Computer Algebra
Independent Integration Tests

Summer 2023 edition

0-Independent-test-suites/9-Stewart-Problems

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September 5, 2023 Compiled on September 5, 2023 at 10:45am

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [376]. This is test number [9].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (376)	0.00 (0)
Mathematica	100.00 (376)	0.00 (0)
Maple	100.00 (376)	0.00 (0)
Fricas	100.00 (376)	0.00 (0)
Giac	99.73 (375)	0.27 (1)
Maxima	99.47 (374)	0.53 (2)
Mupad	98.94 (372)	1.06 (4)
Sympy	96.54 (363)	3.46 (13)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

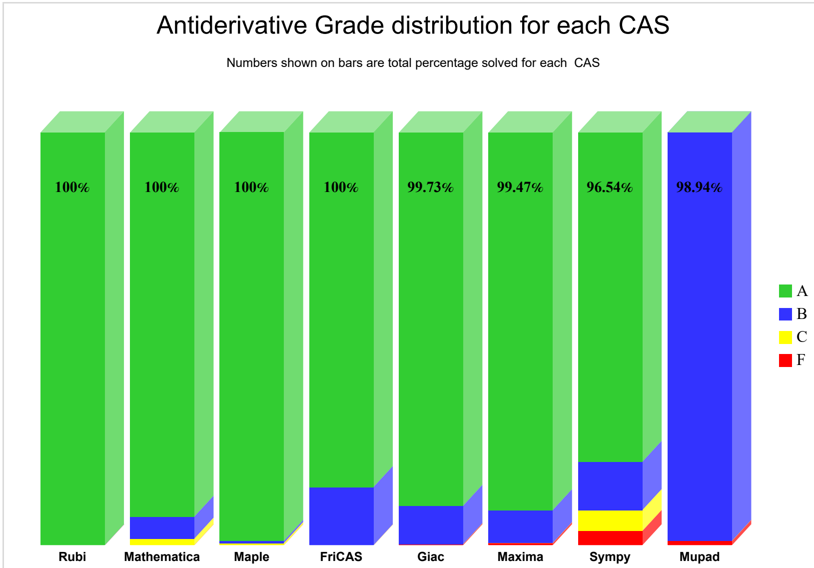
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

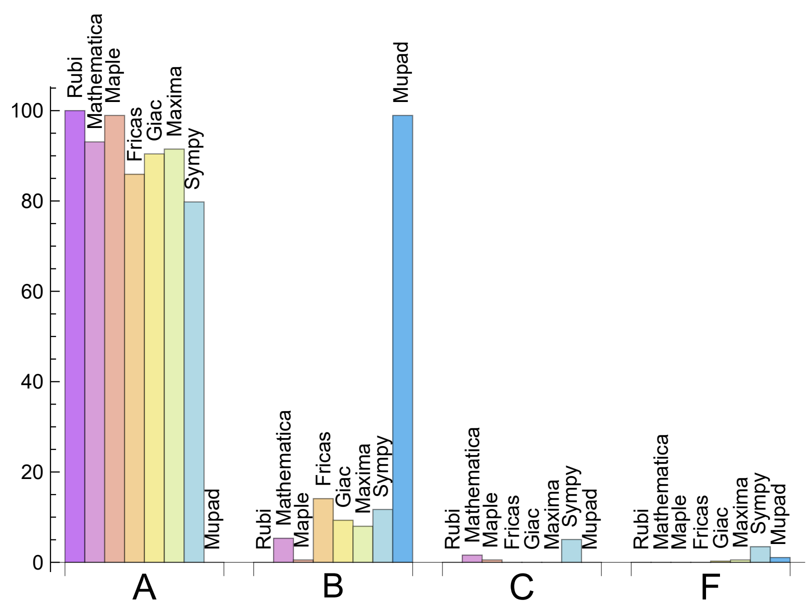
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	98.936	0.532	0.532	0.000
Mathematica	93.085	5.319	1.596	0.000
Maxima	91.489	7.979	0.000	0.532
Giac	90.426	9.309	0.000	0.266
Fricas	85.904	14.096	0.000	0.000
Sympy	79.787	11.702	5.053	3.457
Mupad	0.000	98.936	0.000	1.064

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Giac	1	0.00	100.00	0.00
Maxima	2	100.00	0.00	0.00
Mupad	4	0.00	100.00	0.00
Sympy	13	76.92	23.08	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.02
Mathematica	0.03
Mupad	0.14
Sympy	0.22
Maxima	0.23
Fricas	0.25
Giac	0.29
Maple	0.30

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	18.76	0.89	16.00	0.83
Mupad	19.37	0.90	16.00	0.80
Giac	21.24	1.10	18.00	0.82
Maxima	21.78	1.04	16.00	0.80
Rubi	22.98	1.00	19.00	1.00
Mathematica	23.20	1.12	20.00	1.00
Fricas	24.64	1.10	18.00	0.86
Sympy	223.04	14.37	19.00	0.88

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

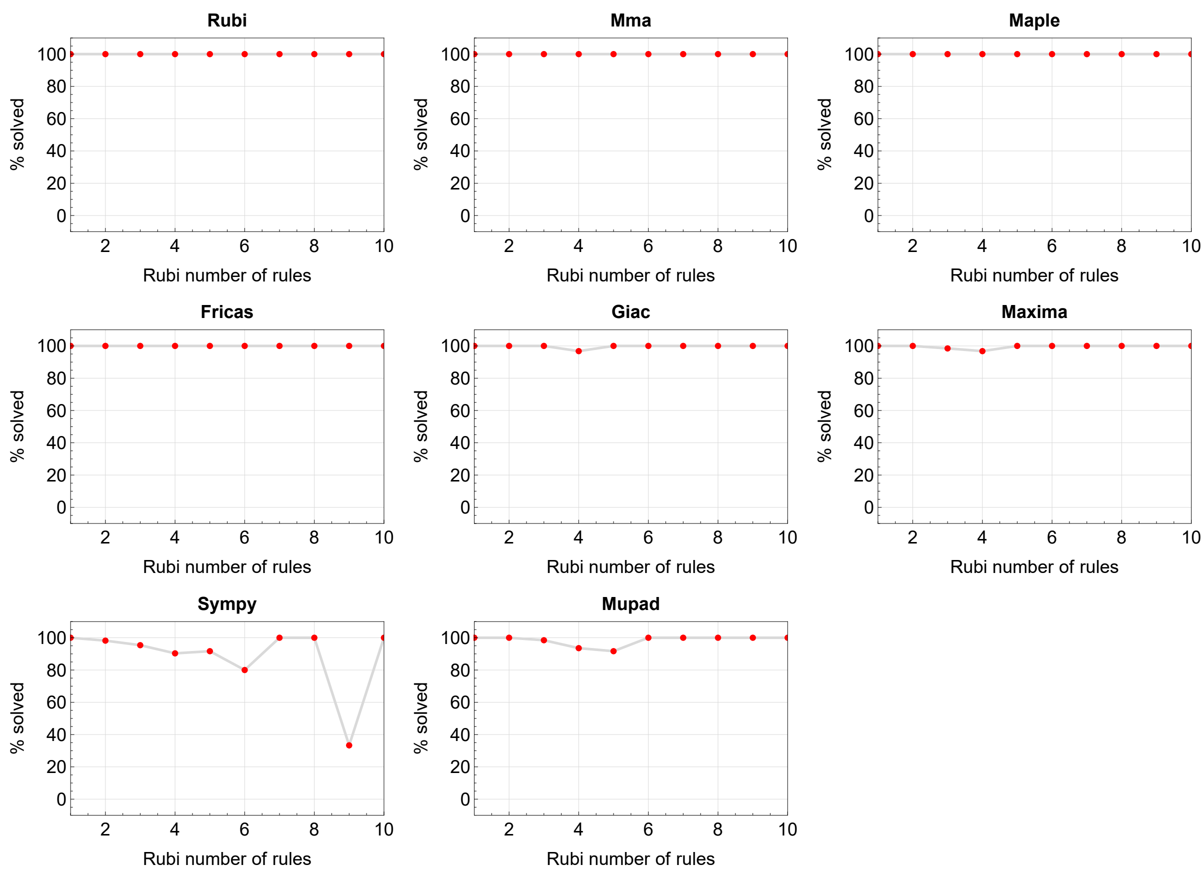


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

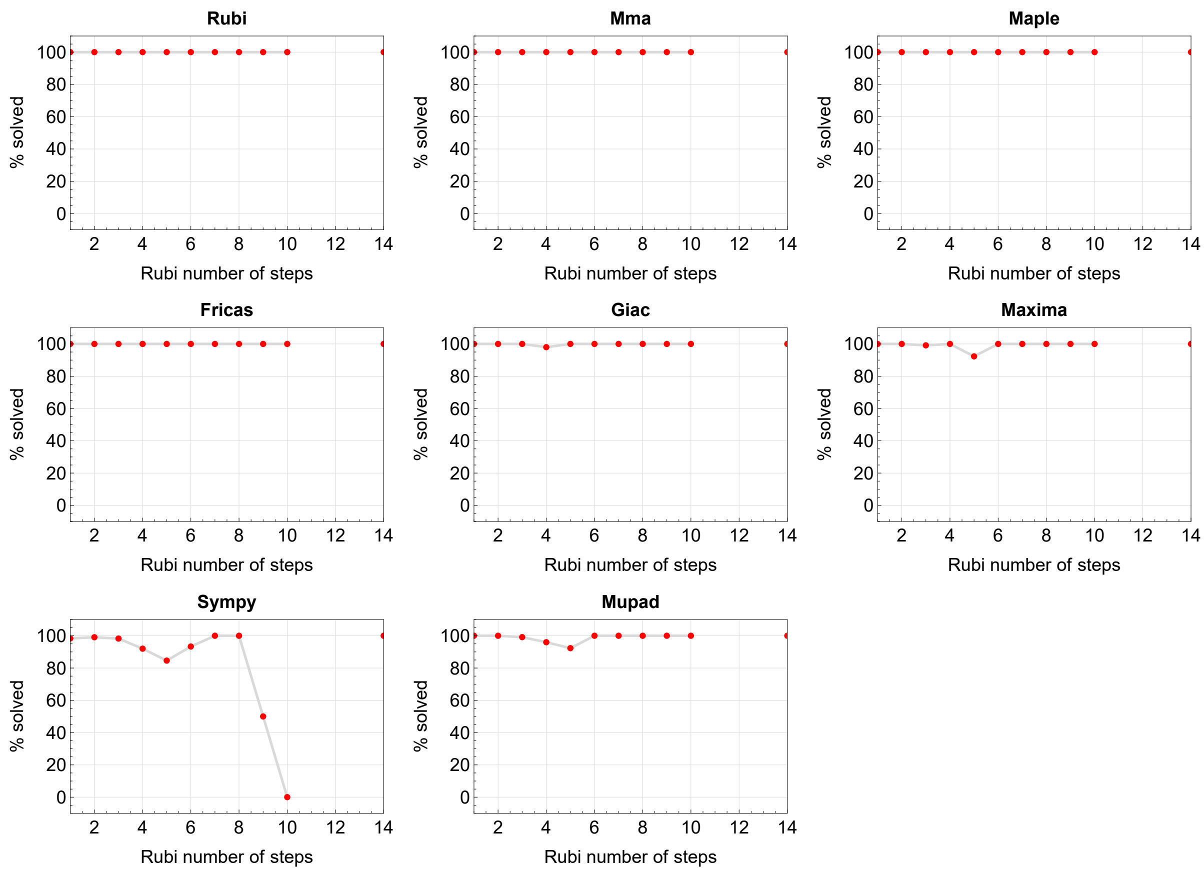


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

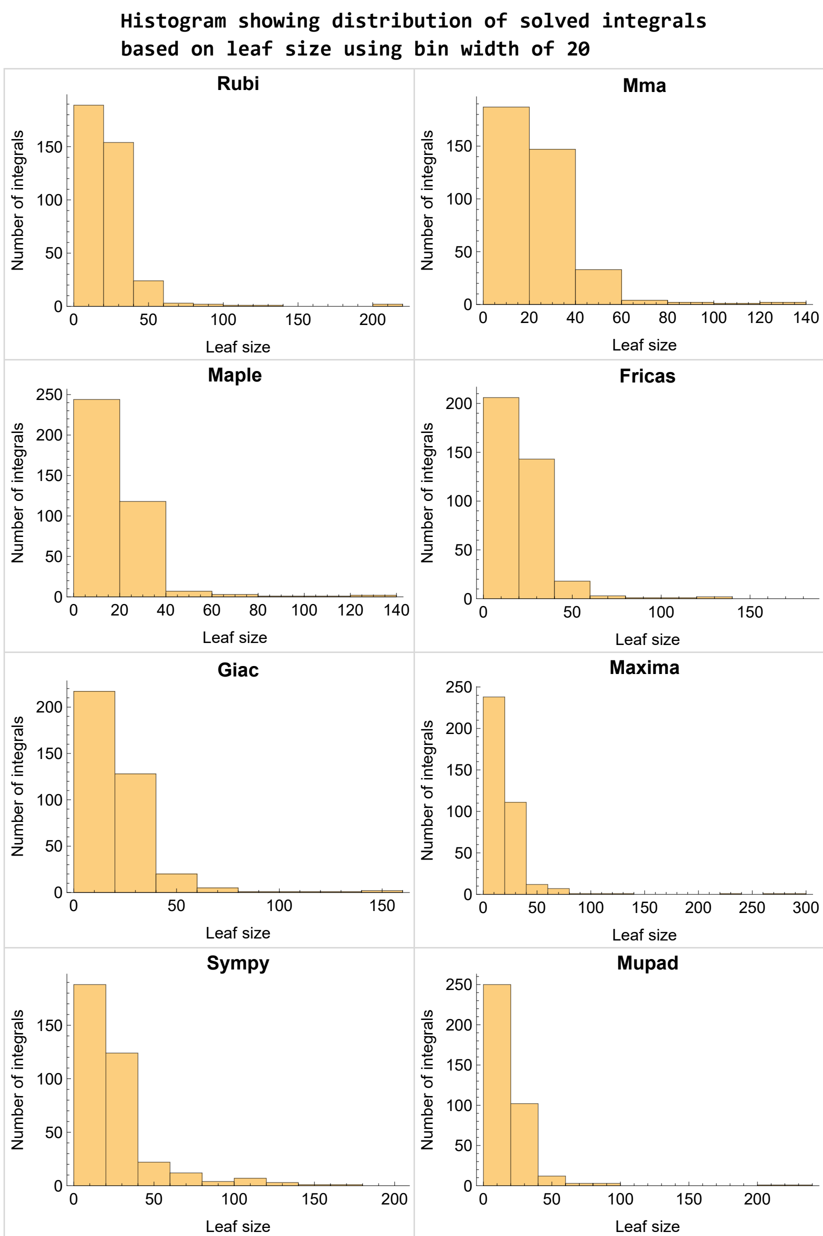


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

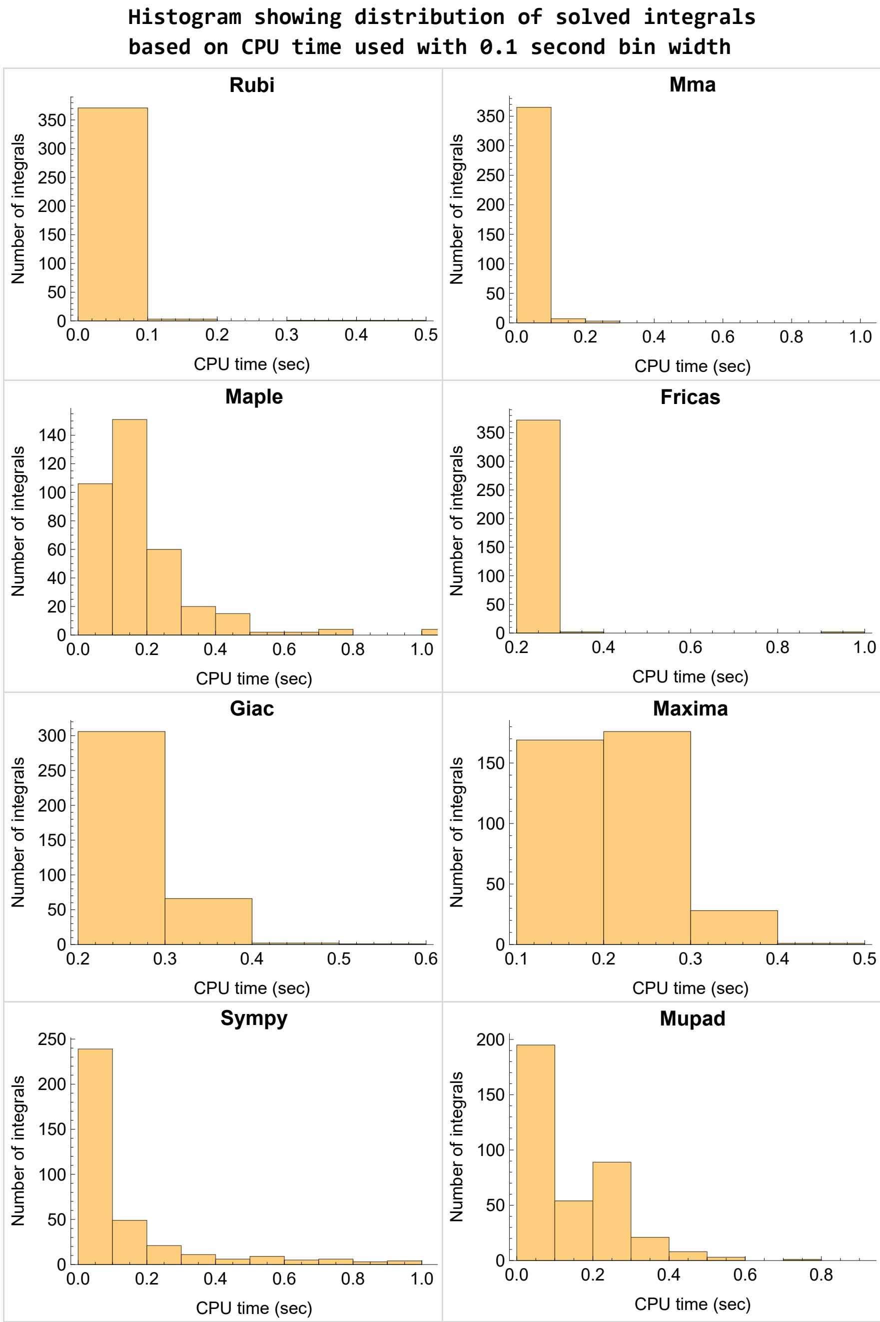


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

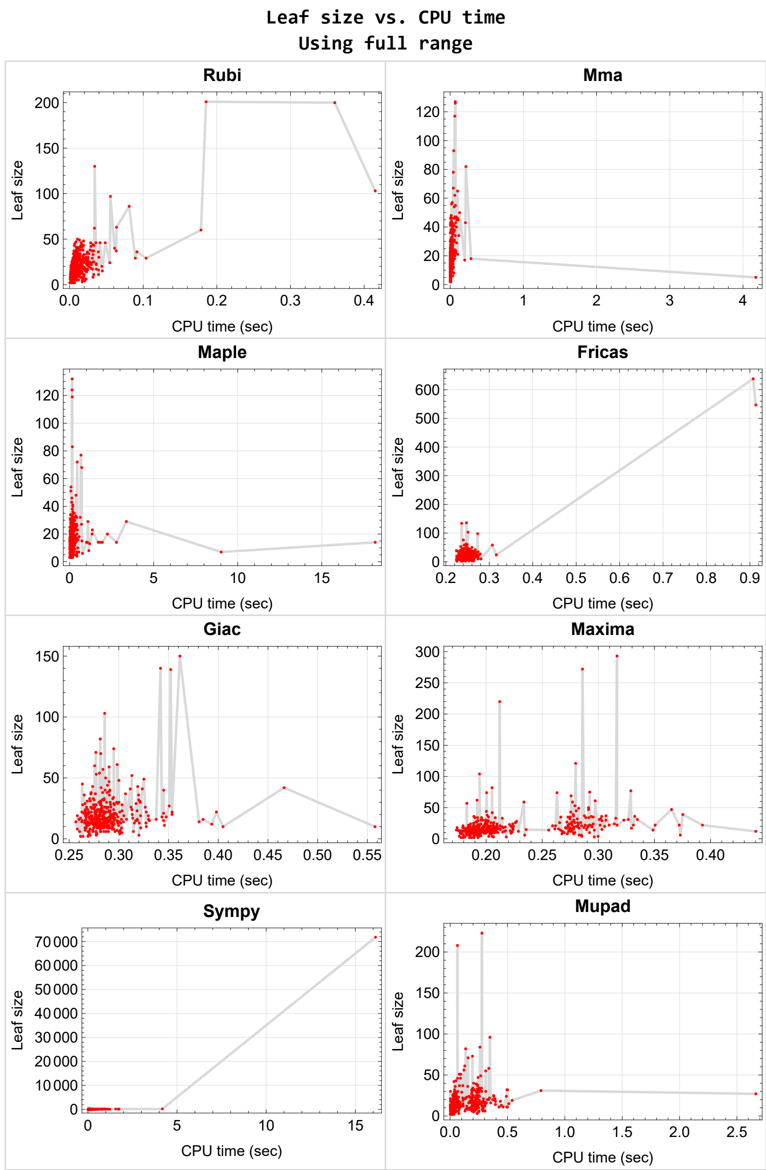


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple $\{235, 323\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

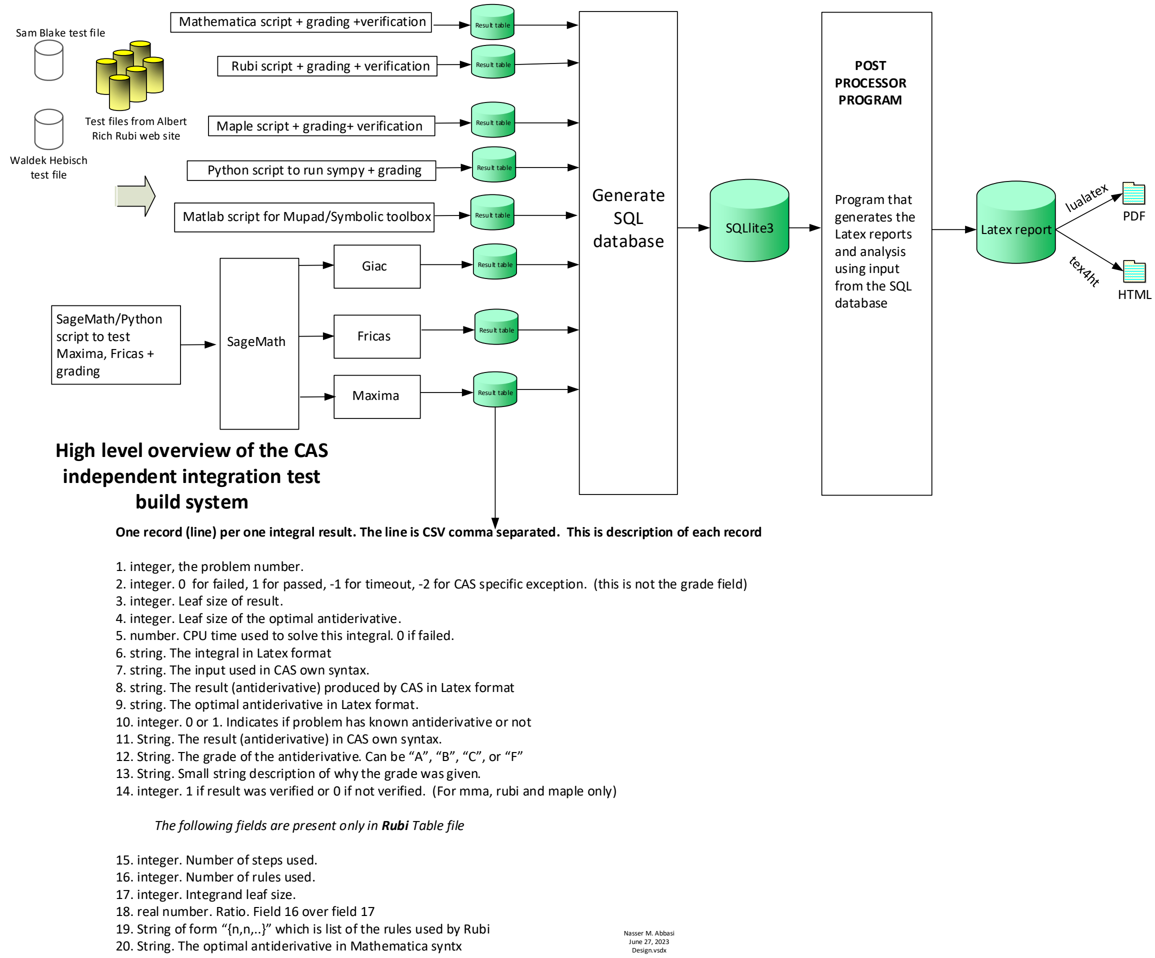
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 101, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 241, 242, 243, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 371, 372, 373, 374, 375, 376 }

B grade { 14, 81, 100, 102, 103, 104, 121, 130, 145, 152, 195, 212, 221, 245, 246, 270, 297, 312, 328, 370 }

C grade { 98, 220, 235, 244, 316, 335 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

B grade { 270, 359 }

C grade { 195, 323 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 106, 107, 109, 110, 111, 112, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 242, 243, 245, 247, 252, 253, 254, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 335, 336, 338, 339, 340, 342, 343, 345, 346, 347, 348, 349, 350, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 371, 372, 373, 374, 375, 376 }

B grade { 7, 13, 14, 29, 41, 67, 79, 86, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 113, 115, 130, 139, 143, 145, 195, 220, 221, 225, 235, 241, 244, 246, 248, 249, 250, 251, 255, 270, 291, 295, 298, 311, 312, 313, 314, 328, 334, 337, 341, 344, 351, 355, 370 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 242, 243, 247, 249, 250, 251, 252, 254, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 272, 273, 274, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

B grade { 29, 34, 35, 41, 95, 97, 103, 104, 112, 113, 220, 221, 225, 235, 241, 244, 245, 246, 248, 253, 255, 271, 276, 291, 298, 313, 328, 329, 355, 363 }

C grade { }

F normal fail { 330, 337 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 122, 123, 125, 126, 127, 128, 129, 131, 132, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 294, 296, 297, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

B grade { 11, 12, 29, 41, 97, 98, 103, 104, 113, 121, 124, 130, 133, 138, 145, 152, 195, 205, 225, 241, 244, 255, 263, 270, 291, 293, 295, 298, 306, 312, 328, 329, 344, 348, 363 }

C grade { }

F normal fail { }

F(-1) timedout fail { 269 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

C grade { }

F normal fail { }

F(-1) timedout fail { 147, 323, 359, 363 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 77, 78, 79, 81, 83, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 103, 105, 107, 109, 110, 112, 114, 115, 116, 117, 118, 119, 120, 122, 123, 125, 126, 128, 129, 130, 131, 137, 138, 140, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 221, 222, 223, 224, 227, 231, 232, 233, 234, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 271, 272, 273, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 323, 325, 326, 327, 328, 329, 331, 332, 333, 335, 338, 339, 340, 342, 343, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 364, 366, 367, 368, 371, 373, 374, 375, 376 }

B grade { 7, 8, 37, 42, 57, 67, 75, 76, 80, 82, 84, 86, 87, 100, 102, 104, 106, 108, 111, 113, 127, 139, 142, 181, 195, 219, 225, 226, 230, 251, 270, 283, 291, 296, 297, 298, 306, 320, 330, 334, 341, 344, 370, 372 }

C grade { 121, 124, 132, 133, 134, 135, 136, 141, 143, 228, 229, 250, 266, 274, 324, 336, 346, 363, 369 }

F normal fail { 149, 220, 235, 238, 247, 248, 249, 301, 322, 359 }

F(-1) timedout fail { 74, 337, 365 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	11	10	12	11	20
N.S.	1	1.00	1.00	1.00	1.00	0.91	1.09	1.00	1.82
time (sec)	N/A	0.003	0.001	0.020	0.193	0.242	0.014	0.279	0.371

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	3	2	2	2	2	2
N.S.	1	1.00	1.00	1.00	0.67	0.67	0.67	0.67	0.67
time (sec)	N/A	0.002	0.000	0.024	0.187	0.236	0.029	0.264	0.006

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	3	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.50	1.00
time (sec)	N/A	0.000	0.000	0.012	0.189	0.238	0.033	0.270	0.002

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	8	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.003	0.001	0.030	0.195	0.235	0.033	0.294	0.209

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.003	0.001	0.045	0.205	0.243	0.033	0.270	0.021

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.002	0.001	0.027	0.193	0.246	0.033	0.269	0.003

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	7	5	2	2
N.S.	1	1.00	1.00	1.50	1.00	3.50	2.50	1.00	1.00
time (sec)	N/A	0.007	0.000	0.174	0.180	0.239	0.038	0.261	0.033

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	6	8	7	6	4
N.S.	1	1.00	1.00	1.25	1.50	2.00	1.75	1.50	1.00
time (sec)	N/A	0.005	0.009	0.175	0.180	0.248	0.035	0.300	0.011

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	4	4	3	4	12
N.S.	1	1.00	1.00	1.50	2.00	2.00	1.50	2.00	6.00
time (sec)	N/A	0.005	0.006	0.078	0.176	0.238	0.021	0.301	0.280

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	6	6	5	6	6
N.S.	1	1.00	1.00	1.25	1.50	1.50	1.25	1.50	1.50
time (sec)	N/A	0.006	0.006	0.053	0.185	0.234	0.038	0.287	0.252

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	11	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	5.50	1.00
time (sec)	N/A	0.003	0.011	0.067	0.185	0.225	0.063	0.290	0.019

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	11	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	5.50	1.00
time (sec)	N/A	0.002	0.008	0.056	0.178	0.225	0.064	0.300	0.016

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	3	11	5	6	5
N.S.	1	1.00	1.00	1.20	0.60	2.20	1.00	1.20	1.00
time (sec)	N/A	0.002	0.001	0.019	0.181	0.240	0.041	0.269	0.029

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	7	4	3	11	3	4	3
N.S.	1	1.00	2.33	1.33	1.00	3.67	1.00	1.33	1.00
time (sec)	N/A	0.002	0.001	0.029	0.181	0.245	0.036	0.278	0.002

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.006	0.003	0.047	0.194	0.248	0.059	0.258	0.023

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	8	6
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.62	1.00	0.75
time (sec)	N/A	0.001	0.000	0.013	0.192	0.229	0.030	0.271	0.019

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	12	12	11	11	10	11	11
N.S.	1	1.00	0.63	0.63	0.58	0.58	0.53	0.58	0.58
time (sec)	N/A	0.011	0.002	0.026	0.203	0.235	0.028	0.267	0.028

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	12	11	13	15	11	11
N.S.	1	1.00	0.74	0.63	0.58	0.68	0.79	0.58	0.58
time (sec)	N/A	0.006	0.008	0.076	0.187	0.243	0.086	0.275	0.003

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	0.87	0.87
time (sec)	N/A	0.003	0.002	0.073	0.186	0.251	0.068	0.288	0.214

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	15	11	11	11	10	11	11
N.S.	1	1.00	0.75	0.55	0.55	0.55	0.50	0.55	0.55
time (sec)	N/A	0.006	0.013	0.037	0.184	0.225	0.032	0.269	0.023

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	7	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.008	0.003	0.064	0.186	0.248	0.063	0.266	0.002

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	14	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.78	0.78	0.78
time (sec)	N/A	0.008	0.033	0.136	0.184	0.248	0.066	0.269	0.028

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.003	0.000	0.017	0.178	0.243	0.030	0.273	0.002

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	22	21	21	27	21	23
N.S.	1	1.00	0.86	0.76	0.72	0.72	0.93	0.72	0.79
time (sec)	N/A	0.024	0.032	0.191	0.179	0.258	0.090	0.268	0.073

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	21	21	21	24	21	24
N.S.	1	1.00	0.86	0.72	0.72	0.72	0.83	0.72	0.83
time (sec)	N/A	0.020	0.031	0.171	0.194	0.247	0.090	0.267	0.032

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	12	15	15	15	12
N.S.	1	1.00	1.00	1.07	0.80	1.00	1.00	1.00	0.80
time (sec)	N/A	0.003	0.000	0.017	0.179	0.236	0.040	0.263	0.032

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	12	14	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.75	0.88	0.88
time (sec)	N/A	0.003	0.001	0.009	0.256	0.243	0.052	0.273	0.002

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	15	14	17	24	14	18
N.S.	1	1.00	0.78	0.65	0.61	0.74	1.04	0.61	0.78
time (sec)	N/A	0.010	0.004	0.102	0.173	0.239	0.086	0.263	0.048

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	74	18	8	103	8
N.S.	1	1.00	1.00	1.12	9.25	2.25	1.00	12.88	1.00
time (sec)	N/A	0.015	0.007	0.230	0.263	0.271	0.304	0.286	0.023

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.006	0.003	0.026	0.176	0.238	0.032	0.278	0.031

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	17	17	16	16	15	16	16
N.S.	1	1.00	0.63	0.63	0.59	0.59	0.56	0.59	0.59
time (sec)	N/A	0.024	0.017	0.032	0.186	0.243	0.032	0.278	0.019

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	19	21	26	19	19
N.S.	1	1.00	0.81	0.74	0.70	0.78	0.96	0.70	0.70
time (sec)	N/A	0.009	0.056	0.124	0.179	0.249	0.090	0.322	0.028

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	18	17	21	20	17	17
N.S.	1	1.00	0.74	0.67	0.63	0.78	0.74	0.63	0.63
time (sec)	N/A	0.010	0.048	0.137	0.186	0.238	0.167	0.270	0.028

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	34	9	7	17	9
N.S.	1	1.00	1.00	1.11	3.78	1.00	0.78	1.89	1.00
time (sec)	N/A	0.009	0.013	0.083	0.196	0.237	0.069	0.289	0.020

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	57	18	20	30	18
N.S.	1	1.00	1.00	1.00	3.00	0.95	1.05	1.58	0.95
time (sec)	N/A	0.012	0.017	0.131	0.183	0.235	0.136	0.280	0.062

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	11	10	9	9	7	9	9
N.S.	1	1.00	0.69	0.62	0.56	0.56	0.44	0.56	0.56
time (sec)	N/A	0.008	0.002	0.026	0.177	0.234	0.028	0.283	0.021

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	15	14	13	14	105	13	9
N.S.	1	1.00	0.71	0.67	0.62	0.67	5.00	0.62	0.43
time (sec)	N/A	0.006	0.005	0.099	0.176	0.247	0.917	0.302	0.035

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	14	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.78	0.78	0.78
time (sec)	N/A	0.009	0.031	0.130	0.187	0.248	0.060	0.294	0.023

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	16	15	14	14	12	14	14
N.S.	1	1.00	0.62	0.58	0.54	0.54	0.46	0.54	0.54
time (sec)	N/A	0.015	0.018	0.031	0.195	0.230	0.031	0.287	0.029

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	12	16	16
N.S.	1	1.00	1.00	0.94	0.89	0.89	0.67	0.89	0.89
time (sec)	N/A	0.004	0.004	0.015	0.277	0.253	0.056	0.279	0.183

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	104	20	8	52	9
N.S.	1	1.00	1.00	1.11	11.56	2.22	0.89	5.78	1.00
time (sec)	N/A	0.012	0.020	0.158	0.194	0.263	0.276	0.313	0.159

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	25	26	13	13
N.S.	1	1.00	1.00	0.82	0.76	1.47	1.53	0.76	0.76
time (sec)	N/A	0.006	0.032	0.257	0.176	0.243	0.129	0.274	0.056

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	9	13	14	22	8	13
N.S.	1	1.00	1.00	0.53	0.76	0.82	1.29	0.47	0.76
time (sec)	N/A	0.007	0.008	0.159	0.183	0.242	0.131	0.294	0.206

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	11	11	10	11	8
N.S.	1	1.00	1.00	0.82	1.00	1.00	0.91	1.00	0.73
time (sec)	N/A	0.009	0.005	0.288	0.183	0.246	0.199	0.280	0.222

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	14	12	11	11	10	11	11
N.S.	1	1.00	0.64	0.55	0.50	0.50	0.45	0.50	0.50
time (sec)	N/A	0.013	0.023	0.042	0.189	0.238	0.030	0.290	0.041

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	9	9	12	8	7	8	8
N.S.	1	1.00	0.60	0.60	0.80	0.53	0.47	0.53	0.53
time (sec)	N/A	0.006	0.027	0.030	0.191	0.224	0.033	0.294	0.035

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	15	14	14	14	14	14
N.S.	1	1.00	0.74	0.79	0.74	0.74	0.74	0.74	0.74
time (sec)	N/A	0.007	0.016	0.046	0.349	0.236	0.039	0.290	0.025

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	11	10	13	15	13	13
N.S.	1	1.00	1.00	0.65	0.59	0.76	0.88	0.76	0.76
time (sec)	N/A	0.004	0.003	0.073	0.208	0.241	0.118	0.277	0.206

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	16	16	11	11	20	11	11
N.S.	1	1.00	0.67	0.67	0.46	0.46	0.83	0.46	0.46
time (sec)	N/A	0.006	0.003	0.026	0.223	0.235	0.068	0.294	0.022

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	12	10	9	9	8	9	7
N.S.	1	1.00	0.86	0.71	0.64	0.64	0.57	0.64	0.50
time (sec)	N/A	0.001	0.000	0.017	0.222	0.245	0.029	0.278	0.030

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	13	12	13	15	13	13
N.S.	1	1.00	1.00	0.76	0.71	0.76	0.88	0.76	0.76
time (sec)	N/A	0.003	0.003	0.066	0.195	0.260	0.117	0.299	0.023

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.009	0.007	0.066	0.207	0.244	0.099	0.286	0.264

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.013	0.014	0.141	0.203	0.261	0.244	0.276	0.217

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	19	17	16	16	15	16	16
N.S.	1	1.00	0.68	0.61	0.57	0.57	0.54	0.57	0.57
time (sec)	N/A	0.025	0.025	0.040	0.278	0.232	0.036	0.295	0.031

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	16	16	15	13	15	15	14
N.S.	1	1.00	0.76	0.76	0.71	0.62	0.71	0.71	0.67
time (sec)	N/A	0.009	0.002	0.029	0.277	0.239	0.084	0.291	0.020

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	15	18	16
N.S.	1	1.00	1.00	0.94	0.89	0.89	0.83	1.00	0.89
time (sec)	N/A	0.010	0.019	0.276	0.191	0.248	0.128	0.280	0.026

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	15	14	13	14	105	13	9
N.S.	1	1.00	0.71	0.67	0.62	0.67	5.00	0.62	0.43
time (sec)	N/A	0.004	0.005	0.100	0.180	0.240	0.930	0.276	0.018

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	14	11	10	14	14	10	10
N.S.	1	1.00	0.78	0.61	0.56	0.78	0.78	0.56	0.56
time (sec)	N/A	0.005	0.038	0.125	0.199	0.237	0.025	0.283	0.052

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.005	0.002	0.056	0.189	0.240	0.017	0.275	0.002

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	17	16	19	24	16	16
N.S.	1	1.00	0.92	0.71	0.67	0.79	1.00	0.67	0.67
time (sec)	N/A	0.015	0.002	0.233	0.187	0.242	0.017	0.282	0.034

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	11	11	8	11	10
N.S.	1	1.00	1.15	0.85	0.85	0.85	0.62	0.85	0.77
time (sec)	N/A	0.005	0.002	0.211	0.183	0.245	0.019	0.282	0.046

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	31	14	13	13	12	13	14
N.S.	1	1.00	1.82	0.82	0.76	0.76	0.71	0.76	0.82
time (sec)	N/A	0.017	0.058	0.202	0.186	0.238	0.020	0.284	0.040

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	31	14	13	22	12	13	14
N.S.	1	1.00	1.82	0.82	0.76	1.29	0.71	0.76	0.82
time (sec)	N/A	0.018	0.057	0.187	0.182	0.244	0.020	0.278	0.202

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	23	18	25	31	22	24
N.S.	1	1.00	0.83	0.64	0.50	0.69	0.86	0.61	0.67
time (sec)	N/A	0.039	0.009	0.172	0.179	0.253	0.019	0.281	0.043

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	14	11	10	19	14	10	18
N.S.	1	1.00	0.58	0.46	0.42	0.79	0.58	0.42	0.75
time (sec)	N/A	0.019	0.001	0.095	0.182	0.249	0.022	0.294	0.045

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	18	15	14	18	37	14	14
N.S.	1	1.00	0.82	0.68	0.64	0.82	1.68	0.64	0.64
time (sec)	N/A	0.008	0.061	0.204	0.181	0.240	0.068	0.295	0.394

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	14	31	37	14	18
N.S.	1	1.00	1.00	0.75	0.70	1.55	1.85	0.70	0.90
time (sec)	N/A	0.015	0.017	0.426	0.184	0.248	0.129	0.305	0.171

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.027	0.034	0.228	0.186	0.264	0.022	0.294	0.054

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	23	24	25	36	22	22
N.S.	1	1.00	0.88	0.68	0.71	0.74	1.06	0.65	0.65
time (sec)	N/A	0.013	0.002	0.254	0.201	0.245	0.017	0.288	0.046

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	23	24	25	36	22	22
N.S.	1	1.00	0.88	0.68	0.71	0.74	1.06	0.65	0.65
time (sec)	N/A	0.014	0.009	0.264	0.186	0.249	0.018	0.281	0.039

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	30	23	18	33	41	22	37
N.S.	1	1.00	0.65	0.50	0.39	0.72	0.89	0.48	0.80
time (sec)	N/A	0.036	0.057	0.312	0.183	0.245	0.020	0.272	0.081

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	17	17	17	17	17
N.S.	1	1.00	1.10	0.81	0.81	0.81	0.81	0.81	0.81
time (sec)	N/A	0.006	0.002	0.230	0.188	0.239	0.021	0.273	0.038

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	22	17	16	31	31	16	32
N.S.	1	1.00	0.48	0.37	0.35	0.67	0.67	0.35	0.70
time (sec)	N/A	0.041	0.007	0.299	0.207	0.258	0.022	0.287	0.040

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	34	14	13	17	0	13	13
N.S.	1	1.00	1.62	0.67	0.62	0.81	0.00	0.62	0.62
time (sec)	N/A	0.016	0.071	0.201	0.209	0.252	0.000	0.272	0.114

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	13	14	170	13	25
N.S.	1	1.00	0.86	0.67	0.62	0.67	8.10	0.62	1.19
time (sec)	N/A	0.016	0.015	0.186	0.189	0.240	4.176	0.292	0.249

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	14	12	13	39	12	12
N.S.	1	1.00	0.95	0.74	0.63	0.68	2.05	0.63	0.63
time (sec)	N/A	0.019	0.034	0.082	0.217	0.249	0.108	0.281	0.303

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	15	15	22	15	14
N.S.	1	1.00	1.00	0.79	0.79	0.79	1.16	0.79	0.74
time (sec)	N/A	0.014	0.022	0.223	0.193	0.243	0.116	0.289	0.203

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	14	10	13	16
N.S.	1	1.00	1.00	0.93	1.14	1.00	0.71	0.93	1.14
time (sec)	N/A	0.017	0.008	1.193	0.197	0.259	0.038	0.276	0.233

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	20	29	20	37	20	28	32
N.S.	1	1.00	0.91	1.32	0.91	1.68	0.91	1.27	1.45
time (sec)	N/A	0.025	0.027	3.375	0.185	0.262	0.044	0.276	0.259

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	7	6	5	5	19	5	5
N.S.	1	1.00	1.40	1.20	1.00	1.00	3.80	1.00	1.00
time (sec)	N/A	0.012	0.008	0.184	0.195	0.253	0.139	0.277	0.203

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	11	15	17	8	10	10
N.S.	1	1.00	2.27	1.00	1.36	1.55	0.73	0.91	0.91
time (sec)	N/A	0.008	0.006	0.095	0.183	0.236	0.183	0.286	0.024

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	8	7	6	6	7	6	6
N.S.	1	1.00	1.33	1.17	1.00	1.00	1.17	1.00	1.00
time (sec)	N/A	0.004	0.001	0.026	0.285	0.257	0.021	0.282	0.027

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	12	12	19	12	12
N.S.	1	1.00	1.14	0.93	0.86	0.86	1.36	0.86	0.86
time (sec)	N/A	0.008	0.001	0.030	0.273	0.244	0.028	0.277	0.028

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	9	16	19	9	17
N.S.	1	1.00	1.00	1.00	0.82	1.45	1.73	0.82	1.55
time (sec)	N/A	0.007	0.005	0.260	0.187	0.246	0.020	0.280	0.029

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	15	22	31	15	27
N.S.	1	1.00	1.00	1.00	0.79	1.16	1.63	0.79	1.42
time (sec)	N/A	0.012	0.006	0.341	0.195	0.243	0.023	0.298	0.037

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	20	29	6	6
N.S.	1	1.00	1.00	0.88	0.75	2.50	3.62	0.75	0.75
time (sec)	N/A	0.015	0.002	0.547	0.189	0.245	0.021	0.315	0.023

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	27	14	13	20	29	13	13
N.S.	1	1.00	1.59	0.82	0.76	1.18	1.71	0.76	0.76
time (sec)	N/A	0.019	0.036	1.950	0.191	0.244	0.021	0.283	0.212

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.009	0.007	0.468	0.183	0.260	0.022	0.301	0.304

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	14	14	14	13
N.S.	1	1.00	1.00	0.82	0.82	0.82	0.82	0.82	0.76
time (sec)	N/A	0.017	0.039	1.060	0.205	0.236	0.044	0.276	0.446

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	34	24	20	22	18
N.S.	1	1.00	1.00	1.05	1.55	1.09	0.91	1.00	0.82
time (sec)	N/A	0.011	0.010	0.063	0.190	0.251	0.052	0.277	0.036

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	24	19	18	18	31	18	18
N.S.	1	1.00	1.09	0.86	0.82	0.82	1.41	0.82	0.82
time (sec)	N/A	0.011	0.006	0.062	0.271	0.252	0.027	0.285	0.036

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	20	22	20	17
N.S.	1	1.00	1.00	0.84	1.05	1.05	1.16	1.05	0.89
time (sec)	N/A	0.013	0.021	0.352	0.190	0.254	0.048	0.295	0.276

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	20	20	22	20	19
N.S.	1	1.00	1.00	0.80	0.80	0.80	0.88	0.80	0.76
time (sec)	N/A	0.019	0.024	2.250	0.188	0.244	0.052	0.313	0.540

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	10	6	7	6	18
N.S.	1	1.00	1.00	0.88	1.25	0.75	0.88	0.75	2.25
time (sec)	N/A	0.015	0.007	9.019	0.207	0.256	0.023	0.296	0.185

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	36	14	14	14	20
N.S.	1	1.00	1.00	0.82	2.12	0.82	0.82	0.82	1.18
time (sec)	N/A	0.017	0.016	18.177	0.189	0.267	0.045	0.280	0.188

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	10	6	7	6	6
N.S.	1	1.00	1.00	0.88	1.25	0.75	0.88	0.75	0.75
time (sec)	N/A	0.009	0.001	0.171	0.187	0.248	0.022	0.277	0.047

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	27	34	27	29	30
N.S.	1	1.00	1.00	1.50	1.69	2.12	1.69	1.81	1.88
time (sec)	N/A	0.011	0.010	0.131	0.192	0.257	0.051	0.295	0.310

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	18	13	10	20	8	18	8
N.S.	1	1.00	2.25	1.62	1.25	2.50	1.00	2.25	1.00
time (sec)	N/A	0.006	0.001	0.039	0.270	0.247	0.021	0.292	0.018

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	19	17	14	28	14	22	18
N.S.	1	1.00	1.36	1.21	1.00	2.00	1.00	1.57	1.29
time (sec)	N/A	0.009	0.001	0.122	0.196	0.254	0.044	0.285	0.025

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	37	14	14	39	41	14	14
N.S.	1	1.00	2.18	0.82	0.82	2.29	2.41	0.82	0.82
time (sec)	N/A	0.020	0.046	0.355	0.188	0.251	0.026	0.285	0.232

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	30	15	14	14
N.S.	1	1.00	1.00	0.82	0.82	1.76	0.88	0.82	0.82
time (sec)	N/A	0.025	0.010	0.241	0.192	0.253	0.048	0.300	0.222

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	17	6	8	19	15	6	5
N.S.	1	1.00	3.40	1.20	1.60	3.80	3.00	1.20	1.00
time (sec)	N/A	0.003	0.003	0.049	0.178	0.251	0.067	0.285	0.047

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	47	18	27	44	27	54	16
N.S.	1	1.00	2.94	1.12	1.69	2.75	1.69	3.38	1.00
time (sec)	N/A	0.007	0.012	0.230	0.191	0.252	0.055	0.280	0.211

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	19	12	17	21	19	19	8
N.S.	1	1.00	2.38	1.50	2.12	2.62	2.38	2.38	1.00
time (sec)	N/A	0.009	0.016	0.108	0.181	0.241	0.044	0.282	0.176

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	17	12	14	25	20	14	17
N.S.	1	1.00	1.31	0.92	1.08	1.92	1.54	1.08	1.31
time (sec)	N/A	0.010	0.010	0.241	0.187	0.244	0.019	0.277	0.034

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	24	26	13	13
N.S.	1	1.00	1.00	0.82	0.76	1.41	1.53	0.76	0.76
time (sec)	N/A	0.007	0.008	0.273	0.185	0.249	0.127	0.276	0.077

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	22	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	1.29	0.76	0.76
time (sec)	N/A	0.007	0.006	0.220	0.185	0.252	0.143	0.288	0.030

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	24	26	11	11
N.S.	1	1.00	1.00	0.80	0.73	1.60	1.73	0.73	0.73
time (sec)	N/A	0.008	0.007	0.232	0.184	0.257	0.136	0.278	0.225

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	9	13	14	24	8	13
N.S.	1	1.00	1.00	0.53	0.76	0.82	1.41	0.47	0.76
time (sec)	N/A	0.007	0.030	0.181	0.176	0.259	0.132	0.271	0.067

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	19
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	2.38
time (sec)	N/A	0.010	0.002	0.151	0.185	0.264	0.021	0.290	0.032

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	25	114	22	22
N.S.	1	1.00	1.00	0.77	0.73	0.83	3.80	0.73	0.73
time (sec)	N/A	0.025	0.002	1.353	0.189	0.260	0.995	0.270	0.343

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	8	6	11	5	7	9	6
N.S.	1	1.00	1.60	1.20	2.20	1.00	1.40	1.80	1.20
time (sec)	N/A	0.017	0.002	0.769	0.185	0.265	0.186	0.282	0.202

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	30	18	69	35	32	29	24
N.S.	1	1.00	2.00	1.20	4.60	2.33	2.13	1.93	1.60
time (sec)	N/A	0.045	0.029	0.569	0.276	0.258	0.525	0.317	0.490

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	14	10	18	16
N.S.	1	1.00	1.00	0.93	1.14	1.00	0.71	1.29	1.14
time (sec)	N/A	0.011	0.001	0.314	0.194	0.263	0.029	0.290	0.161

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	20	29	20	37	20	28	32
N.S.	1	1.00	0.91	1.32	0.91	1.68	0.91	1.27	1.45
time (sec)	N/A	0.023	0.001	1.089	0.185	0.252	0.042	0.296	0.186

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.009	0.002	0.419	0.190	0.236	0.024	0.281	0.002

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	14	14	14	13
N.S.	1	1.00	1.00	0.82	0.82	0.82	0.82	0.82	0.76
time (sec)	N/A	0.024	0.007	1.004	0.175	0.240	0.043	0.281	0.002

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	37	26	21	35	15	39	21
N.S.	1	1.00	1.48	1.04	0.84	1.40	0.60	1.56	0.84
time (sec)	N/A	0.005	0.053	0.429	0.262	0.238	0.096	0.290	0.039

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	14	12	19	12
N.S.	1	1.00	1.00	0.81	0.75	0.88	0.75	1.19	0.75
time (sec)	N/A	0.002	0.030	0.141	0.268	0.233	0.392	0.277	0.030

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	7	7
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.78	0.78	0.78
time (sec)	N/A	0.001	0.001	0.125	0.191	0.244	0.061	0.274	0.035

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	46	15	18	18	19	37	14
N.S.	1	1.00	2.88	0.94	1.12	1.12	1.19	2.31	0.88
time (sec)	N/A	0.002	0.004	0.166	0.174	0.247	0.522	0.276	0.085

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	26	18	27	23	24
N.S.	1	1.00	0.71	0.61	0.84	0.58	0.87	0.74	0.77
time (sec)	N/A	0.010	0.025	0.141	0.276	0.248	0.209	0.276	0.050

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	40	24	21	42	20	23	38
N.S.	1	1.00	1.48	0.89	0.78	1.56	0.74	0.85	1.41
time (sec)	N/A	0.008	0.086	0.234	0.259	0.268	0.280	0.303	0.238

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	27	33	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	1.69	2.06	0.88
time (sec)	N/A	0.003	0.035	0.142	0.266	0.240	0.412	0.294	0.272

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	19	26	23	39	30	23
N.S.	1	1.00	0.87	0.61	0.84	0.74	1.26	0.97	0.74
time (sec)	N/A	0.010	0.021	0.149	0.265	0.230	0.143	0.291	0.037

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	8	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.62	0.85	0.85
time (sec)	N/A	0.002	0.001	0.144	0.196	0.239	0.062	0.282	0.047

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	16	24	11	11
N.S.	1	1.00	1.00	0.80	0.73	1.07	1.60	0.73	0.73
time (sec)	N/A	0.002	0.003	0.138	0.209	0.249	0.088	0.278	0.029

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	41	20	19	32	19	19	18
N.S.	1	1.00	1.64	0.80	0.76	1.28	0.76	0.76	0.72
time (sec)	N/A	0.003	0.057	0.444	0.290	0.233	0.083	0.266	0.034

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	18	15	22	14	24	19	14
N.S.	1	1.00	0.72	0.60	0.88	0.56	0.96	0.76	0.56
time (sec)	N/A	0.008	0.017	0.136	0.302	0.228	0.111	0.277	0.024

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	16	5	4	14	3	25	4
N.S.	1	1.00	2.67	0.83	0.67	2.33	0.50	4.17	0.67
time (sec)	N/A	0.001	0.017	0.141	0.285	0.244	0.060	0.283	0.032

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	33	16	15	25	15	25	15
N.S.	1	1.00	1.57	0.76	0.71	1.19	0.71	1.19	0.71
time (sec)	N/A	0.002	0.028	0.197	0.289	0.241	0.084	0.282	0.033

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	26	22	33	66	25	25
N.S.	1	1.00	1.00	0.74	0.63	0.94	1.89	0.71	0.71
time (sec)	N/A	0.010	0.036	0.332	0.317	0.245	1.144	0.275	0.378

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	23	76	48	19
N.S.	1	1.00	1.00	0.87	0.83	1.00	3.30	2.09	0.83
time (sec)	N/A	0.003	0.050	0.167	0.294	0.242	0.427	0.300	0.388

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	19	28	92	24	24
N.S.	1	1.00	1.00	0.83	0.63	0.93	3.07	0.80	0.80
time (sec)	N/A	0.011	0.026	0.315	0.278	0.242	0.813	0.279	0.379

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	18	37	23	14
N.S.	1	1.00	1.00	0.83	0.78	1.00	2.06	1.28	0.78
time (sec)	N/A	0.003	0.031	0.174	0.296	0.240	0.448	0.284	0.305

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	31	22	58	49	24	34
N.S.	1	1.00	1.00	0.91	0.65	1.71	1.44	0.71	1.00
time (sec)	N/A	0.005	0.118	0.218	0.288	0.242	0.842	0.313	0.272

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	43	23	22	29	24	22	22
N.S.	1	1.00	1.48	0.79	0.76	1.00	0.83	0.76	0.76
time (sec)	N/A	0.005	0.074	0.428	0.277	0.246	0.086	0.398	0.046

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	14	24	15	37	18
N.S.	1	1.00	1.00	0.78	0.61	1.04	0.65	1.61	0.78
time (sec)	N/A	0.008	0.047	0.176	0.274	0.249	0.537	0.272	0.059

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	21	26	9	9
N.S.	1	1.00	1.00	0.77	0.69	1.62	2.00	0.69	0.69
time (sec)	N/A	0.002	0.003	0.134	0.188	0.232	0.398	0.285	0.222

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	26	23	44	32	23
N.S.	1	1.00	0.71	0.61	0.84	0.74	1.42	1.03	0.74
time (sec)	N/A	0.010	0.023	0.153	0.284	0.246	0.149	0.284	0.187

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	46	32	31	39	110	26	27
N.S.	1	1.00	1.02	0.71	0.69	0.87	2.44	0.58	0.60
time (sec)	N/A	0.007	0.104	0.241	0.335	0.237	1.746	0.276	0.047

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	26	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	2.00	0.69	0.69
time (sec)	N/A	0.002	0.002	0.131	0.198	0.235	0.075	0.290	0.035

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	30	34	12	12
N.S.	1	1.00	1.00	0.81	0.75	1.88	2.12	0.75	0.75
time (sec)	N/A	0.002	0.035	0.164	0.440	0.232	0.458	0.394	0.297

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	47	25	36	35	22	23	24
N.S.	1	1.00	1.42	0.76	1.09	1.06	0.67	0.70	0.73
time (sec)	N/A	0.010	0.066	0.208	0.303	0.229	0.217	0.282	0.232

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	20	7	6	18	5	34	14
N.S.	1	1.00	2.50	0.88	0.75	2.25	0.62	4.25	1.75
time (sec)	N/A	0.006	0.066	0.237	0.373	0.240	0.281	0.295	0.249

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	21	22	20	22	41	20
N.S.	1	1.00	0.96	0.84	0.88	0.80	0.88	1.64	0.80
time (sec)	N/A	0.007	0.069	0.197	0.351	0.238	0.278	0.311	0.324

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	55	27	36	35	24	25	0
N.S.	1	1.00	1.25	0.61	0.82	0.80	0.55	0.57	0.00
time (sec)	N/A	0.016	0.072	0.427	0.332	0.262	0.262	0.273	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	23	24	22	28	19	22	23
N.S.	1	1.00	0.88	0.92	0.85	1.08	0.73	0.85	0.88
time (sec)	N/A	0.005	0.009	0.238	0.331	0.244	0.053	0.286	0.184

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	59	49	0	36	29
N.S.	1	1.00	1.00	0.84	1.37	1.14	0.00	0.84	0.67
time (sec)	N/A	0.006	0.208	0.192	0.234	0.246	0.000	0.320	0.056

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	45	23	22	35	24	22	22
N.S.	1	1.00	1.36	0.70	0.67	1.06	0.73	0.67	0.67
time (sec)	N/A	0.020	0.089	0.059	0.372	0.251	0.353	0.283	0.225

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	22	24	22	34
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.80	0.73	1.13
time (sec)	N/A	0.012	0.033	0.088	0.393	0.238	0.291	0.276	0.252

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	42	13	6	16	3	32	12
N.S.	1	1.00	3.00	0.93	0.43	1.14	0.21	2.29	0.86
time (sec)	N/A	0.003	0.003	0.167	0.234	0.242	0.523	0.285	0.203

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	15	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	1.00	0.87
time (sec)	N/A	0.004	0.005	0.168	0.205	0.233	0.040	0.277	0.192

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	21	20	20	19	21	20
N.S.	1	1.00	0.96	0.81	0.77	0.77	0.73	0.81	0.77
time (sec)	N/A	0.014	0.007	0.135	0.222	0.241	0.025	0.295	0.036

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	19	19	19	22	19
N.S.	1	1.00	1.00	0.72	0.76	0.76	0.76	0.88	0.76
time (sec)	N/A	0.033	0.008	0.037	0.214	0.243	0.076	0.287	0.254

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	29	25	24	36	20	26	22
N.S.	1	1.00	0.97	0.83	0.80	1.20	0.67	0.87	0.73
time (sec)	N/A	0.022	0.018	0.042	0.224	0.228	0.041	0.272	0.053

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	18	21
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.91
time (sec)	N/A	0.036	0.007	0.165	0.329	0.252	0.062	0.270	0.056

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	32	31	31	34	31	30
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.89	0.82	0.79
time (sec)	N/A	0.028	0.012	0.424	0.327	0.242	0.053	0.321	0.275

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	93	72	77	136	88	74	96
N.S.	1	1.00	0.90	0.70	0.75	1.32	0.85	0.72	0.93
time (sec)	N/A	0.415	0.048	0.454	0.329	0.247	0.302	0.295	0.347

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	29	44	27	30	33
N.S.	1	1.00	1.00	0.85	0.88	1.33	0.82	0.91	1.00
time (sec)	N/A	0.032	0.022	0.168	0.303	0.237	0.060	0.282	0.238

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	16	15	19	12	15	16
N.S.	1	1.00	0.84	0.84	0.79	1.00	0.63	0.79	0.84
time (sec)	N/A	0.002	0.006	0.144	0.303	0.249	0.042	0.277	0.032

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	13	12	15	12
N.S.	1	1.00	1.00	0.74	0.68	0.68	0.63	0.79	0.63
time (sec)	N/A	0.002	0.004	0.152	0.222	0.238	0.038	0.272	0.100

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	21	14	15	15	17	17	8
N.S.	1	1.00	1.11	0.74	0.79	0.79	0.89	0.89	0.42
time (sec)	N/A	0.005	0.004	0.170	0.224	0.238	0.050	0.261	0.136

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	25	26	37	26	43	22
N.S.	1	1.00	1.00	0.78	0.81	1.16	0.81	1.34	0.69
time (sec)	N/A	0.018	0.019	0.166	0.226	0.253	0.079	0.274	0.110

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	30	34	53	32	31	29
N.S.	1	1.00	0.77	0.70	0.79	1.23	0.74	0.72	0.67
time (sec)	N/A	0.033	0.026	0.152	0.208	0.234	0.086	0.282	0.250

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	19	26	17	21	16
N.S.	1	1.00	1.00	0.81	0.90	1.24	0.81	1.00	0.76
time (sec)	N/A	0.008	0.003	0.148	0.214	0.234	0.042	0.270	0.044

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	23	19	17	26	19
N.S.	1	1.00	1.00	0.80	0.92	0.76	0.68	1.04	0.76
time (sec)	N/A	0.032	0.008	0.160	0.221	0.225	0.037	0.276	0.197

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	13	11	8	13	11
N.S.	1	1.00	1.00	1.09	1.18	1.00	0.73	1.18	1.00
time (sec)	N/A	0.009	0.005	0.142	0.217	0.236	0.037	0.274	0.218

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	31	31	36	31	56
N.S.	1	1.00	1.00	0.89	0.86	0.86	1.00	0.86	1.56
time (sec)	N/A	0.091	0.017	0.235	0.301	0.240	0.095	0.271	0.120

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	33	22	21	23
N.S.	1	1.00	1.00	0.76	0.72	1.14	0.76	0.72	0.79
time (sec)	N/A	0.104	0.018	0.200	0.291	0.239	0.071	0.265	0.192

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	48	47	60	63	60	61
N.S.	1	1.00	0.90	0.80	0.78	1.00	1.05	1.00	1.02
time (sec)	N/A	0.178	0.055	0.377	0.365	0.249	0.131	0.276	0.128

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	28	25	30	39	27	25	30
N.S.	1	1.00	0.76	0.68	0.81	1.05	0.73	0.68	0.81
time (sec)	N/A	0.006	0.015	0.170	0.307	0.224	0.051	0.288	0.191

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	78	68	75	134	88	71	84
N.S.	1	1.00	0.80	0.70	0.77	1.38	0.91	0.73	0.87
time (sec)	N/A	0.055	0.043	0.721	0.292	0.236	0.116	0.277	0.260

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	34	35	39	46	36	49
N.S.	1	1.00	1.00	0.74	0.76	0.85	1.00	0.78	1.07
time (sec)	N/A	0.048	0.028	0.051	0.295	0.233	0.072	0.265	0.266

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	38	37	46	48	38	51
N.S.	1	1.00	1.00	0.79	0.77	0.96	1.00	0.79	1.06
time (sec)	N/A	0.019	0.014	0.164	0.307	0.239	0.069	0.284	0.093

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	19	14	13	13	10	14	13
N.S.	1	1.00	1.27	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.004	0.004	0.128	0.209	0.231	0.023	0.289	0.027

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	8	9	8	8	7	9	8
N.S.	1	1.00	0.80	0.90	0.80	0.80	0.70	0.90	0.80
time (sec)	N/A	0.003	0.002	0.144	0.204	0.231	0.026	0.267	0.030

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	13	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	1.00	0.85
time (sec)	N/A	0.005	0.005	0.145	0.204	0.231	0.046	0.268	0.188

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	13	10
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	1.18	0.91
time (sec)	N/A	0.001	0.005	0.135	0.198	0.238	0.053	0.279	0.081

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	15	11	12	12	10	13	10
N.S.	1	1.00	1.25	0.92	1.00	1.00	0.83	1.08	0.83
time (sec)	N/A	0.004	0.004	0.135	0.195	0.234	0.030	0.271	0.187

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	19	21	26	20	80	28	18
N.S.	1	1.00	0.73	0.81	1.00	0.77	3.08	1.08	0.69
time (sec)	N/A	0.005	0.009	0.193	0.207	0.245	0.116	0.264	0.249

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	14	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	1.00	0.86
time (sec)	N/A	0.016	0.004	0.148	0.229	0.242	0.041	0.273	0.050

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	19	18	18	17	20	14
N.S.	1	1.00	1.00	0.73	0.69	0.69	0.65	0.77	0.54
time (sec)	N/A	0.012	0.006	0.190	0.193	0.241	0.039	0.268	0.041

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	17	10	15	14
N.S.	1	1.00	1.00	1.07	1.00	1.21	0.71	1.07	1.00
time (sec)	N/A	0.004	0.005	0.128	0.191	0.238	0.032	0.266	0.031

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	19	19	19	22	17
N.S.	1	1.00	1.00	0.78	0.83	0.83	0.83	0.96	0.74
time (sec)	N/A	0.007	0.007	0.157	0.198	0.237	0.064	0.266	0.086

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	18	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.031	0.007	0.042	0.191	0.252	0.058	0.267	0.067

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	16	8	13	12
N.S.	1	1.00	1.00	1.08	1.00	1.33	0.67	1.08	1.00
time (sec)	N/A	0.004	0.005	0.167	0.191	0.233	0.028	0.274	0.034

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	22	21	20	26	19	21	22
N.S.	1	1.00	0.73	0.70	0.67	0.87	0.63	0.70	0.73
time (sec)	N/A	0.008	0.010	0.154	0.198	0.235	0.057	0.277	0.063

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	20	27	22	26	22
N.S.	1	1.00	0.93	0.75	0.71	0.96	0.79	0.93	0.79
time (sec)	N/A	0.008	0.019	0.155	0.224	0.237	0.048	0.268	0.238

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	18	14	16	14
N.S.	1	1.00	1.00	1.07	1.00	1.29	1.00	1.14	1.00
time (sec)	N/A	0.023	0.005	0.153	0.195	0.241	0.059	0.267	0.043

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	25	18	17	17	17	20	17
N.S.	1	1.00	1.32	0.95	0.89	0.89	0.89	1.05	0.89
time (sec)	N/A	0.020	0.008	0.037	0.196	0.240	0.056	0.294	0.195

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	0.93	0.87
time (sec)	N/A	0.009	0.006	0.024	0.194	0.235	0.033	0.289	0.052

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	27	40	20	30	27
N.S.	1	1.00	1.00	0.96	1.08	1.60	0.80	1.20	1.08
time (sec)	N/A	0.007	0.013	0.184	0.200	0.236	0.045	0.280	0.051

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	22	31	19	18	21
N.S.	1	1.00	1.00	0.81	1.05	1.48	0.90	0.86	1.00
time (sec)	N/A	0.005	0.011	0.129	0.184	0.244	0.035	0.343	0.026

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	22	16	16	20	15	18	8
N.S.	1	1.00	2.75	2.00	2.00	2.50	1.88	2.25	1.00
time (sec)	N/A	0.005	0.004	0.148	0.206	0.249	0.041	0.273	0.180

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.87	0.87
time (sec)	N/A	0.007	0.006	0.030	0.189	0.244	0.036	0.270	0.046

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	12	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.67	0.78	0.78
time (sec)	N/A	0.006	0.004	0.130	0.207	0.238	0.026	0.278	0.027

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	18	17	18	18
N.S.	1	1.00	1.00	0.95	0.90	0.90	0.85	0.90	0.90
time (sec)	N/A	0.008	0.005	0.235	0.274	0.238	0.047	0.290	0.221

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	26	34	26	28
N.S.	1	1.00	1.00	0.87	0.84	0.84	1.10	0.84	0.90
time (sec)	N/A	0.011	0.008	0.331	0.279	0.230	0.045	0.286	0.188

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	31	22	21	21	22	21	17
N.S.	1	1.00	1.15	0.81	0.78	0.78	0.81	0.78	0.63
time (sec)	N/A	0.023	0.006	0.253	0.294	0.237	0.050	0.281	0.044

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	28	20	19	19	19	20	25
N.S.	1	1.00	1.22	0.87	0.83	0.83	0.83	0.87	1.09
time (sec)	N/A	0.023	0.009	0.181	0.282	0.242	0.066	0.295	0.055

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	24	23	23	29	24	55
N.S.	1	1.00	1.00	0.86	0.82	0.82	1.04	0.86	1.96
time (sec)	N/A	0.017	0.010	0.160	0.282	0.236	0.064	0.281	0.310

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	31	32	32	41	33	46
N.S.	1	1.00	1.00	0.76	0.78	0.78	1.00	0.80	1.12
time (sec)	N/A	0.017	0.006	0.145	0.275	0.256	0.060	0.271	0.065

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	42	34	35	35	42	36	47
N.S.	1	1.00	1.02	0.83	0.85	0.85	1.02	0.88	1.15
time (sec)	N/A	0.016	0.009	0.151	0.273	0.268	0.058	0.290	0.243

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	20	36	20	47	28
N.S.	1	1.00	0.92	0.88	0.83	1.50	0.83	1.96	1.17
time (sec)	N/A	0.024	0.016	0.184	0.280	0.238	0.068	0.290	0.045

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	26	19	18	18	19	20	10
N.S.	1	1.00	1.86	1.36	1.29	1.29	1.36	1.43	0.71
time (sec)	N/A	0.004	0.005	0.180	0.287	0.235	0.054	0.292	0.057

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	25	24	24	29	24	51
N.S.	1	1.00	1.00	0.86	0.83	0.83	1.00	0.83	1.76
time (sec)	N/A	0.089	0.015	0.174	0.295	0.248	0.087	0.295	0.079

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	19	17	33
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.83	0.74	1.43
time (sec)	N/A	0.023	0.010	0.070	0.283	0.254	0.089	0.277	0.201

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	32	39	41	32	36
N.S.	1	1.00	1.00	0.82	0.82	1.00	1.05	0.82	0.92
time (sec)	N/A	0.012	0.025	0.634	0.302	0.239	0.055	0.285	0.044

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	18	8	14	10
N.S.	1	1.00	1.00	1.10	1.40	1.80	0.80	1.40	1.00
time (sec)	N/A	0.013	0.006	0.136	0.193	0.237	0.033	0.293	0.173

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	12	11	15	12	15	11
N.S.	1	1.00	1.55	1.09	1.00	1.36	1.09	1.36	1.00
time (sec)	N/A	0.033	0.199	0.296	0.188	0.261	0.093	0.319	0.088

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	82	18	17	17	19	17	17
N.S.	1	1.00	4.10	0.90	0.85	0.85	0.95	0.85	0.85
time (sec)	N/A	0.044	0.214	0.309	0.272	0.263	0.160	0.314	0.061

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	13	12	15	8
N.S.	1	1.00	1.00	0.74	0.68	0.68	0.63	0.79	0.42
time (sec)	N/A	0.005	0.003	0.175	0.202	0.238	0.045	0.331	0.078

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	11	11	10	13	6
N.S.	1	1.00	1.00	0.71	0.65	0.65	0.59	0.76	0.35
time (sec)	N/A	0.003	0.004	0.147	0.187	0.247	0.038	0.271	0.130

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	17	17	17	19	13
N.S.	1	1.00	1.00	0.67	0.81	0.81	0.81	0.90	0.62
time (sec)	N/A	0.007	0.006	0.181	0.196	0.238	0.045	0.267	0.239

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	27	37	44	46	40	36
N.S.	1	1.00	0.90	0.55	0.76	0.90	0.94	0.82	0.73
time (sec)	N/A	0.013	0.018	0.227	0.288	0.248	0.042	0.291	0.153

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	57	51	50	50	68	53	58
N.S.	1	1.00	0.90	0.81	0.79	0.79	1.08	0.84	0.92
time (sec)	N/A	0.064	0.028	0.074	0.291	0.260	0.199	0.277	0.337

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	67	54	59	103	65	59	71
N.S.	1	1.00	0.78	0.63	0.69	1.20	0.76	0.69	0.83
time (sec)	N/A	0.081	0.044	0.084	0.277	0.251	0.106	0.290	0.156

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	28	28	28	42	29	18
N.S.	1	1.00	1.00	1.17	1.17	1.17	1.75	1.21	0.75
time (sec)	N/A	0.006	0.020	0.142	0.227	0.253	0.506	0.282	0.041

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	126	124	272	638	0	139	223
N.S.	1	1.00	0.63	0.62	1.36	3.19	0.00	0.70	1.12
time (sec)	N/A	0.360	0.070	0.148	0.286	0.907	0.000	0.352	0.277

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	41	22	30	27	20	23	11
N.S.	1	1.00	2.28	1.22	1.67	1.50	1.11	1.28	0.61
time (sec)	N/A	0.015	0.030	0.221	0.200	0.249	0.126	0.290	0.493

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	15	14	15	14	14
N.S.	1	1.00	1.00	0.83	0.83	0.78	0.83	0.78	0.78
time (sec)	N/A	0.004	0.013	0.135	0.207	0.236	0.050	0.285	0.060

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	28	21	28	20	26	20	20
N.S.	1	1.00	0.88	0.66	0.88	0.62	0.81	0.62	0.62
time (sec)	N/A	0.012	0.021	0.143	0.202	0.231	0.052	0.269	0.028

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	14	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.003	0.016	0.183	0.276	0.237	0.070	0.270	0.032

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	19	19	26	20	8
N.S.	1	1.00	1.00	0.90	1.90	1.90	2.60	2.00	0.80
time (sec)	N/A	0.004	0.015	0.147	0.210	0.246	0.346	0.276	0.171

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	25	11	17	8	22	18	8
N.S.	1	1.00	1.79	0.79	1.21	0.57	1.57	1.29	0.57
time (sec)	N/A	0.005	0.015	0.203	0.224	0.226	0.070	0.276	0.155

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	29	22	21	21	36	22	25
N.S.	1	1.00	0.94	0.71	0.68	0.68	1.16	0.71	0.81
time (sec)	N/A	0.018	0.019	0.119	0.204	0.242	0.657	0.278	0.202

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	21	17	22	17	76	22	19
N.S.	1	1.00	0.66	0.53	0.69	0.53	2.38	0.69	0.59
time (sec)	N/A	0.007	0.018	0.151	0.192	0.231	0.731	0.263	0.036

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	25	24	24	75	24	24
N.S.	1	1.00	1.00	0.81	0.77	0.77	2.42	0.77	0.77
time (sec)	N/A	0.006	0.033	0.199	0.314	0.240	0.758	0.269	0.164

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	22	20	19	14	117	19	12
N.S.	1	1.00	0.76	0.69	0.66	0.48	4.03	0.66	0.41
time (sec)	N/A	0.005	0.017	0.124	0.190	0.235	0.504	0.273	0.277

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.004	0.013	0.167	0.283	0.245	0.106	0.274	0.285

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	16	15	15	17	16	15
N.S.	1	1.00	0.90	0.76	0.71	0.71	0.81	0.76	0.71
time (sec)	N/A	0.009	0.016	0.142	0.200	0.261	0.055	0.274	0.216

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	23	22	22	26	23	22
N.S.	1	1.00	0.93	0.77	0.73	0.73	0.87	0.77	0.73
time (sec)	N/A	0.014	0.022	0.154	0.199	0.238	0.063	0.266	0.042

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	16	19	16	26	19	16
N.S.	1	1.00	0.74	0.59	0.70	0.59	0.96	0.70	0.59
time (sec)	N/A	0.013	0.020	0.147	0.210	0.237	0.539	0.258	0.294

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	201	201	127	132	293	547	0	140	208
N.S.	1	1.00	0.63	0.66	1.46	2.72	0.00	0.70	1.03
time (sec)	N/A	0.185	0.070	0.151	0.317	0.913	0.000	0.342	0.063

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	46	45	47	68	45	73
N.S.	1	1.00	1.00	0.74	0.73	0.76	1.10	0.73	1.18
time (sec)	N/A	0.034	0.063	0.135	0.280	0.251	0.180	0.263	0.194

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	117	83	82	76	121	82	82
N.S.	1	1.00	0.90	0.64	0.63	0.58	0.93	0.63	0.63
time (sec)	N/A	0.034	0.063	0.166	0.205	0.240	1.106	0.281	0.135

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	40	37	26	0	28	20
N.S.	1	1.00	1.00	1.67	1.54	1.08	0.00	1.17	0.83
time (sec)	N/A	0.008	0.032	0.190	0.278	0.253	0.000	0.277	0.188

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	13	10	12	9
N.S.	1	1.00	1.00	1.09	1.00	1.18	0.91	1.09	0.82
time (sec)	N/A	0.020	0.010	0.171	0.201	0.255	0.078	0.274	0.145

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	15	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	0.88	0.88
time (sec)	N/A	0.036	0.043	0.067	0.195	0.259	0.064	0.273	0.215

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	21	21	22	21	9
N.S.	1	1.00	1.00	0.83	1.75	1.75	1.83	1.75	0.75
time (sec)	N/A	0.012	0.025	0.069	0.202	0.249	0.231	0.267	0.029

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	27	35	35	32	37	40
N.S.	1	1.00	1.00	0.96	1.25	1.25	1.14	1.32	1.43
time (sec)	N/A	0.012	0.034	0.086	0.202	0.250	0.396	0.272	0.205

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	22	30	27	20	23	11
N.S.	1	1.00	1.00	0.51	0.70	0.63	0.47	0.53	0.26
time (sec)	N/A	0.017	0.022	0.137	0.199	0.279	0.107	0.290	0.462

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	24	19	39	38	39	37	21
N.S.	1	1.00	1.14	0.90	1.86	1.81	1.86	1.76	1.00
time (sec)	N/A	0.010	0.022	0.143	0.375	0.256	0.226	0.307	0.392

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	24	16	25	17	14	17	11
N.S.	1	1.00	2.18	1.45	2.27	1.55	1.27	1.55	1.00
time (sec)	N/A	0.013	0.028	0.231	0.206	0.249	0.114	0.287	0.068

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	43	22	30	27	20	23	11
N.S.	1	1.00	2.39	1.22	1.67	1.50	1.11	1.28	0.61
time (sec)	N/A	0.014	0.028	0.224	0.323	0.271	0.150	0.291	0.499

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	24	25	35	0	28	16
N.S.	1	1.00	1.46	1.00	1.04	1.46	0.00	1.17	0.67
time (sec)	N/A	0.055	0.034	0.217	0.189	0.263	0.000	0.274	0.316

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	39	24	220	35	0	28	16
N.S.	1	1.00	1.62	1.00	9.17	1.46	0.00	1.17	0.67
time (sec)	N/A	0.024	0.035	0.358	0.212	0.274	0.000	0.281	0.353

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	24	23	33	0	25	22
N.S.	1	1.00	1.00	1.33	1.28	1.83	0.00	1.39	1.22
time (sec)	N/A	0.029	0.021	0.267	0.194	0.265	0.000	0.274	0.136

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	61	98	121	61	31
N.S.	1	1.00	1.06	0.97	1.69	2.72	3.36	1.69	0.86
time (sec)	N/A	0.024	0.061	0.282	0.297	0.273	1.557	0.298	0.792

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	43	71839	26	15
N.S.	1	1.00	1.00	1.07	1.00	2.87	4789.27	1.73	1.00
time (sec)	N/A	0.021	0.034	0.492	0.275	0.256	16.133	0.292	0.512

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	8	8	7	9	8
N.S.	1	1.00	0.83	0.75	0.67	0.67	0.58	0.75	0.67
time (sec)	N/A	0.002	0.002	0.144	0.203	0.231	0.031	0.291	0.046

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	26	11	12	11	11
N.S.	1	1.00	1.00	0.71	1.53	0.65	0.71	0.65	0.65
time (sec)	N/A	0.002	0.003	0.029	0.204	0.233	0.050	0.285	0.027

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	10	10	7	8	6
N.S.	1	1.00	0.67	0.75	0.83	0.83	0.58	0.67	0.50
time (sec)	N/A	0.009	0.005	0.065	0.211	0.281	0.162	0.281	0.211

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	27	34	27	29	30
N.S.	1	1.00	1.00	1.50	1.69	2.12	1.69	1.81	1.88
time (sec)	N/A	0.011	0.002	0.104	0.191	0.251	0.051	0.266	0.002

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	14	14	14	13
N.S.	1	1.00	1.00	0.82	0.82	0.82	0.82	0.82	0.76
time (sec)	N/A	0.017	0.008	1.012	0.195	0.238	0.051	0.257	0.002

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	16	16	11	11	20	11	11
N.S.	1	1.00	0.67	0.67	0.46	0.46	0.83	0.46	0.46
time (sec)	N/A	0.007	0.003	0.023	0.209	0.233	0.067	0.283	0.002

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	31	30	30	36	33	30
N.S.	1	1.00	1.00	0.74	0.71	0.71	0.86	0.79	0.71
time (sec)	N/A	0.029	0.008	0.046	0.196	0.256	0.072	0.274	0.225

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.009	0.004	0.042	0.192	0.237	0.121	0.260	0.070

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	11	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.92	0.83
time (sec)	N/A	0.004	0.004	0.147	0.188	0.246	0.031	0.294	0.032

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	4	3	3	3	3	3
N.S.	1	1.00	1.00	0.80	0.60	0.60	0.60	0.60	0.60
time (sec)	N/A	0.005	0.007	0.036	0.218	0.263	0.253	0.271	0.032

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	14	11	10	19	14	10	18
N.S.	1	1.00	0.58	0.46	0.42	0.79	0.58	0.42	0.75
time (sec)	N/A	0.023	0.002	0.096	0.185	0.272	0.022	0.281	0.002

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	11	8	18	32
N.S.	1	1.00	1.00	1.12	1.00	1.38	1.00	2.25	4.00
time (sec)	N/A	0.018	0.050	0.304	0.190	0.274	0.064	0.295	0.500

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	8	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.62	0.85	0.85
time (sec)	N/A	0.003	0.001	0.144	0.189	0.262	0.064	0.288	0.002

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.005	0.003	0.029	0.192	0.258	0.039	0.287	0.228

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	18	18	109	18	18
N.S.	1	1.00	1.00	0.79	0.75	0.75	4.54	0.75	0.75
time (sec)	N/A	0.004	0.023	0.201	0.271	0.246	0.769	0.284	0.047

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	16	8	13	12
N.S.	1	1.00	1.00	1.08	1.00	1.33	0.67	1.08	1.00
time (sec)	N/A	0.004	0.004	0.141	0.189	0.245	0.025	0.286	0.037

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	16	15	16	16
N.S.	1	1.00	1.00	1.06	1.00	1.00	0.94	1.00	1.00
time (sec)	N/A	0.004	0.003	0.108	0.264	0.252	0.048	0.260	0.232

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	30	29	29	32	0	18
N.S.	1	1.00	1.00	1.36	1.32	1.32	1.45	0.00	0.82
time (sec)	N/A	0.036	0.042	0.063	0.192	0.232	0.630	0.000	0.287

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	56	43	19	46	54	42	42
N.S.	1	1.00	2.07	1.59	0.70	1.70	2.00	1.56	1.56
time (sec)	N/A	0.005	0.019	0.122	0.186	0.241	0.132	0.466	0.033

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	30	14	14	14	14
N.S.	1	1.00	1.00	0.82	1.76	0.82	0.82	0.82	0.82
time (sec)	N/A	0.017	0.014	2.787	0.200	0.246	0.050	0.381	0.241

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.68	0.73	0.73
time (sec)	N/A	0.006	0.005	0.376	0.261	0.239	0.040	0.385	0.236

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	24	24	24	26	24
N.S.	1	1.00	0.88	0.78	0.75	0.75	0.75	0.81	0.75
time (sec)	N/A	0.008	0.010	0.017	0.274	0.269	0.087	0.327	0.029

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	28	66	40	30
N.S.	1	1.00	1.00	0.83	1.17	0.93	2.20	1.33	1.00
time (sec)	N/A	0.011	0.029	0.189	0.267	0.241	0.780	0.345	0.066

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	10	15	13
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.77	1.15	1.00
time (sec)	N/A	0.003	0.004	0.182	0.195	0.245	0.035	0.345	0.053

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	14	17	44	14	17	27	16
N.S.	1	1.00	0.88	1.06	2.75	0.88	1.06	1.69	1.00
time (sec)	N/A	0.024	0.017	0.108	0.200	0.252	0.096	0.351	0.209

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	20	14	15	15	14	18	13
N.S.	1	1.00	1.18	0.82	0.88	0.88	0.82	1.06	0.76
time (sec)	N/A	0.013	0.007	0.032	0.192	0.236	0.037	0.348	0.074

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	3	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.017	0.007	0.440	0.296	0.253	0.084	0.320	0.076

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.009	0.007	0.073	0.196	0.250	0.105	0.330	0.297

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	9	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.78	1.00	1.00
time (sec)	N/A	0.004	0.011	0.180	0.191	0.247	0.041	0.322	0.019

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.015	0.003	0.036	0.187	0.237	0.038	0.328	0.049

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	19	21	26	19	19
N.S.	1	1.00	0.81	0.74	0.70	0.78	0.96	0.70	0.70
time (sec)	N/A	0.007	0.049	0.138	0.192	0.239	0.104	0.330	0.031

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	22	26	13	13
N.S.	1	1.00	1.00	0.82	0.76	1.29	1.53	0.76	0.76
time (sec)	N/A	0.006	0.008	0.272	0.199	0.248	0.140	0.321	0.068

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	20	25
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.011	0.006	0.065	0.273	0.268	0.055	0.353	0.228

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	28	28	29	25	29	49	25
N.S.	1	1.00	0.72	0.72	0.74	0.64	0.74	1.26	0.64
time (sec)	N/A	0.012	0.012	0.066	0.202	0.245	0.044	0.325	0.043

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	16	14	13	13	12	13	13
N.S.	1	1.00	0.62	0.54	0.50	0.50	0.46	0.50	0.50
time (sec)	N/A	0.019	0.028	0.052	0.188	0.261	0.041	0.327	0.066

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	11	10	10	12	10	10
N.S.	1	1.00	1.50	0.92	0.83	0.83	1.00	0.83	0.83
time (sec)	N/A	0.005	0.011	0.037	0.292	0.274	0.034	0.405	0.230

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	21	22	20	22	41	20
N.S.	1	1.00	0.96	0.84	0.88	0.80	0.88	1.64	0.80
time (sec)	N/A	0.005	0.069	0.223	0.273	0.254	0.310	0.324	0.336

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	22	21	21	20	21	21
N.S.	1	1.00	0.85	0.81	0.78	0.78	0.74	0.78	0.78
time (sec)	N/A	0.012	0.008	0.076	0.212	0.253	0.107	0.345	0.223

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	12	11	11	15	11	12
N.S.	1	1.00	1.00	0.63	0.58	0.58	0.79	0.58	0.63
time (sec)	N/A	0.003	0.004	0.048	0.215	0.257	0.384	0.346	0.026

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	19	15	19	16	15
N.S.	1	1.00	1.00	1.00	3.17	2.50	3.17	2.67	2.50
time (sec)	N/A	0.008	0.035	0.036	0.209	0.253	0.047	0.338	0.111

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.011	0.006	0.063	0.278	0.232	0.046	0.558	0.247

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	10	32	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.83	2.67	0.67
time (sec)	N/A	0.003	0.017	0.250	0.268	0.250	0.078	0.320	0.278

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	23	18	25	31	22	26
N.S.	1	1.00	0.88	0.68	0.53	0.74	0.91	0.65	0.76
time (sec)	N/A	0.024	0.010	0.315	0.194	0.257	0.027	0.354	0.045

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	23	7	8	29	7	26	6
N.S.	1	1.00	1.92	0.58	0.67	2.42	0.58	2.17	0.50
time (sec)	N/A	0.006	0.078	0.237	0.272	0.247	0.284	0.312	0.190

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	21	48	14	21
N.S.	1	1.00	1.00	0.94	0.88	1.31	3.00	0.88	1.31
time (sec)	N/A	0.039	0.019	0.088	0.206	0.260	1.218	0.301	0.131

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	23	13	5	7	12	7	5
N.S.	1	1.00	3.29	1.86	0.71	1.00	1.71	1.00	0.71
time (sec)	N/A	0.019	0.005	0.159	0.195	0.253	0.798	0.302	0.050

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	15	15	15	16	15
N.S.	1	1.00	1.00	1.00	2.50	2.50	2.50	2.67	2.50
time (sec)	N/A	0.022	0.001	0.044	0.236	0.249	0.053	0.285	0.027

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	33	32	32	41	33	46
N.S.	1	1.00	1.00	0.77	0.74	0.74	0.95	0.77	1.07
time (sec)	N/A	0.019	0.008	0.166	0.281	0.253	0.056	0.285	0.091

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	29	33	74	30	42	57	37
N.S.	1	1.00	0.78	0.89	2.00	0.81	1.14	1.54	1.00
time (sec)	N/A	0.063	0.019	0.136	0.200	0.241	0.279	0.284	0.238

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	12	0	7	27
N.S.	1	1.00	1.00	0.89	0.78	1.33	0.00	0.78	3.00
time (sec)	N/A	0.019	0.008	1.143	0.183	0.259	0.000	0.298	2.664

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	16	16	16	12	16	15
N.S.	1	1.00	1.00	0.80	0.80	0.80	0.60	0.80	0.75
time (sec)	N/A	0.002	0.000	0.020	0.204	0.228	0.019	0.319	0.026

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	13	10	9	9	8	10	9
N.S.	1	1.00	1.08	0.83	0.75	0.75	0.67	0.83	0.75
time (sec)	N/A	0.018	0.023	0.037	0.205	0.259	0.034	0.282	0.184

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	17	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.81	0.81
time (sec)	N/A	0.009	0.006	0.162	0.203	0.244	0.044	0.296	0.053

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	22	30	27	20	23	11
N.S.	1	1.00	1.00	0.51	0.70	0.63	0.47	0.53	0.26
time (sec)	N/A	0.015	0.021	0.134	0.197	0.263	0.120	0.318	0.426

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	18	15	16	22	144	43	14
N.S.	1	1.00	0.75	0.62	0.67	0.92	6.00	1.79	0.58
time (sec)	N/A	0.006	0.020	0.167	0.181	0.257	0.628	0.319	0.212

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	24	20	16	16	34	16	21
N.S.	1	1.00	0.63	0.53	0.42	0.42	0.89	0.42	0.55
time (sec)	N/A	0.015	0.011	0.032	0.202	0.242	0.100	0.291	0.035

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	32	39	30	32
N.S.	1	1.00	1.00	0.84	0.81	0.86	1.05	0.81	0.86
time (sec)	N/A	0.030	0.050	0.234	0.278	0.251	0.601	0.300	0.208

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	21	14	17	15
N.S.	1	1.00	1.00	0.94	0.88	1.24	0.82	1.00	0.88
time (sec)	N/A	0.021	0.005	0.181	0.190	0.247	0.045	0.286	0.042

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	29	26	38	26	39	26	34
N.S.	1	1.00	0.72	0.65	0.95	0.65	0.98	0.65	0.85
time (sec)	N/A	0.061	0.089	0.220	0.201	0.252	0.093	0.272	0.198

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	20	5	5	5
N.S.	1	1.00	1.00	1.20	1.00	4.00	1.00	1.00	1.00
time (sec)	N/A	0.010	4.174	0.201	0.194	0.256	0.154	0.303	0.247

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	20	5	4	18	3	19	4
N.S.	1	1.00	3.33	0.83	0.67	3.00	0.50	3.17	0.67
time (sec)	N/A	0.001	0.035	0.422	0.300	0.240	0.068	0.279	0.008

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	24	22	62	62	61	22	29
N.S.	1	1.00	0.65	0.59	1.68	1.68	1.65	0.59	0.78
time (sec)	N/A	0.011	0.008	0.156	0.192	0.247	0.061	0.291	0.100

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	18	36	19	18	18
N.S.	1	1.00	1.00	0.86	0.86	1.71	0.90	0.86	0.86
time (sec)	N/A	0.024	0.028	0.177	0.215	0.248	0.049	0.299	0.395

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	22	41	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.73	1.37	0.80	0.80
time (sec)	N/A	0.040	0.058	0.172	0.199	0.256	0.074	0.266	0.255

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	27	4	3	3	3	3	3
N.S.	1	1.00	6.75	1.00	0.75	0.75	0.75	0.75	0.75
time (sec)	N/A	0.021	0.007	0.188	0.207	0.260	0.219	0.277	0.236

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	11	10	15	11
N.S.	1	1.00	1.00	0.92	1.15	0.85	0.77	1.15	0.85
time (sec)	N/A	0.004	0.005	0.151	0.200	0.234	0.040	0.317	0.046

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	24	21	21	21	22	21	21
N.S.	1	1.00	0.55	0.48	0.48	0.48	0.50	0.48	0.48
time (sec)	N/A	0.032	0.022	0.046	0.210	0.233	0.042	0.296	0.027

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	42	28	27	25	37	27	27
N.S.	1	1.00	1.02	0.68	0.66	0.61	0.90	0.66	0.66
time (sec)	N/A	0.009	0.039	0.149	0.276	0.230	1.704	0.283	0.031

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	17	114	13	14
N.S.	1	1.00	1.00	0.80	0.76	0.68	4.56	0.52	0.56
time (sec)	N/A	0.023	0.002	1.335	0.185	0.263	0.962	0.316	0.225

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	7	10	8
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.58	0.83	0.67
time (sec)	N/A	0.001	0.003	0.023	0.192	0.234	0.029	0.287	0.186

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	65	41	43	32	0	30	43
N.S.	1	1.00	1.59	1.00	1.05	0.78	0.00	0.73	1.05
time (sec)	N/A	0.012	0.101	0.190	0.275	0.251	0.000	0.322	0.050

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	34	34	27	119	27	27	29	28	0
N.S.	1	1.00	0.79	3.50	0.79	0.79	0.85	0.82	0.00
time (sec)	N/A	0.026	0.015	0.157	0.272	0.268	1.221	0.303	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	42	17	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	2.21	0.89	0.89
time (sec)	N/A	0.005	0.006	0.184	0.275	0.261	0.044	0.294	0.042

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	45	30	39	37	34	31	28
N.S.	1	1.00	1.18	0.79	1.03	0.97	0.89	0.82	0.74
time (sec)	N/A	0.009	0.110	0.311	0.278	0.250	0.224	0.268	0.047

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.58	0.67	0.67
time (sec)	N/A	0.006	0.006	0.210	0.278	0.246	0.048	0.295	0.199

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	26	36	26	28
N.S.	1	1.00	1.00	0.87	0.84	0.84	1.16	0.84	0.90
time (sec)	N/A	0.013	0.013	0.708	0.282	0.237	0.044	0.289	0.040

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	37	16	121	29	12	150	19
N.S.	1	1.00	3.70	1.60	12.10	2.90	1.20	15.00	1.90
time (sec)	N/A	0.011	0.014	0.124	0.280	0.257	0.442	0.361	0.112

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	27	29	26	22	30	13
N.S.	1	1.00	1.00	1.80	1.93	1.73	1.47	2.00	0.87
time (sec)	N/A	0.008	0.006	0.202	0.193	0.256	0.067	0.288	0.061

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	0	13	63	13	21
N.S.	1	1.00	1.00	0.67	0.00	0.62	3.00	0.62	1.00
time (sec)	N/A	0.008	0.103	0.029	0.000	0.232	0.160	0.279	0.237

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	16	19	17	19	18	17
N.S.	1	1.00	0.78	0.70	0.83	0.74	0.83	0.78	0.74
time (sec)	N/A	0.015	0.050	0.040	0.203	0.241	0.060	0.267	0.302

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	17	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.85	0.80	0.80
time (sec)	N/A	0.006	0.009	0.037	0.206	0.261	0.126	0.274	0.313

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	18	19	14	20	18
N.S.	1	1.00	1.00	0.89	1.00	1.06	0.78	1.11	1.00
time (sec)	N/A	0.013	0.006	0.065	0.202	0.256	0.054	0.281	0.216

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	20	19	25	20	20	19
N.S.	1	1.00	1.00	1.67	1.58	2.08	1.67	1.67	1.58
time (sec)	N/A	0.014	0.046	0.052	0.192	0.243	0.051	0.285	0.249

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	23	9	10	15	12	16	8
N.S.	1	1.00	2.88	1.12	1.25	1.88	1.50	2.00	1.00
time (sec)	N/A	0.032	0.007	0.145	0.279	0.262	0.353	0.303	0.228

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	12	44	12	12
N.S.	1	1.00	1.00	0.72	0.67	0.67	2.44	0.67	0.67
time (sec)	N/A	0.003	0.018	0.308	0.274	0.242	0.486	0.295	0.146

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	24	0	9	18
N.S.	1	1.00	1.00	0.91	0.00	2.18	0.00	0.82	1.64
time (sec)	N/A	0.039	0.021	0.479	0.000	0.316	0.000	0.289	0.438

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	46	23	22	30	27	22	22
N.S.	1	1.00	1.53	0.77	0.73	1.00	0.90	0.73	0.73
time (sec)	N/A	0.006	0.077	0.465	0.310	0.256	0.107	0.328	0.177

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	20	23	21	21	26	21	23
N.S.	1	1.00	0.83	0.96	0.88	0.88	1.08	0.88	0.96
time (sec)	N/A	0.030	0.009	0.111	0.206	0.261	0.127	0.285	0.030

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	47	33	49	39	37	40	39
N.S.	1	1.00	0.94	0.66	0.98	0.78	0.74	0.80	0.78
time (sec)	N/A	0.011	0.084	0.373	0.283	0.249	0.250	0.286	0.228

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	46	51	9	9
N.S.	1	1.00	1.00	0.91	0.82	4.18	4.64	0.82	0.82
time (sec)	N/A	0.001	0.002	0.155	0.198	0.244	0.028	0.266	0.224

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	31	20	19	19	20	19	19
N.S.	1	1.00	1.24	0.80	0.76	0.76	0.80	0.76	0.76
time (sec)	N/A	0.024	0.011	0.175	0.201	0.242	0.022	0.279	0.190

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	19	21	26	19	19
N.S.	1	1.00	0.81	0.74	0.70	0.78	0.96	0.70	0.70
time (sec)	N/A	0.008	0.050	0.151	0.191	0.242	0.167	0.272	0.030

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	41	23	34	56	36	70	18
N.S.	1	1.00	1.71	0.96	1.42	2.33	1.50	2.92	0.75
time (sec)	N/A	0.008	0.043	0.295	0.192	0.248	0.057	0.282	0.070

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	35	32	33	35	27	44	32
N.S.	1	1.00	1.03	0.94	0.97	1.03	0.79	1.29	0.94
time (sec)	N/A	0.006	0.042	0.280	0.316	0.232	0.098	0.289	0.495

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	28	73	38	37
N.S.	1	1.00	1.00	0.83	1.17	0.93	2.43	1.27	1.23
time (sec)	N/A	0.012	0.028	0.218	0.275	0.232	0.841	0.288	0.111

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	19	16	16	16	15	16	16
N.S.	1	1.00	0.59	0.50	0.50	0.50	0.47	0.50	0.50
time (sec)	N/A	0.014	0.018	0.047	0.208	0.238	0.036	0.295	0.033

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	31	18	17	12	26	42	12
N.S.	1	1.00	1.35	0.78	0.74	0.52	1.13	1.83	0.52
time (sec)	N/A	0.027	0.025	0.158	0.194	0.254	0.119	0.282	0.113

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	22	21	21	19	21	21
N.S.	1	1.00	0.89	0.81	0.78	0.78	0.70	0.78	0.78
time (sec)	N/A	0.012	0.008	0.023	0.285	0.254	0.072	0.267	0.290

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	31	53	28	29	32	28
N.S.	1	1.00	0.84	0.82	1.39	0.74	0.76	0.84	0.74
time (sec)	N/A	0.018	0.014	0.037	0.279	0.248	0.144	0.275	0.234

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	5	4	16	8	6	6
N.S.	1	1.00	1.00	1.00	0.80	3.20	1.60	1.20	1.20
time (sec)	N/A	0.009	0.008	0.319	0.202	0.244	0.375	0.270	0.048

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	22	21	21	27	21	23
N.S.	1	1.00	0.86	0.76	0.72	0.72	0.93	0.72	0.79
time (sec)	N/A	0.031	0.018	0.194	0.206	0.274	0.088	0.285	0.202

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	45	29	36	47	27	26	27
N.S.	1	1.00	1.25	0.81	1.00	1.31	0.75	0.72	0.75
time (sec)	N/A	0.009	0.090	0.240	0.294	0.251	0.283	0.287	0.187

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	31	23	22	22	24	22	22
N.S.	1	1.00	1.11	0.82	0.79	0.79	0.86	0.79	0.79
time (sec)	N/A	0.014	0.012	0.210	0.292	0.254	0.056	0.301	0.056

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	42	43	46	38	29
N.S.	1	1.00	1.00	0.96	1.62	1.65	1.77	1.46	1.12
time (sec)	N/A	0.016	0.006	0.362	0.205	0.253	0.065	0.272	0.059

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	30	23	24	33	46	22	22
N.S.	1	1.00	0.65	0.50	0.52	0.72	1.00	0.48	0.48
time (sec)	N/A	0.016	0.039	0.316	0.210	0.256	0.025	0.291	0.221

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	15	17	16	24	17	16	11
N.S.	1	1.00	0.75	0.85	0.80	1.20	0.85	0.80	0.55
time (sec)	N/A	0.026	0.017	0.419	0.204	0.274	0.641	0.260	0.286

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	24	18	16	24	14	19	19
N.S.	1	1.00	1.14	0.86	0.76	1.14	0.67	0.90	0.90
time (sec)	N/A	0.017	0.032	0.051	0.211	0.253	0.048	0.282	0.068

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	77	47	58	0	50	0
N.S.	1	1.00	0.89	2.08	1.27	1.57	0.00	1.35	0.00
time (sec)	N/A	0.033	0.024	0.678	0.291	0.307	0.000	0.287	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	46	26	29	27	29	28	23
N.S.	1	1.00	1.64	0.93	1.04	0.96	1.04	1.00	0.82
time (sec)	N/A	0.008	0.058	0.254	0.192	0.247	0.227	0.292	0.145

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	15	18	17	15	21
N.S.	1	1.00	1.00	0.89	0.79	0.95	0.89	0.79	1.11
time (sec)	N/A	0.007	0.008	0.292	0.213	0.250	0.020	0.298	0.035

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	26	25	24	24	22	24	24
N.S.	1	1.00	0.57	0.54	0.52	0.52	0.48	0.52	0.52
time (sec)	N/A	0.029	0.021	0.038	0.195	0.245	0.034	0.278	0.027

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	33	16	32	30	0
N.S.	1	1.00	1.00	0.83	1.83	0.89	1.78	1.67	0.00
time (sec)	N/A	0.006	0.284	0.736	0.214	0.242	0.528	0.298	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	19	21	24	19	19
N.S.	1	1.00	0.81	0.74	0.70	0.78	0.89	0.70	0.70
time (sec)	N/A	0.012	0.074	0.152	0.217	0.254	0.088	0.305	0.220

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	22	18	17	16	15	0	16	18
N.S.	1	1.22	1.00	0.94	0.89	0.83	0.00	0.89	1.00
time (sec)	N/A	0.030	0.009	0.039	0.202	0.266	0.000	0.283	0.248

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	22	21	21	20	21	21
N.S.	1	1.00	0.85	0.81	0.78	0.78	0.74	0.78	0.78
time (sec)	N/A	0.012	0.001	0.030	0.198	0.260	0.096	0.276	0.002

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	20	20	20	31
N.S.	1	1.00	1.00	0.81	0.77	0.77	0.77	0.77	1.19
time (sec)	N/A	0.009	0.031	0.083	0.306	0.251	0.282	0.298	0.245

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	11	14	9	9	15	9	9
N.S.	1	1.00	0.73	0.93	0.60	0.60	1.00	0.60	0.60
time (sec)	N/A	0.014	0.033	1.704	0.205	0.250	0.779	0.275	0.287

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	50	35	34	37	121	29	30
N.S.	1	1.00	1.06	0.74	0.72	0.79	2.57	0.62	0.64
time (sec)	N/A	0.009	0.129	0.250	0.289	0.241	1.713	0.299	0.038

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	36	10	9	26	27	9	26
N.S.	1	1.00	3.27	0.91	0.82	2.36	2.45	0.82	2.36
time (sec)	N/A	0.002	0.003	0.177	0.200	0.238	0.014	0.303	0.024

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	14
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.82
time (sec)	N/A	0.022	0.007	0.167	0.207	0.257	0.021	0.293	0.052

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	27	14	13	20	29	13	13
N.S.	1	1.00	1.59	0.82	0.76	1.18	1.71	0.76	0.76
time (sec)	N/A	0.023	0.012	1.822	0.198	0.274	0.020	0.265	0.199

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	19	17	36	19	14
N.S.	1	1.00	0.67	0.56	0.70	0.63	1.33	0.70	0.52
time (sec)	N/A	0.004	0.012	0.194	0.207	0.242	0.532	0.279	0.036

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	17	16	19	24	16	16
N.S.	1	1.00	0.92	0.71	0.67	0.79	1.00	0.67	0.67
time (sec)	N/A	0.008	0.002	0.274	0.210	0.266	0.017	0.271	0.033

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	20	18	12	16	16
N.S.	1	1.00	1.00	1.42	1.67	1.50	1.00	1.33	1.33
time (sec)	N/A	0.007	0.001	0.027	0.207	0.256	0.037	0.308	0.020

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	30	22	34	26	53	28	25
N.S.	1	1.00	0.75	0.55	0.85	0.65	1.32	0.70	0.62
time (sec)	N/A	0.010	0.021	0.177	0.292	0.251	0.280	0.282	0.024

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [19] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	3	0.333
2	A	1	1	1.00	3	0.333
3	A	1	1	1.00	3	0.333
4	A	1	1	1.00	3	0.333
5	A	1	1	1.00	2	0.500
6	A	1	1	1.00	2	0.500
7	A	2	2	1.00	4	0.500
8	A	2	2	1.00	4	0.500
9	A	2	2	1.00	5	0.400
10	A	2	2	1.00	5	0.400
11	A	1	1	1.00	2	0.500
12	A	1	1	1.00	2	0.500
13	A	1	1	1.00	2	0.500
14	A	1	1	1.00	2	0.500
15	A	2	2	1.00	4	0.500
16	A	1	1	1.00	2	0.500
17	A	3	2	1.00	7	0.286
18	A	1	1	1.00	6	0.167
19	A	2	2	1.00	2	1.000
20	A	2	2	1.00	7	0.286
21	A	2	2	1.00	4	0.500
22	A	2	2	1.00	6	0.333
23	A	1	1	1.00	4	0.250
24	A	3	2	1.00	8	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	2	1.00	8	0.250
26	A	2	2	1.00	4	0.500
27	A	2	2	1.00	2	1.000
28	A	3	3	1.00	6	0.500
29	A	2	2	1.00	6	0.333
30	A	1	1	1.00	6	0.167
31	A	4	2	1.00	7	0.286
32	A	1	1	1.00	10	0.100
33	A	1	1	1.00	10	0.100
34	A	2	2	1.00	4	0.500
35	A	2	2	1.00	6	0.333
36	A	2	2	1.00	7	0.286
37	A	1	1	1.00	8	0.125
38	A	2	2	1.00	6	0.333
39	A	3	2	1.00	9	0.222
40	A	2	2	1.00	2	1.000
41	A	2	2	1.00	6	0.333
42	A	1	1	1.00	9	0.111
43	A	1	1	1.00	9	0.111
44	A	2	2	1.00	6	0.333
45	A	2	2	1.00	9	0.222
46	A	2	2	1.00	9	0.222
47	A	2	2	1.00	5	0.400
48	A	1	1	1.00	3	0.333
49	A	3	3	1.00	7	0.429
50	A	1	1	1.00	6	0.167
51	A	1	1	1.00	3	0.333
52	A	3	3	1.00	6	0.500
53	A	3	3	1.00	8	0.375
54	A	3	2	1.00	9	0.222
55	A	3	3	1.00	4	0.750
56	A	2	2	1.00	6	0.333
57	A	1	1	1.00	8	0.125
58	A	2	2	1.00	6	0.333
59	A	2	2	1.00	4	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	3	2	1.00	4	0.500
61	A	2	1	1.00	4	0.250
62	A	3	2	1.00	9	0.222
63	A	3	2	1.00	9	0.222
64	A	4	3	1.00	9	0.333
65	A	3	3	1.00	9	0.333
66	A	1	1	1.00	10	0.100
67	A	3	2	1.00	11	0.182
68	A	4	3	1.00	9	0.333
69	A	4	2	1.00	4	0.500
70	A	4	2	1.00	4	0.500
71	A	4	3	1.00	13	0.231
72	A	2	1	1.00	4	0.250
73	A	5	3	1.00	9	0.333
74	A	3	2	1.00	11	0.182
75	A	3	2	1.00	11	0.182
76	A	3	3	1.00	14	0.214
77	A	3	2	1.00	8	0.250
78	A	3	2	1.00	7	0.286
79	A	4	3	1.00	9	0.333
80	A	2	2	1.00	9	0.222
81	A	1	1	1.00	8	0.125
82	A	2	2	1.00	4	0.500
83	A	3	2	1.00	4	0.500
84	A	2	1	1.00	4	0.250
85	A	2	1	1.00	4	0.250
86	A	2	2	1.00	9	0.222
87	A	3	2	1.00	9	0.222
88	A	2	2	1.00	7	0.286
89	A	3	2	1.00	9	0.222
90	A	3	2	1.00	4	0.500
91	A	4	2	1.00	4	0.500
92	A	3	2	1.00	7	0.286
93	A	3	2	1.00	9	0.222
94	A	2	2	1.00	7	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	3	2	1.00	9	0.222
96	A	2	2	1.00	7	0.286
97	A	2	2	1.00	7	0.286
98	A	2	2	1.00	4	0.500
99	A	2	2	1.00	4	0.500
100	A	3	2	1.00	9	0.222
101	A	3	2	1.00	9	0.222
102	A	1	1	1.00	2	0.500
103	A	2	2	1.00	4	0.500
104	A	3	3	1.00	5	0.600
105	A	2	1	1.00	4	0.250
106	A	1	1	1.00	9	0.111
107	A	1	1	1.00	7	0.143
108	A	1	1	1.00	9	0.111
109	A	1	1	1.00	9	0.111
110	A	2	2	1.00	7	0.286
111	A	5	2	1.00	11	0.182
112	A	2	2	1.00	13	0.154
113	A	6	4	1.00	10	0.400
114	A	3	2	1.00	7	0.286
115	A	4	3	1.00	9	0.333
116	A	2	2	1.00	7	0.286
117	A	3	2	1.00	9	0.222
118	A	2	2	1.00	15	0.133
119	A	1	1	1.00	13	0.077
120	A	1	1	1.00	11	0.091
121	A	2	2	1.00	13	0.154
122	A	3	2	1.00	15	0.133
123	A	3	3	1.00	16	0.188
124	A	1	1	1.00	15	0.067
125	A	3	2	1.00	15	0.133
126	A	1	1	1.00	13	0.077
127	A	1	1	1.00	13	0.077
128	A	2	2	1.00	11	0.182
129	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	1	1	1.00	9	0.111
131	A	2	2	1.00	9	0.222
132	A	4	4	1.00	13	0.308
133	A	1	1	1.00	17	0.059
134	A	4	4	1.00	15	0.267
135	A	1	1	1.00	15	0.067
136	A	3	3	1.00	17	0.176
137	A	2	2	1.00	15	0.133
138	A	3	3	1.00	13	0.231
139	A	1	1	1.00	11	0.091
140	A	3	2	1.00	15	0.133
141	A	3	3	1.00	15	0.200
142	A	2	2	1.00	12	0.167
143	A	1	1	1.00	11	0.091
144	A	3	3	1.00	13	0.231
145	A	2	2	1.00	12	0.167
146	A	2	2	1.00	14	0.143
147	A	4	4	1.00	17	0.235
148	A	3	3	1.00	10	0.300
149	A	2	2	1.00	14	0.143
150	A	3	3	1.00	17	0.176
151	A	4	4	1.00	11	0.364
152	A	2	2	1.00	11	0.182
153	A	3	2	1.00	12	0.167
154	A	3	2	1.00	11	0.182
155	A	3	2	1.00	25	0.080
156	A	2	1	1.00	29	0.034
157	A	6	5	1.00	20	0.250
158	A	6	5	1.00	23	0.217
159	A	14	10	1.00	32	0.312
160	A	6	5	1.00	26	0.192
161	A	2	2	1.00	7	0.286
162	A	3	2	1.00	11	0.182
163	A	4	3	1.00	14	0.214
164	A	2	1	1.00	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	3	2	1.00	22	0.091
166	A	3	2	1.00	11	0.182
167	A	4	3	1.00	26	0.115
168	A	3	2	1.00	16	0.125
169	A	8	4	1.00	25	0.160
170	A	6	3	1.00	23	0.130
171	A	6	5	1.00	26	0.192
172	A	3	2	1.00	11	0.182
173	A	8	7	1.00	20	0.350
174	A	7	6	1.00	23	0.261
175	A	8	8	1.00	11	0.727
176	A	2	1	1.00	9	0.111
177	A	2	1	1.00	7	0.143
178	A	2	1	1.00	16	0.062
179	A	3	2	1.00	11	0.182
180	A	2	1	1.00	13	0.077
181	A	3	2	1.00	11	0.182
182	A	3	2	1.00	15	0.133
183	A	5	3	1.00	20	0.150
184	A	2	1	1.00	11	0.091
185	A	2	1	1.00	16	0.062
186	A	3	2	1.00	25	0.080
187	A	3	2	1.00	12	0.167
188	A	2	1	1.00	11	0.091
189	A	2	1	1.00	14	0.071
190	A	3	2	1.00	22	0.091
191	A	2	1	1.00	24	0.042
192	A	1	1	1.00	20	0.050
193	A	2	1	1.00	9	0.111
194	A	2	1	1.00	9	0.111
195	A	3	3	1.00	11	0.273
196	A	1	1	1.00	22	0.045
197	A	3	2	1.00	11	0.182
198	A	4	4	1.00	14	0.286
199	A	4	4	1.00	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	6	5	1.00	23	0.217
201	A	5	4	1.00	23	0.174
202	A	6	5	1.00	15	0.333
203	A	6	6	1.00	7	0.857
204	A	7	7	1.00	11	0.636
205	A	5	4	1.00	21	0.190
206	A	4	4	1.00	11	0.364
207	A	6	4	1.00	30	0.133
208	A	7	6	1.00	24	0.250
209	A	3	3	1.00	14	0.214
210	A	3	2	1.00	16	0.125
211	A	2	2	1.00	21	0.095
212	A	3	3	1.00	15	0.200
213	A	3	2	1.00	10	0.200
214	A	1	1	1.00	9	0.111
215	A	3	2	1.00	18	0.111
216	A	3	2	1.00	10	0.200
217	A	6	5	1.00	43	0.116
218	A	7	5	1.00	50	0.100
219	A	3	3	1.00	11	0.273
220	A	9	9	1.00	15	0.600
221	A	2	2	1.00	11	0.182
222	A	3	2	1.00	9	0.222
223	A	3	2	1.00	9	0.222
224	A	3	3	1.00	11	0.273
225	A	2	2	1.00	11	0.182
226	A	2	2	1.00	11	0.182
227	A	4	2	1.00	13	0.154
228	A	2	1	1.00	11	0.091
229	A	3	3	1.00	13	0.231
230	A	3	2	1.00	11	0.182
231	A	3	3	1.00	13	0.231
232	A	3	2	1.00	17	0.118
233	A	4	3	1.00	17	0.176
234	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
235	A	10	9	1.00	21	0.429
236	A	9	9	1.00	13	0.692
237	A	4	3	1.00	13	0.231
238	A	5	5	1.00	13	0.385
239	A	2	2	1.00	12	0.167
240	A	4	3	1.00	20	0.150
241	A	3	3	1.00	9	0.333
242	A	4	4	1.00	11	0.364
243	A	4	3	1.00	8	0.375
244	A	2	2	1.00	7	0.286
245	A	2	2	1.00	10	0.200
246	A	2	2	1.00	11	0.182
247	A	6	6	1.00	7	0.857
248	A	4	2	1.00	11	0.182
249	A	4	3	1.00	9	0.333
250	A	2	2	1.00	11	0.182
251	A	2	1	1.00	19	0.053
252	A	1	1	1.00	9	0.111
253	A	2	1	1.00	13	0.077
254	A	1	1	1.00	8	0.125
255	A	2	2	1.00	7	0.286
256	A	3	2	1.00	9	0.222
257	A	3	3	1.00	7	0.429
258	A	3	2	1.00	20	0.100
259	A	2	2	1.00	10	0.200
260	A	2	1	1.00	11	0.091
261	A	2	2	1.00	7	0.286
262	A	3	3	1.00	9	0.333
263	A	1	1	1.00	15	0.067
264	A	1	1	1.00	13	0.077
265	A	1	1	1.00	6	0.167
266	A	3	3	1.00	13	0.231
267	A	2	1	1.00	7	0.143
268	A	3	3	1.00	6	0.500
269	A	4	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	3	2	1.00	9	0.222
271	A	3	2	1.00	9	0.222
272	A	4	4	1.00	12	0.333
273	A	3	3	1.00	4	0.750
274	A	4	4	1.00	15	0.267
275	A	3	2	1.00	12	0.167
276	A	3	2	1.00	6	0.333
277	A	1	1	1.00	21	0.048
278	A	2	2	1.00	11	0.182
279	A	3	3	1.00	6	0.500
280	A	1	1	1.00	4	0.250
281	A	3	2	1.00	13	0.154
282	A	1	1	1.00	10	0.100
283	A	1	1	1.00	9	0.111
284	A	5	4	1.00	11	0.364
285	A	3	2	1.00	8	0.250
286	A	2	2	1.00	11	0.182
287	A	2	2	1.00	6	0.333
288	A	2	2	1.00	14	0.143
289	A	4	3	1.00	6	0.500
290	A	2	1	1.00	15	0.067
291	A	2	2	1.00	13	0.154
292	A	3	3	1.00	14	0.214
293	A	2	2	1.00	9	0.222
294	A	4	3	1.00	9	0.333
295	A	2	2	1.00	14	0.143
296	A	3	2	1.00	22	0.091
297	A	2	2	1.00	7	0.286
298	A	2	2	1.00	13	0.154
299	A	6	6	1.00	7	0.857
300	A	6	2	1.00	6	0.333
301	A	1	3	1.00	8	0.375
302	A	1	0	1.00	10	0.000
303	A	3	2	1.00	15	0.133
304	A	4	3	1.00	16	0.188

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	4	3	1.00	8	0.375
306	A	2	1	1.00	9	0.111
307	A	4	3	1.00	7	0.429
308	A	5	4	1.00	12	0.333
309	A	3	2	1.00	17	0.118
310	A	8	4	1.00	13	0.308
311	A	2	2	1.00	6	0.333
312	A	1	1	1.00	11	0.091
313	A	2	1	1.00	9	0.111
314	A	3	2	1.00	13	0.154
315	A	6	5	1.00	6	0.833
316	A	1	1	1.00	12	0.083
317	A	4	4	1.00	11	0.364
318	A	4	2	1.00	9	0.222
319	A	7	4	1.00	15	0.267
320	A	5	2	1.00	11	0.182
321	A	1	1	1.00	6	0.167
322	A	3	3	1.00	15	0.200
323	A	5	5	1.00	13	0.385
324	A	3	3	1.00	13	0.231
325	A	3	3	1.00	12	0.250
326	A	2	2	1.00	11	0.182
327	A	4	4	1.00	12	0.333
328	A	2	2	1.00	6	0.333
329	A	2	2	1.00	13	0.154
330	A	3	3	1.00	15	0.200
331	A	4	3	1.00	16	0.188
332	A	2	2	1.00	12	0.167
333	A	4	4	1.00	8	0.500
334	A	3	3	1.00	13	0.231
335	A	4	4	1.00	17	0.235
336	A	2	2	1.00	13	0.154
337	A	5	4	1.00	17	0.235
338	A	2	2	1.00	15	0.133
339	A	4	2	1.00	6	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	4	4	1.00	14	0.286
341	A	1	1	1.00	9	0.111
342	A	3	2	1.00	9	0.222
343	A	1	1	1.00	10	0.100
344	A	2	2	1.00	8	0.250
345	A	3	3	1.00	15	0.200
346	A	4	4	1.00	15	0.267
347	A	3	2	1.00	9	0.222
348	A	3	2	1.00	13	0.154
349	A	3	3	1.00	6	0.500
350	A	5	5	1.00	8	0.625
351	A	2	2	1.00	8	0.250
352	A	3	2	1.00	8	0.250
353	A	3	3	1.00	14	0.214
354	A	3	2	1.00	15	0.133
355	A	3	2	1.00	4	0.500
356	A	4	2	1.00	6	0.333
357	A	4	4	1.00	10	0.400
358	A	3	2	1.00	15	0.133
359	A	4	4	1.00	13	0.308
360	A	3	3	1.00	13	0.231
361	A	2	1	1.00	4	0.250
362	A	5	2	1.00	9	0.222
363	A	3	3	1.00	13	0.231
364	A	1	1	1.00	10	0.100
365	A	4	4	1.22	10	0.400
366	A	4	3	1.00	6	0.500
367	A	4	4	1.00	11	0.364
368	A	4	3	1.00	9	0.333
369	A	3	3	1.00	15	0.200
370	A	1	1	1.00	11	0.091
371	A	3	2	1.00	9	0.222
372	A	3	2	1.00	9	0.222
373	A	2	1	1.00	11	0.091
374	A	3	2	1.00	4	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	2	2	1.00	4	0.500
376	A	3	2	1.00	13	0.154

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^n \, dx$

Optimal result

Integrand size = 3, antiderivative size = 11

$$\int x^n \, dx = \frac{x^{1+n}}{1+n}$$

```
[Out] x^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

```
[In] Int[x^n,x]
```

```
[Out] x^(1 + n)/(1 + n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{x^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int x^n\,dx = \frac{x^{1+n}}{1+n}$$

```
[In] Integrate[x^n,x]
[Out] x^(1 + n)/(1 + n)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{x\,x^{n}}{1+n}$	11
parallelrisch	$\frac{x\,x^{n}}{1+n}$	11
gosper	$\frac{x^{1+n}}{1+n}$	12
default	$\frac{x^{1+n}}{1+n}$	12
norman	$\frac{x\,\mathrm{e}^{n\ln(x)}}{1+n}$	13

```
[In] int(x^n,x,method=_RETURNVERBOSE)
[Out] x/(1+n)*x^n
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int x^n\,dx = \frac{xx^n}{n+1}$$

```
[In] integrate(x^n,x, algorithm="fricas")
[Out] x*x^n/(n + 1)
```

Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int x^n\,dx = \begin{cases} \frac{x^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(x) & \text{otherwise} \end{cases}$$

```
[In] integrate(x**n,x)
[Out] Piecewise((x**(n + 1)/(n + 1), Ne(n, -1)), (log(x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

```
[In] integrate(x^n,x, algorithm="maxima")
[Out] x^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

```
[In] integrate(x^n,x, algorithm="giac")
[Out] x^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int x^n\,dx = \begin{cases} \ln(x) & \text{if } n = -1 \\ \frac{x^{n+1}}{n+1} & \text{if } n \neq -1 \end{cases}$$

```
[In] int(x^n,x)
[Out] piecewise(n == -1, log(x), n ~= -1, x^(n + 1)/(n + 1))
```

3.2 $\int e^x\,dx$

Optimal result

Integrand size = 3, antiderivative size = 3

$$\int e^x\,dx = e^x$$

```
[Out] exp(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{2225\}$

$$\int e^x\,dx = e^x$$

```
[In] Int[E^x,x]
```

```
[Out] E^x
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\text{integral} = e^x$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int e^x\,dx = e^x$$

```
[In] Integrate[E^x,x]
[Out] E^x
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

method	result	size
gosper	e^{x}	3
lookup	e^{x}	3
derivativedivides	e^{x}	3
default	e^{x}	3
norman	e^{x}	3
risch	e^{x}	3
parallelrisch	e^{x}	3
meijerg	$-1+\mathrm{e}^{x}$	5

```
[In] int(exp(x),x,method=_RETURNVERBOSE)
[Out] exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = e^x$$

```
[In] integrate(exp(x),x, algorithm="fricas")
[Out] e^x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = e^x$$

```
[In] integrate(exp(x),x)
[Out] exp(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = e^x$$

```
[In] integrate(exp(x),x, algorithm="maxima")
[Out] e^x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = e^x$$

```
[In] integrate(exp(x),x, algorithm="giac")
[Out] e^x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = \mathrm{e}^x$$

```
[In] int(exp(x),x)
[Out] exp(x)
```

3.3 $\int \frac{1}{x} \, dx$

Optimal result

Integrand size = 3, antiderivative size = 2

$$\int \frac{1}{x} \, dx = \log(x)$$

```
[Out] ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{29\}$

$$\int \frac{1}{x} \, dx = \log(x)$$

```
[In] Int[x^(-1),x]
```

```
[Out] Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\text{integral} = \log(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} \, dx = \log(x)$$

```
[In] Integrate[x^(-1),x]
[Out] Log[x]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\ln(x)$	3
norman	$\ln(x)$	3
risch	$\ln(x)$	3
parallelrisch	$\ln(x)$	3

```
[In] int(1/x,x,method=_RETURNVERBOSE)
[Out] ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} \, dx = \log(x)$$

```
[In] integrate(1/x,x, algorithm="fricas")
[Out] log(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x}\,dx = \log{(x)}$$

```
[In] integrate(1/x,x)
```

```
[Out] log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x}\,dx = \log{(x)}$$

```
[In] integrate(1/x,x, algorithm="maxima")
```

```
[Out] log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{x}\,dx = \log{(|x|)}$$

```
[In] integrate(1/x,x, algorithm="giac")
```

```
[Out] log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x}\,dx = \ln{(x)}$$

```
[In] int(1/x,x)
```

```
[Out] log(x)
```

3.4 $\int a^x\,dx$

Optimal result

Integrand size = 3, antiderivative size = 8

$$\int a^x\,dx = \frac{a^x}{\log(a)}$$

```
[Out] a^x/ln(a)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2225}

$$\int a^x\,dx = \frac{a^x}{\log(a)}$$

```
[In] Int[a^x,x]
```

```
[Out] a^x/Log[a]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\text{integral} = \frac{a^x}{\log(a)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x \, dx = \frac{a^x}{\log(a)}$$

```
[In] Integrate[a^x,x]
[Out] a^x/Log[a]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
gosper	$\frac{a^x}{\ln(a)}$	9
derivativedivides	$\frac{a^x}{\ln(a)}$	9
default	$\frac{a^x}{\ln(a)}$	9
risch	$\frac{a^x}{\ln(a)}$	9
parallelrisch	$\frac{a^x}{\ln(a)}$	9
norman	$\frac{e^{x\ln(a)}}{\ln(a)}$	11
meijerg	$-\frac{1-e^{x\ln(a)}}{\ln(a)}$	16

```
[In] int(a^x,x,method=_RETURNVERBOSE)
[Out] a^x/ln(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x \, dx = \frac{a^x}{\log\left(a\right)}$$

```
[In] integrate(a^x,x, algorithm="fricas")
[Out] a^x/log(a)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x\,dx = \begin{cases} \frac{a^x}{\log(a)} & \text{for } \log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

```
[In] integrate(a**x,x)
[Out] Piecewise((a**x/log(a), Ne(log(a), 0)), (x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x\,dx = \frac{a^x}{\log(a)}$$

```
[In] integrate(a^x,x, algorithm="maxima")
[Out] a^x/log(a)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x\,dx = \frac{a^x}{\log(a)}$$

```
[In] integrate(a^x,x, algorithm="giac")
[Out] a^x/log(a)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x\,dx = \frac{a^x}{\ln(a)}$$

```
[In] int(a^x,x)
[Out] a^x/log(a)
```

3.5 $\int \sin(x)\, dx$

Optimal result

Integrand size = 2, antiderivative size = 4

$$\int \sin(x)\, dx = -\cos(x)$$

```
[Out] -cos(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{2718\}$

$$\int \sin(x)\, dx = -\cos(x)$$

```
[In] Int[Sin[x],x]
```

```
[Out] -Cos[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\cos(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x)\,dx = -\cos(x)$$

```
[In] Integrate[Sin[x],x]
[Out] -Cos[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
lookup	$-\cos\left(x\right)$	5
default	$-\cos\left(x\right)$	5
risch	$-\cos\left(x\right)$	5
parallelrisch	$-\cos\left(x\right)-1$	7
norman	$-\frac{2}{1+\tan^{2}\left(\frac{x}{2}\right)}$	13
meijerg	$\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(x)}{\sqrt{\pi}}\right)$	16

```
[In] int(sin(x),x,method=_RETURNVERBOSE)
[Out] -cos(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x)\,dx = -\cos\left(x\right)$$

```
[In] integrate(sin(x),x, algorithm="fricas")
[Out] -cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \sin(x)\,dx = -\cos{(x)}$$

```
[In] integrate(sin(x),x)
[Out] -cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x)\,dx = -\cos{(x)}$$

```
[In] integrate(sin(x),x, algorithm="maxima")
[Out] -cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x)\,dx = -\cos{(x)}$$

```
[In] integrate(sin(x),x, algorithm="giac")
[Out] -cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x)\,dx = -\cos{(x)}$$

```
[In] int(sin(x),x)
[Out] -cos(x)
```

3.6 $\int \cos(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 2

$$\int \cos(x)\,dx = \sin(x)$$

```
[Out] sin(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{2717\}$

$$\int \cos(x)\,dx = \sin(x)$$

```
[In] Int[Cos[x],x]
```

```
[Out] Sin[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \sin(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

```
[In] Integrate[Cos[x],x]
[Out] Sin[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
lookup	$\sin(x)$	3
default	$\sin(x)$	3
meijerg	$\sin(x)$	3
risch	$\sin(x)$	3
parallelrisch	$\sin(x)$	3
norman	$\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$	17

```
[In] int(cos(x),x,method=_RETURNVERBOSE)
[Out] sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

```
[In] integrate(cos(x),x, algorithm="fricas")
[Out] sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin{(x)}$$

```
[In] integrate(cos(x),x)
[Out] sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin{(x)}$$

```
[In] integrate(cos(x),x, algorithm="maxima")
[Out] sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin{(x)}$$

```
[In] integrate(cos(x),x, algorithm="giac")
[Out] sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin{(x)}$$

```
[In] int(cos(x),x)
[Out] sin(x)
```

3.7 $\int \sec^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 2

$$\int \sec^2(x)\,dx = \tan(x)$$

```
[Out] tan(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3852, 8}

$$\int \sec^2(x)\,dx = \tan(x)$$

```
[In] Int[Sec[x]^2,x]
```

```
[Out] Tan[x]
```

<u>Rule 8</u>

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

<u>Rule 3852</u>

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int 1\,dx, x, -\tan(x)\right)\\ &= \tan(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\,dx = \tan(x)$$

```
[In] Integrate[Sec[x]^2,x]
[Out] Tan[x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\tan(x)$	3
parallelrisch	$\tan(x)$	3
risch	$\frac{2i}{e^{2ix}+1}$	13
norman	$-\frac{2\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}$	17

```
[In] int(sec(x)^2,x,method=_RETURNVERBOSE)
[Out] tan(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. $2(2) = 4$.

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 3.50

$$\int \sec^2(x)\,dx = \frac{\sin(x)}{\cos(x)}$$

```
[In] integrate(sec(x)^2,x, algorithm="fricas")
[Out] sin(x)/cos(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5 vs. $2(2) = 4$.

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 2.50

$$\int \sec^2(x)\,dx = \frac{\sin(x)}{\cos(x)}$$

```
[In] integrate(sec(x)**2,x)
[Out] sin(x)/cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\,dx = \tan(x)$$

```
[In] integrate(sec(x)^2,x, algorithm="maxima")
[Out] tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\,dx = \tan(x)$$

```
[In] integrate(sec(x)^2,x, algorithm="giac")
[Out] tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\,dx = \tan(x)$$

```
[In] int(1/cos(x)^2,x)
[Out] tan(x)
```

3.8 $\int \csc^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 4

$$\int \csc^2(x)\,dx = -\cot(x)$$

```
[Out] -cot(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3852, 8}

$$\int \csc^2(x)\,dx = -\cot(x)$$

```
[In] Int[Csc[x]^2,x]
```

```
[Out] -Cot[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int 1\,dx, x, \cot(x)\right) \\ &= -\cot(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \csc^2(x)\,dx = -\cot(x)$$

```
[In] Integrate[Csc[x]^2,x]
[Out] -Cot[x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$-\cot(x)$	5
parallelrisch	$-\cot(x)$	5
risch	$-\frac{2i}{e^{2ix}-1}$	13
norman	$\frac{-\frac{1}{2}+\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}}{\tan\left(\frac{x}{2}\right)}$	18

```
[In] int(csc(x)^2,x,method=_RETURNVERBOSE)
[Out] -cot(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.00

$$\int \csc^2(x)\,dx = -\frac{\cos(x)}{\sin(x)}$$

```
[In] integrate(csc(x)^2,x, algorithm="fricas")
[Out] -cos(x)/sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.75

$$\int \csc^2(x)\,dx = -\frac{\cos(x)}{\sin(x)}$$

```
[In] integrate(csc(x)**2,x)
[Out] -cos(x)/sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \csc^2(x)\,dx = -\frac{1}{\tan(x)}$$

```
[In] integrate(csc(x)^2,x, algorithm="maxima")
[Out] -1/tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \csc^2(x)\,dx = -\frac{1}{\tan(x)}$$

```
[In] integrate(csc(x)^2,x, algorithm="giac")
[Out] -1/tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \csc^2(x)\,dx = -\cot(x)$$

```
[In] int(1/sin(x)^2,x)
[Out] -cot(x)
```

3.9 $\int \sec(x)\tan(x)\,dx$

Optimal result

Integrand size = 5, antiderivative size = 2

$$\int \sec(x)\tan(x)\,dx = \sec(x)$$

```
[Out] sec(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2686, 8}

$$\int \sec(x)\tan(x)\,dx = \sec(x)$$

```
[In] Int[Sec[x]*Tan[x],x]
```

```
[Out] Sec[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int 1\,dx, x, \sec(x)\right) \\ &= \sec(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec(x)\tan(x)\,dx = \sec(x)$$

```
[In] Integrate[Sec[x]*Tan[x],x]
[Out] Sec[x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
derivativedivides	$\sec(x)$	3
default	$\sec(x)$	3
risch	$\frac{2\,e^{ix}}{e^{2ix}+1}$	17

```
[In] int(sec(x)*tan(x),x,method=_RETURNVERBOSE)
[Out] sec(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

```
[In] integrate(sec(x)*tan(x),x, algorithm="fricas")
[Out] 1/cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

```
[In] integrate(sec(x)*tan(x),x)
[Out] 1/cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

```
[In] integrate(sec(x)*tan(x),x, algorithm="maxima")
[Out] 1/cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

```
[In] integrate(sec(x)*tan(x),x, algorithm="giac")
[Out] 1/cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 6.00

$$\int \sec(x)\tan(x)\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)^2 - 1}$$

```
[In] int(tan(x)/cos(x),x)
[Out] -2/(tan(x/2)^2 - 1)
```

3.10 $\int \cot(x)\csc(x)\,dx$

Optimal result

Integrand size = 5, antiderivative size = 4

$$\int \cot(x)\csc(x)\,dx = -\csc(x)$$

```
[Out] -csc(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2686, 8}

$$\int \cot(x)\csc(x)\,dx = -\csc(x)$$

```
[In] Int[Cot[x]*Csc[x],x]
```

```
[Out] -Csc[x]
```

<u>Rule 8</u>

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

<u>Rule 2686</u>

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int 1\,dx, x, \csc(x)\right)$$
$$= -\csc(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cot(x)\csc(x)\,dx = -\csc(x)$$

```
[In] Integrate[Cot[x]*Csc[x],x]
[Out] -Csc[x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$-\csc(x)$	5
default	$-\csc(x)$	5
risch	$-\frac{2i\,e^{ix}}{e^{2ix}-1}$	18

```
[In] int(csc(x)*cot(x),x,method=_RETURNVERBOSE)
[Out] -csc(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin(x)}$$

```
[In] integrate(cot(x)*csc(x),x, algorithm="fricas")
[Out] -1/sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin(x)}$$

```
[In] integrate(cot(x)*csc(x),x)
[Out] -1/sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin(x)}$$

```
[In] integrate(cot(x)*csc(x),x, algorithm="maxima")
[Out] -1/sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin(x)}$$

```
[In] integrate(cot(x)*csc(x),x, algorithm="giac")
[Out] -1/sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin(x)}$$

```
[In] int(cot(x)/sin(x),x)
[Out] -1/sin(x)
```

3.11 $\int \sinh(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 2

$$\int \sinh(x)\,dx = \cosh(x)$$

```
[Out] cosh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{2718\}$

$$\int \sinh(x)\,dx = \cosh(x)$$

```
[In] Int[Sinh[x],x]
```

```
[Out] Cosh[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\text{integral} = \cosh(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh(x)$$

```
[In] Integrate[Sinh[x],x]
[Out] Cosh[x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
lookup	$\cosh\left(x\right)$	3
default	$\cosh\left(x\right)$	3
risch	$\frac{\mathrm{e}^{x}}{2}+\frac{\mathrm{e}^{-x}}{2}$	12
parallelrisch	$-\frac{2}{\tanh^{2}\left(\frac{x}{2}\right)-1}$	13
meijerg	$-\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(x)}{\sqrt{\pi}}\right)$	17

```
[In] int(sinh(x),x,method=_RETURNVERBOSE)
[Out] cosh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh\left(x\right)$$

```
[In] integrate(sinh(x),x, algorithm="fricas")
[Out] cosh(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh{(x)}$$

```
[In] integrate(sinh(x),x)
[Out] cosh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh{(x)}$$

```
[In] integrate(sinh(x),x, algorithm="maxima")
[Out] cosh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(2) = 4.

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 5.50

$$\int \sinh(x)\,dx = \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(sinh(x),x, algorithm="giac")
[Out] 1/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh(x)$$

```
[In] int(sinh(x),x)
[Out] cosh(x)
```

3.12 $\int \cosh(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 2

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[Out] sinh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{2717\}$

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[In] Int[Cosh[x],x]
```

```
[Out] Sinh[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \sinh(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[In] Integrate[Cosh[x],x]
[Out] Sinh[x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
lookup	$\sinh(x)$	3
default	$\sinh(x)$	3
meijerg	$\sinh(x)$	3
parallelrisch	$\sinh(x)$	3
risch	$-\frac{e^{-x}}{2}+\frac{e^{x}}{2}$	12

```
[In] int(cosh(x),x,method=_RETURNVERBOSE)
[Out] sinh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[In] integrate(cosh(x),x, algorithm="fricas")
[Out] sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[In] integrate(cosh(x),x)
[Out] sinh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[In] integrate(cosh(x),x, algorithm="maxima")
[Out] sinh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(2) = 4.

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 5.50

$$\int \cosh(x)\,dx = -\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(cosh(x),x, algorithm="giac")
[Out] -1/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[In] int(cosh(x),x)
[Out] sinh(x)
```

3.13 $\int \tan(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 5

$$\int \tan(x)\,dx = -\log(\cos(x))$$

```
[Out] -ln(cos(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{3556\}$

$$\int \tan(x)\,dx = -\log(\cos(x))$$

```
[In] Int[Tan[x],x]
```

```
[Out] -Log[Cos[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\log(\cos(x))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \tan(x)\,dx = -\log(\cos(x))$$

```
[In] Integrate[Tan[x],x]
[Out] -Log[Cos[x]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
lookup	$-\ln\left(\cos\left(x\right)\right)$	6
default	$-\ln\left(\cos\left(x\right)\right)$	6
derivativedivides	$\frac{\ln\left(1+\tan^{2}\left(x\right)\right)}{2}$	10
norman	$\frac{\ln\left(1+\tan^{2}\left(x\right)\right)}{2}$	10
parallelrisch	$\frac{\ln\left(1+\tan^{2}\left(x\right)\right)}{2}$	10
risch	$ix-\ln\left(\mathrm{e}^{2ix}+1\right)$	16

```
[In] int(tan(x),x,method=_RETURNVERBOSE)
[Out] -ln(cos(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \tan(x)\,dx = -\frac{1}{2}\log\left(\frac{1}{\tan\left(x\right)^{2}+1}\right)$$

```
[In] integrate(tan(x),x, algorithm="fricas")
[Out] -1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \tan(x)\,dx = -\log\left(\cos\left(x\right)\right)$$

```
[In] integrate(tan(x),x)
[Out] -log(cos(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int \tan(x)\,dx = \log\left(\sec\left(x\right)\right)$$

```
[In] integrate(tan(x),x, algorithm="maxima")
[Out] log(sec(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int \tan(x)\,dx = -\log\left(\left|\cos\left(x\right)\right|\right)$$

```
[In] integrate(tan(x),x, algorithm="giac")
[Out] -log(abs(cos(x)))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \tan(x)\,dx = -\ln\left(\cos\left(x\right)\right)$$

```
[In] int(tan(x),x)
[Out] -log(cos(x))
```

3.14 $\int \cot(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \cot(x)\,dx = \log(\sin(x))$$

```
[Out] ln(sin(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3556}

$$\int \cot(x)\,dx = \log(\sin(x))$$

```
[In] Int[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \log(\sin(x))$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \cot(x)\,dx = \log(\cos(x)) + \log(\tan(x))$$

```
[In] Integrate[Cot[x],x]
[Out] Log[Cos[x]] + Log[Tan[x]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\ln(\sin(x))$	4
default	$\ln(\sin(x))$	4
derivativedivides	$-\frac{\ln(\cot^2(x)+1)}{2}$	10
parallelrisch	$\ln\left(\frac{1}{\sqrt{\sec^2(x)}}\right) + \ln(\tan(x))$	12
norman	$-\frac{\ln(1+\tan^2(x))}{2} + \ln(\tan(x))$	14
risch	$-ix + \ln(e^{2ix} - 1)$	14

```
[In] int(cot(x),x,method=_RETURNVERBOSE)
[Out] ln(sin(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \cot(x)\,dx = \frac{1}{2}\log\left(-\frac{1}{2}\cos(2x) + \frac{1}{2}\right)$$

```
[In] integrate(cot(x),x, algorithm="fricas")
[Out] 1/2*log(-1/2*cos(2*x) + 1/2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \log\left(\sin\left(x\right)\right)$$

```
[In] integrate(cot(x),x)
[Out] log(sin(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \log\left(\sin\left(x\right)\right)$$

```
[In] integrate(cot(x),x, algorithm="maxima")
[Out] log(sin(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int \cot(x)\,dx = \log\left(\left|\sin\left(x\right)\right|\right)$$

```
[In] integrate(cot(x),x, algorithm="giac")
[Out] log(abs(sin(x)))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \ln\left(\sin\left(x\right)\right)$$

```
[In] int(cot(x),x)
[Out] log(sin(x))
```

3.15 $\int x \sin(x)\, dx$

Optimal result

Integrand size = 4, antiderivative size = 8

$$\int x \sin(x)\, dx = -x \cos(x) + \sin(x)$$

```
[Out] -x*cos(x)+sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3377, 2717}

$$\int x \sin(x)\, dx = \sin(x) - x \cos(x)$$

```
[In] Int[x*Sin[x],x]
```

```
[Out] -(x*Cos[x]) + Sin[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -x\cos(x) + \int \cos(x)\,dx \\
&= -x\cos(x) + \sin(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = -x\cos(x) + \sin(x)$$

```
[In] Integrate[x*Sin[x],x]
[Out] -(x*Cos[x]) + Sin[x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$-x\cos\left(x\right)+\sin\left(x\right)$	9
risch	$-x\cos\left(x\right)+\sin\left(x\right)$	9
parallelrisch	$-x\cos\left(x\right)+\sin\left(x\right)$	9
parts	$-x\cos\left(x\right)+\sin\left(x\right)$	9
meijerg	$2\sqrt{\pi}\left(-\frac{x\cos(x)}{2\sqrt{\pi}}+\frac{\sin(x)}{2\sqrt{\pi}}\right)$	22
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)-x+2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$	30

```
[In] int(x*sin(x),x,method=_RETURNVERBOSE)
[Out] -x*cos(x)+sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = -x\cos{(x)} + \sin{(x)}$$

```
[In] integrate(x*sin(x),x, algorithm="fricas")
[Out] -x*cos(x) + sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int x\sin(x)\,dx = -x\cos{(x)} + \sin{(x)}$$

```
[In] integrate(x*sin(x),x)
[Out] -x*cos(x) + sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = -x\cos{(x)} + \sin{(x)}$$

```
[In] integrate(x*sin(x),x, algorithm="maxima")
[Out] -x*cos(x) + sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = -x\cos{(x)} + \sin{(x)}$$

```
[In] integrate(x*sin(x),x, algorithm="giac")
[Out] -x*cos(x) + sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx=\sin\left(x\right)-x\ \cos\left(x\right)$$

```
[In] int(x*sin(x),x)
[Out] sin(x) - x*cos(x)
```

3.16 $\int \log(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 8

$$\int \log(x)\,dx = -x + x\log(x)$$

```
[Out] -x+x*ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2332}

$$\int \log(x)\,dx = x\log(x) - x$$

```
[In] Int[Log[x],x]
```

```
[Out] -x + x*Log[x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\text{integral} = -x + x\log(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x)\,dx = -x + x\log(x)$$

```
[In] Integrate[Log[x],x]
[Out] -x + x*Log[x]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
lookup	$-x+x\ln(x)$	9
default	$-x+x\ln(x)$	9
norman	$-x+x\ln(x)$	9
risch	$-x+x\ln(x)$	9
parallelrisch	$-x+x\ln(x)$	9
parts	$-x+x\ln(x)$	9

```
[In] int(ln(x),x,method=_RETURNVERBOSE)
[Out] -x+x*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x)\,dx = x\log(x) - x$$

```
[In] integrate(log(x),x, algorithm="fricas")
[Out] x*log(x) - x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \log(x)\,dx = x\log(x) - x$$

```
[In] integrate(ln(x),x)
[Out] x*log(x) - x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x)\,dx = x\log(x) - x$$

```
[In] integrate(log(x),x, algorithm="maxima")
[Out] x*log(x) - x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x)\,dx = x\log(x) - x$$

```
[In] integrate(log(x),x, algorithm="giac")
[Out] x*log(x) - x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \log(x)\,dx = x(\ln(x) - 1)$$

```
[In] int(log(x),x)
[Out] x*(log(x) - 1)
```

3.17 $\int e^x x^2 \, dx$

Optimal result

Integrand size = 7, antiderivative size = 19

$$\int e^x x^2 \, dx = 2e^x - 2e^x x + e^x x^2$$

```
[Out] 2*exp(x)-2*exp(x)*x+exp(x)*x^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2207, 2225}

$$\int e^x x^2 \, dx = e^x x^2 - 2e^x x + 2e^x$$

```
[In] Int[E^x*x^2,x]
```

```
[Out] 2*E^x - 2*E^x*x + E^x*x^2
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= e^x x^2 - 2\int e^x x\,dx\\ &= -2e^x x + e^x x^2 + 2\int e^x\,dx\\ &= 2e^x - 2e^x x + e^x x^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^x x^2\,dx = e^x\left(2 - 2x + x^2\right)$$

```
[In] Integrate[E^x*x^2,x]
[Out] E^x*(2 - 2*x + x^2)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
gosper	$\left(x^2-2x+2\right)\mathrm{e}^x$	12
risch	$\left(x^2-2x+2\right)\mathrm{e}^x$	12
default	$2\,\mathrm{e}^x-2\,\mathrm{e}^x x+\mathrm{e}^x x^2$	17
norman	$2\,\mathrm{e}^x-2\,\mathrm{e}^x x+\mathrm{e}^x x^2$	17
meijerg	$-2+\frac{(3x^2-6x+6)\,\mathrm{e}^x}{3}$	17
parallelrisch	$2\,\mathrm{e}^x-2\,\mathrm{e}^x x+\mathrm{e}^x x^2$	17
parts	$2\,\mathrm{e}^x-2\,\mathrm{e}^x x+\mathrm{e}^x x^2$	17

```
[In] int(exp(x)*x^2,x,method=_RETURNVERBOSE)
[Out] (x^2-2*x+2)*exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x x^2 \, dx = \left(x^2 - 2\,x + 2\right)e^x$$

```
[In] integrate(exp(x)*x^2,x, algorithm="fricas")
[Out] (x^2 - 2*x + 2)*e^x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int e^x x^2 \, dx = \left(x^2 - 2x + 2\right) e^x$$

```
[In] integrate(exp(x)*x**2,x)
[Out] (x**2 - 2*x + 2)*exp(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x x^2 \, dx = \left(x^2 - 2\,x + 2\right)e^x$$

```
[In] integrate(exp(x)*x^2,x, algorithm="maxima")
[Out] (x^2 - 2*x + 2)*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x x^2 \, dx = \left(x^2 - 2\,x + 2\right)e^x$$

```
[In] integrate(exp(x)*x^2,x, algorithm="giac")
[Out] (x^2 - 2*x + 2)*e^x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x x^2\,dx = \mathrm{e}^x\,\left(x^2-2\,x+2\right)$$

```
[In] int(x^2*exp(x),x)
[Out] exp(x)*(x^2 - 2*x + 2)
```

3.18 $\int e^x \sin(x) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 19

$$\int e^x \sin(x) \, dx = -\frac{1}{2} e^x \cos(x) + \frac{1}{2} e^x \sin(x)$$

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4517}

$$\int e^x \sin(x) \, dx = \frac{1}{2} e^x \sin(x) - \frac{1}{2} e^x \cos(x)$$

```
[In] Int[E^x*Sin[x],x]
```

```
[Out] -1/2*(E^x*Cos[x]) + (E^x*Sin[x])/2
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{2} e^x \cos(x) + \frac{1}{2} e^x \sin(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int e^x \sin(x)\,dx = \frac{1}{2}e^x(-\cos(x)+\sin(x))$$

```
[In] Integrate[E^x*Sin[x],x]
[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$-\frac{e^{x}(\cos(x)-\sin(x))}{2}$	12
default	$-\frac{e^{x}\cos(x)}{2}+\frac{e^{x}\sin(x)}{2}$	14
norman	$\frac{e^{x}\tan\left(\frac{x}{2}\right)+\frac{e^{x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\frac{e^{x}}{2}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	34
risch	$-\frac{e^{(1+i)x}}{4}-\frac{i e^{(1+i)x}}{4}-\frac{e^{(1-i)x}}{4}+\frac{i e^{(1-i)x}}{4}$	36

```
[In] int(exp(x)*sin(x),x,method=_RETURNVERBOSE)
[Out] -1/2*exp(x)*(cos(x)-sin(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\cos\left(x\right)e^{x} + \frac{1}{2}e^{x}\sin\left(x\right)$$

```
[In] integrate(exp(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int e^x \sin(x)\,dx = \frac{e^x \sin\left(x\right)}{2} - \frac{e^x \cos\left(x\right)}{2}$$

```
[In] integrate(exp(x)*sin(x),x)
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\left(\cos\left(x\right) - \sin\left(x\right)\right)e^x$$

```
[In] integrate(exp(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\left(\cos\left(x\right) - \sin\left(x\right)\right)e^x$$

```
[In] integrate(exp(x)*sin(x),x, algorithm="giac")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{\mathrm{e}^x\left(\cos\left(x\right) - \sin\left(x\right)\right)}{2}$$

```
[In] int(exp(x)*sin(x),x)
[Out] -(exp(x)*(cos(x) - sin(x)))/2
```

3.19 $\int \arctan(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 15

$$\int \arctan(x)\,dx = x\arctan(x) - \frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] x*arctan(x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4930, 266}

$$\int \arctan(x)\,dx = x\arctan(x) - \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] Int[ArcTan[x],x]
```

```
[Out] x*ArcTan[x] - Log[1 + x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= x\arctan(x) - \int \frac{x}{1+x^2}\,dx \\ &= x\arctan(x) - \frac{1}{2}\log\left(1+x^2\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \arctan(x)\,dx = x\arctan(x) - \frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[ArcTan[x],x]
[Out] x*ArcTan[x] - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
lookup	$x\arctan\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$	14
default	$x\arctan\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$	14
parallelrisch	$x\arctan\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$	14
parts	$x\arctan\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$	14
meijerg	$\frac{x^{2}\arctan\left(\sqrt{x^{2}}\right)}{\sqrt{x^{2}}}-\frac{\ln\left(x^{2}+1\right)}{2}$	25
risch	$-\frac{ix\ln\left(ix+1\right)}{2}+\frac{ix\ln\left(-ix+1\right)}{2}-\frac{\ln\left(x^{2}+1\right)}{2}$	32

```
[In] int(arctan(x),x,method=_RETURNVERBOSE)
[Out] x*arctan(x)-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \arctan(x)\,dx = x\arctan(x) - \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate(arctan(x),x, algorithm="fricas")
[Out] x*arctan(x) - 1/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \arctan(x)\,dx = x\,\mathrm{atan}(x) - \frac{\log(x^2+1)}{2}$$

```
[In] integrate(atan(x),x)
[Out] x*atan(x) - log(x**2 + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \arctan(x)\,dx = x\arctan(x) - \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate(arctan(x),x, algorithm="maxima")
[Out] x*arctan(x) - 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \arctan(x)\,dx = x\arctan(x) - \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate(arctan(x),x, algorithm="giac")
[Out] x*arctan(x) - 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \arctan(x)\,dx = x\,\mathrm{atan}(x) - \frac{\ln(x^2+1)}{2}$$

```
[In] int(atan(x),x)
[Out] x*atan(x) - log(x^2 + 1)/2
```

3.20 $\int e^{2x} x \, dx$

Optimal result

Integrand size = 7, antiderivative size = 20

$$\int e^{2x} x \, dx = -\frac{e^{2x}}{4} + \frac{1}{2} e^{2x} x$$

```
[Out] -1/4*exp(2*x)+1/2*exp(2*x)*x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2207, 2225}

$$\int e^{2x} x \, dx = \frac{1}{2} e^{2x} x - \frac{e^{2x}}{4}$$

```
[In] Int[E^(2*x)*x,x]
```

```
[Out] -1/4*E^(2*x) + (E^(2*x)*x)/2
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}e^{2x}x - \frac{1}{2}\int e^{2x}\,dx \\ &= -\frac{e^{2x}}{4} + \frac{1}{2}e^{2x}x \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int e^{2x}x\,dx = e^{2x}\left(-\frac{1}{4} + \frac{x}{2}\right)$$

```
[In] Integrate[E^(2*x)*x,x]
[Out] E^(2*x)*(-1/4 + x/2)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.55

method	result	size
risch	$\left(-\frac{1}{4}+\frac{x}{2}\right)\mathrm{e}^{2x}$	11
gosper	$\frac{(2x-1)\mathrm{e}^{2x}}{4}$	12
meijerg	$\frac{1}{4}-\frac{(2-4x)\mathrm{e}^{2x}}{8}$	14
derivativedivides	$-\frac{\mathrm{e}^{2x}}{4}+\frac{\mathrm{e}^{2x}x}{2}$	15
default	$-\frac{\mathrm{e}^{2x}}{4}+\frac{\mathrm{e}^{2x}x}{2}$	15
norman	$-\frac{\mathrm{e}^{2x}}{4}+\frac{\mathrm{e}^{2x}x}{2}$	15
parallelrisch	$-\frac{\mathrm{e}^{2x}}{4}+\frac{\mathrm{e}^{2x}x}{2}$	15
parts	$-\frac{\mathrm{e}^{2x}}{4}+\frac{\mathrm{e}^{2x}x}{2}$	15

```
[In] int(exp(2*x)*x,x,method=_RETURNVERBOSE)
[Out] (-1/4+1/2*x)*exp(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.55

$$\int e^{2x}x\,dx = \frac{1}{4}\,(2\,x-1)e^{(2\,x)}$$

```
[In] integrate(exp(2*x)*x,x, algorithm="fricas")
[Out] 1/4*(2*x - 1)*e^(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.50

$$\int e^{2x}x\,dx = \frac{(2x-1)\,e^{2x}}{4}$$

```
[In] integrate(exp(2*x)*x,x)
[Out] (2*x - 1)*exp(2*x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.55

$$\int e^{2x}x\,dx = \frac{1}{4}\,(2\,x-1)e^{(2\,x)}$$

```
[In] integrate(exp(2*x)*x,x, algorithm="maxima")
[Out] 1/4*(2*x - 1)*e^(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.55

$$\int e^{2x}x\,dx = \frac{1}{4}\,(2\,x-1)e^{(2\,x)}$$

```
[In] integrate(exp(2*x)*x,x, algorithm="giac")
[Out] 1/4*(2*x - 1)*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.55

$$\int e^{2x}x\,dx = \frac{e^{2\,x}\,(2\,x-1)}{4}$$

```
[In] int(x*exp(2*x),x)
[Out] (exp(2*x)*(2*x - 1))/4
```

3.21 $\int x\cos(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 7

$$\int x\cos(x)\,dx = \cos(x) + x\sin(x)$$

```
[Out] cos(x)+x*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3377, 2718}

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

```
[In] Int[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\sin(x) - \int \sin(x)\,dx \\ &= \cos(x) + x\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = \cos(x) + x\sin(x)$$

```
[In] Integrate[x*Cos[x],x]
[Out] Cos[x] + x*Sin[x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$\cos(x) + x\sin(x)$	8
risch	$\cos(x) + x\sin(x)$	8
parts	$\cos(x) + x\sin(x)$	8
parallelrisch	$x\sin(x) + \cos(x) + 1$	9
norman	$\frac{2x\tan\left(\frac{x}{2}\right)+2}{1+\tan^2\left(\frac{x}{2}\right)}$	21
meijerg	$2\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}} + \frac{\cos(x)}{2\sqrt{\pi}} + \frac{x\sin(x)}{2\sqrt{\pi}}\right)$	27

```
[In] int(x*cos(x),x,method=_RETURNVERBOSE)
[Out] cos(x)+x*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

```
[In] integrate(x*cos(x),x, algorithm="fricas")
[Out] x*sin(x) + cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

```
[In] integrate(x*cos(x),x)
[Out] x*sin(x) + cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

```
[In] integrate(x*cos(x),x, algorithm="maxima")
[Out] x*sin(x) + cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

```
[In] integrate(x*cos(x),x, algorithm="giac")
[Out] x*sin(x) + cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = \cos(x) + x\,\sin(x)$$

```
[In] int(x*cos(x),x)
[Out] cos(x) + x*sin(x)
```

3.22 $\int x\sin(4x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 18

$$\int x\sin(4x)\,dx = -\frac{1}{4}x\cos(4x) + \frac{1}{16}\sin(4x)$$

```
[Out] -1/4*x*cos(4*x)+1/16*sin(4*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3377, 2717}

$$\int x\sin(4x)\,dx = \frac{1}{16}\sin(4x) - \frac{1}{4}x\cos(4x)$$

```
[In] Int[x*Sin[4*x],x]
```

```
[Out] -1/4*(x*Cos[4*x]) + Sin[4*x]/16
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{4}x\cos(4x)+\frac{1}{4}\int\cos(4x)\,dx\\ &= -\frac{1}{4}x\cos(4x)+\frac{1}{16}\sin(4x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x\sin(4x)\,dx = -\frac{1}{4}x\cos(4x)+\frac{1}{16}\sin(4x)$$

```
[In] Integrate[x*Sin[4*x],x]
[Out] -1/4*(x*Cos[4*x]) + Sin[4*x]/16
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{x\cos(4x)}{4}+\frac{\sin(4x)}{16}$	15
default	$-\frac{x\cos(4x)}{4}+\frac{\sin(4x)}{16}$	15
risch	$-\frac{x\cos(4x)}{4}+\frac{\sin(4x)}{16}$	15
parallelrisch	$-\frac{x\cos(4x)}{4}+\frac{\sin(4x)}{16}$	15
parts	$-\frac{x\cos(4x)}{4}+\frac{\sin(4x)}{16}$	15
meijerg	$\frac{\sqrt{\pi}\left(-\frac{2x\cos(4x)}{\sqrt{\pi}}+\frac{\sin(4x)}{2\sqrt{\pi}}\right)}{8}$	26
norman	$\frac{-\frac{x}{4}+\frac{x\left(\tan^2(2x)\right)}{4}+\frac{\tan(2x)}{8}}{1+\tan^2(2x)}$	31

```
[In] int(x*sin(4*x),x,method=_RETURNVERBOSE)
[Out] -1/4*x*cos(4*x)+1/16*sin(4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\sin(4x)\,dx = -\frac{1}{4}\,x\cos\left(4\,x\right)+\frac{1}{16}\,\sin\left(4\,x\right)$$

```
[In] integrate(x*sin(4*x),x, algorithm="fricas")
[Out] -1/4*x*cos(4*x) + 1/16*sin(4*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\sin(4x)\,dx = -\frac{x\cos\left(4x\right)}{4}+\frac{\sin\left(4x\right)}{16}$$

```
[In] integrate(x*sin(4*x),x)
[Out] -x*cos(4*x)/4 + sin(4*x)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\sin(4x)\,dx = -\frac{1}{4}\,x\cos\left(4\,x\right)+\frac{1}{16}\,\sin\left(4\,x\right)$$

```
[In] integrate(x*sin(4*x),x, algorithm="maxima")
[Out] -1/4*x*cos(4*x) + 1/16*sin(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\sin(4x)\,dx = -\frac{1}{4}\,x\cos\left(4\,x\right)+\frac{1}{16}\,\sin\left(4\,x\right)$$

```
[In] integrate(x*sin(4*x),x, algorithm="giac")
[Out] -1/4*x*cos(4*x) + 1/16*sin(4*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\sin(4x)\,dx = \frac{\sin\left(4\,x\right)}{16} - \frac{x\,\cos\left(4\,x\right)}{4}$$

```
[In] int(x*sin(4*x),x)
[Out] sin(4*x)/16 - (x*cos(4*x))/4
```

3.23 $\int x \log(x)\, dx$

Optimal result

Integrand size = 4, antiderivative size = 17

$$\int x \log(x)\, dx = -\frac{x^2}{4} + \frac{1}{2} x^2 \log(x)$$

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2341}

$$\int x \log(x)\, dx = \frac{1}{2} x^2 \log(x) - \frac{x^2}{4}$$

```
[In] Int[x*Log[x],x]
```

```
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{x^2}{4} + \frac{1}{2} x^2 \log(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x \log(x)\, dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

```
[In] Integrate[x*Log[x],x]
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
norman	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
risch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
parallelrisch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
parts	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14

```
[In] int(x*ln(x),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x \log(x)\, dx = \frac{1}{2}\, x^2 \log\left(x\right) - \frac{1}{4}\, x^2$$

```
[In] integrate(x*log(x),x, algorithm="fricas")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x\log(x)\,dx = \frac{x^2\log{(x)}}{2} - \frac{x^2}{4}$$

```
[In] integrate(x*ln(x),x)
[Out] x**2*log(x)/2 - x**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log{(x)} - \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(x),x, algorithm="maxima")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log{(x)} - \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(x),x, algorithm="giac")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int x\log(x)\,dx = \frac{x^2\,\left(\ln{(x)} - \frac{1}{2}\right)}{2}$$

```
[In] int(x*log(x),x)
[Out] (x^2*(log(x) - 1/2))/2
```

3.24 $\int x^2 \cos(3x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 29

$$\int x^2 \cos(3x)\,dx = \frac{2}{9}x\cos(3x) - \frac{2}{27}\sin(3x) + \frac{1}{3}x^2\sin(3x)$$

```
[Out] 2/9*x*cos(3*x)-2/27*sin(3*x)+1/3*x^2*sin(3*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3377, 2717}

$$\int x^2 \cos(3x)\,dx = \frac{1}{3}x^2\sin(3x) - \frac{2}{27}\sin(3x) + \frac{2}{9}x\cos(3x)$$

```
[In] Int[x^2*Cos[3*x],x]
[Out] (2*x*Cos[3*x])/9 - (2*Sin[3*x])/27 + (x^2*Sin[3*x])/3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}x^2\sin(3x) - \frac{2}{3}\int x\sin(3x)\,dx\\ &= \frac{2}{9}x\cos(3x) + \frac{1}{3}x^2\sin(3x) - \frac{2}{9}\int \cos(3x)\,dx\\ &= \frac{2}{9}x\cos(3x) - \frac{2}{27}\sin(3x) + \frac{1}{3}x^2\sin(3x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int x^2\cos(3x)\,dx = \frac{2}{9}x\cos(3x) + \frac{1}{27}\left(-2+9x^2\right)\sin(3x)$$

```
[In] Integrate[x^2*Cos[3*x],x]
[Out] (2*x*Cos[3*x])/9 + ((-2 + 9*x^2)*Sin[3*x])/27
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{2x\cos(3x)}{9}+\frac{(9x^2-2)\sin(3x)}{27}$	22
derivativedivides	$\frac{2x\cos(3x)}{9}-\frac{2\sin(3x)}{27}+\frac{x^2\sin(3x)}{3}$	24
default	$\frac{2x\cos(3x)}{9}-\frac{2\sin(3x)}{27}+\frac{x^2\sin(3x)}{3}$	24
parts	$\frac{2x\cos(3x)}{9}-\frac{2\sin(3x)}{27}+\frac{x^2\sin(3x)}{3}$	24
meijerg	$\frac{4\sqrt{\pi}\left(\frac{3x\cos(3x)}{2\sqrt{\pi}}-\frac{\left(-\frac{27x^2}{2}+3\right)\sin(3x)}{6\sqrt{\pi}}\right)}{27}$	33
norman	$\frac{\frac{2x}{9}-\frac{2x\left(\tan^2\left(\frac{3x}{2}\right)\right)}{9}+\frac{2x^2\tan\left(\frac{3x}{2}\right)}{3}-\frac{4\tan\left(\frac{3x}{2}\right)}{27}}{1+\tan^2\left(\frac{3x}{2}\right)}$	40
parallelrisch	$\frac{18x^2\tan\left(\frac{3x}{2}\right)-6x\left(\tan^2\left(\frac{3x}{2}\right)\right)+6x-4\tan\left(\frac{3x}{2}\right)}{27\left(\tan^2\left(\frac{3x}{2}\right)\right)+27}$	42

```
[In] int(x^2*cos(3*x),x,method=_RETURNVERBOSE)
[Out] 2/9*x*cos(3*x)+1/27*(9*x^2-2)*sin(3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int x^2\cos(3x)\,dx = \frac{2}{9}\,x\cos\left(3\,x\right) + \frac{1}{27}\,\left(9\,x^2 - 2\right)\sin\left(3\,x\right)$$

```
[In] integrate(x^2*cos(3*x),x, algorithm="fricas")
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int x^2\cos(3x)\,dx = \frac{x^2\sin\left(3x\right)}{3} + \frac{2x\cos\left(3x\right)}{9} - \frac{2\sin\left(3x\right)}{27}$$

```
[In] integrate(x**2*cos(3*x),x)
[Out] x**2*sin(3*x)/3 + 2*x*cos(3*x)/9 - 2*sin(3*x)/27
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int x^2\cos(3x)\,dx = \frac{2}{9}\,x\cos\left(3\,x\right) + \frac{1}{27}\,\left(9\,x^2 - 2\right)\sin\left(3\,x\right)$$

```
[In] integrate(x^2*cos(3*x),x, algorithm="maxima")
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int x^2\cos(3x)\,dx = \frac{2}{9}\,x\cos\left(3\,x\right) + \frac{1}{27}\,\left(9\,x^2 - 2\right)\sin\left(3\,x\right)$$

```
[In] integrate(x^2*cos(3*x),x, algorithm="giac")
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int x^2 \cos(3x) \, dx = \frac{2 x \cos(3 x)}{9} - \frac{2 \sin(3 x)}{27} + \frac{x^2 \sin(3 x)}{3}$$

```
[In] int(x^2*cos(3*x),x)
[Out] (2*x*cos(3*x))/9 - (2*sin(3*x))/27 + (x^2*sin(3*x))/3
```

3.25 $\int x^2 \sin(2x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 29

$$\int x^2 \sin(2x)\,dx = \frac{1}{4}\cos(2x) - \frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x)$$

```
[Out] 1/4*cos(2*x)-1/2*x^2*cos(2*x)+1/2*x*sin(2*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3377, 2718}

$$\int x^2 \sin(2x)\,dx = -\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x)$$

```
[In] Int[x^2*Sin[2*x],x]
[Out] Cos[2*x]/4 - (x^2*Cos[2*x])/2 + (x*Sin[2*x])/2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2}x^2\cos(2x) + \int x\cos(2x)\,dx\\ &= -\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x) - \frac{1}{2}\int \sin(2x)\,dx\\ &= \frac{1}{4}\cos(2x) - \frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int x^2\sin(2x)\,dx = -\frac{1}{4}(-1+2x^2)\cos(2x) + \frac{1}{2}x\sin(2x)$$

```
[In] Integrate[x^2*Sin[2*x],x]
[Out] -1/4*((-1 + 2*x^2)*Cos[2*x]) + (x*Sin[2*x])/2
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

method	result	size
risch	$\left(-\frac{x^2}{2}+\frac{1}{4}\right)\cos(2x)+\frac{x\sin(2x)}{2}$	21
derivativedivides	$\frac{\cos(2x)}{4}-\frac{x^2\cos(2x)}{2}+\frac{x\sin(2x)}{2}$	24
default	$\frac{\cos(2x)}{4}-\frac{x^2\cos(2x)}{2}+\frac{x\sin(2x)}{2}$	24
parts	$\frac{\cos(2x)}{4}-\frac{x^2\cos(2x)}{2}+\frac{x\sin(2x)}{2}$	24
parallelrisch	$\frac{1}{4}-\frac{x^2\cos(2x)}{2}+\frac{x\sin(2x)}{2}+\frac{\cos(2x)}{4}$	25
norman	$\frac{x\tan(x)-\frac{x^2}{2}+\frac{x^2\left(\tan^2(x)\right)}{2}+\frac{1}{2}}{1+\tan^2(x)}$	30
meijerg	$\frac{\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-2x^2+1\right)\cos(2x)}{2\sqrt{\pi}}+\frac{x\sin(2x)}{\sqrt{\pi}}\right)}{2}$	37

```
[In] int(x^2*sin(2*x),x,method=_RETURNVERBOSE)
[Out] (-1/2*x^2+1/4)*cos(2*x)+1/2*x*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int x^2 \sin(2x)\, dx = -\frac{1}{4}\left(2\,x^2-1\right)\cos\left(2\,x\right)+\frac{1}{2}\,x\sin\left(2\,x\right)$$

```
[In] integrate(x^2*sin(2*x),x, algorithm="fricas")
[Out] -1/4*(2*x^2 - 1)*cos(2*x) + 1/2*x*sin(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int x^2 \sin(2x)\, dx = -\frac{x^2\cos\left(2x\right)}{2}+\frac{x\sin\left(2x\right)}{2}+\frac{\cos\left(2x\right)}{4}$$

```
[In] integrate(x**2*sin(2*x),x)
[Out] -x**2*cos(2*x)/2 + x*sin(2*x)/2 + cos(2*x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int x^2 \sin(2x)\, dx = -\frac{1}{4}\left(2\,x^2-1\right)\cos\left(2\,x\right)+\frac{1}{2}\,x\sin\left(2\,x\right)$$

```
[In] integrate(x^2*sin(2*x),x, algorithm="maxima")
[Out] -1/4*(2*x^2 - 1)*cos(2*x) + 1/2*x*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int x^2 \sin(2x)\, dx = -\frac{1}{4}\left(2\,x^2-1\right)\cos\left(2\,x\right)+\frac{1}{2}\,x\sin\left(2\,x\right)$$

```
[In] integrate(x^2*sin(2*x),x, algorithm="giac")
[Out] -1/4*(2*x^2 - 1)*cos(2*x) + 1/2*x*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int x^2 \sin(2x)\,dx = \frac{x\,\sin\left(2\,x\right)}{2} + \left(2\sin\left(x\right)^2 - 1\right)\,\left(\frac{x^2}{2} - \frac{1}{4}\right)$$

```
[In] int(x^2*sin(2*x),x)
[Out] (x*sin(2*x))/2 + (2*sin(x)^2 - 1)*(x^2/2 - 1/4)
```

3.26 $\int \log^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 15

$$\int \log^2(x)\,dx = 2x - 2x\log(x) + x\log^2(x)$$

```
[Out] 2*x-2*x*ln(x)+x*ln(x)^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2333, 2332}

$$\int \log^2(x)\,dx = 2x + x\log^2(x) - 2x\log(x)$$

```
[In] Int[Log[x]^2,x]
```

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log^2(x) - 2\int \log(x)\,dx \\
&= 2x - 2x\log(x) + x\log^2(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x)\,dx = 2x - 2x\log(x) + x\log^2(x)$$

```
[In] Integrate[Log[x]^2,x]
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$2x - 2x\ln(x) + x\ln(x)^2$	16
norman	$2x - 2x\ln(x) + x\ln(x)^2$	16
risch	$2x - 2x\ln(x) + x\ln(x)^2$	16
parallelrisch	$2x - 2x\ln(x) + x\ln(x)^2$	16

```
[In] int(ln(x)^2,x,method=_RETURNVERBOSE)
[Out] 2*x-2*x*ln(x)+x*ln(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x)\,dx = x\log(x)^2 - 2\,x\log(x) + 2\,x$$

```
[In] integrate(log(x)^2,x, algorithm="fricas")
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x)\,dx = x\log{(x)}^2 - 2x\log{(x)} + 2x$$

```
[In] integrate(ln(x)**2,x)
[Out] x*log(x)**2 - 2*x*log(x) + 2*x
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \log^2(x)\,dx = \left(\log{(x)}^2 - 2\,\log{(x)} + 2\right)x$$

```
[In] integrate(log(x)^2,x, algorithm="maxima")
[Out] (log(x)^2 - 2*log(x) + 2)*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x)\,dx = x\log{(x)}^2 - 2\,x\log{(x)} + 2\,x$$

```
[In] integrate(log(x)^2,x, algorithm="giac")
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \log^2(x)\,dx = x\left(\ln{(x)}^2 - 2\,\ln{(x)} + 2\right)$$

```
[In] int(log(x)^2,x)
[Out] x*(log(x)^2 - 2*log(x) + 2)
```

3.27 $\int \arcsin(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 16

$$\int \arcsin(x)\,dx = \sqrt{1-x^2} + x\arcsin(x)$$

```
[Out] x*arcsin(x)+(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4715, 267}

$$\int \arcsin(x)\,dx = x\arcsin(x) + \sqrt{1-x^2}$$

```
[In] Int[ArcSin[x],x]
```

```
[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\arcsin(x) - \int \frac{x}{\sqrt{1-x^2}}\,dx\\ &= \sqrt{1-x^2} + x\arcsin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \arcsin(x)\,dx = \sqrt{1-x^2} + x\arcsin(x)$$

```
[In] Integrate[ArcSin[x],x]
[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
lookup	$\arcsin(x)\,x+\sqrt{-x^2+1}$	15
default	$\arcsin(x)\,x+\sqrt{-x^2+1}$	15
parts	$\arcsin(x)\,x+\sqrt{-x^2+1}$	15

```
[In] int(arcsin(x),x,method=_RETURNVERBOSE)
[Out] arcsin(x)*x+(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \arcsin(x)\,dx = x\arcsin(x) + \sqrt{-x^2+1}$$

```
[In] integrate(arcsin(x),x, algorithm="fricas")
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \arcsin(x)\,dx = x\,\mathrm{asin}\,(x) + \sqrt{1-x^2}$$

```
[In] integrate(asin(x),x)
[Out] x*asin(x) + sqrt(1 - x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \arcsin(x)\,dx = x\arcsin\,(x) + \sqrt{-x^2+1}$$

```
[In] integrate(arcsin(x),x, algorithm="maxima")
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \arcsin(x)\,dx = x\arcsin\,(x) + \sqrt{-x^2+1}$$

```
[In] integrate(arcsin(x),x, algorithm="giac")
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \arcsin(x)\,dx = x\,\mathrm{asin}(x) + \sqrt{1-x^2}$$

```
[In] int(asin(x),x)
[Out] x*asin(x) + (1 - x^2)^(1/2)
```

3.28 $\int t\cos(t)\sin(t)\,dt$

Optimal result

Integrand size = 6, antiderivative size = 23

$$\int t\cos(t)\sin(t)\,dt = -\frac{t}{4} + \frac{1}{4}\cos(t)\sin(t) + \frac{1}{2}t\sin^2(t)$$

```
[Out] -1/4*t+1/4*cos(t)*sin(t)+1/2*t*sin(t)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3524, 2715, 8}

$$\int t\cos(t)\sin(t)\,dt = -\frac{t}{4} + \frac{1}{2}t\sin^2(t) + \frac{1}{4}\sin(t)\cos(t)$$

```
[In] Int[t*Cos[t]*Sin[t],t]
```

```
[Out] -1/4*t + (Cos[t]*Sin[t])/4 + (t*Sin[t]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}t\sin^2(t) - \frac{1}{2}\int \sin^2(t)\,dt\\ &= \frac{1}{4}\cos(t)\sin(t) + \frac{1}{2}t\sin^2(t) - \frac{\int 1\,dt}{4}\\ &= -\frac{t}{4} + \frac{1}{4}\cos(t)\sin(t) + \frac{1}{2}t\sin^2(t)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int t\cos(t)\sin(t)\,dt = -\frac{1}{4}t\cos(2t) + \frac{1}{8}\sin(2t)$$

```
[In] Integrate[t*Cos[t]*Sin[t],t]
[Out] -1/4*(t*Cos[2*t]) + Sin[2*t]/8
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

method	result	size
risch	$-\frac{t\cos(2t)}{4}+\frac{\sin(2t)}{8}$	15
parallelrisch	$-\frac{t\cos(2t)}{4}+\frac{\sin(2t)}{8}$	15
default	$-\frac{t(\cos^2(t))}{2}+\frac{\cos(t)\sin(t)}{4}+\frac{t}{4}$	18
meijerg	$\frac{\sqrt{\pi}\left(-\frac{t\cos(2t)}{\sqrt{\pi}}+\frac{\sin(2t)}{2\sqrt{\pi}}\right)}{4}$	26
norman	$\frac{-\frac{t}{4}-\frac{\left(\tan^3\left(\frac{t}{2}\right)\right)}{2}+\frac{3t\left(\tan^2\left(\frac{t}{2}\right)\right)}{2}-\frac{t\left(\tan^4\left(\frac{t}{2}\right)\right)}{4}+\frac{\tan\left(\frac{t}{2}\right)}{2}}{\left(1+\tan^2\left(\frac{t}{2}\right)\right)^2}$	48

```
[In] int(t*cos(t)*sin(t),t,method=_RETURNVERBOSE)
[Out] -1/4*t*cos(2*t)+1/8*sin(2*t)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int t\cos(t)\sin(t)\,dt = -\frac{1}{2}\,t\cos{(t)}^{2} + \frac{1}{4}\,\cos{(t)}\sin{(t)} + \frac{1}{4}\,t$$

```
[In] integrate(t*cos(t)*sin(t),t, algorithm="fricas")
[Out] -1/2*t*cos(t)^2 + 1/4*cos(t)*sin(t) + 1/4*t
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int t\cos(t)\sin(t)\,dt = \frac{t\sin^{2}{(t)}}{4} - \frac{t\cos^{2}{(t)}}{4} + \frac{\sin{(t)}\cos{(t)}}{4}$$

```
[In] integrate(t*cos(t)*sin(t),t)
[Out] t*sin(t)**2/4 - t*cos(t)**2/4 + sin(t)*cos(t)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int t\cos(t)\sin(t)\,dt = -\frac{1}{4}\,t\cos{(2\,t)} + \frac{1}{8}\,\sin{(2\,t)}$$

```
[In] integrate(t*cos(t)*sin(t),t, algorithm="maxima")
[Out] -1/4*t*cos(2*t) + 1/8*sin(2*t)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int t\cos(t)\sin(t)\,dt = -\frac{1}{4}\,t\cos{(2\,t)} + \frac{1}{8}\,\sin{(2\,t)}$$

```
[In] integrate(t*cos(t)*sin(t),t, algorithm="giac")
[Out] -1/4*t*cos(2*t) + 1/8*sin(2*t)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int t\cos(t)\sin(t)\,dt = \frac{\sin\left(2\,t\right)}{8} + \frac{t\,\left(2\sin\left(t\right)^2-1\right)}{4}$$

```
[In] int(t*cos(t)*sin(t),t)
[Out] sin(2*t)/8 + (t*(2*sin(t)^2 - 1))/4
```

3.29 $\int t \sec^2(t)\, dt$

Optimal result

Integrand size = 6, antiderivative size = 8

$$\int t \sec^2(t)\, dt = \log(\cos(t)) + t\tan(t)$$

```
[Out] ln(cos(t))+t*tan(t)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4269, 3556}

$$\int t \sec^2(t)\, dt = t\tan(t) + \log(\cos(t))$$

```
[In] Int[t*Sec[t]^2,t]
```

```
[Out] Log[Cos[t]] + t*Tan[t]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = t\tan(t) - \int \tan(t)\,dt$$
$$= \log(\cos(t)) + t\tan(t)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int t\sec^2(t)\,dt = \log(\cos(t)) + t\tan(t)$$

```
[In] Integrate[t*Sec[t]^2,t]
[Out] Log[Cos[t]] + t*Tan[t]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$\ln(\cos(t)) + t\tan(t)$	9
risch	$-2it + \frac{2it}{e^{2it}+1} + \ln(e^{2it}+1)$	27
parallelrisch	$-\ln\left(\frac{2}{\cos(t)+1}\right) + \ln(\csc(t) - \cot(t) - 1) + \ln(\csc(t) - \cot(t) + 1) + t\tan(t)$	35
norman	$-\frac{2\tan(\frac{t}{2})t}{\tan^2(\frac{t}{2})-1} - \ln\left(1+\tan^2\left(\frac{t}{2}\right)\right) + \ln\left(\tan\left(\frac{t}{2}\right)-1\right) + \ln\left(\tan\left(\frac{t}{2}\right)+1\right)$	44

```
[In] int(t*sec(t)^2,t,method=_RETURNVERBOSE)
[Out] ln(cos(t))+t*tan(t)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(8) = 16.

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int t\sec^2(t)\,dt = \frac{\cos(t)\log(-\cos(t)) + t\sin(t)}{\cos(t)}$$

```
[In] integrate(t*sec(t)^2,t, algorithm="fricas")
[Out] (cos(t)*log(-cos(t)) + t*sin(t))/cos(t)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int t\sec^2(t)\,dt = t\tan(t) + \log(\cos(t))$$

```
[In] integrate(t*sec(t)**2,t)
```

```
[Out] t*tan(t) + log(cos(t))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(8) = 16.

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 9.25

$$\int t\sec^2(t)\,dt = \frac{\left(\cos(2t)^2 + \sin(2t)^2 + 2\cos(2t) + 1\right)\log\left(\cos(2t)^2 + \sin(2t)^2 + 2\cos(2t) + 1\right) + 4t\sin(2t)}{2\left(\cos(2t)^2 + \sin(2t)^2 + 2\cos(2t) + 1\right)}$$

```
[In] integrate(t*sec(t)^2,t, algorithm="maxima")
```

```
[Out] 1/2*((cos(2*t)^2 + sin(2*t)^2 + 2*cos(2*t) + 1)*log(cos(2*t)^2 + sin(2*t)^2
 + 2*cos(2*t) + 1) + 4*t*sin(2*t))/(cos(2*t)^2 + sin(2*t)^2 + 2*cos(2*t) +
1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(8) = 16.

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 12.88

$$\int t\sec^2(t)\,dt = \frac{\log\left(\frac{4\left(\tan\left(\frac{1}{2}t\right)^4 - 2\tan\left(\frac{1}{2}t\right)^2 + 1\right)}{\tan\left(\frac{1}{2}t\right)^4 + 2\tan\left(\frac{1}{2}t\right)^2 + 1}\right)\tan\left(\frac{1}{2}t\right)^2 - 4t\tan\left(\frac{1}{2}t\right) - \log\left(\frac{4\left(\tan\left(\frac{1}{2}t\right)^4 - 2\tan\left(\frac{1}{2}t\right)^2 + 1\right)}{\tan\left(\frac{1}{2}t\right)^4 + 2\tan\left(\frac{1}{2}t\right)^2 + 1}\right)}{2\left(\tan\left(\frac{1}{2}t\right)^2 - 1\right)}$$

```
[In] integrate(t*sec(t)^2,t, algorithm="giac")
```

```
[Out] 1/2*(log(4*(tan(1/2*t)^4 - 2*tan(1/2*t)^2 + 1)/(tan(1/2*t)^4 + 2*tan(1/2*t)
^2 + 1))*tan(1/2*t)^2 - 4*t*tan(1/2*t) - log(4*(tan(1/2*t)^4 - 2*tan(1/2*t)
^2 + 1)/(tan(1/2*t)^4 + 2*tan(1/2*t)^2 + 1)))/(tan(1/2*t)^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int t \sec^2(t)\, dt = \ln\left(\cos\left(t\right)\right) + t \tan(t)$$

```
[In] int(t/cos(t)^2,t)
```

```
[Out] log(cos(t)) + t*tan(t)
```

3.30 $\int t^2 \log(t)\, dt$

Optimal result

Integrand size = 6, antiderivative size = 17

$$\int t^2 \log(t)\, dt = -\frac{t^3}{9} + \frac{1}{3} t^3 \log(t)$$

```
[Out] -1/9*t^3+1/3*t^3*ln(t)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{2341\}$

$$\int t^2 \log(t)\, dt = \frac{1}{3} t^3 \log(t) - \frac{t^3}{9}$$

```
[In] Int[t^2*Log[t],t]
```

```
[Out] -1/9*t^3 + (t^3*Log[t])/3
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{t^3}{9} + \frac{1}{3} t^3 \log(t)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int t^2 \log(t)\, dt = -\frac{t^3}{9} + \frac{1}{3} t^3 \log(t)$$

```
[In] Integrate[t^2*Log[t],t]
[Out] -1/9*t^3 + (t^3*Log[t])/3
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{t^3}{9} + \frac{t^3 \ln(t)}{3}$	14
norman	$-\frac{t^3}{9} + \frac{t^3 \ln(t)}{3}$	14
risch	$-\frac{t^3}{9} + \frac{t^3 \ln(t)}{3}$	14
parallelrisch	$-\frac{t^3}{9} + \frac{t^3 \ln(t)}{3}$	14
parts	$-\frac{t^3}{9} + \frac{t^3 \ln(t)}{3}$	14

```
[In] int(t^2*ln(t),t,method=_RETURNVERBOSE)
[Out] -1/9*t^3+1/3*t^3*ln(t)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int t^2 \log(t)\, dt = \frac{1}{3}\, t^3 \log\left(t\right) - \frac{1}{9}\, t^3$$

```
[In] integrate(t^2*log(t),t, algorithm="fricas")
[Out] 1/3*t^3*log(t) - 1/9*t^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int t^2 \log(t)\, dt = \frac{t^3 \log(t)}{3} - \frac{t^3}{9}$$

```
[In] integrate(t**2*ln(t),t)
[Out] t**3*log(t)/3 - t**3/9
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int t^2 \log(t)\, dt = \frac{1}{3}\, t^3 \log(t) - \frac{1}{9}\, t^3$$

```
[In] integrate(t^2*log(t),t, algorithm="maxima")
[Out] 1/3*t^3*log(t) - 1/9*t^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int t^2 \log(t)\, dt = \frac{1}{3}\, t^3 \log(t) - \frac{1}{9}\, t^3$$

```
[In] integrate(t^2*log(t),t, algorithm="giac")
[Out] 1/3*t^3*log(t) - 1/9*t^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int t^2 \log(t)\, dt = \frac{t^3 \left(\ln(t) - \frac{1}{3}\right)}{3}$$

```
[In] int(t^2*log(t),t)
[Out] (t^3*(log(t) - 1/3))/3
```

3.31 $\int e^t t^3\,dt$

Optimal result

Integrand size = 7, antiderivative size = 27

$$\int e^t t^3\,dt = -6e^t + 6e^t t - 3e^t t^2 + e^t t^3$$

```
[Out] -6*exp(t)+6*exp(t)*t-3*exp(t)*t^2+exp(t)*t^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2207, 2225}

$$\int e^t t^3\,dt = e^t t^3 - 3e^t t^2 + 6e^t t - 6e^t$$

```
[In] Int[E^t*t^3,t]
```

```
[Out] -6*E^t + 6*E^t*t - 3*E^t*t^2 + E^t*t^3
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= e^t t^3 - 3\int e^t t^2 \, dt \\
&= -3e^t t^2 + e^t t^3 + 6\int e^t t \, dt \\
&= 6e^t t - 3e^t t^2 + e^t t^3 - 6\int e^t \, dt \\
&= -6e^t + 6e^t t - 3e^t t^2 + e^t t^3
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int e^t t^3 \, dt = e^t \left(-6 + 6t - 3t^2 + t^3\right)$$

```
[In] Integrate[E^t*t^3,t]
[Out] E^t*(-6 + 6*t - 3*t^2 + t^3)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

method	result	size
gosper	$(t^3 - 3t^2 + 6t - 6)\,\mathrm{e}^t$	17
risch	$(t^3 - 3t^2 + 6t - 6)\,\mathrm{e}^t$	17
meijerg	$6 - \frac{(-4t^3+12t^2-24t+24)\mathrm{e}^t}{4}$	22
default	$-6\,\mathrm{e}^t + 6\,\mathrm{e}^t t - 3\,\mathrm{e}^t t^2 + \mathrm{e}^t t^3$	24
norman	$-6\,\mathrm{e}^t + 6\,\mathrm{e}^t t - 3\,\mathrm{e}^t t^2 + \mathrm{e}^t t^3$	24
parallelrisch	$-6\,\mathrm{e}^t + 6\,\mathrm{e}^t t - 3\,\mathrm{e}^t t^2 + \mathrm{e}^t t^3$	24
parts	$-6\,\mathrm{e}^t + 6\,\mathrm{e}^t t - 3\,\mathrm{e}^t t^2 + \mathrm{e}^t t^3$	24

```
[In] int(exp(t)*t^3,t,method=_RETURNVERBOSE)
[Out] (t^3-3*t^2+6*t-6)*exp(t)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int e^t t^3 \, dt = \left(t^3 - 3\,t^2 + 6\,t - 6\right)e^t$$

```
[In] integrate(exp(t)*t^3,t, algorithm="fricas")
[Out] (t^3 - 3*t^2 + 6*t - 6)*e^t
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

$$\int e^t t^3 \, dt = \left(t^3 - 3t^2 + 6t - 6\right) e^t$$

```
[In] integrate(exp(t)*t**3,t)
[Out] (t**3 - 3*t**2 + 6*t - 6)*exp(t)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int e^t t^3 \, dt = \left(t^3 - 3\,t^2 + 6\,t - 6\right)e^t$$

```
[In] integrate(exp(t)*t^3,t, algorithm="maxima")
[Out] (t^3 - 3*t^2 + 6*t - 6)*e^t
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int e^t t^3 \, dt = \left(t^3 - 3\,t^2 + 6\,t - 6\right)e^t$$

```
[In] integrate(exp(t)*t^3,t, algorithm="giac")
[Out] (t^3 - 3*t^2 + 6*t - 6)*e^t
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int e^t t^3 \, dt = \mathrm{e}^t \left(t^3 - 3\,t^2 + 6\,t - 6\right)$$

```
[In] int(t^3*exp(t),t)
[Out] exp(t)*(6*t - 3*t^2 + t^3 - 6)
```

3.32 $\int e^{2t} \sin(3t) \, dt$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{2t} \sin(3t) \, dt = -\frac{3}{13} e^{2t} \cos(3t) + \frac{2}{13} e^{2t} \sin(3t)$$

```
[Out] -3/13*exp(2*t)*cos(3*t)+2/13*exp(2*t)*sin(3*t)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = $\{4517\}$

$$\int e^{2t} \sin(3t) \, dt = \frac{2}{13} e^{2t} \sin(3t) - \frac{3}{13} e^{2t} \cos(3t)$$

```
[In] Int[E^(2*t)*Sin[3*t],t]
```

```
[Out] (-3*E^(2*t)*Cos[3*t])/13 + (2*E^(2*t)*Sin[3*t])/13
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{3}{13} e^{2t} \cos(3t) + \frac{2}{13} e^{2t} \sin(3t)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{2t}\sin(3t)\,dt=\frac{1}{13}e^{2t}(-3\cos(3t)+2\sin(3t))$$

```
[In] Integrate[E^(2*t)*Sin[3*t],t]
[Out] (E^(2*t)*(-3*Cos[3*t] + 2*Sin[3*t]))/13
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{e^{2t}(-3\cos(3t)+2\sin(3t))}{13}$	20
default	$-\frac{3\,e^{2t}\cos(3t)}{13}+\frac{2\,e^{2t}\sin(3t)}{13}$	22
risch	$-\frac{3\,e^{(2+3i)t}}{26}-\frac{i e^{(2+3i)t}}{13}-\frac{3\,e^{(2-3i)t}}{26}+\frac{i e^{(2-3i)t}}{13}$	36
norman	$\frac{\frac{4\,e^{2t}\tan\left(\frac{3t}{2}\right)}{13}+\frac{3\,e^{2t}\left(\tan^2\left(\frac{3t}{2}\right)\right)}{13}-\frac{3\,e^{2t}}{13}}{1+\tan^2\left(\frac{3t}{2}\right)}$	41

```
[In] int(exp(2*t)*sin(3*t),t,method=_RETURNVERBOSE)
[Out] 1/13*exp(2*t)*(-3*cos(3*t)+2*sin(3*t))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{2t}\sin(3t)\,dt=-\frac{3}{13}\cos\left(3\,t\right)e^{(2\,t)}+\frac{2}{13}\,e^{(2\,t)}\sin\left(3\,t\right)$$

```
[In] integrate(exp(2*t)*sin(3*t),t, algorithm="fricas")
[Out] -3/13*cos(3*t)*e^(2*t) + 2/13*e^(2*t)*sin(3*t)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{2t}\sin(3t)\,dt = \frac{2e^{2t}\sin{(3t)}}{13} - \frac{3e^{2t}\cos{(3t)}}{13}$$

```
[In] integrate(exp(2*t)*sin(3*t),t)
[Out] 2*exp(2*t)*sin(3*t)/13 - 3*exp(2*t)*cos(3*t)/13
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{2t}\sin(3t)\,dt = -\frac{1}{13}\,(3\,\cos{(3\,t)} - 2\,\sin{(3\,t)})e^{(2\,t)}$$

```
[In] integrate(exp(2*t)*sin(3*t),t, algorithm="maxima")
[Out] -1/13*(3*cos(3*t) - 2*sin(3*t))*e^(2*t)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{2t}\sin(3t)\,dt = -\frac{1}{13}\,(3\,\cos{(3\,t)} - 2\,\sin{(3\,t)})e^{(2\,t)}$$

```
[In] integrate(exp(2*t)*sin(3*t),t, algorithm="giac")
[Out] -1/13*(3*cos(3*t) - 2*sin(3*t))*e^(2*t)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{2t}\sin(3t)\,dt = -\frac{\mathrm{e}^{2\,t}\,(3\,\cos{(3\,t)} - 2\,\sin{(3\,t)})}{13}$$

```
[In] int(sin(3*t)*exp(2*t),t)
[Out] -(exp(2*t)*(3*cos(3*t) - 2*sin(3*t)))/13
```

3.33 $\int e^{-t} \cos(3t)\, dt$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{-t} \cos(3t)\, dt = -\frac{1}{10} e^{-t} \cos(3t) + \frac{3}{10} e^{-t} \sin(3t)$$

```
[Out] -1/10*cos(3*t)/exp(t)+3/10*sin(3*t)/exp(t)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{4518\}$

$$\int e^{-t} \cos(3t)\, dt = \frac{3}{10} e^{-t} \sin(3t) - \frac{1}{10} e^{-t} \cos(3t)$$

```
[In] Int[Cos[3*t]/E^t,t]
```

```
[Out] -1/10*Cos[3*t]/E^t + (3*Sin[3*t])/(10*E^t)
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{10} e^{-t} \cos(3t) + \frac{3}{10} e^{-t} \sin(3t)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int e^{-t}\cos(3t)\,dt = -\frac{1}{10}e^{-t}(\cos(3t)-3\sin(3t))$$

```
[In] Integrate[Cos[3*t]/E^t,t]
[Out] -1/10*(Cos[3*t] - 3*Sin[3*t])/E^t
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$-\frac{\mathrm{e}^{-t}(\cos(3t)-3\sin(3t))}{10}$	18
default	$-\frac{\mathrm{e}^{-t}\cos(3t)}{10}+\frac{3\,\mathrm{e}^{-t}\sin(3t)}{10}$	22
norman	$\frac{\left(-\frac{1}{10}+\frac{\left(\tan^2\left(\frac{3t}{2}\right)\right)}{10}+\frac{3\tan\left(\frac{3t}{2}\right)}{5}\right)\mathrm{e}^{-t}}{1+\tan^2\left(\frac{3t}{2}\right)}$	32
risch	$-\frac{\mathrm{e}^{(-1+3i)t}}{20}-\frac{3i\mathrm{e}^{(-1+3i)t}}{20}-\frac{\mathrm{e}^{(-1-3i)t}}{20}+\frac{3i\mathrm{e}^{(-1-3i)t}}{20}$	36

```
[In] int(cos(3*t)/exp(t),t,method=_RETURNVERBOSE)
[Out] -1/10*exp(-t)*(cos(3*t)-3*sin(3*t))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{-t}\cos(3t)\,dt = -\frac{1}{10}\cos(3t)\,e^{(-t)}+\frac{3}{10}\,e^{(-t)}\sin(3t)$$

```
[In] integrate(cos(3*t)/exp(t),t, algorithm="fricas")
[Out] -1/10*cos(3*t)*e^(-t) + 3/10*e^(-t)*sin(3*t)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int e^{-t}\cos(3t)\,dt = \frac{3e^{-t}\sin{(3t)}}{10} - \frac{e^{-t}\cos{(3t)}}{10}$$

```
[In] integrate(cos(3*t)/exp(t),t)
[Out] 3*exp(-t)*sin(3*t)/10 - exp(-t)*cos(3*t)/10
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int e^{-t}\cos(3t)\,dt = -\frac{1}{10}\left(\cos\left(3\,t\right) - 3\,\sin\left(3\,t\right)\right)e^{(-t)}$$

```
[In] integrate(cos(3*t)/exp(t),t, algorithm="maxima")
[Out] -1/10*(cos(3*t) - 3*sin(3*t))*e^(-t)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int e^{-t}\cos(3t)\,dt = -\frac{1}{10}\left(\cos\left(3\,t\right) - 3\,\sin\left(3\,t\right)\right)e^{(-t)}$$

```
[In] integrate(cos(3*t)/exp(t),t, algorithm="giac")
[Out] -1/10*(cos(3*t) - 3*sin(3*t))*e^(-t)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int e^{-t}\cos(3t)\,dt = -\frac{\mathrm{e}^{-t}\,\left(\cos\left(3\,t\right) - 3\,\sin\left(3\,t\right)\right)}{10}$$

```
[In] int(cos(3*t)*exp(-t),t)
[Out] -(exp(-t)*(cos(3*t) - 3*sin(3*t)))/10
```

3.34 $\int y \sinh(y) \, dy$

Optimal result

Integrand size = 4, antiderivative size = 9

$$\int y \sinh(y) \, dy = y \cosh(y) - \sinh(y)$$

```
[Out] y*cosh(y)-sinh(y)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3377, 2717}

$$\int y \sinh(y) \, dy = y \cosh(y) - \sinh(y)$$

```
[In] Int[y*Sinh[y],y]
```

```
[Out] y*Cosh[y] - Sinh[y]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= y\cosh(y) - \int \cosh(y)\,dy \\ &= y\cosh(y) - \sinh(y)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int y\sinh(y)\,dy = y\cosh(y) - \sinh(y)$$

```
[In] Integrate[y*Sinh[y],y]
[Out] y*Cosh[y] - Sinh[y]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$y\cosh\left(y\right)-\sinh\left(y\right)$	10
meijerg	$y\cosh\left(y\right)-\sinh\left(y\right)$	10
parallelrisch	$y\cosh\left(y\right)-\sinh\left(y\right)$	10
parts	$y\cosh\left(y\right)-\sinh\left(y\right)$	10
risch	$\left(-\frac{1}{2}+\frac{y}{2}\right){\mathrm e}^{y}+\left(\frac{1}{2}+\frac{y}{2}\right){\mathrm e}^{-y}$	20

```
[In] int(y*sinh(y),y,method=_RETURNVERBOSE)
[Out] y*cosh(y)-sinh(y)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int y\sinh(y)\,dy = y\cosh\left(y\right) - \sinh\left(y\right)$$

```
[In] integrate(y*sinh(y),y, algorithm="fricas")
[Out] y*cosh(y) - sinh(y)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int y\sinh(y)\,dy = y\cosh(y) - \sinh(y)$$

```
[In] integrate(y*sinh(y),y)
[Out] y*cosh(y) - sinh(y)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(9) = 18.

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.78

$$\int y\sinh(y)\,dy = \frac{1}{2}y^2\sinh(y) + \frac{1}{4}\left(y^2 + 2y + 2\right)e^{(-y)} - \frac{1}{4}\left(y^2 - 2y + 2\right)e^{y}$$

```
[In] integrate(y*sinh(y),y, algorithm="maxima")
[Out] 1/2*y^2*sinh(y) + 1/4*(y^2 + 2*y + 2)*e^(-y) - 1/4*(y^2 - 2*y + 2)*e^y
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int y\sinh(y)\,dy = \frac{1}{2}(y+1)e^{(-y)} + \frac{1}{2}(y-1)e^{y}$$

```
[In] integrate(y*sinh(y),y, algorithm="giac")
[Out] 1/2*(y + 1)*e^(-y) + 1/2*(y - 1)*e^y
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int y\sinh(y)\,dy = y\cosh(y) - \sinh(y)$$

```
[In] int(y*sinh(y),y)
[Out] y*cosh(y) - sinh(y)
```

3.35 $\int y \cosh(ay) \, dy$

Optimal result

Integrand size = 6, antiderivative size = 19

$$\int y \cosh(ay) \, dy = -\frac{\cosh(ay)}{a^2} + \frac{y \sinh(ay)}{a}$$

```
[Out] -cosh(a*y)/a^2+y*sinh(a*y)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3377, 2718}

$$\int y \cosh(ay) \, dy = \frac{y \sinh(ay)}{a} - \frac{\cosh(ay)}{a^2}$$

```
[In] Int[y*Cosh[a*y],y]
```

```
[Out] -(Cosh[a*y]/a^2) + (y*Sinh[a*y])/a
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{y\sinh(ay)}{a} - \frac{\int \sinh(ay)\,dy}{a}\\ &= -\frac{\cosh(ay)}{a^2} + \frac{y\sinh(ay)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int y\cosh(ay)\,dy = -\frac{\cosh(ay)}{a^2} + \frac{y\sinh(ay)}{a}$$

```
[In] Integrate[y*Cosh[a*y],y]
[Out] -(Cosh[a*y]/a^2) + (y*Sinh[a*y])/a
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{ya\sinh(ay)-\cosh(ay)}{a^2}$	19
default	$\frac{ya\sinh(ay)-\cosh(ay)}{a^2}$	19
parts	$-\frac{\cosh(ay)}{a^2}+\frac{y\sinh(ay)}{a}$	20
parallelrisch	$\frac{2-2y\tanh\left(\frac{ay}{2}\right)a}{a^2\left(\tanh^2\left(\frac{ay}{2}\right)-1\right)}$	27
risch	$\frac{(ay-1)e^{ay}}{2a^2}-\frac{(ay+1)e^{-ay}}{2a^2}$	32
meijerg	$-\frac{2\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(ay)}{2\sqrt{\pi}}-\frac{ya\sinh(ay)}{2\sqrt{\pi}}\right)}{a^2}$	35

```
[In] int(y*cosh(a*y),y,method=_RETURNVERBOSE)
[Out] 1/a^2*(y*a*sinh(a*y)-cosh(a*y))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int y\cosh(ay)\,dy = \frac{ay\sinh(ay) - \cosh(ay)}{a^2}$$

```
[In] integrate(y*cosh(a*y),y, algorithm="fricas")
[Out] (a*y*sinh(a*y) - cosh(a*y))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int y\cosh(ay)\,dy = \begin{cases} \frac{y\sinh(ay)}{a} - \frac{\cosh(ay)}{a^2} & \text{for } a \neq 0 \\ \frac{y^2}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(y*cosh(a*y),y)
[Out] Piecewise((y*sinh(a*y)/a - cosh(a*y)/a**2, Ne(a, 0)), (y**2/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(19) = 38.

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.00

$$\int y\cosh(ay)\,dy = \frac{1}{2}\,y^2\cosh(ay) - \frac{1}{4}\,a\left(\frac{(a^2y^2 - 2\,ay + 2)e^{(ay)}}{a^3} + \frac{(a^2y^2 + 2\,ay + 2)e^{(-ay)}}{a^3}\right)$$

```
[In] integrate(y*cosh(a*y),y, algorithm="maxima")
[Out] 1/2*y^2*cosh(a*y) - 1/4*a*((a^2*y^2 - 2*a*y + 2)*e^(a*y)/a^3 + (a^2*y^2 + 2
*a*y + 2)*e^(-a*y)/a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int y\cosh(ay)\,dy = \frac{(ay-1)e^{(ay)}}{2\,a^2} - \frac{(ay+1)e^{(-ay)}}{2\,a^2}$$

```
[In] integrate(y*cosh(a*y),y, algorithm="giac")
[Out] 1/2*(a*y - 1)*e^(a*y)/a^2 - 1/2*(a*y + 1)*e^(-a*y)/a^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int y\cosh(ay)\,dy = -\frac{\cosh(a\,y) - a\,y\sinh(a\,y)}{a^2}$$

```
[In] int(y*cosh(a*y),y)
[Out] -(cosh(a*y) - a*y*sinh(a*y))/a^2
```

3.36 $\int e^{-t}t\,dt$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int e^{-t}t\,dt = -e^{-t} - e^{-t}t$$

```
[Out] -1/exp(t)-t/exp(t)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2207, 2225}

$$\int e^{-t}t\,dt = -e^{-t}t - e^{-t}$$

```
[In] Int[t/E^t,t]
```

```
[Out] -E^(-t) - t/E^t
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -e^{-t}t + \int e^{-t}\,dt\\ &= -e^{-t} - e^{-t}t\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int e^{-t}t\,dt = e^{-t}(-1-t)$$

```
[In] Integrate[t/E^t,t]
[Out] (-1 - t)/E^t
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

method	result	size
gosper	$-(1+t)\,\mathrm{e}^{-t}$	10
norman	$(-1-t)\,\mathrm{e}^{-t}$	11
risch	$(-1-t)\,\mathrm{e}^{-t}$	11
parallelrisch	$(-1-t)\,\mathrm{e}^{-t}$	11
meijerg	$1-\frac{(2+2t)\mathrm{e}^{-t}}{2}$	14
default	$-\mathrm{e}^{-t}-t\,\mathrm{e}^{-t}$	15

```
[In] int(t/exp(t),t,method=_RETURNVERBOSE)
[Out] -(1+t)/exp(t)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-t}t\,dt = -(t+1)e^{(-t)}$$

```
[In] integrate(t/exp(t),t, algorithm="fricas")
[Out] -(t + 1)*e^(-t)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int e^{-t}t\,dt = (-t-1)\,e^{-t}$$

```
[In] integrate(t/exp(t),t)
[Out] (-t - 1)*exp(-t)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-t}t\,dt = -(t+1)e^{(-t)}$$

```
[In] integrate(t/exp(t),t, algorithm="maxima")
[Out] -(t + 1)*e^(-t)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-t}t\,dt = -(t+1)e^{(-t)}$$

```
[In] integrate(t/exp(t),t, algorithm="giac")
[Out] -(t + 1)*e^(-t)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-t}t\,dt = -\mathrm{e}^{-t}\,(t+1)$$

```
[In] int(t*exp(-t),t)
[Out] -exp(-t)*(t + 1)
```

3.37 $\int \sqrt{t} \log(t) \, dt$

Optimal result

Integrand size = 8, antiderivative size = 21

$$\int \sqrt{t} \log(t) \, dt = -\frac{4t^{3/2}}{9} + \frac{2}{3}t^{3/2} \log(t)$$

```
[Out] -4/9*t^(3/2)+2/3*t^(3/2)*ln(t)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2341}

$$\int \sqrt{t} \log(t) \, dt = \frac{2}{3}t^{3/2} \log(t) - \frac{4t^{3/2}}{9}$$

```
[In] Int[Sqrt[t]*Log[t],t]
```

```
[Out] (-4*t^(3/2))/9 + (2*t^(3/2)*Log[t])/3
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{4t^{3/2}}{9} + \frac{2}{3}t^{3/2} \log(t)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{t}\log(t)\,dt = \frac{2}{9}t^{3/2}(-2+3\log(t))$$

```
[In] Integrate[Sqrt[t]*Log[t],t]
[Out] (2*t^(3/2)*(-2 + 3*Log[t]))/9
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$-\frac{4t^{\frac{3}{2}}}{9}+\frac{2t^{\frac{3}{2}}\ln(t)}{3}$	14
default	$-\frac{4t^{\frac{3}{2}}}{9}+\frac{2t^{\frac{3}{2}}\ln(t)}{3}$	14
risch	$-\frac{4t^{\frac{3}{2}}}{9}+\frac{2t^{\frac{3}{2}}\ln(t)}{3}$	14

```
[In] int(ln(t)*t^(1/2),t,method=_RETURNVERBOSE)
[Out] -4/9*t^(3/2)+2/3*t^(3/2)*ln(t)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \sqrt{t}\log(t)\,dt = \frac{2}{9}\,(3\,t\log\left(t\right)-2\,t)\sqrt{t}$$

```
[In] integrate(log(t)*t^(1/2),t, algorithm="fricas")
[Out] 2/9*(3*t*log(t) - 2*t)*sqrt(t)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(19) = 38.

Time = 0.92 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.00

$$\int \sqrt{t}\log(t)\,dt = \begin{cases} -\frac{2t^{\frac{3}{2}}\log\left(\frac{1}{t}\right)}{3} + \frac{2t^{\frac{3}{2}}\log(t)}{3} - \frac{8t^{\frac{3}{2}}}{9} & \text{for } \frac{1}{|t|} < 1 \wedge |t| < 1 \\ \frac{2t^{\frac{3}{2}}\log(t)}{3} - \frac{4t^{\frac{3}{2}}}{9} & \text{for } |t| < 1 \\ -\frac{2t^{\frac{3}{2}}\log\left(\frac{1}{t}\right)}{3} - \frac{4t^{\frac{3}{2}}}{9} & \text{for } \frac{1}{|t|} < 1 \\ -G_{3,3}^{2,1}\left(\begin{matrix} 1 & \frac{5}{2}, \frac{5}{2} \\ \frac{3}{2}, \frac{3}{2} & 0 \end{matrix} \middle| t\right) + G_{3,3}^{0,3}\left(\begin{matrix} \frac{5}{2}, \frac{5}{2}, 1 & \\ & \frac{3}{2}, \frac{3}{2}, 0 \end{matrix} \middle| t\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(t)*t**(1/2),t)
```

```
[Out] Piecewise((-2*t**(3/2)*log(1/t)/3 + 2*t**(3/2)*log(t)/3 - 8*t**(3/2)/9, (Ab
s(t) < 1) & (1/Abs(t) < 1)), (2*t**(3/2)*log(t)/3 - 4*t**(3/2)/9, Abs(t) <
1), (-2*t**(3/2)*log(1/t)/3 - 4*t**(3/2)/9, 1/Abs(t) < 1), (-meijerg(((1,),
 (5/2, 5/2)), ((3/2, 3/2), (0,)), t) + meijerg(((5/2, 5/2, 1), ()), ((), (3
/2, 3/2, 0)), t), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{t}\log(t)\,dt = \frac{2}{3}\,t^{\frac{3}{2}}\log(t) - \frac{4}{9}\,t^{\frac{3}{2}}$$

```
[In] integrate(log(t)*t^(1/2),t, algorithm="maxima")
```

```
[Out] 2/3*t^(3/2)*log(t) - 4/9*t^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{t}\log(t)\,dt = \frac{2}{3}\,t^{\frac{3}{2}}\log(t) - \frac{4}{9}\,t^{\frac{3}{2}}$$

```
[In] integrate(log(t)*t^(1/2),t, algorithm="giac")
```

```
[Out] 2/3*t^(3/2)*log(t) - 4/9*t^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.43

$$\int \sqrt{t}\log(t)\,dt = \frac{2\,t^{3/2}\,\left(\ln\left(t\right)-\frac{2}{3}\right)}{3}$$

```
[In] int(t^(1/2)*log(t),t)
[Out] (2*t^(3/2)*(log(t) - 2/3))/3
```

3.38 $\int x\cos(2x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 18

$$\int x\cos(2x)\,dx=\frac{1}{4}\cos(2x)+\frac{1}{2}x\sin(2x)$$

```
[Out] 1/4*cos(2*x)+1/2*x*sin(2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3377, 2718}

$$\int x\cos(2x)\,dx=\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)$$

```
[In] Int[x*Cos[2*x],x]
```

```
[Out] Cos[2*x]/4 + (x*Sin[2*x])/2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x\sin(2x) - \frac{1}{2}\int \sin(2x)\,dx\\ &= \frac{1}{4}\cos(2x) + \frac{1}{2}x\sin(2x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x\cos(2x)\,dx = \frac{1}{4}\cos(2x) + \frac{1}{2}x\sin(2x)$$

```
[In] Integrate[x*Cos[2*x],x]
[Out] Cos[2*x]/4 + (x*Sin[2*x])/2
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\cos(2x)}{4}+\frac{x\sin(2x)}{2}$	15
default	$\frac{\cos(2x)}{4}+\frac{x\sin(2x)}{2}$	15
risch	$\frac{\cos(2x)}{4}+\frac{x\sin(2x)}{2}$	15
parts	$\frac{\cos(2x)}{4}+\frac{x\sin(2x)}{2}$	15
norman	$\frac{x\tan(x)+\frac{1}{2}}{1+\tan^2(x)}$	16
parallelrisch	$\frac{x\sin(2x)}{2}+\frac{\cos(2x)}{4}+\frac{1}{4}$	16
meijerg	$\frac{\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(2x)}{2\sqrt{\pi}}+\frac{x\sin(2x)}{\sqrt{\pi}}\right)}{2}$	30

```
[In] int(x*cos(2*x),x,method=_RETURNVERBOSE)
[Out] 1/4*cos(2*x)+1/2*x*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\cos(2x)\,dx = \frac{1}{2}\,x\sin\left(2\,x\right) + \frac{1}{4}\,\cos\left(2\,x\right)$$

```
[In] integrate(x*cos(2*x),x, algorithm="fricas")
[Out] 1/2*x*sin(2*x) + 1/4*cos(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\cos(2x)\,dx = \frac{x\sin\left(2x\right)}{2} + \frac{\cos\left(2x\right)}{4}$$

```
[In] integrate(x*cos(2*x),x)
[Out] x*sin(2*x)/2 + cos(2*x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\cos(2x)\,dx = \frac{1}{2}\,x\sin\left(2\,x\right) + \frac{1}{4}\,\cos\left(2\,x\right)$$

```
[In] integrate(x*cos(2*x),x, algorithm="maxima")
[Out] 1/2*x*sin(2*x) + 1/4*cos(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\cos(2x)\,dx = \frac{1}{2}\,x\sin\left(2\,x\right) + \frac{1}{4}\,\cos\left(2\,x\right)$$

```
[In] integrate(x*cos(2*x),x, algorithm="giac")
[Out] 1/2*x*sin(2*x) + 1/4*cos(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\cos(2x)\,dx=\frac{\cos\left(2\,x\right)}{4}+\frac{x\,\sin\left(2\,x\right)}{2}$$

```
[In] int(x*cos(2*x),x)
[Out] cos(2*x)/4 + (x*sin(2*x))/2
```

3.39 $\int e^{-x}x^2\,dx$

Optimal result

Integrand size = 9, antiderivative size = 26

$$\int e^{-x}x^2\,dx=-2e^{-x}-2e^{-x}x-e^{-x}x^2$$

```
[Out] -2/exp(x)-2*x/exp(x)-x^2/exp(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2207, 2225}

$$\int e^{-x}x^2\,dx=-e^{-x}x^2-2e^{-x}x-2e^{-x}$$

```
[In] Int[x^2/E^x,x]
```

```
[Out] -2/E^x - (2*x)/E^x - x^2/E^x
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -e^{-x}x^2 + 2\int e^{-x}x\,dx \\
&= -2e^{-x}x - e^{-x}x^2 + 2\int e^{-x}\,dx \\
&= -2e^{-x} - 2e^{-x}x - e^{-x}x^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int e^{-x}x^2\,dx = e^{-x}\left(-2-2x-x^2\right)$$

```
[In] Integrate[x^2/E^x,x]
[Out] (-2 - 2*x - x^2)/E^x
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\left(x^2+2x+2\right)\mathrm{e}^{-x}$	15
norman	$\left(-x^2-2x-2\right)\mathrm{e}^{-x}$	16
risch	$\left(-x^2-2x-2\right)\mathrm{e}^{-x}$	16
parallelrisch	$\left(-x^2-2x-2\right)\mathrm{e}^{-x}$	16
meijerg	$2-\frac{\left(3x^2+6x+6\right)\mathrm{e}^{-x}}{3}$	19
default	$-2\,\mathrm{e}^{-x}-2x\,\mathrm{e}^{-x}-x^2\mathrm{e}^{-x}$	24

```
[In] int(x^2/exp(x),x,method=_RETURNVERBOSE)
[Out] -(x^2+2*x+2)/exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int e^{-x} x^2 \, dx = -\left(x^2 + 2\,x + 2\right)e^{(-x)}$$

```
[In] integrate(x^2/exp(x),x, algorithm="fricas")
[Out] -(x^2 + 2*x + 2)*e^(-x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.46

$$\int e^{-x} x^2 \, dx = \left(-x^2 - 2x - 2\right) e^{-x}$$

```
[In] integrate(x**2/exp(x),x)
[Out] (-x**2 - 2*x - 2)*exp(-x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int e^{-x} x^2 \, dx = -\left(x^2 + 2\,x + 2\right)e^{(-x)}$$

```
[In] integrate(x^2/exp(x),x, algorithm="maxima")
[Out] -(x^2 + 2*x + 2)*e^(-x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int e^{-x} x^2 \, dx = -\left(x^2 + 2\,x + 2\right)e^{(-x)}$$

```
[In] integrate(x^2/exp(x),x, algorithm="giac")
[Out] -(x^2 + 2*x + 2)*e^(-x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int e^{-x}x^2\,dx = -\mathrm{e}^{-x}\,\left(x^2+2\,x+2\right)$$

```
[In] int(x^2*exp(-x),x)
[Out] -exp(-x)*(2*x + x^2 + 2)
```

3.40 $\int \arccos(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 18

$$\int \arccos(x)\,dx = -\sqrt{1-x^2} + x\arccos(x)$$

```
[Out] x*arccos(x)-(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4716, 267}

$$\int \arccos(x)\,dx = x\arccos(x) - \sqrt{1-x^2}$$

```
[In] Int[ArcCos[x],x]
```

```
[Out] -Sqrt[1 - x^2] + x*ArcCos[x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\text{integral} = x\arccos(x) + \int \frac{x}{\sqrt{1-x^2}}\,dx$$
$$= -\sqrt{1-x^2} + x\arccos(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \arccos(x)\,dx = -\sqrt{1-x^2} + x\arccos(x)$$

```
[In] Integrate[ArcCos[x],x]
[Out] -Sqrt[1 - x^2] + x*ArcCos[x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
lookup	$x\arccos\left(x\right)-\sqrt{-x^2+1}$	17
default	$x\arccos\left(x\right)-\sqrt{-x^2+1}$	17
parts	$x\arccos\left(x\right)-\sqrt{-x^2+1}$	17

```
[In] int(arccos(x),x,method=_RETURNVERBOSE)
[Out] x*arccos(x)-(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \arccos(x)\,dx = x\arccos\left(x\right) - \sqrt{-x^2+1}$$

```
[In] integrate(arccos(x),x, algorithm="fricas")
[Out] x*arccos(x) - sqrt(-x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \arccos(x)\,dx = x\,\mathrm{acos}\,(x) - \sqrt{1-x^2}$$

```
[In] integrate(acos(x),x)
[Out] x*acos(x) - sqrt(1 - x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \arccos(x)\,dx = x\arccos\,(x) - \sqrt{-x^2+1}$$

```
[In] integrate(arccos(x),x, algorithm="maxima")
[Out] x*arccos(x) - sqrt(-x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \arccos(x)\,dx = x\arccos\,(x) - \sqrt{-x^2+1}$$

```
[In] integrate(arccos(x),x, algorithm="giac")
[Out] x*arccos(x) - sqrt(-x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \arccos(x)\,dx = x\,\mathrm{acos}(x) - \sqrt{1-x^2}$$

```
[In] int(acos(x),x)
[Out] x*acos(x) - (1 - x^2)^(1/2)
```

3.41 $\int x \csc^2(x) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 9

$$\int x \csc^2(x) \, dx = -x \cot(x) + \log(\sin(x))$$

```
[Out] -x*cot(x)+ln(sin(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4269, 3556}

$$\int x \csc^2(x) \, dx = \log(\sin(x)) - x \cot(x)$$

```
[In] Int[x*Csc[x]^2,x]
```

```
[Out] -(x*Cot[x]) + Log[Sin[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -x\cot(x) + \int \cot(x)\,dx \\ &= -x\cot(x) + \log(\sin(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x\csc^2(x)\,dx = -x\cot(x) + \log(\sin(x))$$

```
[In] Integrate[x*Csc[x]^2,x]
[Out] -(x*Cot[x]) + Log[Sin[x]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$-x\cot\left(x\right)+\ln\left(\sin\left(x\right)\right)$	10
parallelrisch	$-\ln\left(\frac{2}{\cos(x)+1}\right)+\ln\left(\csc\left(x\right)-\cot\left(x\right)\right)-x\cot\left(x\right)$	26
risch	$-2ix-\frac{2ix}{e^{2ix}-1}+\ln\left(e^{2ix}-1\right)$	27
norman	$\frac{-\frac{x}{2}+\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}}{\tan\left(\frac{x}{2}\right)}-\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	38

```
[In] int(x*csc(x)^2,x,method=_RETURNVERBOSE)
[Out] -x*cot(x)+ln(sin(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int x\csc^2(x)\,dx = -\frac{x\cos\left(x\right)-\log\left(\frac{1}{2}\sin\left(x\right)\right)\sin\left(x\right)}{\sin\left(x\right)}$$

```
[In] integrate(x*csc(x)^2,x, algorithm="fricas")
[Out] -(x*cos(x) - log(1/2*sin(x))*sin(x))/sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int x\csc^2(x)\,dx = -x\cot(x) + \log(\sin(x))$$

```
[In] integrate(x*csc(x)**2,x)
```

```
[Out] -x*cot(x) + log(sin(x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(9) = 18.

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 11.56

$$\int x\csc^2(x)\,dx = \frac{(\cos(2x)^2 + \sin(2x)^2 - 2\cos(2x) + 1)\log(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1) + (\cos(2x)^2 + \sin(2x)^2}{2(\cos(2x)^2 + \sin(2x)^2 - 2\cos(2x) -}$$

```
[In] integrate(x*csc(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2
*cos(x) + 1) + (cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x
) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(9) = 18.

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 5.78

$$\int x\csc^2(x)\,dx = \frac{x\tan\left(\frac{1}{2}x\right)^2 + \log\left(\frac{16\tan\left(\frac{1}{2}x\right)^2}{\tan\left(\frac{1}{2}x\right)^4 + 2\tan\left(\frac{1}{2}x\right)^2 + 1}\right)\tan\left(\frac{1}{2}x\right) - x}{2\tan\left(\frac{1}{2}x\right)}$$

```
[In] integrate(x*csc(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x*tan(1/2*x)^2 + log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 +
1))*tan(1/2*x) - x)/tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x \csc^2(x)\, dx = \ln(\sin(x)) - x\cot(x)$$

```
[In] int(x/sin(x)^2,x)
[Out] log(sin(x)) - x*cot(x)
```

3.42 $\int \cos(5x)\sin(3x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \cos(5x)\sin(3x)\,dx = \frac{1}{4}\cos(2x) - \frac{1}{16}\cos(8x)$$

```
[Out] 1/4*cos(2*x)-1/16*cos(8*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{4369\}$

$$\int \cos(5x)\sin(3x)\,dx = \frac{1}{4}\cos(2x) - \frac{1}{16}\cos(8x)$$

```
[In] Int[Cos[5*x]*Sin[3*x],x]
```

```
[Out] Cos[2*x]/4 - Cos[8*x]/16
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\cos(2x) - \frac{1}{16}\cos(8x)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(5x)\sin(3x)\,dx = \frac{\cos^2(x)}{2} - \frac{1}{16}\cos(8x)$$

```
[In] Integrate[Cos[5*x]*Sin[3*x],x]
[Out] Cos[x]^2/2 - Cos[8*x]/16
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\cos(2x)}{4}-\frac{\cos(8x)}{16}$	14
risch	$\frac{\cos(2x)}{4}-\frac{\cos(8x)}{16}$	14
parallelrisch	$-\frac{3}{16}+\frac{(2-\cos(6x))\cos(2x)}{8}+\frac{\cos(4x)}{16}$	23
norman	$\frac{-\frac{3\left(\tan^2\left(\frac{3x}{2}\right)\right)}{8}-\frac{3\left(\tan^2\left(\frac{5x}{2}\right)\right)}{8}+\frac{5\tan\left(\frac{3x}{2}\right)\tan\left(\frac{5x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{5x}{2}\right)\right)\left(1+\tan^2\left(\frac{3x}{2}\right)\right)}$	49

```
[In] int(cos(5*x)*sin(3*x),x,method=_RETURNVERBOSE)
[Out] 1/4*cos(2*x)-1/16*cos(8*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \cos(5x)\sin(3x)\,dx = -8\cos(x)^8 + 16\cos(x)^6 - 10\cos(x)^4 + \frac{5}{2}\cos(x)^2$$

```
[In] integrate(cos(5*x)*sin(3*x),x, algorithm="fricas")
[Out] -8*cos(x)^8 + 16*cos(x)^6 - 10*cos(x)^4 + 5/2*cos(x)^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \cos(5x)\sin(3x)\,dx = \frac{5\sin(3x)\sin(5x)}{16} + \frac{3\cos(3x)\cos(5x)}{16}$$

```
[In] integrate(cos(5*x)*sin(3*x),x)
[Out] 5*sin(3*x)*sin(5*x)/16 + 3*cos(3*x)*cos(5*x)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(5x)\sin(3x)\,dx = -\frac{1}{16}\cos(8x) + \frac{1}{4}\cos(2x)$$

```
[In] integrate(cos(5*x)*sin(3*x),x, algorithm="maxima")
[Out] -1/16*cos(8*x) + 1/4*cos(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(5x)\sin(3x)\,dx = -\frac{1}{16}\cos(8x) + \frac{1}{4}\cos(2x)$$

```
[In] integrate(cos(5*x)*sin(3*x),x, algorithm="giac")
[Out] -1/16*cos(8*x) + 1/4*cos(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(5x)\sin(3x)\,dx = \frac{\cos(2x)}{4} - \frac{\cos(8x)}{16}$$

```
[In] int(cos(5*x)*sin(3*x),x)
[Out] cos(2*x)/4 - cos(8*x)/16
```

3.43 $\int \sin(2x)\sin(4x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \sin(2x)\sin(4x)\,dx = \frac{1}{4}\sin(2x) - \frac{1}{12}\sin(6x)$$

```
[Out] 1/4*sin(2*x)-1/12*sin(6*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4367}

$$\int \sin(2x)\sin(4x)\,dx = \frac{1}{4}\sin(2x) - \frac{1}{12}\sin(6x)$$

```
[In] Int[Sin[2*x]*Sin[4*x],x]
```

```
[Out] Sin[2*x]/4 - Sin[6*x]/12
```

Rule 4367

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\sin(2x) - \frac{1}{12}\sin(6x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sin(2x)\sin(4x)\,dx = \frac{1}{4}\sin(2x) - \frac{1}{12}\sin(6x)$$

```
[In] Integrate[Sin[2*x]*Sin[4*x],x]
[Out] Sin[2*x]/4 - Sin[6*x]/12
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

method	result	size
derivativedivides	$\frac{\left(\sin^{3}(2x)\right)}{3}$	9
default	$\frac{\left(\sin^{3}(2x)\right)}{3}$	9
risch	$\frac{\sin(2x)}{4} - \frac{\sin(6x)}{12}$	14
parallelrisch	$\frac{\sin(2x)}{4} - \frac{\sin(6x)}{12}$	14
norman	$\frac{\frac{2\tan(x)\left(\tan^{2}(2x)\right)}{3} - \frac{\left(\tan^{2}(x)\right)\tan(2x)}{3} - \frac{2\tan(x)}{3} + \frac{\tan(2x)}{3}}{(1+\tan^{2}(x))(1+\tan^{2}(2x))}$	51

```
[In] int(sin(2*x)*sin(4*x),x,method=_RETURNVERBOSE)
[Out] 1/3*sin(2*x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sin(2x)\sin(4x)\,dx = -\frac{1}{3}\left(\cos\left(2\,x\right)^{2} - 1\right)\sin\left(2\,x\right)$$

```
[In] integrate(sin(2*x)*sin(4*x),x, algorithm="fricas")
[Out] -1/3*(cos(2*x)^2 - 1)*sin(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \sin(2x)\sin(4x)\,dx = -\frac{\sin(2x)\cos(4x)}{3} + \frac{\sin(4x)\cos(2x)}{6}$$

```
[In] integrate(sin(2*x)*sin(4*x),x)
[Out] -sin(2*x)*cos(4*x)/3 + sin(4*x)*cos(2*x)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(2x)\sin(4x)\,dx = -\frac{1}{12}\sin(6x) + \frac{1}{4}\sin(2x)$$

```
[In] integrate(sin(2*x)*sin(4*x),x, algorithm="maxima")
[Out] -1/12*sin(6*x) + 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.47

$$\int \sin(2x)\sin(4x)\,dx = \frac{1}{3}\sin(2x)^3$$

```
[In] integrate(sin(2*x)*sin(4*x),x, algorithm="giac")
[Out] 1/3*sin(2*x)^3
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(2x)\sin(4x)\,dx = \frac{\sin(2x)}{4} - \frac{\sin(6x)}{12}$$

```
[In] int(sin(2*x)*sin(4*x),x)
[Out] sin(2*x)/4 - sin(6*x)/12
```

3.44 $\int \cos(x)\log(\sin(x))\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \cos(x)\log(\sin(x))\,dx = -\sin(x) + \log(\sin(x))\sin(x)$$

```
[Out] -sin(x)+ln(sin(x))*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2717, 2634}

$$\int \cos(x)\log(\sin(x))\,dx = \sin(x)\log(\sin(x)) - \sin(x)$$

```
[In] Int[Cos[x]*Log[Sin[x]],x]
```

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \log(\sin(x))\sin(x) - \int \cos(x)\,dx$$
$$= -\sin(x) + \log(\sin(x))\sin(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos(x)\log(\sin(x))\,dx = -\sin(x) + \log(\sin(x))\sin(x)$$

```
[In] Integrate[Cos[x]*Log[Sin[x]],x]
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result
parallelrisch	$(\ln(\sin(x))-1)\sin(x)$
derivativedivides	$-\sin(x)+\ln(\sin(x))\sin(x)$
default	$-\sin(x)+\ln(\sin(x))\sin(x)$
norman	$\frac{2\tan\left(\frac{x}{2}\right)\ln\left(\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)-2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$
risch	$-\ln\left(e^{ix}\right)\sin(x)-\frac{e^{ix}\pi}{4}+\frac{e^{-ix}\pi}{4}-\frac{e^{-ix}\operatorname{csgn}(\sin(x))^2\operatorname{csgn}\left(ie^{-ix}\right)\pi}{4}-\frac{e^{-ix}\operatorname{csgn}(i\sin(x))\operatorname{csgn}(\sin(x))\pi}{4}-$

```
[In] int(cos(x)*ln(sin(x)),x,method=_RETURNVERBOSE)
[Out] (ln(sin(x))-1)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos(x)\log(\sin(x))\,dx = \log\left(\sin\left(x\right)\right)\sin\left(x\right) - \sin\left(x\right)$$

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="fricas")
[Out] log(sin(x))*sin(x) - sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \cos(x)\log(\sin(x))\,dx = \log\left(\sin\left(x\right)\right)\sin\left(x\right) - \sin\left(x\right)$$

```
[In] integrate(cos(x)*ln(sin(x)),x)
[Out] log(sin(x))*sin(x) - sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos(x)\log(\sin(x))\,dx = \log\left(\sin\left(x\right)\right)\sin\left(x\right) - \sin\left(x\right)$$

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="maxima")
[Out] log(sin(x))*sin(x) - sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos(x)\log(\sin(x))\,dx = \log\left(\sin\left(x\right)\right)\sin\left(x\right) - \sin\left(x\right)$$

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="giac")
[Out] log(sin(x))*sin(x) - sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \cos(x)\log(\sin(x))\,dx = \sin(x)\ (\ln(\sin(x)) - 1)$$

```
[In] int(log(sin(x))*cos(x),x)
[Out] sin(x)*(log(sin(x)) - 1)
```

3.45 $\int e^{x^2} x^3 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 22

$$\int e^{x^2} x^3 \, dx = -\frac{e^{x^2}}{2} + \frac{1}{2} e^{x^2} x^2$$

```
[Out] -1/2*exp(x^2)+1/2*exp(x^2)*x^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2243, 2240}

$$\int e^{x^2} x^3 \, dx = \frac{1}{2} e^{x^2} x^2 - \frac{e^{x^2}}{2}$$

```
[In] Int[E^x^2*x^3,x]
```

```
[Out] -1/2*E^x^2 + (E^x^2*x^2)/2
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
```

```
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}e^{x^2}x^2 - \int e^{x^2}x\,dx \\ &= -\frac{e^{x^2}}{2} + \frac{1}{2}e^{x^2}x^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int e^{x^2}x^3\,dx = \frac{1}{2}e^{x^2}\left(-1+x^2\right)$$

```
[In] Integrate[E^x^2*x^3,x]
[Out] (E^x^2*(-1 + x^2))/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{(x^2-1)\,e^{x^2}}{2}$	12
risch	$\left(\frac{x^2}{2}-\frac{1}{2}\right)e^{x^2}$	13
meijerg	$\frac{1}{2}-\frac{(-2x^2+2)\,e^{x^2}}{4}$	16
derivativedivides	$-\frac{e^{x^2}}{2}+\frac{e^{x^2}x^2}{2}$	17
default	$-\frac{e^{x^2}}{2}+\frac{e^{x^2}x^2}{2}$	17
norman	$-\frac{e^{x^2}}{2}+\frac{e^{x^2}x^2}{2}$	17
parallelrisch	$-\frac{e^{x^2}}{2}+\frac{e^{x^2}x^2}{2}$	17
parts	$\frac{\operatorname{erfi}(x)\sqrt{\pi}\,x^3}{2}-\frac{3\sqrt{\pi}\left(\frac{x^3\operatorname{erfi}(x)}{3}-\frac{2\left(-\frac{e^{x^2}}{2}+\frac{e^{x^2}x^2}{2}\right)}{3\sqrt{\pi}}\right)}{2}$	46

```
[In] int(x^3*exp(x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*(x^2-1)*exp(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int e^{x^2} x^3 \, dx = \frac{1}{2} \left(x^2 - 1\right) e^{\left(x^2\right)}$$

```
[In] integrate(exp(x^2)*x^3,x, algorithm="fricas")
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.45

$$\int e^{x^2} x^3 \, dx = \frac{\left(x^2 - 1\right) e^{x^2}}{2}$$

```
[In] integrate(exp(x**2)*x**3,x)
[Out] (x**2 - 1)*exp(x**2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int e^{x^2} x^3 \, dx = \frac{1}{2} \left(x^2 - 1\right) e^{\left(x^2\right)}$$

```
[In] integrate(exp(x^2)*x^3,x, algorithm="maxima")
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int e^{x^2} x^3 \, dx = \frac{1}{2} \left(x^2 - 1\right) e^{\left(x^2\right)}$$

```
[In] integrate(exp(x^2)*x^3,x, algorithm="giac")
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int e^{x^2} x^3 \, dx = \frac{e^{x^2} \left(x^2 - 1\right)}{2}$$

```
[In] int(x^3*exp(x^2),x)
[Out] (exp(x^2)*(x^2 - 1))/2
```

3.46 $\int e^x(3+2x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 15

$$\int e^x(3+2x)\,dx=-2e^x+e^x(3+2x)$$

```
[Out] -2*exp(x)+exp(x)*(3+2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {2207, 2225}

$$\int e^x(3+2x)\,dx=e^x(2x+3)-2e^x$$

```
[In] Int[E^x*(3 + 2*x),x]
```

```
[Out] -2*E^x + E^x*(3 + 2*x)
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\text{integral} = e^x(3+2x) - 2\int e^x\,dx$$
$$= -2e^x + e^x(3+2x)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int e^x(3+2x)\,dx = e^x(1+2x)$$

```
[In] Integrate[E^x*(3 + 2*x),x]
[Out] E^x*(1 + 2*x)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

method	result	size
gosper	$(1+2x)\,e^x$	9
default	$e^x + 2\,e^x x$	9
norman	$e^x + 2\,e^x x$	9
risch	$(1+2x)\,e^x$	9
parallelrisch	$e^x + 2\,e^x x$	9
parts	$e^x + 2\,e^x x$	9
meijerg	$-1 + 3\,e^x - (-2x+2)\,e^x$	16

```
[In] int(exp(x)*(3+2*x),x,method=_RETURNVERBOSE)
[Out] (1+2*x)*exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.53

$$\int e^x(3+2x)\,dx = (2\,x+1)e^x$$

```
[In] integrate(exp(x)*(3+2*x),x, algorithm="fricas")
[Out] (2*x + 1)*e^x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.47

$$\int e^x(3+2x)\,dx = (2x+1)\,e^x$$

```
[In] integrate(exp(x)*(3+2*x),x)
[Out] (2*x + 1)*exp(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int e^x(3+2x)\,dx = 2\,(x-1)e^x + 3\,e^x$$

```
[In] integrate(exp(x)*(3+2*x),x, algorithm="maxima")
[Out] 2*(x - 1)*e^x + 3*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.53

$$\int e^x(3+2x)\,dx = (2\,x+1)e^x$$

```
[In] integrate(exp(x)*(3+2*x),x, algorithm="giac")
[Out] (2*x + 1)*e^x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.53

$$\int e^x(3+2x)\,dx = \mathrm{e}^x\,(2\,x+1)$$

```
[In] int(exp(x)*(2*x + 3),x)
[Out] exp(x)*(2*x + 1)
```

3.47 $\int 5^x x \, dx$

Optimal result

Integrand size = 5, antiderivative size = 19

$$\int 5^x x \, dx = -\frac{5^x}{\log^2(5)} + \frac{5^x x}{\log(5)}$$

```
[Out] -5^x/ln(5)^2+5^x*x/ln(5)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2207, 2225}

$$\int 5^x x \, dx = \frac{5^x x}{\log(5)} - \frac{5^x}{\log^2(5)}$$

```
[In] Int[5^x*x,x]
```

```
[Out] -(5^x/Log[5]^2) + (5^x*x)/Log[5]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{5^x x}{\log(5)} - \frac{\int 5^x \, dx}{\log(5)} \\ &= -\frac{5^x}{\log^2(5)} + \frac{5^x x}{\log(5)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int 5^x x \, dx = \frac{5^x(-1 + x\log(5))}{\log^2(5)}$$

```
[In] Integrate[5^x*x,x]
[Out] (5^x*(-1 + x*Log[5]))/Log[5]^2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{(x\ln(5)-1)5^x}{\ln(5)^2}$	15
risch	$\frac{(x\ln(5)-1)5^x}{\ln(5)^2}$	15
parallelrisch	$\frac{5^x\ln(5)x-5^x}{\ln(5)^2}$	19
meijerg	$\frac{1-\frac{(2-2x\ln(5))e^{x\ln(5)}}{2}}{\ln(5)^2}$	22
norman	$\frac{x\,e^{x\ln(5)}}{\ln(5)} - \frac{e^{x\ln(5)}}{\ln(5)^2}$	24

```
[In] int(5^x*x,x,method=_RETURNVERBOSE)
[Out] (x*ln(5)-1)*5^x/ln(5)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int 5^x x\,dx = \frac{(x\log(5) - 1)5^x}{\log(5)^2}$$

```
[In] integrate(5^x*x,x, algorithm="fricas")
[Out] (x*log(5) - 1)*5^x/log(5)^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int 5^x x\,dx = \frac{5^x(x\log(5) - 1)}{\log(5)^2}$$

```
[In] integrate(5**x*x,x)
[Out] 5**x*(x*log(5) - 1)/log(5)**2
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int 5^x x\,dx = \frac{(x\log(5) - 1)5^x}{\log(5)^2}$$

```
[In] integrate(5^x*x,x, algorithm="maxima")
[Out] (x*log(5) - 1)*5^x/log(5)^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int 5^x x\,dx = \frac{(x\log(5) - 1)5^x}{\log(5)^2}$$

```
[In] integrate(5^x*x,x, algorithm="giac")
[Out] (x*log(5) - 1)*5^x/log(5)^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int 5^x x\,dx = \frac{5^x\,(x\,\ln(5)-1)}{\ln(5)^2}$$

```
[In] int(5^x*x,x)
[Out] (5^x*(x*log(5) - 1))/log(5)^2
```

3.48 $\int \cos(\log(x)) \, dx$

Optimal result

Integrand size = 3, antiderivative size = 17

$$\int \cos(\log(x)) \, dx = \frac{1}{2} x \cos(\log(x)) + \frac{1}{2} x \sin(\log(x))$$

```
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4564}

$$\int \cos(\log(x)) \, dx = \frac{1}{2} x \sin(\log(x)) + \frac{1}{2} x \cos(\log(x))$$

```
[In] Int[Cos[Log[x]],x]
```

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rule 4564

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2} x \cos(\log(x)) + \frac{1}{2} x \sin(\log(x))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(\log(x))\,dx = \frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

```
[In] Integrate[Cos[Log[x]],x]
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{x(\cos(\ln(x))+\sin(\ln(x)))}{2}$	11
lookup	$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
default	$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
risch	$\left(\frac{1}{4}-\frac{i}{4}\right)x\,x^{i}+\left(\frac{1}{4}+\frac{i}{4}\right)x\,x^{-i}$	22

```
[In] int(cos(ln(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(cos(ln(x))+sin(ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x\cos\left(\log\left(x\right)\right) + \frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

```
[In] integrate(cos(log(x)),x, algorithm="fricas")
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cos(\log(x))\,dx = \frac{x\sin(\log(x))}{2} + \frac{x\cos(\log(x))}{2}$$

```
[In] integrate(cos(ln(x)),x)
[Out] x*sin(log(x))/2 + x*cos(log(x))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x(\cos(\log(x)) + \sin(\log(x)))$$

```
[In] integrate(cos(log(x)),x, algorithm="maxima")
[Out] 1/2*x*(cos(log(x)) + sin(log(x)))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x\cos(\log(x)) + \frac{1}{2}\,x\sin(\log(x))$$

```
[In] integrate(cos(log(x)),x, algorithm="giac")
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{\sqrt{2}\,x\,\sin\left(\frac{\pi}{4} + \ln(x)\right)}{2}$$

```
[In] int(cos(log(x)),x)
[Out] (2^(1/2)*x*sin(pi/4 + log(x)))/2
```

3.49 $\int e^{\sqrt{x}}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 24

$$\int e^{\sqrt{x}}\,dx = -2e^{\sqrt{x}} + 2e^{\sqrt{x}}\sqrt{x}$$

```
[Out] -2*exp(x^(1/2))+2*exp(x^(1/2))*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2238, 2207, 2225}

$$\int e^{\sqrt{x}}\,dx = 2e^{\sqrt{x}}\sqrt{x} - 2e^{\sqrt{x}}$$

```
[In] Int[E^Sqrt[x],x]
```

```
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2238

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Dist[k/d, Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (c
 + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Inte
gerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int e^x x\,dx, x, \sqrt{x}\right) \\
&= 2e^{\sqrt{x}}\sqrt{x} - 2\text{Subst}\left(\int e^x\,dx, x, \sqrt{x}\right) \\
&= -2e^{\sqrt{x}} + 2e^{\sqrt{x}}\sqrt{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int e^{\sqrt{x}}\,dx = 2e^{\sqrt{x}}(-1+\sqrt{x})$$

```
[In] Integrate[E^Sqrt[x],x]
[Out] 2*E^Sqrt[x]*(-1 + Sqrt[x])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
meijerg	$2-\left(-2\sqrt{x}+2\right)\mathrm{e}^{\sqrt{x}}$	16
derivativedivides	$-2\,\mathrm{e}^{\sqrt{x}}+2\,\mathrm{e}^{\sqrt{x}}\sqrt{x}$	17
default	$-2\,\mathrm{e}^{\sqrt{x}}+2\,\mathrm{e}^{\sqrt{x}}\sqrt{x}$	17

```
[In] int(exp(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2-(-2*x^(1/2)+2)*exp(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int e^{\sqrt{x}}\,dx = 2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

```
[In] integrate(exp(x^(1/2)),x, algorithm="fricas")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int e^{\sqrt{x}}\,dx = 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}}$$

```
[In] integrate(exp(x**(1/2)),x)
[Out] 2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int e^{\sqrt{x}}\,dx = 2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

```
[In] integrate(exp(x^(1/2)),x, algorithm="maxima")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int e^{\sqrt{x}}\,dx = 2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

```
[In] integrate(exp(x^(1/2)),x, algorithm="giac")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int e^{\sqrt{x}}\,dx = 2\,\mathrm{e}^{\sqrt{x}}\,\left(\sqrt{x}-1\right)$$

```
[In] int(exp(x^(1/2)),x)
[Out] 2*exp(x^(1/2))*(x^(1/2) - 1)
```

3.50 $\int \log\left(\sqrt{x}\right)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 14

$$\int \log\left(\sqrt{x}\right)\,dx = -\frac{x}{2} + x\log\left(\sqrt{x}\right)$$

```
[Out] -1/2*x+1/2*x*ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2332}

$$\int \log\left(\sqrt{x}\right)\,dx = x\log\left(\sqrt{x}\right) - \frac{x}{2}$$

```
[In] Int[Log[Sqrt[x]],x]
```

```
[Out] -1/2*x + x*Log[Sqrt[x]]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\text{integral} = -\frac{x}{2} + x\log\left(\sqrt{x}\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \log\left(\sqrt{x}\right)\,dx = \frac{1}{2}(-x + x\log(x))$$

```
[In] Integrate[Log[Sqrt[x]],x]
[Out] (-x + x*Log[x])/2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

method	result	size
lookup	$-\frac{x}{2}+\frac{x\ln(x)}{2}$	10
default	$-\frac{x}{2}+\frac{x\ln(x)}{2}$	10
norman	$-\frac{x}{2}+\frac{x\ln(x)}{2}$	10
risch	$-\frac{x}{2}+\frac{x\ln(x)}{2}$	10
parallelrisch	$-\frac{x}{2}+\frac{x\ln(x)}{2}$	10
parts	$-\frac{x}{2}+\frac{x\ln(x)}{2}$	10

```
[In] int(1/2*ln(x),x,method=_RETURNVERBOSE)
[Out] -1/2*x+1/2*x*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int \log\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x\log\left(x\right) - \frac{1}{2}\,x$$

```
[In] integrate(1/2*log(x),x, algorithm="fricas")
[Out] 1/2*x*log(x) - 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \log\left(\sqrt{x}\right)\, dx = \frac{x\log\left(x\right)}{2} - \frac{x}{2}$$

```
[In] integrate(1/2*ln(x),x)
[Out] x*log(x)/2 - x/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int \log\left(\sqrt{x}\right)\, dx = \frac{1}{2}\, x\log\left(x\right) - \frac{1}{2}\, x$$

```
[In] integrate(1/2*log(x),x, algorithm="maxima")
[Out] 1/2*x*log(x) - 1/2*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int \log\left(\sqrt{x}\right)\, dx = \frac{1}{2}\, x\log\left(x\right) - \frac{1}{2}\, x$$

```
[In] integrate(1/2*log(x),x, algorithm="giac")
[Out] 1/2*x*log(x) - 1/2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \log\left(\sqrt{x}\right)\, dx = \frac{x\left(\ln\left(x\right) - 1\right)}{2}$$

```
[In] int(log(x)/2,x)
[Out] (x*(log(x) - 1))/2
```

3.51 $\int \sin(\log(x))\,dx$

Optimal result

Integrand size = 3, antiderivative size = 17

$$\int \sin(\log(x))\,dx = -\frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

```
[Out] -1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4563}

$$\int \sin(\log(x))\,dx = \frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))$$

```
[In] Int[Sin[Log[x]],x]
```

```
[Out] -1/2*(x*Cos[Log[x]]) + (x*Sin[Log[x]])/2
```

Rule 4563

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] - Simp[b*d*n*x*(Cos[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sin(\log(x))\,dx = -\frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

```
[In] Integrate[Sin[Log[x]],x]
[Out] -1/2*(x*Cos[Log[x]]) + (x*Sin[Log[x]])/2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
parallelrisch	$-\frac{x(\cos(\ln(x))-\sin(\ln(x)))}{2}$	13
lookup	$-\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
default	$-\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
risch	$\left(-\frac{1}{4}-\frac{i}{4}\right)x\,x^{i}+\left(-\frac{1}{4}+\frac{i}{4}\right)x\,x^{-i}$	22
norman	$\frac{x\tan\left(\frac{\ln(x)}{2}\right)-\frac{x}{2}+\frac{x\left(\tan^{2}\left(\frac{\ln(x)}{2}\right)\right)}{2}}{1+\tan^{2}\left(\frac{\ln(x)}{2}\right)}$	34

```
[In] int(sin(ln(x)),x,method=_RETURNVERBOSE)
[Out] -1/2*x*(cos(ln(x))-sin(ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(\log(x))\,dx = -\frac{1}{2}\,x\cos\left(\log\left(x\right)\right) + \frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

```
[In] integrate(sin(log(x)),x, algorithm="fricas")
[Out] -1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sin(\log(x))\,dx = \frac{x\sin(\log(x))}{2} - \frac{x\cos(\log(x))}{2}$$

```
[In] integrate(sin(ln(x)),x)
[Out] x*sin(log(x))/2 - x*cos(log(x))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \sin(\log(x))\,dx = -\frac{1}{2}\,x(\cos(\log(x)) - \sin(\log(x)))$$

```
[In] integrate(sin(log(x)),x, algorithm="maxima")
[Out] -1/2*x*(cos(log(x)) - sin(log(x)))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(\log(x))\,dx = -\frac{1}{2}\,x\cos(\log(x)) + \frac{1}{2}\,x\sin(\log(x))$$

```
[In] integrate(sin(log(x)),x, algorithm="giac")
[Out] -1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(\log(x))\,dx = -\frac{\sqrt{2}\,x\,\cos\left(\frac{\pi}{4} + \ln(x)\right)}{2}$$

```
[In] int(sin(log(x)),x)
[Out] -(2^(1/2)*x*cos(pi/4 + log(x)))/2
```

3.52 $\int \sin\left(\sqrt{x}\right)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

```
[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3442, 3377, 2717}

$$\int \sin\left(\sqrt{x}\right)\,dx = 2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

```
[In] Int[Sin[Sqrt[x]],x]
```

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int x\sin(x)\,dx, x, \sqrt{x}\right)\\ &= -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\text{Subst}\left(\int \cos(x)\,dx, x, \sqrt{x}\right)\\ &= -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

```
[In] Integrate[Sin[Sqrt[x]],x]
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$2\sin\left(\sqrt{x}\right) - 2\cos\left(\sqrt{x}\right)\sqrt{x}$	17
default	$2\sin\left(\sqrt{x}\right) - 2\cos\left(\sqrt{x}\right)\sqrt{x}$	17
meijerg	$4\sqrt{\pi}\left(-\frac{\sqrt{x}\cos\left(\sqrt{x}\right)}{2\sqrt{\pi}} + \frac{\sin\left(\sqrt{x}\right)}{2\sqrt{\pi}}\right)$	28

```
[In] int(sin(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin \left(\sqrt{x}\right) \, dx = -2 \, \sqrt{x} \cos \left(\sqrt{x}\right) + 2 \, \sin \left(\sqrt{x}\right)$$

```
[In] integrate(sin(x^(1/2)),x, algorithm="fricas")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sin \left(\sqrt{x}\right) \, dx = -2\sqrt{x} \cos \left(\sqrt{x}\right) + 2 \sin \left(\sqrt{x}\right)$$

```
[In] integrate(sin(x**(1/2)),x)
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin \left(\sqrt{x}\right) \, dx = -2 \, \sqrt{x} \cos \left(\sqrt{x}\right) + 2 \, \sin \left(\sqrt{x}\right)$$

```
[In] integrate(sin(x^(1/2)),x, algorithm="maxima")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin \left(\sqrt{x}\right) \, dx = -2 \, \sqrt{x} \cos \left(\sqrt{x}\right) + 2 \, \sin \left(\sqrt{x}\right)$$

```
[In] integrate(sin(x^(1/2)),x, algorithm="giac")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin \left(\sqrt{x}\right) \, dx = 2 \sin \left(\sqrt{x}\right) - 2 \sqrt{x} \cos \left(\sqrt{x}\right)$$

```
[In] int(sin(x^(1/2)),x)
[Out] 2*sin(x^(1/2)) - 2*x^(1/2)*cos(x^(1/2))
```

3.53 $\int x^5 \cos(x^3)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 20

$$\int x^5 \cos(x^3)\, dx = \frac{\cos(x^3)}{3} + \frac{1}{3}x^3 \sin(x^3)$$

```
[Out] 1/3*cos(x^3)+1/3*x^3*sin(x^3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3461, 3377, 2718}

$$\int x^5 \cos(x^3)\, dx = \frac{1}{3}x^3 \sin(x^3) + \frac{\cos(x^3)}{3}$$

```
[In] Int[x^5*Cos[x^3],x]
```

```
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x\cos(x)\,dx, x, x^3\right)\\ &= \frac{1}{3}x^3\sin\left(x^3\right) - \frac{1}{3}\text{Subst}\left(\int \sin(x)\,dx, x, x^3\right)\\ &= \frac{\cos\left(x^3\right)}{3} + \frac{1}{3}x^3\sin\left(x^3\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^5\cos\left(x^3\right)\,dx = \frac{\cos\left(x^3\right)}{3} + \frac{1}{3}x^3\sin\left(x^3\right)$$

```
[In] Integrate[x^5*Cos[x^3],x]
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\cos\left(x^{3}\right)}{3}+\frac{x^{3} \sin\left(x^{3}\right)}{3}$	17
default	$\frac{\cos\left(x^{3}\right)}{3}+\frac{x^{3} \sin\left(x^{3}\right)}{3}$	17
risch	$\frac{\cos\left(x^{3}\right)}{3}+\frac{x^{3} \sin\left(x^{3}\right)}{3}$	17
norman	$\frac{\frac{2x^{3} \tan\left(\frac{x^{3}}{2}\right)}{3}+\frac{2}{3}}{1+\tan^{2}\left(\frac{x^{3}}{2}\right)}$	27
parallelrisch	$\frac{2x^{3} \tan\left(\frac{x^{3}}{2}\right)+2}{3\left(\tan^{2}\left(\frac{x^{3}}{2}\right)\right)+3}$	29
meijerg	$\frac{2\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos\left(x^{3}\right)}{2\sqrt{\pi}}+\frac{x^{3} \sin\left(x^{3}\right)}{2\sqrt{\pi}}\right)}{3}$	33

```
[In] int(x^5*cos(x^3),x,method=_RETURNVERBOSE)
[Out] 1/3*cos(x^3)+1/3*x^3*sin(x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^5 \cos\left(x^3\right)\, dx = \frac{1}{3}\, x^3 \sin\left(x^3\right) + \frac{1}{3}\, \cos\left(x^3\right)$$

```
[In] integrate(x^5*cos(x^3),x, algorithm="fricas")
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int x^5 \cos\left(x^3\right)\, dx = \frac{x^3 \sin\left(x^3\right)}{3} + \frac{\cos\left(x^3\right)}{3}$$

```
[In] integrate(x**5*cos(x**3),x)
[Out] x**3*sin(x**3)/3 + cos(x**3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^5 \cos\left(x^3\right)\, dx = \frac{1}{3}\, x^3 \sin\left(x^3\right) + \frac{1}{3}\, \cos\left(x^3\right)$$

```
[In] integrate(x^5*cos(x^3),x, algorithm="maxima")
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^5 \cos\left(x^3\right)\, dx = \frac{1}{3}\, x^3 \sin\left(x^3\right) + \frac{1}{3}\, \cos\left(x^3\right)$$

```
[In] integrate(x^5*cos(x^3),x, algorithm="giac")
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^5 \cos\left(x^3\right)\, dx = \frac{\cos\left(x^3\right)}{3} + \frac{x^3\, \sin\left(x^3\right)}{3}$$

```
[In] int(x^5*cos(x^3),x)
[Out] cos(x^3)/3 + (x^3*sin(x^3))/3
```

3.54 $\int e^{x^2} x^5 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 28

$$\int e^{x^2} x^5 \, dx = e^{x^2} - e^{x^2} x^2 + \frac{1}{2} e^{x^2} x^4$$

```
[Out] exp(x^2)-exp(x^2)*x^2+1/2*exp(x^2)*x^4
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2243, 2240}

$$\int e^{x^2} x^5 \, dx = -e^{x^2} x^2 + e^{x^2} + \frac{1}{2} e^{x^2} x^4$$

```
[In] Int[E^x^2*x^5,x]
```

```
[Out] E^x^2 - E^x^2*x^2 + (E^x^2*x^4)/2
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
```

```
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}e^{x^2}x^4 - 2\int e^{x^2}x^3\,dx \\
&= -e^{x^2}x^2 + \frac{1}{2}e^{x^2}x^4 + 2\int e^{x^2}x\,dx \\
&= e^{x^2} - e^{x^2}x^2 + \frac{1}{2}e^{x^2}x^4
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int e^{x^2}x^5\,dx = \frac{1}{2}e^{x^2}\left(2 - 2x^2 + x^4\right)$$

```
[In] Integrate[E^x^2*x^5,x]
[Out] (E^x^2*(2 - 2*x^2 + x^4))/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(x^4-2x^2+2)\mathrm{e}^{x^2}}{2}$	17
risch	$\left(\frac{1}{2}x^4 - x^2 + 1\right)\mathrm{e}^{x^2}$	18
meijerg	$-1 + \frac{(3x^4-6x^2+6)\mathrm{e}^{x^2}}{6}$	21
default	$\mathrm{e}^{x^2} - \mathrm{e}^{x^2}x^2 + \frac{\mathrm{e}^{x^2}x^4}{2}$	24
norman	$\mathrm{e}^{x^2} - \mathrm{e}^{x^2}x^2 + \frac{\mathrm{e}^{x^2}x^4}{2}$	24
parallelrisch	$\mathrm{e}^{x^2} - \mathrm{e}^{x^2}x^2 + \frac{\mathrm{e}^{x^2}x^4}{2}$	24
parts	$\frac{\operatorname{erfi}(x)\sqrt{\pi}\,x^5}{2} - \frac{5\sqrt{\pi}\left(\frac{x^5\operatorname{erfi}(x)}{5} - \frac{2\left(\mathrm{e}^{x^2} - \mathrm{e}^{x^2}x^2 + \frac{\mathrm{e}^{x^2}x^4}{2}\right)}{5\sqrt{\pi}}\right)}{2}$	53

```
[In] int(exp(x^2)*x^5,x,method=_RETURNVERBOSE)
[Out] 1/2*(x^4-2*x^2+2)*exp(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int e^{x^2} x^5 \, dx = \frac{1}{2} \left(x^4 - 2 \, x^2 + 2\right) e^{\left(x^2\right)}$$

```
[In] integrate(exp(x^2)*x^5,x, algorithm="fricas")
[Out] 1/2*(x^4 - 2*x^2 + 2)*e^(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.54

$$\int e^{x^2} x^5 \, dx = \frac{(x^4 - 2x^2 + 2) e^{x^2}}{2}$$

```
[In] integrate(exp(x**2)*x**5,x)
[Out] (x**4 - 2*x**2 + 2)*exp(x**2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int e^{x^2} x^5 \, dx = \frac{1}{2} \left(x^4 - 2 \, x^2 + 2\right) e^{\left(x^2\right)}$$

```
[In] integrate(exp(x^2)*x^5,x, algorithm="maxima")
[Out] 1/2*(x^4 - 2*x^2 + 2)*e^(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int e^{x^2} x^5 \, dx = \frac{1}{2} \left(x^4 - 2\, x^2 + 2\right) e^{\left(x^2\right)}$$

```
[In] integrate(exp(x^2)*x^5,x, algorithm="giac")
[Out] 1/2*(x^4 - 2*x^2 + 2)*e^(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int e^{x^2} x^5 \, dx = \frac{\mathrm{e}^{x^2} \left(x^4 - 2\, x^2 + 2\right)}{2}$$

```
[In] int(x^5*exp(x^2),x)
[Out] (exp(x^2)*(x^4 - 2*x^2 + 2))/2
```

3.55 $\int x \arctan(x) \, dx$

Optimal result

Integrand size = 4, antiderivative size = 21

$$\int x \arctan(x) \, dx = -\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{1}{2}x^2 \arctan(x)$$

```
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4946, 327, 209}

$$\int x \arctan(x) \, dx = \frac{1}{2}x^2 \arctan(x) + \frac{\arctan(x)}{2} - \frac{x}{2}$$

```
[In] Int[x*ArcTan[x],x]
```

```
[Out] -1/2*x + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\arctan(x)-\frac{1}{2}\int\frac{x^2}{1+x^2}\,dx\\ &= -\frac{x}{2}+\frac{1}{2}x^2\arctan(x)+\frac{1}{2}\int\frac{1}{1+x^2}\,dx\\ &= -\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{1}{2}x^2\arctan(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int x\arctan(x)\,dx=\frac{1}{2}\left(-x+\left(1+x^2\right)\arctan(x)\right)$$

```
[In] Integrate[x*ArcTan[x],x]
[Out] (-x + (1 + x^2)*ArcTan[x])/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2}+\frac{(3x^2+3)\arctan(x)}{6}$	16
parallelrisch	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
parts	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
risch	$-\frac{ix^2\ln(ix+1)}{4}+\frac{ix^2\ln(-ix+1)}{4}-\frac{x}{2}+\frac{\arctan(x)}{2}$	35

```
[In] int(x*arctan(x),x,method=_RETURNVERBOSE)
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x \arctan(x)\, dx = \frac{1}{2} \left(x^2 + 1\right) \arctan\left(x\right) - \frac{1}{2}\, x$$

```
[In] integrate(x*arctan(x),x, algorithm="fricas")
[Out] 1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \arctan(x)\, dx = \frac{x^2 \operatorname{atan}\left(x\right)}{2} - \frac{x}{2} + \frac{\operatorname{atan}\left(x\right)}{2}$$

```
[In] integrate(x*atan(x),x)
[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \arctan(x)\, dx = \frac{1}{2}\, x^2 \arctan\left(x\right) - \frac{1}{2}\, x + \frac{1}{2}\, \arctan\left(x\right)$$

```
[In] integrate(x*arctan(x),x, algorithm="maxima")
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \arctan(x)\, dx = \frac{1}{2}\, x^2 \arctan\left(x\right) - \frac{1}{2}\, x + \frac{1}{2}\, \arctan\left(x\right)$$

```
[In] integrate(x*arctan(x),x, algorithm="giac")
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int x \arctan(x)\, dx = \operatorname{atan}(x)\, \left(\frac{x^2}{2} + \frac{1}{2}\right) - \frac{x}{2}$$

```
[In] int(x*atan(x),x)
[Out] atan(x)*(x^2/2 + 1/2) - x/2
```

3.56 $\int x\cos(\pi x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 18

$$\int x\cos(\pi x)\,dx = \frac{\cos(\pi x)}{\pi^2} + \frac{x\sin(\pi x)}{\pi}$$

```
[Out] cos(Pi*x)/Pi^2+x*sin(Pi*x)/Pi
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3377, 2718}

$$\int x\cos(\pi x)\,dx = \frac{x\sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2}$$

```
[In] Int[x*Cos[Pi*x],x]
```

```
[Out] Cos[Pi*x]/Pi^2 + (x*Sin[Pi*x])/Pi
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\sin(\pi x)}{\pi} - \frac{\int \sin(\pi x)\,dx}{\pi}\\ &= \frac{\cos(\pi x)}{\pi^2} + \frac{x\sin(\pi x)}{\pi}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x\cos(\pi x)\,dx = \frac{\cos(\pi x)}{\pi^2} + \frac{x\sin(\pi x)}{\pi}$$

```
[In] Integrate[x*Cos[Pi*x],x]
[Out] Cos[Pi*x]/Pi^2 + (x*Sin[Pi*x])/Pi
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\cos(\pi x)+x\pi\sin(\pi x)}{\pi^2}$	17
default	$\frac{\cos(\pi x)+x\pi\sin(\pi x)}{\pi^2}$	17
risch	$\frac{\cos(\pi x)}{\pi^2}+\frac{x\sin(\pi x)}{\pi}$	19
parts	$\frac{\cos(\pi x)}{\pi^2}+\frac{x\sin(\pi x)}{\pi}$	19
parallelrisch	$\frac{2+2\pi x\tan\left(\frac{\pi x}{2}\right)}{\pi^2\left(1+\tan^2\left(\frac{\pi x}{2}\right)\right)}$	27
norman	$\frac{\frac{2x\tan\left(\frac{\pi x}{2}\right)}{\pi}+\frac{2}{\pi^2}}{1+\tan^2\left(\frac{\pi x}{2}\right)}$	30
meijerg	$\frac{-\frac{1}{\sqrt{\pi}}+\frac{\cos(\pi x)}{\sqrt{\pi}}+\sqrt{\pi}\,x\sin(\pi x)}{\pi^{\frac{3}{2}}}$	31

```
[In] int(x*cos(Pi*x),x,method=_RETURNVERBOSE)
[Out] 1/Pi^2*(cos(Pi*x)+x*Pi*sin(Pi*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int x\cos(\pi x)\,dx = \frac{\pi x\sin(\pi x) + \cos(\pi x)}{\pi^2}$$

```
[In] integrate(x*cos(pi*x),x, algorithm="fricas")
[Out] (pi*x*sin(pi*x) + cos(pi*x))/pi^2
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int x\cos(\pi x)\,dx = \frac{x\sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2}$$

```
[In] integrate(x*cos(pi*x),x)
[Out] x*sin(pi*x)/pi + cos(pi*x)/pi**2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int x\cos(\pi x)\,dx = \frac{\pi x\sin(\pi x) + \cos(\pi x)}{\pi^2}$$

```
[In] integrate(x*cos(pi*x),x, algorithm="maxima")
[Out] (pi*x*sin(pi*x) + cos(pi*x))/pi^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x\cos(\pi x)\,dx = \frac{x\sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2}$$

```
[In] integrate(x*cos(pi*x),x, algorithm="giac")
[Out] x*sin(pi*x)/pi + cos(pi*x)/pi^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int x\cos(\pi x)\,dx = \frac{\cos\left(\Pi\,x\right) + \Pi\,x\,\sin\left(\Pi\,x\right)}{\Pi^2}$$

```
[In] int(x*cos(Pi*x),x)
[Out] (cos(Pi*x) + Pi*x*sin(Pi*x))/Pi^2
```

3.57 $\int \sqrt{x} \log(x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 21

$$\int \sqrt{x} \log(x) \, dx = -\frac{4x^{3/2}}{9} + \frac{2}{3} x^{3/2} \log(x)$$

```
[Out] -4/9*x^(3/2)+2/3*x^(3/2)*ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2341}

$$\int \sqrt{x} \log(x) \, dx = \frac{2}{3} x^{3/2} \log(x) - \frac{4x^{3/2}}{9}$$

```
[In] Int[Sqrt[x]*Log[x],x]
```

```
[Out] (-4*x^(3/2))/9 + (2*x^(3/2)*Log[x])/3
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{4x^{3/2}}{9} + \frac{2}{3} x^{3/2} \log(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{x}\log(x)\,dx = \frac{2}{9}x^{3/2}(-2+3\log(x))$$

```
[In] Integrate[Sqrt[x]*Log[x],x]
[Out] (2*x^(3/2)*(-2 + 3*Log[x]))/9
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$-\frac{4x^{\frac{3}{2}}}{9}+\frac{2x^{\frac{3}{2}}\ln(x)}{3}$	14
default	$-\frac{4x^{\frac{3}{2}}}{9}+\frac{2x^{\frac{3}{2}}\ln(x)}{3}$	14
risch	$-\frac{4x^{\frac{3}{2}}}{9}+\frac{2x^{\frac{3}{2}}\ln(x)}{3}$	14

```
[In] int(ln(x)*x^(1/2),x,method=_RETURNVERBOSE)
[Out] -4/9*x^(3/2)+2/3*x^(3/2)*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \sqrt{x}\log(x)\,dx = \frac{2}{9}\,(3\,x\log\left(x\right)-2\,x)\sqrt{x}$$

```
[In] integrate(log(x)*x^(1/2),x, algorithm="fricas")
[Out] 2/9*(3*x*log(x) - 2*x)*sqrt(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(19) = 38.

Time = 0.93 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.00

$$\int \sqrt{x}\log(x)\,dx = \begin{cases} -\frac{2x^{\frac{3}{2}}\log\left(\frac{1}{x}\right)}{3} + \frac{2x^{\frac{3}{2}}\log(x)}{3} - \frac{8x^{\frac{3}{2}}}{9} & \text{for}\ \frac{1}{|x|} < 1 \wedge |x| < 1 \\ \frac{2x^{\frac{3}{2}}\log(x)}{3} - \frac{4x^{\frac{3}{2}}}{9} & \text{for}\ |x| < 1 \\ -\frac{2x^{\frac{3}{2}}\log\left(\frac{1}{x}\right)}{3} - \frac{4x^{\frac{3}{2}}}{9} & \text{for}\ \frac{1}{|x|} < 1 \\ -G_{3,3}^{2,1}\left(\begin{matrix} 1 & \frac{5}{2},\frac{5}{2} \\ \frac{3}{2},\frac{3}{2} & 0 \end{matrix}\middle| x\right) + G_{3,3}^{0,3}\left(\begin{matrix} \frac{5}{2},\frac{5}{2},1 & \\ & \frac{3}{2},\frac{3}{2},0 \end{matrix}\middle| x\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(x)*x**(1/2),x)
```

```
[Out] Piecewise((-2*x**(3/2)*log(1/x)/3 + 2*x**(3/2)*log(x)/3 - 8*x**(3/2)/9, (Ab
s(x) < 1) & (1/Abs(x) < 1)), (2*x**(3/2)*log(x)/3 - 4*x**(3/2)/9, Abs(x) <
1), (-2*x**(3/2)*log(1/x)/3 - 4*x**(3/2)/9, 1/Abs(x) < 1), (-meijerg(((1,),
 (5/2, 5/2)), ((3/2, 3/2), (0,)), x) + meijerg(((5/2, 5/2, 1), ()), ((), (3
/2, 3/2, 0)), x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{x}\log(x)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\log(x) - \frac{4}{9}\,x^{\frac{3}{2}}$$

```
[In] integrate(log(x)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*log(x) - 4/9*x^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{x}\log(x)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\log(x) - \frac{4}{9}\,x^{\frac{3}{2}}$$

```
[In] integrate(log(x)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)*log(x) - 4/9*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.43

$$\int \sqrt{x}\log(x)\,dx = \frac{2\,x^{3/2}\,\left(\ln\left(x\right)-\frac{2}{3}\right)}{3}$$

```
[In] int(x^(1/2)*log(x),x)
[Out] (2*x^(3/2)*(log(x) - 2/3))/3
```

3.58 $\int \sin^2(3x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 18

$$\int \sin^2(3x)\,dx = \frac{x}{2} - \frac{1}{6}\cos(3x)\sin(3x)$$

```
[Out] 1/2*x-1/6*cos(3*x)*sin(3*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2715, 8}

$$\int \sin^2(3x)\,dx = \frac{x}{2} - \frac{1}{6}\sin(3x)\cos(3x)$$

```
[In] Int[Sin[3*x]^2,x]
```

```
[Out] x/2 - (Cos[3*x]*Sin[3*x])/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{6}\cos(3x)\sin(3x) + \frac{\int 1\,dx}{2}\\ &= \frac{x}{2} - \frac{1}{6}\cos(3x)\sin(3x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \sin^2(3x)\,dx = \frac{x}{2} - \frac{1}{12}\sin(6x)$$

```
[In] Integrate[Sin[3*x]^2,x]
[Out] x/2 - Sin[6*x]/12
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{x}{2}-\frac{\sin(6x)}{12}$	11
parallelrisch	$\frac{x}{2}-\frac{\sin(6x)}{12}$	11
derivativedivides	$\frac{x}{2}-\frac{\sin(3x)\cos(3x)}{6}$	15
default	$\frac{x}{2}-\frac{\sin(3x)\cos(3x)}{6}$	15
meijerg	$\frac{\sqrt{\pi}\left(\frac{6x}{\sqrt{\pi}}-\frac{\sin(6x)}{\sqrt{\pi}}\right)}{12}$	22
norman	$\frac{x\left(\tan^2\left(\frac{3x}{2}\right)\right)+\frac{x}{2}+\frac{\left(\tan^3\left(\frac{3x}{2}\right)\right)}{3}+\frac{x\left(\tan^4\left(\frac{3x}{2}\right)\right)}{2}-\frac{\tan\left(\frac{3x}{2}\right)}{3}}{\left(1+\tan^2\left(\frac{3x}{2}\right)\right)^2}$	47

```
[In] int(sin(3*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x-1/12*sin(6*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \sin^2(3x)\,dx = -\frac{1}{6}\cos(3x)\sin(3x) + \frac{1}{2}x$$

```
[In] integrate(sin(3*x)^2,x, algorithm="fricas")
[Out] -1/6*cos(3*x)*sin(3*x) + 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \sin^2(3x)\,dx = \frac{x}{2} - \frac{\sin(3x)\cos(3x)}{6}$$

```
[In] integrate(sin(3*x)**2,x)
[Out] x/2 - sin(3*x)*cos(3*x)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \sin^2(3x)\,dx = \frac{1}{2}x - \frac{1}{12}\sin(6x)$$

```
[In] integrate(sin(3*x)^2,x, algorithm="maxima")
[Out] 1/2*x - 1/12*sin(6*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \sin^2(3x)\,dx = \frac{1}{2}x - \frac{1}{12}\sin(6x)$$

```
[In] integrate(sin(3*x)^2,x, algorithm="giac")
[Out] 1/2*x - 1/12*sin(6*x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \sin^2(3x)\,dx = \frac{x}{2} - \frac{\sin(6\,x)}{12}$$

```
[In] int(sin(3*x)^2,x)
[Out] x/2 - sin(6*x)/12
```

3.59 $\int \cos^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\sin(x)\cos(x)$$

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\cos(x)\sin(x) + \frac{\int 1\,dx}{2}$$
$$= \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{4}\sin(2x)$$

```
[In] Integrate[Cos[x]^2,x]
[Out] x/2 + Sin[2*x]/4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

```
[In] int(cos(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x) \, dx = \frac{1}{2} \cos(x) \sin(x) + \frac{1}{2} x$$

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x) \, dx = \frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

```
[In] integrate(cos(x)**2,x)
[Out] x/2 + sin(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x) \, dx = \frac{1}{2} x + \frac{1}{4} \sin(2x)$$

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
[Out] 1/2*x + 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x) \, dx = \frac{1}{2} x + \frac{1}{4} \sin(2x)$$

```
[In] integrate(cos(x)^2,x, algorithm="giac")
[Out] 1/2*x + 1/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(2\,x)}{4}$$

```
[In] int(cos(x)^2,x)
[Out] x/2 + sin(2*x)/4
```

3.60 $\int \cos^4(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 24

$$\int \cos^4(x)\,dx=\frac{3x}{8}+\frac{3}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)$$

```
[Out] 3/8*x+3/8*cos(x)*sin(x)+1/4*cos(x)^3*sin(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\int \cos^4(x)\,dx=\frac{3x}{8}+\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{8}\sin(x)\cos(x)$$

```
[In] Int[Cos[x]^4,x]
```

```
[Out] (3*x)/8 + (3*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
```

<u>Rule 8</u>

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

<u>Rule 2715</u>

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\cos^3(x)\sin(x) + \frac{3}{4}\int \cos^2(x)\,dx \\
&= \frac{3}{8}\cos(x)\sin(x) + \frac{1}{4}\cos^3(x)\sin(x) + \frac{3\int 1\,dx}{8} \\
&= \frac{3x}{8} + \frac{3}{8}\cos(x)\sin(x) + \frac{1}{4}\cos^3(x)\sin(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \cos^4(x)\,dx = \frac{3x}{8} + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)$$

```
[In] Integrate[Cos[x]^4,x]
[Out] (3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{3x}{8}+\frac{\sin(4x)}{32}+\frac{\sin(2x)}{4}$	17
parallelrisch	$\frac{3x}{8}+\frac{\sin(4x)}{32}+\frac{\sin(2x)}{4}$	17
default	$\frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{4}+\frac{3x}{8}$	18
norman	$\frac{\frac{3x}{8}-\frac{3\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}+\frac{3\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}-\frac{5\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{9x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}+\frac{5\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	82

```
[In] int(cos(x)^4,x,method=_RETURNVERBOSE)
[Out] 3/8*x+1/32*sin(4*x)+1/4*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \cos^4(x)\,dx = \frac{1}{8}\left(2\cos(x)^3 + 3\cos(x)\right)\sin(x) + \frac{3}{8}x$$

```
[In] integrate(cos(x)^4,x, algorithm="fricas")
[Out] 1/8*(2*cos(x)^3 + 3*cos(x))*sin(x) + 3/8*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cos^4(x)\,dx = \frac{3x}{8} + \frac{\sin(x)\cos^3(x)}{4} + \frac{3\sin(x)\cos(x)}{8}$$

```
[In] integrate(cos(x)**4,x)
[Out] 3*x/8 + sin(x)*cos(x)**3/4 + 3*sin(x)*cos(x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\,dx = \frac{3}{8}x + \frac{1}{32}\sin(4x) + \frac{1}{4}\sin(2x)$$

```
[In] integrate(cos(x)^4,x, algorithm="maxima")
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\,dx = \frac{3}{8}x + \frac{1}{32}\sin(4x) + \frac{1}{4}\sin(2x)$$

```
[In] integrate(cos(x)^4,x, algorithm="giac")
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\,dx = \frac{3\,x}{8} + \frac{\sin(2\,x)}{4} + \frac{\sin(4\,x)}{32}$$

```
[In] int(cos(x)^4,x)
[Out] (3*x)/8 + sin(2*x)/4 + sin(4*x)/32
```

3.61 $\int \sin^3(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 13

$$\int \sin^3(x)\,dx = -\cos(x) + \frac{\cos^3(x)}{3}$$

```
[Out] -cos(x)+1/3*cos(x)^3
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2713}

$$\int \sin^3(x)\,dx = \frac{\cos^3(x)}{3} - \cos(x)$$

```
[In] Int[Sin[x]^3,x]
```

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(x)\right) \\ &= -\cos(x) + \frac{\cos^3(x)}{3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sin^3(x)\,dx = -\frac{3\cos(x)}{4} + \frac{1}{12}\cos(3x)$$

```
[In] Integrate[Sin[x]^3,x]
[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{(2+\sin^2(x))\cos(x)}{3}$	11
risch	$-\frac{3\cos(x)}{4}+\frac{\cos(3x)}{12}$	12
parallelrisch	$-\frac{2}{3}-\frac{3\cos(x)}{4}+\frac{\cos(3x)}{12}$	13
norman	$\frac{-4\left(\tan^2\left(\frac{x}{2}\right)\right)-\frac{4}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$	22

```
[In] int(sin(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/3*(2+sin(x)^2)*cos(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos(x)^3 - \cos(x)$$

```
[In] integrate(sin(x)^3,x, algorithm="fricas")
[Out] 1/3*cos(x)^3 - cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \sin^3(x)\,dx = \frac{\cos^3(x)}{3} - \cos(x)$$

```
[In] integrate(sin(x)**3,x)
[Out] cos(x)**3/3 - cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos(x)^3 - \cos(x)$$

```
[In] integrate(sin(x)^3,x, algorithm="maxima")
[Out] 1/3*cos(x)^3 - cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos(x)^3 - \cos(x)$$

```
[In] integrate(sin(x)^3,x, algorithm="giac")
[Out] 1/3*cos(x)^3 - cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \sin^3(x)\,dx = \frac{\cos(x)\,\left(\cos(x)^2 - 3\right)}{3}$$

```
[In] int(sin(x)^3,x)
[Out] (cos(x)*(cos(x)^2 - 3))/3
```

3.62 $\int \cos^4(x) \sin^3(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \cos^4(x) \sin^3(x) \, dx = -\frac{1}{5} \cos^5(x) + \frac{\cos^7(x)}{7}$$

```
[Out] -1/5*cos(x)^5+1/7*cos(x)^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2645, 14}

$$\int \cos^4(x) \sin^3(x) \, dx = \frac{\cos^7(x)}{7} - \frac{\cos^5(x)}{5}$$

```
[In] Int[Cos[x]^4*Sin[x]^3,x]
```

```
[Out] -1/5*Cos[x]^5 + Cos[x]^7/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int x^4(1-x^2)\ dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int (x^4-x^6)\ dx, x, \cos(x)\right)$$
$$= -\frac{1}{5}\cos^5(x) + \frac{\cos^7(x)}{7}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \cos^4(x)\sin^3(x)\,dx = -\frac{3\cos(x)}{64} - \frac{1}{64}\cos(3x) + \frac{1}{320}\cos(5x) + \frac{1}{448}\cos(7x)$$

```
[In] Integrate[Cos[x]^4*Sin[x]^3,x]
[Out] (-3*Cos[x])/64 - Cos[3*x]/64 + Cos[5*x]/320 + Cos[7*x]/448
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{(\cos^5(x))}{5}+\frac{(\cos^7(x))}{7}$	14
default	$-\frac{(\cos^5(x))}{5}+\frac{(\cos^7(x))}{7}$	14
risch	$-\frac{3\cos(x)}{64}+\frac{\cos(7x)}{448}+\frac{\cos(5x)}{320}-\frac{\cos(3x)}{64}$	24
parallelrisch	$\frac{6}{35}-\frac{3\cos(x)}{64}+\frac{\cos(7x)}{448}+\frac{\cos(5x)}{320}-\frac{\cos(3x)}{64}$	25
norman	$\frac{-8(\tan^6(\frac{x}{2}))-4(\tan^{10}(\frac{x}{2}))+4(\tan^8(\frac{x}{2}))-\frac{4(\tan^2(\frac{x}{2}))}{5}+\frac{8(\tan^4(\frac{x}{2}))}{5}-\frac{4}{35}}{(1+\tan^2(\frac{x}{2}))^7}$	54

```
[In] int(cos(x)^4*sin(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/5*cos(x)^5+1/7*cos(x)^7
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos^4(x)\sin^3(x)\,dx = \frac{1}{7}\cos(x)^7 - \frac{1}{5}\cos(x)^5$$

```
[In] integrate(cos(x)^4*sin(x)^3,x, algorithm="fricas")
[Out] 1/7*cos(x)^7 - 1/5*cos(x)^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \cos^4(x)\sin^3(x)\,dx = \frac{\cos^7(x)}{7} - \frac{\cos^5(x)}{5}$$

```
[In] integrate(cos(x)**4*sin(x)**3,x)
[Out] cos(x)**7/7 - cos(x)**5/5
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos^4(x)\sin^3(x)\,dx = \frac{1}{7}\cos(x)^7 - \frac{1}{5}\cos(x)^5$$

```
[In] integrate(cos(x)^4*sin(x)^3,x, algorithm="maxima")
[Out] 1/7*cos(x)^7 - 1/5*cos(x)^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos^4(x)\sin^3(x)\,dx = \frac{1}{7}\cos(x)^7 - \frac{1}{5}\cos(x)^5$$

```
[In] integrate(cos(x)^4*sin(x)^3,x, algorithm="giac")
[Out] 1/7*cos(x)^7 - 1/5*cos(x)^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cos^4(x) \sin^3(x) \, dx = \frac{\cos(x)^5 \left(5 \cos(x)^2 - 7\right)}{35}$$

```
[In] int(cos(x)^4*sin(x)^3,x)
[Out] (cos(x)^5*(5*cos(x)^2 - 7))/35
```

3.63 $\int \cos^3(x)\sin^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \cos^3(x)\sin^4(x)\,dx = \frac{\sin^5(x)}{5} - \frac{\sin^7(x)}{7}$$

```
[Out] 1/5*sin(x)^5-1/7*sin(x)^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2644, 14}

$$\int \cos^3(x)\sin^4(x)\,dx = \frac{\sin^5(x)}{5} - \frac{\sin^7(x)}{7}$$

```
[In] Int[Cos[x]^3*Sin[x]^4,x]
```

```
[Out] Sin[x]^5/5 - Sin[x]^7/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^4\left(1-x^2\right)\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(x^4-x^6\right)\,dx, x, \sin(x)\right)\\ &= \frac{\sin^5(x)}{5}-\frac{\sin^7(x)}{7}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \cos^3(x)\sin^4(x)\,dx = \frac{3\sin(x)}{64}-\frac{1}{64}\sin(3x)-\frac{1}{320}\sin(5x)+\frac{1}{448}\sin(7x)$$

```
[In] Integrate[Cos[x]^3*Sin[x]^4,x]
[Out] (3*Sin[x])/64 - Sin[3*x]/64 - Sin[5*x]/320 + Sin[7*x]/448
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{(\sin^5(x))}{5}-\frac{(\sin^7(x))}{7}$	14
default	$\frac{(\sin^5(x))}{5}-\frac{(\sin^7(x))}{7}$	14
risch	$\frac{3\sin(x)}{64}+\frac{\sin(7x)}{448}-\frac{\sin(5x)}{320}-\frac{\sin(3x)}{64}$	24
parallelrisch	$\frac{3\sin(x)}{64}+\frac{\sin(7x)}{448}-\frac{\sin(5x)}{320}-\frac{\sin(3x)}{64}$	24
norman	$\frac{\frac{32\left(\tan^5\left(\frac{x}{2}\right)\right)}{5}-\frac{192\left(\tan^7\left(\frac{x}{2}\right)\right)}{35}+\frac{32\left(\tan^9\left(\frac{x}{2}\right)\right)}{5}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^7}$	37

```
[In] int(cos(x)^3*sin(x)^4,x,method=_RETURNVERBOSE)
[Out] 1/5*sin(x)^5-1/7*sin(x)^7
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \cos^3(x)\sin^4(x)\,dx = \frac{1}{35}\left(5\cos(x)^6 - 8\cos(x)^4 + \cos(x)^2 + 2\right)\sin(x)$$

```
[In] integrate(cos(x)^3*sin(x)^4,x, algorithm="fricas")
[Out] 1/35*(5*cos(x)^6 - 8*cos(x)^4 + cos(x)^2 + 2)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \cos^3(x)\sin^4(x)\,dx = -\frac{\sin^7(x)}{7} + \frac{\sin^5(x)}{5}$$

```
[In] integrate(cos(x)**3*sin(x)**4,x)
[Out] -sin(x)**7/7 + sin(x)**5/5
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos^3(x)\sin^4(x)\,dx = -\frac{1}{7}\sin(x)^7 + \frac{1}{5}\sin(x)^5$$

```
[In] integrate(cos(x)^3*sin(x)^4,x, algorithm="maxima")
[Out] -1/7*sin(x)^7 + 1/5*sin(x)^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos^3(x)\sin^4(x)\,dx = -\frac{1}{7}\sin(x)^7 + \frac{1}{5}\sin(x)^5$$

```
[In] integrate(cos(x)^3*sin(x)^4,x, algorithm="giac")
[Out] -1/7*sin(x)^7 + 1/5*sin(x)^5
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cos^3(x) \sin^4(x) \, dx = -\frac{\sin(x)^5 \left(5 \sin(x)^2 - 7\right)}{35}$$

```
[In] int(cos(x)^3*sin(x)^4,x)
[Out] -(sin(x)^5*(5*sin(x)^2 - 7))/35
```

3.64 $\int \cos^2(x)\sin^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 36

$$\int \cos^2(x)\sin^4(x)\,dx = \frac{x}{16} + \frac{1}{16}\cos(x)\sin(x) - \frac{1}{8}\cos^3(x)\sin(x) - \frac{1}{6}\cos^3(x)\sin^3(x)$$

```
[Out] 1/16*x+1/16*cos(x)*sin(x)-1/8*cos(x)^3*sin(x)-1/6*cos(x)^3*sin(x)^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2648, 2715, 8}

$$\int \cos^2(x)\sin^4(x)\,dx = \frac{x}{16} - \frac{1}{6}\sin^3(x)\cos^3(x) - \frac{1}{8}\sin(x)\cos^3(x) + \frac{1}{16}\sin(x)\cos(x)$$

```
[In] Int[Cos[x]^2*Sin[x]^4,x]
```

```
[Out] x/16 + (Cos[x]*Sin[x])/16 - (Cos[x]^3*Sin[x])/8 - (Cos[x]^3*Sin[x]^3)/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{6}\cos^3(x)\sin^3(x)+\frac{1}{2}\int\cos^2(x)\sin^2(x)\,dx\\
&= -\frac{1}{8}\cos^3(x)\sin(x)-\frac{1}{6}\cos^3(x)\sin^3(x)+\frac{1}{8}\int\cos^2(x)\,dx\\
&= \frac{1}{16}\cos(x)\sin(x)-\frac{1}{8}\cos^3(x)\sin(x)-\frac{1}{6}\cos^3(x)\sin^3(x)+\frac{\int 1\,dx}{16}\\
&= \frac{x}{16}+\frac{1}{16}\cos(x)\sin(x)-\frac{1}{8}\cos^3(x)\sin(x)-\frac{1}{6}\cos^3(x)\sin^3(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int\cos^2(x)\sin^4(x)\,dx=\frac{x}{16}-\frac{1}{64}\sin(2x)-\frac{1}{64}\sin(4x)+\frac{1}{192}\sin(6x)$$

```
[In] Integrate[Cos[x]^2*Sin[x]^4,x]
[Out] x/16 - Sin[2*x]/64 - Sin[4*x]/64 + Sin[6*x]/192
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

method	result
risch	$\frac{x}{16}+\frac{\sin(6x)}{192}-\frac{\sin(4x)}{64}-\frac{\sin(2x)}{64}$
parallelrisch	$\frac{x}{16}+\frac{\sin(6x)}{192}-\frac{\sin(4x)}{64}-\frac{\sin(2x)}{64}$
default	$\frac{x}{16}+\frac{\cos(x)\sin(x)}{16}-\frac{(\cos^3(x))\sin(x)}{8}-\frac{(\sin^3(x))(\cos^3(x))}{6}$
norman	$\frac{\frac{x}{16}-\frac{17\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}+\frac{19\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}-\frac{19\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{17\left(\tan^9\left(\frac{x}{2}\right)\right)}{24}+\frac{\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{8}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{15x\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{5x\left(\tan^6\left(\frac{x}{2}\right)\right)}{4}+}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

```
[In] int(sin(x)^4*cos(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*x+1/192*sin(6*x)-1/64*sin(4*x)-1/64*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \cos^2(x)\sin^4(x)\,dx = \frac{1}{48}\left(8\cos(x)^5 - 14\cos(x)^3 + 3\cos(x)\right)\sin(x) + \frac{1}{16}\,x$$

```
[In] integrate(cos(x)^2*sin(x)^4,x, algorithm="fricas")
[Out] 1/48*(8*cos(x)^5 - 14*cos(x)^3 + 3*cos(x))*sin(x) + 1/16*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \cos^2(x)\sin^4(x)\,dx = \frac{x}{16} + \frac{\sin^5(x)\cos(x)}{6} - \frac{\sin^3(x)\cos(x)}{24} - \frac{\sin(x)\cos(x)}{16}$$

```
[In] integrate(cos(x)**2*sin(x)**4,x)
[Out] x/16 + sin(x)**5*cos(x)/6 - sin(x)**3*cos(x)/24 - sin(x)*cos(x)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.50

$$\int \cos^2(x)\sin^4(x)\,dx = -\frac{1}{48}\sin(2x)^3 + \frac{1}{16}\,x - \frac{1}{64}\sin(4x)$$

```
[In] integrate(cos(x)^2*sin(x)^4,x, algorithm="maxima")
[Out] -1/48*sin(2*x)^3 + 1/16*x - 1/64*sin(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \cos^2(x)\sin^4(x)\,dx = \frac{1}{16}\,x + \frac{1}{192}\,\sin(6\,x) - \frac{1}{64}\,\sin(4\,x) - \frac{1}{64}\,\sin(2\,x)$$

```
[In] integrate(cos(x)^2*sin(x)^4,x, algorithm="giac")
[Out] 1/16*x + 1/192*sin(6*x) - 1/64*sin(4*x) - 1/64*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \cos^2(x)\sin^4(x)\,dx = \frac{\cos(x)\,\sin(x)^5}{6} + \frac{x}{16} - \frac{\sin(2\,x)}{24} + \frac{\sin(4\,x)}{192}$$

```
[In] int(cos(x)^2*sin(x)^4,x)
[Out] x/16 - sin(2*x)/24 + sin(4*x)/192 + (cos(x)*sin(x)^5)/6
```

3.65 $\int \cos^2(x) \sin^2(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \cos^2(x) \sin^2(x) \, dx = \frac{x}{8} + \frac{1}{8} \cos(x) \sin(x) - \frac{1}{4} \cos^3(x) \sin(x)$$

```
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2648, 2715, 8}

$$\int \cos^2(x) \sin^2(x) \, dx = \frac{x}{8} - \frac{1}{4} \sin(x) \cos^3(x) + \frac{1}{8} \sin(x) \cos(x)$$

```
[In] Int[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{4}\cos^3(x)\sin(x) + \frac{1}{4}\int \cos^2(x)\,dx \\
&= \frac{1}{8}\cos(x)\sin(x) - \frac{1}{4}\cos^3(x)\sin(x) + \frac{\int 1\,dx}{8} \\
&= \frac{x}{8} + \frac{1}{8}\cos(x)\sin(x) - \frac{1}{4}\cos^3(x)\sin(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{x}{8} - \frac{1}{32}\sin(4x)$$

```
[In] Integrate[Cos[x]^2*Sin[x]^2,x]
[Out] x/8 - Sin[4*x]/32
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

method	result	size
risch	$\frac{x}{8}-\frac{\sin(4x)}{32}$	11
parallelrisch	$\frac{x}{8}-\frac{\sin(4x)}{32}$	11
default	$\frac{x}{8}+\frac{\cos(x)\sin(x)}{8}-\frac{(\cos^3(x))\sin(x)}{4}$	19
norman	$\frac{\frac{x}{8}+\frac{7\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}-\frac{7\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}-\frac{\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	82

```
[In] int(cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/8*x-1/32*sin(4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \cos^2(x)\sin^2(x)\,dx = -\frac{1}{8}\left(2\,\cos\left(x\right)^3 - \cos\left(x\right)\right)\sin\left(x\right) + \frac{1}{8}\,x$$

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{x}{8} - \frac{\sin\left(2x\right)\cos\left(2x\right)}{16}$$

```
[In] integrate(cos(x)**2*sin(x)**2,x)
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.42

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{1}{8}\,x - \frac{1}{32}\,\sin\left(4\,x\right)$$

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
[Out] 1/8*x - 1/32*sin(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.42

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{1}{8}\,x - \frac{1}{32}\,\sin\left(4\,x\right)$$

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
[Out] 1/8*x - 1/32*sin(4*x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{\cos(x)\ \sin(x)^3}{4} - \frac{\cos(x)\ \sin(x)}{8} + \frac{x}{8}$$

```
[In] int(cos(x)^2*sin(x)^2,x)
[Out] x/8 - (cos(x)*sin(x))/8 + (cos(x)*sin(x)^3)/4
```

3.66 $\int (1-\sin(2x))^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 22

$$\int (1-\sin(2x))^2\,dx = \frac{3x}{2}+\cos(2x)-\frac{1}{4}\cos(2x)\sin(2x)$$

```
[Out] 3/2*x+cos(2*x)-1/4*cos(2*x)*sin(2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2723}

$$\int (1-\sin(2x))^2\,dx = \frac{3x}{2}+\cos(2x)-\frac{1}{4}\sin(2x)\cos(2x)$$

```
[In] Int[(1 - Sin[2*x])^2,x]
[Out] (3*x)/2 + Cos[2*x] - (Cos[2*x]*Sin[2*x])/4
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{3x}{2}+\cos(2x)-\frac{1}{4}\cos(2x)\sin(2x)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (1-\sin(2x))^2\,dx=\frac{3x}{2}+\cos(2x)-\frac{1}{8}\sin(4x)$$

```
[In] Integrate[(1 - Sin[2*x])^2,x]
[Out] (3*x)/2 + Cos[2*x] - Sin[4*x]/8
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{3x}{2}-\frac{\sin(4x)}{8}+\cos(2x)$	15
parallelrisch	$\frac{3x}{2}+1-\frac{\sin(4x)}{8}+\cos(2x)$	16
derivativedivides	$\frac{3x}{2}+\cos(2x)-\frac{\sin(2x)\cos(2x)}{4}$	19
default	$\frac{3x}{2}+\cos(2x)-\frac{\sin(2x)\cos(2x)}{4}$	19
parts	$\frac{3x}{2}+\cos(2x)-\frac{\sin(2x)\cos(2x)}{4}$	19
norman	$\frac{2(\tan^2(x))+\frac{3x}{2}+\frac{(\tan^3(x))}{2}+3x(\tan^2(x))+\frac{3x(\tan^4(x))}{2}-\frac{\tan(x)}{2}+2}{(1+\tan^2(x))^2}$	45

```
[In] int((1-sin(2*x))^2,x,method=_RETURNVERBOSE)
[Out] 3/2*x-1/8*sin(4*x)+cos(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (1-\sin(2x))^2\,dx=-\frac{1}{4}\cos(2x)\sin(2x)+\frac{3}{2}x+\cos(2x)$$

```
[In] integrate((1-sin(2*x))^2,x, algorithm="fricas")
[Out] -1/4*cos(2*x)*sin(2*x) + 3/2*x + cos(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int (1-\sin(2x))^2\,dx = \frac{x\sin^2{(2x)}}{2} + \frac{x\cos^2{(2x)}}{2} + x - \frac{\sin{(2x)}\cos{(2x)}}{4} + \cos{(2x)}$$

```
[In] integrate((1-sin(2*x))**2,x)
[Out] x*sin(2*x)**2/2 + x*cos(2*x)**2/2 + x - sin(2*x)*cos(2*x)/4 + cos(2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int (1-\sin(2x))^2\,dx = \frac{3}{2}\,x + \cos\left(2\,x\right) - \frac{1}{8}\,\sin\left(4\,x\right)$$

```
[In] integrate((1-sin(2*x))^2,x, algorithm="maxima")
[Out] 3/2*x + cos(2*x) - 1/8*sin(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int (1-\sin(2x))^2\,dx = \frac{3}{2}\,x + \cos\left(2\,x\right) - \frac{1}{8}\,\sin\left(4\,x\right)$$

```
[In] integrate((1-sin(2*x))^2,x, algorithm="giac")
[Out] 3/2*x + cos(2*x) - 1/8*sin(4*x)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int (1-\sin(2x))^2\,dx = \frac{3\,x}{2} + \cos\left(2\,x\right) - \frac{\sin\left(4\,x\right)}{8}$$

```
[In] int((sin(2*x) - 1)^2,x)
[Out] (3*x)/2 + cos(2*x) - sin(4*x)/8
```

3.67 $\int \cos(x) \sin\left(\frac{\pi}{6}+x\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \cos(x) \sin\left(\frac{\pi}{6}+x\right)\,dx = \frac{x}{4}-\frac{1}{4}\cos\left(\frac{\pi}{6}+2x\right)$$

```
[Out] 1/4*x-1/4*cos(1/6*Pi+2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4670, 2718}

$$\int \cos(x) \sin\left(\frac{\pi}{6}+x\right)\,dx = \frac{x}{4}-\frac{1}{4}\cos\left(2x+\frac{\pi}{6}\right)$$

```
[In] Int[Cos[x]*Sin[Pi/6 + x],x]
```

```
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{4}+\frac{1}{2}\sin\left(\frac{\pi}{6}+2x\right)\right)\,dx\\ &= \frac{x}{4}+\frac{1}{2}\int \sin\left(\frac{\pi}{6}+2x\right)\,dx\\ &= \frac{x}{4}-\frac{1}{4}\cos\left(\frac{\pi}{6}+2x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx = \frac{x}{4}-\frac{1}{4}\cos\left(\frac{\pi}{6}+2x\right)$$

```
[In] Integrate[Cos[x]*Sin[Pi/6 + x],x]
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{x}{4}-\frac{\cos\left(\frac{\pi}{6}+2x\right)}{4}$	15
risch	$\frac{x}{4}-\frac{\sqrt{3}\,\cos\left(2x\right)}{8}+\frac{\sin\left(2x\right)}{8}$	20
parallelrisch	$\frac{\sin\left(\frac{\pi}{3}+2x\right)}{8}-\frac{\cos\left(\frac{\pi}{6}+2x\right)}{8}-\frac{\sqrt{3}\,\cos\left(2x\right)}{8}+\frac{\sqrt{3}}{8}+\frac{x}{4}$	39
norman	$\frac{x\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)+x\tan\left(\frac{x}{2}\right)\left(\tan^{2}\left(\frac{\pi}{12}+\frac{x}{2}\right)\right)+2\tan\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)-x\tan\left(\frac{x}{2}\right)-x\left(\tan^{2}\left(\frac{x}{2}\right)\right)\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\left(1+\tan^{2}\left(\frac{\pi}{12}+\frac{x}{2}\right)\right)}$	91

```
[In] int(cos(x)*sin(1/6*Pi+x),x,method=_RETURNVERBOSE)
[Out] 1/4*x-1/4*cos(1/6*Pi+2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx = -\frac{1}{4}\sqrt{3}\cos\left(\frac{1}{6}\pi+x\right)^2 - \frac{1}{4}\cos\left(\frac{1}{6}\pi+x\right)\sin\left(\frac{1}{6}\pi+x\right) + \frac{1}{4}x$$

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="fricas")
[Out] -1/4*sqrt(3)*cos(1/6*pi + x)^2 - 1/4*cos(1/6*pi + x)*sin(1/6*pi + x) + 1/4*
x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx = -\frac{x\sin(x)\cos\left(x+\frac{\pi}{6}\right)}{2} + \frac{x\sin\left(x+\frac{\pi}{6}\right)\cos(x)}{2} + \frac{\sin(x)\sin\left(x+\frac{\pi}{6}\right)}{2}$$

```
[In] integrate(cos(x)*sin(1/6*pi+x),x)
[Out] -x*sin(x)*cos(x + pi/6)/2 + x*sin(x + pi/6)*cos(x)/2 + sin(x)*sin(x + pi/6)
/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx = \frac{1}{4}x - \frac{1}{4}\cos\left(\frac{1}{6}\pi+2x\right)$$

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="maxima")
[Out] 1/4*x - 1/4*cos(1/6*pi + 2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx = \frac{1}{4}\,x - \frac{1}{4}\,\cos\left(\frac{1}{6}\,\pi + 2\,x\right)$$

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="giac")
[Out] 1/4*x - 1/4*cos(1/6*pi + 2*x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx = \frac{x\,\sin\left(\frac{\Pi}{6}\right)}{2} - \frac{\cos\left(\frac{\Pi}{6}+2\,x\right)}{4}$$

```
[In] int(cos(x)*sin(Pi/6 + x),x)
[Out] (x*sin(Pi/6))/2 - cos(Pi/6 + 2*x)/4
```

3.68 $\int \cos^5(x) \sin^5(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \cos^5(x) \sin^5(x) \, dx = \frac{\sin^6(x)}{6} - \frac{\sin^8(x)}{4} + \frac{\sin^{10}(x)}{10}$$

```
[Out] 1/6*sin(x)^6-1/4*sin(x)^8+1/10*sin(x)^10
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2644, 272, 45}

$$\int \cos^5(x) \sin^5(x) \, dx = \frac{\sin^{10}(x)}{10} - \frac{\sin^8(x)}{4} + \frac{\sin^6(x)}{6}$$

```
[In] Int[Cos[x]^5*Sin[x]^5,x]
```

```
[Out] Sin[x]^6/6 - Sin[x]^8/4 + Sin[x]^10/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^5 (1-x^2)^2 \, dx, x, \sin(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int (1-x)^2 x^2 \, dx, x, \sin^2(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(x^2-2x^3+x^4\right) \, dx, x, \sin^2(x)\right)\\ &= \frac{\sin^6(x)}{6}-\frac{\sin^8(x)}{4}+\frac{\sin^{10}(x)}{10}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cos^5(x)\sin^5(x)\,dx=-\frac{5}{512}\cos(2x)+\frac{5\cos(6x)}{3072}-\frac{\cos(10x)}{5120}$$

```
[In] Integrate[Cos[x]^5*Sin[x]^5,x]
[Out] (-5*Cos[2*x])/512 + (5*Cos[6*x])/3072 - Cos[10*x]/5120
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{(\sin^6(x))}{6}-\frac{(\sin^8(x))}{4}+\frac{(\sin^{10}(x))}{10}$	20
default	$\frac{(\sin^6(x))}{6}-\frac{(\sin^8(x))}{4}+\frac{(\sin^{10}(x))}{10}$	20
risch	$-\frac{\cos(10x)}{5120}+\frac{5\cos(6x)}{3072}-\frac{5\cos(2x)}{512}$	20
parallelrisch	$-\frac{121}{840}-\frac{\cos(10x)}{5120}+\frac{5\cos(6x)}{3072}-\frac{5\cos(2x)}{512}$	21

```
[In] int(cos(x)^5*sin(x)^5,x,method=_RETURNVERBOSE)
[Out] 1/6*sin(x)^6-1/4*sin(x)^8+1/10*sin(x)^10
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos^5(x)\sin^5(x)\,dx = -\frac{1}{10}\cos(x)^{10} + \frac{1}{4}\cos(x)^8 - \frac{1}{6}\cos(x)^6$$

```
[In] integrate(cos(x)^5*sin(x)^5,x, algorithm="fricas")
[Out] -1/10*cos(x)^10 + 1/4*cos(x)^8 - 1/6*cos(x)^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos^5(x)\sin^5(x)\,dx = \frac{\sin^{10}(x)}{10} - \frac{\sin^8(x)}{4} + \frac{\sin^6(x)}{6}$$

```
[In] integrate(cos(x)**5*sin(x)**5,x)
[Out] sin(x)**10/10 - sin(x)**8/4 + sin(x)**6/6
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos^5(x)\sin^5(x)\,dx = \frac{1}{10}\sin(x)^{10} - \frac{1}{4}\sin(x)^8 + \frac{1}{6}\sin(x)^6$$

```
[In] integrate(cos(x)^5*sin(x)^5,x, algorithm="maxima")
[Out] 1/10*sin(x)^10 - 1/4*sin(x)^8 + 1/6*sin(x)^6
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos^5(x)\sin^5(x)\,dx = -\frac{1}{10}\cos(x)^{10} + \frac{1}{4}\cos(x)^8 - \frac{1}{6}\cos(x)^6$$

```
[In] integrate(cos(x)^5*sin(x)^5,x, algorithm="giac")
[Out] -1/10*cos(x)^10 + 1/4*cos(x)^8 - 1/6*cos(x)^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos^5(x)\sin^5(x)\,dx = \frac{\sin(x)^{10}}{10} - \frac{\sin(x)^8}{4} + \frac{\sin(x)^6}{6}$$

```
[In] int(cos(x)^5*sin(x)^5,x)
[Out] sin(x)^6/6 - sin(x)^8/4 + sin(x)^10/10
```

3.69 $\int \sin^6(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 34

$$\int \sin^6(x)\,dx=\frac{5x}{16}-\frac{5}{16}\cos(x)\sin(x)-\frac{5}{24}\cos(x)\sin^3(x)-\frac{1}{6}\cos(x)\sin^5(x)$$

```
[Out] 5/16*x-5/16*cos(x)*sin(x)-5/24*cos(x)*sin(x)^3-1/6*cos(x)*sin(x)^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\int \sin^6(x)\,dx=\frac{5x}{16}-\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{24}\sin^3(x)\cos(x)-\frac{5}{16}\sin(x)\cos(x)$$

```
[In] Int[Sin[x]^6,x]
```

```
[Out] (5*x)/16 - (5*Cos[x]*Sin[x])/16 - (5*Cos[x]*Sin[x]^3)/24 - (Cos[x]*Sin[x]^5
)/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{6}\cos(x)\sin^5(x) + \frac{5}{6}\int \sin^4(x)\,dx \\
&= -\frac{5}{24}\cos(x)\sin^3(x) - \frac{1}{6}\cos(x)\sin^5(x) + \frac{5}{8}\int \sin^2(x)\,dx \\
&= -\frac{5}{16}\cos(x)\sin(x) - \frac{5}{24}\cos(x)\sin^3(x) - \frac{1}{6}\cos(x)\sin^5(x) + \frac{5\int 1\,dx}{16} \\
&= \frac{5x}{16} - \frac{5}{16}\cos(x)\sin(x) - \frac{5}{24}\cos(x)\sin^3(x) - \frac{1}{6}\cos(x)\sin^5(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sin^6(x)\,dx = \frac{5x}{16} - \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) - \frac{1}{192}\sin(6x)$$

```
[In] Integrate[Sin[x]^6,x]
[Out] (5*x)/16 - (15*Sin[2*x])/64 + (3*Sin[4*x])/64 - Sin[6*x]/192
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result
risch	$\frac{5x}{16}-\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}-\frac{15\sin(2x)}{64}$
parallelrisch	$\frac{5x}{16}-\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}-\frac{15\sin(2x)}{64}$
default	$-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}$
norman	$\frac{\frac{5x}{16}-\frac{85\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}-\frac{33\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{33\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{85\left(\tan^9\left(\frac{x}{2}\right)\right)}{24}+\frac{5\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{8}+\frac{15x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{75x\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{25x\left(\tan^6\left(\frac{x}{2}\right)\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

```
[In] int(sin(x)^6,x,method=_RETURNVERBOSE)
[Out] 5/16*x-1/192*sin(6*x)+3/64*sin(4*x)-15/64*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \sin^6(x)\,dx = -\frac{1}{48}\left(8\,\cos\left(x\right)^5 - 26\,\cos\left(x\right)^3 + 33\,\cos\left(x\right)\right)\sin\left(x\right) + \frac{5}{16}\,x$$

```
[In] integrate(sin(x)^6,x, algorithm="fricas")
[Out] -1/48*(8*cos(x)^5 - 26*cos(x)^3 + 33*cos(x))*sin(x) + 5/16*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \sin^6(x)\,dx = \frac{5x}{16} - \frac{\sin^5\left(x\right)\cos\left(x\right)}{6} - \frac{5\sin^3\left(x\right)\cos\left(x\right)}{24} - \frac{5\sin\left(x\right)\cos\left(x\right)}{16}$$

```
[In] integrate(sin(x)**6,x)
[Out] 5*x/16 - sin(x)**5*cos(x)/6 - 5*sin(x)**3*cos(x)/24 - 5*sin(x)*cos(x)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \sin^6(x)\,dx = \frac{1}{48}\,\sin\left(2\,x\right)^3 + \frac{5}{16}\,x + \frac{3}{64}\,\sin\left(4\,x\right) - \frac{1}{4}\,\sin\left(2\,x\right)$$

```
[In] integrate(sin(x)^6,x, algorithm="maxima")
[Out] 1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) - 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \sin^6(x)\,dx = \frac{5}{16}\,x - \frac{1}{192}\,\sin\left(6\,x\right) + \frac{3}{64}\,\sin\left(4\,x\right) - \frac{15}{64}\,\sin\left(2\,x\right)$$

```
[In] integrate(sin(x)^6,x, algorithm="giac")
[Out] 5/16*x - 1/192*sin(6*x) + 3/64*sin(4*x) - 15/64*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \sin^6(x)\,dx = \frac{5\,x}{16} - \frac{15\,\sin(2\,x)}{64} + \frac{3\,\sin(4\,x)}{64} - \frac{\sin(6\,x)}{192}$$

```
[In] int(sin(x)^6,x)
[Out] (5*x)/16 - (15*sin(2*x))/64 + (3*sin(4*x))/64 - sin(6*x)/192
```

3.70 $\int \cos^6(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 34

$$\int \cos^6(x)\,dx = \frac{5x}{16} + \frac{5}{16}\cos(x)\sin(x) + \frac{5}{24}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x)$$

```
[Out] 5/16*x+5/16*cos(x)*sin(x)+5/24*cos(x)^3*sin(x)+1/6*cos(x)^5*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\int \cos^6(x)\,dx = \frac{5x}{16} + \frac{1}{6}\sin(x)\cos^5(x) + \frac{5}{24}\sin(x)\cos^3(x) + \frac{5}{16}\sin(x)\cos(x)$$

```
[In] Int[Cos[x]^6,x]
```

```
[Out] (5*x)/16 + (5*Cos[x]*Sin[x])/16 + (5*Cos[x]^3*Sin[x])/24 + (Cos[x]^5*Sin[x]
)/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{6}\cos^5(x)\sin(x) + \frac{5}{6}\int \cos^4(x)\,dx \\
&= \frac{5}{24}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x) + \frac{5}{8}\int \cos^2(x)\,dx \\
&= \frac{5}{16}\cos(x)\sin(x) + \frac{5}{24}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x) + \frac{5\int 1\,dx}{16} \\
&= \frac{5x}{16} + \frac{5}{16}\cos(x)\sin(x) + \frac{5}{24}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \cos^6(x)\,dx = \frac{5x}{16} + \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) + \frac{1}{192}\sin(6x)$$

```
[In] Integrate[Cos[x]^6,x]
[Out] (5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result
risch	$\frac{5x}{16}+\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}+\frac{15\sin(2x)}{64}$
parallelrisch	$\frac{5x}{16}+\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}+\frac{15\sin(2x)}{64}$
default	$\frac{\left(\cos^5(x)+\frac{5\left(\cos^3(x)\right)}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{6}+\frac{5x}{16}$
norman	$\frac{\frac{5x}{16}-\frac{5\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}+\frac{15\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}-\frac{15\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{5\left(\tan^9\left(\frac{x}{2}\right)\right)}{24}-\frac{11\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{8}+\frac{15x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{75x\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{25x\left(\tan^6\left(\frac{x}{2}\right)\right)}{4}+}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

```
[In] int(cos(x)^6,x,method=_RETURNVERBOSE)
[Out] 5/16*x+1/192*sin(6*x)+3/64*sin(4*x)+15/64*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \cos^6(x)\,dx = \frac{1}{48}\left(8\cos(x)^5 + 10\cos(x)^3 + 15\cos(x)\right)\sin(x) + \frac{5}{16}\,x$$

```
[In] integrate(cos(x)^6,x, algorithm="fricas")
[Out] 1/48*(8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x))*sin(x) + 5/16*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \cos^6(x)\,dx = \frac{5x}{16} + \frac{\sin(x)\cos^5(x)}{6} + \frac{5\sin(x)\cos^3(x)}{24} + \frac{5\sin(x)\cos(x)}{16}$$

```
[In] integrate(cos(x)**6,x)
[Out] 5*x/16 + sin(x)*cos(x)**5/6 + 5*sin(x)*cos(x)**3/24 + 5*sin(x)*cos(x)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \cos^6(x)\,dx = -\frac{1}{48}\sin(2\,x)^3 + \frac{5}{16}\,x + \frac{3}{64}\sin(4\,x) + \frac{1}{4}\sin(2\,x)$$

```
[In] integrate(cos(x)^6,x, algorithm="maxima")
[Out] -1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) + 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \cos^6(x)\,dx = \frac{5}{16}\,x + \frac{1}{192}\sin(6\,x) + \frac{3}{64}\sin(4\,x) + \frac{15}{64}\sin(2\,x)$$

```
[In] integrate(cos(x)^6,x, algorithm="giac")
[Out] 5/16*x + 1/192*sin(6*x) + 3/64*sin(4*x) + 15/64*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \cos^6(x)\,dx = \frac{5\,x}{16} + \frac{15\,\sin\left(2\,x\right)}{64} + \frac{3\,\sin\left(4\,x\right)}{64} + \frac{\sin\left(6\,x\right)}{192}$$

```
[In] int(cos(x)^6,x)
[Out] (5*x)/16 + (15*sin(2*x))/64 + (3*sin(4*x))/64 + sin(6*x)/192
```

3.71 $\int \cos^4(2x)\sin^2(2x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \cos^4(2x)\sin^2(2x)\,dx = \frac{x}{16} + \frac{1}{32}\cos(2x)\sin(2x) + \frac{1}{48}\cos^3(2x)\sin(2x) - \frac{1}{12}\cos^5(2x)\sin(2x)$$

```
[Out] 1/16*x+1/32*cos(2*x)*sin(2*x)+1/48*cos(2*x)^3*sin(2*x)-1/12*cos(2*x)^5*sin(
2*x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2648, 2715, 8}

$$\int \cos^4(2x)\sin^2(2x)\,dx = \frac{x}{16} - \frac{1}{12}\sin(2x)\cos^5(2x) + \frac{1}{48}\sin(2x)\cos^3(2x) + \frac{1}{32}\sin(2x)\cos(2x)$$

```
[In] Int[Cos[2*x]^4*Sin[2*x]^2,x]
```

```
[Out] x/16 + (Cos[2*x]*Sin[2*x])/32 + (Cos[2*x]^3*Sin[2*x])/48 - (Cos[2*x]^5*Sin[
2*x])/12
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
```

```
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{12}\cos^5(2x)\sin(2x) + \frac{1}{6}\int \cos^4(2x)\,dx \\
&= \frac{1}{48}\cos^3(2x)\sin(2x) - \frac{1}{12}\cos^5(2x)\sin(2x) + \frac{1}{8}\int \cos^2(2x)\,dx \\
&= \frac{1}{32}\cos(2x)\sin(2x) + \frac{1}{48}\cos^3(2x)\sin(2x) - \frac{1}{12}\cos^5(2x)\sin(2x) + \frac{\int 1\,dx}{16} \\
&= \frac{x}{16} + \frac{1}{32}\cos(2x)\sin(2x) + \frac{1}{48}\cos^3(2x)\sin(2x) - \frac{1}{12}\cos^5(2x)\sin(2x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

$$\int \cos^4(2x)\sin^2(2x)\,dx = \frac{x}{16} + \frac{1}{128}\sin(4x) - \frac{1}{128}\sin(8x) - \frac{1}{384}\sin(12x)$$

```
[In] Integrate[Cos[2*x]^4*Sin[2*x]^2,x]
[Out] x/16 + Sin[4*x]/128 - Sin[8*x]/128 - Sin[12*x]/384
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

method	result
risch	$\frac{x}{16}-\frac{\sin(12x)}{384}-\frac{\sin(8x)}{128}+\frac{\sin(4x)}{128}$
parallelrisch	$\frac{x}{16}-\frac{\sin(12x)}{384}-\frac{\sin(8x)}{128}+\frac{\sin(4x)}{128}$
derivativedivides	$-\frac{(\cos^5(2x))\sin(2x)}{12}+\frac{\left(\cos^3(2x)+\frac{3\cos(2x)}{2}\right)\sin(2x)}{48}+\frac{x}{16}$
default	$-\frac{(\cos^5(2x))\sin(2x)}{12}+\frac{\left(\cos^3(2x)+\frac{3\cos(2x)}{2}\right)\sin(2x)}{48}+\frac{x}{16}$
norman	$\frac{\frac{x}{16}+\frac{47\left(\tan^3(x)\right)}{48}-\frac{13\left(\tan^5(x)\right)}{8}+\frac{13\left(\tan^7(x)\right)}{8}-\frac{47\left(\tan^9(x)\right)}{48}+\frac{\left(\tan^{11}(x)\right)}{16}+\frac{3x\left(\tan^2(x)\right)}{8}+\frac{15x\left(\tan^4(x)\right)}{16}+\frac{5x\left(\tan^6(x)\right)}{4}+\frac{15x(}{}}{(1+\tan^2(x))^6}$

[In] int(cos(2*x)^4*sin(2*x)^2,x,method=_RETURNVERBOSE)

[Out] 1/16*x-1/384*sin(12*x)-1/128*sin(8*x)+1/128*sin(4*x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \cos^4(2x)\sin^2(2x)\,dx = -\frac{1}{96}\left(8\,\cos\left(2\,x\right)^5-2\,\cos\left(2\,x\right)^3-3\,\cos\left(2\,x\right)\right)\sin\left(2\,x\right)+\frac{1}{16}\,x$$

[In] integrate(cos(2*x)^4*sin(2*x)^2,x, algorithm="fricas")

[Out] -1/96*(8*cos(2*x)^5 - 2*cos(2*x)^3 - 3*cos(2*x))*sin(2*x) + 1/16*x

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \cos^4(2x)\sin^2(2x)\,dx = \frac{x}{16}-\frac{\sin\left(2x\right)\cos^5\left(2x\right)}{12}+\frac{\sin\left(2x\right)\cos^3\left(2x\right)}{48}+\frac{\sin\left(2x\right)\cos\left(2x\right)}{32}$$

[In] integrate(cos(2*x)**4*sin(2*x)**2,x)

[Out] x/16 - sin(2*x)*cos(2*x)**5/12 + sin(2*x)*cos(2*x)**3/48 + sin(2*x)*cos(2*x
)/32

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \cos^4(2x)\sin^2(2x)\,dx = \frac{1}{96}\,\sin\left(4\,x\right)^3 + \frac{1}{16}\,x - \frac{1}{128}\,\sin\left(8\,x\right)$$

```
[In] integrate(cos(2*x)^4*sin(2*x)^2,x, algorithm="maxima")
[Out] 1/96*sin(4*x)^3 + 1/16*x - 1/128*sin(8*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \cos^4(2x)\sin^2(2x)\,dx = \frac{1}{16}\,x - \frac{1}{384}\,\sin\left(12\,x\right) - \frac{1}{128}\,\sin\left(8\,x\right) + \frac{1}{128}\,\sin\left(4\,x\right)$$

```
[In] integrate(cos(2*x)^4*sin(2*x)^2,x, algorithm="giac")
[Out] 1/16*x - 1/384*sin(12*x) - 1/128*sin(8*x) + 1/128*sin(4*x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos^4(2x)\sin^2(2x)\,dx = \frac{x}{16} - \frac{\cos\left(2\,x\right)\,\sin\left(2\,x\right)}{32} + \frac{\sin\left(2\,x\right)^3\,\left(\frac{\cos(2\,x)^3}{6} + \frac{\cos(2\,x)}{8}\right)}{2}$$

```
[In] int(cos(2*x)^4*sin(2*x)^2,x)
[Out] x/16 - (cos(2*x)*sin(2*x))/32 + (sin(2*x)^3*(cos(2*x)/8 + cos(2*x)^3/6))/2
```

3.72 $\int \sin^5(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 21

$$\int \sin^5(x)\,dx = -\cos(x) + \frac{2\cos^3(x)}{3} - \frac{\cos^5(x)}{5}$$

```
[Out] -cos(x)+2/3*cos(x)^3-1/5*cos(x)^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2713}

$$\int \sin^5(x)\,dx = -\frac{1}{5}\cos^5(x) + \frac{2\cos^3(x)}{3} - \cos(x)$$

```
[In] Int[Sin[x]^5,x]
[Out] -Cos[x] + (2*Cos[x]^3)/3 - Cos[x]^5/5
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \cos(x)\right)\\ &= -\cos(x) + \frac{2\cos^3(x)}{3} - \frac{\cos^5(x)}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^5(x)\,dx = -\frac{5\cos(x)}{8} + \frac{5}{48}\cos(3x) - \frac{1}{80}\cos(5x)$$

[In] Integrate[Sin[x]^5,x]

[Out] (-5*Cos[x])/8 + (5*Cos[3*x])/48 - Cos[5*x]/80

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\left(\frac{8}{3}+\sin^4(x)+\frac{4\left(\sin^2(x)\right)}{3}\right)\cos(x)}{5}$	17
risch	$-\frac{5\cos(x)}{8}-\frac{\cos(5x)}{80}+\frac{5\cos(3x)}{48}$	18
parallelrisch	$\frac{8}{15}-\frac{5\cos(x)}{8}+\frac{5\cos(3x)}{48}-\frac{\cos(5x)}{80}$	19
norman	$\frac{-\frac{32\left(\tan^4\left(\frac{x}{2}\right)\right)}{3}-\frac{16\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}-\frac{16}{15}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^5}$	30

[In] int(sin(x)^5,x,method=_RETURNVERBOSE)

[Out] -1/5*(8/3+sin(x)^4+4/3*sin(x)^2)*cos(x)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin^5(x)\,dx = -\frac{1}{5}\cos(x)^5 + \frac{2}{3}\cos(x)^3 - \cos(x)$$

[In] integrate(sin(x)^5,x, algorithm="fricas")

[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin^5(x)\,dx = -\frac{\cos^5(x)}{5} + \frac{2\cos^3(x)}{3} - \cos(x)$$

```
[In] integrate(sin(x)**5,x)
[Out] -cos(x)**5/5 + 2*cos(x)**3/3 - cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin^5(x)\,dx = -\frac{1}{5}\cos(x)^5 + \frac{2}{3}\cos(x)^3 - \cos(x)$$

```
[In] integrate(sin(x)^5,x, algorithm="maxima")
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin^5(x)\,dx = -\frac{1}{5}\cos(x)^5 + \frac{2}{3}\cos(x)^3 - \cos(x)$$

```
[In] integrate(sin(x)^5,x, algorithm="giac")
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin^5(x)\,dx = -\frac{\cos(x)^5}{5} + \frac{2\cos(x)^3}{3} - \cos(x)$$

```
[In] int(sin(x)^5,x)
[Out] (2*cos(x)^3)/3 - cos(x) - cos(x)^5/5
```

3.73 $\int \cos^4(x) \sin^4(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 46

$$\int \cos^4(x) \sin^4(x) \, dx = \frac{3x}{128} + \frac{3}{128} \cos(x) \sin(x) + \frac{1}{64} \cos^3(x) \sin(x) - \frac{1}{16} \cos^5(x) \sin(x) - \frac{1}{8} \cos^5(x) \sin^3(x)$$

```
[Out] 3/128*x+3/128*cos(x)*sin(x)+1/64*cos(x)^3*sin(x)-1/16*cos(x)^5*sin(x)-1/8*c
os(x)^5*sin(x)^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2648, 2715, 8}

$$\int \cos^4(x) \sin^4(x) \, dx = \frac{3x}{128} - \frac{1}{8} \sin^3(x) \cos^5(x) - \frac{1}{16} \sin(x) \cos^5(x) + \frac{1}{64} \sin(x) \cos^3(x) + \frac{3}{128} \sin(x) \cos(x)$$

```
[In] Int[Cos[x]^4*Sin[x]^4,x]
```

```
[Out] (3*x)/128 + (3*Cos[x]*Sin[x])/128 + (Cos[x]^3*Sin[x])/64 - (Cos[x]^5*Sin[x]
)/16 - (Cos[x]^5*Sin[x]^3)/8
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{8}\cos^5(x)\sin^3(x) + \frac{3}{8}\int \cos^4(x)\sin^2(x)\,dx \\
&= -\frac{1}{16}\cos^5(x)\sin(x) - \frac{1}{8}\cos^5(x)\sin^3(x) + \frac{1}{16}\int \cos^4(x)\,dx \\
&= \frac{1}{64}\cos^3(x)\sin(x) - \frac{1}{16}\cos^5(x)\sin(x) - \frac{1}{8}\cos^5(x)\sin^3(x) + \frac{3}{64}\int \cos^2(x)\,dx \\
&= \frac{3}{128}\cos(x)\sin(x) + \frac{1}{64}\cos^3(x)\sin(x) - \frac{1}{16}\cos^5(x)\sin(x) - \frac{1}{8}\cos^5(x)\sin^3(x) + \frac{3\int 1\,dx}{128} \\
&= \frac{3x}{128} + \frac{3}{128}\cos(x)\sin(x) + \frac{1}{64}\cos^3(x)\sin(x) - \frac{1}{16}\cos^5(x)\sin(x) - \frac{1}{8}\cos^5(x)\sin^3(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{3x}{128} - \frac{1}{128}\sin(4x) + \frac{\sin(8x)}{1024}$$

```
[In] Integrate[Cos[x]^4*Sin[x]^4,x]
[Out] (3*x)/128 - Sin[4*x]/128 + Sin[8*x]/1024
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.37

method	result
risch	$\frac{3x}{128}+\frac{\sin(8x)}{1024}-\frac{\sin(4x)}{128}$
parallelrisch	$\frac{3x}{128}+\frac{\sin(8x)}{1024}-\frac{\sin(4x)}{128}$
default	$-\frac{(\cos^5(x))(\sin^3(x))}{8}-\frac{(\cos^5(x))\sin(x)}{16}+\frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{64}+\frac{3x}{128}$
norman	$\frac{\frac{3x}{128}-\frac{23\left(\tan^3\left(\frac{x}{2}\right)\right)}{64}+\frac{333\left(\tan^5\left(\frac{x}{2}\right)\right)}{64}-\frac{671\left(\tan^7\left(\frac{x}{2}\right)\right)}{64}+\frac{671\left(\tan^9\left(\frac{x}{2}\right)\right)}{64}-\frac{333\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{64}+\frac{23\left(\tan^{13}\left(\frac{x}{2}\right)\right)}{64}+\frac{3\left(\tan^{15}\left(\frac{x}{2}\right)\right)}{64}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right.\right.}{16}}{(1+\text{ta}}$

[In] int(cos(x)^4*sin(x)^4,x,method=_RETURNVERBOSE)

[Out] 3/128*x+1/1024*sin(8*x)-1/128*sin(4*x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\sin^4(x)\,dx=\frac{1}{128}\left(16\cos(x)^7-24\cos(x)^5+2\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{3}{128}x$$

[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="fricas")

[Out] 1/128*(16*cos(x)^7 - 24*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 3/128*x

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\sin^4(x)\,dx=\frac{3x}{128}-\frac{\sin^3(2x)\cos(2x)}{128}-\frac{3\sin(2x)\cos(2x)}{256}$$

[In] integrate(cos(x)**4*sin(x)**4,x)

[Out] 3*x/128 - sin(2*x)**3*cos(2*x)/128 - 3*sin(2*x)*cos(2*x)/256

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.35

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{3}{128}\,x + \frac{1}{1024}\,\sin\left(8\,x\right) - \frac{1}{128}\,\sin\left(4\,x\right)$$

```
[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="maxima")
[Out] 3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.35

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{3}{128}\,x + \frac{1}{1024}\,\sin\left(8\,x\right) - \frac{1}{128}\,\sin\left(4\,x\right)$$

```
[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="giac")
[Out] 3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \cos^4(x)\sin^4(x)\,dx = \left(\frac{\cos\left(x\right)^3}{8} + \frac{\cos\left(x\right)}{16}\right)\,\sin\left(x\right)^5 + \frac{3\,x}{128} - \frac{\sin\left(2\,x\right)}{64} + \frac{\sin\left(4\,x\right)}{512}$$

```
[In] int(cos(x)^4*sin(x)^4,x)
[Out] (3*x)/128 - sin(2*x)/64 + sin(4*x)/512 + sin(x)^5*(cos(x)/16 + cos(x)^3/8)
```

3.74 $\int \sqrt{\cos(x)}\sin^3(x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \sqrt{\cos(x)}\sin^3(x)\,dx = -\frac{2}{3}\cos^{\frac{3}{2}}(x) + \frac{2}{7}\cos^{\frac{7}{2}}(x)$$

```
[Out] -2/3*cos(x)^(3/2)+2/7*cos(x)^(7/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2645, 14}

$$\int \sqrt{\cos(x)}\sin^3(x)\,dx = \frac{2}{7}\cos^{\frac{7}{2}}(x) - \frac{2}{3}\cos^{\frac{3}{2}}(x)$$

```
[In] Int[Sqrt[Cos[x]]*Sin[x]^3,x]
[Out] (-2*Cos[x]^(3/2))/3 + (2*Cos[x]^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \sqrt{x}\left(1-x^2\right)\,dx, x, \cos(x)\right)\\
&= -\text{Subst}\left(\int \left(\sqrt{x}-x^{5/2}\right)\,dx, x, \cos(x)\right)\\
&= -\frac{2}{3}\cos^{\frac{3}{2}}(x)+\frac{2}{7}\cos^{\frac{7}{2}}(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$
\int \sqrt{\cos(x)}\sin^3(x)\,dx = \frac{8\sqrt[4]{\cos^2(x)}+\cos^2(x)(-11+3\cos(2x))}{21\sqrt{\cos(x)}}
$$

```
[In] Integrate[Sqrt[Cos[x]]*Sin[x]^3,x]
[Out] (8*(Cos[x]^2)^(1/4) + Cos[x]^2*(-11 + 3*Cos[2*x]))/(21*Sqrt[Cos[x]])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$-\frac{2\left(\cos^{\frac{3}{2}}(x)\right)}{3}+\frac{2\left(\cos^{\frac{7}{2}}(x)\right)}{7}$	14
default	$-\frac{2\left(\cos^{\frac{3}{2}}(x)\right)}{3}+\frac{2\left(\cos^{\frac{7}{2}}(x)\right)}{7}$	14

```
[In] int(sin(x)^3*cos(x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*cos(x)^(3/2)+2/7*cos(x)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sqrt{\cos(x)}\sin^3(x)\,dx = \frac{2}{21}\left(3\,\cos(x)^3 - 7\,\cos(x)\right)\sqrt{\cos(x)}$$

```
[In] integrate(sin(x)^3*cos(x)^(1/2),x, algorithm="fricas")
[Out] 2/21*(3*cos(x)^3 - 7*cos(x))*sqrt(cos(x))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(x)}\sin^3(x)\,dx = \text{Timed out}$$

```
[In] integrate(sin(x)**3*cos(x)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{\cos(x)}\sin^3(x)\,dx = \frac{2}{7}\,\cos(x)^{\frac{7}{2}} - \frac{2}{3}\,\cos(x)^{\frac{3}{2}}$$

```
[In] integrate(sin(x)^3*cos(x)^(1/2),x, algorithm="maxima")
[Out] 2/7*cos(x)^(7/2) - 2/3*cos(x)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{\cos(x)}\sin^3(x)\,dx = \frac{2}{7}\,\cos(x)^{\frac{7}{2}} - \frac{2}{3}\,\cos(x)^{\frac{3}{2}}$$

```
[In] integrate(sin(x)^3*cos(x)^(1/2),x, algorithm="giac")
[Out] 2/7*cos(x)^(7/2) - 2/3*cos(x)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{\cos(x)}\sin^3(x)\,dx = \cos(x)^{3/2}\left(\frac{2\cos(x)^2}{7}-\frac{2}{3}\right)$$

```
[In] int(cos(x)^(1/2)*sin(x)^3,x)
[Out] cos(x)^(3/2)*((2*cos(x)^2)/7 - 2/3)
```

3.75 $\int \cos^3(x)\sqrt{\sin(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = \frac{2}{3}\sin^{\frac{3}{2}}(x) - \frac{2}{7}\sin^{\frac{7}{2}}(x)$$

```
[Out] 2/3*sin(x)^(3/2)-2/7*sin(x)^(7/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2644, 14}

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = \frac{2}{3}\sin^{\frac{3}{2}}(x) - \frac{2}{7}\sin^{\frac{7}{2}}(x)$$

```
[In] Int[Cos[x]^3*Sqrt[Sin[x]],x]
```

```
[Out] (2*Sin[x]^(3/2))/3 - (2*Sin[x]^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \sqrt{x}\left(1-x^2\right)\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \left(\sqrt{x}-x^{5/2}\right)\,dx, x, \sin(x)\right)\\
&= \frac{2}{3}\sin^{\frac{3}{2}}(x)-\frac{2}{7}\sin^{\frac{7}{2}}(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = \frac{1}{21}(11+3\cos(2x))\sin^{\frac{3}{2}}(x)$$

```
[In] Integrate[Cos[x]^3*Sqrt[Sin[x]],x]
[Out] ((11 + 3*Cos[2*x])*Sin[x]^(3/2))/21
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2\left(\sin^{\frac{3}{2}}(x)\right)}{3}-\frac{2\left(\sin^{\frac{7}{2}}(x)\right)}{7}$	14
default	$\frac{2\left(\sin^{\frac{3}{2}}(x)\right)}{3}-\frac{2\left(\sin^{\frac{7}{2}}(x)\right)}{7}$	14

```
[In] int(cos(x)^3*sin(x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*sin(x)^(3/2)-2/7*sin(x)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = \frac{2}{21}\left(3\,\cos\left(x\right)^2+4\right)\sin\left(x\right)^{\frac{3}{2}}$$

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="fricas")
[Out] 2/21*(3*cos(x)^2 + 4)*sin(x)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(19) = 38.

Time = 4.18 (sec) , antiderivative size = 170, normalized size of antiderivative = 8.10

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = \frac{28\sqrt{2}\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^5\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21} + \frac{8\sqrt{2}\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^3\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21} + \frac{28\sqrt{2}\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21}$$

```
[In] integrate(cos(x)**3*sin(x)**(1/2),x)
[Out] 28*sqrt(2)*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**5/(21*tan(x/2)**6 + 6
3*tan(x/2)**4 + 63*tan(x/2)**2 + 21) + 8*sqrt(2)*sqrt(tan(x/2)/(tan(x/2)**2
 + 1))*tan(x/2)**3/(21*tan(x/2)**6 + 63*tan(x/2)**4 + 63*tan(x/2)**2 + 21)
+ 28*sqrt(2)*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)/(21*tan(x/2)**6 + 63
*tan(x/2)**4 + 63*tan(x/2)**2 + 21)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = -\frac{2}{7}\sin(x)^{\frac{7}{2}} + \frac{2}{3}\sin(x)^{\frac{3}{2}}$$

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="maxima")
[Out] -2/7*sin(x)^(7/2) + 2/3*sin(x)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = -\frac{2}{7}\sin(x)^{\frac{7}{2}} + \frac{2}{3}\sin(x)^{\frac{3}{2}}$$

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="giac")
[Out] -2/7*sin(x)^(7/2) + 2/3*sin(x)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = -\frac{\cos(x)^4\sin(x)^{3/2}\,{}_2F_1\left(\frac{1}{4},2;\ 3;\ \cos(x)^2\right)}{4\left(\sin(x)^2\right)^{3/4}}$$

```
[In] int(cos(x)^3*sin(x)^(1/2),x)
[Out] -(cos(x)^4*sin(x)^(3/2)*hypergeom([1/4, 2], 3, cos(x)^2))/(4*(sin(x)^2)^(3/
4))
```

3.76 $\int \frac{\cos^2(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 19

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sqrt{x} + \cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)$$

```
[Out] cos(x^(1/2))*sin(x^(1/2))+x^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3461, 2715, 8}

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sqrt{x} + \sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)$$

```
[In] Int[Cos[Sqrt[x]]^2/Sqrt[x],x]
```

```
[Out] Sqrt[x] + Cos[Sqrt[x]]*Sin[Sqrt[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \cos^2(x)\,dx, x, \sqrt{x}\right)\\ &= \cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right) + \text{Subst}\left(\int 1\,dx, x, \sqrt{x}\right)\\ &= \sqrt{x} + \cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sqrt{x} + \frac{1}{2}\sin\left(2\sqrt{x}\right)$$

```
[In] Integrate[Cos[Sqrt[x]]^2/Sqrt[x],x]
[Out] Sqrt[x] + Sin[2*Sqrt[x]]/2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)+\sqrt{x}$	14
default	$\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)+\sqrt{x}$	14

```
[In] int(cos(x^(1/2))^2/x^(1/2),x,method=_RETURNVERBOSE)
[Out] cos(x^(1/2))*sin(x^(1/2))+x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right) + \sqrt{x}$$

```
[In] integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="fricas")
[Out] cos(sqrt(x))*sin(sqrt(x)) + sqrt(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sqrt{x}\sin^2\left(\sqrt{x}\right) + \sqrt{x}\cos^2\left(\sqrt{x}\right) + \sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x**(1/2))**2/x**(1/2),x)
[Out] sqrt(x)*sin(sqrt(x))**2 + sqrt(x)*cos(sqrt(x))**2 + sin(sqrt(x))*cos(sqrt(x
))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sqrt{x} + \frac{1}{2}\sin\left(2\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="maxima")
[Out] sqrt(x) + 1/2*sin(2*sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sqrt{x} + \frac{1}{2}\sin\left(2\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="giac")
[Out] sqrt(x) + 1/2*sin(2*sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{\sin\left(2\sqrt{x}\right)}{2} + \sqrt{x}$$

```
[In] int(cos(x^(1/2))^2/x^(1/2),x)
[Out] sin(2*x^(1/2))/2 + x^(1/2)
```

3.77 $\int x \sin^3(x^2)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int x \sin^3(x^2)\, dx = -\frac{1}{2}\cos(x^2) + \frac{1}{6}\cos^3(x^2)$$

```
[Out] -1/2*cos(x^2)+1/6*cos(x^2)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3460, 2713}

$$\int x \sin^3(x^2)\, dx = \frac{1}{6}\cos^3(x^2) - \frac{\cos(x^2)}{2}$$

```
[In] Int[x*Sin[x^2]^3,x]
```

```
[Out] -1/2*Cos[x^2] + Cos[x^2]^3/6
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \sin^3(x)\,dx, x, x^2\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int (1-x^2)\,dx, x, \cos(x^2)\right)\right)$$
$$= -\frac{1}{2}\cos(x^2) + \frac{1}{6}\cos^3(x^2)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x\sin^3(x^2)\,dx = -\frac{3}{8}\cos(x^2) + \frac{1}{24}\cos(3x^2)$$

```
[In] Integrate[x*Sin[x^2]^3,x]
[Out] (-3*Cos[x^2])/8 + Cos[3*x^2]/24
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{(2+\sin^2(x^2))\cos(x^2)}{6}$	15
default	$-\frac{(2+\sin^2(x^2))\cos(x^2)}{6}$	15
risch	$-\frac{3\cos(x^2)}{8}+\frac{\cos(3x^2)}{24}$	16
parallelrisch	$-\frac{1}{3}-\frac{3\cos(x^2)}{8}+\frac{\cos(3x^2)}{24}$	17
norman	$\frac{-2\left(\tan^2\left(\frac{x^2}{2}\right)\right)-\frac{2}{3}}{\left(1+\tan^2\left(\frac{x^2}{2}\right)\right)^3}$	26

```
[In] int(x*sin(x^2)^3,x,method=_RETURNVERBOSE)
[Out] -1/6*(2+sin(x^2)^2)*cos(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x\sin^3\left(x^2\right)\,dx = \frac{1}{6}\cos\left(x^2\right)^3 - \frac{1}{2}\cos\left(x^2\right)$$

```
[In] integrate(x*sin(x^2)^3,x, algorithm="fricas")
[Out] 1/6*cos(x^2)^3 - 1/2*cos(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int x\sin^3\left(x^2\right)\,dx = -\frac{\sin^2\left(x^2\right)\cos\left(x^2\right)}{2} - \frac{\cos^3\left(x^2\right)}{3}$$

```
[In] integrate(x*sin(x**2)**3,x)
[Out] -sin(x**2)**2*cos(x**2)/2 - cos(x**2)**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x\sin^3\left(x^2\right)\,dx = \frac{1}{24}\cos\left(3\,x^2\right) - \frac{3}{8}\cos\left(x^2\right)$$

```
[In] integrate(x*sin(x^2)^3,x, algorithm="maxima")
[Out] 1/24*cos(3*x^2) - 3/8*cos(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x\sin^3\left(x^2\right)\,dx = \frac{1}{6}\cos\left(x^2\right)^3 - \frac{1}{2}\cos\left(x^2\right)$$

```
[In] integrate(x*sin(x^2)^3,x, algorithm="giac")
[Out] 1/6*cos(x^2)^3 - 1/2*cos(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int x \sin^3\left(x^2\right) \, dx = \frac{\cos\left(x^2\right) \left(\cos\left(x^2\right)^2 - 3\right)}{6}$$

```
[In] int(x*sin(x^2)^3,x)
```

```
[Out] (cos(x^2)*(cos(x^2)^2 - 3))/6
```

3.78 $\int \sin^2(x)\tan(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \sin^2(x)\tan(x)\,dx = \frac{\cos^2(x)}{2} - \log(\cos(x))$$

```
[Out] 1/2*cos(x)^2-ln(cos(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2670, 14}

$$\int \sin^2(x)\tan(x)\,dx = \frac{\cos^2(x)}{2} - \log(\cos(x))$$

```
[In] Int[Sin[x]^2*Tan[x],x]
```

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1-x^2}{x} \, dx, x, \cos(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{x}-x\right) \, dx, x, \cos(x)\right)\\
&= \frac{\cos^2(x)}{2} - \log(\cos(x))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sin^2(x) \tan(x) \, dx = \frac{\cos^2(x)}{2} - \log(\cos(x))$$

```
[In] Integrate[Sin[x]^2*Tan[x],x]
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{(\sin^2(x))}{2} - \ln(\cos(x))$	13
risch	$ix + \frac{e^{2ix}}{8} + \frac{e^{-2ix}}{8} - \ln(e^{2ix}+1)$	30

```
[In] int(cos(x)^2*tan(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*sin(x)^2-ln(cos(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sin^2(x) \tan(x) \, dx = \frac{1}{2} \cos(x)^2 - \log(-\cos(x))$$

```
[In] integrate(cos(x)^2*tan(x)^3,x, algorithm="fricas")
[Out] 1/2*cos(x)^2 - log(-cos(x))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\tan(x)\,dx = -\log\left(\cos\left(x\right)\right) + \frac{\cos^2\left(x\right)}{2}$$

```
[In] integrate(cos(x)**2*tan(x)**3,x)
[Out] -log(cos(x)) + cos(x)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \sin^2(x)\tan(x)\,dx = -\frac{1}{2}\sin\left(x\right)^2 - \frac{1}{2}\log\left(\sin\left(x\right)^2 - 1\right)$$

```
[In] integrate(cos(x)^2*tan(x)^3,x, algorithm="maxima")
[Out] -1/2*sin(x)^2 - 1/2*log(sin(x)^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \sin^2(x)\tan(x)\,dx = \frac{1}{2}\cos\left(x\right)^2 - \log\left(\left|\cos\left(x\right)\right|\right)$$

```
[In] integrate(cos(x)^2*tan(x)^3,x, algorithm="giac")
[Out] 1/2*cos(x)^2 - log(abs(cos(x)))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \sin^2(x)\tan(x)\,dx = \frac{\cos\left(x\right)^2}{2} + \frac{\ln\left(\tan(x)^2 + 1\right)}{2}$$

```
[In] int(cos(x)^2*tan(x)^3,x)
[Out] log(tan(x)^2 + 1)/2 + cos(x)^2/2
```

3.79 $\int \cos^2(x) \cot^3(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 22

$$\int \cos^2(x) \cot^3(x) \, dx = -\frac{1}{2} \csc^2(x) - 2 \log(\sin(x)) + \frac{\sin^2(x)}{2}$$

```
[Out] -1/2*csc(x)^2-2*ln(sin(x))+1/2*sin(x)^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2670, 272, 45}

$$\int \cos^2(x) \cot^3(x) \, dx = \frac{\sin^2(x)}{2} - \frac{1}{2} \csc^2(x) - 2 \log(\sin(x))$$

```
[In] Int[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] -1/2*Csc[x]^2 - 2*Log[Sin[x]] + Sin[x]^2/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1-x^2)^2}{x^3}\,dx, x, -\sin(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, \sin^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)\,dx, x, \sin^2(x)\right)\\
&= -\frac{1}{2}\csc^2(x) - 2\log(\sin(x)) + \frac{\sin^2(x)}{2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \cos^2(x)\cot^3(x)\,dx = \frac{1}{2}\left(-\csc^2(x) - 4\log(\sin(x)) + \sin^2(x)\right)$$

```
[In] Integrate[Cos[x]^2*Cot[x]^3,x]
[Out] (-Csc[x]^2 - 4*Log[Sin[x]] + Sin[x]^2)/2
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

method	result	size
default	$-\frac{\cos^6(x)}{2\sin(x)^2}-\frac{(\cos^4(x))}{2}-(\cos^2(x))-2\ln(\sin(x))$	29
risch	$2ix-\frac{e^{2ix}}{8}-\frac{e^{-2ix}}{8}+\frac{2\,e^{2ix}}{(e^{2ix}-1)^2}-2\ln(e^{2ix}-1)$	46

```
[In] int(cot(x)^5*sin(x)^2,x,method=_RETURNVERBOSE)
[Out] -1/2/sin(x)^2*cos(x)^6-1/2*cos(x)^4-cos(x)^2-2*ln(sin(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \cos^2(x)\cot^3(x)\,dx = -\frac{2\cos(x)^4 - 3\cos(x)^2 + 8\left(\cos(x)^2 - 1\right)\log\left(\frac{1}{2}\sin(x)\right) - 1}{4\left(\cos(x)^2 - 1\right)}$$

```
[In] integrate(cot(x)^5*sin(x)^2,x, algorithm="fricas")
[Out] -1/4*(2*cos(x)^4 - 3*cos(x)^2 + 8*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos(
x)^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \cos^2(x)\cot^3(x)\,dx = -2\log(\sin(x)) + \frac{\sin^2(x)}{2} - \frac{1}{2\sin^2(x)}$$

```
[In] integrate(cot(x)**5*sin(x)**2,x)
[Out] -2*log(sin(x)) + sin(x)**2/2 - 1/(2*sin(x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \cos^2(x)\cot^3(x)\,dx = \frac{1}{2}\sin(x)^2 - \frac{1}{2\sin(x)^2} - \log\left(\sin(x)^2\right)$$

```
[In] integrate(cot(x)^5*sin(x)^2,x, algorithm="maxima")
[Out] 1/2*sin(x)^2 - 1/2/sin(x)^2 - log(sin(x)^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \cos^2(x)\cot^3(x)\,dx = -\frac{1}{2}\,\cos{(x)}^2 + \frac{1}{2\,\left(\cos{(x)}^2 - 1\right)} - \log\left(-\cos{(x)}^2 + 1\right)$$

```
[In] integrate(cot(x)^5*sin(x)^2,x, algorithm="giac")
[Out] -1/2*cos(x)^2 + 1/2/(cos(x)^2 - 1) - log(-cos(x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \cos^2(x)\cot^3(x)\,dx = \ln\left(\tan(x)^2 + 1\right) - 2\,\ln\left(\tan(x)\right) - \frac{\tan(x)^2 + \frac{1}{2}}{\tan{(x)}^4 + \tan{(x)}^2}$$

```
[In] int(cot(x)^5*sin(x)^2,x)
[Out] log(tan(x)^2 + 1) - 2*log(tan(x)) - (tan(x)^2 + 1/2)/(tan(x)^2 + tan(x)^4)
```

3.80 $\int \sec(x)(1-\sin(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 5

$$\int \sec(x)(1-\sin(x))\,dx = \log(1+\sin(x))$$

```
[Out] ln(1+sin(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2746, 31}

$$\int \sec(x)(1-\sin(x))\,dx = \log(\sin(x)+1)$$

```
[In] Int[Sec[x]*(1 - Sin[x]),x]
```

```
[Out] Log[1 + Sin[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, -\sin(x)\right)\\ &= \log(1+\sin(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \sec(x)(1-\sin(x))\,dx = \text{arctanh}(\sin(x)) + \log(\cos(x))$$

```
[In] Integrate[Sec[x]*(1 - Sin[x]),x]
[Out] ArcTanh[Sin[x]] + Log[Cos[x]]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\ln(\sin(x)+1)$	6
default	$\ln(\sin(x)+1)$	6
risch	$-ix+2\ln(i+e^{ix})$	17
norman	$2\ln\left(1+\tan\left(\frac{x}{2}\right)\right)-\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$	22
parallelrisch	$2\ln(-\cot(x)+1+\csc(x))-\ln\left(\frac{2}{\cos(x)+1}\right)$	24

```
[In] int((-sin(x)+1)/cos(x),x,method=_RETURNVERBOSE)
[Out] ln(sin(x)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \sec(x)(1-\sin(x))\,dx = \log\left(\sin\left(x\right)+1\right)$$

```
[In] integrate((1-sin(x))/cos(x),x, algorithm="fricas")
[Out] log(sin(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(5) = 10.

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 3.80

$$\int \sec(x)(1-\sin(x))\,dx = 2\log\left(\tan\left(\frac{x}{2}\right)+1\right)-\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)$$

```
[In] integrate((1-sin(x))/cos(x),x)
[Out] 2*log(tan(x/2) + 1) - log(tan(x/2)**2 + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \sec(x)(1-\sin(x))\,dx = \log\left(\sin\left(x\right)+1\right)$$

```
[In] integrate((1-sin(x))/cos(x),x, algorithm="maxima")
[Out] log(sin(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \sec(x)(1-\sin(x))\,dx = \log\left(\sin\left(x\right)+1\right)$$

```
[In] integrate((1-sin(x))/cos(x),x, algorithm="giac")
[Out] log(sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \sec(x)(1-\sin(x))\,dx = \ln\left(\sin\left(x\right)+1\right)$$

```
[In] int(-(sin(x) - 1)/cos(x),x)
[Out] log(sin(x) + 1)
```

3.81 $\int \frac{1}{1-\sin(x)} dx$

Optimal result

Integrand size = 8, antiderivative size = 11

$$\int \frac{1}{1-\sin(x)} dx = \frac{\cos(x)}{1-\sin(x)}$$

```
[Out] cos(x)/(1-sin(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2727}

$$\int \frac{1}{1-\sin(x)} dx = \frac{\cos(x)}{1-\sin(x)}$$

```
[In] Int[(1 - Sin[x])^(-1),x]
```

```
[Out] Cos[x]/(1 - Sin[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\cos(x)}{1-\sin(x)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{1}{1-\sin(x)}\,dx = \frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

```
[In] Integrate[(1 - Sin[x])^(-1),x]
[Out] (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
norman	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
parallelrisch	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
risch	$\frac{2}{e^{ix}-i}$	13

```
[In] int(1/(-sin(x)+1),x,method=_RETURNVERBOSE)
[Out] -2/(tan(1/2*x)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1}{1-\sin(x)}\,dx = \frac{\cos(x)+\sin(x)+1}{\cos(x)-\sin(x)+1}$$

```
[In] integrate(1/(1-sin(x)),x, algorithm="fricas")
[Out] (cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{1-\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

```
[In] integrate(1/(1-sin(x)),x)
[Out] -2/(tan(x/2) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1}{1-\sin(x)}\,dx = -\frac{2}{\frac{\sin(x)}{\cos(x)+1}-1}$$

```
[In] integrate(1/(1-sin(x)),x, algorithm="maxima")
[Out] -2/(sin(x)/(cos(x) + 1) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{1-\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{1}{2}\,x\right)-1}$$

```
[In] integrate(1/(1-sin(x)),x, algorithm="giac")
[Out] -2/(tan(1/2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{1-\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

```
[In] int(-1/(sin(x) - 1),x)
[Out] -2/(tan(x/2) - 1)
```

3.82 $\int \tan^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 6

$$\int \tan^2(x)\,dx = -x + \tan(x)$$

```
[Out] -x+tan(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 8}

$$\int \tan^2(x)\,dx = \tan(x) - x$$

```
[In] Int[Tan[x]^2,x]
```

```
[Out] -x + Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \tan(x) - \int 1\,dx \\ &= -x + \tan(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \tan^2(x)\,dx = -\arctan(\tan(x)) + \tan(x)$$

```
[In] Integrate[Tan[x]^2,x]
[Out] -ArcTan[Tan[x]] + Tan[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
norman	$-x+\tan(x)$	7
parallelrisch	$-x+\tan(x)$	7
derivativedivides	$\tan(x)-\arctan(\tan(x))$	9
default	$\tan(x)-\arctan(\tan(x))$	9
risch	$-x+\frac{2i}{e^{2ix}+1}$	17

```
[In] int(tan(x)^2,x,method=_RETURNVERBOSE)
[Out] -x+tan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx = -x + \tan(x)$$

```
[In] integrate(tan(x)^2,x, algorithm="fricas")
[Out] -x + tan(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \tan^2(x)\,dx = -x + \frac{\sin(x)}{\cos(x)}$$

```
[In] integrate(tan(x)**2,x)
[Out] -x + sin(x)/cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx = -x + \tan(x)$$

```
[In] integrate(tan(x)^2,x, algorithm="maxima")
[Out] -x + tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx = -x + \tan(x)$$

```
[In] integrate(tan(x)^2,x, algorithm="giac")
[Out] -x + tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx = \tan(x) - x$$

```
[In] int(tan(x)^2,x)
[Out] tan(x) - x
```

3.83 $\int \tan^4(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \tan^4(x)\,dx = x - \tan(x) + \frac{\tan^3(x)}{3}$$

```
[Out] x-tan(x)+1/3*tan(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$\int \tan^4(x)\,dx = x + \frac{\tan^3(x)}{3} - \tan(x)$$

```
[In] Int[Tan[x]^4,x]
```

```
[Out] x - Tan[x] + Tan[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\tan^3(x)}{3} - \int \tan^2(x)\,dx \\ &= -\tan(x) + \frac{\tan^3(x)}{3} + \int 1\,dx \\ &= x - \tan(x) + \frac{\tan^3(x)}{3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \tan^4(x)\,dx = \arctan(\tan(x)) - \tan(x) + \frac{\tan^3(x)}{3}$$

```
[In] Integrate[Tan[x]^4,x]
[Out] ArcTan[Tan[x]] - Tan[x] + Tan[x]^3/3
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
norman	$x-\tan\left(x\right)+\frac{\left(\tan^{3}\left(x\right)\right)}{3}$	13
parallelrisch	$x-\tan\left(x\right)+\frac{\left(\tan^{3}\left(x\right)\right)}{3}$	13
derivativedivides	$\frac{\left(\tan^{3}\left(x\right)\right)}{3}-\tan\left(x\right)+\arctan\left(\tan\left(x\right)\right)$	15
default	$\frac{\left(\tan^{3}\left(x\right)\right)}{3}-\tan\left(x\right)+\arctan\left(\tan\left(x\right)\right)$	15
risch	$x-\frac{4i\left(3\,\mathrm{e}^{4ix}+3\,\mathrm{e}^{2ix}+2\right)}{3\left(\mathrm{e}^{2ix}+1\right)^{3}}$	31

```
[In] int(tan(x)^4,x,method=_RETURNVERBOSE)
[Out] x-tan(x)+1/3*tan(x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\,dx = \frac{1}{3}\tan\left(x\right)^3 + x - \tan\left(x\right)$$

```
[In] integrate(tan(x)^4,x, algorithm="fricas")
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \tan^4(x)\,dx = x + \frac{\sin^3\left(x\right)}{3\cos^3\left(x\right)} - \frac{\sin\left(x\right)}{\cos\left(x\right)}$$

```
[In] integrate(tan(x)**4,x)
[Out] x + sin(x)**3/(3*cos(x)**3) - sin(x)/cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\,dx = \frac{1}{3}\tan\left(x\right)^3 + x - \tan\left(x\right)$$

```
[In] integrate(tan(x)^4,x, algorithm="maxima")
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\,dx = \frac{1}{3}\tan\left(x\right)^3 + x - \tan\left(x\right)$$

```
[In] integrate(tan(x)^4,x, algorithm="giac")
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\,dx = \frac{\tan(x)^3}{3} - \tan(x) + x$$

```
[In] int(tan(x)^4,x)
[Out] x - tan(x) + tan(x)^3/3
```

3.84 $\int \sec^4(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 11

$$\int \sec^4(x)\,dx=\tan(x)+\frac{\tan^3(x)}{3}$$

```
[Out] tan(x)+1/3*tan(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3852}

$$\int \sec^4(x)\,dx=\frac{\tan^3(x)}{3}+\tan(x)$$

```
[In] Int[Sec[x]^4,x]
```

```
[Out] Tan[x] + Tan[x]^3/3
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=-\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(x)\right)\\&=\tan(x)+\frac{\tan^3(x)}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sec^4(x)\,dx = \tan(x) + \frac{\tan^3(x)}{3}$$

```
[In] Integrate[Sec[x]^4,x]
[Out] Tan[x] + Tan[x]^3/3
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{\tan(x)(2+\sec^2(x))}{3}$	11
default	$-\left(-\frac{2}{3}-\frac{(\sec^2(x))}{3}\right)\tan(x)$	13
risch	$\frac{4i(3\,\mathrm{e}^{2ix}+1)}{3(\mathrm{e}^{2ix}+1)^3}$	22
norman	$\frac{\frac{4\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}-2\left(\tan^5\left(\frac{x}{2}\right)\right)-2\tan\left(\frac{x}{2}\right)}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)^3}$	35

```
[In] int(sec(x)^4,x,method=_RETURNVERBOSE)
[Out] 1/3*tan(x)*(2+sec(x)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

$$\int \sec^4(x)\,dx = \frac{\left(2\,\cos\left(x\right)^2+1\right)\sin\left(x\right)}{3\,\cos\left(x\right)^3}$$

```
[In] integrate(sec(x)^4,x, algorithm="fricas")
[Out] 1/3*(2*cos(x)^2 + 1)*sin(x)/cos(x)^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \sec^4(x)\,dx = \frac{2\sin(x)}{3\cos(x)} + \frac{\sin(x)}{3\cos^3(x)}$$

```
[In] integrate(sec(x)**4,x)
[Out] 2*sin(x)/(3*cos(x)) + sin(x)/(3*cos(x)**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \sec^4(x)\,dx = \frac{1}{3}\tan(x)^3 + \tan(x)$$

```
[In] integrate(sec(x)^4,x, algorithm="maxima")
[Out] 1/3*tan(x)^3 + tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \sec^4(x)\,dx = \frac{1}{3}\tan(x)^3 + \tan(x)$$

```
[In] integrate(sec(x)^4,x, algorithm="giac")
[Out] 1/3*tan(x)^3 + tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \sec^4(x)\,dx = \frac{2\,\sin(x)\,\cos(x)^2 + \sin(x)}{3\cos(x)^3}$$

```
[In] int(1/cos(x)^4,x)
[Out] (sin(x) + 2*cos(x)^2*sin(x))/(3*cos(x)^3)
```

3.85 $\int \sec^6(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 19

$$\int \sec^6(x)\,dx = \tan(x) + \frac{2\tan^3(x)}{3} + \frac{\tan^5(x)}{5}$$

```
[Out] tan(x)+2/3*tan(x)^3+1/5*tan(x)^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{3852\}$

$$\int \sec^6(x)\,dx = \frac{\tan^5(x)}{5} + \frac{2\tan^3(x)}{3} + \tan(x)$$

```
[In] Int[Sec[x]^6,x]
```

```
[Out] Tan[x] + (2*Tan[x]^3)/3 + Tan[x]^5/5
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan(x)\right)\\ &= \tan(x) + \frac{2\tan^3(x)}{3} + \frac{\tan^5(x)}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sec^6(x)\,dx = \tan(x) + \frac{2\tan^3(x)}{3} + \frac{\tan^5(x)}{5}$$

```
[In] Integrate[Sec[x]^6,x]
[Out] Tan[x] + (2*Tan[x]^3)/3 + Tan[x]^5/5
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

method	result	size
default	$-\left(-\frac{8}{15}-\frac{(\sec^4(x))}{5}-\frac{4(\sec^2(x))}{15}\right)\tan(x)$	19
parallelrisch	$\frac{\tan(x)(3(\sec^4(x))+4(\sec^2(x))+8)}{15}$	19
risch	$\frac{16i(10\,e^{4ix}+5\,e^{2ix}+1)}{15(e^{2ix}+1)^5}$	29
norman	$\frac{\frac{8\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}-\frac{116\left(\tan^5\left(\frac{x}{2}\right)\right)}{15}+\frac{8\left(\tan^7\left(\frac{x}{2}\right)\right)}{3}-2\left(\tan^9\left(\frac{x}{2}\right)\right)-2\tan\left(\frac{x}{2}\right)}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)^5}$	51

```
[In] int(sec(x)^6,x,method=_RETURNVERBOSE)
[Out] -(-8/15-1/5*sec(x)^4-4/15*sec(x)^2)*tan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \sec^6(x)\,dx = \frac{\left(8\,\cos(x)^4 + 4\,\cos(x)^2 + 3\right)\sin(x)}{15\,\cos(x)^5}$$

```
[In] integrate(sec(x)^6,x, algorithm="fricas")
[Out] 1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/cos(x)^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \sec^6(x)\,dx = \frac{8\sin(x)}{15\cos(x)} + \frac{4\sin(x)}{15\cos^3(x)} + \frac{\sin(x)}{5\cos^5(x)}$$

```
[In] integrate(sec(x)**6,x)
[Out] 8*sin(x)/(15*cos(x)) + 4*sin(x)/(15*cos(x)**3) + sin(x)/(5*cos(x)**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \sec^6(x)\,dx = \frac{1}{5}\tan(x)^5 + \frac{2}{3}\tan(x)^3 + \tan(x)$$

```
[In] integrate(sec(x)^6,x, algorithm="maxima")
[Out] 1/5*tan(x)^5 + 2/3*tan(x)^3 + tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \sec^6(x)\,dx = \frac{1}{5}\tan(x)^5 + \frac{2}{3}\tan(x)^3 + \tan(x)$$

```
[In] integrate(sec(x)^6,x, algorithm="giac")
[Out] 1/5*tan(x)^5 + 2/3*tan(x)^3 + tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \sec^6(x)\,dx = \frac{8\sin(x)\cos(x)^4 + 4\sin(x)\cos(x)^2 + 3\sin(x)}{15\cos(x)^5}$$

```
[In] int(1/cos(x)^6,x)
[Out] (3*sin(x) + 4*cos(x)^2*sin(x) + 8*cos(x)^4*sin(x))/(15*cos(x)^5)
```

3.86 $\int \sec^2(x) \tan^4(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \sec^2(x) \tan^4(x) \, dx = \frac{\tan^5(x)}{5}$$

```
[Out] 1/5*tan(x)^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2687, 30}

$$\int \sec^2(x) \tan^4(x) \, dx = \frac{\tan^5(x)}{5}$$

```
[In] Int[Sec[x]^2*Tan[x]^4,x]
```

```
[Out] Tan[x]^5/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^4\,dx, x, \tan(x)\right)\\ &= \frac{\tan^5(x)}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\tan^4(x)\,dx = \frac{\tan^5(x)}{5}$$

```
[In] Integrate[Sec[x]^2*Tan[x]^4,x]
[Out] Tan[x]^5/5
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\tan^5(x))}{5}$	7
default	$\frac{(\tan^5(x))}{5}$	7
risch	$\frac{2i(5\,e^{8ix}+10\,e^{4ix}+1)}{5(e^{2ix}+1)^5}$	29

```
[In] int(sec(x)^2*tan(x)^4,x,method=_RETURNVERBOSE)
[Out] 1/5*tan(x)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(6) = 12$.

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \sec^2(x)\tan^4(x)\,dx = \frac{\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\sin(x)}{5\cos(x)^5}$$

```
[In] integrate(sec(x)^2*tan(x)^4,x, algorithm="fricas")
[Out] 1/5*(cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(5) = 10.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 3.62

$$\int \sec^2(x)\tan^4(x)\,dx = \frac{\sin(x)}{5\cos(x)} - \frac{2\sin(x)}{5\cos^3(x)} + \frac{\sin(x)}{5\cos^5(x)}$$

```
[In] integrate(sec(x)**2*tan(x)**4,x)
[Out] sin(x)/(5*cos(x)) - 2*sin(x)/(5*cos(x)**3) + sin(x)/(5*cos(x)**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan^4(x)\,dx = \frac{1}{5}\tan(x)^5$$

```
[In] integrate(sec(x)^2*tan(x)^4,x, algorithm="maxima")
[Out] 1/5*tan(x)^5
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan^4(x)\,dx = \frac{1}{5}\tan(x)^5$$

```
[In] integrate(sec(x)^2*tan(x)^4,x, algorithm="giac")
[Out] 1/5*tan(x)^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan^4(x)\,dx = \frac{\tan(x)^5}{5}$$

```
[In] int(tan(x)^4/cos(x)^2,x)
[Out] tan(x)^5/5
```

3.87 $\int \sec^4(x)\tan^2(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \sec^4(x)\tan^2(x)\,dx=\frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}$$

```
[Out] 1/3*tan(x)^3+1/5*tan(x)^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2687, 14}

$$\int \sec^4(x)\tan^2(x)\,dx=\frac{\tan^5(x)}{5}+\frac{\tan^3(x)}{3}$$

```
[In] Int[Sec[x]^4*Tan[x]^2,x]
```

```
[Out] Tan[x]^3/3 + Tan[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^2(1+x^2)\ dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int (x^2+x^4)\ dx, x, \tan(x)\right)\\ &= \frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \sec^4(x)\tan^2(x)\,dx = -\frac{2\tan(x)}{15}-\frac{1}{15}\sec^2(x)\tan(x)+\frac{1}{5}\sec^4(x)\tan(x)$$

```
[In] Integrate[Sec[x]^4*Tan[x]^2,x]
[Out] (-2*Tan[x])/15 - (Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{(\tan^3(x))}{3}+\frac{(\tan^5(x))}{5}$	14
default	$\frac{(\tan^3(x))}{3}+\frac{(\tan^5(x))}{5}$	14
risch	$-\frac{4i(15\,e^{6ix}-5\,e^{4ix}+5\,e^{2ix}+1)}{15(e^{2ix}+1)^5}$	36

```
[In] int(sec(x)^4*tan(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*tan(x)^3+1/5*tan(x)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sec^4(x)\tan^2(x)\,dx = -\frac{\left(2\,\cos\left(x\right)^4 + \cos\left(x\right)^2 - 3\right)\sin\left(x\right)}{15\,\cos\left(x\right)^5}$$

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="fricas")
[Out] -1/15*(2*cos(x)^4 + cos(x)^2 - 3)*sin(x)/cos(x)^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \sec^4(x)\tan^2(x)\,dx = -\frac{2\sin\left(x\right)}{15\cos\left(x\right)} - \frac{\sin\left(x\right)}{15\cos^3\left(x\right)} + \frac{\sin\left(x\right)}{5\cos^5\left(x\right)}$$

```
[In] integrate(sec(x)**4*tan(x)**2,x)
[Out] -2*sin(x)/(15*cos(x)) - sin(x)/(15*cos(x)**3) + sin(x)/(5*cos(x)**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sec^4(x)\tan^2(x)\,dx = \frac{1}{5}\,\tan\left(x\right)^5 + \frac{1}{3}\,\tan\left(x\right)^3$$

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="maxima")
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sec^4(x)\tan^2(x)\,dx = \frac{1}{5}\,\tan\left(x\right)^5 + \frac{1}{3}\,\tan\left(x\right)^3$$

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="giac")
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sec^4(x)\tan^2(x)\,dx = \frac{\tan(x)^5}{5} + \frac{\tan(x)^3}{3}$$

```
[In] int(tan(x)^2/cos(x)^4,x)
[Out] tan(x)^3/3 + tan(x)^5/5
```

3.88 $\int \sec^3(x) \tan(x) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \sec^3(x) \tan(x) \, dx = \frac{\sec^3(x)}{3}$$

```
[Out] 1/3*sec(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2686, 30}

$$\int \sec^3(x) \tan(x) \, dx = \frac{\sec^3(x)}{3}$$

```
[In] Int[Sec[x]^3*Tan[x],x]
```

```
[Out] Sec[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int x^2\,dx, x, \sec(x)\right)$$
$$= \frac{\sec^3(x)}{3}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sec^3(x)\tan(x)\,dx = \frac{\sec^3(x)}{3}$$

```
[In] Integrate[Sec[x]^3*Tan[x],x]
[Out] Sec[x]^3/3
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sec^3(x))}{3}$	7
default	$\frac{(\sec^3(x))}{3}$	7
risch	$\frac{8\,e^{3ix}}{3(e^{2ix}+1)^3}$	17

```
[In] int(sec(x)^3*tan(x),x,method=_RETURNVERBOSE)
[Out] 1/3*sec(x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\,\cos{(x)}^3}$$

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="fricas")
[Out] 1/3/cos(x)^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\cos^3(x)}$$

```
[In] integrate(sec(x)**3*tan(x),x)
[Out] 1/(3*cos(x)**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\cos(x)^3}$$

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="maxima")
[Out] 1/3/cos(x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\cos(x)^3}$$

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="giac")
[Out] 1/3/cos(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\cos(x)^3}$$

```
[In] int(tan(x)/cos(x)^3,x)
[Out] 1/(3*cos(x)^3)
```

3.89 $\int \sec^3(x) \tan^3(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \sec^3(x) \tan^3(x) \, dx = -\frac{1}{3} \sec^3(x) + \frac{\sec^5(x)}{5}$$

```
[Out] -1/3*sec(x)^3+1/5*sec(x)^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2686, 14}

$$\int \sec^3(x) \tan^3(x) \, dx = \frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

```
[In] Int[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -1/3*Sec[x]^3 + Sec[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^2(-1+x^2)\,dx, x, \sec(x)\right)\\ &= \text{Subst}\left(\int (-x^2+x^4)\,dx, x, \sec(x)\right)\\ &= -\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sec^3(x)\tan^3(x)\,dx = -\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}$$

```
[In] Integrate[Sec[x]^3*Tan[x]^3,x]
[Out] -1/3*Sec[x]^3 + Sec[x]^5/5
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{(\sec^3(x))}{3}+\frac{(\sec^5(x))}{5}$	14
default	$-\frac{(\sec^3(x))}{3}+\frac{(\sec^5(x))}{5}$	14
risch	$-\frac{8(5\,e^{7ix}-2\,e^{5ix}+5\,e^{3ix})}{15(e^{2ix}+1)^5}$	34

```
[In] int(sec(x)^3*tan(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/3*sec(x)^3+1/5*sec(x)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^3(x)\tan^3(x)\,dx = -\frac{5\,\cos(x)^2 - 3}{15\,\cos(x)^5}$$

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="fricas")
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^3(x)\tan^3(x)\,dx = \frac{3 - 5\cos^2(x)}{15\cos^5(x)}$$

```
[In] integrate(sec(x)**3*tan(x)**3,x)
[Out] (3 - 5*cos(x)**2)/(15*cos(x)**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^3(x)\tan^3(x)\,dx = -\frac{5\,\cos(x)^2 - 3}{15\,\cos(x)^5}$$

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="maxima")
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^3(x)\tan^3(x)\,dx = -\frac{5\,\cos(x)^2 - 3}{15\,\cos(x)^5}$$

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="giac")
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sec^3(x)\tan^3(x)\,dx = \frac{1}{5\cos(x)^5} - \frac{1}{3\cos(x)^3}$$

```
[In] int(tan(x)^3/cos(x)^3,x)
[Out] 1/(5*cos(x)^5) - 1/(3*cos(x)^3)
```

3.90 $\int \tan^5(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 22

$$\int \tan^5(x)\,dx = -\log(\cos(x)) - \frac{\tan^2(x)}{2} + \frac{\tan^4(x)}{4}$$

```
[Out] -ln(cos(x))-1/2*tan(x)^2+1/4*tan(x)^4
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 3556}

$$\int \tan^5(x)\,dx = \frac{\tan^4(x)}{4} - \frac{\tan^2(x)}{2} - \log(\cos(x))$$

```
[In] Int[Tan[x]^5,x]
```

```
[Out] -Log[Cos[x]] - Tan[x]^2/2 + Tan[x]^4/4
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tan^4(x)}{4} - \int \tan^3(x)\, dx \\
&= -\frac{1}{2}\tan^2(x) + \frac{\tan^4(x)}{4} + \int \tan(x)\, dx \\
&= -\log(\cos(x)) - \frac{\tan^2(x)}{2} + \frac{\tan^4(x)}{4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$
\int \tan^5(x)\, dx = -\log(\cos(x)) - \frac{\tan^2(x)}{2} + \frac{\tan^4(x)}{4}
$$

```
[In] Integrate[Tan[x]^5,x]
[Out] -Log[Cos[x]] - Tan[x]^2/2 + Tan[x]^4/4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{(\tan^4(x))}{4} - \frac{(\tan^2(x))}{2} + \frac{\ln(1+\tan^2(x))}{2}$	23
default	$\frac{(\tan^4(x))}{4} - \frac{(\tan^2(x))}{2} + \frac{\ln(1+\tan^2(x))}{2}$	23
norman	$\frac{(\tan^4(x))}{4} - \frac{(\tan^2(x))}{2} + \frac{\ln(1+\tan^2(x))}{2}$	23
parallelrisch	$\frac{(\tan^4(x))}{4} - \frac{(\tan^2(x))}{2} + \frac{\ln(1+\tan^2(x))}{2}$	23
risch	$ix - \frac{4(e^{6ix}+e^{4ix}+e^{2ix})}{(e^{2ix}+1)^4} - \ln(e^{2ix}+1)$	43

```
[In] int(tan(x)^5,x,method=_RETURNVERBOSE)
[Out] 1/4*tan(x)^4-1/2*tan(x)^2+1/2*ln(1+tan(x)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \tan^5(x)\,dx = \frac{1}{4}\,\tan{(x)}^4 - \frac{1}{2}\,\tan{(x)}^2 - \frac{1}{2}\,\log\left(\frac{1}{\tan{(x)}^2 + 1}\right)$$

```
[In] integrate(tan(x)^5,x, algorithm="fricas")
[Out] 1/4*tan(x)^4 - 1/2*tan(x)^2 - 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \tan^5(x)\,dx = -\frac{4\cos^2{(x)} - 1}{4\cos^4{(x)}} - \log{(\cos{(x)})}$$

```
[In] integrate(tan(x)**5,x)
[Out] -(4*cos(x)**2 - 1)/(4*cos(x)**4) - log(cos(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \tan^5(x)\,dx = \frac{4\,\sin{(x)}^2 - 3}{4\left(\sin{(x)}^4 - 2\,\sin{(x)}^2 + 1\right)} - \frac{1}{2}\,\log\left(\sin{(x)}^2 - 1\right)$$

```
[In] integrate(tan(x)^5,x, algorithm="maxima")
[Out] 1/4*(4*sin(x)^2 - 3)/(sin(x)^4 - 2*sin(x)^2 + 1) - 1/2*log(sin(x)^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \tan^5(x)\,dx = \frac{1}{4}\,\tan{(x)}^4 - \frac{1}{2}\,\tan{(x)}^2 + \frac{1}{2}\,\log\left(\tan{(x)}^2 + 1\right)$$

```
[In] integrate(tan(x)^5,x, algorithm="giac")
[Out] 1/4*tan(x)^4 - 1/2*tan(x)^2 + 1/2*log(tan(x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^5(x)\,dx = \frac{\tan(x)^4}{4} - \frac{\tan(x)^2}{2} - \ln(\cos(x))$$

```
[In] int(tan(x)^5,x)
[Out] tan(x)^4/4 - tan(x)^2/2 - log(cos(x))
```

3.91 $\int \tan^6(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 22

$$\int \tan^6(x)\,dx = -x + \tan(x) - \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5}$$

```
[Out] -x+tan(x)-1/3*tan(x)^3+1/5*tan(x)^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$\int \tan^6(x)\,dx = -x + \frac{\tan^5(x)}{5} - \frac{\tan^3(x)}{3} + \tan(x)$$

```
[In] Int[Tan[x]^6,x]
```

```
[Out] -x + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\tan^5(x)}{5} - \int \tan^4(x)\,dx \\ &= -\frac{1}{3}\tan^3(x) + \frac{\tan^5(x)}{5} + \int \tan^2(x)\,dx \\ &= \tan(x) - \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5} - \int 1\,dx \\ &= -x + \tan(x) - \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \tan^6(x)\,dx = -\arctan(\tan(x)) + \tan(x) - \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5}$$

```
[In] Integrate[Tan[x]^6,x]
[Out] -ArcTan[Tan[x]] + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
norman	$-x+\tan\left(x\right)-\frac{\left(\tan^{3}\left(x\right)\right)}{3}+\frac{\left(\tan^{5}\left(x\right)\right)}{5}$	19
parallelrisch	$-x+\tan\left(x\right)-\frac{\left(\tan^{3}\left(x\right)\right)}{3}+\frac{\left(\tan^{5}\left(x\right)\right)}{5}$	19
derivativedivides	$\frac{\left(\tan^{5}\left(x\right)\right)}{5}-\frac{\left(\tan^{3}\left(x\right)\right)}{3}+\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)$	21
default	$\frac{\left(\tan^{5}\left(x\right)\right)}{5}-\frac{\left(\tan^{3}\left(x\right)\right)}{3}+\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)$	21
risch	$-x+\frac{2i\left(45\,{\mathrm e}^{8ix}+90\,{\mathrm e}^{6ix}+140\,{\mathrm e}^{4ix}+70\,{\mathrm e}^{2ix}+23\right)}{15\left({\mathrm e}^{2ix}+1\right)^{5}}$	47

```
[In] int(tan(x)^6,x,method=_RETURNVERBOSE)
[Out] -x+tan(x)-1/3*tan(x)^3+1/5*tan(x)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\,dx = \frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

```
[In] integrate(tan(x)^6,x, algorithm="fricas")
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \tan^6(x)\,dx = -x + \frac{\sin^5(x)}{5\cos^5(x)} - \frac{\sin^3(x)}{3\cos^3(x)} + \frac{\sin(x)}{\cos(x)}$$

```
[In] integrate(tan(x)**6,x)
[Out] -x + sin(x)**5/(5*cos(x)**5) - sin(x)**3/(3*cos(x)**3) + sin(x)/cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\,dx = \frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

```
[In] integrate(tan(x)^6,x, algorithm="maxima")
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\,dx = \frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

```
[In] integrate(tan(x)^6,x, algorithm="giac")
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\,dx = \frac{\tan(x)^5}{5} - \frac{\tan(x)^3}{3} + \tan(x) - x$$

```
[In] int(tan(x)^6,x)
[Out] tan(x) - x - tan(x)^3/3 + tan(x)^5/5
```

3.92 $\int \sec(x)\tan^5(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 19

$$\int \sec(x)\tan^5(x)\,dx = \sec(x) - \frac{2\sec^3(x)}{3} + \frac{\sec^5(x)}{5}$$

```
[Out] sec(x)-2/3*sec(x)^3+1/5*sec(x)^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2686, 200}

$$\int \sec(x)\tan^5(x)\,dx = \frac{\sec^5(x)}{5} - \frac{2\sec^3(x)}{3} + \sec(x)$$

```
[In] Int[Sec[x]*Tan[x]^5,x]
```

```
[Out] Sec[x] - (2*Sec[x]^3)/3 + Sec[x]^5/5
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \left(-1+x^2\right)^2 \, dx, x, \sec(x)\right)\\ &= \text{Subst}\left(\int \left(1-2x^2+x^4\right) \, dx, x, \sec(x)\right)\\ &= \sec(x) - \frac{2\sec^3(x)}{3} + \frac{\sec^5(x)}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sec(x)\tan^5(x)\,dx = \sec(x) - \frac{2\sec^3(x)}{3} + \frac{\sec^5(x)}{5}$$

```
[In] Integrate[Sec[x]*Tan[x]^5,x]
[Out] Sec[x] - (2*Sec[x]^3)/3 + Sec[x]^5/5
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\sec\left(x\right)-\frac{2\left(\sec^3\left(x\right)\right)}{3}+\frac{\left(\sec^5\left(x\right)\right)}{5}$	16
default	$\sec\left(x\right)-\frac{2\left(\sec^3\left(x\right)\right)}{3}+\frac{\left(\sec^5\left(x\right)\right)}{5}$	16
risch	$\frac{2\,\mathrm{e}^{9ix}+\frac{8\,\mathrm{e}^{7ix}}{3}+\frac{116\,\mathrm{e}^{5ix}}{15}+\frac{8\,\mathrm{e}^{3ix}}{3}+2\,\mathrm{e}^{ix}}{\left(\mathrm{e}^{2ix}+1\right)^5}$	48

```
[In] int(sec(x)*tan(x)^5,x,method=_RETURNVERBOSE)
[Out] sec(x)-2/3*sec(x)^3+1/5*sec(x)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \sec(x)\tan^5(x)\,dx = \frac{15\,\cos\left(x\right)^4 - 10\,\cos\left(x\right)^2 + 3}{15\,\cos\left(x\right)^5}$$

```
[In] integrate(sec(x)*tan(x)^5,x, algorithm="fricas")
[Out] 1/15*(15*cos(x)^4 - 10*cos(x)^2 + 3)/cos(x)^5
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \sec(x)\tan^5(x)\,dx = -\frac{-15\cos^4(x) + 10\cos^2(x) - 3}{15\cos^5(x)}$$

```
[In] integrate(sec(x)*tan(x)**5,x)
[Out] -(-15*cos(x)**4 + 10*cos(x)**2 - 3)/(15*cos(x)**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \sec(x)\tan^5(x)\,dx = \frac{15\,\cos\left(x\right)^4 - 10\,\cos\left(x\right)^2 + 3}{15\,\cos\left(x\right)^5}$$

```
[In] integrate(sec(x)*tan(x)^5,x, algorithm="maxima")
[Out] 1/15*(15*cos(x)^4 - 10*cos(x)^2 + 3)/cos(x)^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \sec(x)\tan^5(x)\,dx = \frac{15\,\cos\left(x\right)^4 - 10\,\cos\left(x\right)^2 + 3}{15\,\cos\left(x\right)^5}$$

```
[In] integrate(sec(x)*tan(x)^5,x, algorithm="giac")
[Out] 1/15*(15*cos(x)^4 - 10*cos(x)^2 + 3)/cos(x)^5
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \sec(x)\tan^5(x)\,dx = \frac{\cos(x)^4 - \frac{2\cos(x)^2}{3} + \frac{1}{5}}{\cos(x)^5}$$

```
[In] int(tan(x)^5/cos(x),x)
[Out] (cos(x)^4 - (2*cos(x)^2)/3 + 1/5)/cos(x)^5
```

3.93 $\int \sec^3(x) \tan^5(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \sec^3(x) \tan^5(x) \, dx = \frac{\sec^3(x)}{3} - \frac{2\sec^5(x)}{5} + \frac{\sec^7(x)}{7}$$

```
[Out] 1/3*sec(x)^3-2/5*sec(x)^5+1/7*sec(x)^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2686, 276}

$$\int \sec^3(x) \tan^5(x) \, dx = \frac{\sec^7(x)}{7} - \frac{2\sec^5(x)}{5} + \frac{\sec^3(x)}{3}$$

```
[In] Int[Sec[x]^3*Tan[x]^5,x]
```

```
[Out] Sec[x]^3/3 - (2*Sec[x]^5)/5 + Sec[x]^7/7
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \sec(x)\right) \\ &= \text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \sec(x)\right) \\ &= \frac{\sec^3(x)}{3}-\frac{2\sec^5(x)}{5}+\frac{\sec^7(x)}{7} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sec^3(x)\tan^5(x)\,dx = \frac{\sec^3(x)}{3}-\frac{2\sec^5(x)}{5}+\frac{\sec^7(x)}{7}$$

```
[In] Integrate[Sec[x]^3*Tan[x]^5,x]
[Out] Sec[x]^3/3 - (2*Sec[x]^5)/5 + Sec[x]^7/7
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{(\sec^3(x))}{3}-\frac{2(\sec^5(x))}{5}+\frac{(\sec^7(x))}{7}$	20
default	$\frac{(\sec^3(x))}{3}-\frac{2(\sec^5(x))}{5}+\frac{(\sec^7(x))}{7}$	20
risch	$\frac{\frac{8\,e^{11ix}}{3}-\frac{32\,e^{9ix}}{15}+\frac{304\,e^{7ix}}{35}-\frac{32\,e^{5ix}}{15}+\frac{8\,e^{3ix}}{3}}{\left(e^{2ix}+1\right)^7}$	48

```
[In] int(sec(x)^3*tan(x)^5,x,method=_RETURNVERBOSE)
[Out] 1/3*sec(x)^3-2/5*sec(x)^5+1/7*sec(x)^7
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \sec^3(x)\tan^5(x)\,dx = \frac{35\,\cos(x)^4 - 42\,\cos(x)^2 + 15}{105\,\cos(x)^7}$$

```
[In] integrate(sec(x)^3*tan(x)^5,x, algorithm="fricas")
[Out] 1/105*(35*cos(x)^4 - 42*cos(x)^2 + 15)/cos(x)^7
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \sec^3(x)\tan^5(x)\,dx = -\frac{-35\cos^4(x) + 42\cos^2(x) - 15}{105\cos^7(x)}$$

```
[In] integrate(sec(x)**3*tan(x)**5,x)
[Out] -(-35*cos(x)**4 + 42*cos(x)**2 - 15)/(105*cos(x)**7)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \sec^3(x)\tan^5(x)\,dx = \frac{35\,\cos(x)^4 - 42\,\cos(x)^2 + 15}{105\,\cos(x)^7}$$

```
[In] integrate(sec(x)^3*tan(x)^5,x, algorithm="maxima")
[Out] 1/105*(35*cos(x)^4 - 42*cos(x)^2 + 15)/cos(x)^7
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \sec^3(x)\tan^5(x)\,dx = \frac{35\,\cos(x)^4 - 42\,\cos(x)^2 + 15}{105\,\cos(x)^7}$$

```
[In] integrate(sec(x)^3*tan(x)^5,x, algorithm="giac")
[Out] 1/105*(35*cos(x)^4 - 42*cos(x)^2 + 15)/cos(x)^7
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \sec^3(x)\tan^5(x)\,dx = \frac{\frac{\cos(x)^4}{3} - \frac{2\cos(x)^2}{5} + \frac{1}{7}}{\cos(x)^7}$$

```
[In] int(tan(x)^5/cos(x)^3,x)
[Out] (cos(x)^4/3 - (2*cos(x)^2)/5 + 1/7)/cos(x)^7
```

3.94 $\int \sec^6(x) \tan(x) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \sec^6(x) \tan(x) \, dx = \frac{\sec^6(x)}{6}$$

```
[Out] 1/6*sec(x)^6
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2686, 30}

$$\int \sec^6(x) \tan(x) \, dx = \frac{\sec^6(x)}{6}$$

```
[In] Int[Sec[x]^6*Tan[x],x]
```

```
[Out] Sec[x]^6/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int x^5\,dx, x, \sec(x)\right)\\ &= \frac{\sec^6(x)}{6} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sec^6(x)\tan(x)\,dx = \frac{\sec^6(x)}{6}$$

```
[In] Integrate[Sec[x]^6*Tan[x],x]
[Out] Sec[x]^6/6
```

Maple [A] (verified)

Time = 9.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sec^6(x))}{6}$	7
default	$\frac{(\sec^6(x))}{6}$	7
risch	$\frac{32\,e^{6ix}}{3(e^{2ix}+1)^6}$	17

```
[In] int(sec(x)^6*tan(x),x,method=_RETURNVERBOSE)
[Out] 1/6*sec(x)^6
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^6(x)\tan(x)\,dx = \frac{1}{6\,\cos(x)^6}$$

```
[In] integrate(sec(x)^6*tan(x),x, algorithm="fricas")
[Out] 1/6/cos(x)^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \sec^6(x)\tan(x)\,dx = \frac{1}{6\cos^6(x)}$$

```
[In] integrate(sec(x)**6*tan(x),x)
[Out] 1/(6*cos(x)**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \sec^6(x)\tan(x)\,dx = -\frac{1}{6\left(\sin(x)^2-1\right)^3}$$

```
[In] integrate(sec(x)^6*tan(x),x, algorithm="maxima")
[Out] -1/6/(sin(x)^2 - 1)^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^6(x)\tan(x)\,dx = \frac{1}{6\cos(x)^6}$$

```
[In] integrate(sec(x)^6*tan(x),x, algorithm="giac")
[Out] 1/6/cos(x)^6
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \sec^6(x)\tan(x)\,dx = \frac{\tan(x)^2\left(\tan(x)^4+3\tan(x)^2+3\right)}{6}$$

```
[In] int(tan(x)/cos(x)^6,x)
[Out] (tan(x)^2*(3*tan(x)^2 + tan(x)^4 + 3))/6
```

3.95 $\int \sec^6(x)\tan^3(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \sec^6(x)\tan^3(x)\,dx = -\frac{1}{6}\sec^6(x) + \frac{\sec^8(x)}{8}$$

```
[Out] -1/6*sec(x)^6+1/8*sec(x)^8
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2686, 14}

$$\int \sec^6(x)\tan^3(x)\,dx = \frac{\sec^8(x)}{8} - \frac{\sec^6(x)}{6}$$

```
[In] Int[Sec[x]^6*Tan[x]^3,x]
```

```
[Out] -1/6*Sec[x]^6 + Sec[x]^8/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^5(-1+x^2)\ dx, x, \sec(x)\right)\\ &= \text{Subst}\left(\int (-x^5+x^7)\ dx, x, \sec(x)\right)\\ &= -\frac{1}{6}\sec^6(x)+\frac{\sec^8(x)}{8}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sec^6(x)\tan^3(x)\,dx = -\frac{1}{6}\sec^6(x)+\frac{\sec^8(x)}{8}$$

```
[In] Integrate[Sec[x]^6*Tan[x]^3,x]
[Out] -1/6*Sec[x]^6 + Sec[x]^8/8
```

Maple [A] (verified)

Time = 18.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{(\sec^6(x))}{6}+\frac{(\sec^8(x))}{8}$	14
default	$-\frac{(\sec^6(x))}{6}+\frac{(\sec^8(x))}{8}$	14
risch	$-\frac{32(e^{10ix}-e^{8ix}+e^{6ix})}{3(e^{2ix}+1)^8}$	30

```
[In] int(sec(x)^6*tan(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/6*sec(x)^6+1/8*sec(x)^8
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^6(x)\tan^3(x)\,dx = -\frac{4\,\cos\left(x\right)^2-3}{24\,\cos\left(x\right)^8}$$

```
[In] integrate(sec(x)^6*tan(x)^3,x, algorithm="fricas")
[Out] -1/24*(4*cos(x)^2 - 3)/cos(x)^8
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^6(x)\tan^3(x)\,dx = \frac{3-4\cos^2\left(x\right)}{24\cos^8\left(x\right)}$$

```
[In] integrate(sec(x)**6*tan(x)**3,x)
[Out] (3 - 4*cos(x)**2)/(24*cos(x)**8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(13) = 26.

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int \sec^6(x)\tan^3(x)\,dx = \frac{4\,\sin\left(x\right)^2-1}{24\,\left(\sin\left(x\right)^8-4\,\sin\left(x\right)^6+6\,\sin\left(x\right)^4-4\,\sin\left(x\right)^2+1\right)}$$

```
[In] integrate(sec(x)^6*tan(x)^3,x, algorithm="maxima")
[Out] 1/24*(4*sin(x)^2 - 1)/(sin(x)^8 - 4*sin(x)^6 + 6*sin(x)^4 - 4*sin(x)^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^6(x)\tan^3(x)\,dx = -\frac{4\,\cos\left(x\right)^2-3}{24\,\cos\left(x\right)^8}$$

```
[In] integrate(sec(x)^6*tan(x)^3,x, algorithm="giac")
[Out] -1/24*(4*cos(x)^2 - 3)/cos(x)^8
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sec^6(x)\tan^3(x)\,dx = \frac{\tan(x)^4\,\left(3\tan(x)^4+8\tan(x)^2+6\right)}{24}$$

```
[In] int(tan(x)^3/cos(x)^6,x)
[Out] (tan(x)^4*(8*tan(x)^2 + 3*tan(x)^4 + 6))/24
```

3.96 $\int \sec^2(x) \tan(x)\, dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \sec^2(x) \tan(x)\, dx = \frac{\sec^2(x)}{2}$$

```
[Out] 1/2*sec(x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2686, 30}

$$\int \sec^2(x) \tan(x)\, dx = \frac{\sec^2(x)}{2}$$

```
[In] Int[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x\,dx, x, \sec(x)\right)\\ &= \frac{\sec^2(x)}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\tan(x)\,dx = \frac{\sec^2(x)}{2}$$

```
[In] Integrate[Sec[x]^2*Tan[x],x]
[Out] Sec[x]^2/2
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sec^2(x))}{2}$	7
default	$\frac{(\sec^2(x))}{2}$	7
risch	$\frac{2\,e^{2ix}}{(e^{2ix}+1)^2}$	17

```
[In] int(sec(x)^2/cot(x),x,method=_RETURNVERBOSE)
[Out] 1/2*sec(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2\,\cos(x)^2}$$

```
[In] integrate(sec(x)^2/cot(x),x, algorithm="fricas")
[Out] 1/2/cos(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2\cos^2(x)}$$

```
[In] integrate(sec(x)**2/cot(x),x)
[Out] 1/(2*cos(x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \sec^2(x)\tan(x)\,dx = -\frac{1}{2\left(\sin(x)^2 - 1\right)}$$

```
[In] integrate(sec(x)^2/cot(x),x, algorithm="maxima")
[Out] -1/2/(sin(x)^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2\cos(x)^2}$$

```
[In] integrate(sec(x)^2/cot(x),x, algorithm="giac")
[Out] 1/2/cos(x)^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{\tan(x)^2}{2}$$

```
[In] int(1/(cos(x)^2*cot(x)),x)
[Out] tan(x)^2/2
```

3.97 $\int \sec(x)\tan^2(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \sec(x)\tan^2(x)\,dx = -\frac{1}{2}\text{arctanh}(\sin(x)) + \frac{1}{2}\sec(x)\tan(x)$$

```
[Out] -1/2*arctanh(sin(x))+1/2*sec(x)*tan(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2691, 3855}

$$\int \sec(x)\tan^2(x)\,dx = \frac{1}{2}\tan(x)\sec(x) - \frac{1}{2}\text{arctanh}(\sin(x))$$

```
[In] Int[Sec[x]*Tan[x]^2,x]
```

```
[Out] -1/2*ArcTanh[Sin[x]] + (Sec[x]*Tan[x])/2
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\sec(x)\tan(x) - \frac{1}{2}\int \sec(x)\,dx$$
$$= -\frac{1}{2}\text{arctanh}(\sin(x)) + \frac{1}{2}\sec(x)\tan(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sec(x)\tan^2(x)\,dx = -\frac{1}{2}\text{arctanh}(\sin(x)) + \frac{1}{2}\sec(x)\tan(x)$$

```
[In] Integrate[Sec[x]*Tan[x]^2,x]
[Out] -1/2*ArcTanh[Sin[x]] + (Sec[x]*Tan[x])/2
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{\sin^3(x)}{2\cos(x)^2} + \frac{\sin(x)}{2} - \frac{\ln(\sec(x)+\tan(x))}{2}$	24
risch	$-\frac{i(e^{3ix}-e^{ix})}{(e^{2ix}+1)^2} - \frac{\ln(i+e^{ix})}{2} + \frac{\ln(e^{ix}-i)}{2}$	49

```
[In] int(sec(x)*tan(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*sin(x)^3/cos(x)^2+1/2*sin(x)-1/2*ln(sec(x)+tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(12) = 24.

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \sec(x)\tan^2(x)\,dx = -\frac{\cos(x)^2\log(\sin(x)+1) - \cos(x)^2\log(-\sin(x)+1) - 2\sin(x)}{4\cos(x)^2}$$

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="fricas")
[Out] -1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) - 2*sin(x))/cos(
x)^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \sec(x)\tan^2(x)\,dx = \frac{\log(\sin(x)-1)}{4} - \frac{\log(\sin(x)+1)}{4} - \frac{\sin(x)}{2\sin^2(x)-2}$$

```
[In] integrate(sec(x)*tan(x)**2,x)
[Out] log(sin(x) - 1)/4 - log(sin(x) + 1)/4 - sin(x)/(2*sin(x)**2 - 2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(12) = 24.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \sec(x)\tan^2(x)\,dx = -\frac{\sin(x)}{2\left(\sin(x)^2-1\right)} - \frac{1}{4}\log(\sin(x)+1) + \frac{1}{4}\log(\sin(x)-1)$$

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="maxima")
[Out] -1/2*sin(x)/(sin(x)^2 - 1) - 1/4*log(sin(x) + 1) + 1/4*log(sin(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \sec(x)\tan^2(x)\,dx = -\frac{\sin(x)}{2\left(\sin(x)^2-1\right)} - \frac{1}{4}\log(\sin(x)+1) + \frac{1}{4}\log(-\sin(x)+1)$$

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="giac")
[Out] -1/2*sin(x)/(sin(x)^2 - 1) - 1/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \sec(x)\tan^2(x)\,dx = \frac{\tan\left(\frac{x}{2}\right)^3+\tan\left(\frac{x}{2}\right)}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^2} - \operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)$$

```
[In] int(tan(x)^2/cos(x),x)
[Out] (tan(x/2) + tan(x/2)^3)/(tan(x/2)^2 - 1)^2 - atanh(tan(x/2))
```

3.98 $\int \cot^2(x) \, dx$

Optimal result

Integrand size = 4, antiderivative size = 8

$$\int \cot^2(x) \, dx = -x - \cot(x)$$

```
[Out] -x-cot(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$\int \cot^2(x) \, dx = -x - \cot(x)$$

```
[In] Int[Cot[x]^2,x]
```

```
[Out] -x - Cot[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\cot(x) - \int 1\,dx\\ &= -x - \cot(x)\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \cot^2(x)\,dx = -\cot(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(x)\right)$$

```
[In] Integrate[Cot[x]^2,x]
[Out] -(Cot[x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[x]^2])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62

method	result	size
norman	$\frac{-1-x\tan(x)}{\tan(x)}$	13
parallelrisch	$\frac{-1-x\tan(x)}{\tan(x)}$	13
derivativedivides	$-\cot(x)+\frac{\pi}{2}-\operatorname{arccot}(\cot(x))$	14
default	$-\cot(x)+\frac{\pi}{2}-\operatorname{arccot}(\cot(x))$	14
risch	$-x-\frac{2i}{e^{2ix}-1}$	17

```
[In] int(cot(x)^2,x,method=_RETURNVERBOSE)
[Out] (-1-x*tan(x))/tan(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \cot^2(x)\,dx = -\frac{x\sin(2x) + \cos(2x) + 1}{\sin(2x)}$$

```
[In] integrate(cot(x)^2,x, algorithm="fricas")
[Out] -(x*sin(2*x) + cos(2*x) + 1)/sin(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cot^2(x)\,dx = -x - \frac{\cos(x)}{\sin(x)}$$

```
[In] integrate(cot(x)**2,x)
[Out] -x - cos(x)/sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \cot^2(x)\,dx = -x - \frac{1}{\tan(x)}$$

```
[In] integrate(cot(x)^2,x, algorithm="maxima")
[Out] -x - 1/tan(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(8) = 16$.

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \cot^2(x)\,dx = -x - \frac{1}{2\tan\left(\frac{1}{2}x\right)} + \frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

```
[In] integrate(cot(x)^2,x, algorithm="giac")
[Out] -x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cot^2(x) \, dx = -x - \cot(x)$$

```
[In] int(cot(x)^2,x)
[Out] - x - cot(x)
```

3.99 $\int \cot^3(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cot^3(x)\,dx = -\frac{1}{2}\cot^2(x) - \log(\sin(x))$$

```
[Out] -1/2*cot(x)^2-ln(sin(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 3556}

$$\int \cot^3(x)\,dx = -\frac{1}{2}\cot^2(x) - \log(\sin(x))$$

```
[In] Int[Cot[x]^3,x]
```

```
[Out] -1/2*Cot[x]^2 - Log[Sin[x]]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2}\cot^2(x) - \int \cot(x)\,dx \\ &= -\frac{1}{2}\cot^2(x) - \log(\sin(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \cot^3(x)\,dx = -\frac{1}{2}\cot^2(x) - \log(\cos(x)) - \log(\tan(x))$$

```
[In] Integrate[Cot[x]^3,x]
[Out] -1/2*Cot[x]^2 - Log[Cos[x]] - Log[Tan[x]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

method	result	size
derivativedivides	$-\frac{(\cot^2(x))}{2}+\frac{\ln(\cot^2(x)+1)}{2}$	17
default	$-\frac{(\cot^2(x))}{2}+\frac{\ln(\cot^2(x)+1)}{2}$	17
parallelrisch	$-\ln(\tan(x))+\ln\left(\sqrt{\sec^2(x)}\right)-\frac{(\cot^2(x))}{2}$	20
norman	$-\frac{1}{2\tan(x)^2}-\ln(\tan(x))+\frac{\ln(1+\tan^2(x))}{2}$	22
risch	$ix+\frac{2\,e^{2ix}}{(e^{2ix}-1)^2}-\ln(e^{2ix}-1)$	32

```
[In] int(cot(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*cot(x)^2+1/2*ln(cot(x)^2+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \cot^3(x)\,dx = -\frac{(\cos(2x) - 1)\log\left(-\frac{1}{2}\cos(2x) + \frac{1}{2}\right) - 2}{2(\cos(2x) - 1)}$$

```
[In] integrate(cot(x)^3,x, algorithm="fricas")
[Out] -1/2*((cos(2*x) - 1)*log(-1/2*cos(2*x) + 1/2) - 2)/(cos(2*x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\,dx = -\log(\sin(x)) - \frac{1}{2\sin^2(x)}$$

```
[In] integrate(cot(x)**3,x)
[Out] -log(sin(x)) - 1/(2*sin(x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\,dx = -\frac{1}{2\sin(x)^2} - \frac{1}{2}\log\left(\sin(x)^2\right)$$

```
[In] integrate(cot(x)^3,x, algorithm="maxima")
[Out] -1/2/sin(x)^2 - 1/2*log(sin(x)^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \cot^3(x)\,dx = \frac{1}{2\left(\cos(x)^2 - 1\right)} - \frac{1}{2}\log\left(-\cos(x)^2 + 1\right)$$

```
[In] integrate(cot(x)^3,x, algorithm="giac")
[Out] 1/2/(cos(x)^2 - 1) - 1/2*log(-cos(x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \cot^3(x)\,dx = \frac{\sin(x)^2 - 1}{2\sin(x)^2} - \ln(\sin(x))$$

```
[In] int(cot(x)^3,x)
[Out] (sin(x)^2 - 1)/(2*sin(x)^2) - log(sin(x))
```

3.100 $\int \cot^4(x)\csc^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \cot^4(x)\csc^4(x)\,dx = -\frac{1}{5}\cot^5(x) - \frac{\cot^7(x)}{7}$$

```
[Out] -1/5*cot(x)^5-1/7*cot(x)^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2687, 14}

$$\int \cot^4(x)\csc^4(x)\,dx = -\frac{1}{7}\cot^7(x) - \frac{\cot^5(x)}{5}$$

```
[In] Int[Cot[x]^4*Csc[x]^4,x]
```

```
[Out] -1/5*Cot[x]^5 - Cot[x]^7/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int x^4\left(1+x^2\right)\,dx, x, -\cot(x)\right)$$
$$= \text{Subst}\left(\int \left(x^4+x^6\right)\,dx, x, -\cot(x)\right)$$
$$= -\frac{1}{5}\cot^5(x) - \frac{\cot^7(x)}{7}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \cot^4(x)\csc^4(x)\,dx = -\frac{2\cot(x)}{35} - \frac{1}{35}\cot(x)\csc^2(x) + \frac{8}{35}\cot(x)\csc^4(x) - \frac{1}{7}\cot(x)\csc^6(x)$$

```
[In] Integrate[Cot[x]^4*Csc[x]^4,x]
[Out] (-2*Cot[x])/35 - (Cot[x]*Csc[x]^2)/35 + (8*Cot[x]*Csc[x]^4)/35 - (Cot[x]*Cs
c[x]^6)/7
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{\left(\cot^5(x)\right)}{5} - \frac{\left(\cot^7(x)\right)}{7}$	14
default	$-\frac{\left(\cot^5(x)\right)}{5} - \frac{\left(\cot^7(x)\right)}{7}$	14
risch	$\frac{4i\left(35\,\mathrm{e}^{10ix}+35\,\mathrm{e}^{8ix}+70\,\mathrm{e}^{6ix}+14\,\mathrm{e}^{4ix}+7\,\mathrm{e}^{2ix}-1\right)}{35\left(\mathrm{e}^{2ix}-1\right)^7}$	50

```
[In] int(cot(x)^4*csc(x)^4,x,method=_RETURNVERBOSE)
[Out] -1/5*cot(x)^5-1/7*cot(x)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(13) = 26$.

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \cot^4(x)\csc^4(x)\,dx = -\frac{2\,\cos\left(x\right)^7 - 7\,\cos\left(x\right)^5}{35\,\left(\cos\left(x\right)^6 - 3\,\cos\left(x\right)^4 + 3\,\cos\left(x\right)^2 - 1\right)\sin\left(x\right)}$$

```
[In] integrate(cot(x)^4*csc(x)^4,x, algorithm="fricas")
[Out] -1/35*(2*cos(x)^7 - 7*cos(x)^5)/((cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*s
in(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.41

$$\int \cot^4(x)\csc^4(x)\,dx = -\frac{2\cos\left(x\right)}{35\sin\left(x\right)} - \frac{\cos\left(x\right)}{35\sin^3\left(x\right)} + \frac{8\cos\left(x\right)}{35\sin^5\left(x\right)} - \frac{\cos\left(x\right)}{7\sin^7\left(x\right)}$$

```
[In] integrate(cot(x)**4*csc(x)**4,x)
[Out] -2*cos(x)/(35*sin(x)) - cos(x)/(35*sin(x)**3) + 8*cos(x)/(35*sin(x)**5) - c
os(x)/(7*sin(x)**7)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^4(x)\csc^4(x)\,dx = -\frac{7\,\tan\left(x\right)^2 + 5}{35\,\tan\left(x\right)^7}$$

```
[In] integrate(cot(x)^4*csc(x)^4,x, algorithm="maxima")
[Out] -1/35*(7*tan(x)^2 + 5)/tan(x)^7
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^4(x)\csc^4(x)\,dx = -\frac{7\tan(x)^2+5}{35\tan(x)^7}$$

```
[In] integrate(cot(x)^4*csc(x)^4,x, algorithm="giac")
[Out] -1/35*(7*tan(x)^2 + 5)/tan(x)^7
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^4(x)\csc^4(x)\,dx = -\frac{\cot(x)^5\,\left(5\cot(x)^2+7\right)}{35}$$

```
[In] int(cot(x)^4/sin(x)^4,x)
[Out] -(cot(x)^5*(5*cot(x)^2 + 7))/35
```

3.101 $\int \cot^3(x)\csc^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

```
[Out] 1/4*csc(x)^4-1/6*csc(x)^6
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2686, 14}

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

```
[In] Int[Cot[x]^3*Csc[x]^4,x]
```

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^3\left(-1+x^2\right)\,dx,x,\csc(x)\right)\\
&= -\text{Subst}\left(\int \left(-x^3+x^5\right)\,dx,x,\csc(x)\right)\\
&= \frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc^4(x)\,dx=\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

```
[In] Integrate[Cot[x]^3*Csc[x]^4,x]
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{\left(\cot^6(x)\right)}{6}-\frac{\left(\cot^4(x)\right)}{4}$	14
default	$-\frac{\left(\cot^6(x)\right)}{6}-\frac{\left(\cot^4(x)\right)}{4}$	14
risch	$\frac{4\,\mathrm{e}^{8ix}+\frac{8\,\mathrm{e}^{6ix}}{3}+4\,\mathrm{e}^{4ix}}{\left(\mathrm{e}^{2ix}-1\right)^6}$	34

```
[In] int(cot(x)^3*csc(x)^4,x,method=_RETURNVERBOSE)
[Out] -1/6*cot(x)^6-1/4*cot(x)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(13) = 26$.

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\,\cos(x)^2 - 1}{12\,\left(\cos(x)^6 - 3\,\cos(x)^4 + 3\,\cos(x)^2 - 1\right)}$$

```
[In] integrate(cot(x)^3*csc(x)^4,x, algorithm="fricas")
[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cot^3(x)\csc^4(x)\,dx = -\frac{2 - 3\sin^2(x)}{12\sin^6(x)}$$

```
[In] integrate(cot(x)**3*csc(x)**4,x)
[Out] -(2 - 3*sin(x)**2)/(12*sin(x)**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\,\sin(x)^2 - 2}{12\,\sin(x)^6}$$

```
[In] integrate(cot(x)^3*csc(x)^4,x, algorithm="maxima")
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\,\sin(x)^2 - 2}{12\,\sin(x)^6}$$

```
[In] integrate(cot(x)^3*csc(x)^4,x, algorithm="giac")
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^3(x) \csc^4(x) \, dx = -\frac{\cot(x)^4 \left(2 \cot(x)^2 + 3\right)}{12}$$

```
[In] int(cot(x)^3/sin(x)^4,x)
[Out] -(cot(x)^4*(2*cot(x)^2 + 3))/12
```

3.102 $\int \csc(x) \, dx$

Optimal result

Integrand size = 2, antiderivative size = 5

$$\int \csc(x) \, dx = -\operatorname{arctanh}(\cos(x))$$

```
[Out] -arctanh(cos(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{3855\}$

$$\int \csc(x) \, dx = -\operatorname{arctanh}(\cos(x))$$

```
[In] Int[Csc[x],x]
```

```
[Out] -ArcTanh[Cos[x]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\operatorname{arctanh}(\cos(x))$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$.

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 3.40

$$\int \csc(x)\,dx = -\log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Csc[x],x]
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
norman	$\ln\left(\tan\left(\frac{x}{2}\right)\right)$	6
parallelrisch	$\ln\left(\tan\left(\frac{x}{2}\right)\right)$	6
lookup	$-\ln\left(\csc\left(x\right)+\cot\left(x\right)\right)$	9
default	$-\ln\left(\csc\left(x\right)+\cot\left(x\right)\right)$	9
risch	$\ln\left(e^{ix}-1\right)-\ln\left(e^{ix}+1\right)$	20

```
[In] int(csc(x),x,method=_RETURNVERBOSE)
[Out] ln(tan(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 3.80

$$\int \csc(x)\,dx = -\frac{1}{2}\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{2}\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)$$

```
[In] integrate(csc(x),x, algorithm="fricas")
[Out] -1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.00

$$\int \csc(x)\,dx = \frac{\log\left(\cos\left(x\right) - 1\right)}{2} - \frac{\log\left(\cos\left(x\right) + 1\right)}{2}$$

```
[In] integrate(csc(x),x)
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

$$\int \csc(x)\,dx = -\log\left(\cot\left(x\right) + \csc\left(x\right)\right)$$

```
[In] integrate(csc(x),x, algorithm="maxima")
[Out] -log(cot(x) + csc(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int \csc(x)\,dx = \log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

```
[In] integrate(csc(x),x, algorithm="giac")
[Out] log(abs(tan(1/2*x)))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \csc(x)\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right)$$

```
[In] int(1/sin(x),x)
[Out] log(tan(x/2))
```

3.103 $\int \csc^3(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 16

$$\int \csc^3(x)\,dx = -\frac{1}{2}\text{arctanh}(\cos(x)) - \frac{1}{2}\cot(x)\csc(x)$$

```
[Out] -1/2*arctanh(cos(x))-1/2*cot(x)*csc(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3853, 3855}

$$\int \csc^3(x)\,dx = -\frac{1}{2}\text{arctanh}(\cos(x)) - \frac{1}{2}\cot(x)\csc(x)$$

```
[In] Int[Csc[x]^3,x]
```

```
[Out] -1/2*ArcTanh[Cos[x]] - (Cot[x]*Csc[x])/2
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{1}{2}\cot(x)\csc(x) + \frac{1}{2}\int \csc(x)\,dx$$
$$= -\frac{1}{2}\text{arctanh}(\cos(x)) - \frac{1}{2}\cot(x)\csc(x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.94

$$\int \csc^3(x)\,dx = -\frac{1}{8}\csc^2\left(\frac{x}{2}\right) - \frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{1}{8}\sec^2\left(\frac{x}{2}\right)$$

```
[In] Integrate[Csc[x]^3,x]
[Out] -1/8*Csc[x/2]^2 - Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2 + Sec[x/2]^2/8
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

method	result	size
default	$-\frac{\csc(x)\cot(x)}{2} + \frac{\ln(\csc(x)-\cot(x))}{2}$	18
parallelrisch	$-\frac{\csc(x)\cot(x)}{2} + \ln\left(\sqrt{\csc(x) - \cot(x)}\right)$	18
norman	$\frac{-\frac{1}{8}+\frac{\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}}{\tan\left(\frac{x}{2}\right)^2} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2}$	26
risch	$\frac{e^{3ix}+e^{ix}}{(e^{2ix}-1)^2} + \frac{\ln(e^{ix}-1)}{2} - \frac{\ln(e^{ix}+1)}{2}$	43

```
[In] int(csc(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*csc(x)*cot(x)+1/2*ln(csc(x)-cot(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int \csc^3(x)\,dx$$
$$= -\frac{\left(\cos(x)^2 - 1\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) - \left(\cos(x)^2 - 1\right)\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right) - 2\cos(x)}{4\left(\cos(x)^2 - 1\right)}$$

```
[In] integrate(csc(x)^3,x, algorithm="fricas")
[Out] -1/4*((cos(x)^2 - 1)*log(1/2*cos(x) + 1/2) - (cos(x)^2 - 1)*log(-1/2*cos(x)
 + 1/2) - 2*cos(x))/(cos(x)^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \csc^3(x)\,dx = \frac{\log(\cos(x) - 1)}{4} - \frac{\log(\cos(x) + 1)}{4} + \frac{\cos(x)}{2\cos^2(x) - 2}$$

```
[In] integrate(csc(x)**3,x)
[Out] log(cos(x) - 1)/4 - log(cos(x) + 1)/4 + cos(x)/(2*cos(x)**2 - 2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \csc^3(x)\,dx = \frac{\cos(x)}{2\left(\cos(x)^2 - 1\right)} - \frac{1}{4}\log(\cos(x) + 1) + \frac{1}{4}\log(\cos(x) - 1)$$

```
[In] integrate(csc(x)^3,x, algorithm="maxima")
[Out] 1/2*cos(x)/(cos(x)^2 - 1) - 1/4*log(cos(x) + 1) + 1/4*log(cos(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(12) = 24.

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.38

$$\int \csc^3(x)\,dx = -\frac{\left(\frac{2(\cos(x)-1)}{\cos(x)+1}-1\right)(\cos(x)+1)}{8(\cos(x)-1)} - \frac{\cos(x)-1}{8(\cos(x)+1)} + \frac{1}{4}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

```
[In] integrate(csc(x)^3,x, algorithm="giac")
[Out] -1/8*(2*(cos(x) - 1)/(cos(x) + 1) - 1)*(cos(x) + 1)/(cos(x) - 1) - 1/8*(cos
(x) - 1)/(cos(x) + 1) + 1/4*log(-(cos(x) - 1)/(cos(x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \csc^3(x)\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2} - \frac{\cos(x)}{2\sin(x)^2}$$

```
[In] int(1/sin(x)^3,x)
[Out] log(tan(x/2))/2 - cos(x)/(2*sin(x)^2)
```

3.104 $\int \cos(x)\cot(x)\,dx$

Optimal result

Integrand size = 5, antiderivative size = 8

$$\int \cos(x)\cot(x)\,dx = -\operatorname{arctanh}(\cos(x)) + \cos(x)$$

```
[Out] -arctanh(cos(x))+cos(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2672, 327, 212}

$$\int \cos(x)\cot(x)\,dx = \cos(x) - \operatorname{arctanh}(\cos(x))$$

```
[In] Int[Cos[x]*Cot[x],x]
```

```
[Out] -ArcTanh[Cos[x]] + Cos[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(x)\right) \\ &= \cos(x) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right) \\ &= -\operatorname{arctanh}(\cos(x)) + \cos(x) \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \cos(x)\cot(x)\,dx = \cos(x) - \log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Cos[x]*Cot[x],x]
[Out] Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

method	result	size
default	$\cos(x)+\ln(\csc(x)-\cot(x))$	12
parallelrisch	$\cos(x)+\ln(\csc(x)-\cot(x))+1$	13
norman	$\frac{2}{1+\tan^2\left(\frac{x}{2}\right)}+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	19
risch	$\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}+\ln(e^{ix}-1)-\ln(e^{ix}+1)$	34

```
[In] int(cos(x)^2/sin(x),x,method=_RETURNVERBOSE)
[Out] cos(x)+ln(csc(x)-cot(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(8) = 16.

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.62

$$\int \cos(x)\cot(x)\,dx = \cos(x) - \frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

```
[In] integrate(cos(x)^2/sin(x),x, algorithm="fricas")
[Out] cos(x) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \cos(x)\cot(x)\,dx = \frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2} + \cos(x)$$

```
[In] integrate(cos(x)**2/sin(x),x)
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2 + cos(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \cos(x)\cot(x)\,dx = \cos(x) - \frac{1}{2}\log(\cos(x) + 1) + \frac{1}{2}\log(\cos(x) - 1)$$

```
[In] integrate(cos(x)^2/sin(x),x, algorithm="maxima")
[Out] cos(x) - 1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \cos(x)\cot(x)\,dx = \cos(x) - \frac{1}{2}\log(\cos(x) + 1) + \frac{1}{2}\log(-\cos(x) + 1)$$

```
[In] integrate(cos(x)^2/sin(x),x, algorithm="giac")
[Out] cos(x) - 1/2*log(cos(x) + 1) + 1/2*log(-cos(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cos(x)\cot(x)\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right) + \cos(x)$$

```
[In] int(cos(x)^2/sin(x),x)
[Out] log(tan(x/2)) + cos(x)
```

3.105 $\int \csc^4(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 13

$$\int \csc^4(x)\,dx = -\cot(x) - \frac{\cot^3(x)}{3}$$

```
[Out] -cot(x)-1/3*cot(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3852}

$$\int \csc^4(x)\,dx = -\frac{1}{3}\cot^3(x) - \cot(x)$$

```
[In] Int[Csc[x]^4,x]
```

```
[Out] -Cot[x] - Cot[x]^3/3
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, \cot(x)\right)\\ &= -\cot(x) - \frac{\cot^3(x)}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \csc^4(x)\,dx = -\frac{2\cot(x)}{3} - \frac{1}{3}\cot(x)\csc^2(x)$$

```
[In] Integrate[Csc[x]^4,x]
[Out] (-2*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\left(-\frac{2}{3}-\frac{(\csc^2(x))}{3}\right)\cot(x)$	12
parallelrisch	$\frac{2(\cot^3(x))}{3}-\cot(x)(\csc^2(x))$	16
risch	$\frac{4i(3\,e^{2ix}-1)}{3(e^{2ix}-1)^3}$	22
norman	$\frac{-\frac{1}{24}-\frac{3(\tan^2(\frac{x}{2}))}{8}+\frac{3(\tan^4(\frac{x}{2}))}{8}+\frac{(\tan^6(\frac{x}{2}))}{24}}{\tan(\frac{x}{2})^3}$	34

```
[In] int(1/sin(x)^4,x,method=_RETURNVERBOSE)
[Out] (-2/3-1/3*csc(x)^2)*cot(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \csc^4(x)\,dx = -\frac{2\cos(x)^3 - 3\cos(x)}{3\left(\cos(x)^2 - 1\right)\sin(x)}$$

```
[In] integrate(1/sin(x)^4,x, algorithm="fricas")
[Out] -1/3*(2*cos(x)^3 - 3*cos(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \csc^4(x)\,dx = -\frac{2\cos{(x)}}{3\sin{(x)}} - \frac{\cos{(x)}}{3\sin^3{(x)}}$$

```
[In] integrate(1/sin(x)**4,x)
[Out] -2*cos(x)/(3*sin(x)) - cos(x)/(3*sin(x)**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \csc^4(x)\,dx = -\frac{3\,\tan{(x)}^2 + 1}{3\,\tan{(x)}^3}$$

```
[In] integrate(1/sin(x)^4,x, algorithm="maxima")
[Out] -1/3*(3*tan(x)^2 + 1)/tan(x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \csc^4(x)\,dx = -\frac{3\,\tan{(x)}^2 + 1}{3\,\tan{(x)}^3}$$

```
[In] integrate(1/sin(x)^4,x, algorithm="giac")
[Out] -1/3*(3*tan(x)^2 + 1)/tan(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \csc^4(x)\,dx = -\frac{2\,\cos{(x)}\,\sin{(x)}^2 + \cos{(x)}}{3\sin{(x)}^3}$$

```
[In] int(1/sin(x)^4,x)
[Out] -(cos(x) + 2*cos(x)*sin(x)^2)/(3*sin(x)^3)
```

3.106 $\int \sin(2x)\sin(5x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \sin(2x)\sin(5x)\,dx = \frac{1}{6}\sin(3x) - \frac{1}{14}\sin(7x)$$

```
[Out] 1/6*sin(3*x)-1/14*sin(7*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4367}

$$\int \sin(2x)\sin(5x)\,dx = \frac{1}{6}\sin(3x) - \frac{1}{14}\sin(7x)$$

```
[In] Int[Sin[2*x]*Sin[5*x],x]
```

```
[Out] Sin[3*x]/6 - Sin[7*x]/14
```

Rule 4367

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}\sin(3x) - \frac{1}{14}\sin(7x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sin(2x)\sin(5x)\,dx = \frac{1}{6}\sin(3x) - \frac{1}{14}\sin(7x)$$

```
[In] Integrate[Sin[2*x]*Sin[5*x],x]
[Out] Sin[3*x]/6 - Sin[7*x]/14
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sin(3x)}{6} - \frac{\sin(7x)}{14}$	14
risch	$\frac{\sin(3x)}{6} - \frac{\sin(7x)}{14}$	14
parallelrisch	$\frac{\left(-\sin\left(\frac{3x}{2}\right)+3\sin\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{3x}{2}\right)+3\cos\left(\frac{x}{2}\right)\right)\left(11+6\cos(4x)+18\cos(2x)\right)}{21}$	41
norman	$\frac{\frac{10\tan(x)\left(\tan^2\left(\frac{5x}{2}\right)\right)}{21}-\frac{4\left(\tan^2(x)\right)\tan\left(\frac{5x}{2}\right)}{21}-\frac{10\tan(x)}{21}+\frac{4\tan\left(\frac{5x}{2}\right)}{21}}{\left(1+\tan^2(x)\right)\left(1+\tan^2\left(\frac{5x}{2}\right)\right)}$	51

```
[In] int(sin(2*x)*sin(5*x),x,method=_RETURNVERBOSE)
[Out] 1/6*sin(3*x)-1/14*sin(7*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \sin(2x)\sin(5x)\,dx = -\frac{2}{21}\left(48\cos(x)^6 - 60\cos(x)^4 + 11\cos(x)^2 + 1\right)\sin(x)$$

```
[In] integrate(sin(2*x)*sin(5*x),x, algorithm="fricas")
[Out] -2/21*(48*cos(x)^6 - 60*cos(x)^4 + 11*cos(x)^2 + 1)*sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \sin(2x)\sin(5x)\,dx = -\frac{5\sin(2x)\cos(5x)}{21} + \frac{2\sin(5x)\cos(2x)}{21}$$

```
[In] integrate(sin(2*x)*sin(5*x),x)
[Out] -5*sin(2*x)*cos(5*x)/21 + 2*sin(5*x)*cos(2*x)/21
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(2x)\sin(5x)\,dx = -\frac{1}{14}\sin(7x) + \frac{1}{6}\sin(3x)$$

```
[In] integrate(sin(2*x)*sin(5*x),x, algorithm="maxima")
[Out] -1/14*sin(7*x) + 1/6*sin(3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(2x)\sin(5x)\,dx = -\frac{1}{14}\sin(7x) + \frac{1}{6}\sin(3x)$$

```
[In] integrate(sin(2*x)*sin(5*x),x, algorithm="giac")
[Out] -1/14*sin(7*x) + 1/6*sin(3*x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(2x)\sin(5x)\,dx = \frac{\sin(3x)}{6} - \frac{\sin(7x)}{14}$$

```
[In] int(sin(2*x)*sin(5*x),x)
[Out] sin(3*x)/6 - sin(7*x)/14
```

3.107 $\int \cos(x)\sin(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cos(x)\sin(3x)\,dx = -\frac{1}{4}\cos(2x) - \frac{1}{8}\cos(4x)$$

```
[Out] -1/4*cos(2*x)-1/8*cos(4*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{4369\}$

$$\int \cos(x)\sin(3x)\,dx = -\frac{1}{4}\cos(2x) - \frac{1}{8}\cos(4x)$$

```
[In] Int[Cos[x]*Sin[3*x],x]
```

```
[Out] -1/4*Cos[2*x] - Cos[4*x]/8
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{4}\cos(2x) - \frac{1}{8}\cos(4x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(3x)\,dx = -\frac{1}{2}\cos^2(x) - \frac{1}{8}\cos(4x)$$

[In] Integrate[Cos[x]*Sin[3*x],x]

[Out] -1/2*Cos[x]^2 - Cos[4*x]/8

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\cos(2x)}{4}-\frac{\cos(4x)}{8}$	14
risch	$-\frac{\cos(2x)}{4}-\frac{\cos(4x)}{8}$	14
parallelrisch	$-\frac{\cos(4x)}{8}+\frac{3}{8}-\frac{\cos(2x)}{4}$	15
norman	$\frac{\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{4}+\frac{3\left(\tan^2\left(\frac{3x}{2}\right)\right)}{4}-\frac{\tan\left(\frac{x}{2}\right)\tan\left(\frac{3x}{2}\right)}{2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2\left(\frac{3x}{2}\right)\right)}$	49

[In] int(cos(x)*sin(3*x),x,method=_RETURNVERBOSE)

[Out] -1/4*cos(2*x)-1/8*cos(4*x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(3x)\,dx = -\cos(x)^4 + \frac{1}{2}\cos(x)^2$$

[In] integrate(cos(x)*sin(3*x),x, algorithm="fricas")

[Out] -cos(x)^4 + 1/2*cos(x)^2

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \cos(x)\sin(3x)\,dx = -\frac{\sin{(x)}\sin{(3x)}}{8} - \frac{3\cos{(x)}\cos{(3x)}}{8}$$

```
[In] integrate(cos(x)*sin(3*x),x)
[Out] -sin(x)*sin(3*x)/8 - 3*cos(x)*cos(3*x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(3x)\,dx = -\frac{1}{8}\cos{(4x)} - \frac{1}{4}\cos{(2x)}$$

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="maxima")
[Out] -1/8*cos(4*x) - 1/4*cos(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(3x)\,dx = -\cos{(x)}^4 + \frac{1}{2}\cos{(x)}^2$$

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="giac")
[Out] -cos(x)^4 + 1/2*cos(x)^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(3x)\,dx = \frac{\cos{(x)}^2}{2} - \cos{(x)}^4$$

```
[In] int(sin(3*x)*cos(x),x)
[Out] cos(x)^2/2 - cos(x)^4
```

3.108 $\int \cos(3x)\cos(4x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \cos(3x)\cos(4x)\,dx = \frac{\sin(x)}{2} + \frac{1}{14}\sin(7x)$$

```
[Out] 1/2*sin(x)+1/14*sin(7*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{4368\}$

$$\int \cos(3x)\cos(4x)\,dx = \frac{\sin(x)}{2} + \frac{1}{14}\sin(7x)$$

```
[In] Int[Cos[3*x]*Cos[4*x],x]
```

```
[Out] Sin[x]/2 + Sin[7*x]/14
```

Rule 4368

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\sin(x)}{2} + \frac{1}{14}\sin(7x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(3x)\cos(4x)\,dx = \frac{\sin(x)}{2} + \frac{1}{14}\sin(7x)$$

```
[In] Integrate[Cos[3*x]*Cos[4*x],x]
[Out] Sin[x]/2 + Sin[7*x]/14
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sin(x)}{2}+\frac{\sin(7x)}{14}$	12
risch	$\frac{\sin(x)}{2}+\frac{\sin(7x)}{14}$	12
parallelrisch	$\frac{\sin(x)}{2}+\frac{\sin(7x)}{14}$	12
norman	$\frac{-\frac{8\tan(2x)\left(\tan^2\left(\frac{3x}{2}\right)\right)}{7}+\frac{6\left(\tan^2(2x)\right)\tan\left(\frac{3x}{2}\right)}{7}+\frac{8\tan(2x)}{7}-\frac{6\tan\left(\frac{3x}{2}\right)}{7}}{\left(1+\tan^2\left(\frac{3x}{2}\right)\right)\left(1+\tan^2(2x)\right)}$	59

```
[In] int(cos(3*x)*cos(4*x),x,method=_RETURNVERBOSE)
[Out] 1/2*sin(x)+1/14*sin(7*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \cos(3x)\cos(4x)\,dx = \frac{1}{7}\left(32\cos(x)^6 - 40\cos(x)^4 + 12\cos(x)^2 + 3\right)\sin(x)$$

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="fricas")
[Out] 1/7*(32*cos(x)^6 - 40*cos(x)^4 + 12*cos(x)^2 + 3)*sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \cos(3x)\cos(4x)\,dx = -\frac{3\sin{(3x)}\cos{(4x)}}{7} + \frac{4\sin{(4x)}\cos{(3x)}}{7}$$

```
[In] integrate(cos(3*x)*cos(4*x),x)
[Out] -3*sin(3*x)*cos(4*x)/7 + 4*sin(4*x)*cos(3*x)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x)\cos(4x)\,dx = \frac{1}{14}\sin{(7x)} + \frac{1}{2}\sin{(x)}$$

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="maxima")
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x)\cos(4x)\,dx = \frac{1}{14}\sin{(7x)} + \frac{1}{2}\sin{(x)}$$

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="giac")
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x)\cos(4x)\,dx = \frac{\sin{(7x)}}{14} + \frac{\sin{(x)}}{2}$$

```
[In] int(cos(3*x)*cos(4*x),x)
[Out] sin(7*x)/14 + sin(x)/2
```

3.109 $\int \sin(3x)\sin(6x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \sin(3x)\sin(6x)\,dx = \frac{1}{6}\sin(3x) - \frac{1}{18}\sin(9x)$$

```
[Out] 1/6*sin(3*x)-1/18*sin(9*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4367}

$$\int \sin(3x)\sin(6x)\,dx = \frac{1}{6}\sin(3x) - \frac{1}{18}\sin(9x)$$

```
[In] Int[Sin[3*x]*Sin[6*x],x]
```

```
[Out] Sin[3*x]/6 - Sin[9*x]/18
```

Rule 4367

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}\sin(3x) - \frac{1}{18}\sin(9x)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sin(3x)\sin(6x)\,dx = \frac{1}{6}\sin(3x) - \frac{1}{18}\sin(9x)$$

```
[In] Integrate[Sin[3*x]*Sin[6*x],x]
[Out] Sin[3*x]/6 - Sin[9*x]/18
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

method	result	size
derivativedivides	$\frac{2(\sin^3(3x))}{9}$	9
default	$\frac{2(\sin^3(3x))}{9}$	9
risch	$\frac{\sin(3x)}{6} - \frac{\sin(9x)}{18}$	14
parallelrisch	$\frac{\sin(3x)}{6} - \frac{\sin(9x)}{18}$	14
norman	$\frac{-\frac{2\tan(3x)\left(\tan^2\left(\frac{3x}{2}\right)\right)}{9}+\frac{4\left(\tan^2(3x)\right)\tan\left(\frac{3x}{2}\right)}{9}+\frac{2\tan(3x)}{9}-\frac{4\tan\left(\frac{3x}{2}\right)}{9}}{\left(1+\tan^2\left(\frac{3x}{2}\right)\right)\left(1+\tan^2(3x)\right)}$	59

```
[In] int(sin(3*x)*sin(6*x),x,method=_RETURNVERBOSE)
[Out] 2/9*sin(3*x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sin(3x)\sin(6x)\,dx = -\frac{2}{9}\left(\cos\left(3\,x\right)^2 - 1\right)\sin\left(3\,x\right)$$

```
[In] integrate(sin(3*x)*sin(6*x),x, algorithm="fricas")
[Out] -2/9*(cos(3*x)^2 - 1)*sin(3*x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \sin(3x)\sin(6x)\,dx = -\frac{2\sin(3x)\cos(6x)}{9} + \frac{\sin(6x)\cos(3x)}{9}$$

```
[In] integrate(sin(3*x)*sin(6*x),x)
[Out] -2*sin(3*x)*cos(6*x)/9 + sin(6*x)*cos(3*x)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(3x)\sin(6x)\,dx = -\frac{1}{18}\sin(9x) + \frac{1}{6}\sin(3x)$$

```
[In] integrate(sin(3*x)*sin(6*x),x, algorithm="maxima")
[Out] -1/18*sin(9*x) + 1/6*sin(3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.47

$$\int \sin(3x)\sin(6x)\,dx = \frac{2}{9}\sin(3x)^3$$

```
[In] integrate(sin(3*x)*sin(6*x),x, algorithm="giac")
[Out] 2/9*sin(3*x)^3
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(3x)\sin(6x)\,dx = \frac{\sin(3x)}{6} - \frac{\sin(9x)}{18}$$

```
[In] int(sin(3*x)*sin(6*x),x)
[Out] sin(3*x)/6 - sin(9*x)/18
```

3.110 $\int \cos^5(x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \cos^5(x)\sin(x)\,dx = -\frac{1}{6}\cos^6(x)$$

```
[Out] -1/6*cos(x)^6
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2645, 30}

$$\int \cos^5(x)\sin(x)\,dx = -\frac{1}{6}\cos^6(x)$$

```
[In] Int[Cos[x]^5*Sin[x],x]
```

```
[Out] -1/6*Cos[x]^6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int x^5\,dx, x, \cos(x)\right)\\ &= -\frac{1}{6}\cos^6(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cos^5(x)\sin(x)\,dx = -\frac{1}{6}\cos^6(x)$$

```
[In] Integrate[Cos[x]^5*Sin[x],x]
[Out] -1/6*Cos[x]^6
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\left(\cos^6(x)\right)}{6}$	7
default	$-\frac{\left(\cos^6(x)\right)}{6}$	7
risch	$-\frac{\cos(6x)}{192}-\frac{\cos(4x)}{32}-\frac{5\cos(2x)}{64}$	20
parallelrisch	$-\frac{7}{32}-\frac{\cos(6x)}{192}-\frac{\cos(4x)}{32}-\frac{5\cos(2x)}{64}$	21
norman	$\frac{2\left(\tan^2\left(\frac{x}{2}\right)\right)+2\left(\tan^{10}\left(\frac{x}{2}\right)\right)+\frac{20\left(\tan^6\left(\frac{x}{2}\right)\right)}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$	37

```
[In] int(cos(x)^5*sin(x),x,method=_RETURNVERBOSE)
[Out] -1/6*cos(x)^6
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos^5(x)\sin(x)\,dx = -\frac{1}{6}\cos(x)^6$$

```
[In] integrate(cos(x)^5*sin(x),x, algorithm="fricas")
[Out] -1/6*cos(x)^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \cos^5(x)\sin(x)\,dx = -\frac{\cos^6(x)}{6}$$

```
[In] integrate(cos(x)**5*sin(x),x)
[Out] -cos(x)**6/6
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos^5(x)\sin(x)\,dx = -\frac{1}{6}\cos(x)^6$$

```
[In] integrate(cos(x)^5*sin(x),x, algorithm="maxima")
[Out] -1/6*cos(x)^6
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos^5(x)\sin(x)\,dx = -\frac{1}{6}\cos(x)^6$$

```
[In] integrate(cos(x)^5*sin(x),x, algorithm="giac")
[Out] -1/6*cos(x)^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \cos^5(x)\sin(x)\,dx = \frac{\sin(x)^6}{6} - \frac{\sin(x)^4}{2} + \frac{\sin(x)^2}{2}$$

```
[In] int(cos(x)^5*sin(x),x)
[Out] sin(x)^2/2 - sin(x)^4/2 + sin(x)^6/6
```

3.111 $\int \cos(x)\cos(2x)\cos(3x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

```
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4440, 2717}

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

```
[In] Int[Cos[x]*Cos[2*x]*Cos[3*x],x]
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4440

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
```

```
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{4}+\frac{1}{4}\cos(2x)+\frac{1}{4}\cos(4x)+\frac{1}{4}\cos(6x)\right)\,dx\\ &= \frac{x}{4}+\frac{1}{4}\int \cos(2x)\,dx+\frac{1}{4}\int \cos(4x)\,dx+\frac{1}{4}\int \cos(6x)\,dx\\ &= \frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

```
[In] Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$	23
risch	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$	23
parallelrisch	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$	23

```
[In] int(cos(x)*cos(2*x)*cos(3*x),x,method=_RETURNVERBOSE)
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{1}{12}\left(16\,\cos\left(x\right)^{5} - 10\,\cos\left(x\right)^{3} + 3\,\cos\left(x\right)\right)\sin\left(x\right) + \frac{1}{4}\,x$$

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="fricas")
[Out] 1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(22) = 44.

Time = 0.99 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.80

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = -\frac{x\sin\left(x\right)\sin\left(2x\right)\cos\left(3x\right)}{4} + \frac{x\sin\left(x\right)\sin\left(3x\right)\cos\left(2x\right)}{4} + \frac{x\sin\left(2x\right)\sin\left(3x\right)\cos\left(x\right)}{4} + \frac{x\cos\left(x\right)\cos\left(2x\right)\cos\left(3x\right)}{4} - \frac{\sin\left(x\right)\cos\left(2x\right)\cos\left(3x\right)}{24} - \frac{\sin\left(2x\right)\cos\left(x\right)\cos\left(3x\right)}{6} + \frac{3\sin\left(3x\right)\cos\left(x\right)\cos\left(2x\right)}{8}$$

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x)
[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*s
in(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 - sin(x)*cos(2*x)*cos(3*x)/
24 - sin(2*x)*cos(x)*cos(3*x)/6 + 3*sin(3*x)*cos(x)*cos(2*x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{1}{4}\,x + \frac{1}{24}\,\sin\left(6\,x\right) + \frac{1}{16}\,\sin\left(4\,x\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="maxima")
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{1}{4}\,x + \frac{1}{24}\,\sin\left(6\,x\right) + \frac{1}{16}\,\sin\left(4\,x\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="giac")
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{x}{4} + \frac{\sin\left(2\,x\right)}{8} + \frac{\sin\left(4\,x\right)}{16} + \frac{\sin\left(6\,x\right)}{24}$$

```
[In] int(cos(2*x)*cos(3*x)*cos(x),x)
[Out] x/4 + sin(2*x)/8 + sin(4*x)/16 + sin(6*x)/24
```

3.112 $\int \cos^2(x) \left(1-\tan^2(x)\right) dx$

Optimal result

Integrand size = 13, antiderivative size = 5

$$\int \cos^2(x) \left(1-\tan^2(x)\right) dx = \cos(x)\sin(x)$$

```
[Out] cos(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3756, 391}

$$\int \cos^2(x) \left(1-\tan^2(x)\right) dx = \sin(x)\cos(x)$$

```
[In] Int[Cos[x]^2*(1 - Tan[x]^2),x]
```

```
[Out] Cos[x]*Sin[x]
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
```

```
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1-x^2}{(1+x^2)^2}\,dx, x, \tan(x)\right)\\ &= \cos(x)\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

$$\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx = \frac{1}{2}\sin(2x)$$

```
[In] Integrate[Cos[x]^2*(1 - Tan[x]^2),x]
[Out] Sin[2*x]/2
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
default	$\cos(x)\sin(x)$	6
risch	$\frac{\sin(2x)}{2}$	7

```
[In] int((1-tan(x)^2)/sec(x)^2,x,method=_RETURNVERBOSE)
[Out] cos(x)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx = \cos(x)\sin(x)$$

```
[In] integrate((1-tan(x)^2)/sec(x)^2,x, algorithm="fricas")
[Out] cos(x)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx=\frac{\tan(x)}{\sec^2(x)}$$

```
[In] integrate((1-tan(x)**2)/sec(x)**2,x)
[Out] tan(x)/sec(x)**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx=\frac{\tan(x)}{\tan(x)^2+1}$$

```
[In] integrate((1-tan(x)^2)/sec(x)^2,x, algorithm="maxima")
[Out] tan(x)/(tan(x)^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx=\frac{1}{\frac{1}{\tan(x)}+\tan(x)}$$

```
[In] integrate((1-tan(x)^2)/sec(x)^2,x, algorithm="giac")
[Out] 1/(1/tan(x) + tan(x))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx=\frac{\sin(2x)}{2}$$

```
[In] int(-cos(x)^2*(tan(x)^2 - 1),x)
[Out] sin(2*x)/2
```

3.113 $\int \csc(2x)(\cos(x)+\sin(x))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \csc(2x)(\cos(x)+\sin(x))\,dx = -\frac{1}{2}\text{arctanh}(\cos(x))+\frac{1}{2}\text{arctanh}(\sin(x))$$

```
[Out] -1/2*arctanh(cos(x))+1/2*arctanh(sin(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4486, 4372, 3855, 4373}

$$\int \csc(2x)(\cos(x)+\sin(x))\,dx = \frac{1}{2}\text{arctanh}(\sin(x))-\frac{1}{2}\text{arctanh}(\cos(x))$$

```
[In] Int[Csc[2*x]*(Cos[x] + Sin[x]),x]
```

```
[Out] -1/2*ArcTanh[Cos[x]] + ArcTanh[Sin[x]]/2
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (\cos(x)\csc(2x) + \csc(2x)\sin(x))\,dx \\
&= \int \cos(x)\csc(2x)\,dx + \int \csc(2x)\sin(x)\,dx \\
&= \frac{1}{2}\int \csc(x)\,dx + \frac{1}{2}\int \sec(x)\,dx \\
&= -\frac{1}{2}\text{arctanh}(\cos(x)) + \frac{1}{2}\text{arctanh}(\sin(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

$$\int \csc(2x)(\cos(x)+\sin(x))\,dx = \frac{1}{2}\text{arctanh}(\sin(x)) - \frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Csc[2*x]*(Cos[x] + Sin[x]),x]
[Out] ArcTanh[Sin[x]]/2 - Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
parts	$-\frac{\ln(\csc(x)+\cot(x))}{2}+\frac{\ln(\sec(x)+\tan(x))}{2}$	18
default	$\frac{\ln(\sec(x)+\tan(x))}{2}+\frac{\ln(\csc(x)-\cot(x))}{2}$	20
risch	$\frac{\ln(\mathrm{e}^{2ix}+(-1+i)\mathrm{e}^{ix}-i)}{2}-\frac{\ln(\mathrm{e}^{2ix}+(1-i)\mathrm{e}^{ix}-i)}{2}$	42

```
[In] int((cos(x)+sin(x))/sin(2*x),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(csc(x)+cot(x))+1/2*ln(sec(x)+tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(11) = 22.

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \csc(2x)(\cos(x)+\sin(x))\,dx = -\frac{1}{4}\,\log\left(-\frac{1}{2}\,(\cos(x)+1)\sin(x)+\frac{1}{2}\,\cos(x)+\frac{1}{2}\right) + \frac{1}{4}\,\log\left(-\frac{1}{2}\,(\cos(x)-1)\sin(x)-\frac{1}{2}\,\cos(x)+\frac{1}{2}\right)$$

```
[In] integrate((cos(x)+sin(x))/sin(2*x),x, algorithm="fricas")
[Out] -1/4*log(-1/2*(cos(x) + 1)*sin(x) + 1/2*cos(x) + 1/2) + 1/4*log(-1/2*(cos(x
) - 1)*sin(x) - 1/2*cos(x) + 1/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(12) = 24.

Time = 0.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \csc(2x)(\cos(x)+\sin(x))\,dx = -\frac{\log(\sin(x)-1)}{4}+\frac{\log(\sin(x)+1)}{4} + \frac{\log(\cos(x)-1)}{4}-\frac{\log(\cos(x)+1)}{4}$$

```
[In] integrate((cos(x)+sin(x))/sin(2*x),x)
[Out] -log(sin(x) - 1)/4 + log(sin(x) + 1)/4 + log(cos(x) - 1)/4 - log(cos(x) + 1
)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(11) = 22.

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.60

$$\int \csc(2x)(\cos(x)+\sin(x))\,dx = -\frac{1}{4}\,\log\left(\cos(x)^2+\sin(x)^2+2\,\cos(x)+1\right) + \frac{1}{4}\,\log\left(\cos(x)^2+\sin(x)^2-2\,\cos(x)+1\right) + \frac{1}{4}\,\log\left(\cos(x)^2+\sin(x)^2+2\,\sin(x)+1\right) - \frac{1}{4}\,\log\left(\cos(x)^2+\sin(x)^2-2\,\sin(x)+1\right)$$

```
[In] integrate((cos(x)+sin(x))/sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
- 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*log(cos
(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(11) = 22.

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \csc(2x)(\cos(x)+\sin(x))\,dx = \frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right) - \frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right) + \frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

```
[In] integrate((cos(x)+sin(x))/sin(2*x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(tan(1/2*x) + 1)) - 1/2*log(abs(tan(1/2*x) - 1)) + 1/2*log(abs(t
an(1/2*x)))
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \csc(2x)(\cos(x)+\sin(x))\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)\right)}{2} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{2}$$

```
[In] int((cos(x) + sin(x))/sin(2*x),x)
```

```
[Out] log(tan(x/2) + tan(x/2)^2)/2 - log(tan(x/2) - 1)/2
```

3.114 $\int \sin^2(x)\tan(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \sin^2(x)\tan(x)\,dx = \frac{\cos^2(x)}{2} - \log(\cos(x))$$

```
[Out] 1/2*cos(x)^2-ln(cos(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2670, 14}

$$\int \sin^2(x)\tan(x)\,dx = \frac{\cos^2(x)}{2} - \log(\cos(x))$$

```
[In] Int[Sin[x]^2*Tan[x],x]
```

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, \cos(x)\right)\\ &= -\text{Subst}\left(\int \left(\frac{1}{x}-x\right)\,dx, x, \cos(x)\right)\\ &= \frac{\cos^2(x)}{2}-\log(\cos(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sin^2(x)\tan(x)\,dx = \frac{\cos^2(x)}{2}-\log(\cos(x))$$

```
[In] Integrate[Sin[x]^2*Tan[x],x]
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{(\sin^2(x))}{2}-\ln(\cos(x))$	13
risch	$ix+\frac{e^{2ix}}{8}+\frac{e^{-2ix}}{8}-\ln(e^{2ix}+1)$	30

```
[In] int(sin(x)^2*tan(x),x,method=_RETURNVERBOSE)
[Out] -1/2*sin(x)^2-ln(cos(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sin^2(x)\tan(x)\,dx = \frac{1}{2}\cos(x)^2-\log(-\cos(x))$$

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="fricas")
[Out] 1/2*cos(x)^2 - log(-cos(x))
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\tan(x)\,dx = -\log\left(\cos\left(x\right)\right) + \frac{\cos^2\left(x\right)}{2}$$

```
[In] integrate(sin(x)**2*tan(x),x)
[Out] -log(cos(x)) + cos(x)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \sin^2(x)\tan(x)\,dx = -\frac{1}{2}\sin\left(x\right)^2 - \frac{1}{2}\log\left(\sin\left(x\right)^2 - 1\right)$$

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="maxima")
[Out] -1/2*sin(x)^2 - 1/2*log(sin(x)^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \sin^2(x)\tan(x)\,dx = -\frac{1}{2}\sin\left(x\right)^2 - \frac{1}{2}\log\left(-\sin\left(x\right)^2 + 1\right)$$

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="giac")
[Out] -1/2*sin(x)^2 - 1/2*log(-sin(x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \sin^2(x)\tan(x)\,dx = \frac{\cos\left(x\right)^2}{2} + \frac{\ln\left(\tan(x)^2 + 1\right)}{2}$$

```
[In] int(sin(x)^2*tan(x),x)
[Out] log(tan(x)^2 + 1)/2 + cos(x)^2/2
```

3.115 $\int \cos^2(x)\cot^3(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 22

$$\int \cos^2(x)\cot^3(x)\,dx=-\frac{1}{2}\csc^2(x)-2\log(\sin(x))+\frac{\sin^2(x)}{2}$$

```
[Out] -1/2*csc(x)^2-2*ln(sin(x))+1/2*sin(x)^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2670, 272, 45}

$$\int \cos^2(x)\cot^3(x)\,dx=\frac{\sin^2(x)}{2}-\frac{1}{2}\csc^2(x)-2\log(\sin(x))$$

```
[In] Int[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] -1/2*Csc[x]^2 - 2*Log[Sin[x]] + Sin[x]^2/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{(1-x^2)^2}{x^3}\,dx, x, -\sin(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, \sin^2(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)\,dx, x, \sin^2(x)\right)\\ &= -\frac{1}{2}\csc^2(x) - 2\log(\sin(x)) + \frac{\sin^2(x)}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \cos^2(x)\cot^3(x)\,dx = \frac{1}{2}\left(-\csc^2(x) - 4\log(\sin(x)) + \sin^2(x)\right)$$

```
[In] Integrate[Cos[x]^2*Cot[x]^3,x]
[Out] (-Csc[x]^2 - 4*Log[Sin[x]] + Sin[x]^2)/2
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

method	result	size
default	$-\frac{\cos^6(x)}{2\sin(x)^2} - \frac{(\cos^4(x))}{2} - (\cos^2(x)) - 2\ln(\sin(x))$	29
risch	$2ix - \frac{e^{2ix}}{8} - \frac{e^{-2ix}}{8} + \frac{2\,e^{2ix}}{(e^{2ix}-1)^2} - 2\ln(e^{2ix}-1)$	46

```
[In] int(cos(x)^2*cot(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/sin(x)^2*cos(x)^6-1/2*cos(x)^4-cos(x)^2-2*ln(sin(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \cos^2(x)\cot^3(x)\,dx = -\frac{2\,\cos(x)^4 - 3\,\cos(x)^2 + 8\left(\cos(x)^2 - 1\right)\log\left(\frac{1}{2}\,\sin(x)\right) - 1}{4\left(\cos(x)^2 - 1\right)}$$

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="fricas")
[Out] -1/4*(2*cos(x)^4 - 3*cos(x)^2 + 8*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos(
x)^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \cos^2(x)\cot^3(x)\,dx = -2\log(\sin(x)) + \frac{\sin^2(x)}{2} - \frac{1}{2\sin^2(x)}$$

```
[In] integrate(cos(x)**2*cot(x)**3,x)
[Out] -2*log(sin(x)) + sin(x)**2/2 - 1/(2*sin(x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \cos^2(x)\cot^3(x)\,dx = \frac{1}{2}\,\sin(x)^2 - \frac{1}{2\,\sin(x)^2} - \log\left(\sin(x)^2\right)$$

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="maxima")
[Out] 1/2*sin(x)^2 - 1/2/sin(x)^2 - log(sin(x)^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \cos^2(x)\cot^3(x)\,dx = -\frac{1}{2}\cos{(x)}^2 + \frac{1}{2\left(\cos{(x)}^2 - 1\right)} - \log\left(-\cos{(x)}^2 + 1\right)$$

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="giac")
[Out] -1/2*cos(x)^2 + 1/2/(cos(x)^2 - 1) - log(-cos(x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \cos^2(x)\cot^3(x)\,dx = \ln\left(\tan(x)^2 + 1\right) - 2\,\ln\left(\tan(x)\right) - \frac{\tan(x)^2 + \frac{1}{2}}{\tan{(x)}^4 + \tan{(x)}^2}$$

```
[In] int(cos(x)^2*cot(x)^3,x)
[Out] log(tan(x)^2 + 1) - 2*log(tan(x)) - (tan(x)^2 + 1/2)/(tan(x)^2 + tan(x)^4)
```

3.116 $\int \sec^3(x) \tan(x) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \sec^3(x) \tan(x) \, dx = \frac{\sec^3(x)}{3}$$

```
[Out] 1/3*sec(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2686, 30}

$$\int \sec^3(x) \tan(x) \, dx = \frac{\sec^3(x)}{3}$$

```
[In] Int[Sec[x]^3*Tan[x],x]
```

```
[Out] Sec[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int x^2\,dx, x, \sec(x)\right) \\ &= \frac{\sec^3(x)}{3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sec^3(x)\tan(x)\,dx = \frac{\sec^3(x)}{3}$$

```
[In] Integrate[Sec[x]^3*Tan[x],x]
[Out] Sec[x]^3/3
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sec^3(x))}{3}$	7
default	$\frac{(\sec^3(x))}{3}$	7
risch	$\frac{8\,e^{3ix}}{3(e^{2ix}+1)^3}$	17

```
[In] int(sec(x)^3*tan(x),x,method=_RETURNVERBOSE)
[Out] 1/3*sec(x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\cos(x)^3}$$

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="fricas")
[Out] 1/3/cos(x)^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\cos^3(x)}$$

```
[In] integrate(sec(x)**3*tan(x),x)
[Out] 1/(3*cos(x)**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\,\cos(x)^3}$$

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="maxima")
[Out] 1/3/cos(x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\,\cos(x)^3}$$

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="giac")
[Out] 1/3/cos(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\cos(x)^3}$$

```
[In] int(tan(x)/cos(x)^3,x)
[Out] 1/(3*cos(x)^3)
```

3.117 $\int \sec^3(x)\tan^3(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \sec^3(x)\tan^3(x)\,dx = -\frac{1}{3}\sec^3(x) + \frac{\sec^5(x)}{5}$$

```
[Out] -1/3*sec(x)^3+1/5*sec(x)^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2686, 14}

$$\int \sec^3(x)\tan^3(x)\,dx = \frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

```
[In] Int[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -1/3*Sec[x]^3 + Sec[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\sec(x)\right)\\&=\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\sec(x)\right)\\&=-\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sec^3(x)\tan^3(x)\,dx=-\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}$$

```
[In] Integrate[Sec[x]^3*Tan[x]^3,x]
[Out] -1/3*Sec[x]^3 + Sec[x]^5/5
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{(\sec^3(x))}{3}+\frac{(\sec^5(x))}{5}$	14
default	$-\frac{(\sec^3(x))}{3}+\frac{(\sec^5(x))}{5}$	14
risch	$-\frac{8\left(5\,e^{7ix}-2\,e^{5ix}+5\,e^{3ix}\right)}{15\left(e^{2ix}+1\right)^5}$	34

```
[In] int(sec(x)^3*tan(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/3*sec(x)^3+1/5*sec(x)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^3(x)\tan^3(x)\,dx = -\frac{5\,\cos{(x)}^2 - 3}{15\,\cos{(x)}^5}$$

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="fricas")
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^3(x)\tan^3(x)\,dx = \frac{3 - 5\cos^2{(x)}}{15\cos^5{(x)}}$$

```
[In] integrate(sec(x)**3*tan(x)**3,x)
[Out] (3 - 5*cos(x)**2)/(15*cos(x)**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^3(x)\tan^3(x)\,dx = -\frac{5\,\cos{(x)}^2 - 3}{15\,\cos{(x)}^5}$$

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="maxima")
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^3(x)\tan^3(x)\,dx = -\frac{5\,\cos{(x)}^2 - 3}{15\,\cos{(x)}^5}$$

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="giac")
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sec^3(x)\tan^3(x)\,dx = \frac{1}{5\cos(x)^5} - \frac{1}{3\cos(x)^3}$$

```
[In] int(tan(x)^3/cos(x)^3,x)
[Out] 1/(5*cos(x)^5) - 1/(3*cos(x)^3)
```

3.118 $\int \frac{\sqrt{9-x^2}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{\sqrt{9-x^2}}{x^2} dx = -\frac{\sqrt{9-x^2}}{x} - \arcsin\left(\frac{x}{3}\right)$$

```
[Out] -arcsin(1/3*x)-(-x^2+9)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {283, 222}

$$\int \frac{\sqrt{9-x^2}}{x^2} dx = -\arcsin\left(\frac{x}{3}\right) - \frac{\sqrt{9-x^2}}{x}$$

```
[In] Int[Sqrt[9 - x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 - x^2]/x) - ArcSin[x/3]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{9-x^2}}{x} - \int \frac{1}{\sqrt{9-x^2}}\,dx$$
$$= -\frac{\sqrt{9-x^2}}{x} - \arcsin\left(\frac{x}{3}\right)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx = -\frac{\sqrt{9-x^2}}{x} + 2\arctan\left(\frac{\sqrt{9-x^2}}{3+x}\right)$$

```
[In] Integrate[Sqrt[9 - x^2]/x^2,x]
[Out] -(Sqrt[9 - x^2]/x) + 2*ArcTan[Sqrt[9 - x^2]/(3 + x)]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
risch	$\frac{x^2-9}{x\sqrt{-x^2+9}} - \arcsin\left(\frac{x}{3}\right)$	26
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{-x^2+9}}{x}\right)x-\sqrt{-x^2+9}}{x}$	33
default	$-\frac{(-x^2+9)^{\frac{3}{2}}}{9x} - \frac{x\sqrt{-x^2+9}}{9} - \arcsin\left(\frac{x}{3}\right)$	34
meijerg	$-\frac{i\left(-\frac{12i\sqrt{\pi}\sqrt{-\frac{x^2}{9}+1}}{x}-4i\sqrt{\pi}\arcsin\left(\frac{x}{3}\right)\right)}{4\sqrt{\pi}}$	36
trager	$-\frac{\sqrt{-x^2+9}}{x} + \text{RootOf}\left(\_Z^2+1\right)\ln\left(\text{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2+9}\right)$	42

```
[In] int((-x^2+9)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] (x^2-9)/x/(-x^2+9)^(1/2)-arcsin(1/3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx = \frac{2\,x\arctan\left(\frac{\sqrt{-x^2+9}-3}{x}\right)-\sqrt{-x^2+9}}{x}$$

```
[In] integrate((-x^2+9)^(1/2)/x^2,x, algorithm="fricas")
[Out] (2*x*arctan((sqrt(-x^2 + 9) - 3)/x) - sqrt(-x^2 + 9))/x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx = -\operatorname{asin}\left(\frac{x}{3}\right)-\frac{\sqrt{9-x^2}}{x}$$

```
[In] integrate((-x**2+9)**(1/2)/x**2,x)
[Out] -asin(x/3) - sqrt(9 - x**2)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx = -\frac{\sqrt{-x^2+9}}{x}-\arcsin\left(\frac{1}{3}\,x\right)$$

```
[In] integrate((-x^2+9)^(1/2)/x^2,x, algorithm="maxima")
[Out] -sqrt(-x^2 + 9)/x - arcsin(1/3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx = \frac{x}{2\left(\sqrt{-x^2+9}-3\right)}-\frac{\sqrt{-x^2+9}-3}{2\,x}-\arcsin\left(\frac{1}{3}\,x\right)$$

```
[In] integrate((-x^2+9)^(1/2)/x^2,x, algorithm="giac")
[Out] 1/2*x/(sqrt(-x^2 + 9) - 3) - 1/2*(sqrt(-x^2 + 9) - 3)/x - arcsin(1/3*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx=-\mathrm{asin}\left(\frac{x}{3}\right)-\frac{\sqrt{9-x^2}}{x}$$

```
[In] int((9 - x^2)^(1/2)/x^2,x)
[Out] - asin(x/3) - (9 - x^2)^(1/2)/x
```

3.119 $\int \frac{1}{x^2\sqrt{4+x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{x^2\sqrt{4+x^2}}\,dx = -\frac{\sqrt{4+x^2}}{4x}$$

```
[Out] -1/4*(x^2+4)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\int \frac{1}{x^2\sqrt{4+x^2}}\,dx = -\frac{\sqrt{x^2+4}}{4x}$$

```
[In] Int[1/(x^2*Sqrt[4 + x^2]),x]
```

```
[Out] -1/4*Sqrt[4 + x^2]/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{4+x^2}}{4x}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{4+x^2}}\,dx = -\frac{\sqrt{4+x^2}}{4x}$$

```
[In] Integrate[1/(x^2*Sqrt[4 + x^2]),x]
[Out] -1/4*Sqrt[4 + x^2]/x
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{\sqrt{x^2+4}}{4x}$	13
default	$-\frac{\sqrt{x^2+4}}{4x}$	13
trager	$-\frac{\sqrt{x^2+4}}{4x}$	13
risch	$-\frac{\sqrt{x^2+4}}{4x}$	13
pseudoelliptic	$-\frac{\sqrt{x^2+4}}{4x}$	13
meijerg	$-\frac{\sqrt{1+\frac{x^2}{4}}}{2x}$	15

```
[In] int(1/x^2/(x^2+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(x^2+4)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\sqrt{4+x^2}}\,dx = -\frac{x+\sqrt{x^2+4}}{4\,x}$$

```
[In] integrate(1/x^2/(x^2+4)^(1/2),x, algorithm="fricas")
[Out] -1/4*(x + sqrt(x^2 + 4))/x
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^2\sqrt{4+x^2}}\,dx = -\frac{\sqrt{1+\frac{4}{x^2}}}{4}$$

```
[In] integrate(1/x**2/(x**2+4)**(1/2),x)
[Out] -sqrt(1 + 4/x**2)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^2\sqrt{4+x^2}}\,dx = -\frac{\sqrt{x^2+4}}{4\,x}$$

```
[In] integrate(1/x^2/(x^2+4)^(1/2),x, algorithm="maxima")
[Out] -1/4*sqrt(x^2 + 4)/x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^2\sqrt{4+x^2}}\,dx = \frac{2}{\left(x-\sqrt{x^2+4}\right)^2-4}$$

```
[In] integrate(1/x^2/(x^2+4)^(1/2),x, algorithm="giac")
[Out] 2/((x - sqrt(x^2 + 4))^2 - 4)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^2\sqrt{4+x^2}}\,dx = -\frac{\sqrt{x^2+4}}{4\,x}$$

```
[In] int(1/(x^2*(x^2 + 4)^(1/2)),x)
[Out] -(x^2 + 4)^(1/2)/(4*x)
```

3.120 $\int \frac{x}{\sqrt{4+x^2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 9

$$\int \frac{x}{\sqrt{4+x^2}} \, dx = \sqrt{4+x^2}$$

```
[Out] (x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {267}

$$\int \frac{x}{\sqrt{4+x^2}} \, dx = \sqrt{x^2+4}$$

```
[In] Int[x/Sqrt[4 + x^2],x]
```

```
[Out] Sqrt[4 + x^2]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \sqrt{4+x^2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{4+x^2}}\,dx=\sqrt{4+x^2}$$

```
[In] Integrate[x/Sqrt[4 + x^2],x]
[Out] Sqrt[4 + x^2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
gosper	$\sqrt{x^2+4}$	8
derivativedivides	$\sqrt{x^2+4}$	8
default	$\sqrt{x^2+4}$	8
trager	$\sqrt{x^2+4}$	8
risch	$\sqrt{x^2+4}$	8
pseudoelliptic	$\sqrt{x^2+4}$	8
meijerg	$\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1+\frac{x^2}{4}}}{\sqrt{\pi}}$	25

```
[In] int(x/(x^2+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] (x^2+4)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{4+x^2}}\,dx=\sqrt{x^2+4}$$

```
[In] integrate(x/(x^2+4)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{4+x^2}}\,dx = \sqrt{x^2+4}$$

```
[In] integrate(x/(x**2+4)**(1/2),x)
[Out] sqrt(x**2 + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{4+x^2}}\,dx = \sqrt{x^2+4}$$

```
[In] integrate(x/(x^2+4)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{4+x^2}}\,dx = \sqrt{x^2+4}$$

```
[In] integrate(x/(x^2+4)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 + 4)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{4+x^2}}\,dx = \sqrt{x^2+4}$$

```
[In] int(x/(x^2 + 4)^(1/2),x)
[Out] (x^2 + 4)^(1/2)
```

3.121 $\int \frac{1}{\sqrt{-a^2+x^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{\sqrt{-a^2+x^2}} \, dx = \operatorname{arctanh}\left(\frac{x}{\sqrt{-a^2+x^2}}\right)$$

```
[Out] arctanh(x/(-a^2+x^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {223, 212}

$$\int \frac{1}{\sqrt{-a^2+x^2}} \, dx = \operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-a^2}}\right)$$

```
[In] Int[1/Sqrt[-a^2 + x^2],x]
```

```
[Out] ArcTanh[x/Sqrt[-a^2 + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-a^2+x^2}}\right)$$
$$= \text{arctanh}\left(\frac{x}{\sqrt{-a^2+x^2}}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. $2(16) = 32$.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.88

$$\int \frac{1}{\sqrt{-a^2+x^2}}\,dx = -\frac{1}{2}\log\left(1-\frac{x}{\sqrt{-a^2+x^2}}\right)+\frac{1}{2}\log\left(1+\frac{x}{\sqrt{-a^2+x^2}}\right)$$

```
[In] Integrate[1/Sqrt[-a^2 + x^2],x]
[Out] -1/2*Log[1 - x/Sqrt[-a^2 + x^2]] + Log[1 + x/Sqrt[-a^2 + x^2]]/2
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\ln\left(x+\sqrt{-a^2+x^2}\right)$	15
pseudoelliptic	$\text{arctanh}\left(\frac{\sqrt{-a^2+x^2}}{x}\right)$	17

```
[In] int(1/(-a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(x+(-a^2+x^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{-a^2+x^2}}\,dx = -\log\left(-x+\sqrt{-a^2+x^2}\right)$$

```
[In] integrate(1/(-a^2+x^2)^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(-a^2 + x^2))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt{-a^2+x^2}}\,dx = \begin{cases} \operatorname{acosh}\left(\frac{x}{a}\right) & \text{for } \left|\frac{x^2}{a^2}\right| > 1 \\ -i\operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(-a**2+x**2)**(1/2),x)
[Out] Piecewise((acosh(x/a), Abs(x**2/a**2) > 1), (-I*asin(x/a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{-a^2+x^2}}\,dx = \log\left(2\,x+2\,\sqrt{-a^2+x^2}\right)$$

```
[In] integrate(1/(-a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] log(2*x + 2*sqrt(-a^2 + x^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(14) = 28.

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31

$$\int \frac{1}{\sqrt{-a^2+x^2}}\,dx = \frac{1}{2}\,a^2\log\left(\left|-x+\sqrt{-a^2+x^2}\right|\right)+\frac{1}{2}\,\sqrt{-a^2+x^2}x$$

```
[In] integrate(1/(-a^2+x^2)^(1/2),x, algorithm="giac")
[Out] 1/2*a^2*log(abs(-x + sqrt(-a^2 + x^2))) + 1/2*sqrt(-a^2 + x^2)*x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-a^2+x^2}}\,dx=\ln\left(x+\sqrt{x^2-a^2}\right)$$

```
[In] int(1/(x^2 - a^2)^(1/2),x)
[Out] log(x + (x^2 - a^2)^(1/2))
```

3.122 $\int \frac{x^3}{(9+4x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \frac{x^3}{(9+4x^2)^{3/2}}\,dx = \frac{9}{16\sqrt{9+4x^2}} + \frac{1}{16}\sqrt{9+4x^2}$$

```
[Out] 9/16/(4*x^2+9)^(1/2)+1/16*(4*x^2+9)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\int \frac{x^3}{(9+4x^2)^{3/2}}\,dx = \frac{1}{16}\sqrt{4x^2+9} + \frac{9}{16\sqrt{4x^2+9}}$$

```
[In] Int[x^3/(9 + 4*x^2)^(3/2),x]
```

```
[Out] 9/(16*Sqrt[9 + 4*x^2]) + Sqrt[9 + 4*x^2]/16
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(9+4x)^{3/2}}\,dx, x, x^2\right) \\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{9}{4(9+4x)^{3/2}} + \frac{1}{4\sqrt{9+4x}}\right)\,dx, x, x^2\right) \\ &= \frac{9}{16\sqrt{9+4x^2}} + \frac{1}{16}\sqrt{9+4x^2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{(9+4x^2)^{3/2}}\,dx = \frac{9+2x^2}{8\sqrt{9+4x^2}}$$

```
[In] Integrate[x^3/(9 + 4*x^2)^(3/2),x]
[Out] (9 + 2*x^2)/(8*Sqrt[9 + 4*x^2])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{2x^2+9}{8\sqrt{4x^2+9}}$	19
trager	$\frac{2x^2+9}{8\sqrt{4x^2+9}}$	19
risch	$\frac{2x^2+9}{8\sqrt{4x^2+9}}$	19
pseudoelliptic	$\frac{2x^2+9}{8\sqrt{4x^2+9}}$	19
default	$\frac{x^2}{4\sqrt{4x^2+9}}+\frac{9}{8\sqrt{4x^2+9}}$	27
meijerg	$\frac{-\frac{3\sqrt{\pi}}{8}+\frac{3\sqrt{\pi}\left(\frac{16x^2}{9}+8\right)}{64\sqrt{1+\frac{4x^2}{9}}}}{\sqrt{\pi}}$	33

```
[In] int(x^3/(4*x^2+9)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/8*(2*x^2+9)/(4*x^2+9)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int \frac{x^3}{(9+4x^2)^{3/2}}\,dx = \frac{2\,x^2+9}{8\,\sqrt{4\,x^2+9}}$$

```
[In] integrate(x^3/(4*x^2+9)^(3/2),x, algorithm="fricas")
[Out] 1/8*(2*x^2 + 9)/sqrt(4*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{(9+4x^2)^{3/2}}\,dx = \frac{x^2}{4\sqrt{4x^2+9}} + \frac{9}{8\sqrt{4x^2+9}}$$

```
[In] integrate(x**3/(4*x**2+9)**(3/2),x)
[Out] x**2/(4*sqrt(4*x**2 + 9)) + 9/(8*sqrt(4*x**2 + 9))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{(9+4x^2)^{3/2}}\,dx = \frac{x^2}{4\,\sqrt{4\,x^2+9}} + \frac{9}{8\,\sqrt{4\,x^2+9}}$$

```
[In] integrate(x^3/(4*x^2+9)^(3/2),x, algorithm="maxima")
[Out] 1/4*x^2/sqrt(4*x^2 + 9) + 9/8/sqrt(4*x^2 + 9)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{(9+4x^2)^{3/2}}\,dx = \frac{1}{16}\,\sqrt{4\,x^2+9} + \frac{9}{16\,\sqrt{4\,x^2+9}}$$

```
[In] integrate(x^3/(4*x^2+9)^(3/2),x, algorithm="giac")
[Out] 1/16*sqrt(4*x^2 + 9) + 9/16/sqrt(4*x^2 + 9)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{(9+4x^2)^{3/2}}\,dx = \frac{\sqrt{x^2+\frac{9}{4}}\,(2\,x^2+9)}{4\,(4\,x^2+9)}$$

```
[In] int(x^3/(4*x^2 + 9)^(3/2),x)
[Out] ((x^2 + 9/4)^(1/2)*(2*x^2 + 9))/(4*(4*x^2 + 9))
```

3.123 $\int \frac{x}{\sqrt{3-2x-x^2}} dx$

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{x}{\sqrt{3-2x-x^2}} dx = -\sqrt{3-2x-x^2} + \arcsin\left(\frac{1}{2}(-1-x)\right)$$

```
[Out] -arcsin(1/2+1/2*x)-(-x^2-2*x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {654, 633, 222}

$$\int \frac{x}{\sqrt{3-2x-x^2}} dx = \arcsin\left(\frac{1}{2}(-x-1)\right) - \sqrt{-x^2-2x+3}$$

```
[In] Int[x/Sqrt[3 - 2*x - x^2],x]
```

```
[Out] -Sqrt[3 - 2*x - x^2] + ArcSin[(-1 - x)/2]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\sqrt{3-2x-x^2}-\int \frac{1}{\sqrt{3-2x-x^2}}\,dx\\ &= -\sqrt{3-2x-x^2}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx, x, -2-2x\right)\\ &= -\sqrt{3-2x-x^2}+\arcsin\left(\frac{1}{2}(-1-x)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{x}{\sqrt{3-2x-x^2}}\,dx = -\sqrt{3-2x-x^2}+2\arctan\left(\frac{\sqrt{3-2x-x^2}}{3+x}\right)$$

```
[In] Integrate[x/Sqrt[3 - 2*x - x^2],x]
[Out] -Sqrt[3 - 2*x - x^2] + 2*ArcTan[Sqrt[3 - 2*x - x^2]/(3 + x)]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$-\arcsin\left(\frac{1}{2}+\frac{x}{2}\right)-\sqrt{-x^2-2x+3}$
risch	$\frac{x^2+2x-3}{\sqrt{-x^2-2x+3}}-\arcsin\left(\frac{1}{2}+\frac{x}{2}\right)$
trager	$-\sqrt{-x^2-2x+3}+\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)x+\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{-x^2-\right.$

```
[In] int(x/(-x^2-2*x+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -arcsin(1/2+1/2*x)-(-x^2-2*x+3)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{x}{\sqrt{3-2x-x^2}}\,dx = -\sqrt{-x^2-2\,x+3}+\arctan\left(\frac{\sqrt{-x^2-2\,x+3}(x+1)}{x^2+2\,x-3}\right)$$

```
[In] integrate(x/(-x^2-2*x+3)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 - 2*x + 3) + arctan(sqrt(-x^2 - 2*x + 3)*(x + 1)/(x^2 + 2*x - 3)
)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x}{\sqrt{3-2x-x^2}}\,dx = -\sqrt{-x^2-2x+3}-\operatorname{asin}\left(\frac{x}{2}+\frac{1}{2}\right)$$

```
[In] integrate(x/(-x**2-2*x+3)**(1/2),x)
[Out] -sqrt(-x**2 - 2*x + 3) - asin(x/2 + 1/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{3-2x-x^2}}\,dx = -\sqrt{-x^2-2\,x+3}+\arcsin\left(-\frac{1}{2}\,x-\frac{1}{2}\right)$$

```
[In] integrate(x/(-x^2-2*x+3)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 - 2*x + 3) + arcsin(-1/2*x - 1/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{3-2x-x^2}}\,dx = -\sqrt{-x^2-2\,x+3} - \arcsin\left(\frac{1}{2}\,x+\frac{1}{2}\right)$$

```
[In] integrate(x/(-x^2-2*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 - 2*x + 3) - arcsin(1/2*x + 1/2)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{x}{\sqrt{3-2x-x^2}}\,dx = -\sqrt{-x^2-2\,x+3} + \ln\left(x\,1\mathrm{i} + \sqrt{-x^2-2\,x+3} + 1\mathrm{i}\right)\,1\mathrm{i}$$

```
[In] int(x/(3 - x^2 - 2*x)^(1/2),x)
```

```
[Out] log(x*1i + (3 - x^2 - 2*x)^(1/2) + 1i)*1i - (3 - x^2 - 2*x)^(1/2)
```

3.124 $\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}}{x}$$

```
[Out] -(-x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{270\}$

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}}{x}$$

```
[In] Int[1/(x^2*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[1 - x^2]/x)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{1-x^2}}{x}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}}{x}$$

```
[In] Integrate[1/(x^2*Sqrt[1 - x^2]),x]
[Out] -(Sqrt[1 - x^2]/x)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{\sqrt{-x^2+1}}{x}$	15
trager	$-\frac{\sqrt{-x^2+1}}{x}$	15
meijerg	$-\frac{\sqrt{-x^2+1}}{x}$	15
pseudoelliptic	$-\frac{\sqrt{-x^2+1}}{x}$	15
risch	$\frac{x^2-1}{x\sqrt{-x^2+1}}$	19
gosper	$\frac{(-1+x)(1+x)}{x\sqrt{-x^2+1}}$	20

```
[In] int(1/x^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-x^2+1)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx = -\frac{\sqrt{-x^2+1}}{x}$$

```
[In] integrate(1/x^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 + 1)/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx = \begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for } |x^2| > 1 \\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x**2/(-x**2+1)**(1/2),x)
[Out] Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx = -\frac{\sqrt{-x^2+1}}{x}$$

```
[In] integrate(1/x^2/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx = \frac{x}{2\left(\sqrt{-x^2+1}-1\right)} - \frac{\sqrt{-x^2+1}-1}{2\,x}$$

```
[In] integrate(1/x^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*x/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)/x
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}}{x}$$

```
[In] int(1/(x^2*(1 - x^2)^(1/2)),x)
[Out] -(1 - x^2)^(1/2)/x
```

3.125 $\int x^3\sqrt{4-x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int x^3\sqrt{4-x^2}\,dx = -\frac{4}{3}\left(4-x^2\right)^{3/2}+\frac{1}{5}\left(4-x^2\right)^{5/2}$$

```
[Out] -4/3*(-x^2+4)^(3/2)+1/5*(-x^2+4)^(5/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\int x^3\sqrt{4-x^2}\,dx = \frac{1}{5}\left(4-x^2\right)^{5/2}-\frac{4}{3}\left(4-x^2\right)^{3/2}$$

```
[In] Int[x^3*Sqrt[4 - x^2],x]
```

```
[Out] (-4*(4 - x^2)^(3/2))/3 + (4 - x^2)^(5/2)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \sqrt{4-x}x\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(4\sqrt{4-x}-(4-x)^{3/2}\right)\,dx, x, x^2\right)\\
&= -\frac{4}{3}\left(4-x^2\right)^{3/2}+\frac{1}{5}\left(4-x^2\right)^{5/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int x^3\sqrt{4-x^2}\,dx = \frac{1}{15}\sqrt{4-x^2}\left(-32-4x^2+3x^4\right)$$

```
[In] Integrate[x^3*Sqrt[4 - x^2],x]
[Out] (Sqrt[4 - x^2]*(-32 - 4*x^2 + 3*x^4))/15
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$-\frac{\left(3x^2+8\right)\left(-x^2+4\right)^{\frac{3}{2}}}{15}$	19
trager	$\left(\frac{1}{5}x^4-\frac{4}{15}x^2-\frac{32}{15}\right)\sqrt{-x^2+4}$	23
gosper	$\frac{(-2+x)(2+x)\left(3x^2+8\right)\sqrt{-x^2+4}}{15}$	25
default	$-\frac{x^2\left(-x^2+4\right)^{\frac{3}{2}}}{5}-\frac{8\left(-x^2+4\right)^{\frac{3}{2}}}{15}$	27
risch	$-\frac{\left(3x^4-4x^2-32\right)\left(x^2-4\right)}{15\sqrt{-x^2+4}}$	29
meijerg	$-\frac{8\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(1-\frac{x^2}{4}\right)^{\frac{3}{2}}\left(\frac{3x^2}{4}+2\right)}{15}\right)}{\sqrt{\pi}}$	33

```
[In] int(x^3*(-x^2+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/15*(3*x^2+8)*(-x^2+4)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{4-x^2}\,dx = \frac{1}{15}\left(3\,x^4-4\,x^2-32\right)\sqrt{-x^2+4}$$

```
[In] integrate(x^3*(-x^2+4)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*x^4 - 4*x^2 - 32)*sqrt(-x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int x^3\sqrt{4-x^2}\,dx = \frac{x^4\sqrt{4-x^2}}{5} - \frac{4x^2\sqrt{4-x^2}}{15} - \frac{32\sqrt{4-x^2}}{15}$$

```
[In] integrate(x**3*(-x**2+4)**(1/2),x)
[Out] x**4*sqrt(4 - x**2)/5 - 4*x**2*sqrt(4 - x**2)/15 - 32*sqrt(4 - x**2)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int x^3\sqrt{4-x^2}\,dx = -\frac{1}{5}\left(-x^2+4\right)^{\frac{3}{2}}x^2 - \frac{8}{15}\left(-x^2+4\right)^{\frac{3}{2}}$$

```
[In] integrate(x^3*(-x^2+4)^(1/2),x, algorithm="maxima")
[Out] -1/5*(-x^2 + 4)^(3/2)*x^2 - 8/15*(-x^2 + 4)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int x^3\sqrt{4-x^2}\,dx = \frac{1}{5}\left(x^2-4\right)^2\sqrt{-x^2+4} - \frac{4}{3}\left(-x^2+4\right)^{\frac{3}{2}}$$

```
[In] integrate(x^3*(-x^2+4)^(1/2),x, algorithm="giac")
[Out] 1/5*(x^2 - 4)^2*sqrt(-x^2 + 4) - 4/3*(-x^2 + 4)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{4-x^2}\,dx=-\sqrt{4-x^2}\left(-\frac{x^4}{5}+\frac{4\,x^2}{15}+\frac{32}{15}\right)$$

```
[In] int(x^3*(4 - x^2)^(1/2),x)
[Out] -(4 - x^2)^(1/2)*((4*x^2)/15 - x^4/5 + 32/15)
```

3.126 $\int \frac{x}{\sqrt{1-x^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x}{\sqrt{1-x^2}} \, dx = -\sqrt{1-x^2}$$

```
[Out] -(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {267}

$$\int \frac{x}{\sqrt{1-x^2}} \, dx = -\sqrt{1-x^2}$$

```
[In] Int[x/Sqrt[1 - x^2],x]
```

```
[Out] -Sqrt[1 - x^2]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\sqrt{1-x^2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}$$

```
[In] Integrate[x/Sqrt[1 - x^2],x]
[Out] -Sqrt[1 - x^2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\sqrt{-x^2+1}$	12
default	$-\sqrt{-x^2+1}$	12
trager	$-\sqrt{-x^2+1}$	12
pseudoelliptic	$-\sqrt{-x^2+1}$	12
risch	$\frac{x^2-1}{\sqrt{-x^2+1}}$	16
gosper	$\frac{(-1+x)(1+x)}{\sqrt{-x^2+1}}$	17
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^2+1}}{2\sqrt{\pi}}$	26

```
[In] int(1/(-x^2+1)^(1/2)*x,x,method=_RETURNVERBOSE)
[Out] -(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1}$$

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}$$

```
[In] integrate(x/(-x**2+1)**(1/2),x)
[Out] -sqrt(1 - x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1}$$

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1}$$

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}$$

```
[In] int(x/(1 - x^2)^(1/2),x)
[Out] -(1 - x^2)^(1/2)
```

3.127 $\int x\sqrt{4-x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int x\sqrt{4-x^2}\,dx=-\frac{1}{3}\left(4-x^2\right)^{3/2}$$

```
[Out] -1/3*(-x^2+4)^(3/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {267}

$$\int x\sqrt{4-x^2}\,dx=-\frac{1}{3}\left(4-x^2\right)^{3/2}$$

```
[In] Int[x*Sqrt[4 - x^2],x]
```

```
[Out] -1/3*(4 - x^2)^(3/2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral}=-\frac{1}{3}\left(4-x^2\right)^{3/2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x\sqrt{4-x^2}\,dx = -\frac{1}{3}\left(4-x^2\right)^{3/2}$$

```
[In] Integrate[x*Sqrt[4 - x^2],x]
[Out] -1/3*(4 - x^2)^(3/2)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\left(-x^2+4\right)^{\frac{3}{2}}}{3}$	12
default	$-\frac{\left(-x^2+4\right)^{\frac{3}{2}}}{3}$	12
pseudoelliptic	$-\frac{\left(-x^2+4\right)^{\frac{3}{2}}}{3}$	12
gosper	$\frac{(-2+x)(2+x)\sqrt{-x^2+4}}{3}$	18
trager	$\left(\frac{x^2}{3}-\frac{4}{3}\right)\sqrt{-x^2+4}$	18
risch	$-\frac{\left(x^2-4\right)^2}{3\sqrt{-x^2+4}}$	19
meijerg	$\frac{\frac{8\sqrt{\pi}}{3}-\frac{4\sqrt{\pi}\left(-\frac{x^2}{2}+2\right)\sqrt{1-\frac{x^2}{4}}}{3}}{\sqrt{\pi}}$	33

```
[In] int(x*(-x^2+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(-x^2+4)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int x\sqrt{4-x^2}\,dx = \frac{1}{3}\left(x^2-4\right)\sqrt{-x^2+4}$$

```
[In] integrate(x*(-x^2+4)^(1/2),x, algorithm="fricas")
[Out] 1/3*(x^2 - 4)*sqrt(-x^2 + 4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int x\sqrt{4-x^2}\,dx = \frac{x^2\sqrt{4-x^2}}{3} - \frac{4\sqrt{4-x^2}}{3}$$

```
[In] integrate(x*(-x**2+4)**(1/2),x)
[Out] x**2*sqrt(4 - x**2)/3 - 4*sqrt(4 - x**2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x\sqrt{4-x^2}\,dx = -\frac{1}{3}\left(-x^2+4\right)^{\frac{3}{2}}$$

```
[In] integrate(x*(-x^2+4)^(1/2),x, algorithm="maxima")
[Out] -1/3*(-x^2 + 4)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x\sqrt{4-x^2}\,dx = -\frac{1}{3}\left(-x^2+4\right)^{\frac{3}{2}}$$

```
[In] integrate(x*(-x^2+4)^(1/2),x, algorithm="giac")
[Out] -1/3*(-x^2 + 4)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x\sqrt{4-x^2}\,dx = -\frac{\left(4-x^2\right)^{3/2}}{3}$$

```
[In] int(x*(4 - x^2)^(1/2),x)
[Out] -(4 - x^2)^(3/2)/3
```

3.128 $\int \sqrt{1-4x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \sqrt{1-4x^2}\,dx = \frac{1}{2}x\sqrt{1-4x^2} + \frac{1}{4}\arcsin(2x)$$

```
[Out] 1/4*arcsin(2*x)+1/2*x*(-4*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {201, 222}

$$\int \sqrt{1-4x^2}\,dx = \frac{1}{4}\arcsin(2x) + \frac{1}{2}\sqrt{1-4x^2}x$$

```
[In] Int[Sqrt[1 - 4*x^2],x]
```

```
[Out] (x*Sqrt[1 - 4*x^2])/2 + ArcSin[2*x]/4
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x\sqrt{1-4x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1-4x^2}}\,dx \\
&= \frac{1}{2}x\sqrt{1-4x^2}+\frac{1}{4}\arcsin(2x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \sqrt{1-4x^2}\,dx = \frac{1}{2}x\sqrt{1-4x^2}-\frac{1}{2}\arctan\left(\frac{\sqrt{1-4x^2}}{1+2x}\right)$$

```
[In] Integrate[Sqrt[1 - 4*x^2],x]
[Out] (x*Sqrt[1 - 4*x^2])/2 - ArcTan[Sqrt[1 - 4*x^2]/(1 + 2*x)]/2
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\arcsin(2x)}{4}+\frac{x\sqrt{-4x^2+1}}{2}$	20
risch	$-\frac{(4x^2-1)x}{2\sqrt{-4x^2+1}}+\frac{\arcsin(2x)}{4}$	27
pseudoelliptic	$\frac{x\sqrt{-4x^2+1}}{2}-\frac{\arctan\left(\frac{\sqrt{-4x^2+1}}{2x}\right)}{4}$	31
meijerg	$\frac{i\left(-4i\sqrt{\pi}\,x\sqrt{-4x^2+1}-2i\sqrt{\pi}\,\arcsin(2x)\right)}{8\sqrt{\pi}}$	34
trager	$\frac{x\sqrt{-4x^2+1}}{2}-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2+1}+2x\right)}{4}$	44

```
[In] int((-4*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*arcsin(2*x)+1/2*x*(-4*x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \sqrt{1-4x^2}\,dx = \frac{1}{2}\sqrt{-4\,x^2+1}x - \frac{1}{2}\arctan\left(\frac{\sqrt{-4\,x^2+1}-1}{2\,x}\right)$$

```
[In] integrate((-4*x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-4*x^2 + 1)*x - 1/2*arctan(1/2*(sqrt(-4*x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \sqrt{1-4x^2}\,dx = \frac{x\sqrt{1-4x^2}}{2} + \frac{\operatorname{asin}(2x)}{4}$$

```
[In] integrate((-4*x**2+1)**(1/2),x)
[Out] x*sqrt(1 - 4*x**2)/2 + asin(2*x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \sqrt{1-4x^2}\,dx = \frac{1}{2}\sqrt{-4\,x^2+1}x + \frac{1}{4}\arcsin(2\,x)$$

```
[In] integrate((-4*x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-4*x^2 + 1)*x + 1/4*arcsin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \sqrt{1-4x^2}\,dx = \frac{1}{2}\sqrt{-4\,x^2+1}x + \frac{1}{4}\arcsin(2\,x)$$

```
[In] integrate((-4*x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-4*x^2 + 1)*x + 1/4*arcsin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \sqrt{1-4x^2}\,dx = \frac{\operatorname{asin}(2\,x)}{4} + x\,\sqrt{\frac{1}{4}-x^2}$$

```
[In] int((1 - 4*x^2)^(1/2),x)
```

```
[Out] asin(2*x)/4 + x*(1/4 - x^2)^(1/2)
```

3.129 $\int \frac{x^3}{\sqrt{4+x^2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{x^3}{\sqrt{4+x^2}} dx = -4\sqrt{4+x^2} + \frac{1}{3}(4+x^2)^{3/2}$$

```
[Out] 1/3*(x^2+4)^(3/2)-4*(x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\int \frac{x^3}{\sqrt{4+x^2}} dx = \frac{1}{3}(x^2+4)^{3/2} - 4\sqrt{x^2+4}$$

```
[In] Int[x^3/Sqrt[4 + x^2],x]
[Out] -4*Sqrt[4 + x^2] + (4 + x^2)^(3/2)/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{4+x}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{4}{\sqrt{4+x}}+\sqrt{4+x}\right)\,dx, x, x^2\right)\\ &= -4\sqrt{4+x^2}+\frac{1}{3}\left(4+x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{\sqrt{4+x^2}}\,dx = \frac{1}{3}\left(-8+x^2\right)\sqrt{4+x^2}$$

```
[In] Integrate[x^3/Sqrt[4 + x^2],x]
[Out] ((-8 + x^2)*Sqrt[4 + x^2])/3
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{\sqrt{x^2+4}\,\left(x^2-8\right)}{3}$	15
risch	$\frac{\sqrt{x^2+4}\,\left(x^2-8\right)}{3}$	15
pseudoelliptic	$\frac{\sqrt{x^2+4}\,\left(x^2-8\right)}{3}$	15
trager	$\sqrt{x^2+4}\,\left(\frac{x^2}{3}-\frac{8}{3}\right)$	16
default	$\frac{x^2\sqrt{x^2+4}}{3}-\frac{8\sqrt{x^2+4}}{3}$	23
meijerg	$\frac{\frac{16\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(-x^2+8\right)\sqrt{1+\frac{x^2}{4}}}{3}}{\sqrt{\pi}}$	33

```
[In] int(x^3/(x^2+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(x^2+4)^(1/2)*(x^2-8)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{x^3}{\sqrt{4+x^2}}\,dx = \frac{1}{3}\sqrt{x^2+4}\left(x^2-8\right)$$

```
[In] integrate(x^3/(x^2+4)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(x^2 + 4)*(x^2 - 8)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{\sqrt{4+x^2}}\,dx = \frac{x^2\sqrt{x^2+4}}{3} - \frac{8\sqrt{x^2+4}}{3}$$

```
[In] integrate(x**3/(x**2+4)**(1/2),x)
[Out] x**2*sqrt(x**2 + 4)/3 - 8*sqrt(x**2 + 4)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{\sqrt{4+x^2}}\,dx = \frac{1}{3}\sqrt{x^2+4}x^2 - \frac{8}{3}\sqrt{x^2+4}$$

```
[In] integrate(x^3/(x^2+4)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(x^2 + 4)*x^2 - 8/3*sqrt(x^2 + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{\sqrt{4+x^2}}\,dx = \frac{1}{3}\left(x^2+4\right)^{\frac{3}{2}} - 4\sqrt{x^2+4}$$

```
[In] integrate(x^3/(x^2+4)^(1/2),x, algorithm="giac")
[Out] 1/3*(x^2 + 4)^(3/2) - 4*sqrt(x^2 + 4)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{x^3}{\sqrt{4+x^2}}\,dx = \frac{\sqrt{x^2+4}\,(x^2-8)}{3}$$

```
[In] int(x^3/(x^2 + 4)^(1/2),x)
[Out] ((x^2 + 4)^(1/2)*(x^2 - 8))/3
```

3.130 $\int \frac{1}{\sqrt{9+x^2}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 6

$$\int \frac{1}{\sqrt{9+x^2}} \, dx = \operatorname{arcsinh}\left(\frac{x}{3}\right)$$

```
[Out] arcsinh(1/3*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{221\}$

$$\int \frac{1}{\sqrt{9+x^2}} \, dx = \operatorname{arcsinh}\left(\frac{x}{3}\right)$$

```
[In] Int[1/Sqrt[9 + x^2],x]
```

```
[Out] ArcSinh[x/3]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\text{integral} = \operatorname{arcsinh}\left(\frac{x}{3}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{1}{\sqrt{9+x^2}}\,dx = -\log\left(-x+\sqrt{9+x^2}\right)$$

```
[In] Integrate[1/Sqrt[9 + x^2],x]
[Out] -Log[-x + Sqrt[9 + x^2]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

method	result	size
default	$\operatorname{arcsinh}\left(\frac{x}{3}\right)$	5
meijerg	$\operatorname{arcsinh}\left(\frac{x}{3}\right)$	5
pseudoelliptic	$\operatorname{arctanh}\left(\frac{\sqrt{x^2+9}}{x}\right)$	13
trager	$-\ln\left(x-\sqrt{x^2+9}\right)$	15

```
[In] int(1/(x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(1/3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(4) = 8$.

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \frac{1}{\sqrt{9+x^2}}\,dx = -\log\left(-x+\sqrt{x^2+9}\right)$$

```
[In] integrate(1/(x^2+9)^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 + 9))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{9+x^2}}\,dx = \operatorname{asinh}\left(\frac{x}{3}\right)$$

```
[In] integrate(1/(x**2+9)**(1/2),x)
[Out] asinh(x/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{9+x^2}}\,dx = \operatorname{arsinh}\left(\frac{1}{3}\,x\right)$$

```
[In] integrate(1/(x^2+9)^(1/2),x, algorithm="maxima")
[Out] arcsinh(1/3*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(4) = 8.

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 4.17

$$\int \frac{1}{\sqrt{9+x^2}}\,dx = \frac{1}{2}\,\sqrt{x^2+9}x - \frac{9}{2}\,\log\left(-x+\sqrt{x^2+9}\right)$$

```
[In] integrate(1/(x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 + 9)*x - 9/2*log(-x + sqrt(x^2 + 9))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{9+x^2}}\,dx = \operatorname{asinh}\left(\frac{x}{3}\right)$$

```
[In] int(1/(x^2 + 9)^(1/2),x)
[Out] asinh(x/3)
```

3.131 $\int \sqrt{1+x^2}\, dx$

Optimal result

Integrand size = 9, antiderivative size = 21

$$\int \sqrt{1+x^2}\, dx = \frac{1}{2}x\sqrt{1+x^2} + \frac{\operatorname{arcsinh}(x)}{2}$$

```
[Out] 1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {201, 221}

$$\int \sqrt{1+x^2}\, dx = \frac{\operatorname{arcsinh}(x)}{2} + \frac{1}{2}\sqrt{x^2+1}x$$

```
[In] Int[Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\int\frac{1}{\sqrt{1+x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{1+x^2}+\frac{\operatorname{arcsinh}(x)}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int\sqrt{1+x^2}\,dx=\frac{1}{2}x\sqrt{1+x^2}-\frac{1}{2}\log\left(-x+\sqrt{1+x^2}\right)$$

```
[In] Integrate[Sqrt[1 + x^2],x]
[Out] (x*Sqrt[1 + x^2])/2 - Log[-x + Sqrt[1 + x^2]]/2
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\operatorname{arcsinh}(x)}{2}+\frac{x\sqrt{x^2+1}}{2}$	16
risch	$\frac{\operatorname{arcsinh}(x)}{2}+\frac{x\sqrt{x^2+1}}{2}$	16
trager	$\frac{x\sqrt{x^2+1}}{2}+\frac{\ln\left(x+\sqrt{x^2+1}\right)}{2}$	24
meijerg	$-\frac{-2\sqrt{\pi}\,x\sqrt{x^2+1}-2\sqrt{\pi}\,\operatorname{arcsinh}(x)}{4\sqrt{\pi}}$	27
pseudoelliptic	$\frac{x\sqrt{x^2+1}}{2}+\frac{\ln\left(\frac{x+\sqrt{x^2+1}}{x}\right)}{4}-\frac{\ln\left(\frac{\sqrt{x^2+1}-x}{x}\right)}{4}$	46

```
[In] int((x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \sqrt{1+x^2}\,dx = \frac{1}{2}\,\sqrt{x^2+1}x - \frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}\right)$$

```
[In] integrate((x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{1+x^2}\,dx = \frac{x\sqrt{x^2+1}}{2} + \frac{\operatorname{asinh}(x)}{2}$$

```
[In] integrate((x**2+1)**(1/2),x)
[Out] x*sqrt(x**2 + 1)/2 + asinh(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{1+x^2}\,dx = \frac{1}{2}\,\sqrt{x^2+1}x + \frac{1}{2}\,\operatorname{arsinh}(x)$$

```
[In] integrate((x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^2 + 1)*x + 1/2*arcsinh(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \sqrt{1+x^2}\,dx = \frac{1}{2}\,\sqrt{x^2+1}x - \frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}\right)$$

```
[In] integrate((x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{1+x^2}\,dx = \frac{\operatorname{asinh}(x)}{2} + \frac{x\,\sqrt{x^2+1}}{2}$$

```
[In] int((x^2 + 1)^(1/2),x)
[Out] asinh(x)/2 + (x*(x^2 + 1)^(1/2))/2
```

3.132 $\int \frac{1}{x^3\sqrt{-16+x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{1}{x^3\sqrt{-16+x^2}}\,dx = \frac{\sqrt{-16+x^2}}{32x^2} + \frac{1}{128}\arctan\left(\frac{1}{4}\sqrt{-16+x^2}\right)$$

```
[Out] 1/128*arctan(1/4*(x^2-16)^(1/2))+1/32*(x^2-16)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {272, 44, 65, 209}

$$\int \frac{1}{x^3\sqrt{-16+x^2}}\,dx = \frac{1}{128}\arctan\left(\frac{\sqrt{x^2-16}}{4}\right) + \frac{\sqrt{x^2-16}}{32x^2}$$

```
[In] Int[1/(x^3*Sqrt[-16 + x^2]),x]
```

```
[Out] Sqrt[-16 + x^2]/(32*x^2) + ArcTan[Sqrt[-16 + x^2]/4]/128
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-16+x}x^2}\,dx, x, x^2\right)\\ &= \frac{\sqrt{-16+x^2}}{32x^2}+\frac{1}{64}\text{Subst}\left(\int \frac{1}{\sqrt{-16+x}x}\,dx, x, x^2\right)\\ &= \frac{\sqrt{-16+x^2}}{32x^2}+\frac{1}{32}\text{Subst}\left(\int \frac{1}{16+x^2}\,dx, x, \sqrt{-16+x^2}\right)\\ &= \frac{\sqrt{-16+x^2}}{32x^2}+\frac{1}{128}\arctan\left(\frac{1}{4}\sqrt{-16+x^2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{-16+x^2}}\,dx = \frac{\sqrt{-16+x^2}}{32x^2}+\frac{1}{128}\arctan\left(\frac{1}{4}\sqrt{-16+x^2}\right)$$

```
[In] Integrate[1/(x^3*Sqrt[-16 + x^2]),x]
[Out] Sqrt[-16 + x^2]/(32*x^2) + ArcTan[Sqrt[-16 + x^2]/4]/128
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\sqrt{x^{2}-16}}{32 x^{2}}-\frac{\arctan\left(\frac{4}{\sqrt{x^{2}-16}}\right)}{128}$	26
risch	$\frac{\sqrt{x^{2}-16}}{32 x^{2}}-\frac{\arctan\left(\frac{4}{\sqrt{x^{2}-16}}\right)}{128}$	26
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{x^{2}-16}}{4}\right) x^{2}+4\sqrt{x^{2}-16}}{128 x^{2}}$	30
trager	$\frac{\sqrt{x^{2}-16}}{32 x^{2}}-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{4\operatorname{RootOf}\left(\_Z^{2}+1\right)-\sqrt{x^{2}-16}}{x}\right)}{128}$	46
meijerg	$-\frac{\sqrt{-\operatorname{signum}\left(-1+\frac{x^{2}}{16}\right)}\left(\frac{16\sqrt{\pi}}{x^{2}}-\frac{(1-6\ln(2)+2\ln(x)+i\pi)\sqrt{\pi}}{2}-\frac{2\sqrt{\pi}\left(-\frac{x^{2}}{4}+8\right)}{x^{2}}+\frac{16\sqrt{\pi}\sqrt{1-\frac{x^{2}}{16}}}{x^{2}}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{x^{2}}{16}}}{2}\right)\right)}{128\sqrt{\pi}\sqrt{\operatorname{signum}\left(-1+\frac{x^{2}}{16}\right)}}$	106

```
[In] int(1/x^3/(x^2-16)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/32*(x^2-16)^(1/2)/x^2-1/128*arctan(4/(x^2-16)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3\sqrt{-16+x^2}}\,dx=\frac{x^2\arctan\left(-\frac{1}{4}\,x+\frac{1}{4}\,\sqrt{x^2-16}\right)+2\,\sqrt{x^2-16}}{64\,x^2}$$

```
[In] integrate(1/x^3/(x^2-16)^(1/2),x, algorithm="fricas")
[Out] 1/64*(x^2*arctan(-1/4*x + 1/4*sqrt(x^2 - 16)) + 2*sqrt(x^2 - 16))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

$$\int \frac{1}{x^3\sqrt{-16+x^2}}\,dx = \begin{cases} \frac{i\,\mathrm{acosh}\left(\frac{4}{x}\right)}{128} - \frac{i}{32x\sqrt{-1+\frac{16}{x^2}}} + \frac{i}{2x^3\sqrt{-1+\frac{16}{x^2}}} & \text{for } \frac{1}{|x^2|} > \frac{1}{16} \\ -\frac{\mathrm{asin}\left(\frac{4}{x}\right)}{128} + \frac{\sqrt{1-\frac{16}{x^2}}}{32x} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x**3/(x**2-16)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(4/x)/128 - I/(32*x*sqrt(-1 + 16/x**2)) + I/(2*x**3*sqrt(
-1 + 16/x**2)), 1/Abs(x**2) > 1/16), (-asin(4/x)/128 + sqrt(1 - 16/x**2)/(3
2*x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^3\sqrt{-16+x^2}}\,dx = \frac{\sqrt{x^2-16}}{32\,x^2} - \frac{1}{128}\,\arcsin\left(\frac{4}{|x|}\right)$$

```
[In] integrate(1/x^3/(x^2-16)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/32*sqrt(x^2 - 16)/x^2 - 1/128*arcsin(4/abs(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3\sqrt{-16+x^2}}\,dx = \frac{\sqrt{x^2-16}}{32\,x^2} + \frac{1}{128}\,\arctan\left(\frac{1}{4}\,\sqrt{x^2-16}\right)$$

```
[In] integrate(1/x^3/(x^2-16)^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*sqrt(x^2 - 16)/x^2 + 1/128*arctan(1/4*sqrt(x^2 - 16))
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3\sqrt{-16+x^2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{x^2-16}}{4}\right)}{128} + \frac{\sqrt{x^2-16}}{32\,x^2}$$

```
[In] int(1/(x^3*(x^2 - 16)^(1/2)),x)
[Out] atan((x^2 - 16)^(1/2)/4)/128 + (x^2 - 16)^(1/2)/(32*x^2)
```

3.133 $\int \frac{\sqrt{-a^2+x^2}}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{\sqrt{-a^2+x^2}}{x^4}\,dx=\frac{\left(-a^2+x^2\right)^{3/2}}{3a^2x^3}$$

```
[Out] 1/3*(-a^2+x^2)^(3/2)/a^2/x^3
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {270}

$$\int \frac{\sqrt{-a^2+x^2}}{x^4}\,dx=\frac{\left(x^2-a^2\right)^{3/2}}{3a^2x^3}$$

```
[In] Int[Sqrt[-a^2 + x^2]/x^4,x]
```

```
[Out] (-a^2 + x^2)^(3/2)/(3*a^2*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{\left(-a^2+x^2\right)^{3/2}}{3a^2x^3}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-a^2+x^2}}{x^4}\,dx = \frac{(-a^2+x^2)^{3/2}}{3a^2x^3}$$

```
[In] Integrate[Sqrt[-a^2 + x^2]/x^4,x]
[Out] (-a^2 + x^2)^(3/2)/(3*a^2*x^3)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{(-a^2+x^2)^{\frac{3}{2}}}{3a^2x^3}$	20
pseudoelliptic	$\frac{(-a^2+x^2)^{\frac{3}{2}}}{3a^2x^3}$	20
gosper	$-\frac{(a-x)(a+x)\sqrt{-a^2+x^2}}{3x^3a^2}$	28
trager	$-\frac{(a^2-x^2)\sqrt{-a^2+x^2}}{3a^2x^3}$	29
risch	$\frac{(a^2-x^2)^2}{3x^3\sqrt{-a^2+x^2}\,a^2}$	31

```
[In] int((-a^2+x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/3*(-a^2+x^2)^(3/2)/a^2/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-a^2+x^2}}{x^4}\,dx = \frac{x^3+(-a^2+x^2)^{\frac{3}{2}}}{3\,a^2x^3}$$

```
[In] integrate((-a^2+x^2)^(1/2)/x^4,x, algorithm="fricas")
[Out] 1/3*(x^3 + (-a^2 + x^2)^(3/2))/(a^2*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.30

$$\int \frac{\sqrt{-a^2+x^2}}{x^4}\,dx = \begin{cases} -\frac{i\sqrt{\frac{a^2}{x^2}-1}}{3x^2} + \frac{i\sqrt{\frac{a^2}{x^2}-1}}{3a^2} & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ -\frac{\sqrt{-\frac{a^2}{x^2}+1}}{3x^2} + \frac{\sqrt{-\frac{a^2}{x^2}+1}}{3a^2} & \text{otherwise} \end{cases}$$

```
[In] integrate((-a**2+x**2)**(1/2)/x**4,x)
[Out] Piecewise((-I*sqrt(a**2/x**2 - 1)/(3*x**2) + I*sqrt(a**2/x**2 - 1)/(3*a**2)
, Abs(a**2/x**2) > 1), (-sqrt(-a**2/x**2 + 1)/(3*x**2) + sqrt(-a**2/x**2 +
1)/(3*a**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{-a^2+x^2}}{x^4}\,dx = \frac{(-a^2+x^2)^{\frac{3}{2}}}{3\,a^2x^3}$$

```
[In] integrate((-a^2+x^2)^(1/2)/x^4,x, algorithm="maxima")
[Out] 1/3*(-a^2 + x^2)^(3/2)/(a^2*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(19) = 38.

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{-a^2+x^2}}{x^4}\,dx = \frac{2\left(a^4+3\left(x-\sqrt{-a^2+x^2}\right)^4\right)}{3\left(a^2+\left(x-\sqrt{-a^2+x^2}\right)^2\right)^3}$$

```
[In] integrate((-a^2+x^2)^(1/2)/x^4,x, algorithm="giac")
[Out] 2/3*(a^4 + 3*(x - sqrt(-a^2 + x^2))^4)/(a^2 + (x - sqrt(-a^2 + x^2))^2)^3
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{-a^2+x^2}}{x^4}\,dx = \frac{\left(x^2-a^2\right)^{3/2}}{3\,a^2\,x^3}$$

```
[In] int((x^2 - a^2)^(1/2)/x^4,x)
[Out] (x^2 - a^2)^(3/2)/(3*a^2*x^3)
```

3.134 $\int \frac{\sqrt{-4+9x^2}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{\sqrt{-4+9x^2}}{x} \, dx = \sqrt{-4+9x^2} - 2\arctan\left(\frac{1}{2}\sqrt{-4+9x^2}\right)$$

```
[Out] -2*arctan(1/2*(9*x^2-4)^(1/2))+(9*x^2-4)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {272, 52, 65, 209}

$$\int \frac{\sqrt{-4+9x^2}}{x} \, dx = \sqrt{9x^2-4} - 2\arctan\left(\frac{1}{2}\sqrt{9x^2-4}\right)$$

```
[In] Int[Sqrt[-4 + 9*x^2]/x,x]
```

```
[Out] Sqrt[-4 + 9*x^2] - 2*ArcTan[Sqrt[-4 + 9*x^2]/2]
```

<u>Rule 52</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 65</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-4+9x}}{x}\,dx, x, x^2\right) \\
&= \sqrt{-4+9x^2} - 2\text{Subst}\left(\int \frac{1}{x\sqrt{-4+9x}}\,dx, x, x^2\right) \\
&= \sqrt{-4+9x^2} - \frac{4}{9}\text{Subst}\left(\int \frac{1}{\frac{4}{9}+\frac{x^2}{9}}\,dx, x, \sqrt{-4+9x^2}\right) \\
&= \sqrt{-4+9x^2} - 2\arctan\left(\frac{1}{2}\sqrt{-4+9x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-4+9x^2}}{x}\,dx = \sqrt{-4+9x^2} - 2\arctan\left(\frac{1}{2}\sqrt{-4+9x^2}\right)$$

```
[In] Integrate[Sqrt[-4 + 9*x^2]/x,x]
[Out] Sqrt[-4 + 9*x^2] - 2*ArcTan[Sqrt[-4 + 9*x^2]/2]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\sqrt{9x^{2}-4}+2\arctan\left(\frac{2}{\sqrt{9x^{2}-4}}\right)$	25
pseudoelliptic	$-2\arctan\left(\frac{\sqrt{9x^{2}-4}}{2}\right)+\sqrt{9x^{2}-4}$	25
trager	$\sqrt{9x^{2}-4}-2\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^{2}+1\right)+\sqrt{9x^{2}-4}}{x}\right)$	42
meijerg	$-\frac{\sqrt{\operatorname{signum}\left(-1+\frac{9x^{2}}{4}\right)}\left(-2(2-4\ln(2)+2\ln(x)+2\ln(3)+i\pi)\sqrt{\pi}+4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{1-\frac{9x^{2}}{4}}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{9x^{2}}{4}}}{2}\right)\right)}{2\sqrt{\pi}\,\sqrt{-\operatorname{signum}\left(-1+\frac{9x^{2}}{4}\right)}}$	90

```
[In] int((9*x^2-4)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] (9*x^2-4)^(1/2)+2*arctan(2/(9*x^2-4)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-4+9x^2}}{x}\,dx = \sqrt{9\,x^{2}-4}-4\,\arctan\left(-\frac{3}{2}\,x+\frac{1}{2}\,\sqrt{9\,x^{2}-4}\right)$$

```
[In] integrate((9*x^2-4)^(1/2)/x,x, algorithm="fricas")
[Out] sqrt(9*x^2 - 4) - 4*arctan(-3/2*x + 1/2*sqrt(9*x^2 - 4))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.07

$$\int \frac{\sqrt{-4+9x^2}}{x}\,dx = \begin{cases} -\frac{3ix}{\sqrt{-1+\frac{4}{9x^{2}}}}-2i\operatorname{acosh}\left(\frac{2}{3x}\right)+\frac{4i}{3x\sqrt{-1+\frac{4}{9x^{2}}}} & \text{for}\ \frac{1}{\left|x^{2}\right|}>\frac{9}{4} \\ \frac{3x}{\sqrt{1-\frac{4}{9x^{2}}}}+2\operatorname{asin}\left(\frac{2}{3x}\right)-\frac{4}{3x\sqrt{1-\frac{4}{9x^{2}}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((9*x**2-4)**(1/2)/x,x)
[Out] Piecewise((-3*I*x/sqrt(-1 + 4/(9*x**2)) - 2*I*acosh(2/(3*x)) + 4*I/(3*x*sqr
t(-1 + 4/(9*x**2))), 1/Abs(x**2) > 9/4), (3*x/sqrt(1 - 4/(9*x**2)) + 2*asin
(2/(3*x)) - 4/(3*x*sqrt(1 - 4/(9*x**2))), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{-4+9x^2}}{x}\,dx = \sqrt{9\,x^2-4}+2\,\arcsin\left(\frac{2}{3\,|x|}\right)$$

```
[In] integrate((9*x^2-4)^(1/2)/x,x, algorithm="maxima")
[Out] sqrt(9*x^2 - 4) + 2*arcsin(2/3/abs(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{-4+9x^2}}{x}\,dx = \sqrt{9\,x^2-4}-2\,\arctan\left(\frac{1}{2}\,\sqrt{9\,x^2-4}\right)$$

```
[In] integrate((9*x^2-4)^(1/2)/x,x, algorithm="giac")
[Out] sqrt(9*x^2 - 4) - 2*arctan(1/2*sqrt(9*x^2 - 4))
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{-4+9x^2}}{x}\,dx = \sqrt{9\,x^2-4}-2\,\mathrm{atan}\left(\frac{\sqrt{9\,x^2-4}}{2}\right)$$

```
[In] int((9*x^2 - 4)^(1/2)/x,x)
[Out] (9*x^2 - 4)^(1/2) - 2*atan((9*x^2 - 4)^(1/2)/2)
```

3.135 $\int \frac{1}{x^2\sqrt{-9+16x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{1}{x^2\sqrt{-9+16x^2}}\,dx = \frac{\sqrt{-9+16x^2}}{9x}$$

```
[Out] 1/9*(16*x^2-9)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\int \frac{1}{x^2\sqrt{-9+16x^2}}\,dx = \frac{\sqrt{16x^2-9}}{9x}$$

```
[In] Int[1/(x^2*Sqrt[-9 + 16*x^2]),x]
```

```
[Out] Sqrt[-9 + 16*x^2]/(9*x)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{-9+16x^2}}{9x}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{-9+16x^2}}\,dx = \frac{\sqrt{-9+16x^2}}{9x}$$

```
[In] Integrate[1/(x^2*Sqrt[-9 + 16*x^2]),x]
[Out] Sqrt[-9 + 16*x^2]/(9*x)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{16x^2-9}}{9x}$	15
trager	$\frac{\sqrt{16x^2-9}}{9x}$	15
risch	$\frac{\sqrt{16x^2-9}}{9x}$	15
pseudoelliptic	$\frac{\sqrt{16x^2-9}}{9x}$	15
gosper	$\frac{(4x-3)(4x+3)}{9x\sqrt{16x^2-9}}$	25
meijerg	$-\frac{\sqrt{-\operatorname{signum}\left(-1+\frac{16x^2}{9}\right)}\,\sqrt{1-\frac{16x^2}{9}}}{3\sqrt{\operatorname{signum}\left(-1+\frac{16x^2}{9}\right)}\,x}$	37

```
[In] int(1/x^2/(16*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/9*(16*x^2-9)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{-9+16x^2}}\,dx = \frac{4\,x+\sqrt{16\,x^2-9}}{9\,x}$$

```
[In] integrate(1/x^2/(16*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/9*(4*x + sqrt(16*x^2 - 9))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \frac{1}{x^2\sqrt{-9+16x^2}}\,dx = \begin{cases} \frac{4i\sqrt{-1+\frac{9}{16x^2}}}{9} & \text{for } \frac{1}{|x^2|} > \frac{16}{9} \\ \frac{4\sqrt{1-\frac{9}{16x^2}}}{9} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x**2/(16*x**2-9)**(1/2),x)
[Out] Piecewise((4*I*sqrt(-1 + 9/(16*x**2))/9, 1/Abs(x**2) > 16/9), (4*sqrt(1 - 9
/(16*x**2))/9, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{-9+16x^2}}\,dx = \frac{\sqrt{16\,x^2-9}}{9\,x}$$

```
[In] integrate(1/x^2/(16*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/9*sqrt(16*x^2 - 9)/x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^2\sqrt{-9+16x^2}}\,dx = \frac{8}{\left(4\,x-\sqrt{16\,x^2-9}\right)^2+9}$$

```
[In] integrate(1/x^2/(16*x^2-9)^(1/2),x, algorithm="giac")
[Out] 8/((4*x - sqrt(16*x^2 - 9))^2 + 9)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{-9+16x^2}}\,dx = \frac{\sqrt{16\,x^2-9}}{9\,x}$$

```
[In] int(1/(x^2*(16*x^2 - 9)^(1/2)),x)
[Out] (16*x^2 - 9)^(1/2)/(9*x)
```

3.136 $\int \frac{x^2}{\left(a^2-x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 34

$$\int \frac{x^2}{\left(a^2-x^2\right)^{3/2}}\,dx=\frac{x}{\sqrt{a^2-x^2}}-\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[Out] -arctan(x/(a^2-x^2)^(1/2))+x/(a^2-x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {294, 223, 209}

$$\int \frac{x^2}{\left(a^2-x^2\right)^{3/2}}\,dx=\frac{x}{\sqrt{a^2-x^2}}-\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[In] Int[x^2/(a^2 - x^2)^(3/2),x]
```

```
[Out] x/Sqrt[a^2 - x^2] - ArcTan[x/Sqrt[a^2 - x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x}{\sqrt{a^2-x^2}} - \int \frac{1}{\sqrt{a^2-x^2}}\,dx\\ &= \frac{x}{\sqrt{a^2-x^2}} - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{a^2-x^2}}\right)\\ &= \frac{x}{\sqrt{a^2-x^2}} - \arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a^2-x^2)^{3/2}}\,dx = \frac{x}{\sqrt{a^2-x^2}} - \arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[In] Integrate[x^2/(a^2 - x^2)^(3/2),x]
[Out] x/Sqrt[a^2 - x^2] - ArcTan[x/Sqrt[a^2 - x^2]]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result	size
default	$-\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)+\frac{x}{\sqrt{a^2-x^2}}$	31
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{a^2-x^2}}{x}\right)\sqrt{a^2-x^2}+x}{\sqrt{a^2-x^2}}$	43

```
[In] int(x^2/(a^2-x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -arctan(x/(a^2-x^2)^(1/2))+x/(a^2-x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

$$\int \frac{x^2}{(a^2-x^2)^{3/2}}\,dx = \frac{2\,(a^2-x^2)\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)+\sqrt{a^2-x^2}x}{a^2-x^2}$$

```
[In] integrate(x^2/(a^2-x^2)^(3/2),x, algorithm="fricas")
[Out] (2*(a^2 - x^2)*arctan(-(a - sqrt(a^2 - x^2))/x) + sqrt(a^2 - x^2)*x)/(a^2 -
 x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int \frac{x^2}{(a^2-x^2)^{3/2}}\,dx = \begin{cases} i\operatorname{acosh}\left(\frac{x}{a}\right)-\frac{ix}{a\sqrt{-1+\frac{x^2}{a^2}}} & \text{for } \left|\frac{x^2}{a^2}\right|>1 \\ -\operatorname{asin}\left(\frac{x}{a}\right)+\frac{x}{a\sqrt{1-\frac{x^2}{a^2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2/(a**2-x**2)**(3/2),x)
[Out] Piecewise((I*acosh(x/a) - I*x/(a*sqrt(-1 + x**2/a**2)), Abs(x**2/a**2) > 1)
, (-asin(x/a) + x/(a*sqrt(1 - x**2/a**2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{(a^2-x^2)^{3/2}}\,dx = \frac{x}{\sqrt{a^2-x^2}}-\arcsin\left(\frac{x}{a}\right)$$

```
[In] integrate(x^2/(a^2-x^2)^(3/2),x, algorithm="maxima")
[Out] x/sqrt(a^2 - x^2) - arcsin(x/a)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(a^2-x^2)^{3/2}}\,dx = -\arcsin\left(\frac{x}{a}\right)\mathrm{sgn}(a)+\frac{x}{\sqrt{a^2-x^2}}$$

```
[In] integrate(x^2/(a^2-x^2)^(3/2),x, algorithm="giac")
[Out] -arcsin(x/a)*sgn(a) + x/sqrt(a^2 - x^2)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a^2-x^2)^{3/2}}\,dx = \frac{x}{\sqrt{a^2-x^2}}+\ln\left(\sqrt{a^2-x^2}+x\,1\mathrm{i}\right)\,1\mathrm{i}$$

```
[In] int(x^2/(a^2 - x^2)^(3/2),x)
[Out] log(x*1i + (a^2 - x^2)^(1/2))*1i + x/(a^2 - x^2)^(1/2)
```

3.137 $\int \frac{x^2}{\sqrt{5-x^2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{x^2}{\sqrt{5-x^2}} dx = -\frac{1}{2}x\sqrt{5-x^2} + \frac{5}{2}\arcsin\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] 5/2*arcsin(1/5*x*5^(1/2))-1/2*x*(-x^2+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {327, 222}

$$\int \frac{x^2}{\sqrt{5-x^2}} dx = \frac{5}{2}\arcsin\left(\frac{x}{\sqrt{5}}\right) - \frac{1}{2}x\sqrt{5-x^2}$$

```
[In] Int[x^2/Sqrt[5 - x^2],x]
```

```
[Out] -1/2*(x*Sqrt[5 - x^2]) + (5*ArcSin[x/Sqrt[5]])/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2}x\sqrt{5-x^2}+\frac{5}{2}\int\frac{1}{\sqrt{5-x^2}}\,dx\\ &= -\frac{1}{2}x\sqrt{5-x^2}+\frac{5}{2}\arcsin\left(\frac{x}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int\frac{x^2}{\sqrt{5-x^2}}\,dx=-\frac{1}{2}x\sqrt{5-x^2}-5\arctan\left(\frac{x}{\sqrt{5}-\sqrt{5-x^2}}\right)$$

```
[In] Integrate[x^2/Sqrt[5 - x^2],x]
[Out] -1/2*(x*Sqrt[5 - x^2]) - 5*ArcTan[x/(Sqrt[5] - Sqrt[5 - x^2])]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{5\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{2}-\frac{x\sqrt{-x^2+5}}{2}$	23
risch	$\frac{x\left(x^2-5\right)}{2\sqrt{-x^2+5}}+\frac{5\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{2}$	28
pseudoelliptic	$-\frac{x\sqrt{-x^2+5}}{2}-\frac{5\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)}{2}$	30
meijerg	$\frac{5i\left(\frac{i\sqrt{\pi}\,x\sqrt{5}\,\sqrt{-\frac{x^2}{5}+1}}{5}-i\sqrt{\pi}\,\arcsin\left(\frac{x\sqrt{5}}{5}\right)\right)}{2\sqrt{\pi}}$	40
trager	$-\frac{x\sqrt{-x^2+5}}{2}-\frac{5\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+5}+x\right)}{2}$	42

```
[In] int(x^2/(-x^2+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] 5/2*arcsin(1/5*x*5^(1/2))-1/2*x*(-x^2+5)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{5-x^2}}\,dx = -\frac{1}{2}\,\sqrt{-x^2+5}x - \frac{5}{2}\,\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)$$

```
[In] integrate(x^2/(-x^2+5)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-x^2 + 5)*x - 5/2*arctan(sqrt(-x^2 + 5)/x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{\sqrt{5-x^2}}\,dx = -\frac{x\sqrt{5-x^2}}{2} + \frac{5\operatorname{asin}\left(\frac{\sqrt{5}x}{5}\right)}{2}$$

```
[In] integrate(x**2/(-x**2+5)**(1/2),x)
[Out] -x*sqrt(5 - x**2)/2 + 5*asin(sqrt(5)*x/5)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{\sqrt{5-x^2}}\,dx = -\frac{1}{2}\,\sqrt{-x^2+5}x + \frac{5}{2}\,\arcsin\left(\frac{1}{5}\,\sqrt{5}x\right)$$

```
[In] integrate(x^2/(-x^2+5)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-x^2 + 5)*x + 5/2*arcsin(1/5*sqrt(5)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{\sqrt{5-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+5}x + \frac{5}{2}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

```
[In] integrate(x^2/(-x^2+5)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^2 + 5)*x + 5/2*arcsin(1/5*sqrt(5)*x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{\sqrt{5-x^2}}\,dx = \frac{5\,\mathrm{asin}\left(\frac{\sqrt{5}\,x}{5}\right)}{2} - \frac{x\,\sqrt{5-x^2}}{2}$$

```
[In] int(x^2/(5 - x^2)^(1/2),x)
[Out] (5*asin((5^(1/2)*x)/5))/2 - (x*(5 - x^2)^(1/2))/2
```

3.138 $\int \frac{1}{x\sqrt{3+x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{x\sqrt{3+x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{3+x^2}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arctanh(1/3*(x^2+3)^(1/2)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {272, 65, 213}

$$\int \frac{1}{x\sqrt{3+x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2+3}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Int[1/(x*Sqrt[3 + x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[3 + x^2]/Sqrt[3]]/Sqrt[3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{3+x}}\,dx, x, x^2\right) \\ &= \text{Subst}\left(\int \frac{1}{-3+x^2}\,dx, x, \sqrt{3+x^2}\right) \\ &= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{3+x^2}}{\sqrt{3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{3+x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{3+x^2}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Integrate[1/(x*Sqrt[3 + x^2]),x]
[Out] -(ArcTanh[Sqrt[3 + x^2]/Sqrt[3]]/Sqrt[3])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}}{\sqrt{x^{2}+3}}\right)}{3}$	18
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^{2}+3}\,\sqrt{3}}{3}\right)\sqrt{3}}{3}$	19
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^{2}-3\right)+\sqrt{x^{2}+3}}{x}\right)}{3}$	30
meijerg	$\frac{\sqrt{3}\left(\left(-2\ln\left(2\right)+2\ln\left(x\right)-\ln\left(3\right)\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{\frac{x^{2}}{3}+1}}{2}\right)\right)}{6\sqrt{\pi}}$	46

```
[In] int(1/x/(x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*3^(1/2)*arctanh(3^(1/2)/(x^2+3)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\sqrt{3+x^2}}\,dx = \frac{1}{3}\sqrt{3}\log\left(-\frac{\sqrt{3}-\sqrt{x^2+3}}{x}\right)$$

```
[In] integrate(1/x/(x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*log(-(sqrt(3) - sqrt(x^2 + 3))/x)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{x\sqrt{3+x^2}}\,dx = -\frac{\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{3}}{x}\right)}{3}$$

```
[In] integrate(1/x/(x**2+3)**(1/2),x)
[Out] -sqrt(3)*asinh(sqrt(3)/x)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{1}{x\sqrt{3+x^2}}\,dx = -\frac{1}{3}\sqrt{3}\operatorname{arsinh}\left(\frac{\sqrt{3}}{|x|}\right)$$

```
[In] integrate(1/x/(x^2+3)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arcsinh(sqrt(3)/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \frac{1}{x\sqrt{3+x^2}}\,dx = -\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}+\sqrt{x^2+3}\right)+\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}+\sqrt{x^2+3}\right)$$

```
[In] integrate(1/x/(x^2+3)^(1/2),x, algorithm="giac")
[Out] -1/6*sqrt(3)*log(sqrt(3) + sqrt(x^2 + 3)) + 1/6*sqrt(3)*log(-sqrt(3) + sqrt
(x^2 + 3))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\sqrt{3+x^2}}\,dx = -\frac{\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}\sqrt{x^2+3}}{3}\right)}{3}$$

```
[In] int(1/(x*(x^2 + 3)^(1/2)),x)
[Out] -(3^(1/2)*atanh((3^(1/2)*(x^2 + 3)^(1/2))/3))/3
```

3.139 $\int \frac{x}{(4+x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{x}{(4+x^2)^{5/2}} \, dx = -\frac{1}{3(4+x^2)^{3/2}}$$

```
[Out] -1/3/(x^2+4)^(3/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {267}

$$\int \frac{x}{(4+x^2)^{5/2}} \, dx = -\frac{1}{3(x^2+4)^{3/2}}$$

```
[In] Int[x/(4 + x^2)^(5/2),x]
```

```
[Out] -1/3*1/(4 + x^2)^(3/2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{3(4+x^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x}{(4+x^2)^{5/2}}\,dx = -\frac{1}{3\,(4+x^2)^{3/2}}$$

```
[In] Integrate[x/(4 + x^2)^(5/2),x]
[Out] -1/3*1/(4 + x^2)^(3/2)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{1}{3(x^2+4)^{\frac{3}{2}}}$	10
derivativedivides	$-\frac{1}{3(x^2+4)^{\frac{3}{2}}}$	10
default	$-\frac{1}{3(x^2+4)^{\frac{3}{2}}}$	10
trager	$-\frac{1}{3(x^2+4)^{\frac{3}{2}}}$	10
risch	$-\frac{1}{3(x^2+4)^{\frac{3}{2}}}$	10
pseudoelliptic	$-\frac{1}{3(x^2+4)^{\frac{3}{2}}}$	10
meijerg	$\frac{\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(1+\frac{x^2}{4}\right)^{\frac{3}{2}}}}{12\sqrt{\pi}}$	26

```
[In] int(x/(x^2+4)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/3/(x^2+4)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{x}{(4+x^2)^{5/2}}\,dx = -\frac{\sqrt{x^2+4}}{3\,(x^4+8\,x^2+16)}$$

```
[In] integrate(x/(x^2+4)^(5/2),x, algorithm="fricas")
[Out] -1/3*sqrt(x^2 + 4)/(x^4 + 8*x^2 + 16)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \frac{x}{(4+x^2)^{5/2}}\,dx = -\frac{1}{3x^2\sqrt{x^2+4}+12\sqrt{x^2+4}}$$

```
[In] integrate(x/(x**2+4)**(5/2),x)
[Out] -1/(3*x**2*sqrt(x**2 + 4) + 12*sqrt(x**2 + 4))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x}{(4+x^2)^{5/2}}\,dx = -\frac{1}{3\,(x^2+4)^{\frac{3}{2}}}$$

```
[In] integrate(x/(x^2+4)^(5/2),x, algorithm="maxima")
[Out] -1/3/(x^2 + 4)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x}{(4+x^2)^{5/2}}\,dx = -\frac{1}{3\,(x^2+4)^{\frac{3}{2}}}$$

```
[In] integrate(x/(x^2+4)^(5/2),x, algorithm="giac")
[Out] -1/3/(x^2 + 4)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x}{(4+x^2)^{5/2}}\,dx = -\frac{1}{3\,(x^2+4)^{3/2}}$$

```
[In] int(x/(x^2 + 4)^(5/2),x)
[Out] -1/(3*(x^2 + 4)^(3/2))
```

3.140 $\int x^3\sqrt{4-9x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int x^3\sqrt{4-9x^2}\,dx=-\frac{4}{243}\left(4-9x^2\right)^{3/2}+\frac{1}{405}\left(4-9x^2\right)^{5/2}$$

```
[Out] -4/243*(-9*x^2+4)^(3/2)+1/405*(-9*x^2+4)^(5/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\int x^3\sqrt{4-9x^2}\,dx=\frac{1}{405}\left(4-9x^2\right)^{5/2}-\frac{4}{243}\left(4-9x^2\right)^{3/2}$$

```
[In] Int[x^3*Sqrt[4 - 9*x^2],x]
```

```
[Out] (-4*(4 - 9*x^2)^(3/2))/243 + (4 - 9*x^2)^(5/2)/405
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \sqrt{4-9x}x\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{4}{9}\sqrt{4-9x}-\frac{1}{9}(4-9x)^{3/2}\right)\,dx, x, x^2\right)\\ &= -\frac{4}{243}\left(4-9x^2\right)^{3/2}+\frac{1}{405}\left(4-9x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{4-9x^2}\,dx = \frac{(-8-27x^2)\left(4-9x^2\right)^{3/2}}{1215}$$

```
[In] Integrate[x^3*Sqrt[4 - 9*x^2],x]
[Out] ((-8 - 27*x^2)*(4 - 9*x^2)^(3/2))/1215
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$-\frac{(27x^2+8)(-9x^2+4)^{\frac{3}{2}}}{1215}$	19
trager	$\left(\frac{1}{5}x^4-\frac{4}{135}x^2-\frac{32}{1215}\right)\sqrt{-9x^2+4}$	23
default	$-\frac{x^2(-9x^2+4)^{\frac{3}{2}}}{45}-\frac{8(-9x^2+4)^{\frac{3}{2}}}{1215}$	27
gosper	$\frac{(-2+3x)(2+3x)(27x^2+8)\sqrt{-9x^2+4}}{1215}$	29
risch	$-\frac{(243x^4-36x^2-32)(9x^2-4)}{1215\sqrt{-9x^2+4}}$	31
meijerg	$-\frac{8\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(1-\frac{9x^2}{4}\right)^{\frac{3}{2}}\left(\frac{27x^2}{4}+2\right)}{15}\right)}{81\sqrt{\pi}}$	33

```
[In] int(x^3*(-9*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/1215*(27*x^2+8)*(-9*x^2+4)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{4-9x^2}\,dx = \frac{1}{1215}\left(243\,x^4-36\,x^2-32\right)\sqrt{-9\,x^2+4}$$

```
[In] integrate(x^3*(-9*x^2+4)^(1/2),x, algorithm="fricas")
[Out] 1/1215*(243*x^4 - 36*x^2 - 32)*sqrt(-9*x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int x^3\sqrt{4-9x^2}\,dx = \frac{x^4\sqrt{4-9x^2}}{5} - \frac{4x^2\sqrt{4-9x^2}}{135} - \frac{32\sqrt{4-9x^2}}{1215}$$

```
[In] integrate(x**3*(-9*x**2+4)**(1/2),x)
[Out] x**4*sqrt(4 - 9*x**2)/5 - 4*x**2*sqrt(4 - 9*x**2)/135 - 32*sqrt(4 - 9*x**2)
/1215
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int x^3\sqrt{4-9x^2}\,dx = -\frac{1}{45}\left(-9\,x^2+4\right)^{\frac{3}{2}}x^2 - \frac{8}{1215}\left(-9\,x^2+4\right)^{\frac{3}{2}}$$

```
[In] integrate(x^3*(-9*x^2+4)^(1/2),x, algorithm="maxima")
[Out] -1/45*(-9*x^2 + 4)^(3/2)*x^2 - 8/1215*(-9*x^2 + 4)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int x^3\sqrt{4-9x^2}\,dx = \frac{1}{405}\left(9\,x^2-4\right)^2\sqrt{-9\,x^2+4} - \frac{4}{243}\left(-9\,x^2+4\right)^{\frac{3}{2}}$$

```
[In] integrate(x^3*(-9*x^2+4)^(1/2),x, algorithm="giac")
[Out] 1/405*(9*x^2 - 4)^2*sqrt(-9*x^2 + 4) - 4/243*(-9*x^2 + 4)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{4-9x^2}\,dx=-\frac{\sqrt{\frac{4}{9}-x^2}\,\left(-\frac{9\,x^4}{5}+\frac{4\,x^2}{15}+\frac{32}{135}\right)}{3}$$

```
[In] int(x^3*(4 - 9*x^2)^(1/2),x)
[Out] -((4/9 - x^2)^(1/2)*((4*x^2)/15 - (9*x^4)/5 + 32/135))/3
```

3.141 $\int x^2\sqrt{9-x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int x^2\sqrt{9-x^2}\,dx=-\frac{9}{8}x\sqrt{9-x^2}+\frac{1}{4}x^3\sqrt{9-x^2}+\frac{81}{8}\arcsin\left(\frac{x}{3}\right)$$

```
[Out] 81/8*arcsin(1/3*x)-9/8*x*(-x^2+9)^(1/2)+1/4*x^3*(-x^2+9)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {285, 327, 222}

$$\int x^2\sqrt{9-x^2}\,dx=\frac{81}{8}\arcsin\left(\frac{x}{3}\right)-\frac{9}{8}\sqrt{9-x^2}x+\frac{1}{4}\sqrt{9-x^2}x^3$$

```
[In] Int[x^2*Sqrt[9 - x^2],x]
```

```
[Out] (-9*x*Sqrt[9 - x^2])/8 + (x^3*Sqrt[9 - x^2])/4 + (81*ArcSin[x/3])/8
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}x^3\sqrt{9-x^2} + \frac{9}{4}\int \frac{x^2}{\sqrt{9-x^2}}\,dx \\ &= -\frac{9}{8}x\sqrt{9-x^2} + \frac{1}{4}x^3\sqrt{9-x^2} + \frac{81}{8}\int \frac{1}{\sqrt{9-x^2}}\,dx \\ &= -\frac{9}{8}x\sqrt{9-x^2} + \frac{1}{4}x^3\sqrt{9-x^2} + \frac{81}{8}\arcsin\left(\frac{x}{3}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int x^2\sqrt{9-x^2}\,dx = \frac{1}{8}x\sqrt{9-x^2}\left(-9+2x^2\right) - \frac{81}{4}\arctan\left(\frac{\sqrt{9-x^2}}{3+x}\right)$$

```
[In] Integrate[x^2*Sqrt[9 - x^2],x]
[Out] (x*Sqrt[9 - x^2]*(-9 + 2*x^2))/8 - (81*ArcTan[Sqrt[9 - x^2]/(3 + x)])/4
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{x\left(-x^{2}+9\right)^{\frac{3}{2}}}{4}+\frac{9x\sqrt{-x^{2}+9}}{8}+\frac{81\arcsin\left(\frac{x}{3}\right)}{8}$	32
risch	$-\frac{x\left(2x^{2}-9\right)\left(x^{2}-9\right)}{8\sqrt{-x^{2}+9}}+\frac{81\arcsin\left(\frac{x}{3}\right)}{8}$	32
pseudoelliptic	$-\frac{81\arctan\left(\frac{\sqrt{-x^{2}+9}}{x}\right)}{8}+\frac{\left(2x^{3}-9x\right)\sqrt{-x^{2}+9}}{8}$	38
meijerg	$-\frac{81i\left(-\frac{i\sqrt{\pi}\,x\left(-\frac{2x^{2}}{3}+3\right)\sqrt{-\frac{x^{2}}{9}+1}}{18}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{x}{3}\right)}{2}\right)}{4\sqrt{\pi}}$	41
trager	$\frac{x\left(2x^{2}-9\right)\sqrt{-x^{2}+9}}{8}+\frac{81\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+9}+x\right)}{8}$	48

```
[In] int(x^2*(-x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*x*(-x^2+9)^(3/2)+9/8*x*(-x^2+9)^(1/2)+81/8*arcsin(1/3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int x^{2}\sqrt{9-x^{2}}\,dx=\frac{1}{8}\left(2\,x^{3}-9\,x\right)\sqrt{-x^{2}+9}-\frac{81}{4}\arctan\left(\frac{\sqrt{-x^{2}+9}-3}{x}\right)$$

```
[In] integrate(x^2*(-x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/8*(2*x^3 - 9*x)*sqrt(-x^2 + 9) - 81/4*arctan((sqrt(-x^2 + 9) - 3)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.75 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.44

$$\int x^{2}\sqrt{9-x^{2}}\,dx=\begin{cases}\frac{ix^{5}}{4\sqrt{x^{2}-9}}-\frac{27ix^{3}}{8\sqrt{x^{2}-9}}+\frac{81ix}{8\sqrt{x^{2}-9}}-\frac{81i\operatorname{acosh}\left(\frac{x}{3}\right)}{8} & \text{for } |x^{2}|>9\\ -\frac{x^{5}}{4\sqrt{9-x^{2}}}+\frac{27x^{3}}{8\sqrt{9-x^{2}}}-\frac{81x}{8\sqrt{9-x^{2}}}+\frac{81\operatorname{asin}\left(\frac{x}{3}\right)}{8} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*(-x**2+9)**(1/2),x)
[Out] Piecewise((I*x**5/(4*sqrt(x**2 - 9)) - 27*I*x**3/(8*sqrt(x**2 - 9)) + 81*I*
x/(8*sqrt(x**2 - 9)) - 81*I*acosh(x/3)/8, Abs(x**2) > 9), (-x**5/(4*sqrt(9
- x**2)) + 27*x**3/(8*sqrt(9 - x**2)) - 81*x/(8*sqrt(9 - x**2)) + 81*asin(x
/3)/8, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int x^2\sqrt{9-x^2}\,dx = -\frac{1}{4}\left(-x^2+9\right)^{\frac{3}{2}}x + \frac{9}{8}\sqrt{-x^2+9}x + \frac{81}{8}\arcsin\left(\frac{1}{3}x\right)$$

```
[In] integrate(x^2*(-x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/4*(-x^2 + 9)^(3/2)*x + 9/8*sqrt(-x^2 + 9)*x + 81/8*arcsin(1/3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.58

$$\int x^2\sqrt{9-x^2}\,dx = \frac{1}{8}\left(2\,x^2-9\right)\sqrt{-x^2+9}x + \frac{81}{8}\arcsin\left(\frac{1}{3}x\right)$$

```
[In] integrate(x^2*(-x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/8*(2*x^2 - 9)*sqrt(-x^2 + 9)*x + 81/8*arcsin(1/3*x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.60

$$\int x^2\sqrt{9-x^2}\,dx = \frac{81\,\mathrm{asin}\left(\frac{x}{3}\right)}{8} - \sqrt{9-x^2}\left(\frac{9\,x}{8}-\frac{x^3}{4}\right)$$

```
[In] int(x^2*(9 - x^2)^(1/2),x)
[Out] (81*asin(x/3))/8 - (9 - x^2)^(1/2)*((9*x)/8 - x^3/4)
```

3.142 $\int 5x\sqrt{1+x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 13

$$\int 5x\sqrt{1+x^2}\,dx = \frac{5}{3}\left(1+x^2\right)^{3/2}$$

```
[Out] 5/3*(x^2+1)^(3/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {12, 267}

$$\int 5x\sqrt{1+x^2}\,dx = \frac{5}{3}\left(x^2+1\right)^{3/2}$$

```
[In] Int[5*x*Sqrt[1 + x^2],x]
[Out] (5*(1 + x^2)^(3/2))/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 5\int x\sqrt{1+x^2}\,dx\\
&= \frac{5}{3}\left(1+x^2\right)^{3/2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int 5x\sqrt{1+x^2}\,dx = \frac{5}{3}\left(1+x^2\right)^{3/2}$$

```
[In] Integrate[5*x*Sqrt[1 + x^2],x]
[Out] (5*(1 + x^2)^(3/2))/3
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{5\left(x^2+1\right)^{\frac{3}{2}}}{3}$	10
derivativedivides	$\frac{5\left(x^2+1\right)^{\frac{3}{2}}}{3}$	10
default	$\frac{5\left(x^2+1\right)^{\frac{3}{2}}}{3}$	10
risch	$\frac{5\left(x^2+1\right)^{\frac{3}{2}}}{3}$	10
pseudoelliptic	$\frac{5\left(x^2+1\right)^{\frac{3}{2}}}{3}$	10
trager	$5\left(\frac{x^2}{3}+\frac{1}{3}\right)\sqrt{x^2+1}$	17
meijerg	$-\frac{5\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2x^2+2\right)\sqrt{x^2+1}}{3}\right)}{4\sqrt{\pi}}$	31

```
[In] int(5*x*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 5/3*(x^2+1)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int 5x\sqrt{1+x^2}\,dx = \frac{5}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(5*x*(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 5/3*(x^2 + 1)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int 5x\sqrt{1+x^2}\,dx = \frac{5x^2\sqrt{x^2+1}}{3} + \frac{5\sqrt{x^2+1}}{3}$$

```
[In] integrate(5*x*(x**2+1)**(1/2),x)
[Out] 5*x**2*sqrt(x**2 + 1)/3 + 5*sqrt(x**2 + 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int 5x\sqrt{1+x^2}\,dx = \frac{5}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(5*x*(x^2+1)^(1/2),x, algorithm="maxima")
[Out] 5/3*(x^2 + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int 5x\sqrt{1+x^2}\,dx = \frac{5}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(5*x*(x^2+1)^(1/2),x, algorithm="giac")
[Out] 5/3*(x^2 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int 5x\sqrt{1+x^2}\,dx = \frac{5\left(x^2+1\right)^{3/2}}{3}$$

```
[In] int(5*x*(x^2 + 1)^(1/2),x)
[Out] (5*(x^2 + 1)^(3/2))/3
```

3.143 $\int \frac{1}{(-25+4x^2)^{3/2}} dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{1}{(-25+4x^2)^{3/2}} dx = -\frac{x}{25\sqrt{-25+4x^2}}$$

```
[Out] -1/25*x/(4*x^2-25)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {197}

$$\int \frac{1}{(-25+4x^2)^{3/2}} dx = -\frac{x}{25\sqrt{4x^2-25}}$$

```
[In] Int[(-25 + 4*x^2)^(-3/2),x]
```

```
[Out] -1/25*x/Sqrt[-25 + 4*x^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{x}{25\sqrt{-25+4x^2}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-25+4x^2)^{3/2}}\,dx = -\frac{x}{25\sqrt{-25+4x^2}}$$

```
[In] Integrate[(-25 + 4*x^2)^(-3/2),x]
[Out] -1/25*x/Sqrt[-25 + 4*x^2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{x}{25\sqrt{4x^2-25}}$	13
trager	$-\frac{x}{25\sqrt{4x^2-25}}$	13
risch	$-\frac{x}{25\sqrt{4x^2-25}}$	13
pseudoelliptic	$-\frac{x}{25\sqrt{4x^2-25}}$	13
gosper	$-\frac{(2x-5)(5+2x)x}{25(4x^2-25)^{\frac{3}{2}}}$	23
meijerg	$\frac{\left(-\operatorname{signum}\left(-1+\frac{4x^2}{25}\right)\right)^{\frac{3}{2}}x}{125\operatorname{signum}\left(-1+\frac{4x^2}{25}\right)^{\frac{3}{2}}\sqrt{1-\frac{4x^2}{25}}}$	35

```
[In] int(1/(4*x^2-25)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/25*x/(4*x^2-25)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(12) = 24$.

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{1}{(-25+4x^2)^{3/2}}\,dx = -\frac{4\,x^2+2\,\sqrt{4\,x^2-25}x-25}{50\,(4\,x^2-25)}$$

```
[In] integrate(1/(4*x^2-25)^(3/2),x, algorithm="fricas")
[Out] -1/50*(4*x^2 + 2*sqrt(4*x^2 - 25)*x - 25)/(4*x^2 - 25)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \frac{1}{(-25+4x^2)^{3/2}}\,dx = \begin{cases} -\frac{x}{25\sqrt{4x^2-25}} & \text{for } |x^2| > \frac{25}{4} \\ \frac{ix}{25\sqrt{25-4x^2}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(4*x**2-25)**(3/2),x)
[Out] Piecewise((-x/(25*sqrt(4*x**2 - 25)), Abs(x**2) > 25/4), (I*x/(25*sqrt(25 -
 4*x**2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-25+4x^2)^{3/2}}\,dx = -\frac{x}{25\sqrt{4x^2-25}}$$

```
[In] integrate(1/(4*x^2-25)^(3/2),x, algorithm="maxima")
[Out] -1/25*x/sqrt(4*x^2 - 25)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-25+4x^2)^{3/2}}\,dx = -\frac{x}{25\sqrt{4x^2-25}}$$

```
[In] integrate(1/(4*x^2-25)^(3/2),x, algorithm="giac")
[Out] -1/25*x/sqrt(4*x^2 - 25)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-25+4x^2)^{3/2}}\,dx = -\frac{x}{25\,\sqrt{4\,x^2-25}}$$

```
[In] int(1/(4*x^2 - 25)^(3/2),x)
[Out] -x/(25*(4*x^2 - 25)^(1/2))
```

3.144 $\int \sqrt{2x-x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \sqrt{2x-x^2}\,dx = -\frac{1}{2}(1-x)\sqrt{2x-x^2}-\frac{1}{2}\arcsin(1-x)$$

```
[Out] 1/2*arcsin(-1+x)-1/2*(1-x)*(-x^2+2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {626, 633, 222}

$$\int \sqrt{2x-x^2}\,dx = -\frac{1}{2}\arcsin(1-x)-\frac{1}{2}\sqrt{2x-x^2}(1-x)$$

```
[In] Int[Sqrt[2*x - x^2],x]
[Out] -1/2*((1 - x)*Sqrt[2*x - x^2]) - ArcSin[1 - x]/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2}(1-x)\sqrt{2x-x^2}+\frac{1}{2}\int \frac{1}{\sqrt{2x-x^2}}\,dx\\ &= -\frac{1}{2}(1-x)\sqrt{2x-x^2}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx,x,2-2x\right)\\ &= -\frac{1}{2}(1-x)\sqrt{2x-x^2}-\frac{1}{2}\arcsin(1-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \sqrt{2x-x^2}\,dx=\frac{1}{2}\sqrt{-((-2+x)x)}\left(-1+x+\frac{2\log\left(\sqrt{-2+x}-\sqrt{x}\right)}{\sqrt{-2+x}\sqrt{x}}\right)$$

```
[In] Integrate[Sqrt[2*x - x^2],x]
[Out] (Sqrt[-((-2 + x)*x)]*(-1 + x + (2*Log[Sqrt[-2 + x] - Sqrt[x]])/(Sqrt[-2 + x
]*Sqrt[x])))/2
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

method	result	size
risch	$-\frac{(-1+x)x(-2+x)}{2\sqrt{-x(-2+x)}}+\frac{\arcsin(-1+x)}{2}$	25
default	$-\frac{(-2x+2)\sqrt{-x^2+2x}}{4}+\frac{\arcsin(-1+x)}{2}$	26
pseudoelliptic	$-\arctan\left(\frac{\sqrt{-x(-2+x)}}{x}\right)+\frac{(-1+x)\sqrt{-x(-2+x)}}{2}$	30
meijerg	$-\frac{2i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-3x+3)\sqrt{1-\frac{x}{2}}}{12}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)}{2}\right)}{\sqrt{\pi}}$	47
trager	$\left(-\frac{1}{2}+\frac{x}{2}\right)\sqrt{-x^2+2x}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+2x}+x-1\right)}{2}$	49

```
[In] int((-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(-1+x)*x*(-2+x)/(-x*(-2+x))^(1/2)+1/2*arcsin(-1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \sqrt{2x-x^2}\,dx=\frac{1}{2}\sqrt{-x^2+2\,x}(x-1)-\arctan\left(\frac{\sqrt{-x^2+2\,x}}{x}\right)$$

```
[In] integrate((-x^2+2*x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 2*x)*(x - 1) - arctan(sqrt(-x^2 + 2*x)/x)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \sqrt{2x-x^2}\,dx=\left(\frac{x}{2}-\frac{1}{2}\right)\sqrt{-x^2+2x}+\frac{\operatorname{asin}(x-1)}{2}$$

```
[In] integrate((-x**2+2*x)**(1/2),x)
[Out] (x/2 - 1/2)*sqrt(-x**2 + 2*x) + asin(x - 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \sqrt{2x-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+2\,x}x - \frac{1}{2}\sqrt{-x^2+2\,x} - \frac{1}{2}\arcsin\left(-x+1\right)$$

```
[In] integrate((-x^2+2*x)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 2*x)*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*arcsin(-x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \sqrt{2x-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+2\,x}(x-1) + \frac{1}{2}\arcsin\left(x-1\right)$$

```
[In] integrate((-x^2+2*x)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 2*x)*(x - 1) + 1/2*arcsin(x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \sqrt{2x-x^2}\,dx = \frac{\mathrm{asin}(x-1)}{2} + \left(\frac{x}{2}-\frac{1}{2}\right)\sqrt{2\,x-x^2}$$

```
[In] int((2*x - x^2)^(1/2),x)
[Out] asin(x - 1)/2 + (x/2 - 1/2)*(2*x - x^2)^(1/2)
```

3.145 $\int \frac{1}{\sqrt{8+4x+x^2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 8

$$\int \frac{1}{\sqrt{8+4x+x^2}}\,dx = \operatorname{arcsinh}\left(\frac{2+x}{2}\right)$$

```
[Out] arcsinh(1+1/2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {633, 221}

$$\int \frac{1}{\sqrt{8+4x+x^2}}\,dx = \operatorname{arcsinh}\left(\frac{x+2}{2}\right)$$

```
[In] Int[1/Sqrt[8 + 4*x + x^2],x]
```

```
[Out] ArcSinh[(2 + x)/2]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{16}}}\,dx, x, 4+2x\right)$$
$$= \text{arcsinh}\left(\frac{2+x}{2}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{8+4x+x^2}}\,dx = -\log\left(-2-x+\sqrt{8+4x+x^2}\right)$$

```
[In] Integrate[1/Sqrt[8 + 4*x + x^2],x]
[Out] -Log[-2 - x + Sqrt[8 + 4*x + x^2]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\text{arcsinh}\left(1+\frac{x}{2}\right)$	7
trager	$\ln\left(x+2+\sqrt{x^2+4x+8}\right)$	15

```
[In] int(1/(x^2+4*x+8)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(1+1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{8+4x+x^2}}\,dx = -\log\left(-x+\sqrt{x^2+4\,x+8}-2\right)$$

```
[In] integrate(1/(x^2+4*x+8)^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 + 4*x + 8) - 2)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{8+4x+x^2}}\,dx = \operatorname{asinh}\left(\frac{x}{2}+1\right)$$

```
[In] integrate(1/(x**2+4*x+8)**(1/2),x)
[Out] asinh(x/2 + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{8+4x+x^2}}\,dx = \operatorname{arsinh}\left(\frac{1}{2}\,x+1\right)$$

```
[In] integrate(1/(x^2+4*x+8)^(1/2),x, algorithm="maxima")
[Out] arcsinh(1/2*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(6) = 12.

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 4.25

$$\int \frac{1}{\sqrt{8+4x+x^2}}\,dx = \frac{1}{2}\,\sqrt{x^2+4\,x+8}(x+2) - 2\,\log\left(-x+\sqrt{x^2+4\,x+8}-2\right)$$

```
[In] integrate(1/(x^2+4*x+8)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 + 4*x + 8)*(x + 2) - 2*log(-x + sqrt(x^2 + 4*x + 8) - 2)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{8+4x+x^2}}\,dx = \ln\left(x+\sqrt{x^2+4\,x+8}+2\right)$$

```
[In] int(1/(4*x + x^2 + 8)^(1/2),x)
[Out] log(x + (4*x + x^2 + 8)^(1/2) + 2)
```

3.146 $\int \frac{1}{\sqrt{-8+6x+9x^2}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{1}{\sqrt{-8+6x+9x^2}} \, dx = \frac{1}{3}\text{arctanh}\left(\frac{1+3x}{\sqrt{-8+6x+9x^2}}\right)$$

```
[Out] 1/3*arctanh((1+3*x)/(9*x^2+6*x-8)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {635, 212}

$$\int \frac{1}{\sqrt{-8+6x+9x^2}} \, dx = \frac{1}{3}\text{arctanh}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

```
[In] Int[1/Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x + 9*x^2]]/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{1}{36-x^2}\,dx, x, \frac{6+18x}{\sqrt{-8+6x+9x^2}}\right)$$
$$= \frac{1}{3}\text{arctanh}\left(\frac{1+3x}{\sqrt{-8+6x+9x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = -\frac{1}{3}\log\left(-1-3x+\sqrt{-8+6x+9x^2}\right)$$

```
[In] Integrate[1/Sqrt[-8 + 6*x + 9*x^2],x]
[Out] -1/3*Log[-1 - 3*x + Sqrt[-8 + 6*x + 9*x^2]]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
trager	$\frac{\ln\left(\sqrt{9x^2+6x-8}+1+3x\right)}{3}$	21
default	$\frac{\ln\left(\frac{(3+9x)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)\sqrt{9}}{9}$	30

```
[In] int(1/(9*x^2+6*x-8)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*ln((9*x^2+6*x-8)^(1/2)+1+3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = -\frac{1}{3}\log\left(-3x+\sqrt{9x^2+6x-8}-1\right)$$

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="fricas")
[Out] -1/3*log(-3*x + sqrt(9*x^2 + 6*x - 8) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = \frac{\log\left(18x+6\sqrt{9x^2+6x-8}+6\right)}{3}$$

```
[In] integrate(1/(9*x**2+6*x-8)**(1/2),x)
[Out] log(18*x + 6*sqrt(9*x**2 + 6*x - 8) + 6)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = \frac{1}{3}\log\left(18\,x+6\,\sqrt{9\,x^2+6\,x-8}+6\right)$$

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="maxima")
[Out] 1/3*log(18*x + 6*sqrt(9*x^2 + 6*x - 8) + 6)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = \frac{1}{6}\,\sqrt{9\,x^2+6\,x-8}(3\,x+1)+\frac{3}{2}\,\log\left(\left|-3\,x+\sqrt{9\,x^2+6\,x-8}-1\right|\right)$$

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="giac")
[Out] 1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*log(abs(-3*x + sqrt(9*x^2 + 6*x -
 8) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx = \frac{\ln\left(3\,x+\sqrt{9\,x^2+6\,x-8}+1\right)}{3}$$

```
[In] int(1/(6*x + 9*x^2 - 8)^(1/2),x)
[Out] log(3*x + (6*x + 9*x^2 - 8)^(1/2) + 1)/3
```

3.147 $\int \frac{x^2}{\sqrt{4x-x^2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{x^2}{\sqrt{4x-x^2}} dx = -3\sqrt{4x-x^2} - \frac{1}{2}x\sqrt{4x-x^2} - 6\arcsin\left(1-\frac{x}{2}\right)$$

```
[Out] 6*arcsin(-1+1/2*x)-3*(-x^2+4*x)^(1/2)-1/2*x*(-x^2+4*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {684, 654, 633, 222}

$$\int \frac{x^2}{\sqrt{4x-x^2}} dx = -6\arcsin\left(1-\frac{x}{2}\right) - \frac{1}{2}\sqrt{4x-x^2}x - 3\sqrt{4x-x^2}$$

```
[In] Int[x^2/Sqrt[4*x - x^2],x]
```

```
[Out] -3*Sqrt[4*x - x^2] - (x*Sqrt[4*x - x^2])/2 - 6*ArcSin[1 - x/2]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{2}x\sqrt{4x-x^2}+3\int \frac{x}{\sqrt{4x-x^2}}\,dx \\
&= -3\sqrt{4x-x^2}-\frac{1}{2}x\sqrt{4x-x^2}+6\int \frac{1}{\sqrt{4x-x^2}}\,dx \\
&= -3\sqrt{4x-x^2}-\frac{1}{2}x\sqrt{4x-x^2}-\frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx,x,4-2x\right) \\
&= -3\sqrt{4x-x^2}-\frac{1}{2}x\sqrt{4x-x^2}-6\arcsin\left(1-\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{\sqrt{4x-x^2}}\,dx = \frac{x(-24+2x+x^2)-24\sqrt{-4+x}\sqrt{x}\log\left(\sqrt{-4+x}-\sqrt{x}\right)}{2\sqrt{-((-4+x)x)}}$$

```
[In] Integrate[x^2/Sqrt[4*x - x^2],x]
[Out] (x*(-24 + 2*x + x^2) - 24*Sqrt[-4 + x]*Sqrt[x]*Log[Sqrt[-4 + x] - Sqrt[x]])
/(2*Sqrt[-((-4 + x)*x)])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

method	result
risch	$\frac{(6+x)x(x-4)}{2\sqrt{-x(x-4)}}+6\arcsin\left(-1+\frac{x}{2}\right)$
default	$6\arcsin\left(-1+\frac{x}{2}\right)-3\sqrt{-x^{2}+4x}-\frac{x\sqrt{-x^{2}+4x}}{2}$
pseudoelliptic	$-\frac{x\sqrt{-x(x-4)}}{2}-3\sqrt{-x\left(x-4\right)}-12\arctan\left(\frac{\sqrt{-x(x-4)}}{x}\right)$
meijerg	$-\frac{16i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\left(\frac{5x}{2}+15\right)\sqrt{-\frac{x}{4}+1}}{40}+\frac{3i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{x}}{2}\right)}{4}\right)}{\sqrt{\pi}}$
trager	$\left(-3-\frac{x}{2}\right)\sqrt{-x^{2}+4x}-6\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^{2}+1\right)x-2\,\mathrm{RootOf}\left(\_Z^{2}+\right.\right.$

```
[In] int(x^2/(-x^2+4*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(6+x)*x*(x-4)/(-x*(x-4))^(1/2)+6*arcsin(-1+1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\sqrt{4x-x^2}}\,dx=-\frac{1}{2}\,\sqrt{-x^{2}+4\,x}(x+6)-12\,\arctan\left(\frac{\sqrt{-x^{2}+4\,x}}{x}\right)$$

```
[In] integrate(x^2/(-x^2+4*x)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-x^2 + 4*x)*(x + 6) - 12*arctan(sqrt(-x^2 + 4*x)/x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{\sqrt{4x-x^2}}\,dx=\left(-\frac{x}{2}-3\right)\sqrt{-x^{2}+4x}+6\operatorname{asin}\left(\frac{x}{2}-1\right)$$

```
[In] integrate(x**2/(-x**2+4*x)**(1/2),x)
[Out] (-x/2 - 3)*sqrt(-x**2 + 4*x) + 6*asin(x/2 - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{\sqrt{4x-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+4\,x}x - 3\sqrt{-x^2+4\,x} - 6\arcsin\left(-\frac{1}{2}\,x+1\right)$$

```
[In] integrate(x^2/(-x^2+4*x)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-x^2 + 4*x)*x - 3*sqrt(-x^2 + 4*x) - 6*arcsin(-1/2*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.57

$$\int \frac{x^2}{\sqrt{4x-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+4\,x}(x+6) + 6\arcsin\left(\frac{1}{2}\,x-1\right)$$

```
[In] integrate(x^2/(-x^2+4*x)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^2 + 4*x)*(x + 6) + 6*arcsin(1/2*x - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{4x-x^2}}\,dx = \int \frac{x^2}{\sqrt{4\,x-x^2}}\,dx$$

```
[In] int(x^2/(4*x - x^2)^(1/2),x)
[Out] int(x^2/(4*x - x^2)^(1/2), x)
```

3.148 $\int \frac{1}{(2+2x+x^2)^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int \frac{1}{(2+2x+x^2)^2} dx = \frac{1+x}{2(2+2x+x^2)} + \frac{1}{2}\arctan(1+x)$$

```
[Out] 1/2*(1+x)/(x^2+2*x+2)+1/2*arctan(1+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {628, 631, 210}

$$\int \frac{1}{(2+2x+x^2)^2} dx = \frac{1}{2}\arctan(x+1) + \frac{x+1}{2(x^2+2x+2)}$$

```
[In] Int[(2 + 2*x + x^2)^(-2),x]
```

```
[Out] (1 + x)/(2*(2 + 2*x + x^2)) + ArcTan[1 + x]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1+x}{2\left(2+2x+x^2\right)}+\frac{1}{2}\int \frac{1}{2+2x+x^2}\,dx\\ &= \frac{1+x}{2\left(2+2x+x^2\right)}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+x\right)\\ &= \frac{1+x}{2\left(2+2x+x^2\right)}+\frac{1}{2}\arctan(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(2+2x+x^2\right)^2}\,dx=\frac{1}{2}\left(\frac{1+x}{2+2x+x^2}+\arctan(1+x)\right)$$

```
[In] Integrate[(2 + 2*x + x^2)^(-2),x]
[Out] ((1 + x)/(2 + 2*x + x^2) + ArcTan[1 + x])/2
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{\frac{1}{2}+\frac{x}{2}}{x^2+2x+2}+\frac{\arctan(1+x)}{2}$	24
default	$\frac{2x+2}{4x^2+8x+8}+\frac{\arctan(1+x)}{2}$	25
parallelrisch	$-\frac{i\ln(x+1-i)x^2-i\ln(x+1+i)x^2+2i\ln(x+1-i)x-2i\ln(x+1+i)x+2i\ln(x+1-i)-2i\ln(x+1+i)+x^2}{4(x^2+2x+2)}$	79

```
[In] int(1/(x^2+2*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2+1/2*x)/(x^2+2*x+2)+1/2*arctan(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(2+2x+x^2)^2} \, dx = \frac{(x^2+2\,x+2)\arctan(x+1)+x+1}{2\,(x^2+2\,x+2)}$$

```
[In] integrate(1/(x^2+2*x+2)^2,x, algorithm="fricas")
[Out] 1/2*((x^2 + 2*x + 2)*arctan(x + 1) + x + 1)/(x^2 + 2*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{1}{(2+2x+x^2)^2} \, dx = \frac{x+1}{2x^2+4x+4} + \frac{\operatorname{atan}(x+1)}{2}$$

```
[In] integrate(1/(x**2+2*x+2)**2,x)
[Out] (x + 1)/(2*x**2 + 4*x + 4) + atan(x + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{(2+2x+x^2)^2} \, dx = \frac{x+1}{2\,(x^2+2\,x+2)} + \frac{1}{2}\arctan(x+1)$$

```
[In] integrate(1/(x^2+2*x+2)^2,x, algorithm="maxima")
[Out] 1/2*(x + 1)/(x^2 + 2*x + 2) + 1/2*arctan(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{(2+2x+x^2)^2}\,dx = \frac{x+1}{2\,(x^2+2\,x+2)} + \frac{1}{2}\,\arctan\left(x+1\right)$$

```
[In] integrate(1/(x^2+2*x+2)^2,x, algorithm="giac")
[Out] 1/2*(x + 1)/(x^2 + 2*x + 2) + 1/2*arctan(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{1}{(2+2x+x^2)^2}\,dx = \frac{\mathrm{atan}(x+1)}{2} + \frac{\frac{x}{2}+\frac{1}{2}}{x^2+2\,x+2}$$

```
[In] int(1/(2*x + x^2 + 2)^2,x)
[Out] atan(x + 1)/2 + (x/2 + 1/2)/(2*x + x^2 + 2)
```

3.149 $\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx=\frac{2+x}{27\,(5-4x-x^2)^{3/2}}+\frac{2(2+x)}{243\sqrt{5-4x-x^2}}$$

```
[Out] 1/27*(2+x)/(-x^2-4*x+5)^(3/2)+2/243*(2+x)/(-x^2-4*x+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {628, 627}

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx=\frac{2(x+2)}{243\sqrt{-x^2-4x+5}}+\frac{x+2}{27\,(-x^2-4x+5)^{3/2}}$$

```
[In] Int[(5 - 4*x - x^2)^(-5/2),x]
```

```
[Out] (2 + x)/(27*(5 - 4*x - x^2)^(3/2)) + (2*(2 + x))/(243*Sqrt[5 - 4*x - x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2+x}{27\left(5-4x-x^2\right)^{3/2}}+\frac{2}{27}\int\frac{1}{\left(5-4x-x^2\right)^{3/2}}\,dx\\ &= \frac{2+x}{27\left(5-4x-x^2\right)^{3/2}}+\frac{2(2+x)}{243\sqrt{5-4x-x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int\frac{1}{\left(5-4x-x^2\right)^{5/2}}\,dx=\frac{\sqrt{5-4x-x^2}(38+3x-12x^2-2x^3)}{243(-1+x)^2(5+x)^2}$$

```
[In] Integrate[(5 - 4*x - x^2)^(-5/2),x]
[Out] (Sqrt[5 - 4*x - x^2]*(38 + 3*x - 12*x^2 - 2*x^3))/(243*(-1 + x)^2*(5 + x)^2
)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{(5+x)(-1+x)\left(2x^3+12x^2-3x-38\right)}{243\left(-x^2-4x+5\right)^{\frac{5}{2}}}$	36
default	$-\frac{-2x-4}{54\left(-x^2-4x+5\right)^{\frac{3}{2}}}-\frac{-2x-4}{243\sqrt{-x^2-4x+5}}$	40
trager	$-\frac{\left(2x^3+12x^2-3x-38\right)\sqrt{-x^2-4x+5}}{243\left(x^2+4x-5\right)^2}$	40
risch	$\frac{2x^3+12x^2-3x-38}{243\left(x^2+4x-5\right)\sqrt{-x^2-4x+5}}$	40

```
[In] int(1/(-x^2-4*x+5)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/243*(5+x)*(-1+x)*(2*x^3+12*x^2-3*x-38)/(-x^2-4*x+5)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx = -\frac{(2\,x^3+12\,x^2-3\,x-38)\sqrt{-x^2-4\,x+5}}{243\,(x^4+8\,x^3+6\,x^2-40\,x+25)}$$

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="fricas")
[Out] -1/243*(2*x^3 + 12*x^2 - 3*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^4 + 8*x^3 + 6*x^
2 - 40*x + 25)
```

Sympy [F]

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx = \int \frac{1}{(-x^2-4x+5)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(-x**2-4*x+5)**(5/2),x)
[Out] Integral((-x**2 - 4*x + 5)**(-5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx = \frac{2\,x}{243\,\sqrt{-x^2-4\,x+5}} + \frac{4}{243\,\sqrt{-x^2-4\,x+5}} + \frac{x}{27\,(-x^2-4\,x+5)^{\frac{3}{2}}} + \frac{2}{27\,(-x^2-4\,x+5)^{\frac{3}{2}}}$$

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="maxima")
[Out] 2/243*x/sqrt(-x^2 - 4*x + 5) + 4/243/sqrt(-x^2 - 4*x + 5) + 1/27*x/(-x^2 -
4*x + 5)^(3/2) + 2/27/(-x^2 - 4*x + 5)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx = -\frac{((2\,(x+6)x-3)x-38)\sqrt{-x^2-4\,x+5}}{243\,(x^2+4\,x-5)^2}$$

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="giac")
[Out] -1/243*((2*(x + 6)*x - 3)*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^2 + 4*x - 5)^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx = -\frac{(4\,x+8)\,\left(8\,x^2+32\,x-76\right)}{3888\,(-x^2-4\,x+5)^{3/2}}$$

```
[In] int(1/(5 - x^2 - 4*x)^(5/2),x)
[Out] -((4*x + 8)*(32*x + 8*x^2 - 76))/(3888*(5 - x^2 - 4*x)^(3/2))
```

3.150 $\int e^t\sqrt{9-e^{2t}}\,dt$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int e^t\sqrt{9-e^{2t}}\,dt=\frac{1}{2}e^t\sqrt{9-e^{2t}}+\frac{9}{2}\arcsin\left(\frac{e^t}{3}\right)$$

```
[Out] 9/2*arcsin(1/3*exp(t))+1/2*exp(t)*(9-exp(2*t))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2281, 201, 222}

$$\int e^t\sqrt{9-e^{2t}}\,dt=\frac{9}{2}\arcsin\left(\frac{e^t}{3}\right)+\frac{1}{2}e^t\sqrt{9-e^{2t}}$$

```
[In] Int[E^t*Sqrt[9 - E^(2*t)],t]
```

```
[Out] (E^t*Sqrt[9 - E^(2*t)])/2 + (9*ArcSin[E^t/3])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \sqrt{9-t^2}\,dt, t, e^t\right)\\ &= \frac{1}{2}e^t\sqrt{9-e^{2t}} + \frac{9}{2}\text{Subst}\left(\int \frac{1}{\sqrt{9-t^2}}\,dt, t, e^t\right)\\ &= \frac{1}{2}e^t\sqrt{9-e^{2t}} + \frac{9}{2}\arcsin\left(\frac{e^t}{3}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int e^t\sqrt{9-e^{2t}}\,dt = \frac{1}{2}e^t\sqrt{9-e^{2t}} - 9\arctan\left(\frac{\sqrt{9-e^{2t}}}{3+e^t}\right)$$

```
[In] Integrate[E^t*Sqrt[9 - E^(2*t)],t]
[Out] (E^t*Sqrt[9 - E^(2*t)])/2 - 9*ArcTan[Sqrt[9 - E^(2*t)]/(3 + E^t)]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{9\arcsin\left(\frac{e^t}{3}\right)}{2}+\frac{e^t\sqrt{9-e^{2t}}}{2}$	23
risch	$-\frac{e^t\left(-9+e^{2t}\right)}{2\sqrt{9-e^{2t}}}+\frac{9\arcsin\left(\frac{e^t}{3}\right)}{2}$	29

```
[In] int(exp(t)*(9-exp(2*t))^(1/2),t,method=_RETURNVERBOSE)
[Out] 1/2*exp(t)*(9-exp(t)^2)^(1/2)+9/2*arcsin(1/3*exp(t))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int e^t\sqrt{9-e^{2t}}\,dt=\frac{1}{2}\,\sqrt{-e^{(2\,t)}+9}e^t-9\,\arctan\left(\left(\sqrt{-e^{(2\,t)}+9}-3\right)e^{(-t)}\right)$$

```
[In] integrate(exp(t)*(9-exp(2*t))^(1/2),t, algorithm="fricas")
[Out] 1/2*sqrt(-e^(2*t) + 9)*e^t - 9*arctan((sqrt(-e^(2*t) + 9) - 3)*e^(-t))
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int e^t\sqrt{9-e^{2t}}\,dt=\frac{\sqrt{9-e^{2t}}e^t}{2}+\frac{9\,\mathrm{asin}\left(\frac{e^t}{3}\right)}{2}$$

```
[In] integrate(exp(t)*(9-exp(2*t))**(1/2),t)
[Out] sqrt(9 - exp(2*t))*exp(t)/2 + 9*asin(exp(t)/3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int e^t\sqrt{9-e^{2t}}\,dt=\frac{1}{2}\,\sqrt{-e^{(2\,t)}+9}e^t+\frac{9}{2}\,\arcsin\left(\frac{1}{3}\,e^t\right)$$

```
[In] integrate(exp(t)*(9-exp(2*t))^(1/2),t, algorithm="maxima")
[Out] 1/2*sqrt(-e^(2*t) + 9)*e^t + 9/2*arcsin(1/3*e^t)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int e^t\sqrt{9-e^{2t}}\,dt=\frac{1}{2}\,\sqrt{-e^{(2\,t)}+9}e^t+\frac{9}{2}\,\arcsin\left(\frac{1}{3}\,e^t\right)$$

```
[In] integrate(exp(t)*(9-exp(2*t))^(1/2),t, algorithm="giac")
[Out] 1/2*sqrt(-e^(2*t) + 9)*e^t + 9/2*arcsin(1/3*e^t)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int e^t\sqrt{9-e^{2t}}\,dt=\frac{9\,\mathrm{asin}\left(\frac{\mathrm{e}^t}{3}\right)}{2}+\frac{\mathrm{e}^t\,\sqrt{9-\mathrm{e}^{2\,t}}}{2}$$

```
[In] int(exp(t)*(9 - exp(2*t))^(1/2),t)
[Out] (9*asin(exp(t)/3))/2 + (exp(t)*(9 - exp(2*t))^(1/2))/2
```

3.151 $\int \sqrt{-9+e^{2t}}\,dt$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \sqrt{-9+e^{2t}}\,dt=\sqrt{-9+e^{2t}}-3\arctan\left(\frac{1}{3}\sqrt{-9+e^{2t}}\right)$$

```
[Out] -3*arctan(1/3*(-9+exp(2*t))^(1/2))+(-9+exp(2*t))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2320, 52, 65, 209}

$$\int \sqrt{-9+e^{2t}}\,dt=\sqrt{e^{2t}-9}-3\arctan\left(\frac{1}{3}\sqrt{e^{2t}-9}\right)$$

```
[In] Int[Sqrt[-9 + E^(2*t)],t]
```

```
[Out] Sqrt[-9 + E^(2*t)] - 3*ArcTan[Sqrt[-9 + E^(2*t)]/3]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-9+t}}{t}\,dt, t, e^{2t}\right)\\
&= \sqrt{-9+e^{2t}} - \frac{9}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-9+t}t}\,dt, t, e^{2t}\right)\\
&= \sqrt{-9+e^{2t}} - 9\text{Subst}\left(\int \frac{1}{9+t^2}\,dt, t, \sqrt{-9+e^{2t}}\right)\\
&= \sqrt{-9+e^{2t}} - 3\arctan\left(\frac{1}{3}\sqrt{-9+e^{2t}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \sqrt{-9+e^{2t}}\,dt = \sqrt{-9+e^{2t}} - 3\arctan\left(\frac{1}{3}\sqrt{-9+e^{2t}}\right)$$

```
[In] Integrate[Sqrt[-9 + E^(2*t)],t]
[Out] Sqrt[-9 + E^(2*t)] - 3*ArcTan[Sqrt[-9 + E^(2*t)]/3]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-3 \arctan \left(\frac{\sqrt{-9+{\mathrm e}^{2 t}}}{3}\right)+\sqrt{-9+{\mathrm e}^{2 t}}$	23
default	$-3 \arctan \left(\frac{\sqrt{-9+{\mathrm e}^{2 t}}}{3}\right)+\sqrt{-9+{\mathrm e}^{2 t}}$	23
risch	$-3 \arctan \left(\frac{\sqrt{-9+{\mathrm e}^{2 t}}}{3}\right)+\sqrt{-9+{\mathrm e}^{2 t}}$	23

```
[In] int((-9+exp(2*t))^(1/2),t,method=_RETURNVERBOSE)
[Out] -3*arctan(1/3*(-9+exp(2*t))^(1/2))+(-9+exp(2*t))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \sqrt{-9+e^{2t}}\,dt=\sqrt{e^{(2\,t)}-9}-3\,\arctan\left(\frac{1}{3}\,\sqrt{e^{(2\,t)}-9}\right)$$

```
[In] integrate((-9+exp(2*t))^(1/2),t, algorithm="fricas")
[Out] sqrt(e^(2*t) - 9) - 3*arctan(1/3*sqrt(e^(2*t) - 9))
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \sqrt{-9+e^{2t}}\,dt=\sqrt{e^{2t}-9}-3\,\mathrm{atan}\left(\frac{\sqrt{e^{2t}-9}}{3}\right)$$

```
[In] integrate((-9+exp(2*t))**(1/2),t)
[Out] sqrt(exp(2*t) - 9) - 3*atan(sqrt(exp(2*t) - 9)/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \sqrt{-9+e^{2t}}\,dt = \sqrt{e^{(2\,t)}-9} - 3\,\arctan\left(\frac{1}{3}\,\sqrt{e^{(2\,t)}-9}\right)$$

```
[In] integrate((-9+exp(2*t))^(1/2),t, algorithm="maxima")
[Out] sqrt(e^(2*t) - 9) - 3*arctan(1/3*sqrt(e^(2*t) - 9))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \sqrt{-9+e^{2t}}\,dt = \sqrt{e^{(2\,t)}-9} - 3\,\arctan\left(\frac{1}{3}\,\sqrt{e^{(2\,t)}-9}\right)$$

```
[In] integrate((-9+exp(2*t))^(1/2),t, algorithm="giac")
[Out] sqrt(e^(2*t) - 9) - 3*arctan(1/3*sqrt(e^(2*t) - 9))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \sqrt{-9+e^{2t}}\,dt = \left(\frac{3\,\mathrm{e}^{-t}\,\mathrm{asin}\left(3\,\mathrm{e}^{-t}\right)}{\sqrt{1-9\,\mathrm{e}^{-2\,t}}}+1\right)\,\sqrt{\mathrm{e}^{2\,t}-9}$$

```
[In] int((exp(2*t) - 9)^(1/2),t)
[Out] ((3*exp(-t)*asin(3*exp(-t)))/(1 - 9*exp(-2*t))^(1/2) + 1)*(exp(2*t) - 9)^(1
/2)
```

3.152 $\int \frac{1}{\sqrt{a^2+x^2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{1}{\sqrt{a^2+x^2}} \, dx = \operatorname{arctanh}\left(\frac{x}{\sqrt{a^2+x^2}}\right)$$

```
[Out] arctanh(x/(a^2+x^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {223, 212}

$$\int \frac{1}{\sqrt{a^2+x^2}} \, dx = \operatorname{arctanh}\left(\frac{x}{\sqrt{a^2+x^2}}\right)$$

```
[In] Int[1/Sqrt[a^2 + x^2],x]
```

```
[Out] ArcTanh[x/Sqrt[a^2 + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{a^2+x^2}}\right)$$
$$= \text{arctanh}\left(\frac{x}{\sqrt{a^2+x^2}}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{a^2+x^2}}\,dx = -\frac{1}{2}\log\left(1-\frac{x}{\sqrt{a^2+x^2}}\right)+\frac{1}{2}\log\left(1+\frac{x}{\sqrt{a^2+x^2}}\right)$$

```
[In] Integrate[1/Sqrt[a^2 + x^2],x]
[Out] -1/2*Log[1 - x/Sqrt[a^2 + x^2]] + Log[1 + x/Sqrt[a^2 + x^2]]/2
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\ln\left(x+\sqrt{a^2+x^2}\right)$	13
pseudoelliptic	$\text{arctanh}\left(\frac{\sqrt{a^2+x^2}}{x}\right)$	15

```
[In] int(1/(a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(x+(a^2+x^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{a^2+x^2}}\,dx = -\log\left(-x+\sqrt{a^2+x^2}\right)$$

```
[In] integrate(1/(a^2+x^2)^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(a^2 + x^2))
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{a^2 + x^2}}\, dx = \operatorname{asinh}\left(\frac{x}{a}\right)$$

```
[In] integrate(1/(a**2+x**2)**(1/2),x)
[Out] asinh(x/a)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{a^2 + x^2}}\, dx = \operatorname{arsinh}\left(\frac{x}{a}\right)$$

```
[In] integrate(1/(a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] arcsinh(x/a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(12) = 24.

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \frac{1}{\sqrt{a^2 + x^2}}\, dx = -\frac{1}{2}\, a^2 \log\left(-x + \sqrt{a^2 + x^2}\right) + \frac{1}{2}\, \sqrt{a^2 + x^2} x$$

```
[In] integrate(1/(a^2+x^2)^(1/2),x, algorithm="giac")
[Out] -1/2*a^2*log(-x + sqrt(a^2 + x^2)) + 1/2*sqrt(a^2 + x^2)*x
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a^2 + x^2}}\, dx = \ln\left(x + \sqrt{a^2 + x^2}\right)$$

```
[In] int(1/(a^2 + x^2)^(1/2),x)
[Out] log(x + (a^2 + x^2)^(1/2))
```

3.153 $\int \frac{5+x}{-2+x+x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \frac{5+x}{-2+x+x^2} \, dx = 2\log(1-x) - \log(2+x)$$

```
[Out] 2*ln(1-x)-ln(2+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {646, 31}

$$\int \frac{5+x}{-2+x+x^2} \, dx = 2\log(1-x) - \log(x+2)$$

```
[In] Int[(5 + x)/(-2 + x + x^2),x]
```

```
[Out] 2*Log[1 - x] - Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{1}{-1+x}\,dx - \int \frac{1}{2+x}\,dx\\ &= 2\log(1-x) - \log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{5+x}{-2+x+x^2}\,dx = 2\log(1-x) - \log(2+x)$$

```
[In] Integrate[(5 + x)/(-2 + x + x^2),x]
[Out] 2*Log[1 - x] - Log[2 + x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$2\ln(-1+x) - \ln(2+x)$	14
norman	$2\ln(-1+x) - \ln(2+x)$	14
risch	$2\ln(-1+x) - \ln(2+x)$	14
parallelrisch	$2\ln(-1+x) - \ln(2+x)$	14

```
[In] int((5+x)/(x^2+x-2),x,method=_RETURNVERBOSE)
[Out] 2*ln(-1+x)-ln(2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{5+x}{-2+x+x^2}\,dx = -\log\left(x+2\right)+2\,\log\left(x-1\right)$$

```
[In] integrate((5+x)/(x^2+x-2),x, algorithm="fricas")
[Out] -log(x + 2) + 2*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{5+x}{-2+x+x^2}\,dx = 2\log\left(x-1\right)-\log\left(x+2\right)$$

```
[In] integrate((5+x)/(x**2+x-2),x)
[Out] 2*log(x - 1) - log(x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{5+x}{-2+x+x^2}\,dx = -\log\left(x+2\right)+2\,\log\left(x-1\right)$$

```
[In] integrate((5+x)/(x^2+x-2),x, algorithm="maxima")
[Out] -log(x + 2) + 2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{5+x}{-2+x+x^2}\,dx = -\log\left(|x+2|\right)+2\,\log\left(|x-1|\right)$$

```
[In] integrate((5+x)/(x^2+x-2),x, algorithm="giac")
[Out] -log(abs(x + 2)) + 2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{5+x}{-2+x+x^2} \, dx = 2 \ln (x-1) - \ln (x+2)$$

```
[In] int((x + 5)/(x + x^2 - 2),x)
[Out] 2*log(x - 1) - log(x + 2)
```

3.154 $\int \frac{x+x^3}{-1+x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{x+x^3}{-1+x} \, dx = 2x + \frac{x^2}{2} + \frac{x^3}{3} + 2\log(1-x)$$

```
[Out] 2*x+1/2*x^2+1/3*x^3+2*ln(1-x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 786}

$$\int \frac{x+x^3}{-1+x} \, dx = \frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\log(1-x)$$

```
[In] Int[(x + x^3)/(-1 + x),x]
```

```
[Out] 2*x + x^2/2 + x^3/3 + 2*Log[1 - x]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x(1+x^2)}{-1+x}\,dx \\
&= \int \left(2+\frac{2}{-1+x}+x+x^2\right)dx \\
&= 2x+\frac{x^2}{2}+\frac{x^3}{3}+2\log(1-x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$
\int \frac{x+x^3}{-1+x}\,dx = \frac{1}{6}\left(-17+12x+3x^2+2x^3+12\log(-1+x)\right)
$$

```
[In] Integrate[(x + x^3)/(-1 + x),x]
[Out] (-17 + 12*x + 3*x^2 + 2*x^3 + 12*Log[-1 + x])/6
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln\left(-1+x\right)$	21
norman	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln\left(-1+x\right)$	21
risch	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln\left(-1+x\right)$	21
parallelrisch	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln\left(-1+x\right)$	21
meijerg	$\frac{x\left(4x^2+6x+12\right)}{12}+2\ln\left(1-x\right)+x$	24

```
[In] int((x^3+x)/(-1+x),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3+1/2*x^2+2*x+2*ln(-1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + 2\,x + 2\,\log\left(x-1\right)$$

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{x+x^3}{-1+x}\,dx = \frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\log\left(x-1\right)$$

```
[In] integrate((x**3+x)/(-1+x),x)
[Out] x**3/3 + x**2/2 + 2*x + 2*log(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + 2\,x + 2\,\log\left(x-1\right)$$

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="maxima")
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{x+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + 2\,x + 2\,\log\left(|x-1|\right)$$

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="giac")
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x+x^3}{-1+x}\,dx = 2\,x + 2\,\ln\left(x-1\right) + \frac{x^2}{2} + \frac{x^3}{3}$$

```
[In] int((x + x^3)/(x - 1),x)
[Out] 2*x + 2*log(x - 1) + x^2/2 + x^3/3
```

3.155 $\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx = \frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(2+x)$$

```
[Out] 1/10*ln(1-2*x)+1/2*ln(x)-1/10*ln(2+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1608, 1642}

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx = \frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

```
[In] Int[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
```

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{-1+2x+x^2}{x\,(-2+3x+2x^2)}\,dx\\ &= \int \left(\frac{1}{2x}-\frac{1}{10(2+x)}+\frac{1}{5(-1+2x)}\right)dx\\ &= \frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx = \frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(2+x)$$

```
[In] Integrate[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
parallelrisch	$\frac{\ln(x)}{2}-\frac{\ln(2+x)}{10}+\frac{\ln\left(x-\frac{1}{2}\right)}{10}$	18
default	$-\frac{\ln(2+x)}{10}+\frac{\ln(x)}{2}+\frac{\ln(2x-1)}{10}$	20
norman	$-\frac{\ln(2+x)}{10}+\frac{\ln(x)}{2}+\frac{\ln(2x-1)}{10}$	20
risch	$-\frac{\ln(2+x)}{10}+\frac{\ln(x)}{2}+\frac{\ln(2x-1)}{10}$	20

```
[In] int((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x)-1/10*ln(2+x)+1/10*ln(x-1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx = \frac{1}{10}\,\log\left(2\,x-1\right) - \frac{1}{10}\,\log\left(x+2\right) + \frac{1}{2}\,\log\left(x\right)$$

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="fricas")
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx = \frac{\log\left(x\right)}{2} + \frac{\log\left(x-\frac{1}{2}\right)}{10} - \frac{\log\left(x+2\right)}{10}$$

```
[In] integrate((x**2+2*x-1)/(2*x**3+3*x**2-2*x),x)
[Out] log(x)/2 + log(x - 1/2)/10 - log(x + 2)/10
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx = \frac{1}{10}\,\log\left(2\,x-1\right) - \frac{1}{10}\,\log\left(x+2\right) + \frac{1}{2}\,\log\left(x\right)$$

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="maxima")
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx = \frac{1}{10}\,\log\left(|2\,x-1|\right) - \frac{1}{10}\,\log\left(|x+2|\right) + \frac{1}{2}\,\log\left(|x|\right)$$

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="giac")
[Out] 1/10*log(abs(2*x - 1)) - 1/10*log(abs(x + 2)) + 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx = \frac{\operatorname{atanh}\left(\frac{24}{145\,\left(\frac{29\,x}{100}-\frac{11}{50}\right)}+\frac{35}{29}\right)}{5}+\frac{\ln\left(x\right)}{2}$$

```
[In] int((2*x + x^2 - 1)/(3*x^2 - 2*x + 2*x^3),x)
[Out] atanh(24/(145*((29*x)/100 - 11/50)) + 35/29)/5 + log(x)/2
```

3.156 $\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 30

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} \, dx = \frac{2}{1-x}+x+\frac{x^2}{2}+\log(1-x)-\log(1+x)$$

```
[Out] 2/(1-x)+x+1/2*x^2+ln(1-x)-ln(1+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2099}

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} \, dx = \frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

```
[In] Int[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

```
[Out] 2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(1+\frac{1}{-1-x}+\frac{2}{(-1+x)^2}+\frac{1}{-1+x}+x\right) \, dx \\ &= \frac{2}{1-x}+x+\frac{x^2}{2}+\log(1-x)-\log(1+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx=-\frac{2}{-1+x}+\frac{1}{2}(1+x)^2+\log(1-x)-\log(1+x)$$

```
[In] Integrate[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
[Out] -2/(-1 + x) + (1 + x)^2/2 + Log[1 - x] - Log[1 + x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\ln\left(-1+x\right)-\frac{2}{-1+x}+x+\frac{x^{2}}{2}-\ln\left(1+x\right)$	25
risch	$\ln\left(-1+x\right)-\frac{2}{-1+x}+x+\frac{x^{2}}{2}-\ln\left(1+x\right)$	25
norman	$\frac{\frac{1}{2}x^{2}+\frac{1}{2}x^{3}-3}{-1+x}-\ln\left(1+x\right)+\ln\left(-1+x\right)$	30
parallelrisch	$\frac{x^{3}+2\ln(-1+x)x-2\ln(1+x)x+x^{2}-6-2\ln(-1+x)+2\ln(1+x)}{-2+2x}$	42

```
[In] int((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x,method=_RETURNVERBOSE)
[Out] ln(-1+x)-2/(-1+x)+x+1/2*x^2-ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx=\frac{x^{3}+x^{2}-2\,(x-1)\log\left(x+1\right)+2\,(x-1)\log\left(x-1\right)-2\,x-4}{2\,(x-1)}$$

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="fricas")
[Out] 1/2*(x^3 + x^2 - 2*(x - 1)*log(x + 1) + 2*(x - 1)*log(x - 1) - 2*x - 4)/(x
- 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx = \frac{x^2}{2}+x+\log{(x-1)}-\log{(x+1)}-\frac{2}{x-1}$$

```
[In] integrate((x**4-2*x**2+4*x+1)/(x**3-x**2-x+1),x)
[Out] x**2/2 + x + log(x - 1) - log(x + 1) - 2/(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx = \frac{1}{2}\,x^2+x-\frac{2}{x-1}-\log{(x+1)}+\log{(x-1)}$$

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="maxima")
[Out] 1/2*x^2 + x - 2/(x - 1) - log(x + 1) + log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx = \frac{1}{2}\,x^2+x-\frac{2}{x-1}-\log{(|x+1|)}+\log{(|x-1|)}$$

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="giac")
[Out] 1/2*x^2 + x - 2/(x - 1) - log(abs(x + 1)) + log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx = x-\frac{2}{x-1}+\frac{x^2}{2}+\mathrm{atan}(x\,1\mathrm{i})\;2\mathrm{i}$$

```
[In] int(-(4*x - 2*x^2 + x^4 + 1)/(x + x^2 - x^3 - 1),x)
[Out] x + atan(x*1i)*2i - 2/(x - 1) + x^2/2
```

3.157 $\int \frac{4-x+2x^2}{4x+x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx = -\frac{1}{2}\arctan\left(\frac{x}{2}\right)+\log(x)+\frac{1}{2}\log\left(4+x^2\right)$$

```
[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1607, 1816, 649, 209, 266}

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx = -\frac{1}{2}\arctan\left(\frac{x}{2}\right)+\frac{1}{2}\log\left(x^2+4\right)+\log(x)$$

```
[In] Int[(4 - x + 2*x^2)/(4*x + x^3),x]
```

```
[Out] -1/2*ArcTan[x/2] + Log[x] + Log[4 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{4-x+2x^2}{x\left(4+x^2\right)}\,dx \\
&= \int \left(\frac{1}{x}+\frac{-1+x}{4+x^2}\right)dx \\
&= \log(x)+\int \frac{-1+x}{4+x^2}\,dx \\
&= \log(x)-\int \frac{1}{4+x^2}\,dx+\int \frac{x}{4+x^2}\,dx \\
&= -\frac{1}{2}\arctan\left(\frac{x}{2}\right)+\log(x)+\frac{1}{2}\log\left(4+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx = -\frac{1}{2}\arctan\left(\frac{x}{2}\right)+\log(x)+\frac{1}{2}\log\left(4+x^2\right)$$

```
[In] Integrate[(4 - x + 2*x^2)/(4*x + x^3),x]
[Out] -1/2*ArcTan[x/2] + Log[x] + Log[4 + x^2]/2
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln\left(x\right)+\frac{\ln\left(x^{2}+4\right)}{2}$	18
risch	$-\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln\left(x\right)+\frac{\ln\left(x^{2}+4\right)}{2}$	18
meijerg	$\ln\left(x\right)-\ln\left(2\right)+\frac{\ln\left(1+\frac{x^{2}}{4}\right)}{2}-\frac{\arctan\left(\frac{x}{2}\right)}{2}$	24
parallelrisch	$\ln\left(x\right)+\frac{\ln\left(x-2i\right)}{2}+\frac{i\ln\left(x-2i\right)}{4}+\frac{\ln\left(x+2i\right)}{2}-\frac{i\ln\left(x+2i\right)}{4}$	34

```
[In] int((2*x^2-x+4)/(x^3+4*x),x,method=_RETURNVERBOSE)
[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx=-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="fricas")
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx=\log\left(x\right)+\frac{\log\left(x^2+4\right)}{2}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

```
[In] integrate((2*x**2-x+4)/(x**3+4*x),x)
[Out] log(x) + log(x**2 + 4)/2 - atan(x/2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx = -\frac{1}{2}\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="maxima")
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx = -\frac{1}{2}\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(|x|\right)$$

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="giac")
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx = \ln\left(x\right)+\ln\left(x-2\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{1}{4}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\,\left(\frac{1}{2}-\frac{1}{4}\mathrm{i}\right)$$

```
[In] int((2*x^2 - x + 4)/(4*x + x^3),x)
[Out] log(x - 2i)*(1/2 + 1i/4) + log(x + 2i)*(1/2 - 1i/4) + log(x)
```

3.158 $\int \frac{2-3x+4x^2}{3-4x+4x^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 38

$$\int \frac{2-3x+4x^2}{3-4x+4x^2} dx = x + \frac{\arctan\left(\frac{1-2x}{\sqrt{2}}\right)}{4\sqrt{2}} + \frac{1}{8}\log\left(3-4x+4x^2\right)$$

```
[Out] x+1/8*ln(4*x^2-4*x+3)+1/8*arctan(1/2*(1-2*x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1671, 648, 632, 210, 642}

$$\int \frac{2-3x+4x^2}{3-4x+4x^2} dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{2}}\right)}{4\sqrt{2}} + \frac{1}{8}\log\left(4x^2-4x+3\right) + x$$

```
[In] Int[(2 - 3*x + 4*x^2)/(3 - 4*x + 4*x^2),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[2]]/(4*Sqrt[2]) + Log[3 - 4*x + 4*x^2]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(1-\frac{1-x}{3-4x+4x^2}\right) dx \\
&= x-\int \frac{1-x}{3-4x+4x^2} \, dx \\
&= x+\frac{1}{8}\int \frac{-4+8x}{3-4x+4x^2} \, dx-\frac{1}{2}\int \frac{1}{3-4x+4x^2} \, dx \\
&= x+\frac{1}{8}\log\left(3-4x+4x^2\right)+\text{Subst}\left(\int \frac{1}{-32-x^2} \, dx, x, -4+8x\right) \\
&= x-\frac{\arctan\left(\frac{-1+2x}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{1}{8}\log\left(3-4x+4x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{2-3x+4x^2}{3-4x+4x^2}\,dx = x - \frac{\arctan\left(\frac{-1+2x}{\sqrt{2}}\right)}{4\sqrt{2}} + \frac{1}{8}\log\left(3-4x+4x^2\right)$$

```
[In] Integrate[(2 - 3*x + 4*x^2)/(3 - 4*x + 4*x^2),x]
[Out] x - ArcTan[(-1 + 2*x)/Sqrt[2]]/(4*Sqrt[2]) + Log[3 - 4*x + 4*x^2]/8
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

method	result	size
default	$x+\frac{\ln\left(4x^2-4x+3\right)}{8}-\frac{\sqrt{2}\,\arctan\left(\frac{(8x-4)\sqrt{2}}{8}\right)}{8}$	32
risch	$x+\frac{\ln\left(4x^2-4x+3\right)}{8}-\frac{\sqrt{2}\,\arctan\left(\frac{(2x-1)\sqrt{2}}{2}\right)}{8}$	32

```
[In] int((4*x^2-3*x+2)/(4*x^2-4*x+3),x,method=_RETURNVERBOSE)
[Out] x+1/8*ln(4*x^2-4*x+3)-1/8*2^(1/2)*arctan(1/8*(8*x-4)*2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{2-3x+4x^2}{3-4x+4x^2}\,dx = -\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x-1)\right) + x + \frac{1}{8}\log\left(4x^2-4x+3\right)$$

```
[In] integrate((4*x^2-3*x+2)/(4*x^2-4*x+3),x, algorithm="fricas")
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - 1)) + x + 1/8*log(4*x^2 - 4*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{2-3x+4x^2}{3-4x+4x^2}\,dx = x + \frac{\log\left(x^2-x+\frac{3}{4}\right)}{8} - \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-\frac{\sqrt{2}}{2}\right)}{8}$$

```
[In] integrate((4*x**2-3*x+2)/(4*x**2-4*x+3),x)
[Out] x + log(x**2 - x + 3/4)/8 - sqrt(2)*atan(sqrt(2)*x - sqrt(2)/2)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{2-3x+4x^2}{3-4x+4x^2}\,dx = -\frac{1}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(2\,x-1)\right) + x + \frac{1}{8}\,\log\left(4\,x^2-4\,x+3\right)$$

```
[In] integrate((4*x^2-3*x+2)/(4*x^2-4*x+3),x, algorithm="maxima")
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - 1)) + x + 1/8*log(4*x^2 - 4*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{2-3x+4x^2}{3-4x+4x^2}\,dx = -\frac{1}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(2\,x-1)\right) + x + \frac{1}{8}\,\log\left(4\,x^2-4\,x+3\right)$$

```
[In] integrate((4*x^2-3*x+2)/(4*x^2-4*x+3),x, algorithm="giac")
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - 1)) + x + 1/8*log(4*x^2 - 4*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{2-3x+4x^2}{3-4x+4x^2}\,dx = x + \frac{\ln\left(x^2-x+\frac{3}{4}\right)}{8} - \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x-\frac{\sqrt{2}}{2}\right)}{8}$$

```
[In] int((4*x^2 - 3*x + 2)/(4*x^2 - 4*x + 3),x)
[Out] x + log(x^2 - x + 3/4)/8 - (2^(1/2)*atan(2^(1/2)*x - 2^(1/2)/2))/8
```

3.159 $\int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 103

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = \frac{1+x}{8\left(1+x^2\right)^2}-\frac{3(1-x)}{8\left(1+x^2\right)}+\frac{3x}{16\left(1+x^2\right)}+\frac{7\arctan(x)}{16}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{8}\log(1-x)-\log(x)+\frac{15}{16}\log\left(1+x^2\right)-\frac{1}{2}\log\left(1+x+x^2\right)$$

```
[Out] 1/8*(1+x)/(x^2+1)^2-3/8*(1-x)/(x^2+1)+3/16*x/(x^2+1)+7/16*arctan(x)+1/8*ln(
1-x)-ln(x)+15/16*ln(x^2+1)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*
3^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6860, 653, 205, 209, 649, 266, 648, 632, 210, 642}

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = \frac{7\arctan(x)}{16}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{3(1-x)}{8\left(x^2+1\right)}+\frac{3x}{16\left(x^2+1\right)}+\frac{x+1}{8\left(x^2+1\right)^2}+\frac{15}{16}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{1}{8}\log(1-x)-\log(x)$$

```
[In] Int[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] (1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(1 + x^2))
+ (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/8 - Log
[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]/2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{8(-1+x)}-\frac{1}{x}+\frac{1-x}{2\left(1+x^2\right)^3}+\frac{3(1+x)}{4\left(1+x^2\right)^2}+\frac{-1+15x}{8\left(1+x^2\right)}+\frac{-1-x}{1+x+x^2}\right)dx\\
&= \frac{1}{8}\log(1-x)-\log(x)+\frac{1}{8}\int\frac{-1+15x}{1+x^2}\,dx\\
&\quad+\frac{1}{2}\int\frac{1-x}{\left(1+x^2\right)^3}\,dx+\frac{3}{4}\int\frac{1+x}{\left(1+x^2\right)^2}\,dx+\int\frac{-1-x}{1+x+x^2}\,dx\\
&= \frac{1+x}{8\left(1+x^2\right)^2}-\frac{3(1-x)}{8\left(1+x^2\right)}+\frac{1}{8}\log(1-x)-\log(x)-\frac{1}{8}\int\frac{1}{1+x^2}\,dx+\frac{3}{8}\int\frac{1}{\left(1+x^2\right)^2}\,dx\\
&\quad+\frac{3}{8}\int\frac{1}{1+x^2}\,dx-\frac{1}{2}\int\frac{1}{1+x+x^2}\,dx-\frac{1}{2}\int\frac{1+2x}{1+x+x^2}\,dx+\frac{15}{8}\int\frac{x}{1+x^2}\,dx\\
&= \frac{1+x}{8\left(1+x^2\right)^2}-\frac{3(1-x)}{8\left(1+x^2\right)}+\frac{3x}{16\left(1+x^2\right)}+\frac{\arctan(x)}{4}+\frac{1}{8}\log(1-x)-\log(x)+\frac{15}{16}\log\left(1\right.\\
&\quad\left.+x^2\right)-\frac{1}{2}\log\left(1+x+x^2\right)+\frac{3}{16}\int\frac{1}{1+x^2}\,dx+\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= \frac{1+x}{8\left(1+x^2\right)^2}-\frac{3(1-x)}{8\left(1+x^2\right)}+\frac{3x}{16\left(1+x^2\right)}+\frac{7\arctan(x)}{16}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}\\
&\quad+\frac{1}{8}\log(1-x)-\log(x)+\frac{15}{16}\log\left(1+x^2\right)-\frac{1}{2}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = \frac{1}{48}\left(\frac{6(1+x)}{\left(1+x^2\right)^2}+\frac{9(-2+3x)}{1+x^2}+21\arctan(x) - 16\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+20\log(1-x) - 48\log(x)+45\log\left(1+x^2\right) - 10\log\left(1+x+x^2\right)-14\log\left(1-x^3\right)\right)$$

```
[In] Integrate[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] ((6*(1 + x))/(1 + x^2)^2 + (9*(-2 + 3*x))/(1 + x^2) + 21*ArcTan[x] - 16*Sqr
t[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 20*Log[1 - x] - 48*Log[x] + 45*Log[1 + x^2
] - 10*Log[1 + x + x^2] - 14*Log[1 - x^3])/48
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.70

method	result
risch	$\frac{\frac{9}{16}x^3-\frac{3}{8}x^2+\frac{11}{16}x-\frac{1}{4}}{(x^2+1)^2}+\frac{\ln(-1+x)}{8}+\frac{15\ln(49x^2+49)}{16}+\frac{7\arctan(x)}{16}-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{3}-\frac{\ln(x^2+x+1)}{2}-\ln(x)$
default	$\frac{\ln(-1+x)}{8}+\frac{\frac{9}{2}x^3-3x^2+\frac{11}{2}x-2}{8(x^2+1)^2}+\frac{15\ln(x^2+1)}{16}+\frac{7\arctan(x)}{16}-\frac{\ln(x^2+x+1)}{2}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\ln(x)$

```
[In] int((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x,method=_RETURNVERBOSE)
```

```
[Out] (9/16*x^3-3/8*x^2+11/16*x-1/4)/(x^2+1)^2+1/8*ln(-1+x)+15/16*ln(49*x^2+49)+7
/16*arctan(x)-1/3*3^(1/2)*arctan(2/3*3^(1/2)*(x+1/2))-1/2*ln(x^2+x+1)-ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.32

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = \frac{27\,x^3-16\,\sqrt{3}(x^4+2\,x^2+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-18\,x^2+21\,(x^4+2\,x^2+1)\arctan(x)-24\,(x^4+2\,x^{\dots}}{\dots}$$

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/48*(27*x^3 - 16*sqrt(3)*(x^4 + 2*x^2 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) -
 18*x^2 + 21*(x^4 + 2*x^2 + 1)*arctan(x) - 24*(x^4 + 2*x^2 + 1)*log(x^2 + x
 + 1) + 45*(x^4 + 2*x^2 + 1)*log(x^2 + 1) + 6*(x^4 + 2*x^2 + 1)*log(x - 1)
- 48*(x^4 + 2*x^2 + 1)*log(x) + 33*x - 12)/(x^4 + 2*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = -\log{(x)} + \frac{\log{(x-1)}}{8} + \frac{15\log{(x^2+1)}}{16} - \frac{\log{(x^2+x+1)}}{2} + \frac{7\operatorname{atan}{(x)}}{16} - \frac{\sqrt{3}\operatorname{atan}{\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}}{3} + \frac{9x^3-6x^2+11x-4}{16x^4+32x^2+16}$$

```
[In] integrate((x**3+x**2+1)/(-1+x)/x/(x**2+1)**3/(x**2+x+1),x)
```

```
[Out] -log(x) + log(x - 1)/8 + 15*log(x**2 + 1)/16 - log(x**2 + x + 1)/2 + 7*atan
(x)/16 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3 + (9*x**3 - 6*x**2 + 11*
x - 4)/(16*x**4 + 32*x**2 + 16)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{9x^3-6x^2+11x-4}{16\left(x^4+2x^2+1\right)} + \frac{7}{16}\arctan\left(x\right) - \frac{1}{2}\log\left(x^2+x+1\right) + \frac{15}{16}\log\left(x^2+1\right) + \frac{1}{8}\log\left(x-1\right) - \log\left(x\right)$$

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4
)/(x^4 + 2*x^2 + 1) + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2
 + 1) + 1/8*log(x - 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.72

$$\int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{9x^3-6x^2+11x-4}{16(x^2+1)^2} + \frac{7}{16}\arctan(x) - \frac{1}{2}\log(x^2+x+1) + \frac{15}{16}\log(x^2+1) + \frac{1}{8}\log(|x-1|) - \log(|x|)$$

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4
)/(x^2 + 1)^2 + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2 + 1)
+ 1/8*log(abs(x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)}\,dx = \frac{\ln(x-1)}{8} - \ln(x) + \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \frac{\frac{9x^3}{16}-\frac{3x^2}{8}+\frac{11x}{16}-\frac{1}{4}}{x^4+2x^2+1} + \ln(x-\mathrm{i})\left(\frac{15}{16}-\frac{7}{32}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(\frac{15}{16}+\frac{7}{32}\mathrm{i}\right)$$

```
[In] int((x^2 + x^3 + 1)/(x*(x^2 + 1)^3*(x - 1)*(x + x^2 + 1)),x)
```

```
[Out] log(x - 1)/8 + log(x - 1i)*(15/16 - 7i/32) + log(x + 1i)*(15/16 + 7i/32) -
log(x) + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/2) - log(x + (3^
(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/2) + ((11*x)/16 - (3*x^2)/8 + (9*x^3
)/16 - 1/4)/(2*x^2 + x^4 + 1)
```

3.160 $\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 33

$$\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2}\,dx = -\frac{1+2x}{2(1+x^2)} - 2\arctan(x) + \log(x) - \frac{1}{2}\log(1+x^2)$$

```
[Out] 1/2*(-2*x-1)/(x^2+1)-2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1819, 815, 649, 209, 266}

$$\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2}\,dx = -2\arctan(x) - \frac{2x+1}{2(x^2+1)} - \frac{1}{2}\log(x^2+1) + \log(x)$$

```
[In] Int[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
```

```
[Out] -1/2*(1 + 2*x)/(1 + x^2) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1+2x}{2\left(1+x^2\right)}-\frac{1}{2}\int\frac{-2+4x}{x\left(1+x^2\right)}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}-\frac{1}{2}\int\left(-\frac{2}{x}+\frac{2(2+x)}{1+x^2}\right)\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}+\log(x)-\int\frac{2+x}{1+x^2}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}+\log(x)-2\int\frac{1}{1+x^2}\,dx-\int\frac{x}{1+x^2}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}-2\arctan(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx=\frac{-1-2x}{2\left(1+x^2\right)}-2\arctan(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
[Out] (-1 - 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result
default	$-\frac{x+\frac{1}{2}}{x^2+1}-\frac{\ln\left(x^2+1\right)}{2}-2\arctan\left(x\right)+\ln\left(x\right)$
risch	$\frac{-x-\frac{1}{2}}{x^2+1}-\frac{\ln\left(x^2+1\right)}{2}-2\arctan\left(x\right)+\ln\left(x\right)$
meijerg	$-\frac{2x}{2x^2+2}-2\arctan\left(x\right)+\frac{x^2}{x^2+1}+\frac{1}{2}+\ln\left(x\right)-\frac{x^2}{2x^2+2}-\frac{\ln\left(x^2+1\right)}{2}$
parallelrisch	$\frac{2i\ln(x-i)x^2-2i\ln(x+i)x^2+2x^2\ln(x)-x^2\ln(x-i)-\ln(x+i)x^2-1+2i\ln(x-i)-2i\ln(x+i)+2\ln(x)-\ln(x-i)-\ln(x+i)-2x}{2x^2+2}$

```
[In] int((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] -(x+1/2)/(x^2+1)-1/2*ln(x^2+1)-2*arctan(x)+ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx=-\frac{4\left(x^2+1\right)\arctan\left(x\right)+\left(x^2+1\right)\log\left(x^2+1\right)-2\left(x^2+1\right)\log\left(x\right)+2\,x+1}{2\left(x^2+1\right)}$$

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="fricas")
[Out] -1/2*(4*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x) +
 2*x + 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx = -\frac{2x+1}{2x^2+2}+\log\left(x\right)-\frac{\log\left(x^2+1\right)}{2}-2\,\mathrm{atan}\left(x\right)$$

```
[In] integrate((-x**3+2*x**2-3*x+1)/x/(x**2+1)**2,x)
[Out] -(2*x + 1)/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 - 2*atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx = -\frac{2\,x+1}{2\left(x^2+1\right)}-2\,\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^2+1\right)+\log\left(x\right)$$

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx = -\frac{2\,x+1}{2\left(x^2+1\right)}-2\,\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^2+1\right)+\log\left(|x|\right)$$

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx = \ln\left(x\right)-\frac{x+\frac{1}{2}}{x^2+1}+\ln\left(x-\mathrm{i}\right)\,\left(-\frac{1}{2}+1\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(-\frac{1}{2}-\mathrm{i}\right)$$

```
[In] int(-(3*x - 2*x^2 + x^3 - 1)/(x*(x^2 + 1)^2),x)
[Out] log(x) - log(x + 1i)*(1/2 + 1i) - log(x - 1i)*(1/2 - 1i) - (x + 1/2)/(x^2 +
 1)
```

3.161 $\int \frac{1}{(1+x^2)^2}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 19

$$\int \frac{1}{(1+x^2)^2}\,dx = \frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

```
[Out] 1/2*x/(x^2+1)+1/2*arctan(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {205, 209}

$$\int \frac{1}{(1+x^2)^2}\,dx = \frac{\arctan(x)}{2} + \frac{x}{2(x^2+1)}$$

```
[In] Int[(1 + x^2)^(-2),x]
```

```
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx \\ &= \frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{(1+x^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{1+x^2} + \arctan(x)\right)$$

```
[In] Integrate[(1 + x^2)^(-2),x]
[Out] (x/(1 + x^2) + ArcTan[x])/2
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$	16
risch	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$	16
meijerg	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$	17
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2+i\ln(x-i)-i\ln(x+i)-2x}{4(x^2+1)}$	52

```
[In] int(1/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x/(x^2+1)+1/2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)^2}\,dx = \frac{(x^2+1)\arctan(x)+x}{2(x^2+1)}$$

```
[In] integrate(1/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1}{(1+x^2)^2}\,dx = \frac{x}{2x^2+2} + \frac{\operatorname{atan}(x)}{2}$$

```
[In] integrate(1/(x**2+1)**2,x)
[Out] x/(2*x**2 + 2) + atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1+x^2)^2}\,dx = \frac{x}{2(x^2+1)} + \frac{1}{2}\arctan(x)$$

```
[In] integrate(1/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1+x^2)^2}\,dx = \frac{x}{2(x^2+1)} + \frac{1}{2}\arctan(x)$$

```
[In] integrate(1/(x^2+1)^2,x, algorithm="giac")
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{(1+x^2)^2} \, dx = \frac{\operatorname{atan}(x)}{2} + \frac{x}{2 (x^2+1)}$$

```
[In] int(1/(x^2 + 1)^2,x)
```

```
[Out] atan(x)/2 + x/(2*(x^2 + 1))
```

3.162 $\int \frac{1}{(-1+x)(2+x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{1}{(-1+x)(2+x)} dx = \frac{1}{3}\log(1-x) - \frac{1}{3}\log(2+x)$$

```
[Out] 1/3*ln(1-x)-1/3*ln(2+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {36, 31}

$$\int \frac{1}{(-1+x)(2+x)} dx = \frac{1}{3}\log(1-x) - \frac{1}{3}\log(x+2)$$

```
[In] Int[1/((-1 + x)*(2 + x)),x]
```

```
[Out] Log[1 - x]/3 - Log[2 + x]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\int \frac{1}{-1+x}\,dx - \frac{1}{3}\int \frac{1}{2+x}\,dx$$
$$= \frac{1}{3}\log(1-x) - \frac{1}{3}\log(2+x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-1+x)(2+x)}\,dx = \frac{1}{3}\log(1-x) - \frac{1}{3}\log(2+x)$$

```
[In] Integrate[1/((-1 + x)*(2 + x)),x]
[Out] Log[1 - x]/3 - Log[2 + x]/3
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\ln(-1+x)}{3}-\frac{\ln(2+x)}{3}$	14
norman	$\frac{\ln(-1+x)}{3}-\frac{\ln(2+x)}{3}$	14
risch	$\frac{\ln(-1+x)}{3}-\frac{\ln(2+x)}{3}$	14
parallelrisch	$\frac{\ln(-1+x)}{3}-\frac{\ln(2+x)}{3}$	14

```
[In] int(1/(-1+x)/(2+x),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(-1+x)-1/3*ln(2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{(-1+x)(2+x)}\,dx = -\frac{1}{3}\log(x+2) + \frac{1}{3}\log(x-1)$$

```
[In] integrate(1/(-1+x)/(2+x),x, algorithm="fricas")
[Out] -1/3*log(x + 2) + 1/3*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1}{(-1+x)(2+x)}\,dx = \frac{\log(x-1)}{3} - \frac{\log(x+2)}{3}$$

```
[In] integrate(1/(-1+x)/(2+x),x)
[Out] log(x - 1)/3 - log(x + 2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{(-1+x)(2+x)}\,dx = -\frac{1}{3}\,\log(x+2) + \frac{1}{3}\,\log(x-1)$$

```
[In] integrate(1/(-1+x)/(2+x),x, algorithm="maxima")
[Out] -1/3*log(x + 2) + 1/3*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(-1+x)(2+x)}\,dx = -\frac{1}{3}\,\log(|x+2|) + \frac{1}{3}\,\log(|x-1|)$$

```
[In] integrate(1/(-1+x)/(2+x),x, algorithm="giac")
[Out] -1/3*log(abs(x + 2)) + 1/3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1}{(-1+x)(2+x)}\,dx = \frac{\ln\left(\frac{x-1}{x+2}\right)}{3}$$

```
[In] int(1/((x - 1)*(x + 2)),x)
[Out] log((x - 1)/(x + 2))/3
```

3.163 $\int \frac{7}{-12+5x+2x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 19

$$\int \frac{7}{-12+5x+2x^2} \, dx = \frac{7}{11}\log(3-2x) - \frac{7}{11}\log(4+x)$$

```
[Out] 7/11*ln(3-2*x)-7/11*ln(4+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {12, 630, 31}

$$\int \frac{7}{-12+5x+2x^2} \, dx = \frac{7}{11}\log(3-2x) - \frac{7}{11}\log(x+4)$$

```
[In] Int[7/(-12 + 5*x + 2*x^2),x]
```

```
[Out] (7*Log[3 - 2*x])/11 - (7*Log[4 + x])/11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 7\int \frac{1}{-12+5x+2x^2}\,dx\\ &= \frac{14}{11}\int \frac{1}{-3+2x}\,dx - \frac{14}{11}\int \frac{1}{8+2x}\,dx\\ &= \frac{7}{11}\log(3-2x) - \frac{7}{11}\log(4+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{7}{-12+5x+2x^2}\,dx = 7\left(\frac{1}{11}\log(3-2x) - \frac{1}{11}\log(4+x)\right)$$

```
[In] Integrate[7/(-12 + 5*x + 2*x^2),x]
[Out] 7*(Log[3 - 2*x]/11 - Log[4 + x]/11)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$-\frac{7\ln(4+x)}{11}+\frac{7\ln\left(x-\frac{3}{2}\right)}{11}$	14
default	$-\frac{7\ln(4+x)}{11}+\frac{7\ln(2x-3)}{11}$	16
norman	$-\frac{7\ln(4+x)}{11}+\frac{7\ln(2x-3)}{11}$	16
risch	$-\frac{7\ln(4+x)}{11}+\frac{7\ln(2x-3)}{11}$	16

```
[In] int(7/(2*x^2+5*x-12),x,method=_RETURNVERBOSE)
[Out] -7/11*ln(4+x)+7/11*ln(x-3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{7}{-12+5x+2x^2}\,dx=\frac{7}{11}\,\log\left(2\,x-3\right)-\frac{7}{11}\,\log\left(x+4\right)$$

```
[In] integrate(7/(2*x^2+5*x-12),x, algorithm="fricas")
[Out] 7/11*log(2*x - 3) - 7/11*log(x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{7}{-12+5x+2x^2}\,dx=\frac{7\log\left(x-\frac{3}{2}\right)}{11}-\frac{7\log\left(x+4\right)}{11}$$

```
[In] integrate(7/(2*x**2+5*x-12),x)
[Out] 7*log(x - 3/2)/11 - 7*log(x + 4)/11
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{7}{-12+5x+2x^2}\,dx=\frac{7}{11}\,\log\left(2\,x-3\right)-\frac{7}{11}\,\log\left(x+4\right)$$

```
[In] integrate(7/(2*x^2+5*x-12),x, algorithm="maxima")
[Out] 7/11*log(2*x - 3) - 7/11*log(x + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{7}{-12+5x+2x^2}\,dx=\frac{7}{11}\,\log\left(|2\,x-3|\right)-\frac{7}{11}\,\log\left(|x+4|\right)$$

```
[In] integrate(7/(2*x^2+5*x-12),x, algorithm="giac")
[Out] 7/11*log(abs(2*x - 3)) - 7/11*log(abs(x + 4))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{7}{-12+5x+2x^2}\,dx = -\frac{14\,\mathrm{atanh}\left(\frac{4\,x}{11}+\frac{5}{11}\right)}{11}$$

```
[In] int(7/(5*x + 2*x^2 - 12),x)
[Out] -(14*atanh((4*x)/11 + 5/11))/11
```

3.164 $\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = -\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(3+2x)$$

```
[Out] -9/32/(1-2*x)+41/128*ln(1-2*x)-25/128*ln(3+2*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {907}

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = -\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

```
[In] Int[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
```

```
[Out] -9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{9}{16(-1+2x)^2}+\frac{41}{64(-1+2x)}-\frac{25}{64(3+2x)}\right)dx\\ &= -\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(3+2x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = \frac{9}{32(-1+2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(3+2x)$$

```
[In] Integrate[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
[Out] 9/(32*(-1 + 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{9}{64\left(x-\frac{1}{2}\right)}-\frac{25\ln(3+2x)}{128}+\frac{41\ln(2x-1)}{128}$	25
default	$-\frac{25\ln(3+2x)}{128}+\frac{9}{32(2x-1)}+\frac{41\ln(2x-1)}{128}$	27
norman	$\frac{9x}{16(2x-1)}-\frac{25\ln(3+2x)}{128}+\frac{41\ln(2x-1)}{128}$	28
parallelrisch	$\frac{82\ln\left(x-\frac{1}{2}\right)x-50\ln\left(\frac{3}{2}+x\right)x-41\ln\left(x-\frac{1}{2}\right)+25\ln\left(\frac{3}{2}+x\right)+72x}{256x-128}$	40

```
[In] int((x^2+3*x-4)/(2*x-1)^2/(3+2*x),x,method=_RETURNVERBOSE)
[Out] 9/64/(x-1/2)-25/128*ln(3+2*x)+41/128*ln(2*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = -\frac{25\,(2\,x-1)\log\left(2\,x+3\right)-41\,(2\,x-1)\log\left(2\,x-1\right)-36}{128\,(2\,x-1)}$$

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="fricas")
[Out] -1/128*(25*(2*x - 1)*log(2*x + 3) - 41*(2*x - 1)*log(2*x - 1) - 36)/(2*x -
1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = \frac{41\log\left(x-\frac{1}{2}\right)}{128} - \frac{25\log\left(x+\frac{3}{2}\right)}{128} + \frac{9}{64x-32}$$

```
[In] integrate((x**2+3*x-4)/(-1+2*x)**2/(3+2*x),x)
[Out] 41*log(x - 1/2)/128 - 25*log(x + 3/2)/128 + 9/(64*x - 32)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = \frac{9}{32\,(2\,x-1)} - \frac{25}{128}\,\log\left(2\,x+3\right) + \frac{41}{128}\,\log\left(2\,x-1\right)$$

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="maxima")
[Out] 9/32/(2*x - 1) - 25/128*log(2*x + 3) + 41/128*log(2*x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = \frac{9}{32\,(2\,x-1)} - \frac{1}{8}\,\log\left(\frac{|2\,x-1|}{2\,(2\,x-1)^2}\right) - \frac{25}{128}\,\log\left(\left|-\frac{4}{2\,x-1}-1\right|\right)$$

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="giac")
[Out] 9/32/(2*x - 1) - 1/8*log(1/2*abs(2*x - 1)/(2*x - 1)^2) - 25/128*log(abs(-4/
(2*x - 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = \frac{41\,\ln\left(x-\frac{1}{2}\right)}{128} - \frac{25\,\ln\left(x+\frac{3}{2}\right)}{128} + \frac{9}{64\,\left(x-\frac{1}{2}\right)}$$

```
[In] int((3*x + x^2 - 4)/((2*x - 1)^2*(2*x + 3)),x)
[Out] (41*log(x - 1/2))/128 - (25*log(x + 3/2))/128 + 9/(64*(x - 1/2))
```

3.165 $\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = -\frac{12}{1375(3+5x)^2}+\frac{201}{15125(3+5x)}+\frac{20\log(6-x)}{3993}+\frac{1493\log(3+5x)}{499125}$$

```
[Out] -12/1375/(3+5*x)^2+201/15125/(3+5*x)+20/3993*ln(6-x)+1493/499125*ln(3+5*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1607, 153}

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{201}{15125(5x+3)}-\frac{12}{1375(5x+3)^2}+\frac{20\log(6-x)}{3993}+\frac{1493\log(5x+3)}{499125}$$

```
[In] Int[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] -12/(1375*(3 + 5*x)^2) + 201/(15125*(3 + 5*x)) + (20*Log[6 - x])/3993 + (14
93*Log[3 + 5*x])/499125
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(-1+x)x^2}{(-6+x)(3+5x)^3}\,dx\\ &= \int \left(\frac{20}{3993(-6+x)}+\frac{24}{275(3+5x)^3}-\frac{201}{3025(3+5x)^2}+\frac{1493}{99825(3+5x)}\right)dx\\ &= -\frac{12}{1375(3+5x)^2}+\frac{201}{15125(3+5x)}+\frac{20\log(6-x)}{3993}+\frac{1493\log(3+5x)}{499125}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{\frac{99(157+335x)}{(3+5x)^2}+2500\log(-6+x)+1493\log(3+5x)}{499125}$$

```
[In] Integrate[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
[Out] ((99*(157 + 335*x))/(3 + 5*x)^2 + 2500*Log[-6 + x] + 1493*Log[3 + 5*x])/499
125
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

method	result
risch	$\frac{\frac{201x}{3025}+\frac{471}{15125}}{(3+5x)^2}+\frac{20\ln(-6+x)}{3993}+\frac{1493\ln(3+5x)}{499125}$
norman	$\frac{-\frac{113}{3025}x-\frac{157}{1815}x^2}{(3+5x)^2}+\frac{20\ln(-6+x)}{3993}+\frac{1493\ln(3+5x)}{499125}$
default	$\frac{20\ln(-6+x)}{3993}-\frac{12}{1375(3+5x)^2}+\frac{201}{15125(3+5x)}+\frac{1493\ln(3+5x)}{499125}$
parallelrisch	$\frac{187500\ln(-6+x)x^2+111975\ln\left(x+\frac{3}{5}\right)x^2+225000\ln(-6+x)x+134370\ln\left(x+\frac{3}{5}\right)x-129525x^2+67500\ln(-6+x)+40311\ln\left(x+\frac{3}{5}\right.}{1497375(3+5x)^2}$

```
[In] int((x^3-x^2)/(-6+x)/(3+5*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 25*(201/75625*x+471/378125)/(3+5*x)^2+20/3993*ln(-6+x)+1493/499125*ln(3+5*x
)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{1493\,(25\,x^2+30\,x+9)\log(5\,x+3)+2500\,(25\,x^2+30\,x+9)\log(x-6)+33165\,x+15543}{499125\,(25\,x^2+30\,x+9)}$$

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="fricas")
```

```
[Out] 1/499125*(1493*(25*x^2 + 30*x + 9)*log(5*x + 3) + 2500*(25*x^2 + 30*x + 9)*
log(x - 6) + 33165*x + 15543)/(25*x^2 + 30*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{1005x+471}{378125x^2+453750x+136125} + \frac{20\log(x-6)}{3993} + \frac{1493\log\left(x+\frac{3}{5}\right)}{499125}$$

```
[In] integrate((x**3-x**2)/(-6+x)/(3+5*x)**3,x)
```

```
[Out] (1005*x + 471)/(378125*x**2 + 453750*x + 136125) + 20*log(x - 6)/3993 + 149
3*log(x + 3/5)/499125
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{3\,(335\,x+157)}{15125\,(25\,x^2+30\,x+9)} + \frac{1493}{499125}\,\log(5\,x+3) + \frac{20}{3993}\,\log(x-6)$$

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="maxima")
```

```
[Out] 3/15125*(335*x + 157)/(25*x^2 + 30*x + 9) + 1493/499125*log(5*x + 3) + 20/3
993*log(x - 6)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{3\,(335\,x+157)}{15125\,(5\,x+3)^2}+\frac{1493}{499125}\,\log\left(|5\,x+3|\right)+\frac{20}{3993}\,\log\left(|x-6|\right)$$

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="giac")
```

```
[Out] 3/15125*(335*x + 157)/(5*x + 3)^2 + 1493/499125*log(abs(5*x + 3)) + 20/3993
*log(abs(x - 6))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{20\,\ln\left(x-6\right)}{3993}+\frac{1493\,\ln\left(x+\frac{3}{5}\right)}{499125}+\frac{\frac{201\,x}{75625}+\frac{471}{378125}}{x^2+\frac{6\,x}{5}+\frac{9}{25}}$$

```
[In] int(-(x^2 - x^3)/((5*x + 3)^3*(x - 6)),x)
```

```
[Out] (20*log(x - 6))/3993 + (1493*log(x + 3/5))/499125 + ((201*x)/75625 + 471/37
8125)/((6*x)/5 + x^2 + 9/25)
```

3.166 $\int \frac{1}{-x^3+x^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{1}{-x^3+x^4} dx = \frac{1}{2x^2} + \frac{1}{x} + \log(1-x) - \log(x)$$

```
[Out] 1/2/x^2+1/x+ln(1-x)-ln(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 46}

$$\int \frac{1}{-x^3+x^4} dx = \frac{1}{2x^2} + \frac{1}{x} + \log(1-x) - \log(x)$$

```
[In] Int[(-x^3 + x^4)^(-1),x]
```

```
[Out] 1/(2*x^2) + x^(-1) + Log[1 - x] - Log[x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{(-1+x)x^3}\,dx \\
&= \int \left(\frac{1}{-1+x}-\frac{1}{x^3}-\frac{1}{x^2}-\frac{1}{x}\right)dx \\
&= \frac{1}{2x^2}+\frac{1}{x}+\log(1-x)-\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x^3+x^4}\,dx=\frac{1}{2x^2}+\frac{1}{x}+\log(1-x)-\log(x)$$

```
[In] Integrate[(-x^3 + x^4)^(-1),x]
[Out] 1/(2*x^2) + x^(-1) + Log[1 - x] - Log[x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
norman	$\frac{x+\frac{1}{2}}{x^2}-\ln(x)+\ln(-1+x)$	17
risch	$\frac{x+\frac{1}{2}}{x^2}-\ln(x)+\ln(-1+x)$	17
default	$\ln(-1+x)+\frac{1}{2x^2}+\frac{1}{x}-\ln(x)$	18
meijerg	$\frac{1}{2x^2}+\frac{1}{x}-\ln(x)-i\pi+\ln(1-x)$	24
parallelrisch	$-\frac{2x^2\ln(x)-2\ln(-1+x)x^2-1-2x}{2x^2}$	27

```
[In] int(1/(x^4-x^3),x,method=_RETURNVERBOSE)
[Out] (x+1/2)/x^2-ln(x)+ln(-1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{1}{-x^3+x^4}\,dx = \frac{2\,x^2\log\left(x-1\right)-2\,x^2\log\left(x\right)+2\,x+1}{2\,x^2}$$

```
[In] integrate(1/(x^4-x^3),x, algorithm="fricas")
[Out] 1/2*(2*x^2*log(x - 1) - 2*x^2*log(x) + 2*x + 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{-x^3+x^4}\,dx = -\log\left(x\right)+\log\left(x-1\right)+\frac{2x+1}{2x^2}$$

```
[In] integrate(1/(x**4-x**3),x)
[Out] -log(x) + log(x - 1) + (2*x + 1)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{-x^3+x^4}\,dx = \frac{2\,x+1}{2\,x^2}+\log\left(x-1\right)-\log\left(x\right)$$

```
[In] integrate(1/(x^4-x^3),x, algorithm="maxima")
[Out] 1/2*(2*x + 1)/x^2 + log(x - 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x^3+x^4}\,dx = \frac{2\,x+1}{2\,x^2}+\log\left(|x-1|\right)-\log\left(|x|\right)$$

```
[In] integrate(1/(x^4-x^3),x, algorithm="giac")
[Out] 1/2*(2*x + 1)/x^2 + log(abs(x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1}{-x^3+x^4}\,dx = \frac{x+\frac{1}{2}}{x^2} - 2\,\operatorname{atanh}(2\,x-1)$$

```
[In] int(-1/(x^3 - x^4),x)
```

```
[Out] (x + 1/2)/x^2 - 2*atanh(2*x - 1)
```

3.167 $\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 25

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} \, dx = x+\frac{x^2}{2}-\log(x)+\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] x+1/2*x^2-ln(x)+1/2*ln(-x^2+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1607, 1816, 266}

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} \, dx = \frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

```
[In] Int[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
```

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1-x-x^2+x^3+x^4}{x(-1+x^2)}\,dx\\ &= \int \left(1-\frac{1}{x}+x+\frac{x}{-1+x^2}\right)dx\\ &= x+\frac{x^2}{2}-\log(x)+\int \frac{x}{-1+x^2}\,dx\\ &= x+\frac{x^2}{2}-\log(x)+\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx = x+\frac{x^2}{2}-\log(x)+\frac{1}{2}\log\left(1-x^2\right)$$

```
[In] Integrate[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
risch	$x+\frac{x^2}{2}-\ln(x)+\frac{\ln(x^2-1)}{2}$	20
default	$x+\frac{x^2}{2}+\frac{\ln(-1+x)}{2}-\ln(x)+\frac{\ln(1+x)}{2}$	24
norman	$x+\frac{x^2}{2}+\frac{\ln(-1+x)}{2}-\ln(x)+\frac{\ln(1+x)}{2}$	24
parallelrisch	$x+\frac{x^2}{2}+\frac{\ln(-1+x)}{2}-\ln(x)+\frac{\ln(1+x)}{2}$	24
meijerg	$-\ln(x)-\frac{i\pi}{2}+\frac{\ln(-x^2+1)}{2}+\frac{x^2}{2}-\frac{i(2ix-2i\,\text{arctanh}(x))}{2}+\text{arctanh}(x)$	40

```
[In] int((x^4+x^3-x^2-x+1)/(x^3-x),x,method=_RETURNVERBOSE)
[Out] x+1/2*x^2-ln(x)+1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} \, dx = \frac{1}{2} x^2 + x + \frac{1}{2} \log \left(x^2-1\right) - \log \left(x\right)$$

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="fricas")
[Out] 1/2*x^2 + x + 1/2*log(x^2 - 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} \, dx = \frac{x^2}{2} + x - \log \left(x\right) + \frac{\log \left(x^2-1\right)}{2}$$

```
[In] integrate((x**4+x**3-x**2-x+1)/(x**3-x),x)
[Out] x**2/2 + x - log(x) + log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} \, dx = \frac{1}{2} x^2 + x + \frac{1}{2} \log \left(x+1\right) + \frac{1}{2} \log \left(x-1\right) - \log \left(x\right)$$

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="maxima")
[Out] 1/2*x^2 + x + 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} \, dx = \frac{1}{2} x^2 + x + \frac{1}{2} \log \left(|x+1|\right) + \frac{1}{2} \log \left(|x-1|\right) - \log \left(|x|\right)$$

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="giac")
[Out] 1/2*x^2 + x + 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx = x+\frac{\ln\left(x^2-1\right)}{2}-\ln\left(x\right)+\frac{x^2}{2}$$

```
[In] int(-(x^3 - x^2 - x + x^4 + 1)/(x - x^3),x)
[Out] x + log(x^2 - 1)/2 - log(x) + x^2/2
```

3.168 $\int \frac{-2+x^2}{x(2+x^2)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{-2+x^2}{x(2+x^2)} \, dx = -\log(x) + \log(2+x^2)$$

```
[Out] -ln(x)+ln(x^2+2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {457, 78}

$$\int \frac{-2+x^2}{x(2+x^2)} \, dx = \log(x^2+2) - \log(x)$$

```
[In] Int[(-2 + x^2)/(x*(2 + x^2)),x]
```

```
[Out] -Log[x] + Log[2 + x^2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{-2+x}{x(2+x)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x}+\frac{2}{2+x}\right)\,dx, x, x^2\right)\\ &= -\log(x)+\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-2+x^2}{x\left(2+x^2\right)}\,dx = -\log(x)+\log\left(2+x^2\right)$$

```
[In] Integrate[(-2 + x^2)/(x*(2 + x^2)),x]
[Out] -Log[x] + Log[2 + x^2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$-\ln(x)+\ln(x^2+2)$	12
norman	$-\ln(x)+\ln(x^2+2)$	12
risch	$-\ln(x)+\ln(x^2+2)$	12
parallelrisch	$-\ln(x)+\ln(x^2+2)$	12
meijerg	$-\ln(x)+\frac{\ln(2)}{2}+\ln\left(1+\frac{x^2}{2}\right)$	18

```
[In] int((x^2-2)/x/(x^2+2),x,method=_RETURNVERBOSE)
[Out] -ln(x)+ln(x^2+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-2+x^2}{x(2+x^2)}\,dx = \log\left(x^2+2\right) - \log\left(x\right)$$

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="fricas")
[Out] log(x^2 + 2) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{-2+x^2}{x(2+x^2)}\,dx = -\log\left(x\right) + \log\left(x^2+2\right)$$

```
[In] integrate((x**2-2)/x/(x**2+2),x)
[Out] -log(x) + log(x**2 + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{-2+x^2}{x(2+x^2)}\,dx = \log\left(x^2+2\right) - \frac{1}{2}\log\left(x^2\right)$$

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="maxima")
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{-2+x^2}{x(2+x^2)}\,dx = \log\left(x^2+2\right) - \frac{1}{2}\log\left(x^2\right)$$

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="giac")
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-2+x^2}{x\,(2+x^2)}\,dx = \ln\left(x^2+2\right) - \ln\left(x\right)$$

```
[In] int((x^2 - 2)/(x*(x^2 + 2)),x)
[Out] log(x^2 + 2) - log(x)
```

3.169 $\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)} dx$

Optimal result

Integrand size = 25, antiderivative size = 36

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)} dx = 6\arctan(x) - 5\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{2}\log(1+x^2) + \log(2+x^2)$$

```
[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6857, 649, 209, 266}

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)} dx = 6\arctan(x) - 5\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{2}\log(x^2+1) + \log(x^2+2)$$

```
[In] Int[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{6-x}{1+x^2}+\frac{2(-5+x)}{2+x^2}\right) dx\\
&= 2\int \frac{-5+x}{2+x^2}\,dx+\int \frac{6-x}{1+x^2}\,dx\\
&= 2\int \frac{x}{2+x^2}\,dx+6\int \frac{1}{1+x^2}\,dx-10\int \frac{1}{2+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= 6\arctan(x)-5\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(1+x^2\right)+\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx=6\arctan(x)-5\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(1+x^2\right)+\log\left(2+x^2\right)$$

```
[In] Integrate[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$6\arctan\left(x\right)-\frac{\ln\left(x^2+1\right)}{2}+\ln\left(x^2+2\right)-5\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	32
risch	$6\arctan\left(x\right)-\frac{\ln\left(x^2+1\right)}{2}+\ln\left(x^2+2\right)-5\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	32

```
[In] int((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x,method=_RETURNVERBOSE)
[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = -5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="fricas")
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = -\frac{\log(x^2+1)}{2}+\log\left(x^2+2\right)+6\,\mathrm{atan}(x)-5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

```
[In] integrate((x**3-4*x**2+2)/(x**2+1)/(x**2+2),x)
[Out] -log(x**2 + 1)/2 + log(x**2 + 2) + 6*atan(x) - 5*sqrt(2)*atan(sqrt(2)*x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = -5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="maxima")
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = -5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = \ln(x-\mathrm{i})\left(-\frac{1}{2}-3\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(-\frac{1}{2}+3\mathrm{i}\right) + \ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(1+\frac{\sqrt{2}\,5\mathrm{i}}{2}\right)-\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(-1+\frac{\sqrt{2}\,5\mathrm{i}}{2}\right)$$

```
[In] int((x^3 - 4*x^2 + 2)/((x^2 + 1)*(x^2 + 2)),x)
```

```
[Out] log(x - 2^(1/2)*1i)*((2^(1/2)*5i)/2 + 1) - log(x + 1i)*(1/2 - 3i) - log(x -
 1i)*(1/2 + 3i) - log(x + 2^(1/2)*1i)*((2^(1/2)*5i)/2 - 1)
```

3.170 $\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 29

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} \, dx = -\frac{13x}{24(4+x^2)} + \frac{25}{144} \arctan\left(\frac{x}{2}\right) + \frac{\arctan(x)}{9}$$

```
[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6857, 209, 205}

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} \, dx = \frac{25}{144} \arctan\left(\frac{x}{2}\right) + \frac{\arctan(x)}{9} - \frac{13x}{24(x^2+4)}$$

```
[In] Int[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{9(1+x^2)} - \frac{13}{3(4+x^2)^2} + \frac{8}{9(4+x^2)}\right) dx \\
&= \frac{1}{9}\int \frac{1}{1+x^2}\,dx + \frac{8}{9}\int \frac{1}{4+x^2}\,dx - \frac{13}{3}\int \frac{1}{(4+x^2)^2}\,dx \\
&= -\frac{13x}{24(4+x^2)} + \frac{4}{9}\arctan\left(\frac{x}{2}\right) + \frac{\arctan(x)}{9} - \frac{13}{24}\int \frac{1}{4+x^2}\,dx \\
&= -\frac{13x}{24(4+x^2)} + \frac{25}{144}\arctan\left(\frac{x}{2}\right) + \frac{\arctan(x)}{9}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx = -\frac{13x}{24(4+x^2)} + \frac{25}{144}\arctan\left(\frac{x}{2}\right) + \frac{\arctan(x)}{9}$$

```
[In] Integrate[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result
default	$-\frac{13x}{24(x^2+4)}+\frac{25\arctan\left(\frac{x}{2}\right)}{144}+\frac{\arctan(x)}{9}$
risch	$-\frac{13x}{24(x^2+4)}+\frac{25\arctan\left(\frac{x}{2}\right)}{144}+\frac{\arctan(x)}{9}$
parallelrisch	$-\frac{25i\ln(x-2i)x^2+16i\ln(x-i)x^2-16i\ln(x+i)x^2-25i\ln(x+2i)x^2+100i\ln(x-2i)+64i\ln(x-i)-64i\ln(x+i)-100i\ln(x+2i)+156}{288(x^2+4)}$

```
[In] int((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x,method=_RETURNVERBOSE)
[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx = \frac{25\,(x^2+4)\arctan\left(\frac{1}{2}\,x\right)+16\,(x^2+4)\arctan\left(x\right)-78\,x}{144\,(x^2+4)}$$

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="fricas")
[Out] 1/144*(25*(x^2 + 4)*arctan(1/2*x) + 16*(x^2 + 4)*arctan(x) - 78*x)/(x^2 + 4
)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx = -\frac{13x}{24x^2+96}+\frac{25\,\mathrm{atan}\left(\frac{x}{2}\right)}{144}+\frac{\mathrm{atan}\left(x\right)}{9}$$

```
[In] integrate((x**4+x**2+1)/(x**2+1)/(x**2+4)**2,x)
[Out] -13*x/(24*x**2 + 96) + 25*atan(x/2)/144 + atan(x)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx = -\frac{13\,x}{24\,(x^2+4)}+\frac{25}{144}\,\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{9}\,\arctan\left(x\right)$$

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="maxima")
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx = -\frac{13\,x}{24\,(x^2+4)} + \frac{25}{144}\arctan\left(\frac{1}{2}\,x\right) + \frac{1}{9}\arctan\left(x\right)$$

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="giac")
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx = \frac{25\,\mathrm{atan}\left(\frac{x}{2}\right)}{144} + \frac{\mathrm{atan}(x)}{9} - \frac{13\,x}{24\,\left(x^2+4\right)}$$

```
[In] int((x^2 + x^4 + 1)/((x^2 + 1)*(x^2 + 4)^2),x)
[Out] (25*atan(x/2))/144 + atan(x)/9 - (13*x)/(24*(x^2 + 4))
```

3.171 $\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx=-\frac{79}{273(5+x)}+\frac{451\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2793\sqrt{3}}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}-\frac{481\log\left(1+x+x^2\right)}{5586}$$

```
[Out] -79/273/(5+x)+200/3211*ln(3-2*x)+2731/24843*ln(5+x)-481/5586*ln(x^2+x+1)+45
1/8379*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6860, 648, 632, 210, 642}

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx=\frac{451\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}$$

```
[In] Int[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] -79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]) + (200*L
og[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[1 + x + x^2])/5586
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{79}{273(5+x)^2}+\frac{2731}{24843(5+x)}+\frac{400}{3211(-3+2x)}+\frac{-15-481x}{2793\left(1+x+x^2\right)}\right)dx\\
&= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}+\frac{\int \frac{-15-481x}{1+x+x^2}\,dx}{2793}\\
&= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}+\frac{451\int \frac{1}{1+x+x^2}\,dx}{5586}-\frac{481\int \frac{1+2x}{1+x+x^2}\,dx}{5586}\\
&= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}\\
&\quad -\frac{481\log\left(1+x+x^2\right)}{5586}-\frac{451\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)}{2793}\\
&= -\frac{79}{273(5+x)}+\frac{451\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2793\sqrt{3}}+\frac{200\log(3-2x)}{3211}\\
&\quad +\frac{2731\log(5+x)}{24843}-\frac{481\log\left(1+x+x^2\right)}{5586}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\,(1+x+x^2)}\,dx$$
$$=\frac{-\frac{819546}{5+x}+152438\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+176400\log(3-2x)+311334\log(5+x)-243867\log\left(1+x+x^2\right)}{2832102}$$

```
[In] Integrate[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] (-819546/(5 + x) + 152438*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 176400*Log[3
- 2*x] + 311334*Log[5 + x] - 243867*Log[1 + x + x^2])/2832102
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

method	result	si
default	$-\frac{79}{273(5+x)}+\frac{2731\ln(5+x)}{24843}-\frac{481\ln(x^2+x+1)}{5586}+\frac{451\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{8379}+\frac{200\ln(2x-3)}{3211}$	48
risch	$-\frac{79}{273(5+x)}+\frac{2731\ln(5+x)}{24843}-\frac{481\ln(203401x^2+203401x+203401)}{5586}+\frac{451\sqrt{3}\arctan\left(\frac{2\left(451x+\frac{451}{2}\right)\sqrt{3}}{1353}\right)}{8379}+\frac{200\ln(2x-3)}{3211}$	52

```
[In] int((1+16*x)/(5+x)^2/(2*x-3)/(x^2+x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -79/273/(5+x)+2731/24843*ln(5+x)-481/5586*ln(x^2+x+1)+451/8379*arctan(1/3*(
1+2*x)*3^(1/2))*3^(1/2)+200/3211*ln(2*x-3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\,(1+x+x^2)}\,dx$$
$$=\frac{152438\sqrt{3}(x+5)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-243867(x+5)\log\left(x^2+x+1\right)+176400(x+5)\log(2x-3)}{2832102(x+5)}$$

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2832102*(152438*sqrt(3)*(x + 5)*arctan(1/3*sqrt(3)*(2*x + 1)) - 243867*(x
 + 5)*log(x^2 + x + 1) + 176400*(x + 5)*log(2*x - 3) + 311334*(x + 5)*log(x
 + 5) - 819546)/(x + 5)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

$$\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)}\,dx = \frac{200\log\left(x-\frac{3}{2}\right)}{3211} + \frac{2731\log(x+5)}{24843} - \frac{481\log(x^2+x+1)}{5586} + \frac{451\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{8379} - \frac{79}{273x+1365}$$

```
[In] integrate((1+16*x)/(5+x)**2/(-3+2*x)/(x**2+x+1),x)
[Out] 200*log(x - 3/2)/3211 + 2731*log(x + 5)/24843 - 481*log(x**2 + x + 1)/5586
+ 451*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/8379 - 79/(273*x + 1365)
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)}\,dx = \frac{451}{8379}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{79}{273(x+5)} - \frac{481}{5586}\log\left(x^2+x+1\right) + \frac{200}{3211}\log(2x-3) + \frac{2731}{24843}\log(x+5)$$

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="maxima")
[Out] 451/8379*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 79/273/(x + 5) - 481/5586*
log(x^2 + x + 1) + 200/3211*log(2*x - 3) + 2731/24843*log(x + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)}\,dx = \frac{451}{8379}\sqrt{3}\arctan\left(-\sqrt{3}\left(\frac{14}{x+5}-3\right)\right) - \frac{79}{273(x+5)} - \frac{481}{5586}\log\left(-\frac{9}{x+5}+\frac{21}{(x+5)^2}+1\right) + \frac{200}{3211}\log\left(\left|-\frac{13}{x+5}+2\right|\right)$$

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 451/8379*sqrt(3)*arctan(-sqrt(3)*(14/(x + 5) - 3)) - 79/273/(x + 5) - 481/5
586*log(-9/(x + 5) + 21/(x + 5)^2 + 1) + 200/3211*log(abs(-13/(x + 5) + 2))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\,(1+x+x^2)}\,dx = \frac{200\ \ln\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\ \ln\left(x+5\right)}{24843}-\frac{79}{273\ \left(x+5\right)} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{481}{5586}+\frac{\sqrt{3}\,451\mathrm{i}}{16758}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{481}{5586}+\frac{\sqrt{3}\,451\mathrm{i}}{16758}\right)$$

```
[In] int((16*x + 1)/((2*x - 3)*(x + 5)^2*(x + x^2 + 1)),x)
```

```
[Out] (200*log(x - 3/2))/3211 + (2731*log(x + 5))/24843 - 79/(273*(x + 5)) - log(
x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 + 481/5586) + log(x + (3^(1
/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 - 481/5586)
```

3.172 $\int \frac{x^4}{(9+x^2)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{x^4}{(9+x^2)^3} dx = -\frac{x^3}{4(9+x^2)^2} - \frac{3x}{8(9+x^2)} + \frac{1}{8}\arctan\left(\frac{x}{3}\right)$$

```
[Out] -1/4*x^3/(x^2+9)^2-3/8*x/(x^2+9)+1/8*arctan(1/3*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {294, 209}

$$\int \frac{x^4}{(9+x^2)^3} dx = \frac{1}{8}\arctan\left(\frac{x}{3}\right) - \frac{3x}{8(x^2+9)} - \frac{x^3}{4(x^2+9)^2}$$

```
[In] Int[x^4/(9 + x^2)^3,x]
```

```
[Out] -1/4*x^3/(9 + x^2)^2 - (3*x)/(8*(9 + x^2)) + ArcTan[x/3]/8
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{x^3}{4(9+x^2)^2} + \frac{3}{4}\int \frac{x^2}{(9+x^2)^2}\,dx \\ &= -\frac{x^3}{4(9+x^2)^2} - \frac{3x}{8(9+x^2)} + \frac{3}{8}\int \frac{1}{9+x^2}\,dx \\ &= -\frac{x^3}{4(9+x^2)^2} - \frac{3x}{8(9+x^2)} + \frac{1}{8}\arctan\left(\frac{x}{3}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{(9+x^2)^3}\,dx = \frac{1}{8}\left(-\frac{x(27+5x^2)}{(9+x^2)^2} + \arctan\left(\frac{x}{3}\right)\right)$$

```
[In] Integrate[x^4/(9 + x^2)^3,x]
[Out] (-((x*(27 + 5*x^2))/(9 + x^2)^2) + ArcTan[x/3])/8
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

method	result	si
default	$\frac{-\frac{5}{8}x^3-\frac{27}{8}x}{(x^2+9)^2}+\frac{\arctan\left(\frac{x}{3}\right)}{8}$	25
risch	$\frac{-\frac{5}{8}x^3-\frac{27}{8}x}{(x^2+9)^2}+\frac{\arctan\left(\frac{x}{3}\right)}{8}$	25
meijerg	$-\frac{x\left(\frac{25x^2}{9}+15\right)}{360\left(\frac{x^2}{9}+1\right)^2}+\frac{\arctan\left(\frac{x}{3}\right)}{8}$	27
parallelrisch	$-\frac{81i\ln(x-3i)x^4-81i\ln(x+3i)x^4+1458i\ln(x-3i)x^2-1458i\ln(x+3i)x^2+810x^3+6561i\ln(x-3i)-6561i\ln(x+3i)+4374x}{1296(x^2+9)^2}$	79

```
[In] int(x^4/(x^2+9)^3,x,method=_RETURNVERBOSE)
[Out] (-5/8*x^3-27/8*x)/(x^2+9)^2+1/8*arctan(1/3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{x^4}{(9+x^2)^3}\,dx = -\frac{5\,x^3 - (x^4+18\,x^2+81)\arctan\left(\frac{1}{3}\,x\right) + 27\,x}{8\,(x^4+18\,x^2+81)}$$

```
[In] integrate(x^4/(x^2+9)^3,x, algorithm="fricas")
[Out] -1/8*(5*x^3 - (x^4 + 18*x^2 + 81)*arctan(1/3*x) + 27*x)/(x^4 + 18*x^2 + 81)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{x^4}{(9+x^2)^3}\,dx = \frac{-5x^3-27x}{8x^4+144x^2+648} + \frac{\operatorname{atan}\left(\frac{x}{3}\right)}{8}$$

```
[In] integrate(x**4/(x**2+9)**3,x)
[Out] (-5*x**3 - 27*x)/(8*x**4 + 144*x**2 + 648) + atan(x/3)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{(9+x^2)^3}\,dx = -\frac{5\,x^3+27\,x}{8\,(x^4+18\,x^2+81)} + \frac{1}{8}\arctan\left(\frac{1}{3}\,x\right)$$

```
[In] integrate(x^4/(x^2+9)^3,x, algorithm="maxima")
[Out] -1/8*(5*x^3 + 27*x)/(x^4 + 18*x^2 + 81) + 1/8*arctan(1/3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{(9+x^2)^3}\,dx = -\frac{5\,x^3+27\,x}{8\,(x^2+9)^2} + \frac{1}{8}\arctan\left(\frac{1}{3}\,x\right)$$

```
[In] integrate(x^4/(x^2+9)^3,x, algorithm="giac")
[Out] -1/8*(5*x^3 + 27*x)/(x^2 + 9)^2 + 1/8*arctan(1/3*x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{(9+x^2)^3}\,dx = \frac{\operatorname{atan}\left(\frac{x}{3}\right)}{8} - \frac{\frac{5\,x^3}{8}+\frac{27\,x}{8}}{x^4+18\,x^2+81}$$

```
[In] int(x^4/(x^2 + 9)^3,x)
[Out] atan(x/3)/8 - ((27*x)/8 + (5*x^3)/8)/(18*x^2 + x^4 + 81)
```

3.173 $\int \frac{19x}{(-1+x)^3(3+5x+4x^2)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 97

$$\int \frac{19x}{(-1+x)^3(3+5x+4x^2)^2} dx = -\frac{399}{736(1-x)^2} - \frac{1843}{4416(1-x)} + \frac{19(39+44x)}{276(1-x)^2(3+5x+4x^2)} + \frac{114437\arctan\left(\frac{5+8x}{\sqrt{23}}\right)}{52992\sqrt{23}} + \frac{209\log(1-x)}{2304} - \frac{209\log(3+5x+4x^2)}{4608}$$

```
[Out] -399/736/(1-x)^2-1843/4416/(1-x)+19/276*(39+44*x)/(1-x)^2/(4*x^2+5*x+3)+209
/2304*ln(1-x)-209/4608*ln(4*x^2+5*x+3)+114437/1218816*arctan(1/23*(5+8*x)*2
3^(1/2))*23^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {12, 836, 814, 648, 632, 210, 642}

$$\int \frac{19x}{(-1+x)^3(3+5x+4x^2)^2} dx = \frac{114437\arctan\left(\frac{8x+5}{\sqrt{23}}\right)}{52992\sqrt{23}} + \frac{19(44x+39)}{276(1-x)^2(4x^2+5x+3)} - \frac{209\log(4x^2+5x+3)}{4608} - \frac{1843}{4416(1-x)} - \frac{399}{736(1-x)^2} + \frac{209\log(1-x)}{2304}$$

```
[In] Int[(19*x)/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] -399/(736*(1 - x)^2) - 1843/(4416*(1 - x)) + (19*(39 + 44*x))/(276*(1 - x)^
2*(3 + 5*x + 4*x^2)) + (114437*ArcTan[(5 + 8*x)/Sqrt[23]])/(52992*Sqrt[23])
 + (209*Log[1 - x])/2304 - (209*Log[3 + 5*x + 4*x^2])/4608
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
```

```
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 19\int \frac{x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx \\
&= \frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{19}{276}\int \frac{57+132x}{(-1+x)^3\left(3+5x+4x^2\right)}\,dx \\
&= \frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)} \\
&\quad +\frac{19}{276}\int \left(\frac{63}{4(-1+x)^3}-\frac{97}{16(-1+x)^2}+\frac{253}{192(-1+x)}+\frac{2379-1012x}{192\left(3+5x+4x^2\right)}\right)dx \\
&= -\frac{399}{736(1-x)^2}-\frac{1843}{4416(1-x)}+\frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)} \\
&\quad +\frac{209\log(1-x)}{2304}+\frac{19\int \frac{2379-1012x}{3+5x+4x^2}\,dx}{52992} \\
&= -\frac{399}{736(1-x)^2}-\frac{1843}{4416(1-x)}+\frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)} \\
&\quad +\frac{209\log(1-x)}{2304}-\frac{209\int \frac{5+8x}{3+5x+4x^2}\,dx}{4608}+\frac{114437\int \frac{1}{3+5x+4x^2}\,dx}{105984} \\
&= -\frac{399}{736(1-x)^2}-\frac{1843}{4416(1-x)}+\frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{209\log(1-x)}{2304} \\
&\quad -\frac{209\log\left(3+5x+4x^2\right)}{4608}-\frac{114437\text{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,5+8x\right)}{52992} \\
&= -\frac{399}{736(1-x)^2}-\frac{1843}{4416(1-x)}+\frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)} \\
&\quad +\frac{114437\arctan\left(\frac{5+8x}{\sqrt{23}}\right)}{52992\sqrt{23}}+\frac{209\log(1-x)}{2304}-\frac{209\log\left(3+5x+4x^2\right)}{4608}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int \frac{19x}{(-1+x)^3 (3+5x+4x^2)^2} \, dx = \frac{19\left(-\frac{25392}{(-1+x)^2}+\frac{59248}{-1+x}+\frac{184(975+2204x)}{3+5x+4x^2}+36138\sqrt{23}\arctan\left(\frac{5+8x}{\sqrt{23}}\right)+34914\log(1-x)-17457\log(3+5x+\right.}{7312896}$$

```
[In] Integrate[(19*x)/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] (19*(-25392/(-1 + x)^2 + 59248/(-1 + x) + (184*(975 + 2204*x))/(3 + 5*x + 4
*x^2) + 36138*Sqrt[23]*ArcTan[(5 + 8*x)/Sqrt[23]] + 34914*Log[1 - x] - 1745
7*Log[3 + 5*x + 4*x^2]))/7312896
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.70

method	result
default	$-\frac{19}{288(-1+x)^2}+\frac{133}{864(-1+x)}+\frac{209\ln(-1+x)}{2304}-\frac{19\left(-\frac{2204x}{23}-\frac{975}{23}\right)}{6912\left(x^2+\frac{5}{4}x+\frac{3}{4}\right)}-\frac{209\ln\left(4x^2+5x+3\right)}{4608}+\frac{114437\arctan\left(\frac{(5+8x)\sqrt{23}}{23}\right)\sqrt{23}}{1218816}$
risch	$\frac{\frac{1843}{1104}x^3-\frac{7733}{4416}x^2-\frac{95}{184}x-\frac{285}{1472}}{(-1+x)^2(4x^2+5x+3)}+\frac{209\ln(-1+x)}{2304}-\frac{209\ln\left(580424464x^2+725530580x+435318348\right)}{4608}+\frac{114437\sqrt{23}\arctan\left(\frac{2(24092x}{1}\right.}{1218816}$

```
[In] int(19*x/(-1+x)^3/(4*x^2+5*x+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -19/288/(-1+x)^2+133/864/(-1+x)+209/2304*ln(-1+x)-19/6912*(-2204/23*x-975/2
3)/(x^2+5/4*x+3/4)-209/4608*ln(4*x^2+5*x+3)+114437/1218816*arctan(1/23*(5+8
*x)*23^(1/2))*23^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.38

$$\int \frac{19x}{(-1+x)^3 (3+5x+4x^2)^2} \, dx = \frac{19\left(214176\,x^3+12046\,\sqrt{23}(4\,x^4-3\,x^3-3\,x^2-x+3)\arctan\left(\frac{1}{23}\,\sqrt{23}(8\,x+5)\right)-224664\,x^2-5819\,(4\,x^4\right.}{2437632\,(4\,x^4-}$$

```
[In] integrate(19*x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="fricas")
```

```
[Out] 19/2437632*(214176*x^3 + 12046*sqrt(23)*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*arc
tan(1/23*sqrt(23)*(8*x + 5)) - 224664*x^2 - 5819*(4*x^4 - 3*x^3 - 3*x^2 - x
 + 3)*log(4*x^2 + 5*x + 3) + 11638*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*log(x -
1) - 66240*x - 24840)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \frac{19x}{(-1+x)^3(3+5x+4x^2)^2}\,dx = \frac{19\cdot(388x^3-407x^2-120x-45)}{17664x^4-13248x^3-13248x^2-4416x+13248} + \frac{209\log(x-1)}{2304} - \frac{209\log\left(x^2+\frac{5x}{4}+\frac{3}{4}\right)}{4608} + \frac{114437\sqrt{23}\operatorname{atan}\left(\frac{8\sqrt{23}x}{23}+\frac{5\sqrt{23}}{23}\right)}{1218816}$$

```
[In] integrate(19*x/(-1+x)**3/(4*x**2+5*x+3)**2,x)
```

```
[Out] 19*(388*x**3 - 407*x**2 - 120*x - 45)/(17664*x**4 - 13248*x**3 - 13248*x**2
 - 4416*x + 13248) + 209*log(x - 1)/2304 - 209*log(x**2 + 5*x/4 + 3/4)/4608
 + 114437*sqrt(23)*atan(8*sqrt(23)*x/23 + 5*sqrt(23)/23)/1218816
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{19x}{(-1+x)^3(3+5x+4x^2)^2}\,dx = \frac{114437}{1218816}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(8x+5)\right) + \frac{19(388x^3-407x^2-120x-45)}{4416(4x^4-3x^3-3x^2-x+3)} - \frac{209}{4608}\log\left(4x^2+5x+3\right) + \frac{209}{2304}\log(x-1)$$

```
[In] integrate(19*x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="maxima")
```

```
[Out] 114437/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 19/4416*(388*x^3
- 407*x^2 - 120*x - 45)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3) - 209/4608*log(4*x^
2 + 5*x + 3) + 209/2304*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.73

$$\int \frac{19x}{(-1+x)^3(3+5x+4x^2)^2}\,dx = \frac{114437}{1218816}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(8x+5)\right) + \frac{19(388x^3-407x^2-120x-45)}{4416(4x^2+5x+3)(x-1)^2} - \frac{209}{4608}\log\left(4x^2+5x+3\right) + \frac{209}{2304}\log\left(|x-1|\right)$$

```
[In] integrate(19*x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="giac")
```

```
[Out] 114437/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 19/4416*(388*x^3
- 407*x^2 - 120*x - 45)/((4*x^2 + 5*x + 3)*(x - 1)^2) - 209/4608*log(4*x^2
+ 5*x + 3) + 209/2304*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int \frac{19x}{(-1+x)^3(3+5x+4x^2)^2}\,dx = \frac{209\ln(x-1)}{2304} + \frac{-\frac{1843x^3}{4416}+\frac{7733x^2}{17664}+\frac{95x}{736}+\frac{285}{5888}}{-x^4+\frac{3x^3}{4}+\frac{3x^2}{4}+\frac{x}{4}-\frac{3}{4}} - \ln\left(x+\frac{5}{8}-\frac{\sqrt{23}\,1\mathrm{i}}{8}\right)\left(\frac{209}{4608}+\frac{\sqrt{23}\,114437\mathrm{i}}{2437632}\right) + \ln\left(x+\frac{5}{8}+\frac{\sqrt{23}\,1\mathrm{i}}{8}\right)\left(-\frac{209}{4608}+\frac{\sqrt{23}\,114437\mathrm{i}}{2437632}\right)$$

```
[In] int((19*x)/((x - 1)^3*(5*x + 4*x^2 + 3)^2),x)
```

```
[Out] (209*log(x - 1))/2304 + ((95*x)/736 + (7733*x^2)/17664 - (1843*x^3)/4416 +
285/5888)/(x/4 + (3*x^2)/4 + (3*x^3)/4 - x^4 - 3/4) - log(x - (23^(1/2)*1i)
/8 + 5/8)*((23^(1/2)*114437i)/2437632 + 209/4608) + log(x + (23^(1/2)*1i)/8
 + 5/8)*((23^(1/2)*114437i)/2437632 - 209/4608)
```

3.174 $\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx=-\frac{1}{2x}+\frac{\arctan\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)$$

```
[Out] -1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(1+2*x)*7^(1/2))*7^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1608, 1642, 648, 632, 210, 642}

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx=\frac{\arctan\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}+\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}$$

```
[In] Int[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/2*1/x + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1+x^2+x^3}{x^2\left(2+x+x^2\right)}\,dx\\
&= \int \left(\frac{1}{2x^2}-\frac{1}{4x}+\frac{3+5x}{4\left(2+x+x^2\right)}\right)dx\\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{1}{4}\int \frac{3+5x}{2+x+x^2}\,dx\\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{1}{8}\int \frac{1}{2+x+x^2}\,dx+\frac{5}{8}\int \frac{1+2x}{2+x+x^2}\,dx\\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx,x,1+2x\right)\\
&= -\frac{1}{2x}+\frac{\arctan\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx=-\frac{1}{2x}+\frac{\arctan\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)$$

```
[In] Integrate[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
[Out] -1/2*1/x + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{1}{2x}-\frac{\ln(x)}{4}+\frac{5\ln(x^2+x+2)}{8}+\frac{\sqrt{7}\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{7}}{7}\right)}{28}$	34
default	$-\frac{1}{2x}-\frac{\ln(x)}{4}+\frac{5\ln(x^2+x+2)}{8}+\frac{\arctan\left(\frac{(1+2x)\sqrt{7}}{7}\right)\sqrt{7}}{28}$	36

```
[In] int((x^3+x^2+1)/(x^4+x^3+2*x^2),x,method=_RETURNVERBOSE)
[Out] -1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*7^(1/2)*arctan(2/7*(x+1/2)*7^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx=\frac{2\sqrt{7}x\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)+35x\log\left(x^2+x+2\right)-14x\log\left(x\right)-28}{56x}$$

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="fricas")
[Out] 1/56*(2*sqrt(7)*x*arctan(1/7*sqrt(7)*(2*x + 1)) + 35*x*log(x^2 + x + 2) - 1
4*x*log(x) - 28)/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx = -\frac{\log(x)}{4} + \frac{5\log(x^2+x+2)}{8} + \frac{\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}+\frac{\sqrt{7}}{7}\right)}{28} - \frac{1}{2x}$$

```
[In] integrate((x**3+x**2+1)/(x**4+x**3+2*x**2),x)
```

```
[Out] -log(x)/4 + 5*log(x**2 + x + 2)/8 + sqrt(7)*atan(2*sqrt(7)*x/7 + sqrt(7)/7)
/28 - 1/(2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx = \frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right) - \frac{1}{2x} + \frac{5}{8}\log\left(x^2+x+2\right) - \frac{1}{4}\log(x)$$

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="maxima")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) -
 1/4*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx = \frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right) - \frac{1}{2x} + \frac{5}{8}\log\left(x^2+x+2\right) - \frac{1}{4}\log(|x|)$$

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="giac")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) -
 1/4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx = -\frac{\ln(x)}{4} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)\left(-\frac{5}{8}+\frac{\sqrt{7}\,1\mathrm{i}}{56}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)\left(\frac{5}{8}+\frac{\sqrt{7}\,1\mathrm{i}}{56}\right) - \frac{1}{2\,x}$$

```
[In] int((x^2 + x^3 + 1)/(2*x^2 + x^3 + x^4),x)
```

```
[Out] log(x + (7^(1/2)*1i)/2 + 1/2)*((7^(1/2)*1i)/56 + 5/8) - log(x - (7^(1/2)*1i
)/2 + 1/2)*((7^(1/2)*1i)/56 - 5/8) - log(x)/4 - 1/(2*x)
```

3.175 $\int \frac{1}{-x^3+x^6} dx$

Optimal result

Integrand size = 11, antiderivative size = 48

$$\int \frac{1}{-x^3+x^6} dx = \frac{1}{2x^2} - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)$$

```
[Out] 1/2/x^2+1/3*ln(1-x)-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {1607, 331, 206, 31, 648, 632, 210, 642}

$$\int \frac{1}{-x^3+x^6} dx = -\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2x^2} - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log(1-x)$$

```
[In] Int[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^3\,(-1+x^3)}\,dx$$

$$
\begin{aligned}
&= \frac{1}{2x^2} + \int \frac{1}{-1+x^3}\,dx \\
&= \frac{1}{2x^2} + \frac{1}{3}\int \frac{1}{-1+x}\,dx + \frac{1}{3}\int \frac{-2-x}{1+x+x^2}\,dx \\
&= \frac{1}{2x^2} + \frac{1}{3}\log(1-x) - \frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx - \frac{1}{2}\int \frac{1}{1+x+x^2}\,dx \\
&= \frac{1}{2x^2} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) \\
&= \frac{1}{2x^2} - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$
\int \frac{1}{-x^3+x^6}\,dx = \frac{1}{2x^2} - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)
$$

```
[In] Integrate[(-x^3 + x^6)^(-1),x]
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{6}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{1}{2x^2}$	38
risch	$\frac{1}{2x^2}-\frac{\ln(4x^2+4x+4)}{6}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(-1+x)}{3}$	42
meijerg	$-\frac{(-1)^{\frac{2}{3}}\left(\frac{3(-1)^{\frac{1}{3}}}{2x^2}+\frac{x(-1)^{\frac{1}{3}}\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2+\left(x^3\right)^{\frac{1}{3}}}\right)\right)}{\left(x^3\right)^{\frac{1}{3}}}\right)}{3}$	78

```
[In] int(1/(x^6-x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(-1+x)-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/2/x^
2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{1}{-x^3+x^6}\,dx = -\frac{2\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x^2\log\left(x^2+x+1\right)-2x^2\log\left(x-1\right)-3}{6x^2}$$

```
[In] integrate(1/(x^6-x^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) + x^2*log(x^2 + x + 1) -
2*x^2*log(x - 1) - 3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x^3+x^6}\,dx = \frac{\log\left(x-1\right)}{3} - \frac{\log\left(x^2+x+1\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3} + \frac{1}{2x^2}$$

```
[In] integrate(1/(x**6-x**3),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3
)/3 + 1/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{1}{-x^3+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{2x^2} - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log\left(x-1\right)$$

```
[In] integrate(1/(x^6-x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1)
 + 1/3*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \frac{1}{-x^3+x^6}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{1}{2\,x^2}-\frac{1}{6}\,\log\left(x^2+x+1\right)+\frac{1}{3}\,\log\left(|x-1|\right)$$

```
[In] integrate(1/(x^6-x^3),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1)
 + 1/3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{-x^3+x^6}\,dx = \frac{\ln(x-1)}{3}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{1}{2\,x^2}$$

```
[In] int(-1/(x^3 - x^6),x)
[Out] log(x - 1)/3 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x
 + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6) + 1/(2*x^2)
```

3.176 $\int \frac{x^2}{1+x}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \frac{x^2}{1+x}\,dx = -x + \frac{x^2}{2} + \log(1+x)$$

```
[Out] -x+1/2*x^2+ln(1+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{45\}$

$$\int \frac{x^2}{1+x}\,dx = \frac{x^2}{2} - x + \log(x+1)$$

```
[In] Int[x^2/(1 + x),x]
```

```
[Out] -x + x^2/2 + Log[1 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-1 + x + \frac{1}{1+x}\right) dx \\ &= -x + \frac{x^2}{2} + \log(1+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{1+x}\,dx = -2(1+x) + \frac{1}{2}(1+x)^2 + \log(1+x)$$

```
[In] Integrate[x^2/(1 + x),x]
[Out] -2*(1 + x) + (1 + x)^2/2 + Log[1 + x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-x+\frac{x^2}{2}+\ln(1+x)$	14
norman	$-x+\frac{x^2}{2}+\ln(1+x)$	14
meijerg	$-\frac{x(-3x+6)}{6}+\ln(1+x)$	14
risch	$-x+\frac{x^2}{2}+\ln(1+x)$	14
parallelrisch	$-x+\frac{x^2}{2}+\ln(1+x)$	14

```
[In] int(x^2/(1+x),x,method=_RETURNVERBOSE)
[Out] -x+1/2*x^2+ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{1+x}\,dx = \frac{1}{2}\,x^2 - x + \log\left(x+1\right)$$

```
[In] integrate(x^2/(1+x),x, algorithm="fricas")
[Out] 1/2*x^2 - x + log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{1+x}\,dx = \frac{x^2}{2} - x + \log{(x+1)}$$

```
[In] integrate(x**2/(1+x),x)
[Out] x**2/2 - x + log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{1+x}\,dx = \frac{1}{2}\,x^2 - x + \log\left(x+1\right)$$

```
[In] integrate(x^2/(1+x),x, algorithm="maxima")
[Out] 1/2*x^2 - x + log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{1+x}\,dx = \frac{1}{2}\,x^2 - x + \log\left(|x+1|\right)$$

```
[In] integrate(x^2/(1+x),x, algorithm="giac")
[Out] 1/2*x^2 - x + log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{1+x}\,dx = \ln\left(x+1\right) - x + \frac{x^2}{2}$$

```
[In] int(x^2/(x + 1),x)
[Out] log(x + 1) - x + x^2/2
```

3.177 $\int \frac{x}{-5+x} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{x}{-5+x} \, dx = x + 5\log(5-x)$$

```
[Out] x+5*ln(5-x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {45}

$$\int \frac{x}{-5+x} \, dx = x + 5\log(5-x)$$

```
[In] Int[x/(-5 + x),x]
```

```
[Out] x + 5*Log[5 - x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(1+\frac{5}{-5+x}\right) \, dx \\ &= x + 5\log(5-x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{-5+x}\,dx = x + 5\log(-5+x)$$

```
[In] Integrate[x/(-5 + x),x]
[Out] x + 5*Log[-5 + x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
default	$x+5\ln\left(x-5\right)$	9
norman	$x+5\ln\left(x-5\right)$	9
risch	$x+5\ln\left(x-5\right)$	9
parallelrisch	$x+5\ln\left(x-5\right)$	9
meijerg	$x+5\ln\left(1-\frac{x}{5}\right)$	11

```
[In] int(x/(x-5),x,method=_RETURNVERBOSE)
[Out] x+5*ln(x-5)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{-5+x}\,dx = x + 5\,\log\left(x-5\right)$$

```
[In] integrate(x/(-5+x),x, algorithm="fricas")
[Out] x + 5*log(x - 5)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{x}{-5+x}\,dx = x + 5\log{(x - 5)}$$

```
[In] integrate(x/(-5+x),x)
[Out] x + 5*log(x - 5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{-5+x}\,dx = x + 5\,\log\left(x - 5\right)$$

```
[In] integrate(x/(-5+x),x, algorithm="maxima")
[Out] x + 5*log(x - 5)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{x}{-5+x}\,dx = x + 5\,\log\left(|x - 5|\right)$$

```
[In] integrate(x/(-5+x),x, algorithm="giac")
[Out] x + 5*log(abs(x - 5))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{-5+x}\,dx = x + 5\,\ln\left(x - 5\right)$$

```
[In] int(x/(x - 5),x)
[Out] x + 5*log(x - 5)
```

3.178 $\int \frac{-1+4x}{(-1+x)(2+x)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 13

$$\int \frac{-1+4x}{(-1+x)(2+x)} \, dx = \log(1-x) + 3\log(2+x)$$

```
[Out] ln(1-x)+3*ln(2+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{78\}$

$$\int \frac{-1+4x}{(-1+x)(2+x)} \, dx = \log(1-x) + 3\log(x+2)$$

```
[In] Int[(-1 + 4*x)/((-1 + x)*(2 + x)),x]
```

```
[Out] Log[1 - x] + 3*Log[2 + x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{-1+x} + \frac{3}{2+x}\right) \, dx \\ &= \log(1-x) + 3\log(2+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x}{(-1+x)(2+x)}\,dx = \log(1-x)+3\log(2+x)$$

```
[In] Integrate[(-1 + 4*x)/((-1 + x)*(2 + x)),x]
[Out] Log[1 - x] + 3*Log[2 + x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\ln(-1+x)+3\ln(2+x)$	12
norman	$\ln(-1+x)+3\ln(2+x)$	12
risch	$\ln(-1+x)+3\ln(2+x)$	12
parallelrisch	$\ln(-1+x)+3\ln(2+x)$	12

```
[In] int((-1+4*x)/(-1+x)/(2+x),x,method=_RETURNVERBOSE)
[Out] ln(-1+x)+3*ln(2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{-1+4x}{(-1+x)(2+x)}\,dx = 3\,\log\left(x+2\right)+\log\left(x-1\right)$$

```
[In] integrate((-1+4*x)/(-1+x)/(2+x),x, algorithm="fricas")
[Out] 3*log(x + 2) + log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{-1+4x}{(-1+x)(2+x)}\,dx = \log{(x-1)} + 3\log{(x+2)}$$

```
[In] integrate((-1+4*x)/(-1+x)/(2+x),x)
[Out] log(x - 1) + 3*log(x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{-1+4x}{(-1+x)(2+x)}\,dx = 3\,\log\left(x+2\right) + \log\left(x-1\right)$$

```
[In] integrate((-1+4*x)/(-1+x)/(2+x),x, algorithm="maxima")
[Out] 3*log(x + 2) + log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x}{(-1+x)(2+x)}\,dx = 3\,\log\left(|x+2|\right) + \log\left(|x-1|\right)$$

```
[In] integrate((-1+4*x)/(-1+x)/(2+x),x, algorithm="giac")
[Out] 3*log(abs(x + 2)) + log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{-1+4x}{(-1+x)(2+x)}\,dx = \ln\left(x-1\right) + 3\,\ln\left(x+2\right)$$

```
[In] int((4*x - 1)/((x - 1)*(x + 2)),x)
[Out] log(x - 1) + 3*log(x + 2)
```

3.179 $\int \frac{1}{(1+x)(2+x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{1}{(1+x)(2+x)} dx = \log(1+x) - \log(2+x)$$

```
[Out] ln(1+x)-ln(2+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {36, 31}

$$\int \frac{1}{(1+x)(2+x)} dx = \log(x+1) - \log(x+2)$$

```
[In] Int[1/((1 + x)*(2 + x)),x]
```

```
[Out] Log[1 + x] - Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{1+x}\,dx - \int \frac{1}{2+x}\,dx\\ &= \log(1+x) - \log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)(2+x)}\,dx = \log(1+x) - \log(2+x)$$

```
[In] Integrate[1/((1 + x)*(2 + x)),x]
[Out] Log[1 + x] - Log[2 + x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$\ln(1+x) - \ln(2+x)$	12
norman	$\ln(1+x) - \ln(2+x)$	12
risch	$\ln(1+x) - \ln(2+x)$	12
parallelrisch	$\ln(1+x) - \ln(2+x)$	12

```
[In] int(1/(1+x)/(2+x),x,method=_RETURNVERBOSE)
[Out] ln(1+x)-ln(2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)(2+x)}\,dx = -\log(x+2) + \log(x+1)$$

```
[In] integrate(1/(1+x)/(2+x),x, algorithm="fricas")
[Out] -log(x + 2) + log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{(1+x)(2+x)}\,dx = \log(x+1) - \log(x+2)$$

```
[In] integrate(1/(1+x)/(2+x),x)
[Out] log(x + 1) - log(x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)(2+x)}\,dx = -\log(x+2) + \log(x+1)$$

```
[In] integrate(1/(1+x)/(2+x),x, algorithm="maxima")
[Out] -log(x + 2) + log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{1}{(1+x)(2+x)}\,dx = -\log(|x+2|) + \log(|x+1|)$$

```
[In] integrate(1/(1+x)/(2+x),x, algorithm="giac")
[Out] -log(abs(x + 2)) + log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{(1+x)(2+x)}\,dx = \ln\left(1 - \frac{1}{x+2}\right)$$

```
[In] int(1/((x + 1)*(x + 2)),x)
[Out] log(1 - 1/(x + 2))
```

3.180 $\int \frac{-5+6x}{3+2x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{-5+6x}{3+2x} \, dx = 3x - 7\log(3+2x)$$

```
[Out] 3*x-7*ln(3+2*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\int \frac{-5+6x}{3+2x} \, dx = 3x - 7\log(2x+3)$$

```
[In] Int[(-5 + 6*x)/(3 + 2*x),x]
```

```
[Out] 3*x - 7*Log[3 + 2*x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(3 - \frac{14}{3+2x}\right) \, dx \\ &= 3x - 7\log(3+2x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{-5+6x}{3+2x}\,dx=\frac{9}{2}+3x-7\log(3+2x)$$

```
[In] Integrate[(-5 + 6*x)/(3 + 2*x),x]
[Out] 9/2 + 3*x - 7*Log[3 + 2*x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$3x-7\ln\left(\frac{3}{2}+x\right)$	11
default	$3x-7\ln\left(3+2x\right)$	13
norman	$3x-7\ln\left(3+2x\right)$	13
meijerg	$-7\ln\left(1+\frac{2x}{3}\right)+3x$	13
risch	$3x-7\ln\left(3+2x\right)$	13

```
[In] int((6*x-5)/(3+2*x),x,method=_RETURNVERBOSE)
[Out] 3*x-7*ln(3/2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-5+6x}{3+2x}\,dx=3\,x-7\,\log\left(2\,x+3\right)$$

```
[In] integrate((-5+6*x)/(3+2*x),x, algorithm="fricas")
[Out] 3*x - 7*log(2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-5+6x}{3+2x}\,dx = 3x - 7\log\left(2x+3\right)$$

```
[In] integrate((-5+6*x)/(3+2*x),x)
[Out] 3*x - 7*log(2*x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-5+6x}{3+2x}\,dx = 3\,x - 7\,\log\left(2\,x+3\right)$$

```
[In] integrate((-5+6*x)/(3+2*x),x, algorithm="maxima")
[Out] 3*x - 7*log(2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{-5+6x}{3+2x}\,dx = 3\,x - 7\,\log\left(|2\,x+3|\right)$$

```
[In] integrate((-5+6*x)/(3+2*x),x, algorithm="giac")
[Out] 3*x - 7*log(abs(2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-5+6x}{3+2x}\,dx = 3\,x - 7\,\ln\left(x+\frac{3}{2}\right)$$

```
[In] int((6*x - 5)/(2*x + 3),x)
[Out] 3*x - 7*log(x + 3/2)
```

3.181 $\int \frac{1}{(a+x)(b+x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{1}{(a+x)(b+x)}\,dx = -\frac{\log(a+x)}{a-b} + \frac{\log(b+x)}{a-b}$$

```
[Out] -ln(a+x)/(a-b)+ln(b+x)/(a-b)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {36, 31}

$$\int \frac{1}{(a+x)(b+x)}\,dx = \frac{\log(b+x)}{a-b} - \frac{\log(a+x)}{a-b}$$

```
[In] Int[1/((a + x)*(b + x)),x]
```

```
[Out] -(Log[a + x]/(a - b)) + Log[b + x]/(a - b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{1}{a+x}\,dx}{-a+b} - \frac{\int \frac{1}{b+x}\,dx}{-a+b}\\ &= -\frac{\log(a+x)}{a-b} + \frac{\log(b+x)}{a-b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+x)(b+x)}\,dx = \frac{-\log(a+x)+\log(b+x)}{a-b}$$

```
[In] Integrate[1/((a + x)*(b + x)),x]
[Out] (-Log[a + x] + Log[b + x])/(a - b)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$-\frac{\ln(a+x)-\ln(b+x)}{a-b}$	21
default	$-\frac{\ln(a+x)}{a-b}+\frac{\ln(b+x)}{a-b}$	27
norman	$-\frac{\ln(a+x)}{a-b}+\frac{\ln(b+x)}{a-b}$	27
risch	$-\frac{\ln(a+x)}{a-b}+\frac{\ln(b+x)}{a-b}$	27

```
[In] int(1/(a+x)/(b+x),x,method=_RETURNVERBOSE)
[Out] -(ln(a+x)-ln(b+x))/(a-b)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a+x)(b+x)}\,dx = -\frac{\log(a+x)-\log(b+x)}{a-b}$$

```
[In] integrate(1/(a+x)/(b+x),x, algorithm="fricas")
[Out] -(log(a + x) - log(b + x))/(a - b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(15) = 30.

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.08

$$\int \frac{1}{(a+x)(b+x)}\,dx = \frac{\log\left(-\frac{a^2}{2(a-b)}+\frac{ab}{a-b}+\frac{a}{2}-\frac{b^2}{2(a-b)}+\frac{b}{2}+x\right)}{a-b} - \frac{\log\left(\frac{a^2}{2(a-b)}-\frac{ab}{a-b}+\frac{a}{2}+\frac{b^2}{2(a-b)}+\frac{b}{2}+x\right)}{a-b}$$

```
[In] integrate(1/(a+x)/(b+x),x)
[Out] log(-a**2/(2*(a - b)) + a*b/(a - b) + a/2 - b**2/(2*(a - b)) + b/2 + x)/(a
- b) - log(a**2/(2*(a - b)) - a*b/(a - b) + a/2 + b**2/(2*(a - b)) + b/2 +
x)/(a - b)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+x)(b+x)}\,dx = -\frac{\log(a+x)}{a-b}+\frac{\log(b+x)}{a-b}$$

```
[In] integrate(1/(a+x)/(b+x),x, algorithm="maxima")
[Out] -log(a + x)/(a - b) + log(b + x)/(a - b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+x)(b+x)}\,dx = -\frac{\log\left(|a+x|\right)}{a-b} + \frac{\log\left(|b+x|\right)}{a-b}$$

```
[In] integrate(1/(a+x)/(b+x),x, algorithm="giac")
[Out] -log(abs(a + x))/(a - b) + log(abs(b + x))/(a - b)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a+x)(b+x)}\,dx = \frac{\ln\left(\frac{b+x}{a+x}\right)}{a-b}$$

```
[In] int(1/((a + x)*(b + x)),x)
[Out] log((b + x)/(a + x))/(a - b)
```

3.182 $\int \frac{1+x^2}{-x+x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 14

$$\int \frac{1+x^2}{-x+x^2}\,dx = x+2\log(1-x)-\log(x)$$

```
[Out] x+2*ln(1-x)-ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1607, 908}

$$\int \frac{1+x^2}{-x+x^2}\,dx = x+2\log(1-x)-\log(x)$$

```
[In] Int[(1 + x^2)/(-x + x^2),x]
```

```
[Out] x + 2*Log[1 - x] - Log[x]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1+x^2}{(-1+x)x}\,dx \\
&= \int \left(1+\frac{2}{-1+x}-\frac{1}{x}\right)dx \\
&= x+2\log(1-x)-\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{-x+x^2}\,dx = x+2\log(1-x)-\log(x)$$

```
[In] Integrate[(1 + x^2)/(-x + x^2),x]
[Out] x + 2*Log[1 - x] - Log[x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$x+2\ln\left(-1+x\right)-\ln\left(x\right)$	13
norman	$x+2\ln\left(-1+x\right)-\ln\left(x\right)$	13
risch	$x+2\ln\left(-1+x\right)-\ln\left(x\right)$	13
parallelrisch	$x+2\ln\left(-1+x\right)-\ln\left(x\right)$	13
meijerg	$-\ln\left(x\right)-i\pi+2\ln\left(1-x\right)+x$	19

```
[In] int((x^2+1)/(x^2-x),x,method=_RETURNVERBOSE)
[Out] x+2*ln(-1+x)-ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{-x+x^2} \, dx = x + 2 \, \log \left(x - 1\right) - \log \left(x\right)$$

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="fricas")
[Out] x + 2*log(x - 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1+x^2}{-x+x^2} \, dx = x - \log \left(x\right) + 2 \log \left(x - 1\right)$$

```
[In] integrate((x**2+1)/(x**2-x),x)
[Out] x - log(x) + 2*log(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{-x+x^2} \, dx = x + 2 \, \log \left(x - 1\right) - \log \left(x\right)$$

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="maxima")
[Out] x + 2*log(x - 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{-x+x^2} \, dx = x + 2 \, \log \left(|x - 1|\right) - \log \left(|x|\right)$$

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="giac")
[Out] x + 2*log(abs(x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{-x+x^2}\,dx = x + 2\,\ln\left(x-1\right) - \ln\left(x\right)$$

```
[In] int(-(x^2 + 1)/(x - x^2),x)
[Out] x + 2*log(x - 1) - log(x)
```

3.183 $\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 26

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx=\frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(4+x)$$

```
[Out] 1/2*x^2+1/7*ln(3-x)-1/7*ln(4+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1671, 630, 31}

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx=\frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(x+4)$$

```
[In] Int[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
```

```
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(x+\frac{1}{-12+x+x^2}\right) dx \\ &= \frac{x^2}{2}+\int \frac{1}{-12+x+x^2} dx \\ &= \frac{x^2}{2}+\frac{1}{7}\int \frac{1}{-3+x} dx-\frac{1}{7}\int \frac{1}{4+x} dx \\ &= \frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(4+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2} dx = \frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(4+x)$$

```
[In] Integrate[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{x^2}{2}-\frac{\ln(4+x)}{7}+\frac{\ln(-3+x)}{7}$	19
norman	$\frac{x^2}{2}-\frac{\ln(4+x)}{7}+\frac{\ln(-3+x)}{7}$	19
risch	$\frac{x^2}{2}-\frac{\ln(4+x)}{7}+\frac{\ln(-3+x)}{7}$	19
parallelrisch	$\frac{x^2}{2}-\frac{\ln(4+x)}{7}+\frac{\ln(-3+x)}{7}$	19

```
[In] int((x^3+x^2-12*x+1)/(x^2+x-12),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-1/7*ln(4+x)+1/7*ln(-3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{7}\,\log\left(x+4\right) + \frac{1}{7}\,\log\left(x-3\right)$$

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="fricas")
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx = \frac{x^2}{2} + \frac{\log\left(x-3\right)}{7} - \frac{\log\left(x+4\right)}{7}$$

```
[In] integrate((x**3+x**2-12*x+1)/(x**2+x-12),x)
[Out] x**2/2 + log(x - 3)/7 - log(x + 4)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{7}\,\log\left(x+4\right) + \frac{1}{7}\,\log\left(x-3\right)$$

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{7}\,\log\left(|x+4|\right) + \frac{1}{7}\,\log\left(|x-3|\right)$$

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="giac")
[Out] 1/2*x^2 - 1/7*log(abs(x + 4)) + 1/7*log(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx=\frac{x^2}{2}-\frac{2\,\mathrm{atanh}\left(\frac{2\,x}{7}+\frac{1}{7}\right)}{7}$$

```
[In] int((x^2 - 12*x + x^3 + 1)/(x + x^2 - 12),x)
[Out] x^2/2 - (2*atanh((2*x)/7 + 1/7))/7
```

3.184 $\int \frac{3+2x}{(1+x)^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{3+2x}{(1+x)^2} \, dx = -\frac{1}{1+x} + 2\log(1+x)$$

```
[Out] -1/(1+x)+2*ln(1+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\int \frac{3+2x}{(1+x)^2} \, dx = 2\log(x+1) - \frac{1}{x+1}$$

```
[In] Int[(3 + 2*x)/(1 + x)^2,x]
```

```
[Out] -(1 + x)^(-1) + 2*Log[1 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{(1+x)^2} + \frac{2}{1+x}\right) \, dx \\ &= -\frac{1}{1+x} + 2\log(1+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{3+2x}{(1+x)^2} \, dx = -\frac{1}{1+x} + 2\log(1+x)$$

```
[In] Integrate[(3 + 2*x)/(1 + x)^2,x]
[Out] -(1 + x)^(-1) + 2*Log[1 + x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{1}{1+x}+2\ln\left(1+x\right)$	15
norman	$-\frac{1}{1+x}+2\ln\left(1+x\right)$	15
meijerg	$\frac{x}{1+x}+2\ln\left(1+x\right)$	15
risch	$-\frac{1}{1+x}+2\ln\left(1+x\right)$	15
parallelrisch	$\frac{2\ln(1+x)x-1+2\ln(1+x)}{1+x}$	22

```
[In] int((3+2*x)/(1+x)^2,x,method=_RETURNVERBOSE)
[Out] -1/(1+x)+2*ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{3+2x}{(1+x)^2} \, dx = \frac{2\,(x+1)\log\left(x+1\right)-1}{x+1}$$

```
[In] integrate((3+2*x)/(1+x)^2,x, algorithm="fricas")
[Out] (2*(x + 1)*log(x + 1) - 1)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{3+2x}{(1+x)^2}\,dx = 2\log(x+1) - \frac{1}{x+1}$$

```
[In] integrate((3+2*x)/(1+x)**2,x)
[Out] 2*log(x + 1) - 1/(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{3+2x}{(1+x)^2}\,dx = -\frac{1}{x+1} + 2\log(x+1)$$

```
[In] integrate((3+2*x)/(1+x)^2,x, algorithm="maxima")
[Out] -1/(x + 1) + 2*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{3+2x}{(1+x)^2}\,dx = -\frac{1}{x+1} + 2\log(|x+1|)$$

```
[In] integrate((3+2*x)/(1+x)^2,x, algorithm="giac")
[Out] -1/(x + 1) + 2*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{3+2x}{(1+x)^2}\,dx = 2\ln(x+1) - \frac{1}{x+1}$$

```
[In] int((2*x + 3)/(x + 1)^2,x)
[Out] 2*log(x + 1) - 1/(x + 1)
```

3.185 $\int \frac{1}{x(1+x)(3+2x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{1}{x(1+x)(3+2x)}\,dx = \frac{\log(x)}{3} - \log(1+x) + \frac{2}{3}\log(3+2x)$$

```
[Out] 1/3*ln(x)-ln(1+x)+2/3*ln(3+2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{84\}$

$$\int \frac{1}{x(1+x)(3+2x)}\,dx = \frac{\log(x)}{3} - \log(x+1) + \frac{2}{3}\log(2x+3)$$

```
[In] Int[1/(x*(1 + x)*(3 + 2*x)),x]
```

```
[Out] Log[x]/3 - Log[1 + x] + (2*Log[3 + 2*x])/3
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{-1-x} + \frac{1}{3x} + \frac{4}{3(3+2x)}\right) dx \\ &= \frac{\log(x)}{3} - \log(1+x) + \frac{2}{3}\log(3+2x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x)(3+2x)}\,dx = \frac{\log(x)}{3} - \log(1+x) + \frac{2}{3}\log(3+2x)$$

```
[In] Integrate[1/(x*(1 + x)*(3 + 2*x)),x]
[Out] Log[x]/3 - Log[1 + x] + (2*Log[3 + 2*x])/3
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
parallelrisch	$\frac{\ln(x)}{3} - \ln\left(1+x\right) + \frac{2\ln\left(\frac{3}{2}+x\right)}{3}$	18
default	$\frac{\ln(x)}{3} - \ln\left(1+x\right) + \frac{2\ln(3+2x)}{3}$	20
norman	$\frac{\ln(x)}{3} - \ln\left(1+x\right) + \frac{2\ln(3+2x)}{3}$	20
risch	$\frac{\ln(x)}{3} - \ln\left(1+x\right) + \frac{2\ln(3+2x)}{3}$	20

```
[In] int(1/x/(1+x)/(3+2*x),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(x)-ln(1+x)+2/3*ln(3/2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(1+x)(3+2x)}\,dx = \frac{2}{3}\,\log\left(2\,x+3\right) - \log\left(x+1\right) + \frac{1}{3}\,\log\left(x\right)$$

```
[In] integrate(1/x/(1+x)/(3+2*x),x, algorithm="fricas")
[Out] 2/3*log(2*x + 3) - log(x + 1) + 1/3*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(1+x)(3+2x)}\,dx = \frac{\log(x)}{3} - \log(x+1) + \frac{2\log\left(x+\frac{3}{2}\right)}{3}$$

```
[In] integrate(1/x/(1+x)/(3+2*x),x)
[Out] log(x)/3 - log(x + 1) + 2*log(x + 3/2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(1+x)(3+2x)}\,dx = \frac{2}{3}\log(2x+3) - \log(x+1) + \frac{1}{3}\log(x)$$

```
[In] integrate(1/x/(1+x)/(3+2*x),x, algorithm="maxima")
[Out] 2/3*log(2*x + 3) - log(x + 1) + 1/3*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(1+x)(3+2x)}\,dx = \frac{2}{3}\log(|2x+3|) - \log(|x+1|) + \frac{1}{3}\log(|x|)$$

```
[In] integrate(1/x/(1+x)/(3+2*x),x, algorithm="giac")
[Out] 2/3*log(abs(2*x + 3)) - log(abs(x + 1)) + 1/3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{x(1+x)(3+2x)}\,dx = \frac{2\ln\left(x+\frac{3}{2}\right)}{3} - \ln(x+1) + \frac{\ln(x)}{3}$$

```
[In] int(1/(x*(2*x + 3)*(x + 1)),x)
[Out] (2*log(x + 3/2))/3 - log(x + 1) + log(x)/3
```

3.186 $\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 17

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx=2\log(1-x)+\log(x)+3\log(3+x)$$

```
[Out] 2*ln(1-x)+ln(x)+3*ln(3+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1608, 1642}

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx=2\log(1-x)+\log(x)+3\log(x+3)$$

```
[In] Int[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{-3+5x+6x^2}{x\left(-3+2x+x^2\right)}\,dx\\ &= \int \left(\frac{2}{-1+x}+\frac{1}{x}+\frac{3}{3+x}\right)dx\\ &= 2\log(1-x)+\log(x)+3\log(3+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = 2\log(1-x)+\log(x)+3\log(3+x)$$

```
[In] Integrate[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$2\ln\left(-1+x\right)+\ln\left(x\right)+3\ln\left(3+x\right)$	16
norman	$2\ln\left(-1+x\right)+\ln\left(x\right)+3\ln\left(3+x\right)$	16
risch	$2\ln\left(-1+x\right)+\ln\left(x\right)+3\ln\left(3+x\right)$	16
parallelrisch	$2\ln\left(-1+x\right)+\ln\left(x\right)+3\ln\left(3+x\right)$	16

```
[In] int((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x,method=_RETURNVERBOSE)
[Out] 2*ln(-1+x)+ln(x)+3*ln(3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = 3\,\log\left(x+3\right)+2\,\log\left(x-1\right)+\log\left(x\right)$$

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="fricas")
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = \log\left(x\right)+2\log\left(x-1\right)+3\log\left(x+3\right)$$

```
[In] integrate((6*x**2+5*x-3)/(x**3+2*x**2-3*x),x)
[Out] log(x) + 2*log(x - 1) + 3*log(x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = 3\,\log\left(x+3\right)+2\,\log\left(x-1\right)+\log\left(x\right)$$

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="maxima")
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = 3\,\log\left(|x+3|\right)+2\,\log\left(|x-1|\right)+\log\left(|x|\right)$$

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="giac")
[Out] 3*log(abs(x + 3)) + 2*log(abs(x - 1)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = 2\,\ln\left(x-1\right)+3\,\ln\left(x+3\right)+\ln\left(x\right)$$

```
[In] int((5*x + 6*x^2 - 3)/(2*x^2 - 3*x + x^3),x)
[Out] 2*log(x - 1) + 3*log(x + 3) + log(x)
```

3.187 $\int \frac{x}{4+4x+x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{4+4x+x^2} \, dx = \frac{2}{2+x} + \log(2+x)$$

```
[Out] 2/(2+x)+ln(2+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {27, 45}

$$\int \frac{x}{4+4x+x^2} \, dx = \frac{2}{x+2} + \log(x+2)$$

```
[In] Int[x/(4 + 4*x + x^2),x]
```

```
[Out] 2/(2 + x) + Log[2 + x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x}{(2+x)^2}\,dx \\ &= \int \left(-\frac{2}{(2+x)^2}+\frac{1}{2+x}\right)dx \\ &= \frac{2}{2+x}+\log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{4+4x+x^2}\,dx=\frac{2}{2+x}+\log(2+x)$$

```
[In] Integrate[x/(4 + 4*x + x^2),x]
[Out] 2/(2 + x) + Log[2 + x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{2}{2+x}+\ln\left(2+x\right)$	13
norman	$\frac{2}{2+x}+\ln\left(2+x\right)$	13
risch	$\frac{2}{2+x}+\ln\left(2+x\right)$	13
meijerg	$-\frac{x}{2\left(1+\frac{x}{2}\right)}+\ln\left(1+\frac{x}{2}\right)$	18
parallelrisch	$\frac{\ln(2+x)x+2+2\ln(2+x)}{2+x}$	21

```
[In] int(x/(x^2+4*x+4),x,method=_RETURNVERBOSE)
[Out] 2/(2+x)+ln(2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{x}{4+4x+x^2} \, dx = \frac{(x+2)\log(x+2)+2}{x+2}$$

```
[In] integrate(x/(x^2+4*x+4),x, algorithm="fricas")
[Out] ((x + 2)*log(x + 2) + 2)/(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{4+4x+x^2} \, dx = \log(x+2) + \frac{2}{x+2}$$

```
[In] integrate(x/(x**2+4*x+4),x)
[Out] log(x + 2) + 2/(x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{4+4x+x^2} \, dx = \frac{2}{x+2} + \log(x+2)$$

```
[In] integrate(x/(x^2+4*x+4),x, algorithm="maxima")
[Out] 2/(x + 2) + log(x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{x}{4+4x+x^2} \, dx = \frac{2}{x+2} + \log(|x+2|)$$

```
[In] integrate(x/(x^2+4*x+4),x, algorithm="giac")
[Out] 2/(x + 2) + log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{4+4x+x^2} \, dx = \ln(x+2) + \frac{2}{x+2}$$

```
[In] int(x/(4*x + x^2 + 4),x)
[Out] log(x + 2) + 2/(x + 2)
```

3.188 $\int \frac{1}{(-1+x)^2(4+x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \frac{1}{(-1+x)^2(4+x)}\,dx = \frac{1}{5(1-x)} - \frac{1}{25}\log(1-x) + \frac{1}{25}\log(4+x)$$

```
[Out] 1/5/(1-x)-1/25*ln(1-x)+1/25*ln(4+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\int \frac{1}{(-1+x)^2(4+x)}\,dx = \frac{1}{5(1-x)} - \frac{1}{25}\log(1-x) + \frac{1}{25}\log(x+4)$$

```
[In] Int[1/((-1 + x)^2*(4 + x)),x]
[Out] 1/(5*(1 - x)) - Log[1 - x]/25 + Log[4 + x]/25
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{5(-1+x)^2} - \frac{1}{25(-1+x)} + \frac{1}{25(4+x)}\right) dx \\ &= \frac{1}{5(1-x)} - \frac{1}{25}\log(1-x) + \frac{1}{25}\log(4+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1}{(-1+x)^2(4+x)}\,dx = \frac{1}{25}\left(-\frac{5}{-1+x} - \log(-1+x) + \log(4+x)\right)$$

```
[In] Integrate[1/((-1 + x)^2*(4 + x)),x]
[Out] (-5/(-1 + x) - Log[-1 + x] + Log[4 + x])/25
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{1}{5(-1+x)}-\frac{\ln(-1+x)}{25}+\frac{\ln(4+x)}{25}$	21
norman	$-\frac{1}{5(-1+x)}-\frac{\ln(-1+x)}{25}+\frac{\ln(4+x)}{25}$	21
risch	$-\frac{1}{5(-1+x)}-\frac{\ln(-1+x)}{25}+\frac{\ln(4+x)}{25}$	21
parallelrisch	$-\frac{\ln(-1+x)x-\ln(4+x)x+5-\ln(-1+x)+\ln(4+x)}{25(-1+x)}$	33

```
[In] int(1/(-1+x)^2/(4+x),x,method=_RETURNVERBOSE)
[Out] -1/5/(-1+x)-1/25*ln(-1+x)+1/25*ln(4+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{(-1+x)^2(4+x)}\,dx = \frac{(x-1)\log(x+4) - (x-1)\log(x-1) - 5}{25\,(x-1)}$$

```
[In] integrate(1/(-1+x)^2/(4+x),x, algorithm="fricas")
[Out] 1/25*((x - 1)*log(x + 4) - (x - 1)*log(x - 1) - 5)/(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

$$\int \frac{1}{(-1+x)^2(4+x)}\,dx = -\frac{\log(x-1)}{25} + \frac{\log(x+4)}{25} - \frac{1}{5x-5}$$

```
[In] integrate(1/(-1+x)**2/(4+x),x)
[Out] -log(x - 1)/25 + log(x + 4)/25 - 1/(5*x - 5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1}{(-1+x)^2(4+x)}\,dx = -\frac{1}{5(x-1)} + \frac{1}{25}\log(x+4) - \frac{1}{25}\log(x-1)$$

```
[In] integrate(1/(-1+x)^2/(4+x),x, algorithm="maxima")
[Out] -1/5/(x - 1) + 1/25*log(x + 4) - 1/25*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{1}{(-1+x)^2(4+x)}\,dx = -\frac{1}{5(x-1)} + \frac{1}{25}\log\left(\left|-\frac{5}{x-1} - 1\right|\right)$$

```
[In] integrate(1/(-1+x)^2/(4+x),x, algorithm="giac")
[Out] -1/5/(x - 1) + 1/25*log(abs(-5/(x - 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1}{(-1+x)^2(4+x)}\,dx = -\frac{\ln\left(\frac{x-1}{x+4}\right)}{25} - \frac{1}{5\,(x-1)}$$

```
[In] int(1/((x - 1)^2*(x + 4)),x)
[Out] - log((x - 1)/(x + 4))/25 - 1/(5*(x - 1))
```

3.189 $\int \frac{x^2}{(-3+x)(2+x)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 28

$$\int \frac{x^2}{(-3+x)(2+x)^2} dx = \frac{4}{5(2+x)} + \frac{9}{25}\log(3-x) + \frac{16}{25}\log(2+x)$$

```
[Out] 4/5/(2+x)+9/25*ln(3-x)+16/25*ln(2+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {90}

$$\int \frac{x^2}{(-3+x)(2+x)^2} dx = \frac{4}{5(x+2)} + \frac{9}{25}\log(3-x) + \frac{16}{25}\log(x+2)$$

```
[In] Int[x^2/((-3 + x)*(2 + x)^2),x]
[Out] 4/(5*(2 + x)) + (9*Log[3 - x])/25 + (16*Log[2 + x])/25
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{9}{25(-3+x)} - \frac{4}{5(2+x)^2} + \frac{16}{25(2+x)}\right) dx \\ &= \frac{4}{5(2+x)} + \frac{9}{25}\log(3-x) + \frac{16}{25}\log(2+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(-3+x)(2+x)^2}\,dx = \frac{4}{5(2+x)} + \frac{9}{25}\log(-3+x) + \frac{16}{25}\log(2+x)$$

```
[In] Integrate[x^2/((-3 + x)*(2 + x)^2),x]
[Out] 4/(5*(2 + x)) + (9*Log[-3 + x])/25 + (16*Log[2 + x])/25
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{4}{5(2+x)}+\frac{16\ln(2+x)}{25}+\frac{9\ln(-3+x)}{25}$	21
norman	$\frac{4}{5(2+x)}+\frac{16\ln(2+x)}{25}+\frac{9\ln(-3+x)}{25}$	21
risch	$\frac{4}{5(2+x)}+\frac{16\ln(2+x)}{25}+\frac{9\ln(-3+x)}{25}$	21
parallelrisch	$\frac{9\ln(-3+x)x+16\ln(2+x)x+20+18\ln(-3+x)+32\ln(2+x)}{50+25x}$	36

```
[In] int(x^2/(-3+x)/(2+x)^2,x,method=_RETURNVERBOSE)
[Out] 4/5/(2+x)+16/25*ln(2+x)+9/25*ln(-3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(-3+x)(2+x)^2}\,dx = \frac{16\,(x+2)\log\left(x+2\right)+9\,(x+2)\log\left(x-3\right)+20}{25\,(x+2)}$$

```
[In] integrate(x^2/(-3+x)/(2+x)^2,x, algorithm="fricas")
[Out] 1/25*(16*(x + 2)*log(x + 2) + 9*(x + 2)*log(x - 3) + 20)/(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(-3+x)(2+x)^2}\,dx = \frac{9\log{(x-3)}}{25} + \frac{16\log{(x+2)}}{25} + \frac{4}{5x+10}$$

```
[In] integrate(x**2/(-3+x)/(2+x)**2,x)
[Out] 9*log(x - 3)/25 + 16*log(x + 2)/25 + 4/(5*x + 10)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(-3+x)(2+x)^2}\,dx = \frac{4}{5\,(x+2)} + \frac{16}{25}\,\log{(x+2)} + \frac{9}{25}\,\log{(x-3)}$$

```
[In] integrate(x^2/(-3+x)/(2+x)^2,x, algorithm="maxima")
[Out] 4/5/(x + 2) + 16/25*log(x + 2) + 9/25*log(x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(-3+x)(2+x)^2}\,dx = \frac{4}{5\,(x+2)} + \log{(|x+2|)} + \frac{9}{25}\,\log{\left(\left|-\frac{5}{x+2}+1\right|\right)}$$

```
[In] integrate(x^2/(-3+x)/(2+x)^2,x, algorithm="giac")
[Out] 4/5/(x + 2) + log(abs(x + 2)) + 9/25*log(abs(-5/(x + 2) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(-3+x)(2+x)^2}\,dx = \frac{16\,\ln{(x+2)}}{25} + \frac{9\,\ln{(x-3)}}{25} + \frac{4}{5\,(x+2)}$$

```
[In] int(x^2/((x + 2)^2*(x - 3)),x)
[Out] (16*log(x + 2))/25 + (9*log(x - 3))/25 + 4/(5*(x + 2))
```

3.190 $\int \frac{-2+3x+5x^2}{2x^2+x^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 14

$$\int \frac{-2+3x+5x^2}{2x^2+x^3} dx = \frac{1}{x} + 2\log(x) + 3\log(2+x)$$

```
[Out] 1/x+2*ln(x)+3*ln(2+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1607, 907}

$$\int \frac{-2+3x+5x^2}{2x^2+x^3} dx = \frac{1}{x} + 2\log(x) + 3\log(x+2)$$

```
[In] Int[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{-2+3x+5x^2}{x^2(2+x)}\,dx\\ &= \int \left(-\frac{1}{x^2}+\frac{2}{x}+\frac{3}{2+x}\right)dx\\ &= \frac{1}{x}+2\log(x)+3\log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = \frac{1}{x}+2\log(x)+3\log(2+x)$$

```
[In] Integrate[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{1}{x}+2\ln\left(x\right)+3\ln\left(2+x\right)$	15
norman	$\frac{1}{x}+2\ln\left(x\right)+3\ln\left(2+x\right)$	15
risch	$\frac{1}{x}+2\ln\left(x\right)+3\ln\left(2+x\right)$	15
parallelrisch	$\frac{2x\ln(x)+3\ln(2+x)x+1}{x}$	19
meijerg	$\frac{1}{x}+2\ln\left(x\right)-2\ln\left(2\right)+3\ln\left(1+\frac{x}{2}\right)$	21

```
[In] int((5*x^2+3*x-2)/(x^3+2*x^2),x,method=_RETURNVERBOSE)
[Out] 1/x+2*ln(x)+3*ln(2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = \frac{3\,x\log\left(x+2\right)+2\,x\log\left(x\right)+1}{x}$$

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="fricas")
[Out] (3*x*log(x + 2) + 2*x*log(x) + 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = 2\log\left(x\right)+3\log\left(x+2\right)+\frac{1}{x}$$

```
[In] integrate((5*x**2+3*x-2)/(x**3+2*x**2),x)
[Out] 2*log(x) + 3*log(x + 2) + 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = \frac{1}{x}+3\,\log\left(x+2\right)+2\,\log\left(x\right)$$

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="maxima")
[Out] 1/x + 3*log(x + 2) + 2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = \frac{1}{x}+3\,\log\left(|x+2|\right)+2\,\log\left(|x|\right)$$

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="giac")
[Out] 1/x + 3*log(abs(x + 2)) + 2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = 3\,\ln(x+2) + 2\,\ln(x) + \frac{1}{x}$$

```
[In] int((3*x + 5*x^2 - 2)/(2*x^2 + x^3),x)
[Out] 3*log(x + 2) + 2*log(x) + 1/x
```

3.191 $\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 19

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} \, dx = \log(1-x) - 2\log(2+x) - 3\log(3+x)$$

```
[Out] ln(1-x)-2*ln(2+x)-3*ln(3+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {2099}

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} \, dx = \log(1-x) - 2\log(x+2) - 3\log(x+3)$$

```
[In] Int[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
[Out] Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{-1+x} - \frac{2}{2+x} - \frac{3}{3+x}\right) dx \\ &= \log(1-x) - 2\log(2+x) - 3\log(3+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = -2\left(-\frac{1}{2}\log(1-x)+\log(2+x)+\frac{3}{2}\log(3+x)\right)$$

```
[In] Integrate[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
[Out] -2*(-1/2*Log[1 - x] + Log[2 + x] + (3*Log[3 + x])/2)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$\ln(-1+x)-2\ln(2+x)-3\ln(3+x)$	18
norman	$\ln(-1+x)-2\ln(2+x)-3\ln(3+x)$	18
risch	$\ln(-1+x)-2\ln(2+x)-3\ln(3+x)$	18
parallelrisch	$\ln(-1+x)-2\ln(2+x)-3\ln(3+x)$	18

```
[In] int((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x,method=_RETURNVERBOSE)
[Out] ln(-1+x)-2*ln(2+x)-3*ln(3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = -3\,\log(x+3)-2\,\log(x+2)+\log(x-1)$$

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="fricas")
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = \log(x-1)-2\log(x+2)-3\log(x+3)$$

```
[In] integrate((-4*x**2-2*x+18)/(x**3+4*x**2+x-6),x)
[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = -3\,\log(x+3)-2\,\log(x+2)+\log(x-1)$$

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="maxima")
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = -3\,\log(|x+3|)-2\,\log(|x+2|)+\log(|x-1|)$$

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="giac")
[Out] -3*log(abs(x + 3)) - 2*log(abs(x + 2)) + log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = \ln(x-1)-2\,\ln(x+2)-3\,\ln(x+3)$$

```
[In] int(-(2*x + 4*x^2 - 18)/(x + 4*x^2 + x^3 - 6),x)
[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

3.192 $\int \frac{2x+x^2}{4+3x^2+x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 15

$$\int \frac{2x+x^2}{4+3x^2+x^3} \, dx = \frac{1}{3} \log \left(4+3x^2+x^3\right)$$

```
[Out] 1/3*ln(x^3+3*x^2+4)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1601}

$$\int \frac{2x+x^2}{4+3x^2+x^3} \, dx = \frac{1}{3} \log \left(x^3+3x^2+4\right)$$

```
[In] Int[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\text{integral} = \frac{1}{3} \log \left(4+3x^2+x^3\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx=\frac{1}{3}\log\left(4+3x^2+x^3\right)$$

```
[In] Integrate[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
[Out] Log[4 + 3*x^2 + x^3]/3
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(x^3+3x^2+4)}{3}$	14
norman	$\frac{\ln(x^3+3x^2+4)}{3}$	14
risch	$\frac{\ln(x^3+3x^2+4)}{3}$	14
parallelrisch	$\frac{\ln(x^3+3x^2+4)}{3}$	14

```
[In] int((x^2+2*x)/(x^3+3*x^2+4),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(x^3+3*x^2+4)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx=\frac{1}{3}\log\left(x^3+3\,x^2+4\right)$$

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="fricas")
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx=\frac{\log\left(x^3+3x^2+4\right)}{3}$$

```
[In] integrate((x**2+2*x)/(x**3+3*x**2+4),x)
[Out] log(x**3 + 3*x**2 + 4)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx=\frac{1}{3}\,\log\left(x^3+3\,x^2+4\right)$$

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="maxima")
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx=\frac{1}{3}\,\log\left(\left|x^3+3\,x^2+4\right|\right)$$

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="giac")
[Out] 1/3*log(abs(x^3 + 3*x^2 + 4))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx=\frac{\ln\left(x^3+3\,x^2+4\right)}{3}$$

```
[In] int((2*x + x^2)/(3*x^2 + x^3 + 4),x)
[Out] log(3*x^2 + x^3 + 4)/3
```

3.193 $\int \frac{1}{(-1+x)^2x^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{1}{(-1+x)^2x^2}\,dx=\frac{1}{1-x}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

```
[Out] 1/(1-x)-1/x-2*ln(1-x)+2*ln(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {46}

$$\int \frac{1}{(-1+x)^2x^2}\,dx=\frac{1}{1-x}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

```
[In] Int[1/((-1 + x)^2*x^2),x]
```

```
[Out] (1 - x)^(-1) - x^(-1) - 2*Log[1 - x] + 2*Log[x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(\frac{1}{(-1+x)^2}-\frac{2}{-1+x}+\frac{1}{x^2}+\frac{2}{x}\right)\,dx\\&=\frac{1}{1-x}-\frac{1}{x}-2\log(1-x)+2\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-1+x)^2x^2}\,dx = -\frac{1}{-1+x} - \frac{1}{x} - 2\log(1-x) + 2\log(x)$$

```
[In] Integrate[1/((-1 + x)^2*x^2),x]
[Out] -(-1 + x)^(-1) - x^(-1) - 2*Log[1 - x] + 2*Log[x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{1}{-1+x} - 2\ln(-1+x) - \frac{1}{x} + 2\ln(x)$	24
norman	$\frac{1-2x}{x(-1+x)} + 2\ln(x) - 2\ln(-1+x)$	26
risch	$\frac{1-2x}{x(-1+x)} + 2\ln(x) - 2\ln(-1+x)$	26
meijerg	$-\frac{1}{x} + 1 + 2\ln(x) + 2i\pi + \frac{3x}{-3x+3} - 2\ln(1-x)$	34
parallelrisch	$\frac{2x^2\ln(x)-2\ln(-1+x)x^2+1-2x\ln(x)+2\ln(-1+x)x-2x}{x(-1+x)}$	43

```
[In] int(1/(-1+x)^2/x^2,x,method=_RETURNVERBOSE)
[Out] -1/(-1+x)-2*ln(-1+x)-1/x+2*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{1}{(-1+x)^2x^2}\,dx = -\frac{2\,(x^2-x)\log(x-1) - 2\,(x^2-x)\log(x) + 2\,x - 1}{x^2-x}$$

```
[In] integrate(1/(-1+x)^2/x^2,x, algorithm="fricas")
[Out] -(2*(x^2 - x)*log(x - 1) - 2*(x^2 - x)*log(x) + 2*x - 1)/(x^2 - x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(-1+x)^2x^2}\,dx = \frac{1-2x}{x^2-x} + 2\log(x) - 2\log(x-1)$$

```
[In] integrate(1/(-1+x)**2/x**2,x)
[Out] (1 - 2*x)/(x**2 - x) + 2*log(x) - 2*log(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(-1+x)^2x^2}\,dx = -\frac{2\,x-1}{x^2-x} - 2\,\log(x-1) + 2\,\log(x)$$

```
[In] integrate(1/(-1+x)^2/x^2,x, algorithm="maxima")
[Out] -(2*x - 1)/(x^2 - x) - 2*log(x - 1) + 2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{1}{(-1+x)^2x^2}\,dx = -\frac{1}{x-1} + \frac{1}{\frac{1}{x-1}+1} + 2\,\log\left(\left|-\frac{1}{x-1}-1\right|\right)$$

```
[In] integrate(1/(-1+x)^2/x^2,x, algorithm="giac")
[Out] -1/(x - 1) + 1/(1/(x - 1) + 1) + 2*log(abs(-1/(x - 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(-1+x)^2x^2}\,dx = \frac{1}{x\,(x-1)} - \frac{2}{x-1} - 2\,\ln\left(\frac{x-1}{x}\right)$$

```
[In] int(1/(x^2*(x - 1)^2),x)
[Out] 1/(x*(x - 1)) - 2/(x - 1) - 2*log((x - 1)/x)
```

3.194 $\int \frac{x^2}{(1+x)^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 21

$$\int \frac{x^2}{(1+x)^3} dx = -\frac{1}{2(1+x)^2} + \frac{2}{1+x} + \log(1+x)$$

```
[Out] -1/2/(1+x)^2+2/(1+x)+ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{45\}$

$$\int \frac{x^2}{(1+x)^3} dx = \frac{2}{x+1} - \frac{1}{2(x+1)^2} + \log(x+1)$$

```
[In] Int[x^2/(1 + x)^3,x]
[Out] -1/2*1/(1 + x)^2 + 2/(1 + x) + Log[1 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{(1+x)^3} - \frac{2}{(1+x)^2} + \frac{1}{1+x}\right) dx \\ &= -\frac{1}{2(1+x)^2} + \frac{2}{1+x} + \log(1+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+x)^3} dx = -\frac{1}{2(1+x)^2} + \frac{2}{1+x} + \log(1+x)$$

```
[In] Integrate[x^2/(1 + x)^3,x]
[Out] -1/2*1/(1 + x)^2 + 2/(1 + x) + Log[1 + x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
norman	$\frac{2x+\frac{3}{2}}{(1+x)^2} + \ln(1+x)$	17
risch	$\frac{2x+\frac{3}{2}}{(1+x)^2} + \ln(1+x)$	17
meijerg	$-\frac{x(9x+6)}{6(1+x)^2} + \ln(1+x)$	19
default	$-\frac{1}{2(1+x)^2} + \frac{2}{1+x} + \ln(1+x)$	20
parallelrisch	$\frac{2\ln(1+x)x^2+3+4\ln(1+x)x+2\ln(1+x)+4x}{2(1+x)^2}$	35

```
[In] int(x^2/(1+x)^3,x,method=_RETURNVERBOSE)
[Out] (2*x+3/2)/(1+x)^2+ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{x^2}{(1+x)^3} dx = \frac{2(x^2+2x+1)\log(x+1) + 4x + 3}{2(x^2+2x+1)}$$

```
[In] integrate(x^2/(1+x)^3,x, algorithm="fricas")
[Out] 1/2*(2*(x^2 + 2*x + 1)*log(x + 1) + 4*x + 3)/(x^2 + 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{(1+x)^3}\,dx = \frac{4x+3}{2x^2+4x+2} + \log\left(x+1\right)$$

```
[In] integrate(x**2/(1+x)**3,x)
[Out] (4*x + 3)/(2*x**2 + 4*x + 2) + log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{(1+x)^3}\,dx = \frac{4\,x+3}{2\,(x^2+2\,x+1)} + \log\left(x+1\right)$$

```
[In] integrate(x^2/(1+x)^3,x, algorithm="maxima")
[Out] 1/2*(4*x + 3)/(x^2 + 2*x + 1) + log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(1+x)^3}\,dx = \frac{4\,x+3}{2\,(x+1)^2} + \log\left(|x+1|\right)$$

```
[In] integrate(x^2/(1+x)^3,x, algorithm="giac")
[Out] 1/2*(4*x + 3)/(x + 1)^2 + log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+x)^3}\,dx = \ln\left(x+1\right) + \frac{2\,x+\frac{3}{2}}{x^2+2\,x+1}$$

```
[In] int(x^2/(x + 1)^3,x)
[Out] log(x + 1) + (2*x + 3/2)/(2*x + x^2 + 1)
```

3.195 $\int \frac{1}{-x^2+x^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{1}{-x^2+x^4} dx = \frac{1}{x} - \operatorname{arctanh}(x)$$

```
[Out] 1/x-arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1607, 331, 213}

$$\int \frac{1}{-x^2+x^4} dx = \frac{1}{x} - \operatorname{arctanh}(x)$$

```
[In] Int[(-x^2 + x^4)^(-1),x]
```

```
[Out] x^(-1) - ArcTanh[x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^2(-1+x^2)}\,dx \\
&= \frac{1}{x} + \int \frac{1}{-1+x^2}\,dx \\
&= \frac{1}{x} - \operatorname{arctanh}(x)
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 22 vs. 2(8) = 16.

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.75

$$\int \frac{1}{-x^2+x^4}\,dx = \frac{1}{x} + \frac{1}{2}\log(1-x) - \frac{1}{2}\log(1+x)$$

```
[In] Integrate[(-x^2 + x^4)^(-1),x]
[Out] x^(-1) + Log[1 - x]/2 - Log[1 + x]/2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

method	result	size
meijerg	$-\frac{i\left(\frac{2i}{x}-2i\operatorname{arctanh}(x)\right)}{2}$	16
default	$\frac{\ln(-1+x)}{2}+\frac{1}{x}-\frac{\ln(1+x)}{2}$	17
norman	$\frac{\ln(-1+x)}{2}+\frac{1}{x}-\frac{\ln(1+x)}{2}$	17
risch	$\frac{\ln(-1+x)}{2}+\frac{1}{x}-\frac{\ln(1+x)}{2}$	17
parallelrisch	$\frac{\ln(-1+x)x-\ln(1+x)x+2}{2x}$	21

```
[In] int(1/(x^4-x^2),x,method=_RETURNVERBOSE)
[Out] -1/2*I*(2*I/x-2*I*arctanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \frac{1}{-x^2+x^4}\,dx = -\frac{x\log(x+1) - x\log(x-1) - 2}{2\,x}$$

```
[In] integrate(1/(x^4-x^2),x, algorithm="fricas")
[Out] -1/2*(x*log(x + 1) - x*log(x - 1) - 2)/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{1}{-x^2+x^4}\,dx = \frac{\log(x-1)}{2} - \frac{\log(x+1)}{2} + \frac{1}{x}$$

```
[In] integrate(1/(x**4-x**2),x)
[Out] log(x - 1)/2 - log(x + 1)/2 + 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{1}{-x^2+x^4}\,dx = \frac{1}{x} - \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1)$$

```
[In] integrate(1/(x^4-x^2),x, algorithm="maxima")
[Out] 1/x - 1/2*log(x + 1) + 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(8) = 16$.

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{-x^2 + x^4}\, dx = \frac{1}{x} - \frac{1}{2}\log\left(|x+1|\right) + \frac{1}{2}\log\left(|x-1|\right)$$

```
[In] integrate(1/(x^4-x^2),x, algorithm="giac")
[Out] 1/x - 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x^2 + x^4}\, dx = \frac{1}{x} - \operatorname{atanh}(x)$$

```
[In] int(-1/(x^2 - x^4),x)
[Out] 1/x - atanh(x)
```

3.196 $\int \frac{-x+2x^3}{1-x^2+x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 15

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{1}{2}\log\left(1-x^2+x^4\right)$$

```
[Out] 1/2*ln(x^4-x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1601}

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{1}{2}\log\left(x^4-x^2+1\right)$$

```
[In] Int[(-x + 2*x^3)/(1 - x^2 + x^4),x]
```

```
[Out] Log[1 - x^2 + x^4]/2
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\log\left(1-x^2+x^4\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx=\frac{1}{2}\log\left(1-x^2+x^4\right)$$

```
[In] Integrate[(-x + 2*x^3)/(1 - x^2 + x^4),x]
[Out] Log[1 - x^2 + x^4]/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln\left(x^4-x^2+1\right)}{2}$	14
norman	$\frac{\ln\left(x^4-x^2+1\right)}{2}$	14
risch	$\frac{\ln\left(x^4-x^2+1\right)}{2}$	14
parallelrisch	$\frac{\ln\left(x^4-x^2+1\right)}{2}$	14

```
[In] int((2*x^3-x)/(x^4-x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^4-x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx=\frac{1}{2}\log\left(x^4-x^2+1\right)$$

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="fricas")
[Out] 1/2*log(x^4 - x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx=\frac{\log\left(x^4-x^2+1\right)}{2}$$

```
[In] integrate((2*x**3-x)/(x**4-x**2+1),x)
[Out] log(x**4 - x**2 + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx=\frac{1}{2}\log\left(x^4-x^2+1\right)$$

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="maxima")
[Out] 1/2*log(x^4 - x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx=\frac{1}{2}\log\left(x^4-x^2+1\right)$$

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="giac")
[Out] 1/2*log(x^4 - x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx=\frac{\ln\left(x^4-x^2+1\right)}{2}$$

```
[In] int(-(x - 2*x^3)/(x^4 - x^2 + 1),x)
[Out] log(x^4 - x^2 + 1)/2
```

3.197 $\int \frac{x^3}{1+x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{x^3}{1+x^2}\,dx = \frac{x^2}{2} - \frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] 1/2*x^2-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {272, 45}

$$\int \frac{x^3}{1+x^2}\,dx = \frac{x^2}{2} - \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] Int[x^3/(1 + x^2),x]
```

```
[Out] x^2/2 - Log[1 + x^2]/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, x^2\right)\\ &= \frac{x^2}{2}-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+x^2}\,dx = \frac{x^2}{2}-\frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[x^3/(1 + x^2),x]
[Out] x^2/2 - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}$	15
norman	$\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}$	15
meijerg	$\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}$	15
risch	$\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}$	15
parallelrisch	$\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}$	15

```
[In] int(x^3/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate(x^3/(x^2+1),x, algorithm="fricas")
[Out] 1/2*x^2 - 1/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{1+x^2}\,dx = \frac{x^2}{2} - \frac{\log\left(x^2+1\right)}{2}$$

```
[In] integrate(x**3/(x**2+1),x)
[Out] x**2/2 - log(x**2 + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate(x^3/(x^2+1),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate(x^3/(x^2+1),x, algorithm="giac")
[Out] 1/2*x^2 - 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^2}\,dx = \frac{x^2}{2} - \frac{\ln(x^2+1)}{2}$$

```
[In] int(x^3/(x^2 + 1),x)
[Out] x^2/2 - log(x^2 + 1)/2
```

3.198 $\int \frac{-1+x}{2+2x+x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 20

$$\int \frac{-1+x}{2+2x+x^2} \, dx = -2\arctan(1+x)+\frac{1}{2}\log\left(2+2x+x^2\right)$$

```
[Out] -2*arctan(1+x)+1/2*ln(x^2+2*x+2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {648, 631, 210, 642}

$$\int \frac{-1+x}{2+2x+x^2} \, dx = \frac{1}{2}\log\left(x^2+2x+2\right)-2\arctan(x+1)$$

```
[In] Int[(-1 + x)/(2 + 2*x + x^2),x]
```

```
[Out] -2*ArcTan[1 + x] + Log[2 + 2*x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int \frac{2+2x}{2+2x+x^2}\,dx - 2\int \frac{1}{2+2x+x^2}\,dx\\ &= \frac{1}{2}\log\left(2+2x+x^2\right) + 2\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+x\right)\\ &= -2\arctan(1+x) + \frac{1}{2}\log\left(2+2x+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{-1+x}{2+2x+x^2}\,dx = -2\arctan(1+x) + \frac{1}{2}\log\left(2+2x+x^2\right)$$

```
[In] Integrate[(-1 + x)/(2 + 2*x + x^2),x]
[Out] -2*ArcTan[1 + x] + Log[2 + 2*x + x^2]/2
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$-2\arctan\left(1+x\right)+\frac{\ln\left(x^{2}+2x+2\right)}{2}$	19
risch	$-2\arctan\left(1+x\right)+\frac{\ln\left(x^{2}+2x+2\right)}{2}$	19
parallelrisch	$\frac{\ln\left(x+1-i\right)}{2}+i\ln\left(x+1-i\right)+\frac{\ln\left(x+1+i\right)}{2}-i\ln\left(x+1+i\right)$	36

```
[In] int((-1+x)/(x^2+2*x+2),x,method=_RETURNVERBOSE)
[Out] -2*arctan(1+x)+1/2*ln(x^2+2*x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{-1+x}{2+2x+x^2}\,dx=-2\,\arctan\left(x+1\right)+\frac{1}{2}\,\log\left(x^{2}+2\,x+2\right)$$

```
[In] integrate((-1+x)/(x^2+2*x+2),x, algorithm="fricas")
[Out] -2*arctan(x + 1) + 1/2*log(x^2 + 2*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{-1+x}{2+2x+x^2}\,dx=\frac{\log\left(x^{2}+2x+2\right)}{2}-2\operatorname{atan}\left(x+1\right)$$

```
[In] integrate((-1+x)/(x**2+2*x+2),x)
[Out] log(x**2 + 2*x + 2)/2 - 2*atan(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{-1+x}{2+2x+x^2}\,dx=-2\,\arctan\left(x+1\right)+\frac{1}{2}\,\log\left(x^{2}+2\,x+2\right)$$

```
[In] integrate((-1+x)/(x^2+2*x+2),x, algorithm="maxima")
[Out] -2*arctan(x + 1) + 1/2*log(x^2 + 2*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{-1+x}{2+2x+x^2}\,dx = -2\,\arctan\left(x+1\right) + \frac{1}{2}\,\log\left(x^2+2\,x+2\right)$$

```
[In] integrate((-1+x)/(x^2+2*x+2),x, algorithm="giac")
[Out] -2*arctan(x + 1) + 1/2*log(x^2 + 2*x + 2)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{-1+x}{2+2x+x^2}\,dx = \frac{\ln\left(x^2+2\,x+2\right)}{2} - 2\,\mathrm{atan}(x+1)$$

```
[In] int((x - 1)/(2*x + x^2 + 2),x)
[Out] log(2*x + x^2 + 2)/2 - 2*atan(x + 1)
```

3.199 $\int \frac{x}{1+x+x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \frac{x}{1+x+x^2}\,dx = -\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(1+x+x^2\right)$$

```
[Out] 1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {648, 632, 210, 642}

$$\int \frac{x}{1+x+x^2}\,dx = \frac{1}{2}\log\left(x^2+x+1\right) - \frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Int[x/(1 + x + x^2),x]
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx\right)+\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx\\
&= \frac{1}{2}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= -\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x+x^2}\,dx=-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1+x+x^2\right)$$

```
[In] Integrate[x/(1 + x + x^2),x]
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x + x^2]/2
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\ln(x^2+x+1)}{2}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	27
risch	$-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(4x^2+4x+4)}{2}$	31

```
[In] int(x/(x^2+x+1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x}{1+x+x^2}\,dx=-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2}\log\left(x^2+x+1\right)$$

```
[In] integrate(x/(x^2+x+1),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*log(x^2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{x}{1+x+x^2}\,dx=\frac{\log\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate(x/(x**2+x+1),x)
[Out] log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x}{1+x+x^2} \, dx = -\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) + \frac{1}{2} \log\left(x^2+x+1\right)$$

```
[In] integrate(x/(x^2+x+1),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*log(x^2 + x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x}{1+x+x^2} \, dx = -\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) + \frac{1}{2} \log\left(x^2+x+1\right)$$

```
[In] integrate(x/(x^2+x+1),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*log(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{x}{1+x+x^2} \, dx = \frac{\ln\left(x^2+x+1\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] int(x/(x + x^2 + 1),x)
[Out] log(x + x^2 + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3))/3
```

3.200 $\int \frac{7+5x+4x^2}{5+4x+4x^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{7+5x+4x^2}{5+4x+4x^2} dx = x + \frac{3}{8} \arctan\left(\frac{1}{2}+x\right) + \frac{1}{8} \log\left(5+4x+4x^2\right)$$

```
[Out] x+3/8*arctan(1/2+x)+1/8*ln(4*x^2+4*x+5)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1671, 648, 632, 210, 642}

$$\int \frac{7+5x+4x^2}{5+4x+4x^2} dx = \frac{3}{8} \arctan\left(x+\frac{1}{2}\right) + \frac{1}{8} \log\left(4x^2+4x+5\right) + x$$

```
[In] Int[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
```

```
[Out] x + (3*ArcTan[1/2 + x])/8 + Log[5 + 4*x + 4*x^2]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(1+\frac{2+x}{5+4x+4x^2}\right) dx \\
&= x+\int \frac{2+x}{5+4x+4x^2}\,dx \\
&= x+\frac{1}{8}\int \frac{4+8x}{5+4x+4x^2}\,dx+\frac{3}{2}\int \frac{1}{5+4x+4x^2}\,dx \\
&= x+\frac{1}{8}\log\left(5+4x+4x^2\right)-3\text{Subst}\left(\int \frac{1}{-64-x^2}\,dx,x,4+8x\right) \\
&= x+\frac{3}{8}\arctan\left(\frac{1}{2}+x\right)+\frac{1}{8}\log\left(5+4x+4x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx = x+\frac{3}{8}\arctan\left(\frac{1}{2}(1+2x)\right)+\frac{1}{8}\log\left(5+4x+4x^2\right)$$

```
[In] Integrate[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
[Out] x + (3*ArcTan[(1 + 2*x)/2])/8 + Log[5 + 4*x + 4*x^2]/8
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
default	$x+\frac{3\arctan\left(x+\frac{1}{2}\right)}{8}+\frac{\ln\left(4x^{2}+4x+5\right)}{8}$	22
risch	$x+\frac{3\arctan\left(x+\frac{1}{2}\right)}{8}+\frac{\ln\left(4x^{2}+4x+5\right)}{8}$	22
parallelrisch	$x+\frac{\ln\left(x+\frac{1}{2}-i\right)}{8}-\frac{3i\ln\left(x+\frac{1}{2}-i\right)}{16}+\frac{\ln\left(x+\frac{1}{2}+i\right)}{8}+\frac{3i\ln\left(x+\frac{1}{2}+i\right)}{16}$	37

```
[In] int((4*x^2+5*x+7)/(4*x^2+4*x+5),x,method=_RETURNVERBOSE)
[Out] x+3/8*arctan(x+1/2)+1/8*ln(4*x^2+4*x+5)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx = x+\frac{3}{8}\,\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\,\log\left(4\,x^{2}+4\,x+5\right)$$

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="fricas")
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx = x + \frac{\log\left(x^2+x+\frac{5}{4}\right)}{8} + \frac{3\,\mathrm{atan}\left(x+\frac{1}{2}\right)}{8}$$

```
[In] integrate((4*x**2+5*x+7)/(4*x**2+4*x+5),x)
[Out] x + log(x**2 + x + 5/4)/8 + 3*atan(x + 1/2)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx = x + \frac{3}{8}\arctan\left(x+\frac{1}{2}\right) + \frac{1}{8}\log\left(4\,x^2+4\,x+5\right)$$

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="maxima")
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx = x + \frac{3}{8}\arctan\left(x+\frac{1}{2}\right) + \frac{1}{8}\log\left(4\,x^2+4\,x+5\right)$$

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="giac")
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx = x + \frac{\ln\left(x^2+x+\frac{5}{4}\right)}{8} + \frac{3\,\mathrm{atan}\left(x+\frac{1}{2}\right)}{8}$$

```
[In] int((5*x + 4*x^2 + 7)/(4*x + 4*x^2 + 5),x)
[Out] x + log(x + x^2 + 5/4)/8 + (3*atan(x + 1/2))/8
```

3.201 $\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = -3\arctan(x)+2\log(1-x)+\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -3*arctan(x)+2*ln(1-x)+1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1643, 649, 209, 266}

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = -3\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)$$

```
[In] Int[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2}{-1+x}+\frac{-3+x}{1+x^2}\right) dx \\
&= 2\log(1-x)+\int \frac{-3+x}{1+x^2}\,dx \\
&= 2\log(1-x)-3\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx \\
&= -3\arctan(x)+2\log(1-x)+\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)}\,dx = -3\arctan(x)+\frac{1}{2}\log\left(2+2(-1+x)+(-1+x)^2\right)+2\log(-1+x)$$

```
[In] Integrate[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
[Out] -3*ArcTan[x] + Log[2 + 2*(-1 + x) + (-1 + x)^2]/2 + 2*Log[-1 + x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$2\ln\left(-1+x\right)+\frac{\ln\left(x^2+1\right)}{2}-3\arctan\left(x\right)$	20
risch	$2\ln\left(-1+x\right)+\frac{\ln\left(9x^2+9\right)}{2}-3\arctan\left(x\right)$	22
parallelrisch	$2\ln\left(-1+x\right)+\frac{\ln(x-i)}{2}+\frac{3i\ln(x-i)}{2}+\frac{\ln(x+i)}{2}-\frac{3i\ln(x+i)}{2}$	38

```
[In] int((3*x^2-4*x+5)/(-1+x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 2*ln(-1+x)+1/2*ln(x^2+1)-3*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = -3\,\arctan(x) + \frac{1}{2}\,\log\left(x^2+1\right) + 2\,\log(x-1)$$

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="fricas")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = 2\log(x-1) + \frac{\log(x^2+1)}{2} - 3\,\mathrm{atan}(x)$$

```
[In] integrate((3*x**2-4*x+5)/(-1+x)/(x**2+1),x)
[Out] 2*log(x - 1) + log(x**2 + 1)/2 - 3*atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = -3\,\arctan(x) + \frac{1}{2}\,\log\left(x^2+1\right) + 2\,\log(x-1)$$

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="maxima")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = -3\arctan(x) + \frac{1}{2}\log\left(x^2+1\right) + 2\log\left(|x-1|\right)$$

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="giac")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = 2\ln(x-1) + \ln(x-\mathrm{i})\left(\frac{1}{2}+\frac{3}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(\frac{1}{2}-\frac{3}{2}\mathrm{i}\right)$$

```
[In] int((3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)),x)
[Out] 2*log(x - 1) + log(x - 1i)*(1/2 + 3i/2) + log(x + 1i)*(1/2 - 3i/2)
```

3.202 $\int \frac{3+2x}{3x+x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{3+2x}{3x+x^3} \, dx = \frac{2\arctan\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x) - \frac{1}{2}\log\left(3+x^2\right)$$

```
[Out] ln(x)-1/2*ln(x^2+3)+2/3*arctan(1/3*x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1607, 815, 649, 209, 266}

$$\int \frac{3+2x}{3x+x^3} \, dx = \frac{2\arctan\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{2}\log\left(x^2+3\right) + \log(x)$$

```
[In] Int[(3 + 2*x)/(3*x + x^3),x]
```

```
[Out] (2*ArcTan[x/Sqrt[3]])/Sqrt[3] + Log[x] - Log[3 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{3+2x}{x\left(3+x^2\right)}\,dx \\
&= \int \left(\frac{1}{x}+\frac{2-x}{3+x^2}\right)\,dx \\
&= \log(x)+\int \frac{2-x}{3+x^2}\,dx \\
&= \log(x)+2\int \frac{1}{3+x^2}\,dx-\int \frac{x}{3+x^2}\,dx \\
&= \frac{2\arctan\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{2}\log\left(3+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{3+2x}{3x+x^3}\,dx=\frac{2\arctan\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{2}\log\left(3+x^2\right)$$

```
[In] Integrate[(3 + 2*x)/(3*x + x^3),x]
[Out] (2*ArcTan[x/Sqrt[3]])/Sqrt[3] + Log[x] - Log[3 + x^2]/2
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
default	$\ln\left(x\right)-\frac{\ln\left(x^{2}+3\right)}{2}+\frac{2\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}$	24
risch	$\ln\left(x\right)-\frac{\ln\left(x^{2}+3\right)}{2}+\frac{2\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}$	24
meijerg	$\ln\left(x\right)-\frac{\ln(3)}{2}-\frac{\ln\left(\frac{x^{2}}{3}+1\right)}{2}+\frac{2\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}$	30

```
[In] int((3+2*x)/(x^3+3*x),x,method=_RETURNVERBOSE)
[Out] ln(x)-1/2*ln(x^2+3)+2/3*arctan(1/3*x*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{3x+x^3}\,dx=\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right)-\frac{1}{2}\,\log\left(x^2+3\right)+\log\left(x\right)$$

```
[In] integrate((3+2*x)/(x^3+3*x),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/2*log(x^2 + 3) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{3+2x}{3x+x^3}\,dx=\log\left(x\right)-\frac{\log\left(x^2+3\right)}{2}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)}{3}$$

```
[In] integrate((3+2*x)/(x**3+3*x),x)
[Out] log(x) - log(x**2 + 3)/2 + 2*sqrt(3)*atan(sqrt(3)*x/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{3x+x^3}\,dx = \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right) - \frac{1}{2}\,\log\left(x^2+3\right) + \log\left(x\right)$$

```
[In] integrate((3+2*x)/(x^3+3*x),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/2*log(x^2 + 3) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{3+2x}{3x+x^3}\,dx = \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right) - \frac{1}{2}\,\log\left(x^2+3\right) + \log\left(|x|\right)$$

```
[In] integrate((3+2*x)/(x^3+3*x),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/2*log(x^2 + 3) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{3+2x}{3x+x^3}\,dx = \ln\left(x\right) - \frac{\ln\left(x+\sqrt{3}\,1\mathrm{i}\right)}{2} - \frac{\ln\left(x-\sqrt{3}\,1\mathrm{i}\right)}{2} - \frac{\sqrt{3}\,\ln\left(x-\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{3} + \frac{\sqrt{3}\,\ln\left(x+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{3}$$

```
[In] int((2*x + 3)/(3*x + x^3),x)
[Out] log(x) - log(x + 3^(1/2)*1i)/2 - log(x - 3^(1/2)*1i)/2 - (3^(1/2)*log(x - 3
^(1/2)*1i)*1i)/3 + (3^(1/2)*log(x + 3^(1/2)*1i)*1i)/3
```

3.203 $\int \frac{1}{-1+x^3} dx$

Optimal result

Integrand size = 7, antiderivative size = 41

$$\int \frac{1}{-1+x^3} dx = -\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)$$

```
[Out] 1/3*ln(1-x)-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {206, 31, 648, 632, 210, 642}

$$\int \frac{1}{-1+x^3} dx = -\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log(1-x)$$

```
[In] Int[(-1 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\int \frac{1}{-1+x}\,dx + \frac{1}{3}\int \frac{-2-x}{1+x+x^2}\,dx \\
&= \frac{1}{3}\log(1-x) - \frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx - \frac{1}{2}\int \frac{1}{1+x+x^2}\,dx \\
&= \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) \\
&= -\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{-1+x^3}\,dx=-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)$$

```
[In] Integrate[(-1 + x^3)^(-1),x]
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{3}$	31
default	$\frac{\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{6}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	33
meijerg	$\frac{x\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2+\left(x^3\right)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{1}{3}}}$	62

```
[In] int(1/(x^3-1),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(-1+x)-1/6*ln(x^2+x+1)-1/3*3^(1/2)*arctan(2/3*3^(1/2)*(x+1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1}{-1+x^3}\,dx=-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{1}{6}\,\log\left(x^2+x+1\right)+\frac{1}{3}\,\log\left(x-1\right)$$

```
[In] integrate(1/(x^3-1),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log
(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{-1+x^3}\,dx = \frac{\log(x-1)}{3} - \frac{\log(x^2+x+1)}{6} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate(1/(x**3-1),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3
)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1}{-1+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log(x-1)$$

```
[In] integrate(1/(x^3-1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log
(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{1}{-1+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log(|x-1|)$$

```
[In] integrate(1/(x^3-1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log
(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{1}{-1+x^3}\,dx = \frac{\ln(x-1)}{3} + \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

```
[In] int(1/(x^3 - 1),x)
```

```
[Out] log(x - 1)/3 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x
 + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6)
```

3.204 $\int \frac{x^3}{1+x^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 41

$$\int \frac{x^3}{1+x^3}\,dx = x+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)$$

```
[Out] x-1/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {327, 206, 31, 648, 632, 210, 642}

$$\int \frac{x^3}{1+x^3}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(x^2-x+1\right)+x-\frac{1}{3}\log(x+1)$$

```
[In] Int[x^3/(1 + x^3),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x-\int \frac{1}{1+x^3}\,dx\\
&= x-\frac{1}{3}\int \frac{1}{1+x}\,dx-\frac{1}{3}\int \frac{2-x}{1-x+x^2}\,dx\\
&= x-\frac{1}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx
\end{aligned}
$$

$$= x - \frac{1}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right)$$

$$= x - \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{1+x^3}\,dx = x - \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right)$$

```
[In] Integrate[x^3/(1 + x^3),x]
[Out] x - ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

method	result	size
risch	$x-\frac{\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	34
default	$x+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{\ln(1+x)}{3}$	36
meijerg	$x-\frac{x\left(\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}\right)}{\left(x^3\right)^{\frac{1}{3}}}-\frac{\ln\left(1-\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2\left(x^3\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2-\left(x^3\right)^{\frac{1}{3}}}\right)}{\left(x^3\right)^{\frac{1}{3}}}\right)}{3}$	74

```
[In] int(x^3/(x^3+1),x,method=_RETURNVERBOSE)
[Out] x-1/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{1+x^3}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+x+\frac{1}{6}\,\log\left(x^2-x+1\right)-\frac{1}{3}\,\log\left(x+1\right)$$

```
[In] integrate(x^3/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x + 1/6*log(x^2 - x + 1) - 1/3
*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{1+x^3}\,dx = x-\frac{\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate(x**3/(x**3+1),x)
```

```
[Out] x - log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(
3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{1+x^3}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+x+\frac{1}{6}\,\log\left(x^2-x+1\right)-\frac{1}{3}\,\log\left(x+1\right)$$

```
[In] integrate(x^3/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x + 1/6*log(x^2 - x + 1) - 1/3
*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{1+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+x+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(|x+1|\right)$$

```
[In] integrate(x^3/(x^3+1),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x + 1/6*log(x^2 - x + 1) - 1/3
*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{x^3}{1+x^3}\,dx = x-\frac{\ln(x+1)}{3}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

```
[In] int(x^3/(x^3 + 1),x)
[Out] x - log(x + 1)/3 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6) - l
og(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6)
```

3.205 $\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 24

$$\int \frac{-1-2x+x^2}{(-1+x)^2\,(1+x^2)}\,dx=\frac{1}{-1+x}+\arctan(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] 1/(-1+x)+arctan(x)+ln(1-x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1643, 649, 209, 266}

$$\int \frac{-1-2x+x^2}{(-1+x)^2\,(1+x^2)}\,dx=\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)$$

```
[In] Int[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{(-1+x)^2}+\frac{1}{-1+x}+\frac{1-x}{1+x^2}\right) dx \\
&= \frac{1}{-1+x}+\log(1-x)+\int \frac{1-x}{1+x^2}\, dx \\
&= \frac{1}{-1+x}+\log(1-x)+\int \frac{1}{1+x^2}\, dx-\int \frac{x}{1+x^2}\, dx \\
&= \frac{1}{-1+x}+\arctan(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$
\int \frac{-1-2x+x^2}{(-1+x)^2\left(1+x^2\right)}\, dx = \frac{1}{-1+x}+\arctan(x)+\log(-1+x)-\frac{1}{2}\log\left(1+x^2\right)
$$

```
[In] Integrate[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[-1 + x] - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$\ln(-1+x)+\frac{1}{-1+x}-\frac{\ln(x^2+1)}{2}+\arctan(x)$	21
risch	$\ln(-1+x)+\frac{1}{-1+x}-\frac{\ln(x^2+1)}{2}+\arctan(x)$	21
parallelrisch	$\frac{-i\ln(x-i)x+i\ln(x+i)x+2\ln(-1+x)x+i\ln(x-i)-\ln(x-i)x-i\ln(x+i)-\ln(x+i)x+2-2\ln(-1+x)+\ln(x-i)+\ln(x+i)}{-2+2x}$	83

```
[In] int((x^2-2*x-1)/(-1+x)^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] ln(-1+x)+1/(-1+x)-1/2*ln(x^2+1)+arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \frac{2(x-1)\arctan(x)-(x-1)\log(x^2+1)+2(x-1)\log(x-1)+2}{2(x-1)}$$

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
[Out] 1/2*(2*(x - 1)*arctan(x) - (x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x - 1) + 2)
/(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \log(x-1)-\frac{\log(x^2+1)}{2}+\operatorname{atan}(x)+\frac{1}{x-1}$$

```
[In] integrate((x**2-2*x-1)/(-1+x)**2/(x**2+1),x)
[Out] log(x - 1) - log(x**2 + 1)/2 + atan(x) + 1/(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \frac{1}{x-1}+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x-1)$$

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
[Out] 1/(x - 1) + arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(22) = 44.

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \frac{1}{4}\pi-\pi\left\lfloor\frac{\pi+4\arctan(x)}{4\pi}+\frac{1}{2}\right\rfloor+\frac{1}{x-1}+\arctan(x)-\frac{1}{2}\log\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)$$

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="giac")
[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/(x - 1) + arctan(x)
- 1/2*log(2/(x - 1) + 2/(x - 1)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \ln(x-1)+\frac{1}{x-1}+\ln(x-\mathrm{i})\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

```
[In] int(-(2*x - x^2 + 1)/((x^2 + 1)*(x - 1)^2),x)
[Out] log(x - 1) - log(x - 1i)*(1/2 + 1i/2) - log(x + 1i)*(1/2 - 1i/2) + 1/(x - 1
)
```

3.206 $\int \frac{x^4}{-1+x^4} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{x^4}{-1+x^4} \, dx = x - \frac{\arctan(x)}{2} - \frac{\operatorname{arctanh}(x)}{2}$$

```
[Out] x-1/2*arctan(x)-1/2*arctanh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {327, 218, 212, 209}

$$\int \frac{x^4}{-1+x^4} \, dx = -\frac{\arctan(x)}{2} - \frac{\operatorname{arctanh}(x)}{2} + x$$

```
[In] Int[x^4/(-1 + x^4),x]
```

```
[Out] x - ArcTan[x]/2 - ArcTanh[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x + \int \frac{1}{-1+x^4}\,dx\\ &= x - \frac{1}{2}\int \frac{1}{1-x^2}\,dx - \frac{1}{2}\int \frac{1}{1+x^2}\,dx\\ &= x - \frac{\arctan(x)}{2} - \frac{\operatorname{arctanh}(x)}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{x^4}{-1+x^4}\,dx = x - \frac{\arctan(x)}{2} + \frac{1}{4}\log(1-x) - \frac{1}{4}\log(1+x)$$

```
[In] Integrate[x^4/(-1 + x^4),x]
[Out] x - ArcTan[x]/2 + Log[1 - x]/4 - Log[1 + x]/4
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

method	result	size
default	$x+\frac{\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}-\frac{\arctan(x)}{2}$	19
risch	$x+\frac{\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}-\frac{\arctan(x)}{2}$	19
parallelrisch	$x+\frac{i\ln(x-i)}{4}-\frac{i\ln(x+i)}{4}-\frac{\ln(1+x)}{4}+\frac{\ln(-1+x)}{4}$	31
meijerg	$-\frac{(-1)^{\frac{3}{4}}\left(4(-1)^{\frac{1}{4}}x+\frac{x(-1)^{\frac{1}{4}}\left(\ln\left(1-\left(x^4\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^4\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(x^4\right)^{\frac{1}{4}}\right)\right)}{\left(x^4\right)^{\frac{1}{4}}}\right)}{4}$	52

```
[In] int(x^4/(x^4-1),x,method=_RETURNVERBOSE)
[Out] x+1/4*ln(-1+x)-1/4*ln(1+x)-1/2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{x^4}{-1+x^4}\,dx = x-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

```
[In] integrate(x^4/(x^4-1),x, algorithm="fricas")
[Out] x - 1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{x^4}{-1+x^4}\,dx = x+\frac{\log(x-1)}{4}-\frac{\log(x+1)}{4}-\frac{\operatorname{atan}(x)}{2}$$

```
[In] integrate(x**4/(x**4-1),x)
[Out] x + log(x - 1)/4 - log(x + 1)/4 - atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{x^4}{-1+x^4} \, dx = x - \frac{1}{2} \arctan(x) - \frac{1}{4} \log(x+1) + \frac{1}{4} \log(x-1)$$

```
[In] integrate(x^4/(x^4-1),x, algorithm="maxima")
[Out] x - 1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{x^4}{-1+x^4} \, dx = x - \frac{1}{2} \arctan(x) - \frac{1}{4} \log(|x+1|) + \frac{1}{4} \log(|x-1|)$$

```
[In] integrate(x^4/(x^4-1),x, algorithm="giac")
[Out] x - 1/2*arctan(x) - 1/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^4}{-1+x^4} \, dx = x - \frac{\text{atan}(x)}{2} - \frac{\text{atanh}(x)}{2}$$

```
[In] int(x^4/(x^4 - 1),x)
[Out] x - atan(x)/2 - atanh(x)/2
```

3.207 $\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 29

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)} \, dx = -3\arctan(x)+\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(1+x^2\right)$$

```
[Out] -3*arctan(x)+3/2*ln(x^2+1)+arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6857, 649, 209, 266}

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)} \, dx = -3\arctan(x)+\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(x^2+1\right)$$

```
[In] Int[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3(-1+x)}{1+x^2}+\frac{2}{2+x^2}\right) dx \\
&= 2\int \frac{1}{2+x^2}\, dx+3\int \frac{-1+x}{1+x^2}\, dx \\
&= \sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)-3\int \frac{1}{1+x^2}\, dx+3\int \frac{x}{1+x^2}\, dx \\
&= -3\arctan(x)+\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\, dx = -3\arctan(x)+\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

method	result	size
default	$-3\arctan\left(x\right)+\frac{3\ln\left(x^{2}+1\right)}{2}+\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	25
risch	$-3\arctan\left(x\right)+\frac{3\ln\left(x^{2}+1\right)}{2}+\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	25

```
[In] int((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x,method=_RETURNVERBOSE)
[Out] -3*arctan(x)+3/2*ln(x^2+1)+arctan(1/2*x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx = \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3\arctan(x) + \frac{3}{2}\log\left(x^2+1\right)$$

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="fricas")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx = \frac{3\log(x^2+1)}{2} - 3\,\mathrm{atan}(x) + \sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

```
[In] integrate((3*x**3-x**2+6*x-4)/(x**2+1)/(x**2+2),x)
[Out] 3*log(x**2 + 1)/2 - 3*atan(x) + sqrt(2)*atan(sqrt(2)*x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx = \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3\arctan(x) + \frac{3}{2}\log\left(x^2+1\right)$$

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="maxima")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx=\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-3\arctan(x)+\frac{3}{2}\log\left(x^2+1\right)$$

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="giac")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.76

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx=-\sqrt{2}\,\mathrm{atan}\left(\frac{24\sqrt{2}}{24x-64}+\frac{32\sqrt{2}\,x}{24x-64}\right)+\ln(x-\mathrm{i})\left(\frac{3}{2}+\frac{3}{2}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)$$

```
[In] int((6*x - x^2 + 3*x^3 - 4)/((x^2 + 1)*(x^2 + 2)),x)
[Out] log(x - 1i)*(3/2 + 3i/2) + log(x + 1i)*(3/2 - 3i/2) - 2^(1/2)*atan((24*2^(1
/2))/(24*x - 64) + (32*2^(1/2)*x)/(24*x - 64))
```

3.208 $\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 23

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx=-\frac{3}{2}\arctan\left(\frac{x}{2}\right)+\arctan(x)+\frac{1}{2}\log\left(4+x^2\right)$$

```
[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1687, 1180, 209, 1261, 640, 31}

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx=-\frac{3}{2}\arctan\left(\frac{x}{2}\right)+\arctan(x)+\frac{1}{2}\log\left(x^2+4\right)$$

```
[In] Int[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
```

```
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 640

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1-2x^2}{4+5x^2+x^4}\,dx + \int \frac{x(1+x^2)}{4+5x^2+x^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{4+5x+x^2}\,dx, x, x^2\right) - 3\int \frac{1}{4+x^2}\,dx + \int \frac{1}{1+x^2}\,dx \\
&= -\frac{3}{2}\arctan\left(\frac{x}{2}\right) + \arctan(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{4+x}\,dx, x, x^2\right) \\
&= -\frac{3}{2}\arctan\left(\frac{x}{2}\right) + \arctan(x) + \frac{1}{2}\log\left(4+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx=-\frac{3}{2}\arctan\left(\frac{x}{2}\right)+\arctan(x)+\frac{1}{2}\log\left(4+x^2\right)$$

```
[In] Integrate[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{3\arctan\left(\frac{x}{2}\right)}{2}+\arctan\left(x\right)+\frac{\ln\left(x^{2}+4\right)}{2}$	18
risch	$-\frac{3\arctan\left(\frac{x}{2}\right)}{2}+\arctan\left(x\right)+\frac{\ln\left(x^{2}+4\right)}{2}$	18
parallelrisch	$\frac{\ln(x-2i)}{2}+\frac{3i\ln(x-2i)}{4}-\frac{i\ln(x-i)}{2}+\frac{i\ln(x+i)}{2}+\frac{\ln(x+2i)}{2}-\frac{3i\ln(x+2i)}{4}$	48

```
[In] int((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x,method=_RETURNVERBOSE)
[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx=-\frac{3}{2}\arctan\left(\frac{1}{2}\,x\right)+\arctan\left(x\right)+\frac{1}{2}\log\left(x^2+4\right)$$

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="fricas")
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx = \frac{\log\left(x^2+4\right)}{2} - \frac{3\,\mathrm{atan}\left(\frac{x}{2}\right)}{2} + \mathrm{atan}\,(x)$$

```
[In] integrate((x**3-2*x**2+x+1)/(x**4+5*x**2+4),x)
[Out] log(x**2 + 4)/2 - 3*atan(x/2)/2 + atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx = -\frac{3}{2}\arctan\left(\frac{1}{2}\,x\right) + \arctan\,(x) + \frac{1}{2}\log\left(x^2+4\right)$$

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="maxima")
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx = -\frac{3}{2}\arctan\left(\frac{1}{2}\,x\right) + \arctan\,(x) + \frac{1}{2}\log\left(x^2+4\right)$$

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="giac")
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx = -\mathrm{atan}\left(\frac{1305}{4\,(144\,x-162)} + \frac{9}{8}\right) + \ln\,(x-2\mathrm{i})\,\left(\frac{1}{2}+\frac{3}{4}\mathrm{i}\right) + \ln\,(x+2\mathrm{i})\,\left(\frac{1}{2}-\frac{3}{4}\mathrm{i}\right)$$

```
[In] int((x - 2*x^2 + x^3 + 1)/(5*x^2 + x^4 + 4),x)
[Out] log(x - 2i)*(1/2 + 3i/4) + log(x + 2i)*(1/2 - 3i/4) - atan(1305/(4*(144*x -
 162)) + 9/8)
```

3.209 $\int \frac{-3+x}{(4+2x+x^2)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{-3+x}{(4+2x+x^2)^2}\,dx = \frac{-7-4x}{6\,(4+2x+x^2)} - \frac{2\arctan\left(\frac{1+x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 1/6*(-7-4*x)/(x^2+2*x+4)-2/9*arctan(1/3*(1+x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {652, 632, 210}

$$\int \frac{-3+x}{(4+2x+x^2)^2}\,dx = -\frac{2\arctan\left(\frac{x+1}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{4x+7}{6\,(x^2+2x+4)}$$

```
[In] Int[(-3 + x)/(4 + 2*x + x^2)^2,x]
```

```
[Out] -1/6*(7 + 4*x)/(4 + 2*x + x^2) - (2*ArcTan[(1 + x)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{7+4x}{6(4+2x+x^2)}-\frac{2}{3}\int\frac{1}{4+2x+x^2}\,dx\\ &= -\frac{7+4x}{6(4+2x+x^2)}+\frac{4}{3}\text{Subst}\left(\int\frac{1}{-12-x^2}\,dx,x,2+2x\right)\\ &= -\frac{7+4x}{6(4+2x+x^2)}-\frac{2\arctan\left(\frac{1+x}{\sqrt{3}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int\frac{-3+x}{(4+2x+x^2)^2}\,dx=\frac{-7-4x}{6(4+2x+x^2)}-\frac{2\arctan\left(\frac{1+x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[In] Integrate[(-3 + x)/(4 + 2*x + x^2)^2,x]
[Out] (-7 - 4*x)/(6*(4 + 2*x + x^2)) - (2*ArcTan[(1 + x)/Sqrt[3]])/(3*Sqrt[3])
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{-\frac{2x}{3}-\frac{7}{6}}{x^2+2x+4}-\frac{2\arctan\left(\frac{(1+x)\sqrt{3}}{3}\right)\sqrt{3}}{9}$	32
default	$\frac{-8x-14}{12x^2+24x+48}-\frac{2\sqrt{3}\,\arctan\left(\frac{(2x+2)\sqrt{3}}{6}\right)}{9}$	35

```
[In] int((-3+x)/(x^2+2*x+4)^2,x,method=_RETURNVERBOSE)
```

[Out] (-2/3*x-7/6)/(x^2+2*x+4)-2/9*arctan(1/3*(1+x)*3^(1/2))*3^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{-3+x}{(4+2x+x^2)^2}\,dx = -\frac{4\sqrt{3}(x^2+2x+4)\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right)+12x+21}{18(x^2+2x+4)}$$

[In] integrate((-3+x)/(x^2+2*x+4)^2,x, algorithm="fricas")

[Out] -1/18*(4*sqrt(3)*(x^2 + 2*x + 4)*arctan(1/3*sqrt(3)*(x + 1)) + 12*x + 21)/(
x^2 + 2*x + 4)

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{-3+x}{(4+2x+x^2)^2}\,dx = \frac{-4x-7}{6x^2+12x+24} - \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

[In] integrate((-3+x)/(x**2+2*x+4)**2,x)

[Out] (-4*x - 7)/(6*x**2 + 12*x + 24) - 2*sqrt(3)*atan(sqrt(3)*x/3 + sqrt(3)/3)/9

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{-3+x}{(4+2x+x^2)^2}\,dx = -\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right) - \frac{4x+7}{6(x^2+2x+4)}$$

[In] integrate((-3+x)/(x^2+2*x+4)^2,x, algorithm="maxima")

[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/6*(4*x + 7)/(x^2 + 2*x + 4)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{-3+x}{(4+2x+x^2)^2} \, dx = -\frac{2}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(x+1)\right) - \frac{4 x + 7}{6 (x^2 + 2 x + 4)}$$

```
[In] integrate((-3+x)/(x^2+2*x+4)^2,x, algorithm="giac")
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/6*(4*x + 7)/(x^2 + 2*x + 4)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{-3+x}{(4+2x+x^2)^2} \, dx = -\frac{\frac{2 x}{3} + \frac{7}{6}}{x^2 + 2 x + 4} - \frac{2 \sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3} x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] int((x - 3)/(2*x + x^2 + 4)^2,x)
[Out] - ((2*x)/3 + 7/6)/(2*x + x^2 + 4) - (2*3^(1/2)*atan((3^(1/2)*x)/3 + 3^(1/2)
/3))/9
```

3.210 $\int \frac{1+x^4}{x(1+x^2)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 10

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx = \frac{1}{1+x^2}+\log(x)$$

```
[Out] 1/(x^2+1)+ln(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1266, 908}

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx = \frac{1}{x^2+1}+\log(x)$$

```
[In] Int[(1 + x^4)/(x*(1 + x^2)^2),x]
```

```
[Out] (1 + x^2)^(-1) + Log[x]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{x(1+x)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}-\frac{2}{(1+x)^2}\right)\,dx, x, x^2\right)\\ &= \frac{1}{1+x^2}+\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx = \frac{1}{1+x^2}+\log(x)$$

```
[In] Integrate[(1 + x^4)/(x*(1 + x^2)^2),x]
[Out] (1 + x^2)^(-1) + Log[x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{1}{x^2+1}+\ln\left(x\right)$	11
norman	$\frac{1}{x^2+1}+\ln\left(x\right)$	11
risch	$\frac{1}{x^2+1}+\ln\left(x\right)$	11
parallelrisch	$\frac{x^2\ln(x)+1+\ln(x)}{x^2+1}$	19
meijerg	$-\frac{x^2}{2(x^2+1)}+\frac{1}{2}+\ln\left(x\right)-\frac{x^2}{2x^2+2}$	31

```
[In] int((x^4+1)/x/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/(x^2+1)+ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx=\frac{\left(x^2+1\right)\log\left(x\right)+1}{x^2+1}$$

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="fricas")
[Out] ((x^2 + 1)*log(x) + 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx=\log\left(x\right)+\frac{1}{x^2+1}$$

```
[In] integrate((x**4+1)/x/(x**2+1)**2,x)
[Out] log(x) + 1/(x**2 + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx=\frac{1}{x^2+1}+\frac{1}{2}\log\left(x^2\right)$$

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/(x^2 + 1) + 1/2*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx=\frac{1}{x^2+1}+\frac{1}{2}\log\left(x^2\right)$$

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="giac")
[Out] 1/(x^2 + 1) + 1/2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx = \ln\left(x\right) + \frac{1}{x^2+1}$$

```
[In] int((x^4 + 1)/(x*(x^2 + 1)^2),x)
[Out] log(x) + 1/(x^2 + 1)
```

3.211 $\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx = \log\left(2-3\sin(x)+\sin^2(x)\right)$$

```
[Out] ln(2-3*sin(x)+sin(x)^2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4419, 642}

$$\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx = \log\left(\sin^2(x)-3\sin(x)+2\right)$$

```
[In] Int[(Cos[x]*(-3 + 2*Sin[x]))/(2 - 3*Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[2 - 3*Sin[x] + Sin[x]^2]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
```

```
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{-3+2x}{2-3x+x^2}\,dx, x, \sin(x)\right)\\ &= \log\left(2-3\sin(x)+\sin^2(x)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx = 2(\operatorname{arctanh}(3-2\sin(x))+\log(1-\sin(x)))$$

```
[In] Integrate[(Cos[x]*(-3 + 2*Sin[x]))/(2 - 3*Sin[x] + Sin[x]^2),x]
[Out] 2*(ArcTanh[3 - 2*Sin[x]] + Log[1 - Sin[x]])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\ln\left(2-3\sin\left(x\right)+\sin^2\left(x\right)\right)$	12
default	$\ln\left(2-3\sin\left(x\right)+\sin^2\left(x\right)\right)$	12
risch	$-2ix+2\ln\left(e^{ix}-i\right)+\ln\left(-4ie^{ix}+e^{2ix}-1\right)$	33
norman	$2\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-2\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)+\ln\left(\tan^2\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)+1\right)$	37
parallelrisch	$2\ln\left(-\cot\left(x\right)+\csc\left(x\right)-1\right)-2\ln\left(\frac{1}{\cos(x)+1}\right)+\ln\left(\frac{-\sin(x)+2}{4\cos(x)+4}\right)$	38

```
[In] int(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x,method=_RETURNVERBOSE)
[Out] ln(2-3*sin(x)+sin(x)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx = \log\left(-\frac{1}{2}\sin(x)+1\right)+\log(-\sin(x)+1)$$

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="fricas")
[Out] log(-1/2*sin(x) + 1) + log(-sin(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx = \log(\sin(x)-2)+\log(\sin(x)-1)$$

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)**2),x)
[Out] log(sin(x) - 2) + log(sin(x) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx = \log\left(\sin(x)^2-3\sin(x)+2\right)$$

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="maxima")
[Out] log(sin(x)^2 - 3*sin(x) + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx = \log(-\sin(x)+2)+\log(-\sin(x)+1)$$

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="giac")
[Out] log(-sin(x) + 2) + log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx = \ln\left(\sin\left(x\right)^2-3\,\sin\left(x\right)+2\right)$$

```
[In] int((cos(x)*(2*sin(x) - 3))/(sin(x)^2 - 3*sin(x) + 2),x)
[Out] log(sin(x)^2 - 3*sin(x) + 2)
```

3.212 $\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx = \sqrt{5}\arctan\left(\frac{\cos(x)}{\sqrt{5}}\right) - \cos(x)$$

```
[Out] -cos(x)+arctan(1/5*cos(x)*5^(1/2))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4420, 327, 209}

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx = \sqrt{5}\arctan\left(\frac{\cos(x)}{\sqrt{5}}\right) - \cos(x)$$

```
[In] Int[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]
```

```
[Out] Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]] - Cos[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{x^2}{5+x^2}\,dx, x, \cos(x)\right)\\ &= -\cos(x) + 5\text{Subst}\left(\int \frac{1}{5+x^2}\,dx, x, \cos(x)\right)\\ &= \sqrt{5}\arctan\left(\frac{\cos(x)}{\sqrt{5}}\right) - \cos(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 82 vs. 2(20) = 40.

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.10

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx = \frac{1}{20}\left(-\sqrt{5}\arctan\left(\frac{\cos(x)}{\sqrt{5}}\right) + 21\sqrt{5}\arctan\left(\frac{1}{\sqrt{5}} - \sqrt{\frac{6}{5}}\tan\left(\frac{x}{2}\right)\right) + 21\sqrt{5}\arctan\left(\frac{1}{\sqrt{5}} + \sqrt{\frac{6}{5}}\tan\left(\frac{x}{2}\right)\right) - 20\cos(x)\right)$$

```
[In] Integrate[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]
```

```
[Out] (-(Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]]) + 21*Sqrt[5]*ArcTan[1/Sqrt[5] - Sqrt[6/5
]*Tan[x/2]] + 21*Sqrt[5]*ArcTan[1/Sqrt[5] + Sqrt[6/5]*Tan[x/2]] - 20*Cos[x]
)/20
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\cos\left(x\right)+\arctan\left(\frac{\cos\left(x\right)\sqrt{5}}{5}\right)\sqrt{5}$	18
default	$-\cos\left(x\right)+\arctan\left(\frac{\cos\left(x\right)\sqrt{5}}{5}\right)\sqrt{5}$	18
risch	$-\frac{{\mathrm e}^{i x}}{2}-\frac{{\mathrm e}^{-i x}}{2}-\frac{i\sqrt{5}\,\ln\left({\mathrm e}^{2 i x}-2 i\sqrt{5}\,{\mathrm e}^{i x}+1\right)}{2}+\frac{i\sqrt{5}\,\ln\left({\mathrm e}^{2 i x}+2 i\sqrt{5}\,{\mathrm e}^{i x}+1\right)}{2}$	66

```
[In] int(cos(x)^2*sin(x)/(5+cos(x)^2),x,method=_RETURNVERBOSE)
[Out] -cos(x)+arctan(1/5*cos(x)*5^(1/2))*5^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx = \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos\left(x\right)\right)-\cos\left(x\right)$$

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="fricas")
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx = -\cos\left(x\right)+\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\cos\left(x\right)}{5}\right)$$

```
[In] integrate(cos(x)**2*sin(x)/(5+cos(x)**2),x)
[Out] -cos(x) + sqrt(5)*atan(sqrt(5)*cos(x)/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx = \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos(x)\right) - \cos(x)$$

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="maxima")
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx = \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos(x)\right) - \cos(x)$$

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="giac")
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx = \sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\cos(x)}{5}\right) - \cos(x)$$

```
[In] int((cos(x)^2*sin(x))/(cos(x)^2 + 5),x)
[Out] 5^(1/2)*atan((5^(1/2)*cos(x))/5) - cos(x)
```

3.213 $\int \frac{1}{-3+2x+x^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{1}{-3+2x+x^2} \, dx = \frac{1}{4}\log(1-x) - \frac{1}{4}\log(3+x)$$

```
[Out] 1/4*ln(1-x)-1/4*ln(3+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {630, 31}

$$\int \frac{1}{-3+2x+x^2} \, dx = \frac{1}{4}\log(1-x) - \frac{1}{4}\log(x+3)$$

```
[In] Int[(-3 + 2*x + x^2)^(-1),x]
```

```
[Out] Log[1 - x]/4 - Log[3 + x]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\int \frac{1}{-1+x}\,dx - \frac{1}{4}\int \frac{1}{3+x}\,dx$$
$$= \frac{1}{4}\log(1-x) - \frac{1}{4}\log(3+x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{-3+2x+x^2}\,dx = \frac{1}{4}\log(1-x) - \frac{1}{4}\log(3+x)$$

```
[In] Integrate[(-3 + 2*x + x^2)^(-1),x]
[Out] Log[1 - x]/4 - Log[3 + x]/4
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\ln(-1+x)}{4}-\frac{\ln(3+x)}{4}$	14
norman	$\frac{\ln(-1+x)}{4}-\frac{\ln(3+x)}{4}$	14
risch	$\frac{\ln(-1+x)}{4}-\frac{\ln(3+x)}{4}$	14
parallelrisch	$\frac{\ln(-1+x)}{4}-\frac{\ln(3+x)}{4}$	14

```
[In] int(1/(x^2+2*x-3),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(-1+x)-1/4*ln(3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{-3+2x+x^2}\,dx = -\frac{1}{4}\,\log\left(x+3\right) + \frac{1}{4}\,\log\left(x-1\right)$$

```
[In] integrate(1/(x^2+2*x-3),x, algorithm="fricas")
[Out] -1/4*log(x + 3) + 1/4*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1}{-3+2x+x^2}\,dx = \frac{\log(x-1)}{4} - \frac{\log(x+3)}{4}$$

```
[In] integrate(1/(x**2+2*x-3),x)
[Out] log(x - 1)/4 - log(x + 3)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{-3+2x+x^2}\,dx = -\frac{1}{4}\log(x+3) + \frac{1}{4}\log(x-1)$$

```
[In] integrate(1/(x^2+2*x-3),x, algorithm="maxima")
[Out] -1/4*log(x + 3) + 1/4*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{-3+2x+x^2}\,dx = -\frac{1}{4}\log(|x+3|) + \frac{1}{4}\log(|x-1|)$$

```
[In] integrate(1/(x^2+2*x-3),x, algorithm="giac")
[Out] -1/4*log(abs(x + 3)) + 1/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{1}{-3+2x+x^2}\,dx = -\frac{\operatorname{atanh}\left(\frac{x}{2}+\frac{1}{2}\right)}{2}$$

```
[In] int(1/(2*x + x^2 - 3),x)
[Out] -atanh(x/2 + 1/2)/2
```

3.214 $\int \frac{1}{-2x+x^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{1}{-2x+x^2} dx = \frac{1}{2}\log(2-x) - \frac{\log(x)}{2}$$

```
[Out] 1/2*ln(2-x)-1/2*ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {629}

$$\int \frac{1}{-2x+x^2} dx = \frac{1}{2}\log(2-x) - \frac{\log(x)}{2}$$

```
[In] Int[(-2*x + x^2)^(-1),x]
```

```
[Out] Log[2 - x]/2 - Log[x]/2
```

Rule 629

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[Log[x]/b, x] - Simp
[Log[RemoveContent[b + c*x, x]]/b, x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\log(2-x) - \frac{\log(x)}{2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{-2x+x^2}\,dx=\frac{1}{2}\log(2-x)-\frac{\log(x)}{2}$$

```
[In] Integrate[(-2*x + x^2)^(-1),x]
[Out] Log[2 - x]/2 - Log[x]/2
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\ln(x)}{2}+\frac{\ln(-2+x)}{2}$	12
norman	$-\frac{\ln(x)}{2}+\frac{\ln(-2+x)}{2}$	12
risch	$-\frac{\ln(x)}{2}+\frac{\ln(-2+x)}{2}$	12
parallelrisch	$-\frac{\ln(x)}{2}+\frac{\ln(-2+x)}{2}$	12
meijerg	$-\frac{\ln(x)}{2}+\frac{\ln(2)}{2}-\frac{i\pi}{2}+\frac{\ln(1-\frac{x}{2})}{2}$	22

```
[In] int(1/(x^2-2*x),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x)+1/2*ln(-2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{-2x+x^2}\,dx=\frac{1}{2}\log\left(x-2\right)-\frac{1}{2}\log\left(x\right)$$

```
[In] integrate(1/(x^2-2*x),x, algorithm="fricas")
[Out] 1/2*log(x - 2) - 1/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{1}{-2x+x^2}\,dx = -\frac{\log{(x)}}{2} + \frac{\log{(x-2)}}{2}$$

```
[In] integrate(1/(x**2-2*x),x)
[Out] -log(x)/2 + log(x - 2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{-2x+x^2}\,dx = \frac{1}{2}\log{(x-2)} - \frac{1}{2}\log{(x)}$$

```
[In] integrate(1/(x^2-2*x),x, algorithm="maxima")
[Out] 1/2*log(x - 2) - 1/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{-2x+x^2}\,dx = \frac{1}{2}\log{(|x-2|)} - \frac{1}{2}\log{(|x|)}$$

```
[In] integrate(1/(x^2-2*x),x, algorithm="giac")
[Out] 1/2*log(abs(x - 2)) - 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.35

$$\int \frac{1}{-2x+x^2}\,dx = -\mathrm{atanh}(x-1)$$

```
[In] int(-1/(2*x - x^2),x)
[Out] -atanh(x - 1)
```

3.215 $\int \frac{1+2x}{-7+12x+4x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{1+2x}{-7+12x+4x^2} \, dx = \frac{1}{8}\log(1-2x)+\frac{3}{8}\log(7+2x)$$

```
[Out] 1/8*ln(1-2*x)+3/8*ln(7+2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {646, 31}

$$\int \frac{1+2x}{-7+12x+4x^2} \, dx = \frac{1}{8}\log(1-2x)+\frac{3}{8}\log(2x+7)$$

```
[In] Int[(1 + 2*x)/(-7 + 12*x + 4*x^2),x]
```

```
[Out] Log[1 - 2*x]/8 + (3*Log[7 + 2*x])/8
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\int \frac{1}{-2+4x}\,dx + \frac{3}{2}\int \frac{1}{14+4x}\,dx \\ &= \frac{1}{8}\log(1-2x) + \frac{3}{8}\log(7+2x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1+2x}{-7+12x+4x^2}\,dx = \frac{1}{8}\log(1-2x) + \frac{3}{8}\log(7+2x)$$

```
[In] Integrate[(1 + 2*x)/(-7 + 12*x + 4*x^2),x]
[Out] Log[1 - 2*x]/8 + (3*Log[7 + 2*x])/8
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln\left(x-\frac{1}{2}\right)}{8}+\frac{3\ln\left(x+\frac{7}{2}\right)}{8}$	14
default	$\frac{3\ln(7+2x)}{8}+\frac{\ln(2x-1)}{8}$	18
norman	$\frac{3\ln(7+2x)}{8}+\frac{\ln(2x-1)}{8}$	18
risch	$\frac{3\ln(7+2x)}{8}+\frac{\ln(2x-1)}{8}$	18

```
[In] int((1+2*x)/(4*x^2+12*x-7),x,method=_RETURNVERBOSE)
[Out] 1/8*ln(x-1/2)+3/8*ln(x+7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1+2x}{-7+12x+4x^2}\,dx = \frac{3}{8}\,\log\left(2\,x+7\right) + \frac{1}{8}\,\log\left(2\,x-1\right)$$

```
[In] integrate((1+2*x)/(4*x^2+12*x-7),x, algorithm="fricas")
[Out] 3/8*log(2*x + 7) + 1/8*log(2*x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1+2x}{-7+12x+4x^2}\,dx = \frac{\log\left(x-\frac{1}{2}\right)}{8} + \frac{3\log\left(x+\frac{7}{2}\right)}{8}$$

```
[In] integrate((1+2*x)/(4*x**2+12*x-7),x)
[Out] log(x - 1/2)/8 + 3*log(x + 7/2)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1+2x}{-7+12x+4x^2}\,dx = \frac{3}{8}\,\log\left(2\,x+7\right) + \frac{1}{8}\,\log\left(2\,x-1\right)$$

```
[In] integrate((1+2*x)/(4*x^2+12*x-7),x, algorithm="maxima")
[Out] 3/8*log(2*x + 7) + 1/8*log(2*x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1+2x}{-7+12x+4x^2}\,dx = \frac{3}{8}\,\log\left(|2\,x+7|\right) + \frac{1}{8}\,\log\left(|2\,x-1|\right)$$

```
[In] integrate((1+2*x)/(4*x^2+12*x-7),x, algorithm="giac")
[Out] 3/8*log(abs(2*x + 7)) + 1/8*log(abs(2*x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1+2x}{-7+12x+4x^2}\,dx=\frac{\ln\left(x-\frac{1}{2}\right)}{8}+\frac{3\,\ln\left(x+\frac{7}{2}\right)}{8}$$

```
[In] int((2*x + 1)/(12*x + 4*x^2 - 7),x)
[Out] log(x - 1/2)/8 + (3*log(x + 7/2))/8
```

3.216 $\int \frac{x}{-1+x+x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 49

$$\int \frac{x}{-1+x+x^2} dx = \frac{1}{10}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)+\frac{1}{10}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)$$

```
[Out] 1/10*ln(1+2*x-5^(1/2))*(5-5^(1/2))+1/10*ln(1+2*x+5^(1/2))*(5+5^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {646, 31}

$$\int \frac{x}{-1+x+x^2} dx = \frac{1}{10}\left(5-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)+\frac{1}{10}\left(5+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

```
[In] Int[x/(-1 + x + x^2),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/10 + ((5 + Sqrt[5])*Log[1 + Sqrt[5]
+ 2*x])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
```

```
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{10}\left(5-\sqrt{5}\right)\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\,dx+\frac{1}{10}\left(5+\sqrt{5}\right)\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\,dx\\ &= \frac{1}{10}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)+\frac{1}{10}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{x}{-1+x+x^2}\,dx=\frac{1}{10}\left(-\left(\left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x\right)\right)+\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)\right)$$

```
[In] Integrate[x/(-1 + x + x^2),x]
[Out] (-((-5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x]) + (5 + Sqrt[5])*Log[1 + Sqrt[5]
+ 2*x])/10
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\ln\left(x^2+x-1\right)}{2}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)}{5}$	27
risch	$\frac{\ln\left(2x+\sqrt{5}+1\right)}{2}+\frac{\ln\left(2x+\sqrt{5}+1\right)\sqrt{5}}{10}+\frac{\ln\left(2x+1-\sqrt{5}\right)}{2}-\frac{\ln\left(2x+1-\sqrt{5}\right)\sqrt{5}}{10}$	56

```
[In] int(x/(x^2+x-1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2+x-1)+1/5*5^(1/2)*arctanh(1/5*(1+2*x)*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{x}{-1+x+x^2}\,dx = \frac{1}{10}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(2x+1)+2x+3}{x^2+x-1}\right)+\frac{1}{2}\log\left(x^2+x-1\right)$$

```
[In] integrate(x/(x^2+x-1),x, algorithm="fricas")
[Out] 1/10*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1)) + 1/2
*log(x^2 + x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{x}{-1+x+x^2}\,dx = \left(\frac{\sqrt{5}}{10}+\frac{1}{2}\right)\log\left(x+\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+\left(\frac{1}{2}-\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)$$

```
[In] integrate(x/(x**2+x-1),x)
[Out] (sqrt(5)/10 + 1/2)*log(x + 1/2 + sqrt(5)/2) + (1/2 - sqrt(5)/10)*log(x - sq
rt(5)/2 + 1/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{x}{-1+x+x^2}\,dx = -\frac{1}{10}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+1}{2x+\sqrt{5}+1}\right)+\frac{1}{2}\log\left(x^2+x-1\right)$$

```
[In] integrate(x/(x^2+x-1),x, algorithm="maxima")
[Out] -1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) + 1/2*log(x^2 +
x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x}{-1+x+x^2}\,dx = -\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2\,x-\sqrt{5}+1\right|}{\left|2\,x+\sqrt{5}+1\right|}\right)+\frac{1}{2}\,\log\left(\left|x^2+x-1\right|\right)$$

```
[In] integrate(x/(x^2+x-1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) + 1/2*log(
abs(x^2 + x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{x}{-1+x+x^2}\,dx = \ln\left(x+\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\,\left(\frac{\sqrt{5}}{10}+\frac{1}{2}\right)-\ln\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\,\left(\frac{\sqrt{5}}{10}-\frac{1}{2}\right)$$

```
[In] int(x/(x + x^2 - 1),x)
```

```
[Out] log(x + 5^(1/2)/2 + 1/2)*(5^(1/2)/10 + 1/2) - log(x - 5^(1/2)/2 + 1/2)*(5^(
1/2)/10 - 1/2)
```

3.217 $\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5} dx$

Optimal result

Integrand size = 43, antiderivative size = 63

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5} dx = \frac{3988 \arctan\left(\frac{1+2x}{\sqrt{19}}\right)}{13685\sqrt{19}} - \frac{3146 \log(7-3x)}{80155} - \frac{334}{323} \log(1+2x) + \frac{4822 \log(2+5x)}{4879} + \frac{11049 \log(5+x+x^2)}{260015}$$

```
[Out] -3146/80155*ln(7-3*x)-334/323*ln(1+2*x)+4822/4879*ln(2+5*x)+11049/260015*ln
(x^2+x+5)+3988/260015*arctan(1/19*(1+2*x)*19^(1/2))*19^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2099, 648, 632, 210, 642}

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5} dx = \frac{3988 \arctan\left(\frac{2x+1}{\sqrt{19}}\right)}{13685\sqrt{19}} + \frac{11049 \log(x^2+x+5)}{260015} - \frac{3146 \log(7-3x)}{80155} - \frac{334}{323} \log(2x+1) + \frac{4822 \log(5x+2)}{4879}$$

```
[In] Int[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 +
 30*x^5),x]
```

```
[Out] (3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7 - 3*x])/80
155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879 + (11049*Log[5 + x
+ x^2])/260015
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{668}{323(1+2x)}-\frac{9438}{80155(-7+3x)}+\frac{24110}{4879(2+5x)}+\frac{48935+22098x}{260015\,(5+x+x^2)}\right)\,dx\\
&= -\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}+\frac{\int\frac{48935+22098x}{5+x+x^2}\,dx}{260015}\\
&= -\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}\\
&\quad+\frac{11049\int\frac{1+2x}{5+x+x^2}\,dx}{260015}+\frac{1994\int\frac{1}{5+x+x^2}\,dx}{13685}
\end{aligned}$$

$$= -\frac{3146\log(7-3x)}{80155} - \frac{334}{323}\log(1+2x) + \frac{4822\log(2+5x)}{4879} + \frac{11049\log(5+x+x^2)}{260015} - \frac{3988\text{Subst}\left(\int \frac{1}{-19-x^2}\,dx, x, 1+2x\right)}{13685}$$

$$= \frac{3988\arctan\left(\frac{1+2x}{\sqrt{19}}\right)}{13685\sqrt{19}} - \frac{3146\log(7-3x)}{80155} - \frac{334}{323}\log(1+2x) + \frac{4822\log(2+5x)}{4879} + \frac{11049\log(5+x+x^2)}{260015}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \frac{163508\sqrt{19}\arctan\left(\frac{1+2x}{\sqrt{19}}\right) - 418418\log(7-3x) - 11023670\log(1+2x) + 10536070\log(2+5x) + 45300}{10660615}$$

```
[In] Integrate[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13
*x^4 + 30*x^5),x]
```

```
[Out] (163508*Sqrt[19]*ArcTan[(1 + 2*x)/Sqrt[19]] - 418418*Log[7 - 3*x] - 1102367
0*Log[1 + 2*x] + 10536070*Log[2 + 5*x] + 453009*Log[5 + x + x^2])/10660615
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

method	result
default	$\frac{4822\ln(5x+2)}{4879} - \frac{3146\ln(3x-7)}{80155} + \frac{11049\ln(x^2+x+5)}{260015} + \frac{3988\arctan\left(\frac{(1+2x)\sqrt{19}}{19}\right)\sqrt{19}}{260015} - \frac{334\ln(1+2x)}{323}$
risch	$-\frac{3146\ln(3x-7)}{80155} + \frac{4822\ln(5x+2)}{4879} - \frac{334\ln(1+2x)}{323} + \frac{11049\ln(15904144x^2+15904144x+79520720)}{260015} + \frac{3988\sqrt{19}\arctan\left(\frac{(3988}{}\right.}{260015}$

```
[In] int((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,method=
_RETURNVERBOSE)
```

```
[Out] 4822/4879*ln(5*x+2)-3146/80155*ln(3*x-7)+11049/260015*ln(x^2+x+5)+3988/2600
15*arctan(1/19*(1+2*x)*19^(1/2))*19^(1/2)-334/323*ln(1+2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \frac{3988}{260015}\,\sqrt{19}\arctan\left(\frac{1}{19}\,\sqrt{19}(2\,x+1)\right)+\frac{11049}{260015}\,\log\left(x^2+x+5\right) + \frac{4822}{4879}\,\log\left(5\,x+2\right)-\frac{3146}{80155}\,\log\left(3\,x-7\right)-\frac{334}{323}\,\log\left(2\,x+1\right)$$

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="fricas")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(
2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = -\frac{3146\log\left(x-\frac{7}{3}\right)}{80155}+\frac{4822\log\left(x+\frac{2}{5}\right)}{4879}-\frac{334\log\left(x+\frac{1}{2}\right)}{323} + \frac{11049\log\left(x^2+x+5\right)}{260015}+\frac{3988\sqrt{19}\operatorname{atan}\left(\frac{2\sqrt{19}x}{19}+\frac{\sqrt{19}}{19}\right)}{260015}$$

```
[In] integrate((4*x**3-27*x**2+5*x-32)/(30*x**5-13*x**4+50*x**3-286*x**2-299*x-7
0),x)
```

```
[Out] -3146*log(x - 7/3)/80155 + 4822*log(x + 2/5)/4879 - 334*log(x + 1/2)/323 +
11049*log(x**2 + x + 5)/260015 + 3988*sqrt(19)*atan(2*sqrt(19)*x/19 + sqrt(
19)/19)/260015
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \frac{3988}{260015}\,\sqrt{19}\arctan\left(\frac{1}{19}\,\sqrt{19}(2\,x+1)\right)+\frac{11049}{260015}\,\log\left(x^2+x+5\right) + \frac{4822}{4879}\,\log\left(5\,x+2\right)-\frac{3146}{80155}\,\log\left(3\,x-7\right)-\frac{334}{323}\,\log\left(2\,x+1\right)$$

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="maxima")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(
2*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \frac{3988}{260015}\,\sqrt{19}\arctan\left(\frac{1}{19}\,\sqrt{19}(2\,x+1)\right)+\frac{11049}{260015}\,\log\left(x^2+x+5\right) + \frac{4822}{4879}\,\log\left(|5\,x+2|\right)-\frac{3146}{80155}\,\log\left(|3\,x-7|\right)-\frac{334}{323}\,\log\left(|2\,x+1|\right)$$

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="giac")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(abs(5*x + 2)) - 3146/80155*log(abs(3*x - 7)) - 33
4/323*log(abs(2*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \frac{4822\ln\left(x+\frac{2}{5}\right)}{4879} - \frac{334\ln\left(x+\frac{1}{2}\right)}{323} - \frac{3146\ln\left(x-\frac{7}{3}\right)}{80155} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{19}\,1\mathrm{i}}{2}\right)\left(-\frac{11049}{260015}+\frac{\sqrt{19}\,1994\mathrm{i}}{260015}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{19}\,1\mathrm{i}}{2}\right)\left(\frac{11049}{260015}+\frac{\sqrt{19}\,1994\mathrm{i}}{260015}\right)$$

```
[In] int(-(5*x - 27*x^2 + 4*x^3 - 32)/(299*x + 286*x^2 - 50*x^3 + 13*x^4 - 30*x^
5 + 70),x)
```

```
[Out] (4822*log(x + 2/5))/4879 - (334*log(x + 1/2))/323 - (3146*log(x - 7/3))/801
55 - log(x - (19^(1/2)*1i)/2 + 1/2)*((19^(1/2)*1994i)/260015 - 11049/260015
) + log(x + (19^(1/2)*1i)/2 + 1/2)*((19^(1/2)*1994i)/260015 + 11049/260015)
```

3.218 $\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6} dx$

Optimal result

Integrand size = 50, antiderivative size = 86

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6} dx = \frac{5828}{9075(2-5x)} - \frac{313+502x}{1452(1+2x^2)} - \frac{251\arctan\left(\sqrt{2}x\right)}{726\sqrt{2}} + \frac{272\sqrt{2}\arctan\left(\sqrt{2}x\right)}{1331} - \frac{59096\log(2-5x)}{99825} + \frac{2843\log(1+2x^2)}{7986}$$

```
[Out] 5828/9075/(2-5*x)+1/1452*(-313-502*x)/(2*x^2+1)-59096/99825*ln(2-5*x)+2843/
7986*ln(2*x^2+1)+503/15972*arctan(x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2099, 653, 209, 649, 266}

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6} dx = \frac{272\sqrt{2}\arctan\left(\sqrt{2}x\right)}{1331} - \frac{251\arctan\left(\sqrt{2}x\right)}{726\sqrt{2}} - \frac{502x+313}{1452(2x^2+1)} + \frac{2843\log(2x^2+1)}{7986} + \frac{5828}{9075(2-5x)} - \frac{59096\log(2-5x)}{99825}$$

```
[In] Int[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 8
0*x^5 + 100*x^6),x]
```

```
[Out] 5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) - (251*ArcTan[Sqrt
[2]*x])/(726*Sqrt[2]) + (272*Sqrt[2]*ArcTan[Sqrt[2]*x])/1331 - (59096*Log[2
 - 5*x])/99825 + (2843*Log[1 + 2*x^2])/7986
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{5828}{1815(-2+5x)^2}-\frac{59096}{19965(-2+5x)}+\frac{-251+313x}{363\left(1+2x^2\right)^2}+\frac{2(816+2843x)}{3993\left(1+2x^2\right)}\right)dx\\
&= \frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{2\int\frac{816+2843x}{1+2x^2}\,dx}{3993}+\frac{1}{363}\int\frac{-251+313x}{\left(1+2x^2\right)^2}\,dx\\
&= \frac{5828}{9075(2-5x)}-\frac{313+502x}{1452\left(1+2x^2\right)}-\frac{59096\log(2-5x)}{99825}\\
&\quad -\frac{251}{726}\int\frac{1}{1+2x^2}\,dx+\frac{544\int\frac{1}{1+2x^2}\,dx}{1331}+\frac{5686\int\frac{x}{1+2x^2}\,dx}{3993}
\end{aligned}
$$

$$= \frac{5828}{9075(2-5x)} - \frac{313+502x}{1452\left(1+2x^2\right)} - \frac{251\arctan\left(\sqrt{2}x\right)}{726\sqrt{2}} + \frac{272\sqrt{2}\arctan\left(\sqrt{2}x\right)}{1331} - \frac{59096\log(2-5x)}{99825} + \frac{2843\log\left(1+2x^2\right)}{7986}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.78

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \frac{-\frac{33\left(2554+4675x+36458x^2\right)}{-2+5x-4x^2+10x^3} + 12575\sqrt{2}\arctan\left(\sqrt{2}x\right) - 236384\log(2-5x) + 142150\log\left(1+2x^2\right)}{399300}$$

```
[In] Integrate[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x
^4 - 80*x^5 + 100*x^6),x]
```

```
[Out] ((-33*(2554 + 4675*x + 36458*x^2))/(-2 + 5*x - 4*x^2 + 10*x^3) + 12575*Sqrt
[2]*ArcTan[Sqrt[2]*x] - 236384*Log[2 - 5*x] + 142150*Log[1 + 2*x^2])/399300
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{-\frac{2761x}{4}-\frac{3443}{8}}{3993x^2+\frac{3993}{2}}+\frac{2843\ln\left(2x^2+1\right)}{7986}+\frac{503\arctan\left(x\sqrt{2}\right)\sqrt{2}}{15972}-\frac{5828}{9075(5x-2)}-\frac{59096\ln(5x-2)}{99825}$	54
risch	$\frac{-\frac{18229}{60500}x^2-\frac{17}{440}x-\frac{1277}{60500}}{x^3-\frac{2}{5}x^2+\frac{1}{2}x-\frac{1}{5}}+\frac{2843\ln\left(4x^2+2\right)}{7986}+\frac{503\arctan\left(x\sqrt{2}\right)\sqrt{2}}{15972}-\frac{59096\ln(5x-2)}{99825}$	57

```
[In] int((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/3993*(-2761/4*x-3443/8)/(x^2+1/2)+2843/7986*ln(2*x^2+1)+503/15972*arctan(
x*2^(1/2))*2^(1/2)-5828/9075/(5*x-2)-59096/99825*ln(5*x-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.20

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \frac{12575\,\sqrt{2}(10\,x^3-4\,x^2+5\,x-2)\arctan\left(\sqrt{2}x\right)-1203114\,x^2+142150\,(10\,x^3-4\,x^2+5\,x-2)\log\left(2\,x^2+\right.}{399300\,(10\,x^3-4\,x^2+5\,x-2)}$$

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="fricas")
```

```
[Out] 1/399300*(12575*sqrt(2)*(10*x^3 - 4*x^2 + 5*x - 2)*arctan(sqrt(2)*x) - 1203
114*x^2 + 142150*(10*x^3 - 4*x^2 + 5*x - 2)*log(2*x^2 + 1) - 236384*(10*x^3
 - 4*x^2 + 5*x - 2)*log(5*x - 2) - 154275*x - 84282)/(10*x^3 - 4*x^2 + 5*x
- 2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \frac{-36458x^2-4675x-2554}{121000x^3-48400x^2+60500x-24200} - \frac{59096\log\left(x-\frac{2}{5}\right)}{99825} + \frac{2843\log\left(x^2+\frac{1}{2}\right)}{7986} + \frac{503\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x\right)}{15972}$$

```
[In] integrate((12*x**5-7*x**3-13*x**2+8)/(100*x**6-80*x**5+116*x**4-80*x**3+41*
x**2-20*x+4),x)
```

```
[Out] (-36458*x**2 - 4675*x - 2554)/(121000*x**3 - 48400*x**2 + 60500*x - 24200)
- 59096*log(x - 2/5)/99825 + 2843*log(x**2 + 1/2)/7986 + 503*sqrt(2)*atan(s
qrt(2)*x)/15972
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}x\right)-\frac{36458\,x^2+4675\,x+2554}{12100\,(10\,x^3-4\,x^2+5\,x-2)} + \frac{2843}{7986}\,\log\left(2\,x^2+1\right)-\frac{59096}{99825}\,\log\left(5\,x-2\right)$$

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="maxima")
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(
10*x^3 - 4*x^2 + 5*x - 2) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(5*x
- 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}x\right)-\frac{36458\,x^2+4675\,x+2554}{12100\,(2\,x^2+1)(5\,x-2)} + \frac{2843}{7986}\,\log\left(2\,x^2+1\right)-\frac{59096}{99825}\,\log\left(|5\,x-2|\right)$$

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="giac")
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(
(2*x^2 + 1)*(5*x - 2)) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(abs(5*x
 - 2))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx$$
$$= -\frac{59096\ln\left(x-\frac{2}{5}\right)}{99825} - \frac{\frac{18229\,x^2}{60500}+\frac{17\,x}{440}+\frac{1277}{60500}}{x^3-\frac{2\,x^2}{5}+\frac{x}{2}-\frac{1}{5}}$$
$$- \ln\left(x-\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)\left(-\frac{2843}{7986}+\frac{\sqrt{2}\,503\mathrm{i}}{31944}\right)+\ln\left(x+\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)\left(\frac{2843}{7986}+\frac{\sqrt{2}\,503\mathrm{i}}{31944}\right)$$

```
[In] int(-(13*x^2 + 7*x^3 - 12*x^5 - 8)/(41*x^2 - 20*x - 80*x^3 + 116*x^4 - 80*x
^5 + 100*x^6 + 4),x)
```

```
[Out] log(x + (2^(1/2)*1i)/2)*((2^(1/2)*503i)/31944 + 2843/7986) - ((17*x)/440 +
(18229*x^2)/60500 + 1277/60500)/(x/2 - (2*x^2)/5 + x^3 - 1/5) - log(x - (2^
(1/2)*1i)/2)*((2^(1/2)*503i)/31944 - 2843/7986) - (59096*log(x - 2/5))/9982
5
```

3.219 $\int \frac{\sqrt{4+x}}{x} dx$

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{\sqrt{4+x}}{x} dx = 2\sqrt{4+x} - 4\text{arctanh}\left(\frac{\sqrt{4+x}}{2}\right)$$

```
[Out] -4*arctanh(1/2*(4+x)^(1/2))+2*(4+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {52, 65, 213}

$$\int \frac{\sqrt{4+x}}{x} dx = 2\sqrt{x+4} - 4\text{arctanh}\left(\frac{\sqrt{x+4}}{2}\right)$$

```
[In] Int[Sqrt[4 + x]/x,x]
```

```
[Out] 2*Sqrt[4 + x] - 4*ArcTanh[Sqrt[4 + x]/2]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\sqrt{4+x}+4\int \frac{1}{x\sqrt{4+x}}\,dx\\ &= 2\sqrt{4+x}+8\text{Subst}\left(\int \frac{1}{-4+x^2}\,dx,x,\sqrt{4+x}\right)\\ &= 2\sqrt{4+x}-4\text{arctanh}\left(\frac{\sqrt{4+x}}{2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{4+x}}{x}\,dx = 2\sqrt{4+x}-4\text{arctanh}\left(\frac{\sqrt{4+x}}{2}\right)$$

```
[In] Integrate[Sqrt[4 + x]/x,x]
[Out] 2*Sqrt[4 + x] - 4*ArcTanh[Sqrt[4 + x]/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

method	result	size
trager	$2\sqrt{4+x}+2\ln\left(\frac{-8-x+4\sqrt{4+x}}{x}\right)$	28
derivativedivides	$2\sqrt{4+x}-2\ln\left(\sqrt{4+x}+2\right)+2\ln\left(\sqrt{4+x}-2\right)$	29
default	$2\sqrt{4+x}-2\ln\left(\sqrt{4+x}+2\right)+2\ln\left(\sqrt{4+x}-2\right)$	29
meijerg	$-\frac{-2(2-4\ln(2)+\ln(x))\sqrt{\pi}+4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{1+\frac{x}{4}}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x}{4}}}{2}\right)}{\sqrt{\pi}}$	54

```
[In] int((4+x)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 2*(4+x)^(1/2)+2*ln((-8-x+4*(4+x)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int\frac{\sqrt{4+x}}{x}\,dx=2\sqrt{x+4}-2\log\left(\sqrt{x+4}+2\right)+2\log\left(\sqrt{x+4}-2\right)$$

```
[In] integrate((4+x)^(1/2)/x,x, algorithm="fricas")
[Out] 2*sqrt(x + 4) - 2*log(sqrt(x + 4) + 2) + 2*log(sqrt(x + 4) - 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19)=38$.

Time = 0.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int\frac{\sqrt{4+x}}{x}\,dx=\begin{cases}2\sqrt{x+4}-4\operatorname{acoth}\left(\frac{\sqrt{x+4}}{2}\right) & \text{for }|x+4|>4\\ 2\sqrt{x+4}-4\operatorname{atanh}\left(\frac{\sqrt{x+4}}{2}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate((4+x)**(1/2)/x,x)
[Out] Piecewise((2*sqrt(x + 4) - 4*acoth(sqrt(x + 4)/2), Abs(x + 4) > 4), (2*sqrt
(x + 4) - 4*atanh(sqrt(x + 4)/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{4+x}}{x} \, dx = 2 \sqrt{x+4} - 2 \log \left(\sqrt{x+4}+2\right) + 2 \log \left(\sqrt{x+4}-2\right)$$

```
[In] integrate((4+x)^(1/2)/x,x, algorithm="maxima")
[Out] 2*sqrt(x + 4) - 2*log(sqrt(x + 4) + 2) + 2*log(sqrt(x + 4) - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{4+x}}{x} \, dx = 2 \sqrt{x+4} - 2 \log \left(\sqrt{x+4}+2\right) + 2 \log \left(\left|\sqrt{x+4}-2\right|\right)$$

```
[In] integrate((4+x)^(1/2)/x,x, algorithm="giac")
[Out] 2*sqrt(x + 4) - 2*log(sqrt(x + 4) + 2) + 2*log(abs(sqrt(x + 4) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{4+x}}{x} \, dx = 2 \sqrt{x+4} - 4 \operatorname{atanh}\left(\frac{\sqrt{x+4}}{2}\right)$$

```
[In] int((x + 4)^(1/2)/x,x)
[Out] 2*(x + 4)^(1/2) - 4*atanh((x + 4)^(1/2)/2)
```

3.220 $\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 200

$$\begin{aligned}\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx &= 2\sqrt{x}+\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\arctan\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)\\ &\quad-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}+4\sqrt[6]{x}\right)\right)\\ &\quad+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)\\ &\quad-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)\end{aligned}$$

```
[Out] 6/5*ln(1-x^(1/6))-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*(-5^(1/2)+1)
-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*(5^(1/2)+1)+2*x^(1/2)+3/5*arc
tan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-3/5*ar
ctan(1/20*(1+4*x^(1/6)+5^(1/2))*(50+10*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used

$= \{1607, 348, 327, 300, 648, 632, 210, 642, 31\}$

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = \frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\arctan\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right) + 2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right) - \frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)$$

```
[In] Int[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/Sqrt[
2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[5])/
10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 + Sqr
t[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 - Sqrt[5])
*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 300

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*
k - 1)*m*(Pi/n)] + s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k
 - 1)*(Pi/n)]*x + s^2*x^2), x]; (r^(m + 1)/(a*n*s^m))*Int[1/(r - s*x), x] -
 Dist[2*((-r)^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fre
eQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && NegQ[
a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx\\
&= 6\text{Subst}\left(\int \frac{x^7}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)\\
&= 2\sqrt{x} + 6\text{Subst}\left(\int \frac{x^2}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)
\end{aligned}
$$

$$= 2\sqrt{x} - \frac{6}{5}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[6]{x}\right)$$
$$- \frac{12}{5}\text{Subst}\left(\int \frac{\frac{1}{4}(-1-\sqrt{5}) + \frac{1}{4}(1+\sqrt{5})\,x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x + x^2}\,dx, x, \sqrt[6]{x}\right)$$
$$- \frac{12}{5}\text{Subst}\left(\int \frac{\frac{1}{4}(-1+\sqrt{5}) + \frac{1}{4}(1-\sqrt{5})\,x}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x + x^2}\,dx, x, \sqrt[6]{x}\right)$$

$$= 2\sqrt{x} + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) + \frac{3\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{5}}$$
$$- \frac{3\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{5}}$$
$$- \frac{1}{10}\left(3\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{\frac{1}{2}(1+\sqrt{5}) + 2x}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x + x^2}\,dx, x, \sqrt[6]{x}\right)$$
$$- \frac{1}{10}\left(3\left(1+\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{\frac{1}{2}(1-\sqrt{5}) + 2x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x + x^2}\,dx, x, \sqrt[6]{x}\right)$$

$$= 2\sqrt{x} + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) - \frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)$$
$$- \frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)$$
$$- \frac{6\text{Subst}\left(\int \frac{1}{\frac{1}{2}\left(-5-\sqrt{5}\right)-x^2}\,dx, x, \frac{1}{2}(1-\sqrt{5}) + 2\sqrt[6]{x}\right)}{\sqrt{5}}$$
$$+ \frac{6\text{Subst}\left(\int \frac{1}{\frac{1}{2}\left(-5+\sqrt{5}\right)-x^2}\,dx, x, \frac{1}{2}(1+\sqrt{5}) + 2\sqrt[6]{x}\right)}{\sqrt{5}}$$

$$= 2\sqrt{x} + 6\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\arctan\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)$$
$$- \frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}+4\sqrt[6]{x}\right)\right)$$
$$+ \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) - \frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)$$
$$- \frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.63

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = 2\sqrt{x}+\frac{6}{5}\log\left(-1+\sqrt[6]{x}\right)-\frac{6}{5}\text{RootSum}\left[1+\#1+\#1^2+\#1^3+\#1^4\&,\frac{-\log\left(\sqrt[6]{x}-\#1\right)-2\log\left(\sqrt[6]{x}-\#1\right)\#1+2\log\left(\sqrt[6]{x}-\#1\right)\#1^2+\log\left(\sqrt[6]{x}-\#1\right)\#1^3}{1+2\#1+3\#1^2+4\#1^3}\&\right]$$

```
[In] Integrate[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (6*Log[-1 + x^(1/6)])/5 - (6*RootSum[1 + #1 + #1^2 + #1^3 + #1^
4 & , (-Log[x^(1/6) - #1] - 2*Log[x^(1/6) - #1]*#1 + 2*Log[x^(1/6) - #1]*#1
^2 + Log[x^(1/6) - #1]*#1^3)/(1 + 2*#1 + 3*#1^2 + 4*#1^3) & ])/5
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.62

method	result
meijerg	$-\frac{6(-1)^{\frac{2}{5}}\left(\frac{5\sqrt{x}\,(-1)^{\frac{3}{5}}}{3}+(-1)^{\frac{3}{5}}\left(\ln\left(1-x^{\frac{1}{6}}\right)-\cos\left(\frac{\pi}{5}\right)\ln\left(1-2\cos\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)+2\sin\left(\frac{\pi}{5}\right)\arctan\left(\frac{\sin\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}}{1-\cos\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}}\right)+\text{co}\right.\right.}{5}$
derivativedivides	$2\sqrt{x}+\frac{6\ln\left(x^{\frac{1}{6}}-1\right)}{5}+\frac{3\left(\sqrt{5}-1\right)\ln\left(2+x^{\frac{1}{6}}+2x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{5}\right)}{10}+\frac{12\left(-\sqrt{5}+1-\frac{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}{4}\right)\arctan\left(\frac{1+4x^{\frac{1}{6}}+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right.}{5\sqrt{10-2\sqrt{5}}}$
default	$2\sqrt{x}+\frac{6\ln\left(x^{\frac{1}{6}}-1\right)}{5}+\frac{3\left(\sqrt{5}-1\right)\ln\left(2+x^{\frac{1}{6}}+2x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{5}\right)}{10}+\frac{12\left(-\sqrt{5}+1-\frac{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}{4}\right)\arctan\left(\frac{1+4x^{\frac{1}{6}}+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right.}{5\sqrt{10-2\sqrt{5}}}$

```
[In] int(1/(-1/x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -6/5*(-1)^(2/5)*(5/3*x^(1/2)*(-1)^(3/5)+(-1)^(3/5)*(ln(1-x^(1/6))-cos(1/5*P
i)*ln(1-2*cos(2/5*Pi)*x^(1/6)+x^(1/3))+2*sin(1/5*Pi)*arctan(sin(2/5*Pi)*x^(
1/6)/(1-cos(2/5*Pi)*x^(1/6)))+cos(2/5*Pi)*ln(1+2*cos(1/5*Pi)*x^(1/6)+x^(1/3
))-2*sin(2/5*Pi)*arctan(sin(1/5*Pi)*x^(1/6)/(1+cos(1/5*Pi)*x^(1/6)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. 2(133) = 266.

Time = 0.91 (sec) , antiderivative size = 638, normalized size of antiderivative = 3.19

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$$

$$= \frac{1}{10}\left(3\sqrt{5}-\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)\right.\right.$$
$$+\frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2$$
$$+3\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\right.}$$
$$\left.+72\,x^{\frac{1}{6}}+36\right)$$
$$+\frac{1}{10}\left(3\sqrt{5}+\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right.\right.\right.$$
$$+\frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2$$
$$-3\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\right.}$$
$$\left.+72\,x^{\frac{1}{6}}+36\right)$$
$$-\frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)\log\left(-\frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+36\,x^{\frac{1}{6}}\right)$$
$$+\frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)\log\left(-\frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2+36\,x^{\frac{1}{6}}\right)$$
$$+2\sqrt{x}+\frac{6}{5}\log\left(x^{\frac{1}{6}}-1\right)$$

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 +
9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) -
sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2
)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) -
```

```
5) + sqrt(5) + 1)^2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 3*
sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(
sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/
4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5
) + 18*sqrt(5) - 90)*(sqrt(5) - 1) + 72*x^(1/6) + 36) + 1/10*(3*sqrt(5) + s
qrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(s
qrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4
*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5)
 + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 - 3*sqrt(-27/4*(sqrt(2)
*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqr
t(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90
)*(sqrt(5) - 1) + 72*x^(1/6) + 36) - 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) + 1)*log(-9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 36*x^(1/6))
 + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(-9/4*(sqrt(2)*sqrt(sq
rt(5) - 5) - sqrt(5) - 1)^2 + 36*x^(1/6)) + 2*sqrt(x) + 6/5*log(x^(1/6) - 1
)
```

Sympy [F]

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}} + \sqrt{x}}\,dx = \int \frac{\sqrt[3]{x}}{\left(\sqrt[6]{x} - 1\right)\left(\sqrt[6]{x} + x^{\frac{2}{3}} + \sqrt[3]{x} + \sqrt{x} + 1\right)}\,dx$$

```
[In] integrate(1/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(x**(1/3)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3) + sqrt(x)
 + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(133) = 266.

Time = 0.29 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.36

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = -\frac{6}{5}\,(-1)^{\frac{3}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right) - \frac{6\,\sqrt{5}(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\,\sqrt{2\,\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4\,x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\,\sqrt{2\,\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4\,x^{\frac{1}{6}}}\right)}{5\,\sqrt{2\,\sqrt{5}-10}} + \frac{6\,\sqrt{5}(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\,\sqrt{-2\,\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4\,x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\,\sqrt{-2\,\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4\,x^{\frac{1}{6}}}\right)}{5\,\sqrt{-2\,\sqrt{5}-10}} + 2\,\sqrt{x} + \frac{6\,\log\left(-x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\right)+2\,(-1)^{\frac{2}{5}}+2\,x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}\,(-1)^{\frac{2}{5}}+(-1)^{\frac{2}{5}}\right)} - \frac{6\,\log\left(x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\right)+2\,(-1)^{\frac{2}{5}}+2\,x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}\,(-1)^{\frac{2}{5}}-(-1)^{\frac{2}{5}}\right)}$$

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -6/5*(-1)^(3/5)*log((-1)^(1/5) + x^(1/6)) - 6/5*sqrt(5)*(-1)^(3/5)*log((sqr
t(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))
/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(
1/6)))/sqrt(2*sqrt(5) - 10) + 6/5*sqrt(5)*(-1)^(3/5)*log((sqrt(5)*(-1)^(1/5
) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1
)^(1/5) + (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(
-2*sqrt(5) - 10) + 2*sqrt(x) + 6/5*log(-x^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(2/5) + (-1)^(2/5)) - 6/5*
log(x^(1/6)*(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(
sqrt(5)*(-1)^(2/5) - (-1)^(2/5))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.70

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = \frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right) - \frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right) + \frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+1\right) - \frac{3}{10}\sqrt{5}\log\left(-\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}-1\right)+x^{\frac{1}{3}}+1\right) + 2\sqrt{x} - \frac{3}{10}\log\left(x^{\frac{2}{3}}+\sqrt{x}+x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right) + \frac{6}{5}\log\left(\left|x^{\frac{1}{6}}-1\right|\right)$$

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 3/5*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2*
sqrt(5) + 10)) + 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
- 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + 2*sqrt(x) -
3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1/6) -
 1))
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.12

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = \frac{6\ln\left(1296\,x^{1/6}-1296\right)}{5} - \ln\left(\left(-750\,x^{1/6}\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)^3 - 1296\right)\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)\right) + \ln\left(\left(750\,x^{1/6}\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}+\frac{3\sqrt{5}}{10}-\frac{3}{10}\right)^3 - 1296\right)\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}+\frac{3\sqrt{5}}{10}-\frac{3}{10}\right)\right) - \ln\left(-750\right.$$

```
[In] int(1/(x^(1/2) - 1/x^(1/3)),x)
```

```
[Out] (6*log(1296*x^(1/6) - 1296))/5 - log(- 750*x^(1/6)*((3*2^(1/2)*(- 5^(1/2) -
 5)^(1/2))/10 - (3*5^(1/2))/10 + 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(1/2) - 5
```

```
)^(1/2))/10 - (3*5^(1/2))/10 + 3/10) + log(750*x^(1/6)*((3*2^(1/2)*(- 5^(1/
2) - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(1/2)
 - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10) - log(- 750*x^(1/6)*((3*5^(1/2))/1
0 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1/2))/10 -
(3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) - log(- 750*x^(1/6)*((3*5^(1/2))
/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1/2))/10
+ (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) + 2*x^(1/2)
```

3.221 $\int \frac{1}{-4\cos(x)+3\sin(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{-4\cos(x)+3\sin(x)}\,dx = -\frac{1}{5}\text{arctanh}\left(\frac{1}{5}(3\cos(x)+4\sin(x))\right)$$

```
[Out] -1/5*arctanh(3/5*cos(x)+4/5*sin(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3153, 212}

$$\int \frac{1}{-4\cos(x)+3\sin(x)}\,dx = -\frac{1}{5}\text{arctanh}\left(\frac{1}{5}(4\sin(x)+3\cos(x))\right)$$

```
[In] Int[(-4*Cos[x] + 3*Sin[x])^(-1),x]
```

```
[Out] -1/5*ArcTanh[(3*Cos[x] + 4*Sin[x])/5]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{25-x^2}\,dx, x, 3\cos(x)+4\sin(x)\right)$$
$$= -\frac{1}{5}\text{arctanh}\left(\frac{1}{5}(3\cos(x)+4\sin(x))\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 41 vs. $2(18) = 36$.

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{1}{-4\cos(x)+3\sin(x)}\,dx = \frac{1}{5}\log\left(\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right)-\frac{1}{5}\log\left(2\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[(-4*Cos[x] + 3*Sin[x])^(-1),x]
[Out] Log[Cos[x/2] - 2*Sin[x/2]]/5 - Log[2*Cos[x/2] + Sin[x/2]]/5
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{\ln\left(2\tan\left(\frac{x}{2}\right)-1\right)}{5}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)+2\right)}{5}$	22
norman	$\frac{\ln\left(2\tan\left(\frac{x}{2}\right)-1\right)}{5}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)+2\right)}{5}$	22
parallelrisch	$\ln\left(\frac{1}{\left(2\tan\left(\frac{x}{2}\right)+4\right)^{\frac{1}{5}}}\right)+\ln\left(\left(2\tan\left(\frac{x}{2}\right)-1\right)^{\frac{1}{5}}\right)$	24
risch	$-\frac{\ln\left(e^{ix}+\frac{3}{5}+\frac{4i}{5}\right)}{5}+\frac{\ln\left(e^{ix}-\frac{3}{5}-\frac{4i}{5}\right)}{5}$	26

```
[In] int(1/(-4*cos(x)+3*sin(x)),x,method=_RETURNVERBOSE)
[Out] 1/5*ln(2*tan(1/2*x)-1)-1/5*ln(tan(1/2*x)+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{1}{-4\cos(x)+3\sin(x)}\,dx = -\frac{1}{10}\,\log\left(\frac{3}{2}\,\cos\left(x\right)+2\,\sin\left(x\right)+\frac{5}{2}\right) + \frac{1}{10}\,\log\left(-\frac{3}{2}\,\cos\left(x\right)-2\,\sin\left(x\right)+\frac{5}{2}\right)$$

```
[In] integrate(1/(-4*cos(x)+3*sin(x)),x, algorithm="fricas")
[Out] -1/10*log(3/2*cos(x) + 2*sin(x) + 5/2) + 1/10*log(-3/2*cos(x) - 2*sin(x) +
5/2)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{-4\cos(x)+3\sin(x)}\,dx = -\frac{\log\left(\tan\left(\frac{x}{2}\right)+2\right)}{5} + \frac{\log\left(2\tan\left(\frac{x}{2}\right)-1\right)}{5}$$

```
[In] integrate(1/(-4*cos(x)+3*sin(x)),x)
[Out] -log(tan(x/2) + 2)/5 + log(2*tan(x/2) - 1)/5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(12) = 24$.

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{1}{-4\cos(x)+3\sin(x)}\,dx = \frac{1}{5}\,\log\left(\frac{2\,\sin\left(x\right)}{\cos\left(x\right)+1}-1\right) - \frac{1}{5}\,\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+2\right)$$

```
[In] integrate(1/(-4*cos(x)+3*sin(x)),x, algorithm="maxima")
[Out] 1/5*log(2*sin(x)/(cos(x) + 1) - 1) - 1/5*log(sin(x)/(cos(x) + 1) + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{-4\cos(x)+3\sin(x)}\,dx = \frac{1}{5}\,\log\left(\left|2\,\tan\left(\frac{1}{2}\,x\right)-1\right|\right)-\frac{1}{5}\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+2\right|\right)$$

```
[In] integrate(1/(-4*cos(x)+3*sin(x)),x, algorithm="giac")
[Out] 1/5*log(abs(2*tan(1/2*x) - 1)) - 1/5*log(abs(tan(1/2*x) + 2))
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.61

$$\int \frac{1}{-4\cos(x)+3\sin(x)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{4\tan\left(\frac{x}{2}\right)}{5}+\frac{3}{5}\right)}{5}$$

```
[In] int(-1/(4*cos(x) - 3*sin(x)),x)
[Out] -(2*atanh((4*tan(x/2))/5 + 3/5))/5
```

3.222 $\int \frac{1}{1+\sqrt{x}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int \frac{1}{1+\sqrt{x}} \, dx = 2\sqrt{x} - 2\log\left(1+\sqrt{x}\right)$$

```
[Out] -2*ln(1+x^(1/2))+2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {196, 45}

$$\int \frac{1}{1+\sqrt{x}} \, dx = 2\sqrt{x} - 2\log\left(\sqrt{x}+1\right)$$

```
[In] Int[(1 + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] - 2*Log[1 + Sqrt[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
```

```
IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= 2\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, \sqrt{x}\right) \\ &= 2\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx, x, \sqrt{x}\right) \\ &= 2\sqrt{x} - 2\log\left(1+\sqrt{x}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\sqrt{x}}\,dx = 2\sqrt{x} - 2\log\left(1+\sqrt{x}\right)$$

```
[In] Integrate[(1 + Sqrt[x])^(-1),x]
[Out] 2*Sqrt[x] - 2*Log[1 + Sqrt[x]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-2\ln\left(\sqrt{x}+1\right)+2\sqrt{x}$	15
meijerg	$-2\ln\left(\sqrt{x}+1\right)+2\sqrt{x}$	15
trager	$2\sqrt{x}-\ln\left(2\sqrt{x}+1+x\right)$	18
default	$2\sqrt{x}+\ln\left(\sqrt{x}-1\right)-\ln\left(\sqrt{x}+1\right)-\ln\left(-1+x\right)$	27

```
[In] int(1/(x^(1/2)+1),x,method=_RETURNVERBOSE)
[Out] -2*ln(x^(1/2)+1)+2*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{1+\sqrt{x}}\,dx = 2\,\sqrt{x} - 2\,\log\left(\sqrt{x}+1\right)$$

```
[In] integrate(1/(1+x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{1+\sqrt{x}}\,dx = 2\sqrt{x} - 2\log\left(\sqrt{x}+1\right)$$

```
[In] integrate(1/(1+x**(1/2)),x)
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{1+\sqrt{x}}\,dx = 2\,\sqrt{x} - 2\,\log\left(\sqrt{x}+1\right) + 2$$

```
[In] integrate(1/(1+x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1) + 2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{1+\sqrt{x}}\,dx = 2\,\sqrt{x} - 2\,\log\left(\sqrt{x}+1\right)$$

```
[In] integrate(1/(1+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{1+\sqrt{x}}\,dx = 2\sqrt{x} - 2\ln\left(\sqrt{x}+1\right)$$

```
[In] int(1/(x^(1/2) + 1),x)
[Out] 2*x^(1/2) - 2*log(x^(1/2) + 1)
```

3.223 $\int \frac{1}{1+\frac{1}{\sqrt[3]{x}}} dx$

Optimal result

Integrand size = 9, antiderivative size = 32

$$\int \frac{1}{1+\frac{1}{\sqrt[3]{x}}} dx = 3\sqrt[3]{x} - \frac{3x^{2/3}}{2} + x - 3\log\left(1+\frac{1}{\sqrt[3]{x}}\right) - \log(x)$$

```
[Out] 3*x^(1/3)-3/2*x^(2/3)+x-3*ln(1+1/x^(1/3))-ln(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {196, 46}

$$\int \frac{1}{1+\frac{1}{\sqrt[3]{x}}} dx = -\frac{3x^{2/3}}{2} + x + 3\sqrt[3]{x} - 3\log\left(\frac{1}{\sqrt[3]{x}}+1\right) - \log(x)$$

```
[In] Int[(1 + x^(-1/3))^(-1),x]
```

```
[Out] 3*x^(1/3) - (3*x^(2/3))/2 + x - 3*Log[1 + x^(-1/3)] - Log[x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(3\text{Subst}\left(\int \frac{1}{x^4(1+x)}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\
&= -\left(3\text{Subst}\left(\int \left(\frac{1}{x^4}-\frac{1}{x^3}+\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\
&= 3\sqrt[3]{x}-\frac{3x^{2/3}}{2}+x-3\log\left(1+\frac{1}{\sqrt[3]{x}}\right)-\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+\frac{1}{\sqrt[3]{x}}}\,dx = 3\sqrt[3]{x}-\frac{3x^{2/3}}{2}+x-3\log\left(1+\sqrt[3]{x}\right)$$

```
[In] Integrate[(1 + x^(-1/3))^(-1),x]
[Out] 3*x^(1/3) - (3*x^(2/3))/2 + x - 3*Log[1 + x^(1/3)]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$x-\frac{3x^{\frac{2}{3}}}{2}+3x^{\frac{1}{3}}-3\ln\left(x^{\frac{1}{3}}+1\right)$	21
default	$x-\frac{3x^{\frac{2}{3}}}{2}+3x^{\frac{1}{3}}-3\ln\left(x^{\frac{1}{3}}+1\right)$	21
meijerg	$\frac{x^{\frac{1}{3}}\left(4x^{\frac{2}{3}}-6x^{\frac{1}{3}}+12\right)}{4}-3\ln\left(x^{\frac{1}{3}}+1\right)$	27
trager	$-1+x+3x^{\frac{1}{3}}-\frac{3x^{\frac{2}{3}}}{2}-\ln\left(-3x^{\frac{2}{3}}-3x^{\frac{1}{3}}-x-1\right)$	32

```
[In] int(1/(1+1/x^(1/3)),x,method=_RETURNVERBOSE)
[Out] x-3/2*x^(2/3)+3*x^(1/3)-3*ln(x^(1/3)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+\frac{1}{\sqrt[3]{x}}}\,dx = x - \frac{3}{2}\,x^{\frac{2}{3}} + 3\,x^{\frac{1}{3}} - 3\,\log\left(x^{\frac{1}{3}} + 1\right)$$

```
[In] integrate(1/(1+1/x^(1/3)),x, algorithm="fricas")
[Out] x - 3/2*x^(2/3) + 3*x^(1/3) - 3*log(x^(1/3) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{1+\frac{1}{\sqrt[3]{x}}}\,dx = -\frac{3x^{\frac{2}{3}}}{2} + 3\sqrt[3]{x} + x - 3\log\left(\sqrt[3]{x} + 1\right)$$

```
[In] integrate(1/(1+1/x**(1/3)),x)
[Out] -3*x**(2/3)/2 + 3*x**(1/3) + x - 3*log(x**(1/3) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+\frac{1}{\sqrt[3]{x}}}\,dx = -\frac{1}{2}\,x\left(\frac{3}{x^{\frac{1}{3}}} - \frac{6}{x^{\frac{2}{3}}} - 2\right) - \log\left(x\right) - 3\,\log\left(\frac{1}{x^{\frac{1}{3}}} + 1\right)$$

```
[In] integrate(1/(1+1/x^(1/3)),x, algorithm="maxima")
[Out] -1/2*x*(3/x^(1/3) - 6/x^(2/3) - 2) - log(x) - 3*log(1/x^(1/3) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+\frac{1}{\sqrt[3]{x}}}\,dx = x - \frac{3}{2}\,x^{\frac{2}{3}} + 3\,x^{\frac{1}{3}} - 3\,\log\left(x^{\frac{1}{3}} + 1\right)$$

```
[In] integrate(1/(1+1/x^(1/3)),x, algorithm="giac")
[Out] x - 3/2*x^(2/3) + 3*x^(1/3) - 3*log(x^(1/3) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+\frac{1}{\sqrt[3]{x}}}\,dx = x - 3\,\ln\left(x^{1/3} + 1\right) + 3\,x^{1/3} - \frac{3\,x^{2/3}}{2}$$

```
[In] int(1/(1/x^(1/3) + 1),x)
[Out] x - 3*log(x^(1/3) + 1) + 3*x^(1/3) - (3*x^(2/3))/2
```

3.224 $\int \frac{\sqrt{x}}{1+x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{\sqrt{x}}{1+x} \, dx = 2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

```
[Out] -2*arctan(x^(1/2))+2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {52, 65, 209}

$$\int \frac{\sqrt{x}}{1+x} \, dx = 2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

```
[In] Int[Sqrt[x]/(1 + x),x]
```

```
[Out] 2*Sqrt[x] - 2*ArcTan[Sqrt[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\sqrt{x} - \int \frac{1}{\sqrt{x}(1+x)}\,dx \\ &= 2\sqrt{x} - 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right) \\ &= 2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{1+x}\,dx = 2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

```
[In] Integrate[Sqrt[x]/(1 + x),x]
[Out] 2*Sqrt[x] - 2*ArcTan[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-2\arctan\left(\sqrt{x}\right)+2\sqrt{x}$	13
default	$-2\arctan\left(\sqrt{x}\right)+2\sqrt{x}$	13
meijerg	$-2\arctan\left(\sqrt{x}\right)+2\sqrt{x}$	13
risch	$-2\arctan\left(\sqrt{x}\right)+2\sqrt{x}$	13
trager	$2\sqrt{x}+\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{x}-x+1}{1+x}\right)$	38

```
[In] int(x^(1/2)/(1+x),x,method=_RETURNVERBOSE)
[Out] -2*arctan(x^(1/2))+2*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{1+x}\,dx = 2\,\sqrt{x}-2\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(x^(1/2)/(1+x),x, algorithm="fricas")
[Out] 2*sqrt(x) - 2*arctan(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{x}}{1+x}\,dx = 2\sqrt{x}-2\,\mathrm{atan}\left(\sqrt{x}\right)$$

```
[In] integrate(x**(1/2)/(1+x),x)
[Out] 2*sqrt(x) - 2*atan(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{1+x}\,dx = 2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(x^(1/2)/(1+x),x, algorithm="maxima")
[Out] 2*sqrt(x) - 2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{1+x}\,dx = 2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(x^(1/2)/(1+x),x, algorithm="giac")
[Out] 2*sqrt(x) - 2*arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{1+x}\,dx = 2\sqrt{x} - 2\,\mathrm{atan}\left(\sqrt{x}\right)$$

```
[In] int(x^(1/2)/(x + 1),x)
[Out] 2*x^(1/2) - 2*atan(x^(1/2))
```

3.225 $\int \frac{1}{x\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{1}{x\sqrt{1+x}}\,dx = -2\text{arctanh}\left(\sqrt{1+x}\right)$$

```
[Out] -2*arctanh((1+x)^(1/2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {65, 213}

$$\int \frac{1}{x\sqrt{1+x}}\,dx = -2\text{arctanh}\left(\sqrt{x+1}\right)$$

```
[In] Int[1/(x*Sqrt[1 + x]),x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x}\right)\\ &= -2\text{arctanh}\left(\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+x}}\,dx = -2\text{arctanh}\left(\sqrt{1+x}\right)$$

```
[In] Integrate[1/(x*Sqrt[1 + x]),x]
[Out] -2*ArcTanh[Sqrt[1 + x]]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-2\,\text{arctanh}\left(\sqrt{1+x}\right)$	9
default	$-2\,\text{arctanh}\left(\sqrt{1+x}\right)$	9
trager	$-\ln\left(\frac{2\sqrt{1+x}+2+x}{x}\right)$	18
meijerg	$\frac{(-2\ln(2)+\ln(x))\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+x}}{2}\right)}{\sqrt{\pi}}$	32

```
[In] int(1/x/(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((1+x)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{x\sqrt{1+x}}\,dx = -\log\left(\sqrt{x+1}+1\right) + \log\left(\sqrt{x+1}-1\right)$$

```
[In] integrate(1/x/(1+x)^(1/2),x, algorithm="fricas")
[Out] -log(sqrt(x + 1) + 1) + log(sqrt(x + 1) - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.60

$$\int \frac{1}{x\sqrt{1+x}}\,dx = \begin{cases} -2\,\mathrm{acoth}\left(\sqrt{x+1}\right) & \text{for } |x+1|>1 \\ -2\,\mathrm{atanh}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/(1+x)**(1/2),x)
[Out] Piecewise((-2*acoth(sqrt(x + 1)), Abs(x + 1) > 1), (-2*atanh(sqrt(x + 1)),
True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{x\sqrt{1+x}}\,dx = -\log\left(\sqrt{x+1}+1\right) + \log\left(\sqrt{x+1}-1\right)$$

```
[In] integrate(1/x/(1+x)^(1/2),x, algorithm="maxima")
[Out] -log(sqrt(x + 1) + 1) + log(sqrt(x + 1) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{1}{x\sqrt{1+x}}\,dx = -\log\left(\sqrt{x+1}+1\right) + \log\left(\left|\sqrt{x+1}-1\right|\right)$$

```
[In] integrate(1/x/(1+x)^(1/2),x, algorithm="giac")
[Out] -log(sqrt(x + 1) + 1) + log(abs(sqrt(x + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\sqrt{1+x}}\,dx = -2\,\text{atanh}\left(\sqrt{x+1}\right)$$

```
[In] int(1/(x*(x + 1)^(1/2)),x)
[Out] -2*atanh((x + 1)^(1/2))
```

3.226 $\int \frac{1}{-\sqrt[3]{x}+x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{1}{-\sqrt[3]{x}+x} \, dx = \frac{3}{2} \log \left(1-x^{2/3}\right)$$

```
[Out] 3/2*ln(1-x^(2/3))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 266}

$$\int \frac{1}{-\sqrt[3]{x}+x} \, dx = \frac{3}{2} \log \left(1-x^{2/3}\right)$$

```
[In] Int[(-x^(1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 - x^(2/3)])/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\left(-1+x^{2/3}\right)\sqrt[3]{x}}\,dx$$
$$= \frac{3}{2}\log\left(1-x^{2/3}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{-\sqrt[3]{x}+x}\,dx = \frac{3}{2}\log\left(-1+\sqrt[3]{x}\right)+\frac{3}{2}\log\left(1+\sqrt[3]{x}\right)$$

```
[In] Integrate[(-x^(1/3) + x)^(-1),x]
[Out] (3*Log[-1 + x^(1/3)])/2 + (3*Log[1 + x^(1/3)])/2
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
meijerg	$\frac{3\ln\left(1-x^{\frac{2}{3}}\right)}{2}$	11
derivativedivides	$\frac{3\ln\left(x^{\frac{1}{3}}-1\right)}{2}+\frac{3\ln\left(x^{\frac{1}{3}}+1\right)}{2}$	18
trager	$\frac{\ln\left(3x^{\frac{2}{3}}-3x^{\frac{4}{3}}+x^{2}-1\right)}{2}$	19
default	$\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}-\frac{\ln\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}+1\right)}{2}+\ln\left(x^{\frac{1}{3}}-1\right)+\ln\left(x^{\frac{1}{3}}+1\right)-\frac{\ln\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1\right)}{2}$	50

```
[In] int(1/(-x^(1/3)+x),x,method=_RETURNVERBOSE)
[Out] 3/2*ln(1-x^(2/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{-\sqrt[3]{x}+x}\,dx=\frac{3}{2}\log\left(x^{\frac{2}{3}}-1\right)$$

```
[In] integrate(1/(-x^(1/3)+x),x, algorithm="fricas")
[Out] 3/2*log(x^(2/3) - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{1}{-\sqrt[3]{x}+x}\,dx=\frac{3\log\left(\sqrt[3]{x}-1\right)}{2}+\frac{3\log\left(\sqrt[3]{x}+1\right)}{2}$$

```
[In] integrate(1/(-x**(1/3)+x),x)
[Out] 3*log(x**(1/3) - 1)/2 + 3*log(x**(1/3) + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{1}{-\sqrt[3]{x}+x}\,dx=\frac{3}{2}\log\left(x^{\frac{1}{3}}+1\right)+\frac{3}{2}\log\left(x^{\frac{1}{3}}-1\right)$$

```
[In] integrate(1/(-x^(1/3)+x),x, algorithm="maxima")
[Out] 3/2*log(x^(1/3) + 1) + 3/2*log(x^(1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{1}{-\sqrt[3]{x}+x}\,dx = \frac{3}{2}\,\log\left(x^{\frac{1}{3}}+1\right)+\frac{3}{2}\,\log\left(\left|x^{\frac{1}{3}}-1\right|\right)$$

```
[In] integrate(1/(-x^(1/3)+x),x, algorithm="giac")
[Out] 3/2*log(x^(1/3) + 1) + 3/2*log(abs(x^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{-\sqrt[3]{x}+x}\,dx = \frac{3\,\ln\left(x^{2/3}-1\right)}{2}$$

```
[In] int(1/(x - x^(1/3)),x)
[Out] (3*log(x^(2/3) - 1))/2
```

3.227 $\int \frac{1}{x-\sqrt{2+x}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{1}{x-\sqrt{2+x}} \, dx = \frac{4}{3}\log\left(2-\sqrt{2+x}\right)+\frac{2}{3}\log\left(1+\sqrt{2+x}\right)$$

```
[Out] 4/3*ln(2-(2+x)^(1/2))+2/3*ln(1+(2+x)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {646, 31}

$$\int \frac{1}{x-\sqrt{2+x}} \, dx = \frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

```
[In] Int[(x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{x}{-2-x+x^2}\,dx, x, \sqrt{2+x}\right)\\ &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{2+x}\right) + \frac{4}{3}\text{Subst}\left(\int \frac{1}{-2+x}\,dx, x, \sqrt{2+x}\right)\\ &= \frac{4}{3}\log\left(2-\sqrt{2+x}\right) + \frac{2}{3}\log\left(1+\sqrt{2+x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \frac{4}{3}\log\left(-2+\sqrt{2+x}\right) + \frac{2}{3}\log\left(1+\sqrt{2+x}\right)$$

```
[In] Integrate[(x - Sqrt[2 + x])^(-1),x]
[Out] (4*Log[-2 + Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2\ln\left(1+\sqrt{2+x}\right)}{3}+\frac{4\ln\left(\sqrt{2+x}-2\right)}{3}$	22
trager	$\frac{\ln\left(6\sqrt{2+x}\,x^2-x^3+16\sqrt{2+x}\,x-15x^2+8\sqrt{2+x}-24x-12\right)}{3}$	44
default	$\frac{\ln(1+x)}{3}+\frac{2\ln(-2+x)}{3}+\frac{\ln\left(1+\sqrt{2+x}\right)}{3}-\frac{\ln\left(\sqrt{2+x}-1\right)}{3}+\frac{2\ln\left(\sqrt{2+x}-2\right)}{3}-\frac{2\ln\left(\sqrt{2+x}+2\right)}{3}$	54

```
[In] int(1/(x-(2+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/3*ln(1+(2+x)^(1/2))+4/3*ln((2+x)^(1/2)-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \frac{2}{3}\log\left(\sqrt{x+2}+1\right) + \frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="fricas")
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \log\left(x-\sqrt{x+2}\right) + \frac{\log\left(2\sqrt{x+2}-4\right)}{3} - \frac{\log\left(2\sqrt{x+2}+2\right)}{3}$$

```
[In] integrate(1/(x-(2+x)**(1/2)),x)
[Out] log(x - sqrt(x + 2)) + log(2*sqrt(x + 2) - 4)/3 - log(2*sqrt(x + 2) + 2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \frac{2}{3}\log\left(\sqrt{x+2}+1\right) + \frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="maxima")
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \frac{2}{3}\log\left(\sqrt{x+2}+1\right) + \frac{4}{3}\log\left(\left|\sqrt{x+2}-2\right|\right)$$

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="giac")
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(abs(sqrt(x + 2) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \frac{2\ln\left(\frac{2\sqrt{x+2}}{3}+\frac{2}{3}\right)}{3}+\frac{4\ln\left(\frac{4}{3}-\frac{2\sqrt{x+2}}{3}\right)}{3}$$

```
[In] int(1/(x - (x + 2)^(1/2)),x)
[Out] (2*log((2*(x + 2)^(1/2))/3 + 2/3))/3 + (4*log(4/3 - (2*(x + 2)^(1/2))/3))/3
```

3.228 $\int \frac{x^2}{\sqrt{-1+x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{x^2}{\sqrt{-1+x}}\,dx = 2\sqrt{-1+x}+\frac{4}{3}(-1+x)^{3/2}+\frac{2}{5}(-1+x)^{5/2}$$

```
[Out] 4/3*(-1+x)^(3/2)+2/5*(-1+x)^(5/2)+2*(-1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\int \frac{x^2}{\sqrt{-1+x}}\,dx = \frac{2}{5}(x-1)^{5/2}+\frac{4}{3}(x-1)^{3/2}+2\sqrt{x-1}$$

```
[In] Int[x^2/Sqrt[-1 + x],x]
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 + (2*(-1 + x)^(5/2))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{\sqrt{-1+x}}+2\sqrt{-1+x}+(-1+x)^{3/2}\right)dx \\ &= 2\sqrt{-1+x}+\frac{4}{3}(-1+x)^{3/2}+\frac{2}{5}(-1+x)^{5/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{\sqrt{-1+x}}\,dx = \frac{2}{15}\sqrt{-1+x}\left(8+4x+3x^2\right)$$

```
[In] Integrate[x^2/Sqrt[-1 + x],x]
[Out] (2*Sqrt[-1 + x]*(8 + 4*x + 3*x^2))/15
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

method	result	size
trager	$\left(\frac{2}{5}x^2+\frac{8}{15}x+\frac{16}{15}\right)\sqrt{-1+x}$	17
gosper	$\frac{2\sqrt{-1+x}\,(3x^2+4x+8)}{15}$	18
risch	$\frac{2\sqrt{-1+x}\,(3x^2+4x+8)}{15}$	18
derivativedivides	$\frac{4(-1+x)^{\frac{3}{2}}}{3}+\frac{2(-1+x)^{\frac{5}{2}}}{5}+2\sqrt{-1+x}$	23
default	$\frac{4(-1+x)^{\frac{3}{2}}}{3}+\frac{2(-1+x)^{\frac{5}{2}}}{5}+2\sqrt{-1+x}$	23
meijerg	$-\frac{\sqrt{-\operatorname{signum}(-1+x)}\left(-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\,(6x^2+8x+16)\sqrt{1-x}}{15}\right)}{\sqrt{\pi}\,\sqrt{\operatorname{signum}(-1+x)}}$	48

```
[In] int(x^2/(-1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] (2/5*x^2+8/15*x+16/15)*(-1+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \frac{x^2}{\sqrt{-1+x}}\,dx = \frac{2}{15}\left(3\,x^2+4\,x+8\right)\sqrt{x-1}$$

```
[In] integrate(x^2/(-1+x)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*x^2 + 4*x + 8)*sqrt(x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.38

$$\int \frac{x^2}{\sqrt{-1+x}}\,dx = \begin{cases} \frac{2x^2\sqrt{x-1}}{5} + \frac{8x\sqrt{x-1}}{15} + \frac{16\sqrt{x-1}}{15} & \text{for } |x| > 1 \\ \frac{2ix^2\sqrt{1-x}}{5} + \frac{8ix\sqrt{1-x}}{15} + \frac{16i\sqrt{1-x}}{15} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2/(-1+x)**(1/2),x)
```

```
[Out] Piecewise((2*x**2*sqrt(x - 1)/5 + 8*x*sqrt(x - 1)/15 + 16*sqrt(x - 1)/15, A
bs(x) > 1), (2*I*x**2*sqrt(1 - x)/5 + 8*I*x*sqrt(1 - x)/15 + 16*I*sqrt(1 -
x)/15, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{-1+x}}\,dx = \frac{2}{5}\,(x-1)^{\frac{5}{2}} + \frac{4}{3}\,(x-1)^{\frac{3}{2}} + 2\sqrt{x-1}$$

```
[In] integrate(x^2/(-1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(x - 1)^(5/2) + 4/3*(x - 1)^(3/2) + 2*sqrt(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{-1+x}}\,dx = \frac{2}{5}\,(x-1)^{\frac{5}{2}} + \frac{4}{3}\,(x-1)^{\frac{3}{2}} + 2\sqrt{x-1}$$

```
[In] integrate(x^2/(-1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*(x - 1)^(5/2) + 4/3*(x - 1)^(3/2) + 2*sqrt(x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \frac{x^2}{\sqrt{-1+x}}\,dx = \frac{2\,\sqrt{x-1}\,\left(10\,x+3\,{\left(x-1\right)}^2+5\right)}{15}$$

```
[In] int(x^2/(x - 1)^(1/2),x)
[Out] (2*(x - 1)^(1/2)*(10*x + 3*(x - 1)^2 + 5))/15
```

3.229 $\int \frac{\sqrt{-1+x}}{1+x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\sqrt{-1+x}}{1+x} \, dx = 2\sqrt{-1+x} - 2\sqrt{2}\arctan\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)$$

```
[Out] -2*arctan(1/2*(-1+x)^(1/2)*2^(1/2))*2^(1/2)+2*(-1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {52, 65, 209}

$$\int \frac{\sqrt{-1+x}}{1+x} \, dx = 2\sqrt{x-1} - 2\sqrt{2}\arctan\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)$$

```
[In] Int[Sqrt[-1 + x]/(1 + x),x]
```

```
[Out] 2*Sqrt[-1 + x] - 2*Sqrt[2]*ArcTan[Sqrt[-1 + x]/Sqrt[2]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\sqrt{-1+x} - 2\int \frac{1}{\sqrt{-1+x}(1+x)}\,dx\\ &= 2\sqrt{-1+x} - 4\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt{-1+x}\right)\\ &= 2\sqrt{-1+x} - 2\sqrt{2}\arctan\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+x}}{1+x}\,dx = 2\sqrt{-1+x} - 2\sqrt{2}\arctan\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)$$

```
[In] Integrate[Sqrt[-1 + x]/(1 + x),x]
[Out] 2*Sqrt[-1 + x] - 2*Sqrt[2]*ArcTan[Sqrt[-1 + x]/Sqrt[2]]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-2\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{2}+2\sqrt{-1+x}$	25
default	$-2\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{2}+2\sqrt{-1+x}$	25
risch	$-2\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{2}+2\sqrt{-1+x}$	25
trager	$2\sqrt{-1+x}-\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)x-3\operatorname{RootOf}\left(\_Z^2+2\right)-4\sqrt{-1+x}}{1+x}\right)$	49

[In] int((-1+x)^(1/2)/(1+x),x,method=_RETURNVERBOSE)

[Out] -2*arctan(1/2*(-1+x)^(1/2)*2^(1/2))*2^(1/2)+2*(-1+x)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{-1+x}}{1+x}\,dx = -2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\sqrt{x-1}\right)+2\,\sqrt{x-1}$$

[In] integrate((-1+x)^(1/2)/(1+x),x, algorithm="fricas")

[Out] -2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x - 1)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.42

$$\int \frac{\sqrt{-1+x}}{1+x}\,dx = \begin{cases} 2\sqrt{x-1}+2\sqrt{2}\operatorname{asin}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right) & \text{for } |x+1|>2 \\ 2i\sqrt{1-x}+\sqrt{2}i\log\left(x+1\right)-2\sqrt{2}i\log\left(\sqrt{\frac{1}{2}-\frac{x}{2}}+1\right) & \text{otherwise} \end{cases}$$

[In] integrate((-1+x)**(1/2)/(1+x),x)

[Out] Piecewise((2*sqrt(x - 1) + 2*sqrt(2)*asin(sqrt(2)/sqrt(x + 1)), Abs(x + 1) > 2), (2*I*sqrt(1 - x) + sqrt(2)*I*log(x + 1) - 2*sqrt(2)*I*log(sqrt(1/2 - x/2) + 1), True))

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{-1+x}}{1+x}\,dx = -2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\sqrt{x-1}\right) + 2\,\sqrt{x-1}$$

```
[In] integrate((-1+x)^(1/2)/(1+x),x, algorithm="maxima")
[Out] -2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{-1+x}}{1+x}\,dx = -2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\sqrt{x-1}\right) + 2\,\sqrt{x-1}$$

```
[In] integrate((-1+x)^(1/2)/(1+x),x, algorithm="giac")
[Out] -2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{-1+x}}{1+x}\,dx = 2\,\sqrt{x-1} - 2\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{x-1}}{2}\right)$$

```
[In] int((x - 1)^(1/2)/(x + 1),x)
[Out] 2*(x - 1)^(1/2) - 2*2^(1/2)*atan((2^(1/2)*(x - 1)^(1/2))/2)
```

3.230 $\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx = -4\sqrt{1+\sqrt{x}}+\frac{4}{3}\left(1+\sqrt{x}\right)^{3/2}$$

```
[Out] 4/3*(1+x^(1/2))^(3/2)-4*(1+x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {196, 45}

$$\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx = \frac{4}{3}\left(\sqrt{x}+1\right)^{3/2}-4\sqrt{\sqrt{x}+1}$$

```
[In] Int[1/Sqrt[1 + Sqrt[x]],x]
```

```
[Out] -4*Sqrt[1 + Sqrt[x]] + (4*(1 + Sqrt[x])^(3/2))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
```

```
IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx, x, \sqrt{x}\right)\\ &= 2\text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right)\,dx, x, \sqrt{x}\right)\\ &= -4\sqrt{1+\sqrt{x}}+\frac{4}{3}\left(1+\sqrt{x}\right)^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx = \frac{4}{3}\left(-2+\sqrt{x}\right)\sqrt{1+\sqrt{x}}$$

```
[In] Integrate[1/Sqrt[1 + Sqrt[x]],x]
[Out] (4*(-2 + Sqrt[x])*Sqrt[1 + Sqrt[x]])/3
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{4\left(\sqrt{x}+1\right)^{\frac{3}{2}}}{3}-4\sqrt{\sqrt{x}+1}$	20
default	$\frac{4\left(\sqrt{x}+1\right)^{\frac{3}{2}}}{3}-4\sqrt{\sqrt{x}+1}$	20
meijerg	$\frac{\frac{8\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-4\sqrt{x}+8\right)\sqrt{\sqrt{x}+1}}{3}}{\sqrt{\pi}}$	31

```
[In] int(1/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 4/3*(x^(1/2)+1)^(3/2)-4*(x^(1/2)+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx = \frac{4}{3}\sqrt{\sqrt{x}+1}\left(\sqrt{x}-2\right)$$

```
[In] integrate(1/(1+x^(1/2))^(1/2),x, algorithm="fricas")
[Out] 4/3*sqrt(sqrt(x) + 1)*(sqrt(x) - 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(24) = 48.

Time = 0.50 (sec) , antiderivative size = 117, normalized size of antiderivative = 4.03

$$\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx = -\frac{4x^{\frac{5}{2}}\sqrt{\sqrt{x}+1}}{3x^{\frac{5}{2}}+3x^2} + \frac{8x^{\frac{5}{2}}}{3x^{\frac{5}{2}}+3x^2} + \frac{4x^3\sqrt{\sqrt{x}+1}}{3x^{\frac{5}{2}}+3x^2} - \frac{8x^2\sqrt{\sqrt{x}+1}}{3x^{\frac{5}{2}}+3x^2} + \frac{8x^2}{3x^{\frac{5}{2}}+3x^2}$$

```
[In] integrate(1/(1+x**(1/2))**(1/2),x)
[Out] -4*x**(5/2)*sqrt(sqrt(x) + 1)/(3*x**(5/2) + 3*x**2) + 8*x**(5/2)/(3*x**(5/2
) + 3*x**2) + 4*x**3*sqrt(sqrt(x) + 1)/(3*x**(5/2) + 3*x**2) - 8*x**2*sqrt(
sqrt(x) + 1)/(3*x**(5/2) + 3*x**2) + 8*x**2/(3*x**(5/2) + 3*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx = \frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}} - 4\sqrt{\sqrt{x}+1}$$

```
[In] integrate(1/(1+x^(1/2))^(1/2),x, algorithm="maxima")
[Out] 4/3*(sqrt(x) + 1)^(3/2) - 4*sqrt(sqrt(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx = \frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}} - 4\sqrt{\sqrt{x}+1}$$

```
[In] integrate(1/(1+x^(1/2))^(1/2),x, algorithm="giac")
[Out] 4/3*(sqrt(x) + 1)^(3/2) - 4*sqrt(sqrt(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx = x\,{}_2\mathrm{F}_1\left(\frac{1}{2},2;\ 3;\ -\sqrt{x}\right)$$

```
[In] int(1/(x^(1/2) + 1)^(1/2),x)
[Out] x*hypergeom([1/2, 2], 3, -x^(1/2))
```

3.231 $\int \frac{\sqrt{x}}{x+x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{\sqrt{x}}{x+x^2} \, dx = 2 \arctan \left(\sqrt{x}\right)$$

```
[Out] 2*arctan(x^(1/2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {661, 65, 209}

$$\int \frac{\sqrt{x}}{x+x^2} \, dx = 2 \arctan \left(\sqrt{x}\right)$$

```
[In] Int[Sqrt[x]/(x + x^2),x]
```

```
[Out] 2*ArcTan[Sqrt[x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 661

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{x}(1+x)}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\ &= 2\arctan\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{x+x^2}\,dx = 2\arctan\left(\sqrt{x}\right)$$

```
[In] Integrate[Sqrt[x]/(x + x^2),x]
[Out] 2*ArcTan[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$2\arctan\left(\sqrt{x}\right)$	7
default	$2\arctan\left(\sqrt{x}\right)$	7
meijerg	$2\arctan\left(\sqrt{x}\right)$	7
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{x}+x-1}{1+x}\right)$	29

```
[In] int(x^(1/2)/(x^2+x),x,method=_RETURNVERBOSE)
[Out] 2*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{x+x^2}\,dx = 2\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="fricas")
[Out] 2*arctan(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{x}}{x+x^2}\,dx = 2\operatorname{atan}\left(\sqrt{x}\right)$$

```
[In] integrate(x**(1/2)/(x**2+x),x)
[Out] 2*atan(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{x+x^2}\,dx = 2\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="maxima")
[Out] 2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{x+x^2}\,dx = 2\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="giac")
[Out] 2*arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{x+x^2}\,dx = 2\,\mathrm{atan}\left(\sqrt{x}\right)$$

```
[In] int(x^(1/2)/(x + x^2),x)
[Out] 2*atan(x^(1/2))
```

3.232 $\int \frac{1+\sqrt{x}}{-1+\sqrt{x}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}} \, dx = 4\sqrt{x}+x+4\log \left(1-\sqrt{x}\right)$$

```
[Out] x+4*ln(1-x^(1/2))+4*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {383, 78}

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}} \, dx = x+4\sqrt{x}+4\log \left(1-\sqrt{x}\right)$$

```
[In] Int[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
```

```
[Out] 4*Sqrt[x] + x + 4*Log[1 - Sqrt[x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x(1+x)}{-1+x}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(2+\frac{2}{-1+x}+x\right)\,dx, x, \sqrt{x}\right)\\
&= 4\sqrt{x}+x+4\log\left(1-\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx = 4\sqrt{x}+x+4\log\left(-1+\sqrt{x}\right)$$

```
[In] Integrate[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
[Out] 4*Sqrt[x] + x + 4*Log[-1 + Sqrt[x]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$x+4\sqrt{x}+4\ln\left(\sqrt{x}-1\right)$	16
default	$x+4\sqrt{x}+4\ln\left(\sqrt{x}-1\right)$	16
trager	$-2+x+4\sqrt{x}+2\ln\left(2\sqrt{x}-1-x\right)$	22
meijerg	$2\sqrt{x}+4\ln\left(1-\sqrt{x}\right)+\frac{\sqrt{x}\,(3\sqrt{x}+6)}{3}$	29

```
[In] int((x^(1/2)+1)/(x^(1/2)-1),x,method=_RETURNVERBOSE)
[Out] x+4*x^(1/2)+4*ln(x^(1/2)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx = x+4\sqrt{x}+4\,\log\left(\sqrt{x}-1\right)$$

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="fricas")
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx = 4\sqrt{x}+x+4\log\left(\sqrt{x}-1\right)$$

```
[In] integrate((1+x**(1/2))/(-1+x**(1/2)),x)
[Out] 4*sqrt(x) + x + 4*log(sqrt(x) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx = x+4\sqrt{x}+4\,\log\left(\sqrt{x}-1\right)$$

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="maxima")
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx = x+4\sqrt{x}+4\,\log\left(\left|\sqrt{x}-1\right|\right)$$

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="giac")
[Out] x + 4*sqrt(x) + 4*log(abs(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx = x + 4\,\ln\left(\sqrt{x}-1\right) + 4\,\sqrt{x}$$

```
[In] int((x^(1/2) + 1)/(x^(1/2) - 1),x)
[Out] x + 4*log(x^(1/2) - 1) + 4*x^(1/2)
```

3.233 $\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx=-6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(1-\sqrt[3]{x}\right)$$

```
[Out] -6*x^(1/3)-3*x^(2/3)-x-6*ln(1-x^(1/3))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {381, 383, 78}

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx=-3x^{2/3}-x-6\sqrt[3]{x}-6\log\left(1-\sqrt[3]{x}\right)$$

```
[In] Int[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1+\sqrt[3]{x}}{1-\sqrt[3]{x}}\,dx \\
&= 3\text{Subst}\left(\int \frac{x^2(1+x)}{1-x}\,dx, x, \sqrt[3]{x}\right) \\
&= 3\text{Subst}\left(\int \left(-2-\frac{2}{-1+x}-2x-x^2\right)\,dx, x, \sqrt[3]{x}\right) \\
&= -6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(1-\sqrt[3]{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(-1+\sqrt[3]{x}\right)$$

```
[In] Integrate[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[-1 + x^(1/3)]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-x-3x^{\frac{2}{3}}-6x^{\frac{1}{3}}-6\ln\left(x^{\frac{1}{3}}-1\right)$	23
default	$-x-3x^{\frac{2}{3}}-6x^{\frac{1}{3}}-6\ln\left(x^{\frac{1}{3}}-1\right)$	23
trager	$2-x-6x^{\frac{1}{3}}-3x^{\frac{2}{3}}-2\ln\left(-3x^{\frac{2}{3}}+3x^{\frac{1}{3}}+x-1\right)$	32
meijerg	$-\frac{x^{\frac{1}{3}}\left(4x^{\frac{2}{3}}+6x^{\frac{1}{3}}+12\right)}{4}-6\ln\left(1-x^{\frac{1}{3}}\right)-\frac{x^{\frac{1}{3}}\left(3x^{\frac{1}{3}}+6\right)}{2}$	41

```
[In] int((1+1/x^(1/3))/(-1+1/x^(1/3)),x,method=_RETURNVERBOSE)
[Out] -x-3*x^(2/3)-6*x^(1/3)-6*ln(x^(1/3)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx=-x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}-1\right)$$

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="fricas")
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx=-3x^{\frac{2}{3}}-6\sqrt[3]{x}-x-6\log\left(\sqrt[3]{x}-1\right)$$

```
[In] integrate((1+1/x**(1/3))/(-1+1/x**(1/3)),x)
[Out] -3*x**(2/3) - 6*x**(1/3) - x - 6*log(x**(1/3) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -x - 3\,x^{\frac{2}{3}} - 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} - 1\right)$$

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="maxima")
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -x - 3\,x^{\frac{2}{3}} - 6\,x^{\frac{1}{3}} - 6\,\log\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="giac")
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(abs(x^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -x - 6\,\ln\left(x^{1/3} - 1\right) - 6\,x^{1/3} - 3\,x^{2/3}$$

```
[In] int((1/x^(1/3) + 1)/(1/x^(1/3) - 1),x)
[Out] - x - 6*log(x^(1/3) - 1) - 6*x^(1/3) - 3*x^(2/3)
```

3.234 $\int \frac{x^3}{\sqrt[3]{1+x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{x^3}{\sqrt[3]{1+x^2}}\,dx=-\frac{3}{4}\left(1+x^2\right)^{2/3}+\frac{3}{10}\left(1+x^2\right)^{5/3}$$

```
[Out] -3/4*(x^2+1)^(2/3)+3/10*(x^2+1)^(5/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\int \frac{x^3}{\sqrt[3]{1+x^2}}\,dx=\frac{3}{10}\left(x^2+1\right)^{5/3}-\frac{3}{4}\left(x^2+1\right)^{2/3}$$

```
[In] Int[x^3/(1 + x^2)^(1/3),x]
[Out] (-3*(1 + x^2)^(2/3))/4 + (3*(1 + x^2)^(5/3))/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt[3]{1+x}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{\sqrt[3]{1+x}}+(1+x)^{2/3}\right)\,dx, x, x^2\right)\\ &= -\frac{3}{4}\left(1+x^2\right)^{2/3}+\frac{3}{10}\left(1+x^2\right)^{5/3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\sqrt[3]{1+x^2}}\,dx = \frac{3}{20}\left(1+x^2\right)^{2/3}\left(-3+2x^2\right)$$

```
[In] Integrate[x^3/(1 + x^2)^(1/3),x]
[Out] (3*(1 + x^2)^(2/3)*(-3 + 2*x^2))/20
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

method	result	size
trager	$\left(\frac{3x^2}{10}-\frac{9}{20}\right)\left(x^2+1\right)^{\frac{2}{3}}$	16
gosper	$\frac{3\left(x^2+1\right)^{\frac{2}{3}}\left(2x^2-3\right)}{20}$	17
meijerg	$\frac{x^4 {}_2F_1\left(\frac{1}{3},2;3;-x^2\right)}{4}$	17
risch	$\frac{3\left(x^2+1\right)^{\frac{2}{3}}\left(2x^2-3\right)}{20}$	17
pseudoelliptic	$\frac{3\left(x^2+1\right)^{\frac{2}{3}}\left(2x^2-3\right)}{20}$	17

```
[In] int(x^3/(x^2+1)^(1/3),x,method=_RETURNVERBOSE)
[Out] (3/10*x^2-9/20)*(x^2+1)^(2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{x^3}{\sqrt[3]{1+x^2}}\,dx = \frac{3}{20}\left(2\,x^2-3\right)\left(x^2+1\right)^{\frac{2}{3}}$$

```
[In] integrate(x^3/(x^2+1)^(1/3),x, algorithm="fricas")
[Out] 3/20*(2*x^2 - 3)*(x^2 + 1)^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{\sqrt[3]{1+x^2}}\,dx = \frac{3x^2(x^2+1)^{\frac{2}{3}}}{10} - \frac{9(x^2+1)^{\frac{2}{3}}}{20}$$

```
[In] integrate(x**3/(x**2+1)**(1/3),x)
[Out] 3*x**2*(x**2 + 1)**(2/3)/10 - 9*(x**2 + 1)**(2/3)/20
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^3}{\sqrt[3]{1+x^2}}\,dx = \frac{3}{10}\left(x^2+1\right)^{\frac{5}{3}} - \frac{3}{4}\left(x^2+1\right)^{\frac{2}{3}}$$

```
[In] integrate(x^3/(x^2+1)^(1/3),x, algorithm="maxima")
[Out] 3/10*(x^2 + 1)^(5/3) - 3/4*(x^2 + 1)^(2/3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^3}{\sqrt[3]{1+x^2}}\,dx = \frac{3}{10}\left(x^2+1\right)^{\frac{5}{3}} - \frac{3}{4}\left(x^2+1\right)^{\frac{2}{3}}$$

```
[In] integrate(x^3/(x^2+1)^(1/3),x, algorithm="giac")
[Out] 3/10*(x^2 + 1)^(5/3) - 3/4*(x^2 + 1)^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{x^3}{\sqrt[3]{1+x^2}}\,dx = \frac{3\,{\left(x^2+1\right)}^{2/3}\,\left(2\,x^2-3\right)}{20}$$

```
[In] int(x^3/(x^2 + 1)^(1/3),x)
[Out] (3*(x^2 + 1)^(2/3)*(2*x^2 - 3))/20
```

3.235 $\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 201

$$\begin{aligned}\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx &= 6\sqrt[6]{x}+x-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)\\ &\quad-\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}+4\sqrt[6]{x}\right)\right)\\ &\quad+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)\\ &\quad-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)\end{aligned}$$

```
[Out] 6*x^(1/6)+x+6/5*ln(1-x^(1/6))-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*
(-5^(1/2)+1)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*(5^(1/2)+1)-3/5*a
rctan(1/20*(1+4*x^(1/6)+5^(1/2))*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2
)-3/5*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/
2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1598, 348, 308, 208, 648, 632, 210, 642, 31}

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = -\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right)+x + 6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right) - \frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)$$

```
[In] Int[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/S
qrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[
5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 -
 Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 + Sqrt
[5])*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r + s*Cos[(2*k
- 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; (r/(a*
n))*Int[1/(r - s*x), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 1)/2}], x],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^{5/6}}{-1+x^{5/6}}\,dx\\
&= 6\text{Subst}\left(\int \frac{x^{10}}{-1+x^5}\,dx,x,\sqrt[6]{x}\right)\\
&= 6\text{Subst}\left(\int \left(1+x^5+\frac{1}{-1+x^5}\right)\,dx,x,\sqrt[6]{x}\right)\\
&= 6\sqrt[6]{x}+x+6\text{Subst}\left(\int \frac{1}{-1+x^5}\,dx,x,\sqrt[6]{x}\right)
\end{aligned}$$

$$= 6\sqrt[6]{x} + x - \frac{6}{5}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[6]{x}\right)$$
$$- \frac{12}{5}\text{Subst}\left(\int \frac{1+\frac{1}{4}(1-\sqrt{5})\,x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)$$
$$- \frac{12}{5}\text{Subst}\left(\int \frac{1+\frac{1}{4}(1+\sqrt{5})\,x}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\sqrt[6]{x} + x + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right)$$
$$- \frac{1}{10}\left(3\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{\frac{1}{2}(1-\sqrt{5})+2x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)$$
$$- \frac{1}{10}\left(3\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)$$
$$- \frac{1}{10}\left(3\left(1+\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{\frac{1}{2}(1+\sqrt{5})+2x}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)$$
$$- \frac{1}{10}\left(3\left(5+\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\sqrt[6]{x} + x + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) - \frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)$$
$$- \frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)$$
$$+ \frac{1}{5}\left(3\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\left(-5+\sqrt{5}\right)-x^2}\,dx, x, \frac{1}{2}\left(1+\sqrt{5}\right)+2\sqrt[6]{x}\right)$$
$$+ \frac{1}{5}\left(3\left(5+\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\left(-5-\sqrt{5}\right)-x^2}\,dx, x, \frac{1}{2}\left(1-\sqrt{5}\right)+2\sqrt[6]{x}\right)$$
$$= 6\sqrt[6]{x} + x - \frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)$$
$$- \frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}+4\sqrt[6]{x}\right)\right)$$
$$+ \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) - \frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)$$
$$- \frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = 6\sqrt[6]{x}+x+\frac{6}{5}\log\left(-1+\sqrt[6]{x}\right)-\frac{6}{5}\text{RootSum}\Big[1+\#1+\#1^2+\#1^3+\#1^4\&,\frac{4\log\left(\sqrt[6]{x}-\#1\right)+3\log\left(\sqrt[6]{x}-\#1\right)\#1+2\log\left(\sqrt[6]{x}-\#1\right)\#1^2+\log\left(\sqrt[6]{x}-\#1\right)\#1^3}{1+2\#1+3\#1^2+4\#1^3}\&\Big]$$

```
[In] Integrate[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x + (6*Log[-1 + x^(1/6)])/5 - (6*RootSum[1 + #1 + #1^2 + #1^3 +
 #1^4 & , (4*Log[x^(1/6) - #1] + 3*Log[x^(1/6) - #1]*#1 + 2*Log[x^(1/6) - #
1]*#1^2 + Log[x^(1/6) - #1]*#1^3)/(1 + 2*#1 + 3*#1^2 + 4*#1^3) & ])/5
```

Maple [A] (warning: unable to verify)

Time = 0.15 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.66

method	result
meijerg	$\frac{6(-1)^{\frac{4}{5}}\left(-\frac{5x^{\frac{1}{6}}(-1)^{\frac{1}{5}}\left(11x^{\frac{5}{6}}+66\right)}{66}-(-1)^{\frac{1}{5}}\left(\ln\left(1-x^{\frac{1}{6}}\right)+\cos\left(\frac{2\pi}{5}\right)\ln\left(1-2\cos\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)-2\sin\left(\frac{2\pi}{5}\right)\arctan\left(\frac{\sin(\cdots}{1-\cos(\cdots}\right.\right.\right.}{5}$
derivativedivides	$x+6x^{\frac{1}{6}}-\frac{3\ln\left(2+x^{\frac{1}{6}}+2x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{5}\right)\left(-\sqrt{5}+1\right)}{10}-\frac{12\left(4-\frac{\left(-\sqrt{5}+1\right)^2}{4}\right)\arctan\left(\frac{1+4x^{\frac{1}{6}}-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}+\frac{3\left(-\sqrt{5}-1\right)\ln\cdots}{\cdots}$
default	$x+6x^{\frac{1}{6}}-\frac{3\ln\left(2+x^{\frac{1}{6}}+2x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{5}\right)\left(-\sqrt{5}+1\right)}{10}-\frac{12\left(4-\frac{\left(-\sqrt{5}+1\right)^2}{4}\right)\arctan\left(\frac{1+4x^{\frac{1}{6}}-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}+\frac{3\left(-\sqrt{5}-1\right)\ln\cdots}{\cdots}$

```
[In] int(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 6/5*(-1)^(4/5)*(-5/66*x^(1/6)*(-1)^(1/5)*(11*x^(5/6)+66)-(-1)^(1/5)*(ln(1-x
^(1/6))+cos(2/5*Pi)*ln(1-2*cos(2/5*Pi)*x^(1/6)+x^(1/3))-2*sin(2/5*Pi)*arcta
n(sin(2/5*Pi)*x^(1/6)/(1-cos(2/5*Pi)*x^(1/6)))-cos(1/5*Pi)*ln(1+2*cos(1/5*P
i)*x^(1/6)+x^(1/3))-2*sin(1/5*Pi)*arctan(sin(1/5*Pi)*x^(1/6)/(1+cos(1/5*Pi)
*x^(1/6)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(134) = 268$.

Time = 0.91 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.72

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = -\frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)\log\left(\frac{3}{2}\sqrt{2}\sqrt{\sqrt{5}-5}+\frac{3}{2}\sqrt{5}+6\,x^{\frac{1}{6}}+\frac{3}{2}\right)$$
$$+\frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)\log\left(-\frac{3}{2}\sqrt{2}\sqrt{\sqrt{5}-5}+\frac{3}{2}\sqrt{5}+6\,x^{\frac{1}{6}}+\frac{3}{2}\right)$$
$$+\frac{1}{10}\left(3\sqrt{5}-\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^{2}+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right.}\right.$$
$$+\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^{2}+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\sqrt{2}\right.}$$
$$\left.+12\,x^{\frac{1}{6}}+3\right)$$
$$+\frac{1}{10}\left(3\sqrt{5}+\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^{2}+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right.}\right.$$
$$-\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^{2}+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\sqrt{2}\right.}$$
$$\left.+12\,x^{\frac{1}{6}}+3\right)+x+6\,x^{\frac{1}{6}}+\frac{6}{5}\log\left(x^{\frac{1}{6}}-1\right)$$

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(3/2*sqrt(2)*sqrt(sqrt(5
) - 5) + 3/2*sqrt(5) + 6*x^(1/6) + 3/2) + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) -
 sqrt(5) - 1)*log(-3/2*sqrt(2)*sqrt(sqrt(5) - 5) + 3/2*sqrt(5) + 6*x^(1/6)
+ 3/2) + 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5)
+ 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5
) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 +
18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(-3*sqrt(5) + sqrt(
-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(
5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sq
rt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 1
8*sqrt(5) - 90) + 12*x^(1/6) + 3) + 1/10*(3*sqrt(5) + sqrt(-27/4*(sqrt(2)*s
qrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5
) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(
5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90)
```

```
- 3)*log(-3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5
) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sq
rt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) + 12*x^(1/6) + 3) + x + 6*x^(1/6
) + 6/5*log(x^(1/6) - 1)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}} + \sqrt{x}}\,dx = \int \frac{x^{\frac{5}{6}}}{\left(\sqrt[6]{x} - 1\right)\left(\sqrt[6]{x} + x^{\frac{2}{3}} + \sqrt[3]{x} + \sqrt{x} + 1\right)}\,dx$$

```
[In] integrate(x**(1/2)/(-1/x**(1/3)+x**(1/2)),x)
[Out] Integral(x**(5/6)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3) + sqrt(x)
 + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(134) = 268$.

Time = 0.32 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.46

$$\begin{aligned}\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}} + \sqrt{x}}\,dx = & -\frac{3\sqrt{5}(-1)^{\frac{1}{5}}\left(\sqrt{5}-1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}} \\ & -\frac{3\sqrt{5}(-1)^{\frac{1}{5}}\left(\sqrt{5}+1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}\right)}{5\sqrt{-2\sqrt{5}-10}} \\ & -\frac{6}{5}(-1)^{\frac{1}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right)+x \\ & -\frac{3\left(\sqrt{5}+3\right)\log\left(-x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\right)+2(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}(-1)^{\frac{4}{5}}+(-1)^{\frac{4}{5}}\right)} \\ & -\frac{3\left(\sqrt{5}-3\right)\log\left(x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\right)+2(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}(-1)^{\frac{4}{5}}-(-1)^{\frac{4}{5}}\right)}+6x^{\frac{1}{6}}\end{aligned}$$

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
[Out] -3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) - 1)*log((sqrt(5)*(-1)^(1/5) + (-1)^(1/5)*
sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) - (-1)^(
1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6)))/sqrt(2*sqrt(5) - 10) -
```

```
3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) + 1)*log((sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*
sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(-2*sqrt(5) - 10
) - 6/5*(-1)^(1/5)*log((-1)^(1/5) + x^(1/6)) + x - 3/5*(sqrt(5) + 3)*log(-x
^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(
5)*(-1)^(4/5) + (-1)^(4/5)) - 3/5*(sqrt(5) - 3)*log(x^(1/6)*(sqrt(5)*(-1)^(
1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(4/5) - (-1)^(
4/5)) + 6*x^(1/6)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = -\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4\,x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right) - \frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4\,x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right) - \frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}\,x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+1\right) + \frac{3}{10}\sqrt{5}\log\left(-\frac{1}{2}\,x^{\frac{1}{6}}\left(\sqrt{5}-1\right)+x^{\frac{1}{3}}+1\right)+x+6\,x^{\frac{1}{6}} - \frac{3}{10}\log\left(x^{\frac{2}{3}}+\sqrt{x}+x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+\frac{6}{5}\log\left(\left|x^{\frac{1}{6}}-1\right|\right)$$

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -3/5*sqrt(2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(-2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2
*sqrt(5) + 10)) - 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
 + 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + x + 6*x^(1/
6) - 3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1
/6) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = x+\frac{6\,\ln\left(1296\,x^{1/6}-1296\right)}{5} - \ln\left(270\,\sqrt{2}\,\sqrt{-\sqrt{5}-5}-270\,\sqrt{5}+1080\,x^{1/6}+270\right)\,\left(\frac{3\,\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}-\frac{3\,\sqrt{5}}{10}+\frac{3}{10}\right) +\ln\left(270\,\sqrt{2}\,\sqrt{-\sqrt{5}-5}+270\,\sqrt{5}-1080\,x^{1/6}-270\right)\,\left(\frac{3\,\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}+\frac{3\,\sqrt{5}}{10}-\frac{3}{10}\right)+6\,x^{1/6}-\ln\Big(270$$

```
[In] int(x^(1/2)/(x^(1/2) - 1/x^(1/3)),x)
```

```
[Out] x + (6*log(1296*x^(1/6) - 1296))/5 - log(270*2^(1/2)*(- 5^(1/2) - 5)^(1/2)
- 270*5^(1/2) + 1080*x^(1/6) + 270)*((3*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/10 -
 (3*5^(1/2))/10 + 3/10) + log(270*2^(1/2)*(- 5^(1/2) - 5)^(1/2) + 270*5^(1/
2) - 1080*x^(1/6) - 270)*((3*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/10 + (3*5^(1/2)
)/10 - 3/10) + 6*x^(1/6) - log(270*5^(1/2) + 1080*x^(1/6) - 270*2^(1/2)*(5^
(1/2) - 5)^(1/2) + 270)*((3*5^(1/2))/10 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/1
0 + 3/10) - log(270*5^(1/2) + 1080*x^(1/6) + 270*2^(1/2)*(5^(1/2) - 5)^(1/2
) + 270)*((3*5^(1/2))/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)
```

3.236 $\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = 2\sqrt{x}+\frac{4\arctan\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{4}{3}\log\left(1+\sqrt[4]{x}\right)-\frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)$$

```
[Out] 4/3*ln(1+x^(1/4))-2/3*ln(1-x^(1/4)+x^(1/2))+4/3*arctan(1/3*(1-2*x^(1/4))*3^
(1/2))*3^(1/2)+2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {1607, 348, 327, 298, 31, 648, 632, 210, 642}

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = \frac{4\arctan\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}+2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)$$

```
[In] Int[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (4*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log[1 + x^(1/4
)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt[4]{x}}{1+x^{3/4}}\,dx\\
&= 4\text{Subst}\left(\int \frac{x^4}{1+x^3}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x} - 4\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x} + \frac{4}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[4]{x}\right) - \frac{4}{3}\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right)\\
&\quad - 2\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right) + 4\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[4]{x}\right)\\
&= 2\sqrt{x} + \frac{4\arctan\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = \frac{2}{3}\left(3\sqrt{x}+2\sqrt{3}\arctan\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)+2\log\left(1+\sqrt[4]{x}\right)-\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)\right)$$

```
[In] Integrate[(x^(-1/4) + Sqrt[x])^(-1),x]
[Out] (2*(3*Sqrt[x] + 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]] + 2*Log[1 + x^(1/
4)] - Log[1 - x^(1/4) + Sqrt[x]]))/3
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$2\sqrt{x}-\frac{2\ln\left(1-x^{\frac{1}{4}}+\sqrt{x}\right)}{3}-\frac{4\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{4}}-1\right)\sqrt{3}}{3}\right)}{3}+\frac{4\ln\left(1+x^{\frac{1}{4}}\right)}{3}$	46
default	$2\sqrt{x}-\frac{2\ln\left(1-x^{\frac{1}{4}}+\sqrt{x}\right)}{3}-\frac{4\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{4}}-1\right)\sqrt{3}}{3}\right)}{3}+\frac{4\ln\left(1+x^{\frac{1}{4}}\right)}{3}$	46
meijerg	$2\sqrt{x}-\frac{4\sqrt{x}\left(-\frac{\ln\left(1+x^{\frac{1}{4}}\right)}{\sqrt{x}}+\frac{\ln\left(1-x^{\frac{1}{4}}+\sqrt{x}\right)}{2\sqrt{x}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^{\frac{1}{4}}}{2-x^{\frac{1}{4}}}\right)}{\sqrt{x}}\right)}{3}$	65

```
[In] int(1/(1/x^(1/4)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)-2/3*ln(1-x^(1/4)+x^(1/2))-4/3*3^(1/2)*arctan(1/3*(2*x^(1/4)-1)*3^
(1/2))+4/3*ln(1+x^(1/4))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = -\frac{4}{3}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}x^{\frac{1}{4}}-\frac{1}{3}\,\sqrt{3}\right)+2\,\sqrt{x} -\frac{2}{3}\,\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\,\log\left(x^{\frac{1}{4}}+1\right)$$

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -4/3*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/4) - 1/3*sqrt(3)) + 2*sqrt(x) - 2/3*lo
g(sqrt(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = 2\sqrt{x}+\frac{4\log\left(\sqrt[4]{x}+1\right)}{3}-\frac{2\log\left(-4\sqrt[4]{x}+4\sqrt{x}+4\right)}{3} - \frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[4]{x}}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate(1/(1/x**(1/4)+x**(1/2)),x)
[Out] 2*sqrt(x) + 4*log(x**(1/4) + 1)/3 - 2*log(-4*x**(1/4) + 4*sqrt(x) + 4)/3 -
4*sqrt(3)*atan(2*sqrt(3)*x**(1/4)/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = -\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x} - \frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="maxima")
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt
(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = -\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x} - \frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="giac")
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt
(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = \frac{4\ln\left(16\,x^{1/4}+16\right)}{3}+\ln\left(9\left(-\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)^2+16\,x^{1/4}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right) - \ln\left(9\left(\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)^2+16\,x^{1/4}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)+2\sqrt{x}$$

```
[In] int(1/(x^(1/2) + 1/x^(1/4)),x)
```

```
[Out] (4*log(16*x^(1/4) + 16))/3 + log(9*((3^(1/2)*2i)/3 - 2/3)^2 + 16*x^(1/4))*(
(3^(1/2)*2i)/3 - 2/3) - log(9*((3^(1/2)*2i)/3 + 2/3)^2 + 16*x^(1/4))*((3^(1
/2)*2i)/3 + 2/3) + 2*x^(1/2)
```

3.237 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}} dx$

Optimal result

Integrand size = 13, antiderivative size = 130

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}} dx = 12\sqrt[12]{x}-6\sqrt[6]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}+\frac{12x^{5/12}}{5}-2\sqrt{x}+\frac{12x^{7/12}}{7}-\frac{3x^{2/3}}{2}+\frac{4x^{3/4}}{3} -\frac{6x^{5/6}}{5}+\frac{12x^{11/12}}{11}-x+\frac{12x^{13/12}}{13}-\frac{6x^{7/6}}{7}+\frac{4x^{5/4}}{5}-12\log\left(1+\sqrt[12]{x}\right)$$

```
[Out] 12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7/12)-3/2*x
^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*x^(13/12)-6/7*x^(7/6
)+4/5*x^(5/4)-12*ln(1+x^(1/12))-2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1607, 272, 45}

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}} dx = \frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7} +\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-12\log\left(\sqrt[12]{x}+1\right)$$

```
[In] Int[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5 - 2*Sqrt[
x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*
```

```
x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^(7/6))/7 + (4*x^(5/4))/5 - 12*
Log[1 + x^(1/12)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[3]{x}}{1+\sqrt[12]{x}}\,dx \\
&= 12\text{Subst}\left(\int \frac{x^{15}}{1+x}\,dx, x, \sqrt[12]{x}\right) \\
&= 12\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2-x^3+x^4-x^5+x^6-x^7+x^8-x^9+x^{10}-x^{11}\right.\right. \\
&\qquad \left.\left.+x^{12}-x^{13}+x^{14}\right)dx, x, \sqrt[12]{x}\right) \\
&= 12\sqrt[12]{x}-6\sqrt[6]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}+\frac{12x^{5/12}}{5}-2\sqrt{x}+\frac{12x^{7/12}}{7}-\frac{3x^{2/3}}{2}+\frac{4x^{3/4}}{3} \\
&\quad -\frac{6x^{5/6}}{5}+\frac{12x^{11/12}}{11}-x+\frac{12x^{13/12}}{13}-\frac{6x^{7/6}}{7}+\frac{4x^{5/4}}{5}-12\log\left(1+\sqrt[12]{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx$$
$$=\frac{360360\sqrt[12]{x}-180180\sqrt[6]{x}+120120\sqrt[4]{x}-90090\sqrt[3]{x}+72072x^{5/12}-60060\sqrt{x}+51480x^{7/12}-45045x^{2/3}+4}{30030}$$
$$-12\log\left(1+\sqrt[12]{x}\right)$$

```
[In] Integrate[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] (360360*x^(1/12) - 180180*x^(1/6) + 120120*x^(1/4) - 90090*x^(1/3) + 72072*
x^(5/12) - 60060*Sqrt[x] + 51480*x^(7/12) - 45045*x^(2/3) + 40040*x^(3/4) -
 36036*x^(5/6) + 32760*x^(11/12) - 30030*x + 27720*x^(13/12) - 25740*x^(7/6
) + 24024*x^(5/4))/30030 - 12*Log[1 + x^(1/12)]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.64

method	result
derivativedivides	$12x^{\frac{1}{12}}-6x^{\frac{1}{6}}+4x^{\frac{1}{4}}-3x^{\frac{1}{3}}+\frac{12x^{\frac{5}{12}}}{5}+\frac{12x^{\frac{7}{12}}}{7}-\frac{3x^{\frac{2}{3}}}{2}+\frac{4x^{\frac{3}{4}}}{3}-\frac{6x^{\frac{5}{6}}}{5}+\frac{12x^{\frac{11}{12}}}{11}-x+\frac{12x^{\frac{13}{12}}}{13}-\frac{6x}{7}$
default	$12x^{\frac{1}{12}}-6x^{\frac{1}{6}}+4x^{\frac{1}{4}}-3x^{\frac{1}{3}}+\frac{12x^{\frac{5}{12}}}{5}+\frac{12x^{\frac{7}{12}}}{7}-\frac{3x^{\frac{2}{3}}}{2}+\frac{4x^{\frac{3}{4}}}{3}-\frac{6x^{\frac{5}{6}}}{5}+\frac{12x^{\frac{11}{12}}}{11}-x+\frac{12x^{\frac{13}{12}}}{13}-\frac{6x}{7}$
meijerg	$\frac{x^{\frac{1}{12}}\left(48048x^{\frac{7}{6}}-51480x^{\frac{13}{12}}+55440x-60060x^{\frac{11}{12}}+65520x^{\frac{5}{6}}-72072x^{\frac{3}{4}}+80080x^{\frac{2}{3}}-90090x^{\frac{7}{12}}+102960\sqrt{x}-120120x^{\frac{5}{12}}+14\right.}{60060}$

```
[In] int(1/(1/x^(1/3)+1/x^(1/4)),x,method=_RETURNVERBOSE)
```

```
[Out] 12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7/12)-3/2*x
^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*x^(13/12)-6/7*x^(7/6
)+4/5*x^(5/4)-12*ln(1+x^(1/12))-2*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx=\frac{4}{5}\,(x+5)x^{\frac{1}{4}}-\frac{6}{7}\,(x+7)x^{\frac{1}{6}}+\frac{12}{13}\,(x+13)x^{\frac{1}{12}}-x+\frac{12}{11}\,x^{\frac{11}{12}}-\frac{6}{5}\,x^{\frac{5}{6}}+\frac{4}{3}\,x^{\frac{3}{4}}-\frac{3}{2}\,x^{\frac{2}{3}}+\frac{12}{7}\,x^{\frac{7}{12}}-2\,\sqrt{x}+\frac{12}{5}\,x^{\frac{5}{12}}-3\,x^{\frac{1}{3}}-12\,\log\left(x^{\frac{1}{12}}+1\right)$$

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="fricas")
[Out] 4/5*(x + 5)*x^(1/4) - 6/7*(x + 7)*x^(1/6) + 12/13*(x + 13)*x^(1/12) - x + 1
2/11*x^(11/12) - 6/5*x^(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) -
2*sqrt(x) + 12/5*x^(5/12) - 3*x^(1/3) - 12*log(x^(1/12) + 1)
```

Sympy [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.93

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx=\frac{12x^{\frac{13}{12}}}{13}+\frac{12x^{\frac{11}{12}}}{11}+\frac{12x^{\frac{7}{12}}}{7}+\frac{12x^{\frac{5}{12}}}{5}+12\sqrt[12]{x}-\frac{6x^{\frac{7}{6}}}{7}-\frac{6x^{\frac{5}{6}}}{5}-6\sqrt[6]{x}+\frac{4x^{\frac{5}{4}}}{5}+\frac{4x^{\frac{3}{4}}}{3}+4\sqrt[4]{x}-\frac{3x^{\frac{2}{3}}}{2}-3\sqrt[3]{x}-2\sqrt{x}-x-12\log\left(\sqrt[12]{x}+1\right)$$

```
[In] integrate(1/(1/x**(1/3)+1/x**(1/4)),x)
[Out] 12*x**(13/12)/13 + 12*x**(11/12)/11 + 12*x**(7/12)/7 + 12*x**(5/12)/5 + 12*
x**(1/12) - 6*x**(7/6)/7 - 6*x**(5/6)/5 - 6*x**(1/6) + 4*x**(5/4)/5 + 4*x**
(3/4)/3 + 4*x**(1/4) - 3*x**(2/3)/2 - 3*x**(1/3) - 2*sqrt(x) - x - 12*log(x
**(1/12) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx=\frac{4}{5}\,x^{\frac{5}{4}}-\frac{6}{7}\,x^{\frac{7}{6}}+\frac{12}{13}\,x^{\frac{13}{12}}-x+\frac{12}{11}\,x^{\frac{11}{12}}-\frac{6}{5}\,x^{\frac{5}{6}}+\frac{4}{3}\,x^{\frac{3}{4}}-\frac{3}{2}\,x^{\frac{2}{3}}+\frac{12}{7}\,x^{\frac{7}{12}}-2\,\sqrt{x}+\frac{12}{5}\,x^{\frac{5}{12}}-3\,x^{\frac{1}{3}}+4\,x^{\frac{1}{4}}-6\,x^{\frac{1}{6}}+12\,x^{\frac{1}{12}}-12\,\log\left(x^{\frac{1}{12}}+1\right)$$

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="maxima")
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12
) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx = \frac{4}{5}\,x^{\frac{5}{4}} - \frac{6}{7}\,x^{\frac{7}{6}} + \frac{12}{13}\,x^{\frac{13}{12}} - x + \frac{12}{11}\,x^{\frac{11}{12}} - \frac{6}{5}\,x^{\frac{5}{6}} + \frac{4}{3}\,x^{\frac{3}{4}} - \frac{3}{2}\,x^{\frac{2}{3}} + \frac{12}{7}\,x^{\frac{7}{12}} - 2\,\sqrt{x} + \frac{12}{5}\,x^{\frac{5}{12}} - 3\,x^{\frac{1}{3}} + 4\,x^{\frac{1}{4}} - 6\,x^{\frac{1}{6}} + 12\,x^{\frac{1}{12}} - 12\,\log\left(x^{\frac{1}{12}} + 1\right)$$

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="giac")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12
) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx = 4\,x^{1/4} - 12\,\ln\left(x^{1/12}+1\right) - 2\,\sqrt{x} - 3\,x^{1/3} - x - \frac{3\,x^{2/3}}{2} - 6\,x^{1/6} + \frac{4\,x^{3/4}}{3} + \frac{4\,x^{5/4}}{5} - \frac{6\,x^{5/6}}{5} + 12\,x^{1/12} - \frac{6\,x^{7/6}}{7} + \frac{12\,x^{5/12}}{5} + \frac{12\,x^{7/12}}{7} + \frac{12\,x^{11/12}}{11} + \frac{12\,x^{13/12}}{13}$$

```
[In] int(1/(1/x^(1/3) + 1/x^(1/4)),x)
```

```
[Out] 4*x^(1/4) - 12*log(x^(1/12) + 1) - 2*x^(1/2) - 3*x^(1/3) - x - (3*x^(2/3))/
2 - 6*x^(1/6) + (4*x^(3/4))/3 + (4*x^(5/4))/5 - (6*x^(5/6))/5 + 12*x^(1/12)
 - (6*x^(7/6))/7 + (12*x^(5/12))/5 + (12*x^(7/12))/7 + (12*x^(11/12))/11 +
(12*x^(13/12))/13
```

3.238 $\int \sqrt{\frac{1-x}{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \sqrt{\frac{1-x}{x}}\,dx = \sqrt{-1+\frac{1}{x}}x - \arctan\left(\sqrt{-1+\frac{1}{x}}\right)$$

```
[Out] -arctan((-1+1/x)^(1/2))+x*(-1+1/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1997, 248, 43, 65, 209}

$$\int \sqrt{\frac{1-x}{x}}\,dx = \sqrt{\frac{1}{x}-1}x - \arctan\left(\sqrt{\frac{1}{x}-1}\right)$$

```
[In] Int[Sqrt[(1 - x)/x],x]
```

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \sqrt{-1+\frac{1}{x}}\,dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{-1+\frac{1}{x}}x - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{-1+\frac{1}{x}}x - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+\frac{1}{x}}\right) \\
&= \sqrt{-1+\frac{1}{x}}x - \arctan\left(\sqrt{-1+\frac{1}{x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sqrt{\frac{1-x}{x}}\,dx = \sqrt{-1+\frac{1}{x}}x - \arctan\left(\sqrt{-1+\frac{1}{x}}\right)$$

```
[In] Integrate[Sqrt[(1 - x)/x],x]
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

method	result	size
default	$\frac{\sqrt{-\frac{-1+x}{x}}\,x\left(2\sqrt{-x^{2}+x}+\arcsin\left(2x-1\right)\right)}{2\sqrt{-x\left(-1+x\right)}}$	40
risch	$\sqrt{-\frac{-1+x}{x}}\,x-\frac{\arcsin\left(2x-1\right)\sqrt{-\frac{-1+x}{x}}\,\sqrt{-x\left(-1+x\right)}}{2\left(-1+x\right)}$	45
trager	$\sqrt{-\frac{-1+x}{x}}\,x-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(2\sqrt{-\frac{-1+x}{x}}\,x+2\operatorname{RootOf}\left(\_Z^{2}+1\right)x-\operatorname{RootOf}\left(\_Z^{2}+1\right)\right)}{2}$	54

```
[In] int(((1-x)/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(-(-1+x)/x)^(1/2)*x*(2*(-x^2+x)^(1/2)+arcsin(2*x-1))/(-x*(-1+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \sqrt{\frac{1-x}{x}}\,dx = x\sqrt{-\frac{x-1}{x}} - \arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="fricas")
[Out] x*sqrt(-(x - 1)/x) - arctan(sqrt(-(x - 1)/x))
```

Sympy [F]

$$\int \sqrt{\frac{1-x}{x}}\, dx = \int \sqrt{\frac{1-x}{x}}\, dx$$

```
[In] integrate(((1-x)/x)**(1/2),x)
[Out] Integral(sqrt((1 - x)/x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \sqrt{\frac{1-x}{x}}\, dx = -\frac{\sqrt{-\frac{x-1}{x}}}{\frac{x-1}{x}-1} - \arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-(x - 1)/x)/((x - 1)/x - 1) - arctan(sqrt(-(x - 1)/x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \sqrt{\frac{1-x}{x}}\, dx = \frac{1}{4}\,\pi \mathrm{sgn}(x) + \frac{1}{2}\,\arcsin\left(2\,x-1\right)\mathrm{sgn}(x) + \sqrt{-x^2+x}\,\mathrm{sgn}(x)$$

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="giac")
[Out] 1/4*pi*sgn(x) + 1/2*arcsin(2*x - 1)*sgn(x) + sqrt(-x^2 + x)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \sqrt{\frac{1-x}{x}}\, dx = x\,\sqrt{\frac{1}{x}-1} - \mathrm{atan}\left(\sqrt{\frac{1}{x}-1}\right)$$

```
[In] int((-(x - 1)/x)^(1/2),x)
[Out] x*(1/x - 1)^(1/2) - atan((1/x - 1)^(1/2))
```

3.239 $\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 11

$$\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx = \log(\sin(x)) - \log(1+\sin(x))$$

```
[Out] ln(sin(x))-ln(1+sin(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3339, 629}

$$\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx = \log(\sin(x)) - \log(\sin(x)+1)$$

```
[In] Int[Cos[x]/(Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Rule 629

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[Log[x]/b, x] - Simp
[Log[RemoveContent[b + c*x, x]]/b, x] /; FreeQ[{b, c}, x]
```

Rule 3339

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*sin[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*sin[(d_.) + (e_.)*(x_)])^(n2_.))^(p_.), x_Symbol
] :> Module[{g = FreeFactors[Sin[d + e*x], x]}, Dist[g/e, Subst[Int[(1 - g^
2*x^2)^((m - 1)/2)*(a + b*(f*g*x)^n + c*(f*g*x)^(2*n))^p, x], x, Sin[d + e*
```

```
x]/g], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2*n] && Integer
Q[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{x+x^2}\,dx, x, \sin(x)\right)\\ &= \log(\sin(x)) - \log(1+\sin(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx = \log(\sin(x)) - \log(1+\sin(x))$$

```
[In] Integrate[Cos[x]/(Sin[x] + Sin[x]^2),x]
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\ln(\sin(x)) - \ln(\sin(x)+1)$	12
default	$\ln(\sin(x)) - \ln(\sin(x)+1)$	12
norman	$-2\ln\left(1+\tan\left(\frac{x}{2}\right)\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	16
parallelrisch	$-2\ln\left(1+\tan\left(\frac{x}{2}\right)\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	16
risch	$-2\ln\left(i+e^{ix}\right)+\ln\left(e^{2ix}-1\right)$	21

```
[In] int(cos(x)/(sin(x)+sin(x)^2),x,method=_RETURNVERBOSE)
[Out] ln(sin(x))-ln(sin(x)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx = \log\left(\frac{1}{2}\sin\left(x\right)\right) - \log\left(\sin\left(x\right)+1\right)$$

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="fricas")
[Out] log(1/2*sin(x)) - log(sin(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx = -\log\left(\sin\left(x\right)+1\right) + \log\left(\sin\left(x\right)\right)$$

```
[In] integrate(cos(x)/(sin(x)+sin(x)**2),x)
[Out] -log(sin(x) + 1) + log(sin(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx = -\log\left(\sin\left(x\right)+1\right) + \log\left(\sin\left(x\right)\right)$$

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="maxima")
[Out] -log(sin(x) + 1) + log(sin(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx = -\log\left(\sin\left(x\right)+1\right) + \log\left(\left|\sin\left(x\right)\right|\right)$$

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="giac")
[Out] -log(sin(x) + 1) + log(abs(sin(x)))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\cos(x)}{\sin(x) + \sin^2(x)} \, dx = -2\,\mathrm{atanh}(2\,\sin{(x)} + 1)$$

```
[In] int(cos(x)/(sin(x) + sin(x)^2),x)
[Out] -2*atanh(2*sin(x) + 1)
```

3.240 $\int \frac{e^{2x}}{2+3e^x+e^{2x}} dx$

Optimal result

Integrand size = 20, antiderivative size = 17

$$\int \frac{e^{2x}}{2+3e^x+e^{2x}} dx = -\log(1+e^x) + 2\log(2+e^x)$$

```
[Out] -ln(1+exp(x))+2*ln(2+exp(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2320, 646, 31}

$$\int \frac{e^{2x}}{2+3e^x+e^{2x}} dx = 2\log(e^x+2) - \log(e^x+1)$$

```
[In] Int[E^(2*x)/(2 + 3*E^x + E^(2*x)),x]
```

```
[Out] -Log[1 + E^x] + 2*Log[2 + E^x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{x}{2+3x+x^2}\,dx, x, e^x\right)\\ &= 2\text{Subst}\left(\int \frac{1}{2+x}\,dx, x, e^x\right) - \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, e^x\right)\\ &= -\log\left(1+e^x\right) + 2\log\left(2+e^x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx = -\log\left(1+e^x\right) + 2\log\left(2+e^x\right)$$

```
[In] Integrate[E^(2*x)/(2 + 3*E^x + E^(2*x)),x]
[Out] -Log[1 + E^x] + 2*Log[2 + E^x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$-\ln\left(1+e^{x}\right)+2\ln\left(2+e^{x}\right)$	16
norman	$-\ln\left(1+e^{x}\right)+2\ln\left(2+e^{x}\right)$	16
risch	$-\ln\left(1+e^{x}\right)+2\ln\left(2+e^{x}\right)$	16

```
[In] int(exp(2*x)/(2+3*exp(x)+exp(2*x)),x,method=_RETURNVERBOSE)
[Out] -ln(1+exp(x))+2*ln(2+exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx = 2\,\log\left(e^x+2\right) - \log\left(e^x+1\right)$$

```
[In] integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x, algorithm="fricas")
[Out] 2*log(e^x + 2) - log(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx = -\log\left(e^x+1\right) + 2\log\left(e^x+2\right)$$

```
[In] integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x)
[Out] -log(exp(x) + 1) + 2*log(exp(x) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx = 2\,\log\left(e^x+2\right) - \log\left(e^x+1\right)$$

```
[In] integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x, algorithm="maxima")
[Out] 2*log(e^x + 2) - log(e^x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx = 2\,\log\left(e^x+2\right) - \log\left(e^x+1\right)$$

```
[In] integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x, algorithm="giac")
[Out] 2*log(e^x + 2) - log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx = 2\,\ln\left(\mathrm{e}^x+2\right)-\ln\left(\mathrm{e}^x+1\right)$$

```
[In] int(exp(2*x)/(exp(2*x) + 3*exp(x) + 2),x)
[Out] 2*log(exp(x) + 2) - log(exp(x) + 1)
```

3.241 $\int \frac{1}{\sqrt{1+e^x}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{\sqrt{1+e^x}}\,dx = -2\text{arctanh}\left(\sqrt{1+e^x}\right)$$

```
[Out] -2*arctanh((1+exp(x))^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2320, 65, 213}

$$\int \frac{1}{\sqrt{1+e^x}}\,dx = -2\text{arctanh}\left(\sqrt{e^x+1}\right)$$

```
[In] Int[1/Sqrt[1 + E^x],x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + E^x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, e^x\right)\\ &= 2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+e^x}\right)\\ &= -2\text{arctanh}\left(\sqrt{1+e^x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+e^x}}\,dx = -2\text{arctanh}\left(\sqrt{1+e^x}\right)$$

```
[In] Integrate[1/Sqrt[1 + E^x],x]
[Out] -2*ArcTanh[Sqrt[1 + E^x]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-2\,\operatorname{arctanh}\left(\sqrt{1+e^{x}}\right)$	10
default	$-2\,\operatorname{arctanh}\left(\sqrt{1+e^{x}}\right)$	10

```
[In] int(1/(1+exp(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((1+exp(x))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{1+e^x}}\,dx = -\log\left(\sqrt{e^x+1}+1\right)+\log\left(\sqrt{e^x+1}-1\right)$$

```
[In] integrate(1/(1+exp(x))^(1/2),x, algorithm="fricas")
[Out] -log(sqrt(e^x + 1) + 1) + log(sqrt(e^x + 1) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{1}{\sqrt{1+e^x}}\,dx = \log\left(\sqrt{e^x+1}-1\right)-\log\left(\sqrt{e^x+1}+1\right)$$

```
[In] integrate(1/(1+exp(x))**(1/2),x)
[Out] log(sqrt(exp(x) + 1) - 1) - log(sqrt(exp(x) + 1) + 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{1+e^x}}\,dx = -\log\left(\sqrt{e^x+1}+1\right)+\log\left(\sqrt{e^x+1}-1\right)$$

```
[In] integrate(1/(1+exp(x))^(1/2),x, algorithm="maxima")
[Out] -log(sqrt(e^x + 1) + 1) + log(sqrt(e^x + 1) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{1+e^x}}\,dx = -\log\left(\sqrt{e^x+1}+1\right)+\log\left(\sqrt{e^x+1}-1\right)$$

```
[In] integrate(1/(1+exp(x))^(1/2),x, algorithm="giac")
[Out] -log(sqrt(e^x + 1) + 1) + log(sqrt(e^x + 1) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1+e^x}}\,dx = -2\,\mathrm{atanh}\left(\sqrt{\mathrm{e}^x+1}\right)$$

```
[In] int(1/(exp(x) + 1)^(1/2),x)
[Out] -2*atanh((exp(x) + 1)^(1/2))
```

3.242 $\int \sqrt{1-e^x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \sqrt{1-e^x}\,dx = 2\sqrt{1-e^x} - 2\text{arctanh}\left(\sqrt{1-e^x}\right)$$

```
[Out] -2*arctanh((1-exp(x))^(1/2))+2*(1-exp(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2320, 52, 65, 212}

$$\int \sqrt{1-e^x}\,dx = 2\sqrt{1-e^x} - 2\text{arctanh}\left(\sqrt{1-e^x}\right)$$

```
[In] Int[Sqrt[1 - E^x],x]
```

```
[Out] 2*Sqrt[1 - E^x] - 2*ArcTanh[Sqrt[1 - E^x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx, x, e^x\right)\\
&= 2\sqrt{1-e^x} + \text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, e^x\right)\\
&= 2\sqrt{1-e^x} - 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-e^x}\right)\\
&= 2\sqrt{1-e^x} - 2\text{arctanh}\left(\sqrt{1-e^x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \sqrt{1-e^x}\,dx = 2\sqrt{1-e^x} - 2\text{arctanh}\left(\sqrt{1-e^x}\right)$$

```
[In] Integrate[Sqrt[1 - E^x],x]
[Out] 2*Sqrt[1 - E^x] - 2*ArcTanh[Sqrt[1 - E^x]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
risch	$-\frac{2(-1+\mathrm{e}^{x})}{\sqrt{1-\mathrm{e}^{x}}}-2\,\mathrm{arctanh}\left(\sqrt{1-\mathrm{e}^{x}}\right)$	27
derivativedivides	$2\sqrt{1-\mathrm{e}^{x}}+\ln\left(\sqrt{1-\mathrm{e}^{x}}-1\right)-\ln\left(\sqrt{1-\mathrm{e}^{x}}+1\right)$	36
default	$2\sqrt{1-\mathrm{e}^{x}}+\ln\left(\sqrt{1-\mathrm{e}^{x}}-1\right)-\ln\left(\sqrt{1-\mathrm{e}^{x}}+1\right)$	36

```
[In] int((1-exp(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(-1+exp(x))/(1-exp(x))^(1/2)-2*arctanh((1-exp(x))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \sqrt{1-e^x}\,dx = 2\sqrt{-e^x+1}-\log\left(\sqrt{-e^x+1}+1\right)+\log\left(\sqrt{-e^x+1}-1\right)$$

```
[In] integrate((1-exp(x))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(-e^x + 1) - log(sqrt(-e^x + 1) + 1) + log(sqrt(-e^x + 1) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \sqrt{1-e^x}\,dx = 2\sqrt{1-e^x}+\log\left(\sqrt{1-e^x}-1\right)-\log\left(\sqrt{1-e^x}+1\right)$$

```
[In] integrate((1-exp(x))**(1/2),x)
[Out] 2*sqrt(1 - exp(x)) + log(sqrt(1 - exp(x)) - 1) - log(sqrt(1 - exp(x)) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \sqrt{1-e^x}\,dx = 2\sqrt{-e^x+1} - \log\left(\sqrt{-e^x+1}+1\right) + \log\left(\sqrt{-e^x+1}-1\right)$$

```
[In] integrate((1-exp(x))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(-e^x + 1) - log(sqrt(-e^x + 1) + 1) + log(sqrt(-e^x + 1) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \sqrt{1-e^x}\,dx = 2\sqrt{-e^x+1} - \log\left(\sqrt{-e^x+1}+1\right) + \log\left(-\sqrt{-e^x+1}+1\right)$$

```
[In] integrate((1-exp(x))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(-e^x + 1) - log(sqrt(-e^x + 1) + 1) + log(-sqrt(-e^x + 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int \sqrt{1-e^x}\,dx = 2\sqrt{1-e^x} + \frac{2\,e^{-\frac{x}{2}}\,\mathrm{asin}\left(e^{-\frac{x}{2}}\right)\,\sqrt{1-e^x}}{\sqrt{1-e^{-x}}}$$

```
[In] int((1 - exp(x))^(1/2),x)
[Out] 2*(1 - exp(x))^(1/2) + (2*exp(-x/2)*asin(exp(-x/2))*(1 - exp(x))^(1/2))/(1
- exp(-x))^(1/2)
```

3.243 $\int \frac{1}{3-5\sin(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \frac{1}{3-5\sin(x)}\,dx = -\frac{1}{4}\log\left(\cos\left(\frac{x}{2}\right)-3\sin\left(\frac{x}{2}\right)\right)+\frac{1}{4}\log\left(3\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

```
[Out] -1/4*ln(cos(1/2*x)-3*sin(1/2*x))+1/4*ln(3*cos(1/2*x)-sin(1/2*x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2739, 630, 31}

$$\int \frac{1}{3-5\sin(x)}\,dx = \frac{1}{4}\log\left(3\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{4}\log\left(\cos\left(\frac{x}{2}\right)-3\sin\left(\frac{x}{2}\right)\right)$$

```
[In] Int[(3 - 5*Sin[x])^(-1),x]
```

```
[Out] -1/4*Log[Cos[x/2] - 3*Sin[x/2]] + Log[3*Cos[x/2] - Sin[x/2]]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{1}{3-10x+3x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= \frac{3}{4}\text{Subst}\left(\int \frac{1}{-9+3x}\,dx, x, \tan\left(\frac{x}{2}\right)\right) - \frac{3}{4}\text{Subst}\left(\int \frac{1}{-1+3x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= -\frac{1}{4}\log\left(1-3\tan\left(\frac{x}{2}\right)\right) + \frac{1}{4}\log\left(3-\tan\left(\frac{x}{2}\right)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{3-5\sin(x)}\,dx = -\frac{1}{4}\log\left(\cos\left(\frac{x}{2}\right)-3\sin\left(\frac{x}{2}\right)\right)+\frac{1}{4}\log\left(3\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[(3 - 5*Sin[x])^(-1),x]
[Out] -1/4*Log[Cos[x/2] - 3*Sin[x/2]] + Log[3*Cos[x/2] - Sin[x/2]]/4
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.51

method	result	size
default	$-\frac{\ln\left(3\tan\left(\frac{x}{2}\right)-1\right)}{4}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)-3\right)}{4}$	22
norman	$-\frac{\ln\left(3\tan\left(\frac{x}{2}\right)-1\right)}{4}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)-3\right)}{4}$	22
parallelrisch	$\ln\left(\left(3\tan\left(\frac{x}{2}\right)-9\right)^{\frac{1}{4}}\right)+\ln\left(\frac{1}{\left(3\tan\left(\frac{x}{2}\right)-1\right)^{\frac{1}{4}}}\right)$	24
risch	$\frac{\ln\left(\frac{4}{5}-\frac{3i}{5}+e^{ix}\right)}{4}-\frac{\ln\left(e^{ix}-\frac{4}{5}-\frac{3i}{5}\right)}{4}$	26

```
[In] int(1/(3-5*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(3*tan(1/2*x)-1)+1/4*ln(tan(1/2*x)-3)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{1}{3-5\sin(x)}\,dx = \frac{1}{8}\,\log\left(4\,\cos\left(x\right) - 3\,\sin\left(x\right) + 5\right) - \frac{1}{8}\,\log\left(-4\,\cos\left(x\right) - 3\,\sin\left(x\right) + 5\right)$$

```
[In] integrate(1/(3-5*sin(x)),x, algorithm="fricas")
[Out] 1/8*log(4*cos(x) - 3*sin(x) + 5) - 1/8*log(-4*cos(x) - 3*sin(x) + 5)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.47

$$\int \frac{1}{3-5\sin(x)}\,dx = \frac{\log\left(\tan\left(\frac{x}{2}\right) - 3\right)}{4} - \frac{\log\left(3\tan\left(\frac{x}{2}\right) - 1\right)}{4}$$

```
[In] integrate(1/(3-5*sin(x)),x)
[Out] log(tan(x/2) - 3)/4 - log(3*tan(x/2) - 1)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{1}{3-5\sin(x)}\,dx = -\frac{1}{4}\,\log\left(\frac{3\,\sin\left(x\right)}{\cos\left(x\right)+1} - 1\right) + \frac{1}{4}\,\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1} - 3\right)$$

```
[In] integrate(1/(3-5*sin(x)),x, algorithm="maxima")
[Out] -1/4*log(3*sin(x)/(cos(x) + 1) - 1) + 1/4*log(sin(x)/(cos(x) + 1) - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.53

$$\int \frac{1}{3-5\sin(x)}\,dx = -\frac{1}{4}\,\log\left(\left|3\,\tan\left(\frac{1}{2}\,x\right)-1\right|\right)+\frac{1}{4}\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-3\right|\right)$$

```
[In] integrate(1/(3-5*sin(x)),x, algorithm="giac")
[Out] -1/4*log(abs(3*tan(1/2*x) - 1)) + 1/4*log(abs(tan(1/2*x) - 3))
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.26

$$\int \frac{1}{3-5\sin(x)}\,dx = -\frac{\mathrm{atanh}\left(\frac{3\tan\left(\frac{x}{2}\right)}{4}-\frac{5}{4}\right)}{2}$$

```
[In] int(-1/(5*sin(x) - 3),x)
[Out] -atanh((3*tan(x/2))/4 - 5/4)/2
```

3.244 $\int \frac{1}{\cos(x)+\sin(x)} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \frac{1}{\cos(x)+\sin(x)} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(1/2*(cos(x)-sin(x))*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3153, 212}

$$\int \frac{1}{\cos(x)+\sin(x)} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[In] Int[(Cos[x] + Sin[x])^(-1),x]
```

```
[Out] -(ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \cos(x)-\sin(x)\right)$$
$$= -\frac{\operatorname{arctanh}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = (-1-i)(-1)^{3/4}\operatorname{arctanh}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$

```
[In] Integrate[(Cos[x] + Sin[x])^(-1),x]
[Out] (-1 - I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

method	result	size
default	$\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tan\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)$	19
risch	$\frac{\sqrt{2}\,\ln\left(e^{ix}-\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(e^{ix}+\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)}{2}$	48

```
[In] int(1/(cos(x)+sin(x)),x,method=_RETURNVERBOSE)
[Out] 2^(1/2)*arctanh(1/4*(2*tan(1/2*x)-2)*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}-\cos\left(x\right)\right)\sin\left(x\right)-2\sqrt{2}\cos\left(x\right)+3}{2\cos\left(x\right)\sin\left(x\right)+1}\right)$$

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*(sqrt(2) - cos(x))*sin(x) - 2*sqrt(2)*cos(x) + 3)/(2*cos
(x)*sin(x) + 1))
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = \frac{\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{2} - \frac{\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{2}$$

```
[In] integrate(1/(cos(x)+sin(x)),x)
```

```
[Out] sqrt(2)*log(tan(x/2) - 1 + sqrt(2))/2 - sqrt(2)*log(tan(x/2) - sqrt(2) - 1)
/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(18) = 36$.

Time = 0.38 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}+1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}-1}\right)$$

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) + 1)/(sqrt(2) + sin(x)/(co
s(x) + 1) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}{\left|2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}\right)$$

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="giac")
[Out] -1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) - 2)/abs(2*sqrt(2) + 2*tan(1
/2*x) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = -\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}\tan\left(\frac{x}{2}\right)}{2}\right)$$

```
[In] int(1/(cos(x) + sin(x)),x)
[Out] -2^(1/2)*atanh(2^(1/2)/2 - (2^(1/2)*tan(x/2))/2)
```

3.245 $\int \frac{1}{1-\cos(x)+\sin(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \frac{1}{1-\cos(x)+\sin(x)}\,dx = -\log\left(1+\cot\left(\frac{x}{2}\right)\right)$$

```
[Out] -ln(1+cot(1/2*x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3200, 31}

$$\int \frac{1}{1-\cos(x)+\sin(x)}\,dx = -\log\left(\cot\left(\frac{x}{2}\right)+1\right)$$

```
[In] Int[(1 - Cos[x] + Sin[x])^(-1),x]
```

```
[Out] -Log[1 + Cot[x/2]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3200

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, Dist[-f/e
, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a + b, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cot\left(\frac{x}{2}\right)\right)$$
$$= -\log\left(1+\cot\left(\frac{x}{2}\right)\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 24 vs. $2(11) = 22$.

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \frac{1}{1-\cos(x)+\sin(x)}\,dx = \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[(1 - Cos[x] + Sin[x])^(-1),x]
[Out] Log[Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

method	result	size
default	$\ln\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	16
norman	$\ln\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	16
parallelrisch	$\ln\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	16
risch	$\ln\left(e^{ix}-1\right)-\ln\left(i+e^{ix}\right)$	21

```
[In] int(1/(1-cos(x)+sin(x)),x,method=_RETURNVERBOSE)
[Out] ln(tan(1/2*x))-ln(1+tan(1/2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1}{1-\cos(x)+\sin(x)}\,dx = \frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-\frac{1}{2}\log(\sin(x)+1)$$

```
[In] integrate(1/(1-cos(x)+sin(x)),x, algorithm="fricas")
[Out] 1/2*log(-1/2*cos(x) + 1/2) - 1/2*log(sin(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{1}{1-\cos(x)+\sin(x)}\,dx = -\log\left(\tan\left(\frac{x}{2}\right)+1\right)+\log\left(\tan\left(\frac{x}{2}\right)\right)$$

```
[In] integrate(1/(1-cos(x)+sin(x)),x)
[Out] -log(tan(x/2) + 1) + log(tan(x/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(9) = 18.

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{1}{1-\cos(x)+\sin(x)}\,dx = -\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)+\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

```
[In] integrate(1/(1-cos(x)+sin(x)),x, algorithm="maxima")
[Out] -log(sin(x)/(cos(x) + 1) + 1) + log(sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1}{1-\cos(x)+\sin(x)}\,dx = -\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)+\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

```
[In] integrate(1/(1-cos(x)+sin(x)),x, algorithm="giac")
[Out] -log(abs(tan(1/2*x) + 1)) + log(abs(tan(1/2*x)))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-\cos(x)+\sin(x)}\,dx = -2\,\mathrm{atanh}\left(2\tan\left(\frac{x}{2}\right)+1\right)$$

```
[In] int(1/(sin(x) - cos(x) + 1),x)
[Out] -2*atanh(2*tan(x/2) + 1)
```

3.246 $\int \frac{1}{4\cos(x)+3\sin(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{4\cos(x)+3\sin(x)}\,dx = -\frac{1}{5}\text{arctanh}\left(\frac{1}{5}(3\cos(x)-4\sin(x))\right)$$

```
[Out] -1/5*arctanh(3/5*cos(x)-4/5*sin(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3153, 212}

$$\int \frac{1}{4\cos(x)+3\sin(x)}\,dx = -\frac{1}{5}\text{arctanh}\left(\frac{1}{5}(3\cos(x)-4\sin(x))\right)$$

```
[In] Int[(4*Cos[x] + 3*Sin[x])^(-1),x]
```

```
[Out] -1/5*ArcTanh[(3*Cos[x] - 4*Sin[x])/5]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{25-x^2}\,dx, x, 3\cos(x) - 4\sin(x)\right)$$
$$= -\frac{1}{5}\text{arctanh}\left(\frac{1}{5}(3\cos(x) - 4\sin(x))\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(18) = 36.

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int \frac{1}{4\cos(x)+3\sin(x)}\,dx = -\frac{1}{5}\log\left(2\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \frac{1}{5}\log\left(\cos\left(\frac{x}{2}\right) + 2\sin\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[(4*Cos[x] + 3*Sin[x])^(-1),x]
[Out] -1/5*Log[2*Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + 2*Sin[x/2]]/5
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{\ln(2\tan(\frac{x}{2})+1)}{5} - \frac{\ln(\tan(\frac{x}{2})-2)}{5}$	22
norman	$\frac{\ln(2\tan(\frac{x}{2})+1)}{5} - \frac{\ln(\tan(\frac{x}{2})-2)}{5}$	22
parallelrisch	$\ln\left(\frac{1}{(2\tan(\frac{x}{2})-4)^{\frac{1}{5}}}\right) + \ln\left((2\tan\left(\frac{x}{2}\right)+1)^{\frac{1}{5}}\right)$	24
risch	$\frac{\ln(e^{ix}-\frac{3}{5}+\frac{4i}{5})}{5} - \frac{\ln(e^{ix}+\frac{3}{5}-\frac{4i}{5})}{5}$	26

```
[In] int(1/(4*cos(x)+3*sin(x)),x,method=_RETURNVERBOSE)
[Out] 1/5*ln(2*tan(1/2*x)+1)-1/5*ln(tan(1/2*x)-2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{1}{4\cos(x)+3\sin(x)}\,dx = -\frac{1}{10}\log\left(\frac{3}{2}\cos(x)-2\sin(x)+\frac{5}{2}\right)+\frac{1}{10}\log\left(-\frac{3}{2}\cos(x)+2\sin(x)+\frac{5}{2}\right)$$

```
[In] integrate(1/(4*cos(x)+3*sin(x)),x, algorithm="fricas")
[Out] -1/10*log(3/2*cos(x) - 2*sin(x) + 5/2) + 1/10*log(-3/2*cos(x) + 2*sin(x) +
5/2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{4\cos(x)+3\sin(x)}\,dx = -\frac{\log\left(\tan\left(\frac{x}{2}\right)-2\right)}{5}+\frac{\log\left(2\tan\left(\frac{x}{2}\right)+1\right)}{5}$$

```
[In] integrate(1/(4*cos(x)+3*sin(x)),x)
[Out] -log(tan(x/2) - 2)/5 + log(2*tan(x/2) + 1)/5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(12) = 24$.

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{1}{4\cos(x)+3\sin(x)}\,dx = \frac{1}{5}\log\left(\frac{2\sin(x)}{\cos(x)+1}+1\right)-\frac{1}{5}\log\left(\frac{\sin(x)}{\cos(x)+1}-2\right)$$

```
[In] integrate(1/(4*cos(x)+3*sin(x)),x, algorithm="maxima")
[Out] 1/5*log(2*sin(x)/(cos(x) + 1) + 1) - 1/5*log(sin(x)/(cos(x) + 1) - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{4\cos(x)+3\sin(x)}\,dx = \frac{1}{5}\,\log\left(\left|2\,\tan\left(\frac{1}{2}\,x\right)+1\right|\right)-\frac{1}{5}\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-2\right|\right)$$

```
[In] integrate(1/(4*cos(x)+3*sin(x)),x, algorithm="giac")
[Out] 1/5*log(abs(2*tan(1/2*x) + 1)) - 1/5*log(abs(tan(1/2*x) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.61

$$\int \frac{1}{4\cos(x)+3\sin(x)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{4\tan\left(\frac{x}{2}\right)}{5}-\frac{3}{5}\right)}{5}$$

```
[In] int(1/(4*cos(x) + 3*sin(x)),x)
[Out] (2*atanh((4*tan(x/2))/5 - 3/5))/5
```

3.247 $\int \frac{1}{\sin(x)+\tan(x)}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 24

$$\int \frac{1}{\sin(x)+\tan(x)}\,dx=-\frac{1}{2}\text{arctanh}(\cos(x))+\frac{1}{2}\cot(x)\csc(x)-\frac{\csc^2(x)}{2}$$

```
[Out] -1/2*arctanh(cos(x))+1/2*cot(x)*csc(x)-1/2*csc(x)^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4482, 2785, 2686, 30, 2691, 3855}

$$\int \frac{1}{\sin(x)+\tan(x)}\,dx=-\frac{1}{2}\text{arctanh}(\cos(x))-\frac{1}{2}\csc^2(x)+\frac{1}{2}\cot(x)\csc(x)$$

```
[In] Int[(Sin[x] + Tan[x])^(-1),x]
```

```
[Out] -1/2*ArcTanh[Cos[x]] + (Cot[x]*Csc[x])/2 - Csc[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cot(x)}{1+\cos(x)}\,dx \\
&= -\int \cot^2(x)\csc(x)\,dx + \int \cot(x)\csc^2(x)\,dx \\
&= \frac{1}{2}\cot(x)\csc(x) + \frac{1}{2}\int \csc(x)\,dx - \text{Subst}\left(\int x\,dx, x, \csc(x)\right) \\
&= -\frac{1}{2}\text{arctanh}(\cos(x)) + \frac{1}{2}\cot(x)\csc(x) - \frac{\csc^2(x)}{2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sin(x)+\tan(x)}\,dx = -\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{4}\sec^2\left(\frac{x}{2}\right)$$

```
[In] Integrate[(Sin[x] + Tan[x])^(-1),x]
[Out] -1/2*Log[Cos[x/2]] + Log[Sin[x/2]]/2 - Sec[x/2]^2/4
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{1}{2(\cos(x)+1)}-\frac{\ln(\cos(x)+1)}{4}+\frac{\ln(-1+\cos(x))}{4}$	24
risch	$-\frac{e^{ix}}{(e^{ix}+1)^2}-\frac{\ln(e^{ix}+1)}{2}+\frac{\ln(e^{ix}-1)}{2}$	38

```
[In] int(1/(sin(x)+tan(x)),x,method=_RETURNVERBOSE)
[Out] -1/2/(cos(x)+1)-1/4*ln(cos(x)+1)+1/4*ln(-1+cos(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sin(x)+\tan(x)}\,dx = -\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2}{4\,(\cos(x)+1)}$$

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="fricas")
[Out] -1/4*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) + 1
/2) + 2)/(cos(x) + 1)
```

Sympy [F]

$$\int \frac{1}{\sin(x) + \tan(x)}\,dx = \int \frac{1}{\sin(x) + \tan(x)}\,dx$$

```
[In] integrate(1/(sin(x)+tan(x)),x)
[Out] Integral(1/(sin(x) + tan(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sin(x) + \tan(x)}\,dx = -\frac{\sin(x)^2}{4(\cos(x) + 1)^2} + \frac{1}{2}\log\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="maxima")
[Out] -1/4*sin(x)^2/(cos(x) + 1)^2 + 1/2*log(sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sin(x) + \tan(x)}\,dx = \frac{\cos(x) - 1}{4(\cos(x) + 1)} + \frac{1}{4}\log\left(-\frac{\cos(x) - 1}{\cos(x) + 1}\right)$$

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="giac")
[Out] 1/4*(cos(x) - 1)/(cos(x) + 1) + 1/4*log(-(cos(x) - 1)/(cos(x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sin(x) + \tan(x)}\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2} - \frac{\tan\left(\frac{x}{2}\right)^2}{4}$$

```
[In] int(1/(sin(x) + tan(x)),x)
[Out] log(tan(x/2))/2 - tan(x/2)^2/4
```

3.248 $\int \frac{1}{2\sin(x)+\sin(2x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = \frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)+\frac{1}{8}\tan^2\left(\frac{x}{2}\right)$$

```
[Out] 1/4*ln(tan(1/2*x))+1/8*tan(1/2*x)^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 14}

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = \frac{1}{8}\tan^2\left(\frac{x}{2}\right)+\frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)$$

```
[In] Int[(2*Sin[x] + Sin[2*x])^(-1),x]
```

```
[Out] Log[Tan[x/2]]/4 + Tan[x/2]^2/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{1+x^2}{8x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= \frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)+\frac{1}{8}\tan^2\left(\frac{x}{2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$
\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = \frac{1-2\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{4(1+\cos(x))}
$$

```
[In] Integrate[(2*Sin[x] + Sin[2*x])^(-1),x]
[Out] (1 - 2*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(4*(1 + Cos[x]))
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\ln(-1+\cos(x))}{8}+\frac{1}{4\cos(x)+4}-\frac{\ln(\cos(x)+1)}{8}$	24
risch	$\frac{e^{ix}}{2(e^{ix}+1)^2}-\frac{\ln(e^{ix}+1)}{4}+\frac{\ln(e^{ix}-1)}{4}$	38

```
[In] int(1/(2*sin(x)+sin(2*x)),x,method=_RETURNVERBOSE)
[Out] 1/8*ln(-1+cos(x))+1/4/(cos(x)+1)-1/8*ln(cos(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = -\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2}{8(\cos(x)+1)}$$

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="fricas")
[Out] -1/8*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) + 1
/2) - 2)/(cos(x) + 1)
```

Sympy [F]

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = \int \frac{1}{2\sin(x)+\sin(2x)}\,dx$$

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x)
[Out] Integral(1/(2*sin(x) + sin(2*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(16) = 32$.

Time = 0.21 (sec) , antiderivative size = 220, normalized size of antiderivative = 9.17

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = \frac{4\cos(2x)\cos(x)+8\cos(x)^2-\left(2(2\cos(x)+1)\cos(2x)+\cos(2x)^2+4\cos(x)^2+\sin(2x)^2+4\sin(\right.}{}$$

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="maxima")
[Out] 1/8*(4*cos(2*x)*cos(x) + 8*cos(x)^2 - (2*(2*cos(x) + 1)*cos(2*x) + cos(2*x)
^2 + 4*cos(x)^2 + sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) +
1)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (2*(2*cos(x) + 1)*cos(2*x) + c
os(2*x)^2 + 4*cos(x)^2 + sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*co
s(x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*sin(2*x)*sin(x) + 8*s
in(x)^2 + 4*cos(x))/(2*(2*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + 4*cos(x)^2 +
sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = -\frac{\cos(x)-1}{8(\cos(x)+1)} + \frac{1}{8}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="giac")
[Out] -1/8*(cos(x) - 1)/(cos(x) + 1) + 1/8*log(-(cos(x) - 1)/(cos(x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4} + \frac{\tan\left(\frac{x}{2}\right)^2}{8}$$

```
[In] int(1/(sin(2*x) + 2*sin(x)),x)
[Out] log(tan(x/2))/4 + tan(x/2)^2/8
```

3.249 $\int \frac{\sec(x)}{1+\sin(x)} dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int \frac{\sec(x)}{1+\sin(x)} dx = \frac{1}{2}\text{arctanh}(\sin(x)) - \frac{1}{2(1+\sin(x))}$$

```
[Out] 1/2*arctanh(sin(x))-1/2/(1+sin(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2746, 46, 213}

$$\int \frac{\sec(x)}{1+\sin(x)} dx = \frac{1}{2}\text{arctanh}(\sin(x)) - \frac{1}{2(\sin(x)+1)}$$

```
[In] Int[Sec[x]/(1 + Sin[x]),x]
[Out] ArcTanh[Sin[x]]/2 - 1/(2*(1 + Sin[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(1-x)(1+x)^2}\,dx, x, \sin(x)\right) \\
&= \text{Subst}\left(\int \left(\frac{1}{2(1+x)^2} - \frac{1}{2\left(-1+x^2\right)}\right)dx, x, \sin(x)\right) \\
&= -\frac{1}{2(1+\sin(x))} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sin(x)\right) \\
&= \frac{1}{2}\text{arctanh}(\sin(x)) - \frac{1}{2(1+\sin(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)}{1+\sin(x)}\,dx = \frac{1}{2}\text{arctanh}(\sin(x)) - \frac{1}{2(1+\sin(x))}$$

```
[In] Integrate[Sec[x]/(1 + Sin[x]),x]
[Out] ArcTanh[Sin[x]]/2 - 1/(2*(1 + Sin[x]))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

method	result	size
default	$-\frac{\ln(\sin(x)-1)}{4}-\frac{1}{2(\sin(x)+1)}+\frac{\ln(\sin(x)+1)}{4}$	24
norman	$\frac{\tan(\frac{x}{2})}{(1+\tan(\frac{x}{2}))^2}-\frac{\ln(\tan(\frac{x}{2})-1)}{2}+\frac{\ln(1+\tan(\frac{x}{2}))}{2}$	33
parallelrisch	$\ln\left(\frac{1}{\sqrt{-\cot(x)+\csc(x)-1}}\right)+\ln\left(\sqrt{-\cot\left(x\right)+1+\csc\left(x\right)}\right)-\frac{(\tan^2(x))}{2}+\frac{\sec(x)\tan(x)}{2}$	36
risch	$-\frac{ie^{ix}}{(i+e^{ix})^2}-\frac{\ln(e^{ix}-i)}{2}+\frac{\ln(i+e^{ix})}{2}$	42

```
[In] int(sec(x)/(sin(x)+1),x,method=_RETURNVERBOSE)
[Out] -1/4*ln(sin(x)-1)-1/2/(sin(x)+1)+1/4*ln(sin(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.83

$$\int \frac{\sec(x)}{1+\sin(x)}\,dx = \frac{(\sin\left(x\right)+1)\log\left(\sin\left(x\right)+1\right)-(\sin\left(x\right)+1)\log\left(-\sin\left(x\right)+1\right)-2}{4\left(\sin\left(x\right)+1\right)}$$

```
[In] integrate(sec(x)/(1+sin(x)),x, algorithm="fricas")
[Out] 1/4*((sin(x) + 1)*log(sin(x) + 1) - (sin(x) + 1)*log(-sin(x) + 1) - 2)/(sin
(x) + 1)
```

Sympy [F]

$$\int \frac{\sec(x)}{1+\sin(x)}\,dx = \int \frac{\sec\left(x\right)}{\sin\left(x\right)+1}\,dx$$

```
[In] integrate(sec(x)/(1+sin(x)),x)
[Out] Integral(sec(x)/(sin(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{\sec(x)}{1+\sin(x)}\,dx = -\frac{1}{2\left(\sin\left(x\right)+1\right)} + \frac{1}{4}\,\log\left(\sin\left(x\right)+1\right) - \frac{1}{4}\,\log\left(\sin\left(x\right)-1\right)$$

```
[In] integrate(sec(x)/(1+sin(x)),x, algorithm="maxima")
[Out] -1/2/(sin(x) + 1) + 1/4*log(sin(x) + 1) - 1/4*log(sin(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{\sec(x)}{1+\sin(x)}\,dx = -\frac{1}{2\left(\sin\left(x\right)+1\right)} + \frac{1}{4}\,\log\left(\sin\left(x\right)+1\right) - \frac{1}{4}\,\log\left(-\sin\left(x\right)+1\right)$$

```
[In] integrate(sec(x)/(1+sin(x)),x, algorithm="giac")
[Out] -1/2/(sin(x) + 1) + 1/4*log(sin(x) + 1) - 1/4*log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sec(x)}{1+\sin(x)}\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)}{2} - \frac{1}{2\,\left(\sin\left(x\right)+1\right)}$$

```
[In] int(1/(cos(x)*(sin(x) + 1)),x)
[Out] log(tan(x/2 + pi/4))/2 - 1/(2*(sin(x) + 1))
```

3.250 $\int \frac{1}{b\cos(x)+a\sin(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{1}{b\cos(x)+a\sin(x)}\,dx=-\frac{\operatorname{arctanh}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] -arctanh((a*cos(x)-b*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3153, 212}

$$\int \frac{1}{b\cos(x)+a\sin(x)}\,dx=-\frac{\operatorname{arctanh}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[In] Int[(b*Cos[x] + a*Sin[x])^(-1),x]
```

```
[Out] -(ArcTanh[(a*Cos[x] - b*Sin[x])/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, a\cos(x)-b\sin(x)\right)$$
$$= -\frac{\operatorname{arctanh}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{1}{b\cos(x)+a\sin(x)}\,dx = \frac{2\operatorname{arctanh}\left(\frac{-a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[In] Integrate[(b*Cos[x] + a*Sin[x])^(-1),x]
[Out] (2*ArcTanh[(-a + b*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{2\ \operatorname{arctanh}\left(\frac{-2b\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$	35
risch	$\frac{\ln\left(e^{ix}+\frac{ib-a}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{\ln\left(e^{ix}-\frac{ib-a}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$	74

```
[In] int(1/(b*cos(x)+a*sin(x)),x,method=_RETURNVERBOSE)
[Out] -2/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tan(1/2*x)+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(32) = 64$.

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.72

$$\int \frac{1}{b\cos(x) + a\sin(x)}\,dx = \frac{\log\left(-\frac{2\,ab\cos(x)\sin(x) - (a^2-b^2)\cos(x)^2 - a^2 - 2\,b^2 + 2\,\sqrt{a^2+b^2}(a\cos(x) - b\sin(x))}{2\,ab\cos(x)\sin(x) - (a^2-b^2)\cos(x)^2 + a^2}\right)}{2\,\sqrt{a^2+b^2}}$$

```
[In] integrate(1/(b*cos(x)+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 - a^2 - 2*b^2 + 2*sqrt
(a^2 + b^2)*(a*cos(x) - b*sin(x)))/(2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x
)^2 + a^2))/sqrt(a^2 + b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.36

$$\int \frac{1}{b\cos(x) + a\sin(x)}\,dx = \begin{cases} \tilde{\infty}\left(-\log\left(\tan\left(\frac{x}{2}\right) - 1\right) + \log\left(\tan\left(\frac{x}{2}\right) + 1\right)\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{a} & \text{for } b = 0 \\ -\frac{i}{-ib\sin(x) + b\cos(x)} & \text{for } a = -ib \\ \frac{i}{ib\sin(x) + b\cos(x)} & \text{for } a = ib \\ -\frac{\log\left(-\frac{a}{b} + \tan\left(\frac{x}{2}\right) - \frac{\sqrt{a^2+b^2}}{b}\right)}{\sqrt{a^2+b^2}} + \frac{\log\left(-\frac{a}{b} + \tan\left(\frac{x}{2}\right) + \frac{\sqrt{a^2+b^2}}{b}\right)}{\sqrt{a^2+b^2}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(b*cos(x)+a*sin(x)),x)
```

```
[Out] Piecewise((zoo*(-log(tan(x/2) - 1) + log(tan(x/2) + 1)), Eq(a, 0) & Eq(b, 0
)), (log(tan(x/2))/a, Eq(b, 0)), (-I/(-I*b*sin(x) + b*cos(x)), Eq(a, -I*b))
, (I/(I*b*sin(x) + b*cos(x)), Eq(a, I*b)), (-log(-a/b + tan(x/2) - sqrt(a**
2 + b**2)/b)/sqrt(a**2 + b**2) + log(-a/b + tan(x/2) + sqrt(a**2 + b**2)/b)
/sqrt(a**2 + b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\int \frac{1}{b\cos(x) + a\sin(x)}\,dx = -\frac{\log\left(\frac{a-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{a-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[In] integrate(1/(b*cos(x)+a*sin(x)),x, algorithm="maxima")
```

```
[Out] -log((a - b*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(a - b*sin(x)/(cos(x) +
1) - sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\int \frac{1}{b\cos(x) + a\sin(x)}\,dx = -\frac{\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

```
[In] integrate(1/(b*cos(x)+a*sin(x)),x, algorithm="giac")
```

```
[Out] -log(abs(2*b*tan(1/2*x) - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*tan(1/2*x) - 2*a
 + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{1}{b\cos(x) + a\sin(x)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{a-b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[In] int(1/(b*cos(x) + a*sin(x)),x)
```

```
[Out] -(2*atanh((a - b*tan(x/2))/(a^2 + b^2)^(1/2)))/(a^2 + b^2)^(1/2)
```

3.251 $\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 15

$$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx=\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[Out] arctan(a*tan(x)/b)/a/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {211}

$$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx=\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[In] Int[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\text{Subst}\left(\int \frac{1}{b^2+a^2x^2}\,dx,x,\tan(x)\right)\\&=\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx=\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[In] Integrate[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$	16
parallelrisch	$\frac{i\left(\ln\left(\frac{ia\sin(x)-b\cos(x)}{\cos(x)+1}\right)-\ln\left(\frac{-b\cos(x)-ia\sin(x)}{\cos(x)+1}\right)\right)}{2ab}$	53
risch	$\frac{i\ln\left(\mathrm{e}^{2ix}-\frac{a+b}{a-b}\right)}{2ab}-\frac{i\ln\left(\mathrm{e}^{2ix}-\frac{a-b}{a+b}\right)}{2ab}$	58

```
[In] int(1/(b^2*cos(x)^2+a^2*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] arctan(a*tan(x)/b)/a/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.87

$$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx=-\frac{\arctan\left(\frac{(a^2+b^2)\cos(x)^2-a^2}{2\,ab\cos(x)\sin(x)}\right)}{2\,ab}$$

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="fricas")
[Out] -1/2*arctan(1/2*((a^2 + b^2)*cos(x)^2 - a^2)/(a*b*cos(x)*sin(x)))/(a*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71839 vs. $2(10) = 20$.

Time = 16.13 (sec) , antiderivative size = 71839, normalized size of antiderivative = 4789.27

$$\int \frac{1}{b^2 \cos^2(x) + a^2 \sin^2(x)}\, dx = \text{Too large to display}$$

```
[In] integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)
```

```
[Out] Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), ((tan(x/2)
/2 - 1/(2*tan(x/2)))/a**2, Eq(b, 0)), (-2*tan(x/2)/(b**2*(tan(x/2)**2 - 1))
, Eq(a, 0)), (x/(b**2*sin(x)**2 + b**2*cos(x)**2), Eq(a, b) | Eq(a, -b)), (
8192*a**15*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a
**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968
*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**1
0*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a
**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 8192*a**15*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqr
```

t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a
2 - b2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(
a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a
2 - b2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 8192*a**14*sqrt(a**2 - b**2)*sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqr
t(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqr

```
t(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b
**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 8192*a**14*sqrt(a**2 - b**2)
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*
sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a
**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 32768*a**13*b**2*sqrt(-2*a
**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
```

b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2
 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
2 + 2*a*sqrt(a2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2
 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*
a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 32768*a**13*b**2*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**1
3*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*s
qrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a
9*b8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
2 + 2*a*sqrt(a2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a*
*5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
2 + 2*a*sqrt(a2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*s
qrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*s

```
qrt(a**2 - b**2)/b**2 + 1)) + 2048*a**13*b**2*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14
*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a*
*10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a*
*6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b*
*14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)) - 2048*a**13*b**2*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(
x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(
a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b
**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
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1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a*
*2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
 + 1)) + 28672*a**12*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b
2*sqrt(a2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt
(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**1
0*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6
*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**1
4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqr
t(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1)) - 28672*a**12*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192
*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**
2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b*

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*4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33
280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*
b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt
(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5
824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b
**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a
**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1)) - 2048*a**12*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b
**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*
a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2
)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 337
92*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**
2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 240
8*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2
)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
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*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**1
6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1)) + 2048*a**12*b**2*sqrt(a**2 - b**2)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a
**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2
 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqr
t(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**
2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqr
t(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2
 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2
*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 52736*a**11*b**4*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a*
*13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
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b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792
*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*
a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1)) - 52736*a**11*b**4*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**
14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b
2)/b2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*
a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a
2/b2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*
a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*
b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1)) - 7168*a**11*b**4*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b

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**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + t
an(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6
*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a
**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10
*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a
**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt
(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a
**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1)) + 7168*a**11*b**4*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8
192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b
**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a
**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2
```

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- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**
2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) -
 39424*a**10*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan
(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt
(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a
**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a
**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1)) + 39424*a**10*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a
**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b
**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt
```

(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**1
0*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6
*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**1
4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqr
t(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1)) + 6144*a**10*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192
*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**
2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b*
*4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33
280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*
b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt
(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5
824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b
12*sqrt(-2*a2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -

b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a
2*b14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1)) - 6144*a**10*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a
13*b4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 3379
2*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2
)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408
*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1)) - 43520*a**9*b**6*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a*
*14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a
2/b2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280
*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**
2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**

8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824
*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**
2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**1
2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a
2/b2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2
*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a
2/b2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1)) + 43520*a**9*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + t
an(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
2 + 1) + 45568*a11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6
*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a
2/b2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10
*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a
2/b2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
2 + 1) + 170*a3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt
(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a
2/b2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
2 + 1)) + 9728*a9*b**6*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8

192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b
2)*sqrt(-2*a2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
2 + 1) + 19968*a8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a
2 - b2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
2 + 1) + 728*a4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**
2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) -
9728*a**9*b**6*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b*
*2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
2 - b2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 266
24*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*
b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 1
9968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*
b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqr

t(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 26880*a**8*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 26880*a**8*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt

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(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqr
t(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a*
*2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a*
*2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 -
b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
) - 6912*a**8*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + ta
n(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqr
t(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*
sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*
sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(
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a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1)) + 6912*a**8*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b*
*2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b*
*2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10
*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b
**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*
b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b*
*2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14
*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)) + 19392*a**7*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/
2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a*
*2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**
2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**
2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt
(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a
**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b
**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
```

2)/b2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt
(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a
2/b2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2
 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
2 + 2*a*sqrt(a2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1)) - 19392*a**7*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a
15*b2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568
*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**
2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 1299
2*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**
2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a
3*b14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*s
qrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 6400
*a**7*b**8*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*

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sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a
**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968
*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**1
0*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a
**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 6400*a**7*b**8*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
```

1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 9408*a**6*b**8*sqrt(a**2 -
b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**
2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**
2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*
b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8
*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b
12*sqrt(a2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 9408*a**6*b**8*sqrt(
a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b
2)/b2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*
a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**

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6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 1996
8*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**
2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**
10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*
a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 3584*a**6*b**8
*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(
-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**1
5*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a
**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a*
*11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a
**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3
*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt
```

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(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 3584*a*
*6*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(819
2*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**
2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45
568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 -
b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 1
2992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 -
b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 17
0*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 4
480*a**5*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqr
t(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b*
*2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 266
24*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*
b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 1
9968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
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b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*
b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 7
28*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**
14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a
**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 4480*a**5*b
**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b
**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**
2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b*
*4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b
**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt
(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**
12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 2072*a**5*b**10*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt
```

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(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqr
t(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a*
*2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**
2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**
2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a
**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a*
*2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a
**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 2072*a**5*b**10*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a*
*13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792
*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*
a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*
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sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1)) + 1568*a**4*b**10*sqrt(a**2 - b**2)*sqrt(-2*a
2/b2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
2 + 2*a*sqrt(a2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*
a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 1568*a**4*b**10*sqrt(a**2 - b**2)*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqr
t(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -

b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqr
t(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 840*a**4*b**10*sqrt(a**2 - b
2)*sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
2 - b2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b*
*2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b
4*sqrt(a2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt
(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*
b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b*
*12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
2 - b2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s

qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 840*a**4*b**10*sqrt(a
2 - b2)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a
12*b4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968
*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**1
0*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a
4*b12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + 462*a**3*b**12*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*
sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a

```
**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 462*a**3*b**12*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a
**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 307
20*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a
**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b
**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 -
b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b
**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b
**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 292*a**3*b**12*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192
*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**
2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b*
```

*4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33
280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*
b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt
(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5
824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b
12*sqrt(-2*a2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a
2*b14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1)) + 292*a**3*b**12*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
 tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*
sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a
2/b2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**
2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b*
*6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
2 - b2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b*
*2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**
10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
2 - b2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)

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/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1)) - 98*a**2*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14
*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**
2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a*
*10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a*
*6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b*
*14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)) + 98*a**2*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*
a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**
4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 332
80*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
```

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**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b
**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 58
24*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b*
*12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a*
*2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2
*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a
**2 - b**2)/b**2 + 1)) + 72*a**2*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a
**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**
13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b
**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*
a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b
**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a
**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a
**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1)) - 72*a**2*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2
```

/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
 b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 3072
0*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b*
*2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 3
3792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b
2)*sqrt(-2*a2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2
408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b*
*2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b
2)/b2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b*
*16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 14*a*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**1
4*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1
)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*s
qrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a
10*b6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b
2)/b2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a

6*b10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b
14*sqrt(a2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1)) + 14*a*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b
2 + 1)*log(sqrt(-2*a2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2)
)/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2
 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b
2 + 2*a*sqrt(a2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)
/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a
2 - b2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**
2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**
2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*s
qrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a
2 - b2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**
2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**
2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 -
 b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
)) + 12*a*b**14*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b
2*sqrt(-2*a2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(
a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26

624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11
*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)
/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7
*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqr
t(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) +
728*a**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b*
*14*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 +
 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 12*a*b**14
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
 b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a*
*2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*
sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a
2/b2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**
2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*s
qrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -

b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + b**14*sqrt(a**2 - b**2)*sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqr
t(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sq
rt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2
*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 +
1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqr
t(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - b**14*sqrt(a**2 - b**2)*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2
- 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a
2 - b2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*

sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(
a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b
2 - 2*a*sqrt(a2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
2)/b2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a
2 - b2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - b**14*sqrt(a**2 - b**2)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*s
qrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt
(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**
2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a*
*2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2
/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**
2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**
2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sq
rt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 +
1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**
2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) -
2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/
b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) + b**14*sqrt(a**2 - b**2)*sqrt(-2*a
2/b2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2 - 2*

```
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**
2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 3
0720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a**2 -
 b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b
**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(a**2
- b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b*
*2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 -
 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(
-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a**2 -
 b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2 - 2*
a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
- b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 2*a
*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{b^2\cos^2(x) + a^2\sin^2(x)}\,dx = \frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(a*tan(x)/b)/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{1}{b^2 \cos^2(x) + a^2 \sin^2(x)} \, dx = \frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor + \arctan\left(\frac{a \tan(x)}{b}\right)}{ab}$$

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")
[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)/b))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{b^2 \cos^2(x) + a^2 \sin^2(x)} \, dx = \frac{\operatorname{atan}\left(\frac{a \tan(x)}{b}\right)}{a\, b}$$

```
[In] int(1/(b^2*cos(x)^2 + a^2*sin(x)^2),x)
[Out] atan((a*tan(x))/b)/(a*b)
```

3.252 $\int \frac{x}{-1+x^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{x}{-1+x^2} dx = \frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] 1/2*ln(-x^2+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {266}

$$\int \frac{x}{-1+x^2} dx = \frac{1}{2}\log\left(1-x^2\right)$$

```
[In] Int[x/(-1 + x^2),x]
```

```
[Out] Log[1 - x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\log\left(1-x^2\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{-1+x^2} \, dx = \frac{1}{2} \log \left(-1+x^2\right)$$

```
[In] Integrate[x/(-1 + x^2),x]
[Out] Log[-1 + x^2]/2
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\ln(x^2-1)}{2}$	9
risch	$\frac{\ln(x^2-1)}{2}$	9
meijerg	$\frac{\ln(-x^2+1)}{2}$	11
default	$\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14
norman	$\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14
parallelrisch	$\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14

```
[In] int(x/(x^2-1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{-1+x^2} \, dx = \frac{1}{2} \, \log \left(x^2 - 1\right)$$

```
[In] integrate(x/(x^2-1),x, algorithm="fricas")
[Out] 1/2*log(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{x}{-1+x^2}\,dx = \frac{\log(x^2-1)}{2}$$

```
[In] integrate(x/(x**2-1),x)
[Out] log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{-1+x^2}\,dx = \frac{1}{2}\log\left(x^2-1\right)$$

```
[In] integrate(x/(x^2-1),x, algorithm="maxima")
[Out] 1/2*log(x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{x}{-1+x^2}\,dx = \frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

```
[In] integrate(x/(x^2-1),x, algorithm="giac")
[Out] 1/2*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{-1+x^2}\,dx = \frac{\ln(x^2-1)}{2}$$

```
[In] int(x/(x^2 - 1),x)
[Out] log(x^2 - 1)/2
```

3.253 $\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx=\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

```
[Out] 2/3*x^(3/2)+1/2*x^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx=\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

```
[In] Int[(1 + Sqrt[x])*Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3 + x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\sqrt{x}+x\right)\,dx \\
&= \frac{2x^{3/2}}{3}+\frac{x^2}{2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx=\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

```
[In] Integrate[(1 + Sqrt[x])*Sqrt[x],x]
[Out] (2*x^(3/2))/3 + x^2/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3}+\frac{x^2}{2}$	12
default	$\frac{2x^{\frac{3}{2}}}{3}+\frac{x^2}{2}$	12
trager	$\frac{(-1+x)(1+x)}{2}+\frac{2x^{\frac{3}{2}}}{3}$	15

```
[In] int(x^(1/2)*(x^(1/2)+1),x,method=_RETURNVERBOSE)
[Out] 2/3*x^(3/2)+1/2*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx=\frac{1}{2}\,x^2+\frac{2}{3}\,x^{\frac{3}{2}}$$

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="fricas")
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx = \frac{2x^{\frac{3}{2}}}{3} + \frac{x^2}{2}$$

```
[In] integrate(x**(1/2)*(1+x**(1/2)),x)
[Out] 2*x**(3/2)/3 + x**2/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(11) = 22.

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx = \frac{1}{2}\left(\sqrt{x}+1\right)^4 - \frac{4}{3}\left(\sqrt{x}+1\right)^3 + \left(\sqrt{x}+1\right)^2$$

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="maxima")
[Out] 1/2*(sqrt(x) + 1)^4 - 4/3*(sqrt(x) + 1)^3 + (sqrt(x) + 1)^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx = \frac{1}{2}\,x^2 + \frac{2}{3}\,x^{\frac{3}{2}}$$

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="giac")
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx = \frac{x^2}{2} + \frac{2\,x^{3/2}}{3}$$

```
[In] int(x^(1/2)*(x^(1/2) + 1),x)
[Out] x^2/2 + (2*x^(3/2))/3
```

3.254 $\int \frac{1}{1-\cos(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -sin(x)/(1-cos(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2727}

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\sin(x)}{1-\cos(x)}$$

```
[In] Int[(1 - Cos[x])^(-1),x]
```

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx=-\cot\left(\frac{x}{2}\right)$$

```
[In] Integrate[(1 - Cos[x])^(-1),x]
[Out] -Cot[x/2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
norman	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
parallelrisch	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
risch	$-\frac{2i}{e^{ix}-1}$	13

```
[In] int(1/(1-cos(x)),x,method=_RETURNVERBOSE)
[Out] -1/tan(1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)}\,dx=-\frac{\cos(x)+1}{\sin(x)}$$

```
[In] integrate(1/(1-cos(x)),x, algorithm="fricas")
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{x}{2}\right)}$$

```
[In] integrate(1/(1-cos(x)),x)
[Out] -1/tan(x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\cos\left(x\right)+1}{\sin\left(x\right)}$$

```
[In] integrate(1/(1-cos(x)),x, algorithm="maxima")
[Out] -(cos(x) + 1)/sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

```
[In] integrate(1/(1-cos(x)),x, algorithm="giac")
[Out] -1/tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{1-\cos(x)}\,dx = -\cot\left(\frac{x}{2}\right)$$

```
[In] int(-1/(cos(x) - 1),x)
[Out] -cot(x/2)
```

3.255 $\int \sec(x)\tan^2(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \sec(x)\tan^2(x)\,dx = -\frac{1}{2}\text{arctanh}(\sin(x)) + \frac{1}{2}\sec(x)\tan(x)$$

```
[Out] -1/2*arctanh(sin(x))+1/2*sec(x)*tan(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2691, 3855}

$$\int \sec(x)\tan^2(x)\,dx = \frac{1}{2}\tan(x)\sec(x) - \frac{1}{2}\text{arctanh}(\sin(x))$$

```
[In] Int[Sec[x]*Tan[x]^2,x]
```

```
[Out] -1/2*ArcTanh[Sin[x]] + (Sec[x]*Tan[x])/2
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\sec(x)\tan(x) - \frac{1}{2}\int \sec(x)\,dx\\
&= -\frac{1}{2}\text{arctanh}(\sin(x)) + \frac{1}{2}\sec(x)\tan(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$
\int \sec(x)\tan^2(x)\,dx = -\frac{1}{2}\text{arctanh}(\sin(x)) + \frac{1}{2}\sec(x)\tan(x)
$$

```
[In] Integrate[Sec[x]*Tan[x]^2,x]
[Out] -1/2*ArcTanh[Sin[x]] + (Sec[x]*Tan[x])/2
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{\sin^3(x)}{2\cos(x)^2}+\frac{\sin(x)}{2}-\frac{\ln(\sec(x)+\tan(x))}{2}$	24
risch	$-\frac{i(e^{3ix}-e^{ix})}{(e^{2ix}+1)^2}-\frac{\ln(i+e^{ix})}{2}+\frac{\ln(e^{ix}-i)}{2}$	49

```
[In] int(sec(x)*tan(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*sin(x)^3/cos(x)^2+1/2*sin(x)-1/2*ln(sec(x)+tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$
\int \sec(x)\tan^2(x)\,dx = -\frac{\cos(x)^2\log(\sin(x)+1) - \cos(x)^2\log(-\sin(x)+1) - 2\sin(x)}{4\cos(x)^2}
$$

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="fricas")
[Out] -1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) - 2*sin(x))/cos(
x)^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \sec(x)\tan^2(x)\,dx = \frac{\log(\sin(x)-1)}{4} - \frac{\log(\sin(x)+1)}{4} - \frac{\sin(x)}{2\sin^2(x)-2}$$

```
[In] integrate(sec(x)*tan(x)**2,x)
[Out] log(sin(x) - 1)/4 - log(sin(x) + 1)/4 - sin(x)/(2*sin(x)**2 - 2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(12) = 24.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \sec(x)\tan^2(x)\,dx = -\frac{\sin(x)}{2\left(\sin(x)^2-1\right)} - \frac{1}{4}\log(\sin(x)+1) + \frac{1}{4}\log(\sin(x)-1)$$

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="maxima")
[Out] -1/2*sin(x)/(sin(x)^2 - 1) - 1/4*log(sin(x) + 1) + 1/4*log(sin(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \sec(x)\tan^2(x)\,dx = -\frac{\sin(x)}{2\left(\sin(x)^2-1\right)} - \frac{1}{4}\log(\sin(x)+1) + \frac{1}{4}\log(-\sin(x)+1)$$

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="giac")
[Out] -1/2*sin(x)/(sin(x)^2 - 1) - 1/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \sec(x)\tan^2(x)\,dx = \frac{\tan\left(\frac{x}{2}\right)^3+\tan\left(\frac{x}{2}\right)}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^2} - \operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)$$

```
[In] int(tan(x)^2/cos(x),x)
[Out] (tan(x/2) + tan(x/2)^3)/(tan(x/2)^2 - 1)^2 - atanh(tan(x/2))
```

3.256 $\int \sec^3(x)\tan^3(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \sec^3(x)\tan^3(x)\,dx = -\frac{1}{3}\sec^3(x) + \frac{\sec^5(x)}{5}$$

```
[Out] -1/3*sec(x)^3+1/5*sec(x)^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2686, 14}

$$\int \sec^3(x)\tan^3(x)\,dx = \frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

```
[In] Int[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -1/3*Sec[x]^3 + Sec[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\sec(x)\right)\\ &= \text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\sec(x)\right)\\ &= -\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sec^3(x)\tan^3(x)\,dx = -\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}$$

```
[In] Integrate[Sec[x]^3*Tan[x]^3,x]
[Out] -1/3*Sec[x]^3 + Sec[x]^5/5
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{(\sec^3(x))}{3}+\frac{(\sec^5(x))}{5}$	14
default	$-\frac{(\sec^3(x))}{3}+\frac{(\sec^5(x))}{5}$	14
risch	$-\frac{8\left(5\,e^{7ix}-2\,e^{5ix}+5\,e^{3ix}\right)}{15\left(e^{2ix}+1\right)^5}$	34

```
[In] int(sec(x)^3*tan(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/3*sec(x)^3+1/5*sec(x)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^3(x)\tan^3(x)\,dx = -\frac{5\,\cos(x)^2 - 3}{15\,\cos(x)^5}$$

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="fricas")
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^3(x)\tan^3(x)\,dx = \frac{3 - 5\cos^2(x)}{15\cos^5(x)}$$

```
[In] integrate(sec(x)**3*tan(x)**3,x)
[Out] (3 - 5*cos(x)**2)/(15*cos(x)**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^3(x)\tan^3(x)\,dx = -\frac{5\,\cos(x)^2 - 3}{15\,\cos(x)^5}$$

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="maxima")
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^3(x)\tan^3(x)\,dx = -\frac{5\,\cos(x)^2 - 3}{15\,\cos(x)^5}$$

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="giac")
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sec^3(x) \tan^3(x)\, dx = \frac{1}{5\cos(x)^5} - \frac{1}{3\cos(x)^3}$$

```
[In] int(tan(x)^3/cos(x)^3,x)
[Out] 1/(5*cos(x)^5) - 1/(3*cos(x)^3)
```

3.257 $\int e^{\sqrt{x}}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 24

$$\int e^{\sqrt{x}}\,dx=-2e^{\sqrt{x}}+2e^{\sqrt{x}}\sqrt{x}$$

```
[Out] -2*exp(x^(1/2))+2*exp(x^(1/2))*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2238, 2207, 2225}

$$\int e^{\sqrt{x}}\,dx=2e^{\sqrt{x}}\sqrt{x}-2e^{\sqrt{x}}$$

```
[In] Int[E^Sqrt[x],x]
```

```
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2238

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Dist[k/d, Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (c
 + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Inte
gerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int e^x x\,dx, x, \sqrt{x}\right)\\ &= 2e^{\sqrt{x}}\sqrt{x} - 2\text{Subst}\left(\int e^x\,dx, x, \sqrt{x}\right)\\ &= -2e^{\sqrt{x}} + 2e^{\sqrt{x}}\sqrt{x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int e^{\sqrt{x}}\,dx = 2e^{\sqrt{x}}\left(-1+\sqrt{x}\right)$$

```
[In] Integrate[E^Sqrt[x],x]
[Out] 2*E^Sqrt[x]*(-1 + Sqrt[x])
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
meijerg	$2-\left(-2\sqrt{x}+2\right)\mathrm{e}^{\sqrt{x}}$	16
derivativedivides	$-2\,\mathrm{e}^{\sqrt{x}}+2\,\mathrm{e}^{\sqrt{x}}\sqrt{x}$	17
default	$-2\,\mathrm{e}^{\sqrt{x}}+2\,\mathrm{e}^{\sqrt{x}}\sqrt{x}$	17

```
[In] int(exp(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2-(-2*x^(1/2)+2)*exp(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int e^{\sqrt{x}} \, dx = 2 \left(\sqrt{x} - 1\right) e^{\sqrt{x}}$$

```
[In] integrate(exp(x^(1/2)),x, algorithm="fricas")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int e^{\sqrt{x}} \, dx = 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}}$$

```
[In] integrate(exp(x**(1/2)),x)
[Out] 2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int e^{\sqrt{x}} \, dx = 2 \left(\sqrt{x} - 1\right) e^{\sqrt{x}}$$

```
[In] integrate(exp(x^(1/2)),x, algorithm="maxima")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int e^{\sqrt{x}} \, dx = 2 \left(\sqrt{x} - 1\right) e^{\sqrt{x}}$$

```
[In] integrate(exp(x^(1/2)),x, algorithm="giac")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int e^{\sqrt{x}}\,dx = 2\,\mathrm{e}^{\sqrt{x}}\,\left(\sqrt{x}-1\right)$$

```
[In] int(exp(x^(1/2)),x)
[Out] 2*exp(x^(1/2))*(x^(1/2) - 1)
```

3.258 $\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = 19x+\frac{3x^2}{2}+\frac{x^3}{3}+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(2+x)$$

```
[Out] 19*x+3/2*x^2+1/3*x^3+3126/35*ln(5-x)-1/10*ln(x)-31/14*ln(2+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1608, 1642}

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

```
[In] Int[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1+x^5}{x(-10-3x+x^2)}\,dx \\
&= \int \left(19+\frac{3126}{35(-5+x)}-\frac{1}{10x}+3x+x^2-\frac{31}{14(2+x)}\right)dx \\
&= 19x+\frac{3x^2}{2}+\frac{x^3}{3}+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(2+x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = 19x+\frac{3x^2}{2}+\frac{x^3}{3}+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(2+x)$$

```
[In] Integrate[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{31\ln(2+x)}{14}+\frac{3126\ln(x-5)}{35}-\frac{\ln(x)}{10}$	31
norman	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{31\ln(2+x)}{14}+\frac{3126\ln(x-5)}{35}-\frac{\ln(x)}{10}$	31
risch	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{31\ln(2+x)}{14}+\frac{3126\ln(x-5)}{35}-\frac{\ln(x)}{10}$	31
parallelrisch	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{31\ln(2+x)}{14}+\frac{3126\ln(x-5)}{35}-\frac{\ln(x)}{10}$	31

```
[In] int((x^5+1)/(x^3-3*x^2-10*x),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3+3/2*x^2+19*x-31/14*ln(2+x)+3126/35*ln(x-5)-1/10*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{1}{3}\,x^3 + \frac{3}{2}\,x^2 + 19\,x - \frac{31}{14}\,\log\left(x+2\right) + \frac{3126}{35}\,\log\left(x-5\right) - \frac{1}{10}\,\log\left(x\right)$$

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log
(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{x^3}{3} + \frac{3x^2}{2} + 19x - \frac{\log\left(x\right)}{10} + \frac{3126\log\left(x-5\right)}{35} - \frac{31\log\left(x+2\right)}{14}$$

```
[In] integrate((x**5+1)/(x**3-3*x**2-10*x),x)
```

```
[Out] x**3/3 + 3*x**2/2 + 19*x - log(x)/10 + 3126*log(x - 5)/35 - 31*log(x + 2)/1
4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{1}{3}\,x^3 + \frac{3}{2}\,x^2 + 19\,x - \frac{31}{14}\,\log\left(x+2\right) + \frac{3126}{35}\,\log\left(x-5\right) - \frac{1}{10}\,\log\left(x\right)$$

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log
(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{1}{3}\,x^3+\frac{3}{2}\,x^2+19\,x-\frac{31}{14}\,\log\left(|x+2|\right)+\frac{3126}{35}\,\log\left(|x-5|\right)-\frac{1}{10}\,\log\left(|x|\right)$$

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(abs(x + 2)) + 3126/35*log(abs(x - 5))
- 1/10*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = 19\,x-\frac{31\,\ln\left(x+2\right)}{14}+\frac{3126\,\ln\left(x-5\right)}{35}-\frac{\ln\left(x\right)}{10}+\frac{3\,x^2}{2}+\frac{x^3}{3}$$

```
[In] int(-(x^5 + 1)/(10*x + 3*x^2 - x^3),x)
```

```
[Out] 19*x - (31*log(x + 2))/14 + (3126*log(x - 5))/35 - log(x)/10 + (3*x^2)/2 +
x^3/3
```

3.259 $\int \frac{1}{x\sqrt{\log(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 8

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\sqrt{\log(x)}$$

```
[Out] 2*ln(x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2339, 30}

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\sqrt{\log(x)}$$

```
[In] Int[1/(x*Sqrt[Log[x]]),x]
```

```
[Out] 2*Sqrt[Log[x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \log(x)\right)\\
&= 2\sqrt{\log(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\sqrt{\log(x)}$$

```
[In] Integrate[1/(x*Sqrt[Log[x]]),x]
[Out] 2*Sqrt[Log[x]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$2\sqrt{\ln\left(x\right)}$	7
default	$2\sqrt{\ln\left(x\right)}$	7

```
[In] int(1/x/ln(x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*ln(x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\sqrt{\log\left(x\right)}$$

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(log(x))
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\sqrt{\log{(x)}}$$

```
[In] integrate(1/x/ln(x)**(1/2),x)
[Out] 2*sqrt(log(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\,\sqrt{\log{(x)}}$$

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(log(x))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\,\sqrt{\log{(x)}}$$

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(log(x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\,\sqrt{\ln{(x)}}$$

```
[In] int(1/(x*log(x)^(1/2)),x)
[Out] 2*log(x)^(1/2)
```

3.260 $\int \frac{5+2x}{-3+x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{5+2x}{-3+x}\,dx = 2x + 11\log(3-x)$$

```
[Out] 2*x+11*ln(3-x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\int \frac{5+2x}{-3+x}\,dx = 2x + 11\log(3-x)$$

```
[In] Int[(5 + 2*x)/(-3 + x),x]
```

```
[Out] 2*x + 11*Log[3 - x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(2+\frac{11}{-3+x}\right)\,dx \\ &= 2x + 11\log(3-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{5+2x}{-3+x}\,dx = 2(-3+x)+11\log(-3+x)$$

```
[In] Integrate[(5 + 2*x)/(-3 + x),x]
[Out] 2*(-3 + x) + 11*Log[-3 + x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$2x+11\ln\left(-3+x\right)$	11
norman	$2x+11\ln\left(-3+x\right)$	11
risch	$2x+11\ln\left(-3+x\right)$	11
parallelrisch	$2x+11\ln\left(-3+x\right)$	11
meijerg	$11\ln\left(1-\frac{x}{3}\right)+2x$	13

```
[In] int((5+2*x)/(-3+x),x,method=_RETURNVERBOSE)
[Out] 2*x+11*ln(-3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{5+2x}{-3+x}\,dx = 2\,x+11\,\log\left(x-3\right)$$

```
[In] integrate((5+2*x)/(-3+x),x, algorithm="fricas")
[Out] 2*x + 11*log(x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{5+2x}{-3+x}\,dx = 2x + 11\log{(x-3)}$$

```
[In] integrate((5+2*x)/(-3+x),x)
[Out] 2*x + 11*log(x - 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{5+2x}{-3+x}\,dx = 2\,x + 11\,\log\left(x-3\right)$$

```
[In] integrate((5+2*x)/(-3+x),x, algorithm="maxima")
[Out] 2*x + 11*log(x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{5+2x}{-3+x}\,dx = 2\,x + 11\,\log\left(|x-3|\right)$$

```
[In] integrate((5+2*x)/(-3+x),x, algorithm="giac")
[Out] 2*x + 11*log(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{5+2x}{-3+x}\,dx = 2\,x + 11\,\ln\left(x-3\right)$$

```
[In] int((2*x + 5)/(x - 3),x)
[Out] 2*x + 11*log(x - 3)
```

3.261 $\int e^{e^x+x}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 5

$$\int e^{e^x+x}\,dx = e^{e^x}$$

```
[Out] exp(exp(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2320, 2225}

$$\int e^{e^x+x}\,dx = e^{e^x}$$

```
[In] Int[E^(E^x + x),x]
```

```
[Out] E^E^x
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int e^x\,dx, x, e^x\right)$$
$$= e^{e^x}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int e^{e^x+x}\,dx = e^{e^x}$$

```
[In] Integrate[E^(E^x + x),x]
[Out] E^E^x
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.80

method	result	size
default	e^{e^x}	4
risch	e^{e^x}	4

```
[In] int(exp(exp(x)+x),x,method=_RETURNVERBOSE)
[Out] exp(exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int e^{e^x+x}\,dx = e^{(e^x)}$$

```
[In] integrate(exp(exp(x)+x),x, algorithm="fricas")
[Out] e^(e^x)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int e^{e^x+x}\,dx = e^{e^x}$$

```
[In] integrate(exp(exp(x)+x),x)
[Out] exp(exp(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int e^{e^x+x}\,dx = e^{(e^x)}$$

```
[In] integrate(exp(exp(x)+x),x, algorithm="maxima")
[Out] e^(e^x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int e^{e^x+x}\,dx = e^{(e^x)}$$

```
[In] integrate(exp(exp(x)+x),x, algorithm="giac")
[Out] e^(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int e^{e^x+x}\,dx = \mathrm{e}^{e^x}$$

```
[In] int(exp(x + exp(x)),x)
[Out] exp(exp(x))
```

3.262 $\int \cos^2(x)\sin^2(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{x}{8}+\frac{1}{8}\cos(x)\sin(x)-\frac{1}{4}\cos^3(x)\sin(x)$$

```
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2648, 2715, 8}

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

```
[In] Int[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{4}\cos^3(x)\sin(x) + \frac{1}{4}\int \cos^2(x)\,dx \\ &= \frac{1}{8}\cos(x)\sin(x) - \frac{1}{4}\cos^3(x)\sin(x) + \frac{\int 1\,dx}{8} \\ &= \frac{x}{8} + \frac{1}{8}\cos(x)\sin(x) - \frac{1}{4}\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{x}{8} - \frac{1}{32}\sin(4x)$$

```
[In] Integrate[Cos[x]^2*Sin[x]^2,x]
[Out] x/8 - Sin[4*x]/32
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

method	result	size
risch	$\frac{x}{8}-\frac{\sin(4x)}{32}$	11
parallelrisch	$\frac{x}{8}-\frac{\sin(4x)}{32}$	11
default	$\frac{x}{8}+\frac{\cos(x)\sin(x)}{8}-\frac{(\cos^3(x))\sin(x)}{4}$	19
norman	$\frac{\frac{x}{8}+\frac{7\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}-\frac{7\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}-\frac{\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	82

```
[In] int(cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/8*x-1/32*sin(4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \cos^2(x)\sin^2(x)\,dx = -\frac{1}{8}\left(2\,\cos\left(x\right)^3 - \cos\left(x\right)\right)\sin\left(x\right) + \frac{1}{8}\,x$$

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{x}{8} - \frac{\sin\left(2x\right)\cos\left(2x\right)}{16}$$

```
[In] integrate(cos(x)**2*sin(x)**2,x)
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.42

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{1}{8}\,x - \frac{1}{32}\,\sin\left(4\,x\right)$$

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
[Out] 1/8*x - 1/32*sin(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.42

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{1}{8}\,x - \frac{1}{32}\,\sin\left(4\,x\right)$$

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
[Out] 1/8*x - 1/32*sin(4*x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{\cos(x)\ \sin(x)^3}{4} - \frac{\cos(x)\ \sin(x)}{8} + \frac{x}{8}$$

```
[In] int(cos(x)^2*sin(x)^2,x)
[Out] x/8 - (cos(x)*sin(x))/8 + (cos(x)*sin(x)^3)/4
```

3.263 $\int \frac{-\cos(x)+\sin(x)}{\cos(x)+\sin(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 8

$$\int \frac{-\cos(x)+\sin(x)}{\cos(x)+\sin(x)}\,dx = -\log(\cos(x)+\sin(x))$$

```
[Out] -ln(cos(x)+sin(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3212}

$$\int \frac{-\cos(x)+\sin(x)}{\cos(x)+\sin(x)}\,dx = -\log(\sin(x)+\cos(x))$$

```
[In] Int[(-Cos[x] + Sin[x])/(Cos[x] + Sin[x]),x]
```

```
[Out] -Log[Cos[x] + Sin[x]]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\text{integral} = -\log(\cos(x)+\sin(x))$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{-\cos(x)+\sin(x)}{\cos(x)+\sin(x)}\,dx = -\log(\cos(x)+\sin(x))$$

```
[In] Integrate[(-Cos[x] + Sin[x])/(Cos[x] + Sin[x]),x]
[Out] -Log[Cos[x] + Sin[x]]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$-\ln\left(\cos\left(x\right)+\sin\left(x\right)\right)$	9
default	$-\ln\left(\cos\left(x\right)+\sin\left(x\right)\right)$	9
risch	$ix-\ln\left(\mathrm{e}^{2ix}+i\right)$	17
norman	$-\ln\left(\tan^2\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$	28
parallelrisch	$-\ln\left(\frac{-\sin(x)-\cos(x)}{\cos(x)+1}\right)+\ln\left(\frac{1}{\cos(x)+1}\right)$	28

```
[In] int((-cos(x)+sin(x))/(cos(x)+sin(x)),x,method=_RETURNVERBOSE)
[Out] -ln(cos(x)+sin(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int \frac{-\cos(x)+\sin(x)}{\cos(x)+\sin(x)}\,dx = -\frac{1}{2}\,\log\left(2\,\cos\left(x\right)\sin\left(x\right)+1\right)$$

```
[In] integrate((-cos(x)+sin(x))/(cos(x)+sin(x)),x, algorithm="fricas")
[Out] -1/2*log(2*cos(x)*sin(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{-\cos(x)+\sin(x)}{\cos(x)+\sin(x)}\,dx = -\log\left(\sin\left(x\right)+\cos\left(x\right)\right)$$

```
[In] integrate((-cos(x)+sin(x))/(cos(x)+sin(x)),x)
[Out] -log(sin(x) + cos(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{-\cos(x)+\sin(x)}{\cos(x)+\sin(x)}\,dx = -\log\left(\cos\left(x\right)+\sin\left(x\right)\right)$$

```
[In] integrate((-cos(x)+sin(x))/(cos(x)+sin(x)),x, algorithm="maxima")
[Out] -log(cos(x) + sin(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(8) = 16.

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{-\cos(x)+\sin(x)}{\cos(x)+\sin(x)}\,dx = \frac{1}{2}\log\left(\tan\left(x\right)^2+1\right)-\log\left(\left|\tan\left(x\right)+1\right|\right)$$

```
[In] integrate((-cos(x)+sin(x))/(cos(x)+sin(x)),x, algorithm="giac")
[Out] 1/2*log(tan(x)^2 + 1) - log(abs(tan(x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 32, normalized size of antiderivative = 4.00

$$\int \frac{-\cos(x)+\sin(x)}{\cos(x)+\sin(x)}\,dx = -2\,\mathrm{atanh}\left(\frac{128\tan\left(\frac{x}{2}\right)+128}{16\tan\left(\frac{x}{2}\right)^2+32\tan\left(\frac{x}{2}\right)+48}-3\right)$$

```
[In] int(-(cos(x) - sin(x))/(cos(x) + sin(x)),x)
[Out] -2*atanh((128*tan(x/2) + 128)/(32*tan(x/2) + 16*tan(x/2)^2 + 48) - 3)
```

3.264 $\int \frac{x}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x}{\sqrt{1-x^2}}\,dx=-\sqrt{1-x^2}$$

```
[Out] -(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {267}

$$\int \frac{x}{\sqrt{1-x^2}}\,dx=-\sqrt{1-x^2}$$

```
[In] Int[x/Sqrt[1 - x^2],x]
```

```
[Out] -Sqrt[1 - x^2]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral}=-\sqrt{1-x^2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}$$

```
[In] Integrate[x/Sqrt[1 - x^2],x]
[Out] -Sqrt[1 - x^2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\sqrt{-x^{2}+1}$	12
default	$-\sqrt{-x^{2}+1}$	12
trager	$-\sqrt{-x^{2}+1}$	12
pseudoelliptic	$-\sqrt{-x^{2}+1}$	12
risch	$\frac{x^{2}-1}{\sqrt{-x^{2}+1}}$	16
gosper	$\frac{(-1+x)(1+x)}{\sqrt{-x^{2}+1}}$	17
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^{2}+1}}{2\sqrt{\pi}}$	26

```
[In] int(1/(-x^2+1)^(1/2)*x,x,method=_RETURNVERBOSE)
[Out] -(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1}$$

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}$$

```
[In] integrate(x/(-x**2+1)**(1/2),x)
[Out] -sqrt(1 - x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1}$$

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1}$$

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}$$

```
[In] int(x/(1 - x^2)^(1/2),x)
[Out] -(1 - x^2)^(1/2)
```

3.265 $\int x^3 \log(x) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 17

$$\int x^3 \log(x) \, dx = -\frac{x^4}{16} + \frac{1}{4} x^4 \log(x)$$

```
[Out] -1/16*x^4+1/4*x^4*ln(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2341}

$$\int x^3 \log(x) \, dx = \frac{1}{4} x^4 \log(x) - \frac{x^4}{16}$$

```
[In] Int[x^3*Log[x],x]
```

```
[Out] -1/16*x^4 + (x^4*Log[x])/4
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{x^4}{16} + \frac{1}{4} x^4 \log(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^3 \log(x)\,dx = -\frac{x^4}{16} + \frac{1}{4}x^4 \log(x)$$

```
[In] Integrate[x^3*Log[x],x]
[Out] -1/16*x^4 + (x^4*Log[x])/4
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{x^4}{16} + \frac{x^4 \ln(x)}{4}$	14
norman	$-\frac{x^4}{16} + \frac{x^4 \ln(x)}{4}$	14
risch	$-\frac{x^4}{16} + \frac{x^4 \ln(x)}{4}$	14
parallelrisch	$-\frac{x^4}{16} + \frac{x^4 \ln(x)}{4}$	14
parts	$-\frac{x^4}{16} + \frac{x^4 \ln(x)}{4}$	14

```
[In] int(x^3*ln(x),x,method=_RETURNVERBOSE)
[Out] -1/16*x^4+1/4*x^4*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^3 \log(x)\,dx = \frac{1}{4}\,x^4 \log\left(x\right) - \frac{1}{16}\,x^4$$

```
[In] integrate(x^3*log(x),x, algorithm="fricas")
[Out] 1/4*x^4*log(x) - 1/16*x^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^3 \log(x)\,dx = \frac{x^4 \log{(x)}}{4} - \frac{x^4}{16}$$

```
[In] integrate(x**3*ln(x),x)
[Out] x**4*log(x)/4 - x**4/16
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^3 \log(x)\,dx = \frac{1}{4}\,x^4 \log{(x)} - \frac{1}{16}\,x^4$$

```
[In] integrate(x^3*log(x),x, algorithm="maxima")
[Out] 1/4*x^4*log(x) - 1/16*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^3 \log(x)\,dx = \frac{1}{4}\,x^4 \log{(x)} - \frac{1}{16}\,x^4$$

```
[In] integrate(x^3*log(x),x, algorithm="giac")
[Out] 1/4*x^4*log(x) - 1/16*x^4
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int x^3 \log(x)\,dx = \frac{x^4 \left(\ln{(x)} - \frac{1}{4}\right)}{4}$$

```
[In] int(x^3*log(x),x)
[Out] (x^4*(log(x) - 1/4))/4
```

3.266 $\int \frac{\sqrt{-2+x}}{2+x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{\sqrt{-2+x}}{2+x} \, dx = 2\sqrt{-2+x} - 4\arctan\left(\frac{\sqrt{-2+x}}{2}\right)$$

```
[Out] -4*arctan(1/2*(-2+x)^(1/2))+2*(-2+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {52, 65, 209}

$$\int \frac{\sqrt{-2+x}}{2+x} \, dx = 2\sqrt{x-2} - 4\arctan\left(\frac{\sqrt{x-2}}{2}\right)$$

```
[In] Int[Sqrt[-2 + x]/(2 + x),x]
```

```
[Out] 2*Sqrt[-2 + x] - 4*ArcTan[Sqrt[-2 + x]/2]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\sqrt{-2+x}-4\int \frac{1}{\sqrt{-2+x}(2+x)}\,dx\\ &= 2\sqrt{-2+x}-8\text{Subst}\left(\int \frac{1}{4+x^2}\,dx, x, \sqrt{-2+x}\right)\\ &= 2\sqrt{-2+x}-4\arctan\left(\frac{\sqrt{-2+x}}{2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-2+x}}{2+x}\,dx = 2\sqrt{-2+x}-4\arctan\left(\frac{\sqrt{-2+x}}{2}\right)$$

```
[In] Integrate[Sqrt[-2 + x]/(2 + x),x]
[Out] 2*Sqrt[-2 + x] - 4*ArcTan[Sqrt[-2 + x]/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-4\arctan\left(\frac{\sqrt{-2+x}}{2}\right)+2\sqrt{-2+x}$	19
default	$-4\arctan\left(\frac{\sqrt{-2+x}}{2}\right)+2\sqrt{-2+x}$	19
risch	$-4\arctan\left(\frac{\sqrt{-2+x}}{2}\right)+2\sqrt{-2+x}$	19
trager	$2\sqrt{-2+x}+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+1\right)x-6\,\mathrm{RootOf}\left(\_Z^2+1\right)+4\sqrt{-2+x}}{2+x}\right)$	48

```
[In] int((-2+x)^(1/2)/(2+x),x,method=_RETURNVERBOSE)
[Out] -4*arctan(1/2*(-2+x)^(1/2))+2*(-2+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{-2+x}}{2+x}\,dx = 2\sqrt{x-2} - 4\arctan\left(\frac{1}{2}\sqrt{x-2}\right)$$

```
[In] integrate((-2+x)^(1/2)/(2+x),x, algorithm="fricas")
[Out] 2*sqrt(x - 2) - 4*arctan(1/2*sqrt(x - 2))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.54

$$\int \frac{\sqrt{-2+x}}{2+x}\,dx = \begin{cases} -4i\,\mathrm{acosh}\left(\frac{2}{\sqrt{x+2}}\right) - \frac{2i\sqrt{x+2}}{\sqrt{-1+\frac{4}{x+2}}} + \frac{8i}{\sqrt{-1+\frac{4}{x+2}}\sqrt{x+2}} & \text{for } \frac{1}{|x+2|} > \frac{1}{4} \\ 4\,\mathrm{asin}\left(\frac{2}{\sqrt{x+2}}\right) + \frac{2\sqrt{x+2}}{\sqrt{1-\frac{4}{x+2}}} - \frac{8}{\sqrt{1-\frac{4}{x+2}}\sqrt{x+2}} & \text{otherwise} \end{cases}$$

```
[In] integrate((-2+x)**(1/2)/(2+x),x)
[Out] Piecewise((-4*I*acosh(2/sqrt(x + 2)) - 2*I*sqrt(x + 2)/sqrt(-1 + 4/(x + 2))
 + 8*I/(sqrt(-1 + 4/(x + 2))*sqrt(x + 2)), 1/Abs(x + 2) > 1/4), (4*asin(2/s
qrt(x + 2)) + 2*sqrt(x + 2)/sqrt(1 - 4/(x + 2)) - 8/(sqrt(1 - 4/(x + 2))*sq
rt(x + 2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{-2+x}}{2+x}\,dx = 2\sqrt{x-2} - 4\arctan\left(\frac{1}{2}\sqrt{x-2}\right)$$

```
[In] integrate((-2+x)^(1/2)/(2+x),x, algorithm="maxima")
[Out] 2*sqrt(x - 2) - 4*arctan(1/2*sqrt(x - 2))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{-2+x}}{2+x}\,dx = 2\sqrt{x-2} - 4\arctan\left(\frac{1}{2}\sqrt{x-2}\right)$$

```
[In] integrate((-2+x)^(1/2)/(2+x),x, algorithm="giac")
[Out] 2*sqrt(x - 2) - 4*arctan(1/2*sqrt(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{-2+x}}{2+x}\,dx = 2\sqrt{x-2} - 4\,\mathrm{atan}\left(\frac{\sqrt{x-2}}{2}\right)$$

```
[In] int((x - 2)^(1/2)/(x + 2),x)
[Out] 2*(x - 2)^(1/2) - 4*atan((x - 2)^(1/2)/2)
```

3.267 $\int \frac{x}{(2+x)^2} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \frac{x}{(2+x)^2} \, dx = \frac{2}{2+x} + \log(2+x)$$

```
[Out] 2/(2+x)+ln(2+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {45}

$$\int \frac{x}{(2+x)^2} \, dx = \frac{2}{x+2} + \log(x+2)$$

```
[In] Int[x/(2 + x)^2,x]
```

```
[Out] 2/(2 + x) + Log[2 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\frac{2}{(2+x)^2} + \frac{1}{2+x}\right) \, dx \\ &= \frac{2}{2+x} + \log(2+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(2+x)^2}\,dx = \frac{2}{2+x} + \log(2+x)$$

```
[In] Integrate[x/(2 + x)^2,x]
[Out] 2/(2 + x) + Log[2 + x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{2}{2+x}+\ln\left(2+x\right)$	13
norman	$\frac{2}{2+x}+\ln\left(2+x\right)$	13
risch	$\frac{2}{2+x}+\ln\left(2+x\right)$	13
meijerg	$-\frac{x}{2\left(1+\frac{x}{2}\right)}+\ln\left(1+\frac{x}{2}\right)$	18
parallelrisch	$\frac{\ln(2+x)x+2+2\ln(2+x)}{2+x}$	21

```
[In] int(x/(2+x)^2,x,method=_RETURNVERBOSE)
[Out] 2/(2+x)+ln(2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{x}{(2+x)^2}\,dx = \frac{(x+2)\log\left(x+2\right)+2}{x+2}$$

```
[In] integrate(x/(2+x)^2,x, algorithm="fricas")
[Out] ((x + 2)*log(x + 2) + 2)/(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{(2+x)^2}\,dx = \log(x+2) + \frac{2}{x+2}$$

```
[In] integrate(x/(2+x)**2,x)
[Out] log(x + 2) + 2/(x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(2+x)^2}\,dx = \frac{2}{x+2} + \log(x+2)$$

```
[In] integrate(x/(2+x)^2,x, algorithm="maxima")
[Out] 2/(x + 2) + log(x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{x}{(2+x)^2}\,dx = \frac{2}{x+2} + \log(|x+2|)$$

```
[In] integrate(x/(2+x)^2,x, algorithm="giac")
[Out] 2/(x + 2) + log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(2+x)^2}\,dx = \ln(x+2) + \frac{2}{x+2}$$

```
[In] int(x/(x + 2)^2,x)
[Out] log(x + 2) + 2/(x + 2)
```

3.268 $\int \log(1+x^2)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 16

$$\int \log(1+x^2)\,dx = -2x + 2\arctan(x) + x\log(1+x^2)$$

```
[Out] -2*x+2*arctan(x)+x*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2498, 327, 209}

$$\int \log(1+x^2)\,dx = 2\arctan(x) + x\log(x^2+1) - 2x$$

```
[In] Int[Log[1 + x^2],x]
```

```
[Out] -2*x + 2*ArcTan[x] + x*Log[1 + x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log\left(1+x^2\right)-2\int\frac{x^2}{1+x^2}\,dx\\ &= -2x+x\log\left(1+x^2\right)+2\int\frac{1}{1+x^2}\,dx\\ &= -2x+2\arctan(x)+x\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int\log\left(1+x^2\right)\,dx=-2x+2\arctan(x)+x\log\left(1+x^2\right)$$

```
[In] Integrate[Log[1 + x^2],x]
[Out] -2*x + 2*ArcTan[x] + x*Log[1 + x^2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$-2x+2\arctan\left(x\right)+x\ln\left(x^{2}+1\right)$	17
risch	$-2x+2\arctan\left(x\right)+x\ln\left(x^{2}+1\right)$	17
parts	$-2x+2\arctan\left(x\right)+x\ln\left(x^{2}+1\right)$	17
meijerg	$-2x+\frac{2x\arctan\left(\sqrt{x^{2}}\right)}{\sqrt{x^{2}}}+x\ln\left(x^{2}+1\right)$	27
parallelrisch	$-2i\ln\left(x-i\right)+i\ln\left(x^{2}+1\right)+x\ln\left(x^{2}+1\right)-2x$	30

```
[In] int(ln(x^2+1),x,method=_RETURNVERBOSE)
[Out] -2*x+2*arctan(x)+x*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log(1+x^2)\,dx = x\log(x^2+1) - 2\,x + 2\,\arctan(x)$$

```
[In] integrate(log(x^2+1),x, algorithm="fricas")
[Out] x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \log(1+x^2)\,dx = x\log(x^2+1) - 2x + 2\,\mathrm{atan}(x)$$

```
[In] integrate(ln(x**2+1),x)
[Out] x*log(x**2 + 1) - 2*x + 2*atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log(1+x^2)\,dx = x\log(x^2+1) - 2\,x + 2\,\arctan(x)$$

```
[In] integrate(log(x^2+1),x, algorithm="maxima")
[Out] x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log(1+x^2)\,dx = x\log(x^2+1) - 2\,x + 2\,\arctan(x)$$

```
[In] integrate(log(x^2+1),x, algorithm="giac")
[Out] x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log \left(1+x^2\right) \,dx=2\,\mathrm{atan}(x)-2\,x+x\,\ln \left(x^2+1\right)$$

```
[In] int(log(x^2 + 1),x)
[Out] 2*atan(x) - 2*x + x*log(x^2 + 1)
```

3.269 $\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = -2\operatorname{arctanh}\left(\sqrt{1+\log(x)}\right)+2\sqrt{1+\log(x)}$$

```
[Out] -2*arctanh((1+ln(x))^(1/2))+2*(1+ln(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2412, 52, 65, 213}

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = 2\sqrt{\log(x)+1}-2\operatorname{arctanh}\left(\sqrt{\log(x)+1}\right)$$

```
[In] Int[Sqrt[1 + Log[x]]/(x*Log[x]),x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2412

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, \log(x)\right)\\ &= 2\sqrt{1+\log(x)} + \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \log(x)\right)\\ &= 2\sqrt{1+\log(x)} + 2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\log(x)}\right)\\ &= -2\text{arctanh}\left(\sqrt{1+\log(x)}\right) + 2\sqrt{1+\log(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = -2\text{arctanh}\left(\sqrt{1+\log(x)}\right) + 2\sqrt{1+\log(x)}$$

```
[In] Integrate[Sqrt[1 + Log[x]]/(x*Log[x]),x]
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

method	result	size
derivativedivides	$2\sqrt{1+\ln\left(x\right)}+\ln\left(\sqrt{1+\ln\left(x\right)}-1\right)-\ln\left(\sqrt{1+\ln\left(x\right)}+1\right)$	30
default	$2\sqrt{1+\ln\left(x\right)}+\ln\left(\sqrt{1+\ln\left(x\right)}-1\right)-\ln\left(\sqrt{1+\ln\left(x\right)}+1\right)$	30

```
[In] int((1+ln(x))^(1/2)/x/ln(x),x,method=_RETURNVERBOSE)
[Out] 2*(1+ln(x))^(1/2)+ln((1+ln(x))^(1/2)-1)-ln((1+ln(x))^(1/2)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = 2\,\sqrt{\log\left(x\right)+1}-\log\left(\sqrt{\log\left(x\right)+1}+1\right)+\log\left(\sqrt{\log\left(x\right)+1}-1\right)$$

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="fricas")
[Out] 2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x) + 1) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = 2\sqrt{\log\left(x\right)+1}+\log\left(\sqrt{\log\left(x\right)+1}-1\right)-\log\left(\sqrt{\log\left(x\right)+1}+1\right)$$

```
[In] integrate((1+ln(x))**(1/2)/x/ln(x),x)
[Out] 2*sqrt(log(x) + 1) + log(sqrt(log(x) + 1) - 1) - log(sqrt(log(x) + 1) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = 2\sqrt{\log(x)+1} - \log\left(\sqrt{\log(x)+1}+1\right) + \log\left(\sqrt{\log(x)+1}-1\right)$$

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="maxima")
[Out] 2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x) + 1) - 1)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = \text{Timed out}$$

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = 2\sqrt{\ln(x)+1} - 2\,\mathrm{atanh}\left(\sqrt{\ln(x)+1}\right)$$

```
[In] int((log(x) + 1)^(1/2)/(x*log(x)),x)
[Out] 2*(log(x) + 1)^(1/2) - 2*atanh((log(x) + 1)^(1/2))
```

3.270 $\int \left(1+\sqrt{x}\right)^8 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 27

$$\int \left(1+\sqrt{x}\right)^8 \, dx = -\frac{2}{9}\left(1+\sqrt{x}\right)^9+\frac{1}{5}\left(1+\sqrt{x}\right)^{10}$$

```
[Out] -2/9*(1+x^(1/2))^9+1/5*(1+x^(1/2))^10
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {196, 45}

$$\int \left(1+\sqrt{x}\right)^8 \, dx = \frac{1}{5}\left(\sqrt{x}+1\right)^{10}-\frac{2}{9}\left(\sqrt{x}+1\right)^9$$

```
[In] Int[(1 + Sqrt[x])^8,x]
```

```
[Out] (-2*(1 + Sqrt[x])^9)/9 + (1 + Sqrt[x])^10/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
 - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
```

`IntegerQ[1/n]`

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x(1+x)^8\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(-(1+x)^8+(1+x)^9\right)\,dx, x, \sqrt{x}\right)\\
&= -\frac{2}{9}\left(1+\sqrt{x}\right)^9+\frac{1}{5}\left(1+\sqrt{x}\right)^{10}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 56 vs. $2(27) = 54$.

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$
\int \left(1+\sqrt{x}\right)^8\,dx = \frac{1}{45}\left(45x+240x^{3/2}+630x^2+1008x^{5/2}+1050x^3+720x^{7/2}+315x^4+80x^{9/2}+9x^5\right)
$$

```
[In] Integrate[(1 + Sqrt[x])^8,x]
```

```
[Out] (45*x + 240*x^(3/2) + 630*x^2 + 1008*x^(5/2) + 1050*x^3 + 720*x^(7/2) + 315
*x^4 + 80*x^(9/2) + 9*x^5)/45
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

method	result	size
derivativedivides	$\frac{x^5}{5}+\frac{16x^{\frac{9}{2}}}{9}+7x^4+16x^{\frac{7}{2}}+\frac{70x^3}{3}+\frac{112x^{\frac{5}{2}}}{5}+14x^2+\frac{16x^{\frac{3}{2}}}{3}+x$	43
default	$\frac{x^5}{5}+\frac{16x^{\frac{9}{2}}}{9}+7x^4+16x^{\frac{7}{2}}+\frac{70x^3}{3}+\frac{112x^{\frac{5}{2}}}{5}+14x^2+\frac{16x^{\frac{3}{2}}}{3}+x$	43
trager	$\frac{(3x^4+108x^3+458x^2+668x+683)(-1+x)}{15}+\frac{16x^{\frac{3}{2}}(5x^3+45x^2+63x+15)}{45}$	47

```
[In] int((x^(1/2)+1)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5+16/9*x^(9/2)+7*x^4+16*x^(7/2)+70/3*x^3+112/5*x^(5/2)+14*x^2+16/3*x^
(3/2)+x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int \left(1+\sqrt{x}\right)^8 dx = \frac{1}{5}\,x^5 + 7\,x^4 + \frac{70}{3}\,x^3 + 14\,x^2 + \frac{16}{45}\left(5\,x^4 + 45\,x^3 + 63\,x^2 + 15\,x\right)\sqrt{x} + x$$

```
[In] integrate((1+x^(1/2))^8,x, algorithm="fricas")
```

```
[Out] 1/5*x^5 + 7*x^4 + 70/3*x^3 + 14*x^2 + 16/45*(5*x^4 + 45*x^3 + 63*x^2 + 15*x
)*sqrt(x) + x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(20) = 40$.

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \left(1+\sqrt{x}\right)^8 dx = \frac{16x^{\frac{9}{2}}}{9} + 16x^{\frac{7}{2}} + \frac{112x^{\frac{5}{2}}}{5} + \frac{16x^{\frac{3}{2}}}{3} + \frac{x^5}{5} + 7x^4 + \frac{70x^3}{3} + 14x^2 + x$$

```
[In] integrate((1+x**(1/2))**8,x)
```

```
[Out] 16*x**(9/2)/9 + 16*x**(7/2) + 112*x**(5/2)/5 + 16*x**(3/2)/3 + x**5/5 + 7*x
**4 + 70*x**3/3 + 14*x**2 + x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \left(1+\sqrt{x}\right)^8 dx = \frac{1}{5}\left(\sqrt{x}+1\right)^{10} - \frac{2}{9}\left(\sqrt{x}+1\right)^9$$

```
[In] integrate((1+x^(1/2))^8,x, algorithm="maxima")
```

```
[Out] 1/5*(sqrt(x) + 1)^10 - 2/9*(sqrt(x) + 1)^9
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \left(1+\sqrt{x}\right)^8\,dx = \frac{1}{5}\,x^5 + \frac{16}{9}\,x^{\frac{9}{2}} + 7\,x^4 + 16\,x^{\frac{7}{2}} + \frac{70}{3}\,x^3 + \frac{112}{5}\,x^{\frac{5}{2}} + 14\,x^2 + \frac{16}{3}\,x^{\frac{3}{2}} + x$$

```
[In] integrate((1+x^(1/2))^8,x, algorithm="giac")
[Out] 1/5*x^5 + 16/9*x^(9/2) + 7*x^4 + 16*x^(7/2) + 70/3*x^3 + 112/5*x^(5/2) + 14
*x^2 + 16/3*x^(3/2) + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \left(1+\sqrt{x}\right)^8\,dx = x + 14\,x^2 + \frac{70\,x^3}{3} + 7\,x^4 + \frac{16\,x^{3/2}}{3} + \frac{x^5}{5} + \frac{112\,x^{5/2}}{5} + 16\,x^{7/2} + \frac{16\,x^{9/2}}{9}$$

```
[In] int((x^(1/2) + 1)^8,x)
[Out] x + 14*x^2 + (70*x^3)/3 + 7*x^4 + (16*x^(3/2))/3 + x^5/5 + (112*x^(5/2))/5
+ 16*x^(7/2) + (16*x^(9/2))/9
```

3.271 $\int \sec^4(x) \tan^3(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \sec^4(x) \tan^3(x) \, dx = -\frac{1}{4} \sec^4(x) + \frac{\sec^6(x)}{6}$$

```
[Out] -1/4*sec(x)^4+1/6*sec(x)^6
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2686, 14}

$$\int \sec^4(x) \tan^3(x) \, dx = \frac{\sec^6(x)}{6} - \frac{\sec^4(x)}{4}$$

```
[In] Int[Sec[x]^4*Tan[x]^3,x]
```

```
[Out] -1/4*Sec[x]^4 + Sec[x]^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int x^3\left(-1+x^2\right)\,dx, x, \sec(x)\right) \\ &= \text{Subst}\left(\int \left(-x^3+x^5\right)\,dx, x, \sec(x)\right) \\ &= -\frac{1}{4}\sec^4(x)+\frac{\sec^6(x)}{6} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sec^4(x)\tan^3(x)\,dx = -\frac{1}{4}\sec^4(x)+\frac{\sec^6(x)}{6}$$

```
[In] Integrate[Sec[x]^4*Tan[x]^3,x]
[Out] -1/4*Sec[x]^4 + Sec[x]^6/6
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{(\sec^4(x))}{4}+\frac{(\sec^6(x))}{6}$	14
default	$-\frac{(\sec^4(x))}{4}+\frac{(\sec^6(x))}{6}$	14
risch	$-\frac{4(3\,e^{8ix}-2\,e^{6ix}+3\,e^{4ix})}{3(e^{2ix}+1)^6}$	34

```
[In] int(sec(x)^4*tan(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/4*sec(x)^4+1/6*sec(x)^6
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^4(x)\tan^3(x)\,dx = -\frac{3\,\cos\left(x\right)^2 - 2}{12\,\cos\left(x\right)^6}$$

```
[In] integrate(sec(x)^4*tan(x)^3,x, algorithm="fricas")
[Out] -1/12*(3*cos(x)^2 - 2)/cos(x)^6
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^4(x)\tan^3(x)\,dx = \frac{2 - 3\cos^2\left(x\right)}{12\cos^6\left(x\right)}$$

```
[In] integrate(sec(x)**4*tan(x)**3,x)
[Out] (2 - 3*cos(x)**2)/(12*cos(x)**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(13) = 26.

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \sec^4(x)\tan^3(x)\,dx = -\frac{3\,\sin\left(x\right)^2 - 1}{12\,\left(\sin\left(x\right)^6 - 3\,\sin\left(x\right)^4 + 3\,\sin\left(x\right)^2 - 1\right)}$$

```
[In] integrate(sec(x)^4*tan(x)^3,x, algorithm="maxima")
[Out] -1/12*(3*sin(x)^2 - 1)/(sin(x)^6 - 3*sin(x)^4 + 3*sin(x)^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^4(x)\tan^3(x)\,dx = -\frac{3\,\cos\left(x\right)^2 - 2}{12\,\cos\left(x\right)^6}$$

```
[In] integrate(sec(x)^4*tan(x)^3,x, algorithm="giac")
[Out] -1/12*(3*cos(x)^2 - 2)/cos(x)^6
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^4(x)\tan^3(x)\,dx = \frac{\tan(x)^4\left(2\tan(x)^2+3\right)}{12}$$

```
[In] int(tan(x)^3/cos(x)^4,x)
[Out] (tan(x)^4*(2*tan(x)^2 + 3))/12
```

3.272 $\int \frac{x}{2-2x+x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{x}{2-2x+x^2}\,dx = -\arctan(1-x) + \frac{1}{2}\log\left(2-2x+x^2\right)$$

```
[Out] arctan(-1+x)+1/2*ln(x^2-2*x+2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {648, 631, 210, 642}

$$\int \frac{x}{2-2x+x^2}\,dx = \frac{1}{2}\log\left(x^2-2x+2\right) - \arctan(1-x)$$

```
[In] Int[x/(2 - 2*x + x^2),x]
```

```
[Out] -ArcTan[1 - x] + Log[2 - 2*x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{-2+2x}{2-2x+x^2}\,dx + \int \frac{1}{2-2x+x^2}\,dx \\
&= \frac{1}{2}\log\left(2-2x+x^2\right) + \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-x\right) \\
&= -\arctan(1-x) + \frac{1}{2}\log\left(2-2x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{2-2x+x^2}\,dx = -\arctan(1-x) + \frac{1}{2}\log\left(2-2x+x^2\right)$$

```
[In] Integrate[x/(2 - 2*x + x^2),x]
[Out] -ArcTan[1 - x] + Log[2 - 2*x + x^2]/2
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
default	$\arctan\left(-1+x\right)+\frac{\ln\left(x^{2}-2x+2\right)}{2}$	17
risch	$\arctan\left(-1+x\right)+\frac{\ln\left(x^{2}-2x+2\right)}{2}$	17
parallelrisch	$\frac{\ln(x-1-i)}{2}-\frac{i\ln(x-1-i)}{2}+\frac{\ln(x-1+i)}{2}+\frac{i\ln(x-1+i)}{2}$	36

```
[In] int(x/(x^2-2*x+2),x,method=_RETURNVERBOSE)
[Out] arctan(-1+x)+1/2*ln(x^2-2*x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{x}{2-2x+x^2}\,dx=\arctan\left(x-1\right)+\frac{1}{2}\,\log\left(x^{2}-2\,x+2\right)$$

```
[In] integrate(x/(x^2-2*x+2),x, algorithm="fricas")
[Out] arctan(x - 1) + 1/2*log(x^2 - 2*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{x}{2-2x+x^2}\,dx=\frac{\log\left(x^{2}-2x+2\right)}{2}+\operatorname{atan}\left(x-1\right)$$

```
[In] integrate(x/(x**2-2*x+2),x)
[Out] log(x**2 - 2*x + 2)/2 + atan(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{x}{2-2x+x^2}\,dx=\arctan\left(x-1\right)+\frac{1}{2}\,\log\left(x^{2}-2\,x+2\right)$$

```
[In] integrate(x/(x^2-2*x+2),x, algorithm="maxima")
[Out] arctan(x - 1) + 1/2*log(x^2 - 2*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{x}{2-2x+x^2}\,dx = \arctan\left(x-1\right) + \frac{1}{2}\,\log\left(x^2-2\,x+2\right)$$

```
[In] integrate(x/(x^2-2*x+2),x, algorithm="giac")
[Out] arctan(x - 1) + 1/2*log(x^2 - 2*x + 2)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{x}{2-2x+x^2}\,dx = \mathrm{atan}(x-1) + \frac{\ln\left(x^2-2\,x+2\right)}{2}$$

```
[In] int(x/(x^2 - 2*x + 2),x)
[Out] atan(x - 1) + log(x^2 - 2*x + 2)/2
```

3.273 $\int x \arcsin(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 32

$$\int x \arcsin(x)\,dx = \frac{1}{4}x\sqrt{1-x^2} - \frac{\arcsin(x)}{4} + \frac{1}{2}x^2\arcsin(x)$$

```
[Out] -1/4*arcsin(x)+1/2*x^2*arcsin(x)+1/4*x*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4723, 327, 222}

$$\int x \arcsin(x)\,dx = \frac{1}{2}x^2\arcsin(x) - \frac{\arcsin(x)}{4} + \frac{1}{4}\sqrt{1-x^2}x$$

```
[In] Int[x*ArcSin[x],x]
```

```
[Out] (x*Sqrt[1 - x^2])/4 - ArcSin[x]/4 + (x^2*ArcSin[x])/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\arcsin(x) - \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}\,dx \\ &= \frac{1}{4}x\sqrt{1-x^2} + \frac{1}{2}x^2\arcsin(x) - \frac{1}{4}\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= \frac{1}{4}x\sqrt{1-x^2} - \frac{\arcsin(x)}{4} + \frac{1}{2}x^2\arcsin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int x\arcsin(x)\,dx = \frac{1}{4}\left(x\sqrt{1-x^2} + \left(-1+2x^2\right)\arcsin(x)\right)$$

```
[In] Integrate[x*ArcSin[x],x]
[Out] (x*Sqrt[1 - x^2] + (-1 + 2*x^2)*ArcSin[x])/4
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\arcsin(x)}{4}+\frac{x^2\arcsin(x)}{2}+\frac{x\sqrt{-x^2+1}}{4}$	25
parts	$-\frac{\arcsin(x)}{4}+\frac{x^2\arcsin(x)}{2}+\frac{x\sqrt{-x^2+1}}{4}$	25

```
[In] int(arcsin(x)*x,x,method=_RETURNVERBOSE)
[Out] -1/4*arcsin(x)+1/2*x^2*arcsin(x)+1/4*x*(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int x\arcsin(x)\,dx = \frac{1}{4}\left(2\,x^2-1\right)\arcsin\left(x\right)+\frac{1}{4}\,\sqrt{-x^2+1}x$$

```
[In] integrate(x*arcsin(x),x, algorithm="fricas")
[Out] 1/4*(2*x^2 - 1)*arcsin(x) + 1/4*sqrt(-x^2 + 1)*x
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int x\arcsin(x)\,dx = \frac{x^2\,\mathrm{asin}\left(x\right)}{2}+\frac{x\sqrt{1-x^2}}{4}-\frac{\mathrm{asin}\left(x\right)}{4}$$

```
[In] integrate(x*asin(x),x)
[Out] x**2*asin(x)/2 + x*sqrt(1 - x**2)/4 - asin(x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int x\arcsin(x)\,dx = \frac{1}{2}\,x^2\arcsin\left(x\right)+\frac{1}{4}\,\sqrt{-x^2+1}x-\frac{1}{4}\,\arcsin\left(x\right)$$

```
[In] integrate(x*arcsin(x),x, algorithm="maxima")
[Out] 1/2*x^2*arcsin(x) + 1/4*sqrt(-x^2 + 1)*x - 1/4*arcsin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x\arcsin(x)\,dx = \frac{1}{2}\left(x^2-1\right)\arcsin\left(x\right)+\frac{1}{4}\,\sqrt{-x^2+1}x+\frac{1}{4}\,\arcsin\left(x\right)$$

```
[In] integrate(x*arcsin(x),x, algorithm="giac")
[Out] 1/2*(x^2 - 1)*arcsin(x) + 1/4*sqrt(-x^2 + 1)*x + 1/4*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int x \arcsin(x) \, dx = \frac{x \sqrt{1-x^2}}{4} + \frac{\operatorname{asin}(x) \, (2 x^2 - 1)}{4}$$

```
[In] int(x*asin(x),x)
[Out] (x*(1 - x^2)^(1/2))/4 + (asin(x)*(2*x^2 - 1))/4
```

3.274 $\int \frac{\sqrt{9-x^2}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{\sqrt{9-x^2}}{x} \, dx = \sqrt{9-x^2} - 3\operatorname{arctanh}\left(\frac{\sqrt{9-x^2}}{3}\right)$$

```
[Out] -3*arctanh(1/3*(-x^2+9)^(1/2))+(-x^2+9)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 52, 65, 212}

$$\int \frac{\sqrt{9-x^2}}{x} \, dx = \sqrt{9-x^2} - 3\operatorname{arctanh}\left(\frac{\sqrt{9-x^2}}{3}\right)$$

```
[In] Int[Sqrt[9 - x^2]/x,x]
```

```
[Out] Sqrt[9 - x^2] - 3*ArcTanh[Sqrt[9 - x^2]/3]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{9-x}}{x}\,dx, x, x^2\right)\\
&= \sqrt{9-x^2}+\frac{9}{2}\text{Subst}\left(\int \frac{1}{\sqrt{9-x}x}\,dx, x, x^2\right)\\
&= \sqrt{9-x^2}-9\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \sqrt{9-x^2}\right)\\
&= \sqrt{9-x^2}-3\text{arctanh}\left(\frac{\sqrt{9-x^2}}{3}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{9-x^2}}{x}\,dx = \sqrt{9-x^2}-3\text{arctanh}\left(\frac{\sqrt{9-x^2}}{3}\right)$$

```
[In] Integrate[Sqrt[9 - x^2]/x,x]
[Out] Sqrt[9 - x^2] - 3*ArcTanh[Sqrt[9 - x^2]/3]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\sqrt{-x^2+9}-3\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-x^2+9}}\right)$	25
trager	$\sqrt{-x^2+9}-3\ln\left(\frac{\sqrt{-x^2+9}+3}{x}\right)$	29
pseudoelliptic	$\sqrt{-x^2+9}-\frac{3\ln\left(\sqrt{-x^2+9}+3\right)}{2}+\frac{3\ln\left(\sqrt{-x^2+9}-3\right)}{2}$	39
meijerg	$-\frac{3\left(-2(2-2\ln(2)+2\ln(x)-2\ln(3)+i\pi)\sqrt{\pi}+4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{-\frac{x^2}{9}+1}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{x^2}{9}+1}}{2}\right)\right)}{4\sqrt{\pi}}$	68

```
[In] int((-x^2+9)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] (-x^2+9)^(1/2)-3*arctanh(3/(-x^2+9)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{9-x^2}}{x}\,dx = \sqrt{-x^2+9}+3\,\log\left(\frac{\sqrt{-x^2+9}-3}{x}\right)$$

```
[In] integrate((-x^2+9)^(1/2)/x,x, algorithm="fricas")
[Out] sqrt(-x^2 + 9) + 3*log((sqrt(-x^2 + 9) - 3)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.20

$$\int \frac{\sqrt{9-x^2}}{x}\,dx = \begin{cases} i\sqrt{x^2-9}-3\log(x)+\frac{3\log(x^2)}{2}+3i\,\mathrm{asin}\left(\frac{3}{x}\right) & \text{for } |x^2|>9 \\ \sqrt{9-x^2}+\frac{3\log(x^2)}{2}-3\log\left(\sqrt{1-\frac{x^2}{9}}+1\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((-x**2+9)**(1/2)/x,x)
[Out] Piecewise((I*sqrt(x**2 - 9) - 3*log(x) + 3*log(x**2)/2 + 3*I*asin(3/x), Abs
(x**2) > 9), (sqrt(9 - x**2) + 3*log(x**2)/2 - 3*log(sqrt(1 - x**2/9) + 1),
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{9-x^2}}{x}\,dx = \sqrt{-x^2+9} - 3\,\log\left(\frac{6\sqrt{-x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

```
[In] integrate((-x^2+9)^(1/2)/x,x, algorithm="maxima")
[Out] sqrt(-x^2 + 9) - 3*log(6*sqrt(-x^2 + 9)/abs(x) + 18/abs(x))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{9-x^2}}{x}\,dx = \sqrt{-x^2+9} - \frac{3}{2}\,\log\left(\sqrt{-x^2+9}+3\right) + \frac{3}{2}\,\log\left(-\sqrt{-x^2+9}+3\right)$$

```
[In] integrate((-x^2+9)^(1/2)/x,x, algorithm="giac")
[Out] sqrt(-x^2 + 9) - 3/2*log(sqrt(-x^2 + 9) + 3) + 3/2*log(-sqrt(-x^2 + 9) + 3)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{9-x^2}}{x}\,dx = 3\,\ln\left(\sqrt{\frac{9}{x^2}-1} - 3\sqrt{\frac{1}{x^2}}\right) + \sqrt{9-x^2}$$

```
[In] int((9 - x^2)^(1/2)/x,x)
[Out] 3*log((9/x^2 - 1)^(1/2) - 3*(1/x^2)^(1/2)) + (9 - x^2)^(1/2)
```

3.275 $\int \frac{x}{2+3x+x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{x}{2+3x+x^2} dx = -\log(1+x) + 2\log(2+x)$$

```
[Out] -ln(1+x)+2*ln(2+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {646, 31}

$$\int \frac{x}{2+3x+x^2} dx = 2\log(x+2) - \log(x+1)$$

```
[In] Int[x/(2 + 3*x + x^2),x]
```

```
[Out] -Log[1 + x] + 2*Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{1}{2+x}\,dx - \int \frac{1}{1+x}\,dx\\ &= -\log(1+x) + 2\log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x}{2+3x+x^2}\,dx = -\log(1+x) + 2\log(2+x)$$

```
[In] Integrate[x/(2 + 3*x + x^2),x]
[Out] -Log[1 + x] + 2*Log[2 + x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-\ln(1+x)+2\ln(2+x)$	14
norman	$-\ln(1+x)+2\ln(2+x)$	14
risch	$-\ln(1+x)+2\ln(2+x)$	14
parallelrisch	$-\ln(1+x)+2\ln(2+x)$	14

```
[In] int(x/(x^2+3*x+2),x,method=_RETURNVERBOSE)
[Out] -ln(1+x)+2*ln(2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x}{2+3x+x^2}\,dx = 2\,\log\left(x+2\right) - \log\left(x+1\right)$$

```
[In] integrate(x/(x^2+3*x+2),x, algorithm="fricas")
[Out] 2*log(x + 2) - log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{x}{2+3x+x^2}\,dx = -\log\left(x+1\right) + 2\log\left(x+2\right)$$

```
[In] integrate(x/(x**2+3*x+2),x)
[Out] -log(x + 1) + 2*log(x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x}{2+3x+x^2}\,dx = 2\,\log\left(x+2\right) - \log\left(x+1\right)$$

```
[In] integrate(x/(x^2+3*x+2),x, algorithm="maxima")
[Out] 2*log(x + 2) - log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{x}{2+3x+x^2}\,dx = 2\,\log\left(|x+2|\right) - \log\left(|x+1|\right)$$

```
[In] integrate(x/(x^2+3*x+2),x, algorithm="giac")
[Out] 2*log(abs(x + 2)) - log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x}{2+3x+x^2}\,dx = 2\,\ln\left(x+2\right)-\ln\left(x+1\right)$$

```
[In] int(x/(3*x + x^2 + 2),x)
[Out] 2*log(x + 2) - log(x + 1)
```

3.276 $\int x^2 \cosh(x) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 16

$$\int x^2 \cosh(x) \, dx = -2x \cosh(x) + 2 \sinh(x) + x^2 \sinh(x)$$

```
[Out] -2*x*cosh(x)+2*sinh(x)+x^2*sinh(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3377, 2717}

$$\int x^2 \cosh(x) \, dx = x^2 \sinh(x) + 2 \sinh(x) - 2x \cosh(x)$$

```
[In] Int[x^2*Cosh[x],x]
```

```
[Out] -2*x*Cosh[x] + 2*Sinh[x] + x^2*Sinh[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x^2\sinh(x) - 2\int x\sinh(x)\,dx\\ &= -2x\cosh(x) + x^2\sinh(x) + 2\int \cosh(x)\,dx\\ &= -2x\cosh(x) + 2\sinh(x) + x^2\sinh(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2\cosh(x)\,dx = -2x\cosh(x) + (2+x^2)\sinh(x)$$

```
[In] Integrate[x^2*Cosh[x],x]
[Out] -2*x*Cosh[x] + (2 + x^2)*Sinh[x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$-2x\cosh(x)+2\sinh(x)+x^2\sinh(x)$	17
parallelrisch	$-2x\cosh(x)+2\sinh(x)+x^2\sinh(x)$	17
parts	$-2x\cosh(x)+2\sinh(x)+x^2\sinh(x)$	17
risch	$\left(1-x+\frac{1}{2}x^2\right)e^x+\left(-1-x-\frac{1}{2}x^2\right)e^{-x}$	30
meijerg	$4i\sqrt{\pi}\left(\frac{ix\cosh(x)}{2\sqrt{\pi}}-\frac{i\left(\frac{3x^2}{2}+3\right)\sinh(x)}{6\sqrt{\pi}}\right)$	32

```
[In] int(x^2*cosh(x),x,method=_RETURNVERBOSE)
[Out] -2*x*cosh(x)+2*sinh(x)+x^2*sinh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 \cosh(x)\, dx = -2\, x \cosh\left(x\right) + \left(x^2 + 2\right) \sinh\left(x\right)$$

```
[In] integrate(x^2*cosh(x),x, algorithm="fricas")
[Out] -2*x*cosh(x) + (x^2 + 2)*sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int x^2 \cosh(x)\, dx = x^2 \sinh\left(x\right) - 2x \cosh\left(x\right) + 2 \sinh\left(x\right)$$

```
[In] integrate(x**2*cosh(x),x)
[Out] x**2*sinh(x) - 2*x*cosh(x) + 2*sinh(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(16) = 32.

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int x^2 \cosh(x)\, dx = \frac{1}{3}\, x^3 \cosh\left(x\right) - \frac{1}{6}\, \left(x^3 + 3\, x^2 + 6\, x + 6\right) e^{(-x)} - \frac{1}{6}\, \left(x^3 - 3\, x^2 + 6\, x - 6\right) e^x$$

```
[In] integrate(x^2*cosh(x),x, algorithm="maxima")
[Out] 1/3*x^3*cosh(x) - 1/6*(x^3 + 3*x^2 + 6*x + 6)*e^(-x) - 1/6*(x^3 - 3*x^2 + 6
*x - 6)*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int x^2 \cosh(x)\, dx = -\frac{1}{2}\, \left(x^2 + 2\, x + 2\right) e^{(-x)} + \frac{1}{2}\, \left(x^2 - 2\, x + 2\right) e^x$$

```
[In] integrate(x^2*cosh(x),x, algorithm="giac")
[Out] -1/2*(x^2 + 2*x + 2)*e^(-x) + 1/2*(x^2 - 2*x + 2)*e^x
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^2 \cosh(x)\, dx = 2\sinh(x) + x^2 \sinh(x) - 2\,x\cosh(x)$$

```
[In] int(x^2*cosh(x),x)
[Out] 2*sinh(x) + x^2*sinh(x) - 2*x*cosh(x)
```

3.277 $\int \frac{1+x+x^3}{4x+2x^2+x^4} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 17

$$\int \frac{1+x+x^3}{4x+2x^2+x^4} \, dx = \frac{1}{4}\log\left(4x+2x^2+x^4\right)$$

```
[Out] 1/4*ln(x^4+2*x^2+4*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1601}

$$\int \frac{1+x+x^3}{4x+2x^2+x^4} \, dx = \frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

```
[In] Int[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] Log[4*x + 2*x^2 + x^4]/4
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\log\left(4x+2x^2+x^4\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx=\frac{\log(x)}{4}+\frac{1}{4}\log\left(4+2x+x^3\right)$$

```
[In] Integrate[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
[Out] Log[x]/4 + Log[4 + 2*x + x^3]/4
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\ln\left(x\left(x^{3}+2x+4\right)\right)}{4}$	14
risch	$\frac{\ln\left(x^{4}+2x^{2}+4x\right)}{4}$	16
norman	$\frac{\ln(x)}{4}+\frac{\ln\left(x^{3}+2x+4\right)}{4}$	17
parallelrisch	$\frac{\ln(x)}{4}+\frac{\ln\left(x^{3}+2x+4\right)}{4}$	17

```
[In] int((x^3+x+1)/(x^4+2*x^2+4*x),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(x*(x^3+2*x+4))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx=\frac{1}{4}\log\left(x^4+2\,x^2+4\,x\right)$$

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="fricas")
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx=\frac{\log\left(x^4+2x^2+4x\right)}{4}$$

```
[In] integrate((x**3+x+1)/(x**4+2*x**2+4*x),x)
[Out] log(x**4 + 2*x**2 + 4*x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx=\frac{1}{4}\,\log\left(x^4+2\,x^2+4\,x\right)$$

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="maxima")
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx=\frac{1}{4}\,\log\left(4\left|\frac{1}{4}\,x^4+\frac{1}{2}\,x^2+x\right|\right)$$

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="giac")
[Out] 1/4*log(4*abs(1/4*x^4 + 1/2*x^2 + x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx=\frac{\ln\left(x\,\left(x^3+2\,x+4\right)\right)}{4}$$

```
[In] int((x + x^3 + 1)/(4*x + 2*x^2 + x^4),x)
[Out] log(x*(2*x + x^3 + 4))/4
```

3.278 $\int \frac{\cos(x)}{1+\sin^2(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 3

$$\int \frac{\cos(x)}{1+\sin^2(x)} \, dx = \arctan(\sin(x))$$

```
[Out] arctan(sin(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3269, 209}

$$\int \frac{\cos(x)}{1+\sin^2(x)} \, dx = \arctan(\sin(x))$$

```
[In] Int[Cos[x]/(1 + Sin[x]^2),x]
```

```
[Out] ArcTan[Sin[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
```

```
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin(x)\right)$$
$$= \arctan(\sin(x))$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{1+\sin^2(x)}\,dx = \arctan(\sin(x))$$

```
[In] Integrate[Cos[x]/(1 + Sin[x]^2),x]
[Out] ArcTan[Sin[x]]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arctan(\sin(x))$	4
default	$\arctan(\sin(x))$	4
parallelrisch	$-\frac{i\left(-\ln\left(-\frac{2i(\sin(x)+i)}{\cos(x)+1}\right)+\ln\left(\frac{2+2i\sin(x)}{\cos(x)+1}\right)\right)}{2}$	37
risch	$\frac{i\ln(e^{2ix}-2\,e^{ix}-1)}{2}-\frac{i\ln(e^{2ix}+2\,e^{ix}-1)}{2}$	38

```
[In] int(cos(x)/(1+sin(x)^2),x,method=_RETURNVERBOSE)
[Out] arctan(sin(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{1+\sin^2(x)}\,dx = \arctan(\sin(x))$$

```
[In] integrate(cos(x)/(1+sin(x)^2),x, algorithm="fricas")
[Out] arctan(sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{1+\sin^2(x)}\,dx = \operatorname{atan}(\sin(x))$$

```
[In] integrate(cos(x)/(1+sin(x)**2),x)
[Out] atan(sin(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{1+\sin^2(x)}\,dx = \arctan(\sin(x))$$

```
[In] integrate(cos(x)/(1+sin(x)^2),x, algorithm="maxima")
[Out] arctan(sin(x))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{1+\sin^2(x)}\,dx = \arctan(\sin(x))$$

```
[In] integrate(cos(x)/(1+sin(x)^2),x, algorithm="giac")
[Out] arctan(sin(x))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{1+\sin^2(x)}\,dx = \operatorname{atan}(\sin(x))$$

```
[In] int(cos(x)/(sin(x)^2 + 1),x)
[Out] atan(sin(x))
```

3.279 $\int \cos(\sqrt{x})\, dx$

Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \cos(\sqrt{x})\, dx = 2\cos(\sqrt{x}) + 2\sqrt{x}\sin(\sqrt{x})$$

```
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3443, 3377, 2718}

$$\int \cos(\sqrt{x})\, dx = 2\sqrt{x}\sin(\sqrt{x}) + 2\cos(\sqrt{x})$$

```
[In] Int[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3443

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int x\cos(x)\,dx, x, \sqrt{x}\right)\\ &= 2\sqrt{x}\sin\left(\sqrt{x}\right) - 2\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{x}\right)\\ &= 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\sin\left(\sqrt{x}\right)$$

```
[In] Integrate[Cos[Sqrt[x]],x]
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$2\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\sqrt{x}$	17
default	$2\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\sqrt{x}$	17
meijerg	$4\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(\sqrt{x})}{2\sqrt{\pi}}+\frac{\sqrt{x}\,\sin(\sqrt{x})}{2\sqrt{\pi}}\right)$	33

```
[In] int(cos(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\,\sqrt{x}\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x**(1/2)),x)
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\,\sqrt{x}\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\,\sqrt{x}\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos \left(\sqrt{x}\right) \, dx = 2 \cos \left(\sqrt{x}\right) + 2 \sqrt{x} \sin \left(\sqrt{x}\right)$$

```
[In] int(cos(x^(1/2)),x)
[Out] 2*cos(x^(1/2)) + 2*x^(1/2)*sin(x^(1/2))
```

3.280 $\int \sin(\pi x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 9

$$\int \sin(\pi x)\,dx = -\frac{\cos(\pi x)}{\pi}$$

```
[Out] -cos(Pi*x)/Pi
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2718}

$$\int \sin(\pi x)\,dx = -\frac{\cos(\pi x)}{\pi}$$

```
[In] Int[Sin[Pi*x],x]
```

```
[Out] -(Cos[Pi*x]/Pi)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{\cos(\pi x)}{\pi}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \sin(\pi x)\,dx = -\frac{\cos(\pi x)}{\pi}$$

```
[In] Integrate[Sin[Pi*x],x]
[Out] -(Cos[Pi*x]/Pi)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$-\frac{\cos(\pi x)}{\pi}$	10
default	$-\frac{\cos(\pi x)}{\pi}$	10
risch	$-\frac{\cos(\pi x)}{\pi}$	10
parallelrisch	$\frac{-\cos(\pi x)-1}{\pi}$	13
norman	$-\frac{2}{\pi\left(1+\tan^{2}\left(\frac{\pi x}{2}\right)\right)}$	17
meijerg	$\frac{\frac{1}{\sqrt{\pi}}-\frac{\cos(\pi x)}{\sqrt{\pi}}}{\sqrt{\pi}}$	18

```
[In] int(sin(Pi*x),x,method=_RETURNVERBOSE)
[Out] -cos(Pi*x)/Pi
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \sin(\pi x)\,dx = -\frac{\cos\left(\pi x\right)}{\pi}$$

```
[In] integrate(sin(pi*x),x, algorithm="fricas")
[Out] -cos(pi*x)/pi
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \sin(\pi x)\, dx = -\frac{\cos(\pi x)}{\pi}$$

```
[In] integrate(sin(pi*x),x)
[Out] -cos(pi*x)/pi
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \sin(\pi x)\, dx = -\frac{\cos(\pi x)}{\pi}$$

```
[In] integrate(sin(pi*x),x, algorithm="maxima")
[Out] -cos(pi*x)/pi
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \sin(\pi x)\, dx = -\frac{\cos(\pi x)}{\pi}$$

```
[In] integrate(sin(pi*x),x, algorithm="giac")
[Out] -cos(pi*x)/pi
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \sin(\pi x)\, dx = -\frac{\cos(\Pi\, x)}{\Pi}$$

```
[In] int(sin(Pi*x),x)
[Out] -cos(Pi*x)/Pi
```

3.281 $\int \frac{e^{2x}}{1+e^x} dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{e^{2x}}{1+e^x} \, dx = e^x - \log(1+e^x)$$

```
[Out] exp(x)-ln(1+exp(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2280, 45}

$$\int \frac{e^{2x}}{1+e^x} \, dx = e^x - \log(e^x+1)$$

```
[In] Int[E^(2*x)/(1 + E^x),x]
```

```
[Out] E^x - Log[1 + E^x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2280

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
```

```
[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Den
ominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{x}{1+x}\,dx, x, e^x\right)\\ &= \text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx, x, e^x\right)\\ &= e^x - \log(1+e^x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log(1+e^x)$$

```
[In] Integrate[E^(2*x)/(1 + E^x),x]
[Out] E^x - Log[1 + E^x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$e^x - \ln(1+e^x)$	11
norman	$e^x - \ln(1+e^x)$	11
risch	$e^x - \ln(1+e^x)$	11

```
[In] int(exp(2*x)/(1+exp(x)),x,method=_RETURNVERBOSE)
[Out] exp(x)-ln(1+exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x + 1\right)$$

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="fricas")
[Out] e^x - log(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x + 1\right)$$

```
[In] integrate(exp(2*x)/(1+exp(x)),x)
[Out] exp(x) - log(exp(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x + 1\right)$$

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")
[Out] e^x - log(e^x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x + 1\right)$$

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="giac")
[Out] e^x - log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = \mathrm{e}^x - \ln\left(\mathrm{e}^x + 1\right)$$

```
[In] int(exp(2*x)/(exp(x) + 1),x)
[Out] exp(x) - log(exp(x) + 1)
```

3.282 $\int e^{3x}\cos(5x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{3x}\cos(5x)\,dx=\frac{3}{34}e^{3x}\cos(5x)+\frac{5}{34}e^{3x}\sin(5x)$$

```
[Out] 3/34*exp(3*x)*cos(5*x)+5/34*exp(3*x)*sin(5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4518}

$$\int e^{3x}\cos(5x)\,dx=\frac{5}{34}e^{3x}\sin(5x)+\frac{3}{34}e^{3x}\cos(5x)$$

```
[In] Int[E^(3*x)*Cos[5*x],x]
```

```
[Out] (3*E^(3*x)*Cos[5*x])/34 + (5*E^(3*x)*Sin[5*x])/34
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral}=\frac{3}{34}e^{3x}\cos(5x)+\frac{5}{34}e^{3x}\sin(5x)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{3x}\cos(5x)\,dx=\frac{1}{34}e^{3x}(3\cos(5x)+5\sin(5x))$$

```
[In] Integrate[E^(3*x)*Cos[5*x],x]
[Out] (E^(3*x)*(3*Cos[5*x] + 5*Sin[5*x]))/34
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{e^{3x}(3\cos(5x)+5\sin(5x))}{34}$	20
default	$\frac{3\,e^{3x}\cos(5x)}{34}+\frac{5\,e^{3x}\sin(5x)}{34}$	22
risch	$\frac{3\,e^{(3+5i)x}}{68}-\frac{5ie^{(3+5i)x}}{68}+\frac{3\,e^{(3-5i)x}}{68}+\frac{5ie^{(3-5i)x}}{68}$	36
norman	$\frac{\frac{5\,e^{3x}\tan\left(\frac{5x}{2}\right)}{17}-\frac{3\,e^{3x}\left(\tan^2\left(\frac{5x}{2}\right)\right)}{34}+\frac{3\,e^{3x}}{34}}{1+\tan^2\left(\frac{5x}{2}\right)}$	41

```
[In] int(exp(3*x)*cos(5*x),x,method=_RETURNVERBOSE)
[Out] 1/34*exp(3*x)*(3*cos(5*x)+5*sin(5*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{3x}\cos(5x)\,dx=\frac{3}{34}\,\cos\left(5\,x\right)e^{(3\,x)}+\frac{5}{34}\,e^{(3\,x)}\sin\left(5\,x\right)$$

```
[In] integrate(exp(3*x)*cos(5*x),x, algorithm="fricas")
[Out] 3/34*cos(5*x)*e^(3*x) + 5/34*e^(3*x)*sin(5*x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{3x}\cos(5x)\,dx = \frac{5e^{3x}\sin{(5x)}}{34} + \frac{3e^{3x}\cos{(5x)}}{34}$$

```
[In] integrate(exp(3*x)*cos(5*x),x)
[Out] 5*exp(3*x)*sin(5*x)/34 + 3*exp(3*x)*cos(5*x)/34
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{3x}\cos(5x)\,dx = \frac{1}{34}\,(3\,\cos{(5\,x)} + 5\,\sin{(5\,x)})e^{(3\,x)}$$

```
[In] integrate(exp(3*x)*cos(5*x),x, algorithm="maxima")
[Out] 1/34*(3*cos(5*x) + 5*sin(5*x))*e^(3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{3x}\cos(5x)\,dx = \frac{1}{34}\,(3\,\cos{(5\,x)} + 5\,\sin{(5\,x)})e^{(3\,x)}$$

```
[In] integrate(exp(3*x)*cos(5*x),x, algorithm="giac")
[Out] 1/34*(3*cos(5*x) + 5*sin(5*x))*e^(3*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{3x}\cos(5x)\,dx = \frac{\mathrm{e}^{3\,x}\,(3\,\cos{(5\,x)} + 5\,\sin{(5\,x)})}{34}$$

```
[In] int(cos(5*x)*exp(3*x),x)
[Out] (exp(3*x)*(3*cos(5*x) + 5*sin(5*x)))/34
```

3.283 $\int \cos(3x)\cos(5x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \cos(3x)\cos(5x)\,dx = \frac{1}{4}\sin(2x) + \frac{1}{16}\sin(8x)$$

```
[Out] 1/4*sin(2*x)+1/16*sin(8*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{4368\}$

$$\int \cos(3x)\cos(5x)\,dx = \frac{1}{4}\sin(2x) + \frac{1}{16}\sin(8x)$$

```
[In] Int[Cos[3*x]*Cos[5*x],x]
```

```
[Out] Sin[2*x]/4 + Sin[8*x]/16
```

Rule 4368

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\sin(2x) + \frac{1}{16}\sin(8x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(3x)\cos(5x)\,dx = \frac{1}{4}\sin(2x) + \frac{1}{16}\sin(8x)$$

```
[In] Integrate[Cos[3*x]*Cos[5*x],x]
[Out] Sin[2*x]/4 + Sin[8*x]/16
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sin(2x)}{4}+\frac{\sin(8x)}{16}$	14
risch	$\frac{\sin(2x)}{4}+\frac{\sin(8x)}{16}$	14
parallelrisch	$\frac{\sin(2x)}{4}+\frac{\sin(8x)}{16}$	14
norman	$\frac{\frac{3\tan\left(\frac{3x}{2}\right)\left(\tan^2\left(\frac{5x}{2}\right)\right)}{8}-\frac{5\left(\tan^2\left(\frac{3x}{2}\right)\right)\tan\left(\frac{5x}{2}\right)}{8}-\frac{3\tan\left(\frac{3x}{2}\right)}{8}+\frac{5\tan\left(\frac{5x}{2}\right)}{8}}{\left(1+\tan^2\left(\frac{3x}{2}\right)\right)\left(1+\tan^2\left(\frac{5x}{2}\right)\right)}$	59

```
[In] int(cos(3*x)*cos(5*x),x,method=_RETURNVERBOSE)
[Out] 1/4*sin(2*x)+1/16*sin(8*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \cos(3x)\cos(5x)\,dx = \left(8\,\cos\left(x\right)^{7} - 12\,\cos\left(x\right)^{5} + 5\,\cos\left(x\right)^{3}\right)\sin\left(x\right)$$

```
[In] integrate(cos(3*x)*cos(5*x),x, algorithm="fricas")
[Out] (8*cos(x)^7 - 12*cos(x)^5 + 5*cos(x)^3)*sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \cos(3x)\cos(5x)\,dx = -\frac{3\sin{(3x)}\cos{(5x)}}{16} + \frac{5\sin{(5x)}\cos{(3x)}}{16}$$

```
[In] integrate(cos(3*x)*cos(5*x),x)
[Out] -3*sin(3*x)*cos(5*x)/16 + 5*sin(5*x)*cos(3*x)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(3x)\cos(5x)\,dx = \frac{1}{16}\sin{(8x)} + \frac{1}{4}\sin{(2x)}$$

```
[In] integrate(cos(3*x)*cos(5*x),x, algorithm="maxima")
[Out] 1/16*sin(8*x) + 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(3x)\cos(5x)\,dx = \frac{1}{16}\sin{(8x)} + \frac{1}{4}\sin{(2x)}$$

```
[In] integrate(cos(3*x)*cos(5*x),x, algorithm="giac")
[Out] 1/16*sin(8*x) + 1/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(3x)\cos(5x)\,dx = \frac{\sin{(2x)}}{4} + \frac{\sin{(8x)}}{16}$$

```
[In] int(cos(3*x)*cos(5*x),x)
[Out] sin(2*x)/4 + sin(8*x)/16
```

3.284 $\int \frac{1}{1+x+x^2+x^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{1}{1+x+x^2+x^3}\,dx=\frac{\arctan(x)}{2}+\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)$$

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2083, 649, 209, 266}

$$\int \frac{1}{1+x+x^2+x^3}\,dx=\frac{\arctan(x)}{2}-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)$$

```
[In] Int[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2(1+x)}+\frac{1-x}{2\left(1+x^2\right)}\right) dx \\
&= \frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1-x}{1+x^2}\, dx \\
&= \frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1}{1+x^2}\, dx-\frac{1}{2}\int \frac{x}{1+x^2}\, dx \\
&= \frac{\arctan(x)}{2}+\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x+x^2+x^3}\, dx=\frac{\arctan(x)}{2}+\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)$$

```
[In] Integrate[(1 + x + x^2 + x^3)^(-1),x]
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\arctan(x)}{2}+\frac{\ln(1+x)}{2}-\frac{\ln(x^2+1)}{4}$	20
risch	$\frac{\arctan(x)}{2}+\frac{\ln(1+x)}{2}-\frac{\ln(x^2+1)}{4}$	20
parallelrisch	$\frac{\ln(1+x)}{2}-\frac{\ln(x-i)}{4}-\frac{i\ln(x-i)}{4}-\frac{\ln(x+i)}{4}+\frac{i\ln(x+i)}{4}$	38

```
[In] int(1/(x^3+x^2+x+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{1}{2}\,\arctan\left(x\right) - \frac{1}{4}\,\log\left(x^2+1\right) + \frac{1}{2}\,\log\left(x+1\right)$$

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{\log\left(x+1\right)}{2} - \frac{\log\left(x^2+1\right)}{4} + \frac{\operatorname{atan}\left(x\right)}{2}$$

```
[In] integrate(1/(x**3+x**2+x+1),x)
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{1}{2}\,\arctan\left(x\right) - \frac{1}{4}\,\log\left(x^2+1\right) + \frac{1}{2}\,\log\left(x+1\right)$$

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="maxima")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log\left(|x+1|\right)$$

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="giac")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{\ln(x+1)}{2} + \ln(x-\mathrm{i})\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

```
[In] int(1/(x + x^2 + x^3 + 1),x)
[Out] log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)
```

3.285 $\int x^2 \log(1+x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 39

$$\int x^2 \log(1+x) \, dx = -\frac{x}{3} + \frac{x^2}{6} - \frac{x^3}{9} + \frac{1}{3}\log(1+x) + \frac{1}{3}x^3\log(1+x)$$

```
[Out] -1/3*x+1/6*x^2-1/9*x^3+1/3*ln(1+x)+1/3*x^3*ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2442, 45}

$$\int x^2 \log(1+x) \, dx = -\frac{x^3}{9} + \frac{1}{3}x^3\log(x+1) + \frac{x^2}{6} - \frac{x}{3} + \frac{1}{3}\log(x+1)$$

```
[In] Int[x^2*Log[1 + x],x]
```

```
[Out] -1/3*x + x^2/6 - x^3/9 + Log[1 + x]/3 + (x^3*Log[1 + x])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{3}x^3\log(1+x) - \frac{1}{3}\int \frac{x^3}{1+x}\,dx \\ &= \frac{1}{3}x^3\log(1+x) - \frac{1}{3}\int \left(1 + \frac{1}{-1-x} - x + x^2\right)dx \\ &= -\frac{x}{3} + \frac{x^2}{6} - \frac{x^3}{9} + \frac{1}{3}\log(1+x) + \frac{1}{3}x^3\log(1+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int x^2\log(1+x)\,dx = \frac{1}{18}\left(x\left(-6+3x-2x^2\right)+6\left(1+x^3\right)\log(1+x)\right)$$

```
[In] Integrate[x^2*Log[1 + x],x]
[Out] (x*(-6 + 3*x - 2*x^2) + 6*(1 + x^3)*Log[1 + x])/18
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
meijerg	$-\frac{x(4x^2-6x+12)}{36}+\frac{(4x^3+4)\ln(1+x)}{12}$	28
norman	$-\frac{x}{3}+\frac{x^2}{6}-\frac{x^3}{9}+\frac{\ln(1+x)}{3}+\frac{\ln(1+x)x^3}{3}$	30
risch	$-\frac{x}{3}+\frac{x^2}{6}-\frac{x^3}{9}+\frac{\ln(1+x)}{3}+\frac{\ln(1+x)x^3}{3}$	30
parts	$-\frac{x}{3}+\frac{x^2}{6}-\frac{x^3}{9}+\frac{\ln(1+x)}{3}+\frac{\ln(1+x)x^3}{3}$	30
parallelrisch	$\frac{\ln(1+x)x^3}{3}-\frac{x^3}{9}+\frac{x^2}{6}-\frac{x}{3}+\frac{\ln(1+x)}{3}+\frac{1}{3}$	31
derivativedivides	$\frac{(1+x)^3\ln(1+x)}{3}-\frac{(1+x)^3}{9}-\ln(1+x)(1+x)^2+\frac{(1+x)^2}{2}+(1+x)\ln(1+x)-1-x$	50
default	$\frac{(1+x)^3\ln(1+x)}{3}-\frac{(1+x)^3}{9}-\ln(1+x)(1+x)^2+\frac{(1+x)^2}{2}+(1+x)\ln(1+x)-1-x$	50

```
[In] int(ln(1+x)*x^2,x,method=_RETURNVERBOSE)
[Out] -1/36*x*(4*x^2-6*x+12)+1/12*(4*x^3+4)*ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int x^2 \log(1+x)\,dx = -\frac{1}{9}\,x^3 + \frac{1}{6}\,x^2 + \frac{1}{3}\,\left(x^3+1\right)\log\left(x+1\right) - \frac{1}{3}\,x$$

```
[In] integrate(x^2*log(1+x),x, algorithm="fricas")
[Out] -1/9*x^3 + 1/6*x^2 + 1/3*(x^3 + 1)*log(x + 1) - 1/3*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^2 \log(1+x)\,dx = \frac{x^3 \log\left(x+1\right)}{3} - \frac{x^3}{9} + \frac{x^2}{6} - \frac{x}{3} + \frac{\log\left(x+1\right)}{3}$$

```
[In] integrate(x**2*ln(1+x),x)
[Out] x**3*log(x + 1)/3 - x**3/9 + x**2/6 - x/3 + log(x + 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^2 \log(1+x)\,dx = \frac{1}{3}\,x^3 \log\left(x+1\right) - \frac{1}{9}\,x^3 + \frac{1}{6}\,x^2 - \frac{1}{3}\,x + \frac{1}{3}\,\log\left(x+1\right)$$

```
[In] integrate(x^2*log(1+x),x, algorithm="maxima")
[Out] 1/3*x^3*log(x + 1) - 1/9*x^3 + 1/6*x^2 - 1/3*x + 1/3*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int x^2 \log(1+x)\,dx = \frac{1}{3}\,(x+1)^3 \log\left(x+1\right) - \frac{1}{9}\,(x+1)^3 - (x+1)^2 \log\left(x+1\right) + \frac{1}{2}\,(x+1)^2 + (x+1)\log\left(x+1\right) - x - 1$$

```
[In] integrate(x^2*log(1+x),x, algorithm="giac")
[Out] 1/3*(x + 1)^3*log(x + 1) - 1/9*(x + 1)^3 - (x + 1)^2*log(x + 1) + 1/2*(x +
1)^2 + (x + 1)*log(x + 1) - x - 1
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int x^2 \log(1+x)\, dx = \frac{x^2}{6} - \frac{x}{3} - \frac{x^3}{9} + \frac{\ln(x+1)\ (x^3+1)}{3}$$

```
[In] int(x^2*log(x + 1),x)
```

```
[Out] x^2/6 - x/3 - x^3/9 + (log(x + 1)*(x^3 + 1))/3
```

3.286 $\int e^{-x^3} x^5 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int e^{-x^3} x^5 \, dx = -\frac{e^{-x^3}}{3} - \frac{1}{3} e^{-x^3} x^3$$

```
[Out] -1/3/exp(x^3)-1/3*x^3/exp(x^3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2243, 2240}

$$\int e^{-x^3} x^5 \, dx = -\frac{1}{3} e^{-x^3} x^3 - \frac{e^{-x^3}}{3}$$

```
[In] Int[x^5/E^x^3,x]
```

```
[Out] -1/3*1/E^x^3 - x^3/(3*E^x^3)
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
```

```
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{1}{3}e^{-x^3}x^3 + \int e^{-x^3}x^2\,dx \\ &= -\frac{e^{-x^3}}{3} - \frac{1}{3}e^{-x^3}x^3 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int e^{-x^3}x^5\,dx = -\frac{1}{3}e^{-x^3}(1+x^3)$$

```
[In] Integrate[x^5/E^x^3,x]
[Out] -1/3*(1 + x^3)/E^x^3
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{(x^3+1)\,e^{-x^3}}{3}$	14
norman	$\left(-\frac{x^3}{3}-\frac{1}{3}\right)e^{-x^3}$	15
risch	$\left(-\frac{x^3}{3}-\frac{1}{3}\right)e^{-x^3}$	15
parallelrisch	$\frac{(-x^3-1)\,e^{-x^3}}{3}$	16
meijerg	$\frac{1}{3}-\frac{(2x^3+2)\,e^{-x^3}}{6}$	18
derivativedivides	$-\frac{e^{-x^3}}{3}-\frac{x^3 e^{-x^3}}{3}$	21
default	$-\frac{e^{-x^3}}{3}-\frac{x^3 e^{-x^3}}{3}$	21

```
[In] int(x^5/exp(x^3),x,method=_RETURNVERBOSE)
[Out] -1/3*(x^3+1)/exp(x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int e^{-x^3} x^5 \, dx = -\frac{1}{3} \left(x^3 + 1\right) e^{\left(-x^3\right)}$$

```
[In] integrate(x^5/exp(x^3),x, algorithm="fricas")
[Out] -1/3*(x^3 + 1)*e^(-x^3)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.46

$$\int e^{-x^3} x^5 \, dx = \frac{\left(-x^3 - 1\right) e^{-x^3}}{3}$$

```
[In] integrate(x**5/exp(x**3),x)
[Out] (-x**3 - 1)*exp(-x**3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int e^{-x^3} x^5 \, dx = -\frac{1}{3} \left(x^3 + 1\right) e^{\left(-x^3\right)}$$

```
[In] integrate(x^5/exp(x^3),x, algorithm="maxima")
[Out] -1/3*(x^3 + 1)*e^(-x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int e^{-x^3} x^5 \, dx = -\frac{1}{3} \left(x^3 + 1\right) e^{\left(-x^3\right)}$$

```
[In] integrate(x^5/exp(x^3),x, algorithm="giac")
[Out] -1/3*(x^3 + 1)*e^(-x^3)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int e^{-x^3} x^5 \, dx = -\frac{e^{-x^3} (x^3 + 1)}{3}$$

```
[In] int(x^5*exp(-x^3),x)
[Out] -(exp(-x^3)*(x^3 + 1))/3
```

3.287 $\int \tan^2(4x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 12

$$\int \tan^2(4x)\,dx = -x + \frac{1}{4}\tan(4x)$$

```
[Out] -x+1/4*tan(4*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3554, 8}

$$\int \tan^2(4x)\,dx = \frac{1}{4}\tan(4x) - x$$

```
[In] Int[Tan[4*x]^2,x]
```

```
[Out] -x + Tan[4*x]/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\tan(4x) - \int 1\,dx$$
$$= -x + \frac{1}{4}\tan(4x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \tan^2(4x)\,dx = -\frac{1}{4}\arctan(\tan(4x)) + \frac{1}{4}\tan(4x)$$

```
[In] Integrate[Tan[4*x]^2,x]
[Out] -1/4*ArcTan[Tan[4*x]] + Tan[4*x]/4
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
norman	$-x+\frac{\tan(4x)}{4}$	11
parallelrisch	$-x+\frac{\tan(4x)}{4}$	11
derivativedivides	$\frac{\tan(4x)}{4}-\frac{\arctan(\tan(4x))}{4}$	15
default	$\frac{\tan(4x)}{4}-\frac{\arctan(\tan(4x))}{4}$	15
risch	$-x+\frac{i}{2\,\mathrm{e}^{8ix}+2}$	17

```
[In] int(tan(4*x)^2,x,method=_RETURNVERBOSE)
[Out] -x+1/4*tan(4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \tan^2(4x)\,dx = -x + \frac{1}{4}\,\tan\left(4\,x\right)$$

```
[In] integrate(tan(4*x)^2,x, algorithm="fricas")
[Out] -x + 1/4*tan(4*x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \tan^2(4x)\,dx = -x + \frac{\sin\left(4x\right)}{4\cos\left(4x\right)}$$

```
[In] integrate(tan(4*x)**2,x)
[Out] -x + sin(4*x)/(4*cos(4*x))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \tan^2(4x)\,dx = -x + \frac{1}{4}\,\tan\left(4\,x\right)$$

```
[In] integrate(tan(4*x)^2,x, algorithm="maxima")
[Out] -x + 1/4*tan(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \tan^2(4x)\,dx = -x + \frac{1}{4}\,\tan\left(4\,x\right)$$

```
[In] integrate(tan(4*x)^2,x, algorithm="giac")
[Out] -x + 1/4*tan(4*x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \tan^2(4x)\,dx = \frac{\tan(4\,x)}{4} - x$$

```
[In] int(tan(4*x)^2,x)
[Out] tan(4*x)/4 - x
```

3.288 $\int \frac{1}{\sqrt{-5+12x+9x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{1}{\sqrt{-5+12x+9x^2}}\,dx = \frac{1}{3}\text{arctanh}\left(\frac{2+3x}{\sqrt{-5+12x+9x^2}}\right)$$

```
[Out] 1/3*arctanh((2+3*x)/(9*x^2+12*x-5)^(1/2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {635, 212}

$$\int \frac{1}{\sqrt{-5+12x+9x^2}}\,dx = \frac{1}{3}\text{arctanh}\left(\frac{3x+2}{\sqrt{9x^2+12x-5}}\right)$$

```
[In] Int[1/Sqrt[-5 + 12*x + 9*x^2],x]
```

```
[Out] ArcTanh[(2 + 3*x)/Sqrt[-5 + 12*x + 9*x^2]]/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= 2\text{Subst}\left(\int \frac{1}{36-x^2}\,dx, x, \frac{12+18x}{\sqrt{-5+12x+9x^2}}\right) \\ &= \frac{1}{3}\text{arctanh}\left(\frac{2+3x}{\sqrt{-5+12x+9x^2}}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{-5+12x+9x^2}}\,dx = -\frac{1}{3}\log\left(-2-3x+\sqrt{-5+12x+9x^2}\right)$$

```
[In] Integrate[1/Sqrt[-5 + 12*x + 9*x^2],x]
[Out] -1/3*Log[-2 - 3*x + Sqrt[-5 + 12*x + 9*x^2]]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
trager	$-\frac{\ln\left(-2-3x+\sqrt{9x^2+12x-5}\right)}{3}$	21
default	$\frac{\ln\left(\frac{(9x+6)\sqrt{9}}{9}+\sqrt{9x^2+12x-5}\right)\sqrt{9}}{9}$	30

```
[In] int(1/(9*x^2+12*x-5)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*ln(-2-3*x+(9*x^2+12*x-5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{-5+12x+9x^2}}\,dx = -\frac{1}{3}\log\left(-3\,x+\sqrt{9\,x^2+12\,x-5}-2\right)$$

```
[In] integrate(1/(9*x^2+12*x-5)^(1/2),x, algorithm="fricas")
[Out] -1/3*log(-3*x + sqrt(9*x^2 + 12*x - 5) - 2)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-5+12x+9x^2}}\,dx = \frac{\log\left(18x+6\sqrt{9x^2+12x-5}+12\right)}{3}$$

```
[In] integrate(1/(9*x**2+12*x-5)**(1/2),x)
[Out] log(18*x + 6*sqrt(9*x**2 + 12*x - 5) + 12)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-5+12x+9x^2}}\,dx = \frac{1}{3}\,\log\left(18\,x+6\,\sqrt{9\,x^2+12\,x-5}+12\right)$$

```
[In] integrate(1/(9*x^2+12*x-5)^(1/2),x, algorithm="maxima")
[Out] 1/3*log(18*x + 6*sqrt(9*x^2 + 12*x - 5) + 12)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt{-5+12x+9x^2}}\,dx = \frac{1}{6}\,\sqrt{9\,x^2+12\,x-5}(3\,x+2)+\frac{3}{2}\,\log\left(\left|-3\,x+\sqrt{9\,x^2+12\,x-5}-2\right|\right)$$

```
[In] integrate(1/(9*x^2+12*x-5)^(1/2),x, algorithm="giac")
[Out] 1/6*sqrt(9*x^2 + 12*x - 5)*(3*x + 2) + 3/2*log(abs(-3*x + sqrt(9*x^2 + 12*x
 - 5) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{-5+12x+9x^2}}\,dx = \frac{\ln\left(3\,x+\sqrt{9\,x^2+12\,x-5}+2\right)}{3}$$

```
[In] int(1/(12*x + 9*x^2 - 5)^(1/2),x)
[Out] log(3*x + (12*x + 9*x^2 - 5)^(1/2) + 2)/3
```

3.289 $\int x^2 \arctan(x) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 27

$$\int x^2 \arctan(x) \, dx = -\frac{x^2}{6} + \frac{1}{3} x^3 \arctan(x) + \frac{1}{6} \log \left(1 + x^2\right)$$

```
[Out] -1/6*x^2+1/3*x^3*arctan(x)+1/6*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4946, 272, 45}

$$\int x^2 \arctan(x) \, dx = \frac{1}{3} x^3 \arctan(x) - \frac{x^2}{6} + \frac{1}{6} \log \left(x^2 + 1\right)$$

```
[In] Int[x^2*ArcTan[x],x]
```

```
[Out] -1/6*x^2 + (x^3*ArcTan[x])/3 + Log[1 + x^2]/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arctan(x) - \frac{1}{3}\int \frac{x^3}{1+x^2}\,dx \\
&= \frac{1}{3}x^3\arctan(x) - \frac{1}{6}\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, x^2\right) \\
&= \frac{1}{3}x^3\arctan(x) - \frac{1}{6}\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx, x, x^2\right) \\
&= -\frac{x^2}{6} + \frac{1}{3}x^3\arctan(x) + \frac{1}{6}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x^2\arctan(x)\,dx = \frac{1}{6}\left(-x^2+2x^3\arctan(x)+\log\left(1+x^2\right)\right)$$

```
[In] Integrate[x^2*ArcTan[x],x]
[Out] (-x^2 + 2*x^3*ArcTan[x] + Log[1 + x^2])/6
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{x^2}{6}+\frac{x^3\arctan(x)}{3}+\frac{\ln(x^2+1)}{6}$	22
parts	$-\frac{x^2}{6}+\frac{x^3\arctan(x)}{3}+\frac{\ln(x^2+1)}{6}$	22
parallelrisch	$\frac{x^3\arctan(x)}{3}-\frac{x^2}{6}+\frac{\ln(x^2+1)}{6}+\frac{1}{6}$	23
meijerg	$-\frac{x^2}{6}+\frac{x^4\arctan\left(\sqrt{x^2}\right)}{3\sqrt{x^2}}+\frac{\ln(x^2+1)}{6}$	31
risch	$-\frac{ix^3\ln(ix+1)}{6}+\frac{ix^3\ln(-ix+1)}{6}-\frac{x^2}{6}+\frac{\ln(x^2+1)}{6}$	41

```
[In] int(x^2*arctan(x),x,method=_RETURNVERBOSE)
[Out] -1/6*x^2+1/3*x^3*arctan(x)+1/6*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int x^2\arctan(x)\,dx=\frac{1}{3}\,x^3\arctan\left(x\right)-\frac{1}{6}\,x^2+\frac{1}{6}\,\log\left(x^2+1\right)$$

```
[In] integrate(x^2*arctan(x),x, algorithm="fricas")
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^2\arctan(x)\,dx=\frac{x^3\operatorname{atan}\left(x\right)}{3}-\frac{x^2}{6}+\frac{\log\left(x^2+1\right)}{6}$$

```
[In] integrate(x**2*atan(x),x)
[Out] x**3*atan(x)/3 - x**2/6 + log(x**2 + 1)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int x^2 \arctan(x)\,dx = \frac{1}{3}\,x^3 \arctan\left(x\right) - \frac{1}{6}\,x^2 + \frac{1}{6}\,\log\left(x^2 + 1\right)$$

```
[In] integrate(x^2*arctan(x),x, algorithm="maxima")
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int x^2 \arctan(x)\,dx = \frac{1}{3}\,x^3 \arctan\left(x\right) - \frac{1}{6}\,x^2 + \frac{1}{6}\,\log\left(x^2 + 1\right)$$

```
[In] integrate(x^2*arctan(x),x, algorithm="giac")
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int x^2 \arctan(x)\,dx = \frac{\ln\left(x^2+1\right)}{6} + \frac{x^3\,\mathrm{atan}(x)}{3} - \frac{x^2}{6}$$

```
[In] int(x^2*atan(x),x)
[Out] log(x^2 + 1)/6 + (x^3*atan(x))/3 - x^2/6
```

3.290 $\int \frac{1-\sqrt{x}}{\sqrt[3]{x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}} \, dx = \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

```
[Out] 3/2*x^(2/3)-6/7*x^(7/6)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}} \, dx = \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

```
[In] Int[(1 - Sqrt[x])/x^(1/3),x]
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{\sqrt[3]{x}} - \sqrt[6]{x}\right) dx$$

$$= \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx = \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

```
[In] Integrate[(1 - Sqrt[x])/x^(1/3),x]
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$\frac{3x^{\frac{2}{3}}}{2}-\frac{6x^{\frac{7}{6}}}{7}$	12
default	$\frac{3x^{\frac{2}{3}}}{2}-\frac{6x^{\frac{7}{6}}}{7}$	12

```
[In] int((1-x^(1/2))/x^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/2*x^(2/3)-6/7*x^(7/6)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx = -\frac{6}{7}\,x^{\frac{7}{6}} + \frac{3}{2}\,x^{\frac{2}{3}}$$

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="fricas")
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx = -\frac{6x^{\frac{7}{6}}}{7} + \frac{3x^{\frac{2}{3}}}{2}$$

```
[In] integrate((1-x**(1/2))/x**(1/3),x)
[Out] -6*x**(7/6)/7 + 3*x**(2/3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx = -\frac{6}{7}\,x^{\frac{7}{6}} + \frac{3}{2}\,x^{\frac{2}{3}}$$

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="maxima")
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx = -\frac{6}{7}\,x^{\frac{7}{6}} + \frac{3}{2}\,x^{\frac{2}{3}}$$

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="giac")
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx = -\frac{3\,x^{2/3}\,\left(4\,\sqrt{x}-7\right)}{14}$$

```
[In] int(-(x^(1/2) - 1)/x^(1/3),x)
[Out] -(3*x^(2/3)*(4*x^(1/2) - 7))/14
```

3.291 $\int \frac{1}{-e^{-x}+e^x} dx$

Optimal result

Integrand size = 13, antiderivative size = 6

$$\int \frac{1}{-e^{-x}+e^x} dx = -\operatorname{arctanh}(e^x)$$

```
[Out] -arctanh(exp(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2320, 213}

$$\int \frac{1}{-e^{-x}+e^x} dx = -\operatorname{arctanh}(e^x)$$

```
[In] Int[(-E^(-x) + E^x)^(-1),x]
```

```
[Out] -ArcTanh[E^x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^x\right) \\ &= -\text{arctanh}(e^x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{-e^{-x}+e^x}\,dx = -\text{arctanh}(e^x)$$

```
[In] Integrate[(-E^(-x) + E^x)^(-1),x]
[Out] -ArcTanh[E^x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\operatorname{arctanh}\left(\mathrm{e}^{x}\right)$	6
default	$-\operatorname{arctanh}\left(\mathrm{e}^{x}\right)$	6
norman	$\frac{\ln(-1+\mathrm{e}^{x})}{2}-\frac{\ln(1+\mathrm{e}^{x})}{2}$	16
risch	$\frac{\ln(-1+\mathrm{e}^{x})}{2}-\frac{\ln(1+\mathrm{e}^{x})}{2}$	16
parallelrisch	$\frac{\ln(-1+\mathrm{e}^{x})}{2}-\frac{\ln(1+\mathrm{e}^{x})}{2}$	16

```
[In] int(1/(-1/exp(x)+exp(x)),x,method=_RETURNVERBOSE)
[Out] -arctanh(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{1}{-e^{-x}+e^x}\,dx = -\frac{1}{2}\log\left(e^x+1\right)+\frac{1}{2}\log\left(e^x-1\right)$$

```
[In] integrate(1/(-1/exp(x)+exp(x)),x, algorithm="fricas")
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 3.17

$$\int \frac{1}{-e^{-x}+e^x}\,dx = \frac{\log\left(-1+e^{-x}\right)}{2}-\frac{\log\left(1+e^{-x}\right)}{2}$$

```
[In] integrate(1/(-1/exp(x)+exp(x)),x)
[Out] log(-1 + exp(-x))/2 - log(1 + exp(-x))/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 3.17

$$\int \frac{1}{-e^{-x}+e^x}\,dx = -\frac{1}{2}\log\left(e^{(-x)}+1\right)+\frac{1}{2}\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(1/(-1/exp(x)+exp(x)),x, algorithm="maxima")
[Out] -1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(5) = 10.

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{1}{-e^{-x}+e^x}\,dx = -\frac{1}{2}\log\left(e^x+1\right)+\frac{1}{2}\log\left(|e^x-1|\right)$$

```
[In] integrate(1/(-1/exp(x)+exp(x)),x, algorithm="giac")
[Out] -1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{1}{-e^{-x}+e^x}\,dx = \frac{\ln\left(\mathrm{e}^x-1\right)}{2}-\frac{\ln\left(\mathrm{e}^x+1\right)}{2}$$

```
[In] int(-1/(exp(-x) - exp(x)),x)
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

3.292 $\int \frac{x}{10+2x^2+x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{x}{10+2x^2+x^4} \, dx = \frac{1}{6}\arctan\left(\frac{1}{3}(1+x^2)\right)$$

```
[Out] 1/6*arctan(1/3*x^2+1/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1121, 632, 210}

$$\int \frac{x}{10+2x^2+x^4} \, dx = \frac{1}{6}\arctan\left(\frac{1}{3}(x^2+1)\right)$$

```
[In] Int[x/(10 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + x^2)/3]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{10+2x+x^2}\,dx, x, x^2\right)\\ &= -\text{Subst}\left(\int \frac{1}{-36-x^2}\,dx, x, 2(1+x^2)\right)\\ &= \frac{1}{6}\arctan\left(\frac{1}{3}(1+x^2)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x}{10+2x^2+x^4}\,dx = \frac{1}{6}\arctan\left(\frac{1}{3}(1+x^2)\right)$$

```
[In] Integrate[x/(10 + 2*x^2 + x^4),x]
[Out] ArcTan[(1 + x^2)/3]/6
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\arctan\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$	11
risch	$\frac{\arctan\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$	11
parallelrisch	$\frac{i\ln(x^2+3i+1)}{12}-\frac{i\ln(x^2-3i+1)}{12}$	24

```
[In] int(x/(x^4+2*x^2+10),x,method=_RETURNVERBOSE)
[Out] 1/6*arctan(1/3*x^2+1/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x}{10+2x^2+x^4}\,dx = \frac{1}{6}\arctan\left(\frac{1}{3}\,x^2+\frac{1}{3}\right)$$

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="fricas")
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x}{10+2x^2+x^4}\,dx = \frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

```
[In] integrate(x/(x**4+2*x**2+10),x)
[Out] atan(x**2/3 + 1/3)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x}{10+2x^2+x^4}\,dx = \frac{1}{6}\arctan\left(\frac{1}{3}\,x^2+\frac{1}{3}\right)$$

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="maxima")
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

Giac [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x}{10+2x^2+x^4}\,dx = \frac{1}{6}\arctan\left(\frac{1}{3}\,x^2+\frac{1}{3}\right)$$

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="giac")
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x}{10+2x^2+x^4}\,dx=\frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

```
[In] int(x/(2*x^2 + x^4 + 10),x)
[Out] atan(x^2/3 + 1/3)/6
```

3.293 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} dx = \frac{3}{4}\log\left(1+x^{4/3}\right)$$

```
[Out] 3/4*ln(1+x^(4/3))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1607, 266}

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} dx = \frac{3}{4}\log\left(x^{4/3}+1\right)$$

```
[In] Int[(x^(-1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 + x^(4/3)])/4
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sqrt[3]{x}}{1+x^{4/3}}\,dx \\ &= \frac{3}{4}\log\left(1+x^{4/3}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx = \frac{3}{4}\log\left(1+x^{4/3}\right)$$

```
[In] Integrate[(x^(-1/3) + x)^(-1),x]
[Out] (3*Log[1 + x^(4/3)])/4
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{3\ln\left(1+x^{\frac{4}{3}}\right)}{4}$	9
default	$\frac{3\ln\left(1+x^{\frac{4}{3}}\right)}{4}$	9
meijerg	$\frac{3\ln\left(1+x^{\frac{4}{3}}\right)}{4}$	9
trager	$-\frac{\ln\left(\frac{3x^{\frac{20}{3}}-x^{8}-6x^{\frac{16}{3}}-6x^{\frac{8}{3}}+7x^{4}+3x^{\frac{4}{3}}-1}{\left(x^{4}+1\right)^{3}}\right)}{4}$	44

```
[In] int(1/(1/x^(1/3)+x),x,method=_RETURNVERBOSE)
[Out] 3/4*ln(1+x^(4/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} \, dx = \frac{3}{4} \log \left(x^{\frac{4}{3}}+1\right)$$

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="fricas")
[Out] 3/4*log(x^(4/3) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} \, dx = \frac{3 \log \left(x^{\frac{4}{3}}+1\right)}{4}$$

```
[In] integrate(1/(1/x**(1/3)+x),x)
[Out] 3*log(x**(4/3) + 1)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} \, dx = \frac{3}{4} \log \left(x^{\frac{4}{3}}+1\right)$$

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="maxima")
[Out] 3/4*log(x^(4/3) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.67

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}} + x} \, dx = \frac{3}{4} \log \left(\sqrt{2}x^{\frac{1}{3}} + x^{\frac{2}{3}} + 1\right) + \frac{3}{4} \log \left(-\sqrt{2}x^{\frac{1}{3}} + x^{\frac{2}{3}} + 1\right)$$

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="giac")
```

```
[Out] 3/4*log(sqrt(2)*x^(1/3) + x^(2/3) + 1) + 3/4*log(-sqrt(2)*x^(1/3) + x^(2/3)
 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}} + x} \, dx = \frac{3 \ln \left(x^{4/3} + 1\right)}{4}$$

```
[In] int(1/(x + 1/x^(1/3)),x)
```

```
[Out] (3*log(x^(4/3) + 1))/4
```

3.294 $\int \cos^4(x) \sin^2(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 34

$$\int \cos^4(x) \sin^2(x) \, dx = \frac{x}{16} + \frac{1}{16} \cos(x) \sin(x) + \frac{1}{24} \cos^3(x) \sin(x) - \frac{1}{6} \cos^5(x) \sin(x)$$

```
[Out] 1/16*x+1/16*cos(x)*sin(x)+1/24*cos(x)^3*sin(x)-1/6*cos(x)^5*sin(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2648, 2715, 8}

$$\int \cos^4(x) \sin^2(x) \, dx = \frac{x}{16} - \frac{1}{6} \sin(x) \cos^5(x) + \frac{1}{24} \sin(x) \cos^3(x) + \frac{1}{16} \sin(x) \cos(x)$$

```
[In] Int[Cos[x]^4*Sin[x]^2,x]
```

```
[Out] x/16 + (Cos[x]*Sin[x])/16 + (Cos[x]^3*Sin[x])/24 - (Cos[x]^5*Sin[x])/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{6}\cos^5(x)\sin(x)+\frac{1}{6}\int\cos^4(x)\,dx\\
&= \frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)+\frac{1}{8}\int\cos^2(x)\,dx\\
&= \frac{1}{16}\cos(x)\sin(x)+\frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)+\frac{\int 1\,dx}{16}\\
&= \frac{x}{16}+\frac{1}{16}\cos(x)\sin(x)+\frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int\cos^4(x)\sin^2(x)\,dx=\frac{x}{16}+\frac{1}{64}\sin(2x)-\frac{1}{64}\sin(4x)-\frac{1}{192}\sin(6x)$$

```
[In] Integrate[Cos[x]^4*Sin[x]^2,x]
[Out] x/16 + Sin[2*x]/64 - Sin[4*x]/64 - Sin[6*x]/192
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result
risch	$\frac{x}{16}-\frac{\sin(6x)}{192}-\frac{\sin(4x)}{64}+\frac{\sin(2x)}{64}$
parallelrisch	$\frac{x}{16}-\frac{\sin(6x)}{192}-\frac{\sin(4x)}{64}+\frac{\sin(2x)}{64}$
default	$-\frac{(\cos^5(x))\sin(x)}{6}+\frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{24}+\frac{x}{16}$
norman	$\frac{\frac{x}{16}+\frac{47\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}-\frac{13\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{13\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}-\frac{47\left(\tan^9\left(\frac{x}{2}\right)\right)}{24}+\frac{\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{8}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{15x\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{5x\left(\tan^6\left(\frac{x}{2}\right)\right)}{4}+15}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

```
[In] int(sin(x)^2*cos(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*x-1/192*sin(6*x)-1/64*sin(4*x)+1/64*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \cos^4(x)\sin^2(x)\,dx = -\frac{1}{48}\left(8\,\cos\left(x\right)^5 - 2\,\cos\left(x\right)^3 - 3\,\cos\left(x\right)\right)\sin\left(x\right) + \frac{1}{16}\,x$$

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="fricas")
[Out] -1/48*(8*cos(x)^5 - 2*cos(x)^3 - 3*cos(x))*sin(x) + 1/16*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{x}{16} - \frac{\sin\left(x\right)\cos^5\left(x\right)}{6} + \frac{\sin\left(x\right)\cos^3\left(x\right)}{24} + \frac{\sin\left(x\right)\cos\left(x\right)}{16}$$

```
[In] integrate(cos(x)**4*sin(x)**2,x)
[Out] x/16 - sin(x)*cos(x)**5/6 + sin(x)*cos(x)**3/24 + sin(x)*cos(x)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.53

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{1}{48}\,\sin\left(2\,x\right)^3 + \frac{1}{16}\,x - \frac{1}{64}\,\sin\left(4\,x\right)$$

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="maxima")
[Out] 1/48*sin(2*x)^3 + 1/16*x - 1/64*sin(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{1}{16}\,x - \frac{1}{192}\,\sin\left(6\,x\right) - \frac{1}{64}\,\sin\left(4\,x\right) + \frac{1}{64}\,\sin\left(2\,x\right)$$

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="giac")
[Out] 1/16*x - 1/192*sin(6*x) - 1/64*sin(4*x) + 1/64*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \cos^4(x)\sin^2(x)\,dx = \left(\frac{\cos\left(x\right)^3}{6} + \frac{\cos\left(x\right)}{8}\right)\,\sin\left(x\right)^3 - \frac{\cos\left(x\right)\,\sin\left(x\right)}{16} + \frac{x}{16}$$

```
[In] int(cos(x)^4*sin(x)^2,x)
[Out] x/16 - (cos(x)*sin(x))/16 + sin(x)^3*(cos(x)/8 + cos(x)^3/6)
```

3.295 $\int \frac{1}{\sqrt{5-4x-x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 12

$$\int \frac{1}{\sqrt{5-4x-x^2}}\,dx = -\arcsin\left(\frac{1}{3}(-2-x)\right)$$

```
[Out] arcsin(2/3+1/3*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {633, 222}

$$\int \frac{1}{\sqrt{5-4x-x^2}}\,dx = -\arcsin\left(\frac{1}{3}(-x-2)\right)$$

```
[In] Int[1/Sqrt[5 - 4*x - x^2],x]
```

```
[Out] -ArcSin[(-2 - x)/3]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx, x, -4-2x\right)\right)$$
$$= -\arcsin\left(\frac{1}{3}(-2-x)\right)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sqrt{5-4x-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{5-4x-x^2}}{5+x}\right)$$

```
[In] Integrate[1/Sqrt[5 - 4*x - x^2],x]
[Out] -2*ArcTan[Sqrt[5 - 4*x - x^2]/(5 + x)]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

method	result	size
default	$\arcsin\left(\frac{2}{3}+\frac{x}{3}\right)$	7
trager	$\text{RootOf}\left(\_Z^2+1\right)\ln\left(-\text{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2-4x+5}-2\,\text{RootOf}\left(\_Z^2+1\right)\right)$	39

```
[In] int(1/(-x^2-4*x+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(2/3+1/3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{5-4x-x^2}}\,dx = -\arctan\left(\frac{\sqrt{-x^2-4\,x+5}(x+2)}{x^2+4\,x-5}\right)$$

```
[In] integrate(1/(-x^2-4*x+5)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-x^2 - 4*x + 5)*(x + 2)/(x^2 + 4*x - 5))
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{5-4x-x^2}}\,dx = \operatorname{asin}\left(\frac{x}{3}+\frac{2}{3}\right)$$

```
[In] integrate(1/(-x**2-4*x+5)**(1/2),x)
[Out] asin(x/3 + 2/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{5-4x-x^2}}\,dx = -\arcsin\left(-\frac{1}{3}\,x-\frac{2}{3}\right)$$

```
[In] integrate(1/(-x^2-4*x+5)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-1/3*x - 2/3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(6) = 12.

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{5-4x-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2-4\,x+5}(x+2)+\frac{9}{2}\arcsin\left(\frac{1}{3}\,x+\frac{2}{3}\right)$$

```
[In] integrate(1/(-x^2-4*x+5)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 - 4*x + 5)*(x + 2) + 9/2*arcsin(1/3*x + 2/3)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{5-4x-x^2}}\,dx = \operatorname{asin}\left(\frac{x}{3}+\frac{2}{3}\right)$$

```
[In] int(1/(5 - x^2 - 4*x)^(1/2),x)
[Out] asin(x/3 + 2/3)
```

3.296 $\int \frac{x}{1-x^2+\sqrt{1-x^2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{x}{1-x^2+\sqrt{1-x^2}} \, dx = -\log \left(1+\sqrt{1-x^2}\right)$$

```
[Out] -ln(1+(-x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2186, 31}

$$\int \frac{x}{1-x^2+\sqrt{1-x^2}} \, dx = -\log \left(\sqrt{1-x^2}+1\right)$$

```
[In] Int[x/(1 - x^2 + Sqrt[1 - x^2]),x]
```

```
[Out] -Log[1 + Sqrt[1 - x^2]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{1-x}-x}\,dx, x, x^2\right) \\
&= -\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{1-x^2}\right) \\
&= -\log\left(1+\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{1-x^2+\sqrt{1-x^2}}\,dx = -\log\left(1+\sqrt{1-x^2}\right)$$

```
[In] Integrate[x/(1 - x^2 + Sqrt[1 - x^2]),x]
[Out] -Log[1 + Sqrt[1 - x^2]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
trager	$-\ln\left(1+\sqrt{-x^2+1}\right)$	15
default	$-\ln\left(x\right)+\sqrt{-x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)-\frac{\sqrt{-(1+x)^2+2x+2}}{2}-\frac{\sqrt{-(-1+x)^2-2x+2}}{2}$	59

```
[In] int(x/(1-x^2+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -ln(1+(-x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{x}{1-x^2+\sqrt{1-x^2}}\,dx = -\log\left(x\right)+\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate(x/(1-x^2+(-x^2+1)^(1/2)),x, algorithm="fricas")
[Out] -log(x) + log((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(12) = 24$.

Time = 1.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 3.00

$$\int \frac{x}{1-x^2+\sqrt{1-x^2}}\,dx = \frac{\log\left(2\sqrt{1-x^2}\right)}{2} - \frac{\log\left(2\sqrt{1-x^2}+2\right)}{2} - \frac{\log\left(2x^2-2\sqrt{1-x^2}-2\right)}{2}$$

```
[In] integrate(x/(1-x**2+(-x**2+1)**(1/2)),x)
[Out] log(2*sqrt(1 - x**2))/2 - log(2*sqrt(1 - x**2) + 2)/2 - log(2*x**2 - 2*sqrt
(1 - x**2) - 2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{1-x^2+\sqrt{1-x^2}}\,dx = -\log\left(\sqrt{-x^2+1}+1\right)$$

```
[In] integrate(x/(1-x^2+(-x^2+1)^(1/2)),x, algorithm="maxima")
[Out] -log(sqrt(-x^2 + 1) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{1-x^2+\sqrt{1-x^2}}\,dx = -\log\left(\sqrt{-x^2+1}+1\right)$$

```
[In] integrate(x/(1-x^2+(-x^2+1)^(1/2)),x, algorithm="giac")
[Out] -log(sqrt(-x^2 + 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{x}{1-x^2+\sqrt{1-x^2}}\,dx=\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)-\ln\left(x\right)$$

```
[In] int(x/((1 - x^2)^(1/2) - x^2 + 1),x)
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - log(x)
```

3.297 $\int (1+\cos(x))\csc(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 7

$$\int (1+\cos(x))\csc(x)\,dx=\log(1-\cos(x))$$

```
[Out] ln(1-cos(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2746, 31}

$$\int (1+\cos(x))\csc(x)\,dx=\log(1-\cos(x))$$

```
[In] Int[(1 + Cos[x])*Csc[x],x]
```

```
[Out] Log[1 - Cos[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \cos(x)\right)\\ &= \log(1-\cos(x))\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(7) = 14.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 3.29

$$\int (1+\cos(x))\csc(x)\,dx = -\log\left(\cos\left(\frac{x}{2}\right)\right)+\log(\cos(x))+\log\left(\sin\left(\frac{x}{2}\right)\right)+\log(\tan(x))$$

```
[In] Integrate[(1 + Cos[x])*Csc[x],x]
[Out] -Log[Cos[x/2]] + Log[Cos[x]] + Log[Sin[x/2]] + Log[Tan[x]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.86

method	result	size
default	$\ln(\sin(x))+\ln(\csc(x)-\cot(x))$	13
parts	$-\ln(\csc(x))-\ln(\csc(x)+\cot(x))$	15
risch	$-ix+2\ln(\mathrm{e}^{ix}-1)$	16
norman	$2\ln\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$	20
parallelrisch	$2\ln(\csc(x)-\cot(x))-\ln\left(\frac{2}{\cos(x)+1}\right)$	23

```
[In] int((cos(x)+1)*csc(x),x,method=_RETURNVERBOSE)
[Out] ln(sin(x))+ln(csc(x)-cot(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (1+\cos(x))\csc(x)\,dx = \log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="fricas")
[Out] log(-1/2*cos(x) + 1/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(5) = 10.

Time = 0.80 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int (1+\cos(x))\csc(x)\,dx = -\log(\cot(x)+\csc(x))+\log(\sin(x))$$

```
[In] integrate((1+cos(x))*csc(x),x)
[Out] -log(cot(x) + csc(x)) + log(sin(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int (1+\cos(x))\csc(x)\,dx = \log(\cos(x)-1)$$

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="maxima")
[Out] log(cos(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (1 + \cos(x)) \csc(x) \, dx = \log(-\cos(x) + 1)$$

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="giac")
[Out] log(-cos(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int (1 + \cos(x)) \csc(x) \, dx = \ln(\cos(x) - 1)$$

```
[In] int((cos(x) + 1)/sin(x),x)
[Out] log(cos(x) - 1)
```

3.298 $\int \frac{e^x}{-1+e^{2x}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 6

$$\int \frac{e^x}{-1+e^{2x}} \, dx = -\text{arctanh}(e^x)$$

```
[Out] -arctanh(exp(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2281, 213}

$$\int \frac{e^x}{-1+e^{2x}} \, dx = -\text{arctanh}(e^x)$$

```
[In] Int[E^x/(-1 + E^(2*x)),x]
```

```
[Out] -ArcTanh[E^x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
```

```
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^x\right) \\
&= -\text{arctanh}(e^x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{-1+e^{2x}}\,dx = -\text{arctanh}(e^x)$$

```
[In] Integrate[E^x/(-1 + E^(2*x)),x]
[Out] -ArcTanh[E^x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
default	$-\operatorname{arctanh}\left(\mathrm{e}^{x}\right)$	6
norman	$\frac{\ln(-1+\mathrm{e}^{x})}{2}-\frac{\ln(1+\mathrm{e}^{x})}{2}$	16
risch	$\frac{\ln(-1+\mathrm{e}^{x})}{2}-\frac{\ln(1+\mathrm{e}^{x})}{2}$	16

```
[In] int(exp(x)/(exp(2*x)-1),x,method=_RETURNVERBOSE)
[Out] -arctanh(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = -\frac{1}{2}\log\left(e^x+1\right) + \frac{1}{2}\log\left(e^x-1\right)$$

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="fricas")
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = \frac{\log\left(e^x-1\right)}{2} - \frac{\log\left(e^x+1\right)}{2}$$

```
[In] integrate(exp(x)/(-1+exp(2*x)),x)
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = -\frac{1}{2}\log\left(e^x+1\right) + \frac{1}{2}\log\left(e^x-1\right)$$

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="maxima")
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(5) = 10$.

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{e^x}{-1+e^{2x}}\,dx = -\frac{1}{2}\log\left(e^x+1\right) + \frac{1}{2}\log\left(\left|e^x-1\right|\right)$$

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="giac")
[Out] -1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = \frac{\ln\left(\mathrm{e}^x-1\right)}{2} - \frac{\ln\left(\mathrm{e}^x+1\right)}{2}$$

```
[In] int(exp(x)/(exp(2*x) - 1),x)
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

3.299 $\int \frac{1}{-8+x^3} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 43

$$\int \frac{1}{-8+x^3} \, dx = -\frac{\arctan\left(\frac{1+x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{12}\log(2-x) - \frac{1}{24}\log\left(4+2x+x^2\right)$$

```
[Out] 1/12*ln(2-x)-1/24*ln(x^2+2*x+4)-1/12*arctan(1/3*(1+x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {206, 31, 648, 632, 210, 642}

$$\int \frac{1}{-8+x^3} \, dx = -\frac{\arctan\left(\frac{x+1}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{24}\log\left(x^2+2x+4\right) + \frac{1}{12}\log(2-x)$$

```
[In] Int[(-8 + x^3)^(-1),x]
```

```
[Out] -1/4*ArcTan[(1 + x)/Sqrt[3]]/Sqrt[3] + Log[2 - x]/12 - Log[4 + 2*x + x^2]/2
4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{12}\int \frac{1}{-2+x}\,dx + \frac{1}{12}\int \frac{-4-x}{4+2x+x^2}\,dx \\
&= \frac{1}{12}\log(2-x) - \frac{1}{24}\int \frac{2+2x}{4+2x+x^2}\,dx - \frac{1}{4}\int \frac{1}{4+2x+x^2}\,dx \\
&= \frac{1}{12}\log(2-x) - \frac{1}{24}\log\left(4+2x+x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, 2+2x\right) \\
&= -\frac{\arctan\left(\frac{1+x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{12}\log(2-x) - \frac{1}{24}\log\left(4+2x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{-8+x^3}\,dx=-\frac{\arctan\left(\frac{1+x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{12}\log(2-x)-\frac{1}{24}\log\left(4+2x+x^2\right)$$

```
[In] Integrate[(-8 + x^3)^(-1),x]
[Out] -1/4*ArcTan[(1 + x)/Sqrt[3]]/Sqrt[3] + Log[2 - x]/12 - Log[4 + 2*x + x^2]/2
4
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{\ln\left(x^2+2x+4\right)}{24}-\frac{\arctan\left(\frac{(1+x)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\ln(-2+x)}{12}$	33
default	$-\frac{\ln\left(x^2+2x+4\right)}{24}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x+2)\sqrt{3}}{6}\right)}{12}+\frac{\ln(-2+x)}{12}$	35
meijerg	$\frac{x\left(\ln\left(1-\frac{\left(x^3\right)^{\frac{1}{3}}}{2}\right)-\frac{\ln\left(1+\frac{\left(x^3\right)^{\frac{1}{3}}}{2}+\frac{\left(x^3\right)^{\frac{2}{3}}}{4}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{4+\left(x^3\right)^{\frac{1}{3}}}\right)\right)}{12\left(x^3\right)^{\frac{1}{3}}}$	66

```
[In] int(1/(x^3-8),x,method=_RETURNVERBOSE)
[Out] -1/24*ln(x^2+2*x+4)-1/12*arctan(1/3*(1+x)*3^(1/2))*3^(1/2)+1/12*ln(-2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{1}{-8+x^3}\,dx=-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right)-\frac{1}{24}\log\left(x^2+2x+4\right)+\frac{1}{12}\log\left(x-2\right)$$

```
[In] integrate(1/(x^3-8),x, algorithm="fricas")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/24*log(x^2 + 2*x + 4) + 1/12*
log(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{1}{-8+x^3}\,dx = \frac{\log(x-2)}{12} - \frac{\log(x^2+2x+4)}{24} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

```
[In] integrate(1/(x**3-8),x)
```

```
[Out] log(x - 2)/12 - log(x**2 + 2*x + 4)/24 - sqrt(3)*atan(sqrt(3)*x/3 + sqrt(3)
/3)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{1}{-8+x^3}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right) - \frac{1}{24}\log\left(x^2+2x+4\right) + \frac{1}{12}\log(x-2)$$

```
[In] integrate(1/(x^3-8),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/24*log(x^2 + 2*x + 4) + 1/12*
log(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1}{-8+x^3}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right) - \frac{1}{24}\log\left(x^2+2x+4\right) + \frac{1}{12}\log(|x-2|)$$

```
[In] integrate(1/(x^3-8),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/24*log(x^2 + 2*x + 4) + 1/12*
log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{1}{-8+x^3}\,dx=\frac{\ln(x-2)}{12}+\ln\left(x+1-\sqrt{3}\,1\mathrm{i}\right)\left(-\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right)-\ln\left(x+1+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right)$$

```
[In] int(1/(x^3 - 8),x)
[Out] log(x - 2)/12 + log(x - 3^(1/2)*1i + 1)*((3^(1/2)*1i)/24 - 1/24) - log(x +
3^(1/2)*1i + 1)*((3^(1/2)*1i)/24 + 1/24)
```

3.300 $\int x^5 \cosh(x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 37

$$\int x^5 \cosh(x)\,dx = -120\cosh(x) - 60x^2\cosh(x) - 5x^4\cosh(x) + 120x\sinh(x) + 20x^3\sinh(x) + x^5\sinh(x)$$

```
[Out] -120*cosh(x)-60*x^2*cosh(x)-5*x^4*cosh(x)+120*x*sinh(x)+20*x^3*sinh(x)+x^5*
sinh(x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3377, 2718}

$$\int x^5 \cosh(x)\,dx = x^5\sinh(x) - 5x^4\cosh(x) + 20x^3\sinh(x) - 60x^2\cosh(x) + 120x\sinh(x) - 120\cosh(x)$$

```
[In] Int[x^5*Cosh[x],x]
```

```
[Out] -120*Cosh[x] - 60*x^2*Cosh[x] - 5*x^4*Cosh[x] + 120*x*Sinh[x] + 20*x^3*Sinh
[x] + x^5*Sinh[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x^5 \sinh(x) - 5\int x^4 \sinh(x)\,dx \\
&= -5x^4\cosh(x) + x^5\sinh(x) + 20\int x^3\cosh(x)\,dx \\
&= -5x^4\cosh(x) + 20x^3\sinh(x) + x^5\sinh(x) - 60\int x^2\sinh(x)\,dx \\
&= -60x^2\cosh(x) - 5x^4\cosh(x) + 20x^3\sinh(x) + x^5\sinh(x) + 120\int x\cosh(x)\,dx \\
&= -60x^2\cosh(x) - 5x^4\cosh(x) + 120x\sinh(x) + 20x^3\sinh(x) + x^5\sinh(x) - 120\int \sinh(x)\,dx \\
&= -120\cosh(x) - 60x^2\cosh(x) - 5x^4\cosh(x) + 120x\sinh(x) + 20x^3\sinh(x) + x^5\sinh(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int x^5\cosh(x)\,dx = -5\left(24 + 12x^2 + x^4\right)\cosh(x) + x\left(120 + 20x^2 + x^4\right)\sinh(x)$$

```
[In] Integrate[x^5*Cosh[x],x]
[Out] -5*(24 + 12*x^2 + x^4)*Cosh[x] + x*(120 + 20*x^2 + x^4)*Sinh[x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

method	result
parallelrisch	$(-5x^4 - 60x^2 - 120)\cosh(x) - 120 + (x^5 + 20x^3 + 120x)\sinh(x)$
default	$-120\cosh(x) - 60x^2\cosh(x) - 5x^4\cosh(x) + 120x\sinh(x) + 20x^3\sinh(x) + x^5\sinh(x)$
parts	$-120\cosh(x) - 60x^2\cosh(x) - 5x^4\cosh(x) + 120x\sinh(x) + 20x^3\sinh(x) + x^5\sinh(x)$
meijerg	$-32\sqrt{\pi}\left(-\frac{15}{4\sqrt{\pi}} + \frac{\left(\frac{15}{8}x^4+\frac{45}{2}x^2+45\right)\cosh(x)}{12\sqrt{\pi}} - \frac{x\left(\frac{3}{8}x^4+\frac{15}{2}x^2+45\right)\sinh(x)}{12\sqrt{\pi}}\right)$
risch	$\left(10x^3 - 30x^2 + 60x - 60 - \frac{5}{2}x^4 + \frac{1}{2}x^5\right)e^{x} + \left(-10x^3 - 30x^2 - 60x - 60 - \frac{5}{2}x^4 - \frac{1}{2}x^5\right)e^{-x}$

```
[In] int(x^5*cosh(x),x,method=_RETURNVERBOSE)
[Out] (-5*x^4-60*x^2-120)*cosh(x)-120+(x^5+20*x^3+120*x)*sinh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int x^5 \cosh(x)\,dx = -5\left(x^4+12\,x^2+24\right)\cosh\left(x\right)+\left(x^5+20\,x^3+120\,x\right)\sinh\left(x\right)$$

```
[In] integrate(x^5*cosh(x),x, algorithm="fricas")
[Out] -5*(x^4 + 12*x^2 + 24)*cosh(x) + (x^5 + 20*x^3 + 120*x)*sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int x^5 \cosh(x)\,dx = x^5\sinh\left(x\right)-5x^4\cosh\left(x\right)+20x^3\sinh\left(x\right) - 60x^2\cosh\left(x\right)+120x\sinh\left(x\right)-120\cosh\left(x\right)$$

```
[In] integrate(x**5*cosh(x),x)
[Out] x**5*sinh(x) - 5*x**4*cosh(x) + 20*x**3*sinh(x) - 60*x**2*cosh(x) + 120*x*s
inh(x) - 120*cosh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.00

$$\int x^5 \cosh(x)\,dx = \frac{1}{6}\,x^6\cosh\left(x\right) - \frac{1}{12}\left(x^6+6\,x^5+30\,x^4+120\,x^3+360\,x^2+720\,x+720\right)e^{(-x)} - \frac{1}{12}\left(x^6-6\,x^5+30\,x^4-120\,x^3+360\,x^2-720\,x+720\right)e^{x}$$

```
[In] integrate(x^5*cosh(x),x, algorithm="maxima")
[Out] 1/6*x^6*cosh(x) - 1/12*(x^6 + 6*x^5 + 30*x^4 + 120*x^3 + 360*x^2 + 720*x +
720)*e^(-x) - 1/12*(x^6 - 6*x^5 + 30*x^4 - 120*x^3 + 360*x^2 - 720*x + 720)
*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int x^5 \cosh(x)\,dx = -\frac{1}{2}\left(x^5 + 5\,x^4 + 20\,x^3 + 60\,x^2 + 120\,x + 120\right)e^{(-x)} + \frac{1}{2}\left(x^5 - 5\,x^4 + 20\,x^3 - 60\,x^2 + 120\,x - 120\right)e^x$$

```
[In] integrate(x^5*cosh(x),x, algorithm="giac")
[Out] -1/2*(x^5 + 5*x^4 + 20*x^3 + 60*x^2 + 120*x + 120)*e^(-x) + 1/2*(x^5 - 5*x^
4 + 20*x^3 - 60*x^2 + 120*x - 120)*e^x
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x^5 \cosh(x)\,dx = 20\,x^3 \sinh(x) - 60\,x^2 \cosh(x) - 5\,x^4 \cosh(x) - 120\cosh(x) + x^5 \sinh(x) + 120\,x\sinh(x)$$

```
[In] int(x^5*cosh(x),x)
[Out] 20*x^3*sinh(x) - 60*x^2*cosh(x) - 5*x^4*cosh(x) - 120*cosh(x) + x^5*sinh(x)
 + 120*x*sinh(x)
```

3.301 $\int \csc(x)\log(\tan(x))\sec(x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 9

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \frac{1}{2}\log^2(\tan(x))$$

```
[Out] 1/2*ln(tan(x))^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2700, 29, 6818}

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \frac{1}{2}\log^2(\tan(x))$$

```
[In] Int[Csc[x]*Log[Tan[x]]*Sec[x],x]
```

```
[Out] Log[Tan[x]]^2/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\log^2(\tan(x))$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \frac{1}{2}\log^2(\tan(x))$$

```
[In] Integrate[Csc[x]*Log[Tan[x]]*Sec[x],x]
[Out] Log[Tan[x]]^2/2
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\ln(\tan(x))^2}{2}$	8
default	$\frac{\ln(\tan(x))^2}{2}$	8
risch	Expression too large to display	764

```
[In] int(ln(tan(x))/cos(x)/sin(x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(tan(x))^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \frac{1}{2}\,\log\left(\frac{\sin(x)}{\cos(x)}\right)^2$$

```
[In] integrate(log(tan(x))/cos(x)/sin(x),x, algorithm="fricas")
[Out] 1/2*log(sin(x)/cos(x))^2
```

Sympy [F]

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \int \frac{\log(\tan(x))}{\sin(x)\cos(x)}\,dx$$

```
[In] integrate(ln(tan(x))/cos(x)/sin(x),x)
[Out] Integral(log(tan(x))/(sin(x)*cos(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \frac{1}{2}\log(\tan(x))^2$$

```
[In] integrate(log(tan(x))/cos(x)/sin(x),x, algorithm="maxima")
[Out] 1/2*log(tan(x))^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \frac{1}{2}\log(\tan(x))^2$$

```
[In] integrate(log(tan(x))/cos(x)/sin(x),x, algorithm="giac")
[Out] 1/2*log(tan(x))^2
```

Mupad [B] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.00

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \frac{\ln\left(-\frac{e^{x\,2i}\,1i-i}{e^{x\,2i}+1}\right)^2}{2}$$

```
[In] int(log(tan(x))/(cos(x)*sin(x)),x)
[Out] log(-(exp(x*2i)*1i - 1i)/(exp(x*2i) + 1))^2/2
```

3.302 $\int (-2x + x^2 + x^3) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 20

$$\int (-2x + x^2 + x^3) \, dx = -x^2 + \frac{x^3}{3} + \frac{x^4}{4}$$

```
[Out] -x^2+1/3*x^3+1/4*x^4
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (-2x + x^2 + x^3) \, dx = \frac{x^4}{4} + \frac{x^3}{3} - x^2$$

```
[In] Int[-2*x + x^2 + x^3,x]
```

```
[Out] -x^2 + x^3/3 + x^4/4
```

Rubi steps

$$\text{integral} = -x^2 + \frac{x^3}{3} + \frac{x^4}{4}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(-2x + x^2 + x^3\right) dx = -x^2 + \frac{x^3}{3} + \frac{x^4}{4}$$

```
[In] Integrate[-2*x + x^2 + x^3,x]
[Out] -x^2 + x^3/3 + x^4/4
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{x^2(3x^2+4x-12)}{12}$	16
default	$-x^2+\frac{1}{3}x^3+\frac{1}{4}x^4$	17
norman	$-x^2+\frac{1}{3}x^3+\frac{1}{4}x^4$	17
risch	$-x^2+\frac{1}{3}x^3+\frac{1}{4}x^4$	17
parallelrisch	$-x^2+\frac{1}{3}x^3+\frac{1}{4}x^4$	17
parts	$-x^2+\frac{1}{3}x^3+\frac{1}{4}x^4$	17

```
[In] int(x^3+x^2-2*x,x,method=_RETURNVERBOSE)
[Out] 1/12*x^2*(3*x^2+4*x-12)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(-2x + x^2 + x^3\right) dx = \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$$

```
[In] integrate(x^3+x^2-2*x,x, algorithm="fricas")
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \left(-2x+x^2+x^3\right) \, dx = \frac{x^4}{4}+\frac{x^3}{3}-x^2$$

```
[In] integrate(x**3+x**2-2*x,x)
[Out] x**4/4 + x**3/3 - x**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(-2x+x^2+x^3\right) \, dx = \frac{1}{4}\,x^4+\frac{1}{3}\,x^3-x^2$$

```
[In] integrate(x^3+x^2-2*x,x, algorithm="maxima")
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(-2x+x^2+x^3\right) \, dx = \frac{1}{4}\,x^4+\frac{1}{3}\,x^3-x^2$$

```
[In] integrate(x^3+x^2-2*x,x, algorithm="giac")
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \left(-2x+x^2+x^3\right) \, dx = \frac{x^2\,\left(3\,x^2+4\,x-12\right)}{12}$$

```
[In] int(x^2 - 2*x + x^3,x)
[Out] (x^2*(4*x + 3*x^2 - 12))/12
```

3.303 $\int \frac{1+e^x}{1-e^x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{1+e^x}{1-e^x} \, dx = x - 2\log(1-e^x)$$

```
[Out] x-2*ln(1-exp(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2320, 78}

$$\int \frac{1+e^x}{1-e^x} \, dx = x - 2\log(1-e^x)$$

```
[In] Int[(1 + E^x)/(1 - E^x),x]
```

```
[Out] x - 2*Log[1 - E^x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1+x}{(1-x)x}\,dx, x, e^x\right)\\ &= \text{Subst}\left(\int \left(-\frac{2}{-1+x}+\frac{1}{x}\right)\,dx, x, e^x\right)\\ &= x-2\log\left(1-e^x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1+e^x}{1-e^x}\,dx = \log\left(e^x\right)-2\log\left(-1+e^x\right)$$

```
[In] Integrate[(1 + E^x)/(1 - E^x),x]
[Out] Log[E^x] - 2*Log[-1 + E^x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

method	result	size
norman	$x-2\ln\left(-1+e^{x}\right)$	10
risch	$x-2\ln\left(-1+e^{x}\right)$	10
parallelrisch	$x-2\ln\left(-1+e^{x}\right)$	10
derivativedivides	$-2\ln\left(-1+e^{x}\right)+\ln\left(e^{x}\right)$	12
default	$-2\ln\left(-1+e^{x}\right)+\ln\left(e^{x}\right)$	12

```
[In] int((1+exp(x))/(1-exp(x)),x,method=_RETURNVERBOSE)
[Out] x-2*ln(-1+exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1+e^x}{1-e^x}\,dx = x - 2\,\log\left(e^x - 1\right)$$

```
[In] integrate((1+exp(x))/(1-exp(x)),x, algorithm="fricas")
[Out] x - 2*log(e^x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1+e^x}{1-e^x}\,dx = x - 2\log\left(e^x - 1\right)$$

```
[In] integrate((1+exp(x))/(1-exp(x)),x)
[Out] x - 2*log(exp(x) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1+e^x}{1-e^x}\,dx = x - 2\,\log\left(e^x - 1\right)$$

```
[In] integrate((1+exp(x))/(1-exp(x)),x, algorithm="maxima")
[Out] x - 2*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+e^x}{1-e^x}\,dx = x - 2\,\log\left(|e^x - 1|\right)$$

```
[In] integrate((1+exp(x))/(1-exp(x)),x, algorithm="giac")
[Out] x - 2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1+e^x}{1-e^x}\,dx = x - 2\,\ln\left(\mathrm{e}^x - 1\right)$$

```
[In] int(-(exp(x) + 1)/(exp(x) - 1),x)
[Out] x - 2*log(exp(x) - 1)
```

3.304 $\int \frac{x}{(1+x^2)(4+x^2)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \frac{x}{(1+x^2)(4+x^2)} \, dx = \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{6}\log\left(4+x^2\right)$$

```
[Out] 1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {455, 36, 31}

$$\int \frac{x}{(1+x^2)(4+x^2)} \, dx = \frac{1}{6}\log\left(x^2+1\right) - \frac{1}{6}\log\left(x^2+4\right)$$

```
[In] Int[x/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)(4+x)}\,dx, x, x^2\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) - \frac{1}{6}\text{Subst}\left(\int \frac{1}{4+x}\,dx, x, x^2\right)\\ &= \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{6}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x^2)(4+x^2)}\,dx = \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{6}\log\left(4+x^2\right)$$

```
[In] Integrate[x/((1 + x^2)*(4 + x^2)),x]
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\ln(x^2+1)}{6} - \frac{\ln(x^2+4)}{6}$	18
norman	$\frac{\ln(x^2+1)}{6} - \frac{\ln(x^2+4)}{6}$	18
risch	$\frac{\ln(x^2+1)}{6} - \frac{\ln(x^2+4)}{6}$	18
parallelrisch	$\frac{\ln(x^2+1)}{6} - \frac{\ln(x^2+4)}{6}$	18

```
[In] int(x/(x^2+1)/(x^2+4),x,method=_RETURNVERBOSE)
[Out] 1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="fricas")
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{x}{(1+x^2)(4+x^2)}\,dx = \frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^2+4\right)}{6}$$

```
[In] integrate(x/(x**2+1)/(x**2+4),x)
[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="maxima")
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="giac")
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{(1+x^2)(4+x^2)}\,dx = \frac{\operatorname{atanh}\left(\frac{3x^2}{5x^2+8}\right)}{3}$$

```
[In] int(x/((x^2 + 1)*(x^2 + 4)),x)
[Out] atanh((3*x^2)/(5*x^2 + 8))/3
```

3.305 $\int \frac{1}{4-5\sin(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \frac{1}{4-5\sin(x)}\,dx = -\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right)+\frac{1}{3}\log\left(2\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

```
[Out] -1/3*ln(cos(1/2*x)-2*sin(1/2*x))+1/3*ln(2*cos(1/2*x)-sin(1/2*x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2739, 630, 31}

$$\int \frac{1}{4-5\sin(x)}\,dx = \frac{1}{3}\log\left(2\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right)$$

```
[In] Int[(4 - 5*Sin[x])^(-1),x]
```

```
[Out] -1/3*Log[Cos[x/2] - 2*Sin[x/2]] + Log[2*Cos[x/2] - Sin[x/2]]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{1}{4-10x+4x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= \frac{4}{3}\text{Subst}\left(\int \frac{1}{-8+4x}\,dx, x, \tan\left(\frac{x}{2}\right)\right) - \frac{4}{3}\text{Subst}\left(\int \frac{1}{-2+4x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= -\frac{1}{3}\log\left(1-2\tan\left(\frac{x}{2}\right)\right) + \frac{1}{3}\log\left(2-\tan\left(\frac{x}{2}\right)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{4-5\sin(x)}\,dx = -\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right) + \frac{1}{3}\log\left(2\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[(4 - 5*Sin[x])^(-1),x]
[Out] -1/3*Log[Cos[x/2] - 2*Sin[x/2]] + Log[2*Cos[x/2] - Sin[x/2]]/3
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.51

method	result	size
default	$-\frac{\ln\left(2\tan\left(\frac{x}{2}\right)-1\right)}{3}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)-2\right)}{3}$	22
norman	$-\frac{\ln\left(2\tan\left(\frac{x}{2}\right)-1\right)}{3}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)-2\right)}{3}$	22
parallelrisch	$\ln\left(\left(2\tan\left(\frac{x}{2}\right)-4\right)^{\frac{1}{3}}\right)+\ln\left(\frac{1}{\left(2\tan\left(\frac{x}{2}\right)-1\right)^{\frac{1}{3}}}\right)$	24
risch	$-\frac{\ln\left(e^{ix}-\frac{3}{5}-\frac{4i}{5}\right)}{3}+\frac{\ln\left(e^{ix}+\frac{3}{5}-\frac{4i}{5}\right)}{3}$	26

```
[In] int(1/(4-5*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*ln(2*tan(1/2*x)-1)+1/3*ln(tan(1/2*x)-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{1}{4-5\sin(x)}\,dx = \frac{1}{6}\log\left(\frac{3}{2}\cos(x) - 2\sin(x) + \frac{5}{2}\right) - \frac{1}{6}\log\left(-\frac{3}{2}\cos(x) - 2\sin(x) + \frac{5}{2}\right)$$

```
[In] integrate(1/(4-5*sin(x)),x, algorithm="fricas")
[Out] 1/6*log(3/2*cos(x) - 2*sin(x) + 5/2) - 1/6*log(-3/2*cos(x) - 2*sin(x) + 5/2
)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.47

$$\int \frac{1}{4-5\sin(x)}\,dx = \frac{\log\left(\tan\left(\frac{x}{2}\right) - 2\right)}{3} - \frac{\log\left(2\tan\left(\frac{x}{2}\right) - 1\right)}{3}$$

```
[In] integrate(1/(4-5*sin(x)),x)
[Out] log(tan(x/2) - 2)/3 - log(2*tan(x/2) - 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{1}{4-5\sin(x)}\,dx = -\frac{1}{3}\log\left(\frac{2\sin(x)}{\cos(x)+1} - 1\right) + \frac{1}{3}\log\left(\frac{\sin(x)}{\cos(x)+1} - 2\right)$$

```
[In] integrate(1/(4-5*sin(x)),x, algorithm="maxima")
[Out] -1/3*log(2*sin(x)/(cos(x) + 1) - 1) + 1/3*log(sin(x)/(cos(x) + 1) - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.53

$$\int \frac{1}{4-5\sin(x)}\,dx = -\frac{1}{3}\log\left(\left|2\tan\left(\frac{1}{2}x\right)-1\right|\right)+\frac{1}{3}\log\left(\left|\tan\left(\frac{1}{2}x\right)-2\right|\right)$$

```
[In] integrate(1/(4-5*sin(x)),x, algorithm="giac")
[Out] -1/3*log(abs(2*tan(1/2*x) - 1)) + 1/3*log(abs(tan(1/2*x) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.26

$$\int \frac{1}{4-5\sin(x)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{4\tan\left(\frac{x}{2}\right)}{3}-\frac{5}{3}\right)}{3}$$

```
[In] int(-1/(5*sin(x) - 4),x)
[Out] -(2*atanh((4*tan(x/2))/3 - 5/3))/3
```

3.306 $\int x\sqrt[3]{c+x}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int x\sqrt[3]{c+x}\,dx=-\frac{3}{4}c(c+x)^{4/3}+\frac{3}{7}(c+x)^{7/3}$$

```
[Out] -3/4*c*(c+x)^(4/3)+3/7*(c+x)^(7/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\int x\sqrt[3]{c+x}\,dx=\frac{3}{7}(c+x)^{7/3}-\frac{3}{4}c(c+x)^{4/3}$$

```
[In] Int[x*(c + x)^(1/3),x]
```

```
[Out] (-3*c*(c + x)^(4/3))/4 + (3*(c + x)^(7/3))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(-c\sqrt[3]{c+x}+(c+x)^{4/3}\right)dx\\&=-\frac{3}{4}c(c+x)^{4/3}+\frac{3}{7}(c+x)^{7/3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int x\sqrt[3]{c+x}\,dx=\frac{3}{28}(c+x)^{4/3}(-3c+4x)$$

```
[In] Integrate[x*(c + x)^(1/3),x]
[Out] (3*(c + x)^(4/3)*(-3*c + 4*x))/28
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{3(c+x)^{\frac{4}{3}}(3c-4x)}{28}$	15
derivativedivides	$-\frac{3c(c+x)^{\frac{4}{3}}}{4}+\frac{3(c+x)^{\frac{7}{3}}}{7}$	17
default	$-\frac{3c(c+x)^{\frac{4}{3}}}{4}+\frac{3(c+x)^{\frac{7}{3}}}{7}$	17
trager	$\left(-\frac{9}{28}c^{2}+\frac{3}{28}cx+\frac{3}{7}x^{2}\right)(c+x)^{\frac{1}{3}}$	22
risch	$-\frac{3(c+x)^{\frac{1}{3}}(3c^{2}-cx-4x^{2})}{28}$	23

```
[In] int(x*(c+x)^(1/3),x,method=_RETURNVERBOSE)
[Out] -3/28*(c+x)^(4/3)*(3*c-4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x\sqrt[3]{c+x}\,dx=-\frac{3}{28}\left(3\,c^{2}-cx-4\,x^{2}\right)(c+x)^{\frac{1}{3}}$$

```
[In] integrate(x*(c+x)^(1/3),x, algorithm="fricas")
[Out] -3/28*(3*c^2 - c*x - 4*x^2)*(c + x)^(1/3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(20) = 40$.

Time = 0.63 (sec) , antiderivative size = 144, normalized size of antiderivative = 6.00

$$\int x\sqrt[3]{c+x}\,dx = -\frac{9c^{\frac{13}{3}}\sqrt[3]{1+\frac{x}{c}}}{28c^2+28cx} + \frac{9c^{\frac{13}{3}}}{28c^2+28cx} - \frac{6c^{\frac{10}{3}}x\sqrt[3]{1+\frac{x}{c}}}{28c^2+28cx} + \frac{9c^{\frac{10}{3}}x}{28c^2+28cx} + \frac{15c^{\frac{7}{3}}x^2\sqrt[3]{1+\frac{x}{c}}}{28c^2+28cx} + \frac{12c^{\frac{4}{3}}x^3\sqrt[3]{1+\frac{x}{c}}}{28c^2+28cx}$$

```
[In] integrate(x*(c+x)**(1/3),x)
```

```
[Out] -9*c**(13/3)*(1 + x/c)**(1/3)/(28*c**2 + 28*c*x) + 9*c**(13/3)/(28*c**2 + 2
8*c*x) - 6*c**(10/3)*x*(1 + x/c)**(1/3)/(28*c**2 + 28*c*x) + 9*c**(10/3)*x/
(28*c**2 + 28*c*x) + 15*c**(7/3)*x**2*(1 + x/c)**(1/3)/(28*c**2 + 28*c*x) +
 12*c**(4/3)*x**3*(1 + x/c)**(1/3)/(28*c**2 + 28*c*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int x\sqrt[3]{c+x}\,dx = \frac{3}{7}(c+x)^{\frac{7}{3}} - \frac{3}{4}(c+x)^{\frac{4}{3}}c$$

```
[In] integrate(x*(c+x)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/7*(c + x)^(7/3) - 3/4*(c + x)^(4/3)*c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(16) = 32$.

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int x\sqrt[3]{c+x}\,dx = \frac{3}{7}(c+x)^{\frac{7}{3}} - \frac{3}{2}(c+x)^{\frac{4}{3}}c + 3(c+x)^{\frac{1}{3}}c^2 + \frac{3}{4}\left((c+x)^{\frac{4}{3}} - 4(c+x)^{\frac{1}{3}}c\right)c$$

```
[In] integrate(x*(c+x)^(1/3),x, algorithm="giac")
```

```
[Out] 3/7*(c + x)^(7/3) - 3/2*(c + x)^(4/3)*c + 3*(c + x)^(1/3)*c^2 + 3/4*((c + x
)^(4/3) - 4*(c + x)^(1/3)*c)*c
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int x\sqrt[3]{c+x}\,dx=-\frac{3\,{\left(c+x\right)}^{4/3}\,\left(3\,c-4\,x\right)}{28}$$

```
[In] int(x*(c + x)^(1/3),x)
[Out] -(3*(c + x)^(4/3)*(3*c - 4*x))/28
```

3.307 $\int e^{\sqrt[3]{x}}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 38

$$\int e^{\sqrt[3]{x}}\,dx = 6e^{\sqrt[3]{x}} - 6e^{\sqrt[3]{x}}\sqrt[3]{x} + 3e^{\sqrt[3]{x}}x^{2/3}$$

```
[Out] 6*exp(x^(1/3))-6*exp(x^(1/3))*x^(1/3)+3*exp(x^(1/3))*x^(2/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2238, 2207, 2225}

$$\int e^{\sqrt[3]{x}}\,dx = 3e^{\sqrt[3]{x}}x^{2/3} - 6e^{\sqrt[3]{x}}\sqrt[3]{x} + 6e^{\sqrt[3]{x}}$$

```
[In] Int[E^x^(1/3),x]
```

```
[Out] 6*E^x^(1/3) - 6*E^x^(1/3)*x^(1/3) + 3*E^x^(1/3)*x^(2/3)
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2238

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Dist[k/d, Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (c
 + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Inte
gerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 3\text{Subst}\left(\int e^x x^2\,dx, x, \sqrt[3]{x}\right)\\ &= 3e^{\sqrt[3]{x}}x^{2/3} - 6\text{Subst}\left(\int e^x x\,dx, x, \sqrt[3]{x}\right)\\ &= -6e^{\sqrt[3]{x}}\sqrt[3]{x} + 3e^{\sqrt[3]{x}}x^{2/3} + 6\text{Subst}\left(\int e^x\,dx, x, \sqrt[3]{x}\right)\\ &= 6e^{\sqrt[3]{x}} - 6e^{\sqrt[3]{x}}\sqrt[3]{x} + 3e^{\sqrt[3]{x}}x^{2/3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int e^{\sqrt[3]{x}}\,dx = e^{\sqrt[3]{x}}\left(6 - 6\sqrt[3]{x} + 3x^{2/3}\right)$$

```
[In] Integrate[E^x^(1/3),x]
[Out] E^x^(1/3)*(6 - 6*x^(1/3) + 3*x^(2/3))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

method	result	size
meijerg	$-6+\left(3x^{\frac{2}{3}}-6x^{\frac{1}{3}}+6\right)\mathrm{e}^{x^{\frac{1}{3}}}$	20
derivativedivides	$6\,\mathrm{e}^{x^{\frac{1}{3}}}-6\,\mathrm{e}^{x^{\frac{1}{3}}}x^{\frac{1}{3}}+3\,\mathrm{e}^{x^{\frac{1}{3}}}x^{\frac{2}{3}}$	26
default	$6\,\mathrm{e}^{x^{\frac{1}{3}}}-6\,\mathrm{e}^{x^{\frac{1}{3}}}x^{\frac{1}{3}}+3\,\mathrm{e}^{x^{\frac{1}{3}}}x^{\frac{2}{3}}$	26

```
[In] int(exp(x^(1/3)),x,method=_RETURNVERBOSE)
[Out] -6+(3*x^(2/3)-6*x^(1/3)+6)*exp(x^(1/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.42

$$\int e^{\sqrt[3]{x}}\,dx = 3\left(x^{\frac{2}{3}} - 2\,x^{\frac{1}{3}} + 2\right)e^{\left(x^{\frac{1}{3}}\right)}$$

```
[In] integrate(exp(x^(1/3)),x, algorithm="fricas")
[Out] 3*(x^(2/3) - 2*x^(1/3) + 2)*e^(x^(1/3))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int e^{\sqrt[3]{x}}\,dx = 3x^{\frac{2}{3}}e^{\sqrt[3]{x}} - 6\sqrt[3]{x}e^{\sqrt[3]{x}} + 6e^{\sqrt[3]{x}}$$

```
[In] integrate(exp(x**(1/3)),x)
[Out] 3*x**(2/3)*exp(x**(1/3)) - 6*x**(1/3)*exp(x**(1/3)) + 6*exp(x**(1/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.42

$$\int e^{\sqrt[3]{x}}\,dx = 3\left(x^{\frac{2}{3}} - 2\,x^{\frac{1}{3}} + 2\right)e^{\left(x^{\frac{1}{3}}\right)}$$

```
[In] integrate(exp(x^(1/3)),x, algorithm="maxima")
[Out] 3*(x^(2/3) - 2*x^(1/3) + 2)*e^(x^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.42

$$\int e^{\sqrt[3]{x}}\,dx = 3\left(x^{\frac{2}{3}} - 2\,x^{\frac{1}{3}} + 2\right)e^{\left(x^{\frac{1}{3}}\right)}$$

```
[In] integrate(exp(x^(1/3)),x, algorithm="giac")
[Out] 3*(x^(2/3) - 2*x^(1/3) + 2)*e^(x^(1/3))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

$$\int e^{\sqrt[3]{x}}\,dx = 3\,x\,\mathrm{e}^{x^{1/3}}\,\left(\frac{2}{x}+\frac{1}{x^{1/3}}-\frac{2}{x^{2/3}}\right)$$

```
[In] int(exp(x^(1/3)),x)
[Out] 3*x*exp(x^(1/3))*(2/x + 1/x^(1/3) - 2/x^(2/3))
```

3.308 $\int \frac{1}{4+x+\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = -\frac{2\arctan\left(\frac{1+2\sqrt{1+x}}{\sqrt{11}}\right)}{\sqrt{11}} + \log\left(4+x+\sqrt{1+x}\right)$$

```
[Out] ln(4+x+(1+x)^(1/2))-2/11*arctan(1/11*(1+2*(1+x)^(1/2))*11^(1/2))*11^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {648, 632, 210, 642}

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = \log\left(x+\sqrt{x+1}+4\right) - \frac{2\arctan\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

```
[In] Int[(4 + x + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= -\text{Subst}\left(\int \frac{1}{3+x+x^2}\,dx, x, \sqrt{1+x}\right) + \text{Subst}\left(\int \frac{1+2x}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= \log\left(4+x+\sqrt{1+x}\right) + 2\text{Subst}\left(\int \frac{1}{-11-x^2}\,dx, x, 1+2\sqrt{1+x}\right)\\
&= -\frac{2\arctan\left(\frac{1+2\sqrt{1+x}}{\sqrt{11}}\right)}{\sqrt{11}} + \log\left(4+x+\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = -\frac{2\arctan\left(\frac{1+2\sqrt{1+x}}{\sqrt{11}}\right)}{\sqrt{11}} + \log\left(4+x+\sqrt{1+x}\right)$$

```
[In] Integrate[(4 + x + Sqrt[1 + x])^(-1),x]
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\ln\left(4+x+\sqrt{1+x}\right)-\frac{2\arctan\left(\frac{(1+2\sqrt{1+x})\sqrt{11}}{11}\right)\sqrt{11}}{11}$
default	$-\frac{\ln(4+x-\sqrt{1+x})}{2}-\frac{\sqrt{11}\,\arctan\left(\frac{(2\sqrt{1+x}-1)\sqrt{11}}{11}\right)}{11}+\frac{\ln(4+x+\sqrt{1+x})}{2}-\frac{\arctan\left(\frac{(1+2\sqrt{1+x})\sqrt{11}}{11}\right)\sqrt{11}}{11}+\frac{\sqrt{11}}{}$
trager	$-\ln\left(4+x+\sqrt{1+x}\right)\mathrm{RootOf}\left(11\_Z^2-22\_Z+12\right)+\ln\Big(-847\,\mathrm{RootOf}\left(11\_Z^2-2\right.$

[In] int(1/(4+x+(1+x)^(1/2)),x,method=_RETURNVERBOSE)

[Out] ln(4+x+(1+x)^(1/2))-2/11*arctan(1/11*(1+2*(1+x)^(1/2))*11^(1/2))*11^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = -\frac{2}{11}\sqrt{11}\arctan\left(\frac{2}{11}\sqrt{11}\sqrt{x+1}+\frac{1}{11}\sqrt{11}\right)+\log\left(x+\sqrt{x+1}+4\right)$$

[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="fricas")

[Out] -2/11*sqrt(11)*arctan(2/11*sqrt(11)*sqrt(x + 1) + 1/11*sqrt(11)) + log(x + sqrt(x + 1) + 4)

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = \log\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\,\mathrm{atan}\left(\frac{2\sqrt{11}\left(\sqrt{x+1}+\frac{1}{2}\right)}{11}\right)}{11}$$

[In] integrate(1/(4+x+(1+x)**(1/2)),x)

[Out] log(x + sqrt(x + 1) + 4) - 2*sqrt(11)*atan(2*sqrt(11)*(sqrt(x + 1) + 1/2)/1
1)/11

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = -\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="maxima")
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x +
 1) + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = -\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="giac")
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x +
 1) + 4)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = \ln\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\,\mathrm{atan}\left(\frac{\sqrt{11}}{11}+\frac{2\sqrt{11}\sqrt{x+1}}{11}\right)}{11}$$

```
[In] int(1/(x + (x + 1)^(1/2) + 4),x)
[Out] log(x + (x + 1)^(1/2) + 4) - (2*11^(1/2)*atan(11^(1/2)/11 + (2*11^(1/2)*(x
+ 1)^(1/2))/11))/11
```

3.309 $\int \frac{1+x^3}{-x^2+x^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1+x^3}{-x^2+x^3} dx = \frac{1}{x} + x + 2\log(1-x) - \log(x)$$

```
[Out] 1/x+x+2*ln(1-x)-ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1607, 1634}

$$\int \frac{1+x^3}{-x^2+x^3} dx = x + \frac{1}{x} + 2\log(1-x) - \log(x)$$

```
[In] Int[(1 + x^3)/(-x^2 + x^3),x]
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
```

```
xpon[Px, x], 2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1+x^3}{(-1+x)x^2}\,dx \\
&= \int \left(1+\frac{2}{-1+x}-\frac{1}{x^2}-\frac{1}{x}\right)dx \\
&= \frac{1}{x}+x+2\log(1-x)-\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1+x^3}{-x^2+x^3}\,dx=\frac{1}{x}+x+2\log(1-x)-\log(x)$$

```
[In] Integrate[(1 + x^3)/(-x^2 + x^3),x]
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$x+2\ln\left(-1+x\right)+\frac{1}{x}-\ln\left(x\right)$	16
risch	$x+2\ln\left(-1+x\right)+\frac{1}{x}-\ln\left(x\right)$	16
norman	$\frac{x^2+1}{x}-\ln\left(x\right)+2\ln\left(-1+x\right)$	21
meijerg	$\frac{1}{x}-\ln\left(x\right)-i\pi+2\ln\left(1-x\right)+x$	22
parallelrisch	$-\frac{x\ln(x)-2\ln(-1+x)x-x^2-1}{x}$	24

```
[In] int((x^3+1)/(x^3-x^2),x,method=_RETURNVERBOSE)
[Out] x+2*ln(-1+x)+1/x-ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{1+x^3}{-x^2+x^3}\,dx = \frac{x^2+2\,x\log\left(x-1\right)-x\log\left(x\right)+1}{x}$$

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="fricas")
[Out] (x^2 + 2*x*log(x - 1) - x*log(x) + 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3}{-x^2+x^3}\,dx = x-\log\left(x\right)+2\log\left(x-1\right)+\frac{1}{x}$$

```
[In] integrate((x**3+1)/(x**3-x**2),x)
[Out] x - log(x) + 2*log(x - 1) + 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^3}{-x^2+x^3}\,dx = x+\frac{1}{x}+2\,\log\left(x-1\right)-\log\left(x\right)$$

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="maxima")
[Out] x + 1/x + 2*log(x - 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1+x^3}{-x^2+x^3}\,dx = x+\frac{1}{x}+2\,\log\left(|x-1|\right)-\log\left(|x|\right)$$

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="giac")
[Out] x + 1/x + 2*log(abs(x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^3}{-x^2+x^3}\,dx = x+2\,\ln\left(x-1\right)-\ln\left(x\right)+\frac{1}{x}$$

```
[In] int(-(x^3 + 1)/(x^2 - x^3),x)
[Out] x + 2*log(x - 1) - log(x) + 1/x
```

3.310 $\int (-3+4x+x^2)\sin(2x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int (-3+4x+x^2)\sin(2x)\,dx=\frac{7}{4}\cos(2x)-2x\cos(2x)-\frac{1}{2}x^2\cos(2x)+\sin(2x)+\frac{1}{2}x\sin(2x)$$

```
[Out] 7/4*cos(2*x)-2*x*cos(2*x)-1/2*x^2*cos(2*x)+sin(2*x)+1/2*x*sin(2*x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6874, 2718, 3377, 2717}

$$\int (-3+4x+x^2)\sin(2x)\,dx=-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\sin(2x)-2x\cos(2x)+\frac{7}{4}\cos(2x)$$

```
[In] Int[(-3 + 4*x + x^2)*Sin[2*x],x]
```

```
[Out] (7*Cos[2*x])/4 - 2*x*Cos[2*x] - (x^2*Cos[2*x])/2 + Sin[2*x] + (x*Sin[2*x])/
2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-3\sin(2x)+4x\sin(2x)+x^2\sin(2x)\right)\,dx \\
&= -(3\int \sin(2x)\,dx)+4\int x\sin(2x)\,dx+\int x^2\sin(2x)\,dx \\
&= \frac{3}{2}\cos(2x)-2x\cos(2x)-\frac{1}{2}x^2\cos(2x)+2\int \cos(2x)\,dx+\int x\cos(2x)\,dx \\
&= \frac{3}{2}\cos(2x)-2x\cos(2x)-\frac{1}{2}x^2\cos(2x)+\sin(2x)+\frac{1}{2}x\sin(2x)-\frac{1}{2}\int \sin(2x)\,dx \\
&= \frac{7}{4}\cos(2x)-2x\cos(2x)-\frac{1}{2}x^2\cos(2x)+\sin(2x)+\frac{1}{2}x\sin(2x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \left(-3+4x+x^2\right)\sin(2x)\,dx=\frac{1}{4}\left(\left(7-8x-2x^2\right)\cos(2x)+2(2+x)\sin(2x)\right)$$

```
[In] Integrate[(-3 + 4*x + x^2)*Sin[2*x],x]
[Out] ((7 - 8*x - 2*x^2)*Cos[2*x] + 2*(2 + x)*Sin[2*x])/4
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

method	result	size
risch	$\left(-\frac{1}{2}x^2-2x+\frac{7}{4}\right)\cos\left(2x\right)+\frac{(2+x)\sin(2x)}{2}$	26
derivativedivides	$\frac{7\cos(2x)}{4}-2x\cos\left(2x\right)-\frac{x^2\cos(2x)}{2}+\sin\left(2x\right)+\frac{x\sin(2x)}{2}$	35
default	$\frac{7\cos(2x)}{4}-2x\cos\left(2x\right)-\frac{x^2\cos(2x)}{2}+\sin\left(2x\right)+\frac{x\sin(2x)}{2}$	35
parts	$\frac{7\cos(2x)}{4}-2x\cos\left(2x\right)-\frac{x^2\cos(2x)}{2}+\sin\left(2x\right)+\frac{x\sin(2x)}{2}$	35
parallelrisch	$\frac{(x^2+4x)\left(\tan^2(x)\right)+(2x+4)\tan(x)-x^2-4x+7}{2+2\left(\tan^2(x)\right)}$	42
norman	$\frac{x\tan(x)-2x-\frac{x^2}{2}+2x\left(\tan^2(x)\right)+\frac{x^2\left(\tan^2(x)\right)}{2}+2\tan(x)+\frac{7}{2}}{1+\tan^2(x)}$	44
meijerg	$\frac{\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-2x^2+1\right)\cos(2x)}{2\sqrt{\pi}}+\frac{x\sin(2x)}{\sqrt{\pi}}\right)}{2}+2\sqrt{\pi}\left(-\frac{x\cos(2x)}{\sqrt{\pi}}+\frac{\sin(2x)}{2\sqrt{\pi}}\right)-\frac{3\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{2}$	81

```
[In] int((x^2+4*x-3)*sin(2*x),x,method=_RETURNVERBOSE)
[Out] (-1/2*x^2-2*x+7/4)*cos(2*x)+1/2*(2+x)*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int\left(-3+4x+x^2\right)\sin(2x)\,dx=-\frac{1}{4}\left(2\,x^2+8\,x-7\right)\cos\left(2\,x\right)+\frac{1}{2}\left(x+2\right)\sin\left(2\,x\right)$$

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="fricas")
[Out] -1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int\left(-3+4x+x^2\right)\sin(2x)\,dx=-\frac{x^2\cos\left(2x\right)}{2}+\frac{x\sin\left(2x\right)}{2}-2x\cos\left(2x\right)+\sin\left(2x\right)+\frac{7\cos\left(2x\right)}{4}$$

```
[In] integrate((x**2+4*x-3)*sin(2*x),x)
[Out] -x**2*cos(2*x)/2 + x*sin(2*x)/2 - 2*x*cos(2*x) + sin(2*x) + 7*cos(2*x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int (-3+4x+x^2)\sin(2x)\,dx = -\frac{1}{4}\left(2\,x^2-1\right)\cos\left(2\,x\right)-2\,x\cos\left(2\,x\right)+\frac{1}{2}\,x\sin\left(2\,x\right)+\frac{3}{2}\,\cos\left(2\,x\right)+\sin\left(2\,x\right)$$

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="maxima")
[Out] -1/4*(2*x^2 - 1)*cos(2*x) - 2*x*cos(2*x) + 1/2*x*sin(2*x) + 3/2*cos(2*x) +
sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int (-3+4x+x^2)\sin(2x)\,dx = -\frac{1}{4}\left(2\,x^2+8\,x-7\right)\cos\left(2\,x\right)+\frac{1}{2}\left(x+2\right)\sin\left(2\,x\right)$$

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="giac")
[Out] -1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int (-3+4x+x^2)\sin(2x)\,dx = \frac{7\,\cos\left(2\,x\right)}{4}+\sin\left(2\,x\right)-2\,x\,\cos\left(2\,x\right)+\frac{x\,\sin\left(2\,x\right)}{2}-\frac{x^2\,\cos\left(2\,x\right)}{2}$$

```
[In] int(sin(2*x)*(4*x + x^2 - 3),x)
[Out] (7*cos(2*x))/4 + sin(2*x) - 2*x*cos(2*x) + (x*sin(2*x))/2 - (x^2*cos(2*x))/
2
```

3.311 $\int \cos(\cos(x))\sin(x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 5

$$\int \cos(\cos(x))\sin(x)\,dx = -\sin(\cos(x))$$

```
[Out] -sin(cos(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4420, 2717}

$$\int \cos(\cos(x))\sin(x)\,dx = -\sin(\cos(x))$$

```
[In] Int[Cos[Cos[x]]*Sin[x],x]
```

```
[Out] -Sin[Cos[x]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int \cos(x)\,dx, x, \cos(x)\right) \\ &= -\sin(\cos(x)) \end{aligned}$$

Mathematica [A] (verified)

Time = 4.17 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \cos(\cos(x))\sin(x)\,dx = -\sin(\cos(x))$$

```
[In] Integrate[Cos[Cos[x]]*Sin[x],x]
[Out] -Sin[Cos[x]]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$-\sin\left(\cos\left(x\right)\right)$	6
default	$-\sin\left(\cos\left(x\right)\right)$	6
risch	$-\sin\left(\cos\left(x\right)\right)$	6
parallelrisch	$-\sin\left(\cos\left(x\right)\right)$	6
norman	$\frac{-2\left(\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{1-\left(\tan^2\left(\frac{x}{2}\right)\right)}{2+2\left(\tan^2\left(\frac{x}{2}\right)\right)}\right)-2\tan\left(\frac{1-\left(\tan^2\left(\frac{x}{2}\right)\right)}{2+2\left(\tan^2\left(\frac{x}{2}\right)\right)}\right)}{\left(1+\tan^2\left(\frac{1-\left(\tan^2\left(\frac{x}{2}\right)\right)}{2\left(1+\tan^2\left(\frac{x}{2}\right)\right)}\right)\right)\left(1+\tan^2\left(\frac{x}{2}\right)\right)}$	98

```
[In] int(cos(cos(x))*sin(x),x,method=_RETURNVERBOSE)
[Out] -sin(cos(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(5) = 10$.

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 4.00

$$\int \cos(\cos(x))\sin(x)\,dx = \sin\left(\frac{\tan\left(\frac{1}{2}\,x\right)^2 - 1}{\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)$$

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="fricas")
[Out] sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \cos(\cos(x))\sin(x)\,dx = -\sin\left(\cos\left(x\right)\right)$$

```
[In] integrate(cos(cos(x))*sin(x),x)
[Out] -sin(cos(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \cos(\cos(x))\sin(x)\,dx = -\sin\left(\cos\left(x\right)\right)$$

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="maxima")
[Out] -sin(cos(x))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \cos(\cos(x))\sin(x)\,dx = -\sin\left(\cos\left(x\right)\right)$$

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="giac")
[Out] -sin(cos(x))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \cos(\cos(x))\sin(x)\,dx = -\sin\left(\cos\left(x\right)\right)$$

```
[In] int(cos(cos(x))*sin(x),x)
[Out] -sin(cos(x))
```

3.312 $\int \frac{1}{\sqrt{16-x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 6

$$\int \frac{1}{\sqrt{16-x^2}}\,dx = \arcsin\left(\frac{x}{4}\right)$$

```
[Out] arcsin(1/4*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {222}

$$\int \frac{1}{\sqrt{16-x^2}}\,dx = \arcsin\left(\frac{x}{4}\right)$$

```
[In] Int[1/Sqrt[16 - x^2],x]
```

```
[Out] ArcSin[x/4]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\text{integral} = \arcsin\left(\frac{x}{4}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(6) = 12.

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 3.33

$$\int \frac{1}{\sqrt{16-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{16-x^2}}{4+x}\right)$$

```
[In] Integrate[1/Sqrt[16 - x^2],x]
[Out] -2*ArcTan[Sqrt[16 - x^2]/(4 + x)]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

method	result	size
default	$\arcsin\left(\frac{x}{4}\right)$	5
meijerg	$\arcsin\left(\frac{x}{4}\right)$	5
pseudoelliptic	$-\arctan\left(\frac{\sqrt{-x^2+16}}{x}\right)$	17
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+16}+x\right)$	27

```
[In] int(1/(-x^2+16)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(1/4*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(4) = 8.

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{16-x^2}}\,dx = -2\ \arctan\left(\frac{\sqrt{-x^2+16}-4}{x}\right)$$

```
[In] integrate(1/(-x^2+16)^(1/2),x, algorithm="fricas")
[Out] -2*arctan((sqrt(-x^2 + 16) - 4)/x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{16-x^2}}\,dx = \operatorname{asin}\left(\frac{x}{4}\right)$$

```
[In] integrate(1/(-x**2+16)**(1/2),x)
[Out] asin(x/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{16-x^2}}\,dx = \arcsin\left(\frac{1}{4}\,x\right)$$

```
[In] integrate(1/(-x^2+16)^(1/2),x, algorithm="maxima")
[Out] arcsin(1/4*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(4) = 8$.

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 3.17

$$\int \frac{1}{\sqrt{16-x^2}}\,dx = \frac{1}{2}\,\sqrt{-x^2+16}x + 8\,\arcsin\left(\frac{1}{4}\,x\right)$$

```
[In] integrate(1/(-x^2+16)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 16)*x + 8*arcsin(1/4*x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{16-x^2}}\,dx = \operatorname{asin}\left(\frac{x}{4}\right)$$

```
[In] int(1/(16 - x^2)^(1/2),x)
[Out] asin(x/4)
```

3.313 $\int \frac{x^3}{(1+x)^{10}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 37

$$\int \frac{x^3}{(1+x)^{10}} \, dx = \frac{1}{9(1+x)^9} - \frac{3}{8(1+x)^8} + \frac{3}{7(1+x)^7} - \frac{1}{6(1+x)^6}$$

```
[Out] 1/9/(1+x)^9-3/8/(1+x)^8+3/7/(1+x)^7-1/6/(1+x)^6
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\int \frac{x^3}{(1+x)^{10}} \, dx = -\frac{1}{6(x+1)^6} + \frac{3}{7(x+1)^7} - \frac{3}{8(x+1)^8} + \frac{1}{9(x+1)^9}$$

```
[In] Int[x^3/(1 + x)^10,x]
```

```
[Out] 1/(9*(1 + x)^9) - 3/(8*(1 + x)^8) + 3/(7*(1 + x)^7) - 1/(6*(1 + x)^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\frac{1}{(1+x)^{10}} + \frac{3}{(1+x)^9} - \frac{3}{(1+x)^8} + \frac{1}{(1+x)^7}\right) \, dx \\ &= \frac{1}{9(1+x)^9} - \frac{3}{8(1+x)^8} + \frac{3}{7(1+x)^7} - \frac{1}{6(1+x)^6} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \frac{x^3}{(1+x)^{10}}\,dx = -\frac{1+9x+36x^2+84x^3}{504(1+x)^9}$$

```
[In] Integrate[x^3/(1 + x)^10,x]
[Out] -1/504*(1 + 9*x + 36*x^2 + 84*x^3)/(1 + x)^9
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

method	result	size
norman	$\frac{-\frac{1}{6}x^3-\frac{1}{14}x^2-\frac{1}{56}x-\frac{1}{504}}{(1+x)^9}$	22
risch	$\frac{-\frac{1}{6}x^3-\frac{1}{14}x^2-\frac{1}{56}x-\frac{1}{504}}{(1+x)^9}$	22
gosper	$-\frac{84x^3+36x^2+9x+1}{504(1+x)^9}$	23
parallelrisch	$\frac{-84x^3-36x^2-9x-1}{504(1+x)^9}$	23
default	$\frac{1}{9(1+x)^9}-\frac{3}{8(1+x)^8}+\frac{3}{7(1+x)^7}-\frac{1}{6(1+x)^6}$	30
meijerg	$\frac{x^4(x^5+9x^4+36x^3+84x^2+126x+126)}{504(1+x)^9}$	34

```
[In] int(x^3/(1+x)^10,x,method=_RETURNVERBOSE)
[Out] 1/(1+x)^9*(-1/6*x^3-1/14*x^2-1/56*x-1/504)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.68

$$\int \frac{x^3}{(1+x)^{10}}\,dx = -\frac{84\,x^3+36\,x^2+9\,x+1}{504\,(x^9+9\,x^8+36\,x^7+84\,x^6+126\,x^5+126\,x^4+84\,x^3+36\,x^2+9\,x+1)}$$

```
[In] integrate(x^3/(1+x)^10,x, algorithm="fricas")
[Out] -1/504*(84*x^3 + 36*x^2 + 9*x + 1)/(x^9 + 9*x^8 + 36*x^7 + 84*x^6 + 126*x^5
 + 126*x^4 + 84*x^3 + 36*x^2 + 9*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int \frac{x^3}{(1+x)^{10}}\,dx = \frac{-84x^3-36x^2-9x-1}{504x^9+4536x^8+18144x^7+42336x^6+63504x^5+63504x^4+42336x^3+18144x^2+4536x+504}$$

```
[In] integrate(x**3/(1+x)**10,x)
```

```
[Out] (-84*x**3 - 36*x**2 - 9*x - 1)/(504*x**9 + 4536*x**8 + 18144*x**7 + 42336*x
**6 + 63504*x**5 + 63504*x**4 + 42336*x**3 + 18144*x**2 + 4536*x + 504)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.68

$$\int \frac{x^3}{(1+x)^{10}}\,dx = -\frac{84\,x^3+36\,x^2+9\,x+1}{504\,(x^9+9\,x^8+36\,x^7+84\,x^6+126\,x^5+126\,x^4+84\,x^3+36\,x^2+9\,x+1)}$$

```
[In] integrate(x^3/(1+x)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(84*x^3 + 36*x^2 + 9*x + 1)/(x^9 + 9*x^8 + 36*x^7 + 84*x^6 + 126*x^5
 + 126*x^4 + 84*x^3 + 36*x^2 + 9*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int \frac{x^3}{(1+x)^{10}}\,dx = -\frac{84\,x^3+36\,x^2+9\,x+1}{504\,(x+1)^9}$$

```
[In] integrate(x^3/(1+x)^10,x, algorithm="giac")
```

```
[Out] -1/504*(84*x^3 + 36*x^2 + 9*x + 1)/(x + 1)^9
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{(1+x)^{10}}\,dx = \frac{3}{7\,(x+1)^7} - \frac{1}{6\,(x+1)^6} - \frac{3}{8\,(x+1)^8} + \frac{1}{9\,(x+1)^9}$$

```
[In] int(x^3/(x + 1)^10,x)
[Out] 3/(7*(x + 1)^7) - 1/(6*(x + 1)^6) - 3/(8*(x + 1)^8) + 1/(9*(x + 1)^9)
```

3.314 $\int \cot^3(2x)\csc^3(2x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \cot^3(2x)\csc^3(2x)\,dx = \frac{1}{6}\csc^3(2x) - \frac{1}{10}\csc^5(2x)$$

```
[Out] 1/6*csc(2*x)^3-1/10*csc(2*x)^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2686, 14}

$$\int \cot^3(2x)\csc^3(2x)\,dx = \frac{1}{6}\csc^3(2x) - \frac{1}{10}\csc^5(2x)$$

```
[In] Int[Cot[2*x]^3*Csc[2*x]^3,x]
```

```
[Out] Csc[2*x]^3/6 - Csc[2*x]^5/10
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\csc(2x)\right)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\csc(2x)\right)\right)\\ &= \frac{1}{6}\csc^3(2x)-\frac{1}{10}\csc^5(2x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \cot^3(2x)\csc^3(2x)\,dx = \frac{1}{6}\csc^3(2x)-\frac{1}{10}\csc^5(2x)$$

```
[In] Integrate[Cot[2*x]^3*Csc[2*x]^3,x]
[Out] Csc[2*x]^3/6 - Csc[2*x]^5/10
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{(\csc^3(2x))}{6}-\frac{(\csc^5(2x))}{10}$	18
default	$\frac{(\csc^3(2x))}{6}-\frac{(\csc^5(2x))}{10}$	18
risch	$-\frac{4i\left(5\,\mathrm{e}^{14ix}+2\,\mathrm{e}^{10ix}+5\,\mathrm{e}^{6ix}\right)}{15\left(\mathrm{e}^{4ix}-1\right)^5}$	35

```
[In] int(cot(2*x)^3*csc(2*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/6*csc(2*x)^3-1/10*csc(2*x)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.71

$$\int \cot^3(2x)\csc^3(2x)\,dx = -\frac{5\,\cos\left(2\,x\right)^2 - 2}{30\,\left(\cos\left(2\,x\right)^4 - 2\,\cos\left(2\,x\right)^2 + 1\right)\sin\left(2\,x\right)}$$

```
[In] integrate(cot(2*x)^3*csc(2*x)^3,x, algorithm="fricas")
[Out] -1/30*(5*cos(2*x)^2 - 2)/((cos(2*x)^4 - 2*cos(2*x)^2 + 1)*sin(2*x))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \cot^3(2x)\csc^3(2x)\,dx = -\frac{3 - 5\sin^2(2x)}{30\sin^5(2x)}$$

```
[In] integrate(cot(2*x)**3*csc(2*x)**3,x)
[Out] -(3 - 5*sin(2*x)**2)/(30*sin(2*x)**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \cot^3(2x)\csc^3(2x)\,dx = \frac{5\,\sin\left(2\,x\right)^2 - 3}{30\,\sin\left(2\,x\right)^5}$$

```
[In] integrate(cot(2*x)^3*csc(2*x)^3,x, algorithm="maxima")
[Out] 1/30*(5*sin(2*x)^2 - 3)/sin(2*x)^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \cot^3(2x)\csc^3(2x)\,dx = \frac{5\,\sin\left(2\,x\right)^2 - 3}{30\,\sin\left(2\,x\right)^5}$$

```
[In] integrate(cot(2*x)^3*csc(2*x)^3,x, algorithm="giac")
[Out] 1/30*(5*sin(2*x)^2 - 3)/sin(2*x)^5
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \cot^3(2x)\csc^3(2x)\,dx = \frac{5\sin\left(2\,x\right)^2-3}{30\sin\left(2\,x\right)^5}$$

```
[In] int(cot(2*x)^3/sin(2*x)^3,x)
[Out] (5*sin(2*x)^2 - 3)/(30*sin(2*x)^5)
```

3.315 $\int (x+\sin(x))^2 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 30

$$\int (x+\sin(x))^2 \, dx = \frac{x}{2}+\frac{x^3}{3}-2x\cos(x)+2\sin(x)-\frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x+1/3*x^3-2*x*cos(x)+2*sin(x)-1/2*cos(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6874, 3377, 2717, 2715, 8}

$$\int (x+\sin(x))^2 \, dx = \frac{x^3}{3}+\frac{x}{2}+2\sin(x)-2x\cos(x)-\frac{1}{2}\sin(x)\cos(x)$$

```
[In] Int[(x + Sin[x])^2,x]
[Out] x/2 + x^3/3 - 2*x*Cos[x] + 2*Sin[x] - (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(x^2+2x\sin(x)+\sin^2(x)\right)\,dx\\ &= \frac{x^3}{3}+2\int x\sin(x)\,dx+\int \sin^2(x)\,dx\\ &= \frac{x^3}{3}-2x\cos(x)-\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}+2\int \cos(x)\,dx\\ &= \frac{x}{2}+\frac{x^3}{3}-2x\cos(x)+2\sin(x)-\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (x+\sin(x))^2\,dx=\frac{1}{6}x\left(3+2x^2\right)-2x\cos(x)+2\sin(x)-\frac{1}{4}\sin(2x)$$

```
[In] Integrate[(x + Sin[x])^2,x]
[Out] (x*(3 + 2*x^2))/6 - 2*x*Cos[x] + 2*Sin[x] - Sin[2*x]/4
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x}{2}+\frac{x^{3}}{3}-2x\cos\left(x\right)+2\sin\left(x\right)-\frac{\cos\left(x\right)\sin\left(x\right)}{2}$	25
risch	$\frac{x^{3}}{3}+\frac{x}{2}-2x\cos\left(x\right)+2\sin\left(x\right)-\frac{\sin\left(2x\right)}{4}$	25
parallelrisch	$\frac{x^{3}}{3}+\frac{x}{2}-2x\cos\left(x\right)+2\sin\left(x\right)-\frac{\sin\left(2x\right)}{4}$	25
parts	$\frac{x}{2}+\frac{x^{3}}{3}-2x\cos\left(x\right)+2\sin\left(x\right)-\frac{\cos\left(x\right)\sin\left(x\right)}{2}$	25
norman	$\frac{x\left(\tan^{2}\left(\frac{x}{2}\right)\right)-\frac{3x}{2}+\frac{x^{3}}{3}+5\left(\tan^{3}\left(\frac{x}{2}\right)\right)+\frac{5x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{2}+\frac{2x^{3}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{3}+\frac{x^{3}\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{3}+3\tan\left(\frac{x}{2}\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	74

```
[In] int((x+sin(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/3*x^3-2*x*cos(x)+2*sin(x)-1/2*cos(x)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int (x+\sin(x))^2\,dx=\frac{1}{3}\,x^{3}-2\,x\cos\left(x\right)-\frac{1}{2}\left(\cos\left(x\right)-4\right)\sin\left(x\right)+\frac{1}{2}\,x$$

```
[In] integrate((x+sin(x))^2,x, algorithm="fricas")
[Out] 1/3*x^3 - 2*x*cos(x) - 1/2*(cos(x) - 4)*sin(x) + 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int (x+\sin(x))^2\,dx=\frac{x^{3}}{3}+\frac{x\sin^{2}\left(x\right)}{2}+\frac{x\cos^{2}\left(x\right)}{2}-2x\cos\left(x\right)-\frac{\sin\left(x\right)\cos\left(x\right)}{2}+2\sin\left(x\right)$$

```
[In] integrate((x+sin(x))**2,x)
[Out] x**3/3 + x*sin(x)**2/2 + x*cos(x)**2/2 - 2*x*cos(x) - sin(x)*cos(x)/2 + 2*s
in(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (x+\sin(x))^2\,dx = \frac{1}{3}\,x^3 - 2\,x\cos\left(x\right) + \frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right) + 2\,\sin\left(x\right)$$

```
[In] integrate((x+sin(x))^2,x, algorithm="maxima")
[Out] 1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (x+\sin(x))^2\,dx = \frac{1}{3}\,x^3 - 2\,x\cos\left(x\right) + \frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right) + 2\,\sin\left(x\right)$$

```
[In] integrate((x+sin(x))^2,x, algorithm="giac")
[Out] 1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (x+\sin(x))^2\,dx = \frac{x}{2} + 2\,\sin\left(x\right) - \frac{\cos\left(x\right)\,\sin\left(x\right)}{2} - 2\,x\,\cos\left(x\right) + \frac{x^3}{3}$$

```
[In] int((x + sin(x))^2,x)
[Out] x/2 + 2*sin(x) - (cos(x)*sin(x))/2 - 2*x*cos(x) + x^3/3
```

3.316 $\int \frac{e^{\arctan(x)}}{1+x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 4

$$\int \frac{e^{\arctan(x)}}{1+x^2} dx = e^{\arctan(x)}$$

```
[Out] exp(arctan(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5179}

$$\int \frac{e^{\arctan(x)}}{1+x^2} dx = e^{\arctan(x)}$$

```
[In] Int[E^ArcTan[x]/(1 + x^2),x]
```

```
[Out] E^ArcTan[x]
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\text{integral} = e^{\arctan(x)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 6.75

$$\int \frac{e^{\arctan(x)}}{1+x^2}\,dx = (1-ix)^{\frac{i}{2}}(1+ix)^{-\frac{i}{2}}$$

```
[In] Integrate[E^ArcTan[x]/(1 + x^2),x]
[Out] (1 - I*x)^(I/2)/(1 + I*x)^(I/2)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
gosper	$\mathrm{e}^{\arctan(x)}$	4
derivativedivides	$\mathrm{e}^{\arctan(x)}$	4
default	$\mathrm{e}^{\arctan(x)}$	4
parallelrisch	$\mathrm{e}^{\arctan(x)}$	4
risch	$(-ix+1)^{\frac{i}{2}}\,(ix+1)^{-\frac{i}{2}}$	20

```
[In] int(exp(arctan(x))/(x^2+1),x,method=_RETURNVERBOSE)
[Out] exp(arctan(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^{\arctan(x)}}{1+x^2}\,dx = e^{\arctan(x)}$$

```
[In] integrate(exp(arctan(x))/(x^2+1),x, algorithm="fricas")
[Out] e^arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^{\arctan(x)}}{1+x^2}\,dx = e^{\operatorname{atan}(x)}$$

```
[In] integrate(exp(atan(x))/(x**2+1),x)
[Out] exp(atan(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^{\arctan(x)}}{1+x^2}\,dx = e^{\arctan(x)}$$

```
[In] integrate(exp(arctan(x))/(x^2+1),x, algorithm="maxima")
[Out] e^arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^{\arctan(x)}}{1+x^2}\,dx = e^{\arctan(x)}$$

```
[In] integrate(exp(arctan(x))/(x^2+1),x, algorithm="giac")
[Out] e^arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^{\arctan(x)}}{1+x^2}\,dx = \mathrm{e}^{\mathrm{atan}(x)}$$

```
[In] int(exp(atan(x))/(x^2 + 1),x)
[Out] exp(atan(x))
```

3.317 $\int \frac{1}{x(1+x^4)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{1}{x(1+x^4)}\,dx = \log(x) - \frac{1}{4}\log\left(1+x^4\right)$$

```
[Out] ln(x)-1/4*ln(x^4+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {272, 36, 29, 31}

$$\int \frac{1}{x(1+x^4)}\,dx = \log(x) - \frac{1}{4}\log\left(x^4+1\right)$$

```
[In] Int[1/(x*(1 + x^4)),x]
```

```
[Out] Log[x] - Log[1 + x^4]/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^4\right)\\ &= \log(x) - \frac{1}{4}\log\left(1+x^4\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x^4)}\,dx = \log(x) - \frac{1}{4}\log\left(1+x^4\right)$$

```
[In] Integrate[1/(x*(1 + x^4)),x]
[Out] Log[x] - Log[1 + x^4]/4
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\ln(x) - \frac{\ln(x^4+1)}{4}$	12
norman	$\ln(x) - \frac{\ln(x^4+1)}{4}$	12
meijerg	$\ln(x) - \frac{\ln(x^4+1)}{4}$	12
risch	$\ln(x) - \frac{\ln(x^4+1)}{4}$	12
parallelrisch	$\ln(x) - \frac{\ln(x^4+1)}{4}$	12

```
[In] int(1/x/(x^4+1),x,method=_RETURNVERBOSE)
[Out] ln(x)-1/4*ln(x^4+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(1+x^4\right)}\,dx=-\frac{1}{4}\,\log\left(x^4+1\right)+\log\left(x\right)$$

```
[In] integrate(1/x/(x^4+1),x, algorithm="fricas")
[Out] -1/4*log(x^4 + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\left(1+x^4\right)}\,dx=\log\left(x\right)-\frac{\log\left(x^4+1\right)}{4}$$

```
[In] integrate(1/x/(x**4+1),x)
[Out] log(x) - log(x**4 + 1)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\left(1+x^4\right)}\,dx=-\frac{1}{4}\,\log\left(x^4+1\right)+\frac{1}{4}\,\log\left(x^4\right)$$

```
[In] integrate(1/x/(x^4+1),x, algorithm="maxima")
[Out] -1/4*log(x^4 + 1) + 1/4*log(x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{x(1+x^4)}\,dx = -\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

```
[In] integrate(1/x/(x^4+1),x, algorithm="giac")
[Out] -1/4*log(x^4 + 1) + 1/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1+x^4)}\,dx = \ln(x)-\frac{\ln(x^4+1)}{4}$$

```
[In] int(1/(x*(x^4 + 1)),x)
[Out] log(x) - log(x^4 + 1)/4
```

3.318 $\int e^{-2t}t^3\,dt$

Optimal result

Integrand size = 9, antiderivative size = 44

$$\int e^{-2t}t^3\,dt=-\frac{3}{8}e^{-2t}-\frac{3}{4}e^{-2t}t-\frac{3}{4}e^{-2t}t^2-\frac{1}{2}e^{-2t}t^3$$

```
[Out] -3/8/exp(2*t)-3/4*t/exp(2*t)-3/4*t^2/exp(2*t)-1/2*t^3/exp(2*t)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2207, 2225}

$$\int e^{-2t}t^3\,dt=-\frac{1}{2}e^{-2t}t^3-\frac{3}{4}e^{-2t}t^2-\frac{3}{4}e^{-2t}t-\frac{3e^{-2t}}{8}$$

```
[In] Int[t^3/E^(2*t),t]
```

```
[Out] -3/(8*E^(2*t)) - (3*t)/(4*E^(2*t)) - (3*t^2)/(4*E^(2*t)) - t^3/(2*E^(2*t))
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2}e^{-2t}t^3 + \frac{3}{2}\int e^{-2t}t^2\,dt\\ &= -\frac{3}{4}e^{-2t}t^2 - \frac{1}{2}e^{-2t}t^3 + \frac{3}{2}\int e^{-2t}t\,dt\\ &= -\frac{3}{4}e^{-2t}t - \frac{3}{4}e^{-2t}t^2 - \frac{1}{2}e^{-2t}t^3 + \frac{3}{4}\int e^{-2t}\,dt\\ &= -\frac{3}{8}e^{-2t} - \frac{3}{4}e^{-2t}t - \frac{3}{4}e^{-2t}t^2 - \frac{1}{2}e^{-2t}t^3\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

$$\int e^{-2t}t^3\,dt = -\frac{1}{8}e^{-2t}\left(3+6t+6t^2+4t^3\right)$$

```
[In] Integrate[t^3/E^(2*t),t]
[Out] -1/8*(3 + 6*t + 6*t^2 + 4*t^3)/E^(2*t)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

method	result	size
risch	$\left(-\frac{1}{2}t^3-\frac{3}{4}t^2-\frac{3}{4}t-\frac{3}{8}\right)\mathrm{e}^{-2t}$	21
norman	$\left(-\frac{1}{2}t^3-\frac{3}{4}t^2-\frac{3}{4}t-\frac{3}{8}\right)\mathrm{e}^{-2t}$	23
gosper	$-\frac{(4t^3+6t^2+6t+3)\mathrm{e}^{-2t}}{8}$	24
meijerg	$\frac{3}{8}-\frac{(32t^3+48t^2+48t+24)\mathrm{e}^{-2t}}{64}$	24
parallelrisch	$\frac{(-4t^3-6t^2-6t-3)\mathrm{e}^{-2t}}{8}$	24
derivativedivides	$-\frac{3\,\mathrm{e}^{-2t}}{8}-\frac{3t\,\mathrm{e}^{-2t}}{4}-\frac{3t^2\mathrm{e}^{-2t}}{4}-\frac{t^3\mathrm{e}^{-2t}}{2}$	41
default	$-\frac{3\,\mathrm{e}^{-2t}}{8}-\frac{3t\,\mathrm{e}^{-2t}}{4}-\frac{3t^2\mathrm{e}^{-2t}}{4}-\frac{t^3\mathrm{e}^{-2t}}{2}$	41

```
[In] int(t^3/exp(2*t),t,method=_RETURNVERBOSE)
[Out] (-1/2*t^3-3/4*t^2-3/4*t-3/8)*exp(-2*t)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int e^{-2t}t^3\,dt = -\frac{1}{8}\left(4\,t^3 + 6\,t^2 + 6\,t + 3\right)e^{(-2\,t)}$$

```
[In] integrate(t^3/exp(2*t),t, algorithm="fricas")
[Out] -1/8*(4*t^3 + 6*t^2 + 6*t + 3)*e^(-2*t)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int e^{-2t}t^3\,dt = \frac{(-4t^3 - 6t^2 - 6t - 3)\,e^{-2t}}{8}$$

```
[In] integrate(t**3/exp(2*t),t)
[Out] (-4*t**3 - 6*t**2 - 6*t - 3)*exp(-2*t)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int e^{-2t}t^3\,dt = -\frac{1}{8}\left(4\,t^3 + 6\,t^2 + 6\,t + 3\right)e^{(-2\,t)}$$

```
[In] integrate(t^3/exp(2*t),t, algorithm="maxima")
[Out] -1/8*(4*t^3 + 6*t^2 + 6*t + 3)*e^(-2*t)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int e^{-2t}t^3\,dt = -\frac{1}{8}\left(4\,t^3 + 6\,t^2 + 6\,t + 3\right)e^{(-2\,t)}$$

```
[In] integrate(t^3/exp(2*t),t, algorithm="giac")
[Out] -1/8*(4*t^3 + 6*t^2 + 6*t + 3)*e^(-2*t)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int e^{-2t}t^3 \, dt = -\frac{e^{-2\,t}\,\left(8\,t^3+12\,t^2+12\,t+6\right)}{16}$$

```
[In] int(t^3*exp(-2*t),t)
[Out] -(exp(-2*t)*(12*t + 12*t^2 + 8*t^3 + 6))/16
```

3.319 $\int \frac{\sqrt{t}}{1+\sqrt[3]{t}}\,dt$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{\sqrt{t}}{1+\sqrt[3]{t}}\,dt = -6\sqrt[6]{t}+2\sqrt{t}-\frac{6t^{5/6}}{5}+\frac{6t^{7/6}}{7}+6\arctan\left(\sqrt[6]{t}\right)$$

```
[Out] -6*t^(1/6)-6/5*t^(5/6)+6/7*t^(7/6)+6*arctan(t^(1/6))+2*t^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {348, 52, 65, 209}

$$\int \frac{\sqrt{t}}{1+\sqrt[3]{t}}\,dt = 6\arctan\left(\sqrt[6]{t}\right)+\frac{6t^{7/6}}{7}-\frac{6t^{5/6}}{5}+2\sqrt{t}-6\sqrt[6]{t}$$

```
[In] Int[Sqrt[t]/(1 + t^(1/3)),t]
```

```
[Out] -6*t^(1/6) + 2*Sqrt[t] - (6*t^(5/6))/5 + (6*t^(7/6))/7 + 6*ArcTan[t^(1/6)]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{t^{7/2}}{1+t}\,dt, t, \sqrt[3]{t}\right)\\
&= \frac{6t^{7/6}}{7} - 3\text{Subst}\left(\int \frac{t^{5/2}}{1+t}\,dt, t, \sqrt[3]{t}\right)\\
&= -\frac{6t^{5/6}}{5} + \frac{6t^{7/6}}{7} + 3\text{Subst}\left(\int \frac{t^{3/2}}{1+t}\,dt, t, \sqrt[3]{t}\right)\\
&= 2\sqrt{t} - \frac{6t^{5/6}}{5} + \frac{6t^{7/6}}{7} - 3\text{Subst}\left(\int \frac{\sqrt{t}}{1+t}\,dt, t, \sqrt[3]{t}\right)\\
&= -6\sqrt[6]{t} + 2\sqrt{t} - \frac{6t^{5/6}}{5} + \frac{6t^{7/6}}{7} + 3\text{Subst}\left(\int \frac{1}{\sqrt{t}(1+t)}\,dt, t, \sqrt[3]{t}\right)\\
&= -6\sqrt[6]{t} + 2\sqrt{t} - \frac{6t^{5/6}}{5} + \frac{6t^{7/6}}{7} + 6\text{Subst}\left(\int \frac{1}{1+t^2}\,dt, t, \sqrt[6]{t}\right)\\
&= -6\sqrt[6]{t} + 2\sqrt{t} - \frac{6t^{5/6}}{5} + \frac{6t^{7/6}}{7} + 6\arctan\left(\sqrt[6]{t}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{t}}{1+\sqrt[3]{t}}\,dt = \frac{2}{35}\left(-105\sqrt[6]{t}+35\sqrt{t}-21t^{5/6}+15t^{7/6}\right)+6\arctan\left(\sqrt[6]{t}\right)$$

[In] Integrate[Sqrt[t]/(1 + t^(1/3)),t]

[Out] (2*(-105*t^(1/6) + 35*Sqrt[t] - 21*t^(5/6) + 15*t^(7/6)))/35 + 6*ArcTan[t^(1/6)]

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$-6t^{\frac{1}{6}}-\frac{6t^{\frac{5}{6}}}{5}+\frac{6t^{\frac{7}{6}}}{7}+6\arctan\left(t^{\frac{1}{6}}\right)+2\sqrt{t}$	28
default	$-6t^{\frac{1}{6}}-\frac{6t^{\frac{5}{6}}}{5}+\frac{6t^{\frac{7}{6}}}{7}+6\arctan\left(t^{\frac{1}{6}}\right)+2\sqrt{t}$	28
meijerg	$-\frac{2t^{\frac{1}{6}}\left(-45t+63t^{\frac{2}{3}}-105t^{\frac{1}{3}}+315\right)}{105}+6\arctan\left(t^{\frac{1}{6}}\right)$	28

[In] int(t^(1/2)/(1+t^(1/3)),t,method=_RETURNVERBOSE)

[Out] -6*t^(1/6)-6/5*t^(5/6)+6/7*t^(7/6)+6*arctan(t^(1/6))+2*t^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{t}}{1+\sqrt[3]{t}}\,dt = \frac{6}{7}\,(t-7)t^{\frac{1}{6}}-\frac{6}{5}\,t^{\frac{5}{6}}+2\,\sqrt{t}+6\,\arctan\left(t^{\frac{1}{6}}\right)$$

[In] integrate(t^(1/2)/(1+t^(1/3)),t, algorithm="fricas")

[Out] 6/7*(t - 7)*t^(1/6) - 6/5*t^(5/6) + 2*sqrt(t) + 6*arctan(t^(1/6))

Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{t}}{1+\sqrt[3]{t}}\,dt = \frac{6t^{\frac{7}{6}}}{7} - \frac{6t^{\frac{5}{6}}}{5} - 6\sqrt[6]{t} + 2\sqrt{t} + 6\,\mathrm{atan}\left(\sqrt[6]{t}\right)$$

```
[In] integrate(t**(1/2)/(1+t**(1/3)),t)
[Out] 6*t**(7/6)/7 - 6*t**(5/6)/5 - 6*t**(1/6) + 2*sqrt(t) + 6*atan(t**(1/6))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{t}}{1+\sqrt[3]{t}}\,dt = \frac{6}{7}\,t^{\frac{7}{6}} - \frac{6}{5}\,t^{\frac{5}{6}} + 2\,\sqrt{t} - 6\,t^{\frac{1}{6}} + 6\,\arctan\left(t^{\frac{1}{6}}\right)$$

```
[In] integrate(t^(1/2)/(1+t^(1/3)),t, algorithm="maxima")
[Out] 6/7*t^(7/6) - 6/5*t^(5/6) + 2*sqrt(t) - 6*t^(1/6) + 6*arctan(t^(1/6))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{t}}{1+\sqrt[3]{t}}\,dt = \frac{6}{7}\,t^{\frac{7}{6}} - \frac{6}{5}\,t^{\frac{5}{6}} + 2\,\sqrt{t} - 6\,t^{\frac{1}{6}} + 6\,\arctan\left(t^{\frac{1}{6}}\right)$$

```
[In] integrate(t^(1/2)/(1+t^(1/3)),t, algorithm="giac")
[Out] 6/7*t^(7/6) - 6/5*t^(5/6) + 2*sqrt(t) - 6*t^(1/6) + 6*arctan(t^(1/6))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{t}}{1+\sqrt[3]{t}}\,dt = 6\,\mathrm{atan}\left(t^{1/6}\right) + 2\,\sqrt{t} - 6\,t^{1/6} - \frac{6\,t^{5/6}}{5} + \frac{6\,t^{7/6}}{7}$$

```
[In] int(t^(1/2)/(t^(1/3) + 1),t)
[Out] 6*atan(t^(1/6)) + 2*t^(1/2) - 6*t^(1/6) - (6*t^(5/6))/5 + (6*t^(7/6))/7
```

3.320 $\int \sin(x)\sin(2x)\sin(3x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{1}{8}\cos(2x) - \frac{1}{16}\cos(4x) + \frac{1}{24}\cos(6x)$$

```
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4440, 2718}

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{1}{8}\cos(2x) - \frac{1}{16}\cos(4x) + \frac{1}{24}\cos(6x)$$

```
[In] Int[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

<u>Rule 2718</u>

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

<u>Rule 4440</u>

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
```

```
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{4}\sin(2x) + \frac{1}{4}\sin(4x) - \frac{1}{4}\sin(6x)\right) dx\\ &= \frac{1}{4}\int \sin(2x)\,dx + \frac{1}{4}\int \sin(4x)\,dx - \frac{1}{4}\int \sin(6x)\,dx\\ &= -\frac{1}{8}\cos(2x) - \frac{1}{16}\cos(4x) + \frac{1}{24}\cos(6x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{1}{8}\cos(2x) - \frac{1}{16}\cos(4x) + \frac{1}{24}\cos(6x)$$

```
[In] Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]
[Out] -1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\cos(2x)}{8} - \frac{\cos(4x)}{16} + \frac{\cos(6x)}{24}$	20
risch	$-\frac{\cos(2x)}{8} - \frac{\cos(4x)}{16} + \frac{\cos(6x)}{24}$	20
parallelrisch	$-\frac{29}{48} + \frac{\cos(6x)}{24} - \frac{\cos(4x)}{16} - \frac{\cos(2x)}{8}$	21

```
[In] int(sin(x)*sin(2*x)*sin(3*x),x,method=_RETURNVERBOSE)
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = \frac{4}{3}\cos(x)^6 - \frac{5}{2}\cos(x)^4 + \cos(x)^2$$

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
[Out] 4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(19) = 38.

Time = 0.96 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.56

$$\begin{aligned}\int \sin(x)\sin(2x)\sin(3x)\,dx &= \frac{x\sin(x)\sin(2x)\sin(3x)}{4} + \frac{x\sin(x)\cos(2x)\cos(3x)}{4} \\ &+ \frac{x\sin(2x)\cos(x)\cos(3x)}{4} \\ &- \frac{x\sin(3x)\cos(x)\cos(2x)}{4} - \frac{5\sin(x)\sin(2x)\cos(3x)}{24} \\ &- \frac{\sin(2x)\sin(3x)\cos(x)}{8} - \frac{\cos(x)\cos(2x)\cos(3x)}{6}\end{aligned}$$

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x)
[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*co
s(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - 5*sin(x)*sin(2*x)*cos(3*x)
/24 - sin(2*x)*sin(3*x)*cos(x)/8 - cos(x)*cos(2*x)*cos(3*x)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = \frac{1}{24}\cos(6x) - \frac{1}{16}\cos(4x) - \frac{1}{8}\cos(2x)$$

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
[Out] 1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{4}{3}\sin{(x)}^{6} + \frac{3}{2}\sin{(x)}^{4}$$

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
[Out] -4/3*sin(x)^6 + 3/2*sin(x)^4
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{\sin{(x)}^{4}\,\left(8\sin{(x)}^{2}-9\right)}{6}$$

```
[In] int(sin(2*x)*sin(3*x)*sin(x),x)
[Out] -(sin(x)^4*(8*sin(x)^2 - 9))/6
```

3.321 $\int \log \left(\frac{x}{2}\right) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 12

$$\int \log \left(\frac{x}{2}\right) \, dx = -x + x \log \left(\frac{x}{2}\right)$$

```
[Out] -x+x*ln(1/2*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{2332\}$

$$\int \log \left(\frac{x}{2}\right) \, dx = x \log \left(\frac{x}{2}\right) - x$$

```
[In] Int[Log[x/2],x]
```

```
[Out] -x + x*Log[x/2]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\text{integral} = -x + x \log \left(\frac{x}{2}\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{x}{2}\right) \, dx = -x + x \log \left(\frac{x}{2}\right)$$

```
[In] Integrate[Log[x/2],x]
[Out] -x + x*Log[x/2]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-x+x\ln\left(\frac{x}{2}\right)$	11
default	$-x+x\ln\left(\frac{x}{2}\right)$	11
norman	$-x+x\ln\left(\frac{x}{2}\right)$	11
risch	$-x+x\ln\left(\frac{x}{2}\right)$	11
parallelrisch	$-x+x\ln\left(\frac{x}{2}\right)$	11
parts	$-x+x\ln\left(\frac{x}{2}\right)$	11

```
[In] int(ln(1/2*x),x,method=_RETURNVERBOSE)
[Out] -x+x*ln(1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \log \left(\frac{x}{2}\right) \, dx = x \log \left(\frac{1}{2} \, x\right) - x$$

```
[In] integrate(log(1/2*x),x, algorithm="fricas")
[Out] x*log(1/2*x) - x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \log \left(\frac{x}{2}\right) \, dx = x \log \left(\frac{x}{2}\right) - x$$

```
[In] integrate(ln(1/2*x),x)
[Out] x*log(x/2) - x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \log \left(\frac{x}{2}\right) \, dx = x \log \left(\frac{1}{2} \, x\right) - x$$

```
[In] integrate(log(1/2*x),x, algorithm="maxima")
[Out] x*log(1/2*x) - x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \log \left(\frac{x}{2}\right) \, dx = x \log \left(\frac{1}{2} \, x\right) - x$$

```
[In] integrate(log(1/2*x),x, algorithm="giac")
[Out] x*log(1/2*x) - x
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \log \left(\frac{x}{2}\right) \, dx = x \left(\ln \left(\frac{x}{2}\right) - 1\right)$$

```
[In] int(log(x/2),x)
[Out] x*(log(x/2) - 1)
```

3.322 $\int \sqrt{\frac{1+x}{1-x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \sqrt{\frac{1+x}{1-x}}\,dx = -\left((1-x)\sqrt{\frac{1+x}{1-x}}\right) + 2\arctan\left(\sqrt{\frac{1+x}{1-x}}\right)$$

```
[Out] 2*arctan(((1+x)/(1-x))^(1/2))-(1-x)*((1+x)/(1-x))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 294, 209}

$$\int \sqrt{\frac{1+x}{1-x}}\,dx = 2\arctan\left(\sqrt{\frac{x+1}{1-x}}\right) - (1-x)\sqrt{\frac{x+1}{1-x}}$$

```
[In] Int[Sqrt[(1 + x)/(1 - x)],x]
[Out] -((1 - x)*Sqrt[(1 + x)/(1 - x)]) + 2*ArcTan[Sqrt[(1 + x)/(1 - x)]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 4\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2}\,dx, x, \sqrt{\frac{1+x}{1-x}}\right)\\
&= -\left((1-x)\sqrt{\frac{1+x}{1-x}}\right) + 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{1+x}{1-x}}\right)\\
&= -\left((1-x)\sqrt{\frac{1+x}{1-x}}\right) + 2\arctan\left(\sqrt{\frac{1+x}{1-x}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \sqrt{\frac{1+x}{1-x}}\,dx = -\frac{\sqrt{1-x}\sqrt{\frac{1+x}{1-x}}\left(\sqrt{1-x^2}+2\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)\right)}{\sqrt{1+x}}$$

```
[In] Integrate[Sqrt[(1 + x)/(1 - x)],x]
[Out] -((Sqrt[1 - x]*Sqrt[(1 + x)/(1 - x)]*(Sqrt[1 - x^2] + 2*ArcTan[Sqrt[1 - x^2
]/(-1 + x)]))/Sqrt[1 + x])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

method	result
default	$\frac{\sqrt{-\frac{1+x}{-1+x}}\,(-1+x)\left(\sqrt{-x^{2}+1}-\arcsin (x)\right)}{\sqrt{-(-1+x)(1+x)}}$
risch	$(-1+x)\sqrt{-\frac{1+x}{-1+x}}+\frac{\arcsin (x)\sqrt{-\frac{1+x}{-1+x}}\,\sqrt{-(-1+x)(1+x)}}{1+x}$
trager	$(-1+x)\sqrt{-\frac{1+x}{-1+x}}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\frac{1+x}{-1+x}}\,x+\operatorname{RootOf}\left(\_Z^{2}+1\right.\right.$

[In] int(((1+x)/(1-x))^(1/2),x,method=_RETURNVERBOSE)

[Out] (-(1+x)/(-1+x))^(1/2)*(-1+x)/(-(-1+x)*(1+x))^(1/2)*((-x^2+1)^(1/2)-arcsin(x
))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \sqrt{\frac{1+x}{1-x}}\,dx=(x-1)\sqrt{-\frac{x+1}{x-1}}+2\arctan\left(\sqrt{-\frac{x+1}{x-1}}\right)$$

[In] integrate(((1+x)/(1-x))^(1/2),x, algorithm="fricas")

[Out] (x - 1)*sqrt(-(x + 1)/(x - 1)) + 2*arctan(sqrt(-(x + 1)/(x - 1)))

Sympy [F]

$$\int \sqrt{\frac{1+x}{1-x}}\,dx=\int \sqrt{\frac{x+1}{1-x}}\,dx$$

[In] integrate(((1+x)/(1-x))**(1/2),x)

[Out] Integral(sqrt((x + 1)/(1 - x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \sqrt{\frac{1+x}{1-x}}\,dx = \frac{2\sqrt{-\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1} + 2\arctan\left(\sqrt{-\frac{x+1}{x-1}}\right)$$

```
[In] integrate(((1+x)/(1-x))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(-(x + 1)/(x - 1))/((x + 1)/(x - 1) - 1) + 2*arctan(sqrt(-(x + 1)/(x
- 1)))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \sqrt{\frac{1+x}{1-x}}\,dx = \frac{1}{2}\pi\mathrm{sgn}(x-1) - \arcsin(x)\,\mathrm{sgn}(x-1) + \sqrt{-x^2+1}\mathrm{sgn}(x-1)$$

```
[In] integrate(((1+x)/(1-x))^(1/2),x, algorithm="giac")
[Out] 1/2*pi*sgn(x - 1) - arcsin(x)*sgn(x - 1) + sqrt(-x^2 + 1)*sgn(x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \sqrt{\frac{1+x}{1-x}}\,dx = 2\,\mathrm{atan}\left(\sqrt{-\frac{x+1}{x-1}}\right) + \frac{2\sqrt{-\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1}$$

```
[In] int((-(x + 1)/(x - 1))^(1/2),x)
[Out] 2*atan((-(x + 1)/(x - 1))^(1/2)) + (2*(-(x + 1)/(x - 1))^(1/2))/((x + 1)/(x
 - 1) - 1)
```

3.323 $\int \frac{x \log(x)}{\sqrt{-1+x^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{x \log(x)}{\sqrt{-1+x^2}} \, dx = -\sqrt{-1+x^2} + \arctan\left(\sqrt{-1+x^2}\right) + \sqrt{-1+x^2}\log(x)$$

```
[Out] arctan((x^2-1)^(1/2))-(x^2-1)^(1/2)+ln(x)*(x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2376, 272, 52, 65, 209}

$$\int \frac{x \log(x)}{\sqrt{-1+x^2}} \, dx = \arctan\left(\sqrt{x^2-1}\right) - \sqrt{x^2-1} + \sqrt{x^2-1}\log(x)$$

```
[In] Int[(x*Log[x])/Sqrt[-1 + x^2],x]
```

```
[Out] -Sqrt[-1 + x^2] + ArcTan[Sqrt[-1 + x^2]] + Sqrt[-1 + x^2]*Log[x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \sqrt{-1+x^2}\log(x) - \int \frac{\sqrt{-1+x^2}}{x}\,dx \\
&= \sqrt{-1+x^2}\log(x) - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, x^2\right) \\
&= -\sqrt{-1+x^2} + \sqrt{-1+x^2}\log(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x}\,dx, x, x^2\right) \\
&= -\sqrt{-1+x^2} + \sqrt{-1+x^2}\log(x) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^2}\right) \\
&= -\sqrt{-1+x^2} + \arctan\left(\sqrt{-1+x^2}\right) + \sqrt{-1+x^2}\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{x \log(x)}{\sqrt{-1+x^2}}\,dx = -\arctan\left(\frac{1}{\sqrt{-1+x^2}}\right) + \sqrt{-1+x^2}(-1+\log(x))$$

[In] Integrate[(x*Log[x])/Sqrt[-1 + x^2],x]

[Out] -ArcTan[1/Sqrt[-1 + x^2]] + Sqrt[-1 + x^2]*(-1 + Log[x])

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.50

method	result
meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^2-1)}\left(2-2\sqrt{-x^2+1}\right)}{4\sqrt{\operatorname{signum}(x^2-1)}}+\frac{\sqrt{-\operatorname{signum}(x^2-1)}\,\ln(x)\left(2-2\sqrt{-x^2+1}\right)}{2\sqrt{\operatorname{signum}(x^2-1)}}+\frac{\sqrt{-\operatorname{signum}(x^2-1)}\left(-16+16\sqrt{-x^2+1}-32\ln\right.}{32\sqrt{\operatorname{signum}(x^2-1)}}$

[In] int(x*ln(x)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4/signum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*(2-2*(-x^2+1)^(1/2))+1/2/si
gnum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*ln(x)*(2-2*(-x^2+1)^(1/2))+1/32/si
gnum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*(-16+16*(-x^2+1)^(1/2)-32*ln(1/2+1
/2*(-x^2+1)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{x \log(x)}{\sqrt{-1+x^2}}\,dx = \sqrt{x^2-1}(\log(x)-1) + 2\arctan\left(-x+\sqrt{x^2-1}\right)$$

[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="fricas")

[Out] sqrt(x^2 - 1)*(log(x) - 1) + 2*arctan(-x + sqrt(x^2 - 1))

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{x\log(x)}{\sqrt{-1+x^2}}\,dx = \sqrt{x^2-1}\log(x) - \begin{cases} \sqrt{x^2-1} - \operatorname{acos}\left(\frac{1}{x}\right) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

```
[In] integrate(x*ln(x)/(x**2-1)**(1/2),x)
[Out] sqrt(x**2 - 1)*log(x) - Piecewise((sqrt(x**2 - 1) - acos(1/x), (x > -1) & (
x < 1)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{x\log(x)}{\sqrt{-1+x^2}}\,dx = \sqrt{x^2-1}\log(x) - \sqrt{x^2-1} - \arcsin\left(\frac{1}{|x|}\right)$$

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 - 1)*log(x) - sqrt(x^2 - 1) - arcsin(1/abs(x))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{x\log(x)}{\sqrt{-1+x^2}}\,dx = \sqrt{x^2-1}\log(x) - \sqrt{x^2-1} + \arctan\left(\sqrt{x^2-1}\right)$$

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 - 1)*log(x) - sqrt(x^2 - 1) + arctan(sqrt(x^2 - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\log(x)}{\sqrt{-1+x^2}}\,dx = \int \frac{x\ln(x)}{\sqrt{x^2-1}}\,dx$$

```
[In] int((x*log(x))/(x^2 - 1)^(1/2),x)
[Out] int((x*log(x))/(x^2 - 1)^(1/2), x)
```

3.324 $\int \frac{a+x}{a^2+x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{a+x}{a^2+x^2} \, dx = \arctan\left(\frac{x}{a}\right) + \frac{1}{2}\log\left(a^2+x^2\right)$$

```
[Out] arctan(x/a)+1/2*ln(a^2+x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {649, 209, 266}

$$\int \frac{a+x}{a^2+x^2} \, dx = \frac{1}{2}\log\left(a^2+x^2\right) + \arctan\left(\frac{x}{a}\right)$$

```
[In] Int[(a + x)/(a^2 + x^2),x]
```

```
[Out] ArcTan[x/a] + Log[a^2 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= a\int \frac{1}{a^2+x^2}\,dx+\int \frac{x}{a^2+x^2}\,dx\\ &= \arctan\left(\frac{x}{a}\right)+\frac{1}{2}\log\left(a^2+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{a+x}{a^2+x^2}\,dx=\arctan\left(\frac{x}{a}\right)+\frac{1}{2}\log\left(a^2+x^2\right)$$

```
[In] Integrate[(a + x)/(a^2 + x^2),x]
[Out] ArcTan[x/a] + Log[a^2 + x^2]/2
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$\arctan\left(\frac{x}{a}\right)+\frac{\ln\left(a^2+x^2\right)}{2}$	18
risch	$\arctan\left(\frac{x}{a}\right)+\frac{\ln\left(a^2+x^2\right)}{2}$	18
parallelrisch	$\frac{\ln(-ia+x)}{2}-\frac{i\ln(-ia+x)}{2}+\frac{\ln(ia+x)}{2}+\frac{i\ln(ia+x)}{2}$	40

```
[In] int((a+x)/(a^2+x^2),x,method=_RETURNVERBOSE)
[Out] arctan(x/a)+1/2*ln(a^2+x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+x}{a^2+x^2}\,dx = \arctan\left(\frac{x}{a}\right) + \frac{1}{2}\log\left(a^2+x^2\right)$$

```
[In] integrate((a+x)/(a^2+x^2),x, algorithm="fricas")
[Out] arctan(x/a) + 1/2*log(a^2 + x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \frac{a+x}{a^2+x^2}\,dx = \left(\frac{1}{2}-\frac{i}{2}\right)\log\left(-a+2a\left(\frac{1}{2}-\frac{i}{2}\right)+x\right) + \left(\frac{1}{2}+\frac{i}{2}\right)\log\left(-a+2a\left(\frac{1}{2}+\frac{i}{2}\right)+x\right)$$

```
[In] integrate((a+x)/(a**2+x**2),x)
[Out] (1/2 - I/2)*log(-a + 2*a*(1/2 - I/2) + x) + (1/2 + I/2)*log(-a + 2*a*(1/2 +
 I/2) + x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+x}{a^2+x^2}\,dx = \arctan\left(\frac{x}{a}\right) + \frac{1}{2}\log\left(a^2+x^2\right)$$

```
[In] integrate((a+x)/(a^2+x^2),x, algorithm="maxima")
[Out] arctan(x/a) + 1/2*log(a^2 + x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+x}{a^2+x^2}\,dx = \arctan\left(\frac{x}{a}\right) + \frac{1}{2}\,\log\left(a^2+x^2\right)$$

```
[In] integrate((a+x)/(a^2+x^2),x, algorithm="giac")
[Out] arctan(x/a) + 1/2*log(a^2 + x^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+x}{a^2+x^2}\,dx = \frac{\ln\left(a^2+x^2\right)}{2} + \mathrm{atan}\left(\frac{x}{a}\right)$$

```
[In] int((a + x)/(a^2 + x^2),x)
[Out] log(a^2 + x^2)/2 + atan(x/a)
```

3.325 $\int \sqrt{1+x-x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \sqrt{1+x-x^2}\,dx=-\frac{1}{4}(1-2x)\sqrt{1+x-x^2}-\frac{5}{8}\arcsin\left(\frac{1-2x}{\sqrt{5}}\right)$$

```
[Out] -5/8*arcsin(1/5*(1-2*x)*5^(1/2))-1/4*(1-2*x)*(-x^2+x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {626, 633, 222}

$$\int \sqrt{1+x-x^2}\,dx=-\frac{5}{8}\arcsin\left(\frac{1-2x}{\sqrt{5}}\right)-\frac{1}{4}\sqrt{-x^2+x+1}(1-2x)$$

```
[In] Int[Sqrt[1 + x - x^2],x]
```

```
[Out] -1/4*((1 - 2*x)*Sqrt[1 + x - x^2]) - (5*ArcSin[(1 - 2*x)/Sqrt[5]])/8
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{1}{4}(1-2x)\sqrt{1+x-x^2}+\frac{5}{8}\int \frac{1}{\sqrt{1+x-x^2}}\,dx \\ &= -\frac{1}{4}(1-2x)\sqrt{1+x-x^2}-\frac{1}{8}\sqrt{5}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{5}}}\,dx,x,1-2x\right) \\ &= -\frac{1}{4}(1-2x)\sqrt{1+x-x^2}-\frac{5}{8}\arcsin\left(\frac{1-2x}{\sqrt{5}}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \sqrt{1+x-x^2}\,dx = \frac{1}{4}(-1+2x)\sqrt{1+x-x^2}+\frac{5}{4}\arctan\left(\frac{x}{-1+\sqrt{1+x-x^2}}\right)$$

```
[In] Integrate[Sqrt[1 + x - x^2],x]
[Out] ((-1 + 2*x)*Sqrt[1 + x - x^2])/4 + (5*ArcTan[x/(-1 + Sqrt[1 + x - x^2])])/4
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{(1-2x)\sqrt{-x^2+x+1}}{4}+\frac{5\arcsin\left(\frac{2\sqrt{5}\left(x-\frac{1}{2}\right)}{5}\right)}{8}$	30
risch	$-\frac{(x^2-x-1)(2x-1)}{4\sqrt{-x^2+x+1}}+\frac{5\arcsin\left(\frac{2\sqrt{5}\left(x-\frac{1}{2}\right)}{5}\right)}{8}$	38
trager	$\left(-\frac{1}{4}+\frac{x}{2}\right)\sqrt{-x^2+x+1}-\frac{5\,\text{RootOf}\left(\_Z^2+1\right)\ln\left(2\,\text{RootOf}\left(\_Z^2+1\right)x-\text{RootOf}\left(\_Z^2+1\right)+2\sqrt{-x^2+x+1}\right)}{8}$	57

```
[In] int((-x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(1-2*x)*(-x^2+x+1)^(1/2)+5/8*arcsin(2/5*5^(1/2)*(x-1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \sqrt{1+x-x^2}\,dx = \frac{1}{4}\sqrt{-x^2+x+1}(2\,x-1) - \frac{5}{4}\arctan\left(\frac{\sqrt{-x^2+x+1}-1}{x}\right)$$

```
[In] integrate((-x^2+x+1)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(-x^2 + x + 1)*(2*x - 1) - 5/4*arctan((sqrt(-x^2 + x + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \sqrt{1+x-x^2}\,dx = \left(\frac{x}{2}-\frac{1}{4}\right)\sqrt{-x^2+x+1} + \frac{5\,\mathrm{asin}\left(\frac{2\sqrt{5}\left(x-\frac{1}{2}\right)}{5}\right)}{8}$$

```
[In] integrate((-x**2+x+1)**(1/2),x)
[Out] (x/2 - 1/4)*sqrt(-x**2 + x + 1) + 5*asin(2*sqrt(5)*(x - 1/2)/5)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \sqrt{1+x-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+x+1}x - \frac{1}{4}\sqrt{-x^2+x+1} - \frac{5}{8}\arcsin\left(-\frac{1}{5}\sqrt{5}(2\,x-1)\right)$$

```
[In] integrate((-x^2+x+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + x + 1)*x - 1/4*sqrt(-x^2 + x + 1) - 5/8*arcsin(-1/5*sqrt(5)
*(2*x - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \sqrt{1+x-x^2}\,dx = \frac{1}{4}\sqrt{-x^2+x+1}(2x-1) + \frac{5}{8}\arcsin\left(\frac{1}{5}\sqrt{5}(2x-1)\right)$$

```
[In] integrate((-x^2+x+1)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(-x^2 + x + 1)*(2*x - 1) + 5/8*arcsin(1/5*sqrt(5)*(2*x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \sqrt{1+x-x^2}\,dx = \frac{5\,\mathrm{asin}\left(\frac{2\sqrt{5}\left(x-\frac{1}{2}\right)}{5}\right)}{8} + \left(\frac{x}{2}-\frac{1}{4}\right)\sqrt{-x^2+x+1}$$

```
[In] int((x - x^2 + 1)^(1/2),x)
[Out] (5*asin((2*5^(1/2)*(x - 1/2))/5))/8 + (x/2 - 1/4)*(x - x^2 + 1)^(1/2)
```

3.326 $\int \frac{x^4}{16+x^{10}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{x^4}{16+x^{10}} \, dx = \frac{1}{20} \arctan\left(\frac{x^5}{4}\right)$$

```
[Out] 1/20*arctan(1/4*x^5)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {281, 209}

$$\int \frac{x^4}{16+x^{10}} \, dx = \frac{1}{20} \arctan\left(\frac{x^5}{4}\right)$$

```
[In] Int[x^4/(16 + x^10),x]
```

```
[Out] ArcTan[x^5/4]/20
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{16+x^2}\,dx, x, x^5\right)\\ &= \frac{1}{20}\arctan\left(\frac{x^5}{4}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{16+x^{10}}\,dx = \frac{1}{20}\arctan\left(\frac{x^5}{4}\right)$$

```
[In] Integrate[x^4/(16 + x^10),x]
[Out] ArcTan[x^5/4]/20
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\arctan\left(\frac{x^5}{4}\right)}{20}$	9
meijerg	$\frac{\arctan\left(\frac{x^5}{4}\right)}{20}$	9
risch	$\frac{\arctan\left(\frac{x^5}{4}\right)}{20}$	9
parallelrisch	$\frac{i\ln\left(x^5+4i\right)}{40}-\frac{i\ln\left(x^5-4i\right)}{40}$	22

```
[In] int(x^4/(x^10+16),x,method=_RETURNVERBOSE)
[Out] 1/20*arctan(1/4*x^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{16+x^{10}}\,dx = \frac{1}{20}\arctan\left(\frac{1}{4}\,x^5\right)$$

```
[In] integrate(x^4/(x^10+16),x, algorithm="fricas")
[Out] 1/20*arctan(1/4*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{x^4}{16+x^{10}}\,dx = \frac{\operatorname{atan}\left(\frac{x^5}{4}\right)}{20}$$

```
[In] integrate(x**4/(x**10+16),x)
[Out] atan(x**5/4)/20
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{16+x^{10}}\,dx = \frac{1}{20}\arctan\left(\frac{1}{4}\,x^5\right)$$

```
[In] integrate(x^4/(x^10+16),x, algorithm="maxima")
[Out] 1/20*arctan(1/4*x^5)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{16+x^{10}}\,dx = \frac{1}{20}\arctan\left(\frac{1}{4}\,x^5\right)$$

```
[In] integrate(x^4/(x^10+16),x, algorithm="giac")
[Out] 1/20*arctan(1/4*x^5)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{16+x^{10}}\,dx=\frac{\mathrm{atan}\left(\frac{x^5}{4}\right)}{20}$$

```
[In] int(x^4/(x^10 + 16),x)
[Out] atan(x^5/4)/20
```

3.327 $\int \frac{2+x}{2+x+x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{2+x}{2+x+x^2} \, dx = \frac{3\arctan\left(\frac{1+2x}{\sqrt{7}}\right)}{\sqrt{7}} + \frac{1}{2}\log\left(2+x+x^2\right)$$

```
[Out] 1/2*ln(x^2+x+2)+3/7*arctan(1/7*(1+2*x)*7^(1/2))*7^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {648, 632, 210, 642}

$$\int \frac{2+x}{2+x+x^2} \, dx = \frac{3\arctan\left(\frac{2x+1}{\sqrt{7}}\right)}{\sqrt{7}} + \frac{1}{2}\log\left(x^2+x+2\right)$$

```
[In] Int[(2 + x)/(2 + x + x^2),x]
```

```
[Out] (3*ArcTan[(1 + 2*x)/Sqrt[7]])/Sqrt[7] + Log[2 + x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1+2x}{2+x+x^2}\,dx + \frac{3}{2}\int \frac{1}{2+x+x^2}\,dx \\
&= \frac{1}{2}\log\left(2+x+x^2\right) - 3\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, 1+2x\right) \\
&= \frac{3\arctan\left(\frac{1+2x}{\sqrt{7}}\right)}{\sqrt{7}} + \frac{1}{2}\log\left(2+x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{2+x+x^2}\,dx = \frac{3\arctan\left(\frac{1+2x}{\sqrt{7}}\right)}{\sqrt{7}} + \frac{1}{2}\log\left(2+x+x^2\right)$$

```
[In] Integrate[(2 + x)/(2 + x + x^2),x]
[Out] (3*ArcTan[(1 + 2*x)/Sqrt[7]])/Sqrt[7] + Log[2 + x + x^2]/2
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\ln(x^2+x+2)}{2}+\frac{3\arctan\left(\frac{(1+2x)\sqrt{7}}{7}\right)\sqrt{7}}{7}$	27
risch	$\frac{\ln(4x^2+4x+8)}{2}+\frac{3\arctan\left(\frac{(1+2x)\sqrt{7}}{7}\right)\sqrt{7}}{7}$	31

```
[In] int((2+x)/(x^2+x+2),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2+x+2)+3/7*arctan(1/7*(1+2*x)*7^(1/2))*7^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{2+x}{2+x+x^2}\,dx = \frac{3}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2\,x+1)\right)+\frac{1}{2}\log\left(x^2+x+2\right)$$

```
[In] integrate((2+x)/(x^2+x+2),x, algorithm="fricas")
[Out] 3/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) + 1/2*log(x^2 + x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{2+x}{2+x+x^2}\,dx = \frac{\log\left(x^2+x+2\right)}{2}+\frac{3\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x}{7}+\frac{\sqrt{7}}{7}\right)}{7}$$

```
[In] integrate((2+x)/(x**2+x+2),x)
[Out] log(x**2 + x + 2)/2 + 3*sqrt(7)*atan(2*sqrt(7)*x/7 + sqrt(7)/7)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{2+x}{2+x+x^2}\,dx = \frac{3}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2\,x+1)\right) + \frac{1}{2}\log\left(x^2+x+2\right)$$

```
[In] integrate((2+x)/(x^2+x+2),x, algorithm="maxima")
[Out] 3/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) + 1/2*log(x^2 + x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{2+x}{2+x+x^2}\,dx = \frac{3}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2\,x+1)\right) + \frac{1}{2}\log\left(x^2+x+2\right)$$

```
[In] integrate((2+x)/(x^2+x+2),x, algorithm="giac")
[Out] 3/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) + 1/2*log(x^2 + x + 2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{2+x}{2+x+x^2}\,dx = \frac{\ln\left(x^2+x+2\right)}{2} + \frac{3\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}\,x}{7}+\frac{\sqrt{7}}{7}\right)}{7}$$

```
[In] int((x + 2)/(x + x^2 + 2),x)
[Out] log(x + x^2 + 2)/2 + (3*7^(1/2)*atan((2*7^(1/2)*x)/7 + 7^(1/2)/7))/7
```

3.328 $\int x \sec(x) \tan(x) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 10

$$\int x \sec(x) \tan(x) \, dx = -\operatorname{arctanh}(\sin(x)) + x \sec(x)$$

```
[Out] -arctanh(sin(x))+x*sec(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3842, 3855}

$$\int x \sec(x) \tan(x) \, dx = x \sec(x) - \operatorname{arctanh}(\sin(x))$$

```
[In] Int[x*Sec[x]*Tan[x],x]
```

```
[Out] -ArcTanh[Sin[x]] + x*Sec[x]
```

Rule 3842

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\sec(x) - \int \sec(x)\,dx\\ &= -\text{arctanh}(\sin(x)) + x\sec(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. $2(10) = 20$.

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.70

$$\int x\sec(x)\tan(x)\,dx = \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right) + x\sec(x)$$

```
[In] Integrate[x*Sec[x]*Tan[x],x]
[Out] Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + x*Sec[x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

method	result	size
default	$\frac{x}{\cos(x)} - \ln(\sec(x) + \tan(x))$	16
risch	$\frac{2\,e^{ix}x}{e^{2ix}+1} + \ln(e^{ix} - i) - \ln(i + e^{ix})$	39

```
[In] int(x*sec(x)*tan(x),x,method=_RETURNVERBOSE)
[Out] x/cos(x)-ln(sec(x)+tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(10) = 20$.

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int x\sec(x)\tan(x)\,dx = -\frac{\cos(x)\log(\sin(x) + 1) - \cos(x)\log(-\sin(x) + 1) - 2\,x}{2\,\cos(x)}$$

```
[In] integrate(x*sec(x)*tan(x),x, algorithm="fricas")
[Out] -1/2*(cos(x)*log(sin(x) + 1) - cos(x)*log(-sin(x) + 1) - 2*x)/cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x\sec(x)\tan(x)\,dx = x\sec(x) - \log(\tan(x) + \sec(x))$$

```
[In] integrate(x*sec(x)*tan(x),x)
[Out] x*sec(x) - log(tan(x) + sec(x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(10) = 20.

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 12.10

$$\int x\sec(x)\tan(x)\,dx = \frac{4\,x\cos(2\,x)\cos(x) + 4\,x\sin(2\,x)\sin(x) + 4\,x\cos(x) - \left(\cos(2\,x)^2 + \sin(2\,x)^2 + 2\,\cos(2\,x) + 1\right)\log\left(\cos(\right.}{2\left(\cos(2\,x)^2 + \sin\right.}$$

```
[In] integrate(x*sec(x)*tan(x),x, algorithm="maxima")
[Out] 1/2*(4*x*cos(2*x)*cos(x) + 4*x*sin(2*x)*sin(x) + 4*x*cos(x) - (cos(2*x)^2 +
 sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (co
s(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x)
+ 1))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(10) = 20.

Time = 0.36 (sec) , antiderivative size = 150, normalized size of antiderivative = 15.00

$$\int x\sec(x)\tan(x)\,dx = -\frac{2\,x\tan\left(\frac{1}{2}\,x\right)^2 + \log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2 + 2\,\tan\left(\frac{1}{2}\,x\right) + 1\right)}{\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)\tan\left(\frac{1}{2}\,x\right)^2 - \log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2 - 2\,\tan\left(\frac{1}{2}\,x\right) + 1\right)}{\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)\tan\left(\frac{1}{2}\,x\right)^2 + 2\,}{2\left(\tan\left(\frac{1}{2}\,x\right)^2 - 1\right)}$$

```
[In] integrate(x*sec(x)*tan(x),x, algorithm="giac")
[Out] -1/2*(2*x*tan(1/2*x)^2 + log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x
)^2 + 1))*tan(1/2*x)^2 - log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x
)^2 + 1))*tan(1/2*x)^2 + 2*x - log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan
(1/2*x)^2 + 1)) + log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1
)))/(tan(1/2*x)^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int x\sec(x)\tan(x)\,dx = \frac{x}{\cos(x)} + \operatorname{atan}(\cos(x) + \sin(x)\ 1\mathrm{i})\ 2\mathrm{i}$$

```
[In] int((x*tan(x))/cos(x),x)
[Out] atan(cos(x) + sin(x)*1i)*2i + x/cos(x)
```

3.329 $\int \frac{x}{-a^4+x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x}{-a^4+x^4} \, dx = -\frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

```
[Out] -1/2*arctanh(x^2/a^2)/a^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {281, 213}

$$\int \frac{x}{-a^4+x^4} \, dx = -\frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

```
[In] Int[x/(-a^4 + x^4),x]
```

```
[Out] -1/2*ArcTanh[x^2/a^2]/a^2
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{-a^4+x^2}\,dx, x, x^2\right)$$
$$= -\frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{-a^4+x^4}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

```
[In] Integrate[x/(-a^4 + x^4),x]
[Out] -1/2*ArcTanh[x^2/a^2]/a^2
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

method	result	size
parallelrisch	$\frac{\ln(-a+x)+\ln(a+x)-\ln(a^2+x^2)}{4a^2}$	27
default	$\frac{\ln(a^2-x^2)}{4a^2}-\frac{\ln(a^2+x^2)}{4a^2}$	30
risch	$\frac{\ln(-a^2+x^2)}{4a^2}-\frac{\ln(a^2+x^2)}{4a^2}$	30
norman	$\frac{\ln(a-x)}{4a^2}+\frac{\ln(a+x)}{4a^2}-\frac{\ln(a^2+x^2)}{4a^2}$	35

```
[In] int(x/(-a^4+x^4),x,method=_RETURNVERBOSE)
[Out] 1/4*(ln(-a+x)+ln(a+x)-ln(a^2+x^2))/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{x}{-a^4 + x^4}\,dx = -\frac{\log\left(a^2 + x^2\right) - \log\left(-a^2 + x^2\right)}{4\,a^2}$$

```
[In] integrate(x/(-a^4+x^4),x, algorithm="fricas")
[Out] -1/4*(log(a^2 + x^2) - log(-a^2 + x^2))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{x}{-a^4 + x^4}\,dx = \frac{\frac{\log\left(-a^2+x^2\right)}{4} - \frac{\log\left(a^2+x^2\right)}{4}}{a^2}$$

```
[In] integrate(x/(-a**4+x**4),x)
[Out] (log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{x}{-a^4 + x^4}\,dx = -\frac{\log\left(a^2 + x^2\right)}{4\,a^2} + \frac{\log\left(-a^2 + x^2\right)}{4\,a^2}$$

```
[In] integrate(x/(-a^4+x^4),x, algorithm="maxima")
[Out] -1/4*log(a^2 + x^2)/a^2 + 1/4*log(-a^2 + x^2)/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(13) = 26.

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

$$\int \frac{x}{-a^4 + x^4}\,dx = -\frac{\log\left(a^2 + x^2\right)}{4\,a^2} + \frac{\log\left(|-a^2 + x^2|\right)}{4\,a^2}$$

```
[In] integrate(x/(-a^4+x^4),x, algorithm="giac")
[Out] -1/4*log(a^2 + x^2)/a^2 + 1/4*log(abs(-a^2 + x^2))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{-a^4+x^4}\,dx=-\frac{\operatorname{atanh}\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

```
[In] int(-x/(a^4 - x^4),x)
[Out] -atanh(x^2/a^2)/(2*a^2)
```

3.330 $\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx=-\frac{2x^{3/2}}{3}+\frac{2}{3}(1+x)^{3/2}$$

```
[Out] -2/3*x^(3/2)+2/3*(1+x)^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2131, 30, 32}

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx=\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

```
[In] Int[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2131

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Dist[-b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \sqrt{x}\,dx + \int \sqrt{1+x}\,dx\\ &= -\frac{2x^{3/2}}{3} + \frac{2}{3}(1+x)^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = -\frac{2x^{3/2}}{3} + \frac{2}{3}(1+x)^{3/2}$$

```
[In] Integrate[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{2x^{\frac{3}{2}}}{3}+\frac{2(1+x)^{\frac{3}{2}}}{3}$	14
meijerg	$-\frac{\frac{4\sqrt{\pi}\,x^{\frac{3}{2}}}{3}-\frac{2\sqrt{\pi}\,x^{\frac{3}{2}}\left(2+\frac{2}{x}\right)\sqrt{1+\frac{1}{x}}}{3}}{2\sqrt{\pi}}$	37

```
[In] int(1/(x^(1/2)+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -2/3*x^(3/2)+2/3*(1+x)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}} - \frac{2}{3}\,x^{\frac{3}{2}}$$

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(17) = 34.

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \frac{2\sqrt{x}\sqrt{x+1}}{3\sqrt{x}+3\sqrt{x+1}} + \frac{4x}{3\sqrt{x}+3\sqrt{x+1}} + \frac{2}{3\sqrt{x}+3\sqrt{x+1}}$$

```
[In] integrate(1/(x**(1/2)+(1+x)**(1/2)),x)
[Out] 2*sqrt(x)*sqrt(x + 1)/(3*sqrt(x) + 3*sqrt(x + 1)) + 4*x/(3*sqrt(x) + 3*sqrt
(x + 1)) + 2/(3*sqrt(x) + 3*sqrt(x + 1))
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$$

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x + 1) + sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}} - \frac{2}{3}\,x^{\frac{3}{2}}$$

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \frac{2\,x\,\sqrt{x+1}}{3}+\frac{2\,\sqrt{x+1}}{3}-\frac{2\,x^{3/2}}{3}$$

```
[In] int(1/((x + 1)^(1/2) + x^(1/2)),x)
[Out] (2*x*(x + 1)^(1/2))/3 + (2*(x + 1)^(1/2))/3 - (2*x^(3/2))/3
```

3.331 $\int \frac{1}{1-e^{-x}+2e^x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{1}{1-e^{-x}+2e^x}\,dx=\frac{1}{3}\log\left(1-2e^x\right)-\frac{1}{3}\log\left(1+e^x\right)$$

```
[Out] 1/3*ln(1-2*exp(x))-1/3*ln(1+exp(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2320, 630, 31}

$$\int \frac{1}{1-e^{-x}+2e^x}\,dx=\frac{1}{3}\log\left(1-2e^x\right)-\frac{1}{3}\log\left(e^x+1\right)$$

```
[In] Int[(1 - E^(-x) + 2*E^x)^(-1),x]
```

```
[Out] Log[1 - 2*E^x]/3 - Log[1 + E^x]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{-1+x+2x^2}\,dx, x, e^x\right)\\ &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{-1+2x}\,dx, x, e^x\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{2+2x}\,dx, x, e^x\right)\\ &= \frac{1}{3}\log(1-2e^x) - \frac{1}{3}\log(1+e^x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{1}{1-e^{-x}+2e^x}\,dx = \frac{2}{3}\text{arctanh}\left(\frac{1}{3}-\frac{2e^{-x}}{3}\right)$$

```
[In] Integrate[(1 - E^(-x) + 2*E^x)^(-1),x]
[Out] (2*ArcTanh[1/3 - 2/(3*E^x)])/3
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{\ln\left(-\frac{1}{2}+{\mathrm e}^{x}\right)}{3}-\frac{\ln\left(1+{\mathrm e}^{x}\right)}{3}$	16
parallelrisch	$\frac{\ln\left(-\frac{1}{2}+{\mathrm e}^{x}\right)}{3}-\frac{\ln\left(1+{\mathrm e}^{x}\right)}{3}$	16
derivativedivides	$\frac{\ln\left(2\,{\mathrm e}^{x}-1\right)}{3}-\frac{\ln\left(1+{\mathrm e}^{x}\right)}{3}$	18
default	$\frac{\ln\left(2\,{\mathrm e}^{x}-1\right)}{3}-\frac{\ln\left(1+{\mathrm e}^{x}\right)}{3}$	18
norman	$\frac{\ln\left(2\,{\mathrm e}^{x}-1\right)}{3}-\frac{\ln\left(1+{\mathrm e}^{x}\right)}{3}$	18

```
[In] int(1/(1-1/exp(x)+2*exp(x)),x,method=_RETURNVERBOSE)
```

[Out] 1/3*ln(-1/2+exp(x))-1/3*ln(1+exp(x))

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{1-e^{-x}+2e^x} \, dx = \frac{1}{3} \log \left(2 e^x - 1\right) - \frac{1}{3} \log \left(e^x + 1\right)$$

[In] integrate(1/(1-1/exp(x)+2*exp(x)),x, algorithm="fricas")

[Out] 1/3*log(2*e^x - 1) - 1/3*log(e^x + 1)

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-e^{-x}+2e^x} \, dx = \frac{\log \left(-2 + e^{-x}\right)}{3} - \frac{\log \left(1 + e^{-x}\right)}{3}$$

[In] integrate(1/(1-1/exp(x)+2*exp(x)),x)

[Out] log(-2 + exp(-x))/3 - log(1 + exp(-x))/3

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-e^{-x}+2e^x} \, dx = -\frac{1}{3} \log \left(e^{(-x)} + 1\right) + \frac{1}{3} \log \left(e^{(-x)} - 2\right)$$

[In] integrate(1/(1-1/exp(x)+2*exp(x)),x, algorithm="maxima")

[Out] -1/3*log(e^(-x) + 1) + 1/3*log(e^(-x) - 2)

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{1}{1-e^{-x}+2e^x}\,dx = -\frac{1}{3}\,\log\left(e^x+1\right)+\frac{1}{3}\,\log\left(\left|2\,e^x-1\right|\right)$$

```
[In] integrate(1/(1-1/exp(x)+2*exp(x)),x, algorithm="giac")
[Out] -1/3*log(e^x + 1) + 1/3*log(abs(2*e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{1-e^{-x}+2e^x}\,dx = \frac{\ln\left(2\,\mathrm{e}^x-1\right)}{3}-\frac{\ln\left(\mathrm{e}^x+1\right)}{3}$$

```
[In] int(1/(2*exp(x) - exp(-x) + 1),x)
[Out] log(2*exp(x) - 1)/3 - log(exp(x) + 1)/3
```

3.332 $\int \frac{\arctan(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log(1+x)$$

```
[Out] -ln(1+x)+2*arctan(x^(1/2))*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4946, 31}

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log(x+1)$$

```
[In] Int[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
```

```
IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\sqrt{x}\arctan\left(\sqrt{x}\right) - \int \frac{1}{1+x}\,dx\\ &= 2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log(1+x)$$

```
[In] Integrate[ArcTan[Sqrt[x]]/Sqrt[x],x]
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\ln\left(1+x\right)+2\arctan\left(\sqrt{x}\right)\sqrt{x}$	17
default	$-\ln\left(1+x\right)+2\arctan\left(\sqrt{x}\right)\sqrt{x}$	17
meijerg	$-\ln\left(1+x\right)+2\arctan\left(\sqrt{x}\right)\sqrt{x}$	17

```
[In] int(arctan(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -ln(1+x)+2*arctan(x^(1/2))*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right) - \log\left(x+1\right)$$

```
[In] integrate(atan(x**(1/2))/x**(1/2),x)
[Out] 2*sqrt(x)*atan(sqrt(x)) - log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sqrt{x}\,\mathrm{atan}\left(\sqrt{x}\right) - \ln\left(x+1\right)$$

```
[In] int(atan(x^(1/2))/x^(1/2),x)
[Out] 2*x^(1/2)*atan(x^(1/2)) - log(x + 1)
```

3.333 $\int \frac{\log(1+x)}{x^2} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 18

$$\int \frac{\log(1+x)}{x^2} \, dx = \log(x) - \log(1+x) - \frac{\log(1+x)}{x}$$

```
[Out] ln(x)-ln(1+x)-ln(1+x)/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2442, 36, 29, 31}

$$\int \frac{\log(1+x)}{x^2} \, dx = \log(x) - \frac{\log(x+1)}{x} - \log(x+1)$$

```
[In] Int[Log[1 + x]/x^2,x]
```

```
[Out] Log[x] - Log[1 + x] - Log[1 + x]/x
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log(1+x)}{x} + \int \frac{1}{x(1+x)}\,dx \\
&= -\frac{\log(1+x)}{x} + \int \frac{1}{x}\,dx - \int \frac{1}{1+x}\,dx \\
&= \log(x) - \log(1+x) - \frac{\log(1+x)}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\log(1+x)}{x^2}\,dx = \log(x) - \log(1+x) - \frac{\log(1+x)}{x}$$

```
[In] Integrate[Log[1 + x]/x^2,x]
[Out] Log[x] - Log[1 + x] - Log[1 + x]/x
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\ln(x) - \frac{\ln(1+x)(1+x)}{x}$	16
default	$\ln(x) - \frac{\ln(1+x)(1+x)}{x}$	16
meijerg	$\ln(x) - \frac{(2x+2)\ln(1+x)}{2x}$	18
risch	$\ln(x) - \ln(1+x) - \frac{\ln(1+x)}{x}$	19
parts	$\ln(x) - \ln(1+x) - \frac{\ln(1+x)}{x}$	19
norman	$\frac{-\ln(1+x)x-\ln(1+x)}{x} + \ln(x)$	22
parallelrisch	$\frac{x\ln(x)-\ln(1+x)x-\ln(1+x)}{x}$	23

```
[In] int(1/x^2*ln(1+x),x,method=_RETURNVERBOSE)
[Out] ln(x)-ln(1+x)*(1+x)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\log(1+x)}{x^2}\,dx = -\frac{(x+1)\log(x+1) - x\log(x)}{x}$$

```
[In] integrate(log(1+x)/x^2,x, algorithm="fricas")
[Out] -((x + 1)*log(x + 1) - x*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\log(1+x)}{x^2}\,dx = \log(x) - \log(x+1) - \frac{\log(x+1)}{x}$$

```
[In] integrate(ln(1+x)/x**2,x)
[Out] log(x) - log(x + 1) - log(x + 1)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\log(1+x)}{x^2} \, dx = -\frac{\log(x+1)}{x} - \log(x+1) + \log(x)$$

```
[In] integrate(log(1+x)/x^2,x, algorithm="maxima")
[Out] -log(x + 1)/x - log(x + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\log(1+x)}{x^2} \, dx = -\frac{\log(x+1)}{x} - \log(|x+1|) + \log(|x|)$$

```
[In] integrate(log(1+x)/x^2,x, algorithm="giac")
[Out] -log(x + 1)/x - log(abs(x + 1)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\log(1+x)}{x^2} \, dx = -\ln\left(\frac{1}{x}+1\right) - \frac{\ln(x+1)}{x}$$

```
[In] int(log(x + 1)/x^2,x)
[Out] - log(1/x + 1) - log(x + 1)/x
```

3.334 $\int \frac{1}{-e^x+e^{3x}} dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{1}{-e^x+e^{3x}} dx = e^{-x} - \operatorname{arctanh}(e^x)$$

```
[Out] exp(-x)-arctanh(exp(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2320, 331, 213}

$$\int \frac{1}{-e^x+e^{3x}} dx = e^{-x} - \operatorname{arctanh}(e^x)$$

```
[In] Int[(-E^x + E^(3*x))^(-1),x]
```

```
[Out] E^(-x) - ArcTanh[E^x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{x^2\,(-1+x^2)}\,dx, x, e^x\right)\\ &= e^{-x} + \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^x\right)\\ &= e^{-x} - \operatorname{arctanh}(e^x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{-e^x + e^{3x}}\,dx = e^{-x} - \operatorname{arctanh}(e^x)$$

```
[In] Integrate[(-E^x + E^(3*x))^(-1),x]
[Out] E^(-x) - ArcTanh[E^x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

method	result	size
default	$\mathrm{e}^{-x}-\frac{\ln(1+\mathrm{e}^x)}{2}+\frac{\ln(-1+\mathrm{e}^x)}{2}$	20
norman	$\mathrm{e}^{-x}-\frac{\ln(1+\mathrm{e}^x)}{2}+\frac{\ln(-1+\mathrm{e}^x)}{2}$	20
risch	$\mathrm{e}^{-x}-\frac{\ln(1+\mathrm{e}^x)}{2}+\frac{\ln(-1+\mathrm{e}^x)}{2}$	20

```
[In] int(1/(exp(3*x)-exp(x)),x,method=_RETURNVERBOSE)
[Out] 1/exp(x)-1/2*ln(1+exp(x))+1/2*ln(-1+exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int \frac{1}{-e^x + e^{3x}}\, dx = -\frac{1}{2}\left(e^x \log\left(e^x + 1\right) - e^x \log\left(e^x - 1\right) - 2\right)e^{(-x)}$$

```
[In] integrate(1/(-exp(x)+exp(3*x)),x, algorithm="fricas")
[Out] -1/2*(e^x*log(e^x + 1) - e^x*log(e^x - 1) - 2)*e^(-x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{1}{-e^x + e^{3x}}\, dx = \frac{\log\left(e^x - 1\right)}{2} - \frac{\log\left(e^x + 1\right)}{2} + e^{-x}$$

```
[In] integrate(1/(-exp(x)+exp(3*x)),x)
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2 + exp(-x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{1}{-e^x + e^{3x}}\, dx = e^{(-x)} - \frac{1}{2}\log\left(e^x + 1\right) + \frac{1}{2}\log\left(e^x - 1\right)$$

```
[In] integrate(1/(-exp(x)+exp(3*x)),x, algorithm="maxima")
[Out] e^(-x) - 1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{1}{-e^x + e^{3x}}\,dx = e^{(-x)} - \frac{1}{2}\,\log\left(e^x + 1\right) + \frac{1}{2}\,\log\left(\left|e^x - 1\right|\right)$$

```
[In] integrate(1/(-exp(x)+exp(3*x)),x, algorithm="giac")
```

```
[Out] e^(-x) - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{1}{-e^x + e^{3x}}\,dx = \mathrm{e}^{-x} + \frac{\ln\left(\mathrm{e}^x - 1\right)}{2} - \frac{\ln\left(\mathrm{e}^x + 1\right)}{2}$$

```
[In] int(1/(exp(3*x) - exp(x)),x)
```

```
[Out] exp(-x) + log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

3.335 $\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 8

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx = -x-2\cot(x)$$

```
[Out] -x-2*cot(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3250, 3254, 3852, 8}

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx = -x-2\cot(x)$$

```
[In] Int[(1 + Cos[x]^2)/(1 - Cos[x]^2),x]
```

```
[Out] -x - 2*Cot[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3250

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -x + 2\int \frac{1}{1-\cos^2(x)}\,dx\\ &= -x + 2\int \csc^2(x)\,dx\\ &= -x - 2\text{Subst}\left(\int 1\,dx, x, \cot(x)\right)\\ &= -x - 2\cot(x)\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.88

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx = -\cot(x) - \cot(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -\tan^2(x)\right)$$

```
[In] Integrate[(1 + Cos[x]^2)/(1 - Cos[x]^2),x]
[Out] -Cot[x] - Cot[x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[x]^2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
parallelrisch	$-x-2\cot\left(x\right)$	9
default	$-\frac{2}{\tan\left(x\right)}-\arctan\left(\tan\left(x\right)\right)$	13
risch	$-x-\frac{4i}{e^{2ix}-1}$	17
norman	$\frac{-1+\tan^{4}\left(\frac{x}{2}\right)+\tan^{6}\left(\frac{x}{2}\right)-\left(\tan^{2}\left(\frac{x}{2}\right)\right)-x\tan\left(\frac{x}{2}\right)-x\left(\tan^{5}\left(\frac{x}{2}\right)\right)-2\left(\tan^{3}\left(\frac{x}{2}\right)\right)x}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}\tan\left(\frac{x}{2}\right)}$	65

```
[In] int((1+cos(x)^2)/(1-cos(x)^2),x,method=_RETURNVERBOSE)
[Out] -x-2*cot(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx=-\frac{x\sin(x)+2\cos(x)}{\sin(x)}$$

```
[In] integrate((1+cos(x)^2)/(1-cos(x)^2),x, algorithm="fricas")
[Out] -(x*sin(x) + 2*cos(x))/sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx=-x+\tan\left(\frac{x}{2}\right)-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

```
[In] integrate((1+cos(x)**2)/(1-cos(x)**2),x)
[Out] -x + tan(x/2) - 1/tan(x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx=-x-\frac{2}{\tan(x)}$$

```
[In] integrate((1+cos(x)^2)/(1-cos(x)^2),x, algorithm="maxima")
[Out] -x - 2/tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx = -x - \frac{1}{\tan\left(\frac{1}{2}\,x\right)} + \tan\left(\frac{1}{2}\,x\right)$$

```
[In] integrate((1+cos(x)^2)/(1-cos(x)^2),x, algorithm="giac")
[Out] -x - 1/tan(1/2*x) + tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx = -x - 2\cot(x)$$

```
[In] int(-(cos(x)^2 + 1)/(cos(x)^2 - 1),x)
[Out] - x - 2*cot(x)
```

3.336 $\int \frac{1}{x\sqrt{-25+2x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{1}{x\sqrt{-25+2x}}\,dx = \frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{-25+2x}\right)$$

```
[Out] 2/5*arctan(1/5*(-25+2*x)^(1/2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {65, 209}

$$\int \frac{1}{x\sqrt{-25+2x}}\,dx = \frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{2x-25}\right)$$

```
[In] Int[1/(x*Sqrt[-25 + 2*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[-25 + 2*x]/5])/5
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\frac{25}{2}+\frac{x^2}{2}}\,dx, x, \sqrt{-25+2x}\right)$$
$$= \frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{-25+2x}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-25+2x}}\,dx = \frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{-25+2x}\right)$$

```
[In] Integrate[1/(x*Sqrt[-25 + 2*x]),x]
[Out] (2*ArcTan[Sqrt[-25 + 2*x]/5])/5
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{\sqrt{-25+2x}}{5}\right)}{5}$	13
default	$\frac{2\arctan\left(\frac{\sqrt{-25+2x}}{5}\right)}{5}$	13
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{-25+2x}}{5}\right)}{5}$	13
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x-25\operatorname{RootOf}\left(\_Z^2+1\right)+5\sqrt{-25+2x}}{x}\right)}{5}$	40
meijerg	$\frac{\sqrt{-\operatorname{signum}\left(x-\frac{25}{2}\right)}\left(\left(-\ln(2)+\ln(x)-2\ln(5)+i\pi\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{2x}{25}}}{2}\right)\right)}{5\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(x-\frac{25}{2}\right)}}$	57

```
[In] int(1/x/(-25+2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*arctan(1/5*(-25+2*x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{1}{x\sqrt{-25+2x}}\,dx = \frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{2\,x-25}\right)$$

```
[In] integrate(1/x/(-25+2*x)^(1/2),x, algorithm="fricas")
[Out] 2/5*arctan(1/5*sqrt(2*x - 25))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{x\sqrt{-25+2x}}\,dx = \begin{cases} \frac{2i\operatorname{acosh}\left(\frac{5\sqrt{2}}{2\sqrt{x}}\right)}{5} & \text{for } \frac{1}{|x|} > \frac{2}{25} \\ -\frac{2\operatorname{asin}\left(\frac{5\sqrt{2}}{2\sqrt{x}}\right)}{5} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/(-25+2*x)**(1/2),x)
[Out] Piecewise((2*I*acosh(5*sqrt(2)/(2*sqrt(x)))/5, 1/Abs(x) > 2/25), (-2*asin(5
*sqrt(2)/(2*sqrt(x)))/5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{1}{x\sqrt{-25+2x}}\,dx = \frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{2\,x-25}\right)$$

```
[In] integrate(1/x/(-25+2*x)^(1/2),x, algorithm="maxima")
[Out] 2/5*arctan(1/5*sqrt(2*x - 25))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{1}{x\sqrt{-25+2x}}\,dx = \frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{2x-25}\right)$$

```
[In] integrate(1/x/(-25+2*x)^(1/2),x, algorithm="giac")
[Out] 2/5*arctan(1/5*sqrt(2*x - 25))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{1}{x\sqrt{-25+2x}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{2x-25}}{5}\right)}{5}$$

```
[In] int(1/(x*(2*x - 25)^(1/2)),x)
[Out] (2*atan((2*x - 25)^(1/2)/5))/5
```

3.337 $\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 11

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx = -\arcsin\left(\frac{\cos^2(x)}{3}\right)$$

```
[Out] -arcsin(1/3*cos(x)^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {12, 1121, 633, 222}

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx = -\arcsin\left(\frac{\cos^2(x)}{3}\right)$$

```
[In] Int[Sin[2*x]/Sqrt[9 - Cos[x]^4],x]
[Out] -ArcSin[Cos[x]^2/3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{2x}{\sqrt{8+2x^2-x^4}}\,dx, x, \sin(x)\right)\\
&= 2\text{Subst}\left(\int \frac{x}{\sqrt{8+2x^2-x^4}}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{\sqrt{8+2x-x^2}}\,dx, x, \sin^2(x)\right)\\
&= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx, x, 2\cos^2(x)\right)\right)\\
&= -\arcsin\left(\frac{\cos^2(x)}{3}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx = -\arcsin\left(\frac{\cos^2(x)}{3}\right)$$

```
[In] Integrate[Sin[2*x]/Sqrt[9 - Cos[x]^4],x]
[Out] -ArcSin[Cos[x]^2/3]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$-\arcsin\left(\frac{(\cos^2(x))}{3}\right)$	10
default	$-\arcsin\left(\frac{(\cos^2(x))}{3}\right)$	10

```
[In] int(sin(2*x)/(9-cos(x)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -arcsin(1/3*cos(x)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(9) = 18.

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx = \arctan\left(\frac{\sqrt{-\cos(x)^4+9}\cos(x)^2}{\cos(x)^4-9}\right)$$

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="fricas")
[Out] arctan(sqrt(-cos(x)^4 + 9)*cos(x)^2/(cos(x)^4 - 9))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(2*x)/(9-cos(x)**4)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx = \int \frac{\sin(2x)}{\sqrt{-\cos(x)^4+9}}\,dx$$

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(sin(2*x)/sqrt(-cos(x)^4 + 9), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx = -\arcsin\left(\frac{1}{3}\cos(x)^2\right)$$

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="giac")
[Out] -arcsin(1/3*cos(x)^2)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx = -\mathrm{atan}\left(\frac{\cos(x)^2}{\sqrt{9-\cos(x)^4}}\right)$$

```
[In] int(sin(2*x)/(9 - cos(x)^4)^(1/2),x)
[Out] -atan(cos(x)^2/(9 - cos(x)^4)^(1/2))
```

3.338 $\int \frac{x^2}{\sqrt{5-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{x^2}{\sqrt{5-4x^2}}\,dx = -\frac{1}{8}x\sqrt{5-4x^2}+\frac{5}{16}\arcsin\left(\frac{2x}{\sqrt{5}}\right)$$

```
[Out] 5/16*arcsin(2/5*x*5^(1/2))-1/8*x*(-4*x^2+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {327, 222}

$$\int \frac{x^2}{\sqrt{5-4x^2}}\,dx = \frac{5}{16}\arcsin\left(\frac{2x}{\sqrt{5}}\right)-\frac{1}{8}x\sqrt{5-4x^2}$$

```
[In] Int[x^2/Sqrt[5 - 4*x^2],x]
```

```
[Out] -1/8*(x*Sqrt[5 - 4*x^2]) + (5*ArcSin[(2*x)/Sqrt[5]])/16
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{8}x\sqrt{5-4x^2}+\frac{5}{8}\int \frac{1}{\sqrt{5-4x^2}}\,dx\\ &= -\frac{1}{8}x\sqrt{5-4x^2}+\frac{5}{16}\arcsin\left(\frac{2x}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{x^2}{\sqrt{5-4x^2}}\,dx = -\frac{1}{8}x\sqrt{5-4x^2}-\frac{5}{8}\arctan\left(\frac{2x}{\sqrt{5}-\sqrt{5-4x^2}}\right)$$

```
[In] Integrate[x^2/Sqrt[5 - 4*x^2],x]
[Out] -1/8*(x*Sqrt[5 - 4*x^2]) - (5*ArcTan[(2*x)/(Sqrt[5] - Sqrt[5 - 4*x^2])])/8
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{5\arcsin\left(\frac{2x\sqrt{5}}{5}\right)}{16}-\frac{x\sqrt{-4x^2+5}}{8}$	23
risch	$\frac{x(4x^2-5)}{8\sqrt{-4x^2+5}}+\frac{5\arcsin\left(\frac{2x\sqrt{5}}{5}\right)}{16}$	30
pseudoelliptic	$-\frac{x\sqrt{-4x^2+5}}{8}-\frac{5\arctan\left(\frac{\sqrt{-4x^2+5}}{2x}\right)}{16}$	31
meijerg	$\frac{5i\left(\frac{2i\sqrt{\pi}\,x\sqrt{5}\,\sqrt{-\frac{4x^2}{5}+1}}{5}-i\sqrt{\pi}\,\arcsin\left(\frac{2x\sqrt{5}}{5}\right)\right)}{16\sqrt{\pi}}$	40
trager	$-\frac{x\sqrt{-4x^2+5}}{8}+\frac{5\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2+5}+2x\right)}{16}$	43

```
[In] int(x^2/(-4*x^2+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] 5/16*arcsin(2/5*x*5^(1/2))-1/8*x*(-4*x^2+5)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{5-4x^2}}\,dx = -\frac{1}{8}\sqrt{-4\,x^2+5}x - \frac{5}{16}\arctan\left(\frac{\sqrt{-4\,x^2+5}}{2\,x}\right)$$

```
[In] integrate(x^2/(-4*x^2+5)^(1/2),x, algorithm="fricas")
[Out] -1/8*sqrt(-4*x^2 + 5)*x - 5/16*arctan(1/2*sqrt(-4*x^2 + 5)/x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{\sqrt{5-4x^2}}\,dx = -\frac{x\sqrt{5-4x^2}}{8} + \frac{5\operatorname{asin}\left(\frac{2\sqrt{5}x}{5}\right)}{16}$$

```
[In] integrate(x**2/(-4*x**2+5)**(1/2),x)
[Out] -x*sqrt(5 - 4*x**2)/8 + 5*asin(2*sqrt(5)*x/5)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{5-4x^2}}\,dx = -\frac{1}{8}\sqrt{-4\,x^2+5}x + \frac{5}{16}\arcsin\left(\frac{2}{5}\sqrt{5}x\right)$$

```
[In] integrate(x^2/(-4*x^2+5)^(1/2),x, algorithm="maxima")
[Out] -1/8*sqrt(-4*x^2 + 5)*x + 5/16*arcsin(2/5*sqrt(5)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{5-4x^2}}\,dx = -\frac{1}{8}\sqrt{-4\,x^2+5}x + \frac{5}{16}\,\arcsin\left(\frac{2}{5}\sqrt{5}x\right)$$

```
[In] integrate(x^2/(-4*x^2+5)^(1/2),x, algorithm="giac")
[Out] -1/8*sqrt(-4*x^2 + 5)*x + 5/16*arcsin(2/5*sqrt(5)*x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{5-4x^2}}\,dx = \frac{5\,\mathrm{asin}\left(\frac{2\,\sqrt{5}\,x}{5}\right)}{16} - \frac{x\,\sqrt{\frac{5}{4}-x^2}}{4}$$

```
[In] int(x^2/(5 - 4*x^2)^(1/2),x)
[Out] (5*asin((2*5^(1/2)*x)/5))/16 - (x*(5/4 - x^2)^(1/2))/4
```

3.339 $\int x^3 \sin(x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 24

$$\int x^3 \sin(x)\,dx = 6x\cos(x) - x^3\cos(x) - 6\sin(x) + 3x^2\sin(x)$$

```
[Out] 6*x*cos(x)-x^3*cos(x)-6*sin(x)+3*x^2*sin(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3377, 2717}

$$\int x^3 \sin(x)\,dx = x^3(-\cos(x)) + 3x^2\sin(x) - 6\sin(x) + 6x\cos(x)$$

```
[In] Int[x^3*Sin[x],x]
[Out] 6*x*Cos[x] - x^3*Cos[x] - 6*Sin[x] + 3*x^2*Sin[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -x^3\cos(x)+3\int x^2\cos(x)\,dx\\
&= -x^3\cos(x)+3x^2\sin(x)-6\int x\sin(x)\,dx\\
&= 6x\cos(x)-x^3\cos(x)+3x^2\sin(x)-6\int \cos(x)\,dx\\
&= 6x\cos(x)-x^3\cos(x)-6\sin(x)+3x^2\sin(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x^3\sin(x)\,dx=-x\left(-6+x^2\right)\cos(x)+3\left(-2+x^2\right)\sin(x)$$

```
[In] Integrate[x^3*Sin[x],x]
[Out] -(x*(-6 + x^2)*Cos[x]) + 3*(-2 + x^2)*Sin[x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
risch	$(-x^3+6x)\cos(x)+3(x^2-2)\sin(x)$	23
default	$6x\cos(x)-x^3\cos(x)-6\sin(x)+3x^2\sin(x)$	25
parallelrisch	$6x\cos(x)-x^3\cos(x)-6\sin(x)+3x^2\sin(x)$	25
parts	$6x\cos(x)-x^3\cos(x)-6\sin(x)+3x^2\sin(x)$	25
meijerg	$8\sqrt{\pi}\left(\frac{x\left(-\frac{5x^2}{2}+15\right)\cos(x)}{20\sqrt{\pi}}-\frac{\left(-\frac{15x^2}{2}+15\right)\sin(x)}{20\sqrt{\pi}}\right)$	36
norman	$\frac{x^3\left(\tan^2\left(\frac{x}{2}\right)\right)+6x-x^3-6x\left(\tan^2\left(\frac{x}{2}\right)\right)+6x^2\tan\left(\frac{x}{2}\right)-12\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$	55

```
[In] int(x^3*sin(x),x,method=_RETURNVERBOSE)
[Out] (-x^3+6*x)*cos(x)+3*(x^2-2)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int x^3 \sin(x)\,dx = -\left(x^3 - 6\,x\right)\cos\left(x\right) + 3\,\left(x^2 - 2\right)\sin\left(x\right)$$

```
[In] integrate(x^3*sin(x),x, algorithm="fricas")
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^3 \sin(x)\,dx = -x^3\cos\left(x\right) + 3x^2\sin\left(x\right) + 6x\cos\left(x\right) - 6\sin\left(x\right)$$

```
[In] integrate(x**3*sin(x),x)
[Out] -x**3*cos(x) + 3*x**2*sin(x) + 6*x*cos(x) - 6*sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int x^3 \sin(x)\,dx = -\left(x^3 - 6\,x\right)\cos\left(x\right) + 3\,\left(x^2 - 2\right)\sin\left(x\right)$$

```
[In] integrate(x^3*sin(x),x, algorithm="maxima")
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int x^3 \sin(x)\,dx = -\left(x^3 - 6\,x\right)\cos\left(x\right) + 3\,\left(x^2 - 2\right)\sin\left(x\right)$$

```
[In] integrate(x^3*sin(x),x, algorithm="giac")
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int x^3 \sin(x)\,dx = \cos(x)\,\left(6\,x - x^3\right) + \sin(x)\,\left(3\,x^2 - 6\right)$$

```
[In] int(x^3*sin(x),x)
[Out] cos(x)*(6*x - x^3) + sin(x)*(3*x^2 - 6)
```

3.340 $\int x\sqrt{4+2x+x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int x\sqrt{4+2x+x^2}\,dx=-\frac{1}{2}(1+x)\sqrt{4+2x+x^2}+\frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\frac{3}{2}\text{arcsinh}\left(\frac{1+x}{\sqrt{3}}\right)$$

```
[Out] 1/3*(x^2+2*x+4)^(3/2)-3/2*arcsinh(1/3*(1+x)*3^(1/2))-1/2*(1+x)*(x^2+2*x+4)^
(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {654, 626, 633, 221}

$$\int x\sqrt{4+2x+x^2}\,dx=-\frac{3}{2}\text{arcsinh}\left(\frac{x+1}{\sqrt{3}}\right)+\frac{1}{3}\left(x^2+2x+4\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{x^2+2x+4}$$

```
[In] Int[x*Sqrt[4 + 2*x + x^2],x]
```

```
[Out] -1/2*((1 + x)*Sqrt[4 + 2*x + x^2]) + (4 + 2*x + x^2)^(3/2)/3 - (3*ArcSinh[(
1 + x)/Sqrt[3]])/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
```

```
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\int \sqrt{4+2x+x^2}\,dx \\
&= -\frac{1}{2}(1+x)\sqrt{4+2x+x^2}+\frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\frac{3}{2}\int \frac{1}{\sqrt{4+2x+x^2}}\,dx \\
&= -\frac{1}{2}(1+x)\sqrt{4+2x+x^2}+\frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\frac{1}{4}\sqrt{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{12}}}\,dx,x,2+2x\right) \\
&= -\frac{1}{2}(1+x)\sqrt{4+2x+x^2}+\frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\frac{3}{2}\text{arcsinh}\left(\frac{1+x}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int x\sqrt{4+2x+x^2}\,dx=\frac{1}{6}\sqrt{4+2x+x^2}\left(5+x+2x^2\right)+\frac{3}{2}\log\left(-1-x+\sqrt{4+2x+x^2}\right)$$

```
[In] Integrate[x*Sqrt[4 + 2*x + x^2],x]
[Out] (Sqrt[4 + 2*x + x^2]*(5 + x + 2*x^2))/6 + (3*Log[-1 - x + Sqrt[4 + 2*x + x^
2]])/2
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{(2x^2+x+5)\sqrt{x^2+2x+4}}{6}-\frac{3\,\operatorname{arcsinh}\left(\frac{(1+x)\sqrt{3}}{3}\right)}{2}$	33
trager	$\left(\frac{1}{3}x^2+\frac{1}{6}x+\frac{5}{6}\right)\sqrt{x^2+2x+4}-\frac{3\ln\left(1+x+\sqrt{x^2+2x+4}\right)}{2}$	39
default	$\frac{(x^2+2x+4)^{\frac{3}{2}}}{3}-\frac{(2x+2)\sqrt{x^2+2x+4}}{4}-\frac{3\,\operatorname{arcsinh}\left(\frac{(1+x)\sqrt{3}}{3}\right)}{2}$	42

```
[In] int(x*(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*(2*x^2+x+5)*(x^2+2*x+4)^(1/2)-3/2*arcsinh(1/3*(1+x)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int x\sqrt{4+2x+x^2}\,dx=\frac{1}{6}\left(2\,x^2+x+5\right)\sqrt{x^2+2\,x+4}+\frac{3}{2}\log\left(-x+\sqrt{x^2+2\,x+4}-1\right)$$

```
[In] integrate(x*(x^2+2*x+4)^(1/2),x, algorithm="fricas")
[Out] 1/6*(2*x^2 + x + 5)*sqrt(x^2 + 2*x + 4) + 3/2*log(-x + sqrt(x^2 + 2*x + 4)
- 1)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

$$\int x\sqrt{4+2x+x^2}\,dx=\left(\frac{x^2}{3}+\frac{x}{6}+\frac{5}{6}\right)\sqrt{x^2+2x+4}-\frac{3\operatorname{asinh}\left(\frac{\sqrt{3}(x+1)}{3}\right)}{2}$$

```
[In] integrate(x*(x**2+2*x+4)**(1/2),x)
[Out] (x**2/3 + x/6 + 5/6)*sqrt(x**2 + 2*x + 4) - 3*asinh(sqrt(3)*(x + 1)/3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int x\sqrt{4+2x+x^2}\,dx = \frac{1}{3}\left(x^2+2\,x+4\right)^{\frac{3}{2}} - \frac{1}{2}\sqrt{x^2+2\,x+4}x - \frac{1}{2}\sqrt{x^2+2\,x+4} - \frac{3}{2}\,\mathrm{arsinh}\left(\frac{1}{3}\sqrt{3}(x+1)\right)$$

```
[In] integrate(x*(x^2+2*x+4)^(1/2),x, algorithm="maxima")
[Out] 1/3*(x^2 + 2*x + 4)^(3/2) - 1/2*sqrt(x^2 + 2*x + 4)*x - 1/2*sqrt(x^2 + 2*x
+ 4) - 3/2*arcsinh(1/3*sqrt(3)*(x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int x\sqrt{4+2x+x^2}\,dx = \frac{1}{6}\,((2\,x+1)x+5)\sqrt{x^2+2\,x+4} + \frac{3}{2}\,\log\left(-x+\sqrt{x^2+2\,x+4}-1\right)$$

```
[In] integrate(x*(x^2+2*x+4)^(1/2),x, algorithm="giac")
[Out] 1/6*((2*x + 1)*x + 5)*sqrt(x^2 + 2*x + 4) + 3/2*log(-x + sqrt(x^2 + 2*x + 4
) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int x\sqrt{4+2x+x^2}\,dx = \frac{\sqrt{x^2+2\,x+4}\,\left(8\,x^2+4\,x+20\right)}{24} - \frac{3\,\ln\left(x+\sqrt{x^2+2\,x+4}+1\right)}{2}$$

```
[In] int(x*(2*x + x^2 + 4)^(1/2),x)
[Out] ((2*x + x^2 + 4)^(1/2)*(4*x + 8*x^2 + 20))/24 - (3*log(x + (2*x + x^2 + 4)^
(1/2) + 1))/2
```

3.341 $\int x(5+x^2)^8\,dx$

Optimal result

Integrand size = 9, antiderivative size = 11

$$\int x(5+x^2)^8\,dx=\frac{1}{18}(5+x^2)^9$$

```
[Out] 1/18*(x^2+5)^9
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {267}

$$\int x(5+x^2)^8\,dx=\frac{1}{18}(x^2+5)^9$$

```
[In] Int[x*(5 + x^2)^8,x]
```

```
[Out] (5 + x^2)^9/18
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{18}(5+x^2)^9$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int x(5+x^2)^8\,dx=\frac{1}{18}(5+x^2)^9$$

```
[In] Integrate[x*(5 + x^2)^8,x]
```

```
[Out] (5 + x^2)^9/18
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result
default	$\frac{(x^2+5)^9}{18}$
gosper	$\frac{390625}{2}x^2+156250x^4+\frac{218750}{3}x^6+21875x^8+4375x^{10}+\frac{1750}{3}x^{12}+50x^{14}+\frac{5}{2}x^{16}+\frac{1}{18}x^{18}$
norman	$\frac{390625}{2}x^2+156250x^4+\frac{218750}{3}x^6+21875x^8+4375x^{10}+\frac{1750}{3}x^{12}+50x^{14}+\frac{5}{2}x^{16}+\frac{1}{18}x^{18}$
parallelrisch	$\frac{390625}{2}x^2+156250x^4+\frac{218750}{3}x^6+21875x^8+4375x^{10}+\frac{1750}{3}x^{12}+50x^{14}+\frac{5}{2}x^{16}+\frac{1}{18}x^{18}$
risch	$\frac{1}{18}x^{18}+\frac{5}{2}x^{16}+50x^{14}+\frac{1750}{3}x^{12}+4375x^{10}+21875x^8+\frac{218750}{3}x^6+156250x^4+\frac{390625}{2}x^2+\frac{195}{1}$

```
[In] int(x*(x^2+5)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*(x^2+5)^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(9) = 18$.

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 4.18

$$\int x(5+x^2)^8\,dx=\frac{1}{18}\,x^{18}+\frac{5}{2}\,x^{16}+50\,x^{14}+\frac{1750}{3}\,x^{12}+4375\,x^{10}+21875\,x^8+\frac{218750}{3}\,x^6+156250\,x^4+\frac{390625}{2}\,x^2$$

```
[In] integrate(x*(x^2+5)^8,x, algorithm="fricas")
```

```
[Out] 1/18*x^18 + 5/2*x^16 + 50*x^14 + 1750/3*x^12 + 4375*x^10 + 21875*x^8 + 2187
50/3*x^6 + 156250*x^4 + 390625/2*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(7) = 14$.

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 4.64

$$\int x(5+x^2)^8\,dx = \frac{x^{18}}{18} + \frac{5x^{16}}{2} + 50x^{14} + \frac{1750x^{12}}{3} + 4375x^{10} + 21875x^8 + \frac{218750x^6}{3} + 156250x^4 + \frac{390625x^2}{2}$$

```
[In] integrate(x*(x**2+5)**8,x)
[Out] x**18/18 + 5*x**16/2 + 50*x**14 + 1750*x**12/3 + 4375*x**10 + 21875*x**8 +
218750*x**6/3 + 156250*x**4 + 390625*x**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int x(5+x^2)^8\,dx = \frac{1}{18}\left(x^2+5\right)^9$$

```
[In] integrate(x*(x^2+5)^8,x, algorithm="maxima")
[Out] 1/18*(x^2 + 5)^9
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int x(5+x^2)^8\,dx = \frac{1}{18}\left(x^2+5\right)^9$$

```
[In] integrate(x*(x^2+5)^8,x, algorithm="giac")
[Out] 1/18*(x^2 + 5)^9
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int x(5+x^2)^8\,dx=\frac{(x^2+5)^9}{18}$$

```
[In] int(x*(x^2 + 5)^8,x)
[Out] (x^2 + 5)^9/18
```

3.342 $\int \cos^2(x) \sin^5(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \cos^2(x) \sin^5(x) \, dx = -\frac{1}{3} \cos^3(x) + \frac{2 \cos^5(x)}{5} - \frac{\cos^7(x)}{7}$$

```
[Out] -1/3*cos(x)^3+2/5*cos(x)^5-1/7*cos(x)^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2645, 276}

$$\int \cos^2(x) \sin^5(x) \, dx = -\frac{1}{7} \cos^7(x) + \frac{2 \cos^5(x)}{5} - \frac{\cos^3(x)}{3}$$

```
[In] Int[Cos[x]^2*Sin[x]^5,x]
```

```
[Out] -1/3*Cos[x]^3 + (2*Cos[x]^5)/5 - Cos[x]^7/7
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx, x, \cos(x)\right)\\ &= -\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \cos(x)\right)\\ &= -\frac{1}{3}\cos^3(x)+\frac{2\cos^5(x)}{5}-\frac{\cos^7(x)}{7}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \cos^2(x)\sin^5(x)\,dx = -\frac{5\cos(x)}{64}-\frac{1}{192}\cos(3x)+\frac{3}{320}\cos(5x)-\frac{1}{448}\cos(7x)$$

```
[In] Integrate[Cos[x]^2*Sin[x]^5,x]
[Out] (-5*Cos[x])/64 - Cos[3*x]/192 + (3*Cos[5*x])/320 - Cos[7*x]/448
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\left(\cos^3(x)\right)}{3}+\frac{2\left(\cos^5(x)\right)}{5}-\frac{\left(\cos^7(x)\right)}{7}$	20
default	$-\frac{\left(\cos^3(x)\right)}{3}+\frac{2\left(\cos^5(x)\right)}{5}-\frac{\left(\cos^7(x)\right)}{7}$	20
risch	$-\frac{5\cos(x)}{64}-\frac{\cos(7x)}{448}+\frac{3\cos(5x)}{320}-\frac{\cos(3x)}{192}$	24
parallelrisch	$\frac{8}{35}-\frac{5\cos(x)}{64}-\frac{\cos(7x)}{448}+\frac{3\cos(5x)}{320}-\frac{\cos(3x)}{192}$	25
norman	$\frac{-\frac{32\left(\tan^8\left(\frac{x}{2}\right)\right)}{3}-\frac{16\left(\tan^4\left(\frac{x}{2}\right)\right)}{5}-\frac{16\left(\tan^2\left(\frac{x}{2}\right)\right)}{15}+\frac{16\left(\tan^6\left(\frac{x}{2}\right)\right)}{3}-\frac{16}{105}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^7}$	46

```
[In] int(cos(x)^2*sin(x)^5,x,method=_RETURNVERBOSE)
[Out] -1/3*cos(x)^3+2/5*cos(x)^5-1/7*cos(x)^7
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos^2(x)\sin^5(x)\,dx = -\frac{1}{7}\,\cos{(x)}^7 + \frac{2}{5}\,\cos{(x)}^5 - \frac{1}{3}\,\cos{(x)}^3$$

```
[In] integrate(cos(x)^2*sin(x)^5,x, algorithm="fricas")
[Out] -1/7*cos(x)^7 + 2/5*cos(x)^5 - 1/3*cos(x)^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \cos^2(x)\sin^5(x)\,dx = -\frac{\cos^7{(x)}}{7} + \frac{2\cos^5{(x)}}{5} - \frac{\cos^3{(x)}}{3}$$

```
[In] integrate(cos(x)**2*sin(x)**5,x)
[Out] -cos(x)**7/7 + 2*cos(x)**5/5 - cos(x)**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos^2(x)\sin^5(x)\,dx = -\frac{1}{7}\,\cos{(x)}^7 + \frac{2}{5}\,\cos{(x)}^5 - \frac{1}{3}\,\cos{(x)}^3$$

```
[In] integrate(cos(x)^2*sin(x)^5,x, algorithm="maxima")
[Out] -1/7*cos(x)^7 + 2/5*cos(x)^5 - 1/3*cos(x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos^2(x)\sin^5(x)\,dx = -\frac{1}{7}\,\cos{(x)}^7 + \frac{2}{5}\,\cos{(x)}^5 - \frac{1}{3}\,\cos{(x)}^3$$

```
[In] integrate(cos(x)^2*sin(x)^5,x, algorithm="giac")
[Out] -1/7*cos(x)^7 + 2/5*cos(x)^5 - 1/3*cos(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos^2(x)\sin^5(x)\,dx = -\frac{\cos(x)^7}{7} + \frac{2\cos(x)^5}{5} - \frac{\cos(x)^3}{3}$$

```
[In] int(cos(x)^2*sin(x)^5,x)
[Out] (2*cos(x)^5)/5 - cos(x)^3/3 - cos(x)^7/7
```

3.343 $\int e^{-3x}\cos(4x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{-3x}\cos(4x)\,dx=-\frac{3}{25}e^{-3x}\cos(4x)+\frac{4}{25}e^{-3x}\sin(4x)$$

```
[Out] -3/25*cos(4*x)/exp(3*x)+4/25*sin(4*x)/exp(3*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4518}

$$\int e^{-3x}\cos(4x)\,dx=\frac{4}{25}e^{-3x}\sin(4x)-\frac{3}{25}e^{-3x}\cos(4x)$$

```
[In] Int[Cos[4*x]/E^(3*x),x]
[Out] (-3*Cos[4*x])/(25*E^(3*x)) + (4*Sin[4*x])/(25*E^(3*x))
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral}=-\frac{3}{25}e^{-3x}\cos(4x)+\frac{4}{25}e^{-3x}\sin(4x)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{-3x}\cos(4x)\,dx = \frac{1}{25}e^{-3x}(-3\cos(4x)+4\sin(4x))$$

```
[In] Integrate[Cos[4*x]/E^(3*x),x]
[Out] (-3*Cos[4*x] + 4*Sin[4*x])/(25*E^(3*x))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{e^{-3x}(-3\cos(4x)+4\sin(4x))}{25}$	20
default	$-\frac{3\,e^{-3x}\cos(4x)}{25}+\frac{4\,e^{-3x}\sin(4x)}{25}$	22
norman	$\frac{\left(-\frac{3}{25}+\frac{3\left(\tan^2(2x)\right)}{25}+\frac{8\tan(2x)}{25}\right)e^{-3x}}{1+\tan^2(2x)}$	34
risch	$-\frac{3\,e^{(-3+4i)x}}{50}-\frac{2ie^{(-3+4i)x}}{25}-\frac{3\,e^{(-3-4i)x}}{50}+\frac{2ie^{(-3-4i)x}}{25}$	36

```
[In] int(cos(4*x)/exp(3*x),x,method=_RETURNVERBOSE)
[Out] 1/25*exp(-3*x)*(-3*cos(4*x)+4*sin(4*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{-3x}\cos(4x)\,dx = -\frac{3}{25}\cos(4x)e^{(-3x)}+\frac{4}{25}e^{(-3x)}\sin(4x)$$

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="fricas")
[Out] -3/25*cos(4*x)*e^(-3*x) + 4/25*e^(-3*x)*sin(4*x)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{-3x}\cos(4x)\,dx = \frac{4e^{-3x}\sin{(4x)}}{25} - \frac{3e^{-3x}\cos{(4x)}}{25}$$

```
[In] integrate(cos(4*x)/exp(3*x),x)
[Out] 4*exp(-3*x)*sin(4*x)/25 - 3*exp(-3*x)*cos(4*x)/25
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-3x}\cos(4x)\,dx = -\frac{1}{25}\,(3\,\cos{(4\,x)} - 4\,\sin{(4\,x)})e^{(-3\,x)}$$

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="maxima")
[Out] -1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-3x}\cos(4x)\,dx = -\frac{1}{25}\,(3\,\cos{(4\,x)} - 4\,\sin{(4\,x)})e^{(-3\,x)}$$

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="giac")
[Out] -1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-3x}\cos(4x)\,dx = -\frac{\mathrm{e}^{-3\,x}\,(3\,\cos{(4\,x)} - 4\,\sin{(4\,x)})}{25}$$

```
[In] int(cos(4*x)*exp(-3*x),x)
[Out] -(exp(-3*x)*(3*cos(4*x) - 4*sin(4*x)))/25
```

3.344 $\int \csc^3\left(\frac{x}{2}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 24

$$\int \csc^3\left(\frac{x}{2}\right)\,dx = -\text{arctanh}\left(\cos\left(\frac{x}{2}\right)\right) - \cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)$$

```
[Out] -arctanh(cos(1/2*x))-cot(1/2*x)*csc(1/2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3853, 3855}

$$\int \csc^3\left(\frac{x}{2}\right)\,dx = -\text{arctanh}\left(\cos\left(\frac{x}{2}\right)\right) - \cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)$$

```
[In] Int[Csc[x/2]^3,x]
```

```
[Out] -ArcTanh[Cos[x/2]] - Cot[x/2]*Csc[x/2]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right) + \frac{1}{2}\int \csc\left(\frac{x}{2}\right)\,dx$$
$$= -\text{arctanh}\left(\cos\left(\frac{x}{2}\right)\right) - \cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \csc^3\left(\frac{x}{2}\right)\,dx = -\frac{1}{4}\csc^2\left(\frac{x}{4}\right) - \log\left(\cos\left(\frac{x}{4}\right)\right) + \log\left(\sin\left(\frac{x}{4}\right)\right) + \frac{1}{4}\sec^2\left(\frac{x}{4}\right)$$

```
[In] Integrate[Csc[x/2]^3,x]
[Out] -1/4*Csc[x/4]^2 - Log[Cos[x/4]] + Log[Sin[x/4]] + Sec[x/4]^2/4
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
parallelrisch	$\frac{\left(\tan^2\left(\frac{x}{4}\right)\right)}{4}+\ln\left(\tan\left(\frac{x}{4}\right)\right)-\frac{\left(\cot^2\left(\frac{x}{4}\right)\right)}{4}$	23
derivativedivides	$-\cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)+\ln\left(\csc\left(\frac{x}{2}\right)-\cot\left(\frac{x}{2}\right)\right)$	24
default	$-\cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)+\ln\left(\csc\left(\frac{x}{2}\right)-\cot\left(\frac{x}{2}\right)\right)$	24
norman	$\frac{-\frac{1}{4}+\frac{\left(\tan^4\left(\frac{x}{4}\right)\right)}{4}}{\tan\left(\frac{x}{4}\right)^2}+\ln\left(\tan\left(\frac{x}{4}\right)\right)$	24
risch	$\frac{2\,\mathrm{e}^{\frac{3ix}{2}}+2\,\mathrm{e}^{\frac{ix}{2}}}{\left(\mathrm{e}^{ix}-1\right)^2}+\ln\left(\mathrm{e}^{\frac{ix}{2}}-1\right)-\ln\left(\mathrm{e}^{\frac{ix}{2}}+1\right)$	42

```
[In] int(csc(1/2*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/4*tan(1/4*x)^2+ln(tan(1/4*x))-1/4*cot(1/4*x)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(18) = 36$.

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int \csc^3\left(\frac{x}{2}\right)\,dx = -\frac{\left(\cos\left(\frac{1}{2}\,x\right)^2 - 1\right)\log\left(\frac{1}{2}\,\cos\left(\frac{1}{2}\,x\right) + \frac{1}{2}\right) - \left(\cos\left(\frac{1}{2}\,x\right)^2 - 1\right)\log\left(-\frac{1}{2}\,\cos\left(\frac{1}{2}\,x\right) + \frac{1}{2}\right) - 2\,\cos\left(\frac{1}{2}\,x\right)}{2\left(\cos\left(\frac{1}{2}\,x\right)^2 - 1\right)}$$

```
[In] integrate(csc(1/2*x)^3,x, algorithm="fricas")
[Out] -1/2*((cos(1/2*x)^2 - 1)*log(1/2*cos(1/2*x) + 1/2) - (cos(1/2*x)^2 - 1)*log
(-1/2*cos(1/2*x) + 1/2) - 2*cos(1/2*x))/(cos(1/2*x)^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \csc^3\left(\frac{x}{2}\right)\,dx = \frac{\log\left(\cos\left(\frac{x}{2}\right) - 1\right)}{2} - \frac{\log\left(\cos\left(\frac{x}{2}\right) + 1\right)}{2} + \frac{2\cos\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right) - 2}$$

```
[In] integrate(csc(1/2*x)**3,x)
[Out] log(cos(x/2) - 1)/2 - log(cos(x/2) + 1)/2 + 2*cos(x/2)/(2*cos(x/2)**2 - 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \csc^3\left(\frac{x}{2}\right)\,dx = \frac{\cos\left(\frac{1}{2}\,x\right)}{\cos\left(\frac{1}{2}\,x\right)^2 - 1} - \frac{1}{2}\,\log\left(\cos\left(\frac{1}{2}\,x\right) + 1\right) + \frac{1}{2}\,\log\left(\cos\left(\frac{1}{2}\,x\right) - 1\right)$$

```
[In] integrate(csc(1/2*x)^3,x, algorithm="maxima")
[Out] cos(1/2*x)/(cos(1/2*x)^2 - 1) - 1/2*log(cos(1/2*x) + 1) + 1/2*log(cos(1/2*x
) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.92

$$\int \csc^3\left(\frac{x}{2}\right)\,dx = -\frac{\left(\frac{2\left(\cos\left(\frac{1}{2}x\right)-1\right)}{\cos\left(\frac{1}{2}x\right)+1}-1\right)\left(\cos\left(\frac{1}{2}x\right)+1\right)}{4\left(\cos\left(\frac{1}{2}x\right)-1\right)} - \frac{\cos\left(\frac{1}{2}x\right)-1}{4\left(\cos\left(\frac{1}{2}x\right)+1\right)} + \frac{1}{2}\log\left(-\frac{\cos\left(\frac{1}{2}x\right)-1}{\cos\left(\frac{1}{2}x\right)+1}\right)$$

```
[In] integrate(csc(1/2*x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*(cos(1/2*x) - 1)/(cos(1/2*x) + 1) - 1)*(cos(1/2*x) + 1)/(cos(1/2*x)
 - 1) - 1/4*(cos(1/2*x) - 1)/(cos(1/2*x) + 1) + 1/2*log(-(cos(1/2*x) - 1)/(
cos(1/2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \csc^3\left(\frac{x}{2}\right)\,dx = \ln\left(\tan\left(\frac{x}{4}\right)\right) - \frac{\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)^2}$$

```
[In] int(1/sin(x/2)^3,x)
```

```
[Out] log(tan(x/4)) - cos(x/2)/sin(x/2)^2
```

3.345 $\int \frac{\sqrt{-1+9x^2}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{\sqrt{-1+9x^2}}{x^2} dx = -\frac{\sqrt{-1+9x^2}}{x} + 3\text{arctanh}\left(\frac{3x}{\sqrt{-1+9x^2}}\right)$$

```
[Out] 3*arctanh(3*x/(9*x^2-1)^(1/2))-(9*x^2-1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {283, 223, 212}

$$\int \frac{\sqrt{-1+9x^2}}{x^2} dx = 3\text{arctanh}\left(\frac{3x}{\sqrt{9x^2-1}}\right) - \frac{\sqrt{9x^2-1}}{x}$$

```
[In] Int[Sqrt[-1 + 9*x^2]/x^2,x]
```

```
[Out] -(Sqrt[-1 + 9*x^2]/x) + 3*ArcTanh[(3*x)/Sqrt[-1 + 9*x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{-1+9x^2}}{x} + 9\int \frac{1}{\sqrt{-1+9x^2}}\,dx \\
&= -\frac{\sqrt{-1+9x^2}}{x} + 9\text{Subst}\left(\int \frac{1}{1-9x^2}\,dx, x, \frac{x}{\sqrt{-1+9x^2}}\right) \\
&= -\frac{\sqrt{-1+9x^2}}{x} + 3\text{arctanh}\left(\frac{3x}{\sqrt{-1+9x^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{-1+9x^2}}{x^2}\,dx = -\frac{\sqrt{-1+9x^2}}{x} - 3\log\left(-3x+\sqrt{-1+9x^2}\right)$$

```
[In] Integrate[Sqrt[-1 + 9*x^2]/x^2,x]
[Out] -(Sqrt[-1 + 9*x^2]/x) - 3*Log[-3*x + Sqrt[-1 + 9*x^2]]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

method	result	size
trager	$-\frac{\sqrt{9x^2-1}}{x}-3\ln\left(\sqrt{9x^2-1}-3x\right)$	32
risch	$-\frac{\sqrt{9x^2-1}}{x}+\ln\left(\sqrt{9}\,x+\sqrt{9x^2-1}\right)\sqrt{9}$	36
default	$\frac{(9x^2-1)^{\frac{3}{2}}}{x}-9x\sqrt{9x^2-1}+\ln\left(\sqrt{9}\,x+\sqrt{9x^2-1}\right)\sqrt{9}$	47
meijerg	$-\frac{3i\sqrt{\operatorname{signum}(9x^2-1)}\left(-\frac{4i\sqrt{\pi}\,\sqrt{-9x^2+1}}{3x}-4i\sqrt{\pi}\,\arcsin(3x)\right)}{4\sqrt{\pi}\,\sqrt{-\operatorname{signum}(9x^2-1)}}$	58
pseudoelliptic	$\frac{3\ln\left(\frac{\sqrt{9x^2-1}+3x}{x}\right)x-3\ln\left(\frac{\sqrt{9x^2-1}-3x}{x}\right)x-2\sqrt{9x^2-1}}{2x}$	60

```
[In] int((9*x^2-1)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -(9*x^2-1)^(1/2)/x-3*ln((9*x^2-1)^(1/2)-3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{-1+9x^2}}{x^2}\,dx = -\frac{3\,x\log\left(-3\,x+\sqrt{9\,x^2-1}\right)+3\,x+\sqrt{9\,x^2-1}}{x}$$

```
[In] integrate((9*x^2-1)^(1/2)/x^2,x, algorithm="fricas")
[Out] -(3*x*log(-3*x + sqrt(9*x^2 - 1)) + 3*x + sqrt(9*x^2 - 1))/x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{-1+9x^2}}{x^2}\,dx = 3\log\left(3x+\sqrt{9x^2-1}\right)-\frac{\sqrt{9x^2-1}}{x}$$

```
[In] integrate((9*x**2-1)**(1/2)/x**2,x)
[Out] 3*log(3*x + sqrt(9*x**2 - 1)) - sqrt(9*x**2 - 1)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{-1+9x^2}}{x^2}\,dx = -\frac{\sqrt{9\,x^2-1}}{x}+3\,\log\left(18\,x+6\,\sqrt{9\,x^2-1}\right)$$

```
[In] integrate((9*x^2-1)^(1/2)/x^2,x, algorithm="maxima")
[Out] -sqrt(9*x^2 - 1)/x + 3*log(18*x + 6*sqrt(9*x^2 - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{-1+9x^2}}{x^2}\,dx = -\frac{6}{\left(3\,x-\sqrt{9\,x^2-1}\right)^2+1}-\frac{3}{2}\,\log\left(\left(3\,x-\sqrt{9\,x^2-1}\right)^2\right)$$

```
[In] integrate((9*x^2-1)^(1/2)/x^2,x, algorithm="giac")
[Out] -6/((3*x - sqrt(9*x^2 - 1))^2 + 1) - 3/2*log((3*x - sqrt(9*x^2 - 1))^2)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{-1+9x^2}}{x^2}\,dx = -\frac{\left(\frac{3\,x\,\mathrm{asin}(3\,x)}{\sqrt{1-9\,x^2}}+1\right)\sqrt{9\,x^2-1}}{x}$$

```
[In] int((9*x^2 - 1)^(1/2)/x^2,x)
[Out] -(((3*x*asin(3*x))/(1 - 9*x^2)^(1/2) + 1)*(9*x^2 - 1)^(1/2))/x
```

3.346 $\int \frac{\sqrt{4-3x^2}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{\sqrt{4-3x^2}}{x}\,dx = \sqrt{4-3x^2} - 2\text{arctanh}\left(\frac{1}{2}\sqrt{4-3x^2}\right)$$

```
[Out] -2*arctanh(1/2*(-3*x^2+4)^(1/2))+(-3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {272, 52, 65, 212}

$$\int \frac{\sqrt{4-3x^2}}{x}\,dx = \sqrt{4-3x^2} - 2\text{arctanh}\left(\frac{1}{2}\sqrt{4-3x^2}\right)$$

```
[In] Int[Sqrt[4 - 3*x^2]/x,x]
```

```
[Out] Sqrt[4 - 3*x^2] - 2*ArcTanh[Sqrt[4 - 3*x^2]/2]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{4-3x}}{x}\,dx, x, x^2\right) \\
&= \sqrt{4-3x^2} + 2\text{Subst}\left(\int \frac{1}{\sqrt{4-3x}x}\,dx, x, x^2\right) \\
&= \sqrt{4-3x^2} - \frac{4}{3}\text{Subst}\left(\int \frac{1}{\frac{4}{3}-\frac{x^2}{3}}\,dx, x, \sqrt{4-3x^2}\right) \\
&= \sqrt{4-3x^2} - 2\text{arctanh}\left(\frac{1}{2}\sqrt{4-3x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{4-3x^2}}{x}\,dx = \sqrt{4-3x^2} - 2\text{arctanh}\left(\frac{1}{2}\sqrt{4-3x^2}\right)$$

```
[In] Integrate[Sqrt[4 - 3*x^2]/x,x]
[Out] Sqrt[4 - 3*x^2] - 2*ArcTanh[Sqrt[4 - 3*x^2]/2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\sqrt{-3x^2+4}-2\,\operatorname{arctanh}\left(\frac{2}{\sqrt{-3x^2+4}}\right)$	25
trager	$\sqrt{-3x^2+4}-2\ln\left(\frac{\sqrt{-3x^2+4}+2}{x}\right)$	29
pseudoelliptic	$\sqrt{-3x^2+4}+\ln\left(\sqrt{-3x^2+4}-2\right)-\ln\left(\sqrt{-3x^2+4}+2\right)$	37
meijerg	$-\frac{-2(2-4\ln(2)+2\ln(x)+\ln(3)+i\pi)\sqrt{\pi}+4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{1-\frac{3x^2}{4}}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{3x^2}{4}}}{2}\right)}{2\sqrt{\pi}}$	66

```
[In] int((-3*x^2+4)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] (-3*x^2+4)^(1/2)-2*arctanh(2/(-3*x^2+4)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{4-3x^2}}{x}\,dx=\sqrt{-3\,x^2+4}+2\,\log\left(\frac{\sqrt{-3\,x^2+4}-2}{x}\right)$$

```
[In] integrate((-3*x^2+4)^(1/2)/x,x, algorithm="fricas")
[Out] sqrt(-3*x^2 + 4) + 2*log((sqrt(-3*x^2 + 4) - 2)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.43

$$\int \frac{\sqrt{4-3x^2}}{x}\,dx=\begin{cases} i\sqrt{3x^2-4}-2\log(x)+\log(x^2)+2i\operatorname{asin}\left(\frac{2\sqrt{3}}{3x}\right) & \text{for } |x^2|>\frac{4}{3}\\ \sqrt{4-3x^2}+\log(x^2)-2\log\left(\sqrt{1-\frac{3x^2}{4}}+1\right) & \text{otherwise}\end{cases}$$

```
[In] integrate((-3*x**2+4)**(1/2)/x,x)
[Out] Piecewise((I*sqrt(3*x**2 - 4) - 2*log(x) + log(x**2) + 2*I*asin(2*sqrt(3)/(
3*x)), Abs(x**2) > 4/3), (sqrt(4 - 3*x**2) + log(x**2) - 2*log(sqrt(1 - 3*x
**2/4) + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{4-3x^2}}{x}\,dx = \sqrt{-3\,x^2+4} - 2\,\log\left(\frac{4\,\sqrt{-3\,x^2+4}}{|x|} + \frac{8}{|x|}\right)$$

```
[In] integrate((-3*x^2+4)^(1/2)/x,x, algorithm="maxima")
[Out] sqrt(-3*x^2 + 4) - 2*log(4*sqrt(-3*x^2 + 4)/abs(x) + 8/abs(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{4-3x^2}}{x}\,dx = \sqrt{-3\,x^2+4} - \log\left(\sqrt{-3\,x^2+4}+2\right) + \log\left(-\sqrt{-3\,x^2+4}+2\right)$$

```
[In] integrate((-3*x^2+4)^(1/2)/x,x, algorithm="giac")
[Out] sqrt(-3*x^2 + 4) - log(sqrt(-3*x^2 + 4) + 2) + log(-sqrt(-3*x^2 + 4) + 2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{4-3x^2}}{x}\,dx = 2\,\ln\left(\sqrt{\frac{4}{3\,x^2}-1} - \frac{2\,\sqrt{3}\,\sqrt{\frac{1}{x^2}}}{3}\right) + \sqrt{3}\,\sqrt{\frac{4}{3}-x^2}$$

```
[In] int((4 - 3*x^2)^(1/2)/x,x)
[Out] 2*log((4/(3*x^2) - 1)^(1/2) - (2*3^(1/2)*(1/x^2)^(1/2))/3) + 3^(1/2)*(4/3 -
 x^2)^(1/2)
```

3.347 $\int e^{3x} x^2 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 32

$$\int e^{3x} x^2 \, dx = \frac{2e^{3x}}{27} - \frac{2}{9}e^{3x}x + \frac{1}{3}e^{3x}x^2$$

```
[Out] 2/27*exp(3*x)-2/9*exp(3*x)*x+1/3*exp(3*x)*x^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2207, 2225}

$$\int e^{3x} x^2 \, dx = \frac{1}{3}e^{3x}x^2 - \frac{2}{9}e^{3x}x + \frac{2e^{3x}}{27}$$

```
[In] Int[E^(3*x)*x^2,x]
[Out] (2*E^(3*x))/27 - (2*E^(3*x)*x)/9 + (E^(3*x)*x^2)/3
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}e^{3x}x^2 - \frac{2}{3}\int e^{3x}x\,dx \\
&= -\frac{2}{9}e^{3x}x + \frac{1}{3}e^{3x}x^2 + \frac{2}{9}\int e^{3x}\,dx \\
&= \frac{2e^{3x}}{27} - \frac{2}{9}e^{3x}x + \frac{1}{3}e^{3x}x^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int e^{3x}x^2\,dx = \frac{1}{27}e^{3x}\left(2-6x+9x^2\right)$$

```
[In] Integrate[E^(3*x)*x^2,x]
[Out] (E^(3*x)*(2 - 6*x + 9*x^2))/27
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

method	result	size
risch	$\left(\frac{1}{3}x^2-\frac{2}{9}x+\frac{2}{27}\right)\mathrm{e}^{3x}$	16
gosper	$\frac{(9x^2-6x+2)\mathrm{e}^{3x}}{27}$	17
meijerg	$-\frac{2}{27}+\frac{(27x^2-18x+6)\mathrm{e}^{3x}}{81}$	19
derivativedivides	$\frac{2\,\mathrm{e}^{3x}}{27}-\frac{2\,\mathrm{e}^{3x}x}{9}+\frac{\mathrm{e}^{3x}x^2}{3}$	24
default	$\frac{2\,\mathrm{e}^{3x}}{27}-\frac{2\,\mathrm{e}^{3x}x}{9}+\frac{\mathrm{e}^{3x}x^2}{3}$	24
norman	$\frac{2\,\mathrm{e}^{3x}}{27}-\frac{2\,\mathrm{e}^{3x}x}{9}+\frac{\mathrm{e}^{3x}x^2}{3}$	24
parallelrisch	$\frac{2\,\mathrm{e}^{3x}}{27}-\frac{2\,\mathrm{e}^{3x}x}{9}+\frac{\mathrm{e}^{3x}x^2}{3}$	24
parts	$\frac{2\,\mathrm{e}^{3x}}{27}-\frac{2\,\mathrm{e}^{3x}x}{9}+\frac{\mathrm{e}^{3x}x^2}{3}$	24

```
[In] int(exp(3*x)*x^2,x,method=_RETURNVERBOSE)
[Out] (1/3*x^2-2/9*x+2/27)*exp(3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{3x}x^2\,dx = \frac{1}{27}\,\left(9\,x^2 - 6\,x + 2\right)e^{\left(3\,x\right)}$$

```
[In] integrate(exp(3*x)*x^2,x, algorithm="fricas")
[Out] 1/27*(9*x^2 - 6*x + 2)*e^(3*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.47

$$\int e^{3x}x^2\,dx = \frac{(9x^2 - 6x + 2)\,e^{3x}}{27}$$

```
[In] integrate(exp(3*x)*x**2,x)
[Out] (9*x**2 - 6*x + 2)*exp(3*x)/27
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{3x}x^2\,dx = \frac{1}{27}\,\left(9\,x^2 - 6\,x + 2\right)e^{\left(3\,x\right)}$$

```
[In] integrate(exp(3*x)*x^2,x, algorithm="maxima")
[Out] 1/27*(9*x^2 - 6*x + 2)*e^(3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{3x}x^2\,dx = \frac{1}{27}\,\left(9\,x^2 - 6\,x + 2\right)e^{\left(3\,x\right)}$$

```
[In] integrate(exp(3*x)*x^2,x, algorithm="giac")
[Out] 1/27*(9*x^2 - 6*x + 2)*e^(3*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{3x}x^2\,dx = \frac{e^{3\,x}\,\left(9\,x^2-6\,x+2\right)}{27}$$

```
[In] int(x^2*exp(3*x),x)
[Out] (exp(3*x)*(9*x^2 - 6*x + 2))/27
```

3.348 $\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = -2\sqrt{1+\sin(x)}+\frac{2}{3}(1+\sin(x))^{3/2}$$

```
[Out] 2/3*(1+sin(x))^(3/2)-2*(1+sin(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2912, 45}

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = \frac{2}{3}(\sin(x)+1)^{3/2}-2\sqrt{\sin(x)+1}$$

```
[In] Int[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]
```

```
[Out] -2*Sqrt[1 + Sin[x]] + (2*(1 + Sin[x])^(3/2))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
```

```
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}} + \sqrt{1+x}\right)\,dx, x, \sin(x)\right)\\
&= -2\sqrt{1+\sin(x)} + \frac{2}{3}(1+\sin(x))^{3/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = \frac{2\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2(-2+\sin(x))}{3\sqrt{1+\sin(x)}}$$

```
[In] Integrate[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]
[Out] (2*(Cos[x/2] + Sin[x/2])^2*(-2 + Sin[x]))/(3*Sqrt[1 + Sin[x]])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{2(\sin(x)+1)^{\frac{3}{2}}}{3}-2\sqrt{\sin\left(x\right)+1}$	18
default	$\frac{2(\sin(x)+1)^{\frac{3}{2}}}{3}-2\sqrt{\sin\left(x\right)+1}$	18

```
[In] int(cos(x)*sin(x)/(sin(x)+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(sin(x)+1)^(3/2)-2*(sin(x)+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = \frac{2}{3}\sqrt{\sin(x)+1}(\sin(x)-2)$$

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(sin(x) + 1)*(sin(x) - 2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = \frac{2\sqrt{\sin(x)+1}\sin(x)}{3} - \frac{4\sqrt{\sin(x)+1}}{3}$$

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))**(1/2),x)
[Out] 2*sqrt(sin(x) + 1)*sin(x)/3 - 4*sqrt(sin(x) + 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = \frac{2}{3}(\sin(x)+1)^{\frac{3}{2}} - 2\sqrt{\sin(x)+1}$$

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="maxima")
[Out] 2/3*(sin(x) + 1)^(3/2) - 2*sqrt(sin(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(17) = 34.

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = \frac{2\left(2\sqrt{2}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)^3 - 3\sqrt{2}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)}{3\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)}$$

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="giac")
[Out] 2/3*(2*sqrt(2)*cos(-1/4*pi + 1/2*x)^3 - 3*sqrt(2)*cos(-1/4*pi + 1/2*x))/sgn
(cos(-1/4*pi + 1/2*x))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = \frac{2\sqrt{\sin(x)+1}\,(\sin(x)-2)}{3}$$

```
[In] int((cos(x)*sin(x))/(sin(x) + 1)^(1/2),x)
[Out] (2*(sin(x) + 1)^(1/2)*(sin(x) - 2))/3
```

3.349 $\int x \arcsin(x^2)\, dx$

Optimal result

Integrand size = 6, antiderivative size = 27

$$\int x \arcsin(x^2)\, dx = \frac{\sqrt{1-x^4}}{2} + \frac{1}{2}x^2 \arcsin(x^2)$$

```
[Out] 1/2*x^2*arcsin(x^2)+1/2*(-x^4+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6847, 4715, 267}

$$\int x \arcsin(x^2)\, dx = \frac{1}{2}x^2 \arcsin(x^2) + \frac{\sqrt{1-x^4}}{2}$$

```
[In] Int[x*ArcSin[x^2],x]
[Out] Sqrt[1 - x^4]/2 + (x^2*ArcSin[x^2])/2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \arcsin(x)\,dx, x, x^2\right)\\ &= \frac{1}{2}x^2\arcsin\left(x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}}\,dx, x, x^2\right)\\ &= \frac{\sqrt{1-x^4}}{2} + \frac{1}{2}x^2\arcsin\left(x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int x\arcsin\left(x^2\right)\,dx = \frac{1}{2}\left(\sqrt{1-x^4} + x^2\arcsin\left(x^2\right)\right)$$

```
[In] Integrate[x*ArcSin[x^2],x]
[Out] (Sqrt[1 - x^4] + x^2*ArcSin[x^2])/2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{x^2 \arcsin(x^2)}{2}+\frac{\sqrt{-x^4+1}}{2}$	22
default	$\frac{x^2 \arcsin(x^2)}{2}+\frac{\sqrt{-x^4+1}}{2}$	22
parts	$\frac{x^2 \arcsin(x^2)}{2}+\frac{\sqrt{-x^4+1}}{2}$	22

```
[In] int(x*arcsin(x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*arcsin(x^2)+1/2*(-x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int x \arcsin\left(x^2\right) \, dx = \frac{1}{2} \, x^2 \arcsin\left(x^2\right) + \frac{1}{2} \, \sqrt{-x^4 + 1}$$

```
[In] integrate(x*arcsin(x^2),x, algorithm="fricas")
[Out] 1/2*x^2*arcsin(x^2) + 1/2*sqrt(-x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x \arcsin\left(x^2\right) \, dx = \frac{x^2 \operatorname{asin}\left(x^2\right)}{2} + \frac{\sqrt{1 - x^4}}{2}$$

```
[In] integrate(x*asin(x**2),x)
[Out] x**2*asin(x**2)/2 + sqrt(1 - x**4)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int x \arcsin\left(x^2\right) \, dx = \frac{1}{2} \, x^2 \arcsin\left(x^2\right) + \frac{1}{2} \, \sqrt{-x^4 + 1}$$

```
[In] integrate(x*arcsin(x^2),x, algorithm="maxima")
[Out] 1/2*x^2*arcsin(x^2) + 1/2*sqrt(-x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int x \arcsin\left(x^2\right) \, dx = \frac{1}{2} \, x^2 \arcsin\left(x^2\right) + \frac{1}{2} \, \sqrt{-x^4 + 1}$$

```
[In] integrate(x*arcsin(x^2),x, algorithm="giac")
[Out] 1/2*x^2*arcsin(x^2) + 1/2*sqrt(-x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int x \arcsin \left(x^2\right) \, dx = \frac{x^2 \operatorname{asin}(x^2)}{2} + \frac{\sqrt{1 - x^4}}{2}$$

```
[In] int(x*asin(x^2),x)
[Out] (x^2*asin(x^2))/2 + (1 - x^4)^(1/2)/2
```

3.350 $\int x^3 \arcsin(x^2) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 38

$$\int x^3 \arcsin(x^2) \, dx = \frac{1}{8}x^2\sqrt{1-x^4} - \frac{\arcsin(x^2)}{8} + \frac{1}{4}x^4 \arcsin(x^2)$$

```
[Out] -1/8*arcsin(x^2)+1/4*x^4*arcsin(x^2)+1/8*x^2*(-x^4+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4926, 12, 281, 327, 222}

$$\int x^3 \arcsin(x^2) \, dx = -\frac{\arcsin(x^2)}{8} + \frac{1}{4}x^4 \arcsin(x^2) + \frac{1}{8}\sqrt{1-x^4}x^2$$

```
[In] Int[x^3*ArcSin[x^2],x]
```

```
[Out] (x^2*Sqrt[1 - x^4])/8 - ArcSin[x^2]/8 + (x^4*ArcSin[x^2])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4926

```
Int[((a_.) + ArcSin[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcSin[u])/(d*(m + 1))), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\arcsin\left(x^2\right)-\frac{1}{4}\int\frac{2x^5}{\sqrt{1-x^4}}\,dx\\
&= \frac{1}{4}x^4\arcsin\left(x^2\right)-\frac{1}{2}\int\frac{x^5}{\sqrt{1-x^4}}\,dx\\
&= \frac{1}{4}x^4\arcsin\left(x^2\right)-\frac{1}{4}\text{Subst}\left(\int\frac{x^2}{\sqrt{1-x^2}}\,dx,x,x^2\right)\\
&= \frac{1}{8}x^2\sqrt{1-x^4}+\frac{1}{4}x^4\arcsin\left(x^2\right)-\frac{1}{8}\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}}\,dx,x,x^2\right)\\
&= \frac{1}{8}x^2\sqrt{1-x^4}-\frac{\arcsin\left(x^2\right)}{8}+\frac{1}{4}x^4\arcsin\left(x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int x^3 \arcsin\left(x^2\right)\,dx = \frac{1}{8}\left(x^2\sqrt{1-x^4} + \left(-1+2x^4\right)\arcsin\left(x^2\right)\right)$$

```
[In] Integrate[x^3*ArcSin[x^2],x]
[Out] (x^2*Sqrt[1 - x^4] + (-1 + 2*x^4)*ArcSin[x^2])/8
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{\arcsin\left(x^2\right)}{8}+\frac{x^4\arcsin\left(x^2\right)}{4}+\frac{x^2\sqrt{-x^4+1}}{8}$	31
default	$-\frac{\arcsin\left(x^2\right)}{8}+\frac{x^4\arcsin\left(x^2\right)}{4}+\frac{x^2\sqrt{-x^4+1}}{8}$	31
parts	$-\frac{\arcsin\left(x^2\right)}{8}+\frac{x^4\arcsin\left(x^2\right)}{4}+\frac{x^2\sqrt{-x^4+1}}{8}$	31

```
[In] int(x^3*arcsin(x^2),x,method=_RETURNVERBOSE)
[Out] -1/8*arcsin(x^2)+1/4*x^4*arcsin(x^2)+1/8*x^2*(-x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int x^3 \arcsin\left(x^2\right)\,dx = \frac{1}{8}\sqrt{-x^4+1}x^2 + \frac{1}{8}\left(2\,x^4-1\right)\arcsin\left(x^2\right)$$

```
[In] integrate(x^3*arcsin(x^2),x, algorithm="fricas")
[Out] 1/8*sqrt(-x^4 + 1)*x^2 + 1/8*(2*x^4 - 1)*arcsin(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^3 \arcsin(x^2)\ dx = \frac{x^4 \operatorname{asin}(x^2)}{4} + \frac{x^2\sqrt{1-x^4}}{8} - \frac{\operatorname{asin}(x^2)}{8}$$

```
[In] integrate(x**3*asin(x**2),x)
[Out] x**4*asin(x**2)/4 + x**2*sqrt(1 - x**4)/8 - asin(x**2)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int x^3 \arcsin(x^2)\ dx = \frac{1}{4}x^4 \arcsin(x^2) - \frac{\sqrt{-x^4+1}}{8x^2\left(\frac{x^4-1}{x^4}-1\right)} + \frac{1}{8}\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

```
[In] integrate(x^3*arcsin(x^2),x, algorithm="maxima")
[Out] 1/4*x^4*arcsin(x^2) - 1/8*sqrt(-x^4 + 1)/(x^2*((x^4 - 1)/x^4 - 1)) + 1/8*ar
ctan(sqrt(-x^4 + 1)/x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int x^3 \arcsin(x^2)\ dx = \frac{1}{8}\sqrt{-x^4+1}x^2 + \frac{1}{4}(x^4-1)\arcsin(x^2) + \frac{1}{8}\arcsin(x^2)$$

```
[In] integrate(x^3*arcsin(x^2),x, algorithm="giac")
[Out] 1/8*sqrt(-x^4 + 1)*x^2 + 1/4*(x^4 - 1)*arcsin(x^2) + 1/8*arcsin(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int x^3 \arcsin\left(x^2\right)\,dx = \frac{x^2\,\sqrt{1-x^4}}{8}+\frac{\mathrm{asin}(x^2)\,\left(2\,x^4-1\right)}{8}$$

```
[In] int(x^3*asin(x^2),x)
[Out] (x^2*(1 - x^4)^(1/2))/8 + (asin(x^2)*(2*x^4 - 1))/8
```

3.351 $\int e^x \mathbf{sech}(e^x)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 5

$$\int e^x \text{sech}(e^x)\, dx = \arctan(\sinh(e^x))$$

```
[Out] arctan(sinh(exp(x)))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 3855}

$$\int e^x \text{sech}(e^x)\, dx = \arctan(\sinh(e^x))$$

```
[In] Int[E^x*Sech[E^x],x]
```

```
[Out] ArcTan[Sinh[E^x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \text{sech}(x)\,dx, x, e^x\right)$$
$$= \arctan(\sinh(e^x))$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int e^x \text{sech}(e^x)\ dx = \arctan(\sinh(e^x))$$

```
[In] Integrate[E^x*Sech[E^x],x]
[Out] ArcTan[Sinh[E^x]]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\arctan(\sinh(e^x))$	5
default	$\arctan(\sinh(e^x))$	5
risch	$i\ln\left(e^{e^x}+i\right)-i\ln\left(e^{e^x}-i\right)$	22
parallelrisch	$-i\left(\ln\left(\tanh\left(\frac{e^x}{2}\right)-i\right)-\ln\left(\tanh\left(\frac{e^x}{2}\right)+i\right)\right)$	25

```
[In] int(exp(x)*sech(exp(x)),x,method=_RETURNVERBOSE)
[Out] arctan(sinh(exp(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(4) = 8.

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 3.20

$$\int e^x \text{sech}(e^x)\ dx = 2\arctan(\cosh(\cosh(x)+\sinh(x))+\sinh(\cosh(x)+\sinh(x)))$$

```
[In] integrate(exp(x)*sech(exp(x)),x, algorithm="fricas")
[Out] 2*arctan(cosh(cosh(x) + sinh(x)) + sinh(cosh(x) + sinh(x)))
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

$$\int e^x \operatorname{sech}(e^x) \, dx = 2 \operatorname{atan}\left(\tanh\left(\frac{e^x}{2}\right)\right)$$

```
[In] integrate(exp(x)*sech(exp(x)),x)
[Out] 2*atan(tanh(exp(x)/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.80

$$\int e^x \operatorname{sech}(e^x) \, dx = \arctan\left(\sinh\left(e^x\right)\right)$$

```
[In] integrate(exp(x)*sech(exp(x)),x, algorithm="maxima")
[Out] arctan(sinh(e^x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int e^x \operatorname{sech}(e^x) \, dx = 2 \arctan\left(e^{\left(e^x\right)}\right)$$

```
[In] integrate(exp(x)*sech(exp(x)),x, algorithm="giac")
[Out] 2*arctan(e^(e^x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int e^x \operatorname{sech}(e^x) \, dx = 2 \operatorname{atan}\left(\mathrm{e}^{\mathrm{e}^x}\right)$$

```
[In] int(exp(x)/cosh(exp(x)),x)
[Out] 2*atan(exp(exp(x)))
```

3.352 $\int x^2 \cos(3x)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 29

$$\int x^2 \cos(3x)\, dx = \frac{2}{9} x \cos(3x) - \frac{2}{27} \sin(3x) + \frac{1}{3} x^2 \sin(3x)$$

```
[Out] 2/9*x*cos(3*x)-2/27*sin(3*x)+1/3*x^2*sin(3*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3377, 2717}

$$\int x^2 \cos(3x)\, dx = \frac{1}{3} x^2 \sin(3x) - \frac{2}{27} \sin(3x) + \frac{2}{9} x \cos(3x)$$

```
[In] Int[x^2*Cos[3*x],x]
```

```
[Out] (2*x*Cos[3*x])/9 - (2*Sin[3*x])/27 + (x^2*Sin[3*x])/3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}x^2\sin(3x) - \frac{2}{3}\int x\sin(3x)\,dx\\ &= \frac{2}{9}x\cos(3x) + \frac{1}{3}x^2\sin(3x) - \frac{2}{9}\int \cos(3x)\,dx\\ &= \frac{2}{9}x\cos(3x) - \frac{2}{27}\sin(3x) + \frac{1}{3}x^2\sin(3x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int x^2\cos(3x)\,dx = \frac{2}{9}x\cos(3x) + \frac{1}{27}\left(-2+9x^2\right)\sin(3x)$$

```
[In] Integrate[x^2*Cos[3*x],x]
[Out] (2*x*Cos[3*x])/9 + ((-2 + 9*x^2)*Sin[3*x])/27
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{2x\cos(3x)}{9}+\frac{(9x^2-2)\sin(3x)}{27}$	22
derivativedivides	$\frac{2x\cos(3x)}{9}-\frac{2\sin(3x)}{27}+\frac{x^2\sin(3x)}{3}$	24
default	$\frac{2x\cos(3x)}{9}-\frac{2\sin(3x)}{27}+\frac{x^2\sin(3x)}{3}$	24
parts	$\frac{2x\cos(3x)}{9}-\frac{2\sin(3x)}{27}+\frac{x^2\sin(3x)}{3}$	24
meijerg	$\frac{4\sqrt{\pi}\left(\frac{3x\cos(3x)}{2\sqrt{\pi}}-\frac{\left(-\frac{27x^2}{2}+3\right)\sin(3x)}{6\sqrt{\pi}}\right)}{27}$	33
norman	$\frac{\frac{2x}{9}-\frac{2x\left(\tan^2\left(\frac{3x}{2}\right)\right)}{9}+\frac{2x^2\tan\left(\frac{3x}{2}\right)}{3}-\frac{4\tan\left(\frac{3x}{2}\right)}{27}}{1+\tan^2\left(\frac{3x}{2}\right)}$	40
parallelrisch	$\frac{18x^2\tan\left(\frac{3x}{2}\right)-6x\left(\tan^2\left(\frac{3x}{2}\right)\right)+6x-4\tan\left(\frac{3x}{2}\right)}{27\left(\tan^2\left(\frac{3x}{2}\right)\right)+27}$	42

```
[In] int(x^2*cos(3*x),x,method=_RETURNVERBOSE)
[Out] 2/9*x*cos(3*x)+1/27*(9*x^2-2)*sin(3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int x^2 \cos(3x)\, dx = \frac{2}{9}\, x \cos\left(3\, x\right) + \frac{1}{27}\, \left(9\, x^2 - 2\right) \sin\left(3\, x\right)$$

```
[In] integrate(x^2*cos(3*x),x, algorithm="fricas")
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int x^2 \cos(3x)\, dx = \frac{x^2 \sin\left(3x\right)}{3} + \frac{2x \cos\left(3x\right)}{9} - \frac{2 \sin\left(3x\right)}{27}$$

```
[In] integrate(x**2*cos(3*x),x)
[Out] x**2*sin(3*x)/3 + 2*x*cos(3*x)/9 - 2*sin(3*x)/27
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int x^2 \cos(3x)\, dx = \frac{2}{9}\, x \cos\left(3\, x\right) + \frac{1}{27}\, \left(9\, x^2 - 2\right) \sin\left(3\, x\right)$$

```
[In] integrate(x^2*cos(3*x),x, algorithm="maxima")
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int x^2 \cos(3x)\, dx = \frac{2}{9}\, x \cos\left(3\, x\right) + \frac{1}{27}\, \left(9\, x^2 - 2\right) \sin\left(3\, x\right)$$

```
[In] integrate(x^2*cos(3*x),x, algorithm="giac")
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int x^2 \cos(3x)\,dx = \frac{2\,x\,\cos\left(3\,x\right)}{9} - \frac{2\,\sin\left(3\,x\right)}{27} + \frac{x^2\,\sin\left(3\,x\right)}{3}$$

```
[In] int(x^2*cos(3*x),x)
[Out] (2*x*cos(3*x))/9 - (2*sin(3*x))/27 + (x^2*sin(3*x))/3
```

3.353 $\int \sqrt{5-4x-x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 36

$$\int \sqrt{5-4x-x^2}\,dx=\frac{1}{2}(2+x)\sqrt{5-4x-x^2}-\frac{9}{2}\arcsin\left(\frac{1}{3}(-2-x)\right)$$

```
[Out] 9/2*arcsin(2/3+1/3*x)+1/2*(2+x)*(-x^2-4*x+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {626, 633, 222}

$$\int \sqrt{5-4x-x^2}\,dx=\frac{1}{2}(x+2)\sqrt{-x^2-4x+5}-\frac{9}{2}\arcsin\left(\frac{1}{3}(-x-2)\right)$$

```
[In] Int[Sqrt[5 - 4*x - x^2],x]
[Out] ((2 + x)*Sqrt[5 - 4*x - x^2])/2 - (9*ArcSin[(-2 - x)/3])/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}(2+x)\sqrt{5-4x-x^2}+\frac{9}{2}\int\frac{1}{\sqrt{5-4x-x^2}}\,dx\\ &= \frac{1}{2}(2+x)\sqrt{5-4x-x^2}-\frac{3}{4}\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx,x,-4-2x\right)\\ &= \frac{1}{2}(2+x)\sqrt{5-4x-x^2}-\frac{9}{2}\arcsin\left(\frac{1}{3}(-2-x)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int\sqrt{5-4x-x^2}\,dx=\frac{1}{2}(2+x)\sqrt{5-4x-x^2}-9\arctan\left(\frac{\sqrt{5-4x-x^2}}{5+x}\right)$$

```
[In] Integrate[Sqrt[5 - 4*x - x^2],x]
[Out] ((2 + x)*Sqrt[5 - 4*x - x^2])/2 - 9*ArcTan[Sqrt[5 - 4*x - x^2]/(5 + x)]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{(-2x-4)\sqrt{-x^2-4x+5}}{4}+\frac{9\arcsin\left(\frac{2}{3}+\frac{x}{3}\right)}{2}$	29
risch	$-\frac{(2+x)\left(x^2+4x-5\right)}{2\sqrt{-x^2-4x+5}}+\frac{9\arcsin\left(\frac{2}{3}+\frac{x}{3}\right)}{2}$	35
trager	$\left(1+\frac{x}{2}\right)\sqrt{-x^2-4x+5}+\frac{9\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2-4x+5}-2\,\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{2}$	59

```
[In] int((-x^2-4*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-2*x-4)*(-x^2-4*x+5)^(1/2)+9/2*arcsin(2/3+1/3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

$$\int \sqrt{5-4x-x^2}\,dx = \frac{1}{2}\sqrt{-x^2-4\,x+5}(x+2) - \frac{9}{2}\arctan\left(\frac{\sqrt{-x^2-4\,x+5}(x+2)}{x^2+4\,x-5}\right)$$

```
[In] integrate((-x^2-4*x+5)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 - 4*x + 5)*(x + 2) - 9/2*arctan(sqrt(-x^2 - 4*x + 5)*(x + 2)/
(x^2 + 4*x - 5))
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \sqrt{5-4x-x^2}\,dx = \left(\frac{x}{2}+1\right)\sqrt{-x^2-4x+5} + \frac{9\,\mathrm{asin}\left(\frac{x}{3}+\frac{2}{3}\right)}{2}$$

```
[In] integrate((-x**2-4*x+5)**(1/2),x)
[Out] (x/2 + 1)*sqrt(-x**2 - 4*x + 5) + 9*asin(x/3 + 2/3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \sqrt{5-4x-x^2}\,dx = \frac{1}{2}\sqrt{-x^2-4\,x+5}x + \sqrt{-x^2-4\,x+5} - \frac{9}{2}\arcsin\left(-\frac{1}{3}\,x-\frac{2}{3}\right)$$

```
[In] integrate((-x^2-4*x+5)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 - 4*x + 5)*x + sqrt(-x^2 - 4*x + 5) - 9/2*arcsin(-1/3*x - 2/3
)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \sqrt{5-4x-x^2}\,dx = \frac{1}{2}\,\sqrt{-x^2-4\,x+5}(x+2)+\frac{9}{2}\,\arcsin\left(\frac{1}{3}\,x+\frac{2}{3}\right)$$

```
[In] integrate((-x^2-4*x+5)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 - 4*x + 5)*(x + 2) + 9/2*arcsin(1/3*x + 2/3)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \sqrt{5-4x-x^2}\,dx = \frac{9\,\mathrm{asin}\left(\frac{x}{3}+\frac{2}{3}\right)}{2}+\left(\frac{x}{2}+1\right)\,\sqrt{-x^2-4\,x+5}$$

```
[In] int((5 - x^2 - 4*x)^(1/2),x)
[Out] (9*asin(x/3 + 2/3))/2 + (x/2 + 1)*(5 - x^2 - 4*x)^(1/2)
```

3.354 $\int \frac{x^5}{\sqrt{2}+x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{x^5}{\sqrt{2}+x^2} \, dx = -\frac{x^2}{\sqrt{2}} + \frac{x^4}{4} + \log\left(\sqrt{2}+x^2\right)$$

```
[Out] 1/4*x^4+ln(x^2+2^(1/2))-1/2*x^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\int \frac{x^5}{\sqrt{2}+x^2} \, dx = \frac{x^4}{4} - \frac{x^2}{\sqrt{2}} + \log\left(x^2+\sqrt{2}\right)$$

```
[In] Int[x^5/(Sqrt[2] + x^2),x]
[Out] -(x^2/Sqrt[2]) + x^4/4 + Log[Sqrt[2] + x^2]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{2}+x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\sqrt{2}+x+\frac{2}{\sqrt{2}+x}\right)\,dx,x,x^2\right)\\
&= -\frac{x^2}{\sqrt{2}}+\frac{x^4}{4}+\log\left(\sqrt{2}+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{x^5}{\sqrt{2}+x^2}\,dx = \frac{1}{4}\left(-6-2\sqrt{2}x^2+x^4+4\log\left(\sqrt{2}+x^2\right)\right)$$

```
[In] Integrate[x^5/(Sqrt[2] + x^2),x]
[Out] (-6 - 2*Sqrt[2]*x^2 + x^4 + 4*Log[Sqrt[2] + x^2])/4
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^4}{4}+\ln\left(x^2+\sqrt{2}\right)-\frac{x^2\sqrt{2}}{2}$	23
parallelrisch	$\frac{x^4}{4}+\ln\left(x^2+\sqrt{2}\right)-\frac{x^2\sqrt{2}}{2}$	23
risch	$\frac{x^4}{4}-\frac{x^2\sqrt{2}}{2}+\frac{1}{2}+\ln\left(x^2+\sqrt{2}\right)$	24
meijerg	$-\frac{x^2\sqrt{2}\left(-\frac{3x^2\sqrt{2}}{2}+6\right)}{12}+\ln\left(1+\frac{x^2\sqrt{2}}{2}\right)$	31

```
[In] int(x^5/(x^2+2^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4+ln(x^2+2^(1/2))-1/2*x^2*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{\sqrt{2}+x^2} \, dx = \frac{1}{4} \, x^4 - \frac{1}{2} \, \sqrt{2}x^2 + \log\left(x^2 + \sqrt{2}\right)$$

```
[In] integrate(x^5/(x^2+2^(1/2)),x, algorithm="fricas")
[Out] 1/4*x^4 - 1/2*sqrt(2)*x^2 + log(x^2 + sqrt(2))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{\sqrt{2}+x^2} \, dx = \frac{x^4}{4} - \frac{\sqrt{2}x^2}{2} + \log\left(x^2 + \sqrt{2}\right)$$

```
[In] integrate(x**5/(x**2+2**(1/2)),x)
[Out] x**4/4 - sqrt(2)*x**2/2 + log(x**2 + sqrt(2))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{\sqrt{2}+x^2} \, dx = \frac{1}{4} \, x^4 - \frac{1}{2} \, \sqrt{2}x^2 + \log\left(x^2 + \sqrt{2}\right)$$

```
[In] integrate(x^5/(x^2+2^(1/2)),x, algorithm="maxima")
[Out] 1/4*x^4 - 1/2*sqrt(2)*x^2 + log(x^2 + sqrt(2))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{\sqrt{2}+x^2} \, dx = \frac{1}{4} \, x^4 - \frac{1}{2} \, \sqrt{2}x^2 + \log\left(x^2 + \sqrt{2}\right)$$

```
[In] integrate(x^5/(x^2+2^(1/2)),x, algorithm="giac")
[Out] 1/4*x^4 - 1/2*sqrt(2)*x^2 + log(x^2 + sqrt(2))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{\sqrt{2}+x^2}\,dx=\ln\left(x^2+\sqrt{2}\right)-\frac{\sqrt{2}\,x^2}{2}+\frac{x^4}{4}$$

```
[In] int(x^5/(2^(1/2) + x^2),x)
```

```
[Out] log(2^(1/2) + x^2) - (2^(1/2)*x^2)/2 + x^4/4
```

3.355 $\int \sec^5(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 26

$$\int \sec^5(x)\,dx = \frac{3}{8}\text{arctanh}(\sin(x)) + \frac{3}{8}\sec(x)\tan(x) + \frac{1}{4}\sec^3(x)\tan(x)$$

```
[Out] 3/8*arctanh(sin(x))+3/8*sec(x)*tan(x)+1/4*sec(x)^3*tan(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3853, 3855}

$$\int \sec^5(x)\,dx = \frac{3}{8}\text{arctanh}(\sin(x)) + \frac{1}{4}\tan(x)\sec^3(x) + \frac{3}{8}\tan(x)\sec(x)$$

```
[In] Int[Sec[x]^5,x]
```

```
[Out] (3*ArcTanh[Sin[x]])/8 + (3*Sec[x]*Tan[x])/8 + (Sec[x]^3*Tan[x])/4
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\sec^3(x)\tan(x) + \frac{3}{4}\int \sec^3(x)\,dx \\
&= \frac{3}{8}\sec(x)\tan(x) + \frac{1}{4}\sec^3(x)\tan(x) + \frac{3}{8}\int \sec(x)\,dx \\
&= \frac{3}{8}\text{arctanh}(\sin(x)) + \frac{3}{8}\sec(x)\tan(x) + \frac{1}{4}\sec^3(x)\tan(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \sec^5(x)\,dx = \frac{3}{8}\text{arctanh}(\sin(x)) + \frac{3}{8}\sec(x)\tan(x) + \frac{1}{4}\sec^3(x)\tan(x)$$

```
[In] Integrate[Sec[x]^5,x]
[Out] (3*ArcTanh[Sin[x]])/8 + (3*Sec[x]*Tan[x])/8 + (Sec[x]^3*Tan[x])/4
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	siz
default	$-\left(-\frac{(\sec^3(x))}{4} - \frac{3\sec(x)}{8}\right)\tan(x) + \frac{3\ln(\sec(x)+\tan(x))}{8}$	25
parallelrisch	$\ln\left((-\cot(x)+1+\csc(x))^{\frac{3}{8}}\right) + \ln\left(\frac{1}{(-\cot(x)+\csc(x)-1)^{\frac{3}{8}}}\right) + \frac{3\sec(x)\tan(x)}{8} + \frac{(\sec^3(x))\tan(x)}{4}$	38
norman	$\frac{\frac{3\left(\tan^3\left(\frac{x}{2}\right)\right)}{4} + \frac{3\left(\tan^5\left(\frac{x}{2}\right)\right)}{4} + \frac{5\left(\tan^7\left(\frac{x}{2}\right)\right)}{4} + \frac{5\tan\left(\frac{x}{2}\right)}{4}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)^4} - \frac{3\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{8} + \frac{3\ln\left(1+\tan\left(\frac{x}{2}\right)\right)}{8}$	62
risch	$-\frac{i(3\,e^{7ix}+11\,e^{5ix}-11\,e^{3ix}-3\,e^{ix})}{4(e^{2ix}+1)^4} + \frac{3\ln(i+e^{ix})}{8} - \frac{3\ln(e^{ix}-i)}{8}$	65

```
[In] int(sec(x)^5,x,method=_RETURNVERBOSE)
[Out] -(-1/4*sec(x)^3-3/8*sec(x))*tan(x)+3/8*ln(sec(x)+tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(20) = 40$.

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \sec^5(x)\,dx$$
$$= \frac{3 \cos(x)^4 \log(\sin(x) + 1) - 3 \cos(x)^4 \log(-\sin(x) + 1) + 2\left(3 \cos(x)^2 + 2\right)\sin(x)}{16 \cos(x)^4}$$

```
[In] integrate(sec(x)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(3*cos(x)^4*log(sin(x) + 1) - 3*cos(x)^4*log(-sin(x) + 1) + 2*(3*cos(x
)^2 + 2)*sin(x))/cos(x)^4
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \sec^5(x)\,dx = -\frac{3\sin^3(x) - 5\sin(x)}{8\sin^4(x) - 16\sin^2(x) + 8} - \frac{3\log(\sin(x) - 1)}{16} + \frac{3\log(\sin(x) + 1)}{16}$$

```
[In] integrate(sec(x)**5,x)
```

```
[Out] -(3*sin(x)**3 - 5*sin(x))/(8*sin(x)**4 - 16*sin(x)**2 + 8) - 3*log(sin(x) -
 1)/16 + 3*log(sin(x) + 1)/16
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \sec^5(x)\,dx = -\frac{3 \sin(x)^3 - 5 \sin(x)}{8\left(\sin(x)^4 - 2 \sin(x)^2 + 1\right)} + \frac{3}{16} \log(\sin(x) + 1) - \frac{3}{16} \log(\sin(x) - 1)$$

```
[In] integrate(sec(x)^5,x, algorithm="maxima")
```

```
[Out] -1/8*(3*sin(x)^3 - 5*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 3/16*log(sin(x)
+ 1) - 3/16*log(sin(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \sec^5(x)\,dx = -\frac{3\,\sin\left(x\right)^3 - 5\,\sin\left(x\right)}{8\,\left(\sin\left(x\right)^2 - 1\right)^2} + \frac{3}{16}\,\log\left(\sin\left(x\right) + 1\right) - \frac{3}{16}\,\log\left(-\sin\left(x\right) + 1\right)$$

```
[In] integrate(sec(x)^5,x, algorithm="giac")
[Out] -1/8*(3*sin(x)^3 - 5*sin(x))/(sin(x)^2 - 1)^2 + 3/16*log(sin(x) + 1) - 3/16
*log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \sec^5(x)\,dx = \frac{3\,\ln\left(\frac{\sin(x)+1}{\cos(x)}\right)}{8} + \sin\left(x\right)\,\left(\frac{3}{8\cos\left(x\right)^2} + \frac{1}{4\cos\left(x\right)^4}\right)$$

```
[In] int(1/cos(x)^5,x)
[Out] (3*log((sin(x) + 1)/cos(x)))/8 + sin(x)*(3/(8*cos(x)^2) + 1/(4*cos(x)^4))
```

3.356 $\int \sin^6(2x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 46

$$\int \sin^6(2x)\,dx = \frac{5x}{16} - \frac{5}{32}\cos(2x)\sin(2x) - \frac{5}{48}\cos(2x)\sin^3(2x) - \frac{1}{12}\cos(2x)\sin^5(2x)$$

```
[Out] 5/16*x-5/32*cos(2*x)*sin(2*x)-5/48*cos(2*x)*sin(2*x)^3-1/12*cos(2*x)*sin(2*
x)^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2715, 8}

$$\int \sin^6(2x)\,dx = \frac{5x}{16} - \frac{1}{12}\sin^5(2x)\cos(2x) - \frac{5}{48}\sin^3(2x)\cos(2x) - \frac{5}{32}\sin(2x)\cos(2x)$$

```
[In] Int[Sin[2*x]^6,x]
```

```
[Out] (5*x)/16 - (5*Cos[2*x]*Sin[2*x])/32 - (5*Cos[2*x]*Sin[2*x]^3)/48 - (Cos[2*x
]*Sin[2*x]^5)/12
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

*n]

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{12}\cos(2x)\sin^5(2x)+\frac{5}{6}\int\sin^4(2x)\,dx\\
&= -\frac{5}{48}\cos(2x)\sin^3(2x)-\frac{1}{12}\cos(2x)\sin^5(2x)+\frac{5}{8}\int\sin^2(2x)\,dx\\
&= -\frac{5}{32}\cos(2x)\sin(2x)-\frac{5}{48}\cos(2x)\sin^3(2x)-\frac{1}{12}\cos(2x)\sin^5(2x)+\frac{5\int 1\,dx}{16}\\
&= \frac{5x}{16}-\frac{5}{32}\cos(2x)\sin(2x)-\frac{5}{48}\cos(2x)\sin^3(2x)-\frac{1}{12}\cos(2x)\sin^5(2x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

$$\int\sin^6(2x)\,dx=\frac{5x}{16}-\frac{15}{128}\sin(4x)+\frac{3}{128}\sin(8x)-\frac{1}{384}\sin(12x)$$

```
[In] Integrate[Sin[2*x]^6,x]
[Out] (5*x)/16 - (15*Sin[4*x])/128 + (3*Sin[8*x])/128 - Sin[12*x]/384
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

method	result
risch	$\frac{5x}{16}-\frac{\sin(12x)}{384}+\frac{3\sin(8x)}{128}-\frac{15\sin(4x)}{128}$
parallelrisch	$\frac{5x}{16}-\frac{\sin(12x)}{384}+\frac{3\sin(8x)}{128}-\frac{15\sin(4x)}{128}$
derivativedivides	$-\frac{\left(\sin^5(2x)+\frac{5\left(\sin^3(2x)\right)}{4}+\frac{15\sin(2x)}{8}\right)\cos(2x)}{12}+\frac{5x}{16}$
default	$-\frac{\left(\sin^5(2x)+\frac{5\left(\sin^3(2x)\right)}{4}+\frac{15\sin(2x)}{8}\right)\cos(2x)}{12}+\frac{5x}{16}$
norman	$\frac{\frac{5x}{16}-\frac{85\left(\tan^3(x)\right)}{48}-\frac{33\left(\tan^5(x)\right)}{8}+\frac{33\left(\tan^7(x)\right)}{8}+\frac{85\left(\tan^9(x)\right)}{48}+\frac{5\left(\tan^{11}(x)\right)}{16}+\frac{15x\left(\tan^2(x)\right)}{8}+\frac{75x\left(\tan^4(x)\right)}{16}+\frac{25x\left(\tan^6(x)\right)}{4}+7}{(1+\tan^2(x))^6}$

```
[In] int(sin(2*x)^6,x,method=_RETURNVERBOSE)
[Out] 5/16*x-1/384*sin(12*x)+3/128*sin(8*x)-15/128*sin(4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \sin^6(2x)\,dx = -\frac{1}{96}\left(8\,\cos\left(2\,x\right)^5 - 26\,\cos\left(2\,x\right)^3 + 33\,\cos\left(2\,x\right)\right)\sin\left(2\,x\right) + \frac{5}{16}\,x$$

```
[In] integrate(sin(2*x)^6,x, algorithm="fricas")
[Out] -1/96*(8*cos(2*x)^5 - 26*cos(2*x)^3 + 33*cos(2*x))*sin(2*x) + 5/16*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sin^6(2x)\,dx = \frac{5x}{16} - \frac{\sin^5(2x)\cos(2x)}{12} - \frac{5\sin^3(2x)\cos(2x)}{48} - \frac{5\sin(2x)\cos(2x)}{32}$$

```
[In] integrate(sin(2*x)**6,x)
[Out] 5*x/16 - sin(2*x)**5*cos(2*x)/12 - 5*sin(2*x)**3*cos(2*x)/48 - 5*sin(2*x)*c
os(2*x)/32
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \sin^6(2x)\,dx = \frac{1}{96}\,\sin\left(4\,x\right)^3 + \frac{5}{16}\,x + \frac{3}{128}\,\sin\left(8\,x\right) - \frac{1}{8}\,\sin\left(4\,x\right)$$

```
[In] integrate(sin(2*x)^6,x, algorithm="maxima")
[Out] 1/96*sin(4*x)^3 + 5/16*x + 3/128*sin(8*x) - 1/8*sin(4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \sin^6(2x)\,dx = \frac{5}{16}\,x - \frac{1}{384}\,\sin\left(12\,x\right) + \frac{3}{128}\,\sin\left(8\,x\right) - \frac{15}{128}\,\sin\left(4\,x\right)$$

```
[In] integrate(sin(2*x)^6,x, algorithm="giac")
[Out] 5/16*x - 1/384*sin(12*x) + 3/128*sin(8*x) - 15/128*sin(4*x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \sin^6(2x)\, dx = \frac{5\,x}{16} - \frac{15\,\sin\left(4\,x\right)}{128} + \frac{3\,\sin\left(8\,x\right)}{128} - \frac{\sin\left(12\,x\right)}{384}$$

```
[In] int(sin(2*x)^6,x)
[Out] (5*x)/16 - (15*sin(4*x))/128 + (3*sin(8*x))/128 - sin(12*x)/384
```

3.357 $\int \cos(x) \log(\sin(x)) \sin^2(x) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 20

$$\int \cos(x) \log(\sin(x)) \sin^2(x) \, dx = -\frac{1}{9} \sin^3(x) + \frac{1}{3} \log(\sin(x)) \sin^3(x)$$

```
[Out] -1/9*sin(x)^3+1/3*ln(sin(x))*sin(x)^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2644, 30, 2634, 12}

$$\int \cos(x) \log(\sin(x)) \sin^2(x) \, dx = \frac{1}{3} \sin^3(x) \log(\sin(x)) - \frac{\sin^3(x)}{9}$$

```
[In] Int[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]
```

```
[Out] -1/9*Sin[x]^3 + (Log[Sin[x]]*Sin[x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\log(\sin(x))\sin^3(x) - \int \frac{1}{3}\cos(x)\sin^2(x)\,dx\\ &= \frac{1}{3}\log(\sin(x))\sin^3(x) - \frac{1}{3}\int \cos(x)\sin^2(x)\,dx\\ &= \frac{1}{3}\log(\sin(x))\sin^3(x) - \frac{1}{3}\text{Subst}\left(\int x^2\,dx, x, \sin(x)\right)\\ &= -\frac{1}{9}\sin^3(x) + \frac{1}{3}\log(\sin(x))\sin^3(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = \frac{1}{9}(-1 + 3\log(\sin(x)))\sin^3(x)$$

```
[In] Integrate[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]
[Out] ((-1 + 3*Log[Sin[x]])*Sin[x]^3)/9
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{(\sin^3(x))}{9}+\frac{\ln(\sin(x))(\sin^3(x))}{3}$	17
default	$-\frac{(\sin^3(x))}{9}+\frac{\ln(\sin(x))(\sin^3(x))}{3}$	17
parallelrisch	$\frac{(3\ln(\sin(x))-1)(-\sin(3x)+3\sin(x))}{36}$	21
risch	Expression too large to display	577

```
[In] int(cos(x)*ln(sin(x))*sin(x)^2,x,method=_RETURNVERBOSE)
[Out] -1/9*sin(x)^3+1/3*ln(sin(x))*sin(x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = -\frac{1}{3}\left(\cos\left(x\right)^2-1\right)\log\left(\sin\left(x\right)\right)\sin\left(x\right)+\frac{1}{9}\left(\cos\left(x\right)^2-1\right)\sin\left(x\right)$$

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="fricas")
[Out] -1/3*(cos(x)^2 - 1)*log(sin(x))*sin(x) + 1/9*(cos(x)^2 - 1)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = \frac{\log\left(\sin\left(x\right)\right)\sin^3\left(x\right)}{3}-\frac{\sin^3\left(x\right)}{9}$$

```
[In] integrate(cos(x)*ln(sin(x))*sin(x)**2,x)
[Out] log(sin(x))*sin(x)**3/3 - sin(x)**3/9
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = \frac{1}{3}\,\log\left(\sin\left(x\right)\right)\sin\left(x\right)^3 - \frac{1}{9}\,\sin\left(x\right)^3$$

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="maxima")
[Out] 1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = \frac{1}{3}\,\log\left(\sin\left(x\right)\right)\sin\left(x\right)^3 - \frac{1}{9}\,\sin\left(x\right)^3$$

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="giac")
[Out] 1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.55

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = \frac{\sin\left(x\right)^3\,\left(\ln\left(\sin\left(x\right)\right)-\frac{1}{3}\right)}{3}$$

```
[In] int(log(sin(x))*cos(x)*sin(x)^2,x)
[Out] (sin(x)^3*(log(sin(x)) - 1/3))/3
```

3.358 $\int \frac{e^{-x}}{1+2e^x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{e^{-x}}{1+2e^x} \, dx = -e^{-x} - 2x + 2\log(1+2e^x)$$

```
[Out] -1/exp(x)-2*x+2*ln(1+2*exp(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2280, 46}

$$\int \frac{e^{-x}}{1+2e^x} \, dx = -2x - e^{-x} + 2\log(2e^x+1)$$

```
[In] Int[1/(E^x*(1 + 2*E^x)),x]
```

```
[Out] -E^(-x) - 2*x + 2*Log[1 + 2*E^x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2280

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
```

```
[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Den
ominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{x^2(1+2x)}\,dx, x, e^x\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{2}{x}+\frac{4}{1+2x}\right)dx, x, e^x\right)\\ &= -e^{-x}-2x+2\log\left(1+2e^x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{e^{-x}}{1+2e^x}\,dx = -e^{-x}-2\log\left(e^x\right)+2\log\left(1+2e^x\right)$$

```
[In] Integrate[1/(E^x*(1 + 2*E^x)),x]
[Out] -E^(-x) - 2*Log[E^x] + 2*Log[1 + 2*E^x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
risch	$-\mathrm{e}^{-x}-2x+2\ln\left(\frac{1}{2}+\mathrm{e}^x\right)$	18
derivativedivides	$2\ln\left(1+2\,\mathrm{e}^x\right)-\mathrm{e}^{-x}-2\ln\left(\mathrm{e}^x\right)$	22
default	$2\ln\left(1+2\,\mathrm{e}^x\right)-\mathrm{e}^{-x}-2\ln\left(\mathrm{e}^x\right)$	22
parallelrisch	$\left(-1+2\ln\left(\frac{1}{2}+\mathrm{e}^x\right)\mathrm{e}^x-2\,\mathrm{e}^x x\right)\mathrm{e}^{-x}$	22
norman	$\left(-1-2\,\mathrm{e}^x x\right)\mathrm{e}^{-x}+2\ln\left(1+2\,\mathrm{e}^x\right)$	23

```
[In] int(1/exp(x)/(1+2*exp(x)),x,method=_RETURNVERBOSE)
[Out] -exp(-x)-2*x+2*ln(1/2+exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{e^{-x}}{1+2e^x}\,dx = -(2\,xe^x - 2\,e^x\log\left(2\,e^x + 1\right) + 1)e^{(-x)}$$

```
[In] integrate(1/exp(x)/(1+2*exp(x)),x, algorithm="fricas")
[Out] -(2*x*e^x - 2*e^x*log(2*e^x + 1) + 1)*e^(-x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{e^{-x}}{1+2e^x}\,dx = 2\log\left(2 + e^{-x}\right) - e^{-x}$$

```
[In] integrate(1/exp(x)/(1+2*exp(x)),x)
[Out] 2*log(2 + exp(-x)) - exp(-x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{-x}}{1+2e^x}\,dx = -e^{(-x)} + 2\,\log\left(e^{(-x)} + 2\right)$$

```
[In] integrate(1/exp(x)/(1+2*exp(x)),x, algorithm="maxima")
[Out] -e^(-x) + 2*log(e^(-x) + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{e^{-x}}{1+2e^x}\,dx = -2\,x - e^{(-x)} + 2\,\log\left(2\,e^x + 1\right)$$

```
[In] integrate(1/exp(x)/(1+2*exp(x)),x, algorithm="giac")
[Out] -2*x - e^(-x) + 2*log(2*e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{e^{-x}}{1+2e^x}\,dx = 2\,\ln\left(2\,\mathrm{e}^x+1\right)-2\,x-\mathrm{e}^{-x}$$

```
[In] int(exp(-x)/(2*exp(x) + 1),x)
[Out] 2*log(2*exp(x) + 1) - 2*x - exp(-x)
```

3.359 $\int \sqrt{2+3\cos(x)}\tan(x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \sqrt{2+3\cos(x)}\tan(x)\,dx=2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2+3\cos(x)}}{\sqrt{2}}\right)-2\sqrt{2+3\cos(x)}$$

```
[Out] 2*arctanh(1/2*(2+3*cos(x))^(1/2)*2^(1/2))*2^(1/2)-2*(2+3*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2800, 52, 65, 213}

$$\int \sqrt{2+3\cos(x)}\tan(x)\,dx=2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{3\cos(x)+2}}{\sqrt{2}}\right)-2\sqrt{3\cos(x)+2}$$

```
[In] Int[Sqrt[2 + 3*Cos[x]]*Tan[x],x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[2 + 3*Cos[x]]/Sqrt[2]] - 2*Sqrt[2 + 3*Cos[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt{2+x}}{x}\,dx, x, 3\cos(x)\right)\\
&= -2\sqrt{2+3\cos(x)} - 2\text{Subst}\left(\int \frac{1}{x\sqrt{2+x}}\,dx, x, 3\cos(x)\right)\\
&= -2\sqrt{2+3\cos(x)} - 4\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{2+3\cos(x)}\right)\\
&= 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2+3\cos(x)}}{\sqrt{2}}\right) - 2\sqrt{2+3\cos(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \sqrt{2+3\cos(x)}\tan(x)\,dx = 2\sqrt{2}\text{arctanh}\left(\sqrt{1+\frac{3\cos(x)}{2}}\right) - 2\sqrt{2+3\cos(x)}$$

```
[In] Integrate[Sqrt[2 + 3*Cos[x]]*Tan[x],x]
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + (3*Cos[x])/2]] - 2*Sqrt[2 + 3*Cos[x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(30) = 60.

Time = 0.68 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.08

method	result	size
default	$\sqrt{2}\,\operatorname{arctanh}\left(\frac{6\cos\left(\frac{x}{2}\right)-\sqrt{2}}{2\sqrt{-6\left(\sin^{2}\left(\frac{x}{2}\right)\right)+5}}\right)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{6\cos\left(\frac{x}{2}\right)+\sqrt{2}}{2\sqrt{-6\left(\sin^{2}\left(\frac{x}{2}\right)\right)+5}}\right)-2\sqrt{-6\left(\sin^{2}\left(\frac{x}{2}\right)\right)+5}$	77

```
[In] int((2+3*cos(x))^(1/2)*tan(x),x,method=_RETURNVERBOSE)
[Out] 2^(1/2)*arctanh(1/2/(-6*sin(1/2*x)^2+5)^(1/2)*(6*cos(1/2*x)-2^(1/2)))-2^(1/
2)*arctanh(1/2/(-6*sin(1/2*x)^2+5)^(1/2)*(6*cos(1/2*x)+2^(1/2)))-2*(-6*sin(
1/2*x)^2+5)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int \sqrt{2+3\cos(x)}\tan(x)\,dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{9\cos(x)^{2}+4\left(3\sqrt{2}\cos(x)+4\sqrt{2}\right)\sqrt{3\cos(x)+2}+48\cos(x)+32}{\cos(x)^{2}}\right) - 2\sqrt{3\cos(x)+2}$$

```
[In] integrate((2+3*cos(x))^(1/2)*tan(x),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*log(-(9*cos(x)^2 + 4*(3*sqrt(2)*cos(x) + 4*sqrt(2))*sqrt(3*cos(
x) + 2) + 48*cos(x) + 32)/cos(x)^2) - 2*sqrt(3*cos(x) + 2)
```

Sympy [F]

$$\int \sqrt{2+3\cos(x)}\tan(x)\,dx = \int \sqrt{3\cos(x)+2}\tan(x)\,dx$$

```
[In] integrate((2+3*cos(x))**(1/2)*tan(x),x)
[Out] Integral(sqrt(3*cos(x) + 2)*tan(x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \sqrt{2+3\cos(x)}\tan(x)\,dx = -\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{3\cos(x)+2}}{\sqrt{2}+\sqrt{3\cos(x)+2}}\right) - 2\sqrt{3\cos(x)+2}$$

```
[In] integrate((2+3*cos(x))^(1/2)*tan(x),x, algorithm="maxima")
[Out] -sqrt(2)*log(-(sqrt(2) - sqrt(3*cos(x) + 2))/(sqrt(2) + sqrt(3*cos(x) + 2))
) - 2*sqrt(3*cos(x) + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \sqrt{2+3\cos(x)}\tan(x)\,dx = -\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{3\cos(x)+2}\right|}{2\left(\sqrt{2}+\sqrt{3\cos(x)+2}\right)}\right) - 2\sqrt{3\cos(x)+2}$$

```
[In] integrate((2+3*cos(x))^(1/2)*tan(x),x, algorithm="giac")
[Out] -sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(3*cos(x) + 2))/(sqrt(2) + sqrt(3*c
os(x) + 2))) - 2*sqrt(3*cos(x) + 2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+3\cos(x)}\tan(x)\,dx = \int \tan(x)\,\sqrt{3\cos(x)+2}\,dx$$

```
[In] int(tan(x)*(3*cos(x) + 2)^(1/2),x)
[Out] int(tan(x)*(3*cos(x) + 2)^(1/2), x)
```

3.360 $\int \frac{x}{\sqrt{-4x+x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx = \sqrt{-4x+x^2} + 4\text{arctanh}\left(\frac{x}{\sqrt{-4x+x^2}}\right)$$

```
[Out] 4*arctanh(x/(x^2-4*x)^(1/2))+(x^2-4*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {654, 634, 212}

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx = 4\text{arctanh}\left(\frac{x}{\sqrt{x^2-4x}}\right) + \sqrt{x^2-4x}$$

```
[In] Int[x/Sqrt[-4*x + x^2],x]
```

```
[Out] Sqrt[-4*x + x^2] + 4*ArcTanh[x/Sqrt[-4*x + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \sqrt{-4x+x^2}+2\int \frac{1}{\sqrt{-4x+x^2}}\,dx\\ &= \sqrt{-4x+x^2}+4\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-4x+x^2}}\right)\\ &= \sqrt{-4x+x^2}+4\text{arctanh}\left(\frac{x}{\sqrt{-4x+x^2}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx=\frac{(-4+x)x-4\sqrt{-4+x}\sqrt{x}\log\left(\sqrt{-4+x}-\sqrt{x}\right)}{\sqrt{(-4+x)x}}$$

```
[In] Integrate[x/Sqrt[-4*x + x^2],x]
[Out] ((-4 + x)*x - 4*Sqrt[-4 + x]*Sqrt[x]*Log[Sqrt[-4 + x] - Sqrt[x]])/Sqrt[(-4
+ x)*x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
default	$\sqrt{x^2-4x}+2\ln\left(-2+x+\sqrt{x^2-4x}\right)$	26
trager	$\sqrt{x^2-4x}-2\ln\left(2-x+\sqrt{x^2-4x}\right)$	28
risch	$\frac{x(x-4)}{\sqrt{x(x-4)}}+2\ln\left(-2+x+\sqrt{x^2-4x}\right)$	29
pseudoelliptic	$\sqrt{x\left(x-4\right)}+2\ln\left(\frac{\sqrt{x(x-4)}+x}{x}\right)-2\ln\left(\frac{\sqrt{x(x-4)}-x}{x}\right)$	43
meijerg	$\frac{4i\sqrt{-\text{signum}(x-4)}\left(\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{-\frac{x}{4}+1}}{2}-i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{x}}{2}\right)\right)}{\sqrt{\pi}\,\sqrt{\text{signum}(x-4)}}$	50

```
[In] int(x/(x^2-4*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] (x^2-4*x)^(1/2)+2*ln(-2+x+(x^2-4*x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx = \sqrt{x^2-4\,x} - 2\,\log\left(-x+\sqrt{x^2-4\,x}+2\right)$$

```
[In] integrate(x/(x^2-4*x)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^2 - 4*x) - 2*log(-x + sqrt(x^2 - 4*x) + 2)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx = \sqrt{x^2-4x} + 2\log\left(2x+2\sqrt{x^2-4x}-4\right)$$

```
[In] integrate(x/(x**2-4*x)**(1/2),x)
[Out] sqrt(x**2 - 4*x) + 2*log(2*x + 2*sqrt(x**2 - 4*x) - 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx = \sqrt{x^2-4\,x} + 2\,\log\left(2\,x+2\,\sqrt{x^2-4\,x}-4\right)$$

```
[In] integrate(x/(x^2-4*x)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 - 4*x) + 2*log(2*x + 2*sqrt(x^2 - 4*x) - 4)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx = \sqrt{x^2-4\,x} - 2\,\log\left(\left|-x+\sqrt{x^2-4\,x}+2\right|\right)$$

```
[In] integrate(x/(x^2-4*x)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 - 4*x) - 2*log(abs(-x + sqrt(x^2 - 4*x) + 2))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx = 2\,\ln\left(x+\sqrt{x\,\left(x-4\right)}-2\right)+\sqrt{x^2-4\,x}$$

```
[In] int(x/(x^2 - 4*x)^(1/2),x)
[Out] 2*log(x + (x*(x - 4))^(1/2) - 2) + (x^2 - 4*x)^(1/2)
```

3.361 $\int \cos^5(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 19

$$\int \cos^5(x)\,dx = \sin(x) - \frac{2\sin^3(x)}{3} + \frac{\sin^5(x)}{5}$$

```
[Out] sin(x)-2/3*sin(x)^3+1/5*sin(x)^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2713}

$$\int \cos^5(x)\,dx = \frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

```
[In] Int[Cos[x]^5,x]
```

```
[Out] Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \left(1 - 2x^2 + x^4\right)\,dx, x, -\sin(x)\right)\\
&= \sin(x) - \frac{2\sin^3(x)}{3} + \frac{\sin^5(x)}{5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos^5(x)\,dx = \sin(x) - \frac{2\sin^3(x)}{3} + \frac{\sin^5(x)}{5}$$

```
[In] Integrate[Cos[x]^5,x]
[Out] Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\left(\frac{8}{3}+\cos^4(x)+\frac{4\left(\cos^2(x)\right)}{3}\right)\sin(x)}{5}$	17
risch	$\frac{5\sin(x)}{8}+\frac{\sin(5x)}{80}+\frac{5\sin(3x)}{48}$	18
parallelrisch	$\frac{5\sin(x)}{8}+\frac{\sin(5x)}{80}+\frac{5\sin(3x)}{48}$	18

```
[In] int(cos(x)^5,x,method=_RETURNVERBOSE)
[Out] 1/5*(8/3+cos(x)^4+4/3*cos(x)^2)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \cos^5(x)\,dx = \frac{1}{15}\left(3\cos(x)^4 + 4\cos(x)^2 + 8\right)\sin(x)$$

```
[In] integrate(cos(x)^5,x, algorithm="fricas")
[Out] 1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \cos^5(x)\,dx = \frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

```
[In] integrate(cos(x)**5,x)
[Out] sin(x)**5/5 - 2*sin(x)**3/3 + sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cos^5(x)\,dx = \frac{1}{5}\sin(x)^5 - \frac{2}{3}\sin(x)^3 + \sin(x)$$

```
[In] integrate(cos(x)^5,x, algorithm="maxima")
[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cos^5(x)\,dx = \frac{1}{5}\sin(x)^5 - \frac{2}{3}\sin(x)^3 + \sin(x)$$

```
[In] integrate(cos(x)^5,x, algorithm="giac")
[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos^5(x)\,dx = \frac{\sin(x)\cos(x)^4}{5} + \frac{4\sin(x)\cos(x)^2}{15} + \frac{8\sin(x)}{15}$$

```
[In] int(cos(x)^5,x)
[Out] (8*sin(x))/15 + (4*cos(x)^2*sin(x))/15 + (cos(x)^4*sin(x))/5
```

3.362 $\int e^{-x} x^4 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 46

$$\int e^{-x} x^4 \, dx = -24e^{-x} - 24e^{-x}x - 12e^{-x}x^2 - 4e^{-x}x^3 - e^{-x}x^4$$

```
[Out] -24/exp(x)-24*x/exp(x)-12*x^2/exp(x)-4*x^3/exp(x)-x^4/exp(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2207, 2225}

$$\int e^{-x} x^4 \, dx = -e^{-x}x^4 - 4e^{-x}x^3 - 12e^{-x}x^2 - 24e^{-x}x - 24e^{-x}$$

```
[In] Int[x^4/E^x,x]
```

```
[Out] -24/E^x - (24*x)/E^x - (12*x^2)/E^x - (4*x^3)/E^x - x^4/E^x
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -e^{-x}x^4 + 4\int e^{-x}x^3\,dx \\
&= -4e^{-x}x^3 - e^{-x}x^4 + 12\int e^{-x}x^2\,dx \\
&= -12e^{-x}x^2 - 4e^{-x}x^3 - e^{-x}x^4 + 24\int e^{-x}x\,dx \\
&= -24e^{-x}x - 12e^{-x}x^2 - 4e^{-x}x^3 - e^{-x}x^4 + 24\int e^{-x}\,dx \\
&= -24e^{-x} - 24e^{-x}x - 12e^{-x}x^2 - 4e^{-x}x^3 - e^{-x}x^4
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int e^{-x}x^4\,dx = e^{-x}\left(-24 - 24x - 12x^2 - 4x^3 - x^4\right)$$

```
[In] Integrate[x^4/E^x,x]
[Out] (-24 - 24*x - 12*x^2 - 4*x^3 - x^4)/E^x
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.54

method	result	size
gosper	$-(x^4+4x^3+12x^2+24x+24)\,\mathrm{e}^{-x}$	25
norman	$(-x^4-4x^3-12x^2-24x-24)\,\mathrm{e}^{-x}$	26
risch	$(-x^4-4x^3-12x^2-24x-24)\,\mathrm{e}^{-x}$	26
parallelrisch	$(-x^4-4x^3-12x^2-24x-24)\,\mathrm{e}^{-x}$	26
meijerg	$24-\frac{(5x^4+20x^3+60x^2+120x+120)\,\mathrm{e}^{-x}}{5}$	29
default	$-24\,\mathrm{e}^{-x}-24x\,\mathrm{e}^{-x}-12x^2\mathrm{e}^{-x}-4x^3\mathrm{e}^{-x}-x^4\mathrm{e}^{-x}$	42

```
[In] int(x^4/exp(x),x,method=_RETURNVERBOSE)
[Out] -(x^4+4*x^3+12*x^2+24*x+24)/exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int e^{-x}x^4\,dx = -\left(x^4 + 4\,x^3 + 12\,x^2 + 24\,x + 24\right)e^{(-x)}$$

```
[In] integrate(x^4/exp(x),x, algorithm="fricas")
[Out] -(x^4 + 4*x^3 + 12*x^2 + 24*x + 24)*e^(-x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int e^{-x}x^4\,dx = \left(-x^4 - 4x^3 - 12x^2 - 24x - 24\right)e^{-x}$$

```
[In] integrate(x**4/exp(x),x)
[Out] (-x**4 - 4*x**3 - 12*x**2 - 24*x - 24)*exp(-x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int e^{-x}x^4\,dx = -\left(x^4 + 4\,x^3 + 12\,x^2 + 24\,x + 24\right)e^{(-x)}$$

```
[In] integrate(x^4/exp(x),x, algorithm="maxima")
[Out] -(x^4 + 4*x^3 + 12*x^2 + 24*x + 24)*e^(-x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int e^{-x}x^4\,dx = -\left(x^4 + 4\,x^3 + 12\,x^2 + 24\,x + 24\right)e^{(-x)}$$

```
[In] integrate(x^4/exp(x),x, algorithm="giac")
[Out] -(x^4 + 4*x^3 + 12*x^2 + 24*x + 24)*e^(-x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int e^{-x}x^4\,dx=-\mathrm{e}^{-x}\,\left(x^4+4\,x^3+12\,x^2+24\,x+24\right)$$

```
[In] int(x^4*exp(-x),x)
[Out] -exp(-x)*(24*x + 12*x^2 + 4*x^3 + x^4 + 24)
```

3.363 $\int \frac{x^4}{\sqrt{-2+x^{10}}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{x^4}{\sqrt{-2+x^{10}}}\,dx = \frac{1}{5}\text{arctanh}\left(\frac{x^5}{\sqrt{-2+x^{10}}}\right)$$

```
[Out] 1/5*arctanh(x^5/(x^10-2)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {281, 223, 212}

$$\int \frac{x^4}{\sqrt{-2+x^{10}}}\,dx = \frac{1}{5}\text{arctanh}\left(\frac{x^5}{\sqrt{x^{10}-2}}\right)$$

```
[In] Int[x^4/Sqrt[-2 + x^10],x]
```

```
[Out] ArcTanh[x^5/Sqrt[-2 + x^10]]/5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{\sqrt{-2+x^2}}\,dx, x, x^5\right)\\ &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^5}{\sqrt{-2+x^{10}}}\right)\\ &= \frac{1}{5}\text{arctanh}\left(\frac{x^5}{\sqrt{-2+x^{10}}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{\sqrt{-2+x^{10}}}\,dx = \frac{1}{5}\log\left(x^5+\sqrt{-2+x^{10}}\right)$$

```
[In] Integrate[x^4/Sqrt[-2 + x^10],x]
[Out] Log[x^5 + Sqrt[-2 + x^10]]/5
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
trager	$\frac{\ln\left(x^5+\sqrt{x^{10}-2}\right)}{5}$	15
pseudoelliptic	$\frac{\ln\left(x^5+\sqrt{x^{10}-2}\right)}{5}$	15
meijerg	$\frac{\sqrt{-\operatorname{signum}\left(-1+\frac{x^{10}}{2}\right)}\,\arcsin\left(\frac{x^5\sqrt{2}}{2}\right)}{5\sqrt{\operatorname{signum}\left(-1+\frac{x^{10}}{2}\right)}}$	34

```
[In] int(x^4/(x^10-2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/5*ln(x^5+(x^10-2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{\sqrt{-2+x^{10}}}\,dx = -\frac{1}{5}\log\left(-x^5+\sqrt{x^{10}-2}\right)$$

```
[In] integrate(x^4/(x^10-2)^(1/2),x, algorithm="fricas")
[Out] -1/5*log(-x^5 + sqrt(x^10 - 2))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \frac{x^4}{\sqrt{-2+x^{10}}}\,dx = \begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{2}x^5}{2}\right)}{5} & \text{for } |x^{10}| > 2 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{2}x^5}{2}\right)}{5} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**4/(x**10-2)**(1/2),x)
[Out] Piecewise((acosh(sqrt(2)*x**5/2)/5, Abs(x**10) > 2), (-I*asin(sqrt(2)*x**5/
2)/5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.83

$$\int \frac{x^4}{\sqrt{-2+x^{10}}}\,dx = \frac{1}{10}\log\left(\frac{\sqrt{x^{10}-2}}{x^5}+1\right) - \frac{1}{10}\log\left(\frac{\sqrt{x^{10}-2}}{x^5}-1\right)$$

```
[In] integrate(x^4/(x^10-2)^(1/2),x, algorithm="maxima")
[Out] 1/10*log(sqrt(x^10 - 2)/x^5 + 1) - 1/10*log(sqrt(x^10 - 2)/x^5 - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{x^4}{\sqrt{-2+x^{10}}}\,dx = \frac{1}{10}\sqrt{x^{10}-2}x^5 + \frac{1}{5}\log\left(\left|-x^5+\sqrt{x^{10}-2}\right|\right)$$

```
[In] integrate(x^4/(x^10-2)^(1/2),x, algorithm="giac")
[Out] 1/10*sqrt(x^10 - 2)*x^5 + 1/5*log(abs(-x^5 + sqrt(x^10 - 2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{-2+x^{10}}}\,dx = \int \frac{x^4}{\sqrt{x^{10}-2}}\,dx$$

```
[In] int(x^4/(x^10 - 2)^(1/2),x)
[Out] int(x^4/(x^10 - 2)^(1/2), x)
```

3.364 $\int e^x \cos(4+3x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^x \cos(4+3x)\,dx = \frac{1}{10}e^x \cos(4+3x) + \frac{3}{10}e^x \sin(4+3x)$$

```
[Out] 1/10*exp(x)*cos(4+3*x)+3/10*exp(x)*sin(4+3*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{4518\}$

$$\int e^x \cos(4+3x)\,dx = \frac{3}{10}e^x \sin(3x+4) + \frac{1}{10}e^x \cos(3x+4)$$

```
[In] Int[E^x*Cos[4 + 3*x],x]
```

```
[Out] (E^x*Cos[4 + 3*x])/10 + (3*E^x*Sin[4 + 3*x])/10
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{10}e^x \cos(4+3x) + \frac{3}{10}e^x \sin(4+3x)$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^x \cos(4+3x)\,dx = \frac{1}{10}e^x(\cos(4+3x)+3\sin(4+3x))$$

```
[In] Integrate[E^x*Cos[4 + 3*x],x]
[Out] (E^x*(Cos[4 + 3*x] + 3*Sin[4 + 3*x]))/10
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{e^{x}(\cos(3x+4)+3\sin(3x+4))}{10}$	20
default	$\frac{e^{x}\cos(3x+4)}{10}+\frac{3\,e^{x}\sin(3x+4)}{10}$	22
risch	$\left(\frac{1}{20}-\frac{3i}{20}\right)e^{x}e^{3ix}e^{4i}+\left(\frac{1}{20}+\frac{3i}{20}\right)e^{x}e^{-3ix}e^{-4i}$	30
norman	$\frac{\frac{3\,e^{x}\tan\left(\frac{3x}{2}+2\right)}{5}-\frac{e^{x}\left(\tan^{2}\left(\frac{3x}{2}+2\right)\right)}{10}+\frac{e^{x}}{10}}{1+\tan^{2}\left(\frac{3x}{2}+2\right)}$	41

```
[In] int(exp(x)*cos(3*x+4),x,method=_RETURNVERBOSE)
[Out] 1/10*exp(x)*(cos(3*x+4)+3*sin(3*x+4))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^x \cos(4+3x)\,dx = \frac{1}{10}\cos(3x+4)e^x + \frac{3}{10}e^x\sin(3x+4)$$

```
[In] integrate(exp(x)*cos(4+3*x),x, algorithm="fricas")
[Out] 1/10*cos(3*x + 4)*e^x + 3/10*e^x*sin(3*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int e^x \cos(4+3x)\,dx = \frac{3e^x \sin{(3x+4)}}{10} + \frac{e^x \cos{(3x+4)}}{10}$$

```
[In] integrate(exp(x)*cos(4+3*x),x)
[Out] 3*exp(x)*sin(3*x + 4)/10 + exp(x)*cos(3*x + 4)/10
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^x \cos(4+3x)\,dx = \frac{1}{10}\,(\cos{(3\,x+4)} + 3\,\sin{(3\,x+4)})e^x$$

```
[In] integrate(exp(x)*cos(4+3*x),x, algorithm="maxima")
[Out] 1/10*(cos(3*x + 4) + 3*sin(3*x + 4))*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^x \cos(4+3x)\,dx = \frac{1}{10}\,(\cos{(3\,x+4)} + 3\,\sin{(3\,x+4)})e^x$$

```
[In] integrate(exp(x)*cos(4+3*x),x, algorithm="giac")
[Out] 1/10*(cos(3*x + 4) + 3*sin(3*x + 4))*e^x
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^x \cos(4+3x)\,dx = \frac{\mathrm{e}^x\,(\cos{(3\,x+4)} + 3\,\sin{(3\,x+4)})}{10}$$

```
[In] int(exp(x)*cos(3*x + 4),x)
[Out] (exp(x)*(cos(3*x + 4) + 3*sin(3*x + 4)))/10
```

3.365 $\int e^x \log(1+e^x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 18

$$\int e^x \log(1+e^x)\,dx = -e^x + (1+e^x)\log(1+e^x)$$

```
[Out] -exp(x)+(1+exp(x))*ln(1+exp(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2225, 2634, 2280, 45}

$$\int e^x \log(1+e^x)\,dx = -e^x + e^x\log(e^x+1) + \log(e^x+1)$$

```
[In] Int[E^x*Log[1 + E^x],x]
```

```
[Out] -E^x + Log[1 + E^x] + E^x*Log[1 + E^x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2280

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Den
ominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= e^x \log(1+e^x) - \int \frac{e^{2x}}{1+e^x}\,dx \\
&= e^x \log(1+e^x) - \text{Subst}\left(\int \frac{x}{1+x}\,dx, x, e^x\right) \\
&= e^x \log(1+e^x) - \text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx, x, e^x\right) \\
&= -e^x + \log(1+e^x) + e^x \log(1+e^x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^x \log(1+e^x)\,dx = -e^x + (1+e^x)\log(1+e^x)$$

```
[In] Integrate[E^x*Log[1 + E^x],x]
[Out] -E^x + (1 + E^x)*Log[1 + E^x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$(1+e^{x})\ln(1+e^{x})-1-e^{x}$	17
default	$(1+e^{x})\ln(1+e^{x})-1-e^{x}$	17
norman	$e^{x}\ln(1+e^{x})-e^{x}+\ln(1+e^{x})$	19
risch	$e^{x}\ln(1+e^{x})-e^{x}+\ln(1+e^{x})$	19
parallelrisch	$e^{x}\ln(1+e^{x})-e^{x}+\ln(1+e^{x})+1$	20

```
[In] int(exp(x)*ln(1+exp(x)),x,method=_RETURNVERBOSE)
[Out] (1+exp(x))*ln(1+exp(x))-1-exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int e^{x}\log\left(1+e^{x}\right)\,dx=\left(e^{x}+1\right)\log\left(e^{x}+1\right)-e^{x}$$

```
[In] integrate(exp(x)*log(1+exp(x)),x, algorithm="fricas")
[Out] (e^x + 1)*log(e^x + 1) - e^x
```

Sympy [F(-1)]

Timed out.

$$\int e^{x}\log\left(1+e^{x}\right)\,dx=\text{Timed out}$$

```
[In] integrate(exp(x)*ln(1+exp(x)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int e^x \log(1+e^x)\, dx = (e^x+1)\log(e^x+1) - e^x - 1$$

```
[In] integrate(exp(x)*log(1+exp(x)),x, algorithm="maxima")
[Out] (e^x + 1)*log(e^x + 1) - e^x - 1
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int e^x \log(1+e^x)\, dx = (e^x+1)\log(e^x+1) - e^x - 1$$

```
[In] integrate(exp(x)*log(1+exp(x)),x, algorithm="giac")
[Out] (e^x + 1)*log(e^x + 1) - e^x - 1
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^x \log(1+e^x)\, dx = \ln(\mathrm{e}^x+1) - \mathrm{e}^x + \mathrm{e}^x\ \ln(\mathrm{e}^x+1)$$

```
[In] int(exp(x)*log(exp(x) + 1),x)
[Out] log(exp(x) + 1) - exp(x) + exp(x)*log(exp(x) + 1)
```

3.366 $\int x^2 \arctan(x) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 27

$$\int x^2 \arctan(x) \, dx = -\frac{x^2}{6} + \frac{1}{3}x^3 \arctan(x) + \frac{1}{6} \log \left(1 + x^2\right)$$

```
[Out] -1/6*x^2+1/3*x^3*arctan(x)+1/6*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4946, 272, 45}

$$\int x^2 \arctan(x) \, dx = \frac{1}{3}x^3 \arctan(x) - \frac{x^2}{6} + \frac{1}{6} \log \left(x^2 + 1\right)$$

```
[In] Int[x^2*ArcTan[x],x]
```

```
[Out] -1/6*x^2 + (x^3*ArcTan[x])/3 + Log[1 + x^2]/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}x^3\arctan(x) - \frac{1}{3}\int\frac{x^3}{1+x^2}\,dx\\ &= \frac{1}{3}x^3\arctan(x) - \frac{1}{6}\text{Subst}\left(\int\frac{x}{1+x}\,dx, x, x^2\right)\\ &= \frac{1}{3}x^3\arctan(x) - \frac{1}{6}\text{Subst}\left(\int\left(1+\frac{1}{-1-x}\right)\,dx, x, x^2\right)\\ &= -\frac{x^2}{6} + \frac{1}{3}x^3\arctan(x) + \frac{1}{6}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x^2\arctan(x)\,dx = \frac{1}{6}\left(-x^2 + 2x^3\arctan(x) + \log\left(1+x^2\right)\right)$$

```
[In] Integrate[x^2*ArcTan[x],x]
[Out] (-x^2 + 2*x^3*ArcTan[x] + Log[1 + x^2])/6
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{x^2}{6}+\frac{x^3\arctan(x)}{3}+\frac{\ln(x^2+1)}{6}$	22
parts	$-\frac{x^2}{6}+\frac{x^3\arctan(x)}{3}+\frac{\ln(x^2+1)}{6}$	22
parallelrisch	$\frac{x^3\arctan(x)}{3}-\frac{x^2}{6}+\frac{\ln(x^2+1)}{6}+\frac{1}{6}$	23
meijerg	$-\frac{x^2}{6}+\frac{x^4\arctan\left(\sqrt{x^2}\right)}{3\sqrt{x^2}}+\frac{\ln(x^2+1)}{6}$	31
risch	$-\frac{ix^3\ln(ix+1)}{6}+\frac{ix^3\ln(-ix+1)}{6}-\frac{x^2}{6}+\frac{\ln(x^2+1)}{6}$	41

```
[In] int(x^2*arctan(x),x,method=_RETURNVERBOSE)
[Out] -1/6*x^2+1/3*x^3*arctan(x)+1/6*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int x^2\arctan(x)\,dx=\frac{1}{3}\,x^3\arctan\left(x\right)-\frac{1}{6}\,x^2+\frac{1}{6}\,\log\left(x^2+1\right)$$

```
[In] integrate(x^2*arctan(x),x, algorithm="fricas")
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^2\arctan(x)\,dx=\frac{x^3\operatorname{atan}\left(x\right)}{3}-\frac{x^2}{6}+\frac{\log\left(x^2+1\right)}{6}$$

```
[In] integrate(x**2*atan(x),x)
[Out] x**3*atan(x)/3 - x**2/6 + log(x**2 + 1)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int x^2 \arctan(x)\,dx = \frac{1}{3}\,x^3 \arctan\left(x\right) - \frac{1}{6}\,x^2 + \frac{1}{6}\,\log\left(x^2 + 1\right)$$

```
[In] integrate(x^2*arctan(x),x, algorithm="maxima")
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int x^2 \arctan(x)\,dx = \frac{1}{3}\,x^3 \arctan\left(x\right) - \frac{1}{6}\,x^2 + \frac{1}{6}\,\log\left(x^2 + 1\right)$$

```
[In] integrate(x^2*arctan(x),x, algorithm="giac")
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int x^2 \arctan(x)\,dx = \frac{\ln\left(x^2+1\right)}{6} + \frac{x^3\,\mathrm{atan}(x)}{3} - \frac{x^2}{6}$$

```
[In] int(x^2*atan(x),x)
[Out] log(x^2 + 1)/6 + (x^3*atan(x))/3 - x^2/6
```

3.367 $\int \sqrt{-1+e^{2x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \sqrt{-1+e^{2x}}\,dx=\sqrt{-1+e^{2x}}-\arctan\left(\sqrt{-1+e^{2x}}\right)$$

```
[Out] -arctan((-1+exp(2*x))^(1/2))+(-1+exp(2*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2320, 52, 65, 209}

$$\int \sqrt{-1+e^{2x}}\,dx=\sqrt{e^{2x}-1}-\arctan\left(\sqrt{e^{2x}-1}\right)$$

```
[In] Int[Sqrt[-1 + E^(2*x)],x]
```

```
[Out] Sqrt[-1 + E^(2*x)] - ArcTan[Sqrt[-1 + E^(2*x)]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, e^{2x}\right)\\ &= \sqrt{-1+e^{2x}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x}\,dx, x, e^{2x}\right)\\ &= \sqrt{-1+e^{2x}} - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+e^{2x}}\right)\\ &= \sqrt{-1+e^{2x}} - \arctan\left(\sqrt{-1+e^{2x}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \sqrt{-1+e^{2x}}\,dx = \sqrt{-1+e^{2x}} - \arctan\left(\sqrt{-1+e^{2x}}\right)$$

```
[In] Integrate[Sqrt[-1 + E^(2*x)],x]
[Out] Sqrt[-1 + E^(2*x)] - ArcTan[Sqrt[-1 + E^(2*x)]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\arctan\left(\sqrt{\mathrm{e}^{2x}-1}\right)+\sqrt{\mathrm{e}^{2x}-1}$	21
default	$-\arctan\left(\sqrt{\mathrm{e}^{2x}-1}\right)+\sqrt{\mathrm{e}^{2x}-1}$	21
risch	$-\arctan\left(\sqrt{\mathrm{e}^{2x}-1}\right)+\sqrt{\mathrm{e}^{2x}-1}$	21

```
[In] int((exp(2*x)-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -arctan((exp(2*x)-1)^(1/2))+(exp(2*x)-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \sqrt{-1+e^{2x}}\,dx = \sqrt{e^{(2\,x)}-1}-\arctan\left(\sqrt{e^{(2\,x)}-1}\right)$$

```
[In] integrate((-1+exp(2*x))^(1/2),x, algorithm="fricas")
[Out] sqrt(e^(2*x) - 1) - arctan(sqrt(e^(2*x) - 1))
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \sqrt{-1+e^{2x}}\,dx = \sqrt{e^{2x}-1}-\operatorname{atan}\left(\sqrt{e^{2x}-1}\right)$$

```
[In] integrate((-1+exp(2*x))**(1/2),x)
[Out] sqrt(exp(2*x) - 1) - atan(sqrt(exp(2*x) - 1))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \sqrt{-1+e^{2x}}\,dx = \sqrt{e^{(2x)}-1} - \arctan\left(\sqrt{e^{(2x)}-1}\right)$$

```
[In] integrate((-1+exp(2*x))^(1/2),x, algorithm="maxima")
[Out] sqrt(e^(2*x) - 1) - arctan(sqrt(e^(2*x) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \sqrt{-1+e^{2x}}\,dx = \sqrt{e^{(2x)}-1} - \arctan\left(\sqrt{e^{(2x)}-1}\right)$$

```
[In] integrate((-1+exp(2*x))^(1/2),x, algorithm="giac")
[Out] sqrt(e^(2*x) - 1) - arctan(sqrt(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \sqrt{-1+e^{2x}}\,dx = \sqrt{\mathrm{e}^{2x}-1}\left(\frac{\mathrm{e}^{-x}\,\mathrm{asin}(\mathrm{e}^{-x})}{\sqrt{1-\mathrm{e}^{-2x}}}+1\right)$$

```
[In] int((exp(2*x) - 1)^(1/2),x)
[Out] (exp(2*x) - 1)^(1/2)*((exp(-x)*asin(exp(-x)))/(1 - exp(-2*x))^(1/2) + 1)
```

3.368 $\int e^{\sin(x)} \sin(2x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 15

$$\int e^{\sin(x)} \sin(2x) \, dx = -2e^{\sin(x)} + 2e^{\sin(x)} \sin(x)$$

```
[Out] -2*exp(sin(x))+2*exp(sin(x))*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {12, 2207, 2225}

$$\int e^{\sin(x)} \sin(2x) \, dx = 2e^{\sin(x)} \sin(x) - 2e^{\sin(x)}$$

```
[In] Int[E^Sin[x]*Sin[2*x],x]
```

```
[Out] -2*E^Sin[x] + 2*E^Sin[x]*Sin[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
```

```
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int 2e^x x\,dx, x, \sin(x)\right)\\ &= 2\text{Subst}\left(\int e^x x\,dx, x, \sin(x)\right)\\ &= 2e^{\sin(x)}\sin(x) - 2\text{Subst}\left(\int e^x\,dx, x, \sin(x)\right)\\ &= -2e^{\sin(x)} + 2e^{\sin(x)}\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int e^{\sin(x)}\sin(2x)\,dx = e^{\sin(x)}(-2+2\sin(x))$$

```
[In] Integrate[E^Sin[x]*Sin[2*x],x]
[Out] E^Sin[x]*(-2 + 2*Sin[x])
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-2\,\mathrm{e}^{\sin(x)}+2\,\mathrm{e}^{\sin(x)}\sin\left(x\right)$	14
default	$-2\,\mathrm{e}^{\sin(x)}+2\,\mathrm{e}^{\sin(x)}\sin\left(x\right)$	14
risch	$-2\,\mathrm{e}^{\sin(x)}+2\,\mathrm{e}^{\sin(x)}\sin\left(x\right)$	14

```
[In] int(exp(sin(x))*sin(2*x),x,method=_RETURNVERBOSE)
[Out] -2*exp(sin(x))+2*exp(sin(x))*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int e^{\sin(x)}\sin(2x)\,dx = 2\,(\sin{(x)} - 1)e^{\sin(x)}$$

```
[In] integrate(exp(sin(x))*sin(2*x),x, algorithm="fricas")
[Out] 2*(sin(x) - 1)*e^sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int e^{\sin(x)}\sin(2x)\,dx = 2e^{\sin{(x)}}\sin{(x)} - 2e^{\sin{(x)}}$$

```
[In] integrate(exp(sin(x))*sin(2*x),x)
[Out] 2*exp(sin(x))*sin(x) - 2*exp(sin(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int e^{\sin(x)}\sin(2x)\,dx = 2\,(\sin{(x)} - 1)e^{\sin(x)}$$

```
[In] integrate(exp(sin(x))*sin(2*x),x, algorithm="maxima")
[Out] 2*(sin(x) - 1)*e^sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int e^{\sin(x)}\sin(2x)\,dx = 2\,(\sin{(x)} - 1)e^{\sin(x)}$$

```
[In] integrate(exp(sin(x))*sin(2*x),x, algorithm="giac")
[Out] 2*(sin(x) - 1)*e^sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int e^{\sin(x)}\sin(2x)\,dx = 2\,\mathrm{e}^{\sin(x)}\,(\sin{(x)}-1)$$

```
[In] int(sin(2*x)*exp(sin(x)),x)
[Out] 2*exp(sin(x))*(sin(x) - 1)
```

3.369 $\int x^2\sqrt{5-x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int x^2\sqrt{5-x^2}\,dx=-\frac{5}{8}x\sqrt{5-x^2}+\frac{1}{4}x^3\sqrt{5-x^2}+\frac{25}{8}\arcsin\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] 25/8*arcsin(1/5*x*5^(1/2))-5/8*x*(-x^2+5)^(1/2)+1/4*x^3*(-x^2+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {285, 327, 222}

$$\int x^2\sqrt{5-x^2}\,dx=\frac{25}{8}\arcsin\left(\frac{x}{\sqrt{5}}\right)-\frac{5}{8}\sqrt{5-x^2}x+\frac{1}{4}\sqrt{5-x^2}x^3$$

```
[In] Int[x^2*Sqrt[5 - x^2],x]
[Out] (-5*x*Sqrt[5 - x^2])/8 + (x^3*Sqrt[5 - x^2])/4 + (25*ArcSin[x/Sqrt[5]])/8
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{4}x^3\sqrt{5-x^2} + \frac{5}{4}\int \frac{x^2}{\sqrt{5-x^2}}\,dx \\ &= -\frac{5}{8}x\sqrt{5-x^2} + \frac{1}{4}x^3\sqrt{5-x^2} + \frac{25}{8}\int \frac{1}{\sqrt{5-x^2}}\,dx \\ &= -\frac{5}{8}x\sqrt{5-x^2} + \frac{1}{4}x^3\sqrt{5-x^2} + \frac{25}{8}\arcsin\left(\frac{x}{\sqrt{5}}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int x^2\sqrt{5-x^2}\,dx = \frac{1}{8}x\sqrt{5-x^2}(-5+2x^2) + \frac{25}{4}\arctan\left(\frac{-\sqrt{5}+x}{\sqrt{5-x^2}}\right)$$

```
[In] Integrate[x^2*Sqrt[5 - x^2],x]
[Out] (x*Sqrt[5 - x^2]*(-5 + 2*x^2))/8 + (25*ArcTan[(-Sqrt[5] + x)/Sqrt[5 - x^2]]
)/4
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{x(-x^2+5)^{\frac{3}{2}}}{4}+\frac{5x\sqrt{-x^2+5}}{8}+\frac{25\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{8}$	35
risch	$-\frac{x(2x^2-5)(x^2-5)}{8\sqrt{-x^2+5}}+\frac{25\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{8}$	35
pseudoelliptic	$-\frac{25\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)}{8}+\frac{(2x^3-5x)\sqrt{-x^2+5}}{8}$	38
meijerg	$-\frac{25i\left(-\frac{i\sqrt{\pi}\,x\sqrt{5}\left(-\frac{6x^2}{5}+3\right)\sqrt{-\frac{x^2}{5}+1}}{30}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{2}\right)}{4\sqrt{\pi}}$	47
trager	$\frac{x(2x^2-5)\sqrt{-x^2+5}}{8}+\frac{25\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+5}+x\right)}{8}$	48

```
[In] int(x^2*(-x^2+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*x*(-x^2+5)^(3/2)+5/8*x*(-x^2+5)^(1/2)+25/8*arcsin(1/5*x*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int x^2\sqrt{5-x^2}\,dx=\frac{1}{8}\left(2\,x^3-5\,x\right)\sqrt{-x^2+5}-\frac{25}{8}\,\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)$$

```
[In] integrate(x^2*(-x^2+5)^(1/2),x, algorithm="fricas")
[Out] 1/8*(2*x^3 - 5*x)*sqrt(-x^2 + 5) - 25/8*arctan(sqrt(-x^2 + 5)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.57

$$\int x^2\sqrt{5-x^2}\,dx=\begin{cases}\frac{ix^5}{4\sqrt{x^2-5}}-\frac{15ix^3}{8\sqrt{x^2-5}}+\frac{25ix}{8\sqrt{x^2-5}}-\frac{25i\,\mathrm{acosh}\left(\frac{\sqrt{5}x}{5}\right)}{8} & \text{for } |x^2|>5\\ -\frac{x^5}{4\sqrt{5-x^2}}+\frac{15x^3}{8\sqrt{5-x^2}}-\frac{25x}{8\sqrt{5-x^2}}+\frac{25\,\mathrm{asin}\left(\frac{\sqrt{5}x}{5}\right)}{8} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*(-x**2+5)**(1/2),x)
[Out] Piecewise((I*x**5/(4*sqrt(x**2 - 5)) - 15*I*x**3/(8*sqrt(x**2 - 5)) + 25*I*
x/(8*sqrt(x**2 - 5)) - 25*I*acosh(sqrt(5)*x/5)/8, Abs(x**2) > 5), (-x**5/(4
*sqrt(5 - x**2)) + 15*x**3/(8*sqrt(5 - x**2)) - 25*x/(8*sqrt(5 - x**2)) + 2
5*asin(sqrt(5)*x/5)/8, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int x^2\sqrt{5-x^2}\,dx = -\frac{1}{4}\left(-x^2+5\right)^{\frac{3}{2}}x + \frac{5}{8}\sqrt{-x^2+5}x + \frac{25}{8}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

```
[In] integrate(x^2*(-x^2+5)^(1/2),x, algorithm="maxima")
[Out] -1/4*(-x^2 + 5)^(3/2)*x + 5/8*sqrt(-x^2 + 5)*x + 25/8*arcsin(1/5*sqrt(5)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int x^2\sqrt{5-x^2}\,dx = \frac{1}{8}\left(2x^2-5\right)\sqrt{-x^2+5}x + \frac{25}{8}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

```
[In] integrate(x^2*(-x^2+5)^(1/2),x, algorithm="giac")
[Out] 1/8*(2*x^2 - 5)*sqrt(-x^2 + 5)*x + 25/8*arcsin(1/5*sqrt(5)*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

$$\int x^2\sqrt{5-x^2}\,dx = \frac{25\,\mathrm{asin}\left(\frac{\sqrt{5}\,x}{5}\right)}{8} - \sqrt{5-x^2}\left(\frac{5\,x}{8}-\frac{x^3}{4}\right)$$

```
[In] int(x^2*(5 - x^2)^(1/2),x)
[Out] (25*asin((5^(1/2)*x)/5))/8 - (5 - x^2)^(1/2)*((5*x)/8 - x^3/4)
```

3.370 $\int x^2(1+x^3)^4\,dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int x^2(1+x^3)^4\,dx = \frac{1}{15}(1+x^3)^5$$

```
[Out] 1/15*(x^3+1)^5
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {267}

$$\int x^2(1+x^3)^4\,dx = \frac{1}{15}(x^3+1)^5$$

```
[In] Int[x^2*(1 + x^3)^4,x]
```

```
[Out] (1 + x^3)^5/15
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{15}(1+x^3)^5$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 36 vs. 2(11) = 22.

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.27

$$\int x^2(1+x^3)^4\,dx=\frac{x^3}{3}+\frac{2x^6}{3}+\frac{2x^9}{3}+\frac{x^{12}}{3}+\frac{x^{15}}{15}$$

```
[In] Integrate[x^2*(1 + x^3)^4,x]
[Out] x^3/3 + (2*x^6)/3 + (2*x^9)/3 + x^12/3 + x^15/15
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{(x^3+1)^5}{15}$	10
gosper	$\frac{x^3(x^{12}+5x^9+10x^6+10x^3+5)}{15}$	26
norman	$\frac{1}{3}x^3+\frac{2}{3}x^6+\frac{2}{3}x^9+\frac{1}{3}x^{12}+\frac{1}{15}x^{15}$	27
parallelrisch	$\frac{1}{3}x^3+\frac{2}{3}x^6+\frac{2}{3}x^9+\frac{1}{3}x^{12}+\frac{1}{15}x^{15}$	27
risch	$\frac{1}{15}x^{15}+\frac{1}{3}x^{12}+\frac{2}{3}x^9+\frac{2}{3}x^6+\frac{1}{3}x^3+\frac{1}{15}$	28

```
[In] int(x^2*(x^3+1)^4,x,method=_RETURNVERBOSE)
[Out] 1/15*(x^3+1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(9) = 18.

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int x^2(1+x^3)^4\,dx=\frac{1}{15}\,x^{15}+\frac{1}{3}\,x^{12}+\frac{2}{3}\,x^9+\frac{2}{3}\,x^6+\frac{1}{3}\,x^3$$

```
[In] integrate(x^2*(x^3+1)^4,x, algorithm="fricas")
[Out] 1/15*x^15 + 1/3*x^12 + 2/3*x^9 + 2/3*x^6 + 1/3*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(7) = 14$.

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int x^2(1+x^3)^4\,dx = \frac{x^{15}}{15} + \frac{x^{12}}{3} + \frac{2x^9}{3} + \frac{2x^6}{3} + \frac{x^3}{3}$$

```
[In] integrate(x**2*(x**3+1)**4,x)
[Out] x**15/15 + x**12/3 + 2*x**9/3 + 2*x**6/3 + x**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int x^2(1+x^3)^4\,dx = \frac{1}{15}\left(x^3+1\right)^5$$

```
[In] integrate(x^2*(x^3+1)^4,x, algorithm="maxima")
[Out] 1/15*(x^3 + 1)^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int x^2(1+x^3)^4\,dx = \frac{1}{15}\left(x^3+1\right)^5$$

```
[In] integrate(x^2*(x^3+1)^4,x, algorithm="giac")
[Out] 1/15*(x^3 + 1)^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int x^2(1+x^3)^4\,dx = \frac{x^{15}}{15} + \frac{x^{12}}{3} + \frac{2\,x^9}{3} + \frac{2\,x^6}{3} + \frac{x^3}{3}$$

```
[In] int(x^2*(x^3 + 1)^4,x)
[Out] x^3/3 + (2*x^6)/3 + (2*x^9)/3 + x^12/3 + x^15/15
```

3.371 $\int \cos^3(x) \sin^3(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \cos^3(x) \sin^3(x) \, dx = \frac{\sin^4(x)}{4} - \frac{\sin^6(x)}{6}$$

```
[Out] 1/4*sin(x)^4-1/6*sin(x)^6
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2644, 14}

$$\int \cos^3(x) \sin^3(x) \, dx = \frac{\sin^4(x)}{4} - \frac{\sin^6(x)}{6}$$

```
[In] Int[Cos[x]^3*Sin[x]^3,x]
```

```
[Out] Sin[x]^4/4 - Sin[x]^6/6
```

<u>Rule 14</u>

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

<u>Rule 2644</u>

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^3(1-x^2)\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int (x^3-x^5)\,dx, x, \sin(x)\right)\\ &= \frac{\sin^4(x)}{4}-\frac{\sin^6(x)}{6}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos^3(x)\sin^3(x)\,dx = -\frac{3}{64}\cos(2x)+\frac{1}{192}\cos(6x)$$

```
[In] Integrate[Cos[x]^3*Sin[x]^3,x]
[Out] (-3*Cos[2*x])/64 + Cos[6*x]/192
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{(\sin^4(x))}{4}-\frac{(\sin^6(x))}{6}$	14
default	$\frac{(\sin^4(x))}{4}-\frac{(\sin^6(x))}{6}$	14
risch	$\frac{\cos(6x)}{192}-\frac{3\cos(2x)}{64}$	14
parallelrisch	$\frac{7}{40}+\frac{\cos(6x)}{192}-\frac{3\cos(2x)}{64}$	15
norman	$\frac{4(\tan^4(\frac{x}{2}))+4(\tan^8(\frac{x}{2}))-\frac{8(\tan^6(\frac{x}{2}))}{3}}{(1+\tan^2(\frac{x}{2}))^6}$	37

```
[In] int(sin(x)^3*cos(x)^3,x,method=_RETURNVERBOSE)
[Out] 1/4*sin(x)^4-1/6*sin(x)^6
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos^3(x)\sin^3(x)\,dx = \frac{1}{6}\,\cos{(x)}^6 - \frac{1}{4}\,\cos{(x)}^4$$

```
[In] integrate(cos(x)^3*sin(x)^3,x, algorithm="fricas")
[Out] 1/6*cos(x)^6 - 1/4*cos(x)^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \cos^3(x)\sin^3(x)\,dx = -\frac{\sin^6{(x)}}{6} + \frac{\sin^4{(x)}}{4}$$

```
[In] integrate(cos(x)**3*sin(x)**3,x)
[Out] -sin(x)**6/6 + sin(x)**4/4
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos^3(x)\sin^3(x)\,dx = -\frac{1}{6}\,\sin{(x)}^6 + \frac{1}{4}\,\sin{(x)}^4$$

```
[In] integrate(cos(x)^3*sin(x)^3,x, algorithm="maxima")
[Out] -1/6*sin(x)^6 + 1/4*sin(x)^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos^3(x)\sin^3(x)\,dx = \frac{1}{6}\,\cos{(x)}^6 - \frac{1}{4}\,\cos{(x)}^4$$

```
[In] integrate(cos(x)^3*sin(x)^3,x, algorithm="giac")
[Out] 1/6*cos(x)^6 - 1/4*cos(x)^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cos^3(x) \sin^3(x)\,dx = -\frac{\sin(x)^4 \left(2 \sin(x)^2 - 3\right)}{12}$$

```
[In] int(cos(x)^3*sin(x)^3,x)
[Out] -(sin(x)^4*(2*sin(x)^2 - 3))/12
```

3.372 $\int \sec^4(x) \tan^2(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \sec^4(x) \tan^2(x) \, dx = \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5}$$

```
[Out] 1/3*tan(x)^3+1/5*tan(x)^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2687, 14}

$$\int \sec^4(x) \tan^2(x) \, dx = \frac{\tan^5(x)}{5} + \frac{\tan^3(x)}{3}$$

```
[In] Int[Sec[x]^4*Tan[x]^2,x]
```

```
[Out] Tan[x]^3/3 + Tan[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,\tan(x)\right)\\&=\text{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,\tan(x)\right)\\&=\frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \sec^4(x)\tan^2(x)\,dx=-\frac{2\tan(x)}{15}-\frac{1}{15}\sec^2(x)\tan(x)+\frac{1}{5}\sec^4(x)\tan(x)$$

```
[In] Integrate[Sec[x]^4*Tan[x]^2,x]
[Out] (-2*Tan[x])/15 - (Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\left(\tan^3(x)\right)}{3}+\frac{\left(\tan^5(x)\right)}{5}$	14
default	$\frac{\left(\tan^3(x)\right)}{3}+\frac{\left(\tan^5(x)\right)}{5}$	14
risch	$-\frac{4i\left(15\,\mathrm{e}^{6ix}-5\,\mathrm{e}^{4ix}+5\,\mathrm{e}^{2ix}+1\right)}{15\left(\mathrm{e}^{2ix}+1\right)^5}$	36

```
[In] int(sec(x)^4*tan(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*tan(x)^3+1/5*tan(x)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sec^4(x)\tan^2(x)\,dx = -\frac{\left(2\,\cos\left(x\right)^4 + \cos\left(x\right)^2 - 3\right)\sin\left(x\right)}{15\,\cos\left(x\right)^5}$$

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="fricas")
[Out] -1/15*(2*cos(x)^4 + cos(x)^2 - 3)*sin(x)/cos(x)^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \sec^4(x)\tan^2(x)\,dx = -\frac{2\sin\left(x\right)}{15\cos\left(x\right)} - \frac{\sin\left(x\right)}{15\cos^3\left(x\right)} + \frac{\sin\left(x\right)}{5\cos^5\left(x\right)}$$

```
[In] integrate(sec(x)**4*tan(x)**2,x)
[Out] -2*sin(x)/(15*cos(x)) - sin(x)/(15*cos(x)**3) + sin(x)/(5*cos(x)**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sec^4(x)\tan^2(x)\,dx = \frac{1}{5}\,\tan\left(x\right)^5 + \frac{1}{3}\,\tan\left(x\right)^3$$

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="maxima")
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sec^4(x)\tan^2(x)\,dx = \frac{1}{5}\,\tan(x)^5 + \frac{1}{3}\,\tan(x)^3$$

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="giac")
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sec^4(x)\tan^2(x)\,dx = \frac{\tan(x)^5}{5} + \frac{\tan(x)^3}{3}$$

```
[In] int(tan(x)^2/cos(x)^4,x)
[Out] tan(x)^3/3 + tan(x)^5/5
```

3.373 $\int x\sqrt{1+2x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int x\sqrt{1+2x}\,dx = -\frac{1}{6}(1+2x)^{3/2} + \frac{1}{10}(1+2x)^{5/2}$$

```
[Out] -1/6*(1+2*x)^(3/2)+1/10*(1+2*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\int x\sqrt{1+2x}\,dx = \frac{1}{10}(2x+1)^{5/2} - \frac{1}{6}(2x+1)^{3/2}$$

```
[In] Int[x*Sqrt[1 + 2*x],x]
[Out] -1/6*(1 + 2*x)^(3/2) + (1 + 2*x)^(5/2)/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\frac{1}{2}\sqrt{1+2x} + \frac{1}{2}(1+2x)^{3/2}\right) dx \\ &= -\frac{1}{6}(1+2x)^{3/2} + \frac{1}{10}(1+2x)^{5/2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int x\sqrt{1+2x}\,dx = \frac{1}{15}(1+2x)^{3/2}(-1+3x)$$

```
[In] Integrate[x*Sqrt[1 + 2*x],x]
[Out] ((1 + 2*x)^(3/2)*(-1 + 3*x))/15
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{(1+2x)^{\frac{3}{2}}(-1+3x)}{15}$	15
risch	$\frac{(6x^2+x-1)\sqrt{1+2x}}{15}$	18
pseudoelliptic	$\frac{(6x^2+x-1)\sqrt{1+2x}}{15}$	18
trager	$\left(\frac{2}{5}x^2+\frac{1}{15}x-\frac{1}{15}\right)\sqrt{1+2x}$	19
derivativedivides	$-\frac{(1+2x)^{\frac{3}{2}}}{6}+\frac{(1+2x)^{\frac{5}{2}}}{10}$	20
default	$-\frac{(1+2x)^{\frac{3}{2}}}{6}+\frac{(1+2x)^{\frac{5}{2}}}{10}$	20
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(1+2x)^{\frac{3}{2}}(-6x+2)}{15}}{8\sqrt{\pi}}$	29

```
[In] int(x*(1+2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/15*(1+2*x)^(3/2)*(-1+3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int x\sqrt{1+2x}\,dx = \frac{1}{15}\left(6\,x^2+x-1\right)\sqrt{2\,x+1}$$

```
[In] integrate(x*(1+2*x)^(1/2),x, algorithm="fricas")
[Out] 1/15*(6*x^2 + x - 1)*sqrt(2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int x\sqrt{1+2x}\,dx = \frac{2x^2\sqrt{2x+1}}{5} + \frac{x\sqrt{2x+1}}{15} - \frac{\sqrt{2x+1}}{15}$$

```
[In] integrate(x*(1+2*x)**(1/2),x)
[Out] 2*x**2*sqrt(2*x + 1)/5 + x*sqrt(2*x + 1)/15 - sqrt(2*x + 1)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x\sqrt{1+2x}\,dx = \frac{1}{10}\,(2\,x+1)^{\frac{5}{2}} - \frac{1}{6}\,(2\,x+1)^{\frac{3}{2}}$$

```
[In] integrate(x*(1+2*x)^(1/2),x, algorithm="maxima")
[Out] 1/10*(2*x + 1)^(5/2) - 1/6*(2*x + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x\sqrt{1+2x}\,dx = \frac{1}{10}\,(2\,x+1)^{\frac{5}{2}} - \frac{1}{6}\,(2\,x+1)^{\frac{3}{2}}$$

```
[In] integrate(x*(1+2*x)^(1/2),x, algorithm="giac")
[Out] 1/10*(2*x + 1)^(5/2) - 1/6*(2*x + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int x\sqrt{1+2x}\,dx = \frac{(2\,x+1)^{3/2}\,(6\,x-2)}{30}$$

```
[In] int(x*(2*x + 1)^(1/2),x)
[Out] ((2*x + 1)^(3/2)*(6*x - 2))/30
```

3.374 $\int \sin^4(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 24

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{3}{8}\cos(x)\sin(x) - \frac{1}{4}\cos(x)\sin^3(x)$$

```
[Out] 3/8*x-3/8*cos(x)*sin(x)-1/4*cos(x)*sin(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{1}{4}\sin^3(x)\cos(x) - \frac{3}{8}\sin(x)\cos(x)$$

```
[In] Int[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{4}\cos(x)\sin^3(x) + \frac{3}{4}\int \sin^2(x)\,dx \\ &= -\frac{3}{8}\cos(x)\sin(x) - \frac{1}{4}\cos(x)\sin^3(x) + \frac{3\int 1\,dx}{8} \\ &= \frac{3x}{8} - \frac{3}{8}\cos(x)\sin(x) - \frac{1}{4}\cos(x)\sin^3(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)$$

```
[In] Integrate[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result	siz
risch	$\frac{3x}{8}+\frac{\sin(4x)}{32}-\frac{\sin(2x)}{4}$	17
parallelrisch	$\frac{3x}{8}+\frac{\sin(4x)}{32}-\frac{\sin(2x)}{4}$	17
default	$-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}$	18
norman	$\frac{\frac{3x}{8}-\frac{11\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}+\frac{11\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{3\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{9x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}-\frac{3\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	82

```
[In] int(sin(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x+1/32*sin(4*x)-1/4*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \sin^4(x)\,dx = \frac{1}{8}\left(2\,\cos\left(x\right)^3 - 5\,\cos\left(x\right)\right)\sin\left(x\right) + \frac{3}{8}\,x$$

```
[In] integrate(sin(x)^4,x, algorithm="fricas")
[Out] 1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{\sin^3\left(x\right)\cos\left(x\right)}{4} - \frac{3\sin\left(x\right)\cos\left(x\right)}{8}$$

```
[In] integrate(sin(x)**4,x)
[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3}{8}\,x + \frac{1}{32}\,\sin\left(4\,x\right) - \frac{1}{4}\,\sin\left(2\,x\right)$$

```
[In] integrate(sin(x)^4,x, algorithm="maxima")
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3}{8}\,x + \frac{1}{32}\,\sin\left(4\,x\right) - \frac{1}{4}\,\sin\left(2\,x\right)$$

```
[In] integrate(sin(x)^4,x, algorithm="giac")
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3\,x}{8} - \frac{\sin(2\,x)}{4} + \frac{\sin(4\,x)}{32}$$

```
[In] int(sin(x)^4,x)
[Out] (3*x)/8 - sin(2*x)/4 + sin(4*x)/32
```

3.375 $\int \tan^3(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 12

$$\int \tan^3(x)\,dx = \log(\cos(x)) + \frac{\tan^2(x)}{2}$$

```
[Out] ln(cos(x))+1/2*tan(x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 3556}

$$\int \tan^3(x)\,dx = \frac{\tan^2(x)}{2} + \log(\cos(x))$$

```
[In] Int[Tan[x]^3,x]
```

```
[Out] Log[Cos[x]] + Tan[x]^2/2
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\tan^2(x)}{2} - \int \tan(x)\,dx$$
$$= \log(\cos(x)) + \frac{\tan^2(x)}{2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \tan^3(x)\,dx = \log(\cos(x)) + \frac{\tan^2(x)}{2}$$

```
[In] Integrate[Tan[x]^3,x]
[Out] Log[Cos[x]] + Tan[x]^2/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

method	result	size
derivativedivides	$\frac{(\tan^2(x))}{2} - \frac{\ln(1+\tan^2(x))}{2}$	17
default	$\frac{(\tan^2(x))}{2} - \frac{\ln(1+\tan^2(x))}{2}$	17
norman	$\frac{(\tan^2(x))}{2} - \frac{\ln(1+\tan^2(x))}{2}$	17
parallelrisch	$\frac{(\tan^2(x))}{2} - \frac{\ln(1+\tan^2(x))}{2}$	17
risch	$-ix + \frac{2\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^2} + \ln\left(\mathrm{e}^{2ix}+1\right)$	30

```
[In] int(tan(x)^3,x,method=_RETURNVERBOSE)
[Out] 1/2*tan(x)^2-1/2*ln(1+tan(x)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \tan^3(x)\,dx = \frac{1}{2}\,\tan\left(x\right)^2 + \frac{1}{2}\,\log\left(\frac{1}{\tan\left(x\right)^2 + 1}\right)$$

```
[In] integrate(tan(x)^3,x, algorithm="fricas")
[Out] 1/2*tan(x)^2 + 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \tan^3(x)\,dx = \log\left(\cos\left(x\right)\right) + \frac{1}{2\cos^2\left(x\right)}$$

```
[In] integrate(tan(x)**3,x)
[Out] log(cos(x)) + 1/(2*cos(x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \tan^3(x)\,dx = -\frac{1}{2\left(\sin\left(x\right)^2 - 1\right)} + \frac{1}{2}\,\log\left(\sin\left(x\right)^2 - 1\right)$$

```
[In] integrate(tan(x)^3,x, algorithm="maxima")
[Out] -1/2/(sin(x)^2 - 1) + 1/2*log(sin(x)^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan^3(x)\,dx = \frac{1}{2}\,\tan\left(x\right)^2 - \frac{1}{2}\,\log\left(\tan\left(x\right)^2 + 1\right)$$

```
[In] integrate(tan(x)^3,x, algorithm="giac")
[Out] 1/2*tan(x)^2 - 1/2*log(tan(x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan^3(x)\,dx = \ln(\cos(x)) - \frac{\cos(x)^2 - 1}{2\cos(x)^2}$$

```
[In] int(tan(x)^3,x)
[Out] log(cos(x)) - (cos(x)^2 - 1)/(2*cos(x)^2)
```

3.376 $\int x^5\sqrt{1+x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int x^5\sqrt{1+x^2}\,dx=\frac{1}{3}\left(1+x^2\right)^{3/2}-\frac{2}{5}\left(1+x^2\right)^{5/2}+\frac{1}{7}\left(1+x^2\right)^{7/2}$$

```
[Out] 1/3*(x^2+1)^(3/2)-2/5*(x^2+1)^(5/2)+1/7*(x^2+1)^(7/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\int x^5\sqrt{1+x^2}\,dx=\frac{1}{7}\left(x^2+1\right)^{7/2}-\frac{2}{5}\left(x^2+1\right)^{5/2}+\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[In] Int[x^5*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3 - (2*(1 + x^2)^(5/2))/5 + (1 + x^2)^(7/2)/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{1+x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\sqrt{1+x}-2(1+x)^{3/2}+(1+x)^{5/2}\right)\,dx, x, x^2\right)\\ &= \frac{1}{3}\left(1+x^2\right)^{3/2}-\frac{2}{5}\left(1+x^2\right)^{5/2}+\frac{1}{7}\left(1+x^2\right)^{7/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int x^5\sqrt{1+x^2}\,dx = \frac{1}{105}\sqrt{1+x^2}\left(8-4x^2+3x^4+15x^6\right)$$

```
[In] Integrate[x^5*Sqrt[1 + x^2],x]
[Out] (Sqrt[1 + x^2]*(8 - 4*x^2 + 3*x^4 + 15*x^6))/105
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{\left(x^{2}+1\right)^{\frac{3}{2}}\left(15x^{4}-12x^{2}+8\right)}{105}$	22
pseudoelliptic	$\frac{\left(x^{2}+1\right)^{\frac{3}{2}}\left(15x^{4}-12x^{2}+8\right)}{105}$	22
trager	$\left(\frac{1}{7}x^{6}+\frac{1}{35}x^{4}-\frac{4}{105}x^{2}+\frac{8}{105}\right)\sqrt{x^{2}+1}$	26
risch	$\frac{\left(15x^{6}+3x^{4}-4x^{2}+8\right)\sqrt{x^{2}+1}}{105}$	27
default	$\frac{x^{4}\left(x^{2}+1\right)^{\frac{3}{2}}}{7}-\frac{4x^{2}\left(x^{2}+1\right)^{\frac{3}{2}}}{35}+\frac{8\left(x^{2}+1\right)^{\frac{3}{2}}}{105}$	35
meijerg	$-\frac{\frac{32\sqrt{\pi}}{105}-\frac{4\sqrt{\pi}\left(x^{2}+1\right)^{\frac{3}{2}}\left(15x^{4}-12x^{2}+8\right)}{105}}{4\sqrt{\pi}}$	36

```
[In] int(x^5*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/105*(x^2+1)^(3/2)*(15*x^4-12*x^2+8)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int x^5\sqrt{1+x^2}\,dx = \frac{1}{105}\left(15\,x^6+3\,x^4-4\,x^2+8\right)\sqrt{x^2+1}$$

```
[In] integrate(x^5*(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/105*(15*x^6 + 3*x^4 - 4*x^2 + 8)*sqrt(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

$$\int x^5\sqrt{1+x^2}\,dx = \frac{x^6\sqrt{x^2+1}}{7}+\frac{x^4\sqrt{x^2+1}}{35}-\frac{4x^2\sqrt{x^2+1}}{105}+\frac{8\sqrt{x^2+1}}{105}$$

```
[In] integrate(x**5*(x**2+1)**(1/2),x)
[Out] x**6*sqrt(x**2 + 1)/7 + x**4*sqrt(x**2 + 1)/35 - 4*x**2*sqrt(x**2 + 1)/105
+ 8*sqrt(x**2 + 1)/105
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int x^5\sqrt{1+x^2}\,dx = \frac{1}{7}\left(x^2+1\right)^{\frac{3}{2}}x^4-\frac{4}{35}\left(x^2+1\right)^{\frac{3}{2}}x^2+\frac{8}{105}\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x^5*(x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/7*(x^2 + 1)^(3/2)*x^4 - 4/35*(x^2 + 1)^(3/2)*x^2 + 8/105*(x^2 + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int x^5\sqrt{1+x^2}\,dx = \frac{1}{7}\left(x^2+1\right)^{\frac{7}{2}}-\frac{2}{5}\left(x^2+1\right)^{\frac{5}{2}}+\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x^5*(x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/7*(x^2 + 1)^(7/2) - 2/5*(x^2 + 1)^(5/2) + 1/3*(x^2 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.62

$$\int x^5\sqrt{1+x^2}\,dx=\sqrt{x^2+1}\left(\frac{x^6}{7}+\frac{x^4}{35}-\frac{4\,x^2}{105}+\frac{8}{105}\right)$$

```
[In] int(x^5*(x^2 + 1)^(1/2),x)
[Out] (x^2 + 1)^(1/2)*(x^4/35 - (4*x^2)/105 + x^6/7 + 8/105)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):



   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```