arcsin(x)^2)}\,dx = \arctan(\arcsin(x))$$

```
[In] integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] arctan(arcsin(x))
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\,(1+\arcsin(x)^2)}\,dx = \operatorname{atan}(\operatorname{asin}(x))$$

```
[In] integrate(1/(1+asin(x)**2)/(-x**2+1)**(1/2),x)
[Out] atan(asin(x))
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\,(1+\arcsin(x)^2)}\,dx = \int \frac{1}{\sqrt{-x^2+1}\left(\arcsin(x)^2+1\right)}\,dx$$

```
[In] integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 + 1)*(arcsin(x)^2 + 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\,(1+\arcsin(x)^2)}\,dx = \arctan(\arcsin(x))$$

```
[In] integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] arctan(arcsin(x))
```

Mupad [B] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 43, normalized size of antiderivative = 14.33

$$\int \frac{1}{\sqrt{1-x^2}\,(1+\arcsin(x)^2)}\,dx = \frac{\ln\left(\frac{-1+\operatorname{asin}(x)\,1i}{\sqrt{1-x^2}}\right)\,1i}{2} - \frac{\ln\left(\frac{1+\operatorname{asin}(x)\,1i}{\sqrt{1-x^2}}\right)\,1i}{2}$$

```
[In] int(1/((1 - x^2)^(1/2)*(asin(x)^2 + 1)),x)
[Out] (log((asin(x)*1i - 1)/(1 - x^2)^(1/2))*1i)/2 - (log((asin(x)*1i + 1)/(1 - x
^2)^(1/2))*1i)/2
```

3.68 $\int \frac{\sin(x)}{\cos(x)+\sin(x)} dx$

Optimal result 308
Rubi [A] (verified) 308
Mathematica [A] (verified) 309
Maple [C] (verified) 309
Fricas [A] (verification not implemented) 310
Sympy [A] (verification not implemented) 310
Maxima [B] (verification not implemented) 310
Giac [A] (verification not implemented) 311
Mupad [B] (verification not implemented) 311

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{\sin(x)}{\cos(x)+\sin(x)} dx = \frac{x}{2} - \frac{1}{2}\log(\cos(x)+\sin(x))$$

```
[Out] 1/2*x-1/2*ln(cos(x)+sin(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3176, 3212}

$$\int \frac{\sin(x)}{\cos(x)+\sin(x)} dx = \frac{x}{2} - \frac{1}{2}\log(\sin(x)+\cos(x))$$

```
[In] Int[Sin[x]/(Cos[x] + Sin[x]),x]
```

```
[Out] x/2 - Log[Cos[x] + Sin[x]]/2
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
```

```
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x}{2} - \frac{1}{2}\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\,dx \\ &= \frac{x}{2} - \frac{1}{2}\log(\cos(x)+\sin(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx = \frac{x}{2} - \frac{1}{2}\log(\cos(x)+\sin(x))$$

```
[In] Integrate[Sin[x]/(Cos[x] + Sin[x]),x]
[Out] x/2 - Log[Cos[x] + Sin[x]]/2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

method	result	size
risch	$\frac{x}{2}+\frac{ix}{2}-\frac{\ln\left(e^{2ix}+i\right)}{2}$	20
default	$-\frac{\ln(\tan(x)+1)}{2}+\frac{\ln\left(1+\tan^2(x)\right)}{4}+\frac{\arctan(\tan(x))}{2}$	23
parallelrisch	$\frac{x}{2}+\ln\left(\sqrt{\frac{1}{\cos(x)+1}}\right)+\ln\left(\frac{1}{\sqrt{-\frac{\cos(x)+\sin(x)}{\cos(x)+1}}}\right)$	30
norman	$\frac{\frac{x}{2}+\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}}{1+\tan^2\left(\frac{x}{2}\right)}+\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{\ln\left(\tan^2\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)-1\right)}{2}$	54

```
[In] int(sin(x)/(cos(x)+sin(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/2*I*x-1/2*ln(exp(2*I*x)+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx = \frac{1}{2}\,x - \frac{1}{4}\,\log\left(2\,\cos\left(x\right)\sin\left(x\right) + 1\right)$$

```
[In] integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="fricas")
[Out] 1/2*x - 1/4*log(2*cos(x)*sin(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx = \frac{x}{2} - \frac{\log\left(\sin\left(x\right) + \cos\left(x\right)\right)}{2}$$

```
[In] integrate(sin(x)/(cos(x)+sin(x)),x)
[Out] x/2 - log(sin(x) + cos(x))/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

$$\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx = \arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right) - \frac{1}{2}\,\log\left(-\frac{2\,\sin\left(x\right)}{\cos\left(x\right)+1} + \frac{\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}} - 1\right) + \frac{1}{2}\,\log\left(\frac{\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}} + 1\right)$$

```
[In] integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="maxima")
[Out] arctan(sin(x)/(cos(x) + 1)) - 1/2*log(-2*sin(x)/(cos(x) + 1) + sin(x)^2/(co
s(x) + 1)^2 - 1) + 1/2*log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx = \frac{1}{2}\,x + \frac{1}{4}\,\log\left(\tan\left(x\right)^{2} + 1\right) - \frac{1}{2}\,\log\left(\left|\tan\left(x\right) + 1\right|\right)$$

```
[In] integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="giac")
[Out] 1/2*x + 1/4*log(tan(x)^2 + 1) - 1/2*log(abs(tan(x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx = \frac{x}{2} - \frac{\ln\left(\cos\left(x-\frac{\pi}{4}\right)\right)}{2}$$

```
[In] int(sin(x)/(cos(x) + sin(x)),x)
[Out] x/2 - log(cos(x - pi/4))/2
```

3.69 $\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy$

Optimal result 312
Rubi [A] (verified) 312
Mathematica [A] (verified) 314
Maple [A] (verified) 314
Fricas [B] (verification not implemented) 314
Sympy [F] 315
Maxima [B] (verification not implemented) 315
Giac [B] (verification not implemented) 316
Mupad [F(-1)] 316

Optimal result

Integrand size = 30, antiderivative size = 53

$$\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy=-B\arctan\left(\frac{By}{\sqrt{A^2+B^2-B^2y^2}}\right)-A\operatorname{arctanh}\left(\frac{Ay}{\sqrt{A^2+B^2-B^2y^2}}\right)$$

```
[Out] -B*arctan(B*y/(-B^2*y^2+A^2+B^2)^(1/2))-A*arctanh(A*y/(-B^2*y^2+A^2+B^2)^(1
/2))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1999, 399, 223, 209, 385, 212}

$$\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy=-B\arctan\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)-A\operatorname{arctanh}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

```
[In] Int[-(Sqrt[A^2 + B^2*(1 - y^2)]/(1 - y^2)),y]
```

```
[Out] -(B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]]) - A*ArcTanh[(A*y)/Sqrt[A^2 + B
^2 - B^2*y^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1999

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy \\
&= -\left(A^2\int \frac{1}{(1-y^2)\sqrt{A^2+B^2-B^2y^2}}\,dy\right) - B^2\int \frac{1}{\sqrt{A^2+B^2-B^2y^2}}\,dy \\
&= -\left(A^2\text{Subst}\left(\int \frac{1}{1-A^2y^2}\,dy, y, \frac{y}{\sqrt{A^2+B^2-B^2y^2}}\right)\right) \\
&\quad - B^2\text{Subst}\left(\int \frac{1}{1+B^2y^2}\,dy, y, \frac{y}{\sqrt{A^2+B^2-B^2y^2}}\right)
\end{aligned}$$

$$= -B\arctan\left(\frac{By}{\sqrt{A^2+B^2-B^2y^2}}\right) - A\text{arctanh}\left(\frac{Ay}{\sqrt{A^2+B^2-B^2y^2}}\right)$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy = 2B\arctan\left(\frac{By}{\sqrt{A^2+B^2}-\sqrt{A^2+B^2-B^2y^2}}\right) - A\text{arctanh}\left(\frac{\sqrt{A^2+B^2-B^2y^2}}{Ay}\right)$$

```
[In] Integrate[-(Sqrt[A^2 + B^2*(1 - y^2)]/(1 - y^2)),y]
```

```
[Out] 2*B*ArcTan[(B*y)/(Sqrt[A^2 + B^2] - Sqrt[A^2 + B^2 - B^2*y^2])] - A*ArcTanh
[Sqrt[A^2 + B^2 - B^2*y^2]/(A*y)]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68

method	result
pseudoelliptic	$-\frac{A\ln\left(\frac{Ay+\sqrt{-B^2y^2+A^2+B^2}}{y}\right)}{2}+\frac{A\ln\left(\frac{Ay-\sqrt{-B^2y^2+A^2+B^2}}{y}\right)}{2}+B\arctan\left(\frac{\sqrt{-B^2y^2+A^2+B^2}}{By}\right)$
default	$\frac{\sqrt{-B^2(y-1)^2-2B^2(y-1)+A^2}}{2}-\frac{B^2\arctan\left(\frac{\sqrt{B^2}\,y}{\sqrt{-B^2(y-1)^2-2B^2(y-1)+A^2}}\right)}{2\sqrt{B^2}}-\frac{A^2\ln\left(\frac{2A^2-2B^2(y-1)+2\sqrt{A^2}\,\sqrt{-B^2(y-1)^2}}{y-1}\right.}{2\sqrt{A^2}}$

```
[In] int(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A*ln((A*y+(-B^2*y^2+A^2+B^2)^(1/2))/y)+1/2*A*ln((A*y-(-B^2*y^2+A^2+B^2
)^(1/2))/y)+B*arctan(1/B/y*(-B^2*y^2+A^2+B^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(49) = 98$.

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.42

$$\int -\frac{\sqrt{A^2 + B^2\,(1 - y^2)}}{1 - y^2}\,dy$$
$$= B\arctan\left(\frac{\sqrt{-B^2y^2 + A^2 + B^2}}{By}\right)$$
$$- \frac{1}{4}A\log\left(-\frac{(A^2 - B^2)y^2 + 2\sqrt{-B^2y^2 + A^2 + B^2}Ay + A^2 + B^2}{y^2}\right)$$
$$+ \frac{1}{4}A\log\left(-\frac{(A^2 - B^2)y^2 - 2\sqrt{-B^2y^2 + A^2 + B^2}Ay + A^2 + B^2}{y^2}\right)$$

```
[In] integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="fricas")
```

```
[Out] B*arctan(sqrt(-B^2*y^2 + A^2 + B^2)/(B*y)) - 1/4*A*log(-((A^2 - B^2)*y^2 +
2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2) + 1/4*A*log(-((A^2 - B^2
)*y^2 - 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2)
```

Sympy [F]

$$\int -\frac{\sqrt{A^2 + B^2\,(1 - y^2)}}{1 - y^2}\,dy = \int \frac{\sqrt{A^2 - B^2y^2 + B^2}}{(y - 1)\,(y + 1)}\,dy$$

```
[In] integrate(-(A**2+B**2*(-y**2+1))**(1/2)/(-y**2+1),y)
```

```
[Out] Integral(sqrt(A**2 - B**2*y**2 + B**2)/((y - 1)*(y + 1)), y)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(49) = 98$.

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.17

$$\int -\frac{\sqrt{A^2 + B^2\,(1 - y^2)}}{1 - y^2}\,dy = -B\arcsin\left(\frac{B^2y}{\sqrt{A^2B^2 + B^4}}\right) + \frac{1}{2}A\log\left(B^2 + \frac{A^2}{y + 1} + \frac{\sqrt{-B^2y^2 + A^2 + B^2}A}{y + 1}\right) - \frac{1}{2}A\log\left(-B^2 + \frac{2\,A^2}{|2\,y - 2|} + \frac{2\sqrt{-B^2y^2 + A^2 + B^2}A}{|2\,y - 2|}\right)$$

```
[In] integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="maxima")
```

```
[Out] -B*arcsin(B^2*y/sqrt(A^2*B^2 + B^4)) + 1/2*A*log(B^2 + A^2/(y + 1) + sqrt(-
B^2*y^2 + A^2 + B^2)*A/(y + 1)) - 1/2*A*log(-B^2 + 2*A^2/abs(2*y - 2) + 2*s
qrt(-B^2*y^2 + A^2 + B^2)*A/abs(2*y - 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(49) = 98.

Time = 0.35 (sec) , antiderivative size = 295, normalized size of antiderivative = 5.57

$$\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy = \frac{\left(\pi\,\mathrm{sgn}(y)-2\arctan\left(-\frac{B^2y\left(\frac{\left(\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|\right)^2}{B^4y^2}-1\right)}{2\left(\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|\right)}\right)\right)B^2}{2|B|} - \frac{AB\log\left(\left|-\left(\frac{B^2y}{\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|}-\frac{\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|}{B^2y}\right)B+2A\right|\right)}{2|B|} + \frac{AB\log\left(\left|-\left(\frac{B^2y}{\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|}-\frac{\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|}{B^2y}\right)B-2A\right|\right)}{2|B|}$$

```
[In] integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="giac")
```

```
[Out] 1/2*(pi*sgn(y) - 2*arctan(-1/2*B^2*y*((sqrt(A^2 + B^2)*B + sqrt(-B^2*y^2 +
A^2 + B^2)*abs(B))^2/(B^4*y^2) - 1)/(sqrt(A^2 + B^2)*B + sqrt(-B^2*y^2 + A^
2 + B^2)*abs(B))))*B^2/abs(B) - 1/2*A*B*log(abs(-(B^2*y/(sqrt(A^2 + B^2)*B
+ sqrt(-B^2*y^2 + A^2 + B^2)*abs(B)) - (sqrt(A^2 + B^2)*B + sqrt(-B^2*y^2 +
 A^2 + B^2)*abs(B))/(B^2*y))*B + 2*A))/abs(B) + 1/2*A*B*log(abs(-(B^2*y/(sq
rt(A^2 + B^2)*B + sqrt(-B^2*y^2 + A^2 + B^2)*abs(B)) - (sqrt(A^2 + B^2)*B +
 sqrt(-B^2*y^2 + A^2 + B^2)*abs(B))/(B^2*y))*B - 2*A))/abs(B)
```

Mupad [F(-1)]

Timed out.

$$\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy = \begin{cases} \int \frac{\sqrt{-B^2\,y^2}}{y^2-1}\,dy & \text{if } A^2+B^2=0 \\ \ln\left(2\,y\,\sqrt{-B^2}+2\,\sqrt{A^2-B^2\,y^2+B^2}\right)\,\sqrt{-B^2}+\mathrm{atan}\left(\frac{y\,\sqrt{A^2}\,1\mathrm{i}}{\sqrt{A^2-B^2\,y^2+B^2}}\right)\,\sqrt{A^2}\,1\mathrm{i} & \text{if } A^2+B^2\neq 0 \end{cases}$$

```
[In] int((A^2 - B^2*(y^2 - 1))^(1/2)/(y^2 - 1),y)
```

```
[Out] piecewise(A^2 + B^2 == 0, int((-B^2*y^2)^(1/2)/(y^2 - 1), y), A^2 + B^2 ~=
0, atan((y*(A^2)^(1/2)*1i)/(A^2 + B^2 - B^2*y^2)^(1/2))*(A^2)^(1/2)*1i + lo
g(2*y*(-B^2)^(1/2) + 2*(A^2 + B^2 - B^2*y^2)^(1/2))*(-B^2)^(1/2))
```

3.70 $\int \frac{\left(-A^2-B^2\right) \cos^2(z)}{B\left(1-\frac{\left(A^2+B^2\right) \sin^2(z)}{B^2}\right)} dz$

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Optimal result

Integrand size = 39, antiderivative size = 16

$$\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz = -Bz - A\operatorname{arctanh}\left(\frac{A\tan(z)}{B}\right)$$

```
[Out] -B*z-A*arctanh(A*tan(z)/B)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {12, 3270, 400, 209, 212}

$$\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz = -A\operatorname{arctanh}\left(\frac{A\tan(z)}{B}\right) - Bz$$

```
[In] Int[((-A^2 - B^2)*Cos[z]^2)/(B*(1 - ((A^2 + B^2)*Sin[z]^2)/B^2)),z]
```

```
[Out] -(B*z) - A*ArcTanh[(A*Tan[z])/B]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(A^2+B^2)\int \frac{\cos^2(z)}{1-\frac{(A^2+B^2)\sin^2(z)}{B^2}}\,dz}{B} \\
&= -\frac{(A^2+B^2)\,\text{Subst}\left(\int \frac{1}{(1+z^2)\left(1+\left(1-\frac{A^2+B^2}{B^2}\right)z^2\right)}\,dz, z, \tan(z)\right)}{B} \\
&= -\frac{A^2\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{A^2+B^2}{B^2}\right)z^2}\,dz, z, \tan(z)\right)}{B} - B\text{Subst}\left(\int \frac{1}{1+z^2}\,dz, z, \tan(z)\right) \\
&= -Bz - A\,\text{arctanh}\left(\frac{A\tan(z)}{B}\right)
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz = -\frac{B(A^2+B^2)\left(Bz+A\text{arctanh}\left(\frac{A\tan(z)}{B}\right)\right)}{A^2B+B^3}$$

```
[In] Integrate[((-A^2 - B^2)*Cos[z]^2)/(B*(1 - ((A^2 + B^2)*Sin[z]^2)/B^2)),z]
[Out] -((B*(A^2 + B^2)*(B*z + A*ArcTanh[(A*Tan[z])/B]))/(A^2*B + B^3))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(16) = 32$.

Time = 0.62 (sec) , antiderivative size = 71, normalized size of antiderivative = 4.44

method	result
parallelrisch	$\frac{(-A^2-B^2)\left(A\ln\left(\frac{A\sin(z)+B\cos(z)}{\cos(z)+1}\right)-A\ln\left(\frac{A\sin(z)-B\cos(z)}{\cos(z)+1}\right)+2Bz\right)}{2A^2+2B^2}$
default	$(-A^2-B^2)\,B\left(\frac{A\ln(A\tan(z)+B)}{2B(A^2+B^2)}+\frac{\arctan(\tan(z))}{A^2+B^2}-\frac{A\ln(A\tan(z)-B)}{2B(A^2+B^2)}\right)$
norman	$\frac{-Bz-2Bz(\tan^2(\frac{z}{2}))-Bz(\tan^4(\frac{z}{2}))}{(1+\tan^2(\frac{z}{2}))^2}-\frac{A\ln(-B(\tan^2(\frac{z}{2}))+2A\tan(\frac{z}{2})+B)}{2}+\frac{A\ln(B(\tan^2(\frac{z}{2}))+2A\tan(\frac{z}{2})-B)}{2}$
risch	$-\frac{BzA^2}{A^2+B^2}-\frac{B^3z}{A^2+B^2}-\frac{A^3\ln\left(e^{2iz}-\frac{-iB+A}{iB+A}\right)}{2(A^2+B^2)}-\frac{A\ln\left(e^{2iz}-\frac{-iB+A}{iB+A}\right)B^2}{2(A^2+B^2)}+\frac{A^3\ln\left(e^{2iz}-\frac{iB+A}{-iB+A}\right)}{2A^2+2B^2}+\frac{A\ln\left(e^{2iz}-\frac{iB+}{-iB+}\right.}{2A^2+2B^2}$

```
[In] int((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z,method=_RETURNVERBOS
E)
[Out] (-A^2-B^2)*(A*ln((A*sin(z)+B*cos(z))/(cos(z)+1))-A*ln((A*sin(z)-B*cos(z))/(
cos(z)+1))+2*B*z)/(2*A^2+2*B^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 4.19

$$\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz = -Bz-\frac{1}{4}A\log\left(2AB\cos(z)\sin(z)-(A^2-B^2)\cos(z)^2+A^2\right) + \frac{1}{4}A\log\left(-2AB\cos(z)\sin(z)-(A^2-B^2)\cos(z)^2+A^2\right)$$

```
[In] integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="fr
icas")
```

```
[Out] -B*z - 1/4*A*log(2*A*B*cos(z)*sin(z) - (A^2 - B^2)*cos(z)^2 + A^2) + 1/4*A*
log(-2*A*B*cos(z)*sin(z) - (A^2 - B^2)*cos(z)^2 + A^2)
```

Sympy [A] (verification not implemented)

Time = 106.06 (sec) , antiderivative size = 202, normalized size of antiderivative = 12.62

$$\int \frac{(-A^2 - B^2)\cos^2(z)}{B\left(1 - \frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz = \frac{(-A^2 - B^2)\left(\begin{cases} z \\ \frac{z\sin^2(z)}{2} + \frac{z\cos^2(z)}{2} + \frac{\sin(z)\cos(z)}{2} \\ \frac{AB\log\left(-\frac{A}{B} + \tan\left(\frac{z}{2}\right) - \frac{\sqrt{A^2+B^2}}{B}\right)}{2A^2+2B^2} + \frac{AB\log\left(-\frac{A}{B} + \tan\left(\frac{z}{2}\right) + \frac{\sqrt{A^2+B^2}}{B}\right)}{2A^2+2B^2} - \frac{AB\log\left(\frac{A}{B} + \tan\left(\frac{z}{2}\right) - \frac{\sqrt{A^2+B^2}}{B}\right)}{2A^2+2B^2} - \frac{AB}{} \end{cases}\right)}{B}$$

```
[In] integrate((-A**2-B**2)*cos(z)**2/B/(1-(A**2+B**2)*sin(z)**2/B**2),z)
```

```
[Out] (-A**2 - B**2)*Piecewise((z, Eq(A, 0) & Eq(B, 0)), (z*sin(z)**2/2 + z*cos(z
)**2/2 + sin(z)*cos(z)/2, Eq(A, I*B) | Eq(A, -I*B)), (A*B*log(-A/B + tan(z/
2) - sqrt(A**2 + B**2)/B)/(2*A**2 + 2*B**2) + A*B*log(-A/B + tan(z/2) + sqr
t(A**2 + B**2)/B)/(2*A**2 + 2*B**2) - A*B*log(A/B + tan(z/2) - sqrt(A**2 +
B**2)/B)/(2*A**2 + 2*B**2) - A*B*log(A/B + tan(z/2) + sqrt(A**2 + B**2)/B)/
(2*A**2 + 2*B**2) + 2*B**2*z/(2*A**2 + 2*B**2), True))/B
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(16) = 32.

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.31

$$\int \frac{(-A^2 - B^2)\cos^2(z)}{B\left(1 - \frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz = -\frac{(A^2 + B^2)\left(\frac{2B^2z}{A^2+B^2} + \frac{AB\log(A\tan(z)+B)}{A^2+B^2} - \frac{AB\log(A\tan(z)-B)}{A^2+B^2}\right)}{2B}$$

```
[In] integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="ma
xima")
```

```
[Out] -1/2*(A^2 + B^2)*(2*B^2*z/(A^2 + B^2) + A*B*log(A*tan(z) + B)/(A^2 + B^2) -
 A*B*log(A*tan(z) - B)/(A^2 + B^2))/B
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 5.19

$$\int \frac{(-A^2 - B^2)\cos^2(z)}{B\left(1 - \frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz = -\frac{\left(\frac{A^3B\log(|A\tan(z)+B|)}{A^4+A^2B^2} - \frac{A^3B\log(|A\tan(z)-B|)}{A^4+A^2B^2} + \frac{2B^2z}{A^2+B^2}\right)(A^2+B^2)}{2B}$$

```
[In] integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="gi
ac")
```

```
[Out] -1/2*(A^3*B*log(abs(A*tan(z) + B))/(A^4 + A^2*B^2) - A^3*B*log(abs(A*tan(z)
 - B))/(A^4 + A^2*B^2) + 2*B^2*z/(A^2 + B^2))*(A^2 + B^2)/B
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 360, normalized size of antiderivative = 22.50

$$\begin{aligned}\int \frac{(-A^2 - B^2)\cos^2(z)}{B\left(1 - \frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz = &-A\,\mathrm{atanh}\left(\frac{2A^{13}\tan(z)}{2A^{12}B + 6A^{10}B^3 + 6A^8B^5 + 2A^6B^7}\right.\\ &+ \frac{2A^7B^6\tan(z)}{2A^{12}B + 6A^{10}B^3 + 6A^8B^5 + 2A^6B^7}\\ &+ \frac{6A^9B^4\tan(z)}{2A^{12}B + 6A^{10}B^3 + 6A^8B^5 + 2A^6B^7}\\ &\left.+ \frac{6A^{11}B^2\tan(z)}{2A^{12}B + 6A^{10}B^3 + 6A^8B^5 + 2A^6B^7}\right)\\ &- B\,\mathrm{atan}\left(\frac{2A^4B^9\tan(z)}{2A^{10}B^3 + 6A^8B^5 + 6A^6B^7 + 2A^4B^9}\right.\\ &+ \frac{6A^6B^7\tan(z)}{2A^{10}B^3 + 6A^8B^5 + 6A^6B^7 + 2A^4B^9}\\ &+ \frac{6A^8B^5\tan(z)}{2A^{10}B^3 + 6A^8B^5 + 6A^6B^7 + 2A^4B^9}\\ &\left.+ \frac{2A^{10}B^3\tan(z)}{2A^{10}B^3 + 6A^8B^5 + 6A^6B^7 + 2A^4B^9}\right)\end{aligned}$$

```
[In] int((cos(z)^2*(A^2 + B^2))/(B*((sin(z)^2*(A^2 + B^2))/B^2 - 1)),z)
```

```
[Out] - A*atanh((2*A^13*tan(z))/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) +
 (2*A^7*B^6*tan(z))/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^
9*B^4*tan(z))/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^11*B^2
*tan(z))/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3)) - B*atan((2*A^4*B
^9*tan(z))/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^6*B^7*ta
```

```
n(z))/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^8*B^5*tan(z))
/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (2*A^10*B^3*tan(z))/(2*
A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3))
```

3.71 $\int \frac{-A^2-B^2}{B\left(1+w^2\right)^2\left(1-\frac{\left(A^2+B^2\right)w^2}{B^2\left(1+w^2\right)}\right)}\,dw$

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Optimal result

Integrand size = 48, antiderivative size = 16

$$\int \frac{-A^2-B^2}{B\left(1+w^2\right)^2\left(1-\frac{\left(A^2+B^2\right)w^2}{B^2\left(1+w^2\right)}\right)}\,dw = -B\arctan(w) - A\operatorname{arctanh}\left(\frac{Aw}{B}\right)$$

```
[Out] -B*arctan(w)-A*arctanh(A*w/B)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {12, 6820, 400, 209, 214}

$$\int \frac{-A^2-B^2}{B\left(1+w^2\right)^2\left(1-\frac{\left(A^2+B^2\right)w^2}{B^2\left(1+w^2\right)}\right)}\,dw = -A\operatorname{arctanh}\left(\frac{Aw}{B}\right) - B\arctan(w)$$

```
[In] Int[(-A^2 - B^2)/(B*(1 + w^2)^2*(1 - ((A^2 + B^2)*w^2)/(B^2*(1 + w^2)))),w]
```

```
[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A^2+B^2)\int \frac{1}{(1+w^2)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw}{B}\\
&= -\frac{(A^2+B^2)\int \frac{B^2}{(1+w^2)(B^2-A^2w^2)}\,dw}{B}\\
&= -\left(\left(B\left(A^2+B^2\right)\right)\int \frac{1}{(1+w^2)\left(B^2-A^2w^2\right)}\,dw\right)\\
&= -\left(B\int \frac{1}{1+w^2}\,dw\right)-\left(A^2B\right)\int \frac{1}{B^2-A^2w^2}\,dw\\
&= -B\arctan(w)-A\text{arctanh}\left(\frac{Aw}{B}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{-A^2-B^2}{B\,(1+w^2)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw = -\frac{B(A^2+B^2)\left(B\arctan(w)+A\operatorname{arctanh}\left(\frac{Aw}{B}\right)\right)}{A^2B+B^3}$$

```
[In] Integrate[(-A^2 - B^2)/(B*(1 + w^2)^2*(1 - ((A^2 + B^2)*w^2)/(B^2*(1 + w^2)
))),w]
```

```
[Out] -((B*(A^2 + B^2)*(B*ArcTan[w] + A*ArcTanh[(A*w)/B]))/(A^2*B + B^3))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.38

method	result
parallelrisch	$\frac{(-A^2-B^2)(-iB^2\ln(w-i)+iB^2\ln(w+i)-AB\ln(Aw-B)+AB\ln(Aw+B))}{2B(A^2+B^2)}$
default	$(-A^2-B^2)\,B\left(\frac{\arctan(w)}{A^2+B^2}+\frac{A\ln(Aw+B)}{2B(A^2+B^2)}-\frac{A\ln(Aw-B)}{2B(A^2+B^2)}\right)$
risch	$-\frac{A^3\ln(-Aw-B)}{2(A^2+B^2)}-\frac{A\ln(-Aw-B)B^2}{2(A^2+B^2)}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(A^4+2A^2B^2+B^4\right)\_Z^2+B^4\right)}\_R\ln\left(\left(\left(-A^6-B^2A^4+A^2B^4+B^6\right)\right.\right.\right.}{2B}$

```
[In] int((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w,method=_RETURNVE
RBOSE)
```

```
[Out] 1/2*(-A^2-B^2)/B*(-I*B^2*ln(w-I)+I*B^2*ln(w+I)-A*B*ln(A*w-B)+A*B*ln(A*w+B))
/(A^2+B^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{-A^2-B^2}{B\,(1+w^2)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw = -B\arctan(w)-\frac{1}{2}\,A\log(Aw+B)+\frac{1}{2}\,A\log(Aw-B)$$

```
[In] integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorithm
="fricas")
```

```
[Out] -B*arctan(w) - 1/2*A*log(A*w + B) + 1/2*A*log(A*w - B)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 422, normalized size of antiderivative = 26.38

$$\int \frac{-A^2-B^2}{B\left(1+w^2\right)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw$$

$$= \left(A^2B+B^3\right)\left(-\frac{A\log\left(w+\frac{-\frac{A^9}{B(A^2+B^2)^3}-\frac{A^7B}{(A^2+B^2)^3}+\frac{A^5B^3}{(A^2+B^2)^3}+\frac{A^5}{B(A^2+B^2)}+\frac{A^3B^5}{(A^2+B^2)^3}+\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B\left(A^2+B^2\right)} + \frac{A\log\left(w+\frac{\frac{A^9}{B(A^2+B^2)^3}+\frac{A^7B}{(A^2+B^2)^3}-\frac{A^5B^3}{(A^2+B^2)^3}-\frac{A^5}{B(A^2+B^2)}-\frac{A^3B^5}{(A^2+B^2)^3}-\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B\left(A^2+B^2\right)} + \frac{i\log\left(w+\frac{-\frac{iA^6B^2}{(A^2+B^2)^3}-\frac{iA^4B^4}{(A^2+B^2)^3}-\frac{iA^4}{A^2+B^2}+\frac{iA^2B^6}{(A^2+B^2)^3}+\frac{iB^8}{(A^2+B^2)^3}-\frac{iB^4}{A^2+B^2}}{A^2}\right)}{2\left(A^2+B^2\right)} - \frac{i\log\left(w+\frac{\frac{iA^6B^2}{(A^2+B^2)^3}+\frac{iA^4B^4}{(A^2+B^2)^3}+\frac{iA^4}{A^2+B^2}-\frac{iA^2B^6}{(A^2+B^2)^3}-\frac{iB^8}{(A^2+B^2)^3}+\frac{iB^4}{A^2+B^2}}{A^2}\right)}{2\left(A^2+B^2\right)}\right)$$

```
[In] integrate((-A**2-B**2)/B/(w**2+1)**2/(1-(A**2+B**2)*w**2/B**2/(w**2+1)),w)
```

```
[Out] (A**2*B + B**3)*(-A*log(w + (-A**9/(B*(A**2 + B**2)**3) - A**7*B/(A**2 + B*
*2)**3 + A**5*B**3/(A**2 + B**2)**3 + A**5/(B*(A**2 + B**2)) + A**3*B**5/(A
**2 + B**2)**3 + A*B**3/(A**2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + A*log(w
+ (A**9/(B*(A**2 + B**2)**3) + A**7*B/(A**2 + B**2)**3 - A**5*B**3/(A**2 +
B**2)**3 - A**5/(B*(A**2 + B**2)) - A**3*B**5/(A**2 + B**2)**3 - A*B**3/(A*
*2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + I*log(w + (-I*A**6*B**2/(A**2 + B**
2)**3 - I*A**4*B**4/(A**2 + B**2)**3 - I*A**4/(A**2 + B**2) + I*A**2*B**6/(
A**2 + B**2)**3 + I*B**8/(A**2 + B**2)**3 - I*B**4/(A**2 + B**2))/A**2)/(2*
(A**2 + B**2)) - I*log(w + (I*A**6*B**2/(A**2 + B**2)**3 + I*A**4*B**4/(A**
2 + B**2)**3 + I*A**4/(A**2 + B**2) - I*A**2*B**6/(A**2 + B**2)**3 - I*B**8
/(A**2 + B**2)**3 + I*B**4/(A**2 + B**2))/A**2)/(2*(A**2 + B**2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.25

$$\int \frac{-A^2 - B^2}{B\left(1 + w^2\right)^2 \left(1 - \frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw = -\frac{(A^2 + B^2)\left(\frac{2\,B^2 \arctan(w)}{A^2+B^2} + \frac{AB\log(Aw+B)}{A^2+B^2} - \frac{AB\log(Aw-B)}{A^2+B^2}\right)}{2\,B}$$

```
[In] integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorithm
="maxima")
```

```
[Out] -1/2*(A^2 + B^2)*(2*B^2*arctan(w)/(A^2 + B^2) + A*B*log(A*w + B)/(A^2 + B^2
) - A*B*log(A*w - B)/(A^2 + B^2))/B
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 5.12

$$\int \frac{-A^2 - B^2}{B\left(1 + w^2\right)^2 \left(1 - \frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw = -\frac{\left(\frac{A^3B\log(|Aw+B|)}{A^4+A^2B^2} - \frac{A^3B\log(|Aw-B|)}{A^4+A^2B^2} + \frac{2\,B^2\arctan(w)}{A^2+B^2}\right)(A^2 + B^2)}{2\,B}$$

```
[In] integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorithm
="giac")
```

```
[Out] -1/2*(A^3*B*log(abs(A*w + B))/(A^4 + A^2*B^2) - A^3*B*log(abs(A*w - B))/(A^
4 + A^2*B^2) + 2*B^2*arctan(w)/(A^2 + B^2))*(A^2 + B^2)/B
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 352, normalized size of antiderivative = 22.00

$$\int \frac{-A^2-B^2}{B(1+w^2)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw = -A\,\mathrm{atanh}\left(\frac{2A^{13}w}{2A^{12}B+6A^{10}B^3+6A^8B^5+2A^6B^7}+\frac{2A^7B^6w}{2A^{12}B+6A^{10}B^3+6A^8B^5+2A^6B^7}+\frac{6A^9B^4w}{2A^{12}B+6A^{10}B^3+6A^8B^5+2A^6B^7}+\frac{6A^{11}B^2w}{2A^{12}B+6A^{10}B^3+6A^8B^5+2A^6B^7}\right)-B\,\mathrm{atan}\left(\frac{2A^4B^9w}{2A^{10}B^3+6A^8B^5+6A^6B^7+2A^4B^9}+\frac{6A^6B^7w}{2A^{10}B^3+6A^8B^5+6A^6B^7+2A^4B^9}+\frac{6A^8B^5w}{2A^{10}B^3+6A^8B^5+6A^6B^7+2A^4B^9}+\frac{2A^{10}B^3w}{2A^{10}B^3+6A^8B^5+6A^6B^7+2A^4B^9}\right)$$

```
[In] int((A^2 + B^2)/(B*(w^2 + 1)^2*((w^2*(A^2 + B^2))/(B^2*(w^2 + 1)) - 1)),w)
```

```
[Out] - A*atanh((2*A^13*w)/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (2*A
^7*B^6*w)/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^9*B^4*w)/(
2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^11*B^2*w)/(2*A^12*B +
 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3)) - B*atan((2*A^4*B^9*w)/(2*A^4*B^9 + 6
*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^6*B^7*w)/(2*A^4*B^9 + 6*A^6*B^7 +
 6*A^8*B^5 + 2*A^10*B^3) + (6*A^8*B^5*w)/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5
 + 2*A^10*B^3) + (2*A^10*B^3*w)/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10
*B^3))
```

3.72 $\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)} \, dw$

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Optimal result

Integrand size = 31, antiderivative size = 16

$$\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)} \, dw = -B\arctan(w) - A\text{arctanh}\left(\frac{Aw}{B}\right)$$

```
[Out] -B*arctan(w)-A*arctanh(A*w/B)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {12, 400, 209, 214}

$$\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)} \, dw = -A\text{arctanh}\left(\frac{Aw}{B}\right) - B\arctan(w)$$

```
[In] Int[-((B*(A^2 + B^2))/((1 + w^2)*(B^2 - A^2*w^2))),w]
[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\left(B\left(A^2+B^2\right)\right)\int\frac{1}{(1+w^2)\left(B^2-A^2w^2\right)}\,dw\right)\\
&= -\left(B\int\frac{1}{1+w^2}\,dw\right)-\left(A^2B\right)\int\frac{1}{B^2-A^2w^2}\,dw\\
&= -B\arctan(w)-A\text{arctanh}\left(\frac{Aw}{B}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int-\frac{B(A^2+B^2)}{(1+w^2)\left(B^2-A^2w^2\right)}\,dw=-\frac{B(A^2+B^2)\left(B\arctan(w)+A\text{arctanh}\left(\frac{Aw}{B}\right)\right)}{A^2B+B^3}$$

```
[In] Integrate[-((B*(A^2 + B^2))/((1 + w^2)*(B^2 - A^2*w^2))),w]
[Out] -((B*(A^2 + B^2)*(B*ArcTan[w] + A*ArcTanh[(A*w)/B]))/(A^2*B + B^3))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.50

method	result
parallelrisch	$\frac{i\ln(w-i)B}{2}-\frac{i\ln(w+i)B}{2}+\frac{A\ln(Aw-B)}{2}-\frac{A\ln(Aw+B)}{2}$
default	$-(A^2+B^2)\,B\left(\frac{\arctan(w)}{A^2+B^2}+\frac{A\ln(Aw+B)}{2B(A^2+B^2)}-\frac{A\ln(Aw-B)}{2B(A^2+B^2)}\right)$
risch	$-\frac{A\ln(-Aw-B)}{2}+\frac{A\ln(-Aw+B)}{2}-\frac{A^2B\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(A^4+2A^2B^2+B^4\right)\_Z^2\right)}\_R\ln\left(\left(\left(-A^6-B^2A^4+A^2B^4+B^6\right)\_\right.\right.\right.}{2}$

[In] int(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w,method=_RETURNVERBOSE)

[Out] 1/2*I*ln(w-I)*B-1/2*I*ln(w+I)*B+1/2*A*ln(A*w-B)-1/2*A*ln(A*w+B)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw = -B\arctan(w)-\frac{1}{2}A\log(Aw+B)+\frac{1}{2}A\log(Aw-B)$$

[In] integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="fricas")

[Out] -B*arctan(w) - 1/2*A*log(A*w + B) + 1/2*A*log(A*w - B)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 422, normalized size of antiderivative = 26.38

$$\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw$$

$$= (A^2B+B^3)\left(-\frac{A\log\left(w+\frac{-\frac{A^9}{B(A^2+B^2)^3}-\frac{A^7B}{(A^2+B^2)^3}+\frac{A^5B^3}{(A^2+B^2)^3}+\frac{A^5}{B(A^2+B^2)}+\frac{A^3B^5}{(A^2+B^2)^3}+\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B(A^2+B^2)}\right.$$

$$+\frac{A\log\left(w+\frac{\frac{A^9}{B(A^2+B^2)^3}+\frac{A^7B}{(A^2+B^2)^3}-\frac{A^5B^3}{(A^2+B^2)^3}-\frac{A^5}{B(A^2+B^2)}-\frac{A^3B^5}{(A^2+B^2)^3}-\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B(A^2+B^2)}$$

$$+\frac{i\log\left(w+\frac{-\frac{iA^6B^2}{(A^2+B^2)^3}-\frac{iA^4B^4}{(A^2+B^2)^3}-\frac{iA^4}{A^2+B^2}+\frac{iA^2B^6}{(A^2+B^2)^3}+\frac{iB^8}{(A^2+B^2)^3}-\frac{iB^4}{A^2+B^2}}{A^2}\right)}{2(A^2+B^2)}$$

$$\left.-\frac{i\log\left(w+\frac{\frac{iA^6B^2}{(A^2+B^2)^3}+\frac{iA^4B^4}{(A^2+B^2)^3}+\frac{iA^4}{A^2+B^2}-\frac{iA^2B^6}{(A^2+B^2)^3}-\frac{iB^8}{(A^2+B^2)^3}+\frac{iB^4}{A^2+B^2}}{A^2}\right)}{2(A^2+B^2)}\right)$$

```
[In] integrate(-B*(A**2+B**2)/(w**2+1)/(-A**2*w**2+B**2),w)
```

```
[Out] (A**2*B + B**3)*(-A*log(w + (-A**9/(B*(A**2 + B**2)**3) - A**7*B/(A**2 + B*
*2)**3 + A**5*B**3/(A**2 + B**2)**3 + A**5/(B*(A**2 + B**2)) + A**3*B**5/(A
**2 + B**2)**3 + A*B**3/(A**2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + A*log(w
+ (A**9/(B*(A**2 + B**2)**3) + A**7*B/(A**2 + B**2)**3 - A**5*B**3/(A**2 +
B**2)**3 - A**5/(B*(A**2 + B**2)) - A**3*B**5/(A**2 + B**2)**3 - A*B**3/(A*
*2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + I*log(w + (-I*A**6*B**2/(A**2 + B**
2)**3 - I*A**4*B**4/(A**2 + B**2)**3 - I*A**4/(A**2 + B**2) + I*A**2*B**6/(
A**2 + B**2)**3 + I*B**8/(A**2 + B**2)**3 - I*B**4/(A**2 + B**2))/A**2)/(2*
(A**2 + B**2)) - I*log(w + (I*A**6*B**2/(A**2 + B**2)**3 + I*A**4*B**4/(A**
2 + B**2)**3 + I*A**4/(A**2 + B**2) - I*A**2*B**6/(A**2 + B**2)**3 - I*B**8
/(A**2 + B**2)**3 + I*B**4/(A**2 + B**2))/A**2)/(2*(A**2 + B**2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.06

$$\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw$$
$$= -\frac{1}{2}(A^2+B^2)B\left(\frac{A\log(Aw+B)}{A^2B+B^3}-\frac{A\log(Aw-B)}{A^2B+B^3}+\frac{2\arctan(w)}{A^2+B^2}\right)$$

```
[In] integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="maxima")
```

```
[Out] -1/2*(A^2 + B^2)*B*(A*log(A*w + B)/(A^2*B + B^3) - A*log(A*w - B)/(A^2*B +
B^3) + 2*arctan(w)/(A^2 + B^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.94

$$\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw$$
$$= -\frac{1}{2}\left(\frac{A^3\log(|Aw+B|)}{A^4B+A^2B^3}-\frac{A^3\log(|Aw-B|)}{A^4B+A^2B^3}+\frac{2\arctan(w)}{A^2+B^2}\right)(A^2+B^2)B$$

```
[In] integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="giac")
```

```
[Out] -1/2*(A^3*log(abs(A*w + B))/(A^4*B + A^2*B^3) - A^3*log(abs(A*w - B))/(A^4*
B + A^2*B^3) + 2*arctan(w)/(A^2 + B^2))*(A^2 + B^2)*B
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 352, normalized size of antiderivative = 22.00

$$\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw = -A\,\mathrm{atanh}\left(\frac{2A^{13}w}{2A^{12}B+6A^{10}B^3+6A^8B^5+2A^6B^7}+\frac{2A^7B^6w}{2A^{12}B+6A^{10}B^3+6A^8B^5+2A^6B^7}+\frac{6A^9B^4w}{2A^{12}B+6A^{10}B^3+6A^8B^5+2A^6B^7}+\frac{6A^{11}B^2w}{2A^{12}B+6A^{10}B^3+6A^8B^5+2A^6B^7}\right) - B\,\mathrm{atan}\left(\frac{2A^4B^9w}{2A^{10}B^3+6A^8B^5+6A^6B^7+2A^4B^9}+\frac{6A^6B^7w}{2A^{10}B^3+6A^8B^5+6A^6B^7+2A^4B^9}+\frac{6A^8B^5w}{2A^{10}B^3+6A^8B^5+6A^6B^7+2A^4B^9}+\frac{2A^{10}B^3w}{2A^{10}B^3+6A^8B^5+6A^6B^7+2A^4B^9}\right)$$

```
[In] int(-(B*(A^2 + B^2))/((w^2 + 1)*(B^2 - A^2*w^2)),w)
```

```
[Out] - A*atanh((2*A^13*w)/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (2*A
^7*B^6*w)/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^9*B^4*w)/(
2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^11*B^2*w)/(2*A^12*B +
 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3)) - B*atan((2*A^4*B^9*w)/(2*A^4*B^9 + 6
*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^6*B^7*w)/(2*A^4*B^9 + 6*A^6*B^7 +
 6*A^8*B^5 + 2*A^10*B^3) + (6*A^8*B^5*w)/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5
 + 2*A^10*B^3) + (2*A^10*B^3*w)/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10
*B^3))
```

3.73 $\int \frac{x^4}{(1-x^2)^{5/2}}\,dx$

Optimal result 335
Rubi [A] (verified) 335
Mathematica [A] (verified) 336
Maple [A] (verified) 336
Fricas [B] (verification not implemented) 337
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Maxima [A] (verification not implemented) 337
Giac [A] (verification not implemented) 338
Mupad [B] (verification not implemented) 338

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \arcsin(x)$$

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {294, 222}

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \arcsin(x) - \frac{x}{\sqrt{1-x^2}} + \frac{x^3}{3(1-x^2)^{3/2}}$$

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^3}{3(1-x^2)^{3/2}} - \int \frac{x^2}{(1-x^2)^{3/2}}\,dx \\
&= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \arcsin(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{x(-3+4x^2)}{3(1-x^2)^{3/2}} + 2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x^3}{3(-x^2+1)^{\frac{3}{2}}}+\arcsin(x)-\frac{x}{\sqrt{-x^2+1}}$	30
risch	$-\frac{x(4x^2-3)}{3(x^2-1)\sqrt{-x^2+1}}+\arcsin(x)$	30
meijerg	$-\frac{2i\left(-\frac{i\sqrt{\pi}\,x\left(-20x^2+15\right)}{10\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{3i\sqrt{\pi}\,\arcsin(x)}{2}\right)}{3\sqrt{\pi}}$	39
pseudoelliptic	$\frac{(x^2-1)\sqrt{-x^2+1}\,\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right)+\frac{4x^3}{3}-x}{(-x^2+1)^{\frac{3}{2}}}$	49
trager	$\frac{(4x^2-3)x\sqrt{-x^2+1}}{3(x^2-1)^2}+\text{RootOf}\left(\_Z^2+1\right)\ln\left(\text{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)$	54

```
[In] int(x^4/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-1/(-x^2+1)^(1/2)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = -\frac{6\,(x^4-2\,x^2+1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-(4\,x^3-3\,x)\sqrt{-x^2+1}}{3\,(x^4-2\,x^2+1)}$$

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(26) = 52$.

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.00

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3} - \frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3}-\frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3}+\frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{1}{3}\,x\left(\frac{3\,x^2}{(-x^2+1)^{\frac{3}{2}}}-\frac{2}{(-x^2+1)^{\frac{3}{2}}}\right)-\frac{x}{3\,\sqrt{-x^2+1}}+\arcsin(x)$$

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx = \frac{\left(4\,x^2-3\right)\sqrt{-x^2+1}x}{3\,\left(x^2-1\right)^2} + \arcsin\left(x\right)$$

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.60

$$\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx = \mathrm{asin}(x) + \frac{3\sqrt{1-x^2}}{4\,\left(x-1\right)} + \frac{3\sqrt{1-x^2}}{4\,\left(x+1\right)} - \sqrt{1-x^2}\left(\frac{1}{12\,\left(x-1\right)} - \frac{1}{12\,\left(x-1\right)^2}\right) - \sqrt{1-x^2}\left(\frac{1}{12\,\left(x+1\right)} + \frac{1}{12\,\left(x+1\right)^2}\right)$$

```
[In] int(x^4/(1 - x^2)^(5/2),x)
```

```
[Out] asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1))
 - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*(1
/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

3.74 $\int \tan^4(y)\,dy$

Optimal result 339
Rubi [A] (verified) 339
Mathematica [A] (verified) 340
Maple [A] (verified) 340
Fricas [B] (verification not implemented) 341
Sympy [A] (verification not implemented) 341
Maxima [A] (verification not implemented) 341
Giac [A] (verification not implemented) 341
Mupad [B] (verification not implemented) 342

Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \tan^4(y)\,dy = y - \tan(y) + \frac{\tan^3(y)}{3}$$

```
[Out] y-tan(y)+1/3*tan(y)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$\int \tan^4(y)\,dy = y + \frac{\tan^3(y)}{3} - \tan(y)$$

```
[In] Int[Tan[y]^4,y]
```

```
[Out] y - Tan[y] + Tan[y]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\tan^3(y)}{3} - \int \tan^2(y)\,dy \\ &= -\tan(y) + \frac{\tan^3(y)}{3} + \int 1\,dy \\ &= y - \tan(y) + \frac{\tan^3(y)}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \tan^4(y)\,dy = \arctan(\tan(y)) - \tan(y) + \frac{\tan^3(y)}{3}$$

```
[In] Integrate[Tan[y]^4,y]
[Out] ArcTan[Tan[y]] - Tan[y] + Tan[y]^3/3
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$y-\tan\left(y\right)+\frac{\left(\tan^{3}\left(y\right)\right)}{3}$	13
parallelrisch	$y-\frac{4\tan\left(y\right)}{3}+\frac{\tan\left(y\right)\left(\sec^{2}\left(y\right)\right)}{3}$	15
risch	$y-\frac{4i\left(3\,\mathrm{e}^{4iy}+3\,\mathrm{e}^{2iy}+2\right)}{3\left(\mathrm{e}^{2iy}+1\right)^{3}}$	31
norman	$\frac{y\left(\tan^{6}\left(\frac{y}{2}\right)\right)-y-\frac{20\left(\tan^{3}\left(\frac{y}{2}\right)\right)}{3}+2\left(\tan^{5}\left(\frac{y}{2}\right)\right)+3y\left(\tan^{2}\left(\frac{y}{2}\right)\right)-3y\left(\tan^{4}\left(\frac{y}{2}\right)\right)+2\tan\left(\frac{y}{2}\right)}{\left(\tan^{2}\left(\frac{y}{2}\right)-1\right)^{3}}$	64

```
[In] int(sin(y)^4/cos(y)^4,y,method=_RETURNVERBOSE)
[Out] y-tan(y)+1/3*tan(y)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \tan^4(y)\,dy = \frac{3\,y\cos(y)^3 - \left(4\,\cos(y)^2 - 1\right)\sin(y)}{3\,\cos(y)^3}$$

```
[In] integrate(sin(y)^4/cos(y)^4,y, algorithm="fricas")
[Out] 1/3*(3*y*cos(y)^3 - (4*cos(y)^2 - 1)*sin(y))/cos(y)^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \tan^4(y)\,dy = y + \frac{\sin^3(y)}{3\cos^3(y)} - \frac{\sin(y)}{\cos(y)}$$

```
[In] integrate(sin(y)**4/cos(y)**4,y)
[Out] y + sin(y)**3/(3*cos(y)**3) - sin(y)/cos(y)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(y)\,dy = \frac{1}{3}\,\tan(y)^3 + y - \tan(y)$$

```
[In] integrate(sin(y)^4/cos(y)^4,y, algorithm="maxima")
[Out] 1/3*tan(y)^3 + y - tan(y)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(y)\,dy = \frac{1}{3}\,\tan(y)^3 + y - \tan(y)$$

```
[In] integrate(sin(y)^4/cos(y)^4,y, algorithm="giac")
[Out] 1/3*tan(y)^3 + y - tan(y)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(y)\,dy = \frac{\tan(y)^3}{3} - \tan(y) + y$$

```
[In] int(sin(y)^4/cos(y)^4,y)
[Out] y - tan(y) + tan(y)^3/3
```

3.75 $\int \frac{z^4}{1+z^2} dz$

Optimal result 343
Rubi [A] (verified) 343
Mathematica [A] (verified) 344
Maple [A] (verified) 344
Fricas [A] (verification not implemented) 345
Sympy [A] (verification not implemented) 345
Maxima [A] (verification not implemented) 345
Giac [A] (verification not implemented) 345
Mupad [B] (verification not implemented) 346

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{z^4}{1+z^2} \, dz = -z + \frac{z^3}{3} + \arctan(z)$$

```
[Out] -z+1/3*z^3+arctan(z)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {308, 209}

$$\int \frac{z^4}{1+z^2} \, dz = \arctan(z) + \frac{z^3}{3} - z$$

```
[In] Int[z^4/(1 + z^2),z]
```

```
[Out] -z + z^3/3 + ArcTan[z]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-1+z^2+\frac{1}{1+z^2}\right)\,dz\\ &= -z+\frac{z^3}{3}+\int \frac{1}{1+z^2}\,dz\\ &= -z+\frac{z^3}{3}+\arctan(z)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{z^4}{1+z^2}\,dz = -z+\frac{z^3}{3}+\arctan(z)$$

```
[In] Integrate[z^4/(1 + z^2),z]
[Out] -z + z^3/3 + ArcTan[z]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$-z+\frac{z^3}{3}+\arctan\left(z\right)$	12
risch	$-z+\frac{z^3}{3}+\arctan\left(z\right)$	12
meijerg	$-\frac{z\left(-5z^2+15\right)}{15}+\arctan\left(z\right)$	14
parallelrisch	$\frac{z^3}{3}-z+\frac{i\ln(z+i)}{2}-\frac{i\ln(z-i)}{2}$	26

```
[In] int(z^4/(z^2+1),z,method=_RETURNVERBOSE)
[Out] -z+1/3*z^3+arctan(z)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{z^4}{1+z^2}\,dz = \frac{1}{3}\,z^3 - z + \arctan\left(z\right)$$

```
[In] integrate(z^4/(z^2+1),z, algorithm="fricas")
[Out] 1/3*z^3 - z + arctan(z)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{z^4}{1+z^2}\,dz = \frac{z^3}{3} - z + \operatorname{atan}\left(z\right)$$

```
[In] integrate(z**4/(z**2+1),z)
[Out] z**3/3 - z + atan(z)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{z^4}{1+z^2}\,dz = \frac{1}{3}\,z^3 - z + \arctan\left(z\right)$$

```
[In] integrate(z^4/(z^2+1),z, algorithm="maxima")
[Out] 1/3*z^3 - z + arctan(z)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{z^4}{1+z^2}\,dz = \frac{1}{3}\,z^3 - z + \arctan\left(z\right)$$

```
[In] integrate(z^4/(z^2+1),z, algorithm="giac")
[Out] 1/3*z^3 - z + arctan(z)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{z^4}{1+z^2} \, dz = \operatorname{atan}(z) - z + \frac{z^3}{3}$$

```
[In] int(z^4/(z^2 + 1),z)
[Out] atan(z) - z + z^3/3
```

3.76 $\int e^{x^2}(1+2x^2)\,dx$

Optimal result 347
Rubi [A] (verified) 347
Mathematica [A] (verified) 348
Maple [A] (verified) 348
Fricas [A] (verification not implemented) 349
Sympy [A] (verification not implemented) 349
Maxima [A] (verification not implemented) 350
Giac [A] (verification not implemented) 350
Mupad [B] (verification not implemented) 350

Optimal result

Integrand size = 13, antiderivative size = 7

$$\int e^{x^2}\left(1+2x^2\right)\,dx=e^{x^2}x$$

```
[Out] exp(x^2)*x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2258, 2235, 2243}

$$\int e^{x^2}\left(1+2x^2\right)\,dx=e^{x^2}x$$

```
[In] Int[E^x^2*(1 + 2*x^2),x]
```

```
[Out] E^x^2*x
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
```

```
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2258

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(e^{x^2}+2e^{x^2}x^2\right)\,dx \\
&= 2\int e^{x^2}x^2\,dx+\int e^{x^2}\,dx \\
&= e^{x^2}x+\frac{1}{2}\sqrt{\pi}\,\text{erfi}(x)-\int e^{x^2}\,dx \\
&= e^{x^2}x
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int e^{x^2}\left(1+2x^2\right)\,dx=e^{x^2}x$$

```
[In] Integrate[E^x^2*(1 + 2*x^2),x]
[Out] E^x^2*x
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

method	result	size
gosper	$e^{x^2}x$	7
default	$e^{x^2}x$	7
norman	$e^{x^2}x$	7
risch	$e^{x^2}x$	7
parallelrisch	$e^{x^2}x$	7
meijerg	$i\left(-ix\,e^{x^2}+\frac{i\,\mathrm{erfi}(x)\sqrt{\pi}}{2}\right)+\frac{\mathrm{erfi}(x)\sqrt{\pi}}{2}$	29
parts	$\mathrm{erfi}\left(x\right)\sqrt{\pi}\,x^2+\frac{\mathrm{erfi}(x)\sqrt{\pi}}{2}-2\sqrt{\pi}\left(\frac{x^2\,\mathrm{erfi}(x)}{2}-\frac{\frac{e^{x^2}x}{2}-\frac{\mathrm{erfi}(x)\sqrt{\pi}}{4}}{\sqrt{\pi}}\right)$	51

```
[In] int(exp(x^2)*(2*x^2+1),x,method=_RETURNVERBOSE)
[Out] exp(x^2)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int e^{x^2}\left(1+2x^2\right)\,dx=xe^{\left(x^2\right)}$$

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="fricas")
[Out] x*e^(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int e^{x^2}\left(1+2x^2\right)\,dx=xe^{x^2}$$

```
[In] integrate(exp(x**2)*(2*x**2+1),x)
[Out] x*exp(x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int e^{x^2}\left(1+2x^2\right)\,dx = xe^{\left(x^2\right)}$$

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="maxima")
[Out] x*e^(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int e^{x^2}\left(1+2x^2\right)\,dx = xe^{\left(x^2\right)}$$

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="giac")
[Out] x*e^(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int e^{x^2}\left(1+2x^2\right)\,dx = x\,\mathrm{e}^{x^2}$$

```
[In] int(exp(x^2)*(2*x^2 + 1),x)
[Out] x*exp(x^2)
```

3.77 $\int \frac{e^{x^2}\left(1+4x^2+x^3+5x^4+2x^6\right)}{\left(1+x^2\right)^2} \, dx$

Optimal result 351
Rubi [A] (verified) 351
Mathematica [A] (verified) 353
Maple [A] (verified) 353
Fricas [A] (verification not implemented) 353
Sympy [A] (verification not implemented) 354
Maxima [A] (verification not implemented) 354
Giac [A] (verification not implemented) 354
Mupad [B] (verification not implemented) 355

Optimal result

Integrand size = 33, antiderivative size = 24

$$\int \frac{e^{x^2}\left(1+4x^2+x^3+5x^4+2x^6\right)}{\left(1+x^2\right)^2} \, dx = e^{x^2}x+\frac{e^{x^2}}{2\left(1+x^2\right)}$$

```
[Out] exp(x^2)*x+1/2*exp(x^2)/(x^2+1)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6874, 2235, 2243, 6847, 2208, 2209}

$$\int \frac{e^{x^2}\left(1+4x^2+x^3+5x^4+2x^6\right)}{\left(1+x^2\right)^2} \, dx = e^{x^2}x+\frac{e^{x^2}}{2\left(x^2+1\right)}$$

```
[In] Int[(E^x^2*(1 + 4*x^2 + x^3 + 5*x^4 + 2*x^6))/(1 + x^2)^2,x]
```

```
[Out] E^x^2*x + E^x^2/(2*(1 + x^2))
```

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(e^{x^2}+2e^{x^2}x^2-\frac{e^{x^2}x}{\left(1+x^2\right)^2}+\frac{e^{x^2}x}{1+x^2}\right)dx\\
&= 2\int e^{x^2}x^2\,dx+\int e^{x^2}\,dx-\int \frac{e^{x^2}x}{\left(1+x^2\right)^2}\,dx+\int \frac{e^{x^2}x}{1+x^2}\,dx\\
&= e^{x^2}x+\frac{1}{2}\sqrt{\pi}\operatorname{erfi}(x)-\frac{1}{2}\text{Subst}\left(\int \frac{e^x}{(1+x)^2}\,dx,x,x^2\right)\\
&\quad+\frac{1}{2}\text{Subst}\left(\int \frac{e^x}{1+x}\,dx,x,x^2\right)-\int e^{x^2}\,dx\\
&= e^{x^2}x+\frac{e^{x^2}}{2\left(1+x^2\right)}+\frac{\text{ExpIntegralEi}\left(1+x^2\right)}{2e}-\frac{1}{2}\text{Subst}\left(\int \frac{e^x}{1+x}\,dx,x,x^2\right)\\
&= e^{x^2}x+\frac{e^{x^2}}{2\left(1+x^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx = e^{x^2}\left(x+\frac{1}{2(1+x^2)}\right)$$

```
[In] Integrate[(E^x^2*(1 + 4*x^2 + x^3 + 5*x^4 + 2*x^6))/(1 + x^2)^2,x]
[Out] E^x^2*(x + 1/(2*(1 + x^2)))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{(2x^3+2x+1)e^{x^2}}{2x^2+2}$	24
risch	$\frac{(2x^3+2x+1)e^{x^2}}{2x^2+2}$	24
norman	$\frac{x^3e^{x^2}+e^{x^2}x+\frac{e^{x^2}}{2}}{x^2+1}$	30
parallelrisch	$\frac{2x^3e^{x^2}+2\,e^{x^2}x+e^{x^2}}{2x^2+2}$	31

```
[In] int(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*(2*x^3+2*x+1)*exp(x^2)/(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx = \frac{(2\,x^3+2\,x+1)e^{(x^2)}}{2\,(x^2+1)}$$

```
[In] integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="fricas
")
[Out] 1/2*(2*x^3 + 2*x + 1)*e^(x^2)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx = \frac{(2x^3+2x+1)\,e^{x^2}}{2x^2+2}$$

```
[In] integrate(exp(x**2)*(2*x**6+5*x**4+x**3+4*x**2+1)/(x**2+1)**2,x)
[Out] (2*x**3 + 2*x + 1)*exp(x**2)/(2*x**2 + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx = \frac{(2\,x^3+2\,x+1)e^{(x^2)}}{2\,(x^2+1)}$$

```
[In] integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="maxima
")
[Out] 1/2*(2*x^3 + 2*x + 1)*e^(x^2)/(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx = \frac{2\,x^3e^{(x^2)}+2\,xe^{(x^2)}+e^{(x^2)}}{2\,(x^2+1)}$$

```
[In] integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="giac")
[Out] 1/2*(2*x^3*e^(x^2) + 2*x*e^(x^2) + e^(x^2))/(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx=\frac{\mathrm{e}^{x^2}\,\left(2\,x^3+2\,x+1\right)}{2\,\left(x^2+1\right)}$$

```
[In] int((exp(x^2)*(4*x^2 + x^3 + 5*x^4 + 2*x^6 + 1))/(x^2 + 1)^2,x)
[Out] (exp(x^2)*(2*x + 2*x^3 + 1))/(2*(x^2 + 1))
```

3.78 $\int e^{-1-x}\,dx$

Optimal result 356
Rubi [A] (verified) 356
Mathematica [A] (verified) 357
Maple [A] (verified) 357
Fricas [A] (verification not implemented) 357
Sympy [A] (verification not implemented) 358
Maxima [A] (verification not implemented) 358
Giac [A] (verification not implemented) 358
Mupad [B] (verification not implemented) 358

Optimal result

Integrand size = 7, antiderivative size = 9

$$\int e^{-1-x}\,dx = -e^{-1-x}$$

```
[Out] -exp(-1-x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{2225\}$

$$\int e^{-1-x}\,dx = -e^{-x-1}$$

```
[In] Int[E^(-1 - x),x]
```

```
[Out] -E^(-1 - x)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\text{integral} = -e^{-1-x}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int e^{-1-x}\,dx = -e^{-1-x}$$

```
[In] Integrate[E^(-1 - x),x]
[Out] -E^(-1 - x)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\mathrm{e}^{-1-x}$	9
derivativedivides	$-\mathrm{e}^{-1-x}$	9
default	$-\mathrm{e}^{-1-x}$	9
norman	$-\mathrm{e}^{-1-x}$	9
risch	$-\mathrm{e}^{-1-x}$	9
parallelrisch	$-\mathrm{e}^{-1-x}$	9
meijerg	$\mathrm{e}^{-1}(1-\mathrm{e}^{-x})$	12

```
[In] int(exp(-1-x),x,method=_RETURNVERBOSE)
[Out] -exp(-1-x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{-1-x}\,dx = -e^{(-x-1)}$$

```
[In] integrate(exp(-1-x),x, algorithm="fricas")
[Out] -e^(-x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int e^{-1-x}\,dx = -e^{-x-1}$$

```
[In] integrate(exp(-1-x),x)
[Out] -exp(-x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{-1-x}\,dx = -e^{(-x-1)}$$

```
[In] integrate(exp(-1-x),x, algorithm="maxima")
[Out] -e^(-x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{-1-x}\,dx = -e^{(-x-1)}$$

```
[In] integrate(exp(-1-x),x, algorithm="giac")
[Out] -e^(-x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{-1-x}\,dx = -\mathrm{e}^{-x-1}$$

```
[In] int(exp(- x - 1),x)
[Out] -exp(- x - 1)
```

3.79 $\int \left(\frac{1}{x}+x\right) \log(x) \, dx$

Optimal result 359
Rubi [A] (verified) 359
Mathematica [A] (verified) 360
Maple [A] (verified) 361
Fricas [A] (verification not implemented) 361
Sympy [A] (verification not implemented) 361
Maxima [A] (verification not implemented) 362
Giac [A] (verification not implemented) 362
Mupad [B] (verification not implemented) 362

Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \left(\frac{1}{x}+x\right) \log(x) \, dx = -\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}$$

```
[Out] -1/4*x^2+1/2*x^2*ln(x)+1/2*ln(x)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1607, 14, 2393, 2338, 2341}

$$\int \left(\frac{1}{x}+x\right) \log(x) \, dx = -\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}$$

```
[In] Int[(x^(-1) + x)*Log[x],x]
```

```
[Out] -1/4*x^2 + (x^2*Log[x])/2 + Log[x]^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1+x^2)\log(x)}{x}\,dx \\
&= \int \left(\frac{\log(x)}{x} + x\log(x)\right)\,dx \\
&= \int \frac{\log(x)}{x}\,dx + \int x\log(x)\,dx \\
&= -\frac{x^2}{4} + \frac{1}{2}x^2\log(x) + \frac{\log^2(x)}{2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{x}+x\right)\log(x)\,dx = -\frac{x^2}{4} + \frac{1}{2}x^2\log(x) + \frac{\log^2(x)}{2}$$

```
[In] Integrate[(x^(-1) + x)*Log[x],x]
[Out] -1/4*x^2 + (x^2*Log[x])/2 + Log[x]^2/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}+\frac{\ln(x)^2}{2}$	20
norman	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}+\frac{\ln(x)^2}{2}$	20
risch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}+\frac{\ln(x)^2}{2}$	20
parallelrisch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}+\frac{\ln(x)^2}{2}$	20
parts	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}+\frac{\ln(x)^2}{2}$	20

```
[In] int((1/x+x)*ln(x),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2+1/2*x^2*ln(x)+1/2*ln(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(\frac{1}{x}+x\right)\log(x)\,dx=\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2+\frac{1}{2}\,\log\left(x\right)^2$$

```
[In] integrate((1/x+x)*log(x),x, algorithm="fricas")
[Out] 1/2*x^2*log(x) - 1/4*x^2 + 1/2*log(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(\frac{1}{x}+x\right)\log(x)\,dx=\frac{x^2\log\left(x\right)}{2}-\frac{x^2}{4}+\frac{\log\left(x\right)^2}{2}$$

```
[In] integrate((1/x+x)*ln(x),x)
[Out] x**2*log(x)/2 - x**2/4 + log(x)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \left(\frac{1}{x}+x\right)\log(x)\,dx=-\frac{1}{4}\,x^2+\frac{1}{2}\left(x^2+2\,\log\left(x\right)\right)\log\left(x\right)-\frac{1}{2}\,\log\left(x\right)^2$$

```
[In] integrate((1/x+x)*log(x),x, algorithm="maxima")
[Out] -1/4*x^2 + 1/2*(x^2 + 2*log(x))*log(x) - 1/2*log(x)^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(\frac{1}{x}+x\right)\log(x)\,dx=\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2+\frac{1}{2}\,\log\left(x\right)^2$$

```
[In] integrate((1/x+x)*log(x),x, algorithm="giac")
[Out] 1/2*x^2*log(x) - 1/4*x^2 + 1/2*log(x)^2
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(\frac{1}{x}+x\right)\log(x)\,dx=\frac{x^2\,\ln\left(x\right)}{2}-\frac{x^2}{4}+\frac{\ln\left(x\right)^2}{2}$$

```
[In] int(log(x)*(x + 1/x),x)
[Out] (x^2*log(x))/2 + log(x)^2/2 - x^2/4
```

3.80 $\int \frac{x}{1+x^4} dx$

Optimal result . . . 363
Rubi [A] (verified) . . . 363
Mathematica [A] (verified) . . . 364
Maple [A] (verified) . . . 364
Fricas [A] (verification not implemented) . . . 365
Sympy [A] (verification not implemented) . . . 365
Maxima [A] (verification not implemented) . . . 365
Giac [A] (verification not implemented) . . . 365
Mupad [B] (verification not implemented) . . . 366

Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \frac{x}{1+x^4} dx = \frac{\arctan(x^2)}{2}$$

```
[Out] 1/2*arctan(x^2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {281, 209}

$$\int \frac{x}{1+x^4} dx = \frac{\arctan(x^2)}{2}$$

```
[In] Int[x/(1 + x^4),x]
```

```
[Out] ArcTan[x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right)$$
$$= \frac{\arctan(x^2)}{2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x^4}\,dx = \frac{\arctan(x^2)}{2}$$

```
[In] Integrate[x/(1 + x^4),x]
[Out] ArcTan[x^2]/2
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\arctan(x^2)}{2}$	7
meijerg	$\frac{\arctan(x^2)}{2}$	7
risch	$\frac{\arctan(x^2)}{2}$	7
parallelrisch	$\frac{i\ln(x^2+i)}{4}-\frac{i\ln(x^2-i)}{4}$	22

```
[In] int(x/(x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\arctan\left(x^2\right)$$

```
[In] integrate(x/(x^4+1),x, algorithm="fricas")
[Out] 1/2*arctan(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{x}{1+x^4}\,dx = \frac{\operatorname{atan}\left(x^2\right)}{2}$$

```
[In] integrate(x/(x**4+1),x)
[Out] atan(x**2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\arctan\left(x^2\right)$$

```
[In] integrate(x/(x^4+1),x, algorithm="maxima")
[Out] 1/2*arctan(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\arctan\left(x^2\right)$$

```
[In] integrate(x/(x^4+1),x, algorithm="giac")
[Out] 1/2*arctan(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{1+x^4} \, dx = \frac{\text{atan}(x^2)}{2}$$

```
[In] int(x/(x^4 + 1),x)
[Out] atan(x^2)/2
```

3.81 $\int \frac{x^5}{1+x^4} dx$

Optimal result 367
Rubi [A] (verified) 367
Mathematica [A] (verified) 368
Maple [A] (verified) 368
Fricas [A] (verification not implemented) 369
Sympy [A] (verification not implemented) 369
Maxima [A] (verification not implemented) 369
Giac [A] (verification not implemented) 369
Mupad [B] (verification not implemented) 370

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{x^5}{1+x^4} dx = \frac{x^2}{2} - \frac{\arctan(x^2)}{2}$$

```
[Out] 1/2*x^2-1/2*arctan(x^2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {281, 327, 209}

$$\int \frac{x^5}{1+x^4} dx = \frac{x^2}{2} - \frac{\arctan(x^2)}{2}$$

```
[In] Int[x^5/(1 + x^4),x]
```

```
[Out] x^2/2 - ArcTan[x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, x^2\right) \\ &= \frac{x^2}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right) \\ &= \frac{x^2}{2} - \frac{\arctan\left(x^2\right)}{2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{1+x^4}\,dx = \frac{x^2}{2} - \frac{\arctan\left(x^2\right)}{2}$$

```
[In] Integrate[x^5/(1 + x^4),x]
[Out] x^2/2 - ArcTan[x^2]/2
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x^2}{2} - \frac{\arctan(x^2)}{2}$	13
meijerg	$\frac{x^2}{2} - \frac{\arctan(x^2)}{2}$	13
risch	$\frac{x^2}{2} - \frac{\arctan(x^2)}{2}$	13
parallelrisch	$\frac{x^2}{2} + \frac{i\ln(x^2-i)}{4} - \frac{i\ln(x^2+i)}{4}$	27

```
[In] int(x^5/(x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-1/2*arctan(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{1+x^4}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,\arctan\left(x^2\right)$$

```
[In] integrate(x^5/(x^4+1),x, algorithm="fricas")
[Out] 1/2*x^2 - 1/2*arctan(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x^5}{1+x^4}\,dx = \frac{x^2}{2} - \frac{\operatorname{atan}\left(x^2\right)}{2}$$

```
[In] integrate(x**5/(x**4+1),x)
[Out] x**2/2 - atan(x**2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{1+x^4}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,\arctan\left(x^2\right)$$

```
[In] integrate(x^5/(x^4+1),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/2*arctan(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{1+x^4}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,\arctan\left(x^2\right)$$

```
[In] integrate(x^5/(x^4+1),x, algorithm="giac")
[Out] 1/2*x^2 - 1/2*arctan(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{1+x^4}\,dx = \frac{x^2}{2} - \frac{\operatorname{atan}(x^2)}{2}$$

```
[In] int(x^5/(x^4 + 1),x)
[Out] x^2/2 - atan(x^2)/2
```

3.82 $\int \frac{1}{1+\tan^2(x)}\,dx$

Optimal result 371
Rubi [A] (verified) 371
Mathematica [A] (verified) 372
Maple [A] (verified) 372
Fricas [A] (verification not implemented) 373
Sympy [B] (verification not implemented) 373
Maxima [A] (verification not implemented) 373
Giac [A] (verification not implemented) 374
Mupad [B] (verification not implemented) 374

Optimal result

Integrand size = 8, antiderivative size = 14

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3738, 2715, 8}

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[In] Int[(1 + Tan[x]^2)^(-1),x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \cos^2(x)\,dx \\ &= \frac{1}{2}\cos(x)\sin(x) + \frac{\int 1\,dx}{2} \\ &= \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{x}{2} + \frac{1}{4}\sin(2x)$$

```
[In] Integrate[(1 + Tan[x]^2)^(-1),x]
[Out] x/2 + Sin[2*x]/4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
derivativedivides	$\frac{\tan(x)}{2+2(\tan^2(x))}+\frac{\arctan(\tan(x))}{2}$	19
default	$\frac{\tan(x)}{2+2(\tan^2(x))}+\frac{\arctan(\tan(x))}{2}$	19
parallelrisch	$\frac{x(\tan^2(x))+x+\tan(x)}{2+2(\tan^2(x))}$	21
norman	$\frac{\frac{x}{2}+\frac{x\left(\tan^2(x)\right)}{2}+\frac{\tan(x)}{2}}{1+\tan^2(x)}$	25

```
[In] int(1/(1+tan(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/4*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{x\tan(x)^2 + x + \tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

```
[In] integrate(1/(1+tan(x)^2),x, algorithm="fricas")
[Out] 1/2*(x*tan(x)^2 + x + tan(x))/(tan(x)^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(10) = 20.

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.57

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{x\tan^2(x)}{2\tan^2(x)+2} + \frac{x}{2\tan^2(x)+2} + \frac{\tan(x)}{2\tan^2(x)+2}$$

```
[In] integrate(1/(1+tan(x)**2),x)
[Out] x*tan(x)**2/(2*tan(x)**2 + 2) + x/(2*tan(x)**2 + 2) + tan(x)/(2*tan(x)**2 +
 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{1}{2}\,x + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

```
[In] integrate(1/(1+tan(x)^2),x, algorithm="maxima")
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{1}{2}\,x + \frac{\tan\left(x\right)}{2\,\left(\tan\left(x\right)^2+1\right)}$$

```
[In] integrate(1/(1+tan(x)^2),x, algorithm="giac")
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{x}{2} + \frac{\sin\left(2\,x\right)}{4}$$

```
[In] int(1/(tan(x)^2 + 1),x)
[Out] x/2 + sin(2*x)/4
```

3.83 $\int \frac{x^4}{(1-x^2)^{5/2}}\,dx$

Optimal result 375
Rubi [A] (verified) 375
Mathematica [A] (verified) 376
Maple [A] (verified) 376
Fricas [B] (verification not implemented) 377
Sympy [B] (verification not implemented) 377
Maxima [A] (verification not implemented) 377
Giac [A] (verification not implemented) 378
Mupad [B] (verification not implemented) 378

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \arcsin(x)$$

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {294, 222}

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \arcsin(x) - \frac{x}{\sqrt{1-x^2}} + \frac{x^3}{3(1-x^2)^{3/2}}$$

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^3}{3(1-x^2)^{3/2}} - \int \frac{x^2}{(1-x^2)^{3/2}}\,dx \\ &= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \arcsin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{x(-3+4x^2)}{3(1-x^2)^{3/2}} + 2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x^3}{3(-x^2+1)^{\frac{3}{2}}}+\arcsin\left(x\right)-\frac{x}{\sqrt{-x^2+1}}$	30
risch	$-\frac{x(4x^2-3)}{3(x^2-1)\sqrt{-x^2+1}}+\arcsin\left(x\right)$	30
meijerg	$-\frac{2i\left(-\frac{i\sqrt{\pi}\,x\left(-20x^2+15\right)}{10\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{3i\sqrt{\pi}\,\arcsin(x)}{2}\right)}{3\sqrt{\pi}}$	39
pseudoelliptic	$\frac{(x^2-1)\sqrt{-x^2+1}\,\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right)+\frac{4x^3}{3}-x}{(-x^2+1)^{\frac{3}{2}}}$	49
trager	$\frac{(4x^2-3)x\sqrt{-x^2+1}}{3(x^2-1)^2}+\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)$	54

```
[In] int(x^4/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-1/(-x^2+1)^(1/2)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = -\frac{6\,(x^4-2\,x^2+1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-(4\,x^3-3\,x)\sqrt{-x^2+1}}{3\,(x^4-2\,x^2+1)}$$

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(26) = 52$.

Time = 0.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.00

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3} - \frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3}-\frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3}+\frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{1}{3}\,x\left(\frac{3\,x^2}{(-x^2+1)^{\frac{3}{2}}}-\frac{2}{(-x^2+1)^{\frac{3}{2}}}\right)-\frac{x}{3\,\sqrt{-x^2+1}}+\arcsin(x)$$

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{(4\,x^2-3)\sqrt{-x^2+1}x}{3\,(x^2-1)^2} + \arcsin(x)$$

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.60

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \operatorname{asin}(x) + \frac{3\sqrt{1-x^2}}{4\,(x-1)} + \frac{3\sqrt{1-x^2}}{4\,(x+1)} - \sqrt{1-x^2}\left(\frac{1}{12\,(x-1)} - \frac{1}{12\,(x-1)^2}\right) - \sqrt{1-x^2}\left(\frac{1}{12\,(x+1)} + \frac{1}{12\,(x+1)^2}\right)$$

```
[In] int(x^4/(1 - x^2)^(5/2),x)
```

```
[Out] asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1))
 - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*(1
/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

3.84 $\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx$

Optimal result 379
Rubi [A] (verified) 379
Mathematica [B] (verified) 380
Maple [A] (verified) 380
Fricas [B] (verification not implemented) 381
Sympy [B] (verification not implemented) 381
Maxima [A] (verification not implemented) 381
Giac [A] (verification not implemented) 382
Mupad [B] (verification not implemented) 382

Optimal result

Integrand size = 16, antiderivative size = 17

$$\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx = -\frac{x}{\sqrt{1-x^2}} + \arcsin(x)$$

```
[Out] arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {294, 222}

$$\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx = \arcsin(x) - \frac{x}{\sqrt{1-x^2}}$$

```
[In] Int[-(x^2/(1 - x^2)^(3/2)),x]
[Out] -(x/Sqrt[1 - x^2]) + ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x}{\sqrt{1-x^2}}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{x}{\sqrt{1-x^2}}+\arcsin(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx = -\frac{x}{\sqrt{1-x^2}}+2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

```
[In] Integrate[-(x^2/(1 - x^2)^(3/2)),x]
[Out] -(x/Sqrt[1 - x^2]) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\arcsin\left(x\right)-\frac{x}{\sqrt{-x^{2}+1}}$	16
risch	$\arcsin\left(x\right)-\frac{x}{\sqrt{-x^{2}+1}}$	16
meijerg	$-\frac{i\left(-\frac{i\sqrt{\pi}\,x}{\sqrt{-x^{2}+1}}+i\sqrt{\pi}\,\arcsin(x)\right)}{\sqrt{\pi}}$	32
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-x^{2}+1}}{x}\right)\sqrt{-x^{2}+1}+x}{\sqrt{-x^{2}+1}}$	38
trager	$\frac{x\sqrt{-x^{2}+1}}{x^{2}-1}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)$	46

```
[In] int(-1/(-x^2+1)^(3/2)*x^2,x,method=_RETURNVERBOSE)
[Out] arcsin(x)-1/(-x^2+1)^(1/2)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(15) = 30.

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.65

$$\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx = -\frac{2\,(x^2-1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\sqrt{-x^2+1}x}{x^2-1}$$

```
[In] integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="fricas")
[Out] -(2*(x^2 - 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - sqrt(-x^2 + 1)*x)/(x^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(12) = 24.

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

$$\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx = \frac{x^2\,\mathrm{asin}\,(x)}{x^2-1}+\frac{x\sqrt{1-x^2}}{x^2-1}-\frac{\mathrm{asin}\,(x)}{x^2-1}$$

```
[In] integrate(-x**2/(-x**2+1)**(3/2),x)
[Out] x**2*asin(x)/(x**2 - 1) + x*sqrt(1 - x**2)/(x**2 - 1) - asin(x)/(x**2 - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx = -\frac{x}{\sqrt{-x^2+1}}+\arcsin\left(x\right)$$

```
[In] integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="maxima")
[Out] -x/sqrt(-x^2 + 1) + arcsin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx = \frac{\sqrt{-x^2+1}x}{x^2-1} + \arcsin(x)$$

```
[In] integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="giac")
[Out] sqrt(-x^2 + 1)*x/(x^2 - 1) + arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx = \mathrm{asin}(x) + \frac{\sqrt{1-x^2}}{2\,(x-1)} + \frac{\sqrt{1-x^2}}{2\,(x+1)}$$

```
[In] int(-x^2/(1 - x^2)^(3/2),x)
[Out] asin(x) + (1 - x^2)^(1/2)/(2*(x - 1)) + (1 - x^2)^(1/2)/(2*(x + 1))
```

3.85 $\int e^x \sin(x)\, dx$

Optimal result 383
Rubi [A] (verified) 383
Mathematica [A] (verified) 384
Maple [A] (verified) 384
Fricas [A] (verification not implemented) 384
Sympy [A] (verification not implemented) 385
Maxima [A] (verification not implemented) 385
Giac [A] (verification not implemented) 385
Mupad [B] (verification not implemented) 385

Optimal result

Integrand size = 6, antiderivative size = 19

$$\int e^x \sin(x)\, dx = -\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)$$

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4517}

$$\int e^x \sin(x)\, dx = \frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

```
[In] Int[E^x*Sin[x],x]
```

```
[Out] -1/2*(E^x*Cos[x]) + (E^x*Sin[x])/2
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int e^x \sin(x)\,dx = \frac{1}{2}e^x(-\cos(x)+\sin(x))$$

```
[In] Integrate[E^x*Sin[x],x]
[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{e^x(-\cos(x)+\sin(x))}{2}$	12
default	$-\frac{e^x\cos(x)}{2}+\frac{e^x\sin(x)}{2}$	14
norman	$\frac{e^x\tan\left(\frac{x}{2}\right)+\frac{e^x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{e^x}{2}}{1+\tan^2\left(\frac{x}{2}\right)}$	34
risch	$-\frac{e^{(1+i)x}}{4}-\frac{ie^{(1+i)x}}{4}-\frac{e^{(1-i)x}}{4}+\frac{ie^{(1-i)x}}{4}$	36

```
[In] int(exp(x)*sin(x),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x)*(-cos(x)+sin(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\cos(x)e^x + \frac{1}{2}e^x\sin(x)$$

```
[In] integrate(exp(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int e^x \sin(x)\,dx = \frac{e^x \sin(x)}{2} - \frac{e^x \cos(x)}{2}$$

```
[In] integrate(exp(x)*sin(x),x)
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\left(\cos(x) - \sin(x)\right)e^x$$

```
[In] integrate(exp(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\left(\cos(x) - \sin(x)\right)e^x$$

```
[In] integrate(exp(x)*sin(x),x, algorithm="giac")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{\mathrm{e}^x\left(\cos(x) - \sin(x)\right)}{2}$$

```
[In] int(exp(x)*sin(x),x)
[Out] -(exp(x)*(cos(x) - sin(x)))/2
```

3.86 $\int \frac{1}{x} \, dx$

Optimal result 386
Rubi [A] (verified) 386
Mathematica [A] (verified) 387
Maple [A] (verified) 387
Fricas [A] (verification not implemented) 387
Sympy [A] (verification not implemented) 388
Maxima [A] (verification not implemented) 388
Giac [A] (verification not implemented) 388
Mupad [B] (verification not implemented) 388

Optimal result

Integrand size = 3, antiderivative size = 2

$$\int \frac{1}{x} \, dx = \log(x)$$

```
[Out] ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {29}

$$\int \frac{1}{x} \, dx = \log(x)$$

```
[In] Int[x^(-1),x]
```

```
[Out] Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\text{integral} = \log(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} dx = \log(x)$$

```
[In] Integrate[x^(-1),x]
[Out] Log[x]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\ln(x)$	3
norman	$\ln(x)$	3
risch	$\ln(x)$	3
parallelrisch	$\ln(x)$	3

```
[In] int(1/x,x,method=_RETURNVERBOSE)
[Out] ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} dx = \log(x)$$

```
[In] integrate(1/x,x, algorithm="fricas")
[Out] log(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x}\,dx = \log\left(x\right)$$

```
[In] integrate(1/x,x)
[Out] log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x}\,dx = \log\left(x\right)$$

```
[In] integrate(1/x,x, algorithm="maxima")
[Out] log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{x}\,dx = \log\left(|x|\right)$$

```
[In] integrate(1/x,x, algorithm="giac")
[Out] log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x}\,dx = \ln\left(x\right)$$

```
[In] int(1/x,x)
[Out] log(x)
```

3.87 $\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt$

Optimal result . . . 389
Rubi [A] (verified) . . . 389
Mathematica [A] (verified) . . . 390
Maple [A] (verified) . . . 391
Fricas [A] (verification not implemented) . . . 391
Sympy [F] . . . 391
Maxima [B] (verification not implemented) . . . 392
Giac [A] (verification not implemented) . . . 392
Mupad [B] (verification not implemented) . . . 393

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt = -\frac{1}{12}\log(\cos(t)-\sin(t))-\frac{1}{4}\log(\cos(t)+\sin(t)) +\frac{1}{3}\log(2\cos(t)+\sin(t))-\frac{1}{2(1+\tan(t))}$$

```
[Out] -1/12*ln(cos(t)-sin(t))-1/4*ln(cos(t)+sin(t))+1/3*ln(2*cos(t)+sin(t))-1/2/(
1+tan(t))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {723, 814}

$$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt = -\frac{1}{2(\tan(t)+1)}-\frac{1}{12}\log(\cos(t)-\sin(t)) -\frac{1}{4}\log(\sin(t)+\cos(t))+\frac{1}{3}\log(\sin(t)+2\cos(t))$$

```
[In] Int[Sec[2*t]/(1 + Sec[t]^2 + 3*Tan[t]),t]
```

```
[Out] -1/12*Log[Cos[t] - Sin[t]] - Log[Cos[t] + Sin[t]]/4 + Log[2*Cos[t] + Sin[t]
]/3 - 1/(2*(1 + Tan[t]))
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
```

```
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(1+t)^2\,(2-t-t^2)}\,dt, t, \tan(t)\right)\\
&= -\frac{1}{2(1+\tan(t))} + \frac{1}{2}\text{Subst}\left(\int \frac{t}{(1+t)\,(2-t-t^2)}\,dt, t, \tan(t)\right)\\
&= -\frac{1}{2(1+\tan(t))} + \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{6(-1+t)} - \frac{1}{2(1+t)} + \frac{2}{3(2+t)}\right)\,dt, t, \tan(t)\right)\\
&= -\frac{1}{12}\log(\cos(t)-\sin(t)) - \frac{1}{4}\log(\cos(t)+\sin(t)) + \frac{1}{3}\log(2\cos(t)+\sin(t)) - \frac{1}{2(1+\tan(t))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt = -\frac{\cos(t)(\log(\cos(t)-\sin(t))+3\log(\cos(t)+\sin(t))-4\log(2\cos(t)+\sin(t)))+(-6+\log(\cos(t)-\sin(t)}{12(\cos(t)+\sin(t))}$$

```
[In] Integrate[Sec[2*t]/(1 + Sec[t]^2 + 3*Tan[t]),t]
```

```
[Out] -1/12*(Cos[t]*(Log[Cos[t] - Sin[t]] + 3*Log[Cos[t] + Sin[t]] - 4*Log[2*Cos[
t] + Sin[t]]) + (-6 + Log[Cos[t] - Sin[t]] + 3*Log[Cos[t] + Sin[t]] - 4*Log
[2*Cos[t] + Sin[t]])*Sin[t])/(Cos[t] + Sin[t])
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\ln(\tan(t)+2)}{3}-\frac{1}{2(1+\tan(t))}-\frac{\ln(1+\tan(t))}{4}-\frac{\ln(\tan(t)-1)}{12}$	31
risch	$-\frac{1}{2(e^{2it}+i)}-\frac{\ln(e^{2it}-i)}{12}+\frac{\ln(e^{2it}+\frac{3}{5}+\frac{4i}{5})}{3}-\frac{\ln(e^{2it}+i)}{4}$	48

```
[In] int(sec(2*t)/(1+sec(t)^2+3*tan(t)),t,method=_RETURNVERBOSE)
[Out] 1/3*ln(tan(t)+2)-1/2/(1+tan(t))-1/4*ln(1+tan(t))-1/12*ln(tan(t)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

$$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt$$
$$=\frac{4(\cos(t)+\sin(t))\log\left(\frac{3}{4}\cos(t)^2+\cos(t)\sin(t)+\frac{1}{4}\right)-3(\cos(t)+\sin(t))\log(2\cos(t)\sin(t)+1)-(\mathrm{co}}{24(\cos(t)+\sin(t))}$$

```
[In] integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="fricas")
[Out] 1/24*(4*(cos(t) + sin(t))*log(3/4*cos(t)^2 + cos(t)*sin(t) + 1/4) - 3*(cos(
t) + sin(t))*log(2*cos(t)*sin(t) + 1) - (cos(t) + sin(t))*log(-2*cos(t)*sin
(t) + 1) - 6*cos(t) + 6*sin(t))/(cos(t) + sin(t))
```

Sympy [F]

$$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt=\int \frac{\sec(2t)}{3\tan(t)+\sec^2(t)+1}\,dt$$

```
[In] integrate(sec(2*t)/(1+sec(t)**2+3*tan(t)),t)
[Out] Integral(sec(2*t)/(3*tan(t) + sec(t)**2 + 1), t)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(37) = 74$.

Time = 0.36 (sec) , antiderivative size = 256, normalized size of antiderivative = 5.69

$$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt$$
$$= \frac{3\left(\cos(2t)^2+\sin(2t)^2+2\sin(2t)+1\right)\log\left(953674316406250\,(3\cos(2t)+\sin(2t)+4)\cos(4t)+23841\right.}{}$$

```
[In] integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="maxima")
```

```
[Out] 1/48*(3*(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)*log(953674316406250*(3*c
os(2*t) + sin(2*t) + 4)*cos(4*t) + 2384185791015625*cos(4*t)^2 + 9536743164
06250*cos(2*t)^2 - 953674316406250*(cos(2*t) - 3*sin(2*t) + 3)*sin(4*t) + 2
384185791015625*sin(4*t)^2 + 953674316406250*sin(2*t)^2 + 2861022949218750*
cos(2*t) - 953674316406250*sin(2*t) + 2384185791015625) - 6*(cos(2*t)^2 + s
in(2*t)^2 + 2*sin(2*t) + 1)*log(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1) +
 5*(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)*log(1/5*(5*cos(2*t)^2 + 5*sin
(2*t)^2 + 6*cos(2*t) + 8*sin(2*t) + 5)/(cos(2*t)^2 + sin(2*t)^2 - 2*sin(2*t
) + 1)) - 24*cos(2*t))/(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt = -\frac{1}{2\,(\tan(t)+1)}+\frac{1}{3}\log\left(|\tan(t)+2|\right) - \frac{1}{4}\log\left(|\tan(t)+1|\right)-\frac{1}{12}\log\left(|\tan(t)-1|\right)$$

```
[In] integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="giac")
```

```
[Out] -1/2/(tan(t) + 1) + 1/3*log(abs(tan(t) + 2)) - 1/4*log(abs(tan(t) + 1)) - 1
/12*log(abs(tan(t) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

$$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt = \frac{\ln\left(\tan(t)+2\right)}{3} - \frac{\ln\left(\tan(t)+1\right)}{4} - \frac{\ln\left(\tan(t)-1\right)}{12} - \frac{1}{2\,\left(\tan\left(t\right)+1\right)}$$

```
[In] int(1/(cos(2*t)*(3*tan(t) + 1/cos(t)^2 + 1)),t)
[Out] log(tan(t) + 2)/3 - log(tan(t) + 1)/4 - log(tan(t) - 1)/12 - 1/(2*(tan(t) +
 1))
```

3.88 $\int \cos^2(x)\,dx$

Optimal result 394
Rubi [A] (verified) 394
Mathematica [A] (verified) 395
Maple [A] (verified) 395
Fricas [A] (verification not implemented) 396
Sympy [A] (verification not implemented) 396
Maxima [A] (verification not implemented) 396
Giac [A] (verification not implemented) 396
Mupad [B] (verification not implemented) 397

Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\sin(x)\cos(x)$$

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\cos(x)\sin(x) + \frac{\int 1\,dx}{2} \\
&= \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{4}\sin(2x)$$

```
[In] Integrate[Cos[x]^2,x]
[Out] x/2 + Sin[2*x]/4
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

```
[In] int(1/sec(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\, dx = \frac{1}{2} \cos(x) \sin(x) + \frac{1}{2} x$$

```
[In] integrate(1/sec(x)^2,x, algorithm="fricas")
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\, dx = \frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

```
[In] integrate(1/sec(x)**2,x)
[Out] x/2 + sin(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\, dx = \frac{1}{2} x + \frac{1}{4} \sin(2x)$$

```
[In] integrate(1/sec(x)^2,x, algorithm="maxima")
[Out] 1/2*x + 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \cos^2(x)\, dx = \frac{1}{2} x + \frac{\tan(x)}{2 \left(\tan(x)^2 + 1\right)}$$

```
[In] integrate(1/sec(x)^2,x, algorithm="giac")
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(2\,x)}{4}$$

[In] `int(cos(x)^2,x)`

[Out] `x/2 + sin(2*x)/4`

3.89 $\int \frac{1+x^2}{\sqrt{x}} \, dx$

Optimal result . . . 398
Rubi [A] (verified) . . . 398
Mathematica [A] (verified) . . . 399
Maple [A] (verified) . . . 399
Fricas [A] (verification not implemented) . . . 399
Sympy [A] (verification not implemented) . . . 400
Maxima [A] (verification not implemented) . . . 400
Giac [A] (verification not implemented) . . . 400
Mupad [B] (verification not implemented) . . . 400

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1+x^2}{\sqrt{x}} \, dx = 2\sqrt{x} + \frac{2x^{5/2}}{5}$$

```
[Out] 2/5*x^(5/2)+2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\int \frac{1+x^2}{\sqrt{x}} \, dx = \frac{2x^{5/2}}{5} + 2\sqrt{x}$$

```
[In] Int[(1 + x^2)/Sqrt[x],x]
[Out] 2*Sqrt[x] + (2*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{\sqrt{x}} + x^{3/2}\right) \, dx \\ &= 2\sqrt{x} + \frac{2x^{5/2}}{5} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1+x^2}{\sqrt{x}}\,dx = \frac{2}{5}\sqrt{x}(5+x^2)$$

```
[In] Integrate[(1 + x^2)/Sqrt[x],x]
[Out] (2*Sqrt[x]*(5 + x^2))/5
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{2\sqrt{x}\,(x^2+5)}{5}$	11
risch	$\frac{2\sqrt{x}\,(x^2+5)}{5}$	11
derivativedivides	$\frac{2x^{\frac{5}{2}}}{5}+2\sqrt{x}$	12
default	$\frac{2x^{\frac{5}{2}}}{5}+2\sqrt{x}$	12
trager	$\left(\frac{2x^2}{5}+2\right)\sqrt{x}$	12

```
[In] int((x^2+1)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*x^(1/2)*(x^2+5)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{1+x^2}{\sqrt{x}}\,dx = \frac{2}{5}\left(x^2+5\right)\sqrt{x}$$

```
[In] integrate((x^2+1)/x^(1/2),x, algorithm="fricas")
[Out] 2/5*(x^2 + 5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1+x^2}{\sqrt{x}}\, dx = \frac{2x^{\frac{5}{2}}}{5} + 2\sqrt{x}$$

```
[In] integrate((x**2+1)/x**(1/2),x)
[Out] 2*x**(5/2)/5 + 2*sqrt(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1+x^2}{\sqrt{x}}\, dx = \frac{2}{5}\, x^{\frac{5}{2}} + 2\, \sqrt{x}$$

```
[In] integrate((x^2+1)/x^(1/2),x, algorithm="maxima")
[Out] 2/5*x^(5/2) + 2*sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1+x^2}{\sqrt{x}}\, dx = \frac{2}{5}\, x^{\frac{5}{2}} + 2\, \sqrt{x}$$

```
[In] integrate((x^2+1)/x^(1/2),x, algorithm="giac")
[Out] 2/5*x^(5/2) + 2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{1+x^2}{\sqrt{x}}\, dx = \frac{2\, \sqrt{x}\, (x^2+5)}{5}$$

```
[In] int((x^2 + 1)/x^(1/2),x)
[Out] (2*x^(1/2)*(x^2 + 5))/5
```

3.90 $\int \frac{x}{\sqrt{5+2x+x^2}}\,dx$

Optimal result 401
Rubi [A] (verified) 401
Mathematica [A] (verified) 402
Maple [A] (verified) 402
Fricas [A] (verification not implemented) 403
Sympy [A] (verification not implemented) 403
Maxima [A] (verification not implemented) 403
Giac [A] (verification not implemented) 403
Mupad [B] (verification not implemented) 404

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{x}{\sqrt{5+2x+x^2}}\,dx = \sqrt{5+2x+x^2} - \operatorname{arcsinh}\left(\frac{1+x}{2}\right)$$

```
[Out] -arcsinh(1/2+1/2*x)+(x^2+2*x+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {654, 633, 221}

$$\int \frac{x}{\sqrt{5+2x+x^2}}\,dx = \sqrt{x^2+2x+5} - \operatorname{arcsinh}\left(\frac{x+1}{2}\right)$$

```
[In] Int[x/Sqrt[5 + 2*x + x^2],x]
```

```
[Out] Sqrt[5 + 2*x + x^2] - ArcSinh[(1 + x)/2]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \sqrt{5+2x+x^2} - \int \frac{1}{\sqrt{5+2x+x^2}}\,dx \\
&= \sqrt{5+2x+x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{16}}}\,dx, x, 2+2x\right) \\
&= \sqrt{5+2x+x^2} - \text{arcsinh}\left(\frac{1+x}{2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{x}{\sqrt{5+2x+x^2}}\,dx = \sqrt{5+2x+x^2} + \log\left(-1-x+\sqrt{5+2x+x^2}\right)$$

```
[In] Integrate[x/Sqrt[5 + 2*x + x^2],x]
[Out] Sqrt[5 + 2*x + x^2] + Log[-1 - x + Sqrt[5 + 2*x + x^2]]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$-\text{arcsinh}\left(\frac{1}{2}+\frac{x}{2}\right)+\sqrt{x^2+2x+5}$	20
risch	$-\text{arcsinh}\left(\frac{1}{2}+\frac{x}{2}\right)+\sqrt{x^2+2x+5}$	20
trager	$\sqrt{x^2+2x+5}-\ln\left(1+x+\sqrt{x^2+2x+5}\right)$	28

```
[In] int(x/(x^2+2*x+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] -arcsinh(1/2+1/2*x)+(x^2+2*x+5)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x}{\sqrt{5+2x+x^2}}\,dx = \sqrt{x^2+2\,x+5}+\log\left(-x+\sqrt{x^2+2\,x+5}-1\right)$$

```
[In] integrate(x/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^2 + 2*x + 5) + log(-x + sqrt(x^2 + 2*x + 5) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{5+2x+x^2}}\,dx = \sqrt{x^2+2x+5}-\operatorname{asinh}\left(\frac{x}{2}+\frac{1}{2}\right)$$

```
[In] integrate(x/(x**2+2*x+5)**(1/2),x)
[Out] sqrt(x**2 + 2*x + 5) - asinh(x/2 + 1/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{5+2x+x^2}}\,dx = \sqrt{x^2+2\,x+5}-\operatorname{arsinh}\left(\frac{1}{2}\,x+\frac{1}{2}\right)$$

```
[In] integrate(x/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 + 2*x + 5) - arcsinh(1/2*x + 1/2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x}{\sqrt{5+2x+x^2}}\,dx = \sqrt{x^2+2\,x+5}+\log\left(-x+\sqrt{x^2+2\,x+5}-1\right)$$

```
[In] integrate(x/(x^2+2*x+5)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 + 2*x + 5) + log(-x + sqrt(x^2 + 2*x + 5) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x}{\sqrt{5+2x+x^2}}\,dx = \sqrt{x^2+2\,x+5} - \ln\left(x+\sqrt{x^2+2\,x+5}+1\right)$$

```
[In] int(x/(2*x + x^2 + 5)^(1/2),x)
[Out] (2*x + x^2 + 5)^(1/2) - log(x + (2*x + x^2 + 5)^(1/2) + 1)
```

3.91 $\int \cos(x)\sin^2(x)\,dx$

Optimal result . . . 405
Rubi [A] (verified) . . . 405
Mathematica [A] (verified) . . . 406
Maple [A] (verified) . . . 406
Fricas [A] (verification not implemented) . . . 407
Sympy [A] (verification not implemented) . . . 407
Maxima [A] (verification not implemented) . . . 407
Giac [A] (verification not implemented) . . . 407
Mupad [B] (verification not implemented) . . . 408

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \cos(x)\sin^2(x)\,dx = \frac{\sin^3(x)}{3}$$

```
[Out] 1/3*sin(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2644, 30}

$$\int \cos(x)\sin^2(x)\,dx = \frac{\sin^3(x)}{3}$$

```
[In] Int[Cos[x]*Sin[x]^2,x]
```

```
[Out] Sin[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int x^2\,dx, x, \sin(x)\right)$$
$$= \frac{\sin^3(x)}{3}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin^2(x)\,dx = \frac{\sin^3(x)}{3}$$

```
[In] Integrate[Cos[x]*Sin[x]^2,x]
[Out] Sin[x]^3/3
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sin^3(x))}{3}$	7
default	$\frac{(\sin^3(x))}{3}$	7
risch	$\frac{\sin(x)}{4}-\frac{\sin(3x)}{12}$	12
parallelrisch	$\frac{\sin(x)}{4}-\frac{\sin(3x)}{12}$	12
norman	$\frac{8\left(\tan^3\left(\frac{x}{2}\right)\right)}{3\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$	19

```
[In] int(sin(x)^2*cos(x),x,method=_RETURNVERBOSE)
[Out] 1/3*sin(x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \cos(x)\sin^2(x)\,dx = -\frac{1}{3}\left(\cos\left(x\right)^2 - 1\right)\sin\left(x\right)$$

```
[In] integrate(cos(x)*sin(x)^2,x, algorithm="fricas")
[Out] -1/3*(cos(x)^2 - 1)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \cos(x)\sin^2(x)\,dx = \frac{\sin^3\left(x\right)}{3}$$

```
[In] integrate(cos(x)*sin(x)**2,x)
[Out] sin(x)**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin^2(x)\,dx = \frac{1}{3}\sin\left(x\right)^3$$

```
[In] integrate(cos(x)*sin(x)^2,x, algorithm="maxima")
[Out] 1/3*sin(x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin^2(x)\,dx = \frac{1}{3}\sin\left(x\right)^3$$

```
[In] integrate(cos(x)*sin(x)^2,x, algorithm="giac")
[Out] 1/3*sin(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin^2(x)\,dx = \frac{\sin(x)^3}{3}$$

```
[In] int(cos(x)*sin(x)^2,x)
```

```
[Out] sin(x)^3/3
```

3.92 $\int \frac{e^x}{1+e^x} dx$

Optimal result . . . 409
Rubi [A] (verified) . . . 409
Mathematica [A] (verified) . . . 410
Maple [A] (verified) . . . 410
Fricas [A] (verification not implemented) . . . 410
Sympy [A] (verification not implemented) . . . 411
Maxima [A] (verification not implemented) . . . 411
Giac [A] (verification not implemented) . . . 411
Mupad [B] (verification not implemented) . . . 411

Optimal result

Integrand size = 11, antiderivative size = 6

$$\int \frac{e^x}{1+e^x} dx = \log(1+e^x)$$

```
[Out] ln(1+exp(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2278, 31}

$$\int \frac{e^x}{1+e^x} dx = \log(e^x+1)$$

```
[In] Int[E^x/(1 + E^x),x]
```

```
[Out] Log[1 + E^x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2278

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
 e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, e^x\right)\\ &= \log\left(1+e^x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{1+e^x}\,dx = \log\left(1+e^x\right)$$

```
[In] Integrate[E^x/(1 + E^x),x]
[Out] Log[1 + E^x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\ln\left(1+\mathrm{e}^{x}\right)$	6
default	$\ln\left(1+\mathrm{e}^{x}\right)$	6
norman	$\ln\left(1+\mathrm{e}^{x}\right)$	6
risch	$\ln\left(1+\mathrm{e}^{x}\right)$	6
parallelrisch	$\ln\left(1+\mathrm{e}^{x}\right)$	6

```
[In] int(1/(1+exp(x))*exp(x),x,method=_RETURNVERBOSE)
[Out] ln(1+exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x}\,dx = \log\left(e^x+1\right)$$

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="fricas")
[Out] log(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x}\,dx = \log\left(e^x+1\right)$$

```
[In] integrate(exp(x)/(1+exp(x)),x)
[Out] log(exp(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x}\,dx = \log\left(e^x+1\right)$$

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="maxima")
[Out] log(e^x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x}\,dx = \log\left(e^x+1\right)$$

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="giac")
[Out] log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x}\,dx = \ln\left(\mathrm{e}^x+1\right)$$

```
[In] int(exp(x)/(exp(x) + 1),x)
[Out] log(exp(x) + 1)
```

3.93 $\int \frac{e^{2x}}{1+e^x} dx$

Optimal result 412
Rubi [A] (verified) 412
Mathematica [A] (verified) 413
Maple [A] (verified) 413
Fricas [A] (verification not implemented) 414
Sympy [A] (verification not implemented) 414
Maxima [A] (verification not implemented) 414
Giac [A] (verification not implemented) 414
Mupad [B] (verification not implemented) 415

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{e^{2x}}{1+e^x} dx = e^x - \log(1+e^x)$$

```
[Out] exp(x)-ln(1+exp(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2280, 45}

$$\int \frac{e^{2x}}{1+e^x} dx = e^x - \log(e^x + 1)$$

```
[In] Int[E^(2*x)/(1 + E^x),x]
```

```
[Out] E^x - Log[1 + E^x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2280

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
```

```
[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Den
ominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{x}{1+x}\,dx, x, e^x\right)\\ &= \text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx, x, e^x\right)\\ &= e^x - \log\left(1+e^x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(1+e^x\right)$$

```
[In] Integrate[E^(2*x)/(1 + E^x),x]
[Out] E^x - Log[1 + E^x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$\mathrm{e}^{x}-\ln\left(1+\mathrm{e}^{x}\right)$	11
norman	$\mathrm{e}^{x}-\ln\left(1+\mathrm{e}^{x}\right)$	11
risch	$\mathrm{e}^{x}-\ln\left(1+\mathrm{e}^{x}\right)$	11

```
[In] int(exp(2*x)/(1+exp(x)),x,method=_RETURNVERBOSE)
[Out] exp(x)-ln(1+exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x + 1\right)$$

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="fricas")
[Out] e^x - log(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x + 1\right)$$

```
[In] integrate(exp(2*x)/(1+exp(x)),x)
[Out] exp(x) - log(exp(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x + 1\right)$$

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")
[Out] e^x - log(e^x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x + 1\right)$$

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="giac")
[Out] e^x - log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = \mathrm{e}^x - \ln\left(\mathrm{e}^x + 1\right)$$

```
[In] int(exp(2*x)/(exp(x) + 1),x)
[Out] exp(x) - log(exp(x) + 1)
```

3.94 $\int \frac{1}{1-\cos(x)} \, dx$

Optimal result 416
Rubi [A] (verified) 416
Mathematica [A] (verified) 417
Maple [A] (verified) 417
Fricas [A] (verification not implemented) 417
Sympy [A] (verification not implemented) 418
Maxima [A] (verification not implemented) 418
Giac [A] (verification not implemented) 418
Mupad [B] (verification not implemented) 418

Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \frac{1}{1-\cos(x)} \, dx = -\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -sin(x)/(1-cos(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2727}

$$\int \frac{1}{1-\cos(x)} \, dx = -\frac{\sin(x)}{1-\cos(x)}$$

```
[In] Int[(1 - Cos[x])^(-1),x]
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx = -\cot\left(\frac{x}{2}\right)$$

```
[In] Integrate[(1 - Cos[x])^(-1),x]
[Out] -Cot[x/2]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
norman	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
parallelrisch	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
risch	$-\frac{2i}{e^{ix}-1}$	13

```
[In] int(1/(1-cos(x)),x,method=_RETURNVERBOSE)
[Out] -1/tan(1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\cos(x)+1}{\sin(x)}$$

```
[In] integrate(1/(1-cos(x)),x, algorithm="fricas")
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{x}{2}\right)}$$

```
[In] integrate(1/(1-cos(x)),x)
[Out] -1/tan(x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\cos\left(x\right)+1}{\sin\left(x\right)}$$

```
[In] integrate(1/(1-cos(x)),x, algorithm="maxima")
[Out] -(cos(x) + 1)/sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

```
[In] integrate(1/(1-cos(x)),x, algorithm="giac")
[Out] -1/tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{1-\cos(x)}\,dx = -\cot\left(\frac{x}{2}\right)$$

```
[In] int(-1/(cos(x) - 1),x)
[Out] -cot(x/2)
```

3.95 $\int \sec^2(x)\tan(x)\,dx$

Optimal result 419
Rubi [A] (verified) 419
Mathematica [A] (verified) 420
Maple [A] (verified) 420
Fricas [A] (verification not implemented) 420
Sympy [A] (verification not implemented) 421
Maxima [A] (verification not implemented) 421
Giac [A] (verification not implemented) 421
Mupad [B] (verification not implemented) 421

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \sec^2(x)\tan(x)\,dx = \frac{\sec^2(x)}{2}$$

```
[Out] 1/2*sec(x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2686, 30}

$$\int \sec^2(x)\tan(x)\,dx = \frac{\sec^2(x)}{2}$$

```
[In] Int[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int x\,dx, x, \sec(x)\right)$$
$$= \frac{\sec^2(x)}{2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\tan(x)\,dx = \frac{\sec^2(x)}{2}$$

```
[In] Integrate[Sec[x]^2*Tan[x],x]
[Out] Sec[x]^2/2
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sec^2(x))}{2}$	7
default	$\frac{(\sec^2(x))}{2}$	7
risch	$\frac{2\,e^{2ix}}{(e^{2ix}+1)^2}$	17

```
[In] int(sec(x)^2*tan(x),x,method=_RETURNVERBOSE)
[Out] 1/2*sec(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2\,\cos(x)^2}$$

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="fricas")
[Out] 1/2/cos(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2\cos^2{(x)}}$$

```
[In] integrate(sec(x)**2*tan(x),x)
[Out] 1/(2*cos(x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2}\,\tan{(x)}^2$$

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="maxima")
[Out] 1/2*tan(x)^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2\,\cos{(x)}^2}$$

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="giac")
[Out] 1/2/cos(x)^2
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{\tan(x)^2}{2}$$

```
[In] int(tan(x)/cos(x)^2,x)
[Out] tan(x)^2/2
```

3.96 $\int x \log(x) \, dx$

Optimal result 422
Rubi [A] (verified) 422
Mathematica [A] (verified) 423
Maple [A] (verified) 423
Fricas [A] (verification not implemented) 423
Sympy [A] (verification not implemented) 424
Maxima [A] (verification not implemented) 424
Giac [A] (verification not implemented) 424
Mupad [B] (verification not implemented) 424

Optimal result

Integrand size = 4, antiderivative size = 17

$$\int x \log(x) \, dx = -\frac{x^2}{4} + \frac{1}{2} x^2 \log(x)$$

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2341}

$$\int x \log(x) \, dx = \frac{1}{2} x^2 \log(x) - \frac{x^2}{4}$$

```
[In] Int[x*Log[x],x]
```

```
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{x^2}{4} + \frac{1}{2} x^2 \log(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x\log(x)\,dx=-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)$$

```
[In] Integrate[x*Log[x],x]
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
norman	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
risch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
parallelrisch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
parts	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14

```
[In] int(x*ln(x),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx=\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

```
[In] integrate(x*log(x),x, algorithm="fricas")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x\log(x)\,dx = \frac{x^2\log{(x)}}{2} - \frac{x^2}{4}$$

```
[In] integrate(x*ln(x),x)
[Out] x**2*log(x)/2 - x**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log{(x)} - \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(x),x, algorithm="maxima")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log{(x)} - \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(x),x, algorithm="giac")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int x\log(x)\,dx = \frac{x^2\,\left(\ln{(x)} - \frac{1}{2}\right)}{2}$$

```
[In] int(x*log(x),x)
[Out] (x^2*(log(x) - 1/2))/2
```

3.97 $\int \cos(x)\sin(x)\,dx$

Optimal result 425
Rubi [A] (verified) 425
Mathematica [A] (verified) 426
Maple [A] (verified) 426
Fricas [A] (verification not implemented) 427
Sympy [A] (verification not implemented) 427
Maxima [A] (verification not implemented) 427
Giac [A] (verification not implemented) 427
Mupad [B] (verification not implemented) 428

Optimal result

Integrand size = 5, antiderivative size = 8

$$\int \cos(x)\sin(x)\,dx = \frac{\sin^2(x)}{2}$$

```
[Out] 1/2*sin(x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2644, 30}

$$\int \cos(x)\sin(x)\,dx = \frac{\sin^2(x)}{2}$$

```
[In] Int[Cos[x]*Sin[x],x]
```

```
[Out] Sin[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int x\,dx, x, \sin(x)\right)$$
$$= \frac{\sin^2(x)}{2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos^2(x)$$

```
[In] Integrate[Cos[x]*Sin[x],x]
[Out] -1/2*Cos[x]^2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sin^2(x))}{2}$	7
default	$\frac{(\sin^2(x))}{2}$	7
risch	$-\frac{\cos(2x)}{4}$	7
parallelrisch	$\frac{1}{4}-\frac{\cos(2x)}{4}$	9
norman	$\frac{2(\tan^2(\frac{x}{2}))}{(1+\tan^2(\frac{x}{2}))^2}$	19
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{4}$	19

```
[In] int(cos(x)*sin(x),x,method=_RETURNVERBOSE)
[Out] 1/2*sin(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos{(x)}^2$$

```
[In] integrate(cos(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*cos(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \cos(x)\sin(x)\,dx = \frac{\sin^2{(x)}}{2}$$

```
[In] integrate(cos(x)*sin(x),x)
[Out] sin(x)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos{(x)}^2$$

```
[In] integrate(cos(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*cos(x)^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos{(x)}^2$$

```
[In] integrate(cos(x)*sin(x),x, algorithm="giac")
[Out] -1/2*cos(x)^2
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx=\frac{\sin{(x)}^2}{2}$$

```
[In] int(cos(x)*sin(x),x)
[Out] sin(x)^2/2
```

3.98 $\int \frac{1+x}{\sqrt{2x-x^2}}\,dx$

Optimal result 429
Rubi [A] (verified) 429
Mathematica [A] (verified) 430
Maple [A] (verified) 430
Fricas [A] (verification not implemented) 431
Sympy [A] (verification not implemented) 431
Maxima [A] (verification not implemented) 431
Giac [A] (verification not implemented) 432
Mupad [B] (verification not implemented) 432

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \frac{1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{2x-x^2} - 2\arcsin(1-x)$$

```
[Out] 2*arcsin(-1+x)-(-x^2+2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {654, 633, 222}

$$\int \frac{1+x}{\sqrt{2x-x^2}}\,dx = -2\arcsin(1-x) - \sqrt{2x-x^2}$$

```
[In] Int[(1 + x)/Sqrt[2*x - x^2],x]
```

```
[Out] -Sqrt[2*x - x^2] - 2*ArcSin[1 - x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\sqrt{2x-x^2}+2\int \frac{1}{\sqrt{2x-x^2}}\,dx\\ &= -\sqrt{2x-x^2}-\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx,x,2-2x\right)\\ &= -\sqrt{2x-x^2}-2\arcsin(1-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \frac{1+x}{\sqrt{2x-x^2}}\,dx=\frac{(-2+x)x-4\sqrt{-2+x}\sqrt{x}\log\left(\sqrt{-2+x}-\sqrt{x}\right)}{\sqrt{-((-2+x)x)}}$$

```
[In] Integrate[(1 + x)/Sqrt[2*x - x^2],x]
[Out] ((-2 + x)*x - 4*Sqrt[-2 + x]*Sqrt[x]*Log[Sqrt[-2 + x] - Sqrt[x]])/Sqrt[-((-
2 + x)*x)]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$2\arcsin\left(-1+x\right)-\sqrt{-x^2+2x}$	21
risch	$\frac{x(-2+x)}{\sqrt{-x(-2+x)}}+2\arcsin\left(-1+x\right)$	21
pseudoelliptic	$-\sqrt{-x\left(-2+x\right)}-4\arctan\left(\frac{\sqrt{-x(-2+x)}}{x}\right)$	27
trager	$-\sqrt{-x^2+2x}+2\,\text{RootOf}\left(\_Z^2+1\right)\ln\left(\text{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+2x}+x-1\right)$	45
meijerg	$2\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)+\frac{2i\left(\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{1-\frac{x}{2}}}{2}-i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)\right)}{\sqrt{\pi}}$	54

```
[In] int((1+x)/(-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arcsin(-1+x)-(-x^2+2*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{-x^2+2\,x} - 4\,\arctan\left(\frac{\sqrt{-x^2+2\,x}}{x}\right)$$

```
[In] integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 + 2*x) - 4*arctan(sqrt(-x^2 + 2*x)/x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{-x^2+2x} + 2\,\mathrm{asin}\,(x-1)$$

```
[In] integrate((1+x)/(-x**2+2*x)**(1/2),x)
[Out] -sqrt(-x**2 + 2*x) + 2*asin(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{-x^2+2\,x} - 2\,\arcsin\,(-x+1)$$

```
[In] integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 2*x) - 2*arcsin(-x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{-x^2+2\,x} + 2\,\arcsin\left(x-1\right)$$

```
[In] integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 2*x) + 2*arcsin(x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1+x}{\sqrt{2x-x^2}}\,dx = 2\,\mathrm{asin}(x-1) - \sqrt{2\,x-x^2}$$

```
[In] int((x + 1)/(2*x - x^2)^(1/2),x)
[Out] 2*asin(x - 1) - (2*x - x^2)^(1/2)
```

3.99 $\int \frac{2e^x}{2+3e^{2x}}\,dx$

Optimal result 433
Rubi [A] (verified) 433
Mathematica [A] (verified) 434
Maple [A] (verified) 434
Fricas [A] (verification not implemented) 435
Sympy [A] (verification not implemented) 435
Maxima [A] (verification not implemented) 435
Giac [A] (verification not implemented) 435
Mupad [B] (verification not implemented) 436

Optimal result

Integrand size = 16, antiderivative size = 20

$$\int \frac{2e^x}{2+3e^{2x}}\,dx = \sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}e^x\right)$$

```
[Out] 1/3*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {12, 2281, 209}

$$\int \frac{2e^x}{2+3e^{2x}}\,dx = \sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}e^x\right)$$

```
[In] Int[(2*E^x)/(2 + 3*E^(2*x)),x]
```

```
[Out] Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{e^x}{2+3e^{2x}}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, e^x\right)\\ &= \sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}e^x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{2e^x}{2+3e^{2x}}\,dx = \sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}e^x\right)$$

```
[In] Integrate[(2*E^x)/(2 + 3*E^(2*x)),x]
[Out] Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\arctan\left(\frac{e^x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	14
risch	$\frac{i\sqrt{6}\,\ln\left(e^x+\frac{i\sqrt{6}}{3}\right)}{6}-\frac{i\sqrt{6}\,\ln\left(e^x-\frac{i\sqrt{6}}{3}\right)}{6}$	34

```
[In] int(2*exp(x)/(2+3*exp(2*x)),x,method=_RETURNVERBOSE)
[Out] 1/3*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{2e^x}{2+3e^{2x}}\,dx = \frac{1}{3}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}e^x\right)$$

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*e^x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{2e^x}{2+3e^{2x}}\,dx = \operatorname{RootSum}\left(6z^2+1,\left(i\mapsto i\log\left(2i+e^x\right)\right)\right)$$

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x)
[Out] RootSum(6*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{2e^x}{2+3e^{2x}}\,dx = \frac{1}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^x\right)$$

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x, algorithm="maxima")
[Out] 1/3*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{2e^x}{2+3e^{2x}}\,dx = \frac{1}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^x\right)$$

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x, algorithm="giac")
[Out] 1/3*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{2e^x}{2+3e^{2x}}\,dx = \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,\mathrm{e}^x}{2}\right)}{3}$$

```
[In] int((2*exp(x))/(3*exp(2*x) + 2),x)
[Out] (6^(1/2)*atan((6^(1/2)*exp(x))/2))/3
```

3.100 $\int \frac{x^4}{(1-x^2)^{5/2}} \, dx$

Optimal result 437
Rubi [A] (verified) 437
Mathematica [A] (verified) 438
Maple [A] (verified) 438
Fricas [B] (verification not implemented) 439
Sympy [B] (verification not implemented) 439
Maxima [A] (verification not implemented) 439
Giac [A] (verification not implemented) 440
Mupad [B] (verification not implemented) 440

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{x^4}{(1-x^2)^{5/2}} \, dx = \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \arcsin(x)$$

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {294, 222}

$$\int \frac{x^4}{(1-x^2)^{5/2}} \, dx = \arcsin(x) - \frac{x}{\sqrt{1-x^2}} + \frac{x^3}{3(1-x^2)^{3/2}}$$

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^3}{3(1-x^2)^{3/2}} - \int \frac{x^2}{(1-x^2)^{3/2}}\,dx \\ &= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \arcsin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{x(-3+4x^2)}{3(1-x^2)^{3/2}} + 2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x^3}{3(-x^2+1)^{\frac{3}{2}}}+\arcsin(x)-\frac{x}{\sqrt{-x^2+1}}$	30
risch	$-\frac{x(4x^2-3)}{3(x^2-1)\sqrt{-x^2+1}}+\arcsin(x)$	30
meijerg	$-\frac{2i\left(-\frac{i\sqrt{\pi}\,x\left(-20x^2+15\right)}{10\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{3i\sqrt{\pi}\,\arcsin(x)}{2}\right)}{3\sqrt{\pi}}$	39
pseudoelliptic	$\frac{(x^2-1)\sqrt{-x^2+1}\,\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right)+\frac{4x^3}{3}-x}{(-x^2+1)^{\frac{3}{2}}}$	49
trager	$\frac{(4x^2-3)x\sqrt{-x^2+1}}{3(x^2-1)^2}+\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)$	54

```
[In] int(x^4/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-1/(-x^2+1)^(1/2)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = -\frac{6\,(x^4-2\,x^2+1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-(4\,x^3-3\,x)\sqrt{-x^2+1}}{3\,(x^4-2\,x^2+1)}$$

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(26) = 52$.

Time = 0.40 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.00

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3} - \frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3}-\frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3}+\frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{1}{3}\,x\left(\frac{3\,x^2}{(-x^2+1)^{\frac{3}{2}}}-\frac{2}{(-x^2+1)^{\frac{3}{2}}}\right)-\frac{x}{3\,\sqrt{-x^2+1}}+\arcsin(x)$$

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{(4\,x^2-3)\sqrt{-x^2+1}x}{3\,(x^2-1)^2} + \arcsin(x)$$

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.60

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \mathrm{asin}(x) + \frac{3\sqrt{1-x^2}}{4\,(x-1)} + \frac{3\sqrt{1-x^2}}{4\,(x+1)} - \sqrt{1-x^2}\left(\frac{1}{12\,(x-1)} - \frac{1}{12\,(x-1)^2}\right) - \sqrt{1-x^2}\left(\frac{1}{12\,(x+1)} + \frac{1}{12\,(x+1)^2}\right)$$

```
[In] int(x^4/(1 - x^2)^(5/2),x)
[Out] asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1))
 - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*(1
/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

3.101 $\int \frac{e^{6x}}{1+e^{4x}}\,dx$

Optimal result 441
Rubi [A] (verified) 441
Mathematica [A] (verified) 442
Maple [A] (verified) 442
Fricas [A] (verification not implemented) 443
Sympy [A] (verification not implemented) 443
Maxima [A] (verification not implemented) 443
Giac [A] (verification not implemented) 443
Mupad [B] (verification not implemented) 444

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{e^{6x}}{1+e^{4x}}\,dx = \frac{e^{2x}}{2} - \frac{1}{2}\arctan\left(e^{2x}\right)$$

```
[Out] 1/2*exp(2*x)-1/2*arctan(exp(2*x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2280, 327, 209}

$$\int \frac{e^{6x}}{1+e^{4x}}\,dx = \frac{e^{2x}}{2} - \frac{1}{2}\arctan\left(e^{2x}\right)$$

```
[In] Int[E^(6*x)/(1 + E^(4*x)),x]
```

```
[Out] E^(2*x)/2 - ArcTan[E^(2*x)]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2280

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Den
ominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, e^{2x}\right)\\
&= \frac{e^{2x}}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{2x}\right)\\
&= \frac{e^{2x}}{2} - \frac{1}{2}\arctan\left(e^{2x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{e^{6x}}{1+e^{4x}}\,dx = \frac{1}{2}\left(e^{2x} - \arctan\left(e^{2x}\right)\right)$$

```
[In] Integrate[E^(6*x)/(1 + E^(4*x)),x]
[Out] (E^(2*x) - ArcTan[E^(2*x)])/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{e^{2x}}{2} - \frac{\arctan(e^{2x})}{2}$	15
risch	$\frac{e^{2x}}{2} + \frac{i\ln(e^{2x}-i)}{4} - \frac{i\ln(e^{2x}+i)}{4}$	30

```
[In] int(exp(6*x)/(exp(4*x)+1),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x)^2-1/2*arctan(exp(x)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{e^{6x}}{1+e^{4x}}\,dx = -\frac{1}{2}\,\arctan\left(e^{(2\,x)}\right) + \frac{1}{2}\,e^{(2\,x)}$$

```
[In] integrate(exp(6*x)/(1+exp(4*x)),x, algorithm="fricas")
[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{e^{6x}}{1+e^{4x}}\,dx = \frac{e^{2x}}{2} + \operatorname{RootSum}\left(16z^2+1,\left(i\mapsto i\log\left(-4i+e^{2x}\right)\right)\right)$$

```
[In] integrate(exp(6*x)/(1+exp(4*x)),x)
[Out] exp(2*x)/2 + RootSum(16*_z**2 + 1, Lambda(_i, _i*log(-4*_i + exp(2*x))))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{e^{6x}}{1+e^{4x}}\,dx = -\frac{1}{2}\,\arctan\left(e^{(2\,x)}\right) + \frac{1}{2}\,e^{(2\,x)}$$

```
[In] integrate(exp(6*x)/(1+exp(4*x)),x, algorithm="maxima")
[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{e^{6x}}{1+e^{4x}}\,dx = -\frac{1}{2}\,\arctan\left(e^{(2\,x)}\right) + \frac{1}{2}\,e^{(2\,x)}$$

```
[In] integrate(exp(6*x)/(1+exp(4*x)),x, algorithm="giac")
[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{e^{6x}}{1+e^{4x}}\,dx=\frac{\mathrm{e}^{2\,x}}{2}-\frac{\mathrm{atan}\left(\mathrm{e}^{2\,x}\right)}{2}$$

```
[In] int(exp(6*x)/(exp(4*x) + 1),x)
[Out] exp(2*x)/2 - atan(exp(2*x))/2
```

3.102 $\int \log(2+3x^2)\,dx$

Optimal result 445
Rubi [A] (verified) 445
Mathematica [A] (verified) 446
Maple [A] (verified) 446
Fricas [A] (verification not implemented) 447
Sympy [A] (verification not implemented) 447
Maxima [A] (verification not implemented) 447
Giac [A] (verification not implemented) 448
Mupad [B] (verification not implemented) 448

Optimal result

Integrand size = 8, antiderivative size = 33

$$\int \log(2+3x^2)\,dx = -2x + 2\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}x\right) + x\log(2+3x^2)$$

```
[Out] -2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2498, 327, 209}

$$\int \log(2+3x^2)\,dx = 2\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}x\right) + x\log(3x^2+2) - 2x$$

```
[In] Int[Log[2 + 3*x^2],x]
```

```
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log\left(2+3x^2\right)-6\int\frac{x^2}{2+3x^2}\,dx\\&=-2x+x\log\left(2+3x^2\right)+4\int\frac{1}{2+3x^2}\,dx\\&=-2x+2\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}x\right)+x\log\left(2+3x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int\log\left(2+3x^2\right)\,dx=-2x+2\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}x\right)+x\log\left(2+3x^2\right)$$

```
[In] Integrate[Log[2 + 3*x^2],x]
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
default	$-2x+x\ln\left(3x^2+2\right)+\frac{2\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	27
risch	$-2x+x\ln\left(3x^2+2\right)+\frac{2\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	27
parts	$-2x+x\ln\left(3x^2+2\right)+\frac{2\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	27

```
[In] int(ln(3*x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \log \left(2+3x^2\right)\,dx = \frac{2}{3}\,\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{3}\sqrt{2}x\right) + x\log\left(3\,x^2+2\right) - 2\,x$$

```
[In] integrate(log(3*x^2+2),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) + x*log(3*x^2 + 2) - 2*x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \log \left(2+3x^2\right)\,dx = x\log\left(3x^2+2\right) - 2x + \frac{2\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}\right)}{3}$$

```
[In] integrate(ln(3*x**2+2),x)
[Out] x*log(3*x**2 + 2) - 2*x + 2*sqrt(6)*atan(sqrt(6)*x/2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \log \left(2+3x^2\right)\,dx = x\log\left(3\,x^2+2\right) + \frac{2}{3}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right) - 2\,x$$

```
[In] integrate(log(3*x^2+2),x, algorithm="maxima")
[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \log \left(2+3x^2\right)\, dx = x \log \left(3\, x^2+2\right)+\frac{2}{3}\, \sqrt{6} \arctan \left(\frac{1}{2}\, \sqrt{6} x\right)-2\, x$$

```
[In] integrate(log(3*x^2+2),x, algorithm="giac")
[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \log \left(2+3x^2\right)\, dx = \frac{2\, \sqrt{6}\, \mathrm{atan}\left(\frac{\sqrt{6}\, x}{2}\right)}{3}-2\, x+x\, \ln \left(3\, x^2+2\right)$$

```
[In] int(log(3*x^2 + 2),x)
[Out] (2*6^(1/2)*atan((6^(1/2)*x)/2))/3 - 2*x + x*log(3*x^2 + 2)
```

3.103 $\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx$

Optimal result 449
Rubi [A] (verified) 449
Mathematica [A] (verified) 450
Maple [A] (verified) 450
Fricas [A] (verification not implemented) 450
Sympy [A] (verification not implemented) 451
Maxima [A] (verification not implemented) 451
Giac [A] (verification not implemented) 451
Mupad [B] (verification not implemented) 451

Optimal result

Integrand size = 20, antiderivative size = 21

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx=\frac{x}{r\sqrt{-a^2+2Hr^2}}$$

```
[Out] x/r/(2*H*r^2-a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = $\{8\}$

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx=\frac{x}{r\sqrt{2Hr^2-a^2}}$$

```
[In] Int[1/(r*Sqrt[-a^2 + 2*H*r^2]),x]
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\text{integral}=\frac{x}{r\sqrt{-a^2+2Hr^2}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx=\frac{x}{r\sqrt{-a^2+2Hr^2}}$$

```
[In] Integrate[1/(r*Sqrt[-a^2 + 2*H*r^2]),x]
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{x}{r\sqrt{2H\,r^{2}-a^{2}}}$	20
norman	$\frac{x}{r\sqrt{2H\,r^{2}-a^{2}}}$	20
parallelrisch	$\frac{x}{r\sqrt{2H\,r^{2}-a^{2}}}$	20

```
[In] int(1/r/(2*H*r^2-a^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] x/r/(2*H*r^2-a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx=\frac{\sqrt{2\,Hr^2-a^2}x}{2\,Hr^3-a^2r}$$

```
[In] integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(2*H*r^2 - a^2)*x/(2*H*r^3 - a^2*r)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx = \frac{x}{r\sqrt{2Hr^2-a^2}}$$

```
[In] integrate(1/r/(2*H*r**2-a**2)**(1/2),x)
[Out] x/(r*sqrt(2*H*r**2 - a**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx = \frac{x}{\sqrt{2\,Hr^2-a^2}r}$$

```
[In] integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="maxima")
[Out] x/(sqrt(2*H*r^2 - a^2)*r)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx = \frac{x}{\sqrt{2\,Hr^2-a^2}r}$$

```
[In] integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="giac")
[Out] x/(sqrt(2*H*r^2 - a^2)*r)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx = \frac{x}{r\,\sqrt{2\,H\,r^2-a^2}}$$

```
[In] int(1/(r*(2*H*r^2 - a^2)^(1/2)),x)
[Out] x/(r*(2*H*r^2 - a^2)^(1/2))
```

3.104 $\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx$

Optimal result 452
Rubi [A] (verified) 452
Mathematica [A] (verified) 453
Maple [A] (verified) 453
Fricas [A] (verification not implemented) 453
Sympy [A] (verification not implemented) 454
Maxima [A] (verification not implemented) 454
Giac [A] (verification not implemented) 454
Mupad [B] (verification not implemented) 454

Optimal result

Integrand size = 25, antiderivative size = 26

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx=\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

```
[Out] x/r/(2*H*r^2-a^2-e^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {8}

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx=\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

```
[In] Int[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2]),x]
```

```
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\text{integral}=\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx=\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

```
[In] Integrate[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2]),x]
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{x}{r\sqrt{2H\,r^2-a^2-e^2}}$	25
norman	$\frac{x}{r\sqrt{2H\,r^2-a^2-e^2}}$	25
parallelrisch	$\frac{x}{r\sqrt{2H\,r^2-a^2-e^2}}$	25

```
[In] int(1/r/(2*H*r^2-a^2-e^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] x/r/(2*H*r^2-a^2-e^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx=\frac{\sqrt{2\,Hr^2-a^2-e^2}x}{2\,Hr^3-(a^2+e^2)r}$$

```
[In] integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(2*H*r^2 - a^2 - e^2)*x/(2*H*r^3 - (a^2 + e^2)*r)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx = \frac{x}{r\sqrt{2Hr^2-a^2-e^2}}$$

```
[In] integrate(1/r/(2*H*r**2-a**2-e**2)**(1/2),x)
[Out] x/(r*sqrt(2*H*r**2 - a**2 - e**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx = \frac{x}{\sqrt{2\,Hr^2-a^2-e^2}r}$$

```
[In] integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2)*r)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx = \frac{x}{\sqrt{2\,Hr^2-a^2-e^2}r}$$

```
[In] integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="giac")
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2)*r)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx = \frac{x}{r\,\sqrt{-a^2-e^2+2\,H\,r^2}}$$

```
[In] int(1/(r*(2*H*r^2 - a^2 - e^2)^(1/2)),x)
[Out] x/(r*(2*H*r^2 - a^2 - e^2)^(1/2))
```

3.105 $\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx$

Optimal result 455
Rubi [A] (verified) 455
Mathematica [A] (verified) 456
Maple [A] (verified) 456
Fricas [A] (verification not implemented) 456
Sympy [A] (verification not implemented) 457
Maxima [A] (verification not implemented) 457
Giac [A] (verification not implemented) 457
Mupad [B] (verification not implemented) 457

Optimal result

Integrand size = 26, antiderivative size = 27

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

```
[Out] x/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {8}

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

```
[In] Int[1/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4]),x]
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\text{integral} = \frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx=\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

```
[In] Integrate[1/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4]),x]
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{x}{r\sqrt{-2K\,r^4+2H\,r^2-a^2}}$	26
norman	$\frac{x}{r\sqrt{-2K\,r^4+2H\,r^2-a^2}}$	26
parallelrisch	$\frac{x}{r\sqrt{-2K\,r^4+2H\,r^2-a^2}}$	26

```
[In] int(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] x/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx=-\frac{\sqrt{-2\,Kr^4+2\,Hr^2-a^2}x}{2\,Kr^5-2\,Hr^3+a^2r}$$

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*x/(2*K*r^5 - 2*H*r^3 + a^2*r)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx=\frac{x}{r\sqrt{2Hr^2-2Kr^4-a^2}}$$

```
[In] integrate(1/r/(-2*K*r**4+2*H*r**2-a**2)**(1/2),x)
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r**4 - a**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx=\frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2}r}$$

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="maxima")
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*r)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx=\frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2}r}$$

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="giac")
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*r)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx=\frac{x}{r\,\sqrt{-a^2-2\,K\,r^4+2\,H\,r^2}}$$

```
[In] int(1/(r*(2*H*r^2 - 2*K*r^4 - a^2)^(1/2)),x)
[Out] x/(r*(2*H*r^2 - 2*K*r^4 - a^2)^(1/2))
```

3.106 $\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx$

Optimal result 458
Rubi [A] (verified) 458
Mathematica [A] (verified) 459
Maple [A] (verified) 459
Fricas [A] (verification not implemented) 459
Sympy [A] (verification not implemented) 460
Maxima [A] (verification not implemented) 460
Giac [A] (verification not implemented) 460
Mupad [B] (verification not implemented) 460

Optimal result

Integrand size = 31, antiderivative size = 32

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

```
[Out] x/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {8}

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

```
[In] Int[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4]),x]
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\text{integral} = \frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

```
[In] Integrate[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4]),x]
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{x}{r\sqrt{-2K\,r^4+2H\,r^2-a^2-e^2}}$	31
norman	$\frac{x}{r\sqrt{-2K\,r^4+2H\,r^2-a^2-e^2}}$	31
parallelrisch	$\frac{x}{r\sqrt{-2K\,r^4+2H\,r^2-a^2-e^2}}$	31

```
[In] int(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] x/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = -\frac{\sqrt{-2\,Kr^4+2\,Hr^2-a^2-e^2}x}{2\,Kr^5-2\,Hr^3+(a^2+e^2)r}$$

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*x/(2*K*r^5 - 2*H*r^3 + (a^2 + e^2)*r)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{2Hr^2-2Kr^4-a^2-e^2}}$$

```
[In] integrate(1/r/(-2*K*r**4+2*H*r**2-a**2-e**2)**(1/2),x)
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r**4 - a**2 - e**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2-e^2}r}$$

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2-e^2}r}$$

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="giac")
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\,\sqrt{-a^2-e^2-2\,K\,r^4+2\,H\,r^2}}$$

```
[In] int(1/(r*(2*H*r^2 - 2*K*r^4 - a^2 - e^2)^(1/2)),x)
[Out] x/(r*(2*H*r^2 - 2*K*r^4 - a^2 - e^2)^(1/2))
```

3.107 $\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx$

Optimal result 461
Rubi [A] (verified) 461
Mathematica [A] (verified) 462
Maple [A] (verified) 462
Fricas [A] (verification not implemented) 462
Sympy [A] (verification not implemented) 463
Maxima [A] (verification not implemented) 463
Giac [A] (verification not implemented) 463
Mupad [B] (verification not implemented) 463

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

```
[Out] x/r/(-a^2-2*r*(-H*r+K))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{8\}$

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

```
[In] Int[1/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2]),x]
[Out] x/(r*Sqrt[-a^2 - 2*r*(K - H*r)])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\text{integral} = \frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx=\frac{x}{r\sqrt{-a^2-2Kr+2Hr^2}}$$

```
[In] Integrate[1/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2]),x]
[Out] x/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{x}{r\sqrt{2H\,r^2-2Kr-a^2}}$	24
norman	$\frac{x}{r\sqrt{2H\,r^2-2Kr-a^2}}$	24
parallelrisch	$\frac{x}{r\sqrt{2H\,r^2-2Kr-a^2}}$	24

```
[In] int(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/r/(2*H*r^2-2*K*r-a^2)^(1/2)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx=\frac{\sqrt{2\,Hr^2-a^2-2\,Kr}x}{2\,Hr^3-a^2r-2\,Kr^2}$$

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(2*H*r^2 - a^2 - 2*K*r)*x/(2*H*r^3 - a^2*r - 2*K*r^2)
```

Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{2Hr^2-2Kr-a^2}}$$

```
[In] integrate(1/r/(2*H*r**2-2*K*r-a**2)**(1/2),x)
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r - a**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx = \frac{x}{\sqrt{2\,Hr^2-a^2-2\,Kr}r}$$

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="maxima")
[Out] x/(sqrt(2*H*r^2 - a^2 - 2*K*r)*r)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx = \frac{x}{\sqrt{2\,Hr^2-a^2-2\,Kr}r}$$

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="giac")
[Out] x/(sqrt(2*H*r^2 - a^2 - 2*K*r)*r)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\,\sqrt{-a^2+2\,H\,r^2-2\,K\,r}}$$

```
[In] int(1/(r*(2*H*r^2 - 2*K*r - a^2)^(1/2)),x)
[Out] x/(r*(2*H*r^2 - 2*K*r - a^2)^(1/2))
```

3.108 $\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx$

Optimal result 464
Rubi [A] (verified) 464
Mathematica [A] (verified) 465
Maple [A] (verified) 465
Fricas [A] (verification not implemented) 465
Sympy [A] (verification not implemented) 466
Maxima [A] (verification not implemented) 466
Giac [A] (verification not implemented) 466
Mupad [B] (verification not implemented) 466

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2-e^2-2r(K-Hr)}}$$

```
[Out] x/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {8}

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2-e^2-2r(K-Hr)}}$$

```
[In] Int[1/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]),x]
[Out] x/(r*Sqrt[-a^2 - e^2 - 2*r*(K - H*r)])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\text{integral} = \frac{x}{r\sqrt{-a^2-e^2-2r(K-Hr)}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}$$

```
[In] Integrate[1/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]),x]
[Out] x/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{x}{r\sqrt{2H\,r^2-2Kr-a^2-e^2}}$	29
norman	$\frac{x}{r\sqrt{2H\,r^2-2Kr-a^2-e^2}}$	29
parallelrisch	$\frac{x}{r\sqrt{2H\,r^2-2Kr-a^2-e^2}}$	29

```
[In] int(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] x/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{\sqrt{2\,Hr^2-a^2-e^2-2\,Kr}x}{2\,Hr^3-2\,Kr^2-(a^2+e^2)r}$$

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*x/(2*H*r^3 - 2*K*r^2 - (a^2 + e^2)*r)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{2Hr^2-2Kr-a^2-e^2}}$$

```
[In] integrate(1/r/(2*H*r**2-2*K*r-a**2-e**2)**(1/2),x)
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r - a**2 - e**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{x}{\sqrt{2\,Hr^2-a^2-e^2-2\,Kr}r}$$

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="maxima")
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*r)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{x}{\sqrt{2\,Hr^2-a^2-e^2-2\,Kr}r}$$

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="giac")
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*r)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\,\sqrt{-a^2-e^2+2\,H\,r^2-2\,K\,r}}$$

```
[In] int(1/(r*(2*H*r^2 - 2*K*r - a^2 - e^2)^(1/2)),x)
[Out] x/(r*(2*H*r^2 - 2*K*r - a^2 - e^2)^(1/2))
```

3.109 $\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx$

Optimal result . . . 467
Rubi [A] (verified) . . . 467
Mathematica [A] (verified) . . . 468
Maple [A] (verified) . . . 468
Fricas [A] (verification not implemented) . . . 468
Sympy [A] (verification not implemented) . . . 469
Maxima [A] (verification not implemented) . . . 469
Giac [A] (verification not implemented) . . . 469
Mupad [B] (verification not implemented) . . . 469

Optimal result

Integrand size = 18, antiderivative size = 19

$$\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx = \frac{rx}{\sqrt{-a^2+2er^2}}$$

```
[Out] r*x/(-a^2+2*exp(1)*r^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {8}

$$\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx = \frac{rx}{\sqrt{2er^2-a^2}}$$

```
[In] Int[r/Sqrt[-a^2 + 2*E*r^2],x]
```

```
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\text{integral} = \frac{rx}{\sqrt{-a^2+2er^2}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx=\frac{rx}{\sqrt{-a^2+2er^2}}$$

```
[In] Integrate[r/Sqrt[-a^2 + 2*E*r^2],x]
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{rx}{\sqrt{-a^{2}+2\,\mathrm{e}r^{2}}}$	19
norman	$\frac{rx}{\sqrt{-a^{2}+2\,\mathrm{e}r^{2}}}$	19
parallelrisch	$\frac{rx}{\sqrt{-a^{2}+2\,\mathrm{e}r^{2}}}$	19

```
[In] int(r/(-a^2+2*exp(1)*r^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] r*x/(-a^2+2*exp(1)*r^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx=\frac{rx}{\sqrt{2\,r^2e-a^2}}$$

```
[In] integrate(r/(-a^2+2*exp(1)*r^2)^(1/2),x, algorithm="fricas")
[Out] r*x/sqrt(2*r^2*e - a^2)
```

Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{r}{\sqrt{-a^2 + 2er^2}}\,dx = \frac{rx}{\sqrt{-a^2 + 2er^2}}$$

```
[In] integrate(r/(-a**2+2*exp(1)*r**2)**(1/2),x)
[Out] r*x/sqrt(-a**2 + 2*E*r**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{r}{\sqrt{-a^2 + 2er^2}}\,dx = \frac{rx}{\sqrt{2\,r^2e - a^2}}$$

```
[In] integrate(r/(-a^2+2*exp(1)*r^2)^(1/2),x, algorithm="maxima")
[Out] r*x/sqrt(2*r^2*e - a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{r}{\sqrt{-a^2 + 2er^2}}\,dx = \frac{rx}{\sqrt{2\,r^2e - a^2}}$$

```
[In] integrate(r/(-a^2+2*exp(1)*r^2)^(1/2),x, algorithm="giac")
[Out] r*x/sqrt(2*r^2*e - a^2)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{r}{\sqrt{-a^2 + 2er^2}}\,dx = \frac{r\,x}{\sqrt{2\,r^2\,\mathrm{e} - a^2}}$$

```
[In] int(r/(2*r^2*exp(1) - a^2)^(1/2),x)
[Out] (r*x)/(2*r^2*exp(1) - a^2)^(1/2)
```

3.110 $\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx$

Optimal result . . . 470
Rubi [A] (verified) . . . 470
Mathematica [A] (verified) . . . 471
Maple [A] (verified) . . . 471
Fricas [A] (verification not implemented) . . . 471
Sympy [A] (verification not implemented) . . . 472
Maxima [A] (verification not implemented) . . . 472
Giac [A] (verification not implemented) . . . 472
Mupad [B] (verification not implemented) . . . 472

Optimal result

Integrand size = 23, antiderivative size = 24

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx=\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

```
[Out] r*x/(-a^2-e^2+2*exp(1)*r^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {8}

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx=\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

```
[In] Int[r/Sqrt[-a^2 - e^2 + 2*E*r^2],x]
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\text{integral}=\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx=\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

```
[In] Integrate[r/Sqrt[-a^2 - e^2 + 2*E*r^2],x]
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{rx}{\sqrt{-a^2-e^2+2\,\mathrm{e}r^2}}$	24
norman	$\frac{rx}{\sqrt{-a^2-e^2+2\,\mathrm{e}r^2}}$	24
parallelrisch	$\frac{rx}{\sqrt{-a^2-e^2+2\,\mathrm{e}r^2}}$	24

```
[In] int(r/(-a^2-e^2+2*exp(1)*r^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] r*x/(-a^2-e^2+2*exp(1)*r^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx=\frac{rx}{\sqrt{2\,r^2e-a^2-e^2}}$$

```
[In] integrate(r/(-a^2-e^2+2*exp(1)*r^2)^(1/2),x, algorithm="fricas")
[Out] r*x/sqrt(2*r^2*e - a^2 - e^2)
```

Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx=\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

```
[In] integrate(r/(-a**2-e**2+2*exp(1)*r**2)**(1/2),x)
[Out] r*x/sqrt(-a**2 - e**2 + 2*E*r**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx=\frac{rx}{\sqrt{2\,r^2e-a^2-e^2}}$$

```
[In] integrate(r/(-a^2-e^2+2*exp(1)*r^2)^(1/2),x, algorithm="maxima")
[Out] r*x/sqrt(2*r^2*e - a^2 - e^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx=\frac{rx}{\sqrt{2\,r^2e-a^2-e^2}}$$

```
[In] integrate(r/(-a^2-e^2+2*exp(1)*r^2)^(1/2),x, algorithm="giac")
[Out] r*x/sqrt(2*r^2*e - a^2 - e^2)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx=\frac{r\,x}{\sqrt{-a^2-e^2+2\,\mathrm{e}\,r^2}}$$

```
[In] int(r/(2*r^2*exp(1) - a^2 - e^2)^(1/2),x)
[Out] (r*x)/(2*r^2*exp(1) - a^2 - e^2)^(1/2)
```

3.111 $\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx$

Optimal result 473
Rubi [A] (verified) 473
Mathematica [A] (verified) 474
Maple [A] (verified) 474
Fricas [A] (verification not implemented) 474
Sympy [A] (verification not implemented) 475
Maxima [A] (verification not implemented) 475
Giac [A] (verification not implemented) 475
Mupad [B] (verification not implemented) 475

Optimal result

Integrand size = 24, antiderivative size = 25

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx = \frac{rx}{\sqrt{-a^2+2er^2-2Kr^4}}$$

```
[Out] r*x/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {8}

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx = \frac{rx}{\sqrt{-a^2-2Kr^4+2er^2}}$$

```
[In] Int[r/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4],x]
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\text{integral} = \frac{rx}{\sqrt{-a^2+2er^2-2Kr^4}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx=\frac{rx}{\sqrt{-a^2+2er^2-2Kr^4}}$$

```
[In] Integrate[r/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4],x]
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{rx}{\sqrt{-a^{2}+2\,\mathrm{e}\,r^{2}-2K\,r^{4}}}$	25
norman	$\frac{rx}{\sqrt{-a^{2}+2\,\mathrm{e}\,r^{2}-2K\,r^{4}}}$	25
parallelrisch	$\frac{rx}{\sqrt{-a^{2}+2\,\mathrm{e}\,r^{2}-2K\,r^{4}}}$	25

```
[In] int(r/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] r*x/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx=-\frac{\sqrt{-2\,Kr^4+2\,r^2e-a^2}rx}{2\,Kr^4-2\,r^2e+a^2}$$

```
[In] integrate(r/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-2*K*r^4 + 2*r^2*e - a^2)*r*x/(2*K*r^4 - 2*r^2*e + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx=\frac{rx}{\sqrt{-2Kr^4-a^2+2er^2}}$$

```
[In] integrate(r/(-a**2+2*exp(1)*r**2-2*K*r**4)**(1/2),x)
[Out] r*x/sqrt(-2*K*r**4 - a**2 + 2*E*r**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx=\frac{rx}{\sqrt{-2\,Kr^4+2\,r^2e-a^2}}$$

```
[In] integrate(r/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="maxima")
[Out] r*x/sqrt(-2*K*r^4 + 2*r^2*e - a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx=\frac{rx}{\sqrt{-2\,Kr^4+2\,r^2e-a^2}}$$

```
[In] integrate(r/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="giac")
[Out] r*x/sqrt(-2*K*r^4 + 2*r^2*e - a^2)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx=\frac{r\,x}{\sqrt{-a^2-2\,K\,r^4+2\,\mathrm{e}\,r^2}}$$

```
[In] int(r/(2*r^2*exp(1) - 2*K*r^4 - a^2)^(1/2),x)
[Out] (r*x)/(2*r^2*exp(1) - 2*K*r^4 - a^2)^(1/2)
```

3.112 $\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx$

Optimal result 476
Rubi [A] (verified) 476
Mathematica [A] (verified) 477
Maple [A] (verified) 477
Fricas [A] (verification not implemented) 477
Sympy [A] (verification not implemented) 478
Maxima [A] (verification not implemented) 478
Giac [A] (verification not implemented) 478
Mupad [B] (verification not implemented) 478

Optimal result

Integrand size = 29, antiderivative size = 30

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx=\frac{rx}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}$$

```
[Out] r*x/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = $\{8\}$

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx=\frac{rx}{\sqrt{-a^2-e^2-2Kr^4+2er^2}}$$

```
[In] Int[r/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4],x]
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\text{integral}=\frac{rx}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx=\frac{rx}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}$$

```
[In] Integrate[r/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4],x]
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{rx}{\sqrt{-a^2-e^2+2\,\mathrm{e}r^2-2K\,r^4}}$	30
norman	$\frac{rx}{\sqrt{-a^2-e^2+2\,\mathrm{e}r^2-2K\,r^4}}$	30
parallelrisch	$\frac{rx}{\sqrt{-a^2-e^2+2\,\mathrm{e}r^2-2K\,r^4}}$	30

```
[In] int(r/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] r*x/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx=-\frac{\sqrt{-2\,Kr^4+2\,r^2e-a^2-e^2}rx}{2\,Kr^4-2\,r^2e+a^2+e^2}$$

```
[In] integrate(r/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-2*K*r^4 + 2*r^2*e - a^2 - e^2)*r*x/(2*K*r^4 - 2*r^2*e + a^2 + e^2)
```

Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx = \frac{rx}{\sqrt{-2Kr^4-a^2-e^2+2er^2}}$$

```
[In] integrate(r/(-a**2-e**2+2*exp(1)*r**2-2*K*r**4)**(1/2),x)
[Out] r*x/sqrt(-2*K*r**4 - a**2 - e**2 + 2*E*r**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx = \frac{rx}{\sqrt{-2\,Kr^4+2\,r^2e-a^2-e^2}}$$

```
[In] integrate(r/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="maxima")
[Out] r*x/sqrt(-2*K*r^4 + 2*r^2*e - a^2 - e^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx = \frac{rx}{\sqrt{-2\,Kr^4+2\,r^2e-a^2-e^2}}$$

```
[In] integrate(r/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="giac")
[Out] r*x/sqrt(-2*K*r^4 + 2*r^2*e - a^2 - e^2)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx = \frac{r\,x}{\sqrt{-a^2-e^2-2\,K\,r^4+2\,\mathrm{e}\,r^2}}$$

```
[In] int(r/(2*r^2*exp(1) - 2*K*r^4 - a^2 - e^2)^(1/2),x)
[Out] (r*x)/(2*r^2*exp(1) - 2*K*r^4 - a^2 - e^2)^(1/2)
```

3.113 $\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx$

Optimal result 479
Rubi [A] (verified) 479
Mathematica [A] (verified) 480
Maple [A] (verified) 480
Fricas [A] (verification not implemented) 480
Sympy [A] (verification not implemented) 481
Maxima [A] (verification not implemented) 481
Giac [A] (verification not implemented) 481
Mupad [B] (verification not implemented) 481

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx=\frac{rx}{\sqrt{-a^2-e^2-2r(K-Hr)}}$$

[Out] r*x/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {8}

$$\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx=\frac{rx}{\sqrt{-a^2-e^2-2r(K-Hr)}}$$

[In] Int[r/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2],x]

[Out] (r*x)/Sqrt[-a^2 - e^2 - 2*r*(K - H*r)]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\text{integral}=\frac{rx}{\sqrt{-a^2-e^2-2r(K-Hr)}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx=\frac{rx}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}$$

```
[In] Integrate[r/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2],x]
[Out] (r*x)/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{rx}{\sqrt{2H\,r^2-2Kr-a^2-e^2}}$	27
norman	$\frac{rx}{\sqrt{2H\,r^2-2Kr-a^2-e^2}}$	27
parallelrisch	$\frac{rx}{\sqrt{2H\,r^2-2Kr-a^2-e^2}}$	27

```
[In] int(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] r*x/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx=\frac{rx}{\sqrt{2\,Hr^2-a^2-e^2-2\,Kr}}$$

```
[In] integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="fricas")
[Out] r*x/sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)
```

Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{r}{\sqrt{-a^2 - e^2 - 2Kr + 2Hr^2}}\,dx = \frac{rx}{\sqrt{2Hr^2 - 2Kr - a^2 - e^2}}$$

```
[In] integrate(r/(2*H*r**2-2*K*r-a**2-e**2)**(1/2),x)
[Out] r*x/sqrt(2*H*r**2 - 2*K*r - a**2 - e**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2 - e^2 - 2Kr + 2Hr^2}}\,dx = \frac{rx}{\sqrt{2\,Hr^2 - a^2 - e^2 - 2\,Kr}}$$

```
[In] integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="maxima")
[Out] r*x/sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2 - e^2 - 2Kr + 2Hr^2}}\,dx = \frac{rx}{\sqrt{2\,Hr^2 - a^2 - e^2 - 2\,Kr}}$$

```
[In] integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="giac")
[Out] r*x/sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2 - e^2 - 2Kr + 2Hr^2}}\,dx = \frac{r\,x}{\sqrt{-a^2 - e^2 + 2\,H\,r^2 - 2\,K\,r}}$$

```
[In] int(r/(2*H*r^2 - 2*K*r - a^2 - e^2)^(1/2),x)
[Out] (r*x)/(2*H*r^2 - 2*K*r - a^2 - e^2)^(1/2)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 483

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                 end if
             else #result contains complex but optimal is not
                 if debug then
                         print("result contains complex but optimal is not");
                 fi;
                 return "C","Result contains complex when optimal does not.";
             fi;
        else # result do not contain complex
              # this assumes optimal do not as well. No check is needed here.
             if debug then
                    print("result do not contain complex, this assumes optimal do not as well")
             fi;
             if leaf_count_result<=2*leaf_count_optimal then
                 if debug then
                     print("leaf_count_result<=2*leaf_count_optimal");
                 fi;
                 return "A"," ";
             else
                 if debug then
                     print("leaf_count_result>2*leaf_count_optimal");
                 fi;
                 return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
             fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```